

**Fourth Semester B.E. (Computer Science and Engineering)
Examination**

THEORETICAL FOUNDATIONS OF COMPUTER SCIENCE

Time : 3 Hours]

[Max. Marks : 60

Instructions to Candidates :—

- (1) All questions carry marks as indicated against them.
 (2) Assume suitable data and answer with neat diagrams wherever necessary.

1. (a) Compute the reflexive transitive closure (R^*) for given R .
 $R = \{(a,a), (b,b), (b,c), (c,a), (a,d), (b,d)\}$ 2(CO1)
- (b) Describe the Chomsky Hierarchy along with grammar and accepting device. Also Consider the given grammar. Identify the type, name of grammar, Language the grammar can recognize and the accepting device :
 $Sa \rightarrow abc \mid AaB$
 $bB \rightarrow Bb$ 2+2(CO1)
- (c) Prove the following theorem by principle of mathematical induction :
 $1+4+7+\dots+(3n-2) = [n(3n-1)/2]$ 4(CO1)

OR

- (d) Prove the following theorem by principle of mathematical induction that $3^n - 1$ a multiple of 2. 4(CO1)
2. (a) Compute the equivalent DFA for the given NFA with ϵ by first computing NFA without ϵ .
 $NFA = [\{q_0, q_1, q_2, q_3\}, \{a, b, c, \epsilon\}, \delta, q_0, \{q_3\}]$
 $\delta =$

State	a	b	c	ϵ
$\rightarrow q_0$	q_2, q_3	q_2	—	q_1
q_1	q_2	—	q_3	q_2
q_2	—	—	q_2	q_3
$*q_3$	—	—	q_3	—

7(CO2)

- (b) Convert the given Mealy machine to Moore machine :

Present state	a=0		a=1	
	State	Output	State	Output
→q1	q1	1	q3	1
q2	q2	1	q1	1
q3	q2	0	q3	0

3(CO2)

OR

- (c) Construct a DFA for the following on $\Sigma = \{a, b\}$:

- (1) Which accept set of all strings on starting with prefix ab.
- (2) Which accept set of all strings with no more than three a's.

Also show the acceptance of valid string as per the DFA constructed.

3(CO2)

3. (a) Design a CFG for the regular language :

$$(1) L = \{0^m 1^n 0^{m+n} \mid m, n \geq 1\}$$

$$(2) L = \{a^n b^m \mid n \neq m\}$$

Also show that the designed grammar can derive correct string.

3(CO1)

- (b) State how to identify ambiguous grammars.

Also remove left recursion from the following grammar :

$$S \rightarrow aBDh \mid bGa$$

$$B \rightarrow Bb \mid c$$

$$D \rightarrow EF$$

$$E \rightarrow g \mid \epsilon$$

$$F \rightarrow f \mid \epsilon$$

$$G \rightarrow Gb \mid Gt \mid a$$

3(CO1)

OR

- (c) Prove that the language is not regular using Pumping Lemma :

$$L = \{a^n b^n c^n \mid n \geq 1\} \quad 3(\text{CO1})$$

- (d) Convert given Context Free Grammar to its equivalent Greibach Normal Form(GNF).

$$S \rightarrow AB \mid b$$

$$A \rightarrow SB \mid c$$

$$B \rightarrow AB \mid a \quad 4(\text{CO1})$$

4. (a) Solve any **Two** :

Design Push Down Automata for the given language. Also show string acceptance for each language. Consider any valid string of length greater than 5.

$$(1) L = \{a^n b^{2n} \mid n \geq 1\}$$

$$(2) L = \{(a, b)^* \mid \text{number of } a > \text{number of } b\}$$

$$(3) L = \{a^{2n} c b^n \mid n \geq 1\} \quad 5(\text{CO3})$$

- (b) Convert the given Push Down Automata to Context Free Grammar (CFG).

$M = (\{q_0, q_1\}, \{0, 1\}, \{Z_0, X\}, \delta, q_0, Z_0, \Phi)$ where δ is given below :

$$\delta(q_0, 1, Z_0) \rightarrow \{(q_0, XZ_0)\}$$

$$\delta(q_0, 1, X) \rightarrow \{(q_0, XX)\}$$

$$\delta(q_0, 0, X) \rightarrow \{(q_1, X)\}$$

$$\delta(q_0, \epsilon, Z_0) \rightarrow \{(q_0, \epsilon)\}$$

$$\delta(q_1, 1, X) \rightarrow \{(q_1, \epsilon)\}$$

$$\delta(q_1, 0, Z_0) \rightarrow \{(q_0, Z_0)\}$$

Also find the reduced grammar. 5(CO3)

5. (a) Design a Turing machine for the regular expression: aba^*ba . Show that the string "abab" is valid. 3(CO3)

- (b) Design a Turing machine to perform the function $f(x,y) = x*y$. 4(CO3)

OR

- (c) Design a Turing machine that performs mod operation as follows :
 $n \bmod 5$. 4(CO3)
- (d) Write short note on Halting Problem of Turing Machine. 3(CO3)
6. (a) Explain Ackerman function and find : $A[2,2]$ 3(CO4)
- (b) Infer whether the following functions are primitive recursive or not(any **Two**).
- (1) $f(x, y) = x*y$
 - (2) $f(x, y) = xy$
 - (3) $f(x, y) = m^{2n}$ 4(CO4)
- (c) Apply the post correspondence problem to find the solution for the given lists.
- (1) $A = \{001, 0011, 11, 101\}$
 $B = \{01, 111, 111, 010\}$
 - (2) $A = \{0, 01000, 01\}$
 $B = \{000, 01, 1\}$ 3(CO4)