Third Semester B. E. (Computer Science and Engineering / Information Technology) Examination

ENGINEERING MATHEMATICS – III

Time : 3 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions carry equal marks.

(2) Use of Non – programmable calculator is permitted.

(3) Use of normal distribution table is permitted.

1. Solve any **Two** :—

(a) Find the values of λ for which the following system of equations is consistent and has non–trivial solutions. Solve equations for all such values of of λ.

$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$, $(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3) z = 0$, $2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$ 5(CO 1)

(b) Prove that $3 \tan A = A \tan(3)$ where $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$ 5(CO 1)

(c) Determine the largest eigen value and the corresponding eigen vector of the following matrix by iteration method.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$ 5(CO1)

2. Solve :—

(a) Find by Regula–falsi method the root of the equation : $xe^x = \cos x$. 5(CO 1)

(b) Sovle by Euler's modified method : $\frac{dy}{dx} = \log(x+y)$, $y(0) = 2$, at $x = 0.4$ and $x = 0.8$ with $h = 0.4$. 5(CO 1)

3. Solve :—

(a) Using the Z–transform, solve :

$y_{n+2} - 2y_{n+1} + y_n = 3n + 5, \; y_0 = y_1 = 2.$ 5(CO 2)

(b) Determine the Z–transform of following :

(i) $(n+1)^2$ (ii) $n\cos(n\theta)$ 5(CO 2)

4. Solve any **Two** :—

(a) The probability function of a random variable X is zero except at the point $x = 0, 1, 2$. At these points it has the values $P(0) = 3c^3$, $P(1) = 4c - 10c^2$ and $P(2) = 5c-1$ for some $c > 0$.

(i) Determine the value of c.

(ii) Describe the distribution function and draw its graph.

(iii) Find $P(X< 2)$. 5(CO 3)

(b) The diameter of an electric cable, say X, is assumed to be a continuous random variable with $f(x) = 6x(1-x), \; 0 \leqslant x \leqslant 1$.

(i) Check that above f(x) is probability density function.

(ii) Determine a number 'b' such that $P(X < b) = P(X > b)$ 5(CO 3)

(c) Two random variables have joint density function :

$$f(x, y) = \begin{cases} C(xy+e^x), & 0 < x < 1, \; 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

(i) Determine C.

(ii) Examine whether X and Y are independent random variables. 5(CO 3)

5. Solve any **Two** :—

(a) If X is a random variable, then prove that $Var(aX+b) = a^2 Var(X)$, where a and b are constant. 5(CO 3)

(b) Let the joint density function of the random variables X and Y be

$$f(x,\ y) = \begin{cases} 2(x + y - 3xy^2), & 0 < x < 1,\ \ 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the E(X) and E(Y).

(ii) Is E(XY) = E(X)E(Y) ?

(iii) Find E(X+Y) and E(X–Y). 5(CO 3)

(c) A discrete random variable X has probability function :

$$f(x) = \frac{2}{3^x}, \quad x = 1,\ 2,\ 3, \ldots..$$

Find :

(i) The mode.

(ii) The median.

(iii) Compare them with mean. 5(CO 3)

6. Solve :—

(a) The mean and variance of binomial distribution are 4 and 4/3 respectively. Find $P(X \geqslant 1)$. 3(CO 3)

(b) In a book of 520 pages, 390 typo–graphical errors occur. Assuming Poisson law for the number of errors per page, find the probability that a random sample of 5 pages will contain no error. 3(CO 3)

(c) In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution? 4(CO 3)