Course Code : MAT 252 **KOLP/RW – 19 / 9041**

Third Semester B. E. (Computer Science and Engineering, Information Technology) Examination

LINEAR ALGEBRA AND STATISTICS

Time : 3 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions carry equal marks.
(2) Use of Non – programmable calculator is permitted.
(3) Use of area under normal curve table and student's t distribution table is permitted.

1. Solve any **Three** :—

(a) Let V be a set of ordered pairs (a, b) of real numbers, determine whether V is vector space over R or not, with addition and scalar multiplication defined by :

$(a, b) + (c, d) = (a + d, b + c)$ and $k(a, b) = (ka, kb)$. 5 (CO 1, 2)

(b) Determine whether or not W is a subspace of R^3 where W consist of all vectors in R^3 such that $a = 2b = 3c$. 5 (CO 1, 2)

(c) Find the basis as well as dimension of the kernel and the image of linear map $T : R^3 \to R^2$ defined by $T(x, y, z) = (x + y, y + z)$. 5 (CO 1, 2)

(d) Let $F : R^2 \to R^2$ be defined by $F(x, y) = (2x + 7y, x - 3y)$ and consider the following bases of R^2, $S = \{(1, 1), (1, 2)\}$ and $S' = \{(1, 4), (1, 5)\}$. Find the matrix representing F relative to the bases S and S′. 5 (CO 1, 2)

2. Solve any **Three** :—

(a) Consider the following polynomials in **P**(t) with the inner product :

$$\langle f, g\rangle = \int_0^1 f(t)\,g(t)\,dt \quad : \quad f(t) = t + 2, \quad g(t) = 3t - 2, \quad h(t) = t^2 - 2t - 3.$$

(i) Find $\langle f, g\rangle$ and $\langle f, h\rangle$. (ii) Normalize f and g. 5 (CO 1, 2)

(b) Each of the following real matrices defines a linear transformation on R^2 : (i) $A = \begin{bmatrix} 5 & 6 \\ 3 & -2 \end{bmatrix}$ (ii) $B = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$

Find, for each matrix, all eigenvalues and maximum number of linearly independent eigenvectors. Which of these linear operators are diagonalizable ? 5 (CO 1, 2)

(c) Using Gram – Schmidt process, find the orthonormal basis spanned by a set of vectors (2, –5, 1), (4, –1, 2). 5 (CO 1, 2)

(d) Find value of '*a*' for which the matrix $\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2+a \end{bmatrix}$ is positive definite. 5 (CO 1, 2)

3. Solve :—

(a) Let X and Y be two discrete random variables taking integer values and having joint probability function :

$$f(x, y) = \begin{cases} c(x+2y) , & 0 \leq x < 3, \ -1 < y \leq 2 \\ 0 , & \text{otherwise} \end{cases}$$

Find : (i) Constant c, (ii) $P(X \geq 2, Y \leq 1)$.

Also determine whether X and Y are independent or not. 5 (CO 3)

(b) In a certain city if today is sunny, then tomorrow will be sunny 80% of the time. If today is cloudy, tomorrow will be cloudy 60% of the time. Obtain the transition matrix. Supposing today is sunny, what is the probability that it will be cloudy the day after tomorrow ? 5 (CO 3)

(c) Explain the Binomial Process. 5 (CO 3)

4. Solve any **Three** :—

(a) In a sample of 1000 people in Maharashtra 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state at 1% level of significance ? 5 (CO 3)

(b) Before an increase in excise duty on tea, 800 persons out of a sample of 1,000 persons were found to be tea drinkers. After an in duty, 800 people were tea drinkers in a sample of 1200 people. Using standard error of proportion, state whether there is a significant decrease in the consumption of tea after the increase in excise duty. 5 (CO 3)

(c) A normal population has a mean of 6·8 and standard deviation of 1·5. A sample of 400 members gave a mean of 6·75. Is the difference between the means significant ? 5 (CO 3)

(d) A group of boys and girls were given an intelligent test. The mean score, standard deviations and number in each group are as follows :

	Boys	Girls
Mean	124	121
S.D.	12	10
N	18	14

Determine whether the mean score of boys significantly different from that of girls. 5 (CO 3)