Fourth Semester B. E. (Computer Science and Engineering) Examination

THEORETICAL FOUNDATIONS OF COMPUTER SCIENCE

Time : 3 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All Questions carry marks as indicated against them.
(2) Due credit will be given to neatness.
(3) Assume suitable data and Illustrate answers with neat sketches wherever necessary.

1. (a) Recall the principle of mathematical induction. Apply this principle to prove that $11^n - 6$ is divisible by 5 for all $n >= 0$. 4(CO 1)

(b) Solve any **One** :—

(i) Construct grammar for the following language.

(i) $L = \{a^n b^m : n>=3 \text{ and } m>=4\}$

(ii) $L = \{a^n b^{3n} \mid n>=1\}$ 4(CO 1)

(ii) Discuss the Chomsky hierarchy of language. Identify the type of following grammar with justification.

S –> ab | aAbc

Abc –> bA

Ac –> Bbcc

bB –> Bb

aB –> aa | aaA 4(CO 1)

(c) List out the areas where the theory of computation is applicable. 2(CO 1)

2. (a) Construct the minimum state automata for the automata given below using Myhill Nerode Theorem.

States/Σ	a	b
→1	2	4
*2	3	6
3	2	4
*4	6	5
5	2	4
6	6	6

5(CO 2)

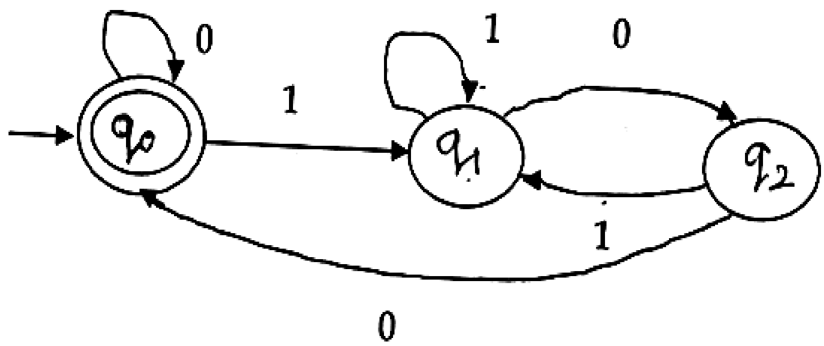
(b) Design DFA over w ε(a, b) * for the following languages.

(i) L = {there are at most two runs of a's of length three}

(ii) L = {w has no more than one pair of consecutive a's and no more than one pair of consecutive b's} 5(CO 2)

OR

(c) Construct the Regular expression for the following Finite Automata.



5(CO 2)

3. (a) For the language L = {a^n:n is either a multiple of three or a multiple of 5} state whether L is regular or not and prove your answer. 4(CO 1)

OR

(b) Why is ambiguity of grammar a problem ?

Let G be the grammar with production rules

E–>E+E E–>E*E E–>(E) E–>id

Show at least 3 different parse trees that can be generated from the grammar for the string w = id + id * id * id 4(CO 1)

(c) Design a Context free grammar for the language

$L = \{a^m b^n c^p d^q \mid m, n, p, q >= 0$ and $m + n = p + q\}$.

Also construct Chomsky normal form from the grammar generated. 6(CO 1)

4. (a) Construct a Push down automata for the language :

$L = \{a^n b^m c^{m-n} \mid m > n$ and $n, m > 1\}$ 5(CO3)

(b) Discriminate the deterministic PDA and non deterministic PDA with example. 5(CO3)

OR

(c) Obtain Context free grammar from the Push down automata A given below:

$A = (\{q1, q2\}, \{0, 1\}, \{0, Z0\}, \delta, q1, Z0, \Phi)$ Where δ is defined by

R1:δ(q1, 0, Z0) = {(q1, 0, Z0) R2: δ(q1, 0, 0) = {q1, 00)

R3:δ(q1, 1 ,0)={(q2,0) R4: δ(q2, 1, 0) = {q2, 0)

R5:δ(q2, 0, 0)= {q2,ε) R4: δ(q2, ε, Z0) = {q2,ε)

Also identify the language for the Push down automata given. 5(CO 3)

5. (a) Analyze the decidability of language using Turing machine. 3(CO3)

(b) Construct a Turing machine for the addition of two binary numbers. Specifically given the input string <x> ; <y> where <x> is binary encoding of natural number x, <y> is binary encoding of natural number y. M should output

<z> where z is binary encoding of x+y For example : on input 101:11 M should output 1000. 7(CO 3)

OR

(c) Design Turing machine for the Language

$L = \{0^n 1^m 0^n \mid m, n >= 1\}$. Also draw the transition diagram. Show Instantaneous description for the string "00011000". 7(CO 3)

6. (a) Consider the following instance of the post correspondence problem. Does it have a solution ? If so, show one.

i	X	Y
1	a	bab
2	bbb	bb
3	aab	ab
4	b	a

3(CO 4)

(b) Explain primitive recursive function. Prove that each of the following function is primitive recursive :—

(i) The proper subtraction function, which is defined as follows:

$$sub(n,m) = \begin{cases} n-m & \text{if } n > m \\ 0 & \text{if } n <= m \end{cases}$$

(ii) The function half which is defined as follows

$$half(n) = \begin{cases} n/2 & \text{if } n \text{ is even} \\ (n-1)/2 & \text{if } n \text{ is odd} \end{cases}$$

5(CO 4)

(c) Prove for the following Ackermann function $A(2, y) = 2y+3$ 2(CO 4)