Course Code : MAT 245 **KOLP/RW – 19 / 9533**

Fourth Semester B. E. (Computer Science and Engineering / Information Technology) Examination

DISCRETE MATHEMATICS

Time : 3 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions carry equal marks.

(2) Use of Non – programmable calculator is permitted.

1. Solve any **Two** :

(a) Among the first 1000 positive integers :

(i) Determine the integers which are not divisible by 5, or by 7, or by 9.

(ii) Determine the integers divissible by 5, but not by 7, but not by 9. 5(CO1)

(b) A set S consist of two types of elements: Type 1 and Type 2. Both Type 1 and Type 2 subsets are non–empty. A relation R on S is defined such that (a, b) $\in$ R only if a and b are of different types. Can such relation be reflexive, symmetric, transitive, and equivalence ? 5(CO1)

(c) Prove that :

(i) $f_{A-B}(x) = f_A(x) - f_{A \cap B}(x)$

(ii) $f_A(x) \leq f_B(x)$ if and only if $A \subseteq B$, where f_A is a characteristic function. 5(CO1)

2. Solve :

(a) Determine the validity of following argument using truth table as well as without truth table. "If the market is free then there is no inflation. If there is no inflation then there are price controls. Since there are price controls, therefore the market is free". 5(CO1)

(b) If $p \to q$ is false, determine the truth value of $\neg(p \wedge q) \to q$. 2(CO1)

(c) Let P(x) : x is a person. F(x, y): x is the father of y. M(x, y) : x is the mother of y. Write the predicate" x is the father of the mother of y". 3(CO1)

3. Solve :

(a) Let $(G, *)$ and $(\bar{G}, \bullet)$ be groups and $f: G \to \bar{G}$ be a homomorphism. Show that kernel of f is a normal subgroup of a group G. 5(CO2)

(b) Find all the distinct left cosets of $H = 6Z$ in the group $(Z, +)$. 3(CO2)

(c) Prove or disprove: The set of all odd integers is a subgroup of group of integers under the operation of addition. 2(CO2)

4. Solve any **Two** :

(a) Let $C = \{a + ib | a, b \in R$ and $i = \sqrt{-1}\}$, show that $(C, +, \bullet)$ is a field. 5(CO2)

(b) Show that the intersection of any two ideals of a ring R is also an ideal of R. 5(CO2)

(c) Define ring homomorphism if $f : R \to R^*$ is a ring homomorphism then prove that :

(i) $f(0) = 0^*$, where 0^* is zero element of R^*.

(ii) $f(-a) = -f(a), \ \forall a \in R$. 5(CO2)

5. Solve :

(a) Let $(L, \leq)$ be any lattice, then for any a, b, c in L, show that :

(i) $a \vee (b \vee c) = (a \vee b) \vee c$

(ii) $a \vee (a \wedge b) = a$ 4(CO2)

(b) Let B be the set of all positive divisors of 30. For any $a, b \in B$, let $a + b = lcm(a,b)$; $a \bullet b = \gcd(a, b)$ and $a' = \frac{a}{30}$

Verify $(B, ,+,\bullet,', 1, 30)$ is a Boolean algebra. 3(CO2)

(c) Simplify the Boolean expression : a'b+a'b'+b'. 3(CO2)

6. Solve :

(a) Show that if we select 151 distinct computer science courses numbered between 1 an 300 inclusive, at least two are consecutive. 3(CO3)

(b) Solve the recurrence relation using generating function :

$a_{n+1} = n+1+a_n, \quad a_0 = 1, \quad n \geqslant 0.$ 4(CO3)

(c) Prove that $\sum_{r=0}^{n} \binom{n}{r}^2 = \binom{2n}{n}$, where n is positive integer. 3(CO3)