Fourth Semester B. E. (Computer Science and Engineering / Information Technology) Examination

DISCRETE MATHEMATICS

Time : 3 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions carry equal marks.

(2) Use of non – programmable calculator is permitted.

1. (a) Let R be an equivalence relation on a set A. Then show that for all $a, b \in A$. either $[a] = [b]$ or $[a] \cap [b] = \phi$ where $[a]$ represents equivalence class of a. 3 (CO 1)

(b) In an examination, 70% of the candidates passed in Mathematics, 73% passed in Physics and 64% passed in both the subjects. If 63 candidates failed in both the subjects, then find the total number of candidates who appeared for the examination. 3 (CO 1)

(c) Let A and B be two finite sets with $n(A) = 5$ and $n(B) = 3$. Find the number of subjective functions from A onto B. 4 (CO 1)

2. (a) Test the validity of the followign arguments using rules of inference.
Hypotheses : Everyone in Discrete Mathematics class loves proofs. Someone in the Discrete Mathematics class has never taken calculus.
Colculsion : Someone who loves proofs has never taken calculus.
4 (CO 1)

(b) Obtain the principal conjunctive normal form of the following formula :
$P \wedge (p \rightarrow q)$
Using PCNF find its PDNF. 4 (CO 1)

(c) Write the negation of the following statement in the symbolic form.
"A number is not even unless it is divisible by 2". 2 (CO 1)

3. (a) Let $G = R \times R$ be the group under the binary operation * defined by $(a, b) * (c, d) = (a + c, b + d)$. Let $H = \{(a, 5a) \mid a \in R\}$. Show that H is a subgroup of G. Describe the left cosets of H in G. 4 (CO 2)

(b) If f is a homomorphism from a group G into a group G_1 and e, e_1 are the identity elements of G and G_1 respectively, then show that

(i) $f(e) = e_1$

(ii) $f(a^{-1}) = \{f(a)\}^{-1}$ 4 (CO 2)

(c) Let G be a group and $a \in G$. If $O(a) = 24$, then find $O(a^4)$. 2 (CO 2)

4. (a) Show that the set of all matrices of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, where $a, b \in R$, is a field with respect to matrix multiplication. 5 (CO 1)

(b) For two ideals A and B of a ring R, show that $A \cup B$ is an ideal of R if and only if $A \subseteq B$ or $B \subseteq A$. 5 (CO 2)

5. (a) Let L be any lattice. Then for any $a, b \in L$, prove that $a \vee (a \wedge b) = a$. 3 (CO 2)

(b) Find two incomparable elemetns in the following Poset $(P(\{0, 1, 2\}), \subseteq)$, where $P(\{0, 1, 2\})$ represents the power set of $\{0, 1, 2\}$. 3 (CO 2)

(c) In a Boolean Algera B, for any a and b prove the following

(i) $a + a'b = a + b$

(ii) $(a + b')(a' + b')(a + b)(a' + b) = 0$ 4 (CO 2)

6. (a) An inventory consists of a list of 80 items, each marked "available" or "unavailable". There are 45 available items. Show that there are at least two available items in the list exactly 9 items apart (For example, available items at position 13 and 22). 3 (CO 3)

(b) Let C (n , r) denotes the number of ways of choosing r objects from n distinct objects.
Show that $C(n+1, r) = C(n, r) + C(n, r-1)$. 2 (CO 3)

(c) Use generating function to solve the recurrence relation
$a_{n+2} - 2a_{n+1} + a_n = 2^n$, $a_0 = 2$, $a_1 = 1$. . 5(CO3)