Course Code : MAT 202 **KOLP/RW – 19 / 9042**

Third Semester B. E. (Computer Science and Engineering / Information Technology) Examination

ENGINEERING MATHEMATICS – III

Time : 3 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions carry equal marks.
(2) Use of Non – programmable calculator is permitted.
(3) Use of area under normal curve table is permitted.

1. Solve any **Two** :

(a) Test the consistency and solve the following set of equtions, if possible:

$x + 2y + z = 2, \quad 3x + y - 2z = 1, \quad 4x - 3y - z = 3, \quad 2x + 4y + 2z = 4.$ 5(CO1)

(b) Reduce the following matrix into diagonal form : $\begin{bmatrix} 5 & 7 & -5 \\ 0 & 4 & -1 \\ 2 & 8 & -3 \end{bmatrix}$

5(CO1)

(c) Use Sylvester's theorem to prove that $\sin^2 A + \cos^2 A = I$,.

where $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ 5(CO1)

2. Solve any **Two** :

(a) Use the method of false position to find the root of the equation $x^4 + x^3 - 7x^2 - x + 5 = 0$, given that it lies between 2 and 3.

5(CO1)

(b) Apply Runge–Kutta method, to find an approximate value of y when $x = 0.2$, given that $\frac{dy}{dx} = x + y^2$ and $y = 1$ when $x = 0$.

5(CO1)

(c) Apply Gauss–Seidel method to solve :

$5x + 2y + z = 12, \quad x + 4y + 2z = 15, \quad x + 2y + 5z = 20.$ 5(CO1)

3. Solve any **Two** :

(a) Find Z–transform of $\cos\left[\frac{n\pi}{8} + \alpha\right]$, $n \geq 0$. 5(CO2)

(b) Find the inverse of Z–transform of $\frac{1}{(z-5)^3}$, $|z|>5$ 5(CO2)

(c) Solve the difference equation by Z–transform method :

$6y_{n+2} - y_{n+1} - y_n = 0, \quad y_0 = 0, \quad y_1 = 1.$ 5(CO2)

4. Solve any **Two** :

(a) A continuous random variable X has a probability density function $f(x) = \begin{cases} 3x^2 & , \ 0 \leq x \leq 1 , \\ 0 & , \ \text{otherwise} \end{cases}$

Find a and b such that :

(i) $P(x \leq a) = P(x > a)$

(ii) $P(x > b) = 0.05$ 5(CO3)

(b) A random variable X has the following probability distribution

x	0	1	2	3	4	5	6	7
f(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Find k

(ii) Evaluate $P(x<6)$, $P(X > 6)$, and $P(0 < X < 5)$,

(iii) If $P(X< c) > 0.5$, find the minimum value of c, and

(iv) Determine the distribution function of X. 5(CO3)

(c) Joint distribution of continuous random variables X and Y is given $f(x, y) = \begin{cases} 4xy\, e^{-(x^2+y^2)} & , \quad x \geq 0 \quad , \quad y \geq 0 \\ 0 & , \quad \text{otherwise} \end{cases}$

Test whether X and Y are independent 5(CO3)

5. Solve any **Two** :

(a) A random process gives measurements X between 0 and 1 with a probability density function

$$f(x) = \begin{cases} 12x^3 - 21x^2 + 10x & , \quad 0 \leq x \leq 1 \\ 0 & , \quad \text{otherwise} \end{cases}$$

Find mean, variance and standard deviation of X. 5(CO3)

(b) The joint probability function of two discrete random variables X and Y is given by $f(x, y) = \begin{cases} \dfrac{2x+y}{42} & , \quad x = 0,1,2, \quad y = 0,1,2,3 \\ 0 & , \quad \text{otherwise} \end{cases}$

Find coefficient of correlation between X and Y 5(CO3)

(c) Find the coefficient of skewness and kurtosis for the distribution having density function $f(x) = \begin{cases} c \quad x & , \quad 0 \leq x \leq 2 \\ 0 & , \quad \text{otherwise} \end{cases}$ where c is appropriate constant. 5(CO3)

6. Solve any **Two** :

(a) The probability that a man aged 60 will live to be 70 is 0.65. What is the probability that out of 10 men, now 60, at least 7 will live to be 70. 5(CO3)

(b) In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find :

(i) How many students score between 12 and 15 ?

(ii) How many score above 18 ?

(iii) How many score 16 ? 5(CO3)

(c) Prove central limit theorem for the independent variables

$$X_k = \begin{cases} 1 & , \text{ prob. } p, \\ -1 & , \text{ prob. } q \end{cases}$$ 5(CO3)