Third Semester B. E. (Computer Science and Engineering / Information Technology) Examination

ENGINEERING MATHEMATICS – III

Time : 3 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions carry equal marks.
(2) Use of Non – programmable calculator is permitted.
(3) Use of area under normal curve table is permitted.

1. Solve any **Two** :—

(a) Find for what values of k the set of equations
$2x - 3y + 6z - 5t = 3, \quad y - 4z + t = 1, \quad 4x - 5y + 8z - 9t = k$
has (i) No solution, (ii) Infinite solutions. 5 (CO 1)

(b) Reduce the following matrix into diagonal form : $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.
5 (CO 1)

(c) Verify Cayley – Hamilton theorem for the matrix A, where
$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$. Hence find A^{-1}. 5 (CO 1)

2. Solve any **Two** :—

(a) Find a real root of the equation $x^3 + x^2 - 1 = 0$ by the method of iteration.
5 (CO 1)

(b) Apply Taylor series method to obtain the solution of
$y' = 2x + 3y, \quad y(0) = 1$ with $h = 0{\cdot}1$.
Find value of y for $x = 0{\cdot}1$, correct to four places of decimals.
5 (CO 1)

(c) Use Gauss – Seidel method to solve the system of equations :

$3x + y + z = 1, \quad x + 3y - z = 11, \quad x - 2y + 4z = 21.$ 5 (CO 1)

3. Solve any **Two** :—

(a) Find the Z – transform of $\left\{ \frac{a^k}{k!} \right\}$, $k \geqslant 0$. 5 (CO 2)

(b) Obtain $Z^{-1}\left\{ \frac{2z^2 - 10z + 13}{(z-3)^2\ (z-2)} \right\}$, when $2 < |z| < 3$. 5 (CO 2)

(c) Solve the difference equation by Z – transform method :

$y_{n+1} + \frac{1}{4} y_n = \left(\frac{1}{4}\right)^n$, $y_0 = 0$, $n \geqslant 0$.

5 (CO 2)

4. Solve any **Two** :—

(a) The diameter of an electric cable ; say X, is assumed to be a continuous random variable with $f(x) = \begin{cases} 6x(1-x) & , \ 0 < x < 1 \\ 0 & , \ \text{otherwise} \end{cases}$

(i) Check that above is probability density function.

(ii) Determine a number b such that $P(X < b) = P(X > b)$.

5 (CO 3)

(b) The following is the distribution function of a discrete random variable X :

x	–3	–1	0	1	2	3	5	8
F(x)	0·1	0·3	0·45	0·5	0·75	0·9	0·95	1

(i) Find the probability distribution of X.

(ii) Find P (X is even) and $P(1 < X < 8)$. 5 (CO 3)

(c) Let the joint probability function of two discrete random variables X and Y be $f(x, y) = \begin{cases} \frac{x+y}{21} & , \ x = 1, 2, 3, \quad y = 1, 2 \\ 0 & , \ \text{otherwise} \end{cases}$

Find marginal probability functions of X and Y. 5 (CO 3)

5. Solve any **Two** :—

(a) (i) Find the expectation of the number on a die when thrown.

(ii) Two unbiased dice are thrown. Find the expected values of the sum of numbers of points on them. 5 (CO 3)

(b) Let X and Y be two continuous random variable having joint density function : $f(x, y) = \begin{cases} \dfrac{3x(x+y)}{5} & , \quad 0 < x < 1, \quad 0 < y < 2 \\ 0 & , \quad \text{otherwise} \end{cases}$

Find Conditional expectation of X and variance of X. 5 (CO 3)

(c) A discrete random variable X has the probability function $f(x) = \dfrac{1}{2^x}$, where $x = 1, 2, 3, 4, \ldots, \infty$, find the mode and the median. 5 (CO 3)

6. Solve any **Two** :—

(a) If on an average one ship in every ten is wrecked, find the probability that out of 5 ships expected to arrive, at least 4 will arrive safely. 5 (CO 3)

(b) Assuming that the probability of an individual coal miner being killed in a mine accident during a year is $\dfrac{1}{2400}$. Use Poisson distribution to calculate the probability that in a mine employing 200 miners, there will be at least one fatal accident in a year. 5 (CO 3)

(c) Find moment generating function of standard normal variable. 5 (CO 3)