Third Semester B. E. (Computer Science and Engineering / Information Technology) Examination

LINEAR ALGEBRA AND STATISTICS

Time : 3 Hours] [Max. Marks : 60

Instructions to Candidates :—

(1) All questions are compulsory.

(2) Use of Non – programmable calculator is permitted.

(3) Use of area under normal curve table and student's t distribution table is permitted.

1. Solve any **Three** :—

(a) Find $F(x, y)$, where the linear map $F : R^2 \to R^2$ is defined by $F(1, 2) = (3, -1)$ and $F(0, 1) = (2, 1)$. 5 (CO 1, 2)

(b) Prove that kernel of a linear mapping is a subspace of domain of linear mapping. 5 (CO 1, 2)

(c) Find the basis as well as dimension of the kernel and the image of linear map $T : R^4 \to R^3$ defined by :

$$T(x, y, z, t) = (x + 2y + 3z + 2t, 2x + 7z + 5t, x + 2y + 6z + 5t).$$

5 (CO 1, 2)

(d) Let $F : R^3 \to R^3$ be defined by $F(x, y, z) = (2y + z, x - 4y, 3x)$. Find the matrix representing F relative to the bases

$$S = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$$

5 (CO 1, 2)

2. Solve any **Three** :—

(a) Let S consist of the following vectors in R^4 :

$u_1 = (1, 2, 1, 3)$, $u_2 = (16, -13, 1, 3)$, $u_3 = (1, 1, -9, 2)$, $u_4 = (1, 1, 0, -1)$

(i) Determine whether S is orthogonal or not.

(ii) Find basis of R^4. 5 (CO 1, 2)

(b) Define inner product space.

Verify that the following defines an inner product in R^2 :

$\langle u, v \rangle = x_1y_1 - x_1y_2 - x_2y_1 + 3x_2y_2$, where $u = (x_1, x_2)$, $v = (y_1, y_2)$.

5 (CO 1, 2)

(c) Suppose $v = (1, 3, 5, 7)$. Find the projection of v onto W or, in other words, find $w \in W$ that minimizes $\|v - w\|$, where W is the subspace of R^4 spanned by : $u_1 = (1, 1, 1, 1)$ and $u_2 = (1, -3, 4, -2)$. 5 (CO 1, 2)

(d) Find the singular value decomposition of matrix $A = \begin{bmatrix} 5 & 5 \\ -1 & 7 \end{bmatrix}$.

5 (CO 1, 2)

3. Solve :—

(a) The joint probability density function of the two continuous random variables X and Y is given by :

$$f(x, y) = \begin{cases} k\ x^3\ y^3 & , \quad 0 \leqslant x \leqslant 2, \quad 0 \leqslant y \leqslant 2 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Find k and the marginal densities of X and Y. 5 (CO 3)

(b) A salesman S sells in only three cities A, B and C. Suppose S never sells in the same city on successive days. If S sells in city A, then the next day S sells in city B. However, if S sells in either B or C, then the next day S is twice as likely to sell in city A as in the other city. Find out how often, in the long run, S sells in each city.

5 (CO 3)

(c) Explain the Poisson process. 5 (CO 3)

4. Solve any **Three** :—

(a) A certain stimulus administered to each of 12 patients resulted in the following change in blood pressure :

5, 2, 8, –1, 3, 0, –2, 1, 5, 0, 4, 6.

Can it be concluded that the stimulus will increase the blood pressure ?

5 (CO 3)

(b) Twenty people were attacked by a disease and only 18 survived. Will you reject the hypothesis that the survival rate, if attacked by this disease, is 85% in favor of the hypothesis that it is more, at 5% level ? (Use Large Sample Test) 5 (CO 3)

(c) A cigarette manufacturing firm claims that its brand A of the cigarettes outsells its brand B by 8%. If it is found that 42 out of a sample of 100 smokers prefer brand A and 18 out of another random sample of 100 smokers prefer brand B, test whether the 8% difference is a valid claim. (Use 5% level of significance). 5 (CO 3)

(d) In a certain factory there are two independent processes manufacturing the same item. The average weight in a sample of 250 items produced from one process is found to be 120 with a standard deviation of 12, while the corresponding figures in a sample of 400 items from the other process are 124 and 14. Obtain the standard error of difference between the two sample means : Is this difference significant ? Also find the 99% confidence limits for the difference in the average weights of items produced by the two processes respectively. 5 (CO 3)