Cylyndrical

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

Jacobian $= r$ $0<\phi$

Spherical

$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$z = \rho\cos\phi$

Jacobian $= \rho^2\sin\phi$

$r = \rho\sin\phi$

$x^2+y^2+z^2 = \rho^2$ $\theta < 2\pi$

Trig Identities

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

Limits

possible subs

$y = mx$ $m \neq 0$

$y = x^2$ use direct sub and hold variables constant

Find critical points

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$D>0$ + $f_{xx}>0$ local min

$D>0$ + $f_{xx}<0$ local max

$D<0$ saddle point

$\nabla T = 0$ or DNE

check bounds by substitution

Error of T

sub in variables?

$|E(x,y)| \leq \frac{M}{(n+1)!}(|x-a|+|y-b|)^{n+1}$

$M \Rightarrow$ greatest upper bound of all partials $(n+1)$

$T(t) \Rightarrow \frac{r'(t)}{\|r'(t)\|}$ unit vector in direction of motion

$N(t) = \frac{T'(t)}{\|T'(t)\|} = \frac{(v\times a)\times v}{\|(v\times a)\times v\|}$

$B(t) = T\times N = \frac{v\times a}{\|v\times a\|}$

$K(t) = \frac{\|T'(t)\|}{\|r'(t)\|} = \frac{\|v(t)\times a(t)\|}{\|v(t)\|^3} = \frac{1}{r}$ for circle

curvature

acc Tangential

$a_T(t) = \frac{d}{dt}\|v(t)\| = \frac{a\cdot T}{\|T\|} = \frac{a\cdot v}{\|v\|}$

$a_N(t) = \sqrt{\|a(t)\|^2 - a_T(t)^2} = \frac{\|v\times a\|}{\|v\|} = \frac{a\cdot N}{\|N\|} = k\|v\|^2$

acc normal

Speed $= \|v(t)\|$

$\text{proj}_A B = \frac{A\cdot B}{\|A\|^2}A$

$a\times b = \|a\|\|b\|\sin\theta$

$a\cdot b = \|a\|\|b\|\cos\theta$

$\text{comp}_{\hat{n}} P_0P \Rightarrow \frac{P_0P\cdot\hat{n}}{\|\hat{n}\|}$ ← shortest distance between point + plane

$\|\vec{AB}\| \Rightarrow$ distance

$\|AB\times BC\|$ = area of parallelgram

$|\vec{AD}\cdot(\vec{AB}\times\vec{AC})|$ = volume of parallelpiped

$\vec{AD}\cdot\vec{BC} > 0$ angle acute else obtuse

$\theta = \cos^{-1}\left(\frac{n_1\cdot n_2}{\|n_1\|\cdot\|n_2\|}\right)$

Line Integrals

arclength of C $\Rightarrow \int_C ds$

for mass input density function

for averge value of $f(x,y,z)$ over

$\int_C f(x,y,z)\,ds \Big/ \int_C ds$

to solve peramiterize in terms of t

$\int_{t=a}^{t=b} f(x(t),y(t),z(t))\,\|\vec{v}(t)\|\,dt$

Directional derivative

$D_uT(x,y) \Rightarrow \nabla T\cdot\hat{u} \rightarrow$ then plug in

$\hat{u}$ = direction   make sure to make a unit vector

$D_uT(x,y) = 0$ for level curves

$\nabla T \Rightarrow$ direction of greatest change

$\|\nabla T\| \Rightarrow$ greatest change possible

Work $= F\cdot D$ peramiterize

$\text{Work}_{total} = \int_a^b F(r(t))\,v(t)$

$F\cdot T$

Line integral in vector feild $\Rightarrow$ $F = \langle P, Q, R\rangle$ $r(t)$ $a<t<b$

$\int_C F\cdot dr\,ds$

$\Rightarrow \int_a^b F\cdot v\,dt$ or $\int_a^b \left(P\frac{dx}{dt} + Q\frac{dy}{dt} + R\frac{dz}{dt}\right)dt$

Surface Integrals

surface area of $S \Rightarrow \iint_S ds$

for mass inport density equation

center of mass

$\bar{x} \Rightarrow \iint_S x\,\delta(x,y,z)\,ds$

$M \rightarrow \iint_S \delta(x,y,z)\,ds$

to solve

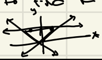
Jacobian $\Rightarrow \|\nabla g\| / |\nabla g\cdot\hat{k}|$ ← whichever im projecting on

$g(x,y,z)$ ← is the surface

$\iint_R f(x,y,z(x,y))\,\frac{\|\nabla g\|}{|\nabla g\cdot\hat{k}|}\,dA$

do not input if not given

to find R figure out lines

Ex  $y=?$ $y=?$ $y=?$

Tangent plane

$L(x,y) = H(a,b) + H_x(a,b)(x-a) + H_y(a,b)(y-b)$

Shapes

Hyperboloid of 1 sheet

$x^2+y^2-z^2=1$

Elyptic Cone

$x^2+y^2-z^2=0$

hyperboloid of 2 sheets

$x^2+y^2-z^2=-1$

Elyptic Parabaloid

$z=x^2+y^2$

Ellipsoid

$x^2+y^2+z^2=1$

Hyperbolic Parabaloid

$z=x^2-y^2$

Cylindr

1 variable missing

only 1 variable squared

FLUX 2D

$\int_C F\cdot n\,ds \Rightarrow \int_{t=a}^{t=b} F(r(t))\cdot\left\langle\frac{dy}{dt}, -\frac{dx}{dt}\right\rangle dt$

$\Rightarrow \int_a^b P\frac{dy}{dt} - Q\frac{dx}{dt}\,dt$ ← how to use? ← plug into values

FLUX 3D

$\iint F\cdot\hat{n}\,ds \Rightarrow \text{Flux} = \iint F\cdot\frac{\nabla g}{\nabla g\cdot p}\,dA$

$\frac{\nabla g}{\nabla g\cdot p}\,dA$

div $F = \frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz}$

Tayler

plug (a,b) in to derivatives

$f(a,b) + f_x(x-a) + f_y(y-b) + \frac{1}{2}(f_{xx}(x-a)^2 + f_{yy}(y-b)^2 + 2f_{xy}(x-a)(y-b)) + \frac{1}{6}(f_{xxx}(x-a)^3 + 3f_{xxy}(x-a)^2(y-b) + 3f_{xyy}(y-b)^2(x-a) + f_{yyy}(y-b)^3)$

Gauses to find Flux in 3d

$\iint \vec{F}\cdot\hat{n}\,ds \Rightarrow \iiint_R \nabla F\,dv$ ← area $= \iint F\cdot ds \times 6$ step 1 for each side

Curl

$\nabla\times F \Rightarrow \begin{vmatrix} dx & dy & dz \\ Q & P & R \end{vmatrix}$

$F \Rightarrow \langle Q, P, R\rangle$

move object from $0\leq t\leq 2\pi$

work $=0$

$(\nabla\times F)\cdot\hat{n} \Rightarrow$ signed rotation speed

$(\nabla\times F)\cdot\hat{n} > 0$ rotates counter clockwise

$F = \langle Q, P\rangle$

Find Circulation $\Rightarrow \iint \frac{dQ}{dx} - \frac{dP}{dy}\,dA$

F must have continuose partials   or $\iint (\nabla\times F)\cdot k\,dA$

Stokes

curl on surface

$\frac{\nabla g}{\nabla g\cdot p}$

$\oint F\cdot dr \Rightarrow \iint(\nabla\times F)\cdot\hat{n}\,ds$

Greens

used for closed curve in 2d space   F has continous partials

Flux divergence form $\oint_C F\cdot\hat{n}\,ds \Rightarrow \iint_R \nabla F\cdot dA$

If asked evaluate $\oint 3y\,dx + 2x\,dy$ Q   P   build orthogonal vector

$F = \langle 2x, -3y\rangle$

$\nabla F = \langle 2, -3\rangle$ multiply into area

$\nabla F = 2-3$

$P-(-Q) \Rightarrow -1$

replace double integral with line integral

$\iint_R 1\,dA$ ← ∇F $F = \langle x, 0\rangle$ or $F = \langle \frac{1}{2}x, \frac{1}{2}y\rangle$

$\text{Area} = \frac{1}{2}\oint x\,dy - y\,dx$

conservative feilds   Curl $(\nabla\times F) = 0$

$\text{FTLI} \Rightarrow \int_C \nabla F\cdot dr \Rightarrow f(b) - f(a)$

only if conservative feild

Plug into potential function

integrate each part of vector feild and add up

$F = \langle 2xy, x^2+1\rangle$

$\frac{df}{dx} = 2xy \Rightarrow x^2y$

$\frac{df}{dy} = x^2+1 \Rightarrow yx^2+y$

$f = x^2y + y$

plug in end - start

Chain rule

$$\frac{df}{dt} = \frac{df}{dx}\cdot\frac{dx}{dt} + \frac{df}{dy}\cdot\frac{dy}{dt}$$

Lagrange Multiplies

use when given 2 functions and asked to find highest or lowest values

$f_x = \lambda g_x$
$f_y = \lambda g_y$

solve for λ and find critical points

plug into $f(x,y)$ and compare

Divergence $= \nabla \cdot F$

For any vector feild $\nabla\cdot(\nabla\times F)=0$

Level Curve Plot

To estimate f_y or f_x use forward difference

$$f_y = \frac{f(a,b_2)-f(a,b_1)}{b_2-b_1}$$

calculate directional derivate

$\nabla g \cdot u$ — make sure to make unit vector

	Flow/ Work/ Circulation	Flux across a curve	Flux through a surface (or across)
	$\int_C F\cdot\hat{T}\,ds$ aka $\int_C P\,dx + Q\,dy + R\,dz$ aka $\int_C F\cdot dr$	$\int_C F\cdot\hat{n}\,ds$ aka $\int_C P\,dy - Q\,dx$	$\iint_S F\cdot\hat{n}\,dS$ aka $\iint_S F\cdot dS$
Analogy and/or Applications	Work done by a force along a curve; Rate at which water flows along a curve	Rate at which water flows across a curve marked on the sidewalk	Rate at which water flows through a net in a river; solar flux, electric flux, magnetic flux, etc

or $\frac{dQ}{dx} - \frac{dP}{dy} = 0$

NO Paramiterize!!!

steps: ① check $\nabla\times F = 0$

yes → use FTLI

no → ③ check closed function with continuous partials

yes → use stokes in 3D, greens in 2D

no → parametize and integrate

Stokes' Thm: $\oint_C \vec{F}\cdot\hat{T}\,ds = \iint_S (\nabla\times\vec{F})\cdot\hat{n}\,dS$ — C curve in 3D

Green's Thm Circulation/Curl form: $\oint_C \vec{F}\cdot\hat{T}\,ds = \iint_R (\nabla\times\vec{F})\cdot\hat{k}\,dA$ — curve in xy-plane

or $\iint_R \left(\frac{dQ}{dx} - \frac{dP}{dy}\right) dA$

Flux across a curve: check closed and continuous partials

yes → greens:

$\oint_C \vec{F}\cdot\hat{n}\,ds = \iint_R \nabla\cdot\vec{F}\,dA$

$\left(\Leftrightarrow \oint_C P\,dy - Q\,dx = \iint_R \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}\right) dA\right)$

no → paramiterize and solve:

$Q(x,y)j$ (Sec 13.2):

$\int_C \vec{F}\cdot\hat{n}\,ds = \int_C \langle P,Q\rangle \cdot \frac{\langle \frac{dy}{dt}, -\frac{dx}{dt}\rangle}{\|\vec{v}\|}\|\vec{v}\|\,dt = \int_a^b \left(P\frac{dy}{dt} - Q\frac{dx}{dt}\right) dt$

Flux through a surface: closed + continuous

$\oiint_S \vec{F}\cdot\hat{n}\,dS = \iiint_E \nabla\cdot\vec{F}\,dV$

2). Option 2: Change variables (Sec 13.7):

$\iint_S \vec{F}\cdot\hat{n}\,dS = \iint_R \vec{F}\cdot\frac{\pm\nabla g}{\|\nabla g\|}\frac{\|\nabla g\|}{|\nabla g\cdot \hat{p}|}\,dA$

$\int_C f(x,y,z)\,ds$ 1D in 2D	$\iint_R f(x,y)\,dA$ 2D in 2D	$\iint_S f(x,y,z)\,dS$ 2D in 3d	$\iiint_D f(x,y,z)\,dV$ 3d in 3d
Length of a wire Mass of a wire Center of mass of a wire Average value of $f(x,y)$ or $f(x,y,z)$ over a path or wire Area of one side of a "curvy fence"	Area of a region in the xy-plane Mass of a 2D lamina/flat region Center of mass of a 2D flat object Average of $f(x,y)$ over a region in the xy-plane	Surface area of a surface Mass of a surface Center of mass of a surface Average of $f(x,y,z)$ on a surface	Volume of a 3D object Mass of a 3D object Center of mass of a 3D object Average of $f(x,y,z)$ on a 3D object
Change variables: $\vec{r}(t) = \langle x(t), y(t), z(t)\rangle, a \le t \le b$ $\int_C f\,ds$ $= \int_{t=a}^{t=b} f(x(t),y(t),z(t))\,\|\vec{v}(t)\|\,dt$	Evaluate iterated integrals If that doesn't work: Try switching order of integration Try changing variables: Switch to polar Try a general uv change	Change variables: Let S be one level surface of $g(x,y,z)$ — project S onto one coordinate plane with normal $\hat{p}$ $\iint_S f(x,y,z)\,dA = \iint_R f(\ )\frac{\|\nabla g\|}{|\nabla g\cdot\hat{p}|}\,dA$ Put in terms of 2 variables of plane where you projected	Evaluate iterated integrals If that doesn't work: Try switching order of integration Try changing variables: Cylindrical Spherical

given $\int_C (2x\cos y + 3)\,dx - (x^2\sin y + 2y)\,dy$

check $\nabla\times F$ — dx dy dz — 0 — $=0$

do FTLI

Solving higest and lowest criticals

$\nabla T = 0$ find criticals

look at boundry ⇒ plug in boundry into function

$T(f(x), y)$
$T(0, y)$
$T(x, 0)$

take derivative to find criticals

plug all criticals into $T(x,y)$

Find equation of plane

create 2 vectors

cross them ⇒ $\langle a_1, b_1, c_1\rangle$

plane equation ⇒ $a_1(x-a) + b_1(y-b) + c_1(z-c)$

rate of change

use chain rule

arclength $= D_uT \Rightarrow \frac{dT}{ds}$

when finding mass $\int_{a_t}^{b_t} k(\text{density})\,\|v(t)\|\,dt$