In [2]:

```
#problem 1
#4. Suppose an interactive system is supporting 100 users with 1
5 second think times and a
#system throughput of 5 interactions/second.
#a. What is the response time of the system?
think = 15  #seconds (Z)
users = 100
rate = 5  #interaction/second

response = users/rate - think
print("Response time for a = ", response, "(s)")

#b. Suppose that the service demands of the workload evolve over
time so that system throughput drops to 50%
#of its former value (i.e., to 2.5 interactions/second). Assumin
g that there still are 100 users with 15 second
#think times, what would their response time be?

rate = 2.5
response = users/rate - think
print("Response time for b = ", response, "(s)")

#c. How do you account for the fact that response time in (b) is
more than twice as large as that in (a)?

print('''There are a lot of users and they form a queue which sl
ows things down.
      As the utilization gets\n near 100 the response time will
increase significantly''')
```

```
Response time for a =  5.0 (s)
Response time for b =  25.0 (s)
There are a lot of users and they form a queue which slows thing
s down.
      As the utilization gets
 near 100 the response time will increase significantly
```

In [3]:
```
#Problem 2
#Part 1 - Visit Count
print("Proxy visits per packet = ", 0.7+2*0.3)
v_proxy = 0.7+2*0.3
print("Router visits per packet = ", 2*0.3)
v_router = 2*0.3
print("A visits per packet = ", 0.3*0.5)
v_A = 0.3*0.5
print("B visits per packet = ", 0.3*0.5)
v_B = 0.3*0.5
```

```
Proxy visits per packet =  1.2999999999999998
Router visits per packet =  0.6
A visits per packet =  0.15
B visits per packet =  0.15
```

In [4]:
```
#Part 2 - Demand
#times in (s)
ser_proxy = 0.010
ser_router = 0.005
ser_A = 0.150
ser_B = 0.100
dem_proxy = ser_proxy*v_proxy
dem_router = ser_router*v_router
dem_A = ser_A*v_A
dem_B = ser_B*v_B

#Add in transmission time to total demand
dem_total = dem_proxy+dem_router+dem_A+dem_B+0.036*2*0.3

#Unit for demand = (visits*seconds)/packet
print("Proxy demand = ", dem_proxy, "(visits*seconds)/packet")
print("Router demand = ", dem_router)
print("A demand = ", dem_A)
print("B demand = ", dem_B)
print("Total demand = ", dem_total)
dem_max = max(dem_proxy,dem_router,dem_A,dem_B)
print("Max demand = ", dem_max)
```

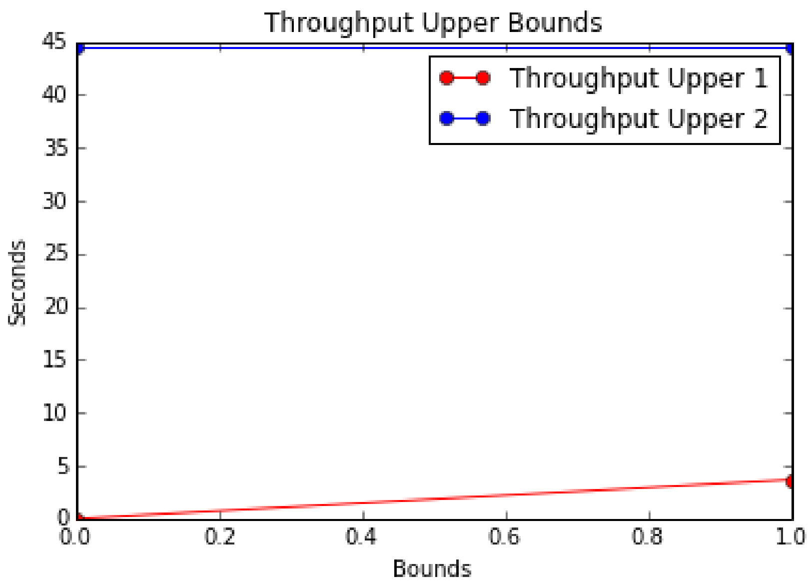
```
Proxy demand =  0.012999999999999998 (visits*seconds)/packet
Router demand =  0.003
A demand =  0.0225
B demand =  0.015
Total demand =  0.07509999999999999
Max demand =  0.0225
```

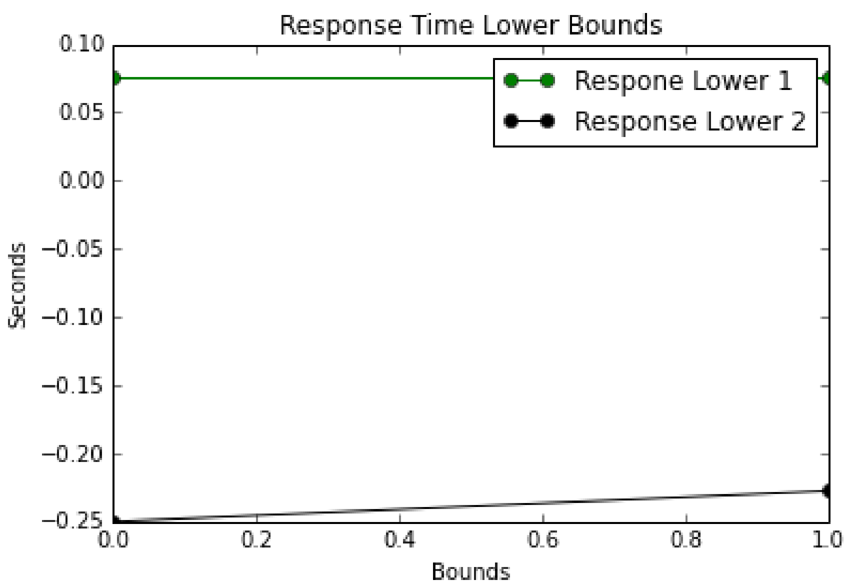
In [5]:

```
#Part 3
#Examine case study 5.3.1 for the process of calculating the job
bounds,
#and calculate the upper bounds on throughput (X) and the lower
bounds on response time (R).
#Write the bounds down and also graph those bounds (two lines fo
r each).
import numpy as np
import scipy as sp
import scipy.stats
import matplotlib.pyplot as plt
import math
%matplotlib inline

#Calculate N*
think_time = 0.250 #(s)
N_star = (dem_total + think_time)/dem_max
print("N* = ", N_star)

#Optimistic bounds throughput
opt_1 = [0,0]
opt_2 = [1,1/(dem_max+think_time)]
line0_x = [0,1]
line0_y = [0,1/(dem_max+think_time)]

opt_3 = [0,1/dem_max]
opt_4 = [1,1/dem_max]
line1_x = [0,1]
line1_y = [1/dem_max, 1/dem_max]

#Response bounds
res_1 = [0, dem_total]
res_2 = [1, dem_total]
line2_x = [0, 1]
line2_y = [dem_total,dem_total]

res_3 = [0, -think_time]
res_4 = [1, dem_max-think_time]
line3_x = [0,1]
line3_y = [-think_time,dem_max-think_time]

line0, = plt.plot(line0_x, line0_y, 'ro-')
line1, = plt.plot(line1_x, line1_y, 'bo-')

#title and axis labels
plt.xlabel('Bounds')
plt.ylabel('Seconds')
plt.title("Throughput Upper Bounds")
#legend
plt.legend([line0, line1],
  ['Throughput Upper 1', 'Throughput Upper 2'], loc=1)
plt.show()
```

```
line2, = plt.plot(line2_x, line2_y, 'go-')
line3, = plt.plot(line3_x, line3_y, 'ko-')

#title and axis labels
plt.xlabel('Bounds')
plt.ylabel('Seconds')
plt.title("Response Time Lower Bounds")
#legend\n",
plt.legend([line2, line3],
  ['Respone Lower 1', 'Response Lower 2'], loc=1)
plt.show()
```

```
N* =  14.448888888888889
```



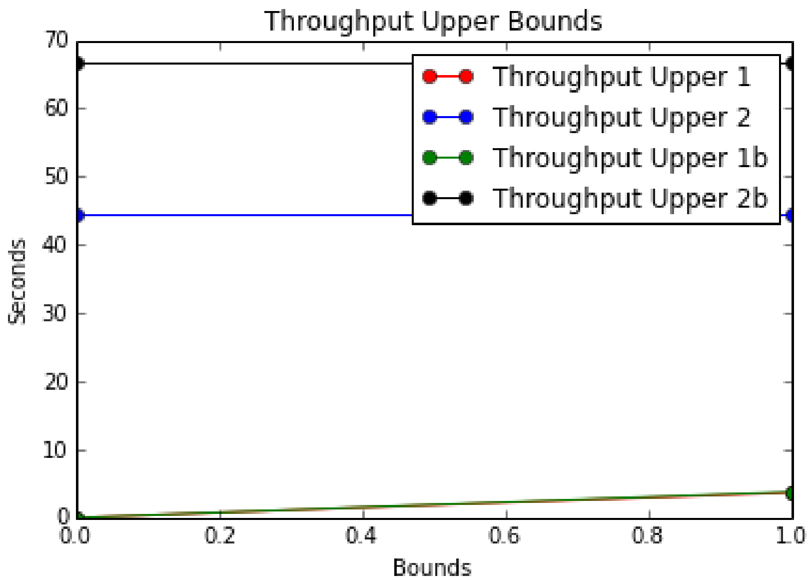


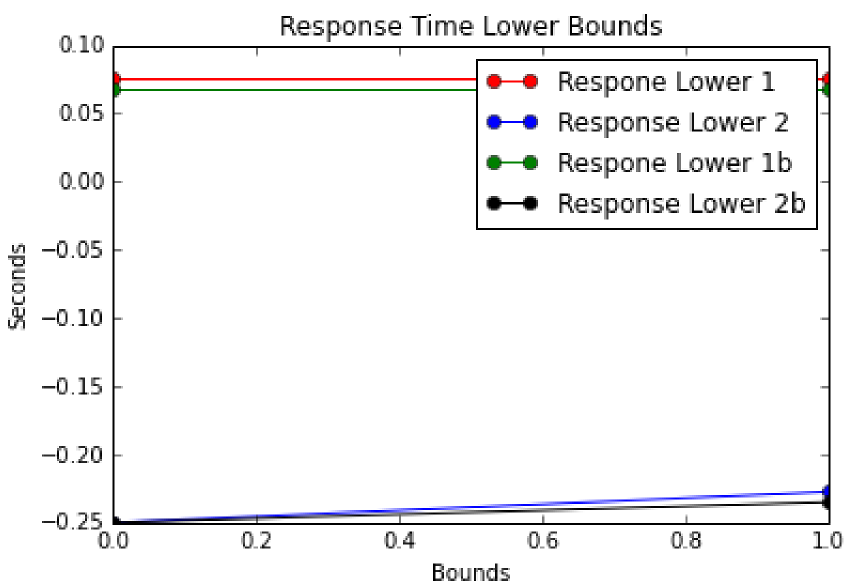
In [6]:

```
#Part 4
#Modify the model in a method similar to 5.3.3 of the QSP text.
#In your case, assume that the two origin servers are of equal p
erformance.
#Write down the bounds equations and graph them again, overlayin
g them with the original bounds from part 3.

#Change response time to 100ms for both A and B
#Part 2 - Demand
#times in (s)
ser_proxy = 0.010
ser_router = 0.005
ser_A = 0.100
ser_B = 0.100
dem_proxy = ser_proxy*v_proxy
dem_router = ser_router*v_router
dem_A = ser_A*v_A
dem_B = ser_B*v_B

#Add in transmission time to total demand
dem_total = dem_proxy+dem_router+dem_A+dem_B+0.036*2*0.3

#Unit for demand = (visits*seconds)/packet
print("Proxy demand = ", dem_proxy, "(visits*seconds)/packet")
print("Router demand = ", dem_router)
print("A demand = ", dem_A)
print("B demand = ", dem_B)
print("Total demand = ", dem_total)
dem_max = max(dem_proxy,dem_router,dem_A,dem_B)
print("Max demand = ", dem_max)

#Calculate N*
think_time = 0.250 #(s)
N_star = (dem_total + think_time)/dem_max
print("N* = ", N_star)

#Optimistic bounds throughput
opt_1 = [0,0]
opt_2 = [1,1/(dem_max+think_time)]
line0b_x = [0,1]
line0b_y = [0,1/(dem_max+think_time)]

opt_3 = [0,1/dem_max]
opt_4 = [1,1/dem_max]
line1b_x = [0,1]
line1b_y = [1/dem_max, 1/dem_max]

#Response bounds
res_1 = [0, dem_total]
res_2 = [1, dem_total]
line2b_x = [0, 1]
line2b_y = [dem_total,dem_total]
```

```
res_3 = [0, -think_time]
res_4 = [1, dem_max-think_time]
line3b_x = [0,1]
line3b_y = [-think_time,dem_max-think_time]

line0, = plt.plot(line0_x, line0_y, 'ro-')
line1, = plt.plot(line1_x, line1_y, 'bo-')
line2, = plt.plot(line0b_x, line0b_y, 'go-')
line3, = plt.plot(line1b_x, line1b_y, 'ko-')

#title and axis labels
plt.xlabel('Bounds')
plt.ylabel('Seconds')
plt.title("Throughput Upper Bounds")
#legend
plt.legend([line0, line1, line2, line3],
  ['Throughput Upper 1', 'Throughput Upper 2', 'Throughput Upper
1b', 'Throughput Upper 2b'], loc=1)
plt.show()

line0, = plt.plot(line2_x, line2_y, 'ro-')
line1, = plt.plot(line3_x, line3_y, 'bo-')
line2, = plt.plot(line2b_x, line2b_y, 'go-')
line3, = plt.plot(line3b_x, line3b_y, 'ko-')

#title and axis labels
plt.xlabel('Bounds')
plt.ylabel('Seconds')
plt.title("Response Time Lower Bounds")
#legend
plt.legend([line0, line1, line2, line3],
  ['Respone Lower 1', 'Response Lower 2', 'Respone Lower 1b', 'R
esponse Lower 2b'], loc=1)
plt.show()
```

```
Proxy demand =  0.012999999999999998 (visits*seconds)/packet
Router demand =  0.003
A demand =  0.015
B demand =  0.015
Total demand =  0.0676
Max demand =  0.015
N* =  21.173333333333332
```



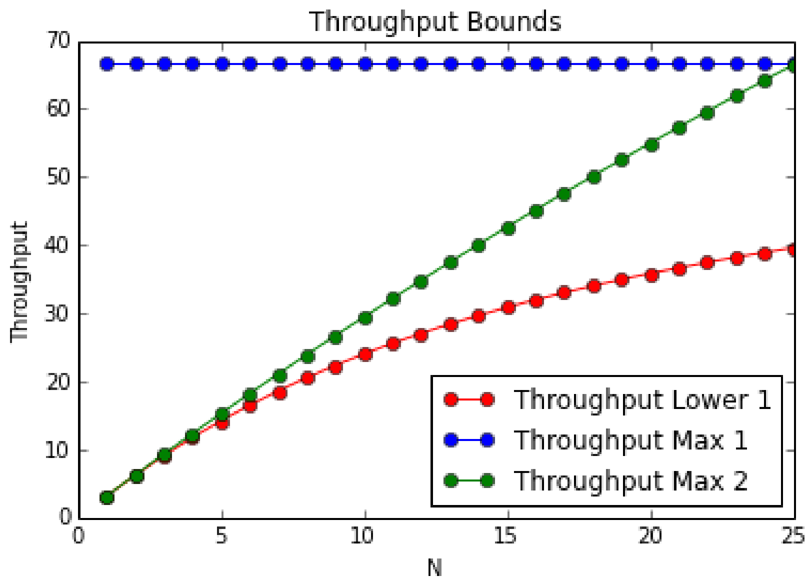


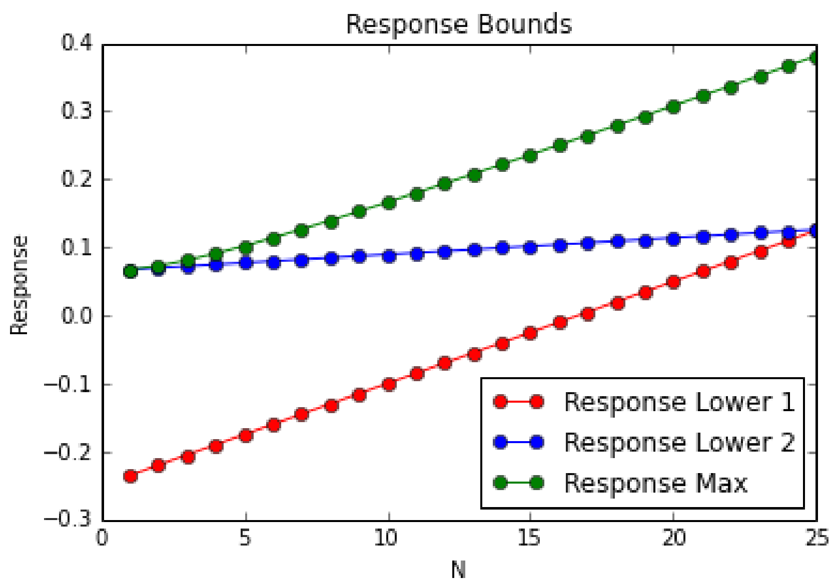
In [7]:

```
#Part 5
#Now, determine the lower bounds on throughput (X) and the upper bounds on response time (R)
#for the original system using the balanced job bounds equations summarized in Table 5.2.
#Produce a single graph with the upper and lower bounds of X and R.

dem_ave = np.mean([dem_proxy,dem_router,dem_A,dem_B])

N_batch = (dem_total-dem_ave)/(dem_max-dem_ave)
N_terminal = ((dem_total+think_time)**2-dem_total*dem_ave)/((dem_total+think_time)*dem_max-dem_total*dem_ave)

print("Dem_ave, N+batch, N+terminal")
print(dem_ave, N_batch, N_terminal)

#Throughput
def terminal_t_min(N):
    output = N/(dem_total+think_time+((N-1)*dem_max)/(1+think_time/(N*dem_total)))
    return output

def terminal_t_max1(N):
    return 1/dem_max

def terminal_t_max2(N):
    output = N/(dem_total+think_time+((N-1)*dem_ave)/(1+think_time/(dem_total)))
    return output

N = []
min1 = []
max1 = []
max2 = []
for ii in range(1,int(N_terminal+1)):
    N.append(ii)
    min1.append(terminal_t_min(ii))
    max1.append(terminal_t_max1(ii))
    max2.append(terminal_t_max2(ii))

line0, = plt.plot(N, min1, 'ro-')
line1, = plt.plot(N, max1, 'bo-')
line2, = plt.plot(N, max2, 'go-')

#title and axis labels
plt.xlabel('N')
plt.ylabel('Throughput')
plt.title("Throughput Bounds")
#legend
plt.legend([line0, line1, line2, line3],
  ['Throughput Lower 1', 'Throughput Max 1', 'Throughput Max 2'], loc=4)
```

```
plt.show()


#Response Time
def terminal_r_min1(N):
    return N*dem_max-think_time

def terminal_r_min2(N):
    return dem_total+((N-1)*dem_ave)/(1+think_time/dem_total)

def terminal_r_max(N):
    return dem_total+((N-1)*dem_max)/(1+think_time/(N*dem_tota
l))

rmin1 = []
rmin2 = []
rmax = []
for ii in range(1,int(N_terminal+1)):
    rmin1.append(terminal_r_min1(ii))
    rmin2.append(terminal_r_min2(ii))
    rmax.append(terminal_r_max(ii))

line0, = plt.plot(N, rmin1, 'ro-')
line1, = plt.plot(N, rmin2, 'bo-')
line2, = plt.plot(N, rmax, 'go-')

#title and axis labels
plt.xlabel('N')
plt.ylabel('Response')
plt.title("Response Bounds")
#legend
plt.legend([line0, line1, line2, line3],
  ['Response Lower 1', 'Response Lower 2', 'Response Max'], loc=
4)
plt.show()
```

```
Dem_ave, N+batch, N+terminal
0.0115 16.0285714286 25.107199117
```





In [8]:

```
#Problem 3
#You will use the Vi and Di from the prior problem.

v_proxy = 0.7+2*0.3
v_router = 2*0.3
v_A = 0.3*0.5
v_B = 0.3*0.5

#times in (s)
ser_proxy = 0.010
ser_router = 0.005
ser_A = 0.150
ser_B = 0.100
dem_proxy = ser_proxy*v_proxy
dem_router = ser_router*v_router
dem_A = ser_A*v_A
dem_B = ser_B*v_B

#Add in transmission time to total demand
dem_total = dem_proxy+dem_router+dem_A+dem_B+0.036*2*0.3

#Unit for demand = (visits*seconds)/packet
dem_max = max(dem_proxy,dem_router,dem_A,dem_B)

#Write down the equations for the Response for the individual co
mponents for N customers in the system,
#using the appropriate service times (again, in the PDF documen
t)
#and the Queue length for N-1 customers in the system

#residence times
#delay center = D_k
#queue center = D_k*(1+A_k(N))

#6.1 system throughput

Q_proxy = []
Q_router = []
Q_A = []
Q_B = []

r_proxy_l = []
r_router_l = []
r_A_l = []
r_B_l = []
r_PR_l = []

XX = []

K=5

#Utilization
def U(N):
```

```
    return (N+1)/(N+K)

#Service Center times
def r_proxy(N):
    if N==0:
        return dem_proxy
    return dem_proxy*(1+Q_proxy[N])

def r_router(N):
    if N==0:
        return dem_router
    return dem_router*(1+Q_router[N])

def r_A(N):
    if N==0:
        return dem_A
    return dem_A*(1+Q_A[N])

def r_B(N):
    if N==0:
        return dem_B
    return dem_B*(1+Q_B[N])

def r_PR_delay(N):
    return 0.036*2*0.3

def X(N):
    output = N/(think_time + (r_proxy_l[N]+r_router_l[N]+r_A_l
[N]+r_B_l[N]+r_PR_l[N]))
    return output

#Queue times for service centers
def Q_proxy_find(N):
    if N==0:
        return U(N)
    else:
        return X(N)*r_proxy(N)

def Q_router_find(N):
    if N==0:
        return U(N)
    else:
        return X(N)*r_router(N)

def Q_A_find(N):
    if N==0:
        return U(N)
    else:
        return X(N)*r_A(N)

def Q_B_find(N):
    if N==0:
        return U(N)
```

```
    else:
        return X(N)*r_B(N)

def Q_PR_find(N):
    if N==0:
        return U(N)
    else:
        return X(N)*r_PR_delay(N)


class prettyfloat(float):
    def __repr__(self):
        return "%0.6f" % self

print("Problem 3 - Original Service Times")
print('''N, Qproxy,   Qrouter,  QA,       QB,       proxy,    ro
uter,   A,       B,       PR_delay, Throughput''')

for N in range(1):
    Q_proxy.append(Q_proxy_find(N))
    Q_router.append(Q_router_find(N))
    Q_A.append(Q_A_find(N))
    Q_B.append(Q_B_find(N))

for N in range(0,31):

    #
    r_proxy_l.append(r_proxy(N))
    r_router_l.append(r_router(N))
    r_A_l.append(r_A(N))
    r_B_l.append(r_B(N))
    r_PR_l.append(r_PR_delay(N))

    XX.append(X(N))

    Q_proxy.append(Q_proxy_find(N))
    Q_router.append(Q_router_find(N))
    Q_A.append(Q_A_find(N))
    Q_B.append(Q_B_find(N))

    print(N, [prettyfloat(x) for x in [Q_proxy[N], Q_router[N],
Q_A[N], Q_B[N], \
          r_proxy_l[N], r_router_l[N], r_A_l[N], r_B_l[N], r_PR_
l[N], XX[N]]] )
```

```
Problem 3 - Original Service Times
N, Qproxy,   Qrouter,  QA,       QB,       proxy,    router,
A,       B,        PR_delay, Throughput
0 [0.200000, 0.200000, 0.200000, 0.200000, 0.013000, 0.003000,
0.022500, 0.015000, 0.021600, 0.000000]
1 [0.200000, 0.200000, 0.200000, 0.200000, 0.015600, 0.003600,
0.027000, 0.018000, 0.021600, 2.977963]
2 [0.046456, 0.010721, 0.080405, 0.053603, 0.013604, 0.003032,
0.024309, 0.015804, 0.021600, 6.091075]
3 [0.082863, 0.018469, 0.148069, 0.096264, 0.014077, 0.003055,
0.025832, 0.016444, 0.021600, 9.063222]
4 [0.127585, 0.027692, 0.234117, 0.149035, 0.014659, 0.003083,
0.027768, 0.017236, 0.021600, 11.963696]
5 [0.175371, 0.036885, 0.332204, 0.206201, 0.015280, 0.003111,
0.029975, 0.018093, 0.021600, 14.790358]
6 [0.225994, 0.046008, 0.443335, 0.267602, 0.015938, 0.003138,
0.032475, 0.019014, 0.021600, 17.535399]
7 [0.279478, 0.055026, 0.569463, 0.333419, 0.016633, 0.003165,
0.035313, 0.020001, 0.021600, 20.189640]
8 [0.335819, 0.063902, 0.712955, 0.403819, 0.017366, 0.003192,
0.038541, 0.021057, 0.021600, 22.743031]
9 [0.394947, 0.072589, 0.876550, 0.478906, 0.018134, 0.003218,
0.042222, 0.022184, 0.021600, 25.184825]
10 [0.456710, 0.081039, 1.063363, 0.558690, 0.018937, 0.003243,
0.046426, 0.023380, 0.021600, 27.503782]
11 [0.520845, 0.089198, 1.276882, 0.643048, 0.019771, 0.003268,
0.051230, 0.024646, 0.021600, 29.688476]
12 [0.586971, 0.097010, 1.520936, 0.731694, 0.020631, 0.003291,
0.056721, 0.025975, 0.021600, 31.727725]
13 [0.654563, 0.104417, 1.799630, 0.824141, 0.021509, 0.003313,
0.062992, 0.027362, 0.021600, 33.611156]
14 [0.722953, 0.111362, 2.117223, 0.919672, 0.022398, 0.003334,
0.070138, 0.028795, 0.021600, 35.329886]
15 [0.791332, 0.117793, 2.477951, 1.017327, 0.023287, 0.003353,
0.078254, 0.030260, 0.021600, 36.877282]
16 [0.858773, 0.123663, 2.885791, 1.115903, 0.024164, 0.003371,
0.087430, 0.031739, 0.021600, 38.249705]
17 [0.924268, 0.128939, 3.344183, 1.213990, 0.025015, 0.003387,
0.097744, 0.033210, 0.021600, 39.447158]
18 [0.986790, 0.133600, 3.855728, 1.310034, 0.025828, 0.003401,
0.109254, 0.034651, 0.021600, 40.473682]
19 [1.045365, 0.137643, 4.421906, 1.402434, 0.026590, 0.003413,
0.121993, 0.036037, 0.021600, 41.337411]
20 [1.099151, 0.141082, 5.042870, 1.489656, 0.027289, 0.003423,
0.135965, 0.037345, 0.021600, 42.050232]
21 [1.147507, 0.143948, 5.717342, 1.570359, 0.027918, 0.003432,
0.151140, 0.038555, 0.021600, 42.627041]
22 [1.190045, 0.146289, 6.442660, 1.643502, 0.028471, 0.003439,
0.167460, 0.039653, 0.021600, 43.084724]
23 [1.226647, 0.148163, 7.214961, 1.708418, 0.028946, 0.003444,
0.184837, 0.040626, 0.021600, 43.440996]
24 [1.257461, 0.149632, 8.029487, 1.764846, 0.029347, 0.003449,
0.203163, 0.041473, 0.021600, 43.713297]
```

```
25 [1.282854, 0.150763, 8.880944, 1.812908, 0.029677, 0.003452,
0.222321, 0.042194, 0.021600, 43.917879]
26 [1.303355, 0.151617, 9.763878, 1.853054, 0.029944, 0.003455,
0.242187, 0.042796, 0.021600, 44.069176]
27 [1.319591, 0.152252, 10.672992, 1.885976, 0.030155, 0.003457,
0.262642, 0.043290, 0.021600, 44.179483]
28 [1.332218, 0.152718, 11.603402, 1.912514, 0.030319, 0.003458,
0.283577, 0.043688, 0.021600, 44.258891]
29 [1.341878, 0.153054, 12.550784, 1.933570, 0.030444, 0.003459,
0.304893, 0.044004, 0.021600, 44.315420]
30 [1.349157, 0.153294, 13.511445, 1.950036, 0.030539, 0.003460,
0.326508, 0.044251, 0.021600, 44.355276]
```

In [9]:

```
#BALANCED SERVICE TIMES

#You will use the Vi and Di from the prior problem.

v_proxy = 0.7+2*0.3
v_router = 2*0.3
v_A = 0.3*0.5
v_B = 0.3*0.5

#times in (s)
ser_proxy = 0.010
ser_router = 0.005
ser_A = 0.100
ser_B = 0.100
dem_proxy = ser_proxy*v_proxy
dem_router = ser_router*v_router
dem_A = ser_A*v_A
dem_B = ser_B*v_B

#Add in transmission time to total demand
dem_total = dem_proxy+dem_router+dem_A+dem_B+0.036*2*0.3

#Unit for demand = (visits*seconds)/packet
dem_max = max(dem_proxy,dem_router,dem_A,dem_B)

#Write down the equations for the Response for the individual co
mponents for N customers in the system,
#using the appropriate service times (again, in the PDF documen
t)
#and the Queue length for N-1 customers in the system

#residence times
#delay center = D_k
#queue center = D_k*(1+A_k(N))

#6.1 system throughput

Q_proxy = []
Q_router = []
Q_A = []
Q_B = []

r_proxy_l = []
r_router_l = []
r_A_l = []
r_B_l = []
r_PR_l = []

XX = []

K=5

for N in range(1):
```

```python
    Q_proxy.append(0)
    Q_router.append(0)
    Q_A.append(0)
    Q_B.append(0)

#Utilization
def U(N):
    return (N+1)/(N+K)

#Service Center times
def r_proxy(N):
    return dem_proxy*(1+Q_proxy[N-1])

def r_router(N):
    return dem_router*(1+Q_router[N-1])

def r_A(N):
    return dem_A*(1+Q_A[N-1])

def r_B(N):
    return dem_B*(1+Q_B[N-1])

def r_PR_delay(N):
    return 0.036*2*0.3

def X(N):
    output = N/(think_time + (r_proxy_l[N]+r_router_l[N]+r_A_l
[N]+r_B_l[N]+r_PR_l[N]))
    return output

#Queue times for service centers
def Q_proxy_find(N):
    if N==0:
        return U(N)
    else:
        return X(N)*r_proxy(N)

def Q_router_find(N):
    if N==0:
        return U(N)
    else:
        return X(N)*r_router(N)

def Q_A_find(N):
    if N==0:
        return U(N)
    else:
        return X(N)*r_A(N)

def Q_B_find(N):
    if N==0:
        return U(N)
    else:
```

```
        return X(N)*r_B(N)

def Q_PR_find(N):
    if N==0:
        return U(N)
    else:
        return X(N)*r_PR_delay(N)


class prettyfloat(float):
    def __repr__(self):
        return "%0.6f" % self

print("Problem 3 - Balanced Service Times (100ms each)")
print('''N, Qproxy,   Qrouter,  QA,       QB,       proxy,    ro
uter,   A,        B,        PR_delay, Throughput''')

for N in range(0,31):
    #
    r_proxy_l.append(r_proxy(N))
    r_router_l.append(r_router(N))
    r_A_l.append(r_A(N))
    r_B_l.append(r_B(N))
    r_PR_l.append(r_PR_delay(N))

    XX.append(X(N))

    Q_proxy.append(Q_proxy_find(N))
    Q_router.append(Q_router_find(N))
    Q_A.append(Q_A_find(N))
    Q_B.append(Q_B_find(N))

    print(N, [prettyfloat(x) for x in [Q_proxy[N], Q_router[N],
Q_A[N], Q_B[N], \
          r_proxy_l[N], r_router_l[N], r_A_l[N], r_B_l[N], r_PR_
l[N], XX[N]]] )
```

```
Problem 3 - Balanced Service Times (100ms each)
N, Qproxy,   Qrouter,  QA,       QB,       proxy,    router,
A,       B,        PR_delay, Throughput
0 [0.000000, 0.000000, 0.000000, 0.000000, 0.013000, 0.003000,
0.015000, 0.015000, 0.021600, 0.000000]
1 [0.200000, 0.200000, 0.200000, 0.200000, 0.013000, 0.003000,
0.015000, 0.015000, 0.021600, 3.148615]
2 [0.040932, 0.009446, 0.047229, 0.047229, 0.015600, 0.003600,
0.018000, 0.018000, 0.021600, 6.119951]
3 [0.095471, 0.022032, 0.110159, 0.110159, 0.013532, 0.003028,
0.015708, 0.015708, 0.021600, 9.387399]
4 [0.127031, 0.028428, 0.147461, 0.147461, 0.014241, 0.003066,
0.016652, 0.016652, 0.021600, 12.414187]
5 [0.176792, 0.038063, 0.206726, 0.206726, 0.014651, 0.003085,
0.017212, 0.017212, 0.021600, 15.443513]
6 [0.226269, 0.047648, 0.265813, 0.265813, 0.015298, 0.003114,
0.018101, 0.018101, 0.021600, 18.392819]
7 [0.281379, 0.057279, 0.332926, 0.332926, 0.015941, 0.003143,
0.018987, 0.018987, 0.021600, 21.298683]
8 [0.339533, 0.066941, 0.404402, 0.404402, 0.016658, 0.003172,
0.019994, 0.019994, 0.021600, 24.138734]
9 [0.402101, 0.076564, 0.482627, 0.482627, 0.017414, 0.003201,
0.021066, 0.021066, 0.021600, 26.918157]
10 [0.468751, 0.086160, 0.567059, 0.567059, 0.018227, 0.003230,
0.022239, 0.022239, 0.021600, 29.626485]
11 [0.540011, 0.095684, 0.658876, 0.658876, 0.019094, 0.003258,
0.023506, 0.023506, 0.021600, 32.261470]
12 [0.615993, 0.105123, 0.758334, 0.758334, 0.020020, 0.003287,
0.024883, 0.024883, 0.021600, 34.815561]
13 [0.697013, 0.114441, 0.866320, 0.866320, 0.021008, 0.003315,
0.026375, 0.026375, 0.021600, 37.284185]
14 [0.783263, 0.123611, 0.983371, 0.983371, 0.022061, 0.003343,
0.027995, 0.027995, 0.021600, 39.660720]
15 [0.874962, 0.132599, 1.110294, 1.110294, 0.023182, 0.003371,
0.029751, 0.029751, 0.021600, 41.939932]
16 [0.972269, 0.141372, 1.247737, 1.247737, 0.024375, 0.003398,
0.031654, 0.031654, 0.021600, 44.115889]
17 [1.075303, 0.149897, 1.396462, 1.396462, 0.025639, 0.003424,
0.033716, 0.033716, 0.021600, 46.183641]
18 [1.184125, 0.158138, 1.557130, 1.557130, 0.026979, 0.003450,
0.035947, 0.035947, 0.021600, 48.138317]
19 [1.298721, 0.166062, 1.730425, 1.730425, 0.028394, 0.003474,
0.038357, 0.038357, 0.021600, 49.976072]
20 [1.419002, 0.173638, 1.916930, 1.916930, 0.029883, 0.003498,
0.040956, 0.040956, 0.021600, 51.693705]
21 [1.544782, 0.180834, 2.117187, 2.117187, 0.031447, 0.003521,
0.043754, 0.043754, 0.021600, 53.289237]
22 [1.675788, 0.187627, 2.331614, 2.331614, 0.033082, 0.003543,
0.046758, 0.046758, 0.021600, 54.761749]
23 [1.811637, 0.193994, 2.560539, 2.560539, 0.034785, 0.003563,
0.049974, 0.049974, 0.021600, 56.111719]
24 [1.951860, 0.199919, 2.804139, 2.804139, 0.036551, 0.003582,
0.053408, 0.053408, 0.021600, 57.340897]
```

```
25 [2.095883, 0.205394, 3.062468, 3.062468, 0.038374, 0.003600,
0.057062, 0.057062, 0.021600, 58.452445]
26 [2.243065, 0.210415, 3.335418, 3.335418, 0.040246, 0.003616,
0.060937, 0.060937, 0.021600, 59.450763]
27 [2.392684, 0.214985, 3.622752, 3.622752, 0.042160, 0.003631,
0.065031, 0.065031, 0.021600, 60.341447]
28 [2.543986, 0.219115, 3.924081, 3.924081, 0.044105, 0.003645,
0.069341, 0.069341, 0.021600, 61.131046]
29 [2.696178, 0.222820, 4.238905, 4.238905, 0.046072, 0.003657,
0.073861, 0.073861, 0.021600, 61.826887]
30 [2.848477, 0.226122, 4.566609, 4.566609, 0.048050, 0.003668,
0.078584, 0.078584, 0.021600, 62.436793]
```

In [10]:

```
#Compare the results from the simulation model, MVA model and the balanced job bounds.
#Do the bounds correctly bound the MVA model? How different are the MVA results than the simulation?

#Simulation results: MeanResp = 0.211 Qrouter = 0.132 Qprox = .57 QA = 0.985 QB = 0.66
balanced_resp_time = 0.048050+0.003668+0.078584+0.021600
print("balanced response time = ", balanced_resp_time, "s")
mva_resp_time = ((0.030539 +0.003460 +0.326508+ 0.021600)+(0.030539+0.003460+ 0.044251+0.021600))/2
print("mva response time = ", mva_resp_time, "s")
sim_resp_time = 0.42
print("simulation response time", sim_resp_time, "s")
print("The bounds correctly bounded the MVA model. The response time and throughput was within the bounded range.")
print('''The MVA analysis was not very close to the actual simluation. It ended up having a response time
         which was about 40% faster. ''')
```

```
balanced response time =  0.151902 s
mva response time =  0.2409785 s
simulation response time 0.42 s
The bounds correctly bounded the MVA model. The response time and throughput was within the bounded range.
The MVA analysis was not very close to the actual simluation. It ended up having a response time
         which was about 40% faster.
```

In [14]:

```
#Problem 4

#Original SERVICE TIMES


v_proxy = 0.7+2*0.3
v_router = 2*0.3
v_A = 0.3*0.5
v_B = 0.3*0.5

#times in (s)
ser_proxy = 0.010
ser_router = 0.005
ser_A = 0.150
ser_B = 0.100
dem_proxy = ser_proxy*v_proxy
dem_router = ser_router*v_router
dem_A = ser_A*v_A
dem_B = ser_B*v_B

#Add in transmission time to total demand
dem_total = dem_proxy+dem_router+dem_A+dem_B+0.036*2*0.3

#Unit for demand = (visits*seconds)/packet
dem_max = max(dem_proxy,dem_router,dem_A,dem_B)

#Write down the equations for the Response for the individual co
mponents for N customers in the system,
#using the appropriate service times (again, in the PDF documen
t)
#and the Queue length for N-1 customers in the system

#residence times
#delay center = D_k
#queue center = D_k*(1+A_k(N))

#6.1 system throughput

Q_proxy = []
Q_router = []
Q_A = []
Q_B = []

r_proxy_l = []
r_router_l = []
r_A_l = []
r_B_l = []
r_PR_l = []

XX = []

K=4
```

```
# h function selected
# h = ((N-1)/N)*Q[N]

#Service Center times
def r_proxy(N):
    return dem_proxy*(1+((N-1)/N)*Q_proxy[N-1])

def r_router(N):
    return dem_router*(1+((N-1)/N)*Q_router[N-1])

def r_A(N):
    return dem_A*(1+((N-1)/N)*Q_A[N-1])

def r_B(N):
    return dem_B*(1+((N-1)/N)*Q_B[N-1])

def r_PR_delay(N):
    return 0.036*2*0.3

def X(jobs, N):
    n = N-1 #account for starting at N=1
    output = jobs/(think_time + (r_proxy_l[n]+r_router_l[n]+r_A_
l[n]+r_B_l[n]+r_PR_l[n]))
    return output

#Queue times for service centers
def Q_proxy_find(jobs, N):
    return X(jobs, N)*r_proxy(N)

def Q_router_find(jobs, N):
    return X(jobs, N)*r_router(N)
def Q_A_find(jobs, N):
    return X(jobs, N)*r_A(N)

def Q_B_find(jobs, N):
    return X(jobs, N)*r_B(N)

def Q_PR_find(jobs, N):
    return X(jobs, N)*r_PR_delay(N)


class prettyfloat(float):
    def __repr__(self):
        return "%0.6f" % self

print("Problem4 - Original Service Times")
print("h function - h=((N-1)/N)*Q")
print('''Iter,Qproxy, Qrouter,  QA,       QB,       proxy,    ro
uter,   A,        B,        PR_delay, Throughput''')


jobs = 30
Q_proxy.append(jobs/K)
```

```
Q_router.append(jobs/K)
Q_A.append(jobs/K)
Q_B.append(jobs/K)


for N in range(1,31):

    r_proxy_l.append(r_proxy(N))
    r_router_l.append(r_router(N))
    r_A_l.append(r_A(N))
    r_B_l.append(r_B(N))
    r_PR_l.append(r_PR_delay(N))

    XX.append(X(jobs, N))

    Q_proxy.append(Q_proxy_find(jobs, N))
    Q_router.append(Q_router_find(jobs, N))
    Q_A.append(Q_A_find(jobs, N))
    Q_B.append(Q_B_find(jobs, N))


    n=N-1 #account for starting at N=1
    print(N, [prettyfloat(x) for x in [Q_proxy[n], Q_router[n],
Q_A[n], Q_B[n], \
          r_proxy_l[n], r_router_l[n], r_A_l[n], r_B_l[n], r_PR_
l[n], XX[n]]] )
```

```
Problem4 - Original Service Times
h function - h=((N-1)/N)*Q
Iter,Qproxy, Qrouter,  QA,       QB,       proxy,    router,
A,        B,        PR_delay, Throughput
1 [7.500000, 7.500000, 7.500000, 7.500000, 0.013000, 0.003000,
0.022500, 0.015000, 0.021600, 92.279299]
2 [1.199631, 0.276838, 2.076284, 1.384189, 0.020798, 0.003415,
0.045858, 0.025381, 0.021600, 81.732183]
3 [1.699833, 0.279136, 3.748091, 2.074479, 0.027732, 0.003558,
0.078721, 0.035745, 0.021600, 71.881027]
4 [1.993397, 0.255772, 5.658572, 2.569372, 0.032436, 0.003575,
0.117988, 0.043905, 0.021600, 63.897091]
5 [2.072542, 0.228463, 7.539116, 2.805430, 0.034554, 0.003548,
0.158204, 0.048665, 0.021600, 58.075159]
6 [2.006754, 0.206069, 9.187727, 2.826237, 0.034740, 0.003515,
0.194770, 0.050328, 0.021600, 54.058646]
7 [1.877989, 0.190025, 10.528996, 2.720661, 0.033926, 0.003489,
0.225559, 0.049980, 0.021600, 51.321183]
8 [1.741131, 0.179041, 11.575966, 2.565029, 0.032805, 0.003470,
0.250402, 0.048666, 0.021600, 49.428021]
9 [1.621504, 0.171514, 12.376867, 2.405464, 0.031737, 0.003457,
0.270037, 0.047073, 0.021600, 48.084248]
10 [1.526068, 0.166245, 12.984542, 2.263463, 0.030855, 0.003449,
0.285437, 0.045557, 0.021600, 47.103334]
11 [1.453373, 0.162453, 13.445033, 2.145875, 0.030176, 0.003443,
0.297512, 0.044262, 0.021600, 46.368335]
12 [1.399221, 0.159649, 13.795138, 2.052352, 0.029674, 0.003439,
0.307025, 0.043220, 0.021600, 45.804488]
13 [1.359205, 0.157523, 14.063110, 1.979663, 0.029310, 0.003436,
0.314580, 0.042411, 0.021600, 45.362626]
14 [1.329599, 0.155876, 14.270174, 1.923861, 0.029050, 0.003434,
0.320645, 0.041797, 0.021600, 45.009515]
15 [1.307534, 0.154573, 14.432063, 1.881246, 0.028865, 0.003433,
0.325573, 0.041337, 0.021600, 44.722165]
16 [1.290894, 0.153522, 14.560344, 1.848700, 0.028733, 0.003432,
0.329632, 0.040997, 0.021600, 44.484372]
17 [1.278159, 0.152661, 14.663483, 1.823741, 0.028639, 0.003431,
0.333021, 0.040747, 0.021600, 44.284531]
18 [1.268249, 0.151942, 14.747671, 1.804459, 0.028571, 0.003431,
0.335888, 0.040563, 0.021600, 44.114211]
19 [1.260400, 0.151334, 14.817435, 1.789412, 0.028523, 0.003430,
0.338345, 0.040428, 0.021600, 43.967206]
20 [1.254069, 0.150812, 14.876098, 1.777528, 0.028488, 0.003430,
0.340477, 0.040330, 0.021600, 43.838888]
21 [1.248871, 0.150359, 14.926115, 1.768012, 0.028462, 0.003430,
0.342345, 0.040257, 0.021600, 43.725757]
22 [1.244532, 0.149962, 14.969309, 1.760282, 0.028444, 0.003429,
0.344000, 0.040204, 0.021600, 43.625138]
23 [1.240852, 0.149610, 15.007044, 1.753907, 0.028430, 0.003429,
0.345478, 0.040165, 0.021600, 43.534952]
24 [1.237687, 0.149295, 15.040355, 1.748570, 0.028420, 0.003429,
0.346808, 0.040136, 0.021600, 43.453569]
25 [1.234929, 0.149012, 15.070030, 1.744039, 0.028412, 0.003429,
```

```
0.348013, 0.040114, 0.021600, 43.379689]
26 [1.232500, 0.148756, 15.096681, 1.740140, 0.028406, 0.003429,
0.349111, 0.040098, 0.021600, 43.312268]
27 [1.230339, 0.148522, 15.120784, 1.736743, 0.028402, 0.003429,
0.350117, 0.040086, 0.021600, 43.250453]
28 [1.228400, 0.148309, 15.142718, 1.733750, 0.028399, 0.003429,
0.351043, 0.040077, 0.021600, 43.193542]
29 [1.226648, 0.148112, 15.162786, 1.731087, 0.028397, 0.003429,
0.351898, 0.040071, 0.021600, 43.140952]
30 [1.225054, 0.147931, 15.181235, 1.728698, 0.028395, 0.003429,
0.352692, 0.040066, 0.021600, 43.092191]
```

In [ ]:

```
```