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Mathematik

Aaron Tsamaltoupis

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Contents

1 Logic

1.1 akkentential logic

1.1.1 Sentence symbols

	symbol	verbose name	remarks
1	(	left parenthesis	
2	)	right parenthesis	
3	$\neg$	negation symbol	not
4	$\wedge$	conjunction symbol	and
5	$\vee$	disjunction symbol	or
6	$\rightarrow$	conditional symbol	if, then
7	$\leftrightarrow$	biconditional symbol	if and only if (iff)
...	A_1	first sentence symbol	
	A_2	second sentence symbol	
	...		
	A_n	nth sentence symbol	
	...		

- The first seven symbols are called the *logical symbols* Their translation is fixed.

  The logical symbols without the parenthesis are the *sentential connective symbols*.
  The sentence symbols A_i are the *parameters (non logical symbols)*.
  Their translation is not fixed.

- No sentence symbol is itself a sequence of other sentence symbols.
  That means:

1.1.2 expressions

An expression is a finite sequence of symbols.
bspw: $(\neg A_1)$
If α and β are expressions $\alpha\beta$ is the sequence of consisting of all the senctence symbols in α followed by all in β.
Sei
$\alpha = (\neg A_1)$
β=A$_2$

$\alpha \rightarrow \beta$ then is $(\neg A_1) \rightarrow A_2$

If parameters are combined with the logical symbols expressions can be used to translate english sentences into the logical language.

Some expressions are nonsense. Bspw: $((\rightarrow A_3$

This is why *grammatically correct* expressions are defined as well formed formulas

1.1.3 well formed formulas

The definition of the well formed formulas aims to exclude the nonsensical expressions:

- Every sentece symbol is a wff.
- If α and β are wffs, then so are $\neg\alpha$, $(\alpha \wedge \beta)$, $(\alpha \vee \beta)$, $(\alpha \rightarrow \beta)$,

the formula-building operations

1. $\varepsilon_{\neg}(\alpha) = (\neg\alpha)$
2. $\varepsilon_{\wedge}(\alpha, \beta) = (\alpha \wedge \beta)$
3. $\varepsilon_{\vee}(\alpha, \beta) = (\alpha \vee \beta)$
4. $\varepsilon_{\rightarrow}(\alpha, \beta) = (\alpha \rightarrow \beta)$
5. $\varepsilon_{\leftrightarrow}(\alpha, \beta) = (\alpha \leftrightarrow \beta)$

1.1.4 Truth Assignments

What does it mean for one wff to *follow logically from other wffs* ? When an expression α follows logically from another expression β then no matter how the sentence symbols in β and α are tanslated, if β is true α will be true as well.
Example: A_1 follows logically from $A_1 \wedge A_2$
That means: $(A_1 \wedge A_2) \rightarrow A_1$ is true.
Clearer Definition of the translation of sentence symbols:

Truth Assignments
Sei S a set of sentence Symbols.
Sei a set $\{F, T\}$ of truth values.
sei a function $v : S \rightarrow \{F, T\}$ the *truth assignment* which determines whether each sentence symbol is true or false.

Sei $\overline{S}$ the set of all wffs that can be built up from S using the formula-building operations.

sei another function $\overline{v} : \overline{S} \rightarrow \{F, T\}$ for which 6 conditions hold:

Conditions for $\overline{v} : \overline{S} \rightarrow \{F, T\}$:

0. $\forall A \in S(\overline{v}(A) = v(A))$
this means $\overline{v}$ is an extension of v

1. ($\overline{v}((\neg\alpha)) = \begin{cases} T if \overline{v}(\alpha) = F \\ F\ otherwise \end{cases}$

2. $\overline{v}((\alpha \wedge \beta)) = \begin{cases} T\ if\ \overline{v}(\alpha) = T\ and\ \overline{v}(\beta) = T \\ F\ otherwise \end{cases}$

3. $\overline{v}((\alpha \vee \beta)) = \begin{cases} T\ if\ \overline{v}(\alpha) = T\ or\ \overline{v}(\beta) = T\ (or\ both) \\ F\ otherwise \end{cases}$

4. $\overline{v}((\alpha \rightarrow \beta)) = \begin{cases} F\ if\ \overline{v}(\alpha) = T\ and\ \overline{v}(\beta) = F \\ T\ otherwise \end{cases}$

5. $\overline{v}((\alpha \leftrightarrow \beta)) = \begin{cases} T \ if \ \overline{v}(\alpha) = \overline{v}(\beta) \\ F \ otherwise \end{cases}$

1.2 Induction

1.2.1 the induction principle

1.2.2 strong induction

1.2.3 the well ordering principle

Every nonempty set of natural numbers has a smallest element.

$\forall S \subseteq \mathbb{N}(S \neq \emptyset \rightarrow S$ has a smallest element)

Proof. Suppose $S \subset \mathbb{N}$ and S does not have a smallest element.
We will prove that $\forall n \in \mathbb{N}(n \notin S)$ meaning that $S = \emptyset$

We will prove this using strong induction.
Suppose that $n \in \mathbb{N}$ and $\forall k < n(k \notin S)$
Goal: $n \notin S$
If $n \in S$ then n would be the smalles element of S since all elements smaller then n are not in S by the induction hypothesis. This is a contradiction to the assumption that S does not have a smalles element.

2 Relations

2.1 functions

2.1.1 extended/restricted functions

A restriction of a function f is a new function $f|_A$ which is the same function as f its Domain is just a subset of the Domain of f:
$Dom(f) \subseteq Dom(f|_A)$

Sei $f : A \to B$
Sei $C \subseteq A$
Sei $f|_C : C \to B$, wobei $f|_C \subseteq f$
$\to f|_C$ is a restriction of f and f is an extension of $f|_C$

In an extension of a function the domain is extended, in an restriction of a function the domain is restricted.

3 Linear algebra

4 Kombinatorik

4.1 Permutationen

4.1.1 Permutationen ohne Wiederholungen

Sei X eine Menge

$$P_x = \{(a_1, ..., a_{|x|}) : \{a_1, ..., a_{|x|}\} = X\}$$

Die Permutationen P_x von X sind also alle Möglichen unterschiedlichen Wege, die Elemente in X anzuordnen

Systematische Notierung der Permutationen von P_{x+1}:

Es gibt genau x+1 Stellen, an denen das "neue" Element a_{x+1} stehen kann. Für jede dieser Konfigurationen gibt es für die anderen stellen P_x Möglichkeiten die anderen Elemente anzuordnen.

P_3:
(1,2,**3**)
(2,1,**3**)

(1,**3**,2)
(2,**3**,1)

(**3**,1,2)
(**3**,2,1)

P_4:
(1,2,3,4) (1,2,4,3) (1,4,2,3) (4,1,2,3)
(2,1,3,4) (2,1,4,3) (1,4,2,3) (4,1,2,3)
(1,3,2,4) (1,3,4,2) (1,4,2,3) (4,1,2,3)
(2,3,1,4) (2,3,4,1) (1,4,2,3) (4,1,2,3)
(3,1,2,4) (3,1,4,2) (1,4,2,3) (4,1,2,3)
(3,2,1,4) (3,2,4,1) (1,4,2,3) (4,1,2,3)

Demnach gilt:

$$P_{x+1} = (x + 1) \cdot P_x$$

Da $P_0 = 1$, gilt $P_n = n!$

4.1.2 Permutationen mit Wiederholungen

Sei ein Tuple (1, 1, 2, 3).
Auf wie viele verschiedene Weisen kann dies angeordnet werden?
4! kann es nicht sein, da wenn die einsen vertauscht werden das selbe Tuple dabei herauskommt.

Sei die beiden einsen seien unterschiedliche Objekte 1_a und 1_b
Die berechnung der Variation ist nun eine Permutation ohne Wiederholung und kann durch 4! berechnet werden.

Für jede Variation in der ursprünglichen Variante kommen hier noch die Variationen innerhalb derselben Objekte hinzu, P_4 muss also durch die Anzahl der Möglichkeiten der Variationen innerhalb der gleichen elemente (P_2) geteilt werden.
$P_{(1,1,2,3)} = \frac{P_4}{P_2}$