Linear Programming: The Diet Problem

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Documentation

Chobani Yogurt Drink

Salmon

Portion: 0.5 fillet (198 g)

Name	Amount	Unit	Deriv. By	n
Water	129	g	Analytical	4
Energy	412	kcal	Calculated	
Energy	1720	kJ	Calculated	
Protein	40.4	g	Analytical	4
Total lipid (fat)	26.5	g	Analytical	4
Ash	2.24	g	Analytical	4
Carbohydrate, by difference	0	g	Assumed zero (Insignificant amount or not naturally occurring in a food, such as fiber in meat)	
Fiber, total dietary	0	g	Assumed zero (Insignificant amount or not naturally occurring in a food, such as fiber in meat)	
Total Sugars	0	g	Based on another form of the food or similar food; Concentration adjustment; No adjustment; Retention factors not used	
Calcium, Ca	17.8	mg	Analytical	4
Iron, Fe	0.673	mg	Analytical	4
Magnesium, Mg	53.5	mg	Analytical	4
Phosphorus, P	475	mg	Analytical	4
Potassium, K	719	mg	Analytical	4
Sodium, Na	117	mg	Analytical	4
Vitamin D (D2 + D3)	21.8	µg	Based on another form of the food or similar food; Concentration adjustment; No adjustment; Retention factors not used	

(USDA Salmon)

Principe Prosciutto

Tomatoes

Portion: 1 large whole (3" dia) (182 g)

Name	Amount	Unit	Deriv. By	n
Water	172	g	Aggregated data involving combinations of data with different source codes when at least one code is not 1, 6, 12, or 13	
Energy	32.8	kcal	Calculated	
Energy	135	kJ	Calculated	
Protein	1.6	g	Aggregated data involving combinations of data with different source codes when at least one code is not 1, 6, 12, or 13	
Total lipid (fat)	0.364	g	Aggregated data involving combinations of data with different source codes when at least one code is not 1, 6, 12, or 13	
Ash	0.91	g	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Carbohydrate, by difference	7.08	g	Calculated	
Fiber, total dietary	2.18	g	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Total Sugars	4.79	g	Calculated	
Sucrose	0	g	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Glucose	2.28	g	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Fructose	2.49	g	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Lactose	0	g	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Maltose	0	g	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Galactose	0	g	Analytical	4
Starch	0	g	Analytical	4
Calcium, Ca	18.2	mg	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Iron, Fe	0.491	mg	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Magnesium, Mg	20	mg	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Phosphorus, P	43.7	mg	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Potassium, K	431	mg	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Sodium, Na	9.1	mg	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Vitamin D (D2 + D3)	0	µg	Assumed zero (Insignificant amount or not naturally occurring in a food, such as fiber in meat)	

(USDA Tomatoes)

Zucchini

Portion: 1 large (323 g)

Name	Amount	Unit	Deriv. By	n
Water	306	g	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Energy	54.9	kcal	Calculated	
Energy	226	kJ	Calculated	
Protein	3.91	g	Aggregated data involving combinations of data with different source codes when at least one code is not 1, 6, 12, or 13	
Total lipid (fat)	1.03	g	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Ash	1.87	g	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Carbohydrate, by difference	10	g	Calculated	
Fiber, total dietary	3.23	g	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Total Sugars	8.08	g	Calculated	
Sucrose	0.162	g	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Glucose	3.46	g	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Fructose	4.46	g	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Lactose	0	g	Analytical	4
Maltose	0	g	Analytical	4
Galactose	0	g	Analytical	4
Starch	0	g	Analytical	4
Calcium, Ca	51.7	mg	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Iron, Fe	1.2	mg	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Magnesium, Mg	58.1	mg	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Phosphorus, P	123	mg	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Potassium, K	843	mg	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Sodium, Na	25.8	mg	Aggregated data involving combinations of data with only source codes 1 and 12 and/or 13	
Vitamin D (D2 + D3)	0	µg	Assumed zero (Insignificant amount or not naturally occurring in a food, such as fiber in meat)	

(USDA Zucchini)

Price Calculations:

- Low-fat yogurt
  - Costco sells 12 bottles for $22.69. Each bottle = $1.89 per serving.
- Prosciutto
  - 9oz is sold for $9.99. 28g (1 serving) / 255g (9oz in g) = 11%
    11% x $9.99 = $1.10 per serving.
- Tomatoes
  - Each is sold for $0.99 individually. $0.99 per serving.
- Zucchini
  - Each is sold for $1.29 individually. $1.29 per serving.
- Salmon
  - 672g is sold for $14.99. $4.42 per serving.

Description

I need to design a daily diet using five food items, being low-fat yogurt, prosciutto, tomatoes, zucchini, and salmon, that ensures I get the required amounts of calories, Protein, Vitamin D, Calcium, Iron, and Potassium while keeping my Sodium intake below the daily limit. The goal is to calculate the right amount of each food item so that I meet nutritional needs while minimizing cost. Each food item provides different amounts of these nutrients. The listed serving sizes in the previous section are my daily serving sizes. For salmon, tomatoes, and zucchini, I don't cook them; I eat them raw as is, so recipes are unneeded.

The variables assigned to each food item is displayed below:

Low-fat yogurt	Prosciutto	Tomatoes	Zucchini	Salmon
x_1	x_2	x_3	x_4	x_5

Objective Function:

- Minimize $Z = 1.89x_1 + 1.10x_2 + 0.99x_3 + 1.29x_4 + 4.42x_5$

Weekly Constraints:

- Sodium: $115x_1 + 546x_2 + 9x_3 + 26x_4 + 117x_5 \leq 35000$
- Calorie: $170x_1 + 70x_2 + 33x_3 + 55x_4 + 412x_5 \geq 14000$
- Protein: $20x_1 + 9x_2 + 2x_3 + 4x_4 + 40x_5 \geq 350$
- Vitamin D: $0x_1 + 0x_2 + 0x_3 + 0x_4 + 22x_5 \geq 140$
- Calcium: $320x_1 + 0x_2 + 18x_3 + 52x_4 + 18x_5 \geq 9100$
- Iron: $0x_1 + 1x_2 + 0.5x_3 + 2x_4 + 1x_5 \geq 324$
- Potassium: $430x_1 + 0x_2 + 431x_3 + 843x_4 + 719x_5 \geq 32900$
- Non-negativity: $x_1, x_2, x_3, x_4, x_5 \geq 0$

Solutions

Weekly minimized cost: $188.83

Low-fat yogurt	Prosciutto	Tomatoes	Zucchini	Salmon
18.62 servings	0 servings	0 servings	53.41 servings	19.11 servings

Weekly minimized cost (minimum 1 serving per food item): $189.35

Low-fat yogurt	Prosciutto	Tomatoes	Zucchini	Salmon
18.74 servings	1 serving	1 serving	52.77 servings	18.95 servings

I have to spend $0.52 more than optimal for minimum 1 serving of each food during the week. I could add further variety to my diet by adding Fiber, Vitamin C, and other nutrients as constraints. I could also require minimum 7 servings every week, min. one serving for each day:

Weekly minimized cost (minimum 7 servings per food item): $192.54

Low-fat yogurt	Prosciutto	Tomatoes	Zucchini	Salmon
19.11 servings	7 servings	7 servings	48.84 servings	17.83 servings

I have to spend $3.71 more than optimal for minimum 7 servings of each food during the week.

LLM-prompted Solution

I selected the o1-mini LLM by The General Mind to solve the Diet Problem. The chat is through OpenAI's UI; The General Mind's URL is thegeneralmind.com. I chose o1-mini as it was recommended to be robust for programming-related problems. I used the following prompt:

Create python code to solve the Diet Problem. I want to minimize cost while meeting nutrient constraints. Lowfat yogurt costs $1.89 per bottle and has 115mg Sodium, 170 cals, 20g Protein, 0mcg VitD, 320mg Calcium, 0mg Iron, 430mg Potassium. Proscuitto costs $1.10 per serving and has 546mg Sodium, 70 cals, 9g Protein, 0mcg VitD, 0mg Calcium, 1mg Iron, 0mg Potassium. Tomatoes cost $.99 per serving and has 9mg Sodium, 33 cals, 2g Protein, 0mcg VitD, 18mg Calcium, 0.5mg Iron, 431mg Potassium. Zucchini costs $1.29 per serving and has 26mg Sodium, 55 cals, 4g Protein, 0mcg VitD, 52mg Calcium, 2mg Iron, 843mg Potassium. Salmon costs $4.42 per serving and has 117mg Sodium, 412 cals, 40g Protein, 22mcg VitD, 18mg Calcium, 1mg Iron, 719mg Potassium. I would like to have at most 35,000mg Sodium, at least 14,000 cals, at least 350g Protein, at least 140mcg VitD, at least 9,100mg Calcium, at least 324mg Iron, and at least 32,900mg Potassium. Serving sizes cannot be negative. I would like the exact serving counts to meet all constraints and the minimized cost.

The conversation with the LLM agent went well, as one prompt was all it needed to formulate the code successfully (as seen in the Ass1_o1-mini_Prompt_Output.txt file). An LLM agent could be used to complete this assignment.

References

1. U.S. Department of Agriculture. "FoodData Central: Salmon." Last modified April 1, 2019. https://fdc.nal.usda.gov/fdc-app.html#/food-details/175167/nutrients.

2. U.S. Department of Agriculture. "FoodData Central: Tomatoes." Last modified April 1, 2019. https://fdc.nal.usda.gov/fdc-app.html#/food-details/170457/nutrients.

3. U.S. Department of Agriculture. "FoodData Central: Zucchini." Last modified April 1, 2019. https://fdc.nal.usda.gov/fdc-app.html#/food-details/169291/nutrients.