Xronos 9/4 #3

①

a) $u \cdot v = (5 \cdot 2 + -1 \cdot 5 + 2 \cdot -4)$

$= -3$

b) $u \cdot u = (5 \cdot 5 + -1 \cdot -1 + 2 \cdot 2)$

$= 30$

c) $|u^2| = (5 \cdot 5 + -1 \cdot -1 + 2 \cdot 2)$

$= 30$

d) $(u \cdot v)v = -3\langle 2, 5, -4 \rangle$

$= \langle -6, -15, 12 \rangle$

② $\cos\theta = \frac{u \cdot v}{|u||v|}$

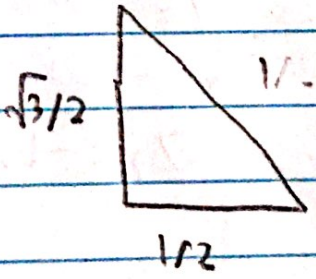
$\cos\frac{\pi}{3} = \frac{a \cdot b}{160}$

$a \cdot b = 80$

③ a) $u = \langle 1, 0 \rangle$

$v = \langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$

$u \cdot v = -.5$



b) $u \cdot v = (1 \cdot \frac{1}{2} + 0 \cdot \frac{\sqrt{3}}{2})$

$= 1/2$

④

$\cos\theta = \frac{a \cdot b}{|a||b|}$

$\theta = \cos^{-1}\left(\frac{0+20-5}{\sqrt{18} \cdot \sqrt{50}}\right)$

$= \cos^{-1}\left(\frac{15}{\sqrt{900}}\right)$

⑤ $\theta = \cos^{-1}\left(\frac{a \cdot b}{|a||b|}\right)$

$= \cos^{-1}\left(\frac{6+4}{\sqrt{25}\cdot\sqrt{9}}\right)$

$= \cos^{-1}\left(\frac{10}{15}\right)$

$= \cos^{-1}\left(\frac{2}{3}\right)$

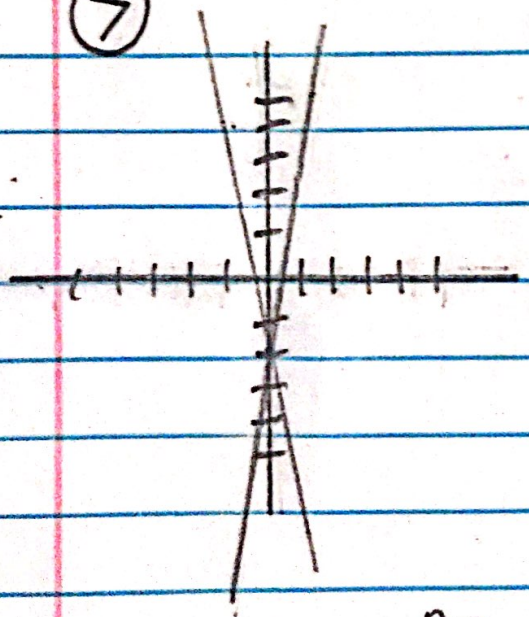
⑥ $a = \langle 1,0,0 \rangle$

$b = \langle 1,1,1 \rangle$

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{1}\cdot\sqrt{3}}\right)$

$= \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

⑦



$y = 7x - 2 \quad (0,-2) \Rightarrow (1,5)$

$y = -6x + 5$

$\Rightarrow y = -6x - 2 \quad (0,-2) = (-1,4)$

$a = \langle 1,7,0 \rangle$

$b = \langle -1,6,0 \rangle$

$\theta = \cos^{-1}\left(\frac{-1+42}{\sqrt{50}\cdot\sqrt{33}}\right)$

$\theta = \cos^{-1}\left(\frac{41}{\sqrt{50}\cdot\sqrt{33}}\right)$

⑧

a) $\text{comp}_u v = \frac{u \cdot v}{|v|} = \frac{-1+10-2}{3} = 7/3$

b) $\text{proj}_v u = \frac{u \cdot v}{|v|^2} \cdot v$

$= \left(\frac{-1+10-2}{9}\right) \cdot \langle 1, 2, -2 \rangle$

$= \langle \frac{7}{9}, \frac{14}{9}, -\frac{14}{9} \rangle$

⑦

$F_1 = x\langle 4, 2, -4 \rangle \qquad u = \langle 4, 2, -4 \rangle$

$\langle 6, 4, 0 \rangle = F_1 + F_2$

$u \cdot F_2 = 0$

$4F_{2x} + 2F_{2y} - 4F_{2z} = 0$

$\langle 6, 4, 0 \rangle = \langle 4x + F_{2x}, 2x + F_{2y}, -4x + F_{2z} \rangle$

$6 = 4x + F_{2x}$

$4 = 2x + F_{2y}$

$4x = F_{2z}$

$4(6-4x) + 2(4-2x) - 4(4x) = 0$

$24 - 16x + 8 - 4x - 16x = 0$

$32 = 36x$

$x = 8/9$

$F_1 = \langle \frac{32}{9}, \frac{16}{9}, -\frac{32}{9} \rangle$

$F_2 = \langle 6 - 32/9, 4 - 16/9, 32/9 \rangle$

(10) $W = Fd$

$= \langle 2,1,3 \rangle \cdot \langle 2-0, -6-1, 0-2 \rangle$

$= (2 \cdot 2 + 1 \cdot -7 + 3 \cdot -2)$

$= -9$

(11) $W = Fd\cos\theta$

$= 1400(1000)(\cos 30°)$

$= 700000\sqrt{3}$ J

(12) $\text{comp}_a b = \text{comp}_b a$

$|b|\cos\theta = |a|\cos\theta$

$\cos\theta = 0$

$\theta = \pi/2$

or

$|b| = |a|$

(13) $|a|(\cos\theta)\left(\frac{b}{|b|}\right) = |b|(\cos\theta)\left(\frac{a}{|a|}\right)$

$\cos\theta = 0$

$\theta = \pi/2$

or

$a = b$