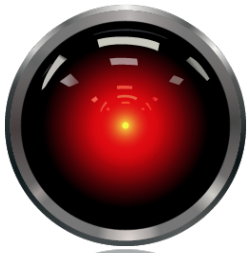

INTELLIGENT COMPUTING

(COMPUTATIONAL INTELLIGENCE)
Introduction to Fuzzy Logic

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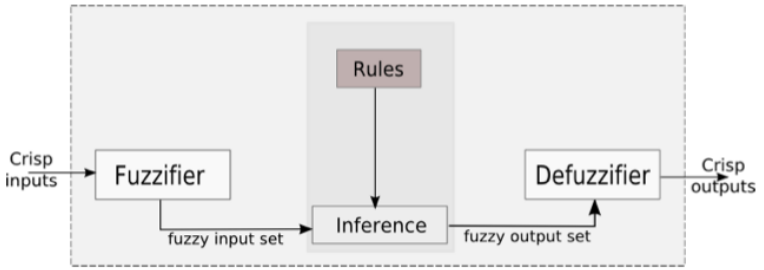
WHAT IS A FUZZY LOGIC SYSTEM?

- A fuzzy logic system (FLS) can be defined as the nonlinear mapping of an input data set to a scalar output data.

- A FLS consists of four main parts: **fuzzifier, rules**, **inference engine**, and **defuzzifier**.



WHAT IS A FUZZY LOGIC SYSTEM?

- The steps involved in a fuzzy system are as follows:

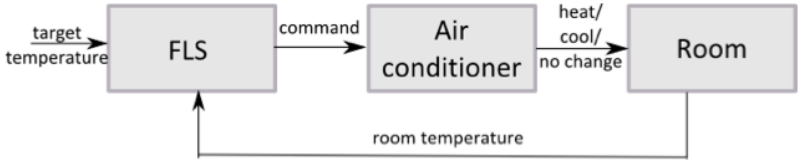
1. Firstly, a crisp set of input data are gathered and converted to a fuzzy set using fuzzy linguistic variables, fuzzy linguistic terms and membership functions. This step is known as fuzzification.

2. Afterwards, an inference is made based on a set of rules.

3. Lastly, the resulting fuzzy output is mapped to a crisp output using the membership functions, in the defuzzification step.

FUZZY LOGIC SYSTEM EXAMPLE

- Consider an air conditioner system controlled by a FLS. The system adjusts the temperature of the room according to the current temperature of the room and the target value. The fuzzy engine periodically compares the room temperature and the target temperature, and produces a command to heat or cool the room.



PSEUDO FUZZY LOGIC ALGORITHM

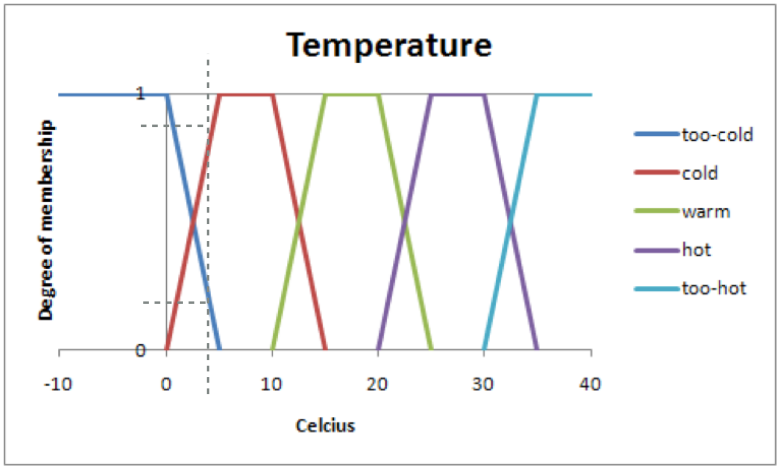
1. Define the linguistic variables and terms (initialization)
2. Construct the membership functions (initialization)
3. Construct the rule base (initialization)
4. Convert crisp input data to fuzzy values using the membership functions (fuzzification)
5. Evaluate the rules in the rule base (inference)
6. Combine the results of each rule (inference)
7. Convert the output data to non-fuzzy values (defuzzification)

FUZZY LOGIC: LINGUISTIC VARIABLES

- Linguistic variables are the input or output variables of the system whose values are words or sentences from a natural language, instead of numerical values. A linguistic variable is generally decomposed into a set of linguistic terms.
- Consider the air conditioner example Let ***temperature (t)*** be the linguistic variable which represents the temperature of a room. To qualify the temperature, terms such as **"hot"** and **"cold"** are used in real life. These are the linguistic values of the temperature.
- Then, ***T(t) = {too-cold, cold, warm, hot, too-hot}*** can be the set of decompositions for the linguistic variable temperature. Each member of this decomposition is called a linguistic ***term*** and can cover a ***portion*** of the overall values of the temperature.

FUZZY LOGIC: MEMBERSHIP FUNCTIONS

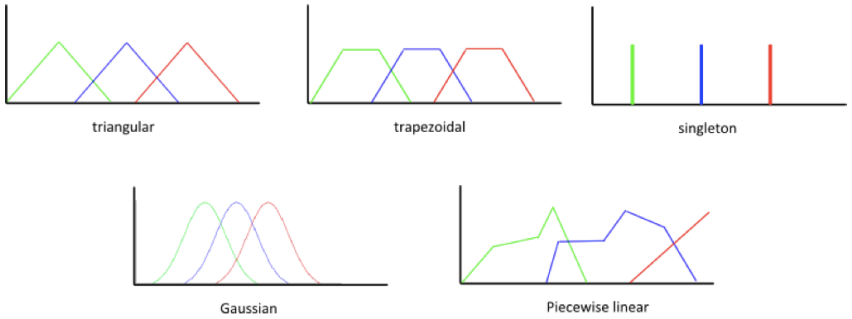
- Membership functions are used in the fuzzification and defuzzification steps of a FLS, to map the non-fuzzy input values to fuzzy linguistic terms and vice versa.
- A membership function is used to quantify a linguistic term. The next slide plots membership functions for the linguistic terms used to describe the idea of temperature as used in the air conditioner example.
- Note that, an important characteristic of fuzzy logic is that a numerical value does not have to be fuzzified using only one membership function. In other words, a value can belong to ***multiple*** sets at the same time.
- So a temperature value can be considered as "cold" and "too-cold" at the same time, with different degree of memberships.



Membership Functions for T(temperature) = {too-cold, cold, warm, hot, too-hot}.

FUZZY LOGIC: MEMBERSHIP FUNCTIONS

- There are different forms of membership functions. The most common types are triangular, trapezoidal, and Gaussian shapes. The type of the membership function can be context dependent and it is generally chosen arbitrarily according to the user experience.



FUZZY LOGIC: FUZZY RULES

- In a FLS, a rule base is constructed to control the output variable. A fuzzy rule is a simple IF-THEN rule with a condition and a conclusion.

1. `IF (temperature is cold OR too-cold) AND (target is warm) THEN command is heat`
2. `IF (temperature is hot OR too-hot) AND (target is warm) THEN command is cool`
3. `IF (temperature is warm) AND (target is warm) THEN command is no-change`

FUZZY LOGIC: FUZZY RULES

- The table below shows the matrix representation of the fuzzy rules for the FLS. Row captions in the matrix contain the values that current room temperature can take, column captions contain the values for target temperature, and each cell is the resulting command when the input variables take the values in that row and column. For instance, the cell <<*heat*>> in the matrix can be read as follows: *If temperature is cold and target is warm then command is heat.*

	Target		
Temperature	**cold**	**warm**	**hot**
too-cold	heat	heat	heat
cold	no-change	***<<heat>>***	heat
warm	cool	no-change	heat
hot	cool	cool	no-change
too-hot	cool	cool	cool

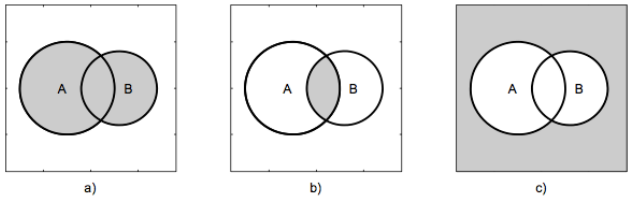
FUZZY LOGIC: FUZZY SET OPERATIONS

- It is clear what the statement X is LOW means in fuzzy logic. But how do we interpret a statement like:

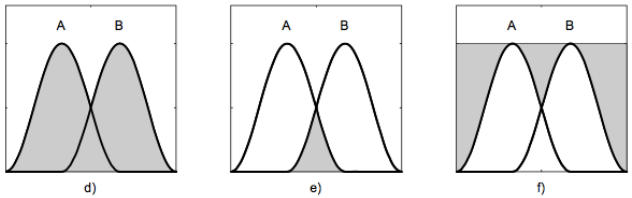
  X is LOW and Y is HIGH or (not Z is MEDIUM)

- The standard definitions in fuzzy logic as suggested by Lotfi are:
  1. Negate(negation criterion) : $\mu A(x) = 1 - \mu A(x)$
  2. Intersection(minimum criterion): $Min\{\mu A(x), \mu B(x)\}$
  3. Union(maximum criterion): $Max\{\mu A(x), \mu B(x)\}$

FUZZY LOGIC: FUZZY SET OPERATIONS



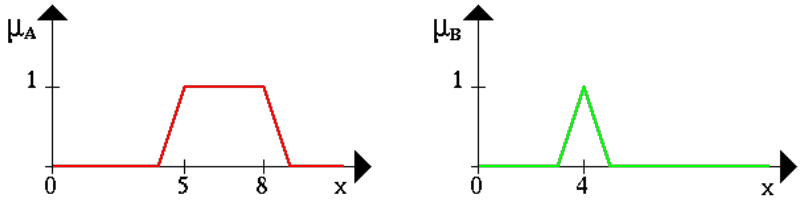
OR *AND* *NOT*



union $A \cup B$ *intersection* $A \cap B$ *complement* $\overline{A \cup B}$

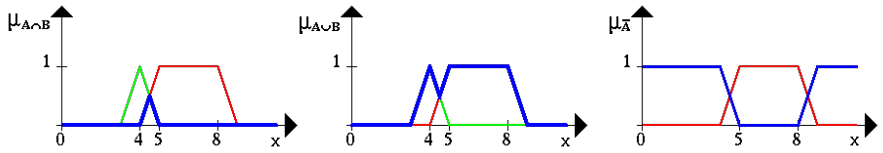
FUZZY LOGIC: FUZZY SET OPERATIONS

- Lets take an example in order to see how the fuzzy set operators work. Let ***A*** represent a fuzzy interval between 5 and 8 and ***B*** represent a fuzzy number about 4.



FUZZY LOGIC: FUZZY SET OPERATIONS

- The following figures show how the fuzzy sets A and B are effected by the operators AND, OR and NOT.



AND *OR* *Negation*

FUZZY LOGIC: INFERENCE

- After evaluating the result of each rule, these results should be combined to obtain a final result. This process is called inference. The results of individual rules can be combined in different ways. The table below contains possible accumulation methods that are used to combine the results of individual rules. The maximum algorithm is generally used for accumulation.

Operation	Formula
Maximum	Max{μA (x), μB (x)}
Bounded sum	Min{1, μA (x) + μB (x)}
Normalized sum	$\frac{\mu A(x)+\mu B(x)}{Max\{1,Max\{\mu A(x'),\mu B(x')\}\}}$

FUZZY LOGIC: DEFUZZIFICATION

- After the inference step, the overall result is a fuzzy value. This result should be defuzzified to obtain a final crisp output. This is the purpose of the defuzzifier component of a FLS.

- Defuzzification is performed according to the membership function of the output variable. For instance, assume that we have the result in the figure below at the end of the inference. In this figure, the shaded areas all belong to the fuzzy result. The purpose is to obtain a crisp value, represented with a dot in the figure, from this fuzzy result.