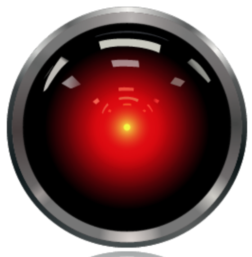


INTELLIGENT COMPUTING

(COMPUTATIONAL INTELLIGENCE)

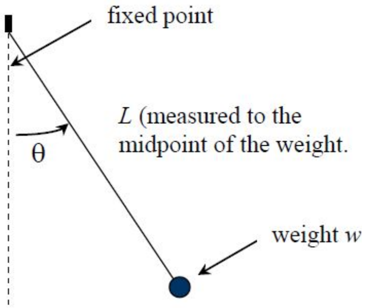
Fuzzy Logic - Inverted Pendulum Example

S. Sheridan

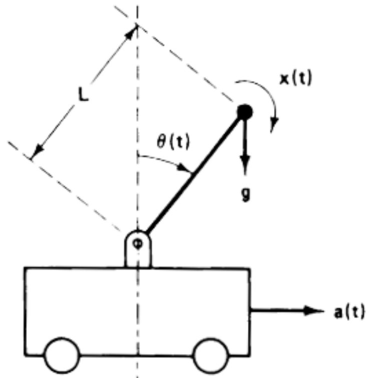
INVERTED PENDULUM PROBLEM

- An inverted pendulum is a pendulum which has its centre of mass above its pivot point. It is often implemented with the pivot point mounted on a cart that can move horizontally and is often referred to as a cart and pole.
- Most applications limit the pendulum to 1 degree of freedom by affixing the pole to an axis of rotation. Whereas a normal pendulum is stable when hanging downwards, an inverted pendulum is inherently unstable, and must be actively balanced in order to remain upright.
- One common approach to balancing an inverted pendulum is to move the pivot point horizontally as part of a feedback system. A simple demonstration of moving the pivot point in a feedback system is achieved by balancing an upturned broomstick on the end of one's finger. The inverted pendulum is a classic problem in dynamics and control theory and is used as a benchmark for testing control strategies.

INVERTED PENDULUM PROBLEM



Classic Pendulum

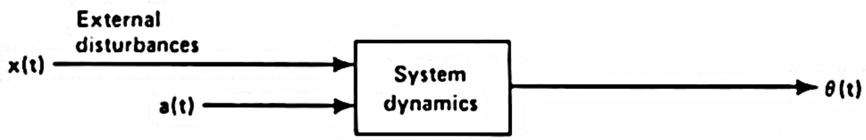


θ(t) = angle pole makes with perpendicular
x(t) = Angular velocity of pole
a(t) = Force applied to cart
L = length of pole
g = gravity

Inverted Pendulum

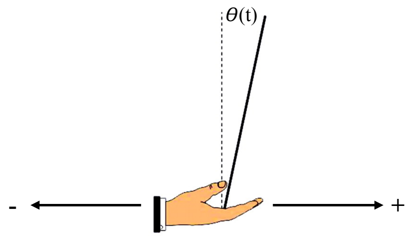
INVERTED PENDULUM PROBLEM

- Looking at the diagram on the previous slide it is easy to see that in order balance the pole we must make the angle between the pole and the perpendicular *θ(t)*, as close to zero as possible.
- If we look at the inverted pendulum diagram we see that there are two external disturbances that effect *θ(t). They are, **x(t)** angular velocity and **a(t)** force on cart.*



INVERTED PENDULUM PROBLEM

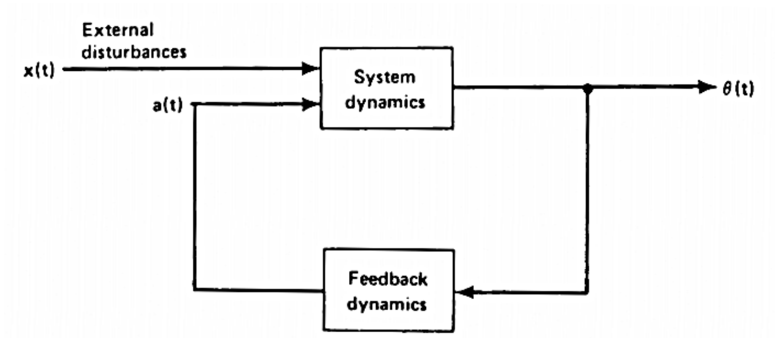
- If we think about balancing a broom on our hand, we know that in order to keep the broom balanced we must move our hand underneath the broom to compensate for any offset in angle the broom makes with the perpendicular.



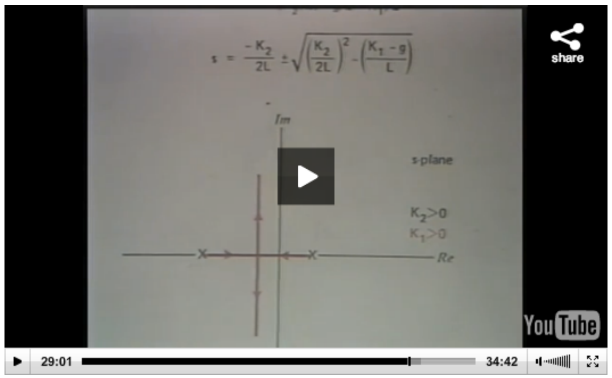
- Therefore, in order to keep the inverted pendulum balanced we need to apply the correct force (+) or (-) to the cart in order to compensate for the angular offset θ(t).

INVERTED PENDULUM PROBLEM

- Ideally then we would like to implement a system to control the force applied to the cart in order to balance the pole. We cannot directly control the angular velocity x(t) because that would be equivalent to cheating (holding the pole in place).



INVERTED PENDULUM PROBLEM

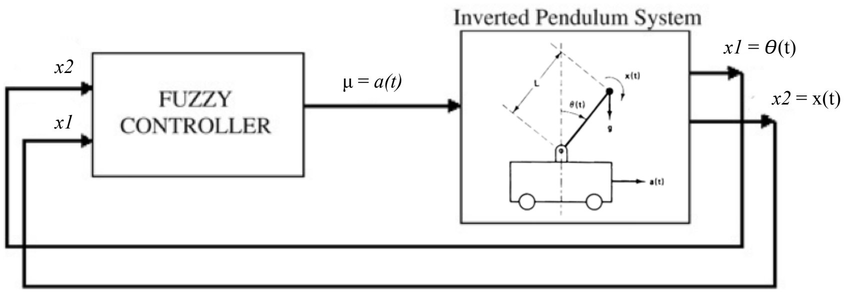


http://ocw.mit.edu/resources/res-6-007-signals-and-systems-spring-2011/video-lectures/lecture-26-feedback-example-the-inverted-pendulum

INVERTED PENDULUM: FUZZY CONTROLLER

- To build a fuzzy controller for the inverted pendulum problem we will need to take as input the angle θ(t) and x(t) in order to calculate an output of a(t).

- **NOTE:** it can be shown mathematically that applying an equal and opposite force to the cart alone will not balance the pendulum. Watch the full MIT video if you are interested in finding out more.



INVERTED PENDULUM: LINGUISTIC VARIABLES

- The next step is to develop a set of linguistic variables and terms that represent our input and output variables. Because we are modelling a physical system it will be import to decide on appropriate domain ranges for each variable. So what possible values could *θ(t)*, *x(t)* and *a(t) have*?

- The answer to this question is that it will really depend on the actual physical system or simulation. If we choose the wrong range of values (domain) for a variable it will have a negative impact on our Fuzzy Control system. For our purposes we will limit the domains of our linguistic variables as follows:

- Let $x1 = \theta(t)$, $x2 = x(t)$ and $\mu = a(t)$

- Let $-4 \leq x1 \leq 4$ and $-8 \leq x2 \leq 8$ and $-32 \leq \mu \leq 32$

INVERTED PENDULUM: LINGUISTIC VARIABLES

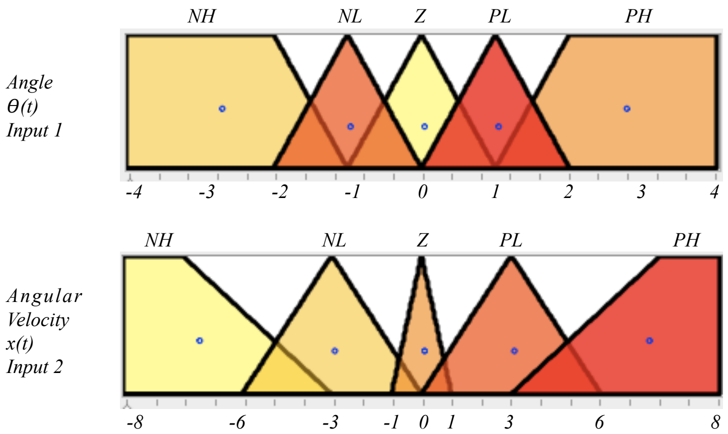
- Now that we have a domain range for each variable we will need to develop a set of terms to describe each variable.

- For each variable we will use the following terms to split the domain values up into more understandable chunks.

Term	Description
PH	Positive High
PL	Positive Low
Z	Zero
NL	Negative Low
NH	Negative High

INVERTED PENDULUM: LINGUISTIC TERMS

- Next we need to choose a membership function for each term and decide what range of values in the domain each term will cover.



INVERTED PENDULUM: LINGUISTIC TERMS

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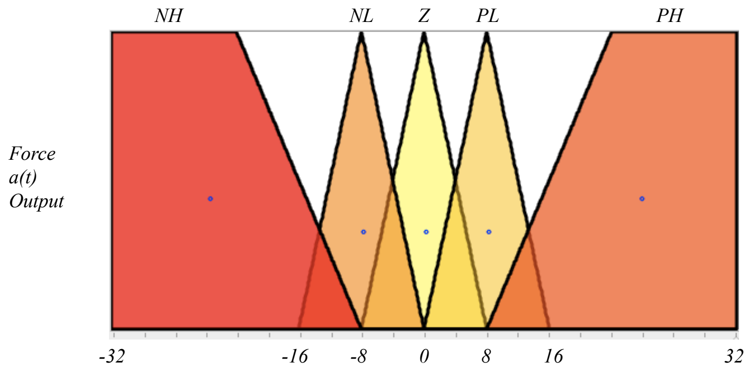


INVERTED PENDULUM: LINGUISTIC TERMS

- It is important to note that the choice of domain for each linguistic variable and terms play a critical role in the performance of a Fuzzy control system.
- The values for each term on the last two slides were chosen based on a Pole balancing simulator which we will use to test the system.
- In our simulator the angle made between the pole and the perpendicular ranges between -4 & 4 which represents and range of $-40° \leq \Theta(t) \leq 40°$.
- The angular velocity ranges between -8 & 8 which represents $-8dps \leq x(t) \leq 8dps$ *(degrees per second)*.
- The force applied to the cart ranges between $-32N \leq \mu \leq 32N$ *(newtons)*

INVERTED PENDULUM: FUZZY RULES

- Now that we have defined a set of linguistic variables and terms we will need to write a set of rules that will use $x1$ and $x2$ as input in order to decide on an output for μ our force variable.
- One approach to writing a set of rules is to use a decision matrix. Our decision matrix will plot out the possible values for $x1$ and $x2$ on a grid. The centre of the grid will contain the desired values for μ our output variable.

	Angle x1				
Angular Velocity x2	**NH**	**NL**	**Z**	**PL**	**PH**
NH	NH	NH	NH	NL	Z
NL	NH	NL	NL	Z	PH
Z	NH	NL	Z	PL	PH
PL	NH	Z	PL	PH	PH
PH	Z	PL	PH	PH	PH

- So if we input $x1$ = PL and $x2$ = PL we should get an output μ = PH

INVERTED PENDULUM: FUZZY RULES

- We can now write a set of rules that capture the contents of our decision matrix.

if angle is Z and angular_velocity is Z then force is Z
if angle is Z and angular_velocity is NH then force is NH
if angle is Z and angular_velocity is NL then force is NL
if angle is Z and angular_velocity is PL then force is PL
if angle is Z and angular_velocity is PH then force is PH
if angle is NH and angular_velocity is Z then force is NH
if angle is NL and angular_velocity is Z then force is NL
if angle is PL and angular_velocity is Z then force is PL
if angle is PH and angular_velocity is Z then force is PH
if angle is NL and angular_velocity is PL then force is Z
if angle is PL and angular_velocity is NL then force is Z

INVERTED PENDULUM: FUZZY RULES

- Our decision matrix on slide 14 was a 5X5 grid giving 25 outcomes.
- The rule list on the previous slide only contains 11 rules. What happened to the other rules?
- If you read the rules you will see that a number of conditions are && together in order to combine different situations.
- Because we are dealing with a Fuzzy system we do not have to implement every rule. However, its important that we pick out the more extreme situations that may occur.

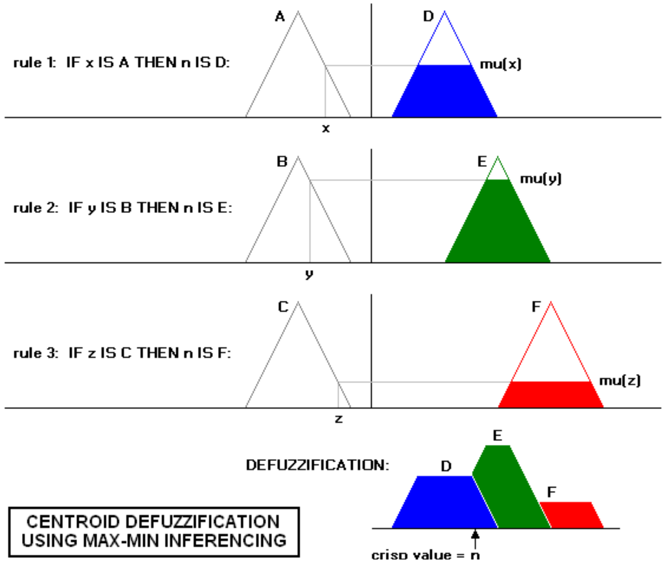
INVERTED PENDULUM: DEFUZZIFICATION

- Since our rule base contains connective statements such as AND we will need to define how these boolean operators are handled by the Fuzzy system. For the purpose of this example we will use the MAX and MIN interpretation for OR and AND respectively.

- We will also need to select an accumulation and defuzzifier function for the output variable. For the purposes of this example we will use MAXIMUM for the accumulation function and COG (centre of gravity) as the defuzzifier.

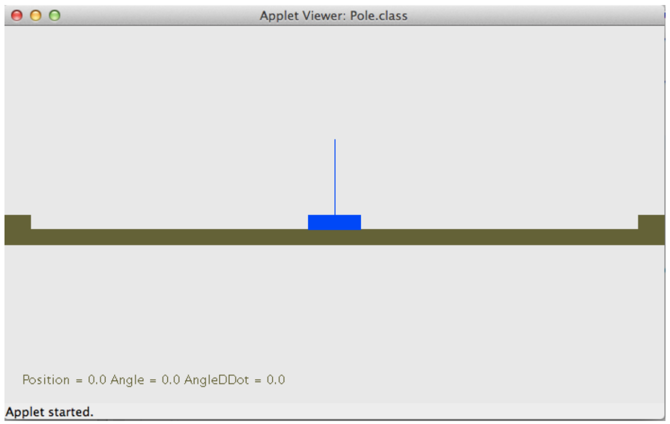
- The choice of accumulation function and defuzzifier is vitally important as this is how our Fuzzy system produces a CRISP value which we can use as the force (μ) on the cart to keep the pole balanced.

INVERTED PENDULUM: DEFUZZIFICATION



INVERTED PENDULUM: SIMULATION & TESTING

- All that is left to do now is test our Fuzzy Logic Controller. Given the nature of the pole and cart system, it would be better to test our Fuzzy Logic with a real cart and pole or at least with a simulated system.



INVERTED PENDULUM: SIMULATION & TESTING

- As part of the practical sessions that accompanies this lecture we will:

- Build the Inverted Pendulum Controller using qtFuzztLite and export the Java Code.

- Install Java SDK 1.7

- Download, compile and test Java Applet code for Pole and Cart Simulation

- Integrate the Fuzzy Logic code with the Pole and Cart simulator

- Test the Fuzzy Logic Controller