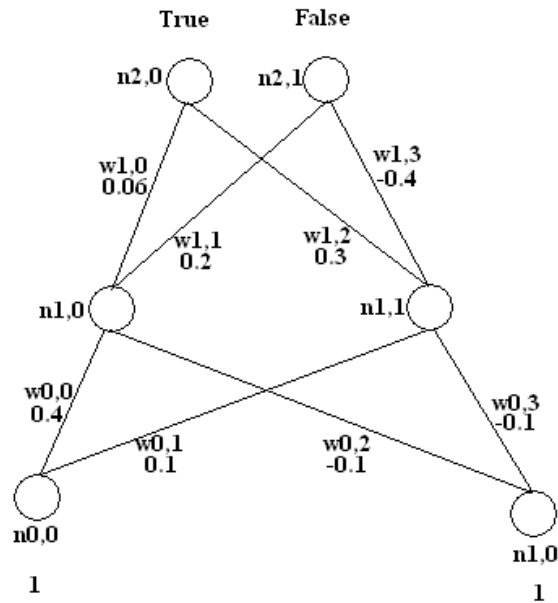


Sample Forward and Backward pass for Backprop Learning



Pattern data for XOR

$n_{0,0}$	$n_{0,1}$	Output $n_{2,0}$	Output $n_{2,1}$
1	1	0	1
1	0	1	0
0	1	1	0
0	0	0	1

β = Learning rate = 0.45

α = Momentum term = 0.9

$$f(x) = 1.0 / (1.0 + \exp(-x))$$

NOTE : XOR problem would not really be encoded with two output nodes. I'm only doing it here for illustration purposes.

Forward Pass

n0,0 = 1 & **n0,1** = 1 {Load the input nodes}

Middle nodes take weighted input so :

$$\begin{aligned}\mathbf{n1,0} &= f((1 * 0.4) + (1 * -0.1)) \\ &= f(0.3) \\ &= 0.5744425\end{aligned}$$

$$\begin{aligned}\mathbf{n1,1} &= f((1 * 0.1) + (1 * -0.2)) \\ &= f(-0.1) \\ &= 0.4750208\end{aligned}$$

Output nodes take weighted input so:

$$\begin{aligned}\mathbf{n2,0} &= f((0.5744425 * 0.06) + (0.4750208 * 0.3)) \\ &= f(0.03446655 + 0.14250624) \\ &= f(0.17697279) \\ &= \mathbf{0.54412806} \quad \text{Output for node } \mathbf{n2,0}\end{aligned}$$

$$\begin{aligned}\mathbf{n2,1} &= f((0.5744425 * 0.2) + (0.4750208 * -0.4)) \\ &= f(0.1148885 + -0.1900083) \\ &= f(-0.0751198) \\ &= \mathbf{0.48122886} \quad \text{Output for node } \mathbf{n2,1}\end{aligned}$$

Backward pass

Calculate the error on each output node

$$\begin{aligned}\mathbf{n2,0_ERROR} &= \mathbf{n2,0} * (1.0 - \mathbf{n2,0}) * (\mathbf{n2,0_DESIRED} - \mathbf{n2,0}) \\ &= 0.54412806 * (1.0 - 0.54412806) * (\mathbf{n2,0_DESIRED} - 0.54412806) \\ &= 0.54412806 * (1.0 - 0.54412806) * (0 - 0.54412806) \\ &= 0.54412806 * 0.45587194 * -0.54412806 \\ &= \mathbf{-0.134972442}\end{aligned}$$

$$\begin{aligned}\mathbf{n2,1_ERROR} &= \mathbf{n2,1} * (1.0 - \mathbf{n2,1}) * (\mathbf{n2,1_DESIRED} - \mathbf{n2,1}) \\ &= 0.48122886 * (1.0 - 0.48122886) * (\mathbf{n2,1_DESIRED} - 0.48122886) \\ &= 0.48122886 * (1.0 - 0.48122886) * (1 - 0.48122886) \\ &= 0.48122886 * 0.51877114 * 0.51877114 \\ &= \mathbf{0.129509993}\end{aligned}$$

Once we have calculated the error on the output nodes we can adjust their weights so:

$$\begin{aligned}w_{1,0}\Delta &= \beta * n_{2,0_ERROR} * n_{1,0} \\&= 0.45 * -0.134972442 * 0.5744425 \\&= \mathbf{-0.034890258}\end{aligned}$$

$$\begin{aligned}w_{1,0} &= w_{1,0} + \Delta + (\alpha * \Delta(t-1)) \quad \{\Delta(t-1) = \text{previous delta change for } w_{1,0}\} \\&= 0.06 + -0.034890258 + (0.9 * 0) \\&= \mathbf{0.025109742} \quad \text{new weight value for } w_{1,0}\end{aligned}$$

NOTE : Store Δ for next time round

$$\begin{aligned}w_{1,2}\Delta &= \beta * n_{2,0_ERROR} * n_{1,1} \\&= 0.45 * -0.134972442 * 0.4750208 \\&= \mathbf{-0.028851623}\end{aligned}$$

$$\begin{aligned}w_{1,2} &= w_{1,2} + \Delta + (\alpha * \Delta(t-1)) \quad \{\Delta(t-1) = \text{previous delta change for } w_{1,2}\} \\&= -0.4 + -0.028851623 + (0.9 * 0) \\&= \mathbf{-0.428851623} \quad \text{new weight value for } w_{1,2}\end{aligned}$$

NOTE : Store Δ for next time round

$$\begin{aligned}w_{1,1}\Delta &= \beta * n_{2,1_ERROR} * n_{1,0} \\&= 0.45 * 0.129509993 * 0.5744425 \\&= \mathbf{0.03347822}\end{aligned}$$

$$\begin{aligned}w_{1,1} &= w_{1,1} + \Delta + (\alpha * \Delta(t-1)) \quad \{\Delta(t-1) = \text{previous delta change for } w_{1,1}\} \\&= 0.2 + 0.03347822 + (0.9 * 0) \\&= \mathbf{0.23347822} \quad \text{new weight value for } w_{1,1}\end{aligned}$$

NOTE : Store Δ for next time round

Now that we have adjusted the weights from the middle layer to the output layer we must calculate the middle layer error values and adjust their weights so :

$$\begin{aligned} \mathbf{n1,0_ERROR} &= (n2,0_ERROR * w1,0) + (n2,1_ERROR * w1,1) \quad \{\text{weighted error from layer 2}\} \\ &= (-0.134972442 * 0.025109742) + (0.1295099993 * 0.23347822) \\ &= 0.026848641 \end{aligned}$$

$$\begin{aligned} \mathbf{n1,1_ERROR} &= (n2,0_ERROR * w1,2) + (n2,1_ERROR * w1,3) \quad \{\text{weighted error from layer 2}\} \\ &= \{\text{follow same method as above}\} \end{aligned}$$

Once we have calculated the error on the middle nodes we can adjust their weights so:

$$\begin{aligned} \mathbf{W0,0\Delta} &= \beta * n1,0_ERROR * n0,0 \\ &= 0.45 * 0.026848641 * 1 \\ &= \mathbf{0.012081888} \end{aligned}$$

$$\begin{aligned} \mathbf{w0,0} &= w0,0 + \Delta + (\alpha * \Delta(t-1)) \quad \{\Delta(t-1) = \text{previous delta change for } w0,0\} \\ &= 0.4 + \mathbf{0.012081888} + (0.9 * 0) \\ &= 0.412081888 \text{ new weight value for } w0,0 \end{aligned}$$

NOTE : Store Δ for next time round

Adjust the rest of the weights for the middle layer using the same method.