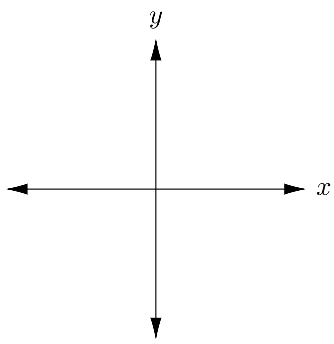
Learning Outcomes: What level of knowledge should you possess by the end of this section?

- Know how to evaluate all six trigonometric functions for a general point on the plane.
- Know how to evaluate all six trigonometric functions using reference angles.
- Use reference angles to determine angles that have the specified values of trigonometric functions.
- Know how to evaluate all six trigonometric functions using algebraic methods.
- Understand some of the ways that the various interpretations of the trigonometric functions are related to each other.

#1) Plot the point and draw the triangle that represents the trigonometric quantities. Indicate the reference angle and calculate the six trigonometric functions for the given point.



$(3, 4)$

$\sin(\theta) =$

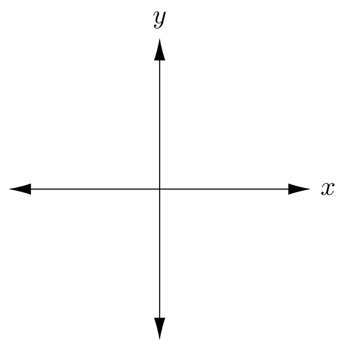
$\cos(\theta) =$

$\tan(\theta) =$

$\csc(\theta) =$

$\sec(\theta) =$

$\cot(\theta) =$



$(-2, 9)$

$\sin(\theta) =$

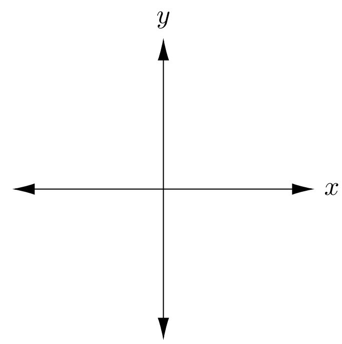
$\cos(\theta) =$

$\tan(\theta) =$

$\csc(\theta) =$

$\sec(\theta) =$

$\cot(\theta) =$



$(\sqrt{3}, -5)$

$\sin(\theta) =$

$\cos(\theta) =$

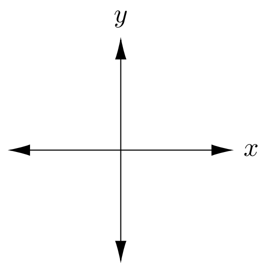
$\tan(\theta) =$

$\csc(\theta) =$

$\sec(\theta) =$

$\cot(\theta) =$

#2) Sketch the given angles in the coordinate axes, labeling both the original angle and the reference angle. Determine the reference angle and compute all six trigonometric functions for the original angle.



$\theta = 135°$

$\rho =$

$\sin(\theta) =$

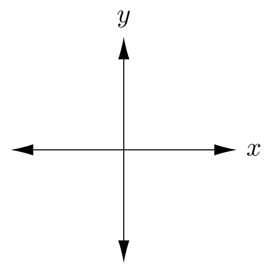
$\cos(\theta) =$

$\tan(\theta) =$

$\csc(\theta) =$

$\sec(\theta) =$

$\cot(\theta) =$



$\theta = 210°$

$\rho =$

$\sin(\theta) =$

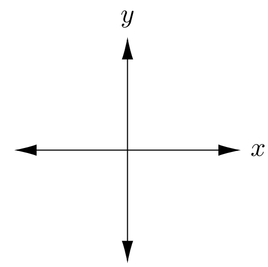
$\cos(\theta) =$

$\tan(\theta) =$

$\csc(\theta) =$

$\sec(\theta) =$

$\cot(\theta) =$



$\theta = 330°$

$\rho =$

$\sin(\theta) =$

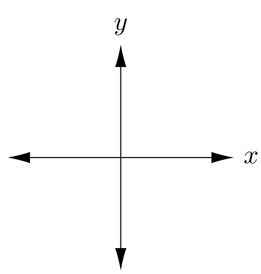
$\cos(\theta) =$

$\tan(\theta) =$

$\csc(\theta) =$

$\sec(\theta) =$

$\cot(\theta) =$



$\theta = \dfrac{11\pi}{6}$

$\rho =$

$\sin(\theta) =$

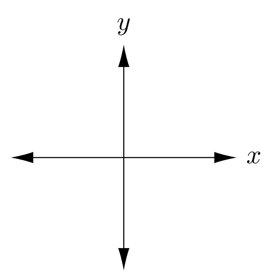
$\cos(\theta) =$

$\tan(\theta) =$

$\csc(\theta) =$

$\sec(\theta) =$

$\cot(\theta) =$



$\theta = \dfrac{2\pi}{3}$

$\rho =$

$\sin(\theta) =$

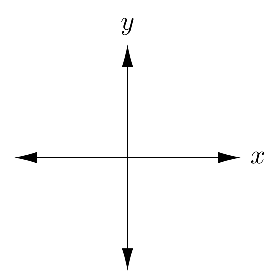
$\cos(\theta) =$

$\tan(\theta) =$

$\csc(\theta) =$

$\sec(\theta) =$

$\cot(\theta) =$



$\theta = \dfrac{5\pi}{4}$

$\rho =$

$\sin(\theta) =$

$\cos(\theta) =$

$\tan(\theta) =$

$\csc(\theta) =$

$\sec(\theta) =$

$\cot(\theta) =$

#3) Suppose θ is in the fourth quadrant and that $\sin(\theta) = -\frac{4}{\sqrt{19}}$. Determine the values of the other trigonometric functions using algebraic methods. Present your work in an organized manner and also write your final answers in the column on the right side of the page.

#4) Suppose θ is in the third quadrant and that $\tan(\theta) = 3$. Determine the values of the other trigonometric functions using algebraic methods. Present your work in an organized manner and also write your final answers in the column on the right side of the page.

#5) Suppose $\cos(\theta) = \frac{5}{7}$ and that $\sin(\theta) < 0$. Determine the values of the other trigonometric functions using algebraic methods. Present your work in an organized manner and also write your final answers in the column on the right side of the page.

In the previous problems, you were asked to compute the output of the trigonometric functions for a given angle. For the next several problems, you are going to try to solve the reverse problem. Given the output of a trigonometric function, can you determine the angle? It is important to recognize that there may be multiple answers. There are two ways to understand the multiplicity of solutions.

- Periodicity: Since the trigonometric functions are periodic, if (for example) $\sin(\theta) = k$, then it's also the case that $\sin(\theta + 2\pi n) = k$. So one solution automatically gives rise to infinitely many others by picking a different coterminal angle.
- Reference Angles: As we have seen in this section, we cannot isolate the quadrant the angle is in from the value of a single trigonometric function. There are (for example) two quadrants in which $\cos(\theta)$ is positive. So we have to allow for both possibilities unless there is information that can eliminate one of the values.

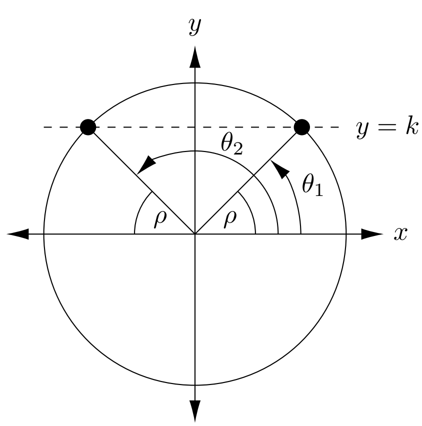
#6) Find two distinct values of θ satisfying $0° < \theta < 360°$ that satisfy the given equation. All of the values come from the special angles and special coordinates of the unit circle.

Condition	$\sin(\theta) = \frac{1}{2}$	$\cos(\theta) = -\frac{\sqrt{2}}{2}$	$\tan(\theta) = \sqrt{3}$	$\csc(\theta) = -2$	$\sec(\theta) = \frac{2}{\sqrt{3}}$	$\cot(\theta) = -\sqrt{3}$
θ_1						
θ_2						

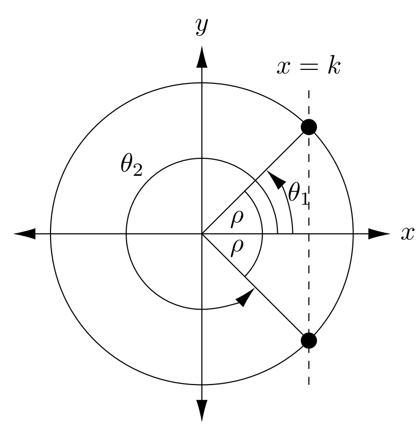
#7) Find two distinct values of θ satisfying $0 < \theta < 2\pi$ that satisfy the given equation. All of the values come from the special angles and special coordinates of the unit circle.

Condition	$\sin(\theta) = \frac{\sqrt{2}}{2}$	$\cos(\theta) = \frac{\sqrt{3}}{2}$	$\tan(\theta) = 1$	$\csc(\theta) = \sqrt{2}$	$\sec(\theta) = -\frac{2}{\sqrt{3}}$	$\cot(\theta) = -\frac{1}{\sqrt{3}}$
θ_1						
θ_2						

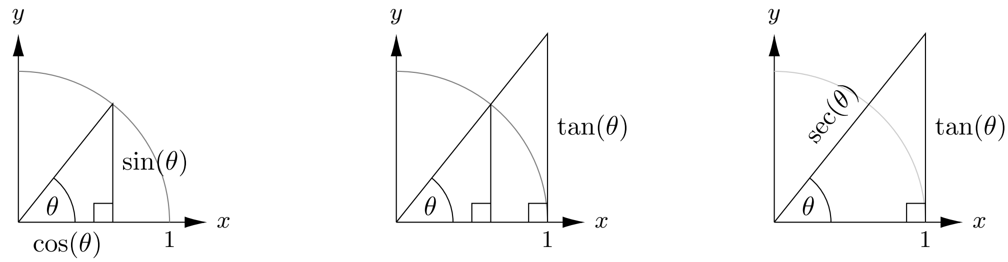
Using reference angles, we can see relationships between the solutions for the sine and cosine equations. The diagrams below demonstrate this idea. The two solutions have the same reference angle and in some cases the sum or difference of the angles results in a nice value.



$\theta_1 + \theta_2 = \pi$



$\theta_1 + \theta_2 = 2\pi$

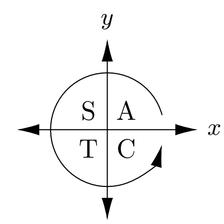


#8) The first diagram above shows the first quadrant of the unit circle. Use sentences and refer to the diagram to explain why the sides labeled $\sin(\theta)$ and $\cos(\theta)$ are accurate. (Hint: Think about the coordinates of the point on the unit circle.)

#9) The second diagram is an extension of the first. Use sentences and refer to the diagram to explain why the side labeled as $\tan(\theta)$ is accurate. (Hint: Think about the side lengths of the smaller triangle in terms of trigonometric functions and use similar triangles.)

#10) The third diagram is an extension of the second. Use sentences and refer to the diagram to explain why the side labeled as $\sec(\theta)$ is accurate. (Hint: Pythagorean identity.)

Mnemonic devices are tools we use to help us remember information. The mnemonic devices don't have to have any separate meaning on its own. For example, SOH-CAH-TOA is just a series of sounds that make us think of certain letters that tell us the right triangle relationships for the sine, cosine, and tangent functions. In the same way, "All Students Take Calculus" is just a series of words whose first letters tell us about the signs of the primary trigonometric functions.



#11) The trigonometric cofunctions are the cosecant, secant, and cotangent functions. Create mnemonic for the signs of the cofunctions. You may make something similar to "All Students Take Calculus" or you can come up with something completely different. One challenge you are facing is that both cosecant and cotangent start with the same letter. So you will need to get creative to find a way to distinguish the two.

#12) Which type of calculation did you find the most difficult in this section? Can you identify why it was difficult?