Learning Outcomes: What level of knowledge should you possess by the end of this section?

- Know the shape of the graphs of basic sine and cosine functions, including the coordinates of the key values.
- Identify the period, amplitude, and midline of sine and cosine functions from a graph.
- Determine the fundamental period and key values of sine and cosine functions.
- Understand the concepts behind transformations of graphs.

#1) Evaluate the sine and cosine functions at the given points.

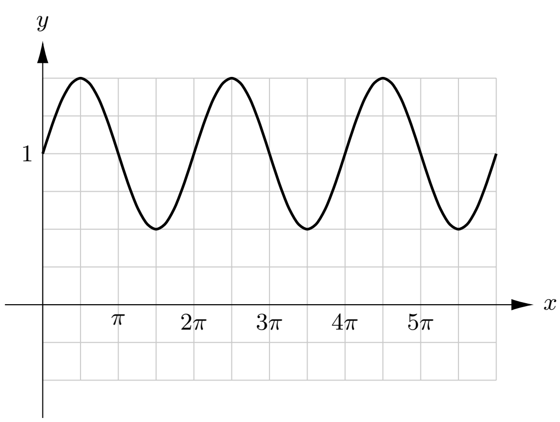
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\sin(x)$									
$\cos(x)$									

x	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
$\sin(x)$									
$\cos(x)$									

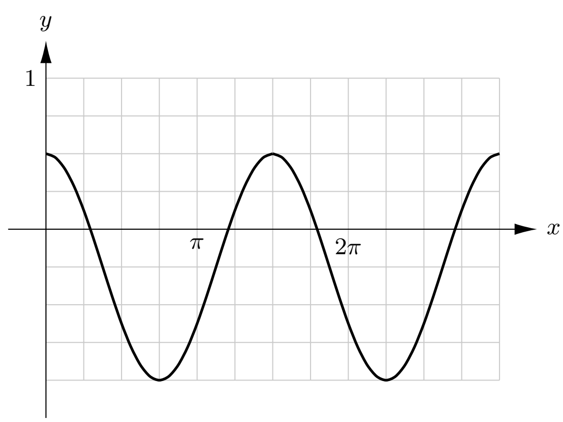
#2) Complete the chart of coordinates for the key values of the sine and cosine functions.

Key Values	1	2	3	4	5
(x, y), $y = \sin(x)$					
(x, y), $y = \cos(x)$					

#3) Identify the period, amplitude, and midline of the given function.



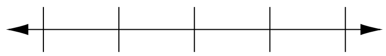
Period	Amplitude	Midline



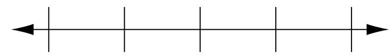
Period	Amplitude	Midline

#4) Determine the fundamental periods and subintervals for the given functions. Plot the x-coordinates on the given number line.

$y = 2\cos(2x - \pi)$



$y = -3\sin\left(\frac{\pi x}{2} + \frac{\pi}{3}\right)$

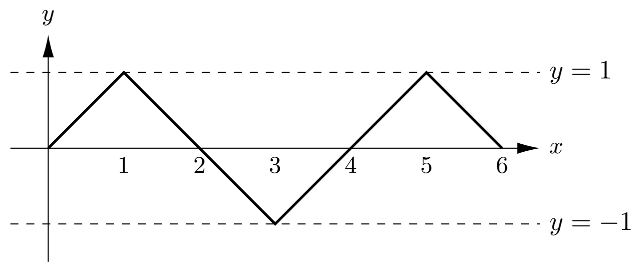


#5) Determine the values of b and c that would cause the function $y = \sin(bx - c)$ to have the specified fundamental period.

Fundamental period: $[\pi, 5\pi]$

Fundamental period: $[4, 6]$

We will spend the next few exercises looking at the transformations of graphs. Rather than using a sine or cosine function, we will use a simpler periodic function known as a triangle wave, which we will call $T(x)$. It is very similar to the sine and cosine functions, but we don't have to worry about multiples of π floating around that make our calculations more complicated.



Key Values	1	2	3	4	5
x	0	1	2	3	4
$T(x)$	0	1	0	-1	0
Location	Mid	Top	Mid	Bot	Mid

#6) Determine the period, amplitude, and midline of $T(x)$.

Period	Amplitude	Midline

Notice that the chart on the right also gives you an explicit value for $T(x)$ evaluated at different points. You can use the fact that this is a repeating pattern to evaluate $T(x)$ at any integer.

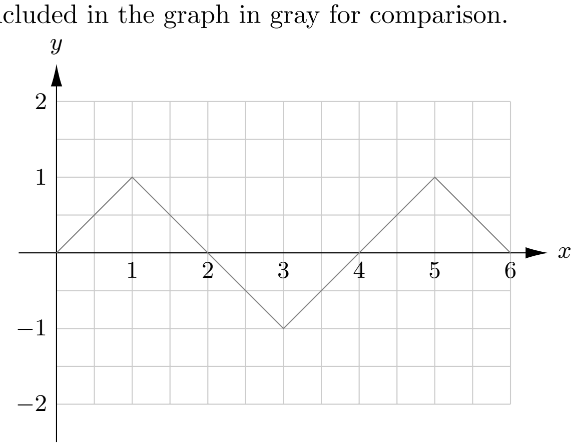
$$T(0) = 0, \qquad T(1) = 1, \qquad T(2) = 0, \qquad T(3) = -1, \qquad T(4) = 0$$

#7) Complete the following chart of values.

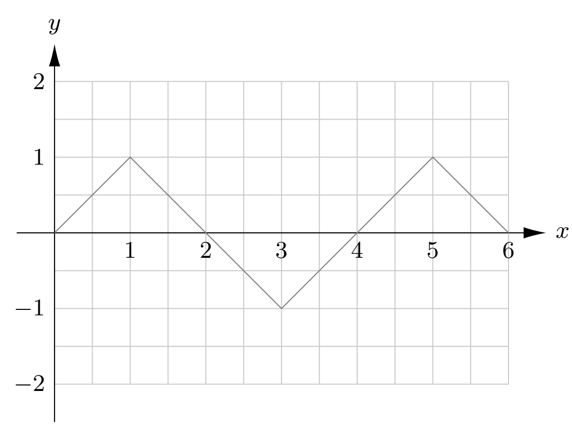
x	0	1	2	3	4	5	6	7	8
$T(x)$									

One of the main features of transformations is whether you are transforming before or after you evaluate the function. Depending on when you do it, it will impact the final graph in different ways. We will start by analyzing what happens if you apply the transformations after you evaluate the function.

#8) Complete the following charts of values. Then graph the functions. The function $T(x)$ has been included in the graph in gray for comparison.



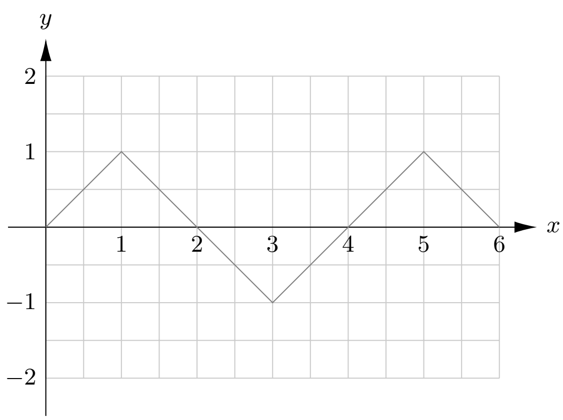
x	0	1	2	3	4	5	6
$T(x)+1$							



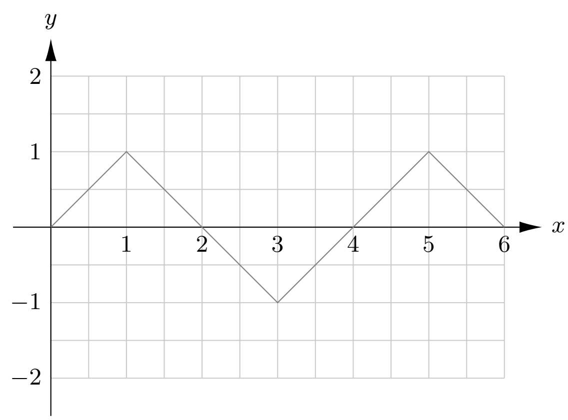
x	0	1	2	3	4	5	6
$T(x)-1$							

#9) Using the previous example as a source of intuition, describe how to graph the function $T(x)+c$ when $c > 0$. Also describe how to graph the function $T(x) - c$ when $c > 0$.

#10) Complete the following charts of values. Then graph the functions. The function $T(x)$ has been included in the graph in gray for comparison.



x	0	1	2	3	4	5	6
$2T(x)$							

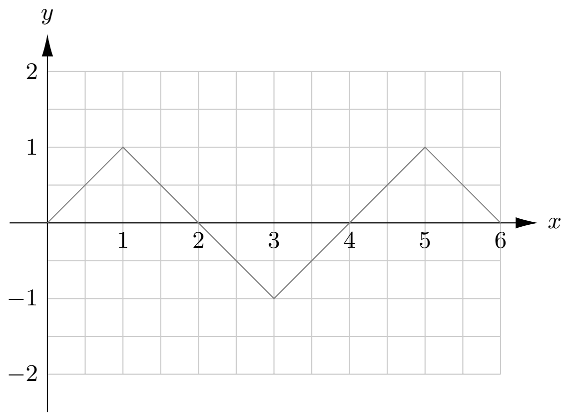


x	0	1	2	3	4	5	6
$-2T(x)$							

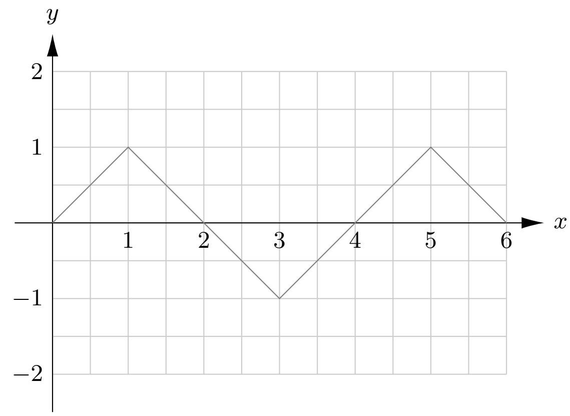
#11) Using the previous example as a source of intuition, describe how to graph the function $cT(x)$ when $c > 0$. Also describe how to graph the function $-cT(x)$ when $c > 0$.

We will now examine the situations where the transformations are applied before the function is evaluated. Notice that this means that the transformations happen inside of the argument of the function.

#12) Complete the following charts of values. Then graph the functions. The function $T(x)$ has been included in the graph in gray for comparison.



x	0	1	2	3	4	5	6
$T(x+1)$							



x	0	1	2	3	4	5	6
$T(x-1)$							

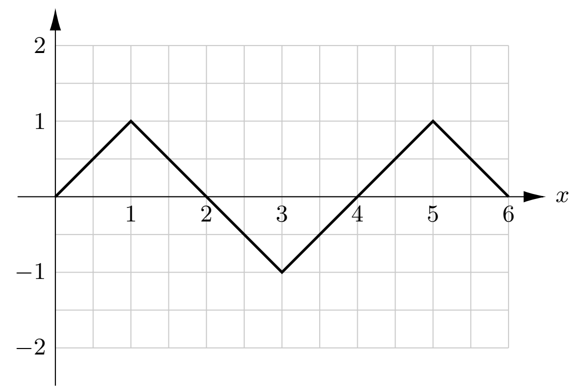
#13) Using the previous example as a source of intuition, describe how to graph the function $T(x+c)$ when $c > 0$. Also describe how to graph the function $T(x-c)$ when $c > 0$.

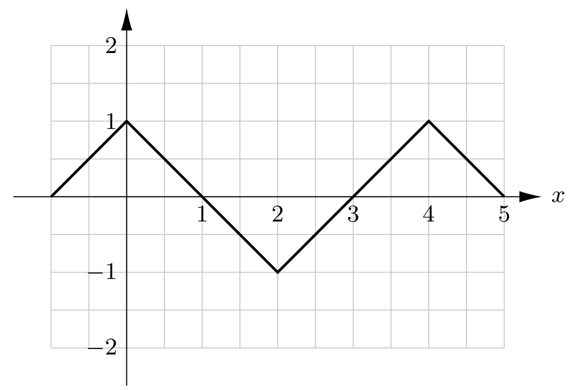
This transformation often leads students to make mistakes because they tend to get the direction of the translation backwards. To understand how this happens, we're going to think a bit about why these transformations impact the horizontal direction and not the vertical direction.

#14) Complete the following chart of values.

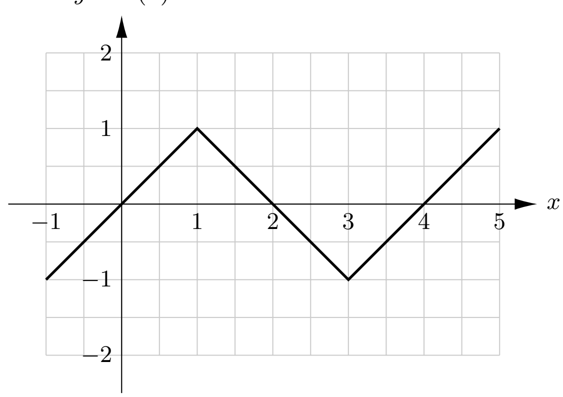
x	0	1	2	3	4	5	6	7	8
$T(x)$									
$x+1$									
$T(x+1)$									
$x-1$									
$T(x-1)$									

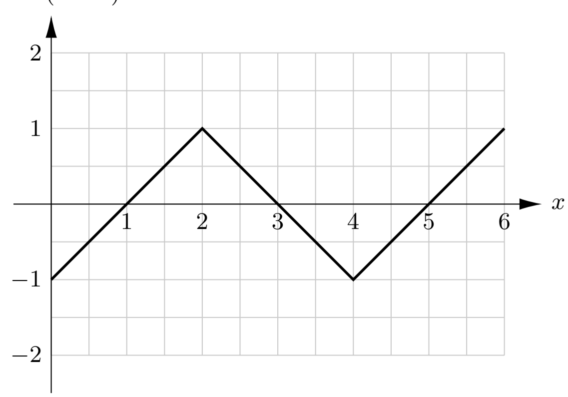
When the transformations are applied to the argument of the function, it impacts how we read the chart of values. The difference between $T(x)$ and $T(x+1)$ is that the function $T(x+1)$ is "reading ahead" by one unit (thinking about this as reading from left to right). Because it's reading ahead, it sees everything sooner, which is why the graph is shifted to the left.



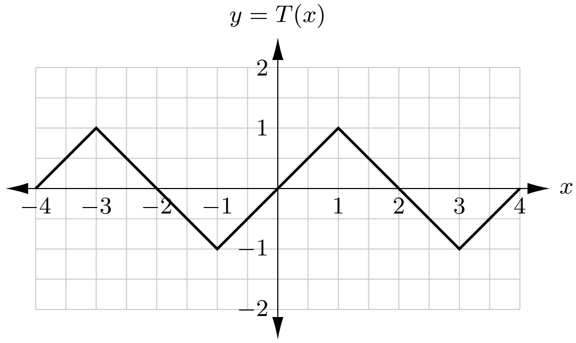


In the same way, the function $T(x-1)$ is "reading behind" by one unit, which pushes the graph to the right.



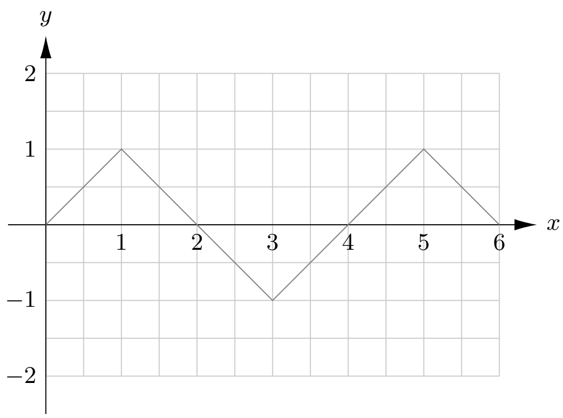


We will do this exercise one last time, this time thinking about what happens when you multiply the argument by a constant. In order to make this one make sense, we're going to need to change the part of the domain that we're looking at.

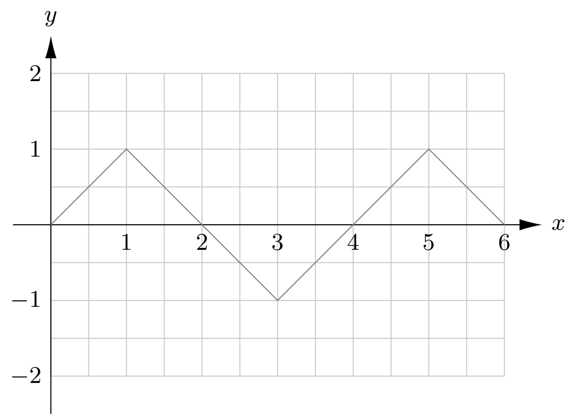


x	-4	-3	-2	-1	0	1	2	3	4
$T(x)$	0	1	0	-1	0	1	0	-1	0

#15) Complete the following charts of values. Then graph the functions. The function $T(x)$ has been included in the graph in gray for comparison.



x	0	1	2	3	4	5	6
$T(2x)$							



x	0	1	2	3	4	5	6
$T(-2x)$							

#16) Using the previous example as a source of intuition, describe how to graph the function $T(cx)$ when $c > 0$. Also describe how to graph the function $T(-cx)$.

#17) Give an explanation for the effect you see for the graph of $T(2x)$ that is similar to the analysis given for the graph of $T(x+1)$ and $T(x-1)$. (This is referring to the "read ahead" and "read behind" descriptions that cause the graphs to shift.)