Learning Outcomes: What level of knowledge should you possess by the end of this section?

- Know how to graph transformed sine and cosine functions using key values.
- Identify and describe transformations by their descriptions, their equations, and their graphs.
- Understand the connection between the general theory of transformations and graphing using key values.

#1) State the process of graphing sine and cosine functions using key values. (Hint: There are four steps.)

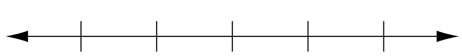
#2) Graph $y = -2\sin\left(2x + \frac{\pi}{2}\right) + 2$.

Step 1: Identify the fundamental period

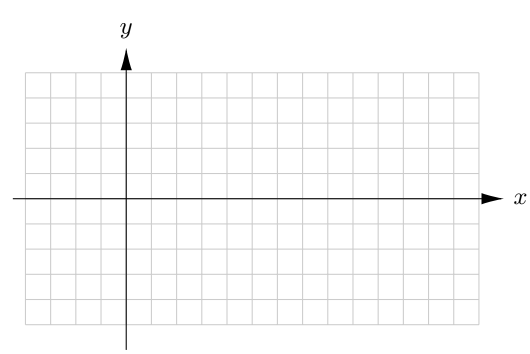
Step 3: Transform the y-coordinates of the key values.

Key Values	1	2	3	4	5
x					
y					

Step 2: Identify the x-coordinates of the key values.



Step 4: Plot the points and sketch the graph.



#3) Graph $y = 3\cos(2\pi x - \pi) - 1$. Organize and present your work in an organized manner.

#4) Graph $y = -2\sin\left(3x - \frac{2\pi}{3}\right) + 1$. Organize and present your work in an organized manner.

#5) Describe the features of each transformation, or state N/A if there is no transformation of that type.

$$y = 3\cos\left(2x - \frac{\pi}{3}\right) - 1$$

Horizontal shift:

Horizontal stretch/compression/flip:

Vertical stretch/compression/flip:

Vertical shift:

$$y = -2\sin\left(\frac{\pi x}{2} + \pi\right) + 3$$

Horizontal shift:

Horizontal stretch/compression/flip:

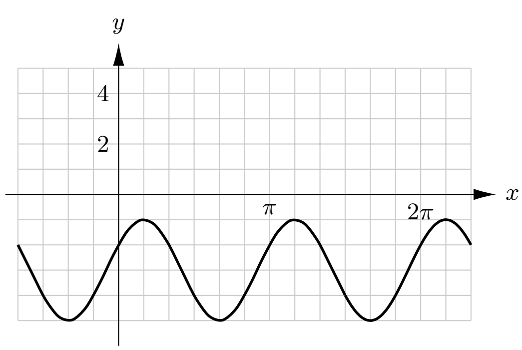
Vertical stretch/compression/flip:

Vertical shift:

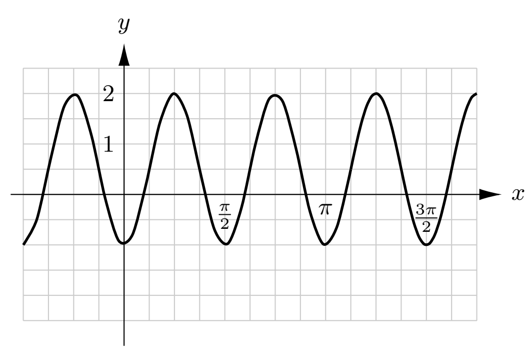
#6) Given the descriptions of the transformations, determine the equation of the function.

- The sine function shifted left by $\frac{\pi}{3}$, horizontally compressed by a factor of 2, vertically flipped and stretched by a factor of 2, and shifted down 3 units.
  - Function:
- The cosine function shifted right by $\frac{3\pi}{4}$, horizontally stretched by a factor of 2, vertically stretched by a factor of 3, and shifted up 1 unit.
  - Function:

#7) Determine an equation of the function from the graph. (Note: There are multiple equations that give the same graph.)



$f(x) =$



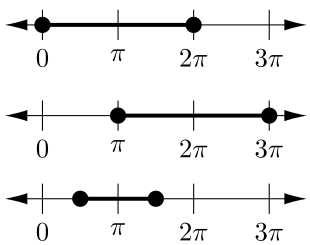
$f(x) =$

Although graphing sine and cosine functions with key values is simpler than graphing by transformations, it is important to recognize the relationships between the techniques. We will start by analyzing the fundamental period.

#8) Determine the fundamental period of the function $f(x) = a\sin(bx - c) + d$.

Notice that when we're working with the fundamental period, we are working inside of the argument of the function. This means that these calculations influence the horizontal stretching and shifting for the graph. Specifically, we can see how the stretching and shifting leads to the same result.

When you solved for the fundamental period, the first step of the calculations was to add c to both sides of the equation. The second step was to divide both sides by b. Both of these steps corresponds to manipulations of the number line. In fact, we are actually manipulating the endpoints of the interval $[0, 2\pi]$. Here is how the calculations look for the function $f(x) = \sin(2x - \pi)$.



$$\begin{aligned} 2x - \pi &= 0 & 2x - \pi &= 2\pi & & \\ 2x &= \pi & 2x &= 3\pi & & \text{Add } \pi \text{ to both sides} \\ x &= \frac{\pi}{2} & x &= \frac{3\pi}{2} & & \text{Divide both sides by 2} \end{aligned}$$

In the process of solving for the endpoints of the fundamental interval, we have also executed the horizontal transformations in the correct order. We can now see why the transformations seem to be in the opposite direction from we expect. In order to cancel out subtracting π, we need to add π. In order to cancel out the multiplication by 2, we need to divide by 2. These algebraic steps of solving for the variable drive the transformation of the argument.

#9) Notice that the vertical transformations follow the order of operations. We apply the transformation corresponding to multiplication (or division) before applying the transformation corresponding to addition (or subtraction). Consider the function $f(x) = a\sin(bx - c) + d$ and explain how the order of operations impacts the order of the transformations.

#10) Use your knowledge of transformations to find a way to write the sine function as a transformed cosine function. Explain in words the logic you applied to reach your goal.

#11) Write your own graphing problem and then graph it. Try to create a problem that is about the same difficulty as the other graphs in this section.

Transformed sine and cosine functions are often used to discuss oscillating behaviors. The next couple problems will apply what you have learned to creating mathematical models that describe the given situation.

#12) A lunar cycle is the amount of time it takes for the moon to pass through all of its phases. This cycle lasts about 29.5 days. Create a sine or cosine function that models the moon's phases. Treat a new moon as having a value of 0 and a full moon as a value of 1, and create your function so that the input time is the number of days since the previous new moon.

#13) The daily weather can be modeled using a sine or cosine function. One can use 3:00 PM as an estimate for the hottest time of the day. Create a model of temperature for day where the hottest temperature was 106° F and the coldest temperature was 78° F. Use the 24-hour clock for your input variable. (For example, $t = 13$ means 1:00 PM.)