Learning Outcomes: What level of knowledge should you possess by the end of this section?

- Represent functions and their inverses using charts and arrow diagrams.
- Understand the connection between the graph of a function and its inverse.
- Sketch the graphs of the inverse sine, inverse cosine, and inverse tangent, and state their primary features.
- Evaluate the inverse sine, inverse cosine, and inverse tangent functions on special values and by using a graph.
- Use right triangles to represent and evaluate expressions involving inverse trigonometric functions.

#1) Complete the chart of values for the function $f(x) = 2x - 1$ and convert the information into an arrow diagram.

(Draw arrow diagram here)

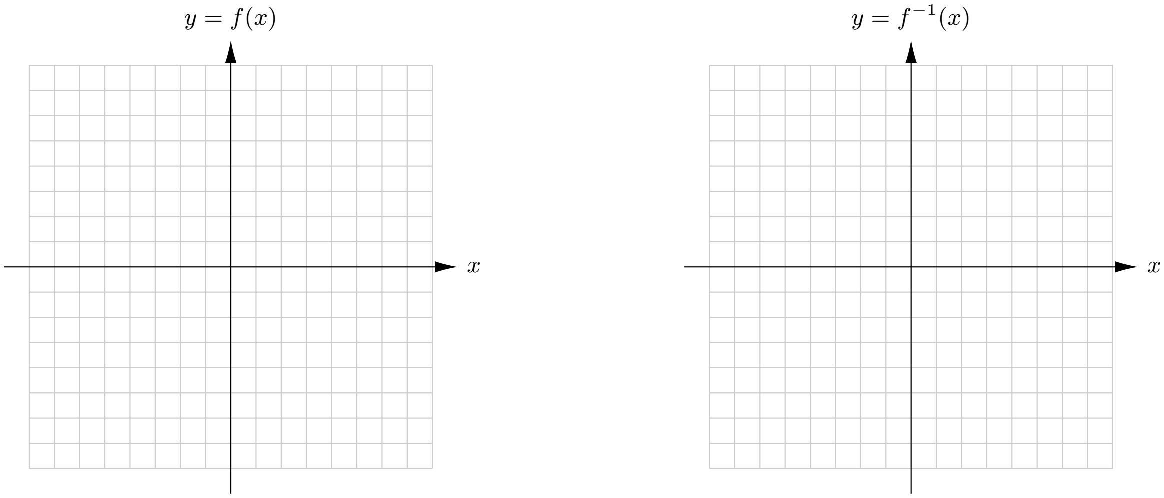
x	-3	-2	-1	0	1	2	3
$f(x)$							

#2) By swapping the rows of the chart and switching the directions of the arrows in the arrow diagram, create a chart of values for the inverse of the function $f(x) = 2x - 1$.

(Draw arrow diagram here)

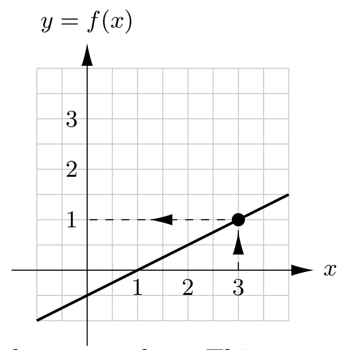
x							
$f^{-1}(x)$							

#3) Graph $f(x)$ and $f^{-1}(x)$ (from the previous two problems) on the coordinate axes. Plot the points from the charts.



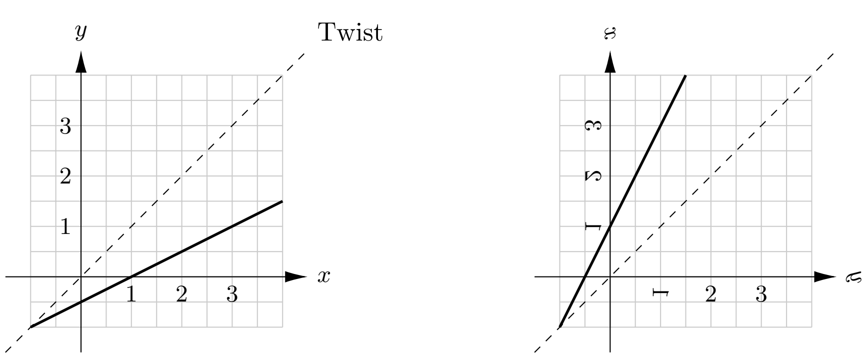
If you have completed the previous few problems correctly, you should be able to identify that the two graphs are mirror images of each other across the line $y = x$. In general, the graph of a function and its inverse will have this property. To understand why this happens, we need to think about how we read and interpret information from a graph.

When reading a graph of a function, the input value is the x-coordinate and the output is the y-coordinate. For the graph to the right, if we wanted to evaluate $f(3)$, we would start from the 3 on the x-axis, draw a vertical line until we hit the graph, and then draw a horizontal line from the intersection to determine the output. In this case, we would be able to see that $f(3) = 1$, and we get this from the point $(3, 1)$ on the graph.



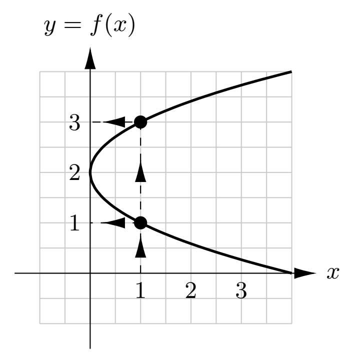
When we look at the inverse, we're switching the roles of the input and output values. This means that if we were starting with a value of 3 and ending up with a value of 1, then the inverse would start with 1 and end up with 3. In other words, the relevant point for the inverse function is $(1, 3)$. In general, if the point (a, b) is on the graph of the function, then the point (b, a) will be on the graph of the inverse.

It is the act of swapping the x and y coordinates that gives us the reflection across the $y = x$ line. Instead of a reflection, you might also think of it as a three dimensional rotation. Imagine you drew the graph on a transparent sheet and twisted it so that the diagonal stays in place, you will achieve the same result as reflecting across the $y = x$ line.



#4) Explain what happens to the vertical line $x = a$ when it is reflected across the $y = x$ line (or rotated in three dimensions around the $y = x$ line).

The "vertical line test" is a test to check whether a graph represents a function. It works by imagining drawing vertical lines and looking to see whether there is a line that crosses the graph in more than one location. If this happens, then the graph does not represent a function. This method works because we're actually going through the process of trying to read information off the graph. If there is a vertical line that passes through the graph in two places, it would correspond to there being two output values for one input value.



#5) The "horizontal line test" is a test to determine whether the inverse of a graph of a function is itself going to be a function. If you draw a horizontal line and it intersects the graph at more than one point, then the graph of the inverse is not going to be a function (unless you restrict the domain). Explain the connection between the horizontal line test and the vertical line test.

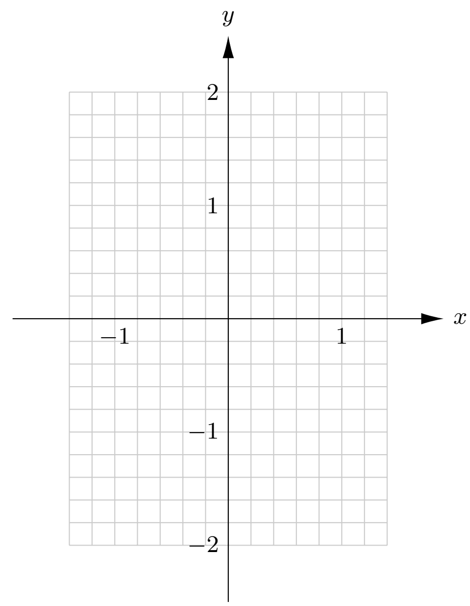
#6) Complete the chart of values for the sine and inverse sine functions, and use that information to sketch the graph of the inverse sine function. You will want to use a calculator to get a numerical approximation of the values.

x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0
$\sin(x)$					

x	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin(x)$				

x	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\sin^{-1}(x)$					

x	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\sin^{-1}(x)$				



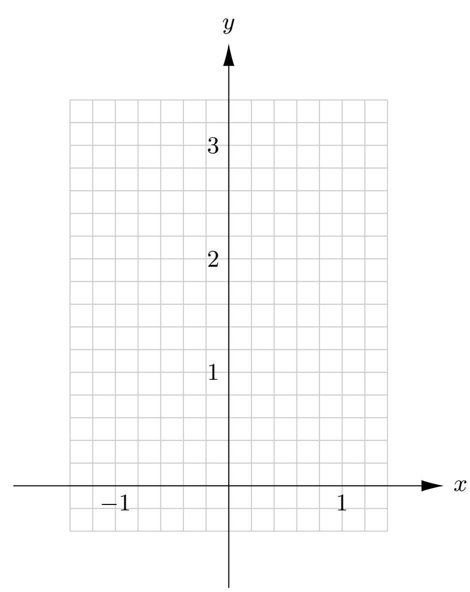
#7) Complete the chart of values for the sine function, and use that information to sketch the graph of the inverse cosine function. You will want to use a calculator to get a numerical approximation of the values.

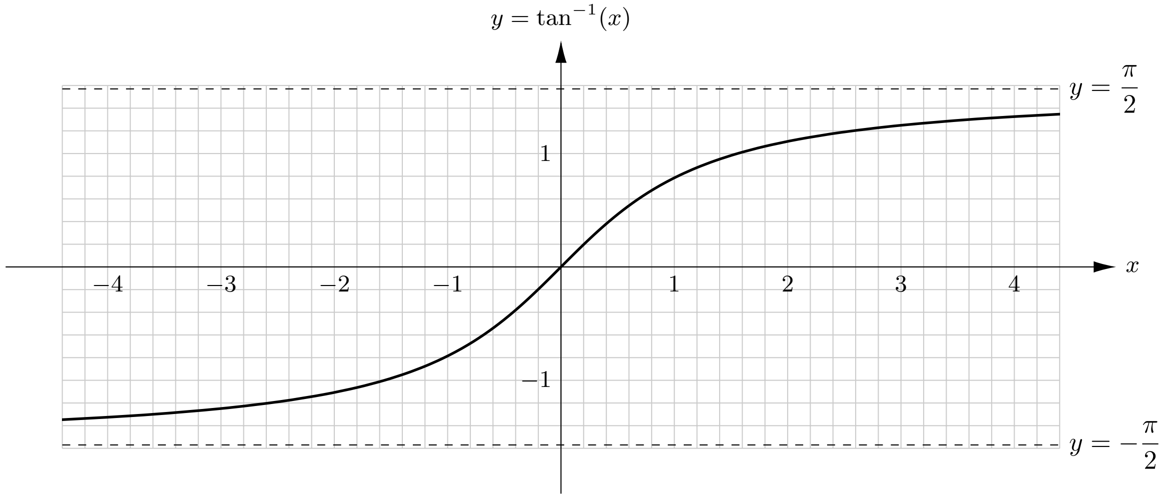
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\cos(x)$					

x	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\cos(x)$				

x	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
$\cos^{-1}(x)$					

x	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos^{-1}(x)$				





#8) Using the graph above, provide numerical estimates for the following special values of the inverse tangent function. You can use a calculator to get a numerical approximation for the x values.

x	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$
$\tan^{-1}(x)$							

#9) Determine the exact values of the following expressions.

$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

$$\tan^{-1}\left(-\sqrt{3}\right)$$

$$\cot^{-1}(1)$$

#10) Determine the exact values of the following expressions. Remember that your final answer must be in the appropriate range for the inverse trigonometric function.

$$\sin^{-1}(\sin(3\pi))$$

$$\tan^{-1}\left(\tan\left(\frac{13\pi}{4}\right)\right)$$

$$\cos^{-1}\left(\cos\left(\frac{17\pi}{6}\right)\right)$$

$$\sec^{-1}\left(\sec\left(-\frac{14\pi}{3}\right)\right)$$

#11) For each inverse trigonometric function, draw a triangle that can be used to represent the angle or explain why such a construction is impossible.

$$\sin^{-1}\left(\frac{2}{9}\right)$$

$$\tan^{-1}\left(\frac{7}{5}\right)$$

$$\sec^{-1}\left(\frac{11}{4}\right)$$

$$\cos^{-1}\left(\frac{7}{6}\right)$$

#12) For each inverse trigonometric function, draw a triangle that can be used to represent the angle.

$$\sin^{-1}\left(\frac{x}{\sqrt{x^2-25}}\right) \qquad \cot^{-1}\left(\frac{x}{3}\right)$$

#13) Compute the six trigonometric functions of the angle $\theta = \cos^{-1}\left(\frac{5\sqrt{3}}{9}\right)$. Show your work and present your final answers in an organized manner.

#14) Compute the six trigonometric functions of the angle $\theta = \cot^{-1}\left(\frac{1}{\sqrt{7}}\right)$. Show your work and present your final answers in an organized manner.

#15) Compute the six trigonometric functions of the angle $\theta = \sin^{-1}\left(\frac{x}{\sqrt{x^2+12}}\right)$. Show your work and present your final answers in an organized manner.

#16) Write your own problem in the form of the previous two problems and solve it.