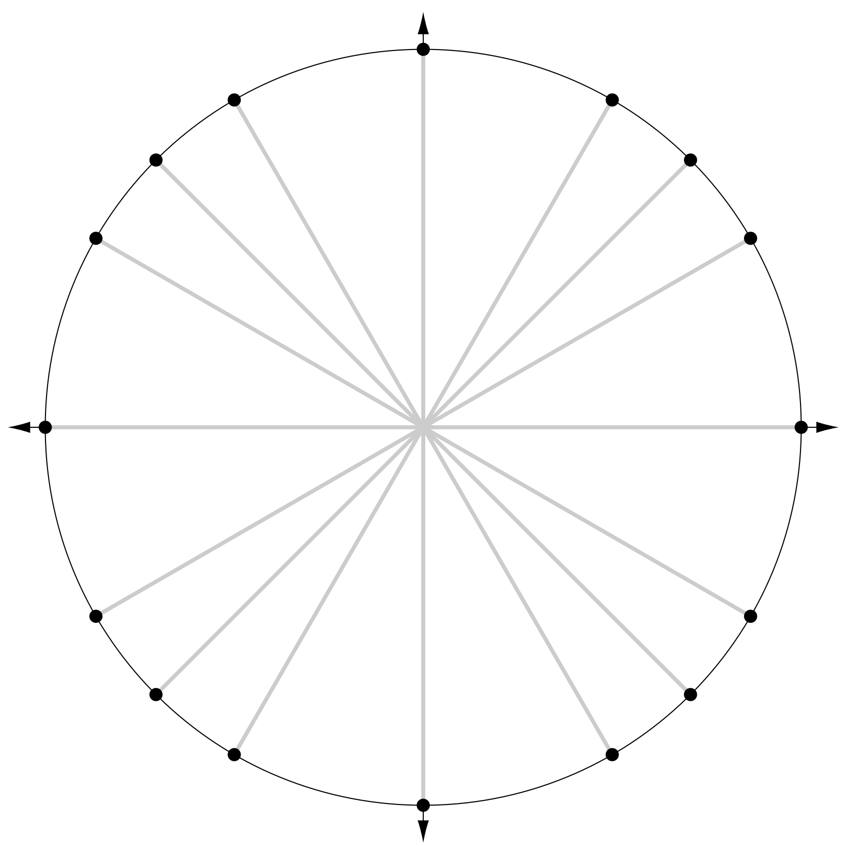
Learning Outcomes: What level of knowledge should you possess by the end of this section?

- Know the degree and radian measures of common angles and the corresponding coordinates on the unit circle, and be able to calculate all six trigonometric functions for these angles and angles coterminal to them
- Understand the relationship between the unit circle and the even/odd properties of the six trigonometric functions, and apply this knowledge to evaluate the trigonometric functions
- Be able to use relationships the evaluate the trigonometric functions and some inverse trigonometric functions.

#1) Identify the angles (in both degrees and radians) and the corresponding coordinates in the diagram below.



#2) State the values of all six trigonometric functions given that the point (x, y) is the point on the unit circle corresponding to the angle θ.

$\sin\theta =$ $\cos\theta =$ $\tan\theta =$

$\csc\theta =$ $\sec\theta =$ $\cot\theta =$

#3) In the preclass preparation, you completed a small chart of values. It is possible to use the unit circle diagram to compute an extended chart of values for the sine and cosine functions. Complete the chart below. Convert the angle measures to radians.

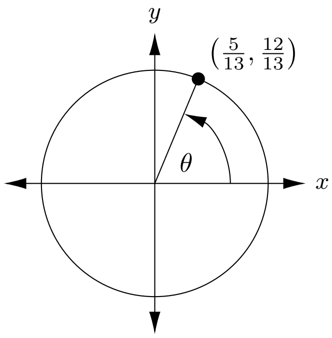
θ (Degrees) θ (Radians)	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
$\sin(\theta)$									
$\cos(\theta)$									
$\tan(\theta)$									

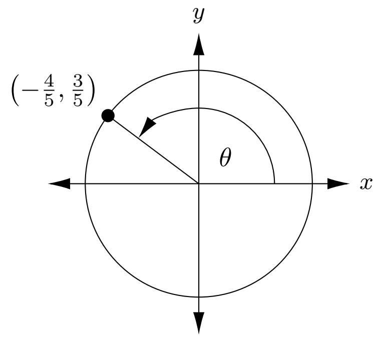
θ (Degrees) θ (Radians)	$180°$	$210°$	$225°$	$240°$	$270°$	$300°$	$315°$	$330°$	$360°$
$\sin(\theta)$									
$\cos(\theta)$									
$\tan(\theta)$									

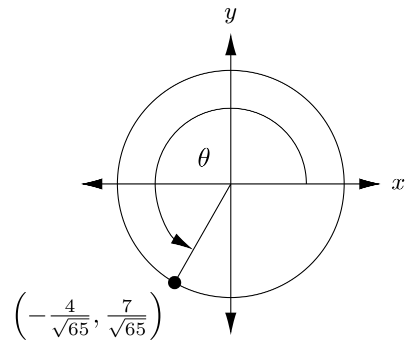
#4) Evaluate the given trigonometric functions for the given angles.

θ (Degrees) θ (Radians)	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
$\csc(\theta)$									
$\sec(\theta)$									
$\cot(\theta)$									

#5) The given diagrams indicate the coordinates of the point on the unit circle the corresponds to the indicated angle. Compute the values of the six trigonometric functions for the angles.



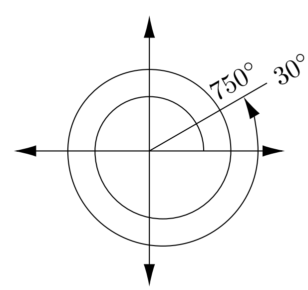




$\sin(\theta) =$	$\sin(\theta) =$	$\sin(\theta) =$
$\cos(\theta) =$	$\cos(\theta) =$	$\cos(\theta) =$
$\tan(\theta) =$	$\tan(\theta) =$	$\tan(\theta) =$
$\csc(\theta) =$	$\csc(\theta) =$	$\csc(\theta) =$
$\sec(\theta) =$	$\sec(\theta) =$	$\sec(\theta) =$
$\cot(\theta) =$	$\cot(\theta) =$	$\cot(\theta) =$

In mathematics, winding number of a curve basically counts the number of times the curve winds its way around a specific point. The idea comes from simply the act of winding strings or ropes around posts. When thinking about angles, we can use a similar concept of counting the number of times an angle requires us to wind around the origin.

For example, the angle 750° has a winding number of 2 and is coterminal with a 30° angle. Notice that the winding number is always an integer.



#6) For each angle, state the winding number and the coterminal angle whose measure is between 0° and 360°. Also, convert the angles from radians to degrees or degrees to radians, as appropriate. Round your answers to 2 decimal places for the last two problems.

θ (Degrees)	500°	800°			$20\pi°$	
θ (Radians)			5π	$\frac{31\pi}{3}$		1
Winding Number						
Coterminal Angle						

#7) Given the winding number and the coterminal angle, determine the angle in both degrees and radians.

Winding Number	2	4	3	6	9	7
Coterminal Angle	$225°$	$\frac{\pi}{12}$	$300°$	$\frac{13\pi}{10}$	$\pi°$	3
θ (Degrees) θ (Radians)						

#8) Use your knowledge of the periodicity of the trigonometric functions to evaluate the given functions. Use the sample calculations as models for how to write up your solutions.

$$\boxed{\begin{aligned}\cos(780°) &= \cos(60° + 720°)\\ &= \cos(60°)\\ &= \frac{1}{2}\end{aligned}}$$

$$\boxed{\begin{aligned}\tan\left(\frac{7\pi}{2}\right) &= \tan\left(\frac{3\pi}{2} + 2\pi\right)\\ &= \tan\left(\frac{3\pi}{2}\right)\\ &\text{Undefined}\end{aligned}}$$

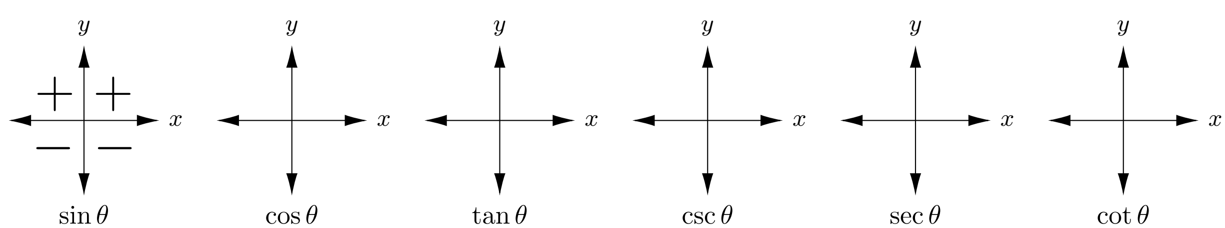
$\sin(390°) =$

$\cot(855°) =$

$\sec\left(\frac{17\pi}{4}\right) =$

$\cos\left(\frac{19\pi}{6}\right) =$

#9) As you continue to work with the trigonometric functions, it will become increasingly important to recognize the relationships between the signs of the functions and the angles. It turns out that the signs are related to the quadrants by just thinking about the definitions of the functions using the unit circle. In the first diagram, we have placed either a + sign or − sign in each of the quadrants based on whether the sine function is positive or negative in that quadrant. Follow the pattern and complete the rest of the diagrams.



#10) From the two conditions given, determine the quadrant in which the angle θ lies.

Condition 1	$\sin(\theta) > 0$	$\cos(\theta) < 0$	$\tan(\theta) > 0$	$\sin(\theta) < 0$	$\sec(\theta) > 0$	$\cot(\theta) < 0$
Condition 2	$\cos(\theta) > 0$	$\tan(\theta) > 0$	$\csc(\theta) < 0$	$\sec(\theta) < 0$	$\cot(\theta) < 0$	$\sin(\theta) < 0$
Quadrant						

#11) Use the unit circle to find two non-coterminal angles θ_1 and θ_2 that have the indicated property. You may state your angles in either degrees or radians.

Condition	$\sin(\theta) = \frac{1}{2}$	$\cos(\theta) = 0$	$\tan(\theta) = -\sqrt{3}$	$\csc(\theta) = \frac{2}{\sqrt{2}}$	$\sec(\theta)$ is undefined	$\cot(\theta) = \frac{1}{\sqrt{3}}$
θ_1						
θ_2						

When working with word problems involving round objects (like wheels) or rotating arms (like fan blades), we often talk about the number of rotations instead of the number of degrees or radians of the rotation angle. Fortunately, the following equality can be used to establish conversion factors.

$$1 \text{ rotation} = 1 \text{ revolution} = 360 \text{ degrees} = 2\pi \text{ radians}$$

#12) Convert 4.5 revolutions to degrees and radians.

#13) The second hand of a clock is 6 inches long. At noon, the tip of the second hand is 10 inches from the ceiling. How far away from the ceiling is the tip of the second hand 30 seconds later? What about 50 seconds later?

#14) A windmill is rotating at 10 rotations per minute. What is the angular velocity in degrees per minute?

#15) A wheel with a 30cm diameter rolls along the ground. After 4.5 rotations, how far has it traveled?

#16) Write a problem that combines two of the ideas from this section and solve it.