

Learning Outcomes: What level of knowledge should you possess by the end of this section?

- Understand the terminology used in the context of discussing angles: vertex, initial side, terminal side, standard position, coterminal, acute, right, obtuse, complementary, supplementary
- Understand the degree and radian measure of angles, including the ability to convert between the two units
- Relate angles and their measures to practical problems involving rotational speeds, circular arc lengths, and areas of sectors

Vocabulary:

- Acute angles: Angles whose measure is between 0° and 90° (or between 0 and $\frac{\pi}{2}$ radians)
- Complementary angles: Two angles whose measures sum to 90° (or $\frac{\pi}{2}$ radians)
- Coterminal angles: Two angles whose initial and terminal sides are the same
- Initial side of an angle: The side from which an angle is measured
- Obtuse angles: Angles whose measure is between 90° and 180° (or between $\frac{\pi}{2}$ and π radians)
- Right angles: Angles whose measure is exactly 90° (or exactly $\frac{\pi}{2}$ radians)
- Standard position: An angle drawn on the xy -plane whose vertex is at the origin and whose initial side is the positive x -axis
- Supplementary angles: Angles whose measures sum to 180° (or π radians)
- Terminal side of an angle: The side to which an angle is measured
- Vertex: The point of intersection of the initial and terminal sides of the angle

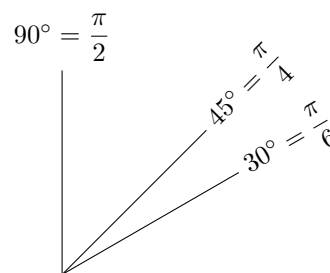
#1) Give two numerical examples each of complementary and supplementary angles. Use degree measures for one pair of angles and radian measures for the other.

#2) Knowing that 15° is $\frac{\pi}{12}$ radians allows you to easily determine the radian measure of angles that are multiples of 15° . For example, 30° is $2 \cdot (15^\circ)$ and so the radian measure is $2 \cdot (\frac{\pi}{12}) = \frac{2\pi}{12}$. (Note that the fraction can be reduced.) Using this idea, determine the radian measure of the following angles. Reduce the fractions whenever possible.

15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°

A protractor is a tool that is used to measure angles. However, if you want to estimate an angle, a sheet of paper can be turned into a makeshift protractor.

- The corner of the paper is a 90° angle.
- By folding a corner in half, you can create a 45° angle.
- By folding the corner into thirds, you can create a 30° angle. (This one may take some practice.)

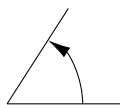


For the next few problems, you will need makeshift protractors. The diagram has “true” angles drawn out so that you can test the quality of your creation.

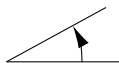
#3) By using your makeshift protractors in combination with each other, you can form many different angles. For example, in the diagram above we’ve actually created a 15° angle (located between the 30° angle and 45° angle). With this in mind, sketch the following angles: 75° , 105° , 285° . Include an indication of how you accomplished this.

#4) Convert 1 radian into degrees. (Round your answer to two decimals.)

#5) The previous calculation should have shown that 1 radian is a little less than 60° . With this in mind, use your makeshift protractor to estimate the following angles to the nearest half radian.



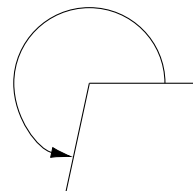
_____ radians



_____ radians

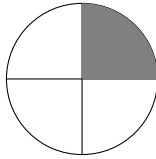


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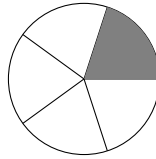
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#6) Each of the circles below have been divided into equally-sized pieces. Determine the measure of the angle created by the wedge in both degrees and radians.



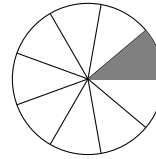
Degrees: _____

Radians: _____



Degrees: _____

Radians: _____



Degrees: _____

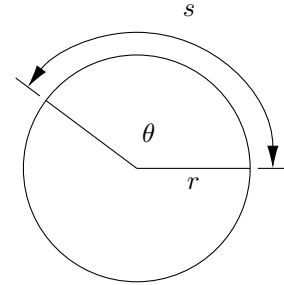
Radians: _____

The angular speed of a rotating object is how much the angle changes per unit of time. The change of angle is usually measured in degrees, radians, or rotations, and the time is usually measured in minutes or seconds.

#7) The hour hand on a clock completes one revolution in 12 hours. Determine the angular speed in degrees per second and radians per second. Explain your calculation using both words and equations.

In the preclass activity, we computed the radian measure of angles by thinking about the relationship between the length of an arc and the radius of the circle used to create that arc. Recall that an angle of 1 radian means that the arc length is equal to the radius and that an angle of 2 radians means that the arc is twice as long as the radius. From this idea, we can get the following formula for the length of a circular arc if we know the radius and the angle.

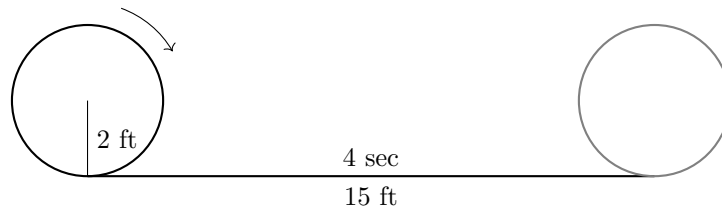
$$s = r\theta$$



#8) A decorator wants to wrap ribbon around a circular disc. She wants to use five different colors of ribbon that are all the same length. If the radius of the disc has a 10 inch radius, how long must each piece of ribbon be? Explain your thinking in both words and equations. (Hint: Draw a picture.)

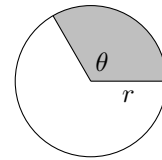
#9) A man is walking around a circular path whose radius is 500 meters. In 12 minutes, he has traveled a third of the way around the track. Determine the man's speed in meters per second. Explain your thinking in both words and equations. (Hint: The formula for speed is distance divided by time. Pay attention to your units and draw a picture.)

#10) A wheel with a 2 foot radius is rolling along the ground. In 4 seconds, the wheel has traveled 15 feet. Determine how fast the wheel is rotating in revolutions per second. Explain your thinking in both words and equations. Round your answer to two decimal places.



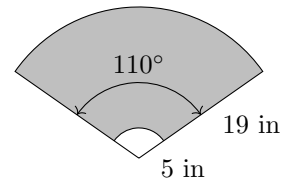
A sector of a circle is the area enclosed between two radii and the arc that connects them. Radian measure can be used to calculate the area of a sector using the following formula:

$$A = \frac{1}{2}r^2\theta$$



#11) Verify that when $\theta = 2\pi$ (the whole circle) that the formula above gives the correct result for the area of a circle.

#12) A 19-inch windshield wiper blade has a 110° range of motion. The inner edge of the blade is 5 inches from the motor. (See the diagram.) Determine the area that is swept by the blade.



There is another notation that is used for subdividing degrees. This is known as degrees-minutes-seconds (DMS) notation. There are 60 minutes in a degree and 60 seconds in a minute. This should remind you of the relationship between hours, minutes, and seconds. The notation $D^{\circ}M'S''$ means D degrees, M minutes, and S seconds. This notation is often used in cartography. For example, Google maps will give latitude and longitude using DMS. When using this notation, there are restrictions on the values.

- D must be an integer.
- S must be an integer with $0 \leq S < 60$.
- M is any number (integer or decimal) with $0 \leq M < 60$.

The following formula can be used to convert from DMS to degrees.

$$D^{\circ}M'S'' = \left(D + \frac{M}{60} + \frac{S}{3600} \right)^{\circ}$$

#13) Convert the following angles from DMS to degrees. (Round your answer to four decimals.)

DMS	$26^{\circ}22'43''$	$133^{\circ}15'21''$	$43^{\circ}6'9''$
Degrees			

#14) The radius of the earth is approximately 4000 miles. If you were to travel enough to change your longitude by one degree, what is the distance that you would travel? How far would you travel to change your latitude by a minute? By a second? (This is why DMS is often used with maps. The earth is large enough that you need smaller subdivisions to have manageable distances.)

#15) Write a problem that combines two of the ideas from this section and solve it.