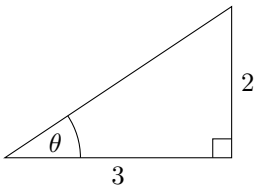
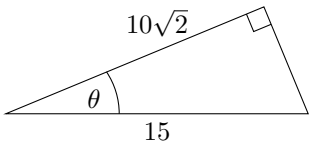
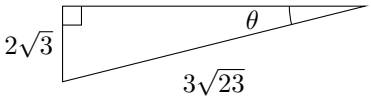
Learning Outcomes: What level of knowledge should you possess by the end of this section?

- Know the right triangle relationships for all six trigonometric functions and be able to calculate them from a triangle given only the measurements of two of the sides or from the value of a trigonometric function of an acute angle.
- Determine the side lengths of 30-60-90 and 45-45-90 triangles given only the measurement of one side.
- Use trigonometric identities to calculate the values of trigonometric functions and verify trigonometric identities.
- Use right triangle trigonometric relationships to solve geometric word problems, including problems involving the angle of elevation and the angle of depression.

#1) Determine all six trigonometric functions for the indicated angle.







$\sin(\theta) =$ $\sin(\theta) =$ $\sin(\theta) =$

$\cos(\theta) =$ $\cos(\theta) =$ $\cos(\theta) =$

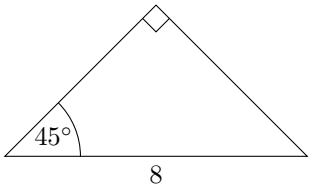
$\tan(\theta) =$ $\tan(\theta) =$ $\tan(\theta) =$

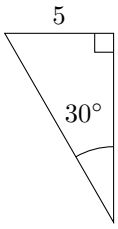
$\csc(\theta) =$ $\csc(\theta) =$ $\csc(\theta) =$

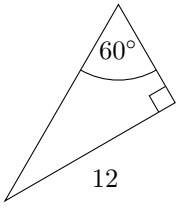
$\sec(\theta) =$ $\sec(\theta) =$ $\sec(\theta) =$

$\cot(\theta) =$ $\cot(\theta) =$ $\cot(\theta) =$

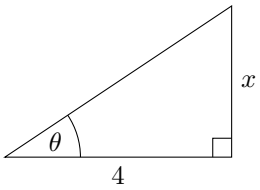
#2) Identify the lengths of the remaining sides of the triangle.

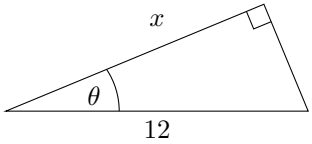


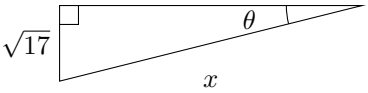




#3) Determine all six trigonometric functions for the indicated angle.







$\sin(\theta) =$ $\sin(\theta) =$ $\sin(\theta) =$

$\cos(\theta) =$ $\cos(\theta) =$ $\cos(\theta) =$

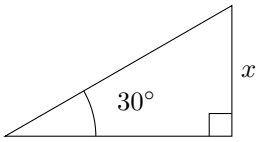
$\tan(\theta) =$ $\tan(\theta) =$ $\tan(\theta) =$

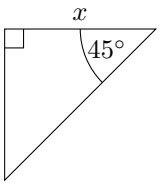
$\csc(\theta) =$ $\csc(\theta) =$ $\csc(\theta) =$

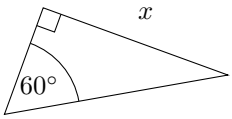
$\sec(\theta) =$ $\sec(\theta) =$ $\sec(\theta) =$

$\cot(\theta) =$ $\cot(\theta) =$ $\cot(\theta) =$

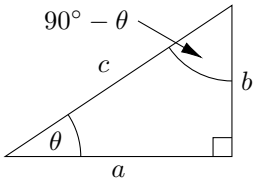
#4) Identify the lengths of the remaining sides of the triangle.







#5) In a right triangle, there are two acute angles. If one of the angles is θ, then the other one is $90° - \theta$ (or $\frac{\pi}{2} - \theta$). Determine an expression for the given trigonometric functions for the angle $90° - \theta$ in terms of a, b, and c, and then relate those values back to the six trigonometric functions of the angle θ. (These are known as the cofunction identities.) One relationship has been given to you as an example.



$\sin(90° - \theta) = \dfrac{a}{c} = \cos(\theta)$ $\csc(90° - \theta) =$

$\cos(90° - \theta) =$ $\sec(90° - \theta) =$

$\tan(90° - \theta) =$ $\cot(90° - \theta) =$

#6) Given that the angle θ is an acute angle and that $\sin(\theta) = \frac{2}{3}$, use trigonometric identities to determine the values of the remaining five trigonometric functions. State the formulas you use for each calculation before substituting. (Hint: Since θ is in the first quadrant, all of your trigonometric functions will be positive. This fact will be useful when taking the square root of both sides of the equation in some situations.)

#7) Given that the angle θ is an acute angle and that $\sec(\theta) = \frac{5}{2}$, use trigonometric identities to determine the values of the remaining five trigonometric functions. State the formulas you use for each calculation before substituting.

#8) Given that the angle θ is an acute angle and that $\cos(\theta) = \frac{\sqrt{5}}{7}$, use a right triangle to determine the values of the remaining five trigonometric functions.

#9) Given that the angle θ is an acute angle and that $\cot(\theta) = \frac{\sqrt{6}}{9}$, use a right triangle to determine the values of the remaining five trigonometric functions.

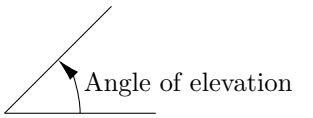
#10) Verify the identity $\tan(\theta)\cot(\theta) = 1$. (Be sure to indicate your substitutions.)

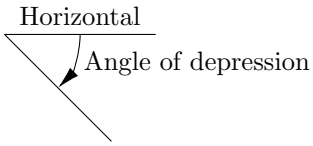
#11) Verify the identity $\cot(\theta)\sin(\theta) = \cos(\theta)$.

#12) Verify the identity $\dfrac{\sin(\theta)}{\cos(\theta)} + \dfrac{\cos(\theta)}{\sin(\theta)} = \csc(\theta)\sec(\theta)$.

When discussing angles in real-life applications, you will come across the terms *angle of elevation* and *angle of depression*. These are words that indicate the measure of an angle relative to the horizontal.

When working with word problems, it is important to draw the horizontal into the picture and ensure that your angle of elevation or angle of depression is measured from that angle. A common mistake for students is to use the angle of elevation or angle of depression relative to other directions in the problem (or blindly plug the angle into a trigonometric function), which leads to incorrect answers. If you take the time to draw a good diagram for word problems, you will be less likely to makes these types of mistakes.





#13) From the top of a 1000 foot tall building, a superhero looks down and sees someone breaking into a car. The angle of depression to the car is 80°. How far away from the base of the building is the car?

#14) ADA compliance of a wheelchair ramp requires a maximum 1:12 slope ratio, meaning that it can only rise at most 1 foot for each 12 feet of horizontal distance. (Think of 1:12 as "rise over run.") Determine the maximum angle of elevation of an ADA compliant wheelchair ramp.

#15) As you drive across a stretch of level (no slope) road, you see a mountain directly in front of you. The angle of elevation to the peak of the mountain is 4°. The road continues towards the mountain for another 15 miles. At that point, the angle of elevation to the peak of the mountain is 10°. Determine the height of the mountain. Be sure to draw and label a diagram and explain your calculations.

#16) Write a word problem using the ideas from this section that incorporates real-life data. (For example, you might use the the height of an object or the distance between two locations in your word problem.)

Extra Credit Activity: You are an ADA compliance officer and you are investigating the wheelchair ramp on the east side of the Rogers Student Center. Your goal is to assess whether the ramp is in compliant with the 1:12 slope ratio. Write a 3-5 page report that outlines your investigation. You are given both an outline for the report as well as a framework for how to conduct your investigation.

Successful completion of this activity is worth the equivalent of a single homework assignment. You will be graded on the clarity and quality of your presentation. You are encouraged to submit a rough draft for review before your final report for feedback.

Report Outline:

- Introduction: This section lays out the goal of your report. Define the underlying question and the standard that is to be met. You will want to describe the ramp you are measuring, using either pictures or diagrams so that a person unfamiliar with our campus would be able to understand the layout of the ramp under consideration. You will also want to cite the appropriate codes that are relevant to the question.
- Analysis: This section is where you actually describe your calculations to explain your ultimate conclusion. You will want to be sure in this section that you explain what you're doing. There will be two parts.
  - Measurements: In real life, you would be able to use fancy equipment (laser range finders and angle measuring tools) to do your measurements. But you don't have access to these. Instead, you're going to make estimates based on the physical structures. To determine the height of the ramp, measure the height of a single step on the staircase adjacent to the ramp and extrapolate the total height from that information. You will next want to compute the ramp length. This isn't the horizontal run of the ramp, but the physical distance that you would travel as someone going up the ramp. You can measure this directly with a yard stick or meter stick. You may also be able to get access to a long measuring tape.
  - Calculations: Using diagrams, words, and equations, explain to someone how you've pieced together the measurements in order to determine the horizontal run.
- Conclusion: Explain what you can conclude from your calculations. Is the ramp in compliance? Be sure to state your conclusion using the final numbers from your analysis section. You only need a few sentences here.