# OVERVIEW

There are 16 problem types that address the following learning objectives. Six (6) of these are labeled as **CORE** problems:

**Problem 0** Compute limits and determine continuity and differentiability of a function given a graph or table.

**Problem 1** Estimate the average and instantaneous rate of change in a function and interpret the values of a first derivative and a second derivative.

**Problem 2 (CORE)** Find the equation for the tangent line to the graph of a function at a point, and find the formula for the derivative of a function, using the limit-based definition of the derivative.

**Problem 3** Use a linear approximation of a function to estimate quantities that would be difficult to compute without a calculator.

**Problem 4** Find and use the derivative of polynomial, power, and exponential functions.

**Problem 5 (CORE)** Find and use the derivative of a function using the Product and Quotient Rules.

**Problem 6** Find and use the derivative of functions involving trigonometric functions.

**Problem 7 (CORE)** Find and use the derivative of a function using the Chain Rule.

**Problem 8** Find and use the derivative of functions involving logarithmic and inverse trigonometric functions.

**Problem 9** Determine the derivative of an implicitly-defined function.

**Problem 10 (CORE)** Use algebraic methods to determine critical values, intervals of increase and decrease, local extrema, intervals of concavity, and inflection points of a function.

**Problem 11** Use algebraic methods to find the global extrema of a continuous function on a closed interval.

**Problem 12 (CORE)** Set up and solve applied optimization problems.

**Problem 13** Set up and solve applied related rates problems.

**Problem 14** Find the total change of a changing quantity over an interval using a Riemann sum.

**Problem 15 (CORE)** Find the area between the graph of a function and the horizontal axis over a closed interval using the definite integral.

**Problem 16** Use the First and Second Fundamental Theorems of Calculus to evaluate derivatives and integrals.

Each midterm exam as well as the final exam will contain new instances of these basic types. The first midterm will contain instances of Problems 0–3. The second midterm will contain Problems 0–7. The third midterm will contain Problem 0 through Problem 13, and the fourth midterm and final exam will contain instances of all 17 of these. In this way, you can get repeated tries at each problem until you attain mastery on these.

A sample of all 17 problems (i.e. a sample final exam) begins on the next page.

MTH 201: Calculus
Sample Final Exam

0. (a) $\lim_{x\to 2} f(x)$

   (b) $\lim_{x\to 4} f(x)$

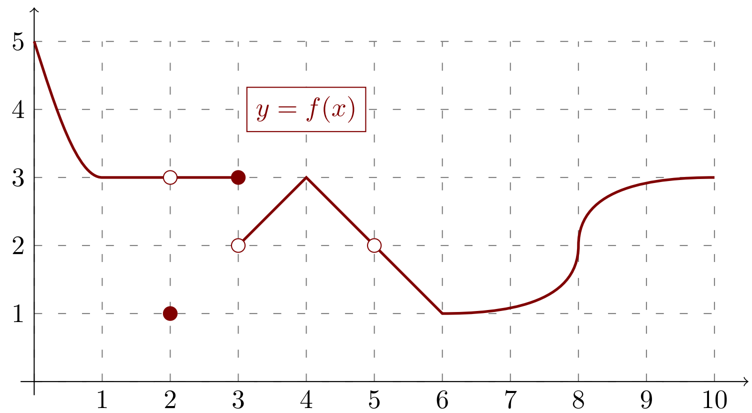
   (c) $\lim_{x\to 5} f(x)$

   (d) Identify all values of $a$ such that $\lim_{x\to a} f(x)$ is undefined.

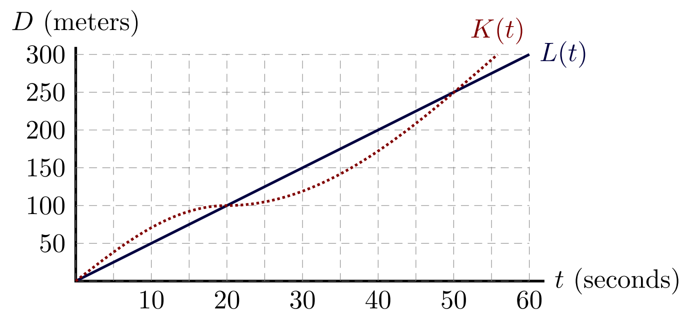
   (e) Identify all values of $a$ such that $f(x)$ is undefined at $x = a$.

   (f) Identify all values of $a$ such that $f(x)$ is discontinuous at $x = a$.

   (g) Identify all values of $a$ such that $f(x)$ is not differentiable at $x = a$.




1. Lenny and Karl decide to have a foot race around the LAS building parking lot (a distance of 300 meters). Let $L(t)$ and $K(t)$ be Lenny's and Karl's respective distances along the course $t$ seconds after the race began. A graph of $L(t)$ and $K(t)$ is shown below.




   (a) Who won the race? (Circle your answer.)

   Lenny Karl

   (b) Estimate the times at which Lenny and Karl were running the same speed.

   (c) Estimate Karl's average velocity over the first 35 seconds of the race.

   (d) Estimate Karl's instantaneous velocity exactly 35 seconds into the race.

   (e) At exactly 35 seconds into the race is Karl's acceleration positive, negative, or zero? (Circle one.)

   positive negative zero

2. **(CORE)** Consider the function $f(x) = 3x^2 - 5x + 10$.

   (a) Find the equation for the tangent line to the graph of this function at $x = 5$. Use only the definition of the derivative.

   (b) Find the formula for the derivative of this function at any point. Use only the definition of the derivative.

3. (a) Use the formula $f'(x) = \dfrac{1}{3\sqrt[3]{x^2}}$ to find the linear approximation of $f(x) = \sqrt[3]{x}$ at $x = 1000$.

   (b) Use the linear approximation to estimate the value of $\sqrt[3]{1003}$.

4. For each of the following functions, use the algebraic rules for differentiation to perform the indicated tasks.
   (a) $f(x) = x^2 + 3x - 7$; Find all the points where the tangent line to the graph of the function is horizontal.
   (b) $g(w) = 6w^4 + 7w^{2/3}$; Find the equation of the tangent line to the graph of the function when $w = 1$.
   (c) $s(t) = 3e^t - t^3$; If this function represents the position of a moving object, find the velocity and acceleration of the object when $t = 1$. The units of $t$ are seconds; the units of $s$ are feet.

5. 5 **(CORE)** For each of the following functions, use the algebraic rules for differentiation to perform the indicated tasks.
   (a) $f(x) = \dfrac{3x-2}{4x-9}$; Find the equation of the tangent line to the graph of the function when $x = 3$.
   (b) $g(t) = (t^3 + t^2 + t + 1)e^t$; Find formulas for the first and second derivatives.
   (c) $h(z) = \dfrac{z^4 + e^z}{z+1}$; Find the exact value (no decimal approximations) of the slope of the tangent line to the graph of the function when $z = 5$.

6. For each of the following functions, use the algebraic rules for differentiation to perform the indicated tasks.
   (a) $y = \dfrac{\tan\theta}{\theta^2}$; Find formulas for the first, second, and third derivatives.
   (b) $y = e^\theta \sec\theta$; find an equation for the tangent line to this function when $\theta = \pi/4$.
   (c) $s(t) = 300 + 40\sin(t)$; If this function represent the height ($s$, in meters) at time $t$ seconds of a weight oscillating up and down on the end of a spring, find the velocity and acceleration of the weight at $t = \pi/3$ seconds.

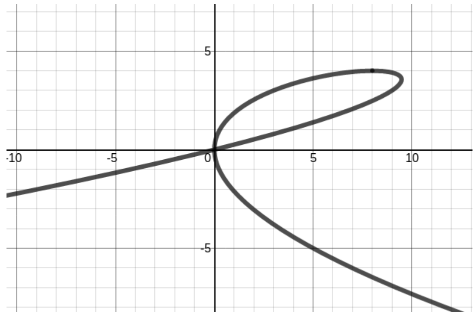
7. **(CORE)**
   (a) According to the U.S. standard atmospheric model, atmospheric temperature $T$ (in degrees Celsius), pressure $P$ (in kilopascals), and altitude $h$ (in meters) are related by the following formulas (valid whenever $h \leq 11000$):
   $$T = 15.04 - 0.000649h \qquad P = 101.29 + \left(\frac{T + 273.1}{288.08}\right)^{5.256}$$
   Use the Chain Rule to calculate a formula for $dP/dh$. Then find the instantaneous rate of change in atmospheric pressure with respect to altitude, at the point where $h = 3000$.
   (b) Consider the following table of data:

| $x$ | 1 | 4 | 6 |
|---|---|---|---|
| $f(x)$ | 4 | 0 | 6 |
| $g(x)$ | 5 | 7 | 4 |
| $f'(x)$ | 4 | 1 | 6 |
| $g'(x)$ | 5 | 1/2 | 3 |

   Let $a(x) = f(g(x))$ and $b(x) = e^{f(x)}$. Calculate the values of $a'(6)$ and $b'(4)$.

8. For each of the following functions, use the algebraic rules for differentiation to perform the indicated tasks.
   (a) $f(t) = \ln(\sin t)$; Find an equation for the tangent line to the function when $t = \pi/4$.
   (b) $g(x) = \arcsin(e^x)$; Find the exact values (no decimal approximations) of the first and second derivative when $x = -1$
   (c) The energy $E$ (in joules) radiated as seismic waves from an earthquake of Richter magnitude $M$ is given by the formula $\log_{10} E = 4.8 + 1.5M$. Express $E$ as a function of $M$; then find a formula for $dE/dM$ and find the instantaneous rate of change in energy when $M = 2$.

9. Consider the equation $4xy = x^2 + y^3$, which traces out the curve below:




(a) Find an equation for the line tangent to the curve at $(5, -5)$. Show all your work and do not merely estimate values from the graph.

(b) From the graph you can see that there are two places where the tangent line to the curve is vertical: at the origin, and at a point in the first quadrant. Find the exact coordinates of the one in the first quadrant. Show all work and make no estimations.

10. **(CORE)** Consider the function

$$f(t) = -\frac{1}{4}e^{-2t}(2t^2 + 2t - 9)$$

(a) Find the exact values of all critical values of $f$.

(b) Find the intervals on which $f$ is increasing and the intervals on which $f$ is decreasing. Show all work and do not merely estimate from a graph.

(c) Classify each critical value you found as a local maximum, local minimum, or neither. Show all work and do not merely estimate from a graph.

(d) Find all the intervals on which $f$ is concave up and all the intervals on which $f$ is concave down. Show all work and do not merely estimate from a graph.

(e) Give the coordinates of all the inflection points of $f$. Show all work and do not merely estimate from a graph.

11. For each of the following continuous functions, find the global minimum and global maximum value of the function on the given interval.

(a) $y = x^3 + 3x^2 - 9x + 2$, $[0, 2]$

(b) $y = \sqrt{2}\theta - \sec\theta$, $[0, \pi/3]$

12. **(CORE)**

(a) A homeowner is putting up a pen area for his two dogs. The pen will be a rectangular fenced-in area, with a row of fence dividing the pen into two equal halves (one for each dog). He has enough money to afford 200 feet of fencing. What dimensions $x$ and $y$ of the fence maximize the area contained in the pens?

(b) Find the point $(x, y)$ on the line $6x + y = 9$ that is closest to the point $(9, -5)$.

13. (a) An electrical voltage $V$ across a resistance $R$ generates an electrical current $I$ given by the formula $V = IR$. If a constant voltage of 9 volts is put across a resistance that is increasing at a rate of 0.2 ohms per second when the resistance is 0.5 ohms, at what rate is the current changing?

(b) A hot air balloon rising vertically is tracked by an observer located 5 miles from the lift-off point. At a certain moment, the angle between the observer's line-of-sight and the horizontal is $\frac{\pi}{3}$, and it is changing at a rate of 0.1 rad/min. How fast is the balloon rising at this moment?

14. At a testing facility, a prototype electric car was monitored for performance as it drove down a straight track. Its velocity $v$, in meters per second, at time $t$ seconds, was measured using every 0.5 seconds using a radar gun. The resulting data are given in a table:

| $t$ | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 | 5.5 | 6 | 6.5 | 7 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $v$ | 0 | 10.2 | 22.1 | 29.3 | 34 | 35 | 35.1 | 35.2 | 34.3 | 25 | 21.1 | 14.3 | 10.2 | 9 | 8.9 |

Estimate the distance that the car travelled over the entire 7-second interval. Show work and put correct units on your answer.

15. **(CORE)** For each of the functions below, find the *exact* value of the area between the graph of the function and the horizontal axis on the given interval. Show all work, and do not approximate.

    (a) $y = 3x^5 + x^2 - 2x$, $[0, 4]$

    (b) $y = 8/x^3$, $[4, 9]$

    (c) $y = \csc^2 x$, $[\pi/6, \pi/4]$

16. Evaluate each expression below.

    (a) $\dfrac{d}{dx}\left[\displaystyle\int_{-1}^{16x^2} \cos(e^t)\,dt\right]$

    (b) $\displaystyle\int_2^x \frac{d}{dt}\left[\frac{t}{1-t^2}\right] dt$

    (c) $\dfrac{d}{dx}\left[\displaystyle\int_x^5 t\sqrt{t^2 - 6t + 2}\,dt\right]$

    (d) $\displaystyle\int_0^1 \frac{1}{1+x^2}\,dx$