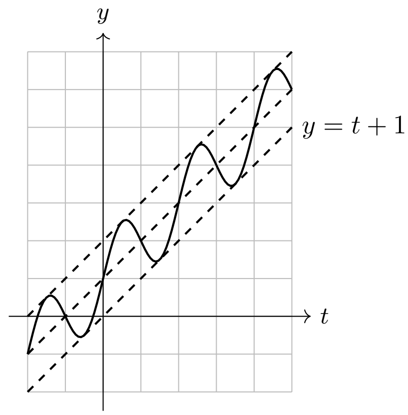
Group Practice Problems #1 - Changing Midlines and Amplitudes: Sketch the graph. Clearly indicate the midline and amplitude.

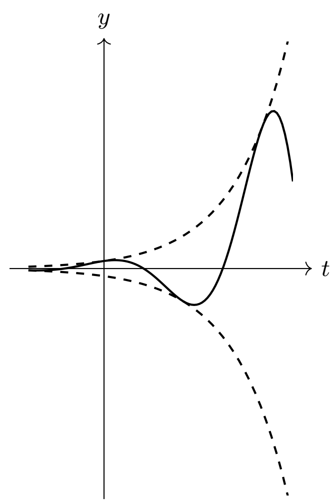
- $y = \sin(\pi t) + (t + 1)$

The midline of this function is the equation $y = t + 1$ and the amplitude is 1. We can also see that the period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$ and that there is no horizontal shift.



- $y = e^t \cos(t)$

The midline of this function is $y = 0$ and the amplitude changes with t as an exponential function. The period is 2π and there is no horizontal shift.



Comments and Observations:

- These exercises are mostly about establishing the general sense of how the variable midline and amplitude behave.
- The exponential function grows very quickly, and so the y scale on the graph has been omitted. When the cosine function completes its first period at 2π, the amplitude is $e^{2\pi} \approx 535.5$.

Group Practice Problems #2 - Population Models: Solve the word problems.

- A population of elk currently averages 2000 elk, and that average has been growing by 4% per year. Due to seasonal fluctuations, the population oscillates from 50 below average in the winter up to 50 above average in the summer. Find a function that models the number of elk after t years, starting in the winter.

Since the population grows by a fixed percent each year, we know that the population is growing exponentially. Furthermore, since the initial population is 2000 and the growth rate is 4%, we can see that the population is modeled as $2000 \cdot (1.04)^t$. This is the midline.

The seasonal fluctuations of 50 above and below gives us a fixed amplitude of 50. Since the model starts in the winter, when the population is at its lowest, we will want to use a negative cosine function to model the population. Lastly, we note that the period is 1 year, so that $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{1} = 2\pi$. This gives us the information we need to create the full model.

$$y = -50\cos(2\pi t) + 2000 \cdot (1.04)^t$$

- A population of fish oscillates 30 above and below average during the year, reaching the lowest value in January. The average population starts at 1000 fish and increases by 20 per month. Find a function that models the population P in terms of the months since January t.

The population is growing by a fixed amount per month, which means that the midline is can be modeled as a line. The rate of change in the population is 20 per month with an initial population of 1000, which means the midline is represented by $1000 + 20t$.

The seasonal fluctuation of 30 above and below gives us a fixed amplitude of 30. Since the model starts in January, which corresponds to the low point of the year, we will use a negative cosine function to model the population. The period is 12 months, so that we have $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{12} = \frac{\pi}{6}$. This means that the full model is the following:

$$y = -30\cos\left(\frac{\pi}{6} \cdot t\right) + 1000 + 20t$$

Comments and Observations:

- These problems draw significantly from the experience of determining the equation from a graph in Section 6.1. It will be helpful to review that section.

Group Practice Problems #3 - Damped Harmonic Oscillation: Solve the word problems.

- A spring is attached to the ceiling and pulled 5 cm down from equilibrium and released. The amplitude decreases by 7% each second. The spring oscillates once every 2 seconds. Find a function that models the distance D the end of the spring is below equilibrium in terms of the seconds t since the spring was released.

Since D is measured relative to equilibrium, we must have that the midline is $D = 0$. The amplitude starts at 5 and decays by 7% per second, which means that the amplitude is modeled by $5 \cdot (0.93)^t$. Since the spring oscillates onces every two second, so that $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{2} = \pi$. We can also see that the initial position is $D = 5$ since D is the distance below equilibrium. This means that we can model the behavior with a positive cosine function. Putting the pieces together gives the following model:

$$D = 5 \cdot (0.93)^t \cos(\pi t)$$

- A spring is attached to the ceiling and pulled 15 cm down from equilibrium and released. After 4 seconds the amplitude has decreased to 10 cm. The spring oscillates 3 times per second. Find a function that models the distance D that the end of the spring is below equilibrium in terms of the seconds t since the spring was released.

Since D is measured relative to equilibrium, we must have that the midline is $D = 0$. Initially, the amplitude is 15 cm, but it drops to 10 cm after 4 seconds. If we model the amplitude as $15 \cdot r^t$, then we would have $10 = 15 \cdot r^4$, so that $r = \sqrt[4]{\frac{2}{3}} \approx 0.904$. So the amplitude is modeled by $15 \cdot (0.904)^t$.

Since the spring oscillates three times per second, the period is $\frac{1}{3}$ seconds, so that $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{1/3} = 6\pi$. We can model the behavior with a positive cosine function since the initial positive is below equilibrium. Putting the pieces together gives the following model:

$$D = 15 \cdot (0.904)^t \cos(6\pi t)$$

Comments and Observations:

- Both of these problems define how D is to be measured. In problems where this is not defined, you need to determine what makes the most sense for the context of the problem you're doing.