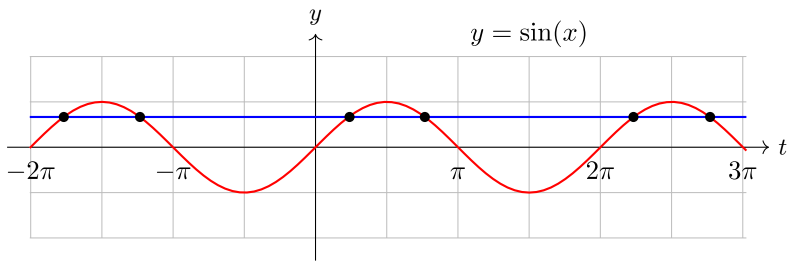
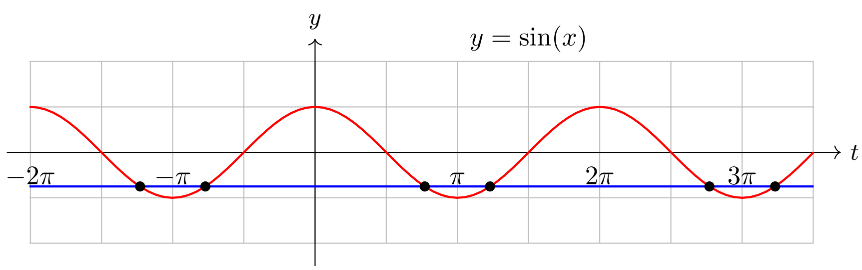
Group Practice Problems #1 - Solving by Graphing: Use Desmos or a graphing calculator to approximate the first four positive and first two negative solutions of equation by graphing.

- $\sin(t) = \frac{2}{3}$



- First four positive solutions: $t = 0.75, 2.41, 7.01, 8.70$
- First two negative solutions: $t = -3.87, -5.55$

- $\cos(t) = -\frac{3}{4}$



- First four positive solutions: $t = 2.42, 3.86, 8.70, 10.15$
- First two negative solutions: $t = -2.42, -3.86$

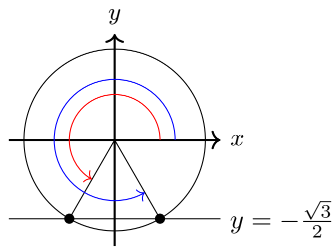
Comments and Observations:

- The presentation here is to use two graphs and find the points of intersection. Depending on the program or calculator being used, it may be easier to rearrange the equation and search for zeros. For example in the first problem, instead of graphing $y = \sin(t)$ and $y = \frac{2}{3}$ separately and searching for intersections, you can graph $y = \sin(t) - \frac{2}{3}$ and search for zeros.
- The steps and presentation will look different depending on the program or calculator used. The main thing for this problem is that you are able to get the correct answers and visualize them on a graph.
- A common error is to be working in degrees when you should be working in radians. Unless otherwise indicated, you should be working in radians.

Group Practice Problems #2 - Exact Solutions on the Real Line: Determine the exact solutions (in radians) of the equation. Draw a unit circle diagram and indicate the quadrants in which the solutions lie, and identify the reference angle.

- $\sin(t) = -\frac{\sqrt{3}}{2}$

We will start with a unit circle diagram to determine the quadrants of the solutions.



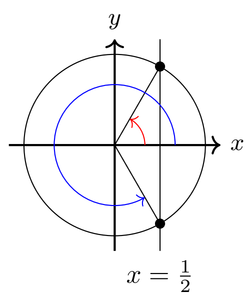
This shows us that the solutions will be in the third and fourth quadrants.

We can also identify that the reference angle is $\frac{\pi}{3}$ since $\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$. We can combine this with the geometry of the unit circle to get all solutions.

$$t = \begin{cases} \pi + \frac{\pi}{3} + 2\pi k \\ 2\pi - \frac{\pi}{3} + 2\pi k \end{cases}$$
$$= \begin{cases} \frac{4\pi}{3} + 2\pi k \\ \frac{5\pi}{3} + 2\pi k \end{cases}, \text{ where } k \text{ is any integer}$$

- $\cos(t) = \frac{1}{2}$

We will start with a unit circle diagram to determine the quadrants of the solutions.



This shows us that the solutions will be in the first and fourth quadrants.

We can also identify that the reference angle is $\frac{\pi}{3}$ since $\cos(\frac{\pi}{3}) = \frac{1}{2}$. We can combine this with the geometry of the unit circle to get all solutions.

$$t = \begin{cases} \frac{\pi}{3} + 2\pi k \\ 2\pi - \frac{\pi}{3} + 2\pi k \end{cases}$$
$$= \begin{cases} \frac{\pi}{3} + 2\pi k \\ \frac{5\pi}{3} + 2\pi k \end{cases}, \text{ where } k \text{ is any integer}$$

Comments and Observations:

- It is possible to choose negative angles for the initial solutions. For example in the first problem, the second angle can be chosen as $-\frac{\pi}{3}$ instead of working with $2\pi - \frac{\pi}{3}$. This has both advantages and disadvantages. An advantage is that it is computationally simpler to not have to worry about the additional 2π term. However, a disadvantage is that we are often in pursuit of positive solutions, and so if the initial solution is negative then work will need to be done later to find a coterminal angle to it that has a positive measure.

- Notice that the instructions do not constrain the solutions to be within a certain interval, so that we have to include the $2\pi k$ term to account for all of the coterminal angles.

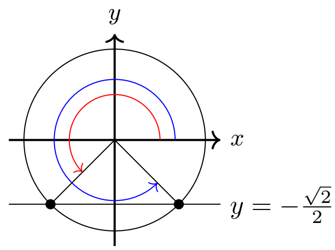
- Although the restriction of k to being integer values is implied due to the nature of the problem, it's important to be in the habit of clearly defining variables that are introduced and explicitly stating any conditions that there may be on them.

- You will also see $k \in \mathbb{Z}$ to indicate that k can be any integer. For example, instead of "where k is any integer" you might see "where $k \in \mathbb{Z}$."

Group Practice Problems #3 - Exact Solutions on an Interval: Determine the exact solutions (in degrees) of the equation on the interval $[0, 360°)$. Draw a unit circle diagram and indicate the quadrants in which the solutions lie, and identify the reference angle.

- $\sin(t) = -\frac{\sqrt{2}}{2}$

We will start with a unit circle diagram to determine the quadrants of the solutions.



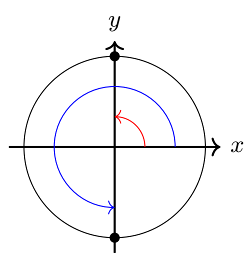
This shows us that the solutions will be in the third and fourth quadrants.

We can also identify that the reference angle is $45°$ since $\sin(45°) = \frac{\sqrt{2}}{2}$. We can combine this with the geometry of the unit circle to get all solutions.

$$t = \begin{cases} 180° + 45° \\ 360° - 45° \end{cases}$$
$$= \begin{cases} 225° \\ 315° \end{cases}$$

- $\cos(t) = 0$

We will start with a unit circle diagram to determine the quadrants of the solutions.



This shows us that the solutions will be on the y-axis. Therefore, the solutions are

$$t = \begin{cases} 90° \\ 270° \end{cases}$$

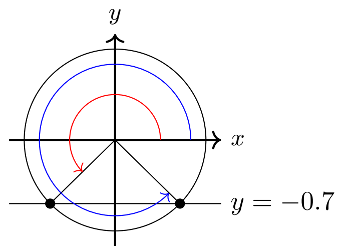
Comments and Observations:

- In this problem, we did not include the coterminal angles because of the restricted range of the solutions.

Group Practice Problems #4 - Approximate Solutions on the Real Line: Determine the approximate solutions (in degrees) of the equation. Draw a unit circle diagram and indicate the quadrants in which the solutions lie, and identify the reference angle.

- $\sin(t) = -0.7$

We will start with a unit circle diagram to determine the quadrants of the solutions.



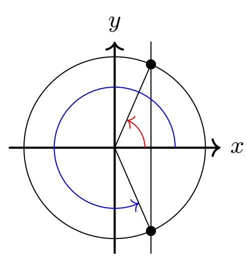
This shows us that the solutions will be in the third and fourth quadrants.

Using a calculator, we can determine that $\sin^{-1}(-0.7) \approx -44.42°$, which is the angle in the fourth quadrant. This implies that the reference angle is $44.42°$, which allows us to calculate the solution in the third quadrant.

$$t \approx \begin{cases} 180° + 44.42° + k \cdot 360° \\ -44.42° + k \cdot 360° \end{cases}$$
$$\approx \begin{cases} 224.42° + k \cdot 360° \\ -44.42° + k \cdot 360° \end{cases}, \text{ where } k \text{ is any integer}$$

- $\cos(t) = 0.4$

We will start with a unit circle diagram to determine the quadrants of the solutions.



Using a calculator, we can determine that $\cos^{-1}(0.4) \approx 66.42°$, which is the angle in the first quadrant. This implies that the reference angle is $44.42°$, which allows us to determine the solution in the fourth quadrant.

$$t \approx \begin{cases} 66.42° + k \cdot 360° \\ -66.42° + k \cdot 360° \end{cases}, \text{ where } k \text{ is any integer}$$

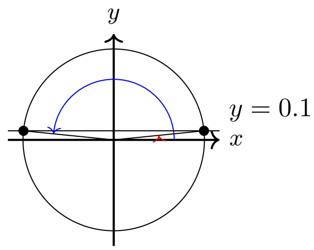
Comments and Observations:

- Pay attention to the quadrant of the output of the inverse trigonometric function and focus on that geometry to help you to understand the meaning of the result.

Group Practice Problems #5 - Approximate Solutions on the an Interval: Determine the approximate solutions (in radians) of the equation on the interval $[0, 2\pi)$. Draw a unit circle diagram and indicate the quadrants in which the solutions lie, and identify the reference angle.

- $\sin(t) = 0.1$

We will start with a unit circle diagram to determine the quadrants of the solutions.



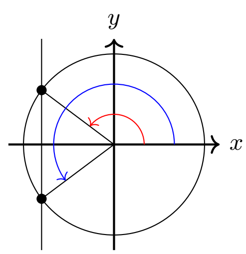
This shows us that the solutions will be in the first and second quadrants.

Using a calculator, we can determine that $\sin^{-1}(0.1) \approx 0.10$, which is the angle in the first quadrant. This is the reference angle, which allows us to calculate the angle in the second quadrant.

$$t \approx \begin{cases} 0.10 \\ \pi - 0.10 \end{cases} \approx \begin{cases} 0.10 \\ 3.04 \end{cases}$$

- $\cos(t) = -0.8$

We will start with a unit circle diagram to determine the quadrants of the solutions.



This shows us that the solutions will be in the second and third quadrants.

Using a calculator, we can determine that $\cos^{-1}(-0.8) \approx 2.50$, which is the angle in the second quadrant. This shows us that the reference angle is $\pi - 2.50 = 0.64$. And we can use this value to calculate the solution in the third quadrant.

$$t \approx \begin{cases} 2.50 \\ \pi + 2.50 \end{cases} \approx \begin{cases} 2.50 \\ 5.64 \end{cases}$$

Comments and Observations:

- Typically, we try to avoid mixing decimals and exact values (such as $\pi - 0.10$). The reason for this is that the initial value is already a decimal approximation, and so there is less value when mixing it with an exact value. An exception to this is when we refer to coterminal angles (the "$+2\pi k$" terms) because we want to communicate the exact periodicity. This is important when the variable has a coefficient, which can lead to compounding rounding errors.

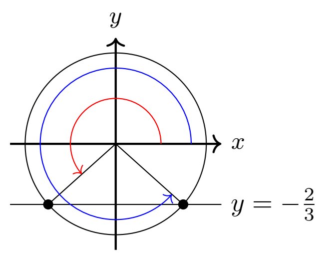
Group Practice Problems #6 - Solving Linear Trigonometric Equations: Determine the solutions (in radians) of the equation.

- $3\sin(2t) + 1 = -1$

We begin by isolating the trigonometric function.

$$\begin{aligned} 3\sin(2t) + 1 &= -1 && \\ 3\sin(2t) &= -2 && \text{Subtract 1 from both sides} \\ \sin(2t) &= -\frac{2}{3} && \text{Divide both sides by 3} \end{aligned}$$

We have reduced the problem to a trigonometric equation. To solve this, we will draw a unit circle diagram.



This shows us that the solutions will be in the third and fourth quadrants. Using a calculator, we get $\sin^{-1}(-\frac{2}{3}) \approx -0.7297$, which implies that the reference angle is 0.73. This means that we have the following:

$$2t \approx \begin{cases} -0.7297 + 2\pi k \\ \pi + 0.7297 + 2\pi k \end{cases}$$

$$2t \approx \begin{cases} -0.7297 + 2\pi k \\ 3.8713 + 2\pi k \end{cases}$$

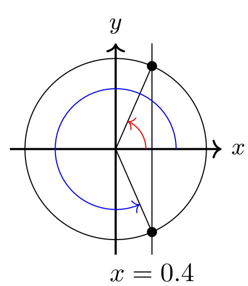
$$t \approx \begin{cases} -0.36 + \pi k \\ 1.94 + \pi k \end{cases}, \text{ where } k \text{ is any integer}$$

- $4\cos(3t) - 1 = -\cos(3t) + 1$

We begin by isolating the trigonometric function.

$$\begin{aligned} 4\cos(3t) - 1 &= -\cos(3t) + 1 && \\ 5\cos(3t) - 1 &= 1 && \text{Add } \cos(3t) \text{ to both sides} \\ 5\cos(3t) &= 2 && \text{Add 1 to both sides} \\ \cos(3t) &= \frac{2}{5} && \text{Divide both sides by 5} \end{aligned}$$

We have reduced the problem to a trigonometric equation. To solve this, we will draw a unit circle diagram.



This shows us that the solutions will be in the third and fourth quadrants. Using a calculator, we get $\cos^{-1}(-\frac{2}{5}) \approx 1.1593$, which is also the reference angle. This means that we have the following:

$$3t \approx \begin{cases} 1.1593 + 2\pi k \\ 2\pi - 1.1593 + 2\pi k \end{cases}$$

$$3t \approx \begin{cases} 1.1593 + 2\pi k \\ 5.1239 + 2\pi k \end{cases}$$

$$t \approx \begin{cases} 0.39 + \frac{2\pi k}{3} \\ 1.71 + \frac{2\pi k}{3} \end{cases}, \text{ where } k \text{ is any integer}$$

Comments and Observations:

- Even though the end goal is to have two decimal places, it's good to carry an extra decimal or two in the middle of the calculation to avoid rounding errors.