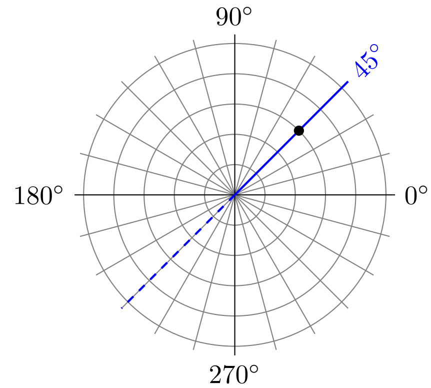
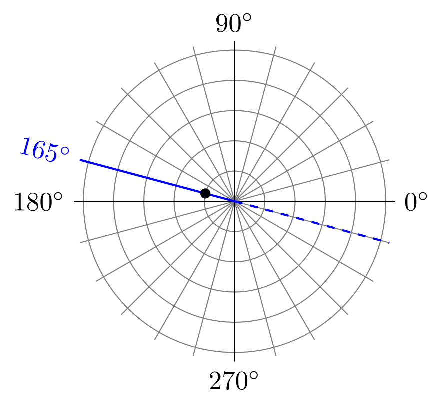
Group Practice Problems #1 - Polar Graphing: Plot the points on a polar graph. (Feel free to plot multiple points on a single graph as long as they are clearly labeled.)

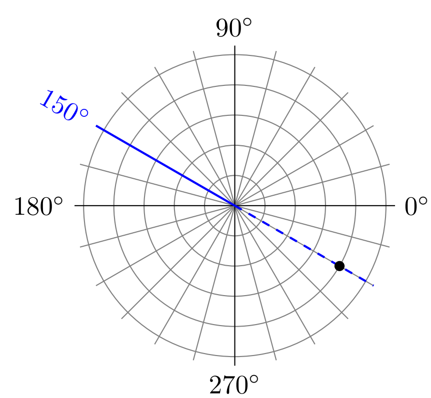
- $(3, 45°)$



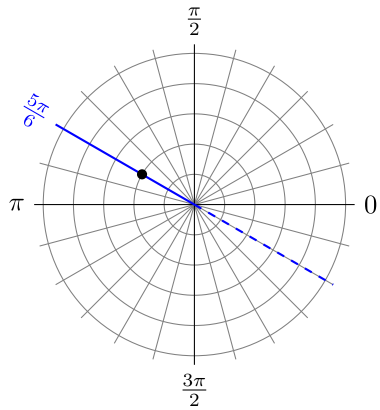
- $(1, 165°)$



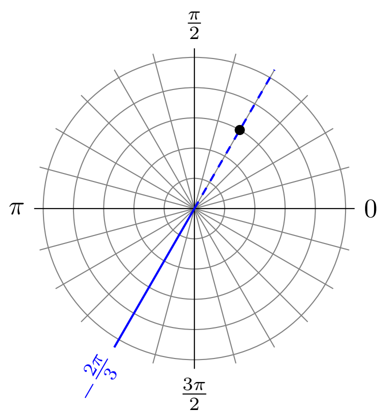
- $(-4, 150°)$



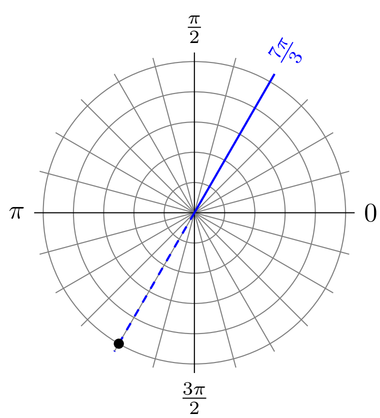
- $(2, \frac{5\pi}{6})$



- $(-3, -\frac{2\pi}{3})$



- $(-5, \frac{7\pi}{3})$

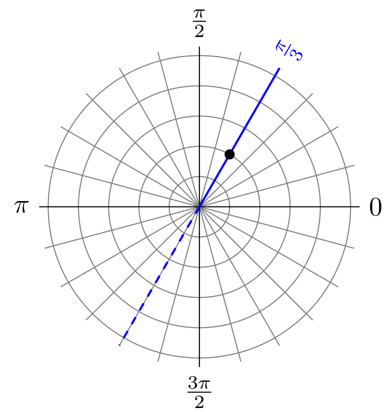


Comments and Observations:

- By this point in the semester, you should be comfortable enough with radians that you can convert between degrees and radians for multiples of $30°$and $45°$, and with a little work you can also get all multiples of $15°(\frac{\pi}{12}$ radians).

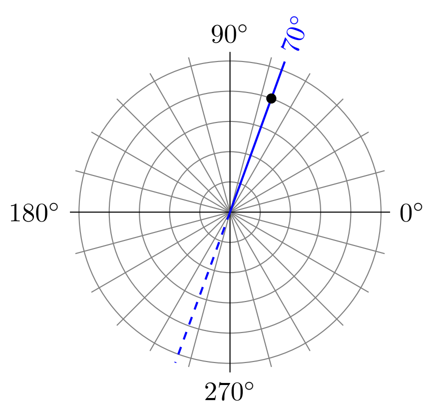
Group Practice Problems #2 - Converting Points from Polar to Cartesian: Plot the point then convert it from polar coordinates to Cartesian coordinates.

- $(2, \frac{\pi}{3})$



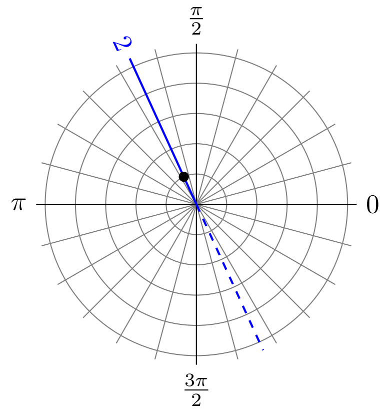
$$x = r\cos(\theta) = 2\cos\left(\frac{\pi}{3}\right) = 2 \cdot \frac{1}{2} = 1$$
$$y = r\sin(\theta) = 2\sin\left(\frac{\pi}{3}\right) = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$$

- $(4, 70^\circ)$



$$x = r\cos(\theta) = 4\cos(70^\circ) \approx 4 \cdot (0.3420) \approx 1.37$$
$$y = r\sin(\theta) = 4\sin(70^\circ) \approx 4 \cdot (0.9397) \approx 3.76$$

- $(1, 2)$ (Hint: The angle is in radians and you will need a calculator.)



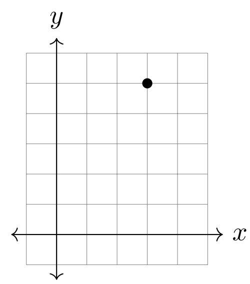
$$\begin{aligned} x &= r\cos(\theta) = 1\cos(2) \approx 1 \cdot (-0.4161) \approx -0.42 \\ y &= r\sin(\theta) = 1\sin(2) \approx 1 \cdot (0.9093) \approx 0.91 \end{aligned}$$

Comments and Observations:

- The last problem is a reminder that angles don't need to have a π in them in order to be in radians. Notice that $\frac{\pi}{2} \approx 1.57$ and $\pi \approx 3.14$, so that 2 is about a third of the way from up to left.

Group Practice Problems #3 - Converting Points from Cartesian to Polar: Plot the point then convert it from Cartesian coordinates to polar coordinates.

- $(3, 5)$

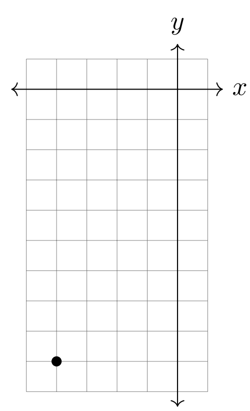


$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 5^2} = \sqrt{34}$$

To get the angle, we first note that θ must satisfy $\tan(\theta) = \frac{y}{x}$. So we start by calculating $\tan^{-1}(\frac{y}{x}) = \tan^{-1}(\frac{5}{3}) \approx 1.03$ (radians). Since the point and angle are both in the first quadrant, this is the correct angle to use.

The point in polar coordinates is $(r, \theta) \approx (\sqrt{34}, 1.03)$.

- $(-4, -9)$

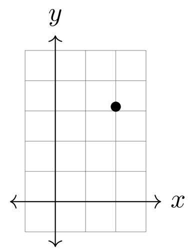


$$r = \sqrt{x^2 + y^2} = \sqrt{(-4)^2 + (-9)^2} = \sqrt{97}$$

To get the angle, we first note that θ must satisfy $\tan(\theta) = \frac{y}{x}$. So we start by calculating $\tan^{-1}(\frac{y}{x}) = \tan^{-1}(\frac{-9}{-4}) \approx 1.15$ (radians). However, this angle is in the first quadrant but the point is in the third quadrant, so we need to add π to the angle to get the appropriate angle.

The point in polar coordinates is $(r, \theta) \approx (\sqrt{97}, \pi + 1.15) = (\sqrt{97}, 4.29)$.

- $(2, \pi)$ (Hint: Remember that this is given in Cartesian coordinates!)



$$r = \sqrt{x^2 + y^2} = \sqrt{2^2 + \pi^2} \approx \sqrt{13.8696} \approx 3.72$$

To get the angle, we first note that θ must satisfy $\tan(\theta) = \frac{y}{x}$. So we start by calculating $\tan^{-1}(\frac{y}{x}) = \tan^{-1}(\frac{\pi}{2}) = 1$ (radians). Since the point and angle are both in the first quadrant, this is the correct angle to use.

The point in polar coordinates is $(r, \theta) \approx (3.72, 1)$.

Comments and Observations:

- The adjustment of the angle by π for points in the second and third quadrants stem from the fact that the range of the inverse tangent function is $(-\frac{\pi}{2}, \frac{\pi}{2})$, so that this only gives values in the first and fourth quadrants.

Group Practice Problems #4 - Converting Equations from Polar to Cartesian: Convert the equation from polar coordinates to Cartesian coordinates.

- $r = 4$

$$\begin{aligned} r &= 4 && \text{Original equation} \\ \sqrt{x^2+y^2} &= 4 && \text{Substitute} \\ x^2+y^2 &= 16 && \text{Square both sides of the equation} \end{aligned}$$

- $r\cos(\theta) + 3r\sin(\theta) = 5$

$$\begin{aligned} r\cos(\theta) + 3r\sin(\theta) &= 5 && \text{Original equation} \\ x + 3y &= 5 && \text{Substitute} \end{aligned}$$

- $\frac{1}{r^2} = \sin(2\theta)$ (Hint: Use the double angle formula.)

$$\begin{aligned} \frac{1}{r^2} &= \sin(2\theta) && \text{Original equation} \\ \frac{1}{r^2} &= 2\sin(\theta)\cos(\theta) && \text{Substitute} \\ 1 &= 2r^2\sin(\theta)\cos(\theta) && \text{Multiply both sides by } r^2 \\ 1 &= 2(r\cos(\theta))(r\sin(\theta)) && \\ 1 &= 2xy && \text{Substitute} \end{aligned}$$

Comments and Observations:

- These exercises are more about recognizing expressions and practicing some basic algebraic manipulations.

Group Practice Problems #5 - Converting Equations from Cartesian to polar: Convert the equation from Cartesian coordinates to polar coordinates.

- $x^2 + y^2 = 9$

$$\begin{aligned}
x^2 + y^2 &= 9 && \text{Original equation} \\
(r\cos(\theta))^2 + (r\sin(\theta))^2 &= 9 && \text{Substitute} \\
r^2\cos^2(\theta) + r^2\sin^2(\theta) &= 9 \\
r^2(\cos^2(\theta) + \sin^2(\theta)) &= 9 \\
r^2 \cdot 1 &= 9 && \text{Pythagorean identity} \\
r &= \pm 3 && \text{Take the square root of both sides} \\
r &= 3 && \text{Select the positive value only}
\end{aligned}$$

The graphs of $r = 3$ and $r = -3$ are identical, so we can select just the positive root.

- $x^2 + (y-2)^2 = 4$

$$\begin{aligned}
x^2 + (y-2)^2 &= 4 && \text{Original equation} \\
(r\cos(\theta))^2 + (r\sin(\theta) - 2)^2 &= 4 && \text{Substitute} \\
r^2\cos^2(\theta) + r^2\sin^2(\theta) - 4r\sin(\theta) + 4 &= 4 \\
r^2(\cos^2(\theta) + \sin^2(\theta)) - 4r\sin(\theta) &= 0 \\
r^2 \cdot 1 - 4r\sin(\theta) &= 0 && \text{Pythagorean identity} \\
r(r - 4\sin(\theta)) &= 0
\end{aligned}$$

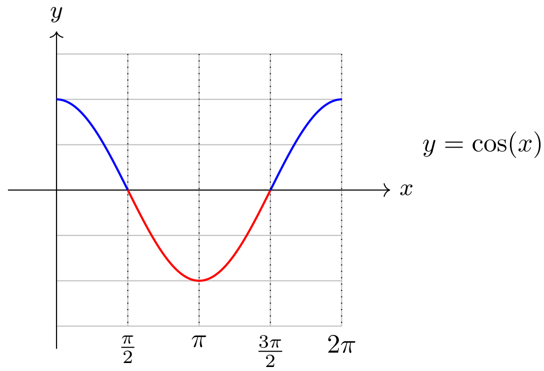
By the zero product property, we must have $r = 0$ or $r = 4\sin(\theta)$. Since the graph of $r = 0$ is contained within the graph of $r = 4\sin(\theta)$, we can select just $r = 4\sin(\theta)$ as the formula.

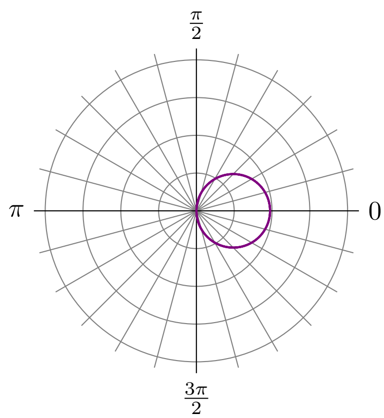
Comments and Observations:

- When finding intersections of polar graphs, the specific representation might matter, depending on the interpretation of the problem. For example, some might say that the graphs of $r = 3$ and $r = -3$ do not intersect (since r cannot be both 3 and -3 at the same time), even though the graphs appear to be identical. But these ideas go beyond the scope of the course, so we won't concern ourselves with it.

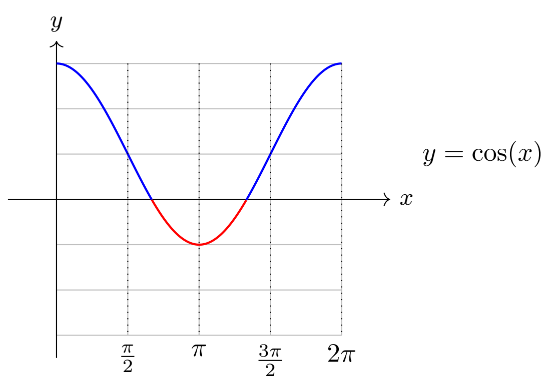
Group Practice Problems #6 - Polar Graphing: Plot the graph as an rectangular graph and use that to graph the polar graph.

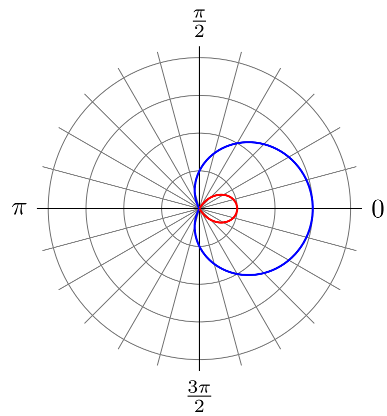
- $r = 2\cos(\theta)$



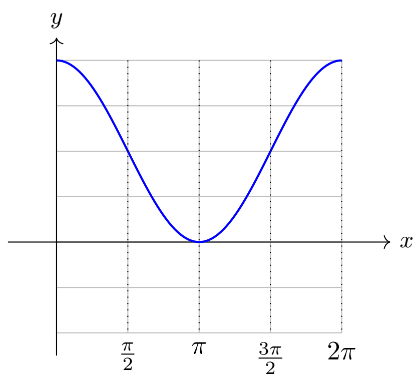


- $r = 2\cos(\theta) + 1$

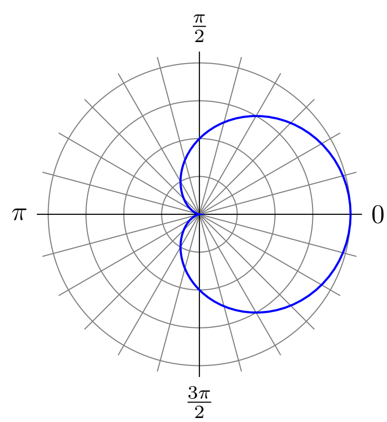




- $r = 2\cos(\theta) + 2$



$y = \cos(x)$



Comments and Observations:

- Polar graphs are difficult to describe in writing without using a whole lot of words. But you can at least verify the graphs by checking that the values in each quadrant make sense.
- The first graph feels a little confusing because it actually draws out the circle twice. So the red part of the graph (the negative part of the graph) completely overlaps the blue part (the positive part of the graph).