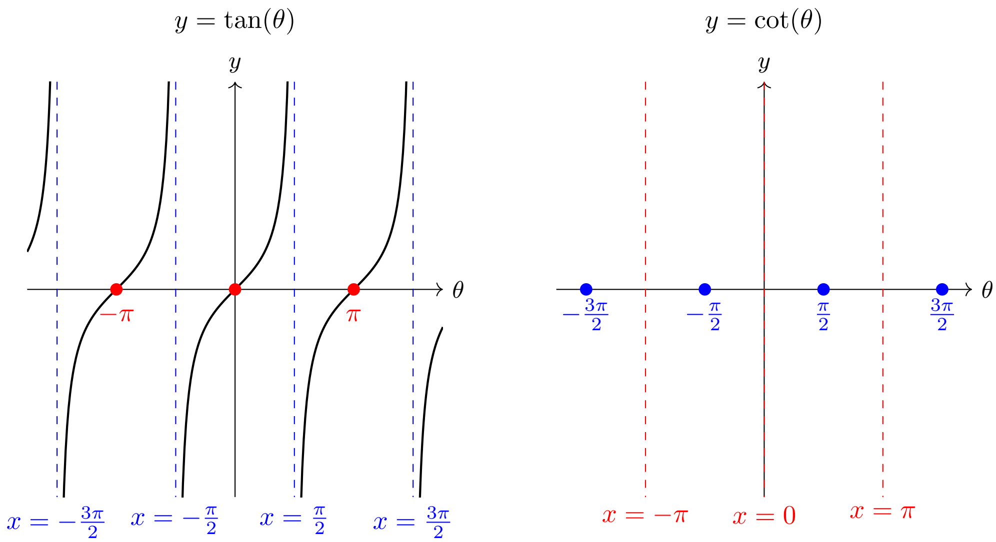
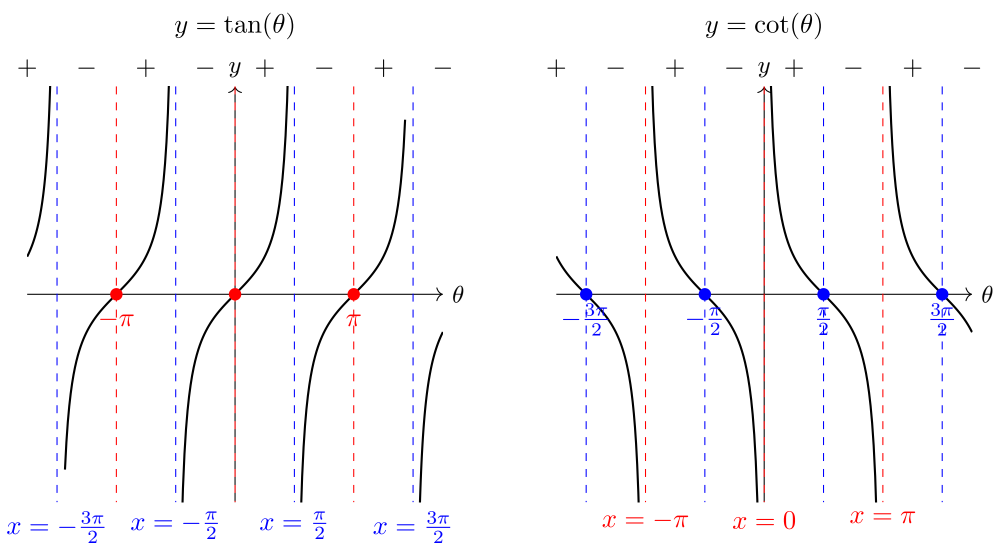
Group Practice Problems #1 - The Tangent and Cotangent Graphs: Starting with the graph of the tangent function, use the fact that $\cot(\theta) = \frac{1}{\tan(\theta)}$ to graph the cotangent function. Be sure to explain the relationships between the zeros and the vertical asymptotes.

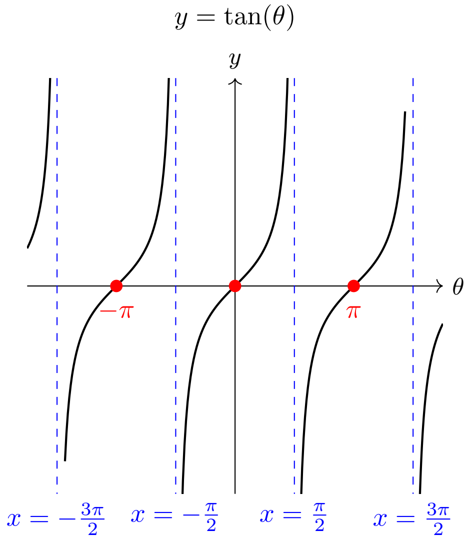
When taking the reciprocal of a function, the zeros turn into vertical asymptotes and the vertical asymptotes turn into zeros. We will start with the graph of the tangent function and convert the appropriate components.

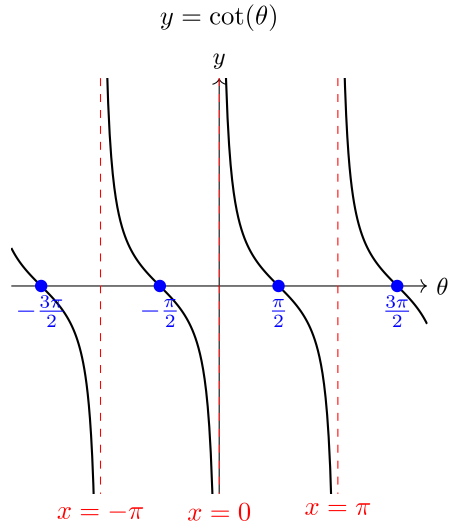


Next, we will look at whether the tangent function is positive or negative on each interval. Since taking the reciprocal does not change the sign, we can use this information to determine the signs of the cotangent function.



Lastly, we will clean up the result and present the final graphs.





Comments and Observations:

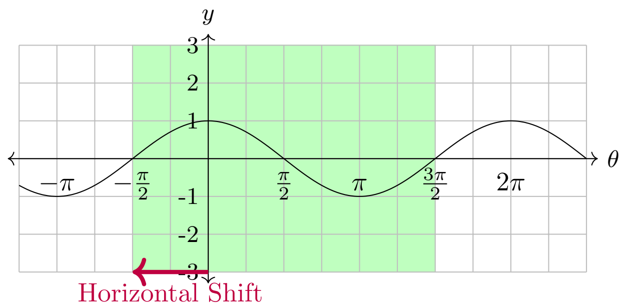
- The value of this exercise is to focus on how the graph of a reciprocal is related to the original function. While memorizing the tangent and cotangent graphs separately is an option, it does not take much to think through the logic of reciprocals and get the correct result without memorization.

Group Practice Problems #2 - The Secant and Cosecant Graphs: Recall the cofunction identity $\sin\left(\frac{\pi}{2}-\theta\right) = \cos(\theta)$. (Do you remember why this is true?) Use this to prove that $\csc\left(\frac{\pi}{2}-\theta\right) = \sec(\theta)$. Using this equation and the transformation of graphs, determine the graph of the secant function starting from the graph of the cosecant function. (Hint: You will want to rearrange the terms inside the argument of the cosecant function so that it fits the pattern we used in the previous section.)

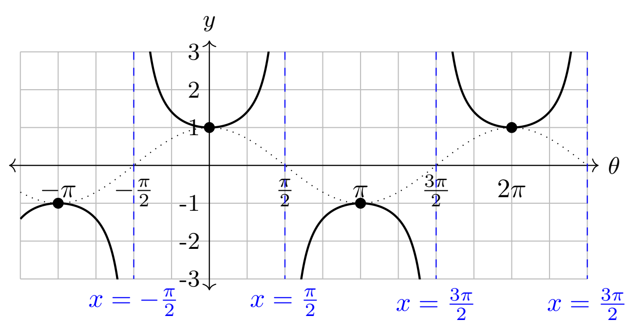
We will first prove the identity is true.

$$\csc\left(\frac{\pi}{2}-\theta\right) = \frac{1}{\sin\left(\frac{\pi}{2}-\theta\right)} = \frac{1}{\cos(\theta)} = \sec(\theta)$$

For the graph of $y = \csc(\frac{\pi}{2}-\theta) = \csc(-\theta+\frac{\pi}{2})$, we will first graph $y = \csc(\theta+\frac{\pi}{2})$. To obtain that graph, we will start by graphing $y = \sin(\theta+\frac{\pi}{2})$ and then invert it relative to the midline. Notice that $y = \sin(\theta+\frac{\pi}{2})$ is the basic sine graph but shifted $\frac{\pi}{2}$ to the left.



We will now invert the graph so that zeros become vertical asymptotes. Also, points with y-coordinate ± 1 remain fixed in the inversion.



Lastly, replacing θ with $-\theta$ reflects the graph across the y-axis. In this case, it causes no change to the graph, and so this is the final result. Notice that this is identical to the cosecant graph.

Comments and Observations:

- The cofunction identity can be obtained by thinking about the opposite and adjacent labels for the two acute angles of a right triangle and noting that the sum of those two angles is $\frac{\pi}{2}$. See Section 5.5 for more details.
- Notice that the shifted sine graph is identical to the cosine graph, and so when we already know that when we invert it we will get the cosecant graph.

Group Practice Problems #3 - Graphing Trigonometric Functions: Graph at least two periods of the following functions. Be sure that the vertical asymptotes are clearly identified and labeled, and indicate the midlines.

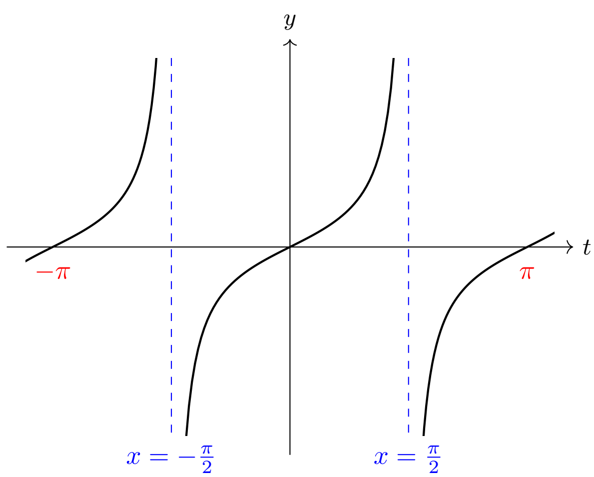
- $y = \tan(2t) + 1$

We can see that there is a vertical shift of 1 with no vertical stretch. We can also calculate the fundamental period:

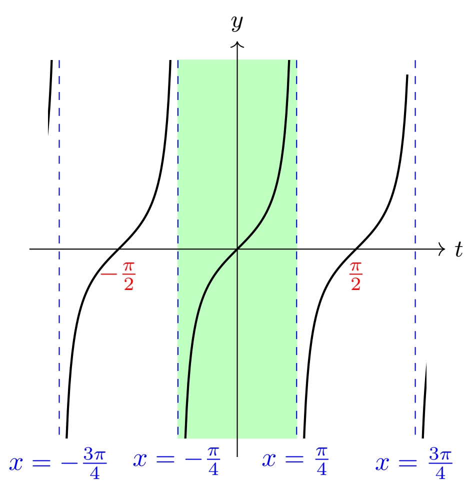
$$2t = -\frac{\pi}{2} \implies t = -\frac{\pi}{4}$$
$$2t = \frac{\pi}{2} \implies t = \frac{\pi}{4}$$

This shows us that the fundamental period is the interval $(-\frac{\pi}{4}, \frac{\pi}{4})$ and that the period has length $\frac{\pi}{2}$. (We can also calculate the period using $\frac{\pi}{B} = \frac{\pi}{2}$.)

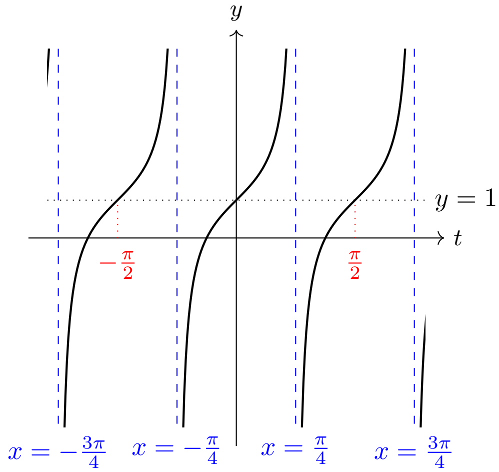
We will show the process of obtaining the end result as a sequence of transformations. We start with the basic tangent function:



Then we adjust the horizontal stretch to match the fundamental period.

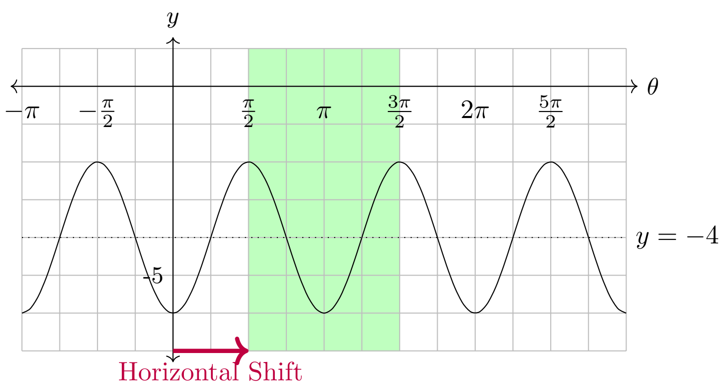


Then we apply the vertical shift.

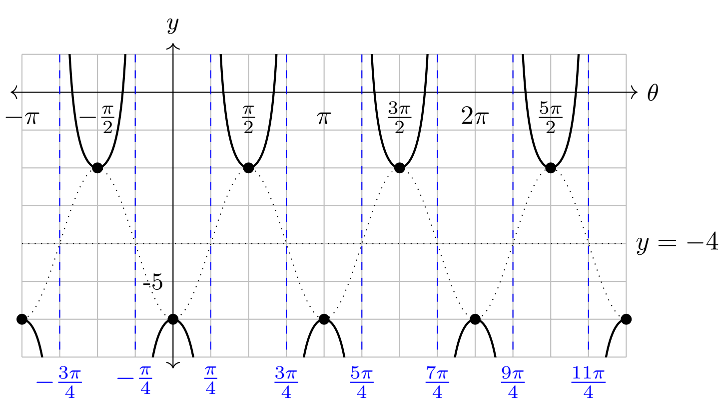


- $y = 2\sec\left(2\left(t - \frac{\pi}{2}\right)\right) - 4$

We will graph the underlying cosine function. Notice that the midline is at $y = -4$, the period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$, the amplitude is 2 and the horizontal shift is $\frac{\pi}{2}$.



From here, we convert the points at which the curve crosses the midline into vertical asymptotes and invert the shape while keeping the minimum and maximum values fixed.



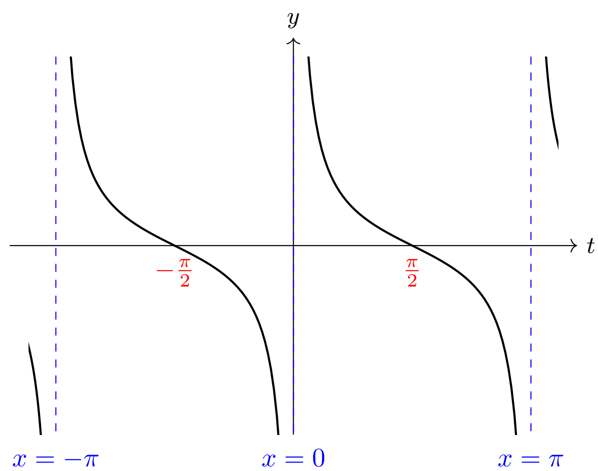
- $y = -3\cot\left(\pi\left(t+2\right)\right) + 2$

We can see that this is a cotangent graph with a midline of $y = 2$. There is vertical stretch of 3 and the graph is flipped. We will now calculate the fundamental period.

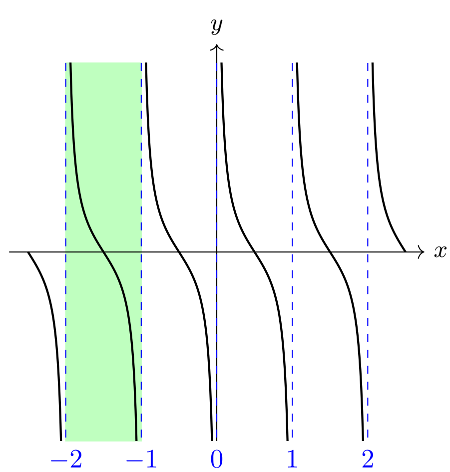
$$\pi(t+2) = 0 \implies t = -2$$
$$\pi(t+2) = \pi \implies t = -1$$

This shows us that the fundamental period is $[-2, -1]$ and that the period has length 1. (Alternatively, we can calculate the period using $\frac{\pi}{B} = \frac{\pi}{\pi} = 1$.)

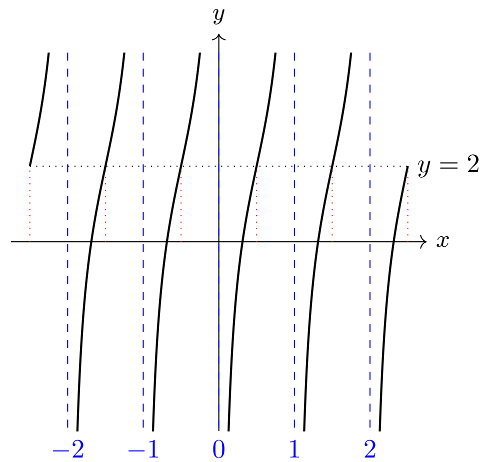
We will show the process of obtaining the end result as a sequence of transformations. We start with the basic cotangent function:



Then we adjust the horizontal stretch and shift to match the fundamental period.



Then we adjust the vertical stretch and shift the midline.



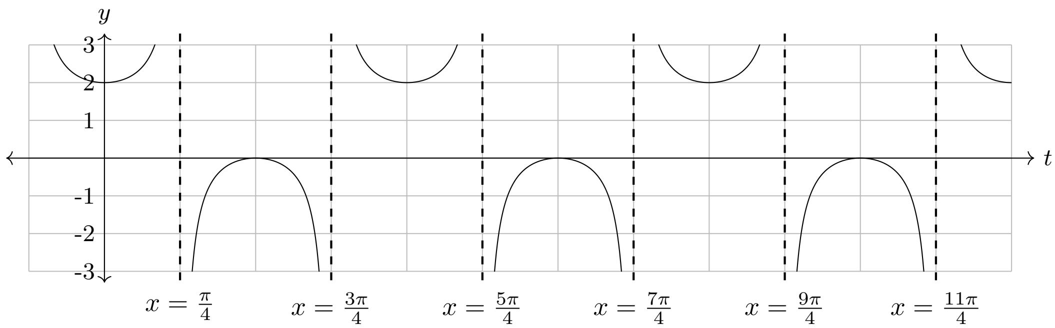
Comments and Observations:

- In the graphs of the tangent and cotangent function, it's possible to be more precise by noting specifically how the vertical stretch is made using the fact that the quarter-period values for the base function are ± 1. In fact, the key value approach can be used with the following charts (U stands for "Undefined" and refers to the locations of the vertical asymptotes):

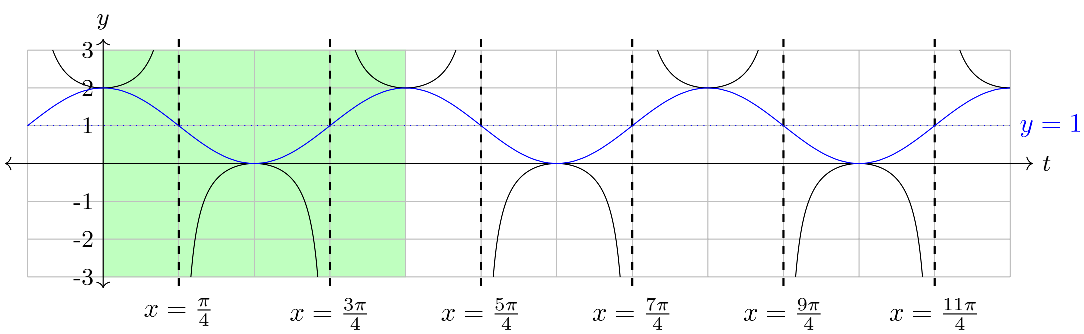
Key Value	1	2	3	4	5
x	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$\tan(x)$	U	-1	0	1	U

Key Value	1	2	3	4	5
x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$\tan(x)$	U	1	0	-1	U

Group Practice Problems #4 - Determining the Equations of Graphs: Determine two equations for each of the graphs. One of the equations should be a secant function and the other should be a cosecant function. (Hint: You may find it helpful to graph the underlying sinusoidal function.)

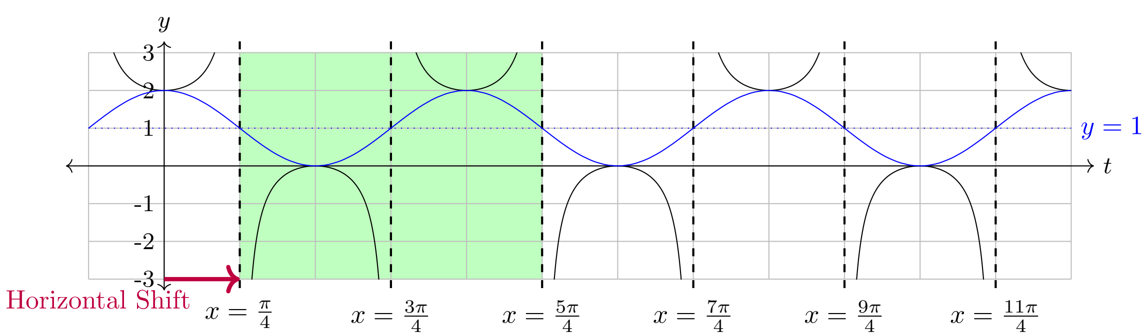


We begin by drawing the underlying sinusoidal function, indicating its midline and the period that will be used to get the equation of the function.

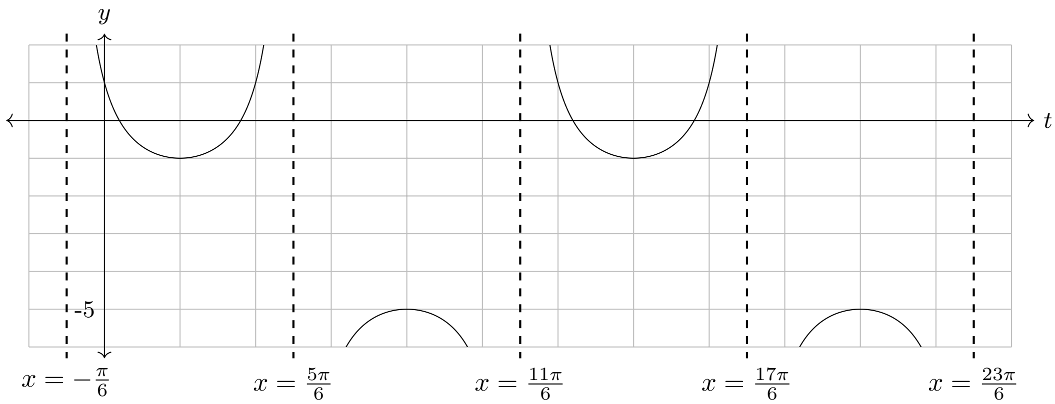


We can see that this graph is a positive cosine graph with no horizontal shift. The midline is at $y = 1$ and the amplitude is 1. Since the period is π, we know that $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi} = 2$. Combining these, we get that the equation of the cosine function is $y = \cos(2t) + 1$. Therefore, the graph of the original function is described by $y = \sec(2t) + 1$.

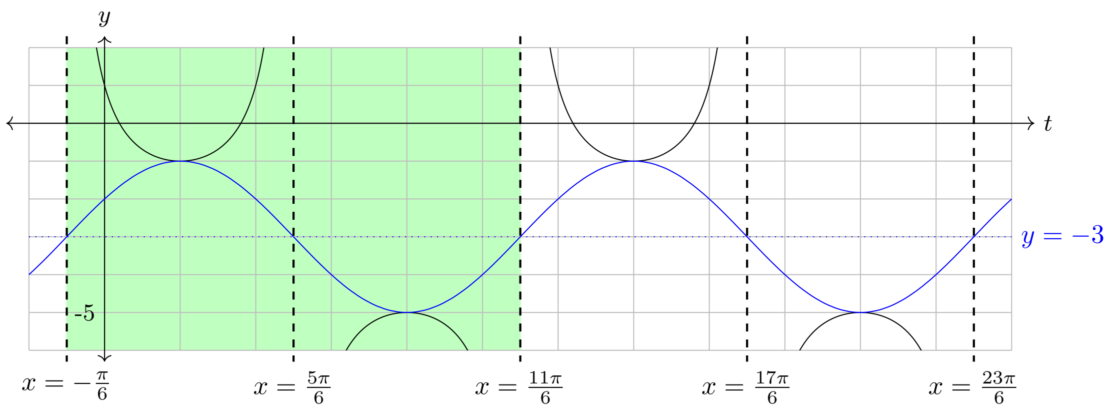
If we were to pick a different period, we could write this as a cosecant function.



This has a horizontal shift of $\frac{\pi}{4}$ to the right and this is a negative sine curve. The equation of this sinusoidal is $y = \sin(2(t - \frac{\pi}{4})) + 1$, so that the original graph is $y = -\csc(2(t - \frac{\pi}{4})) + 1$.

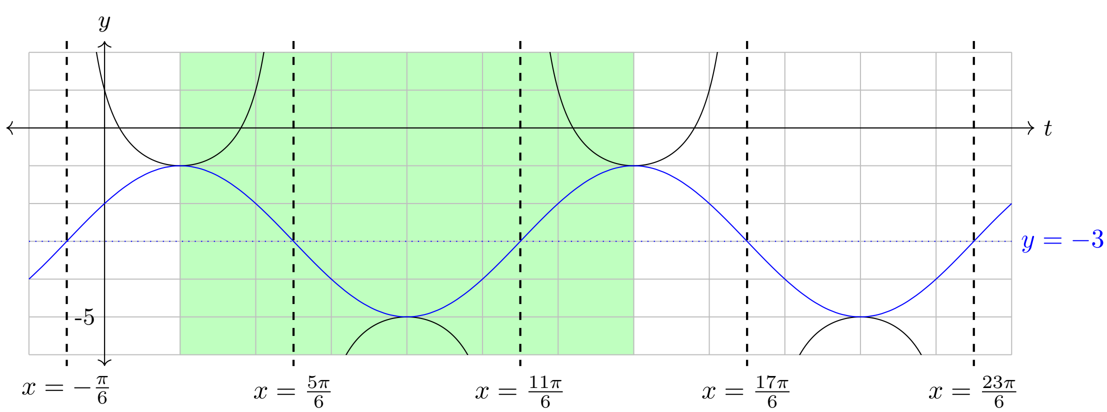


We begin by drawing the underlying sinusoidal function, indicating its midline and the period that will be used to get the equation of the function.



We can see that this graph is a positive sine graph with a horizontal shift of $\frac{\pi}{6}$ to the left. The midline is at $y = -3$ and the amplitude is 2. Since the period is 2π, we know that $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{2\pi} = 1$. Combining these, we get that the equation of the sine function is $y = 2\sin(t + \frac{\pi}{6}) - 3$. Therefore, the graph of the original function is $y = 2\csc(t + \frac{\pi}{6}) - 3$.

Here is an alternative choice:



This is a positive sine curve with a horizontal shift of $\frac{\pi}{3}$ to the right. The equation for this sinusoidal is $y = 2\cos(t - \frac{\pi}{3}) - 3$, so that the original graph can be described by $y = 2\sec(t - \frac{\pi}{3}) - 3$.

Comments and Observations:

- It is possible to identify these graphs without graphing the underlying sinusoidal function. You would still need to identify the same pieces of information (midline, amplitude, and period), so it's just a matter of preference.

Group Practice Problems #5 - Negative Angle Identities: Simplify the expressions.

- $\tan(-\theta)\cos(\theta)$

$$\begin{aligned}\tan(-\theta)\cos(\theta) &= \frac{\sin(-\theta)}{\cos(-\theta)}\cdot\cos(\theta)\\ &= \frac{-\sin(\theta)}{\cos(\theta)}\cdot\cos(\theta)\\ &= -\sin(\theta)\end{aligned}$$

- $\sqrt{1-\sin(-\theta)^2}$

$$\begin{aligned}\sqrt{1-\sin(-\theta)^2} &= \sqrt{1-(-\sin(\theta))^2}\\ &= \sqrt{1-\sin^2(\theta)}\\ &= \sqrt{\cos^2(\theta)}\\ &= |\cos(\theta)|\end{aligned}$$

Comments and Observations:

- This is more practice with the algebra of trigonometric functions.
- In the second problem, it is not uncommon to forget the absolute value signs. However, it's important to keep in mind that the square root function returns a non-negative value, and so the absolute value signs are essential.