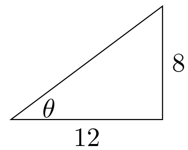
Group Practice Problems #1 - Determining Trigonometric Functions from Triangles: For each triangle, calculate all six trigonometric functions of the angle θ.

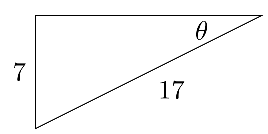


We can determine the length of the hypotenuse using the Pythagorean Theorem:

$$\begin{aligned} a^2 + b^2 &= c^2 && \text{Pythagorean Theorem} \\ 12^2 + 8^2 &= c^2 && \text{Substitute} \\ 144 + 64 &= c^2 \\ c &= \sqrt{208} = 4\sqrt{13} && \text{Take the square root of both sides} \\ & && \text{Keep the positive value because this is a distance} \end{aligned}$$

From this, we can calculate the trigonometric functions.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{4\sqrt{13}} = \frac{2}{\sqrt{13}} \qquad \csc(\theta) = \frac{1}{\sin(\theta)} = \frac{\sqrt{13}}{2}$$
$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{4\sqrt{13}} = \frac{3}{\sqrt{13}} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{\sqrt{13}}{3}$$
$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{8}{12} = \frac{2}{3} \qquad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{3}{2}$$

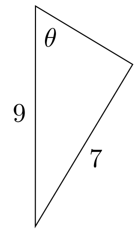


We can determine the length of the remaining side using the Pythagorean Theorem:

$$\begin{aligned} a^2 + b^2 &= c^2 && \text{Pythagorean Theorem} \\ a^2 + 7^2 &= 17^2 && \text{Substitute} \\ a^2 + 49 &= 289 \\ a^2 &= 240 && \text{Subtract 49 from both sides} \\ a &= \sqrt{240} = 4\sqrt{15} && \text{Take the square root of both sides} \\ & && \text{Keep the positive value because this is a distance} \end{aligned}$$

From this, we can calculate the trigonometric functions.

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{7}{17} \qquad \csc(\theta) = \frac{1}{\sin(\theta)} = \frac{17}{7}$$
$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4\sqrt{15}}{17} \qquad \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{17}{4\sqrt{15}}$$
$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{7}{4\sqrt{15}} \qquad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{4\sqrt{15}}{7}$$



We can determine the length of the remaining side using the Pythagorean Theorem:

$$
\begin{aligned}
a^2 + b^2 &= c^2 && \text{Pythagorean Theorem} \\
a^2 + 7^2 &= 9^2 && \text{Substitute} \\
a^2 + 49 &= 81 && \\
a^2 &= 32 && \text{Subtract 49 from both sides} \\
a &= \sqrt{32} = 4\sqrt{2} && \text{Take the square root of both sides} \\
&&& \text{Keep the positive value because this is a distance}
\end{aligned}
$$

From this, we can calculate the trigonometric functions.

$$
\begin{aligned}
\sin(\theta) &= \frac{\text{opposite}}{\text{hypotenuse}} = \frac{7}{9} & \csc(\theta) &= \frac{1}{\sin(\theta)} = \frac{9}{7} \\
\cos(\theta) &= \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4\sqrt{2}}{9} & \sec(\theta) &= \frac{1}{\cos(\theta)} = \frac{9}{4\sqrt{2}} \\
\tan(\theta) &= \frac{\text{opposite}}{\text{adjacent}} = \frac{7}{4\sqrt{2}} & \cot(\theta) &= \frac{1}{\tan(\theta)} = \frac{4\sqrt{2}}{7}
\end{aligned}
$$

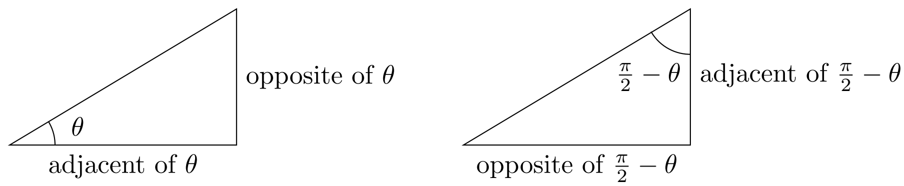
Comments and Observations:

- As with previous sections, it is not necessary to rationalize the denominator.
- These problems are just about practicing using the formulas and connecting the ideas to the triangles. This familiarity will be helpful when setting up word problems.

Group Practice Problems #2 - Cofunction Identities:

- Write a short paragraph that explains why the cofunction identities are true. You may find it helpful to include a diagram as part of your explanation.

There are two key facts that make the cofunction identities true. The first is that the non-right angles of a right triangle are complementary (meaning that they add up to $\frac{\pi}{2}$ radians). This means that if one angle has measure θ, then the other has measure $\frac{\pi}{2} - \theta$. The second is that the opposite and adjacent sides swap when taking the perspective of the other angle.



Observe that the adjacent side of the angle θ is the same as the opposite side of the angle $\frac{\pi}{2}$ and that the opposite side of the angle θ is the adjacent side of the angle $\frac{\pi}{2} - \theta$. This allows us to set up the following equalities:

$$\sin(\theta) = \frac{\text{opposite of } \theta}{\text{hypotenuse}} = \frac{\text{adjacent of } \frac{\pi}{2} - \theta}{\text{hypotenuse}} = \cos\left(\frac{\pi}{2} - \theta\right)$$
$$\cos(\theta) = \frac{\text{adjacent of } \theta}{\text{hypotenuse}} = \frac{\text{opposite of } \frac{\pi}{2} - \theta}{\text{hypotenuse}} = \sin\left(\frac{\pi}{2} - \theta\right)$$

- Using the logic of your explanation, find two more pairs of cofunction identities.

Following the logic from above, we can work out two more pairs of cofunction identities by working with the remaining trigonometric functions and using the same labels.

$$\csc(\theta) = \frac{\text{hypotenuse}}{\text{opposite of } \theta} = \frac{\text{hypotenuse}}{\text{adjacent of } \frac{\pi}{2} - \theta} = \sec\left(\frac{\pi}{2} - \theta\right)$$
$$\sec(\theta) = \frac{\text{hypotenuse}}{\text{adjacent of } \theta} = \frac{\text{hypotenuse}}{\text{opposite of } \frac{\pi}{2} - \theta} = \csc\left(\frac{\pi}{2} - \theta\right)$$
$$\tan(\theta) = \frac{\text{opposite of } \theta}{\text{adjacent of } \theta} = \frac{\text{adjacent of } \frac{\pi}{2} - \theta}{\text{opposite of } \frac{\pi}{2} - \theta} = \cot\left(\frac{\pi}{2} - \theta\right)$$
$$\cot(\theta) = \frac{\text{adjacent of } \theta}{\text{opposite of } \theta} = \frac{\text{opposite of } \frac{\pi}{2} - \theta}{\text{adjacent of } \frac{\pi}{2} - \theta} = \tan\left(\frac{\pi}{2} - \theta\right)$$

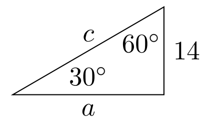
Comments and Observations:

- The additional forms of the cofunction identity can also be derived directly from the original cofunction identities.

$$\begin{aligned}
\csc(\theta) &= \frac{1}{\sin(\theta)} = \frac{1}{\cos\left(\frac{\pi}{2}-\theta\right)} = \sec\left(\frac{\pi}{2}-\theta\right) \\
\sec(\theta) &= \frac{1}{\cos(\theta)} = \frac{1}{\sin\left(\frac{\pi}{2}-\theta\right)} = \csc\left(\frac{\pi}{2}-\theta\right) \\
\tan(\theta) &= \frac{\sin(\theta)}{\cos(\theta)} = \frac{\cos\left(\frac{\pi}{2}-\theta\right)}{\sin\left(\frac{\pi}{2}-\theta\right)} = \cot\left(\frac{\pi}{2}-\theta\right) \\
\cot(\theta) &= \frac{\cos(\theta)}{\sin(\theta)} = \frac{\sin\left(\frac{\pi}{2}-\theta\right)}{\cos\left(\frac{\pi}{2}-\theta\right)} = \tan\left(\frac{\pi}{2}-\theta\right)
\end{aligned}$$

Group Practice Problems #3 - Solving Triangles: Draw the triangle that matches the given description. Then solve the triangle.

- A 30-60-90 triangle where the short leg has length 14.



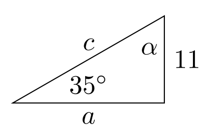
Since a is adjacent to the 30° angle and 14 is opposite of it, we will use the tangent function to establish the relationship.

$$\begin{aligned}
\tan(\theta) &= \frac{\text{opposite}}{\text{adjacent}} && \text{Definition} \\
\tan(30^\circ) &= \frac{14}{a} && \text{Substitute} \\
\frac{1}{\sqrt{3}} &= \frac{14}{a} && \text{Evaluate the trigonometric function} \\
1 \cdot a &= \sqrt{3} \cdot 14 && \text{Cross-multiply to solve the proportion} \\
a &= 14\sqrt{3}
\end{aligned}$$

To determine c, we will use the 60° angle and its adjacent side.

$$\begin{aligned}
\cos(\theta) &= \frac{\text{adjacent}}{\text{hypotenuse}} && \text{Definition} \\
\cos(60^\circ) &= \frac{14}{c} && \text{Substitute} \\
\frac{1}{2} &= \frac{14}{c} && \text{Evaluate the trigonometric function} \\
1 \cdot c &= 2 \cdot 14 && \text{Cross-multiply to solve the proportion} \\
c &= 28
\end{aligned}$$

- A right triangle with an acute angle of 35° whose opposite side has length 11.



Since the sum of the acute angles of a right triangle is 90°, we get that $\alpha = 90^\circ - 35^\circ = 55^\circ$.

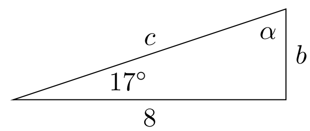
Since a is adjacent to the 35° angle and 11 is opposite of it, we will use the tangent function to establish the relationship.

$$\begin{aligned}
\tan(\theta) &= \frac{\text{opposite}}{\text{adjacent}} && \text{Definition} \\
\tan(35^\circ) &= \frac{11}{a} && \text{Substitute} \\
0.700 &\approx \frac{11}{a} && \text{Evaluate the trigonometric function} \\
0.7 \cdot a &= 1 \cdot 11 && \text{Cross-multiply to solve the proportion} \\
a &\approx \frac{11}{0.7} \approx 15.71 && \text{Divide both sides by 0.7}
\end{aligned}$$

To determine c, we will use the $35°$ angle and its opposite side.

$$\begin{aligned}
\sin(\theta) &= \frac{\text{opposite}}{\text{hypotenuse}} && \text{Definition} \\
\sin(35°) &= \frac{11}{c} && \text{Substitute} \\
0.574 &\approx \frac{11}{c} && \text{Evaluate the trigonometric function} \\
0.574 \cdot c &\approx 1 \cdot 11 && \text{Cross-multiply to solve the proportion} \\
c &\approx \frac{11}{0.574} \approx 19.16 && \text{Divide both sides by 0.0.574}
\end{aligned}$$

- A right triangle with an acute angle of $17°$ whose adjacent side has length 8.



Since the sum of the acute angles of a right triangle is $17°$, we get that $\alpha = 90° - 17° = 73°$.

Since b is opposite of the $17°$ angle and 8 is adjacent to it, we will use the tangent function to establish the relationship.

$$\begin{aligned}
\tan(\theta) &= \frac{\text{opposite}}{\text{adjacent}} && \text{Definition} \\
\tan(17°) &= \frac{b}{8} && \text{Substitute} \\
0.306 &\approx \frac{b}{8} && \text{Evaluate the trigonometric function} \\
2.45 &\approx b && \text{Multiply both sides by 8}
\end{aligned}$$

To determine c, we will use the $17°$ angle and its adjacent side.

$$\begin{aligned}
\cos(\theta) &= \frac{\text{adjacent}}{\text{hypotenuse}} && \text{Definition} \\
\cos(17°) &= \frac{8}{c} && \text{Substitute} \\
0.956 &\approx \frac{8}{c} && \text{Evaluate the trigonometric function} \\
0.956 \cdot c &\approx 1 \cdot 8 && \text{Cross-multiply to solve the proportion} \\
c &\approx \frac{8}{0.956} \approx 8.37 && \text{Divide both sides by 0.956}
\end{aligned}$$

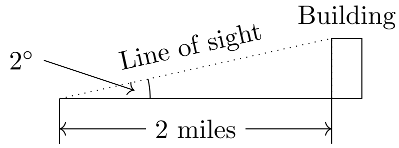
Comments and Observations:

- There are many paths to the answer as you can set up the trigonometric relationships relative to either angle. For example, the third side can be determined using the Pythagorean Theorem instead of a trigonometric relationship. However, all paths will lead to the same final results.
- The choice of labels does not matter as long as you are internally consistent with your usage.

- It is possible for some rounding error to enter into the calculations. If you're off about 0.01 or so, it's probably just a rounding error. If you're off by more than that, there's likely something else going on. You can double check by keeping more decimal places in the earlier parts of your calculations to see if the error decreases or remains the same.
- To avoid compounding errors, it's generally best to reference the values given in the problem as often as possible rather than referencing your calculated values.

Group Practice Problems #4 - Word Problems

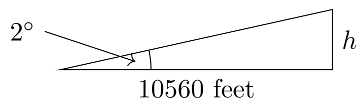
- The angle of elevation to the top of a building is found to be 2 degrees from the ground at a distance of 2 miles from the base of the building. Using this information, find the height of the building in feet.



We start by converting the distances to feet.

$$2 \text{ miles} = 2 \text{ miles} \cdot \frac{5280 \text{ feet}}{1 \text{ mile}} = 10560 \text{ feet}$$

We will now simplify the diagram to the core components.

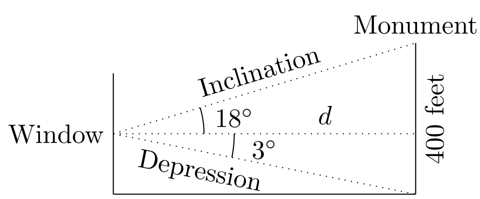


We can use the tangent function to relate these parts of the diagram together.

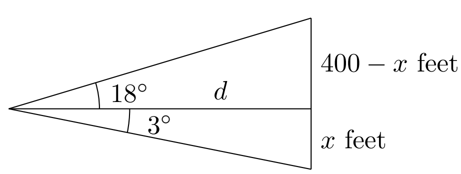
$$\begin{aligned}
\tan(\theta) &= \frac{\text{opposite}}{\text{adjacent}} && \text{Definition} \\
\tan(2^\circ) &= \frac{h}{10560} && \text{Substitute} \\
0.03492 &\approx \frac{h}{10560} && \text{Evaluate the trigonometric function} \\
0.03492 \cdot 10560 &\approx h && \text{Multiply both sides by 10560} \\
h &\approx 368.76
\end{aligned}$$

The building is approximately 369 feet tall.

- A 400-foot tall monument is located in the distance. From a window in a building, a person determines that the angle of elevation to the top of the monument is 18° and the angle of depression to the bottom of it is 3°. How far is the person from the monument?



We will first simplify the diagram to the core components. We will also add a distance x that represents the height of the window.



From the bottom triangle, we can use the tangent function to get the following relationship:

$$\begin{aligned} \tan(\theta) &= \frac{\text{opposite}}{\text{adjacent}} && \text{Definition} \\ \tan(3^\circ) &= \frac{x}{d} && \text{Substitute} \\ 0.052 &\approx \frac{x}{d} && \text{Evaluate the trigonometric function} \\ 0.052d &\approx x && \text{Multiply both sides by } d \end{aligned}$$

And from the top triangle, we have the following:

$$\begin{aligned} \tan(\theta) &= \frac{\text{opposite}}{\text{adjacent}} && \text{Definition} \\ \tan(18^\circ) &= \frac{400-x}{d} && \text{Substitute} \\ 0.325 &\approx \frac{400-x}{d} && \text{Evaluate the trigonometric function} \\ 0.325d &\approx 400-x && \text{Multiply both sides by } d \end{aligned}$$

We will plug the first equation into the second and solve for d.

$$\begin{aligned} 0.325d &\approx 400 - x && \\ 0.325d &\approx 400 - 0.052d && \text{Substitute} \\ 0.377d &\approx 400 && \text{Add } 0.052d \text{ to both sides} \\ d &\approx 1061.01 && \text{Divide both sides by } 0.377 \end{aligned}$$

Therefore, the person is approximately 1061 feet from the monument.

Comments and Observations:

- Drawing an accurate diagram is key to solving these problems.
- Be sure to answer the question! Lots of students make the mistake of getting to the end of a part of the calculation and then just stopping. It is best with a word problem to answer the question with a sentence that accurately communicates that you've reached the correct conclusion.