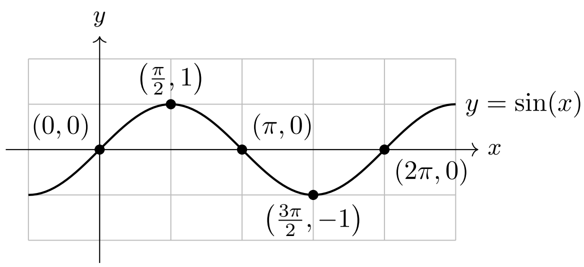
Group Practice Problems #1 - The Sine and Cosine Graphs: Sketch the graphs of $y = \sin(t)$ and $y = \cos(t)$. State the domain and range of these functions and locate all of the key values for both functions. Determine the coordinates of the next two minimum values, next two maximum values, and next four zeros to the left and to the right of the fundamental period.

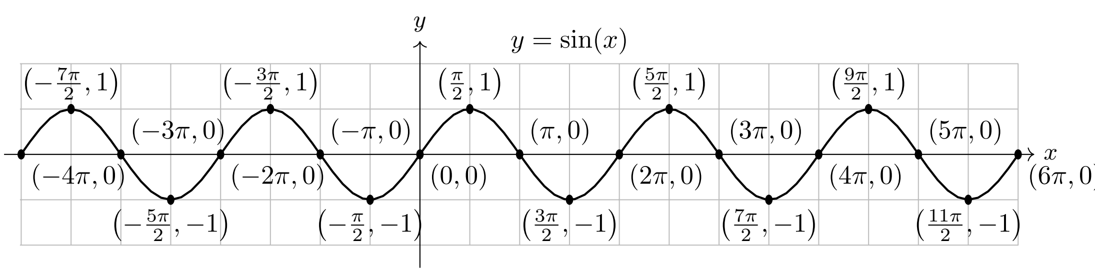
Properties of $y = \sin(x)$:



- Domain: All real numbers
- Range: $[-1, 1]$
- Key Values:

Key Value	1	2	3	4	5
x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin(x)$	0	1	0	-1	0

The extended graph of $y = \sin(x)$:



- Minimum values: $x = \ldots, -\frac{5\pi}{2}, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots = \frac{3\pi}{2} + 2\pi k$ for any integer k
- Maximum values: $x = \ldots, -\frac{7\pi}{2}, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots = \frac{\pi}{2} + 2\pi k$ for any integer k
- Zeros: $x = \ldots, -4\pi, -3\pi, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, \ldots = \pi k$ for any integer k

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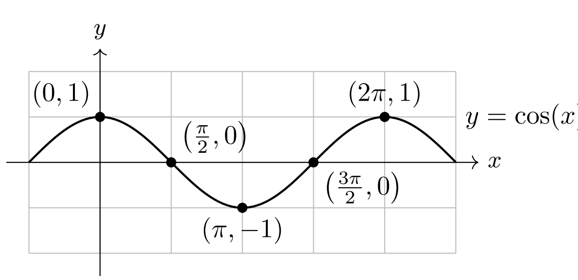
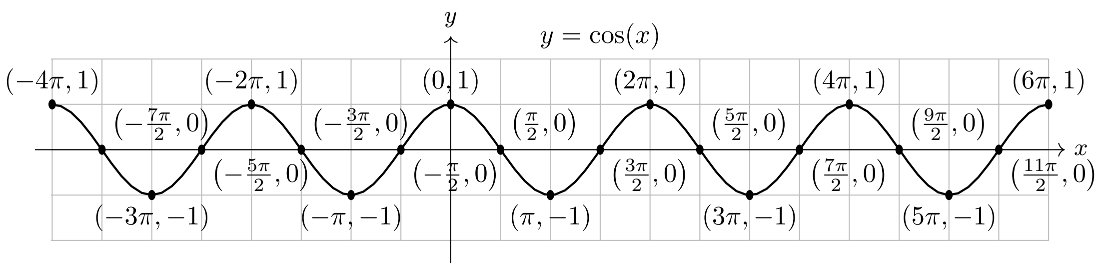
Properties of $y = \cos(x)$:



- Domain: All real numbers
- Range: $[-1, 1]$
- Key Values:

Key Value	1	2	3	4	5
x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos(x)$	1	0	-1	0	1

The extended graph of $y = \cos(x)$:



- Minimum values: $x = \ldots, -3\pi, -\pi, \pi, 3\pi, 5\pi, \ldots = \pi + 2\pi k$ for any integer k
- Maximum values: $x = \ldots, -4\pi, -2\pi, 0, 2\pi, 4\pi, 6\pi, \ldots = 2\pi k$ for any integer k
- Zeros: $x = \ldots, -\frac{7\pi}{2}, -\frac{5\pi}{2}, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}, \ldots = \frac{\pi}{2} + \pi k$ for any integer k

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Comments and Observations:

- The reason for finding values beyond the interval $[0, 2\pi)$ is to emphasize the periodicity of the sine and cosine functions as well as provide an opportunity to get more comfortable with working in intervals of multiples of π.
- Notice that the maximum and minimum values occur every 2π but the zeros occur every π.

Group Practice Problems #2 - Negative Angle Identities: Starting from the negative angle identities for the sine and cosine functions, derive the negative angle identities for the tangent, cotangent, secant, and cosecant functions. Determine which of the six trigonometric functions are even and which of them are odd.

Negative angle identity for tangent:

$$\tan(-\theta) = \frac{\sin(-\theta)}{\cos(-\theta)} = \frac{-\sin(\theta)}{\cos(\theta)} = -\tan(\theta)$$

Negative angle identity for cotangent:

$$\cot(-\theta) = \frac{\cos(-\theta)}{\sin(-\theta)} = \frac{\cos(\theta)}{-\sin(\theta)} = -\cot(\theta)$$

Negative angle identity for secant:

$$\sec(-\theta) = \frac{1}{\cos(-\theta)} = \frac{1}{\cos(\theta)} = \sec(\theta)$$

Negative angle identity for cosecant:

$$\csc(-\theta) = \frac{1}{\sin(-\theta)} = \frac{1}{-\sin(\theta)} = -\csc(\theta)$$

The sine, tangent, cosecant, and cotangent functions are odd, and the cosine and secant functions are even.

Comments and Observations:

- These problems are just algebra practice and practice with the basic trigonometric relationships.

Group Practice Problems #3 - Graphing Sinusoidal Functions: State the amplitude, midline, period, and horizontal shift of the following functions. Then sketch the graphs of the following functions, including the midline. Your graph should include both the fundamental period and at least half of the period to the left and to the right of the fundamental period. Identify the coordinates of the key values.

- $y = -2\sin\left(2\left(t - \frac{\pi}{3}\right)\right) + 1$

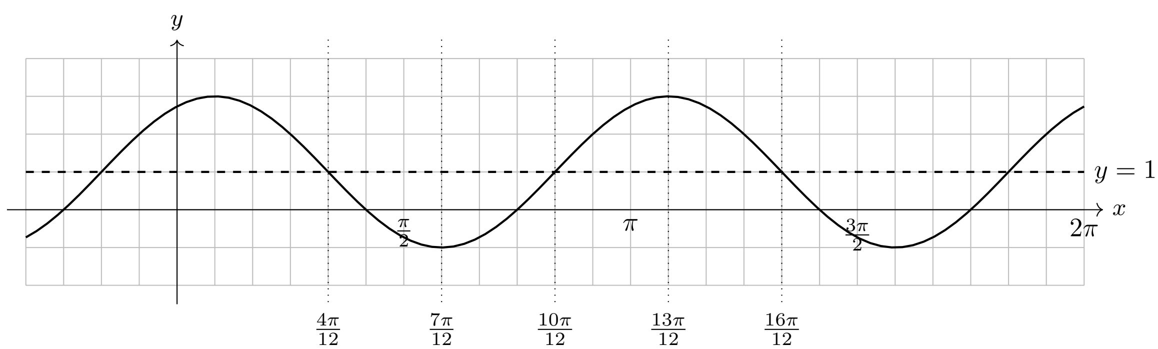
We will first analyze the function's properties:

- Amplitude: Since $A = -2$, the amplitude is $|A| = 2$.
- Midline: Since $D = 1$, the midline is $y = D = 1$.
- Period: Since $B = 2$, the period is $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$.
- Horizontal Shift: Since $C = \frac{\pi}{3}$, the horizontal shift is $\frac{\pi}{3}$ to the right.

The x-coordinates of the key values start from the horizontal shift and occur every quarter-period. Since the period is π, a quarter-period is $\frac{\pi}{4}$. We will write these values with a denominator of 12 for clarity. To get the y-coordinates, we first note that the shape of the function is a negative sine, meaning that it starts at the middle and goes down. Note that the midline is at 1 and the amplitude is 2.

Key Value	1	2	3	4	5
t	$\frac{4\pi}{12}$	$\frac{7\pi}{12}$	$\frac{10\pi}{12}$	$\frac{13\pi}{12}$	$\frac{16\pi}{12}$
$-2\sin(2(t - \frac{\pi}{3}))$	1	-1	1	3	1

We will now plot the graph.



- $y = 3\cos\left(\pi\left(t + 2\right)\right) - 4$

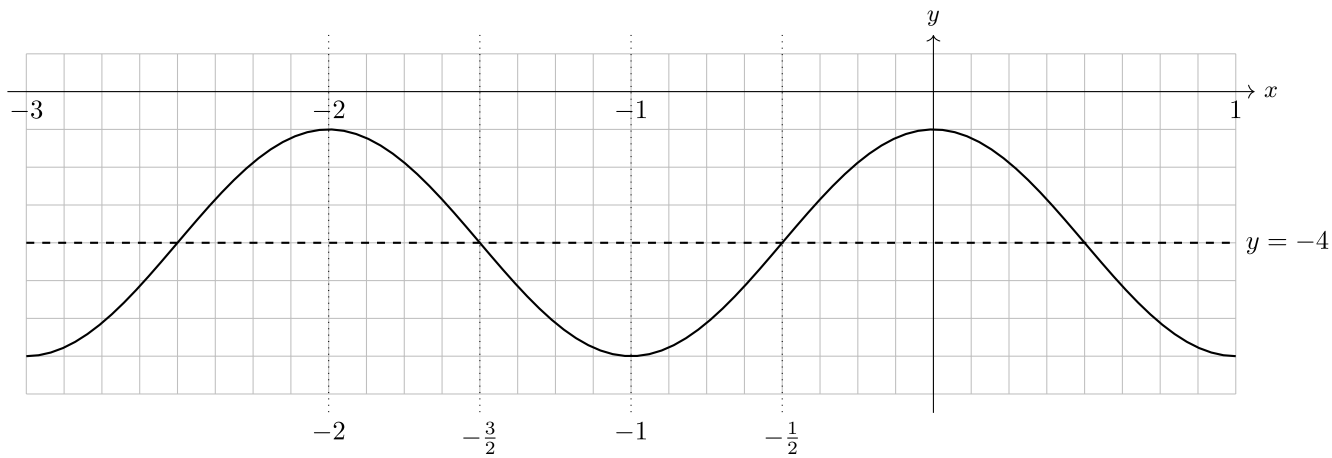
We will first analyze the function's properties:

- Amplitude: Since $A = 3$, the amplitude is $|A| = 3$.
- Midline: Since $D = -4$, the midline is $y = D = -4$.
- Period: Since $B = \pi$, the period is $\frac{2\pi}{B} = \frac{2\pi}{\pi} = 2$.
- Horizontal Shift: Since $C = -2$, the horizontal shift is 2 to the left.

The x-coordinates of the key values start from the horizontal shift and occur every quarter-period. Since the period is 2, a quarter-period is $\frac{1}{2}$. To get the y-coordinates, we first note that the shape of the function is a positive cosine, meaning that it starts at the top and goes down. Note that the midline is at -4 and the amplitude is 3.

Key Value	1	2	3	4	5
t	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0
$3\cos(\pi(t+2))-4$	-1	-4	-7	-4	-1

We will now plot the graph.



- $y = 5\sin\left(\frac{1}{2}\left(t+\frac{3\pi}{4}\right)\right)+2$

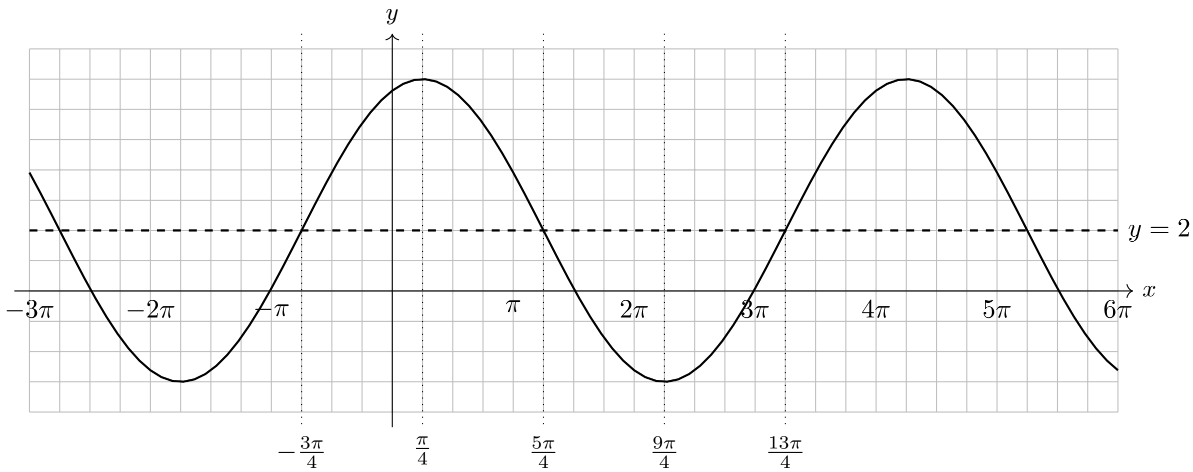
We will first analyze the function's properties:

- Amplitude: Since $A = 5$, the amplitude is $|A| = 5$.
- Midline: Since $D = 2$, the midline is $y = D = 2$.
- Period: Since $B = \frac{1}{2}$, the period is $\frac{2\pi}{B} = \frac{2\pi}{1/2} = 4\pi$.
- Horizontal Shift: Since $C = -\frac{3\pi}{4}$, the horizontal shift is $\frac{3\pi}{4}$ to the left.

The x-coordinates of the key values start from the horizontal shift and occur every quarter-period. Since the period is 4π, a quarter-period is π. To get the y-coordinates, we first note that the shape of the function is a positive sine, meaning that it starts at the middle and goes up. Note that the midline is at 2 and the amplitude is 5.

Key Value	1	2	3	4	5
t	$-\frac{3\pi}{4}$	$\frac{\pi}{4}$	$\frac{5\pi}{4}$	$\frac{9\pi}{4}$	$\frac{13\pi}{4}$
$5\sin(\frac{1}{2}(t+\frac{3\pi}{4}))+2$	2	7	2	-3	2

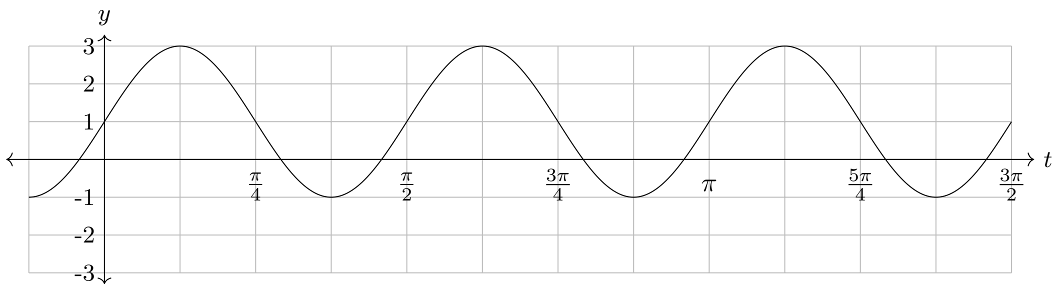
We will now plot the graph.



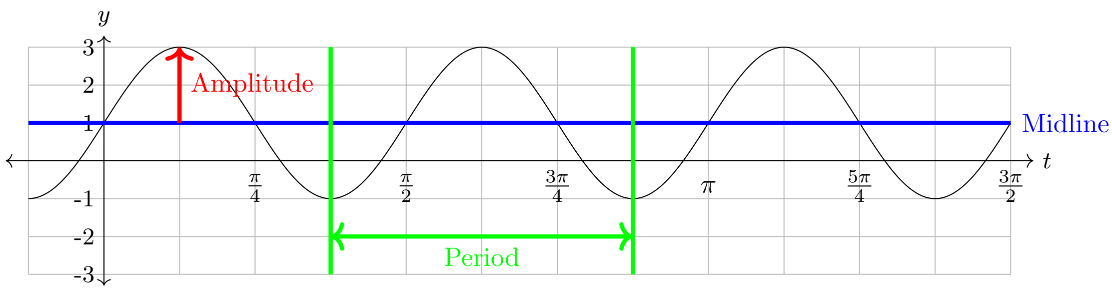
Comments and Observations:

- It is important to be methodical as there are many steps to these problems. Drawing the start and end of the period with the quarter periods marked out is a good way to stay organized when graphing.
- It is helpful to pick the grid spacing to work with the values you get in your problem.

Group Practice Problems #4 - Determining the Equations of Graphs: Determine two equations for each of the graphs. One of the equations should be a sine function and the other should be a cosine function.

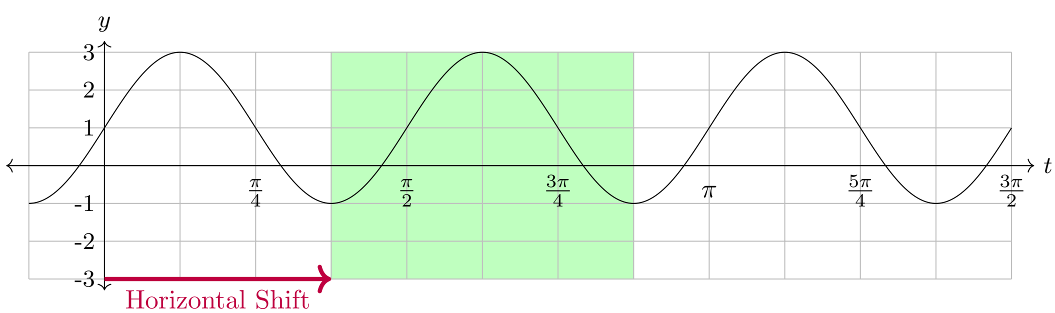


We will first identify the midline, amplitude, and period of the function from the graph:



This shows us that the midline is $y = 1$, the amplitude is 2, and the period is $\frac{\pi}{2}$. Notice that since the period is $\frac{\pi}{2}$, we have $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{\pi/2} = 4$.

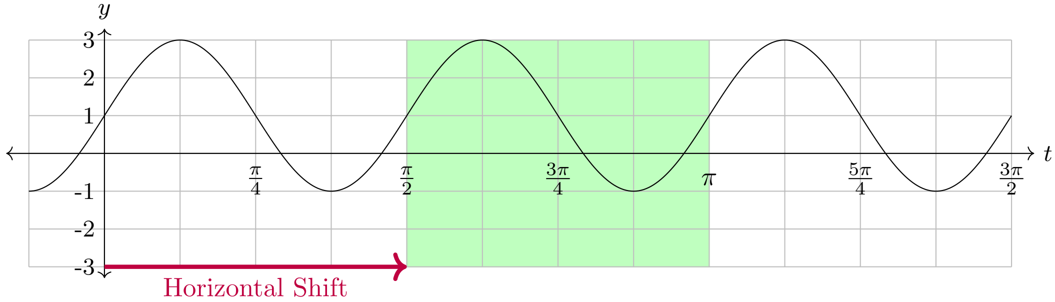
We now need to determine an equation of this graph using a cosine function. We will choose to look for a negative cosine function, which means that we will look for a period that starts and ends at a low point. We will then use this to determine the horizontal shift.



This shows us that for this equation, the horizontal shift will be $\frac{3\pi}{8}$. We can now write down an equation for the graph:

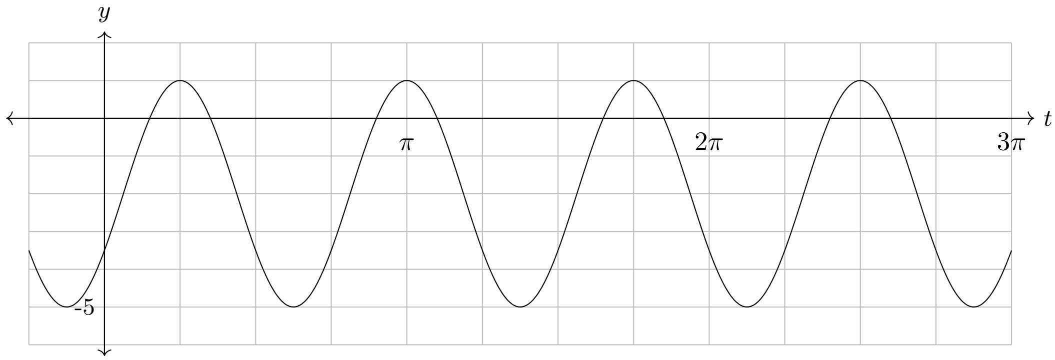
$$y = -2\cos\left(4\left(t - \frac{3\pi}{8}\right)\right) + 1$$

If we were to use a positive sine function, we would need to look for a period that starts at the midline and moves up.

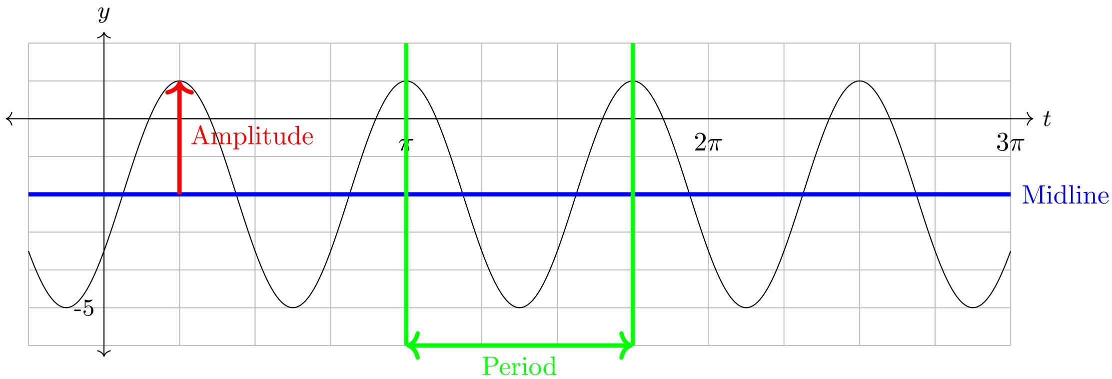


This shows us that for this equation, the horizontal shift will be $\frac{\pi}{2}$. We can now write down an equation for the graph:

$$y = 2\sin\left(4\left(t - \frac{\pi}{2}\right)\right) + 1$$

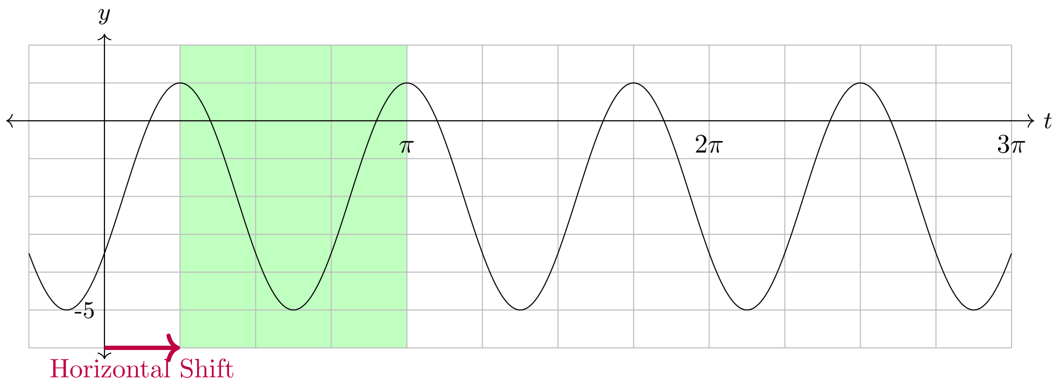


We will first identify the midline, amplitude, and period of the function from the graph:



This shows us that the midline is $y = -2$, the amplitude is 3, and the period is $\frac{3\pi}{4}$. Notice that since the period is $\frac{3\pi}{4}$, we have $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{3\pi/4} = \frac{8}{3}$.

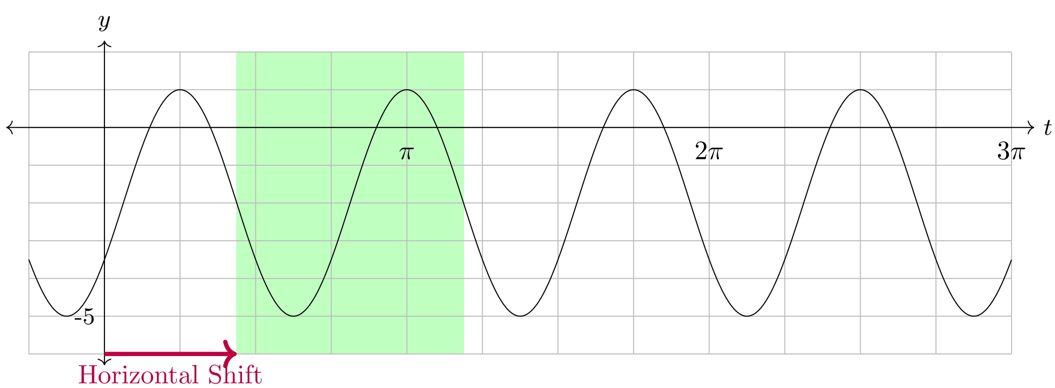
We now need to determine an equation of this graph using a cosine function. We will choose to look for a positive cosine function, which means that we will look for a period that starts and ends at a high point. We will then use this to determine the horizontal shift.



This shows us that for this equation, the horizontal shift will be $\frac{\pi}{4}$. We can now write down an equation for the graph:

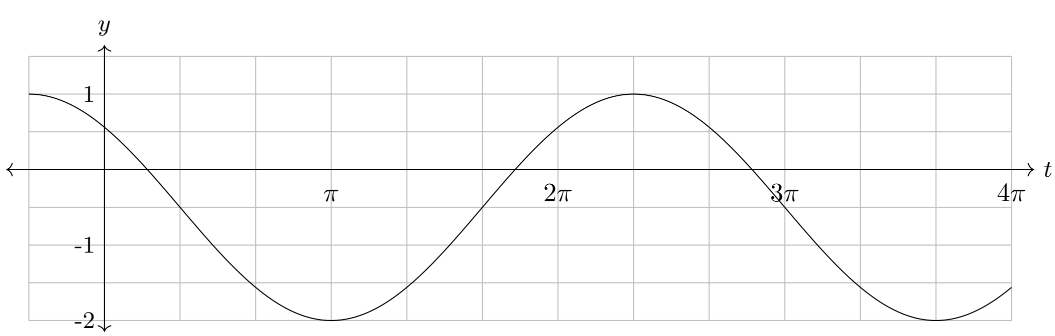
$$y = 3\cos\left(\frac{8}{3}\left(t - \frac{\pi}{4}\right)\right) - 2$$

If we were to use a negative sine function, we would need to look for a period that starts at the midline and moves down. In this case, the midline crossings does not happen at a grid marking, and so we need to use the fact that it is a quarter period away from a peak or valley. Since the period is $\frac{3\pi}{4}$, a quarter period is $\frac{3\pi}{16}$. And so the horizontal shift is $\frac{\pi}{4} + \frac{3\pi}{16} = \frac{7\pi}{16}$.

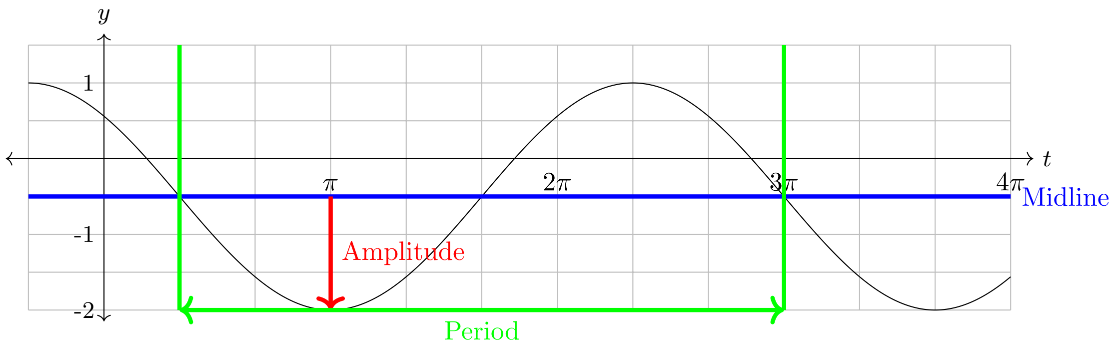


This shows us that for this equation, the horizontal shift will be $\frac{\pi}{2}$. We can now write down an equation for the graph:

$$y = -3\sin\left(\frac{8}{3}\left(t - \frac{7\pi}{16}\right)\right) - 2$$

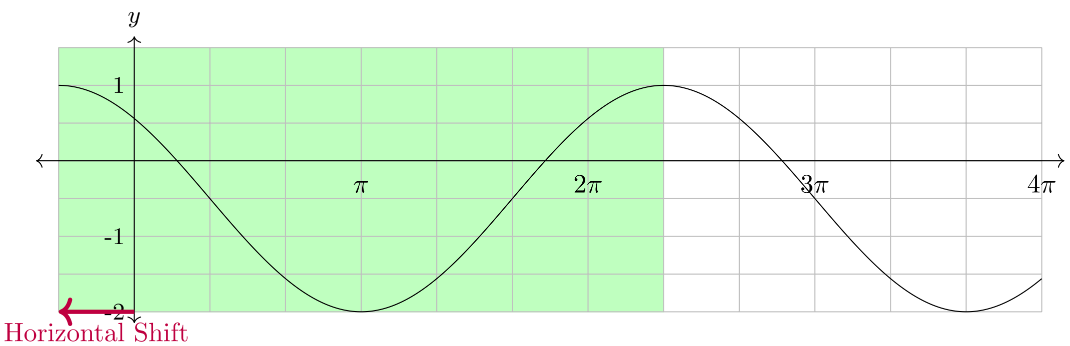


We will first identify the midline, amplitude, and period of the function from the graph:



This shows us that the midline is $y = -\frac{1}{2}$, the amplitude is $\frac{3}{2}$, and the period is $\frac{8\pi}{3}$. Notice that since the period is $\frac{8\pi}{3}$, we have $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{8\pi/3} = \frac{3}{4}$.

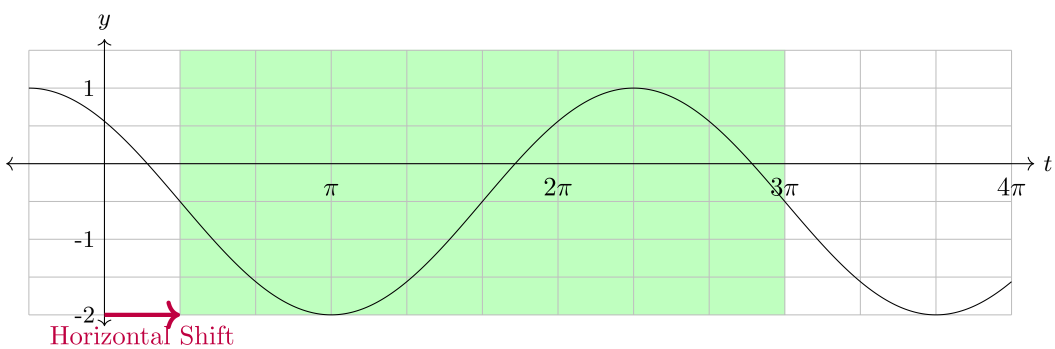
We now need to determine an equation of this graph using a cosine function. We will choose to look for a positive cosine function, which means that we will look for a period that starts and ends at a low point. We will then use this to determine the horizontal shift.



This shows us that for this equation, the horizontal shift will be $-\frac{\pi}{3}$. We can now write down an equation for the graph:

$$y = \frac{3}{2}\cos\left(\frac{3}{4}\left(t + \frac{\pi}{3}\right)\right) - \frac{1}{2}$$

If we were to use a negative sine function, we would need to look for a period that starts at the midline and moves down.



This shows us that for this equation, the horizontal shift will be $\frac{\pi}{2}$. We can now write down an equation for the graph:

$$y = \frac{3}{2}\sin\left(\frac{3}{4}\left(t - \frac{\pi}{3}\right)\right) - \frac{1}{2}$$

Comments and Observations:

- As much as possible, use points on the grid. But when you can't, guessing at the values is often an error. For example, in the second problem, students might estimate the that the graph crosses the midline is halfway or one third of the way across the spacing, and both estimates would be wrong. It is better to use logic and the known grid points than to just guess.