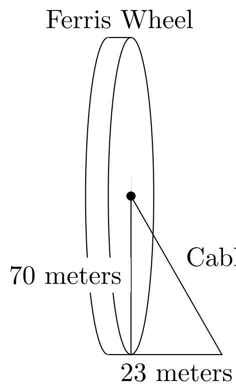
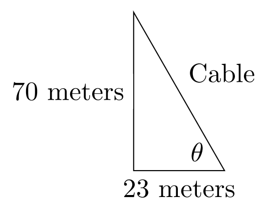
Group Practice Problems #1 - Geometric Word Problem: Sketch a diagram and solve the problem.

- A cable that anchors the center of the London Eye Ferris wheel to the ground must be replaced. The center of the Ferris wheel is 70 meters above the ground and the second anchor on the ground is 23 meters from the base of the wheel. What is the angle from the ground up to the center of the Ferris wheel and how long is the cable?

We begin by drawing a general diagram and then drawing a diagram that focuses on the relevant geometry.





Notice that the angle θ satisfies the relationship $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{70}{23}$, so that

$$\theta = \tan^{-1}\left(\frac{70}{23}\right) = 71.81°.$$

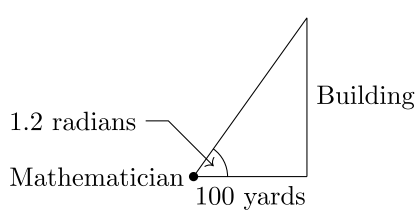
To determine the length of the cable, we will use the Pythagorean Theorem.

$$\begin{aligned}
a^2 + b^2 &= c^2 && \text{Pythagorean Theorem} \\
23^2 + 70^2 &= c^2 && \text{Substitute} \\
529 + 4900 &= c^2 \\
5429 &= c^2 \\
c &= \sqrt{5429} \approx 73.68
\end{aligned}$$

The cable is about 73.68 meters long and forms a 71.81°angle with the ground.

- A mathematician is standing 100 yards from the base of a building. The angle of elevation to the top of the building is 1.2 radians. (They're a mathematician. Of course they're using radians!) Determine the height of the building.

We will begin with a diagram.



We will let h represent the height of the building. To determine h, we will use the tangent function.

$$\begin{aligned} \tan(\theta) &= \frac{\text{opposite}}{\text{adjacent}} && \text{Definition of tangent} \\ \tan(1.2) &= \frac{h}{100} && \text{Substitute} \\ h &= 100\tan(1.2) \approx 257.22 && \text{Multiply both sides by 100} \end{aligned}$$

Therefore, the building is approximately 257.22 yards tall.

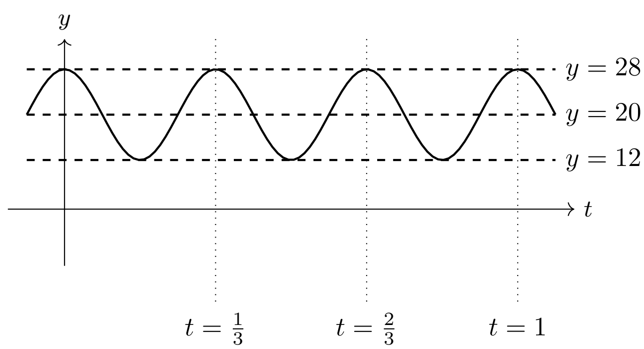
Comments and Observations:

- Typically, practical application problems are done in degrees. The only reason to make the person a mathematician was to justify giving the measurement in radians instead of degrees.

Group Practice Problems #2 - Algebraic Word Problem: Sketch the graph of the model and solve the problem.

- An object is connected to the wall with a spring that has a natural length of 20 cm. The object is pulled back 8 cm past the natural length and released. The object oscillates 3 times per second. Find an equation for the horizontal position of the object ignoring the effects of friction. How much time during each cycle is the object more than 27 cm from the wall?

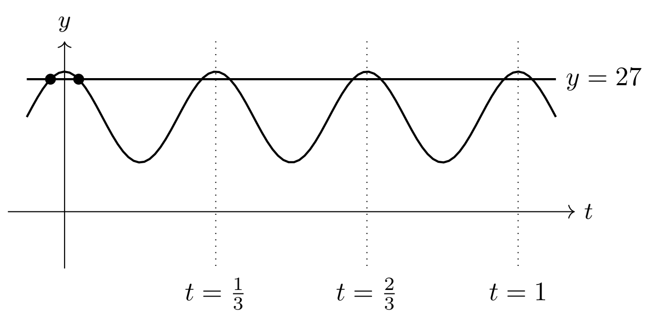
We will begin by sketching a graph that meets the requirements. Let y be the distance from the wall. Since the natural length of the spring is 20 cm, we know that is where we should draw the midline. The initial position is 8 cm beyond the midline, so that $y(0) = 28$. If the object oscillates 3 times per second, the period is $\frac{1}{3}$ of a second.



To get the equation, we can see that this is an unshifted positive cosine graph, with midline $y = 20$ and amplitude 8. We can also calculate $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{1/3} = 6\pi$. Therefore, our model is

$$y = 8\cos(6\pi t) + 20.$$

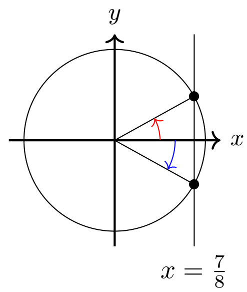
To determine the amount of time in each cycle spent more than 27 cm from the wall, we need to find the intersection of the curve with the line $y = 27$.



From the graph, we can see that the time between the first negative solution and the first positive solution encompasses the time spent beyond 27 cm from the wall. We now solve the equation algebraically:

$$\begin{aligned} 8\cos(6\pi t) + 20 &= 27 && \text{Set the model equation equal to 27} \\ 8\cos(6\pi t) &= 7 && \text{Subtract 20 from both sides} \\ \cos(6\pi t) &= \frac{7}{8} && \text{Divide both sides by 8} \end{aligned}$$

The equation has been reduced to a basic sinusoidal equation. We will draw the unit circle diagram to determine the quadrants of the solutions.



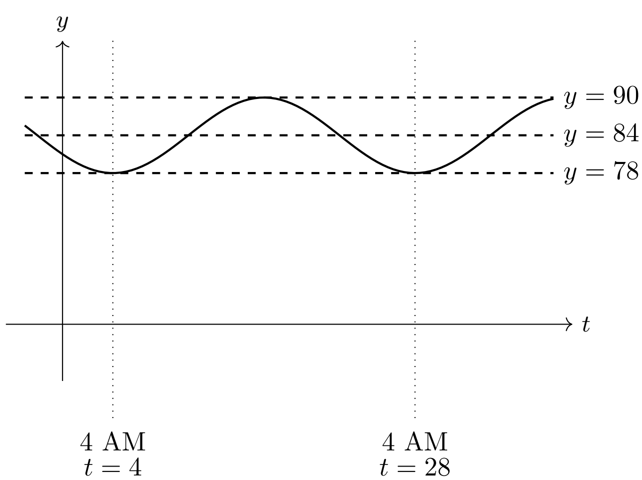
From this, we can see that the solutions will be in the first and fourth quadrants. We can also calculate that $\cos^{-1}(\frac{7}{8}) \approx 0.5053$. Therefore, we have

$$\begin{aligned} \cos(6\pi t) &= \frac{7}{8} && \text{From above} \\ 6\pi t &\approx \begin{cases} 0.5053 \\ -0.5053 \end{cases} \\ t &\approx \begin{cases} 0.027 \\ -0.027 \end{cases} \end{aligned}$$

Therefore, the object spends about 0.054 seconds per cycle further than 27 cm from the wall.

- Outside temperature over the course of a day can be modeled as a sinusoidal function. Suppose you know the high temperature for the day is 92 degrees and the low temperature of 78 degrees happens at 4 AM. Assuming t is the number of hours since midnight, find an equation for the temperature D in terms of t.

We will begin by sketching a graph that meets the requirements. Since minimum temperature is 78 and the maximum temperature is 90, we can see that the midline will be at 84 degrees with an amplitude of 6 degrees. We also need the low temperature to be at 4 AM, which corresponds to $t = 4$ (4 hours after midnight). The period is 24 hours.



The graph is a negative cosine curve with a horizontal shift of 4 hours to the right and an amplitude of 6 degrees. The midline is $y = 84$ and $B = \frac{2\pi}{\text{period}} = \frac{2\pi}{24} = \frac{\pi}{12}$. Therefore, the model is

$$y = -6\cos\left(\frac{\pi}{12}(t-4)\right) + 84.$$

Comments and Observations:

- Be careful to read the whole problem! It's easy to finish one part of the problem without rereading it to see if there's another part to do.
- Be sure to read the problem carefully! A lot of errors occur when students read a problem too quickly and make a mistake with the details. For example, a similarly-worded second problem might have given the high temperature at 4 PM, and many students still use $t = 4$ for that because the number 4 jumps out at them.