Butterworth requires design specifications:

Choose the following:

$$0.95 \leq |H(e^{j\omega})| \leq 1, \quad 0 \leq \omega \leq 0.45\pi$$

$$|H(e^{j\omega})| \leq 0.05, \quad 0.55\pi \leq \omega \leq \pi$$

Continuous time equivalents

$$0.95 \leq |H(e^{j\omega})| < 1, \quad 0 \leq \Omega \leq \frac{2}{T_d}\tan\left(\frac{0.45\pi}{2}\right)$$ let $T_d = 1$ to make design easy

$$|H(e^{j\omega})| \leq 0.05, \quad \frac{2}{T_d}\tan\left(\frac{0.55\pi}{2}\right) \leq \Omega \leq \infty$$

$$|H_c(j\Omega)|^2 = \frac{1}{1+(\Omega/\Omega_c)^{2N}}$$

$$(1 + (2\tan(0.225\pi)/\Omega_c)^{2N}) \leq 0.95^{-2}; \quad (1+(2\tan(0.275\pi)/\Omega_c)^{2N}) \geq 0.05^{-2}$$

$$2N[\log(2\tan(0.225\pi)) - \log(\Omega_c)] \leq \log(0.95^{-2}-1); \quad 2N[\log(2\tan(0.275\pi)) - \log\Omega_c] \geq \log(0.05^{-2}-1)$$

Switch to equality to use both equations → will meet specifications still.

$$2N[\log(2\tan(0.225\pi)) - \log(2\tan(0.275\pi))] = \log(0.95^{-2}-1) - \log(0.05^{-2}-1)$$

$$2N(-0.137002199) = -3.567415491$$

$$N = 13.01955\ldots \rightarrow \text{use } N = 14$$

$$\log\Omega_c = \left(\frac{1}{28}\log(0.05^{-2}-1) - \log(2\tan(0.275\pi))\right)\times -1 \qquad \Omega_c = 1.890772191$$

$$|H_c(j\Omega)|^2 = \frac{1}{1+(\Omega/1.890772191)^{28}}$$

use MATLAB to finish the design...

- Take 10 LHP roots of $1/(1+(\Omega/1.890772191)^{28})$ to get C.T. T.F.
- perform Bilinear transform on the T.F.