Finding Frequent Itemsets (Chapter 6)

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Thanks for source slides and material to:
J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets
http://www.mmds.org

Frequent Itemsets and Association Rules

- **Family of techniques for characterizing data: discovery of frequent itemsets**
  - **e.g., identify sets of items that are frequently purchased together**

Outline:

- Introduce market-basket model of data
- Define frequent itemsets
- Discover association rules
  - Confidence and interest of rules
- A-Priori Algorithm and variations

THE MARKET-BASKET MODEL

Association Rule Discovery

Supermarket shelf management – Market-basket model:

- **Goal:** Identify items that are bought together by **sufficiently many customers**
- **Approach:** Process the sales data to find dependencies among items
  - Brick and mortar stores: data collected with barcode scanners
  - Online retailers: transaction records for sales
- **A classic rule:**
  - If someone buys diaper and milk, then he/she is likely to buy beer
  - Don’t be surprised if you find six-packs next to diapers!

The Market-Basket Model

- A **large set** of **items**
  - e.g., things sold in a supermarket
- A **large set** of **baskets**
- Each basket is a **small subset of items**
  - e.g., the things one customer buys on one day
- **Want to discover Association Rules**
  - People who bought {x,y,z} tend to buy {v,w}
    - **Brick and mortar stores**: Influences setting of prices, what to put on sale when, product placement on store shelves
    - **Recommender systems**: Amazon, Netflix, etc.

Input:

TID	*Items*
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Output:

Rules Discovered:

{Milk} --> {Coke}

{Diaper, Milk} --> {Beer}

Market-Baskets

- Really a **general many-many mapping** (association) between two kinds of things: **items** and **baskets**
  - But we ask about **connections among "items,"** not "baskets."
- The technology focuses on common events, not rare events
  - Don't need to focus on identifying *all* association rules
  - Want to focus on **common events**, focus pricing strategies or product recommendations on those items or association rules

Market Basket Applications: Identify items bought together

- **Items** = products
- **Baskets** = sets of products someone bought in one trip to the store
- **Real market baskets:** Stores (Walmart, Target, Ralphs, etc.) keep terabytes of data about what items customers buy together
  - Tells how typical customers navigate stores
  - Lets them position tempting items
  - Suggests tie-in "tricks", e.g., run sale on diapers and raise the price of beer
  - **Need the rule to occur frequently, or no profits!**
- **Amazon's people who bought *X* also bought *Y***
  - Recommendation Systems

Market Basket Applications: Plagiarism detection

- **Baskets** = sentences? documents containing those sentences?
- **items** = sentences? documents containing those sentences?

Market Basket Applications: Plagiarism detection

- **Baskets** = sentences
- **items** = documents containing those sentences
  - Item/document is "in" a basket if sentence is in the document
  - May seem backward, but relationship between baskets and items is many-to-many
- Look for items that appear together in several baskets
  - Multiple documents share sentence(s)
- **Items (documents) that appear together too often could represent plagiarism.**

Market Basket Applications: Identify related concepts in web documents

- **Baskets** = words? Web pages?
- **items** = words? Web pages?

Market Basket Applications: Identify related concepts in web documents

- **Baskets** = Web pages
- **items** = words
- Baskets/documents contain items/words in the document
- Look for sets of words that appear together in many documents
- Ignore most common words
- Unusual words appearing together in a large number of documents, e.g., "World" and "Cup," may indicate an interesting relationship or joint concept

Market Basket Applications: Drug interactions

- **Baskets** = patients
- **items** = drugs and side effects
- Has been used to detect combinations of drugs that result in particular side-effects
- **But requires extension:** Absence of an item needs to be observed as well as presence!!

Scale of the Problem

- WalMart sells 100,000 items and can store billions of baskets.
- The Web has billions of words and many billions of pages.

DEFINE FREQUENT ITEMSETS

Support

- **Simplest question: Find sets of items that appear "frequently" in the baskets**

- ***Support* for itemset *I*** = the number of baskets containing all items in *I*
  - Sometimes given as a percentage

- **Given a *support threshold s*, sets of items that appear in at least *s* baskets are called *frequent itemsets***

Example: Frequent Itemsets

- Items={milk, coke, pepsi, beer, juice}.
- **Support = 3 baskets**.

B_1 = {m, c, b}	B_2 = {m, p, j}
B_3 = {m, b}	B_4 = {c, j}
B_5 = {m, p, b}	B_6 = {m, c, b, j}
B_7 = {c, b, j}	B_8 = {b, c}

- Frequent itemsets of size 1: {m}, {c}, {b}, {j}

{m,b}, {b,c}, {c,j}.

ASSOCIATION RULES

Association Rules

- **If-then rules about the contents of baskets**
- Basket $\boldsymbol{I}$ contains $\{\boldsymbol{i_1}, \boldsymbol{i_2}, \ldots, \boldsymbol{i_k}\}$
- **Rule** $\{\boldsymbol{i_1}, \boldsymbol{i_2}, \ldots, \boldsymbol{i_k}\} \rightarrow \boldsymbol{j}$ means: "if a basket contains all of $i_1,\ldots,i_k$ then it is ***likely*** to contain j."

- ***Confidence*** **of this association rule is the probability of** $\boldsymbol{j}$ **given** $\boldsymbol{i_1,\ldots,i_k}$
  - **Ratio of support for** $\boldsymbol{I \cup \{j\}}$ **with support for** $\boldsymbol{I}$
  - Support for $\boldsymbol{I}$: number of baskets containing $\boldsymbol{I}$

Example: Confidence

+ $B_1 = \{m, c, b\}$ — $B_2 = \{m, p, j\}$

– $B_3 = \{m, b\}$ — $B_4 = \{c, j\}$

– $B_5 = \{m, p, b\}$ — + $B_6 = \{m, c, b, j\}$

$B_7 = \{c, b, j\}$ — $B_8 = \{b, c\}$

- **An association rule: $\{m, b\} \rightarrow c$**
  - **Confidence: Ratio of support for I U {j} with support for I**
  - Ratio of support for {m,b} U {c} to support for {m,b}
  - Confidence = 2/4 = 50%

- **Want to identify association rules with high confidence**

Interesting Association Rules

- **Not all high-confidence rules are interesting**
  - The rule $X \rightarrow$ ***milk*** may have high confidence for many itemsets X
    - because milk is just purchased very often (independent of X)
- **Interest of an association rule $I \rightarrow j$: difference between its confidence and the fraction of baskets that contain j**

$$\text{Interest}(I \rightarrow j) = \text{conf}(I \rightarrow j) - \Pr[j]$$

  - **Interesting rules are those with high positive or negative interest values (usually above 0.5)**
  - High **positive**/**negative** interest means presence of I **encourages** or **discourages** presence of j
  - Example: {coke} -> pepsi should have high negative interest

Example: Confidence and Interest

B_1 = {m, c, b} **B_2 = {m, p, j}**
B_3 = {m, b} **B_4= {c, j}**
B_5 = {m, p, b} **B_6 = {m, c, b, j}**
B_7 = {c, b, j} **B_8 = {b, c}**

- **Association rule: {m, b} →c**
  - Confidence: Ratio of support for I ∪ {j} with support for I
  - **Confidence** = 2/4 = 0.5
  - Interest: $\text{Interest}(I \to j) = \text{conf}(I \to j) - \Pr[j]$
  - **Difference between its confidence and the fraction of baskets that contain *j***
  - **Interest** = |0.5 – 5/8| = 1/8
    - Item *c* appears in 5/8 of the baskets
    - Rule is not very interesting!

Finding Useful Association Rules

- **Question: "find all association rules with support ≥ s and confidence ≥ c "**

- **Hard part: finding the frequent itemsets**
  - Note: **if $\{i_1, i_2,\ldots,i_k\} \rightarrow j$ has high support and confidence, then both $\{i_1, i_2,\ldots,i_k\}$ and $\{i_1, i_2,\ldots,i_k, j\}$ will be "frequent"**

- **Assume: not too many frequent itemsets or candidates for high support, high confidence association rules**
  - Not so many that they can't be acted upon
  - Adjust support threshold to avoid too many frequent itemsets

Example: Find Association Rules with support ≥ s and confidence ≥ c

$B_1 = \{m, c, b\}$ $B_2 = \{m, p, j\}$
$B_3 = \{m, c, b, n\}$ $B_4 = \{c, j\}$
$B_5 = \{m, p, b\}$ $B_6 = \{m, c, b, j\}$
$B_7 = \{c, b, j\}$ $B_8 = \{b, c\}$

- **Support threshold $s = 3$, confidence $c = 0.75$**
- **1) Frequent itemsets:**
  - {b} {c} {j} {m} {b,m} {b,c} {c,m} {c,j} {m,c,b}
- **2) Generate rules:**
  - ~~b→m: c=4/6~~ b→c: c=5/6 ~~b,c→m: c=3/5~~
  - m→b: c=4/5 … b,m→c: c=3/4
  - ~~b→c,m: c=3/6~~

$$\operatorname{conf}(I \rightarrow j) = \frac{\operatorname{support}(I \cup j)}{\operatorname{support}(I)}$$

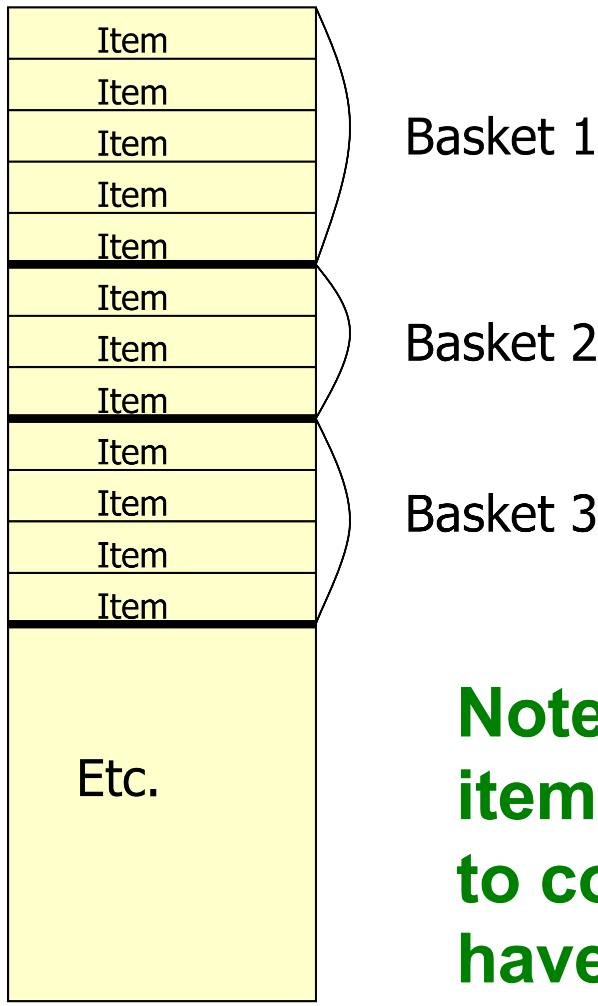
Difficult part is identifying frequent itemsets: algorithms to find them are the focus of this chapter

FIND FREQUENT ITEMSETS

Computation Model

- Typically, market basket data are kept in **flat files** rather than in a database system
  - Stored **on disk because they are very large files**
  - Stored **basket-by-basket**
  - **Goal: Expand baskets into pairs, triples, etc. as you read baskets**
    - Use k nested loops to generate all sets of size k

File Organization



Example: items are positive integers, and boundaries between baskets are –1

Note: We want to find frequent itemsets. To find them, we have to count them. To count them, we have to generate them.

Computation Model – (2)

- **The true cost of mining disk-resident data is usually the number of disk I/O' s**
- **In practice, association-rule algorithms read the data in *passes* – all baskets read in turn**
- Thus, we measure the cost by the **number of passes** an algorithm takes

Main-Memory Bottleneck

- **For many frequent-itemset algorithms, main memory is the critical resource**
  - As we read baskets, **we need to count something, e.g., occurrences of pairs**
  - **The number of different things we can count is limited by main memory**
  - Swapping counts in/out is a disaster
  - **Algorithms are designed so that counts can fit into main memory**

Finding Frequent Pairs

- **The hardest problem often turns out to be finding the frequent pairs**
  - Why? Often frequent pairs are common, frequent triples are rare
    - Why? Probability of being frequent drops exponentially with size; number of sets grows more slowly with size
- **We'll concentrate on pairs, then extend to larger itemsets**

Naïve Algorithm

- **Read file once, counting in main memory the occurrences of each pair**
  - Number of pairs in a basket of n items: n choose 2

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

  - From each basket of ***n*** items, **generate its *n**(*n* -1)/2 pairs using two nested loops, add to the count for each pair**
  - **First basket: (a,b), (a,c), (a,y), (b,c), (b,y), (c,y)**
  - **Second basket: (a,b), (a,x), (a,y), (a,z), (b,x), (b,y), (b,z), ...**
- **Total possible number of pairs in all baskets: (#items)(#items -1)/2**
- **Fails if $(\#items)^2$ exceeds main memory**
  - Remember: #items can be 100K (Wal-Mart) or 10B (Web pages)

Example: Counting Pairs

- Suppose 10^5 items
- Suppose counts are 4-byte integers
- Number of pairs of items: $10^5(10^5-1)/2 = 5*10^9$ (approximately)
- Therefore, $2*10^{10}$ (20 gigabytes) of main memory needed

Details of Main-Memory Counting

- **Two approaches:**
  1. **Count all pairs, using a triangular matrix**
  2. **Keep a table of triples [i, j, c] = "the count of the pair of items {i, j } is c"**

(1) requires only 4 bytes/pair, but requires a count for each pair

Note: assume integers are 4 bytes

(2) requires 12 bytes, but only for those pairs with count > 0

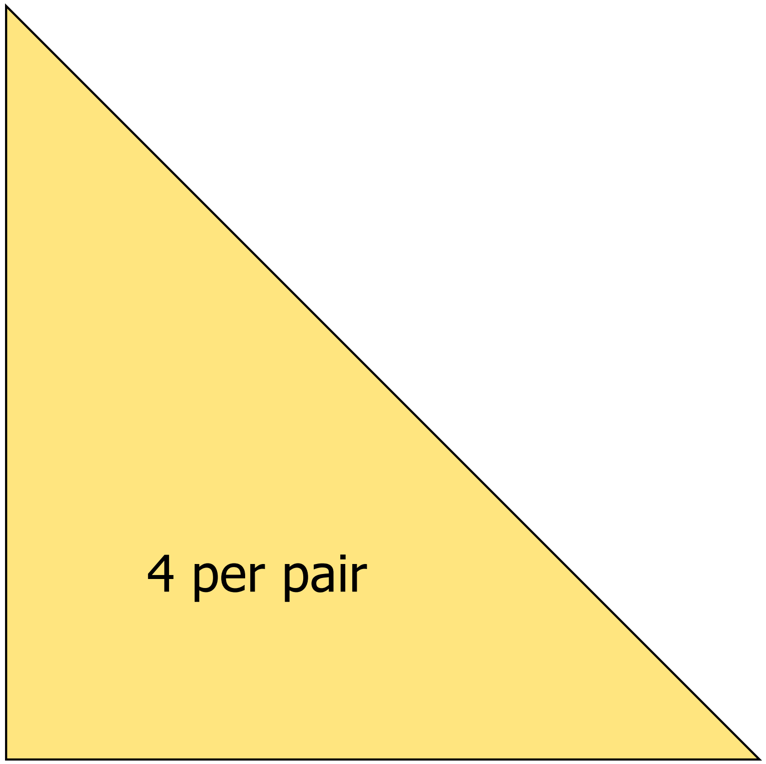
Plus some additional overhead for a hashtable

Triangular Matrix

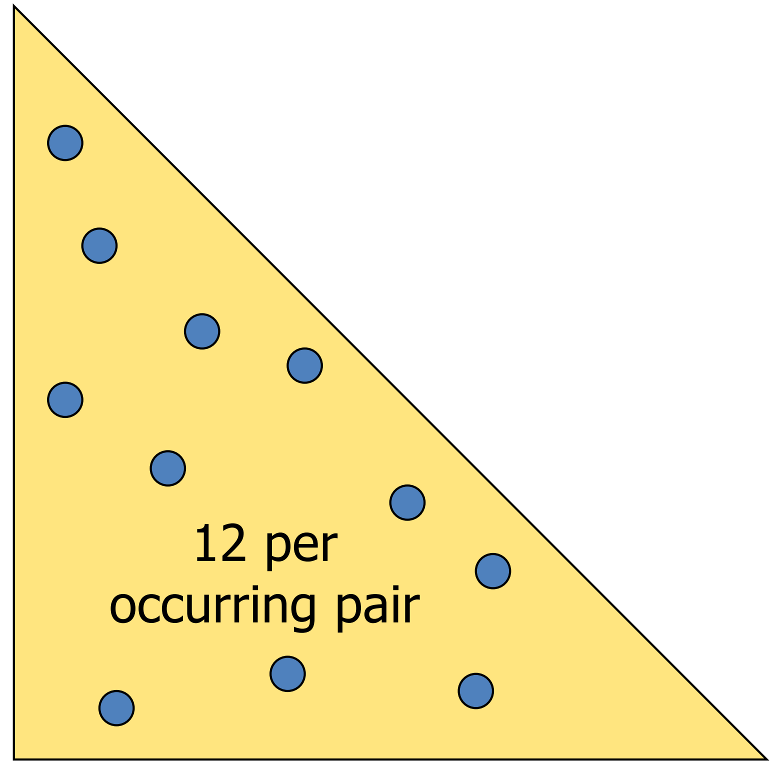
a11	a12	a13	a14	a15	…
a21	a22	a23	a24	a25	…
a31	a32	a33	a34	a35	…
a41	a42	a43	a44	a45	…
…					

Triangular-Matrix Approach – (1)

- **n** = total number of items
- Order each pair of items $\{i, j\}$ so that $i<j$
- Keep pair counts in lexicographic order:
  - **{1,2}, {1,3},…, {1,*n*}, {2,3}, {2,4},…,{2,*n*}, {3,4},…**
- Pair $\{i, j\}$ is at position $\boldsymbol{(i-1)(n-i/2)+j-i}$
  - ***Every time you see a pair {i,j} from a basket, increment the count at the corresponding position in triangular matrix***
- Total number of pairs $\boldsymbol{n(n-1)/2}$; total bytes= $\boldsymbol{2n^2}$
- **Triangular Matrix** requires 4 bytes per pair



Method (1)



Method (2)

Comparing the two approaches

- **Approach 1: Triangular Matrix**
  - **n** = total number items
  - **Count pair of items {*i*, *j*} only if *i*<*j***
  - Keep pair counts in lexicographic order:
    - {1,2}, {1,3},…, {1,*n*}, {2,3}, {2,4},…,{2,*n*}, {3,4},…
  - **Pair {*i*, *j*} is at position** $(i-1)(n-i/2)+j-i$
  - Total number of pairs $n(n-1)/2$; total bytes= $2n^2$
  - **Triangular Matrix** requires 4 bytes per pair
- **Approach 2** uses **12 *bytes*** per occurring pair *(but only for pairs with count > 0)*
  - **Beats Approach 1 if fewer than 1/3 of possible pairs actually occur in the market basket data**

Comparing the two approaches

- **Approach 1: Triangular Matrix**
  - **n** = total number items
  - Co
  - K
  - F
  - T
  - T
- **App**

  (but
  - B

    possible pairs actually occur

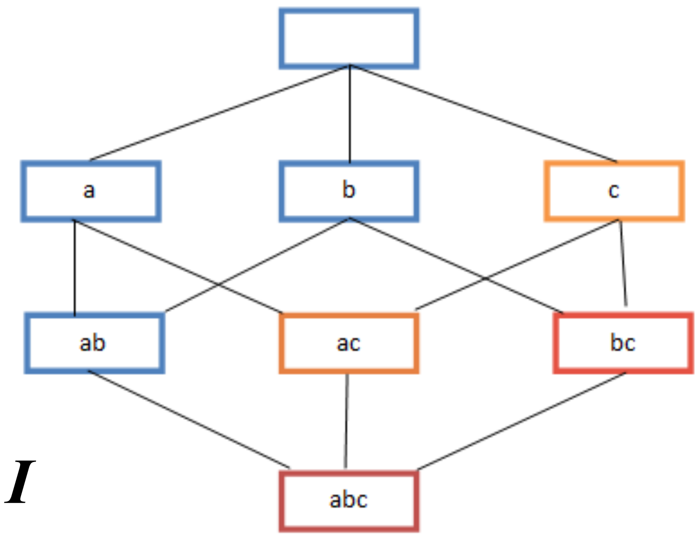
Problem is if we have too many items so the pairs do not fit into memory.

Can we do better?

A-Priori Algorithm

A-Priori Algorithm – (1)

- A **two-pass** approach called ***A-Priori*** limits the need for main memory
- **Key idea: *monotonicity***
  - If a set of items ***I*** appears at least ***s*** times, so does every **subset *J*** of ***I***
- **Contrapositive for pairs:** If item ***i*** does not appear in ***s*** baskets, then no pair including ***i*** can appear in ***s*** baskets



- **So, how does A-Priori find freq. pairs?**

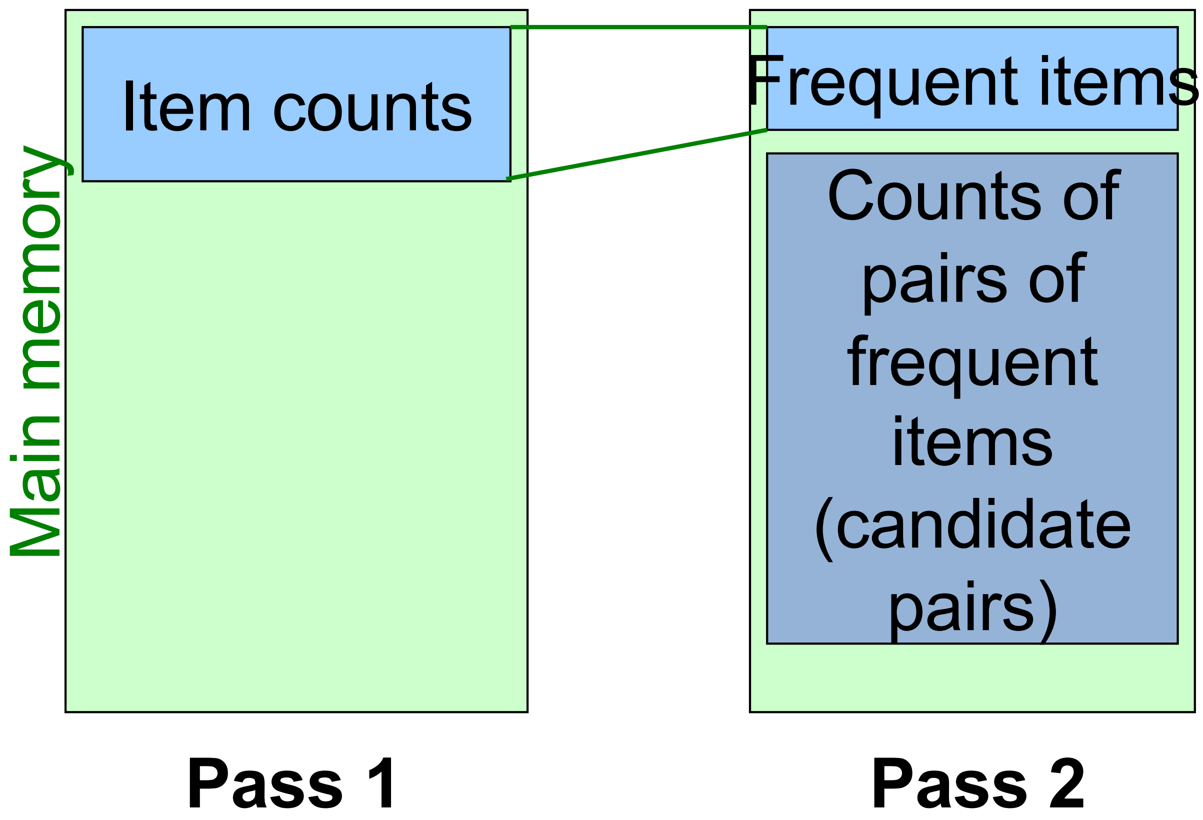
A-Priori Algorithm

- **Pass 1: Read baskets and count in main memory the occurrences of each item**
  - Requires only memory proportional to #items
- **Items that appear at least *s* times are the *frequent items***
  - **At the end of pass 1, after the complete input file has been processed, check the count for each item**
  - **If count > s, then that item is frequent: saved for the next pass**
- **Pass 1 identifies frequent itemsets of size 1**

A-Priori Algorithm

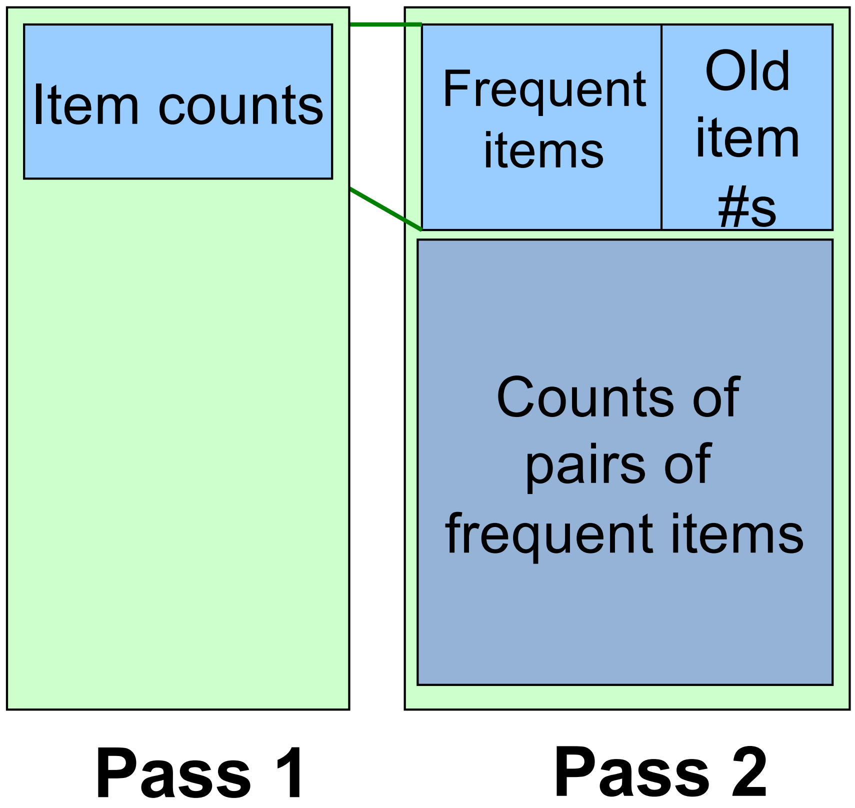
- **Pass 2: Read baskets again and count in main memory only those pairs of items where both were found in Pass 1 to be frequent**
  - **Requires:**
    - **Memory proportional to square of *frequent* items only** (to hold counts of pairs)
    - **List of the frequent items from the first pass** (so you know what must be counted)
- **Pairs of items that appear at least *s* times are the *frequent pairs***
  - **At the end of pass 2, check the count for each pair**
  - **If count > s, then that pair is frequent**
- **Pass 2 identifies frequent pairs: itemsets of size 2**

Main-Memory: Picture of A-Priori



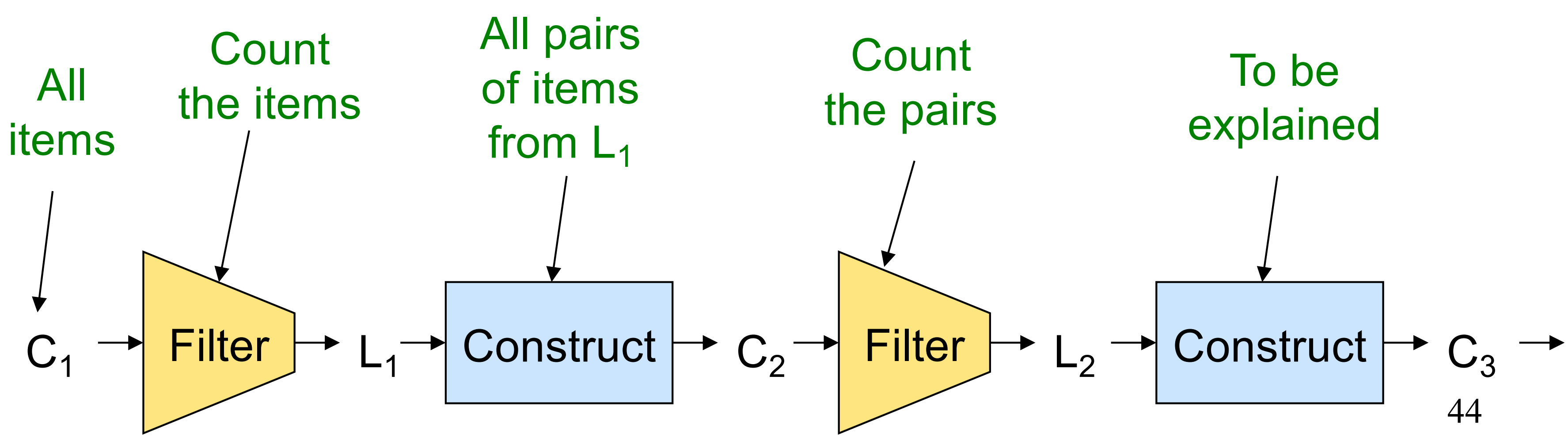
Detail for A-Priori

- **You can use the triangular matrix method with *n* = number of frequent items**
  - May save space compared with storing triples
- **Trick: re-number frequent items 1,2,…** and keep a table relating new numbers to original item numbers



What About Larger Frequent Itemsets? Frequent Triples, Etc.

- **For each *k*, we construct two sets of *k-tuples*** (sets of size *k*):
  - C_k = ***candidate k-tuples*** = those that might be frequent sets (support ≥ **s**) based on information from the pass for ***k*–1**
  - L_k = the set of **truly frequent *k*-tuples**



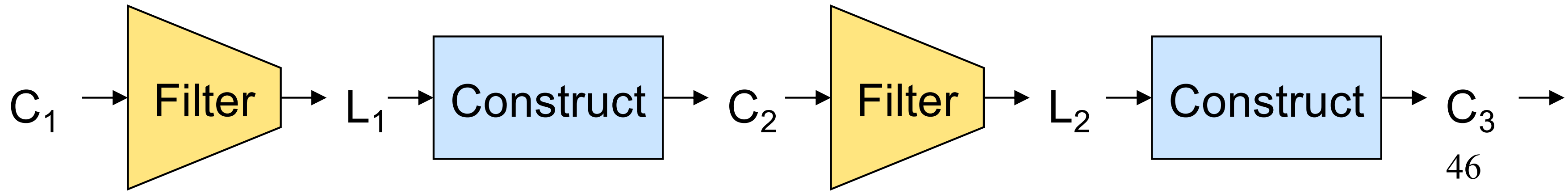
Recall: Example

$B_1 = \{m, c, b\}$ $B_2 = \{m, p, j\}$
$B_3 = \{m, c, b, n\}$ $B_4 = \{c, j\}$
$B_5 = \{m, p, b\}$ $B_6 = \{m, c, b, j\}$
$B_7 = \{c, b, j\}$ $B_8 = \{b, c\}$

- **Frequent itemsets (s=3):**
  - {b}, {c}, {j}, {m}
  - {b,m} {b,c} {c,m} {c,j}
  - {m,c,b}

Example

- **Hypothetical steps of the A-Priori algorithm**
  - C_1 = { {b} {c} {j} {m} {n} {p} }: all candidate items
  - Count the support of itemsets in C_1
  - Prune non-frequent: L_1 = { b, c, j, m }
  - Generate C_2 = { {b,c} {b,j} {b,m} {c,j} {c,m} {j,m} }
  - Count the support of itemsets in C_2
  - Prune non-frequent: L_2 = { {b,m} {b,c} {c,m} {c,j} }
  - Generate C_3 = { {b,c,m} }
  - Count the support of itemsets in C_3
  - Prune non-frequent: L_3 = { {b,c,m} }



A-Priori for All Frequent Itemsets

- **One pass for each *k* (itemset size)**
- Needs room in main memory to count each candidate ***k***–tuple
- For typical market-basket data and reasonable support (e.g., 1%), ***k* = 2** requires the most memory