Collaborative Filtering

Harnessing quality judgments of other users

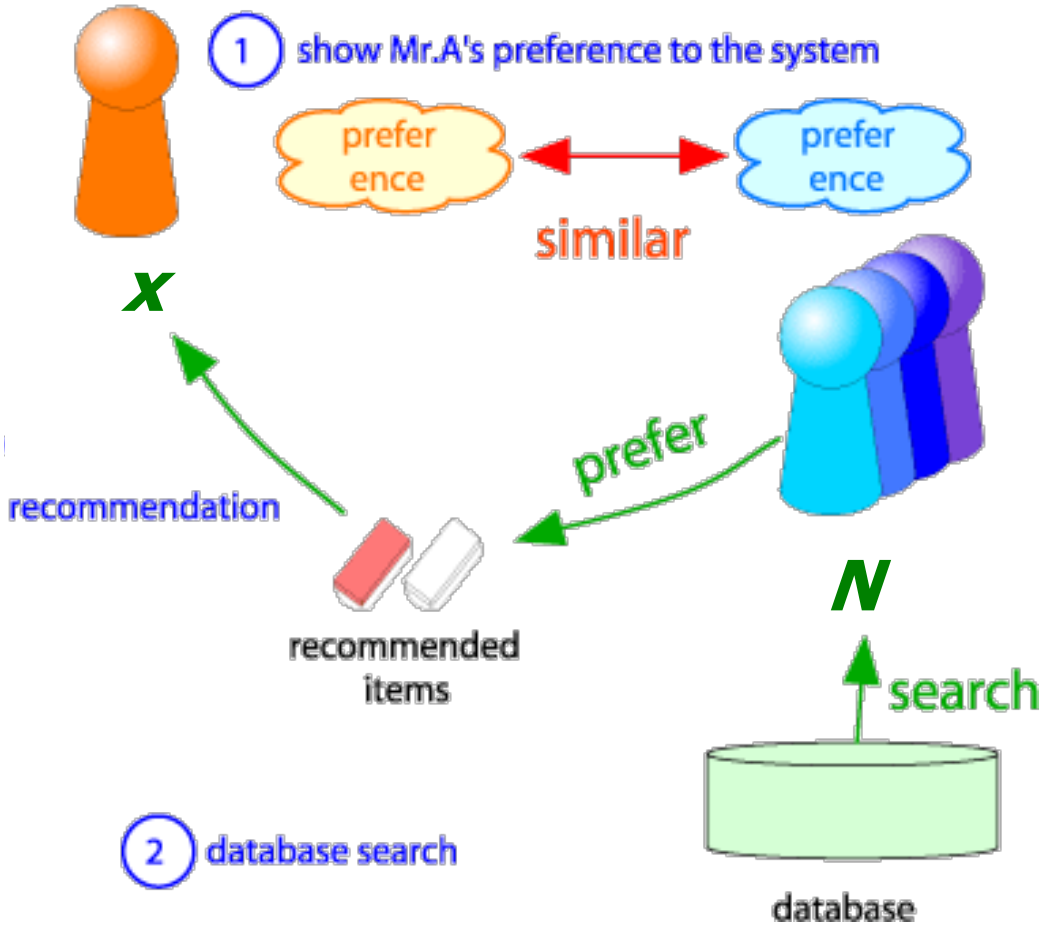
Thanks for source slides and material to: J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets
http://www.mmds.org

Three Approaches to Recommendation Systems

- **1) Content-based**
  - Use characteristics of an item
  - Recommend items that
    - have similar content to items user liked in the past
    - match pre-defined attributes of the user
- **2) Collaborative filtering**
  - Build a model from
    - a user's past behavior (e.g., items previously purchased or rated), and
    - similar decisions made by other users
  - Use the model to predict items that the user may like
  - Collaborative: suggestions made to a user utilizing information across the entire user base
- **3) Hybrid approaches**

Collaborative Filtering Example

- **User-based** collaborative filtering
- Consider user ***x***
  - Find set ***N*** of other users whose ratings are "**similar**" to ***x***'s ratings
  - Estimate ***x***'s ratings based on ratings of users in ***N***



Collaborative Filtering: Overview

- CF works by **collecting user feedback**: e.g., **ratings for items**
  - Exploit similarities in rating behavior among users in determining recommendations
- **Two classes of CF algorithms:**

1. **Neighborhood-based or Memory-based approaches**
   - User-based CF
   - Item-based CF
2. **Model-based approaches**
   - Estimate parameters of statistical models for user ratings
   - Latent factor and matrix factorization models

NEIGHBORHOOD-BASED OR MEMORY-BASED COLLABORATIVE FILTERING

USER-BASED CF

Neighborhood-based / Memory-based Collaborative Filtering

- Active user: the user we want to make predictions for
- **User-based CF:** A subset of other users is chosen based on their similarity to the active user
- A weighted combination of their ratings is used to make predictions for the active user
- **Steps:**

1. Assign a weight to all users w.r.t. **similarity with the active user**
2. **Select k users that have the highest similarity** with active user (the neighborhood)
3. **Compute a prediction from a weighted combination of the selected neighbors' ratings**

Similarity between users: by what measure?

- **Weight $w_{x,y}$ is measure of similarity between user *x* and active user *y***

Let r_x be the vector of user ***x***'s ratings

Possible similarity metrics:

- **Jaccard similarity**
  - (Intersection / union) of two sets
  - Doesn't use non-boolean values: e.g., ratings
- **Cosine similarity**
  - Treat ratings as vectors to points
  - Cosine similarity between points

$$\text{similarity} = \cos(\theta) = \frac{A \cdot B}{\|A\|\|B\|} = \frac{\sum_{i=1}^{n} A_i \times B_i}{\sqrt{\sum_{i=1}^{n} (A_i)^2} \times \sqrt{\sum_{i=1}^{n} (B_i)^2}}$$

r_x, r_y as sets:

$r_x = \{1, 4, 5\}$

$r_y = \{1, 3, 4\}$

r_x, r_y as points:

$r_x = \{1, 0, 0, 1, 3\}$

$r_y = \{1, 0, 2, 2, 0\}$

Similarity between users:
by what measure?

JACCARD SIMILARITY

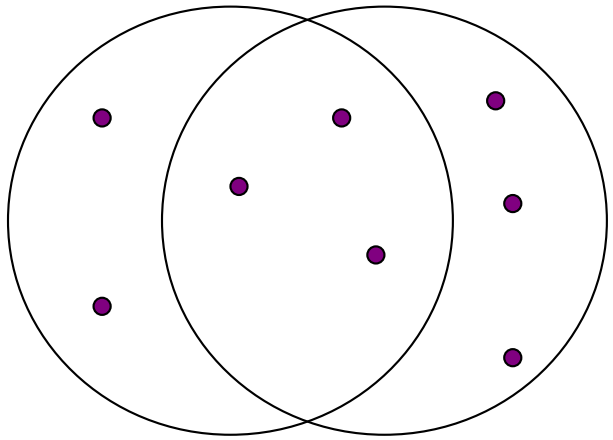
Jaccard Similarity and Distance of Sets

- The ***Jaccard similarity*** of two sets is the size of their intersection divided by the size of their union

$$Sim\ (C_1, C_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$$

- ***Jaccard distance*** **= 1 – Jaccard Similarity**

Example:
Jaccard Similarity and Distance



3 in intersection
8 in union

Jaccard similarity = 3/8

- **Jaccard distance** = 1 – Jaccard Similarity or 5/8 in this example

How well do these similarity metrics work?

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

- **Intuitively we want: sim(*A*, *B*) > sim(*A*, *C*) (why?)**
  - For A and B: One movie rated in common with similar ratings
  - For A and C: Two movies rated in common but with dissimilar ratings
- **Jaccard similarity (Example 9.7)**
  - Ignores values (rates) in matrix
  - Only look at which items are rated in matrix
- Intersection / union: sim(A,B) = 1/5, sim(A,C) = 2/4
- 1/5 < 2/4 indicates sim(A,C) > sim(A,B)
- **Not a good similarity metric for ratings data!**

Similarity between users:
by what measure?

COSINE SIMILARITY

Same matrix with cosine similarity (Example 9.8)

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

- Treat blanks as 0 value: **questionable, since it treats no rating as more similar to disliking a movie than liking it**
- Cosine similarity of A and B is:

$$\frac{4 \times 5}{\sqrt{4^2 + 5^2 + 1^2}\sqrt{5^2 + 5^2 + 4^2}} = 0.380$$

- Cosine similarity of A and C is: 0.322
- 0.380 > 0.322: **Indicates A,B slightly more similar than A,C**
- **in this case, cosine similarity better than Jaccard similarity**

Normalizing ratings (Example 9.9)

- To deal better with non-rated items
- Subtract the average rating of that user from each rating
  - low ratings -> negative numbers; high ratings -> positive numbers

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

Avg. Rating
10/3
14/3
11/3
6/2

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	$2/3$			$5/3$	$-7/3$		
B	$1/3$	$1/3$	$-2/3$				
C				$-5/3$	$1/3$	$4/3$	
D		0					0

Cosine sim A,B vs. A,C: 0.092 **>** -0.559 (85 degrees > 124 degrees)
A, C are much further apart than A, B, but neither is close

Similarity between users:
by what measure?

PEARSON CORRELATION

Most commonly used measure of similarity: Pearson Correlation Coefficient

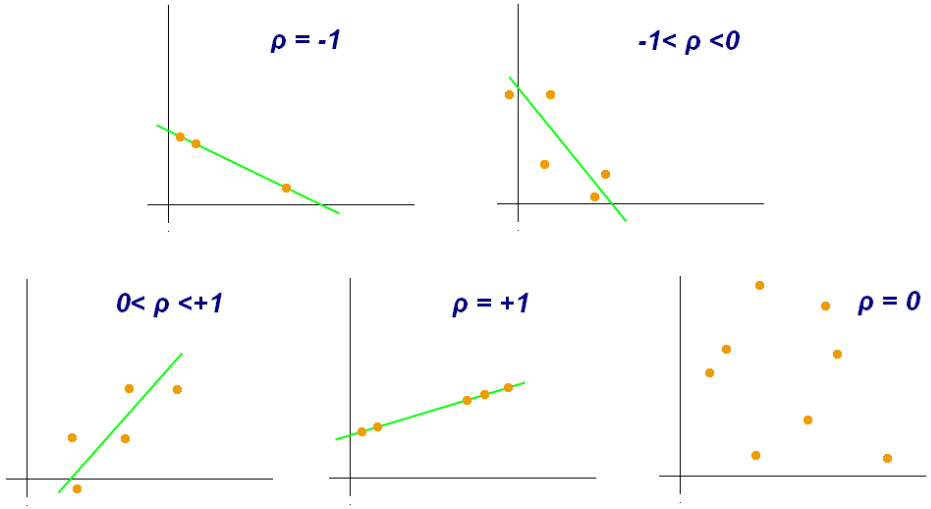
- **Pearson correlation measures extent to which two variables linearly relate**
- For users u, v: Pearson correlation is

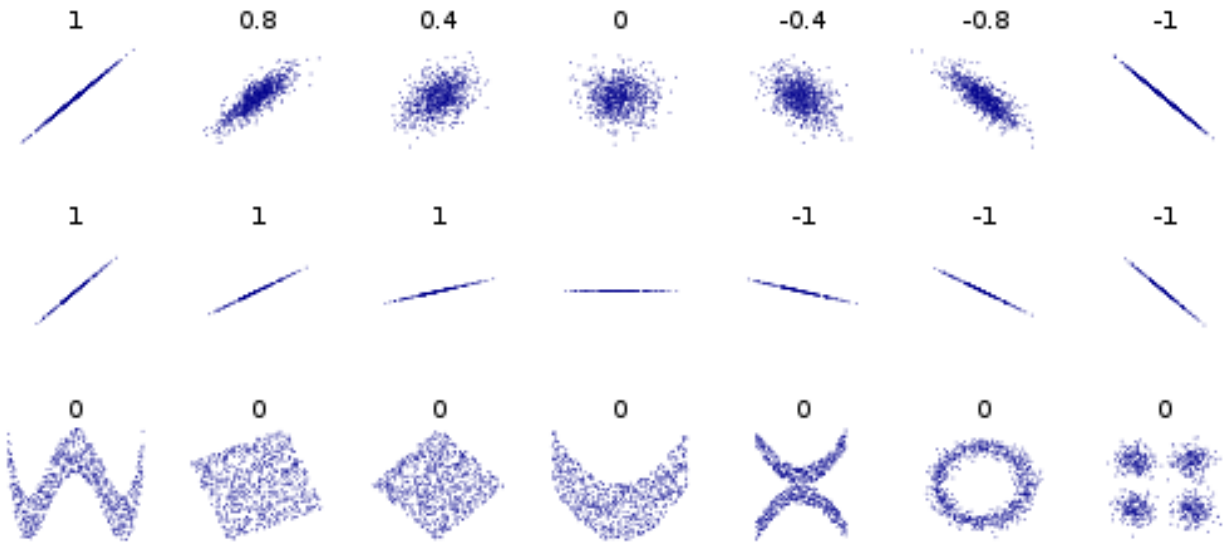
$$w_{u,v} = \frac{\sum_{i \in I} (r_{u,i} - \bar{r}_u)(r_{v,i} - \bar{r}_v)}{\sqrt{\sum_{i \in I} (r_{u,i} - \bar{r}_u)^2} \sqrt{\sum_{i \in I} (r_{v,i} - \bar{r}_v)^2}}$$

where the $i \in I$ summations are over the items that both the users u and v have rated and $\bar{r}_u$ is the average rating of the co-rated items of the uth user.

  - **And $r_{u,i}$ is rating of item i by user u**
- **Note: When calculating these similarities, look only at the co-rated items**

Pearson Correlation Examples





Source: https://en.wikipedia.org/wiki/Pearson_product-moment_correlation_coefficient

Example: Pearson Correlation Coefficient

$$w_{u,v} = \frac{\sum_{i \in I} (r_{u,i} - \bar{r}_u)(r_{v,i} - \bar{r}_v)}{\sqrt{\sum_{i \in I} (r_{u,i} - \bar{r}_u)^2} \sqrt{\sum_{i \in I} (r_{v,i} - \bar{r}_v)^2}}$$

	I_1	I_2	I_3	I_4
U_1	4	?	5	5
U_2	4	2	1	
U_3	3		2	4
U_4	4	4		
U_5	2	1	3	5

- Want correlation of user 1, user 5: $w_{1,5}$ **Set *I* of co-rated movies = {I_1, I_3, I_4}**
- For user 1: **average rating on co-rated movies** is 14/3; For user 5: 10/3
- Number: (4 - 14/3)(2 - 10/3) + (5- 14/3)(3- 10/3) + (5-14/3)(5-10/3) = 12/9 = 1.3333
- Denominator: sqrt((4-14/3)^2 + (5-14/3)^2 + (5-14/3)^2)*sqrt((2-10/3)^2 + (3-10/3)^2 + (5-10/3)^2) = 1.76383
- **Pearson correlation $w_{1,5}$= 1.3333 / 1.76383 = 0.756**

Making User-based CF Predictions with Pearson: Weighted Sum of Other Users' Ratings

- Weighted average of their ratings is used to generate predictions
- To make a prediction for an active user a on an item i:

$$P_{a,i} = \bar{r}_a + \frac{\sum_{u \in U} (r_{u,i} - \bar{r}_u) \cdot w_{a,u}}{\sum_{u \in U} | w_{a,u} |}$$

where $\bar{r}_a$ and $\bar{r}_u$ are the average ratings for the user a and user u on all other rated items, and $w_{a,u}$ is the weight between the user a and user u. The summations are over all the users $u \in U$ who have rated the item i.

- Note: When making predictions, calculate average of ALL rated items for users a and u
- Summation is over all users who rated item i

Continued Example: User-Based CF Prediction with Pearson Correlation Coefficient

	I_1	I_2	I_3	I_4
U_1	4	?	5	5
U_2	4	2	1	
U_3	3		2	4
U_4	4	4		
U_5	2	1	3	5

- **Want to predict rating for user U1 on item I2:** users U2, U4 and U5 rated I2
- **Similarity of U1 to these users:** $w_{1,5} = 0.756$, $w_{1,4} = 0$, $w_{1,2} = -1$

$$P_{1,2} = \bar{r}_1 + \frac{\sum_u (r_{u,2} - \bar{r}_u) \cdot w_{1,u}}{\sum_u |w_{1,u}|}$$

14/3 = 4.67

$$= \bar{r}_1 + \frac{(r_{2,2} - \bar{r}_2) w_{1,2} + (r_{4,2} - \bar{r}_4) w_{1,4} + (r_{5,2} - \bar{r}_5) w_{1,5}}{|w_{1,2}| + |w_{1,4}| + |w_{1,5}|}$$

5/2=2.5

4/1=4

$$= 4.67 + \frac{(2 - 2.5)(-1) + (4 - 4)0 + (1 - 3.33)0.756}{1 + 0 + 0.756}$$

10/3=3.33

$$= 3.95.$$

Neighborhood-Based algorithms

- In neighborhood-based CF algorithms, **a subset of nearest neighbors** of the active user are **chosen based on their similarity with active user**
- Use these for predictions rather than all users who have rated the item

ITEM-BASED CF

Item-based Collaborative Filtering

- Neighborhood-based CF algorithms do not scale well when applied to millions of users & items
  - Due to computational complexity of search for similar users (possible solutions?)
- **Item-to-item collaborative filtering**
  - Rather than matching similar users
  - **Match user's rated items to similar items**
- In practice, often leads to faster online systems and better recommendations
- **Similarities between pairs of items i and j are computed off-line**
- Predict rating of user **a** on item **i** with a simple weighted average

Pearson Correlation between items i, j (Cont'd)

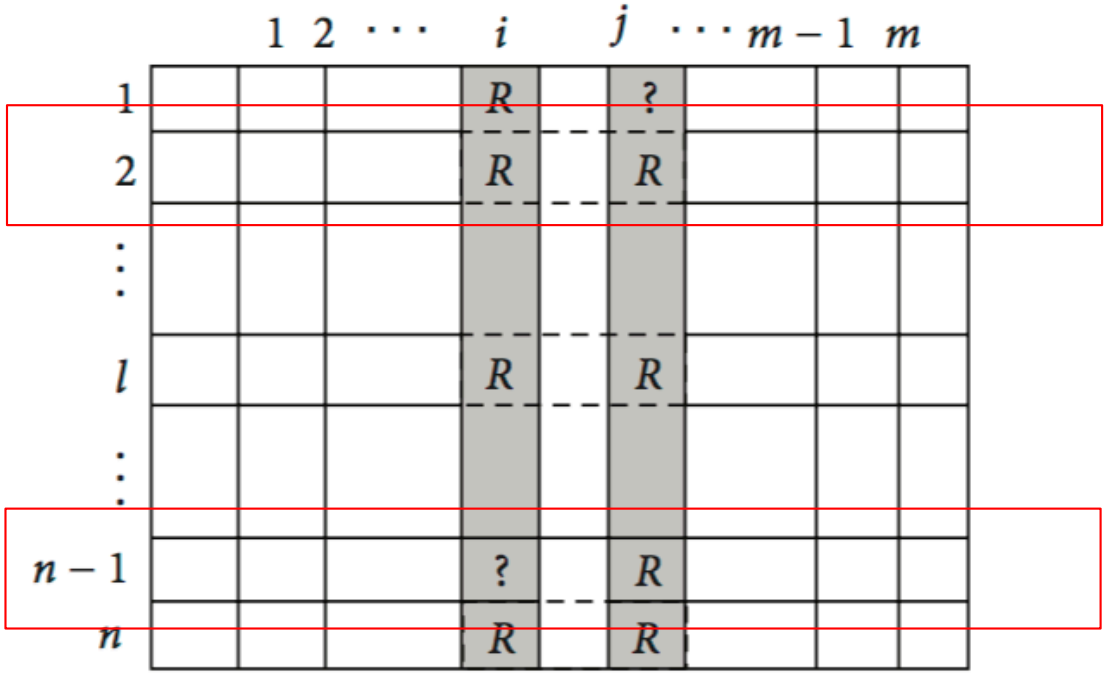


FIGURE 2: item-based similarity ($w_{i,j}$) calculation based on the co-rated items i and j from users 2, l and n.

Source: Su, X., & Khoshgoftaar, T. M. (2009). A survey of collaborative filtering techniques. *Advances in artificial intelligence*, *2009*, 4.

Pearson Correlation between items i, j

For the item-based algorithm, denote the set of users $u \in U$ who rated both items i and j, then the *Pearson Correlation* will be

$$w_{i,j} = \frac{\sum_{u \in U} \left(r_{u,i} - \overline{r}_i\right)\left(r_{u,j} - \overline{r}_j\right)}{\sqrt{\sum_{u \in U} \left(r_{u,i} - \overline{r}_i\right)^2} \sqrt{\sum_{u \in U} \left(r_{u,j} - \overline{r}_j\right)^2}}, \qquad (2)$$

where $r_{u,i}$ is the rating of user u on item i, $\overline{r}_i$ is the average rating of the ith item by those users, see Figure 2 [40].

- **Note: Sum over set of users *U* who rated both items *i, j***
- $r_{u,i}$ is rating of user *u* on item *i*
- $\overline{r_i}$ is average rating of i^{th} item by those users

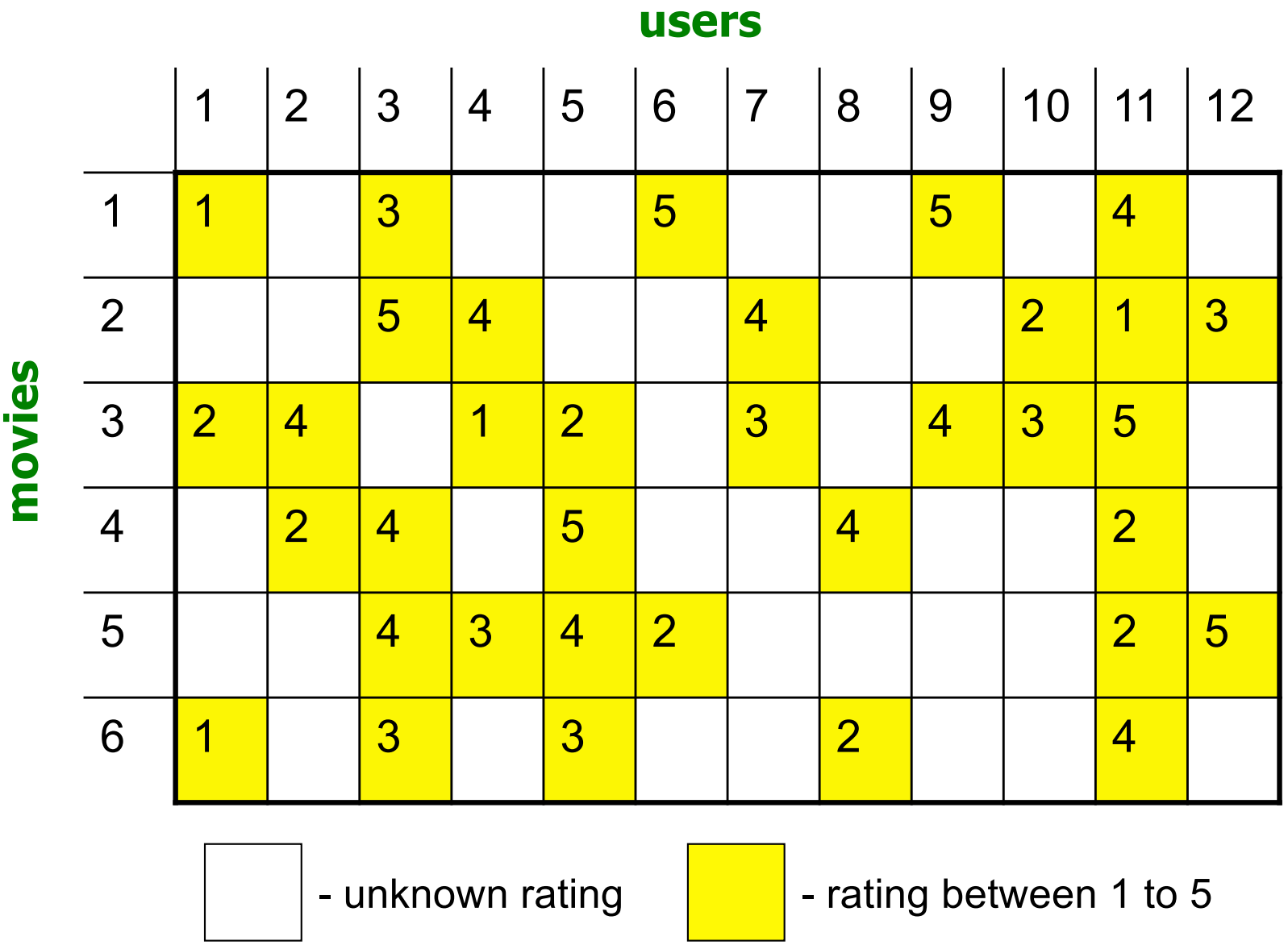
Make Item-Based Predictions Using a Simple Weighted Average

- Predict rating for user *u* on item *i*
- $w_{i,n}$ is weight between items *i and n*
- $r_{u,n}$ is rating for user *u* on item *n*
- Summation over **neighborhood set *N* of items** rated by *u* that are most similar to *i*

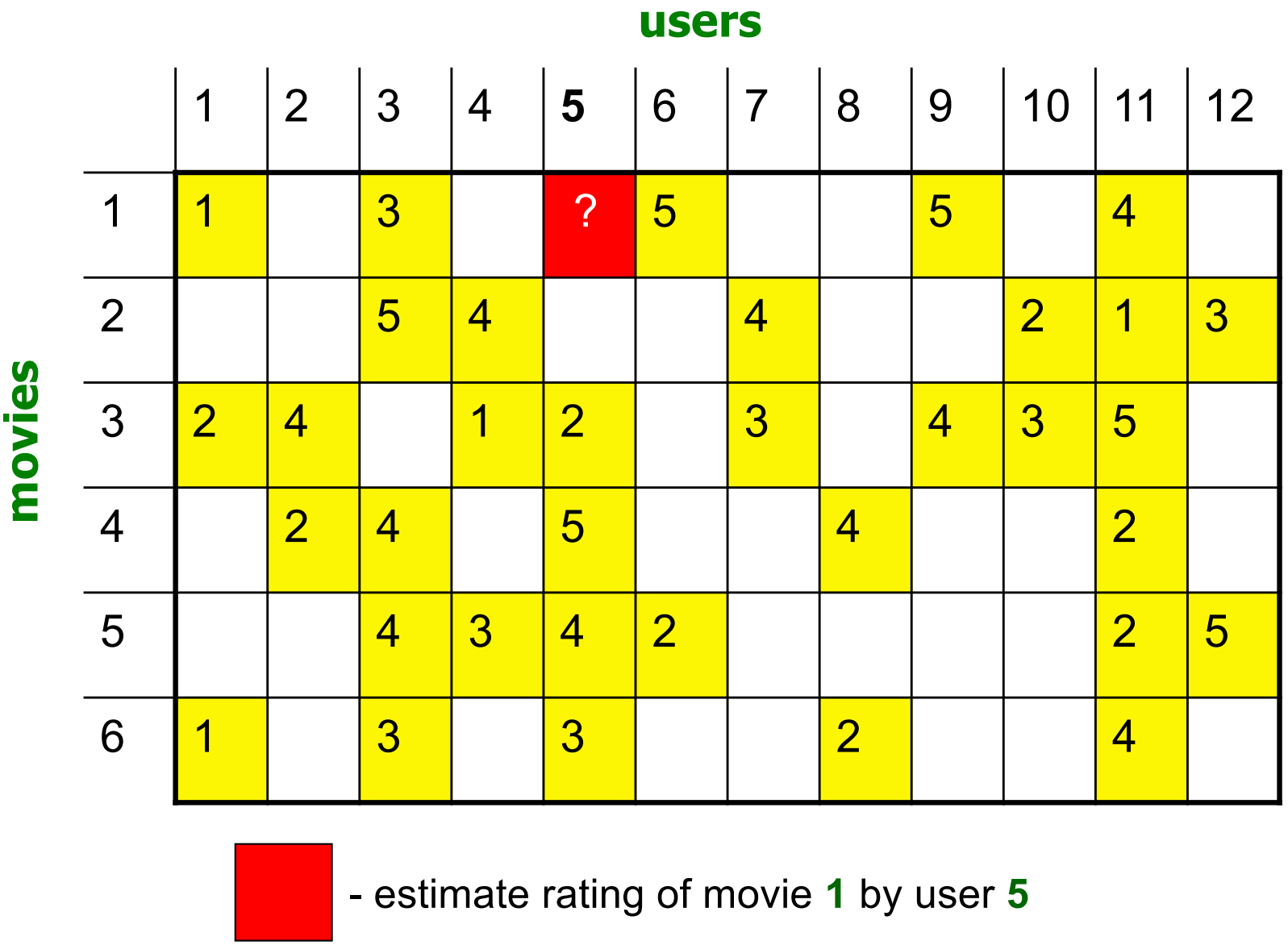
$$P_{u,i} = \frac{\sum_{n \in N} r_{u,n} w_{i,n}}{\sum_{n \in N} | w_{i,n} |}$$

where the summations are over all other rated items $n \in N$ for user u, $w_{i,n}$ is the weight between items i and n, $r_{u,n}$ is the rating for user u on item n.

Item-Item CF (|N|=2)

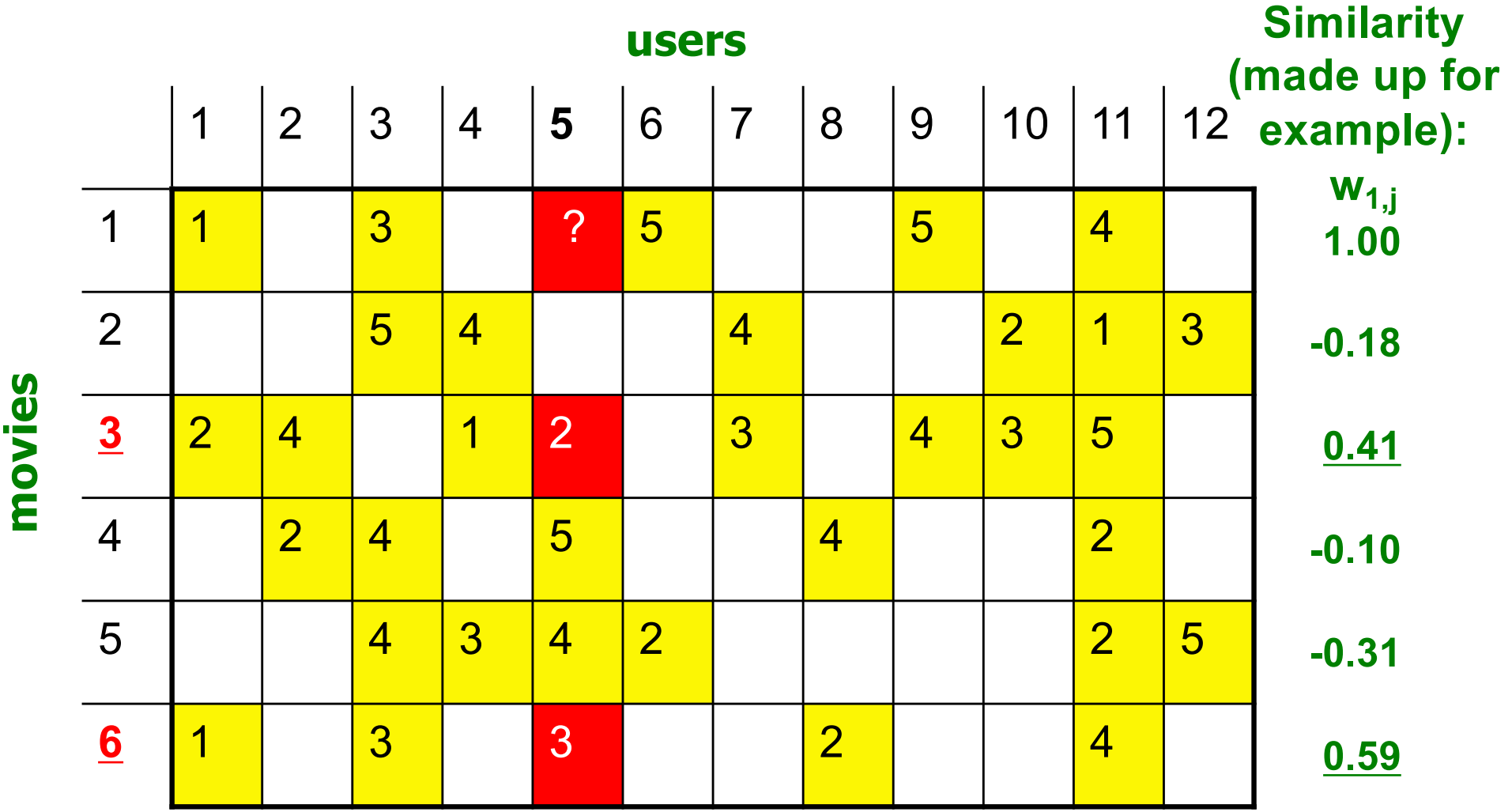


Item-Item CF (|N|=2)



Item-Item CF (|N|=2)

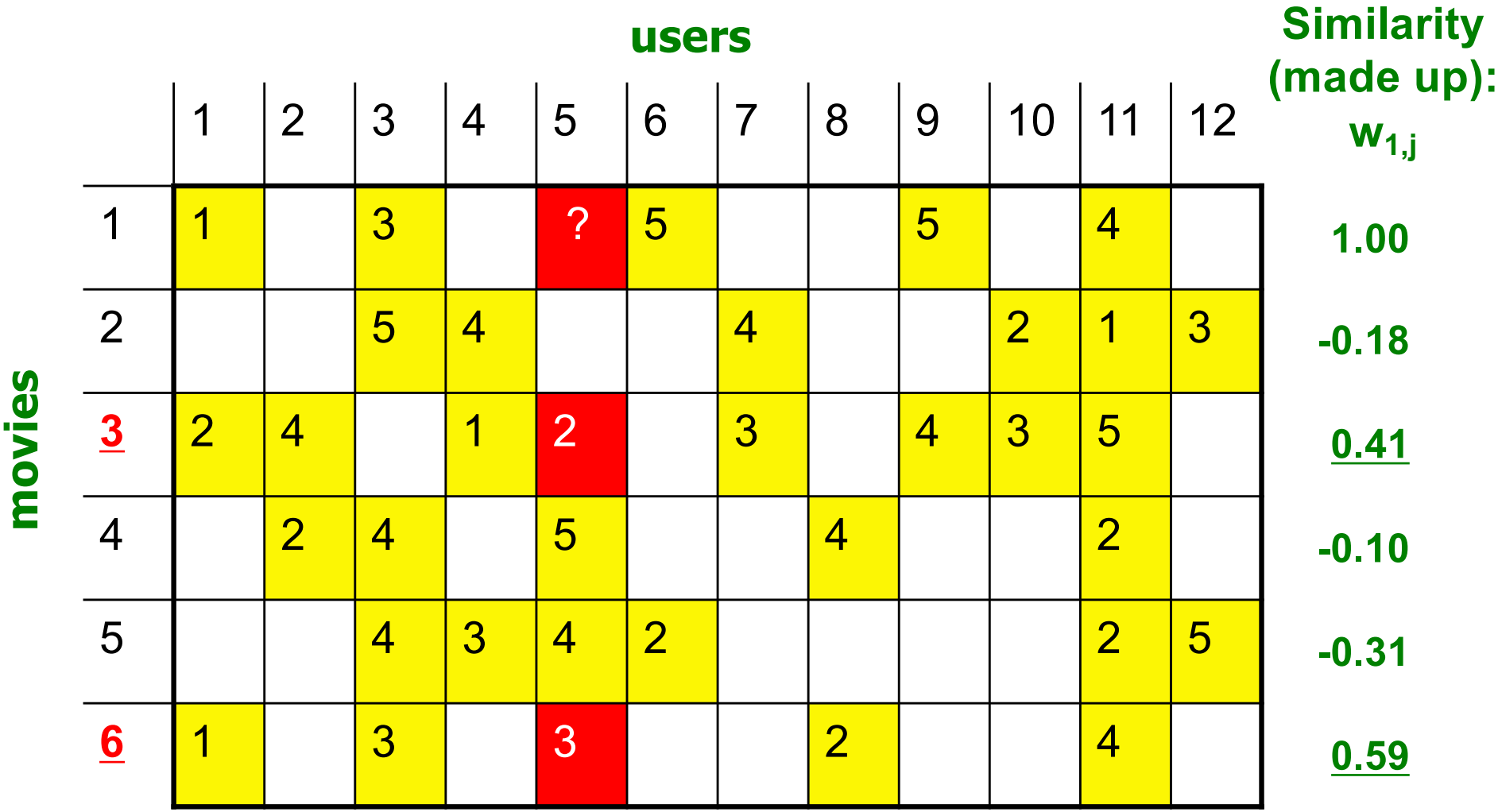
First: what is similarity between items?



Neighbor selection: Identify movies **most similar to movie 1** and **rated by user 5**

Neighborhood size is 2: pick movies 3 and 6

Item-Item CF (|N|=2)



Similarity weights:

$w_{1,3}$=0.41, $w_{1,6}$=0.59

Item-Item CF (|N|=2)

users

movies

	1	2	3	4	5	6	7	8	9	10	11	12	Similarity: $w_{1,j}$
1	1		3		2.6	5			5		4		1.00
2			5	4			4			2	1	3	-0.18
3	2	4		1	2		3		4	3	5		0.41
4		2	4		5			4			2		-0.10
5			4	3	4	2					2	5	-0.31
6	1		3		3			2			4		0.59

Predict by taking weighted average:

$$P_{u,i} = \frac{\sum_{n \in N} r_{u,n} w_{i,n}}{\sum_{n \in N} | w_{i,n} |}$$

$P_{1.5}$ = (0.41*2 + 0.59*3) / (0.41+0.59) = 2.6

Example: Item-to-Item Collaborative Filtering with Pearson Similarity

	I1	I2	I3	I4
U1	2	1		3
U2	3	?	5	2
U3		4	2	3
U4	5	3	1	

$$w_{i,j} = \frac{\sum_{u \in U} (r_{u,i} - \bar{r}_i)(r_{u,j} - \bar{r}_j)}{\sqrt{\sum_{u \in U} (r_{u,i} - \bar{r}_i)^2} \sqrt{\sum_{u \in U} (r_{u,j} - \bar{r}_j)^2}}$$

- What does set U represent?
  - *Set of users U who rated both items i, j*
- What are the members of the set U for $w_{1,2}$?
  - *U1 and U4 rated items I1 and I2*
- When calculating average ratings for item i, which ratings do we use? All ratings or just for co-rated items?
  - *We will show examples of* co-rated items
  - *We can use all ratings as well: based on all ratings: avg(r_1) = 10/3, avg(r_2) = 8/3*

Example: Item-to-Item Collaborative Filtering with Pearson Similarity

	I1	I2	I3	I4
U1	2	1		3
U2	3	?	5	2
U3		4	2	3
U4	5	3	1	

$$w_{i,j} = \frac{\sum_{u \in U} (r_{u,i} - \bar{r}_i)(r_{u,j} - \bar{r}_j)}{\sqrt{\sum_{u \in U} (r_{u,i} - \bar{r}_i)^2} \sqrt{\sum_{u \in U} (r_{u,j} - \bar{r}_j)^2}}$$

- *Based on all ratings: average ratings for items I1, I2: $r_1 = 10/3$, $r_2 = 8/3$*
- Pearson correlation for $w_{1,2}$: *Similarity between items 1 and 2*
- *Set U includes U1 and U4*
- $w_{1,2}$ = ((r$_{U1,I1}$ – 10/3)(r$_{U1,I2}$ – 8/3) + (r$_{U4,I1}$ – 10/3)(r$_{U4,I2}$ – 8/3)) /
  (sqrt((r$_{U1,I1}$ – 10/3)^2 + (r$_{U4,I1}$ – 10/3)^2) * sqrt((r$_{U1,I2}$ – 8/3)^2) + (r$_{U4,I2}$ – 8/3)^2))
- $w_{1,2}$ = (2-10/3)(1-8/3) + (5-10/3)(3-8/3)
  sqrt((2-10/3)^2 + (5-10/3)^2) * sqrt((1-8/3)^2 + (3-8/3)^2)

=2.778/3.628=0.765

Example: Item-to-Item Collaborative Filtering with Pearson Similarity

	I1	I2	I3	I4
U1	2	1		3
U2	3	?	5	2
U3		4	2	3
U4	5	3	1	

$$w_{i,j} = \frac{\sum_{u \in U} (r_{u,i} - \bar{r}_i)(r_{u,j} - \bar{r}_j)}{\sqrt{\sum_{u \in U} (r_{u,i} - \bar{r}_i)^2} \sqrt{\sum_{u \in U} (r_{u,j} - \bar{r}_j)^2}}$$

co-rated items only

- *Based on co-ratings: average ratings for items I1, I2: $r_1 = 7/2$, $r_2 = 4/2$*
- Pearson correlation for $w_{1,2}$: *Similarity between items 1 and 2*
- *Set U includes U1 and U4*
- $w_{1,2}$ = ((r$_{U1,I1}$ – 7/2)(r$_{U1,I2}$ – 4/2) + (r$_{U4,I1}$ – 7/2)(r$_{U4,I2}$ – 4/2)) /
  (sqrt((r$_{U1,I1}$ – 7/2)^2 + (r$_{U4,I1}$ – 7/2)^2) * sqrt((r$_{U1,I2}$ – 4/2)^2) + (r$_{U4,I2}$ – 4/2)^2))
- $w_{1,2}$ = (2-7/2)(1-4/2) + (5-7/2)(3-4/2)
  sqrt((2-7/2)^2 + (5-7/2)^2) * sqrt((1-4/2)^2 + (3-4/2)^2)

=3/3=1

Item-Based Prediction

	I1	I2	I3	I4
U1	2	1		3
U2	3	?	5	2
U3		4	2	3
U4	5	3	1	

$$P_{u,i} = \frac{\sum_{n \in N} r_{u,n} w_{i,n}}{\sum_{n \in N} | w_{i,n} |}$$

- If we have the following item similarities: $w_{2,1} = 0.5$, $w_{2,3} = 0.2$, $w_{2,4} = 0.3$
- Which items are in the neighborhood N for item 2 if |N| = 2?
  - Items I1 and I4
- **Predict the rating of user U2 on I2:** user = 2, item = 2, |N| = items 1,4

$$P_{2,2} = \frac{(r_{2,1} * w_{2,1}) + (r_{2,4} * w_{2,4})}{0.5 + 0.3} = \frac{3*0.5 + 2*0.3}{0.8} = 2.625$$

EXTENSIONS TO MEMORY-BASED ALGORITHMS

Extensions to Memory-Based Algorithms

- A variety of approaches/extensions have been studied to improve the performance of CF predictions
- Typically involve **modifying the similarity weights** or the **ratings** used in predictions or **guessing missing ratings**

- **User-based CF:**

$$w_{u,v} = \frac{\sum_{i \in I} (r_{u,i} - \bar{r}_u)(r_{v,i} - \bar{r}_v)}{\sqrt{\sum_{i \in I} (r_{u,i} - \bar{r}_u)^2} \sqrt{\sum_{i \in I} (r_{v,i} - \bar{r}_v)^2}} \qquad P_{a,i} = \bar{r}_a + \frac{\sum_{u \in U} (r_{u,i} - \bar{r}_u) \cdot w_{a,u}}{\sum_{u \in U} | w_{a,u} |}$$

- **Item-Based CF:**

$$w_{i,j} = \frac{\sum_{u \in U} (r_{u,i} - \bar{r}_i)(r_{u,j} - \bar{r}_j)}{\sqrt{\sum_{u \in U} (r_{u,i} - \bar{r}_i)^2} \sqrt{\sum_{u \in U} (r_{u,j} - \bar{r}_j)^2}} \qquad P_{u,i} = \frac{\sum_{n \in N} r_{u,n} w_{i,n}}{\sum_{n \in N} | w_{i,n} |}$$

Extensions to Memory-Based Algorithms: Default Voting

- In many collaborative filters, **pairwise similarity is computed only from the ratings in the intersection of the items both users have rated ("co-rated items")**
  - **Not reliable when there are too few votes** to generate similarity values (U is small)
  - Focusing on co-rated items ("intersection set similarity") also **neglects global rating behavior reflected in a user's entire rating history**
- Assuming some default voting values for the missing ratings: **can improve CF prediction performance**

Extensions to Memory-Based Algorithms: Default Voting (cont.)

Approaches to default voting values:

- Herlocker et al. accounts for small intersection sets (small number of co-rated items) by **reducing the weight of users that have fewer than 50 items in common**

$$P_{u,i} = \frac{\sum_{n \in N} r_{u,n} w_{i,n}}{\sum_{n \in N} | w_{i,n} |}$$

- Chee et al. **use average of the clique (small group of co-rated items) as a default voting** to extend a user's rating history
- Breese et al. **use a neutral or somewhat negative preference for the unobserved ratings** and then computes similarity between users on the resulting ratings data.

Extensions to Memory-Based Algorithms: Inverse User Frequency

- **Universally liked items are not as useful in capturing similarity as less common items**
- **Inverse frequency**
  - $f_j = log\ (n/n_j)$
  - **n_j is number of users who have rated item *j***
  - **n is total number of users**
- If everyone has rated item *j*, then f_j is zero
  - (Note: looks a lot like Inverse Document Frequency (IDF))
- **Approach: transform the ratings**
  - For vector similarity-based CF: **new rating = original rating multiplied by f_j**

$$P_{a,i} = \bar{r}_a + \frac{\sum_{u \in U} (r_{u,i} - \bar{r}_u) \cdot w_{a,u}}{\sum_{u \in U} |w_{a,u}|}$$

  - **For very popular items, ratings $r_{u,i}$ will be greatly reduced**
  - **Less popular items will have greater effect on prediction**

Extensions to Memory-Based Algorithms: Case Amplification

- **Transform applied to weights used in CF prediction**
- Emphasizes high weights and punishes low weights

$$w'_{i,j} = w_{i,j} \cdot \left| w_{i,j} \right|^{\rho-1}, \qquad (8)$$

where ρ is the *case amplification* power, $\rho \geq 1$, and a typical choice of ρ is 2.5 [65].

- **Reduces noise in the data**
- **Favors high weights**

$$P_{u,i} = \frac{\sum_{n \in N} r_{u,n} w_{i,n}}{\sum_{n \in N} \left| w_{i,n} \right|}$$

- Small values raised to a power become negligible
- Example:
  - For $w_{i,j} = 0.9$, weight it remains high ($0.9^{2.5} \approx 0.8$)
  - For $w_{i,j} = 0.1$, weight becomes negligible ($0.1^{2.5} \approx 0.003$)

Extensions to Memory-Based Algorithms: Imputation-Boosted CF

- **When the rating data for CF tasks are extremely sparse: hard to produce accurate predictions using the Pearson correlation-based CF**
- Su et al. proposed imputation-boosted collaborative filtering (IBCF)
- **First uses an imputation technique to fill in missing data**
- **Then use traditional Pearson correlation-based CF algorithm** on this completed data to predict a user rating for a specified item
  - **Example imputation techniques:** mean imputation, linear regression imputation, predictive mean matching imputation, Bayesian multiple imputation, and machine learning classifiers (including naıve Bayes, SVM, neural network, decision tree, lazy Bayesian rules)

Extensions to Memory-Based Algorithms: Imputation-Boosted CF (Cont'd)

Simple mean imputation

The data on the left below has one missing observation on variable 2, unit 10.

We replace this with the arithmetic average of the observed data *for that variable*. This value is shown in red in the table below.

Unit	Variables 1	2
1	3.4	5.67
2	3.9	4.81
3	2.6	4.93
4	1.9	6.21
5	2.2	6.83
6	3.3	5.61
7	1.7	5.45
8	2.4	4.94
9	2.8	5.73
10	3.6	5.58

- This approach is clearly inappropriate for categorical variables.
- It does not lead to proper estimates of measures of association or regression coefficients. Rather, associations tend to be diluted.
- In addition, variances will be wrongly estimated (typically under estimated) if the imputed values are treated as real. Thus inferences will be wrong too.

Imputation Example

Source: http://missingdata.lshtm.ac.uk/index.php?option=com_content&view=article&id=68:simple-mean-imputation&catid=39:simple-ad-hoc-methods-for-coping-with-missing-data&Itemid=96