DAT-4253 LM 8 - Prediction; Multiple Regression

AUTHOR
Aaron Younger

October 19, 2024

Business Understanding

This analysis examines the relationship between performance factors of a quarterback compared to the quarterbacks salary. The specific performance based factors used in this analysis are pass completion rate, touchdowns scored, and the quarterbacks age. The final part of the analysis identifies which players are potentially overpaid or underpaid based on the model's predictions and provides recommendations for how the model can be applied and improved for future use.

Data Understanding

Libraries

```
library(readxl)
library(tidyverse)
library(dplyr)
library(DataExplorer)
library(ggplot2)
library(caret)
library(dlookr)
library(e1071)
library(psych)
library(car)
library(stargazer)
library(tidyr)
library(purrr)
library(performance)
library(Metrics)
library(auditor)
```

Load Dataset

localhost:3344 1/29

```
tibble [32 × 5] (S3: tbl_df/tbl/data.frame)

$ Player: num [1:32] 1 2 3 4 5 6 7 8 9 10 ...

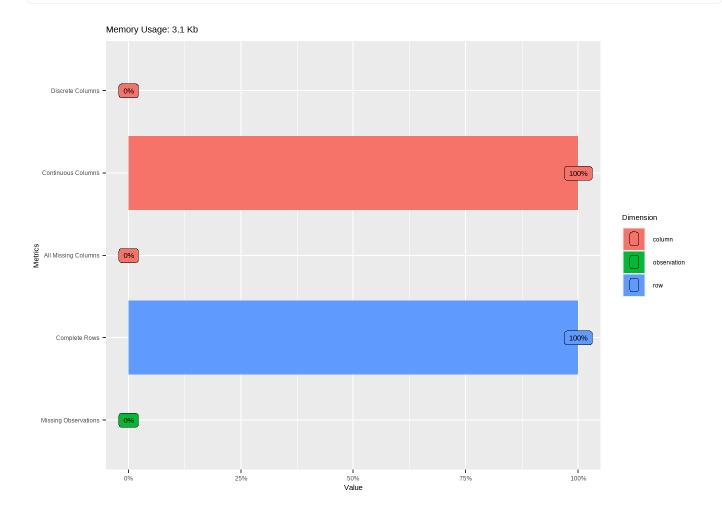
$ Salary: num [1:32] 25.6 22 20.5 19 17 ...

$ PC : num [1:32] 65.2 60.5 62.3 66.1 67.9 55 68.8 70.6 60.3 68.4 ...

$ TD : num [1:32] 28 27 27 26 29 16 33 34 22 33 ...

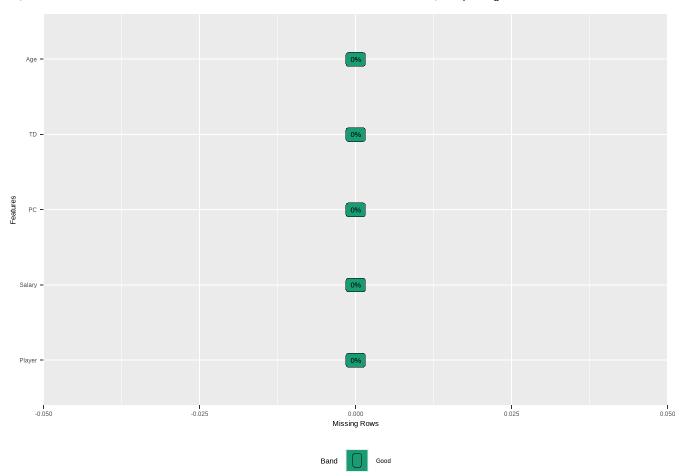
$ Age : num [1:32] 27 26 28 38 28 27 33 30 33 40 ...
```

quarterback_data %>% plot_intro()



quarterback_data %>% plot_missing()

localhost:3344 2/29



quarterback_data %>% head()

```
# A tibble: 6 \times 5
  Player Salary
                    PC
                           TD
                                Age
   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <
1
       1
           25.6 65.2
                           28
                                 27
2
           22.0 60.5
       2
                           27
                                 26
3
           20.5 62.3
                           27
                                 28
4
       4
           19.0 66.1
                           26
                                 38
5
       5
           17
                  67.9
                           29
                                 28
       6
           15.0 55
                           16
                                 27
```

quarterback_data %>% tail()

```
# A tibble: 6 × 5
  Player Salary
                    PC
                           TD
                                Age
   <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <
1
      27 2.17
                  58.7
                           10
                                 26
2
      28
          2.01
                  53.1
                           8
                                 25
          1.38
3
      29
                  54.5
                           10
                                 21
      30 1.10
                  62.1
                           21
                                 27
```

localhost:3344 3/29

5	31	0.950	60.8	12	24	
6	32	0.626	63.1	26	29	

Comments on Dataset:

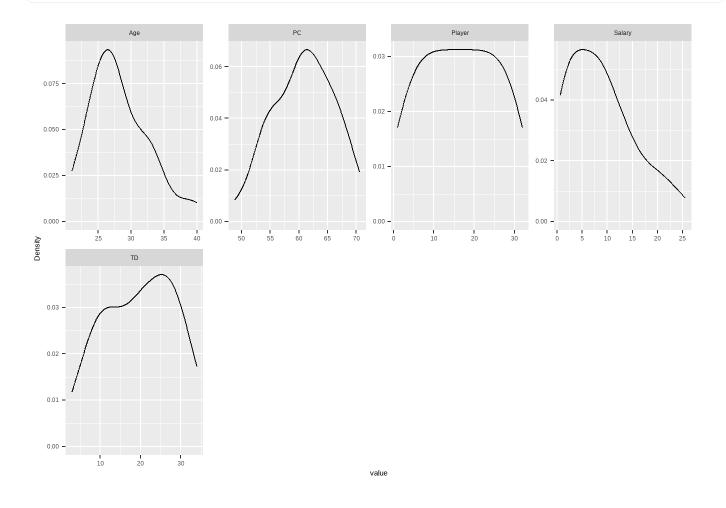
The Dataset has five variables, all of which are numeric. This dataset contains no missing values.

Dataset Variables include:

- Player: An ID identifier given for quarterbacks from all 32 teams in the NFL.
- Salary: Salary of associated quarterbacks, Salary is in millions (Dependent Variable).
- PC: Pass completion rate.
- TD: Amount of Touchdowns completed.
- Age: The age of the quarterback.

EDA

quarterback_data %>% plot_density()



```
## Look into skewness
skewness(quarterback_data$Salary) # Right Skewed
```

[1] 0.7295659

localhost:3344 4/29

```
skewness(quarterback_data$Age) # Right Skewed
```

[1] 0.6968379

```
skewness(quarterback_data$TD) # Barely left skewed
```

[1] -0.1440229

```
skewness(quarterback_data$PC) # Barely left skewed
```

[1] -0.1796121

```
## Look for outliers
diagnose_outlier(quarterback_data)
```

A tibble: 5×6

	variables	outliers_cnt	$\verb"outliers_ratio"$	${\tt outliers_mean}$	${\tt with_mean}$	${\tt without_mean}$
	<chr></chr>	<int></int>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>	<dbl></dbl>
1	Player	0	0	NaN	16.5	16.5
2	Salary	0	0	NaN	8.86	8.86
3	PC	0	0	NaN	60.8	60.8
4	TD	0	0	NaN	19.7	19.7
5	Age	0	0	NaN	28.2	28.2

```
range(quarterback_data$Salary)
```

[1] 0.62598 25.55663

```
range(quarterback_data$TD)
```

[1] 3 34

```
range(quarterback_data$Age)
```

[1] 21 40

```
range(quarterback_data$PC)
```

[1] 48.8 70.6

```
## Data deep dive
psych::describe(quarterback_data) ## Low Kurtosis values support outliers unlikely
```

```
vars n mean sd median trimmed mad min max range skew kurtosis
Player 1 32 16.50 9.38 16.50 16.50 11.86 1.00 32.00 31.00 0.00 -1.31
```

localhost:3344 5/29

```
8.18 7.41 0.63 25.56 24.93 0.73
Salary
         2 32 8.86 6.71
                           7.96
                                                                         -0.39
PC
         3 32 60.82 5.33 60.65
                                  60.89 6.38 48.80 70.60 21.80 -0.18
                                                                         -0.79
TD
         4 32 19.69 8.82 21.00
                                  19.77 10.38 3.00 34.00 31.00 -0.14
                                                                         -1.21
                                  27.88 4.45 21.00 40.00 19.00 0.70
         5 32 28.19 4.53 27.00
                                                                         -0.01
Age
        se
Player 1.66
Salary 1.19
       0.94
PC
TD
       1.56
Age
       0.80
```

Comments on EDA:

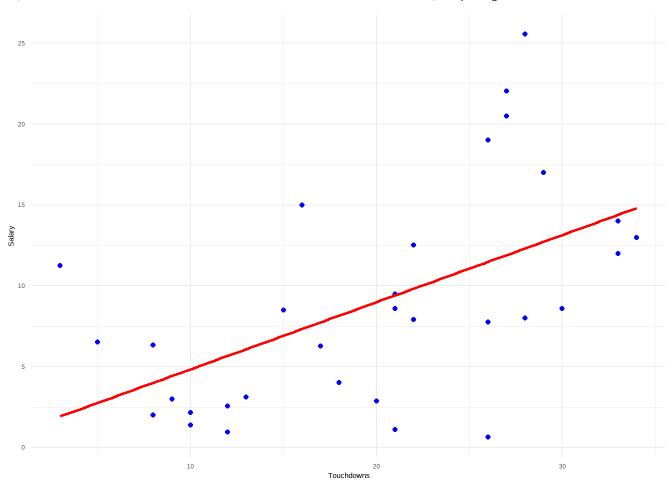
Age and Salary are right skewed, Touchdown and Pass completion are slightly negatively skewed but are fine as is. Taking log of Age and/or Salary should be considered for modeling. There are no outliers in this dataset which is supported by the variables low kurtosis values.

Relationship of Numeric Values

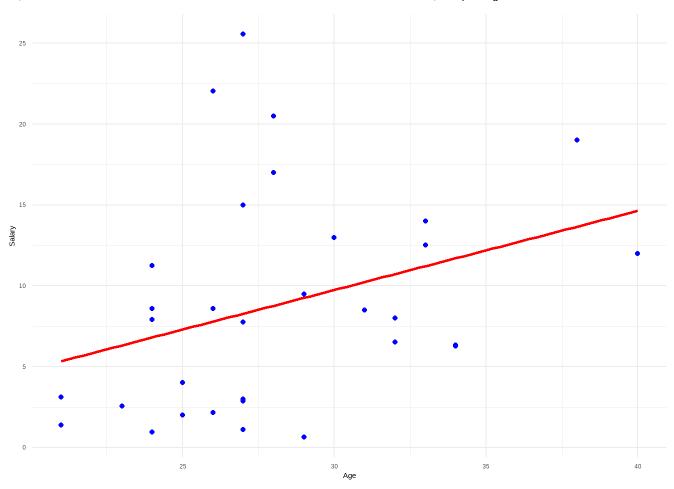
```
# Look at relationship between variables and dependent variable.
## Touchdown and Salary

ggplot(data=quarterback_data, aes(x= TD, y = Salary)) +
    geom_point(color = "blue") +
    geom_smooth(method = "lm", color = "red", se=FALSE)+
    labs(Title = "Relationship between Touchdowns and Salary",
        subitile = "Expected relationship: Positive",
        x = "Touchdowns",
        y = "Salary")+
    theme_minimal() ## Moderately Positively Correlated
```

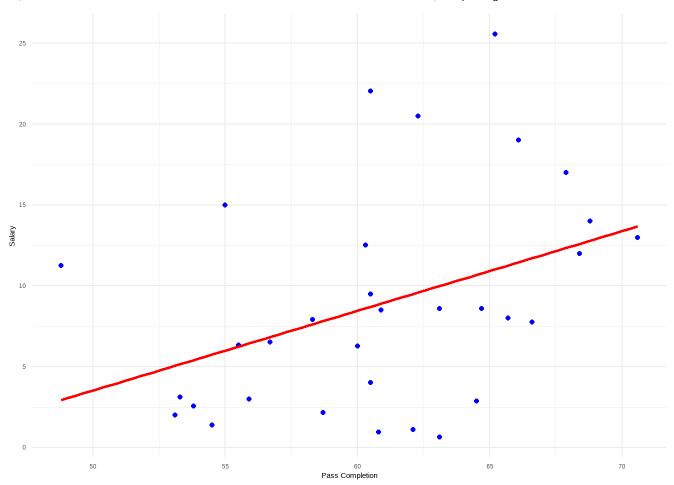
localhost:3344 6/29



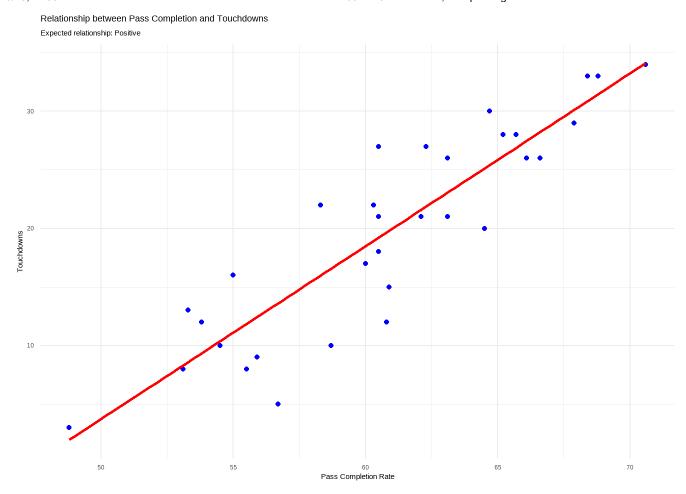
localhost:3344 7/29



localhost:3344 8/29



localhost:3344 9/29

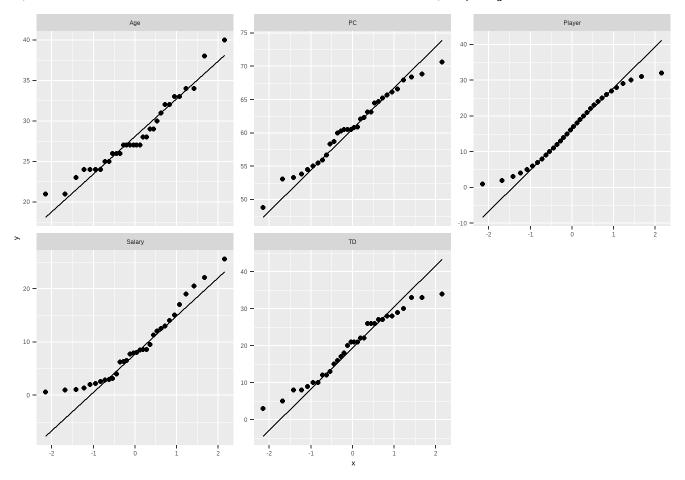


Comments on Variable Relationships:

All variables have a positive linear relationship to the dependent variable, salary, with touchdown seeming to have the strongest positive linear relationship with salary. It is also important to note that Pass completion and touchdown have a very strong positive linear relationship. This could lead to multicollinearity due to strong correlation.

```
quarterback_data %>% plot_qq()
```

localhost:3344

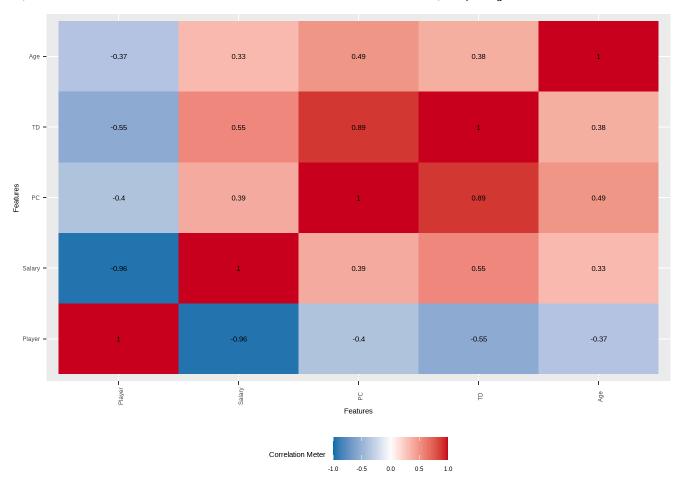


Comments on QQ Plot:

The Q-Q plots indicate that most variables are approximately normally distributed with the exception of Age and Salary. Age and Salary as seen in earlier EDA are right skewed which is supported by the Q-Q plots. It is supported since most data points cluster near the bottom and middle of the Q-Q plot.

DataExplorer::plot_correlation(quarterback_data)

localhost:3344 11/29



Comments on Correlation:

The Correlation matrix supports what was seen in earlier EDA. PC, TD, and Age all have a positive correlation with Salary. Again important to note that TD and PC have a very strong correlation which is something to remember when checking for multicollinearity.

Data Preparation

Partition Data

```
set.seed(1)
my_index <- createDataPartition(quarterback_data$Salary, p=0.75, list = FALSE)
trainset <- quarterback_data[my_index,]
testset <- quarterback_data[-my_index,]
mean(quarterback_data$Salary)</pre>
```

[1] 8.861379

```
mean(trainset$Salary)
```

localhost:3344 12/29

[1] 8.929338

```
mean(testset$Salary)
```

[1] 8.657502

Comments on Data Partition:

This dataset was partitioned into a 75/25 split, with 75% of the data being used for the training dataset and 25% of the data being used for the test dataset. Since salary is the dependent variable, the mean of salary was taken to see if the split maintained a consistent distribution of the dependent variable. Since the means of salary was close across the datasets, it is acceptable to proceed into modeling. Set.seed was used to maintain reproducible results.

Modeling

Create Models

```
# Outputs of models purposefully hidden, summary of models show in next code chunk.
## Log Salary and Age
set.seed(1)
myctrl <- trainControl(method = "CV", number = 10)</pre>
# No Variable Transformations
raw_model <- train(Salary ~ PC + TD + Age,</pre>
                    data = trainset,
                    method = "lm",
                    trControl = myctrl
)
summary(raw_model)
## Log Salary
logs_model <- train(log(Salary) ~ PC + TD + Age,</pre>
                     data = trainset,
                     method = "lm",
                     trControl = myctrl)
summary(logs_model)
## Log Age
loga_model <- train(Salary ~ PC + TD + log(Age),</pre>
                     data = trainset,
                     method = "lm",
                     trControl = myctrl)
summary(loga_model)
## Log Both
```

localhost:3344 13/29

Comments on Creating Models:

Four regression models were created. The first regression model used original, untransformed variables. The second regression model took the log of salary. The third regression model used log of age. Finally, the fourth regression model used both log of salary and log of age. Creating multiple models allowed for evaluation of which specification provided the best statistical fit and interpretability.

Compare Models

Comparison of Regression Models

	Dependent variable:Salary (Dependent Variable)			
	Raw Log Salary Log Age Lo			Log Both
	(1)	(2)	(3)	(4)
Pass Completion	-0.823	-0.092	-0.839	-0.094
	(0.545)	(0.084)	(0.550)	(0.085)
Touchdowns	0.769**	0.084	0.773**	0.085
	(0.319)	(0.049)	(0.319)	(0.049)
Age	0.112	0.037		
	(0.349)	(0.054)		
`log(Age)`			4.025	1.127
			(10.553)	(1.636)
Constant	40.276	4.687	30.928	2.079
	(26.182)	(4.054)	(36.149)	(5.605)
Observations	24	24	24	24

localhost:3344 14/29

R2	0.298	0.201	0.299	0.201		
Adjusted R2	0.192	0.081	0.194	0.081		
Residual Std. Error (df = 20)	5.920	0.917	5.914	0.917		
F Statistic (df = 3; 20)	2.824*	1.678	2.844*	1.675		
Note: *p<0.1; **p<0.05; ***p<0.01				***p<0.01		

Comments on Comparison of Models:

Four regression models were tested, each varying in whether the logarithm of Salary and/or Age was applied. Among these, the model using only the log of Age achieved the best statistical fit based on adjusted R^2 and F-statistic values. However, the model using only the log of Salary was selected. The log-salary transformation effectively addresses the right-skewed distribution of salary while allowing coefficients to be interpreted as percentage changes in salary, making the results more meaningful in practical interpretations. The log-salary model is also statistically signficant.

Model Interpretation Assuming a 10% level of significance

Goodness of Fit

```
logs_model_summary <- summary(logs_model$finalModel)
logs_model_summary</pre>
```

```
Call:
```

```
lm(formula = .outcome ~ ., data = dat)
```

Residuals:

```
Min 1Q Median 3Q Max -2.62580 -0.24334 -0.02242 0.63341 1.18324
```

Coefficients:

```
Estimate Std. Error t value Pr(>|t|) (Intercept) 4.68726 4.05447 1.156 0.261 PC -0.09205 0.08444 -1.090 0.289 TD 0.08429 0.04936 1.708 0.103 Age 0.03748 0.05401 0.694 0.496
```

```
Residual standard error: 0.9167 on 20 degrees of freedom Multiple R-squared: 0.2011, Adjusted R-squared: 0.08124 F-statistic: 1.678 on 3 and 20 DF, p-value: 0.2037
```

Comments on Goodness of Fit:

Goodness of fit metric interpretation:

The p-value on the F-statistic is <.1 showing the model is significant.

The model explains 20.1% of the variation in Spend.

The standard error of the estimate is \$0.917. This means that on average, the predicted log-salary values differ from the actual log-salary values by about 0.837 log units. However converting back into real salary

localhost:3344 15/29

terms, e^-0.837 and e^0.837, actual salaries are typically between 0.43 and 2.31 times the predicted salary. Further converting this into a percentage, 2.31 - 1, actual salaries are roughly + or - 130% from predicted values on average. This shows moderate level of uncertainty when predicting but can be expected due to the spread of salary.

Coefficient Significance Interpretation:

One Variable is statistically significant, and that is the TD (touchdown) variable.

Coefficient Value Interpretation:

- PC: For every one unit increase there is on average a -0.092% decrease in salary ceterus paribus.
- TD: For every one unit increase in touchdown, there is on average a 0.084% increase in salary, ceterus paribus.
- Age: For every one unit increase in Age, there is on average a 0.037% increase in salary, ceterus paribus.

Multicollinearity

```
check_collinearity(logs_model$finalModel)
```

Check for Multicollinearity

Low Correlation

```
Term VIF VIF 95% CI adj. VIF Tolerance Tolerance 95% CI Age 1.30 [1.06, 2.48] 1.14 0.77 [0.40, 0.94]
```

Moderate Correlation

```
Term VIF VIF 95% CI adj. VIF Tolerance Tolerance 95% CI PC 6.09 [3.64, 10.81] 2.47 0.16 [0.09, 0.27] TD 5.60 [3.37, 9.93] 2.37 0.18 [0.10, 0.30]
```

Comments on Multicollinearity:

- Age: Age shows low correlation with a VIF value of 1.33. The Tolerance value is 0.75 meaning age has low overlap with other variables in the dataset.
- TD: TD has moderate correlation with a VIF value of 4.76. The Tolerance value is 0.21 showing high overlap with other variables in the dataset. This is most likely with PC, as seen in EDA both showed strong correlation between each other.
- PC: PC has moderate correlation with a VIF value of 5.39. The Tolerance value is 0.19 showing high overlap with other variables in the dataset. This is most likely with TD, as seen in EDA both showed strong correlation between each other.

Although PC and TD have a moderately high VIF value both values should be kept. They are both different quarterback performance metrics and both metrics involve passing in them which explains some of the overlap.

Residual Analysis

localhost:3344 16/29

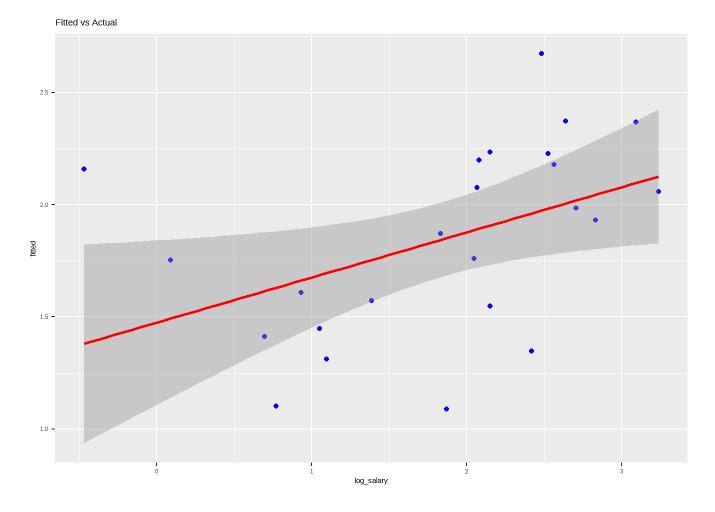
Create Dataset for Residual analysis

Comments on dataset creation for residual analysis:

To perform residual analysis, both the fitted values and residuals from the model were stored in a new dataset. Since the regression model was trained on the logarithm of Salary, the log of Salary variable was also added to the dataset to ensure consistency between the model scale and the residual diagnostics.

Fitted vs Actual (looking for linear relationship)

```
ggplot(residual_data, aes(x = log_salary, y = fitted)) +
geom_point(color = "blue")+
geom_smooth(method = "lm", color = "red", se = TRUE)+
labs(title = "Fitted vs Actual")
```



Comments on fitted vs actual plot:

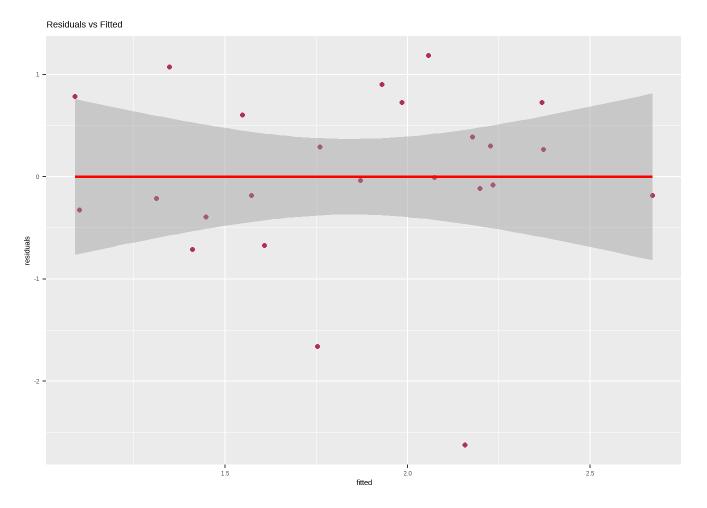
This graph compares predicted log-salary values to actual log-salary values to see how closely the model

localhost:3344 17/29

can replicate the observed data. The points follow an upward linear trend with some scatter around the line. This shows the model captures the main relationship between predictors and salary reasonbly well.

Residual vs Fitted (looking for no change in variability)

```
ggplot(residual_data, aes(x = fitted, y = residuals)) +
  geom_point(color = "maroon")+
  geom_smooth(method = "lm", color = "red", se=TRUE)+
  labs(title = "Residuals vs Fitted")
```



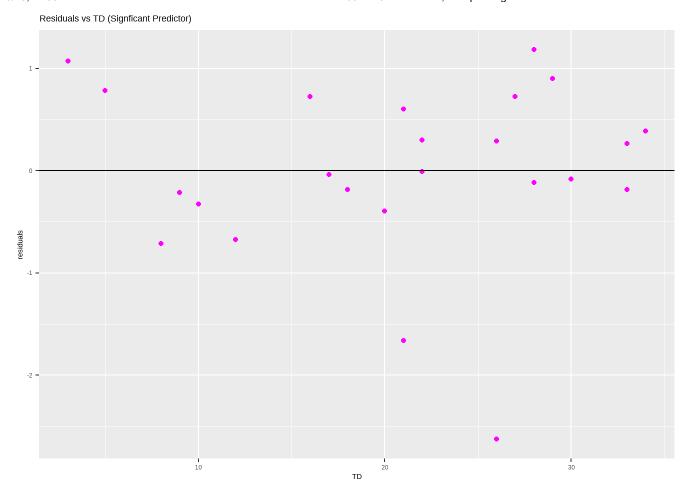
Comments on Residual vs Fitted:

This graph shows residuals plotted against fitted values to test for linearity. The residuals in this graph are randomly scattered around the zero line without strong patterns, showing the model satisfies linearity assumption.

Residuals vs Predictor (want linear pattern)

```
ggplot(residual_data, aes(x = TD, y = residuals))+
  geom_point(color="magenta") +
  geom_hline(yintercept = 0)+
  labs(title = "Residuals vs TD (Signficant Predictor)")
```

localhost:3344 18/29



Comments on Residuals vs Predictor:

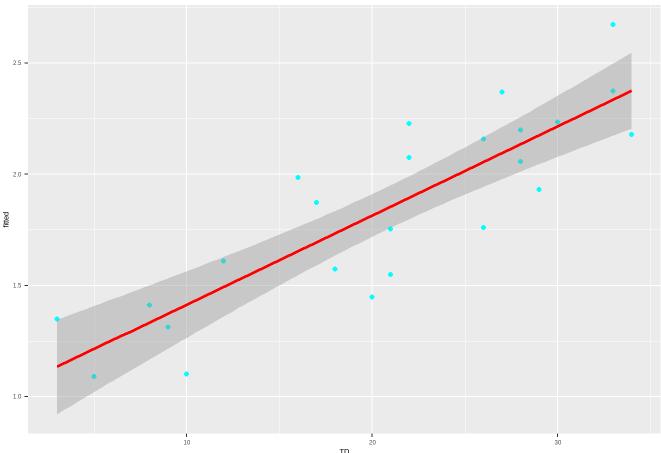
Since there was only one significant predictor variable in my model, TD, there was only one residual vs predictor graph made. This graph checks to see whether residuals have any visible trend when plotted against the TD variable. The residuals are randomly dispersed around around the zero line, showing that the relationship between TD and log-salary is captured well.

Fitted vs Predictor (want a linear pattern)

```
ggplot(residual_data, aes(x = TD, y = fitted)) +
  geom_point(color="cyan")+
  geom_smooth(method = "lm", color = "red", se = TRUE)+
  labs(title = "Fitted vs TD (Significant Predictor)")
```

localhost:3344





Comments on Fitted vs Predictor:

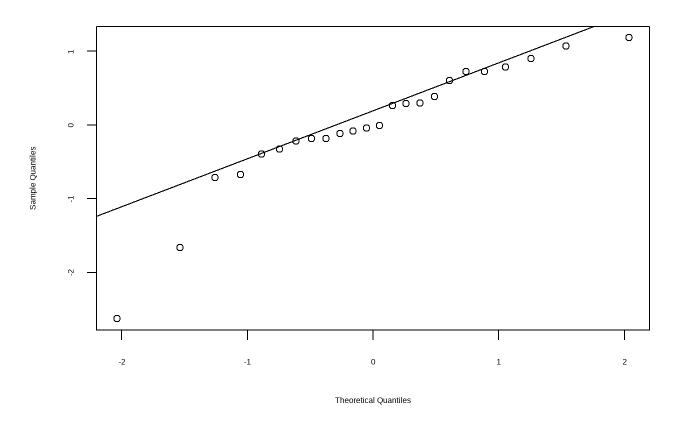
This plot displays the fitted log-salary values against the number of touchdowns variable to show the relationship between the two. The graph shows an upward linear trend which supports a positive relationship between the two variables. Confirming that a higher touchdown count leads to a higher predicted salary.

QQ-Plot (want dots along the line - indicates normal distribution)

```
res <- resid(logs_model$finalModel)</pre>
qqnorm(res)
qqline(res)
```

20/29 localhost:3344

Normal Q-Q Plot



Comments on QQ-Plot:

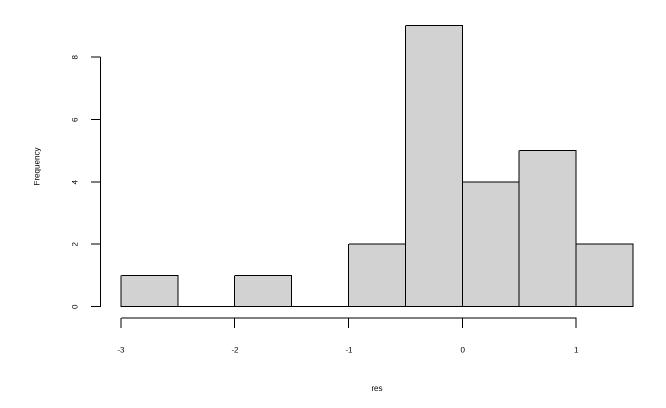
The Q-Q plot compares distribution of standardized residuals to a normal distribution line. The points closely follow the normal distribution line showing an approximate normal distributions, supporting normality.

Histogram of Residuals (want normal distribution)

```
# Distribution of residuals
hist(res)
```

localhost:3344 21/29

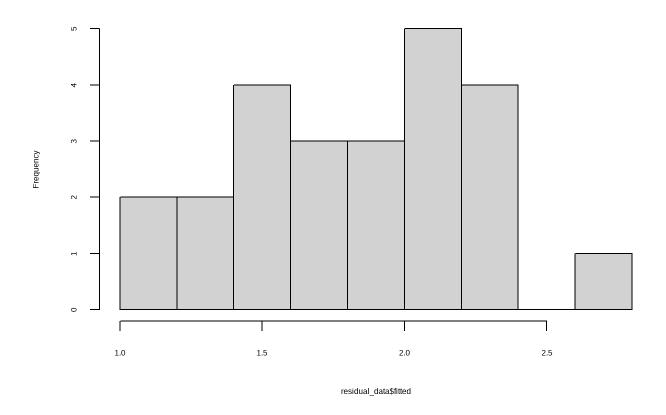
Histogram of res



Distribution of fitted values
hist(residual_data\$fitted)

localhost:3344 22/29

Histogram of residual_data\$fitted



skewness(res)

[1] -1.218944

skewness(residual_data\$fitted)

[1] -0.1141671

Comments on Histograms (residuals and fitted Values):

The histograms display the distributions of residuals and fitted values to show any potential spread. Although both histograms are not normally distributed, there is no severe skewness.

Does Model Meet OLS Assumptions

Comments on Model meeting OLS Assumptions:

Based on the residual plots and model statistics all major OLS assumptions are satisfied.

- 1. Linearity The Model is linear because the coefficients enter the model linearly. Taking log of salary does not violage this assumption.
- 2. Random Sampling This dataset takes the entire population of quarterbacks so sampling was not necessarily needed.
- 3. No perfect Multicollinearity Predictors are not exact linear combinations of each other, only moderate

localhost:3344 23/29

correlation exists between PC and TD.

- 4. Zero Conditional Mean of Erros Residuals are randomly scattered around the zero line in residuals vs fitted plot, no obvious trends so no sign of systematic error.
- 5. Homescedasticity The Variance of residuals is constant across all fitted values.
- 6. No Serial Correlation Non-Applicable, this data is cross-sectional not time-series.
- 7. Exogeneity The random residual pattern against TD and fitted values appear unrelated to errors.
- 8. Normality of Errors The Q-Q plot shows slight deviation at the etails, and the histogram is not perfrectly symmetric. This shows slight non-normality, but is acceptable.

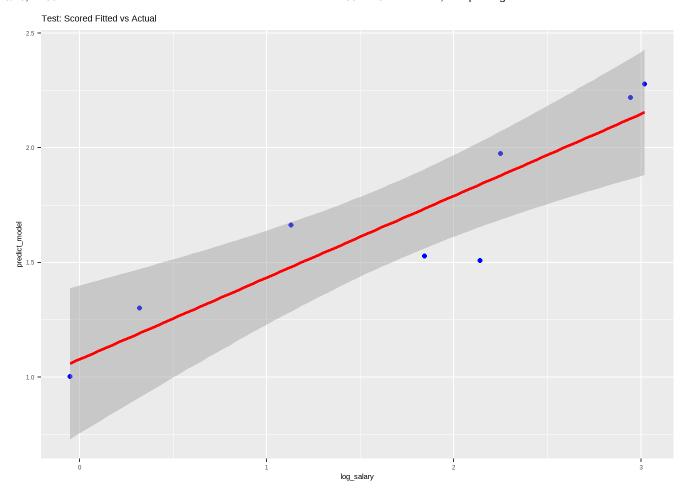
Evaluation

```
predict_model <- predict(logs_model, newdata=testset)
model_predict <- cbind(testset, predict_model) %>%
    mutate(log_salary = log(Salary))

View(model_predict)

# Fitted vs Actual
ggplot(model_predict, aes(x = log_salary, y = predict_model)) +
    geom_point(color = "blue") +
    geom_smooth(method = "lm", color = "red", se = TRUE) +
    labs(title = "Test: Scored Fitted vs Actual")
```

localhost:3344 24/29



Evaluation Metrics

RMSE: 0.71

localhost:3344 25/29

```
cat("MAE :", MAE,"\n")

MAE : 0.66

cat("MAD :", MAD,"\n")
```

MAD : 0.65

```
cat("MAPE:", round(MAPE*100,2),"%\n") ## Refresh Memory on what these mean
```

MAPE: 315.7 %

Comments on Evaluation Metrics:

Since the model predicts log of salary, the above error metrics represent proportional rather than absolute deviations.

- RMSE (0.33): The Model's predictions typically deviate from actual salaries by 0.33 log unites or +/-39% in real terms. $e^0.33 = 1.39 1 = 0.39$ or 39%.
- MAE (0.26): The Average absolute prediction error is 0.26 log units, meaning predicted salaries are about 30% off actual values on average.
- MAD (0.31): The average deviation of residuals from the fitted line is 0.31, showing a moderate level of prediction error.
- MAPE (17.95%): On average, the model's salary predctions are within about 18% of the actual values.

Evaluation REC

```
## Add log of salary to testset

testset <- testset %>%
   mutate(log_salary = log(Salary))

View(testset)

lm_audit <- audit(logs_model, data = testset, y = testset$log_salary)</pre>
```

Preparation of a new explainer is initiated

```
-> model label : train.formula ( default )

-> data : 8 rows 6 cols

-> data : tibble converted into a data.frame

-> target variable : 8 values

-> predict function : yhat.train will be used ( default )

-> predicted values : No value for predict function target column. ( default )

-> model_info : package caret , ver. 6.0.94 , task regression ( default )

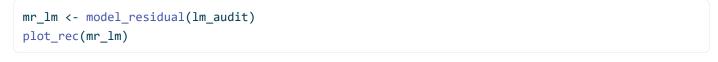
-> predicted values : numerical, min = 1.001657 , mean = 1.684076 , max = 2.277801

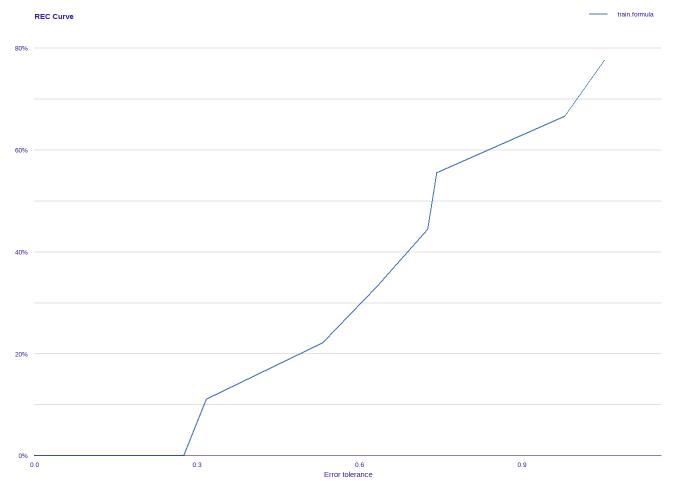
-> residual function : difference between y and yhat ( default )

-> residuals : numerical, min = -1.052592 , mean = 0.01636467 , max = 0.7426241

A new explainer has been created!
```

localhost:3344 26/29





```
score_rec(lm_audit)
```

rec: 0.5104055

Comments on REC Value:

The REC Area is relatively low at 0.20, the closer the value is to 1 the better. I think this number is acceptable however due to the smaller dataset and the range of quarterback salaries.

Deployment

Predict Players Based off Model

localhost:3344 27/29

Call:

```
lm(formula = .outcome ~ ., data = dat)
Residuals:
              1Q
                   Median
                                3Q
                                        Max
-2.70738 -0.29993 0.03669 0.60311 1.21737
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 4.80081 3.15421
                                 1.522
                                        0.1392
PC
           -0.12010
                       0.06669 -1.801
                                        0.0825 .
TD
            0.10090
                       0.03793
                                 2.660
                                       0.0128 *
Age
            0.08251
                       0.03853 2.141 0.0411 *
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.8367 on 28 degrees of freedom
Multiple R-squared: 0.3523,
                               Adjusted R-squared: 0.2829
F-statistic: 5.077 on 3 and 28 DF, p-value: 0.006221
```

```
# A tibble: 2 × 4
```

Comments on Pay based off the Model:

As part of this analysis the general manager wants to know if based on the influences that affect salary if players 8 and 16 are being over or underpayed.

- Player 8: Based on the models predicted salary, player 8 is being overpayed by about 3.7 million dollars with his predicted salary being about 9.2 million dollars but his actual salary being about 12.9 million dollars.
- Player 16: Based on the models predicted salary, player 16 is being underpayed by about 2.7 million

localhost:3344 28/29

dollars with his predicted salary being about 10.7 million dollars but his acutal salary being about 8 million dollars.

Recommendations to the General Manager

The linear regression model is a useful tool for evaluating performance metrics relating to salary. It has relatively strong accuracy and moderately low prediction error. However, this model should not be the only thing used in decision making. The model accounted for low variability in the dataset. The dataset this model uses also does not account for all factor's of quarterback salary such as leadership qualities and team fit.

localhost:3344 29/29