PT Symmetry Breaking and Nonlinear Optical Isolation in Coupled Microcavities

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Outline

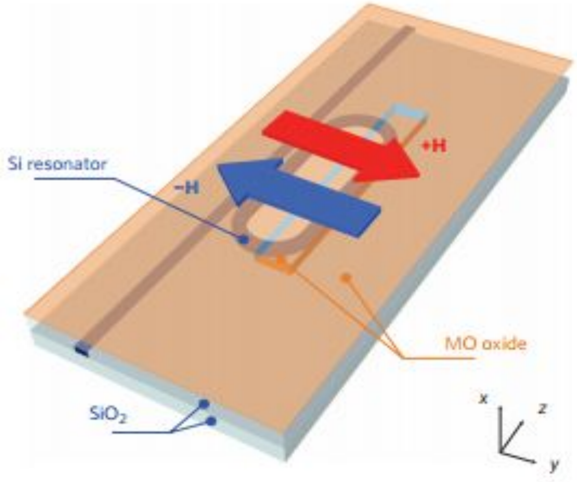
- Optical isolator and PT symmetry
- Optical isolation in PT symmetric coupled microcavities
- Stability of the PT symmetric coupled microcavities
- Relation between PT symmetry and isolaton ratio in PT symmetric coupled microcavities

Optical isolator

Optical isolator

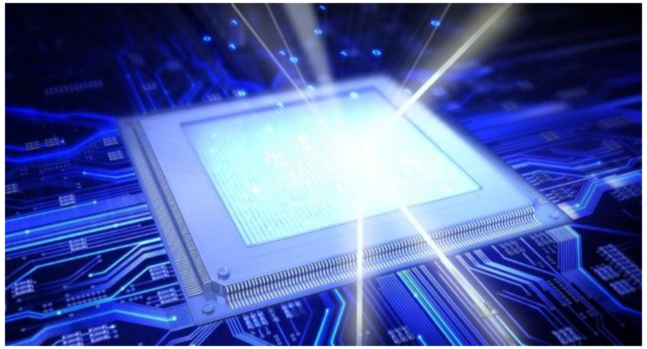
Commercial optical isolator (Faraday rotator)

On-chip optical isolator

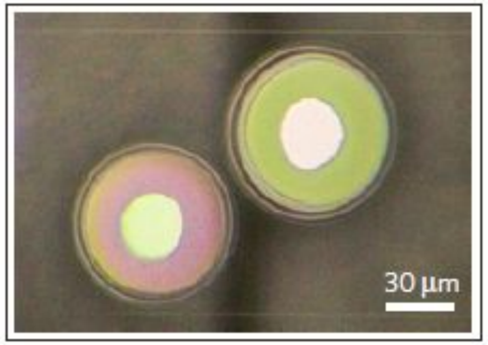


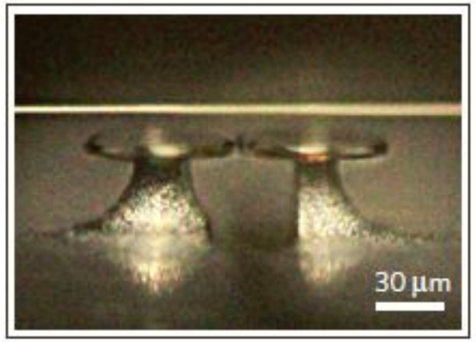
A schematic picture of on-chip isolator

Possible Application:Optical computer

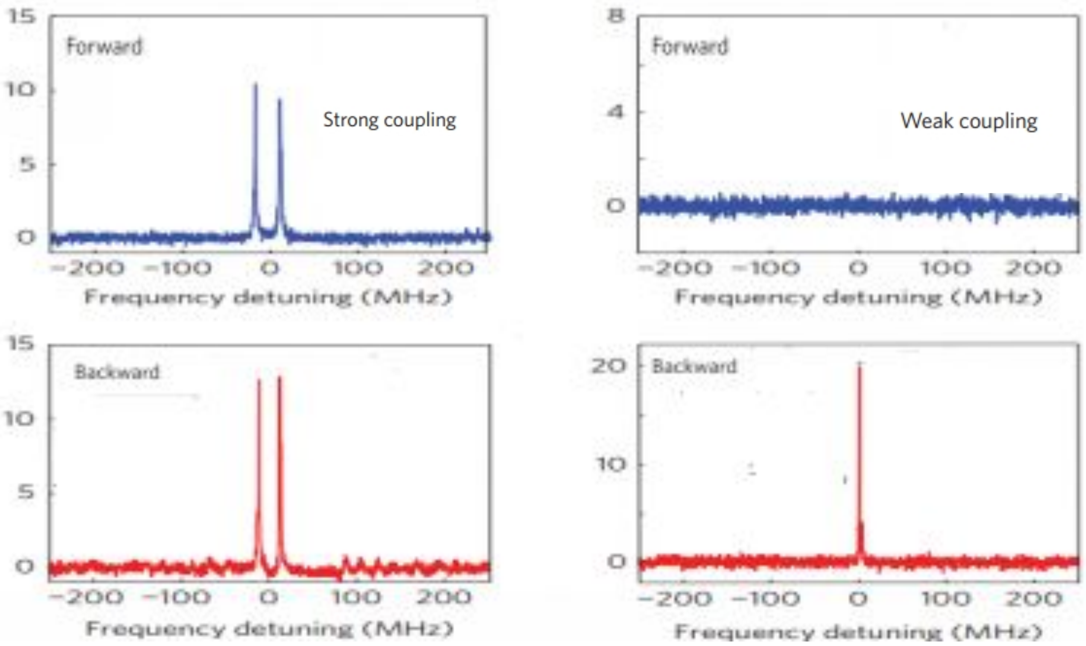


On-chip Optical Isolation





B. Peng et al.:Nature Phys. 10, 5 (2014).



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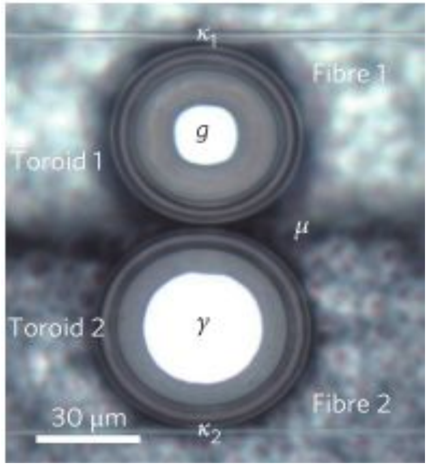
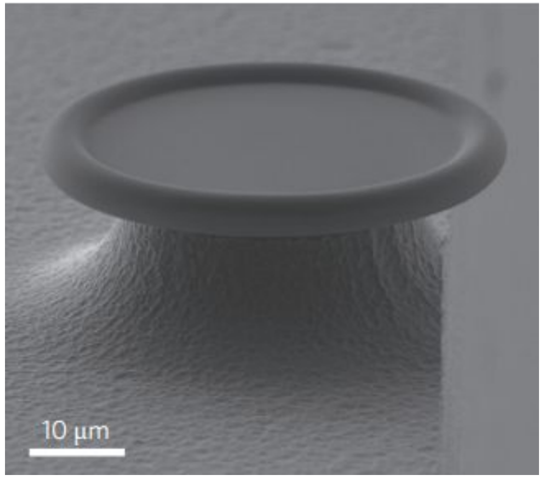
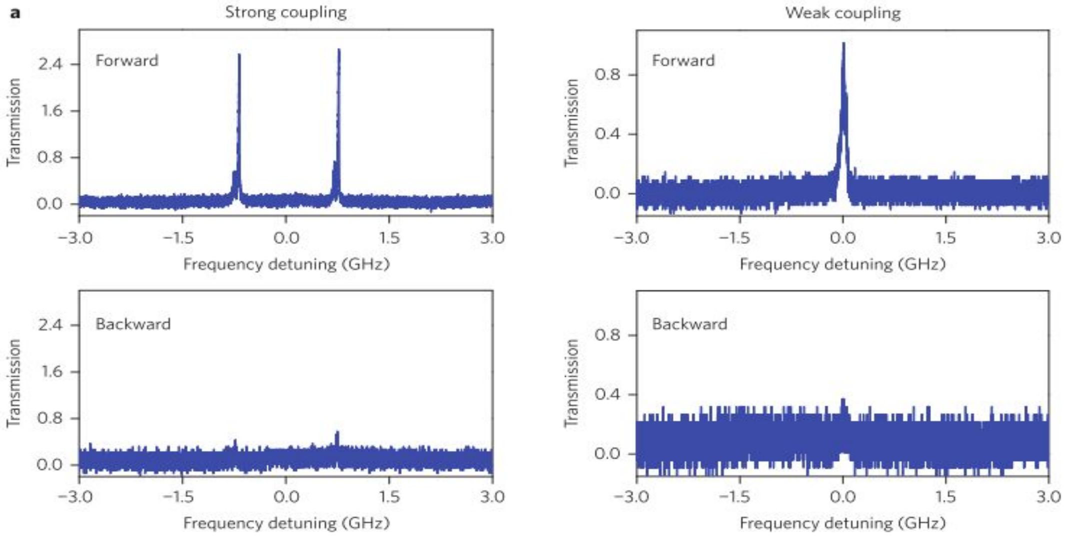




Long Chang et al.: Nature Photonics 8.7 (2014), pp. 524–529.



Long Chang et al.: Nature Photonics 8.7 (2014), pp. 524–529.

- record low power of 1 μW for observing isolation
- extend PT-symmtric optics from cetimeter/meter

-sclae to on-chip micro-scale structure

Parity-Time Symmetry

$$PT\hat{H} = \hat{H}PT.$$

P(space reflection): $\tilde{p} \to -\tilde{p}$ and $\tilde{x} \to -\tilde{x}$
T(time reversal): $\tilde{p} \to -\tilde{p}$, $\tilde{x} \to \tilde{x}$ and $i \to -i$

$$H = \begin{pmatrix} e^{i\theta} & g \\ g & e^{-i\theta} \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

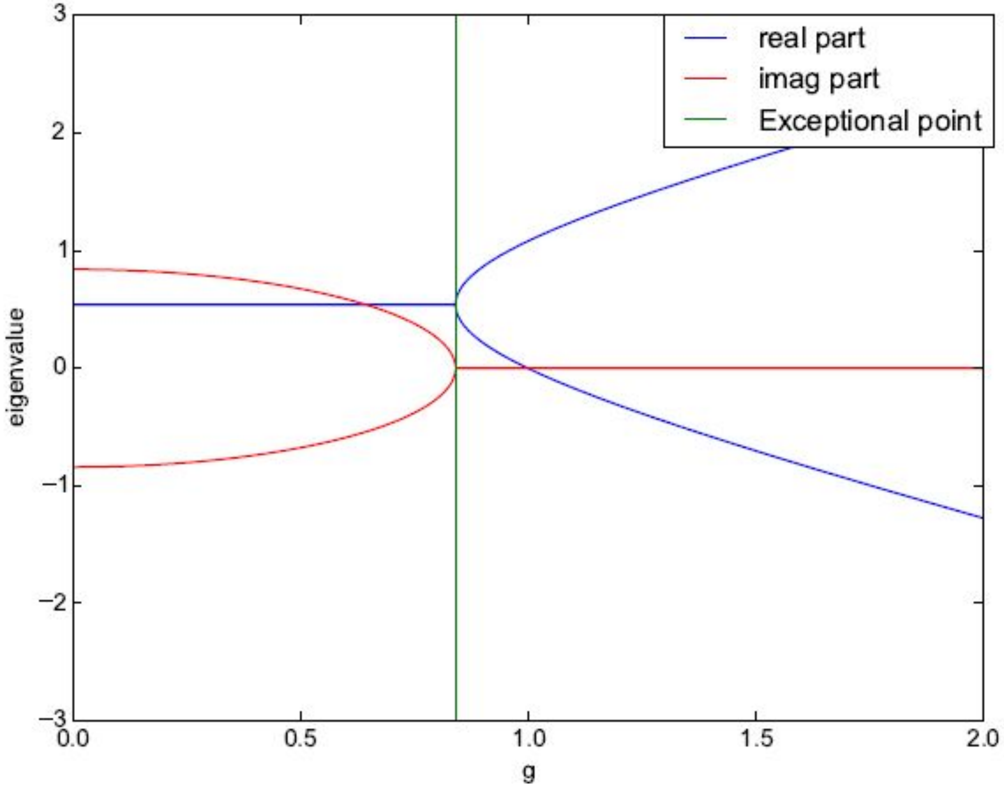
T performs complex conjugation

C. M. Bender, M. V. Berry and A. Mandilara, J. Phys. A: Math. Gen. 35, L467 (2002).

$$\lambda = \cos\theta \pm \sqrt{g^2 - \sin^2\theta}$$

Real eigenvalue

Spontaneous symmetry breaking



Application in optics

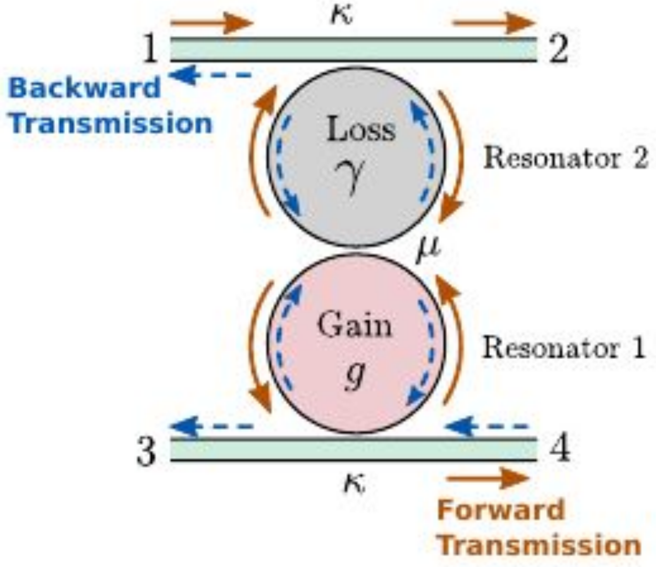
- Loss-induced transparency
- Power oscillations violating left-right symmetry
- Unidirectional invisibility
- Coherent perfect laser absorbtion
- Single mode laser

nonlinearity → Not PT-symmetric

How will the PT-symmetric property of the underlying linear system influence the isolation phenomena？

Model

Using coupled mode theory



Forward transmission

$$\frac{da_1}{dt} = (i\Delta\omega + g)a_1 - i\mu a_2$$
$$\frac{da_2}{dt} = (i\Delta\omega - \gamma)a_2 - i\mu a_1 + \sqrt{\kappa}s_{\text{in}}$$
$$I_F = \kappa|a_1|^2$$

Backward transmission

$$\frac{da_1}{dt} = (i\Delta\omega + g)a_1 - i\mu a_2 + \sqrt{\kappa}\, s_{\text{in}}$$
$$\frac{da_2}{dt} = (i\Delta\omega - \gamma)a_2 - i\mu a_1$$
$$I_B = \kappa\, |a_2|^2$$

Gain saturation term $$g = \frac{1}{2}\left(\frac{g_0}{1 + |a_1/a_s|^2} - \gamma_0 - \kappa\right)$$
$$\gamma = \frac{1}{2}(\gamma_0 + \kappa)$$

At steady state

Forward transmission

$$(i\Delta\omega + g)a_1 - i\mu a_2 = 0$$
$$(i\Delta\omega - \gamma)a_2 - i\mu a_1 + \sqrt{\kappa}s_{\text{in}} = 0$$

Backward transmission

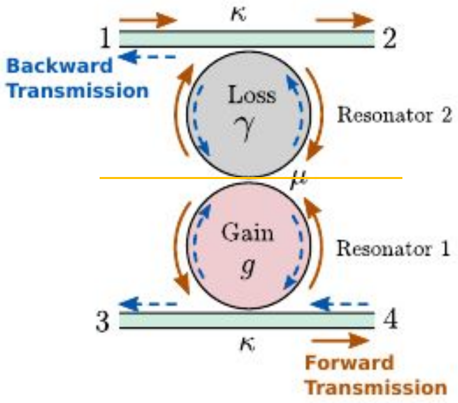
$$(i\Delta\omega + g)a_1 - i\mu a_2 + \sqrt{\kappa}\, s_{\text{in}} = 0$$
$$(i\Delta\omega - \gamma)a_2 - i\mu a_1 = 0$$

Isolation Ratio: $$R = \frac{I_F(I_{\text{in}})}{I_B(I_{\text{in}})}$$

Linear operation

$a_s \to \infty \quad g = \gamma$

the gain and loss resonators are to be PT symmetric



Eigenfrequency $$\Delta\omega = i\frac{g-\gamma}{2} \pm \sqrt{\mu^2 - \gamma g - \left(\frac{g-\gamma}{2}\right)^2}$$ $$\Delta\omega = \pm\sqrt{\mu^2 - \gamma^2}$$

$\mu > \gamma$

$\Delta\omega = \pm\sqrt{\mu^2 - \gamma^2}$

PT-symmetric phase

$\mu = \gamma$

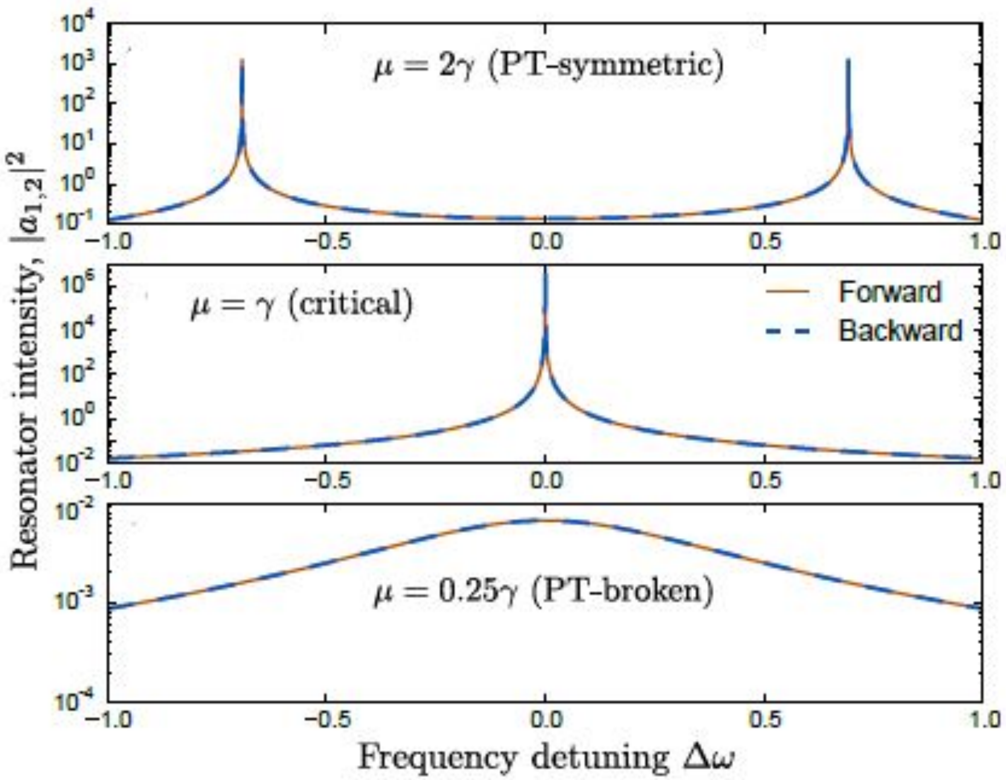
$\Delta\omega = 0$

Exceptional point

$\mu < \gamma$

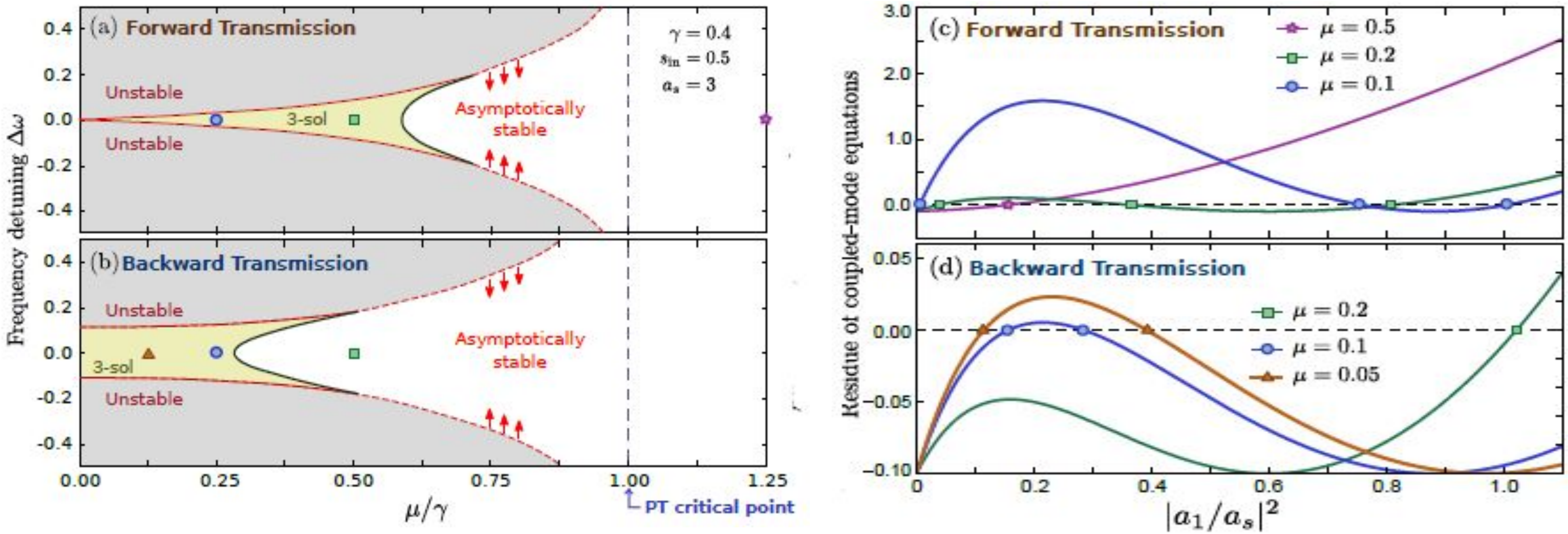
$\Delta\omega = \pm i\sqrt{\gamma^2 - \mu^2}$

PT-symmetric broken phase



Nonlinear Operation

Uniqueness of the steady state solution



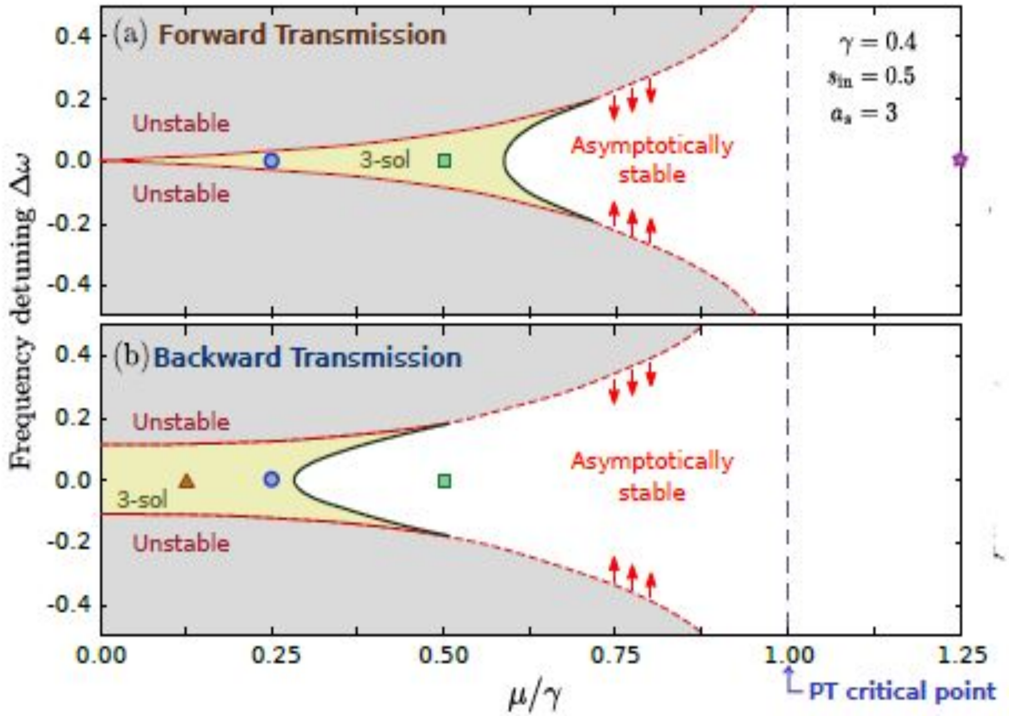
Yellow region: More than one steady state solution

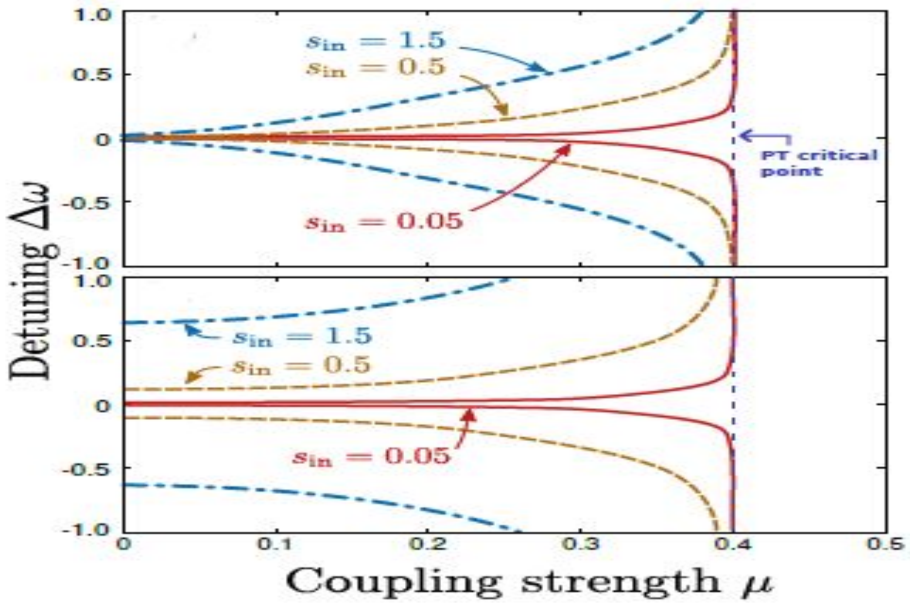
Grey and white region: Only one steady state solution

Emergence of multi-solution

Stability of the steady state solution

Using the Lyapunov exponent analysis, stability of the steady state solution can be determined .





Yellow region: Three solutions, the highest solution is stable
White region: One solution and it is stable
Grey region: One solution and it is not stable stable

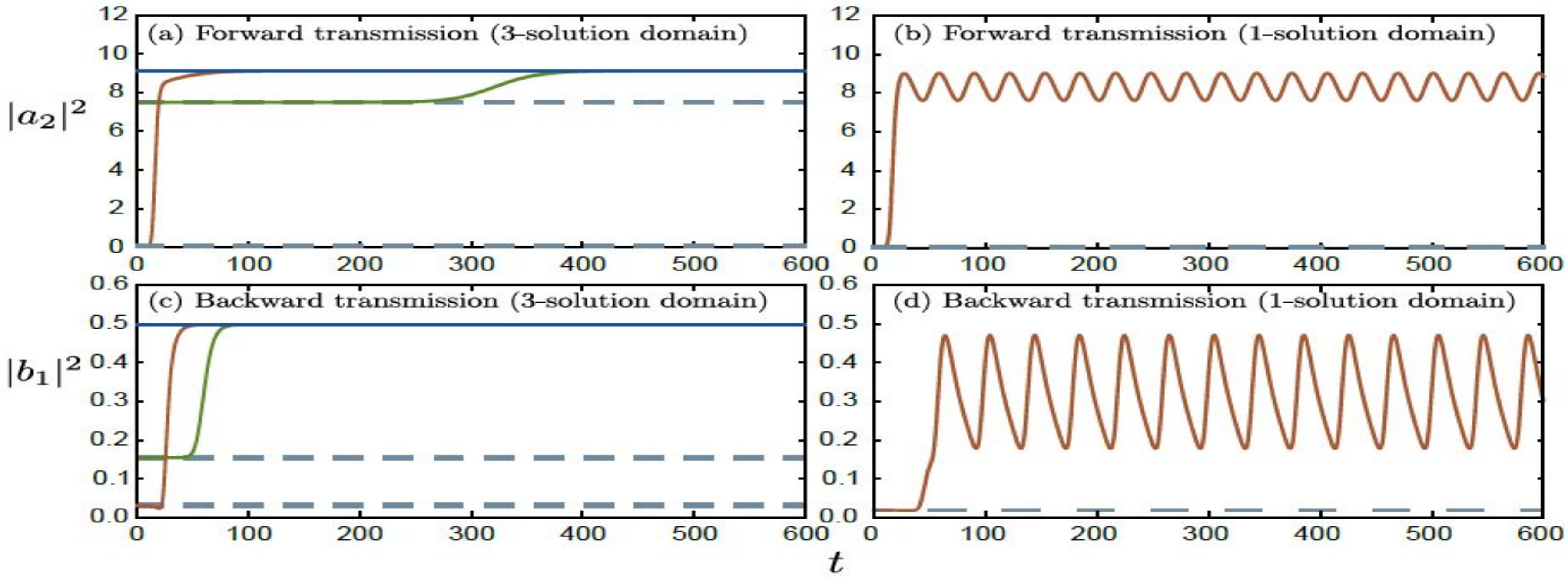
Always stable at zero detuning

Boundary of asymptotically stable region diverge at PT critical point of the underlying PT-symmetric linear system

Stability of the steady state solution

$$\frac{da_1}{dt} = \left(i\Delta\omega - \gamma + \frac{2\gamma}{1+|a_1/a_s|^2}\right) a_1(t) - i\mu a_2(t),$$

$$\frac{da_2}{dt} = (i\Delta\omega - \gamma)\, a_2(t) - i\mu a_1(t) + \sqrt{\kappa_2}\, s_{\text{in}}.$$



Relation between Isolation Ratio and PT-symmetry

Influence of the PT-symmetric property of the underlying linear system to the isolation ratio.

At zero detuning $\Delta\omega = 0$ and equal coupling $\kappa_1 = \kappa_2$

$$|\alpha|^2 x^3 + \left(2|\alpha-1|^2 - 2 - \beta\right)x^2 + \left(|\alpha-2|^2 - 2\beta\right)x - \beta = 0$$

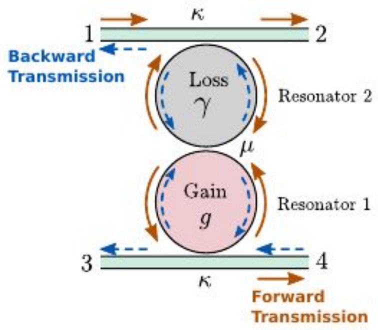
$$\alpha = 1 + \left(\frac{\mu}{\gamma}\right)^2$$

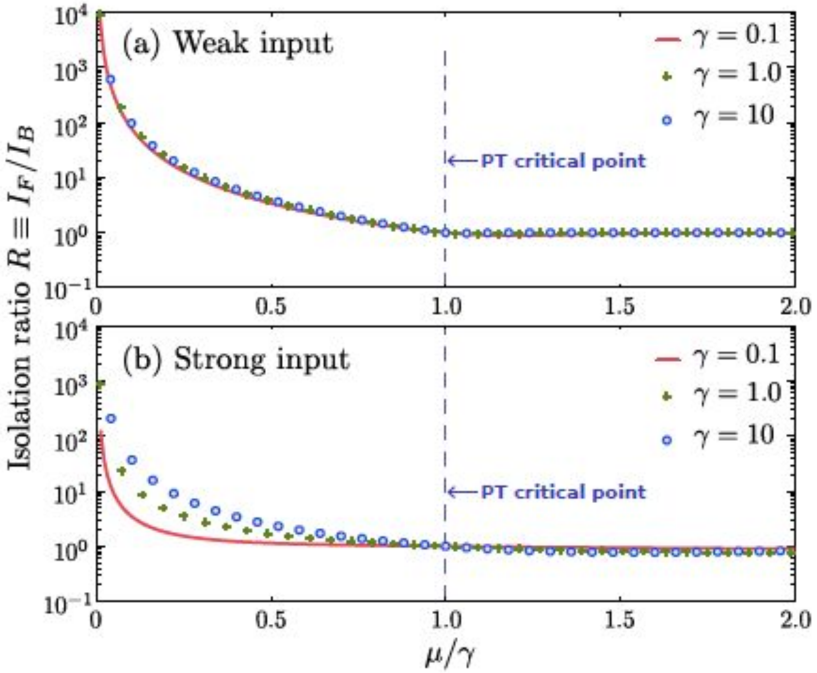
$$\beta = \frac{|s_{\text{in}}/a_s|^2}{\gamma} \times \begin{cases} \left(\frac{\mu}{\gamma}\right)^2, & \text{(Forward)} \\ 1, & \text{(Backward).} \end{cases}$$

$$x = \left|\frac{a_1}{a_s}\right|^2$$

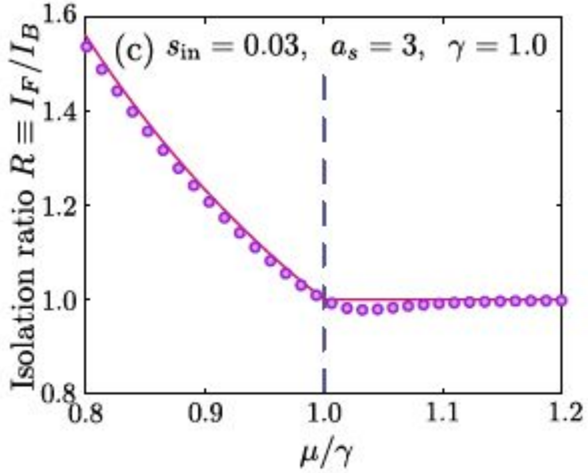
At weak input limit: $s_{\text{in}} \ll \sqrt{\gamma} a_s$

the steady state behaviour will be principally determined by μ/γ .





Kink behaviour happens around the PT critical point



Relation between Isolation Ratio and PT-symmetry

why kink happens

$$|\alpha|^2 x^3 + \left(2|\alpha-1|^2 - 2 - \beta\right)x^2 + \left(|\alpha-2|^2 - 2\beta\right)x - \beta = 0$$

$$R = \frac{I_F}{I_B} = (\mu/\gamma)^{-2}\,\frac{x_F}{x_B}$$

At weak input limit: $\beta \to 0$

$$x\left(x - \frac{1-(\mu/\gamma)^2}{1+(\mu/\gamma)^2}\right)^2 \approx 0$$

For $\mu/\gamma < 1$ and $s_{\text{in}} \ll \sqrt{\gamma} a_s$

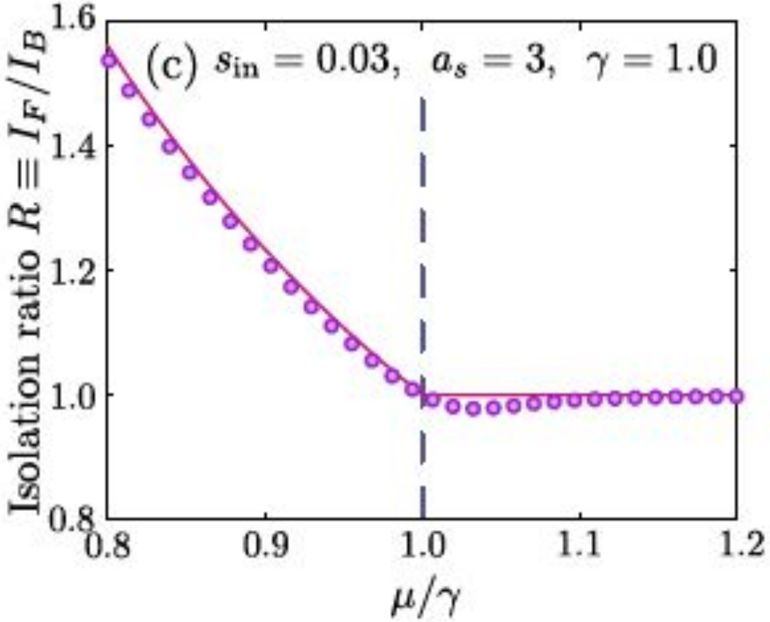
Roots for forward and backward transmission case are equal

$$R \approx (\mu/\gamma)^{-2}$$

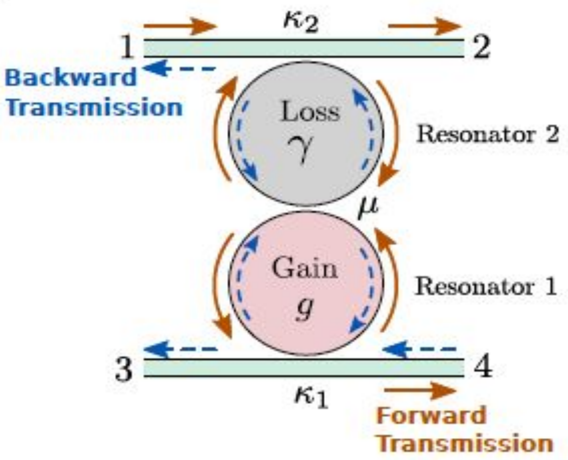
For $\mu/\gamma > 1$ and $s_{\text{in}} \ll \sqrt{\gamma} a_s$

Root becomes $\mathscr{O}(\beta)$ which implies $x_F/x_B \approx (\mu/\gamma)^2$

$$R \approx 1$$



Relation between Isolation Ratio and PT-symmetry



Unequal coupling

$$|\alpha|^2x^3+\left(2|\alpha-1|^2-2-\beta\right)x^2+\left(|\alpha-2|^2-2\beta\right)x-\beta=0$$

$$R=\frac{I_F}{I_B}=(\kappa_1/\kappa_2)(\mu/\gamma)^{-2}x_F/x_B \qquad \beta\to\frac{\kappa_2}{\gamma}\beta \quad \text{(Forward)}$$

$$\beta\to\frac{\kappa_1}{\gamma}\beta \quad \text{(Backward)}.$$

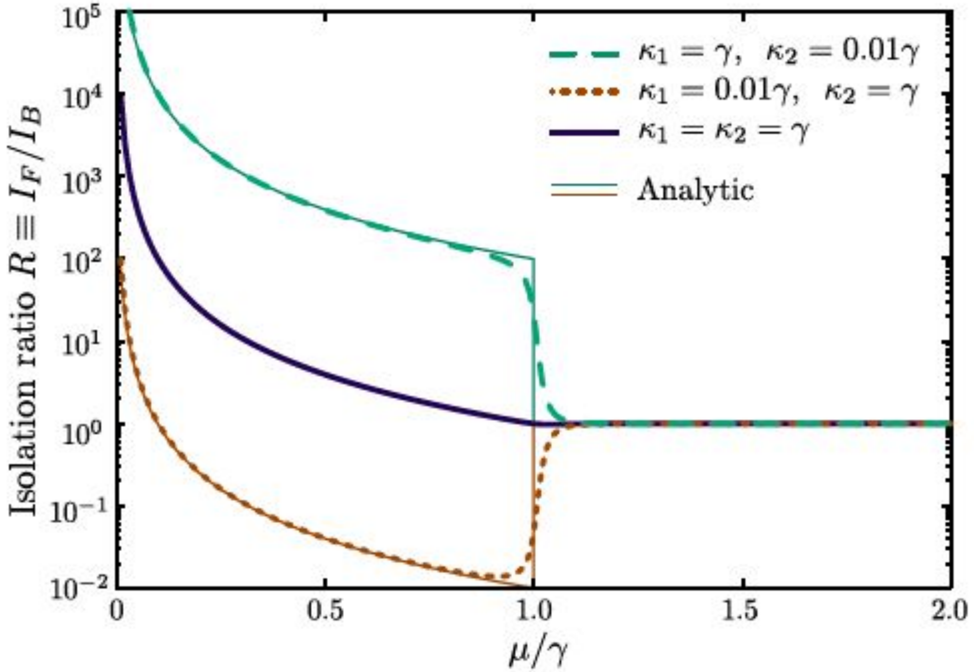
At weak input limit: $\beta\to 0$

For $\mu/\gamma<1$ and $s_{\text{in}}\ll\sqrt{\gamma}a_s$

$$x\left(x-\frac{1-(\mu/\gamma)^2}{1+(\mu/\gamma)^2}\right)^2\approx 0$$

Roots for forward and backward transmission case are equal

$$x_F/x_B\approx 1$$



For $\mu/\gamma>1$ and $s_{\text{in}}\ll\sqrt{\gamma}a_s$

$$x_F/x_B\approx\beta_F/\beta_B=(\kappa_2/\kappa_1)(\mu/\gamma)^2.$$

$$R=(\kappa_1/\kappa_2)(\mu/\gamma)^{-2}x_F/x_B \approx\begin{cases}(\kappa_1/\kappa_2)\,(\mu/\gamma)^{-2} & \text{for } \mu/\gamma<1\\ 1 & \text{for } \mu/\gamma>1\end{cases}$$

Conclusion

- The PT transition of the linear system is shown to correspond closely with the dynamical and steady-state behaviors of the gain-saturated nonlinear system.

- In the linear system’s “PT-symmetric” phase, the resonances are always asymptotically stable, and the isolation ratio approaches unity.

- In the linear system’s “PT-broken” phase, the coupled-mode dynamics are unstable at sufficiently large frequency detunings and the isolation ratio starts to grow.

Thanks!

Quantum mechanical
Schrodinger equation

Optical wave equation

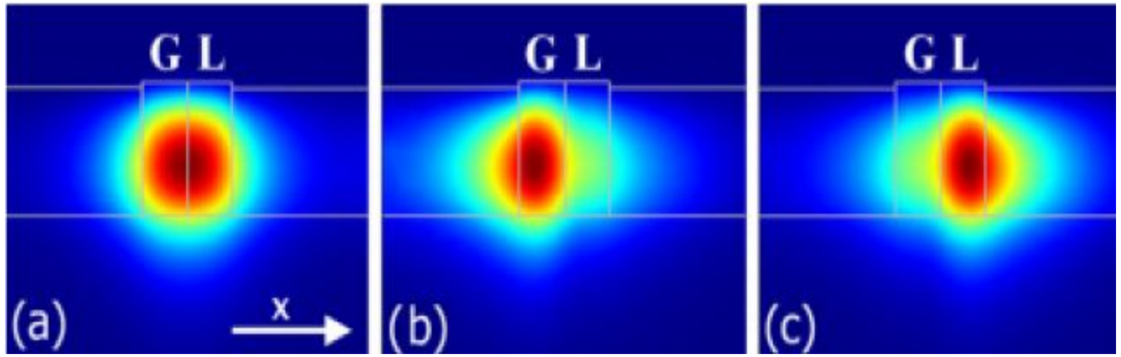
$$\frac{d^2\psi(x)}{dx^2} + \frac{2m}{\hbar^2}[E - V(x)]\psi(x) = 0$$

$$[\frac{d^2}{dx^2} + (n^2(x)\frac{\omega^2}{c^2} - \kappa^2)]\xi(x) = 0$$

$$V(x) = V^*(-x)$$

$$n(x) = n_R(x) + in_I(x)$$

$$n(x) = n^*(-x)$$



A. Guo, A. et. al., Phys. Rev. Lett. 103, 093902 (2009).