Week 4: Coursework 1 (and stoic wisdom)

Trisolaris: the three body problem

Dr K Clough, Topics in Scientific computing, Autumn term 2025

Coursework - Trisolaris, the three body problem

1. Introduce the theory behind the coursework
2. Discuss the problems
3. Discuss strategy and common errors
4. Q&A
5. Complete feedback questionnaire

“I judge you unfortunate because you have never lived through misfortune. You have passed through life without an opponent—no one can ever know what you are capable of, not even you.”

Seneca, Essays and Letters

Coursework - Trisolaris, the three body problem

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Annals of Mathematics, **152** (2000), 881–901

A remarkable periodic solution of the three-body problem in the case of equal masses

By Alain Chenciner and Richard Montgomery

Figure 1 (Initial conditions computed by Carles Simó)

$x_1 = -x_2 = 0.97000436 - 0.24308753i, x_3 = 0;\ \vec{V} = \dot{x}_3 = -2\dot{x}_1 = -2\dot{x}_2 = -0.93240737 - 0.86473146i$

$\overline{T} = 12T = 6.32591398,\ I(0) = 2,\ m_1 = m_2 = m_3 = 1$

Let's learn Newtonian mechanics

Newton's Law of Universal Gravitation

$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

For any two objects, the gravitational force of attraction (*F*) is proportional to the mass (*m*) of the objects. Force decreases with the sguare of the distance (*r*) between the center of the two objects. *G* is the *Universal Gravitational Constant*.

Let’s learn Newtonian mechanics

The ODE for the x position of each star is:

$$\frac{d^2 x_i}{dt^2} = \sum_{j=1, j\neq i}^{j=N} \frac{M_j}{r_{ij}^2} \cos(\theta) \qquad (1)$$

and for the y position:

$$\frac{d^2 y_i}{dt^2} = \sum_{j=1, j\neq i}^{j=N} \frac{M_j}{r_{ij}^2} \sin(\theta) \qquad (2)$$

where

$r_{ij} = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}$

$\cos\theta = (x_j - x_i)/r_{ij}$

$\sin\theta = (y_j - y_i)/r_{ij}$

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Problem 1: Binary system (35 marks)

Binary stars orbit each other such that the center of mass of the combined system is located closest to the more massive star

Coursework - Problem 2a stable triple (15 marks)

2. Three body problem

Raffi discovers a stable octuple system that contains a hidden message (Star Trek: Picard)

Coursework - Problem 2b chaotic triple (10 marks)

Coursework Problem 3 - modified gravity (10 marks)

$$\frac{d^2x_i}{dt^2} = \sum_{j=1, j\neq i}^{j=N} \left[1 + \frac{3\lambda^2 L_{ij}^2}{r_{ij}^2}\right] \frac{M_j}{r_{ij}^2}\cos(\theta) \qquad (4)$$

$$\frac{d^2y_i}{dt^2} = \sum_{j=1, j\neq i}^{j=N} \left[1 + \frac{3\lambda^2 L_{ij}^2}{r_{ij}^2}\right] \frac{M_j}{r_{ij}^2}\sin(\theta) \qquad (5)$$

where $L_{ij} = \left(\frac{dy_j}{dt} - \frac{dy_i}{dt}\right)(x_j - x_i) - \left(\frac{dx_j}{dt} - \frac{dx_i}{dt}\right)(y_j - y_i)$

Not in the coursework - build a universe!

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Coursework - strategy

You have to read the full instructions. I will make you do this at the end of the lecture.
Here I will only highlight the key points.

Coursework - strategy

Do not put this coursework in your GitHub repository, or any public place where others could see and copy it.

TurnItIn will detect if your solutions are very similar.

Coursework - strategy

Marking scheme

The marks available for each section are indicated below. The marks available will be split as follows:

- 70% for working code that correctly implements all of the requested components
- 10% for use of defensive programming techniques - asserts and tests implemented to prevent user error and check the code is functioning correctly.
- 10% for readability of code, following the agreed naming conventions of the course, appropriate commenting
- 10% for appropriate documentation of the code implemented in markdown format around the code.

“Men are disturbed not by the things that happen, but by their opinion about those things.”

Epictetus, Enchiridion 5

Do not disturb your marker

1. First impressions count - if I open your file and it contains mess I will be annoyed (I am not as stoic as Epictetus).

2. Better to be honest if something isn't working and explain what you would do to debug it, rather than try to hide it in a confusing plot. Also do not say things that are manifestly not true. e.g. "Here we see that the convergence test has worked perfectly" when it hasn't. If you aren't sure, best to say it.

3. Spelling check for markdown cells https://pypi.org/project/jupyterlab-spellchecker/ can be conda installed

Why was Socrates the wisest person in Athens?

Coursework - strategy

Check the required components for each part - print a copy of the coursework and physically tick them off

Required Components

1. The masses of the stars should be $M_1 = 0.6,\ M_2 = 0.3$
2. The first star should start at position $[x_1, y_1] = [2.0, 0.0]$ with velocity $\left[\frac{dx_1}{dt}, \frac{dy_1}{dt}\right] = [0.0, -0.1]$. Put the second star at $[x_1, y_1] = [-2.0, 0.0]$ and use equations (3) and (4) above to find its initial velocity so that the stars orbits are fixed with respect to the coordinate axes
3. You should write (at least) one class to represent stars and one for star systems
4. Provide plots of the trajectories for the stars

NB Do not provide animations, as these sometimes don't work on different systems and require additional python packages, you must provide static plots that illustrate the results. It is an important and useful skill to be able to represent time varying data using static plots.

5. Provide a phase diagram for the x-position and x-velocity of the first star. What does this tell you about the system?
6. Compare the solutions using scipy's `solve_ivp()` and the midpoint method (which you should implement yourself)
7. For the midpoint method, provide a convergence test for the x-position of the second star, confirming that the solution converges at the required order.
8. Give two reasons why it is better to use the midpoint method than Euler's method

Coursework - strategy

Take the hint(s)

If there is a HINT there is probably a mark attached to it

(HINT 1: you may want to investigate the rtol parameter for solve_ivp())

(HINT 2: try to think ahead - what features of problem 1 might you want to reuse in problems 2 and 3? You can save time coding by making it sufficiently general from the start)

Coursework - strategy

NEVER NEVER NEVER leave a part blank

If you can’t do something, you can still give me something to work with. Describing a convergence test might give you marks even if you can’t do it.

The project consists of 3 problems, each containing a single goal with multiple component parts. You are expected and encouraged to attempt every problem in this assignment. Fractional marks may be awarded for meaningful partial solutions, for example, a description in words/pseudocode of what you would do to solve the problem, or implement a test. It will always be better to write something rather than nothing, since you can only gain (you won't lose marks for wrong answers).

Coursework - strategy

No animations are allowed - you cannot assume I have installed the same packages as you, so you need static plots - often animations are not the clearest way to communicate data anyway.

One nice method of seeing evolution is to use a series of snapshots.

Coursework - strategy

Other methods to visualise motion:

How could you give a star a “tail”? Investigate the parameter “alpha” that changes the opacity of the line, noting that it can be a vector…

Make sure you are plotting frequently enough to show smooth curves - remember the role of time vector in solve_ivp() is not related to solution but to outputs

Asserts and testing

1. Asserts require **judgement** and **humility**

2. The usual mistake is to underestimate your/the user's ability to screw things up

3. Every time your code doesn't work, think "can I make this an assert?"

4. **Each assert should be tested with a unit test**. e.g. if you should not be able to set the mass attribute of the star class negative, create a class inputting a negative mass parameter and check that it triggers the assert. This may seem overkill, but I refer you back to point (2).

5. Consider also **using print statements**, which can be helpful to check the code reflects back the values you expect. e.g. "Initial x value of star 1 is set to 2.0."

6. It is definitely possible to have too many asserts, but in the context of the coursework at least I am unlikely to penalise for it.

Code readability and structure

1. Use the naming conventions of the course, as discussed in lectures 1 and 2
2. Use classes where asked for
3. Think about the structure, avoid repeating code
4. Comments can be useful, but code should “self document” where possible

What is “documentation”?

- Markdown formatting: https://www.markdownguide.org/cheat-sheet/
- The documentation required depends heavily on the expected user. Usually people make the mistake of **assuming too much prior knowledge of the user**.
- Don’t just repeat the question back to me in new words!
- Check lecture notes for buzz phrases for any “comment on” questions, rather than using ChatGPT which tends to make things up.
- Say the obvious things! “So we can see that X is bigger than Y, which is what we expected.”

What is “documentation”?

- The **target user** is important - in this case the documentation is for a new student on this course, who may be rusty on maths or python or both.

- Possible components of documentation:

  A list of capabilities of the code, and overview of the structure, prerequisites (did you tell the user to active the python environment? What python modules do you need?), list of coding conventions (what order are your variables in the state vector?), what do the parameters do? Do they have expected ranges? What would break the code? How do I know it has worked? What is a phase plot? How does the convergence test work? …

“They are not going to admire you for your quick wittedness. So be it! Show them those qualities that are entirely up to you: sincerity, dignity, endurance of hardship… Don’t you see how many you could display immediately… yet you willingly fall short?”

Marcus Aurelius, Meditations 5.5

It is unforgivable to lose marks because you have no documentation, inappropriate naming and no asserts/tests.

This is not about ticking boxes - in any real world job or research position these things will be mandatory, and for good reason. Good researchers and coders do not work in isolation, and need to consider and communicate to others.

“The mind is not like a bucket that needs filling, it is like wood that needs igniting - nothing more - to produce an impulse to discovery and a longing for truth.”

Plutarch

“Imagine that someone needing fire from their neighbours, and finding there a big blazing one, just stayed warming themselves until the fire burned out…”

Plutarch

“… it is the same if someone who comes to ChatGPT to get their thinking does not realise that they ought to strike some light of their own and kindle their own ideas, but just sits there enchanted.”

Plutarch

ChatGPT << Your brain

- You are a free human being, with infinite capacity to change, evolve, love, make choices, read and digest information, and use your judgement.
- YOU ARE BETTER THAN CHATGPT
- Take pride in your human ability to not know, to make mistakes and to improve (remember: a mistake is not a sign that you aren’t good enough)

ChatGPT can of course be useful

- If you already know the answer roughly but you forgot it
- If you want to find the right python function for a specific use
- If you have an error message you don’t understand
- If you use it to improve your grammar and spelling for pre written text
- …
- AI is good for reducing effort, but for the coursework ***effort is the point***

“The impediment to action advances action. What stands in the way becomes the way.”

Marcus Aurelius, Meditations

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Q&A

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1. ~~Revise Newtonian mechanics~~
2. ~~Discuss problems~~
3. ~~Discuss strategy~~
4. ~~Q&A~~
5. Complete feedback questionnaire

“If someone is able to show me that what I think or do is not right, I will happily change, for I seek the truth, by which no one was ever truly harmed. It is the person who continues in his self-deception and ignorance who is harmed.”

Marcus Aurelius, Meditations