CSC242: Homework 2.2
AIMA Chapter 7.0–7.4

1. Complete truth tables for the following formulas:

   (a) $\neg P$

   ANSWER:

P	$\neg P$
T	F
F	T

   (b) $P \Rightarrow Q$

   ANSWER:

P	Q	$P \Rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

   (c) $\neg Q \Rightarrow \neg P$

   ANSWER:

P	Q	$\neg Q$	$\neg P$	$\neg Q \Rightarrow \neg P$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

(d) $\neg P \Rightarrow \neg Q$

ANSWER:

P	Q	$\neg P$	$\neg Q$	$\neg P \Rightarrow \neg Q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(e) $P \wedge (Q \vee R)$

ANSWER:

P	Q	R	$Q \vee R$	$P \wedge (Q \vee R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

(f) $(P \wedge Q) \vee (P \wedge R)$

ANSWER:

P	Q	R	$P \wedge Q$	$P \wedge R$	$(P \wedge Q) \vee (P \wedge R)$
T	T	T	T	T	T
T	T	F	T	F	T
T	F	T	F	T	T
T	F	F	F	F	F
F	T	T	F	F	F
F	T	F	F	F	F
F	F	T	F	F	F
F	F	F	F	F	F

2. Briefly and specifically define *entailment*. Why is it important?

   ANSWER: A sentence or set of sentences α entails a sentence or set of sentences β if and only if:

   - Every model of α is also a model of β.
   - In every possible world in which α is true, β is also true.
   - Every assignment to the propositional variables that makes α true also makes β true.
   - $Models(\alpha) \subseteq Models(\beta)$

   Entailment is crucial because it allows us to compute what "follows from" our knowledge. That is, it is a form of *inference*: making implicit information explicit.

3. Establish by model checking whether $(P \Rightarrow Q) \models (\neg Q \Rightarrow \neg P)$.

   ANSWER:

P	Q	$P \Rightarrow Q$	$\neg Q$	$\neg P$	$\neg Q \Rightarrow \neg P$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

   Rows 1, 3, and 4 are models of $P \Rightarrow Q$ (*i.e.*, they make it true). And for these rows, $\neg Q \Rightarrow \neg P$ is also true. Hence yes $(P \Rightarrow Q) \models (\neg Q \Rightarrow \neg P)$.

4. Establish by model checking whether $\{P, P \Rightarrow Q\} \models Q$.

   ANSWER:

P	Q	$P \Rightarrow Q$	$\{P, P \Rightarrow Q\}$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	F

   Only the first row describes a model that satisfies $\{P, P \Rightarrow Q\}$ (*i.e.*, that makes them both true). And that model also satisfies Q. Thus every model of $\{P, P \Rightarrow Q\}$ is also a model of Q, thus yes $\{P, P \Rightarrow Q\} \models Q$.

5. Suppose you are using model checking to determine whether $\alpha \models \beta$ for some sentences α and β. You find that there are no models of α. What should you conclude?

   ANSWER: $\alpha \models \beta$ if and only if every model of α is also a model of β. If there are no models of α, then α entails *any* sentence β.

   But think about it. If there are no models of α, then α cannot be true in the real world (which is one of the possible worlds). In other words, α itself is always false. You generally wouldn't want to believe what you can infer from falsehoods, right?