CSC242: Introduction to Artificial Intelligence

Lecture 3.1

Please put away all electronic devices

242 TAs Wanted

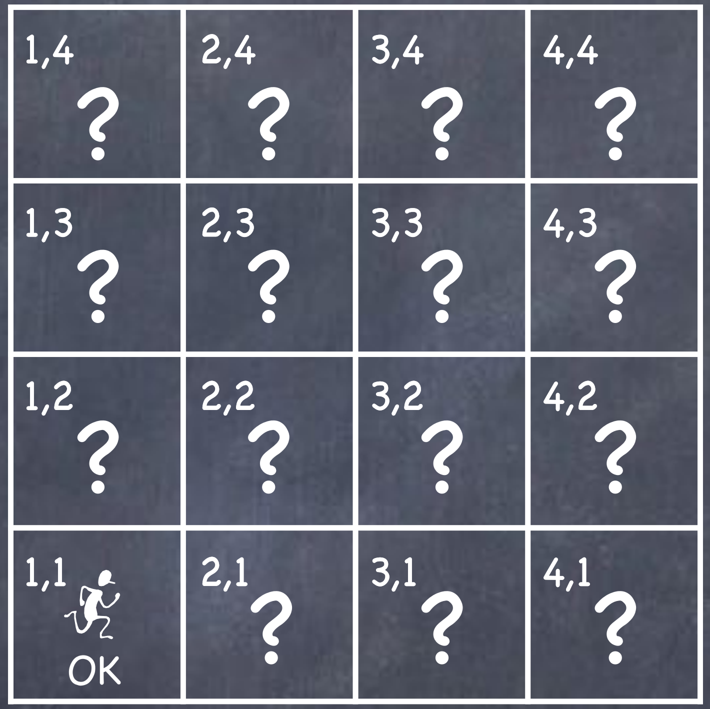
CSC242 TA Application for Spring 2018

https://goo.gl/forms/Od7rvfA8g4PfQU8T2

Uncertainty

Hunt The Wumpus

$At_{1,1} \quad \neg P_{1,1}$



$At_{1,1}$ $\neg P_{1,1}$

$\neg B_{1,1}$

$At_{1,1} \quad \neg P_{1,1}$

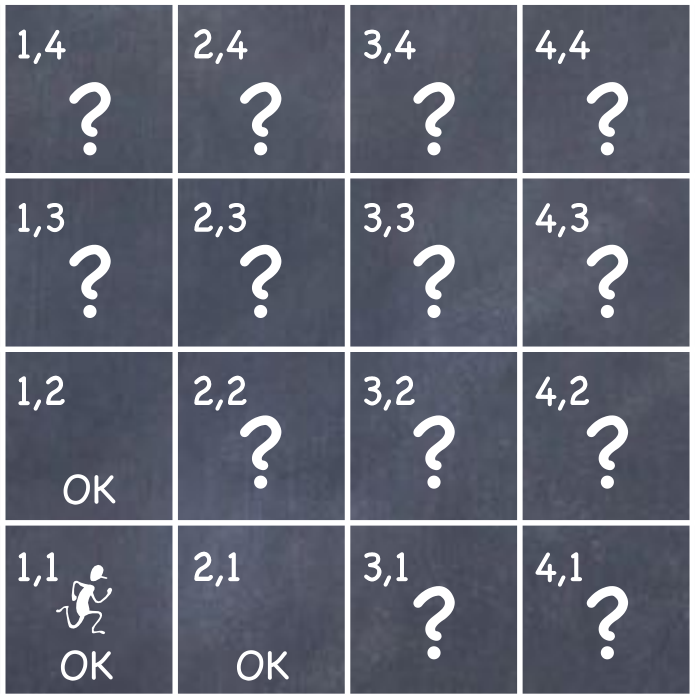
$\neg B_{1,1}$

$$B_{1,1} \Leftrightarrow P_{2,1} \vee P_{1,2}$$

$At_{1,1}$ $\neg P_{1,1}$

$\neg P_{2,1}$

$\neg P_{1,2}$



$\neg B_{1,1}$

$At_{2,1}$ $\neg P_{1,1}$

$\neg P_{2,1}$

$\neg P_{1,2}$

$\neg B_{1,1}$

$At_{2,1}$ $\neg P_{1,1}$

$\neg P_{2,1}$

$\neg P_{1,2}$

$\neg B_{1,1}$

$B_{2,1}$

$At_{2,1}$ $\neg P_{1,1}$

$\neg P_{2,1}$

$\neg P_{1,2}$

$\neg B_{1,1}$

$B_{2,1}$

$$B_{2,1} \Leftrightarrow P_{1,1} \lor P_{2,2} \lor P_{3,1}$$

$At_{2,1}$ $\neg P_{1,1}$

$\neg P_{2,1}$

$\neg P_{1,2}$

$P_{2,2} \vee P_{3,1}$

$\neg B_{1,1}$

$B_{2,1}$

$At_{1,2}$ $\neg P_{1,1}$

$\neg P_{2,1}$

$\neg P_{1,2}$

$P_{2,2} \vee P_{3,1}$

1,4 ?	2,4 ?	3,4 ?	4,4 ?
1,3 ?	2,3 ?	3,3 ?	4,3 ?
1,2 OK	2,2 P?	3,2 ?	4,2 ?
1,1 OK	2,1 OK B	3,1 P?	4,1 ?

$\neg B_{1,1}$

$B_{2,1}$

$At_{1,2}$ $\neg P_{1,1}$

$\neg P_{2,1}$

$\neg P_{1,2}$

$P_{2,2} \vee P_{3,1}$

$\neg B_{1,1}$

$B_{2,1}$

$B_{1,2}$

$At_{1,2}$ $\neg P_{1,1}$

$\neg P_{2,1}$

$\neg P_{1,2}$

$P_{2,2} \vee P_{3,1}$

$P_{2,2} \vee P_{1,3}$

$\neg B_{1,1}$

$B_{2,1}$

$B_{1,2}$

$$B_{1,2} \Leftrightarrow P_{1,1} \vee P_{2,2} \vee P_{1,3}$$

$At_{1,2}$ $\neg P_{1,1}$

$\neg P_{2,1}$

$\neg P_{1,2}$

$P_{2,2} \vee P_{3,1}$

$P_{2,2} \vee P_{1,3}$

$\neg B_{1,1}$

$B_{2,1}$

$B_{1,2}$

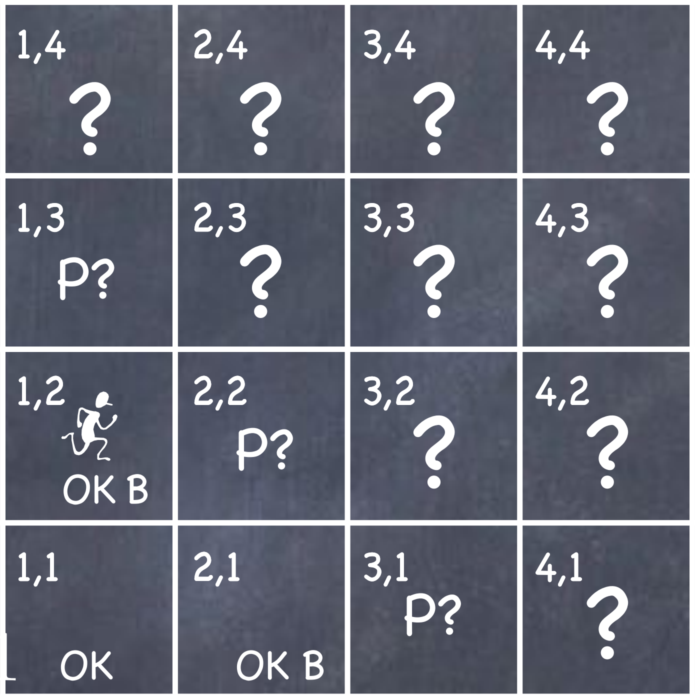
$At_{1,2}$ $\neg P_{1,1}$

$\neg P_{2,1}$

$\neg P_{1,2}$

$P_{2,2} \vee P_{3,1}$

$P_{2,2} \vee P_{1,3}$



$\neg B_{1,1}$

$B_{2,1}$

$B_{1,2}$

Which room is least likely to contain a pit?

Diagnosis

- Reasoning from symptoms to causes
  - What disease is most likely to be causing the observed symptoms

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Diagnostic rule: $Toothache \Rightarrow Cavity$

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  - What disease is most likely to be causing the observed symptoms

~~Diagnostic rule:~~ $\cancel{Toothache \Rightarrow Cavity}$

$$Toothache \Rightarrow Cavity \lor GumProblem \lor Abcess \lor \ldots$$

Diagnosis

- Reasoning from symptoms to causes
  - What disease is most likely to be causing the observed symptoms

~~Diagnostic rule:~~ $\cancel{Toothache \Rightarrow Cavity}$

$$Toothache \Rightarrow Cavity \vee GumProblem \vee Abcess \vee \ldots$$

Causal rule: $Cavity \Rightarrow Toothache$

Approach

- Representing uncertain information
- Reasoning with uncertain information
  - Uncertain inference

Uncertainty

Sources of Uncertainty

Partial Observability

Nondeterminism

Sources of Uncertainty

Partial Observability and
Nondeterminism

Probability

WARNING!

MATH AHEAD

Probability

Possible Worlds

Hungry=true,
Cranky=false

Hungry=false,
Cranky=true

Hungry=true,
Cranky=true

Hungry=false,
Cranky=false

Possible Worlds

Hungry ∨ Cranky

Hungry=true,
Cranky=false

Hungry=false,
Cranky=true

Hungry=true,
Cranky=true

Hungry=false,
Cranky=false

Possible Worlds

Hungry $\Rightarrow$ Cranky

~~Hungry=true, Cranky=false~~

Hungry=false, Cranky=true

Hungry=true, Cranky=true

Hungry=false, Cranky=false

Possible Worlds

- Logical knowledge rules out possible worlds

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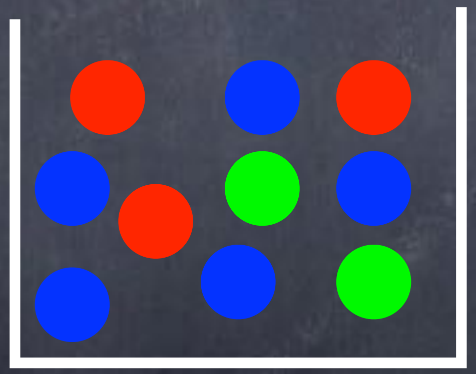
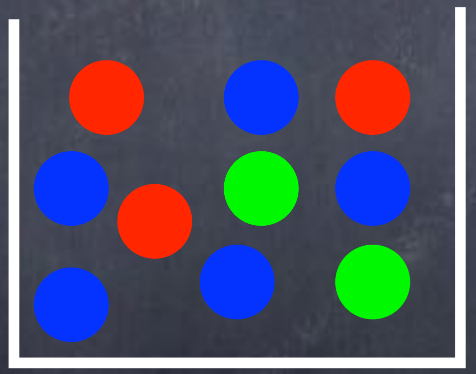
- Logical knowledge rules out possible worlds
- Probabilistic assertions talk about how probable (likely) the possible worlds are

Experiments	Outcomes
	Heads or Tails
	Red, Green, or Blue

Experiments

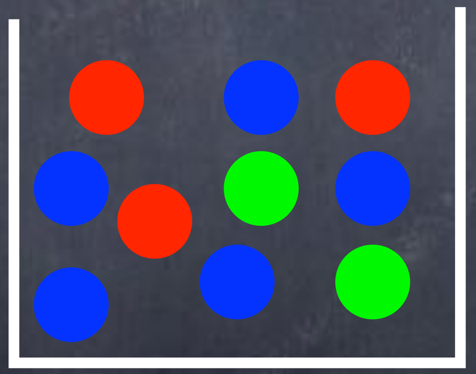
Outcomes

Heads or Tails



Red, Green, or Blue

Conceptualization
(Ontology)

Experiments	Outcomes
	Heads or Tails
	Red, Green, or Blue
Conceptualization (Ontology)	Possible World

Sample Space

- Set of all possible outcomes:

$$\Omega = \{ \ \omega_i \ \}$$

- Possible worlds ω_i are:
  - Mutually exclusive
  - Exhaustive

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$$\Omega = \{ \omega_i \}$$

- Possible worlds ω_i are:
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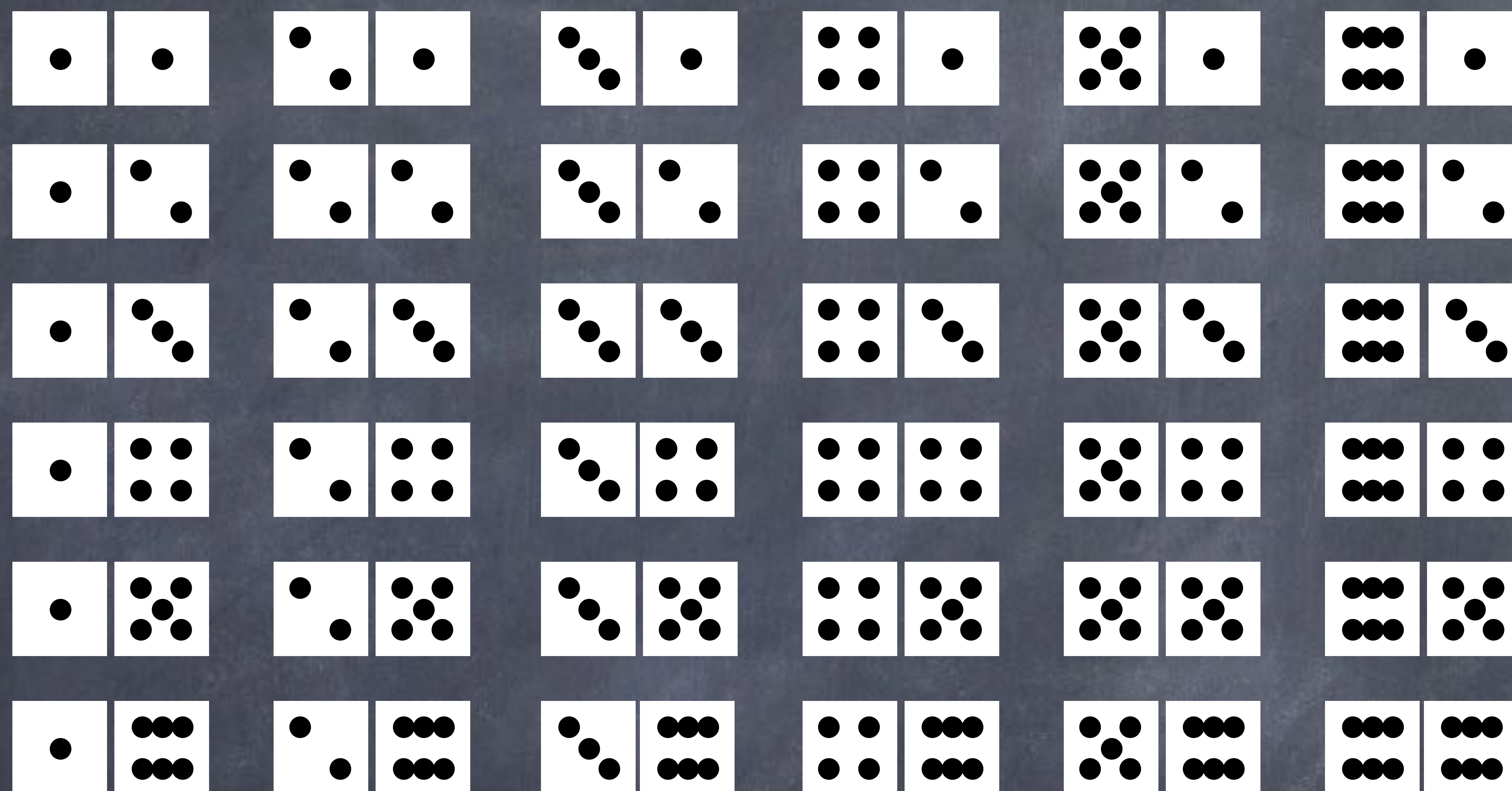
Same as in logic



$$\Omega = \{\ \omega_i\ \} = \{\ (1,1),\ (1,2),\ (2,1),\ (3,1),\ \ldots\ \}$$

Probability Model

- Assigns a numerical probability $P(\omega)$ to each outcome*, such that:

$$0 \leq P(\omega) \leq 1$$

$$\sum_{\omega \in \Omega} P(\omega) = 1$$

*Finite, countable set

Where Do Probabilities Come From?

Degrees of Belief

- The degree to which an agent believes a possible world is the actual world
  - 0: certainly not the case (i.e., false)
  - 1: certainly is the case (i.e., true)

Degrees of Belief

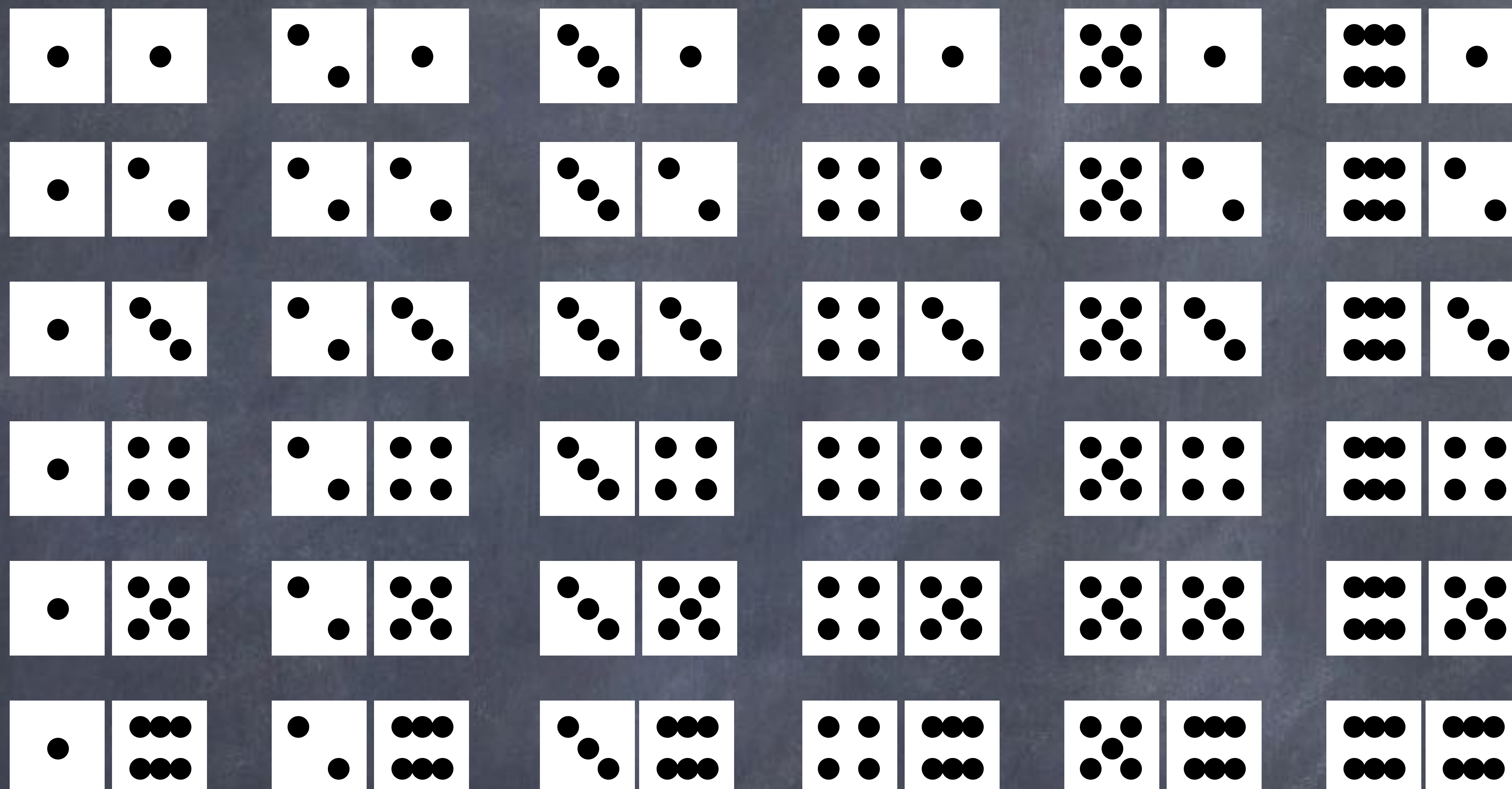
- The degree to which an agent believes a possible world is the actual world
  - 0: certainly not the case (i.e., false)
  - 1: certainly is the case (i.e., true)
- Could come from statistical data, general principles, combination of evidence, ...

Probability Model

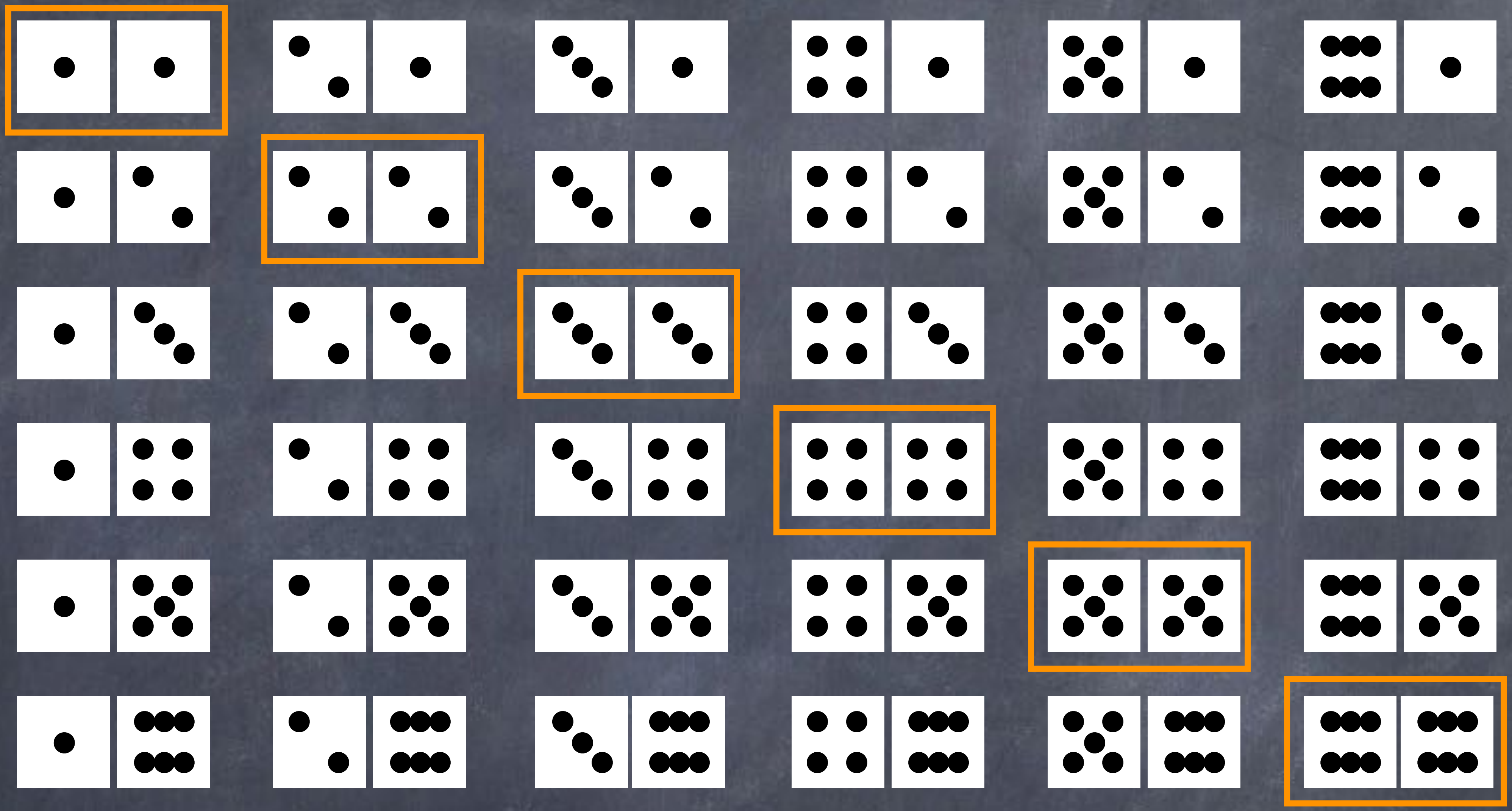
- Assigns a degree of belief $P(\omega)$ to each possible world, such that:

$$0 \leq P(\omega) \leq 1$$

$$\sum_{\omega \in \Omega} P(\omega) = 1$$



$$P(\omega_i) = 1/36 \text{ for all } \omega_i \in \Omega$$



$$P(\omega_i) = 2/36 \text{ if } \omega_i \in \{[1,1], [2,2], [3,3], [4,4], [5,5], [6,6]\}$$

$$P(\omega_i) = 2/90 \text{ otherwise}$$

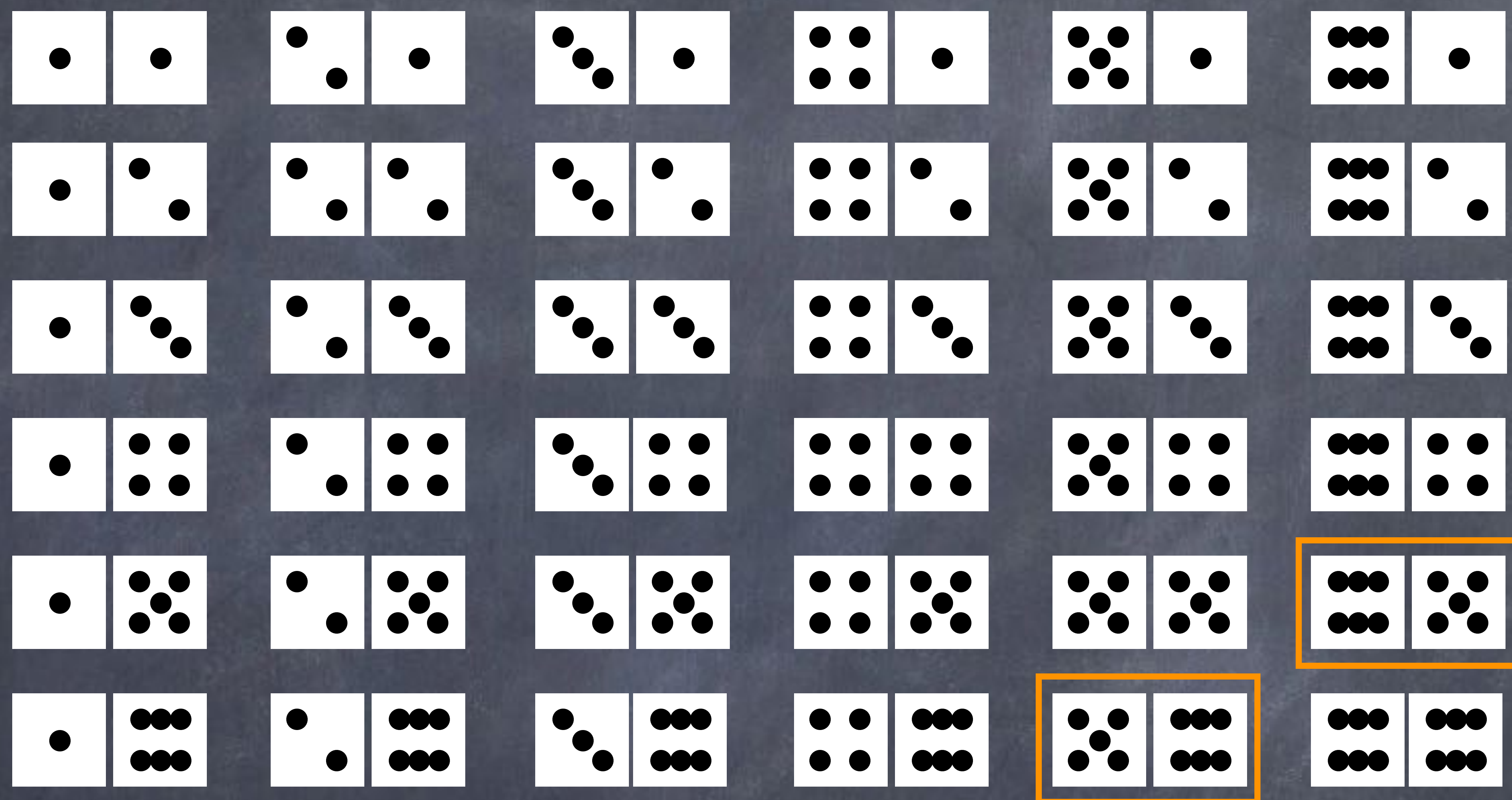
Events

- An event is a subset of the sample space (that is, a set of outcomes a.k.a. possible worlds)
  - "Coming up heads"
  - "Throwing doubles"
  - "Picking one red and one green ball"

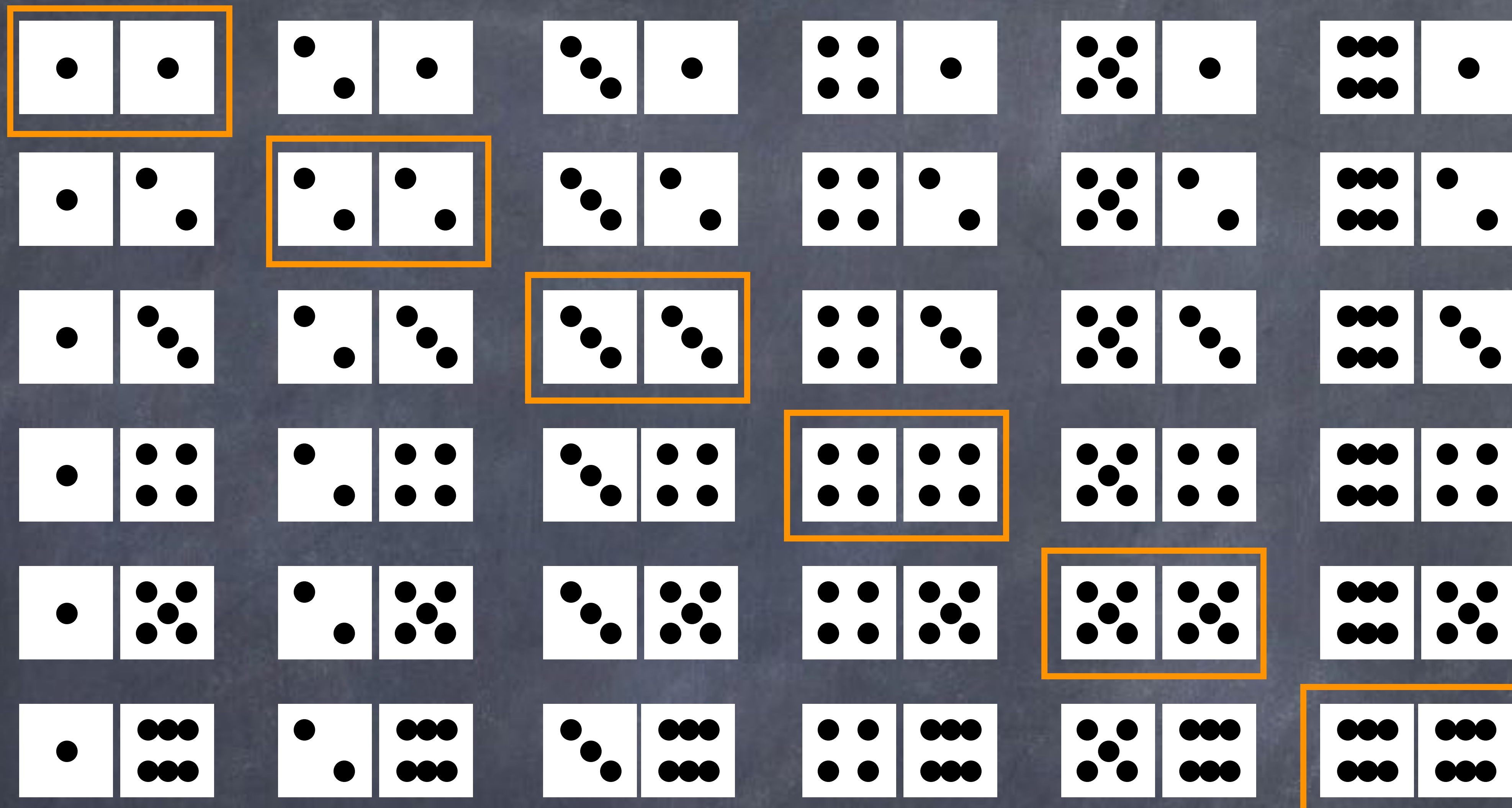
Events

- An event is a subset of the sample space (that is, a set of outcomes a.k.a. possible worlds)

$$P(e) = \sum_{\omega \in e} P(\omega)$$

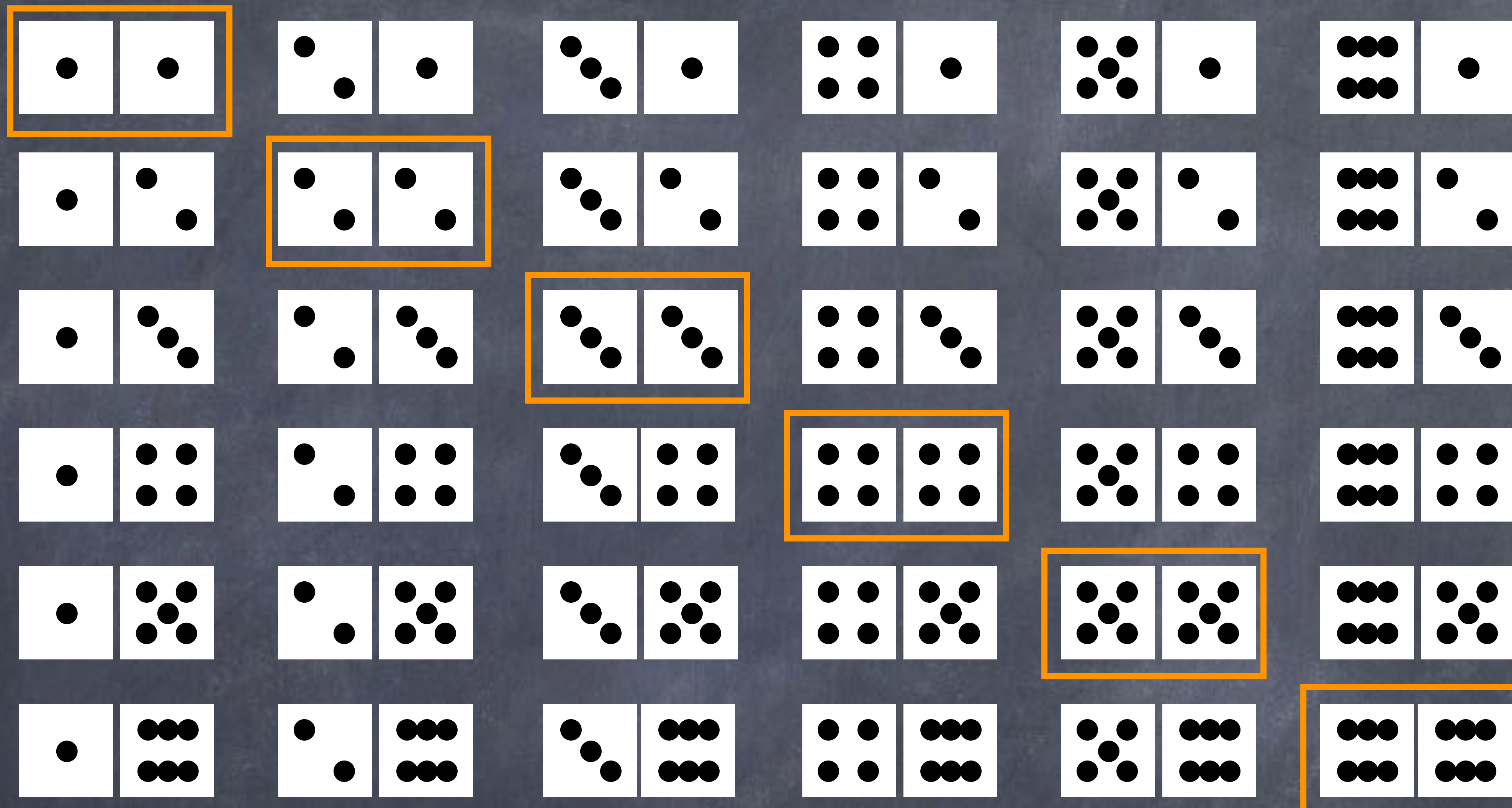


$$P(Total{=}11) = P([5{,}6]){+}P([6{,}5]) = 1/36{+}1/36 = 1/18$$



$$P(Doubles) = 1/6$$

$$P([1,1])+P([2,2])+P([3,3])+P([4,4])+P([5,5])+P([6,6]) = 1/6$$



$$P(Doubles) = 1/4$$

$$P([1,1])+P([2,2])+P([3,3])+P([4,4])+P([5,5])+P([6,6]) = 1/4$$

Probability

- Sample space: possible worlds
- Probability model: degrees of belief in possible worlds
- Events: subsets of possible worlds

Probability Statements

- “Heads and tails are equally likely”
- “Doubles are more likely than non-doubles”
- “The probability of picking one red and one green ball is 30% (0.3)”

Random Variable

- A function from outcomes to values
  - “the number of heads in three flips”
  - “the total of the two dice”
  - “the dice are matched doubles”
  - “a red ball was picked”

Factored Representation

- Represents a state (outcome, possible world) as a set of variable/value pairs
- Each random variable represents a feature or aspect of the world that we care about

Random Variable

"the number of heads in three flips"

$X(HHH) = 3,$

$X(HHT) = X(HTH) = X(THH) = 2,$

$X(HTT) = X(THT) = X(TTH) = 1,$

$X(TTT) = 0$

Random Variable

"the total of the two dice"

$$X([1,1]) = 2,$$

$$X([1,2]) = X([2,1]) = 3,$$

$$X([1,3]) = X([2,2]) = X([3,1]) = 4,$$

$$\ldots$$

$$X([6,6]) = 12$$

Random Variable

"the dice are matched doubles"

$$X([1,1]) = X([2,2]) = X([3,3]) =$$
$$X([4,4]) = X([5,5]) = X([6,6]) = true,$$
$$X([1,2]) = X([1,3]) = X([1,4]) = \ldots$$
$$\ldots\ X([6,3]) = X([6,4]) = X([6,5]) = false$$

Domain (of Random Variable)

- Finite
  - Set of values
    - Two values: Boolean
- Infinite
  - Discrete (e.g. integers)
  - Continuous (e.g. reals)
- May be ordered (or not)

Domain (of Random Variable)

- Finite
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  - Discrete (e.g. integers)
  - Continuous (e.g. reals)
- May be ordered (or not)

Domain
(of Random Variable)

$$Die_1 = \{\ 1,\ ...,\ 6\ \}$$

$$Total = \{\ 2,\ ...,\ 12\ \}$$

$$Doubles = \{\ true,\ false\ \}$$

$$Weather = \{\ sunny,\ rain,\ cloudy,\ snow\ \}$$

Statements

- Elementary (atomic) statement: $RV = value$

$$Die_1 = 3$$

$$Total = 7$$

$$Doubles = true \longrightarrow doubles$$

$$Weather = sunny \longrightarrow sunny$$

Statements

- Elementary (atomic) statement: $RV = value$

$$Die_1 = 3$$

$$Total = 7$$

$$Doubles = true \longrightarrow doubles$$

$$Weather = sunny \longrightarrow sunny$$

- Ordered domains: $<$, $>$, $\leq$, $\geq$

Statements

- Elementary (atomic) statement: $RV = value$

$$Die_1 = 3$$

$$Total = 7$$

$$Doubles = true \longrightarrow doubles$$

$$Weather = sunny \longrightarrow sunny$$

- Ordered domains: $<$, $>$, $\leq$, $\geq$

Can be true or false

Statements

- Elementary (atomic) statement: $RV = value$
- Combine with connectives from PL

$$Die_1 = 3 \wedge Total = 7$$

$$Total = 7 \vee doubles$$

$$Die_1 = 3 \wedge \neg doubles$$

Probability Statements

- Assign a probability to a statement

$$P(Die_1 = 3 \wedge Total = 7) = 1/36$$

$$P(Total = 7 \vee doubles) = 12/36$$

$$P(Die_1 = 3 \wedge \neg doubles) = 5/36$$

Probability Language

- Elementary (atomic) statement: $RV = value$
- Combine with connectives from PL
- Assign probabilities that statements are true: $P(statement) = p$

Probability Distribution

- Assign a probability to every possible value of a random variable

$$Weather\text{: } \{\ sunny,\ rain,\ cloudy,\ snow\ \}$$

$$P(Weather{=}sunny) = P(sunny) = 0.6$$

$$P(Weather{=}rain) = P(rain) = 0.1$$

$$P(Weather{=}cloudy) = P(cloudy) = 0.29$$

$$P(Weather{=}snow) = P(snow) = 0.01$$

Probability Distribution

- Assign a probability to every possible value of a random variable

$$Weather\colon \{\ sunny,\ rain,\ cloudy,\ snow\ \}$$
$$P(Weather{=}sunny) = P(sunny) = 0.6$$
$$P(Weather{=}rain) = P(rain) = 0.1$$
$$P(Weather{=}cloudy) = P(cloudy) = 0.29$$
$$P(Weather{=}snow) = P(snow) = 0.01$$
$$\mathbf{P}(Weather) = \langle\ 0.6,\ 0.1,\ 0.29,\ 0.01\rangle$$

Bold

Vector

Probability Distribution

- Assign a probability to every possible value of a random variable

$$Weather\text{: } \{\ sunny,\ rain,\ cloudy,\ snow\ \}$$

$$P(Weather{=}sunny) = P(sunny) = 0.6$$

$$P(Weather{=}rain) = P(rain) = 0.1$$

$$P(Weather{=}cloudy) = P(cloudy) = 0.29$$

$$P(Weather{=}snow) = P(snow) = 0.01$$

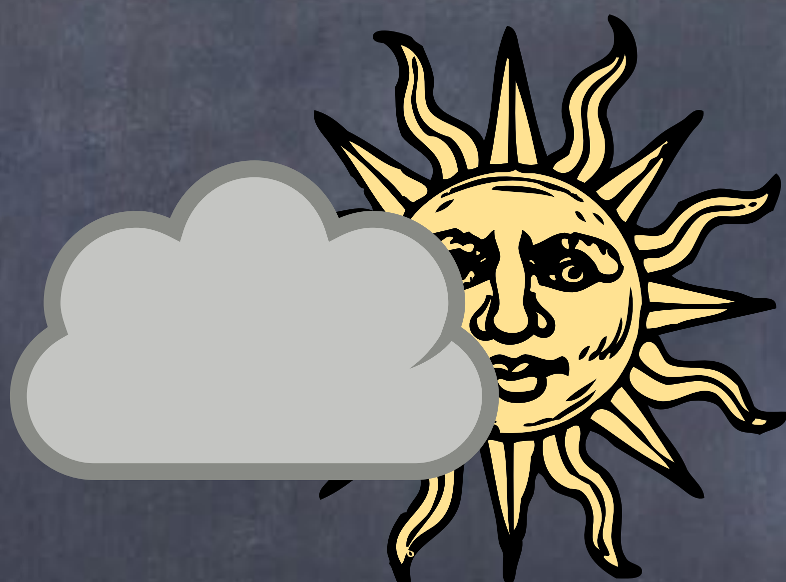
$$\mathbf{P}(Weather) = \langle\ 0.6,\ 0.1,\ 0.29,\ 0.01 \rangle$$

Probabilities of a distribution must sum to 1

Probability Statements

- Random variables
- Domains
- Probability statements
  - Elementary (atomic) and compound
- Probability distribution
  - $\mathrm{P}(Weather) = \langle 0.6, 0.1, 0.29, 0.01 \rangle$

Multiple Variables



$Weather$

$Cavity$

Joint Distribution

- Distribution over multiple variables
- Gives probabilities of all combinations of the values of the variables

Joint Distributions

$$\mathbf{P}(Weather, Cavity)$$

		Cavity	
		true	*false*
Weather	*sunny*		
	rain		
	cloudy		
	snow		

Joint Distributions

$$\mathbf{P}(sunny, Cavity)$$

		$Cavity$	
		$true$	$false$
$Weather$	$sunny$		

Joint Distributions

$$\mathbf{P}(sunny, cavity)$$

		$Cavity$
		$true$
$Weather$	$sunny$	

$$\mathbf{P}(sunny, cavity) = P(sunny, cavity) = \mathrm{P}(sunny \wedge cavity)$$

Joint Distributions

- Distributions over multiple variables
- Describe probabilities of all combinations of the values of the variables

Full Joint Probability Distribution

- Joint probability distribution over all the random variables
- Probabilities for every possible combination of values assigned to random variables
- Probabilities for every possible world

Full Joint Probability Distribution

$$\mathbf{P}(Cavity, Toothache, Weather)$$

	$toothache$		$\neg toothache$	
	$cavity$	$\neg cavity$	$cavity$	$\neg cavity$
$sunny$				
$rain$				
$cloudy$				
$snow$				

Representing Uncertainty

- Probability: Sample space, probabilities, events
- Random variables, domains
- Language of probability statements
- Probability distributions, joint distributions, full joint distribution

For Next Time:

AIMA 13.3 - 13.6