CSC242: Introduction to Artificial Intelligence

Lecture 2.5

Please put away all electronic devices

Announcements

- Unit 2 Exam next class
  - Bring your envelope if using one
- Unit 2 Project due that night 1159PM
  - Understand Academic Honesty policy!

Pickup old exams before
the new one is returned!

- Rooms adjacent to pits are breezy
- Socrates is a person
  All people are mortal
- Anybody's grandmother is either their mother's or their father's mother

First-Order Predicate Logic

Ontology:

- Objects
- Relations
- Functions

First-Order Predicate Logic

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Language:

- Constant symbols
- Predicate symbols
- Function symbols

First-Order Predicate Logic

Ontology:

- Objects
- Relations
- Functions

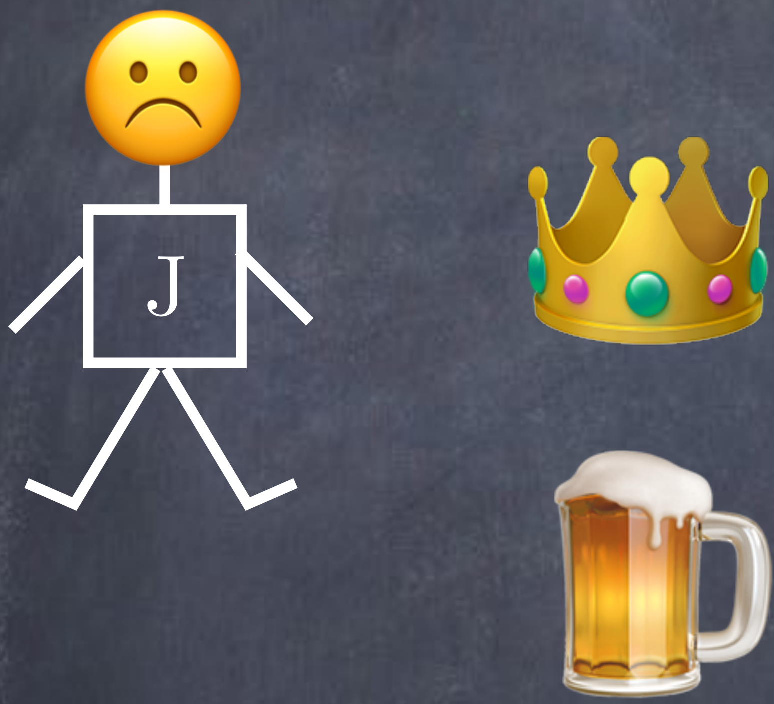
Language:

- Constant symbols
- Predicate symbols
- Function symbols
- Terms
- Atomic Sentences
- Complex Sentences

Conceptualization

Domain of Discourse

Ontology:



Conceptualization

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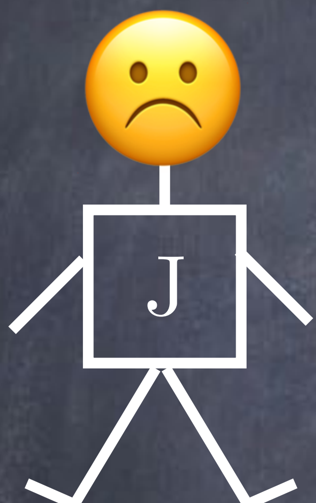

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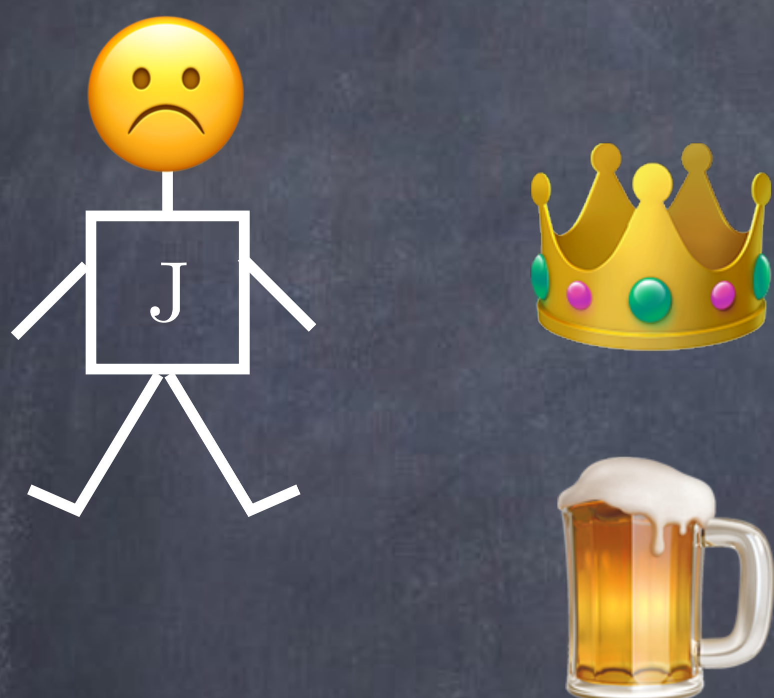

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Language:

$John$

$Crown(\cdot)$

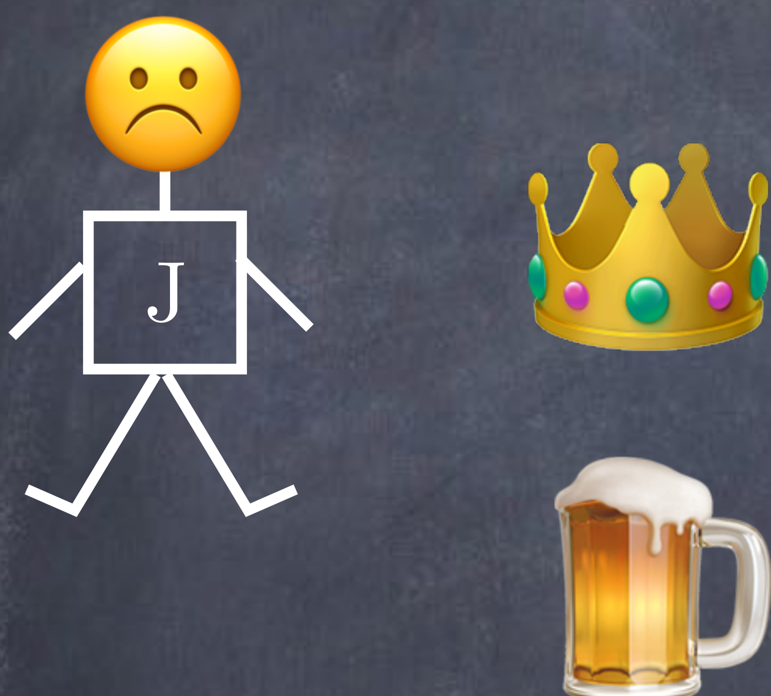
$King(\cdot)$

$OnHead(\cdot,\cdot)$

Conceptualization

Domain of Discourse

Ontology:



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Conceptualization

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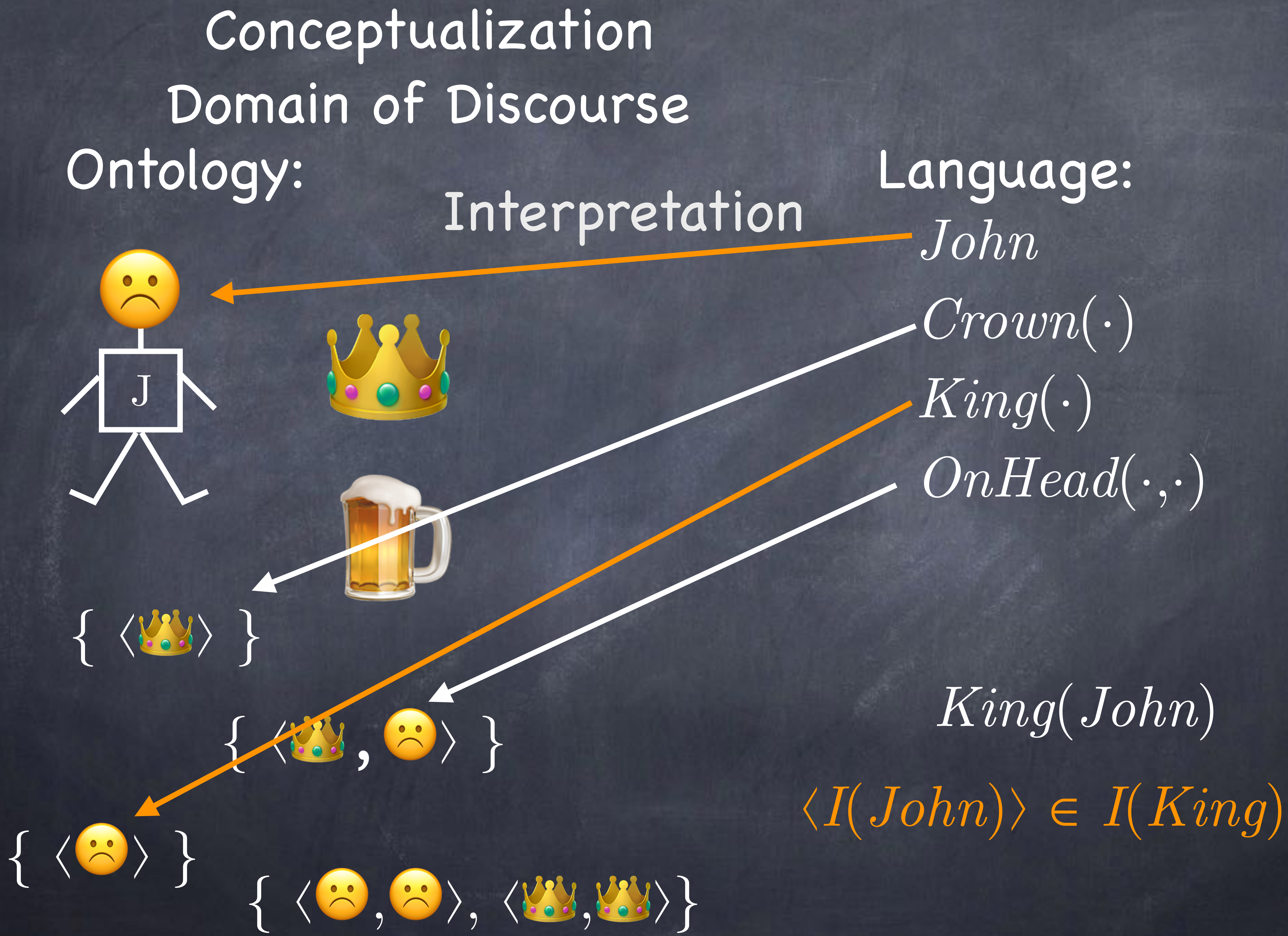
Conceptualization
Domain of Discourse
Ontology:
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J
$King(John)$
$\langle I(John) \rangle \in I(King)$

Conceptualization

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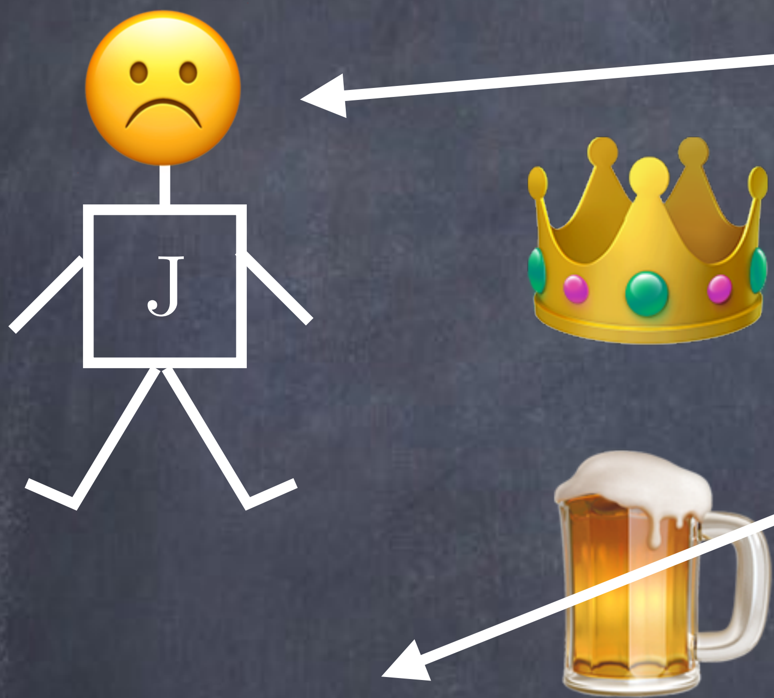

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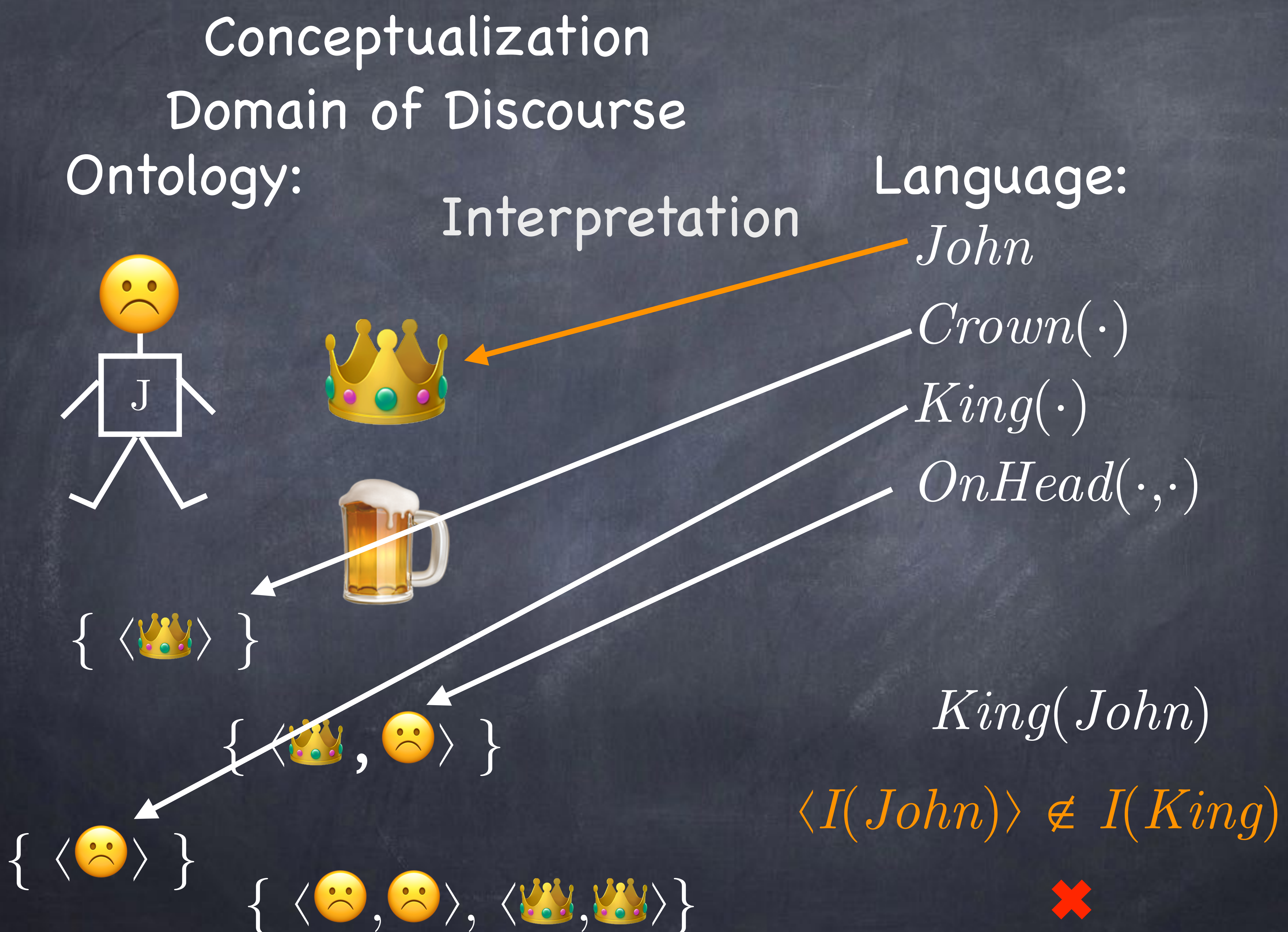
$King(John)$

$\langle I(John)\rangle \in I(King)$

✔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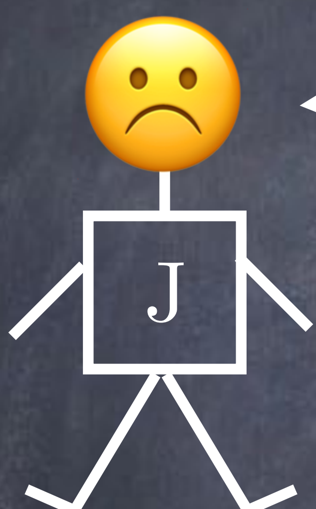

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$King(John)$

$\langle I(John)\rangle \notin I(King)$

✖

Semantics of First-Order Logic

- For ontology (objects, relations, functions)
- Interpretation function I
  - Constant symbols $\rightarrow$ objects
  - Predicate symbols $\rightarrow$ relations (tuples)
  - Function symbols $\rightarrow$ functions (mappings)
- A model <u>satisfies</u> a sentence if it makes the sentence true

- Rooms adjacent to pits are breezy
- Socrates is a person
  All people are mortal
- Anybody's grandmother is either their mother's or their father's mother

$$Person(Socrates)$$

- Rooms adjacent to pits are breezy
- Socrates is a person
  All people are mortal
- Anybody's grandmother is either their mother's or their father's mother

Universal Quantification

- Syntax: $\forall x\, \varphi$
- Semantics: φ is true for every object x
  - Extended interpretation maps every variable to an object in the domain
  - $\forall x\, \varphi$ is true if φ is true in every extended interpretation

$$\forall x\ King(x) \Rightarrow Person(x)$$ True!

false

Richard is a king ⇒ Richard is a person True

true true

John is a king ⇒ John is a person True

false

Richard's left leg is a king ⇒ Richard's left leg is a person True

false

John's left leg is a king ⇒ John's left leg is a person True

false

the crown is a king ⇒ the crown is a person True

All people are mortal.

$$\forall x \; Person(x) \Rightarrow Mortal(x)$$

Rooms adjacent to pits are breezy.

$$\forall x \forall y \; Room(x) \wedge Pit(y) \wedge Adjacent(x, y) \Rightarrow Breezy(x)$$

Anybody's grandmother is either their mother's or their father's mother

$$\forall x \forall y \; Grandmother(x, y) \Rightarrow \\ x = mother(mother(y)) \vee x = mother(father(y)))$$

Existential Quantification

- Syntax: $\exists x\ \varphi$
- Semantics: φ is true for <u>some</u> object x
  - <u>Extended interpretation</u> maps every variable to an object in the domain
  - $\exists x\ \varphi$ is true if φ is true in <u>some</u> extended interpretation

John has a crown on his head.

$$\exists x \; Crown(x) \wedge OnHead(x, John)$$

John has a crown on his head.

$$\exists x \; Crown(x) \wedge OnHead(x, John)$$

$x \rightarrow$ Richard

$x \rightarrow$ John

$x \rightarrow$ Richard's left leg

$x \rightarrow$ John's left leg

$x \rightarrow$ the crown

John has a crown on his head.

$\exists x \; Crown(x) \wedge OnHead(x, John)$ True!

$x \rightarrow$ Richard

$x \rightarrow$ John

$x \rightarrow$ Richard's left leg

$x \rightarrow$ John's left leg

$x \rightarrow$ the crown True

Existential Quantification

- Syntax: $\exists x \; \varphi$
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  - $\exists x \; \varphi$ is true if φ is true in <u>some</u> extended interpretation

Nested Quantifiers

Brothers are siblings

$$\forall x \forall y \; Brother(x,y) \Rightarrow Sibling(x,y)$$

Being a sibling is a symmetric relationship

$$\forall x \forall y \; Sibling(x,y) \Rightarrow Sibling(y,x)$$

Nested Quantifiers

Everyone (every person) loves someone

$$\forall x\ Person(x) \Rightarrow \exists y\ Person(y) \wedge Loves(x,y)$$

Nested Quantifiers

Everyone (every person) loves someone

$$\forall x\ Person(x) \Rightarrow \exists y\ Person(y) \land Loves(x,y)$$

Someone is loved by everyone

$$\exists x\ Person(x) \land \forall y\ Person(y) \Rightarrow Loves(y,x)$$

Nested Quantifiers

Everyone (every person) loves someone

$$\forall x \; Person(x) \Rightarrow \exists y \; Person(y) \wedge Loves(x,y)$$

Someone is loved by everyone

$$\exists x \; Person(x) \wedge \forall y \; Person(y) \Rightarrow Loves(y,x)$$

Someone loves everyone

$$\exists x \; Person(x) \wedge \forall y \; Person(y) \Rightarrow Loves(x,y)$$

$$\exists x \; \forall y \; Person(x) \wedge Person(y) \Rightarrow Loves(x,y)$$

First-Order Predicate Logic

- Syntax:
  - Constant, predicate, and function symbols
  - Terms, atomic sentences, connectives
  - Quantifiers and variables
- Semantics:
  - Domain of objects, relations, functions
  - First-order interpretation
  - Extended interpretation
  - Satisfaction (sentence true in a possible world)

Entailment

- α entails β ($\alpha \models \beta$) when:
  - β is true in **every** world considered possible by α
  - Every model of α is also a model of β
  - Models(α) $\subseteq$ Models(β)

All Possible Models

- # of objects in the world from 1 to ∞
- Some constants refer to the same object
- Some objects are not referred to by any constant ("unnamed")
- Relations and functions defined over sets of subsets of objects
- Variables range over all possible objects in extended interpretations

Constant symbols: $\{ R, J \}$
Relation symbol: $P(\cdot,\cdot)$

Constant symbols: $\{\ R,\ J\ \}$

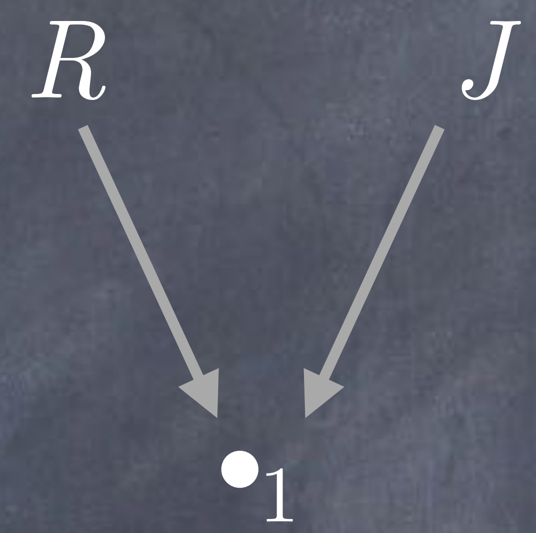
Relation symbol: $P(\cdot,\cdot)$

$$\Omega_I = \{\ \bullet_1\ \}$$

$$I(R) = \bullet_1$$

$$I(J) = \bullet_1$$

$$I(P) = \{\ \}$$



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$$\Omega_I = \{\ \bullet_1\ \}$$

$$I(R) = \bullet_1$$

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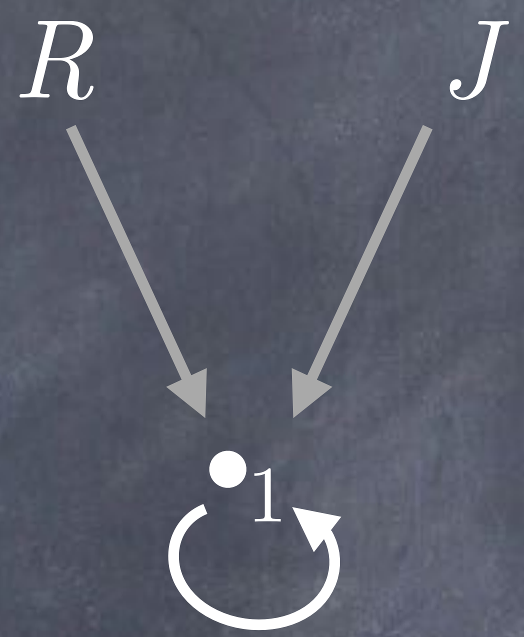
$$I(P) = \{\ \langle \bullet_1, \bullet_1 \rangle \}$$



Constant symbols: $\{ R, J \}$

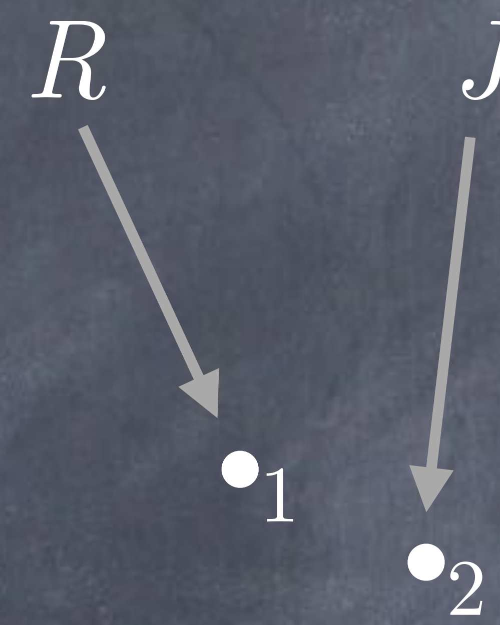
Relation symbol: $P(\cdot,\cdot)$

$$\Omega_I = \{ \bullet_1, \bullet_2 \}$$

$$I(R) = \bullet_1$$

$$I(J) = \bullet_2$$

$$I(P) = \dots$$



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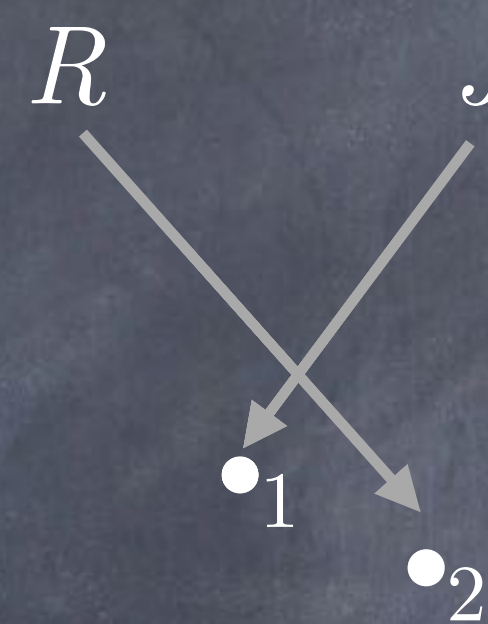
$$\Omega_I = \{\ \bullet_1,\ \bullet_2\ \}$$

$$I(R) = \bullet_2$$

$$I(J) = \bullet_1$$

$$I(P) = \ldots$$



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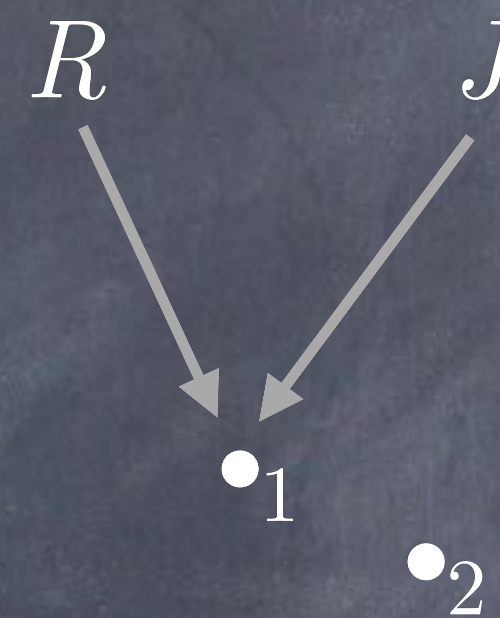

Constant symbols: $\{ R, J \}$

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$$\Omega_I = \{ \bullet_1, \bullet_2 \}$$

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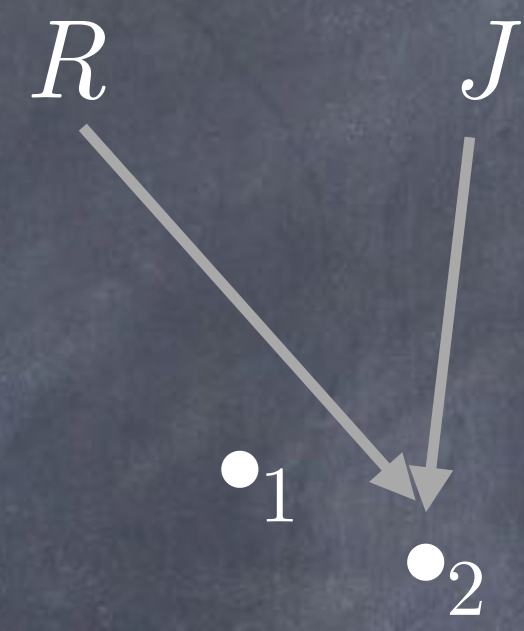


Constant symbols: $\{\ R,\ J\ \}$

Relation symbol: $P(\cdot,\cdot)$

$$\Omega_I = \{\ \bullet_1,\ \bullet_2\ \}$$

$$I(R) = \bullet_1$$

$$I(J) = \bullet_2$$

$$I(P) = \ldots$$

$\langle\bullet_1,\bullet_1\rangle$, $\langle\bullet_1,\bullet_2\rangle$, $\langle\bullet_2,\bullet_1\rangle$, $\langle\bullet_2,\bullet_2\rangle$: 2^2=4 binary tuples

2^{2^2}=16 interpretations of P

64 possible interpretations

Constant symbols: $\{ R, J \}$

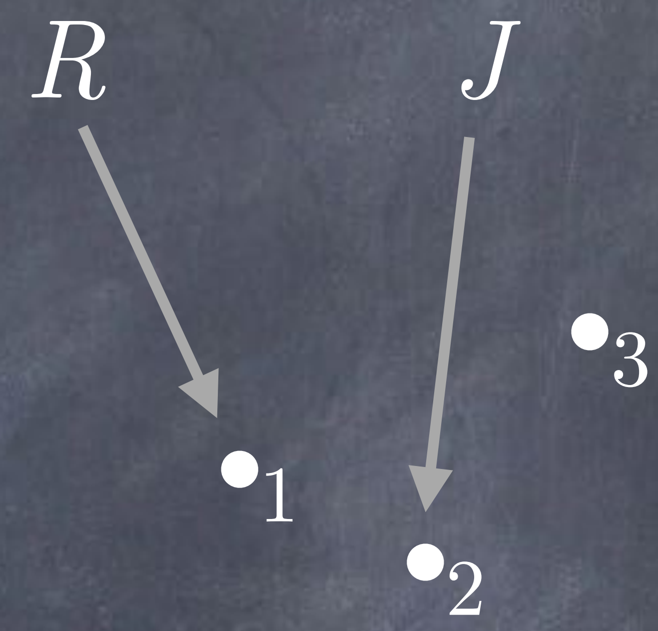
Relation symbol: $P(\cdot,\cdot)$

$$\Omega_I = \{ \bullet_1, \bullet_2, \bullet_3 \}$$

$$I(R) = \bullet_1$$

$$I(J) = \bullet_2$$

$$I(P) = \ldots$$



$2^3=8$ interpretations of R and J

$2^{2^3}=2^8=256$ interpretations of P

2048 possible interpretations

Constant symbols: $\{ R, J \}$

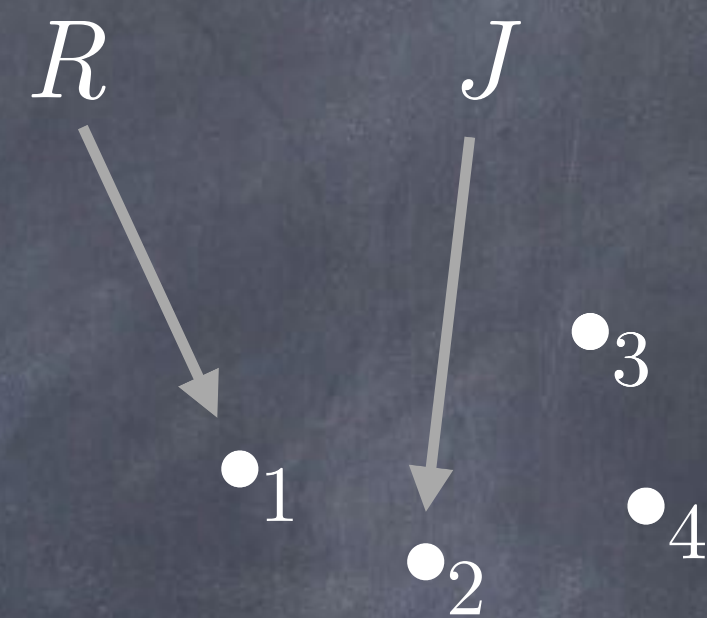
Relation symbol: $P(\cdot,\cdot)$

$$\Omega_I = \{ \bullet_1, \bullet_2, \bullet_3, \bullet_4 \}$$

$$I(R) = \bullet_1$$

$$I(J) = \bullet_2$$

$$I(P) = \ldots$$



1,048,576 possible interpretations

Computing Entailment

- Number of models HUGE (possibly unbounded: infinite # of objects)
- Can't do model checking

Computing Entailment

- Number of models HUGE (possibly unbounded: infinite # of objects)
- Can't do model checking
- Look for inference rules, do theorem proving

First-Order Inference

$$\forall x \; King(x) \Rightarrow Evil(x)$$

$$King(John)$$

$$\forall x \; King(x) \Rightarrow Evil(x)$$

$$King(John)$$

Conclude: $Evil(John)$

“Modus Ponens”

$\forall x \; King(x) \Rightarrow Evil(x)$ Applies to any object x

$King(John)$

$\forall x\ King(x) \Rightarrow Evil(x)$ Applies to any object x

$King(John)$ $John$ denotes an object

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$King(John) \Rightarrow Evil(John)$

So the rule applies to $John$

$\forall x \; King(x) \Rightarrow Evil(x)$ Applies to any object x

$King(John)$ $John$ denotes an object

$King(John) \Rightarrow Evil(John)$

So the rule applies to $John$

Conclude: $Evil(John)$ By Modus Ponens

Universal Instantiation

$$\frac{\forall v\ \alpha}{\textsc{Subst}(\{v/g\},\ \alpha)}$$

for any variable v and ground term g

Universal Instantiation

$$\frac{\forall v\ \alpha}{\textsc{Subst}(\{v/g\},\ \alpha)}$$

for any variable v and ground term g

Sound? If $\alpha \vdash \beta$ then $\alpha \models \beta$ ✔

Complete? If $\alpha \models \beta$ then $\alpha \vdash \beta$ ✖

Existential Instantiation

$$\frac{\exists v\ \alpha}{\textsc{Subst}(\{v/k\},\ \alpha)}$$

for any variable v and new constant k

Existential Instantiation

$$\frac{\exists v\ \alpha}{\textsc{Subst}(\{v/k\},\ \alpha)}$$

for any variable v and new constant k

Sound? ✔

Complete? ✖

Propositionalization

- Use UI to eliminate ∀
- Use EI to eliminate ∃
- Result is a set of sentences with no variables (ground sentences)
  - Atomic sentences with no variables that can be true or false
  - Propositions!

$$\forall x\ King(x) \land Greedy(x) \Rightarrow Evil(x)$$

$$King(John)$$

$$Greedy(John)$$

$$Brother(Richard, John)$$

$$King(John) \land Greedy(John) \Rightarrow Evil(John)$$

$$King(Richard) \land Greedy(Richard) \Rightarrow Evil(Richard)$$

$$King(John)$$

$$Greedy(John)$$

$$Brother(Richard, John)$$

$$King(John) \land Greedy(John) \Rightarrow Evil(John)$$

$$King(Richard) \land Greedy(Richard) \Rightarrow Evil(Richard)$$

$$King(John)$$

$$Greedy(John)$$

$$Brother(Richard, John)$$

Conclude: $Evil(John)$

Propositionalization

- Convert FOL sentences to PL sentences and do PL inference on them

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- Function f: $f(a)$, $f(f(a))$, $f(f(f(a)$, ...
  - Infinitely many ground terms for UI

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- Convert FOL sentences to PL sentences and do PL inference on them
- Function f: $f(\mathrm{a})$, $f(f(a))$, $f(f(f(a)$, ...
  - Infinitely many ground terms for UI
- Herbrand's Theorem (see book)

Propositionalization

- Convert FOL sentences to PL sentences and do PL inference on them
- Function f: $f(\mathrm{a})$, $f(f(a))$, $f(f(f(a)$, ...
  - Infinitely many ground terms for UI
- Herbrand's Theorem (see book)
- May never stop generating ground sentences if query not entailed by KB

First-Order Logic is Semi-Decidable

- Algorithms exist that say yes to every entailed sentence, but no algorithm exists that also says no to every nonentailed sentence

Propositionalization

- Convert FOL sentences to PL sentences and do PL inference on them
- Universal Instantiation (UI) for $\forall$
- Existential Instantiation (EI) for $\exists$

$$\forall x \; King(x) \Rightarrow Evil(x)$$

$$King(John)$$

$$King(John) \Rightarrow Evil(John)$$

$$King(John)$$

$$King(John) \Rightarrow Evil(John)$$

$$King(John)$$

Conclude: $Evil(John)$

$$\forall x\ King(x) \land Greedy(x) \Rightarrow Evil(x)$$

$$King(John)$$

$$Greedy(John)$$

$$King(Richard)$$

$$\forall x \; King(x) \land Greedy(x) \Rightarrow Evil(x)$$

$$King(John)$$

$$Greedy(John)$$

$$King(Richard)$$

$$King(John) \land Greedy(John) \Rightarrow Evil(John)$$

$$King(John)$$

$$Greedy(John)$$

$$King(Richard)$$

$$King(John) \wedge Greedy(John) \Rightarrow Evil(John)$$

$$King(John)$$

$$Greedy(John)$$

$$King(Richard)$$

$$King(John) \land Greedy(John) \Rightarrow Evil(John)$$

$$King(John)$$

$$Greedy(John)$$

$$King(Richard)$$

Conclude: $Evil(John)$

$$\forall x \; King(x) \land Greedy(x) \Rightarrow Evil(x)$$

$$King(John)$$

$$Greedy(John)$$

$$King(Richard)$$

$$King(Richard) \land Greedy(Richard) \Rightarrow Evil(Richard)$$

$$King(John)$$

$$Greedy(John)$$

?

$$King(Richard)$$

Can't conclude: $Evil(Richard)$

Substitution

Replacement of variables by terms (a.k.a. binding)

$\{\ x/John\ \}$

$\{\ y/father(x),\ x/Richard\ \}$

Substitution

Replacement of variables by terms (a.k.a. binding)

$\{\ x/John\ \}$

$\{\ y/father(x),\ x/Richard\ \}$

$$\textsc{Subst}(\Theta,\ \alpha)$$

$$\textsc{Subst}(\{x/John\},\ King(x) \Rightarrow Evil(x)) =$$

$$King(John) \Rightarrow Evil(John)$$

$$\forall x \ King(x) \land Greedy(x) \Rightarrow Evil(x)$$

$$King(John)$$

$$\Theta = \{ \ x/John \ \}$$

$$Greedy(John)$$

$$King(Richard)$$

If there is some substitution that makes the premises true, then the conclusion with the same substitution is also true

Generalized Modus Ponens

$$\frac{p_1', p_2', \cdots, p_n',\ (p_1 \wedge p_2 \wedge \cdots \wedge p_n \Rightarrow q)}{Subst(\Theta, q)}$$

p_i, p_i', q are atomic sentences

Θ is a substitution such that:

$$Subst(\Theta, p_i') = Subst(\Theta, p_i)$$

Generalized Modus Ponens

$$\frac{p_1', p_2', \cdots, p_n', \ (p_1 \wedge p_2 \wedge \cdots \wedge p_n \Rightarrow q)}{Subst(\Theta, q)}$$

p_i, p_i', q are atomic sentences

Θ is a substitution such that:

$$Subst(\Theta, p_i') = Subst(\Theta, p_i)$$

Sound? ✔

Complete? ✖

Lifted Inference Rule

- Inference rule lifted from ground (variable-free) propositional logic to first-order logic
- "The key advantage of lifted inference rules over propositionalization is that they make only those substitutions that are required to allow particular inferences to succeed"

Lifted Inference Rules

$$\frac{\alpha \wedge \beta}{\alpha}$$

And-elimination

$$\frac{\neg\neg\alpha}{\alpha}$$

Double negation

$$\frac{\neg(\alpha \wedge \beta)}{\neg\alpha \vee \neg\beta} \qquad \frac{\neg(\alpha \vee \beta)}{\neg\alpha \wedge \neg\beta}$$

DeMorgan's Laws

$$\frac{\alpha \Rightarrow \beta, \alpha}{\beta}$$

Modus Ponens

$$\frac{\alpha \Leftrightarrow \beta}{(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)} \qquad \frac{(\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)}{\alpha \Leftrightarrow \beta}$$

Definition of biconditional

Unit Resolution (PL)

$$\frac{l_1 \vee \cdots \vee l_i \vee \cdots \vee l_k, \quad m}{l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \cdots \vee l_k}$$

$l_1, \ldots, l_k$ and m are literals

l_i and m are complementary

Resolution (PL)

$$\frac{l_1 \vee \cdots \vee l_i \vee \cdots \vee l_k, \quad m_1 \vee \cdots \vee m_j \vee \cdots \vee m_n}{l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \cdots \vee m_n}$$

l_1, ..., l_k , m_1, ..., m_n are <u>literals</u>

l_i and m_j are <u>complementary</u>

Technical note: Resulting clause must be <u>factored</u> to contain only one copy of each literal.

Resolution (PL)

$$\frac{l_1 \vee \cdots \vee l_i \vee \cdots \vee l_k, \quad m_1 \vee \cdots \vee m_j \vee \cdots \vee m_n}{l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \cdots \vee m_n}$$

l_1, ..., l_k , m_1, ..., m_n are literals

l_i and m_j are complementary

Technical note: Resulting clause must be factored to contain only one copy of each literal.

Sound? ✔ Complete? ✔

Resolution

- Propositional resolution is sound and complete
- Can it be lifted to first-order logic?

First-Order CNF

- Every sentence of first-order logic can be converted into an inferentially equivalent sentence in conjunctive normal form (CNF)

First-Order CNF

- Eliminate implications
- Move negation inwards
- Standardize variables apart
- Skolemize existential variables
- Drop universal variables
- Distribute $\vee$ over $\wedge$

First-Order CNF

- Every sentence of first-order logic can be converted into an inferentially equivalent sentence in conjunctive normal form (CNF)
- Not logically equivalent
- Inferentially equivalent: (un)satisfiable iff original sentence is (un)satisfiable

Resolution (PL)

$$\frac{l_1 \vee \cdots \vee l_i \vee \cdots \vee l_k, \quad m_1 \vee \cdots \vee m_j \vee \cdots \vee m_n}{l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \cdots \vee m_n}$$

l_1, ..., l_k , m_1, ..., m_n are <u>literals</u>

l_i and m_j are <u>complementary</u>

Technical note: Resulting clause must be <u>factored</u> to contain only one copy of each literal.

$$\forall x \; King(x) \wedge Greedy(x) \Rightarrow Evil(x)$$

$$King(John)$$

$$\Theta = \{ \; x/John \; \}$$

$$Greedy(John)$$

$$King(Richard)$$

Unification

$unify(\alpha, \beta) = \Theta$ **where** $subst(\Theta,\alpha) = subst(\Theta,\beta)$

$$Knows(John, x),\ Knows(John, Jane)$$

$$\{\ x/Jane\ \}$$

$$Knows(John, x),\ Knows(y,\ Bill)$$

$$\{\ x/Bill,\ y/John\ \}$$

$$Knows(John, x),\ Knows(y,\ mother(y))$$

$$\{\ y/John,\ x/mother(John)\ \}$$

$$Knows(John, x),\ Knows(x, Elizabeth)$$

No unifier

Unification

$$\mathit{unify}(\alpha, \beta) = \Theta \;\; \text{where} \;\; \mathit{subst}(\Theta,\alpha) = \mathit{subst}(\Theta,\beta)$$

- Variables need to be standardized apart
- Occurs check
- Most general unifier: places the fewest restrictions on the variables
  - Is unique
  - Algorithm: AIMA Fig 9.1

First-Order Resolution

$$\frac{l_1 \vee \cdots \vee l_i \vee \cdots \vee l_k, \quad m_1 \vee \cdots \vee m_j \vee \cdots \vee m_n}{Subst(\Theta, l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \cdots \vee m_n)}$$

$$\Theta = Unify(l_i, \neg m_j)$$

First-Order Resolution

$$\frac{l_1 \vee \cdots \vee l_i \vee \cdots \vee l_k, \quad m_1 \vee \cdots \vee m_j \vee \cdots \vee m_n}{Subst(\Theta, l_1 \vee \cdots \vee l_{i-1} \vee l_{i+1} \cdots \vee l_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \cdots \vee m_n)}$$

$$\Theta = Unify(l_i, \neg m_j)$$

Sound? ✔

Complete? ✔ (Refutation-complete, with factoring or non-binary resolution; see AIMA)

Proof by Resolution

- Convert KB to CNF
- Convert $\neg\beta$ to CNF and add to KB
- Apply resolution rule to complementary clauses (with unification)
- Until ...

Resolution Refutation

- Convert KB to CNF
- Convert $\neg\beta$ to CNF and add to KB
- Apply resolution rule to complementary clauses (with unification)
- Until you derive the empty clause

Resolution Refutation

- If by applying the resolution inference rule to a set of clauses you can derive the empty clause...
- Then the set of clauses is unsatisfiable

The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all its missiles were sold to it by Colonel West, who is American.

To prove: West is a criminal

The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all its missiles were sold to it by Colonel West, who is American.

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To prove: West is a criminal

The law says that it is a crime for an American to sell weapons to hostile nations. The country Nono, an enemy of America, has some missiles, and all its missiles were sold to it by Colonel West, who is American.

To prove: West is a criminal

- Objects:
  - $Nono,\ America,\ West$
- Relations:
  - $Criminal(\cdot),\ American(\cdot),\ Weapon(\cdot),$ $Hostile(\cdot),\ Missile(\cdot)$
  - $Enemy(\cdot,\cdot)$
  - $Sells(\cdot,\cdot,\cdot)$

It is a crime for an American to sell weapons to hostile nations.

$$\forall x, y, z\ American(x) \land Weapon(y) \land Sells(x, y, z) \land Hostile(z) \Rightarrow Criminal(x)$$

Nono ... has some missiles.

$$\exists x\ Owns(Nono, x) \land Missile(x)$$

All Nono's missiles were sold to it by Colonel West.

$$\forall x\ Missile(x) \land Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$$

Missiles are weapons.

$$\forall x\ Missile(x) \Rightarrow Weapon(x)$$

An enemy of America counts as "hostile."

$$\forall x\ Enemy(x, America) \Rightarrow Hostile(x)$$

West is an American.

$$American(West)$$

Nono is an enemy of America.

$$Enemy(Nono, America)$$

$$\forall x, y, z\ American(x) \land Weapon(y) \land Sells(x, y, z) \land Hostile(z) \Rightarrow Criminal(x)$$

$$\exists x\ Owns(Nono, x) \land Missile(x)$$

$$\forall x\ Missile(x) \land Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$$

$$\forall x\ Missile(x) \Rightarrow Weapon(x)$$

$$\forall x\ Enemy(x, America) \Rightarrow Hostile(x)$$

$American(West)$ $\qquad$ $Enemy(Nono, America)$

Proof by Resolution

- Convert KB to CNF
- Convert $\neg\beta$ to CNF and add to KB
- Apply resolution rule to complementary clauses (with unification)
- Until you derive the empty clause

Convert to CNF

- Eliminate implications
- Move negation inwards
- Standardize variables apart
- Skolemize existential variables
- Drop universal variables
- Distribute $\vee$ over $\wedge$

$$\forall x, y, z\ American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z) \Rightarrow Criminal(x)$$

Eliminate implications:

$$\forall x, y, z\ \neg[American(x) \wedge Weapon(y) \wedge Sells(x, y, z) \wedge Hostile(z)] \vee Criminal(x)$$

Move negation inwards (DeMorgan's Laws)

$$\forall x, y, z\ [\neg American(x) \vee \neg Weapon(y) \vee \neg Sells(x, y, z) \vee \neg Hostile(z)] \vee Criminal(x)$$

Standardize variables apart

Skolemize existential variables

Drop universal quantifiers

$$\neg American(x) \vee \neg Weapon(y) \vee \neg Sells(x, y, z) \vee \neg Hostile(z) \vee Criminal(x)$$

Distribute $\vee$ over $\wedge$

$$\forall x, y, z\ American(x) \land Weapon(y) \land Sells(x, y, z) \land Hostile(z) \Rightarrow Criminal(x)$$

$$\neg American(x) \lor \neg Weapon(y) \lor \neg Sells(x, y, z) \lor \neg Hostile(z) \lor Criminal(x)$$

Clause

Horn clause

Definite clause

$$\forall x, y, z\ American(x) \land Weapon(y) \land Sells(x, y, z) \land Hostile(z) \Rightarrow Criminal(x)$$

$$\neg American(x) \lor \neg Weapon(y) \lor \neg Sells(x, y, z) \lor \neg Hostile(z) \lor Criminal(x)$$

$$\exists x\ Owns(Nono, x) \land Missile(x)$$

$$Owns(Nono, M_1), Missile(M_1)$$

$$\forall x, y, z\ American(x) \land Weapon(y) \land Sells(x, y, z) \land Hostile(z) \Rightarrow Criminal(x)$$

$$\neg American(x) \lor \neg Weapon(y) \lor \neg Sells(x, y, z) \lor \neg Hostile(z) \lor Criminal(x)$$

$$\exists x\ Owns(Nono, x) \land Missile(x)$$

$$Owns(Nono, M_1), Missile(M_1)$$

$$\forall x\ Missile(x) \land Owns(Nono, x) \Rightarrow Sells(West, x, Nono)$$

$$\neg Missile(x) \lor \neg Owns(Nono, x) \lor Sells(West, x, Nono)$$

$$\forall x\ Missile(x) \Rightarrow Weapon(x)$$

$$\neg Missile(x) \lor Weapon(x)$$

$$\forall x\ Enemy(x, America) \Rightarrow Hostile(x)$$

$$\neg Enemy(x, America) \lor Hostile(x)$$

$American(West)$ $\qquad$ $Enemy(Nono, America)$

$American(West)$ $\qquad$ $Enemy(Nono, America)$

$$\neg American(x) \lor \neg Weapon(y) \lor \neg Sells(x, y, z) \lor \neg Hostile(z) \lor Criminal(x)$$

$$Owns(Nono, M_1), Missile(M_1)$$

$$\neg Missile(x) \lor \neg Owns(Nono, x) \lor Sells(West, x, Nono)$$

$$\neg Missile(x) \lor Weapon(x)$$

$$\neg Enemy(x, America) \lor Hostile(x)$$

$$American(West)$$

$$Enemy(Nono, America)$$

Proof by Resolution

- Convert KB to CNF
- Convert $\neg\beta$ to CNF and add to KB
- Apply resolution rule to complementary clauses (with unification)
- Until you derive the empty clause

"West is a criminal"

$$Criminal(West)$$

Negate and convert to CNF:

$$\neg Criminal(West)$$

$$\neg American(x) \lor \neg Weapon(y) \lor \neg Sells(x, y, z) \lor \neg Hostile(z) \lor Criminal(x)$$

$$Owns(Nono, M_1), Missile(M_1)$$

$$\neg Missile(x) \lor \neg Owns(Nono, x) \lor Sells(West, x, Nono)$$

$$\neg Missile(x) \lor Weapon(x)$$

$$\neg Enemy(x, America) \lor Hostile(x)$$

$$American(West)$$

$$Enemy(Nono, America)$$

$$\neg Criminal(West)$$

Proof by Resolution

- Convert KB to CNF
- Convert $\neg\beta$ to CNF and add to KB
- Apply resolution rule to complementary clauses (with unification)
- Until you derive the empty clause

$$\neg American(x) \vee \neg Weapon(y) \vee \neg Sells(x, y, z) \vee \neg Hostile(z) \vee Criminal(x)$$

$$Owns(Nono, M_1), Missile(M_1)$$

$$\neg Missile(x) \vee \neg Owns(Nono, x) \vee Sells(West, x, Nono)$$

$$\neg Missile(x) \vee Weapon(x)$$

$$\neg Enemy(x, America) \vee Hostile(x)$$

$$American(West)$$

$$Enemy(Nono, America)$$

$$\neg Criminal(West)$$

$$\neg American(x) \lor \neg Weapon(y) \lor \neg Sells(x, y, z) \lor \neg Hostile(z) \lor \boxed{Criminal(x)}$$

$$Owns(Nono, M_1), Missile(M_1)$$

$$\neg Missile(x) \lor \neg Owns(Nono, x) \lor Sells(West, x, Nono)$$

$$\neg Missile(x) \lor Weapon(x)$$

$$\neg Enemy(x, America) \lor Hostile(x)$$

$$American(West)$$

$$\{ x/West \}$$

$$Enemy(Nono, America)$$

$$\boxed{\neg Criminal(West)}$$

$$\neg American(x) \vee \neg Weapon(y) \vee \neg Sells(x,y,z) \vee \neg Hostile(z) \vee \underline{Criminal(x)}$$

$$\underline{\neg Criminal(West)}$$

$$\{x/West\}$$

$$\neg American(West) \vee \neg Weapon(y) \vee \neg Sells(West,y,z) \vee \neg Hostile(z)$$

$$\neg American(x) \lor \neg Weapon(y) \lor \neg Sells(x,y,z) \lor \neg Hostile(z) \lor Criminal(x)$$

$$\neg Criminal(West)$$

$$\underline{\neg American(West)} \lor \neg Weapon(y) \lor \neg Sells(West,y,z) \lor \neg Hostile(z)$$

$$\{\}$$

$$\underline{American(West)}$$

$$\neg Weapon(y) \lor \neg Sells(West,y,z) \lor \neg Hostile(z)$$

$$\neg American(x) \lor \neg Weapon(y) \lor \neg Sells(x,y,z) \lor \neg Hostile(z) \lor Criminal(x)$$

$$\neg Criminal(West)$$

$$\neg American(West) \lor \neg Weapon(y) \lor \neg Sells(West,y,z) \lor \neg Hostile(z)$$

$$American(West)$$

$$\underline{\neg Weapon(y)} \lor \neg Sells(West,y,z) \lor \neg Hostile(z) \qquad \neg Missile(x) \lor \underline{Weapon(x)}$$

$$\{x/y\}$$

$$\neg Missile(y) \lor \neg Sells(West,y,z) \lor \neg Hostile(z)$$

$$\neg American(x) \vee \neg Weapon(y) \vee \neg Sells(x,y,z) \vee \neg Hostile(z) \vee Criminal(x)$$

$$\neg Criminal(West)$$

$$\neg American(West) \vee \neg Weapon(y) \vee \neg Sells(West,y,z) \vee \neg Hostile(z)$$

$$American(West)$$

$$\neg Weapon(y) \vee \neg Sells(West,y,z) \vee \neg Hostile(z)$$

$$\neg Missile(x) \vee Weapon(x)$$

$$\underline{\neg Missile(y)} \vee \neg Sells(West,y,z) \vee \neg Hostile(z)$$

$$\underline{Missile(M_1)}$$

$$\{y/M_1\}$$

$$\neg Sells(West,M_1,z) \vee \neg Hostile(z)$$

$$\neg American(x) \vee \neg Weapon(y) \vee \neg Sells(x,y,z) \vee \neg Hostile(z) \vee Criminal(x)$$

$$\neg Criminal(West)$$

$$\neg American(West) \vee \neg Weapon(y) \vee \neg Sells(West,y,z) \vee \neg Hostile(z)$$

$$American(West)$$

$$\neg Weapon(y) \vee \neg Sells(West,y,z) \vee \neg Hostile(z)$$

$$\neg Missile(x) \vee Weapon(x)$$

$$\neg Missile(y) \vee \neg Sells(West,y,z) \vee \neg Hostile(z)$$

$$Missile(M_1)$$

$$\underline{\neg Sells(West,M_1,z)} \vee \neg Hostile(z)$$

$$\neg Missile(x) \vee \neg Owns(Nono,x) \vee \underline{\neg Sells(West,x,Nono)}$$

$$\{x/M_1, z/Nono\}$$

$$\neg Missile(M_1) \vee \neg Owns(Nono,M_1) \vee \neg Hostile(Nono)$$

$$\neg American(x) \vee \neg Weapon(y) \vee \neg Sells(x,y,z) \vee \neg Hostile(z) \vee Criminal(x)$$

$$\neg Criminal(West)$$

$$\neg American(West) \vee \neg Weapon(y) \vee \neg Sells(West,y,z) \vee \neg Hostile(z)$$

$$American(West)$$

$$\neg Weapon(y) \vee \neg Sells(West,y,z) \vee \neg Hostile(z) \qquad \neg Missile(x) \vee Weapon(x)$$

$$\neg Missile(y) \vee \neg Sells(West,y,z) \vee \neg Hostile(z)$$

$$Missile(M_1)$$

$$\neg Sells(West,M_1,z) \vee \neg Hostile(z)$$

$$\neg Missile(x) \vee \neg Owns(Nono,x) \vee \neg Sells(West,x,Nono)$$

$$\underline{\neg Missile(M_1)} \vee \neg Owns(Nono,M_1) \vee \neg Hostile(Nono) \qquad \underline{Missile(M_1)}$$

$$\{\}$$

$$\neg Owns(Nono,M_1) \vee \neg Hostile(Nono)$$

$$\neg American(x) \lor \neg Weapon(y) \lor \neg Sells(x,y,z) \lor \neg Hostile(z) \lor Criminal(x)$$

$$\neg Criminal(West)$$

$$\neg American(West) \lor \neg Weapon(y) \lor \neg Sells(West,y,z) \lor \neg Hostile(z)$$

$$American(West)$$

$$\neg Weapon(y) \lor \neg Sells(West,y,z) \lor \neg Hostile(z) \qquad \neg Missile(x) \lor Weapon(x)$$

$$\neg Missile(y) \lor \neg Sells(West,y,z) \lor \neg Hostile(z)$$

$$Missile(M_1)$$

$$\neg Sells(West,M_1,z) \lor \neg Hostile(z)$$

$$\neg Missile(x) \lor \neg Owns(Nono,x) \lor \neg Sells(West,x,Nono)$$

$$\neg Missile(M_1) \lor \neg Owns(Nono,M_1) \lor \neg Hostile(Nono) \qquad Missile(M_1)$$

$$\underline{\neg Owns(Nono,M_1)} \lor \neg Hostile(Nono) \qquad \underline{Owns(Nono,M_1)}$$

$$\{\}$$

$$\neg Hostile(Nono)$$

$$\neg American(x) \lor \neg Weapon(y) \lor \neg Sells(x,y,z) \lor \neg Hostile(z) \lor Criminal(x)$$

$$\neg Criminal(West)$$

$$\neg American(West) \lor \neg Weapon(y) \lor \neg Sells(West,y,z) \lor \neg Hostile(z)$$

$$American(West)$$

$$\neg Weapon(y) \lor \neg Sells(West,y,z) \lor \neg Hostile(z) \qquad \neg Missile(x) \lor Weapon(x)$$

$$\neg Missile(y) \lor \neg Sells(West,y,z) \lor \neg Hostile(z)$$

$$Missile(M_1)$$

$$\neg Sells(West,M_1,z) \lor \neg Hostile(z)$$

$$\neg Missile(x) \lor \neg Owns(Nono,x) \lor \neg Sells(West,x,Nono)$$

$$\neg Missile(M_1) \lor \neg Owns(Nono,M_1) \lor \neg Hostile(Nono) \qquad Missile(M_1)$$

$$\neg Owns(Nono,M_1) \lor \neg Hostile(Nono) \qquad Owns(Nono,M_1)$$

$$\underline{\neg Hostile(Nono)} \qquad \neg Enemy(x,America) \lor \underline{Hostile(x)}$$

$$\{x/Nono\}$$

$$\neg Enemy(Nono,America)$$

$$\neg American(x) \vee \neg Weapon(y) \vee \neg Sells(x,y,z) \vee \neg Hostile(z) \vee Criminal(x)$$

$$\neg Criminal(West)$$

$$\neg American(West) \vee \neg Weapon(y) \vee \neg Sells(West,y,z) \vee \neg Hostile(z)$$

$$American(West)$$

$$\neg Weapon(y) \vee \neg Sells(West,y,z) \vee \neg Hostile(z) \qquad \neg Missile(x) \vee Weapon(x)$$

$$\neg Missile(y) \vee \neg Sells(West,y,z) \vee \neg Hostile(z) \qquad Missile(M_1)$$

$$\neg Sells(West,M_1,z) \vee \neg Hostile(z)$$

$$\neg Missile(x) \vee \neg Owns(Nono,x) \vee \neg Sells(West,x,Nono)$$

$$\neg Missile(M_1) \vee \neg Owns(Nono,M_1) \vee \neg Hostile(Nono) \qquad Missile(M_1)$$

$$\neg Owns(Nono,M_1) \vee \neg Hostile(Nono) \qquad Owns(Nono,M_1)$$

$$\neg Hostile(Nono) \qquad \neg Enemy(x,America) \vee Hostile(x)$$

$$\underline{\neg Enemy(Nono,America)} \qquad \underline{Enemy(Nono,America)}$$

$$\square \quad \{\}$$

Resolution Proof

Since $KB \wedge \neg Criminal(West)$ is unsatisfiable,

$$KB \models Criminal(West)$$

Forward Chaining

- Knowledge base of definite clauses
- Starting from known facts, trigger rules whose premises are satisfied
    - Using substitution to match
- Add their conclusions to the KB
- Until the query is answered or no new facts can be generated

Backward Chaining

- Work backward from the goal, chaining through rules to find known facts that support the proof
  - Allow substitutions when matching facts and rules
- DFS => incomplete

Logic Programming (Prolog)

- Horn clause KB (facts & rules)
- Match using unification
- Backwards chaining from conclusions to premises (DFS)
- Special search strategy for selecting which clauses (rules) to match
- Includes non-logical statements also

First-Order Inference

- Semantics of first-order sentences: Interpretations, extended interpretations
- Entailment
- Propositionalization
  - First order logic is semi-decidable (R.E.)
- Lifted inference rules
- Resolution (first-order CNF, unification)
  - Proof by contradiction

Representation

- Factored representations of state
- Constraint Satisfaction Problems
- Propositional Logic and Inference
- First-Order Logic and Inference

For Next Time:

Unit Exam