CSC242: Homework 3.2
AIMA Chapter 13.4–13.6

1. Consider the following full joint probability distribution over the random variables $Cavity$, $Toothache$, and $Catch$:

	$toothache$		$\neg toothache$	
$Cavity$	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

   (a) What are the most and least likely situations, according to the distribution?
   (b) Compute the prior probability that the patient has a cavity and a toothache and the dentist's probe catches on the tooth.
   (c) Compute the prior probability that the patient has a cavity and a toothache.
   (d) Compute the prior probability that the patient does not have a cavity.

2. Give a formal definition of the conditional probability of event a given event b.

3. Using the dentistry distribution given above, compute the conditional probability of not having a cavity given the patient does not have a toothache.

4. Compute the conditional distribution of $Cavity$ given $\neg toothache$.

5. Explain how to compute, in general, a posterior distribution for a random variable given some evidence, using the full joint distribution. Include the formula.

6. Suppose we know that having a cavity usually causes a toothache. A dentist sees a patient who is complaining of a toothache. Is it correct to conclude that they probably have a cavity? Explain your answer briefly.

7. What does it mean for two random variables to be independent? Give a formal definition.

8. Prove that the definition of independence (AIMA p. 494) is correct.

9. What does it mean for two random variables to be conditionally independent? Give a formal definition.

10. Why are independence assertions, particularly conditional independence assumptions, useful for inference?