CSC242: Homework 3.4
AIMA 14.5

Project 3 gives you hands-on experience with both exact and approximate inference in Bayesian Networks. These questions just complement what you learn by doing the project.

1. What does it mean for a probability estimate to be *consistent*. Be brief.

   ANSWER: The estimated probability becomes exact in the large-sample limit:

   $$\lim_{N\to\infty} \frac{N_{PS}(x_1,\ldots,x_n)}{N} = P(x_1,\ldots,x_n). \qquad \text{(AIMA Eq. 14.5)}$$

2. What are the main strength and main weakness of rejection sampling?

   ANSWER: Strength: Easy to implement (sampling is easy, linear in number of variables, rejection is easy).

   Weakness: May reject many samples. The fraction of samples consistent with the evidence drops exponentially as the number of evidence variables grows. That is, there are an exponential number of combinations of possible values for the evidence variables, but we will only accept the one of them that matches our evidence (AIMA p. 532).

3. How does likelihood weighting improve on rejection sampling?

   ANSWER: It never rejects a sample, but rather accumulates weighted samples where the weights are proportional to the probability of the sample given the distribution encoded in the network. Note however that it will still degrade in performance as the number of evidence variables grows (AIMA p. 535).

4. Describe briefly how Gibbs sampling works.

   ANSWER:

   - Start with an arbitrary sample (event, possible world, state) consistent with the evidence.
   - Generate the next sample by randomly sampling a value for one of the non-evidence variables X_i, conditioned on the current values of the variables in the Markov blanket of X_i (which are its parents, its children, and its children's parents). While you can't read this right out of the Bayesian Network, AIMA Eq. 14.12 and the description preceding it explain how to sample this distribution.
   - Accumulate the samples for the different values of the query variable to estimate its distribution (and then normalize).