CSC242: Homework 2.3
AIMA Chapter 7.5

1. Briefly define the following properties of a sentence or set of sentences:

   (a) Satisfiable
   ANSWER: There exists an assignment of true or false to the propositions making up the sentence or set of sentences that makes it/them true. Equivalently, at least one model of the sentence(s) exists. There is at least one possible world in which the sentence is true.

   (b) Unsatisatisfiable
   ANSWER: There are no models of the sentence or set of sentences. The sentence cannot be true in any possible world. ("Not satisfiable" would be a poor answer.)

   (c) Tautology
   ANSWER: True in any possible world. All possible assignments make the sentence true.

2. Briefly define the following properties of inference rules:

   (a) Soundness
   ANSWER: If $\alpha \vdash \beta$ then $\alpha \models \beta$.

   (b) Completeness
   ANSWER: If $\alpha \models \beta$ then $\alpha \vdash \beta$.

3. Prove that the following inference rules are sound:

   (a) Double Negation: $\frac{\neg\neg P}{P}$
   ANSWER:

P	$\neg P$	$\neg\neg P$
F	T	F
T	F	T

   Every model of $\neg\neg P$ (second row only) is also a model of P, so the inference rule is sound.

(b) *Modus Tollens*: $\dfrac{P \Rightarrow Q, \neg Q}{\neg P}$

ANSWER:

P	Q	$P \Rightarrow Q$	$\neg Q$	$\neg P$
F	F	T	T	T
F	T	T	F	T
T	F	F	T	F
T	T	T	F	F

Models of the premises: first row only. And it is also a model of the consequent (conclusion), so the rule is sound.

4. One rule of thumb for faculty hiring might be that a person who is not sociable ($\neg S$) is tenurable (T) if he or she is brilliant (B), but otherwise is not tenurable. Which of the following are correct representations of this assertion?

   (a) $(\neg S \wedge T) \iff B$

   ANSWER: No; this sentence asserts, among other things, that all brilliant people are not sociable, which is not what was stated (although it may be true ;-)).

   (b) $\neg S \Rightarrow (T \iff B)$

   ANSWER: Yes, this says that if a person is not sociable, then they are tenurable if and only if they are brilliant.

   (c) $\neg S \Rightarrow ((B \Rightarrow T) \vee \neg T)$

   ANSWER: No, this is equivalent to $S \vee \neg B \vee T \vee \neg T$ which is a tautology (always true, that is, true in all possible worlds, which isn't a useful definition).

5. Use resolution to prove the sentence $\neg A \wedge \neg B$ from the following knowledge base:

   S1: $A \iff (B \vee E)$

   S2: $E \Rightarrow D$

   S3: $C \wedge F \Rightarrow \neg B$

   S4: $E \Rightarrow B$

   S5: $B \Rightarrow F$

   S6: $B \Rightarrow C$

   ANSWER: Convert knowledge base to clauses (CNF):

   1a. $\neg A \vee B \vee E$

   1b. $\neg B \vee A$

   1c. $\neg E \vee A$

   2. $\neg E \vee D$

   3. $\neg C \vee \neg F \vee \neg B$

   4. $\neg E \vee B$

   5. $\neg B \vee F$

   6. $\neg B \vee C$

   To prove: $\neg A \wedge \neg B$

   Negated: $\neg(\neg A \wedge \neg B)$

   Converted to CNF: 7. $A \vee B$

   Here's one possible resolution refutation:

   1b & 7: 8. $A \vee A = A$ (after factoring)

   1a & 8: 9. $B \vee E$

   9 & 4: 10. $B \vee B = B$ (after factoring)

   10 & 5: 11. F

   10 & 6: 12. C

   11 & 3: 13. $\neg C \vee \neg B$

   12 & 13: 14. $\neg B$

   10 & 14: 15. $\square$

   For what it's worth, my very naive implementation of a resolution theorem prover generated over 700 new clauses before finding the empty clause for this problem.

It is also possible to solve this problem using reasoning by cases. That is, if the sentence $(\neg A \wedge \neg B)$ is true, then both conjuncts, $\neg A$ and $\neg B$, are individually true. That's what conjunction means. Therefore you can prove $\neg A$ and $\neg B$ seperately and then conclude that the conjunction is true. This would not be a "pure" resolution proof, so I wouldn't do it on an exam, but you can try it and see.

To prove $\neg B$, add the negated goal: 7. B

7 & 5: 8. F

7 & 6: 9. C

8 & 3: 10. $\neg C \vee \neg B$

9 & 10: 11. $\neg B$

7 & 11: 12. $\square$

To prove $\neg A$, add the negated goal: 7. A

7 & 1a: 8. $(B \vee E)$

8 & 4: 9. B

And then you can proceed as above to derive the empty clause.

6. Briefly explain why a knowledge base that can be expressed entirely as Horn clauses might be A Good Thing.

   ANSWER: Horn clauses have a natural reading as facts and rules, making their use convenient for expressing knowledge about the world. Furthermore, inference in Horn logic is tractable. Horn logic is the basis of the logic programming language Prolog.