CSC242: Introduction to Artificial Intelligence

Lecture 3.2

Please put away all electronic devices

$At_{1,2}$ $\neg P_{1,1}$

$\neg P_{2,1}$

$\neg P_{1,2}$

$P_{2,2} \vee P_{3,1}$

$P_{2,2} \vee P_{1,3}$

$\neg B_{1,1}$

$B_{2,1}$

$B_{1,2}$

Which room is least likely to contain a pit?

Diagnostic rule: $Toothache \Rightarrow Cavity$

Causal rule: $Cavity \Rightarrow Toothache$

Probability

- Sample space: possible worlds
- Probability model: degrees of belief in possible worlds
- Events: subsets of possible worlds

Random Variable

- A function from outcomes to values

- “the number of heads in three flips”
- “the total of the two dice”
- “the dice are matched doubles”
- “a red ball was picked”

Statements

- Elementary (atomic) statement

  $RV = value$

- Combine with connectives from PL

$$Die_1 = 3 \wedge Total = 7$$

$$Total = 7 \vee doubles$$

$$Die_1 = 3 \wedge \neg doubles$$

Probability Statements

- Assign a probability to a statement

$$P(Die_1 = 3 \wedge Total = 7) = 1/36$$

$$P(Total = 7 \vee doubles) = 12/36$$

$$P(Die_1 = 3 \wedge \neg doubles) = 5/36$$

Probability Distribution

- Assign a probability to every possible value of a random variable

$$Weather : \{sunny, rain, cloudy, snow\}$$

$$P(Weather = sunny) = 0.6$$

$$P(Weather = rain) = 0.1$$

$$P(Weather = cloudy) = 0.29$$

$$P(Weather = snow) = 0.01$$

$$\mathbf{P}(Weather) = \langle 0.6, 0.1, 0.29, 0.01 \rangle$$

Bold

Vector

Joint Distribution

- Distribution over multiple variables
- Gives probabilities of all combinations of the values of the variables

Joint Distributions

$$\mathbf{P}(Weather, Cavity)$$

		$Cavity$	
		$true$	$false$
$Weather$	$sunny$		
	$rain$		
	$cloudy$		
	$snow$		

Full Joint Probability Distribution

- Joint probability distribution over all the random variables
- Probabilities for every possible combination of values assigned to random variables
- Probabilities for every possible world

Full Joint Probability Distribution

$$\mathbf{P}(Cavity, Toothache, Weather)$$

	toothache		*¬toothache*	
	cavity	*¬cavity*	*cavity*	*¬cavity*
sunny				
rain				
cloudy				
snow				

Representing Uncertainty

- Probability: Sample space, probabilities, events
- Random variables, domains
- Language of probability statements
- Probability distributions, joint distributions, full joint distribution


THE
OFFICIAL
METALLICA
SO WHAT!
THE GOOD, THE MAD AND THE UGLY
EDITED BY STEFFAN CHIRAZI

Inference

- Compute what "follows from" our knowledge
- Make implicit knowledge explicit

Now with Probabilities!

Probabilistic Inference

- Compute what “follows from” our (uncertain) knowledge
- Make implicit knowledge explicit

Cavity

Toothache

Catch

$$\mathbf{P}(Cavity, Toothache, Catch)$$

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

$$\mathrm{P}(Cavity, Toothache, Catch)$$

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

$$P(Cavity, Toothache, Catch)$$

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

Probability of a Statement

$\mathrm{P}(cavity \vee toothache)$

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavit$	0.016	0.064	0.144	0.576

Probability of a Statement

$$P(cavity \vee toothache) = 0.28$$

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavit$	0.016	0.064	0.144	0.576

Probability of a Statement

$$P(cavity \wedge toothache) = 0.12$$

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavit$	0.016	0.064	0.144	0.576

Background Knowledge + Evidence ⇨ Conclusions

Unconditional (Prior) Probabilities

- Degrees of belief in propositions in the <u>absence</u> of any other information

$$P(doubles) = 1/3$$

$$P(cavity) = 0.2$$

Conditional (Posterior) Probability

- Degree of belief in a proposition given some information (evidence)

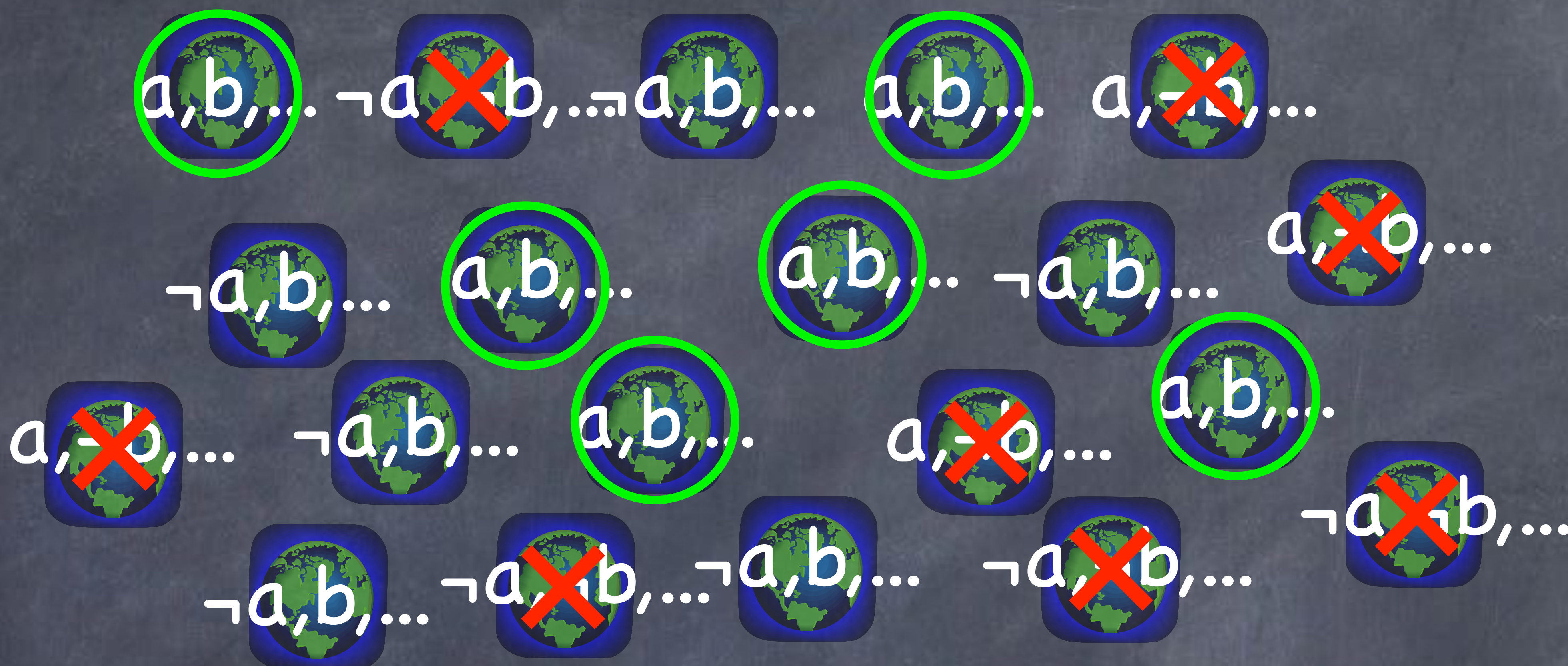
$$P(SnakeEyes \mid Die_1 = 1) = 1/6$$

$$P(cavity \mid toothache) = 0.6$$

- Whenever evidence is true and we have no further information, conclude probability of proposition

Conditional (Posterior) and Unconditional (Prior) Probability

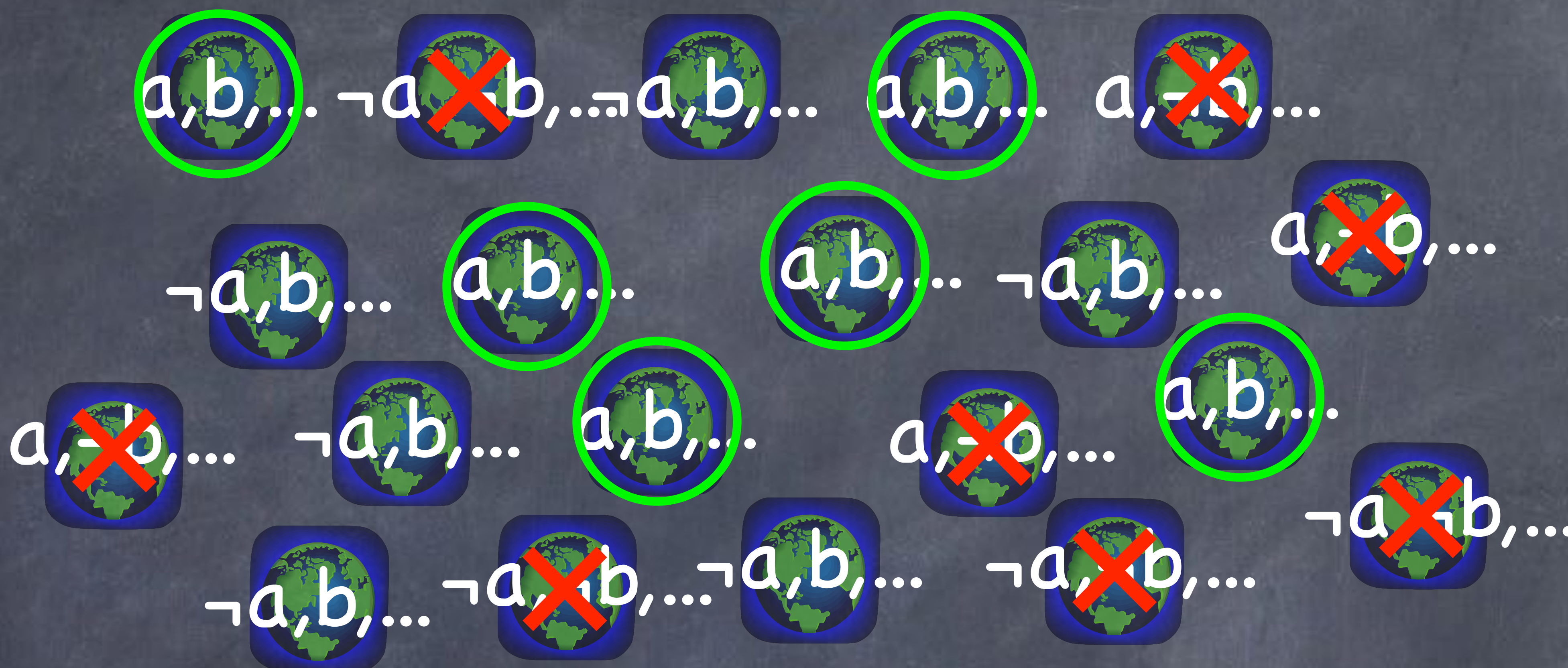
$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)} \text{ when } P(b) > 0$$



20 equally likely possible worlds

12 possible worlds where b is true $\qquad P(b) = 12/20$

a is true in 6 of those 12 $\qquad P(a \mid b) = 6/12 = 1/2$



20 equally likely possible worlds

12 possible worlds where b is true $\quad P(b) = 12/20$

6 worlds where $a \wedge b$ is true: $\quad P(a \wedge b) = 6/20$

$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)} = \frac{6/20}{12/20} = 1/2$$

Conditional (Posterior) and Unconditional (Prior) Probability

$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)} \text{ when } P(b) > 0$$

Conditional (Posterior) and Unconditional (Prior) Probability

$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)} \text{ when } P(b) > 0$$

Product Rule:

$$P(a \wedge b) = P(a \mid b)P(b)$$

Background Knowledge + Evidence ⇨ Conclusions

Prior Probabilities

Posterior (Conditional) Probabilities given evidence

Probabilistic Inference

- Computing posterior probabilities for statements given prior probabilities and observed evidence
- Given priors and evidence, compute probabilities given evidence (posteriors)

Probabilistic Inference

$$P(cavity \mid toothache) = \frac{P(cavity \wedge toothache)}{P(toothache)}$$

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

Probabilistic Inference

$$P(cavity \mid toothache) = \frac{P(cavity \wedge toothache)}{P(toothache)} = \underline{0.12}$$

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	$\boxed{0.108}$	$\boxed{0.012}$	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

Probabilistic Inference

$$P(cavity \mid toothache) = \frac{P(cavity \wedge toothache)}{P(toothache)} = \frac{0.12}{0.2}$$

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

Probabilistic Inference

$$P(cavity \mid toothache) = \frac{P(cavity \land toothache)}{P(toothache)} = 0.6$$

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

Probabilistic Inference

$$P(cavity \mid toothache) = \frac{P(cavity \wedge toothache)}{P(toothache)} = 0.6$$

$$P(\neg cavity \mid toothache) = \frac{P(\neg cavity \wedge toothache)}{P(toothache)}$$

Probabilistic Inference

$$P(\neg cavity \mid toothache) = \frac{P(\neg cavity \wedge toothache)}{P(toothache)} = 0.4$$

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

Probabilistic Inference

$$P(cavity \mid toothache) = \frac{P(cavity \land toothache)}{P(toothache)} = 0.6$$

$$P(\neg cavity \mid toothache) = \frac{P(\neg cavity \land toothache)}{P(toothache)} = 0.4$$

$$\mathbf{P}(Cavity \mid toothache) = \langle 0.6, 0.4 \rangle$$

Probabilistic Inference

$$P(cavity \mid toothache) = \frac{P(cavity \wedge toothache)}{P(toothache)} = 0.6$$

$$P(\neg cavity \mid toothache) = \frac{P(\neg cavity \wedge toothache)}{P(toothache)} = 0.4$$

$$\mathbf{P}(Cavity \mid toothache) = \langle 0.6, 0.4 \rangle$$

Probabilistic Inference

$$P(cavity \mid toothache) = \frac{P(cavity \land toothache)}{P(toothache)} = 0.6$$

$$P(\neg cavity \mid toothache) = \frac{P(\neg cavity \land toothache)}{P(toothache)} = 0.4$$

Probabilistic Inference

$$P(cavity \mid toothache) = \alpha\, P(cavity \wedge toothache) = \alpha\, 0.12$$

$$P(\neg cavity \mid toothache) = \alpha\, P(\neg cavity \wedge toothache) = \alpha\, 0.08$$

Probabilistic Inference

$$P(cavity \mid toothache) = \alpha\, P(cavity \wedge toothache) = \alpha\, 0.12$$

$$P(\neg cavity \mid toothache) = \alpha\, P(\neg cavity \wedge toothache) = \alpha\, 0.08$$

$$\mathbf{P}(Cavity \mid toothache) = \alpha\, \langle 0.12, 0.08 \rangle$$

Probabilistic Inference

$$P(cavity \mid toothache) = \alpha\, P(cavity \wedge toothache) = \alpha\, 0.12$$

$$P(\neg cavity \mid toothache) = \alpha\, P(\neg cavity \wedge toothache) = \alpha\, 0.08$$

$$\mathbf{P}(Cavity \mid toothache) = \frac{1}{0.12 + 0.08}\, \langle 0.12, 0.08 \rangle$$

S

$$P(cavity \mid toothache) = \alpha\, P(cavity \wedge toothache) = \alpha\, 0.12$$

$$P(\neg cavity \mid toothache) = \alpha\, P(\neg cavity \wedge toothache) = \alpha\, 0.08$$

$$\mathbf{P}(Cavity \mid toothache) = \langle 0.6, 0.4 \rangle$$

Normalization constant

We didn't know $P(toothache)$!

$$P(cavity) = 0.108 + 0.012 + 0.072 + 0.008 = 0.2$$

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

$$P(cavity) = \sum_{y_1 \in Toothache} \sum_{y_2 \in Catch} P(cavity, y_1, y_2)$$

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

Marginalization

$$\mathbf{P}(X) = \sum_{\mathbf{y} \in \mathbf{Y}} \mathbf{P}(X, \mathbf{y})$$

$X : \ Cavity$

$\mathbf{Y} : \{ \ Toothache, \ Catch \ \}$

$$\mathbf{P}(Cavity) = \sum_{y_1 \in Toothache} \sum_{y_2 \in Catch} \mathbf{P}(Cavity, y_1, y_2)$$

Probabilistic Inference (Single Variable)

$$\mathbf{P}(X \mid \mathbf{e})$$

Query variable $X : Domain(X) = \{x_1, \ldots, x_m\}$

Evidence variables $\mathbf{E} : \{E_1, \ldots, E_k\}$

Observations $\mathbf{e} : \{e_1, \ldots, e_k\}$ s.t. $E_i = e_i$

Unobserved variables $\mathbf{Y} : \{Y_1, \ldots, Y_l\}$

$Domain(Y_i) = \{y_{i,1}, \ldots, y_{i,n_i}\}$

Probabilistic Inference (Single Variable)

$$\mathbf{P}(X \mid \mathbf{e}) = \alpha\, \mathbf{P}(X, \mathbf{e})$$

Query variable $X : Domain(X) = \{x_1, \ldots, x_m\}$

Evidence variables $\mathbf{E} : \{E_1, \ldots, E_k\}$

Observations $\mathbf{e} : \{e_1, \ldots, e_k\}$ s.t. $E_i = e_i$

Unobserved variables $\mathbf{Y} : \{Y_1, \ldots, Y_l\}$

$Domain(Y_i) = \{y_{i,1}, \ldots, y_{i,n_i}\}$

Probabilistic Inference (Single Variable)

$$\mathbf{P}(X \mid \mathbf{e}) = \alpha\, \mathbf{P}(X, \mathbf{e}) = \alpha \sum_{\mathbf{y}} \mathbf{P}(X, \mathbf{e}, \mathbf{y})$$

Query variable $X : Domain(X) = \{x_1, \ldots, x_m\}$

Evidence variables $\mathbf{E} : \{E_1, \ldots, E_k\}$

Observations $\mathbf{e} : \{e_1, \ldots, e_k\}$ s.t. $E_i = e_i$

Unobserved variables $\mathbf{Y} : \{Y_1, \ldots, Y_l\}$

$Domain(Y_i) = \{y_{i,1}, \ldots, y_{i,n_i}\}$

Probabilistic Inference (Single Variable)

$$\mathbf{P}(X \mid \mathbf{e}) = \alpha\, \mathbf{P}(X, \mathbf{e}) = \alpha \sum_{\mathbf{y}} \mathbf{P}(X, \mathbf{e}, \mathbf{y})$$

Given values $\mathbf{e}$ for evidence variables $\mathbf{E}$:

For each possible value x_i for query X

For each possible combination of values $\mathbf{y}$ for $\mathbf{Y}$

Sum up $P(x_i, \mathbf{e}, \mathbf{y})$

Result: $\mathbf{P}(X | \mathbf{e}) = \langle P(x_i, \mathbf{e}, \mathbf{y}) \rangle$

Probabilistic Inference (Single Variable)

$$\mathbf{P}(X \mid \mathbf{e}) = \alpha\, \mathbf{P}(X, \mathbf{e}) = \alpha \sum_{\mathbf{y}} \mathbf{P}(X, \mathbf{e}, \mathbf{y})$$

Given values $\mathbf{e}$ for evidence variables $\mathbf{E}$:

For each possible value x_i for query X

For each possible combination of values $\mathbf{y}$ for $\mathbf{Y}$

Sum up $P(x_i, \mathbf{e}, \mathbf{y}) = P(x_i, e_1, \ldots, e_k, y_{1,\mathrm{i}_1}, \ldots, y_{l,\mathrm{i}_l})$

Result: $\mathbf{P}(X \mid \mathbf{e}) = \langle P(x_i, \mathbf{e}, \mathbf{y}) \rangle$

Probabilistic Inference (Single Variable)

$$\mathbf{P}(X \mid \mathbf{e}) = \alpha\,\mathbf{P}(X, \mathbf{e}) = \alpha \sum_{\mathbf{y}} \mathbf{P}(X, \mathbf{e}, \mathbf{y})$$

Given values $\mathbf{e}$ for evidence variables $\mathbf{E}$:

For each possible value x_i for query X

For each possible combination of values $\mathbf{y}$ for $\mathbf{Y}$

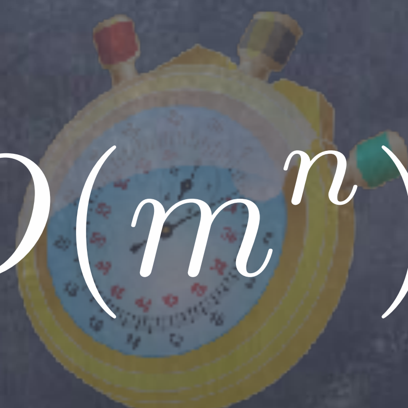
Sum up $P(x_i, \mathbf{e}, \mathbf{y}) = P(x_i, e_1, \ldots, e_k, y_{1,\mathrm{i}_1}, \ldots, y_{l,\mathrm{i}_l})$

$= P(X{=}x_i, E_1{=}e_1, \ldots, E_k{=}e_k, Y_1{=}y_{1,\mathrm{i}_1}, \ldots, Y_l{=}y_{l,\mathrm{i}_l})$

Result: $\mathbf{P}(X \mid \mathbf{e}) = \langle P(x_i, \mathbf{e}, \mathbf{y}) \rangle$

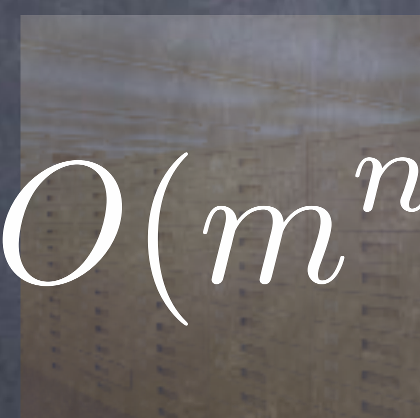
Probabilistic Inference (Single Variable)

$$\mathbf{P}(X \mid \mathbf{e}) = \alpha\, \mathbf{P}(X, \mathbf{e}) = \alpha \sum_{\mathbf{y}} \mathbf{P}(X, \mathbf{e}, \mathbf{y})$$



$$O(m^n)$$

Time Complexity



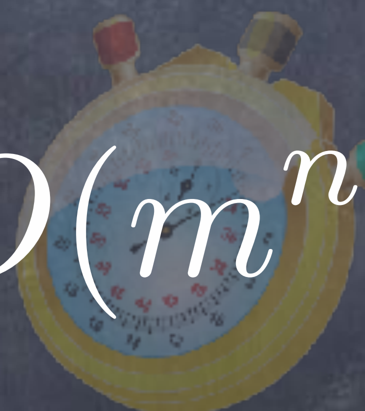
$$O(m^n)$$

Space Complexity

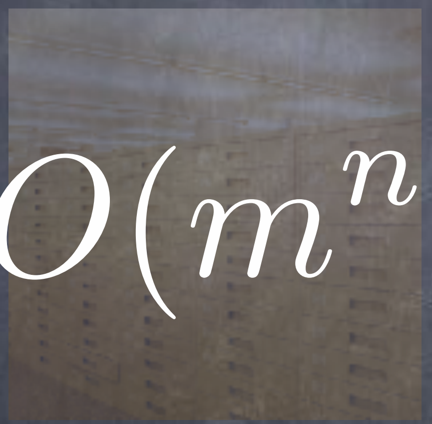
For variables with at most m values

Probabilistic Inference (Single Variable)

$$\mathbf{P}(X \mid \mathbf{e}) = \alpha\, \mathbf{P}(X, \mathbf{e}) = \alpha \sum_{\mathbf{y}} \mathbf{P}(X, \mathbf{e}, \mathbf{y})$$



$O(m^n)$



$O(m^n)$

Time Complexity

Space Complexity

Intractable!

Conditional Probability

$$P(a \mid b) = \frac{P(a \wedge b)}{P(b)} \text{ when } P(b) > 0$$

Product Rule

$$P(a \wedge b) = P(a \mid b)P(b) \qquad P(b \wedge a) = P(b \mid a)P(a)$$

$$P(a \mid b)P(b) = P(b \mid a)P(a)$$

$$P(b \mid a)P(a) = P(a \mid b)P(b)$$

$$P(b \mid a) = \frac{P(a \mid b)P(b)}{P(a)}$$

Bayes' Rule

$$P(b \mid a) = \frac{P(a \mid b)P(b)}{P(a)}$$

Thomas Bayes

(c. 1702 – 1761)

Bayes' Rule

$$\mathbf{P}(Y \mid X) = \frac{\mathbf{P}(X \mid Y)\mathbf{P}(Y)}{\mathbf{P}(X)}$$

$$\mathbf{P}(Y \mid X) = \alpha \; \mathbf{P}(X \mid Y)\mathbf{P}(Y)$$

Causal and Diagnostic Knowledge

Causal knowledge: $P(effect \mid cause)$

Diagnostic knowledge: $P(cause \mid effect)$

Causal and Diagnostic Knowledge

Causal knowledge: $P(symptom \mid disease)$

Diagnostic knowledge: $P(disease \mid symptom)$

Bayesian Diagnosis

$$P(disease \mid symptom) = \frac{P(symptom \mid disease)P(disease)}{P(symptom)}$$

Meningitis causes a stiff neck 70% of the time

$$P(stiffneck \mid meningitis) = 0.7$$

Meningitis causes a stiff neck 70% of the time

$$P(stiffneck \mid meningitis) = 0.7$$

Prior probability of meningitis $P(meningitis) = 0.00002$

Prior probability of stiff neck $P(stiffneck) = 0.01$

Meningitis causes a stiff neck 70% of the time

$$P(stiffneck \mid meningitis) = 0.7$$

Prior probability of meningitis $P(meningitis) = 0.00002$

Prior probability of stiff neck $P(stiffneck) = 0.01$

$$P(meningitis \mid stiffneck) =$$

Meningitis causes a stiff neck 70% of the time

$$P(stiffneck \mid meningitis) = 0.7$$

Prior probability of meningitis $P(meningitis) = 0.00002$

Prior probability of stiff neck $P(stiffneck) = 0.01$

$$P(meningitis \mid stiffneck) = \frac{P(stiffneck \mid meningitis)P(meningitis)}{P(stiffneck)}$$

Meningitis causes a stiff neck 70% of the time

$$P(stiffneck \mid meningitis) = 0.7$$

Prior probability of meningitis $P(meningitis) = 0.00002$

Prior probability of stiff neck $P(stiffneck) = 0.01$

$$P(meningitis \mid stiffneck) = \frac{P(stiffneck \mid meningitis)P(meningitis)}{P(stiffneck)}$$

$$= \frac{0.7 \times 0.00002}{0.01}$$

$$= 0.0014$$

Bayesian Diagnosis

$$P(disease \mid symptom) = \frac{P(symptom \mid disease)P(disease)}{P(symptom)}$$

$$toothache \qquad\qquad catch$$

$$(Toothache = true) \qquad (Catch = true)$$

$$\mathbf{P}(Cavity \mid toothache \wedge catch)$$

Combining Evidence

$$\mathbf{P}(Cavity \mid toothache \wedge catch)$$

Combining Evidence

$$\mathbf{P}(Cavity \mid toothache \land catch) \\ = \alpha \langle 0.180, 0.016 \rangle \approx \langle 0.871, 0.129 \rangle$$

	$toothache$		$\neg toothache$	
	$catch$	$\neg catch$	$catch$	$\neg catch$
$cavity$	0.108	0.012	0.072	0.008
$\neg cavity$	0.016	0.064	0.144	0.576

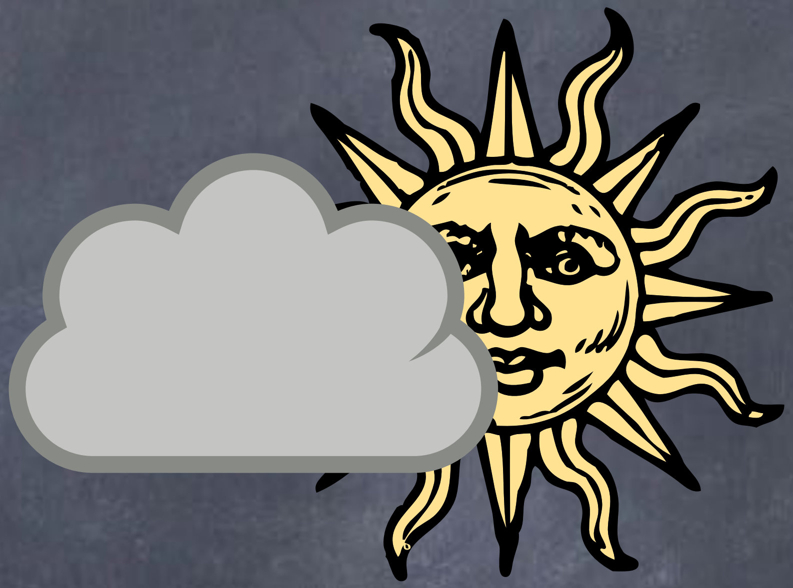
Combining Evidence

$$\mathbf{P}(Cavity \mid toothache \land catch)$$
$$= \alpha\, \mathbf{P}(toothache \land catch \mid Cavity)\, \mathbf{P}(Cavity)$$

Combining Evidence

- In general, if there are n evidence variables, then there are $O(2^n)$ possible combinations of observed values for which we would need to know the conditional probabilities

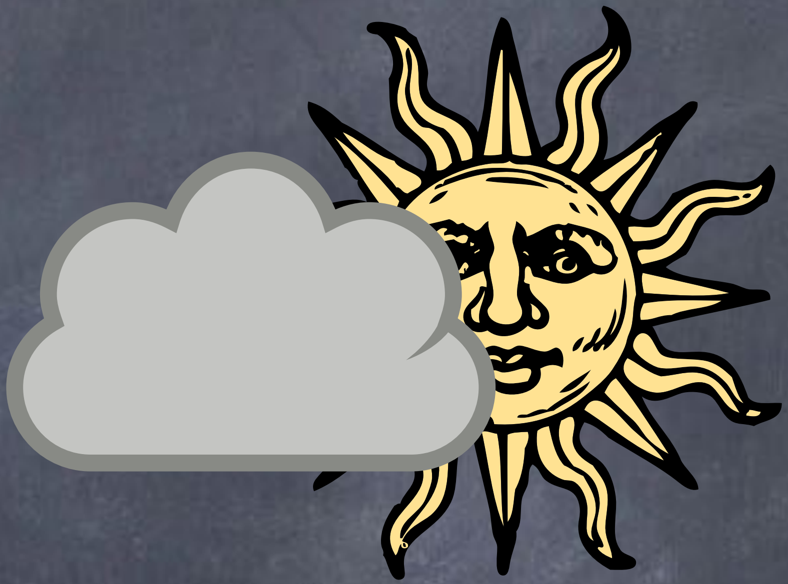
$Cavity$



$Weather$

Independence

$Cavity$



$Weather$

Independence

$$P(cavity \mid sunny) = P(cavity)$$

$$P(rain \mid cavity) = P(rain)$$

$$\mathbf{P}(Cavity \mid Weather) = \mathbf{P}(Cavity)$$

$$\mathbf{P}(Weather \mid Cavity) = \mathbf{P}(Weather)$$

Independence

$$P(a \mid b) = P(a)$$
$$P(b \mid a) = P(b)$$
$$P(a \wedge b) = P(a)P(b)$$

Independence

$$\mathbf{P}(X \mid Y) = \mathbf{P}(X)$$
$$\mathbf{P}(Y \mid X) = \mathbf{P}(Y)$$
$$\mathbf{P}(X, Y) = \mathbf{P}(X)\mathbf{P}(Y)$$

Independence

$$\mathbf{P}(X) = \langle p_{x_1}, p_{x_2}, \ldots, p_{x_n} \rangle$$

$$\mathbf{P}(Y) = \langle p_{y_1}, p_{y_2}, \ldots, p_{y_m} \rangle$$

$\mathbf{P}(X, Y)$	x_1	x_2	...	x_n
y_1	$p_{x_1}p_{y_1}$	$p_{x_2}p_{y_1}$	...	$p_{x_n}p_{y_1}$
y_2	$p_{x_1}p_{y_2}$	$p_{x_2}p_{y_2}$	...	$p_{x_n}p_{y_2}$
...	...	...	...	...
y_m	$p_{x_1}p_{y_m}$	$p_{x_2}p_{y_m}$	...	$p_{x_n}p_{y_m}$

Independence

$$\mathbf{P}(X \mid Y) = \mathbf{P}(X)$$

$$\mathbf{P}(Y \mid X) = \mathbf{P}(Y)$$

$$\mathbf{P}(X, Y) = \mathbf{P}(X)\mathbf{P}(Y)$$

Can compute $n \times m$ probabilities

from $n + m$ probabilities

(if random variables are independent)

$$toothache \qquad\qquad catch$$

$$(Toothache = true) \qquad (Catch = true)$$

$$\mathbf{P}(Cavity \mid toothache \wedge catch)$$

$$toothache \qquad\qquad catch$$

$$(Toothache = true) \qquad (Catch = true)$$

$$\mathbf{P}(Cavity \mid toothache \wedge catch)$$
$$= \alpha\, \mathbf{P}(toothache \wedge catch \mid Cavity)\, \mathbf{P}(Cavity)$$

$toothache$ $catch$

$(Toothache = true)$ $(Catch = true)$

Independent?

$$\mathbf{P}(Cavity \mid toothache \wedge catch)$$

$$= \alpha\, \mathbf{P}(toothache \wedge catch \mid Cavity)\, \mathbf{P}(Cavity)$$

Conditional Independence

- Both $toothache$ and $catch$ are caused by a cavity, but neither has a direct effect on the other
- The variables are independent <u>given the presence or absence of a cavity</u>

Conditional Independence

$$\mathbf{P}(X \mid Y, Z) = \mathbf{P}(X \mid Z)$$
$$\mathbf{P}(Y \mid X, Z) = \mathbf{P}(Y \mid Z)$$
$$\mathbf{P}(X, Y \mid Z) = \mathbf{P}(X \mid Z)\mathbf{P}(Y \mid Z)$$

Conditional Independence

$$\mathbf{P}(Toothache, Catch \mid Cavity) =$$
$$\mathbf{P}(Toothache \mid Cavity)\mathbf{P}(Catch \mid Cavity)$$

Combining Evidence

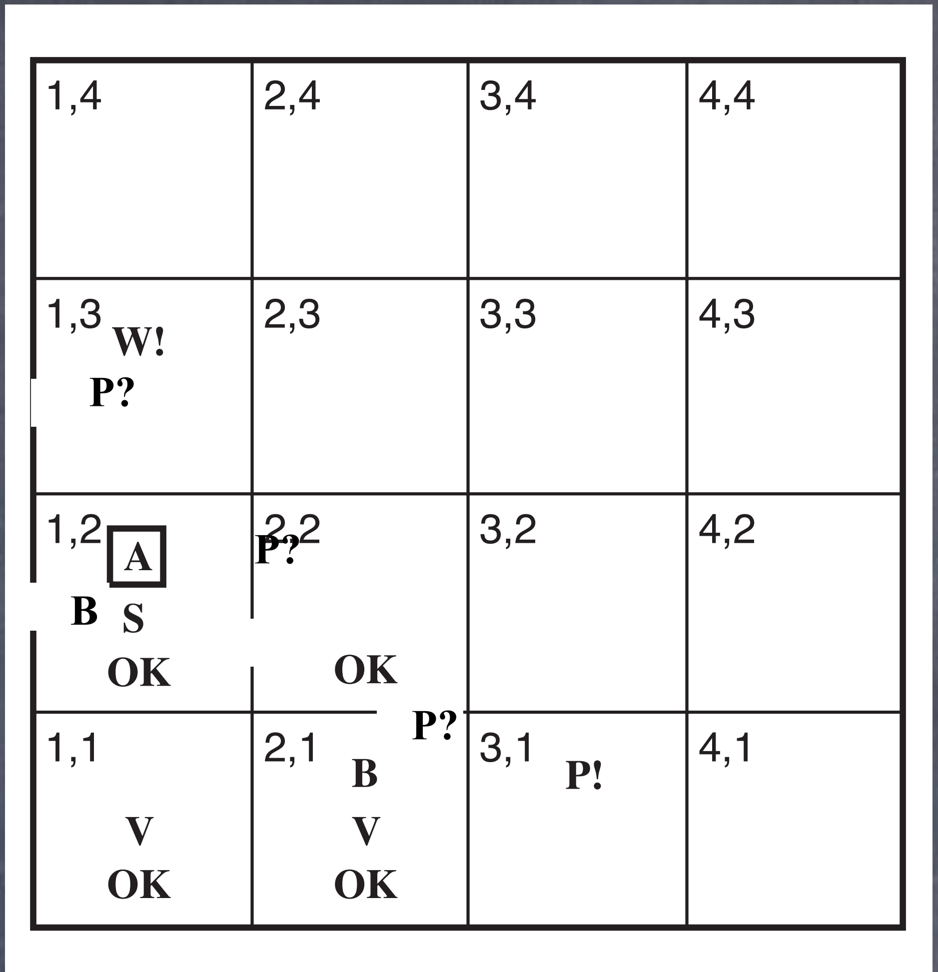
$$\mathbf{P}(Cavity \mid toothache \wedge catch)$$

$$= \alpha\, \mathbf{P}(toothache \wedge catch \mid Cavity)\, \mathbf{P}(Cavity)$$

$$= \alpha\, \mathbf{P}(toothache|Cavity)\, \mathbf{P}(catch|Cavity)\, \mathbf{P}(Cavity)$$

Combining Evidence

- For n symptoms (e.g., $Toothache$, $Catch$) that are all conditionally independent given a disease (e.g., $Cavity$), we need $O(n)$ probabilities rather than $O(2^n)$
  - Representation scales to larger problems
  - Conditional probabilities more likely to be available than absolute independence assumptions



Example Section 13.6

You can do it!

Probabilistic Inference

- Computing posterior distribution given evidence
  - Normalization, Marginalization
- Full joint distribution: intractable as problem grows
- Bayes' Rule
- Independence assumptions
  - Factor FJPD into smaller distributions

For Next Time:

AIMA 14 - 14.4