

CSC242: Homework 2.4

AIMA Chapter 80–8.3

1. Briefly define the following terms related to first-order logic:
 - (a) Domain (of discourse)
 - (b) Term
 - (c) Atomic sentence or atomic formula
2. Describe the components of a first-order interpretation.
3. Translate the following sentence of first-order logic into a *reasonable* sentence of English:

$$\forall x, y, l \text{ SpeaksLanguage}(x, l) \wedge \text{SpeaksLanguage}(y, l) \Rightarrow \text{Understands}(x, y) \wedge \text{Understands}(y, x). \quad (1)$$

4. Explain why this sentence is entailed by the sentence

$$\forall x, y, l \text{ SpeaksLanguage}(x, l) \wedge \text{SpeaksLanguage}(y, l) \Rightarrow \text{Understands}(x, y). \quad (2)$$

5. Translate the following English sentences into first-order logic using the predicates *Understands* and *FriendOf*:
 - (a) Mutual understanding leads to mutual friendship.
 - (b) My friend's friends are also my friends (that is, friendship is transitive).
6. Explain what is wrong with the following proposed definition of adjacent squares in the wumpus world:

$$\forall x, y \text{ Adjacent}([x, y], [x + 1, y]) \wedge \text{Adjacent}([x, y], [x, y + 1]).$$

7. Write out the axioms for reasoning about the wumpus' location, using a constant symbol *Wumpus*, unary predicate *Smelly*, and binary predicates *In*, *Adjacent*, and equality if you need it. Remember that there is only one wumpus.

8. Assuming predicates $Manager(p, q)$ and $Friendly(p)$, and constants $Juanita$ and $Kyle$, express each of the following as sentences of first-order logic. You may use the abbreviation “ $\exists!$ ” to mean “there exists exactly one.”
- (a) Juanita manages a friendly person (perhaps more than one, and perhaps unfriendly people also).
 - (b) Juanita manages exactly one friendly person, but may manage unfriendly people also).
 - (c) Juanita manages exactly one person and that person is friendly.
 - (d) Juanita and Kyle manage exactly one person together.
 - (e) Juanita manages at least one person with Kyle, and no people with anybody else.
9. Arithmetic statements can be written using first-order logic with the binary predicate symbol $<$, the function symbols $+$ and \times , and the constant symbols 0 and 1 (and equality). Additional predicates can be defined using biconditionals (as you might recall from CSC173).
- (a) Represent the statement “ x is an even number” by defining a unary predicate *Even*.
 - (b) Represent the statement “ x is prime” by defining a unary predicate *Prime*.
 - (c) Goldbach’s conjecture (Wikipedia) is the conjecture (unproven as yet) that every even number is the sum of two primes. Represent this conjecture as a logical sentence.