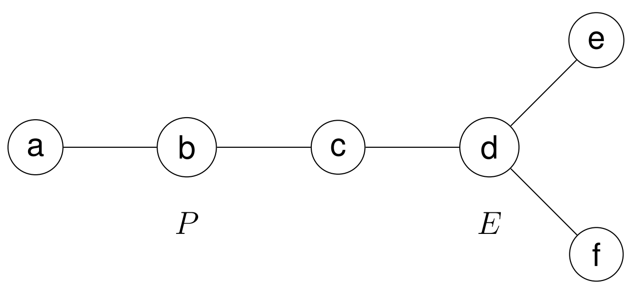
CSC242: Homework 1.4
AIMA Chapter 5.3–5.4.2, 5.5–5.6

1. Suppose two people are playing a game where one player, the pursuer, is trying to catch the other player, the evader. They are in a house with many rooms. Some rooms are connected with doors or passages between them. The pursuer starts in some room and the evader in another. The two players take turns moving to an adjacent room. The pursuer wins if they get to the same room as the evader. The terminal payoff to the pursuer is minus the number of moves taken. The evader "wins" by not getting caught. Importantly, both players have an app on their phones that displays which room the other player is in.

   For example, here's an adjacency graph representation of the rooms of some house, with the positions of the pursuer and evader marked by P and E:

   (a) Formalize this game as an adversarial search problem.

   (b) Draw the first two levels of the game tree from the situation given above with P to move next.

   (c) From the non-terminal states with P in room **c**, and P to move, there is an obvious move for P to end the game after E's following move. Expand the tree to include these moves.

   (d) Mark the values of the terminal nodes.

   (e) From either of the non-terminal states with P in room **c**, there is another move for P. We suspect that it must be worse than the move considered above. Prove it.

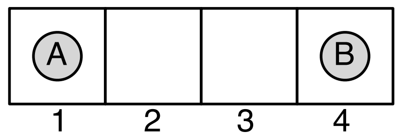
   (f) Mark these bounds on your tree. What can you now conclude about the value of the non-terminal states with P in room **c**?

   (g) How far can you push this score up the tree?

   (h) What can you say about the search on the other side of the tree, where P moves to **a** on their first move?

(i) Can you prove who must win the game on a map that is a finite tree?

2. The following diagram shows the starting position for a simple game:



Player A moves first. The two players take turns moving and on their turn can move their token to an open adjacent space in either direction. If the opponent occupies an adjacent space, then a player may jump over them to the next open space (only if there is such a space, of course). The game ends when one player reaches the opposite end of the board. If player A reaches space 4 first, the value to A is +1. If player B reaches space 1 first, the value to A is -1.

(a) Formalize this game as an adversarial search problem.

(b) Draw the complete game tree (search tree) for this game, starting at the initial state with A to move as given in the description. Mark terminal states and give the value of the state for A. Mark "loopy" (repeated) states and give their value as "?".

(c) Mark the values of the terminal states.

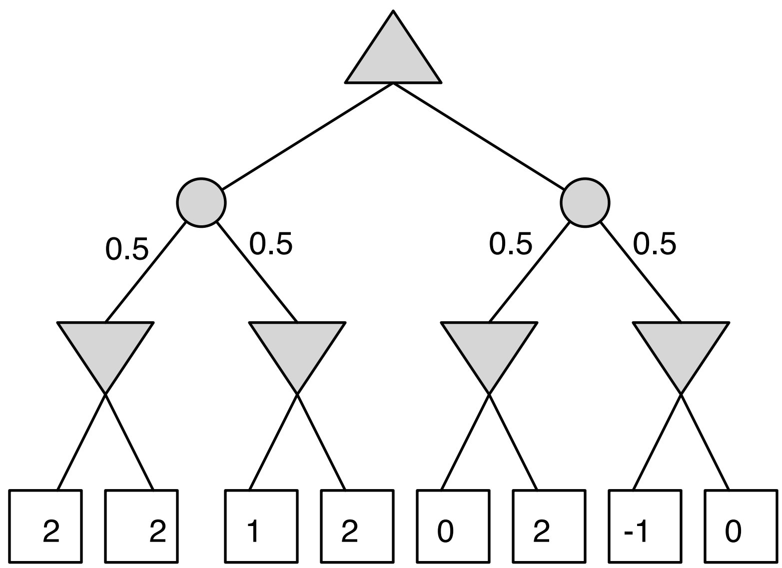
(d) Compute the minimax value of the non-terminal states. Explain how you handle the value "?".

(e) Does the standard MINIMAX algorithm compute these values properly (assuming your handling of "?" values)? If not, can you fix it for this game?

(f) Does your solution work to compute optimal solutions for arbitrary games with loopy states?

(g) Bonus: This 4-square game can be generalized to n squares for any $n > 2$. Can you prove anything about who wins and loses as a function of n?

3. The following figure shows the complete game tree for a trivial two-player game with chance nodes (circles). Assume leaf nodes are evaluated in left-to-right order.



(a) Compute the value of all internal nodes and indicate the best move at the root.

(b) With no bounds on the values of the leaf nodes, do we have to evaluate the seventh and eighth nodes given the value of the first six leaves? What about after computing the value of the seventh leaf?

(c) Suppose we know that leaf node values are between -2 and +2. What is the value range for the left-hand chance node after the first two leaves have been evaluated?

(d) Circle all the leaves that do not need to be evaluated under the assumption from part (c).

4. Which of the following statements are true and which are false? Explain briefly.

(a) In a fully observable, turn-taking, zero-sum game between two perfectly rational players, it does not help the first player to know what strategy the second player is using—that is, what move the second player will make, given the first player's move.

(b) In a partially observable, turn-taking, zero-sum game between two perfectly rational players, it does not help the first player to know what move the second player will make, given the first player's move.

(c) A perfectly rational backgammon agent never loses.