CSC242: Homework 1.2
AIMA Chapter 3.3.2–3.6

1. Consider a state space where the start state is number 1 and each state k has two successors numbered $2k$ and $2k+1$ respectively.
   (a) Draw the portion of the state space for states 1 to 15.
   (b) Suppose the goal state is 11. List the order the nodes will be visited for breadth-first search, depth-first search to a depth limit of 3, and iterative deepening depth-first search.
2. For a search tree, let b be the branching factor, d the depth of the shallowest ("best") solution, and m the maximum depth of the tree. Complete the following table:

	Breadth-first search	Depth-first graph search	Depth-first tree search	Iterative-deepening depth-first tree search
Time Complexity				
Space Complexity				
Complete?				
Optimal?				

3. Define what it means for a heuristic to be *admissible*. Give a simple example of an admissible heuristic for a problem and explain why it's admissible.
4. Trace the operation of A* search applied to the problem of getting to Bucharest from Lugoj using the straight-line distance heuristic. Show the sequence of nodes and the g, h, and f scores for each node. Show the solution found by the search.
5. Consider the following heuristic functions for the 8-puzzle:

   h_1: Number of misplaced tiles

   h_2: Sum of Manhattan distances of tiles from final positions

(a) Are these heuristics admissible? Why or why not?

(b) Consider the heuristic

$$h = h_1 + h_2$$

Is this heuristic admissible? Prove it or give a counterexample.

6. (Harder) Prove that if h never overestimates by more than c, then A* using h returns a solution whose cost does not exceed that of the optimal solution by more than c. This is a very useful thing to know in practice.

   Hint: After you think about yourself, if you want some help, try the following approach:

   (a) Start with the definition of the evaluation function $f(n)$ for best-first search (of which A* is one flavor). See AIMA p. 93 if you don't remember what this is.

   (b) Suppose you had an optimal heuristic function $h^*(n)$ that always returned *exactly* the cost to a goal node. Using the condition given in the question, give an expression relating your heuristic evaluation function $h(n)$ to $h^*(n)$.

   (c) Now consider an optimal (cheapest) goal node, call it G^*. What is the path cost from the root to G^*? See AIMA p. 83 if you don't remember what this is (and it's also used in the definition of f).

   (d) Now suppose that there was some other goal node, call it G, that is suboptimal by more than c. Give an expression for the path cost from the root to G in terms of quantities you have previously defined.

   (e) Put the pieces together to show that G will never be expanded before an optimal goal is expanded, which is what the question asks you to prove.

7. Suppose that you need to search a state space that is a regular (evenly-spaced), unbounded, discrete, 2D grid of points. States (points) are connected to their four neighbors (north, south, east, and west). The start state is the origin $(0, 0)$. The goal state is (x, y). Note that since the state space is discrete, these coordinates are *integers*.

   (a) What is the branching factor b in this state space?

   (b) How many distinct states are there at depth k (for $k > 0$)? Hint: Draw a picture and mark a couple of generations of states in different colors to see the pattern, then generalize.

   (c) What is the maximum depth of search for goal (x, y)?

   (d) Give a big-O expression for the maximum number of nodes expanded by breadth-first tree search?

   (e) Give a big-O expression for the maximum number of nodes expanded by breadth-first graph search?

   (f) For a state (u, v), consider the heuristic $h = |u - x| + |v - y|$. Is h admissible? Why or why not?

   (g) How many nodes are expanded by an A* search using h?

   (h) Is h still admissible if some links (edges) are removed?

   (i) Is h still admissible if some links (edges) are added between non-adjacent states?