

# CSC242: Introduction to Artificial Intelligence

Lecture 1.1

Please put away all electronic devices

# Announcements

- Sorry for running late last class!
  - Will not happen again
- Study Sessions:
  - Mon & Wed 745PM-9PM, room TBA
- Project 1 will be posted after class

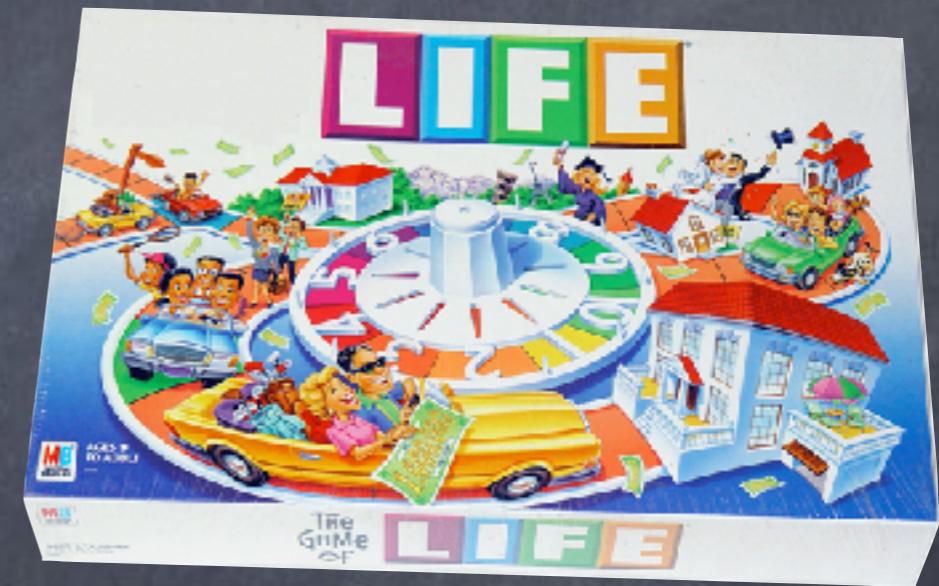
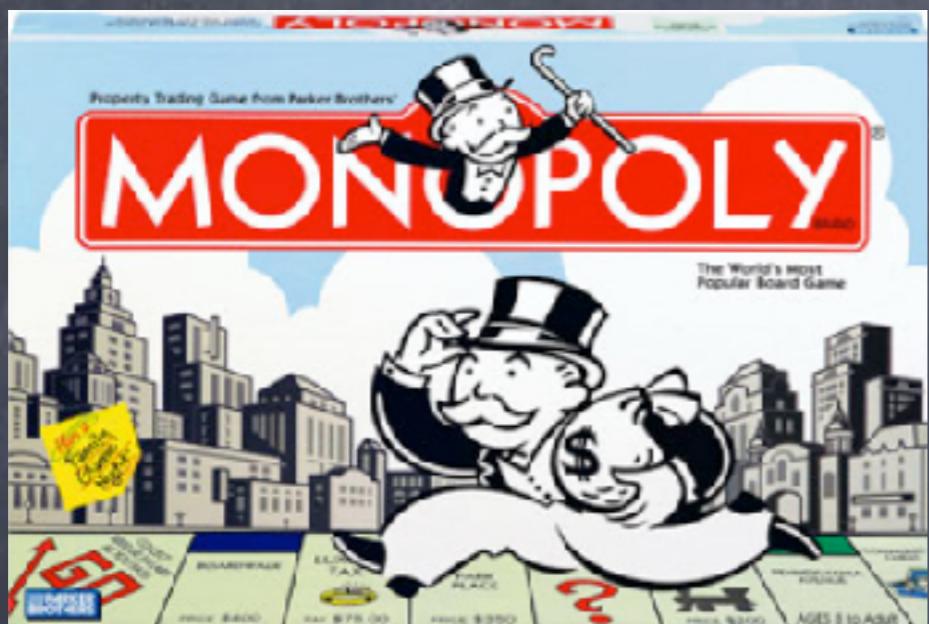


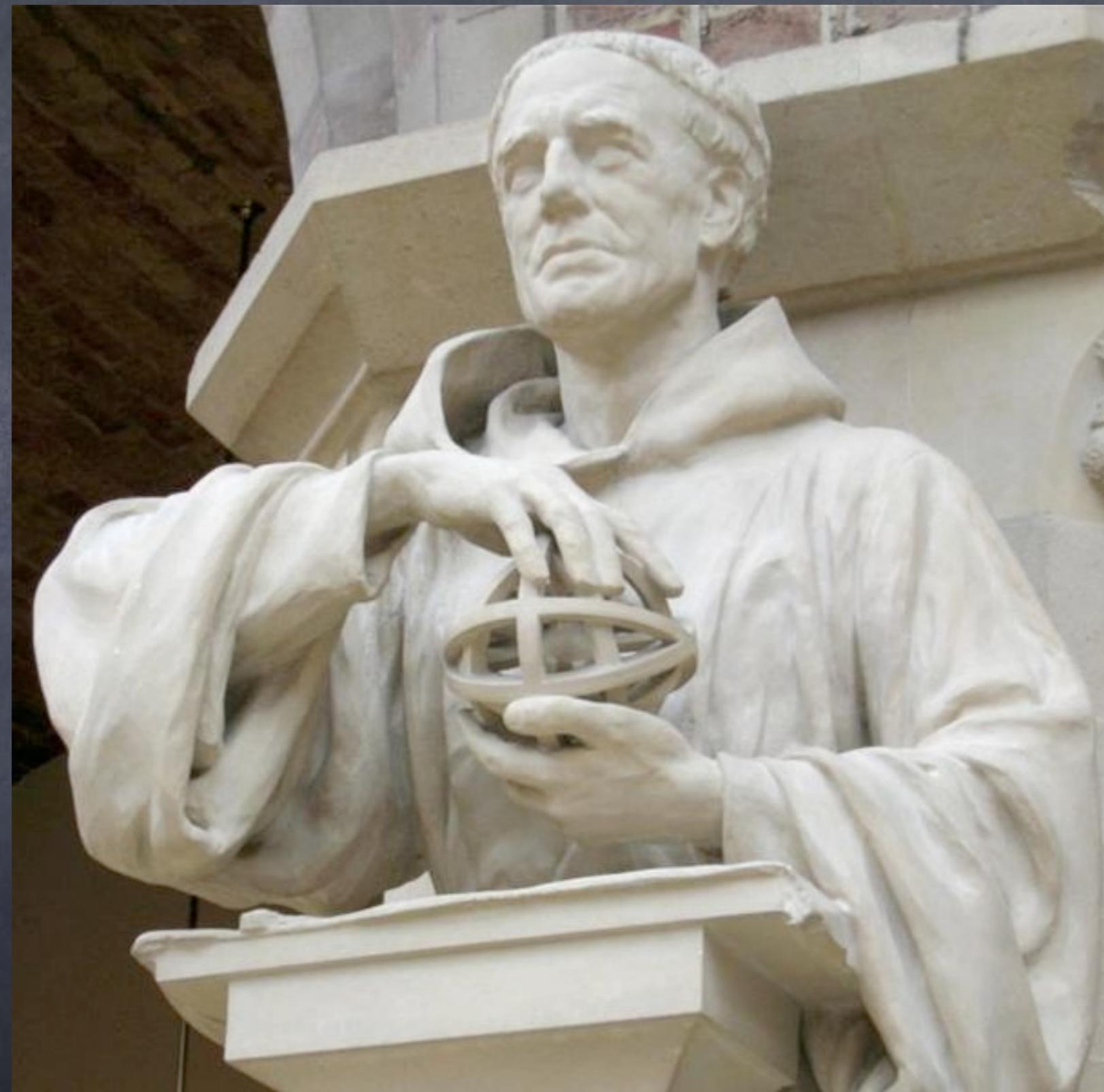
# Problem Solving

# Toy Problems



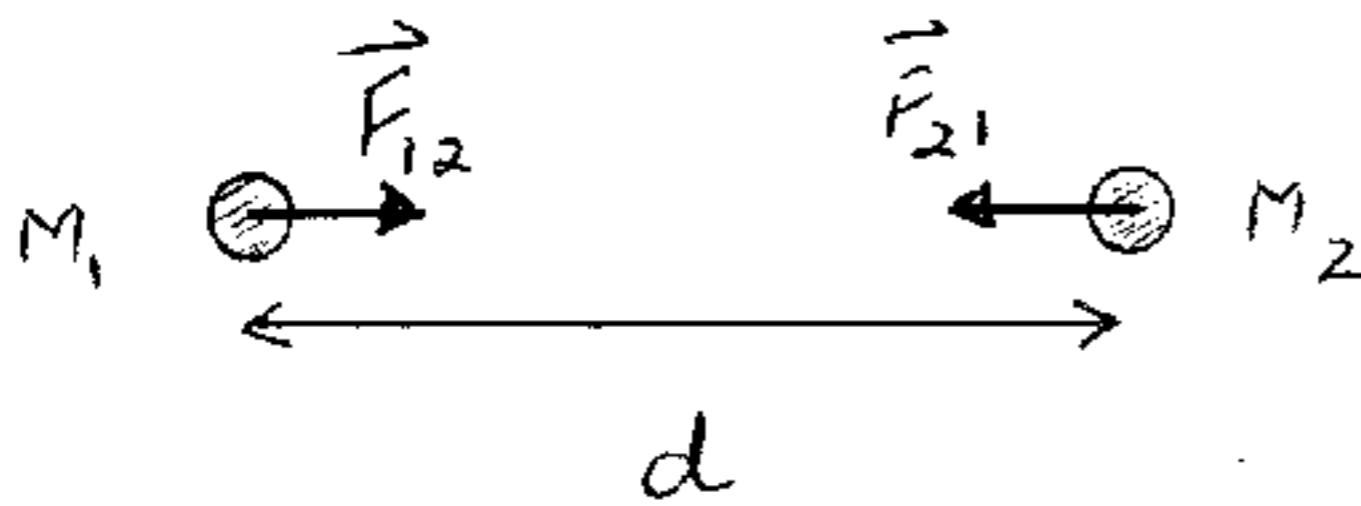
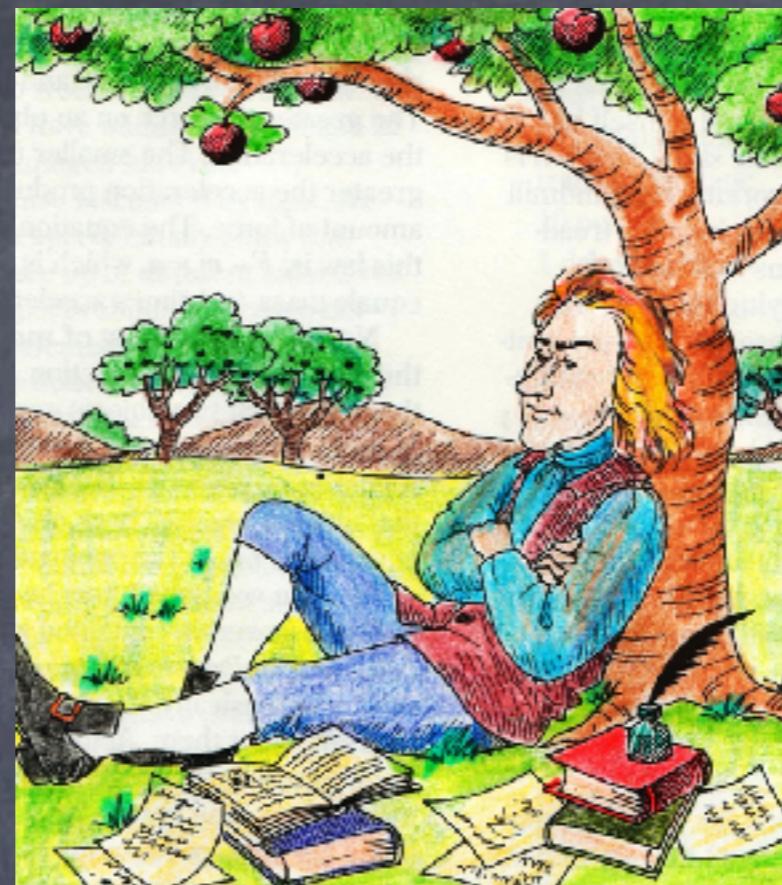
# Toy Problems





Roger Bacon (c. 1214–1294)





$$|\vec{F}_{1,2}| = |\vec{F}_{2,1}| = \frac{G M_1 M_2}{d^2}$$

# Abstraction

Simplification and generalization of  
real-world phenomena

# Abstraction

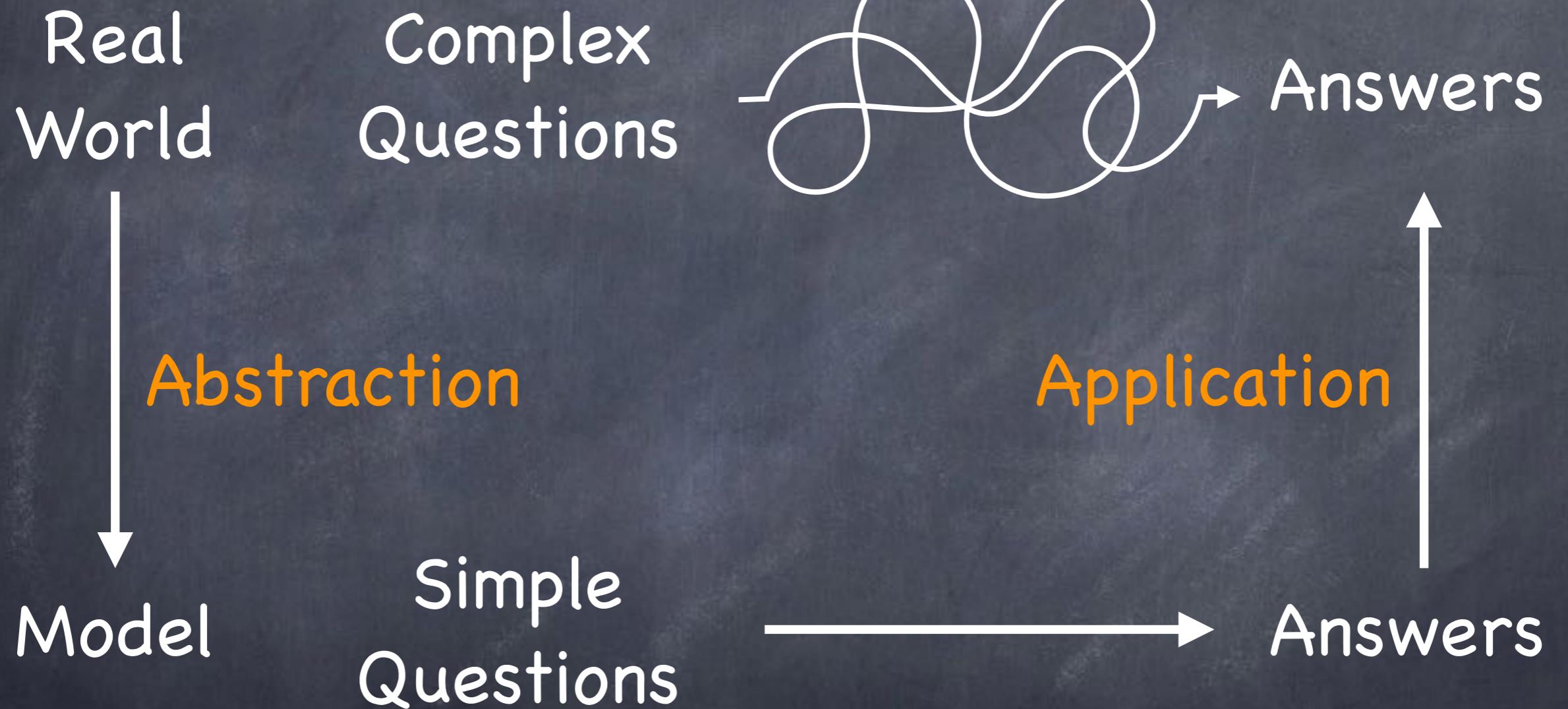
We “abstract” away the details whose effect on the solution to a problem is minimal or nonexistent, thereby creating a model that lets us deal with the essence of the problem.

– Aho & Ullman,  
Foundations of Computer Science

# Abstraction

Fundamentally, Computer Science is a science of abstraction – creating the right model for thinking about a problem and devising the appropriate mechanizable techniques to solve it.

– Aho & Ullman,  
Foundations of Computer Science





# Our First Problem

# RETEAUA DRUMURILOR NATIONALE DIN ROMANIA



M.T.

Legendă	
Drumuri nationale europene din care:	6.167 Km
Autostrăzi	211 Km
Drumuri naționale principale	4.388 Km
Drumuri naționale secundare	5.442 Km
Total	15.997 Km

CF





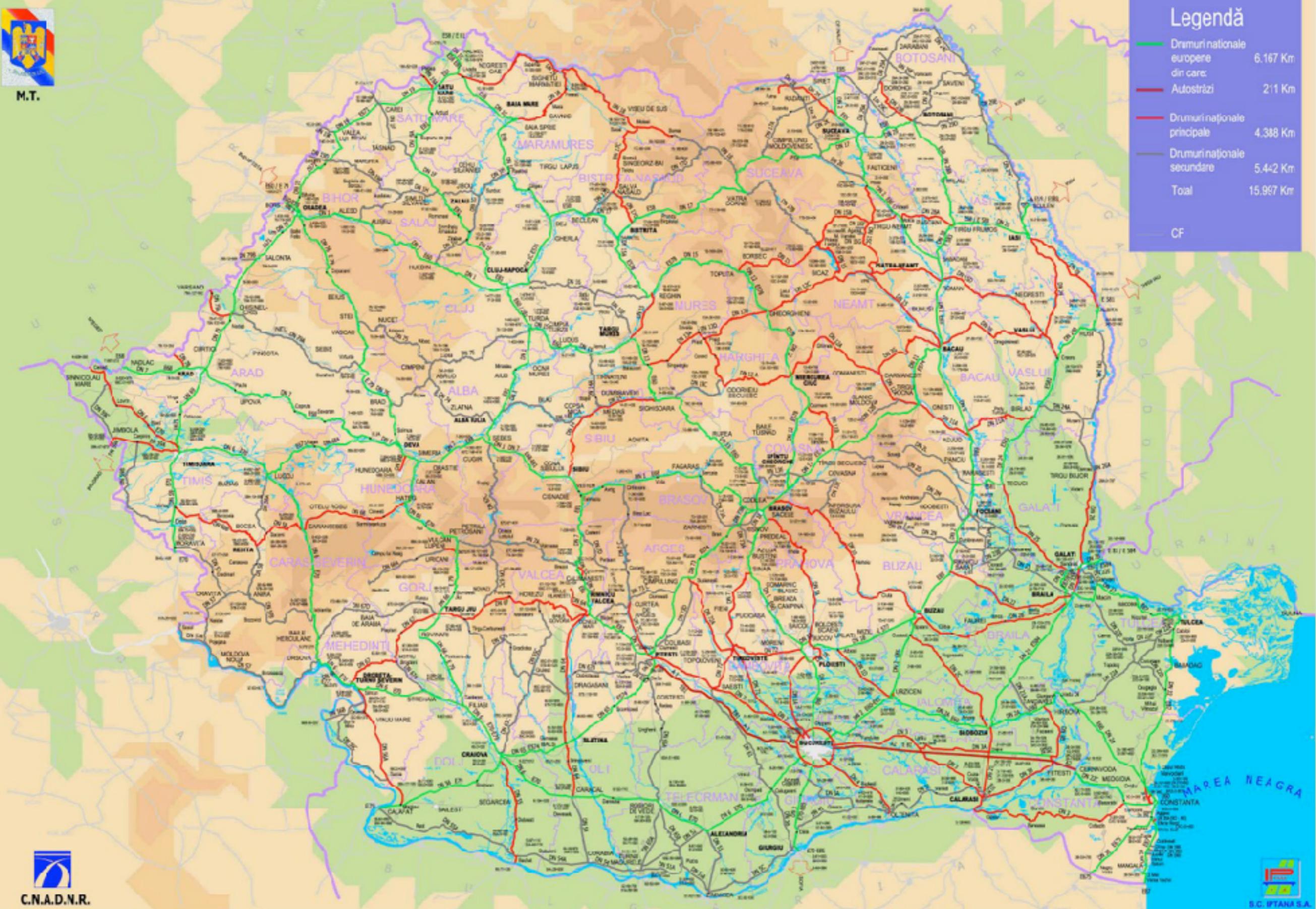
# RETEAUA DRUMURILOR NATIONALE DIN ROMANIA

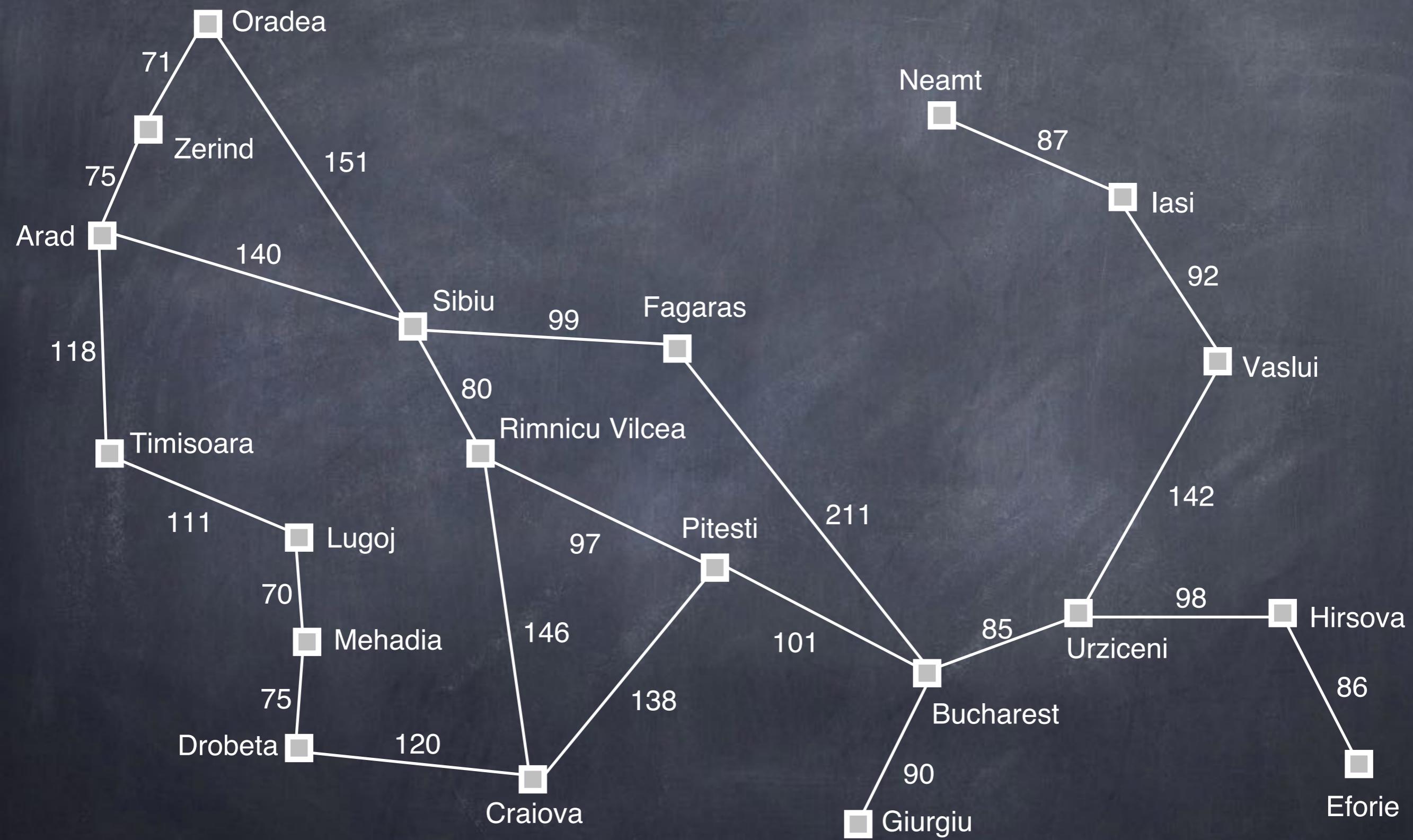


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Total	15.997 Km

CF





Oradea

Zerind

Arad

Neamt

Iasi

Vaslui

How do I get from A to B?

118

Timisoara

111

Lugoj

70

Mehadia

75

Drobeta

120

Craiova

Sibiu

80

Rimnicu Vilcea

146

138

Fagaras

Pitesti

97

101

211

90

Bucharest

Giurgiu

85

Urziceni

98

86

Eforie

142

Given start city A and destination city B:

Can I get from A to B?

Oradea

Zerind

Arad

Timisoara

111

Lugoj

70

Mehadia

75

Drobeta

120

Craiova

Deadline

Mileage limit

How do I get from A to B?

71

151

140

118

Sibiu

Rimnicu Vilcea

80

Fagaras

97

Pitesti

211

101

90

Giurgiu

Neamt

87

Iasi

92

Vaslui

142

Hirsova

98

Urziceni

Bucharest

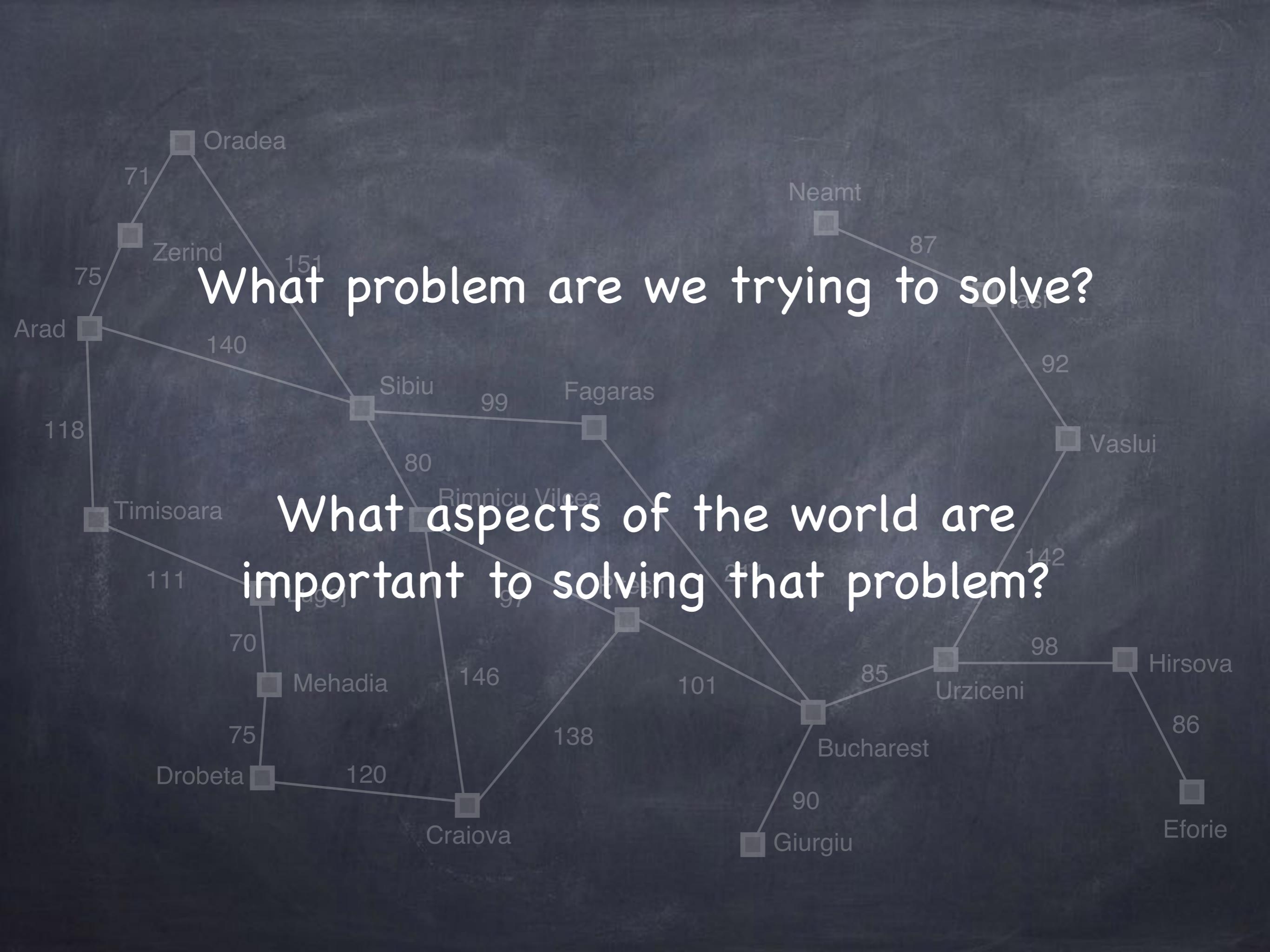
86

Eforie

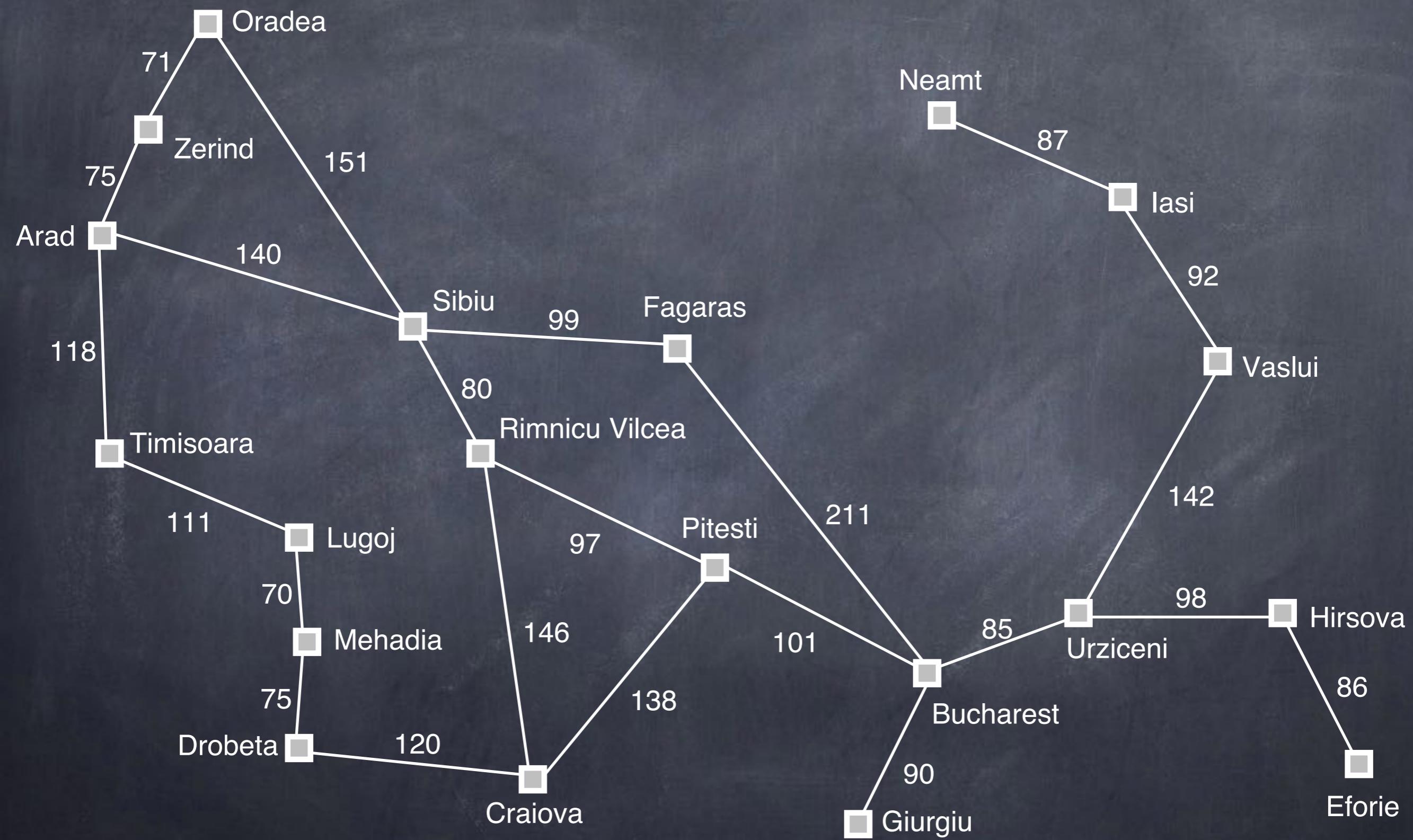
Minimize tolls

Stay close to  
rest areas

Given start city A and destination city B:  
Can I get from A to B?



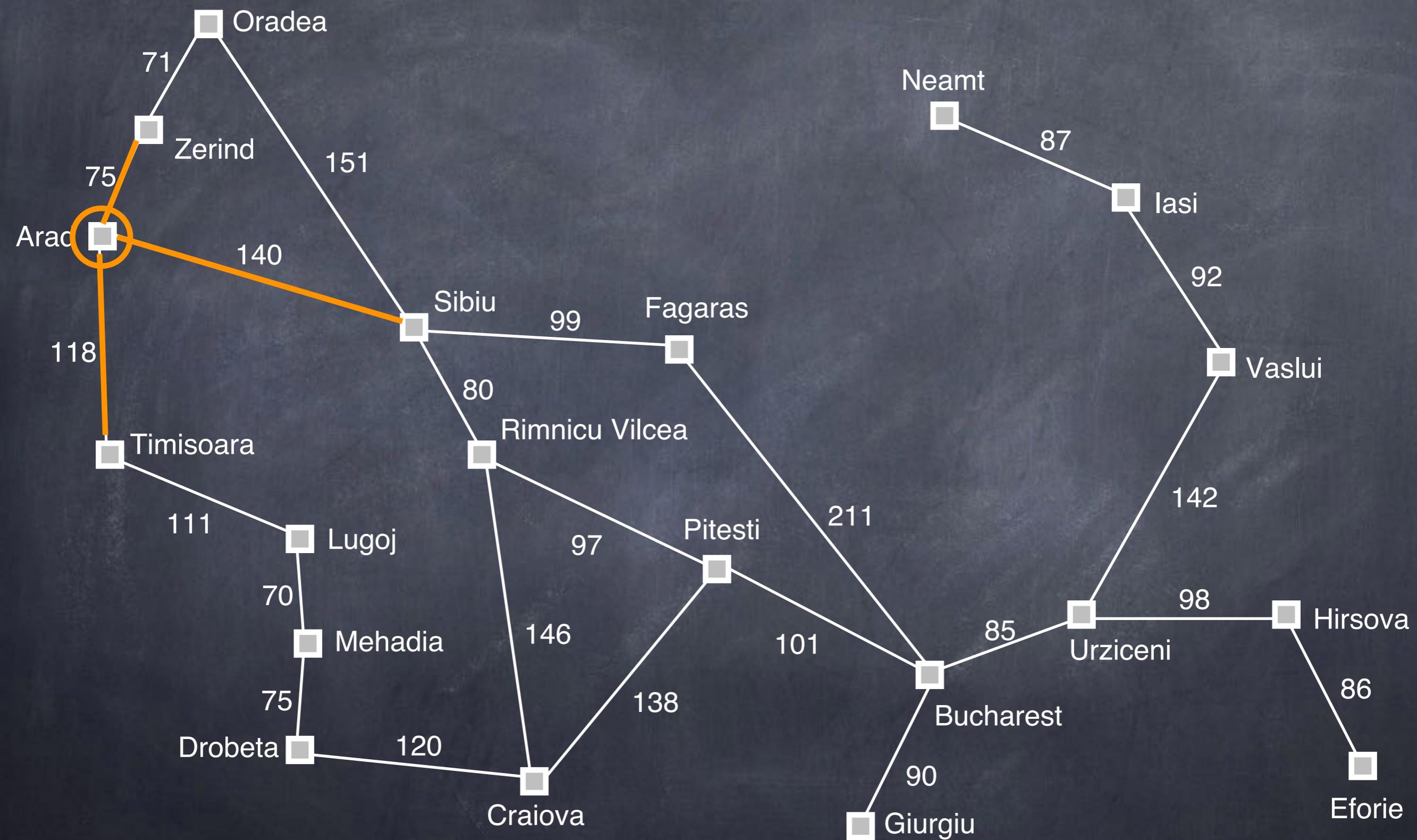
# Problem Solving by Computers





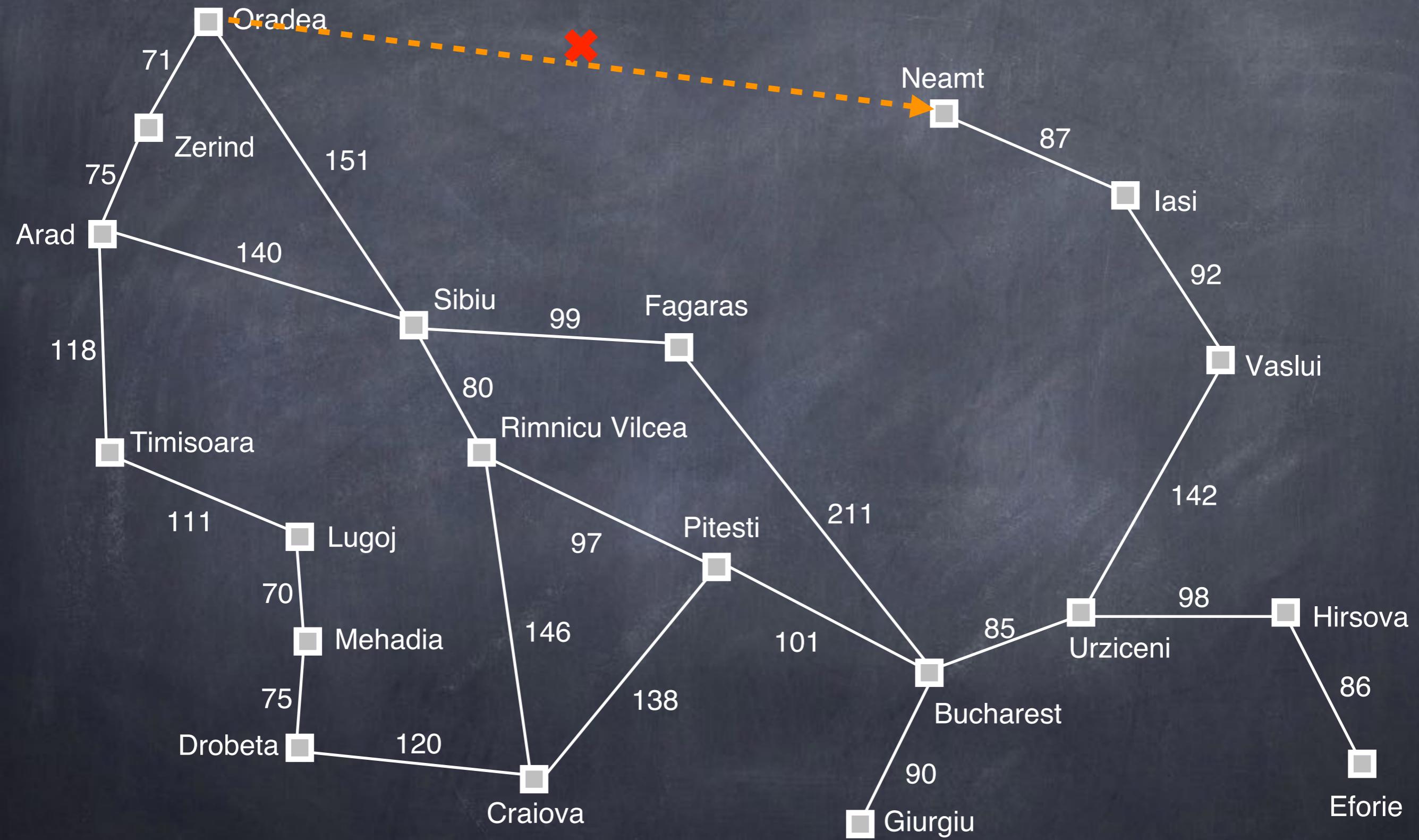
# State (of the World)

- A representation of those aspects of the world that matter to solving the problem



# Actions

- Change the state of the world
  - Cause a transition from one state to another



# Actions

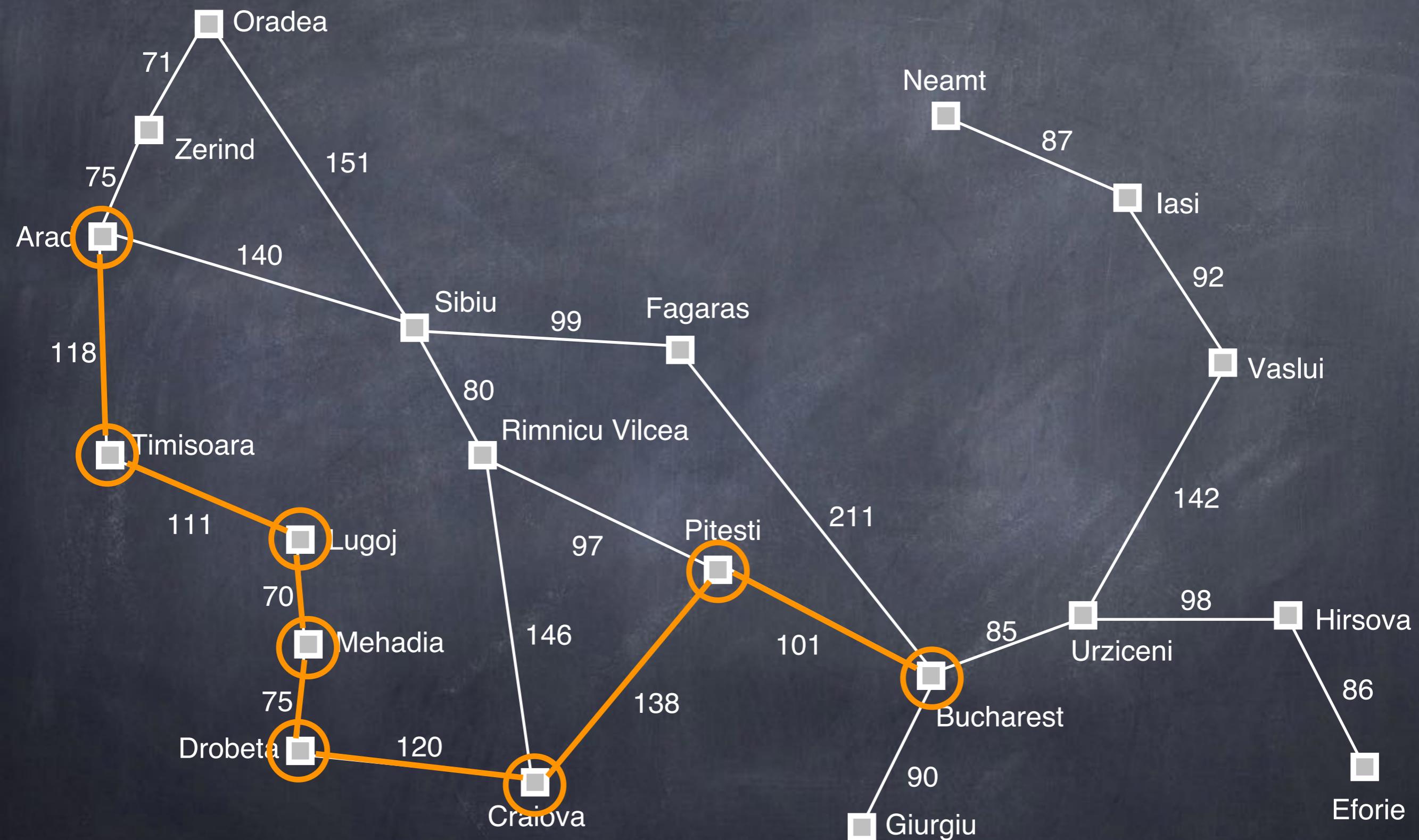
- Change the state of the world
  - Cause a transition from one state to another
  - Not all actions are **applicable** in every state



# Solutions

- Solutions are sequences of actions that lead from the initial state to a goal state





$$118+111+70+75+120+138+101=733$$

# Modeling Problems (and their Solutions)

- Model the world as a set of states
- Actions change the state of the world
- Cost of solution (path cost)
  - Cost of individual actions (step cost)
- Solutions are sequences of actions that lead from the initial state to a goal state

**Problem (Domain):**  $\langle S, A, \text{ACTIONS}, \text{RESULT}, \text{COST} \rangle$

$\text{ACTIONS} : s \in S \rightarrow$

$\{ a \in A : a \text{ can be executed (is applicable) in } s \}$

$\text{RESULT} : s \in S, a \in A \rightarrow$

$s' \in S$  s.t.  $s'$  is the result of performing  $a$  in  $s$

$\text{COST}$  : Assigns a cost to each path/step  $c(s, a, s')$

**Problem (Domain):**  $\langle S, A, \text{ACTIONS}, \text{RESULT}, \text{COST} \rangle$

Transition  
Model

$\text{ACTIONS} : s \in S \rightarrow$

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$s' \in S$  s.t.  $s'$  is the result of performing  $a$  in  $s$

$\text{COST} : \text{Assigns a cost to each path/step } c(s, a, s')$

**Problem (Domain):**  $\langle S, A, \text{ACTIONS}, \text{RESULT}, \text{COST} \rangle$

**Transition Model**

$\text{ACTIONS} : s \in S \rightarrow$ $\{ a \in A : a \text{ can be executed (is applicable) in } s \}$
$\text{RESULT} : s \in S, a \in A \rightarrow$ $s' \in S \text{ s.t. } s' \text{ is the result of performing } a \text{ in } s$
$\text{COST} : \text{Assigns a cost to each path/step } c(s, a, s')$

**Problem (Instance):**  $\langle I \in S, G \subseteq S \rangle$

**Problem (Domain):**  $\langle S, A, \text{ACTIONS}, \text{RESULT}, \text{COST} \rangle$

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**Problem (Instance):**  $\langle I \in S, G \subseteq S \rangle$

**Solution:**  $\langle a_1, a_2, \dots, a_n \rangle \in A^n$  s.t.

$\text{RESULT}(\dots \text{ RESULT}(\text{RESULT}(I, a_1), a_2) \dots, a_n) \in G$

**Problem (Domain):**  $\langle S, A, \text{ACTIONS}, \text{RESULT}, \text{COST} \rangle$

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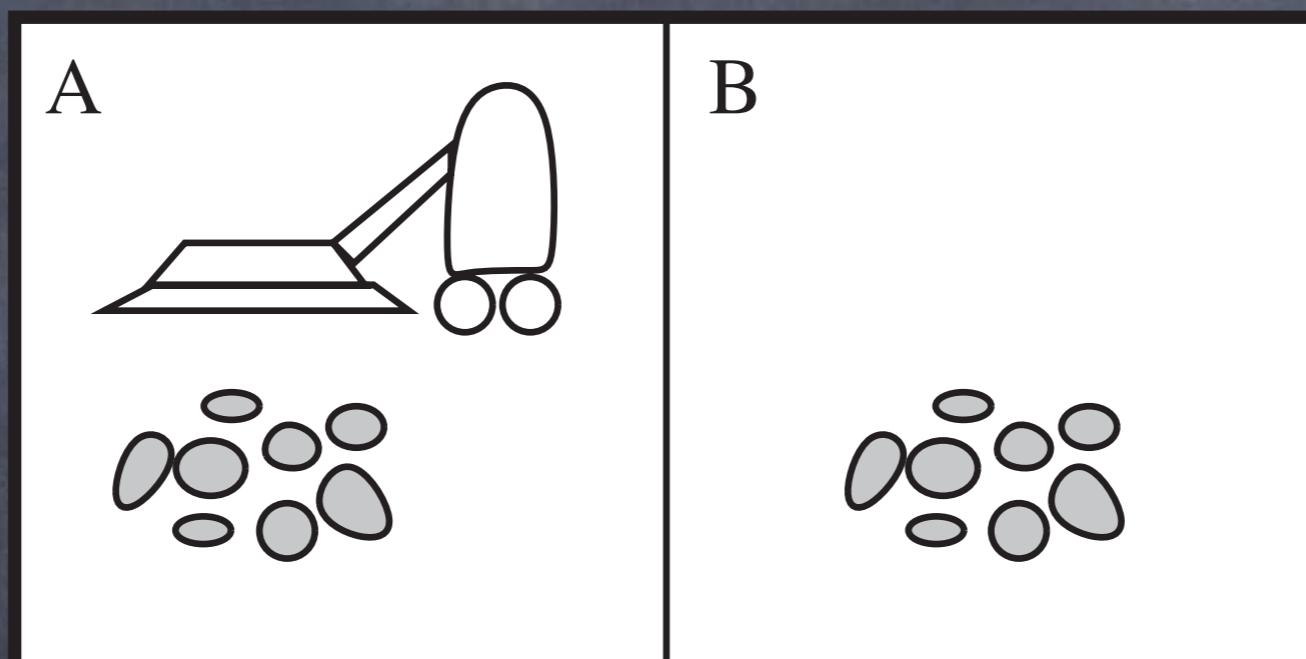
# Romania Driving

- States: What city are we in
- Actions: Drive to another city
- Transition model: Adjacency per map
- Cost: sum of distances between cities
- Initial state: any city
- Goal states: any city

# Romania Driving

- $S = \{A, B, C, D, E, F, G, H, I, L, M, N, O, P, R, S, T, U, V, Z\}$
- $A = \{a, b, c, d, e, f, g, h, i, l, m, n, o, p, r, s, t, u, v, z\}$
- Let  $\uparrow a = A, \uparrow b = B, \dots, \uparrow z = Z$
- ACTIONS( $s$ ) includes  $a$  iff  $s$  is adjacent to  $\uparrow a$  on the map
- RESULT( $s, a$ ) =  $\uparrow a$
- COST:  $c(s, a, s') = \text{map dist btw } s \text{ and } s'$
- $I = \text{any } x \in S$
- $G = \{y\} \text{ for any } y \in S$

# Vacuum World



# Vacuum World

- States: Where is the vacuum, Room A clean/dirty, Room B clean/dirty
- Actions: Left, Right, Suck
- Transition model: move, clean, or no effect
- Cost: 1 per action
- Initial state: any
- Goal state: both rooms clean

# Vacuum World

- $S = \{ \langle v, r_A, r_B \rangle \}$  where
  - $v \in \{ A, B \}$  = location of vacuum
  - $r_i \in \{ \text{dirty}, \text{clean} \}$  = room  $i$  is dirty/clean

# Vacuum World

- $S = \{ \langle v, r_1, r_2 \rangle \}$
- $A = \{ Left, Right, Suck \}$

# Vacuum World

- $S = \{ \langle v, r_A, r_B \rangle \}$
- $A = \{ Left, Right, Suck \}$
- $\text{ACTIONS}(\langle v, r_A, r_B \rangle) = A$
- $\text{RESULT}(\langle v, r_A, r_B \rangle, a) = \langle A, r_A, r_B \rangle \text{ if } a=Left$   
 $\text{RESULT}(\langle v, r_A, r_B \rangle, a) = \langle B, r_B, r_B \rangle \text{ if } a=Right$   
 $\text{RESULT}(\langle v, r_A, r_B \rangle, Suck) = \langle v, clean, r_B \rangle \text{ if } v=A$   
 $\qquad\qquad\qquad \langle v, r_A, clean \rangle \text{ if } v=B$

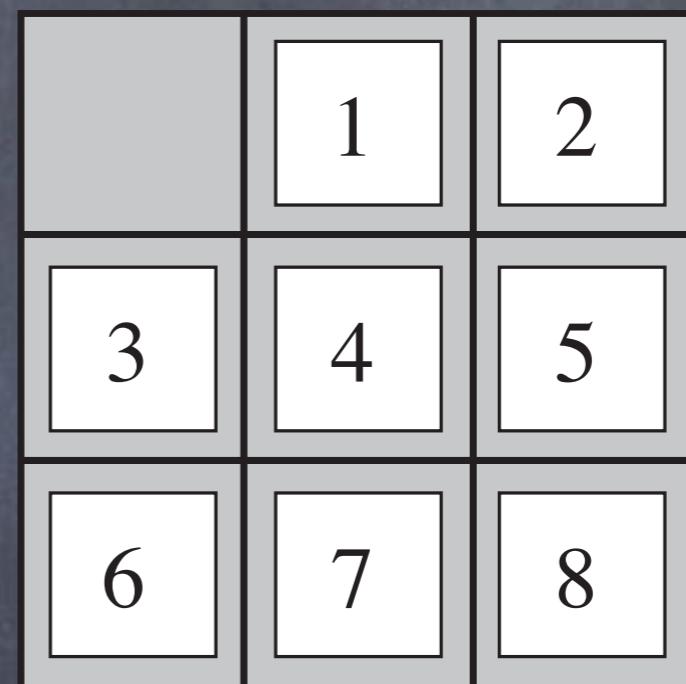
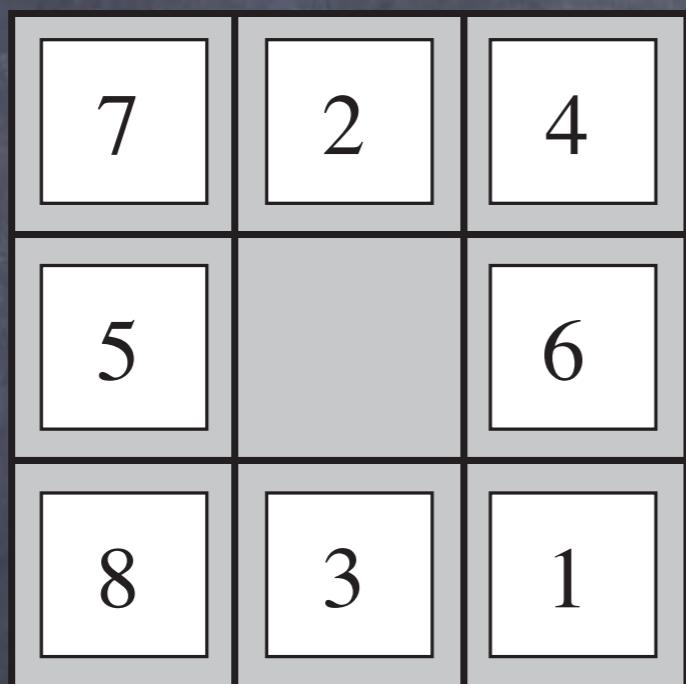
# Vacuum World

- $S = \{ \langle v, r_A, r_B \rangle \}$
- $A = \{ Left, Right, Suck \}$
- $\text{ACTIONS}(\langle v, r_A, r_B \rangle) = A$
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 $\text{RESULT}(\langle v, r_A, r_B \rangle, a) = \langle B, r_A, r_B \rangle \text{ if } a=Right$   
 $\text{RESULT}(\langle v, r_A, r_B \rangle, Suck) = \langle v, clean, r_B \rangle \text{ if } v=A$   
 $\qquad\qquad\qquad \langle v, r_A, clean \rangle \text{ if } v=B$
- COST:  $c(s, a, s') = 1$
- $I = \langle v, r_A, r_B \rangle$  for any  $v, r_A, r_B$
- $G = \{ \langle v, clean, clean \rangle \}$  for any  $v$

# 15-Puzzle



# 8-Puzzle



# 8-Puzzle

- States: 3x3 grid of numbers 0-8 (0 for blank)
- Actions: Move a tile into the blank  
alternative: Move the blank
- Transition model: Swap moved tile with blank
- Cost: 1 per action
- Initial state: any
- Goal state: tiles in order 0-8 (or whatever)

# 8-Puzzle

- $S = \{ M_k \}$  where  $M_k = [m_{i,j}]$  for  
 $1 \leq i, j \leq 3$ ,  $m_{i,j} \in \{0, \dots, 8\}$ ,  $m_{i,j}$  unique

# 8-Puzzle

- $S = \{ M_k \}$  where  $M_k = [m_{i,j}] \dots$
- $A = \{ 1, \dots, 8 \}$

# 8-Puzzle

- $S = \{ M_k \}$  where  $M_k = [m_{i,j}] \dots$
- $A = \{ 1, \dots, 8 \}$
- Let  $loc(M,i) = \langle x, y \rangle$  be such that  $m_{x,y} = i$   
(location of tile  $i$  in state  $M$ )
- ACTIONS( $M$ ) includes  $a$  iff  
 $loc(M,a) = \langle x_a, y_a \rangle$  and  $loc(M,0) = \langle x_0, y_0 \rangle$   
and  $|x_a - x_0| = 1$  and  $|y_a - y_0| = 0$  or  
 $|x_a - x_0| = 0$  and  $|y_a - y_0| = 1$

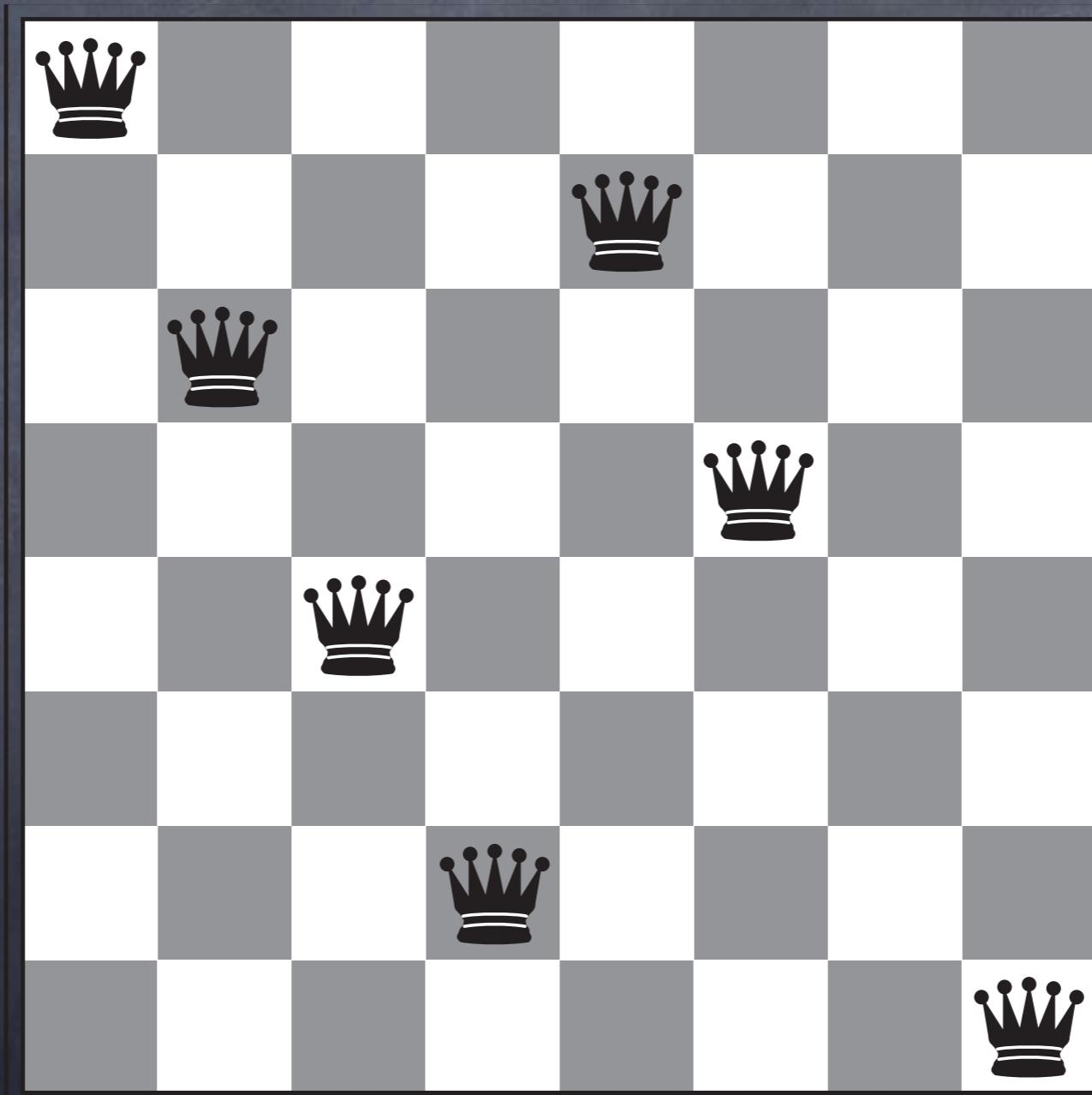
# 8-Puzzle

- $S = \{ M_k \}$  where  $M_k = [m_{i,j}] \dots$
- $A = \{ 1, \dots, 8 \}$
- Let  $loc(M,i) = \langle x, y \rangle \dots$
- ACTIONS( $M$ ) ...
- RESULT( $M, a$ ):  
Let  $loc(M,a) = \langle x_a, y_a \rangle$  and  $loc(M,0) = \langle x_0, y_0 \rangle$   
Then in RESULT( $M,a$ ):  $m_{x_0,y_0} = a$  and  $m_{x_a,y_a} = 0$

# 8-Puzzle

- $S = \{ M_k \}$  where  $M_k = [m_{i,j}] \dots$
- $A = \{ 1, \dots, 8 \}$
- Let  $loc(M,i) = \langle x,y \rangle \dots$
- ACTIONS( $M$ ) ...
- RESULT( $M, a$ ) ...
- COST:  $c(s,a,s') = 1$
- $I = \text{any } M_k$
- $G = \{ [0, \dots, 8] \}$  (or whatever)

# 8-Queens



# Knuth's Conjecture

$$\lfloor \sqrt{\sqrt{\sqrt{\sqrt{(4!)!}}}} \rfloor = 5$$

**Problem (Domain):**  $\langle S, A, \text{ACTIONS}, \text{RESULT}, \text{COST} \rangle$

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**Problem (Instance):**  $\langle I \in S, G \subseteq S \rangle$

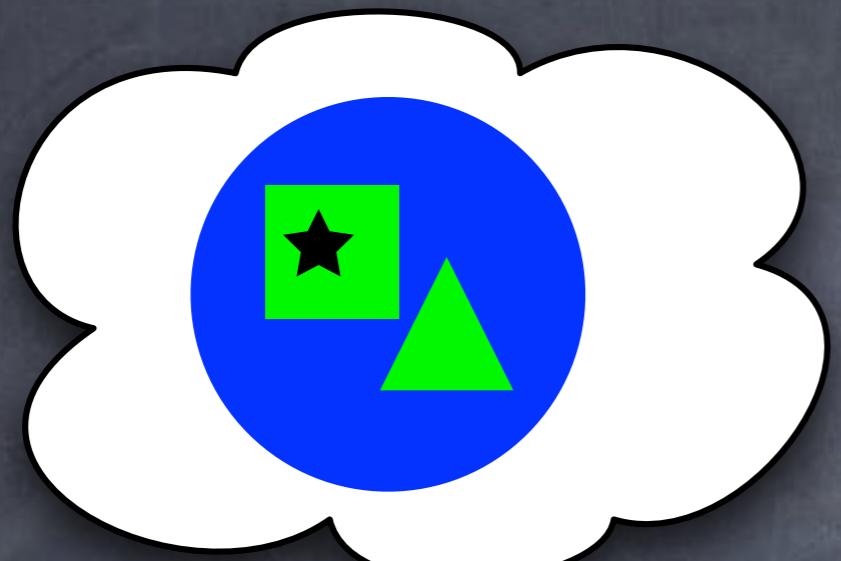
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$\text{RESULT}(\dots \text{ RESULT}(\text{RESULT}(I, a_1), a_2) \dots, a_n) \in G$





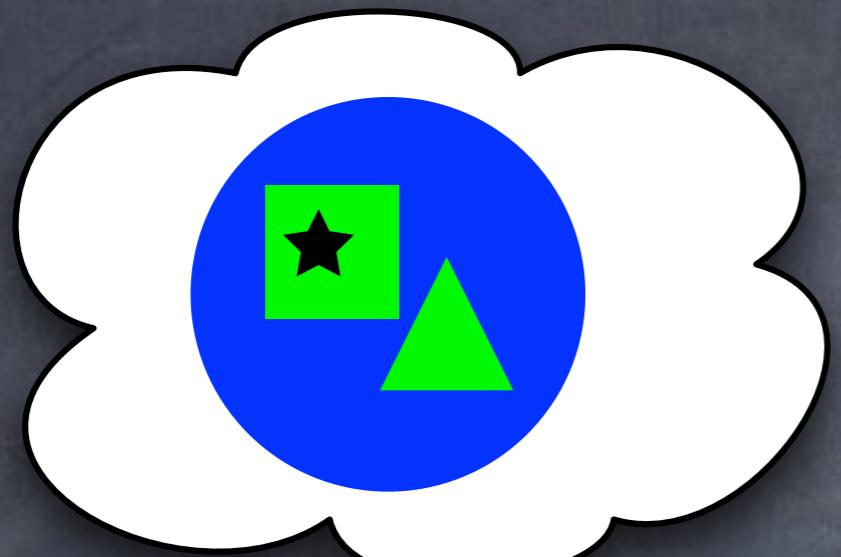
World state



Problem-solving state



World state



Problem-solving state



World state  
Action



Problem-solving state  
Transition Model



World state  
Action

# State Space

- The set of all states reachable from the initial state by some sequence of actions

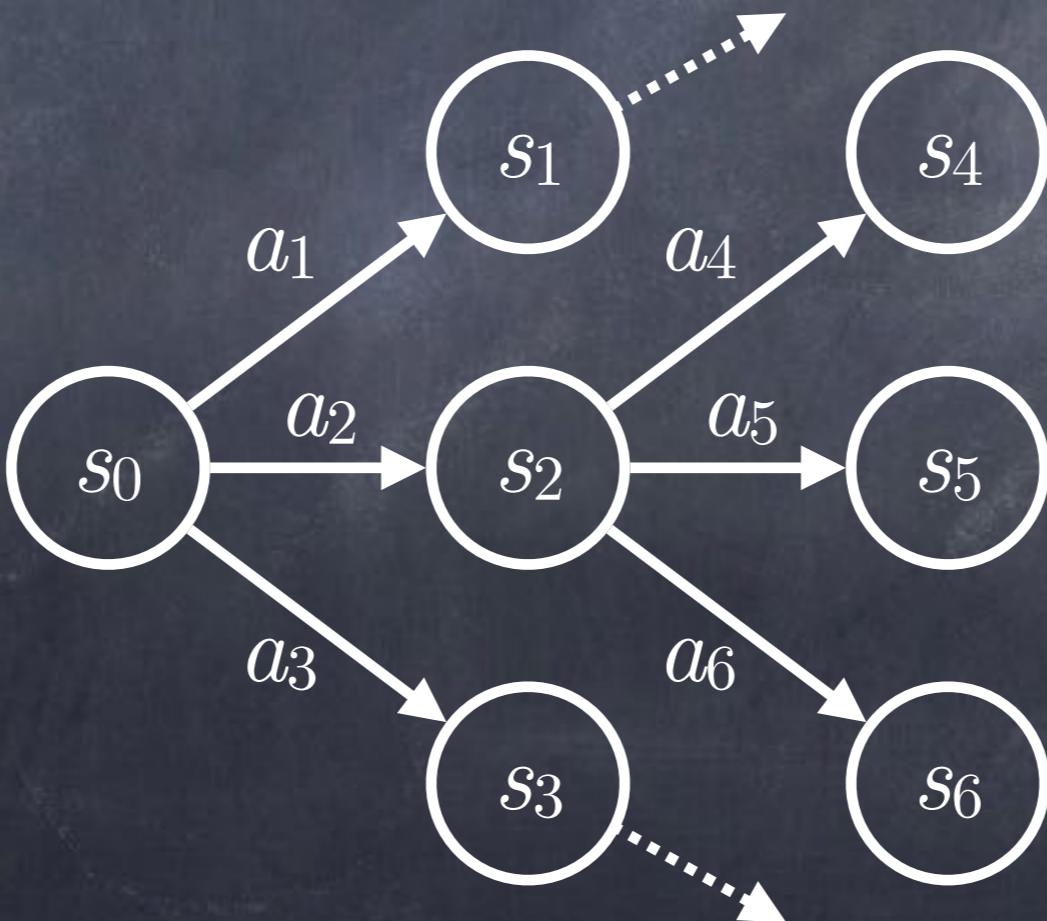
States  
Actions      —————> State Space  
Transition Model

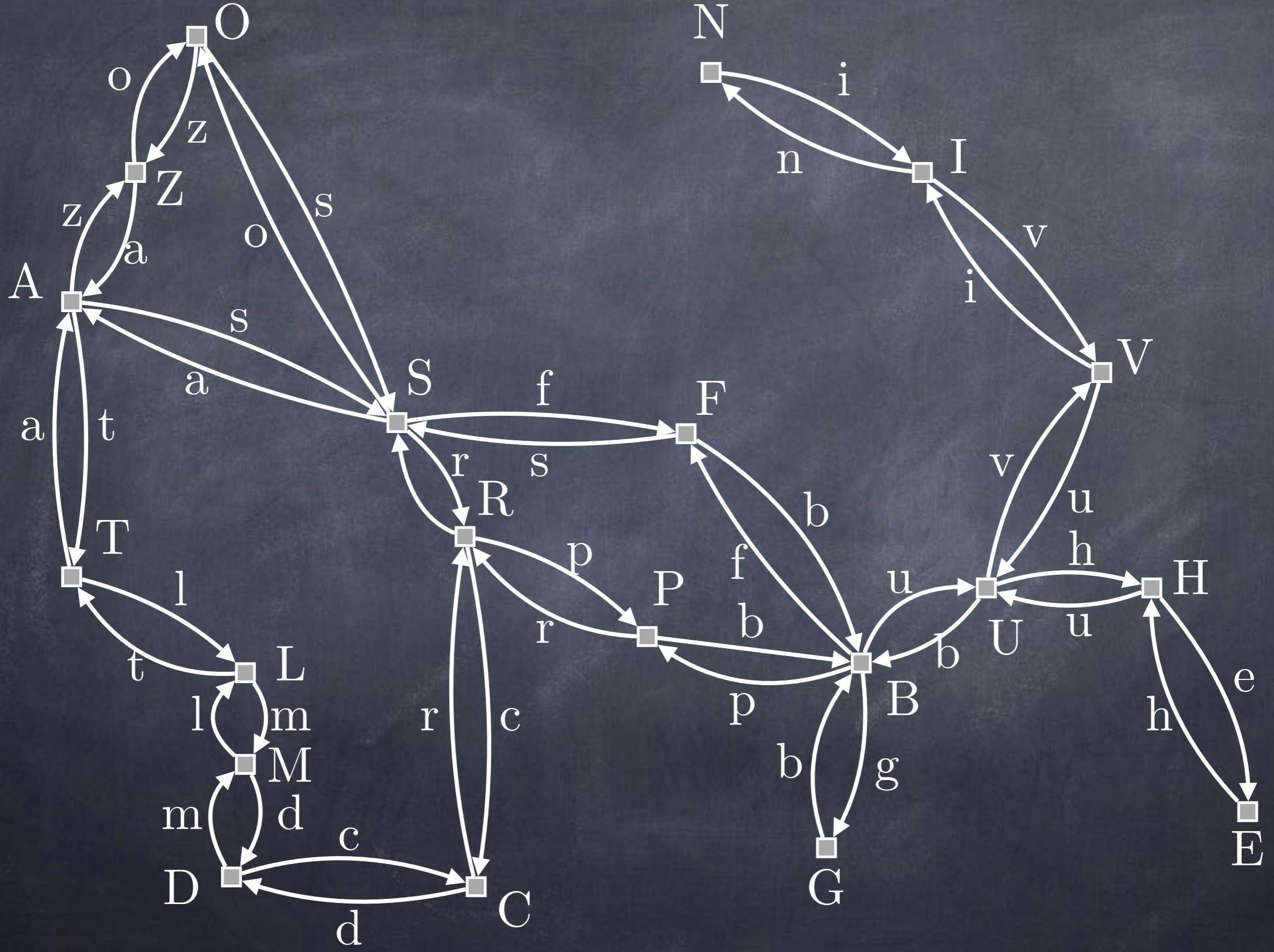
# State Space

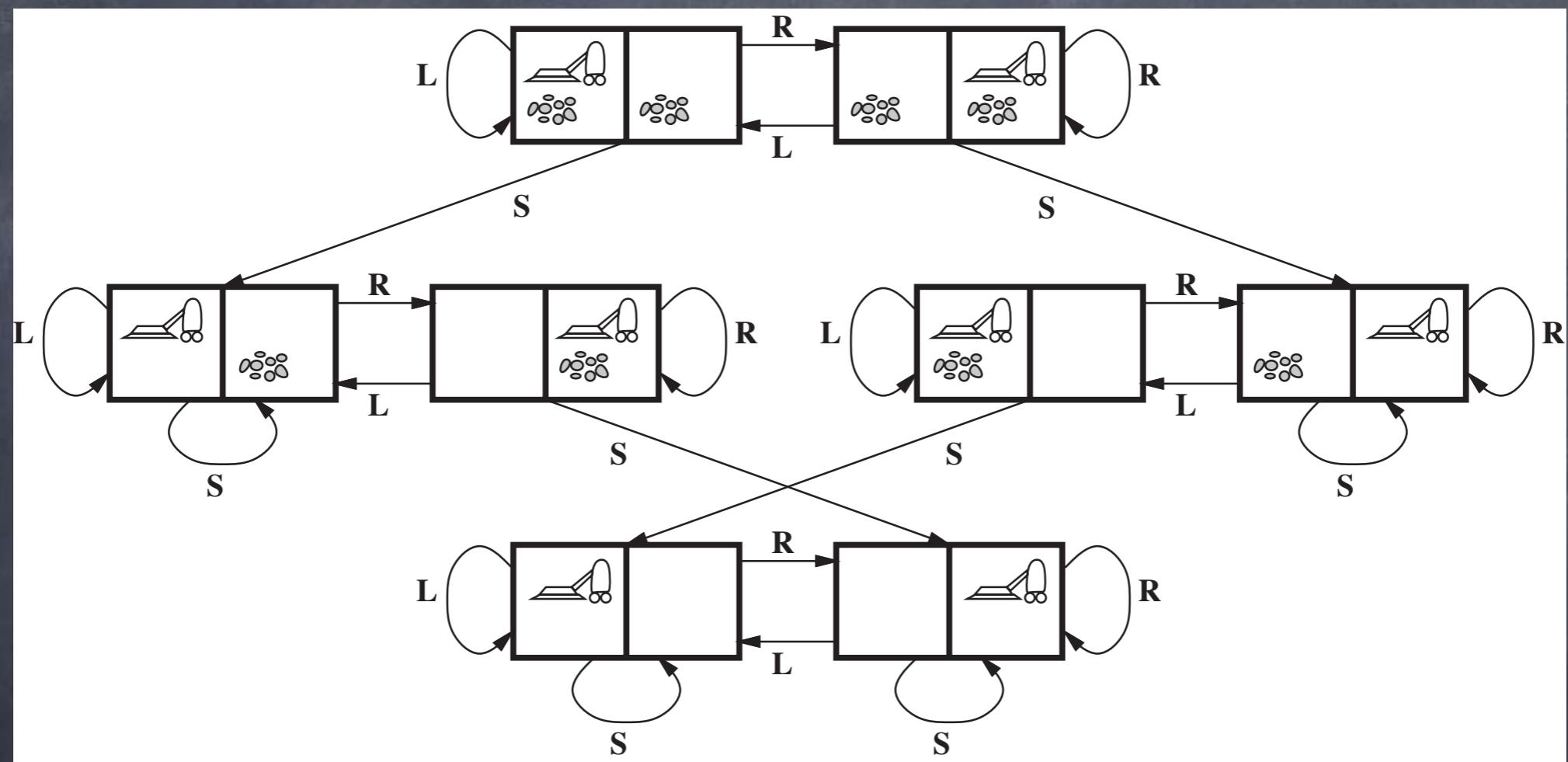
$\langle V, E \rangle :$

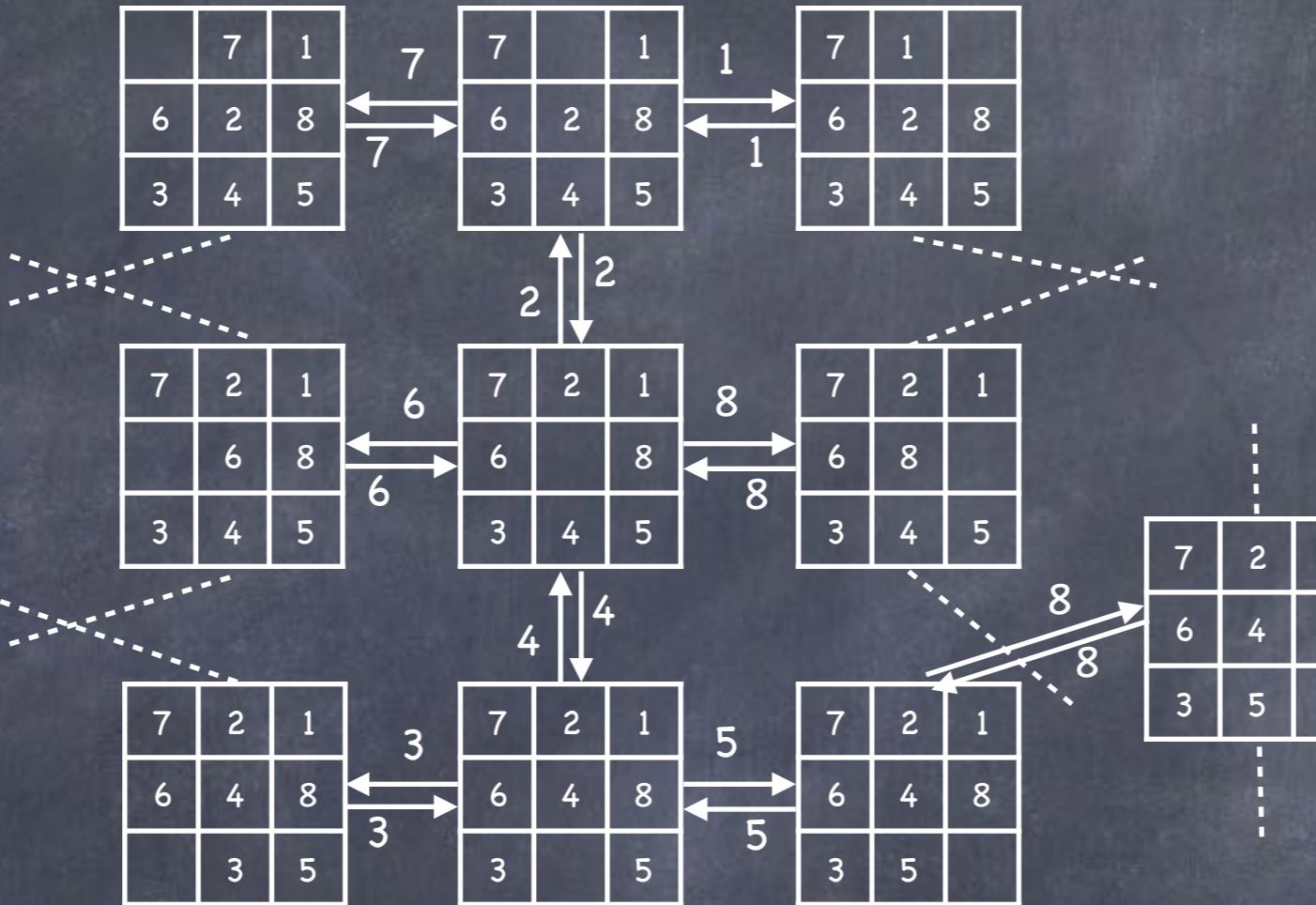
$$V = \{ v_i \mid s_i \in S \}$$

$$E = \{ \langle v_i, v_j, a \rangle \mid s_j = \text{RESULT}(s_i, a) \}$$

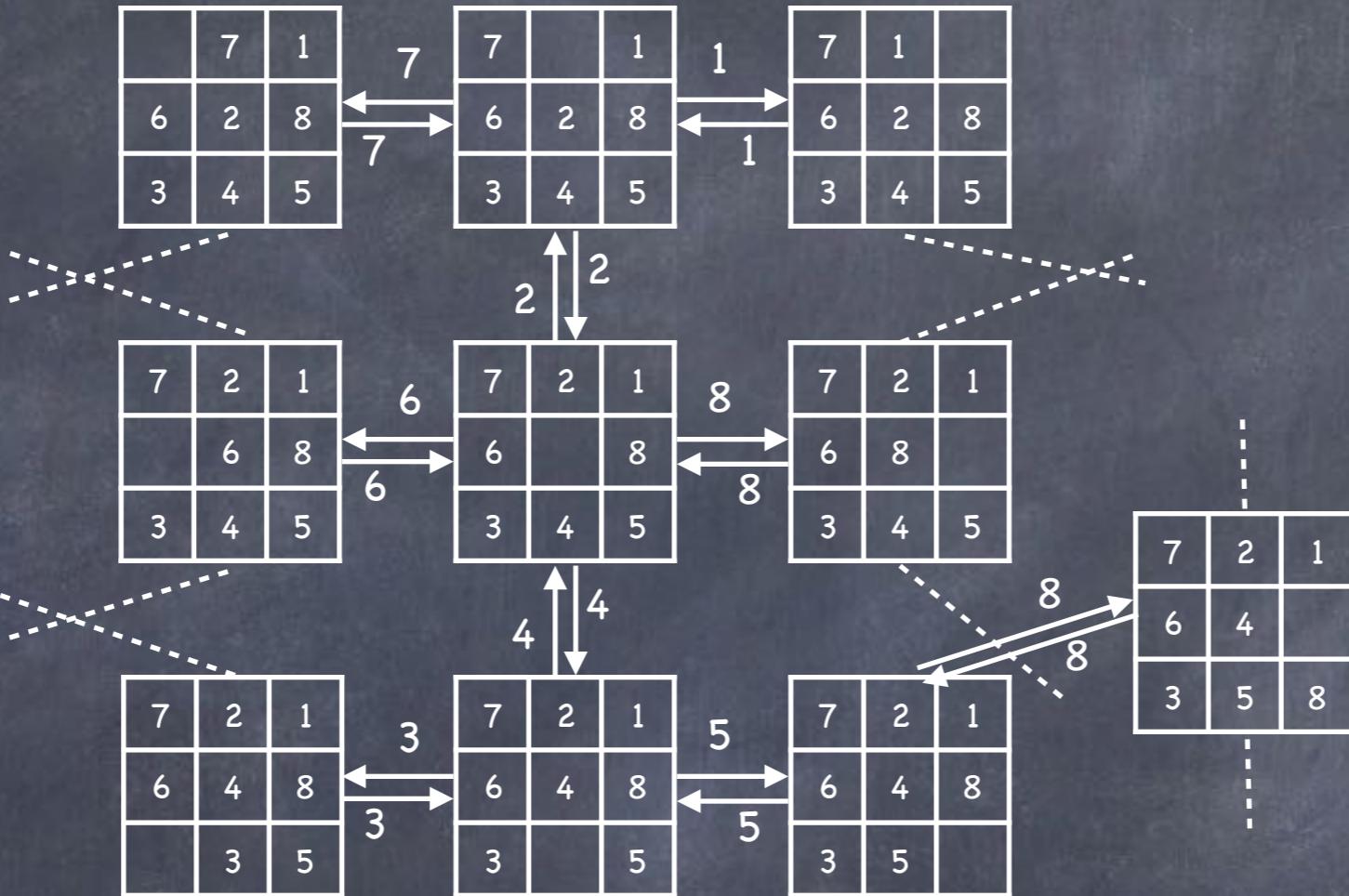








	1	2
3	4	5
6	7	8

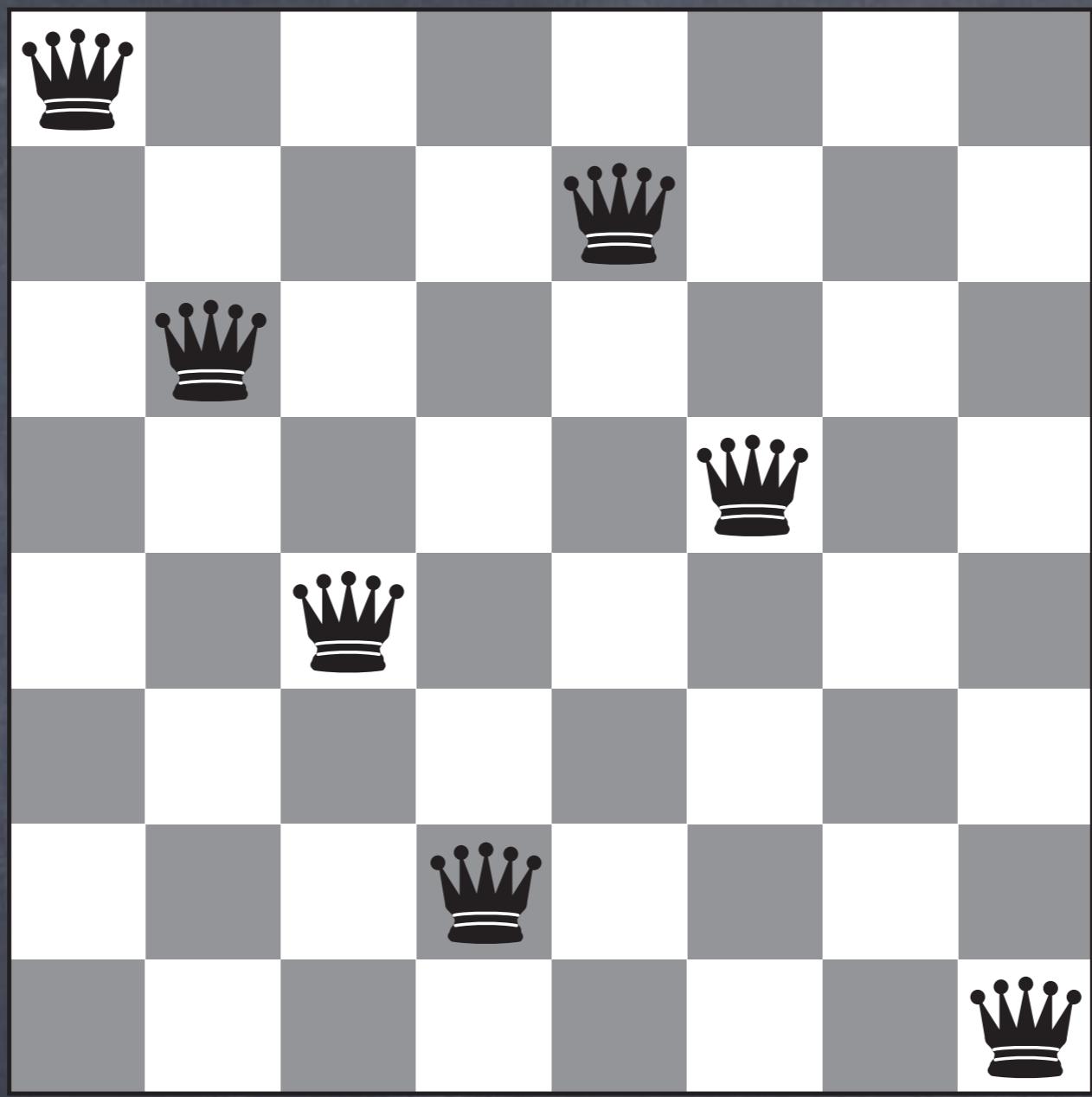


**8-puzzle:**  $9!/2 = 181,440$

**15-puzzle:**  $16!/2 = \sim 1.3$  trillion

**24-puzzle:**  $25!/2 = \sim 10^{25}$

	1	2
3	4	5
6	7	8



8-queens:  $63 \times 62 \times \dots \times 57 = \sim 1.8 \times 10^{14}$

Can be improved...

$$\lfloor \sqrt{\sqrt{\sqrt{\sqrt{(4!)!}}}} \rfloor = 5$$

State space is infinite  
(as far as we know)

# State-Based Model

- Model the world as a set of states
  - Actions change the state of the world
  - State Space
    - Defined by states, actions, and transition model
    - Small, large, or even infinite
- Abstraction!



# Romania



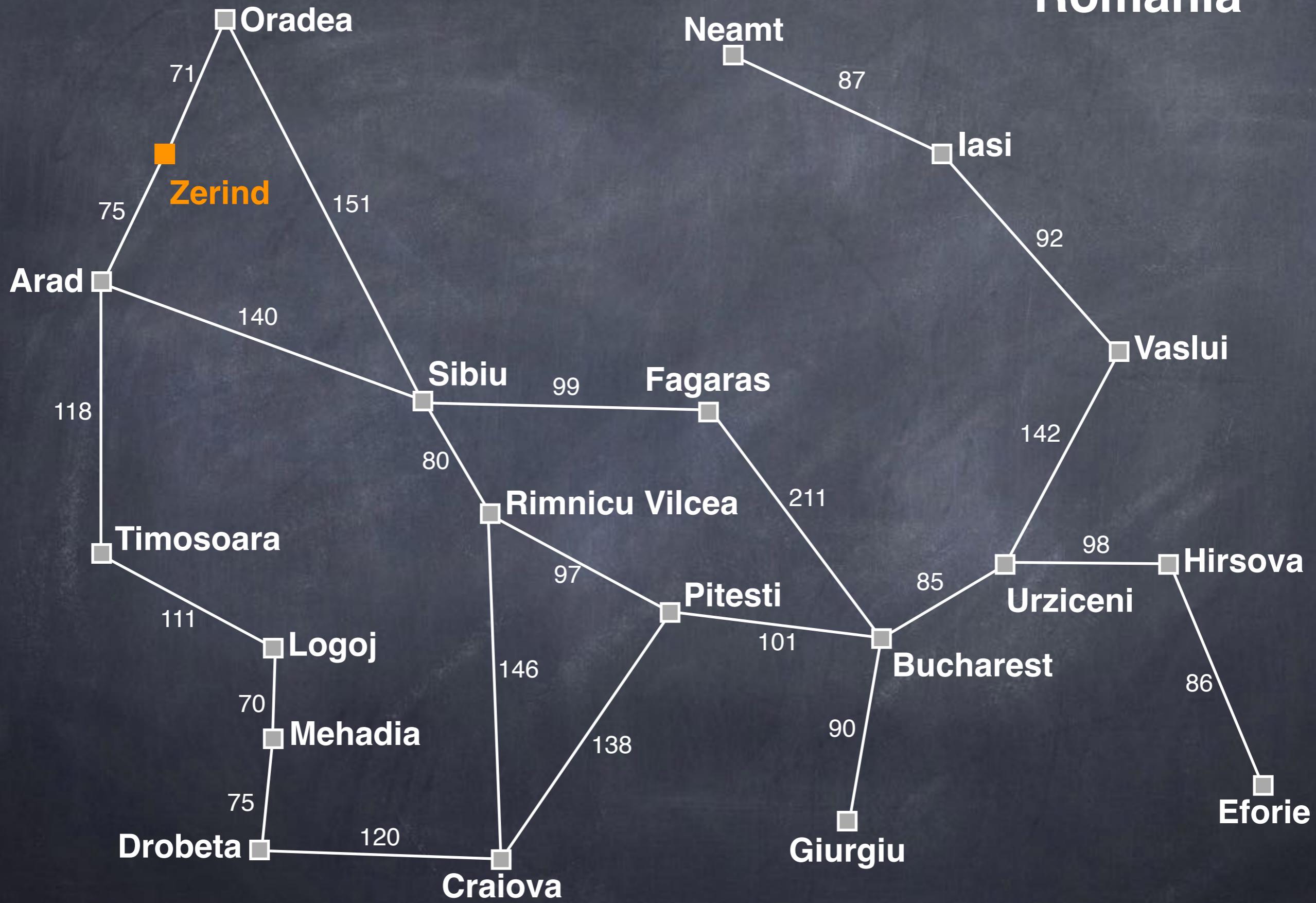
# Romania



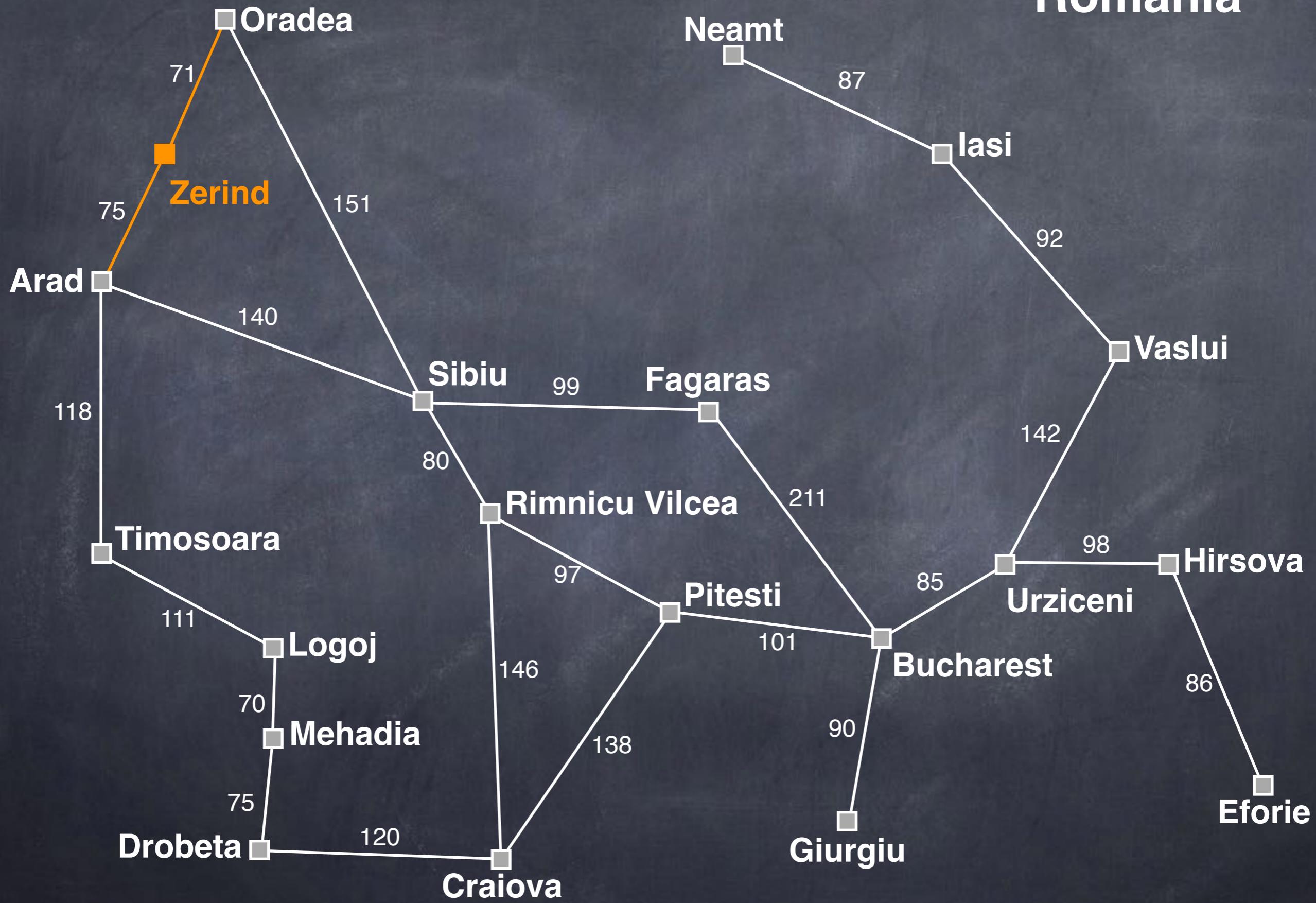
# Romania



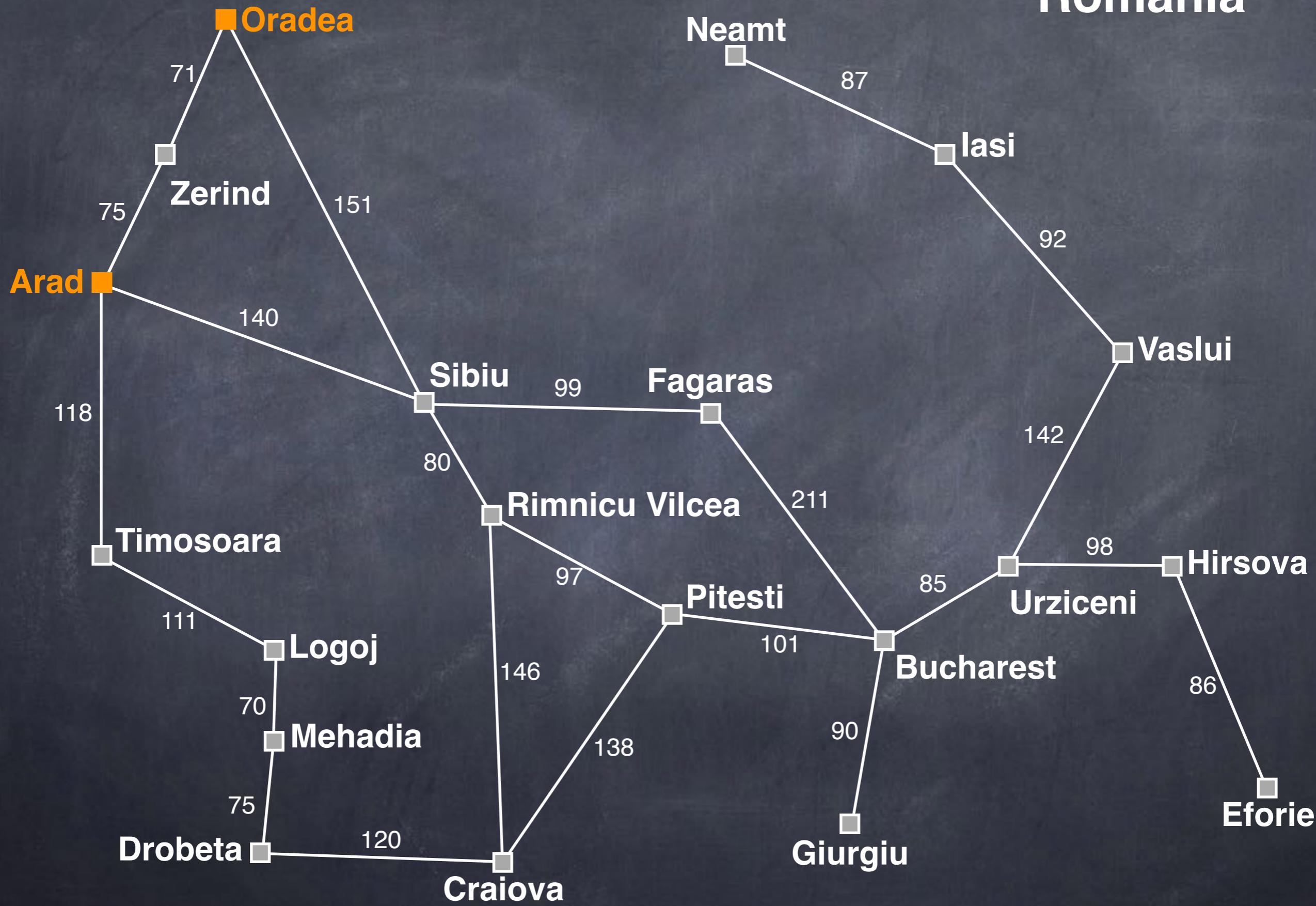
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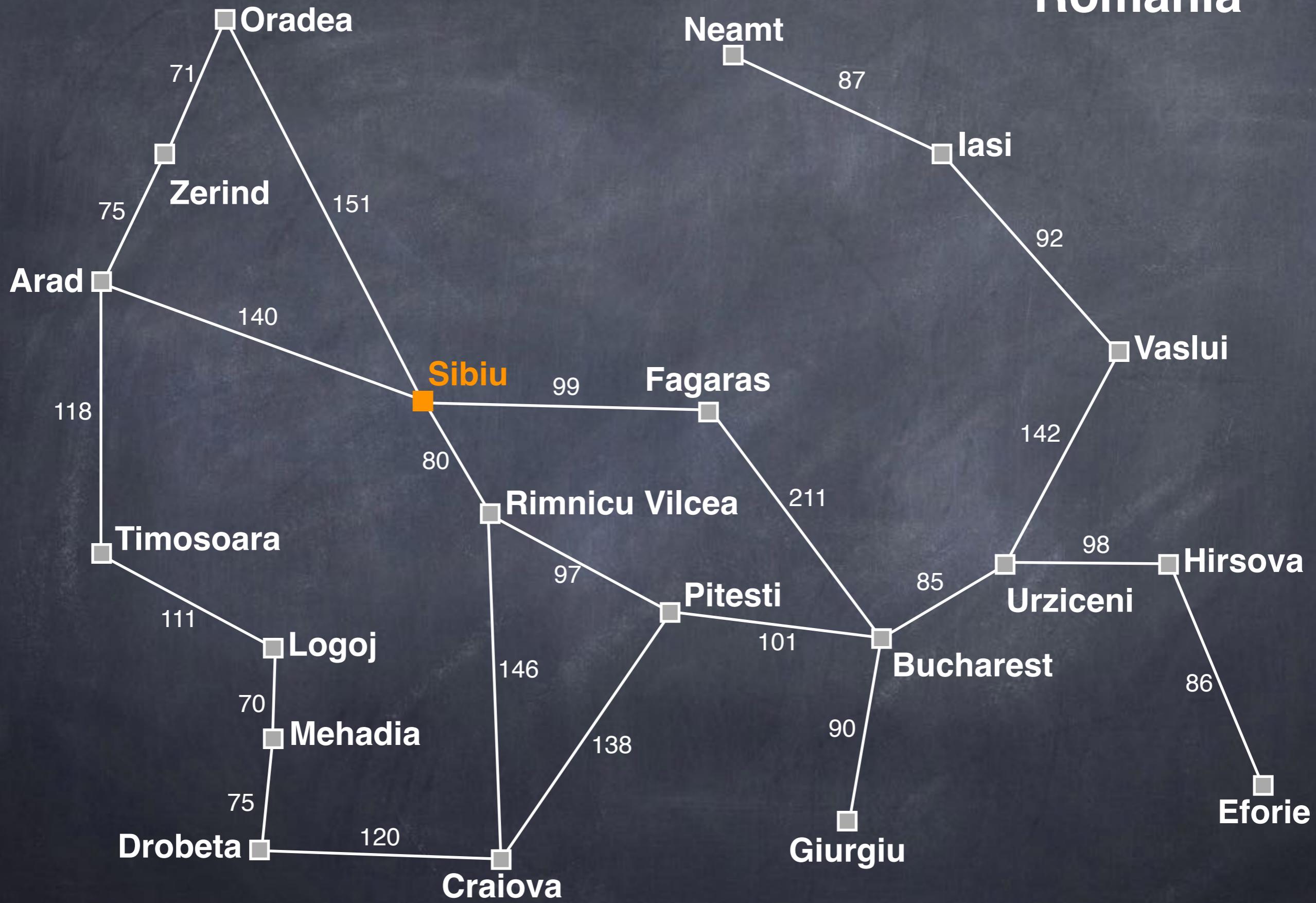
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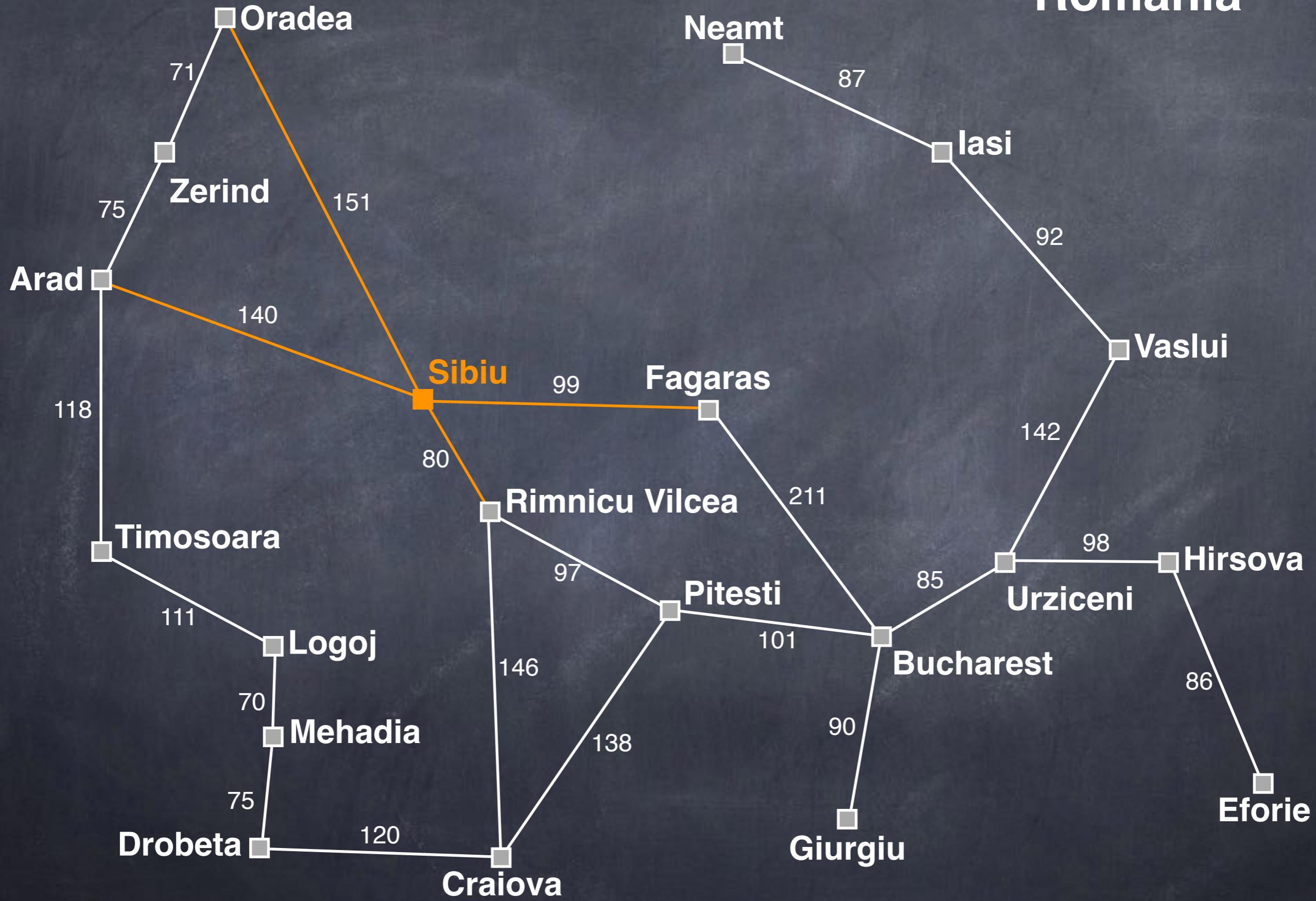
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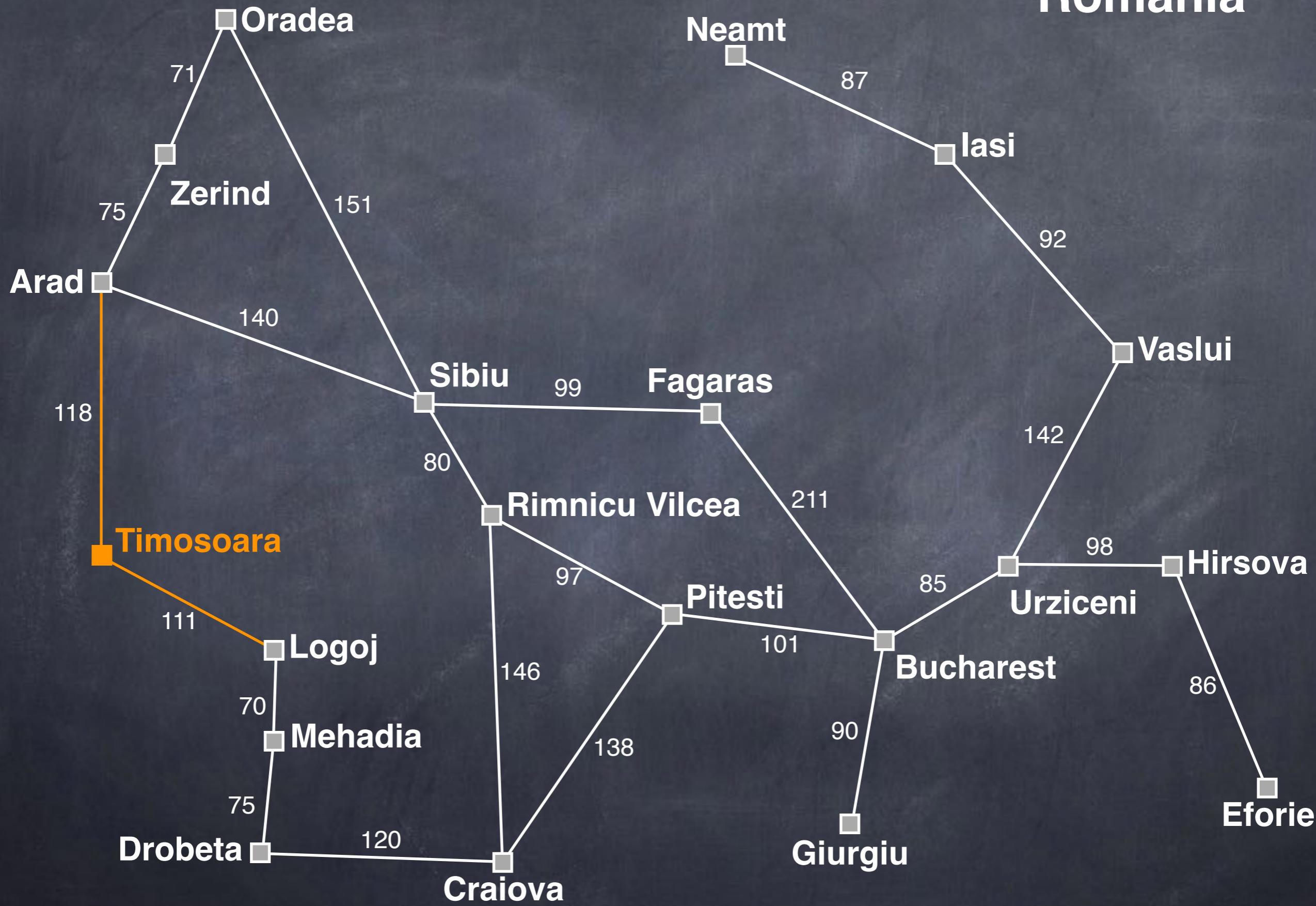
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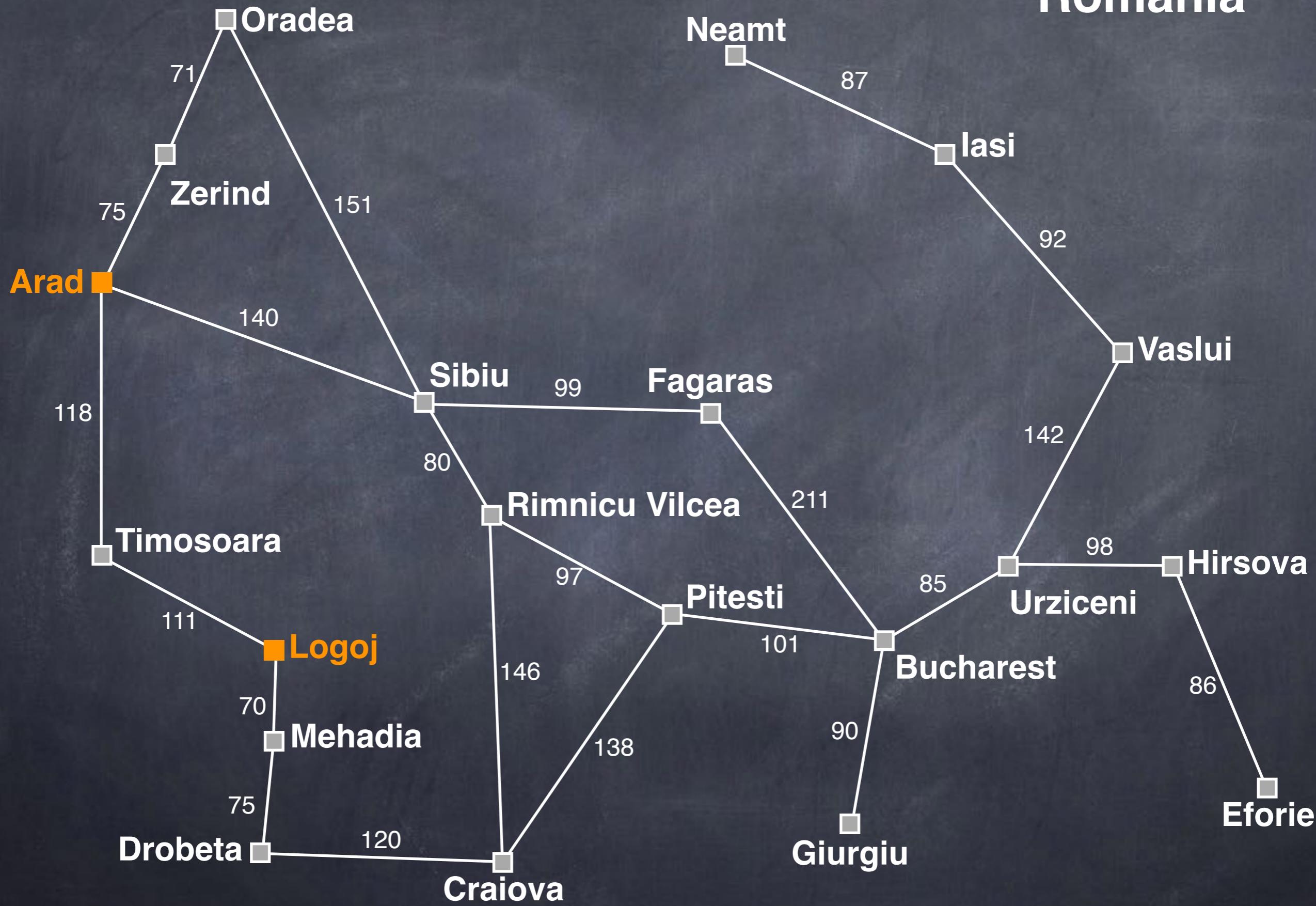
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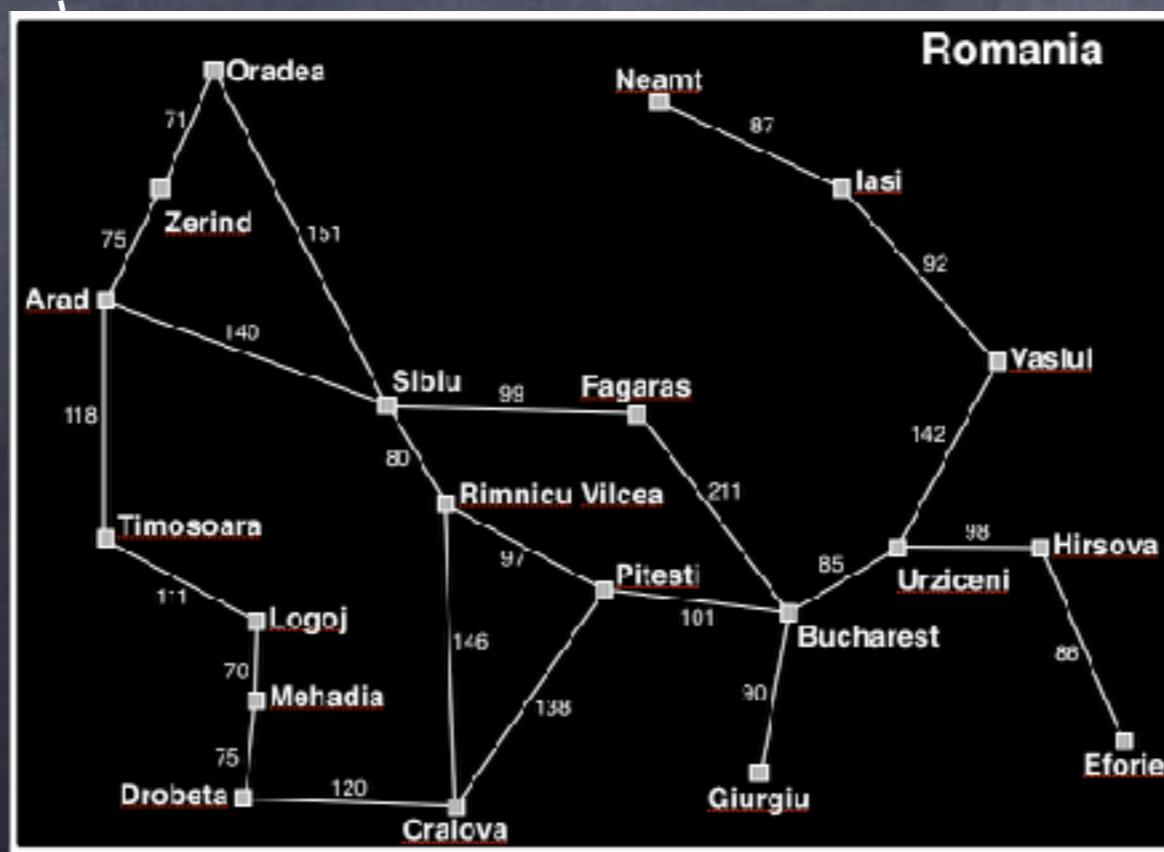
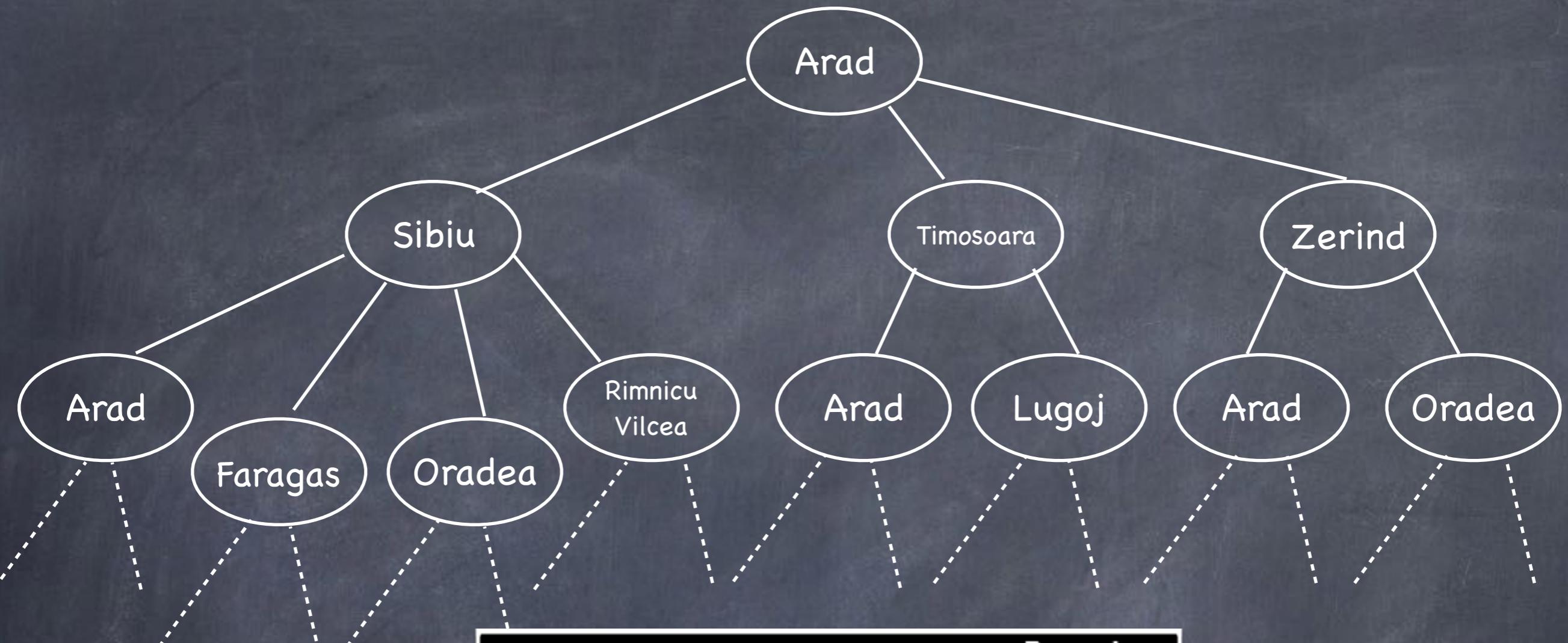


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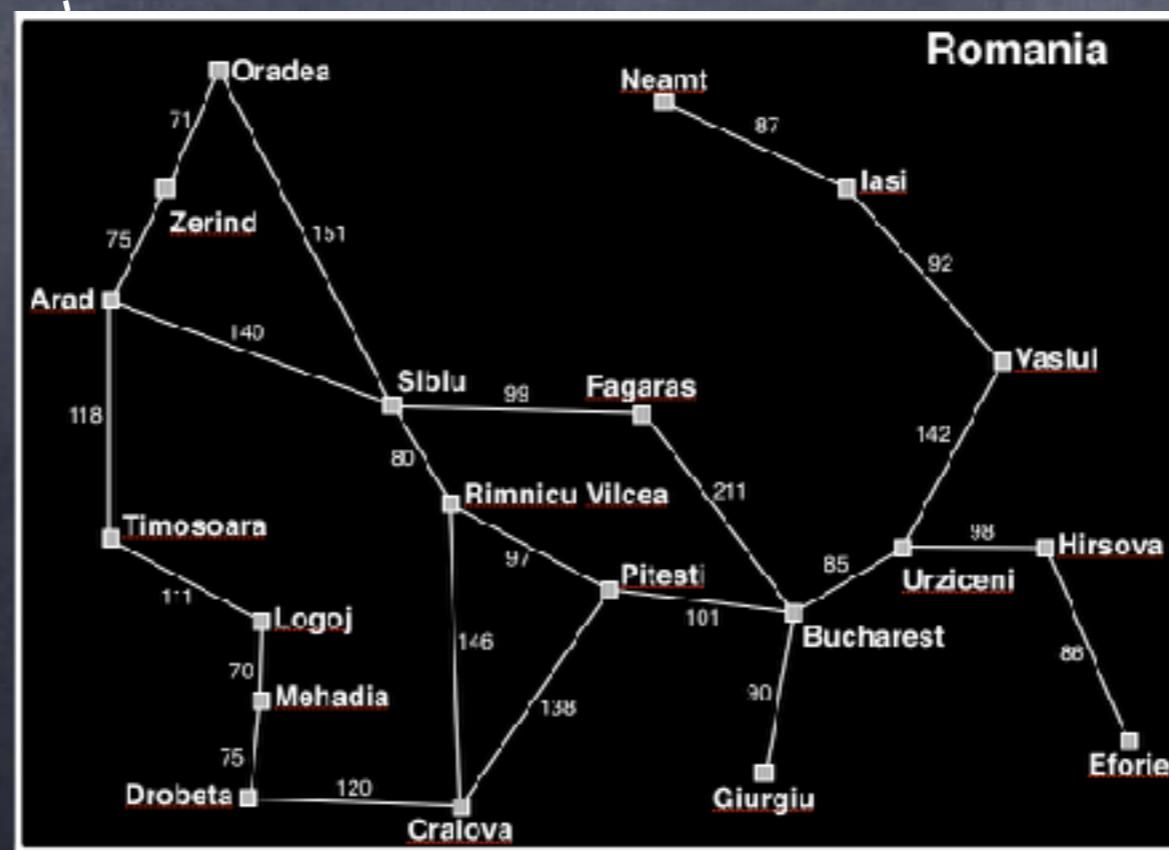
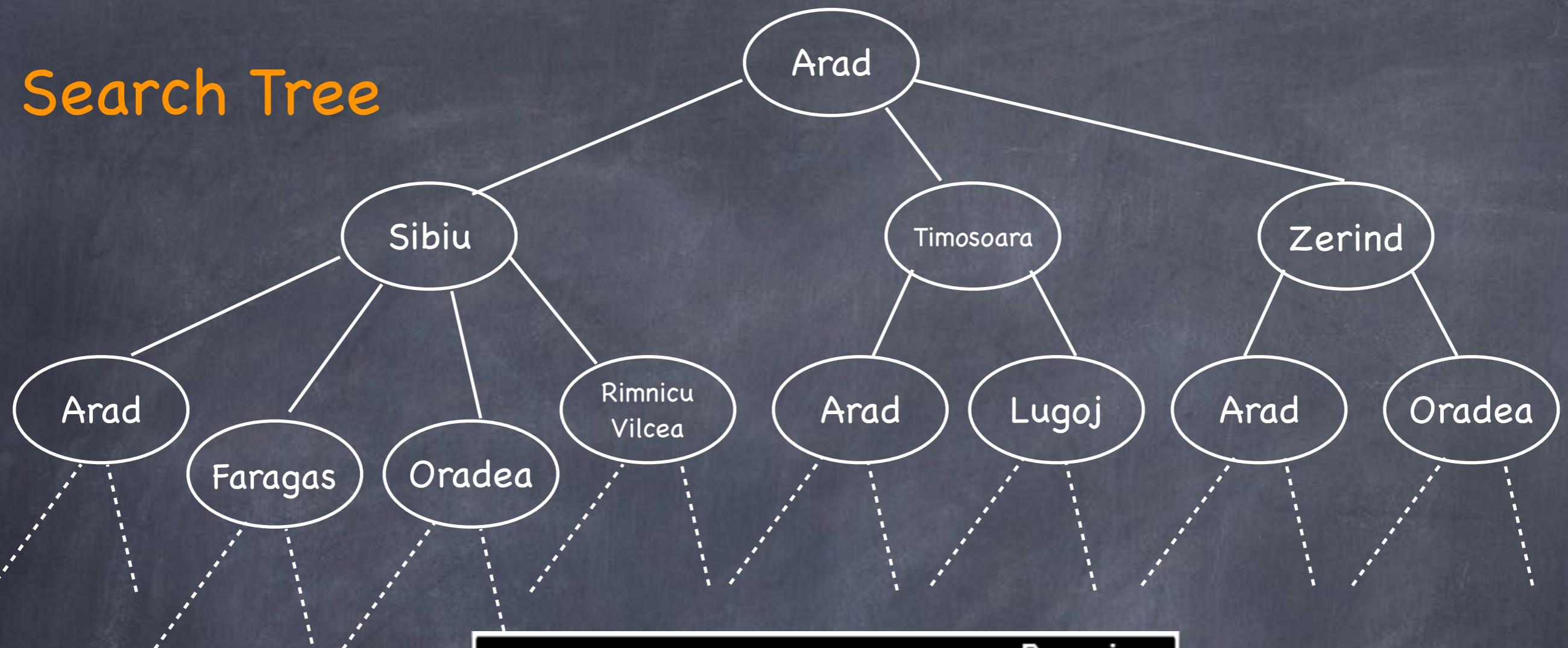


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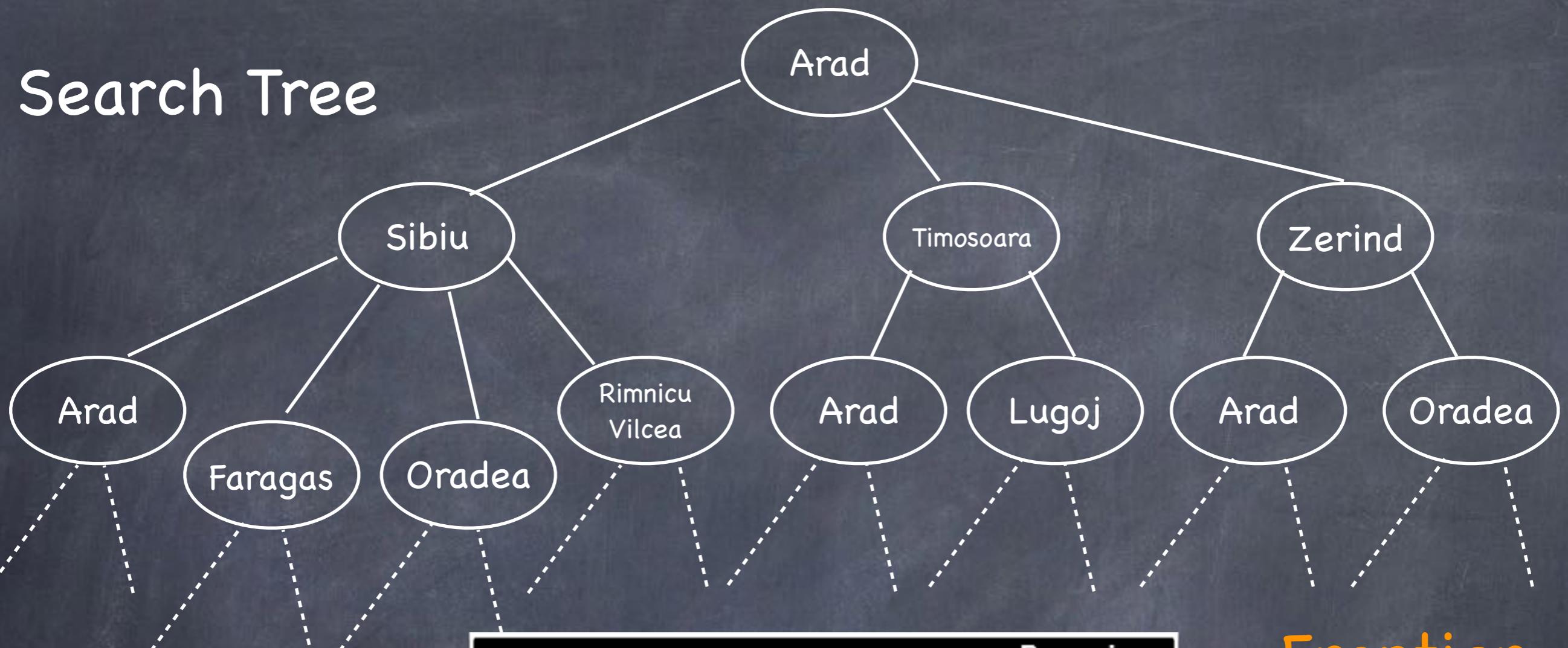




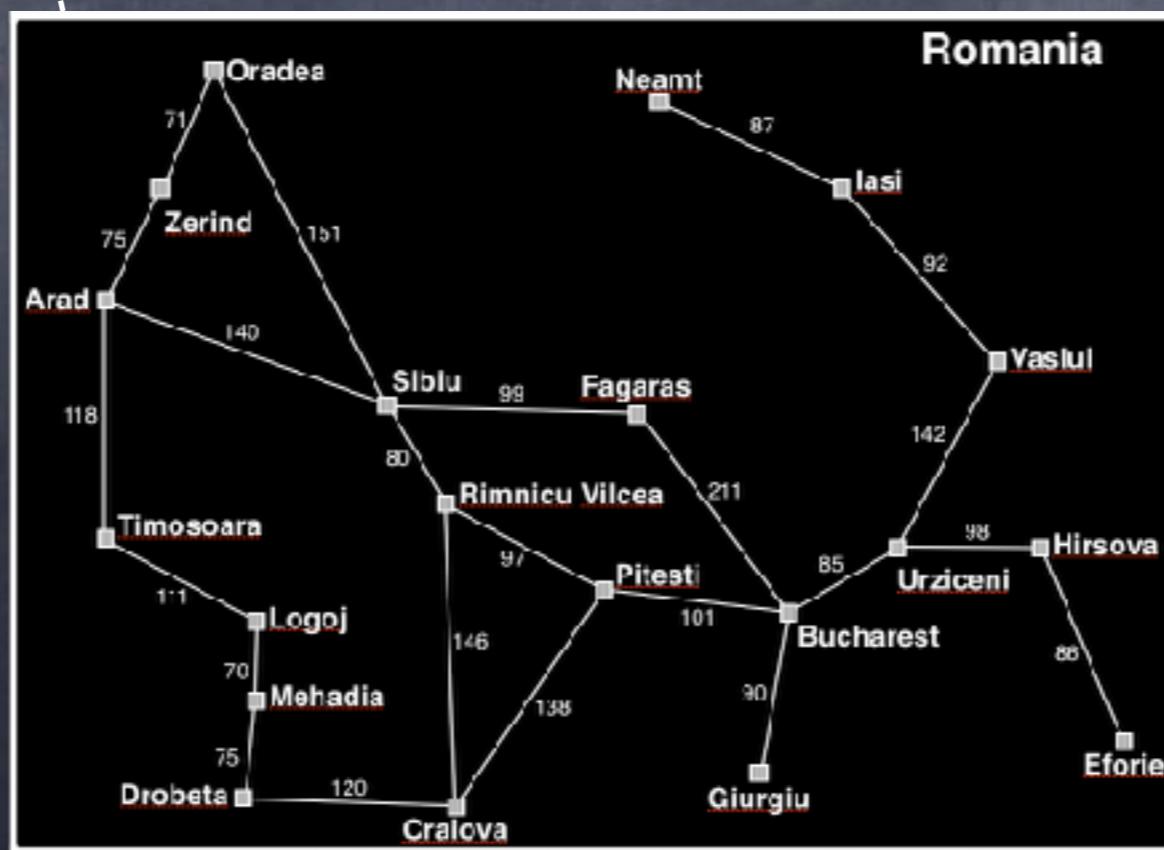
# Search Tree



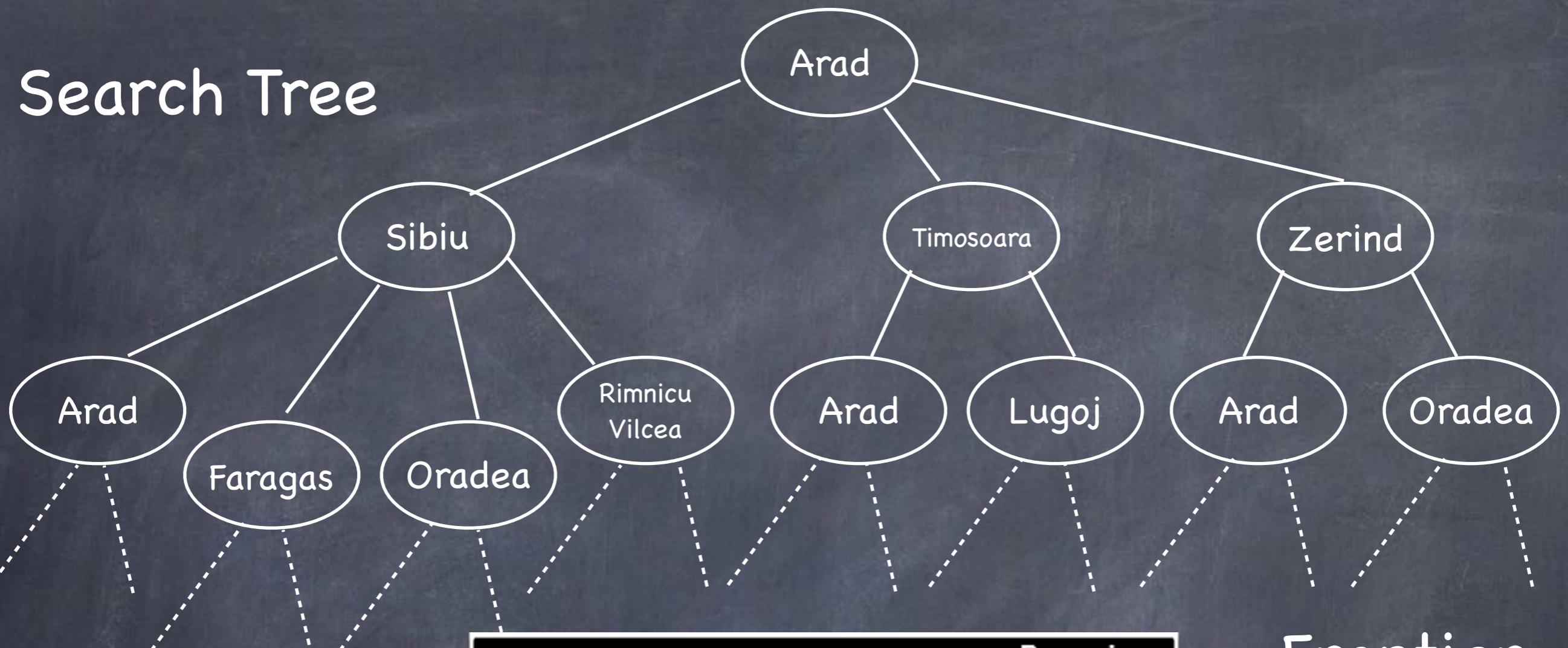
# Search Tree



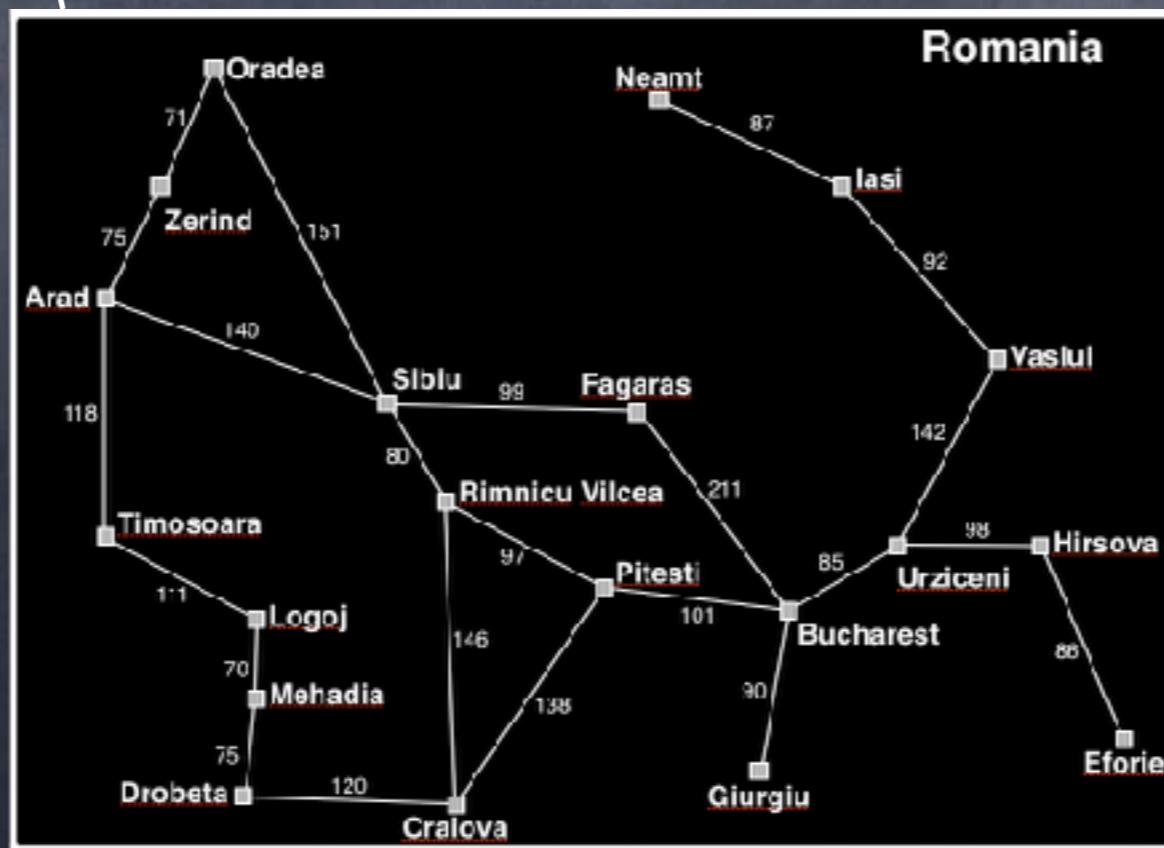
Frontier

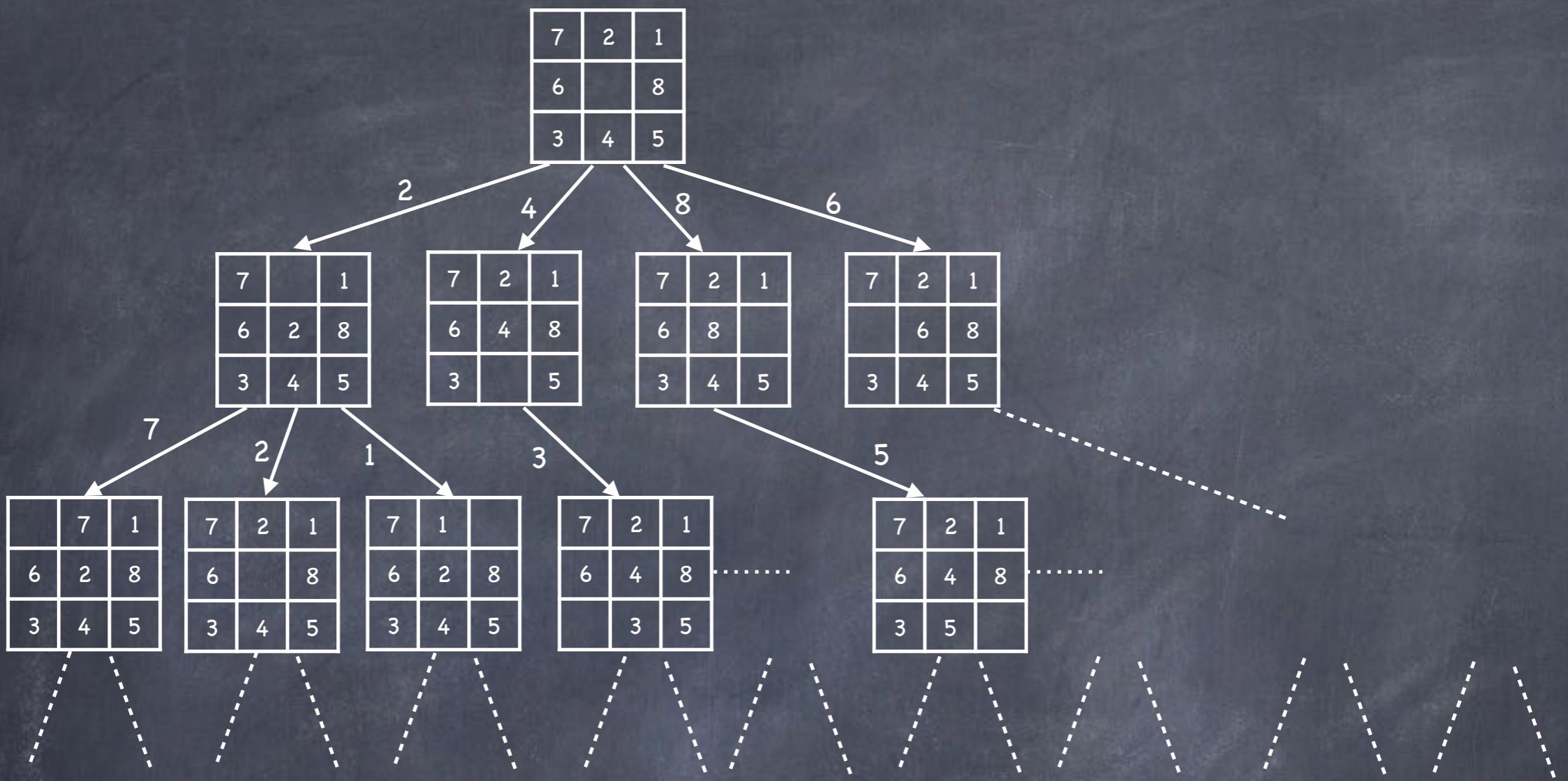


# Search Tree



Frontier





# State Space

- The set of all states reachable from the initial state by some sequence of actions

States  
Actions      —————> State Space  
Transition Model

# State-Space Search Tree

- The tree formed by starting with the initial state and using applicable actions to generate successor (child) states
- May be infinite (even for finite search spaces)
- Not stored explicitly

# State-Space Search

- Start with initial state
- Generate successor states by applying applicable actions
- Until you find a goal state

Initialize the frontier to just  $I$

While the frontier is not empty:

    Remove a state  $s$  from the frontier

    If  $s \in G$ :

        Return solution to  $s$

    else:

        Add  $\text{successors}(s)$  to the frontier

# Universal Problem-Solving Procedure

Initialize the frontier to just  $I$

While the frontier is not empty:

    Remove a state  $s$  from the frontier

    If  $s \in G$ :

        Return solution to  $s$

    else:

        Add  $\text{successors}(s)$  to the frontier

# Coolest Program Ever

Initialize the frontier to just  $I$

While the frontier is not empty:

    Remove a state  $s$  from the frontier

    If  $s \in G$ :

        Return solution to  $s$

    else:

        Add  $\text{successors}(s)$  to the frontier

NO CODE  
REQUIRED!

# AIMA Fig. 3.7

```
Solution treeSearch(Problem p) {
    Set<Node> frontier = new Set<Node>(p.getInitialState());

    while (true) {
        if (frontier.isEmpty()) {
            return null;
        }
        Node node = frontier.selectOne();
        if (p.isGoalState(node.getState())) {
            return n.getSolution();
        }

        for (Node n : node.expand()) {
            frontier.add(n);
        }
    }
}
```

# AIMA Fig. 3.7

```
Solution graphSearch(Problem p) {
    Set<Node> frontier = new Set<Node>(p.getInitialState());
    Set<Node> explored = new Set<Node>();
    while (true) {
        if (frontier.isEmpty()) {
            return null;
        }
        Node node = frontier.selectOne();
        if (p.isGoalState(node.getState())) {
            return n.getSolution();
        }
        explored.add(node);
        for (Node n : node.expand()) {
            if (!explored.contains(n)) {
                frontier.add(n);
            }
        }
    }
}
```

THE  
OFFICIAL  
**METALLICA**  
ILLUSTRATED  
CHRONICLE

# SWHAT!

THE GOOD, THE MAD AND THE UGLY



EDITED BY STEPHEN CHIRAZI

# Summary

- Formal model of problems and solutions based on states and actions that transition between them
- General-purpose algorithm for solving any problem that can be represented using the model
- State-space search framework will allow us to explore and compare different problem-solving strategies

# State-Space Search

```
Solution graphSearch(Problem p) {  
    Set<Node> frontier = new Set<Node>(p.getInitialState());  
    Set<Node> explored = new Set<Node>();  
    while (true) {  
        if (frontier.isEmpty()) {  
            return null;  
        }  
        Node node = frontier.selectOne();  
        if (p.isGoalState(node.getState())) {  
            return n.getSolution();  
        }  
        explored.add(node);  
        for (Node n : node.expand()) {  
            if (!explored.contains(n)) {  
                frontier.add(n);  
            }  
        }  
    }  
}
```

For Next Time:  
Finish Chapter 3