CSC242: Homework 2.5
AIMA Chapter 9

1. One of the early successes of AI was using rule-based systems (a.k.a. production systems or "expert" systems) for clinical diagnosis. These systems did not do logical inference, strictly speaking, but they did apply rules to derive conclusions. Let's take a look at a small example based on the real-life system MYCIN (Wikipedia) that diagnosed bacterial infections.

   MYCIN used diagnostic rules like the following:

$$\forall x \; \mathit{Gram}(x, \mathit{Negative}) \land \mathit{Morph}(x, \mathit{Rod}) \land \mathit{Anaerobic}(x) \Rightarrow \mathit{Identity}(x, \mathit{Bacteroides}, 0.6) \tag{1}$$

   That is, if an organism is "gram-negative" (a staining test), has a "rod" morphology (shape), and is anerobic (does not use oxygen), then with 60% confidence it is *Bacteroides*.

   MYCIN used its own inference procedure to draw conclusions. We will do first-order inference, but with only the following inference rules:

$$\text{UI:} \quad \frac{\forall v \; \alpha}{\text{Subst}(\{v/g\}, \alpha)} \quad \text{for any variable } v \text{ and ground term } g$$

$$\text{AI:} \quad \frac{\alpha, \beta}{\alpha \land \beta}$$

$$\text{MP:} \quad \frac{\alpha \Rightarrow \beta, \alpha}{\beta}$$

   The expression Subst(Θ, α) denotes the result of applying subtitution Θ to sentence α, yielding a new sentence.

   Now suppose that you have the following information about an infectious organism denoted by the constant symbol $\mathit{ORG1}$:

$$\mathit{Gram}(\mathit{ORG1}, \mathit{Negative}) \tag{2}$$
$$\mathit{Morph}(\mathit{ORG1}, \mathit{Rod}) \tag{3}$$
$$\mathit{Anaerobic}(\mathit{ORG1}) \tag{4}$$

   Give a proof of the organism's identity using *only* the given inference rules. Be sure to show which rule is being used at each step and any substitutions.

ANSWER:

First use rule UI on (1) with substitution $\{x/\mathit{ORG1}\}$ to get:

$$\begin{aligned} &\mathit{Gram}(\mathit{ORG1}, \mathit{Negative}) \land \mathit{Morph}(\mathit{ORG1}, \mathit{Rod}) \land \mathit{Anaerobic}(\mathit{ORG1}) \Rightarrow \\ &\quad \mathit{Identity}(\mathit{ORG1}, \mathit{Bacteroides}, 0.6) \end{aligned} \tag{5}$$

So now we have a *propositionalized* version of the rule. That's good, because our other two inference rules are propositional (don't do anything special about quantifiers).

We don't have generalized *modus ponens*, just the basic *modus ponens* rule MP. So we need to first construct the antecedent of the rule using the AI rule (twice), then apply MP.

Using AI on (2) and (3):

$$\mathit{Gram}(\mathit{ORG1}, \mathit{Negative}) \land \mathit{Morph}(\mathit{ORG1}, \mathit{Rod}) \tag{6}$$

Using AI on (6) and (4):

$$\mathit{Gram}(\mathit{ORG1}, \mathit{Negative}) \land \mathit{Morph}(\mathit{ORG1}, \mathit{Rod}) \land \mathit{Anaerobic}(\mathit{ORG1}) \tag{7}$$

Finally using MP on (5) and (7) yields:

$$\mathit{Identity}(\mathit{ORG1}, \mathit{Bacteroides}, 0.6)$$

It's straightforward, but you must be able to understand and strictly apply the inference rules that you're given. That's what a computer would have to do, after all.

2. Suppose we introduce a new quantifier, $\Box$, and the following inference rule:

$$\text{BX:} \quad \frac{\Box v\ \alpha}{\text{Subst}(\{v/n\}, \alpha)} \quad \text{for any variable } v \text{ and natural number } n$$

Now suppose we have the following knowledge base:

$$\Box v\ Num(v) \tag{1}$$
$$\forall n, m\ Num(n) \wedge Num(m) \Rightarrow Num(n+m) \tag{2}$$

Use the BX inference rule with the inference rules from the previous question to prove that $Num(3+4)$.

ANSWER: Again, the key is to use the inference rules very precisely, like a machine would have to.

First use UI on (2) with substitution $\{n/3\}$ to get

$$\forall m\ Num(3) \wedge Num(m) \Rightarrow Num(3+m) \tag{3}$$

Then use UI on (5) with substitution $\{m/4\}$ to get

$$Num(3) \wedge Num(4) \Rightarrow Num(3+4) \tag{4}$$

Note that you can't do it in one step with the inference rule UI since it only allows you to substitute one variable at a time.

Now use BX twice on (1), once with substitution $\{n/3\}$ and once with $\{n/4\}$ to produce

$$Num(3) \tag{5}$$
$$Num(4) \tag{6}$$

As before, we need to use AI on (5) and (6) to produce the antecedent:

$$Num(3) \wedge Num(4) \tag{7}$$

And then finally we can use MP on (4) and (7) to get

$$Num(3+4)$$

3. Do you think the following first-order knowledge base is consistent?

$$\exists x\; P(x) \tag{1}$$
$$\forall x\; \neg P(x) \tag{2}$$

Use the UI inference rule from before and the following inference rule to prove that it's not, by deriving a contradiction.

EI: $\dfrac{\exists v\; \alpha}{\text{Subst}(\{v/k\}, \alpha)}$ where k is a new constant symbol not already occurring in the knowledge base

ANSWER: Using EI on (1) with substitution $\{x/X\}$ where X is a new constant, we get $P(X)$. This inference rule is related to Skolemization (AIMA p. 323).

Using UI on (2) with substitution $\{x/X\}$, we get $\neg P(X)$. This substitution is possible after X is introduced in the first step.

Clearly $P(X) \wedge \neg P(X)$ is a contradiction.

4. For each pair of atomic sentences, give the most general unifier if one exists:

   (a) $P(A, B, B)$ and $P(x, y, z)$
   ANSWER: $\{x/A, y/B, z/B\}$ (or some permutation of this).

   (b) $Q(y, g(A, B))$ and $Q(g(x, x), y)$
   ANSWER: No unifier (x cannot bind to both A and B).

   (c) $Older(Father(y), y)$ and $Older(Father(x), John)$
   ANSWER: $\{y/John, x/John\}$.

   (d) $Knows(Father(y), y)$ and $Knows(x, x)$
   ANSWER: No unifier (because the occurs-check prevents unification of y with $Father(y)$).

5. From “Horses are animals,” it follows that “The head of a horse is the head of an animal.” How would you demonstrate that this inference is valid? Think about it. . . then read my suggestion for how to proceed:

   (a) Translate both the premise and the conclusion into first-order logic using the predicates $Horse(x)$ (“x is a horse”), $Animal(x)$ (“x is an animal”), and $HeadOf(h, x)$ (“h is the head of x”).
   (b) Negate the conclusion, and convert the premise and the negated conclusion into conjunctive normal form.
   (c) Use resolution in the appropriate way to show that the conclusion logically follows from the premise.

ANSWER:

   (a) Write down the premise and conclusion in first-order logic.
   The premise is clearly a universally-quantified rule:

$$\forall x\ Horse(x) \Rightarrow Animal(x)$$

   The conclusion says that if something (anything) is the head of a (any) horse, then it is also the head of some animal. That is, there is some animal of which it is the head, right? So I would write this as:

$$\forall h, x\ Horse(x) \wedge HeadOf(h, x) \Rightarrow \exists y\ Animal(y) \wedge HeadOf(h, y)$$

   Of course there are infinitely many logically equivalent sentences one could write, but you want one that demonstrates your understanding clearly.

   (b) Convert to CNF. The premise turns into a single clause:

$$\neg Horse(x) \vee Animal(x) \tag{1}$$

   I'll let you do the steps.
   The negation of conclusion is:

$$\neg\forall h \forall x\ [Horse(x) \wedge HeadOf(h, x) \Rightarrow \exists y\ Animal(y) \wedge HeadOf(h, y)]$$

   Conversion to CNF with all the steps:

   i. Eliminate biconditionals and implications:

$$\neg\forall h \forall x\ \neg[Horse(x) \wedge HeadOf(h, x)] \vee [\exists y\ Animal(y) \wedge HeadOf(h, y)]]$$

   ii. Move negations in:

$$\exists h \neg\forall x\ \neg[Horse(x) \wedge HeadOf(h, x)] \vee [\exists y\ Animal(y) \wedge HeadOf(h, y)]]$$
$$\exists h \exists x\ \neg[\neg[Horse(x) \wedge HeadOf(h, x)] \vee [\exists y\ Animal(y) \wedge HeadOf(h, y)]]]$$
$$\exists h \exists x\ \neg\neg[Horse(x) \wedge HeadOf(h, x)] \wedge \neg[\exists y\ Animal(y) \wedge HeadOf(h, y)]]$$
$$\exists h \exists x\ [Horse(x) \wedge HeadOf(h, x)] \wedge [\forall y\ \neg[Animal(y) \wedge HeadOf(h, y)]]$$
$$\exists h \exists x\ [Horse(x) \wedge HeadOf(h, x)] \wedge [\forall y\ \neg Animal(y) \vee \neg HeadOf(h, y)]$$

iii. Standardize variables apart: nothing to do

iv. Move quantifiers out:

$$\exists h \exists x \forall y\ [Horse(x) \wedge HeadOf(h, x)] \wedge [\neg Animal(y) \vee \neg HeadOf(h, y)]$$

v. Skolemize: Introduce H for h and X for x:

$$\forall y\ [Horse(X) \wedge HeadOf(H, X)] \wedge [\neg Animal(y) \vee \neg HeadOf(H, y)]$$

vi. Drop universal quantifiers:

$$[Horse(X) \wedge HeadOf(H, X)] \wedge [\neg Animal(y) \vee \neg HeadOf(H, y)]$$

vii. Distribute OR over AND: nothing to do

viii. Break into clauses:

$$Horse(X) \tag{2}$$

$$HeadOf(H, X) \tag{3}$$

$$\neg Animal(y) \vee \neg HeadOf(H, y) \tag{4}$$

(c) You must use a resolution refutation to show this. That is, from the premises and the negation of the conclusion, derive a contradiction (the empty clause). Here's one possible such refutation that is short and sweet:

Resolve (3) and (4) with substitution y/X to get

$$\neg Animal(X) \tag{5}$$

Resolve (5) with (1) with substitution x/X to get

$$\neg Horse(X) \tag{6}$$

Resolve (6) with (2) to get the empty clause.

6. Suppose a knowledge base contains just the following first-order Horn clauses:

$$Ancestor(Mother(x), x)$$
$$Ancestor(x, y) \land Ancestor(y, z) \Rightarrow Ancestor(x, z)$$

Consider a forward-chaining algorithm that, on the jth iteration, terminates if the KB contains a sentence that unifies with the query, and otherwise adds to the KB every atomic sentence that can be inferred from the sentences already in the KB after iteration $j - 1$.

(a) For each of the following queries, say whether the algorithm will (1) give an answer (if so, give that answer); or (2) terminate with no answer; or (3) not terminate.

i. $Ancestor(Mother(y), John)$
ANSWER: Yes, $\{y/John\}$ (immediate).

ii. $Ancestor(Mother(Mother(y)), John)$
ANSWER: Yes, $\{y/John\}$ (second iteration).

iii. $Ancestor(Mother(Mother(Mother(y))), y)$
ANSWER: Yes, $\{\}$ (second iteration).

iv. $Ancestor(Mother(John), Mother(Mother(John)))$
ANSWER: Does not terminate.

(b) Can a resolution algorithm prove the sentence $\neg Ancestor(John, John)$?

ANSWER: Although resolution is complete, it cannot prove this because it does not follow. Nothing in the axioms rules out the possibility of everything being the ancestor of everything else.