

5

Histogram Modeling

5.1 Introduction :

MU - May 2012

Histogram of images provide a global description of the appearance of an image. The information obtained from histograms is enormous and hence histogram modelling; though a spatial domain technique is introduced as a separate chapter.

By definition, histogram of an image represents the relative frequency of occurrence of the various grey levels in an image. Histogram of an image can be plotted in two ways.

In the first method, the x-axis has the grey levels and the y-axis has the number of pixels in each grey level, while in the second method, the x-axis represents the grey levels, while the y-axis represents the probability of the occurrence of that grey level.

Method 1 :

| Grey level | Number of pixels (n_k) |
|------------|----------------------------|
| 0 | 40 |
| 1 | 20 |
| 2 | 10 |
| 3 | 15 |
| 4 | 10 |
| 5 | 3 |
| 6 | 2 |

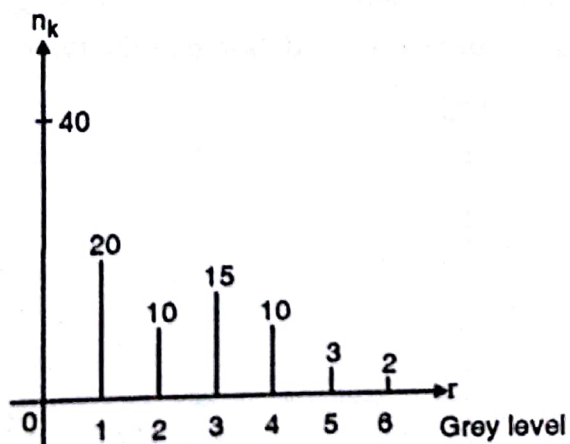


Fig. 5.1.1

Method 2 :

In the second method, instead of plotting the number of pixels directly, we plot their probability of occurrence i.e.,

$$p(r_k) = \frac{n_k}{n}$$

$r_k \rightarrow k^{\text{th}}$ grey level

$n_k \rightarrow$ Number of pixels in the k^{th} grey level

$n \rightarrow$ Total number of pixels in an image

| Grey level | Number of pixels (n_k) | $p(r_k) = \frac{n_k}{n}$ |
|------------|----------------------------|--------------------------|
| 0 | 40 | 0.4 |
| 1 | 20 | 0.2 |
| 2 | 10 | 0.1 |
| 3 | 15 | 0.15 |
| 4 | 10 | 0.1 |
| 5 | 3 | 0.03 |
| 6 | 2 | 0.02 |
| $n = 100$ | | |

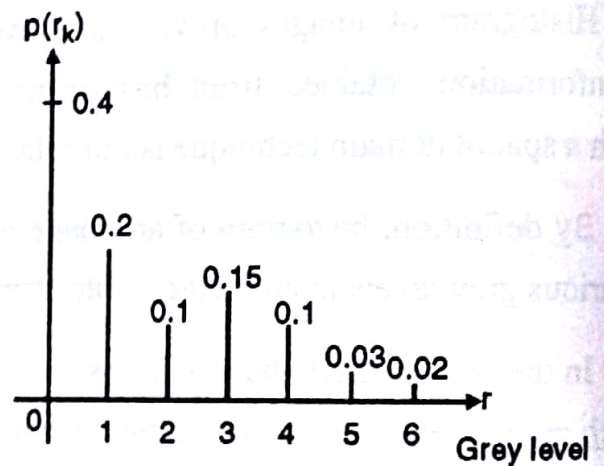
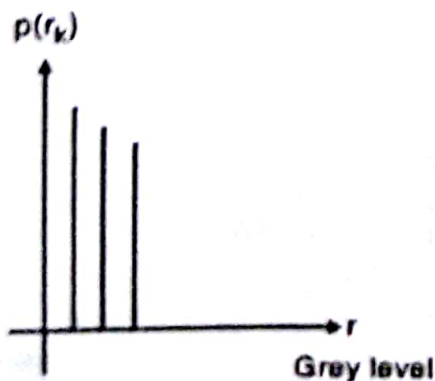
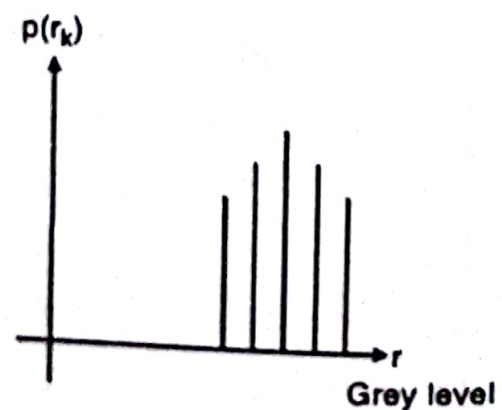


Fig. 5.1.2

This is known as a normalised histogram. The advantage of the second method is that the maximum value to be plotted will always be 1. Generally black is considered as grey level 0 and white as the maximum. Just by looking at the histogram of the image, a great deal of information can be obtained. Some of the typical histograms are shown in Fig. 5.1.3.



(a)



(b)

Fig. 5.1.3

Ex. 5.3.3 : Equalize the given histogram

| Grey level | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|------------------|-----|------|-----|-----|-----|-----|-----|----|
| Number of pixels | 790 | 1023 | 850 | 656 | 329 | 245 | 122 | 81 |

Soln. :

$L = 8$ (Number of grey levels)

We first plot the original histogram

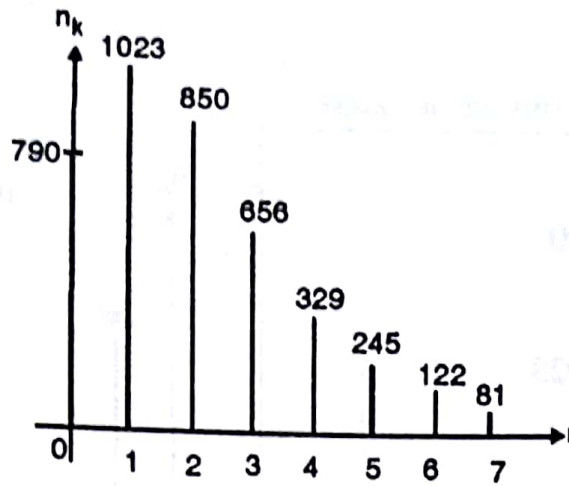


Fig. P. 5.3.3 : Original dark histogram

| Grey level | n_k | PDF $p_r(r_k) = \frac{n_k}{n}$ | CDF $s_k = \sum p_r(r_k)$ | $(L-1) \times s_k$ $7 \times s_k$ | Rounding off |
|------------|------------|-----------------------------------|------------------------------|--------------------------------------|-----------------|
| 0 | 790 | 0.19 | 0.19 | 1.33 | 1 |
| 1 | 1023 | 0.25 | 0.44 | 3.08 | 3 |
| 2 | 850 | 0.21 | 0.65 | 4.55 | 5 |
| 3 | 656 | 0.16 | 0.81 | 5.67 | 6 |
| 4 | 329 | 0.08 | 0.89 | 6.23 | 6 |
| 5 | 245 | 0.06 | 0.95 | 6.65 | 7 |
| 6 | 122 | 0.03 | 0.98 | 6.86 | 7 |
| 7 | 81 | 0.02 | 1 | 7 | 7 |
| | $N = 4096$ | | | | |

We take 1st, 2nd and the last column

| Old grey level | Equalized grey level | New grey level |
|----------------|----------------------|----------------|
| 0 | 790 | → 1 |
| 1 | 1023 | → 3 |
| 2 | 850 | → 5 |
| 3 | 656 | → 6 |
| 4 | 329 | → 6 |
| 5 | 245 | → 7 |
| 6 | 122 | → 7 |
| 7 | 81 | → 7 |

We notice that the new grey levels have pixels only at 1, 3, 5, 6, 7. There are no pixels in grey levels 0, 2 and 4.

| Equalized grey level | Number of pixels |
|----------------------|------------------------|
| 0 | 0 |
| 1 | 790 |
| 2 | 0 |
| 3 | 1023 |
| 4 | 0 |
| 5 | 850 |
| 6 | $656 + 329 = 985$ |
| 7 | $245 + 122 + 81 = 448$ |

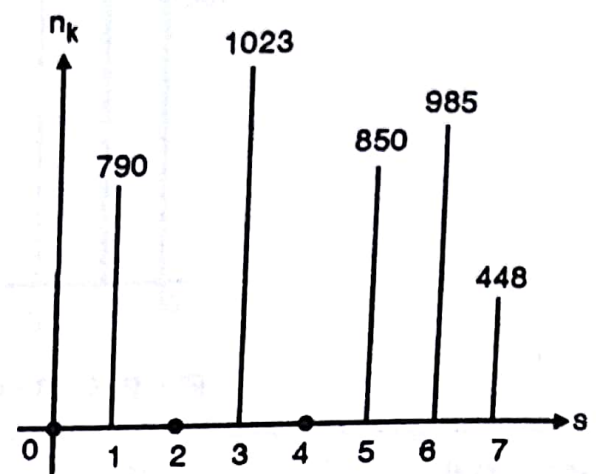


Fig. P. 5.3.3(a) : Equalized histogram

So here we are. A dark histogram becomes an evenly spaced histogram. But wait a minute, we had started out to get a flat histogram as in the continuous case but the histogram obtained is not flat; why?

Discrete domain is an approximation of the continuous domain i.e., values between 0 and 1, 1 and 2 and so on are not known. Due to this reason, perfectly flat results are never obtained.