COLUMBIA UNIVERSITY

DEPARTMENT OF BIOSTATISTICS

P8109 – STATISTICAL INFERENCE

MIDTERM EXAMINATIONS - SPRING 2021

Question 1

Consider the i.i.d. r.v.'s $X_1,...,X_n$, where $X_i \sim \text{Bernoulli}(p)$. Let the sample mean of $X_1,...,X_n$ be $\bar{X}_n$.

(a) Interpret the meaning of $\bar{X}_n$.

(b) Use the delta method to show that

$$\bar{X}_n\left(1-\bar{X}_n\right) \dot{\sim} N\left[p(1-p), \frac{(1-2p)^2\, p(1-p)}{n}\right]$$

[1+4=5 marks]

Question 2

Let $X_1,...,X_n$ be a random sample from a $N(0,\ \sigma^2)$ population and consider the parameter $\theta = \sigma^r$, where *r* is a positive integer.

(a) Develop an explicit expression for the MLE of the parameter θ.

(b) Obtain the CRLB for the variance of any unbiased estimator of the parameter θ

(c) Find the particular value of *r* for which the CRLB is actually achieved in (b) above.

[5+5+5=15 marks]

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