COLUMBIA UNIVERSITY

DEPARTMENT OF BIOSTATISTICS

P8109 – STATISTICAL INFERENCE

MIDTERM EXAMINATIONS - SPRING 2020

INSTRUCTIONS:

- **Time allowed: 60 minutes**
- **Answer ALL questions.**

Question 1

Let *X* and *Y* be independent r.v.'s, each with a normal distribution, such that $\mathrm{E}X = \theta$ $\mathrm{E}Y = 3\theta$, var *X* = var *Y* = 1.

(a) Find the MLE of θ.

(b) If an estimator of the form $aX + bY$ is to be used to estimate θ, what restrictions must the constants *a* and *b* satisfy in order that the estimator be unbiased?

[8+4 =12 marks]

Question 2

A sample $Y_1, ..., Y_n$ is drawn from a gamma distribution with parameters α_0 (known) and β (unknown):

$$f_{Y_i}(y) = \frac{y^{\alpha_0 - 1} e^{-y/\beta},}{\beta^{\alpha_0}\Gamma(\alpha_0)} \qquad y > 0$$

Obtain a sufficient statistic for β

[8 marks]
