# Bachelor’s Thesis

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Dr. Dale Miller, LIX

# Top-Down and Bottom-Up Approaches to Parsing

# Motivation

x * 7 + 61 * 7

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The grammar:

Expr -> Lb Expr Rb | Num | Expr Sym Expr

Num -> Minus Dig Dig* | Dig Dig* | x

Sym -> + | - | / | *

Minus -> -

Rb -> )

Lb -> (

Dig -> 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

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(x*7) + (61*7) is 3 operations

However

7*(61+x) is just 2

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(x*7) + (61*7) is 3 operations

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7*(61+x) is just 2

Lastly say x = 9 and we will have the answer as 490

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# A Good Parser

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1. Sound

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# Parse Trees

Consider: 4+23-(7*8)

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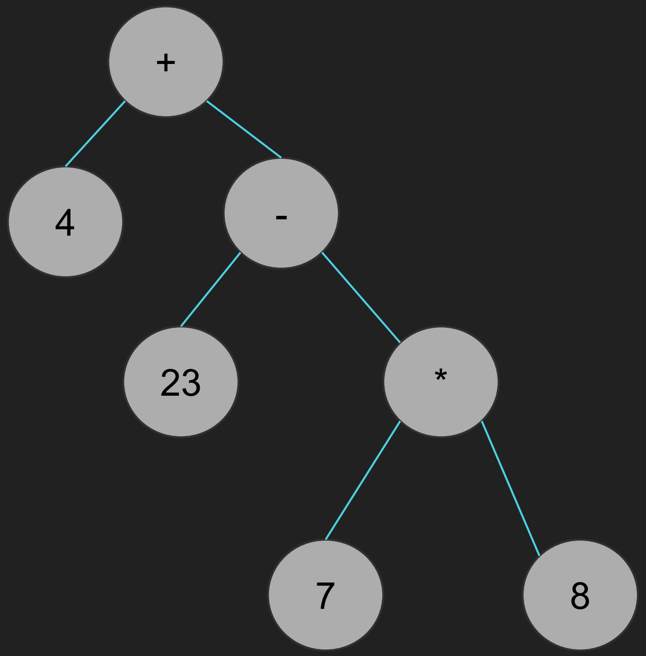

# Parse Trees

- Prefer deterministic grammars

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- Same parse tree

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- Prefer deterministic grammars
- Same parse tree
- Super important and precise - hence automated

# A Good Parser

1. Sound
2. Complete
3. Guaranteed to terminate
4. Fast

# Our POV

Start with (●S, N)

Length left to parse

What is left to parse

Current position

# Our POV

Start with (•S, N)

Want (•e, 0)

# Our POV

Start with (●S, N)

Want (●e, 0)

Rule A -> BC

Then

$$\frac{(B, i, k), (C, k, j)}{(A,i,j)}$$

# Our POV

Start with (●S, N)

Want (●e, 0)

Rule A -> a

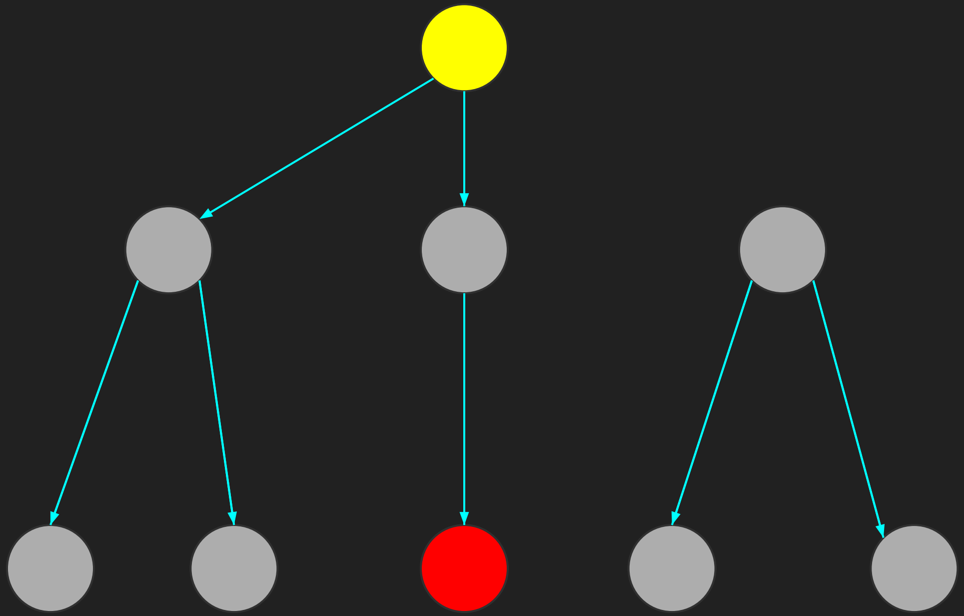
Then

ABCD… (●abcd…, N)

—-------------------------------------

BCD… (a●bcd , N-1)

# Top-Down



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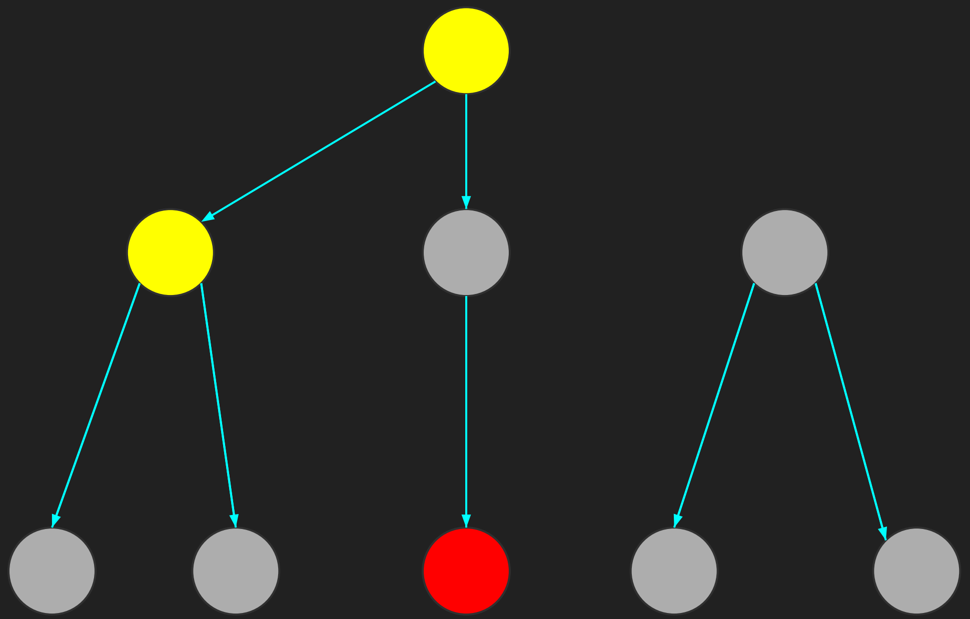

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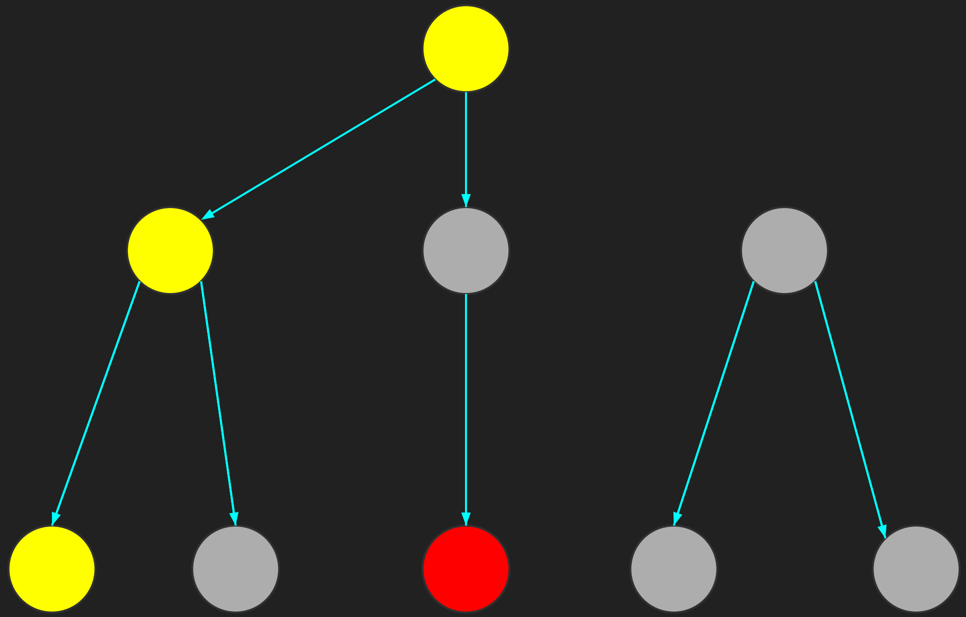

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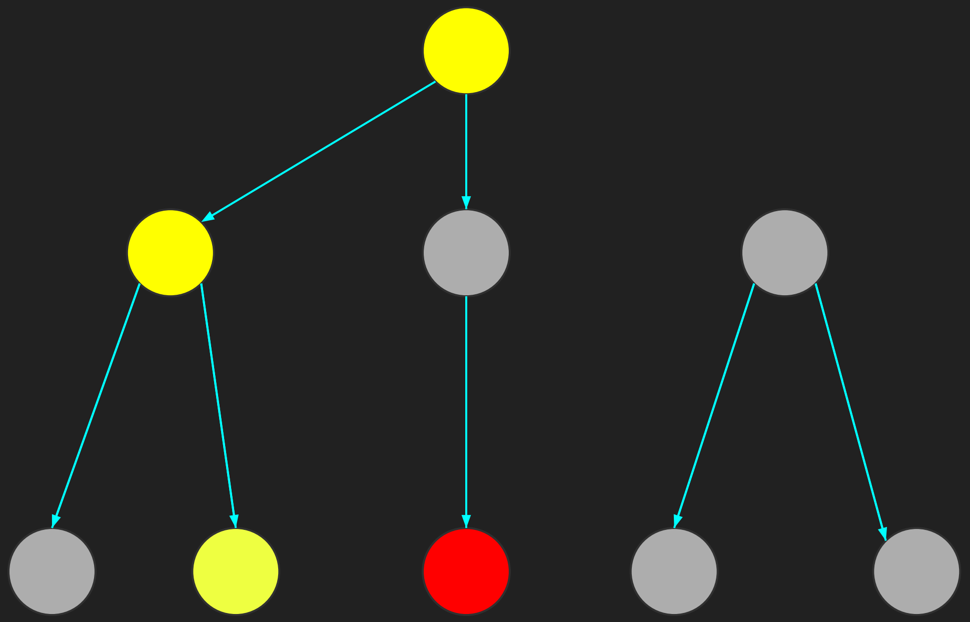

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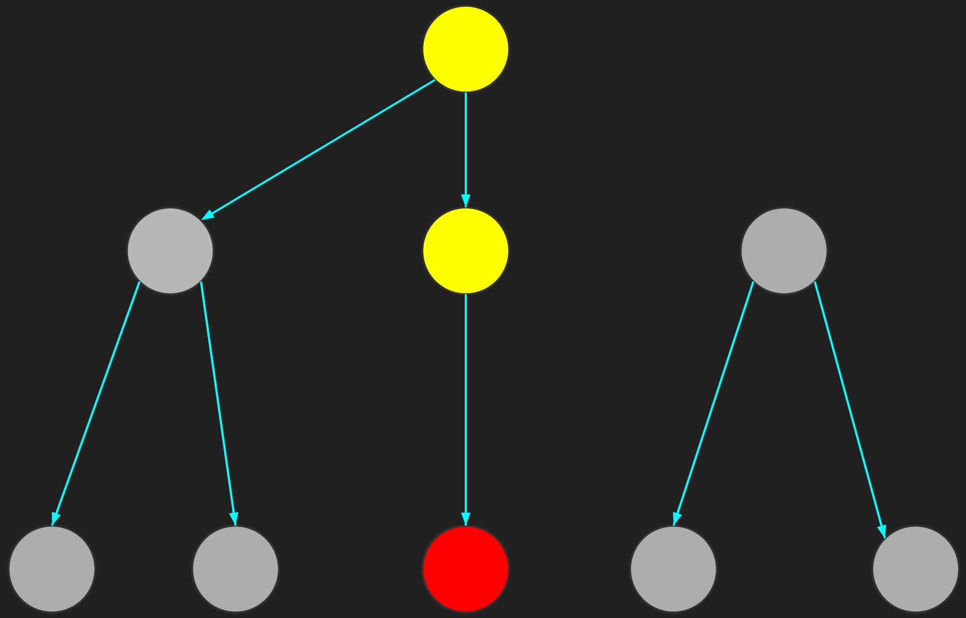

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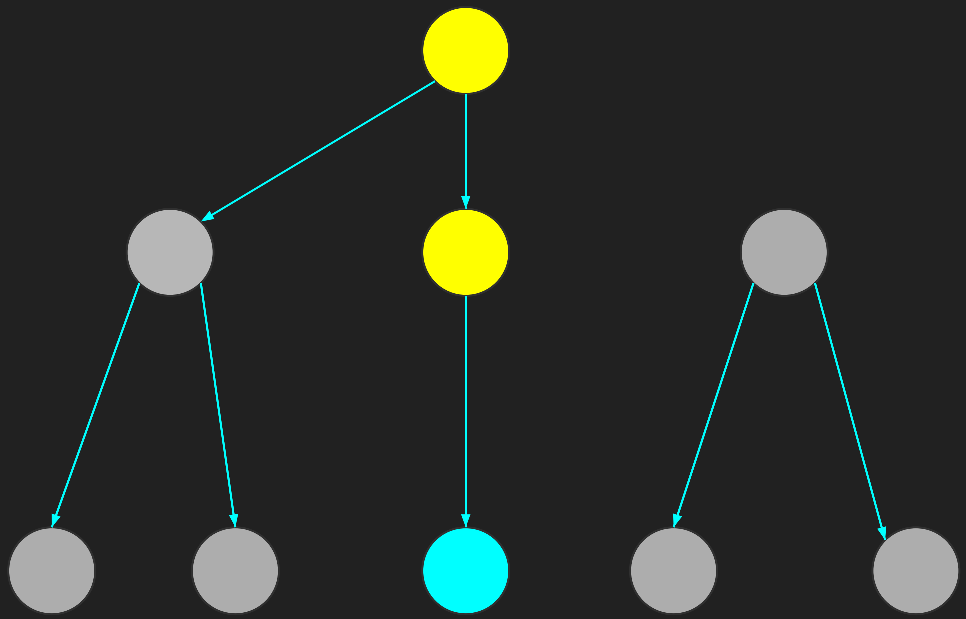
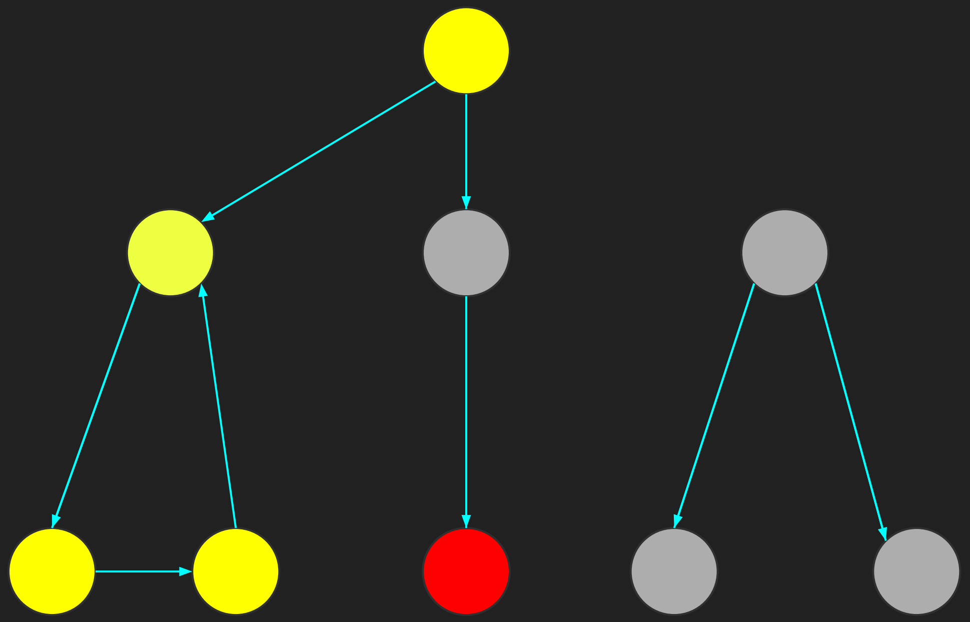


# Top-Down - but what if?



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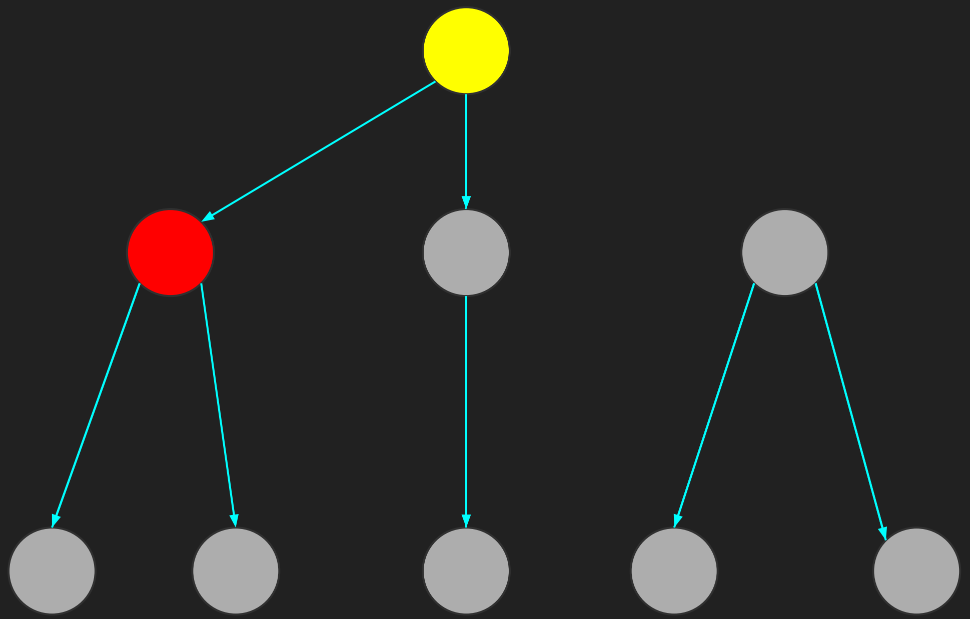

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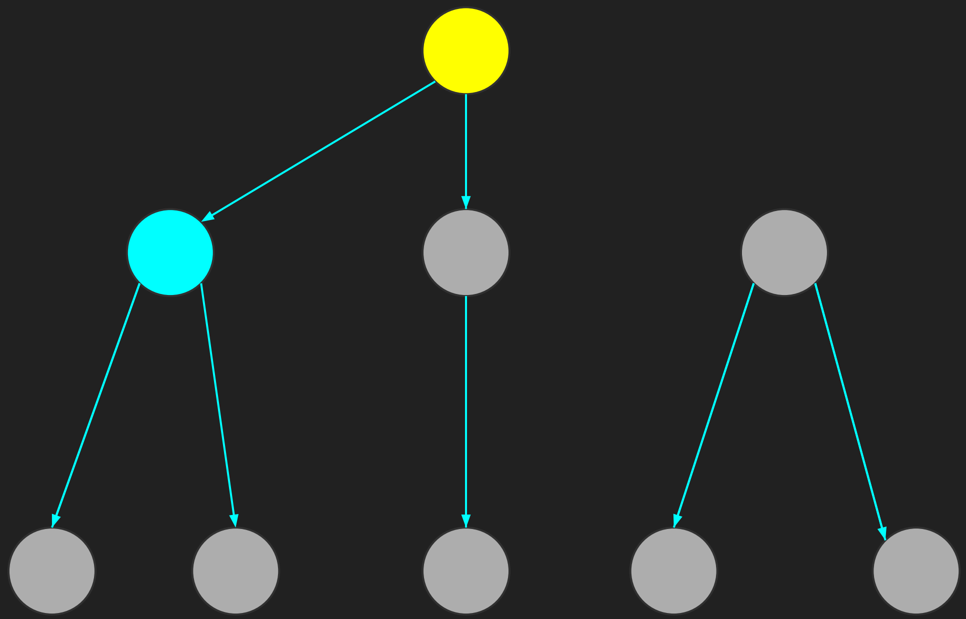

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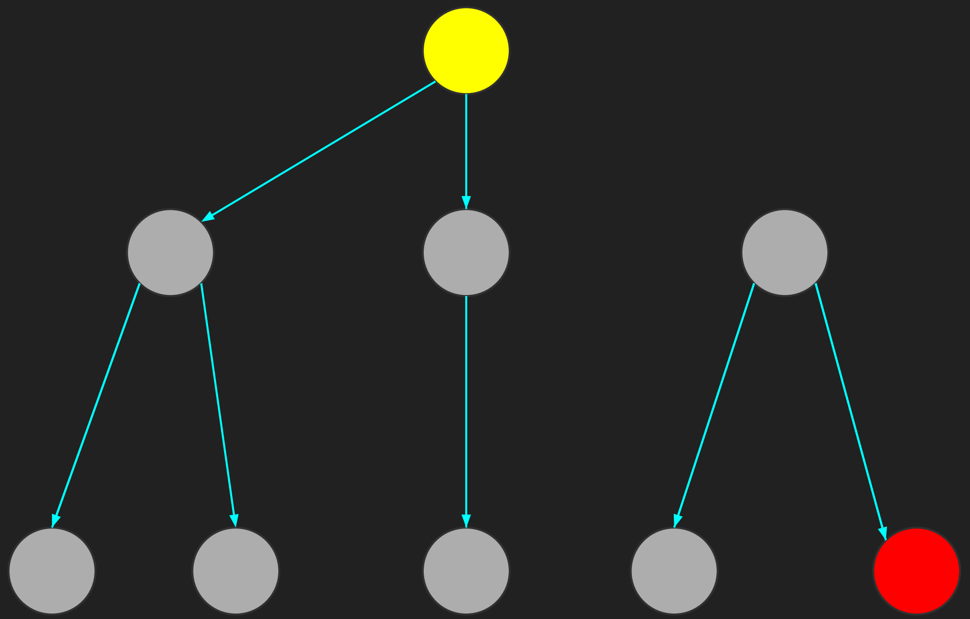
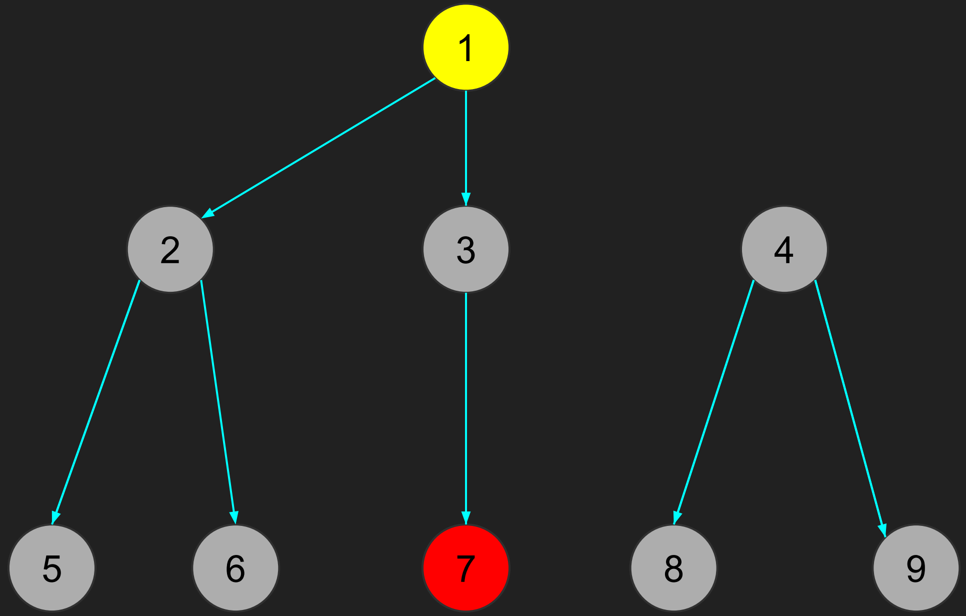


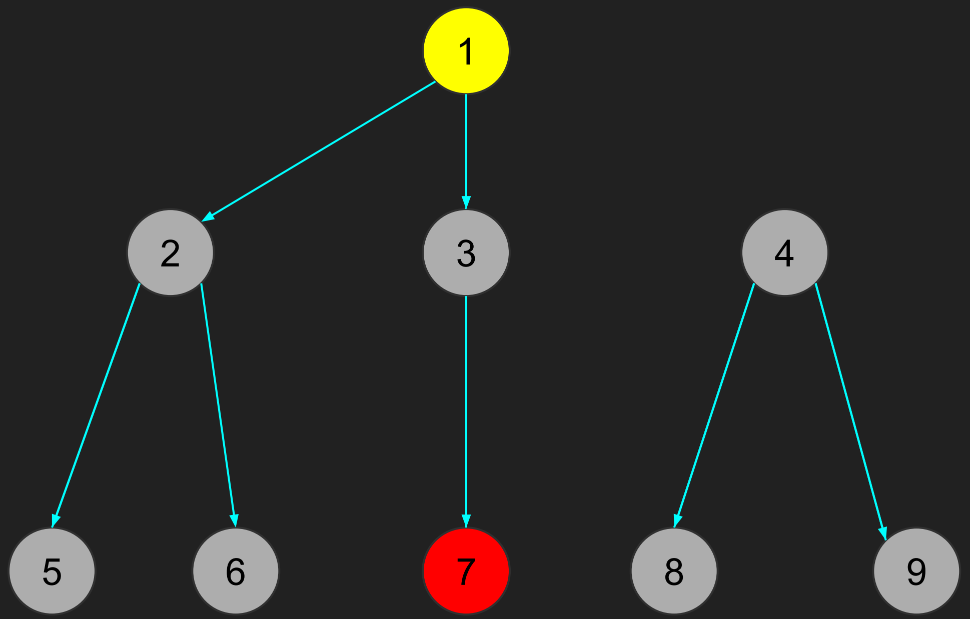
Worst case is O(E) where E is number of edges

# Bottom-Up



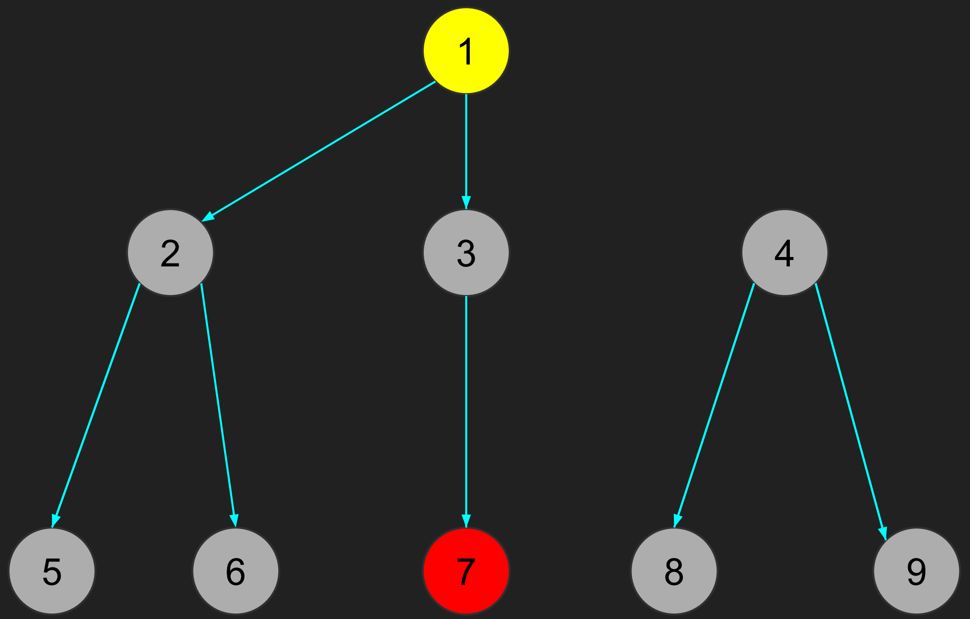

[]

# Bottom-Up




[5, 6, 7, 8, 9]

# Bottom-Up




[5, 6, 7, (2,5), (2,6), (3,7), 8, 9, (4,8), (4,9)]

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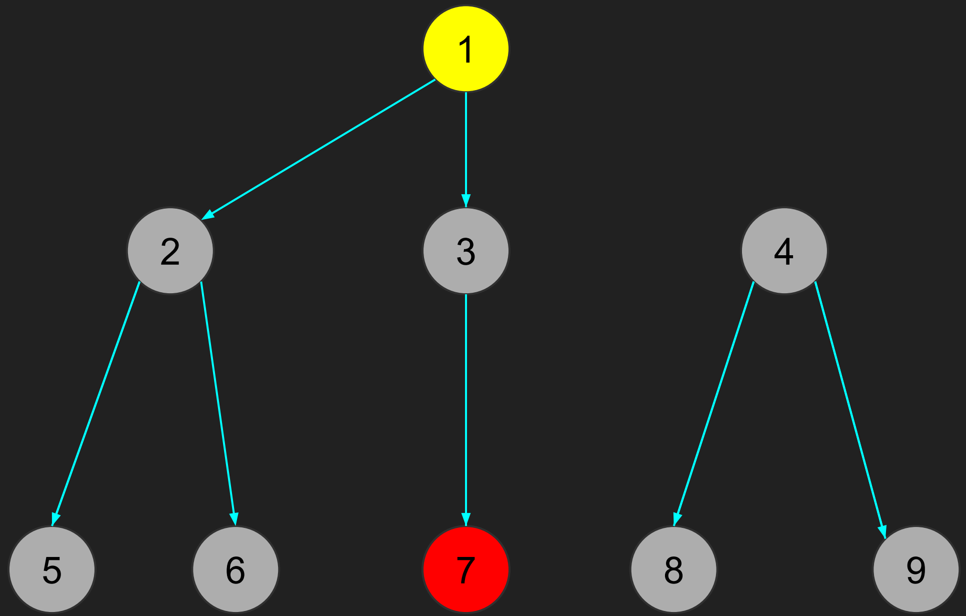


[5, 6, 7, (2,5), (2,6), (3,7), (1,2) (1,3), 8, 9, (4,8), (4,9)]

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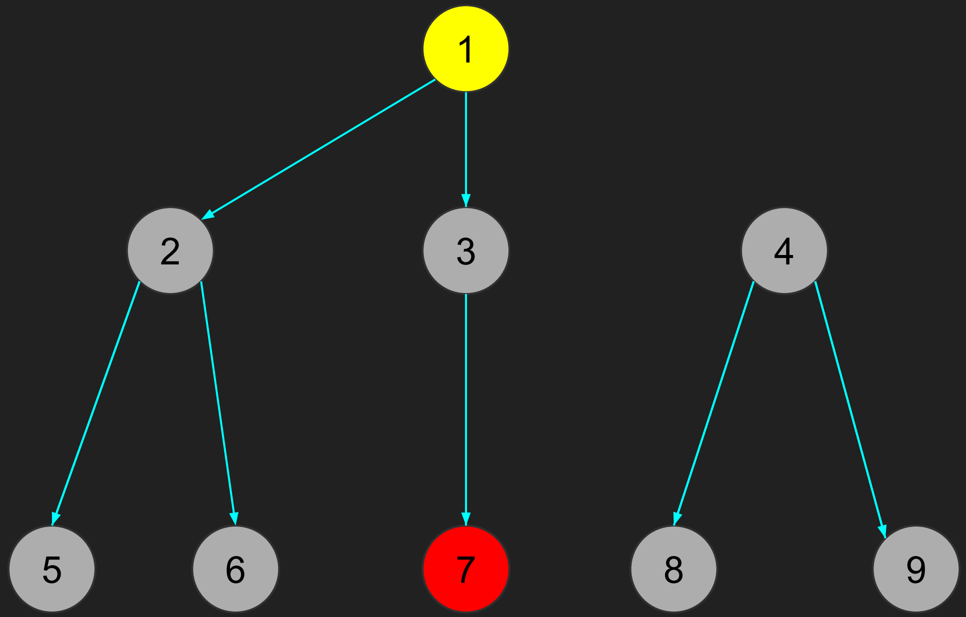


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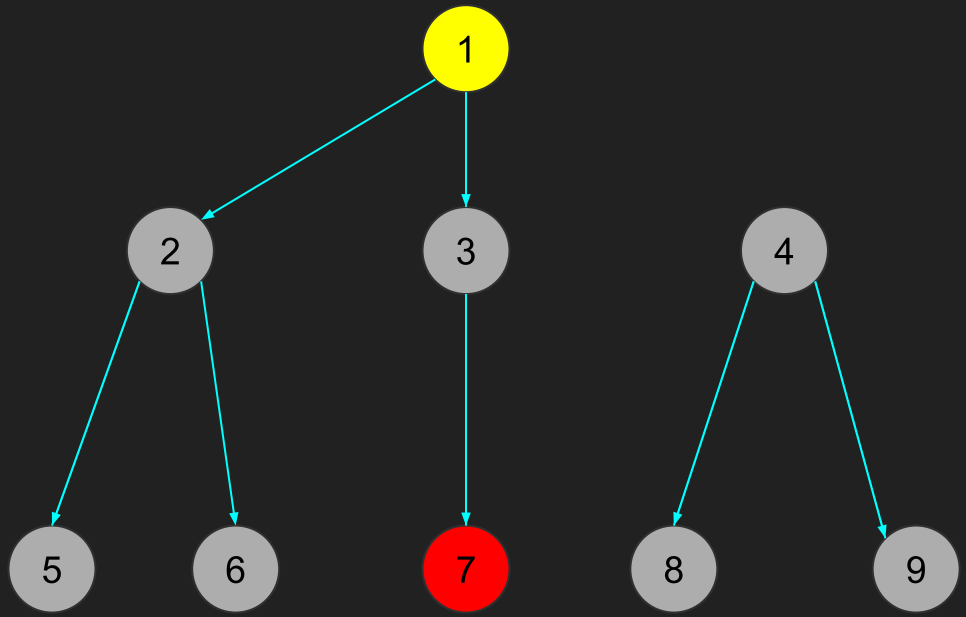
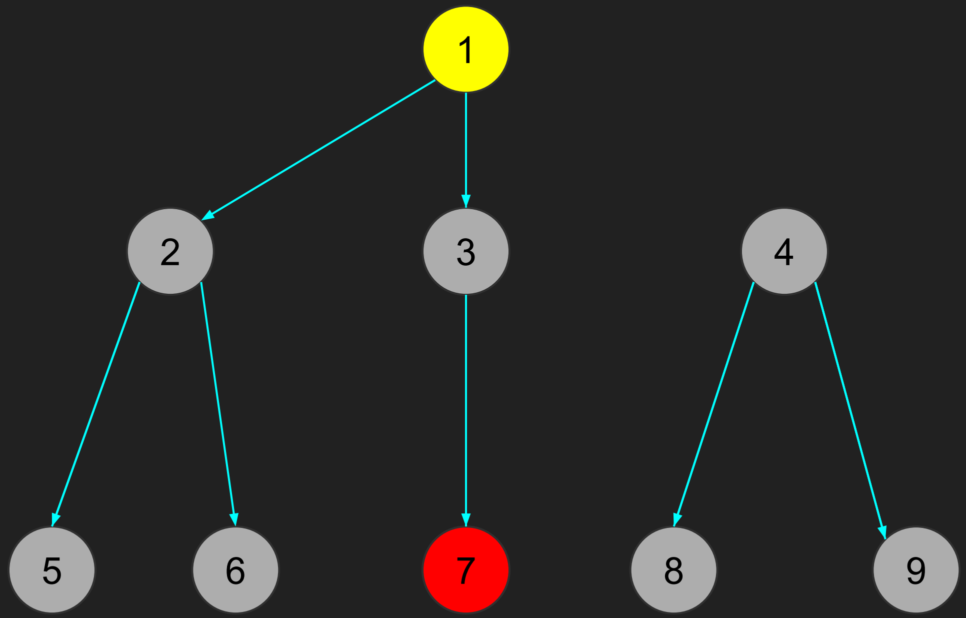



[5, 6, 7, (2,5), (2,6), (3,7), (1,2) (1,3), (1,5), (1,6), (1,7), 8, 9, (4,8), (4,9)]

Now Query (1,7) in the list

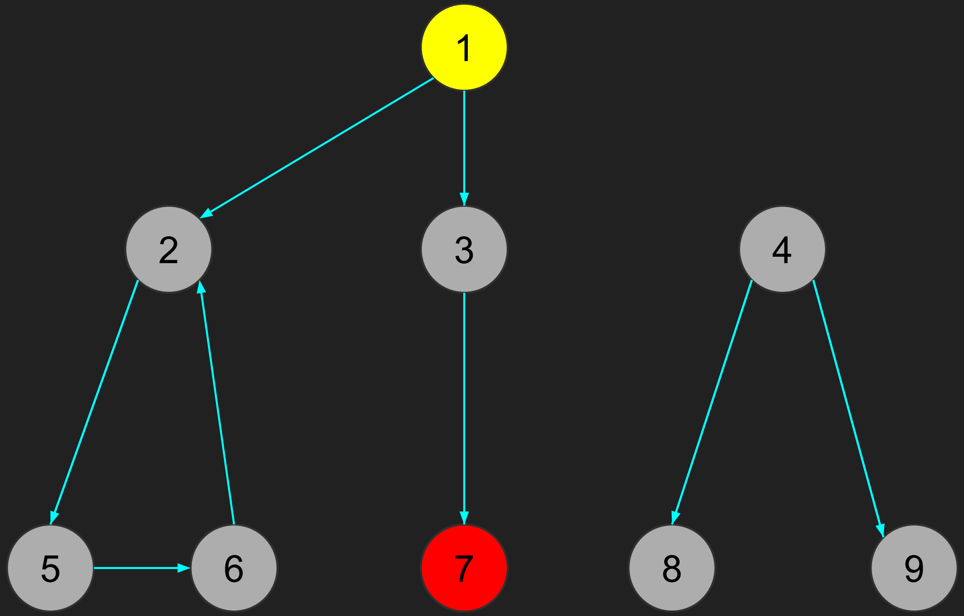
# Bottom-Up




[5, 6, 7, (2,5), (2,6), (3,7), (1,2) (1,3), (1,5), (1,6), (1,7), 8, 9, (4,8), (4,9)]

Now Query (1,7) in the list - O(E) once but then quick

# Bottom-Up - But what if?




[6, 7, (5,6), (2,6), (3,7), (1,2) (1,3), (1,5), (1,6), (1,7), 8, 9, (4,8), (4,9)]

No Difference

# Top-Down: Stack-Based parsing

S -> (SRS | (SR | (RS | (R

R -> )

( ( ) )

STACK - []

# Top-Down: Stack-Based parsing

S -> (SRS | (SR | (RS | (R

R -> )

( ( ) )

▲

STACK - [S, R, S]

# Top-Down: Stack-Based parsing

S -> (SRS | (SR | (RS | (R

R -> )

( ( ) )

▲

STACK - [S, R, S, R, S] and fail

Eventually…

# Top-Down: Stack-Based parsing

S -> (SRS | (SR | (RS | (R

R -> )

( ( ) )

STACK - [S, R]

# Top-Down: Stack-Based parsing

S -> (SRS | (SR | (RS | (R

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STACK - [R, R]

# Top-Down: Stack-Based parsing

Key Observations (after fixing a grammar)

- Grammar in Greibach Normal Form for convenience

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- Complexity of $O(R^n)$ with n size of input and R the number of rules

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Key Observations  (after fixing a grammar)

- Grammar in Greibach Normal Form for convenience
- Complexity of O(R^n) with n size of input and R the number of rules
- Cannot have useful error messages  (but this can be helped with look ahead features)

# Idea of Look-Ahead - Recall

S -> (SRS | (SR | (RS | (R

R -> )

( ( ) )

STACK - [S, R, S]

# Idea of Look-Ahead

S -> (SRS | (SR | (RS | (R

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STACK - [S, R, S]

We look one step ahead in our set of rules and see if S can parse ( and ), S can parse )).

# Bottom-Up: CKY Algorithm

S -> AB | c

A -> AB | a

B -> BA | b

“ a b a b a ”

0 1 2 3 4 5

| | | | | |
|---|---|---|---|---|
| A | | | | |
| | | | | |
| | | | | |
| | | | | |
| | | | | |

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|---|---|---|---|---|
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| | B | | | |
| | | A | | |
| | | | B | |
| | | | | A |

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|---|---|---|---|---|
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| | B | B | | |
| | | A | S, A | |
| | | | B | B |
| | | | | A |

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0 1 2 3 4 5

| | | | | |
|---|---|---|---|---|
| A | S, A | S, A, – | | |
| | B | B | | |
| | | A | S, A | |
| | | | B | B |
| | | | | A |

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0 1 2 3 4 5

| | | | | |
|---|---|---|---|---|
| A | S, A | S, A | S, A | S, A |
| | B | B | B | B |
| | | A | S, A | S, A |
| | | | B | B |
| | | | | A |

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- O(n^3) for CNF

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- O(n^3) for CNF
- In general, it is O(n^(k+1)) where k is the maximum number of non terminals in a rule
- This implementation is not natural to prolog

# CKY Algorithm - Our implementation

S -> AB | c

A -> AB | a

B -> BA | b

“ a b a b a ”

0 1 2 3 4 5

```
[]

Infer ((S, X, Y), L):-
        member ((A, X, Z), L),
        member ((B, Z, Y), L).
infer((S, X, X+1), L):-
        member((‘c’, 0, 1), L)
```

# CKY Algorithm - Our implementation

S -> AB | c

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" a b a b a "

0 1 2 3 4 5

```
[(A, 0, 1)]
Infer ((S, X, Y), L):-
        member ((A, X, Z), L),
        member ((B, Z, Y), L).
infer((S, X, X+1), L):-
        member(('c', 0, 1), L)
```

# CKY Algorithm - Our implementation

S -> AB | c

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“ a b a b a ”

0 1 2 3 4 5

[(A, 0, 1), (B, 1, 2), (A, 2, 3), (B, 3, 4), (A, 4, 5)]

```
Infer ((S, X, Y), L):-
        member ((A, X, Z), L),
        member ((B, Z, Y), L).
infer((S, X, X+1), L):-
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# CKY Algorithm - Our implementation

S -> AB | c

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0 1 2 3 4 5

```
[(A, 0, 1), (B, 1, 2), (A, 2, 3), (B, 3, 4), (A, 4, 5), (S, 0, 2), (A, 0, 2)...]
Infer ((S, X, Y), L):-
        member ((A, X, Z), L),
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infer((S, X, X+1), L):-
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```

# CKY Algorithm - Our implementation

- Idea is same as before!

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- By changing the nature of inference, we have made the algorithm more free

# Outlook

- Combining Bottom-Up and Top-Down - Earley’s Algorithm

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- runs in $O(n^3)$ for all

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- runs in O(n^3) for all

- No obvious and simple implementation in Prolog (to me at least)

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- Combining Bottom-Up and Top-Down - Earley's Algorithm

- A better Parsing Algorithm and connections to Matrices

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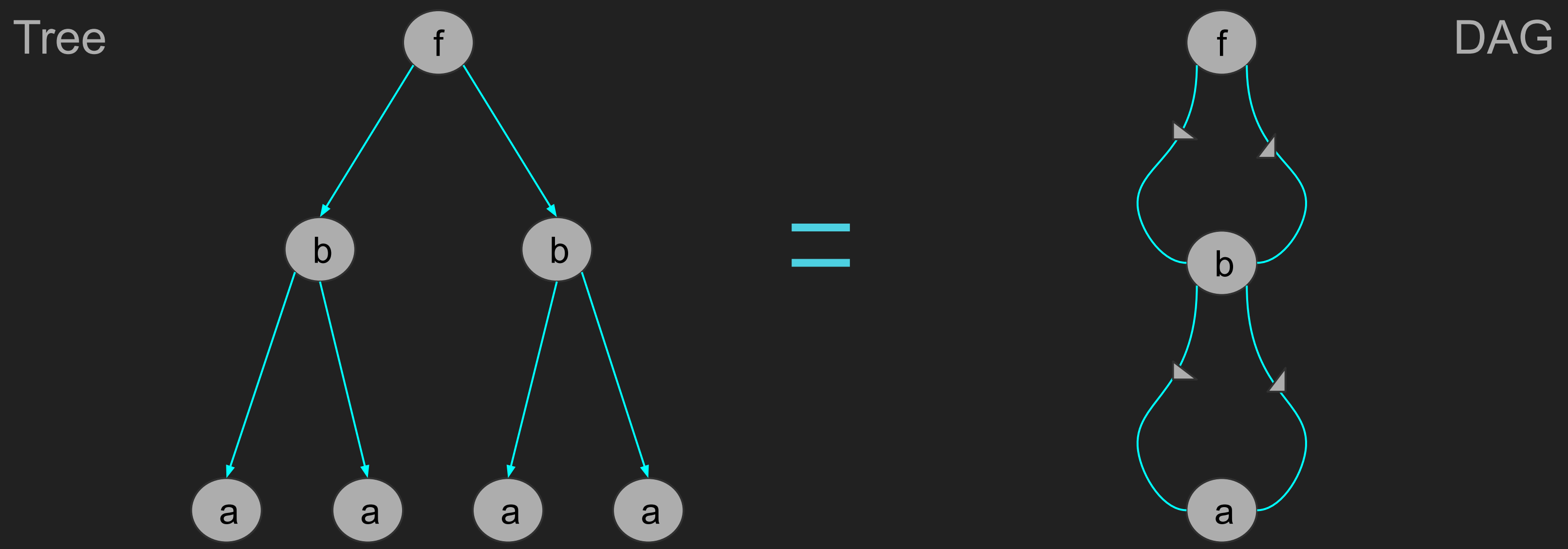
- Theoretical lower bound unknown

- Current best is ~ O(n^2.37)

# Outlook

- Combining Bottom-Up and Top-Down - Earley’s Algorithm
- A better Parsing Algorithm and connections to Matrices
- Term sharing and common subformulae elimination

# Term Sharing




Much easier to do with a Bottom-Up Implementation

# Common SubFormulae Elimination

A(B(C), D(B(C)))  = A(K, D(K)), K = B(C)

Again much easier with Bottom-Up Approaches!

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- Analyzed top down and bottom up approaches
- Found a new prolog implementation - generalising the algorithm
- Based on this work we can do a better logical analysis of Earley’s algorithm
- This may lead to better algorithms
- Also helpful in CSFE and Term-Sharing

# Thank You!