



**SECOND SEMESTER 2022-2023**

**16-01-2023**

Course Handout - Part II

In addition to Part-I (General Handout for all courses appended to the time table), this portion gives further specific details regarding the course.

Course No. : MATH F244  
Course Title : Measure & Integration  
Instructor-in-Charge : SHARAN GOPAL  
Instructors : SHARAN GOPAL, Aleena Philip and Sri Sakti Swarup Anupindi

**1. Scope and Objective of the Course:** The objective of this course is to give a comprehensive and sound introduction to Lebesgue measure theory and integration. The concepts of several notions of convergence and convergence theorems are also covered in this course. The classical theory of Riemann integration has some obvious draw backs: Firstly, the class of Riemann integrable functions is relatively small and secondly the limiting operations often lead to insurmountable difficulties. In this course, the students will be taught, how to resolve these problems in the case of Lebesgue measure theory.

**2. Textbook:**

H. L. Royden, P. M. Fitzpatrick, Real Analysis, 4<sup>th</sup> Edition, Pearson Education India, 2015.

**3. Reference books**

1. G. de Barra, *Measure Theory and integration*, New Age International Ltd, Delhi, 2003.
2. P.K. Jain, V.P. Gupta, P. Jain, *Lebesgue Measure And Integration*, New Age International Ltd, Delhi, 2nd ed., 2011.
3. Inder Kumar Rana : *Introduction to Measure & Integration*, Narosa, Delhi 1997.

**4. Course Plan:**

Lecture No.	Learning objectives	Topics to be covered	Chapter in the Text Book
1-2	To make the students understand that it is impossible to define a measure for all subsets of real numbers.	Length of an interval, Outer measure.	<b>2.1-2.2</b>
3 -5	To introduce the concepts of	Lebesgue measurable sets	<b>2.3 - 2.4</b>



	measurable sets and study the properties of Measurable sets	and its properties, Borel sets and their measurability, Approximation of measurable sets.	
6-7	To introduce Lebesgue measure, study its properties and introduce the idea of “almost everywhere”.	Lebesgue measure and its properties, The Borel-Cantelli Lemma	<b>2.5</b>
8	To prove the existence of non-measurable sets.	Non-measurable sets	<b>2.6</b>
9-11	To define the Cantor set and show the existence of a non Borel subset of the Cantor set	The Cantor Set and the Cantor-Lebesgue Function	<b>2.7</b>
12-14	To study measurable functions	Definition and properties of measurable functions, Operations on measurable functions.	<b>3.1</b>
15-17	To study the measurability of limits of sequence of functions under various notions of convergence and then different approximations of measurable functions.	Pointwise limits and simple approximation. Littlewood's three principles, Egoroff's theorem, and Lusin's theorem	<b>3.2-3.3</b>
		Proofs for Egoroff's theorem, and Lusin's theorem – Self study	
18-23	To study the Lebesgue Integral in various forms and its properties.	Review of Riemann integral, Lebesgue integral of a bounded function and its properties, Integrals of a non-negative measurable functions, General Lebesgue integrals and its properties	<b>4.1-4.5</b>
24-25	To give a characterization of Riemann and Lebesgue integral functions	Characterizations of Riemann and Lebesgue Integrability	<b>5.3</b>
26-28	To study the concept of a new	Uniform integrability, The	<b>4.6, 5.1</b>

	notion of integrability, namely the uniform integrability	Vitali convergence theorem, A general Vitali convergence theorem	
29-30	To study a new notion of convergence of sequence of functions	Convergence in Measure	<b>5.2</b>
31-37	To define differentiability and study the relationship between Integration and Differentiation	Continuity of Monotone Functions, Differentiability of Monotone Functions, Functions of Bounded Variation, Absolutely Continuous Functions, Integrating Derivatives	<b>6.1-6.5</b>
38-40	To define $L^p$ spaces and study its completeness property.	Youngs inequality, Holder inequality, Minikowski inequality, and Reisz-Fischer theorem	<b>7.1-7.4</b>
		Approximations and Separability – Self study	

## 5. Evaluation Scheme:

EC No.	Evaluation Component	Duration	Weightage*	Date, Time	Nature of Component
1.	Quiz 1	To be announced	5%	To be announced	Closed Book
2.	Assignment 1	To be announced	10%	To be announced	Open Book
3.	Mid-Semester Test	90 min	30%	17/03 2.00 - 3.30PM	Closed Book
4.	Quiz 2	To be announced	5%	To be announced	Closed Book
5.	Assignment 2	To be announced	10%	To be announced	Open Book
6.	Comprehensive Examination	3 hours	40%	18/05 FN	Closed Book

**6. Chamber Consultation Hour:** To be announced in the class.

**7. Notices:** All notices concerning this course will be displayed in CMS or through e-mail only.

## 8. Make-up Policy:



- Makeup will be given only for very genuine cases and prior permission has to be obtained from the Instructor-in-charge.

**9. Academic Honesty and Integrity Policy:** Academic honesty and integrity are to be maintained by all the students throughout the semester and no type of academic dishonesty is acceptable.

**INSTRUCTOR-IN-CHARGE**

