Nirma University
Institute of Technology
2CSDE56 – Graph Theory
A.Y. 2023-2024
Question bank 1

In a graph G let p1 and p2 be two different paths between two given vertices. Prove that p1 $\oplus$ p2 is a circuit or a set of circuits in G.

Give step wise representation to find distance between two spanning trees T1 and T2 of a graph G as shown in Figure 1.

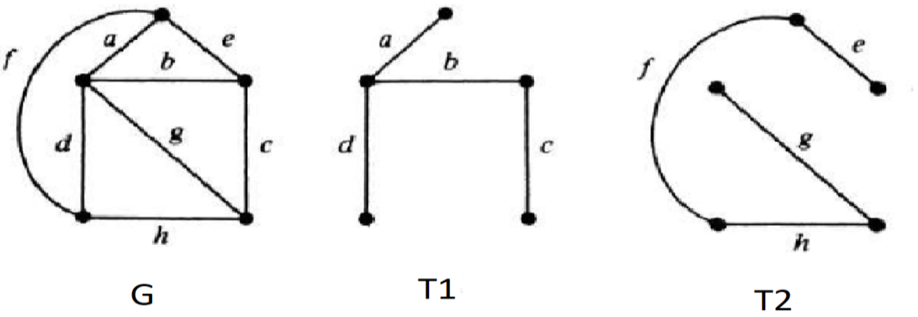


Figure 1

Obtain the Prufer sequence 'S' for the graph shown in Figure 2:

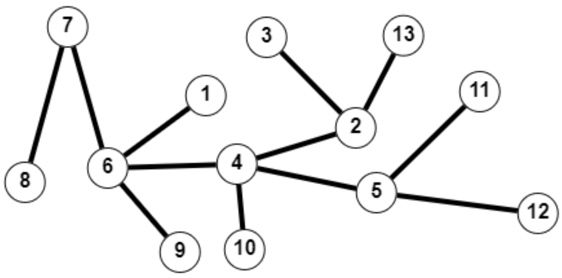


Figure 2

Find all the fundamental circuits and fundamental cutsets for the graph given in Figure 3(a) w.r.t. the spanning tree given in Figure 3(b).

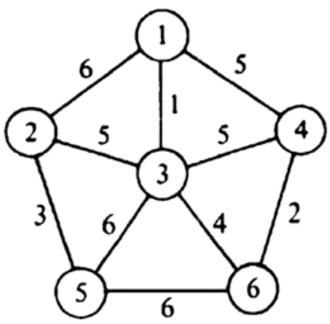


Figure 3(a)

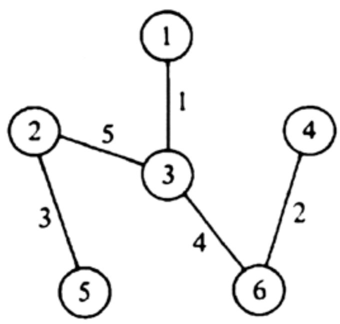


Figure 3 (b)

An undirected graph has 8 vertices labelled 1, 2, 3…, 8 and 31 edges. Vertices 1, 3, 5, 7 have degree 8 and vertices 2, 4, 6 have degree 7. What is the degree of vertex 8?

What is the number of vertices in undirected connected graph with 45 edges, 7 vertices of degree 2, 2 vertices of degree 5 and remaining of degree 6?

Find the maximum possible distance between any two spanning trees T_i and T_j of a connected graph having 5 vertices and 9 edges?

How many edge-disjoint Hamiltonian circuits are there in K_7? List all the edge disjoint Hamiltonian circuits. Is it Eulerian graph? State necessary and sufficient condition for a graph to be Euler graph. (**Note:** Hamiltonian graph is a graph which contains Hamiltonian path in it)

Define the steps to generate the Prufer code from the spanning tree. Generate the Prufer code for the spanning tree shown in Figure 4.

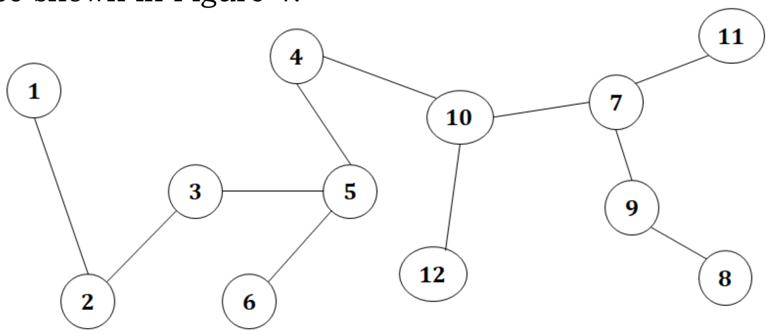


Figure 4

Calculate weighted path length for the tree as shown in Figure 5. **Note:** leaf node values represent its weight.

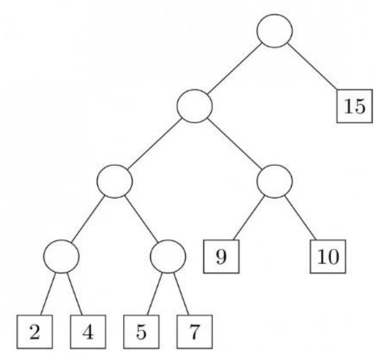


Figure 5

Answer the following questions:

Draw the graph for degree sequence S=(6,6,6,5,5,3,3,3,3,2) if realization of S is possible.

Find out all longest monotonically decreasing subsequence(s) from the sequence: 6,14,8,11,3,7,4,9,1 by constructing a tree.

Find eccentricity of each vertex, center, radius, and diameter for the tree as shown in Figure 6.

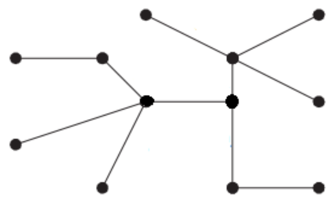
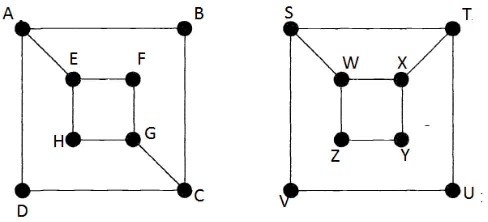
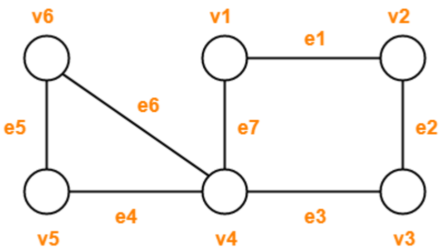


Figure 6

Define following terms and give proper example:

1. Isomorphic graphs
2. Unicursal graph
3. Arbitrarily traceable graph from vertex v
4. Maximal tree subgraphs
5. Rank and Nullity of a graph

Prove that a simple graph (i.e., a graph without parallel edges or self-loops) with n vertices and k components can have at most **(n−k)(n−k+1)/2** edges.
Prove that a pendant edge (an edge whose one end vertex is of degree one) in a connected graph G is contained in every spanning tree of G.
Compute the eccentricity of every vertex of a regular pentagon. How many centers it will have? Justify your answer.
Prove with all necessary steps "A tree has one or two centers".
Check whether below given graphs are isomorphic or not. Give proper justification for your answer. 
Twenty members of a new club meet each day for lunch at a round table. They decide to sit such that every member has different neighbours at each lunch. How many days can this arrangement last?
For the given degree sequences, check whether it is graphical or not. Show all necessary steps. 1) 7, 6, 5, 4, 4, 4, 3, 3, 2 2) 6, 6, 5, 4, 4, 3, 2, 2, 1
A graph G contains 8 edges. Find out number of different ways in which G can be decomposed into pairs of subgraphs G1 and G2.
For the below given graph, find 1) Circuit with longest path length starting from v1. Walk from vertex v1 to vertex v6 with path length 5. 
A networking company uses a compression technique to encode the message before transmitting over the network. Suppose the message contains the following characters with their probabilities. Construct Huffman code for every character.
Prove with contradiction that 'A graph G with n vertices, n-1 edges and no circuits, is connected'.

Character	Probability
A	0.05
B	0.09
C	0.12
D	0.13
E	0.16
F	0.45