Miscellaneous notes for *Tape and Tape* (2012a):
"A geometric setting for moment tensors"

Carl Tape (ctape@alaska.edu)

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See webpage with links here:
http://www.giseis.alaska.edu/input/carl/research/beachball.html

1 Computing source type coordinates on the fundamental lune

Here we present a guide for converting moment tensors to source types on the fundamental lune, then plotting. All equation references are for *Tape and Tape* (2012a).

1. Start with a moment tensor represented as a 3×3 symmetric matrix in some orthonormal basis (it does not matter which one). Compute the eigenvalues.

2. Sort the eigenvalues as $\lambda_1 \geq \lambda_2 \geq \lambda_3$. Then compute the (γ, β, ρ) coordinates with Eq. 21:

$$\begin{aligned} \tan\gamma &= \frac{-\lambda_1 + 2\lambda_2 - \lambda_3}{\sqrt{3}(\lambda_1 - \lambda_3)} \\ \cos\beta &= \frac{\lambda_1 + \lambda_2 + \lambda_3}{\sqrt{3}\|\mathbf{\Lambda}\|} \\ \rho &= \|\mathbf{\Lambda}\| \end{aligned}$$

   (Here ρ is not needed, but it represents the seismic moment: $M_0 = \rho/\sqrt{2}$.) For latitude on the lune, use $\delta = \pi/2 - \beta$.

If you are starting with eigenvalues, rather than the moment tensor, then start with step 2.

1.1 Plotting in Mathematica

All 3D plots in *Tape and Tape* (2012a) were generated using Mathematica. Some Mathematica plotting scripts are available from the webpage link at the top. Some scripts are related to figures in *Tape and Tape* (2013).

1.2 Plotting in GMT

Generic Mapping Tools (GMT: *Wessel and Smith* (1991)) is open-source software. It provides numerous options for global projections. One example, the equal-area Hammer projection, is shown in Figure 3. The key command in GMT is `-JH0/3i -R-30/30/-90/90`; this specifies the projection and also the region of the fundamental lune (γ-δ coordinates). A script to generate these plots, `lune.pl`, is available from the webpage link above.

The 2D plotting option was intentionally not included in *Tape and Tape* (2012a) for two reasons: (1) the projection distracts from the key point that the lune is part of the sphere; (2) the projection, while equal-area, distorts areas that are undistorted on the sphere. Nevertheless, a comparison between Figures 2 and 4 shows that the 2D version might be preferred for representing points that span the full lune. In either case, we note that the lune is preferred over the T-k source-type plots (*Hudson et al.*, 1989; *Tape and Tape*, 2012b).

References

Hudson, J. A., R. G. Pearce, and R. M. Rogers (1989), Source type plot for inversion of the moment tensor, *J. Geophys. Res.*, *94*(B1), 765–774.

Tape, W., and C. Tape (2012a), A geometric setting for moment tensors, *Geophys. J. Int.*, *190*, 476–498, doi:10.1111/j.1365-246X.2012.05491.x.

Tape, W., and C. Tape (2012b), A geometric comparison of source-type plots for moment tensors, *Geophys. J. Int.*, *190*, 499–510, doi:10.1111/j.1365-246X.2012.05490.x.

Tape, W., and C. Tape (2013), The classical model for moment tensors, *Geophys. J. Int.*, *195*, 1701–1720, doi:10.1093/gji/ggt302.

Wessel, P., and W. H. F. Smith (1991), Free software helps map and display data, *Eos Trans. Am. Geophys. Un.*, *72*(41), 441 ff.

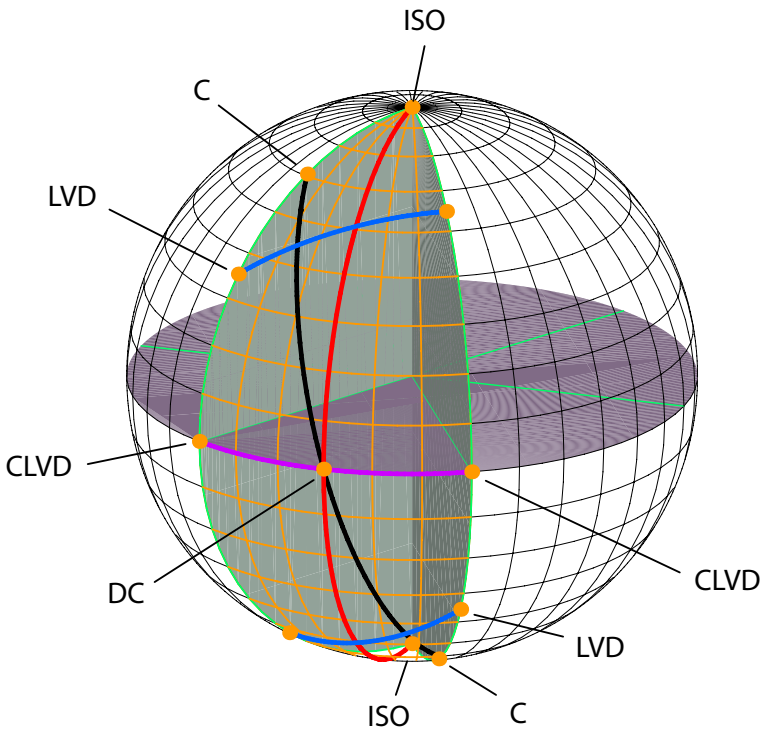

Figure 1: Fundamental lune representation of source types (*Tape and Tape*, 2012a, Figure 1). See 2D version in Figure 3.

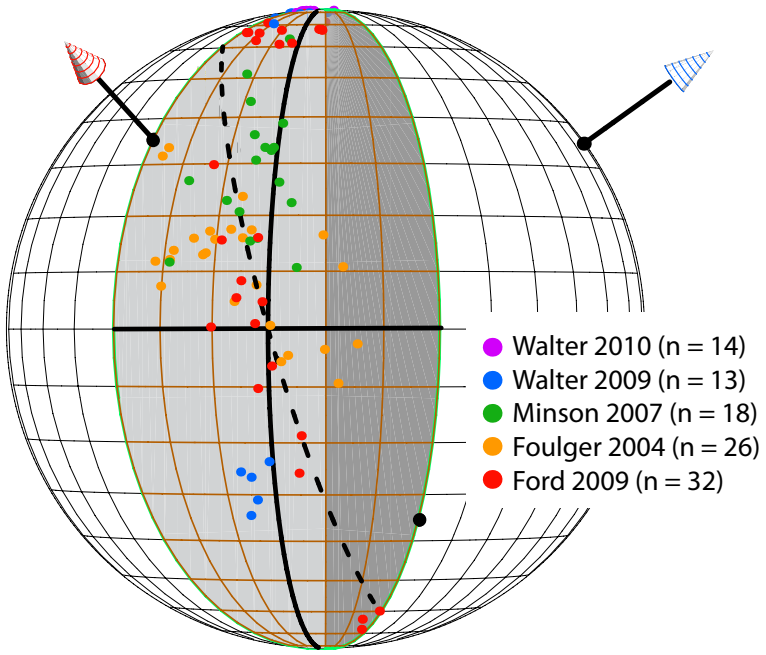

Figure 2: Five full moment tensor data sets plotted on the fundamental lune (*Tape and Tape*, 2012a, Figure 25). See 2D version in Figure 4.

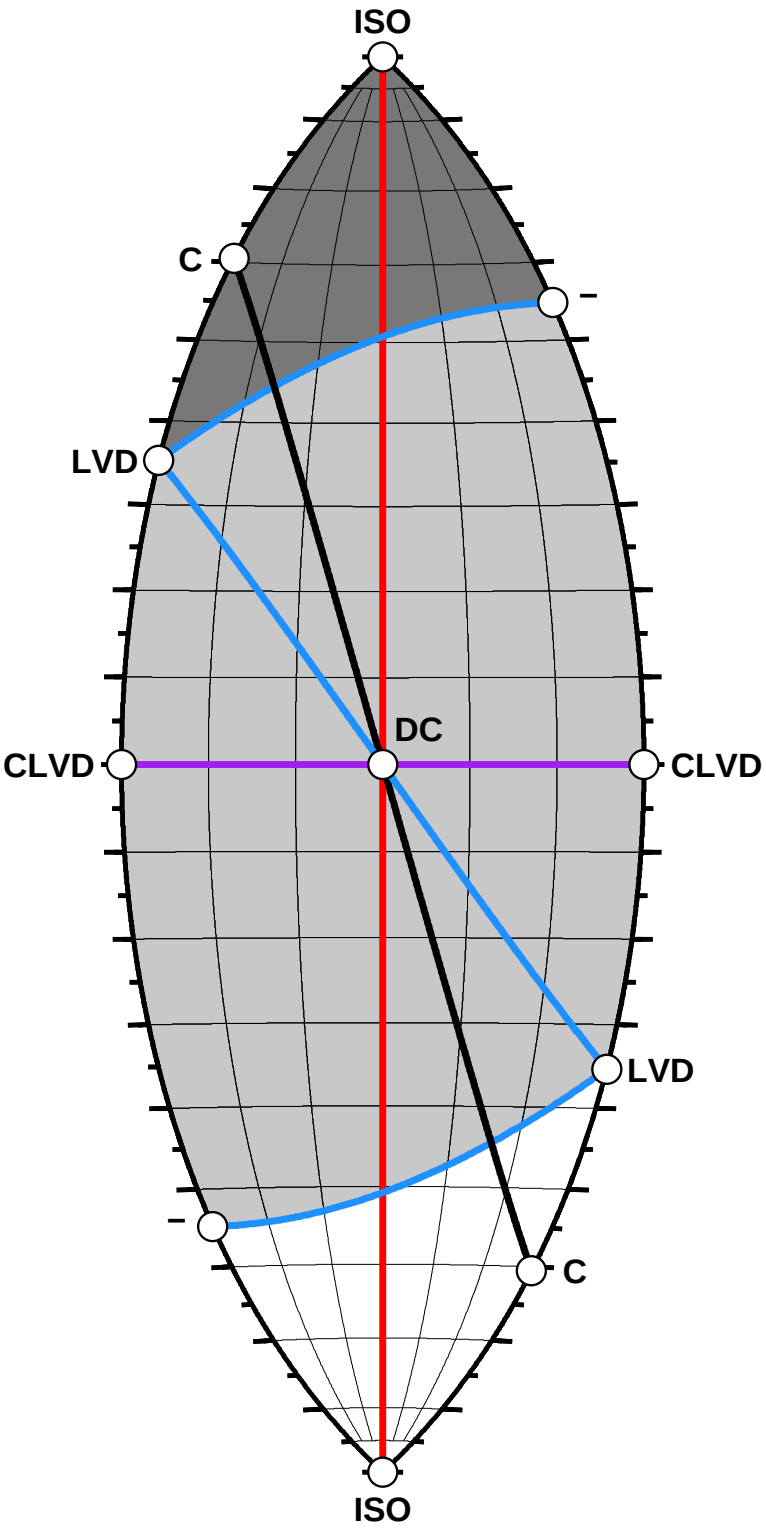

Figure 3: Home for source types, 2D version. See 3D version in Figure 1. The dark gray region at the top represents source types for which all vectors point outward. The white region at the bottom represents source types for which all vectors point inward.

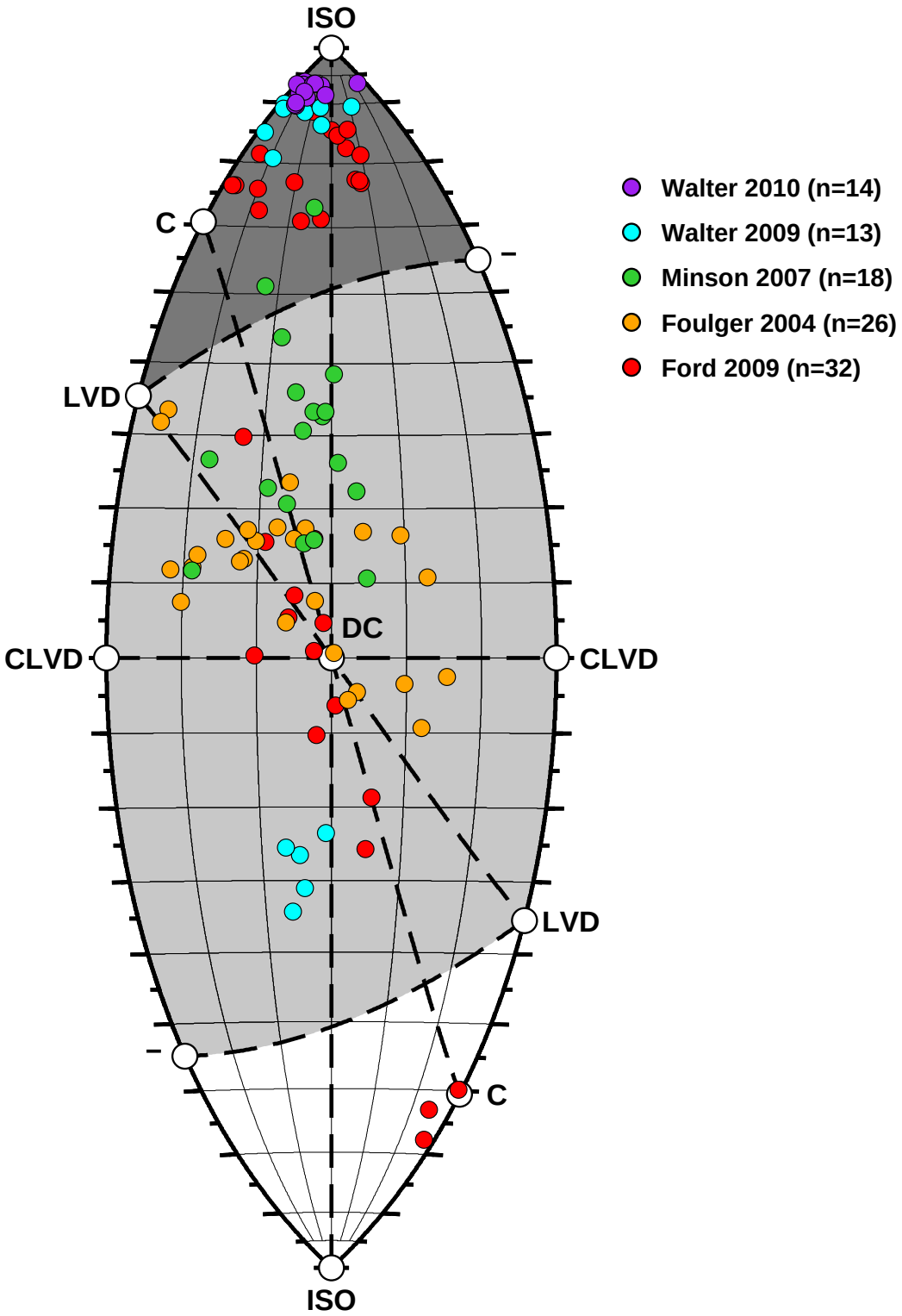

Figure 4: Five full moment tensor data sets plotted on the fundamental lune. See 3D version in Figure 2.

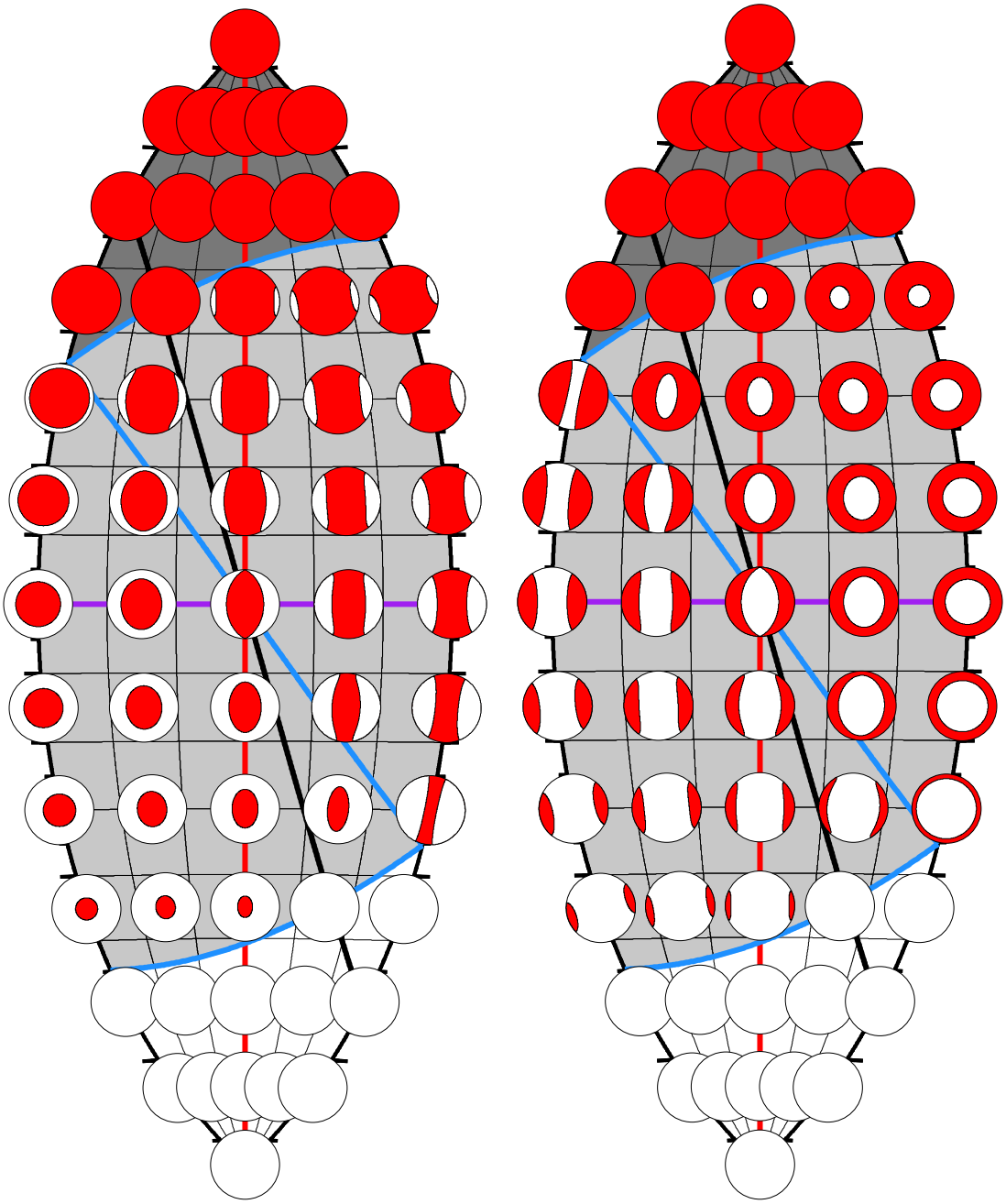

Figure 5: Beachballs on fundamental lune. All beachballs within each lune have the same basis U, which is easiest to identify in the double couple beachball at the center of the lune. Two additional examples are shown in Figure 6.

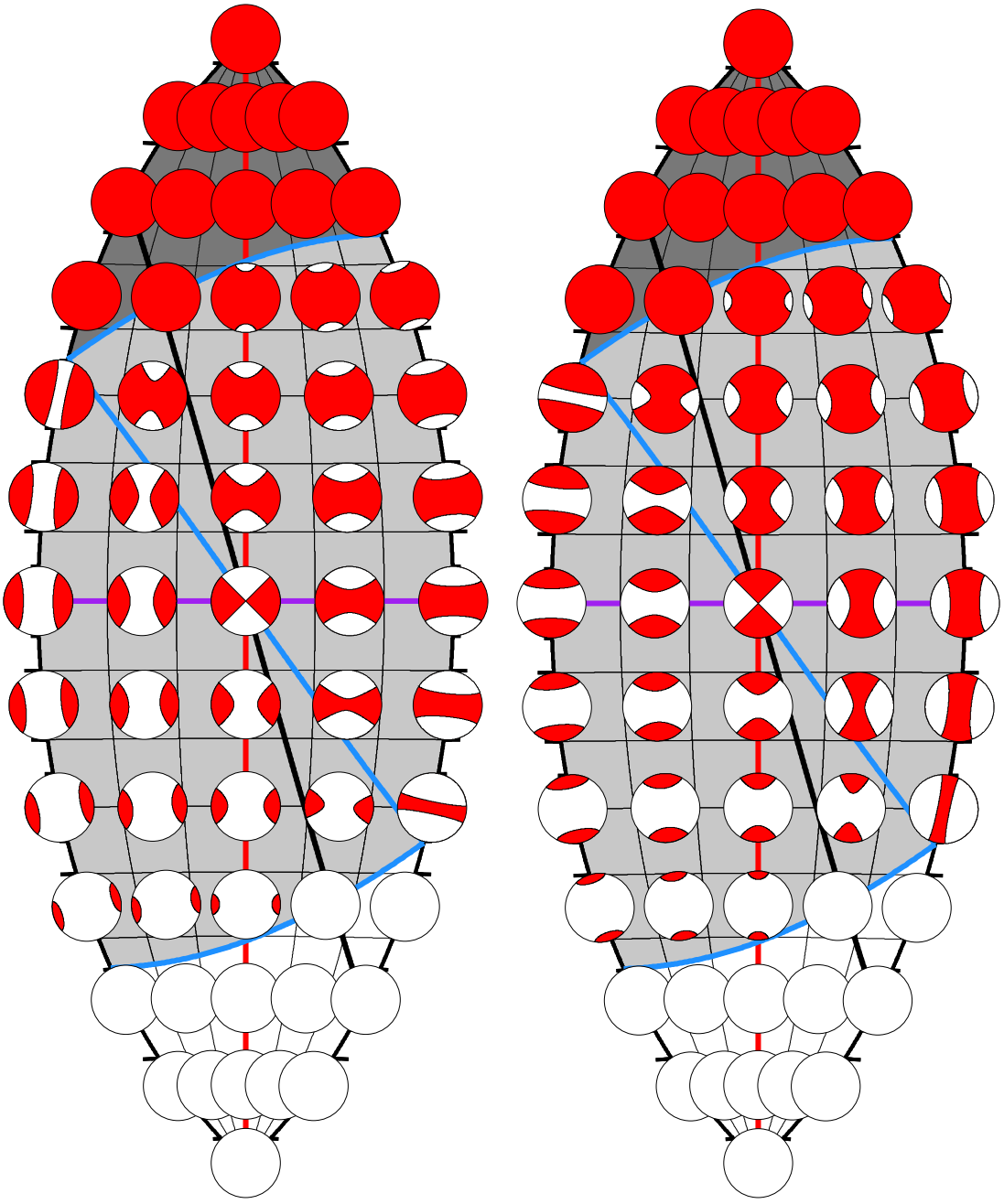

Figure 6: Same as Figure 5, but for two different reference orientations.