

# Assignment 3

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Question:

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes—large, medium, and small—that yield a net unit profit of \$420, \$360, and \$300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

**1. Solve the problem using `lpSolve`, or any other equivalent library in R.**

```
# Loading packages

library(lpSolve)
library(lpSolveAPI)

# creating lp with 9 variables and 0 constraints

lp <- make.lp(0,9)

#Specify the objective function

set.objfn(lp,c(420,420,420,360,360,360,300,300,300))
lp.control(lp, sense = 'max')

## $anti.degen
## [1] "fixedvars" "stalling"
##
## $basis.crash
## [1] "none"
##
## $bb.depthlimit
```

```

## [1] -50
##
## $bb.floorfirst
## [1] "automatic"
##
## $bb.rule
## [1] "pseudononint" "greedy"      "dynamic"      "rcostfixing"
##
## $break.at.first
## [1] FALSE
##
## $break.at.value
## [1] 1e+30
##
## $epsilon
##      epsb      epsd      epsel      epsint  epsperturb  epspivot
##      1e-10      1e-09      1e-12      1e-07      1e-05      2e-07
##
## $improve
## [1] "dualfeas" "thetagap"
##
## $infinite
## [1] 1e+30
##
## $maxpivot
## [1] 250
##
## $mip.gap
## absolute relative
##      1e-11      1e-11
##
## $negrange
## [1] -1e+06
##
## $obj.in.basis
## [1] TRUE
##
## $pivoting
## [1] "devex"      "adaptive"
##
## $presolve
## [1] "none"
##
## $scalelimit
## [1] 5
##
## $scaling
## [1] "geometric"  "equilibrate" "integers"
##
## $sense
## [1] "maximize"
##
## $simplextype
## [1] "dual"      "primal"

```

```

##
## $timeout
## [1] 0
##
## $verbose
## [1] "neutral"

#Adding Constraints

add.constraint(lp, c(1,1,1,0,0,0,0,0,0), "<=", 750 )
add.constraint(lp, c(0,0,0,1,1,1,0,0,0), "<=", 900)
add.constraint(lp, c(0,0,0,0,0,0,1,1,1), "<=", 450)

add.constraint(lp, c(20,15,12,0,0,0,0,0,0), "<=", 13000)
add.constraint(lp, c(0,0,0,20,15,12,0,0,0), "<=", 12000)
add.constraint(lp, c(0,0,0,0,0,0,20,15,12), "<=", 5000)

add.constraint(lp, c(1,1,1,0,0,0,0,0,0), "<=", 900)
add.constraint(lp, c(0,0,0,1,1,1,0,0,0), "<=", 1200)
add.constraint(lp, c(0,0,0,0,0,0,1,1,1), "<=", 750)

add.constraint(lp, c(900,-750,0,900,-750,0,900,-750,0), "=", 0)
add.constraint(lp, c(0,450,-900,0,450,-900,0,450,-900), "=", 0)
add.constraint(lp, c(450,0,-750,450,0,-750,450,0,-750), "=", 0)

RowNames <-c("1-ProductionCapacity", "2-ProductionCapacity", "3-ProductionCapacity",
             "1-StorageSpace", "2-StorageSpace", "3-StorageSpace",
             "ForecastLarge", "ForecastMedium", "ForecastSmall",
             "PercentCapP1andP2", "PercentCapP2andP3", "PercentCapP1andP3")

ColNames <- c("1-PlantLarge", "2-PlantLarge", "3-PlantLarge",
             "1-PlantMedium", "2-PlantMedium", "3-PlantMedium",
             "1-PlantSmall", "2-PlantSmall", "3-PlantSmall")

solve(lp)

## [1] 0

get.objective(lp)

## [1] 699026.5

get.constraints(lp)

## [1] 750.0000 858.4071 250.0000 13000.0000 12000.0000 5000.0000
## [7] 750.0000 858.4071 250.0000 0.0000 0.0000 0.0000

dimnames(lp) <- list(RowNames, ColNames)
lp

## Model name:
## a linear program with 9 decision variables and 12 constraints

```

## 2. Identify the shadow prices, dual solution, and reduced costs

### *# Reduced Costs*

```
get.sensitivity.obj(lp)
```

```
## $objfrom
## [1] 4.200e+02 4.125e+02 4.200e+02 -1.000e+30 3.600e+02 3.480e+02 2.800e+02
## [8] -1.000e+30 -1.000e+30
##
## $objtill
## [1] 4.40e+02 4.20e+02 4.32e+02 3.60e+02 3.75e+02 3.60e+02 1.00e+30 3.15e+02
## [9] 3.24e+02
```

### *#Shadow Prices*

```
get.sensitivity.rhs(lp)
```

```
## $duals
## [1] 60.0000000 0.0000000 0.0000000 22.3008850 22.3008850 19.3008850
## [7] 0.0000000 0.0000000 0.0000000 -0.0339823 0.0000000 -0.1231858
## [13] 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## [19] 0.0000000 -15.0000000 -24.0000000
##
## $dualsfrom
## [1] 7.287611e+02 -1.000000e+30 -1.000000e+30 1.216129e+04 9.120000e+03
## [6] 1.904762e+03 -1.000000e+30 -1.000000e+30 -1.000000e+30 0.000000e+00
## [11] -1.000000e+30 0.000000e+00 -1.000000e+30 -1.000000e+30 -1.000000e+30
## [16] -8.628319e+01 -1.000000e+30 -1.000000e+30 -1.000000e+30 -8.495575e+01
## [21] -1.039823e+02
##
## $dualstill
## [1] 8.075221e+02 1.000000e+30 1.000000e+30 1.335821e+04 1.267143e+04
## [6] 7.285714e+03 1.000000e+30 1.000000e+30 1.000000e+30 0.000000e+00
## [11] 1.000000e+30 0.000000e+00 1.000000e+30 1.000000e+30 1.000000e+30
## [16] 6.371681e+01 1.000000e+30 1.000000e+30 1.000000e+30 1.150442e+02
## [21] 1.438053e+02
```

### *#Dual solution*

```
get.dual.solution(lp)
```

```
## [1] 1.0000000 60.0000000 0.0000000 0.0000000 22.3008850 22.3008850
## [7] 19.3008850 0.0000000 0.0000000 0.0000000 -0.0339823 0.0000000
## [13] -0.1231858 0.0000000 0.0000000 0.0000000 0.0000000 0.0000000
## [19] 0.0000000 0.0000000 -15.0000000 -24.0000000
```

3. Further, identify the sensitivity of the above prices and costs. That is, specify the range of shadow prices and reduced cost within which the optimal solution will not change.

```
Sensitivity<-data.frame(get.sensitivity.rhs(lp)$duals[1:21],get.sensitivity.rhs(lp)$dualsfrom[1:21],get.s
names(Sensitivity)<-c("Price","low","High")
Sensitivity
```

##	Price	low	High
## 1	60.0000000	7.287611e+02	8.075221e+02
## 2	0.0000000	-1.000000e+30	1.000000e+30
## 3	0.0000000	-1.000000e+30	1.000000e+30
## 4	22.3008850	1.216129e+04	1.335821e+04
## 5	22.3008850	9.120000e+03	1.267143e+04
## 6	19.3008850	1.904762e+03	7.285714e+03
## 7	0.0000000	-1.000000e+30	1.000000e+30
## 8	0.0000000	-1.000000e+30	1.000000e+30
## 9	0.0000000	-1.000000e+30	1.000000e+30
## 10	-0.0339823	0.000000e+00	0.000000e+00
## 11	0.0000000	-1.000000e+30	1.000000e+30
## 12	-0.1231858	0.000000e+00	0.000000e+00
## 13	0.0000000	-1.000000e+30	1.000000e+30
## 14	0.0000000	-1.000000e+30	1.000000e+30
## 15	0.0000000	-1.000000e+30	1.000000e+30
## 16	0.0000000	-8.628319e+01	6.371681e+01
## 17	0.0000000	-1.000000e+30	1.000000e+30
## 18	0.0000000	-1.000000e+30	1.000000e+30
## 19	0.0000000	-1.000000e+30	1.000000e+30
## 20	-15.0000000	-8.495575e+01	1.150442e+02
## 21	-24.0000000	-1.039823e+02	1.438053e+02

4. Formulate the dual of the above problem and solve it. Does the solution agree with what you observed for the primal problem?

```
lpdual <- make.lp(0,12)
set.objfn(lpdual, c(750,900,450,13000,12000,5000,900,1200,750,0,0,0))

lp.control(lpdual,sense='min',simplextype="dual")
```

```
## $anti.degen
## [1] "fixedvars" "stalling"
##
## $basis.crash
## [1] "none"
##
## $bb.depthlimit
## [1] -50
##
## $bb.floorfirst
## [1] "automatic"
##
## $bb.rule
## [1] "pseudononint" "greedy" "dynamic" "rcostfixing"
##
```

```

## $break.at.first
## [1] FALSE
##
## $break.at.value
## [1] -1e+30
##
## $epsilon
##      epsb      epsd      epsel      epsint  epsperturb  epspivot
##      1e-10      1e-09      1e-12      1e-07      1e-05      2e-07
##
## $improve
## [1] "dualfeas" "thetagap"
##
## $infinite
## [1] 1e+30
##
## $maxpivot
## [1] 250
##
## $mip.gap
## absolute relative
##      1e-11      1e-11
##
## $negrange
## [1] -1e+06
##
## $obj.in.basis
## [1] TRUE
##
## $pivoting
## [1] "devex"      "adaptive"
##
## $presolve
## [1] "none"
##
## $scalelimit
## [1] 5
##
## $scaling
## [1] "geometric"  "equilibrate" "integers"
##
## $sense
## [1] "minimize"
##
## $simplextype
## [1] "dual" "dual"
##
## $timeout
## [1] 0
##
## $verbose
## [1] "neutral"

```

```

add.constraint(lp dual ,c(1,0,0,20,0,0,1,0,0,900,0,450), ">=", 420)
add.constraint(lp dual ,c(0,1,0,0,20,0,1,0,0,-750,450,0), ">=", 420)
add.constraint(lp dual ,c(0,0,1,0,0,20,1,0,0,0,-900,-750), ">=", 420)
add.constraint(lp dual ,c(1,0,0,15,0,0,0,1,0,900,0,450), ">=", 360)
add.constraint(lp dual ,c(0,1,0,0,15,0,0,1,0,-750,450,0), ">=", 360)
add.constraint(lp dual ,c(0,0,1,0,0,15,0,1,0,0,-900,-750), ">=", 360)
add.constraint(lp dual ,c(1,0,0,12,0,0,0,0,1,900,0,450), ">=", 300)
add.constraint(lp dual ,c(0,1,0,0,12,0,0,0,1,-750,450,0), ">=", 300)
add.constraint(lp dual ,c(0,0,1,0,0,12,0,0,1,0,-900,-750), ">=", 300)

```

```

solve(lp dual)

```

```

## [1] 0

```

```

get.objective(lp dual)

```

```

## [1] 696000

```

```

get.variables(lp dual)

```

```

## [1] 0.0000000 0.0000000 0.0000000 12.0000000 20.0000000 60.0000000
## [7] 0.0000000 0.0000000 0.0000000 0.2000000 0.4666667 0.0000000

```

```

get.constraints(lp dual)

```

```

## [1] 420 460 780 360 360 480 324 300 300

```