# Calculus Practices Week 1

Doral Academy Math Club – Aarush Chaubey

October 24, 2023

## 1 Questions

1. Prove
$$\lim_{x\to 0} \frac{\sin(x)}{x}$$
2. Prove
$$\lim_{x\to 0} \frac{\sin(ax)}{x}$$
3. Prove
$$\lim_{x\to \infty} \left(1+\frac{1}{x}\right)^x$$
4. Prove
$$\lim_{x\to \infty} \left[\frac{(1+\frac{1}{x})^x}{e}\right]^x$$
5. Define
$$\zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}$$
Find
$$\lim_{x\to \infty} (\zeta(x)-1)^{\frac{1}{x}}$$
6. Define the $nth$ Fibonacci number as
$$F_n = F_{n-1} + F_{n-2}$$
where
$$F_1 = F_2 = 1$$
Find
$$\lim_{n\to \infty} \frac{F_{n+1}}{F_n}$$
7. Find
$$\lim_{x\to 0} \left(\frac{x}{\sin^3(x)} - \frac{1}{x^2}\right)$$
8. Bonus Question! Define the double factorial (!!) as
$$n!! = n*(n-2)*(n-4)\cdots$$
where the last term is 2 for even $n$ and 1 for odd $n$ Find
$$\frac{(2n-1)!!\sqrt{n}}{(2n)!!}$$

## 2 Answers

1. $\boxed{1}$
2. $\boxed{a}$
3. $\boxed{e}$
4. $\boxed{\frac{1}{\sqrt{e}}}$
5. $\boxed{\frac{1}{2}}$
6. $\boxed{\frac{1+\sqrt{5}}{2}}$
7. $\boxed{\frac{1}{2}}$
8. $\boxed{\frac{1}{\sqrt{\pi}}}$