Aarush's Super Awesome Precalculus Test

Traditional restrictions on the domains of the arctrig(θ) functions are used. Re(z) and Im(z) are defined as the real and imaginary components of z. The last 10 questions are arbitrarily ordered, so you should try and attempt #30 even if you don't know #21. Good luck!

1. Find the period of $2\pi\csc(4x+6)+40$
2. Find the amplitude of $2\pi\csc(4x+6)+40$
3. Find the phase shift of $2\pi\csc(4x+6)+40$
4. Find the area of a regular dodecagon with circumradius 1.
5. Find sin(18) using a trigonometric identity.
6. Using the value you got from the previous question and the fact that $\cos(18°)$ is $\frac{\sqrt{10+2\sqrt{5}}}{4}$, find $\sin(36°)$.
7. Ordóñez forgot precalculus! Help him remember the smallest angle between the planes $x+2y+3z=76$ and $4x+5y-z=14$
8. Given an isosceles trapezoid with main diagonal of length 5 and legs of length 2, find the value of the product of the bases.
9. When rotating the conic $2x^2+2y^2-2xy=69$ by 45 degrees clockwise find the sum of the coefficients on the x^2, y^2, xy terms.
10. Aarush rolls a die(not necessarily fair) which has sides of $\arcsin(x), \arccos(x), \arctan(x), \operatorname{arccsc}(x), \operatorname{arcsec}(x), \operatorname{arccot}(x)$ where $0 \le x \le 2\pi$. Find the probability that the value Aarush rolls is undefined.
11. Find
$$\log_2\left[\prod_{n=1}^{44} 1+\tan(n°)\right]$$
12. As n tends to infinity, what does the area bounded by the roots of the equation $z^n=69$ approach, where $z \in \mathbb{C}$?
13. Tim isn't happy about the lack of American Heritage names on this test. To cheer him up, Aarush gave him a heart in the equation $r = \frac{\sin(\theta)\sqrt{|\cos(\theta)|}}{\sin(\theta)+\frac{7}{5}} - 2\sin(\theta)+2$ where $-\pi \le \theta \le \pi$. Find the period of the function on $0 \le \theta \le 2\pi$.
14. Find the minimum value of Re(z) if
$$\sum_{n=1}^{2022} |z-n|^2 - |z-(n+1)|^2 \ge 0$$

15. If

$$\frac{\log x}{2022a + 2b - 2024c} = \frac{\log y}{2022c + 2a - 2024b} = \frac{\log z}{2022b + 2c - 2024a}$$

for $x, y, z, a, b, c \in \mathbb{R}$, find xyz.

16. Ordóñez forgot precalculus! He needs you to find $z^{2022} + \frac{1}{z^{2022}}$ where $z + \frac{1}{z} = \sqrt{3}$.

The following information will be used for questions 15-18

The Lorentz Transformation is an often used transformation in modern physics that describes the motion of a particle(in only the x-direction for the purposes of this problem) in a separate coordinate system from a stationary frame of reference. Namely,

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where x, y, z describe the initial positions of the particle, x', y', z' describe the positions of the particle in motion with respect to the frame of reference, and t, t' describe the times with respect to the initial coordinate system and the moving coordinate system. The constant c is the speed of light. For the following problems, assume traditional physical restrictions to be lifted e.g. velocities can go beyond the speed of light.

17. Find

$$\lim_{v \to c} x' + vt'$$

18. As v approaches 0, if $x' + \frac{1}{t'}$ is at a minimum, find x in terms of t, given that $x, t \geq 0$. Hint: *inequalities*

19. Find

$$\sum_{x=1}^{\infty} \frac{16x}{9x^4 + 15x^2 + 16}$$

20. Find the area bounded by $x^4 - 100x^2 + y^4 + 100y^2 + 2x^2y^2 = 0$

21. Find the sum of the solutions to

$$\sum_{n=0}^{2022} \cos(n\theta) = -\frac{1}{2}$$

for $0 \leq \theta \leq 2\pi$

22. Find the number of intersections between the two functions:

$$y = \sin(x) \text{ and } y = \frac{x}{100}$$

23. Find

$$\sum_{x=0}^{\infty} \arctan\left(\frac{6x^2+2}{1+(x^2-1)^3}\right)$$

24. Find the period of

$$\lim_{n\to\infty} \underbrace{\sin(\sin(\sin(\ldots.\sin(x))))}_{\text{composed n times}}$$

25. If the roots of a function $y = 4x^3 + 2x^2 - 3x - 1$ can be expressed as $\cos(a), \cos(b), \cos(c)$ where $0 \leq a, b, c \leq 2\pi$ are in simplest form, find $a + b + c$. Hint: *substitute* $x = \cos(t)$

26. Find the minimum value of

$$x^4 + y^6 + \frac{1}{z^2} - 2x^2 - 2y^3 - \frac{6}{z}$$

. Hint: $factor$

27. Let γ be the circumcircle of triangle ABC. If K is constructed perpendicular to BC and tangent to γ at A, D is constructed so that $KD \parallel AB$, and $\angle ACK = 30°$, what is the measure of $\angle BAD$?

28. For $x, y \in \mathbb{R}$ and $x, y \neq 0$, if $x^2 + y^2 - 138xy\sin^2\theta = 0, y = ax$ where a is a constant. Find a.

29. Find

$$\sum_{n=0}^{\infty} \arcsin\left(\frac{1}{n\sqrt{n+1} - (n^2-1)\sqrt{n}}\right)$$

Hint: *Divide both numerator and denominator by* $\sqrt{n} + \sqrt{n-1}$.

30. When listed in numerical order, the first, second, and third solutions described by question 21 are α, β, γ. If x has an equal chance of being either α, β or γ, find the expected value for the period of

$$\sum_{n=0}^{2022} \sin^{2n}(x) = 0$$