Calculus/AMC PSET Week 1

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September 2023

1. $$\int_{69}^{420} x \, dn$$

2. $$\int_{-6}^{6} \sqrt{36 - x^2} \, dx$$

3. $$\int_{-\infty}^{\infty} \arctan(x) \, dx$$

4. $$\int \frac{3x^2 - 10x + 2}{x^3 - 5x^2 + 2x - 6} \, dx$$

5. $$\int_{0}^{\frac{\pi}{2}} \frac{1}{1 + \tan(x)} \, dx$$

6. $$\int_{0}^{5} \lfloor x \rfloor \, dx$$

7. For certain real values of a, b, c, and d, the equation $x^4+ax^3+bx^2+cx+d = 0$ has four non-real roots. The product of two of these roots is $13 + i$ and the sum of the other two roots is $3 + 4i$, where $i = \sqrt{-1}$. Find b.

8. $$\int_{0}^{1} \frac{x^n}{x^n + (x - 1)^n}$$

9. Isosceles trapezoid $ABCD$ has parallel sides $\overline{AD}$ and $\overline{BC}$, with $BC < AD$ and $AB = CD$. There is a point P in the plane such that $PA = 1, PB = 2, PC = 3$, and $PD = 4$. What is $\frac{BC}{AD}$?

10. The real numbers m and c are such that the equation

$$x^2 + (mx + c)^2 = 1$$

has a double root at $x = a$ and the equation

$$x^2 + (mx + c - 1)^2 = 1$$

has a double root at $x = b$ where a, b are not necessarily distinct. Find the number of possible pairs of (m, c) such that these constraints are satisfied.

11.

$$\lim_{x \to \infty} \frac{x^2}{2x + 3} \sin(\frac{\pi}{x})$$