Hawking Area Theorem Intuition

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Because black holes are so massive, they are extremely, if not actually, perfect spheres. This means we can approximate with very high accuracy $SA = 4\pi r^2$. We also know that $F_g = \frac{GMm}{r^2}$, so we know that the force due to gravity and surface area are related(made obvious by substitution).

If we say $SA \propto G^\alpha M^\beta c^\gamma$, we can use dimensional analysis to arrive at the following

$$m^2 = (\frac{m^3}{s^2 * kg})^\alpha (kg)^\beta (\frac{m}{s})^\gamma$$

The units of G can be found by $F_g = \frac{GMm}{r^2} = ma$ and solving for G in terms of a, M, r

Rearranging:

$$\begin{aligned} m^2 &= m^{3\alpha+\gamma} \\ s^0 &= s^{-2\alpha-\gamma} \\ kg^0 &= kg^{\beta-\alpha} \end{aligned}$$

Solving the system gives: $\alpha = 2, \beta = 2, \gamma = -4$, yielding $\boxed{SA \propto \frac{G^2 M^2}{c^4}}$ The actual formula for surface area is $\boxed{SA = \frac{16\pi G^2 M^2}{c^4}}$

Any increase in mass will result in a greater surface area, thus:

$$\frac{d(SA)}{dt} \geq 0$$