Problems

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1 Precalculus

1. The real numbers m and c are such that the equation

$$x^2 + (mx + c)^2 = 1$$

has a double root at $x = a$ and the equation

$$x^2 + (mx + c - 1)^2 = 1$$

has a double root at $x - b$ where a, b are not necessarily distinct. Find the number of possible pairs of (m, c) such that these constraints are satisfied.

2. Let a, b, c be complex numbers with magnitude 1. Find the maximum value of $|a - b| + |a - c| + b - c|$

3. How many values of z satisfy the equation $z^{2000} = \overline{z}$?

4. Find the minimum integer value of $\Re(z)$ if $|z - 4| < |z - 2|$

5. For certain real values of a, b, c, and d, the equation $x^4+ax^3+bx^2+cx+d = 0$ has four non-real roots. The product of two of these roots is $13 + i$ and the sum of the other two roots is $3 + 4i$, where $i = \sqrt{-1}$. Find b.

2 Calculus

1.

$$\int_0^1 \frac{x^n}{x^n + (x - 1)^n}$$

2. Let x, y, and z be positive real numbers such that $xyz = 32$. Find the minimum value of

$$x^2 + 4xy + 4y^2 + 2z^2.$$

3 Precalculus Answers

1. $\boxed{4}$ (Oxford MAT 2022)
2. $\boxed{3\sqrt{3}}$ (AoPS)
3. $\boxed{2002}$ (AoPS)
4. $\boxed{4}$ (AoPS)
5. $\boxed{51}$ (AoPS)

4 Calculus Answers

$\boxed{\frac{1}{2}}$(Selfmade -¿ u=1-x)

$\boxed{96}$(MSE)