# MAST Application

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## Algebra

### A1

Let $x = 2020$. Then we can rewrite the expression to be

$$\frac{(2x-1)(2x)(2x+1)+(x-1)(x)(x+1)}{(x-1)(x)+x(x+1)+(x-1)(x)} = \frac{9x^3-3x}{3x^2-1} = 3x = \boxed{6060}$$

### A2

$$\begin{aligned}\sum_{a=1}^{\infty} \frac{32a}{16a^4+14a^2+25} &= \sum_{a-1}^{\infty} \frac{32a}{(4a+3)^2+16} \\ &= 32\left(\frac{1}{5\cdot 13} + \frac{2}{13\cdot 29} + \frac{3}{29\cdot 53} + \frac{4}{85\cdot 53}\cdots\right)\end{aligned}$$

Through common differences, we see that the difference of the difference of factors in the denominators is constant i.e.

$$5 \quad 13 \quad 29 \quad 53 \quad 85...$$

$$8 \quad 16 \quad 24 \quad 32...$$

$$8 \quad 8 \quad 8...$$

We can rewrite the top sequence to be:

$$\begin{aligned}8\binom{n}{2} + 8\binom{n}{1} + 5\binom{n}{0} &= \frac{8(n)(n-1)}{2} + 8n + 5 \\ &= 4n^2 + 4n + 5\end{aligned}$$

where $n$ is a positive integer. Now we can rewrite the original series as:

$$\sum_{a=1}^{\infty} \frac{32a}{16a^4+14a^2+25} = \sum_{n=1}^{\infty} \frac{32n}{(4n^2+4n+5)(4n^2-4n+5)}$$

Because there are two quadratic factors in the denominator of the series, we can use partial fraction decomposition. We can also factor out the 32 and multiply it later.

$$\frac{n}{(4n^2+4n+5)(4n^2-4n+5)} = \frac{An+B}{4n^2+4n+5} + \frac{Cn+D}{4n^2-4n+5}$$

Multiplying both sides by $(4n^2+4n+5)(4n^2-4n+5)$ gives $A=0, B=\frac{-1}{8}, C=0, D=\frac{1}{8}$. Expanding out the sequence shows that it telescopes and all but $\frac{C\cdot 0+\frac{1}{8}}{4\cdot 1^2-4\cdot 1+5}$ are cancelled. Multiplying by 32 gives $32\cdot\frac{1}{40}=\boxed{\frac{4}{5}}$