DPP No. # 1 (JEE–ADVANCED)
Special DPP on "Number System"

Total Marks: 33		**Max. Time: 30 min.**
Comprehension ('–1' negative marking) Q.1 to Q.6	**(3 marks 3 min.)**	**[18, 18]**
Single choice Objective Questions ('–1' negative marking) Q.7	**(3 marks 3 min.)**	**[03, 03]**
Multiple choice Objective Questions ('–1' negative marking) Q.8 to Q.10	**(4 marks 3 min.)**	**[12, 09]**

Question No.	1	2	3	4	5	6	7	8	9	10	Total
Marks Obtained											

Comprehension-1 (Q. No. 1 to 3)

Counting numbers have fascinated human mind from time immemorial. The first set he seems to have pondered about is the set of natural numbers, N. Various subsets of this set were defined. Note worthy among them are

Prime Number:- If a natural number has exactly two divisors it is called a prime number. Yet another way to define it is as a natural number, other than 1, which is divisible by 1 & it self only.
Simple examples are 2, 3, 5, 7, ...
{2, 3} in the only set of consecutive primes.

Composite numbers:- A natural number having more than 2 divisors is called a composite number.
Simple examples are 4, 6, 8, 9, 10, ...
Note that 1 is neither prime nor composite.

Coprime or relatively prime numbers:- A pair of natural numbers is called a set of coprime numbers if their highest common factor (HCF) or greatest common divisor (g.c.d.) is 1.
For example 8 & 5 are co-prime
Note that these two numbers need not be prime.
More over 1 is coprime with every natural numbers.
A prime number is coprime with all natural numbers which are not it's multiple.

Twin Prime:- A pair of primes is called twin primes if their non-negative difference is '2'
For example {3, 5}, {5, 7}, {11, 13},..........

Based on above definitions solve the following problems

1. Number of prime numbers less than 10 is
(A) 2 (B) 3 (C) 4 (D) 5

2. Number of composite numbers less than 15 is
(A) 10 (B) 8 (C) 9 (D) 7

3. Let p & q be the number of natural numbers which are less than or equal 20 and are prime & composite respectively, then 20 – p – q is equal to
(A) 1 (B) 0 (C) 2 (D) 3

Comprehension-2 (Q. No. 4 to 6)

The natural numbers were not sufficient to deal with various equations that mathematicians encountered so some new sets of numbers were defined

Whole Numbers (W) = {0, 1, 2, 3, 4, ...}

Integers (Z or I) = {......, –3, –2, – 1, 0, 1, 2, 3, 4, ...}

Even Integers:- Integers divisible by 2, they are expressed as 2n, $n \in Z$.

Odd Integers:- Integers not divisible by 2, they are expressed as 2n + 1 or 2n – 1, $n \in Z$.

4. If $m^2 - n^2 = 7$, where $m, n \in Z$, then number of ordered pairs (m, n) is
(A) 1 (B) 2 (C) 3 (D) 4

5. Difference of squares of two odd integers is always divisible by
(A) 3 (B) 5 (C) 16 (D) 8

6. If $m, n \in N$ and $m^2 - n^2 = 13$, then $(m + 1)(n + 1)$ is equal to
(A) 42 (B) 56 (C) 50 (D) None of these

7. Number of ordered pairs of integers (n, m) for which $n^2 - m^2 = 14$ is
(A) 0 (B) 1 (C) 2 (D) 4

8. Identify the correct statement
(A) If a, b, c are odd integers a + b + c cannot be zero
(B) If a, b, c are odd integers $a^2 + b^2 - c^2 \neq 0$
(C) If $a^2 + b^2 = c^2$, then at least one of a, b, c is even, given that a, b, c are integers
(D) If $a^2 + b^2 = c^2$ where a, b, c are integers then $c > a + b$

9. If $n^2 + 2n - 8$ is a prime number where $n \in N$, then n is
(A) also a prime number (B) relatively prime to 10
(C) relatively prime to 6 (D) a composite number

10. If $n^2 - 11n + 24 = 0$ is satisfied by n_1 & n_2 where $n_2 > n_1$ then
(A) ${n_1}^2 + n_2$ is prime number (B) n_1 & $n_2 - n_1$ are co-prime
(C) n_1 & $n_2 - n_1$ are twin primes (D) $n_1 + n_2 + n_1 n_2$ has 2 prime divisors

DPP No. # 1													
1.	(C)	**2.**	(D)	**3.**	(A)	**4.**	(D)	**5.**	(D)	**6.**	(B)	**7.**	(A)
8.	(ABC)	**9.**	(AB)	**10.**	(ABCD)								

Solutions

1. Prime No. {2, 3, 5, 7}

2. Composite number < 15 = {4, 6, 8, 9, 10, 12, 14}

3. Except 1 every natural number is either prime or composite.

4. $(m + n)(m - n) = 7 \times 1 = (-7) \times (-1)$

$m + n = 7$ or $m + n = 1$

$m - n = 1$ or $m - n = 7$ $\Rightarrow$ $m = 4, n = 3$ $m = 4, n = +3$

$m + n = -7$ or $m + n = -1$

$m - n = -1$ or $m - n = -7$ $\Rightarrow$ $m = -4, n = -3$ $m = -4, n = +3$

5. Difference = $(2m + 1)^2 - (2n + 1)^2$

$= (2m + 2n + 2)(2m + 1 - 2n - 1)$

$= 2 \times 2\,(m - n)$ $(m + n + 1)$

even odd

$\Rightarrow$ divisible by 8.

6. $n_1 = 3$, $n_2 = 8$

7. $(n + m)(n - m)$ = 7.2 or $(-7) \times (-2)$

= 14.1 or $(-14) \times (-1)$

On solving

We do not get any integer value of n & m.

8. Obvious

9. $n^2 + 2n - 8 = p \Rightarrow$ $(n + 1) = p + 9$ $\therefore$ $n \in N$ so $p + 9$ is a perfect square

So p can only be 7 $\Rightarrow$ $n = 3$

10. $n^2 - 11n + 24 = 0$

$\Rightarrow (n - 8)(n - 3) = 0$

$\Rightarrow n = 3, 8$

$\Rightarrow n_2 = 8, n_1 = 3$