1.11.2

EE25BTECH11065 - Yoshita

Question:

Unit vector along PQ, where coordinates of P and Q respectively are $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 4 \\ -7 \end{pmatrix}$ is.

Solution:
Let the coordinates of the points be $\mathbf{P}(2, 1, -1)$ and $\mathbf{Q}(4, 4, -7)$.

Point	**Name**
$\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$	P
$\begin{pmatrix} 4 \\ 4 \\ -7 \end{pmatrix}$	Q

TABLE 0: Vectors

To find the vector **PQ**, we subtract the matrix for P from the matrix for Q:

$$\mathbf{PQ} = \mathbf{Q} - \mathbf{P} \tag{1}$$

$$= \begin{pmatrix} 4 \\ 4 \\ -7 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix} \tag{2}$$

$$= \begin{pmatrix} 4-2 \\ 4-1 \\ -7-(-1) \end{pmatrix} \tag{3}$$

$$= \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix} \tag{4}$$

This resulting vector can also be written in the standard basis notation shown in the image:

$$\mathbf{PQ} = 2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$$

If we represent the vector **PQ** as a column vector **a**:

$$\mathbf{a} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$$

The norm is the square root of the dot product of the vector with itself, which can be expressed as the matrix product of its transpose $\mathbf{a}^T$ and $\mathbf{a}$.

$$\|\mathbf{a}\| = \sqrt{\mathbf{a}^T\mathbf{a}} \tag{5}$$

$$= \sqrt{\begin{pmatrix} 2 & 3 - 6 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}} \tag{6}$$

$$= \sqrt{49} \tag{7}$$

$$= 7 \tag{8}$$

The unit vector in the direction of $\mathbf{PQ}$, denoted as $\hat{\mathbf{u}}$, is found by dividing the vector by its magnitude.

$$\hat{\mathbf{u}} = \frac{\mathbf{PQ}}{\|\mathbf{PQ}\|} \tag{9}$$

$$= \frac{1}{7}(2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}) \tag{10}$$

$$= \frac{2}{7}\mathbf{i} + \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k} \tag{11}$$

Thus, the unit vector along PQ is $\begin{pmatrix} 2/7 \\ 3/7 \\ -6/7 \end{pmatrix}$ or $\frac{\mathbf{1}}{\mathbf{7}}(\mathbf{2i} + \mathbf{3j} - \mathbf{6k})$.

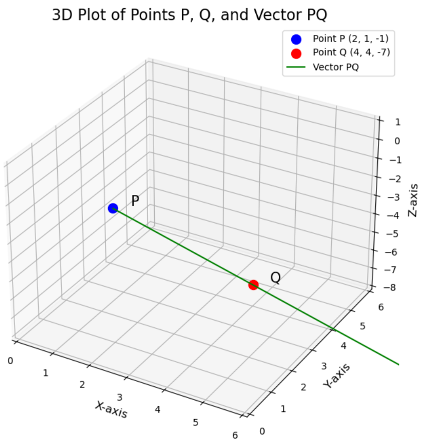

Fig. 0