

Question 5.2.3

AI25BTECH11040 - Vivaan Parashar

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Question:

Solve the following system of linear equations:

$$9x + 3y + 12 = 0$$

$$18x + 6y + 24 = 0$$

Solution:

The given equations can be rewritten as:

$$9x + 3y = -12 \quad (1)$$

$$18x + 6y = -24 \quad (2)$$

We can represent this system of equations in matrix form as:

$$\mathbf{MX} = \mathbf{D} \quad (3)$$

$$\Rightarrow \begin{pmatrix} 9 & 3 \\ 18 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -12 \\ -24 \end{pmatrix} \quad (4)$$

To obtain values of x and y , we can multiply both sides by the inverse of the coefficient matrix on the left side, but in this case, the coefficient matrix has a rank of 1 ($R_2 = 2R_1$). This implies that either the system has no solutions or infinite solutions.

If the system has infinite solution, then the rank of the augmented matrix ($\mathbf{M} \quad \mathbf{D}$) should also have a rank of 1.

$$\text{rank}((\mathbf{M} \ \mathbf{D})) = \text{rank}\left(\begin{pmatrix} 9 & 3 & -12 \\ 18 & 6 & -24 \end{pmatrix}\right) \quad (5)$$

$$\begin{pmatrix} 9 & 3 & -12 \\ 18 & 6 & -24 \end{pmatrix} \xrightarrow{R_2=R_2-2R_1} \begin{pmatrix} 9 & 3 & -12 \\ 0 & 0 & 0 \end{pmatrix} \quad (6)$$

$$\therefore \text{rank}((\mathbf{M} \ \mathbf{D})) = 1 \quad (7)$$

Therefore, this system of equations has infinite solutions, which are all values of $(x, y) \in \mathbb{R}^2$ that satisfy the equation $3x + y = -4$.

Plot:

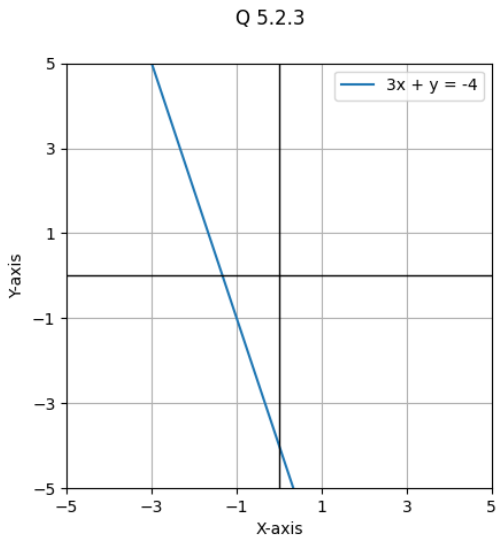


Figure: Graph of line representing all possible solutions