

# 4.3.32

EE25BTECH11042 - Nipun Dasari

## Question:

Find the slope of a line which cuts off intercepts of equal length on the axes is. Solve using matrices.

## Solution:

Let the line cut the x-axis at an intercept 'a' and the y-axis at an intercept 'b'. The points where the line intersects the axes can be represented by position vectors (column matrices).

The point of x-intercept is  $P_1 = (a, 0)$ . Its position vector is:

$$\mathbf{p}_1 = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (0.1)$$

The point of y-intercept is  $P_2 = (0, b)$ . Its position vector is:

$$\mathbf{p}_2 = \begin{pmatrix} 0 \\ b \end{pmatrix} \quad (0.2)$$

A direction vector for the line can be found by taking the difference between the two position vectors:

$$\mathbf{v} = \mathbf{p}_2 - \mathbf{p}_1 = \begin{pmatrix} 0 \\ b \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} -a \\ b \end{pmatrix} \quad (0.3)$$

The direction vector  $\mathbf{v}$  can be written as  $\mathbf{v} = \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$ . The slope,  $m$ , is defined as the ratio of the change in y to the change in x.

$$m = \frac{\Delta y}{\Delta x} = \frac{b}{-a} \quad (0.4)$$

The problem states that the intercepts have equal length, which means their magnitudes are equal:

$$|a| = |b| \quad (0.5)$$

### Case 1: The intercepts are equal ( $b = a$ )

Substituting  $b = a$  into the slope equation (assuming  $a \neq 0$ ): By (0.4)

$$m_1 = \frac{a}{-a} = -1 \quad (0.6)$$

### Case 2: The intercepts are opposite ( $b = -a$ )

Substituting  $b = -a$  into the slope equation (assuming  $a \neq 0$ ): By (0.4)

$$m_2 = \frac{-a}{-a} = 1 \quad (0.7)$$

Thus, using a matrix representation for the points, we find that the two possible slopes are -1 and 1.

