1.6.26

AI25BTECH11011-VARUN

Question:
Show that the points P(-2,3,5),Q(1,2,3) and R(7,0,-1) are collinear.
Solution:
Let the points are $\mathbf{P}\begin{pmatrix}-2\\3\\5\end{pmatrix}$,$\mathbf{Q}\begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\mathbf{R}\begin{pmatrix}7\\0\\-1\end{pmatrix}$.

$$\mathbf{Q}-\mathbf{P}=\begin{pmatrix}1\\2\\3\end{pmatrix}-\begin{pmatrix}-2\\3\\5\end{pmatrix} \tag{0.1}$$

$$\mathbf{Q}-\mathbf{P}=\begin{pmatrix}1-(-2)\\2-3\\3-5\end{pmatrix} \tag{0.2}$$

$$\mathbf{Q}-\mathbf{P}=\begin{pmatrix}3\\-1\\-2\end{pmatrix} \tag{0.3}$$

$$\mathbf{R}-\mathbf{P}=\begin{pmatrix}7\\0\\-1\end{pmatrix}-\begin{pmatrix}-2\\3\\5\end{pmatrix} \tag{0.4}$$

$$\mathbf{R}-\mathbf{P}=\begin{pmatrix}7-(-2)\\0-3\\-1-5\end{pmatrix} \tag{0.5}$$

$$\mathbf{R}-\mathbf{P}=\begin{pmatrix}9\\-3\\-6\end{pmatrix} \tag{0.6}$$

$$\tag{0.7}$$

If $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$ are collinear, then the Rank of matrix $(\mathbf{Q}-\mathbf{P},\mathbf{R}-\mathbf{P})$ should be 1.

$$(\mathbf{Q}-\mathbf{P},\mathbf{R}-\mathbf{P})=\begin{pmatrix}3&9\\-1&-3\\-2&-6\end{pmatrix} \tag{0.8}$$

$$R_3\rightarrow(\frac{R_1}{3}\times 2)+R_3 \tag{0.9}$$

$$R_2\rightarrow\frac{R_1}{3}+R_2 \tag{0.10}$$

$$=\begin{pmatrix}3&9\\0&0\\0&0\end{pmatrix} \tag{0.11}$$

$$\tag{0.12}$$

Since all elements of R_2 and R_3 are 0, The Rank of matrix $(\mathbf{Q}-\mathbf{P},\mathbf{R}-\mathbf{P})$ is 1.
$\implies$ $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$ are collinear.

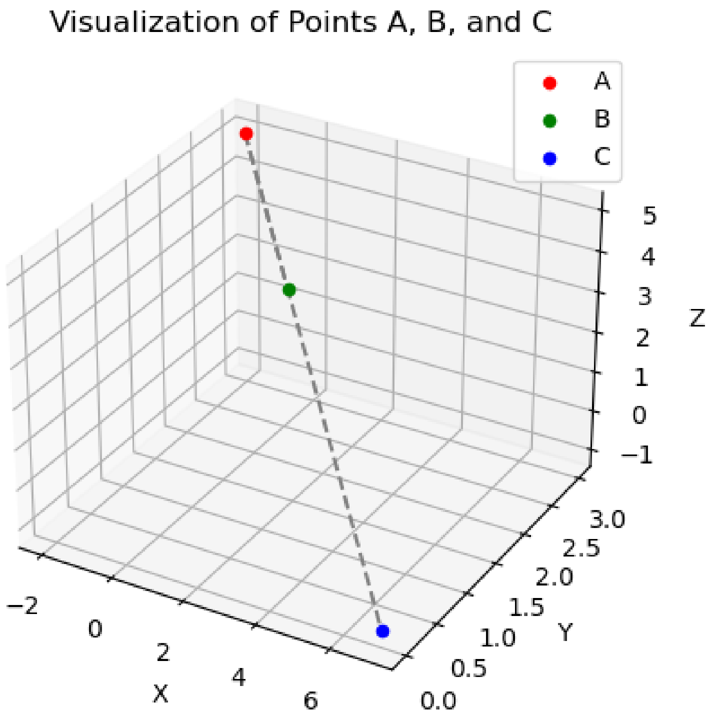
Visualization of Points A, B, and C
A
B
C
X
Y
Z
−2
0
2
4
6
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−1
0
1
2
3
4
5

Fig. 0.1