4.4.17

AI25BTECH11021 - Abhiram Reddy N

Question

A point $\mathbf{P}$ divides the line segment joining the points $\mathbf{A}(3, -5)$ and $\mathbf{B}(-4, 8)$ such that $\frac{AP}{PB} = \frac{K}{1}$. If $\mathbf{P}$ lies on the line $x + y = 0$, then find the value of K.

Answer

Step 1: Represent points as column vectors

$$\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}.$$

Step 2: Express $\mathbf{P}$ *using section formula in vector form*

Since $\mathbf{P}$ divides $\mathbf{AB}$ in the ratio $K : 1$,

$$\mathbf{P} = \frac{K\mathbf{B} + \mathbf{A}}{K + 1}.$$

Step 3: Use the line equation condition

Suppose the line is given as

$$\mathbf{n}^\top \mathbf{x} = c, \quad \text{where } \mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}.$$

Since $\mathbf{P}$ lies on this line,

$$\mathbf{n}^\top \mathbf{P} = c.$$

Substitute for $\mathbf{P}$:

$$\mathbf{n}^\top \left(\frac{K\mathbf{B} + \mathbf{A}}{K + 1} \right) = c.$$

Step 4: Derive general formula for K

Multiplying through,

$$\mathbf{n}^\top (K\mathbf{B} + \mathbf{A}) = c(K + 1).$$

$$K(\mathbf{n}^\top \mathbf{B} - c) = c - \mathbf{n}^\top \mathbf{A}.$$

$$K = \frac{c - \mathbf{n}^\top \mathbf{A}}{\mathbf{n}^\top \mathbf{B} - c}.$$

Step 5: Substitute given values

Here,

$$\mathbf{A} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -4 \\ 8 \end{pmatrix}, \quad \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad c = 0.$$

Thus,

$$K = \frac{0 - (1 \cdot 3 + 1 \cdot (-5))}{(1 \cdot (-4) + 1 \cdot 8) - 0} = \frac{-(-2)}{4} = \frac{2}{4} = \frac{1}{2}.$$

Final answer

$$\boxed{K = \frac{1}{2}}$$

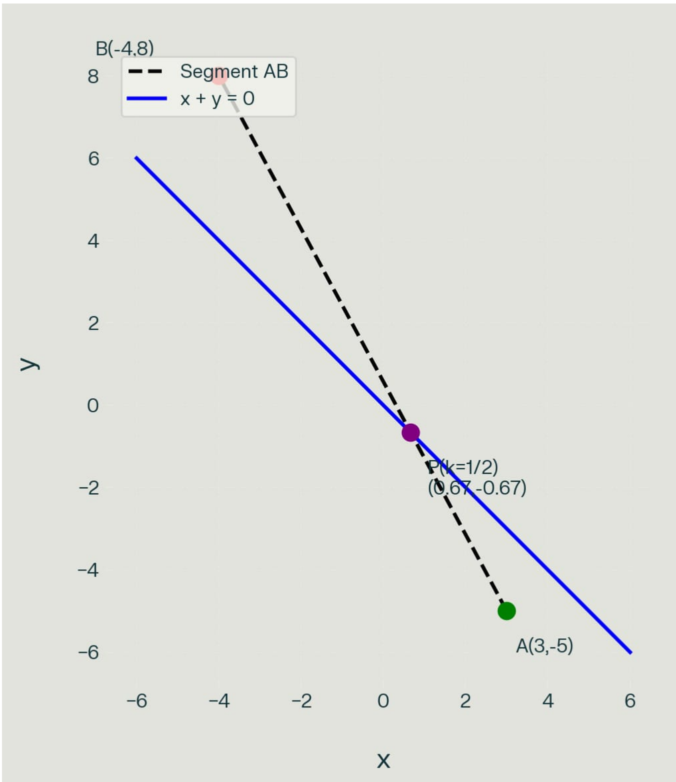

Fig. 0.1: Plot of the points and line showing the division ratio.