

12.150

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Question

Two sides of a triangle are represented by vectors $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$ and $\mathbf{b} = -\hat{i} + -\hat{j} + \hat{k}$. The area (magnitude) of the triangle is

- ① $\frac{1}{\sqrt{2}}$
- ② 1
- ③ $\sqrt{2}$
- ④ $2\sqrt{2}$

Theoretical Solution

Given ,

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \quad (1)$$

Area of Trianle

$$\frac{1}{2} \|\mathbf{a} \times \mathbf{b}\| \quad (2)$$

Also,

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} \begin{vmatrix} \mathbf{a}_{23} & \mathbf{b}_{23} \\ \mathbf{a}_{31} & \mathbf{b}_{31} \end{vmatrix} \\ \begin{vmatrix} \mathbf{a}_{31} & \mathbf{b}_{31} \\ \mathbf{a}_{12} & \mathbf{b}_{12} \end{vmatrix} \\ \begin{vmatrix} \mathbf{a}_{12} & \mathbf{b}_{12} \\ \mathbf{a}_{23} & \mathbf{b}_{23} \end{vmatrix} \end{pmatrix} \quad (3)$$

$$= \begin{pmatrix} 1 \cdot 1 - 1 \cdot (-1) \\ 1 \cdot (-1) - 1 \cdot 1 \\ 1 \cdot (-1) - 1 \cdot (-1) \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} \quad (4)$$

Theoretical Solution

$$Ar(\Delta) = \frac{1}{2} \left\| \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} \right\| \quad (5)$$

$$= \frac{1}{2} 2\sqrt{2} \quad (6)$$

$$= \sqrt{2} \quad (7)$$

Hence, Answer : Option 3

