1.8.18

EE25BTECH11001 - Aarush Dilawri

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Question

Find the values of y for which the distance between the points $\mathbf{P}(2, -3)$ and $\mathbf{Q}(10, y)$ is 10 units.

Solution

We are given the points

$$\mathbf{P} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 10 \\ y \end{pmatrix} \tag{1}$$

The distance between them is 10 units, so

$$\|\mathbf{P} - \mathbf{Q}\| = 10 \tag{2}$$

Squaring both sides,

$$\|\mathbf{P} - \mathbf{Q}\|^2 = (\mathbf{P} - \mathbf{Q})^\top (\mathbf{P} - \mathbf{Q}) = 10^2 \tag{3}$$

Solution (continued)

Substituting,

$$\left(\begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 10 \\ y \end{pmatrix}\right)^{\top} \left(\begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 10 \\ y \end{pmatrix}\right) = 100 \tag{4}$$

$$\begin{pmatrix} -8 \\ -3 - y \end{pmatrix}^{\top} \begin{pmatrix} -8 \\ -3 - y \end{pmatrix} = 100 \tag{5}$$

$$(-8)^2 + (-3 - y)^2 = 100 \tag{6}$$

$$64 + (y + 3)^2 = 100 \tag{7}$$

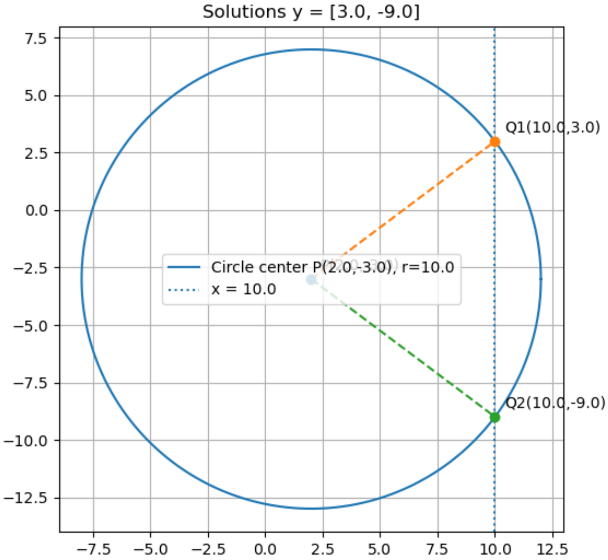
$$(y + 3)^2 = 36 \tag{8}$$

$$y + 3 = \pm 6 \tag{9}$$

Final Answer

$$y = 3 \quad \text{or} \quad y = -9 \tag{10}$$

Graphical Representation



C Code (code.c)

```
#include<math.h>

int find_y(double x1, double y1, double x2, double d, double *y_out1,
    double *y_out2) {
    double dx = x1 - x2;
    double inside = d * d - dx * dx; /* (y1 - y)^2 = inside */

    if (inside < 0.0) {
        /* No real solutions */
        return 0;
    } else if (inside == 0.0) {
        /* exactly one solution (double root) */
        *y_out1 = y1;
        *y_out2 = y1;
        return 1;
```

C Code (code.c)

```
    } else {
        double r = sqrt(inside);
        *y_out1 = y1 + r;
        *y_out2 = y1 - r;
        return 2;
    }
}
```

Python Code (code.py)

```
import numpy as np
import matplotlib.pyplot as plt

# Given values
x1, y1 = 2, -3
x2 = 10
d = 10

# Solve for y values
dx = x1 - x2
inside = d**2 - dx**2
if inside < 0:
    y_solutions = []
else:
    r = np.sqrt(inside)
    y_solutions = [y1 + r, y1 - r]
```

Python Code (code.py)

```
print("Solutions for y:", y_solutions)

# Circle centered at P with radius d
theta = np.linspace(0, 2*np.pi, 400)
circ_x = x1 + d * np.cos(theta)
circ_y = y1 + d * np.sin(theta)

# Plot
plt.figure(figsize=(6,6))
plt.plot(circ_x, circ_y, label=f'Circle: center P({x1},{y1}), r={d}')
plt.axvline(x=x2, linestyle=':', color='gray', label=f'x = {x2}')

# Point P
plt.scatter([x1], [y1], color='red', zorder=5)
plt.annotate(f'P({x1},{y1})', (x1,y1), textcoords="offset points", xytext
    =(6,6))
```

Python Code (code.py)

```
for i, yq in enumerate(y_solutions):
    plt.scatter([x2], [yq], color='blue', zorder=6)
    plt.annotate(f'Q{i+1}({x2},{yq:.1f})', (x2,yq), textcoords="offset-
        points", xytext=(6,6))
    plt.plot([x1, x2], [y1, yq], linestyle='--', color='green')

plt.gca().set_aspect('equal', 'box')
plt.grid(True)
plt.legend()
plt.title('Specific Case: P(2,-3), Q(10,y), Distance=10')
plt.show()
```

Python Code (nativecode.py)

```
import ctypes
from ctypes import c_double, POINTER, byref, c_int
import numpy as np
import matplotlib.pyplot as plt
from pathlib import Path
import sys

lib_path = Path(__file__).with_name('libfindy.so')
lib = ctypes.CDLL(str(lib_path))

lib.find_y.argtypes = [c_double, c_double, c_double, c_double, POINTER(
    c_double), POINTER(c_double)]
lib.find_y.restype = c_int
```

Python Code (nativecode.py)

```
def find_y_py(x1, y1, x2, d):
    """Wrapper around the C function. Returns a list of real solutions (
        possibly empty)."""
    out1 = c_double()
    out2 = c_double()
    n = lib.find_y(x1, y1, x2, d, byref(out1), byref(out2))
    if n == 0:
        return []
    elif n == 1:
        return [out1.value]
    else:
        return [out1.value, out2.value]
```

Python Code (nativecode.py)

```
def plot_solution(x1, y1, x2, sols, d, savefile='findy_plot.png'):
    """Plot circle (center P, radius d), the vertical x=x2 line and solution
        points."""
    theta = np.linspace(0, 2*np.pi, 400)
    circ_x = x1 + d * np.cos(theta)
    circ_y = y1 + d * np.sin(theta)

    plt.figure(figsize=(6,6))
    plt.plot(circ_x, circ_y, label=f'Circle center P({x1},{y1}), r={d}')
    plt.axvline(x=x2, linestyle=':', label=f'x = {x2}')

    # plot P
    plt.scatter([x1], [y1], zorder=5)
    plt.annotate(f'P({x1},{y1})', (x1, y1), textcoords="offset points",
        xytext=(6,6))
```

Python Code (nativecode.py)

```
# plot Q solutions
for i, yq in enumerate(sols):
    plt.scatter([x2], [yq], zorder=6)
    plt.annotate(f'Q{i+1}({x2},{yq})', (x2, yq), textcoords="offset-
        points", xytext=(6,6))
    # line between P and Q
    plt.plot([x1, x2], [y1, yq], linestyle='--')

plt.gca().set_aspect('equal', 'box')
plt.grid(True)
plt.legend()
plt.title(f'Solutions y = {sols}' if sols else 'No real solution')
plt.savefig(savefile, dpi=150)
plt.show()
```

Python Code (nativecode.py)

```
if __name__ == '__main__':
    # Usage:
    # python3 findy_plot.py x1 y1 x2 d
    # If no args supplied, a default example is used: P(2,-3), x2=10, d
        =10
    if len(sys.argv) == 5:
        x1 = float(sys.argv[1])
        y1 = float(sys.argv[2])
        x2 = float(sys.argv[3])
        d = float(sys.argv[4])
    else:
        x1, y1, x2, d = 2.0, -3.0, 10.0, 10.0
        print("No arguments provided - using default example P(2,-3), x2
            =10, d=10.")
        print("To specify custom values run: python3 findy_plot.py x1 y1
            x2 d")
```

Python Code (nativecode.py)

```
sols = find_y_py(x1, y1, x2, d)
if not sols:
    print("No real solutions for the given inputs.")
else:
    print("y solutions:", sols)

plot_solution(x1, y1, x2, sols, d)
```