2.5.4

EE25BTECH11050-Hema Havil

Question:

If $\mathbf{a} = 2\hat{i} + y\hat{j} + \hat{k}$ and $\mathbf{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are two vectors for which the vector $(\mathbf{a} + \mathbf{b})$ is perpendicular to the vector $(\mathbf{a} - \mathbf{b})$, then find all the possible values of y.

Solution:
Let the given vectors be :

$$\mathbf{a} = \begin{pmatrix} 2 \\ y \\ 1 \end{pmatrix} \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \tag{0.1}$$

Given that $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ are perpendicular, then

$$(\mathbf{a} + \mathbf{b})^T (\mathbf{a} - \mathbf{b}) = 0 \tag{0.2}$$

$$\mathbf{a}^T \mathbf{a} - \mathbf{b}^T \mathbf{b} = 0 \tag{0.3}$$

$$\mathbf{a}^T \mathbf{a} = \mathbf{b}^T \mathbf{b} \tag{0.4}$$

The values of $\mathbf{a}^T \mathbf{a}$ and $\mathbf{b}^T \mathbf{b}$ can be calculated by,

$$\mathbf{a}^T \mathbf{a} = \begin{pmatrix} 2 & y & 1 \end{pmatrix} \begin{pmatrix} 2 \\ y \\ 1 \end{pmatrix} = 4 + y^2 + 1 = 5 + y^2 \tag{0.5}$$

$$\mathbf{b}^T \mathbf{b} = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 1 + 4 + 9 = 14 \tag{0.6}$$

From equation 0.4,

$$5 + y^2 = 14 \tag{0.7}$$

$$y^2 = 9 \tag{0.8}$$

$$y = \pm 3 \tag{0.9}$$

Therefore the values of y are 3 and -3

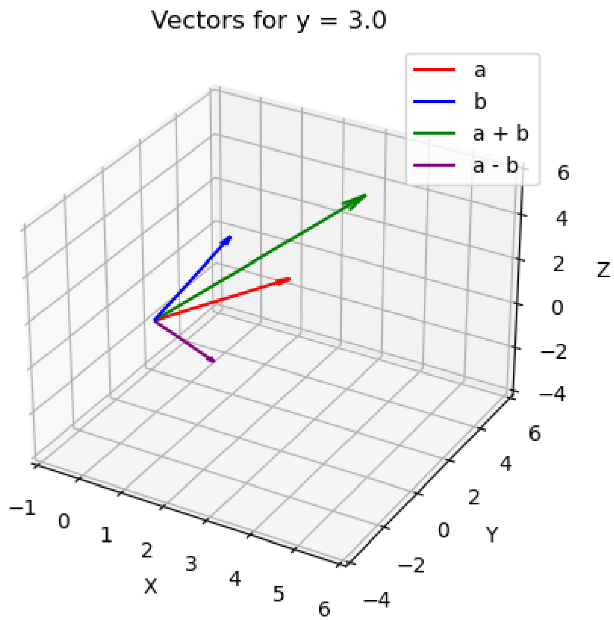

Fig. 0.1: Plot of vectors when y=3

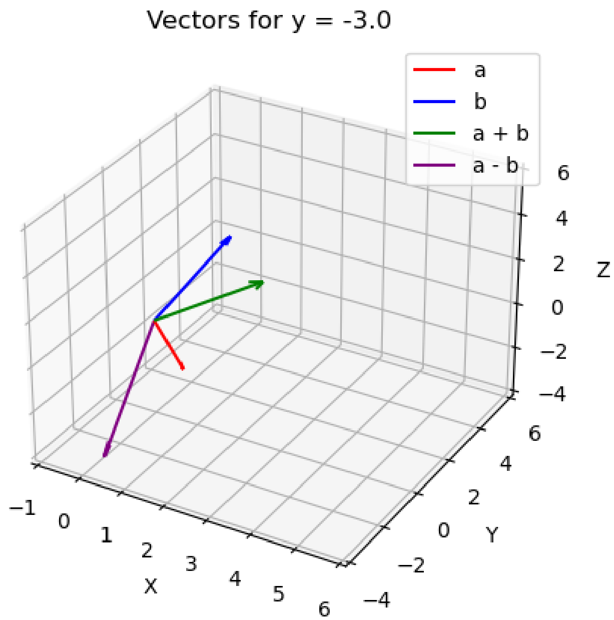

Fig. 0.2: Plot of the vectors when y=-3