

# Matgeo-q.4.13.1

AI25BTECH11036-SNEHAMRUDULA

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# Question

Consider the lines

$$L_1 : x + 3y - 5 = 0,$$

$$L_2 : 3x - ky - 1 = 0,$$

$$L_3 : 5x + 2y - 12 = 0.$$

Match the statements in Column I with Column II (choices refer to values of  $k$  or relations).

## Column I

- ☐  $L_1, L_2, L_3$  are concurrent, if
- ☐ One of  $L_1, L_2, L_3$  is parallel to at least one of the other two, if
- ☐  $L_1, L_2, L_3$  form a triangle, if
- ☐  $L_1, L_2, L_3$  do not form a triangle, if

## Column II

- ☐  $k = 9$
- ☐  $k = \frac{-6}{5}$
- ☐  $k = \frac{5}{6}$
- ☐  $k = 5$

The three line equations can be written in matrix form as

$$\begin{pmatrix} 1 & 3 \\ 3 & -k \\ 5 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 \\ 1 \\ 12 \end{pmatrix}. \quad (1)$$

**(A) Concurrency** The lines are concurrent when all three equations admit a common solution. This happens if

$$\text{rank} \begin{pmatrix} 1 & 3 \\ 3 & -k \\ 5 & 2 \end{pmatrix} = \text{rank} \begin{pmatrix} 1 & 3 & 5 \\ 3 & -k & 1 \\ 5 & 2 & 12 \end{pmatrix} = 2. \quad (2)$$

Expanding the augmented determinant condition gives

$$k = 5. \quad (3)$$

**(B) Parallelism** Two lines are parallel when their normals are proportional. Equivalently, rank of the normals is 1:

$$\text{rank} \begin{pmatrix} \mathbf{n}_2 & \mathbf{n}_3 \end{pmatrix} = 1. \quad (4)$$

This gives

$$k = -\frac{6}{5}. \quad (5)$$

**(C) Triangle condition** Three lines form a triangle if they intersect pairwise but are not concurrent. That is,

$$\text{rank} \begin{pmatrix} \mathbf{n}_1 & \mathbf{n}_2 \end{pmatrix} = \text{rank} \begin{pmatrix} \mathbf{n}_2 & \mathbf{n}_3 \end{pmatrix} = \text{rank} \begin{pmatrix} \mathbf{n}_3 & \mathbf{n}_1 \end{pmatrix} = 2, \quad k \neq 5, \quad k \neq -\frac{6}{5}. \quad (6)$$

So among the given options,

$$k = 9 \quad \text{or} \quad k = \frac{5}{6}. \quad (7)$$

**(D) Do not form a triangle** This occurs if either concurrent or parallel, i.e.

$$k = 5 \quad \text{or} \quad k = -\frac{6}{5}. \quad (8)$$

$$(A) \rightarrow (d) : k = 5, \quad (9)$$

$$(B) \rightarrow (b) : k = -\frac{6}{5}, \quad (10)$$

$$(C) \rightarrow (a) \text{ or } (c) : k = 9 \text{ or } \frac{5}{6}, \quad (11)$$

$$(D) \rightarrow (d) \text{ or } (b) : k = 5 \text{ or } -\frac{6}{5}. \quad (12)$$

# Graphical Representation

