Question 2.7.12

AI25BTECH11040 - Vivaan Parashar

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Question:

Find the area of the triangle formed by joining the midpoints of the sides of the triangle ABC, whose vertices are $A(0, -1)$, $B(2, 1)$, and $C(0, 3)$

Solution:

Let us start by finding the midpoints, let's call them D, E and F. The midpoint formula is: (Here the vectors represent position vectors of the points from the origin)

$$\mathbf{D} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{1}$$

$$\mathbf{E} = \frac{\mathbf{B} + \mathbf{C}}{2} \tag{2}$$

$$\mathbf{F} = \frac{\mathbf{C} + \mathbf{A}}{2} \tag{3}$$

$$\therefore \mathbf{D} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{F} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4}$$

Now the area formula for a triangle with vertices at **P**, **Q** and **R** is given by:

$$\text{Area} = \frac{1}{2}\left|(\mathbf{P}-\mathbf{Q})\times(\mathbf{P}-\mathbf{R})\right| \tag{5}$$

$$\therefore \text{Area of } \triangle DEF = \frac{1}{2}\left|(\mathbf{D}-\mathbf{E})\times(\mathbf{D}-\mathbf{F})\right| \tag{6}$$

$$= \frac{1}{2}\left|\left(\frac{\mathbf{A}+\mathbf{B}}{2}-\frac{\mathbf{B}+\mathbf{C}}{2}\right)\times\left(\frac{\mathbf{A}+\mathbf{B}}{2}-\frac{\mathbf{C}+\mathbf{A}}{2}\right)\right| \tag{7}$$

$$= \frac{1}{2}\left|\left(\begin{pmatrix}1\\0\end{pmatrix}-\begin{pmatrix}1\\2\end{pmatrix}\right)\times\left(\begin{pmatrix}1\\0\end{pmatrix}-\begin{pmatrix}0\\1\end{pmatrix}\right)\right| \tag{8}$$

$$= \frac{1}{2}\left|\left(\begin{pmatrix}0\\-2\end{pmatrix}\right)\times\left(\begin{pmatrix}1\\-1\end{pmatrix}\right)\right| \tag{9}$$

$$= \frac{1}{2}\left|0-2\right| = 1 \tag{10}$$

Diagram:

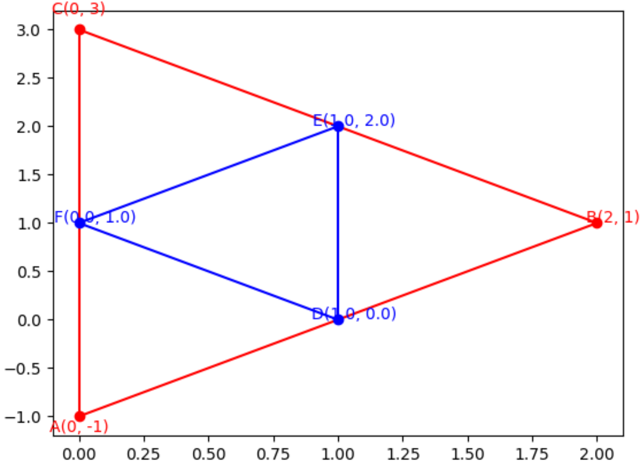

Figure: Diagram showing the triangle ABC and the triangle DEF.