5.5.29 Matgeo

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Question

If the inverse of the matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$,

then find the value of λ .

Solution

Let :

$$\mathbf{A} = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

The characteristic equation for a matrix $\vec{A}$ is

$$f(\lambda) = |\mathbf{A} - \lambda\mathbf{I}| = 0 \tag{1}$$

$$f(\lambda) = \begin{vmatrix} 7-\lambda & -3 & -3 \\ -1 & 1-\lambda & 0 \\ -1 & 0 & 1-\lambda \end{vmatrix} = 0 \tag{2}$$

Solving the above equation we get :

$$\lambda^3 - 9\lambda^2 + 9\lambda - 1 = 0 \tag{3}$$

By Cayley-Hamilton theorem :

$$f(\lambda) = f(\mathbf{A}) = 0 \tag{4}$$

$$\mathbf{A}^3 - 9\mathbf{A}^2 + 9\mathbf{A} - 1 = 0 \tag{5}$$

Solution

Multiplying the equation 5 by $\mathbf{A}^{-}1$ we get :

$$\mathbf{A}^2 - 9\mathbf{A} + 9\mathbf{I} - \mathbf{A}^{-1} = 0 \tag{6}$$

$$\mathbf{A}^{-1} = \mathbf{A}^2 - 9\mathbf{A} + 9\mathbf{I} \tag{7}$$

Solving the above equation we get :

$$\mathbf{A}^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \tag{8}$$

Hence ,

$$\lambda = 4 \tag{9}$$