AI25BTECH11003 - Bhavesh Gaikwad

Question: The matrix $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$, one of the eigen values is 1. The eigen vectors corresponding to the eigne value 1 are: (CS 2016)

a)
$$\alpha \begin{pmatrix} 4 & -2 & 1 \end{pmatrix}$$
, $\alpha \neq 0$, $\alpha \epsilon \mathbb{R}$
b) $\alpha \begin{pmatrix} -4 & 2 & 1 \end{pmatrix}$, $\alpha \neq 0$, $\alpha \epsilon \mathbb{R}$
c) $\alpha \begin{pmatrix} -2 & 0 & 1 \end{pmatrix}$, $\alpha \neq 0$, $\alpha \epsilon \mathbb{R}$
d) $\alpha \begin{pmatrix} 2 & 0 & 1 \end{pmatrix}$, $\alpha \neq 0$, $\alpha \epsilon \mathbb{R}$

c)
$$\alpha (-2 \quad 0 \quad 1), \alpha \neq 0, \alpha \in \mathbb{R}$$

d)
$$\alpha (2 \ 0 \ 1), \alpha \neq 0, \alpha \in \mathbb{R}$$

Solution:

Given:
$$\lambda = 1$$
, Let $\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$

Let v be the corresponding eigenvector.

$$\Rightarrow \mathbf{A}\mathbf{v} = (1)\mathbf{v} \tag{0.1}$$

$$(\mathbf{A} - \mathbf{I})\mathbf{v} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \tag{0.2}$$

$$\begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{pmatrix} \mathbf{v} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \tag{0.3}$$

$$\therefore \mathbf{v} = \alpha \begin{pmatrix} 4 & -2 & 1 \end{pmatrix} \tag{0.4}$$

Thus, Option-A is correct.