

5.13.30

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September 30,2025

Question

Let \mathbf{A} be a square matrix all of whose entries are integers. Then which of the following is true?

- ① If $\det(\mathbf{A}) \neq \pm 1$, then \mathbf{A}^{-1} exists but all its entries are not necessarily integers
- ② If $\det(\mathbf{A}) \neq \pm 1$, then \mathbf{A}^{-1} exists and all its entries are non-integers
- ③ If $\det(\mathbf{A}) = \pm 1$, then \mathbf{A}^{-1} exists but all its entries are integers
- ④ If $\det(\mathbf{A}) = \pm 1$, then \mathbf{A}^{-1} need not exist

Solution

We will proceed by checking each option.

A)

Let us take a square matrix \mathbf{A} having all integer entries. Let rows R_1 and R_2 be equal. By performing row operation $R_1 \rightarrow R_1 - R_2$, all elements in R_1 become 0. Therefore, $|\mathbf{A}| = 0$. We know that if $|\mathbf{A}| = 0$, \mathbf{A}^{-1} does not exist. Therefore, this option is wrong.

B)

This option is wrong according to the previous reasoning.

D)

We know that if $|\mathbf{A}| \neq 0$, \mathbf{A}^{-1} exists. By this logic, this option is wrong.

Therefore, the correct answer is C).