## 7.4.11

## EE25BTECH11004 - Aditya Appana

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## Question

If the chord y = mx + 1 of the circle  $x^2 + y^2 = 1$  subtends an angle of measure  $45^\circ$ at the major segment of the circle then the value of m is

a) 
$$2 \pm \sqrt{2}$$

b) 
$$-2 \pm \sqrt{2}$$

c) 
$$-1 \pm \sqrt{2}$$

a)  $2 \pm \sqrt{2}$  b)  $-2 \pm \sqrt{2}$  c)  $-1 \pm \sqrt{2}$  d) none of these

## **Solution**

The given line subtends an angle 45° at the major segment of the circle. Therefore, it will subtend  $2 \times 45^{\circ} = 90^{\circ}$  at the centre of the circle.

The line y = mx + 1 can be expressed as:

This line always passes through  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ , which lies on the circle  $x^2 + y^2 = 1$  (since  $0^2 + 1^2 = 1$ ). Therefore one point of intersection is  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ .

Let the other point of intersection be **P**. **P** will be a  $\pm 90^{\circ}$  rotation of  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$  about the origin.

The rotation matrix is:

$$\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \tag{2}$$

Therefore:

$$\mathbf{P} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} \cos(\pm 90^\circ) & \sin(\pm 90^\circ) \\ -\sin(\pm 90^\circ) & \cos(\pm 90^\circ) \end{pmatrix}$$
(3)

$$\mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} -1 \\ 0 \end{pmatrix} \tag{4}$$

If 
$$\mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, m = \frac{0-1}{1-0} = -1$$
 (5)

If 
$$\mathbf{P} = \begin{pmatrix} -1\\0 \end{pmatrix}, m = \frac{0-1}{-1-0} = 1$$
 (6)

$$m = \pm 1 \tag{7}$$

Therefore, **d**) is the correct answer.

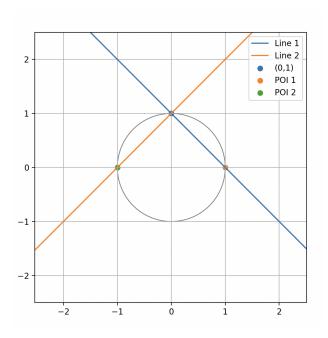


Figure 1: Plot