3.2.30

AI25BTECH11014 - Gooty Suhas

Problem

Construct a triangle $\triangle ABC$ given:

$$\angle B = 105^\circ, \quad \angle C = 90^\circ, \quad AB + BC + CA = 10 \text{ cm}$$

Matrix Formulation

Let the side lengths be:

$$\mathbf{x} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Define the system:

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & \cos C & \cos B \\ 0 & \sin C & -\sin B \end{bmatrix} \mathbf{x} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

Substitute:

$$\cos C = 0, \quad \sin C = 1, \quad \cos B = \cos(105^\circ), \quad \sin B = \sin(105^\circ)$$

Numerically:

$$\cos(105^\circ) \approx -0.2588, \quad \sin(105^\circ) \approx 0.9659$$

So the system becomes:

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & -0.2588 \\ 0 & 1 & -0.9659 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

Solution

Solving the matrix system:

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -1.52 \\ 5.66 \\ 5.86 \end{bmatrix}$$

Conclusion

Since $a < 0$, the triangle is not physically constructible.

Construction is not possible.