1.8.19

EE25BTECH11002 - Achat Parth Kalpesh

Question:
If $\mathbf{Q}(0,1)$ is equidistant from $\mathbf{P}(5,-3)$ and $\mathbf{R}(x,6)$, find the values of x. Also find the distances **QR** and **PR**.

Solution:

Let the given points be represented by the column vectors **P**, **Q**, and **R**.

$$\mathbf{P} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} x \\ 6 \end{pmatrix} \tag{0.1}$$

According to given condition;

$$\|\mathbf{P}-\mathbf{Q}\|^2 = \|\mathbf{R}-\mathbf{Q}\|^2 \tag{0.2}$$

The squared norm of a vector $\mathbf{v}$ is given by the matrix product $\mathbf{v}^T\mathbf{v}$.

$$(\mathbf{P}-\mathbf{Q})^T(\mathbf{P}-\mathbf{Q}) = (\mathbf{R}-\mathbf{Q})^T(\mathbf{R}-\mathbf{Q}) \tag{0.3}$$

$$\mathbf{P}-\mathbf{Q} = \begin{pmatrix} 5-0 \\ -3-1 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix} \tag{0.4}$$

$$\mathbf{R}-\mathbf{Q} = \begin{pmatrix} x-0 \\ 6-1 \end{pmatrix} = \begin{pmatrix} x \\ 5 \end{pmatrix} \tag{0.5}$$

Substituting these into Equation (0.3) and performing the matrix multiplication:

$$\begin{pmatrix} 5 & -4 \end{pmatrix}\begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} x & 5 \end{pmatrix}\begin{pmatrix} x \\ 5 \end{pmatrix} \tag{0.6}$$

$$(5)(5)+(-4)(-4) = (x)(x)+(5)(5) \tag{0.7}$$

$$25+16 = x^2+25 \tag{0.8}$$

$$x^2 = 16 \tag{0.9}$$

$$\implies x = \pm 4 \tag{0.10}$$

Therefore, the two possible vectors for **R** are:

$$\mathbf{R}_1 = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \tag{0.11}$$

$$\mathbf{R}_2 = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{0.12}$$

Distance **QR**:

$$\|\mathbf{Q} - \mathbf{R}\| = \|\mathbf{P} - \mathbf{Q}\| = \sqrt{5^2 + (-4)^2} = \sqrt{41} \approx 6.40 \tag{0.13}$$

Distance **PR**:

- For $\mathbf{R}_1$:

$$\|\mathbf{R_1} - \mathbf{P}\| = \left\| \begin{pmatrix} 4-5 \\ 6-(-3) \end{pmatrix} \right\| = \left\| \begin{pmatrix} -1 \\ 9 \end{pmatrix} \right\| \tag{0.14}$$

$$= \sqrt{(-1)^2 + 9^2} = \sqrt{82} \approx 9.06 \tag{0.15}$$

- For $\mathbf{R}_2$:

$$\|\mathbf{R}_2 - \mathbf{P}\| = \left\| \begin{pmatrix} -4-5 \\ 6-(-3) \end{pmatrix} \right\| = \left\| \begin{pmatrix} -9 \\ 9 \end{pmatrix} \right\| \tag{0.16}$$

$$= \sqrt{(-9)^2 + 9^2} = \sqrt{162} = 9\sqrt{2} \approx 12.73 \tag{0.17}$$

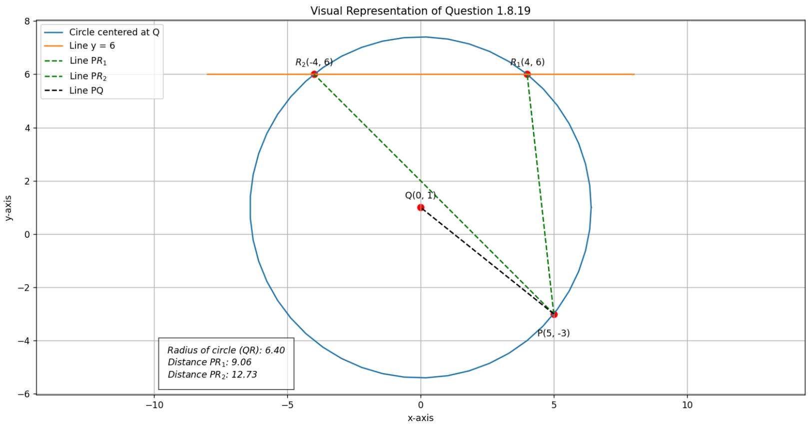

Fig. 0.1: Visual representation of the solution. The points R_1 and R_2 are the intersections of the circle centered at Q and the line $y = 6$.