2.10.41

EE25BTECH11012-BEERAM MADHURI

Question:
Let the vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and $\mathbf{d}$ be such that $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) = \mathbf{0}$. Let A and B be planes determined by the pairs of vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}, \mathbf{d}$ respectively. Then the angle between A and B is

a) 0
b) $\frac{\pi}{4}$
c) $\frac{\pi}{3}$
d) $\frac{\pi}{2}$

Solution:
given,

$$(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{d}) = 0 \tag{0.1}$$

$\Rightarrow$ angle between $\mathbf{a} \times \mathbf{b}$ and $\mathbf{c} \times \mathbf{d}$ is 0

$$\therefore \mathbf{a} \times \mathbf{b} \parallel \mathbf{c} \times \mathbf{d} \tag{0.2}$$

Given that,
plane A is determined by $\mathbf{a}, \mathbf{b}$
plane B is determined by $\mathbf{c}, \mathbf{d}$

normals to planes A and B:

$$n_A = \mathbf{a} \times \mathbf{b} \tag{0.3}$$
$$n_B = \mathbf{c} \times \mathbf{d} \tag{0.4}$$

Angle between Planes A and B = Angle between Normals n_A and n_B

$$\mathbf{a} \times \mathbf{b} \parallel \mathbf{c} \times \mathbf{d} \tag{0.5}$$
$$\therefore n_A \parallel n_B \tag{0.6}$$
$$\therefore planeA \parallel planeB \tag{0.7}$$

Hence, Angle between the planes is 0.
option (a).

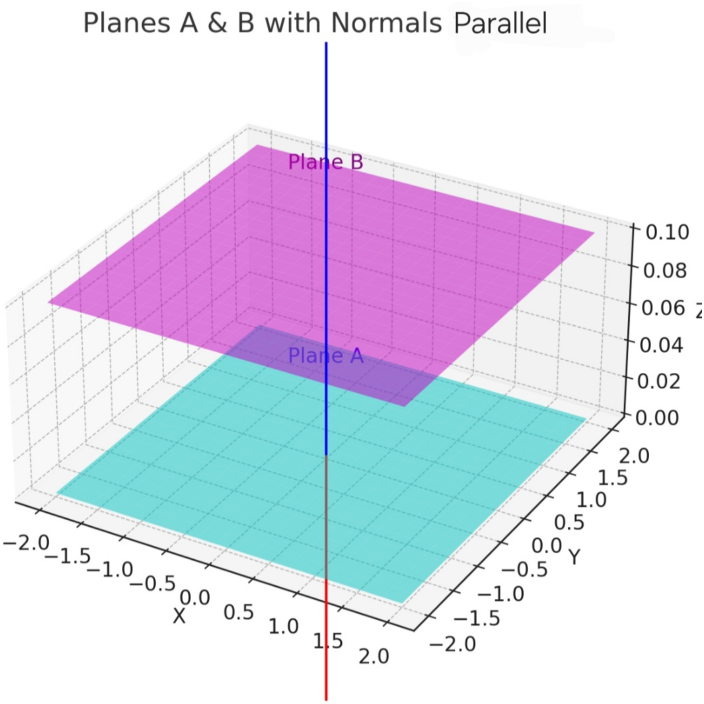

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Fig. 0.1: Planes A and B