Matgeo Presentation - Problem 12.69

ee25btech11056 - Suraj.N

September 30, 2025

Problem Statement

Find the **condition number** for the matrix

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$$

Data

Name	Value
$\mathbf{A}$	$\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}$

Table : Matrix

The **condition number** of a matrix measures how sensitive the solution of a linear system involving that matrix is to small changes or errors in the input data. More precisely, it is the ratio of the largest singular value of the matrix to the smallest singular value

$$\kappa(\mathbf{A}) = \frac{\sigma_{\text{max}}(\mathbf{A})}{\sigma_{\text{min}}(\mathbf{A})} \tag{0.1}$$

Solution

SVD / singular-value method

Calculate $\mathbf{A}^\top\mathbf{A}$

$$\mathbf{A}^\top = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix} \tag{0.2}$$

$$\mathbf{A}^\top\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 2 & 10 \end{pmatrix} \tag{0.3}$$

Then , find the eigen values of $\mathbf{A}^\top\mathbf{A}$

$$\left|\mathbf{A}^\top\mathbf{A} - \lambda\mathbf{I}\right| = 0 \tag{0.4}$$

$$\begin{vmatrix} 4-\lambda & 2 \\ 2 & 10-\lambda \end{vmatrix} = 0 \tag{0.5}$$

$$\begin{vmatrix} 4-\lambda & 2 \\ 2 & 10-\lambda \end{vmatrix} \xleftrightarrow{R_2 \to R_2 - \frac{2}{(4-\lambda)}R_1} \begin{vmatrix} 4-\lambda & 2 \\ 0 & \frac{(4-\lambda)(10-\lambda)-4}{(4-\lambda)} \end{vmatrix} = 0 \tag{0.6}$$

Solution

By calculating the determinant

$$(4-\lambda)(10-\lambda)-4=0 \tag{0.7}$$

$$\lambda^2-14\lambda+36=0 \tag{0.8}$$

The eigenvalues are

$$\lambda_i=\frac{14\pm\sqrt{196-144}}{2}=\frac{14\pm\sqrt{52}}{2}=7\pm\sqrt{13} \tag{0.9}$$

So,

$$\lambda_1=7+\sqrt{13} \qquad \lambda_2=7-\sqrt{13} \tag{0.10}$$

The singular values are

$$\sigma_{\text{max}}=\sqrt{7+\sqrt{13}} \qquad \sigma_{\text{min}}=\sqrt{7-\sqrt{13}} \tag{0.11}$$

Solution

Finally, the **condition number** is

$$\kappa(\mathbf{A}) = \frac{\sigma_{\max}(\mathbf{A})}{\sigma_{\min}(\mathbf{A})} = \sqrt{\frac{7+\sqrt{13}}{7-\sqrt{13}}} = 1.768 \tag{0.12}$$

The **condition number** of **A** is

$$\kappa(\mathbf{A}) = 1.768 \tag{0.13}$$