

# 4.3.25

EE25BTECH11036 - M Chanakya Srinivas

## PROBLEM

Find the ratio in which the  $YZ$  plane divides the line segment joining the points

$$\mathbf{A} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 3 \\ -5 \\ 8 \end{pmatrix}.$$

## SOLUTION

### Step 1: Vector and Matrix Forms

a) *Line segment in vector form::* The line joining  $\mathbf{A}$  and  $\mathbf{B}$  can be expressed as

$$\mathbf{R} = \mathbf{A} + \lambda(\mathbf{B} - \mathbf{A}) \quad (1)$$

b) *Parametric form as a vector::* Using  $\lambda$  as a parameter,

$$\mathbf{R} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (2)$$

c)  *$YZ$ -plane as a matrix equation::*

$$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \mathbf{R} = 0 \quad (3)$$

### Step 2: Symbolic intersection using vectors

d) *Intersection point  $\mathbf{P}$ ::* Solve symbolically using

$$\mathbf{P} = \mathbf{A} + \lambda(\mathbf{B} - \mathbf{A}) \quad (4)$$

$$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \mathbf{P} = 0 \quad (5)$$

e) *Ratio along the line::* Using parameter  $\lambda$ , the ratio is

$$AP : PB = \lambda : (1 - \lambda) \quad (6)$$

*Step 3: Algebraic Substitution (Numerical)*

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ -5 \\ 8 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 5 \\ -9 \\ 1 \end{pmatrix} \quad (7)$$

$$\mathbf{R} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -9 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 + 5\lambda \\ 4 - 9\lambda \\ 7 + \lambda \end{pmatrix} \quad (8)$$

$$(1 \ 0 \ 0)\mathbf{R} = -2 + 5\lambda = 0 \quad (9)$$

$$\lambda = \frac{2}{5} \quad (10)$$

$$\mathbf{P} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} + \frac{2}{5} \begin{pmatrix} 5 \\ -9 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2/5 \\ 37/5 \end{pmatrix} \approx \begin{pmatrix} 0 \\ 0.4 \\ 7.4 \end{pmatrix} \quad (11)$$

$$AP : PB = 2 : 3 \quad (12)$$

**ANSWER**

The  $YZ$ -plane divides the line segment  $AB$  internally in the ratio

$$\boxed{2 : 3}$$

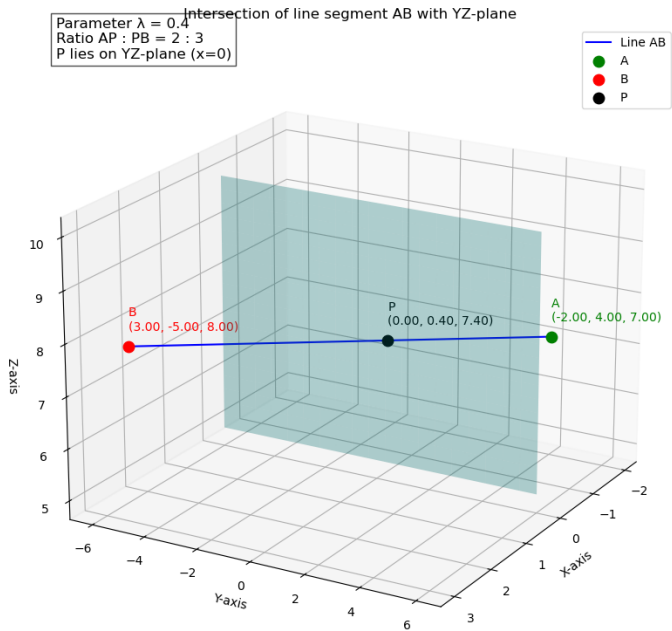


Fig. 1: 2D representation of the line segment and intersection with YZ-plane

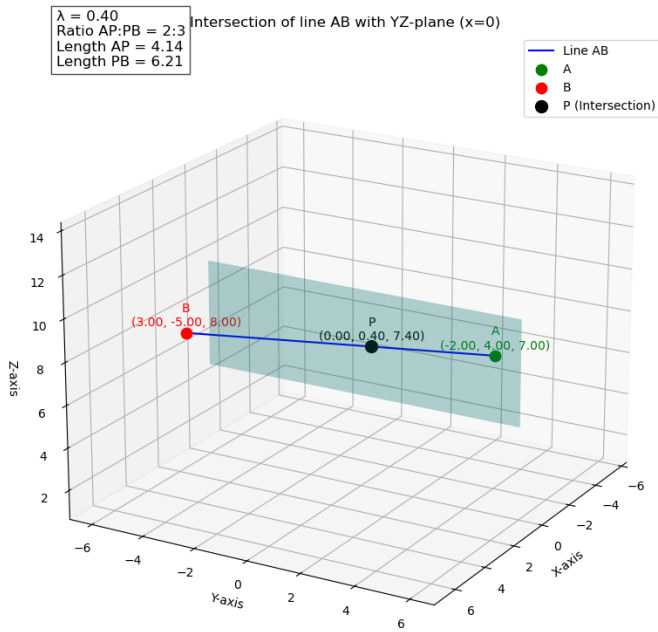


Fig. 2: 2D plot of intersection point on the YZ-plane