8.2.27

EE25BTECH11060 - V.Namaswi

Question

Find Equation of curve whose Focus is $(0,-3)$ and Directrix y=3

Solution

General Equation of a conic is given by

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

where

$$\mathbf{V} = \|\mathbf{n}\|^2\mathbf{I} - e^2\mathbf{n}\mathbf{n}^\top \tag{2}$$

$$\mathbf{u} = ce^2\mathbf{n} - \|\mathbf{n}\|^2\mathbf{F} \tag{3}$$

$$f = \|\mathbf{n}\|^2\|\mathbf{F}\|^2 - c^2e^2 \tag{4}$$

Given,

$$e = 1 \quad \mathbf{F} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$$

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad c = 3$$

From given equations,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{5}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{6}$$

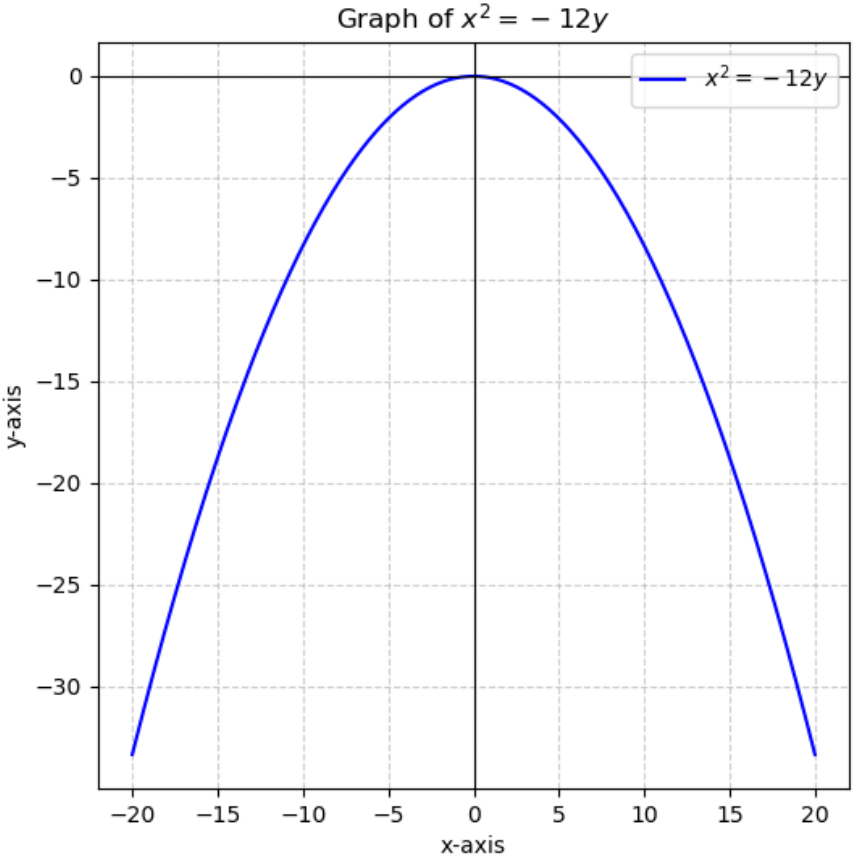
$$\mathbf{u} = 3\begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ -3 \end{pmatrix} \tag{7}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \tag{8}$$

$$f = 9 - 9 = 0 \tag{9}$$

From equation 1,

$$\mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\mathbf{x} + 2\begin{pmatrix} 0 \\ 6 \end{pmatrix}^\top \mathbf{x} = 0 \tag{10}$$


Graph of $x^2 = -12y$
$x^2 = -12y$
0
−5
−10
−15
−20
−25
−30
y-axis
−20
−15
−10
−5
0
5
10
15
20
x-axis

(11)