4.11.18

EE25BTECH11041 - Naman Kumar

Question:
Find the equation of the plane which contains the line of intersection of the planes $\mathbf{r}\cdot(\imath - 2\jmath + 3\hat{k}) - 4 = 0$ and $\mathbf{r}\cdot(-2\imath + \jmath + \hat{k}) + 5 = 0$ and whose intercept on X axis is equal to that of on Y axis.

Solution:
Given Planes,

$$\mathbf{n_1}^T\mathbf{x} = c_1, \mathbf{n_2}^T\mathbf{x} = c_2 \tag{1}$$

Where

$$\mathbf{n_1} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \mathbf{n_2} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, c_1 = 4, c_2 = -5 \tag{2}$$

Let Required equation of plane

$$\mathbf{n_3}^T\mathbf{x} = c_3 \tag{3}$$

Since we can write,

$$P_3 = P_1 - \lambda P_2 \text{ (Where } P_1, P_2, P_3 \text{ are equation of planes)} \tag{4}$$

Because All three planes intersect at same line,Therefore

$$(\mathbf{n_1} - \lambda\mathbf{n_2})^T\mathbf{x} = c_1 - \lambda c_2 \tag{5}$$

$$\tag{6}$$

Given,

$$X - intercept = Y - intercept \tag{7}$$

$$\tag{8}$$

for X-intercept

$$(\mathbf{n_1} - \lambda\mathbf{n_2})^T\begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} = c_1 - \lambda c_2 \tag{9}$$

$$(\mathbf{n_1} - \lambda\mathbf{n_2})^T x\mathbf{e_1} = c_1 - \lambda c_2 \tag{10}$$

Therefore,

$$X - intercept = \frac{c_1 - \lambda c_2}{(\mathbf{n_1} - \lambda\mathbf{n_2})^T\mathbf{e_1}} \tag{11}$$

Similarly

$$Y - intercept = \frac{c_1 - \lambda c_2}{(\mathbf{n_1} - \lambda\mathbf{n_2})^T\mathbf{e_2}} \tag{12}$$

Comparing equations (11) and (12)

$$\frac{c_1 - \lambda c_2}{(\mathbf{n_1} - \lambda \mathbf{n_2})^T \mathbf{e_1}} = \frac{c_1 - \lambda c_2}{(\mathbf{n_1} - \lambda \mathbf{n_2})^T \mathbf{e_2}} \tag{13}$$

$$(\mathbf{n_1} - \lambda \mathbf{n_2})^T \mathbf{e_1} = (\mathbf{n_1} - \lambda \mathbf{n_2})^T \mathbf{e_2} \tag{14}$$

$$\mathbf{n_1}^T \mathbf{e_1} - \lambda \mathbf{n_2}^T \mathbf{e_1} = \mathbf{n_1}^T \mathbf{e_2} - \lambda \mathbf{n_2}^T \mathbf{e_2} \tag{15}$$

$$\lambda \mathbf{n_2}^T \mathbf{e_2} - \lambda \mathbf{n_2}^T \mathbf{e_1} = \mathbf{n_1}^T \mathbf{e_2} - \mathbf{n_1}^T \mathbf{e_1} \tag{16}$$

$$\lambda = \frac{\mathbf{n_1}^T \mathbf{e_2} - \mathbf{n_1}^T \mathbf{e_1}}{\mathbf{n_2}^T \mathbf{e_2} - \lambda \mathbf{n_2}^T \mathbf{e_1}} \tag{17}$$

$$\lambda = \frac{\mathbf{n_1^T}(\mathbf{e_2} - \mathbf{e_1})}{n_2^T(\mathbf{e_2} - \mathbf{e_1})} \tag{18}$$

$$\lambda = \frac{\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}^T \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}}{\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}^T \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}} \tag{19}$$

$$\lambda = \frac{-1 - 2}{2 + 1} \tag{20}$$

$$\lambda = -1 \tag{21}$$

Therefore equation of required plane is

$$\begin{pmatrix} 1 + 2(-1) \\ -2 - 1(-1) \\ 3 - 1(-1) \end{pmatrix}^T \mathbf{x} = 4 + 5(-1) \tag{22}$$

$$\begin{pmatrix} -1 \\ -1 \\ 4 \end{pmatrix}^T x = -1 \tag{23}$$

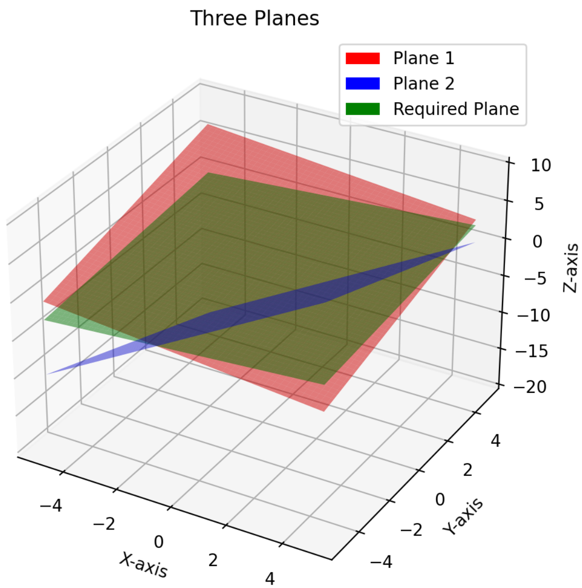
Three Planes
Plane 1
Plane 2
Required Plane
X-axis
Y-axis
Z-axis
−4
−2
0
2
4
10
5
0
−5
−10
−15
−20

Fig. 1