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EE25BTECH11032 - Kartik Lahoti

Question:

The two vectors $[1, 1, 1]$ and $\left[1, a, a^2\right]$, where $a = \left(\frac{-1}{2} + j\frac{\sqrt{3}}{2}\right)$

1) orthonormal
2) orthogonal
3) paralle
4) collinear

Solution:

Given ,

$$\mathbf{P} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{4.1}$$

$$\mathbf{Q} = \begin{pmatrix} 1 \\ a \\ a^2 \end{pmatrix} \tag{4.2}$$

Let,

$$\mathbf{z_1} = x_1 + jy_1 \longrightarrow \begin{pmatrix} x_1 & -y_1 \\ y_1 & x_1 \end{pmatrix} \tag{4.3}$$

$$\mathbf{z_2} = x_2 + jy_2 \longrightarrow \begin{pmatrix} x_2 & -y_2 \\ y_2 & x_2 \end{pmatrix} \tag{4.4}$$

Look at

$$\mathbf{z_1} + \mathbf{z_2} = (x_1 + x_2) + j\,(y_1 + y_2) \tag{4.5}$$

Which is equivalent to

$$\begin{pmatrix} x_1 & -y_1 \\ y_1 & x_1 \end{pmatrix} + \begin{pmatrix} x_2 & -y_2 \\ y_2 & x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 & -y_1 - y_2 \\ y_1 + y_2 & x_1 + x_1 \end{pmatrix} \tag{4.6}$$

Also,

$$\mathbf{z_1 z_2} = (x_1x_2 - y_1y_2) + j\,(x_1y_2 + x_2y_1) \tag{4.7}$$

This is equivalent to

$$\begin{pmatrix} x_1 & -y_1 \\ y_1 & x_1 \end{pmatrix}\begin{pmatrix} x_2 & -y_2 \\ y_2 & x_2 \end{pmatrix} = \begin{pmatrix} (x_1x_2 - y_1y_2) & -(x_1y_2 + x_2y_1) \\ (x_1y_2 + x_2y_1) & (x_1x_2 - y_1y_2) \end{pmatrix} \tag{4.8}$$

$\therefore$ Complex Numbers can be represented as this matrix form since it satisfies Addition and Multiplication properties.

$$x + jy \longrightarrow \begin{pmatrix} x & -y \\ y & x \end{pmatrix} \tag{4.9}$$

$$\therefore a = \left(\frac{-1}{2} + j\frac{\sqrt{3}}{2}\right) \longrightarrow \mathbf{A} = \begin{pmatrix} \frac{-1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \tag{4.10}$$

Similarly

$$a^2 = \left(\frac{-1}{2} - j\frac{\sqrt{3}}{2}\right) \longrightarrow \mathbf{A}^2 = \begin{pmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \tag{4.11}$$

$$1 \longrightarrow \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{4.12}$$

Now,

$$\mathbf{I} + \mathbf{A} + \mathbf{A^2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \frac{-1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} + \begin{pmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \tag{4.13}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{4.14}$$

$$\implies 1 + a + a^2 = 0 \tag{4.15}$$

Now, Look At ,

$$\mathbf{P}^\top\mathbf{Q} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ a \\ a^2 \end{pmatrix} = 1 + a + a^2 = 0 \tag{4.16}$$

Hence **P** and **Q** are orthogonal.
Answer : Option (2)