4.11.11

EE25BTECH11033 - Kavin

Question:
Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points $(-2, -5)$ and $(6, 3)$. Find the coordinates of the point of intersection.

Solution:
Given the points,

$$\mathbf{A} = \begin{pmatrix} -2 \\ -5 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 6 \\ 3 \end{pmatrix} \tag{1}$$

and the line L_1,

$$L_1 : \begin{pmatrix} 1 & -3 \end{pmatrix} \mathbf{x} = 0 \tag{2}$$

$$\implies \mathbf{n}^\top \mathbf{x} = 0 \tag{3}$$

Let the vector $\mathbf{P}$ be a point on the line $x-3y = 0$ wihch divides the line segment joining the points $\mathbf{A}$ and $\mathbf{B}$.
Section formula for a vector $\mathbf{P}$ which divides the line formed by vectors $\mathbf{A}$ and $\mathbf{B}$ in the ratio $k : 1$ is given by

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \tag{4}$$

$$\mathbf{P} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \frac{1}{k+1} \\ \frac{k}{k+1} \end{pmatrix} \tag{5}$$

Since $\mathbf{P}$ lies on line L_1,

$$\mathbf{n}^\top \mathbf{P} = 0 \tag{6}$$

$$\implies \begin{pmatrix} 1 & -3 \end{pmatrix} \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \frac{1}{k+1} \\ \frac{k}{k+1} \end{pmatrix} = 0 \tag{7}$$

$$\implies \begin{pmatrix} 1 & -3 \end{pmatrix} \begin{pmatrix} -2 & 6 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} \frac{1}{k+1} \\ \frac{k}{k+1} \end{pmatrix} = 0 \tag{8}$$

$$\implies \begin{pmatrix} 13 & -3 \end{pmatrix} \begin{pmatrix} \frac{1}{k+1} \\ \frac{k}{k+1} \end{pmatrix} = 0 \tag{9}$$

$$\implies \frac{13-3k}{k+1} = 0 \tag{10}$$

$$\implies k = \frac{13}{3} \tag{11}$$

Therefore the ratio in which **P** divides the line segment joining the points **A** and **B** is 13 : 3
On substituting the value of k in equation (5) we will get,

$$\mathbf{P} = \begin{pmatrix} -2 & 6 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} 3/16 \\ 13/16 \end{pmatrix} \tag{12}$$

$$\implies \mathbf{P} = \begin{pmatrix} 9/2 \\ 3/2 \end{pmatrix} \tag{13}$$