2.10.32

EE25BTECH11003 - Adharvan Kshathriya Bommagani

Question:
Let $\mathbf{p}$ and $\mathbf{q}$ be the position vectors of **P** and **Q** respectively, with respect to **O** and $|\mathbf{p}| = p, |\mathbf{q}| = q$. The points **R** and **S** divide PQ internally and externally in the ratio 2:3 respectively. If OR and OS are perpendicular then

a) $9p^2 = 4q^2$
b) $4p^2 = 9q^2$
c) $9p = 4q$
d) $4p = 9q$

Solution:

Since R divides PQ internally in the ratio $2 : 3$, its position vector is

$$\mathbf{R} = \frac{3\mathbf{p} + 2\mathbf{q}}{5}.$$

Since S divides PQ externally in the ratio $2 : 3$, its position vector is

$$\mathbf{S} = 3\mathbf{p} - 2\mathbf{q}.$$

Given $OR \perp OS$, we have

$$\mathbf{R}^T\mathbf{S} = 0.$$

Substitute the expressions for **R** and **S**:

$$\left(\frac{3\mathbf{p} + 2\mathbf{q}}{5}\right)^T (3\mathbf{p} - 2\mathbf{q}) = 0.$$

Multiply both sides by 5:

$$(3\mathbf{p} + 2\mathbf{q})^T (3\mathbf{p} - 2\mathbf{q}) = 0.$$

Expand:

$$9\mathbf{p}^T\mathbf{p} - 6\mathbf{p}^T\mathbf{q} + 6\mathbf{q}^T\mathbf{p} - 4\mathbf{q}^T\mathbf{q} = 0.$$

$$9\mathbf{p}^T\mathbf{p} - 4\mathbf{q}^T\mathbf{q} = 0.$$

That is,

$$9\|\mathbf{p}\|^2 - 4\|\mathbf{q}\|^2 = 0 \qquad \Longrightarrow \qquad 9p^2 = 4q^2.$$

Answer: **(a)** $9p^2 = 4q^2$

Vectors OR and OS with OR and OS:

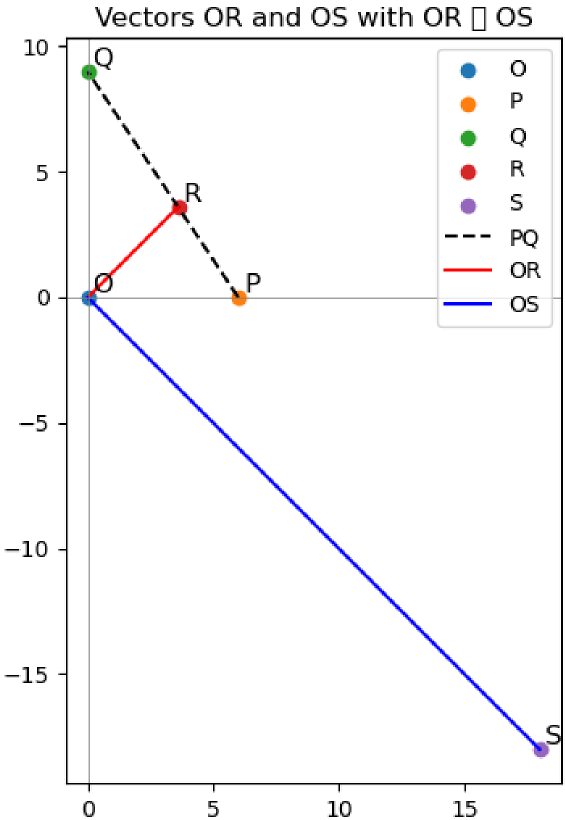

Fig. 4: Figure for 2.10.32