frame=single, breaklines=true, columns=fullflexible

Matrix 4.8.11

ai25btech11015 – M Sai Rithik

Question

Find the vector equation of the plane determined by the points

$$A(3, -1, 2), \quad B(5, 2, 4), \quad C(-1, -1, 6),$$

and hence find the distance of this plane from the origin.

Step 1: Position vectors

$$\mathbf{A} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} -1 \\ -1 \\ 6 \end{pmatrix}.$$

Step 2: Plane form

We write the plane in normalized form:

$$\mathbf{N}^{\top} x = 1,$$

where $\mathbf{N} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$.

Since A, B, C lie on the plane:

$$\mathbf{N}^{\top}\mathbf{A} = 1, \ \mathbf{N}^{\top}\mathbf{B} = 1, \ \mathbf{N}^{\top}\mathbf{C} = 1.$$

Step 3: Linear system

This gives the system

$$\begin{bmatrix} 3 & -1 & 2 \\ 5 & 2 & 4 \\ -1 & -1 & 6 \end{bmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Or in augmented form:

$$\left[\begin{array}{ccc|c} 3 & -1 & 2 & 1 \\ 5 & 2 & 4 & 1 \\ -1 & -1 & 6 & 1 \end{array}\right].$$

Step 4: Solution

Row reducing, we obtain

$$\mathbf{N} = \begin{pmatrix} \frac{1}{19} \\ -\frac{5}{57} \\ \frac{3}{19} \end{pmatrix}.$$

Hence the plane equation is

$$\frac{1}{19}x - \frac{5}{57}y + \frac{3}{19}z = 1.$$

Step 5: Distance from origin

Distance of origin $(0, 0, 0)$ from the plane

$$\mathbf{N}^\top x = 1$$

is

$$d = \frac{|\,1\,|}{\|\mathbf{N}\|} = \frac{1}{\sqrt{\left(\frac{1}{19}\right)^2 + \left(-\frac{5}{57}\right)^2 + \left(\frac{3}{19}\right)^2}}.$$

$$d = \frac{57}{\sqrt{187}}.$$