

## 5.5.18

AI25BTECH11001 - ABHISEK MOHAPATRA

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**Question:** Find the inverse of the following matrix, using elementary transformations

$$\begin{pmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{pmatrix}$$

**Solution:** Given:

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I} \quad (0.1)$$

$$\begin{pmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{pmatrix} \mathbf{A}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (0.2)$$

Augumented Matrix:

$$\left( \begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ 3 & 7 & 2 & 0 & 0 & 1 \end{array} \right) \quad (0.3)$$

$$\xrightarrow{R_2 \rightarrow R_2 - R_1} \left( \begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 3 & 7 & 2 & 0 & 0 & 1 \end{array} \right) \quad (0.4)$$

$$\xrightarrow{R_3 \rightarrow R_3 - \frac{3}{2}R_1} \left( \begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & \frac{5}{2} & \frac{1}{2} & -\frac{3}{2} & 0 & 1 \end{array} \right) \quad (0.5)$$

$$\xrightarrow{R_3 \rightarrow R_3 - \frac{5}{2}R_2} \left( \begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & \frac{1}{2} & 1 & -\frac{5}{2} & 1 \end{array} \right) \quad (0.6)$$

$$\xrightarrow{R_1 \rightarrow R_1 - 3R_2 - 2R_3} \left( \begin{array}{ccc|ccc} 2 & 0 & 0 & 2 & 2 & -2 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & \frac{1}{2} & 1 & -\frac{5}{2} & 1 \end{array} \right) \quad (0.7)$$

$$\xrightarrow{R_1 \rightarrow R_1 - 3R_2 - 2R_3} \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -5 & 2 \end{array} \right) \quad (0.8)$$

So,

$$\mathbf{A}^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{pmatrix} \quad (0.9)$$