10.6.8

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Question

Construct a pair of tangents to a circle of radius 4cm from a point P lying outside the circle at a distance of 6cm from the centre. (10, 2023)

Solution

Let the center of the circle be at origin,The equation is $x^2 + y^2 = 16$ and Point P (at distance 6 from center along x-axis)

$$O = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1}$$

$$P = \begin{pmatrix} 6 \\ 0 \end{pmatrix} \tag{2}$$

$$\vec{x}^\top \vec{V} \vec{x} + 2\vec{u}^\top \vec{x} + f = 0 \tag{3}$$

where

$$\vec{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \vec{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad f = -16 \tag{4}$$

The center and radius are:

$$\vec{c} = -\vec{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad r = r = \sqrt{\|\vec{u}\|^2 - f} = \sqrt{0 + 16} = 4 \tag{5}$$

The equation of a tangent line at a point of contact $\vec{q}$ on the circle is given by:

Solution

$$\left(\vec{V}\vec{q}+\vec{u}\right)^{\top}\vec{x}+\vec{u}^{\top}\vec{q}+f=0 \tag{6}$$

For this tangent to pass through the external point $P=\begin{pmatrix}6\\0\end{pmatrix}$, the equation must hold when $\vec{x}=P$. Since $\vec{V}=\vec{I}$ and $\vec{u}=\vec{0}$:

$$\left(\vec{I}\vec{q}\right)^{\top}P+f=0 \tag{7}$$

$$\vec{q}^{\top}P-16=0 \tag{8}$$

Let $\vec{q}=\begin{pmatrix}q_1\\q_2\end{pmatrix}$ be a point of contact. It must satisfy two conditions:

(a) $\vec{q}$ lies on the circle: $q_1^2+q_2^2=16$

(b) The tangent at $\vec{q}$ passes through P: $\vec{q}^{\top}P=16$

Solution

From condition (a) & (b):

$$\begin{pmatrix} q_1 & q_2 \end{pmatrix} \begin{pmatrix} 6 \\ 0 \end{pmatrix} = 16 \implies 6q_1 = 16 \implies q_1 = \frac{8}{3} \tag{9}$$

$$\left(\frac{8}{3}\right)^2 + q_2^2 = 16 \implies \frac{64}{9} + q_2^2 = 16 \implies q_2^2 = \frac{80}{9} \tag{10}$$

$$q_2 = \pm\frac{4\sqrt{5}}{3} \tag{11}$$

The two points of contact are:

$$\vec{q}_1 = \begin{pmatrix} \frac{8}{3} \\ \frac{4\sqrt{5}}{3} \end{pmatrix}, \quad \vec{q}_2 = \begin{pmatrix} \frac{8}{3} \\ -\frac{4\sqrt{5}}{3} \end{pmatrix} \tag{12}$$

The equation of the tangent at a point $\vec{q}$ is
$\left(\vec{V}\vec{q} + \vec{u}\right)^\top \vec{x} + \vec{u}^\top \vec{q} + f = 0.$
Tangent 1 at $\vec{q}_1$:

Solution

$$\vec{V}\vec{q}_1 = \begin{pmatrix} \frac{8}{3} \\ \frac{4\sqrt{5}}{3} \end{pmatrix} \tag{13}$$

$$\begin{pmatrix} \frac{8}{3} & \frac{4\sqrt{5}}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - 16 = 0 \tag{14}$$

$$\frac{8}{3}x + \frac{4\sqrt{5}}{3}y = 16 \tag{15}$$

$$2x + \sqrt{5}y = 12 \tag{16}$$

Tangent 2 at $\vec{q}_2$:

Solution

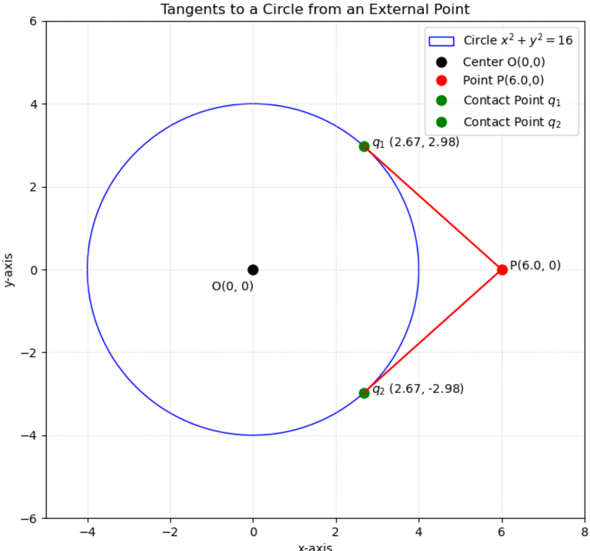
$$\begin{pmatrix} \frac{8}{3} & -\frac{4\sqrt{5}}{3} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - 16 = 0 \tag{17}$$

$$2x - \sqrt{5}y = 12 \tag{18}$$

The equations of tangents are:

$$\boxed{2x + \sqrt{5}y = 12 \quad \text{and} \quad 2x - \sqrt{5}y = 12} \tag{19}$$

Plot



Tangents to a Circle from an External Point
Circle $x^2 + y^2 = 16$
Center O(0,0)
Point P(6.0,0)
Contact Point q_1
Contact Point q_2
q_1 (2.67, 2.98)
q_2 (2.67, -2.98)
P(6.0, 0)
O(0, 0)
y-axis
x-axis