ee25btech11063-vejith

Question:

The inverse of the matrix $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ is (CH 2010)

Solution:

Let

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \tag{1}$$

The augmented matrix is

$$\begin{pmatrix} \mathbf{A} \mid \mathbf{I} \end{pmatrix} \Longrightarrow \begin{pmatrix} 1 & 2 \mid 1 & 0 \\ 3 & 4 \mid 0 & 1 \end{pmatrix} \stackrel{R_2 \to R_2 - 3R_1}{\longleftrightarrow} \begin{pmatrix} 1 & 2 \mid 1 & 0 \\ 0 & -2 \mid -3 & 1 \end{pmatrix}$$

$$\stackrel{R_1 \to R_1 + R_2}{\longleftrightarrow} \begin{pmatrix} 1 & 0 \mid -2 & 1 \\ 0 & -2 \mid -3 & 1 \end{pmatrix}$$

$$\stackrel{R_2 \to \frac{-1}{2} \times R_2}{\longleftrightarrow} \begin{pmatrix} 1 & 0 \mid -2 & 1 \\ 0 & 1 \mid \frac{3}{2} & \frac{-1}{2} \end{pmatrix}$$

$$(2)$$

$$(3)$$

$$\stackrel{R_1 \to R_1 + R_2}{\longleftrightarrow} \left(\begin{array}{cc|c} 1 & 0 & -2 & 1 \\ 0 & -2 & -3 & 1 \end{array} \right) \tag{3}$$

$$\xrightarrow{R_2 \to \frac{-1}{2} \times R_2} \begin{pmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & \frac{3}{2} & \frac{-1}{2} \end{pmatrix} \tag{4}$$

As the left block of the Augmented matrix is \boldsymbol{I} the right block is $\boldsymbol{A}^{-1}.$

$$\mathbf{A}^{-1} = \begin{pmatrix} -2 & 1\\ \frac{3}{2} & \frac{-1}{2} \end{pmatrix} \tag{5}$$