4.11.25

Revanth Siva Kumar.D - EE25BTECH11048

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Question

Find the distance of the point $\left(1, -2, 9\right)$ from the point of intersection of the line

$$\mathbf{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$$

and the plane

$$\mathbf{r} \cdot \left(\hat{i} - \hat{j} + \hat{k}\right) = 10.$$

General Formulation

The line can be written as

$$\mathbf{r} = \mathbf{r}_0 + \lambda \mathbf{d}, \tag{1}$$

where $\mathbf{r}_0$ is a point on the line, $\mathbf{d}$ is the direction vector.
The plane equation is

$$\mathbf{n}^T \mathbf{r} = c, \tag{2}$$

where $\mathbf{n}$ is the normal vector and c is a constant.

Intersection of Line and Plane

Substitute $\mathbf{r} = \mathbf{r}_0 + \lambda \mathbf{d}$ into the plane:

$$\mathbf{n}^T (\mathbf{r}_0 + \lambda \mathbf{d}) = c \tag{3}$$

$$\implies \lambda = \frac{c - \mathbf{n}^T \mathbf{r}_0}{\mathbf{n}^T \mathbf{d}}. \tag{4}$$

The intersection point is

$$\mathbf{P} = \mathbf{r}_0 + \frac{c - \mathbf{n}^T \mathbf{r}_0}{\mathbf{n}^T \mathbf{d}} \mathbf{d}. \tag{5}$$

Distance Formula

Let the external point be $\mathbf{A}$.
Then displacement vector is

$$\mathbf{v} = \mathbf{P} - \mathbf{A}.\mathbf{v} \qquad = \mathbf{r}_0 + \frac{c - \mathbf{n}^T\mathbf{r}_0}{\mathbf{n}^T\mathbf{d}}\mathbf{d} - \mathbf{A}. \tag{6}$$

Thus the required distance is

$$d = \|\mathbf{v}\| = \sqrt{\mathbf{v}^T\mathbf{v}}. \tag{7}$$

Substitution from Question

From the question:

$$\mathbf{r}_0 = \begin{pmatrix} 4 \\ 2 \\ 7 \end{pmatrix}, \qquad \mathbf{d} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}, \tag{8}$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \qquad c = 10, \tag{9}$$

$$\mathbf{A} = \begin{pmatrix} 1 \\ -2 \\ 9 \end{pmatrix}. \tag{10}$$

Substitution from Question

Compute:

$$\mathbf{n}^T\mathbf{d} = \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} = 1, \tag{11}$$

$$\mathbf{n}^T\mathbf{r}_0 = \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 7 \end{pmatrix} = 9, \tag{12}$$

$$\lambda = \frac{10-9}{1} = 1. \tag{13}$$

Final Calculation

Intersection point:

$$\mathbf{P} = \begin{pmatrix} 4 \\ 2 \\ 7 \end{pmatrix} + 1 \cdot \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \\ 9 \end{pmatrix}. \tag{14}$$

Displacement:

$$\mathbf{v} = \mathbf{P} - \mathbf{A} = \begin{pmatrix} 7 \\ 6 \\ 9 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 9 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix}. \tag{15}$$

Distance:

$$d = \sqrt{6^2 + 8^2 + 0^2} = \sqrt{100} = 10. \tag{16}$$

Conclusion

Final Answer: The required distance is

$$\boxed{10}$$

C Code

```c
#include <stdio.h>
#include <math.h>

// Function to compute distance between A(1,-2,9) and P(7,6,9)
double compute_distance() {
    double A[3] = {1, -2, 9};
    double P[3] = {7, 6, 9};
    double v[3];
    double sum = 0.0;

    for(int i=0; i<3; i++) {
        v[i] = P[i] - A[i];
        sum += v[i]*v[i];
    }

    return sqrt(sum);
}
```

Python Code (C Shared Library)

```
import ctypes

# Load the shared C library
lib = ctypes.CDLL(./points.so)

# Specify return type
lib.compute_distance.restype = ctypes.c_double

# Call the C function
distance = lib.compute_distance()

print(Distance from (1,-2,9) to intersection point (7,6,9):,
    distance)
```

Python Code (Visualization)

```
import numpy as np
import matplotlib.pyplot as plt

# Given point
A = np.array([1, -2, 9])

# Line: r = r0 + lambda*d
r0 = np.array([4, 2, 7])
d = np.array([3, 4, 2])
lambda_vals = np.linspace(-1, 2, 100)
line_points = r0.reshape(3,1) + d.reshape(3,1) * lambda_vals

# Intersection point
P = r0 + d

# Plane: x - y + z = 10
x_plane = np.linspace(-5, 10, 20)
y_plane = np.linspace(-5, 10, 20)
```

Python code

```
X, Y = np.meshgrid(x_plane, y_plane)
Z = 10 - X + Y
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Plot plane
ax.plot_surface(X, Y, Z, alpha=0.3, color='cyan', rstride=1,
    cstride=1, edgecolor='none')

# Plot line
ax.plot(line_points[0,:], line_points[1,:], line_points[2,:],
    color='blue', label=Line r=r0+d)
```

Python code

```
# Plot points
ax.scatter(A[0], A[1], A[2], color='red', s=50, label=Point A
    (1,-2,9))
ax.scatter(P[0], P[1], P[2], color='green', s=50, label=
    Intersection P(7,6,9))

# Dotted line from A to P
ax.plot([A[0], P[0]], [A[1], P[1]], [A[2], P[2]], color='magenta'
    , linestyle='--', label=Distance d)

ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title(Distance from Point to Line-Plane Intersection)
ax.legend()
plt.show()
```

Plot

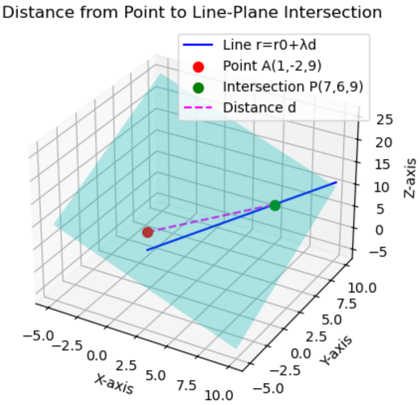

Figure: 3D visualization of point, line, plane, and distance