1.11.9

AI25BTECH11007

Question:
If

$$\mathbf{a} = \hat{\imath} - 7\hat{\jmath} + 7\hat{k} \quad \text{and} \quad \mathbf{b} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k},$$

find a unit vector perpendicular to both the vectors **a** and **b**.
Solution:
We want **n** such that

$$\mathbf{a}^T \mathbf{n} = 0, \tag{0.1}$$
$$\mathbf{b}^T \mathbf{n} = 0. \tag{0.2}$$

This system can be written as

$$\begin{pmatrix} 1 & -7 & 7 \\ 3 & -2 & 2 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{0.3}$$

The solution is given by the **null space** of the coefficient matrix. Equivalently, **n** can be expressed as the cross product of **a** and **b**:

$$\mathbf{n} = \mathbf{a} \times \mathbf{b}. \tag{0.4}$$

Using the **transpose method**, we write

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 1 \\ -7 \\ 7 \end{pmatrix} \times \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} \tag{0.5}$$

$$= \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \end{bmatrix} \begin{bmatrix} 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix}^T \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{pmatrix} 1 \\ -7 \\ 7 \end{pmatrix}. \tag{0.6}$$

Simplifying (**??**), we obtain

$$\mathbf{n} = \begin{pmatrix} 0 \\ 19 \\ 19 \end{pmatrix}. \tag{0.7}$$

Now, the unit vector is

$$\hat{n} = \frac{\mathbf{n}}{\|\mathbf{n}\|} \tag{0.8}$$

$$= \frac{1}{\sqrt{0^2 + 19^2 + 19^2}} \begin{pmatrix} 0 \\ 19 \\ 19 \end{pmatrix} \tag{0.9}$$

$$= \frac{1}{\sqrt{722}} \begin{pmatrix} 0 \\ 19 \\ 19 \end{pmatrix}. \tag{0.10}$$

Hence, the required unit vectors are

$$\hat{n} = \pm \frac{1}{\sqrt{722}} \begin{pmatrix} 0 \\ 19 \\ 19 \end{pmatrix}. \tag{0.11}$$

$$\hat{n} = \pm \frac{1}{\sqrt{2}} (\hat{j} + \hat{k}) \tag{0.12}$$

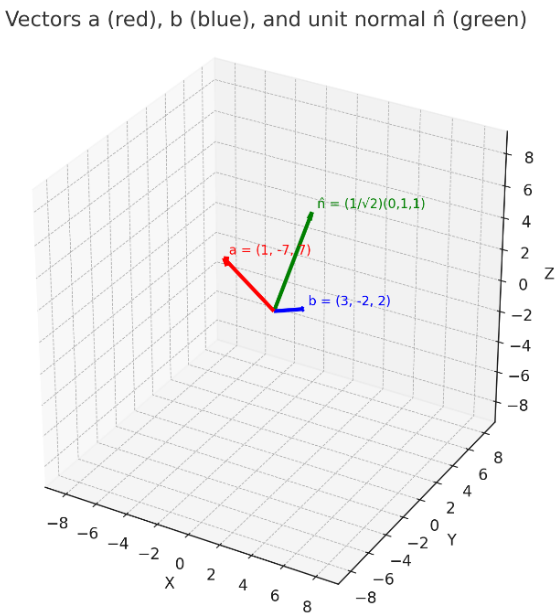

Fig. 0.1: Image Visual