

7.2.18

Pratik R-AI25BTECH11023

October 3, 2025

Question

\mathbf{A} is 2×2 with $\text{tr}(\mathbf{A}) = 5$, $\det(\mathbf{A}) = 6$. Let the characteristic polynomial of $(\mathbf{A} + \mathbf{I}_2)^{-1}$ be $x^2 - bx + c$. Find $b/c = (\text{integer})$.

Solution

Given:

$$\text{tr}(\mathbf{A}) = 5 \quad (1)$$

$$\det(\mathbf{A}) = 6 \quad (2)$$

Solution

Let the eigenvalues of \mathbf{A} be λ_1 and λ_2 :

$$\lambda_1 + \lambda_2 = 5 \quad (3)$$

$$\lambda_1 \lambda_2 = 6 \quad (4)$$

Eigenvalues of $\mathbf{A} + \mathbf{I}_2$ are:

$$\lambda_1 + 1, \quad \lambda_2 + 1 \quad (5)$$

Eigenvalues of $(\mathbf{A} + \mathbf{I}_2)^{-1}$ are:

$$\frac{1}{\lambda_1 + 1}, \quad \frac{1}{\lambda_2 + 1} \quad (6)$$

Thus, its characteristic polynomial is:

$$x^2 - \left(\frac{1}{\lambda_1 + 1} + \frac{1}{\lambda_2 + 1} \right) x + \frac{1}{(\lambda_1 + 1)(\lambda_2 + 1)} \quad (7)$$

Calculate:

$$(\lambda_1 + 1)(\lambda_2 + 1) = \lambda_1 \lambda_2 + (\lambda_1 + \lambda_2) + 1 = 12 \quad (8)$$

$$\frac{1}{\lambda_1 + 1} + \frac{1}{\lambda_2 + 1} = \frac{(\lambda_1 + 1) + (\lambda_2 + 1)}{(\lambda_1 + 1)(\lambda_2 + 1)} = \frac{7}{12} \quad (9)$$

Therefore:

$$b = \frac{7}{12}, \quad c = \frac{1}{12} \quad (10)$$

$$\frac{b}{c} = \frac{\frac{7}{12}}{\frac{1}{12}} = 7 \quad (11)$$

Hence, the answer is 7.