

2.7.17 Solution

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Problem

Show that the points $\mathbf{A} = 2\hat{i} - \hat{j} + \hat{k}$, $\mathbf{B} = \hat{i} - 3\hat{j} - 5\hat{k}$, and $\mathbf{C} = 3\hat{i} - 4\hat{j} - 4\hat{k}$ are vertices of a right-angled triangle. Find the area of the triangle.

Given Vectors

$$\mathbf{A} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 3 \\ -4 \\ -4 \end{pmatrix}$$

Check Right Angle at **A**

Side vectors:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -1 \\ -2 \\ -6 \end{pmatrix}, \quad \mathbf{C} - \mathbf{A} = \begin{pmatrix} 1 \\ -3 \\ -5 \end{pmatrix}$$

Dot product:

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{A}) = -1 + 6 + 30 = 35 \neq 0$$

No right angle at **A**.

Check Right Angle at **B**

Side vectors:

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix}, \quad \mathbf{C} - \mathbf{B} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

Dot product:

$$(\mathbf{A} - \mathbf{B})^T (\mathbf{C} - \mathbf{B}) = 2 - 2 + 6 = 6 \neq 0$$

No right angle at **B**.

Right Angle at **C**

Side vectors:

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}, \quad \mathbf{B} - \mathbf{C} = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$$

Dot product:

$$(\mathbf{A} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C}) = 0$$

Right angle at **C**.

Area Calculation

Use identity:

$$\|\mathbf{a} \times \mathbf{b}\|^2 + (\mathbf{a}^\top \mathbf{b})^2 = \|\mathbf{a}\|^2 \|\mathbf{b}\|^2$$

$$\mathbf{a} = \mathbf{A} - \mathbf{C} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}, \quad \mathbf{b} = \mathbf{B} - \mathbf{C} = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}$$

Calculate:

$$\|\mathbf{a}\|^2 = 35, \quad \|\mathbf{b}\|^2 = 6, \quad (\mathbf{a}^\top \mathbf{b})^2 = 0$$

Therefore,

$$\|\mathbf{a} \times \mathbf{b}\| = \sqrt{35 \times 6} = \sqrt{210}$$

Area:

$$= \frac{\sqrt{210}}{2}$$

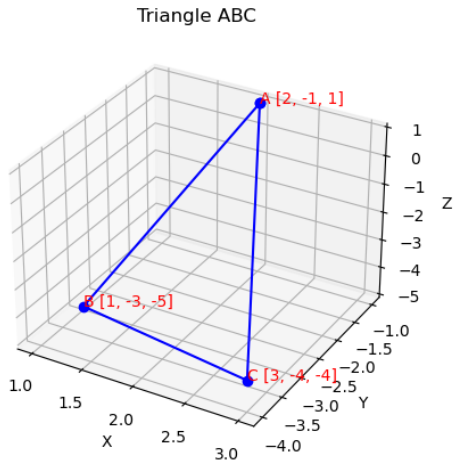


Figure: Plot

For Codes refer to the URL given below:

`https://github.com/Aditya-Mishra11005/ee1030-2025/tree/main/ee25btech11005/matgeo/2.7.17/Codes`