MATGEO Presentation: 2.9.6

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Problem Statement

$|\mathbf{a}| = 8, |\mathbf{b}| = 3,$ and $\mathbf{a} \cdot \mathbf{b} = 12\sqrt{3}$, then the value $|\mathbf{a} \times \mathbf{b}|$ is

Given data

Given:

$$\|\mathbf{a}\| = 8 \tag{3.1}$$

$$\|\mathbf{b}\| = 3 \tag{3.2}$$

$$\mathbf{a}^\top \mathbf{b} = 12\sqrt{3} \tag{3.3}$$

Formulae

We know:

$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\|\|\mathbf{b}\| \sin\theta \tag{3.4}$$

$$\cos\theta = \frac{\mathbf{a}^\top \mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|} \tag{3.5}$$

Solving

Thus

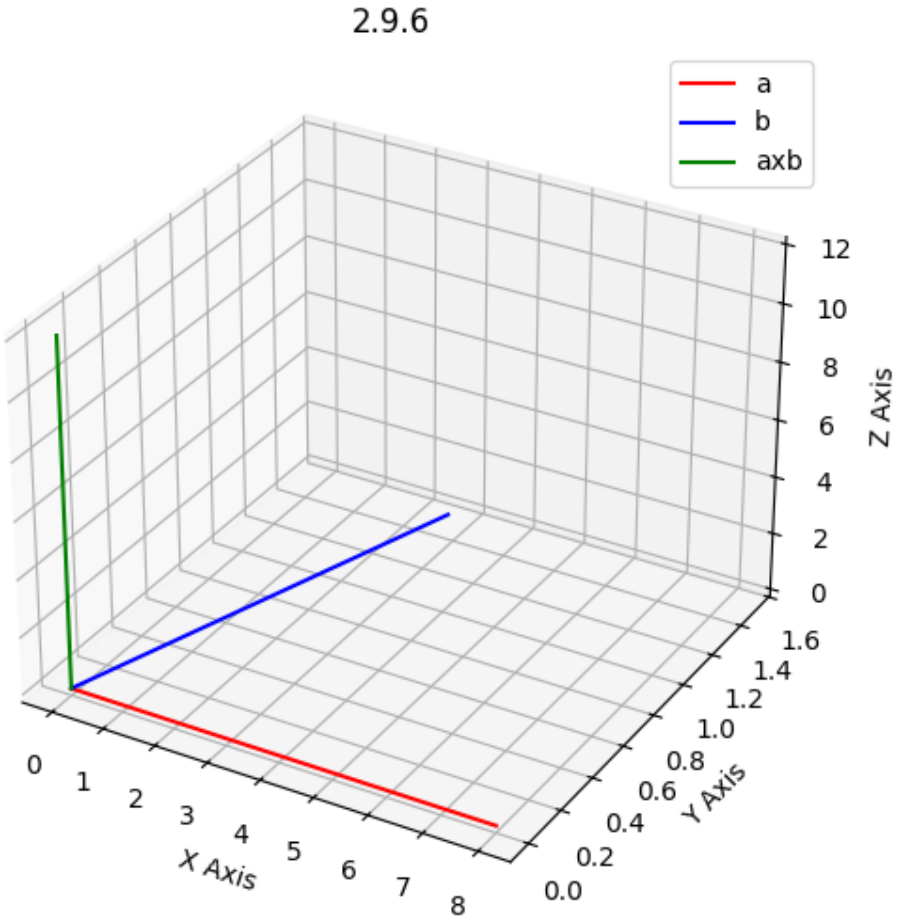
$$\left(\mathbf{a}^{\top}\mathbf{b}\right)^{2} + (\|\mathbf{a} \times \mathbf{b}\|)^{2} = \|\mathbf{a}\|^{2}\|\mathbf{b}\|^{2} \tag{3.6}$$

Substituting values

$$\|\mathbf{a} \times \mathbf{b}\| = \sqrt{64 \times 9 - 144 \times 3} \tag{3.7}$$

$$\|\mathbf{a} \times \mathbf{b}\| = 12 \tag{3.8}$$

Plot


2.9.6
a
b
axb
12
10
8
6
4
2
0
Z Axis
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Y Axis
0
1
2
3
4
5
6
7
8
X Axis

C code for generating points on line

```
void point_gen(const double* P1, const double* P2, double t, double
    * result_point) {
    result_point[0] = P1[0] + t * (P2[0] - P1[0]);
    result_point[1] = P1[1] + t * (P2[1] - P1[1]);
    result_point[2] = P1[2] + t * (P2[2] - P1[2]);
}
```

Python code for plotting using C

```
import ctypes
import numpy as np
import numpy.linalg as LA
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

libline = ctypes.CDLL("./line.so")

get_point = libline.point_gen
get_point.argtypes = [
    ctypes.POINTER(ctypes.c_double), # P1
    ctypes.POINTER(ctypes.c_double), # P2
    ctypes.c_double, # t
    ctypes.POINTER(ctypes.c_double), # result_point
]
get_point.restype = None
```

```
DoubleArray3 = ctypes.c_double * 3
o = DoubleArray3(0, 0, 0)
a = DoubleArray3(8, 0, 0)
b = DoubleArray3((27**0.5) / 2, 1.5, 0)

vec1 = np.array([8, 0, 0]).T
vec2 = np.array([(27**0.5) / 2, 1.5, 0]).T
cross = np.cross(vec1,vec2)

c = DoubleArray3(cross[0],cross[1],cross[2])
```

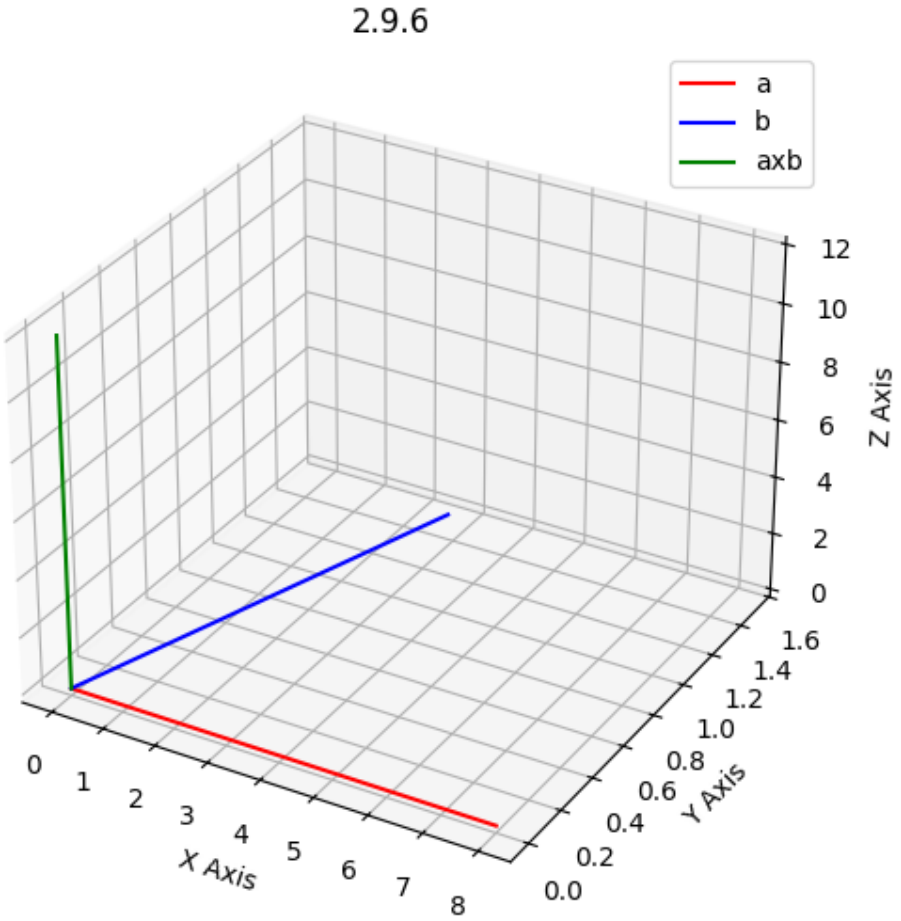
```
fig = plt.figure(figsize=(8, 6))
ax = fig.add_subplot(111, projection="3d")
t_values = np.linspace(0, 1, 100)
line_points_x, line_points_y, line_points_z = [], [], []
for t in t_values:
    result_arr = DoubleArray3()

    get_point(o, a, t, result_arr)

    line_points_x.append(result_arr[0])
    line_points_y.append(result_arr[1])
    line_points_z.append(result_arr[2])
ax.plot(
    line_points_x,
    line_points_y,
    line_points_z,
    color="red",
    label="a",
)
```

```
t_values = np.linspace(0, 1, 100)
line_points_x, line_points_y, line_points_z = [], [], []

for t in t_values:
    result_arr = DoubleArray3()

    get_point(o, b, t, result_arr)

    line_points_x.append(result_arr[0])
    line_points_y.append(result_arr[1])
    line_points_z.append(result_arr[2])

ax.plot(
    line_points_x,
    line_points_y,
    line_points_z,
    color="blue",
    label="b",
)
```

```
  t_values = np.linspace(0, 1, 100)
line_points_x, line_points_y, line_points_z = [], [], []

for t in t_values:
    result_arr = DoubleArray3()

    get_point(o, c, t, result_arr)

    line_points_x.append(result_arr[0])
    line_points_y.append(result_arr[1])
    line_points_z.append(result_arr[2])

ax.plot(
    line_points_x,
    line_points_y,
    line_points_z,
    color="green",
    label="axb",
)
```

```
ax.set_xlabel("X Axis")
ax.set_ylabel("Y Axis")
ax.set_zlabel("Z Axis")
ax.set_title("2.9.6")
ax.legend()
ax.grid(True)

plt.savefig("../figs/plot.png")
plt.show()
```

Plot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2
3
4
5
6
7
8
X Axis

Pure Python code

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

a = np.array([8, 0, 0]).T
b = np.array([(27**0.5) / 2, 1.5, 0]).T

# Solving
c = np.cross(a,b)
```

Pure Python code

```
# Plotting
fig = plt.figure(figsize=(8, 8))
ax = fig.add_subplot(111, projection="3d")
ax.quiver(0, 0, 0, a[0], a[1], a[2], color="red", label="a")
ax.quiver(0, 0, 0, b[0], b[1], b[2], color="blue", label="b")
ax.quiver(0, 0, 0, c[0], c[1], c[2], color="green", label="axb")
ax.set_xlabel("X-axis")
ax.set_ylabel("Y-axis")
ax.set_zlabel("Z-axis")
ax.set_title("2.9.6")
ax.set_xlim([-5, 5])
ax.set_ylim([-5, 5])
ax.set_zlim([-5, 5])
ax.legend()
ax.grid(True)
plt.savefig("../figs/python.png")
plt.show()
```

Plot