Assignment : GATE 2022 MA

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1) As you grow older, an injury to your __________ may take longer to __________.
(GATE MA 2022)

a) heel / heel
b) heal / heel
c) heal / heal
d) heel / heal

2) In a 500 m race, P and Q have speeds in the ratio $3:4$. Q starts the race when P has already covered 140 m. What is the distance between P and Q (in m) when P wins the race?
(GATE MA 2022)

a) 20
b) 40
c) 60
d) 140

3) Three bells P, Q, and R are rung periodically in a school. P is rung every 20 minutes; Q is rung every 30 minutes and R is rung every 50 minutes. If all the three bells are rung at $12:00$ PM, when will the three bells ring together again the next time?
(GATE MA 2022)

a) $5:00$ PM
b) $5:30$ PM
c) $6:00$ PM
d) $6:30$ PM

4) Given below are two statements and four conclusions drawn based on the statements.
   a) Statement 1: Some bottles are cups.
   b) Statement 2: All cups are knives.
   c) Conclusion I: Some bottles are knives.
   d) Conclusion II: Some knives are cups.
   e) Conclusion III: All cups are bottles.
   f) Conclusion IV: All knives are cups.

   Which one of the following options can be logically inferred?
(GATE MA 2022)

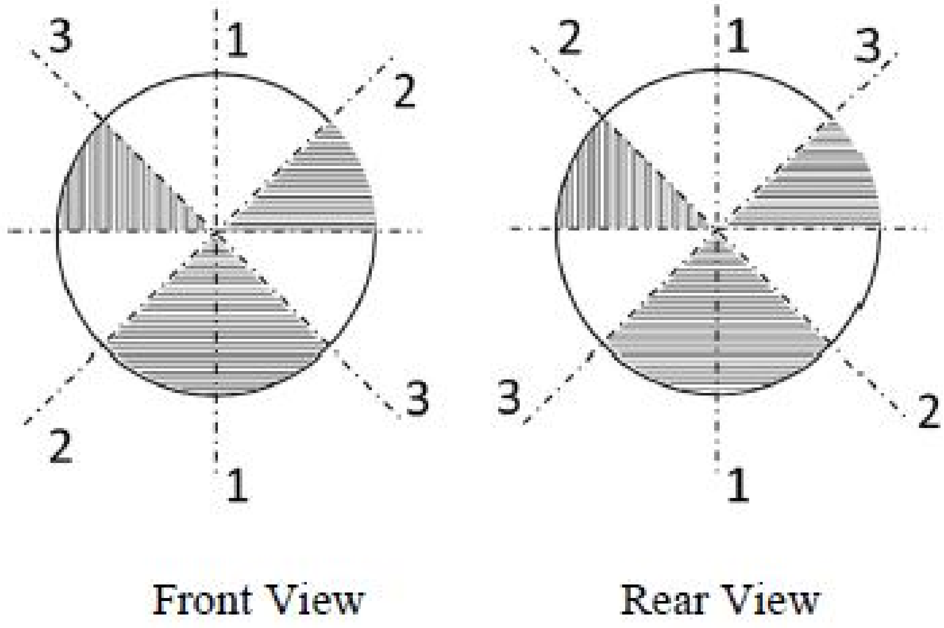
a) Only conclusion I and conclusion II are correct
b) Only conclusion II and conclusion III are correct
c) Only conclusion II and conclusion IV are correct
d) Only conclusion III and conclusion IV are correct

5) The figure below shows the front and rear view of a disc, which is shaded with identical patterns. The disc is flipped once with respect to any one of the fixed axes (1-1, 2-2 or 3-3) chosen uniformly at random. What is the probability that the disc DOES NOT retain the same front and rear views after the flipping operation?
(GATE MA 2022)

a) 0
b) $\frac{1}{3}$
c) $\frac{2}{3}$
d) 1

6) Altruism is the human concern for the wellbeing of others. Altruism has been shown to be motivated more by social bonding, familiarity and identification of belongingness to a group. The notion that altruism may be attributed to empathy or guilt has now been rejected. Which one of the following is the CORRECT logical inference based on the information in the above passage?
(GATE MA 2022)

a) Humans engage in altruism due to guilt but not empathy



b) Humans engage in altruism due to empathy but not guilt
c) Humans engage in altruism due to group identification but not empathy
d) Humans engage in altruism due to empathy but not familiarity

7) There are two identical dice with a single letter on each of the faces. The following six letters: Q, R, S, T, U, and V, one on each of the faces. Any of the six outcomes are equally likely. The two dice are thrown once independently at random. What is the probability that the outcomes on the dice were composed only of any combination of the following possible outcomes: Q, U and V?

(GATE MA 2022)

a) $\frac{1}{4}$ b) $\frac{3}{4}$ c) $\frac{1}{6}$ d) $\frac{5}{36}$

8) The price of an item is 10% cheaper in an online store S compared to the price at another online store M. Store S charges 150 for delivery. There are no delivery charges for orders from the store M. A person bought the item from the store S and saved 100. What is the price of the item at the online store S (in rupees) if there are no other charges than what is described above?

(GATE MA 2022)

a) 2500 b) 2250 c) 1750 d) 1500

9) The letters P, Q, R, S, T and U are to be placed one per vertex on a regular convex hexagon, but not necessarily in the same order.
Consider the following statements:
- The line segment joining R and S is longer than the line segment joining P and Q.
- The line segment joining R and S is perpendicular to the line segment joining P and Q.
- The line segment joining R and U is parallel to the line segment joining T and Q.

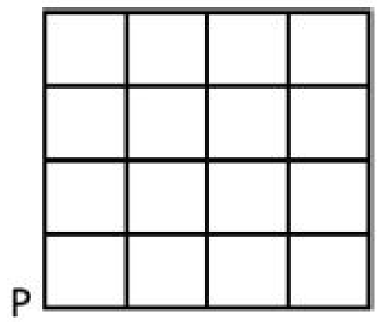
Based on the above statements, which one of the following options is CORRECT?

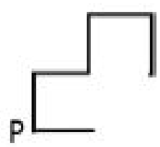
(GATE MA 2022)

a) The line segment joining R and T is parallel to the line segment joining Q and S.
b) The line segment joining T and Q is parallel to the line joining P and U.
c) The line segment joining R and P is perpendicular to the line segment joining U and Q.
d) The line segment joining Q and S is perpendicular to the line segment joining R and P.

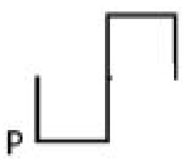
10) An ant is at the bottom-left corner of a grid (point P) as shown below. It aims to move to the top-right corner of the grid. The ant moves only along the lines marked in the grid such that the current distance to the top-right corner strictly decreases. Which one of the following is a part of a possible trajectory of the ant during the movement?

(GATE MA 2022)

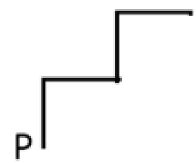




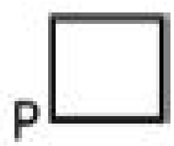
a)



b)



c)



d)

11) Suppose that the characteristic equation of $M \in \mathbb{C}^{3\times 3}$ is $\lambda^3 + \alpha\lambda^2 + \beta\lambda - 1 = 0$, where $\alpha, \beta \in \mathbb{C}$ with $\alpha + \beta \neq 0$. Which of the following statements is TRUE?

(GATE MA 2022)

a) $M(I - \beta M) = M^{-1}(M + \alpha I)$
b) $M(I + \beta M) = M^{-1}(M - \alpha I)$
c) $M^{-1}\left(M^{-1} + \beta I\right) = M - \alpha I$
d) $M^{-1}\left(M^{-1} - \beta I\right) = M + \alpha I$

12) Consider
P : Let $M \in \mathbb{R}^{m\times n}$ with $m > n \geq 2$. If rank $(M) = n$, then the system of linear equations $Mx = 0$ has $x = 0$ as the only solution.
Q : Let $E \in \mathbb{R}^{n\times n}$, $n \geq 2$ be a non-zero matrix such that $E^3 = 0$. Then $I + E^2$ is a singular matrix.
Which of the following statements is TRUE?

(GATE MA 2022)

a) Both P and Q are TRUE
b) Both P and Q are FALSE
c) P is TRUE and Q is FALSE
d) P is FALSE and Q is TRUE

13) Consider the real function of two real variables given by

$$u(x, y) = e^{2x} (\sin 3x \cos 2y \cosh 3y - \cos 3x \sin 2y \sinh 3y).$$

Let $v(x, y)$ be the harmonic conjugate of $u(x, y)$ such that $v(0, 0) = 2$. Let $z = x + iy$ and $f(z) = u(x, y) + i\,v(x, y)$, then the value of $4 + 2i\,f(i\pi)$ is

(GATE MA 2022)

a) $e^{3\pi} + e^{-3\pi}$
b) $e^{3\pi} - e^{-3\pi}$
c) $-e^{3\pi} + e^{-3\pi}$
d) $-e^{3\pi} - e^{-3\pi}$

14) The value of the integral

$$\int_C \frac{z^{100}}{z^{101} + 1}\, dz$$

where C is the circle of radius 2 centred at the origin taken in the anti-clockwise direction, is

(GATE MA 2022)

a) $-2\pi i$
b) 2π
c) 0
d) $2\pi i$

15) Let X be a real normed linear space. Let $X_0 = \{x \in X : ||x|| = 1\}$. If X_0 contains two distinct points x and y and the line segment joining them, which of the following statements is TRUE?

(GATE MA 2022)

a) $||x + y|| = ||x|| + ||y||$ and x, y are linearly independent
b) $||x + y|| = ||x|| + ||y||$ and x, y are linearly dependent
c) $||x + y||^2 = ||x||^2 + ||y||^2$ and x, y are linearly independent
d) $||x + y|| = 2||x||||y||$ and x, y are linearly dependent

16) Let $\{e_k : k \in \mathbb{N}\}$ be an orthonormal basis for a Hilbert space H. Define $f_k = e_k + e_{k+1}$ for $k \in \mathbb{N}$ and $g_j = \sum_{n=1}^{j} (-1)^{n+1} e_n$ for $j \in \mathbb{N}$. Then

$$\sum_{k=1}^{\infty} \left|\langle g_j, f_k\rangle\right|^2 =$$

(GATE MA 2022)

a) 0
b) j^2
c) $4j^2$
d) 1

17) Consider $\mathbb{R}^2$ with the usual metric. Let $A = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ and $B = \{(x, y) \in \mathbb{R}^2 : (x-2)^2 + y^2 \leq 1\}$. Let $M = A \cup B$ and $N = \text{interior}(A) \cup \text{interior}(B)$. Then, which of the following statements is TRUE?

(GATE MA 2022)

a) M and N are connected
b) Neither M nor N is connected
c) M is connected and N is not connected
d) M is not connected and N is connected

18) The real sequence generated by the iterative scheme

$$x_n = \frac{x_{n-1}}{2} + \frac{1}{x_{n-1}}, \quad n \geq 1$$

(GATE MA 2022)

a) converges to $\sqrt{2}$, for all $x_0 \in \mathbb{R} \setminus \{0\}$
b) converges to $\sqrt{2}$, whenever $x_0 > \sqrt{\frac{2}{3}}$
c) converges to $\sqrt{2}$, whenever $x_0 \in (-1, 1) \setminus \{0\}$
d) diverges for any $x_0 \neq 0$

19) The initial value problem

$$\frac{dy}{dx} = \cos(xy), \quad x \in \mathbb{R}, \quad y(0) = y_0,$$

where y_0 is a real constant, has

(GATE MA 2022)

a) a unique solution
b) exactly two solutions
c) infinitely many solutions
d) no solution

20) If eigenfunctions corresponding to distinct eigenvalues λ of the Sturm–Liouville problem

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} = \lambda y, \quad 0 < x < \pi,$$
$$y(0) = y(\pi) = 0$$

are orthogonal with respect to the weight function $w(x)$, then $w(x)$ is

(GATE MA 2022)

a) e^{-3x}
b) e^{-2x}
c) e^{2x}
d) e^{3x}

21) The steady state solution for the heat equation

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = 0, \quad (0 < x < 2,\, t > 0),$$

with the initial condition $u(x, 0) = 0$, $(0 < x < 2)$ and the boundary conditions $u(0, t) = 1$ and $u(2, t) = 3$, $t > 0$, at $x = 1$ is

(GATE MA 2022)

a) 1
b) 2
c) 3
d) 4

22) Consider $([0,1],T_1)$, where T_1 is the subspace topology induced by the Euclidean topology on $\mathbb{R}$, and let T_2 be any topology on $[0,1]$. Consider the following statements.
P : If T_1 is a proper subset of T_2, then $([0,1],T_2)$ is not compact.
Q : If T_2 is a proper subset of T_1, then $([0,1],T_2)$ is not Hausdorff.
Then

(GATE MA 2022)

a) P is TRUE and Q is FALSE
b) Both P and Q are TRUE
c) Both P and Q are FALSE
d) P is FALSE and Q is TRUE

23) Let $p\colon ([0,1],T_1) \to (\{0,1\},T_2)$ be the quotient map arising from the characteristic function on $\left[\frac{1}{2},1\right]$, where T_1 is the subspace topology induced by the Euclidean topology on $\mathbb{R}$. Which of the following statements is TRUE?

(GATE MA 2022)

a) p is an open map but not a closed map
b) p is a closed map but not an open map
c) p is a closed map as well as an open map
d) p is neither an open map nor a closed map

24) Set $X_n := \mathbb{R}$ for each $n \in \mathbb{N}$. Define $Y := \prod_{n\in\mathbb{N}} X_n$. Endow Y with the product topology, where the topology on each X_n is the Euclidean topology. Consider the set

$$\Delta = \{(x,x,x,\cdots) \mid x \in \mathbb{R}\}$$

with the subspace topology induced from Y. Which of the following statements is TRUE?

(GATE MA 2022)

a) Δ is open in Y
b) Δ is locally compact
c) Δ is dense in Y
d) Δ is disconnected

25) Consider the linear system of equations $Ax = b$ with

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 4 & 1 \\ 2 & 0 & 3 \end{pmatrix}, \quad b = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}.$$

Which of the following statements are TRUE?

(GATE MA 2022)

a) The Jacobi iterative matrix is $\begin{pmatrix} 0 & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{2}{3} & 0 & 0 \end{pmatrix}$
b) The Jacobi iterative method converges for any initial vector
c) The Gauss–Seidel iterative method converges for any initial vector
d) The spectral radius of the Jacobi iterative matrix is less than 1

26) The number of non-isomorphic abelian groups of order $2^2 \cdot 3^3 \cdot 5^4$ is __________.

(GATE MA 2022)

27) The number of subgroups of a cyclic group of order 12 is __________.

(GATE MA 2022)

28) The radius of convergence of the series

$$\sum_{n\geq 0} 3^{n+1} z^{2n}, \quad z \in \mathbb{C}$$

is (round off to TWO decimal places) __________.

(GATE MA 2022)

29) The number of zeros of the polynomial $2z^7 - 7z^5 + 2z^3 - z + 1$ in the unit disc $\{z \in \mathbb{C} : |z| < 1\}$ is __________.

(GATE MA 2022)

30) If $P(x)$ is a polynomial of degree 5 and

$$\alpha = \sum_{i=0}^{6} P(x_i) \left(\prod_{\substack{j=0 \\ j\neq i}}^{6} \left(x_i - x_j\right)^{-1} \right),$$

where $x_0, x_1, \ldots, x_6$ are distinct points in the interval $[2, 3]$, then the value of $\alpha^2 - \alpha + 1$ is __________.

(GATE MA 2022)

31) The maximum value of $f(x, y) = 49 - x^2 - y^2$ on the line $x + 3y = 10$ is __________.

(GATE MA 2022)

32) If the function $f(x, y) = x^2 + xy + y^2 + \dfrac{1}{x} + \dfrac{1}{y}$ for $x \neq 0$, $y \neq 0$ attains its local minimum value at the point (a, b), then the value of $a^3 + b^3$ is (round off to TWO decimal places) __________.

(GATE MA 2022)

33) If the ordinary differential equation

$$x^2 \frac{d^2\phi}{dx^2} + x \frac{d\phi}{dx} + x^2 \phi = 0, \quad x > 0$$

has a solution of the form $\phi(x) = x^r \sum_{n=0}^{\infty} a_n x^n$, where a_n are constants and $a_0 \neq 0$, then the value of $r^2 + 1$ is __________.

(GATE MA 2022)

34) The Bessel functions $J_\alpha(x)$ for $x > 0$, $\alpha \in \mathbb{R}$ satisfy $J_{\alpha-1}(x) + J_{\alpha+1}(x) = \frac{2\alpha}{x} J_\alpha(x)$. Then, the value of $(\pi J_{3/2}(\pi))^2$ is __________.

(GATE MA 2022)

35) The partial differential equation

$$7 \frac{\partial^2 u}{\partial x^2} + 16 \frac{\partial^2 u}{\partial x \partial y} + 4 \frac{\partial^2 u}{\partial y^2} = 0$$

is transformed to

$$A \frac{\partial^2 u}{\partial \xi^2} + B \frac{\partial^2 u}{\partial \xi \partial \eta} + C \frac{\partial^2 u}{\partial \eta^2} = 0,$$

using $\xi = y - 2x$ and $\eta = 7y - 2x$. Then, the value of $\dfrac{1}{123}\left(B^2 - 4AC\right)$ is __________.

(GATE MA 2022)

36) Let $\mathbb{R}[X]$ denote the ring of polynomials in X with real coefficients. Then, the quotient ring $\mathbb{R}[X]/\left(X^4+4\right)$ is

(GATE MA 2022)

a) a field
b) an integral domain, but not a field
c) not an integral domain, but has 0 as the only nilpotent element
d) a ring which contains non–zero nilpotent elements

37) Consider the following conditions on two proper non–zero ideals J_1 and J_2 of a non–zero commutative ring R.
P : For any $r_1, r_2 \in R$, there exists a unique $r \in R$ such that $r - r_1 \in J_1$ and $r - r_2 \in J_2$.
Q : $J_1 + J_2 = R$.
Then, which of the following statements is TRUE?

(GATE MA 2022)

a) P implies Q but Q does not imply P
b) Q implies P but P does not imply Q
c) P implies Q and Q implies P
d) P does not imply Q and Q does not imply P

38) Let $f\colon [-\pi,\pi] \to \mathbb{R}$ be a continuous function such that $f(x) > \frac{f(0)}{2}$ for $|x| < \delta$, where $0 < \delta < \pi$. Define $P_{n,\delta}(x) = (1+\cos x - \cos\delta)^n$, for $n = 1, 2, 3, \ldots$ Then, which of the following statements is TRUE?

(GATE MA 2022)

a) $\lim\limits_{n\to\infty} \int_0^{2\delta} f(x)\, P_{n,\delta}(x)\, dx = 0$
b) $\lim\limits_{n\to\infty} \int_{-2\delta}^{0} f(x)\, P_{n,\delta}(x)\, dx = 0$
c) $\lim\limits_{n\to\infty} \int_{-\delta}^{\delta} f(x)\, P_{n,\delta}(x)\, dx = 0$
d) $\lim\limits_{n\to\infty} \int_{[-\pi,\pi]\setminus[-\delta,\delta]} f(x)\, P_{n,\delta}(x)\, dx = 0$

39) P : Suppose that $\sum\limits_{n=0}^{\infty} a_n x^n$ converges at $x = -3$ and diverges at $x = 6$. Then $\sum\limits_{n=0}^{\infty} (-1)^n a_n$ converges.
Q : The interval of convergence of the series $\sum\limits_{n=2}^{\infty} \frac{(-1)^n x^n}{4^n \ln n}$ is $[-4, 4]$.
Which of the following statements is TRUE?

(GATE MA 2022)

a) P is TRUE and Q is TRUE
b) P is FALSE and Q is FALSE
c) P is TRUE and Q is FALSE
d) P is FALSE and Q is TRUE

40) Let

$$f_n(x) = \frac{x^2}{x^2 + (1-nx)^2}, \quad x \in [0,1],\ n = 1, 2, 3, \ldots$$

Then, which of the following statements is TRUE?

(GATE MA 2022)

a) $\{f_n\}$ is not equicontinuous on $[0,1]$
b) $\{f_n\}$ is uniformly convergent on $[0,1]$
c) $\{f_n\}$ is equicontinuous on $[0,1]$
d) $\{f_n\}$ is uniformly bounded and has a subsequence converging uniformly on $[0,1]$

41) Let $(\mathbb{Q}, d)$ be the metric space with $d(x, y) = |x - y|$. Let $E = \{ p \in \mathbb{Q} \colon 2 < p^2 < 3 \}$. Then, the set E is

(GATE MA 2022)

a) closed but not compact
b) not closed but compact
c) compact
d) neither closed nor compact

42) Let $T \colon \mathrm{L}^2[-1, 1] \to \mathrm{L}^2[-1, 1]$ be defined by $Tf = \tilde{f}$, where $\tilde{f}(x) = f(-x)$ (a.e.) If M is the kernel of $I - T$, then the distance between the function $\phi(t) = e^t$ and M is

(GATE MA 2022)

a) $\frac{1}{2}\sqrt{e^2 - e^{-2} + 4}$
b) $\frac{1}{2}\sqrt{e^2 - e^{-2} - 2}$
c) $\frac{1}{2}\sqrt{e^2 - 4}$
d) $\frac{1}{2}\sqrt{e^2 - e^{-2} - 4}$

43) Let X, Y and Z be Banach spaces. Suppose that $T \colon X \to Y$ is linear and $S : Y \to Z$ is linear, bounded and injective. In addition, if $S \circ T \colon X \to Z$ is bounded, then which of the following statements is TRUE?

(GATE MA 2022)

a) T is surjective
b) T is bounded but not continuous
c) T is bounded
d) T is not bounded

44) The first derivative of a function $f \in \mathrm{C}^\infty(-3, 3)$ is approximated by an interpolating polynomial of degree 2, using the data $(-1, f(-1)), (0, f(0))$ and $(2, f(2))$. It is found that

$$f'(0) \approx -\frac{2}{3} f(-1) + \alpha f(0) + \beta f(2).$$

Then, the value of $\frac{1}{\alpha\beta}$ is

(GATE MA 2022)

a) 3
b) 6
c) 9
d) 12

45) The work done by the force

$$\mathbf{F} = (x + y)\,\hat{\imath} - \left(x^2 + y^2\right)\hat{\jmath},$$

where $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors along the $\overrightarrow{OX}$ and $\overrightarrow{OY}$ directions, respectively along the upper half of the circle $x^2 + y^2 = 1$ from $(1, 0)$ to $(-1, 0)$ in the xy-plane is

(GATE MA 2022)

a) $-\pi$
b) $-\frac{\pi}{2}$
c) $\frac{\pi}{2}$
d) π

46) Let $u(x,t)$ be the solution of the wave equation

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, \qquad 0 < x < \pi,\ t > 0,$$

with the initial conditions

$$u(x,0) = \sin x + \sin 2x + \sin 3x, \qquad \frac{\partial u}{\partial t}(x,0) = 0, \quad 0 < x < \pi,$$

and the boundary conditions $u(0,t) = u(\pi,t) = 0,\ t \geq 0$. Then, the value of $u\left(\frac{\pi}{2}, \pi\right)$ is (GATE MA 2022)

a) $-\frac{1}{2}$ b) 0 c) $\frac{1}{2}$ d) 1

47) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by

$$T((1,2)) = (1,0) \quad \text{and} \quad T((2,1)) = (1,1).$$

For $p, q \in \mathbb{R}$, let $T^{-1}((p,q)) = (x,y)$. Which of the following statements is TRUE? (GATE MA 2022)

a) $x = p - q;\ y = 2p - q$
b) $x = p + q;\ y = 2p - q$
c) $x = p + q;\ y = 2p + q$
d) $x = p - q;\ y = 2p + q$

48) Let $y = \left(\alpha, -1\right)^T$, $\alpha \in \mathbb{R}$ be a feasible solution for the dual problem of the linear programming problem

$$\begin{aligned} \text{Maximize } &: 5x_1 + 12x_2 \\ \text{subject to } &: x_1 + 2x_2 + x_3 \leq 10, \\ & \quad 2x_1 - x_2 + 3x_3 = 8, \\ & \quad x_1, x_2, x_3 \geq 0. \end{aligned}$$

Which of the following statements is TRUE?

(GATE MA 2022)

a) $\alpha < 3$ b) $3 \leq \alpha < 5.5$ c) $5.5 \leq \alpha < 7$ d) $\alpha \geq 7$

49) Let K denote the subset of $\mathbb{C}$ consisting of elements algebraic over $\mathbb{Q}$. Then, which of the following statements are TRUE?

(GATE MA 2022)

a) No element of $\mathbb{C} \setminus K$ is algebraic over $\mathbb{Q}$
b) K is an algebraically closed field
c) For any bijective ring homomorphism $f\colon \mathbb{C} \to \mathbb{C}$, we have $f(K) = K$
d) There is no bijection between K and $\mathbb{Q}$

50) Let T be a MÃ¶bius transformation such that $T(0) = \alpha$, $T(\alpha) = 0$ and $T(\infty) = -\alpha$, where $\alpha = \frac{-1+i}{\sqrt{2}}$. Let L denote the straight line passing through the origin with slope -1, and let C denote the circle of unit radius centred at the origin. Then, which of the following statements are TRUE? (GATE MA 2022)

a) T maps L to a straight line
b) T maps L to a circle
c) T^{-1} maps C to a straight line
d) T^{-1} maps C to a circle

51) Let $a > 0$. Define $D_a\colon L_2(\mathbb{R}) \to L_2(\mathbb{R})$ by $(D_a f)(x) = \frac{1}{\sqrt{a}} f\left(\frac{x}{a}\right)$, almost everywhere, for $f \in L_2(\mathbb{R})$. Then, which of the following statements are TRUE?

(GATE MA 2022)

a) D_a is a linear isometry
b) D_a is a bijection
c) $D_a \circ D_b = D_{a+b}$, $b > 0$
d) D_a is bounded from below

52) Let $\{\phi_0, \phi_1, \phi_2, \dots\}$ be an orthonormal set in $L^2[-1,1]$ such that $\phi_n = C_n P_n$, where C_n is a constant and P_n is the Legendre polynomial of degree n, for each $n \in \mathbb{N} \cup \{0\}$. Then, which of the following statements are TRUE?

(GATE MA 2022)

a) $\phi_6(1) = 1$
b) $\phi_7(-1) = 1$
c) $\phi_7(1) = \dfrac{\sqrt{15}}{2}$
d) $\phi_6(-1) = \dfrac{\sqrt{13}}{2}$

53) Let $X = (\mathbb{R}, T)$, where T is the smallest topology on $\mathbb{R}$ in which all the singleton sets are closed. Then, which of the following statements are TRUE?

(GATE MA 2022)

a) $\{x \in \mathbb{R} \mid 0 \le x < 1\}$ is compact in X.
b) X is not first countable.
c) X is second countable.
d) X is first countable.

54) Consider (Z, T), where T is the topology generated by sets of the form $A_{m,n} = \{\, m + nk \mid k \in Z \,\}$, for $m, n \in Z$ and $n \neq 0$. Then, which of the following statements are TRUE?

(GATE MA 2022)

a) (Z, T) is connected.
b) Each $A_{m,n}$ is a closed subset of (Z, T).
c) (Z, T) is Hausdorff.
d) (Z, T) is metrizable.

55) Let $A \in \mathbb{R}^{m \times n}$, $c \in \mathbb{R}^n$ and $b \in \mathbb{R}^m$. Consider the linear programming primal problem

$$\begin{aligned} \text{Minimize:}\ & c^\top x \\ \text{subject to:}\ & Ax = b, \\ & x \ge 0. \end{aligned}$$

Let x^0 and y^0 be feasible solutions of the primal and its dual, respectively. Which of the following statements are TRUE?

(GATE MA 2022)

a) $c^\top x^0 \ge b^\top y^0$
b) $c^\top x^0 = b^\top y^0$
c) If $c^\top x^0 = b^\top y^0$, then x^0 is optimal for the primal
d) If $c^\top x^0 = b^\top y^0$, then y^0 is optimal for the dual

56) Consider $\mathbb{R}^3$ as a vector space with the usual operations of vector addition and scalar multiplication. Let $x \in \mathbb{R}^3$ be denoted by $x = (x_1, x_2, x_3)$. Define subspaces W_1 and W_2 by

$$\begin{aligned} W_1 &:= \{x \in \mathbb{R}^3 \colon x_1 + 2x_2 - x_3 = 0\}, \\ W_2 &:= \{x \in \mathbb{R}^3 \colon 2x_1 + 3x_3 = 0\}. \end{aligned}$$

Let $\dim(U)$ denote the dimension of the subspace U. Which of the following statements are TRUE?

(GATE MA 2022)

a) $\dim(W_1) = \dim(W_2)$

b) $\dim(W_1) + \dim(W_2) - \dim\left(\mathbb{R}^3\right) = 1$
c) $\dim(W_1 + W_2) = 2$
d) $\dim(W_1 \cap W_2) = 1$

57) Three companies C_1, C_2 and C_3 submit bids for three jobs J_1, J_2 and J_3. The costs involved per unit are given in the table below.

	J_1	J_2	J_3
C_1	10	12	8
C_2	9	15	10
C_3	15	10	9

Then, the cost of the optimal assignment is __________.

(GATE MA 2022)

58) The initial value problem $\frac{dy}{dx} = f(x, y),\ y(x_0) = y_0$ is solved by using the following second order Runge-Kutta method :

$$K_1 = h\, f(x_i, y_i),$$
$$K_2 = h\, f(x_i + \alpha h,\ y_i + \beta K_1),$$
$$y_{i+1} = y_i + \frac{1}{4}(K_1 + 3K_2), \quad i \geq 0,$$

where h is the uniform step length between the points $x_0, x_1, \cdots, x_n$ and $y_i = y(x_i)$. The value of the product $\alpha\beta$ is __________ (round off to TWO decimal places).

(GATE MA 2022)

59) The surface area of the paraboloid $z = x^2 + y^2$ between the planes $z = 0$ and $z = 1$ is __________ (round off to ONE decimal place).

(GATE MA 2022)

60) The rate of change of $f(x, y, z) = x + x\cos z - y\sin z + y$ at P_0 in the direction from $P_0(2, -1, 0)$ to $P_1(0, 1, 2)$ is __________.

(GATE MA 2022)

61) Consider the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ 1 < x < 2,\ 1 < y < 2$$

with the boundary conditions

$$\frac{\partial u}{\partial x}(1, y) = y, \quad \frac{\partial u}{\partial x}(2, y) = 5, \quad 1 < y < 2,$$
$$\frac{\partial u}{\partial y}(x, 1) = \frac{\alpha x^2}{7}, \quad \frac{\partial u}{\partial y}(x, 2) = x, \quad 1 < x < 2$$

If the problem has a solution, then the constant α is __________.

(GATE MA 2022)

62) Let $u(x, y)$ be the solution of the first order partial differential equation

$$x\frac{\partial u}{\partial x} + \left(x^2 + y\right)\frac{\partial u}{\partial y} = u$$

for all $x, y \in \mathbb{R}$, satisfying $u(2, y) = y - 4$, $y \in \mathbb{R}$. Then, the value of $u(1, 2)$ is __________.

(GATE MA 2022)

63) The optimal value for the linear programming problem

$$\begin{aligned} \text{Maximize:}\quad & 6x_1 + 5x_2 \\ \text{subject to:}\quad & 3x_1 + 2x_2 \leq 12, \\ & -x_1 + x_2 \leq 1, \\ & x_1, x_2 \geq 0 \end{aligned}$$

is __________.

(GATE MA 2022)

64) A certain product is manufactured by plants P_1, P_2 and P_3 whose capacities are 15, 25 and 10 units, respectively. The product is shipped to markets M_1, M_2, M_3 and M_4, whose requirements are 10, 10, 10 and 20, respectively. The transportation costs per unit are given in the table below.

	M_1	M_2	M_3	M_4	
P_1	1	3	1	3	15
P_2	2	2	4	1	25
P_3	2	1	1	2	10
	10	10	10	20	

Then the cost corresponding to the starting basic solution by the Northwest-corner method is __________.

(GATE MA 2022)

65) Let M be a 3×3 real matrix such that $M^2 = 2M + 3I$. If $\det M = -9$, then the trace of M equals __________.

(GATE MA 2022)