5.6.10

AI25BTECH11024 - Pratyush Panda

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Question:

If $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, show that $\mathbf{A}^2 - 5\mathbf{A} + 7\mathbf{I} = 0$.

Solution:

Given matrix $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$.

We can write the character equation for the matrix by,

$$f(\lambda) = |\mathbf{A} - \lambda \mathbf{I}| \tag{0.1}$$

$$f(\lambda) = \left| \begin{pmatrix} 3-\lambda & 1 \\ -1 & 2-\lambda \end{pmatrix} \right| \tag{0.2}$$

$$f(\lambda) = \lambda^2 - 5\lambda + 7 \tag{0.3}$$

Since we know by the Caley Hamilton theorem, the matrix itself satisfies its own characteristic equation. Thus, we get;

$$\mathbf{A}^2 - 5\mathbf{A} + 7\mathbf{I} = 0 \tag{0.4}$$

Hence proved.