

4.11.3

EE25BTECH11025 - Ganachari Vishwambhar

Question:

Find the equation of the line passing through (2,-1,2) and (5,3,4) and the equation of the plane passing through (2,0,3), (1,1,5), and (3,2,4). Also, find their point of intersection.

Solution:

Let:

$$\mathbf{P}_1 = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}; \mathbf{P}_2 = \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix} \quad (1)$$

$$\mathbf{A} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}; \mathbf{B} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}; \mathbf{C} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} \quad (2)$$

Direction vector of the line:

$$\mathbf{m} = \mathbf{P}_2 - \mathbf{P}_1 = \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \quad (3)$$

Vector form of the line can be written as:

$$\mathbf{x} = \mathbf{P}_1 + \kappa \mathbf{m} \quad (4)$$

$$\mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \kappa \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \quad (5)$$

Vector form of the line can be written as:

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}^T \mathbf{n} = \mathbf{1} \quad (6)$$

$$\begin{pmatrix} 2 & 0 & 3 \\ 1 & 1 & 5 \\ 3 & 2 & 4 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (7)$$

Augmented matrix can be written as:

$$\left(\begin{array}{ccc|c} 2 & 0 & 3 & 1 \\ 1 & 1 & 5 & 1 \\ 3 & 2 & 4 & 1 \end{array}\right) R_2 \leftrightarrow R_1 \left(\begin{array}{ccc|c} 1 & 1 & 5 & 1 \\ 2 & 0 & 3 & 1 \\ 3 & 2 & 4 & 1 \end{array}\right) \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1 \end{array} \quad (8)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 5 & 1 \\ 0 & -2 & -7 & -1 \\ 0 & -1 & -11 & -2 \end{array}\right) \begin{array}{l} R_2 \leftrightarrow R_3 \\ R_2 \rightarrow -R_2 \end{array} \left(\begin{array}{ccc|c} 1 & 1 & 5 & 1 \\ 0 & 1 & 11 & 2 \\ 0 & -2 & -7 & -1 \end{array}\right) \quad (9)$$

$$\begin{array}{l} R_1 \rightarrow R_1 - R_2 \\ R_3 \rightarrow R_3 + 2R_2 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & -6 & -1 \\ 0 & 1 & 11 & 2 \\ 0 & 0 & 15 & 3 \end{array}\right) R_3 \rightarrow \frac{1}{15}R_3 \quad (10)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -6 & -1 \\ 0 & 1 & 11 & 2 \\ 0 & 0 & 1 & \frac{1}{5} \end{array}\right) \begin{array}{l} R_1 \rightarrow R_1 + 6R_3 \\ R_2 \rightarrow R_2 - 11R_3 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{5} \\ 0 & 1 & 0 & \frac{-1}{5} \\ 0 & 0 & 1 & \frac{1}{5} \end{array}\right) \quad (11)$$

Therefore, the plane equation is:

$$(1 \quad -1 \quad 1)\mathbf{x} = 5 \quad (12)$$

General form of a point on the line can be written as:

$$\mathbf{x} = \begin{pmatrix} 2 + 3\kappa \\ -1 + 6\kappa \\ 2 + 2\kappa \end{pmatrix} \quad (13)$$

Substituting (13) in (12), we get:

$$(1 \quad -1 \quad 1) \begin{pmatrix} 2 + 3\kappa \\ -1 + 6\kappa \\ 2 + 2\kappa \end{pmatrix} = 5 \quad (14)$$

$$\kappa = 0 \quad (15)$$

Hence the point of intersection of the line and the plane can be found by substituting (15) in (13):

$$\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \quad (16)$$

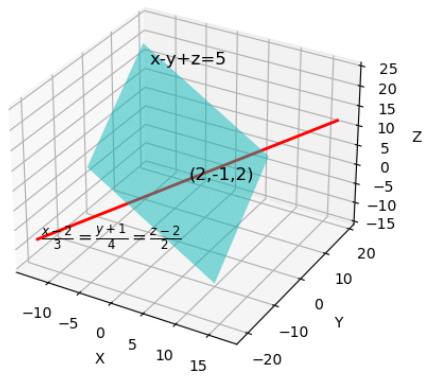


Fig. 1: Plot of the given plane and line