2.6.38

AI25BTECH11027 - NAGA BHUVANA

Question:
If $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$ and $\mathbf{b} = \hat{j} - \hat{k}$, find a vector $\mathbf{c}$ such that $\mathbf{a} \times \mathbf{c} = \mathbf{b}$ and $\mathbf{a} \cdot \mathbf{c} = 3$
Solution:
Given that

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{a}^T \mathbf{c} = 3 \tag{2}$$
$$\mathbf{b}^T \mathbf{c} = 0 \tag{3}$$

$$\begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix}^T \mathbf{c} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{5}$$

By solving

$$\mathbf{c} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 - 2\lambda \\ \lambda \\ \lambda \end{pmatrix} \tag{6}$$

$$\lambda = \frac{2}{3} \quad \text{Satisfies the cross product condition} \tag{7}$$

$$\mathbf{c} = \begin{pmatrix} \frac{5}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{pmatrix} \tag{8}$$

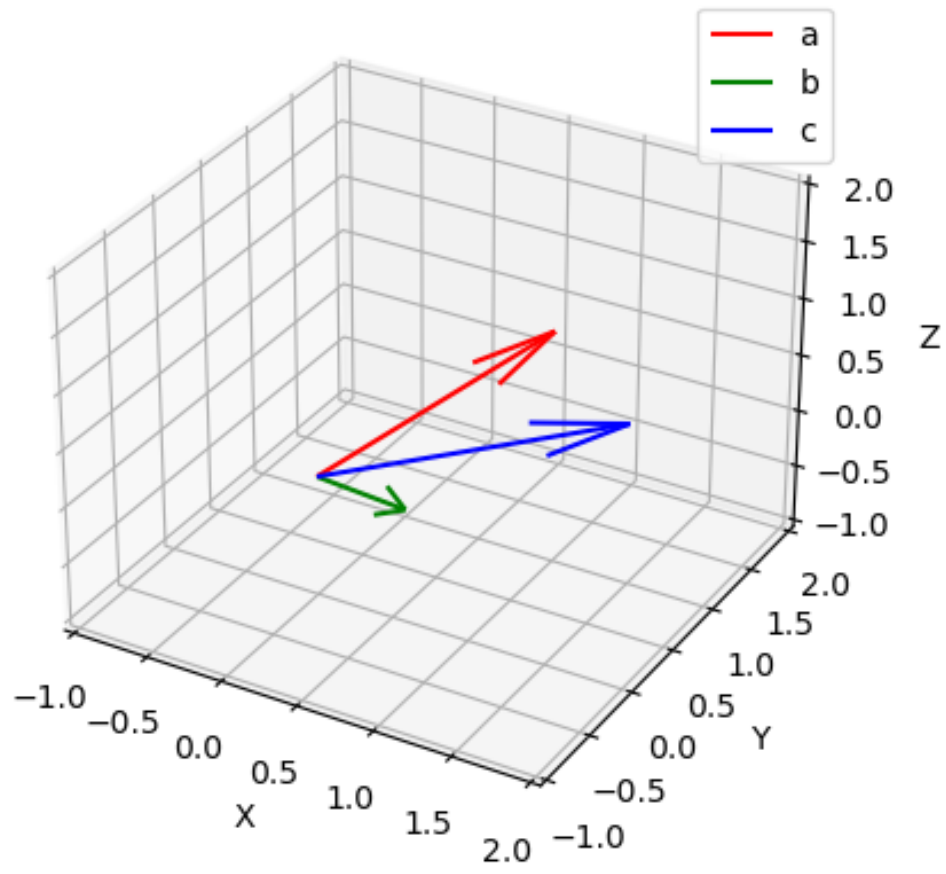$$\therefore \mathbf{c} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$$

Fig. 1