# 1.7.10

AI25BTECH11024 - Pratyush Panda

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**Question:**

The value of $\hat{\mathbf{i}}.\left(\hat{\mathbf{j}} \times \hat{\mathbf{k}}\right) + \hat{\mathbf{j}}.\left(\hat{\mathbf{i}} \times \hat{\mathbf{k}}\right) + \hat{\mathbf{k}}.\left(\hat{\mathbf{i}} \times \hat{\mathbf{j}}\right)$ is __________

**Solution:**

Given:

$$\hat{\mathbf{i}} = \mathbf{e_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \hat{\mathbf{j}} = \mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \hat{\mathbf{k}} = \mathbf{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{0.1}$$

Each term of the expression in the question can be found using the scalar triple product determinant.

The first term can be written as:

$$\left(\hat{\mathbf{i}}.\left(\hat{\mathbf{j}} \times \hat{\mathbf{k}}\right)\right) = \begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{0.2}$$

Determinant of this matrix is 1. Thus, the value of first term is 1.

The second term can be written as:

$$\left(\hat{\mathbf{j}}.\left(\hat{\mathbf{i}} \times \hat{\mathbf{k}}\right)\right) = \begin{pmatrix} e_2 & e_3 & e_1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \tag{0.3}$$

Determinant of this matrix is -1. Thus, the value of first term is -1.
The third term can be written as:

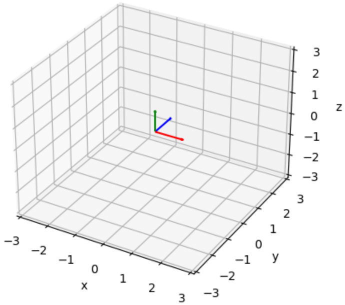
$$\left(\hat{\mathbf{k}}.\left(\hat{\mathbf{i}} \times \hat{\mathbf{j}}\right)\right) = \begin{pmatrix} e_3 & e_1 & e_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \tag{0.4}$$

Determinant of this matrix is 1. Thus, the value of first term is 1.

So, the sum of all the terms is;

$$\hat{\mathbf{i}}.\left(\hat{\mathbf{j}}\times\hat{\mathbf{k}}\right)+\hat{\mathbf{j}}.\left(\hat{\mathbf{i}}\times\hat{\mathbf{k}}\right)+\hat{\mathbf{k}}.\left(\hat{\mathbf{i}}\times\hat{\mathbf{j}}\right)=1+(-1)+1 \tag{0.5}$$

$$or,\ \hat{\mathbf{i}}.\left(\hat{\mathbf{j}}\times\hat{\mathbf{k}}\right)+\hat{\mathbf{j}}.\left(\hat{\mathbf{i}}\times\hat{\mathbf{k}}\right)+\hat{\mathbf{k}}.\left(\hat{\mathbf{i}}\times\hat{\mathbf{j}}\right)=1 \tag{0.6}$$



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