EE25BTECH11001 - Aarush Dilawri

Question:

Find the values of y for which the distance between the points P(2, -3) and Q(10,y) is 10 units.

Solution:

We are given the points

$$\mathbf{P} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 10 \\ \mathbf{y} \end{pmatrix} \tag{1}$$

The distance between them is 10 units, so

$$\|\mathbf{P} - \mathbf{Q}\| = 10\tag{2}$$

1

Squaring both sides,

$$\|\mathbf{P} - \mathbf{Q}\|^2 = (\mathbf{P} - \mathbf{Q})^{\mathsf{T}} (\mathbf{P} - \mathbf{Q}) = 10^2$$
(3)

Substituting,

$$\left(\begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 10 \\ y \end{pmatrix} \right)^{\mathsf{T}} \left(\begin{pmatrix} 2 \\ -3 \end{pmatrix} - \begin{pmatrix} 10 \\ y \end{pmatrix} \right) = 100 \tag{4}$$

$$\begin{pmatrix} -8 \\ -3 - y \end{pmatrix}^{\mathsf{T}} \begin{pmatrix} -8 \\ -3 - y \end{pmatrix} = 100 \tag{5}$$

$$(-8)^2 + (-3 - y)^2 = 100$$
 (6)

$$64 + (y+3)^2 = 100 (7)$$

$$(y+3)^2 = 36 (8)$$

$$y + 3 = \pm 6 \tag{9}$$

$$y = 3$$
 or $y = -9$ (10)

See Fig. 0,

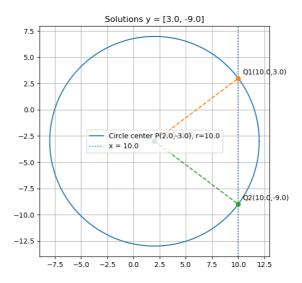


Fig. 0