1.8.27

EE25BTECH11010 - Arsh Dhoke

Question:
Find the equation of set of points $\mathbf{P}$ such that $||\mathbf{A}-\mathbf{P}||^2 + ||\mathbf{B}-\mathbf{P}||^2 = 2k^2$, where $\mathbf{A}$ and $\mathbf{B}$ are the points (3,4,5) and (-1,3,-7), respectively.
Solution:

The input parameters for the problem are given in the table below.

Vectors	**Points**
$\mathbf{A}$	$\begin{pmatrix}3\\4\\5\end{pmatrix}$
$\mathbf{B}$	$\begin{pmatrix}-1\\3\\-7\end{pmatrix}$

TABLE 0: Vectors and their corresponding points

The condition is:

$$||\mathbf{A}-\mathbf{P}||^2 + ||\mathbf{B}-\mathbf{P}||^2 = 2k^2 \tag{0.1}$$

$$(\mathbf{P}-\mathbf{A})^T(\mathbf{P}-\mathbf{A}) + (\mathbf{P}-\mathbf{B})^T(\mathbf{P}-\mathbf{B}) = 2k^2 \tag{0.2}$$

$$\mathbf{P}^T\mathbf{P} - (\mathbf{A}+\mathbf{B})^T\mathbf{P} + \frac{\mathbf{A}^T\mathbf{A}+\mathbf{B}^T\mathbf{B}}{2} = k^2 \tag{0.3}$$

Now by completing the square we get:

$$\left\|\mathbf{P}-\frac{\mathbf{A}+\mathbf{B}}{2}\right\|^2 - \frac{(\mathbf{A}+\mathbf{B})^T(\mathbf{A}+\mathbf{B})}{4} + \frac{\mathbf{A}^T\mathbf{A}+\mathbf{B}^T\mathbf{B}}{2} = k^2 \tag{0.4}$$

$$(\mathbf{A}+\mathbf{B})^T(\mathbf{A}+\mathbf{B}) = 57,\ \ \mathbf{A}^T\mathbf{A} = 50,\ \ \mathbf{B}^T\mathbf{B} = 59 \tag{0.5}$$

Rearranging and substituting values we get:

$$\left\|\mathbf{P}-\frac{\mathbf{A}+\mathbf{B}}{2}\right\|^2 = k^2 - \frac{161}{4} \tag{0.6}$$

$$\left(\mathbf{P}-\frac{\mathbf{A}+\mathbf{B}}{2}\right)^T\left(\mathbf{P}-\frac{\mathbf{A}+\mathbf{B}}{2}\right) = k^2 - \frac{161}{4} \tag{0.7}$$

$$k^2 > \frac{161}{4} \tag{0.8}$$

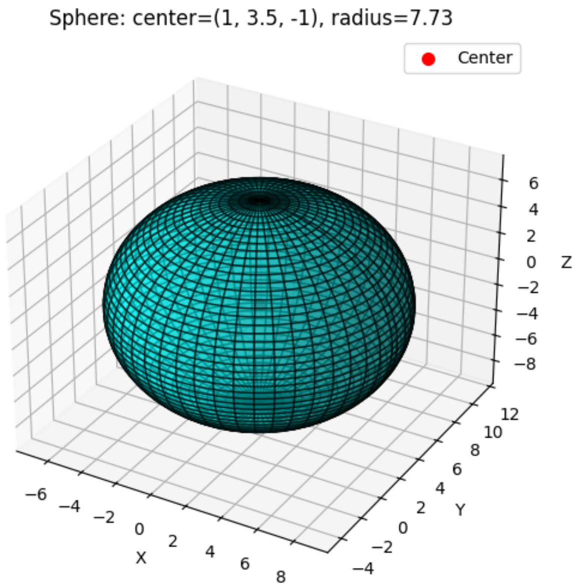

Fig. 0.1: Graph plotted by taking k=10 as example.