4.3.38

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Question : Solve the system of equations

$$5x - y + 4z = 5$$
$$2x + 3y + 5z = 2$$
$$5x - 2y + 6z = -1$$

Solution :

Name	Equation
Equation 1	$5x - y + 4z = 5 \iff \begin{pmatrix} 5 & -1 & 4 \end{pmatrix} \mathbf{x}_1 = 5$
Equation 2	$2x + 3y + 5z = 2 \iff \begin{pmatrix} 2 & 3 & 5 \end{pmatrix} \mathbf{x}_2 = 2$
Equation 3	$5x - 2y + 6z = -1 \iff \begin{pmatrix} 5 & -2 & 6 \end{pmatrix} \mathbf{x}_3 = -1$

Table : Equations

The system of equations in matrix form is :

$$\begin{pmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix} \tag{1}$$

Forming the augmented matrix,

$$\left(\begin{array}{ccc|c} 5 & -1 & 4 & 5 \\ 2 & 3 & 5 & 2 \\ 5 & -2 & 6 & -1 \end{array}\right) \tag{2}$$

Using Gaussian elimination,

$$\left(\begin{array}{ccc|c} 5 & -1 & 4 & 5 \\ 2 & 3 & 5 & 2 \\ 5 & -2 & 6 & -1 \end{array}\right) \xleftarrow[R_2 \to R_2 - \frac{2}{5}R_1]{R_3 \to R_3 - R_1} \left(\begin{array}{ccc|c} 5 & -1 & 4 & 5 \\ 0 & \frac{17}{5} & \frac{17}{5} & 0 \\ 0 & -1 & 2 & -6 \end{array}\right) \tag{3}$$

$$\xleftarrow{R_3 \to R_3 + \frac{5}{17}R_2} \left(\begin{array}{ccc|c} 5 & -1 & 4 & 5 \\ 0 & \frac{17}{5} & \frac{17}{5} & 0 \\ 0 & 0 & 3 & -6 \end{array}\right) \tag{4}$$

Using back substitution we get :

$$3z = -6 \tag{5}$$
$$z = -2 \tag{6}$$
$$\frac{17}{5}y + \frac{17}{5}z = 0 \quad \Rightarrow \quad y + z = 0 \tag{7}$$
$$y = -z = 2 \tag{8}$$
$$5x - y + 4z = 5 \tag{9}$$
$$5x - 2 + 4(-2) = 5 \tag{10}$$
$$5x - 10 = 5 \tag{11}$$
$$x = 3 \tag{12}$$

Therefore the solution for the system of equations is :

$$\begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \tag{13}$$

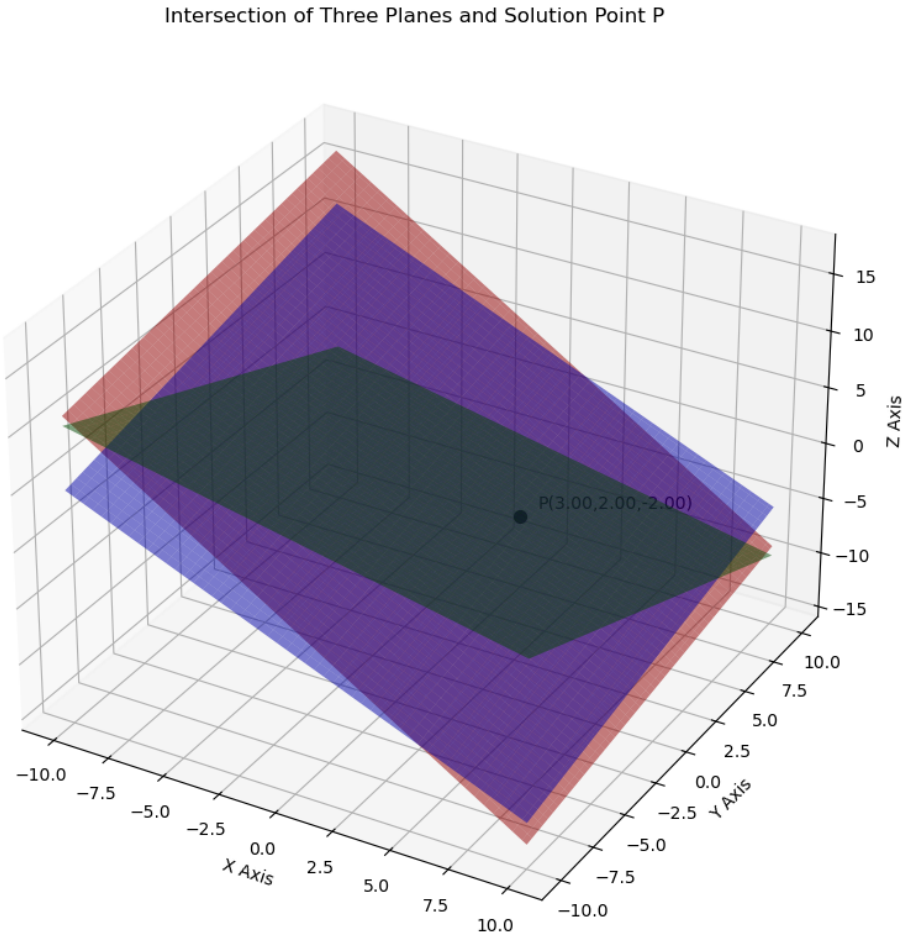
Intersection of Three Planes and Solution Point P
P(3.00,2.00,-2.00)
X Axis
Y Axis
Z Axis
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
15
10
5
0
−5
−10
−15

Fig : Planes