4.7.25

EE25BTECH11033 - Kavin

Question:
Find the points on the line $x+y = 4$ which lie at a unit distance from the line $4x+3y = 10$.

Solution:
According to the question,

$$\text{Equation of line } L_1\text{: } \begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = 4 \tag{1}$$

and

$$\text{Equation of line } L_2\text{: } \begin{pmatrix}4 & 3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = 10 \tag{2}$$

Any point $\mathbf{P}$ on line L_1 is given by ,

$$\mathbf{P} = \begin{pmatrix}k\\4-k\end{pmatrix} \tag{3}$$

The distance λ of a vector $\mathbf{P}$ from the line $\mathbf{n}^\top\mathbf{x} = c$ is given by ,

$$\lambda = \frac{\left|\mathbf{n}^\top\mathbf{P} - c\right|}{\|\mathbf{n}\|} \tag{4}$$

where,

$$\mathbf{n}^\top = \begin{pmatrix}4 & 3\end{pmatrix}\ , c = 10 \text{ and } \lambda = 1$$

$$\implies \lambda\|\mathbf{n}\| = \left|\mathbf{n}^\top\mathbf{P} - c\right| \tag{5}$$

Also,

$$\|\mathbf{n}\| = \sqrt{\mathbf{n}^\top\mathbf{n}} = \sqrt{25} = 5 \tag{6}$$

$$\mathbf{n}^\top\mathbf{P} = k + 12 \tag{7}$$

$$\implies 5 = |k + 12 - 10| \tag{8}$$

$$\implies 5 = |k + 2| \tag{9}$$

$$\implies k = 3, -7 \tag{10}$$

Therefore the points on L_1 which lie at a unit distance from the line L_2 are ,

$$\begin{pmatrix} 3 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} -7 \\ 11 \end{pmatrix}$$