3.2.4

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Question:
Construct the triangle $BD'C'$ similar to $\triangle BDC$ with scale factor $\frac{4}{3}$.Draw the line segment $D'A'$. parallel to DA where A^p prime lies on extended side BA.Is $A'BC'D'$ a parallelogram?
solution

Vector	Name
$\begin{pmatrix}0\\0\end{pmatrix}$	Vector $\mathbf{A}$
$\begin{pmatrix}4\\0\end{pmatrix}$	Vector $\mathbf{B}$
$\begin{pmatrix}4\\3\end{pmatrix}$	Vector $\mathbf{C}$
$\begin{pmatrix}0\\3\end{pmatrix}$	Vector $\mathbf{D}$

TABLE 0: Variables Used

consider $\triangle BDC$.constructs a $\triangle BD'C'$ with scale factor $\frac{4}{3}$.
This means

$$\triangle BD'C' \sim \triangle BDC. \tag{1}$$

$$\frac{\mathbf{D'}-\mathbf{B}}{\mathbf{D}-\mathbf{B}} = \frac{\mathbf{C'}-\mathbf{B}}{\mathbf{C}-\mathbf{B}} = \frac{\mathbf{C'}-\mathbf{D'}}{\mathbf{C}-\mathbf{D}} = \frac{4}{3}. \tag{2}$$

$$\mathbf{D'} = \mathbf{B} + \frac{4}{3}(\mathbf{D}-\mathbf{B}) \tag{3}$$

$$\mathbf{D'} = \begin{pmatrix}-4/3\\4\end{pmatrix} \tag{4}$$

$$\mathbf{C'} = \mathbf{B} + \frac{4}{3}(\mathbf{C}-\mathbf{B}) \tag{5}$$

$$\mathbf{C'} = \begin{pmatrix}4\\4\end{pmatrix} \tag{6}$$

Construct $\mathbf{A'}$
Mark $\mathbf{D'}$ and $\mathbf{A'}$ parallel to $\mathbf{D}-\mathbf{A}$ with $\mathbf{A'}$ along the direction of $\mathbf{B}-\mathbf{A}$.

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix}4\\0\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}4\\0\end{pmatrix} \tag{7}$$

$$\mathbf{C}-\mathbf{D} = \begin{pmatrix}4\\3\end{pmatrix} - \begin{pmatrix}0\\3\end{pmatrix} = \begin{pmatrix}4\\0\end{pmatrix} \tag{8}$$

$$\implies \mathbf{B}-\mathbf{A} = \mathbf{C}-\mathbf{D} \tag{9}$$

$$\mathbf{D}-\mathbf{A} = \begin{pmatrix}0\\3\end{pmatrix} - \begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}0\\3\end{pmatrix} \tag{10}$$

$$\mathbf{C}-\mathbf{B} = \begin{pmatrix}4\\3\end{pmatrix} - \begin{pmatrix}4\\0\end{pmatrix} = \begin{pmatrix}0\\3\end{pmatrix} \tag{11}$$

$$\implies \mathbf{D}-\mathbf{A} = \mathbf{C}-\mathbf{B} \tag{12}$$

$\implies$ ABCD is a Parallelogram
Check the parallelogram property of $A'BC'D'$

$$\mathbf{B}-\mathbf{A'} = -t(\mathbf{A}-\mathbf{B}) \tag{13}$$

$$\mathbf{D'}-\mathbf{C'} = k(\mathbf{C}-\mathbf{D}) \tag{14}$$

$$\text{From Equation (9) } \mathbf{B}-\mathbf{A} = \mathbf{C}-\mathbf{D} \tag{15}$$

$$\implies \mathbf{B}-\mathbf{A'} = -t(\mathbf{A}-\mathbf{B}) = t(\mathbf{C}-\mathbf{D}) = \frac{t}{k}\mathbf{D'}-\mathbf{C'} \tag{16}$$

$$\implies \mathbf{B}-\mathbf{A'} \parallel \mathbf{D'}-\mathbf{C'} \tag{17}$$

By construction of A′

$$\mathbf{D}' - \mathbf{A}' \parallel \mathbf{D} - \mathbf{A} \tag{18}$$
$$\mathbf{D} - \mathbf{A} \parallel \mathbf{C} - \mathbf{B} \tag{19}$$
$$\mathbf{C} - \mathbf{B} \parallel \mathbf{C}' - \mathbf{B} \tag{20}$$
$$\implies \mathbf{D}' - \mathbf{A}' \parallel \mathbf{C}' - \mathbf{B} \tag{21}$$

$\implies$ A′BC′D′ a parallelogram