

# 9.4.34

EE25BTECH11021 - Dhanush sagar

## Question:

Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

## Solution:

Let the present ages be represented as the vector:

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (1)$$

where  $x$  and  $y$  denote Rohan's and his mother's present ages respectively.

given,

eq 1 : Since the mother is 26 years older than Rohan,

$$y = x + 26 \quad (2)$$

eq 2 : The product of their ages three years from now is given as

$$(x + 3)(y + 3) = 360 \quad (3)$$

Expanding the above equation:

$$xy + 3x + 3y - 351 = 0 \quad (4)$$

This can be written in quadratic (matrix) form as

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (5)$$

where

$$\mathbf{V} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} \frac{3}{2} \\ \frac{3}{2} \end{pmatrix}, \quad f = -351 \quad (6)$$

The line  $y = x + 26$  can be expressed parametrically as

$$\mathbf{x} = \mathbf{h} + \kappa \mathbf{m}, \quad \kappa \in \mathbb{R} \quad (7)$$

where

$$\mathbf{h} = \begin{pmatrix} 0 \\ 26 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (8)$$

Substituting  $\mathbf{x} = \mathbf{h} + \kappa \mathbf{m}$  in the conic equation:

$$(\mathbf{h} + \kappa \mathbf{m})^T \mathbf{V} (\mathbf{h} + \kappa \mathbf{m}) + 2\mathbf{u}^T (\mathbf{h} + \kappa \mathbf{m}) + f = 0 \quad (9)$$

Grouping powers of  $\kappa$ , we get:

$$\kappa^2 (\mathbf{m}^T \mathbf{V} \mathbf{m}) + 2\kappa \mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u}) + g(\mathbf{h}) = 0 \quad (10)$$

where

$$g(\mathbf{h}) = \mathbf{h}^T \mathbf{V} \mathbf{h} + 2\mathbf{u}^T \mathbf{h} + f \quad (11)$$

Now compute each term:

$$\mathbf{m}^T \mathbf{V} \mathbf{m} = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1 \quad (12)$$

$$\mathbf{V} \mathbf{h} + \mathbf{u} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 26 \end{pmatrix} + \begin{pmatrix} \frac{3}{2} \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} 14.5 \\ 1.5 \end{pmatrix} \quad (13)$$

$$\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u}) = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 14.5 \\ 1.5 \end{pmatrix} = 16 \quad (14)$$

$$g(\mathbf{h}) = \mathbf{h}^T \mathbf{V} \mathbf{h} + 2\mathbf{u}^T \mathbf{h} + f = 0 + 2\left(\frac{3}{2} \times 26\right) - 351 = -273 \quad (15)$$

Substituting these results gives:

$$\kappa^2 + 32\kappa - 273 = 0 \quad (16)$$

The general quadratic solution is

$$\kappa = \frac{-\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h}) (\mathbf{m}^T \mathbf{V} \mathbf{m})}}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \quad (17)$$

Substituting numerical values:

$$\kappa = \frac{-16 \pm \sqrt{16^2 - (-273)}}{1} = -16 \pm \sqrt{529} = -16 \pm 23 \quad (18)$$

Thus,

$$\kappa_1 = -39, \quad \kappa_2 = 7 \quad (19)$$

The intersection points are

$$\mathbf{x}_i = \mathbf{h} + \kappa_i \mathbf{m}, \quad i = 1, 2 \quad (20)$$

Hence,

$$\mathbf{x}_1 = \begin{pmatrix} 0 \\ 26 \end{pmatrix} - 39 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -39 \\ -13 \end{pmatrix}, \quad \mathbf{x}_2 = \begin{pmatrix} 0 \\ 26 \end{pmatrix} + 7 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 33 \end{pmatrix} \quad (21)$$

The physically meaningful (non-negative) intersection corresponds to

$$\boxed{x = 7, \quad y = 33} \quad (22)$$

Therefore, Rohan's present age is 7 years and his mother's present age is 33 years.

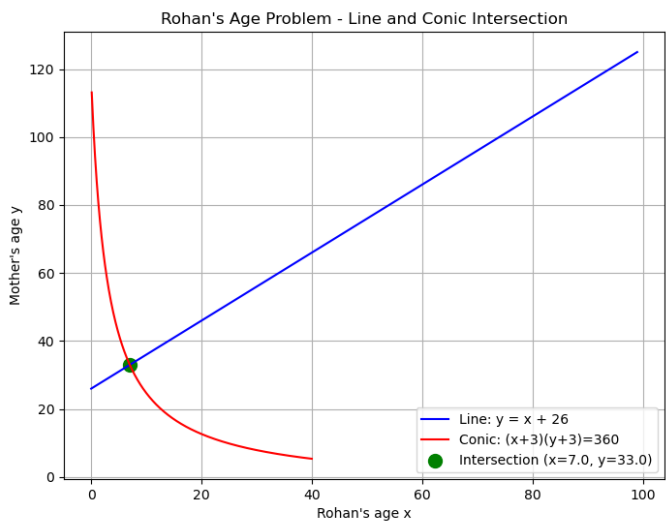


Fig. 0.1