5.13.1

AI25BTECH11014 - Gooty Suhas

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Problem

If the system of equations

$$x + 2ay + az = 0$$
$$x + 3by + bz = 0$$
$$x + 4cy + cz = 0$$

has a non-zero solution, then what is the relation among a, b, c?

Options:

a) $a + 2b + 3c = 0$

b) a, b, c are in arithmetic progression (A.P.)

c) a, b, c are in geometric progression (G.P.)

d) a, b, c are in harmonic progression (H.P.)

Matrix Form

We write the system as:

$$\mathbf{M}_0 = \begin{bmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{bmatrix}$$

Since the system has a non-zero solution, the rows of $\mathbf{M}_0$ must be linearly dependent.

Row Operations

Subtract Row 1 from Rows 2 and 3:

$$\mathbf{M}_1 = \begin{bmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-2a & c-a \end{bmatrix}$$

Now subtract Row 2 from Row 3:

$$\mathbf{M}_2 = \begin{bmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 4c-3b & c-b \end{bmatrix}$$

Linear Dependence Condition

For linear dependence:

$$\frac{4c-3b}{3b-2a} = \frac{c-b}{b-a}$$

Cross-multiplying:

$$(4c-3b)(b-a) = (c-b)(3b-2a)$$

Algebraic Expansion

Expand both sides:

$$\text{LHS} = 4bc - 4ac - 3b^2 + 3ab$$
$$\text{RHS} = 3bc - 2ac - 3b^2 + 2ab$$

Subtract RHS from LHS:

$$(4bc - 3bc) + (3ab - 2ab) + (-4ac + 2ac) = 0 \Rightarrow \boxed{ab + bc = 2ac}$$

Verification of Option d

If a, b, c are in H.P., then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.:

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

Multiply both sides by abc:

$$2ac = bc + ab \Rightarrow \boxed{ab + bc = 2ac}$$

This matches our derived condition.

Final Answer

$$\boxed{ab + bc = 2ac} \quad \Rightarrow \quad \boxed{\text{Option d) } a, b, c \text{ are in H.P.}}$$

independent_verify.py

```
from sympy import symbols, Eq, simplify

a, b, c = symbols('a b c')

# Harmonic progression condition: 2/b = 1/a + 1/c
hp = Eq(2/b, 1/a + 1/c)

# Multiply both sides by abc
lhs = simplify(2*a*c)
rhs = simplify(a*b + b*c)

print("ab + bc =", rhs)
print("2ac =", lhs)
print("Condition holds:", lhs == rhs)
```

verify_condition.c

```c
#include <math.h>

void verify_condition(float a, float b, float c, float* result) {
    float lhs = a*b + b*c;
    float rhs = 2*a*c;

    if (fabs(lhs - rhs) < 1e-6)
        *result = 1.0;
    else
        *result = 0.0;
}
```

call_verify.py (Part 1)

```
import ctypes

lib = ctypes.CDLL('./libverify.so')

lib.verify_condition.argtypes = [
    ctypes.c_float, ctypes.c_float,
    ctypes.c_float,
    ctypes.POINTER(ctypes.c_float)
]
lib.verify_condition.restype = None
```

call_verify.py (Part 2)

```
a = ctypes.c_float(1.0)
b = ctypes.c_float(2.0)
c = ctypes.c_float(0.5)
result = ctypes.c_float()

lib.verify_condition(a, b, c,
                     ctypes.byref(result))

if result.value == 1.0:
    print("Verified: ab + bc = 2ac")
else:
    print("Condition fails")
```