5.7.2

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Question

If $\mathbf{A} = \begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find the scalar k so that $\mathbf{A}^2 + \mathbf{I} = k\mathbf{A}$.

Solution

Given:

$$A = \begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{1}$$

Characteristic Polynomial of A

The characteristic polynomial is obtained from:

$$\det(A - \lambda I) = 0 \tag{2}$$

Solution

$$\det\begin{pmatrix} -3-\lambda & 2 \\ 1 & -1-\lambda \end{pmatrix} = (-3-\lambda)(-1-\lambda) - (2)(1) \tag{3}$$

$$= (\lambda+3)(\lambda+1) - 2 = \lambda^2 + 4\lambda + 3 - 2 = \lambda^2 + 4\lambda + 1 \tag{4}$$

So the characteristic equation is:

$$\lambda^2 + 4\lambda + 1 = 0 \tag{5}$$

Solution

By Cayley-Hamilton theorem, matrix A satisfies its own characteristic equation:

$$A^2 + 4A + I = 0 \tag{6}$$

From the Cayley-Hamilton result:

$$A^2 + I = -4A \tag{7}$$

Solution

Comparing with the target equation $A^2 + I = kA$, we get:

$$kA = -4A \Rightarrow \boxed{k = -4} \tag{8}$$

Final Answer

$$\boxed{k = -4} \tag{9}$$