9.2.31

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Question
Find the area of the region bounded by the curve y^2=4x and x^2=4y
Solution:

Variable	Name
$\mathbf{x_1},\mathbf{x_2}$	points of intersection
A	vector Area of the desired region

TABLE 0: Variables Used

The equation of a parabola in Matrix form is

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

For y^2=4x

$$\mathbf{V_1} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{2}$$

$$\mathbf{u_1} = -2\mathbf{e_1} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{3}$$

$$f_1 = 0 \tag{4}$$

For x^2=4y

$$\mathbf{V_2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{5}$$

$$\mathbf{u_2} = -2\mathbf{e_2} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{6}$$

$$f_2 = 0 \tag{7}$$

The intersection of two conics with parameters $\mathbf{v_i}$,$\mathbf{u_i}$,f_i is defined as

$$\mathbf{X}^T \left(\mathbf{V}_1 + \mu\mathbf{V}_2\right)\mathbf{X} + 2(\mathbf{u_1} + \mu\mathbf{u_2})^T\mathbf{X} \; + \; (f_1 + \mu f_2) \; = \; 0 \tag{8}$$

$$\implies \begin{vmatrix} \mathbf{V}_1 + \mu\mathbf{V}_2 & \mathbf{u}_1 + \mu\mathbf{u}_2 \\ (\mathbf{u}_1 + \mu\mathbf{u}_2)^\top & f_1 + \mu f_2 \end{vmatrix} = 0 \tag{9}$$

$$\implies \begin{vmatrix} \mu & 0 & -2 \\ 0 & 1 & -2\mu \\ -2 & -2\mu & 0 \end{vmatrix} = 0 \tag{10}$$

$$\implies \begin{vmatrix} \mu & 0 & -2 \\ 0 & 1 & -2\mu \\ -2 & -2\mu & 0 \end{vmatrix} \xleftrightarrow{R_3 \leftrightarrow R_3 + \frac{2}{\mu}\times R_1} \begin{vmatrix} \mu & 0 & -2 \\ 0 & 1 & -2\mu \\ 0 & -2\mu & -\frac{4}{\mu} \end{vmatrix} \tag{11}$$

$$\xleftrightarrow{R_3 \leftrightarrow R_3 + 2\mu\times R_2} \begin{vmatrix} \mu & 0 & -2 \\ 0 & 1 & -2\mu \\ 0 & 0 & -(\frac{4}{\mu} + 4\mu^2) \end{vmatrix} = 0 \tag{12}$$

$$\implies -(4 + +4\mu^3) = 0 \tag{13}$$

$$\implies \mu = -1 \tag{14}$$

substituting the value of μ=-1 in (8) we get points of intersection as

$$\mathbf{x_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{15}$$

$$\mathbf{x_2} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{16}$$

Area of the desired region is given by

$$A = \int_0^4 2\sqrt{x} - \frac{x^2}{4} dx \tag{17}$$

$$A = \frac{32}{3} - \frac{16}{3} \tag{18}$$

$$A = \frac{16}{3} \tag{19}$$

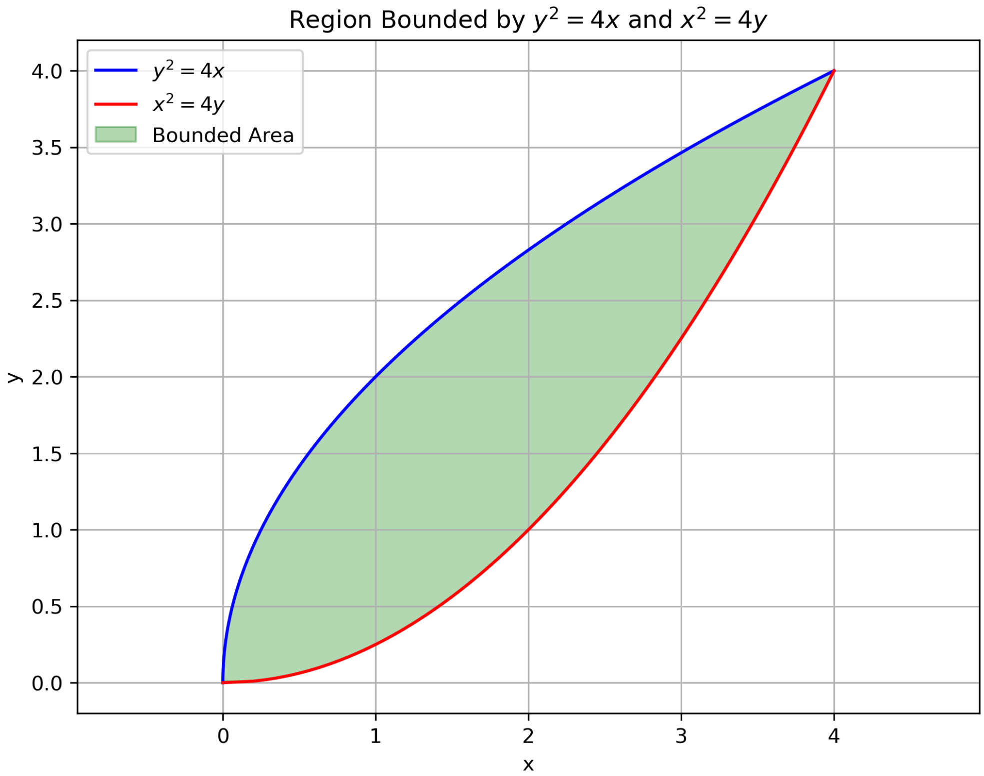

Fig. 0: Area