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EE25BTECH11023 - Venkata Sai

Question:
If the system of equations

$$x - ky - z = 0 \tag{1}$$
$$kx - y - z = 0 \tag{2}$$
$$x + y + z = 0 \tag{3}$$

has a non-zero solution, then the possible values of k are

Solution:
For the given homogeneous system

$$\mathbf{Ax} = 0 \tag{4}$$

Augmented matrix of $\left(\mathbf{A} \mid 0\right)$ is given by

$$\left(\begin{array}{ccc|c} 1 & -k & -1 & 0 \\ k & -1 & -1 & 0 \\ 1 & 1 & -1 & 0 \end{array}\right) \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - kR_1} \left(\begin{array}{ccc|c} 1 & -k & -1 & 0 \\ 0 & k^2-1 & k-1 & 0 \\ 0 & 1+k & 0 & 0 \end{array}\right) \xrightarrow{R_2 - (k-1)R_3} \left(\begin{array}{ccc|c} 1 & -k & -1 & 0 \\ 0 & k+1 & 0 & 0 \\ 0 & 0 & k-1 & 0 \end{array}\right) \tag{5}$$

For a non-zero solution,The rank of the matrix must be less than the number of variables From (5), In order to be Rank<3

$$k + 1 = 0 \ \text{(or)} \ k - 1 = 0 \tag{6}$$
$$k = -1 \ \text{(or)} \ k = 1 \tag{7}$$