2.10.20

AI25BTECH11030 -Sarvesh Tamgade

Question: Which of the following expressions are meaningful?

(a) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$
(b) $(\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w}$
(c) $(\mathbf{u} \cdot \mathbf{v})\ \mathbf{w}$
(d) $\mathbf{u} \times (\mathbf{v} \cdot \mathbf{w})$

Solution:

Let

$$\mathbf{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}. \tag{4.1}$$

(4.2)

a) $\mathbf{u}^\top(\mathbf{v} \times \mathbf{w})$

$$\mathbf{v} \times \mathbf{w} = \begin{pmatrix} v_{23} & w_{23} \\ v_{31} & w_{31} \\ v_{12} & w_{12} \end{pmatrix} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix} = \begin{pmatrix} 1 \times 0 - 0 \times 5 \\ 0 \times 0 - 4 \times 0 \\ 4 \times 5 - 1 \times 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 20 \end{pmatrix}$$

$$\mathbf{u}^\top(\mathbf{v} \times \mathbf{w}) = \begin{bmatrix} 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix} = 0$$

Since the scalar (dot) product of two vectors is defined, the expression $\mathbf{u}^\top(\mathbf{v} \times \mathbf{w})$ is meaningful.

b) $(\mathbf{u}^\top \mathbf{v})^\top \mathbf{w}$

$$\mathbf{u}^\top \mathbf{v} = \begin{bmatrix} 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} = 2 \times 4 + 3 \times 1 = 11,$$

$$(\mathbf{u}^\top \mathbf{v})^\top \mathbf{w} = 11^\top \mathbf{w} \quad \text{(scalar dot vector)} \quad \text{undefined.}$$

c) $(\mathbf{u}^\top \mathbf{v})\mathbf{w}$

$$(\mathbf{u}^\top \mathbf{v})\mathbf{w} = 11 \times \begin{bmatrix} 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 55 \end{bmatrix}.$$

This is meaningful scalar multiplication.

d) $\mathbf{u} \times (\mathbf{v}^\top \mathbf{w})$

$$\mathbf{v}^\top \mathbf{w} = \begin{bmatrix} 4 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} = 0 + 5 = 5,$$

$$\mathbf{u} \times 5 = \text{cross product of vector and scalar – undefined.}$$

Answer: Only (a) and (c) are meaningful

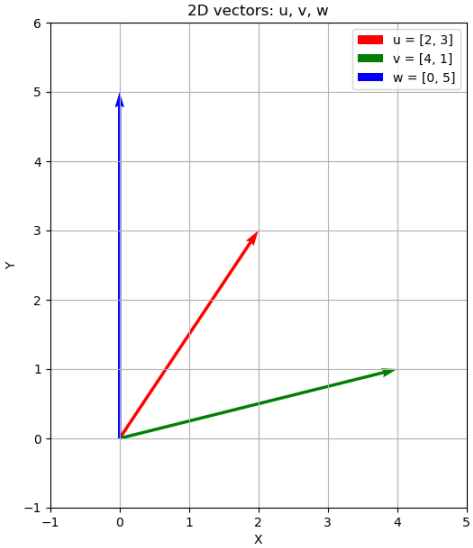

Fig. 4.1: Vector Representation