

12.547

EE25BTECH11013 - Bhargav

Question:

Consider \mathbf{R}^3 with the usual inner product. If d is the distance from $(1,1,1)$ to the subspace $\text{span} \{(1, 1, 0), (0, 1, 1)\}$ of \mathbf{R}^3 , then $3d^2$ is

Solution:

Let $\mathbf{W} = \text{span} \{u_1, u_2\}$

Where $\mathbf{U} = \begin{pmatrix} u_1 & u_2 \end{pmatrix}$

The distance from $\mathbf{P} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ to the subspace $\text{span } \mathbf{W}$ can be found by finding the projection of \mathbf{P} onto \mathbf{W} .

Let \mathbf{Ux} be the projection of \mathbf{P} on the span \mathbf{W}

$$\mathbf{U}^T (\mathbf{P} - \mathbf{Ux}) = 0 \quad (0.1)$$

(since \mathbf{U} is perpendicular to $\mathbf{P} - \mathbf{Ux}$)

$$\implies \mathbf{U}^T \mathbf{Ux} = \mathbf{U}^T \mathbf{P} \quad (0.2)$$

Since the columns of \mathbf{U} are Linearly independent, so are the columns of $\mathbf{U}^T \mathbf{U}$ and hence $\mathbf{U}^T \mathbf{U}$ is invertible

$$\mathbf{x} = (\mathbf{U}^T \mathbf{U})^{-1} \mathbf{U}^T \mathbf{P} \quad (0.3)$$

Hence the projection of \mathbf{P} on the span \mathbf{W} is

$$\mathbf{Ux} = \mathbf{U} (\mathbf{U}^T \mathbf{U})^{-1} \mathbf{U}^T \mathbf{P} \quad (0.4)$$

The distance of \mathbf{P} from the span \mathbf{W} is:

$$d = \|\mathbf{P} - \mathbf{Ux}\| \quad (0.5)$$

$$d = \left\| \mathbf{P} - \mathbf{U} (\mathbf{U}^T \mathbf{U})^{-1} \mathbf{U}^T \mathbf{P} \right\| \quad (0.6)$$

$$\mathbf{P} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{U} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} \quad (0.7)$$

Substituting the values in (0.6):

$$d = \frac{1}{\sqrt{3}} \quad (0.8)$$

$3d^2 = 1$

Distance from Point to Plane

