7.2.12

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Question

If the lines $2x - 3y = 5$ and $3x - 4y = 7$ are the diameters of a circle of area 154 square units, then obtain the equation of the circle.

Theoretical Solution

Solution:

Let :

$$\mathbf{r_1} = \begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{k} = 5 \tag{1}$$

$$\mathbf{r_2} = \begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{k} = 7 \tag{2}$$

The augmented matrix of the above equations is given by,

$$\begin{pmatrix} 2 & -3 & 5 \\ 3 & -4 & 7 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow 2R_2 - 3R_1} \begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -1 \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -1 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 + 3R_2} \begin{pmatrix} 2 & 0 & 2 \\ 0 & 1 & -1 \end{pmatrix} \tag{4}$$

Theoretical Solution

$$2x = 2 \qquad x = 1 \tag{5}$$

$$y = -1 \tag{6}$$

Point of intersection of diameters of circle is the center of circle $\mathbf{k} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Given

$$\text{Area of circle} = \pi r^2 = 154 \text{ sq. units}$$
$$\text{Using } \pi = \frac{22}{7} \qquad \text{r=7 units}$$

$$\text{Equation of circle is } ||\mathbf{x}||^2 + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{7}$$

$$\mathbf{u} = -\mathbf{k} \qquad f = ||\mathbf{u}||^2 - r^2 \tag{8}$$

Theoretical Solution

$$\mathbf{u} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \qquad f = (\sqrt{2})^2 - 7^2 = -47 \tag{9}$$

$$\text{Equation of circle is } ||\mathbf{x}||^2 + 2\begin{pmatrix} -1 & 1 \end{pmatrix}\mathbf{x} - 47 = 0 \tag{10}$$

C Code

```c
#include <stdio.h>
#include <math.h>

int main() {
    // Define the coefficient matrix A and the constant vector B
    // Corresponding to the system:
    // 2x - 3y = 5
    // 3x - 4y = 7
    double A[2][2] = {{2.0, -3.0}, {3.0, -4.0}};
    double B[2] = {5.0, 7.0};

    // Calculate the determinant of the coefficient matrix A
    double determinant = A[0][0] * A[1][1] - A[0][1] * A[1][0];

    // Check if a unique solution exists
    if (determinant == 0) {
        printf("The lines are parallel or coincident; no unique
            solution exists.\n");
        return 1;
```

C Code

```c
// Solve for x and y using Cramer's Rule
   // Determinant for x (replace first column with B)
   double det_x = B[0] * A[1][1] - B[1] * A[0][1];

   // Determinant for y (replace second column with B)
   double det_y = A[0][0] * B[1] - A[1][0] * B[0];

   double center_x = det_x / determinant;
   double center_y = det_y / determinant;

   // --- Part 2: Calculate radius and find the circle's
       equation ---

   // Given area of the circle
   double area = 154.0;
   const double PI = 22.0 / 7.0;
```

C Code

```
// Calculate the radius squared from the area formula: Area = PI
    * r^2
  double r_squared = area / PI;

  // The equation of a circle can be expressed as: x^2 + y^2 -
      2hx - 2ky + h^2 + k^2 - r^2 = 0
  // where (h, k) is the center of the circle.

  // The general form parameters (as used in the provided
      solution) are:
  // 2gx = -2hx => g = -h
  // 2fy = -2ky => f = -k
  // c = h^2 + k^2 - r^2

  double h = center_x;
  double k = center_y;
  double c = h * h + k * k - r_squared;
```

C Code

```
    // Print the final equation
      printf("The equation of the circle is:\n");
      printf("x^2 + y^2 - %.0fx - %.0fy + %.0f = 0\n", 2*h, 2*k, c)
          ;

      return 0;
}
```

Python Code

```python
import numpy as np
import matplotlib.pyplot as plt
import math

def plot_circle_solution():
    """
    Plots the two diameter lines and the resulting circle,
        highlighting the center.
    """

    ## 1. Solve for the center of the circle (intersection of the
         diameters)
    # The system of equations is:
    # 2x - 3y = 5
    # 3x - 4y = 7
    # Use Cramer's Rule to solve for x and y
```

Python Code

```
A = np.array([[2, -3], [3, -4]])
  B = np.array([5, 7])

  det_A = np.linalg.det(A)

  if det_A == 0:
      print("The lines are parallel or coincident; no unique
          solution exists.")
      return

  det_x = np.linalg.det(np.array([[5, -3], [7, -4]]))
  det_y = np.linalg.det(np.array([[2, 5], [3, 7]]))

  center_x = det_x / det_A
  center_y = det_y / det_A
```

Python Code

```
print(f"The center of the circle is at ({center_x:.2f}, {center_y
    :.2f})")

    ## 2. Calculate the radius from the area
    area = 154.0
    r_squared = area / math.pi
    radius = math.sqrt(r_squared)

    print(f"The radius of the circle is: {radius:.2f}")

    ## 3. Plot the solution
    fig, ax = plt.subplots(figsize=(8, 8))

    # Plot the diameter lines
    x_vals = np.linspace(center_x - 5, center_x + 5, 400)
    y_line1 = (2 * x_vals - 5) / 3
    y_line2 = (3 * x_vals - 7) / 4
```

Python Code

```
ax.plot(x_vals, y_line1, label='$2x - 3y = 5$')
  ax.plot(x_vals, y_line2, label='$3x - 4y = 7$')

  # Plot the circle
  circle = plt.Circle((center_x, center_y), radius, color='
      green', fill=False, linewidth=2, label='Circle')
  ax.add_patch(circle)

  # Plot the center point
  ax.plot(center_x, center_y, 'o', color='red', markersize=8,
      label='Center')
  ax.annotate(f'({center_x:.2f}, {center_y:.2f})', (center_x,
      center_y),
              textcoords="offset points", xytext=(0,10), ha='
                  center')
```

Python Code

```
    # Add labels, title, and legend
    ax.set_title('Equation of a Circle from its Diameters')
    ax.set_xlabel('x')
    ax.set_ylabel('y')
    ax.set_aspect('equal', adjustable='box')
    ax.grid(True, linestyle='--', alpha=0.6)
    ax.legend()

    plt.show()

# Run the plotting function
plot_circle_solution()
```

Plot

Beamer/figs/circle.png

Figure: