12.456

EE25BTECH11026-Harsha

Question:
If a rectangle is deformed into a parallelogram of equal area by simple shear deformation (with shear strain γ) parallel to the abscissa, the displacement matrix is ___________.

1) $\begin{pmatrix} 1 & \gamma \\ 0 & 1 \end{pmatrix}$
2) $\begin{pmatrix} 1 & 0 \\ \gamma & 1 \end{pmatrix}$
3) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
4) $\begin{pmatrix} 0 & \gamma \\ 1 & 0 \end{pmatrix}$

Solution:
Let us solve the given question theoretically and then verify the solution computationally.

Due to the shear deformation, let x', y' be the new coordinates. As the deformation is along the direction of abscissa,

$$\therefore y' = y \tag{4.1}$$

Let the displacement due to the shear deformation be Δh.

$$\gamma = \frac{\Delta h}{y} \tag{4.2}$$

$$\therefore \Delta h = \gamma y \tag{4.3}$$

$$\implies x' = x + \Delta h = x + \gamma y \tag{4.4}$$

From (4.1) and (4.4),

$$\therefore \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x + \gamma y \\ y \end{pmatrix} = \begin{pmatrix} 1 & \gamma \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{4.5}$$

From the figure, it is clearly verified that the theoretical solution matches with the computational solution.