# 4.8.35

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**Question.** $y = 10^x$ is the reflection of $y = \log_{10} x$ in the line whose equation is ____________________.

**Solution.**

Let the mirror line be written in normal form

$$\mathbf{n}^\top \mathbf{x} = c, \tag{1}$$

with normal $\mathbf{n} \in \mathbb{R}^2$ and variable vector $\mathbf{x} \in \mathbb{R}^2$.

A point on the curve $y = \log_{10} u$ is

$$\mathbf{Q}(t) = \begin{pmatrix} t \\ \log_{10} t \end{pmatrix}, \qquad t > 0. \tag{2}$$

Its reflection in the line (1) is

$$\mathbf{R}(t) = \mathbf{Q}(t) - \frac{2(\mathbf{n}^\top \mathbf{Q}(t) - c)}{\|\mathbf{n}\|^2}\,\mathbf{n}. \tag{3}$$

For the image curve to be $y = 10^x$ we require

$$\mathbf{R}(t) = \begin{pmatrix} \log_{10} t \\ t \end{pmatrix} \quad \text{for all } t > 0. \tag{4}$$

Equating components in (3)–(4) and collecting the independent functions $t$ and $\log_{10} t$ yields

$$a^2 = b^2, \qquad a\,b = -a\,b, \qquad c = 0. \tag{5}$$

Hence $a = -b \neq 0$ and $c = 0$. Thus $\mathbf{n} \propto \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and the mirror line is

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 0. \tag{6}$$

Therefore, the required line of reflection is

$$\boxed{\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = 0} \quad \text{(scalar form } y = x\text{)}.$$

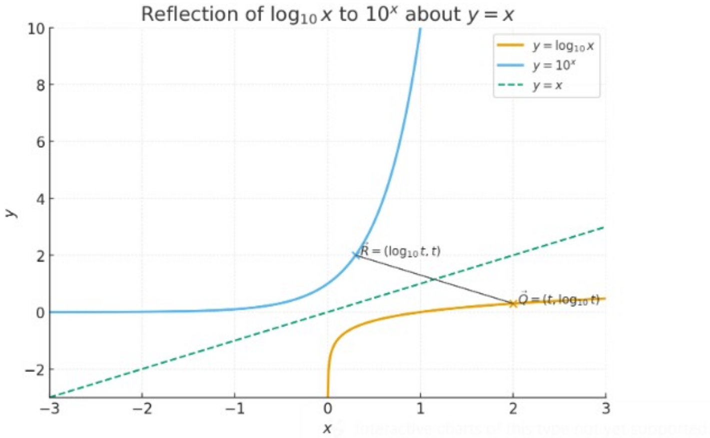


Fig. 0.1: $y = \log_{10} x$ and $y = 10^x$ with mirror line $y = x$.