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Question

The area of the triangle formed by the intersection of line parallel to X axis and passing through $\mathbf{p}(h,k)$ with the lines y=x and x+y=2 is $4h^2$. Find the locus of point \mathbf{p}

Solution

line parallel to X axis is of the form

$$y = c. (1)$$

which can be expressed in the form of

$$\mathbf{n}^T \mathbf{x} = c. \tag{2}$$

$$\implies (0 \quad 1) \begin{pmatrix} x \\ y \end{pmatrix} = c. \tag{3}$$

As the above line passes through $\mathbf{p}(h,k)$

$$(0 1) \binom{h}{k} = c. \implies c = k. (4)$$

The three lines are as follows

$$y = k \implies (0 \quad 1) \begin{pmatrix} x \\ y \end{pmatrix} = k. \tag{5}$$

$$-x + y = 0 \implies (-1 \quad 1) \begin{pmatrix} x \\ y \end{pmatrix} = 0. \tag{6}$$

$$x + y = 2(1 1) {x \choose y} = 2.$$
 (7)

Let **A,B,C** be the point of intersection of above 3 lines By solving equation (5) and (6) we get

$$\mathbf{A} = \begin{pmatrix} k \\ k \end{pmatrix}. \tag{8}$$

By solving equation (6) and (7) we get

$$\mathbf{B} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{9}$$

By solving equation (5) and (7) we get

$$\mathbf{C} = \begin{pmatrix} 2 - k \\ k \end{pmatrix}. \tag{10}$$

area of
$$\triangle ABc = \frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{B} - \mathbf{C})\|$$
 (11)

$$= \frac{1}{2} \left\| \begin{pmatrix} k-1 \\ k-1 \end{pmatrix} \times \begin{pmatrix} k-1 \\ 1-k \end{pmatrix} \right\| \tag{12}$$

$$= \frac{1}{2}(2(k-1)^2) = (k-1)^2.$$
 (13)

Given area of the triangle formed by the intersection of above 3 lines is 4h².

$$\implies (k-1)^2 = 4h^2. \tag{14}$$

$$\implies (y-1)^2 = 4x^2 \tag{15}$$

$$\implies (y - 1 - 2x)(y - 1 + 2x) = 0 \tag{16}$$

 \implies The locus of **p** is union of 2 straight lines

$$y - 1 - 2x = 0. \implies (-2 \quad 1) {x \choose y} = 1.$$
 (17)

$$y - 1 + 2x = 0. \implies (2 \quad 1) \binom{x}{y} = 1.$$
 (18)



