5.13.53

INDHIRESH S- EE25BTECH11027

Question Given

$$2x - y + 2z = 2$$
$$x - 2y + z = -4$$
$$x + y + \lambda z = 4$$

then the value of λ such that the given system of equation has NO solution, is

1) 3
2) 1
3) 0
4) -3

Solution:
Let us solve the given equation theoretically and then verify the solution computationally.
The given equation can be combined as:

$$\mathbf{A}\mathbf{x} = \mathbf{C} \tag{1}$$

$$\begin{pmatrix} 2 & -1 & 2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{pmatrix} \mathbf{x} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} \tag{2}$$

Where,

$$\mathbf{A} = \begin{pmatrix} 2 & -1 & 2 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{pmatrix} \quad and \quad \mathbf{C} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix} \tag{3}$$

Now forming the augmented matrix:

$$[\mathbf{A}|\mathbf{C}] = \left(\begin{array}{ccc|c} 2 & -1 & 2 & 2 \\ 1 & -2 & 1 & -4 \\ 1 & 1 & \lambda & 4 \end{array}\right) \tag{4}$$

$$\left(\begin{array}{ccc|c} 2 & -1 & 2 & 2 \\ 1 & -2 & 1 & -4 \\ 1 & 1 & \lambda & 4 \end{array}\right) \xleftrightarrow[R_1 \leftarrow R_1 - 2R_2]{R_3 \leftarrow R_3 - R_2} \left(\begin{array}{ccc|c} 0 & 3 & 0 & 10 \\ 1 & -2 & 1 & -4 \\ 1 & 1 & \lambda & 4 \end{array}\right) \tag{5}$$

$$\left(\begin{array}{ccc|c} 0 & 3 & 0 & 10 \\ 1 & -2 & 1 & -4 \\ 1 & 1 & \lambda & 4 \end{array}\right) \xleftrightarrow{R_3 \leftarrow R_3 - R_2} \left(\begin{array}{ccc|c} 0 & 3 & 0 & 10 \\ 1 & -2 & 1 & -4 \\ 0 & 3 & \lambda - 1 & 8 \end{array}\right) \tag{6}$$

$$\left(\begin{array}{ccc|c} 0 & 3 & 0 & 10 \\ 1 & -2 & 1 & -4 \\ 0 & 3 & \lambda-1 & 8 \end{array}\right) \xleftrightarrow{R_3 \longleftarrow R_3 - R_1} \left(\begin{array}{ccc|c} 0 & 3 & 0 & 10 \\ 1 & -2 & 1 & -4 \\ 0 & 0 & \lambda-1 & -2 \end{array}\right) \tag{7}$$

Given that the system of equation has NO solution . So,

$$\lambda = 1 \tag{8}$$

From the figure it is clearly verified that the theoretical solution matches with the computational solution.