

Matgeo Presentation - Problem 2.9.20

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Question

X and **Y** are two points with position vectors $3\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - 3\mathbf{b}$, respectively. Write the position vector of a point **Z** which divides the line segment **XY** in the ratio 2:1 externally.

solution

Given,

$$\mathbf{X} = \begin{pmatrix} 3\mathbf{a} \\ \mathbf{b} \end{pmatrix}, \mathbf{Y} = \begin{pmatrix} \mathbf{a} \\ -3\mathbf{b} \end{pmatrix} \quad (0.1)$$

If \mathbf{Z} divides XY in the ratio k:1 externally, Then

$$\mathbf{Z} = \frac{k\mathbf{Y} - \mathbf{X}}{k - 1} \quad (0.2)$$

So,

$$\mathbf{Z} = \frac{2\mathbf{Y} - \mathbf{X}}{1} \quad (0.3)$$

$$\mathbf{Z} = (\mathbf{X} \ \mathbf{Y}) \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad (0.4)$$

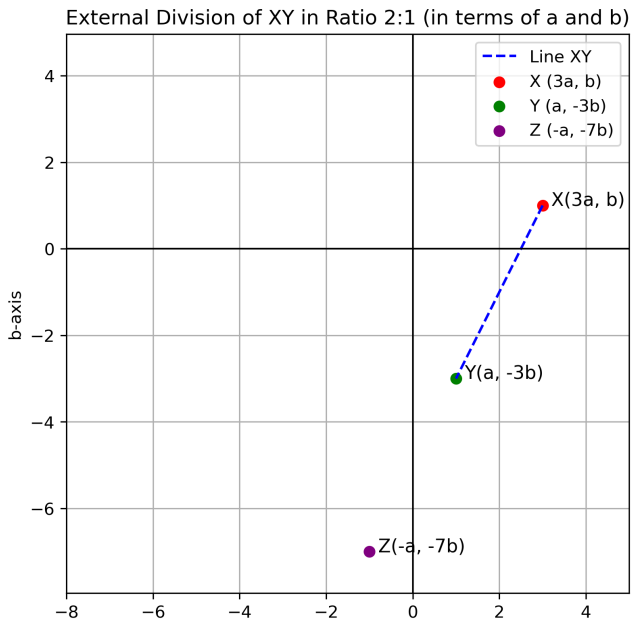
$$= \begin{pmatrix} 3\mathbf{a} & \mathbf{a} \\ \mathbf{b} & -3\mathbf{b} \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -\mathbf{a} \\ -7\mathbf{b} \end{pmatrix} \quad (0.5)$$

Conclusion

Therefore,

$$\mathbf{z} = \begin{pmatrix} -\mathbf{a} \\ -7\mathbf{b} \end{pmatrix} \quad (0.6)$$

Plot



C Code: Code.c

```
#include <stdio.h>

int main() {
    FILE *fp;
    float a, b;
    float X1, X2, Y1, Y2, Z1, Z2;
    int k = 2; // Ratio 2:1 (external division)

    // Input a and b
    printf("Enter values of a and b: ");
    scanf("%f%f", &a, &b);

    // Position vectors
    X1 = 3 * a;
    X2 = b;
    Y1 = a;
    Y2 = -3 * b;

    // External division formula: Z = (kY - X) / (k - 1)
    Z1 = (k * Y1 - X1) / (k - 1);
    Z2 = (k * Y2 - X2) / (k - 1);

    // Open file to write output
    fp = fopen("solution.dat", "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }
}
```

C Code: Code.c

```
// Write result to file
fprintf(fp, "External_Division_of_XY_in_ratio_2:1\n");
fprintf(fp, "Given:\n");
fprintf(fp, "X=(3a,b)\nY=(a,-3b)\n");
fprintf(fp, "Computed_position_vector_of_Z:\n");
fprintf(fp, "Z=(%.2fa,%.2fb)\n", Z1/a, Z2/b); // symbolic form
fprintf(fp, "Or_numerically:_(%.2f,_.2f)\n", Z1, Z2);

fclose(fp);

printf("_Output_written_to_'solution.dat'\n");
return 0;
}
```

Python: plot.py

```
import numpy as np
import matplotlib.pyplot as plt

# Symbolic vectors (set a = 1, b = 1 for proportional plotting)
a, b = 1, 1

# Given position vectors
X = np.array([3*a, b]) # (3a, b)
Y = np.array([a, -3*b]) # (a, -3b)
k = 2 # ratio 2:1 externally

# External division formula:  $Z = (kY - X) / (k - 1)$ 
Z = (k*Y - X) / (k - 1) # should give (-a, -7b)

# Plot
plt.figure(figsize=(6,6))
plt.plot([X[0], Y[0]], [X[1], Y[1]], 'b--', label='Line_XY')
plt.scatter(*X, color='red', label='X(3a, b)')
plt.scatter(*Y, color='green', label='Y(a, -3b)')
plt.scatter(*Z, color='purple', label='Z(-a, -7b)')

# Annotate points
plt.text(X[0]+0.2, X[1], 'X(3a, b)', fontsize=11)
plt.text(Y[0]+0.2, Y[1], 'Y(a, -3b)', fontsize=11)
plt.text(Z[0]+0.2, Z[1], 'Z(-a, -7b)', fontsize=11)

# Axes styling
plt.axhline(0, color='black', linewidth=1)
plt.axvline(0, color='black', linewidth=1)
plt.grid(True)
plt.axis('equal')
plt.xlim(-8, 5)
plt.ylim(-8, 5)
```


Python: plot.py

```
plt.xlabel('a-axis')
plt.ylabel('b-axis')
plt.title('External_Division_of_XY_in_Ratio_2:1_(in_terms_of_a_and_b)')
plt.legend()

# Save figure
plt.savefig("external_division.png", dpi=300, bbox_inches='tight')
plt.show()

print("_Figure_saved_as_'external_division.png'")
```