

# Matgeo Presentation - Problem 12.180

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## Question

The system of linear equations

$$4x+2y=7$$

$$2x+y=6 \text{ has}$$

- a) a unique solution
- b) no solution
- c) infinite number of solutions
- d) exactly two distinct solutions

## Solution

Given linear equations are

$$(4 \quad 2) \begin{pmatrix} x \\ y \end{pmatrix} = 7 \quad (0.1)$$

$$(2 \quad 1) \begin{pmatrix} x \\ y \end{pmatrix} = 6 \quad (0.2)$$

Equations (0.1) and (0.2) can be written as

$$\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} \quad (0.3)$$

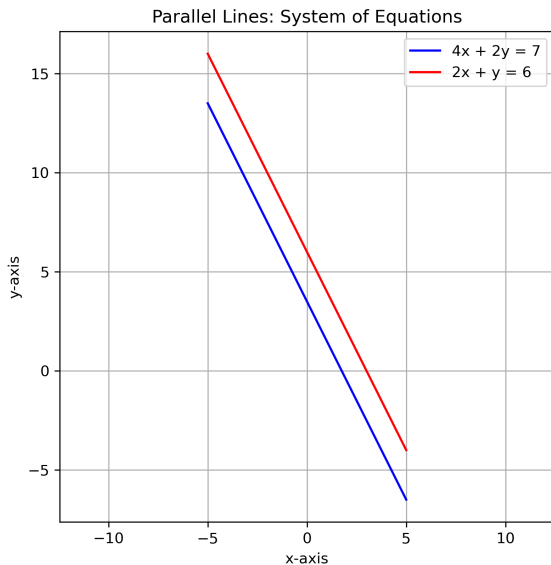
Forming the augmented matrix

$$\left( \begin{array}{cc|c} 4 & 2 & 7 \\ 2 & 1 & 6 \end{array} \right) \xleftrightarrow{R_2 \rightarrow R_2 - \frac{1}{2} \times R_1} \left( \begin{array}{cc|c} 4 & 2 & 7 \\ 0 & 0 & \frac{5}{2} \end{array} \right) \quad (0.4)$$

As in the augmented matrix the entries of second row are 0 their linear combination should also give 0 but it is  $\frac{5}{2}$  which is a contradiction

$\implies$  So, the given system of linear equations has no solution

# Plot



# C Code: Solution.c

```
#include <stdio.h>

int main() {
    // Coefficients of equations:
    // Equation 1:  $4x + 2y = 7$ 
    // Equation 2:  $2x + y = 6$ 
    int a1 = 4, b1 = 2, c1 = 7;
    int a2 = 2, b2 = 1, c2 = 6;

    // File pointer
    FILE *fp;
    fp = fopen("solution.dat", "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }

    // Calculate determinants
    int det = a1*b2 - a2*b1; // determinant of coefficients
    int detx = c1*b2 - c2*b1; // determinant replacing x-column
    int dety = a1*c2 - a2*c1; // determinant replacing y-column

    if (det != 0) {
        // Unique solution exists
        double x = (double)detx / det;
        double y = (double)dety / det;
        fprintf(fp, "The system has a unique solution: x=%.2f, y=%.2f\n", x, y);
    }
}
```

## C Code: Solution.c

```
else {
    if (dety == 0 && dety == 0) {
        // Infinite solutions
        fprintf(fp, "The system has infinite number of solutions.\n");
    } else {
        // No solution
        fprintf(fp, "The system has no solution.\n");
    }
}

fclose(fp);
return 0;
}
```

# Python: plot.py

```
import numpy as np
import matplotlib.pyplot as plt

# Define x range
x = np.linspace(-5, 5, 400)

# Line 1:  $4x + 2y = 7 \rightarrow y = (7 - 4x) / 2$ 
y1 = (7 - 4*x) / 2

# Line 2:  $2x + y = 6 \rightarrow y = 6 - 2x$ 
y2 = 6 - 2*x

# Plot the lines
plt.figure(figsize=(6,6))
plt.plot(x, y1, label="4x+2y=7", color="blue")
plt.plot(x, y2, label="2x+y=6", color="red")

# Formatting
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.title("Parallel Lines: System of Equations")
plt.legend()
plt.grid(True)
plt.axis("equal")

# Save the figure
plt.savefig("parallel_lines.png", dpi=300)

# Show the plot
plt.show()
```