4.13.1

AI25BTECH11036-SNEHAMRUDULA

Question:

Consider the lines given by

$$L_1 : x + 3y - 5 = 0,$$
$$L_2 : 3x - ky - 1 = 0,$$
$$L_3 : 5x + 2y - 12 = 0.$$

Match the Statements/Expressions in Column I with the Statements/Expressions in Column II.

Column I	**Column II**
(A) L_1, L_2, L_3 are concurrent, if	(a) $k = 9$
(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(b) $k = \frac{-6}{5}$
(C) L_1, L_2, L_3 form a triangle, if	(c) $k = \frac{5}{6}$
(D) L_1, L_2, L_3 do not form a triangle, if	(d) $k = 5$

Solution.

(A) Concurrency. Intersection of L_1 and L_3:

$$\begin{pmatrix} 1 & 3 \\ 5 & 2 \end{pmatrix} \mathbf{x_0} = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \tag{4.1}$$

From row reduction $\mathbf{x}_0 = (2, 1)^\top$ lies on L_2 iff

$$\mathbf{n}_2^\top \mathbf{x}_0 = 1 \iff k = 5.$$

Thus concurrency $\iff k = 5$.

(B) Parallelism. Normals proportional:

$$\mathbf{n}_2 = \lambda \mathbf{n}_3 \implies 3 = 5\lambda,\ -k = 2\lambda \Rightarrow \lambda = \tfrac{3}{5},\ k = -\tfrac{6}{5}.$$

(Other proportionalities give $k = -9$, impossible among options.) Hence parallel pair $\iff k = -\frac{6}{5}$.

(C) Triangle. No two parallel and not concurrent $\iff k \neq 5,\ k \neq -\frac{6}{5}$. Among given choices $k = 9$ and $k = \frac{5}{6}$.

(D) Do not form triangle. Occurs when concurrent or a parallel pair $\Rightarrow k = 5$ or $k = -\frac{6}{5}$.

Final match (concise):

$(A) \to (d)\ k = 5, \quad (B) \to (b)\ k = -\frac{6}{5}, \quad (C) \to (a) \text{ or } (c)\ (k = 9 \text{ or } k = \frac{5}{6}), \quad (D) \to (d) \text{ or } (b).$

If a strict one-to-one choice is required, use:

$$(A) \to (d),\ (B) \to (b),\ (C) \to (a),\ (D) \to (d).$$

Graphical Representation