

# 1.8.24

EE25BTECH11007- Aniket

## Question:

If  $(a, b)$  is the mid-point of the line segment joining the point **A**  $(10, -6)$  and **B**  $(k, 4)$  and  $a - 2b = 18$ , find the value of  $a, b$  and the distance **AB**.

## Solution:

Let **A** =  $\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$  and **B** =  $\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$ . By the (matrix) section formula, the point dividing **AB** in the ratio  $k : 1$  is

$$R_{\text{int}} = \frac{1}{k+1} [\mathbf{A} \ \mathbf{B}] \begin{pmatrix} 1 \\ k \end{pmatrix}. \quad (1)$$

With **A** =  $\begin{pmatrix} 10 \\ -6 \end{pmatrix}$  and **B** =  $\begin{pmatrix} k \\ 4 \end{pmatrix}$  and **O** =  $\begin{pmatrix} a \\ b \end{pmatrix}$  the midpoint ( $k = 1$ ) is

$$\mathbf{O} = \frac{1}{2} [\mathbf{A} \ \mathbf{B}] \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 10 & k \\ -6 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{10+k}{2} \\ -1 \end{pmatrix}. \quad (2)$$

Thus,

$$a = \frac{10+k}{2}, \quad b = -1 \quad (3)$$

Using the given condition  $a - 2b = 18$ :

$$\frac{10+k}{2} - 2(-1) = 18 \quad (4)$$

$$k = 22 \quad (5)$$

So,

$$a = \boxed{16}, \quad b = \boxed{-1} \quad (6)$$

## Distance Between AB:

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{(\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{B})}. \quad (7)$$

Given,

$$\mathbf{A} = \begin{pmatrix} 10 \\ -6 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 22 \\ 4 \end{pmatrix}. \quad (8)$$

$$(\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{B}) = \begin{pmatrix} -12 & -10 \end{pmatrix} \begin{pmatrix} -12 \\ -10 \end{pmatrix}. \quad (9)$$

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{(-12)^2 + (-10)^2} \quad (10)$$

$$= \boxed{2\sqrt{61}} \quad (11)$$

