4.11.21

Sai Hasini Pappula - EE25BTECH11044

October 3, 2025

Question

Problem

Find the equation of the plane passing through the line of intersection of:

$$r \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 7 \tag{1}$$

$$r \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) = 9 \tag{2}$$

such that the intercepts made by the plane on the x-axis and z-axis are equal.

Solution

Let the plane intercepts on x and z axes be equal. The x-intercept occurs when $y = 0, z = 0$ and the z-intercept occurs when $x = 0, y = 0$.
This gives the system

$$\begin{bmatrix} 2+2\lambda & 0 & 0 \\ 0 & 0 & -3+3\lambda \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} 7+9\lambda \\ 7+9\lambda \end{bmatrix}. \tag{3}$$

Let coefficients be represented as:

$$[a\ b\ c] = [2+2\lambda,\ 2+5\lambda,\ -3+3\lambda], \quad d = 7+9\lambda$$

we can write:

$$(2+2\lambda)x_0 = 7+9\lambda, \quad (-3+3\lambda)z_0 = 7+9\lambda. \tag{4}$$

Solution

Equal intercepts condition:

$$x_0 = z_0 \quad \Rightarrow \quad 2 + 2\lambda = -3 + 3\lambda \quad \Rightarrow \quad \lambda = 5. \tag{5}$$

Substitute in (4),

$$\begin{aligned} a &= 2 + 2(5) = 12, \\ b &= 2 + 5(5) = 27, \\ c &= -3 + 3(5) = 12, \\ d &= 7 + 9(5) = 52. \end{aligned} \tag{6}$$

Solution

Substitute $\lambda = 5$

$$\begin{bmatrix} a & b & c \end{bmatrix} = \begin{bmatrix} 2+10 & 2+25 & -3+15 \end{bmatrix} = \begin{bmatrix} 12 & 27 & 12 \end{bmatrix}, \tag{7}$$

$$\mathrm{d} = 7 + 45 = 52.$$

Hence, the required plane is:

$$12x + 27y + 12z = 52 \tag{8}$$

C Code (1/2)

```c
#include <stdio.h>

int main() {
    // Given plane coefficients
    float a1 = 2, b1 = 2, c1 = -3, d1 = 7;
    float a2 = 2, b2 = 5, c2 = 3, d2 = 9;

    float lambda; // scalar parameter
    // Step 1: Equal intercepts condition -> a/c must be same for
         x and z intercepts
    // From: (2 + 2)x = 7 + 9 and (-3 + 3)z = 7 + 9, with x = z
    // => (2 + 2) = (-3 + 3)
    lambda = 5.0;
    // Step 2: Substitute  = 5 in general plane coefficients
    float a = a1 + lambda * a2; // 2 + 2
    float b = b1 + lambda * b2; // 2 + 5
    float c = c1 + lambda * c2; // -3 + 3
    float d = d1 + lambda * d2; // 7 + 9
```

C Code (2/2)

```c
    // Step 3: Display result
    printf("The required plane equation is:\n");
    printf("(%.0f)x + (%.0f)y + (%.0f)z = %.0f\n", a, b, c, d);

    // Optional: print intercepts for verification
    float x_intercept = d / a;
    float y_intercept = d / b;
    float z_intercept = d / c;

    printf("\nIntercepts on axes:\n");
    printf("x-intercept = %.2f\n", x_intercept);
    printf("y-intercept = %.2f\n", y_intercept);
    printf("z-intercept = %.2f\n", z_intercept);

    return 0;
}
```

Python Code (1/3)

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load shared library
lib = ctypes.CDLL('./libplane.so')

# Define argument and return types
lib.find_plane.argtypes = [
    ctypes.c_float, ctypes.c_float, ctypes.c_float, ctypes.
        c_float,
    ctypes.c_float, ctypes.c_float, ctypes.c_float, ctypes.
        c_float,
    ctypes.POINTER(ctypes.c_float)
]
```

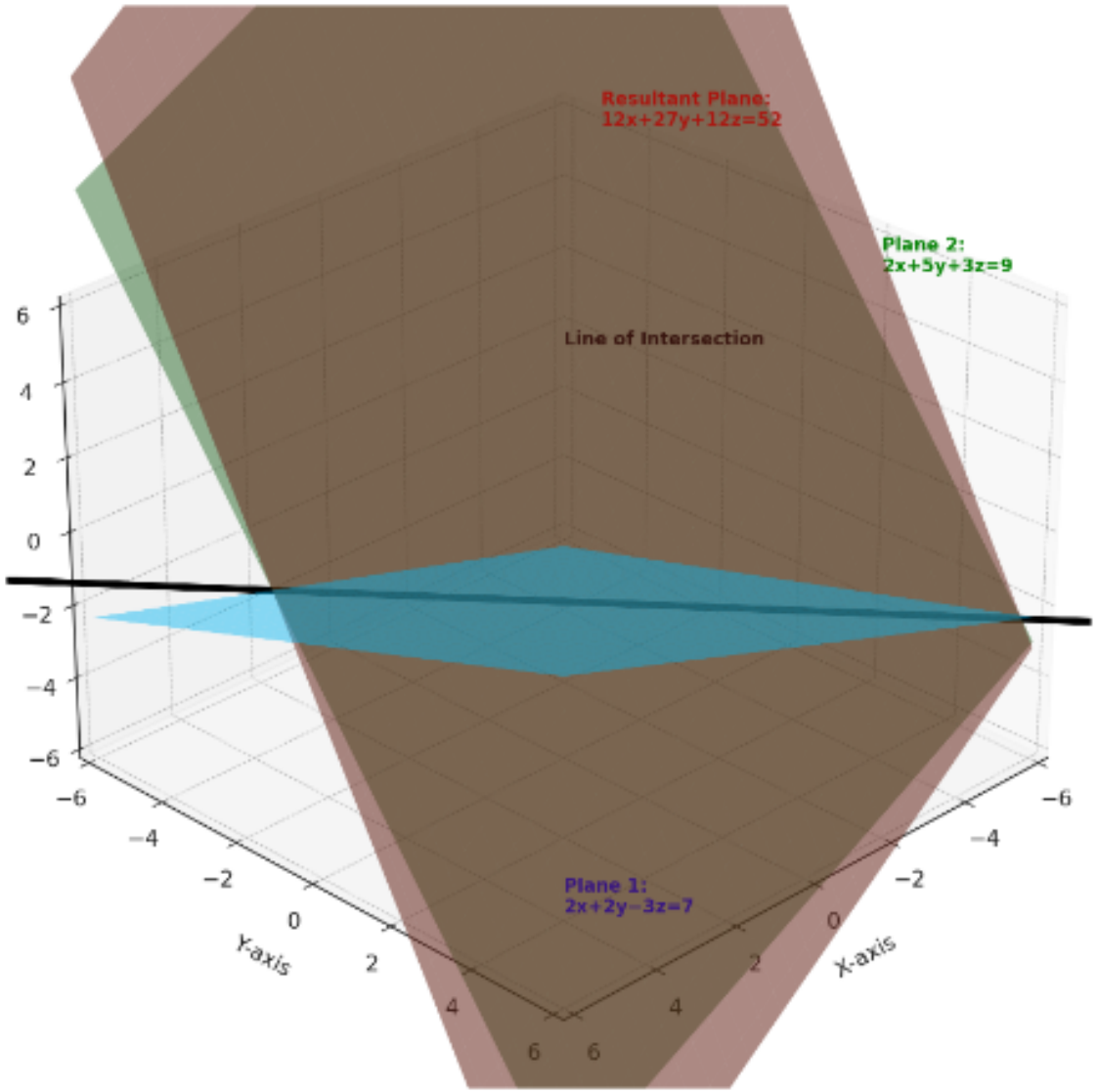
Python Code (2/3)

```
# Input from user
print("Enter coefficients of Plane 1 (a1 b1 c1 d1): ")
a1, b1, c1, d1 = map(float, input().split())
print("Enter coefficients of Plane 2 (a2 b2 c2 d2): ")
a2, b2, c2, d2 = map(float, input().split())

# Output array
res = (ctypes.c_float * 5)()
lib.find_plane(a1, b1, c1, d1, a2, b2, c2, d2, res)

lam, a, b, c, d = [res[i] for i in range(5)]
print(f"\n = {lam:.2f}")
print(f"Required plane: {a:.2f}x + {b:.2f}y + {c:.2f}z = {d:.2f}"
    )

# ---- Plotting Section ----
rng = np.linspace(-6, 6, 60)
X, Y = np.meshgrid(rng, rng)
```

Python Code (3/3)

```
Z1 = (a1 * X + b1 * Y - d1) / (-c1)
Z2 = (a2 * X + b2 * Y - d2) / (-c2)
Z3 = (a * X + b * Y - d) / (-c)
fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')
ax.plot_surface(X, Y, Z1, color='skyblue', alpha=0.5)
ax.plot_surface(X, Y, Z2, color='lightgreen', alpha=0.5)
ax.plot_surface(X, Y, Z3, color='salmon', alpha=0.6)

ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title('Plane through Intersection (Equal X and Z
    Intercepts)')
ax.view_init(elev=28, azim=45)
plt.tight_layout()
plt.show()
```

Plot


Resultant Plane:
12x+27y+12z=52
Plane 2:
2x+5y+3z=9
Line of Intersection
Plane 1:
2x+2y−3z=7
6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6
Y-axis
X-axis