Matgeo Presentation - Problem 12.173

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Problem Statement

Consider the system

$$
\begin{aligned}
x + 10y &= 5 \\
y + 5z &= 1 \\
10x - y + z &= 0
\end{aligned}
$$

Data

Name	Value (normal form)
Equation 1	$x + 10y = 5$ $\begin{pmatrix}1 & 10 & 0\end{pmatrix} \mathbf{x} = 5$
Equation 2	y + 5z = 1 $\begin{pmatrix}0 & 1 & 5\end{pmatrix} \mathbf{x} = 1$
Equation 3	$10x - y + z = 0$ $\begin{pmatrix}10 & -1 & 1\end{pmatrix} \mathbf{x} = 0$

Table : Equations

Solution

Using **Gauss-Seidel** method

We reorder equations for diagonal dominance:

$$10x - y + z = 0 \tag{0.1}$$

$$x + 10y = 5 \tag{0.2}$$

$$y + 5z = 1 \tag{0.3}$$

$$\begin{pmatrix} 10 & -1 & 1 \\ 1 & 10 & 0 \\ 0 & 1 & 5 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix} \tag{0.4}$$

Gauss–Seidel iteration formulas:

$$x^{(k+1)} = \frac{1}{10}\left(y^{(k)} - z^{(k)}\right) \tag{0.5}$$

$$y^{(k+1)} = \frac{1}{10}\left(5 - x^{(k+1)}\right) \tag{0.6}$$

$$z^{(k+1)} = \frac{1}{5}\left(1 - y^{(k+1)}\right) \tag{0.7}$$

Solution

Initial guess:

$$\mathbf{x}^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{0.8}$$

Iterations:

$$\mathbf{x}^{(1)} = \begin{pmatrix} 0 \\ 0.5 \\ 0.1 \end{pmatrix} \tag{0.9}$$

$$\mathbf{x}^{(2)} = \begin{pmatrix} 0.04 \\ 0.496 \\ 0.1008 \end{pmatrix} \tag{0.10}$$

$$\mathbf{x}^{(3)} = \begin{pmatrix} 0.03952 \\ 0.496048 \\ 0.1007904 \end{pmatrix} \tag{0.11}$$

Solution

$$\mathbf{x}^{(4)} = \begin{pmatrix} 0.03952576 \\ 0.49604742 \\ 0.10079052 \end{pmatrix} \tag{0.12}$$

$$\mathbf{x}^{(5)} = \begin{pmatrix} 0.03952569 \\ 0.49604743 \\ 0.10079051 \end{pmatrix} \tag{0.13}$$

Thus, the first component is

$$x \approx 0.03952569 \tag{0.14}$$

Correct to four decimal places:

$$x \approx 0.0395 \tag{0.15}$$

Answer: $x = 0.0395$

Solution

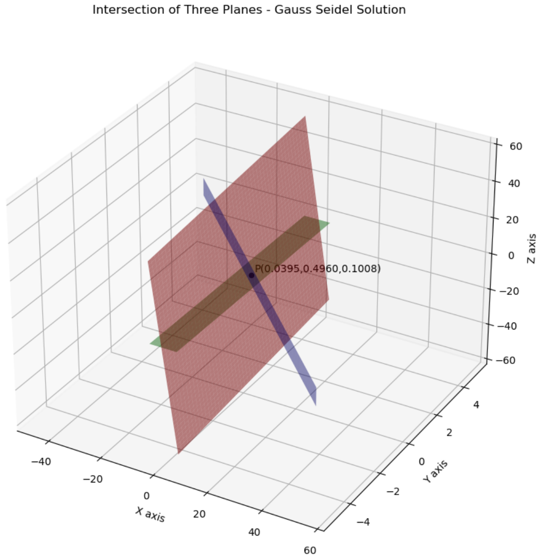

Fig : Planes