2.4.22

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Question Find the equation of a plane which bisects perpendicularly the line joining the points $A(2, 3, 4)$ and $B(4, 5, 8)$ at right angles.
Solution:
Let us solve the given equation theoretically and then verify the solution computationally.
Let,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad and \quad \mathbf{B} = \begin{pmatrix} 4 \\ 5 \\ 8 \end{pmatrix} \tag{1}$$

Given that the plane is a perpendicular bisector to the line joining points A and B. Since it is a perpendicular bisector to the line joining points A and B , the midpoint of the line joining points A and B lies on the plane.
Let the midpoint of points A and B be C. Then

$$norm(\mathbf{C} - \mathbf{A}) = norm(\mathbf{C} - \mathbf{B}) \tag{2}$$

$$\sqrt{(\mathbf{C} - \mathbf{A})^T(\mathbf{C} - \mathbf{A})} = \sqrt{(\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{B})} \tag{3}$$

$$(\mathbf{C} - \mathbf{A})^T(\mathbf{C} - \mathbf{A}) = (\mathbf{C} - \mathbf{B})^T(\mathbf{C} - \mathbf{B}) \tag{4}$$

Let,

$$C = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{5}$$

$$\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}\right)^T \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}\right) = \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 8 \end{pmatrix}\right)^T \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 8 \end{pmatrix}\right) \tag{6}$$

$$\begin{pmatrix} x-2 \\ y-3 \\ z-4 \end{pmatrix}^T \begin{pmatrix} x-2 \\ y-3 \\ z-4 \end{pmatrix} = \begin{pmatrix} x-4 \\ y-5 \\ z-8 \end{pmatrix}^T \begin{pmatrix} x-4 \\ y-5 \\ z-8 \end{pmatrix} \tag{7}$$

$$(x-2)^2 + (y-3)^2 + (z-4)^2 = (x-4)^2 + (y-5)^2 + (z-8)^2 \tag{8}$$

$$x^2 + 4 - 4x + y^2 + 9 - 6y + z^2 + 16 - 8z = x^2 + 16 - 8x + y^2 + 25 - 10y + z^2 + 64 - 16z \tag{9}$$

$$4x + 4y + 8z = 76 \tag{10}$$

$$x + y + 2z = 19 \tag{11}$$

Now the equation of plane is :

$$x + y + 2z = 19 \tag{12}$$

In matrix form:

$$\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}^T \mathbf{R} = 19 \tag{13}$$

Where $\mathbf{R}$ is the equation of the plane

From the figure it is clearly verified that the theoretical solution matches with the computational solution.