

12.358

EE25BTECH11032 - Kartik Lahoti

Question:

The inverse of the 2×2 matrix.

$$\begin{pmatrix} 1 & 2 \\ 5 & 7 \end{pmatrix}$$

$$1) \frac{1}{3} \begin{pmatrix} -7 & 2 \\ 5 & -1 \end{pmatrix}$$

$$2) \frac{1}{3} \begin{pmatrix} 7 & 2 \\ 5 & 1 \end{pmatrix}$$

$$3) \frac{1}{3} \begin{pmatrix} 7 & -2 \\ -5 & 1 \end{pmatrix}$$

$$4) \frac{1}{3} \begin{pmatrix} -7 & -2 \\ -5 & -1 \end{pmatrix}$$

Solution:

Given the matrix,

$$\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 5 & 7 \end{pmatrix} \quad (1)$$

Let \mathbf{A}^{-1} be the inverse of the matrix \mathbf{A}

We know that,

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I} \quad (2)$$

The augmented matrix of $(\mathbf{A} \mid \mathbf{I})$ is given by

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 5 & 7 & 0 & 1 \end{array} \right) \quad (3)$$

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 5 & 7 & 0 & 1 \end{array} \right) \xrightarrow{R_2 \rightarrow R_2 - 5R_1} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -3 & -5 & 1 \end{array} \right) \quad (4)$$

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -3 & -5 & 1 \end{array} \right) \xrightarrow{R_2 \rightarrow -\frac{1}{3}R_2} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{5}{3} & -\frac{1}{3} \end{array} \right) \quad (5)$$

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{5}{3} & -\frac{1}{3} \end{array} \right) \xrightarrow{R_1 \rightarrow R_1 - 2R_2} \left(\begin{array}{cc|cc} 1 & 0 & -\frac{7}{3} & \frac{2}{3} \\ 0 & 1 & \frac{5}{3} & -\frac{1}{3} \end{array} \right) \quad (6)$$

Hence,

$$\mathbf{A}^{-1} = \frac{1}{3} \begin{pmatrix} -7 & 2 \\ 5 & -1 \end{pmatrix} \quad (7)$$

Answer: Option (2)