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Matrix 4.4.11

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Question

The line segment joining the points $A(2, 1)$ and $B(5, -8)$ is trisected at the points P and Q, where P is nearer to A. If P lies on the line

$$2x - y + k = 0,$$

find the value of k. Use matrix / linear-algebra concepts only.

Solution

Write the position vectors of the points using column matrices:

$$\mathbf{A} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 5 \\ -8 \end{pmatrix}. \tag{1}$$

The vector from A to B is

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 5 \\ -8 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -9 \end{pmatrix}. \tag{2}$$

Trisecting the segment AB means the first trisection point P (closer to A) is obtained by moving one third of the way from A toward B. In vector form

$$\mathbf{P} = \mathbf{A} + \tfrac{1}{3}(\mathbf{B} - \mathbf{A}). \tag{3}$$

Substitute (1) and (2) into (3):

$$\mathbf{P} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 3 \\ -9 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}. \tag{4}$$

Thus the co-ordinates of P are $(3, -2)$. Since P lies on the line $2x - y + k = 0$, substitute $x = 3,\ y = -2$ into the line equation:

$$2(3) - (-2) + k = 0. \tag{5}$$

Solve (5) for k:

$$6 + 2 + k = 0 \quad \Longrightarrow \quad k = -8. \tag{6}$$

Final Answer

$$\boxed{k = -8} \tag{7}$$

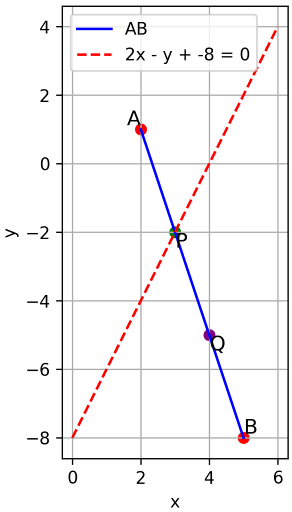

Figure 1: