5.7.12

AI25BTECH11039-Harichandana Varanasi

Question: If

$$\mathbf{A} = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix},$$

show that $(\mathbf{A} - 2\mathbf{I})(\mathbf{A} - 3\mathbf{I}) = \mathbf{0}$.

Solution:

From the characteristic equation definition and the Cayley-Hamilton theorem,

$$f(\lambda) = \det(\mathbf{A} - \lambda\mathbf{I}) = 0, \qquad f(\mathbf{A}) = \mathbf{0}.$$

For the given matrix,

$$\det(\mathbf{A} - \lambda\mathbf{I}) = \det\begin{pmatrix} 4-\lambda & 2 \\ -1 & 1-\lambda \end{pmatrix} = (4-\lambda)(1-\lambda) - (-2)$$
$$= \lambda^2 - 5\lambda + 6 = (\lambda - 2)(\lambda - 3). \tag{1}$$

Hence, by Cayley-Hamilton,

$$f(\mathbf{A}) = \mathbf{A}^2 - 5\mathbf{A} + 6\mathbf{I} = (\mathbf{A} - 2\mathbf{I})(\mathbf{A} - 3\mathbf{I}) = \mathbf{0}. \tag{2}$$

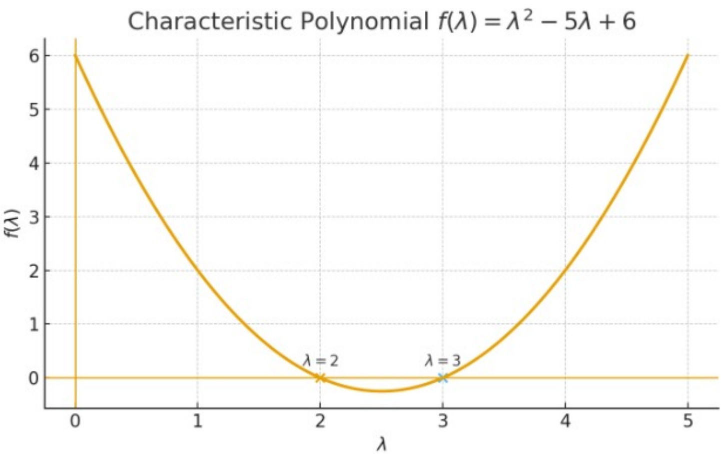

Fig. 0.1: Characteristic polynomial $f(\lambda) = \lambda^2 - 5\lambda + 6$ with roots $\lambda = 2, 3$.