

Matgeo-4.13.4

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Question

Q-4.13.4 $y = 10^x$ is the reflection of $y = \log_{10} x$ in the line whose equation is _____.

Solution

Let the mirror line be written in normal form

$$\mathbf{n}^\top \mathbf{x} = c, \quad (1)$$

with normal $\mathbf{n} \in \mathbb{R}^2$ and variable vector $\mathbf{x} \in \mathbb{R}^2$.

A point on the curve $y = \log_{10} u$ is

$$\mathbf{Q}(t) = \begin{pmatrix} t \\ \log_{10} t \end{pmatrix}, \quad t > 0. \quad (2)$$

Its reflection in the line (1) is

$$\mathbf{R}(t) = \mathbf{Q}(t) - \frac{2(\mathbf{n}^\top \mathbf{Q}(t) - c)}{\|\mathbf{n}\|^2} \mathbf{n}. \quad (3)$$

For the image curve to be $y = 10^x$ we require

$$\mathbf{R}(t) = \begin{pmatrix} \log_{10} t \\ t \end{pmatrix} \quad \text{for all } t > 0. \quad (4)$$

Equating components in (3)–(4) and collecting the independent functions t and $\log_{10} t$ yields

$$a^2 = b^2, \quad a b = -a b, \quad c = 0. \quad (5)$$

Hence $a = -b \neq 0$ and $c = 0$. Thus $\mathbf{n} \propto \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and the mirror line is

$$(1 \quad -1) \mathbf{x} = 0. \quad (6)$$

Therefore, the required line of reflection is

$$\boxed{(1 \quad -1) \mathbf{x} = 0} \quad (\text{scalar form } y = x).$$

Plot

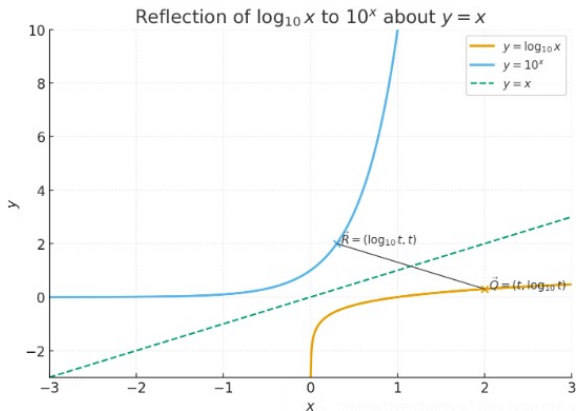


Figure: $y = \log_{10} x$ and $y = 10^x$ with mirror line $y = x$.