2.10.30

EE25BTECH11001 - Aarush Dilawri

Question:
The points with position vectors $60\mathbf{i} + 3\mathbf{j}$, $40\mathbf{i} - 6\mathbf{j}$, $a\mathbf{i} - 52\mathbf{j}$ are collinear if

(a) a = -40

(b) a = 40

(c) a = 20

(d) None of these

Solution:
We have position vectors

$$\mathbf{A} = \begin{pmatrix} 60 \\ 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 40 \\ -6 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} a \\ -52 \end{pmatrix}. \tag{1}$$

Now,

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -20 \\ -9 \end{pmatrix}, \tag{2}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} a - 60 \\ -55 \end{pmatrix}. \tag{3}$$

For collinearity, we require

$$\text{rank} \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix} = 1 \tag{4}$$

$$\text{rank} \begin{pmatrix} -20 & a - 60 \\ -9 & -55 \end{pmatrix} = 1 \tag{5}$$

$$R_2 \to R_1 - \tfrac{20}{9} R_2 \tag{6}$$

$$\begin{pmatrix} -20 & a - 60 \\ -9 & -55 \end{pmatrix} \xleftarrow{R_2 \to R_1 - \frac{20}{9} R_2} \begin{pmatrix} -20 & a - 60 \\ 0 & a + \frac{560}{9} \end{pmatrix} \tag{7}$$

$$\text{rank} \begin{pmatrix} -20 & a - 60 \\ 0 & a + \frac{560}{9} \end{pmatrix} = 1 \tag{8}$$

Therefore, equating the last row to 0, we have

$$a + \frac{560}{9} = 0 \implies a = -\frac{560}{9} \tag{9}$$
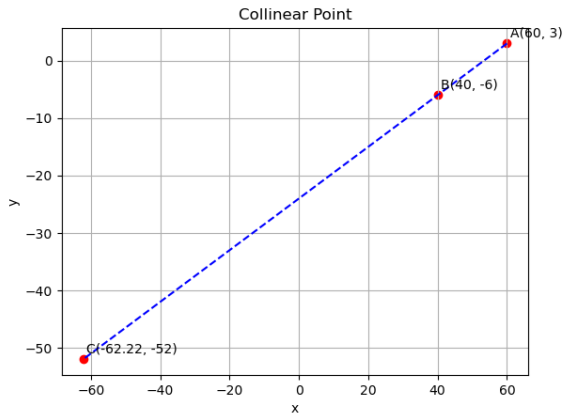
Therefore, the answer is (d) None of these.

See Fig. 0 ,

Fig. 4