

2.9.11

EE25BTECH11060 - V.Namaswi

Question

If \vec{a} and \vec{b} are unit vectors and θ is angle between them then prove that $\sin \frac{\theta}{2} = \frac{1}{2} |\vec{a} - \vec{b}|$

Solution

Squaring on both sides,

$$\sin^2 \frac{\theta}{2} = \frac{1}{4} |\vec{a} - \vec{b}|^2$$

consider RHS,

$$\Rightarrow \frac{1}{4} |\vec{a} - \vec{b}|^2 \quad (1)$$

$$= \frac{1}{4} (|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}^\top \vec{b}) \quad (2)$$

$$= \frac{1}{4} (2 - 2 \cos \theta) \quad (3)$$

$$= \frac{1}{2} (2 \sin^2 \frac{\theta}{2}) \quad (4)$$

$$(5)$$

$$= \sin^2 \frac{\theta}{2} \quad (6)$$

$$= LHS \quad (7)$$

Hence, $\sin \frac{\theta}{2} = \frac{1}{2} |\vec{a} - \vec{b}|$