

2.10.33

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Question

Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$, $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$:

- ① are collinear
- ② form an equilateral triangle
- ③ form a scalene triangle
- ④ form a right angled triangle

Solution

To answer this question, we need to find the distance between each of these points.

Let **A** be $\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$, **B** be $\begin{pmatrix} \beta \\ \gamma \\ \alpha \end{pmatrix}$, and **C** be $\begin{pmatrix} \gamma \\ \alpha \\ \beta \end{pmatrix}$.

$$\bullet \mathbf{A} - \mathbf{B} \text{ is } \begin{pmatrix} \alpha - \beta \\ \beta - \gamma \\ \gamma - \alpha \end{pmatrix} \quad \bullet \mathbf{B} - \mathbf{C} \text{ is } \begin{pmatrix} \beta - \gamma \\ \gamma - \alpha \\ \alpha - \beta \end{pmatrix} \quad \bullet \mathbf{C} - \mathbf{A} \text{ is } \begin{pmatrix} \gamma - \alpha \\ \alpha - \beta \\ \beta - \gamma \end{pmatrix}$$

Solution

The norms of $\mathbf{A} - \mathbf{B}$, $\mathbf{B} - \mathbf{C}$, $\mathbf{C} - \mathbf{A}$ are all equal, and equal to

$$\sqrt{(\alpha - \beta)^2 + (\beta - \gamma)^2 + (\gamma - \alpha)^2}$$

The three points therefore form an equilateral triangle, so option (2) is correct.

```
import numpy as np

vector = np.zeros(3)
vector[0] = input()
vector[1] = input()
vector[2] = input()

print(np.linalg.norm(vector))
```

```
#include<stdio.h>
#include<math.h>

float norm(float a, float b, float c){

float answer;
answer = pow(a,2) + pow(b,2) + pow(c,2);
answer = sqrt(answer);

return answer;

}
```

Python and C Code

```
import numpy as np
import ctypes
c_lib=ctypes.CDLL('./5c.so')

c_lib.norm.argtypes = [ctypes.c_float, ctypes.c_float, ctypes.c_float]
c_lib.norm.restype = ctypes.c_float

vector = np.zeros(3)
vector[0] = input()
vector[1] = input()
vector[2] = input()

answer = c_lib.norm(
    ctypes.c_float(vector[0]),
    ctypes.c_float(vector[1]),
    ctypes.c_float(vector[2]))
print(answer)
```