Matgeo-q 2.3.2

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Question

Q. Find the angle between unit vectors $\mathbf{a}$ and $\mathbf{b}$ such that $\sqrt{3}\,\mathbf{a} - \mathbf{b}$ is also a unit vector.

Solution

Solution. Given: $\| (a) \| = \| (b) \| = 1$ and $\left\| \sqrt{3}\ (a) - (b) \right\| = 1$.

Use the length definition $\|x\|^2 = x^\top x$ and the scalar–product relation $(a)^\top (b) = \| (a) \| \| (b) \| \cos\theta$.

$$\left\| \sqrt{3}\ (a) - (b) \right\|^2 = \left(\sqrt{3}\ (a) - (b) \right)^\top \left(\sqrt{3}\ (a) - (b) \right)$$
$$= 3\ (a)^\top (a) + (b)^\top (b) - 2\sqrt{3}\ (a)^\top (b)$$
$$= 3\| (a) \|^2 + \| (b) \|^2 - 2\sqrt{3} \| (a) \| \| (b) \| \cos\theta$$
$$= 3 + 1 - 2\sqrt{3}\cos\theta.$$

Since $\left\| \sqrt{3}\ (a) - (b) \right\| = 1$, we get

$$1 = 4 - 2\sqrt{3}\cos\theta \implies 3 = 2\sqrt{3}\cos\theta \implies \cos\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}.$$

Hence,

$$\boxed{\theta = 30^\circ}.$$

Plot

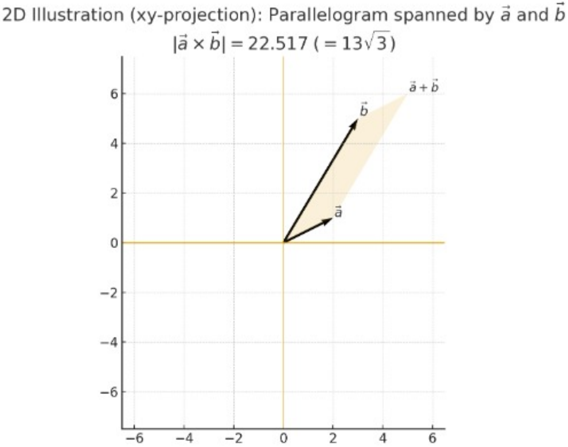

Figure: xy-projection of **a** and **b**; $|\mathbf{a} \times \mathbf{b}| = 13\sqrt{3}$.