4.3.13

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Question

Find the distance of the line $4x - y = 0$ from the point $\mathrm{P}(4, 1)$ measured along the line making an angle of $135°$ with the positive x-axis.

Theoretical Solution

According to the question,

$$\textit{Equation of target line} : \begin{pmatrix} 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

and

$$\mathbf{P} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

As the direction of line makes an angle of $135°$ with the $+x$ axis, the unit direction vector of the line is given by

$$\mathbf{m} = \begin{pmatrix} \cos 135° \\ \sin 135° \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

Theoretical Solution

Parametrize the required line using **P**,yielding

$$\mathbf{x} = \mathbf{P} + \kappa \mathbf{m}$$

Inserting the parametric form in the equation of target line,

$$\begin{pmatrix} 4 & -1 \end{pmatrix} (\mathbf{P} + \kappa \mathbf{m}) = 0$$

$$\therefore \kappa = \frac{-\begin{pmatrix} 4 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix}}{\begin{pmatrix} 4 & -1 \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}}$$

$$\implies \kappa = 3\sqrt{2}$$

Theoretical Solution

Since **m** is a unit vector, the norm of vector **P** from the given line along the line with $\mathbf{m} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}^\top$ is

$$\kappa = 3\sqrt{2} \; units$$

C Code -Finding Equations of diagonals of a square

```c
#include <stdio.h>
#include <math.h>

// Function to compute intersection point Q
// Inputs: Px, Py = point P
// Outputs: Qx, Qy (via pointers)
void find_intersection(double Px, double Py, double *Qx, double *
    Qy) {
    // direction vector for 135 = (-1, 1)
    double dx = -1.0, dy = 1.0;
```

C Code -Finding Equations of diagonals of a square

```
    // Parametric line: (x,y) = (Px + t dx, Py + t dy)
    // Line: 4x - y = 0 -> y = 4x
    // Substitute: Py + t dy = 4(Px + t dx)
    // => Py + t = 4Px + 4t dx
    // => Py + t = 4Px + 4t(-1)
    // => Py + t = 4Px - 4t
    // => t + 4t = 4Px - Py
    // => 5t = 4Px - Py
    double t = (4*Px - Py) / 5.0;

    *Qx = Px + t*dx;
    *Qy = Py + t*dy;
}

// Function to compute distance between two points
double distance(double x1, double y1, double x2, double y2) {
    return sqrt((x2-x1)*(x2-x1) + (y2-y1)*(y2-y1));
}
```

Python + C code

```python
import ctypes
import math
import matplotlib.pyplot as plt
import matplotlib as mp
mp.use("TkAgg")
# Load shared library
lib = ctypes.CDLL("./libnorm.so")
# Define function signatures
lib.find_intersection.argtypes = [ctypes.c_double, ctypes.c_double,
                                  ctypes.POINTER(ctypes.c_double), ctypes.POINTER(ctypes.c_double)]
lib.distance.argtypes = [ctypes.c_double, ctypes.c_double, ctypes.c_double, ctypes.c_double]
lib.distance.restype = ctypes.c_double
```

Python + C code

```python
# Given point P
Px, Py = 4.0, 1.0

# Call C function to get Q
Qx = ctypes.c_double()
Qy = ctypes.c_double()
lib.find_intersection(Px, Py, ctypes.byref(Qx), ctypes.byref(Qy))
Qx, Qy = Qx.value, Qy.value

# Distance from C
dist = lib.distance(Px, Py, Qx, Qy)
print(f"distance: {dist:.3f} units")
```

Python + C code

```
# ------------------------------
# Plot same as before
# ------------------------------
fig, ax = plt.subplots(figsize=(6, 6))

# Line y = 4x
x_line = [-1, 5]
y_line = [4*x for x in x_line]
ax.plot(x_line, y_line, label="Line: 4x - y = 0 (y=4x)")

# Direction line through P (slope -1)
x_dir = [Px - 4, Px + 4]
y_dir = [Py + 4, Py - 4]
ax.plot(x_dir, y_dir, linestyle="--", label="Direction 135 (slope
     -1)")
```

Python + C code

```python
# Points P and Q
ax.plot(Px, Py, 'ro')
ax.annotate(f'P({Px:.0f},{Py:.0f})', xy=(Px,Py), xytext=(Px+0.2,
    Py-0.5))
ax.plot(Qx, Qy, 'mo')
ax.annotate(f'Q({Qx:.0f},{Qy:.0f})', xy=(Qx,Qy), xytext=(Qx+0.2,
    Qy+0.2))

# Segment PQ
ax.plot([Px, Qx], [Py, Qy], 'r-', linewidth=2)

# Distance label
mid = ((Px+Qx)/2, (Py+Qy)/2)
ax.text(mid[0], mid[1]+0.3, f'distance = {dist:.3f}', ha='center'
    , color='red')
```

Python + C code

```
# Formatting
ax.set_aspect('equal', 'box')
ax.set_xlabel("x")
ax.set_ylabel("y")
ax.grid(True, alpha=0.4)
ax.legend(loc="upper right")


plt.savefig("/home/user/Matrix/Matgeo_assignments/4.7.12/figs/
    Figure_1.png")
plt.show()
```

Python code

```
import math
import matplotlib.pyplot as plt
import matplotlib as mp
mp.use("TkAgg")
# ------------------------------
# Data
# ------------------------------
P = (4.0, 1.0) # given point
m_dir = -1.0 # slope for 135 direction (tan 135 = -1)
d = (-1.0, 1.0) # unit direction up to scale
# Given line: 4x - y = 0 -> y = 4x
# Parametric line through P in direction d: (x, y) = P + t d = (4
    - t, 1 + t)
# Intersect with y = 4x:
# 1 + t = 4(4 - t) -> 1 + t = 16 - 4t -> 5t = 15 -> t = 3
t = 3.0
Q = (P[0] + t*d[0], P[1] + t*d[1]) # (1, 4)
```

Python code

```python
# Distance along the 135 direction
DX = Q[0] - P[0]
DY = Q[1] - P[1]
distance = math.hypot(DX, DY) # 3*sqrt(2)

print(f"distance: {distance:.3f} units")

# ------------------------------
# Plot
# ------------------------------
fig, ax = plt.subplots(figsize=(6, 6))

# Plot given line y = 4x
x_line = [ -1, 5 ]
y_line = [ 4*x for x in x_line ]
ax.plot(x_line, y_line, label='Line: 4x - y = 0 (y=4x)')
```

Python code

```python
# Plot direction line through P (slope -1)
x_dir = [P[0] - 4, P[0] + 4]
y_dir = [P[1] + 4, P[1] - 4]
ax.plot(x_dir, y_dir, linestyle='--', label='Direction 135 (slope
     -1)')
# Points P and Q
ax.plot(P[0], P[1], 'ro')
ax.annotate('P(4,1)', xy=P, xytext=(P[0]+0.2, P[1]-0.5))
ax.plot(Q[0], Q[1], 'mo')
ax.annotate(f'Q{Q}', xy=Q, xytext=(Q[0]+0.2, Q[1]+0.2))
# Segment PQ (the measured distance)
ax.plot([P[0], Q[0]], [P[1], Q[1]], 'r-', linewidth=2)
# Annotate distance directly above the segment
mid = ((P[0]+Q[0])/2, (P[1]+Q[1])/2)
ax.text(mid[0], mid[1]+0.3, f'distance = {distance:.3f}', ha='
    center', va='bottom', fontsize=10, color='red')
```

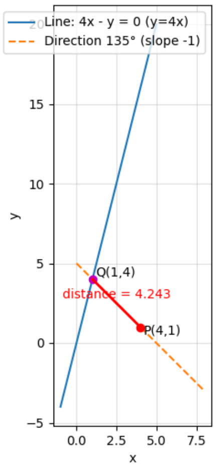
Python code

```
# Axes formatting
ax.set_aspect('equal', 'box')
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.grid(True, alpha=0.4)
ax.legend(loc='upper right')


# Set sensible limits around all geometry
xs = [-1, 5, P[0], Q[0]]
ys = [min(y_line), max(y_line), P[1], Q[1]]
ax.set_xlim(min(xs)-1, max(xs)+1)
ax.set_ylim(min(ys)-1, max(ys)+1)

plt.savefig("/home/user/Matrix/Matgeo_assignments/4.7.12/figs/
    Figure_1.png")
plt.show()
```

Plot


Line: 4x - y = 0 (y=4x)
Direction 135° (slope -1)
Q(1,4)
distance = 4.243
P(4,1)
y
x
20
15
10
5
0
−5
0.0
2.5
5.0
7.5