2.4.41

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Question

Problem

Determine whether the points $A(3, 6, 9)$, $B(10, 20, 30)$, $C(24, -41, 5)$ are the vertices of a right-angled triangle using matrices.

Process

Step 1: Represent points as vectors

$$\mathbf{A} = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 24 \\ -41 \\ 5 \end{bmatrix}$$

Step 2: Compute side vectors

$$\mathbf{AB} = \mathbf{B} - \mathbf{A}, \quad \mathbf{BC} = \mathbf{C} - \mathbf{B}, \quad \mathbf{AC} = \mathbf{C} - \mathbf{A}$$

Step 3: Use dot product test $\mathbf{u} \cdot \mathbf{v} = 0$ $\Rightarrow$ Vectors are perpendicular

Conclusion

Dot product results:

$$\mathbf{AB} \cdot \mathbf{AC} = -595, \quad \mathbf{AB} \cdot \mathbf{BC} = -1281, \quad \mathbf{AC} \cdot \mathbf{BC} = 3261$$

Since none are zero, the points **do not form a right-angled triangle**.

C Code

```c
#include <stdio.h>

int main() {
    double AB[3] = {B[0]-A[0], B[1]-A[1], B[2]-A[2]};
    double AC[3] = {C[0]-A[0], C[1]-A[1], C[2]-A[2]};
    double BC[3] = {C[0]-B[0], C[1]-B[1], C[2]-B[2]};
    double ABdotAC = AB[0]*AC[0]+AB[1]*AC[1]+AB[2]*AC[2];
    double ABdotBC = AB[0]*BC[0]+AB[1]*BC[1]+AB[2]*BC[2];
    double ACdotBC = AC[0]*BC[0]+AC[1]*BC[1]+AC[2]*BC[2];

    if(ABdotAC==0 || ABdotBC==0 || ACdotBC==0)
        printf("Right-angled triangle\n");
    else
        printf("Not right-angled\n");
    return 0;
}
```

Python Code (1/2)

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

A = np.array([3, 6, 9])
B = np.array([10, 20, 30])
C = np.array([24, -41, 5])
AB = B - A
AC = C - A
BC = C - B
AB_dot_AC = np.dot(AB, AC)
AB_dot_BC = np.dot(AB, BC)
AC_dot_BC = np.dot(AC, BC)

if AB_dot_AC==0 or AB_dot_BC==0 or AC_dot_BC==0:
    print("Right-angled triangle")
else:
    print("Not right-angled")
```

Python Code (2/2: Plot)

```
fig = plt.figure(figsize=(8,6))
ax = fig.add_subplot(111, projection='3d')

ax.scatter(*A, color='black', s=80)
ax.text(A[0]+0.5, A[1]+0.5, A[2]+0.5, "A(3,6,9)")

ax.scatter(*B, color='blue', s=80)
ax.text(B[0]+0.5, B[1]+0.5, B[2]+0.5, "B(10,20,30)")

ax.scatter(*C, color='red', s=80)
ax.text(C[0]+0.5, C[1]+0.5, C[2]+0.5, "C(24,-41,5)")

ax.plot([A[0],B[0]], [A[1],B[1]], [A[2],B[2]], color='blue')
ax.plot([A[0],C[0]], [A[1],C[1]], [A[2],C[2]], color='green')
ax.plot([B[0],C[0]], [B[1],C[1]], [B[2],C[2]], color='red',
    linestyle='--')

plt.show()
```

Triangle in 3D