2.9.16

EE25BTECH11065 - Yoshita

Question:
Prove that three points A, B, and C with position vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ respectively are collinear if and only if $(\mathbf{b} \times \mathbf{c}) + (\mathbf{c} \times \mathbf{a}) + (\mathbf{a} \times \mathbf{b}) = \mathbf{0}$.

Solution:
The three points A, B, and C are collinear if and only if the vectors $\mathbf{AB}$ and $\mathbf{AC}$ are parallel. The position vectors for these are:

$$\mathbf{AB} = \mathbf{b} - \mathbf{a}$$
$$\mathbf{AC} = \mathbf{c} - \mathbf{a}$$

If two vectors are collinear,

$$\mathbf{AB} \times \mathbf{AC} = \mathbf{0} \tag{1}$$
$$(\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a}) = \mathbf{0} \tag{2}$$

Expanding the expression,

$$(\mathbf{b} \times \mathbf{c}) - (\mathbf{b} \times \mathbf{a}) - (\mathbf{a} \times \mathbf{c}) + (\mathbf{a} \times \mathbf{a}) = \mathbf{0} \tag{3}$$

Rearranging the equation we get,

$$(\mathbf{a} \times \mathbf{b}) + (\mathbf{b} \times \mathbf{c}) + (\mathbf{c} \times \mathbf{a}) = \mathbf{0} \tag{4}$$

Hence we proved that that three points A, B, and C with position vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ respectively are collinear if and only if $(\mathbf{b} \times \mathbf{c}) + (\mathbf{c} \times \mathbf{a}) + (\mathbf{a} \times \mathbf{b}) = \mathbf{0}$

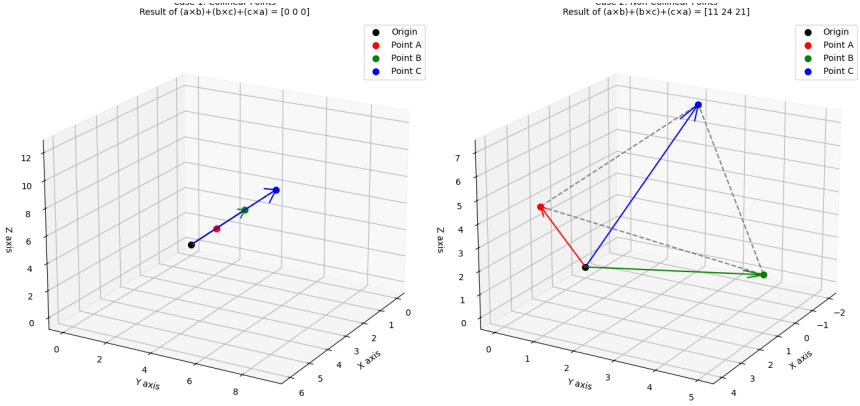
Result of (a×b)+(b×c)+(c×a) = [0 0 0]
Origin
Point A
Point B
Point C
Z axis
Y axis
X axis
Result of (a×b)+(b×c)+(c×a) = [11 24 21]
Origin
Point A
Point B
Point C
Z axis
Y axis
X axis

Fig. 0