7.4.27

EE25BTECH11020 - Darsh Pankaj Gajare

Question:
The triangle PQR is inscribed in the circle $x^2 + y^2 = 25$.If $\mathbf{Q}$ and $\mathbf{R}$ have co-ordinates $(3, 4)$ and $(-4, 3)$ respectively then $\angle QPR$ is equal to

1) $\frac{\pi}{2}$
2) $\frac{\pi}{3}$
3) $\frac{\pi}{4}$
4) $\frac{\pi}{6}$

Solution:

TABLE I

$\mathbf{Q}$	$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$
$\mathbf{R}$	$\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

$$\mathbf{x}^\top \mathbf{x} = 25 \tag{1}$$

The given points (position vectors) are

$$\mathbf{q} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \qquad \mathbf{r} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \tag{2}$$

Verify they lie on the circle:

$$\mathbf{q}^\top \mathbf{q} = 3^2 + 4^2 = 25, \tag{3}$$
$$\mathbf{r}^\top \mathbf{r} = (-4)^2 + 3^2 = 25. \tag{4}$$

Compute the inner product (matrix/dot product):

$$\mathbf{q}^\top \mathbf{r} = \begin{pmatrix} 3 & 4 \end{pmatrix} \begin{pmatrix} -4 \\ 3 \end{pmatrix} \tag{5}$$
$$= 3 \cdot (-4) + 4 \cdot 3 = -12 + 12 = 0. \tag{6}$$

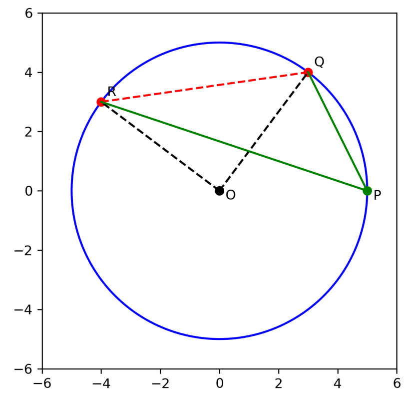
Compute norms (using matrix notation) and the central angle θ:

$$\|\mathbf{q}\| = \sqrt{\mathbf{q}^\top \mathbf{q}} = 5, \qquad \|\mathbf{r}\| = \sqrt{\mathbf{r}^\top \mathbf{r}} = 5, \tag{7}$$
$$\cos\theta = \frac{\mathbf{q}^\top \mathbf{r}}{\|\mathbf{q}\| \, \|\mathbf{r}\|} = \frac{0}{5 \cdot 5} = 0 \tag{8}$$
$$\implies \theta = \frac{\pi}{2}. \tag{9}$$

Since $\angle QPR$ is the angle subtended at the circumference by chord QR, it equals half the central angle:

$$\angle QPR = \frac{\theta}{2} = \frac{\pi}{4}. \tag{10}$$

Plot using C libraries:



Plot using Python: