

4.13.25

EE25BTECH11020 - Darsh Pankaj Gajare

Question:

Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus?

1) $\left(\frac{1}{3}, -\frac{8}{3}\right)$

2) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$

3) $(-3, -9)$

4) $(-3, -8)$

Solution:

TABLE I

Line	Normal \mathbf{n}	Constant c
$\mathbf{n}_1^T \mathbf{x} = -1$	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	-1
$\mathbf{n}_2^T \mathbf{x} = 5$	$\begin{pmatrix} 7 \\ -1 \end{pmatrix}$	5
Centre	-	$\mathbf{O} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

$$A\mathbf{V}_A = b, \quad A = \begin{pmatrix} 1 & -1 \\ 7 & -1 \end{pmatrix}, \quad b = \begin{pmatrix} -1 \\ 5 \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} 1 & -1 & | & -1 \\ 7 & -1 & | & 5 \end{pmatrix} \xrightarrow{R_2 - 7R_1} \begin{pmatrix} 1 & -1 & | & -1 \\ 0 & 6 & | & 12 \end{pmatrix} \quad (2)$$

$$\mathbf{V}_A = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (3)$$

$$\mathbf{V}_C = 2\mathbf{O} - \mathbf{V}_A = 2\begin{pmatrix} -1 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -3 \\ -6 \end{pmatrix} \quad (4)$$

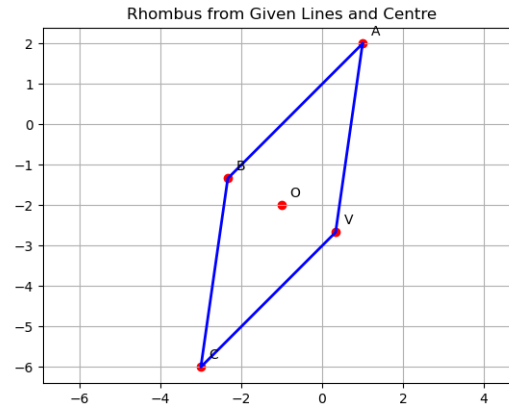
$$k = -\mathbf{n}_1^T \mathbf{V}_C = -3, \quad m = -\mathbf{n}_2^T \mathbf{V}_C = 15 \quad (5)$$

$$\begin{pmatrix} 1 & -1 & | & 3 \\ 7 & -1 & | & 5 \end{pmatrix} \xrightarrow{R_2 - 7R_1} \begin{pmatrix} 1 & -1 & | & 3 \\ 0 & 6 & | & -16 \end{pmatrix} \quad (6)$$

$$\mathbf{V} = \begin{pmatrix} \frac{1}{3} \\ -\frac{8}{3} \end{pmatrix} \quad (7)$$

$$(8)$$

Plot using C libraries:



Plot using Python:

