1.5.4

EE25BTECH11016 - Taraka Abhinav

Question:
A circle has its center at $(4, 4)$. If one end of a diameter is $(4, 0)$, then find the coordinates of the other end.
Solution:
Let the position vectors for the center, the known end, and the unknown end of the diameter be $\mathbf{C}$, $\mathbf{B}$, and $\mathbf{A}$ respectively. Let the coordinates of the unknown end $\mathbf{A}$ be (a, b).

The given vectors are:

$$\mathbf{A} = \begin{pmatrix} a \\ b \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{1}$$

The center of the circle is the midpoint of the diameter. Therefore, the center vector is the average of the endpoint vectors.

$$\mathbf{C} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{2}$$

To find the unknown vector $\mathbf{A}$, we rearrange the equation:

$$2\mathbf{C} = \mathbf{A} + \mathbf{B} \tag{3}$$

$$\mathbf{A} = 2\mathbf{C} - \mathbf{B} \tag{4}$$

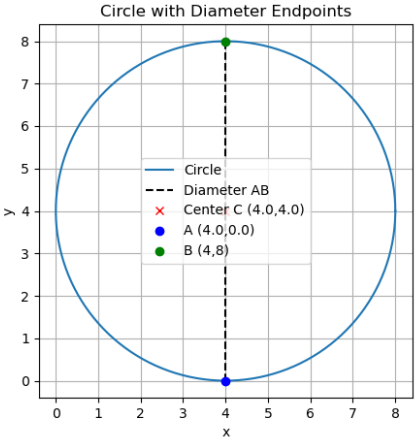
Substituting the given vector values:

$$\begin{pmatrix} a \\ b \end{pmatrix} = 2\begin{pmatrix} 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 8 \\ 8 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{5}$$

$$= \begin{pmatrix} 4 \\ 8 \end{pmatrix} \tag{6}$$

$\therefore$ The other end of the diameter is $(4, 8)$.


Circle with Diameter Endpoints
Circle
Diameter AB
Center C (4.0,4.0)
A (4.0,0.0)
B (4,8)
x
y
0
1
2
3
4
5
6
7
8