4.7.34

M Chanakya Srinivas- EE25BTECH11036

Problem Statement

Find the equation of a plane at a distance $3\sqrt{3}$ from the origin, whose normal is equally inclined to the coordinate axes.

Normal Vector

If the normal is equally inclined to all coordinate axes:

$$\mathbf{n} = \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \lambda \neq 0 \tag{1}$$

Equation of Plane

The general equation of a plane is:

$$\mathbf{n}^\top \mathbf{x} = p \tag{2}$$

Distance Condition

Distance of the plane from the origin:

$$d = \frac{|p|}{\|\mathbf{n}\|} \tag{3}$$

$$\|\mathbf{n}\| = |\lambda|\sqrt{1^2 + 1^2 + 1^2} = |\lambda|\sqrt{3} \tag{4}$$

Substitute $d = 3\sqrt{3}$:

$$3\sqrt{3} = \frac{|p|}{|\lambda|\sqrt{3}} \tag{5}$$

$$|p| = 9|\lambda| \tag{6}$$

Simplification

Divide through by λ:

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{x} = \frac{p}{\lambda} \tag{7}$$

Since $\frac{p}{\lambda} = \pm 9$:

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{x} = 9 \tag{8}$$

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{x} = -9 \tag{9}$$

Final Answer

Vector Form:

$$\mathbf{n}^\top \mathbf{x} = \pm 9, \quad \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{10}$$

Algebraic Form:

$$x + y + z = 9 \tag{11}$$

$$x + y + z = -9 \tag{12}$$

C CODE

```c
#include <math.h>

void equal_inclined_planes(double distance, double *coeffs_out)
{
    if (!coeffs_out) return;
    if (distance < 0) distance = -distance;

    double a = 1.0, b = 1.0, c = 1.0;
    double rhs = distance * sqrt(3.0);

    coeffs_out[0] = a; coeffs_out[1] = b; coeffs_out[2] = c;
        coeffs_out[3] = rhs;
    coeffs_out[4] = a; coeffs_out[5] = b; coeffs_out[6] = c;
        coeffs_out[7] = -rhs;
}
```

Python code through shared output

```
# call_planes.py
from ctypes import CDLL, c_double
import math
import numpy as np
import matplotlib.pyplot as plt

# Load library
lib = CDLL('./libplanes.so')
lib.equal_inclined_planes.argtypes = (c_double, c_double * 8)
lib.equal_inclined_planes.restype = None

# Prepare output array
coeffs = (c_double * 8)()
distance = 3.0 * math.sqrt(3.0)
```

Python code through shared output

```python
# Call C function
lib.equal_inclined_planes(distance, coeffs)

# Extract coefficients
a1,b1,c1,d1 = coeffs[0], coeffs[1], coeffs[2], coeffs[3]
a2,b2,c2,d2 = coeffs[4], coeffs[5], coeffs[6], coeffs[7]

print(fPlane 1: {a1}x + {b1}y + {c1}z = {d1})
print(fPlane 2: {a2}x + {b2}y + {c2}z = {d2})

# -------------------
# Plotting
# -------------------
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Grid for plotting planes
xx, yy = np.meshgrid(np.linspace(-10,10,20), np.linspace
    (-10,10,20))
```

Python code through shared output

```
# Plane 1: z = (d1 - a1*xx - b1*yy)/c1
zz1 = (d1 - a1*xx - b1*yy)/c1
ax.plot_surface(xx, yy, zz1, alpha=0.3, color='cyan')

# Plane 2: z = (d2 - a2*xx - b2*yy)/c2
zz2 = (d2 - a2*xx - b2*yy)/c2
ax.plot_surface(xx, yy, zz2, alpha=0.3, color='magenta')

# Origin
ax.scatter(0,0,0, color='black', s=60, label=Origin)

ax.set_xlabel(X)
ax.set_ylabel(Y)
ax.set_zlabel(Z)
ax.legend()
plt.show()
```

only Python code

```
import sys
sys.path.insert(0, '/sdcard/github/matgeo/codes/CoordGeo') # Your
    custom path
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
# Local imports (keeping as requested)
from line.funcs import *
from triangle.funcs import *
from conics.funcs import circ_gen
# Plane coefficients: a = b = c = 1, distance from origin = 3 *
    sqrt(3)
a, b, c = 1, 1, 1
d1 = -9 # Plane 1: x + y + z = 9 (rewrite as x + y + z - 9 = 0)
d2 = 9 # Plane 2: x + y + z = -9 (rewrite as x + y + z + 9 = 0)
```

only Python code

```
# Create grid for x and y
x = np.linspace(-10, 10, 50)
y = np.linspace(-10, 10, 50)
X, Y = np.meshgrid(x, y)
# Calculate corresponding z for both planes
Z1 = (-a * X - b * Y - d1) / c
Z2 = (-a * X - b * Y - d2) / c
# Create figure and 3D axis
fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')
# Plot both planes with transparency and different colors
ax.plot_surface(X, Y, Z1, alpha=0.5, color='cyan', rstride=1,
    cstride=1, edgecolor='none', label='Plane 1: x+y+z=9')
ax.plot_surface(X, Y, Z2, alpha=0.5, color='orange', rstride=1,
    cstride=1, edgecolor='none', label='Plane 2: x+y+z=-9')
```

only Python code

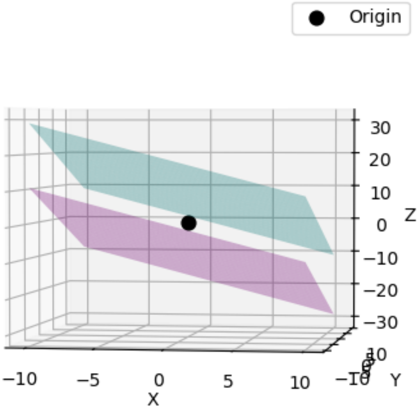
```
# Mark the origin
ax.scatter(0, 0, 0, color='red', s=80, label='Origin (0,0,0)')
# Normal vector (same for both planes)
normal_vec = np.array([a, b, c])
origin = np.array([0, 0, 0])
ax.quiver(*origin, *normal_vec, length=7, color='black',
    linewidth=2, label='Normal Vector (1,1,1)')
# Annotate the intercepts of Plane 1 (where x=0,y=0 -> z=9 etc)
intercepts_p1 = np.array([[9, 0, 0], [0, 9, 0], [0, 0, 9]])
for i, point in enumerate(intercepts_p1):
    ax.scatter(*point, color='blue', s=60)
    ax.text(*point, f'P{i+1}\n({point[0]}, {point[1]}, {point
        [2]})', color='blue', fontsize=10, ha='left')
```

only Python code

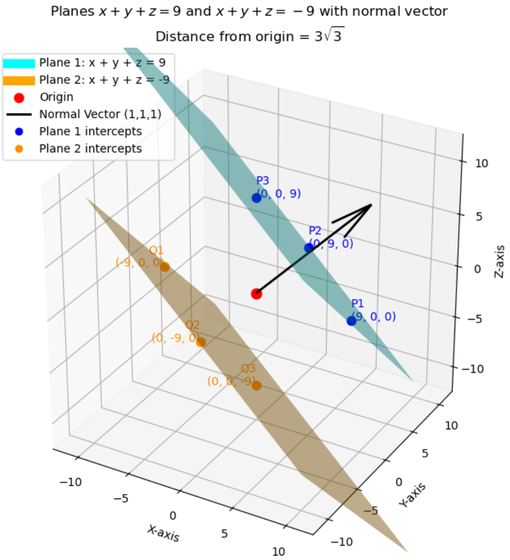
```python
# Annotate the intercepts of Plane 2
intercepts_p2 = np.array([[-9, 0, 0], [0, -9, 0], [0, 0, -9]])
for i, point in enumerate(intercepts_p2):
    ax.scatter(*point, color='darkorange', s=60)
    ax.text(*point, f'Q{i+1}\n({point[0]}, {point[1]}, {point[2]})', color='darkorange', fontsize=10, ha='right')
# Set labels and title
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title('Planes $x + y + z = 9$ and $x + y + z = -9$ with normal vector\nDistance from origin = $3\\sqrt{3}$')
# Set limits and equal aspect ratio for better view
ax.set_xlim(-12, 12)
ax.set_ylim(-12, 12)
```

```
ax.set_zlim(-12, 12)
ax.set_box_aspect([1, 1, 1])
# Add legend manually because plot_surface does not support label
    param well
from matplotlib.lines import Line2D
legend_elements = [
    Line2D([0], [0], color='cyan', lw=8, label='Plane 1: x + y +
        z = 9'),
    Line2D([0], [0], color='orange', lw=8, label='Plane 2: x + y
        + z = -9'),
    Line2D([0], [0], marker='o', color='w', label='Origin',
        markerfacecolor='red', markersize=10),
    Line2D([0], [0], color='black', lw=2, label='Normal Vector
        (1,1,1)'),
    Line2D([0], [0], marker='o', color='w', label='Plane 1
        intercepts', markerfacecolor='blue', markersize=8),
    Line2D([0], [0], marker='o', color='w', label='Plane 2
        intercepts', markerfacecolor='darkorange', markersize=8),
]
```

PLOTS



PLOTS



Planes $x + y + z = 9$ and $x + y + z = -9$ with normal vector
Distance from origin = $3\sqrt{3}$
Plane 1: x + y + z = 9
Plane 2: x + y + z = -9
Origin
Normal Vector (1,1,1)
Plane 1 intercepts
Plane 2 intercepts
P3
(0, 0, 9)
P2
(0, 9, 0)
P1
(9, 0, 0)
Q1
(-9, 0, 0)
Q2
(0, -9, 0)
Q3
(0, 0, -9)
X-axis
Y-axis
Z-axis
−10
−5
0
5
10