4.11.25

EE25BTECH11048 - Revanth Siva Kumar.D

Question

Find the distance of the point $\left(1,-2,9\right)$ from the point of intersection of the line

$$\mathbf{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$$

and the plane

$$\mathbf{r} \cdot \left(\hat{i} - \hat{j} + \hat{k}\right) = 10.$$

Solution:

The line can be written as

$$\mathbf{r} = \mathbf{r}_0 + \lambda\mathbf{d}, \tag{1}$$

$$\mathbf{r}_0 = \begin{pmatrix}4\\2\\7\end{pmatrix}, \quad \mathbf{d} = \begin{pmatrix}3\\4\\2\end{pmatrix}. \tag{2}$$

The plane equation is

$$\mathbf{n}^T\mathbf{r} = c, \quad \mathbf{n} = \begin{pmatrix}1\\-1\\1\end{pmatrix}, \; c = 10. \tag{3}$$

Step 1: Intersection point of line and plane

Substitute $\mathbf{r} = \mathbf{r}_0 + \lambda\mathbf{d}$:

$$\mathbf{n}^T(\mathbf{r}_0 + \lambda\mathbf{d}) = c \tag{4}$$

$$\implies \lambda = \frac{c - \mathbf{n}^T\mathbf{r}_0}{\mathbf{n}^T\mathbf{d}}. \tag{5}$$

Thus, the intersection point is

$$\mathbf{P} = \mathbf{r}_0 + \frac{c - \mathbf{n}^T\mathbf{r}_0}{\mathbf{n}^T\mathbf{d}}\mathbf{d}. \tag{6}$$

Step 2: Distance formula

Let the given point be

$$\mathbf{A} = \begin{pmatrix}1\\-2\\9\end{pmatrix}. \tag{7}$$

Then the displacement vector is

$$\mathbf{v} = \mathbf{P} - \mathbf{A}. \tag{8}$$

So the distance is

$$d = \|\mathbf{v}\| = \sqrt{\mathbf{v}^T\mathbf{v}}. \tag{9}$$

Step 3: Substitution of values

Now substituting:

$$\mathbf{n}^T\mathbf{d} = \begin{pmatrix}1 & -1 & 1\end{pmatrix}\begin{pmatrix}3\\4\\2\end{pmatrix} = 1, \tag{10}$$

$$\mathbf{n}^T\mathbf{r}_0 = \begin{pmatrix}1 & -1 & 1\end{pmatrix}\begin{pmatrix}4\\2\\7\end{pmatrix} = 9, \tag{11}$$

$$\lambda = \frac{10-9}{1} = 1. \tag{12}$$

Hence

$$\mathbf{P} = \mathbf{r}_0 + \mathbf{d} = \begin{pmatrix}4\\2\\7\end{pmatrix} + \begin{pmatrix}3\\4\\2\end{pmatrix} = \begin{pmatrix}7\\6\\9\end{pmatrix}. \tag{13}$$

$$\mathbf{v} = \mathbf{P} - \mathbf{A} = \begin{pmatrix}7\\6\\9\end{pmatrix} - \begin{pmatrix}1\\-2\\9\end{pmatrix} = \begin{pmatrix}6\\8\\0\end{pmatrix}. \tag{14}$$

$$d = \sqrt{6^2 + 8^2 + 0^2} = \sqrt{100} = 10. \tag{15}$$

Final Answer:

$$\boxed{10}$$

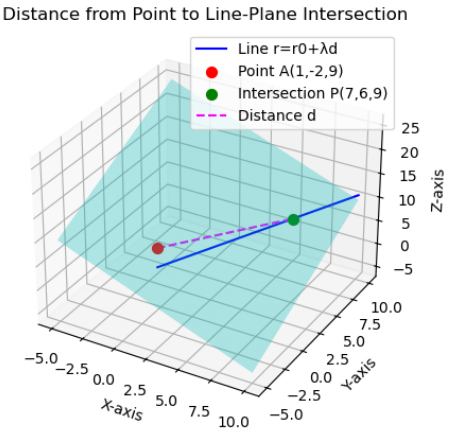

Fig. 1: Intersection point P, given point A, and distance AP.