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Question

Find the angle between two vectors **a** and **b** with magnitudes 1 and 2 respectively and when $\mathbf{a} \cdot \mathbf{b} = 1$.

Solution

Given two vectors **a** and **b** with magnitudes:

$$\|\mathbf{a}\| = 1, \quad \|\mathbf{b}\| = 2 \tag{1}$$

and dot product:

$$\mathbf{a} \cdot \mathbf{b} = 1 \tag{2}$$

We use the matrix formulation of the dot product:

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{a}^T \mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\| \cos\theta \tag{3}$$

Substituting the known values:

$$1 = (1)(2)\cos\theta \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow \theta = 60^\circ \tag{4}$$

Matrix Representation

Let:

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{5}$$

Then:

$$\mathbf{a}^T \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = 1 \tag{6}$$

$$\|\mathbf{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2} = 1, \quad \|\mathbf{b}\| = \sqrt{b_1^2 + b_2^2 + b_3^2} = 2 \tag{7}$$

So the angle is:

$$\theta = \cos^{-1}\left(\frac{\mathbf{a}^T \mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ \tag{8}$$

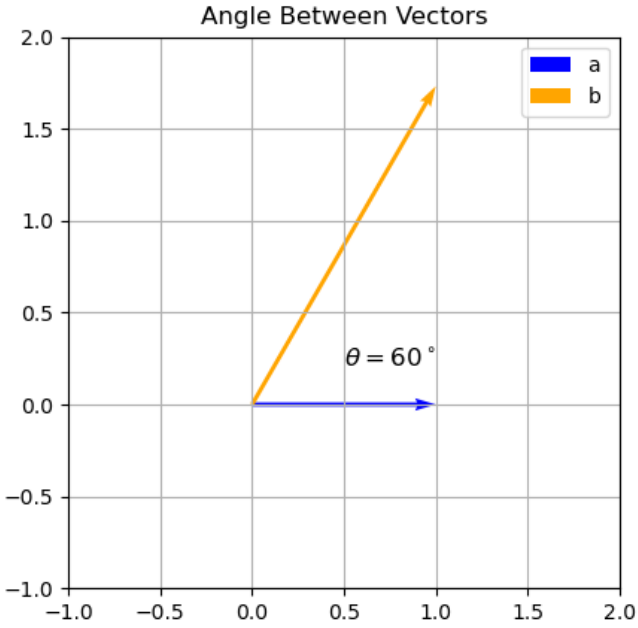
Angle Between Vectors
a
b
$\theta = 60^\circ$
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0

Figure 1