

4.13.28

EE25BTECH11023 - Venkata Sai

Question:

Slope of a line passing through $\mathbf{P}(2, 3)$ and intersecting the line $x + y = 7$ at a distance of 4 units from \mathbf{P} , is

Solution: Given

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (1)$$

Equation of a line through \mathbf{P} and having slope m is

$$\begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} x-2 \\ y-3 \end{pmatrix} = 0 \quad (2)$$

$$\begin{pmatrix} -m & 1 \end{pmatrix} \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right) = 0 \implies \begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (3)$$

$$\begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 3 - 2m \quad (4)$$

$$x + y = 7 \implies \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 7 \quad (5)$$

$$\begin{pmatrix} -m & 1 & 3-2m \\ 1 & 1 & 7 \end{pmatrix} \xleftrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 1 & 7 \\ -m & 1 & 3-2m \end{pmatrix} \xleftrightarrow{R_2 \rightarrow R_2 + mR_1} \begin{pmatrix} 1 & 1 & 7 \\ 0 & 1+m & 3+5m \end{pmatrix} \quad (6)$$

$$y = \frac{3+5m}{1+m} \quad (7)$$

Given the point is at a distance of 4 units from point \mathbf{P}

$$\left\| \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\| = 4 \implies \left\| \begin{pmatrix} x-2 \\ y-3 \end{pmatrix} \right\| = 4 \quad (8)$$

$$\sqrt{(x-2)^2 + (y-3)^2} = 4 \quad (9)$$

$$\sqrt{(7-y-2)^2 + (y-3)^2} = 4 \quad (10)$$

$$(5-y)^2 + (y-3)^2 = 4^2 = 16 \quad (11)$$

$$25 + y^2 - 10y + y^2 + 9 - 6y = 16 \quad (12)$$

$$2y^2 - 16y + 18 = 0 \implies y^2 - 8y + 9 = 0 \quad (13)$$

$$y^2 - 8y + 9 + 16 = 16 \implies (y-4)^2 = 7 \quad (14)$$

$$y - 4 = \pm \sqrt{7} \quad (15)$$

$$y = 4 - \sqrt{7} \text{ (or) } 4 + \sqrt{7} \quad (16)$$

$$\frac{3+5m}{1+m} = \frac{3+5m+2-2}{1+m} = \frac{5+5m-2}{1+m} = \frac{5(m+1)-2}{1+m} = 5 - \frac{2}{1+m} \quad (17)$$

$$5 - \frac{2}{1+m} = 4 - \sqrt{7} \text{ (or) } 5 - \frac{2}{1+m} = 4 + \sqrt{7} \quad (18)$$

$$\frac{2}{1+m} = 1 + \sqrt{7} \text{ (or) } \frac{2}{1+m} = 1 - \sqrt{7} \quad (19)$$

$$1+m = \frac{2}{1+\sqrt{7}} \text{ (or) } 1+m = \frac{2}{1-\sqrt{7}} \quad (20)$$

$$m = \frac{2-1-\sqrt{7}}{1+\sqrt{7}} \text{ (or) } m = \frac{2-1+\sqrt{7}}{1-\sqrt{7}} \quad (21)$$

$$m = \frac{1-\sqrt{7}}{1+\sqrt{7}} \text{ (or) } m = \frac{1+\sqrt{7}}{1-\sqrt{7}} \quad (22)$$

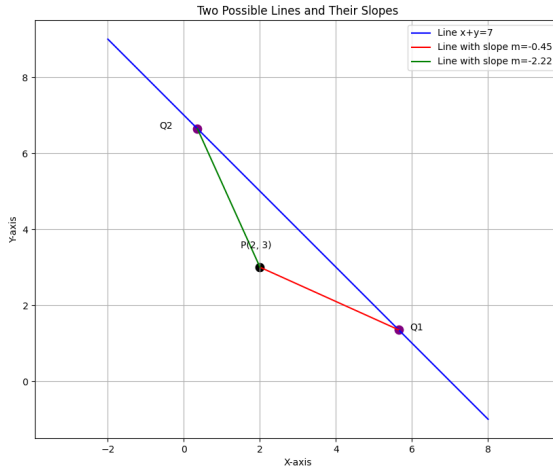


Fig. 0.1