4.11.27

EE25BTECH11050-Hema Havil

Question:

Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$

Solution:

Let the given points be P(4,-3,-4) and Q(3,-2,2) then the direction vector along pq be d,

$$\mathbf{d} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \\ -4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \tag{0.1}$$

equation of line passing through P,Q be

$$r(t) = r_0 + td \tag{0.2}$$

where t is a parameter

$$\mathbf{r(t)} = \begin{pmatrix} 4 \\ -3 \\ -4 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \tag{0.3}$$

Let the given plane equation be

$$\mathbf{n}^T \mathbf{x} = c \tag{0.4}$$

where,

$$\mathbf{n} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \tag{0.5}$$

$$c = 6 \tag{0.6}$$

Consider a point with parameter t_1 which is the intersection point then, it satisfies line equation and plane equation

$$\mathbf{r(t_1)} = \begin{pmatrix} 4 \\ -3 \\ -4 \end{pmatrix} + t_1 \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \tag{0.7}$$

Substitute this point in the plane equation

$$\mathbf{n}^T \mathbf{r_{t_1}} = c \tag{0.8}$$

$$\begin{pmatrix} 2 & 1 & 1 \end{pmatrix}\left(\begin{pmatrix} 4 \\ -3 \\ -4 \end{pmatrix} + t_1 \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix}\right) = 6 \tag{0.9}$$

$$1 + t_1 (5) = 6 \tag{0.10}$$

$$5t_1 = 5 \tag{0.11}$$

$$t_1 = 1 \tag{0.12}$$

then the intersection point be,

$$\mathbf{r_{t_1}} = \begin{pmatrix} 4 \\ -3 \\ -4 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \tag{0.13}$$

$$\mathbf{r_{t_1}} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} \tag{0.14}$$

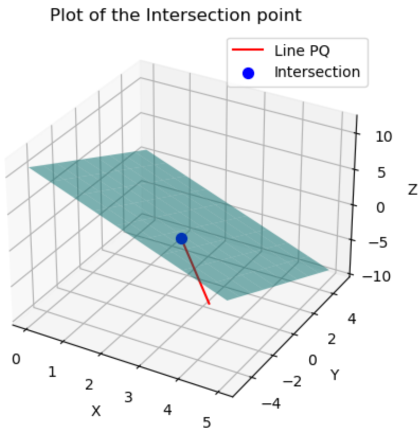

Fig. 0.1: Plot of the Intersection point