4.13.41

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Problem

Find the area of the parallelogram formed by the lines

$$y = mx, \quad y = mx + 1, \quad y = nx, \quad y = nx + 1.$$

Step 1: Represent lines in parametric vector form

$$\mathbf{r}_1 = \kappa_1 \begin{pmatrix} 1 \\ m \end{pmatrix}, \qquad \mathbf{r}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \kappa_2 \begin{pmatrix} 1 \\ m \end{pmatrix},$$

$$\mathbf{r}_3 = \mu_1 \begin{pmatrix} 1 \\ n \end{pmatrix}, \qquad \mathbf{r}_4 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \mu_2 \begin{pmatrix} 1 \\ n \end{pmatrix}.$$

Step 1: Represent lines in parametric vector form

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$$\mathbf{r}_3 = \mu_1 \begin{pmatrix} 1 \\ n \end{pmatrix}, \qquad \mathbf{r}_4 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \mu_2 \begin{pmatrix} 1 \\ n \end{pmatrix}.$$

Here, $\mathbf{r}_1, \mathbf{r}_2$ represent the pair of lines with slope m, and $\mathbf{r}_3, \mathbf{r}_4$ represent the pair of lines with slope n.

Step 2: Compute vertices by intersection

Intersection of $\mathbf{r}_1$ and $\mathbf{r}_3$: $\kappa_1 \begin{pmatrix} 1 \\ m \end{pmatrix} = \mu_1 \begin{pmatrix} 1 \\ n \end{pmatrix} \quad \Rightarrow \quad \mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Intersection of $\mathbf{r}_2$ and $\mathbf{r}_3$: $\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \kappa_2 \begin{pmatrix} 1 \\ m \end{pmatrix} = \mu_1 \begin{pmatrix} 1 \\ n \end{pmatrix}$

Expressed as matrix equation:

$$\underbrace{\begin{pmatrix} 1 & -1 \\ m & -n \end{pmatrix}}_{M} \underbrace{\begin{pmatrix} \kappa_2 \\ \mu_1 \end{pmatrix}}_{\mathbf{z}} = \underbrace{\begin{pmatrix} 0 \\ -1 \end{pmatrix}}_{\mathbf{b}}$$

Step 3: Solve system for κ_2, μ_1

Row operations:

$$R_2 \to R_2 - mR_1 : \quad (m-n)\mu_1 = m-1 \quad \Rightarrow \quad \mu_1 = \frac{1}{m-n}$$

$$R_1 \to R_1 : \quad \kappa_2 - \mu_1 = 0 \quad \Rightarrow \quad \kappa_2 = \frac{1}{m-n}$$

Hence,

$$\mathbf{Q} = \begin{pmatrix} \kappa_2 \\ 1 + m\kappa_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{m-n} \\ 1 + \frac{m}{m-n} \end{pmatrix}$$

Other vertices:

$$\mathbf{R} = \begin{pmatrix} \frac{1}{m-n} \\ \frac{m}{m-n} \end{pmatrix}, \quad \mathbf{S} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Step 4: Area of the parallelogram

$$\mathbf{PQ} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} \frac{1}{m-n} \\ 1 + \frac{m}{m-n} \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{m-n} \\ 1 + \frac{m}{m-n} \end{pmatrix}$$

$$\mathbf{PR} = \mathbf{R} - \mathbf{P} = \begin{pmatrix} \frac{1}{m-n} \\ \frac{m}{m-n} \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{m-n} \\ \frac{m}{m-n} \end{pmatrix}$$

Area is magnitude of the vector cross product:

$$\begin{aligned} \text{Area} &= \left| \mathbf{PQ} \times \mathbf{PR} \right| \\ &= \left| \frac{1}{m-n} \left(\frac{m}{m-n} \right) - \left(1 + \frac{m}{m-n} \right) \left(\frac{1}{m-n} \right) \right| \\ &= \frac{1}{|m-n|} \end{aligned}$$

Answer

$$\boxed{\text{Area of the parallelogram} = \frac{1}{|m-n|}}$$

C code

```
#include <math.h>

// Make the symbol visible in the shared object
__attribute__((visibility(default)))
double parallelogram_area(double x1, double y1, double x2, double
     y2) {
    // Cross product magnitude
    return fabs(x1 * y2 - x2 * y1);
}
```

Python code through shared output

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load the compiled shared library
lib = ctypes.CDLL('./libpara.so')
lib.parallelogram_area.argtypes = [ctypes.c_double, ctypes.
    c_double, ctypes.c_double, ctypes.c_double]
lib.parallelogram_area.restype = ctypes.c_double

# Slopes
m = 1
n = -1
```

Python code through shared output

```
# Function to get intersection of two lines: y = m1 x + c1 and y
    = m2 x + c2
def line_intersection(m1, c1, m2, c2):
    x = (c2 - c1) / (m1 - m2)
    y = m1 * x + c1
    return x, y

# Calculate all four vertices (intersections)
P1 = line_intersection(m, 0, n, 0) # y=mx and y=nx
P2 = line_intersection(m, 0, n, 1) # y=mx and y=nx+1
P3 = line_intersection(m, 1, n, 1) # y=mx+1 and y=nx+1
P4 = line_intersection(m, 1, n, 0) # y=mx+1 and y=nx

# Convert to numpy arrays for vector operations
P1 = np.array(P1)
P2 = np.array(P2)
```

Python code through shared output

```
P3 = np.array(P3)
P4 = np.array(P4)

# Compute side vectors for area calculation (two adjacent sides
    from P1)
vec1 = P2 - P1
vec2 = P4 - P1

# Call the C function for area
area = lib.parallelogram_area(vec1[0], vec1[1], vec2[0], vec2[1])

# ----- Plotting Section -----

# X range for lines plotting
x_vals = np.linspace(-2, 2, 400)
```

Python code through shared output

```
# Lines y = mx and y = mx + 1
y_m = m * x_vals
y_m1 = m * x_vals + 1

# Lines y = nx and y = nx + 1
y_n = n * x_vals
y_n1 = n * x_vals + 1

plt.figure(figsize=(8, 8))
plt.plot(x_vals, y_m, label=r'$y = mx$', color='blue')
plt.plot(x_vals, y_m1, label=r'$y = mx + 1$', linestyle='--',
    color='blue')
plt.plot(x_vals, y_n, label=r'$y = nx$', color='red')
plt.plot(x_vals, y_n1, label=r'$y = nx + 1$', linestyle='--',
    color='red')
```

Python code through shared output

```
# Parallelogram vertices for plotting (close the polygon by
    adding P1 again)
vertices_x = [P1[0], P2[0], P3[0], P4[0], P1[0]]
vertices_y = [P1[1], P2[1], P3[1], P4[1], P1[1]]

# Plot parallelogram
plt.plot(vertices_x, vertices_y, 'k-', linewidth=2, label='
    Parallelogram')
plt.fill(vertices_x, vertices_y, color='gray', alpha=0.3)

# Label vertices
for i, (xv, yv) in enumerate(zip(vertices_x[:-1], vertices_y
    [:-1]), 1):
    plt.text(xv, yv, f'P{i}', fontsize=12, ha='right', va='bottom
        ')
```

```
# Show area on plot
plt.text(-1.5, 1.5, f'Area = {area:.4f}', fontsize=14, color='
    green',
        bbox=dict(facecolor='white', alpha=0.8))

# Axis formatting
plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
```

Python code through shared output

```
plt.grid(True, linestyle='--', alpha=0.5)
plt.legend()
plt.title('Parallelogram formed by lines $y=mx$, $y=mx+1$, $y=nx$
    , $y=nx+1$\n(Area via C shared library)')
plt.xlabel('x-axis')
plt.ylabel('y-axis')
plt.axis('equal')
plt.xlim(-2, 2)
plt.ylim(-2, 2)

plt.show()
```

Only Python code

```python
import numpy as np
import matplotlib.pyplot as plt

def line_intersect(n1, c1, n2, c2):

    Find intersection point of two lines:
    n1.T * x = c1
    n2.T * x = c2
    Inputs:
        n1, n2: (2,) or (2,1) normal vectors to lines
        c1, c2: scalars
    Returns:
        p: (2,1) intersection point

    N = np.column_stack((n1.flatten(), n2.flatten())) # 2x2 matrix
    C = np.array([c1, c2])
    p = np.linalg.solve(N, C)
    return p.reshape(2,1)
```

Only Python code

```
def line_dir_pt(direction, point, k1, k2, num=100):

    Generate points on a line given direction and a point.
    direction: (2,) array
    point: (2,) array
    k1, k2: scalar parameters along the line
    num: number of points
    Returns: 2 x num array of points

    k = np.linspace(k1, k2, num)
    line_pts = point.reshape(2,1) + direction.reshape(2,1) * k
    return line_pts

# Slopes
m = 1
n = -1
```

Only Python code

```python
# Normals (for line: y = m x + c => -m x + y = c)
n1 = np.array([-m, 1])
n2 = np.array([-m, 1])
n3 = np.array([-n, 1])
n4 = np.array([-n, 1])

# Constants c
c1 = 0
c2 = 1
c3 = 0
c4 = 1

# Find intersection points
P1 = line_intersect(n1, c1, n3, c3)
P2 = line_intersect(n1, c1, n4, c4)
P3 = line_intersect(n2, c2, n4, c4)
P4 = line_intersect(n2, c2, n3, c3)

# Parallelogram points array
```

Only Python code

```
# Calculate area via cross product
vec1 = P2 - P1
vec2 = P4 - P1
area = abs(np.cross(vec1.flatten(), vec2.flatten()))

# Direction vectors for lines (x direction)
dir_m = np.array([1, m])
dir_n = np.array([1, n])

# Generate line points for plotting
k1, k2 = -5, 5

line_m0 = line_dir_pt(dir_m, np.array([0,0]), k1, k2)
line_m1 = line_dir_pt(dir_m, np.array([0,1]), k1, k2)
line_n0 = line_dir_pt(dir_n, np.array([0,0]), k1, k2)
line_n1 = line_dir_pt(dir_n, np.array([0,1]), k1, k2)
```

Only Python code

```python
# Plotting
plt.figure(figsize=(8,8))
plt.plot(line_m0[0], line_m0[1], label='y = m x', color='blue')
plt.plot(line_m1[0], line_m1[1], label='y = m x + 1', linestyle='
    --', color='blue')
plt.plot(line_n0[0], line_n0[1], label='y = n x', color='red')
plt.plot(line_n1[0], line_n1[1], label='y = n x + 1', linestyle='
    --', color='red')

plt.plot(parallelogram[0], parallelogram[1], 'k-', linewidth=2,
    label='Parallelogram')
plt.fill(parallelogram[0], parallelogram[1], 'grey', alpha=0.3)

for i, P in enumerate([P1, P2, P3, P4], 1):
    plt.plot(P[0], P[1], 'ko')
    plt.text(P[0] + 0.1, P[1] + 0.1, f'P{i}', fontsize=12)
```

Only Python code

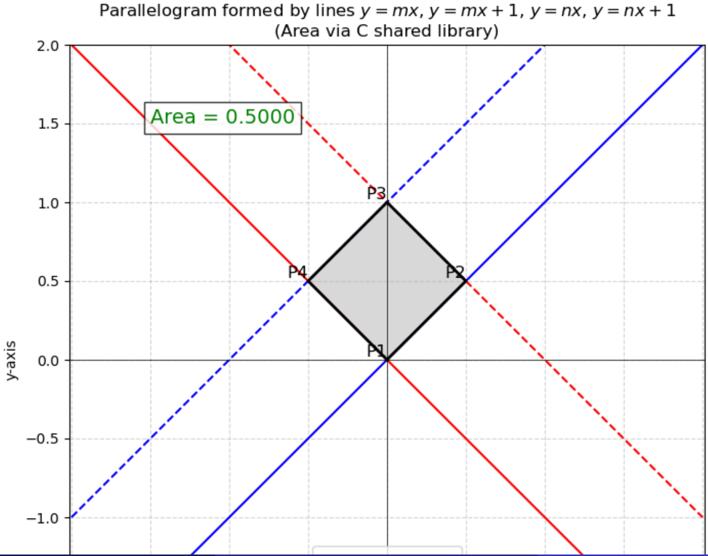
```
plt.text(-4.5, 4, f'Area = {area:.3f}', fontsize=14, color='green
    ',
        bbox=dict(facecolor='white', alpha=0.8))

plt.axhline(0, color='black', lw=0.5)
plt.axvline(0, color='black', lw=0.5)
plt.grid(True, linestyle='--', alpha=0.6)
plt.axis('equal')
```

Only Python code

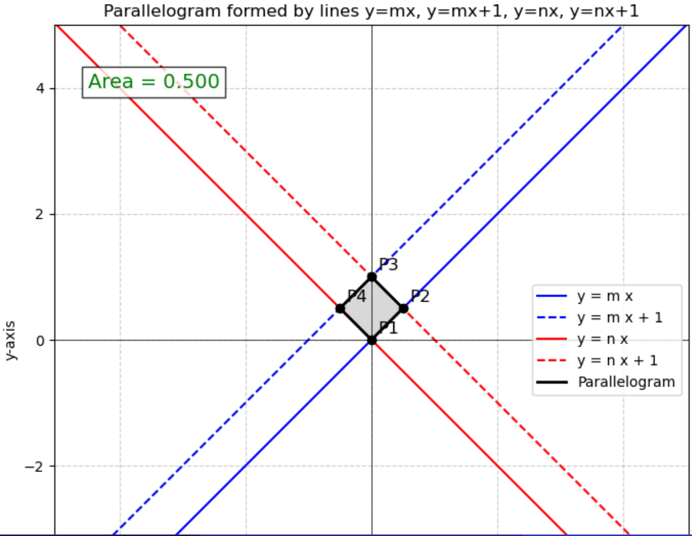
```
plt.xlim(-5, 5)
plt.ylim(-5, 5)
plt.xlabel('x-axis')
plt.ylabel('y-axis')
plt.title('Parallelogram formed by lines y=mx, y=mx+1, y=nx, y=nx
    +1')
plt.legend()
plt.show()
```

PLOTS



Parallelogram formed by lines $y = mx$, $y = mx + 1$, $y = nx$, $y = nx + 1$
(Area via C shared library)
Area = 0.5000
P1
P2
P3
P4
y-axis
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0

PLOTS



Parallelogram formed by lines y=mx, y=mx+1, y=nx, y=nx+1
Area = 0.500
P1
P2
P3
P4
y-axis
4
2
0
−2
y = m x
y = m x + 1
y = n x
y = n x + 1
Parallelogram