

Problem 9.4.36

ee25btech11023-Venkata Sai

September 29, 2025

1 Problem

2 Solution

- Formula
- Equation
- Formula
- Finding the variables
- Finding the solutions
- Plot

3 C Code

4 Python Code

Problem

The sum of the reciprocals of Ram's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age

Formula

Variable	Description
x	Present Age of Ram
$x - 3$	Age of Ram 3 years ago
$x + 5$	Age of Ram 5 years from now

Equation

Given sum of reciprocal of Ram's ages 3 years ago and 5 years from now is $\frac{1}{3}$

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3} \quad (1.1)$$

$$\frac{(x+5) + (x-3)}{(x-3)(x+5)} = \frac{1}{3} \quad (1.2)$$

$$\frac{2x+2}{x^2+5x-3x-15} = \frac{1}{3} \quad (1.3)$$

$$\frac{2x+2}{x^2+2x-15} = \frac{1}{3} \quad (1.4)$$

$$(2x+2) = x^2+2x-15 \quad (1.5)$$

$$6x+6 = x^2+2x-15 \quad (1.6)$$

$$x^2+2x-15-6x-6=0 \quad (1.7)$$

$$x^2-4x-21=0 \quad (1.8)$$

$$\implies y = x^2 - 4x - 21 \quad (1.9)$$

Formula

which can be expressed as the conic

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (1.10)$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -2 \\ -\frac{1}{2} \end{pmatrix}, f = -21 \quad (1.11)$$

To find the roots of (9), we find the points of intersection of the conic with the x-axis

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \quad (1.12)$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (1.13)$$

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{\{\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u})\}^2 - g(\mathbf{h}) (\mathbf{m}^\top \mathbf{V} \mathbf{m})} \right) \quad (1.14)$$

Finding the variables

where

$$g(\mathbf{h}) = \mathbf{h}^\top \mathbf{V} \mathbf{h} + 2\mathbf{u}^\top \mathbf{h} + f \quad (1.15)$$

$$g(\mathbf{h}) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} -2 \\ -\frac{1}{2} \end{pmatrix}^\top \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 21 \quad (1.16)$$

$$g(\mathbf{h}) = \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} -2 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 21 \quad (1.17)$$

$$g(\mathbf{h}) = \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2(0) - 21 \quad (1.18)$$

$$g(\mathbf{h}) = 0 + 0 - 21 = -21 \quad (1.19)$$

Finding the solutions

$$\mathbf{m}^\top \mathbf{V} \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (1 \quad 0) \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = (1 \quad 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1 \quad (1.20)$$

$$\begin{aligned} \mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u}) &= \begin{pmatrix} 1 \\ 0 \end{pmatrix}^\top \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ -\frac{1}{2} \end{pmatrix} \right) \\ &= (1 \quad 0) \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ -\frac{1}{2} \end{pmatrix} \right) = (1 \quad 0) \begin{pmatrix} -2 \\ -\frac{1}{2} \end{pmatrix} = -2 \quad (1.21) \end{aligned}$$

From equation (1.14)

$$\kappa_i = \frac{1}{1} \left(-(-2) \pm \sqrt{(-2)^2 + 21} \right) \quad (1.22)$$

$$= 2 \pm \sqrt{25} = 2 \pm 5 \quad (1.23)$$

$$= 7, -3 \quad (1.24)$$

Conclusion

Hence the points of intersection are

$$\mathbf{h} + k\mathbf{m} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad (1.25)$$

Hence the solutions are $x = -3$ and $x = 7$. We reject $x = -3$ as the Age cannot be negative. Hence, the present age of Ram will be 7 years

Plot

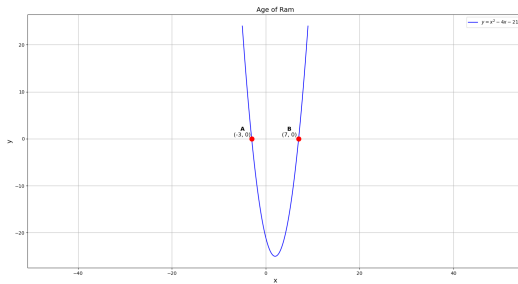


Figure:

C Code

```
#include <math.h>

void get_parabola_data(double* out_data) {
    double a = 1.0, b = -4.0, c = -21.0;
    double discriminant = sqrt(b*b - 4*a*c);
    double root1 = (-b + discriminant) / (2 * a);
    double root2 = (-b - discriminant) / (2 * a);
    out_data[0] = root1;
    out_data[1] = root2;
    int num_points = 101;
    out_data[2] = (double)num_points;
    int index = 3;
    for (int i = 0; i < num_points; i++) {
        double x = -5.0 + (14.0 * i) / (num_points - 1);
        double y = a*x*x + b*x + c;
        out_data[index] = x;
        out_data[index + 1] = y;
        index += 2;
    }
}
```

Python Code for Solving

```
import ctypes
import numpy as np

def get_data_from_c():

    lib = ctypes.CDLL('./code.so')

    data_size = 3 + 101 * 2
    double_array = ctypes.c_double * data_size
    lib.get_parabola_data.argtypes = [ctypes.POINTER(ctypes.
        c_double)]

    out_data_c = double_array()
    lib.get_parabola_data(out_data_c)

    return np.array(out_data_c)
```

Python Code for Plotting

```
# Code by /sdcard/github/matgeo/codes/CoordGeoVV Sharma
# September 12, 2023
# Revised July 21, 2024
# Released under GNU GPL
# Section Formula

import sys
sys.path.insert(0, '/workspaces/urban-potato/matgeo/codes/
    CoordGeo/')

import numpy as np
import matplotlib.pyplot as plt

from call import get_data_from_c
all_data = get_data_from_c()
num_points = int(all_data[2])
roots = all_data[0:2]
parabola_points = all_data[3:].reshape((num_points, 2))

positive_root = max(roots)
fig, ax = plt.subplots(figsize=(8, 8))
```

Python Code for Plotting

```
ax.plot(parabola_points[:, 0], parabola_points[:, 1], 'b-', label
        = '$y = x^2 - 4x - 21$')
ax.scatter(roots, [0, 0], color='red', s=100, zorder=5)
pointA = np.array([min(roots), 0])
pointB = np.array([max(roots), 0])

label_A = f'$\\mathbf{{A}}$\\n({pointA[0]:.0f}, {pointA[1]:.0f})'
ax.annotate(label_A,
            xy=pointA,
            xytext=(-20, 5),
            textcoords='offset points',
            ha='center',
            fontsize=12)

label_B = f'$\\mathbf{{B}}$\\n({pointB[0]:.0f}, {pointB[1]:.0f})'
ax.annotate(label_B,
            xy=pointB,
            xytext=(-20, 5),
            textcoords='offset points',
            ha='center',
```

Python Code for Plotting

```
    fontsize=12)
    x.set_title(Age of Ram,fontsize=12)
ax.set_xlabel(x,fontsize=12)
ax.set_ylabel('y'.fontsize=12)

ax.grid(True)
ax.axis('equal')
ax.legend(loc='best')
plt.show()
```