

Question

Find the coordinates of the foot of the perpendicular drawn from the point $\mathbf{A}(-1, 8, 4)$ to the line joining the points $\mathbf{B}(0, -1, 3)$ and $\mathbf{C}(2, -3, -1)$. Hence find the image of the point \mathbf{A} in the line BC .

Solution

Direction Vector of Line BC

$$\mathbf{d} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} - \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix} \quad (1)$$

Parametric Form of Line BC

$$\mathbf{r}(t) = \mathbf{B} + t\mathbf{d} = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix} = \begin{pmatrix} 2t \\ -1 - 2t \\ 3 - 4t \end{pmatrix} \quad (2)$$

Orthogonality Condition

$$\mathbf{r}(t) - \mathbf{A} = \begin{pmatrix} 2t + 1 \\ -2t - 9 \\ -4t - 1 \end{pmatrix} \quad (3)$$

$$(\mathbf{r}(t) - \mathbf{A}) \cdot \mathbf{d} = 0$$

$$(2t + 1)(2) + (-2t - 9)(-2) + (-4t - 1)(-4) = 0 \quad (4)$$

$$4t + 2 + 4t + 18 + 16t + 4 = 0 \quad (5)$$

$$24t + 24 = 0 \Rightarrow t = -1 \quad (6)$$

Foot of Perpendicular

$$\mathbf{r}(-1) = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix} + (-1) \begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 7 \end{pmatrix} \quad (7)$$

Image of A in Line BC

$$\mathbf{A}_{\text{image}} = 2\mathbf{r}_{\perp} - \mathbf{A} = 2 \begin{pmatrix} -2 \\ 1 \\ 7 \end{pmatrix} - \begin{pmatrix} -1 \\ 8 \\ 4 \end{pmatrix} \quad (8)$$

$$= \begin{pmatrix} -4 \\ 2 \\ 14 \end{pmatrix} + \begin{pmatrix} 1 \\ -8 \\ -4 \end{pmatrix} = \begin{pmatrix} -3 \\ -6 \\ 10 \end{pmatrix} \quad (9)$$

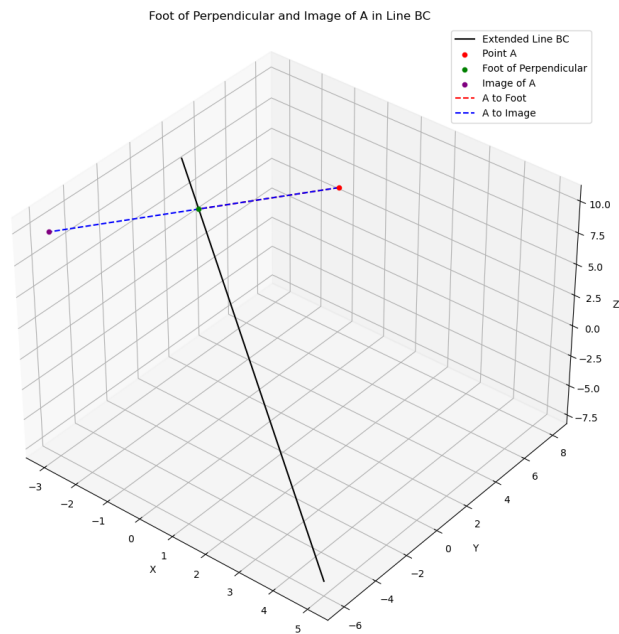


Figure 1