4.2.12

EE25BTECH11009 - Anshu Kumar Ram

Question:
Find the direction and normal vector for the line

$$3 = 2x + y \tag{0.1}$$

Solution:

The line can be written as:

$$2x + y = 3 \tag{0.2}$$

This equation can be expressed in terms of matrices
Let

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{0.3}$$

$$\mathbf{n^T} = \begin{pmatrix} 2 & 1 \end{pmatrix} \tag{0.4}$$

$$c = 3 \tag{0.5}$$

The line equation can be written as:

$$\mathbf{n^T x} = c \tag{0.6}$$

Where $\mathbf{n}$ is the normal vector of the given line.
The direction vector of the line can be found by observing the normal vector.

$$\mathbf{m} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \tag{0.7}$$

This is true because if the direction vector is represented as

$$\mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{0.8}$$

then the normal vector can be represented as

$$\mathbf{n} = \begin{pmatrix} -m \\ 1 \end{pmatrix} \tag{0.9}$$

This can be verified by the following equation:

$$\mathbf{n^T m} = 0 \tag{0.10}$$

$$\begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \end{pmatrix} = 0 \tag{0.11}$$

The normal vector of the line is $\mathbf{n} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

The direction vector of the line is $\mathbf{m} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$