10.2.7

EE25BTECH11001 - Aarush Dilawri

October 12, 2025

Question

Question:
At what points on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangents are parallel to the y-axis?

Solution

Solution:

$$\text{The general equation of a conic is } \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

Comparing with $x^2 + y^2 - 2x - 4y + 1 = 0$, we get

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \quad f = 1 \tag{2}$$

Solution

The centre and radius of the circle are given by

$$\mathbf{c} = -\mathbf{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad r = \sqrt{\left\|\mathbf{u}\right\|^2 - f} = \sqrt{(-1)^2 + (-2)^2 - 1} = \sqrt{4} = 2 \tag{3}$$

The points of contact of the tangent are given by

$$\mathbf{q}_{i,j} = \pm r \frac{\mathbf{n}_j}{\left\|\mathbf{n}_j\right\|} - \mathbf{u} \quad i, j = 1, 2 \tag{4}$$

Solution

Since the tangents are parallel to the y-axis, the normal vector is

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{5}$$

Substituting in (0.4),

$$\mathbf{q}_{1,1} = \pm 2 \frac{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}{\left\| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\|} - \begin{pmatrix} -1 \\ -2 \end{pmatrix} \tag{6}$$

$$= \pm 2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{7}$$

Solution

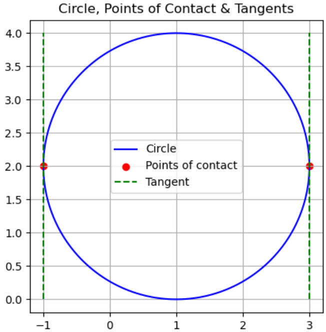
Therefore, the two points of contact are

$$\mathbf{q}_1 = \begin{pmatrix} 1+2 \\ 2+0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{8}$$

$$\mathbf{q}_2 = \begin{pmatrix} 1-2 \\ 2+0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \tag{9}$$

Hence, the required points are $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

Figure



Circle, Points of Contact & Tangents
Circle
Points of contact
Tangent
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−1
0
1
2
3

Codes

`https://github.com/AarushDilawri/ee1030-2025/tree/main/ee25btech11001/MATGEO/10.2.7/codes`