2.10.65

EE25BTECH11036 - M Chanakya Srinivas

Problem Statement

Let $OACB$ be a parallelogram with O at the origin and OC a diagonal. Let D be the midpoint of OA. Using vector methods, prove that BD and CO intersect in the same ratio. Determine this ratio.

Solution

Step 1: Define position vectors of vertices

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{1}$$

$$\mathbf{A} = \mathbf{a}, \tag{2}$$

$$\mathbf{C} = \mathbf{c}, \tag{3}$$

$$\mathbf{B} = \mathbf{A} + \mathbf{C}. \tag{4}$$

Midpoint D of OA:

$$\mathbf{D} = \frac{\mathbf{O} + \mathbf{A}}{2} = \frac{1}{2}\mathbf{A}. \tag{5}$$

Step 2: Represent lines in vector form

Line BD:

$$\mathbf{R_1} = \mathbf{B} + \lambda(\mathbf{D} - \mathbf{B}) \tag{6}$$

$$= \mathbf{B} + \lambda\left(\frac{1}{2}\mathbf{A} - (\mathbf{A} + \mathbf{C})\right) \tag{7}$$

$$= \mathbf{B} - \lambda\left(\frac{1}{2}\mathbf{A} + \mathbf{C}\right) \tag{8}$$

Line CO:

$$\mathbf{R_2} = \mathbf{C} + \mu(\mathbf{O} - \mathbf{C}) \tag{9}$$

$$= \mathbf{C} - \mu\mathbf{C} = (1 - \mu)\mathbf{C} \tag{10}$$

Step 3: Intersection condition

$$\mathbf{R_1} = \mathbf{R_2} \quad \Rightarrow \quad \mathbf{B} - \lambda\left(\frac{1}{2}\mathbf{A} + \mathbf{C}\right) = (1 - \mu)\mathbf{C}$$

Substitute $\mathbf{B} = \mathbf{A} + \mathbf{C}$:

$$\mathbf{A} + \mathbf{C} - \lambda\left(\frac{1}{2}\mathbf{A} + \mathbf{C}\right) = (1 - \mu)\mathbf{C} \tag{11}$$

Step 4: Equate coefficients of $\mathbf{A}$ *and* $\mathbf{C}$

$$\text{Coefficient of } \mathbf{A}: \quad 1 - \frac{\lambda}{2} = 0 \quad \Rightarrow \quad \lambda = 2 \tag{12}$$

$$\text{Coefficient of } \mathbf{C}: \quad 1 - \lambda = 1 - \mu \quad \Rightarrow \quad \mu = 2 \tag{13}$$

Step 5: Interpret the ratio

- On BD, $\lambda = 2$ implies the intersection divides BD in ratio $2:1$. - On CO, $\mu = 2$ implies the intersection divides CO in ratio $2:1$.

$$\boxed{\text{The lines } BD \text{ and } CO \text{ intersect in the ratio } 2:1.} \tag{14}$$

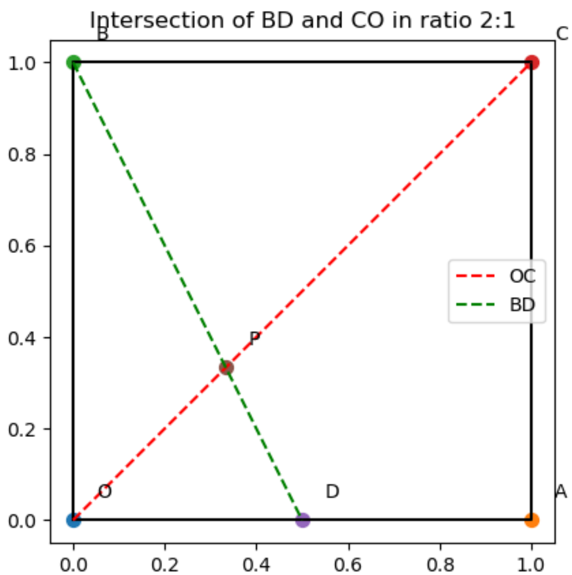

Fig. 1

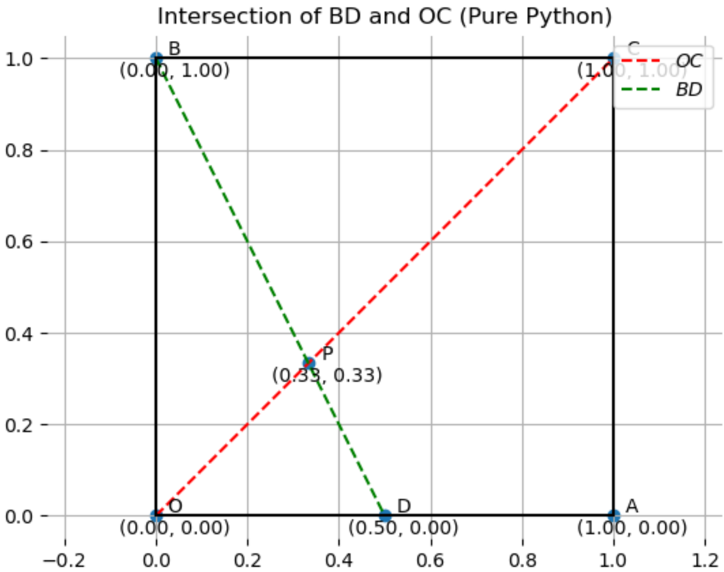
Intersection of BD and OC (Pure Python)
B
(0.00, 1.00)
C
(1.00, 1.00)
OC
BD
P
(0.33, 0.33)
O
(0.00, 0.00)
D
(0.50, 0.00)
A
(1.00, 0.00)
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2

Fig. 2