5.3.36

EE25BTECH11002 - Achat Parth Kalpesh

Question:
Solve the system of equations

$$\frac{bx}{a} - \frac{ay}{b} + a + b = 0 \tag{0.1}$$
$$bx - ay + 2ab = 0 \tag{0.2}$$

Solution:
The above equation can be written as

$$\mathbf{n_1}^\top \mathbf{x} = c_1 \tag{0.3}$$
$$\mathbf{n_2}^\top \mathbf{x} = c_2 \tag{0.4}$$
$$\begin{pmatrix} \mathbf{n_1} \\ \mathbf{n_2} \end{pmatrix}^\top \mathbf{x} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \tag{0.5}$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{n_1} \\ \mathbf{n_2} \end{pmatrix}^\top \quad \mathbf{b} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \tag{0.6}$$

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{0.7}$$
$$\begin{pmatrix} \frac{b}{a} & -\frac{a}{b} \\ b & -a \end{pmatrix} \mathbf{x} = \begin{pmatrix} -a-b \\ -2ab \end{pmatrix} \tag{0.8}$$

$$\mathbf{A}^\top \mathbf{A} \neq \mathbf{I} \tag{0.9}$$

Performing row operations:

$$\left(\begin{array}{cc|c} \frac{b}{a} & -\frac{a}{b} & -a-b \\ b & -a & -2ab \end{array}\right) \xleftrightarrow{R_1 \leftarrow R_1 - \frac{R_2}{b}} \left(\begin{array}{cc|c} \frac{b-a}{a} & 0 & a-b \\ b & -a & -2ab \end{array}\right) \tag{0.10}$$
$$\left(\begin{array}{cc|c} \frac{b-a}{a} & 0 & a-b \\ b & -a & -2ab \end{array}\right) \xleftrightarrow{R_2 \leftarrow -\frac{ab}{b-a} R_1 + R_2} \left(\begin{array}{cc|c} \frac{b-a}{a} & 0 & a-b \\ 0 & -a & -ab \end{array}\right) \tag{0.11}$$
$$\left(\begin{array}{cc|c} \frac{b-a}{a} & 0 & a-b \\ 0 & -a & -ab \end{array}\right) \xleftrightarrow{R_2 \leftarrow -\frac{R_2}{a}} \left(\begin{array}{cc|c} \frac{b-a}{a} & 0 & a-b \\ 0 & 1 & b \end{array}\right) \tag{0.12}$$
$$\left(\begin{array}{cc|c} \frac{b-a}{a} & 0 & a-b \\ 0 & 1 & -b \end{array}\right) \xleftrightarrow{R_1 \leftarrow \frac{a}{b-a} R_1} \left(\begin{array}{cc|c} 1 & 0 & -a \\ 0 & 1 & b \end{array}\right) \tag{0.13}$$

Thus,

$$\mathbf{x} = \begin{pmatrix} -a \\ b \end{pmatrix} \tag{0.14}$$

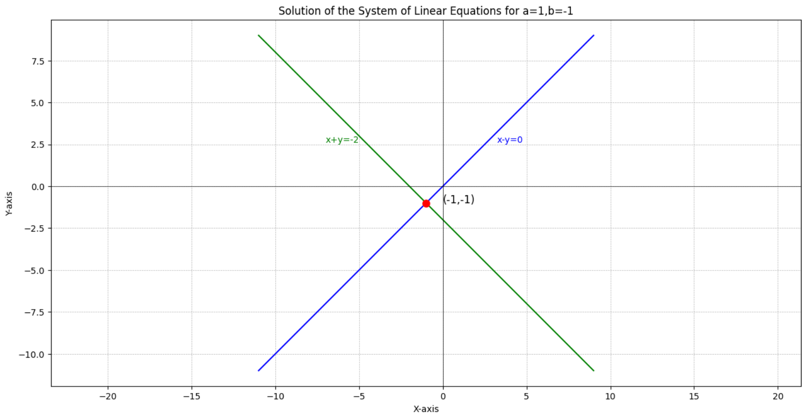
Solution of the System of Linear Equations for a=1,b=-1
x+y=-2
x-y=0
(-1,-1)
Y-axis
X-axis
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−20
−15
−10
−5
0
5
10
15
20

Fig. 0.1: Graph