Matgeo Presentation - Problem 1.6.6

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Problem Statement

In each of the following, find the value of k for which the points are collinear:

$(7, -2),\ (5, 1),\ (3, k)$
$(8, 1),\ (k, -4),\ (2, -5)$

Method

Condition for Collinearity:

Three points A, B, C are collinear iff vectors $\mathbf{B} - \mathbf{A},\ \mathbf{C} - \mathbf{A}$ are linearly dependent.

Equivalently, the collinearity matrix

$$M = \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix}^\top$$

must satisfy $\text{rank}(M) = 1$.

Part (a) Setup

Let

$$A = \begin{pmatrix} 7 \\ -2 \end{pmatrix}, \quad B = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 3 \\ k \end{pmatrix}.$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 5-7 \\ 1-(-2) \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}, \quad \mathbf{C} - \mathbf{A} = \begin{pmatrix} 3-7 \\ k-(-2) \end{pmatrix} = \begin{pmatrix} -4 \\ k+2 \end{pmatrix}.$$

Thus,

$$M = \begin{pmatrix} -2 & 3 \\ -4 & k+2 \end{pmatrix}.$$

Part (a) Row Reduction

Apply row transformation:

$$R_2 = R_2 - 2R_1 \;\Rightarrow\; \begin{pmatrix} -2 & 3 \\ 0 & k-4 \end{pmatrix}.$$

For collinearity: $k - 4 = 0 \Rightarrow k = \boxed{4}$.

Part (b) Setup

Let

$$A = \begin{pmatrix} 8 \\ 1 \end{pmatrix}, \quad B = \begin{pmatrix} k \\ -4 \end{pmatrix}, \quad C = \begin{pmatrix} 2 \\ -5 \end{pmatrix}.$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} k-8 \\ -5 \end{pmatrix}, \quad \mathbf{C} - \mathbf{A} = \begin{pmatrix} -6 \\ -6 \end{pmatrix}.$$

Thus,

$$M = \begin{pmatrix} k-8 & -5 \\ -6 & -6 \end{pmatrix}.$$

Part (b) Row Reduction

Row operation:

$$R_2 = (k-8)R_2 + 6R_1 \;\Rightarrow\; \begin{pmatrix} k-8 & -5 \\ 0 & 18-6k \end{pmatrix}.$$

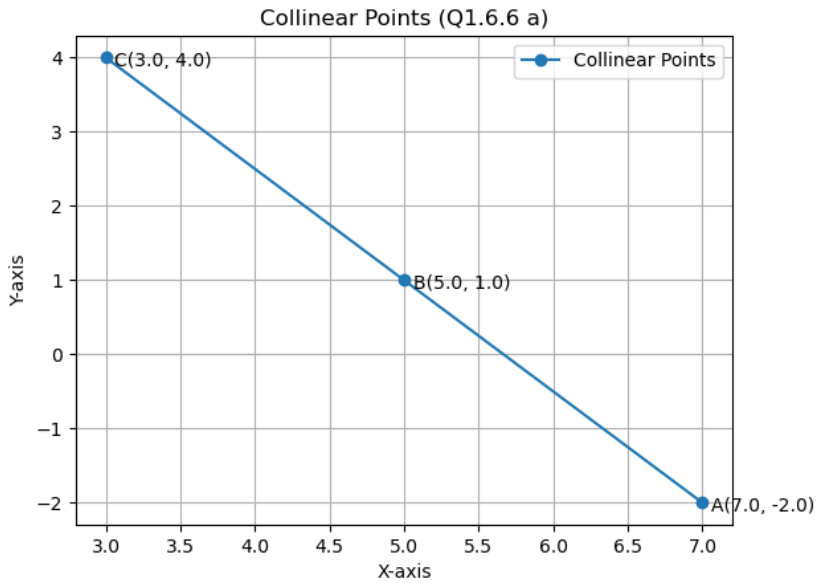
For collinearity: $18 - 6k = 0 \Rightarrow k = \boxed{3}$.

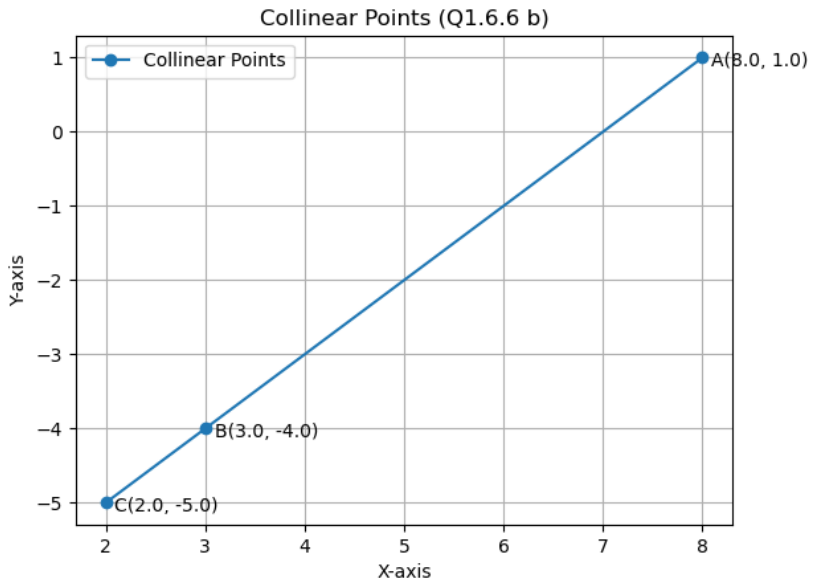
Final Answer

(a) $k = 4$

(b) $k = 3$

Plots


Collinear Points (Q1.6.6 a)
Collinear Points
C(3.0, 4.0)
B(5.0, 1.0)
A(7.0, -2.0)
Y-axis
X-axis
4
3
2
1
0
−1
−2
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0


Collinear Points (Q1.6.6 b)
Collinear Points
A(8.0, 1.0)
B(3.0, -4.0)
C(2.0, -5.0)
Y-axis
X-axis
1
0
−1
−2
−3
−4
−5
2
3
4
5
6
7
8

C Code: points.c (Part 1)

```c
#include <stdio.h>

int main() {
  FILE *fp;

  // Question 1.6.6 (a)
  int k_a = 4; // Final answer
  printf("Q1.6.6␣(a):␣k␣=␣%d\n", k_a);

  fp = fopen("points_a.dat", "w");
  fprintf(fp, "%d,%d,%d\n", 7, -2, 0); // A
  fprintf(fp, "%d,%d,%d\n", 5, 1, 0); // B
  fprintf(fp, "%d,%d,%d\n", 3, k_a, 0); // C
  fclose(fp);
```

C Code: points.c (Part 2)

```
  // Question 1.6.6 (b)
  int k_b = 3; // Final answer
  printf("Q1.6.6␣(b):␣k␣=␣%d\n", k_b);

  fp = fopen("points_b.dat", "w");
  fprintf(fp, "%d,%d,%d\n", 8, 1, 0); // A
  fprintf(fp, "%d,%d,%d\n", k_b, -4, 0); // B
  fprintf(fp, "%d,%d,%d\n", 2, -5, 0); // C
  fclose(fp);

  return 0;
}
```

Python: call_c.py

```
import subprocess

# Compile the C program
subprocess.run(["gcc", "points.c", "-o", "points"], check=True)

# Run the compiled C program
result = subprocess.run(
    ["./points"], capture_output=True, text=True, check=True
)

# Print the output from the C program
print(result.stdout)
```

Python: plot.py (Part 1)

```
import numpy as np
import matplotlib.pyplot as plt

def plot_points(filename, labels, title, output_file):
    points = np.loadtxt(filename, delimiter=',', usecols=(0,1))
    x = points[:,0]
    y = points[:,1]

    plt.plot(x, y, 'o-', label='Collinear Points')

    for i, txt in enumerate(labels):
        plt.annotate(f"{txt}{tuple(points[i])}", (x[i], y[i]),
                     xytext=(5,-5), textcoords="offset points")
```

Python: plot.py (Part 2)

```
    plt.xlabel("X-axis")
    plt.ylabel("Y-axis")
    plt.title(title)
    plt.legend()
    plt.grid(True)

    plt.savefig(output_file)
    print(f"Saved␣figure␣as␣{output_file}")
    plt.close()

# Part (a)
plot_points("points_a.dat", ["A","B","C"],
           "Collinear␣Points␣(Q1.6.6␣a)", "fig_a.png")

# Part (b)
plot_points("points_b.dat", ["A","B","C"],
           "Collinear␣Points␣(Q1.6.6␣b)", "fig_b.png")
```