12.683

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Question

The unit normal vector to the surface $X^2 + Y^2 + Z^2 - 48 = 0$ at the point (4, 4, 4) is _____.

Theoretical Solution

Let the surface be defined by the level set of a function $f(\mathbf{x}) = 0$.

$$f(X, Y, Z) = X^2 + Y^2 + Z^2 - 48 \tag{1}$$

The normal vector to the surface at any point is given by the gradient of the function, ∇f.

$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial X} \\ \frac{\partial f}{\partial Y} \\ \frac{\partial f}{\partial Z} \end{pmatrix} = \begin{pmatrix} 2X \\ 2Y \\ 2Z \end{pmatrix} \tag{2}$$

Theoretical Solution

Substituting

$$\mathbf{p} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix} \tag{3}$$

in (2)

$$\mathbf{n} = \nabla f(\mathbf{p}) = \begin{pmatrix} 2(4) \\ 2(4) \\ 2(4) \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \\ 8 \end{pmatrix} \tag{4}$$

Theoretical Solution

To find the unit normal vector $\hat{\mathbf{n}}$,

$$\|\mathbf{n}\| = \sqrt{8^2 + 8^2 + 8^2} = \sqrt{3 \times 64} = 8\sqrt{3} \tag{5}$$

$$\hat{\mathbf{n}} = \frac{\mathbf{n}}{\|\mathbf{n}\|} = \frac{1}{8\sqrt{3}} \begin{pmatrix} 8 \\ 8 \\ 8 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{6}$$

Plot

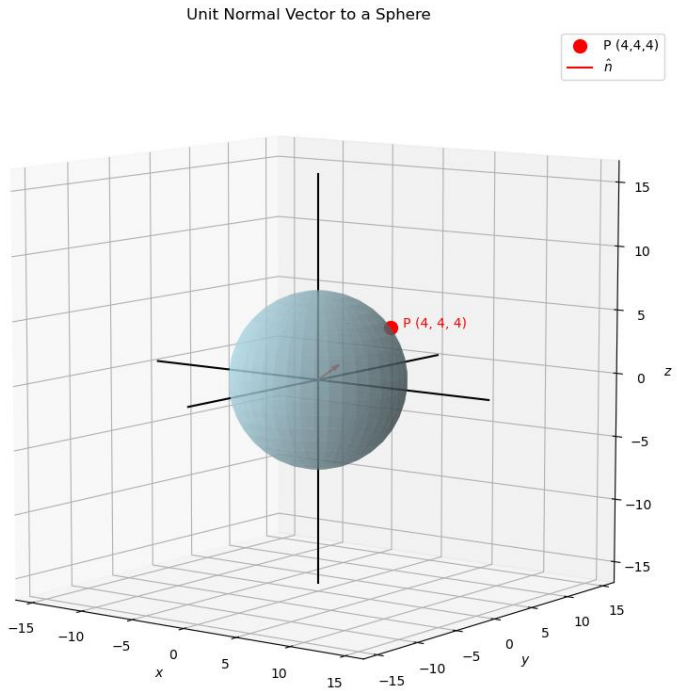

Figure: Plot