2.9.1

EE25BTECH11050-Hema Havil

Question:
Jagdish has a field which is in the shape of a right-angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining space for growing vegetables. In the field, there is a pole marked as O. Based on the above information, answer the following equations

a) Taking O as the origin, P = (-200, 0) and Q = (200, 0). PQRS being a square, what are the coordinates of R and S?
b) i) What is the area of square PQRS ?
   ii) What is the length of diagonal PR in PQRS ?
c) If S divides CA in the ratio K : 1, what is the value of K, where A = (200, 800)?

Solution:
Given that,
AQC is a right angled triangle at point Q and PQRS is a square inside the ΔAQC,

(a) We were given two points

$$P = (-200, 0), Q = (200, 0) \tag{(a).1}$$

Let,
X be the vector along the side PQ,
Y be the vector along the side QR,
Z be the vector along the side PS then,

$$\mathbf{X} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} 200 \\ 0 \end{pmatrix} - \begin{pmatrix} -200 \\ 0 \end{pmatrix} \tag{(a).2}$$

$$\mathbf{X} = \begin{pmatrix} 400 \\ 0 \end{pmatrix} \tag{(a).3}$$

Rotation vector for 2x2 matrix is

$$\mathbf{R}_\theta = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \tag{(a).4}$$

Rotate the vector **X** by 90° anticlockwise to get Y

$$\mathbf{Y} = \mathbf{R_{90}}\mathbf{X} \tag{(a).5}$$

$$\mathbf{Y} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 400 \\ 0 \end{pmatrix} \tag{(a).6}$$

$$\mathbf{Y} = \begin{pmatrix} 0 \\ 400 \end{pmatrix} \tag{(a).7}$$

So the vector along the side QR is $\mathbf{Y} = \begin{pmatrix} 0 \\ 400 \end{pmatrix}$ then,

$$\mathbf{Y} = \mathbf{R} - \mathbf{Q} \tag{(a).8}$$

$$\mathbf{R} = \mathbf{Y} + \mathbf{Q} \tag{(a).9}$$

$$\mathbf{R} = \begin{pmatrix} 0 \\ 400 \end{pmatrix} + \begin{pmatrix} 200 \\ 0 \end{pmatrix} \tag{(a).10}$$

$$\mathbf{R} = \begin{pmatrix} 200 \\ 400 \end{pmatrix} \tag{(a).11}$$

Since the sides QR and PS are parallel, vectors $\mathbf{Y} = \mathbf{Z}$ then

$$\mathbf{Z} = \mathbf{S} - \mathbf{P} \tag{(a).12}$$

$$\mathbf{S} = \mathbf{Z} + \mathbf{P} \tag{(a).13}$$

$$\mathbf{S} = \begin{pmatrix} 0 \\ 400 \end{pmatrix} + \begin{pmatrix} -200 \\ 0 \end{pmatrix} \tag{(a).14}$$

$$\mathbf{S} = \begin{pmatrix} -200 \\ 400 \end{pmatrix} \tag{(a).15}$$

Therefore the coordinates of the points R and S are (200,400) and (-200,400)

(b)(i) We know the points P(-200,0) and Q(200,0)
Let length of the side of the square PQRS be x then,

$$x = \|\mathbf{Q} - \mathbf{P}\| \tag{(b).1}$$

$$x = \left\| \begin{pmatrix} 400 \\ 0 \end{pmatrix} \right\| = 400 \tag{(b).2}$$

Area of the square $= x^2 = (400)^2 = 160000$ sq units

(ii) Length of diagnol of the square $= x\sqrt{2} = 400\sqrt{2}$ units

(c) Given the point A=(200,800)
Since it was given that point S divides CA in the ratio K:1, this shows that points A,C and S are collinear. Since AQC is a right angled triangle, from this we can say that point C lies on X axis
Let point C be (t,0), Consider the matrix M

$$M = \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} \tag{(c).1}$$

Where the points in the matrix are A(200,800),S(-200,400) and C(t,0) then, substitute in ((c).1)

$$M = \begin{pmatrix} 200 & 800 & 1 \\ -200 & 400 & 1 \\ t & 0 & 1 \end{pmatrix} \tag{(c).2}$$

For the points to be collinear rank of matrix M should be equal to 2,
By applying row transformations,

$R_1 \rightarrow \frac{1}{200}R_1$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ -200 & 400 & 1 \\ t & 0 & 1 \end{pmatrix} \tag{(c).3}$$

$R_2 \rightarrow R_2 + 200R_1$ $\qquad$ $R_3 \rightarrow R_3 - tR_1$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ 0 & 1200 & 2 \\ 0 & -4t & 1 - \frac{t}{200} \end{pmatrix} \tag{(c).4}$$

$R_2 \rightarrow \frac{1}{200}R_2$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ 0 & 1 & \frac{1}{600} \\ 0 & -4t & 1 - \frac{t}{200} \end{pmatrix} \tag{(c).5}$$

$R_3 \rightarrow R_3 + 4tR_2$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ 0 & 1 & \frac{1}{600} \\ 0 & 0 & 1 - \frac{t}{200} + \frac{4t}{600} \end{pmatrix} \tag{(c).6}$$

Since the rank of matrix is 2,

$$1 - \frac{t}{200} + \frac{4t}{600} = 0 \tag{(c).7}$$

$$1 + \frac{t}{600} = 0 \tag{(c).8}$$

$$\frac{t}{600} = -1 \tag{(c).9}$$

$$t = -600 \tag{(c).10}$$

Therefore point C=(-600,0), Now S divides CA in the ratio K:1,

$$S = \frac{KA + C}{K + 1} \tag{(c).11}$$

$$K = \frac{(S - A)^T (C - S)}{\|S - A\|^2} \tag{(c).12}$$

$$K = \frac{1}{(400)^2 + (400)^2} \begin{pmatrix} -400 & -400 \end{pmatrix} \begin{pmatrix} -400 \\ -400 \end{pmatrix} \tag{(c).13}$$

By solving ((c).13) we get K=1

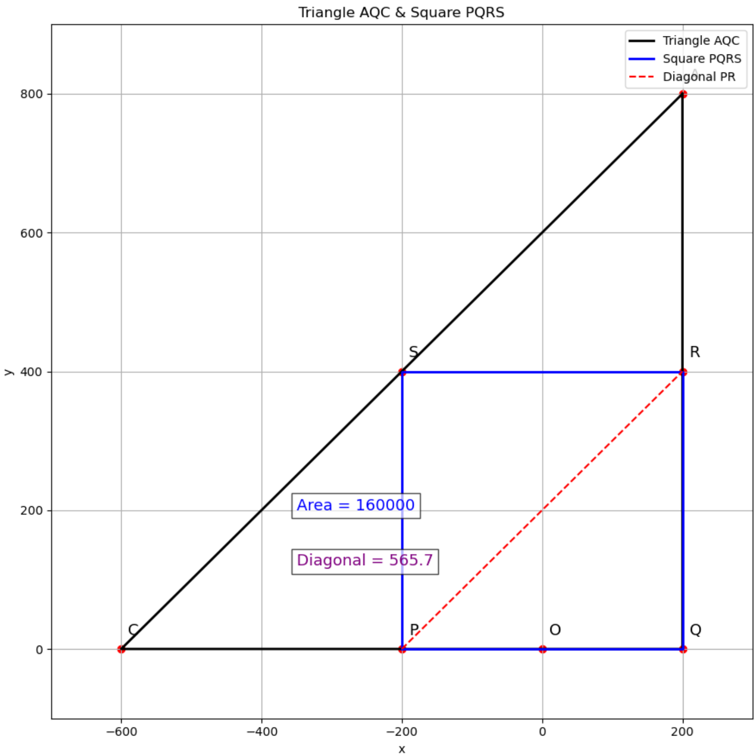

Fig. 3.1: Plot of the square and triangle