4.12.17

AI25BTECH11012 - GARIGE UNNATHI

Question:
P1,**P2** are points on either of the two lines y - $\sqrt{3}|x|$ = 2 at a distance of 5 units from their point of intersection. Find the coordinates of the foot of the perpendiculars drawn from **P1**,**P2** on the bisector of the angle between the given lines.

Solution:

The equation of the lines is :

$$y - \sqrt{3}x = \begin{pmatrix} -\sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 2 \tag{0.1}$$

$$y + \sqrt{3}x = \begin{pmatrix} \sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 2 \tag{0.2}$$

Combining both the equations 0.1 and 0.2 ,we get :

$$\begin{pmatrix} -\sqrt{3} & 1 \\ \sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \tag{0.3}$$

Solving by row reduction we get :

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{0.4}$$

The equation for the point $\mathbf{P_1}$ are:

$$\begin{pmatrix} -\sqrt{3} & 1 \end{pmatrix}\mathbf{P_1} = 2 \tag{0.5}$$

$$\|x \quad y - 2\| = 5 \tag{0.6}$$

The equation for the point $\mathbf{P_2}$ are:

$$\begin{pmatrix} \sqrt{3} & 1 \end{pmatrix}\mathbf{P_1} = 2 \tag{0.7}$$

$$\|x \quad y - 2\| = 5 \tag{0.8}$$

Solving the equations we get :

$$\mathbf{P_1} = \begin{pmatrix} \frac{5}{2} \\ 2 + \frac{5\sqrt{3}}{2} \end{pmatrix} \tag{0.9}$$

$$\mathbf{P_2} = \begin{pmatrix} -\frac{5}{2} \\ 2 - \frac{5\sqrt{3}}{2} \end{pmatrix} \tag{0.10}$$

The equation of the angle bisector is given by
Let us take a point $\mathbf{P}$ on the angle bisector , substitution it in the line equtions and equating the angles we get the equation :

$$\frac{|n_1\mathbf{P}-2|}{||n_1||} = \frac{|n_2\mathbf{P}-2|}{||n_1||} \tag{0.11}$$

$$\frac{n_1\mathbf{P}-2}{||n_1||} \pm \frac{n_2\mathbf{P}-2}{||n_1||} = 0 \tag{0.12}$$

solving the above equation we get locus of $\mathbf{P}$ as two lines which are the angle bisectors :

$$\begin{pmatrix}1\\0\end{pmatrix}^T \mathbf{x} = 0 \tag{0.13}$$

$$\begin{pmatrix}0\\1\end{pmatrix}^T \mathbf{x} = 2 \tag{0.14}$$

Let Q be the foot of the perpendicular from P to the line

$$\mathbf{n}^T\mathbf{x} = c \tag{0.15}$$

Then :

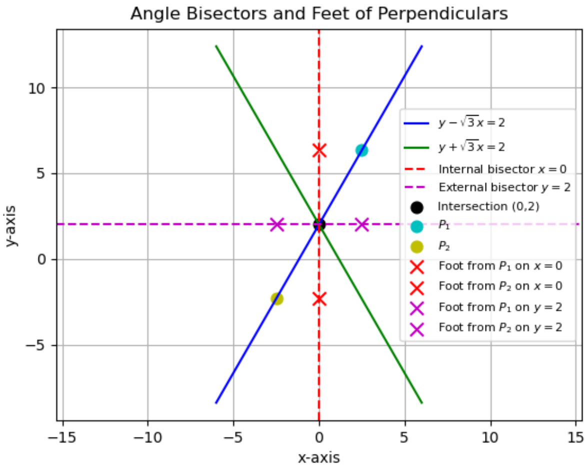
$$\begin{pmatrix}\mathbf{m} & \mathbf{n}\end{pmatrix}^T \mathbf{Q} = \begin{pmatrix}\mathbf{m}^T\mathbf{P}\\ c\end{pmatrix} \tag{0.16}$$

solving this equation for the line $\begin{pmatrix}1\\0\end{pmatrix}^T \mathbf{x} = 0$,we get :

$$\begin{pmatrix}0\\2+\frac{5\sqrt{3}}{2}\end{pmatrix} \quad and \quad \begin{pmatrix}0\\2-\frac{5\sqrt{3}}{2}\end{pmatrix} \tag{0.17}$$

and solving it for the line $\begin{pmatrix}0\\1\end{pmatrix}^T \mathbf{x} = 2$, we get :

$$\begin{pmatrix}\frac{5}{2}\\2\end{pmatrix} \quad and \quad \begin{pmatrix}-\frac{5}{2}\\2\end{pmatrix} \tag{0.18}$$



Angle Bisectors and Feet of Perpendiculars
$y-\sqrt{3}x=2$
$y+\sqrt{3}x=2$
Internal bisector $x=0$
External bisector $y=2$
Intersection (0,2)
P_1
P_2
Foot from P_1 on $x=0$
Foot from P_2 on $x=0$
Foot from P_1 on $y=2$
Foot from P_2 on $y=2$
y-axis
x-axis

Fig. 0.1