

4.6.8

EE25BTECH11020 - Darsh Pankaj Gajare

Question:

Find the equation of the plane passing through the points $(2, 2, -1)$, $(3, 4, 2)$ and $(7, 0, 6)$. Also find the equation of the plane passing through $(4, 3, 1)$ and parallel to the plane obtained above.

Solution:

TABLE I

A	$\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$
B	$\begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$
C	$\begin{pmatrix} 7 \\ 0 \\ 6 \end{pmatrix}$
D	$\begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$

Let the equation of plane be

$$\mathbf{n}^T \mathbf{x} = C_1 \quad (1)$$

A,B,C satisfies this equation,

$$\mathbf{n}^T \mathbf{A} = C_1, \mathbf{n}^T \mathbf{B} = C_1, \mathbf{n}^T \mathbf{C} = C_1 \quad (2)$$

$$\begin{pmatrix} \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \end{pmatrix}^T \mathbf{n} = \begin{pmatrix} C_1 \\ C_1 \\ C_1 \end{pmatrix} \quad (3)$$

Using augmented matrix,

$$\left(\begin{array}{ccc|c} 2 & 2 & -1 & C_1 \\ 3 & 4 & 2 & C_1 \\ 7 & 0 & 6 & C_1 \end{array} \right) \quad (4)$$

$$R_1 = R_1/2$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -0.5 & \frac{C_1}{2} \\ 3 & 4 & 2 & C_1 \\ 7 & 0 & 6 & C_1 \end{array} \right) \quad (5)$$

$$R_2 = R_2 - 3R_1$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -0.5 & \frac{C_1}{2} \\ 0 & 1 & 3.5 & \frac{-C_1}{2} \\ 7 & 0 & 6 & C_1 \end{array} \right) \quad (6)$$

$$R_3 = R_3 - 7R_1$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -0.5 & \frac{C_1}{2} \\ 0 & 1 & 3.5 & \frac{-C_1}{2} \\ 0 & -7 & 9.5 & \frac{-5C_1}{2} \end{array} \right) \quad (7)$$

$$R_3 = R_3 + 7R_2$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -0.5 & \frac{C_1}{2} \\ 0 & 1 & 3.5 & \frac{-C_1}{2} \\ 0 & 0 & 34 & -6C_1 \end{array} \right) \quad (8)$$

$$34z + 6C_1 = 0 \implies z = \frac{-3C_1}{17} \quad (9)$$

$$y + 3.5z + 0.5C_1 = 0 \implies y = \frac{2C_1}{17} \quad (10)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -0.5 & \frac{C_1}{2} \\ 0 & 1 & 3.5 & \frac{-C_1}{2} \\ 0 & -7 & 9.5 & \frac{-5C_1}{2} \end{array} \right) \quad (11)$$

$$x + y - 0.5z = 0.5C_1 \implies x = \frac{5C_1}{17} \quad (12)$$

Let $C_1 = 17$

$$\mathbf{n} = \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}, C_1 = 17 \quad (13)$$

Equation of plane parallel to given plane passing through D,

$$\mathbf{n}^T \mathbf{x} = C_2 \quad (14)$$

$$\mathbf{n}^T \mathbf{D} = C_2 \quad (15)$$

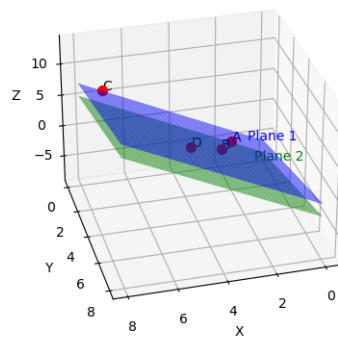
$$C_2 = (5 \quad 2 \quad -3) \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} \quad (16)$$

$$C_2 = 23 \quad (17)$$

Equation of plane

$$\begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}^T \mathbf{x} = 23 \quad (18)$$

Plot using C libraries:



Plot using Python:

