4.7.64

AI25BTECH11003 - Bhavesh Gaikwad

Question: Find the distance between the point **P**(6, 5, 9) and the plane determined by the points **A**(3, -1, 2), **B**(5, 2, 4) and **C**(-1, -1, 6).

Solution:
Given:

$$P = \begin{pmatrix} 6 \\ 5 \\ 9 \end{pmatrix}, A = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, B = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}, C = \begin{pmatrix} -1 \\ -1 \\ 6 \end{pmatrix}. \tag{0.1}$$

First, form two direction vectors on the plane using the given points.

$$\mathbf{u} = B - A = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}, \qquad \mathbf{v} = C - A = \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix}. \tag{0.2}$$

Next, find a nonzero normal $\mathbf{n}$ orthogonal to both $\mathbf{u}$ and $\mathbf{v}$ by solving the homogeneous system.

$$\begin{pmatrix} 2 & 3 & 2 \\ -4 & 0 & 4 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{0.3}$$

Row-reduce and solve for a convenient integer solution.

$$\begin{pmatrix} 2 & 3 & 2 \\ -4 & 0 & 4 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 + 2R_1} \begin{pmatrix} 2 & 3 & 2 \\ 0 & 6 & 8 \end{pmatrix}, \tag{0.4}$$

$$6n_2 + 8n_3 = 0 \implies 3n_2 + 4n_3 = 0, \quad n_3 = 3, \; n_2 = -4, \; 2n_1 + 3n_2 + 2n_3 = 0 \implies n_1 = 3, \tag{0.5}$$

$$\therefore \mathbf{n} = \begin{pmatrix} 3 \\ -4 \\ 3 \end{pmatrix}. \tag{0.6}$$

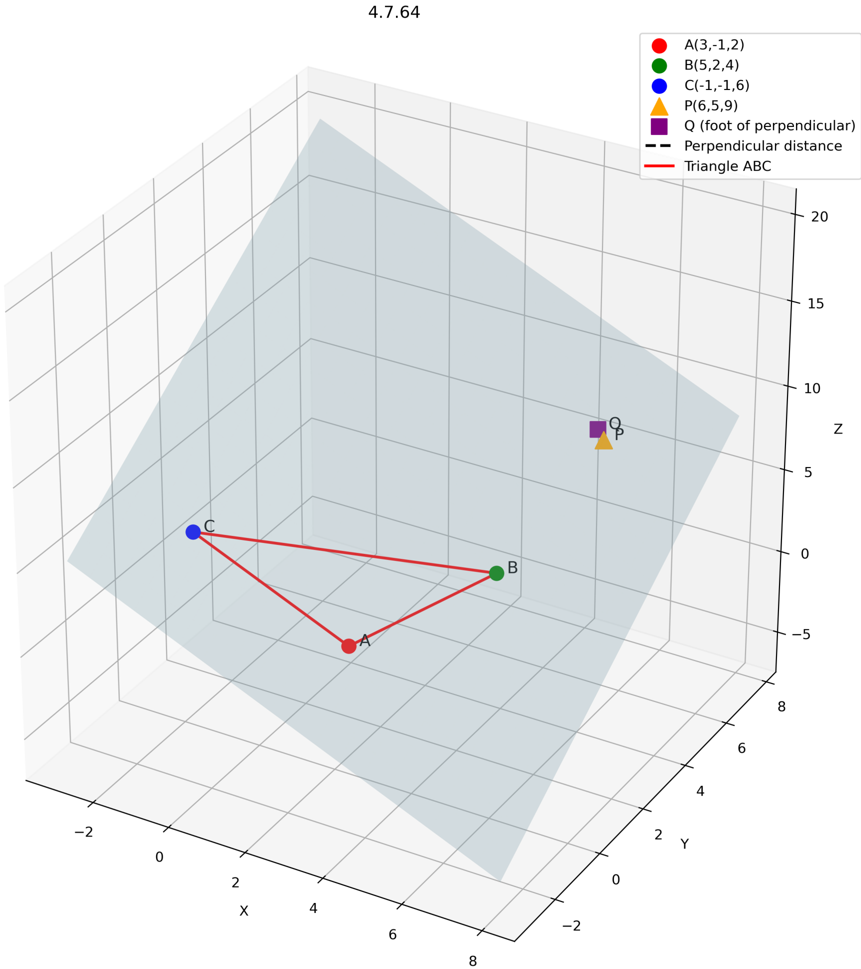
Writing the plane in normal form $\mathbf{n}^\top x = c$ with $c = \mathbf{n}^\top A$.

$$c = \mathbf{n}^\top A = 3 \cdot 3 + (-4)(-1) + 3 \cdot 2 = 19. \tag{0.7}$$

Finally, applying the point-to-plane distance formula and simplify.

$$d = \frac{\left|\mathbf{n}^\top P - c\right|}{||\mathbf{n}||} \tag{0.8}$$
$$= \frac{|3 \cdot 6 + (-4) \cdot 5 + 3 \cdot 9 - 19|}{\sqrt{3^2 + (-4)^2 + 3^2}} \tag{0.9}$$
$$= \frac{|25 - 19|}{\sqrt{34}} \tag{0.10}$$
$$= \frac{6}{\sqrt{34}} = \frac{3\sqrt{34}}{17}. \tag{0.11}$$

$$\boxed{\text{The Distance between the Plane and } \mathbf{P}\, is\, \frac{3\sqrt{34}}{17}\, units.} \tag{0.12}$$



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A(3,-1,2)
B(5,2,4)
C(-1,-1,6)
P(6,5,9)
Q (foot of perpendicular)
Perpendicular distance
Triangle ABC
A
B
C
P
Q
X
Y
Z

Fig. 0.1: Plane