4.13.30

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Question

If $\mathbf{P} = (1, 0)$, $\mathbf{Q} = (-1, 0)$ and $\mathbf{R} = (2, 0)$ are three given points, then the locus of point $\mathbf{S}$ satisfying the relation $(SQ)^2 + (SR)^2 = 2(SP)^2$, is:

1. a straight line parallel to X axis
2. a circle passing through the origin
3. a circle with the center at the origin
4. a straight line parallel to Y axis

Given

$$\mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \mathbf{Q} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}; \mathbf{R} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{1}$$

$$\mathbf{S} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{2}$$

Solving

$$||\mathbf{Q}-\mathbf{S}||^2+||\mathbf{R}-\mathbf{S}||^2=2||\mathbf{P}-\mathbf{S}||^2 \tag{3}$$

$$\left(\mathbf{Q}-\mathbf{S}\right)^\top\left(\mathbf{Q}-\mathbf{S}\right)+\left(\mathbf{R}-\mathbf{S}\right)^\top\left(\mathbf{R}-\mathbf{S}\right)=2\left(\mathbf{P}-\mathbf{S}\right)^\top\left(\mathbf{P}-\mathbf{S}\right) \tag{4}$$

$$||\mathbf{Q}||^2+||\mathbf{R}||^2-2||\mathbf{P}||^2=\mathbf{S}^\top+\mathbf{Q}^\top\mathbf{S}+\mathbf{S}^\top\mathbf{R}+\mathbf{R}^\top\mathbf{S}-2\mathbf{S}^\top\mathbf{P}-2\mathbf{P}^\top\mathbf{S} \tag{5}$$

$$||\mathbf{Q}||^2+||\mathbf{R}||^2-2||\mathbf{P}||^2=\mathbf{S}^\top\left(\mathbf{Q}+\mathbf{R}-2\mathbf{P}\right)+\mathbf{S}\left(\mathbf{Q}+\mathbf{R}-2\mathbf{P}\right)^\top \tag{6}$$

$$||\mathbf{Q}||^2+||\mathbf{R}||^2-2||\mathbf{P}||^2=2\left(\mathbf{Q}+\mathbf{R}-2\mathbf{P}\right)^\top\mathbf{S} \tag{7}$$

Equation (7) is of the form:

$$\mathbf{n}^\top \mathbf{x} = c \tag{8}$$

$$\left(\mathbf{Q} + \mathbf{R} - 2\mathbf{P}\right)^\top \mathbf{S} = \frac{||\mathbf{Q}||^2 + ||\mathbf{R}||^2 - 2||\mathbf{P}||^2}{2} \tag{9}$$

Substituting values:

$$\left(\begin{pmatrix}-1\\0\end{pmatrix}+\begin{pmatrix}2\\0\end{pmatrix}-2\begin{pmatrix}1\\0\end{pmatrix}\right)^\top \mathbf{s}=\frac{\left((-1)^2+0^2\right)+\left(2^2+0^2\right)-2\left(1^2+0^2\right)}{2} \tag{10}$$

$$\begin{pmatrix}-1\\0\end{pmatrix}^\top \mathbf{s}=\frac{3}{2} \tag{11}$$

Hence the locus of **s** is a line parallel to Y-axis.

C Code

```c
#include<stdio.h>

int arrP[2] = {1,0};
int arrQ[2] = {-1,0};
int arrR[2] = {2,0};

int get_pointP(int index){
    return arrP[index];
}

int get_pointQ(int index){
    return arrQ[index];
}

int get_pointR(int index){
    return arrR[index];
}
```

Python Code 1

```
import ctypes
import numpy as np


lib = ctypes.CDLL("./problem.so")

pointP = [0.00,0.00]
pointQ = [0.00,0.00]
pointR = [0.00,0.00]

for i in range(0,2):
    pointP[i] = lib.get_pointP(i)
for i in range(0,2):
    pointQ[i] = lib.get_pointQ(i)
for i in range(0,2):
    pointR[i] = lib.get_pointR(i)
```

Python Code 1

```
normal = [0,0]
print(pointP)
print(pointQ)
print(pointR)

for i in range(0,2):
    normal[i] = pointQ[i] + pointR[i] - (2*pointP[i])
z = np.array(['x','y'])
z_t = z.T
k = 0.00
for i in range(0,2):
    k += ((pointQ[i]**2)+(pointR[i]**2)-(2*(pointP[i]**2)))/2
print(normal,z_t,'=',k,"\nHence the locus of S is a line.")
```

Python Code 2

```
import matplotlib.pyplot as plt
import numpy as np

x = [-3/2, -3/2]
y = [5, -5]

X = [1, -1, 2]
Y = [0, 0, 0]

plt.plot(x, y, '-r')
plt.plot(X, Y, 'ko')

plt.text(0.6, 0.1, "(1,0)", fontsize=10, color="black")
plt.text(-1.1, 0.1, "(-1,0)", fontsize=10, color="black")
```

Python Code 2

```
plt.text(2.1, 0.1, "(2,0)", fontsize=10, color="black")
plt.text(-1.51, 3.20, r"$x=\frac{3}{2}$", fontsize=13, color="
    black")

plt.axvline(x=0, color='k', linewidth=1.5)
plt.axhline(y=0, color='k', linewidth=1.5)

plt.xlabel("X-axis")
plt.ylabel("Y-axis")
plt.grid(True)
plt.axis("equal")
plt.savefig("../figs/plot.png")
plt.show()
```

Plot

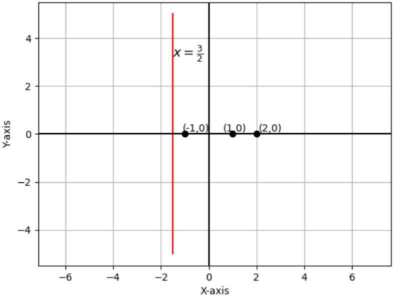

Figure: Plot of the given points and locus of **S**