4.7.64

AI25BTECH11003 - Bhavesh Gaikwad

Question: Find the distance between the point **P**(6, 5, 9) and the plane determined by the points **A**(3, -1, 2), **B**(5, 2, 4) and **C**(-1, -1, 6).

Solution: Let the Equation of the Plane be $\mathbf{n}^\top\mathbf{x} = 1$.

Since, **A**, **B**, **C** lie on the Plane.

$$\therefore \mathbf{n}^\top\mathbf{A} = 1,\ \mathbf{n}^\top\mathbf{B} = 1,\ \mathbf{n}^\top\mathbf{C} = 1 \tag{0.1}$$

OR

$$\mathbf{A}^\top\mathbf{n} = 1,\ \mathbf{B}^\top\mathbf{n} = 1,\ \mathbf{C}^\top\mathbf{n} = 1 \tag{0.2}$$

Let $\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$

From Equation 0.2,

$$(\mathbf{A}\,\mathbf{B}\,\mathbf{C})^\top n = 1 \tag{0.3}$$

$$\begin{pmatrix} 3 & -1 & 2 \\ 5 & 2 & 4 \\ -1 & -1 & 6 \end{pmatrix}\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = 1 \tag{0.4}$$

From Equation 0.4,

$$n_1 = 3/19,\ n_2 = -4/19,\ n_3 = 3/19 \tag{0.5}$$

$$\therefore \text{ The Equation of the Plane is: } \frac{1}{19}\begin{pmatrix} 3 & -4 & 3 \end{pmatrix}\mathbf{x} = 1 \tag{0.6}$$

The Distance of Point from a Plane is given by the formula:

$$d = \frac{\left\|\mathbf{n}^\top\mathbf{Q} - 1\right\|}{\|\mathbf{n}\|} \tag{0.7}$$

Let d be the distance between Point **P** and the Plane.

$$Then,\ d = \frac{\left\|\mathbf{n}^\top\mathbf{P} - 1\right\|}{\|\mathbf{n}\|} \tag{0.8}$$

$$d = \frac{\left\| \frac{18-20+27}{19} - 1 \right\|}{(\frac{\sqrt{34}}{19})} \tag{0.9}$$

$$\therefore d = \frac{3\sqrt{34}}{17} \tag{0.10}$$

$$\boxed{\text{The Distance between the Plane and } \mathbf{P}\, is\, \frac{3\sqrt{34}}{17}\, units.} \tag{0.11}$$

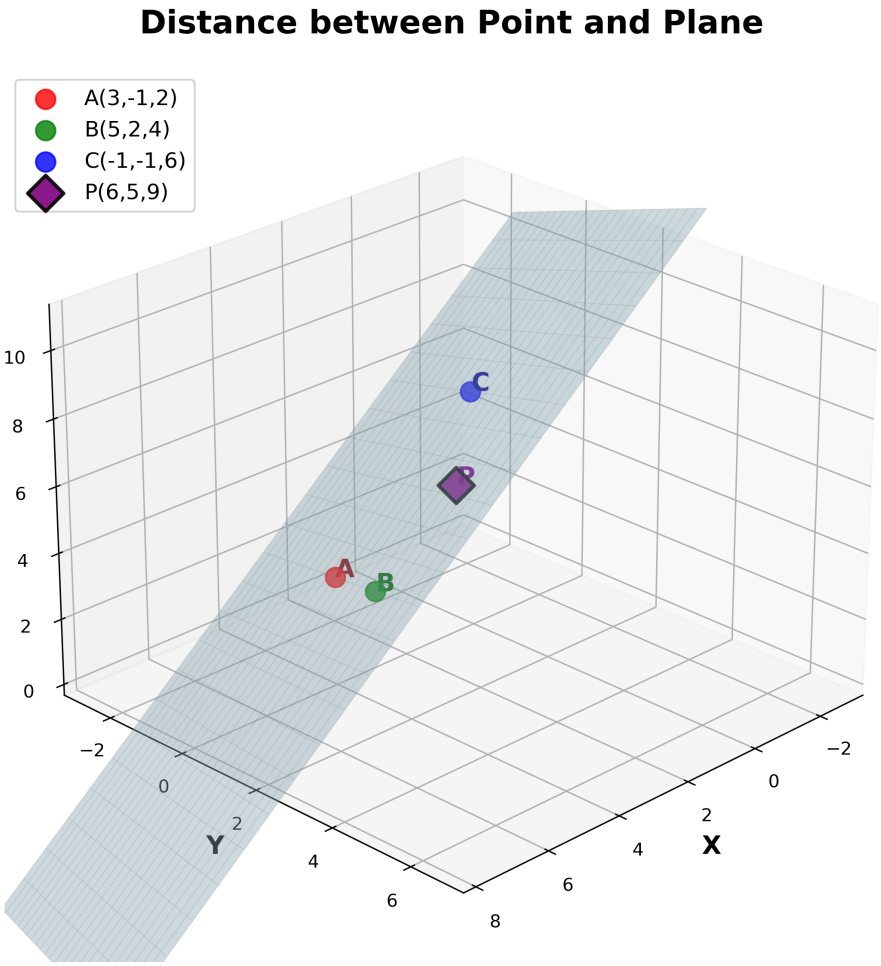

Fig. 0.1: Plane