Problem 4.12.46 :

Find the values of θ and p, if the equation

$$x\cos\theta + y\sin\theta = p \tag{1}$$

is the normal form of the line

$$\sqrt{3}x + y + 2 = 0. \tag{2}$$

Solution

We first identify the given data:

Quantity	Value
Normal vector $\mathbf{n}$	$\begin{pmatrix}\sqrt{3}\\1\end{pmatrix}$
Constant c	2

Table 1

The line can be expressed as

$$\mathbf{n}^T\mathbf{u} = -c. \tag{3}$$

The length of the normal is

$$\|\mathbf{n}\| = \sqrt{(\sqrt{3})^2 + 1^2} = 2. \tag{4}$$

Thus, the unit normal becomes

$$\hat{\mathbf{n}} = \frac{\mathbf{n}}{\|\mathbf{n}\|} = \begin{pmatrix}\frac{\sqrt{3}}{2}\\\frac{1}{2}\end{pmatrix}. \tag{5}$$

Dividing (3) by $\|\mathbf{n}\|$ gives the normal form:

$$\hat{\mathbf{n}}^T\mathbf{u} = \frac{-c}{\|\mathbf{n}\|} = -1. \tag{6}$$

Comparing with the standard normal form

$$x\cos\theta + y\sin\theta = p, \tag{7}$$

we identify

$$\cos\theta = \frac{\sqrt{3}}{2}, \quad \sin\theta = \frac{1}{2}. \tag{8}$$

Hence,

$$\theta = \frac{\pi}{6}, \quad p = -1. \tag{9}$$

Final Answer:

$$\boxed{\theta = \frac{\pi}{6}, \quad p = -1.} \tag{10}$$

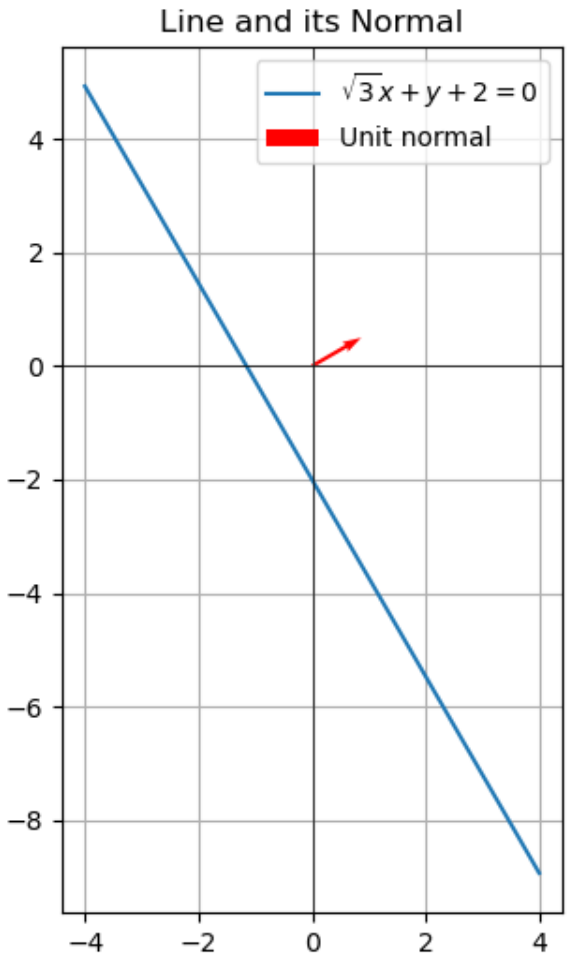

Figure 1