

MatGeo Assignment 1.2.13

AI25BTECH11007

September 9, 2025

Question

Using vectors, find the area of $\triangle ABC$ with vertices $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$.

Solution

Compute vectors $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}, \quad (1)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix}. \quad (2)$$

Recall the identity:

$$\|\mathbf{B} - \mathbf{A} \times \mathbf{C} - \mathbf{A}\|^2 = \|\mathbf{B} - \mathbf{A}\|^2 \|\mathbf{C} - \mathbf{A}\|^2 - (\mathbf{B} - \mathbf{A}^T \mathbf{C} - \mathbf{A})^2. \quad (3)$$

Compute the inner products:

$$\|\mathbf{B} - \mathbf{A}\|^2 = (\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{A}) = 1^2 + (-3)^2 + 1^2 = 11, \quad (4)$$

$$\|\mathbf{C} - \mathbf{A}\|^2 = (\mathbf{C} - \mathbf{A})^T (\mathbf{C} - \mathbf{A}) = 3^2 + 3^2 + (-4)^2 = 34, \quad (5)$$

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{C} - \mathbf{A}) = (1)(3) + (-3)(3) + (1)(-4) = -10. \quad (6)$$

Substitute into (3):

$$\begin{aligned} \|\mathbf{B} - \mathbf{A} \times \mathbf{C} - \mathbf{A}\|^2 &= (11)(34) - (-10)^2 \\ &= 374 - 100 \\ &= 274. \end{aligned} \quad (7)$$

Hence

$$\|\mathbf{B} - \mathbf{A} \times \mathbf{C} - \mathbf{A}\| = \sqrt{274}. \quad (8)$$

The area of

$$\triangle ABC$$

is

$$\text{Area}(\triangle ABC) = \frac{1}{2} \|\mathbf{B} - \mathbf{A} \times \mathbf{C} - \mathbf{A}\| = \frac{\sqrt{274}}{2}. \quad (9)$$

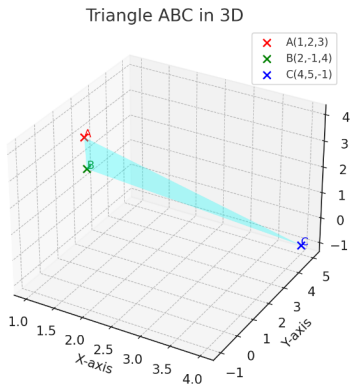


Figure: Image Visual