

# 2.9.11

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## Question

If  $\vec{a}$  and  $\vec{b}$  are unit vectors and  $\theta$  is angle between them then prove that  $\sin \frac{\theta}{2} = \frac{1}{2} |\vec{a} - \vec{b}|$

## Solution

Squaring on both sides,

$$\sin^2 \frac{\theta}{2} = \frac{1}{4} |\vec{a} - \vec{b}|^2$$

consider RHS,

$$\Rightarrow \frac{1}{4} \|\hat{a} - \hat{b}\|^2 \quad (1)$$

$$= \frac{1}{4} (\|\hat{a}\|^2 + \|\hat{b}\|^2 - 2\hat{a}^\top \hat{b}) \quad (2)$$

$$= \frac{1}{4} (2 - 2 \cos \theta) \quad (3)$$

$$= \frac{1}{2} \left( 2 \sin^2 \frac{\theta}{2} \right) \quad (4)$$

$$(5)$$

$$= \sin^2 \frac{\theta}{2} \quad (6)$$

$$= LHS \quad (7)$$

Hence,  $\sin \frac{\theta}{2} = \frac{1}{2} |\vec{a} - \vec{b}|$