4.4.10

AI25BTECH11014 - Gooty Suhas

Problem

Point $\mathbf{P}$ divides the line segment joining $\mathbf{A} = \begin{pmatrix}2\\1\end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix}k\\8\end{pmatrix}$ such that:

$$\frac{AP}{PB} = \frac{1}{3} \tag{0.1}$$

and lies on the line:

$$\begin{pmatrix}2 & -1\end{pmatrix}\mathbf{P} = -1 \tag{0.2}$$

Solution

Let the ratio be $m : n = 1 : 3$. Then the section formula in matrix form is:

$$\mathbf{P} = \frac{n\mathbf{A} + m\mathbf{B}}{m+n} = \frac{3\mathbf{A} + 1\mathbf{B}}{4} \tag{0.3}$$

Substitute:

$$\mathbf{A} = \begin{pmatrix}2\\1\end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix}k\\8\end{pmatrix} \Rightarrow \mathbf{P} = \frac{1}{4}\left(3\begin{pmatrix}2\\1\end{pmatrix} + \begin{pmatrix}k\\8\end{pmatrix}\right) = \frac{1}{4}\begin{pmatrix}6+k\\3+8\end{pmatrix} = \begin{pmatrix}\frac{6+k}{4}\\ \frac{11}{4}\end{pmatrix}$$

Substitute into the line equation:

$$\begin{pmatrix}2 & -1\end{pmatrix}\begin{pmatrix}\frac{6+k}{4}\\ \frac{11}{4}\end{pmatrix} = -1 \tag{0.4}$$

Simplify:

$$\frac{2(6+k)-11}{4} = -1 \Rightarrow \frac{12+2k-11}{4} = -1 \Rightarrow \frac{1+2k}{4} = -1 \Rightarrow 1+2k = -4 \Rightarrow k = -\frac{5}{2}$$

Final Answer

$$\boxed{k = -\frac{5}{2}} \tag{0.5}$$

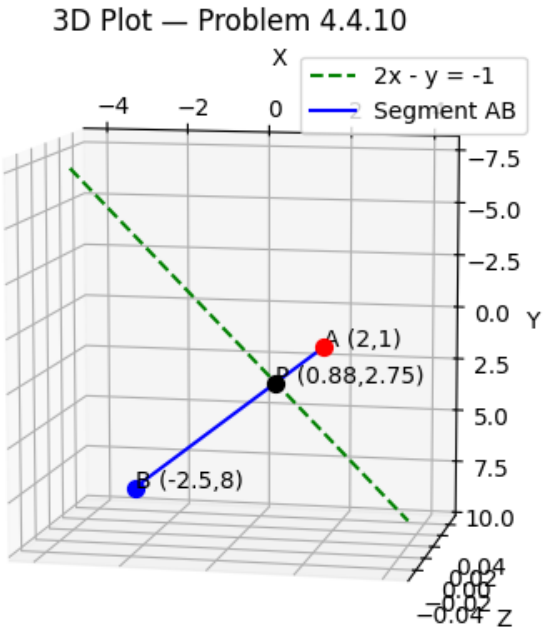

Fig. 0.1: Point **P** dividing **AB** and lying on the line