# 4.8.35

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**Question.** Find the coordinates of the foot of the perpendicular drawn from the point $\begin{pmatrix}0\\1\\2\end{pmatrix}$ on the $x$-axis .

**Solution.** Let the $x$-axis be represented as the intersection of the two planes

$$\mathbf{e}_2^T\mathbf{x} = 0, \quad \mathbf{e}_3^T\mathbf{x} = 0,$$

where

$$\mathbf{e}_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}, \ \mathbf{e}_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}, \ \mathbf{e}_3 = \begin{pmatrix}0\\0\\1\end{pmatrix}.$$

The direction vector of the $x$-axis is $\mathbf{m} = \mathbf{e}_1$ and the given point is

$$\mathbf{P} = \begin{pmatrix}0\\1\\2\end{pmatrix}.$$

By the foot-of-perpendicular relation,

$$\begin{pmatrix}\mathbf{m} & \mathbf{e}_2 & \mathbf{e}_3\end{pmatrix}^T \mathbf{Q} = \begin{pmatrix}\mathbf{m}^T\mathbf{P}\\0\\0\end{pmatrix}. \tag{1}$$

Substituting $\mathbf{m} = \mathbf{e}_1$ and $\mathbf{P} = \begin{pmatrix}0\\1\\2\end{pmatrix}$,

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\mathbf{Q} = \begin{pmatrix}0\\0\\0\end{pmatrix} \implies \mathbf{Q} = \begin{pmatrix}0\\0\\0\end{pmatrix}. \tag{2}$$

Thus, the foot of the perpendicular from $\begin{pmatrix}0\\1\\2\end{pmatrix}$ to the $x$-axis is

$$\boxed{\begin{pmatrix}0\\0\\0\end{pmatrix}}.$$

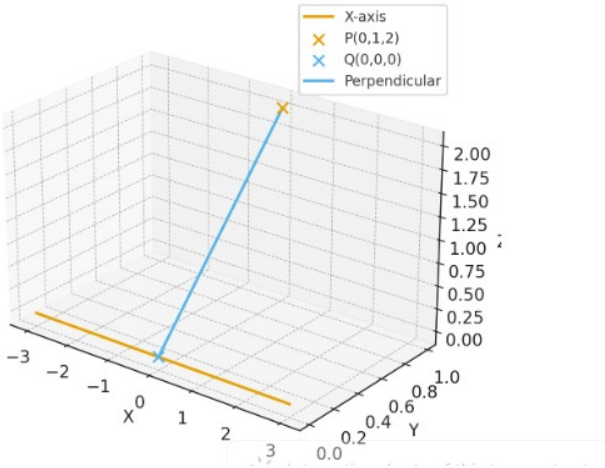


Fig. 0.1: Perpendicular from $\mathbf{P} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ to thethe $x$-axis with foot $\mathbf{Q} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.