7.2.12

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Question:

If the lines 2x - 3y = 5 and 3x - 4y = 7 are the diameters of a circle of area 154 square units, then obtain the equation of the circle.
Solution:

Let :

$$\mathbf{r_1} = \begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{k} = 5 \tag{0.1}$$

$$\mathbf{r_2} = \begin{pmatrix} 3 & -4 \end{pmatrix} \mathbf{k} = 7 \tag{0.2}$$

The augmented matrix of the above equations is given by,

$$\begin{pmatrix} 2 & -3 & 5 \\ 3 & -4 & 7 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow 2R_2 - 3R_1} \begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -1 \end{pmatrix} \tag{0.3}$$

$$\begin{pmatrix} 2 & -3 & 5 \\ 0 & 1 & -1 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 + 3R_2} \begin{pmatrix} 2 & 0 & 2 \\ 0 & 1 & -1 \end{pmatrix} \tag{0.4}$$

$$2x = 2 \qquad x = 1 \tag{0.5}$$

$$y = -1 \tag{0.6}$$

Point of intersection of diameters of circle is the center of circle $\mathbf{k} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
Given

$$\text{Area of circle} = \pi r^2 = 154 \text{ sq. units}$$
$$\text{Using } \pi = \frac{22}{7} \qquad \text{r=7 units}$$

$$\text{Equation of circle is } \|\mathbf{x}\|^2 + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{0.7}$$

$$\mathbf{u} = -\mathbf{k} \qquad f = \|\mathbf{u}\|^2 - r^2 \tag{0.8}$$

$$\mathbf{u} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \qquad f = (\sqrt{2})^2 - 7^2 = -47 \tag{0.9}$$

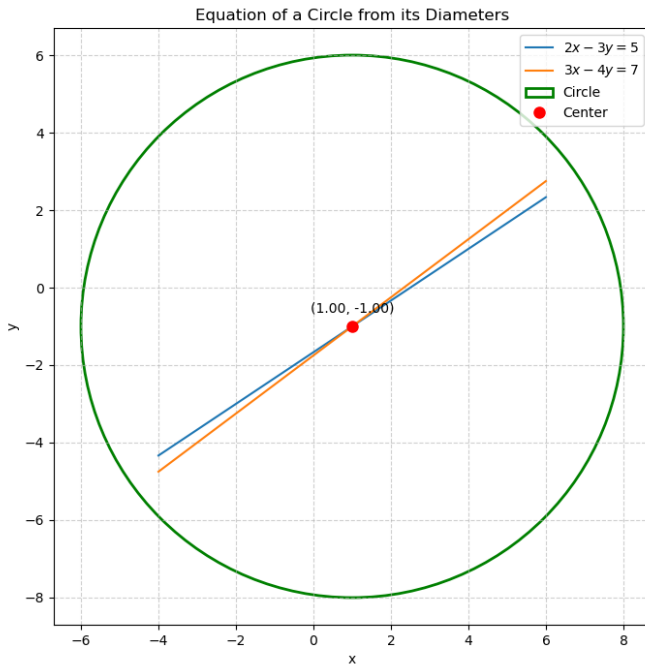Equation of circle is $\|\mathbf{x}\|^2 + 2\begin{pmatrix} -1 & 1 \end{pmatrix}\mathbf{x} - 47 = 0$ (0.10)

Fig. 0.1