5.8.13

EE25BTECH11015 - Bhoomika V

Question :-
The difference between two numbers is 26 and one number is three times the other. Find them.
Solution:
Let the two numbers be x and y ($x > y$).
Define equations From the problem:

$$x - y = 26$$

$$x = 3y$$

Rewriting in standard form $Ax = b$:

$$\begin{cases} x - y = 26 \\ x - 3y = 0 \end{cases}$$

Matrices A and b

$$A = \begin{bmatrix} 1 & -1 \\ 1 & -3 \end{bmatrix}, \quad b = \begin{bmatrix} 26 \\ 0 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

So the system is:

$$A\mathbf{x} = b$$

Reduce A to RREF (only A)
Start with:

$$A = \begin{bmatrix} 1 & -1 \\ 1 & -3 \end{bmatrix}$$

Eliminate first column in row 2

$$R_2 \to R_2 - R_1 \implies \begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix}$$

$$R_2 \to -\frac{1}{2}R_2 \implies \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$R_1 \to R_1 + R_2 \implies \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So the RREF of A is the identity matrix:

$$\text{RREF}(A) = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Solve $A\mathbf{x} = b$
Using the original b:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 39 \\ 13 \end{bmatrix}$$

Thus:

$$x = 39, \quad y = 13$$

Answer

x = 39, y = 13

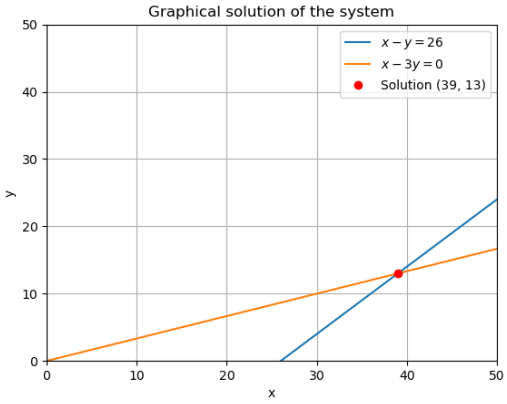

Fig. 0.1