

4.13.17

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Question:

Three distinct points A , B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point $(1, 0)$ to the distance from the point $(-1, 0)$ is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point:

1) $(\frac{5}{4}, 0)$

2) $(\frac{5}{2}, 0)$

3) $(\frac{5}{3}, 0)$

4) $(0, 0)$

Solution: let \mathbf{F}_1 , \mathbf{F}_2 be the vectors such that:

Point	Vector
\mathbf{F}_1	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
\mathbf{F}_2	$\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

TABLE 4: Variables used

Let $\mathbf{P} \begin{pmatrix} x \\ y \end{pmatrix}$ be any vector in the plane of A,B,C.
given,

$$\frac{\|\mathbf{PF}_1\|}{\|\mathbf{PF}_2\|} = \frac{1}{3} \quad (4.1)$$

$$\frac{\sqrt{(\mathbf{P} - \mathbf{F}_1)^\top (\mathbf{P} - \mathbf{F}_1)}}{\sqrt{(\mathbf{P} - \mathbf{F}_2)^\top (\mathbf{P} - \mathbf{F}_2)}} = \frac{1}{3} \quad (4.2)$$

By Substituting Values and Simplifying:

$$2x^2 + 2y^2 - 5x + 2 = 0 \quad (4.3)$$

$$\left\| \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} \frac{5}{4} \\ 0 \end{pmatrix} \right\| = \sqrt{\frac{21}{4}} \quad (4.4)$$

On comparing the equation with general form:-

$$\|\mathbf{P} - \mathbf{C}\| = r \quad (4.5)$$

$$\text{where , } \mathbf{P} = \text{any point on the circle} \quad (4.6)$$

$$\mathbf{C} = \text{Center of circle} \quad (4.7)$$

$$r = \text{radius of circle} \quad (4.8)$$

$$\text{Center of circle} = \left(\frac{5}{4}, 0 \right) \quad (4.9)$$

Hence, the circumcenter of the triangle is $\left(\frac{5}{4}, 0\right)$

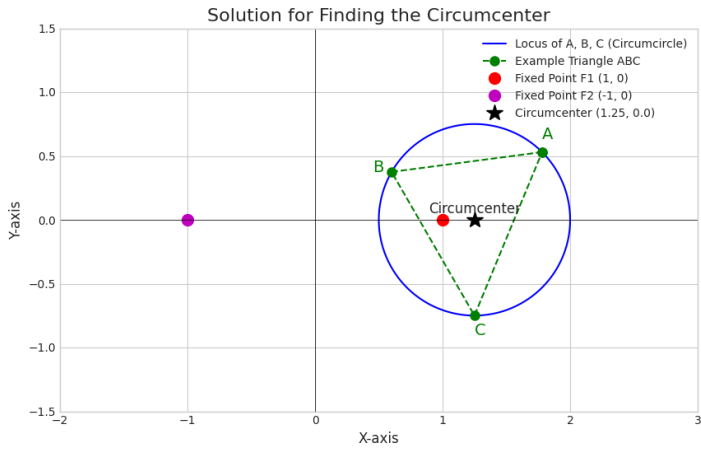


Fig. 4.1: 4.13.17