Problem 12.6

Compute the 4-point Discrete Fourier Transform (DFT) of the sequence

$$x[n] = \{1, 0, 2, 3\}, \quad n = 0, 1, 2, 3. \tag{1}$$

Input Variables

Symbol	Description	Value
N	Length of sequence	4
$x[n]$	Input sequence	$\{1, 0, 2, 3\}$
W_N	Twiddle factor	$e^{-j2\pi/N}$

Table 1

Solution

The N-point DFT is defined as

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}, \quad k = 0, 1, \ldots, N-1, \tag{2}$$

where

$$W_N = e^{-j\frac{2\pi}{N}}. \tag{3}$$

For $N = 4$,

$$W_4 = e^{-j\frac{2\pi}{4}} = -j, \tag{4}$$

so that

$$W_4^0 = 1, \quad W_4^1 = -j, \quad W_4^2 = -1, \quad W_4^3 = j. \tag{5}$$

The DFT matrix is

$$F_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix}. \tag{6}$$

The input vector is

$$\mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 3 \end{pmatrix}. \tag{7}$$

Thus,

$$\mathbf{X} = F_4 \, \mathbf{x}. \tag{8}$$

Row-by-row computation:

$$X[0] = 1 + 0 + 2 + 3 = 6, \tag{9}$$
$$X[1] = 1 + 0(-j) - 2 + 3j = -1 + 3j, \tag{10}$$
$$X[2] = 1 + 0(-1) + 2 - 3 = 0, \tag{11}$$
$$X[3] = 1 + 0(j) - 2 - 3j = -1 - 3j. \tag{12}$$

Therefore, the DFT vector is

$$\mathbf{X} = \begin{pmatrix} 6 \\ -1 + 3j \\ 0 \\ -1 - 3j \end{pmatrix}. \tag{13}$$

Final Answer

$$\mathbf{X} = \begin{pmatrix} 6 \\ -1 + 3j \\ 0 \\ -1 - 3j \end{pmatrix} \tag{14}$$

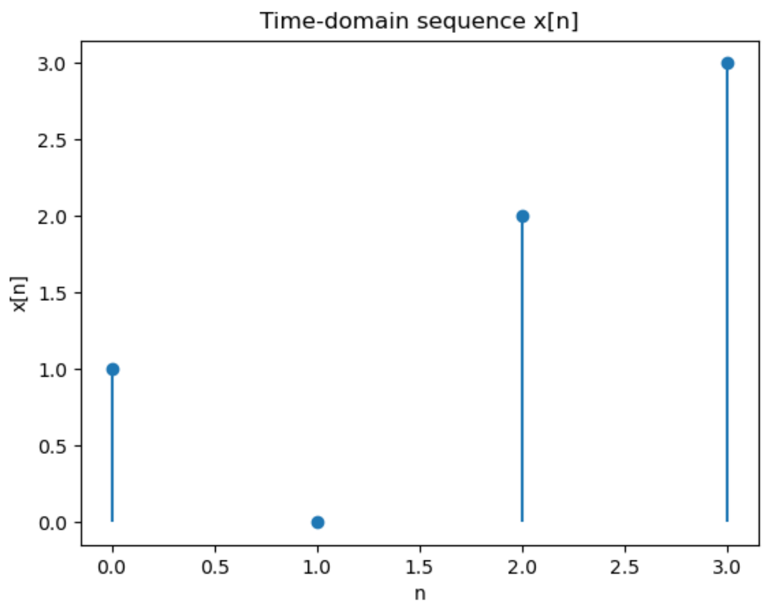

Figure 1

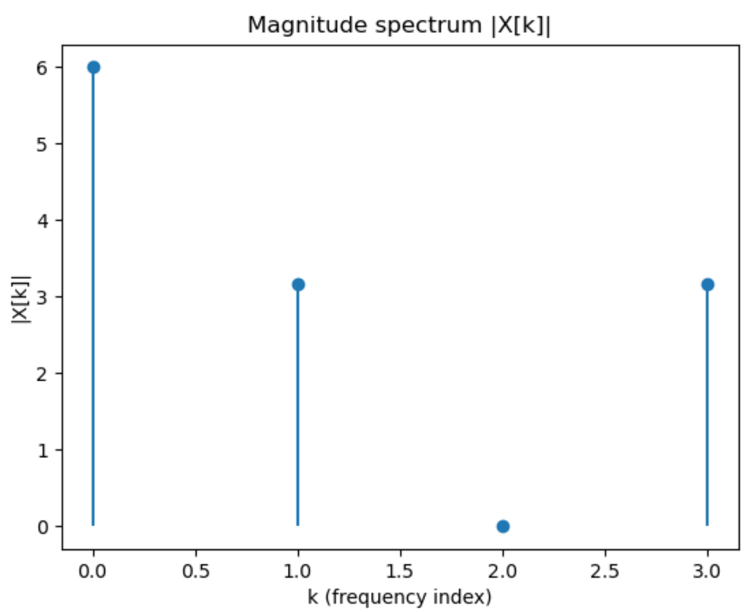

Figure 2

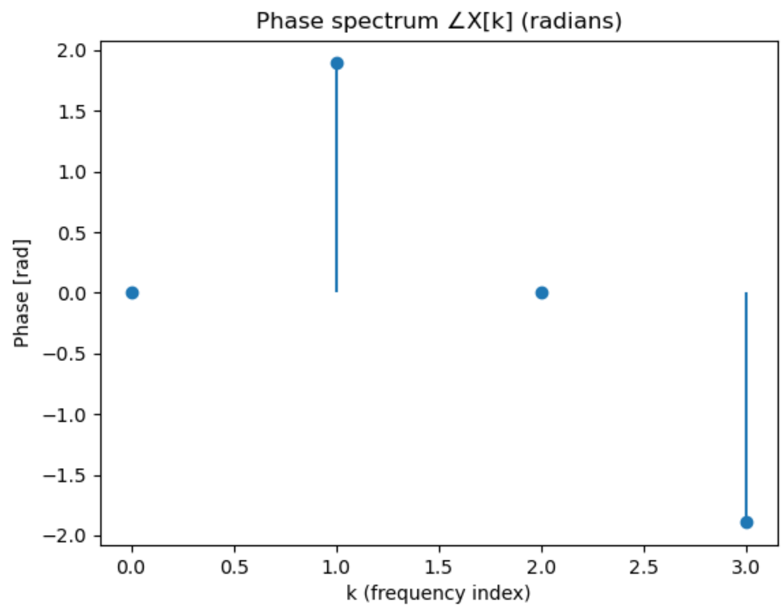
Phase spectrum ∠X[k] (radians)
Phase [rad]
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
k (frequency index)

Figure 3