1.3.6

AI25BTECH11027 - NAGA BHUVANA

Question:
Show that the points $\mathbf{A}(6,2)$,$\mathbf{B}(2,1)$,$\mathbf{C}(1,5)$ and $\mathbf{D}(5,6)$ are vertices of a square.
Solution:
From the given information,

$$\mathbf{A} = \begin{pmatrix}6\\2\end{pmatrix}, \mathbf{B} = \begin{pmatrix}2\\1\end{pmatrix}, \mathbf{C} = \begin{pmatrix}1\\5\end{pmatrix}, \mathbf{D} = \begin{pmatrix}5\\6\end{pmatrix} \tag{1}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix}2-6\\1-2\end{pmatrix} = \begin{pmatrix}-4\\-1\end{pmatrix} \tag{2}$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix}1-5\\5-6\end{pmatrix} = \begin{pmatrix}-4\\-1\end{pmatrix} \tag{3}$$

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \tag{4}$$

By the above property we can say that **ABCD** is a parallelogram.
Now

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix}2-6\\1-2\end{pmatrix} = \begin{pmatrix}-4\\-1\end{pmatrix} \tag{5}$$

$$\implies (\mathbf{B} - \mathbf{A})^T = \begin{pmatrix}-4 & -1\end{pmatrix} \tag{6}$$

$$\tag{7}$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix}1-2\\5-1\end{pmatrix} = \begin{pmatrix}-1\\4\end{pmatrix} \tag{8}$$

$$\implies (\mathbf{C} - \mathbf{B})^T = \begin{pmatrix}-1 & 4\end{pmatrix} \tag{9}$$

$$\tag{10}$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix}5-1\\6-5\end{pmatrix} = \begin{pmatrix}4\\1\end{pmatrix} \tag{11}$$

$$\implies (\mathbf{D} - \mathbf{C})^T = \begin{pmatrix}4 & 1\end{pmatrix} \tag{12}$$

$$\tag{13}$$

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix}6-5\\2-6\end{pmatrix} = \begin{pmatrix}1\\-4\end{pmatrix} \tag{14}$$

$$\implies (\mathbf{A} - \mathbf{D})^T = \begin{pmatrix}1 & -4\end{pmatrix} \tag{15}$$

$$\tag{16}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix}1-6\\5-2\end{pmatrix} = \begin{pmatrix}-5\\-3\end{pmatrix} \tag{17}$$
$$\implies (\mathbf{C}-\mathbf{A})^T = \begin{pmatrix}-5 & -3\end{pmatrix} \tag{18}$$
$$\tag{19}$$

$$\mathbf{D}-\mathbf{B} = \begin{pmatrix}5-2\\6-1\end{pmatrix} = \begin{pmatrix}3\\6\end{pmatrix} \tag{20}$$
$$\implies (\mathbf{D}-\mathbf{B})^T = \begin{pmatrix}3 & 6\end{pmatrix} \tag{21}$$
$$\tag{22}$$

The magnitude of the sides and the diagonals of the parallelogram are

$$||\mathbf{B}-\mathbf{A}||^2 = \left(B-A\right)^T\left(B-A\right) \tag{23}$$
$$\tag{24}$$

$$||\mathbf{B}-\mathbf{A}||^2 = \begin{pmatrix}-4 & -1\end{pmatrix}\begin{pmatrix}-4\\-1\end{pmatrix} \tag{25}$$
$$||\mathbf{B}-\mathbf{A}||^2 = (-4)^2 + (-1)^2 = 17 \tag{26}$$
$$\therefore ||\mathbf{B}-\mathbf{A}|| = \sqrt{17} \tag{27}$$

$$||\mathbf{C}-\mathbf{B}||^2 = \left(C-B\right)^T\left(C-B\right) \tag{28}$$

$$||\mathbf{C}-\mathbf{B}||^2 = \begin{pmatrix}-1 & 4\end{pmatrix}\begin{pmatrix}-1\\4\end{pmatrix} \tag{29}$$
$$||\mathbf{C}-\mathbf{B}||^2 = (-1)^2 + (4)^2 = 17 \tag{30}$$
$$\therefore ||\mathbf{C}-\mathbf{B}|| = \sqrt{17} \tag{31}$$

$$||\mathbf{D}-\mathbf{C}||^2 = \left(D-C\right)^T\left(D-C\right) \tag{32}$$

$$||\mathbf{D}-\mathbf{C}||^2 = \begin{pmatrix}4 & 1\end{pmatrix}\begin{pmatrix}4\\1\end{pmatrix} \tag{33}$$
$$||\mathbf{D}-\mathbf{C}||^2 = (4)^2 + (1)^2 = 17 \tag{34}$$
$$\therefore ||\mathbf{D}-\mathbf{C}|| = \sqrt{17} \tag{35}$$

$$||\mathbf{A}-\mathbf{D}||^2 = \left(A-D\right)^T\left(A-D\right) \tag{36}$$

$$||\mathbf{A}-\mathbf{D}||^2 = \begin{pmatrix}1 & -4\end{pmatrix}\begin{pmatrix}1\\-4\end{pmatrix} \tag{37}$$
$$||\mathbf{A}-\mathbf{D}||^2 = (1)^2 + (-4)^2 = 17 \tag{38}$$
$$\therefore ||\mathbf{A}-\mathbf{D}|| = \sqrt{17} \tag{39}$$
$$\tag{40}$$

$$||\mathbf{B}-\mathbf{A}|| = ||\mathbf{C}-\mathbf{B}|| = ||\mathbf{D}-\mathbf{C}|| = ||\mathbf{A}-\mathbf{D}|| = \sqrt{17} \tag{41}$$

From the above all the sides of the parallelogram are equal
Now consider the diagonals of the parallelogram

$$||\mathbf{C}-\mathbf{A}||^2 = \left(C-A\right)^T\left(C-A\right) \tag{42}$$

$$\tag{43}$$

$$||\mathbf{C}-\mathbf{A}||^2 = \begin{pmatrix}-5 & -3\end{pmatrix}\begin{pmatrix}-5\\-3\end{pmatrix} ||\mathbf{C}-\mathbf{A}||^2 = (-5)^2+(-3)^2 = 34 \tag{44}$$

$$||\mathbf{C}-\mathbf{A}|| = \sqrt{34} \tag{45}$$

$$\tag{46}$$

$$||\mathbf{D}-\mathbf{B}||^2 = \left(D-B\right)^T\left(D-B\right) \tag{47}$$

$$\tag{48}$$

$$||\mathbf{D}-\mathbf{B}||^2 = \begin{pmatrix}3 & 5\end{pmatrix}\begin{pmatrix}3\\5\end{pmatrix} ||\mathbf{D}-\mathbf{B}||^2 = (3)^2+(5)^2 = 34 \tag{49}$$

$$||\mathbf{D}-\mathbf{B}|| = \sqrt{34} \tag{50}$$

$$\tag{51}$$

$$||\mathbf{C}-\mathbf{A}|| = ||\mathbf{D}-\mathbf{B}|| = \sqrt{34} \tag{52}$$

From the above the diagonals of the parallelogram are equal
Property:
A parallelogram with all the sides of equal length and the diagonals of equal length must be a square.

The given Points forms a Square

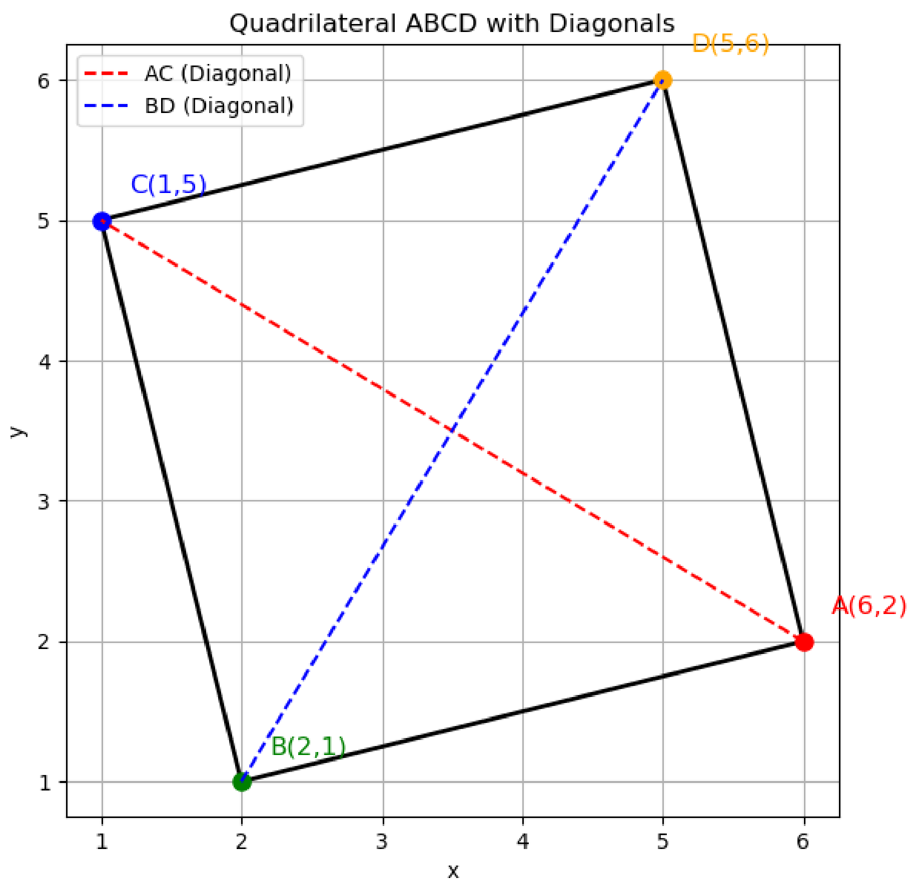
Quadrilateral ABCD with Diagonals
AC (Diagonal)
BD (Diagonal)
D(5,6)
C(1,5)
A(6,2)
B(2,1)
y
x
1
2
3
4
5
6
1
2
3
4
5
6

Fig. 1