

# 4.11.1

EE25BTECH11023 - Venkata Sai

**Question:**

Find the coordinates of the point where the line  $\frac{x-1}{3} = \frac{y+4}{7} = \frac{z+4}{2}$  cuts the XY-plane

**Solution:** The line equation is

$$\mathbf{r} = \mathbf{a} + t\mathbf{b} \quad (1)$$

where  $\mathbf{a}$  is the point on line and  $\mathbf{b}$  is the direction vector

$$\mathbf{a} = \begin{pmatrix} 1 \\ -4 \\ -4 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 3 \\ 7 \\ 2 \end{pmatrix} \quad (2)$$

The normal vector to XY plane is

$$\mathbf{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (3)$$

The plane equation of the XY-plane is

$$\mathbf{n}^T \mathbf{x} = 0 \implies \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \mathbf{x} = 0 \quad (4)$$

Substituting the line into the plane equation gives

$$\mathbf{n}^T (\mathbf{a} + t\mathbf{b}) = 0 \quad (5)$$

$$\mathbf{n}^T \mathbf{a} + t(\mathbf{n}^T \mathbf{b}) = 0 \quad (6)$$

$$t(\mathbf{n}^T \mathbf{b}) = -\mathbf{n}^T \mathbf{a} \quad (7)$$

$$t = -\frac{\mathbf{n}^T \mathbf{a}}{\mathbf{n}^T \mathbf{b}} \quad (8)$$

$$t = -\frac{\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -4 \\ -4 \end{pmatrix}}{\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 7 \\ 2 \end{pmatrix}} \quad (9)$$

$$t = \frac{4}{2} \quad (10)$$

$$t = 2 \quad (11)$$

The intersection point is

$$\mathbf{r} = \mathbf{a} + t\mathbf{b} = \begin{pmatrix} 1 \\ -4 \\ -4 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ 7 \\ 2 \end{pmatrix} \quad (12)$$

$$\mathbf{r} = \begin{pmatrix} 7 \\ 10 \\ 0 \end{pmatrix} \quad (13)$$

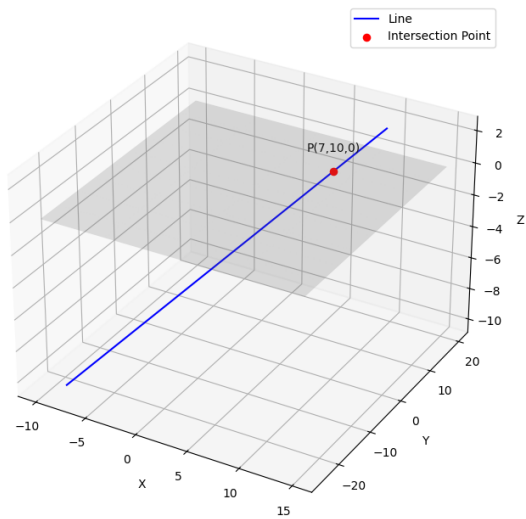


Fig. 0.1