4.8.14

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Question:

Let $\mathbf{P}(3, 2, 6)$ be a point in space and $\mathbf{Q}$ be a point on the line $\mathbf{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(-3\hat{i} + \hat{j} + 5\hat{k})$. Then the value of μ for which the vector $\overrightarrow{PQ}$ is parallel to the plane $x - 4y + 3z = 1$ is

Solution:

$$\text{The position vector of point } \mathbf{P} \text{ is } \begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix} \tag{0.1}$$

Point $\mathbf{Q}$ lies on line $\mathbf{r}$ .So

$$\text{The position vector of point } \mathbf{Q} \text{ is } \begin{pmatrix} 1 - 3\mu \\ -1 + \mu \\ 2 + 5\mu \end{pmatrix} \tag{0.2}$$

$$\overrightarrow{PQ} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} 1 - 3\mu - 3 \\ -1 + \mu - 2 \\ 2 + 5\mu - 6 \end{pmatrix} = \begin{pmatrix} -2 - 3\mu \\ -3 + \mu \\ -4 + 5\mu \end{pmatrix} \tag{0.3}$$

Equation of plane is $x - 4y + 3z = 1$

$$\text{Normal of plane is} \quad \mathbf{n} = \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix} \tag{0.4}$$

$\overrightarrow{PQ}$ is parallel to the plane ,So $\mathbf{n}^T(\mathbf{PQ}) = 0$

$$\mathbf{n}^T(\mathbf{PQ}) = \begin{pmatrix} 1 & -4 & 3 \end{pmatrix} \begin{pmatrix} -2 - 3\mu \\ -3 + \mu \\ -4 + 5\mu \end{pmatrix} = 0 \tag{0.5}$$

$$-2 - 3\mu + 12 - 4\mu - 12 + 15\mu = 0 \tag{0.6}$$

$$-2 + 8\mu = 0 \tag{0.7}$$

$$\mu = \frac{1}{4} \tag{0.8}$$

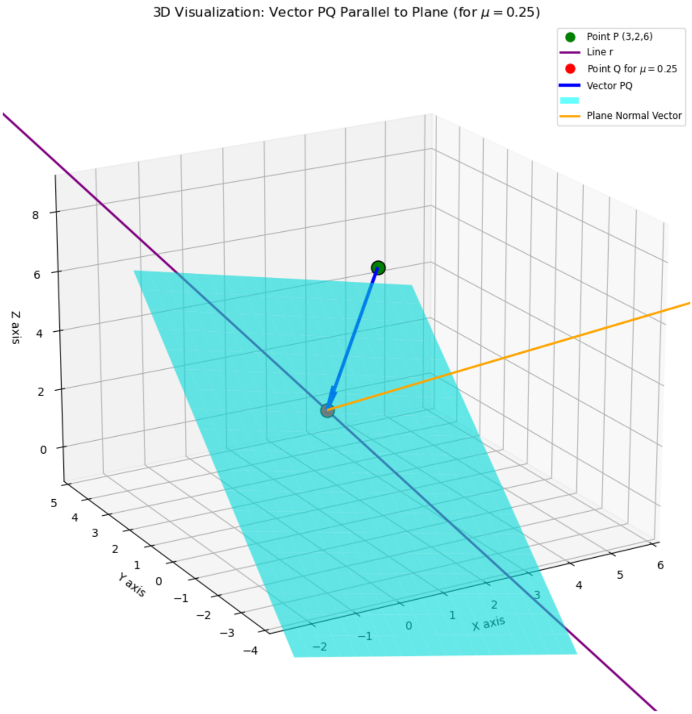
3D Visualization: Vector PQ Parallel to Plane (for $\mu = 0.25$)
Point P (3,2,6)
Line r
Point Q for $\mu = 0.25$
Vector PQ
Plane Normal Vector
Z axis
Y axis
X axis

Fig. 0.1