

5.8.2

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Question

10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

Solution

Let the number of girls in the class be g , and the number of boys be b .
Let the vector representing this data be

$$\mathbf{x} = \begin{pmatrix} g \\ b \end{pmatrix} \quad (1)$$

Since the total number of students in the class is 10, $g + b = 10$ which can be expressed as:

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}^T \mathbf{x} = 10 \quad (2)$$

Since there are 4 more girls than boys, $b + 4 = g$, which can be expressed as:

$$\begin{pmatrix} -1 \\ 1 \end{pmatrix}^T \mathbf{x} = -4 \quad (3)$$

Solution

Organising these two equations into the form $\mathbf{Ax} = \mathbf{b}$:

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 10 \\ -4 \end{pmatrix} \quad (4)$$

Normalising \mathbf{A} :

$$\sqrt{2} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \mathbf{x} = \begin{pmatrix} 10 \\ -4 \end{pmatrix} \quad (5)$$

Solution

The normalised \mathbf{A} is orthogonal, so $\mathbf{A}^T \mathbf{A} = \mathbf{I}$

Multiplying by normalised \mathbf{A}^T on both the sides:

$$\sqrt{2}\mathbf{x} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 10 \\ -4 \end{pmatrix} \quad (6)$$

$$\mathbf{x} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 10 \\ -4 \end{pmatrix} \quad (7)$$

Solving we get:

$$\mathbf{x} = \begin{pmatrix} 7 \\ 3 \end{pmatrix} \quad (8)$$

$$g = 7 \quad (9)$$

$$b = 3 \quad (10)$$

Python Code

```
import numpy as np
import numpy.linalg
import matplotlib.pyplot as plt

answer = numpy.linalg.solve([[1,1],[-1,1]], [10,-4])

answer[0] = round(answer[0],2)
answer[1] = round(answer[1],2)
print(answer)
```

Python Code

```
fig = plt.figure(figsize =(6,6))
ax = fig.add_subplot(111)

X = np.linspace(-20,20,2)

Y1 = (10-X)
Y2 = (X-4)

ax.plot(X, Y1, label='Line 1')
ax.plot(X, Y2, label='Line 2')
ax.scatter(answer[0], answer[1], label=f'({answer[0]}, {answer
[1]})')
ax.grid(True)
ax.legend()
plt.show()
```

Plot

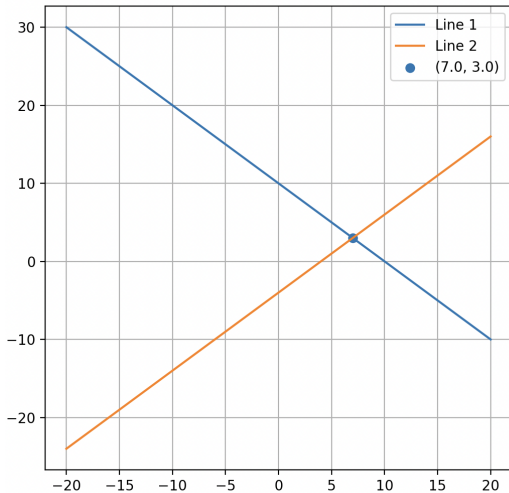


Figure: Plot