

2.10.74

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Question

If $\mathbf{X} \cdot \mathbf{A} = 0$, $\mathbf{X} \cdot \mathbf{B} = 0$, and $\mathbf{X} \cdot \mathbf{C} = 0$ for some non-zero vector \mathbf{X} , then $[\mathbf{A} \ \mathbf{B} \ \mathbf{C}] = 0$.

Theoretical Solution

$$\mathbf{X} \cdot \mathbf{A} = 0 \quad (1)$$

$$\mathbf{X} \cdot \mathbf{B} = 0 \quad (2)$$

$$\mathbf{X} \cdot \mathbf{C} = 0 \quad (3)$$

From (1), (2) and (3),

$$\mathbf{A}^\top \mathbf{X} = \mathbf{B}^\top \mathbf{X} = \mathbf{C}^\top \mathbf{X} = 0 \quad (4)$$

Theoretical Solution

This forms the set of equations

$$\begin{pmatrix} \mathbf{A}^\top \\ \mathbf{B}^\top \\ \mathbf{C}^\top \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (5)$$

For a homogeneous set of equations to have non-trivial solution \mathbf{x} , the coefficient matrix is singular.

$$\implies [\mathbf{A}^\top \ \mathbf{B}^\top \ \mathbf{C}^\top] = 0 \quad (6)$$

From (6)

$$[\mathbf{A} \ \mathbf{B} \ \mathbf{C}] = 0 \quad (7)$$

Hence, the given statement is true.

Example

Let

$$\mathbf{X} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

$$\mathbf{A}^\top \mathbf{X} = \mathbf{B}^\top \mathbf{X} = \mathbf{C}^\top \mathbf{X} = 0 \text{ for this example.} \quad (8)$$

$$[\mathbf{A} \ \mathbf{B} \ \mathbf{C}] = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \left(\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right) \quad (9)$$

$$[\mathbf{A} \ \mathbf{B} \ \mathbf{C}] = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (10)$$

$$[\mathbf{A} \ \mathbf{B} \ \mathbf{C}] = 0 \quad (11)$$

Plot

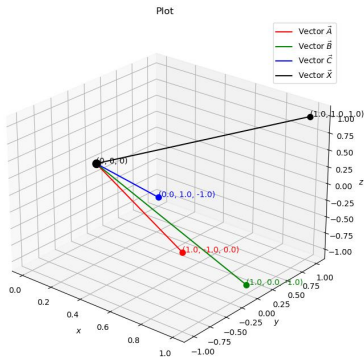


Figure: Example