1.2.16

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Question

Show that $(-1, 2, 1)$, $(1, -2, 5)$, $(4, -7, 8)$ and $(2, -3, 4)$ are the vertices of a parallelogram.

Variables used

Name	**Point**
Point A	$\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$
Point B	$\begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$
Point C	$\begin{pmatrix} 4 \\ -7 \\ 8 \end{pmatrix}$
Point D	$\begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$

Table: Variables Used

Solution

Now,

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}1-(-1)\\-2-2\\5-1\end{pmatrix}=\begin{pmatrix}2\\-4\\4\end{pmatrix}, \tag{1}$$

$$\mathbf{C}-\mathbf{D}=\begin{pmatrix}4-2\\-7-(-3)\\8-4\end{pmatrix}=\begin{pmatrix}2\\-4\\4\end{pmatrix}. \tag{2}$$

Hence,

$$\mathbf{B}-\mathbf{A}=\mathbf{C}-\mathbf{D}=\begin{pmatrix}2\\-4\\4\end{pmatrix}. \tag{3}$$

Hence,

$$\mathbf{B}-\mathbf{A}||\mathbf{C}-\mathbf{D}$$

Solution

Also,

$$\mathbf{C}-\mathbf{B}=\begin{pmatrix}4-1\\-7-(-2)\\8-5\end{pmatrix}=\begin{pmatrix}3\\-5\\3\end{pmatrix}, \tag{4}$$

$$\mathbf{D}-\mathbf{A}=\begin{pmatrix}2-(-1)\\-3-2\\4-1\end{pmatrix}=\begin{pmatrix}3\\-5\\3\end{pmatrix}. \tag{5}$$

Thus,

$$\mathbf{C}-\mathbf{B}=\mathbf{D}-\mathbf{A}=\begin{pmatrix}3\\-5\\3\end{pmatrix}. \tag{6}$$

Hence,

$$\mathbf{C}-\mathbf{B}||\mathbf{D}-\mathbf{A}$$

Therefore, A, B, C, D are the vertices of a parallelogram.

Python code- Checking whether the points are vertices of parallelogram

```
import numpy as np
 import itertools

 def is_parallel(v, w, tol=1e-9):
    """Check if vectors v and w are parallel (cross pro duct = 0)
        ."""
     v = np.array(v, dtype=float)
   w = np.array(w, dtype=float)
   if np.allclose(v, 0, atol=tol) or np.allclose(w, 0, atol=tol):
       return np.allclose(v, w, atol=tol)
    return np.allclose(np.cross(v, w), 0, atol=tol)

 def is_parallelogram(points):
     """
   Check if 4 points form a parallelogram using only the parallel
       -sides test.
     Returns True if yes, else False.
"""
```

Python code - Checking whether the points ar e vertices of parallelogram

```

    P = [np.array(p, dtype=float) for p in points]

   for perm in itertools.permutations(range(4)):
        A, B, C, D = [P[i] for i in perm]

      AB, BC, CD, DA = B - A, C - B, D - C, A - D

  # Check opposite sides are parallel and adjacen t sides not
      parallel
   if is_parallel(AB, CD) and is_parallel(BC, DA) and not
       is_parallel(AB, BC):
         return True
    return False
```

Python code - Checking whether the points ar e vertices of parallelogram

```
 # Example points
 A = (-1, 2, 1)
B = ( 1, -2, 5)
 C = ( 4, -7, 8)
D = ( 2, -3, 4)

points = [A, B, C, D]

if is_parallelogram(points):
  print("The given points form a parallelogram.")
 else:
  print(" The given points do NOT form a parallelogra m.")
```

Python code - plotting the points

```
        import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d.art3d import Poly3DCollection
import os

# Given points as column vectors (x,y,z)
A = np.array([-1, 2, 1]).reshape(-1,1)
B = np.array([1, -2, 5]).reshape(-1,1)
C = np.array([4, -7, 8]).reshape(-1,1)
D = np.array([2, -3, 4]).reshape(-1,1)

# Stack coordinates
coords = np.block([A,B,C,D])

# Create 3D plot
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
```

Python code - plotting the points

```python
# Scatter points
ax.scatter(coords[0,:], coords[1,:], coords[2,:], color='r', s
    =50)

# Draw parallelogram edges
edges = [(A,B), (B,C), (C,D), (D,A)]
for edge in edges:
    pts = np.hstack(edge)
    ax.plot(pts[0,:], pts[1,:], pts[2,:], color='b')

# Fill parallelogram face
verts = [[A.flatten(), B.flatten(), C.flatten(), D.flatten()]]
ax.add_collection3d(Poly3DCollection(verts, alpha=0.3, facecolor=
    'cyan'))
```

Python code - plotting the points

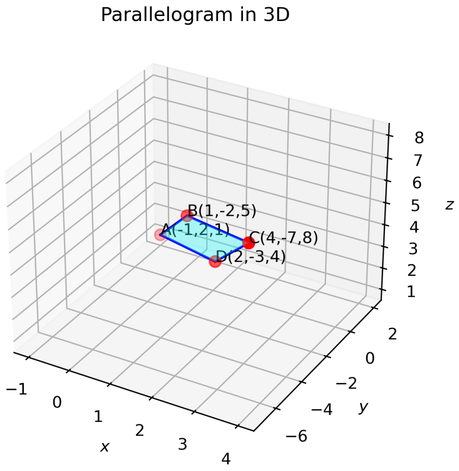
```python
# Labels
ax.text(A[0,0], A[1,0], A[2,0], "A(-1,2,1)")
ax.text(B[0,0], B[1,0], B[2,0], "B(1,-2,5)")
ax.text(C[0,0], C[1,0], C[2,0], "C(4,-7,8)")
ax.text(D[0,0], D[1,0], D[2,0], "D(2,-3,4)")

# Axes
ax.set_xlabel('$x$')
ax.set_ylabel('$y$')
ax.set_zlabel('$z$')
plt.title("Parallelogram in 3D")

# Save figure
save_path = '../figs/img.png'
os.makedirs(os.path.dirname(save_path), exist_ok=True)
plt.savefig(save_path, dpi=300)

print(f"Image saved at: {save_path}")
```

Plot-Using Python


Parallelogram in 3D
A(-1,2,1)
B(1,-2,5)
C(4,-7,8)
D(2,-3,4)
x
y
z