2.9.22

AI25BTECH11006 - Nikhila

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Question

Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be three vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$, and $|\vec{c}| = 3$. If the projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$, and $\vec{b}$ and $\vec{c}$ are perpendicular to each other, then find $|3\vec{a} - 2\vec{b} + 2\vec{c}|$.

Theoretical Solution

Given:

$$\|\mathbf{a}\| = 1,\ \|\mathbf{b}\| = 2,\ \|\mathbf{c}\| = 3 \tag{1}$$

$$\text{The projection of } \mathbf{b} \text{ along } \mathbf{a} = \mathbf{b}^T \frac{\mathbf{a}}{\|\mathbf{a}\|^2}\mathbf{a} \tag{2}$$

$$\text{The projection of } \mathbf{c} \text{ along } \mathbf{a} = \mathbf{c}^T \frac{\mathbf{a}}{\|\mathbf{a}\|^2}\mathbf{a} \tag{3}$$

$$\mathbf{b}^T \frac{\mathbf{a}}{\|\mathbf{a}\|}\mathbf{a} = \mathbf{c}^T \frac{\mathbf{a}}{\|\mathbf{a}\|}\mathbf{a} \tag{4}$$

$$\text{Since, } \|\mathbf{a}\| = 1 \Rightarrow \quad \therefore\ \mathbf{b}^T\mathbf{a} = \mathbf{c}^T\mathbf{a} \tag{5}$$

Since $\mathbf{b}$ and $\mathbf{c}$ are perpendicular:

$$\mathbf{b}^T\mathbf{c} = 0 \tag{6}$$

Theoretical Solution

$$\text{Let } \mathbf{v} = 3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c} \tag{7}$$

$$\|\mathbf{v}\|^2 = (3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c})^T(3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c}) \tag{8}$$

$$\|\mathbf{v}\|^2 = 9(\mathbf{a}^T\mathbf{a}) - 6(\mathbf{a}^T\mathbf{b}) + 6(\mathbf{a}^T\mathbf{c}) - 6(\mathbf{b}^T\mathbf{a}) + 4(\mathbf{b}^T\mathbf{b}) - 4(\mathbf{b}^T\mathbf{c}) + 6(\mathbf{c}^T\mathbf{a}) - 4(\mathbf{c}^T\mathbf{b} \tag{9}$$

$$\text{Since } \mathbf{a}^T\mathbf{b} = \mathbf{b}^T\mathbf{a} \,\&\, \mathbf{a}^T\mathbf{c} = \mathbf{c}^T\mathbf{a} \tag{10}$$

$$\|\mathbf{v}\|^2 = 9(\mathbf{a}^T\mathbf{a}) + 4(\mathbf{b}^T\mathbf{b}) + 4(\mathbf{c}^T\mathbf{c}) - 12(\mathbf{a}^T\mathbf{b}) + 12(\mathbf{a}^T\mathbf{c}) - 8(\mathbf{b}^T\mathbf{c}) \tag{11}$$

Theoretical Solution

From Equation 1 & 6,

$$\mathbf{a}^T\mathbf{a} = \|\mathbf{a}\|^2 = 1,\ \mathbf{b}^T\mathbf{b} = \|\mathbf{b}\|^2 = 4,\ \mathbf{c}^T\mathbf{c} = \|\mathbf{c}\|^2 = 9,\ \mathbf{b}^T\mathbf{c} = 0 \tag{12}$$

$$\|\mathbf{v}\|^2 = 9 + 16 + 36 \tag{13}$$

$$\|\mathbf{v}\|^2 = 61 \quad \Rightarrow \quad \|\mathbf{v}\| = \sqrt{61} \tag{14}$$

$$\boxed{\|3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c}\| = \sqrt{61}} \tag{15}$$

Graphical Representation