5.4.10

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Question

Find the area of the triangle ABC whose vertices are $\mathbf{A}(2,5),\ \mathbf{B}(4,7),\ \mathbf{C}(6,2$

solution

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}.$$

We form the augmented matrix $[A \mid I]$:

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 2 & 3 & 5 & 0 & 1 & 0 \\ -2 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3+2R_1]{R_2 \to R_2-2R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 5 & 1 & -2 & 1 & 0 \\ 0 & -2 & 5 & 2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \to \frac{1}{5}R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{5} & -\frac{2}{5} & \frac{1}{5} & 0 \\ 0 & -2 & 5 & 2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 + 2R_2]{R_1 \to R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{11}{5} & \frac{3}{5} & \frac{1}{5} & 0 \\ 0 & 1 & \frac{1}{5} & -\frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 0 & \frac{27}{5} & \frac{6}{5} & \frac{2}{5} & 1 \end{array}\right]$$

$$\xrightarrow{R_3 \to \frac{5}{27} R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{11}{5} & \frac{3}{5} & \frac{1}{5} & 0 \\ 0 & 1 & \frac{1}{5} & -\frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & \frac{2}{9} & \frac{2}{27} & \frac{5}{27} \end{array}\right]$$

$$\xrightarrow[R_2 \to R_2 - \frac{1}{5}R_3]{R_1 \to R_1 - \frac{11}{5}R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{9} & \frac{1}{27} & -\frac{11}{27} \\ 0 & 1 & 0 & -\frac{4}{9} & \frac{5}{27} & -\frac{1}{27} \\ 0 & 0 & 1 & \frac{2}{9} & \frac{2}{27} & \frac{5}{27} \end{array}\right]$$

Thus the inverse is

$$A^{-1} = \begin{bmatrix} \frac{1}{9} & \frac{1}{27} & -\frac{11}{27} \\ -\frac{4}{9} & \frac{5}{27} & -\frac{1}{27} \\ \frac{2}{9} & \frac{2}{27} & \frac{5}{27} \end{bmatrix}.$$

```c
#include <stdio.h>

// Compute determinant of 3x3 matrix
float determinant(float A[3][3]) {
    return A[0][0]*(A[1][1]*A[2][2] - A[1][2]*A[2][1])
         - A[0][1]*(A[1][0]*A[2][2] - A[1][2]*A[2][0])
         + A[0][2]*(A[1][0]*A[2][1] - A[1][1]*A[2][0]);
}

// Function to compute inverse of 3x3 matrix
int matrix_inverse(float A[3][3], float inverse[3][3]) {
    float det = determinant(A);
    if (det == 0) return 0; // singular, no inverse

    float adj[3][3];

    // Cofactor matrix
```

```
   adj[1][0] = -(A[0][1]*A[2][2] - A[0][2]*A[2][1]);
    adj[1][1] = (A[0][0]*A[2][2] - A[0][2]*A[2][0]);
    adj[1][2] = -(A[0][0]*A[2][1] - A[0][1]*A[2][0]);

    adj[2][0] = (A[0][1]*A[1][2] - A[0][2]*A[1][1]);
    adj[2][1] = -(A[0][0]*A[1][2] - A[0][2]*A[1][0]);
    adj[2][2] = (A[0][0]*A[1][1] - A[0][1]*A[1][0]);

    // Transpose adjoint and divide by determinant  inverse
    for (int i = 0; i < 3; i++) {
        for (int j = 0; j < 3; j++) {
            inverse[i][j] = adj[j][i] / det;
        }
    }
    return 1;
}
```

Python Code

```
import ctypes
import os
import numpy as np

# Load C library
lib = ctypes.CDLL(os.path.abspath("./matrix_inverse.so"))

# Define function signatures
ArrayType = ctypes.c_float * 9 # 3x3 = 9 elements

lib.matrix_inverse.argtypes = [ArrayType, ArrayType]
lib.matrix_inverse.restype = ctypes.c_int

def c_matrix_inverse(A):
    A = np.array(A, dtype=np.float32).reshape(9)
    A_c = ArrayType(*A)
    inv_c = ArrayType()
```

Python Code

```
success = lib.matrix_inverse(A_c, inv_c)
    if not success:
        raise ValueError("Matrix is singular, no inverse exists."
            )

    inv = np.array(list(inv_c), dtype=np.float32).reshape(3, 3)
    return inv

# --- Example ---
A = [
    [1, -1, 2],
    [2, 3, 5],
    [-2, 0, 1]
]
```

Python Code

```python
inv = c_matrix_inverse(A)

print("Original matrix A:")
print(np.array(A, dtype=np.float32))

print("\nInverse of A (from C function):")
print(inv)

print("\nCheck with NumPy:")
print(np.linalg.inv(np.array(A, dtype=np.float32)))
```