Matgeo Presentation - System of 3 Equations

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Problem Statement

Solve the system of equations

$$
\begin{aligned}
5x - y + 4z &= 5 \\
2x + 3y + 5z &= 2 \\
5x - 2y + 6z &= -1
\end{aligned}
$$

Data

Name	Equation
Equation 1	$5x - y + 4z = 5 \iff \begin{pmatrix} 5 & -1 & 4 \end{pmatrix} \mathbf{x}_1 = 5$
Equation 2	$2x + 3y + 5z = 2 \iff \begin{pmatrix} 2 & 3 & 5 \end{pmatrix} \mathbf{x}_2 = 2$
Equation 3	$5x - 2y + 6z = -1 \iff \begin{pmatrix} 5 & -2 & 6 \end{pmatrix} \mathbf{x}_3 = -1$

Table : Equations

Solution - Matrix Form

The system of equations in matrix form is :

$$\begin{pmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix} \tag{0.1}$$

Forming the augmented matrix:

$$\left(\begin{array}{ccc|c} 5 & -1 & 4 & 5 \\ 2 & 3 & 5 & 2 \\ 5 & -2 & 6 & -1 \end{array}\right) \tag{0.2}$$

Solution - Gaussian Elimination

Using Gaussian elimination:

$$\left(\begin{array}{ccc|c} 5 & -1 & 4 & 5 \\ 2 & 3 & 5 & 2 \\ 5 & -2 & 6 & -1 \end{array}\right) \xleftrightarrow[R_2 \to R_2 - \frac{2}{5}R_1]{R_3 \to R_3 - R_1} \left(\begin{array}{ccc|c} 5 & -1 & 4 & 5 \\ 0 & \frac{17}{5} & \frac{17}{5} & 0 \\ 0 & -1 & 2 & -6 \end{array}\right) \tag{0.3}$$

$$\xleftrightarrow{R_3 \to R_3 + \frac{5}{17}R_2} \left(\begin{array}{ccc|c} 5 & -1 & 4 & 5 \\ 0 & \frac{17}{5} & \frac{17}{5} & 0 \\ 0 & 0 & 3 & -6 \end{array}\right) \tag{0.4}$$

Solution - Back Substitution

Using back substitution we get:

$$
\begin{aligned}
3z &= -6 & &\Rightarrow z = -2 & & \quad (0.5)\\
\tfrac{17}{5}y + \tfrac{17}{5}z &= 0 & &\Rightarrow y + z = 0 & \Rightarrow y = 2 & \quad (0.6)\\
5x - y + 4z &= 5 & \Rightarrow 5x - 2 + 4(-2) = 5 & & \Rightarrow 5x - 10 = 5 \quad \Rightarrow x = 3 & \quad (0.7)
\end{aligned}
$$

Solution - Final Answer

Therefore the solution for the system of equations is :

$$\begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}$$

Plot

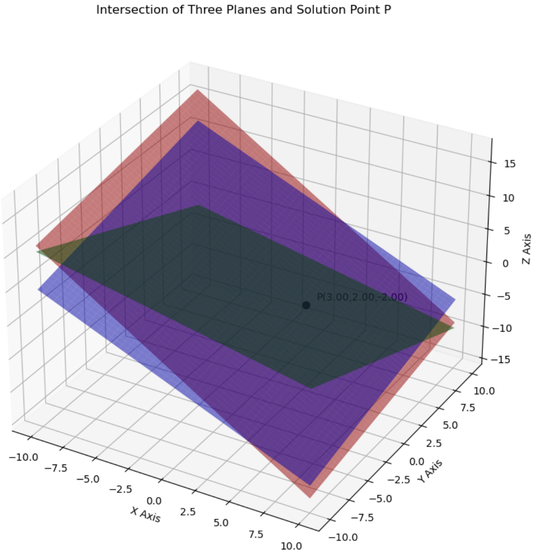

Fig : Planes

C Code - points.c

```
#include <stdio.h>

double *gaussian_elimination(double a[3][4]) {
    static double sol[3];
    int i, j, k;
    double ratio;
    int n = 3;

    // Forward elimination
    for (i = 0; i < n - 1; i++) {
        for (j = i + 1; j < n; j++) {
            ratio = a[j][i] / a[i][i];
            for (k = i; k < n + 1; k++) {
                a[j][k] -= ratio * a[i][k];
            }
        }
    }

    // Back substitution
    for (i = n - 1; i >= 0; i--) {
        sol[i] = a[i][3];
        for (j = i + 1; j < n; j++)
            sol[i] -= a[i][j] * sol[j];
        sol[i] /= a[i][i];
    }

    return sol;
}
```

Python Code - callc.py

```
import ctypes, os
import numpy as np
import matplotlib.pyplot as plt

figs_folder = os.path.join("..","figs")
os.makedirs(figs_folder, exist_ok=True)

lib = ctypes.CDLL("./points.so")
lib.gaussian_elimination.restype = ctypes.POINTER(ctypes.c_double)
lib.gaussian_elimination.argtypes = [((ctypes.c_double*4)*3)]

aug_matrix = ((ctypes.c_double*4)*3)()
aug_matrix[0][:] = [5, -1, 4, 5]
aug_matrix[1][:] = [2, 3, 5, 2]
aug_matrix[2][:] = [5, -2, 6, -1]

sol = lib.gaussian_elimination(aug_matrix)
solution = [sol[i] for i in range(3)]
x, y, z = solution
print("Solution␣from␣C:", solution)
```

Python Plot Code - plot.py

```
import os
import numpy as np
import matplotlib.pyplot as plt

figs_folder = os.path.join("..", "figs")
os.makedirs(figs_folder, exist_ok=True)

A = np.array([[5,-1,4],[2,3,5],[5,-2,6]], dtype=float)
b = np.array([5,2,-1], dtype=float)
solution = np.linalg.solve(A,b)
x,y,z = solution
print("Solution from NumPy:",solution)

x_vals = np.linspace(-10,10,100)
y_vals = np.linspace(-10,10,100)
X,Y = np.meshgrid(x_vals,y_vals)

Z1 = (5 - 5*X + Y)/4
Z2 = (2 - 2*X - 3*Y)/5
Z3 = (-1 - 5*X + 2*Y)/6

fig = plt.figure(figsize=(10,8))
ax = fig.add_subplot(111,projection="3d")
ax.plot_surface(X,Y,Z1,alpha=0.5,color="red")
ax.plot_surface(X,Y,Z2,alpha=0.5,color="green")
ax.plot_surface(X,Y,Z3,alpha=0.5,color="blue")
ax.scatter(x,y,z,color="black",s=50)
ax.text(x+0.5,y+0.5,z+0.5,f"P({x:.2f},{y:.2f},{z:.2f})",fontsize=10)
ax.set_xlabel("X Axis"); ax.set_ylabel("Y Axis"); ax.set_zlabel("Z Axis")
ax.set_title("Intersection of Three Planes and Solution Point P")
plt.tight_layout()
fig.savefig(os.path.join(figs_folder,"solution.png"))
plt.show()
```