Matgeo Presentation - Problem 2.4.16

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Problem Statement

Given two points

$$\mathbf{A} = \begin{pmatrix} 0 \\ 7 \\ -10 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 1 \\ 6 \\ -6 \end{pmatrix}, \qquad \mathbf{C} = \begin{pmatrix} 4 \\ 9 \\ -6 \end{pmatrix}.$$

(a) prove the given points forms isosceles triangle

(b) prove the given points forms right angled triangle

solution

solution : We consider the vectors

$$\mathbf{A} = \begin{pmatrix} 0 \\ 7 \\ -10 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 1 \\ 6 \\ -6 \end{pmatrix}, \qquad \mathbf{C} = \begin{pmatrix} 4 \\ 9 \\ -6 \end{pmatrix}.$$

Form the difference vectors $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$.

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 - 0 \\ 6 - 7 \\ -6 - (-10) \end{pmatrix} \tag{1.1}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \tag{1.2}$$

solution

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 - 0 \\ 9 - 7 \\ -6 - (-10) \end{pmatrix} \tag{1.3}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} \tag{1.4}$$

Place these as columns in the 3×2 matrix M.

$$M = \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix} \tag{1.5}$$

$$M = \begin{pmatrix} 1 & 4 \\ -1 & 2 \\ 4 & 4 \end{pmatrix} \tag{1.6}$$

solution

Compute the 2×2 minor using rows 1 and 2.

$$\det \begin{pmatrix} 1 & 4 \\ -1 & 2 \end{pmatrix} = 1 \cdot 2 - 4 \cdot (-1) \tag{1.7}$$

$$\det \begin{pmatrix} 1 & 4 \\ -1 & 2 \end{pmatrix} = 2 + 4 = 6 \neq 0 \tag{1.8}$$

Hence $\text{rank}(M) = 2$, so $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$ are linearly independent. Therefore $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are not collinear and determine a triangle.

solution A

verification of Isosceles triangle via perpendicular bisector:

$$\text{Midpoint } \mathbf{M} \text{ of } \overline{AC} = \frac{\mathbf{A}+\mathbf{C}}{2} = \frac{1}{2}\begin{pmatrix} 0+4 \\ 7+9 \\ -10+(-6) \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ -8 \end{pmatrix}, \tag{1.9}$$

$$\mathbf{MB} = \mathbf{B} - \mathbf{M} = \begin{pmatrix} 1-2 \\ 6-8 \\ -6-(-8) \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix}, \tag{1.10}$$

$$\mathbf{AC} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 4-0 \\ 9-7 \\ -6-(-10) \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix}. \tag{1.11}$$

$$\mathbf{MB}^\top \mathbf{AC} = \begin{pmatrix} -1 & -2 & 2 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} = (-1)\cdot 4 + (-2)\cdot 2 + 2\cdot 4 \tag{1.12}$$

$$= -4 - 4 + 8 = 0. \tag{1.13}$$

solution

Thus $\mathbf{MB} \perp \mathbf{AC}$ and $\mathbf{M}$ is the midpoint of AC, so the line through $\mathbf{M}$ in the direction of $\mathbf{MB}$ is the *perpendicular bisector* of AC and passes through $\mathbf{B}$.

By the perpendicular-bisector property, points on it are equidistant from A and C, hence

$$\|\mathbf{AB}\| = \|\mathbf{BC}\|. \tag{1.14}$$

Therefore, the triangle is *isosceles* with equal sides AB and BC.

$$\mathbf{MB}^\top \mathbf{AC} = \begin{pmatrix} -1 & -2 & 2 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} = (-1) \cdot 4 + (-2) \cdot 2 + 2 \cdot 4 = -4 - 4 + 8 = \tag{1.15}$$

Thus $\mathbf{MB} \perp \mathbf{AC}$ and $\mathbf{M}$ is the midpoint of AC, so the line through $\mathbf{M}$ in the direction of $\mathbf{MB}$ is the *perpendicular bisector* of AC and passes through $\mathbf{B}$. By the perpendicular-bisector property, points on it are equidistant from A and C, hence

solution

$$\|\mathbf{AB}\| = \|\mathbf{BC}\|. \tag{1.16}$$

Therefore, the triangle is *isosceles* with equal sides AB and BC.

B)veification for right angled triangle (matrix / inner-product test)

To show the triangle is right-angled, compute the inner product of two adjacent side vectors **AB** and **BC**.

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}, \tag{1.17}$$

$$\mathbf{BC} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 4-1 \\ 9-6 \\ -6-(-6) \end{pmatrix} \tag{1.18}$$

solution

$$\mathbf{BC} = \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}, \tag{1.19}$$

$$(\mathbf{AB})^T(\mathbf{BC}) = \begin{pmatrix} 1 & -1 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix} \tag{1.20}$$

$$(\mathbf{AB})^T(\mathbf{BC}) = 1 \cdot 3 + (-1) \cdot 3 + 4 \cdot 0 \tag{1.21}$$

$$(\mathbf{AB})^T(\mathbf{BC}) = 3 - 3 + 0 = 0. \tag{1.22}$$

Since the inner product is zero, $\mathbf{AB} \perp \mathbf{BC}$ and therefore the angle $\angle ABC$ is a right angle; the triangle is **right-angled at** B.

C Source Code:gen point.c

```
#include <stdio.h>
// Function to write points into a file
void generate_points(const char *filename) {
    FILE *fp = fopen(filename, "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return;
    }// Points A, B, C
    double A[3] = {0, 7, -10};
    double B[3] = {1, 6, -6};
    double C[3] = {4, 9, -6};
    fprintf(fp, "%lf %lf %lf\n", A[0], A[1], A[2]);
    fprintf(fp, "%lf %lf %lf\n", B[0], B[1], B[2]);
    fprintf(fp, "%lf %lf %lf\n", C[0], C[1], C[2]);
    fclose(fp);
}
```

Python Script: solve triangle.py

```
import ctypes
import numpy as np

# Load the shared object
lib = ctypes.CDLL("./gen_points.so")

# Call the C function to generate points.dat
lib.generate_points(b"points.dat")

# Load points from file
points = np.loadtxt("points.dat")
A, B, C = points

# Function to compute squared distance
def dist2(P, Q):
    return np.sum((P - Q) ** 2)
```

Python Script: solve triangle.py

```
# Squared lengths
AB2 = dist2(A, B)
BC2 = dist2(B, C)
CA2 = dist2(C, A)

print("Squared lengths:")
print("AB^2 =", AB2, " BC^2 =", BC2, " CA^2 =", CA2)

# Check isosceles (two sides equal)
isosceles = (AB2 == BC2) or (BC2 == CA2) or (CA2 == AB2)
print("Isosceles Triangle:", isosceles)

# Check right angle (Pythagoras theorem)
right_angle = (AB2 + BC2 == CA2) or (BC2 + CA2 == AB2) or (CA
print("Right Angled Triangle:", right_angle)
```

Python Script: plot triangle.py

```
import sys
sys.path.insert(0, '/home/dhanush-sagar/matgeo/codes/CoordGeo
import numpy as np
import matplotlib.pyplot as plt

# Local imports
from line.funcs import *
from triangle.funcs import *
from conics.funcs import circ_gen

# Load points
points = np.loadtxt("points.dat")
A, B, C = points
# Plot triangle
tri_coords = np.vstack((A, B, C, A))  # close loop
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
```

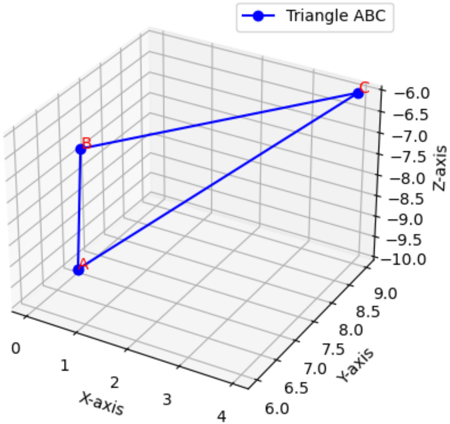
Python Script: plot triangle.py

```
ax.plot(tri_coords[:,0], tri_coords[:,1], tri_coords[:,2], 'b-

# Mark points
ax.text(A[0], A[1], A[2], "A", color='red')
ax.text(B[0], B[1], B[2], "B", color='red')
ax.text(C[0], C[1], C[2], "C", color='red')

ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.legend()
plt.savefig("triangle_plot.png")
plt.show()
```

Result Plot


Triangle ABC
A
B
C
0
1
2
3
4
X-axis
6.0
6.5
7.0
7.5
8.0
8.5
9.0
Y-axis
−6.0
−6.5
−7.0
−7.5
−8.0
−8.5
−9.0
−9.5
−10.0
Z-axis