4.7.24

EE25BTECH11032 - Kartik Lahoti

Question:

Find the equation of line passing through the point $(5, 2)$ and perpendicular to the line joining the points $(2, 3)$ and $(3, -1)$

Solution:

Given :

Symbol	Value	Description
A	$\begin{pmatrix}2\\3\end{pmatrix}$	Given Point
B	$\begin{pmatrix}3\\-1\end{pmatrix}$	Given Point
P	$\begin{pmatrix}5\\2\end{pmatrix}$	Given Point

Let , $\mathbf{X}$ be a vector on the Required Line

Direction Vector for the Line **AB** ,

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix}3\\-1\end{pmatrix} - \begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}1\\-4\end{pmatrix} \tag{0.1}$$

Direction Vector for the Required Line in terms of $\mathbf{X}$,

$$\mathbf{X} - \mathbf{P} = \left(\mathbf{X} - \begin{pmatrix}5\\2\end{pmatrix}\right) \tag{0.2}$$

Direction Vector for the Line **AB** is perpendicular to the required line

$$\therefore (\mathbf{B} - \mathbf{A})^{\top}\left(\mathbf{X} - \begin{pmatrix}5\\2\end{pmatrix}\right) = 0 \tag{0.3}$$

$$\begin{pmatrix}1 & -4\end{pmatrix}\left(\mathbf{X} - \begin{pmatrix}5\\2\end{pmatrix}\right) = 0 \tag{0.4}$$

Hence, the desired equation is

$$\begin{pmatrix}1 & -4\end{pmatrix}\mathbf{X} = -3 \tag{0.5}$$

Fig:4.7.24