

12.768

EE25BTECH11026-Harsha

Question:

In the figure, the vectors \mathbf{u} and \mathbf{v} are related as $\mathbf{A}\mathbf{u} = \mathbf{v}$ by a transformation matrix \mathbf{A} . The correct choice of \mathbf{A} is

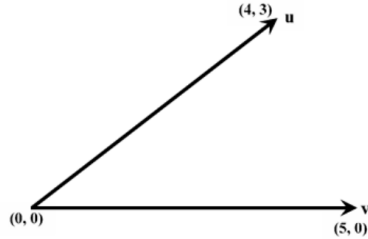


Fig. 0.1: Figure-1

- 1) $\begin{pmatrix} 4 & 3 \\ 5 & -4 \end{pmatrix}$ 2) $\begin{pmatrix} 4 & -3 \\ 5 & 5 \end{pmatrix}$ 3) $\begin{pmatrix} 4 & 3 \\ 5 & -3 \end{pmatrix}$ 4) $\begin{pmatrix} 4 & -3 \\ 5 & -5 \end{pmatrix}$

Solution:

Let us solve the given question theoretically and then verify the solution computationally.

Given,

$$\mathbf{u} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad \mathbf{v} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad (4.1)$$

From 0.1,

$$\|\mathbf{u}\| = \|\mathbf{v}\| = 5 \text{ units} \quad (4.2)$$

This implies, \mathbf{A} is a rotation matrix.

Rotation matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \quad (4.3)$$

where θ is the angle between the vectors in counter-clockwise sense.

$$\therefore \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad (4.4)$$

$$\begin{pmatrix} 4 \cos \theta - 3 \sin \theta \\ 4 \sin \theta + 3 \cos \theta \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad (4.5)$$

The above equation can be re-arranged as,

$$\begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad (4.6)$$

We need to solve for $\cos \theta$ and $\sin \theta$ to get the transformation matrix \mathbf{A} .

Forming the augmented matrix,

$$\Rightarrow \left(\begin{array}{cc|c} 4 & -3 & 5 \\ 3 & 4 & 0 \end{array} \right) \xleftrightarrow{R_2 \leftarrow R_2 - \frac{3}{4} \times R_1} \left(\begin{array}{cc|c} 4 & -3 & 5 \\ 0 & \frac{25}{4} & -\frac{15}{4} \end{array} \right) \xleftrightarrow[R_1 \leftarrow \frac{1}{4}(R_1 + 3 \times R_2)]{R_2 \leftarrow \frac{4}{25} \times R_2} \left(\begin{array}{cc|c} 1 & 0 & \frac{4}{5} \\ 0 & 1 & -\frac{3}{5} \end{array} \right) \quad (4.7)$$

$$\therefore \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} \frac{4}{5} \\ -\frac{3}{5} \end{pmatrix} \quad (4.8)$$

$$\Rightarrow \mathbf{A} = \begin{pmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{pmatrix} \quad (4.9)$$