12.601

AI25BTECH11003 - Bhavesh Gaikwad

Question: The matrix $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$, one of the eigen values is 1. The eigen vectors corresponding to the eigne value 1 are: (CS 2016)

a) $\alpha \begin{pmatrix} 4 & -2 & 1 \end{pmatrix}, \alpha \neq 0,\ \alpha \epsilon \mathbb{R}$
b) $\alpha \begin{pmatrix} -4 & 2 & 1 \end{pmatrix}, \alpha \neq 0,\ \alpha \epsilon \mathbb{R}$
c) $\alpha \begin{pmatrix} -2 & 0 & 1 \end{pmatrix}, \alpha \neq 0,\ \alpha \epsilon \mathbb{R}$
d) $\alpha \begin{pmatrix} 2 & 0 & 1 \end{pmatrix}, \alpha \neq 0,\ \alpha \epsilon \mathbb{R}$

Solution:

Given: $\lambda = 1$, *Let* $\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$

$$\mathbf{A} - \mathbf{I} = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{pmatrix} \tag{0.1}$$

Row Transformation-1: $R_1 \leftrightarrow R_3$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 2 \end{pmatrix} \tag{0.2}$$

Row Transformation-2: $R_2 \leftrightarrow R_3$

$$\mathbf{A} - \mathbf{I} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \tag{0.3}$$

Let $\mathbf{v}$ be the corresponding eigenvector.

$$\implies \mathbf{A}\mathbf{v} = (1)\mathbf{v} \tag{0.4}$$

$$(\mathbf{A} - \mathbf{I})\mathbf{v} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \tag{0.5}$$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{v} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \tag{0.6}$$

Let $\mathbf{v} = \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$

Substituting value of $\mathbf{v}$ in Equation 0.6,

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \tag{0.7}$$

$$Row - 1 \rightarrow \; v_2 + 2v_2 = 0 \tag{0.8}$$

$$Row - 2 \rightarrow \; v_1 + 2v_3 = 0 \tag{0.9}$$

$$Row - 3 \rightarrow \; 0 + 0 + 0 = 0 \text{ (Always true)} \tag{0.10}$$

Let $v_3 = \alpha$ (Free parameter)
Substituting value of v_3 in Equations 0.8 and 0.9

$$\therefore \; v_2 = -2\alpha \,\&\, v_1 = 4\alpha \tag{0.11}$$

$$\therefore \; \mathbf{v} = \alpha \begin{pmatrix} 4 & -2 & 1 \end{pmatrix} \tag{0.12}$$

$$\boxed{\text{Thus, Option-A is correct.}}$$