4.8.8

AI25BTECH11012 - GARIGE UNNATHI

Question:
Find the equation of the plane passing through the point (-1,3,2) and perpendicular to the planes x + 2y + 3z = 5 and 3x + 3y + z = 0. .

Solution:
The equation of a plane can be given by the formula :

$$\mathbf{n^T x} = c$$

From the above formula we can write :

$$x + 2y + 3z = 5 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}^T \mathbf{x} = 5 \tag{0.1}$$

$$3x + 3y + z = 0 = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}^T \mathbf{x} = 0 \tag{0.2}$$

Variable	**Value**
$\mathbf{n_1}$	$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$
$\mathbf{n_2}$	$\begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}$
$\mathbf{A}$	$\begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$

TABLE 0: Variables Used

Let us assume the equation of the plane to be

$$\mathbf{n^T x} = 1 \tag{0.3}$$

$$or \tag{0.4}$$

$$\mathbf{x^T n} = 1 \tag{0.5}$$

As point $\mathbf{A}$ lies on the plane we can write :

$$\mathbf{A^T n} = 1 \tag{0.6}$$

If two planes are perpendicular then there normal vectors must also be perpendicular ,using this we can write :

$$\mathbf{n_1^T n} = 0 \tag{0.7}$$

$$\mathbf{n_2^T n} = 0 \tag{0.8}$$

Combining equations 0.6,0.7 and 0.8 ,we get :

$$\begin{pmatrix}\mathbf{A} & \mathbf{n_1} & \mathbf{n_2}\end{pmatrix}^{\mathbf{T}}\mathbf{n} = \begin{pmatrix} -1 & 3 & 2 \\ 1 & 2 & 3 \\ 3 & 3 & 1 \end{pmatrix}\mathbf{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{0.9}$$

Solving the above equation by row reduction we get :

$$\mathbf{n} = \begin{pmatrix} -\frac{7}{25} \\ \frac{8}{25} \\ -\frac{3}{25} \end{pmatrix} = \frac{1}{25}\begin{pmatrix} -7 \\ 8 \\ -3 \end{pmatrix} \tag{0.10}$$

From the equation 0.3 we can write the plane euation as :

$$\begin{pmatrix} -7 \\ 8 \\ -3 \end{pmatrix}^T \mathbf{x} = 25 \tag{0.11}$$

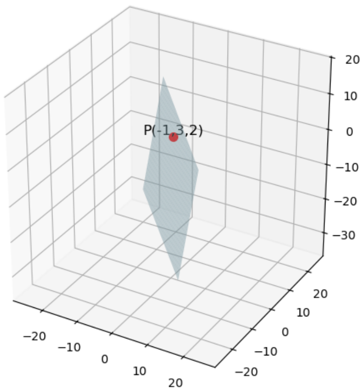

Fig. 0.1