5.4.16

EE25BTECH11021 - Dhanush sagar

Question:
Using elementary transformations, find the inverse of the following matrix.

$$\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$$

Solution: Given

$$\mathbf{A} = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \tag{1}$$

Let $\mathbf{A}^{-1}$ be the inverse of $\mathbf{A}$. Then

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I} \tag{2}$$

Augmented matrix of $\left(\mathbf{A} \mid \mathbf{I}\right)$ is given by

$$\left(\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right) \tag{3}$$

Perform the elementary row operation $R_2 \to 2R_2 - R_1$ to eliminate the first column entry of R_2:

$$\left(\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right) \xrightarrow{R_2 \to 2R_2 - R_1} \left(\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 0 & 1 & -1 & 2 \end{array}\right) \tag{4}$$

Now eliminate the 5 above the (2,2) pivot by $R_1 \to R_1 - 5R_2$:

$$\left(\begin{array}{cc|cc} 2 & 5 & 1 & 0 \\ 0 & 1 & -1 & 2 \end{array}\right) \xrightarrow{R_1 \to R_1 - 5R_2} \left(\begin{array}{cc|cc} 2 & 0 & 6 & -10 \\ 0 & 1 & -1 & 2 \end{array}\right) \tag{5}$$

Finally make the leading entry of R_1 unity by $R_1 \to \frac{1}{2}R_1$:

$$\left(\begin{array}{cc|cc} 2 & 0 & 6 & -10 \\ 0 & 1 & -1 & 2 \end{array}\right) \xrightarrow{R_1 \to \frac{1}{2}R_1} \left(\begin{array}{cc|cc} 1 & 0 & 3 & -5 \\ 0 & 1 & -1 & 2 \end{array}\right) \tag{6}$$

Hence the inverse of the matrix $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$ is

$$\mathbf{A}^{-1} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}.$$