9.4.33

EE25BTECH11020 - Darsh Pankaj Gajare

Question:
Find the roots of the following quadratic equation graphically. $x^2 - 4x + 3$

Solution:

The parabola can be expressed in matrix form as

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + \mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

where

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}, \quad f = 3. \tag{2}$$

The line $y = 0$ is expressed as:

$$\mathbf{x} = \mathbf{q} + \lambda \mathbf{m}, \quad \mathbf{q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{3}$$

Substituting

$$(\mathbf{q} + \lambda \mathbf{m})^\top \mathbf{V}(\mathbf{q} + \lambda \mathbf{m}) + \mathbf{u}^\top (\mathbf{q} + \lambda \mathbf{m}) + f = 0. \tag{4}$$

$$\lambda^2 (\mathbf{m}^\top \mathbf{V}\mathbf{m}) + \lambda (2\mathbf{q}^\top \mathbf{V}\mathbf{m} + \mathbf{u}^\top \mathbf{m}) + (\mathbf{q}^\top \mathbf{V}\mathbf{q} + \mathbf{u}^\top \mathbf{q} + f) = 0. \tag{5}$$

$$\mathbf{m}^\top \mathbf{V}\mathbf{m} = 1, \tag{6}$$

$$2\mathbf{q}^\top \mathbf{V}\mathbf{m} + \mathbf{u}^\top \mathbf{m} = -4, \tag{7}$$

$$\mathbf{q}^\top \mathbf{V}\mathbf{q} + \mathbf{u}^\top \mathbf{q} + f = 3. \tag{8}$$

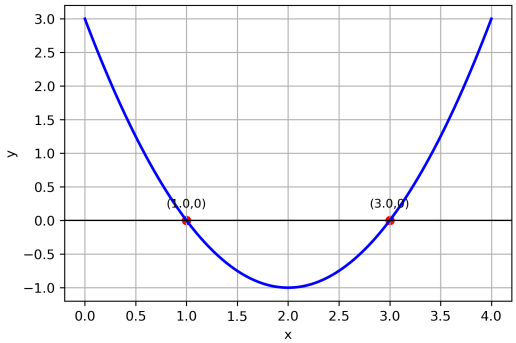
$$\lambda^2 - 4\lambda + 3 = 0. \tag{9}$$

$$\lambda_1 = 1, \quad \lambda_2 = 3. \tag{10}$$

Thus, the intersection points are

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathbf{x}_2 = \begin{pmatrix} 3 \\ 0 \end{pmatrix}. \tag{11}$$

Plot using C libraries:



Plot using Python: