2.6.32

AI25BTECH11021 - Abhiram Reddy N

Question 2.6.32

Find the area of the triangle whose vertices are

$$(1,-1), \quad (-4,6), \quad (-3,5).$$

Solution

Step 1: Define the vertices as vectors

$$A = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad B = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \quad C = \begin{pmatrix} -3 \\ 5 \end{pmatrix}$$

Step 2: Calculate the vectors $A - B$ and $B - C$

$$A - B = \begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \end{pmatrix} = \begin{pmatrix} 5 \\ -7 \end{pmatrix}$$

$$B - C = \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

Step 3: Calculate the 2D cross product magnitude

For vectors $\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$, the 2D cross product is

$$\mathbf{u} \times \mathbf{v} = u_1 v_2 - u_2 v_1.$$

Applying this,

$$(A - B) \times (B - C) = 5 \times 1 - (-7) \times (-1) = 5 - 7 = -2$$

$$\Rightarrow \|(A - B) \times (B - C)\| = 2$$

Step 4: Calculate the area of the triangle

$$ar(ABC) = \frac{1}{2} \times \|(A - B) \times (B - C)\| = \frac{1}{2} \times 2 = 1$$

$$\boxed{ar(ABC) = 1 \text{ square unit}}$$

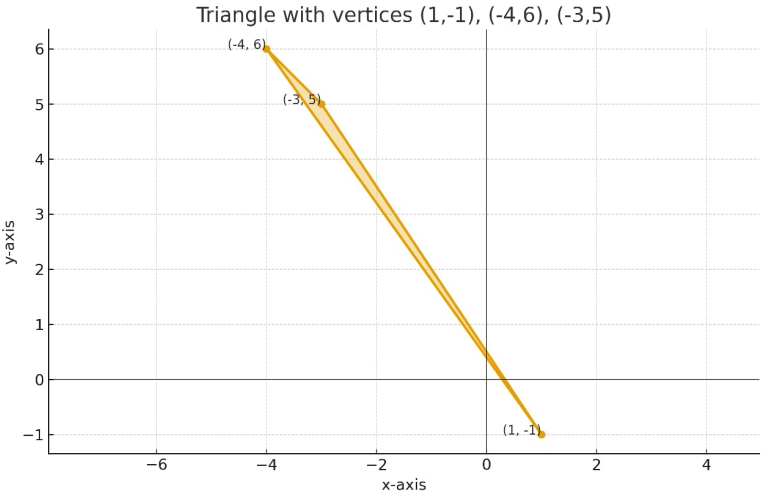
Triangle with vertices (1,-1), (-4,6), (-3,5)
(-4, 6)
(-3, 5)
(1, -1)
y-axis
x-axis
6
5
4
3
2
1
0
−1
−6
−4
−2
0
2
4

Fig. 0.1: plot