4.12.48

AI25BTECH11034 - Sujal Chauhan

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Prove that if a plane has the intercept a,b,c and is at a distance of p units from the origin, then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$

Solution

Point	Positon vector
a	$\begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}$
b	$\begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix}$
c	$\begin{pmatrix} 0 \\ 0 \\ c \end{pmatrix}$

(1)

Equation of plane in vector form is given by

$$\mathbf{n}^T \mathbf{P} = d \qquad (2)$$

Solution

where **n** is normal unit vector to the plane and d is the distance of the plane from the origin.

Now given plane satisfys the vectors

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \mathbf{n} = \begin{pmatrix} p \\ p \\ p \end{pmatrix} \tag{3}$$

Now multiply both side with inverse of the matrix

$$\mathbf{n} = \frac{1}{abc} \begin{pmatrix} bc & 0 & 0 \\ 0 & ac & 0 \\ 0 & 0 & ab \end{pmatrix} \begin{pmatrix} p \\ p \\ p \end{pmatrix} \tag{4}$$

$$\mathbf{n} = \begin{pmatrix} \frac{p}{a} \\ \frac{p}{b} \\ \frac{p}{c} \end{pmatrix} \tag{5}$$

Now since **n** is a unit vector then

$$\mathbf{n}^T \mathbf{n} = 1 \tag{6}$$

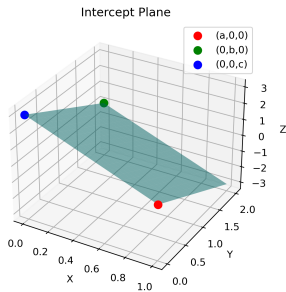$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2} \tag{7}$$

Figure

Figure: Ploting