3.2.3

EE25BTECH11062 - Vivek K Kumar

Question:
Draw a parallelogram ABCD in which $BC = 5cm, AB = 3cm$ and $\angle ABC = 60°$, divide it into triangles ACB and ABD by the diagonal BD

Solution:

Let **A**, **B**, **C** and **D** represent position vectors of the vertices of parallelogram.

Given information,

$$\|\mathbf{A} - \mathbf{B}\| = 3 \tag{0.1}$$

$$\|\mathbf{C} - \mathbf{B}\| = 5 \tag{0.2}$$

$$\angle B = \frac{\pi}{3} \tag{0.3}$$

The coordinates of $\mathbf{A}, \mathbf{B}, \mathbf{C}$ can be expressed as

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.4}$$

$$\mathbf{C} = \|\mathbf{C} - \mathbf{B}\| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.5}$$

$$\mathbf{A} = \|\mathbf{A} - \mathbf{B}\| \begin{pmatrix} cosB \\ sinB \end{pmatrix} \tag{0.6}$$

Since $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ form vertices of a parallelogram,

$$\frac{\mathbf{A} + \mathbf{C}}{2} = \frac{\mathbf{B} + \mathbf{D}}{2} \tag{0.7}$$

$$\mathbf{D} = \mathbf{A} + \mathbf{C} - \mathbf{B} \tag{0.8}$$

$$\mathbf{D} = \|\mathbf{A} - \mathbf{B}\| \begin{pmatrix} cosB \\ sinB \end{pmatrix} + \|\mathbf{C} - \mathbf{B}\| \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.9}$$

Substituting values,

$$\mathbf{A} = \begin{pmatrix} 3/2 \\ 3\sqrt{3}/2 \end{pmatrix} \tag{0.10}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.11}$$

$$\mathbf{C} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{0.12}$$

$$\mathbf{D} = \begin{pmatrix} 13/2 \\ 3\sqrt{3}/2 \end{pmatrix} \tag{0.13}$$

Name	**Point**
A	$\begin{pmatrix} 3/2 \\ 3\sqrt{3}/2 \end{pmatrix}$
B	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
C	$\begin{pmatrix} 5 \\ 0 \end{pmatrix}$
D	$\begin{pmatrix} 13/2 \\ 3\sqrt{3}/2 \end{pmatrix}$

TABLE 0: Coordinates of the vertices of parallelogram

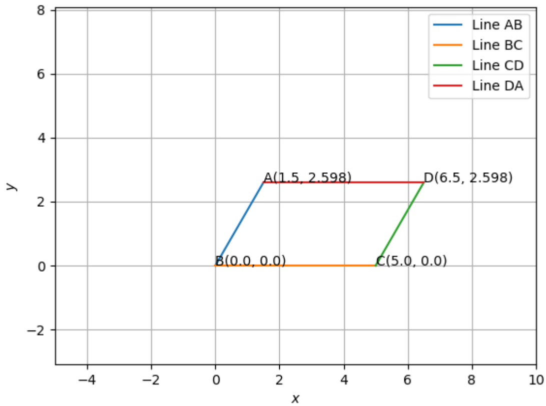

Fig. 0.1: Vectors $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{A} + \mathbf{B} + \mathbf{C}$