5.2.28

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19th september, 2025

Question

Solve the system of linear equations:

$$5x - 8y = -1 \tag{1}$$

$$3x - \frac{24}{5}y = \frac{-3}{5} \tag{2}$$

Given

Given

$$\begin{pmatrix} 5 & -8 \end{pmatrix} \mathbf{x} = -1; \begin{pmatrix} 3 & \left(\frac{-24}{5}\right) \end{pmatrix} \mathbf{x} = \frac{-3}{5} \tag{3}$$

$$A = \begin{pmatrix} 5 & -8 \\ 3 & \left(\frac{-24}{5}\right) \end{pmatrix}; \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}; \mathbf{b} = \begin{pmatrix} -1 \\ \left(\frac{-3}{5}\right) \end{pmatrix} \tag{4}$$

$$A\mathbf{x} = \mathbf{b} \tag{5}$$

Given

Let:
Rank of coefficient matrix $= r$
Rank of Augmented matrix $= r_a$
Order of coefficient matrix $= n$

Solving

Augmented Matrix:

$$\left(\begin{array}{cc|c} 5 & -8 & -1 \\ 3 & \left(\frac{-24}{5}\right) & \left(\frac{-3}{5}\right) \end{array}\right) \tag{6}$$

$$R_2 \to R_2 - \frac{3}{5}R_1 \tag{7}$$

$$\left(\begin{array}{cc|c} 5 & -8 & -1 \\ 0 & 0 & 0 \end{array}\right) \tag{8}$$

$$r = 1; r_a = 1; n = 2 \tag{9}$$

$$\because r = r_a < n \tag{10}$$

Infinite solutions exist for the given system of linear equations.

C Code

```c
#include<stdio.h>

void make_data(double *points){
    double n = 2;
    points[0] = 5;
    points[1] = -8;
    points[2] = -1;
    points[3] = 3;
    points[4] = -4.8;
    points[5] = -0.6;
    points[6] = 2;
}
```

C Code

```
void processing(double rA, double rB, double n, double X, double
    Y){
    if(rA==rB&&rB==n){
        printf("Unique solution exists for the given system of
            linear equations.\n");
        printf("The solution for the given system of linear
            equations is: x=%.2lf, y=%.2lf", X, Y);
    }
    else if(rA==rB&&rA!=n){
        printf("Infinite solutions exist for the given system of
            linear equations in 2 variables.");
    }
    else{
        printf("No solution exists for the given system of linear
             equations in 2 variables");
    }
}
```

Python Code 1

```
import ctypes as ct
import sympy as sp

lib = ct.CDLL("./problem.so")
entry = ct.c_double*7
lib.make_data.argtypes = [ct.POINTER(ct.c_double)]
lib.processing.argtypes = [ct.c_double, ct.c_double, ct.c_double,
     ct.c_double, ct.c_double]
data = entry()
lib.make_data(data)
A = sp.Matrix([[data[0], data[1], data[2]],
                  [data[3], data[4], data[5]]])
B = sp.Matrix([[data[0], data[1]],
                  [data[3], data[4]]])
```

Python Code 1

```
C = ([data[0], data[3]])
D = ([data[1], data[4]])
E = data[2]
F = data[5]
n = data[6]
def get_data():
    return C, D, E, F
rA = A.rank()
rB = B.rank()
rref_matrix, pivots = A.rref()
lib.processing(rA, rB, n, rref_matrix[0,2], rref_matrix[1,2])
```

Python Code 2

```
import matplotlib.pyplot as plt
import numpy as np
from call import get_data

C, D, E, F = get_data()

x = np.linspace(-10, 10, 100)
y = -((C[0]*x)-E)/D[0]

X = np.linspace(-15, 15, 100)
Y = -((C[0]*X)-E)/D[0]
```

Python Code 2

```
plt.plot(X, Y, '-k')
plt.plot(x, y, '-r')
plt.text(-13.64, -8.96, r'$5x-8y=-1$', fontsize=10, color='black'
    )
plt.text(1.06, 1.08, r'$3x-\frac{24}{5}y=-\frac{3}{5}$', fontsize
    =10, color='black')
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.axis('equal')
plt.grid(True)
plt.savefig("../figs/plot.png")
plt.show()
```

Plot

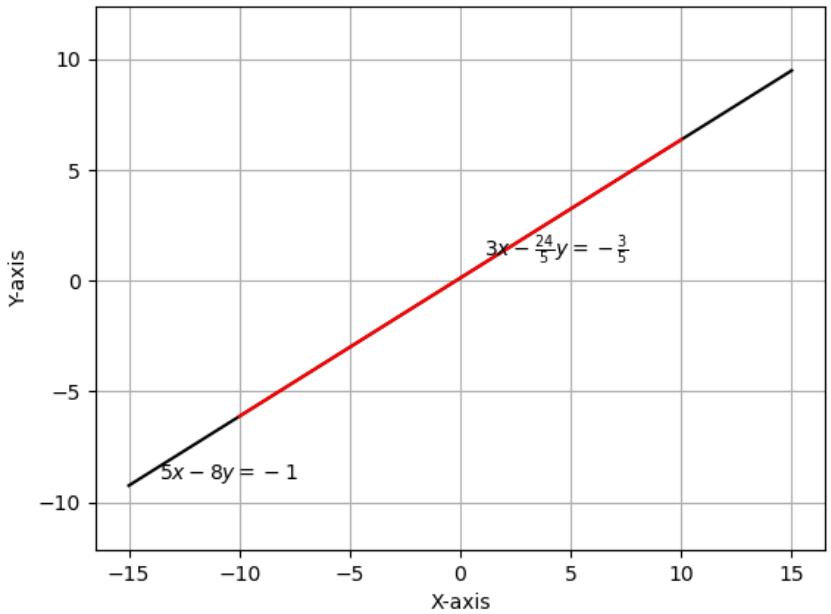

Figure: Plot of the given line equations