

# Matrices in Geometry 1.5.25

EE25BTECH11037 - Divyansh

**Question:** In what ratio does the point  $\left(\frac{24}{11}, y\right)$  divide the line segment joining the points  $\mathbf{P} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$  and  $\mathbf{Q} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ ? Also find the value of  $y$ .

**Given:**  $\mathbf{P} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ ,  $\mathbf{Q} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$  and a point  $\mathbf{R} = \begin{pmatrix} \frac{24}{11} \\ y \end{pmatrix}$  on  $PQ$ .

Let  $R$  divide  $PQ$  internally in the ratio  $k : 1$ .

Therefore, they are defined to be collinear if,

$$\text{rank}(\mathbf{R} - \mathbf{P} \quad \mathbf{Q} - \mathbf{R}) = 1$$

$$\mathbf{R} - \mathbf{P} = \begin{pmatrix} \frac{2}{11} \\ y + 2 \end{pmatrix}$$

$$\mathbf{Q} - \mathbf{R} = \begin{pmatrix} \frac{9}{11} \\ 7 - y \end{pmatrix}$$

$$\text{rank} \begin{pmatrix} \frac{2}{11} & \frac{9}{11} \\ y + 2 & 7 - y \end{pmatrix} = 1$$

$$\begin{pmatrix} \frac{2}{11} & \frac{9}{11} \\ y + 2 & 7 - y \end{pmatrix} \xrightarrow{C_1 \rightarrow C_1 + C_2} \begin{pmatrix} \frac{2}{11} & 1 \\ y + 2 & 9 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 - 9R_1} \begin{pmatrix} \frac{2}{11} & 1 \\ y + \frac{4}{11} & 0 \end{pmatrix}$$

for the rank of this matrix to be 1, its 2<sup>nd</sup> row has to be 0

$$y + \frac{4}{11} = 0$$

$$\therefore y = \frac{-4}{11}$$

We know that  $k$  is the ratio in which  $\mathbf{R}$  divides  $\mathbf{P}$  and  $\mathbf{Q}$ ,

$$\mathbf{R} = \frac{k\mathbf{Q} + \mathbf{P}}{1 + k}$$

$$k(\mathbf{R} - \mathbf{Q}) = \mathbf{P} - \mathbf{R}$$

$$\Rightarrow k = \frac{(\mathbf{P} - \mathbf{R})^\top (\mathbf{R} - \mathbf{Q})}{\|\mathbf{R} - \mathbf{Q}\|^2}$$

$$(\mathbf{P} - \mathbf{R})^\top = \begin{pmatrix} -\frac{2}{11} & -\frac{18}{11} \end{pmatrix}$$

$$(\mathbf{R} - \mathbf{Q}) = \begin{pmatrix} -\frac{9}{11} \\ -\frac{81}{11} \end{pmatrix}$$

$$\|\mathbf{R} - \mathbf{Q}\|^2 = \frac{81}{121} + \frac{6561}{121} = \frac{6642}{121}$$

$$\therefore k = \frac{\begin{pmatrix} -\frac{2}{11} & -\frac{18}{11} \end{pmatrix} \begin{pmatrix} -\frac{9}{11} \\ -\frac{81}{11} \end{pmatrix}}{\frac{6642}{121}}$$

$$\Rightarrow k = \frac{\frac{18}{121} + \frac{1458}{121}}{\frac{6642}{121}} \Rightarrow k = \frac{1476}{6624} = \frac{2}{9}$$

Hence, the final answer is  $k = \frac{2}{9}$  and  $y = \frac{-4}{11}$

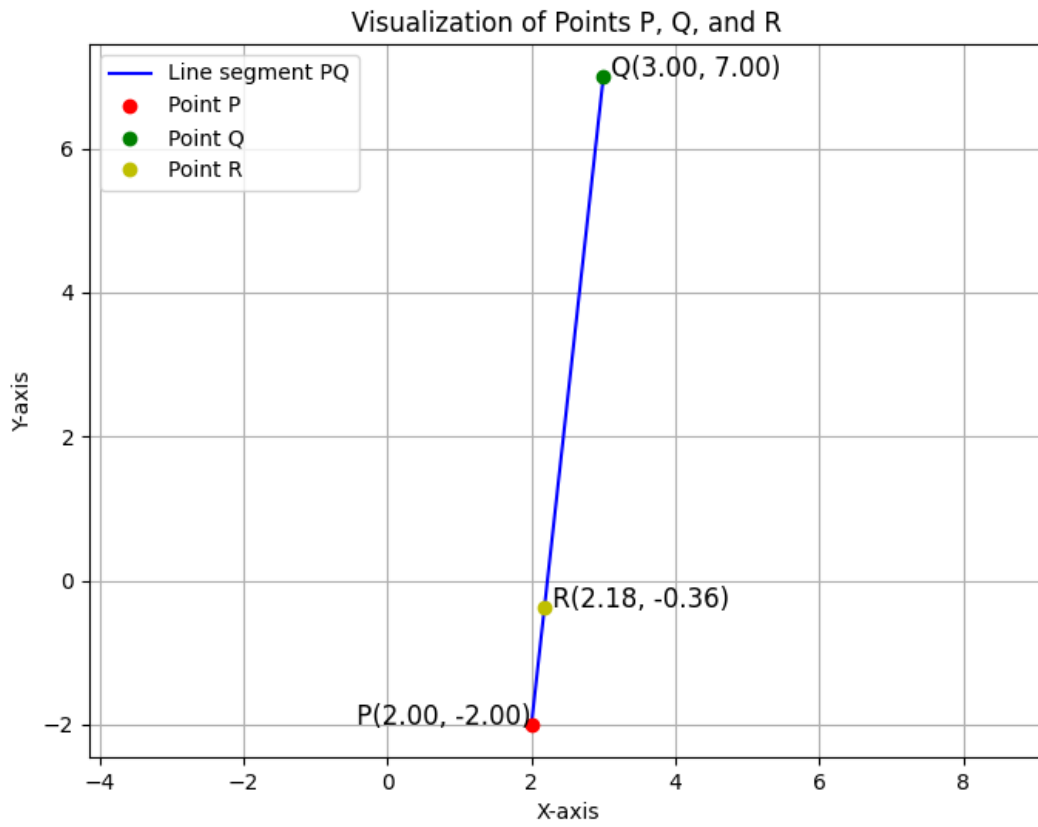


Fig. 1: Plot for 1.5.25