1.4.12

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Problem:

In what ratio does the point $P(-4, 6)$ divide the line segment joining the points $A(-6, 0)$ and $C(3, -8)$?

Solution:

Given Points

$$\mathbf{A} = \begin{pmatrix} -6 \\ 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 3 \\ -8 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} -4 \\ 6 \end{pmatrix},$$

the ratio k in which P divides AC is

$$k = \frac{(\mathbf{A} - \mathbf{P})^T (\mathbf{P} - \mathbf{C})}{\|\mathbf{P} - \mathbf{C}\|^2}$$

$$= \frac{\begin{pmatrix} -6+4 \\ 0-6 \end{pmatrix}^T \begin{pmatrix} -4-3 \\ 6+8 \end{pmatrix}}{(-4-3)^2 + (6+8)^2}$$

$$= \frac{\begin{pmatrix} -2 \\ -6 \end{pmatrix}^T \begin{pmatrix} -7 \\ 14 \end{pmatrix}}{(-7)^2 + 14^2}$$

Therefore,

$$k = \frac{-70}{245} = -\frac{2}{7}.$$

Since $k = -\frac{2}{7}$, Point P divides the segment AC **externally** in the ratio $2 : 7$.