

Matgeo Presentation - Problem 1.6.6

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Problem Statement

In each of the following, find the value of k for which the points are collinear:

$$(7, -2), (5, 1), (3, k)$$

$$(8, 1), (k, -4), (2, -5)$$

Method

Condition for Collinearity:

Three points A, B, C are collinear iff vectors

$$\mathbf{B} - \mathbf{A}, \quad \mathbf{C} - \mathbf{A}$$

are linearly dependent.

Equivalently, the *collinearity matrix*

$$M = [\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A}]^T$$

must satisfy $\text{rank}(M) = 1$.

Part (a) Setup

Let

$$\mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 3 \\ k \end{pmatrix}.$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 5 - 7 \\ 1 - (-2) \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix},$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 3 - 7 \\ k - (-2) \end{pmatrix} = \begin{pmatrix} -4 \\ k + 2 \end{pmatrix}.$$

Thus,

$$\mathbf{M} = \begin{pmatrix} -2 & 3 \\ -4 & k + 2 \end{pmatrix}.$$

Part (a) Row Reduction

Apply row transformation:

$$R_2 \leftarrow R_2 - 2R_1 \quad \implies \quad \begin{pmatrix} -2 & 3 \\ 0 & k - 4 \end{pmatrix}.$$

$$\text{For collinearity: } k - 4 = 0 \quad \implies \quad k = \boxed{4}.$$

Part (b) Setup

Let

$$A = \begin{pmatrix} 8 \\ 1 \end{pmatrix}, \quad B = \begin{pmatrix} k \\ -4 \end{pmatrix}, \quad C = \begin{pmatrix} 2 \\ -5 \end{pmatrix}.$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} k - 8 \\ -5 \end{pmatrix}, \quad \mathbf{C} - \mathbf{A} = \begin{pmatrix} -6 \\ -6 \end{pmatrix}.$$

Thus,

$$M = \begin{pmatrix} k - 8 & -5 \\ -6 & -6 \end{pmatrix}.$$

Part (b) Row Reduction

Row operation:

$$\begin{aligned} R_2 &\leftarrow (k - 8)R_2 + 6R_1 \\ \Rightarrow &\begin{pmatrix} k - 8 & -5 \\ 0 & 18 - 6k \end{pmatrix}. \end{aligned}$$

$$\text{For collinearity: } 18 - 6k = 0 \quad \Rightarrow \quad k = \boxed{3}.$$

Final Answer

(a) $k = 4$

(b) $k = 3$

Plot of line1

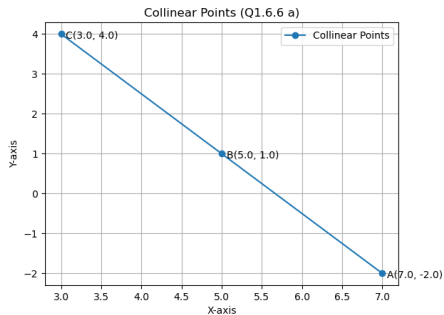


Fig 1 : Line through the given points

Plot of line2

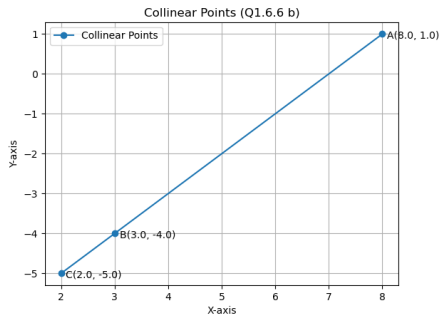


Fig 2 : Line through the given points