# 2.3.2

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## Question

**Q.** Find the angle between unit vectors **a** and **b** such that $\sqrt{3}\,\mathbf{a}-\mathbf{b}$ is also a unit vector.

## Solution

**Given:** $\|\left(a\right)\| = \|\left(b\right)\| = 1$ and $\left\|\sqrt{3}\left(a\right)-\left(b\right)\right\| = 1.$

Use the length definition $\|x\|^2 = x^\top x$ and the scalarâproduct relation $\left(a\right)^\top\left(b\right) = \|\left(a\right)\|\,\|\left(b\right)\|\cos\theta.$

$$\begin{aligned}
\left\|\sqrt{3}\left(a\right)-\left(b\right)\right\|^2 &= \left(\sqrt{3}\left(a\right)-\left(b\right)\right)^\top\left(\sqrt{3}\left(a\right)-\left(b\right)\right)\\
&= 3\left(a\right)^\top\left(a\right)+\left(b\right)^\top\left(b\right)-2\sqrt{3}\left(a\right)^\top\left(b\right)\\
&= 3\|\left(a\right)\|^2+\|\left(b\right)\|^2-2\sqrt{3}\,\|\left(a\right)\|\,\|\left(b\right)\|\cos\theta\\
&= 3+1-2\sqrt{3}\cos\theta.
\end{aligned}$$

Since $\left\|\sqrt{3}\left(a\right)-\left(b\right)\right\| = 1$, we get

$$1 = 4-2\sqrt{3}\cos\theta \implies 3 = 2\sqrt{3}\cos\theta \implies \cos\theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}.$$

Hence,

$$\boxed{\theta = 30^\circ}.$$

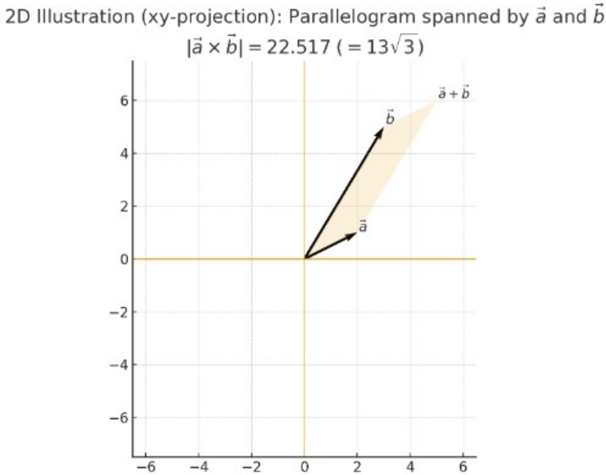


Fig. 0.1: xy-projection of **a** and **b**; $|\mathbf{a} \times \mathbf{b}| = 13\sqrt{3}$.