4.11.14

EE25BTECH11036 - M Chanakya Srinivas

Question

Find the value of λ for which the following lines are perpendicular to each other. Hence determine whether the lines intersect or not.

$$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}, \tag{1}$$

$$\frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}. \tag{2}$$

Solution

Vector form and perpendicularity

Take

$$\mathbf{A}_1 = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}, \qquad \mathbf{m}_1 = \begin{pmatrix} 5\lambda+2 \\ -5 \\ 1 \end{pmatrix}, \tag{3}$$

$$\mathbf{A}_2 = \begin{pmatrix} 0 \\ -\frac{1}{2} \\ 1 \end{pmatrix}, \qquad \mathbf{m}_2 = \begin{pmatrix} 1 \\ 2\lambda \\ 3 \end{pmatrix}. \tag{4}$$

Perpendicularity: $\mathbf{m}_1^\top \mathbf{m}_2 = 0$.

$$\mathbf{m}_1^\top \mathbf{m}_2 = \begin{pmatrix} 5\lambda+2 & -5 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2\lambda \\ 3 \end{pmatrix} \tag{5}$$

$$= (5\lambda+2) - 10\lambda + 3 = -5\lambda + 5, \tag{6}$$

so

$$-5\lambda + 5 = 0 \implies \boxed{\lambda = 1}. \tag{7}$$

Intersection: set up linear system

Intersection requires κ_1, κ_2 with

$$\kappa_1 \mathbf{m}_1 - \kappa_2 \mathbf{m}_2 = \mathbf{A}_2 - \mathbf{A}_1.$$

Augmented matrix (written explicitly):

$$\mathcal{M}_0 = \left[\begin{array}{cc|c} 5\lambda+2 & -1 & -5 \\ -5 & -2\lambda & -\frac{5}{2} \\ 1 & -3 & 0 \end{array}\right]. \tag{8}$$

We now reduce $\mathcal{M}_0$ to RREF using explicit row operations.

RREF — Step 1: make a leading 1 in row 1: Swap $R_1 \leftrightarrow R_3$:

$$\mathcal{M}_1 = \left[\begin{array}{cc|c} 1 & -3 & 0 \\ -5 & -2\lambda & -\frac{5}{2} \\ 5\lambda+2 & -1 & -5 \end{array}\right]. \tag{9}$$

RREF — Step 2: eliminate column 1 in R2 and R3: Perform

$$R_2 \leftarrow R_2 + 5R_1, \qquad R_3 \leftarrow R_3 - (5\lambda+2)R_1.$$

This yields

$$\mathcal{M}_2 = \left[\begin{array}{cc|c} 1 & -3 & 0 \\ 0 & -2\lambda-15 & -\frac{5}{2} \\ 0 & 15\lambda+5 & -5 \end{array}\right]. \tag{10}$$

RREF — Step 3: make the pivot in row 2 equal to 1 (when nonzero): Pivot element in row 2 is $p_2 = -2\lambda - 15$. If $p_2 \neq 0$ we scale:

$$R_2 \leftarrow \frac{1}{p_2} R_2.$$

Write the scaled row explicitly (keeping symbol p_2):

$$\mathcal{M}_3 = \left[\begin{array}{cc|c} 1 & -3 & 0 \\ 0 & 1 & \dfrac{-\frac{5}{2}}{p_2} \\ 0 & 15\lambda+5 & -5 \end{array}\right], \quad p_2 = -2\lambda - 15. \tag{11}$$

RREF — Step 4: eliminate column 2 entries using row 2: Eliminate entry in row 1 and row 3:

$$R_1 \leftarrow R_1 + 3R_2, \qquad R_3 \leftarrow R_3 - (15\lambda+5)R_2.$$

After these operations we obtain

$$\mathcal{M}_4 = \left[\begin{array}{cc|c} 1 & 0 & 3 \cdot \dfrac{-\frac{5}{2}}{p_2} \\ 0 & 1 & \dfrac{-\frac{5}{2}}{p_2} \\ 0 & 0 & -5 - (15\lambda+5)\dfrac{-\frac{5}{2}}{p_2} \end{array}\right]. \tag{12}$$

RREF — Step 5: interpret last row for consistency: Compute the bottom-right expression (call it r):

$$r = -5 - (15\lambda+5)\frac{-\frac{5}{2}}{p_2} = -5 + \frac{(15\lambda+5)(5/2)}{p_2}. \tag{13}$$

Substitute $p_2 = -2\lambda - 15$ and simplify:

$$r = -5 + \frac{(15\lambda+5)(5/2)}{-2\lambda-15} = -5 + \frac{(15\lambda+5)(5)}{2(-2\lambda-15)}. \tag{14}$$

Factor common 5:

$$r = -5 + \frac{5(15\lambda + 5)}{2(-2\lambda - 15)} = -5 + \frac{25(3\lambda + 1)}{2(-2\lambda - 15)}. \tag{15}$$

Set numerator to zero (consistency) — equivalently require $r = 0$. Solving $r = 0$ leads to (the same algebraic condition as equating the two expressions for κ_2 below), which simplifies to

$$\boxed{\lambda = -\frac{35}{19}}. \tag{16}$$

Check perpendicular value $\lambda = 1$

Set $\lambda = 1$

$$(-17)\kappa_2 = -\tfrac{5}{2} \quad \Rightarrow \quad \kappa_2 = \tfrac{5}{34}, \tag{17}$$

$$20\kappa_2 = -5 \quad \Rightarrow \quad \kappa_2 = -\tfrac{1}{4}. \tag{18}$$

Contradiction $\Rightarrow$ last row of $\mathcal{M}_4$ is nonzero ($r \neq 0$), hence inconsistent. Thus $\lambda = 1$ gives perpendicular direction vectors but no intersection (skew lines).

Final conclusions

$$\begin{aligned} \boxed{\lambda = 1} \quad & \text{direction vectors perpendicular; lines are skew (no intersection),} \\ \boxed{\lambda = -\frac{35}{19}} \quad & \text{system consistent; lines intersect (not perpendicular).} \end{aligned}$$

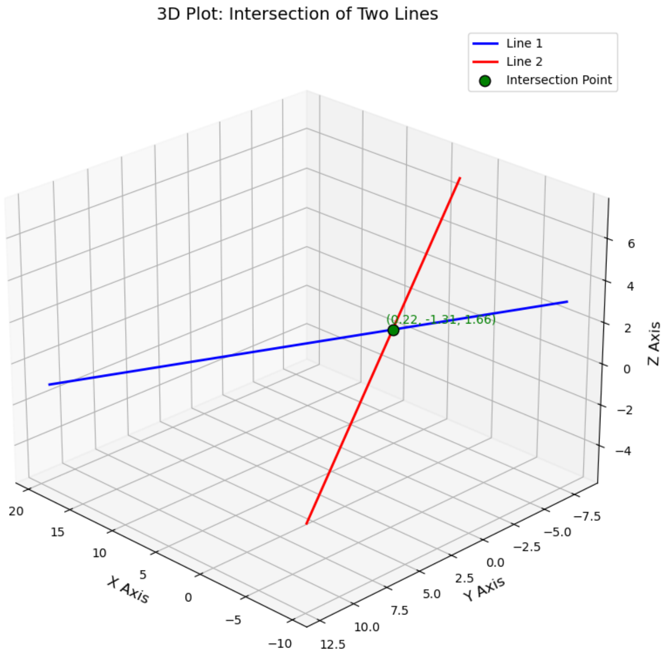
3D Plot: Intersection of Two Lines
Line 1
Line 2
Intersection Point
(0.22, -1.31, 1.66)
X Axis
Y Axis
Z Axis
20
15
10
5
0
−5
−10
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
6
4
2
0
−2
−4

Fig. 1

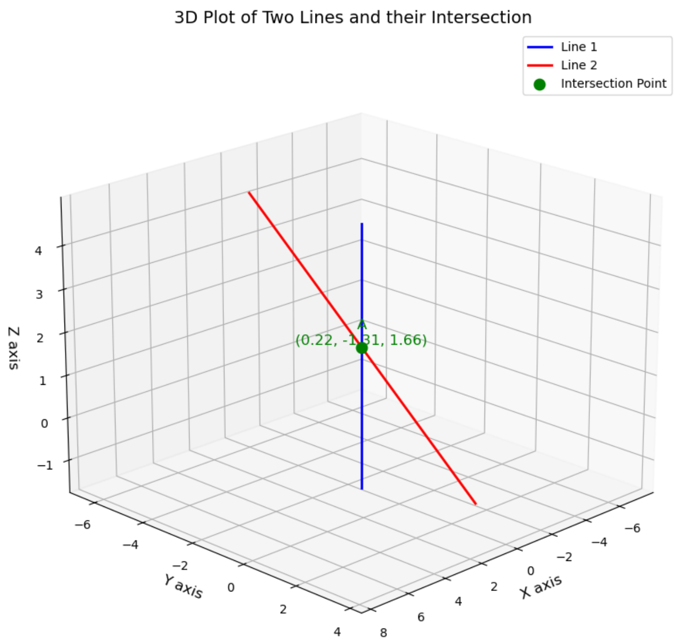
3D Plot of Two Lines and their Intersection
Line 1
Line 2
Intersection Point
(0.22, -1.31, 1.66)
Z axis
Y axis
X axis
4
3
2
1
0
−1
−6
−4
−2
0
2
4
8
6
4
2
0
−2
−4
−6

Fig. 2