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EE25BTECH11032 - Kartik Lahoti

Question:

The two vectors $[1,1,1]$ and $\left[1,a,a^2\right]$, where $a=\left(\frac{-1}{2}+j\frac{\sqrt{3}}{2}\right)$

1) orthonormal
2) orthogonal
3) paralle
4) collinear

Solution:

Given ,

$$\mathbf{P}=\begin{pmatrix}1\\1\\1\end{pmatrix} \tag{4.1}$$

$$\mathbf{Q}=\begin{pmatrix}1\\a\\a^2\end{pmatrix} \tag{4.2}$$

we know,

$$x+jy \longrightarrow \begin{pmatrix}x & -y\\ y & x\end{pmatrix} \tag{4.3}$$

$$a=\left(\frac{-1}{2}+j\frac{\sqrt{3}}{2}\right) \longrightarrow \mathbf{A}=\begin{pmatrix}\frac{-1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{-1}{2} & \frac{\sqrt{3}}{2}\end{pmatrix} \tag{4.4}$$

Similarly

$$a^2=\left(\frac{-1}{2}-j\frac{\sqrt{3}}{2}\right) \longrightarrow \mathbf{A}^2=\begin{pmatrix}\frac{-1}{2} & \frac{\sqrt{3}}{2}\\ \frac{-1}{2} & -\frac{\sqrt{3}}{2}\end{pmatrix} \tag{4.5}$$

$$1 \longrightarrow \mathbf{I}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \tag{4.6}$$

Now,

$$\mathbf{I}+\mathbf{A}+\mathbf{A^2}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+\begin{pmatrix}\frac{-1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{-1}{2} & \frac{\sqrt{3}}{2}\end{pmatrix}+\begin{pmatrix}\frac{-1}{2} & \frac{\sqrt{3}}{2}\\ \frac{-1}{2} & -\frac{\sqrt{3}}{2}\end{pmatrix} \tag{4.7}$$

$$=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix} \tag{4.8}$$

$$\implies 1 + a + a^2 = 0 \tag{4.9}$$

Now, Look At ,

$$\mathbf{P}^\top \mathbf{Q} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ a \\ a^2 \end{pmatrix} = 1 + a + a^2 = 0 \tag{4.10}$$

Hence **P** and **Q** are orthogonal.
Answer : Option (2)