

2.4.41

EE25BTECH11044 - Sai Hasini Pappula

Question:

Determine whether the points $A(3, 6, 9)$, $B(10, 20, 30)$, $C(24, -41, 5)$ are the vertices of a right-angled triangle using matrices.

Solution:

$$\mathbf{A} = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 24 \\ -41 \\ 5 \end{bmatrix}. \quad (1)$$

$$\mathbf{B} - \mathbf{A} = \begin{bmatrix} 10 - 3 \\ 20 - 6 \\ 30 - 9 \end{bmatrix} = \begin{bmatrix} 7 \\ 14 \\ 21 \end{bmatrix}, \quad \mathbf{C} - \mathbf{B} = \begin{bmatrix} 24 - 10 \\ -41 - 20 \\ 5 - 30 \end{bmatrix} = \begin{bmatrix} 14 \\ -61 \\ -25 \end{bmatrix}, \quad \mathbf{C} - \mathbf{A} = \begin{bmatrix} 24 - 3 \\ -41 - 6 \\ 5 - 9 \end{bmatrix} = \begin{bmatrix} 21 \\ -47 \\ -4 \end{bmatrix}. \quad (2)$$

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{C} - \mathbf{A}) = \begin{bmatrix} 7 & 14 & 21 \end{bmatrix} \begin{bmatrix} 21 \\ -47 \\ -4 \end{bmatrix} = 7 \cdot 21 + 14 \cdot (-47) + 21 \cdot (-4) = -595 \neq 0. \quad (3)$$

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{C} - \mathbf{B}) = \begin{bmatrix} 7 & 14 & 21 \end{bmatrix} \begin{bmatrix} 14 \\ -61 \\ -25 \end{bmatrix} = 7 \cdot 14 + 14 \cdot (-61) + 21 \cdot (-25) = -1281 \neq 0. \quad (4)$$

$$(\mathbf{C} - \mathbf{A})^T (\mathbf{C} - \mathbf{B}) = \begin{bmatrix} 21 & -47 & -4 \end{bmatrix} \begin{bmatrix} 14 \\ -61 \\ -25 \end{bmatrix} = 21 \cdot 14 + (-47) \cdot (-61) + (-4) \cdot (-25) = 3261 \neq 0. \quad (5)$$

Since none of the products is zero, no angle of the triangle is 90° .

Conclusion:

The points A , B , and C do **not** form a right-angled triangle.

Triangle formed by points A, B, C

