Matrices in Geometry - 5.8.35

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Problem Statement

The cost of 4 *kg* onion, 3 *kg* wheat and 2 *kg* rice is ₹60. The cost of 2 *kg* onion, 4 *kg* wheat, and 6 *kg* rice is ₹90. The cost of 6 *kg* onion, 2 *kg* wheat, and 3 *kg* rice is ₹70. Find the cost of each item per kilogram.

Solution

Let the cost of 1 kg of onion, wheat and rice be ₹x, ₹y and ₹z, respectively.

The given information is:

$$\begin{pmatrix} 4 & 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 60 \tag{1}$$

$$\begin{pmatrix} 2 & 4 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 90 \tag{2}$$

$$\begin{pmatrix} 6 & 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 70 \tag{3}$$

Stacking them in a single matrix:

$$\begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 4 \\ 6 & 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 60 \\ 90 \\ 70 \end{pmatrix} \tag{4}$$

Solution

Writing the augmented matrix

$$\left(\begin{array}{ccc|c} 4 & 3 & 2 & 60 \\ 2 & 4 & 6 & 90 \\ 6 & 2 & 3 & 70 \end{array}\right) \overset{R_1 \to R_1/4, R_2 \to R_2/2}{\longleftrightarrow} \left(\begin{array}{ccc|c} 1 & 3/4 & 1/2 & 15 \\ 1 & 2 & 3 & 45 \\ 6 & 2 & 3 & 70 \end{array}\right) \tag{5}$$

$$\overset{R_2 \to R_2 - R_1, R_3 \to R_3 - 6R_1}{\longleftrightarrow} \left(\begin{array}{ccc|c} 1 & 3/4 & 1/2 & 15 \\ 0 & 5/4 & 5/2 & 30 \\ 0 & -5/2 & 0 & -20 \end{array}\right) \overset{R_2 \to 4R_2/5}{\longleftrightarrow} \tag{6}$$

$$\left(\begin{array}{ccc|c} 1 & 3/4 & 1/2 & 15 \\ 0 & 1 & 2 & 24 \\ 0 & -5/2 & 0 & -20 \end{array}\right) \overset{R_1 \to R_1 - 3R_2/4, R_3 \to R_3 + 5R_2/2}{\longleftrightarrow} \tag{7}$$

Solution

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & -3 \\ 0 & 1 & 2 & 24 \\ 0 & 0 & 5 & 40 \end{array}\right) \overset{R_3 \to R_3/5}{\longleftrightarrow} \left(\begin{array}{ccc|c} 1 & 0 & -1 & -3 \\ 0 & 1 & 2 & 24 \\ 0 & 0 & 1 & 8 \end{array}\right) \tag{8}$$

$$\overset{R_1 \to R_1 + R_3, R_2 \to R_2 - 2R_3}{\longleftrightarrow} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 8 \end{array}\right) \tag{9}$$

This implies that

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \\ 8 \end{pmatrix} \tag{10}$$

Therefore, the cost of 1 kg of onion, wheat, rice is ₹5, ₹8 and ₹8.

Solution

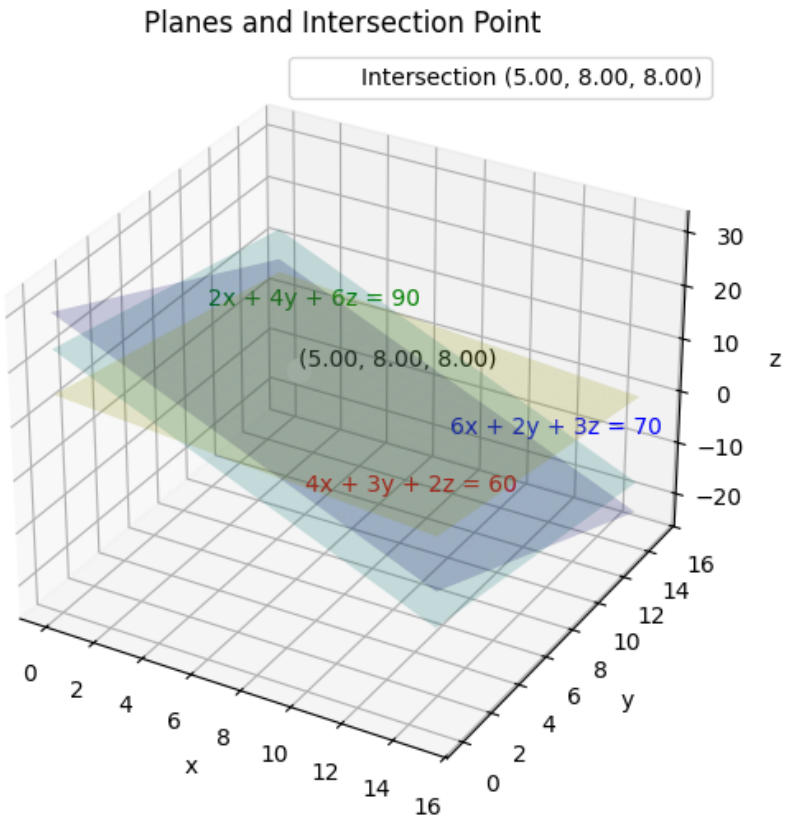

Figure: Graph for 5.8.35