

4.2.22

EE25BTECH11019 - Darji Vivek M.

Question:

Show that the two lines

$$a_1x + b_1y + c_1 = 0, \quad a_2x + b_2y + c_2 = 0$$

with $b_1b_2 \neq 0$ are parallel iff $\frac{a_1}{b_1} = \frac{a_2}{b_2}$.

Solution:

First form the matrix equation

$$\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = - \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}. \quad (1)$$

Augmented matrix:

$$\left(\begin{array}{cc|c} a_1 & b_1 & -c_1 \\ a_2 & b_2 & -c_2 \end{array} \right). \quad (2)$$

Assume $\frac{a_1}{b_1} = \frac{a_2}{b_2} \Rightarrow a_2 = ka_1, b_2 = kb_1$.

Row reduction:

$$\left(\begin{array}{cc|c} a_1 & b_1 & -c_1 \\ ka_1 & kb_1 & -c_2 \end{array} \right) \xrightarrow{R_2 \rightarrow R_2 - kR_1} \left(\begin{array}{cc|c} a_1 & b_1 & -c_1 \\ 0 & 0 & -c_2 + kc_1 \end{array} \right). \quad (3)$$

$\text{rank}(\mathbf{M}) = 1$ since only one nonzero row.

Conversely, if the lines are parallel,

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = 0 \Rightarrow a_1b_2 - a_2b_1 = 0 \Rightarrow \frac{a_1}{b_1} = \frac{a_2}{b_2}. \quad (4)$$

\therefore The two lines are parallel iff $\frac{a_1}{b_1} = \frac{a_2}{b_2}$.

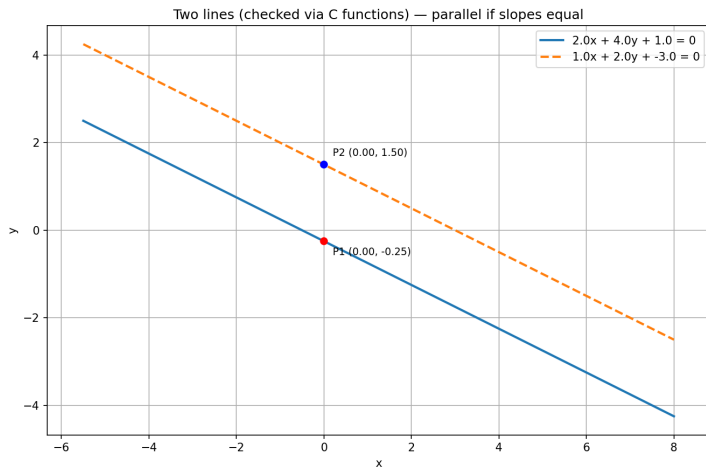


Fig. 0.1: plot