4.13.9

EE25BTECH11004 - Aditya Appana

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Question

Let the algebraic sum of the perpendicular distances from the points (2,0),(0,2) and (1,1) to a variable straight line be zero; then the line passes through a fixed point whose coordinates are ________.

Solution

The normal form of a line is:

$$\mathbf{n}^T x = c \tag{1}$$

The perpendicular distance of a point from a line is:

$$\frac{|\mathbf{n}^T x - c|}{||n||} \tag{2}$$

It is given that the algebraic sum of the perpendicular distances of three points (2,0), (0,2), and (1,1) to a line $\mathbf{n}^T x = c$ is 0. Therefore:

$$\frac{\mathbf{n}^T x_1 - c}{||n||} + \frac{\mathbf{n}^T x_2 - c}{||n||} + \frac{\mathbf{n}^T x_3 - c}{||n||} = 0 \tag{3}$$

Substituting the points:

$$\frac{\mathbf{n}^T\begin{pmatrix}2\\0\end{pmatrix}-c}{||n||}+\frac{\mathbf{n}^T\begin{pmatrix}0\\2\end{pmatrix}-c}{||n||}+\frac{\mathbf{n}^T\begin{pmatrix}1\\1\end{pmatrix}-c}{||n||}=0 \tag{4}$$

$$\frac{\mathbf{n}^T\begin{pmatrix}3\\3\end{pmatrix}-3c}{||n||}=0 \tag{5}$$

$$\frac{\mathbf{n}^T\begin{pmatrix}1\\1\end{pmatrix}-c}{||n||}=0 \tag{6}$$

$$\mathbf{n}^T\begin{pmatrix}1\\1\end{pmatrix}=c \tag{7}$$

Therefore the line passes through the fixed point (1,1).

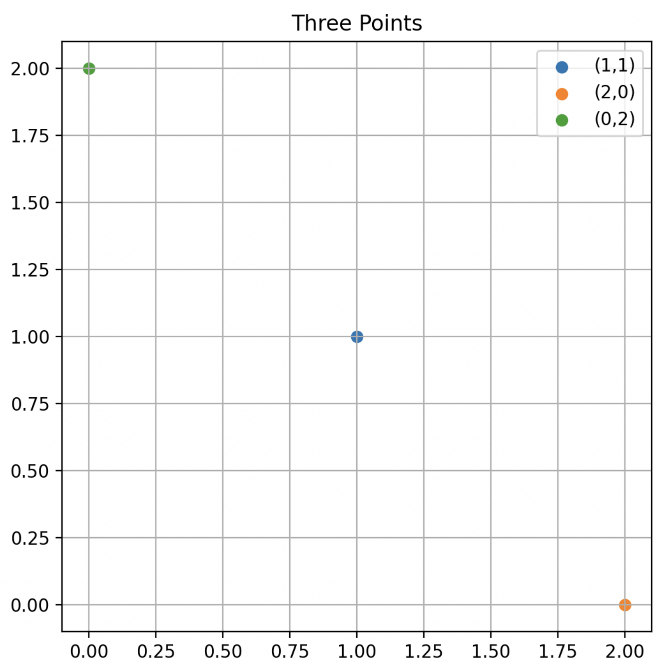

Figure 1: Plot