9.8.5

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Question

Let **S** be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is

Theoretical Solution

Given:
Circle: $x^2 + y^2 - 2x - 4y = 0$
Parabola: $y^2 = 8x$

Parameters of the Circle:

$$\mathbf{V}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u}_1 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, f_1 = 0 \tag{1}$$

Parameters of the Parabola:

$$\mathbf{V}_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, f_2 = 0, \mathbf{S} = \begin{pmatrix} 2e \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{2}$$

Points of Intersection of Circle and Parabola can be given as:

$$\mathbf{X}^\top(\mathbf{V}_1 + \mu\mathbf{V}_2)\mathbf{X} + 2(\mathbf{u}_1 + \mu\mathbf{u}_2)^\top\mathbf{X} + (f_1 + \mu f_2) \tag{3}$$

Theoretical Solution

$$\mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 1+\mu \end{pmatrix} \mathbf{x} - 2 \begin{pmatrix} 1+4\mu & 2 \end{pmatrix} \mathbf{x} = 0 \tag{4}$$

From Equation 4, We get

$$\mathbf{x} = \begin{pmatrix} 1+4\mu \\ \dfrac{2}{1+\mu} \end{pmatrix} \tag{5}$$

Putting Value of $\mathbf{x}$ in Equation 4, We get points of intersection as:

$$\mathbf{x}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \& \qquad \mathbf{x}_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{6}$$

Therefore, Let $\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

Theoretical Solution

The Area of Triangle PQS is:

$$Area(\triangle PQS) = \frac{1}{2} \left\| \mathbf{SP} \times \mathbf{QP} \right\| = 4 \tag{7}$$

$$\boxed{\text{The Area of } \triangle PQS \text{ is 4 sq.units.}}$$

Intersection of Two Conics and Triangle PQS