4.3.25

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Problem Statement

Find the ratio in which the YZ plane divides the line segment joining the points

$$\mathbf{A} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 3 \\ -5 \\ 8 \end{pmatrix}.$$

Step 1: Vector and Matrix Forms

Line segment in vector form:

$$\mathbf{R} = \mathbf{A} + \lambda(\mathbf{B} - \mathbf{A}) \tag{1}$$

Parametric vector:

$$\mathbf{R} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{2}$$

YZ**-plane as a matrix equation:**

$$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \mathbf{R} = 0 \tag{3}$$

Step 2: Intersection Symbolically

Intersection point P:

$$\mathbf{P} = \mathbf{A} + \lambda(\mathbf{B} - \mathbf{A}) \tag{4}$$

$$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \mathbf{P} = 0 \tag{5}$$

Ratio along the line:

$$AP : PB = \lambda : (1 - \lambda) \tag{6}$$

Step 3: Numerical Algebra

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ -5 \\ 8 \end{pmatrix} - \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 5 \\ -9 \\ 1 \end{pmatrix} \tag{7}$$

$$\mathbf{R} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -9 \\ 1 \end{pmatrix} = \begin{pmatrix} -2+5\lambda \\ 4-9\lambda \\ 7+\lambda \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \mathbf{R} = -2 + 5\lambda = 0 \tag{9}$$

$$\lambda = \frac{2}{5} \tag{10}$$

$$\mathbf{P} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} + \frac{2}{5} \begin{pmatrix} 5 \\ -9 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2/5 \\ 37/5 \end{pmatrix} \approx \begin{pmatrix} 0 \\ 0.4 \\ 7.4 \end{pmatrix} \tag{11}$$

$$AP : PB = 2 : 3 \tag{12}$$

Answer

The YZ-plane divides the line segment **AB** internally in the ratio

$$\boxed{2 : 3}$$

C Code

```
// ratio.c
#include <stdio.h>
#include <math.h>
void yz_plane_ratio(double x1, double y1, double z1,
                    double x2, double y2, double z2,
                    double *t_out, double *one_minus_t_out){
    if (x2 == x1) {
        // line parallel to YZ-plane in x-direction -> x constant
        *t_out = NAN;
        *one_minus_t_out = NAN;
        return;
    }
    double t = -x1 / (x2 - x1); // from (1-t)*x1 + t*x2 = 0 -> t
        = -x1/(x2-x1)
    *t_out = t;
    *one_minus_t_out = 1.0 - t;
}
```

Python code through shared output

```
import numpy as np
import math
import matplotlib.pyplot as plt
from ctypes import CDLL, c_double, POINTER, byref
from math import gcd

# Load the C library
lib = CDLL('./libratio.so')

lib.yz_plane_ratio.argtypes = (c_double, c_double, c_double,
                               c_double, c_double, c_double,
                               POINTER(c_double), POINTER(c_double))
lib.yz_plane_ratio.restype = None

# Points A and B
A = (-2.0, 4.0, 7.0)
B = (3.0, -5.0, 8.0)
```

```
t = c_double()
omt = c_double()

# Call the C function to get t and 1-t
lib.yz_plane_ratio(A[0], A[1], A[2],
                   B[0], B[1], B[2],
                   byref(t), byref(omt))

if math.isnan(t.value):
    print(No finite intersection with YZ-plane (x=0))
else:
    print(f (t) = {t.value:.4f})
    print(f1 -  = {omt.value:.4f})

    # Correct ratio conversion from floats to simplest integer
        ratio
    def ratio_to_ints(t1, t2):
        precision = 10**6 # High precision to convert float to
            int safely
```

Python code through shared output

```
a, b = ratio_to_ints(t.value, omt.value)
print(fRatio AP:PB = {a}:{b})

# Calculate intersection point P
P = tuple((1 - t.value)*A[i] + t.value*B[i] for i in range(3)
    )
print(fIntersection point P on YZ-plane: ({P[0]:.2f}, {P
    [1]:.2f}, {P[2]:.2f}))

# Calculate lengths AP and PB
A_np = np.array(A)
B_np = np.array(B)
P_np = np.array(P)

AP_len = np.linalg.norm(P_np - A_np)
PB_len = np.linalg.norm(B_np - P_np)
print(fLength AP = {AP_len:.4f})
print(fLength PB = {PB_len:.4f})
print(fLength ratio AP:PB  {AP_len/PB_len:.4f} (should be
```

```
# Generate points for line AB (for plotting)
points = np.array([np.linspace(A[i], B[i], 100) for i in
    range(3)])

# Plot setup
fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')

# Plot line AB
ax.plot(points[0], points[1], points[2], label='Line AB',
    color='blue')

# Plot points A, B, P
ax.scatter(*A, color='green', s=80, label='A')
ax.scatter(*B, color='red', s=80, label='B')
ax.scatter(*P, color='black', s=100, label='P (Intersection)'
    )

# Annotate points with coordinates
```

Python code through shared output

```
    annotate_point(ax, A, 'A', 'green')
    annotate_point(ax, B, 'B', 'red')
    annotate_point(ax, P, 'P', 'black')

    # Plot YZ-plane (x=0)
    y = np.linspace(-6, 6, 30)
    z = np.linspace(5, 10, 30)
    Y, Z = np.meshgrid(y, z)
    X = np.zeros_like(Y)
    ax.plot_surface(X, Y, Z, color='cyan', alpha=0.3)

    # Add text box with ratio and lengths
    ratio_text = (
        f = {t.value:.2f}\n
        fRatio AP:PB = {a}:{b}\n
        fLength AP = {AP_len:.2f}\n
        fLength PB = {PB_len:.2f}
    )
    ax.text2D(0.05, 0.95, ratio_text, transform=ax.transAxes,
```

Python code through shared output

```
# Equal aspect ratio (very important!)
def set_axes_equal(ax):
    '''Set 3D plot axes to equal scale.'''
    limits = np.array([
        ax.get_xlim3d(),
        ax.get_ylim3d(),
        ax.get_zlim3d(),
    ])
    spans = limits[:,1] - limits[:,0]
    centers = np.mean(limits, axis=1)
    radius = 0.5 * max(spans)
    ax.set_xlim3d(centers[0] - radius, centers[0] + radius)
    ax.set_ylim3d(centers[1] - radius, centers[1] + radius)
    ax.set_zlim3d(centers[2] - radius, centers[2] + radius)
```

Python code through shared output

```
    set_axes_equal(ax)

    ax.set_xlabel('X-axis')
    ax.set_ylabel('Y-axis')
    ax.set_zlabel('Z-axis')
    ax.set_title('Intersection of line AB with YZ-plane (x=0)')
    ax.legend()
    ax.grid(True)
    ax.view_init(elev=20, azim=30)

    plt.tight_layout()
    plt.show()
```

```
import sys
sys.path.insert(0, '/sdcard/github/matgeo/codes/CoordGeo')

import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from line.funcs import line_gen

# Points A and B
A = np.array([-2, 4, 7]).reshape(-1, 1)
B = np.array([3, -5, 8]).reshape(-1, 1)

# Compute parameter lambda (t) where line intersects x=0 plane (
    YZ-plane)
t = 2/5 # from solution

# Intersection point P
P = (1 - t) * A + t * B
```

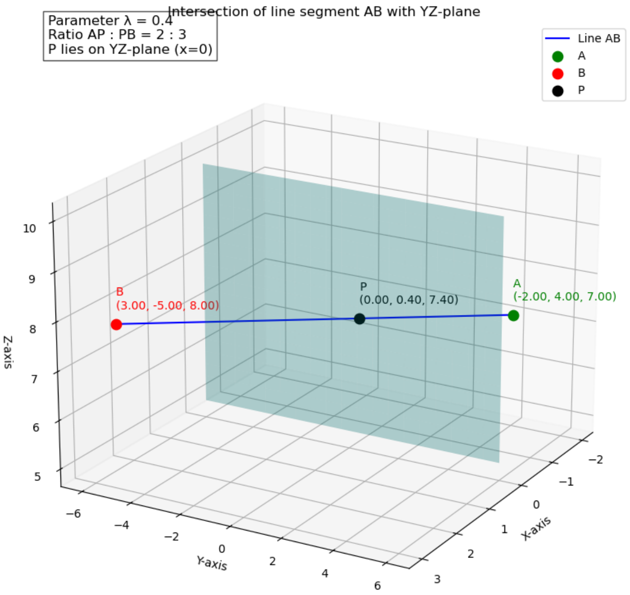
```
# Generate line segment AB points for plotting
line_AB = line_gen(A, B)

# Plotting setup
fig = plt.figure(figsize=(10,8))
ax = fig.add_subplot(111, projection='3d')

# Plot line AB
ax.plot(line_AB[0,:], line_AB[1,:], line_AB[2,:], label='Line AB'
    , color='blue')

# Function to annotate points
def annotate_3d_point(ax, point, label, color):
    ax.scatter(point[0], point[1], point[2], color=color, s=80,
        label=label)
    ax.text(point[0], point[1], point[2] + 0.3,
            f{label}\n({point[0]:.2f}, {point[1]:.2f}, {point
                [2]:.2f}),
```

only Python code

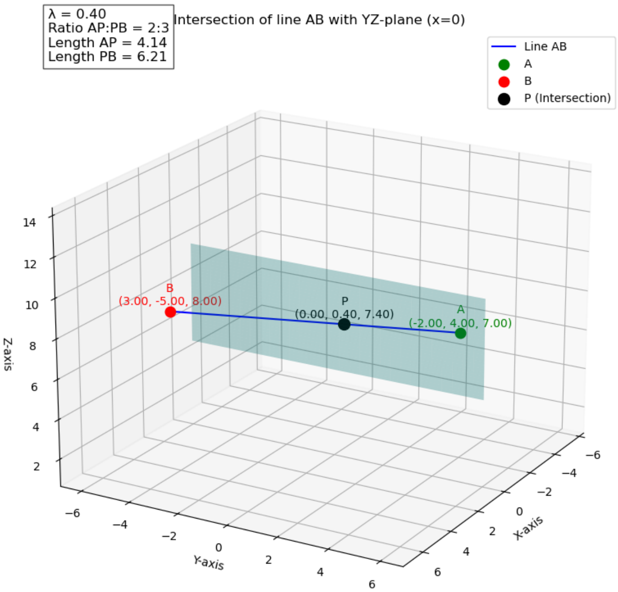
```
# Plot and annotate points A, B, and P
annotate_3d_point(ax, A.flatten(), 'A', 'green')
annotate_3d_point(ax, B.flatten(), 'B', 'red')
annotate_3d_point(ax, P.flatten(), 'P', 'black')

# Plot YZ-plane (x=0)
y = np.linspace(-6, 6, 20)
z = np.linspace(5, 10, 20)
Y, Z = np.meshgrid(y, z)
X = np.zeros_like(Y)
ax.plot_surface(X, Y, Z, alpha=0.3, color='cyan')

# Add text box with ratio info
ax.text2D(0.05, 0.95,
          fParameter  = {t}\nRatio AP : PB = {ratio_str}\nP lies
              on YZ-plane (x=0),
          transform=ax.transAxes, fontsize=12,
          bbox=dict(facecolor='white', alpha=0.7))
```

```
# Labels and title
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title('Intersection of line segment AB with YZ-plane')

ax.legend()
ax.grid(True)
ax.view_init(elev=20, azim=30)

plt.tight_layout()
plt.show()
```

PLOTS


Intersection of line segment AB with YZ-plane
Parameter λ = 0.4
Ratio AP : PB = 2 : 3
P lies on YZ-plane (x=0)
Line AB
A
B
P
B
(3.00, -5.00, 8.00)
P
(0.00, 0.40, 7.40)
A
(-2.00, 4.00, 7.00)
Z-axis
Y-axis
X-axis

PLOTS



λ = 0.40
Ratio AP:PB = 2:3
Length AP = 4.14
Length PB = 6.21
Intersection of line AB with YZ-plane (x=0)
Line AB
A
B
P (Intersection)
B
(3.00, -5.00, 8.00)
P
(0.00, 0.40, 7.40)
A
(-2.00, 4.00, 7.00)
Z-axis
Y-axis
X-axis