Matrices in Geometry - 1.5.25

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Problem Statement

In what ratio does the point $\vec{R} = \begin{pmatrix} \frac{24}{11} \\ y \end{pmatrix}$ divide the line segment joining the points $\vec{P} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and $\vec{Q} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$? Also find the value of y. ;

Solution

$\vec{P} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $\vec{Q} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ and a point $\vec{R} = \begin{pmatrix} \frac{24}{11} \\ y \end{pmatrix}$ on PQ.

Let R divide PQ internally in the ratio $k : 1$.

Therefore, they are defined to be collinear if,

$$\text{rank} \begin{pmatrix} \vec{R} - \vec{P} & \vec{Q} - \vec{R} \end{pmatrix} = 1$$

$$\vec{R} - \vec{P} = \begin{pmatrix} \frac{2}{11} \\ y + 2 \end{pmatrix}$$

$$\vec{Q} - \vec{R} = \begin{pmatrix} \frac{9}{11} \\ 7 - y \end{pmatrix}$$

$$\implies \text{rank} \begin{pmatrix} \frac{2}{11} & \frac{9}{11} \\ y + 2 & 7 - y \end{pmatrix} = 1$$

Solution

$$\implies \Delta = 0$$

$$\frac{2}{11}(7-y) - \frac{9}{11}(y+2) = 0$$

$$14 - 2y - 18 - 9y = 0$$

$$\implies y = \frac{-4}{11}$$

We know that k is the ratio in which $\vec{R}$ divides $\vec{P}$ and $\vec{Q}$,

$$k = \frac{\|\overline{PR}\|}{\|\overline{RQ}\|}$$

$$\overline{PR} = \begin{pmatrix} -2/11 \\ -18/11 \end{pmatrix}$$

Solution

$$\implies \|\overline{PR}\| = \sqrt{4/121 + 324/121} = \sqrt{328/21}$$

$$\implies \|\overline{PR}\| = 2\sqrt{82}/11$$

$$\overline{QR} = \begin{pmatrix} 9/11 \\ 81/11 \end{pmatrix}$$

$$\implies \|\overline{QR}\| = \sqrt{81/121 + 6561/121} = \sqrt{6642/121}$$

$$\implies \|\overline{QR}\| = 9\sqrt{82}/11$$

$$\therefore k = \frac{\|\overline{PR}\|}{\|\overline{RQ}\|} = \frac{2}{9}$$

Final Answer

Hence, the final answer is $\boxed{k = \frac{2}{9}}$ and $\boxed{y = \frac{-4}{11}}$