

# 4.3.32

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## Question:

Find the slope of a line which cuts off intercepts of equal length on the axes is. Solve using matrices.

## Solution:

Consider normal form of a line:

$$\mathbf{n}^\top \mathbf{x} = 1 \quad (0.1)$$

Given that equal intercepts are cut off we get 2 cases:

On substituting the intercepts in place of  $\mathbf{x}$ :

**Case 1: The intercepts are equal** ( $b = a$ )

$$\Rightarrow \mathbf{n}^\top \begin{pmatrix} a \\ 0 \end{pmatrix} = 1 \Rightarrow an_1 = 1 \quad (0.2)$$

$$\Rightarrow \mathbf{n}^\top \begin{pmatrix} 0 \\ a \end{pmatrix} = 1 \Rightarrow an_2 = 1 \quad (0.3)$$

$$(0.4)$$

from (0.2) and from (0.3)

$$\mathbf{n}^\top \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} = \begin{pmatrix} 1 & 1 \end{pmatrix} \quad (0.5)$$

$$\Rightarrow a\mathbf{n}^\top \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \end{pmatrix} \quad (0.6)$$

$$\Rightarrow a\mathbf{n}^\top \mathbf{I} = \begin{pmatrix} 1 & 1 \end{pmatrix} \Rightarrow a\mathbf{n}^\top = \begin{pmatrix} 1 & 1 \end{pmatrix} \Rightarrow \mathbf{n} = \frac{1}{a} \begin{pmatrix} 1/1 \end{pmatrix} \quad (0.7)$$

Thus, direction vector  $\mathbf{n} = (1/1)$

**Case 2: The intercepts are negatives of each other** ( $-b = a$ )

$$\Rightarrow \mathbf{n}^\top \begin{pmatrix} a \\ 0 \end{pmatrix} = 1 \quad (0.8)$$

$$\Rightarrow \mathbf{n}^\top \begin{pmatrix} 0 \\ -a \end{pmatrix} = 1 \quad (0.9)$$

$$(0.10)$$

By (0.8), (0.9):

$$\mathbf{n}^\top \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix} = \begin{pmatrix} 1 & 1 \end{pmatrix} \quad (0.11)$$

$$\Rightarrow a\mathbf{n}^\top \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \end{pmatrix} \quad (0.12)$$

Let  $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

For this matrix  $\mathbf{A} = \mathbf{A}^{-1}$

$$\Rightarrow a\mathbf{n}^\top = \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{A}^{-1} \Rightarrow a\mathbf{n}^\top = \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{A}^{-1} \Rightarrow \mathbf{n} = \frac{1}{a} \begin{pmatrix} 1 & 1 \end{pmatrix} \quad (0.13)$$

Thus, direction vector  $\mathbf{n} = \begin{pmatrix} 1 & 1 \end{pmatrix}$  The direction vector is given (in general) by:

$$\mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \text{ where } m \text{ is slope of given line} \quad (0.14)$$

On comparing with the obtained direction vectors

$$\therefore m = \pm 1 \quad (0.15)$$

