

4.3.32

EE25BTECH11042 - Nipun Dasari

Question:

Find the slope of a line which cuts off intercepts of equal length on the axes is. Solve using matrices.

Solution:

Consider normal form of a line:

$$\mathbf{n}^T \mathbf{x} = c, \text{ where } \mathbf{n} = \begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix} \quad (0.1)$$

Given that equal intercepts are cut off we get 2 cases:

Case 1: The intercepts are equal ($b = a$)

$$\Rightarrow \mathbf{n}^T \begin{pmatrix} a \\ 0 \end{pmatrix} = c \Rightarrow a \cos \alpha = c \quad (0.2)$$

$$\Rightarrow \mathbf{n}^T \begin{pmatrix} 0 \\ a \end{pmatrix} = c \Rightarrow a \sin \alpha = c \quad (0.3)$$

$$(0.2)/(0.3) \Rightarrow \tan \alpha = 1 \quad (0.4)$$

Case 2: The intercepts are negatives of each other ($-b = a$)

$$\Rightarrow \mathbf{n}^T \begin{pmatrix} a \\ 0 \end{pmatrix} = c \Rightarrow a \cos \alpha = c \quad (0.5)$$

$$\Rightarrow \mathbf{n}^T \begin{pmatrix} 0 \\ -a \end{pmatrix} = c \Rightarrow -a \sin \alpha = c \quad (0.6)$$

$$(0.5)/(0.6) \Rightarrow \tan \alpha = -1 \quad (0.7)$$

This gives us two values of slope $m = \tan \alpha$

$$\therefore m = \pm 1 \quad (0.8)$$

