Problem 4.13.28

ee25btech11023-Venkata Sai

September 13, 2025

Problem

Slope of a line passing through $\mathbf{P}\,(2,3)$ and intersecting the line $x+y=7$ at a distance of 4 units from $\mathbf{P}$, is

Finding values

Given

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{2.1}$$

Equation of a line through **P** and having slope m is

$$\begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} x-2 \\ y-3 \end{pmatrix} = 0 \tag{2.2}$$

$$\begin{pmatrix} -m & 1 \end{pmatrix} \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right) \implies \begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{2.3}$$

$$\begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 3 - 2m \tag{2.4}$$

$$x + y = 7 \implies \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = 7 \tag{2.5}$$

Solving

$$\begin{pmatrix} -m & 1 & 3-2m \\ 1 & 1 & 7 \end{pmatrix} \xleftrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 1 & 7 \\ -m & 1 & 3-2m \end{pmatrix} \tag{2.6}$$

$$\begin{pmatrix} 1 & 1 & 7 \\ -m & 1 & 3-2m \end{pmatrix} \xleftrightarrow{R_2 \rightarrow R_2 + mR_1} \begin{pmatrix} 1 & 1 & 7 \\ 0 & 1+m & 3+5m \end{pmatrix} \tag{2.7}$$

$$y = \frac{3+5m}{1+m} \tag{2.8}$$

Given the point is at a distance of 4 units from point **P**

$$\left\| \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\| = 4 \implies \left\| \begin{pmatrix} x-2 \\ y-3 \end{pmatrix} \right\| = 4 \tag{2.9}$$

$$\sqrt{(x-2)^2 + (y-3)^2} = 4 \tag{2.10}$$

$$\sqrt{(7-y-2)^2 + (y-3)^2} = 4 \tag{2.11}$$

Substitution

$$(5-y)^2 + (y-3)^2 = 4^2 = 16 \tag{2.12}$$

$$25 + y^2 - 10y + y^2 + 9 - 6y = 16 \tag{2.13}$$

$$2y^2 - 16y + 18 = 0 \implies y^2 - 8y + 9 = 0 \tag{2.14}$$

$$y^2 - 8y + 9 + 16 = 16 \implies (y-4)^2 = 7 \tag{2.15}$$

$$y - 4 = \pm\sqrt{7} \tag{2.16}$$

$$y = 4 - \sqrt{7} \ \text{(or)} \ 4 + \sqrt{7} \tag{2.17}$$

$$\frac{3+5m}{1+m} = \frac{3+5m+2-2}{1+m} = \frac{5+5m-2}{1+m} = \frac{5\,(m+1)-2}{1+m} = 5 - \frac{2}{1+m} \tag{2.18}$$

Conclusion

$$5 - \frac{2}{1+m} = 4 - \sqrt{7} \quad \text{(or)} \quad 5 - \frac{2}{1+m} = 4 + \sqrt{7} \tag{2.19}$$

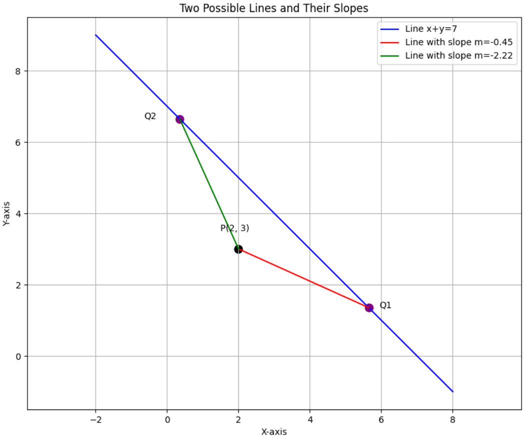
$$\frac{2}{1+m} = 1 + \sqrt{7} \quad \text{(or)} \quad \frac{2}{1+m} = 1 - \sqrt{7} \tag{2.20}$$

$$1 + m = \frac{2}{1+\sqrt{7}} \quad \text{(or)} \quad 1 + m = \frac{2}{1-\sqrt{7}} \tag{2.21}$$

$$m = \frac{2 - 1 - \sqrt{7}}{1+\sqrt{7}} \quad \text{(or)} \quad m = \frac{2 - 1 + \sqrt{7}}{1-\sqrt{7}} \tag{2.22}$$

$$m = \frac{1 - \sqrt{7}}{1+\sqrt{7}} \quad \text{(or)} \quad m = \frac{1 + \sqrt{7}}{1-\sqrt{7}} \tag{2.23}$$

Plot



Two Possible Lines and Their Slopes
Line x+y=7
Line with slope m=-0.45
Line with slope m=-2.22
Q2
P(2, 3)
Q1
Y-axis
X-axis
8
6
4
2
0
−2
0
2
4
6
8

Figure:

C Code

```c
#include <math.h>
void calculate_slope_data(double* out_data) {
    double px = 2.0, py = 3.0;
    double a = 1.0, b = -6.0, c = 2.0;
    double discriminant = sqrt(b*b - 4*a*c);

    double q1_x = (-b + discriminant) / (2 * a); // 3 + sqrt(7)
    double q2_x = (-b - discriminant) / (2 * a); // 3 - sqrt(7)

    double q1_y = 7 - q1_x;
    double q2_y = 7 - q2_x;

    double slope1 = (q1_y - py) / (q1_x - px);
    double slope2 = (q2_y - py) / (q2_x - px);
    out_data[0] = px; out_data[1] = py;
    out_data[2] = q1_x; out_data[3] = q1_y;
    out_data[4] = q2_x; out_data[5] = q2_y;
    out_data[6] = slope1; out_data[7] = slope2;
}
```

Python Code for Calling

```
import ctypes
import numpy as np

def get_data_from_c():
    lib = ctypes.CDLL('./code.so')

    double_array_8 = ctypes.c_double * 8
    lib.calculate_slope_data.argtypes = [ctypes.POINTER(ctypes.
        c_double)]
    out_data_c = double_array_8()
    lib.calculate_slope_data(out_data_c)
    all_data = np.array(out_data_c)
    # Unpack the data
    point_p = all_data[0:2]
    point_q1 = all_data[2:4]
    point_q2 = all_data[4:6]
    slopes = all_data[6:8]

    return point_p, point_q1, point_q2, slopes
```

Python Code for Plotting

```
#Code by GVV Sharma
#September 12, 2023
#Revised July 21, 2024
#released under GNU GPL
import sys #for path to external scripts
sys.path.insert(0, '/workspaces/urban-potato/matgeo/codes/
    CoordGeo/')
import numpy as np
import matplotlib.pyplot as plt

from call import get_data_from_c

# Get the points and slopes from the C library
P, Q1, Q2, slopes = get_data_from_c()
slope1, slope2 = slopes
```

Python Code for Plotting

```python
print(f"The two possible slopes are: {slope1:.4f} and {slope2:.4f}")
x_line_given = np.array([-2, 8])
y_line_given = 7 - x_line_given

# Create points for the two possible solution lines
x_line_1 = np.array([P[0], Q1[0]])
y_line_1 = np.array([P[1], Q1[1]])
x_line_2 = np.array([P[0], Q2[0]])
y_line_2 = np.array([P[1], Q2[1]])

fig, ax = plt.subplots(figsize=(10, 8))

ax.plot(x_line_given, y_line_given, 'b-', label='Line x+y=7')
ax.plot(x_line_1, y_line_1, 'r-', label=f'Line with slope m={slope1:.2f}')
ax.plot(x_line_2, y_line_2, 'g-', label=f'Line with slope m={slope2:.2f}')
```

Python Code for Plotting

```python
ax.scatter(P[0], P[1], color='black', s=80)
ax.scatter(Q1[0], Q1[1], color='purple', s=80)
ax.scatter(Q2[0], Q2[1], color='purple', s=80)

ax.text(P[0] - 0.5, P[1] + 0.5, f'P({P[0]:.0f}, {P[1]:.0f})')
ax.text(Q1[0] + 0.3, Q1[1], 'Q1')
ax.text(Q2[0] - 1.0, Q2[1], 'Q2')

ax.set_title('Two Possible Lines and Their Slopes')
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')

ax.grid(True)
ax.axis('equal')
ax.legend()
plt.show()
plt.savefig('../figs/fig1.png')
```