2.9.19

AI25BTECH11003 - Bhavesh Gaikwad

Question: Let $\vec{a}$, $\vec{b}$, and $\vec{c}$ be three vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$, and $|\vec{c}| = 3$. If the projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$, and $\vec{b}$ and $\vec{c}$ are perpendicular to each other, then find $|3\vec{a} - 2\vec{b} + 2\vec{c}|$.

Solution:

Given: $\|\mathbf{a}\| = 1$, $\|\mathbf{b}\| = 2$, $\|\mathbf{c}\| = 3$

$$\mathbf{b}^T \frac{\mathbf{a}}{\|\mathbf{a}\|} = \mathbf{c}^T \frac{\mathbf{a}}{\|\mathbf{a}\|}$$

Since $\mathbf{b}$ and $\mathbf{c}$ are perpendicular: $\mathbf{b}^T\mathbf{c} = 0$

Let $\mathbf{v} = 3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c}$

$\|\mathbf{v}\|^2 = (3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c})^T(3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c})$

$\Rightarrow \quad 9\|\mathbf{a}\|^2 + 4\|\mathbf{b}\|^2 + 4\|\mathbf{c}\|^2 - 12(\mathbf{a}^T\mathbf{b}) + -12(\mathbf{a}^T\mathbf{b}) - 8(\mathbf{b}^T\mathbf{c}) = 9 + 16 + 36 = 61$

$$\boxed{\|3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c}\| = \sqrt{61}} \tag{0.1}$$

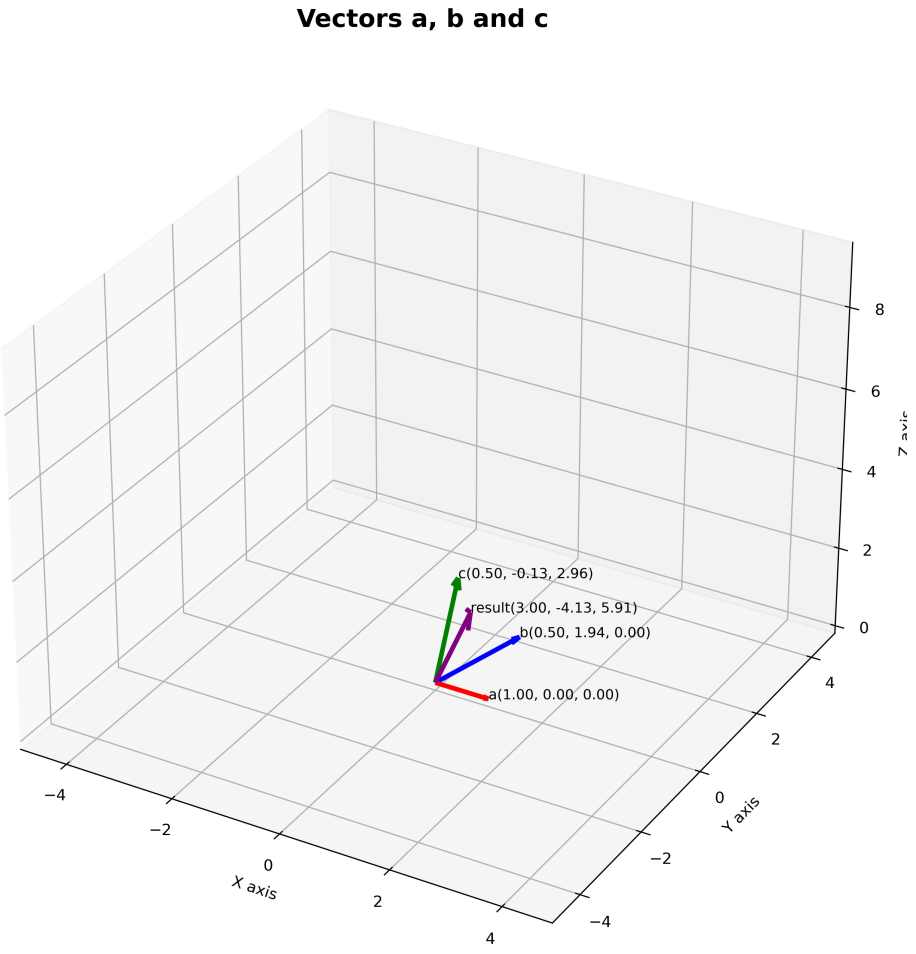

Fig. 0.1: Vector Representation