

9.4.39

Harsha-EE25BTECH11026

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Question

The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Theoretical Solution

Let x and y be the 2 numbers such that $x > y$.

The given equations are,

$$x^2 - y^2 = 180 \quad (1)$$

$$y^2 = 8x \quad (2)$$

As the given equations are homogeneous, converting them into quadratic form,

$$\implies \mathbf{x}^T \mathbf{V}_1 \mathbf{x} + c = 0 \quad (3)$$

where $\mathbf{x}^T = \begin{pmatrix} x & y \end{pmatrix}$ and $\mathbf{V}_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $c = -180$

Theoretical Solution

And also,

$$\mathbf{x}^\top \mathbf{V}_2 \mathbf{x} + 2\mathbf{b}^\top \mathbf{x} = 0 \quad (4)$$

where $\mathbf{x}^\top = (x \quad y)^\top$, $\mathbf{V}_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$.

To identify the intersection of conics, we can employ the approach of degenerating the conics.

To work with degeneracy in matrix form we form the standard augmented 3×3 matrix for each conic:

$$\mathbf{M}_i = \begin{pmatrix} \mathbf{V}_i & \mathbf{b}_i \\ \mathbf{b}_i^\top & c_i \end{pmatrix} \quad (5)$$

Theoretical Solution

From (??),

$$\Rightarrow \mathbf{M}_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -180 \end{pmatrix} \quad \mathbf{M}_2 = \begin{pmatrix} 0 & 0 & -4 \\ 0 & 1 & 0 \\ -4 & 0 & 0 \end{pmatrix} \quad (6)$$

$$\therefore \mathbf{x}^\top (\mathbf{M}_1 + \lambda \mathbf{M}_2) \mathbf{x} = 0 \quad (7)$$

To degenerate the conic into a line, we can find the solutions of λ when $\|\mathbf{M}_1 + \lambda \mathbf{M}_2\| = 0$

$$\therefore \|\mathbf{M}_1 + \lambda \mathbf{M}_2\| = 0 \quad (8)$$

$$\Rightarrow (\lambda - 1)(4\lambda^2 + 45) = 0 \quad (9)$$

$$\therefore \lambda = 1 \quad (10)$$

Theoretical Solution

Substituting λ in (??),

$$\implies \mathbf{x}^T (\mathbf{M}_1 + \mathbf{M}_2) \mathbf{x} \quad (11)$$

$$\implies x^2 - 8x - 180 = 0 \quad (12)$$

$$\implies x = 18, -10 \quad (13)$$

for $x = -10$, there is no real solution of y in (??),

$$\implies y = \pm 12 \quad (14)$$

$$\therefore \text{The two numbers are } (18, 12) \text{ and } (18, -12) \quad (15)$$

C Code -Finding the intersection of conics

```
#include <stdio.h>
#include <math.h>

void solve_conics(double *results) {
    // Quadratic in x:  $x^2 - 8x - 180 = 0$ 
    double a = 1, b = -8, c = -180;
    double disc = b*b - 4*a*c;

    if (disc < 0) {
        // No real solution
        results[0] = results[1] = results[2] = results[3] = NAN;
        return;
    }

    // Roots of quadratic
    double sqrt_disc = sqrt(disc);
    double x1 = (-b + sqrt_disc) / (2*a);
    double x2 = (-b - sqrt_disc) / (2*a);
```

C Code -Finding the intersection of conics

```
// Solutions
int idx = 0;
double xs[2] = {x1, x2};
for (int i=0; i<2; i++) {
    double x = xs[i];
    if (x < 0) continue; //  $y^2 = 8x$  requires  $x \geq 0$ 
    double y2 = 8*x;
    double y = sqrt(y2);
    results[idx++] = x;
    results[idx++] = y;
    results[idx++] = x;
    results[idx++] = -y;
}

while (idx < 4) {
    results[idx++] = NAN;
}
}
```


Python+C code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load C shared library
lib = ctypes.CDLL("./libconics.so")

# Define return type
lib.solve_conics.argtypes = [ctypes.POINTER(ctypes.c_double)]

# Prepare results array
results = (ctypes.c_double * 4)()
lib.solve_conics(results)

vals = list(results)
points = [(vals[i], vals[i+1]) for i in range(0, len(vals), 2) if
          not np.isnan(vals[i])]
print("Solutions :", points)
```

```
# PLOT
fig, ax = plt.subplots(figsize=(6,6))
ax.set_xlabel("x")
ax.set_ylabel("y")
ax.set_title("Intersection of Hyperbola and Parabola")
ax.grid(True)

# Hyperbola:  $x^2 - y^2 = 180 \rightarrow y = \sqrt{x^2 - 180}$ 
xh = np.linspace(-40, 40, 800)
for sign in [1, -1]:
    yh = sign*np.sqrt(np.maximum(xh**2 - 180, 0))
    ax.plot(xh, yh, 'r', label="Hyperbola" if sign==1 else "")

# Parabola:  $y^2 = 8x \rightarrow y = \sqrt{8x}$ 
xp = np.linspace(0, 40, 400)
for sign in [1, -1]:
    yp = sign*np.sqrt(8*xp)
    ax.plot(xp, yp, 'b', label="Parabola" if sign==1 else "")
```

```
# Intersection points from C
for (px, py) in points:
    ax.plot(px, py, 'ko', markersize=8)
    ax.text(px+0.5, py+0.5, f"({px:.0f},{py:.0f})")

ax.legend()
plt.savefig("/home/user/Matrix Theory: workspace/
    Matgeo_assignments/9.4.39/figs/Figure_1.png")
plt.show()
```

Python code

```
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt

# Variables
x, y = sp.symbols('x y', real=True)
# Equations
eq1 = sp.Eq(x**2 - y**2, 180) # Hyperbola
eq2 = sp.Eq(y**2, 8*x) # Parabola
# Solve system
solutions = sp.solve([eq1, eq2], [x, y], dict=True)
print("Solutions:")
for sol in solutions:
    print(sol)
# Extract real solutions
real_solutions = [(float(sol[x]), float(sol[y])) for sol in
    solutions if sol[y].is_real]
```

Python code

```
# Setup plot
fig, ax = plt.subplots(figsize=(6,6))
ax.set_xlabel("x")
ax.set_ylabel("y")
ax.set_title("Intersection of Hyperbola and Parabola")
ax.grid(True)

# Range for plotting
xx = np.linspace(-20, 40, 400)

# Hyperbola:  $x^2 - y^2 = 180 \rightarrow y = \sqrt{x^2 - 180}$ 
xh = np.linspace(-40, 40, 800)
for sign in [1, -1]:
    yh = sign*np.sqrt(np.maximum(xh**2 - 180, 0)) # avoid
        negatives under sqrt
    ax.plot(xh, yh, 'r', label="Hyperbola" if sign==1 else "")
```

Python code

```
# Parabola:  $y^2 = 8x \rightarrow y = \sqrt{8x}$ 
xp = np.linspace(0, 40, 400) # parabola domain  $x \geq 0$ 
for sign in [1, -1]:
    yp = sign*np.sqrt(8*xp)
    ax.plot(xp, yp, 'b', label="Parabola" if sign==1 else "")
# Mark intersection points
for (px, py) in real_solutions:
    ax.plot(px, py, 'ko', markersize=8)
    ax.text(px+0.5, py+0.5, f"({px:.0f},{py:.0f})")

ax.legend()
plt.savefig("/home/user/Matrix Theory: workspace/
Matgeo_assignments/9.4.39/figs/Figure_1.png")
plt.show()
```

