

1.6.25

AI25BTECH11010 - Dhanush Kumar

Question:

Three points $P(h, k)$, $Q(x_1, y_1)$ and $R(x_2, y_2)$ lie on a line. Show that $(h - x_1)(y_2 - y_1) = (k - y_1)(x_2 - x_1)$.

Solution:

Let

$$\mathbf{P} = \begin{pmatrix} h \\ k \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}. \quad (1)$$

$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix} h - x_1 \\ k - y_1 \end{pmatrix} \quad (2)$$

$$\mathbf{R} - \mathbf{Q} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix} \quad (3)$$

Now form the matrix:

$$\mathbf{M} = (\mathbf{P} - \mathbf{Q} \quad \mathbf{R} - \mathbf{Q}) = \begin{pmatrix} h - x_1 & x_2 - x_1 \\ k - y_1 & y_2 - y_1 \end{pmatrix} \quad (4)$$

Apply row reduction:

$$\begin{pmatrix} h - x_1 & x_2 - x_1 \\ k - y_1 & y_2 - y_1 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - \frac{k - y_1}{h - x_1} R_1} \begin{pmatrix} h - x_1 & x_2 - x_1 \\ 0 & (y_2 - y_1) - \frac{k - y_1}{h - x_1} (x_2 - x_1) \end{pmatrix} \quad (5)$$

Since P, Q and R lie on line the rank of matrix M is 1

For rank = 1, the second entry in the last row must vanish:

$$(y_2 - y_1)(h - x_1) - (k - y_1)(x_2 - x_1) = 0 \quad (6)$$

Thus,

$$(h - x_1)(y_2 - y_1) = (k - y_1)(x_2 - x_1). \quad (7)$$

Hence proved.