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Puni Aditya - EE25BTECH11046

Question: Given matrix

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

The eigenvalue corresponding to the eigenvector

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

is ____________.

Solution: Let the eigenvalue λ have $\mathbf{v}$ as its corresponding eigenvector for the matrix $\mathbf{A}$.

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{v} \tag{1}$$

$$\mathbf{v}^\top\mathbf{A}\mathbf{v} = \lambda\mathbf{v}^\top\mathbf{v} \tag{2}$$

$$\lambda = \frac{\mathbf{v}^\top\mathbf{A}\mathbf{v}}{\mathbf{v}^\top\mathbf{v}} \tag{3}$$

Here,

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \text{ and } \mathbf{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{4}$$

Using (3),

$$\lambda = \frac{\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}}{\begin{pmatrix} 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}} \tag{5}$$

$$\therefore \lambda = 3 \tag{6}$$