Matrices in Geometry - 5.2.40

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Problem Statement

Solve

$$\frac{4}{x} + 3y = 14$$
$$\frac{3}{x} - 4y = 23$$

Solution

We have the following two equations as:

$$\begin{pmatrix} 4 & 3 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ y \end{pmatrix} = \begin{pmatrix} 14 \\ 23 \end{pmatrix} \tag{1}$$

Writing the augmented matrix for these equations,

$$\left(\begin{array}{cc|c} 4 & 3 & 14 \\ 3 & -4 & 23 \end{array}\right) \overset{R_1 \to R_1/4}{\longleftrightarrow} \left(\begin{array}{cc|c} 1 & 3/4 & 7/2 \\ 3 & -4 & 23 \end{array}\right) \overset{R_2 \to R_2 - 3R_1}{\longleftrightarrow} \tag{2}$$

$$\left(\begin{array}{cc|c} 1 & 3/4 & 7/2 \\ 0 & -25/4 & 25/2 \end{array}\right) \overset{R_2 \to \frac{-4}{25} R_2}{\longleftrightarrow} \left(\begin{array}{cc|c} 1 & 3/4 & 7/2 \\ 0 & 1 & -2 \end{array}\right) \overset{R_1 \to R_1 - \frac{3}{4} R_2}{\longleftrightarrow} \tag{3}$$

$$\left(\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & -2 \end{array}\right) \tag{4}$$

Solution

This implies that

$$\begin{pmatrix} \frac{1}{x} \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \implies x = \frac{1}{5}\ ,\ y = -2 \tag{5}$$

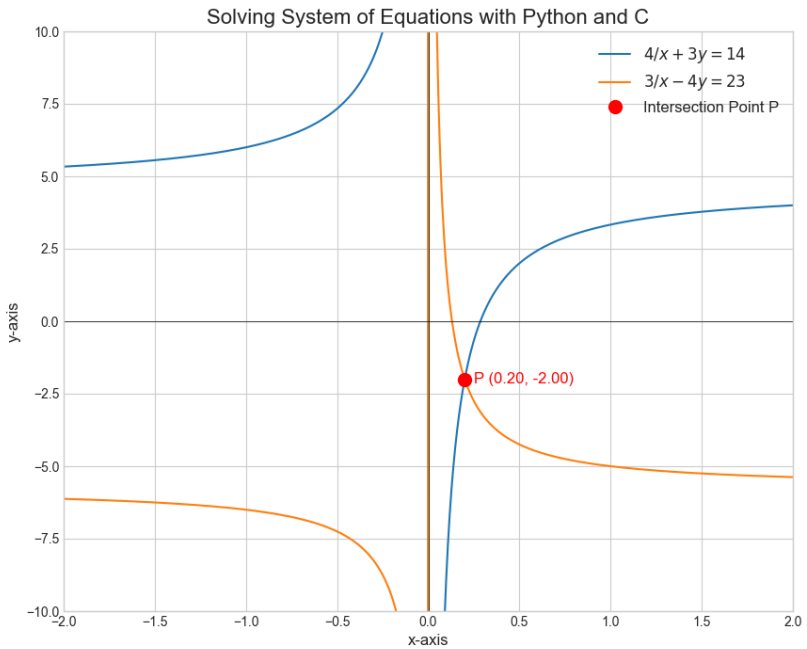

Figure: Graph for 5.2.40