4.7.42

EE25BTECH11045 - P.Navya Priya

Question:

Find the length and the foot of perpendicular from the point $\left(1, \frac{3}{2}, 2\right)$ to the plane $2x - 2y + 4z + 5 = 0$.

Solution:

Given plane equation $2x - 2y + 4z + 5 = 0$ can be written as

$$\mathbf{n}^\top \mathbf{x} = c \tag{1}$$

Where

$$\mathbf{n} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} \text{ and } c = -5$$

Let the point be $\mathbf{p}\begin{pmatrix} 1 \\ \frac{3}{2} \\ 2 \end{pmatrix}$ and the point on the plane be $\mathbf{x_o}$. The equation of the line joining $\mathbf{p}$ and $\mathbf{x_o}$ is

$$\mathbf{x_o} = \mathbf{p} + \lambda \mathbf{n} \tag{2}$$

Multiply equation(2) on both sides by $\mathbf{n}^\top$

$$\mathbf{n}^\top (\mathbf{x_o}) = \mathbf{n}^\top (\mathbf{p} + \lambda \mathbf{n}) \tag{3}$$

$$\lambda = \frac{\mathbf{n}^\top \mathbf{x}_o}{\mathbf{n}^\top \mathbf{p} + \lambda \mathbf{n}^\top \mathbf{n}} \tag{4}$$

$$\lambda = \frac{-5}{\begin{pmatrix} 2 & -2 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ \frac{3}{2} \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 2 & -2 & 4 \end{pmatrix}\begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}} (\because \mathbf{n}^\top \mathbf{x_o} = -5) \tag{5}$$

$$\lambda = -\frac{1}{2} \tag{6}$$

Substitute the value of λ in equation(2) to get $\mathbf{x}_o$

$$\mathbf{x}_o = \begin{pmatrix} 0 \\ \frac{5}{2} \\ 0 \end{pmatrix} \tag{7}$$

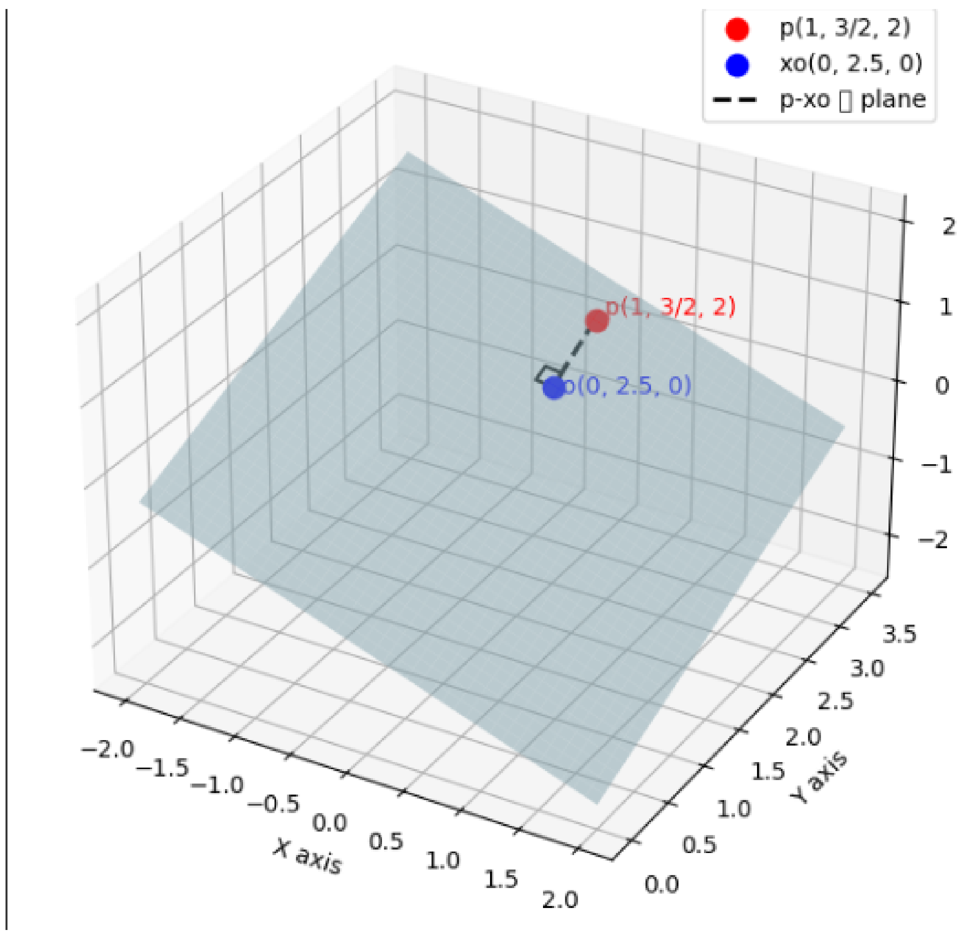
$$\therefore \textbf{Foot of perpendicular} \text{ is } \mathbf{x}_o = \begin{pmatrix} 0 \\ \frac{5}{2} \\ 0 \end{pmatrix}$$

The length of point $\begin{pmatrix} 1 \\ \frac{3}{2} \\ 2 \end{pmatrix}$ to the plane $2x - 2y + 4z + 5 = 0$ is

$$\|\mathbf{x_o} - \mathbf{p}\| = \sqrt{(\mathbf{x_o} - \mathbf{p})^\top (\mathbf{x_o} - \mathbf{p})} \tag{8}$$
$$= \sqrt{6} \tag{9}$$

$$\therefore \|\mathbf{x_o} - \mathbf{p}\| = \sqrt{6}$$

From the graph, theoretical solution matches with the computational solution.



Perpendicular to the plane (length= $\sqrt{6}$)