

12.27

EE25BTECH11013 - Bhargav

Question:

1200 men and 500 women can build a bridge in 2 weeks. 900 men and 250 women will take 3 weeks to build the same bridge. How many men will be needed to build the bridge in one week?

Solution:

Let one man complete work in x weeks and one woman complete work in y weeks

In one week a man can complete $\frac{1}{x}$ work and woman can complete $\frac{1}{y}$

$$\frac{1200}{x} + \frac{500}{y} = \frac{1}{2} \quad (0.1)$$

$$\frac{900}{x} + \frac{250}{y} = \frac{1}{3} \quad (0.2)$$

$$\begin{pmatrix} 1200 & 500 \\ 900 & 250 \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{3} \end{pmatrix} \quad (0.3)$$

This can be converted into an augmented matrix and can be solved by Gaussian elimination:

$$\left(\begin{array}{cc|c} 1200 & 500 & \frac{1}{2} \\ 900 & 250 & \frac{1}{3} \end{array} \right) \xrightarrow[R_2 \leftarrow R_2 - 3R_1/4]{R_2 \leftarrow R_2 - 125} \left(\begin{array}{cc|c} 1200 & 500 & \frac{1}{2} \\ 0 & 1 & \frac{1}{3000} \end{array} \right) \quad (0.4)$$

$$\xrightarrow[R_1 \leftarrow R_1 - 500R_2]{R_1 \leftarrow R_1/1200} \left(\begin{array}{cc|c} 1 & 0 & \frac{1}{3600} \\ 0 & 1 & \frac{1}{3000} \end{array} \right) \quad (0.5)$$

$$\begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} \frac{1}{3600} \\ \frac{1}{3000} \end{pmatrix} \quad (0.6)$$

A man can finish the work in 3600 weeks, a woman can finish the work in 3000 weeks
Therefore 3600 men are required for completing the task in 1 week