2.2.24

AI25BTECH110031
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Question(2.2.24) If $\hat{i} + \hat{j} + \hat{k}, 2\hat{i} + 5\hat{j}, 3\hat{i} + 2\hat{j} - 3\hat{k}$ and $\hat{i} - 6\hat{j} - \hat{k}$ are the position vectors of the point **A**,**B**,**C** and **D** respectively, then find the angle between **AB** and **CD**. Deduce that **AB** and **CD** are collinear.

Solution:

Given points are

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 1 \\ -6 \\ -1 \end{pmatrix}. \tag{0.1}$$

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}, \qquad \mathbf{CD} = \mathbf{D} - \mathbf{C} = \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix}. \tag{0.2}$$

The angle θ between **AB** and **CD** is given by

$$\cos\theta = \frac{\mathbf{AB}^\top \mathbf{CD}}{\|AB\| \, \|CD\|}. \tag{0.3}$$

$$\mathbf{AB}^T \mathbf{CD} = \begin{pmatrix} 1 & 4 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix} = (1)(-2) + (4)(-8) + (-1)(2) = -36. \tag{0.4}$$

$$\|AB\| = 3\sqrt{2}, \qquad \|CD\| = 6\sqrt{2}. \tag{0.5}$$

$$\cos\theta = \frac{-36}{(3\sqrt{2})(6\sqrt{2})} = \frac{-36}{36} = -1. \tag{0.6}$$

Hence,

$$\theta = \cos^{-1}(-1) = \pi \ (180°). \tag{0.7}$$

Since, angle between vectors is 180° the given vectors are collinear

Proof of collinearity by rank method
Let,

$$\mathbf{P} = \begin{pmatrix} B - A & D - C \end{pmatrix} \tag{0.8}$$

$$\mathbf{P} = \begin{pmatrix} 1 & -2 \\ 4 & -8 \\ -1 & 2 \end{pmatrix} \tag{0.9}$$

$$\mathbf{P}^T = \begin{pmatrix} 1 & 4 & -1 \\ -2 & -8 & 2 \end{pmatrix} \tag{0.10}$$

$$R_2 \rightarrow R_2 - 2R_1 \tag{0.11}$$

$$\mathbf{P}^T = \begin{pmatrix} 1 & 4 & -1 \\ 0 & 0 & 0 \end{pmatrix} \tag{0.12}$$

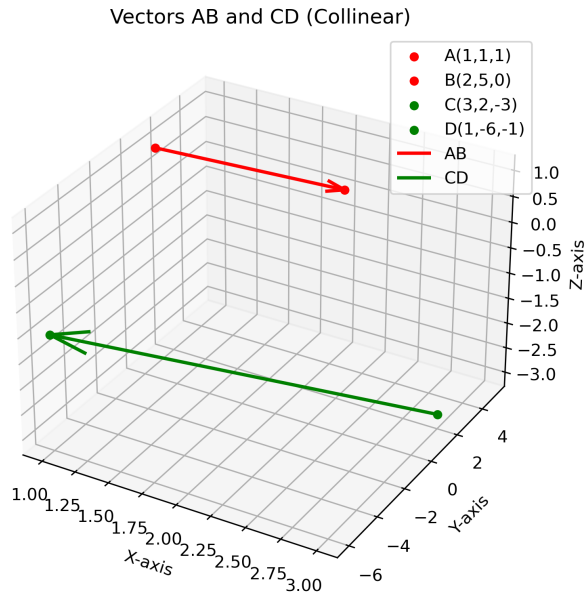$$rank\mathbf{P} = rank\mathbf{P}^T = 1 \tag{0.13}$$

$$\tag{0.14}$$

Thus the given vectors are collinear.

Fig. 0.1

NOTE: Parallel vectors are collinear.