

1.5.16

EE25btech11028 - J.Navya sri

Question: Find the point A if AB is a diameter of the circle with center $C = (3, -1)$ and point $B = (2, 6)$.

Solution:

Point	Vector
B	$\begin{pmatrix} 2 \\ 6 \end{pmatrix}$
C	$\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

Section Formula:

If a point P divides the line joining A and B internally in the ratio $m : n$, then

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k+1} = (\mathbf{A} \quad \mathbf{B}) \begin{pmatrix} \frac{1}{k+1} \\ \frac{k}{k+1} \end{pmatrix}$$

Here, C is the midpoint of AB , i.e. ratio $1 : 1$.

$$\mathbf{C} = \frac{\mathbf{A} + \mathbf{B}}{2} = (\mathbf{A} \quad \mathbf{B}) \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}.$$

Substitute values:

$$\begin{pmatrix} 3 \\ -1 \end{pmatrix} = (\mathbf{A} \quad \begin{pmatrix} 2 \\ 6 \end{pmatrix}) \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}.$$

$$2 \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \mathbf{A} + \begin{pmatrix} 2 \\ 6 \end{pmatrix} \Rightarrow \mathbf{A} = 2 \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}.$$

Rank Verification:

Check collinearity of A, B, C :

$$(\mathbf{C} - \mathbf{A} \quad \mathbf{B} - \mathbf{A}) = \begin{pmatrix} 3-4 & 2-4 \\ -1-(-8) & 6-(-8) \end{pmatrix} = \begin{pmatrix} -1 & -2 \\ 7 & 14 \end{pmatrix} = 0$$

Thus, rank = 1 and points are collinear.

$$\mathbf{A} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$$

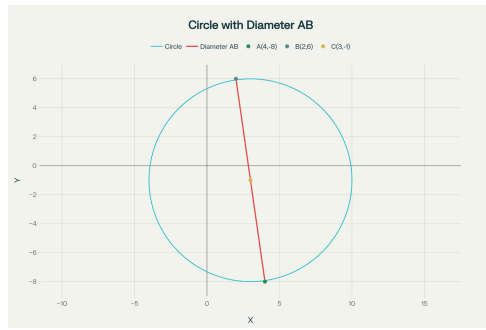


Fig. 0