

10.7.75

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Question

Find the equations of tangents drawn from origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$, are

- ① $x = 0$
- ② $y = 0$
- ③ $(h^2 - r^2)x - 2rhy = 0$
- ④ $(h^2 - r^2)x + 2rhy = 0$

Theoretical Solution

Given the equation of circle,

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (1)$$

where, $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$, $\mathbf{u} = \begin{pmatrix} r \\ h \end{pmatrix}$ and $f = h^2$.

It is given that the tangents pass through the origin.

$$\therefore \mathbf{n}^\top \mathbf{x} = 0 \quad (2)$$

where \mathbf{n} is the direction vector of the tangent. From (1),

$$\therefore \frac{|n^\top \mathbf{u}|}{\|\mathbf{n}\|} = r_c \quad (3)$$

where r_c is the radius of the circle. For the given equation of circle,

$$\therefore r_c = r \quad (4)$$

Theoretical Solution

$$\implies \frac{|\mathbf{n}^T \mathbf{u}|}{\|\mathbf{n}\|} = r \quad (5)$$

Let $\mathbf{n} = \begin{pmatrix} a \\ b \end{pmatrix}$, substituting in the equation (5),

$$\therefore \frac{|ar + bh|}{\sqrt{a^2 + b^2}} = r \quad (6)$$

Squaring on both sides,

$$a^2 r^2 + b^2 h^2 + 2arbh = r^2 a^2 + r^2 b^2 \implies b(h^2 b + 2arh - br^2) = 0 \quad (7)$$

Theoretical Solution

From (7),

$$b = 0 \quad (8)$$

$$a = \frac{r^2 - h^2}{2rh} b \quad (9)$$

∴ The required equations of tangents are,

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 0 \quad (10)$$

$$\begin{pmatrix} h^2 - r^2 & -2rh \end{pmatrix} \mathbf{x} = 0 \quad (11)$$

C Code -Finding the equation of tangents

```
#include <stdio.h>

void solve_tangents(double r, double h, double tangents[4]) {
    // First tangent: x = 0 -> line (1,0)
    tangents[0] = 1.0;
    tangents[1] = 0.0;
    // Second tangent:  $(h^2 - r^2)x - 2rh y = 0$ 
    double a = h*h - r*r;
    double b = -2.0*r*h;

    if (a == 0 && b == 0) {
        // Special case -> y=0
        tangents[2] = 0.0;
        tangents[3] = 1.0;
    } else {
        tangents[2] = a;
        tangents[3] = b;
    }
}
```

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load shared C library
lib = ctypes.CDLL("./libtangent_solver.so")

# Define C function signature
lib.solve_tangents.argtypes = [ctypes.c_double, ctypes.c_double,
                                ctypes.POINTER(ctypes.c_double)]
lib.solve_tangents.restype = None

def get_tangents_from_c(r, h):
    results = (ctypes.c_double * 4)()
    lib.solve_tangents(r, h, results)
    vals = list(results)
    tangents = [(vals[0], vals[1]), (vals[2], vals[3])]
```

```
tangent_eqs = []
    for a, b in tangents:
        if b == 0: # x=0
            tangent_eqs.append("x = 0")
        elif a == 0: # y=0
            tangent_eqs.append("y = 0")
        else:
            tangent_eqs.append(f"{a:.0f}x {b:+.0f}y = 0")

    return tangents, tangent_eqs

def plot_tangents(r, h):
    tangents, tangent_eqs = get_tangents_from_c(r, h)

    print("Tangents from origin:")
    for eq in tangent_eqs:
        print(" ", eq)
```


Circle

```
theta = np.linspace(0, 2*np.pi, 400)
x_circ = r + r*np.cos(theta)
y_circ = h + r*np.sin(theta)

plt.figure(figsize=(6,6))
plt.plot(x_circ, y_circ, 'b', label="Circle")
```

Tangents

```
x_vals = np.linspace(-2*r, 2*r, 400)
for (a,b), eq in zip(tangents, tangent_eqs):
    if b == 0:
        plt.axvline(0, color='r', linestyle='--', label=eq)
    else:
        y_vals = -(a/b)*x_vals
        plt.plot(x_vals, y_vals, 'r--', label=eq)
```

Origin & center

```
plt.scatter([0],[0], color='k', marker='o', label="Origin
(0,0)")
plt.scatter([r],[h], color='g', marker='x', label=f"Center ({
r},{h})")

plt.gca().set_aspect('equal', adjustable='box')
plt.axhline(0, color='gray', linewidth=0.5)
plt.axvline(0, color='gray', linewidth=0.5)
plt.legend()
plt.title(f"Tangents from Origin to Circle (r={r}, h={h})")
plt.savefig("/home/user/Matrix Theory: workspace/
Matgeo_assignments/10.7.75/figs/figure_1.png")
plt.show()
```

Example

```
plot_tangents(r=3, h=4)
```

Python code

```
1 import numpy as np
2 import matplotlib.pyplot as plt
3
4 def tangents_from_origin(r, h):
5     """
6     Finds tangent lines from origin to the circle
7      $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ .
8
9     Returns:
10         tangents: list of (a,b) representing  $ax+by=0$ 
11         tangent_eqs: list of pretty string equations
12     """
13     tangents = []
14     tangent_eqs = []
15     # Case 1:  $x=0$ 
16     tangents.append((1, 0))
17     tangent_eqs.append("x = 0")
```

Python code

```
# Case 2
```

```
if (h**2 - r**2) != 0:
    a, b = (h**2 - r**2, -2*r*h)
    tangents.append((a, b))
    tangent_eqs.append(f"{a}x {b:+}y = 0")
else:
    tangents.append((0, 1))
    tangent_eqs.append("y = 0")
```

```
return tangents, tangent_eqs
```

```
def plot_tangents(r, h):
    tangents, tangent_eqs = tangents_from_origin(r, h)

    # Print immediately
    print("Tangents from origin:")
    for eq in tangent_eqs:
        print(" ", eq)
```

Python code

```
# Circle: center = (r,h), radius = r
theta = np.linspace(0, 2*np.pi, 400)
x_circ = r + r*np.cos(theta)
y_circ = h + r*np.sin(theta)

plt.figure(figsize=(6,6))
plt.plot(x_circ, y_circ, 'b', label="Circle")

# Tangents
x_vals = np.linspace(-2*r, 2*r, 400)
for (a,b), eq in zip(tangents, tangent_eqs):
    if b == 0:
        plt.axvline(0, color='r', linestyle='--', label=eq)
    else:
        y_vals = -(a/b)*x_vals
        plt.plot(x_vals, y_vals, 'r--', label=eq)
```

Python code

```
# Mark origin & center
```

```
plt.scatter([0],[0], color='k', marker='o', label="Origin  
(0,0)")  
plt.scatter([r],[h], color='g', marker='x', label=f"Center ({  
r},{h})")
```

```
plt.gca().set_aspect('equal', adjustable='box')  
plt.axhline(0, color='gray', linewidth=0.5)  
plt.axvline(0, color='gray', linewidth=0.5)  
plt.legend()  
plt.title(f"Tangents from Origin to Circle (r={r}, h={h})")  
plt.savefig("/home/user/Matrix Theory: workspace/  
Matgeo_assignments/10.7.75/figs/Figure_1.png")  
plt.show()
```

```
# Example usage
```

```
plot_tangents(r=3, h=4)
```

