4.5.14

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Question

Solve the system of equations

$$2x + y = 5$$
$$3x + 2y = 8$$

finding the solution of given equations :

The equation of line:

$$n^\top x = c \tag{1}$$

Line L:

$$\begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 5 \tag{2}$$

Line K:

$$\begin{pmatrix} 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 8 \tag{3}$$

Writing in matrix form:

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \tag{4}$$

The following augmented matrix can be solved by gaussian elimination

$$\left(\begin{array}{cc|c} 2 & 1 & 5 \\ 3 & 2 & 8 \end{array}\right) \xrightarrow{R_2 \to R_2 - \frac{3}{2}R_1} \left(\begin{array}{cc|c} 2 & 1 & 5 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{array}\right) \tag{5}$$

Since,

$$\text{rank}(A) = \text{rank}(A|b) = 2 \tag{6}$$

the system has a unique solution.
from 2^{nd} row,

$$y = 1 \Rightarrow x = 2 \tag{7}$$

$\therefore$ Solution of given system of equations is: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Python Code

```
import numpy as np
import matplotlib.pyplot as plt

# Create a range of x-values for plotting
x = np.linspace(-2, 6, 400)

# Rearrange the equations to solve for y
# Equation 1: 2x + y = 5 => y = 5 - 2x
y1 = 5 - 2 * x
```

Python Code

```
# Equation 2: 3x + 2y = 8 => 2y = 8 - 3x => y = (8 - 3x) / 2
y2 = (8 - 3 * x) / 2
# --- Create the Plot ---
plt.figure(figsize=(10, 8))
# Plot the two lines
plt.plot(x, y1, label=r'$2x + y = 5$')
plt.plot(x, y2, label=r'$3x + 2y = 8$')
```

Python Code

```
# The solution is the intersection point (2, 1)
solution_x = 2
solution_y = 1

# Plot and annotate the intersection point
plt.plot(solution_x, solution_y, 'ro', markersize=8, label=f'
    Solution ({solution_x}, {solution_y})')
plt.annotate(f'({solution_x}, {solution_y})',
             xy=(solution_x, solution_y),
             xytext=(solution_x + 0.2, solution_y + 0.2))
```

Python Code

```
# --- Formatting the Graph ---
plt.title('Solution of the System of Equations', fontsize=16)
plt.xlabel('x-axis', fontsize=12)
plt.ylabel('y-axis', fontsize=12)
plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
plt.grid(True, which='both', linestyle='--', linewidth=0.5)
plt.legend(fontsize=12)
plt.show()
```

C Code

```c
#include <stdio.h>

int main() {
// The given system of equations is:
// 2x + 1y = 5
// 3x + 2y = 8

// Define the coefficients and constants from the equations
// Equation 1: a1*x + b1*y = c1
```

C Code

```c
double a1 = 2.0, b1 = 1.0, c1 = 5.0;
// Equation 2: a2*x + b2*y = c2
double a2 = 3.0, b2 = 2.0, c2 = 8.0;

// Variables to store the solution
double x, y;
// Calculate the determinant of the coefficient matrix
// D = (a1 * b2) - (a2 * b1)
double determinant = a1 * b2 - a2 * b1;
```

C Code

```
if (determinant != 0) {
x = (c1 * b2 - c2 * b1) / determinant;
y = (a1 * c2 - a2 * c1) / determinant;
printf("The solution is:\n");
printf("x = %.2f\n", x);
printf("y = %.2f\n", y);
    } else {
printf("The system does not have a unique solution.\n");
    }
    return 0;
}
```

Python and C Code

```
from ctypes import c_double

def solve_linear_system():
    # Coefficients and constants from the equations
    a1 = c_double(2.0)
    b1 = c_double(1.0)
    c1 = c_double(5.0)

    a2 = c_double(3.0)
    b2 = c_double(2.0)
    c2 = c_double(8.0)
```

Python and C Code

```
    # Calculate the determinant
    determinant = c_double(a1.value * b2.value - a2.value * b1.
        value)

    if determinant.value != 0:
        # Use Cramer's Rule
        x = c_double((c1.value * b2.value - c2.value * b1.value)
            / determinant.value)
        y = c_double((a1.value * c2.value - a2.value * c1.value)
            / determinant.value)
```

Python and C Code

```
        # Print the results
        print("The solution is:")
        print(f"x = {x.value:.2f}")
        print(f"y = {y.value:.2f}")
    else:
        print("The system does not have a unique solution.")
# Run the function
solve_linear_system()
```

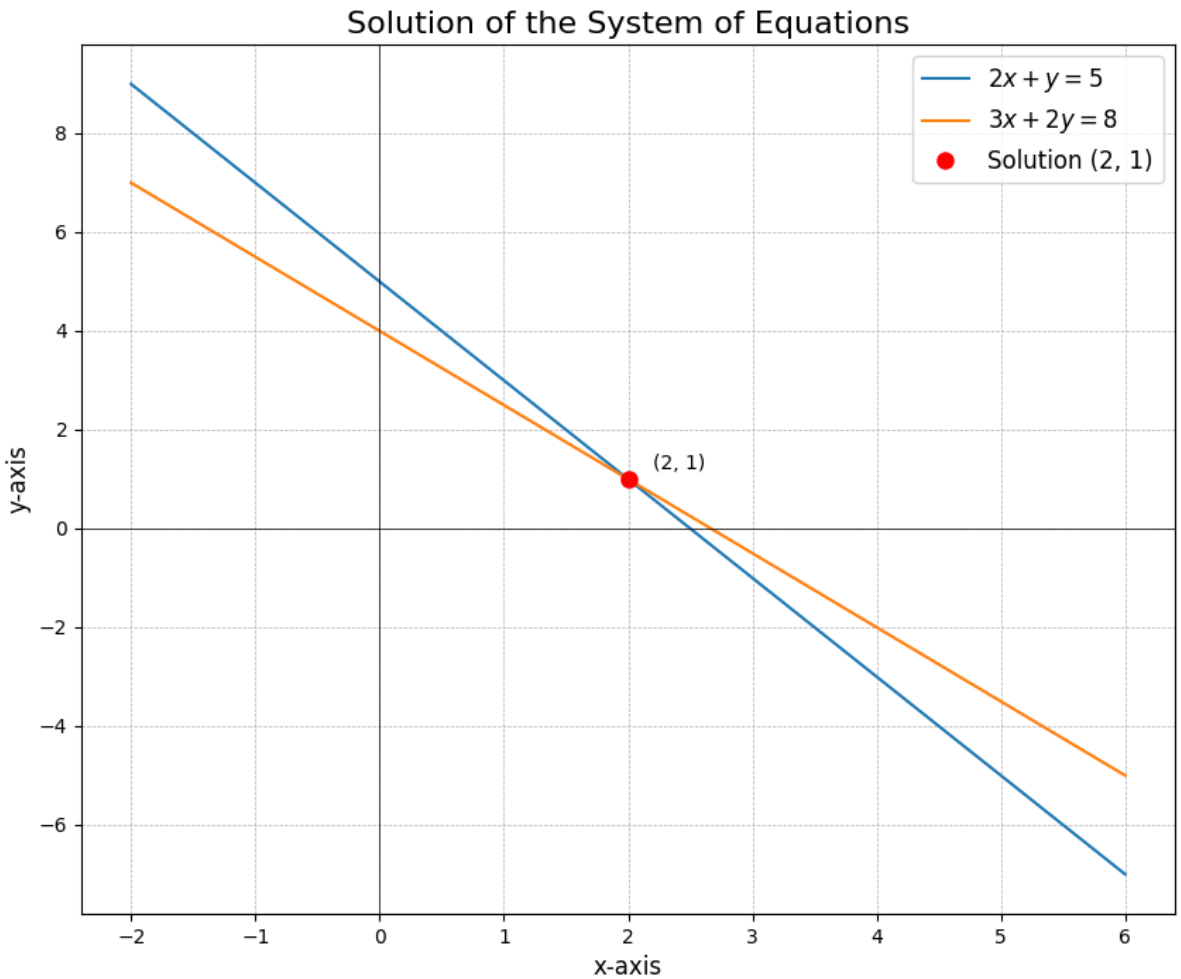
Solution of the System of Equations
$2x + y = 5$
$3x + 2y = 8$
Solution (2, 1)
(2, 1)
y-axis
x-axis
8
6
4
2
0
−2
−4
−6
−2
−1
0
1
2
3
4
5
6

Figure: Plot