2.8.16

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Question

Prove that the lines $x = py + q, z = ry + s$ and $x = p'y + q', z = r'y + s'$ are perpendicular if $pp' + rr' + 1 = 0$..

Theoretical Solution

Let line L_1 be the intersection of the planes,

$$x - py - q = 0\ ,\ z - ry - s = 0$$

Let line L_2 be the intersection of the planes,

$$x - p'y - q' = 0\ ,\ z - r'y - s' = 0$$

The direction vectors of the lines L_1 and L_2 are given by the cross product of the direction vectors of the normals of the intersecting planes.

Theoretical Solution

Let $\mathbf{n_1}$, $\mathbf{n_2}$ be the normals for the planes
$x - py - q = 0$ and $z - ry - s = 0$ respectively.

$$\text{direction vector of } \mathbf{n_1} = \begin{pmatrix} 1 \\ -p \\ 0 \end{pmatrix} \tag{1}$$

$$\text{direction vector of } \mathbf{n_2} = \begin{pmatrix} 0 \\ -r \\ 1 \end{pmatrix} \tag{2}$$

Let $\mathbf{n_3}$, $\mathbf{n_4}$ be the normals for the planes
$x - p'y - q' = 0$ and $z - r'y - s' = 0$ respectively.

$$\text{direction vector of } \mathbf{n_3} = \begin{pmatrix} 1 \\ -p' \\ 0 \end{pmatrix} \tag{3}$$

Theoretical Solution

$$\textit{direction vector of } \mathbf{n_4} = \begin{pmatrix} 0 \\ -r' \\ 1 \end{pmatrix} \tag{4}$$

$$\therefore \textit{direction vector of } L_1 = \mathbf{n_1} \times \mathbf{n_2} \tag{5}$$

$$\textit{direction vector of } L_2 = \mathbf{n_3} \times \mathbf{n_4} \tag{6}$$

Formulae

The *cross product* or *vector product* of $\mathbf{n_1}, \mathbf{n_2}$ is defined as

$$\mathbf{n_1} \times \mathbf{n_2} = \begin{pmatrix} \begin{vmatrix} (\mathbf{n_1})_{23} & (\mathbf{n_2})_{23} \end{vmatrix} \\ \begin{vmatrix} (\mathbf{n_1})_{31} & (\mathbf{n_2})_{31} \end{vmatrix} \\ \begin{vmatrix} (\mathbf{n_1})_{12} & (\mathbf{n_2})_{12} \end{vmatrix} \end{pmatrix} \tag{7}$$

$$\begin{vmatrix}(\mathbf{n_1})_{23} & (\mathbf{n_2})_{23}\end{vmatrix} = \begin{vmatrix}-p & -r\\ 0 & 1\end{vmatrix} = -p \tag{8}$$

$$\begin{vmatrix}(\mathbf{n_1})_{31} & (\mathbf{n_2})_{31}\end{vmatrix} = \begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix} = -1 \tag{9}$$

$$\begin{vmatrix}(\mathbf{n_1})_{12} & (\mathbf{n_2})_{12}\end{vmatrix} = \begin{vmatrix}1 & 0\\ -p & -r\end{vmatrix} = -r \tag{10}$$

$$\mathbf{n_1} \times \mathbf{n_2} = \begin{pmatrix}\begin{vmatrix}(\mathbf{n_1})_{23} & (\mathbf{n_2})_{23}\end{vmatrix}\\ \begin{vmatrix}(\mathbf{n_1})_{31} & (\mathbf{n_2})_{31}\end{vmatrix}\\ \begin{vmatrix}(\mathbf{n_1})_{12} & (\mathbf{n_2})_{12}\end{vmatrix}\end{pmatrix} = \begin{pmatrix}-p\\ -1\\ -r\end{pmatrix} \tag{11}$$

$$\implies \textit{direction vector of } L_1 = \begin{pmatrix}-p\\ -1\\ -r\end{pmatrix} \tag{12}$$

Formulae

The *cross product* or *vector product* of $\mathbf{n_3}, \mathbf{n_4}$ is defined as

$$\mathbf{n_3} \times \mathbf{n_4} = \begin{pmatrix} \begin{vmatrix} (\mathbf{n_3})_{23} & (\mathbf{n_4})_{23} \end{vmatrix} \\ \begin{vmatrix} (\mathbf{n_3})_{31} & (\mathbf{n_4})_{31} \end{vmatrix} \\ \begin{vmatrix} (\mathbf{n_3})_{12} & (\mathbf{n_4})_{12} \end{vmatrix} \end{pmatrix} \tag{13}$$

Theoretical Solution

$$\begin{vmatrix}(\mathbf{n_3})_{23} & (\mathbf{n_4})_{23}\end{vmatrix} = \begin{vmatrix}-p' & -r' \\ 0 & 1\end{vmatrix} = -p' \tag{14}$$

$$\begin{vmatrix}(\mathbf{n_3})_{31} & (\mathbf{n_4})_{31}\end{vmatrix} = \begin{vmatrix}0 & 1 \\ 1 & 0\end{vmatrix} = -1 \tag{15}$$

$$\begin{vmatrix}(\mathbf{n_3})_{12} & (\mathbf{n_4})_{12}\end{vmatrix} = \begin{vmatrix}1 & 0 \\ -p' & -r'\end{vmatrix} = -r', \tag{16}$$

$$\mathbf{n_3} \times \mathbf{n_4} = \begin{pmatrix}\begin{vmatrix}(\mathbf{n_3})_{23} & (\mathbf{n_4})_{23}\end{vmatrix} \\ \begin{vmatrix}(\mathbf{n_3})_{31} & (\mathbf{n_4})_{31}\end{vmatrix} \\ \begin{vmatrix}(\mathbf{n_3})_{12} & (\mathbf{n_4})_{12}\end{vmatrix}\end{pmatrix} = \begin{pmatrix}-p' \\ -1 \\ -r'\end{pmatrix} \tag{17}$$

$$\implies \textit{direction vector of } L_2 = \begin{pmatrix}-p' \\ -1 \\ -r'\end{pmatrix} \tag{18}$$

Theoretical Solution

If the lines are perpendicular, then their dot product of direction vectors must be zero.

$$\implies (direction\ vector\ of\ L_1)^\top (direction\ vector\ of\ L_2) = 0 \tag{19}$$

$$\implies \begin{pmatrix} -p & -1 & -r \end{pmatrix} \begin{pmatrix} -p' \\ -1 \\ -r' \end{pmatrix} = 0 \tag{20}$$

$$\implies pp' + rr' + 1 = 0 \tag{21}$$

$\therefore$the lines $x = py + q, z = ry + s$ and $x = p'y + q', z = r'y + s'$ are perpendicular if $pp' + rr' + 1 = 0$

C Code - A function to check whether they are perpendicular

```c
#include <stdio.h>

int is_perpendicular(double p, double r, double p_prime, double
    r_prime) {
    if ((p * p_prime) + (r * r_prime) + 1 == 0) {
        return 1; // True, the lines are perpendicular
    }
    return 0; // False, the lines are not perpendicular
}
```

Python Code

```python
import ctypes
import os

# Load the shared library
lib = ctypes.CDLL('./code.so')
# Define the argument types for the C function
lib.is_perpendicular.argtypes = [ctypes.c_double, ctypes.c_double
    , ctypes.c_double, ctypes.c_double]
# Define the return type for the C function
lib.is_perpendicular.restype = ctypes.c_int

def check_perpendicular(p, r, p_prime, r_prime):
    result = lib.is_perpendicular(p, r, p_prime, r_prime)
    return bool(result)
```