

1.5.13

EE25BTECH11025 - Ganachari Vishwambhar

Question:

Find the ratio in which the Y axis divides the line segment joining the points $\mathbf{A}(-1, -4)$ and $\mathbf{B}(5, -6)$. Also find the coordinates of the point of intersection.

Solution:

Given points are:

$$\mathbf{A} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 5 \\ -6 \end{pmatrix} \quad (1)$$

Let the line segment joining \mathbf{A} and \mathbf{B} intersect the $X=0$ at point \mathbf{C} . Let the ratio in which the $X=0$ divide the line segment $\mathbf{B} - \mathbf{A}$ be $m : 1$:

$$\mathbf{C} = \begin{pmatrix} 0 \\ a \end{pmatrix} \quad (2)$$

Points \mathbf{A} , \mathbf{B} and \mathbf{C} are collinear, so vectors $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$ are parallel. Collinearity leads to:

$$\text{rank}(\mathbf{B} - \mathbf{A}, \mathbf{C} - \mathbf{A}) \quad (3)$$

Matrix setup:

$$\begin{pmatrix} -6 & -5 \\ 2 & a+6 \end{pmatrix} \quad (4)$$

$$R_2 \rightarrow R_2 + \frac{1}{3}R_3 \quad (5)$$

$$(6)$$

Echelon form:

$$\begin{pmatrix} -6 & -5 \\ 0 & a + \frac{13}{3} \end{pmatrix} \quad (7)$$

For rank = 1, the second row must be all zeroes:

$$a + \frac{13}{3} = 0 \quad (8)$$

$$a = \left(\frac{-13}{3} \right) \quad (9)$$

Thus the coordinates of point \mathbf{C} :

$$\mathbf{C} = \begin{pmatrix} 0 \\ \frac{-13}{3} \end{pmatrix} \quad (10)$$

Finding m by projecting $\mathbf{C} - \mathbf{A}$ onto the direction of $\mathbf{B} - \mathbf{A}$:

$$m = \frac{\begin{pmatrix} 5 & \frac{-5}{3} \end{pmatrix} \begin{pmatrix} 1 \\ \frac{-1}{3} \end{pmatrix}}{\left\| \begin{pmatrix} 1 \\ \frac{-1}{3} \end{pmatrix} \right\|^2} = 5 \quad (11)$$

Thus, the ratio in which the point \mathbf{C} divides the line segment $\mathbf{B} - \mathbf{A}$ is **5:1**.

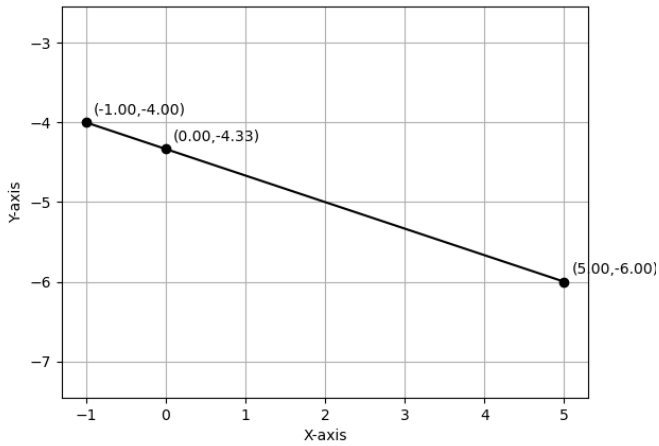


Fig. 1: Plot of line segment $\mathbf{B}-\mathbf{A}$