

# 4.13.38

EE25BTECH11033 - Kavın

## Question:

Let  $PS$  be the median of the triangle with vertices  $\mathbf{P}(2, 2)$ ,  $\mathbf{Q}(6, -1)$  and  $\mathbf{R}(7, 3)$ . The equation of the line passing through  $(1, -1)$  and parallel to  $PS$  is

$$1) 4x + 7y + 3 = 0$$

$$2) 2x - 9y - 11 = 0$$

$$3) 4x - 7y - 11 = 0$$

$$4) 2x + 9y + 7 = 0$$

## Solution:

Given the points,

$$\mathbf{P} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 6 \\ -1 \end{pmatrix} \quad \mathbf{R} = \begin{pmatrix} 7 \\ 3 \end{pmatrix} \quad (1)$$

$\mathbf{S}$  is the midpoint of the line segment joining points  $\mathbf{Q}$  and  $\mathbf{R}$ .

If  $\mathbf{S}$  divides  $QR$  in the ratio  $k : 1$ ,

$$\mathbf{S} = \frac{k\mathbf{R} + \mathbf{Q}}{k + 1} \quad (2)$$

where,

$$k = 1 \quad (3)$$

$$\mathbf{S} = \frac{\mathbf{R} + \mathbf{Q}}{2} \quad (4)$$

$$\Rightarrow \mathbf{S} = \begin{pmatrix} 13/2 \\ 1 \end{pmatrix} \quad (5)$$

The direction vector of line  $PS$  is given by,

$$\mathbf{d} = \mathbf{S} - \mathbf{P} \equiv \begin{pmatrix} 9/2 \\ -1 \end{pmatrix} \quad (6)$$

The slope of the line is given by,

$$\mathbf{m} = \begin{pmatrix} 1 \\ \mathbf{d} \end{pmatrix} = \begin{pmatrix} -1 \\ 9/2 \end{pmatrix} \quad (7)$$

Therefore, the normal vector of the desired line is given by,

$$\mathbf{n} = \begin{pmatrix} \mathbf{m} \\ -1 \end{pmatrix} \quad (8)$$

$$\Rightarrow \mathbf{n} = \begin{pmatrix} 1 \\ 9/2 \end{pmatrix} \quad (9)$$

$\therefore$  The equation of the line passing through  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$  and parallel to  $PS$  is given by

$$\mathbf{n}^\top \left( \mathbf{x} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right) = 0 \quad (10)$$

$$(1 \quad 9/2) \begin{pmatrix} x-1 \\ y+1 \end{pmatrix} = 0 \quad (11)$$

$$\Rightarrow x - 1 + \frac{9}{2}(y + 1) = 0 \quad (12)$$

$$\Rightarrow 2x + 9y + 7 = 0 \quad (13)$$

