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# Question

Find the equation of the conic, that satisfies the given conditions:  
Vertex  $(0,0)$  passing through  $(2,3)$  and axis is along X axis.

# Solution

The general conic is

$$g(\vec{x}) = \vec{x}^\top \vec{V} \vec{x} + 2\vec{u}^\top \vec{x} + f = 0 \quad (1)$$

For axis along the  $x$ -axis,

$$\vec{V} = \begin{pmatrix} A & 0 \\ 0 & C \end{pmatrix}, \quad \vec{u} = \begin{pmatrix} D \\ 0 \end{pmatrix} \quad (2)$$

Since the vertex is at the origin,

$$\nabla g(\vec{0}) = 2\vec{u} = \vec{0} \implies \vec{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (3)$$

If  $\det(\vec{V}) = AC \neq 0$ , then the conic has a center at the origin, which corresponds to an ellipse or hyperbola.

But the problem specifies a single vertex at the origin, not a center, so this case is invalid.

$$\therefore \det(\vec{V}) = 0 \implies \text{The conic is a parabola.} \quad (4)$$

$$g(\vec{x}) = \vec{x}^\top \vec{V} \vec{x} + 2\vec{u}^\top \vec{x} + f = 0 \quad (5)$$

For a parabola with axis along the  $x$ -axis, vertex at origin, focus  $\vec{F} = \begin{pmatrix} p \\ 0 \end{pmatrix}$  and directrix  $\vec{n}^\top \vec{x} = c$ , we have  $\vec{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ,  $c = -p$ .

$$\vec{V} = \|\vec{n}\|^2 \vec{I} - \vec{n} \vec{n}^\top = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (6)$$

$$\vec{u} = c\vec{n} - \vec{F} = (-p) \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} p \\ 0 \end{pmatrix} = \begin{pmatrix} -2p \\ 0 \end{pmatrix} \quad (7)$$

$$f = \|\vec{F}\|^2 - c^2 = p^2 - (-p)^2 = 0 \quad (8)$$

Thus, the parabola equation becomes:

$$\vec{x}^\top \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \vec{x} + 2 \begin{pmatrix} -2p & 0 \end{pmatrix} \vec{x} + 0 = 0 \quad (9)$$

$$y^2 - 4px = 0 \quad (10)$$

Since (2, 3) lies on the parabola:

$$3^2 - 4p(2) = 0 \quad (11)$$

$$9 - 8p = 0 \quad (12)$$

$$p = \frac{9}{8} \quad (13)$$

Therefore,

Therefore, the equation of the required parabola is

$$y^2 = \frac{9}{2}x \quad (14)$$

# Plot

