# 2.9.4

Shriyansh Chawda-EE25BTECH11052 August 23, 2025

# Question

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{a} \cdot \vec{b} = 1, \quad \text{and} \quad \vec{a} \times \vec{b} = \hat{j} - \hat{k}$, then find $|\vec{b}|$.
(12, 2022)

# Solution

We are given the vectors in component form:

$$\vec{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{1}$$

$$\vec{a} \times \vec{b} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}. \tag{2}$$

$$\vec{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \tag{3}$$

From the dot product:

# Solution

$$\vec{a}^\top \vec{b} = 1 \implies \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = 1 \tag{4}$$

$$b_1 + b_2 + b_3 = 1 \tag{5}$$

From the cross product:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ b_1 & b_2 & b_3 \end{vmatrix} = (b_3 - b_2)\,\hat{i} + (b_1 - b_3)\,\hat{j} + (b_2 - b_1)\,\hat{k} \tag{6}$$

Comparing Equation (0.2) and (0.6)

$$b_3 - b_2 = 0 \tag{7}$$

$$b_1 - b_3 = 1 \tag{8}$$

Substituting values in (0.5):

# Solution

$$(1 + b_3) + (b_3) + b_3 = 1 \tag{9}$$
$$1 + 3b_3 = 1 \tag{10}$$
$$3b_3 = 0 \implies b_3 = 0 \tag{11}$$

So, now for $b_2$ and $b_1$

$$b_2 = b_3 = 0 \tag{12}$$
$$b_1 = 1 + b_3 = 1 + 0 = 1 \tag{13}$$

So, $\vec{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$,

To find magnitude,

# Solution

$$\vec{b}^\top \vec{b} = 1 \tag{14}$$

$$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = 1 \tag{15}$$

The magnitude of vector $\vec{b}$ is $\mathbf{1}$.