1.11.9

AI25BTECH11007

Question:
If

$$\mathbf{a} = \hat{\imath} - 7\hat{\jmath} + 7\hat{k} \quad \text{and} \quad \mathbf{b} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k},$$

find a unit vector perpendicular to both the vectors **a** and **b**.
Solution:
We want $\mathbf{n} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ such that

$$\mathbf{a}^T \mathbf{n} = 0, \tag{0.1}$$

$$\mathbf{b}^T \mathbf{n} = 0. \tag{0.2}$$

This gives the linear system

$$\begin{bmatrix} 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{0.3}$$

Step 1: Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & -7 & 7 & 0 \\ 3 & -2 & 2 & 0 \end{array} \right]. \tag{0.4}$$

Step 2: Row operations

$$R_2 \to R_2 - 3R_1 : \left[\begin{array}{ccc|c} 1 & -7 & 7 & 0 \\ 0 & 19 & -19 & 0 \end{array} \right], \tag{0.5}$$

$$R_2 \to \tfrac{1}{19}R_2 : \left[\begin{array}{ccc|c} 1 & -7 & 7 & 0 \\ 0 & 1 & -1 & 0 \end{array} \right], \tag{0.6}$$

$$R_1 \to R_1 + 7R_2 : \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{array} \right]. \tag{0.7}$$

Step 3: Solution
From RREF:

$$x = 0, \tag{0.8}$$

$$y - z = 0 \quad \Rightarrow \quad y = z. \tag{0.9}$$

Thus the general solution is

$$\mathbf{n} = t \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad t \in \mathbb{R}. \tag{0.10}$$

Step 4: Unit vector
Since

$$\|(0, 1, 1)\| = \sqrt{2}, \tag{0.11}$$

the unit vectors are

$$\hat{n} = \pm \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \tag{0.12}$$

$$= \pm \frac{1}{\sqrt{2}} (\hat{j} + \hat{k}). \tag{0.13}$$

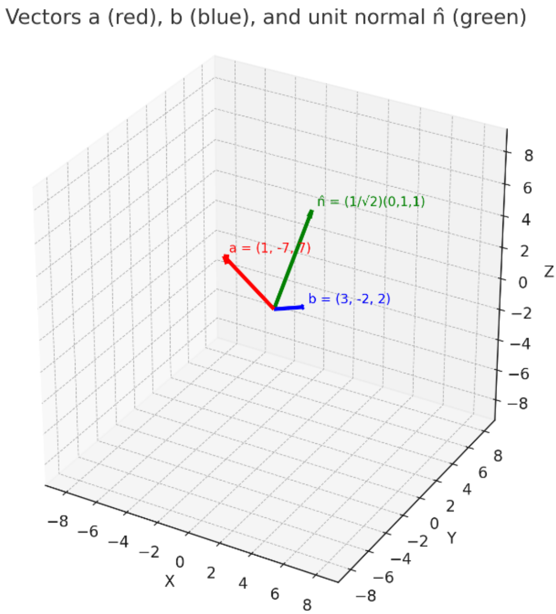

Fig. 0.1: Image Visual