4.11.4

EE25BTECH11026-Harsha

Question:
Find the equation of the plane which contains the line of intersection of the planes $\mathbf{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0$ and $\mathbf{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)+5=0$ and which is perpendicular to the plane $\mathbf{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$.
Solution:
Let us solve the given question theoretically and then verify the solution computationally.

According to the question,

$$\mathbf{n_1}=\begin{pmatrix}1\\2\\3\end{pmatrix}\quad\mathbf{n_2}=\begin{pmatrix}2\\1\\-1\end{pmatrix}\quad c_1=4\quad c_2=-5 \tag{0.1}$$

The equation of intersection of planes is given by

$$\mathbf{n_1}^\top\mathbf{r}-c_1+\lambda\left(\mathbf{n_2}^\top\mathbf{r}-c_2\right)=0 \tag{0.2}$$

$$\implies\left(\mathbf{n_1}^\top+\lambda\mathbf{n_2}^\top\right)\mathbf{r}=c_1+\lambda c_2 \tag{0.3}$$

Let the direction vector of the plane perpendicular to intersection of planes be $\mathbf{n_3}$

$$\therefore\left(\mathbf{n_1}^\top+\lambda\mathbf{n_2}^\top\right)\mathbf{n_3}=0 \tag{0.4}$$

$$\implies\lambda=-\frac{\mathbf{n_1}^\top\mathbf{n_3}}{\mathbf{n_2}^\top\mathbf{n_3}} \tag{0.5}$$

$$\therefore\lambda=-\frac{\begin{pmatrix}1&2&3\end{pmatrix}\begin{pmatrix}5\\3\\-6\end{pmatrix}}{\begin{pmatrix}2&1&-1\end{pmatrix}\begin{pmatrix}5\\3\\-6\end{pmatrix}}=\frac{7}{19} \tag{0.6}$$

$$\implies\text{equation of the plane}:\left(\mathbf{n_1}^\top+\frac{7}{19}\mathbf{n_2}^\top\right)\mathbf{r}=c_1+\frac{7}{19}c_2 \tag{0.7}$$

From the figure, it is clearly verified that the theoretical solution matches with the computational solution.