

# MCQ

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# Problem Statement

For  $3 \times 3$  matrices  $\mathbf{M}$  and  $\mathbf{N}$ , which of the following statement(s) is (are) **NOT** correct?

- (a)  $\mathbf{N}^T \mathbf{M} \mathbf{N}$  is symmetric or skew symmetric, according as  $\mathbf{M}$  is symmetric or skew symmetric.
- (b)  $\mathbf{M} \mathbf{N} - \mathbf{N} \mathbf{M}$  is skew symmetric for all matrices  $\mathbf{M}$  and  $\mathbf{N}$ .
- (c)  $\mathbf{M} \mathbf{N}$  is symmetric for all symmetric matrices  $\mathbf{M}$  and  $\mathbf{N}$ .
- (d)  $(\text{adj } \mathbf{M})(\text{adj } \mathbf{N}) = \text{adj}(\mathbf{M} \mathbf{N})$  for all invertible matrices  $\mathbf{M}$  and  $\mathbf{N}$ .

## Solution

$$(\mathbf{N}^T \mathbf{M} \mathbf{N})^T = \mathbf{N}^T \mathbf{M}^T \mathbf{N} \quad (1)$$

If  $\mathbf{M}^T = \mathbf{M}$ ,

$$(\mathbf{N}^T \mathbf{M} \mathbf{N})^T = \mathbf{N}^T \mathbf{M} \mathbf{N} \quad (2)$$

Hence symmetric.

If  $\mathbf{M}^T = -\mathbf{M}$ ,

$$(\mathbf{N}^T \mathbf{M} \mathbf{N})^T = -\mathbf{N}^T \mathbf{M} \mathbf{N} \quad (3)$$

Hence skew symmetric.

Thus, (a) is **correct**.

## Solution (cont..)

Assume  $\mathbf{MN} - \mathbf{NM}$  is skew symmetric  $\forall \mathbf{M}, \mathbf{N}$ .

Then,

$$(\mathbf{MN} - \mathbf{NM})^T = -(\mathbf{MN} - \mathbf{NM}) \quad (4)$$

$$\Rightarrow \mathbf{N}^T \mathbf{M}^T - \mathbf{M}^T \mathbf{N}^T = -\mathbf{MN} + \mathbf{NM} \quad (5)$$

Let  $\mathbf{M}$  and  $\mathbf{N}$  be symmetric:

$$\mathbf{M}^T = \mathbf{M}, \quad \mathbf{N}^T = \mathbf{N} \quad (6)$$

## Solution (cont..)

Then,

$$\mathbf{NM} - \mathbf{MN} = -\mathbf{MN} + \mathbf{NM} \quad (7)$$

$$\Rightarrow 2(\mathbf{NM} - \mathbf{MN}) = 0 \quad (8)$$

$$\Rightarrow \mathbf{MN} = \mathbf{NM} \quad (9)$$

Thus, this requires all symmetric matrices to commute, which is not true in general. Hence, (b) is **not correct**.

## Solution (cont..)

$$(\mathbf{MN})^T = \mathbf{N}^T \mathbf{M}^T \quad (10)$$

If  $\mathbf{M}$  and  $\mathbf{N}$  are symmetric,

$$(\mathbf{MN})^T = \mathbf{NM} \quad (11)$$

For  $\mathbf{MN}$  to be symmetric,

$$\mathbf{MN} = \mathbf{NM} \quad (12)$$

Thus,  $\mathbf{MN}$  is symmetric  $\iff \mathbf{M}$  and  $\mathbf{N}$  commute. Since all symmetric matrices do not commute, (c) is **not correct**.

## Solution (cont..)

For any invertible matrix  $\mathbf{A}$ ,

$$\text{adj}(\mathbf{A}) = |\mathbf{A}| \mathbf{A}^{-1} \quad (13)$$

Hence,

$$(\text{adj } \mathbf{M})(\text{adj } \mathbf{N}) = |\mathbf{M}| |\mathbf{N}| \mathbf{M}^{-1} \mathbf{N}^{-1}, \quad (14)$$

$$\text{adj}(\mathbf{MN}) = |\mathbf{MN}| (\mathbf{MN})^{-1} = |\mathbf{M}| |\mathbf{N}| \mathbf{N}^{-1} \mathbf{M}^{-1}. \quad (15)$$

## Solution (cont..)

Equality holds only if

$$\mathbf{M}^{-1}\mathbf{N}^{-1} = \mathbf{N}^{-1}\mathbf{M}^{-1} \iff \mathbf{MN} = \mathbf{NM}. \quad (16)$$

Since this does not hold for all invertible matrices, (d) is **not correct**.

Statements (b), (c), and (d) are NOT correct.



# C Code (Computations)

https:

`//github.com/Anirudh-EE25BTECH11008/ee1030-2025/tree/  
main/EE25BTECH11008/MATGEO/5.13.34/Codes/verify.c`

# Python Code (Verifying)

https:

`//github.com/Anirudh-EE25BTECH11008/ee1030-2025/tree/  
main/EE25BTECH11008/MATGEO/5.13.34/Codes/verify.py`