2.3.8

Aditya Mishra - EE25BTECH11005

September 15, 2025

Question

If $\hat{i}+\hat{j}+\hat{k},\ 2\hat{i}+5\hat{j},\ 3\hat{i}+2\hat{j}-3\hat{k},\ \hat{i}-6\hat{j}-\hat{k}$ respectively are the position vectors of points A, B, C, and D, then find the angle between the straight lines $(\mathbf{B}-\mathbf{A})$ and $(\mathbf{D}-\mathbf{C})$. Find whether $(\mathbf{B}-\mathbf{A})$ and $(\mathbf{D}-\mathbf{C})$ are collinear or not.

Given Information

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 1 \\ -6 \\ -1 \end{pmatrix}$$

Direction vectors:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}, \quad \mathbf{D} - \mathbf{C} = \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix}$$

Angle Formula

The angle θ between $(\mathbf{B}-\mathbf{A})$ and $(\mathbf{D}-\mathbf{C})$ is

$$\cos\theta = \frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{D}-\mathbf{C})}{\|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{D}-\mathbf{C}\|} \tag{1}$$

$$\theta = \cos^{-1}\left(\frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{D}-\mathbf{C})}{\|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{D}-\mathbf{C}\|}\right) \tag{2}$$

Angle Calculation

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{D}-\mathbf{C}) = \begin{pmatrix} 1 & 4 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix} = -2 - 32 - 2 = -36$$

$$\|\mathbf{B}-\mathbf{A}\| = \sqrt{1^2 + 4^2 + (-1)^2} = \sqrt{18}$$

$$\|\mathbf{D}-\mathbf{C}\| = \sqrt{(-2)^2 + (-8)^2 + 2^2} = \sqrt{72}$$

$$\cos\theta = \frac{-36}{\sqrt{18}\,\sqrt{72}} = -1$$

$$\theta = \cos^{-1}(-1) = 180^\circ$$

Collinearity using Rank

For collinearity, form the matrix

$$M = \begin{pmatrix} 1 & 4 & -1 \\ -2 & -8 & 2 \end{pmatrix}$$

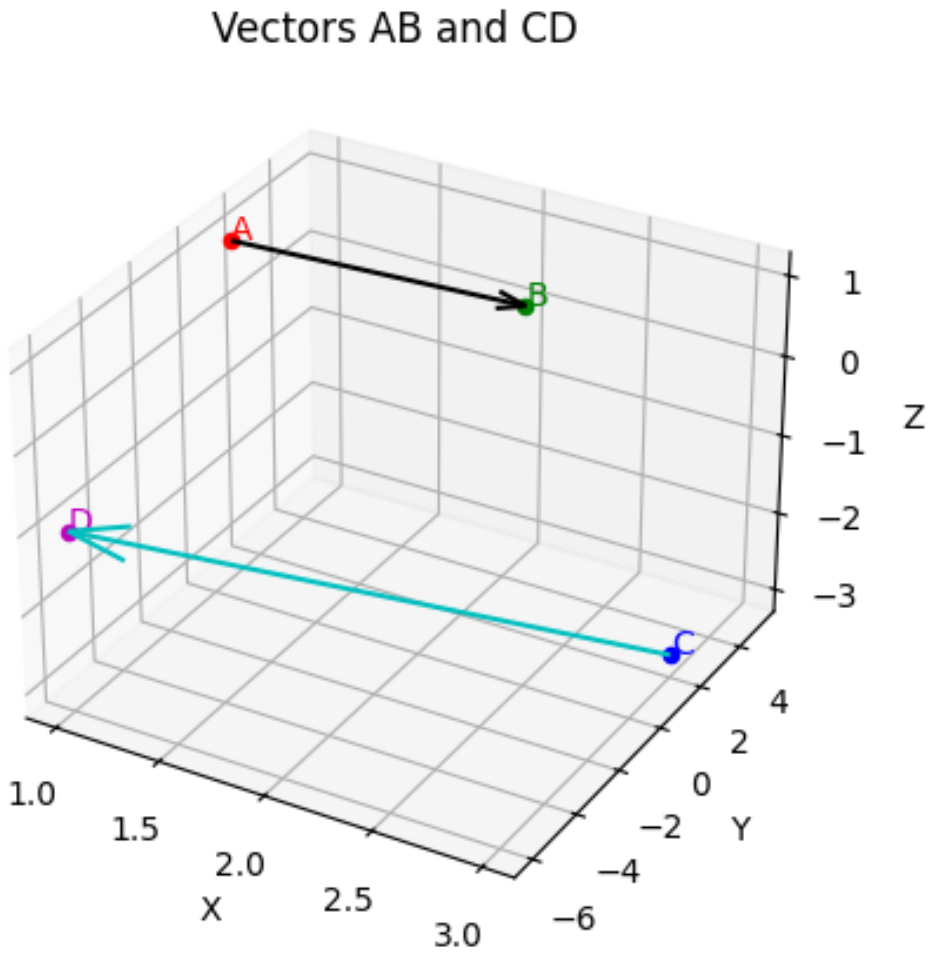
Row-reducing:

$$R_2 \to R_2 + 2R_1 \implies \begin{pmatrix} 1 & 4 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{rank}(M) = 1$$

Thus, $(\mathbf{B} - \mathbf{A})$ and $(\mathbf{D} - \mathbf{C})$ are collinear. Since $\theta = 180^\circ$, they are anti-parallel.

Plot



Codes

For Codes, refer to the URL below:
https://github.com/Aditya-Mishra11005/ee1030-2025/tree/main/ee25btech11005/matgeo/2.3.8/Codes