5.8.34

EE25BTECH11036 - M Chanakya Srinivas

Problem: Triangle formed by lines and axes

Given lines

$$\mathbf{L_1} : x - y + 1 = 0, \tag{1}$$
$$\mathbf{L_2} : 3x + 2y - 12 = 0. \tag{2}$$

Step 1: Represent lines in matrix form

A line $\mathbf{L}$ can be written as

$$\mathbf{n}^\top \mathbf{X} = c, \tag{3}$$

where $\mathbf{n}$ is the normal vector and $\mathbf{X} = (\mathbf{x}, \mathbf{y})^\top$.

$$\mathbf{L_1} : \mathbf{n_1}^\top \mathbf{X} = -1, \quad \mathbf{n_1} = (\mathbf{1}, -\mathbf{1})^\top, \quad \mathbf{X} = (\mathbf{x}, \mathbf{y})^\top, \tag{4}$$
$$\mathbf{L_2} : \mathbf{n_2}^\top \mathbf{X} = 12, \quad \mathbf{n_2} = (\mathbf{3}, \mathbf{2})^\top. \tag{5}$$

Step 2: Intersections with axes

a) *Intersection of* $\mathbf{L_1}$ *with x-axis::* let $\mathbf{Y} = (\mathbf{x}, \mathbf{0})^\top$, then

$$\mathbf{n_1}^\top \mathbf{Y} = -1$$
$$(\mathbf{1}, -\mathbf{1}) \begin{pmatrix} x \\ 0 \end{pmatrix} = -1$$
$$x = -1$$
$$\Rightarrow \mathbf{A} = (-\mathbf{1}, \mathbf{0}). \tag{6}$$

b) *Intersection of* $\mathbf{L_1}$ *with y-axis::* let $\mathbf{Y} = (\mathbf{0}, \mathbf{y})^\top$, then

$$\mathbf{n_1}^\top \mathbf{Y} = -1$$
$$(-\mathbf{1}) \begin{pmatrix} y \end{pmatrix} = -1$$
$$y = 1$$
$$\Rightarrow \mathbf{B} = (\mathbf{0}, \mathbf{1}). \tag{7}$$

c) Intersection of $\mathbf{L_2}$ *with x-axis::* $\mathbf{Y} = (\mathbf{x}, \mathbf{0})^\top$,

$$\begin{aligned} \mathbf{n_2}^\top \mathbf{Y} &= 12 \\ (\mathbf{3}, \mathbf{2}) \begin{pmatrix} x \\ 0 \end{pmatrix} &= 12 \\ x &= 4 \\ \Rightarrow \mathbf{C} &= (\mathbf{4}, \mathbf{0}). \end{aligned} \tag{8}$$

d) Intersection of $\mathbf{L_2}$ *with y-axis::* $\mathbf{Y} = (\mathbf{0}, \mathbf{y})^\top$,

$$\begin{aligned} \mathbf{n_2}^\top \mathbf{Y} &= 12 \\ (\mathbf{3}, \mathbf{2}) \begin{pmatrix} 0 \\ y \end{pmatrix} &= 12 \\ y &= 6 \\ \Rightarrow \mathbf{D} &= (\mathbf{0}, \mathbf{6}). \end{aligned} \tag{9}$$

Step 3: Intersection of lines using matrices

The intersection point $\mathbf{P}$ satisfies

$$\mathbf{NP} = \mathbf{C_0}, \tag{10}$$

where

$$\mathbf{N} = \begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}, \quad \mathbf{C_0} = \begin{pmatrix} -1 \\ 12 \end{pmatrix}. \tag{11}$$

Solving using the inverse of $\mathbf{N}$:

$$\begin{aligned} \mathbf{P} &= \mathbf{N}^{-1} \mathbf{C_0} \\ &= \frac{1}{5} \begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 12 \end{pmatrix} \\ &= (\mathbf{2}, \mathbf{3}). \end{aligned} \tag{12}$$

Step 4: Vertices of the triangle

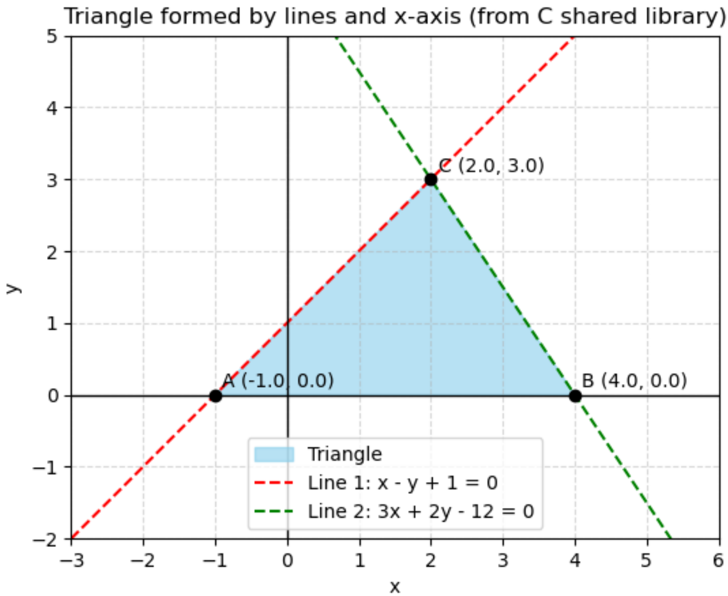
The triangle formed by the lines and axes has vertices

$$\mathbf{B} = (\mathbf{0}, \mathbf{1}), \quad \mathbf{C} = (\mathbf{4}, \mathbf{0}), \quad \mathbf{P} = (\mathbf{2}, \mathbf{3}). \tag{13}$$

Conclusion

All intersections and the triangle vertices have been determined **strictly using matrices and vectors**. The triangular region is bounded by:

$$\boxed{\mathbf{B} = (\mathbf{0}, \mathbf{1}),\ \mathbf{C} = (\mathbf{4}, \mathbf{0}),\ \mathbf{P} = (\mathbf{2}, \mathbf{3})}.$$



Triangle formed by lines and x-axis (from C shared library)
C (2.0, 3.0)
A (-1.0, 0.0)
B (4.0, 0.0)
Triangle
Line 1: x - y + 1 = 0
Line 2: 3x + 2y - 12 = 0
x
y

Fig. 1

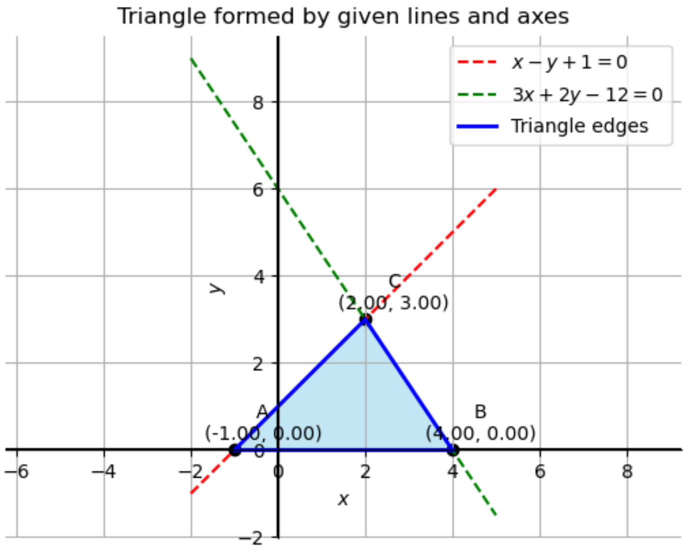

Triangle formed by given lines and axes
$x - y + 1 = 0$
$3x + 2y - 12 = 0$
Triangle edges
C
(2.00, 3.00)
A
(-1.00, 0.00)
B
(4.00, 0.00)
x
y
−6
−4
−2
0
2
4
6
8
−2
0
2
4
6
8

Fig. 2