4.4.26

AI25BTECH11030 -Sarvesh Tamgade

Question: Find the equation of the median through vertex **A** of the triangle ABC, having vertices
$$\mathbf{A}(2, 5), \quad \mathbf{B}(-4, 9), \quad \mathbf{C}(-2, -1).$$

Solution:
Using the section formula, the midpoint **M** of the side BC is
$$\mathbf{M} = \frac{\mathbf{B} + \mathbf{C}}{2} = \frac{1}{2}\begin{bmatrix} -4 \\ 9 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} -2 \\ -1 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}.$$

The median passes through points $\mathbf{A}(2, 5)$ and $\mathbf{M}(-3, 4)$. The direction vector is
$$\mathbf{d} = \mathbf{M} - \mathbf{A} = \begin{bmatrix} -3 - 2 \\ 4 - 5 \end{bmatrix} = \begin{bmatrix} -5 \\ -1 \end{bmatrix}.$$

Following the matrix approach :
$$\begin{pmatrix} -5 & -1 \end{pmatrix}\mathbf{x} = c,$$

where c is found by substituting point $\mathbf{A}(2, 5)$:
$$-5 \times 2 - 1 \times 5 = -10 - 5 = -15.$$

Thus, the equation of the median is
$$-5x - y = -15,$$

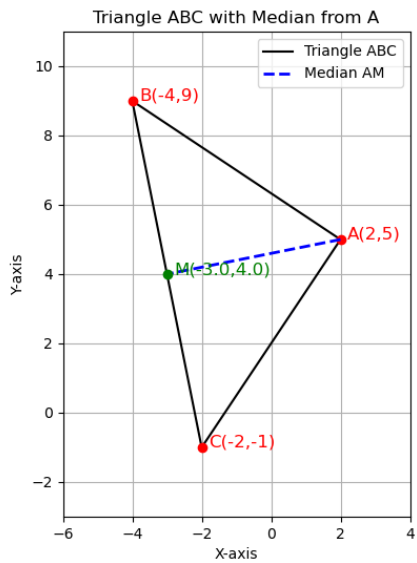or equivalently,
$$5x + y = 15.$$

Therefore, equation of required line is :
$$\boxed{5x + y = 15}$$

Fig. 0.1: Vector Representation