2.10.76

EE25BTECH11048 - Revanth Siva Kumar.D

Question Find the equation of the line joining the points $(3, 1)$ and $(9, 3)$.
Solution :
Given

$$\mathbf{A} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 9 \\ 3 \end{pmatrix} \tag{1}$$

Let us assume line equation to be:

$$\mathbf{n}^T \mathbf{x} = c \tag{2}$$

We get the line equation on solving

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix}^T \mathbf{n} = c \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

The line passes through the points from (1) substituting, we get:

$$\begin{pmatrix} 3 & 9 \\ 1 & 3 \end{pmatrix}^T \mathbf{n} = c \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} 3 & 1 \\ 9 & 3 \end{pmatrix} \mathbf{n} = c \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{4}$$

Now by Gaussian Elimination solve:

$$\left(\begin{array}{cc|c} 3 & 1 & 1 \\ 9 & 3 & 1 \end{array}\right) \tag{5}$$

$$R_1 \leftarrow \frac{1}{3} R_1$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{1}{3} \\ 9 & 3 & 1 \end{array}\right) \tag{6}$$

$$R_2 \leftarrow R_2 - 9R_1$$

$$\Rightarrow \left(\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & -2 \end{array}\right) \tag{7}$$

By the assumption that line equation is $\mathbf{n}^T \mathbf{x} = 1$ which doesn't pass through origin we are not getting any solution.So our assumption is wrong and origin lies on the line . So consider

$$\mathbf{n}^T \mathbf{x} = 0 \tag{8}$$

$c = 0$ because origin lies on the line and solving: so now,Assume the line equation:

$$\mathbf{n}^T\mathbf{x} = 0, \quad \mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

Line passes through points **A** and **B**

$$\mathbf{n}^T\mathbf{A} = 0 \implies 3n_1 + 1n_2 = 0 \tag{9}$$

$$\mathbf{n}^T\mathbf{B} = 0 \implies 9n_1 + 3n_2 = 0 \tag{10}$$

Matrix form:

$$\begin{pmatrix} 3 & 1 \\ 9 & 3 \end{pmatrix}\begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{11}$$

Augmented matrix:

$$\left(\begin{array}{cc|c} 3 & 1 & 0 \\ 9 & 3 & 0 \end{array}\right) \tag{12}$$

$$R_1 \leftarrow \frac{1}{3}R_1$$
$$\Rightarrow \left(\begin{array}{cc|c} 1 & \frac{1}{3} & 0 \\ 9 & 3 & 0 \end{array}\right) \tag{13}$$

$$R_2 \leftarrow R_2 - 9R_1$$
$$\Rightarrow \left(\begin{array}{cc|c} 1 & \frac{1}{3} & 0 \\ 0 & 0 & 0 \end{array}\right) \tag{14}$$

From first row:

$$n_1 + \frac{1}{3}n_2 = 0 \implies n_1 = -\frac{1}{3}n_2 \tag{15}$$

$$\tag{16}$$

Let,

$$n_2 = 3 \implies n_1 = -1 \tag{17}$$

$$\mathbf{n} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \tag{18}$$

$$\mathbf{n}^T\mathbf{x} = 0 \implies \begin{pmatrix} -1 & 3 \end{pmatrix}\mathbf{x} = 0 \tag{19}$$

The equation of the line passing through $(3, 1)$ and $(9, 3)$ is:

$$\boxed{\begin{pmatrix} -1 & 3 \end{pmatrix}\mathbf{x} = 0}$$

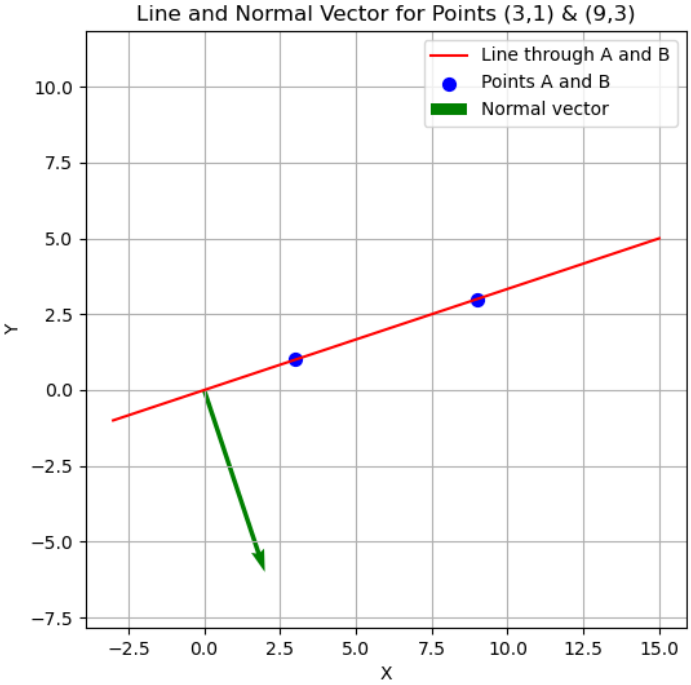
Line and Normal Vector for Points (3,1) & (9,3)
Line through A and B
Points A and B
Normal vector
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
Y
−2.5
0.0
2.5
5.0
7.5
10.0
12.5
15.0
X

Fig. 1: PLOT