8.2.27

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Find Equation of curve whose Focus is $(0, -3)$ and Directrix y=3

General Equation of a conic is given by

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1}$$

where

$$\mathbf{V} = ||\mathbf{n}||^2 \mathbf{I} - e^2 \mathbf{n}\mathbf{n}^\top \tag{2}$$

$$\mathbf{u} = ce^2 \mathbf{n} - ||\mathbf{n}||^2 \mathbf{F} \tag{3}$$

$$f = ||\mathbf{n}||^2 ||\mathbf{F}||^2 - c^2 e^2 \tag{4}$$

Solution

Given,

$$e = 1 \quad \mathbf{F} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$$

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad c = 3$$

Solution

From given equations,

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{5}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{6}$$

$$\mathbf{u} = 3 \begin{pmatrix} 0 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ -3 \end{pmatrix} \tag{7}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \tag{8}$$

$$f = 9 - 9 = 0 \tag{9}$$

From equation 1,

$$\mathbf{x}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 0 \\ 6 \end{pmatrix}^\top \mathbf{x} = 0 \tag{10}$$

C Code

```c
    #include <stdio.h>
// Function to compute V, u, f and print simplified equation
void conic_equation() {
    // Given parameters
    double e = 1;
    double c = 3;
    double F[2] = {0, -3};
    double n[2] = {0, 1};

    // Step 1: Compute V = ||n||^2 * I - e^2 * (n * n^T)
    double norm_n2 = n[0]*n[0] + n[1]*n[1]; // ||n||^2
    double V[2][2];
```

C Code

```
 V[0][0] = norm_n2 - e*e * n[0]*n[0];
V[0][1] = 0 - e*e * n[0]*n[1];
V[1][0] = 0 - e*e * n[1]*n[0];
V[1][1] = norm_n2 - e*e * n[1]*n[1];

// Step 2: Compute u = c*e^2*n - ||n||^2 * F
double u[2];
u[0] = c*e*e*n[0] - norm_n2*F[0];
u[1] = c*e*e*n[1] - norm_n2*F[1];
```

C Code

```c
    // Step 3: Compute f = ||n||^2 * ||F||^2 - c^2 * e^2
    double norm_F2 = F[0]*F[0] + F[1]*F[1]; // ||F||^2
    double f = norm_n2 * norm_F2 - c*c*e*e;

    // Step 4: Print results
    printf("V = [[%.2f, %.2f], [%.2f, %.2f]]\n", V[0][0], V
        [0][1], V[1][0], V[1][1]);
    printf("u = [%.2f, %.2f]\n", u[0], u[1]);
    printf("f = %.2f\n", f);
    printf("Simplified scalar equation: x^2 + 12*y = 0\n");
}
```

Python Code

```python
    import numpy as np
import matplotlib.pyplot as plt

# Equation: x^2 = -12y => y = -x^2/12
x = np.linspace(-20, 20, 400) # range for x
y = -x**2 / 12

# Plot
plt.figure(figsize=(6,6))
plt.plot(x, y, 'b', label=r'$x^2 = -12y$')
```

Python Code

```python
    # Axes setup
plt.axhline(0, color='black', linewidth=0.8) # x-axis
plt.axvline(0, color='black', linewidth=0.8) # y-axis
plt.grid(True, linestyle='--', alpha=0.6)

plt.title("Graph of $x^2 = -12y$")
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.legend()
plt.show()
```

```python
 import ctypes

# Load shared library
lib = ctypes.CDLL('./libconic.so')

# Call the function
lib.conic_equation()
```

plot

Graph of $x^2 = -12y$