4.3.53

Vivek K Kumar - EE25BTECH11062

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Question

The Fahrenheit temperature F and absolute temperature K satisfy a linear equation. Given that $K = 273$ when $F = 32$ and that $K = 373$ when $F = 212$. Express K in terms of F and find the value of F, when $K = 0$.

Variables used

(1)

Name	Point
x	$\begin{pmatrix} K \\ F \end{pmatrix}$
A	$\begin{pmatrix} 273 \\ 32 \end{pmatrix}$
B	$\begin{pmatrix} 373 \\ 212 \end{pmatrix}$
C	$\begin{pmatrix} 0 \\ F \end{pmatrix}$

Table: Variables used

Solution

Since there is a linear relation, the equation of the straight line can be expressed as

$$\mathbf{n}^\top \mathbf{x} = c \tag{2}$$

$$\mathbf{A}^\top \mathbf{n} = c \tag{3}$$

$$\mathbf{B}^\top \mathbf{n} = c \tag{4}$$

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix}^\top \mathbf{n} = c \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} 273 & 32 \\ 373 & 212 \end{pmatrix} \mathbf{n} = c \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{6}$$

As rank $\begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix}^\top \neq 1$ from above equation, $c \neq 0$.
Taking $c = 1$,

Solution

$$\begin{pmatrix} 273 & 32 \\ 373 & 212 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{7}$$

$$\implies \left(\begin{array}{cc|c} 273 & 32 & 1 \\ 373 & 212 & 1 \end{array}\right) \xleftrightarrow{R_2 \leftarrow R_2 - 373/273 R_1} \left(\begin{array}{cc|c} 273 & 32 & 1 \\ 0 & 45940/273 & -100/273 \end{array}\right) \tag{8}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 8736/45940 R_2} \left(\begin{array}{cc|c} 273 & 0 & 2457/2297 \\ 0 & 45940/273 & -100/273 \end{array}\right) \tag{9}$$

$$\mathbf{n} = \frac{1}{2297} \begin{pmatrix} 9 \\ -5 \end{pmatrix} \tag{10}$$

Solution

Substituting in line equation

$$\mathbf{n}^\top \mathbf{x} = 1 \tag{11}$$

$$\begin{pmatrix} 9 & -5 \end{pmatrix} \begin{pmatrix} K \\ F \end{pmatrix} = 2297 \tag{12}$$

Solving for point $\mathbf{C}$, $\begin{pmatrix} 0 \\ F \end{pmatrix}$

We have,

$$\begin{pmatrix} 9 & -5 \end{pmatrix} \begin{pmatrix} 0 \\ F \end{pmatrix} = 2297 \tag{13}$$

$$F = -\frac{2297}{5} \tag{14}$$

Python - Importing libraries and checking system

```
import sys
import numpy as np
import numpy.linalg as LA
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
import math

from libs.line.funcs import *
from libs.triangle.funcs import *
from libs.conics.funcs import circ_gen

import subprocess
import shlex

print('Using termux?(y/n)')
y = input()
```

Python - Solving for equation of line and finding point C

```
A = np.array([273, 32]).reshape(-1, 1)
B = np.array([373, 212]).reshape(-1, 1)

K = np.block([A, B]).T
N = LA.solve(K, np.array([1, 1]).reshape(-1,1)).reshape(-1, 1)
R = np.array([1, 0]).reshape(-1, 1)
Q = np.block([N, R]).T
C = LA.solve(Q, np.array([1,0]).T).reshape(-1, 1)
```

Python - Generating points and plotting

```
p_AB = line_gen(A, B)
p_BC = line_gen(B, C)
p_CA = line_gen(C, A)

fig = plt.figure()
ax = fig.add_subplot(111)
ax.plot(p_BC[0, :], p_BC[1, :])
ax.plot(p_CA[0, :], p_CA[1, :], label = 'Line CA')
ax.plot(p_AB[0, :], p_AB[1, :], label = 'Line AB')
```

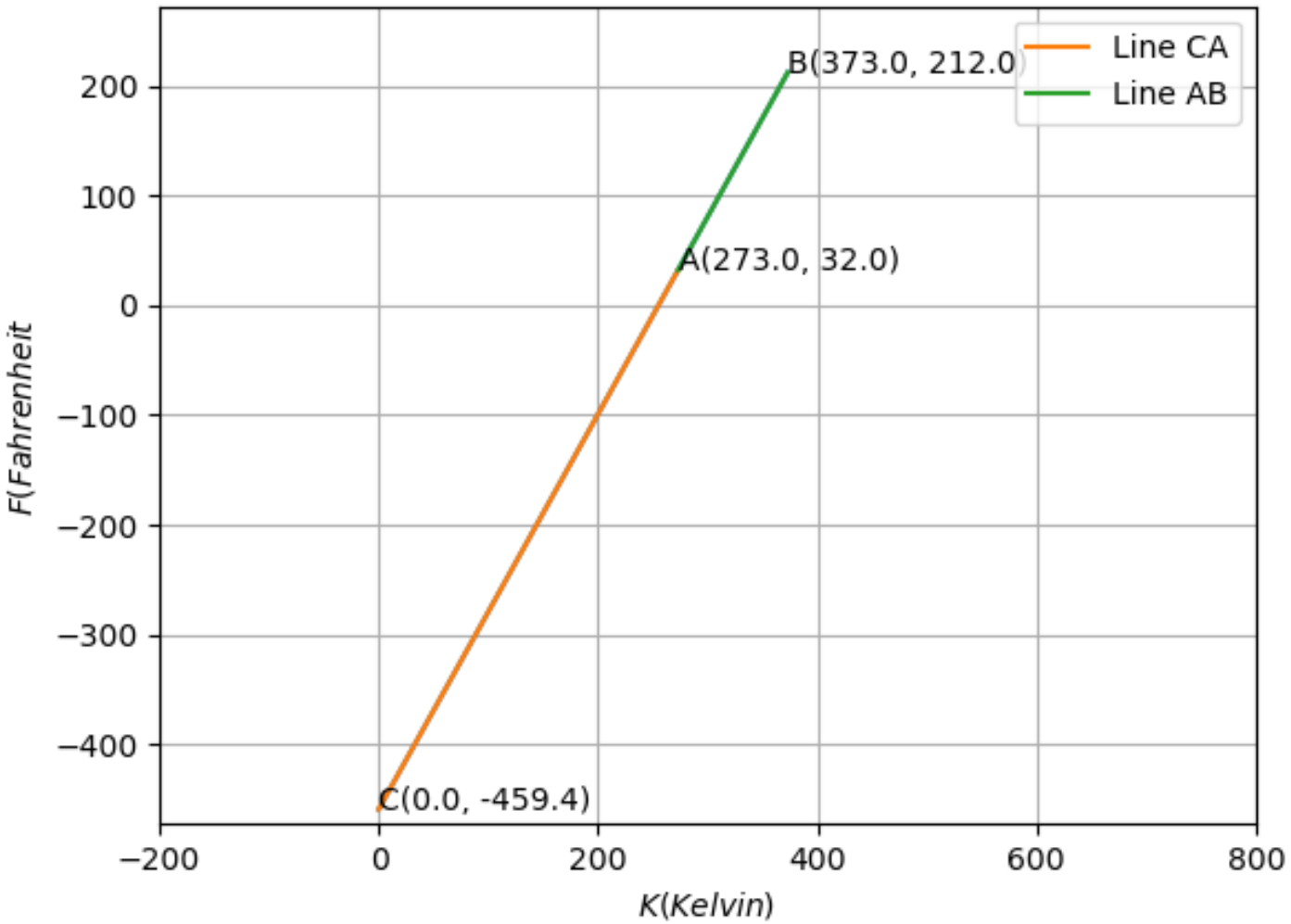
Python - Labelling points

```
pts = np.block([A, B, C])
names = ['A', 'B', 'C']
for i in range(3):
    Z = pts[:, i]
    ax.text(Z[0], Z[1], s=f'{names[i]}({round(Z[0], 3)}, {round(Z
        [1],3)})')

ax.set_xlabel('$K(Kelvin)$')
ax.set_ylabel('$F(Fahrenheit$')
ax.legend(loc='best')
ax.grid(True)
ax.axis('equal')
ax.set_xlim([-200, 800])
ax.set_ylim([-500, 300])
```

Python - Saving figure and opening it

```
fig.savefig('../figs/fig.png')
print('Saved figure to ../figs/fig.png')

if(y == 'y'):
    subprocess.run(shlex.split('termux-open ../figs/fig.png'))
else:
    subprocess.run(["open", "../figs/fig.png"])
```

Plot-Using only Python



C Code (0) - Importing libraries

```c
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <math.h>
#include <sys/socket.h>
#include <netinet/in.h>
#include <unistd.h>
#include "libs/matfun.h"
#include "libs/geofun.h"
```

C Code (1) - Function to Generate Points on a Line

```c
void point_gen(FILE *p_file, double **A, double **B, int rows,
    int cols, int npts){
    for(int i = 0; i <= npts; i++){
     double **output = Matadd(A, Matscale(Matsub(B, A, rows, cols
         ), rows, cols, (double)i/npts), rows, cols);
     fprintf(p_file, "%lf, %lf\n", output[0][0], output[1][0]);
     freeMat(output, rows);
    }
}
```

C Code (2) - Function to write points b/w given points to a file

```c
void write_points(double x1, double y1, double x2, double y2,
    double x3, double y3, int npts){
    int m = 2;
    int n = 1;

    double **A = createMat(m, n);
    double **B = createMat(m, n);
    double **C = createMat(m, n);

    B[0][0] = x2;
    B[1][0] = y2;
```

C Code (2) - Function to write points b/w given 2 points to a file

```
    A[0][0] = x1;
    A[1][0] = y1;

    C[0][0] = x3;
    C[1][0] = y3;

    FILE *p_file;
    p_file = fopen("plot.dat", "w");

    if(p_file == NULL)
        printf("Error opening one of the data files\n");
```

C Code (2) - Function to write points b/w given 2 points to a file

```
    point_gen(p_file, C, B, m, n, npts);
    point_gen(p_file, C, A, m, n, npts);
    point_gen(p_file, A, B, m, n, npts);

    freeMat(A, m);
    freeMat(B, m);
    freeMat(C, m);

    fclose(p_file);
}
}
```

Python Code (0) - Importing libraries and checking system

```
import numpy as np
import matplotlib.pyplot as plt
import ctypes
import os
import sys
import subprocess
import math

print('Using termux? (y/n)')
termux = input()
```

Python Code (1) - Using Shared Object

```
lib_path = os.path.join(os.path.dirname(__file__), 'plot.so')
my_lib = ctypes.CDLL(lib_path)

my_lib.write_points.argtypes = [ctypes.c_double, ctypes.c_double,
     ctypes.c_double, ctypes.c_double, ctypes.c_double, ctypes.
    c_double, ctypes.c_int]
my_lib.write_points.restype = None
A = np.array([273, 32]).reshape(-1, 1)
B = np.array([373, 212]).reshape(-1, 1)
C = np.array([0, -459.4]).reshape(-1, 1)
npts = 20000
```

Python Code (2) - Loading points and plotting them

```
my_lib.write_points(A[0][0], A[1][0], B[0][0], B[1][0], C[0][0],
    C[1][0], npts)

fig = plt.figure()
ax = fig.add_subplot(111)
labels = ['CB', 'CA', 'AB']
pts = np.block([A, B, C])
vertices = ['A', 'B', 'C']
for i,label in enumerate(labels):
    points = np.loadtxt('plot.dat', delimiter = ',', usecols
        =(0,1))[i*(npts+1):(i+1)*(npts+1)]
    if(i > 0):
        ax.plot(points[:, 0], points[:, 1], label = (f'Line {
            label}'))
    else:
        ax.plot(points[:, 0], points[:, 1])
    ax.text(pts[:, i][0], pts[:, i][1], s=f'{vertices[i]}({round(
        pts[:, i][0],3)}, {round(pts[:, i][1],3)})')
```

Python Code (3) - Labelling plot

```
ax.set_xlabel('$K(Kelvin)$')
ax.set_ylabel('$F(Fahrenheit)$')
ax.legend(loc='best')
ax.grid()
ax.axis('equal')
ax.set_xlim([-200, 800])
ax.set_ylim([-500, 300])
```

Python Code (4) - Saving and displaying plot

```
fig.savefig('../figs/fig2.png')
print('Saved figure to ../figs/fig2.png')

if(termux == 'y'):
    subprocess.run(shlex.split('termux-open ../figs/fig2.png'))
else:
    subprocess.run(["open", "../figs/fig2.png"])
```

Plot-Using Both C and Python