7.2.18

AI25BTECH11023 - Pratik R

QUESTION

Find the equation of the circle passing through $(0,0)$ and making intercepts a and b on the coordinate axes.

Solution

Let:

$$\mathbf{x_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{x_2} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \quad \mathbf{x_3} = \begin{pmatrix} 0 \\ b \end{pmatrix} \tag{0.1}$$

We use the general matrix form of a circle:

$$\begin{pmatrix} 2x_1 & 2x_2 & 2x_3 \\ 1 & 1 & 1 \end{pmatrix}^T \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = -\begin{pmatrix} \|x_1\|^2 \\ \|x_2\|^2 \\ \|x_3\|^2 \end{pmatrix} \tag{0.2}$$

Substituting the values:

$$\begin{pmatrix} 0 & 0 & 1 \\ 2a & 0 & 1 \\ 0 & 2b & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ f \end{pmatrix} = -\begin{pmatrix} 0 \\ a^2 \\ b^2 \end{pmatrix} \tag{0.3}$$

Using augmented matrix and
applying $R_1 \leftrightarrow R_2$
and $R_2 \leftrightarrow R_3$

$$\begin{pmatrix} 2a & 0 & 1 & | & -a^2 \\ 0 & 2b & 1 & | & -b^2 \\ 0 & 0 & 1 & | & 0 \end{pmatrix} \xleftrightarrow{R_1 = R_1 - R_3} \xleftrightarrow{R_2 = R_2 - R_3} \begin{pmatrix} 2a & 0 & 0 & | & -a^2 \\ 0 & 2b & 0 & | & -b^2 \\ 0 & 0 & 1 & | & 0 \end{pmatrix} \tag{0.4}$$

Solving the system:

$$f = 0$$
$$2au_1 = -a^2 \Rightarrow u_1 = -\frac{a}{2}$$
$$2bu_2 = -b^2 \Rightarrow u_2 = -\frac{b}{2}$$

we get

$$\mathbf{u} = \begin{pmatrix} -\frac{a}{2} \\ -\frac{b}{2} \end{pmatrix}, \quad f = 0 \tag{0.5}$$

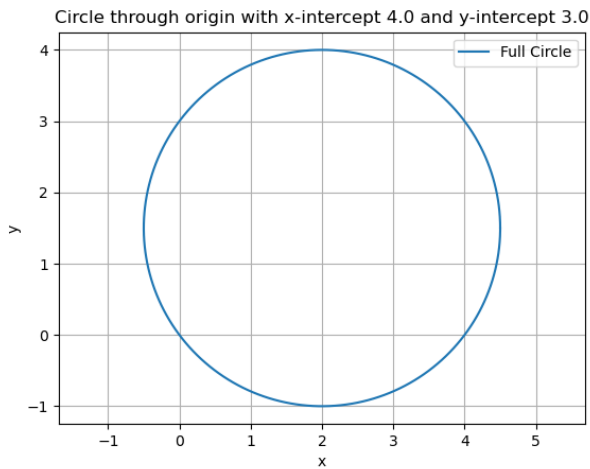So the equation of the circle becomes:

$$x^2 + y^2 + ax + by = 0 \tag{0.6}$$

Fig. 0.1: plane