4.3.34

EE25BTECH11044 - Sai Hasini Pappula

Question: If the line

$$\frac{x}{a} + \frac{y}{b} = 1$$

passes through the points $(2, -3)$ and $(4, -5)$, then find (a, b)

Solution (using $\mathbf{n}^T\mathbf{x} = c$):

The equation of a line can be expressed as

$$\mathbf{n}^T\mathbf{x} = c,$$

where $\mathbf{n}$ is the normal vector to the line.

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Step 1: Direction vector of the line

The line passes through

$$\mathbf{x}_1 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 4 \\ -5 \end{bmatrix}.$$

Hence, its direction vector is

$$\mathbf{m} = \mathbf{x}_2 - \mathbf{x}_1 = \begin{bmatrix} 4 - 2 \\ -5 - (-3) \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}.$$

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Step 2: Find the normal vector n

The normal vector $\mathbf{n} = \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$ must satisfy

$$\mathbf{n}^T\mathbf{m} = 0.$$

That is,

$$\begin{bmatrix} n_1 & n_2 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix} = 0,$$

$$2n_1 - 2n_2 = 0 \quad \Rightarrow \quad n_1 = n_2.$$

So, a valid choice is

$$\mathbf{n} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

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Step 3: Find c

Using the equation

$$\mathbf{n}^T\mathbf{x} = c,$$

substitute $\mathbf{x}_1 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$:

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \end{bmatrix} = 2 - 3 = -1.$$

Thus,

$$c = -1.$$

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Final Answer:

$$\mathbf{n}^T\mathbf{x} = -1 \quad \Rightarrow \quad x + y = -1.$$