

# 1.5.36

EE25BTECH11049 - Sai Krishna Bakki

## Question:

Point  $P(x, 4)$  lies on the line segment joining the points  $A(-5, 8)$  and  $B(4, -10)$ . Find the ratio in which point  $P$  divides the line segment  $AB$ . Also, find the value of  $x$ .

## Solution:

Let

$$\mathbf{A} = \begin{pmatrix} -5 \\ 8 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ -10 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} x \\ 4 \end{pmatrix} \quad (0.1)$$

Since  $\mathbf{P}$  lies on  $\mathbf{A}$  and  $\mathbf{B}$ , they must be collinear

$$\therefore \text{rank}(\mathbf{P} - \mathbf{A} \quad \mathbf{B} - \mathbf{A}) = 1 \quad (0.2)$$

$$\text{rank} \begin{pmatrix} x+5 & 9 \\ -4 & -18 \end{pmatrix} = 1 \quad (0.3)$$

By transformation  $R_1 \rightarrow R_1 + \frac{1}{2}R_2$

$$\text{rank} \begin{pmatrix} x+3 & 0 \\ -4 & -18 \end{pmatrix} = 1 \quad (0.4)$$

Thus for rank to be 1

$$x + 3 = 0 \quad (0.5)$$

$$\therefore x = -3 \quad (0.6)$$

Thus  $\mathbf{P}$  is :

$$\mathbf{P} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \quad (0.7)$$

Let  $\mathbf{P} = \lambda\mathbf{A} + \mu\mathbf{B}$  with  $\lambda + \mu = 1$ . Using the y-coordinates:

$$\begin{pmatrix} 8 & -10 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad (0.8)$$

Hence the internal division ratio

$$AP : PB = \mu : \lambda = 2 : 7 \quad (0.9)$$

