

4.4.12

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Question:

Find the equation of the plane passing through the points $(2, 5, -3)$, $(-2, -3, 5)$ and $(5, 3, -3)$. Also find the point of intersection of this plane with the line passing through points $(3, 1, 5)$ and $(-1, -3, -1)$.

Solution:

The points are

$$\mathbf{A} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ -3 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 3 \\ -3 \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} 2 & 5 & -3 \\ -2 & -3 & 5 \\ 5 & 3 & -3 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (2)$$

$$\left(\begin{array}{ccc|c} 2 & 5 & -3 & 1 \\ -2 & -3 & 5 & 1 \\ 5 & 3 & -3 & 1 \end{array} \right) \xrightarrow{R_2 \leftarrow R_2 + R_1, R_3 \leftarrow 2R_3 - 5R_1} \left(\begin{array}{ccc|c} 2 & 5 & -3 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & -19 & 9 & -3 \end{array} \right)$$

$$\xrightarrow{R_3 \leftarrow 2R_3 + 19R_2} \left(\begin{array}{ccc|c} 2 & 5 & -3 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 56 & 22 \end{array} \right)$$

$$\xrightarrow{R_1 \leftarrow \frac{1}{2}R_1, R_2 \leftarrow \frac{1}{2}R_2, R_3 \leftarrow \frac{1}{56}R_3} \left(\begin{array}{ccc|c} 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & \frac{11}{28} \end{array} \right)$$

$$\xrightarrow{R_2 \leftarrow R_2 - R_3} \left(\begin{array}{ccc|c} 1 & \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 0 & \frac{17}{28} \\ 0 & 0 & 1 & \frac{11}{28} \end{array} \right)$$

$$\xrightarrow{R_1 \leftarrow R_1 + \frac{3}{2}R_3, R_1 \leftarrow R_1 - \frac{5}{2}R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2}{7} \\ 0 & 1 & 0 & \frac{3}{7} \\ 0 & 0 & 1 & \frac{4}{7} \end{array} \right)$$

Hence the equation of the plane is

$$\left(\frac{2}{7} \quad \frac{3}{7} \quad \frac{4}{7} \right) \mathbf{x} = 1 \quad (3)$$

The equation of the line passing through:

$$\mathbf{A} = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ -3 \\ -1 \end{pmatrix} \quad (4)$$

(5)

The direction vector of the line

$$\mathbf{m} = \mathbf{A} - \mathbf{B} \quad (6)$$

$$= \begin{pmatrix} 4 \\ 4 \\ 6 \end{pmatrix} \quad (7)$$

Vector equation of the line is

$$\mathbf{x} = \mathbf{A} + \lambda \mathbf{m} \quad (8)$$

$$\mathbf{x} = \begin{pmatrix} 3 + 4\lambda \\ 1 + 4\lambda \\ 5 + 6\lambda \end{pmatrix}$$

Solving the equation of the plane and the line,

$$\left(\frac{2}{7} \quad \frac{3}{7} \quad \frac{4}{7} \right) \begin{pmatrix} 3 + 4\lambda \\ 1 + 4\lambda \\ 5 + 6\lambda \end{pmatrix} = 1 \quad (9)$$

$$\frac{2}{7}(3 + 4\lambda) + \frac{3}{7}(1 + 4\lambda) + \frac{4}{7}(5 + 6\lambda) = 1 \quad (10)$$

$$\frac{6 + 8\lambda}{7} + \frac{3 + 12\lambda}{7} + \frac{20 + 24\lambda}{7} = 1 \quad (11)$$

$$\frac{29 + 44\lambda}{7} = 1 \quad (12)$$

$$\lambda = -1/2 \quad (13)$$

Therefore,

the point of intersection is

$$\mathbf{x} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \quad (14)$$

Therefore,

the equation of the plane passing through the points (2, 5, -3), (-2, -3, 5) and (5, 3, -3) is $\left(\frac{2}{7} \quad \frac{3}{7} \quad \frac{4}{7} \right) \mathbf{x} = 1$

and the point of intersection of the line with the plane is $\mathbf{x} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

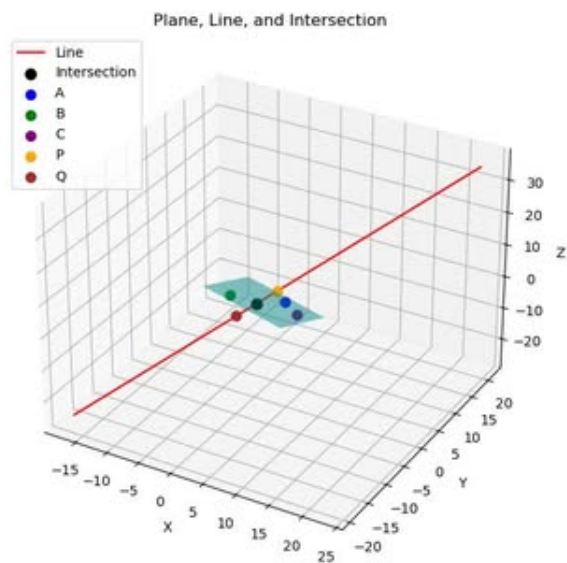


Fig. 0.1