2.8.24

EE25BTECH11042 - Nipun Dasari

Question:
The $\mathbf{a}+\mathbf{b}$ bisects the angle between $\mathbf{a}$ and $\mathbf{b}$ if ___________
Solution:
Theorem: The $\mathbf{a}+\mathbf{b}$ bisects the angle between $\mathbf{a}$ and $\mathbf{b}$ if $\|\mathbf{a}\| = \|\mathbf{b}\|$
Given :

$$\|\mathbf{a}\| = \|\mathbf{b}\| \tag{0.1}$$

To prove : $\mathbf{a}+\mathbf{b}$ bisects the angle between $\mathbf{a}$ and $\mathbf{b}$
Proof:
Assume a $\mathbf{c}$ such that

$$\mathbf{c} = \mathbf{a}+\mathbf{b} \tag{0.2}$$

Let α and β be angle made by $\mathbf{c}$ with $\mathbf{a}$ and $\mathbf{b}$
We need to prove

$$\cos\alpha = \cos\beta \tag{0.3}$$

The angle θ between 2 vectors $\mathbf{p}$ and $\mathbf{q}$ can be found by the following formula:

$$\cos\theta = \frac{\mathbf{p}^T\mathbf{q}}{\|\mathbf{p}\|\,\|\mathbf{q}\|} \tag{0.4}$$

By (0.4)

$$\cos\alpha = \frac{\mathbf{a}^T\mathbf{c}}{\|\mathbf{a}\|\,\|\mathbf{c}\|} \tag{0.5}$$

$$\cos\beta = \frac{\mathbf{a}^T\mathbf{c}}{\|\mathbf{a}\|\,\|\mathbf{c}\|} \tag{0.6}$$

For $\alpha = \beta$:

$$\cos\beta = \cos\alpha \tag{0.7}$$

By (0.2)

$$\implies \cos\alpha = \frac{\mathbf{a}^T(\mathbf{a}+\mathbf{b})}{\|\mathbf{a}\|\,\|\mathbf{a}+\mathbf{b}\|} \tag{0.8}$$

$$\implies \cos\beta = \frac{\mathbf{b}^T(\mathbf{a}+\mathbf{b})}{\|\mathbf{b}\|\,\|\mathbf{a}+\mathbf{b}\|} \tag{0.9}$$

But $||\mathbf{a}|| = ||\mathbf{b}||$. So for the angles to be equal:

$$\mathbf{a}^T\mathbf{c} = \mathbf{b}^T\mathbf{c} \tag{0.10}$$

L.H.S:

$$\mathbf{a}^T\mathbf{a} + \mathbf{a}^T\mathbf{b} \tag{0.11}$$

R.H.S:

$$\mathbf{b}^T\mathbf{a} + \mathbf{b}^T\mathbf{b} \tag{0.12}$$

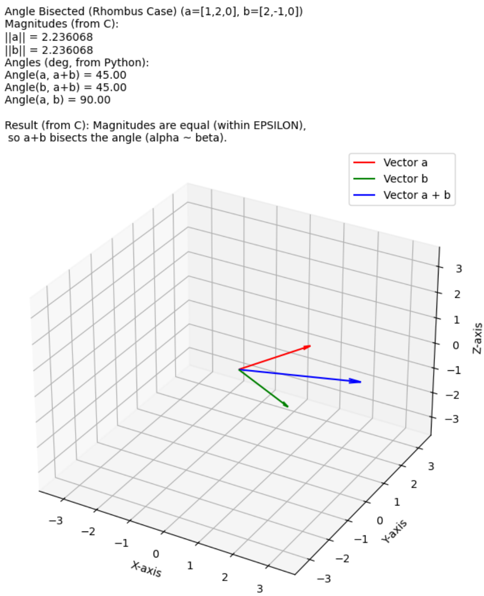
$$\mathbf{b}^T\mathbf{a} = \mathbf{a}^T\mathbf{b} \tag{0.13}$$

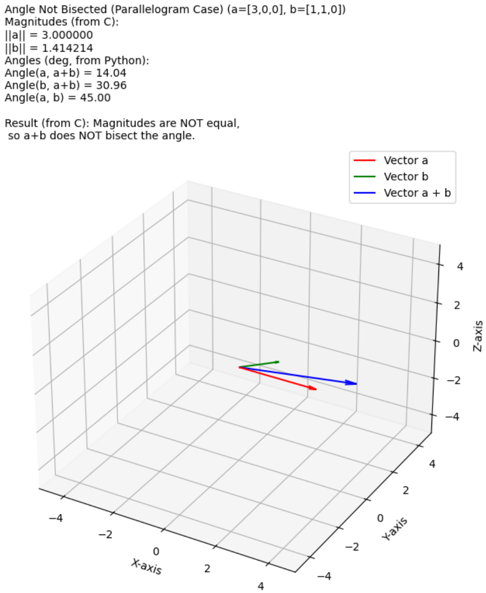
By (0.1)

$$\mathbf{a}^T\mathbf{a} = \mathbf{b}^T\mathbf{b} \tag{0.14}$$

Thus we have proved:

$$\beta = \alpha \tag{0.15}$$




Angle Not Bisected (Parallelogram Case) (a=[3,0,0], b=[1,1,0])
Magnitudes (from C):
||a|| = 3.000000
||b|| = 1.414214
Angles (deg, from Python):
Angle(a, a+b) = 14.04
Angle(b, a+b) = 30.96
Angle(a, b) = 45.00
Result (from C): Magnitudes are NOT equal,
so a+b does NOT bisect the angle.
Vector a
Vector b
Vector a + b
X-axis
Y-axis
Z-axis
−4
−2
0
2
4