

5.13.51

EE25BTECH11025 - Ganachari Vishwambhar

Question:

If $A = \begin{pmatrix} \alpha & 0 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$, then value of α for which $A^2 = B$, is

1) 1

3) 2

2) 4

4) infinite

Solution:

Given:

$$A = \begin{pmatrix} \alpha & 0 \\ 1 & 1 \end{pmatrix}; B = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix} \quad (1)$$

Using outer product,

$$\begin{pmatrix} \alpha \\ 1 \end{pmatrix} \begin{pmatrix} \alpha & 0 \end{pmatrix} = \begin{pmatrix} \alpha^2 & 0 \\ \alpha & 0 \end{pmatrix} \quad (2)$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \quad (3)$$

Adding (2) and (3):

$$\begin{pmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{pmatrix} \quad (4)$$

Equating (4) to B :

$$\alpha = \pm 1; \alpha = 4 \quad (5)$$

No finite α satisfies the above conditions. Hence α is infinite.