

Matgeo-q.1.4.6

AI25BTECH11036-SNEHAMRUDULA

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Question

If the point $P(2, 1)$ lies on the line segment joining points $A(4, 2)$ and $B(8, 4)$, then which of the following is true?

$$AP = \frac{1}{3}AB$$

$$AP = PB$$

$$PB = \frac{1}{3}AB$$

$$AP = \frac{1}{2}AB$$

Solution

Given:

$$\mathbf{A} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Direction vector:

$$\mathbf{m} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

Parameter:

$$k = \frac{(\mathbf{P} - \mathbf{A})^\top (\mathbf{B} - \mathbf{A})}{\|\mathbf{B} - \mathbf{A}\|^2} = \frac{\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix}\right)^\top \begin{pmatrix} 4 \\ 2 \end{pmatrix}}{4^2 + 2^2} = \frac{\begin{pmatrix} -2 \\ -1 \end{pmatrix}^\top \begin{pmatrix} 4 \\ 2 \end{pmatrix}}{20} = \frac{-8 - 2}{20} = -\frac{1}{2}.$$

Hence,

$$\mathbf{P} = \mathbf{A} + k(\mathbf{B} - \mathbf{A}) = \mathbf{A} - \frac{1}{2}(\mathbf{B} - \mathbf{A})$$

Since $k < 0$, point P lies on the line AB but *outside* the segment (beyond A).

Lengths:

$$AB = \|\mathbf{B} - \mathbf{A}\| = \sqrt{4^2 + 2^2} = 2\sqrt{5}, \quad AP = \|\mathbf{P} - \mathbf{A}\| = \left\| \begin{pmatrix} -2 \\ -1 \end{pmatrix} \right\| = \sqrt{5}.$$

solution

Therefore,

$$AP = \frac{1}{2}AB.$$

Conclusion: The correct relation is

$$AP = \frac{1}{2}AB$$

and P does not lie on the segment AB . Hence, the given statement is **false**.

Graphical Representation

