4.2.16

EE25BTECH11018 - Darisy Sreetej

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Question

Let a, b, c, d be non-zero numbers. If the point of intersection of the lines $4a\mathbf{x} + 2a\mathbf{y} + c = 0$ and $5b\mathbf{x} + 2b\mathbf{y} + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then

1. $3bc - 2ad = 0$
2. $2bc - 3ad = 0$
3. $3bc + 2ad = 0$
4. $2bc + 3ad = 0$

Solution

The two lines are

$$4a\mathbf{x} + 2a\mathbf{y} + c = 0, \tag{1}$$
$$5b\mathbf{x} + 2b\mathbf{y} + d = 0 \tag{2}$$

This equation can be expressed in terms of matrices

$$\begin{pmatrix} 4a \\ 2a \end{pmatrix}^\top \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = -c \tag{3}$$
$$\begin{pmatrix} 5b \\ 2b \end{pmatrix}^\top \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = -d \tag{4}$$

They can be represented as,

$$\begin{pmatrix} 4a & 5b \\ 2a & 2b \end{pmatrix}^\top \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = \begin{pmatrix} -c \\ -d \end{pmatrix} \tag{5}$$

Solution

Using augmented matrix,

$$\left(\begin{array}{cc|c} 4a & 2a & -c \\ 5b & 2b & -d \end{array}\right) \tag{6}$$

$R_1 = \frac{R_1}{4a}$

$$\left(\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{-c}{4a} \\ 5b & 2b & -d \end{array}\right) \tag{7}$$

$R_2 = R_2 - 5bR_1$

$$\left(\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{-c}{4a} \\ 0 & \frac{-b}{2} & \frac{-4ad+5bc}{4a} \end{array}\right) \tag{8}$$

$$\mathbf{y}\frac{-b}{2} = \frac{-4ad + 5bc}{4a} \tag{9}$$

$$\mathbf{y} = \frac{4ad - 5bc}{2ab} \tag{10}$$

Solution

Also,

$$\mathbf{x} + \frac{1}{2}\mathbf{y} = \frac{-c}{4a} \tag{11}$$

$$\mathbf{x} = \frac{bc - ad}{ab} \tag{12}$$

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = \begin{pmatrix} \frac{bc-ad}{ab} \\ \frac{4ad-5bc}{2ab} \end{pmatrix} \tag{13}$$

Therefore , the point of intersection is $\left(\frac{bc-ad}{ab}, \frac{4ad-5bc}{2ab}\right)$

Solution

According to the condition,the intersection point is equidistant from the axes and lies in the fourth quadrant, so its coordinates satisfy $y = -x$
Therefore,

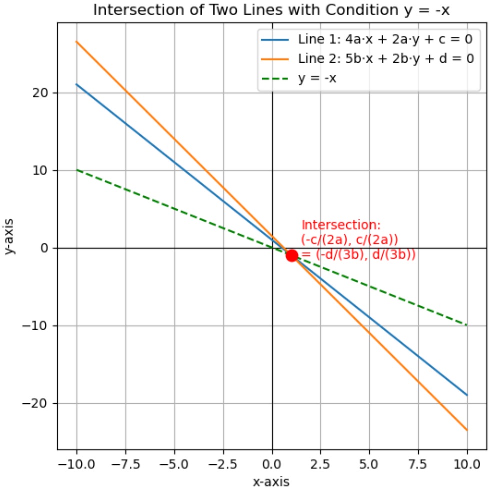
$$\frac{4ad - 5bc}{2ab} = -\frac{bc - ad}{ab} \tag{14}$$

$$2ad - 2bc = 4ad - 5bc \tag{15}$$

$$3bc = 2ad \tag{16}$$

Therefore, option(a) is correct

Plot



Intersection of Two Lines with Condition y = -x
Line 1: 4a·x + 2a·y + c = 0
Line 2: 5b·x + 2b·y + d = 0
y = -x
Intersection:
(-c/(2a), c/(2a))
= (-d/(3b), d/(3b))
20
10
0
−10
−20
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
x-axis
y-axis

C code

```c
#include <stdio.h>

// Function to compute x from first line
double computeX1(double a, double c) {
    return -c / (2.0 * a);
}

// Function to compute x from second line
double computeX2(double b, double d) {
    return -d / (3.0 * b);
}
```

C Code

```c
// Function to compute y (since y = -x for equidistant from axes
    in 4th quadrant)
double computeY(double x) {
    return -x;
}

// Function to check relation 3bc - 2ad = 0
int checkRelation(double a, double b, double c, double d) {
    double relation = 3*b*c - 2*a*d;
    return (relation == 0) ? 1 : 0;
}
```

Python + C Code

```
import ctypes
import matplotlib.pyplot as plt
import numpy as np

# Load the compiled C shared library
lib = ctypes.CDLL("./point.so")

# Set argument and return types
lib.computeX1.argtypes = [ctypes.c_double, ctypes.c_double]
lib.computeX1.restype = ctypes.c_double

lib.computeX2.argtypes = [ctypes.c_double, ctypes.c_double]
lib.computeX2.restype = ctypes.c_double

lib.computeY.argtypes = [ctypes.c_double]
lib.computeY.restype = ctypes.c_double

lib.checkRelation.argtypes = [ctypes.c_double, ctypes.c_double,
    ctypes.c_double, ctypes.c_double]
```

Python + C code

```
lib.checkRelation.restype = ctypes.c_int

# Example values (you can change these)
a, b, c, d = 1.0, 1.0, -2.0, -3.0

# Call C functions
x1 = lib.computeX1(a, c)
x2 = lib.computeX2(b, d)

if abs(x1 - x2) > 1e-6:
    print("Inconsistent intersection: x1 != x2")
    exit()

x = x1
y = lib.computeY(x)

print(f"Intersection Point: ({x:.3f}, {y:.3f})")
```

Python + C code

```
# Check relation
if lib.checkRelation(a, b, c, d):
    print("Relation satisfied: 3bc - 2ad = 0 ")
    else:
    print("Relation NOT satisfied ")

# ---------- Plotting ----------
X = np.linspace(-10, 10, 400)

# Line1: 4ax + 2ay + c = 0 -> y = -(4aX + c)/(2a)
Y1 = -(4*a*X + c) / (2*a)

# Line2: 5bx + 2by + d = 0 -> y = -(5*bX + d)/(2*b)
Y2 = -(5*b*X + d) / (2*b)

plt.figure(figsize=(6,6))
plt.axhline(0, color="black", linewidth=0.8)
plt.axvline(0, color="black", linewidth=0.8)
```

Python + C code

```
plt.plot(X, Y1, label="Line 1")
plt.plot(X, Y2, label="Line 2")
plt.scatter([x], [y], color="red", s=80, label="Intersection
    Point")

plt.title("Intersection of Two Lines")
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.legend()
plt.grid(True)
plt.show()
```

Python code

```
import matplotlib.pyplot as plt
import numpy as np

# Example coefficients (you can change these)
a, b, c, d = 1.0, 1.0, -2.0, -3.0

# Intersection point (from condition y = -x)
x = -c / (2 * a)
y = -x

# Create range of x values for plotting
X = np.linspace(-10, 10, 400)

# Line1: 4ax + 2ay + c = 0 -> y = -(4aX + c)/(2a)
Y1 = -(4 * a * X + c) / (2 * a)

# Line2: 5bx + 2by + d = 0 -> y = -(5 * b * X + d) / (2 * b)
Y2 = -(5 * b * X + d) / (2 * b)
```

Python code

```
# y = -x line
Y_diag = -X

# ---------- Plot ----------
plt.figure(figsize=(6, 6))
plt.axhline(0, color="black", linewidth=0.8) # x-axis
plt.axvline(0, color="black", linewidth=0.8) # y-axis

plt.plot(X, Y1, label="Line 1: 4ax + 2ay + c = 0")
plt.plot(X, Y2, label="Line 2: 5bx + 2by + d = 0")
plt.plot(X, Y_diag, "g--", label="y = -x")

# Mark intersection point with symbolic label
plt.scatter([x], [y], color="red", s=80, zorder=5)
plt.text(x + 0.5, y - 0.5,
        "Intersection:\n(-c/(2a), c/(2a))\n= (-d/(3b), d/(3b))",
        fontsize=10, color="red")
```

Python code

```
plt.title("Intersection of Two Lines with Condition y = -x")
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.legend(loc="best")
plt.grid(True)
plt.show()
```