Matrices in Geometry - 5.2.38

EE25BTECH11035 Kushal B N

Oct, 2025

Problem Statement

Solve the following system of equations.

$$\frac{1}{2x} + \frac{1}{3y} = 2$$

$$\frac{1}{3x} + \frac{1}{2y} = \frac{13}{6}$$

Solution

Let

$$\mathbf{x} = \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} \tag{1}$$

So that the given equations, after multiplying by 6 on both sides, can be represented in the matrix form as

$$\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 12 \\ 13 \end{pmatrix} \tag{2}$$

Which can be represented as the augmented matrix

$$\left(\begin{array}{cc|c} 3 & 2 & 12 \\ 2 & 3 & 13 \end{array}\right) \tag{3}$$

$$\left(\begin{array}{cc|c} 3 & 2 & 12 \\ 2 & 3 & 13 \end{array}\right) \xleftrightarrow{R_2 \leftarrow R_2 - \frac{2}{3} R_1 0} \left(\begin{array}{cc|c} 3 & 2 & 12 \\ 0 & \frac{5}{3} & 5 \end{array}\right) \tag{4}$$

Solution

$$\xleftarrow{R_1 \leftarrow R_1 - \frac{6}{5}R_2} \left(\begin{array}{cc|c} 3 & 0 & 6 \\ 0 & \frac{5}{3} & 5 \end{array}\right) \tag{5}$$

So, by this, we get

$$\frac{1}{y} = 3 \implies y = \frac{1}{3} \tag{6}$$

$$\frac{1}{x} = 2 \implies x = \frac{1}{2} \tag{7}$$

Conclusion

$\therefore$ The solution for the given system of linear equations is $x = \frac{1}{2}$ and $y = \frac{1}{3}$.

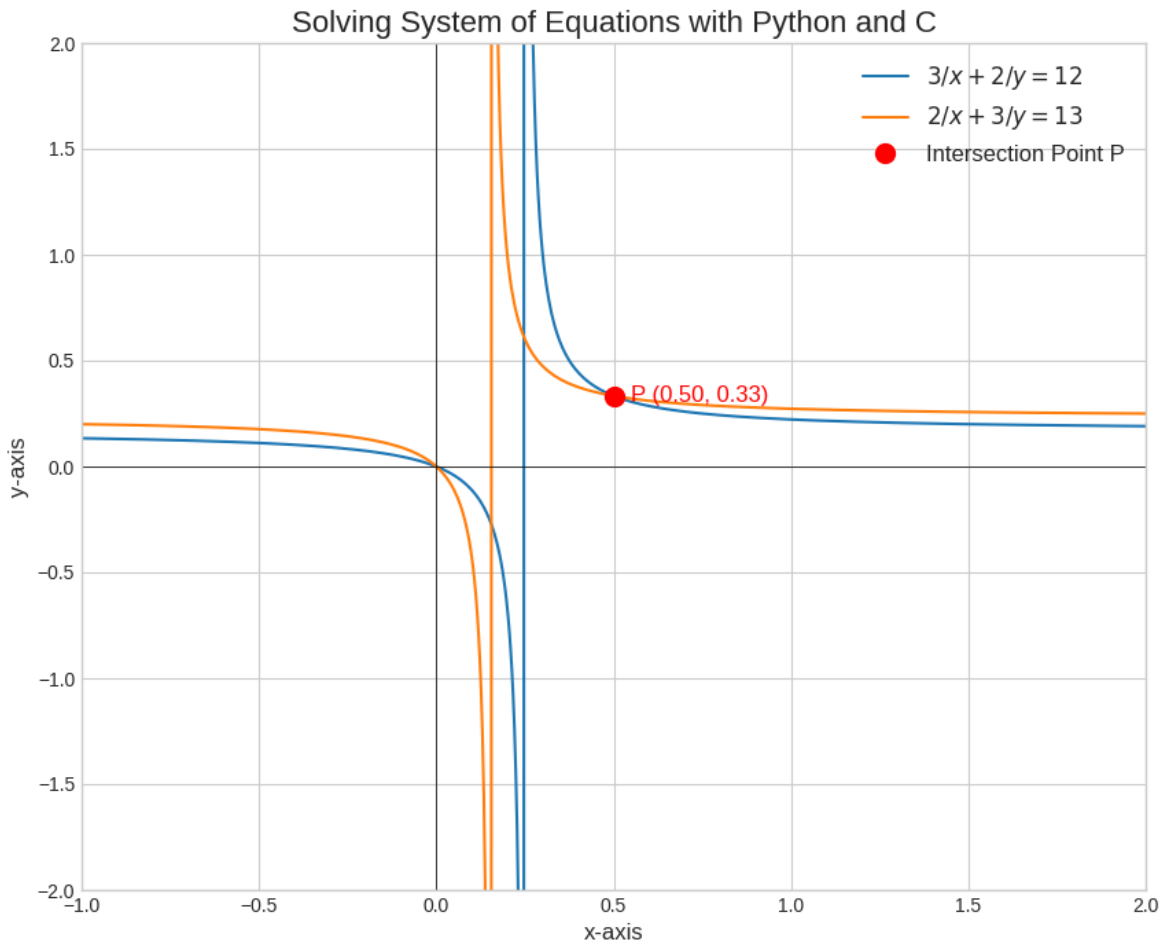

Figure: Figure for 5.2.38