AI25BTECH11034 - SUJAL CHAUHAN
2.10.23

Question:
The vector(s) which is/are coplanar with the vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to vector $\hat{i}+\hat{j}+\hat{k}$ is/are.

a) $\hat{\mathbf{j}}-\hat{\mathbf{k}}$

b) $\hat{\mathbf{i}}+\hat{\mathbf{j}}$

c) $\hat{\mathbf{i}}-\hat{\mathbf{j}}$

d) $\hat{\mathbf{j}}+\hat{\mathbf{k}}$

Variable	Vector
$\mathbf{A}$	$\begin{pmatrix}1\\1\\2\end{pmatrix}$
$\mathbf{B}$	$\begin{pmatrix}1\\2\\1\end{pmatrix}$
$\mathbf{C}$	$\begin{pmatrix}1\\1\\1\end{pmatrix}$

Listing options as vectors $\mathbf{D_i}$:

Input	Vector
$\mathbf{D_1}$	$\begin{pmatrix}0\\1\\-1\end{pmatrix}$
$\mathbf{D_2}$	$\begin{pmatrix}1\\1\\1\end{pmatrix}$
$\mathbf{D_3}$	$\begin{pmatrix}1\\-1\\0\end{pmatrix}$
$\mathbf{D_4}$	$\begin{pmatrix}0\\1\\1\end{pmatrix}$

Checking coplanarity

If the given vector $\mathbf{D_i}$ is coplanar with $\mathbf{A}$ and $\mathbf{B}$:

$$[\mathbf{A}\ \mathbf{B}\ \mathbf{D_i}] = 0 \quad \iff \quad [\mathbf{A}\ \mathbf{B}\ \mathbf{D_i}]^2 = 0 \tag{1}$$

The determinant test via Gram matrix:

$$\mathbf{G_i} = \begin{pmatrix} \mathbf{A}^T\mathbf{A} & \mathbf{A}^T\mathbf{B} & \mathbf{A}^T\mathbf{D_i} \\ \mathbf{B}^T\mathbf{A} & \mathbf{B}^T\mathbf{B} & \mathbf{B}^T\mathbf{D_i} \\ \mathbf{D_i}^T\mathbf{A} & \mathbf{D_i}^T\mathbf{B} & \mathbf{D_i}^T\mathbf{D_i} \end{pmatrix} \tag{2}$$

$$[\mathbf{A}\ \mathbf{B}\ \mathbf{D_i}]^2 = \det(\mathbf{G_i}) \tag{3}$$

Checking coplanarity for all four vectors:

Vector	$\det(G)$	Coplanar?
$\mathbf{D_1}$	0	Yes
$\mathbf{D_2}$	4	No
$\mathbf{D_3}$	16	No
$\mathbf{D_4}$	4	No

Checking perpendicular to $\mathbf{C}$

If a given vector is perpendicular to $\mathbf{C}$:

$$\mathbf{C}^T\mathbf{D_i} = 0 \tag{4}$$

$$\mathbf{C}^T = \begin{pmatrix}1 & 1 & 1\end{pmatrix} \tag{5}$$

Vector	$\mathbf{C}^T\mathbf{D_i}$	Perpendicular?
$\mathbf{D_1}$	$\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}0 \\ 1 \\ -1\end{pmatrix} = 0$	Yes
$\mathbf{D_2}$	$\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}1 \\ 1 \\ 1\end{pmatrix} = 3$	No
$\mathbf{D_3}$	$\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}1 \\ -1 \\ 0\end{pmatrix} = 0$	Yes
$\mathbf{D_4}$	$\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}0 \\ 1 \\ 1\end{pmatrix} = 2$	No

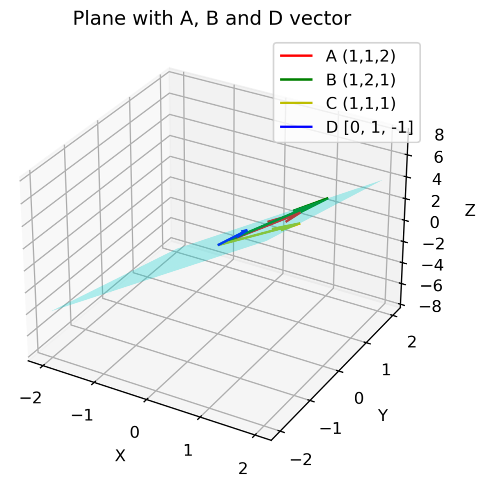

Figure 1: Vector $\mathbf{D_1}$ in plane

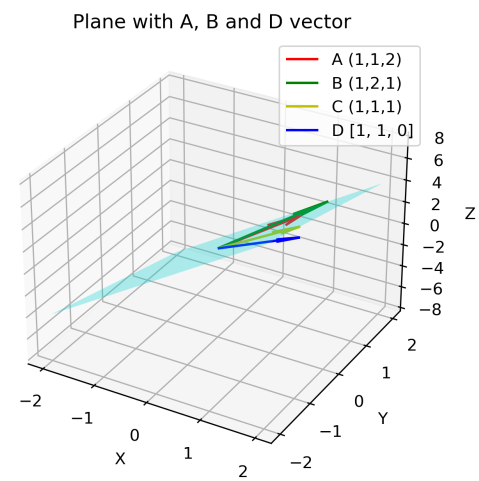

Figure 2: Vector $\mathbf{D_2}$ not coplanar

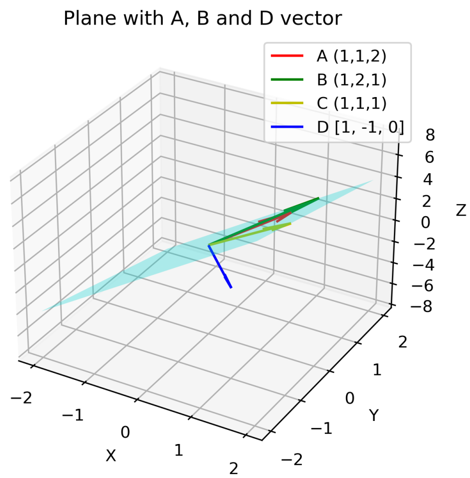

Figure 3: Vector $\mathbf{D_3}$ not coplanar

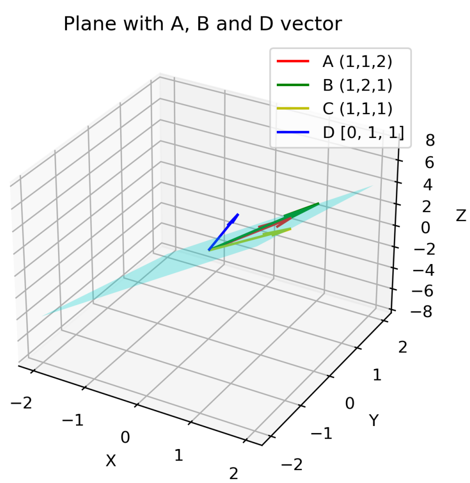

Figure 4: Vector $\mathbf{D_4}$ not coplanar