

## 3.2.19

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# Question

For which values of  $p$  does the pair of equations given below have a unique solution.

$$4x + py + 8 = 0$$

$$2x + 2y + 2 = 0$$

# Theoretical Solution

Given:

$$4x + py + 8 = 0 \quad (1)$$

$$2x + 2y + 2 = 0 \quad (2)$$

Standard Form:  $\mathbf{Ax} = \mathbf{b}$  where:

Coefficient Matrix:  $\mathbf{A} = \begin{pmatrix} 4 & p \\ 2 & 2 \end{pmatrix}$

Constant Vector:  $\mathbf{b} = \begin{pmatrix} -8 \\ -2 \end{pmatrix}$

Augmented Matrix:  $[\mathbf{A}|\mathbf{b}] = \begin{pmatrix} 4 & p & -8 \\ 2 & 2 & -2 \end{pmatrix}$

# Theoretical Solution

For Unique Solution:

Unique Solution:  $\text{rank}(\mathbf{A}) = \text{rank}([\mathbf{A}|\mathbf{b}]) = n$  (number of variables)

For our system:  $n = 2$  variables

Finding  $\text{rank}(\mathbf{A})$  - Rank of Coefficient Matrix

Initial Matrix  $\mathbf{A}$ :

$$\mathbf{A} = \begin{pmatrix} 4 & p \\ 2 & 2 \end{pmatrix} \quad (3)$$

Row Operations on  $\mathbf{A}$ :

$$R_2 \rightarrow R_2 - \frac{1}{2}R_1 \quad (4)$$

Row Echelon Form of  $\mathbf{A}$ :

$$\begin{pmatrix} 4 & p \\ 0 & 2 - \frac{p}{2} \end{pmatrix} \quad (5)$$

# Theoretical Solution

Rank Analysis:

Case 1: If  $2 - \frac{p}{2} \neq 0$  (i.e.,  $p \neq 4$ )

Both rows are non-zero and linearly independent

$\Rightarrow \text{rank}(\mathbf{A}) = 2$

Case 2: If  $2 - \frac{p}{2} = 0$  (i.e.,  $p = 4$ )

Second row is zero, only first row is non-zero

$\Rightarrow \text{rank}(\mathbf{A}) = 1$

Finding  $\text{rank}([\mathbf{A}|\mathbf{b}])$  - Rank of Augmented Matrix

Initial Augmented Matrix:

$$\begin{pmatrix} 4 & p & -8 \\ 2 & 2 & -2 \end{pmatrix} \quad (6)$$

# Theoretical Solution

Row Operation on  $[\mathbf{A}|\mathbf{b}]$ :

$$R_2 \rightarrow R_2 - \frac{1}{2}R_1 \quad (7)$$

Row Echelon Form of Augmented Matrix:

$$\begin{pmatrix} 4 & p & -8 \\ 0 & 2 - \frac{p}{2} & 2 \end{pmatrix} \quad (8)$$

Rank Analysis:

Case 1: If  $p \neq 4$ :

$$\Rightarrow \text{rank}([\mathbf{A}|\mathbf{b}]) = 2$$

Case 2: If  $p = 4$ :

$$\Rightarrow \text{rank}([\mathbf{A}|\mathbf{b}]) = 2$$

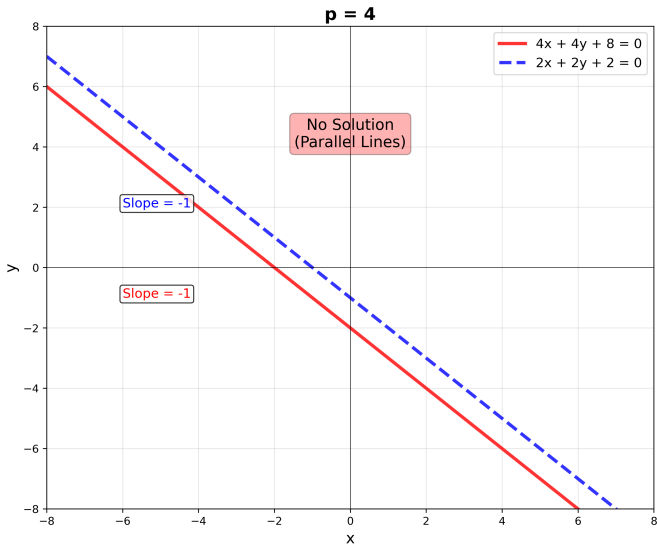
# Theoretical Solution

Comparing Ranks and Providing Solution Type

Value of $p$	$\text{rank}(\mathbf{A})$	$\text{rank}([\mathbf{A} \mathbf{b}])$	Solution Type
$p \neq 4$	2	2	UNIQUE
$p = 4$	1	2	NO SOLUTION

$\therefore$  For  $p \in \mathbb{R} - \{4\}$ , the pair of equations has an Unique Solution. (9)

# Graph of Pair of Equations (When $p=4$ )





# Graph of Pair of Equations (When $p \neq 4$ )

