

1.8.26

EE25BTECH11009-Anshu kumar ram

Question:

Find a point on the Y axis which is equidistant from the points $A(6, 5)$ and $B(-4, 3)$.

Solution:

The input parameters for this problem are available in Table

Symbol	Value	Description
A	$\begin{pmatrix} 6 \\ 5 \end{pmatrix}$	First point
B	$\begin{pmatrix} -4 \\ 3 \end{pmatrix}$	Second point
O	$\begin{pmatrix} 0 \\ y \end{pmatrix}$	Desired point

TABLE 0: Parameters for the problem

If \mathbf{O} lies on the y-axis and is equidistant from the points \mathbf{A} and \mathbf{B} ,

$$\|\mathbf{O} - \mathbf{A}\| = \|\mathbf{O} - \mathbf{B}\| \quad (0.1)$$

$$\implies \|\mathbf{O} - \mathbf{A}\|^2 = \|\mathbf{O} - \mathbf{B}\|^2 \quad (0.2)$$

$$\implies \|\mathbf{O}\|^2 - 2\mathbf{O}^\top \mathbf{A} + \|\mathbf{A}\|^2 = \|\mathbf{O}\|^2 - 2\mathbf{O}^\top \mathbf{B} + \|\mathbf{B}\|^2, \quad (0.3)$$

which can be simplified to obtain

$$(\mathbf{A} - \mathbf{B})^\top \mathbf{O} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2}. \quad (0.4)$$

$$\therefore \mathbf{O} = y\mathbf{e}_2, \quad (0.5)$$

$$y = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2(\mathbf{A} - \mathbf{B})^\top \mathbf{e}_2}. \quad (0.6)$$

Substituting from table we get , $y = 9$. Thus,

$$\mathbf{O} = \begin{pmatrix} 0 \\ 9 \end{pmatrix}. \quad (0.7)$$

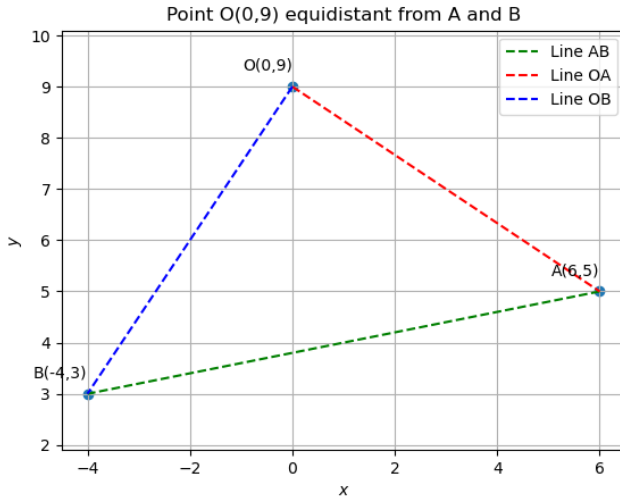


Fig. 0.1: Point $O(0,9)$ on the y-axis is equidistant from A and B.