2.2.20

AI25BTECH11027 - NAGA BHUVANA

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Question:

If the co-ordinates of the points $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ be (1,2,3),(4,5,7),(-4,3,-6) and (2,9,2) respectively, then find the angle between lines AB and CD.

Solution:

Let

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 5 \\ 7 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -4 \\ 3 \\ -6 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} 2 \\ 9 \\ 2 \end{pmatrix} \tag{0.1}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 4 \\ 5 \\ 7 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} \tag{0.2}$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} 2 \\ 9 \\ 2 \end{pmatrix} - \begin{pmatrix} -4 \\ 3 \\ -6 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \\ 8 \end{pmatrix} \tag{0.3}$$

Let the angle between $\mathbf{B} - \mathbf{A}$ and $\mathbf{D} - \mathbf{C}$ be θ

$$\cos \theta = \frac{(B - A)^T (D - C)}{\|\mathbf{B} - \mathbf{A}\| \|\mathbf{D} - \mathbf{C}\|} \tag{0.4}$$

$$\cos \theta = \frac{\begin{pmatrix} 3 & 3 & 4 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \\ 8 \end{pmatrix}}{\sqrt{34}\sqrt{136}} \tag{0.5}$$

$$\cos \theta = \frac{(3)(6) + (3)(6) + (4)(8)}{68} \tag{0.6}$$

$$\cos \theta = \frac{68}{68} \tag{0.7}$$

$$\cos \theta = 1 \tag{0.8}$$

$$\theta = 0° \tag{0.9}$$

$\therefore$ The angle between lines $(\mathbf{B} - \mathbf{A})$ and $(\mathbf{D} - \mathbf{C})$ is $0°$ (Collinear lines)

Lines AB and CD in 3D