AI25BTECH11034 - SUJAL CHAUHAN

2.10.23

question

The vector(s) which is/are coplanar with with the vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$, and perpentdicular to vector $\hat{i} + \hat{j} + \hat{k}$ is/are.

a) $\hat{\mathbf{j}} - \hat{\mathbf{k}}$

b) $\hat{\mathbf{i}} + \hat{\mathbf{j}}$

c) $\hat{\mathbf{i}} - \hat{\mathbf{j}}$

d) $\hat{\mathbf{j}} + \hat{\mathbf{k}}$

Variable	Vector
$\mathbf{A}$	$\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$
$\mathbf{B}$	$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$
$\mathbf{C}$	$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Listing Options as vectors $\mathbf{D_i}$

Input	Vector
$\mathbf{D_1}$	$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$
$\mathbf{D_2}$	$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
$\mathbf{D_3}$	$\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$
$\mathbf{D_4}$	$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

Checking coplanarity

Let's first check conditions for coplanarity of each vectors

If the given vector $\mathbf{D_i}$ is coplanar with $\mathbf{A}$ and $\mathbf{B}$

$$[\mathbf{A}\ \mathbf{B}\ \mathbf{D_i}] = 0 = (\mathbf{A} \times \mathbf{B})^T \mathbf{D_i} \tag{1}$$

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} \tag{2}$$

$$(\mathbf{A} \times \mathbf{B})^T = \begin{pmatrix} -3 & 1 & 1 \end{pmatrix} \tag{3}$$

Now for checking coplanarity for all four points:

Vector	$(\mathbf{A} \times \mathbf{B})^T \mathbf{D_i}$	Is coplanar
$\mathbf{D_1}$	$\begin{pmatrix} -3 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = 0$	Yes
$\mathbf{D_2}$	$\begin{pmatrix} -3 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = -1$	No
$\mathbf{D_3}$	$\begin{pmatrix} -3 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = -4$	No
$\mathbf{D_4}$	$\begin{pmatrix} -3 & 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 2$	No

Checking perpendicular to $\mathbf{C}$

If given vector is perpendicular to $\mathbf{C}$ then :

$$\mathbf{C}^T\mathbf{D_i} = 0 \tag{4}$$

$$\mathbf{C}^T = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \tag{5}$$

Vector	$\mathbf{C}^T\mathbf{D_i}$	Is perpendicular
$\mathbf{D_1}$	$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = 0$	Yes
$\mathbf{D_2}$	$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 3$	No
$\mathbf{D_3}$	$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = 0$	Yes
$\mathbf{D_4}$	$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = 2$	No

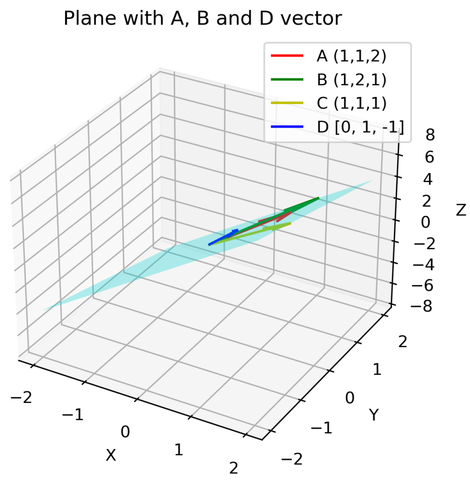

Figure 1: 1

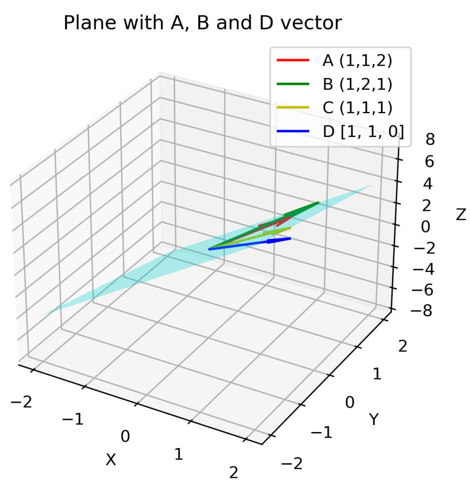

Figure 2: 2

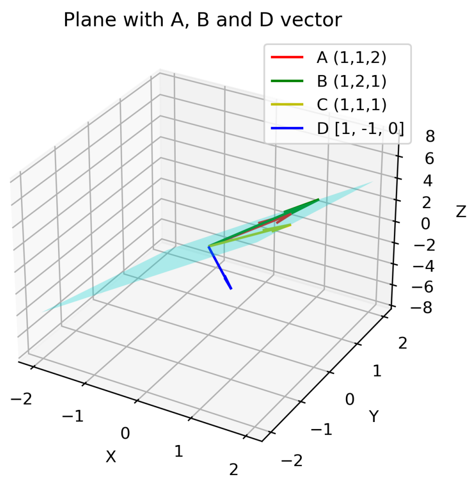

Figure 3: 3

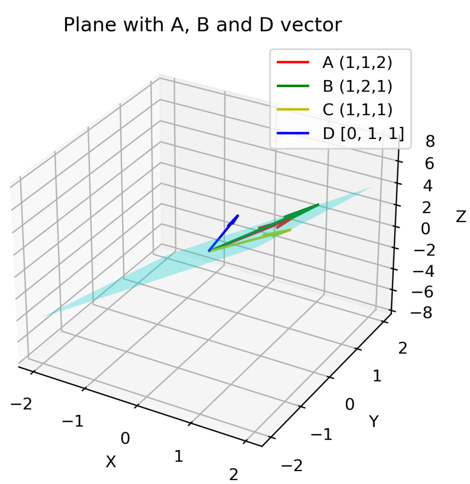

Figure 4: 4