4.4.20

AI25BTECH11024 - Pratyush Panda

Question:
The pair of linear equations $2x = 5y + 6$ and $15y = 6x - 18$ represents two lines which are:

1) intersecting
2) parallel
3) coincident
4) either intersecting or parallel

Solution:
First, let us rewrite the equation of lines as;

$$ax + by = c \tag{4.1}$$

$$2x - 5y = 6 \tag{4.2}$$

$$6x - 15y = 18 \ or \ 2x - 5y = 6 \tag{4.3}$$

Normal vectors of the given lines can be written as:

$$\mathbf{n} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{4.4}$$

$$\mathbf{n_1} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \qquad \mathbf{n_2} = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \tag{4.5}$$

Let the matrix $\mathbf{M}$ be;

$$\mathbf{M} = \begin{pmatrix} n_1 & n_2 \end{pmatrix}^T = \begin{pmatrix} 2 & -5 \\ 2 & -5 \end{pmatrix} \tag{4.6}$$

After reducing it to its Echelon form;

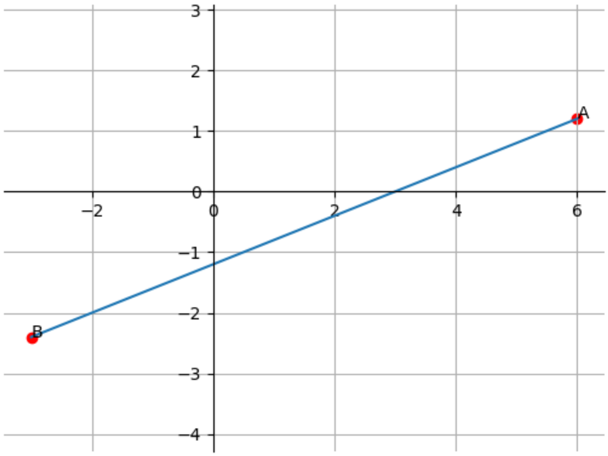
$$\begin{pmatrix} 2 & -5 \\ 2 & -5 \end{pmatrix} \xrightarrow{R_2 \longleftrightarrow R_2 - R_1} \begin{pmatrix} 2 & -5 \\ 0 & 0 \end{pmatrix} \tag{4.7}$$

We can see that the rank of $\mathbf{M}$ is 1. Therefore, the given lines can be either parallel or coincident.

Now, consider the matrices $\mathbf{P}$ and $\mathbf{Q}$;

$$\mathbf{P} = \begin{pmatrix} a_1 & c_1 \\ a_2 & c_2 \end{pmatrix} \qquad \mathbf{Q} = \begin{pmatrix} b_1 & c_1 \\ b_2 & c_2 \end{pmatrix} \tag{4.8}$$

Since, the rank of both the matrices is 1. Thus, we can conclude that the the given lines are coincident.



A
B
3
2
1
0
−1
−2
−3
−4
−2
0
2
4
6