# 2.8.9

J.NAVYASRI- EE25BTECH11028

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# Question

Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three vectors such that $|\mathbf{a}| = 3,\ |\mathbf{b}| = 4,\ |\mathbf{c}| = 5$, and each one of them is perpendicular to the sum of the other two. Find $|\mathbf{a} + \mathbf{b} + \mathbf{c}|$.

# Solution:

From the identity:

$$\mathbf{a}^\top(\mathbf{b}+\mathbf{c}) = 0, \tag{1}$$

we expand:

$$\mathbf{a}^\top\mathbf{b} + \mathbf{a}^\top\mathbf{c} = 0. \tag{2}$$

Similarly, from the symmetry of dot products:

$$\mathbf{b}^\top\mathbf{c} + \mathbf{b}^\top\mathbf{a} = 0, \tag{3}$$

$$\mathbf{c}^\top\mathbf{a} + \mathbf{c}^\top\mathbf{b} = 0. \tag{4}$$

Let

$$x = \mathbf{a}^\top \mathbf{b}, \quad y = \mathbf{b}^\top \mathbf{c}, \quad z = \mathbf{c}^\top \mathbf{a}. \tag{5}$$

Then equations (2), (3), and (4) become:

$$x + z = 0, \tag{6}$$

$$x + y = 0, \tag{7}$$

$$y + z = 0. \tag{8}$$

From equation (7):

$$y = -x,$$

# Solution:

and from equation (6):

$$z = -x.$$

Substitute into equation (8):

$$y + z = -x + (-x) = -2x = 0 \Rightarrow x = 0.$$

Therefore:

$$x = y = z = 0, \tag{9}$$

so $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are **pairwise orthogonal**.

The **Gram matrix** of $(\mathbf{a}, \mathbf{b}, \mathbf{c})$ is:

$$G = \begin{bmatrix} \mathbf{a}^\top\mathbf{a} & \mathbf{a}^\top\mathbf{b} & \mathbf{a}^\top\mathbf{c} \\ \mathbf{b}^\top\mathbf{a} & \mathbf{b}^\top\mathbf{b} & \mathbf{b}^\top\mathbf{c} \\ \mathbf{c}^\top\mathbf{a} & \mathbf{c}^\top\mathbf{b} & \mathbf{c}^\top\mathbf{c} \end{bmatrix} = \begin{bmatrix} \|\mathbf{a}\|^2 & 0 & 0 \\ 0 & \|\mathbf{b}\|^2 & 0 \\ 0 & 0 & \|\mathbf{c}\|^2 \end{bmatrix} = \begin{pmatrix} 9 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 25 \end{pmatrix}. \quad (10)$$

Let

$$\mathbf{u} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Then

$$\|\mathbf{a}+\mathbf{b}+\mathbf{c}\|^2 = (\mathbf{a}+\mathbf{b}+\mathbf{c})^\top(\mathbf{a}+\mathbf{b}+\mathbf{c}) = \mathbf{u}^\top G\mathbf{u}. \quad (11)$$

# Solution:

Now compute:

$$\mathbf{u}^\top G\mathbf{u} = \begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}9 & 0 & 0\\0 & 16 & 0\\0 & 0 & 25\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}$$
$$= 9 + 16 + 25 = 50. \tag{12}$$

Therefore:

$$\|\mathbf{a}+\mathbf{b}+\mathbf{c}\| = \sqrt{50} = 5\sqrt{2}. \tag{13}$$

**Final Answer:**

$$\boxed{5\sqrt{2}}$$

# Python Code

```
import numpy as np
import matplotlib.pyplot as plt

# Define mutually perpendicular vectors
a = np.array([3, 0, 0])
b = np.array([0, 4, 0])
c = np.array([0, 0, 5])
s = a + b + c # resultant (3,4,5)

fig = plt.figure(figsize=(8, 6))
ax = fig.add_subplot(111, projection='3d')
```

# Python Code

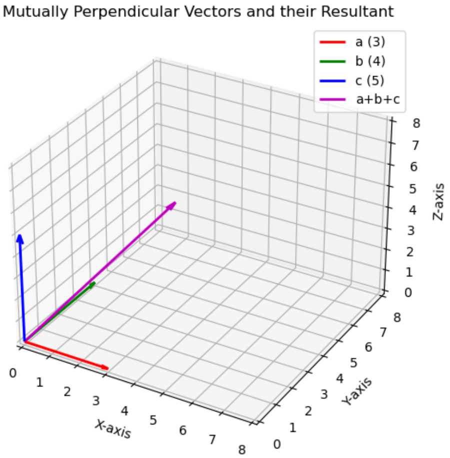
```
# Plot main vectors
ax.quiver(0, 0, 0, *a, color='r', linewidth=2,
          arrow_length_ratio=0.08, normalize=False, label='a (3)')
ax.quiver(0, 0, 0, *b, color='g', linewidth=2,
          arrow_length_ratio=0.08, normalize=False, label='b (4)')
ax.quiver(0, 0, 0, *c, color='b', linewidth=2,
          arrow_length_ratio=0.08, normalize=False, label='c (5)')

# Plot resultant
ax.quiver(0, 0, 0, *s, color='m', linewidth=2,
          arrow_length_ratio=0.05, normalize=False,
          label='a+b+c')
```

# Python Code

```python
# Axis limits
ax.set_xlim(0, 8)
ax.set_ylim(0, 8)
ax.set_zlim(0, 8)

# Axis labels
ax.set_xlabel("X-axis")
ax.set_ylabel("Y-axis")
ax.set_zlabel("Z-axis")
ax.set_title("Mutually Perpendicular Vectors and their Resultant"
    )

ax.legend()
plt.show()
```

```c
#include <stdio.h>
#include <math.h>

int main() {
    // Given magnitudes
    int a = 3, b = 4, c = 5;
    // Since a, b, c are mutually perpendicular (proved in
        solution),
    // |a + b + c|^2 = |a|^2 + |b|^2 + |c|^2
    int sum_sq = a*a + b*b + c*c;

    double magnitude = sqrt(sum_sq);

    // Print result
    printf("The magnitude |a + b + c| = %.2f\n", magnitude);

    return 0;
}
```

# Plot-Using by Python



# Python and C Code

```python
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load the compiled C library
lib = ctypes.CDLL("./vector_calc.so") # use "vector_calc.dll" on
    Windows

# Call the C function
lib.vector_magnitude.restype = ctypes.c_double
magnitude = lib.vector_magnitude()
print("Result from C code |a+b+c| =", magnitude)
```

# Python and C Code

```python
# ---- Plotting in Python ----
a = np.array([3, 0, 0])
b = np.array([0, 4, 0])
c = np.array([0, 0, 5])
resultant = a + b + c

fig = plt.figure(figsize=(8, 6))
ax = fig.add_subplot(111, projection="3d")

# plot vectors
origin = np.array([0, 0, 0])
ax.quiver(*origin, *a, color='r', label='a (3)')
ax.quiver(*origin, *b, color='g', label='b (4)')
ax.quiver(*origin, *c, color='b', label='c (5)')
ax.quiver(*origin, *resultant, color='m', label='a+b+c')
```

# Python and C Code

```
ax.set_xlim([0, 8])
ax.set_ylim([0, 8])
ax.set_zlim([0, 8])

ax.set_xlabel("X axis")
ax.set_ylabel("Y axis")
ax.set_zlabel("Z axis")
ax.set_title("C code calculation + Python plot")

ax.legend()
plt.show()
```

# Plot-Using by both C and Python