4.11.4

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Question

Find the equation of the plane which contains the line of intersection of the planes $\mathbf{r}.\left(\hat{i}+2\hat{j}+3\hat{k}\right)-4=0$ and $\mathbf{r}.\left(2\hat{i}+\hat{j}-\hat{k}\right)+5=0$ and which is perpendicular to the plane $\mathbf{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$.

Theoretical Solution

According to the question,

$$\mathbf{n_1} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \mathbf{n_2} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \quad c_1 = 4 \quad c_2 = -5 \tag{1}$$

Equation

The equation of intersection of planes is given by

$$\mathbf{n_1}^\top \mathbf{r} - c_1 + \lambda \left(\mathbf{n_2}^\top \mathbf{r} - c_2 \right) = 0 \tag{2}$$

$$\implies \left(\mathbf{n_1}^\top + \lambda \mathbf{n_2}^\top \right) \mathbf{r} = c_1 + \lambda c_2 \tag{3}$$

Theoretical Solution

Let the direction vector of the plane perpendicular to intersection of planes be $\mathbf{n_3}$

$$\therefore \left(\mathbf{n_1}^\top + \lambda \mathbf{n_2}^\top\right) \mathbf{n_3} = 0 \tag{4}$$

$$\implies \lambda = -\frac{\mathbf{n_1}^\top \mathbf{n_3}}{\mathbf{n_2}^\top \mathbf{n_3}} \tag{5}$$

Theoretical Solution

$$\therefore \lambda = -\frac{\begin{pmatrix} 1 & 2 & 3 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ -6 \end{pmatrix}}{\begin{pmatrix} 2 & 1 & -1 \end{pmatrix}\begin{pmatrix} 5 \\ 3 \\ -6 \end{pmatrix}} = \frac{7}{19} \tag{6}$$

$$\implies \text{equation of the plane}: \left(\mathbf{n_1}^\top + \frac{7}{19}\mathbf{n_2}^\top\right)\mathbf{r} = c_1 + \frac{7}{19}c_2 \tag{7}$$

C Code -Finding Equation of the plane

```
#include <stdio.h>
// Function to compute required plane coefficients
void required_plane(double n1[3], double d1,
                    double n2[3], double d2,
                    double n3[3], double d3,
                    double n_req[3], double *d_req) {

    // Find λsuch that (n1 + λn2)·n3 = 0
    double dot_n1n3 = n1[0]*n3[0] + n1[1]*n3[1] + n1[2]*n3[2];
    double dot_n2n3 = n2[0]*n3[0] + n2[1]*n3[1] + n2[2]*n3[2];
    double lam = -dot_n1n3 / dot_n2n3;
    // Required normal = n1 + λn2
    for(int i=0; i<3; i++) {
        n_req[i] = n1[i] + lam * n2[i];
    }
    // Required constant term
    *d_req = d1 + lam * d2;
}
```

Python+C code

```
import ctypes
import numpy as np
import sympy as sp
import matplotlib.pyplot as plt
import matplotlib as mp
from fractions import Fraction
import math
mp.use("TkAgg")

lib = ctypes.CDLL("./libplane.so")
# Define argtypes and restype
lib.required_plane.argtypes = [
    ctypes.POINTER(ctypes.c_double), ctypes.c_double,
    ctypes.POINTER(ctypes.c_double), ctypes.c_double,
    ctypes.POINTER(ctypes.c_double), ctypes.c_double,
    ctypes.POINTER(ctypes.c_double), ctypes.POINTER(ctypes.
        c_double)]
lib.required_plane.restype = None
```

Python+C code

```python
# Input planes
n1 = (ctypes.c_double * 3)(1, 2, 3)
d1 = -4.0
n2 = (ctypes.c_double * 3)(2, 1, -1)
d2 = 5.0
n3 = (ctypes.c_double * 3)(5, 3, -6)
d3 = 8.0
# Output storage
n_req = (ctypes.c_double * 3)()
d_req = ctypes.c_double()
# Call C function
lib.required_plane(n1, d1, n2, d2, n3, d3, n_req, ctypes.byref(d_req))
# Convert to numpy + sympy
n_req_np = np.array([n_req[i] for i in range(3)], dtype=float)
d_req_val = float(d_req.value)
n_vec = sp.Matrix(n_req_np)
```

Python+C code

```
# Formatting helpers
def format_plane_integer(n, d):
    # Fractions for exactness
    n_frac = [Fraction(str(float(v))).limit_denominator() for v
        in n]
    d_frac = Fraction(str(float(d))).limit_denominator()
    # LCM of denominators
    denoms = [f.denominator for f in n_frac] + [d_frac.
        denominator]
    lcm = math.lcm(*denoms)
    n_int = [int(f * lcm) for f in n_frac]
    d_int = int(d_frac * lcm)
    # Fix sign convention
    # Equation is n·r + d = 0 →n·r = -d
    rhs = -d_int
```

Python+C code

```python
# Make gcd simplification
g = math.gcd(math.gcd(abs(n_int[0]), abs(n_int[1])), math.gcd(abs
    (n_int[2]), abs(rhs)))
n_int = [ni // g for ni in n_int]
rhs //= g
return f"r ·({n_int[0]}, {n_int[1]}, {n_int[2]}) = {rhs}"
x, y, z = sp.symbols('x y z')
plane_eq = n_vec[0]*x + n_vec[1]*y + n_vec[2]*z + d_req_val
print("\nEquation of required plane:")
print(format_plane_integer(n_req_np, d_req_val))
```

Python+C code

```
# Step 6: Plot planes
fig = plt.figure(figsize=(8,8))
ax = fig.add_subplot(111, projection='3d')

def plot_plane(ax, n, d, color, alpha=0.4):
    xx, yy = np.meshgrid(np.linspace(-5,5,15), np.linspace
        (-5,5,15))
    zz = (-n[0]*xx - n[1]*yy - d) / n[2]
    ax.plot_surface(xx, yy, zz, color=color, alpha=alpha)

# Convert sympy / ctypes →float numpy
n1f, d1f = np.array([1,2,3], dtype=float), -4.0
n2f, d2f = np.array([2,1,-1], dtype=float), 5.0
n3f, d3f = np.array([5,3,-6], dtype=float), 8.0
nrf = n_req_np
drf = d_req_val
```

Python+C code

```python

# Plot given planes and required plane
plot_plane(ax, n1f, d1f, "red", 0.2) # 1
plot_plane(ax, n2f, d2f, "blue", 0.2) # 2
plot_plane(ax, n3f, d3f, "green", 0.2) # 3
plot_plane(ax, nrf, drf, "purple", 0.6) # Required plane

ax.set_xlabel("X")
ax.set_ylabel("Y")
ax.set_zlabel("Z")
ax.set_title(r"Required Plane through Intersection, $\perp$ to
    Given Plane")

plt.savefig("/home/user/Matrix/Matgeo_assignments/4.11.4/figs/
    Figure_1", dpi=300, bbox_inches="tight")
plt.show()
```

Python code

```python
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
import matplotlib as mp
mp.use("TkAgg")
# Step 1: Define symbols
lam = sp.symbols('lam')
# Given plane normals and constants
n1, d1 = sp.Matrix([1,2,3]), -4
n2, d2 = sp.Matrix([2,1,-1]), 5
n3, d3 = sp.Matrix([5,3,-6]), 8
# Step 2: General plane through intersection
n = n1 + lam*n2 # normal vector depends on λ
d = d1 + lam*d2 # constant term
# Step 3: Perpendicular condition
eq = sp.Eq(n.dot(n3), 0)
lam_val = sp.solve(eq, lam)[0]
```

Python code

```
# Substitute λinto plane equation
n_req = (n.subs(lam, lam_val))
d_req = d.subs(lam, lam_val)

# Step 4: Scale to integers (fix applied here)
all_vals = list(n_req) + [d_req] # avoid .tolist()
mult = sp.lcm([sp.denom(val) for val in all_vals])
n_req = (n_req * mult).applyfunc(sp.simplify)
d_req = sp.simplify(d_req * mult)

# Step 5: Vector form
x, y, z = sp.symbols('x y z')
plane_eq = n_req[0]*x + n_req[1]*y + n_req[2]*z + d_req
point_on_plane = sp.solve(plane_eq.subs({y:0, z:0}), x)
```

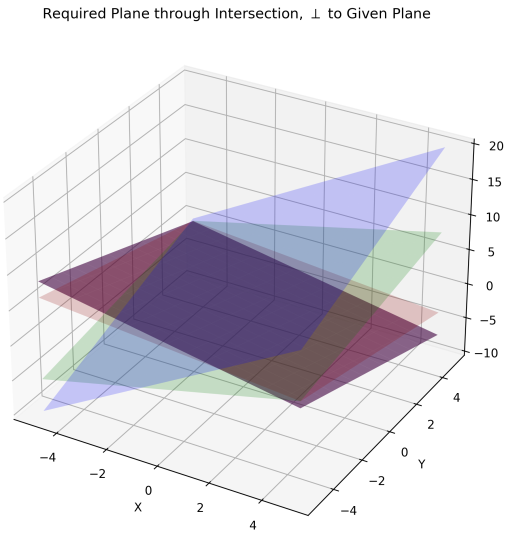
Python code

```
# Step 6: Plot planes
fig = plt.figure(figsize=(8,8))
ax = fig.add_subplot(111, projection='3d')
def plot_plane(ax, n, d, color, alpha=0.4):
    xx, yy = np.meshgrid(np.linspace(-5,5,15), np.linspace
        (-5,5,15))
    zz = (-n[0]*xx - n[1]*yy - d) / n[2]
    ax.plot_surface(xx, yy, zz, color=color, alpha=alpha)
# Convert sympy →float numpy
n1f, d1f = np.array(n1, dtype=float), float(d1)
n2f, d2f = np.array(n2, dtype=float), float(d2)
n3f, d3f = np.array(n3, dtype=float), float(d3)
nrf, drf = np.array([float(v) for v in n_req]), float(d_req)
# Plot given planes and required plane
plot_plane(ax, n1f, d1f, "red", 0.2)
plot_plane(ax, n2f, d2f, "blue", 0.2)
plot_plane(ax, n3f, d3f, "green", 0.2)
plot_plane(ax, nrf, drf, "purple", 0.6) # Required plane
```

Python code

```
ax.set_xlabel("X")
ax.set_ylabel("Y")
ax.set_zlabel("Z")
ax.set_title(r"Required Plane through Intersection, $\perp$ to
    Given Plane")
plt.savefig("/home/user/Matrix/Matgeo_assignments/4.11.4/figs/
    Figure_1")
plt.show()
```

Plot


Required Plane through Intersection, ⊥ to Given Plane
20
15
10
5
0
−5
−10
−4
−2
0
2
4
X
4
2
0
−2
−4
Y