5.8.39

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Question

The cost of 2 pencils and 3 erasers is $9rs$ and the cost of 4 pencils and 6 erasers is $18rs$. Find the cost of each pencil and each eraser.

Theoretical Solution

Let x and y denote the cost of each pencil and eraser respectively.
By forming the equations we get

$$\mathbf{n_1}^\top \mathbf{x} = c_1 \tag{1}$$

$$\mathbf{n_2}^\top \mathbf{x} = c_2 \tag{2}$$

Stacking these gives:

$$\begin{pmatrix} \mathbf{n_1}^\top \\ \mathbf{n_2}^\top \end{pmatrix} \mathbf{x} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \tag{3}$$

$$\mathbf{n_1} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \mathbf{n_1} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}, c_1 = 9, c_2 = 18 \tag{4}$$

Theoretical Solution

Thus,

$$\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 18 \end{pmatrix} \tag{5}$$

The augmented matrix is

$$\left(\begin{array}{cc|c} 2 & 3 & 9 \\ 4 & 6 & 18 \end{array}\right) \tag{6}$$

By row transformations:

$$\left(\begin{array}{cc|c} 2 & 3 & 9 \\ 4 & 6 & 18 \end{array}\right) R_2 \leftarrow R_2 - 2R_1 \left(\begin{array}{cc|c} 2 & 3 & 9 \\ 0 & 0 & 0 \end{array}\right) \tag{7}$$

This implies that there exist infinitely many solutions as one row is a linear factor of the other. Both equations lead to the same line.

C Code

```c
double calculate_y(double x, double a, double b,
    double c) {
        if (b == 0) {
                // Avoid division by zero for
                    vertical lines
                return 0.0;
        }
        return (c - a * x) / b;
}
```

Python Code using shared output

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt
# --- Load the Shared C Library ---
# Assumes 'line_solver.so' is in the same
    directory.
try:
line_lib = ctypes.CDLL('./5.8.39.so')
except OSError as e:
print(Error: Could not load the shared library '
    line_solver.so'.)
print(Please ensure you have compiled 'line_solver
    .c' correctly.)
print(e)
exit()
```

Python Code using shared output

```
# --- Define the C function's signature for Python ---
calculate_y_c = line_lib.calculate_y
calculate_y_c.argtypes = [ctypes.c_double, ctypes.c_double, ctypes.c_double, ctypes.c_double]
calculate_y_c.restype = ctypes.c_double
# --- Define coefficients for BOTH equations ---
# Equation 1: 2x + 3y = 9
a1, b1, c1 = 2.0, 3.0, 9.0
# Equation 2: 4x + 6y = 18
a2, b2, c2 = 4.0, 6.0, 18.0
# --- Generate data points using the C function ---
x_values = np.linspace(0, 5, 100)
y_values_1 = []
y_values_2 = []
```

Python Code using shared output

```
# Call the C function for each x value for both
    lines
for x in x_values:
y1 = calculate_y_c(x, a1, b1, c1)
y_values_1.append(y1)

y2 = calculate_y_c(x, a2, b2, c2)
y_values_2.append(y2)
# --- Plot the Results ---
plt.figure(figsize=(8, 6))
# Plot the first line
plt.plot(x_values, y_values_1,
label=f'{int(a1)}x + {int(b1)}y = {int(c1)} (via C
    )',
color='blue',
linewidth=2)
```

Python Code using shared output

```
# Plot the second line on top with a
    different style
plt.plot(x_values, y_values_2,
label=f'{int(a2)}x + {int(b2)}y = {int(c2)}
     (via C)',
color='red',
linestyle='--',
linewidth=4)
plt.title('Plotting Two Identical Lines
    Using a C Function')
plt.xlabel('Cost of Pencil (x)')
plt.ylabel('Cost of Eraser (y)')
plt.grid(True)
plt.legend()
plt.show()
```

Python Code

```
# Plot the first line
plt.plot(x, y1, label='2x + 3y = 9', color='blue', linewidth=4)
# Plot the second line on top of the first one with a different
    style to show they are identical
plt.plot(x, y2, label='4x + 6y = 18', color='red', linestyle='--'
    , linewidth=2)
# Add titles and labels
plt.title(Both Equations Represent the Same Line)
plt.xlabel(Cost of Pencil (x))
plt.ylabel(Cost of Eraser (y))
plt.grid(True)
plt.legend()
plt.tight_layout()
# Save the plot to a file
plt.savefig(Figure_2.png)
```

Plot by python using shared output from c

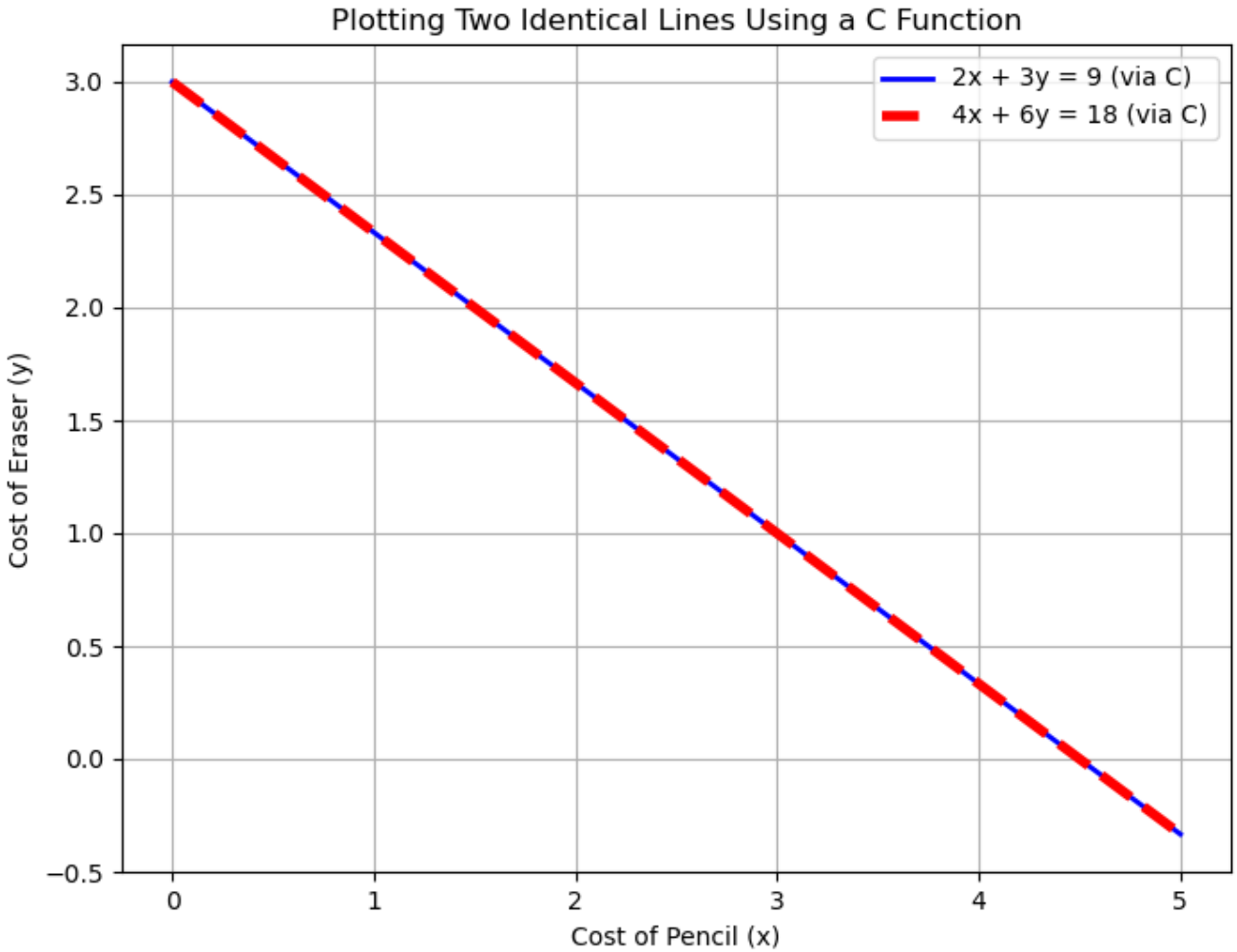

Figure: *

Plot by python

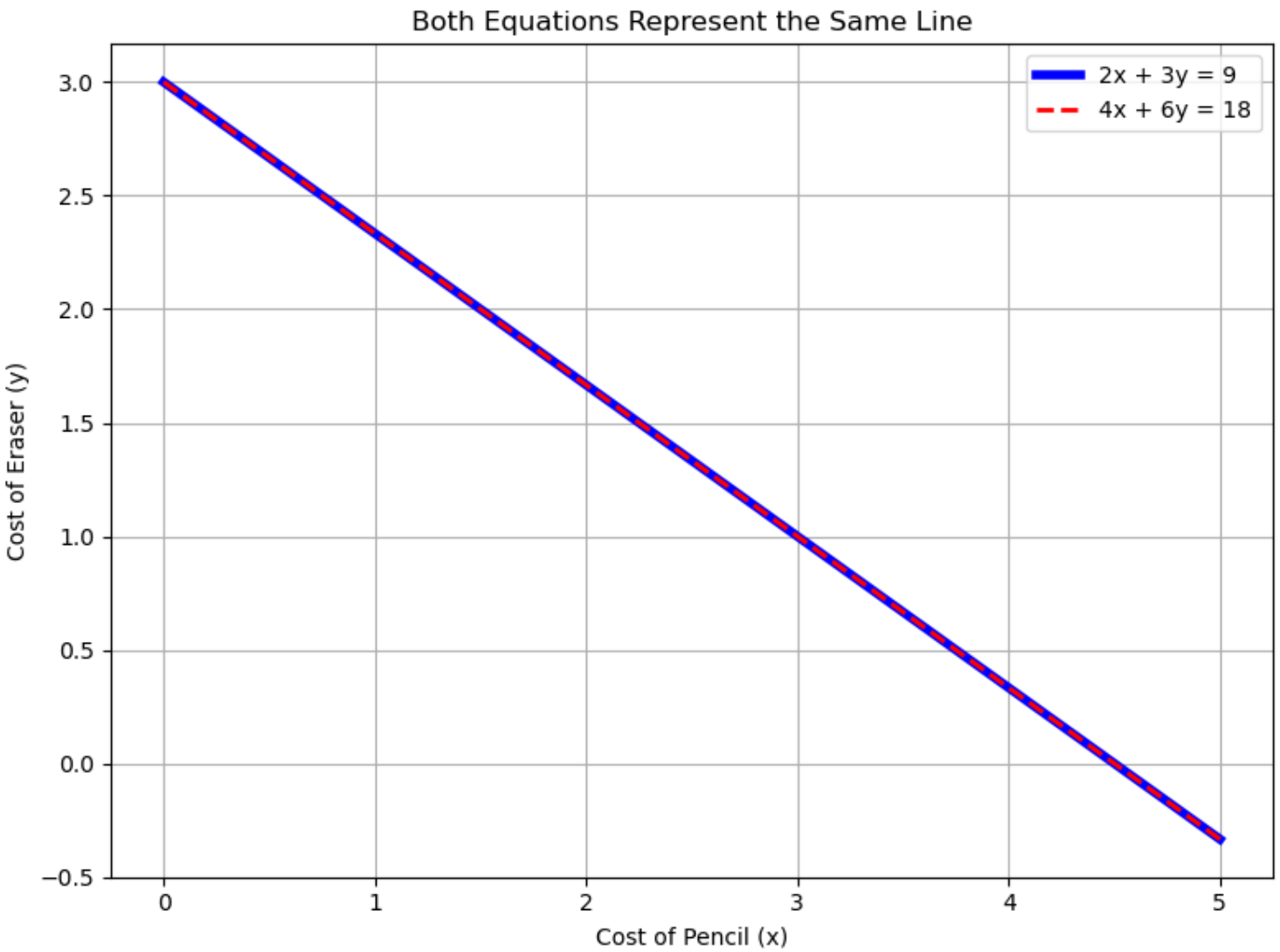

Figure: *