4.13.92

AI25BTECH11023-Pratik R

Question

The equation of a plane passing through the line of intersection of the planes $x+2y+3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point $(3, 1, -1)$ is

Solution

According to the question,

$$\mathbf{n_1} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \mathbf{n_2} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad c_1 = 2 \quad c_2 = 3 \tag{0.1}$$

The equation of plane which contains the line of intersection of the two planes is given by

$$\mathbf{n_1}^\top \mathbf{x} - c_1 + \lambda \left(\mathbf{n_2}^\top \mathbf{x} - c_2 \right) = 0 \tag{0.2}$$

$$\implies \left(\mathbf{n_1}^\top + \lambda \mathbf{n_2}^\top \right) \mathbf{x} = c_1 + \lambda c_2 \tag{0.3}$$

Let $d = \frac{2}{\sqrt{3}}$ be the distance of the plane from the point $P(3, 1, -1)$

$$\therefore d = \frac{|\left(\mathbf{n_1} + \lambda \mathbf{n_2} \right)^\top \mathbf{P} - (c_1 + \lambda c_2)|}{\|\mathbf{n_1} + \lambda \mathbf{n_2}\|} \tag{0.4}$$

simplifying RHS

$$\frac{|2\lambda|}{\sqrt{3\lambda^2 + 4\lambda + 14}} \tag{0.5}$$

$$\therefore d^2 = \frac{4\lambda^2}{3\lambda^2 + 4\lambda + 14} \tag{0.6}$$

solving this, we get

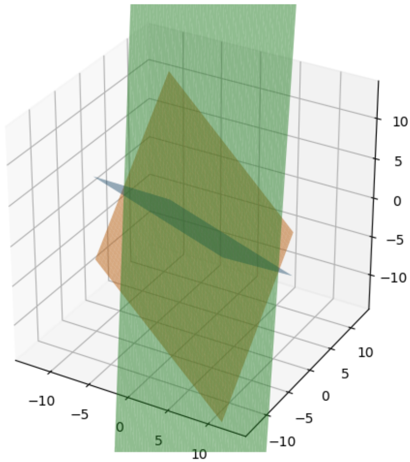
$$\lambda = \frac{-7}{2} \text{ or} \tag{0.7}$$

$$\lambda = \infty \tag{0.8}$$

Hence the Equation of plane is given by

$$\begin{pmatrix} -5 & 11 & -1 \end{pmatrix} \mathbf{x} = -17 \tag{0.9}$$

or since λ is ∞ the plane can be $\mathbf{n_2}^\top \mathbf{x} = 3$ itself.



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