12.185

AI25BTECH11003 - Bhavesh Gaikwad

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Question

If the rank of a (5×6) matrix $\mathbf{Q}$ is 4, then which one of the following statements is correct?

(EE 2008)

a) $\mathbf{Q}$ will have four linearly independent rows and four linearly independent columns.
b) $\mathbf{Q}$ will have four linearly independent rows and five linearly independent columns.
c) $\mathbf{Q}\mathbf{Q}^\top$ will be invertible.
d) $\mathbf{Q}^\top\mathbf{Q}$ will be invertible

Primary Analysis:
Since rank($\mathbf{Q}$)=4 $\Rightarrow \therefore$ $\mathbf{Q}$ will have four linearly independent rows and four linearly independent columns.

Option-A:
Correct Option by Primary Analysis itself.

Option-B:
Incorrect Option by Primary Analysis itself.

Option C:
$\mathbf{QQ}^\top$ is a 5×5 matrix.
Since $\text{rank}(\mathbf{Q}^\top) = \text{rank}(\mathbf{Q}) = 4$,
By the Gram matrix rank theorem, $\text{rank}(\mathbf{AA}^\top) = \text{rank}(\mathbf{A})$ for any matrix $\mathbf{A}$.
Applying this theorem,

$$\text{rank}(\mathbf{QQ}^\top) = \text{rank}(\mathbf{Q}) = 4 \tag{1}$$

Since $\mathbf{QQ}^\top$ is a 5×5 matrix with rank $4 < 5$, it is not full rank and therefore sdet $(\mathbf{QQ}^\top) = 0$. A square matrix is invertible if and only if it has full rank. Therefore, $\mathbf{QQ}^\top$ is NOT invertible.
Thus, Incorrect Option.

Theoretical Solution

Option D:
$\mathbf{Q}^\top \mathbf{Q}$ is a 6×6 matrix.
Since $\text{rank}(\mathbf{Q}^\top) = \text{rank}(\mathbf{Q}) = 4$,
By the Gram matrix rank theorem, $\text{rank}(\mathbf{A}^\top \mathbf{A}) = \text{rank}(\mathbf{A})$ for any matrix $\mathbf{A}$.
Applying this theorem,

$$\text{rank}(\mathbf{Q}^\top \mathbf{Q}) = \text{rank}(\mathbf{Q}) = 4 \tag{2}$$

Since $\mathbf{Q}^\top \mathbf{Q}$ is a 6×6 matrix with rank $4 < 6$, it is not full rank and therefore $\det\left(\mathbf{Q}^\top \mathbf{Q}\right) = 0$. A square matrix is invertible if and only if it has full rank. Therefore, $\mathbf{Q}^\top \mathbf{Q}$ is NOT invertible.
Thus, Incorrect Option.

$$\boxed{\text{Only Option-A is Correct.}}$$