Question 4.13.5

AI25BTECH11040 - Vivaan Parashar

September 30, 2025

1 Question:

The set of lines $ax+by+c=0$, where $3a+2b+4c=0$ are concurrent at the point __________.

2 Solution:

We are given the fact that $3a+2b+4c=0$. This can be written as:

$$\implies \frac{3a}{4}+\frac{b}{2}=-c \tag{1}$$

$$\implies \begin{pmatrix} \frac{3}{4} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = -c \tag{2}$$

Any arbitrary point on the line $ax+by+c=0$ satisfies the equation:

$$\mathbf{x}^{\mathrm{T}} \begin{pmatrix} a \\ b \end{pmatrix} = -c \tag{3}$$

From equations (2) and (3), we can clearly see that for all values of $(a,b,c) \in \mathbb{R}^3$, the point $\mathbf{P} = \begin{pmatrix} \frac{3}{4} \\ \frac{1}{2} \end{pmatrix}$ satisfies the equation of the line $ax+by+c=0$ (by substituting $\mathbf{x}=\mathbf{P}$ in equation 3). Thus, all lines of the form $ax+by+c=0$ where $3a+2b+4c=0$ are concurrent at the point $\begin{pmatrix} \frac{3}{4} \\ \frac{1}{2} \end{pmatrix}$.

3 Plot:

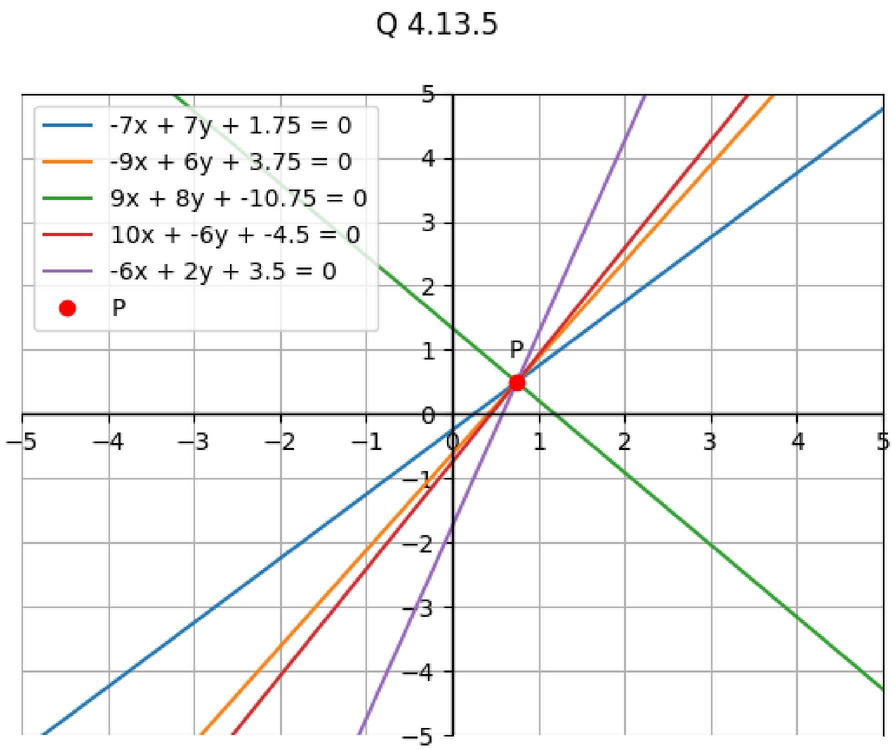

Figure 1: Graph of lines with randomly generated values of a and b satisfying $3a + 2b + 4c = 0$. All lines are concurrent at the point $\begin{pmatrix} \frac{3}{4} \\ \frac{1}{2} \end{pmatrix}$ (marked in red).