4.13.9

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Question

Let the algebraic sum of the perpendicular distances from the points (2,0),(0,2) and (1,1) to a variable straight line be zero; then the line passes through a fixed point whose coordinates are _____.

Formulae

The normal form of a line is:

$$\mathbf{n}^T x = c \tag{1}$$

The perpendicular distance of a point from a line is:

$$\frac{|\mathbf{n}^T x - c|}{||n||} \tag{2}$$

Solution

It is given that the algebraic sum of the perpendicular distances of three points (2,0), (0,2), and (1,1) to a line $\mathbf{n}^T x = c$ is 0. Therefore:

$$\frac{\mathbf{n}^T x_1 - c}{||n||} + \frac{\mathbf{n}^T x_2 - c}{||n||} + \frac{\mathbf{n}^T x_3 - c}{||n||} = 0 \tag{3}$$

Solution

Substituting the points:

$$\frac{\mathbf{n}^T \begin{pmatrix} 2 \\ 0 \end{pmatrix} - c}{||n||} + \frac{\mathbf{n}^T \begin{pmatrix} 0 \\ 2 \end{pmatrix} - c}{||n||} + \frac{\mathbf{n}^T \begin{pmatrix} 1 \\ 1 \end{pmatrix} - c}{||n||} = 0 \tag{4}$$

$$\frac{\mathbf{n}^T \begin{pmatrix} 3 \\ 3 \end{pmatrix} - 3c}{||n||} = 0 \tag{5}$$

$$\frac{\mathbf{n}^T \begin{pmatrix} 1 \\ 1 \end{pmatrix} - c}{||n||} = 0 \tag{6}$$

$$\mathbf{n}^T \begin{pmatrix} 1 \\ 1 \end{pmatrix} = c \tag{7}$$

Therefore the line passes through the fixed point (1,1).

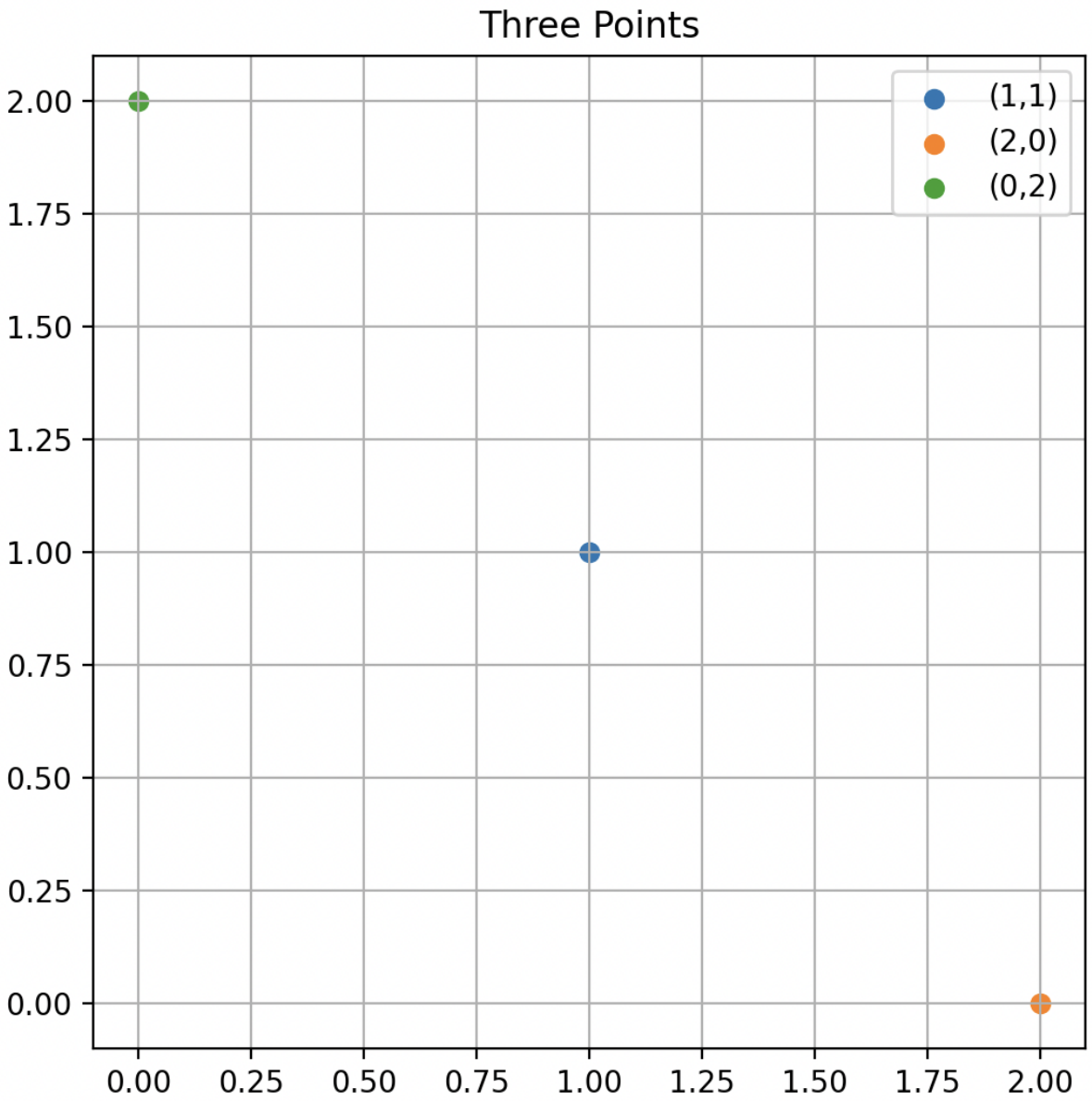
Python Code

```
import numpy as np
import matplotlib.pyplot as plt

fig = plt.figure(figsize = (6,6))
ax = fig.add_subplot(111)
ax.set_aspect('equal', adjustable='box')
ax.set_title(Three Points)

vector = str(input(Input the vectors, and input X when you are
    done))

counter = 0;
sum = np.zeros(2)
```

Python Code

```
while(vector != X):
    vector.strip()
    vector0, vector1 = vector.split( )
    vector0 = int(vector0)
    vector1= int(vector1)
    vectorA = np.array([vector0,vector1])
    sum += vectorA
    counter = counter + 1
    ax.scatter(vector0, vector1, label = f'({vector0}, {vector1})
        ')
    vector = str(input(Input the next vector))

commonpoint = sum/counter
print(fThe line passes through ( {commonpoint[0]}, {commonpoint
    [1]} ))
plt.legend()
ax.grid(True)
plt.show()
```

Python Code


Three Points
(1,1)
(2,0)
(0,2)
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00