Matrices in Geometry - 2.4.34

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Problem Statement

What type of quadrilateral do the points **A** $(2, -2)$, **B** $(7, 3)$, **C** $(11, -1)$ and **D** $(6, -6)$ taken in that order, form?

Solution

Given: $\mathbf{A}\begin{pmatrix}2\\-2\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}7\\3\end{pmatrix}$, $\mathbf{C}\begin{pmatrix}11\\-1\end{pmatrix}$ and $\mathbf{D}\begin{pmatrix}6\\-6\end{pmatrix}$.

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}5\\5\end{pmatrix} \tag{1}$$

$$\mathbf{C}-\mathbf{B}=\begin{pmatrix}4\\-4\end{pmatrix} \tag{2}$$

$$\mathbf{D}-\mathbf{C}=\begin{pmatrix}-5\\-5\end{pmatrix} \tag{3}$$

$$\mathbf{A}-\mathbf{D}=\begin{pmatrix}-4\\4\end{pmatrix} \tag{4}$$

Solution

Checking opposite sides, $\mathbf{D}-\mathbf{C}=-(\mathbf{A}-\mathbf{B})$ and $\mathbf{A}-\mathbf{D}=-(\mathbf{B}-\mathbf{C})$ (5)

Each pair of opposite sides are parallel and equal in length; this implies that the quadrilateral is a parallelogram.
Now, checking for right angle, we check for inner product.

$$(\mathbf{B}-\mathbf{A})^{\top}(\mathbf{C}-\mathbf{B})=\begin{pmatrix}5 & 5\end{pmatrix}\begin{pmatrix}4\\-4\end{pmatrix}=0 \tag{6}$$

Solution

This implies that the angle at **B** is one right angle. A parallelogram with a right angle is a rectangle.
Checking for a square:

$$\|\mathbf{B}-\mathbf{A}\| = 5\sqrt{2} \text{ and } \|\mathbf{C}-\mathbf{B}\| = 4\sqrt{2} \tag{7}$$

We can see that $\|\mathbf{B}-\mathbf{A}\| \neq \|\mathbf{C}-\mathbf{B}\|$, and therefore it is not a square.
Therefore, the quadrilateral **ABCD** is a rectangle.

Final Answer

The quadrilateral **ABCD** is a rectangle (8)