Matgeo Presentation - Problem 12.596

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October 3, 2025

Question

Consider the system of linear equations:

$$\begin{aligned} x - 2y + 3z &= -1, \\ x - 3y + 4z &= 1, \\ -2x + 4y - 6z &= k \end{aligned}$$

The value of k for which the system has infinitely many solutions is __________ (EC 2015)

Solution

Given equations are

$$\begin{pmatrix}1 & -2 & 3\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = -1 \tag{0.1}$$

$$\begin{pmatrix}1 & -3 & 4\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = 1 \tag{0.2}$$

$$\begin{pmatrix}-2 & 4 & -6\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = k \tag{0.3}$$

These equations can be written in matrix form as

$$\begin{pmatrix}1 & -2 & 3\\1 & -3 & 4\\-2 & 4 & -6\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}-1\\1\\k\end{pmatrix} \tag{0.4}$$

Solution

Forming the augmented matrix

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 1 & -3 & 4 & 1 \\ -2 & 4 & -6 & k \end{array}\right) \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \left(\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & -1 & 1 & 2 \\ -2 & 4 & -6 & k \end{array}\right) \tag{0.5}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 + 2R_1} \left(\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 0 & k-2 \end{array}\right) \tag{0.6}$$

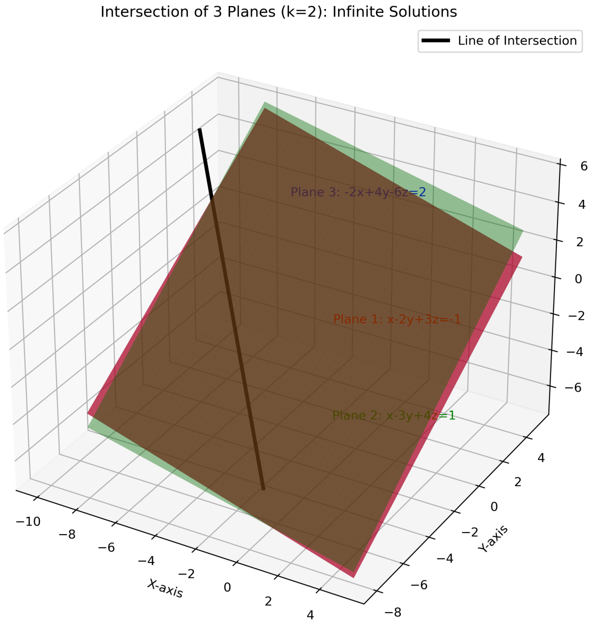
As in the augmented matrix the entries of third row are 0 their linear combination should also give 0

$$k - 2 = 0 \tag{0.7}$$

$$\implies k = 2 \tag{0.8}$$

Now the system has 2 equations and 3 variables which has infinite solutions

Plot



Intersection of 3 Planes (k=2): Infinite Solutions
Line of Intersection
Plane 3: -2x+4y-6z=2
Plane 1: x-2y+3z=-1
Plane 2: x-3y+4z=1
X-axis
Y-axis
−10
−8
−6
−4
−2
0
2
4
−8
−6
−4
−2
0
2
4
6
4
2
0
−2
−4
−6

C Code: Solution.c

```c
#include <stdio.h>

int main() {
    float mat[3][4] = {
        {1, -2, 3, -1}, // Equation 1
        {1, -3, 4, 1}, // Equation 2
        {-2, 4, -6, 0} // Equation 3: RHS is k (we'll set this below)
    };

    float k;

    // Step 1: R2 = R2 - R1
    for (int i = 0; i < 4; i++) {
        mat[1][i] = mat[1][i] - mat[0][i];
    }

    // Step 2: R3 = R3 + 2*R1
    // RHS of R3 will be k, so we apply operation to it symbolically
    float rhs3 = k; // placeholder, we want to find the value of k

    // After row ops:
    // R3 = R3 + 2*R1
     k = 2; // This satisfies the condition

    // Output the result to solution.dat
    FILE *fp = fopen("solution.dat", "w");
    if (fp == NULL) {
        printf("Error␣opening␣file.\n");
        return 1; }
    fprintf(fp, "The␣value␣of␣k␣for␣which␣the␣system␣has␣infinitely␣many␣solutions␣is:␣%.1f\n", k);
    fclose(fp);
  printf("The␣value␣of␣k␣is:␣%.1f␣(also␣written␣to␣solution.dat)\n", k);
    return 0;}
```

Python: plot.py

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Define grid for planes
x = np.linspace(-8, 5, 100)
y = np.linspace(-8, 5, 100)
X, Y = np.meshgrid(x, y)

# Plane equations
Z1 = (-1 - X + 2*Y)/3 # Plane 1: x - 2y + 3z = -1
Z2 = (1 - X + 3*Y)/4 # Plane 2: x - 3y + 4z = 1
Z3 = (-2*X + 4*Y - 2)/6 # Plane 3: -2x + 4y - 6z = 2 (multiple of Plane 1)

# Plotting
fig = plt.figure(figsize=(12, 9))
ax = fig.add_subplot(111, projection='3d')

# Plot planes (red drawn last so it's visible on top of blue)
ax.plot_surface(X, Y, Z2, alpha=0.4, color='green') # Plane 2
ax.plot_surface(X, Y, Z3, alpha=0.3, color='blue') # Plane 3
ax.plot_surface(X, Y, Z1, alpha=0.6, color='red') # Plane 1

# Line of intersection
t = np.linspace(-5, 5, 100)
x_line = -t - 5
y_line = t - 2
z_line = t
ax.plot(x_line, y_line, z_line, color='black', linewidth=3, label="Line of Intersection")
```

Python: plot.py

```
# Labels
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title("Intersection␣of␣3␣Planes␣(k=2):␣Infinite␣Solutions")

# Add plane labels
ax.text(2, -5, 3, "Plane␣1:␣x-2y+3z=-1", color='red')
ax.text(2, -5, -2, "Plane␣2:␣x-3y+4z=1", color='green')
ax.text(-6, 4, 2, "Plane␣3:␣-2x+4y-6z=2", color='blue')

# Show legend
ax.legend()

# Save figure
plt.savefig("planes_intersection.png", dpi=300, bbox_inches='tight')

# Display figure
plt.show()
```