2.3.8

EE25BTECH11005 - Aditya Mishra

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Question

If $\mathbf{A} = \hat{\imath} + \hat{\jmath} + \hat{k},\ \mathbf{B} = 2\hat{\imath} + 5\hat{\jmath},\ \mathbf{C} = 3\hat{\imath} + 2\hat{\jmath} - 3\hat{k},\ \mathbf{D} = \hat{\imath} - 6\hat{\jmath} - \hat{k}$ are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether $(\mathbf{B} - \mathbf{A})$ and $(\mathbf{D} - \mathbf{C})$ are collinear or not.

Solution

Let the direction vectors be

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} 1 \\ -6 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix} \tag{2}$$

The angle θ between $(\mathbf{B} - \mathbf{A})$ and $(\mathbf{D} - \mathbf{C})$ is given by

$$\cos\theta = \frac{(\mathbf{B} - \mathbf{A})^\top (\mathbf{D} - \mathbf{C})}{|(\mathbf{B} - \mathbf{A})|\,|(\mathbf{D} - \mathbf{C})|} \tag{3}$$

Substitute the values:

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{D} - \mathbf{C}) = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}^\top \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix} \tag{4}$$

$$= (1)(-2) + (4)(-8) + (-1)(2) \tag{5}$$

$$= -2 - 32 - 2 = -36 \tag{6}$$

Magnitudes:

$$|(\mathbf{B}-\mathbf{A})| = \sqrt{1^2+4^2+(-1)^2} = \sqrt{1+16+1} = \sqrt{18} \tag{7}$$

$$|(\mathbf{D}-\mathbf{C})| = \sqrt{(-2)^2+(-8)^2+2^2} = \sqrt{4+64+4} = \sqrt{72} \tag{8}$$

Thus,

$$\cos\theta = \frac{-36}{\sqrt{18}\sqrt{72}} = \frac{-36}{\sqrt{1296}} = \frac{-36}{36} = -1 \tag{9}$$

$$\theta = \cos^{-1}(-1) = \pi \text{ radians } = 180^\circ \tag{10}$$

So, $(\mathbf{B}-\mathbf{A})$ and $(\mathbf{D}-\mathbf{C})$ are collinear but point in opposite directions, i.e., they are anti-parallel.

The lines are collinear (anti-parallel).

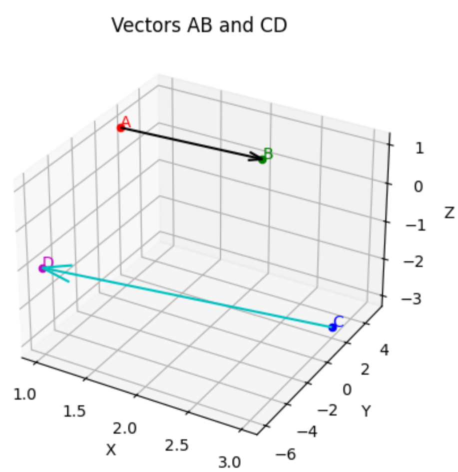

Figure 1: Line directions