2.10.12

AI25BTECH11023-Pratik R

Question:
A unit vector perpendicular to the plane determined by the points $P(1,-1,2)$,$Q(2,0,-1)$ and $R(0,2,1)$ is
Solution:
According to the question,
Given the position vectors,

$$\mathbf{P} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}; \mathbf{Q} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}; \mathbf{R} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \tag{0.1}$$

$$\mathbf{A} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} \tag{0.2}$$

$$\mathbf{B} = \mathbf{R} - \mathbf{P} = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix} \tag{0.3}$$

we need to find the unit vector which is perpendicular to the vectors **A** and **B**.The vector perpendicular to **A** and **B** is given by their cross-product.
Let the perpendicular vector be $\mathbf{X}^T = \begin{pmatrix} X_1 & X_2 & X_3 \end{pmatrix}$

$$\because \mathbf{A}^T\mathbf{X} = 0 \tag{0.4}$$

$$\mathbf{B}^T\mathbf{X} = 0\ , \tag{0.5}$$

$$\therefore \begin{pmatrix} \mathbf{A}^T \\ \mathbf{B}^T \end{pmatrix} \mathbf{X} = 0 \tag{0.6}$$

$$\begin{pmatrix} 1 & 1 & -3 \\ -1 & 3 & -1 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = 0 \tag{0.7}$$

This can be represented as,

$$\begin{pmatrix} 1 & 1 & -3 \\ -1 & 3 & -1 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 + R_1} \begin{pmatrix} 1 & 1 & -3 \\ 0 & 4 & -4 \end{pmatrix} \tag{0.8}$$

yielding,

$$x_1 + x_2 - 3x_3 = 0 \tag{0.9}$$
$$4x_2 - 4x_3 = 0 \tag{0.10}$$
$$\implies x_2 = x_3 \tag{0.11}$$
$$x_1 = 2x_3 \tag{0.12}$$

$$\mathbf{x} = x_3 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \tag{0.13}$$

The unit vector perpendicular to the plane is given by

$$\mathbf{x} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \tag{0.14}$$