

Question

Find the length of the median of the triangle with vertices **A**(0,0,6),**B**(0,4,0) and **C**(6,0,0).

Step 1: Midpoints of Opposite Sides

$$M_{BC} = \frac{1}{2}(\mathbf{B} + \mathbf{C}) = \frac{1}{2} \begin{pmatrix} 0+6 \\ 4+0 \\ 0+0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} \quad (1)$$

$$M_{AC} = \frac{1}{2}(\mathbf{A} + \mathbf{C}) = \frac{1}{2} \begin{pmatrix} 0+6 \\ 0+0 \\ 6+0 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} \quad (2)$$

$$M_{AB} = \frac{1}{2}(\mathbf{A} + \mathbf{B}) = \frac{1}{2} \begin{pmatrix} 0+0 \\ 0+4 \\ 6+0 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} \quad (3)$$

Step 2: Vectors Representing Medians

$$\mathbf{AM} = M_{BC} - \mathbf{A} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ -6 \end{pmatrix} \quad (4)$$

$$\mathbf{BM} = M_{AC} - \mathbf{B} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ 3 \end{pmatrix} \quad (5)$$

$$\mathbf{CM} = M_{AB} - \mathbf{C} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \\ 3 \end{pmatrix} \quad (6)$$

Step 3: Lengths of Medians

Using the Euclidean norm:

$$\|\mathbf{v}\| = \sqrt{x^2 + y^2} \quad (7)$$

$$\|\mathbf{AM}\| = \sqrt{3^2 + 2^2 + (-6)^2} = \sqrt{9 + 4 + 36} = \sqrt{49} = 7 \quad (8)$$

$$\|\mathbf{BM}\| = \sqrt{3^2 + (-4)^2 + 3^2} = \sqrt{9 + 16 + 9} = \sqrt{34} \quad (9)$$

$$\|\mathbf{CM}\| = \sqrt{(-6)^2 + 2^2 + 3^2} = \sqrt{36 + 4 + 9} = \sqrt{49} = 7 \quad (10)$$

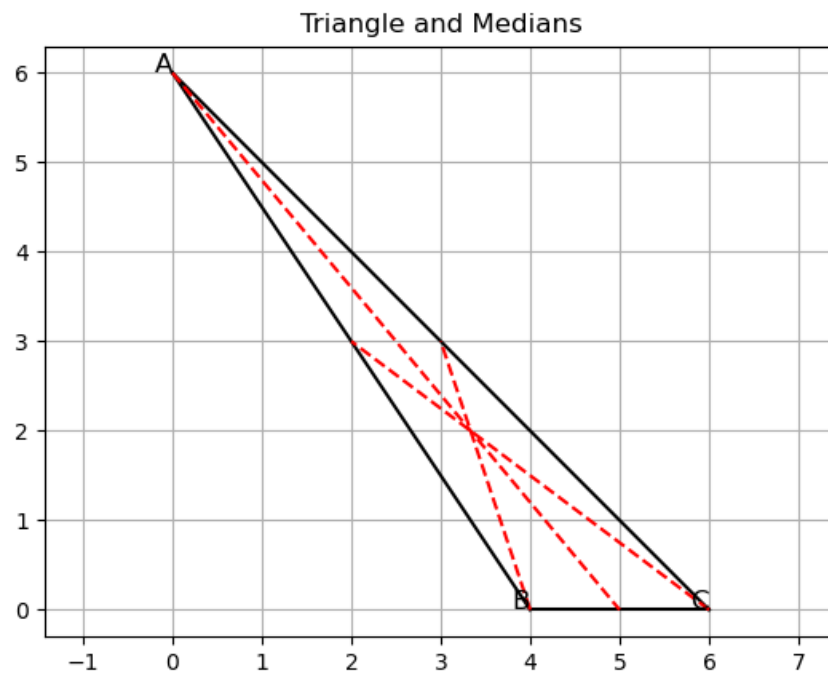


Figure 1

Final Answer

$$\|\mathbf{AM}\| = \sqrt{61}, \quad \|\mathbf{BM}\| = \sqrt{10}, \quad \|\mathbf{CM}\| = 5$$

(11)