4.12.19

AI25BTECH11014 - Gooty Suhas

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Question

The point $\mathbf{P} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ undergoes:

1. Reflection about the line $y = x$
2. Translation 2 units along the positive x-axis

Find the final coordinates of the point.

Options:

- $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$

- $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

- $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

- $\begin{pmatrix} 3.5 \\ 3.5 \end{pmatrix}$

Original point:

$$\mathbf{P} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

Reflection about $y = x$:

$$\mathbf{R} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

Let final point be:

$$\mathbf{Q} = \begin{pmatrix} x \\ 4 \end{pmatrix}$$

Collinearity via Rank

Vectors:
$$\mathbf{R} - \mathbf{P} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}, \quad \mathbf{Q} - \mathbf{R} = \begin{pmatrix} x-1 \\ 0 \end{pmatrix}$$

Matrix:
$$\mathbf{M} = \begin{pmatrix} -3 & x-1 \\ 3 & 0 \end{pmatrix}$$

Row operations:
$$R_1 \leftarrow R_1 + R_2 = \begin{pmatrix} 0 & x-1 \end{pmatrix}$$

Echelon form:
$$\begin{pmatrix} 3 & 0 \\ 0 & x-1 \end{pmatrix} \Rightarrow x - 1 = 0 \Rightarrow x = 1$$

Translation vector:

$$\mathbf{T} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

Final point:

$$\mathbf{F} = \mathbf{Q} + \mathbf{T} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\boxed{\text{Final coordinates: } \begin{pmatrix} 3 \\ 4 \end{pmatrix}} \quad \Rightarrow \quad \boxed{\text{Option (b)}}$$

Python Code — SymPy

```python
from sympy import Matrix

# Original point
P = Matrix([4, 1])

# Reflection about y = x
R = Matrix([[0, 1], [1, 0]]) * P

# Translation vector
T = Matrix([2, 0])
F = R + T

print("Reflected point:", R)
print("Final point:", F)
```

C Code — Matrix Only (1/2)

```c
#include <stdio.h>

int main() {
    // Original point
    double P[2] = {4, 1};

    // Reflection about y = x
    double R[2];
    R[0] = P[1];
    R[1] = P[0];

    // Translation vector
    double T[2] = {2, 0};
    double F[2];
```

```c
    // Apply translation
    F[0] = R[0] + T[0];
    F[1] = R[1] + T[1];

    // Output results
    printf("Reflected point: (%.1f, %.1f)\n", R[0], R[1]);
    printf("Final point: (%.1f, %.1f)\n", F[0], F[1]);

    return 0;
}
```

Python Code — With .so or Executable

```python
import subprocess

# Input for C program
input_str = "4 1"

result = subprocess.run(
    ['./reflect_translate'], # compiled C binary
    input=input_str,
    capture_output=True,
    text=True
)

print(result.stdout.strip())
```

Plot

Transformations in 3D

Legend:
- A (4,1) (red)
- B (1,4) (blue)
- C (3,4) (green)
- Line y = x (dashed)