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Matgeo-q.5.13.12

AI25BTECH11036-SNEHAMRUDULA

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question

Find the set of all values of λ for which the system

$$\begin{aligned} 2x_1 - 2x_2 + x_3 &= \lambda x_1, \\ 2x_1 - 3x_2 + 2x_3 &= \lambda x_2, \\ -x_1 + 2x_2 &= \lambda x_3 \end{aligned}$$

has a non-trivial solution. Which of the following is true?

A)contains two elements

B) contains more than two elements

C)is an empty set

D)is a singlet

solution

Bring all terms to one side:

$$(2-\lambda)x_1 - 2x_2 + x_3 = 0$$
$$2x_1 + (-3-\lambda)x_2 + 2x_3 = 0$$
$$-x_1 + 2x_2 - \lambda x_3 = 0$$

$$\begin{pmatrix} 2-\lambda & -2 & 1 \\ 2 & -3-\lambda & 2 \\ -1 & 2 & -\lambda \end{pmatrix} \mathbf{x} = \mathbf{0} \quad \text{where} \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad \mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

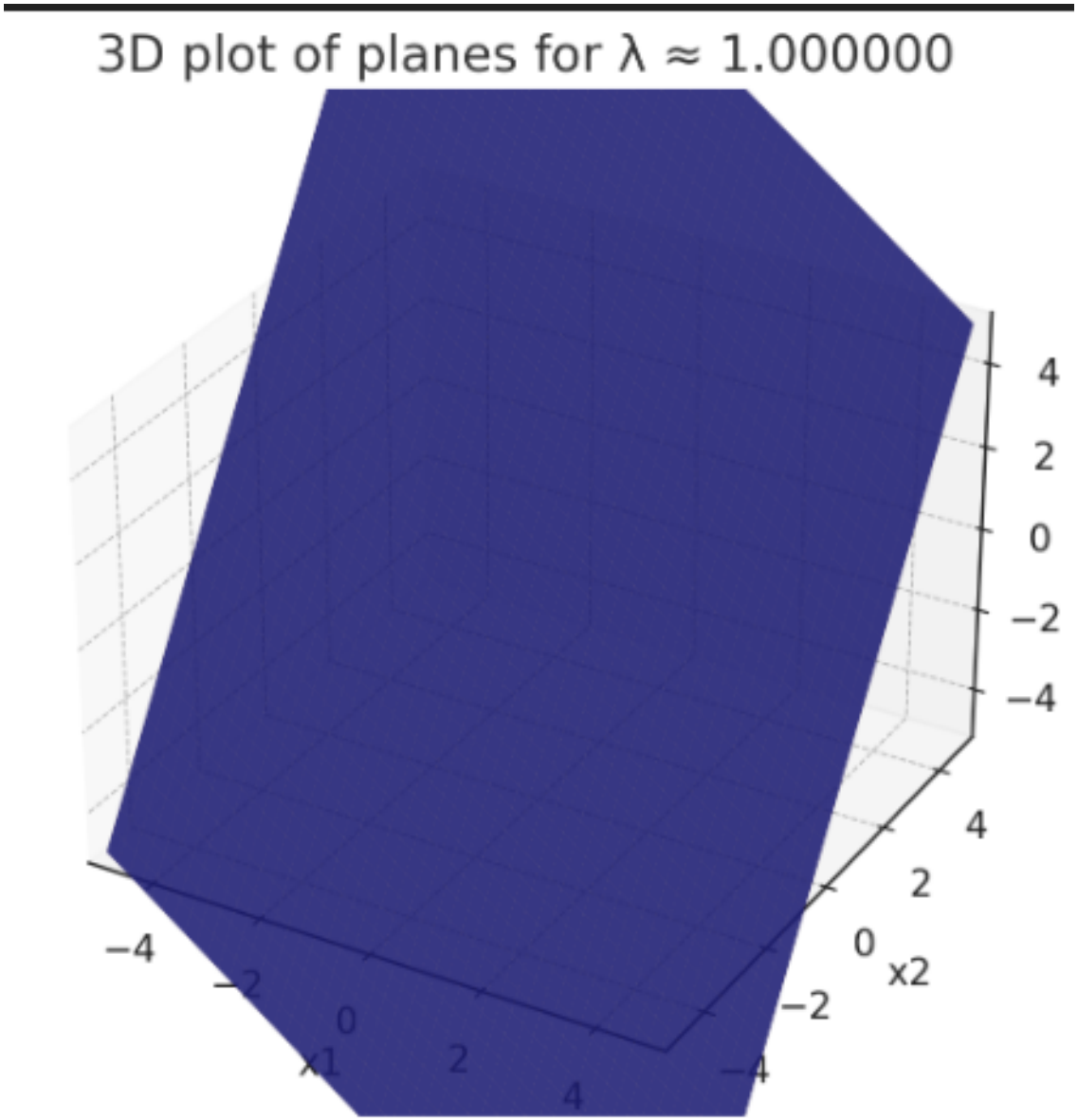
$$\text{rank}(\text{Coefficient Matrix}) < 3$$

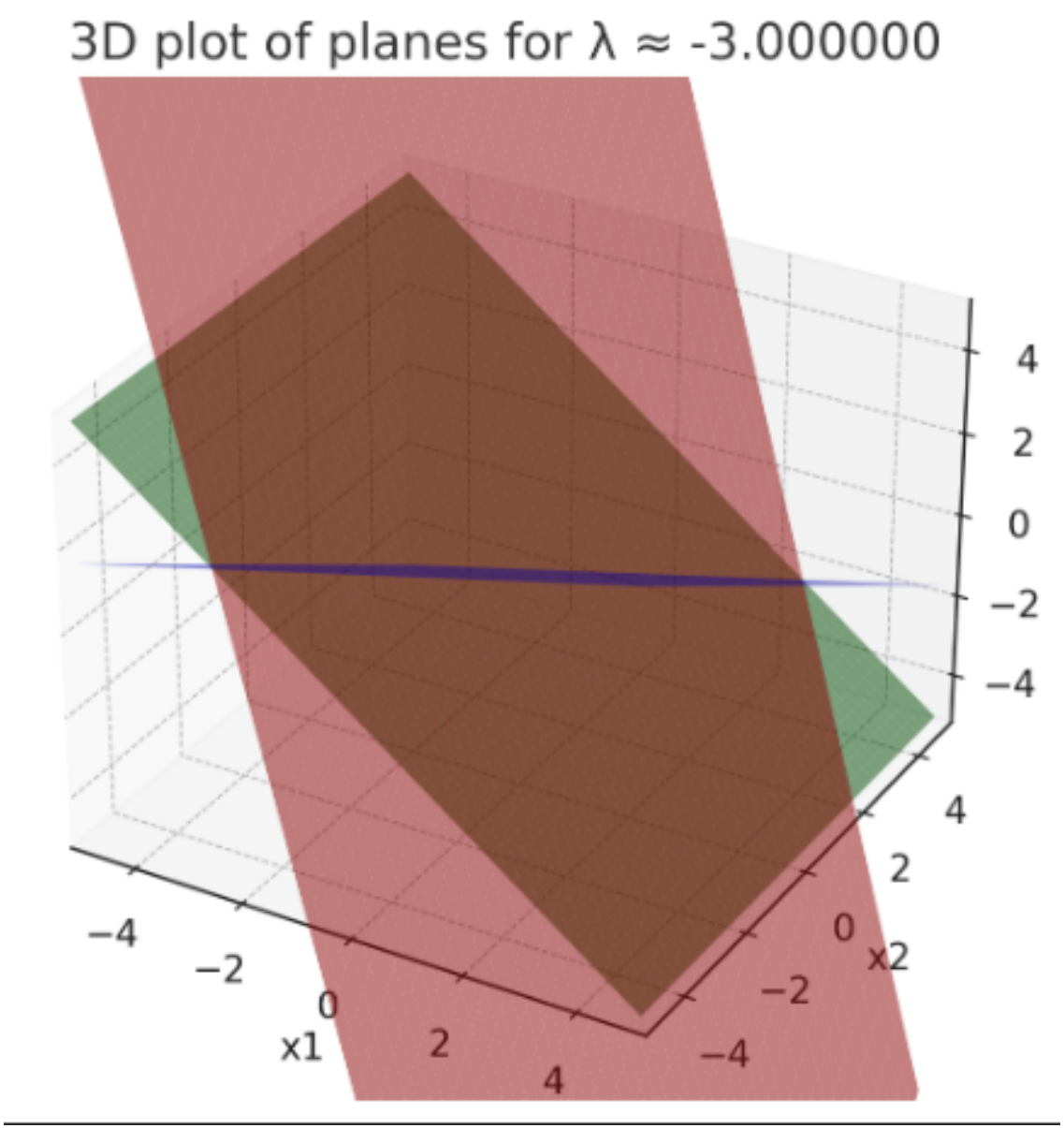
$$\text{rank}\left(\begin{pmatrix} 2-\lambda & -2 & 1 \\ 2 & -3-\lambda & 2 \\ -1 & 2 & -\lambda \end{pmatrix}\right) = 2$$

So, the system has a non-trivial solution for exactly two values of λ.

Correct answer: (a) contains two elements

Graphical Representation




3D plot of planes for λ ≈ -3.000000
4
2
0
−2
−4
−4
−2
0
2
4
x1
4
2
0
−2
−4
x2