# 4.8.20

AI25BTECH11024 - Pratyush Panda

**Question:**

Find the distance between the point $(2, 3, 4)$ measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$

**Solution:**

Let the vector $\mathbf{A}$ be $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$, and the direction vector of the line $\mathbf{b} = \begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix}$.

The equation of the plane can be written as;

$$\mathbf{n}^T\mathbf{X} = 1 \qquad where,\ \mathbf{n} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}\ and\,\mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{0.1}$$

The equation of the line passing through $\mathbf{A}$ and with the direction vector $\mathbf{b}$ is;

$$\mathbf{x} = \mathbf{A} + \lambda\mathbf{b} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix} \tag{0.2}$$

The point on the plane lying on this line can be found out by substituting the parametric point in the equation of the plane and find out the value of $\lambda$.

$$3(2 + 3\lambda) + 2(3 + 6\lambda) + 2(4 + 2\lambda) + 5 = 0 \tag{0.3}$$

$$\lambda = \frac{-25}{25} \tag{0.4}$$

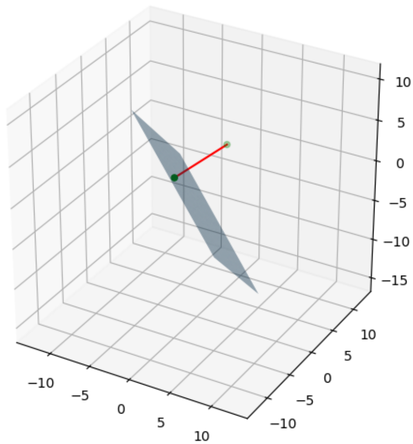
$$or,\ \lambda = -1 \tag{0.5}$$

After solving for $\lambda$ we got $\lambda = -1$. Thus, the point is $\mathbf{B}$ would be $\begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix}$.

Thus, the final distance along the line can be written as;

$$d = \mathbf{A}^T.\mathbf{B} = 7 \tag{0.6}$$

Thus, the distance between the point $(2, 3, 4)$ measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$ is 7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