10.7.76

INDHIRESH S- EE25BTECH11027

Question.The number of common tangents to the circles $x^2+y^2 = 4$ and $x^2+y^2-6x-8y = 24$ is

1) 0
2) 1
3) 2
4) 3

Solution:
Let us solve the given equation theoretically and then verify the solution computationally.
Let the equation of 1st circle be:

$$\|\mathbf{x}\|^2 + 2\mathbf{u_1}^T\mathbf{x} + f_1 = 0 \tag{1}$$

Let the equation of 2nd circle be

$$\|\mathbf{x}\|^2 + 2\mathbf{u_2}^T\mathbf{x} + f_2 = 0 \tag{2}$$

From the given information:

$$\mathbf{u_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad and \quad f_1 = -4 \tag{3}$$

$$\mathbf{u_2} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \quad and \quad f_2 = -24 \tag{4}$$

The intersection of two curves can be given as :

$$\mathbf{x}^T(\mathbf{V_1} + \mu\mathbf{V_2})\mathbf{x} + 2(\mathbf{u_1} + \mu\mathbf{u_2})^T\mathbf{x} + (f_1 + \mu f_2) = 0 \tag{5}$$

Given conic is a circle. So,

$$\mathbf{V_1} = \mathbf{V_2} = \mathbf{I} \tag{6}$$

Now subdtituitng the given values:

$$(\mu + 1)\mathbf{x}^T\mathbf{x} + 2\mu\begin{pmatrix} -3 \\ -4 \end{pmatrix}^T \mathbf{x} + (-4 - 24\mu) = 0 \tag{7}$$

$$(\mu + 1)\|\mathbf{x}\|^2 - 2\mu\begin{pmatrix} 3 \\ 4 \end{pmatrix}^T \mathbf{x} - 4(1 + 6\mu) = 0 \tag{8}$$

$\mathbf{x}$ lies on the circle 1. So,

$$\|\mathbf{x}\|^2 = 4 \tag{9}$$

$$4(\mu+1) - 2\mu \begin{pmatrix} 3 \\ 4 \end{pmatrix}^T \mathbf{x} - 4(1+6\mu) = 0 \tag{10}$$

$$4\mu - 2\mu \begin{pmatrix} 3 \\ 4 \end{pmatrix}^T \mathbf{x} - 24\mu = 0; \tag{11}$$

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix}^T \mathbf{x} = -10 \tag{12}$$

Which is the equation of a single line
So the number of common tangents is 1

From the figure it is clearly verified that the theoretical solution matches with the computational solution.