4.13.45

EE25BTECH11041 - Naman Kumar

Question b):
Find the equation of the line which bisects the obtuse angle between the lines $x-2y+4=0$ and $4x-3y+2=0$.
Solution:
Given,

$\mathbf{n_1}$	Normal vector of line 1
$\mathbf{n_2}$	Normal vector of line 2
c_1	constant of line 1
c_2	constant of line 2
$\mathbf{B}$	vector on bisector
$\mathbf{n_{B_1}}$	Normal vector Bisector of line 1 and 2
$\mathbf{n_{B_2}}$	Normal vector Bisector of line 1 and 2
θ_1	angle between line 1 and bisector 1
θ_2	angle between line 1 and bisector 2

TABLE I

$$\mathbf{n_1}^T\mathbf{x} = c_1, \mathbf{n_2}^T\mathbf{x} \tag{1}$$

Where,

$$\mathbf{n_1} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \mathbf{n_2} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}, c_1 = -4 \text{ and } c_2 = -2 \tag{2}$$

Equation for bisectors is

$$\left|\frac{\mathbf{n_1}^T\mathbf{B} - c_1}{\| \mathbf{n_1} \|}\right| = \left|\frac{\mathbf{n_2}^T\mathbf{B} - c_2}{\| \mathbf{n_2} \|}\right| \tag{3}$$

$$\frac{\mathbf{n_1}^T\mathbf{B} - c_1}{\| \mathbf{n_1} \|} = \pm\frac{\mathbf{n_2}^T\mathbf{B} - c_2}{\| \mathbf{n_2} \|} \tag{4}$$

$$\frac{\mathbf{n_1}^T\mathbf{B_1} - c_1}{\| \mathbf{n_1} \|} = \frac{\mathbf{n_2}^T\mathbf{B_1} - c_2}{\| \mathbf{n_2} \|}, \text{ and } \frac{\mathbf{n_1}^T\mathbf{B_2} - c_1}{\| \mathbf{n_1} \|} = -\frac{\mathbf{n_2}^T\mathbf{B_2} - c_2}{\| \mathbf{n_2} \|} \tag{5}$$

Can be written as

$$\left(\frac{\mathbf{n_1}^T}{\| \mathbf{n_1} \|} - \frac{\mathbf{n_2}^T}{\| \mathbf{n_2} \|}\right)\mathbf{B_1} = \left(\frac{c_1}{\| \mathbf{n_1} \|} - \frac{c_2}{\| \mathbf{n_2} \|}\right) \tag{6}$$

$$\mathbf{n_{B_1}}^T B_1 = c_{B_1} \tag{7}$$

and

$$\left(\frac{\mathbf{n_1}^T}{\| \mathbf{n_1} \|} + \frac{\mathbf{n_2}^T}{\| \mathbf{n_2} \|}\right)\mathbf{B_2} = \left(\frac{c_2}{\| \mathbf{n_2} \|} + \frac{c_1}{\| \mathbf{n_1} \|}\right)b2 \tag{8}$$

$$\mathbf{n_{B_2}}^T B_2 = c_{B_2} \tag{9}$$

Now for obtuse angle bisector

$$\cos\theta_1 = \frac{\mathbf{n_{B_1}}^T \mathbf{n_1}}{\| \mathbf{n_1} \|\| \mathbf{n_{B_1}} \|} \tag{10}$$

$$\cos\theta_2 = \frac{\mathbf{n_{B_2}}^T \mathbf{n_1}}{\| \mathbf{n_1} \|\| \mathbf{n_{B_2}} \|} \tag{11}$$

Solving with (7) and (9)

$$\cos\theta_1 = \frac{\left(\frac{\mathbf{n_1}^T}{\|\mathbf{n_1}\|} - \frac{\mathbf{n_2}^T}{\|\mathbf{n_2}\|}\right)\mathbf{n_1}}{\| \mathbf{n_1} \|\| \mathbf{n_{B_1}} \|} \tag{12}$$

$$\cos\theta_1 = \frac{\left(\frac{\mathbf{n_1}^T}{\|\mathbf{n_1}\|}\mathbf{n_1} - \frac{\mathbf{n_2}^T}{\|\mathbf{n_2}\|}\mathbf{n_1}\right)}{\| \mathbf{n_1} \|\| \mathbf{n_{B_1}} \|} \tag{13}$$

$$\cos\theta_1 = \frac{\left(\sqrt{5} - \frac{10}{5}\right)}{\| \mathbf{n_1} \|\| \mathbf{n_{B_1}} \|} \tag{14}$$

$$\cos\theta_1 = \frac{\sqrt{5} - 2}{\sqrt{5}\sqrt{\left(\frac{1}{\sqrt{5}} - \frac{4}{5}\right)^2 + \left(\frac{3}{5} - \frac{2}{\sqrt{5}}\right)^2}} \approx 0.22 \tag{15}$$

Similarly

$$\cos\theta_2 = \frac{\sqrt{5} + 2}{\sqrt{5}\sqrt{\left(\frac{1}{\sqrt{5}} + \frac{4}{5}\right)^2 + \left(\frac{3}{5} + \frac{2}{\sqrt{5}}\right)^2}} \approx 0.97 \tag{16}$$

by comparing (15) and (16)

$$\theta_1 > \theta_2 \tag{17}$$

So B_1 is obtuse angle bisector

$$\mathbf{n_{B_1}}^T B_1 = c_{B_1} \tag{18}$$

$$\mathbf{n_{B_1}} = \begin{pmatrix} \frac{1}{\sqrt{5}} - \frac{4}{5} \\ \frac{3}{5} - \frac{2}{\sqrt{5}} \end{pmatrix} \tag{19}$$

and

$$c_{B_1} = \frac{2}{5} + \frac{-4}{\sqrt{5}} \tag{20}$$

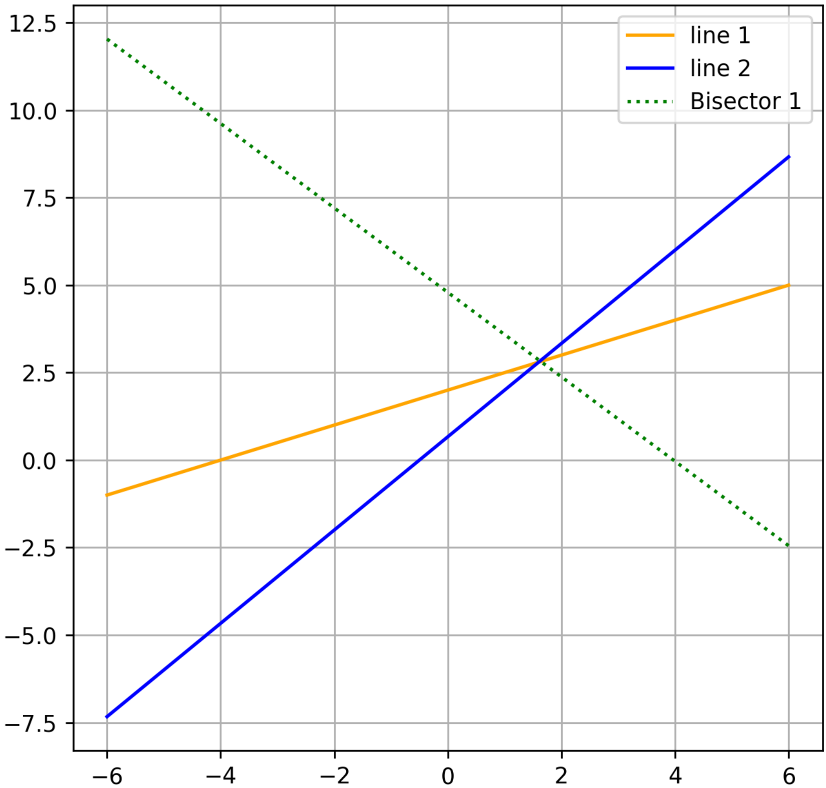

Fig. 1