

4.13.28

EE25BTECH11023 - Venkata Sai

Question:

Slope of a line passing through **P**(2,3) and intersecting the line $x + y = 7$ at a distance of 4 units from **P**, is

Solution: Given

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (1)$$

Equation of a line through **P** and having slope m is

$$(-m \quad 1) \begin{pmatrix} x-2 \\ y-3 \end{pmatrix} = 0 \quad (2)$$

$$(-m \quad 1) \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right) = 0 \implies (-m \quad 1) \begin{pmatrix} x \\ y \end{pmatrix} = (-m \quad 1) \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (3)$$

$$(-m \quad 1) \begin{pmatrix} x \\ y \end{pmatrix} = 3 - 2m \quad (4)$$

$$x + y = 7 \implies (1 \quad 1) \begin{pmatrix} x \\ y \end{pmatrix} = 7 \quad (5)$$

$$\begin{pmatrix} -m & 1 & 3-2m \\ 1 & 1 & 7 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 1 & 7 \\ -m & 1 & 3-2m \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 + mR_1} \begin{pmatrix} 1 & 1 & 7 \\ 0 & 1+m & 3+5m \end{pmatrix} \quad (6)$$

$$y = \frac{3+5m}{1+m} \quad (7)$$

$$x + y = 7 \implies x = 7 - y \implies x = 7 - \frac{3+5m}{1+m} \quad (8)$$

$$x = \frac{7+7m-3-5m}{1+m} = \frac{4+2m}{1+m} \quad (9)$$

Given the point is at a distance of 4 units from point **P**

$$\left\| \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\| = 4 \implies \left\| \begin{pmatrix} \frac{4+2m}{1+m} - 2 \\ \frac{3+5m}{1+m} - 3 \end{pmatrix} \right\| = 4 \quad (10)$$

$$\left\| \begin{pmatrix} \frac{4+2m-2-2m}{1+m} \\ \frac{3+5m-3-3m}{1+m} \end{pmatrix} \right\| = \left\| \begin{pmatrix} \frac{2}{1+m} \\ \frac{2m}{1+m} \end{pmatrix} \right\| = 4 \quad (11)$$

$$\sqrt{\left(\frac{2}{1+m} \right)^2 + \left(\frac{2m}{1+m} \right)^2} = 4 \quad (12)$$

$$\frac{4 + 4m^2}{(1 + m)^2} = 4^2 = 16 \quad (13)$$

$$4(1 + m^2) = 16(1 + m^2 + 2m) \implies (1 + m^2) = 4(1 + m^2 + 2m) \quad (14)$$

$$4 + 4m^2 + 8m = 1 + m^2 \implies 3m^2 + 8m + 3 = 0 \quad (15)$$

$$m^2 + \frac{8m}{3} + 1 = 0 \quad (16)$$

$$m^2 + \frac{8m}{3} + 1 + \left(\frac{4}{3}\right)^2 = \left(\frac{4}{3}\right)^2 \quad (17)$$

$$\left(m + \frac{4}{3}\right)^2 = \frac{16}{9} - 1 = \frac{7}{9} \quad (18)$$

$$m + \frac{4}{3} = \pm \frac{\sqrt{7}}{3} \quad (19)$$

$$m = \frac{-4 + \sqrt{7}}{3} \text{ or } \frac{-4 - \sqrt{7}}{3} \quad (20)$$

According to options

$$\frac{-4 + \sqrt{7}}{3} = \frac{-8 + 2\sqrt{7}}{6} = \frac{1 - \sqrt{7}}{1 + \sqrt{7}} \quad (21)$$

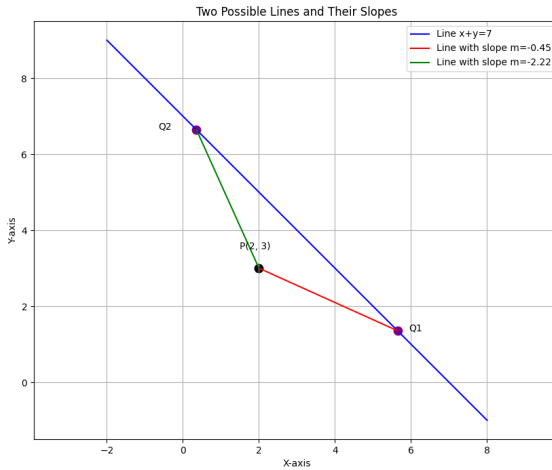


Fig. 0.1