4.4.27

AI25BTECH110031
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Question(4.4.27) Find the value of x such that the points $A(3,2,1)$, $B(4,x,5)$, $C(4,2,-2)$ and $D(6,5,-1)$ are coplanar.

Solution:

Let the plane (not passing through the origin) be given by

$$\mathbf{n}^\top \mathbf{x} = 1, \qquad \mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \tag{0.1}$$

Since the points

$$\mathbf{A} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 6 \\ 5 \\ -1 \end{pmatrix} \tag{0.2}$$

lie on the plane, they satisfy

$$\mathbf{n}^\top \mathbf{A} = 1 \tag{0.3}$$

$$\mathbf{n}^\top \mathbf{B} = 1 \tag{0.4}$$

$$\mathbf{n}^\top \mathbf{C} = 1 \tag{0.5}$$

$$\begin{pmatrix} 3 & 2 & 1 \\ 4 & 2 & -2 \\ 6 & 5 & -1 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{0.6}$$

Thus

$$n_1 = \frac{9}{16}, \qquad n_2 = -\frac{7}{16}, \qquad n_3 = \frac{3}{16}. \tag{0.7}$$

Now require B to lie on the same plane:

$$\mathbf{n}^\top \mathbf{B} = 1 \tag{0.8}$$

$$\begin{pmatrix} \frac{9}{16} & \frac{-7}{16} & \frac{3}{16} \end{pmatrix} \begin{pmatrix} 4 \\ x \\ 5 \end{pmatrix} = 1 \tag{0.9}$$

$$\frac{36}{16} - \frac{7}{16}x + \frac{15}{16} = 1 \tag{0.10}$$

$$51 - 7x = 16 \tag{0.11}$$

$$\boxed{x = 5} \tag{0.12}$$

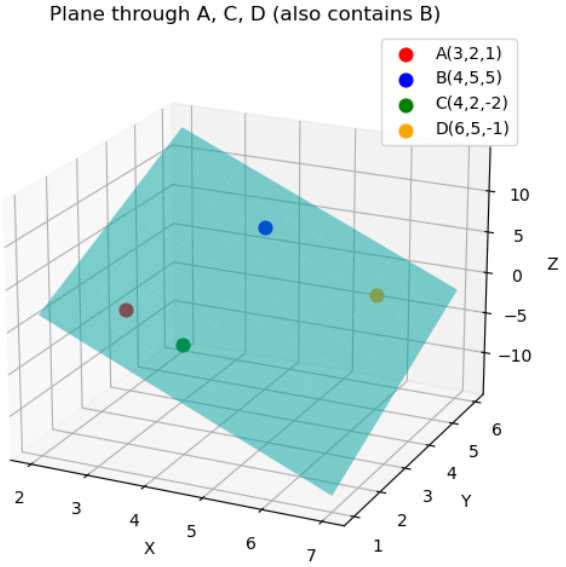

Fig. 0.1