1.8.27

EE25BTECH11010 - Arsh Dhoke

Question:
Find the equation of set of points $\mathbf{P}$ such that $||\mathbf{A} - \mathbf{P}||^2 + ||\mathbf{B} - \mathbf{P}||^2 = 2k^2$, where $\mathbf{A}$ and $\mathbf{B}$ are the points (3,4,5) and (-1,3,-7), respectively.
Solution:

The input parameters for the problem are given in the table below.

Vectors	**Points**
$\mathbf{A}$	$\begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}$
$\mathbf{B}$	$\begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix}$

TABLE 0: Vectors and their corresponding points

Let $\mathbf{P} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

The condition is:

$$||\mathbf{A} - \mathbf{P}||^2 + ||\mathbf{B} - \mathbf{P}||^2 = 2k^2 \tag{0.1}$$

$$(\mathbf{P} - \mathbf{A})^T(\mathbf{P} - \mathbf{A}) + (\mathbf{P} - \mathbf{B})^T(\mathbf{P} - \mathbf{B}) = 2k^2 \tag{0.2}$$

$$\begin{pmatrix} x-3 & y-4 & z-5 \end{pmatrix}\begin{pmatrix} x-3 \\ y-4 \\ z-5 \end{pmatrix} + \begin{pmatrix} x+1 & y-3 & z+7 \end{pmatrix}\begin{pmatrix} x+1 \\ y-3 \\ z+7 \end{pmatrix} = 2k^2 \tag{0.3}$$

$$(x-3)^2 + (y-4)^2 + (z-5)^2 + (x+1)^2 + (y-3)^2 + (z+7)^2 = 2k^2 \tag{0.4}$$

$$2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 = 2k^2 \tag{0.5}$$

$$(x-1)^2 + \left(y - \frac{7}{2}\right)^2 + (z+1)^2 = k^2 - \frac{161}{4} \tag{0.6}$$

$$k^2 > \frac{161}{4} \tag{0.7}$$

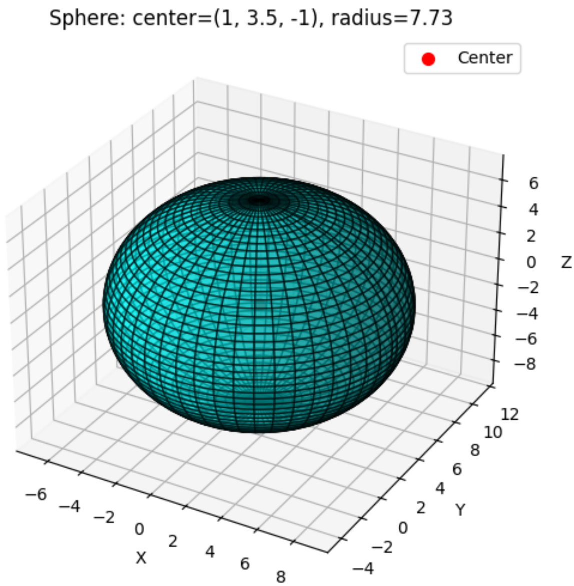

Fig. 0.1: Graph plotted by taking k=10 as example.