4.9.5

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Question

Find the equations of the lines that pass through the point $(3, 2)$ and make an angle of 40° with the line $x - 2y = 3$.

Theoretical Solution

First, we express the given point and line using column vectors.

The line passes through the point $(3, 2)$. The position vector $\mathbf{h}$ is:

$$\mathbf{h} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

The given line is $x - 2y = 3$. From the formula $\mathbf{n}^\top \mathbf{x} = c$, we can identify the **normal vector** to this line, $\mathbf{n_1}$:

$$\mathbf{n_1} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

The **direction vector** of the line, $\mathbf{m_1}$, is orthogonal to its normal vector ($\mathbf{m_1}^\top \mathbf{n_1} = 0$). A simple choice is:

$$\mathbf{m_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Theoretical Solution

We find the direction vectors, $\mathbf{m_2}$ and $\mathbf{m_3}$, for the new lines by rotating $\mathbf{m_1}$ by $+40^\circ$ and -40°. The rotation matrix $R(\theta)$ is:

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Rotation by +40°:

$$\mathbf{m_2} = R(40^\circ)\mathbf{m_1} = \begin{pmatrix} 2\cos(40^\circ) - \sin(40^\circ) \\ 2\sin(40^\circ) + \cos(40^\circ) \end{pmatrix}$$

Rotation by -40°:

$$\mathbf{m_3} = R(-40^\circ)\mathbf{m_1} = \begin{pmatrix} 2\cos(40^\circ) + \sin(40^\circ) \\ -2\sin(40^\circ) + \cos(40^\circ) \end{pmatrix}$$

Theoretical Solution

To find the equation in normal form $\mathbf{n}^\top \mathbf{x} = c$, we need the normal vector $\mathbf{n_2}$ and the constant $c_2 = \mathbf{n_2}^\top \mathbf{h}$.

For a direction vector $\mathbf{m} = \begin{pmatrix} u \\ v \end{pmatrix}$, a normal vector is $\mathbf{n} = \begin{pmatrix} -v \\ u \end{pmatrix}$.

Normal Vector $\mathbf{n_2}$:

$$\mathbf{n_2} = \begin{pmatrix} -(2\sin(40^\circ) + \cos(40^\circ)) \\ 2\cos(40^\circ) - \sin(40^\circ) \end{pmatrix}$$

Constant c_2:

$$\begin{aligned} c_2 &= \mathbf{n_2}^\top \mathbf{h} \\ &= -3(2\sin(40^\circ) + \cos(40^\circ)) + 2(2\cos(40^\circ) - \sin(40^\circ)) \\ &= \cos(40^\circ) - 8\sin(40^\circ) \end{aligned}$$

Equation:
$-(2\sin(40^\circ) + \cos(40^\circ))x + (2\cos(40^\circ) - \sin(40^\circ))y = \cos(40^\circ) - 8\sin(40^\circ)$

Theoretical Solution

Similarly, we find the normal vector $\mathbf{n_3}$ and constant $c_3 = \mathbf{n_3}^\top \mathbf{h}$.

Normal Vector $\mathbf{n_3}$:

$$\mathbf{n_3} = \begin{pmatrix} 2\sin(40^\circ) - \cos(40^\circ) \\ 2\cos(40^\circ) + \sin(40^\circ) \end{pmatrix}$$

Constant c_3:

$$\begin{aligned} c_3 &= \mathbf{n_3}^\top \mathbf{h} \\ &= 3(2\sin(40^\circ) - \cos(40^\circ)) + 2(2\cos(40^\circ) + \sin(40^\circ)) \\ &= \cos(40^\circ) + 8\sin(40^\circ) \end{aligned}$$

Equation:
$(2\sin(40^\circ) - \cos(40^\circ))x + (2\cos(40^\circ) + \sin(40^\circ))y = \cos(40^\circ) + 8\sin(40^\circ)$

Plot of the Lines