

1.4.6

AI25BTECH11033–SNEHAMRUDULA

If the point $P(2, 1)$ lies on the line segment joining points $A(4, 2)$ and $B(8, 4)$, then

1) $AP = \frac{1}{4}AB$

2) $AP = PE$

3) $PB = \frac{1}{3}AB$

4) $AP = \frac{3}{5}AB$

Solution:

Let the points be represented as vectors:

$$\mathbf{A} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Since P lies on the line joining A and B , we have:

$$\mathbf{P} = \mathbf{A} + t(\mathbf{B} - \mathbf{A})$$

Compute:

$$\mathbf{B} - \mathbf{A} = \begin{bmatrix} 8 - 4 \\ 4 - 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

So,

$$\mathbf{P} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} + t \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 + 4t \\ 2 + 2t \end{bmatrix}$$

But $\mathbf{P} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, so:

$$4 + 4t = 2 \quad \text{and} \quad 2 + 2t = 1$$

Solving:

$$4t = -2 \implies t = -0.5$$

$$2t = -1 \implies t = -0.5$$

Thus, $t = -0.5$.

The distance $AP = |t| \cdot AB = 0.5 \cdot AB = \frac{1}{2}AB$.

Therefore, the correct option is **d)** $AP = \frac{1}{2}AB$.