1.10.7

EE25BTECH11048-Revanth Siva Kumar.D

Question:

Find a unit vector in the direction of the vector PQ, where P and Q have co-ordinates (5,0,8) and (3,3,2),respectively.

Solution:

Given,

The points:

$$\mathbf{P} = \begin{pmatrix} 5 \\ 0 \\ 8 \end{pmatrix} \mathbf{Q} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} \tag{0.1}$$

Let the required unit vector be $\mathbf{x}$, then

The formula for unit vector along a line joining two points

$$\mathbf{x} = \frac{\mathbf{X}}{\|\mathbf{X}\|} \tag{0.2}$$

The vector along $\mathbf{P}$ and $\mathbf{Q}$ is

$$\mathbf{X} = \mathbf{Q} - \mathbf{P} \tag{0.3}$$

By (0.3)

$$\mathbf{X} = \begin{pmatrix} 5 \\ 0 \\ 8 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} \tag{0.4}$$

$$\mathbf{X} = \begin{pmatrix} 5-3 \\ 0-3 \\ 8-2 \end{pmatrix} \tag{0.5}$$

$$\mathbf{X} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} \tag{0.6}$$

Magnitude of the vector $\mathbf{X}$ is

$$\|\mathbf{X}\| = \sqrt{X^T X} \tag{0.7}$$

$$\|\mathbf{X}\| = \sqrt{(2,-3,6)\begin{pmatrix}2\\-3\\6\end{pmatrix}} \tag{0.8}$$

$$\|\mathbf{X}\| = \sqrt{(2)^2 + (-3)^2 + (6)^2} \tag{0.9}$$

$$\|\mathbf{X}\| = \sqrt{49} \tag{0.10}$$

$$\|\mathbf{X}\| = 7 \tag{0.11}$$

Then the unit vector by (0.2),

$$\mathbf{x} = \frac{1}{7}\mathbf{X} = \mathbf{x} = \frac{1}{7}\begin{pmatrix}2\\-3\\6\end{pmatrix} \tag{0.12}$$

$$\mathbf{x} = \frac{1}{7}\begin{pmatrix}2\\-3\\6\end{pmatrix} \tag{0.13}$$

$$\mathbf{x} = \left(\tfrac{2}{7}, \tfrac{-3}{7}, \tfrac{6}{7}\right) \tag{0.14}$$

Therefore, the required unit vector is

$$\mathbf{x} = \left(\tfrac{2}{7}, \tfrac{-3}{7}, \tfrac{6}{7}\right) \tag{0.15}$$

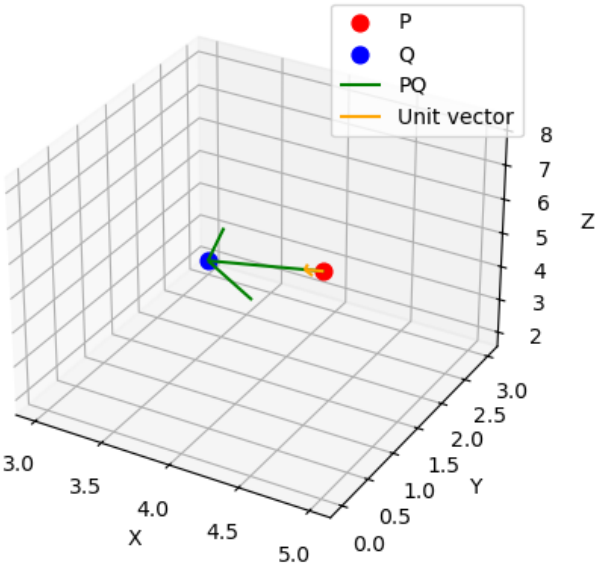

Fig. 0.1: Plot for the unit vector along PQ using shared output

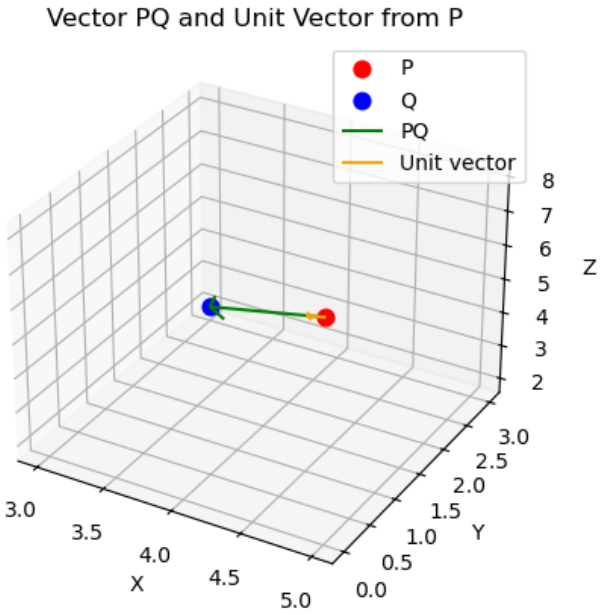

Fig. 0.2: Plot for the unit vector along PQ using direct python code