9.4.33

EE25BTECH11020 - Darsh Pankaj Gajare

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Question:
Find the roots of the following quadratic equation graphically. $x^2 - 4x + 3$
Solution:
The parabola can be expressed in matrix form as

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + \mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.1}$$

where

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}, \quad f = 3. \tag{0.2}$$

The line $y = 0$ is expressed as:

$$\mathbf{x} = \mathbf{q} + \lambda \mathbf{m}, \quad \mathbf{q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{0.3}$$

Substituting

$$(\mathbf{q} + \lambda \mathbf{m})^\top \mathbf{V}(\mathbf{q} + \lambda \mathbf{m}) + \mathbf{u}^\top (\mathbf{q} + \lambda \mathbf{m}) + f = 0. \tag{0.4}$$

$$\lambda^2 \left(\mathbf{m}^\top \mathbf{V}\mathbf{m}\right) + \lambda \left(2\mathbf{q}^\top \mathbf{V}\mathbf{m} + \mathbf{u}^\top \mathbf{m}\right) + \left(\mathbf{q}^\top \mathbf{V}\mathbf{q} + \mathbf{u}^\top \mathbf{q} + f\right) = 0. \quad (0.5)$$

$$\mathbf{m}^\top \mathbf{V}\mathbf{m} = 1, \quad (0.6)$$

$$2\mathbf{q}^\top \mathbf{V}\mathbf{m} + \mathbf{u}^\top \mathbf{m} = -4, \quad (0.7)$$

$$\mathbf{q}^\top \mathbf{V}\mathbf{q} + \mathbf{u}^\top \mathbf{q} + f = 3. \quad (0.8)$$

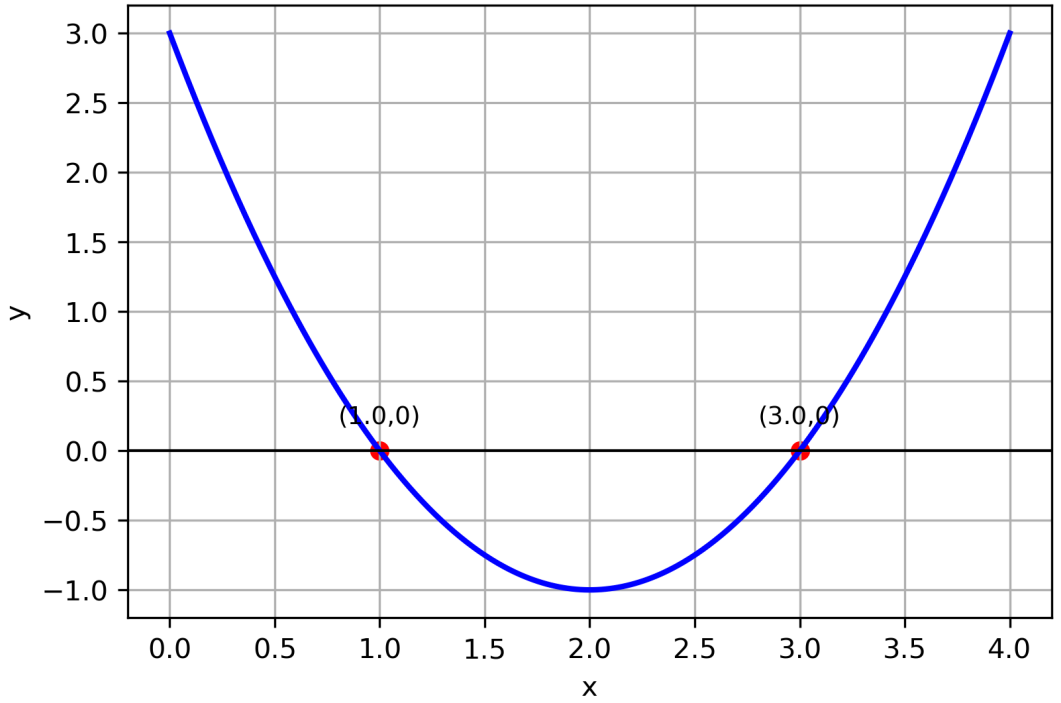
$$\lambda^2 - 4\lambda + 3 = 0. \quad (0.9)$$

$$\lambda_1 = 1, \quad \lambda_2 = 3. \quad (0.10)$$

Thus, the intersection points are

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \mathbf{x}_2 = \begin{pmatrix} 3 \\ 0 \end{pmatrix}. \quad (0.11)$$

Plot using C libraries:



Plot using Python: