5.8.40

EE25BTECH11043 - Nishid Khandagre

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Question

The ratio of incomes of two persons is 9:7 and the ratio of their expenditures is 4:3. If each of them manages to save rupees 2000 per month, find their monthly incomes.

Solution

Let the monthly incomes be x and y.
Given ratio of their incomes:

$$\frac{x}{y} = \frac{9}{7} \tag{1}$$

$$7x - 9y = 0 \tag{2}$$

Expenditures are Income − Savings. Expenditures of the two persons are $x - 2000$ and $y - 2000$.
Given expenditure ratio:

$$\frac{x - 2000}{y - 2000} = \frac{4}{3} \tag{3}$$

$$3x - 6000 = 4y - 8000 \tag{4}$$

$$3x - 4y = -2000 \tag{5}$$

Solution

$$7x - 9y = 0 \tag{6}$$
$$3x - 4y = -2000 \tag{7}$$

Matrix form:

$$\begin{pmatrix} 7 & -9 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -2000 \end{pmatrix} \tag{8}$$

Augmented matrix:

$$\left(\begin{array}{cc|c} 7 & -9 & 0 \\ 3 & -4 & -2000 \end{array}\right) \tag{9}$$

Solution

Then $R_2 \to 7R_2 - 3R_1$:

$$\left(\begin{array}{cc|c} 7 & -9 & 0 \\ 0 & -1 & -14000 \end{array}\right) \tag{10}$$

Then $R_2 \to -R_2$:

$$\left(\begin{array}{cc|c} 7 & -9 & 0 \\ 0 & 1 & 14000 \end{array}\right) \tag{11}$$

Then $R_1 \to R_1 + 9R_2$:

$$\left(\begin{array}{cc|c} 7 & 0 & 126000 \\ 0 & 1 & 14000 \end{array}\right) \tag{12}$$

Then $R_1 \to \frac{1}{7}R_1$:

$$\left(\begin{array}{cc|c} 1 & 0 & 18000 \\ 0 & 1 & 14000 \end{array}\right) \tag{13}$$

Solution

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18000 \\ 14000 \end{pmatrix} \tag{14}$$

$$x = 18000 \tag{15}$$

$$y = 14000 \tag{16}$$

The monthly incomes are 18000 and 14000.

C Code

```c
#include <stdio.h>

// Function to solve a 2x2 system of linear equations using
    Cramer's Rule
// a1*x + b1*y = c1
// a2*x + b2*y = c2
// a1, b1, c1: Coefficients and constant for the first equation
// a2, b2, c2: Coefficients and constant for the second equation
// *x_solution: Pointer to store the solution for x
// *y_solution: Pointer to store the solution for y
int solve_2x2_system(double a1, double b1, double c1,
                     double a2, double b2, double c2,
                     double *x_solution, double *y_solution) {

    double determinant = a1 * b2 - a2 * b1;
```

C Code

```c
    // Check if a unique solution exists
    if (determinant == 0) {
        // No unique solution (parallel lines or same line)
        return 0;
    }

    double det_x = c1 * b2 - c2 * b1;
    double det_y = a1 * c2 - a2 * c1;

    *x_solution = det_x / determinant;
    *y_solution = det_y / determinant;

    return 1; // Unique solution found
}
```

Python Code using C shared library

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt
# Load the shared library
lib_solver = ctypes.CDLL(./code12.so)

# Define the argument types and return type for the C function
# int solve_2x2_system(double a1, double b1, double c1,
# double a2, double b2, double c2,
# double *x_solution, double *y_solution)
lib_solver.solve_2x2_system.argtypes = [
    ctypes.c_double, ctypes.c_double, ctypes.c_double, # a1, b1,
        c1
    ctypes.c_double, ctypes.c_double, ctypes.c_double, # a2, b2,
        c2
    ctypes.POINTER(ctypes.c_double), # x_solution
    ctypes.POINTER(ctypes.c_double) # y_solution
]
```

Python Code using C shared library

```python
lib_solver.solve_2x2_system.restype = ctypes.c_int

# --- Problem: Income and Expenditure ---
# Equations:
# 1) 9x - 4y = 2000
# 2) 7x - 3y = 2000

# Coefficients for the C solver
a1, b1, c1 = 9.0, -4.0, 2000.0
a2, b2, c2 = 7.0, -3.0, 2000.0

# Create ctypes doubles to hold the results for x and y
    multipliers
x_multiplier_result = ctypes.c_double()
y_multiplier_result = ctypes.c_double()
```

Python Code using C shared library

```
# Call the C function to solve the system
print(Solving the system of equations for income and expenditure multipliers using C function:)
print(f Equation 1: {a1}x + {b1}y = {c1})
print(f Equation 2: {a2}x + {b2}y = {c2})

success = lib_solver.solve_2x2_system(
    a1, b1, c1,
    a2, b2, c2,
    ctypes.byref(x_multiplier_result),
    ctypes.byref(y_multiplier_result)
)

if success:
    x_solution = x_multiplier_result.value
    y_solution = y_multiplier_result.value
```

Python Code using C shared library

```
print(f\nSolution found (intersection point):)
print(f x (income multiplier) = {x_solution:.2f})
print(f y (expenditure multiplier) = {y_solution:.2f})

# --- Plotting the two lines and their intersection ---
plt.figure(figsize=(10, 8))

# Generate points for the lines
# We'll use a range around the solution to make the
    intersection clear
x_vals_range = np.linspace(x_solution - 1000, x_solution +
    1000, 400) # Extend range for visualization
```

Python Code using C shared library

```
# Plotting Eq 1: a1*x + b1*y = c1 => y = (c1 - a1*x) / b1
if b1 != 0:
    y1_vals = (c1 - a1 * x_vals_range) / b1
    plt.plot(x_vals_range, y1_vals, label=f'{a1:.0f}x + {b1:.0f}y = {c1:.0f} (Eq 1)', color='blue')
elif a1 != 0: # Handle vertical line case: x = c1/a1
    plt.axvline(x=c1/a1, label=f'x = {c1/a1:.0f} (Eq 1)', color='blue', linestyle='--')
else:
    print(Equation 1 is trivial (0=C). Not plotted.)
# Plotting Eq 2: a2*x + b2*y = c2 => y = (c2 - a2*x) / b2
if b2 != 0:
    y2_vals = (c2 - a2 * x_vals_range) / b2
    plt.plot(x_vals_range, y2_vals, label=f'{a2:.0f}x + {b2:.0f}y = {c2:.0f} (Eq 2)', color='red')
elif a2 != 0: # Handle vertical line case: x = c2/a2
    plt.axvline(x=c2/a1, label=f'x = {c2/a2:.0f} (Eq 2)', color='red', linestyle='--')
```

Python Code using C shared library

```
    else:
        print(Equation 2 is trivial (0=C). Not plotted.)

    # Plot the intersection point
    plt.scatter(x_solution, y_solution, color='green', s=150,
        zorder=5,
                label=f'Intersection ({x_solution:.0f}, {
                    y_solution:.0f})')
    plt.annotate(f'({x_solution:.0f}, {y_solution:.0f})',
                 (x_solution, y_solution), textcoords=offset
                     points, xytext=(5,5), ha='left',
                 bbox=dict(boxstyle=round,pad=0.3, fc=yellow, ec=b
                     , lw=1, alpha=0.7))

    plt.xlabel('Income Multiplier (x)')
```

Python Code using C shared library

```
    plt.ylabel('Expenditure Multiplier (y)')
    plt.title('Graphical Solution of Income and Expenditure
        Equations')
    plt.grid(True)
    plt.legend()
    plt.gca().set_aspect('auto', adjustable='box')
    plt.xlim(min(x_vals_range), max(x_vals_range))
    plt.ylim(min(y1_vals.min(), y2_vals.min(), y_solution) - 500,
         max(y1_vals.max(), y2_vals.max(), y_solution) + 500)
    plt.show()

else:
    print(\nError: No unique solution exists for this system (
        determinant is zero).)
    print(The lines are either parallel or the same line, which
        should not happen for this problem.)
```

Direct Python Code

```
import numpy as np
import numpy.linalg as LA
import matplotlib.pyplot as plt

# --- Problem: Income and Expenditure ---
# The ratio of incomes of two persons is 9:7 => Incomes: 9x, 7x
# The ratio of their expenditures is 4:3 => Expenditures: 4y, 3y
# Each saves 2000 per month.
#
# Equations:
# 1) Income1 - Expenditure1 = Savings => 9x - 4y = 2000
# 2) Income2 - Expenditure2 = Savings => 7x - 3y = 2000

# Represent the system as Ax = B
# A = [[9, -4],
# [7, -3]]
# x = [x_multiplier, y_multiplier]
# B = [2000, 2000]
```

Direct Python Code

```
A_matrix = np.array([[9.0, -4.0],
                     [7.0, -3.0]])

B_vector = np.array([2000.0, 2000.0])

print(Solving the system of equations for income and expenditure multipliers:)
print(f Equation 1: {A_matrix[0,0]}x + {A_matrix[0,1]}y = {B_vector[0]})
print(f Equation 2: {A_matrix[1,0]}x + {A_matrix[1,1]}y = {B_vector[1]})

 # Solve the system of linear equations using numpy.linalg.solve
solution = LA.solve(A_matrix, B_vector)
x_solution = solution[0]
y_solution = solution[1]
```

Direct Python Code

```
print(f x (income multiplier) = {x_solution:.2f})
print(f y (expenditure multiplier) = {y_solution:.2f})
   # --- Plotting the two lines and their intersection ---
plt.figure(figsize=(10, 8))

# Define a generous range for x_vals for plotting purposes.
# Knowing the solution (x=2000, y=4000), we can set a reasonable
    range.
x_plot_min = x_solution - 1000
x_plot_max = x_solution + 1000
x_vals_range = np.linspace(x_plot_min, x_plot_max, 400)

# Plotting Equation 1: a1*x + b1*y = c1 => y = (c1 - a1*x) / b1
# Coefficients from A_matrix and B_vector
y1_vals = (B_vector[0] - A_matrix[0,0] * x_vals_range) / A_matrix
    [0,1]
plt.plot(x_vals_range, y1_vals, b-, label=f'{A_matrix[0,0]:.0f}x
    {A_matrix[0,1]:+.0f}y = {B_vector[0]:.0f} (Person 1)')
```

Direct Python Code

```
# Plotting Equation 2: a2*x + b2*y = c2 => y = (c2 - a2*x) / b2
y2_vals = (B_vector[1] - A_matrix[1,0] * x_vals_range) / A_matrix[1,1]
plt.plot(x_vals_range, y2_vals, r-, label=f'{A_matrix[1,0]:.0f}x{A_matrix[1,1]:+.0f}y = {B_vector[1]:.0f} (Person 2)')

 # Plot the intersection point
plt.scatter(x_solution, y_solution, color='green', s=150, zorder=5,
           label=f'Intersection ({x_solution:.0f}, {y_solution:.0f})')
plt.annotate(f'({x_solution:.0f}, {y_solution:.0f})',
            (x_solution, y_solution), textcoords=offset points, xytext=(5,5), ha='left',
            bbox=dict(boxstyle=round,pad=0.3, fc=yellow, ec=b, lw=1, alpha=0.7))
```

Direct Python Code

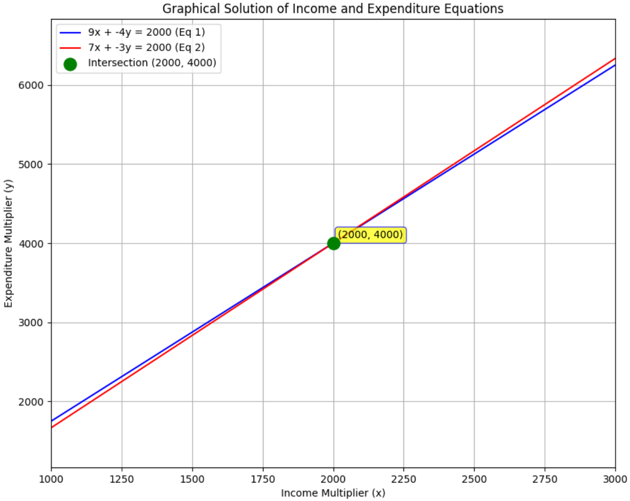
```
plt.xlabel('Income Multiplier (x)')
plt.ylabel('Expenditure Multiplier (y)')
plt.title('Graphical Solution of Income and Expenditure Equations
    ')
plt.grid(True)
plt.legend(loc='best')

# Set plot limits based on data for good visualization
y_plot_min = min(y1_vals.min(), y2_vals.min(), y_solution) - 500
y_plot_max = max(y1_vals.max(), y2_vals.max(), y_solution) + 500
plt.xlim(x_plot_min, x_plot_max)
plt.ylim(y_plot_min, y_plot_max)
plt.gca().set_aspect('auto', adjustable='box')

plt.savefig(fig2.png)
plt.show()

print(Figure saved as fig2.png)
```

Plot by Python using shared output from C



Plot by Python only