2.9.2

EE25BTECH11051 - Shreyas Goud Burra

Question If (-5, 3) and (5, 3) are two vertices of an equilateral triangle, then the coordinates of the third vertex, given that the origin lies inside the triangle (take $\sqrt{3} = 1.7$), are

Solution:

Let us find the solution theoretically first and then verify it computationally.
Let the two given points be represented as vectors, **A** and **B**, respectively

$$\mathbf{A} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \tag{0.1}$$

Let us assume the third point be **C**.
We have to first find the line equation of the line joining the points A and B.

$$\mathbf{x} = \mathbf{A} + t(\mathbf{B} - \mathbf{A}) \tag{0.2}$$

This gives,

$$\mathbf{x} = \begin{pmatrix} -5 \\ 3 \end{pmatrix} + t \begin{pmatrix} 10 \\ 0 \end{pmatrix} \tag{0.3}$$

We have to find the lines aligned at 60° to this line at both **A** and **B**.
We can get this by multiplying a rotation vector to this vector, this is given by,

$$\mathbf{V}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{0.4}$$

By multiplying this to 0.3 with $\theta = \pm 60°$, we get the lines,

$$\mathbf{x} = \begin{pmatrix} -5 \\ 3 \end{pmatrix} + t(\mathbf{V}(\pm 60°)) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.5}$$

The lines we get from this equation are,

$$\mathbf{x} = \begin{pmatrix} -5 \\ 3 \end{pmatrix} + t \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \tag{0.6}$$

$$\mathbf{x} = \begin{pmatrix} -5 \\ 3 \end{pmatrix} + t \begin{pmatrix} \frac{1}{2} \\ \frac{-\sqrt{3}}{2} \end{pmatrix} \tag{0.7}$$

By doing the same thing taking point B,

$$\mathbf{x} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + t \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \tag{0.8}$$

$$\mathbf{x} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + t \begin{pmatrix} \frac{1}{2} \\ \frac{-\sqrt{3}}{2} \end{pmatrix} \tag{0.9}$$

We can get two possible points that fit the given conditions for an equilateral triangle, let us assume these to be **C1** and **C2**

We can get **C1** by finding the point of intersection of 0.6 and 0.9

$$\begin{pmatrix} -5 \\ 3 \end{pmatrix} + t \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + t \begin{pmatrix} \frac{1}{2} \\ \frac{-\sqrt{3}}{2} \end{pmatrix} \tag{0.10}$$

On further solving, we get the point to be,

$$\mathbf{C1} = \begin{pmatrix} 0 \\ 3 + 5\sqrt{3} \end{pmatrix} = \begin{pmatrix} 0 \\ 11.5 \end{pmatrix} \tag{0.11}$$

Similarly, on solving for the other two lines, 0.7 and 0.8, we get,

$$\mathbf{C2} = \begin{pmatrix} 0 \\ 3 - 5\sqrt{3} \end{pmatrix} = \begin{pmatrix} 0 \\ -5.5 \end{pmatrix} \tag{0.12}$$

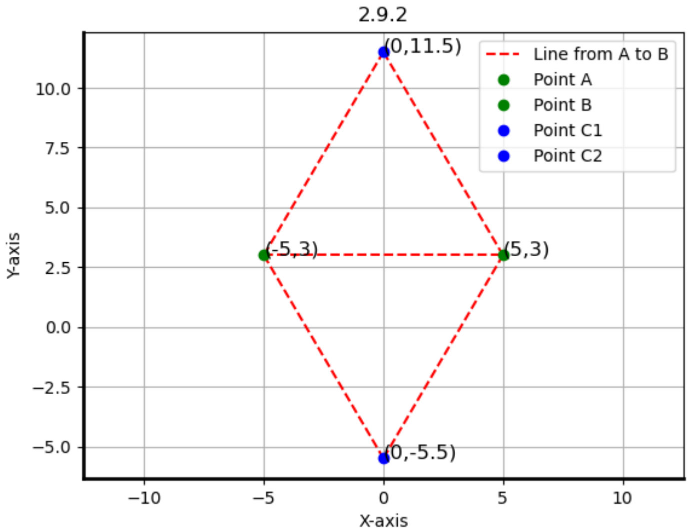

Fig. 0.1: 2D Plot