2.10.33

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Question

Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$, $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$

1. are collinear
2. form an equilateral triangle
3. form a scalene triangle
4. form a right angled triangle

Solution

Let $\mathbf{A}$ be $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\mathbf{B}$ be $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$, and $\mathbf{C}$ be $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$.

Then

- $\mathbf{A} - \mathbf{B}$ is $(\alpha - \beta)\hat{i} + (\beta - \gamma)\hat{j} + (\gamma - \alpha)\hat{k}$
- $\mathbf{B} - \mathbf{C}$ is $(\beta - \gamma)\hat{i} + (\gamma - \alpha)\hat{j} + (\alpha - \beta)\hat{k}$
- $\mathbf{C} - \mathbf{A}$ is $(\gamma - \alpha)\hat{i} + (\alpha - \beta)\hat{j} + (\beta - \gamma)\hat{k}$

Then norms of $\mathbf{A} - \mathbf{B}, \mathbf{B} - \mathbf{C}, \mathbf{C} - \mathbf{A}$ are all equal, and equal to

$$\sqrt{(\alpha - \beta)^2 + (\beta - \gamma)^2 + (\gamma - \alpha)^2}$$

The three points therefore form an equilateral triangle, so option (2) is correct.