

# 8.2.33

EE25BTECH11065 - Yoshita J

## Question

Find the equation of the conic with length of major axis 26, foci  $(\pm 5, 0)$ .

## Solution

The equation of a conic is

$$\mathbf{x}^T V \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (1)$$

where

$$V = \|\mathbf{n}\|^2 I - e^2 \mathbf{n} \mathbf{n}^T \quad (2)$$

The foci are

$$\mathbf{F}_1 = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \quad \mathbf{F}_2 = \begin{pmatrix} -5 \\ 0 \end{pmatrix} \quad (3)$$

The centre is

$$\mathbf{u} = \frac{\mathbf{F}_1 + \mathbf{F}_2}{2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (4)$$

The axis vector is

$$\mathbf{n} = \mathbf{F}_1 - \mathbf{F}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (5)$$

Therefore, substituting  $\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  in (2), we get

$$V = \begin{pmatrix} 1 - e^2 & 0 \\ 0 & 1 \end{pmatrix} \quad (6)$$

From the formula for the length of the major axis,

$$2\sqrt{\frac{|f|}{\lambda_1}} \quad (7)$$

where  $\lambda_1 = 1 - e^2$ . Hence

$$26 = 2\sqrt{\frac{|f|}{1 - e^2}} \quad (8)$$

The relation between focus and eccentricity is

$$\pm c e^2 = 5 \quad (9)$$

The distance  $c$  is

$$c = \pm \frac{1}{e} \sqrt{\frac{|f|}{|e^2 - 1|}} \quad (10)$$

Thus from (8)–(10), solving the unknowns  $(c, e, f)$  we get

$$e = \frac{5}{13}, \quad c = \pm 5, \quad |f| = 144. \quad (11)$$

Let  $\mathbf{x} = \begin{pmatrix} 0 \\ \alpha \end{pmatrix}$  be a vertex on the minor axis. Substituting in (1):

$$\frac{12^2}{1} + f = 0 \implies f = -144. \quad (12)$$

Hence the equation of the conic is

$$\mathbf{x}^T \begin{pmatrix} \frac{144}{169} & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - 144 = 0. \quad (13)$$

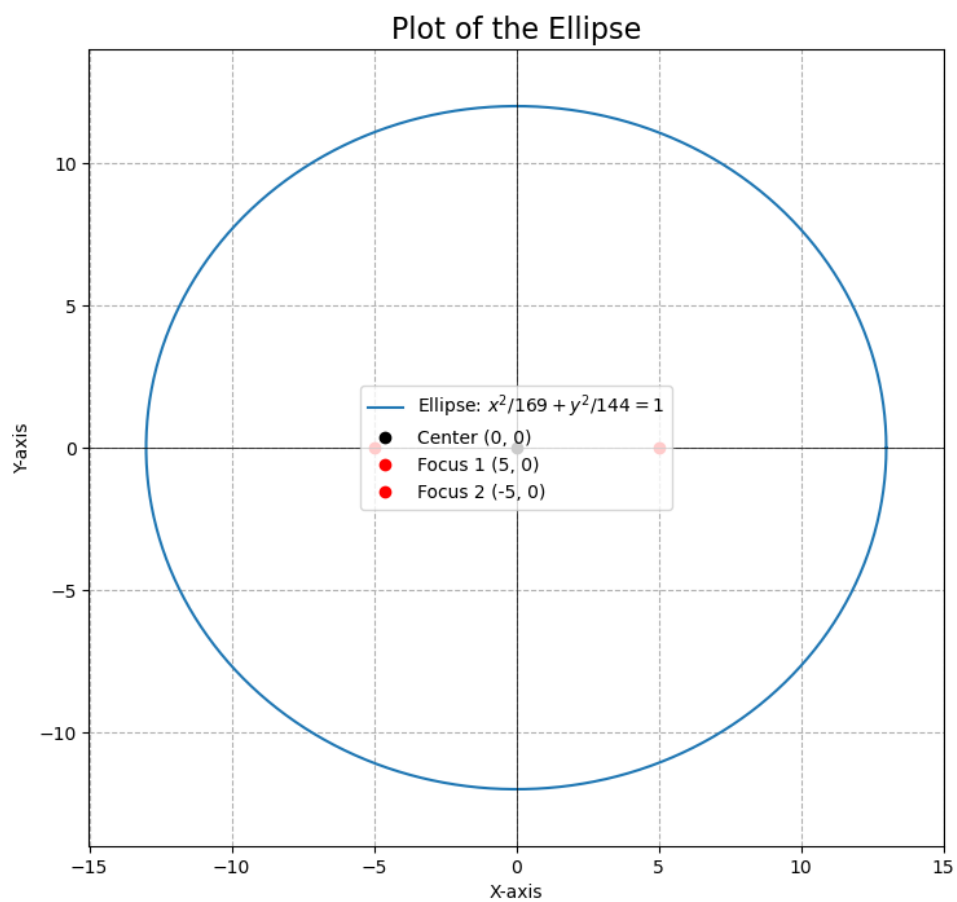


Fig. 0: Ellipse with major axis 26 and foci at  $(\pm 5, 0)$