Question 4.4.36

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1 Question:

The area of the triangle formed by the lines $\frac{x}{a} + \frac{y}{b} = 1$ and the coordinate axes is _____.

2 Solution:

Let the origin be **O**, the x-intercept be **A**, and the y-intercept be **B**. We then need the area of triangle OAB. The x-intercept is found by setting $y = 0$ in the equation of the line, written as $\mathbf{m}^{\mathrm{T}}\mathbf{x} = 1$, where $\mathbf{m} = \begin{pmatrix} \frac{1}{a} \\ \frac{1}{b} \end{pmatrix}$ and $\mathbf{x}$ represents a point on the line. Similarly, the y-intercept is found by setting $x = 0$.

$$\therefore \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} x \\ 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 \\ y \end{pmatrix} \tag{1}$$

Putting **A** and **B** in the equation of the line, we get

$$x = a, \quad y = b \tag{2}$$

Clearly, the triangle formed is a right angled triangle, with the right angle at **O** (due to the axes being perpendicular). The area of the triangle is given by

$$\Delta\mathrm{OAB} = \frac{1}{2} \times \text{base} \times \text{height, in this case} \tag{3}$$

$$\Delta\mathrm{OAB} = \frac{1}{2} \times OA \times OB \tag{4}$$

$$\Delta\mathrm{OAB} = \frac{1}{2} \times |a| \times |b| \tag{5}$$

$$\therefore \Delta\mathrm{OAB} = \left| \frac{ab}{2} \right| \tag{6}$$

3 Plot:

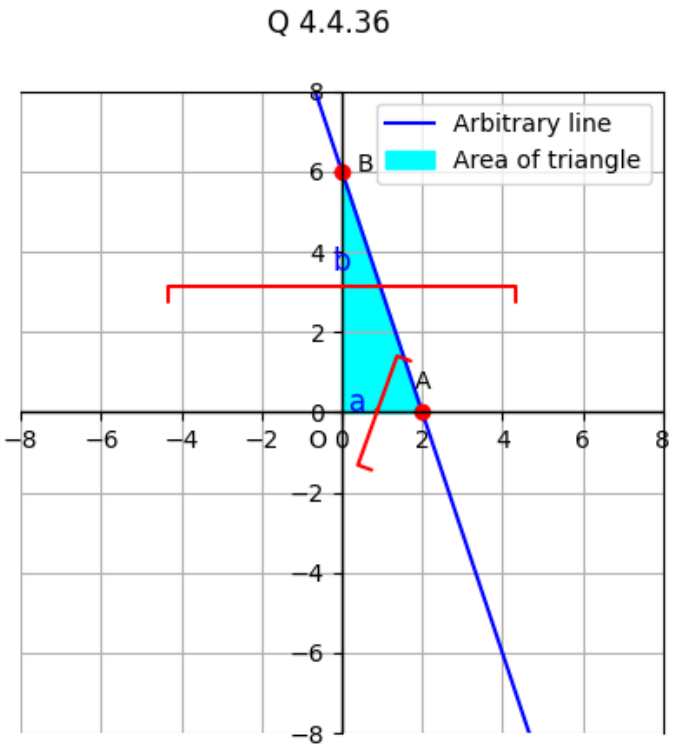

Figure 1: Graph of line with direction and normal vectors