1.9.14

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27th August, 2025

Question

If $\mathbf{P} = (2, 2)$, $\mathbf{Q} = (-4, -4)$, and $\mathbf{R} = (5, -8)$ are the vertices of a triangle ΔPQR, then find the length of the median through $\mathbf{R}$.

Vectors

Given position vectors of the points are:

$$\mathbf{P} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} -4 \\ -4 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 5 \\ -8 \end{pmatrix} \tag{1}$$

Let the position vectors of $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ be the columns of the 2×3 matrix:

$$V = \begin{pmatrix} \mathbf{R} & \mathbf{Q} & \mathbf{P} \end{pmatrix} \tag{2}$$

$$V = \begin{pmatrix} 5 & -4 & 2 \\ -8 & -4 & 2 \end{pmatrix} \tag{3}$$

Midpoint

The midpoint of PQ is:

$$\mathbf{M} = \frac{1}{2}\mathbf{P} + \frac{1}{2}\mathbf{Q} = V \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \tag{4}$$

$$\mathbf{RM} = \mathbf{M} - \mathbf{R} = V \begin{pmatrix} -1 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \tag{5}$$

Let

$$\mathbf{c_R} = \begin{pmatrix} -1 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \tag{6}$$

Length

Let the gram matrix:

$$G = V^T V \tag{7}$$

$$G = \begin{pmatrix} 89 & 12 & -6 \\ 12 & 32 & -16 \\ -6 & -16 & 8 \end{pmatrix} \tag{8}$$

Then the squares length of the median from **R** is :

$$||\mathbf{RM}||^2 = (V\mathbf{c_R})^T (V\mathbf{c_R}) \tag{9}$$

$$= \mathbf{c_R}^T \left(V^T V\right) \mathbf{c_R} = \mathbf{c_R}^T G \mathbf{c_R} \tag{10}$$

$$||\mathbf{RM}|| = \sqrt{85} \approx 9.2195 \tag{11}$$

C Code

```c
#include <stdio.h>

void get_points(double *points) {
    points[0] = 5; points[1] = -8; // R
    points[2] = -4; points[3] = -4; // Q
    points[4] = 2; points[5] = 2; // P
}
```

Python Code

```
import sys
import math
import numpy as np
import matplotlib.pyplot as plt
import ctypes

problem = ctypes.CDLL('/home/ganachari-vishwmabhar/ee1030-2025/
    EE25BTECH11025/ASSIGNMENTS/matgeo/1.5.13/codes/problem.so')
```

Python Code

```
P = np.array([2, 2])
Q = np.array([-4, -4])
R = np.array([5, -8])

# Calculate the midpoint M of PQ (for the median through R)
M = (P + Q) / 2

# Prepare plot
plt.figure()
# Plot the triangle
xs = [P[0], Q[0], R[0], P[0]]
ys = [P[1], Q[1], R[1], P[1]]
plt.plot(xs, ys, 'k-', label='Triangle')
```

Python Code

```
plt.plot([R[0], M[0]], [R[1], M[1]], 'r--', label='Median from R'
    )

# Mark vertices
plt.scatter([P[0], Q[0], R[0], M[0]], [P[1], Q[1], R[1], M[1]], c
    =['b','g','r','m'])
plt.text(P[0], P[1], 'P', fontsize=12)
plt.text(Q[0], Q[1], 'Q', fontsize=12)
plt.text(R[0], R[1], 'R', fontsize=12)
plt.text(M[0], M[1], 'M', fontsize=12)
```

Python Code

```
plt.axis('equal')
plt.grid(True)
plt.legend()
plt.title("Triangle PQR and Median through R")
plt.savefig("../figs/plot.png")
plt.show()
```

Plot

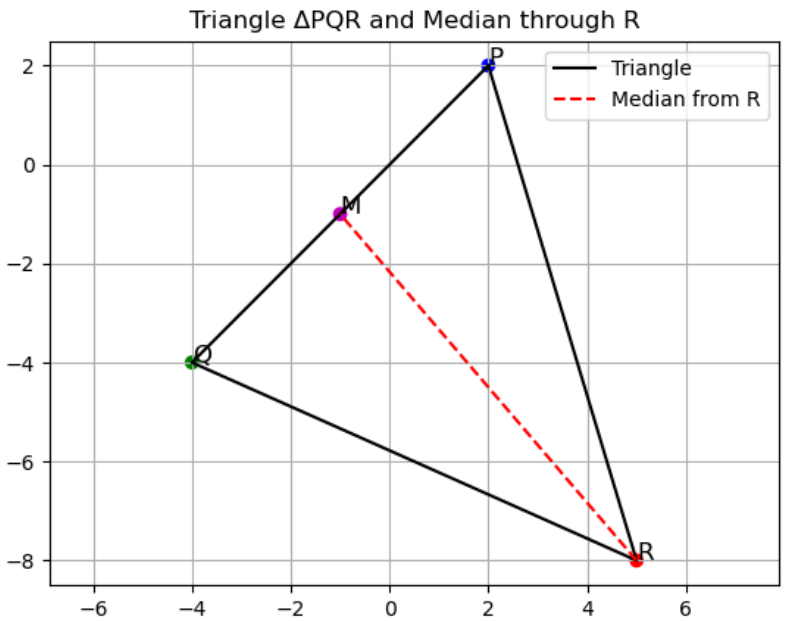

Figure: Plot of along with median