2.7.21

EE25BTECH11009-Anshu kumar ram

Question:
Find the values of k so that the area of the triangle with vertices $A(1, -1),\ B(-4, 2k),\ C(-k, -5)$ is 24 sq. units.
Solution:

The given vertices are

Point	Coordinates
A	(1, −1)
B	(−4, 2k)
C	(−k, −5)

TABLE 0: Vertices of $\triangle ABC$ before substituting k

$$\mathbf{u} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} -5 \\ 2k+1 \end{pmatrix}, \tag{0.1}$$

$$\mathbf{v} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} -k-1 \\ -4 \end{pmatrix}. \tag{0.2}$$

The area of $\triangle ABC$ is

$$\Delta = \frac{1}{2}\left\|\mathbf{u} \times \mathbf{v}\right\|. \tag{0.3}$$

Using the identity

$$\left\|\mathbf{u} \times \mathbf{v}\right\|^2 = \left\|\mathbf{u}\right\|^2 \left\|\mathbf{v}\right\|^2 - \left(\mathbf{u}^\top \mathbf{v}\right)^2 \tag{0.4}$$

Hence,

$$\left\|\mathbf{u} \times \mathbf{v}\right\|^2 = \left(2k^2 + 3k + 21\right)^2 \tag{0.5}$$

$$\implies \left\|\mathbf{u} \times \mathbf{v}\right\| = \left|2k^2 + 3k + 21\right|. \tag{0.6}$$

So,

$$\Delta = \frac{1}{2}\left|2k^2 + 3k + 21\right|. \tag{0.7}$$

Given $\Delta = 24$,

$$\left|2k^2 + 3k + 21\right| = 48 \tag{0.8}$$

Case 1:

$$2k^2 + 3k + 21 = 48 \tag{0.9}$$
$$\implies 2k^2 + 3k - 27 = 0 \tag{0.10}$$

$$k = \frac{-3 \pm 15}{4} = \{3, \ -\tfrac{9}{2}\} \tag{0.11}$$

Case 2:

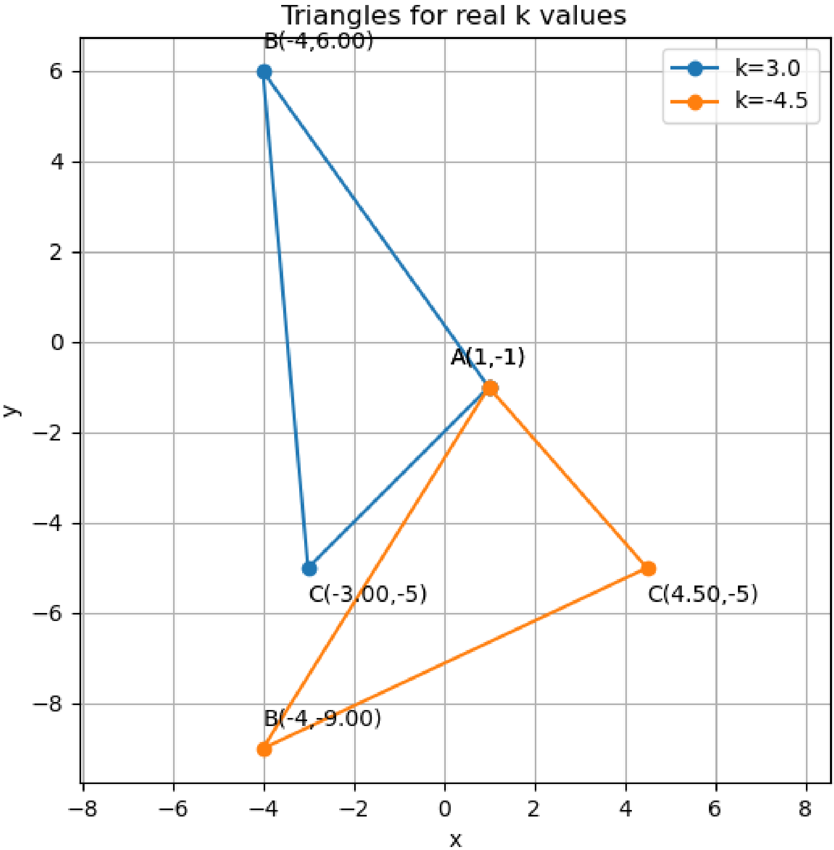
$$2k^2 + 3k + 21 = -48 \tag{0.12}$$
$$\implies 2k^2 + 3k + 69 = 0 \tag{0.13}$$

This has no real roots.

$$\therefore k \in \left\{3, \ -\tfrac{9}{2}\right\} \tag{0.14}$$

Point	For $k = 3$	For $k = -\frac{9}{2}$
A	$(1, -1)$	$(1, -1)$
B	$(-4, 6)$	$(-4, -9)$
C	$(-3, -5)$	$\left(\frac{9}{2}, -5\right)$

TABLE 0: Vertices of $\triangle ABC$ after substituting k values


Triangles for real k values
B(-4,6.00)
k=3.0
k=-4.5
A(1,-1)
C(-3.00,-5)
C(4.50,-5)
B(-4,-9.00)
y
x
6
4
2
0
−2
−4
−6
−8
−8
−6
−4
−2
0
2
4
6
8