4.11.1

EE25BTECH11023 - Venkata Sai

Question:
Find the coordinates of the point where the line $\frac{x-1}{3} = \frac{y+4}{7} = \frac{z+4}{2}$ cuts the XY-plane

Solution: The line equation is

$$\mathbf{r} = \mathbf{a} + t\mathbf{b} \tag{1}$$

where $\mathbf{a}$ is the point on line and $\mathbf{b}$ is the direction vector

$$\mathbf{a} = \begin{pmatrix} 1 \\ -4 \\ -4 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 3 \\ 7 \\ 2 \end{pmatrix} \tag{2}$$

The normal vector to XY plane is

$$\mathbf{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{3}$$

The plane equation of the XY-plane is

$$\mathbf{n}^T\mathbf{x} = 0 \implies \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\mathbf{x} = 0 \tag{4}$$

Substituting the line into the plane equation gives

$$\mathbf{n}^T(\mathbf{a} + t\mathbf{b}) = 0 \tag{5}$$

$$\mathbf{n}^T\mathbf{a} + t\left(\mathbf{n}^T\mathbf{b}\right) = 0 \tag{6}$$

$$t\left(\mathbf{n}^T\mathbf{b}\right) = -\mathbf{n}^T\mathbf{a} \tag{7}$$

$$t = -\frac{\mathbf{n}^T\mathbf{a}}{\mathbf{n}^T\mathbf{b}} \tag{8}$$

$$t = -\frac{\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -4 \\ -4 \end{pmatrix}}{\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 7 \\ 2 \end{pmatrix}} \tag{9}$$

$$t = \frac{4}{2} \tag{10}$$

$$t = 2 \tag{11}$$

The intersection point is

$$\mathbf{r} = \mathbf{a} + t\mathbf{b} = \begin{pmatrix} 1 \\ -4 \\ -4 \end{pmatrix} + 2 \begin{pmatrix} 3 \\ 7 \\ 2 \end{pmatrix} \tag{12}$$

$$\mathbf{r} = \begin{pmatrix} 7 \\ 10 \\ 0 \end{pmatrix} \tag{13}$$

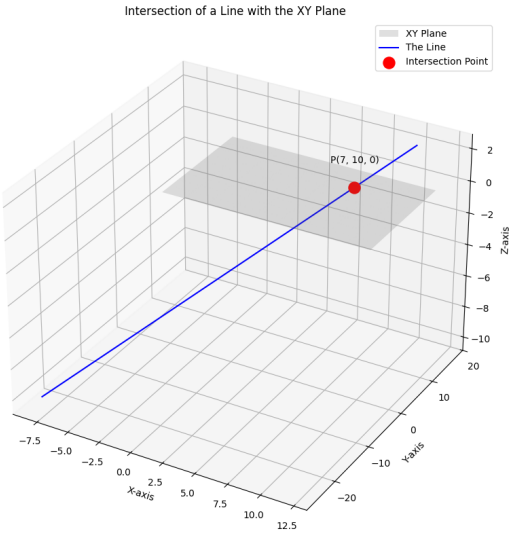

Fig. 0.1