

4.8.3

AI25BTECH11008 - Chiruvella Harshith Sharan

Question:

Find the equation of the plane passing through the points $A(2, 5, -3)$, $B(-2, -3, 5)$ and $C(5, 3, -3)$.

Solution:

$$\mathbf{A} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -2 \\ -3 \\ 5 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 5 \\ 3 \\ -3 \end{pmatrix}. \quad (1)$$

Let the equation of the plane be

$$\mathbf{n}^T \mathbf{x} = 1. \quad (2)$$

Since $\mathbf{A}, \mathbf{B}, \mathbf{C}$ lie in the plane:

$$\mathbf{n}^T \mathbf{A} = 1, \quad \mathbf{n}^T \mathbf{B} = 1, \quad \mathbf{n}^T \mathbf{C} = 1, \quad (3)$$

or equivalently

$$\mathbf{A}^T \mathbf{n} = 1, \quad \mathbf{B}^T \mathbf{n} = 1, \quad \mathbf{C}^T \mathbf{n} = 1. \quad (4)$$

Hence,

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}^T \mathbf{n} = 1. \quad (5)$$

$$\begin{pmatrix} 2 & 5 & -3 \\ -2 & -3 & 5 \\ 5 & 3 & -3 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \quad (6)$$

Performing row operations:

$$R_2 \leftarrow R_2 + R_1, \quad (7)$$

$$\begin{pmatrix} 2 & 5 & -3 \\ 0 & 2 & 2 \\ 5 & 3 & -3 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad (8)$$

$$R_3 \leftarrow 2R_3 - 5R_1, \quad (9)$$

$$\begin{pmatrix} 2 & 5 & -3 \\ 0 & 2 & 2 \\ 0 & -19 & 9 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \quad (10)$$

$$R_3 \leftarrow 19R_2 + 2R_3, \quad (11)$$

$$\begin{pmatrix} 2 & 5 & -3 \\ 0 & 2 & 2 \\ 0 & 0 & 56 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ 32 \end{pmatrix}. \quad (12)$$

Thus, solving we get

$$\mathbf{n} = \begin{pmatrix} \frac{2}{7} \\ \frac{3}{7} \\ \frac{4}{7} \end{pmatrix}. \quad (13)$$

Therefore, The equation of plane is

$$\begin{pmatrix} \frac{2}{7} \\ \frac{3}{7} \\ \frac{4}{7} \end{pmatrix}^T \mathbf{x} = 1. \quad (14)$$

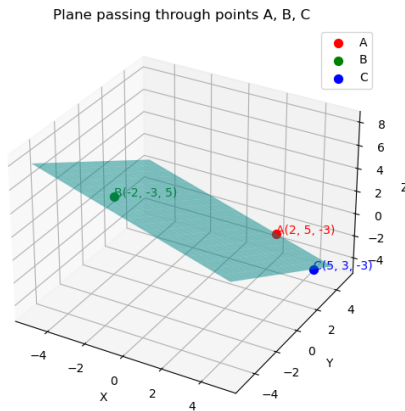


Fig. 0