5.8.25

INDHIRESH S- EE25BTECH11027

Question.One says, "Give me a hundred, Friend! I shall then become twice as rich as you ". The other "if you give me ten, i shall be six times as rich as you ". Tell me What is the amount of their (respective) capital? [From the bijaganita of Bhaskara II].

Solution:

Let us solve the given equation theoretically and then verify the solution computationally.
Let an amount with Friend 1 be a and amount with Friend 2 be b
From given information:

$$a + 100 = 2(b - 100) \tag{1}$$

$$a - 2b = -300 \tag{2}$$

And

$$b + 10 = 6(a - 10) \tag{3}$$

$$b + 10 = 6a - 60; \tag{4}$$

$$6a - b = 70 \tag{5}$$

By combining the Eq.2 and Eq.5 we get

$$\begin{pmatrix} 1 & -2 \\ 6 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -300 \\ 70 \end{pmatrix} \tag{6}$$

Where

$$\mathbf{x} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{7}$$

$$\left(\begin{array}{cc|c} 1 & -2 & -300 \\ 6 & -1 & 70 \end{array}\right) \xleftrightarrow{R_2 \leftarrow R_2 - 6R_1} \left(\begin{array}{cc|c} 1 & -2 & -300 \\ 0 & 11 & 1870 \end{array}\right) \tag{8}$$

$$\left(\begin{array}{cc|c} 1 & -2 & -300 \\ 0 & 11 & 1870 \end{array}\right) \xleftrightarrow{R_2 \leftarrow \frac{1}{11} R_2} \left(\begin{array}{cc|c} 1 & -2 & -300 \\ 0 & 1 & 170 \end{array}\right) \tag{9}$$

$$\left(\begin{array}{cc|c} 1 & -2 & -300 \\ 0 & 1 & 170 \end{array}\right) \xleftrightarrow{R_1 \leftarrow R_1 + 2R_2} \left(\begin{array}{cc|c} 1 & 0 & 40 \\ 0 & 1 & 170 \end{array}\right) \tag{10}$$

$$\mathbf{x} = \begin{pmatrix} 40 \\ 170 \end{pmatrix} \tag{11}$$

$$a = 40 \quad and \quad b = 170 \tag{12}$$

The amount with Friend 1 = 40
The amount with Friend 2 = 170

From the figure it is clearly verified that the theoretical solution matches with the computational solution.