4.2.23

EE25BTECH11020 - Darsh Pankaj Gajare

Question:
Find the equation of the plane passing through the points $(2,2,-1)$,$(3,4,2)$ and $(7,0,6)$. Also find the equation of the plane passing through $(4,3,1)$ and parallel to the plane obtained above.
Solution:

TABLE I

A	$\begin{pmatrix}2\\2\\-1\end{pmatrix}$
B	$\begin{pmatrix}3\\4\\2\end{pmatrix}$
C	$\begin{pmatrix}7\\0\\6\end{pmatrix}$
D	$\begin{pmatrix}4\\3\\1)\end{pmatrix}$

Let the equation of plane be

$$\mathbf{n}^\top \mathbf{x} = C_1 \tag{1}$$

A,B,C satisfies this equation,

$$\mathbf{n}^\top \mathbf{A} = C_1, \mathbf{n}^\top \mathbf{B} = C_1, \mathbf{n}^\top \mathbf{C} = C_1 \tag{2}$$

$$\begin{pmatrix}\mathbf{A}\\\mathbf{B}\\\mathbf{C}\end{pmatrix}^\top \mathbf{n} = \begin{pmatrix}C_1\\C_1\\C_1\end{pmatrix} \tag{3}$$

Using augmented matrix,

$$\left(\begin{array}{ccc|c}2 & 2 & -1 & C_1\\3 & 4 & 2 & C_1\\7 & 0 & 6 & C_1\end{array}\right) \tag{4}$$

$R_1 = R_1/2$

$$\left(\begin{array}{ccc|c}1 & 1 & -0.5 & \frac{C_1}{2}\\3 & 4 & 2 & C_1\\7 & 0 & 6 & C_1\end{array}\right) \tag{5}$$

$R_2 = R_2 - 3R_1$

$$\left(\begin{array}{ccc|c}1 & 1 & -0.5 & \frac{C_1}{2}\\0 & 1 & 3.5 & \frac{-C_1}{2}\\7 & 0 & 6 & C_1\end{array}\right) \tag{6}$$

$R_3 = R_3 - 7R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & -0.5 & \frac{C_1}{2} \\ 0 & 1 & 3.5 & \frac{-C_1}{2} \\ 0 & -7 & 9.5 & \frac{-5C_1}{2} \end{array}\right) \tag{7}$$

$R_3 = R_3 + 7R_2$

$$\left(\begin{array}{ccc|c} 1 & 1 & -0.5 & \frac{C_1}{2} \\ 0 & 1 & 3.5 & \frac{-C_1}{2} \\ 0 & 0 & 34 & -6C_1 \end{array}\right) \tag{8}$$

$$34z + 6C_1 = 0 \implies z = \frac{-3C_1}{17} \tag{9}$$

$$y + 3.5z + 0.5C_1 = 0 \implies y = \frac{2C_1}{17} \tag{10}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -0.5 & \frac{C_1}{2} \\ 0 & 1 & 3.5 & \frac{-C_1}{2} \\ 0 & -7 & 9.5 & \frac{-5C_1}{2} \end{array}\right) \tag{11}$$

$$x + y - 0.5z = 0.5C_1 \implies x = \frac{5C_1}{17} \tag{12}$$

Let $C_1 = 17$

$$\mathbf{n} = \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}, C_1 = 17 \tag{13}$$

Equation of plane parallel to given plane passing through D,

$$\mathbf{n}^\top \mathbf{x} = C_2 \tag{14}$$

$$\mathbf{n}^\top \mathbf{D} = C_2 \tag{15}$$

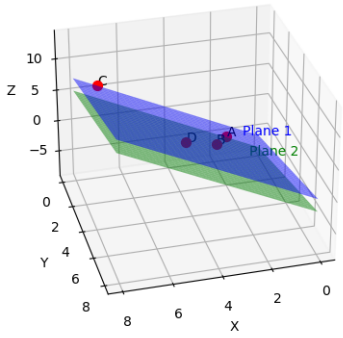
$$C_2 = \begin{pmatrix} 5 & 2 & -3 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} \tag{16}$$

$$C_2 = 23 \tag{17}$$

Equation of plane

$$\begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}^\top \mathbf{x} = 23 \tag{18}$$

Plot using C libraries:



Plot using Python: