4.13.72

AI25BTECH11003 - Bhavesh Gaikwad

Question: A non-zero vector **a** is parallel to the line of intersection of the plane determined by the vectors $\hat{i}$, $\hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}$, $\hat{i}+\hat{k}$. The angle between **a** and the vector $\hat{i}-2\hat{j}+2\hat{k}$ is _____.

(1996)

Solution:

First plane is determined by the vectors $\begin{pmatrix}1\\0\\0\end{pmatrix}$ and $\begin{pmatrix}1\\1\\0\end{pmatrix}$, so a normal is

$$\mathbf{n}_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} \times \begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix}. \tag{0.1}$$

Second plane is determined by $\begin{pmatrix}1\\-1\\0\end{pmatrix}$ and $\begin{pmatrix}1\\0\\1\end{pmatrix}$, so a normal is

$$\mathbf{n}_2 = \begin{pmatrix}1\\-1\\0\end{pmatrix} \times \begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\-1\\1\end{pmatrix}. \tag{0.2}$$

Let vector $\mathbf{n}_3$ be the parallel vector of the intersection line.

$$\mathbf{n}_3 = \mathbf{n}_1 \times \mathbf{n}_2 = \begin{pmatrix}1\\-1\\0\end{pmatrix} \tag{0.3}$$

Thus any vector **a** parallel to the intersection line is parallel to $\begin{pmatrix}1\\-1\\0\end{pmatrix}$.

$$\therefore\ \mathbf{a} = \begin{pmatrix}1\\-1\\0\end{pmatrix}. \tag{0.4}$$

Given vector in the question:

$$\text{Let } \mathbf{u} = \begin{pmatrix}1\\-2\\2\end{pmatrix}. \text{ (Already Given in the question)} \tag{0.5}$$

Using the scalar product formula

$$\cos\theta = \frac{\mathbf{a}^\top \mathbf{u}}{\|\mathbf{a}\|\ \|\mathbf{u}\|}, \tag{0.6}$$

We compute

$$\mathbf{a}^\top \mathbf{u} = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = 1 + 2 + 0 = 3, \tag{0.7}$$

$$\|\mathbf{a}\| = \sqrt{\mathbf{a}^\top \mathbf{a}} = \sqrt{1^2 + (-1)^2 + 0^2} = \sqrt{2}, \tag{0.8}$$

$$\|\mathbf{u}\| = \sqrt{\mathbf{u}^\top \mathbf{u}} = \sqrt{1^2 + (-2)^2 + 2^2} = 3. \tag{0.9}$$

Substituting value from Equation 0.7 and 0.8 in Equation 0.5,

$$\cos\theta = \frac{3}{\sqrt{2} \cdot 3} = \frac{1}{\sqrt{2}} \ \Rightarrow\ \theta = 45^\circ. \tag{0.10}$$

$$\boxed{\text{The angle between } \mathbf{a} \textit{ and } 1\hat{i} - 2\hat{j} + 2\hat{k} \textit{ is } 45^\circ.} \tag{0.11}$$

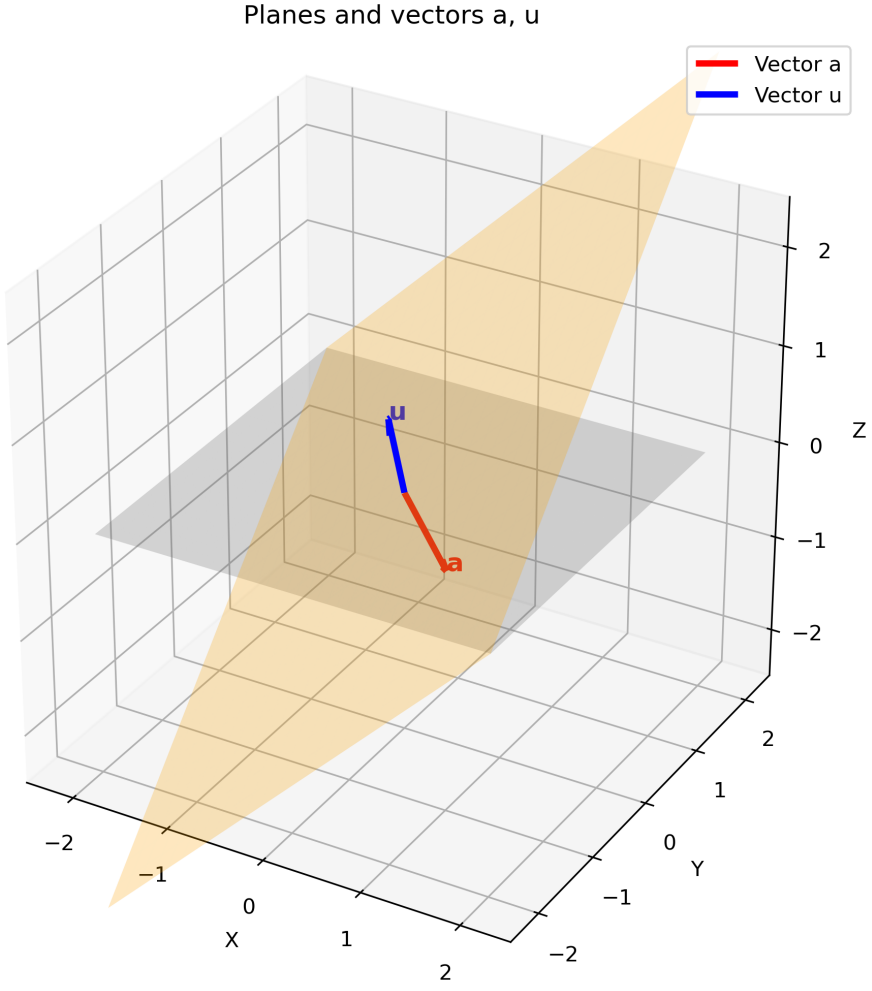

Fig. 0.1: Plane