

2.3.15

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Question: The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ .

Solution: let **A**, **B** and **C** be the vectors such that:

Variable	value
A	$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
B	$\begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}$
C	$\begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix}$

TABLE 0: Variables used

given,

$$\frac{\mathbf{A}^\top (\mathbf{B} + \mathbf{C})}{\|\mathbf{B} + \mathbf{C}\|} = 1 \quad (0.1)$$

$$\mathbf{A}^\top (\mathbf{B} + \mathbf{C}) = \|\mathbf{B} + \mathbf{C}\| \quad (0.2)$$

squaring on both sides:

$$(\mathbf{A}^\top (\mathbf{B} + \mathbf{C}))^2 = \|\mathbf{B} + \mathbf{C}\|^2 \quad (0.3)$$

$$(\mathbf{A}^\top \mathbf{B} + \mathbf{A}^\top \mathbf{C})^2 = (\mathbf{B} + \mathbf{C})^\top (\mathbf{B} + \mathbf{C}) \quad (0.4)$$

Substituting the values of **A**, **B** and **C**:

$$\left((1 \ 1 \ 1) \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} + (1 \ 1 \ 1) \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix} \right)^2 = (2 + \lambda \ 6 \ -2) \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix} \quad (0.5)$$

$$(\lambda + 6)^2 = \lambda^2 + 4\lambda + 44 \quad (0.6)$$

$$\lambda^2 + 36 + 12\lambda = \lambda^2 + 4\lambda + 44 \quad (0.7)$$

$$8\lambda = 8 \quad (0.8)$$

$$\lambda = 1 \quad (0.9)$$

Hence value of λ is 1.

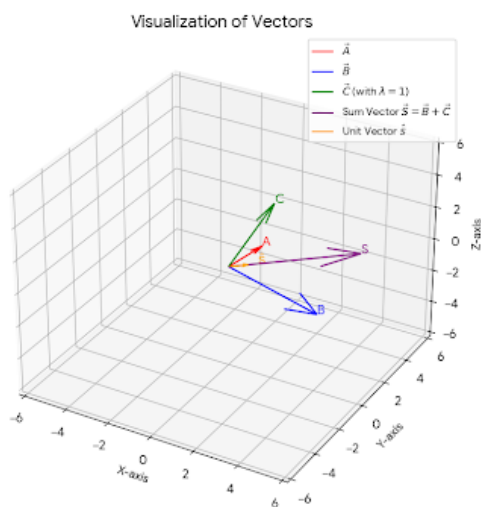


Fig. 0.1