5.4.27

EE25BTECH11032 - Kartik Lahoti

Question:

Using elementary transformations, find the inverse of the following matrix.

$$\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

Solution:
Given the matrix,

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} \tag{0.1}$$

Let $\mathbf{A}^{-1}$ be the inverse of the matrix $\mathbf{A}$
We know that,

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I} \tag{0.2}$$

The augmented matrix of $\left(\mathbf{A} \mid \mathbf{I}\right)$ is given by ,

$$\left(\begin{array}{ccc|ccc} 2 & 0 & -1 & 1 & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right) \tag{0.3}$$

$$\left(\begin{array}{ccc|ccc} 2 & 0 & -1 & 1 & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right) \xleftrightarrow{R_1 \to \frac{1}{2}R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{-1}{2} & \frac{1}{2} & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right) \tag{0.4}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{-1}{2} & \frac{1}{2} & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right) \xleftrightarrow{R_2 \to R_2 - 5R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{-1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{-5}{2} & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right) \tag{0.5}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{-1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{-5}{2} & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right) \xleftrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{-1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{-5}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{5}{2} & -1 & 1 \end{array}\right) \tag{0.6}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{-1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{-5}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{5}{2} & -1 & 1 \end{array}\right) \xleftrightarrow{R_3 \to 2R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{-1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{-5}{2} & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{array}\right) \tag{0.7}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{-1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{-5}{2} & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{array}\right) \xleftrightarrow{R_2 \to R_2 - \frac{5}{2} R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{-1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -15 & 6 & -5 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{array}\right) \tag{0.8}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & \frac{-1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -15 & 6 & -5 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{array}\right) \xleftrightarrow{R_1 \to R_1 + \frac{1}{2} R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & 1 \\ 0 & 1 & 0 & -15 & 6 & -5 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{array}\right) \tag{0.9}$$

Hence ,

$$\mathbf{A}^{-1} = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix} \tag{0.10}$$