1.10.20

EE25BTECH11057 - Rushil Shanmukha Srinivas

Question : Find the direction cosines of the line passing through the two points (-2,4,-5) and (1,2,3).

Solution :

Variable	Description	Values
A	Point	$(-2, 4, -5)$
B	Point	$(1, 2, 3)$

TABLE 0: Variables used

Let

$$\mathbf{A} = \begin{pmatrix} -2 \\ 4 \\ -5 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}. \tag{0.1}$$

Form the 3×2 matrix with these as columns:

$$\mathbf{M} = \begin{pmatrix} -2 & 1 \\ 4 & 2 \\ -5 & 3 \end{pmatrix}. \tag{0.2}$$

Apply the column operation $C_2 \leftarrow C_2 - C_1$ to extract the difference vector as the second column:

$$\mathbf{M} \xrightarrow{C_2 \leftarrow C_2 - C_1} \begin{pmatrix} -2 & 3 \\ 4 & -2 \\ -5 & 8 \end{pmatrix}. \tag{0.3}$$

Thus the direction (difference) vector of the line is

$$\mathbf{v} = \mathbf{AB} = \begin{pmatrix} 3 \\ -2 \\ 8 \end{pmatrix}. \tag{0.4}$$

The length of $\mathbf{v}$ is

$$\mathbf{v}^\top \mathbf{v} = \begin{pmatrix} 3 & -2 & 8 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \\ 8 \end{pmatrix}$$
$$= 3^3 + (-2)^2 + (8)^2$$
$$= 9 + 4 + 64 = 77$$

Therefore, the norm of $\mathbf{v}$ is

$$\|\mathbf{v}\| \triangleq \sqrt{\mathbf{v}^\top \mathbf{v}} = \sqrt{77}$$

The unit vector in the direction of $\mathbf{v}$ is

$$\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\sqrt{77}} \begin{pmatrix} 3 \\ -2 \\ 8 \end{pmatrix}$$

Let α, β, γ be the angles made by the line with the x, y, z axes respectively.Then, the direction cosines are the elements of the above direction vector

$$\cos\alpha = \frac{3}{\sqrt{77}}, \quad \cos\beta = -\frac{2}{\sqrt{77}}, \quad \cos\gamma = \frac{8}{\sqrt{77}}$$

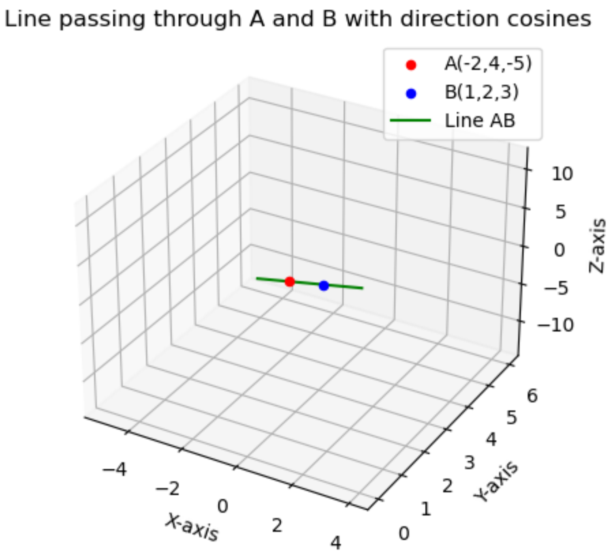

Fig : Vector v