12.287

AI25BTECH11001 - ABHISEK MOHAPATRA

Question: If

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

then $\mathbf{A}^{50}$ is

a) $\begin{pmatrix} 1 & 0 & 0 \\ 50 & 1 & 0 \\ 50 & 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} 1 & 0 & 0 \\ 49 & 1 & 0 \\ 49 & 0 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{pmatrix}$

d) $\begin{pmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 24 & 0 & 1 \end{pmatrix}$

Solution:

Given

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{1}$$

$$\mathbf{A}^2 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} = \mathbf{I} + \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{2}$$

Let $\mathbf{P} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

$$\mathbf{P}^2 = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \mathbf{0} \tag{3}$$

so,

$$\mathbf{A}^{50} = (\mathbf{I} + \mathbf{P})^{25} = \mathbf{I} + 25\mathbf{P} + {}^{25}C_2\mathbf{P}^2 + \ldots + \mathbf{P}^{25} = \mathbf{I} + 25\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{pmatrix}(C) \tag{4}$$