Problem 4.4.26

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Question

Question: Find the equation of the median through vertex **A** of the triangle ABC, having vertices

$$\mathbf{A}(2,5), \quad \mathbf{B}(-4,9), \quad \mathbf{C}(-2,-1).$$

Solution

Using the section formula, the midpoint $\mathbf{M}$ of the side BC is

$$\mathbf{M} = \frac{\mathbf{B}+\mathbf{C}}{2} = \frac{1}{2}\begin{pmatrix}-4 & 9\end{pmatrix} + \frac{1}{2}\begin{pmatrix}-2 & -1\end{pmatrix} = \begin{pmatrix}-3 & 4\end{pmatrix}.$$

The median passes through points $\mathbf{A}(2,5)$ and $\mathbf{M}(-3,4)$. Let the required line have the equation

$$\mathbf{n}^\top \mathbf{x} = 1$$

where $\mathbf{n} = \begin{pmatrix}n_1 & n_2\end{pmatrix}$ is the direction vector. Since both the points A and M lie on the median, they satisfy the line equation. That is,

$$\mathbf{n}^\top \mathbf{A} = 1, \quad \mathbf{n}^\top \mathbf{M} = 1$$

or, writing explicitly for the points $A(2,5)$, $M(-3,4)$:

$$\begin{pmatrix}2 & 5\\ -3 & 4\end{pmatrix}\begin{pmatrix}n_1\\ n_2\end{pmatrix} = \begin{pmatrix}1\\ 1\end{pmatrix}$$

Solution

We want to find the vector $\mathbf{n}$ satisfying

$$\begin{pmatrix} 2 & 5 \\ -3 & 4 \end{pmatrix} \mathbf{n} = \mathbf{c}.$$

Set up the augmented matrix with right-hand side 1:

$$\left(\begin{array}{cc|c} 2 & 5 & 1 \\ -3 & 4 & 1 \end{array}\right) \tag{2.1}$$

Perform row operation $R_2 \to R_2 + \frac{3}{2}R_1$:

$$\left(\begin{array}{cc|c} 2 & 5 & 1 \\ 0 & \frac{23}{2} & \frac{5}{2} \end{array}\right) \tag{2.2}$$

Perform row operation $R_1 \to R_1 - \frac{10}{23}R_2$:

$$\left(\begin{array}{cc|c} 2 & 0 & -\frac{2}{23} \\ 0 & \frac{23}{2} & \frac{5}{2} \end{array}\right) \tag{2.3}$$

Solution

The final augmented matrix is:

$$\left(\begin{array}{cc|c} 2 & 0 & -\frac{2}{23} \\ 0 & \frac{23}{2} & \frac{5}{2} \end{array}\right) \tag{2.4}$$

Solve the system:

$$2n_1 = -\frac{2}{23} \quad \Rightarrow \quad n_1 = -\frac{1}{23}$$
$$\frac{23}{2}n_2 = \frac{5}{2} \quad \Rightarrow \quad n_2 = \frac{5}{23}$$

$$\mathbf{n} = \frac{1}{23}\begin{pmatrix} -1 \\ 5 \end{pmatrix}$$

$$\mathbf{n}^\top \mathbf{x} = 1$$

Substitute $\mathbf{n}$:

$$\left(\frac{1}{23}\begin{pmatrix} -1 \\ 5 \end{pmatrix}\right)^\top \mathbf{x} = 1$$

Solution

$$\begin{pmatrix} -1 & 5 \end{pmatrix} \mathbf{x} = 23$$

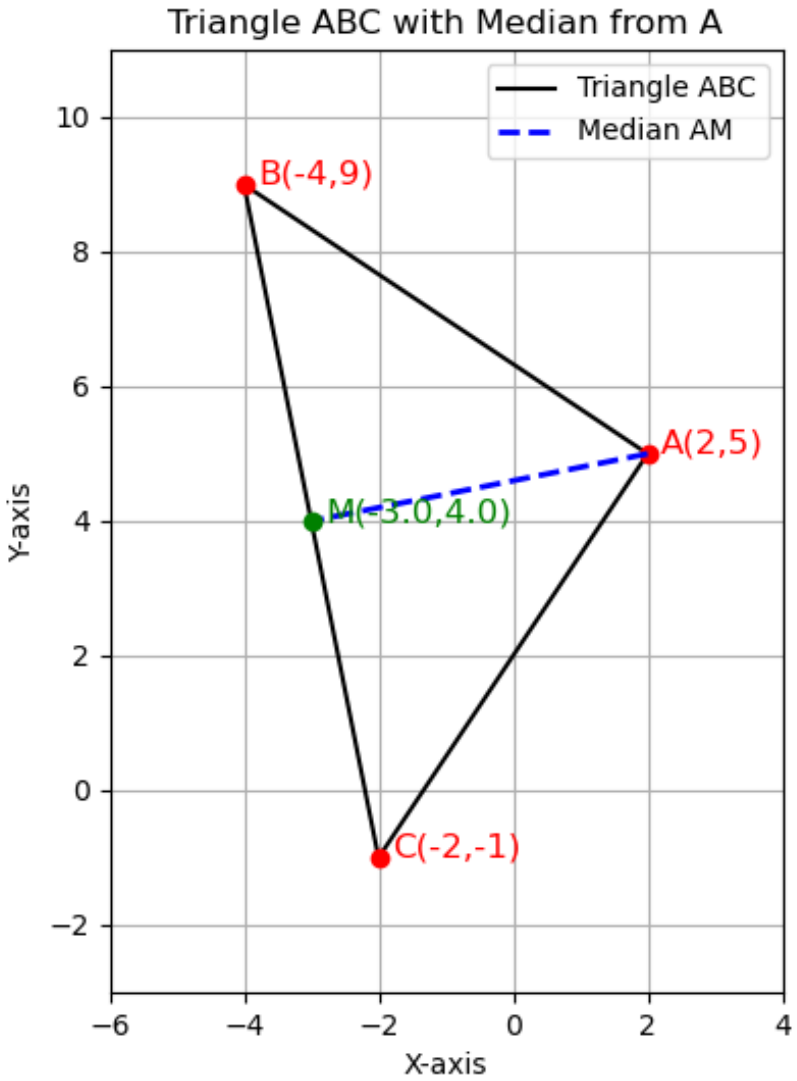
or equivalently,

$$5y - x = 23.$$

Therefore, equation of required line is:

$$\boxed{5y - x = 23.}$$

Graph


Triangle ABC with Median from A
Triangle ABC
Median AM
B(-4,9)
A(2,5)
M(-3.0,4.0)
C(-2,-1)
Y-axis
X-axis
10
8
6
4
2
0
−2
−6
−4
−2
0
2
4

C Code

```c
#include <stdio.h>
#include "trianglefun.h"

int main() {
    // Vertices of triangle
    int Ax = 2, Ay = 5;
    int Bx = -4, By = 9;
    int Cx = -2, Cy = -1;

    char equation[50];

    // Calculate the median equation and store as string
    median_equation(Ax, Ay, Bx, By, Cx, Cy, equation);

    // Print the equation
    printf("Equation of the median from A: %s\n", equation);

    return 0;
}
```

Python Code for Plotting

```
import matplotlib.pyplot as plt
import numpy as np

# Vertices of the triangle
A = np.array([2, 5])
B = np.array([-4, 9])
C = np.array([-2, -1])

# Calculate midpoint M of BC
M = (B + C) / 2

# Plot triangle
plt.figure(figsize=(6,6))
triangle_points = np.array([A, B, C, A])
plt.plot(triangle_points[:,0], triangle_points[:,1], 'k-', label=
    'Triangle ABC')

# Plot vertices
plt.plot(A[0], A[1], 'ro')
```

Python Code for Plotting

```
plt.plot(B[0], B[1], 'ro')
plt.plot(C[0], C[1], 'ro')

# Label vertices
plt.text(A[0]+0.2, A[1], 'A(2,5)', fontsize=12, color='red')
plt.text(B[0]+0.2, B[1], 'B(-4,9)', fontsize=12, color='red')
plt.text(C[0]+0.2, C[1], 'C(-2,-1)', fontsize=12, color='red')

# Plot median from A to midpoint M
plt.plot([A[0], M[0]], [A[1], M[1]], 'b--', linewidth=2, label='
    Median AM')

# Label midpoint M
plt.plot(M[0], M[1], 'go')
plt.text(M[0]+0.2, M[1], f'M({M[0]:.1f},{M[1]:.1f})', fontsize
    =12, color='green')

# Position to place equation on the median line midpoint
mid_x = (A[0] + M[0]) / 2
```

Python Code for Plotting

```
mid_y = (A[1] + M[1]) / 2

# Settings
plt.gca().set_aspect('equal', adjustable='box')
plt.grid(True)
plt.legend()
plt.title('Triangle ABC with Median from A')
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.xlim(-6, 4)
plt.ylim(-3, 11)

# Save the figure as PNG
filename = 'triangle_median_eqonline.png'
plt.savefig(filename)
plt.close()
```