1.3.6

AI25BTECH11027 - NAGA BHUVANA

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Question:
Show that the points $\mathbf{A}\,(6,2)$,$\mathbf{B}\,(2,1)$,$\mathbf{C}\,(1,5)$ and $\mathbf{D}\,(5,6)$ are vertices of a square.
Solution:
From the given information,

$$\mathbf{A} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} \tag{0.1}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2-6 \\ 1-2 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \tag{0.2}$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} 1-5 \\ 5-6 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \tag{0.3}$$

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \tag{0.4}$$

By the above property we can say that **ABCD** is a parallelogram. Now

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2-6 \\ 1-2 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \tag{0.5}$$

$$\implies (\mathbf{B} - \mathbf{A})^T = \begin{pmatrix} -4 & -1 \end{pmatrix} \tag{0.6}$$

$$\tag{0.7}$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 1-2 \\ 5-1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} \tag{0.8}$$

$$\implies (\mathbf{C} - \mathbf{B})^T = \begin{pmatrix} -1 & 4 \end{pmatrix} \tag{0.9}$$

$$\tag{0.10}$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} 5-1 \\ 6-5 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \tag{0.11}$$

$$\implies (\mathbf{D} - \mathbf{C})^T = \begin{pmatrix} 4 & 1 \end{pmatrix} \tag{0.12}$$

$$\tag{0.13}$$

$$\mathbf{A}-\mathbf{D}=\begin{pmatrix}6-5\\2-6\end{pmatrix}=\begin{pmatrix}1\\-4\end{pmatrix} \tag{0.14}$$

$$\implies (\mathbf{A}-\mathbf{D})^T=\begin{pmatrix}1 & -4\end{pmatrix} \tag{0.15}$$

$$\tag{0.16}$$

$$\mathbf{C}-\mathbf{A}=\begin{pmatrix}1-6\\5-2\end{pmatrix}=\begin{pmatrix}-5\\-3\end{pmatrix} \tag{0.17}$$

$$\implies (\mathbf{C}-\mathbf{A})^T=\begin{pmatrix}-5 & -3\end{pmatrix} \tag{0.18}$$

$$\tag{0.19}$$

$$\mathbf{D}-\mathbf{B}=\begin{pmatrix}5-2\\6-1\end{pmatrix}=\begin{pmatrix}3\\6\end{pmatrix} \tag{0.20}$$

$$\implies (\mathbf{D}-\mathbf{B})^T=\begin{pmatrix}3 & 6\end{pmatrix} \tag{0.21}$$

$$\tag{0.22}$$

The magnitude of the sides and the diagonals of the parallelogram are

$$\|\mathbf{B}-\mathbf{A}\|^2 = \left(B-A\right)^T \left(B-A\right) \tag{0.23}$$

$$\tag{0.24}$$

$$\|\mathbf{B}-\mathbf{A}\|^2 = \begin{pmatrix}-4 & -1\end{pmatrix} \begin{pmatrix}-4 \\ -1\end{pmatrix} \tag{0.25}$$

$$\|\mathbf{B}-\mathbf{A}\|^2 = (-4)^2 + (-1)^2 = 17 \tag{0.26}$$

$$\therefore \|\mathbf{B}-\mathbf{A}\| = \sqrt{17} \tag{0.27}$$

$$\|\mathbf{C}-\mathbf{B}\|^2 = \left(C-B\right)^T \left(C-B\right) \tag{0.28}$$

$$\|\mathbf{C}-\mathbf{B}\|^2 = \begin{pmatrix}-1 & 4\end{pmatrix} \begin{pmatrix}-1 \\ 4\end{pmatrix} \tag{0.29}$$

$$\|\mathbf{C}-\mathbf{B}\|^2 = (-1)^2 + (4)^2 = 17 \tag{0.30}$$

$$\therefore \|\mathbf{C}-\mathbf{B}\| = \sqrt{17} \tag{0.31}$$

$$\|\mathbf{D}-\mathbf{C}\|^2 = \left(D-C\right)^T \left(D-C\right) \tag{0.32}$$

$$\|\mathbf{D}-\mathbf{C}\|^2 = \begin{pmatrix} 4 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix} \tag{0.33}$$

$$\|\mathbf{D}-\mathbf{C}\|^2 = (4)^2 + (1)^2 = 17 \tag{0.34}$$

$$\therefore \|\mathbf{D}-\mathbf{C}\| = \sqrt{17} \tag{0.35}$$

$$\|\mathbf{A}-\mathbf{D}\|^2 = \left(A-D\right)^T \left(A-D\right) \tag{0.36}$$

$$\|\mathbf{A}-\mathbf{D}\|^2 = \begin{pmatrix} 1 & -4 \end{pmatrix} \begin{pmatrix} 1 \\ -4 \end{pmatrix} \tag{0.37}$$

$$\|\mathbf{A}-\mathbf{D}\|^2 = (1)^2 + (-4)^2 = 17 \tag{0.38}$$

$$\therefore \|\mathbf{A}-\mathbf{D}\| = \sqrt{17} \tag{0.39}$$

$$\tag{0.40}$$

$$\|\mathbf{B}-\mathbf{A}\| = \|\mathbf{C}-\mathbf{B}\| = \|\mathbf{D}-\mathbf{C}\| = \|\mathbf{A}-\mathbf{D}\| = \sqrt{17} \tag{0.41}$$

From the above all the sides of the parallelogram are equal
Now consider the diagonals of the parallelogram

$$\|\mathbf{C}-\mathbf{A}\|^2 = \left(C-A\right)^T \left(C-A\right) \tag{0.42}$$
$$\tag{0.43}$$

$$\|\mathbf{C}-\mathbf{A}\|^2 = \begin{pmatrix}-5 & -3\end{pmatrix}\begin{pmatrix}-5\\-3\end{pmatrix} \tag{0.44}$$
$$\|\mathbf{C}-\mathbf{A}\|^2 = (-5)^2+(-3)^2 = 34 \tag{0.45}$$
$$\|\mathbf{C}-\mathbf{A}\| = \sqrt{34} \tag{0.46}$$
$$\tag{0.47}$$

$$\|\mathbf{D}-\mathbf{B}\|^2 = \left(D-B\right)^T \left(D-B\right) \tag{0.48}$$
$$\tag{0.49}$$

$$\|\mathbf{D}-\mathbf{B}\|^2 = \begin{pmatrix} 3 & 5 \end{pmatrix} \begin{pmatrix} 3 \\ 5 \end{pmatrix} \tag{0.50}$$

$$\|\mathbf{D}-\mathbf{B}\|^2 = (3)^2 + (5)^2 = 34 \tag{0.51}$$

$$\|\mathbf{D}-\mathbf{B}\| = \sqrt{34} \tag{0.52}$$

$$\tag{0.53}$$

$$\|\mathbf{C}-\mathbf{A}\| = \|\mathbf{D}-\mathbf{B}\| = \sqrt{34} \tag{0.54}$$

From the above the diagonals of the parallelogram are equal
Property:
A parallelogram with all the sides of equal length and the diagonals of equal length must be a square.

Graphical Representation

Hence **ABCD** forms a square