5.3.8

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Question

Solve the following system of linear equations using matrix row operations.

$$217a + 131b = 912$$
$$131a + 217b = 827$$

Matrix Form

Augmented matrix:

$$\left[\begin{array}{cc|c} 217 & 131 & 912 \\ 131 & 217 & 827 \end{array}\right]$$

Row Operation 1

Apply $R_1 \leftarrow R_1 \div 217$:

$$R_1 = \begin{bmatrix} 1 & \frac{131}{217} & \frac{912}{217} \end{bmatrix}$$

Now:

$$\left[\begin{array}{cc|c} 1 & \frac{131}{217} & \frac{912}{217} \\ 131 & 217 & 827 \end{array}\right]$$

Row Operation 2

Apply $R_2 \leftarrow R_2 - 131 \cdot R_1$:

$$R_2 = \begin{bmatrix} 0 & 217 - 131 \cdot \frac{131}{217} & 827 - 131 \cdot \frac{912}{217} \end{bmatrix}$$

Simplify:

$$\left[\begin{array}{cc|c} 1 & \frac{131}{217} & \frac{912}{217} \\ 0 & \frac{29928}{217} & \frac{59987}{217} \end{array}\right]$$

Row Operation 3

Apply $R_2 \leftarrow R_2 \div \dfrac{29928}{217}$:

$$R_2 = \begin{bmatrix} 0 & 1 & \dfrac{59987}{29928} \end{bmatrix}$$

Now:

$$\left[\begin{array}{cc|c} 1 & \dfrac{131}{217} & \dfrac{912}{217} \\ 0 & 1 & \dfrac{59987}{29928} \end{array}\right]$$

Row Operation 4

Apply $R_1 \leftarrow R_1 - \frac{131}{217} \cdot R_2$:

$$R_1 = \begin{bmatrix} 1 & 0 & \frac{912}{217} - \frac{131}{217} \cdot \frac{59987}{29928} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{650}{217} \end{bmatrix}$$

Final matrix:

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right]$$

Final Result

After performing matrix row operations on the augmented system, we arrive at the reduced form:

$$\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right]$$

This corresponds to the unique solution of the system:

$$\boxed{\begin{pmatrix} 3 \\ 2 \end{pmatrix}} \quad \text{(Row-reduced form)}$$

Conclusion: The system is consistent and has a unique solution. The values satisfy both equations exactly.

Python Code — SymPy (1/2)

```
from sympy import Matrix

# Augmented matrix
M = Matrix([
    [217, 131, 912],
    [131, 217, 827]
])

# Row 1 normalization
M[0, :] = M[0, :] / 217

# Row 2 elimination
M[1, :] = M[1, :] - 131 * M[0, :]

# Row 2 normalization
M[1, :] = M[1, :] / M[1, 1]
```

Python Code — SymPy (2/2)

```
# Row 1 elimination
M[0, :] = M[0, :] - M[0, 1] * M[1, :]

# Final matrix
print("Reduced matrix:")
print(M)

# Extract solution
sol = M[:, 2]
print("Solution:")
print(sol)
```

C Code — Matrix Logic (1/2)

```
#include <stdio.h>

int main() {
    double M[2][3] = {
        {217, 131, 912},
        {131, 217, 827}
    };

    // Normalize R1
    for (int i = 0; i < 3; i++)
        M[0][i] /= 217;

    // Eliminate R2
    for (int i = 0; i < 3; i++)
        M[1][i] -= 131 * M[0][i];
```

C Code — Matrix Logic (2/2)

```
    // Normalize R2
    for (int i = 0; i < 3; i++)
        M[1][i] /= M[1][1];

    // Eliminate R1
    for (int i = 0; i < 3; i++)
        M[0][i] -= M[0][1] * M[1][i];

    printf("Solution:\n");
    printf("a = %.0f\n", M[0][2]);
    printf("b = %.0f\n", M[1][2]);

    return 0;
}
```

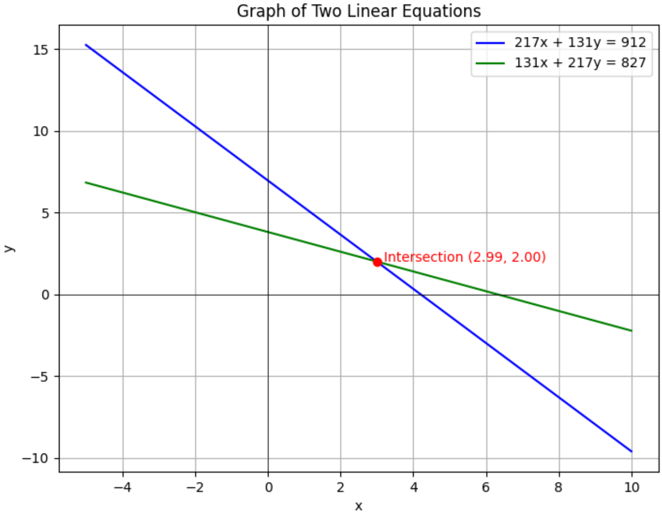
Python Code — Executable Runner (1/2)

```
import subprocess

# Prepare input
input_data = "217 131 912\n131 217 827\n"

# Run C binary
result = subprocess.run(
    ['./solve_538'],
    input=input_data,
    capture_output=True,
    text=True
)

# Output
print("C Output:")
print(result.stdout.strip())
```

Python Code — Executable Runner (2/2)

```
# Optional: check return code
if result.returncode != 0:
    print("Execution failed")
else:
    print("Execution successful")
```

Figure


Graph of Two Linear Equations
217x + 131y = 912
131x + 217y = 827
Intersection (2.99, 2.00)
y
x
15
10
5
0
−5
−10
−4
−2
0
2
4
6
8
10