2.10.3

AI25BTECH11014 - Gooty Suhas

Problem

Find the unit vector perpendicular to the plane determined by the points:

$$\mathbf{P} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

Solution

To find a unit vector perpendicular to the plane, we compute two direction vectors and take their cross product.

Step 1: Direction Vectors

$$\mathbf{PQ} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}, \quad \mathbf{PR} = \mathbf{R} - \mathbf{P} = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$$

Step 2: Cross Product

The cross product of two vectors $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ is:

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

Applying this:

$$\mathbf{N} = \mathbf{PQ} \times \mathbf{PR} = \begin{pmatrix} (1)(-1) - (-3)(3) \\ (-3)(-1) - (1)(1) \\ (1)(3) - (1)(-1) \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \\ 4 \end{pmatrix}$$

Step 3: Magnitude

$$||\mathbf{N}|| = \sqrt{8^2 + 2^2 + 4^2} = \sqrt{84}$$

Step 4: Unit Vector

$$\hat{n} = \frac{1}{\sqrt{84}}\mathbf{N} = \begin{pmatrix} \frac{8}{\sqrt{84}} \\ \frac{2}{\sqrt{84}} \\ \frac{4}{\sqrt{84}} \end{pmatrix}$$

$$\boxed{\hat{n} = \begin{pmatrix} \frac{8}{\sqrt{84}} \\ \frac{2}{\sqrt{84}} \\ \frac{4}{\sqrt{84}} \end{pmatrix}}$$

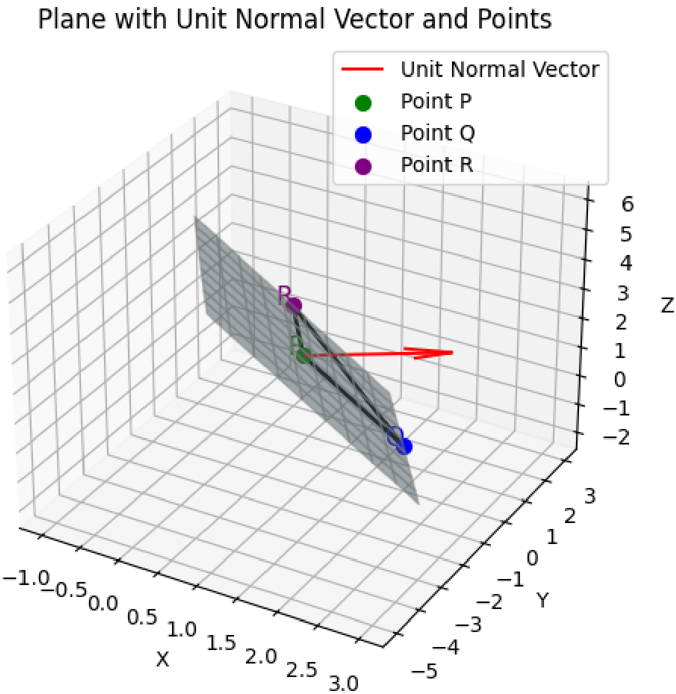

Fig. 0.1: Plane and its normal