Matgeo-2.10.28

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Question

Q 2.10.28. For non-zero vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$, the relation

$$\left|(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}\right| = \|\mathbf{a}\| \, \|\mathbf{b}\| \, \|\mathbf{c}\|$$

holds if and only if

1. $\mathbf{a} \cdot \mathbf{b} = 0, \ \mathbf{b} \cdot \mathbf{c} = 0$
2. $\mathbf{b} \cdot \mathbf{c} = 0, \ \mathbf{c} \cdot \mathbf{a} = 0$
3. $\mathbf{c} \cdot \mathbf{a} = 0, \ \mathbf{a} \cdot \mathbf{b} = 0$
4. $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a} = 0$

Solution

We need the condition for

$$|(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}| = \|\mathbf{a}\| \, \|\mathbf{b}\| \, \|\mathbf{c}\|.$$

Now,

$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \, \|\mathbf{b}\| \sin\theta,$$

where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.
So,

$$|(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}| \le \|\mathbf{a}\| \, \|\mathbf{b}\| \, \|\mathbf{c}\|.$$

Equality holds iff 1. $\sin\theta = 1 \quad \Rightarrow \quad \mathbf{a} \perp \mathbf{b}$*, and 2.*
$\mathbf{c} \parallel (\mathbf{a} \times \mathbf{b}) \quad \Rightarrow \quad \mathbf{c} \perp \mathbf{a}, \; \mathbf{c} \perp \mathbf{b}$*.*
Thus the conditions are

$$\mathbf{a} \cdot \mathbf{b} = 0, \quad \mathbf{b} \cdot \mathbf{c} = 0, \quad \mathbf{c} \cdot \mathbf{a} = 0.$$

Hence, the correct option is

$$\boxed{\text{(d)}}$$

Plot

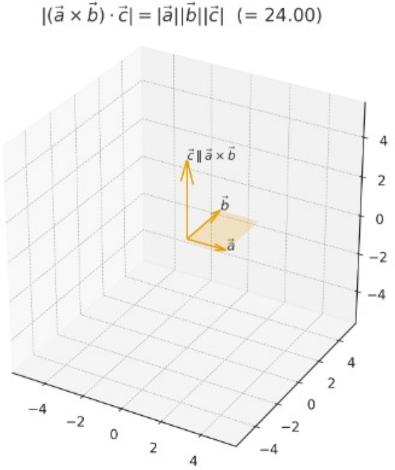

Figure: Illustration of $|(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}| = |\mathbf{a}|\,|\mathbf{b}|\,|\mathbf{c}|$ with $\mathbf{a} \perp \mathbf{b}$ and $\mathbf{c} \parallel (\mathbf{a} \times \mathbf{b})$.