

Matgeo-5.7.12

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Question

Q-5.7.12 If

$$\mathbf{A} = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix},$$

show that $(\mathbf{A} - 2\mathbf{I})(\mathbf{A} - 3\mathbf{I}) = \mathbf{0}$.

Solution

From the characteristic equation definition and the Cayley-Hamilton theorem,

$$f(\lambda) = \det(\mathbf{A} - \lambda \mathbf{I}) = 0, \quad f(\mathbf{A}) = \mathbf{0}.$$

For the given matrix,

$$\begin{aligned} \det(\mathbf{A} - \lambda \mathbf{I}) &= \det \begin{pmatrix} 4 - \lambda & 2 \\ -1 & 1 - \lambda \end{pmatrix} = (4 - \lambda)(1 - \lambda) - (-2) \\ &= \lambda^2 - 5\lambda + 6 = (\lambda - 2)(\lambda - 3). \end{aligned} \tag{1}$$

Hence, by Cayley-Hamilton,

$$f(\mathbf{A}) = \mathbf{A}^2 - 5\mathbf{A} + 6\mathbf{I} = (\mathbf{A} - 2\mathbf{I})(\mathbf{A} - 3\mathbf{I}) = \mathbf{0}. \tag{2}$$

Plot

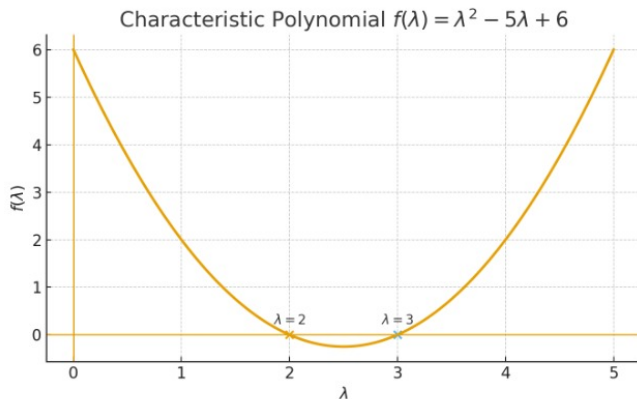


Figure: Characteristic polynomial $f(\lambda) = \lambda^2 - 5\lambda + 6$ with roots $\lambda = 2, 3$.