12.41

INDHIRESH S- EE25BTECH11027

Question. The matrix $A = \begin{pmatrix} 4 & 3 \\ 9 & -2 \end{pmatrix}$ has eigenvalues -5 and 7. The eigenvector(s) is/are

1) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
2) $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$
3) $\begin{pmatrix} 2 \\ -6 \end{pmatrix}$
4) $\begin{pmatrix} 2 \\ 8 \end{pmatrix}$

Solution:
Let us solve the given equation theoretically and then verify the solution computationally. Given

$$\mathbf{A} = \begin{pmatrix} 4 & 3 \\ 9 & -2 \end{pmatrix} , \lambda_1 = -5 \;\; and \;\; \lambda_2 = 7 \tag{1}$$

Now

$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x} \tag{2}$$

$$\left(\mathbf{A} - \lambda\mathbf{I}\right)\mathbf{x} = 0 \tag{3}$$

Here $\mathbf{x}$ is eigen vector.

$$\left(\begin{pmatrix} 4 & 3 \\ 9 & -2 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right)\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{4}$$

For $\lambda = -5$

$$\begin{pmatrix} 9 & 3 \\ 9 & 3 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{5}$$

Let

$$\mathbf{x} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{6}$$

By substituting Eq.6 in Eq.5 we get

$$9a = -3b \tag{7}$$

$$3a = -b \tag{8}$$

$$\mathbf{x} = \begin{pmatrix} a \\ -3a \end{pmatrix} \tag{9}$$

$$\mathbf{x} = a\begin{pmatrix} 1 \\ -3 \end{pmatrix} \tag{10}$$

Where a is a scalar. So the eigen vector will be the scalar multiple of $\mathbf{x}$.
For $a = 2$

$$\mathbf{x} = \begin{pmatrix} 2 \\ -6 \end{pmatrix} \tag{11}$$

Option (3) is correct
For $\lambda = 7$

$$\begin{pmatrix} -3 & 3 \\ 9 & -9 \end{pmatrix}\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{12}$$

By substituting Eq.6 in Eq.11 we get

$$a = b \tag{13}$$

$$\mathbf{x} = \begin{pmatrix} a \\ a \end{pmatrix} \tag{14}$$

$$\mathbf{x} = a\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{15}$$

Where a is a scalar. So the eigen vector will be the scalar multiple of $\mathbf{x}$.
For $a = 1$

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{16}$$

Option (1) is also correct
From the figure it is clearly verified that the theoretical solution matches with the computational solution.

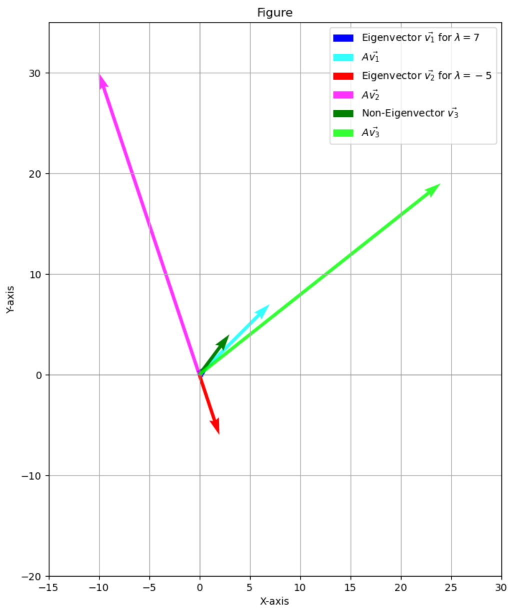

Figure
Eigenvector $\vec{v_1}$ for $\lambda = 7$
$A\vec{v_1}$
Eigenvector $\vec{v_2}$ for $\lambda = -5$
$A\vec{v_2}$
Non-Eigenvector $\vec{v_3}$
$A\vec{v_3}$
30
20
10
0
−10
−20
−15
−10
−5
0
5
10
15
20
25
30
X-axis
Y-axis