

2.10.78

EE25BTECH11050-Hema Havil

Question:

The point (4, 1) undergoes the following three transformations successively.

- (a) Reflection about the line $y = x$.
- (b) Translation through a distance 2 units along the positive direction of x-axis.
- (c) Rotation through an angle $\frac{\pi}{4}$ about the origin in the counter clockwise direction.

Then the final position of the point is given by the coordinates.

- (a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$
- (b) $\left(-\sqrt{2}, 7\sqrt{2}\right)$
- (c) $\left(\sqrt{2}, 7\sqrt{2}\right)$
- (d) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$

Solution:

Let the given point be $P=(4,1)$ and vector be $\mathbf{P} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$,

- (a) Reflection matrix for $y = x$ is,

$$\mathbf{M} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad ((a).1)$$

then the reflection of P about $y = x$ is,

$$\mathbf{P}_1 = \mathbf{MP} \quad ((a).2)$$

$$\mathbf{P}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad ((a).3)$$

$$\mathbf{P}_1 = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad ((a).4)$$

- (b) Translate point $P_1 = (1, 4)$, 2 units along the positive direction of x axis,

$$\mathbf{P}_2 = \mathbf{P}_1 + \mathbf{X} \quad ((b).1)$$

where $\mathbf{X}=(2,0)$

$$\mathbf{P}_2 = \begin{pmatrix} 1 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad ((b).2)$$

$$\mathbf{P}_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad ((b).3)$$

(c) Rotation matrix is given as,

$$\mathbf{R} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \quad ((c).1)$$

Rotation of P_2 by an angle $\frac{\pi}{4}$ about the origin in the counter clockwise direction, substitute $\frac{\pi}{4}$ in \mathbf{R}

$$\mathbf{P}_3 = \mathbf{R}\mathbf{P}_2 \quad ((c).2)$$

$$\mathbf{P}_3 = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad ((c).3)$$

$$\mathbf{P}_3 = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{7}{\sqrt{2}} \end{pmatrix} \quad ((c).4)$$

Therefore the final position of the point is $P_3 = (-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}})$, option (d) is correct

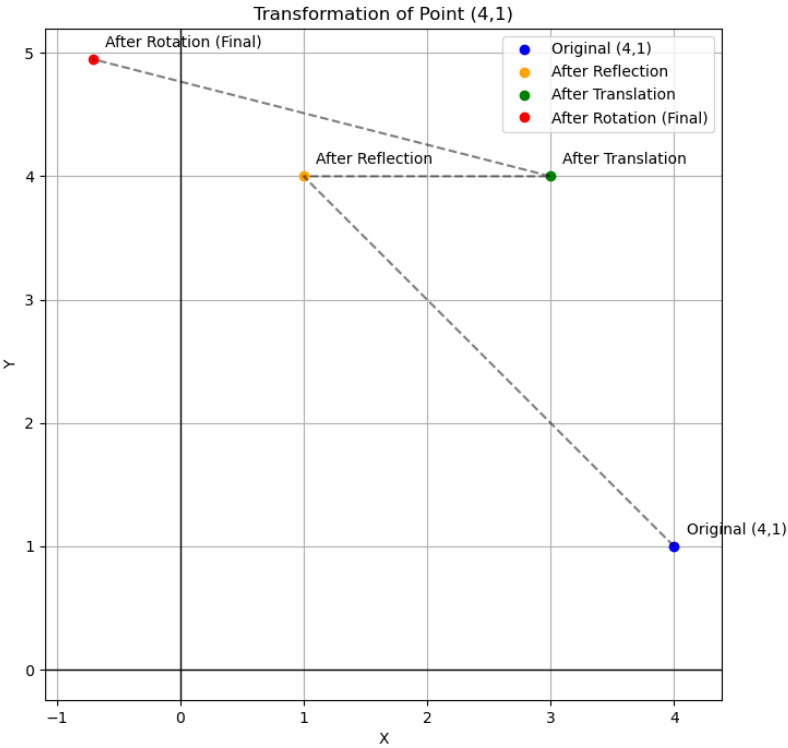


Fig. 3.1: Plot for the above Transformations