12.872

EE25BTECH11026-Harsha

Question:
Let $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 1 & 3 \\ -2 & -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$. For $\mathbf{Ax} = \mathbf{b}$ to be solvable, which one of the following options is the correct condition on $b_1, b_2,$ *and* b_3.

1) $b_1 + b_2 + b_3 = 1$
2) $3b_1 + b_2 + 2b_3 = 0$
3) $b_1 + 3b_2 + b_3 = 2$
4) $b_1 + b_2 + b_3 = 2$

Solution:
Let us solve the given question theoretically and then verify the solution computationally.

Let $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. Forming the augmented matrix of $\mathbf{A}$ and $\mathbf{b}$,

$$\left(\begin{array}{cc|c} 1 & 1 & b_1 \\ 1 & 3 & b_2 \\ -2 & -3 & b_3 \end{array}\right) \xleftrightarrow[R_2 \leftarrow R_2 - R_1]{R_3 \leftarrow R_3 + R_2} \left(\begin{array}{cc|c} 1 & 1 & b_1 \\ 0 & 2 & b_2 - b_1 \\ -1 & 0 & b_2 + b_3 \end{array}\right) \tag{4.1}$$

$$\implies \begin{pmatrix} x_1 + x_2 \\ 2x_2 \\ -x_1 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 - b_1 \\ b_2 + b_3 \end{pmatrix} \tag{4.2}$$

$$\therefore x_1 = -(b_2 + b_3) \qquad x_2 = \frac{b_2 - b_1}{2} \tag{4.3}$$

Substituting the above, yielding,

$$\therefore -(b_2 + b_3) + \frac{b_2 - b_1}{2} = b_1 \tag{4.4}$$

$$\implies 3b_1 + b_2 + 2b_3 = 0 \tag{4.5}$$