4.3.20

Sai Sreevallabh - EE25BTECH11031

September 17, 2025

Question

Find the ratio in which the Y-axis divides the line segment joining the points $(5, -6)$ and $(-1, -4)$. Also find the point of intersection.

Theoretical Solution

Given points are

$$\mathbf{A} = \begin{pmatrix} 5 \\ -6 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} \tag{1}$$

Let $\mathbf{P}$ be a point on the Y-axis. We can assume it to be

$$\mathbf{P} = \begin{pmatrix} 0 \\ y \end{pmatrix} \tag{2}$$

Theoretical Solution

A, **B** and **P** are collinear.

$$\mathbf{P}-\mathbf{A}=\begin{pmatrix}-5\\ y+6\end{pmatrix}\ ,\ \mathbf{B}-\mathbf{A}=\begin{pmatrix}-6\\ 2\end{pmatrix} \tag{3}$$

$$\begin{pmatrix}\mathbf{P}-\mathbf{A} & \mathbf{B}-\mathbf{A}\end{pmatrix}^{T}=\begin{pmatrix}-5 & -6\\ y+6 & 2\end{pmatrix}^{T} \tag{4}$$

$$=\begin{pmatrix}-5 & y+6\\ -6 & 2\end{pmatrix} \tag{5}$$

Theoretical Solution

Converting into echelon form using row operations

$$\begin{pmatrix} x-1 & -3 \\ 3 & 2 \end{pmatrix} \xleftrightarrow{R_2 \to R_2 - \frac{6}{5}R_1} \begin{pmatrix} -5 & y+6 \\ 0 & \frac{-6y-26}{5} \end{pmatrix} \tag{6}$$

The points are collinear. Hence the rank of the above matrix must be 1. So,

$$\frac{6y+26}{5} = 0 \tag{7}$$

$$\implies y = -\frac{13}{3} \tag{8}$$

Theoretical Solution

Let $\mathbf{P}$ divide the line joining points $\mathbf{A}$ and $\mathbf{B}$ in the ratio $k : 1$.

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k+1} \tag{9}$$

$$k\left(\mathbf{P} - \mathbf{B}\right) = \mathbf{A} - \mathbf{P} \tag{10}$$

$$k = \frac{\left(\mathbf{P} - \mathbf{B}\right)^{T}\left(\mathbf{A} - \mathbf{P}\right)}{\left|\left|\left(\mathbf{P} - \mathbf{B}\right)\right|\right|^{2}} \tag{11}$$

Theoretical Solution

$$k = \frac{\begin{pmatrix} 1 & y+4 \end{pmatrix} \begin{pmatrix} 5 \\ -y-6 \end{pmatrix}}{\left\| \begin{pmatrix} 1 \\ y+4 \end{pmatrix} \right\|^2} \tag{12}$$

Substituting the value of y as $-\frac{13}{3}$, we get the value of k as

$$k = 5 \tag{13}$$

$\therefore$ The point $\mathbf{P} \begin{pmatrix} 0 \\ -\frac{13}{3} \end{pmatrix}$ on the X-axis divides the line segment in the ratio $5:1$.

C Code - Function to Find y Coordinate of P

```c
#include <stdio.h>
#include <math.h>

double Solve_for_y(double A[2], double B[2]){

        double k = -A[0]/B[0];
        double y = (A[1]+(k*B[1]))/(k+1);

        return y;
}
```

Python Code - Using Shared Object

```
import numpy as np
import numpy.linalg as LA
import matplotlib.pyplot as plt
import ctypes

c_lib=ctypes.CDLL('./code.so')

c_lib.Solve_for_y.argtypes = [
        ctypes.c_double*2,
        ctypes.c_double*2
        ]

c_lib.Solve_for_y.restype = ctypes.c_double

A = (ctypes.c_double*2)(5.0, -6.0)
B = (ctypes.c_double*2)(-1.0, -4.0)
```

Python Code - Using Shared Object

```
y = c_lib.Solve_for_y(A,B)

y = np.round(y,2)

A = np.array([5,-6]).reshape(-1,1)
B = np.array([-1,-4]).reshape(-1,1)

P = np.array([0.0,y]).reshape(-1,1)

plt.plot([A[0,0], B[0,0]], [A[1,0], B[1,0]], label="Line Segment
    $AB$")

x = np.array([A[0,0], B[0,0], P[0,0]])
y = np.array([A[1,0], B[1,0], P[1,0]])
```

Python Code - Using Shared Object

```
plt.scatter(x,y, c='red')

vert_labels = ['A', 'B', 'P']

for i,txt in enumerate(vert_labels):
    plt.annotate(f'{txt}\n({x[i]},{y[i]})',
                 (x[i],y[i]),
                 textcoords = "offset points",
                 xytext = (20,5),
                 ha = 'center')
```

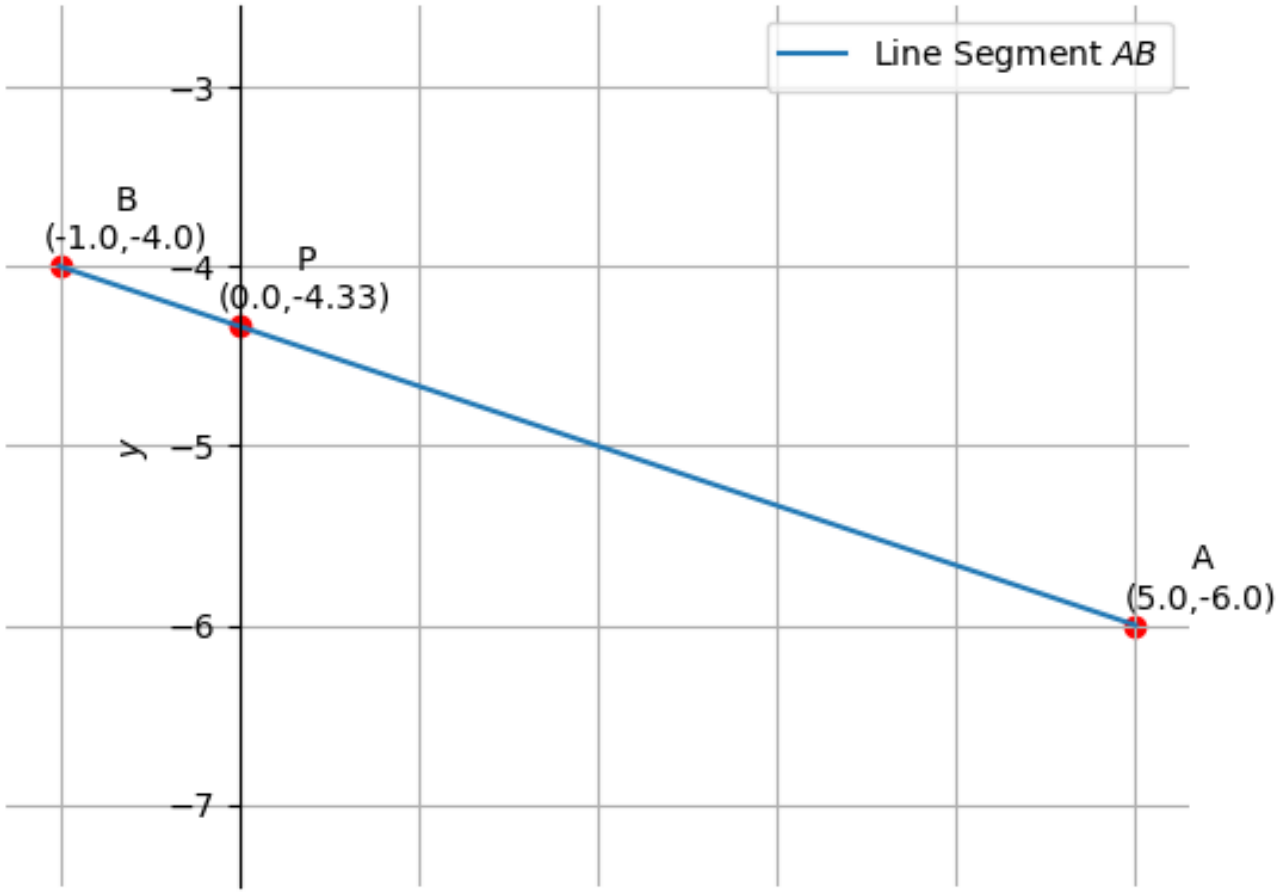
Python Code - Using Shared Object

```
ax = plt.gca()
ax.spines['top'].set_color('none')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.spines['bottom'].set_position('zero')
plt.xlabel('$x$')
plt.ylabel('$y$')
plt.legend(loc='best')
plt.grid()
plt.axis('equal')

plt.savefig("../Figs/plot(py+C).png")

plt.show()
```

Plot-Using Both C and Python



Python Code

```
import math
import numpy as np
import matplotlib.pyplot as plt
import numpy.linalg as LA

A = np.array([5,-6])
B = np.array([-1,-4])

k = -(A[0])/(B[0])

y = (A[1] + k*B[1])/(k+1)

y = np.round(y,2)
```

Python Code

```
P = np.array([0.0,y]).reshape(-1,1)

A = A.reshape(-1,1)
B = B.reshape(-1,1)

plt.plot([A[0,0], B[0,0]], [A[1,0],B[1,0]], label = "Line Segment
     $AB$")

x = np.array([A[0,0], B[0,0], P[0,0]])
y = np.array([A[1,0], B[1,0], P[1,0]])

plt.scatter(x,y, c='red')
```

Python Code

```
vert_labels = ['A', 'B', 'P']

for i,txt in enumerate(vert_labels):
    plt.annotate(f'{txt}({x[i]},{y[i]})',
                 (x[i],y[i]),
                 textcoords = "offset points",
                 xytext = (20,5),
                 ha = 'center')
```

Python Code

```
ax = plt.gca()
ax.spines['top'].set_color('none')
ax.spines['bottom'].set_position('zero')
ax.spines['right'].set_color('none')
ax.spines['left'].set_position('zero')
plt.xlabel('x')
plt.ylabel('y')
plt.legend(loc='best')
plt.grid()
plt.axis('equal')

plt.savefig("../Figs/plot(py).png")
plt.show()
```

Plot-Using Python only