

# 5.8.19

EE25BTECH11021 - Dhanush sagar

## Question:

Let  $a, b, c$  be real numbers. Consider the following system of equations in  $x, y, z$ :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1,$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

The system has:

- 1) no solution
- 2) unique solution
- 3) infinitely many solutions
- 4) finitely many solutions

**Solution:** Let

$$A = \frac{x^2}{a^2}, \tag{1}$$

$$B = \frac{y^2}{b^2}, \tag{2}$$

$$C = \frac{z^2}{c^2}. \tag{3}$$

Then the system becomes

$$A + B - C = 1, \tag{4}$$

$$A - B + C = 1, \tag{5}$$

$$-A + B + C = 1. \tag{6}$$

The augmented matrix is

$$\begin{aligned}
 \begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{pmatrix} &\xrightarrow{R_2 \rightarrow R_2 - R_1} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 2 & 0 \\ -1 & 1 & 1 & 1 \end{pmatrix} \xrightarrow{R_3 \rightarrow R_3 + R_1} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 2 & 0 \\ 0 & 2 & 0 & 2 \end{pmatrix} \\
 &\xrightarrow{R_3 \rightarrow R_3 + R_2} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix} \xrightarrow{R_2 \rightarrow -\frac{1}{2}R_2} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 2 \end{pmatrix} \\
 &\xrightarrow{R_3 \rightarrow \frac{1}{2}R_3} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 + R_3} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \\
 &\xrightarrow{R_1 \rightarrow R_1 + R_3} \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \xrightarrow{R_1 \rightarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}. \tag{7}
 \end{aligned}$$

From the final matrix we read

$$A = 1, \quad B = 1, \quad C = 1. \tag{8}$$

Therefore,

$$\frac{x^2}{a^2} = 1, \quad \frac{y^2}{b^2} = 1, \quad \frac{z^2}{c^2} = 1, \tag{9}$$

which gives

$$x = \pm a, \quad y = \pm b, \quad z = \pm c. \tag{10}$$

Hence there are  $2^3 = 8$  distinct solutions for  $(x, y, z)$ , so the correct choice is

$$\boxed{\text{(d) finitely many solutions}} \tag{11}$$

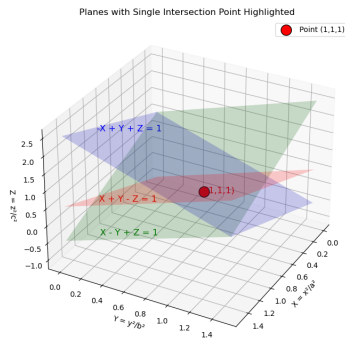


Fig. 4.1