

# 12.254

EE25BTECH11032 - Kartik Lahoti

*Question:*

The two vectors  $[1, 1, 1]$  and  $[1, a, a^2]$ , where  $a = \left(\frac{-1}{2} + j\frac{\sqrt{3}}{2}\right)$

1) orthonormal      2) orthogonal      3) parallel      4) collinear

**Solution:**

Given ,

$$\mathbf{P} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (4.1)$$

$$\mathbf{Q} = \begin{pmatrix} 1 \\ a \\ a^2 \end{pmatrix} \quad (4.2)$$

Let,

$$\mathbf{z}_1 = x_1 + jy_1 \longrightarrow \begin{pmatrix} x_1 & -y_1 \\ y_1 & x_1 \end{pmatrix} \quad (4.3)$$

$$\mathbf{z}_2 = x_2 + jy_2 \longrightarrow \begin{pmatrix} x_2 & -y_2 \\ y_2 & x_2 \end{pmatrix} \quad (4.4)$$

Look at

$$\mathbf{z}_1 + \mathbf{z}_2 = (x_1 + x_2) + j(y_1 + y_2) \quad (4.5)$$

Which is equivalent to

$$\begin{pmatrix} x_1 & -y_1 \\ y_1 & x_1 \end{pmatrix} + \begin{pmatrix} x_2 & -y_2 \\ y_2 & x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 & -y_1 - y_2 \\ y_1 + y_2 & x_1 + x_1 \end{pmatrix} \quad (4.6)$$

Also,

$$\mathbf{z}_1 \mathbf{z}_2 = (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1) \quad (4.7)$$

This is equivalent to

$$\begin{pmatrix} x_1 & -y_1 \\ y_1 & x_1 \end{pmatrix} \begin{pmatrix} x_2 & -y_2 \\ y_2 & x_2 \end{pmatrix} = \begin{pmatrix} (x_1x_2 - y_1y_2) & -(x_1y_2 + x_2y_1) \\ (x_1y_2 + x_2y_1) & (x_1x_2 - y_1y_2) \end{pmatrix} \quad (4.8)$$

$\therefore$  Complex Numbers can be represented as this matrix form since it satisfies Addition and Multiplication properties.

$$x + jy \longrightarrow \begin{pmatrix} x & -y \\ y & x \end{pmatrix} \quad (4.9)$$

$$a = \left( \frac{-1}{2} + j\frac{\sqrt{3}}{2} \right) \longrightarrow \mathbf{A} = \begin{pmatrix} \frac{-1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \quad (4.10)$$

Similarly

$$a^2 = \left( \frac{-1}{2} - j\frac{\sqrt{3}}{2} \right) \longrightarrow \mathbf{A}^2 = \begin{pmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \quad (4.11)$$

$$1 \longrightarrow \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (4.12)$$

Now,

$$\mathbf{I} + \mathbf{A} + \mathbf{A}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \frac{-1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} + \begin{pmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \quad (4.13)$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad (4.14)$$

$$\implies 1 + a + a^2 = 0 \quad (4.15)$$

Now, Look At ,

$$\mathbf{P}^T \mathbf{Q} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ a \\ a^2 \end{pmatrix} = 1 + a + a^2 = 0 \quad (4.16)$$

Hence  $\mathbf{P}$  and  $\mathbf{Q}$  are orthogonal.

Answer : Option (2)