1.9.4

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Question

If $\|\mathbf{a}\| = 4$ and $-3 \leq \lambda \leq 2$, then $\|\lambda\mathbf{a}\|$ lies in

1. $[0, 12]$
2. $[2, 3]$
3. $[8, 12]$
4. $[-12, 8]$

Theoretical Solution

$$\|\mathbf{a}\| = \sqrt{\mathbf{a}^T\mathbf{a}}, \qquad \text{hence} \qquad \mathbf{a}^T\mathbf{a} = \|\mathbf{a}\|^2 = 4^2 = 16. \tag{1}$$

Theoretical Solution

The squared norm of $\lambda\mathbf{a}$ using matrix notation is:

$$\|\lambda\mathbf{a}\|^2 = (\lambda\mathbf{a})^T(\lambda\mathbf{a}) = \lambda^2(\mathbf{a}^T\mathbf{a}).$$

Substituting from Equation (1):

$$\|\lambda\mathbf{a}\|^2 = 16\lambda^2.$$

Taking square roots (norms are nonnegative) gives

$$\|\lambda\mathbf{a}\| = \sqrt{16\lambda^2} = 4\,|\lambda|.$$

Answer

The range of $|\lambda|$ given $-3 \leq \lambda \leq 2$.

$$0 \leq |\lambda| \leq \max\{|-3|, |2|\} = 3.$$

Multiplying by 4 yields

$$0 \leq 4|\lambda| \leq 12.$$

Therefore

$$\|\lambda \mathbf{a}\| = 4|\lambda| \in [0, 12].$$

$$\boxed{\|\lambda \mathbf{a}\| \in [0, 12]}$$

C Code - A function to find the value of $\|\lambda\mathbf{a}\|$

```c
#include <stdio.h>
#include <math.h>

int main() {
    float norm_a = 4; // ||a|| = 4
    float lambda_min = -3, lambda_max = 2;

    // Compute max |lambda|
    float max_abs_lambda = fmax(fabs(lambda_min), fabs(lambda_max
        ));
    float min_abs_lambda = 0; // since lambda can be 0 in [-3, 2]
```

C Code - A function to find the value of $\|\lambda \mathbf{a}\|$

```
// Corresponding ||lambda * a|| values
float min_norm = norm_a * min_abs_lambda;
float max_norm = norm_a * max_abs_lambda;
printf( "||lambda * a|| lies in [%.0f, %.0f]\n " , min_norm,
    max_norm);
return 0;
```

Python Code

```
    import numpy as np
import matplotlib.pyplot as plt
import ctypes
import os

# --- Load the C library ---
try:
    c_lib = ctypes.CDLL('./code.so')
except OSError:
    print("Error: 'code.so' not found.")
    print("Please compile code.c using: gcc -shared -o code.so -
        fPIC code.c")
    exit()

# Define argument and return types for the C function
c_lib.norm_lambda_a.argtypes = [ctypes.c_float, ctypes.c_float]
c_lib.norm_lambda_a.restype = ctypes.c_float
```

Python Code

```
    # --- Given ---
norm_a = 4.0
lam_min, lam_max = -3.0, 2.0

# --- Generate λvalues and call C function ---
lambdas = np.linspace(lam_min, lam_max, 200)
y = np.array([c_lib.norm_lambda_a(ctypes.c_float(l), ctypes.
    c_float(norm_a)) for l in lambdas])

# --- Range ---
y_min, y_max = np.min(y), np.max(y)
print(f" The values of ||λa|| lie in the interval [{y_min:.0f}, {
    y_max:.0f}]")

# --- Plotting ---
plt.plot(lambdas, y, label="||λa|| = 4|λ|", color="blue")
```

Python Code

```
    # Mark endpoints
plt.scatter([lam_min, lam_max],
            [c_lib.norm_lambda_a(lam_min, norm_a), c_lib.
                norm_lambda_a(lam_max, norm_a)],
            color=['red','green'], zorder=5)

# Labels
plt.text(lam_min, c_lib.norm_lambda_a(lam_min, norm_a)+0.5,
         f"({lam_min:.0f},{c_lib.norm_lambda_a(lam_min, norm_a):.0
             f})")
plt.text(lam_max, c_lib.norm_lambda_a(lam_max, norm_a)+0.5,
         f"({lam_max:.0f},{c_lib.norm_lambda_a(lam_max, norm_a):.0
             f})")
```

Python Code

```
    # Axes and grid
plt.axhline(0, color='gray', linewidth=1)
plt.axvline(0, color='gray', linewidth=1)
plt.xlabel("lambda")
plt.ylabel("||lambda a||")
plt.title("Graph of ||lambda a|| = 4|lambda|")
plt.legend(loc='best')
plt.grid(True)
plt.show()
```

Plot