8.2.48

AI25BTECH11012 - GARIGE UNNATHI

Question:
Find the equation of the conic, that satisfies the given conditions.
focus (-1,-2) and directrix x - 2y + 3 = 0.

Solution: Let :

$$\mathbf{F} = \begin{pmatrix} -1 \\ -2 \end{pmatrix} \tag{0.1}$$

$$directrix \quad equation \quad is : \begin{pmatrix} 1 \\ -2 \end{pmatrix}^T \mathbf{x} = -3 \tag{0.2}$$

The equation of a conic with directrix $\mathbf{n}^T\mathbf{x}$ = c , eccentricity e and focus $\mathbf{F}$ is given by:

$$g(\mathbf{x}) = \mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{0.3}$$

where :

$$\mathbf{V} = ||\mathbf{n}||^2\mathbf{I} - e^2\mathbf{n}\mathbf{n}^T,$$
$$\mathbf{u} = ce^2\mathbf{n} - ||\mathbf{n}||^2\mathbf{F},$$
$$f = ||\mathbf{n}||^2||\mathbf{F}||^2 - c^2e^2$$

From the question we can say that the conic is a parabola that is e = 1 ;
Calculating $\mathbf{V}$,$\mathbf{u}$ and f by using the above equations we get :

$$\mathbf{V} = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \tag{0.4}$$

$$\mathbf{u} = \begin{pmatrix} 2 \\ 16 \end{pmatrix} \tag{0.5}$$

$$f = 16 \tag{0.6}$$

Finding eigen values of $\mathbf{V}$:

$$det|\mathbf{V} - \lambda\mathbf{I}| = 0 \tag{0.7}$$

$$\begin{vmatrix} 4-\lambda & 2 \\ 2 & 1-\lambda \end{vmatrix} = 0 \tag{0.8}$$

$$\lambda^2 - 5\lambda = 0 \tag{0.9}$$

$$\lambda = 5 \quad and \quad 0 \tag{0.10}$$

Eigen vectors **v** for any any square matrix **A** is defined as :

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{v} \tag{0.11}$$

$$(\mathbf{A} - \lambda)\mathbf{v} = 0 \tag{0.12}$$

$$for \quad \lambda = 0 \quad v_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \tag{0.13}$$

$$for \quad \lambda = 5 \quad v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{0.14}$$

Substituting in the equation 0.3 we get the equation of the conic to be :

$$\mathbf{x}^T \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 2 \\ 16 \end{pmatrix}^T \mathbf{x} + 16 = 0 \tag{0.15}$$

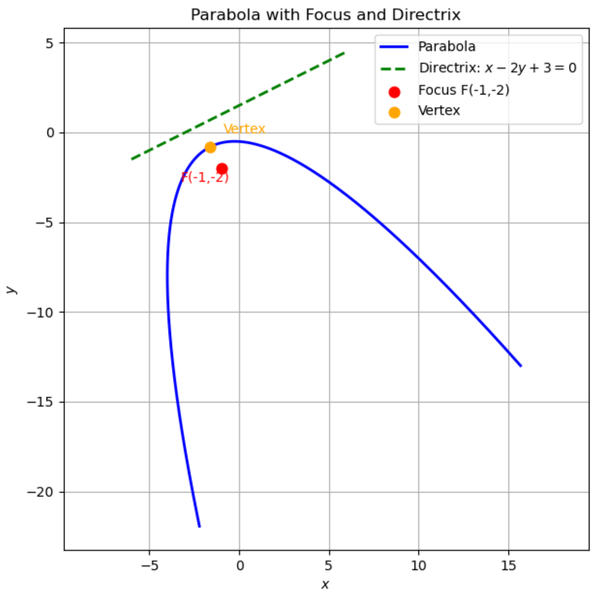

Fig. 0.1