1.5.19

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Find the ratio in which the segment joining the points $(1, 3)$ and $(4, 5)$ is divided by the X-axis. Also find the coordinates of this point on the X-axis.

Theoretical Solution

Given points are

$$\mathbf{A} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \tag{1}$$

Let $\mathbf{P}$ be a point on the x-axis. We can assume it to be

$$\mathbf{P} = \begin{pmatrix} x \\ 0 \end{pmatrix} \tag{2}$$

A, **B** and **P** are collinear.

$$\mathbf{P} - \mathbf{A} = \begin{pmatrix} x - 1 \\ -3 \end{pmatrix} \ , \ \mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{3}$$

$$\left(\mathbf{P} - \mathbf{A} \quad \mathbf{B} - \mathbf{A}\right)^T = \begin{pmatrix} x - 1 & 3 \\ -3 & 2 \end{pmatrix}^T \tag{4}$$

$$= \begin{pmatrix} x - 1 & -3 \\ 3 & 2 \end{pmatrix} \tag{5}$$

Theoretical Solution

Converting into echelon form using row operations

$$\begin{pmatrix} x-1 & -3 \\ 3 & 2 \end{pmatrix} \xleftarrow{R_2 \to R_2 - \frac{3}{x-1}R_1} \begin{pmatrix} x-1 & -3 \\ 0 & \frac{2x+7}{x-1} \end{pmatrix}$$

Since the points are collinear, we can say that the rank of the matrix is 1 i.e.

$$\frac{2x+7}{x-1} = 0 \tag{6}$$

$$\implies x = -\frac{7}{2} \tag{7}$$

Let **P** divide the line joining points **A** and **B** in the ratio $k : 1$.

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \tag{8}$$

$$k(\mathbf{P} - \mathbf{B}) = \mathbf{A} - \mathbf{P} \tag{9}$$

$$k = \frac{(\mathbf{P} - \mathbf{B})^T (\mathbf{A} - \mathbf{P})}{\|(\mathbf{P} - \mathbf{B})\|^2} \tag{10}$$

$$k = \frac{\begin{pmatrix} x - 4 & -5 \end{pmatrix} \begin{pmatrix} 1 - x \\ 3 \end{pmatrix}}{\left\| \begin{pmatrix} x - 4 \\ -5 \end{pmatrix} \right\|^2} \tag{11}$$

Substituting the value of x as $-\frac{7}{2}$, we get the value of k as

$$k = -\frac{3}{5} \tag{12}$$

Therefore,

The point $\mathbf{P}\begin{pmatrix} -\frac{7}{2} \\ 0 \end{pmatrix}$ on the X-axis divides the line segment in the ratio $-3:5$ i.e. externally in the ratio $3:5$.

C Code - Function to Find x Coordinate of P

```c
#include <stdio.h>
#include <math.h>

float Solve_for_x(float x1, float y1, float x2, float y2){

//assuming that the point divides the line in ratio k:1

        float k = -y1/y2;
        float x = (x1+k*x2)/(k+1);

        return x;
}
```

Python Code - Using Shared Object

```python
import sys
import math
import numpy as np
import matplotlib.pyplot as plt
import ctypes
import numpy.linalg as LA

c_lib=ctypes.CDLL("./code.so")

c_lib.Solve_for_x.argtypes = [
        ctypes.c_float,
        ctypes.c_float,
        ctypes.c_float,
        ctypes.c_float
        ]
c_lib.Solve_for_x.restype = ctypes.c_float
```

Python Code - Using Shared Object

```
A= np.array([1,3]).reshape(-1,1)
B= np.array([4,5]).reshape(-1,1)

x = c_lib.Solve_for_x(
        ctypes.c_float(A[0]),
        ctypes.c_float(A[1]),
        ctypes.c_float(B[0]),
        ctypes.c_float(B[1])
        )

#P is the point on X-axis that divides the given line segment in
    the ratio k:1

P = np.array([x,0]).reshape(-1,1)
```

```python
plt.plot([P[0,0], B[0,0]], [P[1,0], B[1,0]], label="Line Segment
    AB (Extended to P)")

plot_coords = np.block([[A, B, P]])
plt.scatter(plot_coords[0,:], plot_coords[1,:], color="red")


vert_labels = [
    f'A({A[0,0]}, {A[1,0]})',
    f'B({B[0,0]}, {B[1,0]})',
    f'P({P[0,0]:.2f}, {P[1,0]})'
]
```

Python Code - Using Shared Object
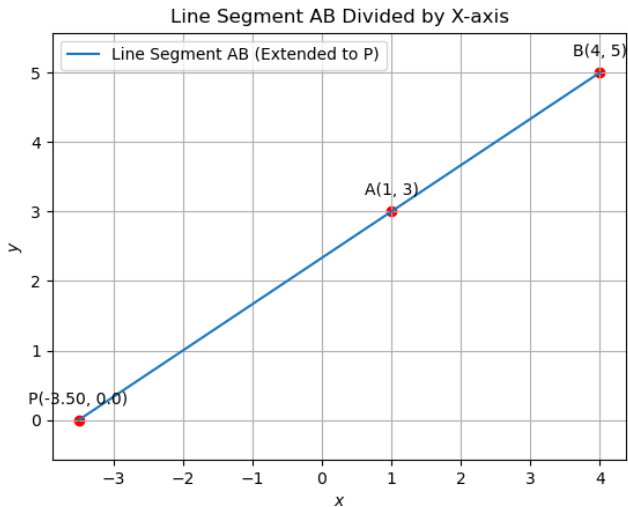
```python
for i, txt in enumerate(vert_labels):
    plt.annotate(txt,
            (plot_coords[0,i],plot_coords[1,i]),
            textcoords="offset points",
            xytext=(0,10),
            ha='center')

plt.xlabel('$x$')
plt.ylabel('$y$')
plt.title("Line Segment AB Divided by X-axis")
plt.legend(loc='upper left')
plt.grid()
plt.axis('equal')

plt.savefig("../Figs/plot(py+C).png")

plt.show()
```

Plot-Using Both C and Python

Line Segment AB Divided by X-axis

Python Code

```python
import sys
import math
sys.path.insert(0, '/home/sai-sreevallabh/Matrix_Theory/Matgeo/
    codes/CoordGeo')
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
import numpy.linalg as LA

#local imports
from line.funcs import *
from triangle.funcs import *

#if using termux
import subprocess
import shlex
```

Python Code

```python
A = np.array([1,3])
B = np.array([4,5])

k = -(A[1])/(B[1])

x = (A[0] + k*B[0])/(k+1)
x = np.round(x,1)

P = np.array([x,0.0])

A = A.reshape(-1,1)
B = B.reshape(-1,1)
P = P.reshape(-1,1)
```

Python Code

```
x_PB = line_gen_num(P, B, 20)
plt.plot(x_PB[0,:],x_PB[1,:], color='green', label="Line Segment
    AB (Extended to P)")

plot_coords = np.block([[A, B, P]])
plt.scatter(plot_coords[0,:], plot_coords[1,:], color='red')

vert_labels = [
    f'A({A[0,0]}, {A[1,0]})',
    f'B({B[0,0]}, {B[1,0]})',
    f'P({P[0,0]:.2f}, {P[1,0]})'
]
```

```python
for i, txt in enumerate(vert_labels):
    plt.annotate(txt,
            (plot_coords[0,i], plot_coords[1,i]),
            textcoords="offset points",
            xytext=(0,10),
            ha='center')

plt.xlabel('$x$')
plt.ylabel('$y$')
plt.title("Line Segment AB Divided by X-axis")
plt.legend(loc='upper left')
plt.grid()
plt.axis('equal')

plt.savefig("../Figs/plot(py).png")
plt.show()
```

Plot-Using Python only

Line Segment AB Divided by X-axis