4.3.57

AI25BTECH11001 - ABHISEK MOHAPATRA

Question: Show that the lines

$$\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha-\delta} \tag{1}$$

$$\frac{x-b+c}{\beta-\delta} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta-\delta} \tag{2}$$

are coplanar.

Solution: Given:

$$\mathbf{L_1} = \mathbf{A} + \lambda\mathbf{m_1} \tag{3}$$

$$\mathbf{L_1} = \begin{pmatrix} a-d \\ a \\ a+d \end{pmatrix} + \lambda \begin{pmatrix} \alpha-\delta \\ \alpha \\ \alpha+\delta \end{pmatrix} \tag{4}$$

And,

$$\mathbf{L_2} = \mathbf{B} + \lambda\mathbf{m_2} \tag{5}$$

$$\mathbf{L_2} = \begin{pmatrix} b-c \\ b \\ b+c \end{pmatrix} + \lambda\mathbf{m_2} \begin{pmatrix} \beta-\delta \\ \beta \\ \beta+\delta \end{pmatrix} \tag{6}$$

If the lines lie in a plane, then they satisfy,

$$nullity\begin{pmatrix} \mathbf{m_1} & \mathbf{m_2} & \mathbf{B}-\mathbf{A} \end{pmatrix} \geq 1 \tag{7}$$

$$nullity\begin{pmatrix} \alpha-\delta & \beta-\delta & a-b+c-d \\ \alpha & \beta & a-b \\ \alpha+\delta & \beta+\delta & a-b-c+d \end{pmatrix} \geq 1 \tag{8}$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} \alpha-\delta & \beta-\delta & a-b+c-d \\ \alpha+\delta & \beta+\delta & a-b-c+d \\ \alpha & \beta & a-b \end{pmatrix} \tag{9}$$

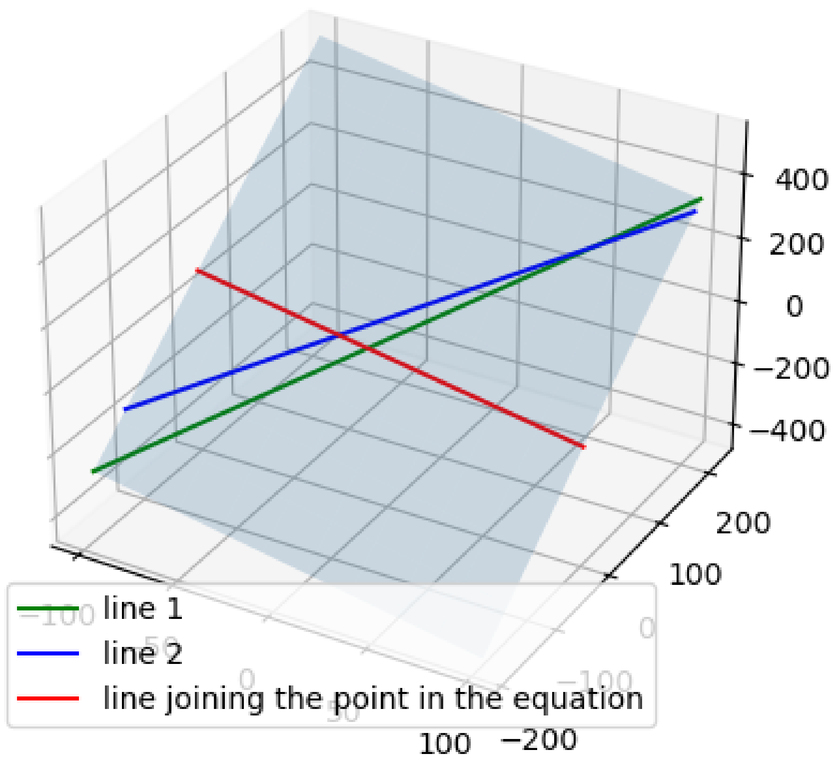
$$\xrightarrow{R_3 \to R_3 - \frac{R_1+R_2}{2}} \begin{pmatrix} \alpha-\delta & \beta-\delta & a-b+c-d \\ \alpha+\delta & \beta+\delta & a-b-c+d \\ 0 & 0 & 0 \end{pmatrix} \tag{10}$$

$$\xrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix} \alpha-\delta & \beta-\delta & a-b+c-d \\ 2\delta & 2\delta & -2c+2d \\ 0 & 0 & 0 \end{pmatrix} \tag{11}$$

$$\xrightarrow{C_1 \to C_1 - C_2} \begin{pmatrix} \alpha-\beta & \beta-\delta & a-b+c-d \\ 0 & 2\delta & -2c+2d \\ 0 & 0 & 0 \end{pmatrix} \tag{12}$$

The matrix is in echelon form and the rank of the matrix is two. And, thus the lines are co-planer.

Graph(using some random values for the variables):



400
200
0
−200
−400
200
100
0
−100
−200
−100
−50
0
50
100
line 1
line 2
line joining the point in the equation