# 5.2.18

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# Question

Solve the following system of linear equation.

$8x + 5y = 9$

$3x + 2y = 4$

# Solution

We are solving the system:

$$8x + 5y = 9, \quad 3x + 2y = 4$$

Coefficient matrix and vector

$$A = \begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 9 \\ 4 \end{bmatrix}$$

# Solution

Performing row operations (RREF)
$R_1 \to \frac{R_1}{8}$

$$\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix} \xrightarrow{R_1 \to R_1/8} \begin{bmatrix} 1 & 5/8 \\ 3 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 9 \\ 4 \end{bmatrix} \xrightarrow{R_1 \to R_1/8} \begin{bmatrix} 9/8 \\ 4 \end{bmatrix}$$

# Solution

Eliminate first column in $R_2$: $R_2 \to R_2 - 3R_1$

$$\begin{bmatrix} 1 & 5/8 \\ 3 & 2 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 3R_1} \begin{bmatrix} 1 & 5/8 \\ 0 & 1/8 \end{bmatrix}, \quad b = \begin{bmatrix} 9/8 \\ 4 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 3R_1} \begin{bmatrix} 9/8 \\ 5/8 \end{bmatrix}$$

$R_2 \to 8R_2$

$$\begin{bmatrix} 1 & 5/8 \\ 0 & 1/8 \end{bmatrix} \xrightarrow{R_2 \to 8R_2} \begin{bmatrix} 1 & 5/8 \\ 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 9/8 \\ 5/8 \end{bmatrix} \xrightarrow{R_2 \to 8R_2} \begin{bmatrix} 9/8 \\ 5 \end{bmatrix}$$

# Solution

Eliminate second column in $R_1$: $R_1 \to R_1 - (5/8)R_2$

$$\begin{bmatrix} 1 & 5/8 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_1 \to R_1 - (5/8)R_2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 9/8 \\ 5 \end{bmatrix} \xrightarrow{R_1 \to R_1 - (5/8)R_2} \begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

Answer

$$x = -1, \quad y = 5$$

```c
 // linear.c
#include <stdio.h>

// Function to solve system of 2 equations:
// a1*x + b1*y = c1
// a2*x + b2*y = c2
// Returns solution in x_out and y_out
void solve_linear(float a1, float b1, float c1,
                  float a2, float b2, float c2,
                  float *x_out, float *y_out) {
    float det = a1*b2 - a2*b1;
    if (det == 0) {
        *x_out = 0;
        *y_out = 0;
        return;
    }
    *x_out = (c1*b2 - c2*b1) / det;
    *y_out = (a1*c2 - a2*c1) / det;
}
```

# Python Code

```
import numpy as np
import matplotlib.pyplot as plt
import ctypes
import os

# --- Load the C library ---
try:
    c_lib = ctypes.CDLL('./linear.so') # use linear.dll on
        Windows
except OSError:
    print("Error: 'linear.so' not found. Compile using: gcc -
        shared -o linear.so -fPIC linear.c")
    exit()
```

# Python Code

```
# Define argument and return types
c_lib.solve_linear.argtypes = [ctypes.c_float, ctypes.c_float,
    ctypes.c_float,
                               ctypes.c_float, ctypes.c_float,
                                   ctypes.c_float,
                               ctypes.POINTER(ctypes.c_float),
                               ctypes.POINTER(ctypes.c_float)]
c_lib.solve_linear.restype = None

# --- Coefficients of system ---
# 8x + 5y = 9
# 3x + 2y = 4
a1, b1, c1 = 8.0, 5.0, 9.0
a2, b2, c2 = 3.0, 2.0, 4.0
```

# Python Code

```
# Allocate memory for outputs
x_out = ctypes.c_float()
y_out = ctypes.c_float()

# Call C function
c_lib.solve_linear(a1, b1, c1,
                   a2, b2, c2,
                   ctypes.byref(x_out),
                   ctypes.byref(y_out))

x_sol, y_sol = x_out.value, y_out.value
print(f"Solution: x = {x_sol:.2f}, y = {y_sol:.2f}")

# --- Plotting ---
x = np.linspace(-5, 5, 400)
y1 = (c1 - a1*x)/b1 # from 8x + 5y = 9
y2 = (c2 - a2*x)/b2 # from 3x + 2y = 4
```

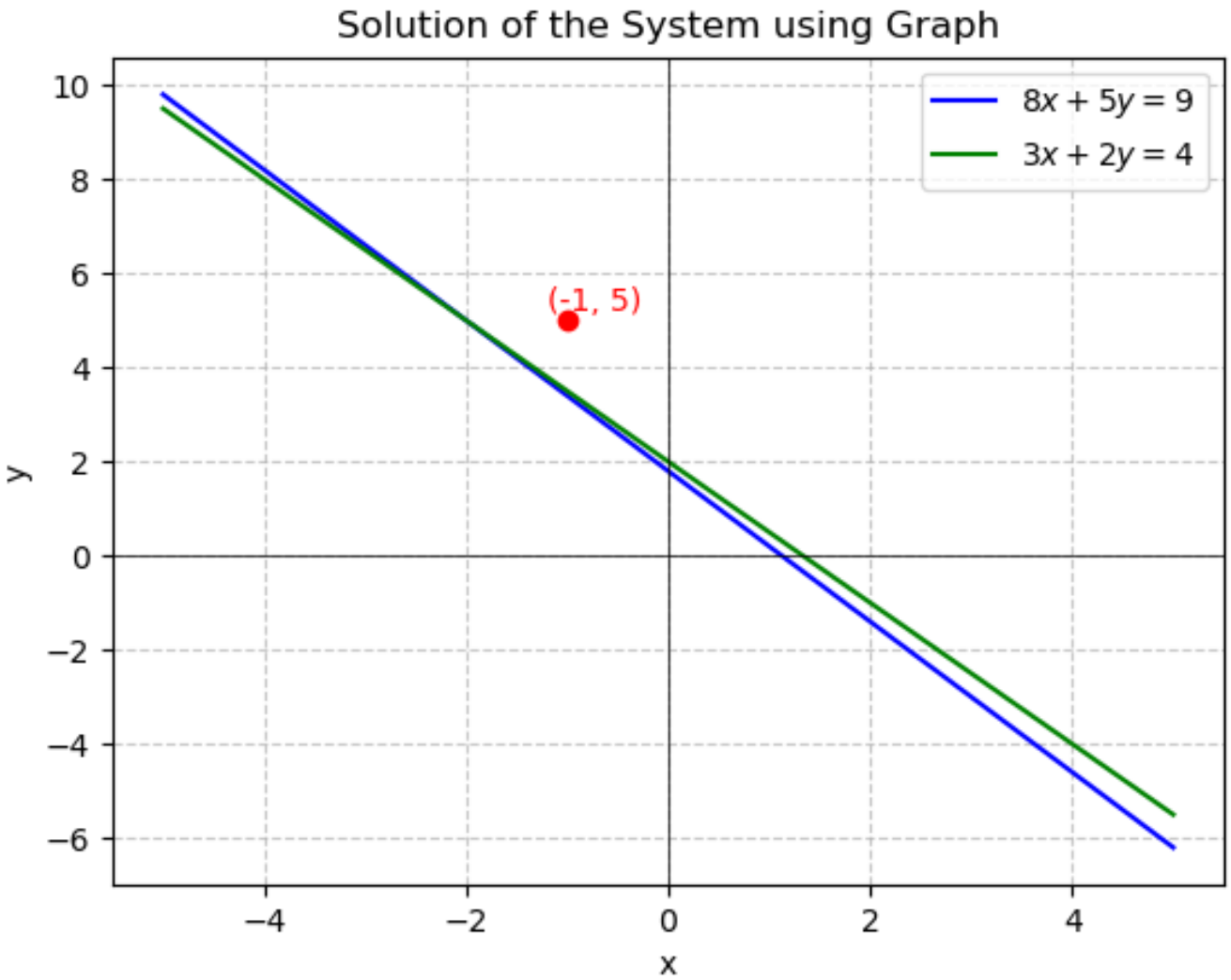
# Python Code

```
plt.plot(x, y1, label=r'$8x + 5y = 9$', color="blue")
plt.plot(x, y2, label=r'$3x + 2y = 4$', color="green")

# Intersection point
plt.plot(x_sol, y_sol, 'ro')
plt.text(x_sol+0.2, y_sol, f"({x_sol:.0f},{y_sol:.0f})", color="
    red")

# Formatting
plt.xlabel("x")
plt.ylabel("y")
plt.title("Solution of Linear System")
plt.grid(True)
plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
plt.legend()
plt.show()
```

# Plot


Solution of the System using Graph
8x + 5y = 9
3x + 2y = 4
(-1, 5)
10
8
6
4
2
0
−2
−4
−6
y
−4
−2
0
2
4
x