

2.4.40

EE25BTECH11043 - Nishid Khandagre

Question: Find the angle between the lines $-\sqrt{3}x + y - 5 = 0$ and $-x + \sqrt{3}y + 6 = 0$.

Solution: Given lines:

$$L_1 : -\sqrt{3}x + y - 5 = 0 \quad (0.1)$$

$$L_2 : -x + \sqrt{3}y + 6 = 0 \quad (0.2)$$

The matrix form of a line can be written as

$$\mathbf{n}^T \mathbf{x} = C \quad (0.3)$$

Where \mathbf{n} is the normal vector and $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ is the position vector.

$$L_1 : \mathbf{n}_1^T \mathbf{x} = c_1 \quad (0.4)$$

$$L_2 : \mathbf{n}_2^T \mathbf{x} = c_2 \quad (0.5)$$

Where \mathbf{n}_1 and \mathbf{n}_2 are the normal vectors to the lines L_1 and L_2 respectively.

$$\mathbf{n}_1 = \begin{pmatrix} -\sqrt{3} \\ 1 \end{pmatrix} \quad (0.6)$$

$$\mathbf{n}_2 = \begin{pmatrix} -1 \\ \sqrt{3} \end{pmatrix} \quad (0.7)$$

The angle θ between the lines is the angle between their normal vectors.

$$\cos \theta = \frac{\mathbf{n}_1^T \mathbf{n}_2}{\|\mathbf{n}_1\| \|\mathbf{n}_2\|} \quad (0.8)$$

$$\mathbf{n}_1^T \mathbf{n}_2 = \begin{pmatrix} -\sqrt{3} & 1 \end{pmatrix} \begin{pmatrix} -1 \\ \sqrt{3} \end{pmatrix} \quad (0.9)$$

$$= (-\sqrt{3})(-1) + (1)(\sqrt{3}) \quad (0.10)$$

$$= 2\sqrt{3} \quad (0.11)$$

$$\|\mathbf{n}_1\| = \sqrt{\mathbf{n}_1^T \mathbf{n}_1} \quad (0.12)$$

$$= \sqrt{(-\sqrt{3})^2 + (1)^2} \quad (0.13)$$

$$= \sqrt{4} \quad (0.14)$$

$$= 2 \quad (0.15)$$

$$\|\mathbf{n}_2\| = \sqrt{\mathbf{n}_2^T \mathbf{n}_2} \quad (0.16)$$

$$= \sqrt{(-1)^2 + (\sqrt{3})^2} \quad (0.17)$$

$$= \sqrt{4} \quad (0.18)$$

$$= 2 \quad (0.19)$$

Now, substitute these values into the formula (??)

$$\cos \theta = \frac{2\sqrt{3}}{(2)(2)} \quad (0.20)$$

$$= \frac{\sqrt{3}}{2} \quad (0.21)$$

$$\theta = \frac{\pi}{6} \text{ radians} \quad (0.22)$$

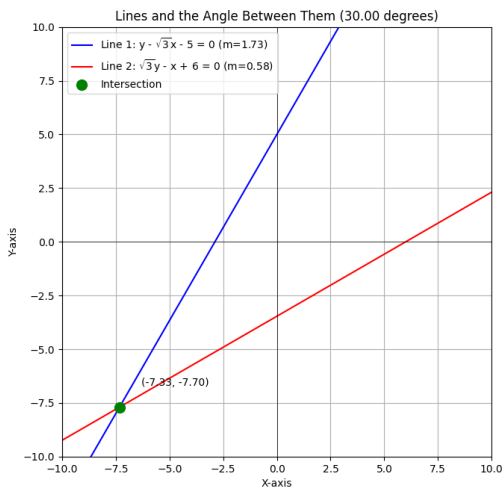


Fig. 0.1