

2.9.11

EE25BTECH11060 - V.Namaswi

Question

If \vec{a} and \vec{b} are unit vectors and θ is angle between them then prove that $\sin \frac{\theta}{2} = \frac{1}{2} |\vec{a} - \vec{b}|$

Solution

Squaring on both sides,

$$\sin^2 \frac{\theta}{2} = \frac{1}{4} |\mathbf{a} - \mathbf{b}|^2$$

consider RHS,

$$\Rightarrow \frac{1}{4} \|\mathbf{a} - \mathbf{b}\|^2 \quad (1)$$

$$= \frac{1}{4} (\mathbf{a} - \mathbf{b})^\top (\mathbf{a} - \mathbf{b}) \quad (2)$$

$$= \frac{1}{4} (\mathbf{a}^\top \mathbf{a} - 2\mathbf{a}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{b}) \quad (3)$$

$$= \frac{1}{4} (1 - 2\mathbf{a}^\top \mathbf{b} + 1) \quad (4)$$

$$= \frac{1}{2} (1 - \mathbf{a}^\top \mathbf{b}) \quad (5)$$

$$= \frac{1}{2} (1 - \cos \theta) \quad (6)$$

$$= \sin^2 \frac{\theta}{2} \quad (7)$$

$$= LHS \quad (8)$$

Hence, $\sin \frac{\theta}{2} = \frac{1}{2} |\vec{a} - \vec{b}|$