Application Problem

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Problem Statement

A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to the denominator. Find the fraction.

Solution

$$\frac{x-1}{y} = \frac{1}{3}, \tag{1}$$

$$\frac{x}{y+8} = \frac{1}{4} \tag{2}$$

$$3(x-1) - y = 0 \implies 3x - y - 3 = 0, \tag{3}$$

$$4x - (y+8) = 0 \implies 4x - y - 8 = 0 \tag{4}$$

$$\begin{pmatrix} 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 3, \tag{5}$$

$$\begin{pmatrix} 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 8 \tag{6}$$

Solution (cont..)

Augmented matrix:

$$\left(\begin{array}{cc|c} 3 & -1 & 3 \\ 4 & -1 & 8 \end{array}\right) \tag{7}$$

RREF using row operations:

$$R_2 \to R_2 - \frac{4}{3}R_1 \implies \left(\begin{array}{cc|c} 3 & -1 & 3 \\ 0 & 1/3 & 4 \end{array}\right) \implies \left(\begin{array}{cc|c} 3 & -1 & 3 \\ 0 & 1 & 12 \end{array}\right), \tag{8}$$

$$R_1 \to R_1 + R_2 \implies \left(\begin{array}{cc|c} 3 & 0 & 15 \\ 0 & 1 & 12 \end{array}\right) \implies \left(\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 12 \end{array}\right) \tag{9}$$

Solution (cont..)

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \tag{10}$$

Hence, the fraction is:

$$\boxed{\frac{5}{12}}$$

Python Code (Plotting Line and Vectors)

```
import numpy as np
import matplotlib.pyplot as plt

m1, c1 = 1.5, 2
m2, c2 = −0.5, 5
A = np.array([0, c1])
B = np.array([0, c2])
x_intersect = (c2 − c1) / (m1 − m2)
y_intersect = m1 ∗ x_intersect + c1
C = np.array([x_intersect, y_intersect])
triangle = np.array([A, B, C, A])
```

Python Code (cont..)

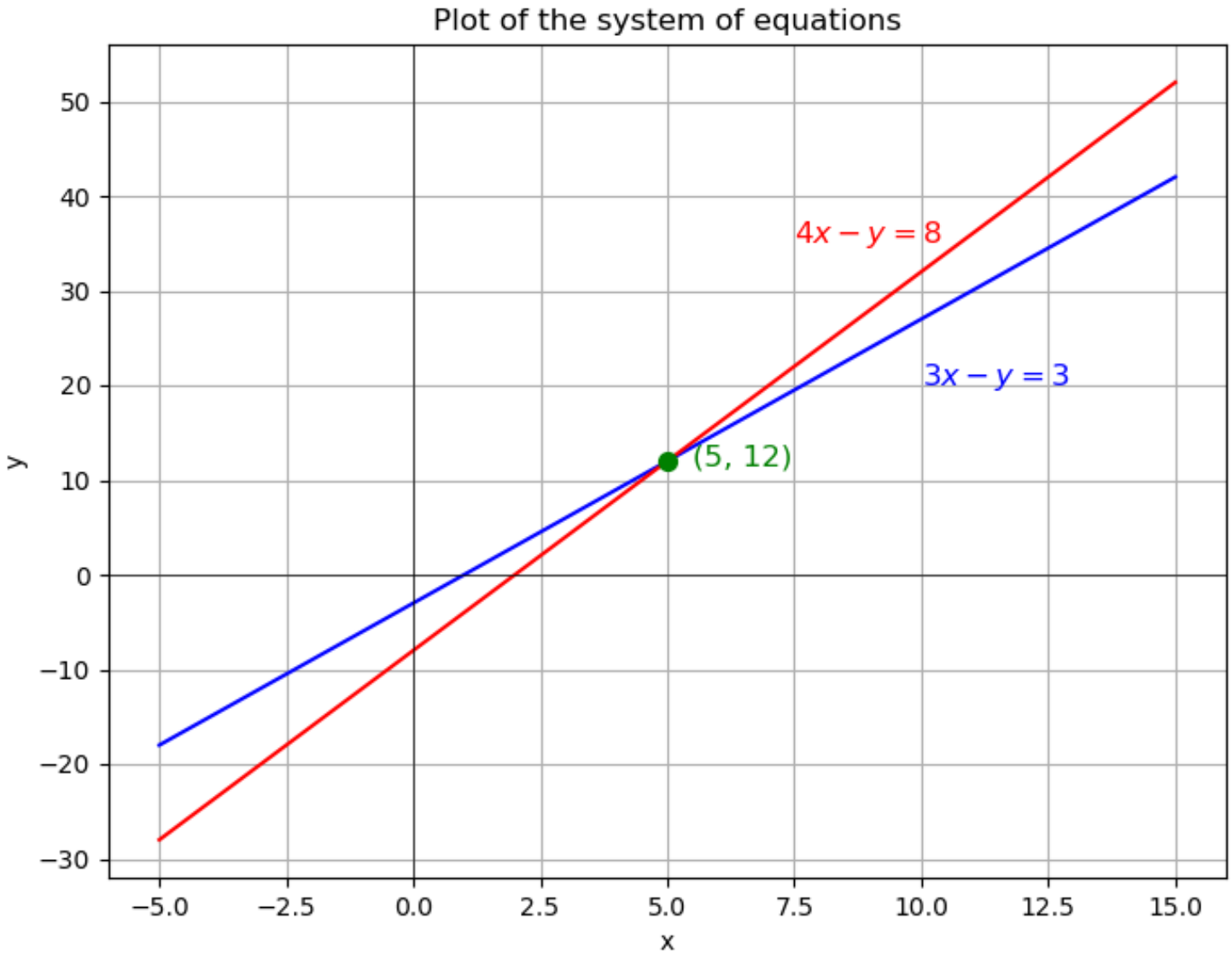
```
plt.figure(figsize=(6,6))
plt.plot(triangle[:,0], triangle[:,1], 'b--o', label='Triangle')
plt.axvline(0, color='k', linewidth=0.8)
plt.axhline(0, color='k', linewidth=0.8)
x_vals = np.linspace(min(0, C[0])-1, max(C[0]+1, 2), 100)
plt.plot(x_vals, m1*x_vals + c1, 'r--', label=f'y={m1}x+{c1}')
plt.plot(x_vals, m2*x_vals + c2, 'g--', label=f'y={m2}x+{c2}')
```

Python Code (cont..)

```
plt.scatter([A[0], B[0], C[0]], [A[1], B[1], C[1]], color='black')
plt.text(A[0]-0.3, A[1]+0.1, 'A')
plt.text(B[0]-0.3, B[1]+0.1, 'B')
plt.text(C[0]+0.1, C[1]+0.1, 'C')
plt.grid(True)
plt.axis('equal')
plt.title('Triangle formed by lines and x=0')

plt.show()
```

Plot


Plot of the system of equations
50
40
30
20
10
0
−10
−20
−30
y
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
12.5
15.0
x
$4x - y = 8$
$3x - y = 3$
(5, 12)

C Code (Computations)

```
#include <stdio.h>

void get_lines(double* x, double* y1, double* y2, int n) {
    for (int i = 0; i < n; i++) {
        y1[i] = 3*x[i] - 3.0;
        y2[i] = 4*x[i] - 8.0;
    }
}
```

Python Code (Using C)

```
import numpy as np
import matplotlib.pyplot as plt
import ctypes

lines_lib = ctypes.CDLL('./points.so')

n = 100
x = np.linspace(−5, 15, n)
y1 = np.zeros(n, dtype=np.float64)
y2 = np.zeros(n, dtype=np.float64)
```

Python Code (Cont..)

```
lines_lib.get_lines.argtypes = [
    np.ctypeslib.ndpointer(dtype=np.float64, ndim=1, flags="
        C_CONTIGUOUS"),
    np.ctypeslib.ndpointer(dtype=np.float64, ndim=1, flags="
        C_CONTIGUOUS"),
    np.ctypeslib.ndpointer(dtype=np.float64, ndim=1, flags="
        C_CONTIGUOUS"),
    ctypes.c_int
]
lines_lib.get_lines(x, y1, y2, n)

plt.figure(figsize=(8, 6))
plt.plot(x, y1, color='blue')
plt.plot(x, y2, color='red')
```

Python Code (Cont..)

```
plt.text(10, 20, r'$3x - y = 3$', color='blue', fontsize=12)
plt.text(7.5, 35, r'$4x - y = 8$', color='red', fontsize=12)

plt.title("System of Equations")
plt.xlabel("x")
plt.ylabel("y")
plt.grid(True)
plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
plt.show()
```