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Question

If $\mathbf{A} = \begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find the scalar k so that $\mathbf{A}^2 + \mathbf{I} = k\mathbf{A}$.

Solution

Given:

$$\mathbf{A} = \begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix}, \quad \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{1}$$

We are asked to find scalar k such that:

$$\mathbf{A}^2 + \mathbf{I} = k\mathbf{A} \tag{2}$$

$$\mathbf{A}^2 = \mathbf{A} \cdot \mathbf{A} = \begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix} \tag{3}$$

$$= \begin{pmatrix} (-3)(-3) + (2)(1) & (-3)(2) + (2)(-1) \\ (1)(-3) + (-1)(1) & (1)(2) + (-1)(-1) \end{pmatrix} = \begin{pmatrix} 11 & -8 \\ -4 & 3 \end{pmatrix} \tag{4}$$

$$\mathbf{A}^2 + \mathbf{I} = \begin{pmatrix} 11 & -8 \\ -4 & 3 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 12 & -8 \\ -4 & 4 \end{pmatrix} \tag{5}$$

Let:

$$k\mathbf{A} = k \cdot \begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} -3k & 2k \\ k & -k \end{pmatrix} \tag{6}$$

Equating both sides:

$$\begin{pmatrix} 12 & -8 \\ -4 & 4 \end{pmatrix} = \begin{pmatrix} -3k & 2k \\ k & -k \end{pmatrix} \tag{7}$$

Compare corresponding entries:

$$-3k = 12 \Rightarrow k = -4 \tag{8}$$

$$2k = -8 \Rightarrow k = -4 \tag{9}$$

Final Answer

$$\boxed{k = -4} \tag{10}$$