4.3.25

EE25BTECH11036 - M Chanakya Srinivas

Question

Find the ratio in which the line joining the points

$$\mathbf{A} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 3 \\ -5 \\ 8 \end{pmatrix}$$

is divided by the YZ-plane.

Solution

Step 1: Algebraic Derivation

Let the direction vector be

$$\mathbf{d} = \mathbf{B} - \mathbf{A}. \tag{1}$$

The parametric equation of the line is

$$\mathbf{R}(\lambda) = \mathbf{A} + \lambda\mathbf{d}. \tag{2}$$

The equation of the YZ-plane is

$$\mathbf{n}^\top \mathbf{x} = 0, \quad \mathbf{n} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \tag{3}$$

Substituting $\mathbf{R}(\lambda)$,

$$\mathbf{n}^\top (\mathbf{A} + \lambda\mathbf{d}) = 0. \tag{4}$$

Thus,

$$\lambda = -\frac{\mathbf{n}^\top \mathbf{A}}{\mathbf{n}^\top \mathbf{d}}. \tag{5}$$

If P is the intersection point, then the section ratio is

$$AP : PB = \lambda : (1 - \lambda). \tag{6}$$

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Step 2: Numerical Substitution

Now substitute the given coordinates:

$$\mathbf{A} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 3 \\ -5 \\ 8 \end{pmatrix}, \tag{7}$$

$$\mathbf{d} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 5 \\ -9 \\ 1 \end{pmatrix}. \tag{8}$$

$$\lambda = -\frac{\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\mathbf{A}}{\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\mathbf{d}} = -\frac{\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}}{\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 5 \\ -9 \\ 1 \end{pmatrix}} \tag{9}$$

$$= \frac{2}{5}. \tag{10}$$

Hence,

$$AP : PB = \frac{2}{5} : \left(1 - \frac{2}{5}\right) = 2 : 3. \tag{11}$$

$$\boxed{2 : 3}$$

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Step 3: Verification

The intersection point is

$$\mathbf{P} = \mathbf{A} + \frac{2}{5}\mathbf{d} \tag{12}$$

$$= \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} + \frac{2}{5}\begin{pmatrix} 5 \\ -9 \\ 1 \end{pmatrix} \tag{13}$$

$$= \begin{pmatrix} 0 \\ \frac{2}{5} \\ \frac{37}{5} \end{pmatrix}. \tag{14}$$

Clearly, $x = 0$, so P lies on the YZ-plane. Thus, the ratio is verified as $2 : 3$.

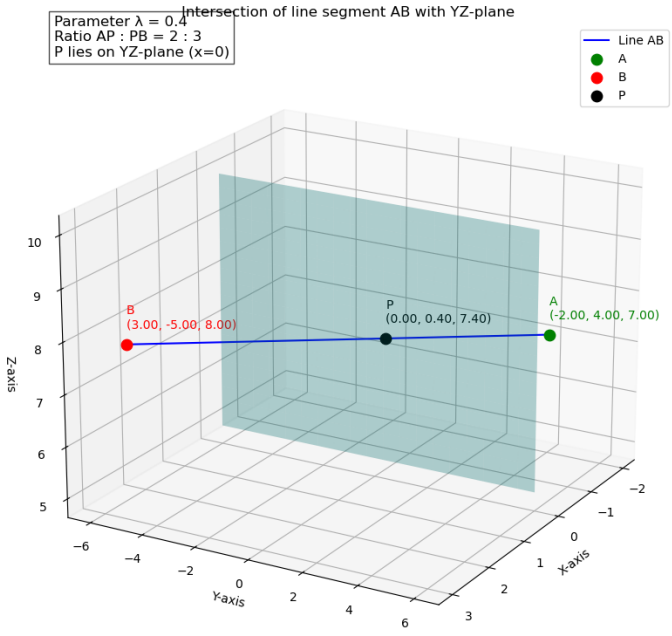

Fig. 1: Line segment AB intersecting the YZ-plane

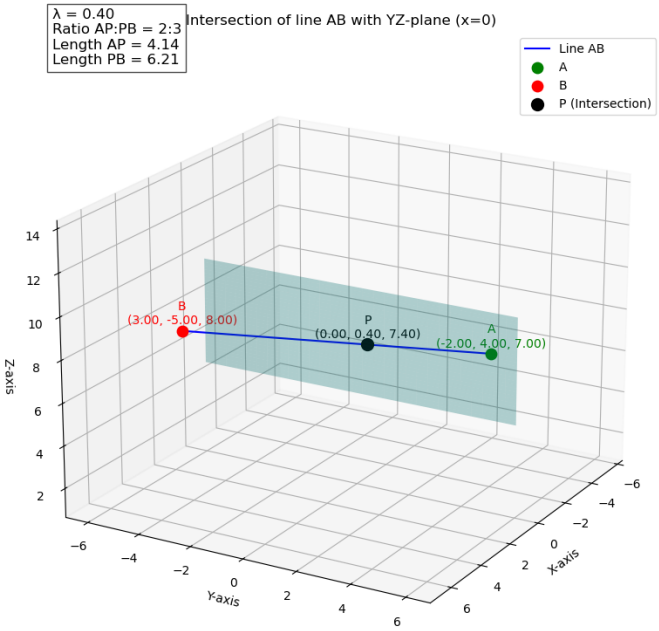
λ = 0.40
Ratio AP:PB = 2:3
Length AP = 4.14
Length PB = 6.21
Intersection of line AB with YZ-plane (x=0)
Line AB
A
B
P (Intersection)
B
(3.00, -5.00, 8.00)
P
(0.00, 0.40, 7.40)
A
(-2.00, 4.00, 7.00)
Z-axis
Y-axis
X-axis

Fig. 2