2.3.15

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Question: The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with the unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

Solution: let **A**, **B** and **C** be the vectors such that:

Variable	value
A	$\begin{pmatrix}1\\1\\1\end{pmatrix}$
B	$\begin{pmatrix}2\\4\\-5\end{pmatrix}$
C	$\begin{pmatrix}\lambda\\2\\3\end{pmatrix}$

TABLE 0: Variables used

given,

$$\frac{\mathbf{A}^\top(\mathbf{B}+\mathbf{C})}{\|\mathbf{B}+\mathbf{C}\|} = 1 \tag{0.1}$$

$$\mathbf{A}^\top(\mathbf{B}+\mathbf{C}) = \|\mathbf{B}+\mathbf{C}\| \tag{0.2}$$

squaring on both sides:

$$(\mathbf{A}^\top(\mathbf{B}+\mathbf{C}))^2 = \|\mathbf{B}+\mathbf{C}\|^2 \tag{0.3}$$

$$(\mathbf{A}^\top\mathbf{B}+\mathbf{A}^\top\mathbf{C})^2 = (\mathbf{B}+\mathbf{C})^\top(\mathbf{B}+\mathbf{C}) \tag{0.4}$$

Substituting the values of **A**,**B** and **C**:

$$\left(\begin{pmatrix}1&1&1\end{pmatrix}\begin{pmatrix}2\\4\\-5\end{pmatrix}+\begin{pmatrix}1&1&1\end{pmatrix}\begin{pmatrix}\lambda\\2\\3\end{pmatrix}\right)^2 = \begin{pmatrix}2+\lambda&6&-2\end{pmatrix}\begin{pmatrix}2+\lambda\\6\\-2\end{pmatrix} \tag{0.5}$$

$$(\lambda+6)^2 = \lambda^2+4\lambda+44 \tag{0.6}$$

$$\lambda^2+36+12\lambda = \lambda^2+4\lambda+44 \tag{0.7}$$

$$8\lambda = 8 \tag{0.8}$$

$$\lambda = 1 \tag{0.9}$$

Hence value of λ is 1.

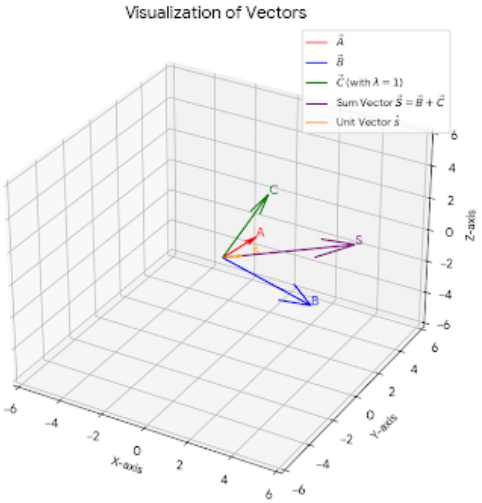

Fig. 0.1