

# 4.13.1

AI25BTECH11036-SNEHAMRUDULA

## Question:

Consider the lines given by

$$L_1 : x + 3y - 5 = 0,$$

$$L_2 : 3x - ky - 1 = 0,$$

$$L_3 : 5x + 2y - 12 = 0.$$

Match the Statements/Expressions in Column I with the Statements/Expressions in Column II.

### Column I

- (A)  $L_1, L_2, L_3$  are concurrent, if
- (B) One of  $L_1, L_2, L_3$  is parallel to at least one of the other two, if
- (C)  $L_1, L_2, L_3$  form a triangle, if
- (D)  $L_1, L_2, L_3$  do not form a triangle, if

### Column II

- (a)  $k = 9$
- (b)  $k = \frac{-6}{5}$
- (c)  $k = \frac{5}{6}$
- (d)  $k = 5$

## Solution.

(A) **Concurrency.** Intersection of  $L_1$  and  $L_3$ :

$$\begin{cases} x + 3y = 5, \\ 5x + 2y = 12, \end{cases} \Rightarrow 13y = 13, \quad y = 1, \quad x = 2.$$

$\mathbf{x}_0 = (2, 1)^\top$  lies on  $L_2$  iff

$$\mathbf{n}_2^\top \mathbf{x}_0 = p_2 \iff 3 \cdot 2 + (-k) \cdot 1 = 1 \iff k = 5.$$

Thus concurrency  $\iff k = 5$ .

(B) **Parallelism.** Normals proportional:

$$\mathbf{n}_2 = \lambda \mathbf{n}_3 \Rightarrow 3 = 5\lambda, \quad -k = 2\lambda \Rightarrow \lambda = \frac{3}{5}, \quad k = -\frac{6}{5}.$$

(Other proportionalities give  $k = -9$ , impossible among options.) Hence parallel pair  $\iff k = -\frac{6}{5}$ .

(C) **Triangle.** No two parallel and not concurrent  $\iff k \neq 5, \quad k \neq -\frac{6}{5}$ . Among given choices  $k = 9$  and  $k = \frac{5}{6}$ .

(D) **Do not form triangle.** Occurs when concurrent or a parallel pair  $\Rightarrow k = 5$  or  $k = -\frac{6}{5}$ .

**Final match (concise):**

$(A) \rightarrow (d) \ k = 5, \quad (B) \rightarrow (b) \ k = -\frac{6}{5}, \quad (C) \rightarrow (a) \text{ or } (c) \ (k = 9 \text{ or } k = \frac{5}{6}), \quad (D) \rightarrow (d) \text{ or } (b).$

If a strict one-to-one choice is required, use:

$(A) \rightarrow (d), \ (B) \rightarrow (b), \ (C) \rightarrow (a), \ (D) \rightarrow (d).$

**Graphical Representation**

