4.3.13

EE25BTECH11026-Harsha

Question:
Equations of the diagonals of the square formed by the lines x = 0, y = 0, x = 1 and y = 1 are ___________.
Solution:
Let us solve the given equation theoretically and then verify the solution computationally.

According to the question,
The vertices of the square are ,

$$\mathbf{a} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \mathbf{d} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

To compute the equation of the diagnols , we can use the normal form of the equation,which is given by

$$\mathbf{n}^T\mathbf{x} = 0 \; for \; the \; lines \; passing \; through \; the \; origin$$

$$\mathbf{n}^T\mathbf{x} = 1 \; for \; the \; lines \; not \; passing \; through \; the \; origin$$

where,

$$\mathbf{n} - vector \; orthogonal \; to \; the \; direction \; vector$$

$$\mathbf{x} = \begin{pmatrix} x & y \end{pmatrix}^T$$

For diagonal $\mathbf{c} - \mathbf{a}$, as it passes through the origin,

$$\therefore \mathbf{n}^T\mathbf{x} = 0$$

By substituting the vector through which it passes through,

$$\mathbf{n}^T \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0$$

$$\implies \mathbf{n} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

But, for diagonal $\mathbf{d} - \mathbf{b}$, as the diagonal doesn't pass through the origin,

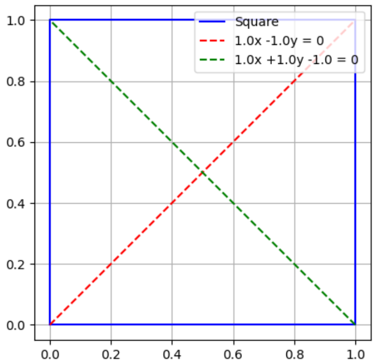
$$\mathbf{n}^T \mathbf{x} = 1$$

$$\therefore \mathbf{n}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\implies \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1$$

From the figure, it is clearly verified that the theoretical solution matches with the computational solution.



Plot of Square with diagonals