MA: MATHEMATICS

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1) Consider the subspace $W = \{[a_{ij}]; a_{ij} = 0 \text{ if } i \text{ is even}\}$ of all 10×10 real matrices. Then the dimension of W is

[GATE EE 2025]

(A) 25 (B) 50 (C) 75 (D) 100

2) Let S be the open unit disk and $f : S \to \mathbb{C}$ be a real-valued analytic function with $f(0) = 1$. Then the set $\{z \in S : f(z) \neq 1\}$ is

[GATE EE 2025]

(A) empty (B) nonempty finite (C) countably infinite (D) uncountable

3) Let $E = \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq x\}$. Then

$$\iint_E (x + y)\, dx\, dy$$

is equal to

[GATE EE 2025]

(A) -1 (B) 0 (C) $\frac{1}{2}$ (D) 1

4) For $(x, y) \in \mathbb{R}^2$, let

$$f(x, y) = \begin{cases} \dfrac{2xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

Then

[GATE EE 2025]

(A) f_x and f_y exist at $(0, 0)$, and f is continuous at $(0, 0)$
(B) f_x and f_y exist at $(0, 0)$, and f is discontinuous at $(0, 0)$
(C) f_x and f_y do not exist at $(0, 0)$, and f is continuous at $(0, 0)$
(D) f_x and f_y do not exist at $(0, 0)$, and f is discontinuous at $(0, 0)$

5) Let y be a solution of $y' = e^{x^2} - 1$ on $[0, 1]$ which satisfies $y(0) = 0$. Then

[GATE EE 2025]

(A) $y(x) > 0$ for $x > 0$
(B) $y(x) < 0$ for $x > 0$
(C) y changes sign in $[0, 1]$
(D) $y \equiv 0$ for $x > 0$

6) For the equation $x(x - 1)y'' + \sin(x)y' + 2x(x - 1)y = 0$, consider the statements: P: $x = 0$ is a regular singular point. Q: $x = 1$ is a regular singular point. Then

[GATE EE 2025]

(A) both P and Q are true
(B) P is false but Q is true
(C) P is true but Q is false
(D) both P and Q are false

7) Let $G = \mathbb{R} \setminus \{0\}$ and $H = \{-1, 1\}$ be groups under multiplication. Then the map $\varphi : G \to H$ defined by $\varphi(x) = \frac{x}{|x|}$ is

[GATE EE 2025]

(A) not a homomorphism
(B) a one-one homomorphism, which is not onto
(C) an onto homomorphism, which is not one-one
(D) an isomorphism

8) The number of maximal ideals in $\mathbb{Z}_{27}$ is

[GATE EE 2025]

a) 0
b) 1
c) 2
d) 3

9) For $1 \leq p \leq \infty$, let $\|\cdot\|_p$ denote the p-norm on $\mathbb{R}^2$. If $\|\cdot\|_p$ satisfies the parallelogram law, then p is equal to

[GATE EE 2025]

a) 1
b) 2
c) 3
d) ∞

10) Consider the initial value problem $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$. The aim is to compute the value of $y_1 = y(x_1)$, where $x_1 = x_0 + h$ $(h > 0)$. At $x = x_1$, if the value of y_1 is equated to the corresponding value of the straight line passing through (x_0, y_0) and having the slope equal to the slope of the curve $y(x)$ at $x = x_0$, then the method is called

[GATE EE 2025]

a) Euler's method
b) Improved Euler's method
c) Backward Euler's method
d) Taylor series method of order 2

11) The solution of $xu_x + yu_y = 0$ is of the form

[GATE EE 2025]

a) $f\left(\frac{y}{x}\right)$
b) $f(x + y)$
c) $f(x - y)$
d) $f(xy)$

12) If the partial differential equation $(x-1)^2u_{xx} - (y-2)^2u_{yy} + 2xu_x + 2yu_y + 2xyu = 0$ is parabolic in $S \subset \mathbb{R}^2$ but not in $\mathbb{R}^2 \setminus S$, then S is

[GATE EE 2025]

a) $\{(x, y) \in \mathbb{R}^2 : x = 1 \text{ or } y = 2\}$
b) $\{(x, y) \in \mathbb{R}^2 : x = 1 \text{ and } y = 2\}$
c) $\{(x, y) \in \mathbb{R}^2 : x = 1\}$
d) $\{(x, y) \in \mathbb{R}^2 : y = 2\}$

13) Let E be a connected subset of $\mathbb{R}$ with at least two elements. Then the number of elements in E is

[GATE EE 2025]

a) exactly two
b) more than two but finite
c) countably infinite
d) uncountable

14) Let X be a non-empty set. Let T_1 and T_2 be two topologies on X such that T_1 is strictly contained in T_2. If $I : (X, T_3) \to (X, T_3)$ is the identity map, then

[GATE EE 2025]

a) both I and I^{-1} are continuous
b) both I and I^{-1} are not continuous
c) I is continuous but I^{-1} is not continuous

d) I is not continuous but I^{-1} is continuous

15) Let $X_1, X_2, \ldots, X_{10}$ be a random sample from $N(80, 3^2)$ distribution. Define

$$S = \sum_{i=1}^{10} U_i \quad \text{and} \quad T = \sum_{i=1}^{10} \left(U_i - \frac{S}{10}\right)^2,$$

where $U_i = \frac{X_i - 80}{3}$, $i = 1, 2, \ldots, 10$. Then the value of $E(ST)$ is equal to

[GATE EE 2025]

a) 0
b) 1
c) 10
d) $\frac{80}{3}$

16) Two (indistinguishable) fair coins are tossed simultaneously. Given that ONE of them lands up head, the probability of the OTHER to land up tail is equal to

[GATE EE 2025]

a) $\frac{1}{3}$
b) $\frac{1}{2}$
c) $\frac{2}{3}$
d) $\frac{3}{4}$

17) Let $c_{ij} \geq 2$ be the cost of the $(i, j)^{\text{th}}$ cell of an assignment problem. If a new cost matrix is generated by the elements $c'_{ij} = \frac{1}{2}c_{ij} + 1$, then

[GATE EE 2025]

a) optimal assignment plan remains unchanged and cost of assignment decreases
b) optimal assignment plan changes and cost of assignment decreases
c) optimal assignment plan remains unchanged and cost of assignment increases
d) optimal assignment plan changes and cost of assignment increases

18) Let a primal linear programming problem admit an optimal solution. Then the corresponding dual problem

[GATE EE 2025]

a) does not have a feasible solution
b) has a feasible solution but does not have any optimal solution
c) does not have a convex feasible region
d) has an optimal solution

19) In any system of particles, suppose we do not assume that the internal forces come in pairs. Then the fact that the sum of internal forces is zero follows from

[GATE EE 2025]

a) Newton's second law
b) conservation of angular momentum
c) conservation of energy
d) principle of virtual displacement

20) Let $q_1, q_2, \ldots, q_n$ be the generalized coordinates and $\dot{q}_1, \dot{q}_2, \ldots, \dot{q}_n$ be the generalized velocities in a conservative force field. If under a transformation φ, the new coordinate system has the generalized coordinates $Q_1, Q_2, \ldots, Q_n$ and velocities $\dot{Q}_1, \dot{Q}_2, \ldots, \dot{Q}_n$, then the equation

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)$$

takes the form

[GATE EE 2025]

a) $\varphi \frac{\partial L}{\partial Q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{Q}_i}\right)$
b) $\varphi \frac{\partial L}{\partial Q_i} + \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{Q}_i}\right)$

c) $-\varphi\frac{\partial L}{\partial Q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{Q}_i}\right)$
d) $\frac{\partial L}{\partial Q_i} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{Q}_i}\right)$

21) Let $T : \mathbb{R}^4 \to \mathbb{R}^4$ be the linear map satisfying

$$T(e_1) = e_2, \quad T(e_2) = e_3, \quad T(e_3) = 0, \quad T(e_4) = e_3,$$

where $\{e_1, e_2, e_3, e_4\}$ is the standard basis of $\mathbb{R}^4$. Then

[GATE EE 2025]

a) T is idempotent
b) T is invertible
c) $\text{Rank}\, T = 3$
d) T is nilpotent

22) Let

$$M = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \qquad V = \{Mx : x \in \mathbb{R}^3\}.$$

Then an orthonormal basis for V is

[GATE EE 2025]

(A) $\{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}\}$

(B) $\{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}\}$

(C) $\{\begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}, \begin{pmatrix} \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{pmatrix}\}$

(D) $\{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\}$

23) For any $n \in \mathbb{N}$, let P_n denote the vector space of all real polynomials of degree at most $n-1$. Define the linear map $T : P_n \to P_{n+1}$ by

$$T(p)(x) = p(x) - \int_0^x p(t)\, dt.$$

Then $\dim(\ker T)$ is

[GATE EE 2025]

(A) 0
(B) 1
(C) n
(D) $n + 1$

24) Let

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}, \qquad 0 < \theta < \frac{\pi}{2},$$

and let $V = \{u \in \mathbb{R}^3 : Mu = u\}$. Then $\dim(V)$ is

[GATE EE 2025]

(A) 0
(B) 1
(C) 2
(D) 3

25) The number of linearly independent eigenvectors of the matrix

$$\begin{pmatrix} 2 & 2 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 \end{pmatrix}$$

is

[GATE EE 2025]

(A) 1
(B) 2
(C) 3
(D) 4

26) Let f be a bilinear transformation mapping $-1 \mapsto 1$, $i \mapsto 0$, and $-i \mapsto 0$. Then $f(i)$ is equal to *[GATE EE 2025]*

(A) -2
(B) -1
(C) i
(D) $-i$

27) Which one of the following does NOT hold for all continuous functions $f : [-\pi, \pi] \to \mathbb{C}$? *[GATE EE 2025]*

(A) If $f(-t) = f(t)$ for each $t \in [-\pi, \pi]$, then $\int_{-\pi}^{\pi} f(t)dt = 2\int_{0}^{\pi} f(t)dt$
(B) If $f(-t) = -f(t)$ for each $t \in [-\pi, \pi]$, then $\int_{-\pi}^{\pi} f(t)dt = 0$
(C) $\int_{-\pi}^{\pi} f(-t)dt = -\int_{-\pi}^{\pi} f(t)dt$
(D) There is an α with $-\pi < \alpha < \pi$ such that $\int_{-\pi}^{\pi} f(t)dt = 2\pi f(\alpha)$

28) Let S be the positively oriented circle given by $|z - 3i| = 2$. Then the value of $\int_S \frac{dz}{z^2+4}$ is *[GATE EE 2025]*

(A) $-\frac{\pi}{2}$
(B) $\frac{\pi}{2}$
(C) $-\frac{i\pi}{2}$
(D) $\frac{i\pi}{2}$

29) Let T be the closed unit disk and ∂T be the unit circle. Then which one of the following holds for every analytic function $f : T \to \mathbb{C}$?

[GATE EE 2025]

(A) $|f|$ attains its minimum and its maximum on ∂T
(B) $|f|$ attains its minimum on ∂T but need not attain its maximum on ∂T
(C) $|f|$ attains its maximum on ∂T but need not attain its minimum on ∂T
(D) $|f|$ need not attain its maximum on ∂T and also it need not attain its minimum on ∂T

30) Let S be the disk $|z| < 3$ in the complex plane and let $f : S \to \mathbb{C}$ be an analytic function such that

$$f\left(1 + \frac{\sqrt{2}}{n}i\right) = \frac{2}{n^2}$$

for each natural number n. Then $f(\sqrt{2}i)$ is equal to

[GATE EE 2025]

(A) $3 - 2\sqrt{2}$
(B) $3 + 2\sqrt{2}$
(C) $2 - 3\sqrt{2}$
(D) $2 + 3\sqrt{2}$

31) Which one of the following statements holds?

[GATE EE 2025]

(A) The series $\sum_{n=0}^{\infty} x^n$ converges for each $x \in [-1, 1]$
(B) The series $\sum_{n=0}^{\infty} x^n$ converges uniformly in $(-1, 1)$
(C) The series $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ converges for each $x \in [-1, 1]$
(D) The series $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converges uniformly in $(-1, 1)$

32) For $x \in [-\pi, \pi]$, let

$$f(x) = (\pi + x)(\pi - x), \quad g(x) = \begin{cases} \cos\left(\frac{1}{x}\right), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Consider the statements:
P: The Fourier series of f converges uniformly to f on $[-\pi, \pi]$. Q: The Fourier series of g converges uniformly to g on $[-\pi, \pi]$.
Then

[GATE EE 2025]

(A) P and Q are true
(B) P is true but Q is false
(C) P is false but Q is true
(D) Both P and Q are false

33) Let $W = \left\{(x, y, z) \in \mathbb{R}^3 : 1 \leq x^2 + y^2 + z^2 \leq 4\right\}$ and $F : W \to \mathbb{R}^3$ be defined by

$$F(x, y, z) = \frac{(x, y, z)}{\left[x^2 + y^2 + z^2\right]^{3/2}}$$

for $(x, y, z) \in W$. If ∂W denotes the boundary of W oriented by the outward normal n to W, then

$$\iint_{\partial W} F \cdot n \, dS$$

is equal to

[GATE EE 2025]

(A) 0
(B) 4π
(C) 8π
(D) 12π

34) For each $n \in \mathbb{N}$, let $f_n : [0, 1] \to \mathbb{R}$ be a measurable function such that $|f_n(t)| \leq \frac{1}{\sqrt{t}}$ for all $t \in (0, 1]$. Let $f : [0, 1] \to \mathbb{R}$ be defined by $f(t) = 1$ if t is irrational and $f(t) = -1$ if t is rational. Assume that $f_n(t) \to f(t)$ as $n \to \infty$ for all $t \in [0, 1]$. Then

[GATE EE 2025]

(A) f is not measurable
(B) f is measurable and $\int_{[0,1]} f_n \, d\mu \to 1$ as $n \to \infty$
(C) f is measurable and $\int_{[0,1]} f_n \, d\mu \to 0$ as $n \to \infty$
(D) f is measurable and $\int_{[0,1]} f_n \, d\mu \to -1$ as $n \to \infty$

35) Let y_1 and y_2 be two linearly independent solutions of

$$y'' + (\sin x)y = 0, \quad 0 \leq x \leq 1.$$

Let $g(x) = W(y_1, y_2)(x)$ be the Wronskian of y_1 and y_2. Then

[GATE EE 2025]

(A) $g' > 0$ on $[0, 1]$
(B) $g' < 0$ on $[0, 1]$
(C) g' vanishes at only one point of $[0, 1]$
(D) g' vanishes at all points of $[0, 1]$

36) One particular solution of $y''' - y'' + y' + y = e^x$ is a constant multiple of

[GATE EE 2025]

(A) xe^x
(B) xe^{-x}
(C) x^2e^x
(D) x^2e^{-x}

37) Let $a, b \in \mathbb{R}$. Let $y = (y_1, y_2)^\top$ be a solution of the system

$$y_1' = y_2, \quad y_2' = ay_1 + by_2.$$

Every solution $y(x) \to 0$ as $x \to \infty$ if

[GATE EE 2025]

(A) $a < 0,\ b < 0$
(B) $a < 0,\ b > 0$
(C) $a > 0,\ b > 0$
(D) $a > 0,\ b < 0$

38) Let G be a group of order 45. Let H be a 3-Sylow subgroup of G and K be a 5-Sylow subgroup of G. Then

[GATE EE 2025]

(A) Both H and K are normal in G
(B) H is normal in G but K is not normal in G
(C) H is not normal in G but K is normal in G
(D) Both H and K are not normal in G

39) The ring $\mathbb{Z}[\sqrt{-11}]$ is

[GATE EE 2025]

(A) A Euclidean Domain
(B) A Principal Ideal Domain, but not a Euclidean Domain
(C) A Unique Factorization Domain, but not a Principal Ideal Domain
(D) Not a Unique Factorization Domain

40) Let R be a Principal Ideal Domain and a, b be any two non-unit elements of R. Then the ideal generated by a and b is also generated by

[GATE EE 2025]

(A) $a + b$
(B) ab
(C) $\gcd(a, b)$
(D) $\mathrm{lcm}(a, b)$

41) Consider the action of S_4, the symmetric group of order 4, on $\mathbb{Z}[x_1, x_2, x_3, x_4]$ given by

$$\sigma \cdot p(x_1, x_2, x_3, x_4) = p\left(x_{\sigma^{-1}(1)}, x_{\sigma^{-1}(2)}, x_{\sigma^{-1}(3)}, x_{\sigma^{-1}(4)}\right)$$

for $\sigma \in S_4$.

Let $H \subseteq S_4$ denote the cyclic subgroup generated by $(1\ 4\ 2\ 3)$. Then the cardinality of the orbit $O_H(x_1x_3 + x_2x_4)$ of H on the polynomial $x_1x_3 + x_2x_4$ is

[GATE EE 2025]

(A) 1
(B) 2
(C) 3
(D) 4

42) Let $f : \ell^2 \to \mathbb{R}$ be defined by

$$f(x_1, x_2, \dots) = \sum_{n=1}^{\infty} \frac{x_n}{2^{n/2}}$$

for $(x_1, x_2, \dots) \in \ell^2$. Then $\|f\|$ is equal to

[GATE EE 2025]

(A) $\frac{1}{2}$
(B) 1
(C) 2
(D) $\frac{1}{\sqrt{2}-1}$

43) Consider $\mathbb{R}^3$ with norm $\|\cdot\|$ and the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by the 3×3 matrix

$$\begin{pmatrix} 1 & 1 & 3 \\ 2 & 2 & 2 \\ 1 & 3 & -3 \end{pmatrix}$$

Then the operator norm $\|T\|$ of T is equal to

[GATE EE 2025]

(A) 6
(B) 7
(C) 8
(D) $\sqrt{42}$

44) Consider $\mathbb{R}^2$ with norm $\|\cdot\|$, and let

$$Y = \{(y_1, y_2) \in \mathbb{R}^2 : y_1 + y_2 = 0\}.$$

If $g : Y \to \mathbb{R}$ is defined by $g(y_1, y_2) = y_2$ for $(y_1, y_2) \in Y$, then

[GATE EE 2025]

(A) g has no Hahnanach extension to $\mathbb{R}^2$
(B) g has a unique HahnBanach extension to $\mathbb{R}^2$
(C) Every linear functional $f : \mathbb{R}^2 \to \mathbb{R}$ satisfying $f(-1, 1) = 1$ is a HahnBanach extension of g to $\mathbb{R}^2$
(D) The functionals $f_1, f_2 : \mathbb{R}^2 \to \mathbb{R}$ given by $f_1(x_1, x_2) = x_2$ and $f_2(x_1, x_2) = -x_1$ are both HahnBanach extensions of g to $\mathbb{R}^2$

45) Let X be a Banach space and Y be a normed linear space. Consider a sequence (F_n) of bounded linear maps from X to Y such that for each fixed $x \in X$, the sequence $(F_n(x))$ is bounded in Y. Then

[GATE EE 2025]

(A) For each fixed $x \in X$, the sequence $(F_n(x))$ is convergent in Y
(B) For each fixed $n \in \mathbb{N}$, the set $\{F_n(x) : x \in X\}$ is bounded in Y
(C) The sequence $(\|F_n\|)$ is bounded in $\mathbb{R}$
(D) The sequence (F_n) is uniformly bounded on X

46) Let $H = L^2\,[[0, \pi]]$ with the usual inner product. For $n \in \mathbb{N}$, let

$$u_n(t) = \frac{\sqrt{2}}{\sqrt{\pi}} \sin nt, \quad t \in [0, \pi],$$

and $E = \{u_n : n \in \mathbb{N}\}$. Then

[GATE EE 2025]

(A) E is not a linearly independent subset of H
(B) E is a linearly independent subset of H, but is not an orthonormal subset of H
(C) E is an orthonormal subset of H, but is not an orthonormal basis for H
(D) E is an orthonormal basis for H

47) Let $X = \mathbb{R}$ and let $\mathfrak{T} = \{U \subset X : X \setminus U \text{ is finite}\} \cup \{\phi, X\}$. The sequence

$$1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots$$

in $(X, \mathfrak{T})$

[GATE EE 2025]

(A) Converges to 0 and not to any other point of X

(B) Does not converge to 0
(C) Converges to each point of X
(D) Is not convergent in X

48) Let $E = \{(x, y) \in \mathbb{R}^2 : |x| \leq 1,\ |y| \leq 1\}$. Define $f : E \to \mathbb{R}$ by

$$f(x, y) = \frac{x + y}{1 + x^2 + y^2}.$$

Then the range of f is a

[GATE EE 2025]

(A) Connected open set
(B) Connected closed set
(C) Bounded open set
(D) Closed and unbounded set

49) Let $X = \{1, 2, 3\}$ and $T = \{\phi, \{1\}, \{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$. The topological space (X, T) is said to have the property P if for any two proper disjoint closed subsets Y and Z of X, there exist disjoint open sets U, V such that $Y \subseteq U$ and $Z \subseteq V$. Then the topological space (X, T)

[GATE EE 2025]

(A) is T_1 and satisfies P
(B) is T_1 and does not satisfy P
(C) is not T_1 and satisfies P
(D) is not T_1 and does not satisfy P

50) Which one of the following subsets of $\mathbb{R}$ (with the usual metric) is NOT complete?

[GATE EE 2025]

(A) $[1, 2] \cup [3, 4]$
(B) $[0, \infty]$
(C) $[0, 1]$
(D) $\{0\} \cup \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$

51) Consider the function

$$f(x) = \begin{cases} k\,(x - \lfloor x \rfloor), & 0 \leq x < 2 \\ 0, & \text{otherwise} \end{cases}$$

where $\lfloor x \rfloor$ is the integral part of x. The value of k for which the above function is a probability density function of some random variable is

[GATE EE 2025]

(A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) 1
(D) 2

52) For two random variables X and Y, the regression lines are given by

$$Y = 5X - 15 \quad \text{and} \quad Y = 10X - 35$$

Then the regression coefficient of X on Y is

[GATE EE 2025]

(A) 0.1
(B) 0.2
(C) 5
(D) 10

53) In an examination there are 80 questions each having four choices. Exactly one of these four choices is correct and the other three are wrong. A student is awarded 1 mark for each correct answer, and -0.25 for each wrong answer. If a student ticks the answer of each question randomly, then the

expected value of his/her total marks in the examination is

[GATE EE 2025]

(A) -15
(B) 0
(C) 5
(D) 20

54) Let $X_1, X_2, \ldots, X_n$ be a random sample from the uniform distribution on $[0, \theta]$. Then the maximum likelihood estimator (MLE) of θ based on the above random sample is

[GATE EE 2025]

(A) $\frac{2}{n}\sum_{i=1}^{n} X_i$
(B) $\frac{1}{n}\sum_{i=1}^{n} X_i$
(C) $\min\{X_1, X_2, \ldots, X_n\}$
(D) $\max\{X_1, X_2, \ldots, X_n\}$

55) The cost matrix of a transportation problem is given by

$$\begin{pmatrix} 4 & 1 & 2 & 3 \\ 3 & 2 & 3 & 2 \\ 2 & 2 & 1 & 4 \end{pmatrix}$$

The following are the values of variables in a feasible solution:

$$x_{12} = 6, \quad x_{13} = 2, \quad x_{24} = 6, \quad x_{31} = 4, \quad x_{33} = 6$$

Then which of the following is correct?

[GATE EE 2025]

(A) The solution is degenerate and basic
(B) The solution is non-degenerate and basic
(C) The solution is degenerate and non-basic
(D) The solution is non-degenerate and non-basic

56) The maximum value of $z = 3x_1 - x_2$ subject to $2x_1 - x_2 \leq 1$, $x_1 \leq 3$ and $x_1, x_2 \geq 0$ is

[GATE EE 2025]

(A) 0
(B) 4
(C) 6
(D) 9

57) Consider the problem of maximizing $z = 2x_1 + 3x_2 - 4x_3 + x_4$ subject to

$$\begin{aligned} x_1 + x_2 + x_3 &= 2, \\ x_2 + x_4 &= 3, \\ 2x_1 + 3x_2 + 2x_5 - x_6 &= 0, \\ x_1, x_2, x_3, x_4 &\geq 0. \end{aligned}$$

Then

[GATE EE 2025]

(A) $(1, 0, 1, 4)$ is a basic feasible solution but $(2, 0, 0, 4)$ is not
(B) $(1, 0, 1, 4)$ is not a basic feasible solution but $(2, 0, 0, 4)$ is
(C) Neither $(1, 0, 1, 4)$ nor $(2, 0, 0, 4)$ are basic feasible solutions
(D) Both $(1, 0, 1, 4)$ and $(2, 0, 0, 4)$ are basic feasible solutions

58) In the closed system of a simple harmonic motion of a pendulum, let H denote the Hamiltonian and E be the total energy. Then

[GATE EE 2025]

(A) H is a constant and $H = E$

(B) H is a constant but $H \neq E$
(C) H is not constant but $H = E$
(D) H is not constant and $H \neq E$

59) The possible values of α for which the variational problem

$$J[y(x)] = \int_0^1 \left[(3y')^2 + 2x^\alpha y^2\right] dx, \quad y(\alpha) = 1$$

has extremals are

[GATE EE 2025]

(A) $-1, 0$
(B) $0, 1$
(C) $-1, 1$
(D) $-1, 0, 1$

60) The functional

$$\int_0^1 \left[y'^2 + x^4\right] dx, \quad y(1) = 1$$

achieves its

[GATE EE 2025]

(A) Weak maximum on all its extremals
(B) Weak minimum on all its extremals
(C) Weak maximum on some, but not on all, of its extremals
(D) Weak minimum on some, but not on all, of its extremals

61) The integral equation

$$x(t) = \sin t + \lambda \int_0^t \left[\left(s^3 + e^{s^2}\right) x(s)\right] ds, \quad 0 \leq t \leq 1, \ \lambda \in \mathbb{R}, \ \lambda \neq 0$$

has a solution for

[GATE EE 2025]

(A) All non-zero values of λ
(B) No value of λ
(C) Only countably many positive values of λ
(D) Only countably many negative values of λ

62) The integral equation

$$x(t) - \int_0^t [\cos t \ \sec s \ x(s)] \ ds = \sinh t, \quad 0 \leq t \leq 1$$

has

[GATE EE 2025]

(A) No solution
(B) A unique solution
(C) More than one but finitely many solutions
(D) Infinitely many solutions

63) If $y_{i+1} = y_i + h\,\varphi(f, x_i, y_i, h), \ i = 1, 2, \ldots,$ where

$$\varphi(f, x, y, h) = af(x, y) + bf(x + h, y + hf(x, y)),$$

is a second-order accurate scheme to solve the initial value problem

$$\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0,$$

then a and b, respectively, are

[GATE EE 2025]

(A) $\frac{h}{2},\ \frac{h}{2}$
(B) $1,\ -1$
(C) $\frac{1}{2},\ \frac{1}{2}$
(D) $h,\ -h$

64) If a quadrature formula

$$\frac{3}{2}f\left(-\frac{1}{3}\right) + K f\left(\frac{1}{3}\right) + \frac{1}{2}f(1)$$

that approximates $\int_0^1 f(x)\,dx$ is found to be exact for quadratic polynomials, then the value of K is
[GATE EE 2025]

(A) 2
(B) 1
(C) 0
(D) -1

65) If

$$\begin{pmatrix} 1 & 4 & 3 \\ 1 & 2 & 0 \\ 5 & 8 & a \end{pmatrix}\begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix}\begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} = \begin{pmatrix} 1 & * & * \\ 0 & -53 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

then the value of a is
[GATE EE 2025]

(A) -2
(B) -1
(C) 1
(D) 2

66) Using the least squares method, if a curve $y = ax^2 + bx + c$ is fitted to the collinear data points $(1, 2)$, $(1, 1)$, $(3, 5)$ and $(7, 13)$, then the triplet (a, b, c) is equal to
[GATE EE 2025]

(A) $(-1, 2, 0)$
(B) $(0, 2, -1)$
(C) $(2, -1, 0)$
(D) $(0, 1, 2)$

67) A quadratic polynomial $p(x)$ is constructed by interpolating the data points $(0, 1)$, $(1, e)$ and $(2, e^2)$. If $\sqrt{e}$ is approximated by using $p(x)$, then its approximate value is
[GATE EE 2025]

(A) $\frac{1}{8}(3 + 6e - e^2)$
(B) $\frac{1}{8}(3 - 6e + 2e^2)$
(C) $\frac{1}{8}(3 - 6e - e^2)$
(D) $\frac{1}{8}(3 + 6e - 2e^2)$

68) The characteristic curve of $2yu_{tt} + (2x + y^2)u_x = 0$ passing through $(0, 0)$ is
[GATE EE 2025]

(A) $y^2 = 2\,(e^x + x - 1)$
(B) $y^2 = 2\,(e^x - x + 1)$
(C) $y^2 = 2\,(e^{-x} - x - 1)$
(D) $y^2 = 2\,(e^{-x} + x + 1)$

69) The initial value problem $u_t + u_y = 1,\ u(x, s) = \sin s,\ 0 \le s \le 1$, has
[GATE EE 2025]

(A) Two solutions
(B) A unique solution
(C) No solution
(D) Infinitely many solutions

70) Let $u(x, t)$ be the solution of $u_{tt} - u_{xx} = 1,\ x \in \mathbb{R},\ t > 0$, with $u(x, 0) = 0,\ u_t(x, 0) = 0,\ x \in \mathbb{R}$. Then $u\left(\frac{1}{2}, \frac{1}{2}\right)$ is equal to
[GATE EE 2025]

(A) $\frac{1}{8}$
(B) $-\frac{1}{8}$
(C) $\frac{1}{4}$
(D) $-\frac{1}{4}$

Let $X = C([0,1])$ with sup norm $\|\cdot\|$.

71) Let $S = \{x \in X : \|x\|_\infty \leq 1\}$. Then

[GATE EE 2025]

(A) S is convex and compact
(B) S is not convex but compact
(C) S is convex but not compact
(D) S is neither convex nor compact

72) Which one of the following is true?

[GATE EE 2025]

(A) $C^\infty([0,1])$ is dense in X
(B) X is dense in $L^\infty([0,1])$
(C) X has a countable basis
(D) There is a sequence in X which is uniformly Cauchy on $[0,1]$ but does not converge uniformly on $[0,1]$

73) Let $I = \{x \in X : x(0) = 0\}$. Then

[GATE EE 2025]

(A) I is not an ideal of X
(B) I is an ideal, but not a prime ideal of X
(C) I is a prime ideal, but not a maximal ideal of X
(D) I is a maximal ideal of X

74) Let $X = C^1([0,1])$ and $Y = C([0,1])$, both with the sup norm. Define $F : X \to Y$ by $F(x) = x + x'$ and $f(x) = x(1) + x'(1)$ for $x \in X$.
Then

[GATE EE 2025]

(A) F and f are continuous
(B) F is continuous and f is discontinuous
(C) F is discontinuous and f is continuous
(D) F and f are discontinuous

75) Then

[GATE EE 2025]

(A) F and f are closed maps
(B) F is a closed map and f is not a closed map
(C) F is not a closed map and f is a closed map
(D) Neither F nor f is a closed map

Linked Answer Questions: Q.76 to Q.85 carry two marks each.

76) Let

$$N = \begin{pmatrix} \frac{3}{5} & -\frac{4}{5} & 0 \\ \frac{4}{5} & \frac{3}{5} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Then N is

[GATE EE 2025]

(A) Non-invertible
(B) Skew-symmetric
(C) Symmetric
(D) Orthogonal

77) If M is any 3×3 real matrix, then $\operatorname{trace}(NMN^T)$ is equal to

[GATE EE 2025]

(A) $[\text{trace}(N)]^2\,\text{trace}(M)$
(B) $2\,\text{trace}(N) + \text{trace}(M)$
(C) $\text{trace}(M)$
(D) $[\text{trace}(N)]^2 + \text{trace}(M)$

78) Let $f(z) = \frac{\cos z - \frac{\sin z}{z}}{z}$ for non-zero $z \in \mathbb{C}$ and $f(0) = 0$. Also, let $g(z) = \sinh z$ for $z \in \mathbb{C}$. Then $f(z)$ has a zero at $z = 0$ of order

[GATE EE 2025]

(A) 0
(B) 1
(C) 2
(D) Greater than 2

79) Then $\frac{g(z)}{zf(z)}$ has a pole at $z = 0$ of order

[GATE EE 2025]

(A) 1
(B) 2
(C) 3
(D) Greater than 3

80) Let $n \geq 3$ be an integer. Let y be the polynomial solution of

$$(1 - x^2)y'' - 2xy' + n(n-1)y = 0$$

satisfying $y(1) = 1$.
Then the degree of y is

[GATE EE 2025]

(A) n
(B) $n - 1$
(C) Less than $n - 1$
(D) Greater than $n + 1$

81) If $I = \int_{-1}^{1} y(x)x^{n-3}\,dx$ and $J = \int_{-1}^{1} y(x)x^n\,dx$, then

[GATE EE 2025]

(A) $I \neq 0, J \neq 0$
(B) $I \neq 0, J = 0$
(C) $I = 0, J \neq 0$
(D) $I = 0, J = 0$

Statement for Linked Answer Questions 82 & 83:
Consider the boundary value problem

$$u_{xx} + u_{yy} = 0, \quad x \in (0,\pi),\ y \in (0,\pi),$$
$$u(x,0) = u(x,\pi) = u(0,y) = 0.$$

82) Any solution of this boundary value problem is of the form

[GATE EE 2025]

(A) $\sum_{n=1}^{\infty} a_n \sinh nx \sin ny$
(B) $\sum_{n=1}^{\infty} a_n \cosh nx \sin ny$
(C) $\sum_{n=1}^{\infty} a_n \sinh nx \cos ny$
(D) $\sum_{n=1}^{\infty} a_n \cosh nx \cos ny$

83) If an additional boundary condition $u_x(\pi, y) = \sin y$ is satisfied, then $u(x, \pi/2)$ is equal to

[GATE EE 2025]

(A) $\frac{\pi}{2}\frac{(e^x - e^{-x})(e^x + e^{-x})}{}$
(B) $\frac{\pi(e^x + e^{-x})}{(e^x - e^{-x})}$
(C) $\frac{\pi(e^x - e^{-x})}{(e^x + e^{-x})}$
(D) $\frac{\pi}{2}(e^x + e^{-x})(e^x + e^{-x})$

Statement for Linked Answer Questions 84 & 85: Let a random variable X follow the exponential distribution with mean 2. Define

$$Y = [X - 2 \,|\, X > 2].$$

84) The value of $P(Y \geq t)$ is

[GATE EE 2025]

(A) $e^{-t/2}$
(B) e^{-2t}
(C) $\frac{1}{2}e^{-t/2}$
(D) $\frac{1}{2}e^{-t}$

85) The value of $E(Y)$ is equal to

[GATE EE 2025]

(A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) 1
(D) 2