4.4.10

AI25BTECH11014 - Gooty Suhas

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Question

Point **P** divides the line segment joining:

$$\mathbf{A} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} k \\ 8 \end{pmatrix}$$

in the ratio:

$$\frac{AP}{PB} = \frac{1}{3}$$

and lies on the line:

$$\begin{pmatrix} 2 & -1 \end{pmatrix} \mathbf{P} = -1$$

Section Formula (Matrix Form)

Let the ratio be $m : n = 1 : 3$. Then the section formula becomes:

$$\mathbf{P} = \frac{n\mathbf{A} + m\mathbf{B}}{m+n} = \frac{3\mathbf{A} + 1\mathbf{B}}{4}$$

Substitute:

$$\mathbf{A} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} k \\ 8 \end{pmatrix} \Rightarrow \mathbf{P} = \frac{1}{4}\left(3\begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} k \\ 8 \end{pmatrix}\right) = \frac{1}{4}\begin{pmatrix} 6+k \\ 11 \end{pmatrix} = \begin{pmatrix} \frac{6+k}{4} \\ \frac{11}{4} \end{pmatrix}$$

Substitute into Line Equation

Given:

$$\begin{pmatrix} 2 & -1 \end{pmatrix} \mathbf{P} = -1$$

Substitute:

$$\begin{pmatrix} 2 & -1 \end{pmatrix} \begin{pmatrix} \frac{6+k}{4} \\ \frac{11}{4} \end{pmatrix} = -1 \Rightarrow \frac{2(6+k)-11}{4} = -1$$

$$\Rightarrow \frac{12+2k-11}{4} = -1 \Rightarrow \frac{1+2k}{4} = -1 \Rightarrow 1+2k = -4 \Rightarrow k = -\frac{5}{2}$$

Final Answer

$$\boxed{k = -\frac{5}{2}}$$

This is the value of k such that point **P** lies on the line and divides **AB** in the ratio $1 : 3$.

Python Code

```python
from sympy import symbols, Matrix, Eq, solve

k = symbols('k')
A = Matrix([2, 1])
B = Matrix([k, 8])
P = (3 * A + B) / 4
n = Matrix([[2, -1]])
eq = Eq(n @ P, -1)
sol = solve(eq, k)
print(sol[0])
```

C Code

```c
#include <stdio.h>

int main() {
    double A_x, A_y, B_y, m, n, rhs;

    scanf("%lf %lf %lf %lf %lf %lf", &A_x, &A_y, &B_y, &m, &n, &
        rhs);

    double P_y = (n * A_y + m * B_y) / (m + n);
    double k = ((rhs + P_y) * (m + n) - 2.0 * n * A_x) / (2.0 * m
        );

    printf("%.6f\n", k);
    return 0;
}
```

Python Code — With .so

```python
import subprocess

inputs = [2.0, 1.0, 8.0, 1.0, 3.0, -1.0]
input_str = ' '.join(map(str, inputs))

result = subprocess.run(
    ["./k_solver"],
    input=input_str,
    capture_output=True,
    text=True
)

k_value = float(result.stdout.strip())
print(k_value)
```

Plot

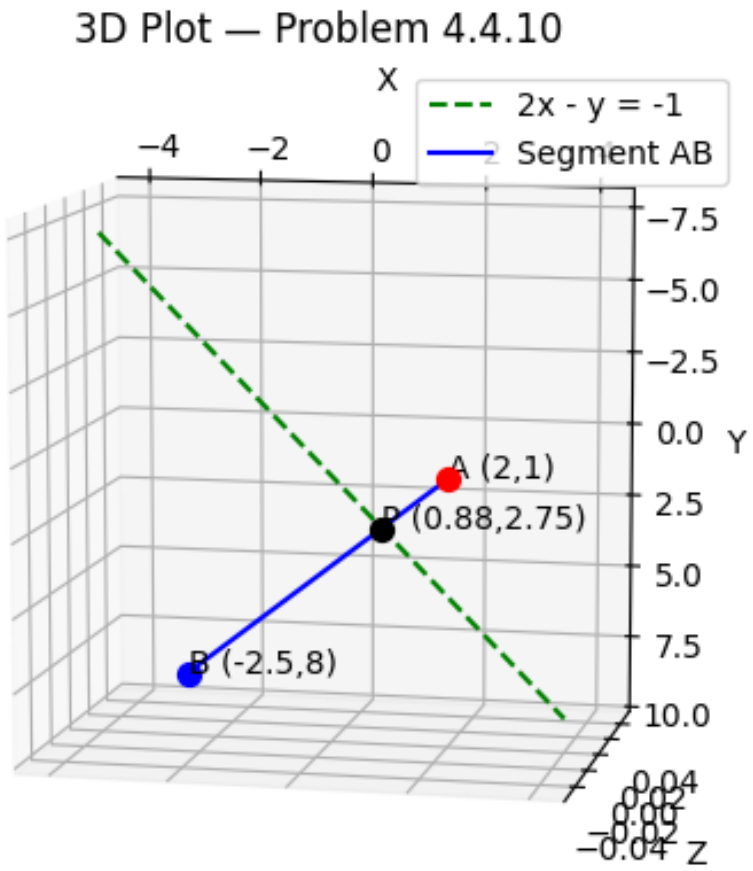

Figure: Point **P** on the line and dividing **AB**