

11.2.7

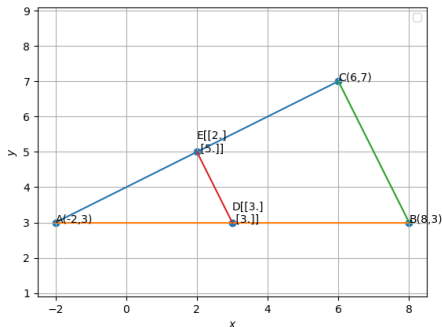
AI25BTECH11001 - ABHISEK MOHAPATRA

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Question: A line through the mid-point of a side of a triangle parallel to another side bisects the third side.

Solution:

Graph:



Consider a triangle $\triangle ABC$. Let D and E are midpoints on the sides

opposite to **C** and **B**. So,

$$\mathbf{D} = \frac{\mathbf{A} + \mathbf{B}}{2}, \mathbf{E} = \frac{\mathbf{A} + \mathbf{C}}{2} \quad (0.1)$$

so the line joining the midpoints is

$$\mathbf{D} - \mathbf{E} = \frac{\mathbf{A} + \mathbf{B}}{2} - \frac{\mathbf{A} + \mathbf{C}}{2} = \frac{\mathbf{B} - \mathbf{C}}{2} = \frac{1}{2}(\mathbf{B} - \mathbf{C}) = \lambda(\mathbf{B} - \mathbf{C}) \quad (0.2)$$

So, the line is parallel to the third side as it $\lambda(\mathbf{B} - \mathbf{C})$.

Hence, proved.