

Matgeo Presentation - Problem 5.13.81

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Problem Statement

Let $S = \left\{ \mathbf{A} = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a, b, c, d, e \in \{0, 1\} \text{ and } |\mathbf{A}| \in \{-1, 1\} \right\}$.

Find the number of elements in S .

| Name | Matrix |
|----------|---|
| A | $\begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} \quad \text{with } a, b, c, d, e \in \{0, 1\}$ |

Table : Matrix

Solution

Applying row operation to **A**

$$\begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} \xleftrightarrow{R_2 \rightarrow R_2 - R_3} \begin{pmatrix} 0 & 1 & c \\ 0 & a - b & d - e \\ 1 & b & e \end{pmatrix} \quad (0.1)$$

Finding the determinant by the first column

$$|\mathbf{A}| = d - e - c(a - b) \quad (0.2)$$

Taking cases to find the possibilities of matrix **A**

Solution

Case 1 : $|\mathbf{A}| = 1$

if $c = 0$

the value of b and a can be 0 or 1.

$$d - e = 1 \quad (0.3)$$

So,

$$d = 1 \quad (0.4)$$

$$e = 0 \quad (0.5)$$

By permutation we get ,

$$2 \times 2 \times 1 \times 1 = 4 \quad (0.6)$$

Solution

if $c = 1$, we get 4 possibilities

$$d - e - (a - b) = 1 \quad (0.7)$$

So,

$$d = 1 \quad e = 0 \quad (0.8)$$

$$b = a = 1 \quad b = a = 0 \quad (0.9)$$

$$a = 0 \quad b = 1 \quad (0.10)$$

$$d = e = 1 \quad d = e = 0 \quad (0.11)$$

Solution

Case 2 : $|\mathbf{A}| = -1$

if $c = 0$

the value of b and a can be 0 or 1.

$$d - e = -1 \quad (0.12)$$

So,

$$d = 0 \quad (0.13)$$

$$e = 1 \quad (0.14)$$

By permutation we get ,

$$2 \times 2 \times 1 \times 1 = 4 \quad (0.15)$$

Solution

if $c = 1$, we get 4 possibilities

$$d - e - (a - b) = -1 \quad (0.16)$$

So,

$$d = 0 \quad e = 1 \quad (0.17)$$

$$b = a = 1 \quad b = a = 0 \quad (0.18)$$

$$a = 1 \quad b = 0 \quad (0.19)$$

$$d = e = 1 \quad d = e = 0 \quad (0.20)$$

By adding all the possibilities , we get

$$4 + 4 + 4 + 4 = 16 \quad (0.21)$$

Therefore, the number of elements in $S = 16$.