4.11.40

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Question

Find the area of the region bounded by line y=3x+2, the X axis and the ordinates x=-2 and x=1.

Solution:

let

$$\mathbf{A} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{1}$$

$$\mathbf{C} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2}$$

$$\tag{3}$$

let **D** and **E** be the vectors on the line corresponding to x= -2 and x=1

Given line equation is

$$-3x + y = 2 \tag{4}$$

which can be expressed as

$$\mathbf{n}^T\mathbf{x} = c \tag{5}$$

$$\implies (-3 \quad 1)\begin{pmatrix} x \\ y \end{pmatrix} = 2 \tag{6}$$

$$\mathbf{n} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \text{ and } \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \text{ and c} = 2 \tag{7}$$

let us find the vector **D**

$$(-3 \quad 1)\begin{pmatrix} -2 \\ y \end{pmatrix} = 2 \quad \implies y = -4 \tag{8}$$

$$\implies \mathbf{D} = \begin{pmatrix} -2 \\ -4 \end{pmatrix} \tag{9}$$

as y<0 we should find the **B** where the line meets the x axis

$$(-3 \quad 1)\begin{pmatrix} x \\ 0 \end{pmatrix} = 2 \quad \implies 3x = -2 \tag{10}$$

$$\implies \mathbf{B} = \begin{pmatrix} \frac{-2}{3} \\ 0 \end{pmatrix} \tag{11}$$

let us find the vector **E**

$$(-3 \quad 1)\begin{pmatrix} 1 \\ y \end{pmatrix} = 2 \quad \implies y = 5 \tag{12}$$

$$\implies \mathbf{E} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \tag{13}$$

The area to be computed is area of △EBC+area of △ABD

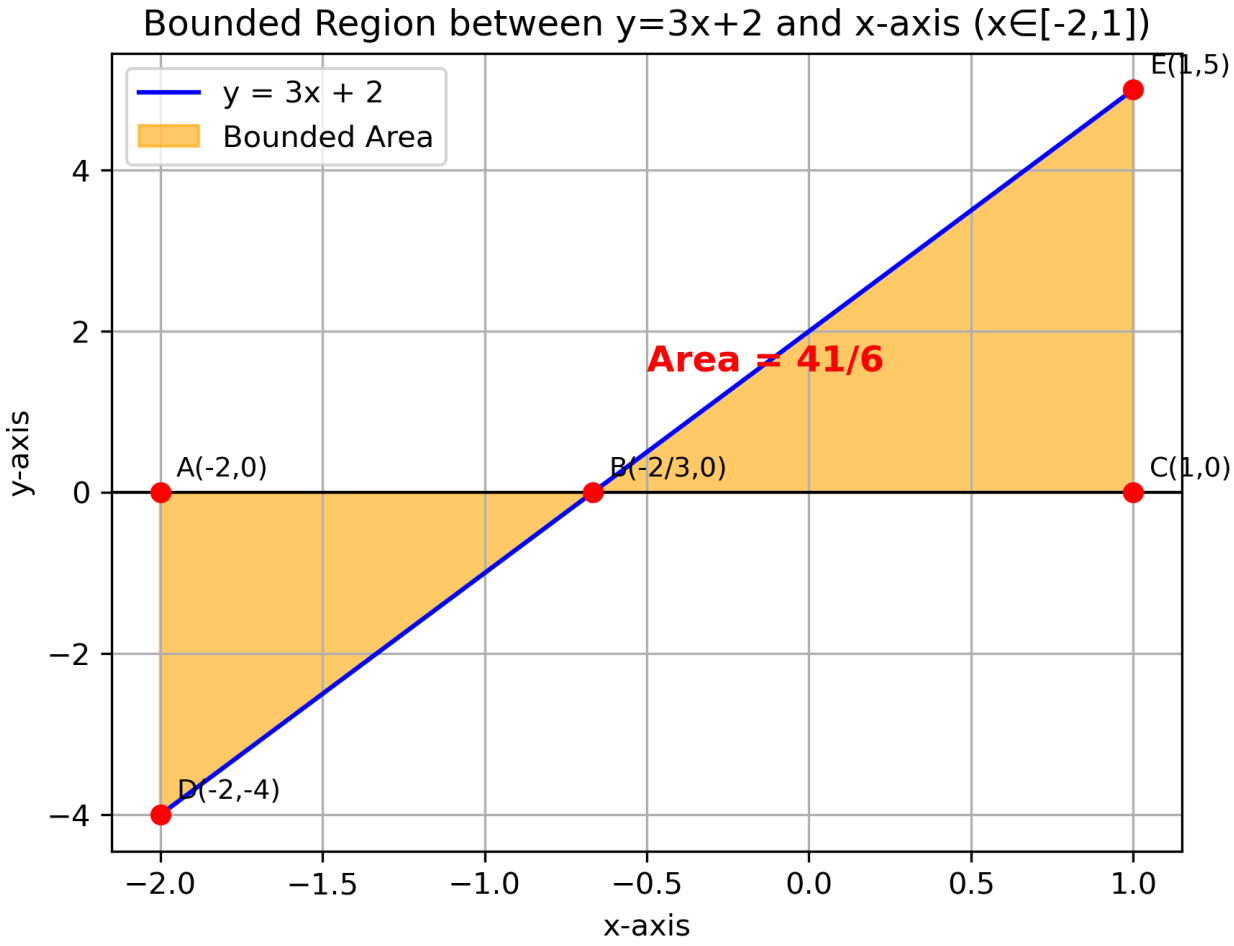
$$ar(\triangle ABD) = \frac{1}{2}\left\|(A-B)\times(A-D)\right\| \tag{14}$$

$$= \frac{1}{2}\left\|\begin{pmatrix} \frac{4}{3} \end{pmatrix}\times\begin{pmatrix} 0 \\ 4 \end{pmatrix}\right\| = \frac{8}{3} \tag{15}$$

$$ar(\triangle EBC) = \frac{1}{2}\left\|(E-c)\times(B-c)\right\| \tag{16}$$

$$= \frac{1}{2}\left\|\begin{pmatrix} 0 \\ 5 \end{pmatrix}\times\begin{pmatrix} -5/3 \\ 0 \end{pmatrix}\right\| = \frac{25}{6} \tag{17}$$

$$\implies \text{area of the region is } = \frac{8}{3} + \frac{25}{6} = \frac{41}{6} \tag{18}$$


Bounded Region between y=3x+2 and x-axis (x∈[-2,1])
y = 3x + 2
Bounded Area
Area = 41/6
A(-2,0)
B(-2/3,0)
C(1,0)
D(-2,-4)
E(1,5)
y-axis
x-axis
4
2
0
−2
−4
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0