

8.2.31

ee25btech11063-vejith

Question

Find the equation of conic if ends of the major axis are $(\pm 3, 0)$ and ends of the minor axis are $(0, \pm 2)$

solution

The equation of conic is represented as

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (1)$$

$$\mathbf{V} = \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n} \mathbf{n}^T \quad (2)$$

As the major axis is along the X-axis

$$\mathbf{n} = \mathbf{e}_1 \quad (3)$$

$$\Rightarrow \mathbf{V} = \begin{pmatrix} 1 - e^2 & 0 \\ 0 & 1 \end{pmatrix} \quad (4)$$

as the centre of ellipse is $\mathbf{c} = \mathbf{0}$

$$\Rightarrow \mathbf{u} = \mathbf{0} \quad (5)$$

let

$$\mathbf{P} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (6)$$

\mathbf{P} satisfy (1)

$$\mathbf{P}^T \mathbf{V} \mathbf{P} + 2\mathbf{u}^T \mathbf{P} + f = 0 \quad (7)$$

$$(0 \quad 2) \begin{pmatrix} 1 - e^2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} + f = 0 \quad (8)$$

$$4 + f = 0 \quad (9)$$

$$\Rightarrow f = -4 \quad (10)$$

End of the ellipse $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ also satisfy (1)

$$(3 \quad 0) \begin{pmatrix} 1 - e^2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix} + f = 0 \quad (11)$$

$$\Rightarrow 9(1 - e^2) + f = 0 \quad (12)$$

from (10)

$$1 - e^2 = \frac{4}{9} \quad (13)$$

$$\Rightarrow e^2 = \frac{5}{9} \quad (14)$$

$$\Rightarrow \mathbf{V} = \begin{pmatrix} \frac{4}{9} & 0 \\ 0 & 1 \end{pmatrix} \quad (15)$$

Equation of conic is

$$\mathbf{x}^T \begin{pmatrix} \frac{4}{9} & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - 4 = 0 \quad (16)$$

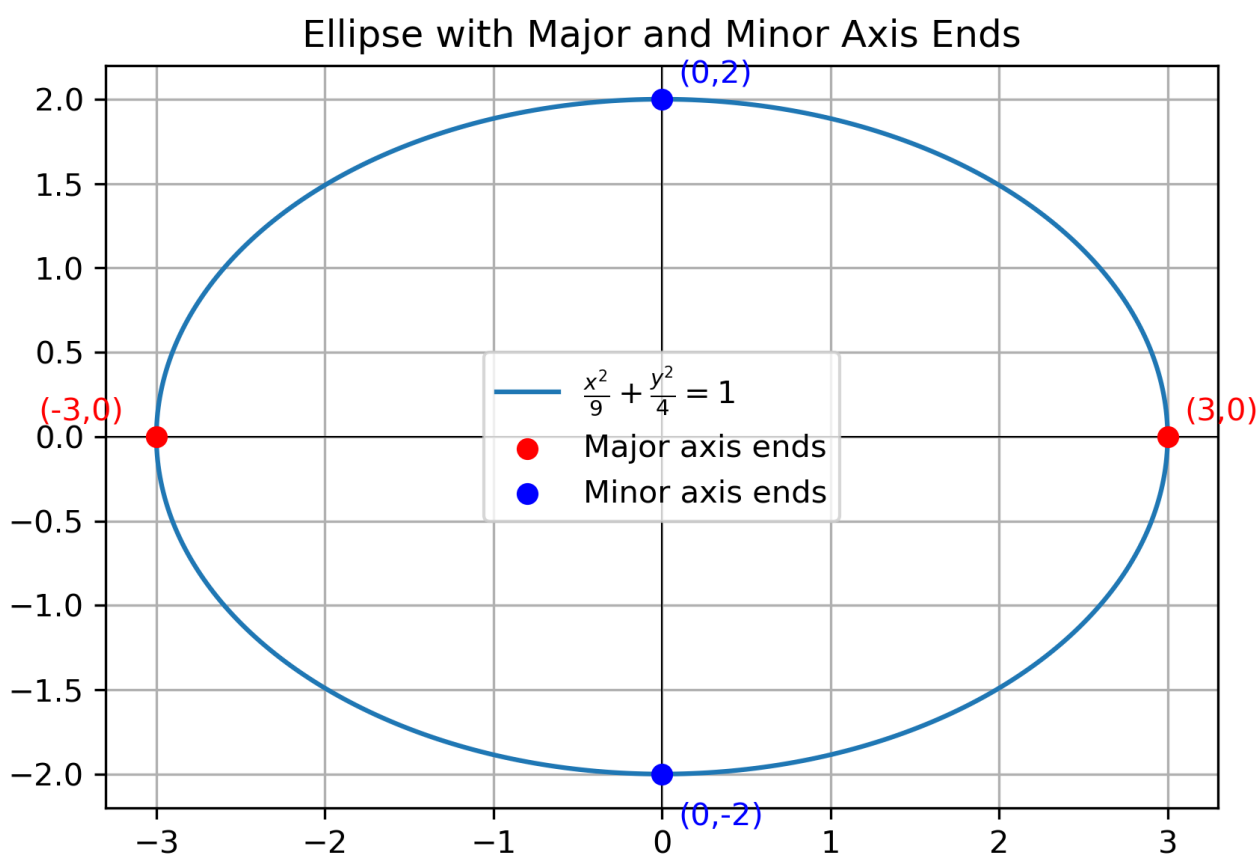


Fig. 0: Caption