# 5.13.28

EE25BTECH11002 - Achat Parth Kalpesh

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# Question

Let $\mathbf{A}$ be a $2 \times 2$ matrix with real entries. Let $\mathbf{I}$ be the $2 \times 2$ identity matrix. Denote by tr($\mathbf{A}$),the sum of diagonal entries of $\mathbf{A}$. Assume that $\mathbf{A}^2 = \mathbf{I}$.

Statement-1 : If $\mathbf{A} \neq \mathbf{I}$ and $\mathbf{A} \neq -\mathbf{I}$,then det($\mathbf{A}$)=-1

Statement-2 : If $\mathbf{A} \neq \mathbf{I}$ and $\mathbf{A} \neq -\mathbf{I}$,then $tr\left(\mathbf{A}\right) \neq 0$.

1. Statement-1 is false, Statement-2 is true
2. Statement-1 is true, Statement-2 is true;Statement-2 is a correct explanation for Statement-1
3. Statement-1 is true, Statement-2 is true;Statement-2 is not a correct explanation for Statement-1
4. Statement-1 is true, Statement-2 is false

# Solution

Let the eigenvalue of $\mathbf{A}$ be $\lambda$.

$$\mathbf{A}\mathbf{v} = \lambda \mathbf{v} \tag{1}$$

$$\mathbf{A}^2\mathbf{v} = \lambda^2 \mathbf{v} \tag{2}$$

$$\mathbf{A}^2 = \mathbf{I} \tag{3}$$

$$\lambda^2 = 1 \tag{4}$$

$$\lambda = \pm 1 \tag{5}$$

Thus, eigenvalues $\lambda_1$ , $\lambda_2$ of $\mathbf{A}$ are chosen from $\{1, -1\}$
As it is given as $\mathbf{A} \neq \mathbf{I}$ and $\mathbf{A} \neq -\mathbf{I}$ , so the possible case is

$$\lambda_1 = 1, \lambda_2 = -1 \tag{6}$$

# Solution

Thereby,

$$\begin{aligned} det\left(\mathbf{A}\right) &= \lambda_1\lambda_2 && (7)\\ &= -1 && (8)\\ tr\left(\mathbf{A}\right) &= \lambda_1 + \lambda_2 && (9)\\ &= 0 && (10)\end{aligned}$$

Thus Statement-1 is true, Statement-2 is false