4.13.1

AI25BTECH11036-SNEHAMRUDULA

Question:

Consider the lines given by

$$L_1 : x + 3y - 5 = 0,$$
$$L_2 : 3x - ky - 1 = 0,$$
$$L_3 : 5x + 2y - 12 = 0.$$

Match the Statements/Expressions in Column I with the Statements/Expressions in Column II.

Column I	Column II
(A) L_1, L_2, L_3 are concurrent, if	(a) $k = 9$
(B) One of L_1, L_2, L_3 is parallel to at least one of the other two, if	(b) $k = \dfrac{-6}{5}$
(C) L_1, L_2, L_3 form a triangle, if	(c) $k = \dfrac{5}{6}$
(D) L_1, L_2, L_3 do not form a triangle, if	(d) $k = 5$

Solution. The three line equations can be written in matrix form as

$$\begin{pmatrix} 1 & 3 \\ 3 & -k \\ 5 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 \\ 1 \\ 12 \end{pmatrix}. \tag{4.1}$$

(A) Concurrency: The lines are concurrent when all three equations admit a common solution. This happens if

$$\text{rank} \begin{pmatrix} 1 & 3 \\ 3 & -k \\ 5 & 2 \end{pmatrix} = \text{rank} \begin{pmatrix} 1 & 3 & 5 \\ 3 & -k & 1 \\ 5 & 2 & 12 \end{pmatrix} = 2. \tag{4.2}$$

Expanding the augmented determinant condition gives

$$k = 5. \tag{4.3}$$

(B) Parallelism: Two lines are parallel when their normals are proportional. Equivalently, rank of the normals is 1:

$$\text{rank} \begin{pmatrix} \mathbf{n}_2 & \mathbf{n}_3 \end{pmatrix} = 1. \tag{4.4}$$

This gives

$$k = -\tfrac{6}{5}. \tag{4.5}$$

(C) Triangle condition: Three lines form a triangle if they intersect pairwise but are not concurrent. That is,

$$\text{rank}\begin{pmatrix} \mathbf{n}_1 & \mathbf{n}_2 \end{pmatrix} = \text{rank}\begin{pmatrix} \mathbf{n}_2 & \mathbf{n}_3 \end{pmatrix} = \text{rank}\begin{pmatrix} \mathbf{n}_3 & \mathbf{n}_1 \end{pmatrix} = 2, \quad k \neq 5,\ k \neq -\tfrac{6}{5}. \tag{4.6}$$

So among the given options,

$$k = 9 \quad \text{or} \quad k = \tfrac{5}{6}. \tag{4.7}$$

(D) Do not form a triangle: This occurs if either concurrent or parallel, i.e.

$$k = 5 \quad \text{or} \quad k = -\tfrac{6}{5}. \tag{4.8}$$

Final Match:

$$(A) \rightarrow (d) : k = 5, \tag{4.9}$$

$$(B) \rightarrow (b) : k = -\tfrac{6}{5}, \tag{4.10}$$

$$(C) \rightarrow (a) \text{ or } (c) : k = 9 \text{ or } \tfrac{5}{6}, \tag{4.11}$$

$$(D) \rightarrow (d) \text{ or } (b) : k = 5 \text{ or } -\tfrac{6}{5}. \tag{4.12}$$

Graphical Representation