

5.2.44

EE25BTECH11042 - Nipun Dasari

Question:

Solve the following system of rational equations

$$\frac{10}{x+y} + \frac{2}{x-y} = 4 \quad (0.1)$$

$$\frac{15}{x+y} - \frac{5}{x-y} = -2 \quad (0.2)$$

Solution:

Introduce a and b as follows:

$$a = \frac{1}{x+y}, \quad b = \frac{1}{x-y} \quad (0.3)$$

Also define the vector $\mathbf{a} = \begin{pmatrix} a \\ b \end{pmatrix}$. This gives us the simplified linear system:

$$\begin{pmatrix} 10 & 2 \end{pmatrix} \mathbf{a} = 4 \quad (0.4)$$

$$\begin{pmatrix} 15 & -5 \end{pmatrix} \mathbf{a} = -2 \quad (0.5)$$

The augmented matrix for this system is:

$$\begin{aligned} \left(\begin{array}{cc|c} 10 & 2 & 4 \\ 15 & -5 & -2 \end{array} \right) & \xrightarrow{R_2 \leftarrow R_2 - \frac{3}{2}R_1} \left(\begin{array}{cc|c} 10 & 2 & 4 \\ 0 & -8 & -8 \end{array} \right) \\ & \xrightarrow{R_1 \leftarrow R_1 + \frac{1}{4}R_2} \left(\begin{array}{cc|c} 10 & 0 & 2 \\ 0 & -8 & -8 \end{array} \right) \\ & \xrightarrow{\begin{array}{l} R_1 \leftarrow \frac{1}{10}R_1 \\ R_2 \leftarrow -\frac{1}{8}R_2 \end{array}} \left(\begin{array}{cc|c} 1 & 0 & \frac{1}{5} \\ 0 & 1 & 1 \end{array} \right) \end{aligned} \quad (0.6)$$

From the reduced matrix, we have the solution:

$$\mathbf{a} = \begin{pmatrix} \frac{1}{5} \\ 1 \end{pmatrix} \quad (0.7)$$

Substituting back for x and y gives:

$$\begin{pmatrix} \frac{1}{x+y} \\ \frac{1}{x-y} \end{pmatrix} = \begin{pmatrix} \frac{1}{5} \\ 1 \end{pmatrix} \implies \begin{pmatrix} x+y \\ x-y \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \quad (0.8)$$

This is another linear system, $\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$, where $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$. We can solve for \mathbf{x} by

multiplying by the inverse of the matrix:

$$\mathbf{x} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 5 \\ 1 \end{pmatrix} \quad (0.9)$$

$$= \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 5 \\ 1 \end{pmatrix} \quad (0.10)$$

$$= \begin{pmatrix} \frac{1}{2}(5) + \frac{1}{2}(1) \\ \frac{1}{2}(5) - \frac{1}{2}(1) \end{pmatrix} \quad (0.11)$$

$$= \begin{pmatrix} \frac{6}{2} \\ \frac{4}{2} \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad (0.12)$$

Thus, the solution is $\mathbf{x} = 3$ and $\mathbf{y} = 2$.



