8.4.23

EE25BTECH11020 - Darsh Pankaj Gajare

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Question:
The curve described parametrically by $x = t^2 + t + 1$ and $y = t^2 - t + 1$ represents:

(A) a pair of straight lines

(B) an ellipse

(C) a parabola

(D) a hyperbola

Solution:

Table

x	$\begin{pmatrix} x \\ y \end{pmatrix}$
a	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
b	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
c	$\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

The parametric form can be written as

$$\mathbf{x} = \mathbf{a}t^2 + \mathbf{b}t + \mathbf{c}. \tag{0.1}$$

$$\mathbf{x} = \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix}^{\top} \begin{pmatrix} t^2 \\ t \end{pmatrix} + \mathbf{c} \tag{0.2}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} t^2 \\ t \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{0.3}$$

Solving for $\begin{pmatrix} t^2 \\ t \end{pmatrix}$ using the inverse matrix,

$$\begin{pmatrix} t^2 \\ t \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right) \tag{0.4}$$

$$= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right). \tag{0.5}$$

Multiplying,

$$\begin{pmatrix} t^2 \\ t \end{pmatrix} = \frac{1}{2} \begin{pmatrix} x + y - 2 \\ x - y \end{pmatrix}. \tag{0.6}$$

Eliminating t:

$$\begin{pmatrix} t^2 \\ t \end{pmatrix} = \frac{1}{2} \begin{pmatrix} x + y - 2 \\ x - y \end{pmatrix} \implies \frac{1}{2}(x + y - 2) = \left(\frac{1}{2}(x - y)\right)^2 \tag{0.7}$$

$$\Rightarrow (x - y)^2 = 2(x + y - 2) \tag{0.8}$$

$$\text{Write as quadratic form:} \quad \mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.9}$$

$$\mathbf{V} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \qquad \mathbf{u} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \qquad f = 4 \tag{0.10}$$

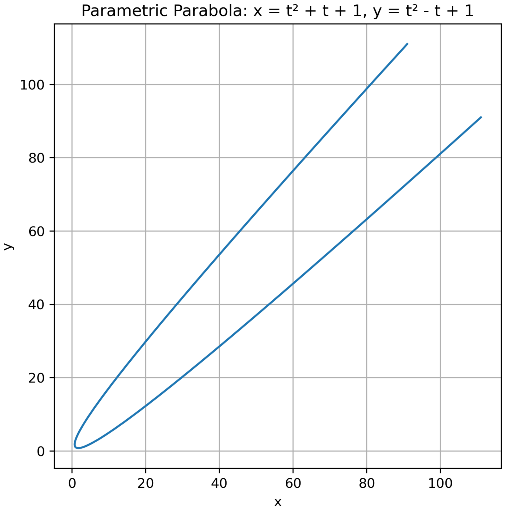
Extract quadratic coefficients:

$$A = V_{11} = 1, \qquad B = 2V_{12} = -2, \qquad C = V_{22} = 1 \tag{0.11}$$

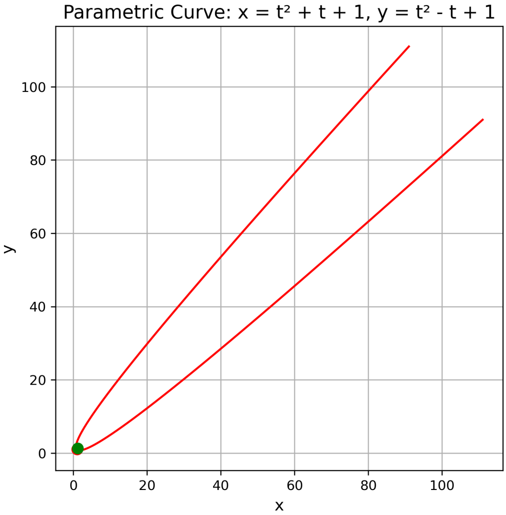
Discriminant:

$$\Delta = B^2 - 4AC = (-2)^2 - 4(1)(1) = 0 \tag{0.12}$$

Since $\Delta = 0$ the conic is a parabola. Plot using C libraries:



Plot using Python:


Parametric Curve: x = t² + t + 1, y = t² - t + 1
y
x
0
20
40
60
80
100