1.5.22

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Question

X and **Y** are two points with position vectors $3\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - 3\mathbf{b}$ respectively. Write the position vector of a point **V** which divides the line segment XY in the ratio $2 : 1$ externally.

Solution

Vectors $\mathbf{A}$ and $\mathbf{B}$ are given. Let $\mathbf{A} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Then,

$$\mathbf{X} = 3 * \mathbf{A} + \mathbf{B} \tag{1}$$

$$\mathbf{Y} = \mathbf{A} - 3\mathbf{B} \tag{2}$$

Or,

$$\mathbf{X} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{3}$$

$$\mathbf{Y} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} 1 \\ -3 \end{pmatrix} \tag{4}$$

Formula

Section formula for a vector **P** which divides the line formed by vectors **A** and **B** in the ratio k:1 externally is given by

$$\mathbf{P} = \frac{k\mathbf{B} - \mathbf{A}}{k - 1} \tag{5}$$

It is given that k=2.

Solution

$$\mathbf{V} = \frac{k\mathbf{Y} - \mathbf{X}}{k-1} \tag{6}$$

$$\implies \mathbf{V} = \frac{2\mathbf{Y} - \mathbf{X}}{1} \tag{7}$$

$$\implies \mathbf{V} = \frac{-2\begin{pmatrix}\mathbf{A} & \mathbf{B}\end{pmatrix}\begin{pmatrix}1\\-3\end{pmatrix} - \begin{pmatrix}\mathbf{A} & \mathbf{B}\end{pmatrix}\begin{pmatrix}3\\1\end{pmatrix}}{1} \tag{8}$$

$$\implies \mathbf{V} = \frac{\begin{pmatrix}\mathbf{A} & \mathbf{B}\end{pmatrix}\begin{pmatrix}2\\-6\end{pmatrix} - \begin{pmatrix}\mathbf{A} & \mathbf{B}\end{pmatrix}\begin{pmatrix}3\\1\end{pmatrix}}{1} \tag{9}$$

Solution

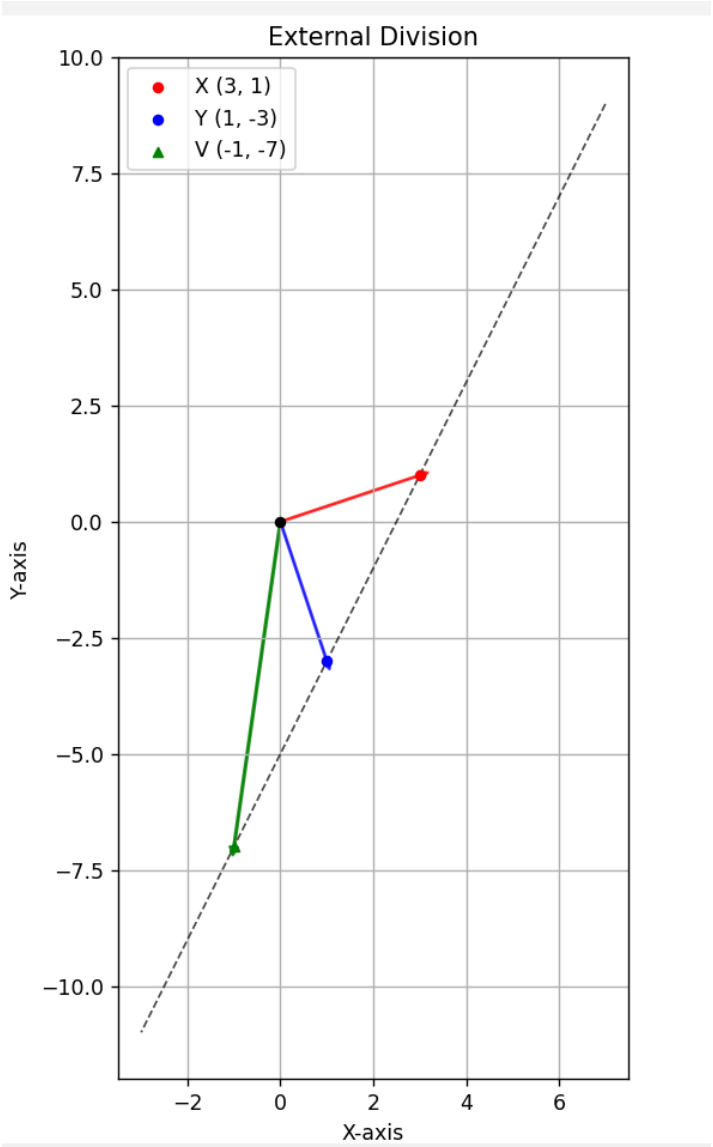
$$\implies \mathbf{V} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} -1 \\ -7 \end{pmatrix} \tag{10}$$

$$\implies \mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ -7 \end{pmatrix} \tag{11}$$

$$\implies \mathbf{V} = \begin{pmatrix} -1 \\ -7 \end{pmatrix} \tag{12}$$

Therefore, the vector $\mathbf{V} = \begin{pmatrix} -1 \\ -7 \end{pmatrix}$.

Plot


External Division
X (3, 1)
Y (1, -3)
V (-1, -7)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
Y-axis
−2
0
2
4
6
X-axis