Matgeo Presentation - Problem 5.13.81

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Problem Statement

Let $S = \{\mathbf{A} = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a, b, c, d, e \in \{0, 1\} \text{ and } |\mathbf{A}| \in \{-1, 1\}\}$.

Find the number of elements in S.

Data

Name	**Matrix**
A	$\begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix}$ with $a, b, c, d, e \in \{0, 1\}$

Table : Matrix

Solution

Rearranging the rows of **A**

$$\begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} \xleftrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 0 & 1 & c \\ 1 & b & e \\ 1 & a & d \end{pmatrix} \xleftrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & b & e \\ 0 & 1 & c \\ 1 & a & d \end{pmatrix} \tag{0.1}$$

Applyting row operation to **A** to reduce it into Echelon form

$$\begin{pmatrix} 1 & b & e \\ 0 & 1 & c \\ 1 & a & d \end{pmatrix} \xleftrightarrow{R_3 \to R_3 - R_1} \begin{pmatrix} 1 & b & e \\ 0 & 1 & c \\ 0 & a-b & d-e \end{pmatrix} \xleftrightarrow{R_3 \to R_3 - (a-b)R_2} \tag{0.2}$$

$$\begin{pmatrix} 1 & \mathrm{b} & \mathrm{e} \\ 0 & 1 & \mathrm{c} \\ 0 & 0 & d-e-c(a-b) \end{pmatrix} \tag{0.3}$$

Finding the determinant by the first column

$$\left|\mathbf{A}\right| = d - e - c(a-b) \tag{0.4}$$

Taking cases to find the possibilities of matrix **A**

Solution

Case 1 : $\left|\mathbf{A}\right| = 1$
if c = 0
the value of b and a can be 0 or 1.

$$d - e = 1 \tag{0.5}$$

So,

$$d = 1 \tag{0.6}$$
$$e = 0 \tag{0.7}$$

By permutation we get ,

$$2 \times 2 \times 1 \times 1 = 4 \tag{0.8}$$

Solution

if c $= 1$, we get 4 possibilities

$$d - e - (a - b) = 1 \tag{0.9}$$

So,

$$d = 1 \qquad e = 0 \tag{0.10}$$

$$b = a = 1 \qquad b = a = 0 \tag{0.11}$$

$$a = 0 \qquad b = 1 \tag{0.12}$$

$$d = e = 1 \qquad d = e = 0 \tag{0.13}$$

Solution

Case 2 : $\left|\mathbf{A}\right| = -1$
if c = 0
the value of b and a can be 0 or 1.

$$d - e = -1 \tag{0.14}$$

So,

$$d = 0 \tag{0.15}$$
$$e = 1 \tag{0.16}$$

By permutation we get ,

$$2 \times 2 \times 1 \times 1 = 4 \tag{0.17}$$

Solution

if c = 1, we get 4 possibilities

$$d - e - (a - b) = -1 \tag{0.18}$$

So,

$$d = 0 \qquad e = 1 \tag{0.19}$$

$$b = a = 1 \qquad b = a = 0 \tag{0.20}$$

$$a = 1 \qquad b = 0 \tag{0.21}$$

$$d = e = 1 \qquad d = e = 0 \tag{0.22}$$

By adding all the possibilities , we get

$$4 + 4 + 4 + 4 = 16 \tag{0.23}$$

Therefore, the number of elements in $S = 16$.