4.13.38

EE25BTECH11033 - Kavin

Question:
Let PS be the median of the triangle with vertices $\mathbf{P}(2,2)$, $\mathbf{Q}(6,-1)$ and $\mathbf{R}(7,3)$. The equation of the line passing through $(1,-1)$ and parallel to PS is

1) $4x+7y+3=0$
2) $2x-9y-11=0$
3) $4x-7y-11=0$
4) $2x+9y+7=0$

Solution:
Given the points,

$$\mathbf{P} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 6 \\ -1 \end{pmatrix} \quad \mathbf{R} = \begin{pmatrix} 7 \\ 3 \end{pmatrix} \tag{1}$$

$\mathbf{S}$ is the midpoint of the line segment joining points $\mathbf{Q}$ and $\mathbf{R}$.
If $\mathbf{S}$ divides QR in the ratio $k:1$,

$$\mathbf{S} = \frac{k\mathbf{R}+\mathbf{Q}}{k+1} \tag{2}$$

where,

$$k = 1 \tag{3}$$

$$\mathbf{S} = \frac{\mathbf{R}+\mathbf{Q}}{2} \tag{4}$$

$$\implies \mathbf{S} = \begin{pmatrix} 13/2 \\ 1 \end{pmatrix} \tag{5}$$

The direction vector of line PS is given by,

$$\mathbf{m} = \mathbf{S} - \mathbf{P} \equiv \begin{pmatrix} 9/2 \\ -1 \end{pmatrix} \tag{6}$$

Therefore, the normal vector of the desired line is given by,

$$\mathbf{n} = \begin{pmatrix} 1 \\ 9/2 \end{pmatrix} \tag{7}$$

$\therefore$ The equation of the line passing through $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and parallel to PS is given by

$$\mathbf{n}^\top \left(\mathbf{x} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right) = 0 \tag{8}$$

$$\begin{pmatrix} 1 & 9/2 \end{pmatrix} \begin{pmatrix} x-1 \\ y+1 \end{pmatrix} = 0 \tag{9}$$

$$\implies x - 1 + \frac{9}{2}(y+1) = 0 \tag{10}$$

$$\implies 2x + 9y + 7 = 0 \tag{11}$$