2.9.2

EE25BTECH11051 - Shreyas Goud Burra

Question If (-5, 3) and (5, 3) are two vertices of an equilateral triangle, then the coordinates of the third vertex, given that the origin lies inside the triangle (take $\sqrt{3} = 1.7$), are

Solution:

Let us find the solution theoretically first and then verify it computationally.
Let the two given points be represented as vectors, **A** and **B**, respectively

$$\mathbf{A} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \tag{0.1}$$

Let us assume the third point be **C**.
C must be equidistant from both **A** and **B**, and it lies on the perpendicular bisector to both **A** and **B**.

The distance between **A** and **B**, is given by

$$||\mathbf{A} - \mathbf{B}|| = \left\| \begin{pmatrix} -10 \\ 0 \end{pmatrix} \right\| \tag{0.2}$$

We know that the norm of a vector is given by

$$||\mathbf{A} - \mathbf{B}||^2 = (\mathbf{A} - \mathbf{B})^{\mathrm{T}}(\mathbf{A} - \mathbf{B}) \implies \begin{pmatrix} -10 & 0 \end{pmatrix} . \begin{pmatrix} -10 \\ 0 \end{pmatrix} = 100 \tag{0.3}$$

As the norm of a vector is always greater than or equal to zero. From 0.3 we get

$$||\mathbf{A} - \mathbf{B}|| = 10 \tag{0.4}$$

The midpoint to the line segment **AB** is given by

$$\frac{\mathbf{A} + \mathbf{B}}{2} = \frac{\begin{pmatrix} -5 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ 3 \end{pmatrix}}{2} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{0.5}$$

Slope of line segment **AB** is given by

$$\mathbf{B} - \mathbf{A} = k \begin{pmatrix} 1 \\ m \end{pmatrix}, \text{ where m is the slope of the line segment} \tag{0.6}$$

On further solving

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 10 \\ 0 \end{pmatrix} \implies m = 0 \tag{0.7}$$

Therefore the perpendicular bisector for this line segment is a vertical line passing through the midpoint (0, 3).

In parametric form

$$\mathbf{C} = \begin{pmatrix} 0 \\ t+3 \end{pmatrix}, \text{ where t is the distance between the point } \mathbf{C} \text{ and the line segment } \mathbf{AB} \tag{0.8}$$

We know for an equilateral triangle, distance between a point and the opposite edge is $\frac{\sqrt{3}}{2}$ times the length of an edge of that triangle.

$$t = \pm\frac{\sqrt{3}}{2}\left\|\mathbf{A} - \mathbf{B}\right\| \implies t = \pm 5\sqrt{3} \tag{0.9}$$

Therefore the required points for $\mathbf{C}$ are given by

$$\mathbf{C} = \begin{pmatrix} 0 \\ \pm 5\sqrt{3} + 3 \end{pmatrix} \tag{0.10}$$

On plotting this gives us

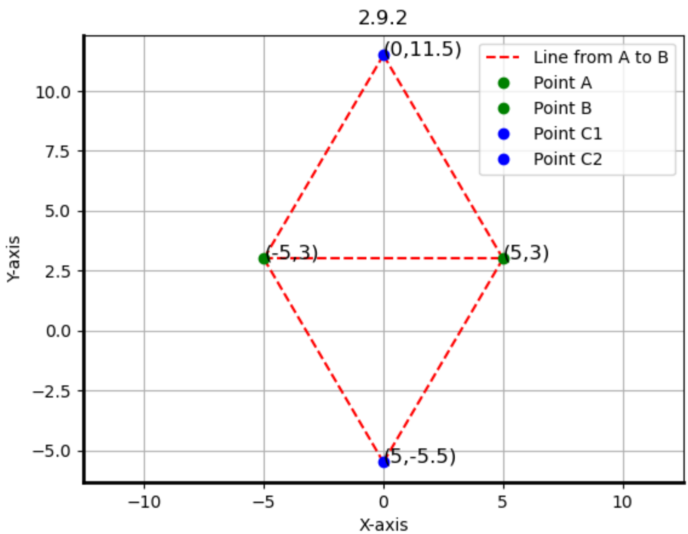

Fig. 0.1: 2D Plot