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Matrix 4.4.11

ai25btech11015 – M Sai Rithik

Question

The line segment joining the points $A(2, 1)$ and $B(5, -8)$ is trisected at the points P and Q, where P is nearer to A. If P lies on the line

$$2x - y + k = 0,$$

find the value of k. (Use matrix/linear algebra concepts only.)

Step 1: Vectors

Write the position vectors:

$$\mathbf{A} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 5 \\ -8 \end{pmatrix} \tag{1}$$

Difference:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ -9 \end{pmatrix} \tag{2}$$

Step 2: Trisection point P

First trisection point:

$$\mathbf{P} = \mathbf{A} + \tfrac{1}{3}(\mathbf{B} - \mathbf{A}) \tag{3}$$

Substitution:

$$\mathbf{P} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \tfrac{1}{3}\begin{pmatrix} 3 \\ -9 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{4}$$

Step 3: Line condition

Coordinates of P are $(3, -2)$. Substituting into the line:

$$2(3) - (-2) + k = 0 \tag{5}$$

Simplify:

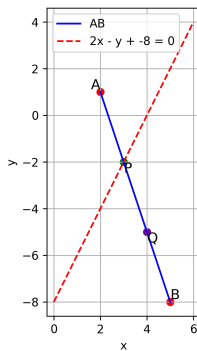$$8 + k = 0 \implies k = -8 \tag{6}$$

Final Answer

$$k = -8$$

Figure: