Matgeo Presentation - Problem 1.6.6

ee25btech11056 - Suraj.N

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Problem Statement

In each of the following, find the value of k for which the points are collinear:

$(7, -2),\ (5, 1),\ (3, k)$
$(8, 1),\ (k, -4),\ (2, -5)$

Method

Condition for Collinearity:

Three points A, B, C are collinear iff vectors

$$\mathbf{B} - \mathbf{A}, \quad \mathbf{C} - \mathbf{A}$$

are linearly dependent.

Equivalently, the *collinearity matrix*

$$M = \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix}^\top$$

must satisfy $\text{rank}(M) = 1$.

Part (a) Setup

Let

$$A = \begin{pmatrix} 7 \\ -2 \end{pmatrix}, \quad B = \begin{pmatrix} 5 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 3 \\ k \end{pmatrix}.$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 5-7 \\ 1-(-2) \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix},$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 3-7 \\ k-(-2) \end{pmatrix} = \begin{pmatrix} -4 \\ k+2 \end{pmatrix}.$$

Thus,

$$M = \begin{pmatrix} -2 & 3 \\ -4 & k+2 \end{pmatrix}.$$

Part (a) Row Reduction

Apply row transformation:

$$R_2 = R_2 - 2R_1 \quad \Longrightarrow \quad \begin{pmatrix} -2 & 3 \\ 0 & k-4 \end{pmatrix}.$$

$$\text{For collinearity: } k - 4 = 0 \quad \Longrightarrow \quad k = \boxed{4}.$$

Part (b) Setup

Let

$$A = \begin{pmatrix} 8 \\ 1 \end{pmatrix}, \quad B = \begin{pmatrix} k \\ -4 \end{pmatrix}, \quad C = \begin{pmatrix} 2 \\ -5 \end{pmatrix}.$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} k-8 \\ -5 \end{pmatrix}, \quad \mathbf{C} - \mathbf{A} = \begin{pmatrix} -6 \\ -6 \end{pmatrix}.$$

Thus,

$$M = \begin{pmatrix} k-8 & -5 \\ -6 & -6 \end{pmatrix}.$$

Part (b) Row Reduction

Row operation:

$$R_2 = (k-8)R_2 + 6R_1$$

$$\Longrightarrow \quad \begin{pmatrix} k-8 & -5 \\ 0 & 18-6k \end{pmatrix}.$$

For collinearity: $18 - 6k = 0 \quad \Longrightarrow \quad k = \boxed{3}$.

Final Answer

(a) $k = 4$

(b) $k = 3$

Plot of line1

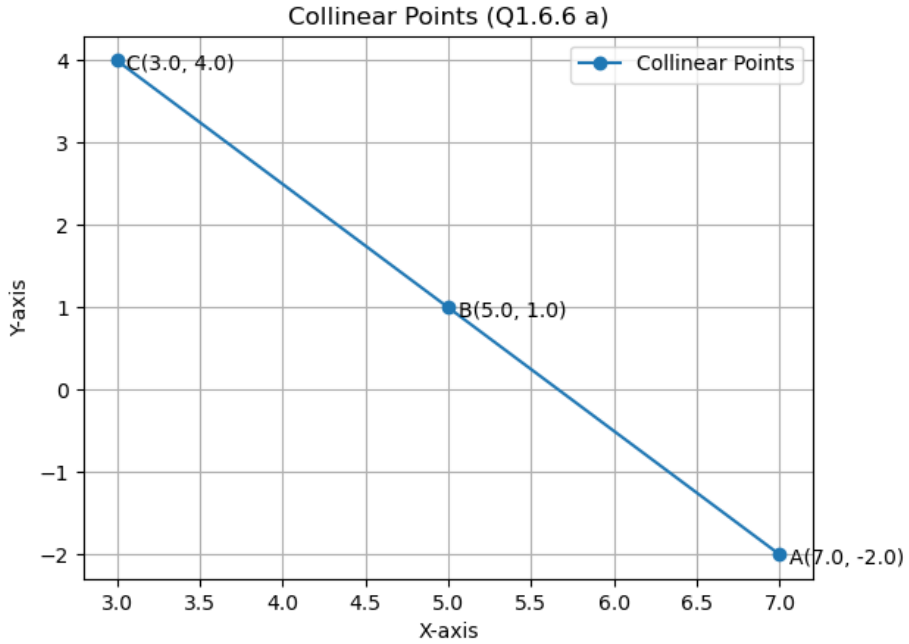

Fig 1 : Line through the given points

Plot of line2

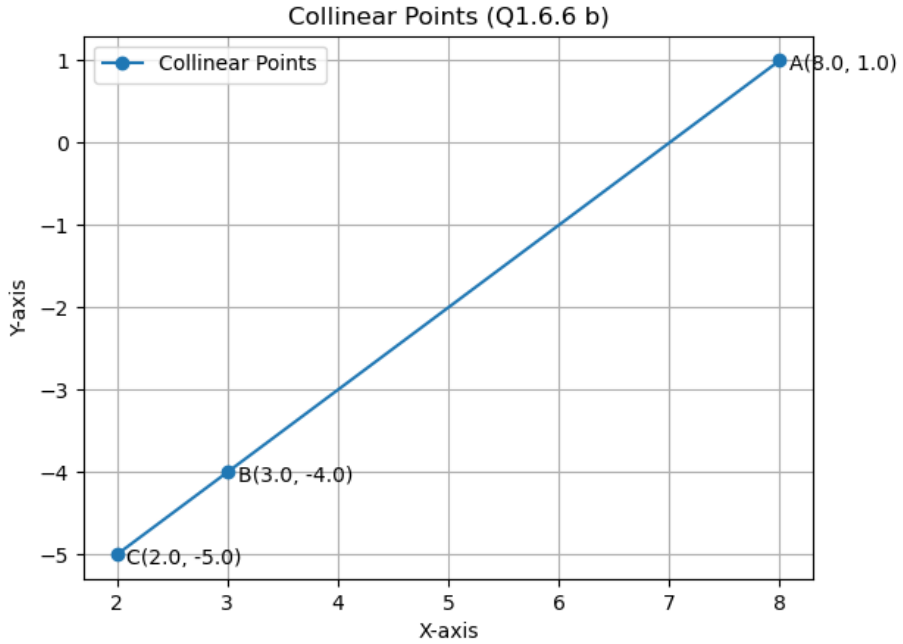

Fig 2 : Line through the given points