4.6.1

EE25BTECH11013 - Bhargav

Question:
The distance between the parallel planes

$$2x + y - 2z - 6 = 0 \tag{0.1}$$

$$4x + 2y - 4z = 0 \tag{0.2}$$

Solution:

The second plane equation can be written as:

$$2x + y - 2z = 0 \tag{0.3}$$

The 2 given planes are parallel since their normal vectors are the same

The normal vector of the planes

$$\mathbf{n} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \tag{0.4}$$

The distance between the planes is given by this formula

$$\text{Distance} = \frac{|\mathbf{n}^{\mathbf{T}}\mathbf{p} - d|}{\|\mathbf{n}\|} \tag{0.5}$$

Where $\mathbf{p}$ represents a point on the second plane

$$\mathbf{p} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, d = 6 \tag{0.6}$$

$$\mathbf{n}^{\mathbf{T}}\mathbf{p} - d = \begin{pmatrix} 2 & 1 & -2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - 6 \tag{0.7}$$

$$\mathbf{n}^{\mathbf{T}}\mathbf{p} - d = -6 \tag{0.8}$$

$$\|\mathbf{n}\|^2 = \mathbf{n}^{\mathbf{T}}\mathbf{n} = \begin{pmatrix} 2 & 1 & -2 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} \tag{0.9}$$

$$\|\mathbf{n}\|^2 = 2 \times 2 + 1 \times 1 + (-2) \times (-2) \tag{0.10}$$

$$\|\mathbf{n}\|^2 = 9 \tag{0.11}$$

$$\therefore \|\mathbf{n}\| = 3 \tag{0.12}$$

Substituting these values in the Distance formula, we get

$$\therefore \text{Distance} = \frac{|-6|}{3} \tag{0.13}$$

$$\text{Distance} = 2 \tag{0.14}$$

Therefore, the distance between the planes is 2