

# Matrices in Geometry 1.9.26

EE25BTECH11037 - Divyansh

**Question:** Find the value of  $k$ , if the point  $\mathbf{P}(2, 4)$  is equidistant from point  $\mathbf{A}(5, k)$  and  $\mathbf{B}(k, 7)$

**Given:**  $\mathbf{P}\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ ,  $\mathbf{A}\begin{pmatrix} 5 \\ k \end{pmatrix}$  and a point  $\mathbf{B}\begin{pmatrix} k \\ 7 \end{pmatrix}$  such that  $\mathbf{P}$  is equidistant from  $\mathbf{A}$  and  $\mathbf{B}$ .

$$\therefore \|\mathbf{A} - \mathbf{P}\| = \|\mathbf{B} - \mathbf{P}\|$$

On squaring both the sides, we get

$$\|\mathbf{A} - \mathbf{P}\|^2 = \|\mathbf{B} - \mathbf{P}\|^2$$

$$(\mathbf{A} - \mathbf{P})^\top (\mathbf{A} - \mathbf{P}) = (\mathbf{B} - \mathbf{P})^\top (\mathbf{B} - \mathbf{P})$$

$$(\mathbf{A} - \mathbf{P}) = \begin{pmatrix} 3 \\ k - 4 \end{pmatrix}$$

$$(\mathbf{B} - \mathbf{P}) = \begin{pmatrix} k - 2 \\ 3 \end{pmatrix}$$

$$(\mathbf{A} - \mathbf{P})^\top (\mathbf{A} - \mathbf{P}) = \begin{pmatrix} 3 & k - 4 \end{pmatrix} \begin{pmatrix} 3 \\ k - 4 \end{pmatrix} = 9 + (k - 4)^2 = k^2 - 8k + 25$$

$$(\mathbf{B} - \mathbf{P})^\top (\mathbf{B} - \mathbf{P}) = \begin{pmatrix} k - 2 & 3 \end{pmatrix} \begin{pmatrix} k - 2 \\ 3 \end{pmatrix} = (k - 2)^2 + 9 = k^2 - 4k + 13$$

$$\text{From above, } k^2 - 8k + 25 = k^2 - 4k + 13$$

$$\implies 4k = 12 \implies k = 3$$

Hence, the final answer is  $\boxed{k = 3}$

Point P is Equidistant from A and B for  $k = 3.00$

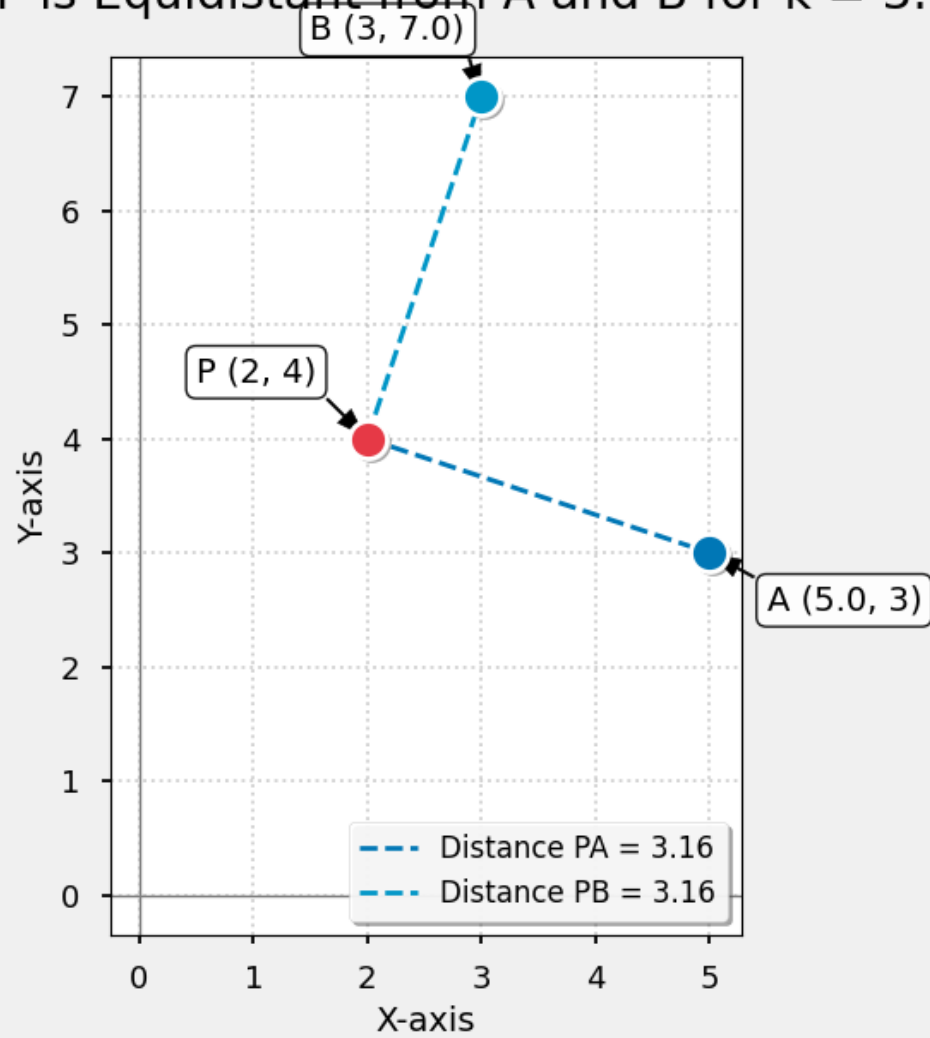


Fig. 1: Plot for 1.9.26