Matrices in Geometry 4.3.24

EE25BTECH11035 - Kushal B N

Question: Find the ratio in which the line $2x + 3y - 5 = 0$ divides the line segment joining the points $(8, -9)$ and $(2, 1)$. Also find the coordinates of the point of division.

Given:
Line $\begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 5$
Points $\mathbf{A}\begin{pmatrix} 8 \\ -9 \end{pmatrix}$ and $\mathbf{B}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Solution:
Let $\mathbf{P}\begin{pmatrix} h \\ k \end{pmatrix}$ be the point on the given line dividing the line segment joining the given points.
So, the point $\mathbf{P}$ also lies on the line joining the given points $\mathbf{A}$ and $\mathbf{B}$.
The direction vector for this line would be

$$\mathbf{d} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} -6 \\ 10 \end{pmatrix} \tag{1}$$

So that the normal vector for the line(after dividing by common factor 2) will be

$$\mathbf{n} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \tag{2}$$

and in the line equation $\mathbf{n}^T \mathbf{P} = c$,

$$c = \begin{pmatrix} 5 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 13 \tag{3}$$

Thus, the line equation is

$$\begin{pmatrix} 5 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 13 \tag{4}$$

Solving for the intersection of the two lines, and by forming the augmented matrix,

$$\begin{pmatrix} 2 & 3 \\ 5 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 13 \end{pmatrix} \tag{5}$$

$$\implies \left(\begin{array}{cc|c} 2 & 3 & 5 \\ 5 & 3 & 13 \end{array}\right) \xrightarrow{R_1 \to \frac{1}{2}R_1} \left(\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{5}{2} \\ 5 & 3 & 13 \end{array}\right) \tag{6}$$

$$\left(\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{5}{2} \\ 5 & 3 & 13 \end{array}\right) \xrightarrow{R_2 \to R_2 - 5R_1} \left(\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{5}{2} \\ 0 & \frac{-9}{2} & \frac{1}{2} \end{array}\right) \tag{7}$$

$$\left(\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{5}{2} \\ 0 & \frac{-9}{2} & \frac{1}{2} \end{array}\right) \xrightarrow{R_2 \to \frac{-2}{9}R_2} \left(\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{5}{2} \\ 0 & 1 & \frac{-1}{9} \end{array}\right) \tag{8}$$

$$\implies \mathbf{P} = \begin{pmatrix} \frac{8}{3} \\ \frac{-1}{9} \end{pmatrix} \tag{9}$$

Section Formula for a point $\mathbf{P}$ which divides the line segment formed by points $\mathbf{A}$ and $\mathbf{B}$ in the ratio $k : 1$ is given by

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k+1} \tag{10}$$

$$k\left(\mathbf{P} - \mathbf{B}\right) = \mathbf{A} - \mathbf{P} \tag{11}$$

$$\implies k = \frac{\left(\mathbf{A} - \mathbf{P}\right)^{\top}\left(\mathbf{P} - \mathbf{B}\right)}{\|\mathbf{P} - \mathbf{B}\|^2} \tag{12}$$

$$\left(\mathbf{A} - \mathbf{P}\right)^{\top}\left(\mathbf{P} - \mathbf{B}\right) = \begin{pmatrix} \frac{16}{3} & \frac{-80}{9} \end{pmatrix}\begin{pmatrix} \frac{2}{3} \\ \frac{-10}{9} \end{pmatrix} = \frac{1088}{81} \tag{13}$$

$$\|\mathbf{P} - \mathbf{B}\|^2 = \left(\mathbf{P} - \mathbf{B}\right)^{\top}\left(\mathbf{P} - \mathbf{B}\right) = \frac{136}{81} \tag{14}$$

$$\implies \boxed{\text{k=8}} \tag{15}$$

Final Answer:

$\therefore$ The ratio in which the line divides the two given points is 8:1 and the coordinates of the point of division is $\begin{pmatrix} \frac{8}{3} \\ \frac{-1}{9} \end{pmatrix}$.

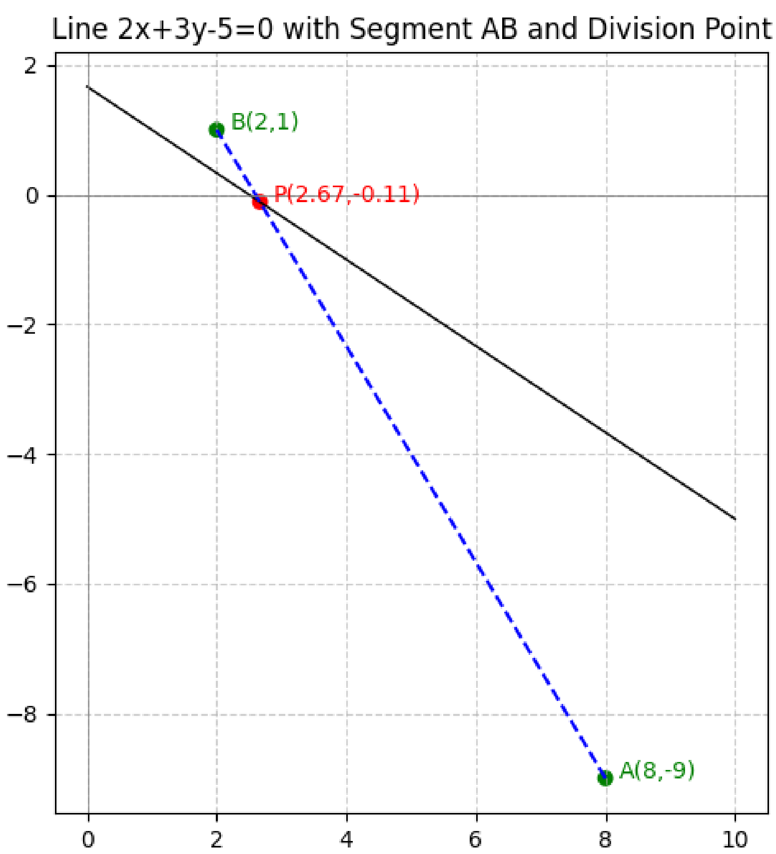

Fig. 1