Matrices in Geometry - 4.3.26

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Problem Statement

Find the ratio in which the line segment joining the points $\mathbf{A} = (4, 8, 10)$ and $\mathbf{B} = (6, 10, -8)$ is divided bv the YZ plane.

Solution

We have two points $\mathbf{A} = \begin{pmatrix} 4 \\ 8 \\ 10 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 6 \\ 10 \\ -8 \end{pmatrix}$

Let $\mathbf{P}$ be the point on the Y-Z plane. Since it is collinear to $\mathbf{A}$ and $\mathbf{B}$,

Since $\mathbf{P}$ lies on Y-Z plane, $\mathbf{P} = \begin{pmatrix} 0 \\ P_y \\ P_z \end{pmatrix}$.

From (4.1.2.5)

$$\begin{pmatrix} \mathbf{B} & \mathbf{A} & \mathbf{P} \end{pmatrix}^{\top} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{1}$$

$$\begin{pmatrix} 6 & 10 & -8 \\ 4 & 8 & 10 \\ 0 & P_y & P_z \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \implies \left(\begin{array}{ccc|c} 6 & 10 & -8 & 1 \\ 4 & 8 & 10 & 1 \\ 0 & P_y & P_z & 1 \end{array}\right) \tag{2}$$

Solution

$$\xrightarrow[2]{R_1 \to \frac{(R_1 - R_2)}{2}} \left(\begin{array}{ccc|c} 1 & 1 & -9 & 0 \\ 4 & 8 & 10 & 1 \\ 0 & P_y & P_z & 1 \end{array}\right) \xrightarrow{R_2 \to R_2 - 4R_1} \tag{3}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -9 & 0 \\ 0 & 4 & 46 & 1 \\ 0 & P_y & P_z & 1 \end{array}\right) \xrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 1 & -9 & 0 \\ 0 & 4 & 46 & 1 \\ 0 & P_y - 4 & P_z - 46 & 0 \end{array}\right) \tag{4}$$

Since **P**, **A** and **B** are collinear, the rank of this matrix must be less than or equal to 2.. Therefore, the third row should be a zero row and therefore, $\mathbf{P} = \begin{pmatrix} 0 \\ 4 \\ 46 \end{pmatrix}$

Solution

Let **P** divide **A** and **B** in the ratio $k : 1$

Using the formula (1.1.5.2)

$$k = \frac{(\mathbf{A} - \mathbf{P})^{\top} (\mathbf{P} - \mathbf{B})}{\|\mathbf{P} - \mathbf{B}\|^2} \tag{5}$$

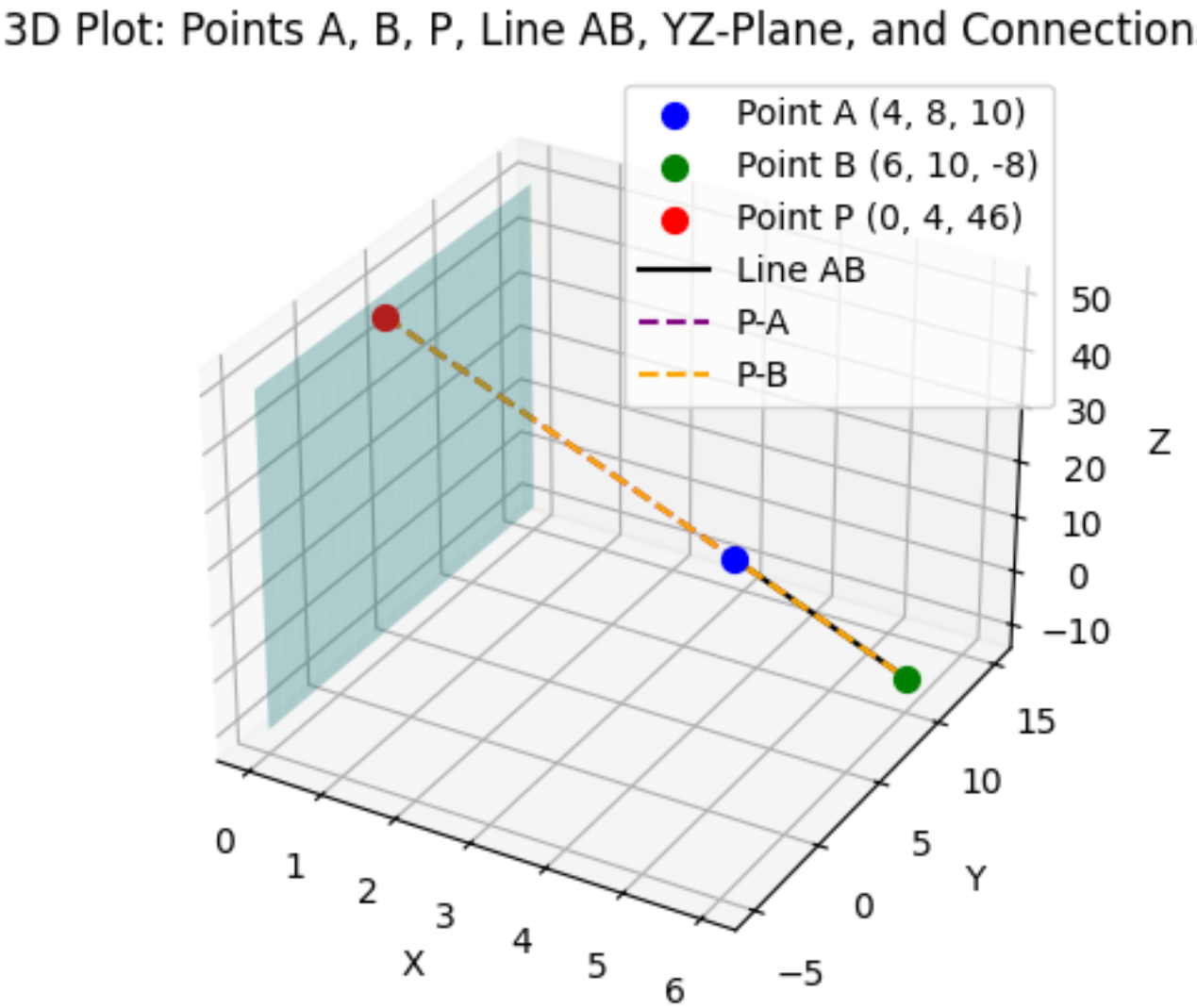
$$\implies k = \frac{\begin{pmatrix} 4 & 4 & -36 \end{pmatrix} \begin{pmatrix} -6 \\ -6 \\ 54 \end{pmatrix}}{\left\| \begin{pmatrix} -6 \\ -6 \\ 54 \end{pmatrix} \right\|} \tag{6}$$

$$\implies k = \frac{-24 - 24 - 1944}{36 + 36 + 2916} = \frac{-1992}{2988} \tag{7}$$

$$\implies k = \frac{-2}{3} \tag{8}$$

Hence, the Y-Z plane divides the line segment that joins the points **A** and **B** in the external ratio $2 : 3$.

Solution


3D Plot: Points A, B, P, Line AB, YZ-Plane, and Connections
Point A (4, 8, 10)
Point B (6, 10, -8)
Point P (0, 4, 46)
Line AB
P-A
P-B
X
Y
Z
0
1
2
3
4
5
6
−5
0
5
10
15
−10
0
10
20
30
40
50