9.3.3 Matgeo

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Question

Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

Solution

The points of intersection of the line :

$$L : \mathbf{x} = \mathbf{h} + \kappa \mathbf{m} \tag{1}$$

with the conic is given by

$$\mathbf{x}_i = \mathbf{h} + \kappa_i \mathbf{m} \tag{2}$$

Solution

where :

$$g(\mathbf{x}) = \mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f \quad (3)$$

$$\kappa_i = \frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}}(-\mathbf{m}^T(\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T(\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^T\mathbf{V}\mathbf{m})}) \quad (4)$$

Solution

For the parabola $3x^2$ - 4y $= 0$

$$\mathbf{V} = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix} \tag{5}$$

$$\mathbf{u} = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \tag{6}$$

For the line 2y $=$ 3x $+$ 12.

$$\mathbf{X} = \begin{bmatrix} 0 \\ 6 \end{bmatrix} + \kappa \begin{bmatrix} 0 \\ \frac{3}{2} \end{bmatrix} \tag{7}$$

$$\mathbf{h} = \begin{bmatrix} 0 \\ 6 \end{bmatrix} \tag{8}$$

$$\mathbf{m} = \begin{bmatrix} 0 \\ \frac{3}{2} \end{bmatrix} \tag{9}$$

Solution

Substituting and solving we get :

$$\kappa = \begin{bmatrix} 2 \\ -1 \end{bmatrix} \tag{10}$$

so the points of intersection after solving using the equation 0.2 are :

$$\mathbf{X} = \begin{bmatrix} 4 \\ 12 \end{bmatrix} \quad and \quad \begin{bmatrix} -2 \\ 3 \end{bmatrix} \tag{11}$$

Calculating the area :

$$\int_{-2}^{4} \frac{3}{2}x + 6 - \frac{3}{4}x^2 \, dx = 27 \tag{12}$$

Graphical Representation