12.755

EE25BTECH11013 - Bhargav

Question:
Which one of the following vectors is an eigenvector corresponding to the eigenvalue $\lambda = 1$ for the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 1 & -1 & 1 \end{pmatrix} \tag{0.1}$$

is
Solution:

The eigenvalue of the the the matrix **A** can be found out by (where $\lambda = 1$ is the eigenvalue, **x** is the eigenvector, **I** is the identity matrix)

$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x} \implies \mathbf{A}\mathbf{x} = \mathbf{x} \tag{0.2}$$

$$(\mathbf{A} - \mathbf{I})\,\mathbf{x} = \mathbf{0} \tag{0.3}$$

$$\implies \begin{pmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ 1 & -1 & 0 \end{pmatrix} \mathbf{x} = \mathbf{0} \tag{0.4}$$

This can be solved by representing it as an augmented matrix and using row elimination

$$\left(\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 1 & -2 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{array}\right) \xleftrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{array}\right) \xleftrightarrow{R_3 \leftarrow R_3 - R_1} \tag{0.5}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right) \xleftrightarrow{R_3 \leftarrow R_3 - R_2} \left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \tag{0.6}$$

Thus $\mathbf{x} = t \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ where $t \in \mathbf{R}$

So, the eigenvector of **A** is $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

This can be further verified by the intersection of planes

$$x - 2y = 0 \tag{0.7}$$

$$y = 0 \tag{0.8}$$

The intersection of the 2 planes is $x = y = 0$