1.5.16

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Question: Find the point A if AB is a diameter of the circle with center C = (3, -1) and point B = (2, 6).

Solution:

Point	Vector
В	$\begin{pmatrix} 2 \\ 6 \end{pmatrix}$
С	$\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

Section Formula: If a point P divides the line joining A and B internally in the ratio m:n, then

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k+1} = (\mathbf{A} \quad \mathbf{B}) \begin{pmatrix} \frac{1}{k+1} \\ \frac{k}{k+1} \end{pmatrix}.$$

Here, C is the midpoint of AB, i.e. ratio 1:1.

$$\mathbf{C} = \frac{\mathbf{A} + \mathbf{B}}{2} = (\mathbf{A} \quad \mathbf{B}) \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}.$$

Express A in terms of B and C:

$$C = \frac{A + B}{2}$$
 \Rightarrow $2C = A + B$ \Rightarrow $A = 2C - B$.

Using matrix notation,

$$\mathbf{A} = 2\mathbf{C} - \mathbf{B} = \begin{pmatrix} \mathbf{B} & \mathbf{C} \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix}.$$

Substitute values:

Given

$$\mathbf{B} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 3 \\ -1 \end{pmatrix},$$

we have

$$\mathbf{A} = 2 \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}.$$

$$\mathbf{A} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$$

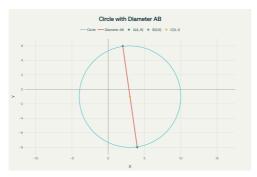


Fig. 0