7.4.17

Circle equation

EE25BTECH11010 - Arsh Dhoke

Question

If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along the diameter of a circle of circumference 10π, then the equation of the circle is:

1. $x^2 + y^2 + 2x - 2y - 23 = 0$
2. $x^2 + y^2 - 2x - 2y - 23 = 0$
3. $x^2 + y^2 + 2x + 2y - 23 = 0$
4. $x^2 + y^2 - 2x + 2y - 23 = 0$

Equation of Circle

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{1}$$

where $\mathbf{V}$ is an identity matrix of order 2.

$$2x + 3y + 1 = 0, \quad 3x - y - 4 = 0 \tag{2}$$

Finding Intersection (Centre)

$$\begin{pmatrix} 2 & 3 \\ 3 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -1 \\ 4 \end{pmatrix} \tag{3}$$

Augmented form:

$$\left(\begin{array}{cc|c} 2 & 3 & -1 \\ 3 & -1 & 4 \end{array}\right) \tag{4}$$

Performing row operations:

$$\left(\begin{array}{cc|c} 2 & 3 & -1 \\ 3 & -1 & 4 \end{array}\right) \xrightarrow[R_2 \to R_2 - \frac{3}{2}R_1]{} \left(\begin{array}{cc|c} 2 & 3 & -1 \\ 0 & -\frac{11}{2} & \frac{11}{2} \end{array}\right) \tag{5}$$

$$\xrightarrow[R_2 \to \frac{2}{-11}R_2]{} \left(\begin{array}{cc|c} 2 & 3 & -1 \\ 0 & 1 & -1 \end{array}\right) \tag{6}$$

$$\xrightarrow[R_1 \to R_1 - 3R_2]{} \left(\begin{array}{cc|c} 2 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right) \tag{7}$$

Centre and Radius

$$\xrightarrow[R_1 \to \frac{1}{2}R_1]{} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right) \tag{8}$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{9}$$

$$\Rightarrow \mathbf{c} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{10}$$

Given circumference $10\pi \Rightarrow r = 5 \Rightarrow r^2 = 25$.

Finding Constants

$$\mathbf{V} = \mathbf{I}, \quad \mathbf{c} = -\mathbf{u} \tag{11}$$

$$\Rightarrow \mathbf{u} = -\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{12}$$

$$f = \mathbf{c}^T\mathbf{V}\mathbf{c} - r^2 = 2 - 25 = -23 \tag{13}$$

Final Equation of Circle

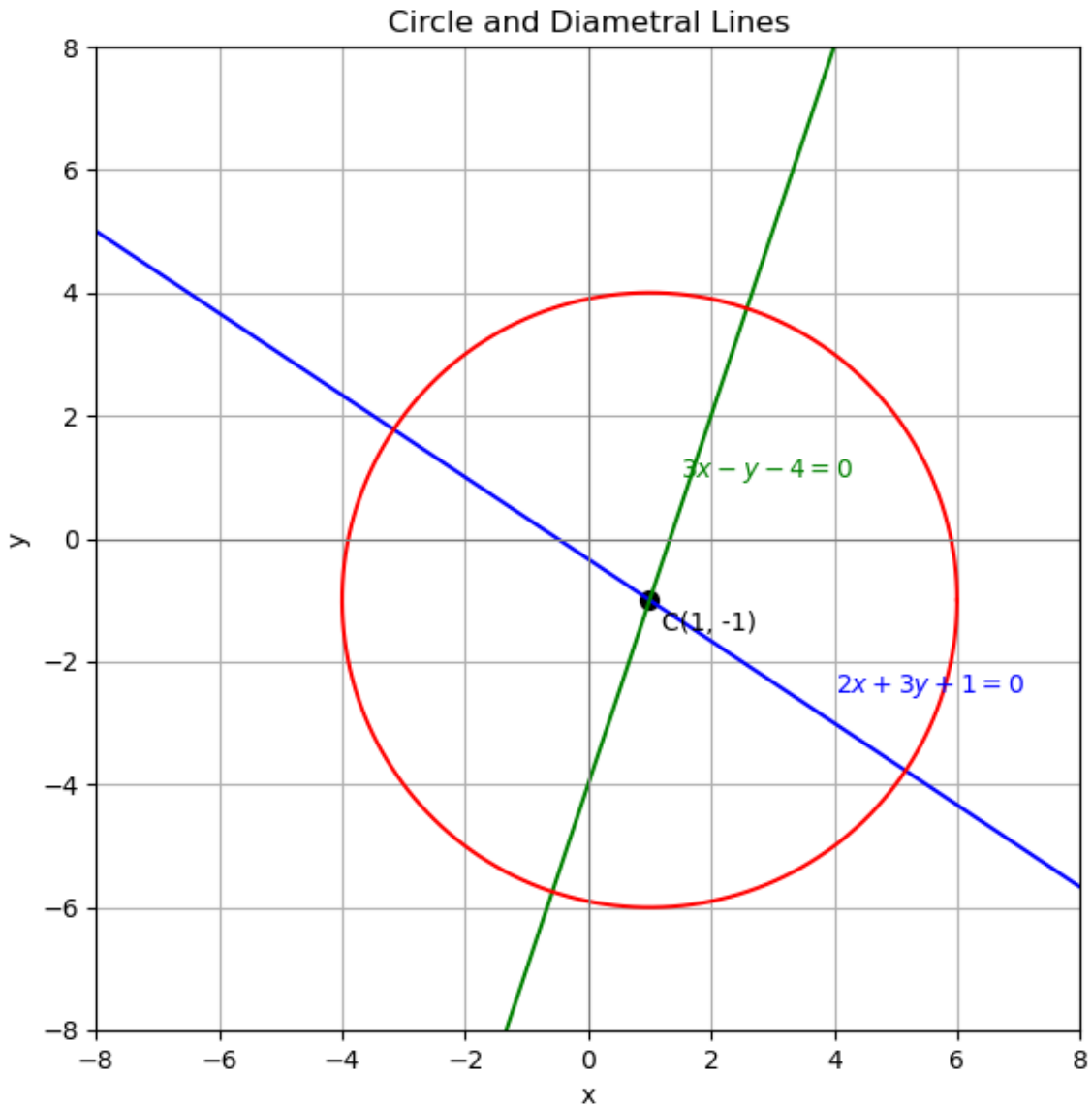
$$(\mathbf{x})^T \mathbf{I}\mathbf{x} + 2\begin{pmatrix}-1 & 1\end{pmatrix}\mathbf{x} - 23 = 0 \tag{14}$$

$$(\mathbf{x})^T \mathbf{x} + 2\begin{pmatrix}-1 & 1\end{pmatrix}\mathbf{x} - 23 = 0 \tag{15}$$

Substituting $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$,

$$x^2 + y^2 - 2x + 2y - 23 = 0 \tag{16}$$

Graphical Representation



C Code

```c
#include <math.h>

// Function to solve circle given two lines on diameter and
    circumference
void circle_from_diameter_lines(double a1, double b1, double c1,
                                double a2, double b2, double c2,
                                double circumference,
                                double coeffs[5]) {
    // Find intersection point of the two lines (midpoint of
        diameter)
    double det = a1*b2 - a2*b1;
    double x0 = (b1*c2 - b2*c1)/det;
    double y0 = (a2*c1 - a1*c2)/det;

    // Radius from circumference: 2*pi*r = circumference
    double r = circumference / (2*M_PI);
    double r2 = r*r;
```

C Code

```
    // Circle equation: (x - x0)^2 + (y - y0)^2 = r^2
    // Expand: x^2 + y^2 - 2*x0*x - 2*y0*y + (x0^2 + y0^2 - r^2)
        = 0
    coeffs[0] = -2*x0; // D
    coeffs[1] = -2*y0; // E
    coeffs[2] = x0*x0 + y0*y0 - r2; // F
    coeffs[3] = 1.0; // Coefficient of x^2
    coeffs[4] = 1.0; // Coefficient of y^2
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt

# Circle parameters
h, k = 1, -1
r = 5

# Line equations
x = np.linspace(-8, 8, 400)
y1 = (-2*x - 1)/3 # 2x + 3y + 1 = 0
y2 = 3*x - 4 # 3x - y - 4 = 0

# Circle
theta = np.linspace(0, 2*np.pi, 400)
xc = h + r * np.cos(theta)
yc = k + r * np.sin(theta)

# Plot setup
plt.figure(figsize=(7,7))
```

Python Code

```
plt.plot(x, y1, color='blue')
plt.plot(x, y2, color='green')
plt.plot(xc, yc, color='red')

# Centre
plt.scatter(h, k, color='black', s=50)
plt.text(h+0.2, k-0.5, 'C(1, -1)', fontsize=10)

# Annotate equations beside lines
# For 2x + 3y + 1 = 0
plt.text(4, (-2*4 - 1)/3 + 0.5, r'$2x + 3y + 1 = 0$', color='blue
    ', fontsize=10)

# For 3x - y - 4 = 0
plt.text(1.5, 3*(1.5) - 4 + 0.5, r'$3x - y - 4 = 0$', color='
    green', fontsize=10)
```

Python Code

```
# Axes and styling
plt.axhline(0, color='gray', linewidth=0.8)
plt.axvline(0, color='gray', linewidth=0.8)
plt.gca().set_aspect('equal', adjustable='box')

plt.xlim(-8, 8)
plt.ylim(-8, 8)
plt.xlabel('x')
plt.ylabel('y')
plt.title('Circle and Diametral Lines')
plt.grid(True)
plt.savefig("/home/arsh-dhoke/ee1030-2025/ee25btech11010/matgeo
    /7.4.17/figs/circle.png")
plt.show()
```

Python+ C Code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load the shared library
lib = ctypes.CDLL('./code.so')

# Define argument and return types
lib.circle_from_diameter_lines.argtypes = [
    ctypes.c_double, ctypes.c_double, ctypes.c_double, # line1 a,
        b,c
    ctypes.c_double, ctypes.c_double, ctypes.c_double, # line2 a,
        b,c
    ctypes.c_double, # circumference
    ctypes.POINTER(ctypes.c_double) # coeffs array
]
lib.circle_from_diameter_lines.restype = None
```

Python+ C Code

```
# Prepare coefficients array
coeffs = (ctypes.c_double * 5)()

# Call the function: example lines 2x+3y+1=0, 3x-y-4=0,
    circumference=10*pi
lib.circle_from_diameter_lines(2, 3, 1, 3, -1, -4,
    10*3.14159265358979 3, coeffs)

coeffs_list = list(coeffs)
print("Circle coefficients [D, E, F, x^2 coeff, y^2 coeff]:",
    coeffs_list)

# Extract center and radius
D, E, F, _, _ = coeffs_list
x0 = -D/2
y0 = -E/2
r = np.sqrt(x0**2 + y0**2 - F)
```

Python+ C Code

```
# Plot the circle
theta = np.linspace(0, 2*np.pi, 500)
x = x0 + r*np.cos(theta)
y = y0 + r*np.sin(theta)

plt.figure(figsize=(6,6))
plt.plot(x, y, label='Circle')
plt.scatter([x0], [y0], color='red', label='Center')
plt.gca().set_aspect('equal', 'box')
plt.title('Circle from two diameter lines')
plt.xlabel('x')
plt.ylabel('y')
plt.grid(True)
plt.legend()
plt.savefig("/home/arsh-dhoke/ee1030-2025/ee25btech11010/matgeo
    /7.4.17/figs/circle.png")
plt.show()
```