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AI25BTECH11030 -Sarvesh Tamgade

Question: Which of the following expressions are meaningful?

(a) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$

(b) $(\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w}$

(c) $(\mathbf{u} \cdot \mathbf{v}) \, \mathbf{w}$

(d) $\mathbf{u} \times (\mathbf{v} \cdot \mathbf{w})$

Solution:

Let $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ be vectors in $\mathbb{R}^3$.

(a) $\mathbf{u}(\mathbf{v} \times \mathbf{w})$:

The expression $\mathbf{v} \times \mathbf{w}$ is a vector (cross product), and the expression $\mathbf{u}(\mathbf{v} \times \mathbf{w})$ denotes the scalar triple product (sometimes written as the inner product of $\mathbf{u}$ and the vector $\mathbf{v} \times \mathbf{w}$).

Meaningful.

(b) $(\mathbf{u}^\top \mathbf{v})\mathbf{w}$:

Here, $(\mathbf{u}^\top \mathbf{v})$ represents the inner (dot) product, which is a scalar. Multiplying a scalar by a vector $\mathbf{w}$ is valid. However, if it is interpreted as $(\mathbf{u}^\top \mathbf{v})^\top \mathbf{w}$ having a dot between scalar and vector, that is not defined.

Not meaningful if interpreted as scalar dot vector.

(c) $\langle \mathbf{u}^\top \mathbf{v} \rangle^\top \mathbf{w}$:

$\langle \mathbf{u}, \mathbf{v} \rangle$ denotes the inner product (a scalar) and multiplying this scalar by vector $\mathbf{w}$ is valid scalar multiplication of a vector.

Meaningful.

(d) $\mathbf{u} \times (\mathbf{v}^\top \mathbf{w})$:

$\mathbf{v}^\top \mathbf{w}$ inside parentheses denotes the inner product (scalar), and cross product between a vector and scalar is undefined.
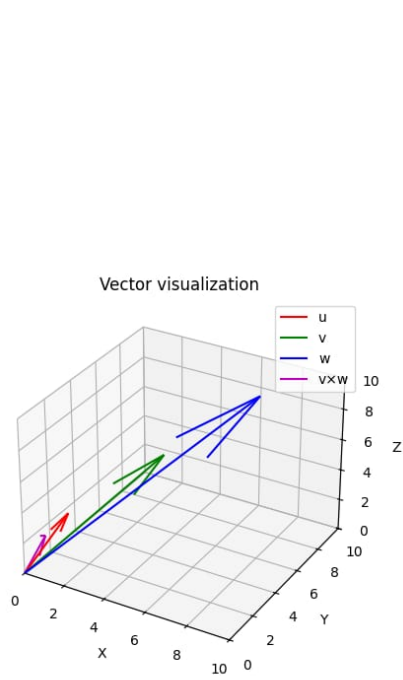
Not meaningful.

Answer: | Only (a) and (c) are meaningful |

Fig. 4.1: Vector Representation