

# 12.475

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## Question:

Consider a triangle PQR with initial coordinates of the vertices as  $P \begin{pmatrix} 1 & 3 \end{pmatrix}^T$ ,  $Q \begin{pmatrix} 4 & 5 \end{pmatrix}^T$  and  $R \begin{pmatrix} 5 & 3.5 \end{pmatrix}^T$ . The triangle is rotated in the X-Y plane about the vertex P by angle  $\theta$  in clockwise direction. If  $\sin \theta = 0.6$  and  $\cos \theta = 0.8$ , the new coordinates of the vertex Q are

- 1)  $\begin{pmatrix} 4.6 & 2.8 \end{pmatrix}^T$
- 2)  $\begin{pmatrix} 3.2 & 4.6 \end{pmatrix}^T$
- 3)  $\begin{pmatrix} 7.9 & 5.5 \end{pmatrix}^T$
- 4)  $\begin{pmatrix} 5.5 & 7.9 \end{pmatrix}^T$

## Solution:

Let the coordinates of the vertices be represented by vectors  $\mathbf{p} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$  and  $\mathbf{q} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ . The rotation of a point  $\mathbf{q}$  about a pivot point  $\mathbf{p}$  is given by:

$$\mathbf{q}_{\text{new}} = \mathbf{R}(\mathbf{q} - \mathbf{p}) + \mathbf{p} \quad (1)$$

The matrix for a clockwise rotation by an angle  $\theta$  is:

$$\mathbf{R} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad (2)$$

Substituting the given values and vectors into (1)

$$\mathbf{q}_{\text{new}} = \begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix} \left( \begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right) + \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (3)$$

$$= \begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (4)$$

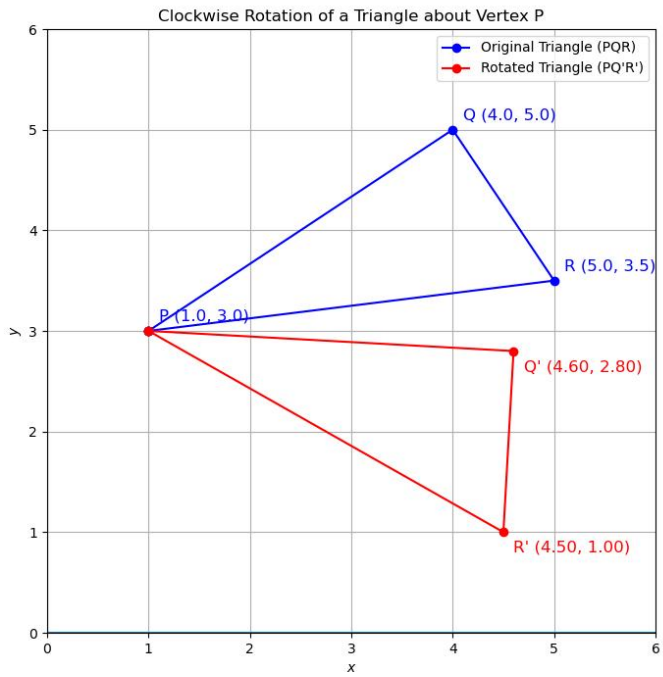
$$= \begin{pmatrix} 0.8(3) + 0.6(2) \\ -0.6(3) + 0.8(2) \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (5)$$

$$= \begin{pmatrix} 2.4 + 1.2 \\ -1.8 + 1.6 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (6)$$

$$= \begin{pmatrix} 3.6 \\ -0.2 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4.6 \\ 2.8 \end{pmatrix} \quad (7)$$

The new coordinates of the vertex Q are  $\begin{pmatrix} 4.6 \\ 2.8 \end{pmatrix}$ .

The correct option is **1**).



Plot