Matrices in Geometry 12.259

EE25BTECH11037 - Divyansh

Question: Consider the system of equations

$$\begin{pmatrix} 5 & 2 & 1 \\ -2 & 5 & 2 \\ -1 & 2 & 8 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 13 \\ -22 \\ 14 \end{pmatrix}$$

With an initial guess of the solution $\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}^\top = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}^\top$, the approximate value of the solution $\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}^\top$ after one iteration of the Gauss-Seidel method is

1) $\begin{pmatrix} 2 & -4.4 & 1.625 \end{pmatrix}^\top$
2) $\begin{pmatrix} 2 & 4.4 & 1.625 \end{pmatrix}^\top$
3) $\begin{pmatrix} 2 & -4 & -3 \end{pmatrix}^\top$
4) $\begin{pmatrix} 2 & -4 & 3 \end{pmatrix}^\top$

Solution: Let the initial guess of the solution be

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{1}$$

Isolating each variable from the given equations

$$x_1 = \frac{13 - 2x_2 - x_3}{5} \;,\; x_2 = \frac{-22 + 2x_1 - 2x_3}{5} \;,\; x_3 = \frac{14 + x_1 - 2x_2}{8} \tag{2}$$

Substituting $x_2 = 1, x_3 = 1$ in the first equation

$$x_1 = \frac{13 - 2 - 1}{5} = \frac{10}{5} = 2 \tag{3}$$

Substituting $x_1 = 2, x_3 = 1$ in the second equation

$$x_2 = \frac{-22 + 4 - 2}{5} = \frac{-20}{5} = -4 \tag{4}$$

Substituting $x_1 = 2, x_2 = -4$

$$x_3 = \frac{14 + 2 + 8}{8} = \frac{24}{8} = 3 \tag{5}$$

After one iteration of the Gauss-Seidel method, we get

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 3 \end{pmatrix} \tag{6}$$

which is option 4)
Plotting these points in a graph

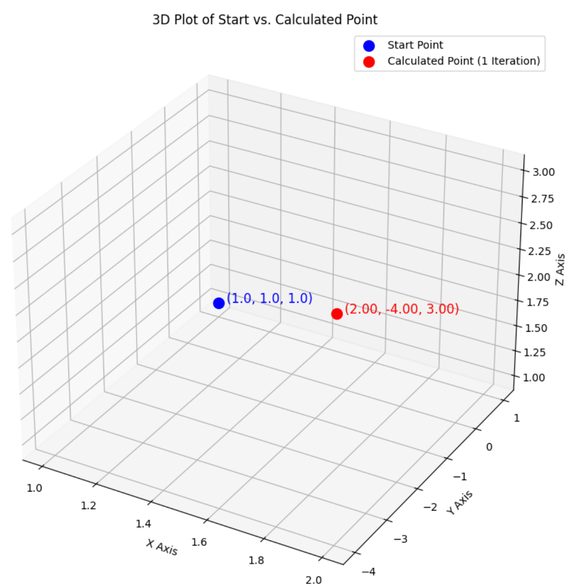

Fig. 1: Graph for 12.259