Assignment 1: GATE 2018 MA

EE25BTECH11061– Vankudoth Sainadh

1) "The dress _________ her so well that they all immediately _________ her on her appearance." (GATE MA 2018)

   The word that best fill blanks in the above sentence are

   a) complemented, complemented
   b) complimented, complemented
   c) complimented, complimented
   d) complemented, complimented

2) "The judge's standing in the legal community, though shaken by false allegations of wrongdoing, remained _________." (GATE MA 2018)

   The word that best fill blanks in the above sentence are

   a) undiminished
   b) damaged
   c) illegal
   d) uncertain

3) Find the missing group of letters in the following series: BC, FGH, LMNO, _________ (GATE MA 2018)

   a) UVWXY
   b) TUVWX
   c) STUVW
   d) RSTUV

4) The perimeters of a circle, a square and an equilateral triangle are equal. Which one of the following statements is true? (GATE MA 2018)

   a) The circle has the largest area.
   b) The square has the largest area.
   c) The equilateral triangle has the largest area.
   d) All the three shapes have the same area.

5) The value of the expression $\frac{1}{1+\log_u vw} + \frac{1}{1+\log_v wu} + \frac{1}{1+\log_w uv}$ is _________. (GATE MA 2018)

   a) -1
   b) 0
   c) 1
   d) 3

6) Forty students watched films A, B and C over a week. Each student watched either only one film or all three. Thirteen students watched film A, sixteen students watched film B and nineteen students watched film C. How many students watched all three films? (GATE MA 2018)

   a) 0
   b) 2
   c) 4
   d) 8

7) A wire would enclose an area of $1936\,\mathrm{m}^2$, if it is bent into a square. The wire is cut into two pieces. The longer piece is thrice as long as the shorter piece. The long and the short pieces are bent into a square and a circle, respectively. Which of the following choices is closest to the sum of the areas enclosed by the two pieces in square meters? (GATE MA 2018)

   a) 1096
   b) 1111
   c) 1243
   d) 2486

8) A contract is to be completed in 52 days and 125 identical robots were employed, each operational for 7 hours a day. After 39 days, five-seventh of the work was completed. How many additional robots would be required to complete the work on time, if each robot is now operational for 8 hours a day?

(GATE MA 2018)

a) 50 b) 89 c) 146 d) 175

9) A house has a number which needs to be identified. The following three statements are given that can help in identifying the house number.

(GATE MA 2018)

i. If the house number is a multiple of 3, then it is a number from 50 to 59.
ii. If the house number is NOT a multiple of 4, then it is a number from 60 to 69.
iii. If the house number is NOT a multiple of 6, then it is a number from 70 to 79.

a) 54 b) 65 c) 66 d) 76

10) An unbiased coin is tossed six times in a row and four different such trials are conducted. One trial implies six tosses of the coin. If H stands for head and T stands for tail, the following are the observations from the four trials:
(1) HTHTHT (2) TTHHHT (3) HTTHHT (4) HHHT__ __.

Which statement describing the last two coin tosses of the fourth trial has the highest probability of being correct?

(GATE MA 2018)

a) Two T will occur.
b) One H and one T will occur.
c) Two H will occur.
d) One H will be followed by one T.

11) The principal value of $(-1)^{(-2i/\pi)}$ is

(GATE MA 2018)

a) e^2 b) e^{2i} c) e^{-2i} d) e^{-2}

12) Let $f : \mathbb{C} \to \mathbb{C}$ be an entire function with $f(0) = 1$, $f(1) = 2$ and $f'(0) = 0$. If there exists $M > 0$ such that $|f''(z)| \leq M$ for all $z \in \mathbb{C}$, then $f(2) =$

(GATE MA 2018)

a) 2 b) 5 c) $2 + 5i$ d) $5 + 2i$

13) In the Laurent series expansion of $f(z) = \dfrac{1}{z(z-1)}$ valid for $|z - 1| > 1$, the coefficient of $\dfrac{1}{z-1}$ is

(GATE MA 2018)

a) -2 b) -1 c) 0 d) 1

14) Let X and Y be metric spaces, and let $f : X \to Y$ be a continuous map. For any subset S of X, which one of the following statements is true? (GATE MA 2018)

a) If S is open, then $f(S)$ is open
b) If S is connected, then $f(S)$ is connected
c) If S is closed, then $f(S)$ is closed
d) If S is bounded, then $f(S)$ is bounded

15) The general solution of the differential equation $xy' = y + \sqrt{x^2 + y^2}$ for $x > 0$ is given by (with an arbitrary positive constant k)

(GATE MA 2018)

a) $ky^2 = x + \sqrt{x^2 + y^2}$
b) $kx^2 = x + \sqrt{x^2 + y^2}$
c) $kx^2 = y + \sqrt{x^2 + y^2}$
d) $ky^2 = y + \sqrt{x^2 + y^2}$

16) Let $p_n(x)$ be the polynomial solution of the differential equation

$$\frac{d}{dx}\left(\left(1 - x^2\right) y'\right) + n(n+1) y = 0$$

with $p_n(1) = 1$ for $n = 1, 2, 3, \ldots$. If

$$\frac{d}{dx}\left(p_{n+2}(x) - p_n(x)\right) = \alpha_n \, p_{n+1}(x),$$

then α_n is

(GATE MA 2018)

a) $2n$ b) $2n + 1$ c) $2n + 2$ d) $2n + 3$

17) In the permutation group S_6, the number of elements of order 8 is

(GATE MA 2018)

a) 0 b) 1 c) 2 d) 4

18) Let R be a commutative ring with 1 (unity) which is not a field. Let $I \subset R$ be a proper ideal such that every element of R not in I is invertible in R. Then the number of maximal ideals of R is (GATE MA 2018)

a) 1
b) 2
c) 3
d) infinite

19) Let $f : \mathbb{R} \to \mathbb{R}$ be a twice continuously differentiable function. The order of convergence of the secant method for finding root of the equation $f(x) = 0$ is (GATE MA 2018)

a) $\frac{1+\sqrt{5}}{2}$
b) $\frac{2}{1+\sqrt{5}}$
c) $\frac{1+\sqrt{5}}{3}$
d) $\frac{3}{1+\sqrt{5}}$

20) The Cauchy problem $u\,u_x + y\,u_y = x$ with $u(x,1) = 2x$, when solved using its characteristic equations with an independent variable t, is found to admit of a solution in the form

$$x = \frac{3}{2}se^t - \frac{1}{2}se^{-t}, \quad y = e^t, \quad u = f(s,t).$$

Then $f(s,t)$ is (GATE MA 2018)

a) $\frac{3}{2}se^t + \frac{1}{2}se^{-t}$
b) $\frac{1}{2}se^t + \frac{3}{2}se^{-t}$
c) $\frac{1}{2}se^t - \frac{3}{2}se^{-t}$
d) $\frac{3}{2}se^t - \frac{1}{2}se^{-t}$

21) An urn contains four balls, each ball having equal probability of being white or black. Three black balls are added to the urn. The probability that five balls in the urn are black is (GATE MA 2018)

a) $\frac{2}{7}$
b) $\frac{3}{8}$
c) $\frac{1}{2}$
d) $\frac{5}{7}$

22) For a linear programming problem, which one of the following statements is FALSE? (GATE MA 2018)

a) If a constraint is an equality, then the corresponding dual variable is unrestricted in sign
b) Both primal and its dual can be infeasible
c) If primal is unbounded, then its dual is infeasible
d) Even if both primal and dual are feasible, the optimal values of the primal and the dual can differ

23) Let $A = \begin{pmatrix} a & 2f & 0 \\ 2f & b & 3f \\ 0 & 3f & c \end{pmatrix}$, where a, b, c, f are real numbers and $f \neq 0$. The geometric multiplicity of the largest eigenvalue of A equals __________. (GATE MA 2018)

24) Consider the subspaces

$$W_1 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 = x_2 + 2x_3\},$$
$$W_2 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 = 3x_2 + 2x_3\}$$

of $\mathbb{R}^3$. Then the dimension of $W_1 + W_2$ equals __________. (GATE MA 2018)

25) Let V be the real vector space of all polynomials of degree less than or equal to 2 with real coefficients. Let $T: V \to V$ be the linear transformation given by

$$T(p) = 2p + p', \quad \text{for } p \in V,$$

where p' is the derivative of p. Then the number of nonzero entries in the Jordan canonical form of a matrix of T equals __________. (GATE MA 2018)

26) Let $I = [2,3)$, J be the set of all rational numbers in the interval $[4,6]$, K be the Cantor (ternary) set, and let $L = \{7 + x : x \in K\}$. Then the Lebesgue measure of the set $I \cup J \cup L$ equals __________. (GATE MA 2018)

27) Let $u(x,y,z) = x^2 - 2y + 4z^2$ for $(x,y,z) \in \mathbb{R}^3$. Then the directional derivative of u in the direction $\frac{3}{5}\hat{\imath} - \frac{4}{5}\hat{k}$ at the point $(5,1,0)$ is __________. (GATE MA 2018)

28) If the Laplace transform of $y(t)$ is given by $Y(s) = \mathcal{L}(y(t)) = \frac{5}{2(s-1)} - \frac{2}{s-2} + \frac{1}{2(s-3)}$, then $y(0) + y'(0) =$ __________. (GATE MA 2018)

29) The number of regular singular points of the differential equation

$$[(x-1)^2 \sin x]\, y'' + [\cos x \, \sin(x-1)]\, y' + (x-1)\, y = 0.$$

in the interval $\left(0, \frac{\pi}{2}\right)$ is equal to __________. (GATE MA 2018)

30) Let F be a field with 49 elements and let K be a subfield of F with 7 elements. Then the dimension of F as a vector space over K is __________. (GATE MA 2018)

31) Let $C([0,1])$ be the real vector space of all continuous real valued functions on $[0,1]$, and let T be the linear operator on $C([0,1])$ given by

$$(Tf)(x) = \int_0^1 \sin(x+y)\, f(y)\, dy, \quad x \in [0,1].$$

Then the dimension of the range space of T equals __________. (GATE MA 2018)

32) Let $a \in (-1,1)$ be such that the quadrature rule $\int_{-1}^{1} f(x)\, dx \approx f(-a) + f(a)$ is exact for all polynomials of degree less than or equal to 3. Then $3a^2 =$ __________. (GATE MA 2018)

33) Let X and Y have joint probability density function given by

$$f_{X,Y}(x,y) = \begin{cases} 2, & 0 \le x \le 1-y,\ 0 \le y \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

If f_Y denotes the marginal probability density function of Y, then $f_Y(1/2) =$ __________. (GATE MA 2018)

34) Let the cumulative distribution function of the random variable X be given by

$$F_X(x) = \begin{cases} 0, & x < 0, \\ x, & 0 \le x < 1/2, \\ (1+x)/2, & 1/2 \le x < 1, \\ 1, & x \ge 1. \end{cases}$$

Then $P(X = 1/2) =$ __________. (GATE MA 2018)

35) Let $\{X_j\}$ be a sequence of independent Bernoulli random variables with $P\left(X_j = 1\right) = 1/4$ and let $Y_n = \frac{1}{n}\sum_{j=1}^{n} X_j^2$. Then Y_n converges, in probability, to __________. (GATE MA 2018)

36) Let Γ be the circle given by $z = 4e^{i\theta}$, where θ varies from 0 to 2π. Then

$$\oint_{\Gamma} \frac{e^z}{z^2 - 2z}\, dz =$$

(GATE MA 2018)

a) $2\pi i\left(e^2 - 1\right)$
b) $\pi i\left(1 - e^2\right)$
c) $\pi i\left(e^2 - 1\right)$
d) $2\pi i\left(1 - e^2\right)$

37) The image of the half plane $\Re(z) + \Im(z) > 0$ under the map $w = \dfrac{z-1}{z+i}$ is given by (GATE MA 2018)

a) $Re(w) > 0$
b) $Im(w) > 0$
c) $|w| > 1$
d) $|w| < 1$

38) Let $D \subset \mathbb{R}^2$ denote the closed disc with center at the origin and radius 2. Then

$$\iint_D e^{-\left(x^2+y^2\right)}\, dx\, dy =$$

(GATE MA 2018)

a) $\pi\left(1 - e^{-4}\right)$
b) $\frac{\pi}{2}\left(1 - e^{-4}\right)$
c) $\pi\left(1 - e^{-2}\right)$
d) $\frac{\pi}{2}\left(1 - e^{-2}\right)$

39) Consider the polynomial $p(X) = X^4 + 4$ in the ring $\mathbb{Q}[X]$ of polynomials in the variable X with coefficients in the field $\mathbb{Q}$ of rational numbers. Then (GATE MA 2018)

a) the set of zeros of $p(X)$ in $\mathbb{C}$ forms a group under multiplication
b) $p(X)$ is reducible in the ring $\mathbb{Q}[X]$
c) the splitting field of $p(X)$ has degree 3 over $\mathbb{Q}$
d) the splitting field of $p(X)$ has degree 4 over $\mathbb{Q}$

40) Which one of the following statements is true?

(GATE MA 2018)

a) Every group of order 12 has a non-trivial proper normal subgroup
b) Some group of order 12 does not have a non-trivial proper normal subgroup
c) Every group of order 12 has a subgroup of order 6
d) Every group of order 12 has an element of order 12

41) For an odd prime p, consider the ring $\mathbb{Z}\left(\sqrt{-p}\right) = \{a + b\sqrt{-p} : a, b \in \mathbb{Z}\} \subseteq \mathbb{C}$. Then the element 2 in $\mathbb{Z}\left(\sqrt{-p}\right)$ is

(GATE MA 2018)

a) a unit
b) a square
c) a prime
d) irreducible

42) Consider the following two statements:

(GATE MA 2018)

P: $\begin{pmatrix} 0 & 5 \\ 0 & 7 \end{pmatrix}$ has infinitely many LU factorizations, where L is lower triangular with each diagonal entry 1 and U is upper triangular.

Q: $\begin{pmatrix} 0 & 0 \\ 2 & 5 \end{pmatrix}$ has no LU factorization, where L is lower triangular with each diagonal entry 1 and U is upper triangular.

Then which one of the following options is correct?

a) P is TRUE and Q is FALSE
b) Both P and Q are TRUE
c) P is FALSE and Q is TRUE
d) Both P and Q are FALSE

43) If the characteristic curves of the partial differential equation $x\,u_{xx} + 2x^2 u_{xy} = u_x - 1$ are $\mu(x, y) = c_1$ and $\nu(x, y) = c_2$, where c_1 and c_2 are constants, then

(GATE MA 2018)

a) $\mu(x, y) = x^2 - y,\ \nu(x, y) = y$
b) $\mu(x, y) = x^2 + y,\ \nu(x, y) = y$
c) $\mu(x, y) = x^2 + y,\ \nu(x, y) = x^2$
d) $\mu(x, y) = x^2 - y,\ \nu(x, y) = x^2$

44) Let $f : X \to Y$ be a continuous map from a Hausdorff topological space X to a metric space Y. Consider the following two statements:

(GATE MA 2018)

P: f is a closed map and the inverse image $f^{-1}(y) = \{x \in X : f(x) = y\}$ is compact for each $y \in Y$.

Q: For every compact subset $K \subset Y$, the inverse image $f^{-1}(K)$ is a compact subset of X.

Which one of the following is true?

a) Q implies P but P does NOT imply Q
b) P implies Q but Q does NOT imply P
c) P and Q are equivalent
d) neither P implies Q nor Q implies P

45) Let X denote $\mathbb{R}^2$ endowed with the usual topology. Let Y denote $\mathbb{R}$ endowed with the co-finite topology. If Z is the product topological space $Y \times Y$, then

(GATE MA 2018)

a) the topology of X is the same as the topology of Z
b) the topology of X is strictly coarser (weaker) than that of Z
c) the topology of Z is strictly coarser (weaker) than that of X
d) the topology of X cannot be compared with that of Z

46) Consider $\mathbb{R}^n$ with the usual topology for $n = 1, 2, 3$. Each of the following options gives topological spaces X and Y with respective induced topologies. In which option is X homeomorphic to Y?

(GATE MA 2018)

a) $X = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\},\ Y = \{(x, y, z) \in \mathbb{R}^3 : z = 0,\ x^2 + y^2 \neq 0\}$
b) $X = \{(x, y) \in \mathbb{R}^2 : y = \sin(1/x),\ 0 < x \leq 1\} \cup \{(x, y) \in \mathbb{R}^2 : x = 0,\ -1 \leq y \leq 1\},\ Y = [0, 1] \subset \mathbb{R}$
c) $X = \{(x, y) \in \mathbb{R}^2 : y = x\sin(1/x),\ 0 < x \leq 1\},\ Y = [0, 1] \subset \mathbb{R}$
d) $X = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = 1\},\ Y = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 = z^2 \neq 0\}$

47) Let $\{X_i\}$ be a sequence of independent Poisson (λ) variables and let $W_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then the limiting distribution of $\sqrt{n}\,(W_n - \lambda)$ is the normal distribution with zero mean and variance given by (GATE MA 2018)

a) 1
b) $\sqrt{\lambda}$
c) λ
d) λ^2

48) Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed random variables with probability density function given by

$$f_X(x;\theta) = \begin{cases} \theta e^{-\theta(x-1)}, & x \geq 1, \\ 0, & \text{otherwise.} \end{cases}$$

Also, let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then the maximum likelihood estimator of θ is (GATE MA 2018)

a) $1/\bar{X}$
b) $\left(1/\bar{X}\right) - 1$
c) $1/\left(\bar{X} - 1\right)$
d) $\bar{X}$

49) Consider the Linear Programming Problem (LPP):
Maximize $\alpha x_1 + x_2$
Subject to $2x_1 + x_2 \leq 6,\ -x_1 + x_2 \leq 1,\ x_1 + x_2 \leq 4,\ x_1 \geq 0,\ x_2 \geq 0$,
where α is a constant. If $(3, 0)$ is the only optimal solution, then (GATE MA 2018)

a) $\alpha < -2$
b) $-2 < \alpha < 1$
c) $1 < \alpha < 2$
d) $\alpha > 2$

50) Let $M_2(\mathbb{R})$ be the vector space of all 2×2 real matrices over the field $\mathbb{R}$. Define the linear transformation $S : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ by $S(X) = 2X + X^T$, where X^T denotes the transpose of the matrix X. Then the trace of S equals __________. (GATE MA 2018)

51) Consider $\mathbb{R}^3$ with the usual inner product. If d is the distance from $(1, 1, 1)$ to the subspace span$\{(1, 1, 0), (0, 1, 1)\}$ of $\mathbb{R}^3$, then $3d^2 =$ __________. (GATE MA 2018)

52) Consider the matrix $A = I_9 - 2uu^T$ with $u = \frac{1}{3}[1, 1, 1, 1, 1, 1, 1, 1, 1]$, where I_9 is the 9×9 identity matrix and u^T is the transpose of u. If λ and μ are two distinct eigenvalues of A, then $|\lambda - \mu| =$ __________. (GATE MA 2018)

53) Let $f(z) = z^3 e^{z^2}$ for $z \in \mathbb{C}$ and let Γ be the circle $z = e^{i\theta}$, where θ varies from 0 to 4π. Then

$$\frac{1}{2\pi i}\oint_{\Gamma} \frac{f'(z)}{f(z)}\,dz = \_\_\_\_\_\_\_\_\_\_.$$

(GATE MA 2018)

54) Let S be the surface of the solid $V = \{(x, y, z) : 0 \leq x \leq 1,\ 0 \leq y \leq 2,\ 0 \leq z \leq 3\}$. Let $\hat{n}$ denote the unit outward normal to S and let $\mathbf{F}(x, y, z) = x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$, $(x, y, z) \in V$. Then the surface integral $\iint_S \mathbf{F} \cdot \hat{n}\, dS$ equals __________. (GATE MA 2018)

55) Let A be a 3×3 matrix with real entries. If three solutions of the linear system of differential equations $\dot{x}(t) = Ax(t)$ are given by

$$\begin{pmatrix} e^{t} - e^{2t} \\ -e^{t} + e^{2t} \\ e^{t} + e^{2t} \end{pmatrix}, \quad \begin{pmatrix} -e^{2t} - e^{-t} \\ e^{2t} - e^{-t} \\ e^{2t} + e^{-t} \end{pmatrix}, \quad \begin{pmatrix} e^{-t} + 2e^{t} \\ e^{-t} - 2e^{t} \\ -e^{-t} + 2e^{t} \end{pmatrix},$$

then the sum of the diagonal entries of A is equal to __________.

(GATE MA 2018)

56) If $y_1(x) = e^{-x^2}$ is a solution of the differential equation $xy'' + \alpha y' + \beta x^3 y = 0$ for some real numbers α and β, then $\alpha\beta =$ __________.

(GATE MA 2018)

57) Let $L^2([0,1])$ be the Hilbert space of all real valued square integrable functions on $[0,1]$ with the usual inner product. Let ϕ be the linear functional on $L^2([0,1])$ defined by

$$\phi(f) = \int_{1/4}^{3/4} 3\sqrt{2}\, f\, d\mu,$$

where μ denotes the Lebesgue measure on $[0,1]$. Then $\|\phi\| =$ __________.

(GATE MA 2018)

58) Let U be an orthonormal set in a Hilbert space H and let $x \in H$ be such that $\|x\| = 2$. Consider the set $E = \{u \in U : |\langle x, u\rangle| \geq 1/4\}$. Then the maximum possible number of elements in E is __________.

(GATE MA 2018)

59) If $p(x) = 2 - (x+1) + x(x+1) - \beta x(x+1)(x-\alpha)$ interpolates the points (x, y) in the table

x	-1	0	1	2
y	2	1	2	-7

then $\alpha + \beta =$ __________.

(GATE MA 2018)

60) If $\sin(\pi x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x)$ for $0 < x < 1$, then $(a_0 + a_1)\pi =$ __________.

(GATE MA 2018)

61) For $n = 1, 2, \ldots$, let $f_n(x) = \dfrac{2^n x^{n-1}}{1+x}$, $x \in [0,1]$. Then $\displaystyle\lim_{n\to\infty} \int_0^1 f_n(x)\, dx =$ __________.

(GATE MA 2018)

62) Let X_1, X_2, X_3, X_4 be independent exponential random variables with mean 1, 1/2, 1/3, 1/4, respectively. Then $Y = \min(X_1, X_2, X_3, X_4)$ has exponential distribution with mean equal to __________.

(GATE MA 2018)

63) Let X be the number of heads in 4 tosses of a fair coin by Person 1 and let Y be the number of heads in 4 tosses of a fair coin by Person 2. Assume that all the tosses are independent. Then the value of $P(X = Y)$ correct up to three decimal places is __________.

(GATE MA 2018)

64) Let X_1 and X_2 be independent geometric random variables with the same probability mass function given by $P(X = k) = p(1-p)^{k-1}$, $k = 1, 2, \ldots$. Then the value of $P(X_1 = 2 \mid X_1 + X_2 = 4)$ correct up to three decimal places is __________.

(GATE MA 2018)

65) A certain commodity is produced by the manufacturing plants P_1 and P_2 whose capacities are 6 and 5 units, respectively. The commodity is shipped to markets M_1, M_2, M_3 and M_4 whose requirements are 1, 2, 3 and 5 units, respectively. The transportation cost per unit from plant P_i to market M_j is as follows:

	M_1	M_2	M_3	M_4	
P_1	1	3	5	8	6
P_2	2	5	6	7	5
	1	2	3	5	

Then the optimal cost of transportation is _________.

(GATE MA 2018)