

2.3.9

EE25BTECH11006 - ADUDOTLA SRIVIDYA

QUESTION

If vectors \mathbf{a} and \mathbf{b} are such that $|\mathbf{a}| = \frac{1}{2}$, $|\mathbf{b}| = \frac{4}{\sqrt{3}}$ and $|\mathbf{a} \times \mathbf{b}| = \frac{1}{\sqrt{3}}$, then find $\mathbf{a} \cdot \mathbf{b}$.

SOLUTION

We know that

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin \theta \quad (1)$$

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta \quad (2)$$

where θ is the angle between \mathbf{a} and \mathbf{b} .

Substitute the values:

$$\frac{1}{\sqrt{3}} = \left(\frac{1}{2}\right)\left(\frac{4}{\sqrt{3}}\right) \sin \theta \quad (3)$$

$$\frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}} \sin \theta \quad (4)$$

$$\sin \theta = \frac{1}{2} \quad (5)$$

Thus,

$$\theta = 30^\circ \quad \text{or} \quad 150^\circ \quad (6)$$

Now,

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta \quad (7)$$

$$= \left(\frac{1}{2}\right)\left(\frac{4}{\sqrt{3}}\right) \cos \theta \quad (8)$$

$$= \frac{2}{\sqrt{3}} \cos \theta \quad (9)$$

So, the possible values are:

$$\mathbf{a} \cdot \mathbf{b} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{2} = 1 \quad (\theta = 30^\circ) \quad (10)$$

$$\mathbf{a} \cdot \mathbf{b} = \frac{2}{\sqrt{3}} \cdot \left(-\frac{\sqrt{3}}{2}\right) = -1 \quad (\theta = 150^\circ) \quad (11)$$

Therefore, $\mathbf{a} \cdot \mathbf{b} = \pm 1$.

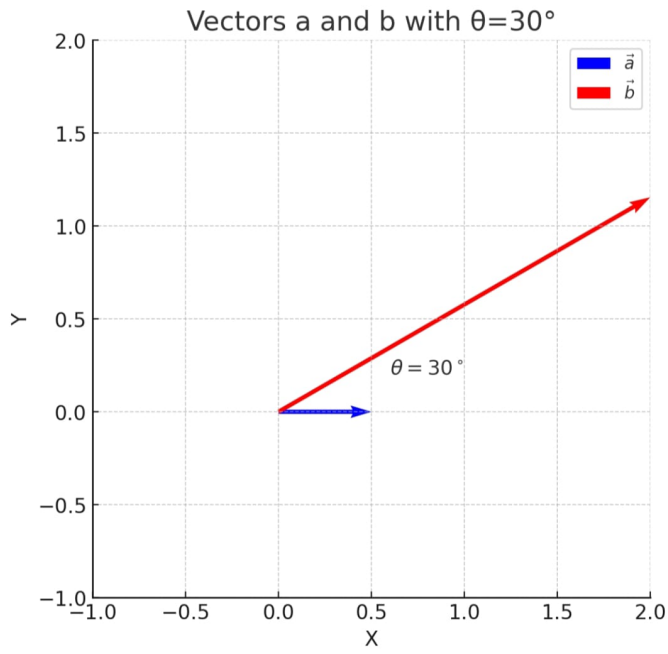


Fig. 1: Vectors **a** and **b** with angle θ