4.5.6

Aditya Appana - EE25BTECH11004

September 12,2025

Question

Find the equations of the line that passes through the point (3,0,1) and parallel to the planes $x + 2y = 0$ and $3y - z = 0$.

Solution

We know that the normal form of a plane is $\mathbf{n}^T\mathbf{x} = 0$
The plane $x + 2y = 0$ can be expressed in vector form as:

$$\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}^T \mathbf{x} = 0 \tag{1}$$

therefore,

$$\mathbf{n}_1 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \tag{2}$$

The plane $3y - z = 0$ can be expressed in vector form as:

$$\begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix}^T \mathbf{x} = 0 \tag{3}$$

Solution

therefore,

$$\mathbf{n}_2 = \begin{pmatrix} 0 \\ 3 \\ -1 \end{pmatrix} \tag{4}$$

The direction vector of the line is given by:

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \end{pmatrix} \mathbf{m} = 0 \tag{5}$$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \end{pmatrix} \mathbf{m} = 0 \xrightarrow{\mathrm{R}_1 \to \mathrm{R}_1 - \frac{2}{3} R_2} \begin{pmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 3 & -1 \end{pmatrix} \xrightarrow{\mathrm{R}_1 \to \mathrm{R}_1/3} \begin{pmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & \frac{-1}{3} \end{pmatrix} \tag{6}$$

Solution

$$\mathbf{m} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \tag{7}$$

Therefore the equation of the line is:

$$\mathbf{x} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \tag{8}$$

Python, C, Python+C Codes

codes permalink

Plot

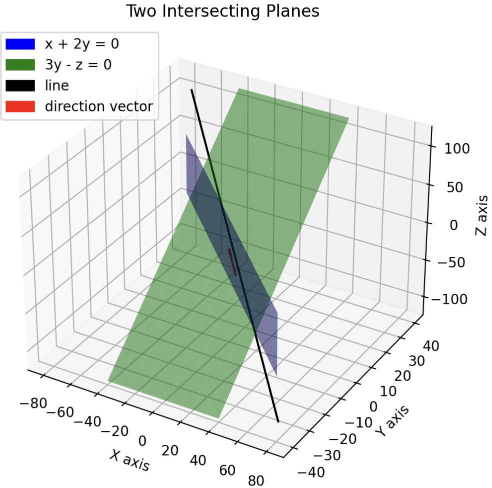

Figure: Plot