4.7.41

EE25BTECH11044 - Sai Hasini Pappula

Question

Find the distance of the point $\mathbf{P} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$ from the line

$$\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}. \tag{0.1}$$

Solution

Write the line in parametric form:

$$\mathbf{Q} = \begin{pmatrix} -5 \\ -3 \\ 6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ -9 \end{pmatrix}, \qquad \lambda \in \mathbb{R}, \tag{0.2}$$

so that a general point on the line is

$$\mathbf{Q} = \begin{pmatrix} -5+\lambda \\ -3+4\lambda \\ 6-9\lambda \end{pmatrix}. \tag{0.3}$$

The direction vector of the line is

$$\begin{pmatrix} 1 \\ 4 \\ -9 \end{pmatrix}. \tag{0.4}$$

The perpendicularity condition for the foot of the perpendicular from $\mathbf{P}$ to the line is

$$\begin{pmatrix} 1 \\ 4 \\ -9 \end{pmatrix} \cdot (\mathbf{P} - \mathbf{Q}) = 0. \tag{0.5}$$

Expanding and collecting the linear equations in the unknowns x, y, z, λ (where x, y, z are the coordinates of $\mathbf{Q}$) gives the system

$$x - \lambda = -5, \tag{0.6}$$

$$y - 4\lambda = -3, \tag{0.7}$$

$$z + 9\lambda = 6, \tag{0.8}$$

$$-x - 4y + 9z = -27. \tag{0.9}$$

AUGMENTED MATRIX AND FULL ROW-REDUCTION STEPS

Write the augmented matrix for the linear system in the unknown order (x, y, z, λ):

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & -4 & -3 \\ 0 & 0 & 1 & 9 & 6 \\ -1 & -4 & 9 & 0 & -27 \end{array}\right]. \tag{0.10}$$

We perform elementary row operations step by step.

1. Eliminate the 1st column entry of row 4 by adding row 1 to row 4:

$$R_4 \leftarrow R_4 + R_1 \qquad \Longrightarrow \qquad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & -4 & -3 \\ 0 & 0 & 1 & 9 & 6 \\ 0 & -4 & 9 & -1 & -32 \end{array}\right] \tag{0.11}$$

2. Eliminate the 2nd column entry of row 4 using row 2:

$$R_4 \leftarrow R_4 + 4R_2 \qquad \Longrightarrow \qquad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & -4 & -3 \\ 0 & 0 & 1 & 9 & 6 \\ 0 & 0 & 9 & -17 & -44 \end{array}\right] \tag{0.12}$$

3. Eliminate the 3rd column entry of row 4 using row 3:

$$R_4 \leftarrow R_4 - 9R_3 \qquad \Longrightarrow \qquad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & -4 & -3 \\ 0 & 0 & 1 & 9 & 6 \\ 0 & 0 & 0 & -98 & -98 \end{array}\right] \tag{0.13}$$

4. Scale row 4 to make a leading 1 (divide by −98):

$$R_4 \leftarrow \frac{1}{-98} R_4 \qquad \Longrightarrow \qquad \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & -4 & -3 \\ 0 & 0 & 1 & 9 & 6 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \tag{0.14}$$

5. Use the pivot in row 4 to eliminate the λ-entries above it:

$$\begin{aligned} R_1 &\leftarrow R_1 + R_4, \\ R_2 &\leftarrow R_2 + 4R_4, \\ R_3 &\leftarrow R_3 - 9R_4, \end{aligned} \qquad \Longrightarrow \qquad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]. \tag{0.15}$$

This matrix is now in RREF.

Solution from RREF

from (0.15):

$$x = -4, \tag{0.16}$$
$$y = 1, \tag{0.17}$$
$$z = -3, \tag{0.18}$$
$$\lambda = 1. \tag{0.19}$$

Thus the foot of the perpendicular (point on the line closest to **P**) is

$$\mathbf{Q} = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix}. \tag{0.20}$$

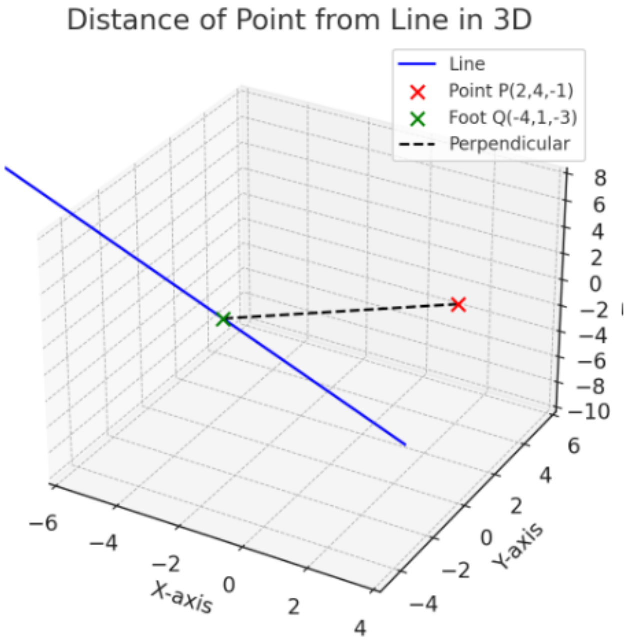
Distance

Compute $\mathbf{P} - \mathbf{Q}$ and its norm:

$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} - \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix}, \tag{0.21}$$

$$||\mathbf{P} - \mathbf{Q}|| = \sqrt{6^2 + 3^2 + 2^2} = \sqrt{36 + 9 + 4} = \sqrt{49} = 7. \tag{0.22}$$

Final Answer: The distance from $\mathbf{P} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$ to the given line is $\boxed{7}$.



Distance of Point from Line in 3D
Line
Point P(2,4,-1)
Foot Q(-4,1,-3)
Perpendicular
8
6
4
2
0
−2
−4
−6
−8
−10
−6
−4
−2
0
2
4
X-axis
6
4
2
0
−2
−4
Y-axis