frame=single, breaklines=true, columns=fullflexible
remark
frame=single, breaklines=true, columns=fullflexible

# 1.5.24

EE25BTECH11036 - M Chanakya Srinivas

**1.5.24** A line intersects the $Y$-axis and $X$-axis at the points $P = (0, b)$ and $Q = (c, 0)$ respectively. If $(2, -5)$ is the midpoint of $\overline{PQ}$, then find the coordinates of $P$ and $Q$.

**Coordinates:** $P = (0, -10),\ Q = (4, 0)$

MATRIX SOLUTION

1) **Vector Midpoint Formula**

Let

$$\mathbf{P} = \begin{pmatrix} 0 \\ b \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} c \\ 0 \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}.$$

The midpoint formula is

$$\mathbf{M} = \tfrac{1}{2}(\mathbf{P} + \mathbf{Q})$$

$$\begin{pmatrix} 2 \\ -5 \end{pmatrix} = \tfrac{1}{2}\left(\begin{pmatrix} 0 \\ b \end{pmatrix} + \begin{pmatrix} c \\ 0 \end{pmatrix}\right)$$

$$= \tfrac{1}{2}\begin{pmatrix} c \\ b \end{pmatrix}.$$

Multiplying through by 2:

$$\begin{pmatrix} 4 \\ -10 \end{pmatrix} = \begin{pmatrix} c \\ b \end{pmatrix}.$$

—

2) **System in Matrix Form ($A\mathbf{x} = \mathbf{B}$)**

This gives the system

$$\begin{cases} b = -10, \\ c = 4. \end{cases}$$

Equivalently,

$$\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} b \\ c \end{pmatrix}}_{\mathbf{x}} = \underbrace{\begin{pmatrix} -10 \\ 4 \end{pmatrix}}_{\mathbf{B}}.$$

—

3) **Solving the Matrix Equation**

Since $A$ is the identity,

$$\mathbf{x} = A^{-1}\mathbf{B} = I\mathbf{B} = \begin{pmatrix} -10 \\ 4 \end{pmatrix}.$$

So,

$$b = -10, \quad c = 4.$$

4) **Final Coordinates**

$$P = \begin{pmatrix} 0 \\ -10 \end{pmatrix}, \qquad Q = \begin{pmatrix} 4 \\ 0 \end{pmatrix}.$$

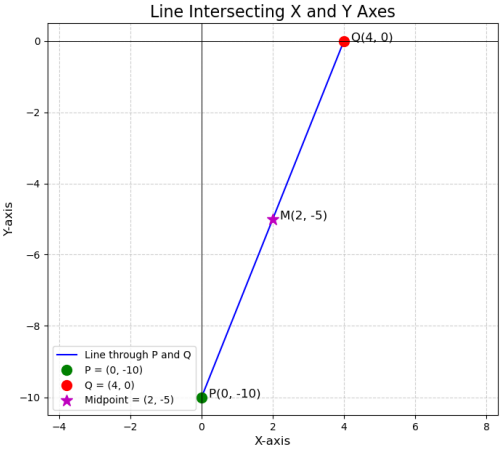


Fig. 4.1. Plot using Shared output

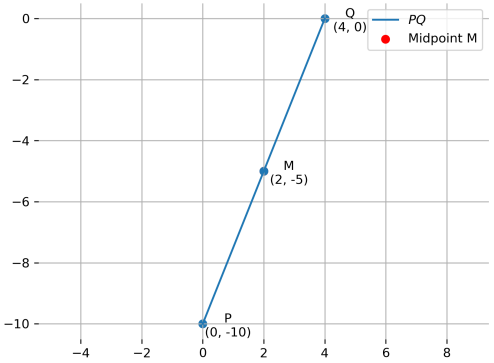


Fig. 4.2. Plot using Python