

12.664

EE25BTECH11026-Harsha

Question:

A real, invertible 3×3 matrix \mathbf{M} has eigenvalues λ_i , ($i = 1, 2, 3$) and the corresponding eigenvectors are \mathbf{e}_i , ($i = 1, 2, 3$) respectively. Which one of the following is correct?

- 1) $\mathbf{M}\mathbf{e}_i = \frac{1}{\lambda_i}\mathbf{e}_i$, for $i=1,2,3$
- 2) $\mathbf{M}^{-1}\mathbf{e}_i = \frac{1}{\lambda_i}\mathbf{e}_i$, for $i=1,2,3$
- 3) $\mathbf{M}^{-1}\mathbf{e}_i = \lambda_i\mathbf{e}_i$, for $i=1,2,3$
- 4) The eigenvalues of \mathbf{M} and \mathbf{M}^{-1} are not related.

Solution:

Let us solve the given question theoretically and then verify the solution computationally.

According to the definition of eigen-vector,

$$\mathbf{M}\mathbf{e}_i = \lambda_i\mathbf{e}_i \quad (4.1)$$

Pre-multiplying \mathbf{M}^{-1} on both sides,

$$\therefore (\mathbf{M}^{-1}\mathbf{M})\mathbf{e}_i = \mathbf{M}^{-1}\lambda_i\mathbf{e}_i \quad (4.2)$$

$$\implies \mathbf{e}_i = \lambda_i\mathbf{M}^{-1}\mathbf{e}_i \quad (4.3)$$

$$\therefore \mathbf{M}^{-1}\mathbf{e}_i = \frac{1}{\lambda_i}\mathbf{e}_i \quad (4.4)$$