4.13.45

Naman Kumar-EE25BTECH11041

25 September,2025

Question a)

Two vertices of a triangle are $(5, -1)$ and $(2, -3)$. If the orthocentre of the triangle is the origin, find the coordinates of the third point.

Solution

Given,

$$\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \tag{1}$$

Where,

$\mathbf{O}$	Poiint vector of orthocenter
$\mathbf{A}$ and $\mathbf{B}$	known vector of Points of triangle
$\mathbf{m}_1$	Direction vector of line from $\mathbf{C}$ to $\mathbf{B}$
$\mathbf{m}_2$	Direction vector of line from $\mathbf{C}$ to $\mathbf{A}$
$\mathbf{A}_1$	Altitude from $\mathbf{A}$ to $\mathbf{O}$
$\mathbf{A}_2$	Altitude from $\mathbf{B}$ to $\mathbf{O}$
$\mathbf{L}$	Line from $\mathbf{B}$ to $\mathbf{A}$
$\mathbf{C}$	Required point

Table:

Solution

From General Triangle Properties.

$$m_1^T A_1 = 0, m_2^T A_2 = 0 \tag{2}$$

$$(\mathbf{A} - \mathbf{O})^T (\mathbf{C} - \mathbf{B}) = 0; \tag{3}$$

$$\mathbf{A}^T (\mathbf{C} - \mathbf{B}) = 0, \text{Since } \mathbf{O} \text{ is origin} \tag{4}$$

and

$$(\mathbf{B} - \mathbf{O})^T (\mathbf{C} - \mathbf{A}) = 0; \tag{5}$$

$$(\mathbf{B})^T (\mathbf{C} - \mathbf{A}) = 0, \text{Since } \mathbf{O} \text{ is origin} \tag{6}$$

Solution

Modifying (4) and (6)

$$\mathbf{A}^T\mathbf{C} - \mathbf{A}^T\mathbf{B} = 0, \mathbf{B}^T\mathbf{C} - \mathbf{B}^T\mathbf{A} = 0 \tag{7}$$

$$\mathbf{A}^T\mathbf{C} = \mathbf{A}^T\mathbf{B} \tag{8}$$

$$\mathbf{B}^T\mathbf{C} = \mathbf{B}^T\mathbf{A} \tag{9}$$

This can be written as

$$\begin{pmatrix} \mathbf{A}^T \\ \mathbf{B}^T \end{pmatrix} \mathbf{C} = \begin{pmatrix} \mathbf{A}^T\mathbf{B} \\ \mathbf{B}^T\mathbf{A} \end{pmatrix} \tag{10}$$

$$\begin{pmatrix} 5 & -1 \\ 2 & -3 \end{pmatrix} \mathbf{C} = \begin{pmatrix} \mathbf{A}^T\mathbf{B} \\ \mathbf{B}^T\mathbf{A} \end{pmatrix} \tag{11}$$

Solution

Let

$$\left(\begin{array}{cc|c} 5 & -1 & \mathbf{A}^T\mathbf{B} \\ 2 & -3 & \mathbf{B}^T\mathbf{A} \end{array}\right) = \left(\begin{array}{cc|c} 5 & -1 & 13 \\ 2 & -3 & 13 \end{array}\right) \tag{12}$$

By Gaussian Elimination

$$\left(\begin{array}{cc|c} 5 & -1 & 13 \\ 2 & -3 & 13 \end{array}\right) \xrightarrow{R_2 - \frac{2}{5}R_1} \left(\begin{array}{cc|c} 5 & -1 & 13 \\ 0 & \frac{-13}{5} & \frac{39}{5} \end{array}\right) \tag{13}$$

$$\xrightarrow{-\frac{5}{13}R_2} \left(\begin{array}{cc|c} 5 & -1 & 13 \\ 0 & 1 & -3 \end{array}\right) \tag{14}$$

Solution

In equation (11)

$$\begin{pmatrix} 5 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13 \\ -3 \end{pmatrix} \tag{15}$$

Therefore, **C** is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \tag{16}$$

Solution

But,Now

$$\mathbf{B} = \mathbf{C} \tag{17}$$

Which is not possible for a triangle,
Slope of line **A** to **B** or **L** be **m**

$$\mathbf{m} = \mathbf{A} - \mathbf{B} \tag{18}$$

$$\mathbf{m} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{19}$$

Solution

We can see, that

$$\mathbf{m}^T\mathbf{A_2} = 0, \mathbf{m_2}^T\mathbf{A_2} = 0 \tag{20}$$

Therefore, for this to be possible when

$$\mathbf{L} \parallel \mathbf{m_2}, \tag{21}$$

$$\mathbf{A} - \mathbf{C} \parallel \mathbf{A} - \mathbf{B} \tag{22}$$

Since both lines have a point in common $\mathbf{A}$, therefore they must be collinear.
So, $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ is just a straight line

Figure

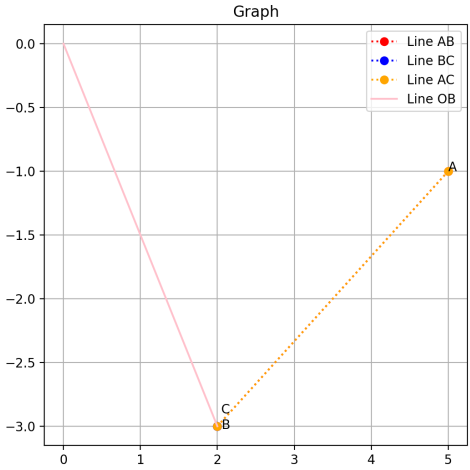

Figure:

direct python

```
import numpy as np
import matplotlib.pyplot as plt

plt.figure(figsize=(6,6), dpi = 200)


abX= np.array([5,2])
abY= np.array([-1,-3])
acX= np.array([5,2])
acY=np.array([-1,-3])
bcX=np.array([2,2])
bcY=np.array([-3,-3])
obx=np.array([0,2])
oby=np.array([0,-3])
```

direct python

```
plt.plot(abX,abY, ':r',marker='o', label="Line AB")
plt.plot(bcX,bcY, ':b',marker='o', label="Line BC")
plt.plot(acX,acY, ':',marker='o', color='orange', label="Line AC"
    )
plt.plot(obx,oby, '-', color='pink', label="Line OB")


plt.annotate("A",xy=(abX[0], abY[0]))
plt.annotate("B",xy=(abX[1]+0.05, abY[1]-0.02))
plt.annotate("C",xy=(abX[1]+0.05, abY[1]+0.1))
```

direct python

```
plt.title("Graph")
plt.legend()
plt.grid()
plt.savefig("Figure.png", dpi=200)
plt.show()
```

C code

```c
#include <stdio.h>

typedef struct {
    double x;
    double y;
} Point;

// Function to find third vertex given two vertices and
    orthocenter
Point third_vertex(double x1, double y1, double x2, double y2,
    double hx, double hy){
    Point C;
    double m_alt_A, m_alt_B;
    double m_perp_BC, m_perp_AC;

    // Slope of altitude from A passing through H
    if(x1 != hx)
        m_alt_A = (hy - y1)/(hx - x1);
    else
```

C code

```c
        m_alt_B = 1e9;
    // Equation for perpendicular slope relation
    if(m_alt_A != 1e9)
        m_perp_BC = -1.0 / m_alt_A;
    else
        m_perp_BC = 0; // horizontal BC

    if(m_alt_B != 1e9)
        m_perp_AC = -1.0 / m_alt_B;
    else
        m_perp_AC = 0; // horizontal AC
```

C code

```c
    // Solve system: slope formula
    // y3 - y2 = m_perp_BC * (x3 - x2)
    // y3 - y1 = m_perp_AC * (x3 - x1)
    double x3 = (m_perp_BC * x2 - m_perp_AC * x1 + y1 - y2) / (
        m_perp_BC - m_perp_AC);
    double y3 = m_perp_BC * (x3 - x2) + y2;

    C.x = x3;
    C.y = y3;
    return C;
}
```

Python code with shared object

```
import ctypes
from ctypes import Structure, c_double
import matplotlib.pyplot as plt

# Define Point struct
class Point(Structure):
    _fields_ = [("x", c_double), ("y", c_double)]

# Load shared object
lib = ctypes.CDLL("./libtriangle.so")
lib.third_vertex.restype = Point
lib.third_vertex.argtypes = [c_double, c_double, c_double,
    c_double, c_double, c_double]
```

Python code with shared object

```python
# Given vertices and orthocenter
A_x, A_y = 5, -1
B_x, B_y = 2, -3
H_x, H_y = 0, 0

# Call C function
C = lib.third_vertex(A_x, A_y, B_x, B_y, H_x, H_y)
print(f"Third vertex: ({C.x:.2f}, {C.y:.2f})")

# Plot triangle
x_coords = [A_x, B_x, C.x, A_x]
y_coords = [A_y, B_y, C.y, A_y]
```

Python code with shared object

```python
plt.figure(figsize=(6,6))
plt.plot(x_coords, y_coords, 'b-o', label='Triangle')
plt.plot(H_x, H_y, 'r*', markersize=12, label='Orthocenter')
plt.text(A_x, A_y, 'A', fontsize=12, ha='right')
plt.text(B_x, B_y, 'B', fontsize=12, ha='right')
plt.text(C.x, C.y, 'C', fontsize=12, ha='right')
plt.text(H_x, H_y, 'H', fontsize=12, ha='right')
plt.grid(True)
plt.legend()
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.title('Triangle with Orthocenter')
plt.show()
```