10.4.8

Kartik Lahoti - EE25BTECH11032

October 2, 2025

Question

For which value of m is the line

$$y = mx + 1 \tag{1}$$

a tangent to the curve

$$y^2 = 4x \tag{2}$$

Theoretical Solution

Given parabola

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{3}$$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, f = 0. \tag{4}$$

Given Line equation

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{5}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix}. \tag{6}$$

Theoretical Solution

For the line to be a tangent , all the solution for k should be equal

$$k_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \pm \sqrt{\left[\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 - g\left(\mathbf{h}\right) \left(\mathbf{m}^\top \mathbf{V}\mathbf{m} \right)} \right) \tag{7}$$

From 1

$$\left[\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 = g\left(\mathbf{h}\right) \left(\mathbf{m}^\top \mathbf{V}\mathbf{m} \right) \tag{8}$$

Theoretical Solution

$$g\left(\mathbf{h}\right) = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} -2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 0 = 1 \tag{9}$$

Using 2 in 9

$$\left[\begin{pmatrix} 1 & m \end{pmatrix} \left(\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix}\right)\right]^2 = 1 \left(\begin{pmatrix} 1 & m \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ m \end{pmatrix}\right) \tag{10}$$

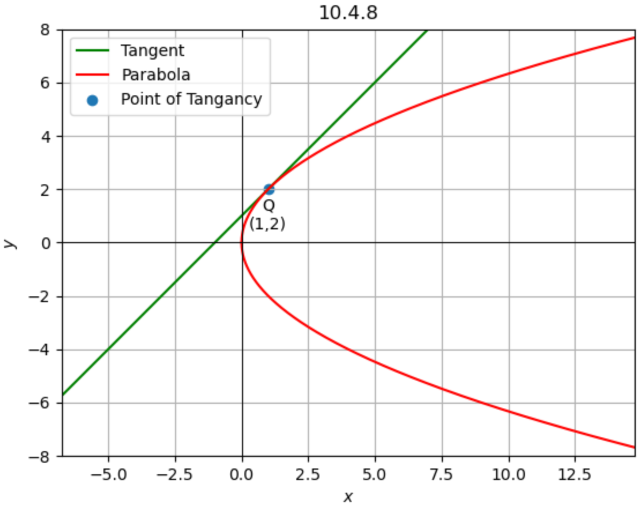
Theoretical Solution

$$\implies (m-2)^2 = m^2 \tag{11}$$

$$\implies 4 - 4m = 0 \tag{12}$$

$$\implies m = 1 \tag{13}$$

Plot



10.4.8
Tangent
Parabola
Point of Tangancy
Q
(1,2)
8
6
4
2
0
−2
−4
−6
−8
y
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
12.5
x