1.6.10

EE25BTECH11060 - V.Namaswi

Question:
Show that the points $\mathbf{A}(1,-2,-8)$,$\mathbf{B}(5,0,-2)$ and $\mathbf{C}(11,3,7)$ are collinear and find the ratio in which B divides AC.

Solution:

Point	**x**	**y**	**z**
A	1	-2	-8
B	5	0	-2
C	11	3	7

collinearity matrix can be expressed as

$$\begin{pmatrix} B-A & C-A \end{pmatrix} = \begin{pmatrix} 4 & 10 \\ 2 & 5 \\ 6 & 15 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 10 \\ 2 & 5 \\ 6 & 15 \end{pmatrix} \xleftrightarrow{R_3 \leftarrow R_3-(R_1+R_2)} \begin{pmatrix} 4 & 10 \\ 2 & 5 \\ 0 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2-(R_1/2)} \begin{pmatrix} 4 & 10 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow (R_1/4)} \begin{pmatrix} 1 & 2.5 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Which is a Rank 1 matrix , Hence $\mathbf{A}(1,-2,-8)$,$\mathbf{B}(5,0,-2)$ and $\mathbf{C}(11,3,7)$ are collinear.

Section formula for a vector $\mathbf{B}$ which divides the line formed by vectors $\mathbf{A}$ and $\mathbf{C}$ in the ratio k:1 is given by

$$\mathbf{B} = \frac{k\mathbf{C} + \mathbf{A}}{k+1} \tag{1}$$

$$\tag{2}$$

$$k\left(\mathbf{B} - \mathbf{C}\right) = \mathbf{A} - \mathbf{B} \tag{3}$$

Taking dot product on both sides with $(\mathbf{B} - \mathbf{C})$

$$k = \frac{\left(\mathbf{A} - \mathbf{B}\right)^\top \left(\mathbf{B} - \mathbf{C}\right)}{\|\mathbf{B} - \mathbf{C}\|^2} \tag{4}$$

$$(\mathbf{A}-\mathbf{B})^\top(\mathbf{B}-\mathbf{C}) = \begin{pmatrix}-4 & -2 & -6\end{pmatrix}\begin{pmatrix}-6\\-3\\-9\end{pmatrix} = 84 \tag{5}$$

$$\|\mathbf{B}-\mathbf{C}\|^2 = \left(\sqrt{(-6^2)+(-3)^2+(-9)^2}\right)^2 = 126 \tag{6}$$

B which divides **AC** in the ratio 2:3
Refer to Fig. 0

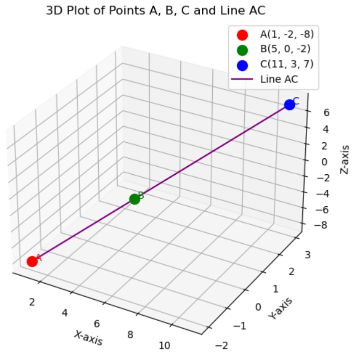

Fig. 0