1.5.2

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Question

Find the ratio in which the Y axis divides the line segment joining the points $(6, -4)$ and $(-2, -7)$. Also find the point of intersection.

Theoretical Solution

Given the points,

$$\mathbf{A} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \mathbf{B} = \begin{pmatrix} -2 \\ -7 \end{pmatrix} \tag{1}$$

Let the vector $\mathbf{P}$ be

$$\mathbf{P} = \begin{pmatrix} 0 \\ y \end{pmatrix} \ , \tag{2}$$

The points $\mathbf{A}$,$\mathbf{P}$,$\mathbf{B}$ are collinear.

Formulae

The points to be collinear,

$$\text{rank}\begin{pmatrix}\mathbf{P}-\mathbf{A} & \mathbf{B}-\mathbf{A}\end{pmatrix} = 1 \tag{3}$$

(4)

Theoretical Solution

$$\mathbf{P}-\mathbf{A}=\begin{pmatrix}-6\\ y+4\end{pmatrix} \tag{5}$$

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}-8\\ -3\end{pmatrix} \tag{6}$$

$$\begin{pmatrix}\mathbf{P}-\mathbf{A} & \mathbf{B}-\mathbf{A}\end{pmatrix}=\begin{pmatrix}-6 & -8\\ y+4 & -3\end{pmatrix} \tag{7}$$

Conversion to Row Echelon form, $R_2 \to R_2 + \frac{y+4}{6}R_1$:

$$\begin{pmatrix}-6 & -8\\ 0 & -3+\frac{y+4}{6}(-8)\end{pmatrix} \Longrightarrow \begin{pmatrix}-6 & -8\\ 0 & \frac{-4y-25}{3}\end{pmatrix}$$

$$\frac{-4y-25}{3}=0 \implies y=-\frac{25}{4} \tag{8}$$

$$\therefore \mathbf{P}=\begin{pmatrix}0\\ -\frac{25}{4}\end{pmatrix}$$

Formulae

Section formula for a vector P which divides the line formed by vectors A and B in the ratio k:1 is given by

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k+1} \tag{9}$$

$$k\left(\mathbf{P} - \mathbf{B}\right) = \mathbf{A} - \mathbf{P} \tag{10}$$

$$\implies k = \frac{\left(\mathbf{A} - \mathbf{P}\right)^{\top}\left(\mathbf{P} - \mathbf{B}\right)}{\left\|\mathbf{P} - \mathbf{B}\right\|^{2}} \tag{11}$$

Theoretical Solution

$$(\mathbf{A}-\mathbf{P})^{\top}(\mathbf{P}-\mathbf{B}) = \begin{pmatrix} 6 & \frac{9}{4} \end{pmatrix} \begin{pmatrix} 2 \\ \frac{3}{4} \end{pmatrix} = \frac{219}{16} \tag{12}$$

$$\|\mathbf{P}-\mathbf{B}\|^2 = \left(\sqrt{2^2 + \left(\frac{3}{4}\right)^2} \right)^2 = \frac{73}{16} \tag{13}$$

$$k = \frac{\frac{219}{16}}{\frac{73}{16}} \tag{14}$$

$$\implies k = 3 \tag{15}$$

Therefore the ratio in which point **P** divides the line segment joining **A** and **B** is 3:1

C Code

```c
#include <stdio.h>
int main() {
    int x1 = 6, y1 = -4;
    int x2 = -2, y2 = -7;
    int m, n;
    m = x1;
    n = -x2;
    printf(The Y-axis divides the line in the ratio %d:%d\n, m, n
        );
    float x = (m * x2 + n * x1) / (float)(m + n);
    float y = (m * y2 + n * y1) / (float)(m + n);
    printf(Point of intersection: (%.2f, %.2f)\n, x, y);
    return 0;
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
import ctypes
import os

# Load compiled C library
c_lib = ctypes.CDLL('./code.so')

# Define C function signature: takes 5 floats, returns float
# (Ax, Ay, Bx, By, Px) and returns Py
c_lib.findM.argtypes = [ctypes.c_float, ctypes.c_float, ctypes.
    c_float,
                        ctypes.c_float, ctypes.c_float]
c_lib.findM.restype = ctypes.c_float
```

Python Code

```
    # Define points A and B
A = np.array([6.0, -4.0])
B = np.array([-2.0, -7.0])
Px = 0.0 # x = 0 (Y-axis)

# Call C function to get Py (y-coordinate of intersection with Y-
    axis)
Py = c_lib.findM(
    ctypes.c_float(A[0]),
    ctypes.c_float(A[1]),
    ctypes.c_float(B[0]),
    ctypes.c_float(B[1]),
    ctypes.c_float(Px)
)
```

Python Code

```
# The dividing point on the Y-axis
P_dividing = np.array([Px, Py])

def find_ratio(point_A, point_B, dividing_point):
    A_vec = np.array(point_A)
    B_vec = np.array(point_B)
    P_vec = np.array(dividing_point)

    epsilon = 1e-9
    ratio_vector = (P_vec - A_vec) / (B_vec - P_vec + epsilon)
    return ratio_vector
```

Python Code

```
# Calculate and print the ratio
ratio = find_ratio(A, B, P_dividing)
print(f'Point {tuple(P_dividing)} divides the line AB in the
    ratio: {round(ratio[0])}:{round(ratio[1])}')

def generate_line_segment(point1, point2, num_points=10):
    dim = point1.shape[0]
    line_segment = np.zeros((dim, num_points))
    lambda_vals = np.linspace(0, 1, num_points)
    for i in range(num_points):
        temp = point1 + lambda_vals[i] * (point2 - point1)
        line_segment[:, i] = temp.T
    return line_segment

# Generate line segment for plotting
x_AB = generate_line_segment(A, B)
```

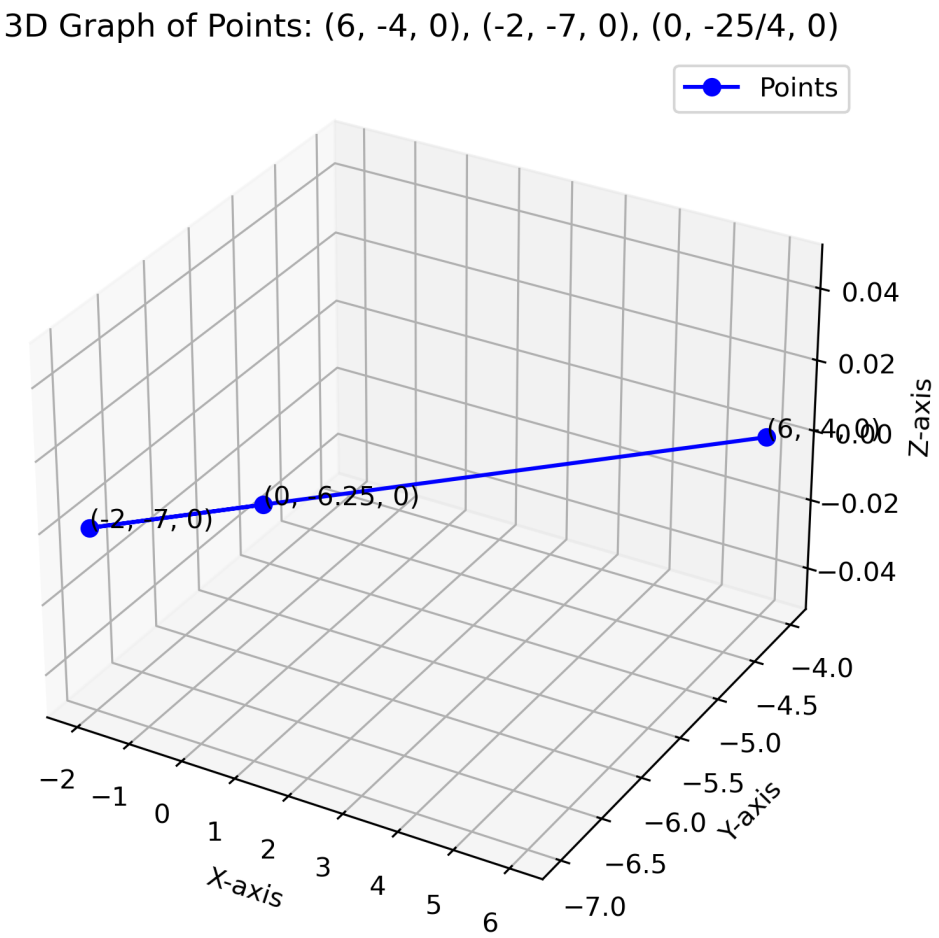
Python Code

```
# Plotting
plt.plot(x_AB[0, :], x_AB[1, :], label='$AB$')

# Plot points A, B, and P
all_points = np.vstack((A, B, P_dividing)).T
plt.scatter(all_points[0, :], all_points[1, :], color='red')

# Add labels
point_labels = [f'A {tuple(A)}', f'B {tuple(B)}', f'P {tuple(
    P_dividing)}']
for i, txt in enumerate(point_labels):
    plt.annotate(txt,
                 (all_points[0, i], all_points[1, i]),
                 textcoords=offset points,
                 xytext=(10, 5),
                 ha='center')
```

Python Code

```
# Set plot details
plt.xlabel('$x$')
plt.ylabel('$y$')
plt.title(f'Point P{tuple(P_dividing)} divides AB in ratio of {
    round(ratio[0])}:{round(ratio[1])}')
plt.legend(loc='best')
plt.grid(True)
plt.axis('equal')

# Save and show plot
plt.savefig('../Figs/graph3d.png')
plt.show()
```

Plot


3D Graph of Points: (6, -4, 0), (-2, -7, 0), (0, -25/4, 0)
Points
(-2, -7, 0)
(0, -6.25, 0)
(6, -4, 0)
X-axis
Y-axis
Z-axis
−2
−1
0
1
2
3
4
5
6
−7.0
−6.5
−6.0
−5.5
−5.0
−4.5
−4.0
0.04
0.02
0.00
−0.02
−0.04