Matgeo Presentation - Problem 12.76

ee25btech11063 - Vejith

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Question

Four points **P** $(0, 1)$, **Q** $(0, -3)$, **R** $(-2, -1)$, **S** $(2, -1)$ represent the vertices of a quadrilateral. What is the area enclosed by the quadrilateral ? (ST 2022)

(a) 4
(b) $4\sqrt{2}$
(c) 8
(d) $8\sqrt{2}$

Solution

$$\mathbf{P} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \mathbf{Q} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} \qquad \mathbf{R} = \begin{pmatrix} -2 \\ -1 \end{pmatrix} \qquad \mathbf{S} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \tag{0.1}$$

let PSQR be the quadrilateral then it's diagonals are $\mathbf{P} - \mathbf{Q}$ and $\mathbf{R} - \mathbf{S}$

$$\|\mathbf{P} - \mathbf{Q}\| = \left\| \begin{pmatrix} 0 \\ 4 \end{pmatrix} \right\| = 4 \tag{0.2}$$

$$\|\mathbf{R} - \mathbf{S}\| = \left\| \begin{pmatrix} -4 \\ 0 \end{pmatrix} \right\| = 4 \tag{0.3}$$

$$(\mathbf{P} - \mathbf{Q})^\top (\mathbf{R} - \mathbf{S}) = \begin{pmatrix} 0 & 4 \end{pmatrix} \begin{pmatrix} -4 \\ 0 \end{pmatrix} \tag{0.4}$$

$$= 0 \tag{0.5}$$

$\implies$ diagonals of the quadrilateral are of equal length and they bisect each other perpendicularly

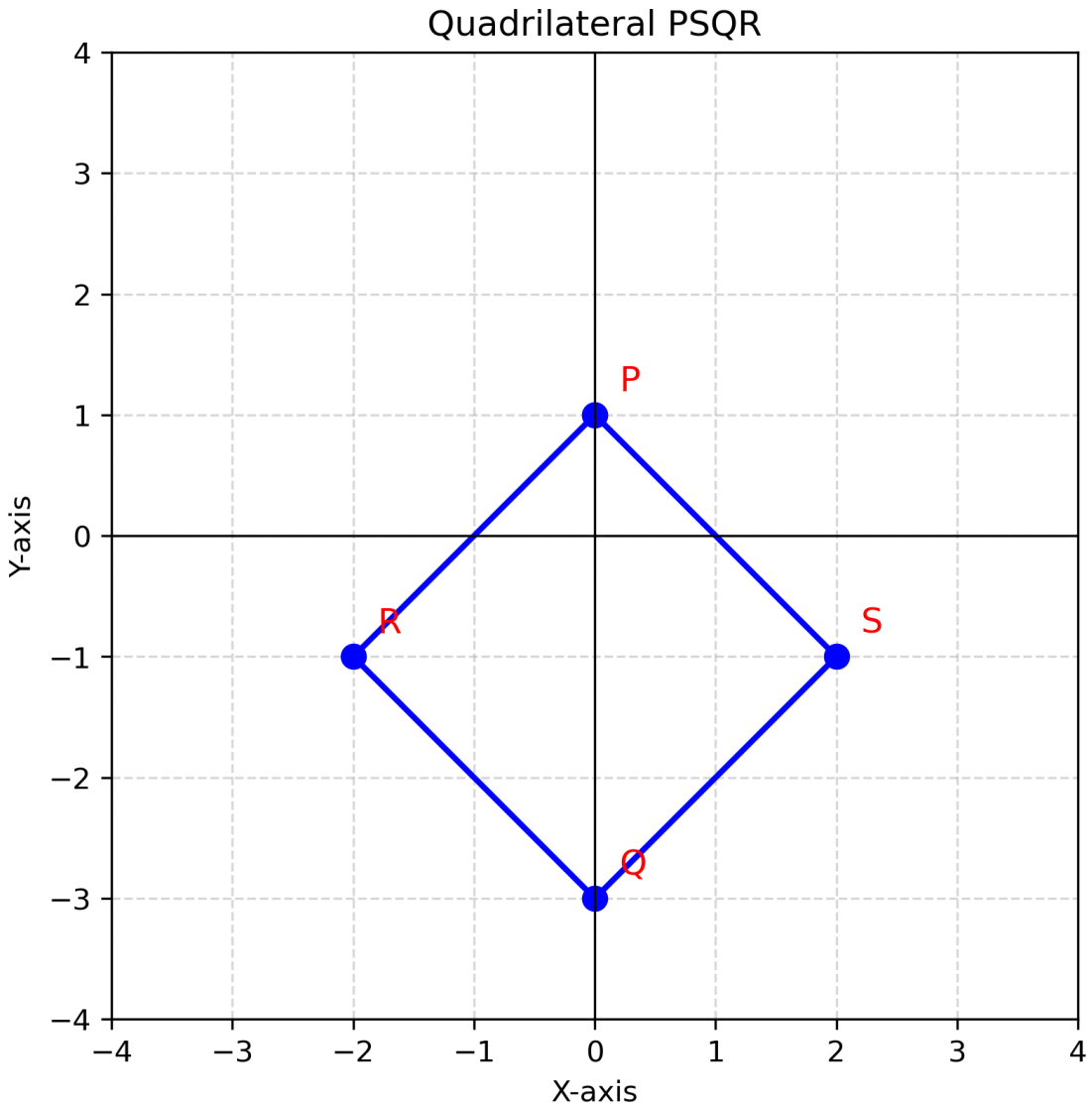
$\implies$ the given quadrilateral is a square

Conclusion

$$\text{area of the quadrilateral PSQR} = \frac{1}{2}\|\mathbf{P}-\mathbf{Q}\|^2 \tag{0.6}$$

$$= \frac{1}{2} \times 16 = 8 \tag{0.7}$$

Plot



Figure

C Code: area.c

```c
#include <stdio.h>
#include <math.h>

int main() {
    // Coordinates
    float x[4] = {0, 0, -2, 2};
    float y[4] = {1, -3, -1, -1};
    float area;
    float sum1 = 0, sum2 = 0;

    // Shoelace formula: sum over vertices
    for(int i = 0; i < 4; i++) {
        int j = (i + 1) % 4;
        sum1 += x[i] * y[j];
        sum2 += y[i] * x[j];
    }
    area = fabs(sum1 - sum2) / 2.0;

    // Calculate side lengths
    float pq = sqrt(pow(x[1]-x[0],2) + pow(y[1]-y[0],2));
    float qr = sqrt(pow(x[2]-x[1],2) + pow(y[2]-y[1],2));
    float rs = sqrt(pow(x[3]-x[2],2) + pow(y[3]-y[2],2));
    float sp = sqrt(pow(x[0]-x[3],2) + pow(y[0]-y[3],2));

    // Check type
    char type[20];
    if (fabs(pq - qr) < 1e-3 && fabs(qr - rs) < 1e-3 && fabs(rs - sp) < 1e-3)
        sprintf(type, "Square");
    else if (fabs(pq - rs) < 1e-3 && fabs(qr - sp) < 1e-3)
        sprintf(type, "Rectangle");
    else
        sprintf(type, "Other␣Quadrilateral");
```

C Code: area.c

```
    // Write to file
    FILE *fp = fopen("area.dat", "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }

    fprintf(fp, "Area of the quadrilateral = %.2f\n", area);
    fprintf(fp, "Type of quadrilateral = %s\n", type);

    fclose(fp);

    printf("Output written to area.dat successfully.\n");
    return 0;
}
```

Python: plot.py

```
import numpy as np
import matplotlib.pyplot as plt

# Points
P = (0, 1)
Q = (0, -3)
R = (-2, -1)
S = (2, -1)

# Order: P  S  Q  R  P
x = [P[0], S[0], Q[0], R[0], P[0]]
y = [P[1], S[1], Q[1], R[1], P[1]]

# Plot
plt.figure(figsize=(6,6))
plt.plot(x, y, 'b-o', linewidth=2, markersize=8)
points = {'P': P, 'Q': Q, 'R': R, 'S': S}
for name, (x_pt, y_pt) in points.items():
    plt.text(x_pt + 0.2, y_pt + 0.2, name, fontsize=12, color='red')
limit = 4
plt.xlim(-limit, limit)
plt.ylim(-limit, limit)
plt.gca().set_aspect('equal', adjustable='box') # equal scaling

# Axes & grid
plt.axhline(0, color='black', linewidth=0.8)
plt.axvline(0, color='black', linewidth=0.8)
plt.grid(True, linestyle='--', alpha=0.5)
plt.xlabel("X-axis")
plt.ylabel("Y-axis")
plt.title("Quadrilateral␣PSQR")
plt.savefig("plot.png", dpi=300, bbox_inches='tight')
plt.show()
```