1.5.16

J.NAVYASRI- EE25BTECH11028

August 2025

Question

Question: Find the point A if AB is a diameter of the circle with center $C = (3, -1)$ and point $B = (2, 6)$.

given data

Point	Vector
B	$\begin{pmatrix} 2 \\ 6 \end{pmatrix}$
C	$\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

Section Formula

If a point P divides the line joining A and B internally in the ratio $m : n$, then

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k+1} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \frac{1}{k+1} \\ \frac{k}{k+1} \end{pmatrix} \tag{1}$$

Here, C is the midpoint of AB, i.e. ratio $1 : 1$.

$$\mathbf{C} = \frac{\mathbf{A} + \mathbf{B}}{2} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}. \tag{2}$$

Express A in terms of B and C :

$$\mathbf{C} = \frac{\mathbf{A}+\mathbf{B}}{2} \quad \Rightarrow \quad 2\mathbf{C} = \mathbf{A}+\mathbf{B} \quad \Rightarrow \quad \mathbf{A} = 2\mathbf{C}-\mathbf{B}. \tag{3}$$

Using matrix notation,

$$\mathbf{A} = 2\mathbf{C}-\mathbf{B} = \begin{pmatrix}\mathbf{B} & \mathbf{C}\end{pmatrix}\begin{pmatrix}-1\\ 2\end{pmatrix}. \tag{4}$$

substitute values :

Given

$$\mathbf{B} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 3 \\ -1 \end{pmatrix},$$

we have,

$$\mathbf{A} = 2\begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}. \tag{5}$$

$$\boxed{\mathbf{A} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}}. \tag{6}$$

Figure

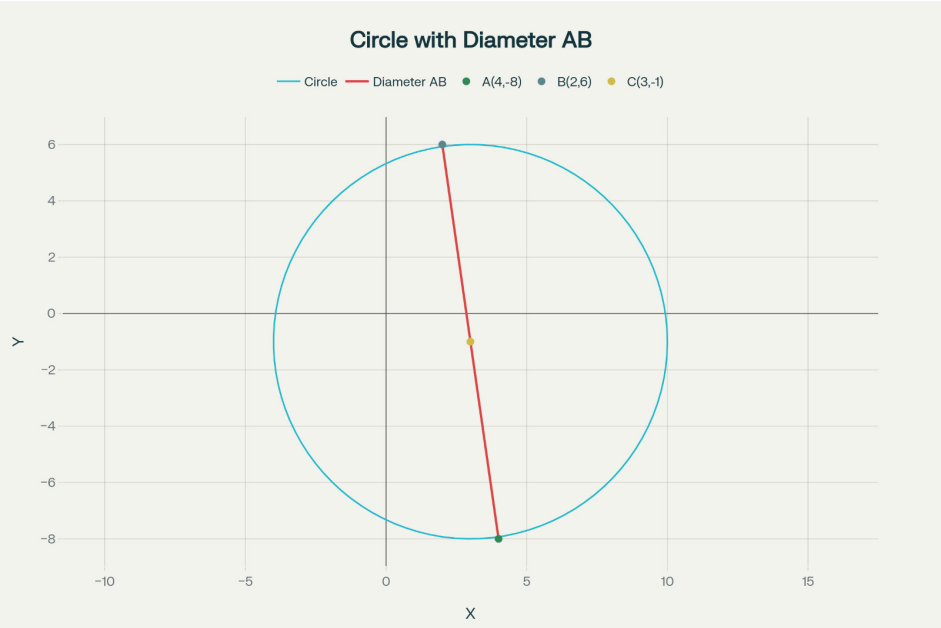

Figure:

Python Code

```
    import matplotlib.pyplot as plt

# Center C = (3, -1)
# B = (2, 6)
# Let A = (x, y). Midpoint formula: center = (A + B) / 2 =>
# 3 = (x + 2) / 2, -1 = (y + 6) / 2
# Solve for (x, y):
# x = 2*3 - 2 = 4
# y = 2*(-1) - 6 = -8

A = np.array([4, -8])
B = np.array([2, 6])
C = np.array([3, -1])
```

Python Code

```
# For the circle, radius = distance(center, B)
import numpy as np
def dist(P, Q):
    return np.sqrt((P[0] - Q[0])**2 + (P[1] - Q[1])**2)
radius = dist(C, B)

fig, ax = plt.subplots(figsize=(7,7))

# Plot the circle
circle = plt.Circle(C, radius, color='blue', fill=False,
    linestyle='dotted', label='Circle')
ax.add_patch(circle)

# Plot A, B, C
ax.scatter(*A, color='red', label='A (unknown, solved)')
ax.scatter(*B, color='green', label='B (2, 6)')
ax.scatter(*C, color='orange', label='Center (3, -1)')
```

Python Code

```
 # Plot line AC
ax.plot([A[0], C[0]], [A[1], C[1]], [A[2], C[2]], color='purple',
     label='Line AC')

# Annotate points
ax.text(*A, ' A', color='red', fontsize=10)
ax.text(*B, ' B', color='green', fontsize=10)
ax.text(*C, ' C', color='blue', fontsize=10)
```

Python Code

```
 # Draw diameter AB
ax.plot([A[0], B[0]], [A[1], B[1]], color='purple', linewidth=2,
    linestyle='--', label='Diameter AB')

# Annotate
ax.annotate('A'+str(A), (A[0], A[1]), xytext=(10, -10),
    textcoords='offset points')
ax.annotate('B'+str(B), (B[0], B[1]), xytext=(-40, 10),
    textcoords='offset points')
ax.annotate('C'+str(C), (C[0], C[1]), xytext=(5, -10), textcoords
    ='offset points')

ax.set_xlim(C[0] - radius - 2, C[0] + radius + 2)
ax.set_ylim(C[1] - radius - 2, C[1] + radius + 2)
ax.set_aspect('equal')
ax.grid(True)
plt.legend()
plt.title('Circle with Diameter AB')
plt.xlabel('x')
```

C Code

```c
#include <stdio.h>

int main() {
    // Given values
    int xB = 2, yB = 6;
    int xC = 3, yC = -1; // Center of the circle

    // Calculate coordinates of A using midpoint formula
    int xA = 2 * xC - xB;
    int yA = 2 * yC - yB;

    // Print result
    printf("Coordinates of point A are: (%d, %d)\n", xA, yA);

    // Verify midpoint
    float midX = (xA + xB) / 2.0;
    float midY = (yA + yB) / 2.0;
    printf("Midpoint of A and B is: (%.1f, %.1f)\n", midX, midY);
```

Python and C Code

```
 import subprocess

# Compile the C program
subprocess.run(["gcc", "midpoint.c", "-o", "midpoint"])

# Run the compiled C program
result = subprocess.run(["./midpoint"], capture_output=True, text
    =True)

# Print the output from the C program
print(result.stdout)
```

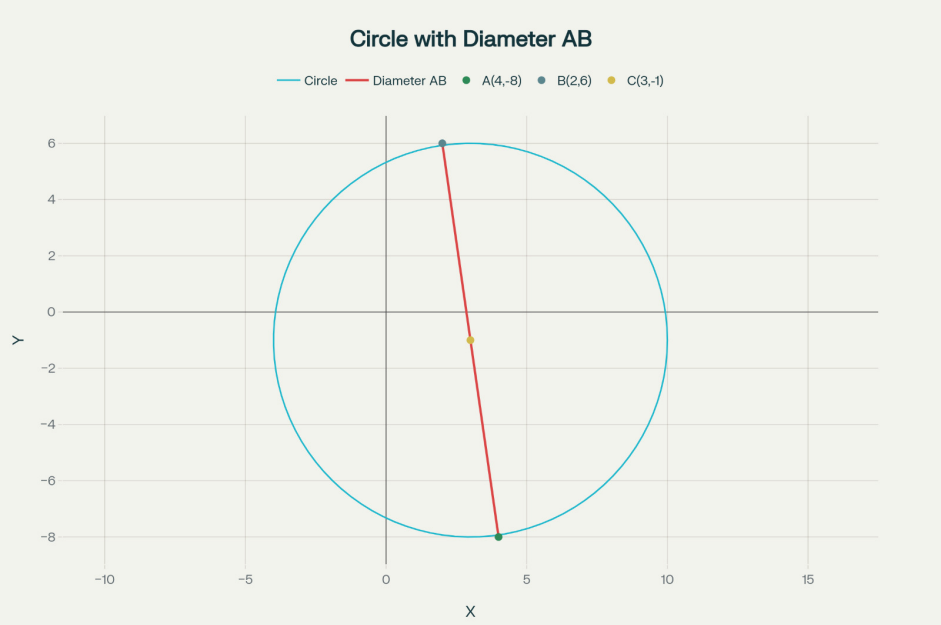
Circle with Diameter AB
Circle
Diameter AB
A(4,-8)
B(2,6)
C(3,-1)
6
4
2
0
−2
−4
−6
−8
Y
−10
−5
0
5
10
15
X

Figure: