

6.4.12 Matgeo

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Question

Find the shortest distance between the lines

$$\mathbf{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\mathbf{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} - \hat{j} + 2\hat{k})$$

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \quad (1)$$

$$x_2 = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} + \mu \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix} \quad (2)$$

$$\mathbf{M} = \begin{bmatrix} 1 & 2 \\ -1 & -1 \\ 1 & 2 \end{bmatrix} \quad (3)$$

$$\mathbf{B} - \mathbf{A} = \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} \quad (4)$$

$$\begin{bmatrix} \vec{M} & \vec{B} - \vec{A} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ -1 & -1 & -3 \\ 1 & 2 & -2 \end{bmatrix} \quad (5)$$

$$R_3 = R_3 + R_2 \quad \text{and} \quad R_2 = R_2 + R_1 \quad (6)$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -3 \\ 0 & 1 & -5 \end{bmatrix} \quad (7)$$

$$R_3 = R_3 - R_2 \quad (8)$$

$$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -3 \\ 0 & 0 & -2 \end{bmatrix} \quad (9)$$

Solution

as the rank of above matrix is 3 the lines are skew lines

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \\ 1 & 2 \end{bmatrix} \kappa = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} 3 & 5 \\ 5 & 9 \end{bmatrix} \kappa = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \quad (11)$$

The augmented matrix of the above matrix is

$$= \begin{bmatrix} 3 & 5 & 2 \\ 5 & 9 & 1 \end{bmatrix} \quad (12)$$

$$R_2 = R_2 - \frac{5}{3}R_1 \quad \text{and} \quad R_1 = R_1 - \frac{15}{2}R_2 \quad (13)$$

$$= \begin{bmatrix} 3 & 0 & \frac{39}{2} \\ 0 & \frac{2}{3} & -\frac{7}{3} \end{bmatrix} \quad (14)$$

yeilding

$$\begin{bmatrix} \lambda \\ -\mu \end{bmatrix} = \begin{bmatrix} \frac{13}{2} \\ 7 \\ -\frac{7}{2} \end{bmatrix} \quad (15)$$

$$x_1 = \frac{1}{2} \begin{bmatrix} 15 \\ -9 \\ 15 \end{bmatrix}, x_2 = \frac{1}{2} \begin{bmatrix} -10 \\ -1 \\ -4 \end{bmatrix} \quad (16)$$

The minimum distance between the lines is given by

$$\|x_2 - x_1\| = \left\| \frac{1}{2} \begin{bmatrix} 25 \\ -8 \\ 19 \end{bmatrix} \right\| = \frac{5\sqrt{42}}{2} \quad (17)$$

Graphical Representation

