4.7.45

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Question The equation of the line passing through the point $(1, 2)$ and perpendicular to the line $x + y + 1 = 0$ is

Solution Let desired line :

$$\mathbf{n}^T \mathbf{x} = c \tag{1}$$

Given line equation and point say A:

$$x + y + 1 = 0 \tag{2}$$

$$y = -x - 1 \tag{3}$$

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{4}$$

Since, the line from eq (2) is perpendicular to (1)
We get the normal vector which is equal to:

$$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{5}$$

Because line (2) is perpendicular, the equation of the line can be changed as:

$$\mathbf{n}^T \left(\mathbf{x} - \mathbf{A} \right) = 0 \tag{6}$$

Thus the equation of line:

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right) = 0 \tag{7}$$

$$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = 0 \tag{8}$$

$$\implies \begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -1 \tag{9}$$

Final Answer The desired line equation is as follows
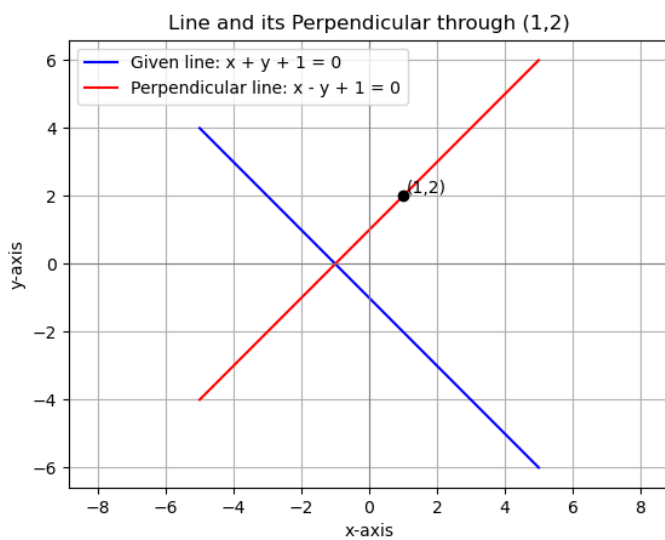
$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{x} = -1$$

Fig. 1: Plot