# 2.10.10

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## Question

Given that

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad \mathbf{a} \cdot \mathbf{b} = 3, \quad \mathbf{a} \times \mathbf{b} = \mathbf{c},$$

find $\mathbf{b}$.

## Solution

*Step 1: Express vectors as column matrices*

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}.$$

*Step 2: Use the dot product condition*

$$\mathbf{a}^\top \mathbf{b} = x + y + z = 3.$$

*Step 3: Use the cross product condition*

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} z - y \\ x - z \\ y - x \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}.$$

From which we get:

$$\begin{cases} z - y = 0, \\ x - z = 1, \\ y - x = -1. \end{cases}$$

*Step 4: Solve the system*

From $z - y = 0$, we have

$$z = y.$$

From $y - x = -1$, we get

$$y = x - 1.$$

Substitute into $x - z = 1$ (with $z = y$):

$$x - y = 1 \implies x - (x - 1) = 1 \implies 1 = 1,$$

which is consistent.

*Step 5: Use the dot product to find x*

$$x + y + z = x + (x-1) + (x-1) = 3x - 2 = 3 \implies 3x = 5 \implies x = \frac{5}{3}.$$

Then,

$$y = \frac{2}{3}, \quad z = \frac{2}{3}.$$

*Final answer*

$$\boxed{\mathbf{b} = \begin{pmatrix} \frac{5}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{pmatrix}.}$$

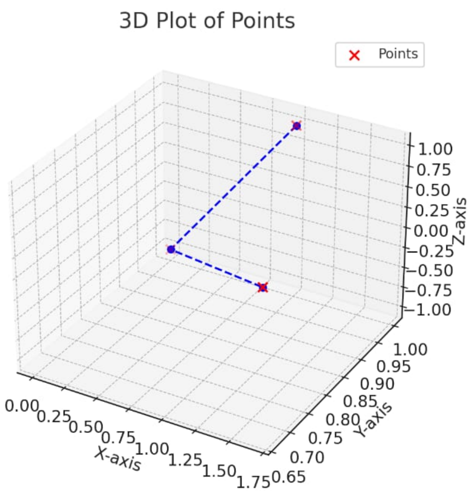


Fig. 0.1: plot