Question 2.6.37:

The vector from origin to the points A and B are

$$\mathbf{a} = 2\hat{i} - 3\hat{j} + 2\hat{k} \quad \text{and} \quad \mathbf{b} = 2\hat{i} + 3\hat{j} + \hat{k}, \tag{1}$$

respectively, then the area of $\triangle OAB$ is ___________.

Solution: Given

The area of the triangle OAB is given by

$$\text{Area}(OAB) = \frac{1}{2}\|\mathbf{a} \times \mathbf{b}\|. \tag{2}$$

We have

$$\mathbf{a} = (2, -3, 2), \quad \mathbf{b} = (2, 3, 1). \tag{3}$$

Using the cross product definition,

$$\begin{pmatrix} \mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2 & b_2 \\ a_3 & b_3 \end{pmatrix} \\ \\ \begin{pmatrix} a_3 & b_3 \\ a_1 & b_1 \end{pmatrix} \\ \\ \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \end{pmatrix} \tag{4}$$

Substituting values:

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} \begin{pmatrix} -3 & 3 \\ 2 & 1 \end{pmatrix} \\ \\ \begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix} \\ \\ \begin{pmatrix} 2 & 2 \\ -3 & 3 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} (-3)(1) - (3)(2) \\ (2)(2) - (1)(2) \\ (2)(3) - (2)(-3) \end{pmatrix}. \tag{5}$$

$$\mathbf{a} \times \mathbf{b} = (-9,\ 2,\ 12). \tag{6}$$

Now, its magnitude is

$$\|\mathbf{a} \times \mathbf{b}\| = \sqrt{(-9)^2 + (2)^2 + (12)^2} = \sqrt{81 + 4 + 144} = \sqrt{229}. \tag{7}$$

Therefore, the required area is

$$\text{Area}(OAB) = \frac{1}{2}\|\mathbf{a} \times \mathbf{b}\| = \frac{1}{2}\sqrt{229}. \tag{8}$$

$$\boxed{\text{Area}(OAB) = \frac{\sqrt{229}}{2}} \tag{9}$$

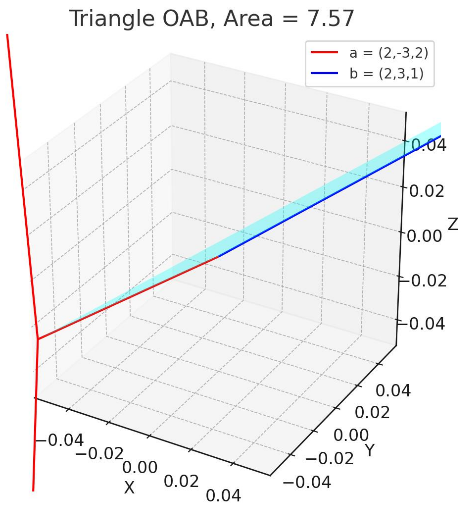

Fig. 1: Caption