4.3.41

EE25BTECH11051 - Shreyas Goud Burra

Question

The cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$. Write its vector form.

Solution:

Given cartesian equation of line is

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2} = \lambda \tag{0.1}$$

We know the vector form of a line is given by,

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{0.2}$$

Where $\mathbf{x}$ is a point on the given line, $\mathbf{h}$ is a known point on that line, $\mathbf{m}$ is the slope of the line and k is an arbitrary real constant.

From 0.1, we can determine a point on the line taking $\lambda = 0$

$$\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2} = 0 \implies x = 5, y = -4, z = 6 \tag{0.3}$$

$$\implies \mathbf{h} = \begin{pmatrix} 5 \\ -4 \\ 6 \end{pmatrix} \tag{0.4}$$

We can get the ratio of direction cosines from 0.1

$$\text{ratio} = 3 : 7 : 2 \implies \mathbf{m} = \begin{pmatrix} 3 \\ 7 \\ 2 \end{pmatrix} \tag{0.5}$$

Substituting 0.4 and 0.5 in 0.2, we get

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix} + k \begin{pmatrix} 3 \\ 7 \\ 2 \end{pmatrix} \tag{0.6}$$
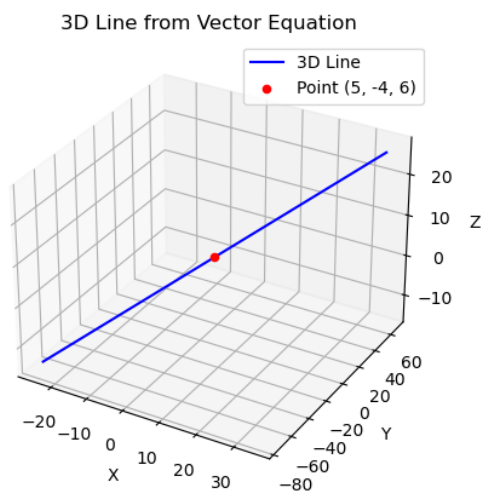
The plot for the given question is given below,

Fig. 0.1: 3D Plot