Matgeo Presentation - Problem 10.7.111

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Question

a,**b**,**c** are three orthogonal vectors. Given that **a**$=\hat{i}+2\hat{j}+5\hat{k}$ and **b**$=\hat{i}+2\hat{j}-\hat{k}$, the vector **c** is parallel to (IN 2019)

(a) $\hat{i}+2\hat{j}+3\hat{k}$ (b) $2\hat{i}+\hat{j}$ (c) $2\hat{i}-\hat{j}$ (d) $4\hat{k}$

Solution

Given

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} \tag{0.1}$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \tag{0.2}$$

$$\mathbf{a}^\top \mathbf{c} = 0 \tag{0.3}$$

$$\mathbf{b}^\top \mathbf{c} = 0 \tag{0.4}$$

(0.3) and (0.4) can be written as

$$\begin{pmatrix} \mathbf{a}^\top \\ \mathbf{b}^\top \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.5}$$

$$\implies \begin{pmatrix} 1 & 2 & 5 \\ 1 & 2 & -1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.6}$$

Solution

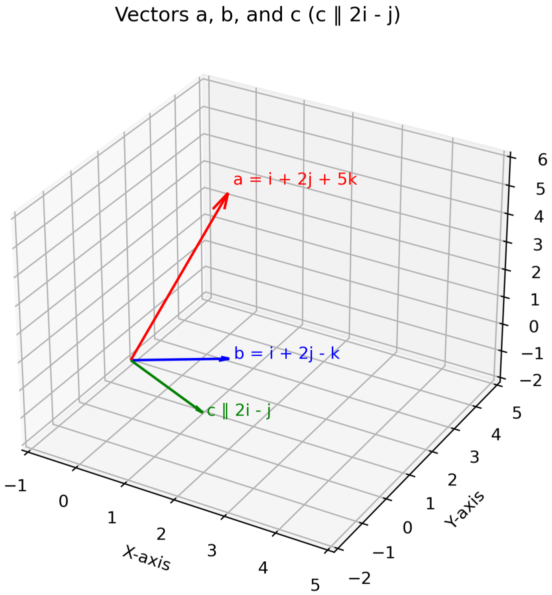
Forming the augmented matrix

$$\left(\begin{array}{ccc|c} 1 & 2 & 5 & 0 \\ 1 & 2 & -1 & 0 \end{array}\right) \xleftrightarrow{R_2 \to R_2 - R_1} \left(\begin{array}{ccc|c} 1 & 2 & 5 & 0 \\ 0 & 0 & -6 & 0 \end{array}\right) \tag{0.7}$$

$\implies$ vector $\mathbf{c}$ can be written in general as $\mathbf{c} = \begin{pmatrix} 2k \\ -k \\ 0 \end{pmatrix}$ (for some scalar k)

$\implies$ vector $\mathbf{c}$ is parallel to $2\hat{i} - \hat{j}$

Plot



Vectors a, b, and c (c ∥ 2i - j)
a = i + 2j + 5k
b = i + 2j - k
c ∥ 2i - j
X-axis
Y-axis
−1
0
1
2
3
4
5
−2
−1
0
1
2
3
4
5
6

C Code: vector.c

```
#include <stdio.h>

int main() {
    // Given vectors
    float a[3] = {1, 2, 5};
    float b[3] = {1, 2, -1};

    // Variables for the unknown vector c = (x, y, z)
    // We will solve the system:
    // ac = 0 => x + 2y + 5z = 0
    // bc = 0 => x + 2y - z = 0

    float A[2][3] = {
        {1, 2, 5},
        {1, 2, -1}
    };
    float x, y, z;

    // Using elimination (matrix method):
    // Subtract equation 2 from equation 1
    // (1 - 1)x + (2 - 2)y + (5 - (-1))z = 0 - 0
    // => 6z = 0 => z = 0

    z = 0;

    // Substitute z = 0 in first equation:
    // x + 2y + 5*0 = 0 => x = -2y
    // Let y = 1 (for direction vector)

    y = 1;
    x = -2 * y;
```

C Code: vector.c

```
    // Vector c is proportional to (-2, 1, 0)
    // Parallel vector can be written as (2, -1, 0)

    FILE *fp;
    fp = fopen("vector.dat", "w");

    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }

    fprintf(fp, "Vector c is parallel to: 2i - j\n");
    fprintf(fp, "Hence, c is parallel to (2, -1, 0)\n");

    fclose(fp);

    printf("Result written to vector.dat successfully.\n");

    return 0;
}
```

Python: plot.py

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Given vectors
a = np.array([1, 2, 5])
b = np.array([1, 2, -1])

# Vector c is parallel to 2i - j
c = np.array([2, -1, 0])

# Verify orthogonality (dot products)
dot_a_c = np.dot(a, c)
dot_b_c = np.dot(b, c)

print("a␣␣c␣=", dot_a_c)
print("b␣␣c␣=", dot_b_c)
print("\nSince␣both␣dot␣products␣are␣0,␣c␣is␣orthogonal␣to␣both␣a␣and␣b.")
print("Hence,␣c␣is␣parallel␣to␣2i␣-␣j.\n")

# Plot setup
fig = plt.figure(figsize=(8, 6))
ax = fig.add_subplot(111, projection='3d')

# Plot vectors
ax.quiver(0, 0, 0, a[0], a[1], a[2], color='r', arrow_length_ratio=0.1)
ax.quiver(0, 0, 0, b[0], b[1], b[2], color='b', arrow_length_ratio=0.1)
ax.quiver(0, 0, 0, c[0], c[1], c[2], color='g', arrow_length_ratio=0.1)

# Label each vector beside its arrow
ax.text(a[0]*1.05, a[1]*1.05, a[2]*1.05, 'a␣=␣i␣+␣2j␣+␣5k', color='r', fontsize=10)
```

Python: plot.py

```
ax.text(b[0]*1.05, b[1]*1.05, b[2]*1.05, 'b␣=␣i␣+␣2j␣-␣k', color='b', fontsize=10)
ax.text(c[0]*1.05, c[1]*1.05, c[2]*1.05, 'c␣␣2i␣-␣j', color='g', fontsize=10)

# Axis labels and style
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_xlim([-1, 5])
ax.set_ylim([-2, 5])
ax.set_zlim([-2, 6])
ax.set_title('Vectors␣a,␣b,␣and␣c␣(c␣␣2i␣-␣j)')
ax.grid(True)

# Save the figure
plt.savefig("vectors_plot.png", dpi=300, bbox_inches='tight')
print("Figure␣saved␣as␣'vectors_plot.png'␣successfully.")

# Show the plot
plt.show()
```