5.4.20

EE25BTECH11025 - Ganachari Vishwambhar

Question:
Using elementary transformations, find the inverse of the following matrix.

$$\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix} \tag{1}$$

Solution:
Given:

$$A = \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix} \tag{2}$$

Let A^{-1} be the inverse of the given matrix A:

$$AA^{-1} = I \tag{3}$$

The augmented matrix $A|I$:

$$\left(\begin{array}{cc|cc} 3 & -1 & 1 & 0 \\ -4 & 2 & 0 & 1 \end{array}\right) R_1 \to \frac{R_1}{3} \tag{4}$$

$$\left(\begin{array}{cc|cc} 1 & \frac{-1}{3} & \frac{1}{3} & 0 \\ -4 & 2 & 0 & 1 \end{array}\right) R_2 \to R_2 + 4R_1 \tag{5}$$

$$\left(\begin{array}{cc|cc} 1 & \frac{-1}{3} & \frac{1}{3} & 0 \\ 0 & \frac{2}{3} & \frac{4}{3} & 1 \end{array}\right) R_2 \to \frac{3}{2}R_2 \tag{6}$$

$$\left(\begin{array}{cc|cc} 1 & \frac{-1}{3} & \frac{1}{3} & 0 \\ 0 & 1 & 2 & \frac{3}{2} \end{array}\right) R_1 \to R_1 + \frac{1}{3}R_2 \tag{7}$$

$$\left(\begin{array}{cc|cc} 1 & 0 & 1 & \frac{1}{2} \\ 0 & 1 & 2 & \frac{3}{2} \end{array}\right) \tag{8}$$

Therefore,

$$A^{-1} = \begin{pmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{pmatrix} \tag{9}$$