Question:

The position vector of the point which divides the join of points $2\mathbf{a} - 3\mathbf{b}$ and $\mathbf{a} + \mathbf{b}$ in the ratio 3:1 is _____.

Solution:

Let
$$\mathbf{a} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$
 and $\mathbf{b} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.
Then,

$$\mathbf{P} = 2\mathbf{a} - 3\mathbf{b} \tag{0.1}$$

$$\mathbf{Q} = \mathbf{a} + \mathbf{b} \tag{0.2}$$

Now, the matrix form for **P** and **Q** is:

$$\begin{pmatrix} \mathbf{P} & \mathbf{Q} \end{pmatrix} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix} \tag{0.3}$$

Using the section formula, the point \mathbf{R} dividing \mathbf{PQ} in ratio 3:1 is:

$$\mathbf{R} = \frac{3\mathbf{Q} + 1\mathbf{P}}{3+1}.\tag{0.4}$$

$$\mathbf{R} = \begin{pmatrix} \mathbf{Q} & \mathbf{P} \end{pmatrix} \begin{pmatrix} \frac{3}{4} \\ \frac{1}{4} \end{pmatrix} \tag{0.5}$$

$$\mathbf{R} = \begin{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} & \begin{pmatrix} 2\mathbf{a} \\ -3\mathbf{b} \end{pmatrix} \end{pmatrix} \begin{pmatrix} \frac{3}{4} \\ \frac{1}{4} \end{pmatrix}$$
 (0.6)

$$\mathbf{R} = \frac{1}{4} \left(3 \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} + \begin{pmatrix} 2\mathbf{a} \\ -3\mathbf{b} \end{pmatrix} \right) \tag{0.7}$$

$$= \frac{1}{4} \begin{pmatrix} 3\mathbf{a} + 2\mathbf{a} \\ 3\mathbf{b} - 3\mathbf{b} \end{pmatrix} \tag{0.8}$$

$$=\frac{1}{4} \begin{pmatrix} 5\mathbf{a} \\ 0 \end{pmatrix} \tag{0.9}$$

$$= \begin{pmatrix} \frac{5\mathbf{a}}{4} \\ 0 \end{pmatrix}. \tag{0.10}$$

1

$$\mathbf{R} = \begin{pmatrix} \frac{5\mathbf{a}}{4} \\ 0 \end{pmatrix} \tag{0.11}$$

Let $\mathbf{a}=1$ and $\mathbf{b}=0$.

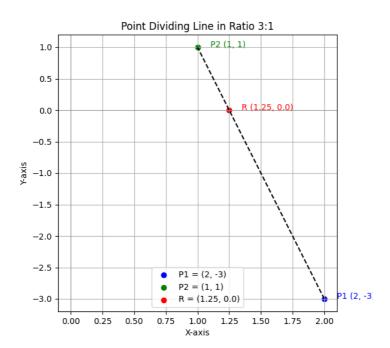


Fig. 0.1. Graph for question 1