7.4.9

EE25BTECH11002 - Achat Parth Kalpesh

Question:
The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \leq 6$ into two parts. If S is $\{\ (2, 3/4), (5/2, 3/4), (1/4, -1/4), (1/8, 1/4)\ \}$ then the number of point(s) in S lying inside the smaller part is ______.

Solution:
Let the points be

$$\mathbf{p_1} = \begin{pmatrix} 2 \\ \frac{3}{4} \end{pmatrix} \quad \mathbf{p_2} = \begin{pmatrix} \frac{5}{2} \\ \frac{3}{4} \end{pmatrix} \quad \mathbf{p_3} = \begin{pmatrix} \frac{1}{4} \\ -\frac{1}{4} \end{pmatrix} \quad \mathbf{p_4} = \begin{pmatrix} \frac{1}{8} \\ \frac{1}{4} \end{pmatrix} \tag{0.1}$$

The circular region is

$$\mathbf{x}^\top \mathbf{x} \leq 6 \tag{0.2}$$

The line is

$$\mathbf{n}^\top \mathbf{x} = 1 \tag{0.3}$$

$$\mathbf{n} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \tag{0.4}$$

Since the origin $\mathbf{0}$ lies inside the circle, checking which side of the line it belongs to:

$$\mathbf{n}^\top \mathbf{0} - 1 = -1 < 0 \tag{0.5}$$

Thus, the smaller part of the circle is the region

$$R = \left\{ \mathbf{x} : \mathbf{x}^\top \mathbf{x} \leq 6, \mathbf{n}^\top \mathbf{x} - 1 > 0 \right\} \tag{0.6}$$

For $\mathbf{p_1}$

$$\mathbf{p_1}^\top \mathbf{p_1} = 4 + \frac{9}{16} = \frac{73}{16} \leq 6 \tag{0.7}$$

$$\mathbf{n}^\top \mathbf{p_1} - 1 = 4 - \frac{9}{4} - 1 = \frac{3}{4} > 0 \tag{0.8}$$

For $\mathbf{p_2}$

$$\mathbf{p_2}^\top \mathbf{p_2} = \frac{109}{16} > 6 \tag{0.9}$$

$$\mathbf{n}^\top \mathbf{p_2} - 1 = 5 - \frac{9}{4} - 1 = \frac{7}{4} > 0 \tag{0.10}$$

For $\mathbf{p_3}$

$$\mathbf{p_3}^\top \mathbf{p_3} = \frac{1}{8} \leq 6 \tag{0.11}$$

$$\mathbf{n}^\top \mathbf{p_3} - 1 = \frac{1}{2} + \frac{3}{4} - 1 = \frac{1}{4} > 0 \tag{0.12}$$

For $\mathbf{p_4}$

$$\mathbf{p_4}^\top \mathbf{p_4} = \frac{5}{64} \leq 6 \tag{0.13}$$

$$\mathbf{n}^\top \mathbf{p_4} - 1 = \frac{1}{4} - \frac{3}{4} - 1 = -\frac{3}{2} < 0 \tag{0.14}$$

Thus, the points lying in the smaller part of the circle are

$$\mathbf{p_1} = \begin{pmatrix} 2 \\ \frac{3}{4} \end{pmatrix} \quad \mathbf{p_3} = \begin{pmatrix} \frac{1}{4} \\ -\frac{1}{4} \end{pmatrix} \tag{0.15}$$

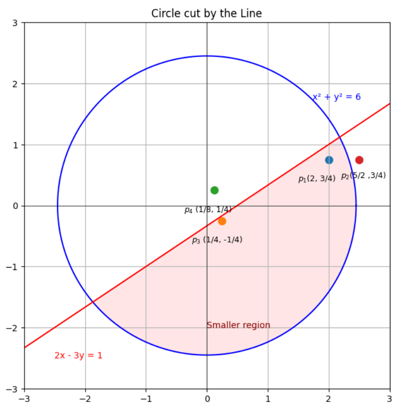

Fig. 0.1: Graph