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Question

Find the equation of the circle passing through $(0, 0)$ and making intercepts a and b on the coordinate axes.

Solution

We use the general matrix form of a conic:

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{1}$$

where:

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}, \quad \mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad f \in \mathbb{R} \tag{2}$$

Substitute the three points:

- At $(0, 0)$: $f = 0$
- At $(a, 0)$: $a^2 + 2u_1a = 0 \Rightarrow u_1 = -\frac{a}{2}$
- At $(0, b)$: $b^2 + 2u_2b = 0 \Rightarrow u_2 = -\frac{b}{2}$

So,

$$\mathbf{u} = \begin{pmatrix} -\frac{a}{2} \\ -\frac{b}{2} \end{pmatrix}, \quad f = 0 \tag{3}$$

Substitute back into the general form:

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -\frac{a}{2} & -\frac{b}{2} \end{pmatrix} \mathbf{x} = 0 \tag{4}$$

Simplify:

$$x^2 + y^2 - ax - by = 0 \tag{5}$$

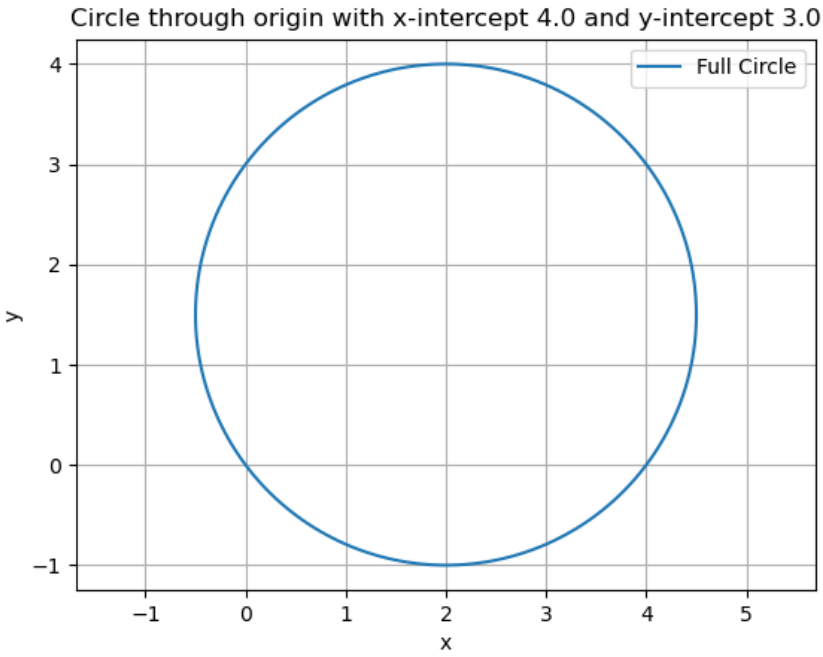
Circle through origin with x-intercept 4.0 and y-intercept 3.0
Full Circle
4
3
2
1
0
−1
y
−1
0
1
2
3
4
5
x

Figure 1