1.3.6

AI25BTECH11027 - NAGA BHUVANA

Question:
Show that the points $\mathbf{A}\,(6,2)$,$\mathbf{B}\,(2,1)$,$\mathbf{C}\,(1,5)$ and $\mathbf{D}\,(5,6)$ are vertices of a square.
Solution:
Given that

$$\mathbf{A} = \begin{pmatrix}6\\2\end{pmatrix}, \mathbf{B} = \begin{pmatrix}2\\1\end{pmatrix}, \mathbf{C} = \begin{pmatrix}1\\5\end{pmatrix}, \mathbf{D} = \begin{pmatrix}5\\6\end{pmatrix} \tag{1}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix}2-6\\1-2\end{pmatrix} = \begin{pmatrix}-4\\-1\end{pmatrix} \tag{2}$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix}1-5\\5-6\end{pmatrix} = \begin{pmatrix}-4\\-1\end{pmatrix} \tag{3}$$

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \tag{4}$$

By the above property we can say that **ABCD** is a parallelogram.
Consider the sides

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix}6-5\\2-6\end{pmatrix} = \begin{pmatrix}1\\-4\end{pmatrix} \tag{5}$$

$$(\mathbf{B} - \mathbf{A})^T = \begin{pmatrix}-4 & -1\end{pmatrix} \tag{6}$$

Consider the angle θ between the sides $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{B}$ of the parallelogram

$$\cos\theta = \frac{\left(B - A\right)^T \left(C - B\right)}{\|\mathbf{B} - \mathbf{A}\|\|\mathbf{C} - \mathbf{B}\|} \tag{7}$$

$$\cos\theta = \frac{\begin{pmatrix}-4 & -1\end{pmatrix}\begin{pmatrix}-1\\4\end{pmatrix}}{\sqrt{17}\sqrt{17}} \tag{8}$$

$$\cos\theta = \frac{(-4)(-1) + (-1)(4)}{17} \tag{9}$$

$$\tag{10}$$

$$\cos\theta = 0 \tag{11}$$

$$\tag{12}$$

$$\theta = 90^\circ$$

Property:
A parallelogram with one angle 90° is a rectangle
Hence the parallelogram is a rectangle

$$\mathbf{A}-\mathbf{C}=\begin{pmatrix}5\\-3\end{pmatrix} \tag{13}$$

$$\implies (\mathbf{A}-\mathbf{C})^T=\begin{pmatrix}5 & -3\end{pmatrix} \tag{14}$$

$$\mathbf{B}-\mathbf{D}=\begin{pmatrix}-3\\-5\end{pmatrix} \tag{15}$$

Let the angle between the diagonals of the rectangle be α
Now Consider the inner product of the diagonals of rectangle $\mathbf{A}-\mathbf{C}$ and $\mathbf{B}-\mathbf{D}$

$$\cos\alpha=\frac{\left(A-C\right)^T\left(B-D\right)}{\|\mathbf{A}-\mathbf{C}\|\|\mathbf{B}-\mathbf{D}\|}=\frac{\begin{pmatrix}5 & -3\end{pmatrix}\begin{pmatrix}-3\\-5\end{pmatrix}}{\sqrt{34}\sqrt{34}} \tag{16}$$

$$\cos\alpha=0 \tag{17}$$

$$\cos\alpha=90^\circ \tag{18}$$

Property:
Rectangle with diagonals at right angle is a square
Hence given points forms a square

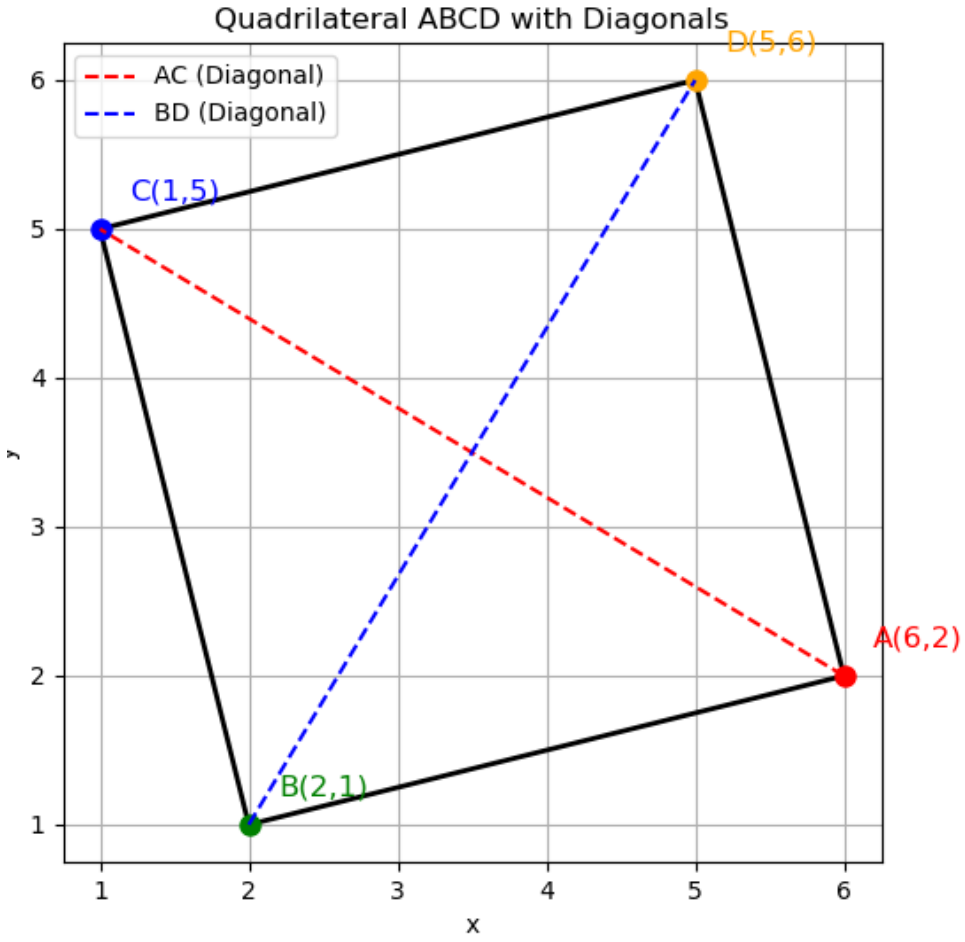
Quadrilateral ABCD with Diagonals
AC (Diagonal)
BD (Diagonal)
D(5,6)
C(1,5)
A(6,2)
B(2,1)
x
y
1
2
3
4
5
6

Fig. 1