

2.10.15

RATHLAVATH JEEVAN -AI25BTECH11026

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# Question

The number of vectors of unit length perpendicular to vectors

$$\mathbf{a} = (1, 1, 0) \quad \text{and} \quad \mathbf{b} = (0, 1, 1) \quad (1)$$

is

- (a) one    (b) two    (c) three    (d) infinite    (e) None of these    (2)

# Theoretical Solution

Let

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}. \quad (3)$$

A vector  $\mathbf{x}$  perpendicular to both  $\mathbf{a}$  and  $\mathbf{b}$  satisfies

$$\mathbf{a}^T \mathbf{x} = 0 \quad (4)$$

$$\mathbf{b}^T \mathbf{x} = 0 \quad (5)$$

$$\begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix}^T \mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (6)$$

From row reduction

$$\mathbf{x} = \lambda \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}. \quad (7)$$

# Theoretical Solution

## Solution:

Thus a direction vector is

$$\mathbf{n} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, \quad \|\mathbf{n}\| = \sqrt{3}. \quad (8)$$

Hence the *unit* vectors perpendicular to both **a** and **b** are

$$\mathbf{u} = \pm \frac{1}{\sqrt{3}} \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}. \quad (9)$$

Therefore, the number of such unit vectors is  $\boxed{2}$ .

# C Code

```
#include <stdio.h>
#include <math.h>

typedef struct { double x, y, z; } Vec;

Vec cross(Vec a, Vec b) {
    Vec c = {
        a.y*b.z - a.z*b.y,
        a.z*b.x - a.x*b.z,
        a.x*b.y - a.y*b.x
    };
    return c;
}

double norm(Vec v) {
    return sqrt(v.x*v.x + v.y*v.y + v.z*v.z);
}
```

# C Code

```
Vec scale(Vec v, double s) {
    Vec r = { v.x * s, v.y * s, v.z * s };
    return r;
}

int main(void) {
    // Given vectors
    Vec a = {1, 1, 0};
    Vec b = {0, 1, 1};

    // Vector perpendicular to both is a * b
    Vec n = cross(a, b);
    double m = norm(n);

    // If cross product is zero, vectors are parallel ->
    // infinitely many unit normals
    const double EPS = 1e-12;
```

```
if (m < EPS) {  
    printf("Number of unit vectors perpendicular to both:  
        infinite\n");  
    return 0;  
}  
  
// Two unit vectors:+- (a * b) / ||a * b||  
Vec u = scale(n, 1.0 / m);  
Vec v = scale(u, -1.0);  
  
printf("Number of unit vectors perpendicular to both: 2\n");  
printf("u1 = (%.6f, %.6f, %.6f)\n", u.x, u.y, u.z);  
printf("u2 = (%.6f, %.6f, %.6f)\n", v.x, v.y, v.z);  
  
return 0;  
}
```

# Python Code

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Given vectors
a = np.array([1, 1, 0])
b = np.array([0, 1, 1])

# Cross product gives a vector perpendicular to both
v = np.cross(a, b)
v = v / np.linalg.norm(v) # Unit vector

# The two perpendicular unit vectors are +-v
v1 = v
v2 = -v
```



# Python Code

```
# Create figure
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Plot given vectors
ax.quiver(0, 0, 0, a[0], a[1], a[2], color='r', label='a =
(1,1,0)')
ax.quiver(0, 0, 0, b[0], b[1], b[2], color='g', label='b =
(0,1,1)')

# Plot perpendicular unit vectors
ax.quiver(0, 0, 0, v1[0], v1[1], v1[2], color='b', label='Unit
perp vector +v')
ax.quiver(0, 0, 0, v2[0], v2[1], v2[2], color='orange', label='
Unit perp vector -v')
```

```
# Set labels
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('Unit vectors perpendicular to a and b')
ax.legend()

# Save image
plt.savefig("perpendicular_vectors.png")
plt.show()
```

## Unit vectors perpendicular to a and b

