4.8.23

AI25BTECH11027 - NAGA BHUVANA

Question:
Find the values of λ,for which the distance of point $(2, 1, \lambda)$ from plane $3x + 5y + 4z = 11$ is $2\sqrt{2}$ units.
Solution:
The normal vector of the plane is $\begin{pmatrix}3\\5\\4\end{pmatrix}$ and $\mathbf{P} = \begin{pmatrix}2\\1\\\lambda\end{pmatrix}$
The equation of the plane be $\mathbf{n}^T\mathbf{x} = 1$

$$distance = \frac{|\mathbf{n}^T\mathbf{p} - 1|}{\|\mathbf{n}\|} \tag{1}$$

$$2\sqrt{2} = \frac{|\begin{pmatrix}3 & 5 & 4\end{pmatrix}\begin{pmatrix}2\\1\\\lambda\end{pmatrix} - 1|}{5\sqrt{2}} \tag{2}$$

$$\frac{|10 + 4\lambda|}{5\sqrt{2}} = 2\sqrt{2} \tag{3}$$

$$|10 + 4\lambda| = 20 \tag{4}$$

$$10 + 4\lambda = 20 \quad \text{or} \quad 10 + 4\lambda = -20 \tag{5}$$

$$4\lambda = 10 \quad \text{or} \quad 4\lambda = -30 \tag{6}$$

$$\lambda = \frac{5}{2} \quad \text{or} \quad -\frac{15}{2} \tag{7}$$

$\therefore$The values of $\lambda = \frac{5}{2}$ or $-\frac{15}{2}$