4.11.27

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Question

Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$

Theoretical Solution

Let the given points be P(4,-3,-4) and Q(3,-2,2) then the direction vector along PQ be d,

$$\mathbf{d} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ -3 \\ -4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \tag{1}$$

equation of line passing through P,Q be

$$r(t) = r_0 + td \tag{2}$$

where t is a parameter

$$\mathbf{r(t)} = \begin{pmatrix} 4 \\ -3 \\ -4 \end{pmatrix} + t \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \tag{3}$$

Let the given plane equation be

$$\mathbf{n}^T\mathbf{x} = c \tag{4}$$

Theoretical Solution

where,

$$\mathbf{n} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \tag{5}$$

$$c = 6 \tag{6}$$

Consider a point with parameter t_1 which is the intersection point then, it satisfies line equation and plane equation

$$\mathbf{r(t_1)} = \begin{pmatrix} 4 \\ -3 \\ -4 \end{pmatrix} + t_1 \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \tag{7}$$

Theoretical Solution

Substitute this point in the plane equation

$$\mathbf{n}^T \mathbf{r}_{\mathbf{t}_1} = c \tag{8}$$

$$\begin{pmatrix} 2 & 1 & 1 \end{pmatrix} \left(\begin{pmatrix} 4 \\ -3 \\ -4 \end{pmatrix} + t_1 \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \right) = 6 \tag{9}$$

$$1 + t_1 (5) = 6 \tag{10}$$

$$5t_1 = 5 \tag{11}$$

$$t_1 = 1 \tag{12}$$

Theoretical Solution

then the intersection point be,

$$\mathbf{r_{t_1}} = \begin{pmatrix} 4 \\ -3 \\ -4 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \tag{13}$$

$$\mathbf{r_{t_1}} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} \tag{14}$$

C Code- Computing the unit vector

```c
#include <stdio.h>
void find_intersection(double *result) {
    double P[3] = {4, -3, -4};
    double Q[3] = {3, -2, 2};
    double n[3] = {2, 1, 1};
    double c = 6;
    double d[3];
    for (int i = 0; i < 3; i++) d[i] = Q[i] - P[i];

    double num = c - (n[0]*P[0] + n[1]*P[1] + n[2]*P[2]);
    double den = n[0]*d[0] + n[1]*d[1] + n[2]*d[2];
    double t = num / den;

    for (int i = 0; i < 3; i++)
        result[i] = P[i] + t * d[i];
}
```

Python Code using shared output

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Load the compiled C library
lib = ctypes.CDLL('./4.11.27.so') # or './intersection.dll' on
    Windows

# Prepare result array
result = (ctypes.c_double * 3)()
lib.find_intersection(result)

intersection = np.array([result[0], result[1], result[2]])
print(Intersection point:, intersection)
```

Python Code using shared output

```
    # Define line points
P = np.array([4, -3, -4])
Q = np.array([3, -2, 2])
line_points = np.array([P, Q])

# Plane setup
xx, yy = np.meshgrid(np.linspace(0, 5, 10), np.linspace(-5, 5,
    10))
zz = 6 - 2*xx - yy # from plane equation 2x + y + z = 6

# Plot
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
```

Python Code using shared output

```
        ax.plot_surface(xx, yy, zz, alpha=0.5, color='cyan')
ax.plot(line_points[:,0], line_points[:,1], line_points[:,2],
    color='red', label='Line PQ')
ax.scatter(intersection[0], intersection[1], intersection[2],
    color='blue', s=50, label='Intersection')

ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('Plot of the Intersection point ')
ax.legend()
plt.show()
```

Plot by python using shared output from c

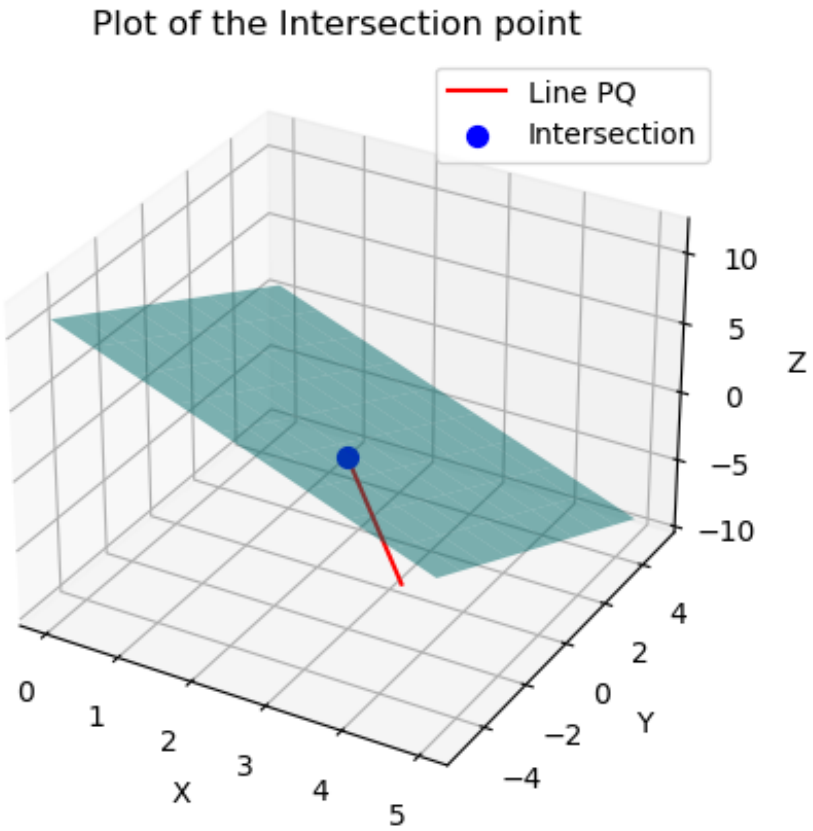

Figure: Plot of the Intersection point

Python code for the plot

```
    import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Given data
P = np.array([4, -3, -4]) # Point 1
Q = np.array([3, -2, 2]) # Point 2
# Plane: 2x + y + z = 6
n = np.array([2, 1, 1]) # Normal vector
c = 6
# Direction vector of the line
d = Q - P
# Parameter t for intersection point
t = (c - np.dot(n, P)) / np.dot(n, d)
# Intersection point
R = P + t * d
print(Intersection point:, R)
```

Python code for the plot

```
# --- Plotting the line, plane, and intersection point ---
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Generate the plane surface
xx, yy = np.meshgrid(np.linspace(0, 5, 10), np.linspace(-5, 5,
    10))
zz = 6 - 2*xx - yy # from plane equation 2x + y + z = 6

# Plot the plane
ax.plot_surface(xx, yy, zz, alpha=0.5, color='cyan')

# Plot the line passing through P and Q
line_points = np.array([P, Q])
ax.plot(line_points[:,0], line_points[:,1], line_points[:,2],
        color='red', label='Line PQ')
```

Python code for the plot

```
# Plot the intersection point
ax.scatter(R[0], R[1], R[2], color='blue', s=50, label='
    Intersection Point')

# Annotate points
ax.text(P[0], P[1], P[2], 'P(4,-3,-4)', color='black')
ax.text(Q[0], Q[1], Q[2], 'Q(3,-2,2)', color='black')
ax.text(R[0], R[1], R[2], f'R({R[0]:.1f},{R[1]:.1f},{R[2]:.1f})',
     color='blue')

# Labels
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title('Intersection of Line and Plane')

ax.legend()
plt.show()
```

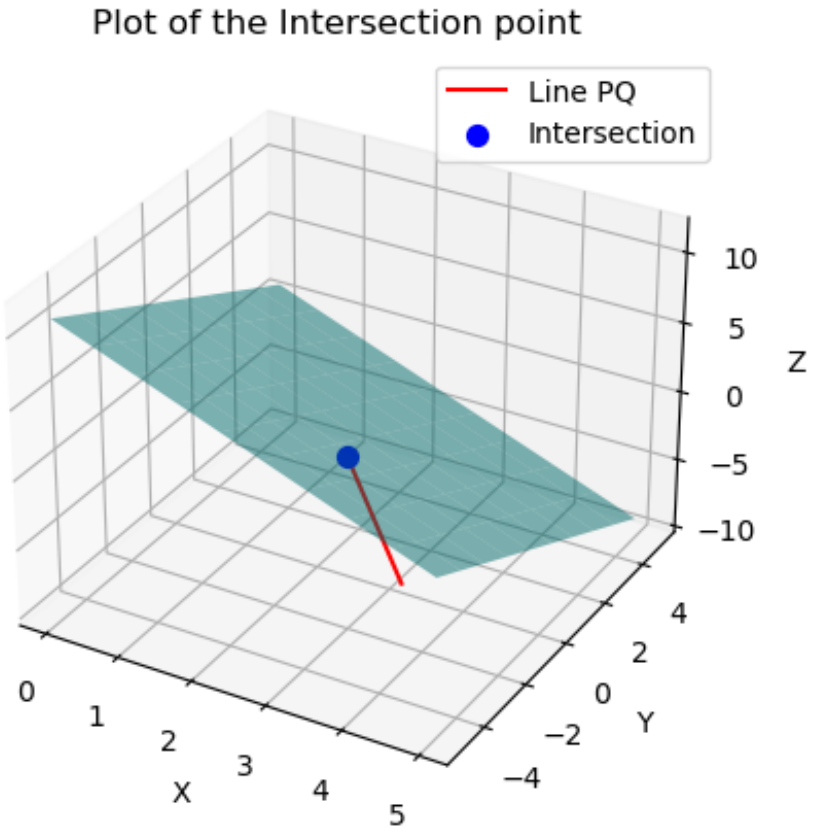

Figure: Plot of the Intersection point