2.10.73

EE25BTECH11045 - P.Navya Priya

Question:

Let $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ be unit vectors. Suppose that $\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{C} = 0$, and that the angle between $\mathbf{B}$ and $\mathbf{C}$ is $\frac{\pi}{6}$. Then $\mathbf{A} = \pm 2(\mathbf{B} \times \mathbf{C})$

Solution:

Let us solve the given equation theoretically and then verify the solution computationally.

Since $\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{C} = 0$, it follows that $\mathbf{A}$ is perpendicular to both $\mathbf{B}$ and $\mathbf{C}$. Therefore A is parallel(or anti-parallel) to the cross product of $\mathbf{B}$ and $\mathbf{C}$.

$$\mathbf{A} = \lambda(\mathbf{B} \times \mathbf{C}) \tag{1}$$

From the given question,

$$\mathbf{B}^\top \mathbf{C} = \cos\left(\frac{\pi}{6}\right) \tag{2}$$

We know that,

$$\mathbf{B}^\top \mathbf{C}^2 + \|\mathbf{B} \times \mathbf{C}\|^2 = \|\mathbf{B}\|^2 \|\mathbf{C}\|^2 \tag{3}$$

$$\implies \|\mathbf{B} \times \mathbf{C}\|^2 = \frac{1}{4} \tag{4}$$

$$\implies \|\mathbf{B} \times \mathbf{C}\| = \frac{1}{2} \tag{5}$$

As $\mathbf{A}$ is a unit vector,
from(1)

$$\|\mathbf{A}\| = \|\lambda(\mathbf{B} \times \mathbf{C})\| \tag{6}$$

$$1 = |\lambda| \frac{1}{2} \tag{7}$$

Hence

$$\lambda = \pm 2 \tag{8}$$

$$\therefore \mathbf{A} = \pm 2(\mathbf{B} \times \mathbf{C}) \tag{9}$$

To verify the solution computationally let us assume the vectors **B** and **C** as

$$\mathbf{B} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$$