9.4.25

EE25BTECH11013 - Bhargav

Question:
Find the roots of the quadratic equation graphically.

$$5x^2 - 6x - 2 = 0 \tag{0.1}$$

Solution:

$$y = 5x^2 - 6x - 2 = 0 \tag{0.2}$$

This equation can be represented as the conic

$$\mathbf{x}^{\mathbf{T}}\mathbf{V}\mathbf{x} + 2\mathbf{u}^{\mathbf{T}}\mathbf{x} + f = 0 \tag{0.3}$$

$$\mathbf{V} = \begin{pmatrix} 5 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}, f = -2 \tag{0.4}$$

To find the roots, we find the points of intersection of the conic with the x-axis.

$$\mathbf{x} = \mathbf{h} + k_i\mathbf{m} \tag{0.5}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.6}$$

The value of k_i can be found out by solving the line and conic equation

$$(\mathbf{h} + k_i\mathbf{m})^\top \mathbf{V}(\mathbf{h} + k_i\mathbf{m}) + 2\mathbf{u}^\top(\mathbf{h} + k_i\mathbf{m}) + f = 0 \tag{0.7}$$

$$\implies k_i^2\mathbf{m}^\top\mathbf{V}\mathbf{m} + 2k_i\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) + \mathbf{h}^\top\mathbf{V}\mathbf{h} + 2\mathbf{u}^\top\mathbf{h} + f = 0 \tag{0.8}$$

$$\text{or, } k_i^2\mathbf{m}^\top\mathbf{V}\mathbf{m} + 2k_i\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) + g(\mathbf{h}) = 0 \tag{0.9}$$

Solving the above quadratic gives the equation

$$k_i = \frac{1}{\mathbf{m}^\top\mathbf{V}\mathbf{m}}\left(-\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top\mathbf{V}\mathbf{m})}\right) \tag{0.10}$$

Substituting the values in the above equation gives

$$\therefore k_i = \frac{3}{5} \pm \frac{\sqrt{19}}{5} \tag{0.11}$$

$$\implies k_1 = \frac{3}{5} + \frac{\sqrt{19}}{5}, k_2 = \frac{3}{5} - \frac{\sqrt{19}}{5} \tag{0.12}$$

$$\therefore \mathbf{x} = \mathbf{h} + k_i \mathbf{m} = \begin{pmatrix} \frac{3}{5} + \frac{\sqrt{19}}{5} \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{3}{5} - \frac{\sqrt{19}}{5} \\ 0 \end{pmatrix} \tag{0.13}$$