7.4.17

EE25BTECH11010 - Arsh Dhoke

Question:
If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along the diameter of a circle of circumference 10π, then the equation of the circle is (2004)

1) $x^2 + y^2 + 2x - 2y - 23 = 0$
2) $x^2 + y^2 - 2x - 2y - 23 = 0$
3) $x^2 + y^2 + 2x + 2y - 23 = 0$
4) $x^2 + y^2 - 2x + 2y - 23 = 0$

Solution:
The equation of the circle is:(**V** is an identity matrix of order = 2)

$$\mathbf{x^T V x} + 2\mathbf{u^T x} + f = 0 \tag{4.1}$$

$$2x + 3y + 1 = 0 \tag{4.2}$$

$$3x - y - 4 = 0 \tag{4.3}$$

Line Equation	$\mathbf{n}_i$	c_i
$\mathbf{n}_1^T\mathbf{x} + c_1 = 0$	$\begin{pmatrix}2\\3\end{pmatrix}$	1
$\mathbf{n}_2^T\mathbf{x} + c_2 = 0$	$\begin{pmatrix}3\\-1\end{pmatrix}$	-4

$$\begin{pmatrix}2 & 3\\3 & -1\end{pmatrix}\mathbf{x} = \begin{pmatrix}-1\\4\end{pmatrix} \tag{4.4}$$

$$\tag{4.5}$$

In augmented form:

$$\left(\begin{array}{cc|c}2 & 3 & -1\\3 & -1 & 4\end{array}\right) \tag{4.6}$$

Performing row operations:

$$\left(\begin{array}{cc|c} 2 & 3 & -1 \\ 3 & -1 & 4 \end{array}\right) \xrightarrow[R_2 \to R_2 - \frac{3}{2}R_1]{} \left(\begin{array}{cc|c} 2 & 3 & -1 \\ 0 & -\frac{11}{2} & \frac{11}{2} \end{array}\right) \tag{4.7}$$

$$\xrightarrow[R_2 \to \frac{2}{-11}R_2]{} \left(\begin{array}{cc|c} 2 & 3 & -1 \\ 0 & 1 & -1 \end{array}\right) \tag{4.8}$$

$$\xrightarrow[R_1 \to R_1 - 3R_2]{} \left(\begin{array}{cc|c} 2 & 0 & 2 \\ 0 & 1 & -1 \end{array}\right) \tag{4.9}$$

$$\xrightarrow[R_1 \to \frac{1}{2}R_1]{} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right) \tag{4.10}$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{4.11}$$

$$\text{Hence, the centre of the circle is } \mathbf{c} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{4.12}$$

Given circumference $10\pi \Rightarrow r = 5 \Rightarrow r^2 = 25$.

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{4.13}$$

$$\mathbf{V} = \mathbf{I}, \quad \mathbf{c} = -\mathbf{u} \tag{4.14}$$

$$\Rightarrow \mathbf{u} = -\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{4.15}$$

$$f = \mathbf{c}^T\mathbf{V}\mathbf{c} - r^2 = 2 - 25 = -23 \tag{4.16}$$

Final equation of the circle:

$$(\mathbf{x})^T\mathbf{I}\mathbf{x} + 2\begin{pmatrix} -1 & 1 \end{pmatrix}\mathbf{x} - 23 = 0 \tag{4.17}$$

$$(\mathbf{x})^T\mathbf{x} + 2\begin{pmatrix} -1 & 1 \end{pmatrix}\mathbf{x} - 23 = 0 \tag{4.18}$$

Put $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$, option d will be the answer.

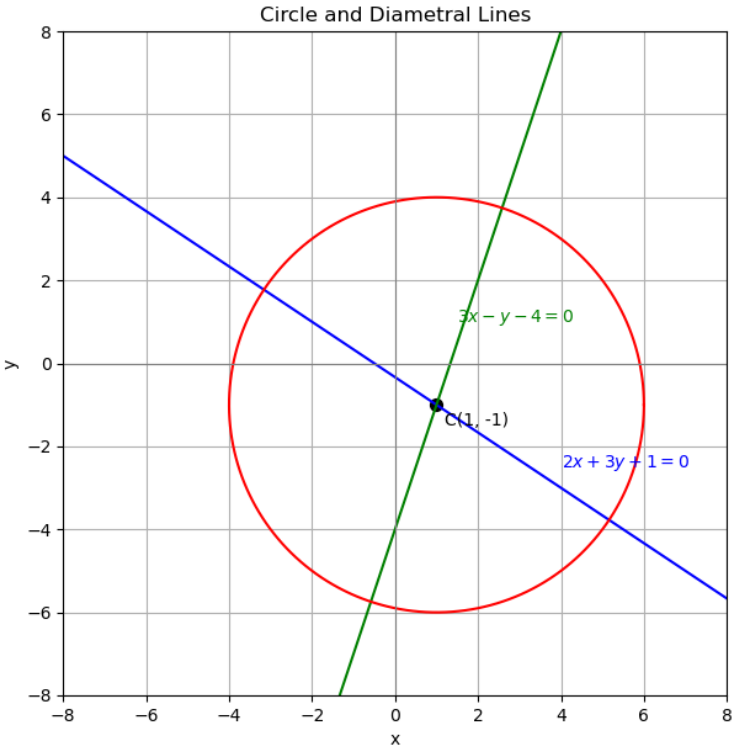
Circle and Diametral Lines
8
6
4
2
0
−2
−4
−6
−8
y
−8
−6
−4
−2
0
2
4
6
8
x
3x − y − 4 = 0
C(1, -1)
2x + 3y + 1 = 0

Fig. 4.1: Graph