12.443

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Question:

The positive eigenvalue of $\begin{pmatrix} 2 & 1 \\ 5 & 2 \end{pmatrix}$ is

Solution

$$\mathbf{Ax} = \lambda\mathbf{x} \implies (\mathbf{A} - \lambda\mathbf{I})\,\mathbf{x} = 0 \tag{1}$$

$$|\mathbf{A} - \lambda\mathbf{I}| = 0 \tag{2}$$

where λ is the eigenvalue, $\mathbf{x}$ is the eigenvector, $\mathbf{I}$ is the identity matrix

$$\left|\begin{pmatrix} 2-\lambda & 1 \\ 5 & 2-\lambda \end{pmatrix}\right| = 0 \tag{3}$$

$$(2-\lambda)^2 - 5 = 0 \implies \lambda^2 - 4\lambda - 1 = 0 \tag{4}$$

Solution

Using the quadratic formula,

$$\lambda = \frac{4 \pm \sqrt{16+4}}{2} \tag{5}$$

$$\lambda = 2 \pm \sqrt{5} \tag{6}$$

The positive eigenvalue of $\begin{pmatrix} 2 & 1 \\ 5 & 2 \end{pmatrix}$ is $2 + \sqrt{5}$

C Code

```c
#include <math.h>

double positiveeigenvalue(double a, double b, double c, double d) {
    double trace = a + d;
    double det = a*d - b*c;
    double discriminant = trace*trace - 4*det;
    double lambda1 = (trace + sqrt(discriminant)) / 2.0;
    double lambda2 = (trace - sqrt(discriminant)) / 2.0;

    if (lambda1 > 0) {
        return lambda1;
    }
    return lambda2;
}
```

Python + C Code

```python
import ctypes

lib = ctypes.CDLL("./code.so")

lib.positiveeigenvalue.argtypes = [ctypes.c_double, ctypes.c_double,
                                    ctypes.c_double, ctypes.c_double]
lib.positiveeigenvalue.restype = ctypes.c_double
a, b, c, d = 2.0, 1.0, 5.0, 2.0

pos_eig = lib.positiveeigenvalue(a, b, c, d)
print("The positive eigenvalue is:", pos_eig)
```

Python Code

```python
import numpy as np

a = np.array([[2,1], [5,2]])
x,y = np.linalg.eig(a)
if(x[0]>0):
    print("The positive eigen value: ", x[0])
else:
    print("The positive eigen value: ", x[1])
```