

# 4.12.17

AI25BTECH11012 - GARIGE UNNATHI

## Question:

**P1,P2** are points on either of the two lines  $y - \sqrt{3}|x| = 2$  at a distance of 5 units from their point of intersection. Find the coordinates of the foot of the perpendiculars drawn from **P1,P2** on the bisector of the angle between the given lines.

## Solution:

The equation of the lines is :

$$y - \sqrt{3}x = (-\sqrt{3} \ 1) \begin{pmatrix} x \\ y \end{pmatrix} = 2 \quad (0.1)$$

$$y + \sqrt{3}x = (\sqrt{3} \ 1) \begin{pmatrix} x \\ y \end{pmatrix} = 2 \quad (0.2)$$

Combining both the equations 0.1 and 0.2 ,we get :

$$\begin{pmatrix} -\sqrt{3} & 1 \\ \sqrt{3} & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad (0.3)$$

Solving by row reduction we get :

$$\mathbf{Q} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (0.4)$$

The equation for the point **P1** are:

$$(\sqrt{3} \ 1) \mathbf{P}_1 = 2 \quad (0.5)$$

$$\|\mathbf{P}_1 - \mathbf{Q}\| = 5 \quad (0.6)$$

The equation for the point **P2** are:

$$(-\sqrt{3} \ 1) \mathbf{P}_2 = 2 \quad (0.7)$$

$$\|\mathbf{P}_2 - \mathbf{Q}\| = 5 \quad (0.8)$$

Solving the equations we get :

$$\mathbf{P}_1 = \left( 2 + \frac{\frac{5}{2}\sqrt{3}}{2} \right) \quad (0.9)$$

$$\mathbf{P}_2 = \left( 2 - \frac{\frac{5}{2}\sqrt{3}}{2} \right) \quad (0.10)$$

The equation of the angle bisector is given by

Let us take a point  $\mathbf{P}$  on the angle bisector, substitution it in the line equations and equating the angles we get the equation :

$$\frac{|n_1\mathbf{P} - 2|}{\|n_1\|} = \frac{|n_2\mathbf{P} - 2|}{\|n_2\|} \quad (0.11)$$

$$\frac{n_1\mathbf{P} - 2}{\|n_1\|} \pm \frac{n_2\mathbf{P} - 2}{\|n_2\|} = 0 \quad (0.12)$$

solving the above equation we get locus of  $\mathbf{P}$  as two lines which are the angle bisectors :

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \mathbf{x} = 0 \quad (0.13)$$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}^T \mathbf{x} = 2 \quad (0.14)$$

Let Q be the foot of the perpendicular from P to the line

$$\mathbf{n}^T \mathbf{x} = c \quad (0.15)$$

Then :

$$\begin{pmatrix} \mathbf{m} & \mathbf{n} \end{pmatrix}^T \mathbf{Q} = \begin{pmatrix} \mathbf{m}^T \mathbf{P} \\ c \end{pmatrix} \quad (0.16)$$

solving this equation for the line  $\begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \mathbf{x} = 0$ , we get :

$$\begin{pmatrix} 0 \\ 2 + \frac{5\sqrt{3}}{2} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 2 - \frac{5\sqrt{3}}{2} \end{pmatrix} \quad (0.17)$$

and solving it for the line  $\begin{pmatrix} 0 \\ 1 \end{pmatrix}^T \mathbf{x} = 2$ , we get :

$$\begin{pmatrix} \frac{5}{2} \\ 2 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -\frac{5}{2} \\ 2 \end{pmatrix} \quad (0.18)$$

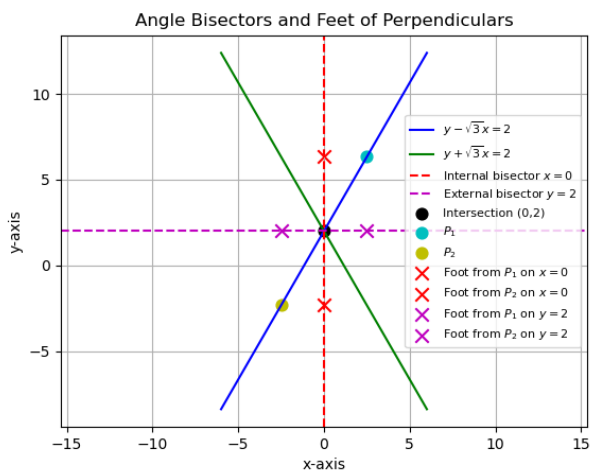


Fig. 0.1