Matgeo Presentation - Problem 12.485

ee25btech11056 - Suraj.N

October 2, 2025

Problem Statement

Let

$$\mathbf{M} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$$

Which of the following is correct

(1) Rank of $\mathbf{M}$ is 1 and $\mathbf{M}$ is diagonalizable

(2) Rank of $\mathbf{M}$ is 2 and $\mathbf{M}$ is diagonalizable

(3) 1 is the only eigenvalue and $\mathbf{M}$ is diagonalizable

(4) 1 is the only eigenvalue and $\mathbf{M}$ is not diagonalizable

Data

Name	**Value**
M	$\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$

Table : Matrix

Solution

First convert $\mathbf{M}$ into echelon form by applying row reduction

$$\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \xleftrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{0.1}$$

From the echelon form we see that there is one nonzero row, hence

$$\mathrm{rank}(\mathbf{M}) = 1 \tag{0.2}$$

Next find the eigenvalues. Because $\mathbf{M}$ is upper triangular, its eigenvalues are the diagonal entries:

$$\lambda_1 = 0 \qquad\qquad \lambda_2 = 1 \tag{0.3}$$

Now find eigenvectors by solving

$$(\mathbf{M} - \lambda \mathbf{I})\mathbf{v} = \mathbf{0} \tag{0.4}$$

Solution

For $\lambda = 0$ solve

$$\mathbf{Mv} = \mathbf{0} \tag{0.5}$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \mathbf{v} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.6}$$

$$\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.7}$$

This gives

$$y = 0 \tag{0.8}$$

And x can be anything
Thus an eigenvector for $\lambda = 0$ is

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.9}$$

Solution

For $\lambda = 1$ solve

$$(\mathbf{M} - \mathbf{I})\mathbf{v} = \mathbf{0} \tag{0.10}$$

$$\begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} \mathbf{v} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.11}$$

$$\begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.12}$$

This gives

$$-x + y = 0 \tag{0.13}$$

$$y = x \tag{0.14}$$

Thus an eigenvector for $\lambda = 1$ is

$$\mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.15}$$

Solution

Since the eigenvalues λ_1 and λ_2 are distinct, the matrix $\mathbf{M}$ is diagonalizable.

Form the matrix $\mathbf{P}$ with eigenvectors as columns and the diagonal matrix $\mathbf{D}$ of eigenvalues:

$$\mathbf{P} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \tag{0.16}$$

$$\mathbf{D} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{0.17}$$

Compute $\mathbf{P}^{-1}$

$$\left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right) \xleftrightarrow{R_1 \to R_1 - R_2} \left(\begin{array}{cc|cc} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1 \end{array}\right) \tag{0.18}$$

The right block gives

$$\mathbf{P}^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \tag{0.19}$$

Solution

Finally, the diagonalization:

$$\mathbf{M} = \mathbf{PDP}^{-1} \tag{0.20}$$

$$\mathbf{M} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \tag{0.21}$$

Conclusion : The matrix $\mathbf{M}$ has $rank(\mathbf{M}) = 1$ and is **diagonizable**. Therefore the correct option is (1).