

1.3.6

AI25BTECH11027 - NAGA BHUVANA

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Question:

Show that the points **A** (6, 2), **B** (2, 1), **C** (1, 5) and **D** (5, 6) are vertices of a square.

Solution:

Given that

$$\mathbf{A} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} \quad (0.1)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 - 6 \\ 1 - 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \quad (0.2)$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} 1 - 5 \\ 5 - 6 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \quad (0.3)$$

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \quad (0.4)$$

By the above property we can say that **ABCD** is a parallelogram.
Consider the inner product of the vectors **(B – A)** and **(C – B)**

$$\implies (\mathbf{B} - \mathbf{A})^T(\mathbf{C} - \mathbf{B}) = (-4)(-1) + (-1)(4) = 0 \quad (0.5)$$

Hence the angle at the vertex B is 90°

Property:

A parallelogram with one angle 90° is a rectangle

Hence the parallelogram is a rectangle

Now consider the inner product of the diagonals of the rectangle **(C – A)**
and **(D – B)**

$$\implies (\mathbf{C} - \mathbf{A})^T(\mathbf{D} - \mathbf{B}) = (-5)(3) + (3)(5) = 0 \quad (0.6)$$

Hence the angle between the diagonals of the rectangle is 90°

Property:

Rectangle with diagonals at right angle is a square

Hence given points forms a square

Graphical Representation

