

# 5.13.49

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**Question:**

If the system of equations

$$x - ky - z = 0 \quad (1)$$

$$kx - y - z = 0 \quad (2)$$

$$x + y + z = 0 \quad (3)$$

has a non-zero solution, then the possible values of  $k$  are

**Solution:**

For the given homogeneous system

$$\mathbf{Ax} = 0 \quad (4)$$

Augmented matrix of  $(\mathbf{A} \mid 0)$  is given by

$$\left( \begin{array}{ccc|c} 1 & -k & -1 & 0 \\ k & -1 & -1 & 0 \\ 1 & 1 & -1 & 0 \end{array} \right) \xrightarrow[R_3 \rightarrow R_3 - R_1]{R_2 \rightarrow R_2 - kR_1} \left( \begin{array}{ccc|c} 1 & -k & -1 & 0 \\ 0 & k^2 - 1 & k - 1 & 0 \\ 0 & 1 + k & 0 & 0 \end{array} \right) \xrightarrow{R_2 - (k-1)R_3} \left( \begin{array}{ccc|c} 1 & -k & -1 & 0 \\ 0 & k + 1 & 0 & 0 \\ 0 & 0 & k - 1 & 0 \end{array} \right) \quad (5)$$

For a non-zero solution, The rank of the matrix must be less than the number of variables  
From (5), In order to be Rank < 3

$$k + 1 = 0 \quad (\text{or}) \quad k - 1 = 0 \quad (6)$$

$$k = -1 \quad (\text{or}) \quad k = 1 \quad (7)$$