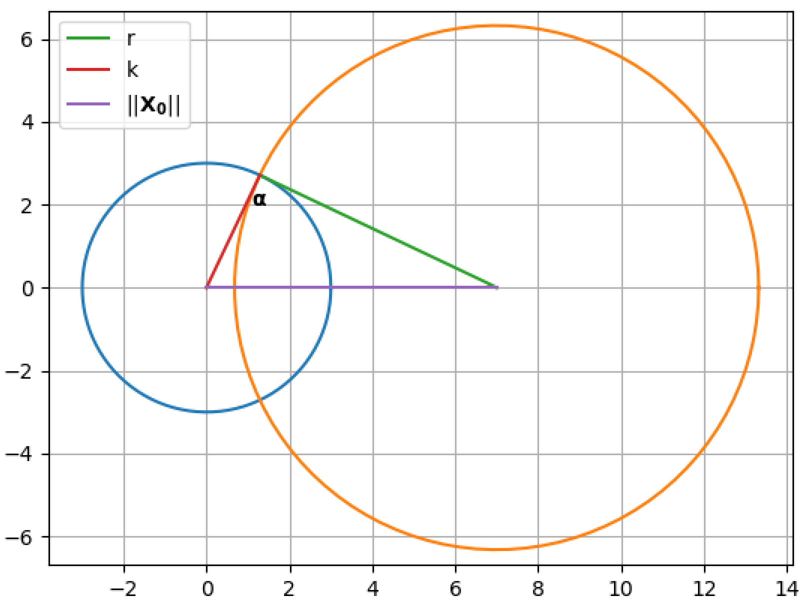
9.8.3

AI25BTECH11001 - ABHISEK MOHAPATRA

Question: If a circle is passing through the point (a, b) and it is cutting the circle $x^2 + y^2 = k^2$ orthogonally, then the equation of the locus of its centre is

Solution:

Graph:



Let center of the circle be $\mathbf{X_0}$ and radius of the circle be r. So, equation of the circle be

$$\|\mathbf{X} - \mathbf{X_0}\| = r \tag{1}$$

$$\|\mathbf{X} - \mathbf{X_0}\|^2 = r^2 \tag{2}$$

$$(\mathbf{X} - \mathbf{X_0})^\top (\mathbf{X} - \mathbf{X_0}) = r^2 \tag{3}$$

$$\|\mathbf{X}\|^2 - 2\mathbf{X_0}^\top\mathbf{X} + \|\mathbf{X_0}\|^2 - r^2 = 0 \tag{4}$$

And the other given circle be with center $\mathbf{0}$ and radius k.

As evident from the fig, for the circle to be orthogonal, $\angle\alpha = 90°$and

$$r^2 + k^2 = \|\mathbf{X_0} - 0\|^2 = \|\mathbf{X_0}\|^2 \tag{5}$$

substituing in the equation,

$$\|\mathbf{X}\|^2 - 2\mathbf{X_0}^\top\mathbf{X} + k^2 = 0 \tag{6}$$

Putting th given point $\beta = \begin{pmatrix} a \\ b \end{pmatrix}$

$$\|\beta\|^2 - 2\mathbf{X_0}^\top \beta + k^2 = 0 \tag{7}$$

$$a^2 + b^2 - 2(ax + by) + k^2 = 0 \tag{8}$$

So, option (a) is correct. Graph: