

4.13.38

EE25BTECH11033 - Kevin

Question:

Let PS be the median of the triangle with vertices $\mathbf{P}(2, 2)$, $\mathbf{Q}(6, -1)$ and $\mathbf{R}(7, 3)$. The equation of the line passing through $(1, -1)$ and parallel to PS is

$$1) 4x + 7y + 3 = 0$$

$$3) 4x - 7y - 11 = 0$$

$$2) 2x - 9y - 11 = 0$$

$$4) 2x + 9y + 7 = 0$$

Solution:

Given the points,

$$\mathbf{P} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 6 \\ -1 \end{pmatrix} \quad \mathbf{R} = \begin{pmatrix} 7 \\ 3 \end{pmatrix} \quad (1)$$

\mathbf{S} is the midpoint of the line segment joining points \mathbf{Q} and \mathbf{R} .

If \mathbf{S} divides QR in the ratio $k : 1$,

$$\mathbf{S} = \frac{k\mathbf{R} + \mathbf{Q}}{k + 1} \quad (2)$$

where,

$$k = 1 \quad (3)$$

$$\mathbf{S} = \frac{\mathbf{R} + \mathbf{Q}}{2} \quad (4)$$

$$\Rightarrow \mathbf{S} = \begin{pmatrix} 13/2 \\ 1 \end{pmatrix} \quad (5)$$

As \mathbf{P} and \mathbf{S} are collinear,

$$\mathbf{n}^\top \mathbf{P} = c \quad (6)$$

$$\mathbf{n}^\top \mathbf{S} = c \quad (7)$$

which can be expressed as

$$(\mathbf{P} \quad \mathbf{S})^\top \mathbf{n} = c \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (8)$$

$$\equiv (\mathbf{P} \quad \mathbf{S})^\top \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (9)$$

$$\Rightarrow \begin{pmatrix} 2 & 2 \\ 13/2 & 1 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (10)$$

$$\Rightarrow \left(\begin{array}{cc|c} 2 & 2 & 1 \\ 13/2 & 1 & 1 \end{array} \right) \xleftrightarrow{R_2 \rightarrow 2R_2} \left(\begin{array}{cc|c} 2 & 2 & 1 \\ 13 & 2 & 2 \end{array} \right) \quad (11)$$

$$\xleftrightarrow{R_2 \rightarrow 2R_2 - 13R_1} \left(\begin{array}{cc|c} 2 & 2 & 1 \\ 0 & -22 & -9 \end{array} \right) \xleftrightarrow{R_1 \rightarrow 1/2R_1} \left(\begin{array}{cc|c} 1 & 1 & 1/2 \\ 0 & -22 & -9 \end{array} \right) \quad (12)$$

$$\xleftrightarrow{R_2 \rightarrow -1/22R_1} \left(\begin{array}{cc|c} 1 & 1 & 1/2 \\ 0 & 1 & 9/22 \end{array} \right) \xleftrightarrow{R_1 \rightarrow R_1 - R_2} \left(\begin{array}{cc|c} 1 & 0 & 1/11 \\ 0 & 1 & 9/22 \end{array} \right) \quad (13)$$

$$\Rightarrow n = \begin{pmatrix} 1/11 \\ 9/22 \end{pmatrix} \quad (14)$$

\therefore The equation of the line passing through $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and parallel to PS is given by

$$\mathbf{n}^\top \left(\mathbf{x} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right) = 0 \quad (15)$$

$$\begin{pmatrix} 1/11 & 9/22 \end{pmatrix} \begin{pmatrix} x-1 \\ y+1 \end{pmatrix} = 0 \quad (16)$$

$$\Rightarrow 2x + 9y + 7 = 0 \quad (17)$$

