2.4.31

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Question

Check if the point $\mathbf{A}(2, 7)$ lies on the perpendicular bisector of line segment joining the points $\mathbf{P}(6, 5)$ and $\mathbf{Q}(0, -4)$.

Solution

The equation of the perpendicular bisector of PQ is

$$\left(\mathbf{A} - \frac{\mathbf{P}+\mathbf{Q}}{2}\right)^{\top} (\mathbf{P}-\mathbf{Q}) = 0 \tag{1}$$

The given points are,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 7 \end{pmatrix} \tag{2}$$

$$\mathbf{P} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} \tag{3}$$

$$\mathbf{Q} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \tag{4}$$

$$\mathbf{P}-\mathbf{Q} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ -4 \end{pmatrix} \tag{5}$$

Solution

$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix} 6 \\ 9 \end{pmatrix} \tag{7}$$

$$\frac{\mathbf{P} + \mathbf{Q}}{2} = \frac{\begin{pmatrix} 6 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix}}{2} \tag{8}$$

$$\frac{\mathbf{P} + \mathbf{Q}}{2} = \frac{\begin{pmatrix} 6 \\ 1 \end{pmatrix}}{2} \tag{9}$$

$$\frac{\mathbf{P} + \mathbf{Q}}{2} = \begin{pmatrix} 3 \\ 0.5 \end{pmatrix} \tag{10}$$

$$\mathbf{A} - \frac{\mathbf{P} + \mathbf{Q}}{2} = \begin{pmatrix} 2 \\ 7 \end{pmatrix} - \begin{pmatrix} 3 \\ 0.5 \end{pmatrix} \tag{11}$$

Solution

$$\mathbf{A} - \frac{\mathbf{P}+\mathbf{Q}}{2} = \begin{pmatrix} -1 \\ 6.5 \end{pmatrix} \tag{12}$$

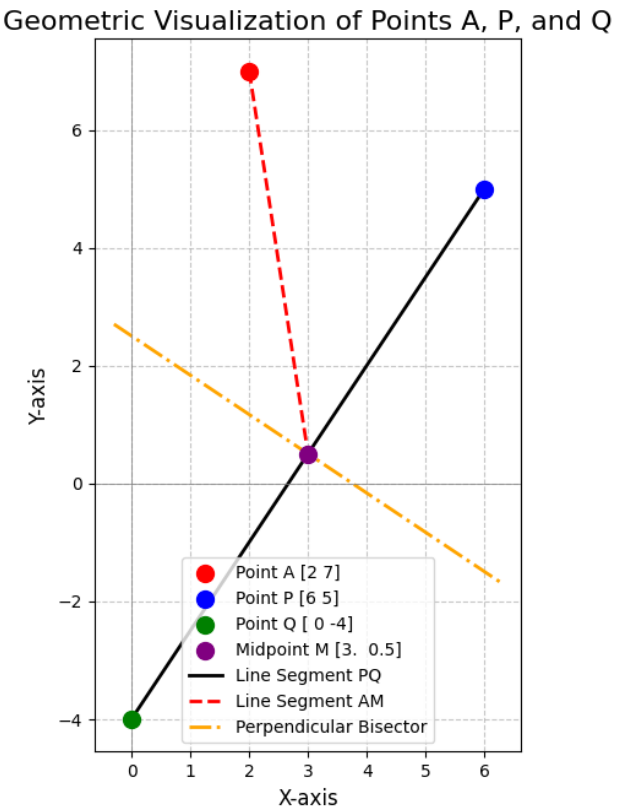
$$\left(\mathbf{A} - \frac{\mathbf{P}+\mathbf{Q}}{2}\right)^{\top} (\mathbf{P}-\mathbf{Q}) = \begin{pmatrix} -1 & 6.5 \end{pmatrix} \begin{pmatrix} 6 \\ 9 \end{pmatrix} \tag{13}$$

$$\left(\mathbf{A} - \frac{\mathbf{P}+\mathbf{Q}}{2}\right)^{\top} (\mathbf{P}-\mathbf{Q}) = (-1)(6) + (6.5)(9) \tag{14}$$

$$\left(\mathbf{A} - \frac{\mathbf{P}+\mathbf{Q}}{2}\right)^{\top} (\mathbf{P}-\mathbf{Q}) = 52.5 \neq 0 \tag{15}$$

The equation of perpendicular bisector is not satisfied. Therefore, point **A** does not lie on the perpendicular bisector of line segment joining the points **P** and **Q**.

Plot


Geometric Visualization of Points A, P, and Q
Y-axis
X-axis
Point A [2 7]
Point P [6 5]
Point Q [0 -4]
Midpoint M [3. 0.5]
Line Segment PQ
Line Segment AM
Perpendicular Bisector