2.8.19

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Problem Statement

Suppose for some non-zero vector $\mathbf{r}$ we have:

$$\mathbf{r} \cdot \mathbf{a} = 0, \quad \mathbf{r} \cdot \mathbf{b} = 0, \quad \mathbf{r} \cdot \mathbf{c} = 0$$

Show that the scalar triple product $(\mathbf{a}\ \mathbf{b}\ \mathbf{c}) = 0$.

Step 1: Write as Matrix Equation

The three scalar equations can be written as:

$$\mathbf{r}^\top \mathbf{a} = 0 \tag{1}$$

$$\mathbf{r}^\top \mathbf{b} = 0 \tag{2}$$

$$\mathbf{r}^\top \mathbf{c} = 0 \tag{3}$$

Stacked together:

$$\begin{pmatrix} \mathbf{a}^\top \\ \mathbf{b}^\top \\ \mathbf{c}^\top \end{pmatrix} \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

Step 2: Define the Matrix

Define the 3×3 matrix with columns $\mathbf{a}, \mathbf{b}, \mathbf{c}$:

$$A = \begin{bmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{bmatrix} \tag{5}$$

Then the stacked matrix equation becomes:

$$A^\top \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{6}$$

Step 3: Deduce Singularity

Since $\mathbf{r} \neq \mathbf{0}$ and

$$A^\top \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix},$$

the matrix $A^\top$ is singular. Therefore:

$$\det(A^\top) = 0 \tag{7}$$

But $\det(A^\top) = \det(A)$, and the determinant of A is the scalar triple product:

$$(\mathbf{a}\ \mathbf{b}\ \mathbf{c}) = \det\begin{bmatrix}\mathbf{a} & \mathbf{b} & \mathbf{c}\end{bmatrix} = \det(A) = 0 \tag{8}$$

Conclusion

$$\boxed{(\mathbf{a}\ \mathbf{b}\ \mathbf{c}) = 0}$$

This completes the proof **non-zero $\mathbf{r}$ orthogonal to all three vectors**.

C Code

```c
#include <stdio.h>

double scalar_triple(double a[3], double b[3], double c[3]) {
    return a[0]*(b[1]*c[2] - b[2]*c[1])
         - a[1]*(b[0]*c[2] - b[2]*c[0])
         + a[2]*(b[0]*c[1] - b[1]*c[0]);
}
```

Python code through shared output

```python
import ctypes
import matplotlib.pyplot as plt
import numpy as np
from mpl_toolkits.mplot3d import Axes3D
# Load the shared library
lib = ctypes.CDLL(./libstp.so)
lib.scalar_triple.restype = ctypes.c_double
# Define vectors
a = (ctypes.c_double * 3)(1, 0, 0)
b = (ctypes.c_double * 3)(0, 1, 0)
c = (ctypes.c_double * 3)(1, 1, 0)
# Call the C function
result = lib.scalar_triple(a, b, c)
print(Scalar triple product =, result)
# Convert to numpy arrays for plotting
a_vec = np.array([a[0], a[1], a[2]])
b_vec = np.array([b[0], b[1], b[2]])
c_vec = np.array([c[0], c[1], c[2]])
```

```
# Plot vectors in 3D
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
origin = np.array([0, 0, 0])
ax.quiver(*origin, *a_vec, color='r', label='a')
ax.quiver(*origin, *b_vec, color='g', label='b')
ax.quiver(*origin, *c_vec, color='b', label='c')
ax.set_xlim([0, 1.5])
ax.set_ylim([0, 1.5])
ax.set_zlim([0, 1.5])
ax.set_xlabel(X)
ax.set_ylabel(Y)
ax.set_zlabel(Z)
ax.set_title(fScalar Triple Product = {result})
ax.legend()
plt.show()
```

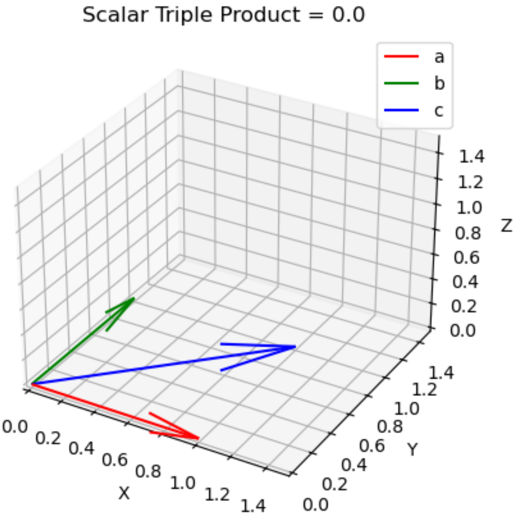
Only Python code

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
# Define 3 coplanar vectors (lying in xy-plane)
a = np.array([1, 1, 0])
b = np.array([1, 0, 0])
c = np.array([0, 1, 0])
# Cross product b x c
b_cross_c = np.cross(b, c)
# Scalar triple product a . (b x c)
scalar_triple = np.dot(a, b_cross_c)
# Print the scalar triple product (should be 0)
print(fScalar triple product: {scalar_triple:.2f})
# Plotting
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
```

Only Python code

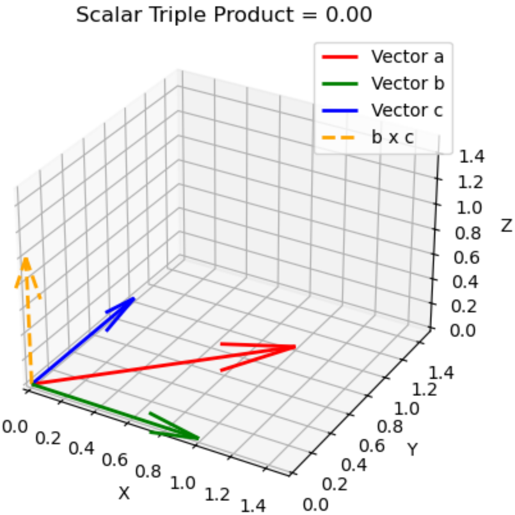
```
# Plot origin
origin = np.array([0, 0, 0])
# Plot vectors
ax.quiver(*origin, *a, color='r', label='Vector a', linewidth=2)
ax.quiver(*origin, *b, color='g', label='Vector b', linewidth=2)
ax.quiver(*origin, *c, color='b', label='Vector c', linewidth=2)
# Plot b x c
ax.quiver(*origin, *b_cross_c, color='orange', linestyle='dashed'
    , label='b x c', linewidth=2)
ax.set_xlim([0, 1.5])
ax.set_ylim([0, 1.5])
ax.set_zlim([0, 1.5])
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title(f'Scalar Triple Product = {scalar_triple:.2f}')
ax.legend()
ax.grid(True)
plt.show()
```

PLOTS



Scalar Triple Product = 0.0
a
b
c
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Z
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4
X
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4
Y

PLOTS


Scalar Triple Product = 0.00
Vector a
Vector b
Vector c
b x c
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Z
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
X
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
Y