5.2.43

EE25BTECH11041 - Naman Kumar

Question:
Solve the linear equation:

$$6x + 3y = 6xy \tag{1}$$

$$2x + 4y = 5xy \tag{2}$$

Solution:
General equation of conic

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f \tag{3}$$

Given set of equations in the form of general conic can be written as

$$\mathbf{x}^T\begin{pmatrix}0 & 3\\ 3 & 0\end{pmatrix}\mathbf{x} + 2\begin{pmatrix}-3\\ -1.5\end{pmatrix}^T\mathbf{x} = 0 \tag{4}$$

$$\mathbf{x}^T\mathbf{V_1}\mathbf{x} + 2\mathbf{u_1}^T\mathbf{x} = 0 \tag{5}$$

Similarly

$$\mathbf{x}^T\begin{pmatrix}0 & 2.5\\ 2.5 & 0\end{pmatrix}\mathbf{x} + 2\begin{pmatrix}-1\\ -2\end{pmatrix}^T\mathbf{x} = 0 \tag{6}$$

$$\mathbf{x}^T\mathbf{V_2}\mathbf{x} + 2\mathbf{u_2}^T\mathbf{x} = 0 \tag{7}$$

Intersection of two conic

$$\mathbf{x}^T(\mathbf{V_1} + \mu\mathbf{V_2})\mathbf{x} + 2(\mathbf{u_1} + \mu\mathbf{u_2})^T\mathbf{x} = 0 \tag{8}$$

General equation of conic represent pair of lines if

$$\begin{vmatrix}\mathbf{V} & \mathbf{u}\\ \mathbf{u}^T & f\end{vmatrix} = 0 \tag{9}$$

From (8)

$$\begin{vmatrix}\mathbf{V_1} + \mu\mathbf{V_2} & \mathbf{u_1} + \mu\mathbf{u_2}\\ (\mathbf{u_1} + \mu\mathbf{u_2})^T & 0\end{vmatrix} = 0 \tag{10}$$

Here

$$\mathbf{A} = \mathbf{V_1} + \mu \mathbf{V_2} = \begin{pmatrix} 0 & 3 + 2.5\mu \\ 3 + 2.5\mu & 0 \end{pmatrix} \tag{11}$$

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \tag{12}$$

$$\tag{13}$$

$$\mathbf{B} = \mathbf{u_1} + \mu \mathbf{u_2} = \begin{pmatrix} -3 + \mu(-1) \\ -1.5 + \mu(-2) \end{pmatrix} \tag{14}$$

$$\mathbf{B} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \tag{15}$$

Putting values in (10)

$$\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ b_1 & b_2 & 0 \end{vmatrix} \tag{16}$$

$$-b_2(b_2 a_{11} - b_1 a_{21}) + b_1(b_2 a_{12} - b_1 a_{22}) \tag{17}$$

Putting values from (11) (14)

$$(-3 - 2.5\mu)(3 + \mu)(1.5 + 2\mu) \tag{18}$$

$$\mu = \frac{-6}{5}, -3, \frac{-3}{4} \tag{19}$$

Case 1:$\mu = -3$ in (8)

$$\mathbf{x}^T \begin{pmatrix} 0 & 4.5 \\ 4.5 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 0 \\ -4.5 \end{pmatrix}^T \mathbf{x} = 0 \tag{20}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} 0 & 4.5 \\ 4.5 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} 0 \\ -4.5 \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} = 0 \tag{21}$$

$$2 \times 4.5xy + 2(0 - 4.5y) \tag{22}$$

$$= 9xy - 9y = 9y(x - 1) = 0 \tag{23}$$

$$y = 0, x = 1 \tag{24}$$

Case 2: $\mu = \frac{-6}{5}$ in (8)

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 1.8 \\ -0.9 \end{pmatrix}^T \mathbf{x} = 0 \tag{25}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} 1.8 \\ -0.9 \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} = 0 \tag{26}$$

$$2x - y = 0 \tag{27}$$

Case 3: $\mu = \frac{-3}{4}$ in (8)

$$\mathbf{x}^T \begin{pmatrix} 0 & -1.125 \\ -1.125 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 2.25 \\ 0 \end{pmatrix}^T \mathbf{x} = 0 \tag{28}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} 0 & -1.125 \\ -1.125 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} 2.25 \\ 0 \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} = 0 \tag{29}$$

$$-2.25xy + 4.5 = 2.25x(2-y) = 0 \tag{30}$$

$$x = 0, y = 2 \tag{31}$$

Now checking point of intersection with conic
from $\mu = -3$ factors y=0 and x=1

- y=0 in (1) 6x=6x.0 $\implies$ x=0 and in (2) 2x=0, so point (0,0)
- x=1 in (1) 6+3y=6y $\implies$ y=2, so (1,2)

similarly for $\mu = \frac{-3}{4}$ factors are x=0 and y=2

- x=0 gives (0,0)
- y=2 gives (1,2)

And for $\mu = \frac{-6}{5}$ line y=2x

- put y=2x in (1) $12x = 12x^2 \implies x = 0, 1$ so (0,0),(1,2)

All three cases have same points ,
so Points are (0,0) and (1,2)

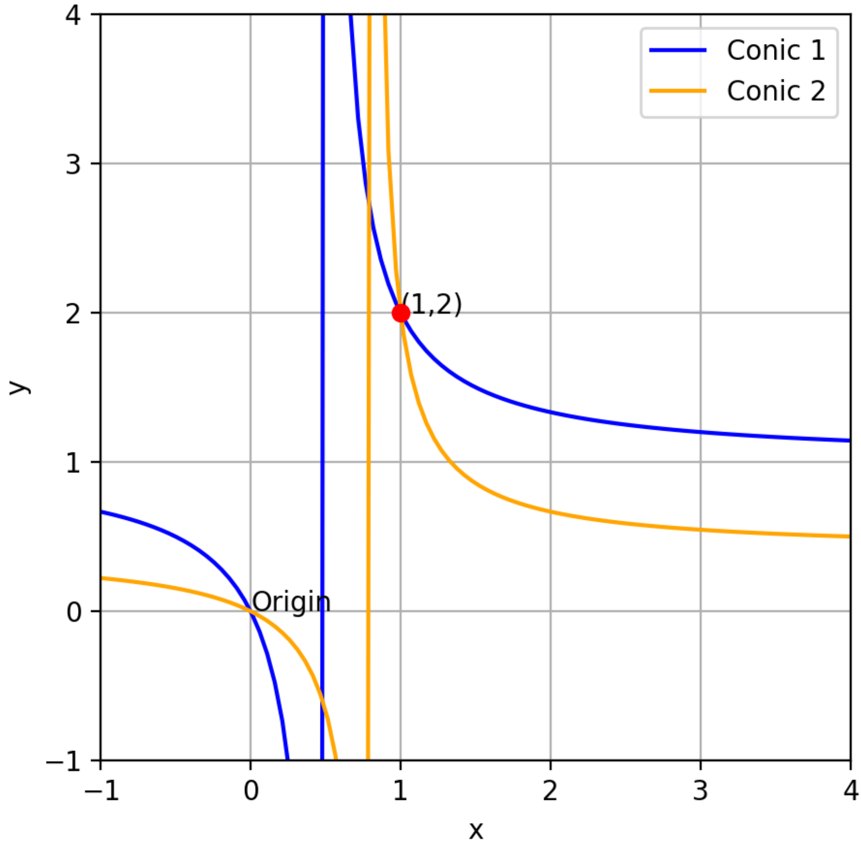

Figure 1