2.6.12

AI25BTECH11006 - Nikhila

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Find the sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + -2\hat{j} + 4\hat{k}$.

Solution

The given vectors are $\vec{a} \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ and $\vec{b} \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$

We know that

$$|\vec{a} \times \vec{b}| = |\vec{a}| \ |\vec{b}| \sin \theta \tag{1}$$

$$\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}. \tag{2}$$

$$\vec{a} \times \vec{b} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} = 8 \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \tag{3}$$

$$\|\vec{a} \times \vec{b}\| = 8\sqrt{3} \tag{4}$$

Solution

$$|\vec{a}| = \sqrt{(3)^2 + (1)^2 + (2)^2} = \sqrt{14}, \tag{5}$$

$$|\vec{b}| = \sqrt{(2)^2 + (-2)^2 + (4)^2} = \sqrt{24}. \tag{6}$$

$$\sin \theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} \tag{7}$$

$$= \frac{8\sqrt{3}}{\sqrt{14} \cdot \sqrt{24}} \tag{8}$$

$$= \frac{2}{\sqrt{7}}. \tag{9}$$

Graphical Representation

Angle Between Vectors a and b