4.3.32

EE25BTECH11042 - Nipun Dasari

Question:
Find the slope of a line which cuts off intercepts of equal length on the axes is. Solve using matrices.
Solution:

Consider normal form of a line:

$$\mathbf{n}^\top \mathbf{x} = c, \text{where } \mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} \tag{0.1}$$

Given that equal intercepts are cut off we get 2 cases:
On substituting the intercepts in place of $\mathbf{x}$:
Case 1: The intercepts are equal $(b = a)$

$$\implies \mathbf{n}^\top \begin{pmatrix} a \\ 0 \end{pmatrix} = 1 \implies an_1 = 1 \tag{0.2}$$

$$\implies \mathbf{n}^\top \begin{pmatrix} 0 \\ a \end{pmatrix} = 1 \implies an_2 = 1 \tag{0.3}$$

$$\tag{0.4}$$

from (0.2)

$$n_1 = \frac{1}{a} \tag{0.5}$$

from (0.3)

$$n_2 = \frac{1}{a} \therefore \mathbf{n} = \begin{pmatrix} \frac{1}{a} \\ \frac{1}{a} \end{pmatrix} \tag{0.6}$$

A simpler direction vector would be

$$\mathbf{n}' = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.7}$$

Case 2: The intercepts are negatives of each other $(-b = a)$

$$\implies \mathbf{n}^\top \begin{pmatrix} a \\ 0 \end{pmatrix} = 1 \implies an_1 = 1 \tag{0.8}$$

$$\implies \mathbf{n}^\top \begin{pmatrix} 0 \\ -a \end{pmatrix} = 1 \implies -an_2 = 1 \tag{0.9}$$

$$\tag{0.10}$$

(0.8)

$$n_1 = \frac{1}{a} \tag{0.11}$$

(0.9)

$$n_2 = \frac{-1}{a} \tag{0.12}$$

$$\therefore \mathbf{n} = \begin{pmatrix} \frac{1}{a} \\ \frac{-1}{a} \end{pmatrix} \tag{0.13}$$

A simpler direction vector would be

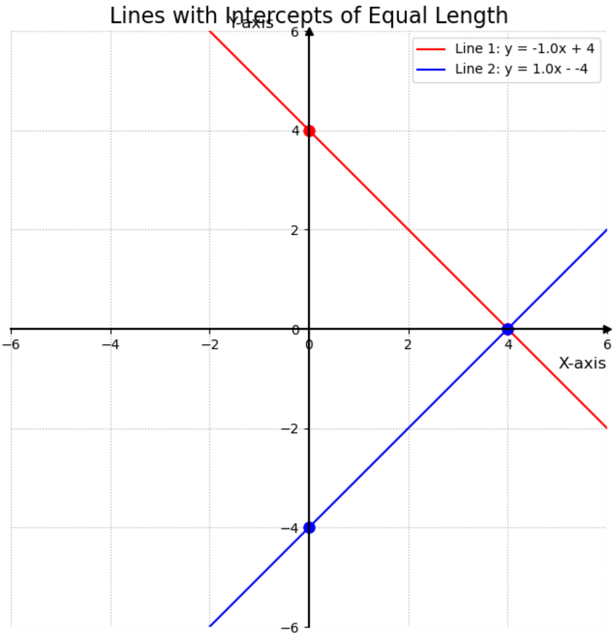
$$\mathbf{n}' = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{0.14}$$

The direction vector is given (in general) by:

$$\mathbf{n}' = \begin{pmatrix} 1 \\ m \end{pmatrix} \text{ where } m \text{ is slope of given line} \tag{0.15}$$

On comparing with the obtained direction vectors

$$\therefore m = \pm 1 \tag{0.16}$$


Lines with Intercepts of Equal Length
Y-axis
X-axis
Line 1: y = -1.0x + 4
Line 2: y = 1.0x - -4
6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6