1.5.3

EE25BTECH11015 - Bhoomika V

Question :-
In what ratio does the X axis divide line segment joining the points **A**(3,6) and **B**(-12,-3)?

Solution:
Let **A**(3,6) , **B**(-12,-3) and the point on X axis be **X**(t,0)

Point	Vector
A	$\begin{bmatrix}3\\6\\0\end{bmatrix}$
B	$\begin{bmatrix}-12\\-3\\0\end{bmatrix}$
X	$\begin{bmatrix}t\\0\\0\end{bmatrix}$

TABLE 0: Vectors

Using the collinearity (*rank*) test, form the matrix with difference vectors:

$$\begin{aligned}(\mathbf{B}-\mathbf{A} \quad \mathbf{X}-\mathbf{A}) &= \begin{pmatrix}-12-3 & t-3\\ -3-6 & 0-6\end{pmatrix}\\ &= \begin{pmatrix}-15 & t-2\\ -9 & -6\end{pmatrix}.\end{aligned}$$

The three points are collinear $\iff$ this matrix has rank 1 (*its rows are linearly dependent*).

$$R_2 \leftarrow 5R_2 - 3R_1 \implies \begin{pmatrix}-45 & 3t-9\\ 0 & -3t-21\end{pmatrix}.$$

For rank 1, the second row must be zero:

$$-3t - 21 = 0 \implies t = -7$$

let **X** divide **A** and **B** in the ratio k:1 then

$$k = \frac{(A-X)^T(X-B)}{\|X-B\|^2} \tag{0.1}$$

$$\implies k = 2$$

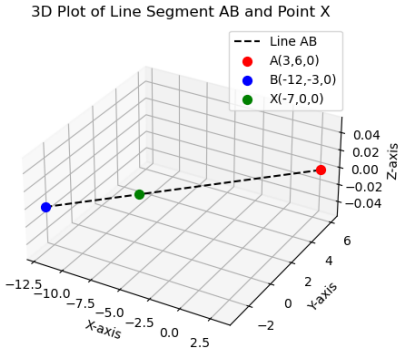

Fig. 0.1