8.4.4

EE25BTECH11001 - Aarush Dilawri

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Question

Question:
Equation of the ellipse whose axes are the coordinates and which passes through the point $(-3, 1)$ and has eccentricity $\sqrt{\frac{2}{5}}$ is

Solution

Solution:
The general equation of a conic can be written as:

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{1}$$

Since the ellipse is centered at origin, we have

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad f = -1 \tag{2}$$

Solution

Let the major axis be along the X-axis:

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad e = \sqrt{\frac{2}{5}} \tag{3}$$

Then, using the formula:

$$\mathbf{V} = \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n}\mathbf{n}^T \tag{4}$$

we get

$$\mathbf{V} = \begin{pmatrix} 1 - \frac{2}{5} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & 0 \\ 0 & 1 \end{pmatrix} \tag{5}$$

Solution

The ellipse passes through the point $(-3, 1)$, so scale $\mathbf{V}$ such that:

$$\mathbf{x}_0^T \mathbf{V} \mathbf{x}_0 = 1, \quad \mathbf{x}_0 = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \tag{6}$$

This gives

$$\mathbf{V} = \frac{5}{32} \begin{pmatrix} \frac{3}{5} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{32} & 0 \\ 0 & \frac{5}{32} \end{pmatrix} \tag{7}$$

Solution

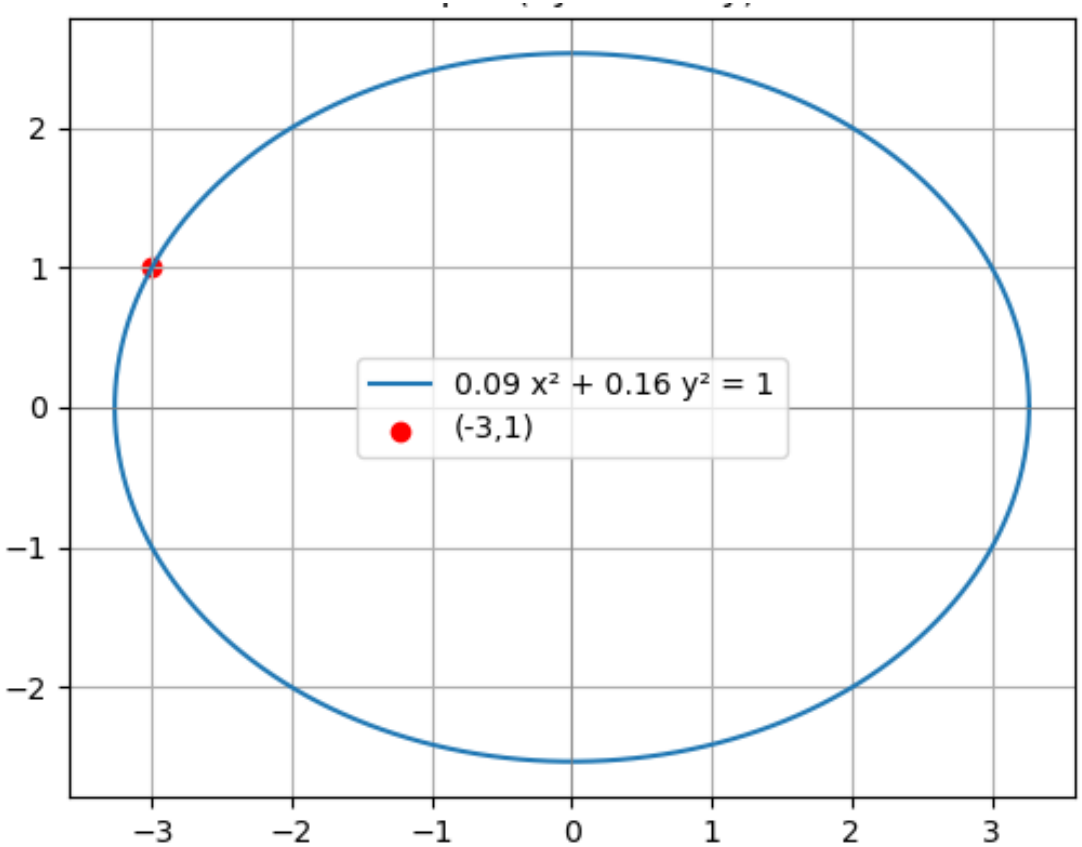
Hence, the equation of the ellipse is:

$$\mathbf{x}^T \begin{pmatrix} \frac{3}{32} & 0 \\ 0 & \frac{5}{32} \end{pmatrix} \mathbf{x} = 1 \tag{8}$$

Or equivalently:

$$3x^2 + 5y^2 = 32 \tag{9}$$

Figure



C Code (code.c)

```c
#include <math.h>
int ellipse_equation(double *A, double *B, double *C,
                     double x0, double y0, double e)
{
    double V11 = 1.0 - e*e;
    double V22 = 1.0;

    double val = V11*x0*x0 + V22*y0*y0;
    double scale = 1.0 / val;

    *A = V11 * scale;
    *B = V22 * scale;
    *C = 1.0;

    return 0;
}
```

Python Code (code.py)

```
import numpy as np
import matplotlib.pyplot as plt

# Given parameters
x0, y0 = -3.0, 1.0
e = np.sqrt(2.0/5.0)

V11 = 1 - e**2
V22 = 1

val = V11*x0**2 + V22*y0**2
scale = 1.0 / val
A = V11 * scale
B = V22 * scale
C = 1.0

print(f"Ellipse equation (Python): {A: 6f} x^2 + {B: 6f} y^2 = {C: 6f}")
```

Python Code (code.py)

```
theta = np.linspace(0, 2*np.pi, 400)
a = np.sqrt(C/A)
b = np.sqrt(C/B)
x = a * np.cos(theta)
y = b * np.sin(theta)

plt.figure(figsize=(6,6))
plt.plot(x, y, label=f"{A:.2f}x + {B:.2f}y = {C:.0f}")
plt.scatter(x0, y0, color='red', label='(−3,1)')
plt.axhline(0, color='gray', linewidth=0.5)
plt.axvline(0, color='gray', linewidth=0.5)
plt.gca().set_aspect('equal', adjustable='box')
plt.legend()
plt.grid(True)
plt.title('Ellipse (Python only)')
plt.show()
```

Python Code (nativecode.py)

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load the shared library
lib = ctypes.CDLL('./code.so')

# Define argument and return types
lib.ellipse_equation.argtypes = [ctypes.POINTER(ctypes.c_double),
                                 ctypes.POINTER(ctypes.c_double),
                                 ctypes.POINTER(ctypes.c_double),
                                 ctypes.c_double,
                                 ctypes.c_double,
                                 ctypes.c_double]
lib.ellipse_equation.restype = ctypes.c_int # returning int now
```

Python Code (nativecode.py)

```
A = ctypes.c_double()
B = ctypes.c_double()
C = ctypes.c_double()
x0 = -3.0
y0 = 1.0
e = np.sqrt(2.0/5.0)

ret = lib.ellipse_equation(ctypes.byref(A), ctypes.byref(B), ctypes.byref(C)
    , x0, y0, e)

if ret != 0:
    raise RuntimeError("Error computing ellipse coefficients in C.")

print(f"Ellipse equation from C: {A.value:.6f} x^2 + {B.value:.6f} y^2 = {C.
    value:.6f}")

theta = np.linspace(0, 2*np.pi, 400)
```

Python Code (nativecode.py)

```
a = np.sqrt(C.value/A.value)
b = np.sqrt(C.value/B.value)
x = a * np.cos(theta)
y = b * np.sin(theta)

plt.figure(figsize=(6,6))
plt.plot(x, y, label=f"{A.value:.2f}x + {B.value:.2f}y = {C.value:.0f}")
plt.scatter(x0, y0, color='red', label='(−3,1)')
plt.axhline(0, color='gray', linewidth=0.5)
plt.axvline(0, color='gray', linewidth=0.5)
plt.gca().set_aspect('equal', adjustable='box')
plt.legend()
plt.grid(True)
plt.title('Ellipse from C Library')
plt.show()
```