4.3.46

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Question : Find the coordinates of the point where the line through $(3,-4,-5)$ and $(2,-3,1)$ crosses the plane $2x+y+z=7$.

Solution :

Description	Value
Line	$\mathbf{x}=\begin{pmatrix}2\\-3\\1\end{pmatrix}+k\begin{pmatrix}-1\\1\\6\end{pmatrix}$
Plane	$\mathbf{n}^{\top}\mathbf{x}=7 \quad$ where $\quad \mathbf{n}=\begin{pmatrix}2\\1\\1\end{pmatrix}$

Table : Line and Plane

Let the point of intersection be $\mathbf{I}$.
The line is written as

$$\mathbf{x}=\mathbf{h}+k\,\mathbf{m} \tag{1}$$

$$\mathbf{h}=\begin{pmatrix}2\\-3\\1\end{pmatrix} \quad \mathbf{m}=\begin{pmatrix}-1\\1\\6\end{pmatrix} \tag{2}$$

So,

$$\mathbf{I}=\mathbf{h}+k\,\mathbf{m} \tag{3}$$

$$\mathbf{I}=\begin{pmatrix}2\\-3\\1\end{pmatrix}+k\begin{pmatrix}-1\\1\\6\end{pmatrix}=\begin{pmatrix}2-k\\k-3\\1+6k\end{pmatrix} \tag{4}$$

The plane equation is

$$\mathbf{n}^{\top}\mathbf{x}=c \tag{5}$$

$$\mathbf{n}=\begin{pmatrix}2\\1\\1\end{pmatrix}, \quad c=7 \tag{6}$$

Substitute $\mathbf{I}$ into the plane:

$$\mathbf{n}^{\top}\mathbf{I}=c \tag{7}$$

$$\begin{pmatrix}2 & 1 & 1\end{pmatrix}\begin{pmatrix}2-k\\k-3\\1+6k\end{pmatrix}=7 \tag{8}$$

$$4-2k+k-3+1+6k=7 \tag{9}$$

$$k=1 \tag{10}$$

Substitute $k = 1$ back:

$$\mathbf{I} = \mathbf{h} + 1 \cdot \mathbf{m} \tag{11}$$

$$\mathbf{I} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} = \begin{pmatrix} 2 - 1 \\ -3 + 1 \\ 1 + 6 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix}. \tag{12}$$

Answer:

$$\mathbf{I} = \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix} \tag{13}$$

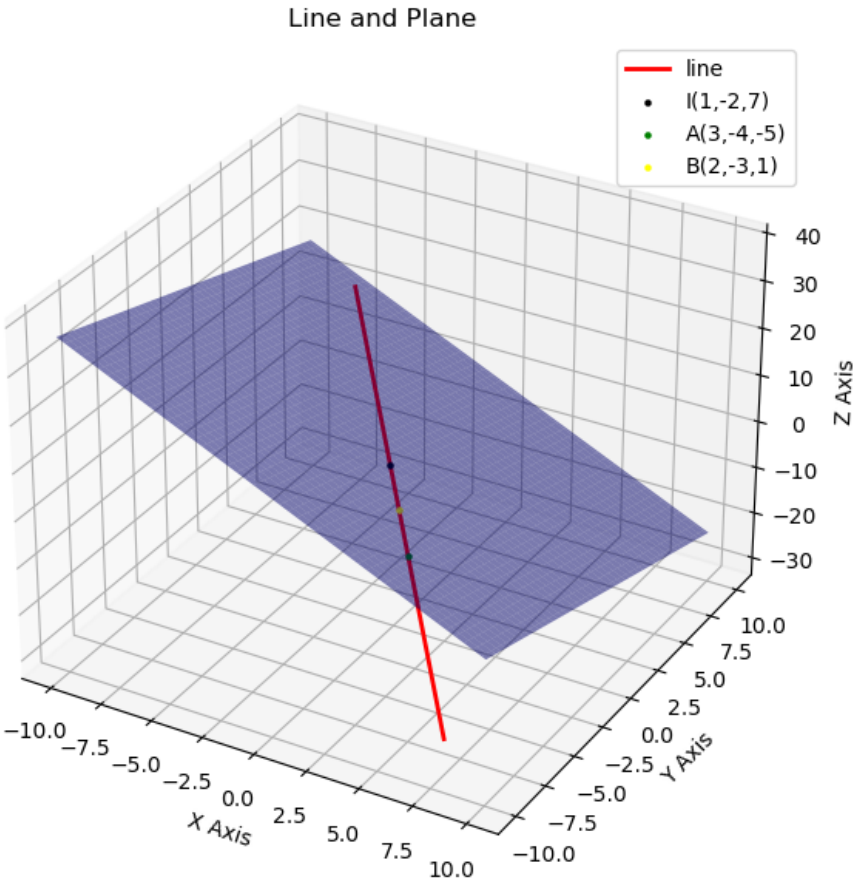

Fig : Line and Plane