Presentation - Matgeo

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Problem Statement

Given that $\mathbf{P}(3, 2, -4)$, $\mathbf{Q}(5, 4, -6)$ and $\mathbf{R}(9, 8, -10)$ are collinear. Find the ratio in which $\mathbf{Q}$ divides PR.

Solution:

Given that, P , Q and R are 3 coincident points

$$P = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}, \qquad Q = \begin{pmatrix} 5 \\ 4 \\ -6 \end{pmatrix}, \qquad R = \begin{pmatrix} 9 \\ 8 \\ -10 \end{pmatrix} \tag{1.1}$$

From the section formula,

$$\mathbf{Q} = \frac{k\mathbf{P} + \mathbf{R}}{k+1} \tag{1.2}$$

for some scalar k. Where $\mathbf{Q}$ divides PR in the ratio $k : 1$.
From equation 1.2 :

$$(\mathbf{R} - \mathbf{P})\, k = (\mathbf{Q} - \mathbf{P}) \tag{1.3}$$

$$k = \frac{(\mathbf{Q} - \mathbf{P})\,(\mathbf{R} - \mathbf{P})^T}{\|\mathbf{R} - \mathbf{P}\|^2} \tag{1.4}$$

Calculation

$$(\mathbf{R}-\mathbf{P}) = \begin{pmatrix} 6 \\ 6 \\ -6 \end{pmatrix}, \qquad (\mathbf{Q}-\mathbf{P}) = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}. \tag{1.5}$$

$$(\mathbf{Q}-\mathbf{P})(\mathbf{R}-\mathbf{P})^T = \begin{pmatrix} 2 & 2 & -2 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \\ -6 \end{pmatrix} = 2 \cdot 6 + 2 \cdot 6 + (-2)(-6) = 36 \tag{1.6}$$

$$(\mathbf{R}-\mathbf{P})(\mathbf{R}-\mathbf{P})^T = \begin{pmatrix} 6 & 6 & -6 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \\ -6 \end{pmatrix} = 6^2 + 6^2 + (-6)^2 = 108 \tag{1.7}$$

$$\therefore \quad k = \frac{36}{108} = \frac{1}{3} \tag{1.8}$$

Result

Thus,

$$PQ : QR = k : (1-k) = \frac{1}{3} : \frac{2}{3} = 1 : 2 \tag{1.9}$$

The line PR is divided in the ratio 1 : 2 by the point Q.

Plot

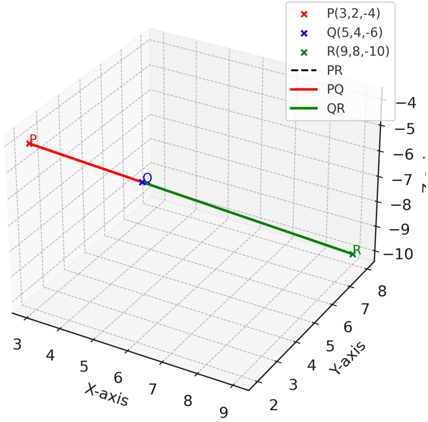

Figure: Plot of the points P, Q and R

Code - C

Helper Functions

```
#include <stdio.h>
#include <math.h>

float dotProduct(float a[3], float b[3]) {
    return a[0]*b[0] + a[1]*b[1] + a[2]*b[2];
}

float normSquared(float a[3]) {
    return dotProduct(a, a);
}
```

Code-C

findK Function

```
float findK(float P[3], float Q[3], float R[3]) {
    float QP[3], RP[3];

    for (int i = 0; i < 3; i++) {
        QP[i] = Q[i] - P[i];
        RP[i] = R[i] - P[i];
    }
    float numerator = dotProduct(QP, RP);
    float denominator = normSquared(RP);

    if (denominator == 0) {
        return 0.0f; // if P and R are same point, k is undefined -
            return 0
    }
    return numerator / denominator;
}
```

Code - Python

```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Points
P = (3, 2, −4)
Q = (5, 4, −6)
R = (9, 8, −10)

# Plotting
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Scatter points
ax.scatter(*P, color='red', label='P(3,2,−4)')
ax.scatter(*Q, color='blue', label='Q(5,4,−6)')
ax.scatter(*R, color='green', label='R(9,8,−10)')
```

Code-Python

```
# Line PR (whole line)
ax.plot([P[0], R[0]], [P[1], R[1]], [P[2], R[2]], color='black', linestyle='--'
    , label='PR')

# Subsegments PQ and QR
ax.plot([P[0], Q[0]], [P[1], Q[1]], [P[2], Q[2]], color='red', linewidth=2,
    label='PQ')
ax.plot([Q[0], R[0]], [Q[1], R[1]], [Q[2], R[2]], color='green', linewidth=2,
    label='QR')

# Labels on points
ax.text(*P, "P", color='red')
ax.text(*Q, "Q", color='blue')
ax.text(*R, "R", color='green')
```

Code-Python

```
# Axis labels
ax.set_xlabel("X-axis")
ax.set_ylabel("Y-axis")
ax.set_zlabel("Z-axis")
ax.legend()

plt.show()

plt.savefig('../figs/img.png')
```