2.10.17

AI25BTECH11028-R.Manohar

Question:
The vector

$$\frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k})$$

is

1) a unit vector
2) parallel to the vector $(-\hat{i} + \hat{j} - \frac{1}{2}\hat{k})$
3) perpendicular to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

Solution:
Given

$$\mathbf{v} = \frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k}) = \begin{pmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{pmatrix}.$$

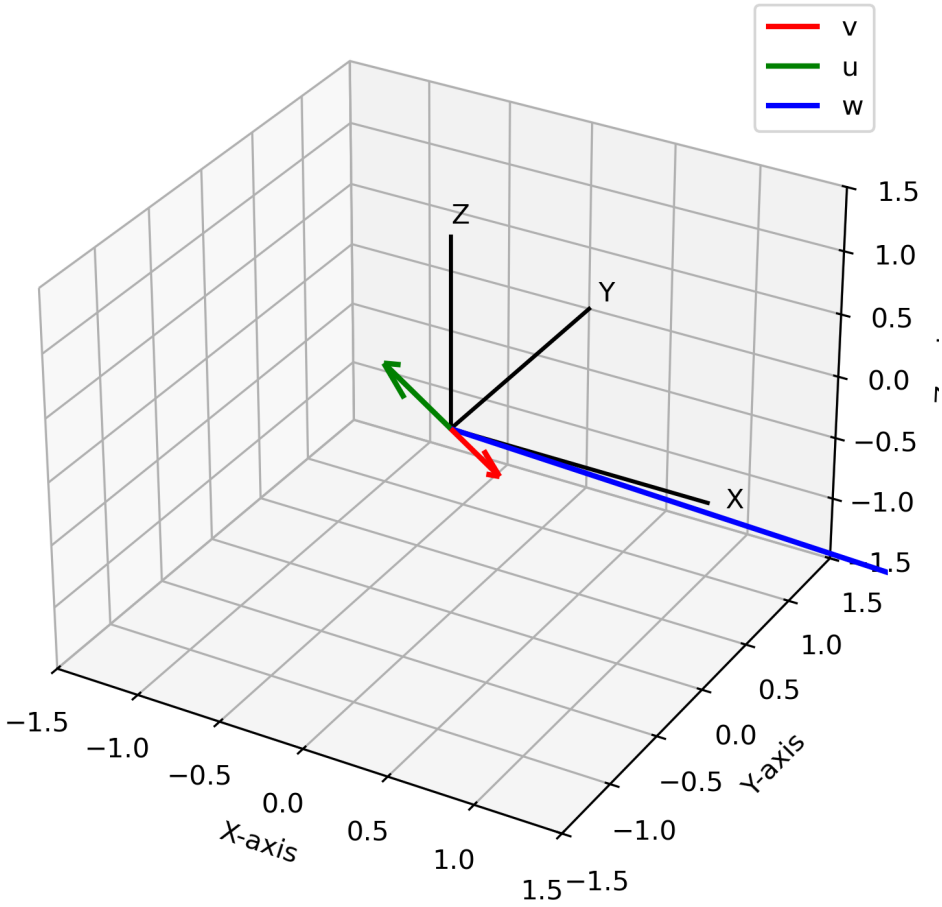
$$\begin{aligned} \|\mathbf{v}\| &= \sqrt{\mathbf{v}^T\mathbf{v}} \\ &= \sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2} \\ &= 1 \end{aligned}$$

Hence, $\mathbf{v}$ is a unit vector.
Let

$$\mathbf{u} = \begin{pmatrix} -1 \\ 1 \\ -\frac{1}{2} \end{pmatrix}, \mathbf{w} = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}.$$

$$\mathbf{v} = -\frac{2}{3}\mathbf{u} \implies \mathbf{v} \parallel \mathbf{u},$$

$$\begin{aligned} \mathbf{v}^T\mathbf{w} &= \begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \\ &= \frac{2}{3} \times 3 + \left(-\frac{2}{3}\right) \times 2 + \frac{1}{3} \times (-2) \\ &= 0 \implies \mathbf{v} \perp \mathbf{w}. \end{aligned}$$


v
u
w
Z
Y
X
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
X-axis
Y-axis