3.3.11

AI25BTECH11027 - NAGA BHUVANA

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Question:
Construct a triangle in which $AB = 6cm$,$\angle A = 30^\circ$ and $\angle B = 60^\circ$
Solution:
Let **A** be $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ as $AB = 6cm$ position vector of **B** be $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$
Property:
Sum of angles in a triangle is 180°

$$\angle A + \angle B + \angle C = 180^\circ \tag{0.1}$$

$$30^\circ + 60^\circ + \angle C = 180^\circ \tag{0.2}$$

$$\angle C = 90^\circ \tag{0.3}$$

$$a\cos B + b\cos A = c \tag{0.4}$$

$$a\sin B - b\sin A = 0 \tag{0.5}$$

$$\mathbf{P} = \begin{pmatrix} \cos B & \cos A \\ \sin B & -\sin A \end{pmatrix}, \mathbf{x} = \begin{pmatrix} a \\ b \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} c \\ 0 \end{pmatrix} \tag{0.6}$$

Consider the augmented matrix for solving $\mathbf{Px} = \mathbf{Q}$

$$\begin{pmatrix} \cos B & \cos A & c \\ \sin B & -\sin A & 0 \end{pmatrix} \tag{0.7}$$

$$\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 6 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \end{pmatrix} \tag{0.8}$$

By doing Row operations

$$\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 6 \\ 0 & -2 & -6\sqrt{3} \end{pmatrix} \tag{0.9}$$

On solving

$$BC = a = 3, AC = b = 3\sqrt{3} \tag{0.10}$$

$$\mathbf{C} = \begin{pmatrix} 3\sqrt{3}\cos 30^\circ \\ 3\sqrt{3}\sin 30^\circ \end{pmatrix} \tag{0.11}$$

$$\mathbf{C} = \begin{pmatrix} \frac{9}{2} \\ \frac{3\sqrt{3}}{2} \end{pmatrix} \tag{0.12}$$