

4.8.12

AI25BTECH11016-Varun

Question:

If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then the value of k is

Solution:

The vector equation of a line $\frac{x-\alpha}{a} = \frac{y-\beta}{b} = \frac{z-\gamma}{c}$ is $\begin{pmatrix} \alpha + a\lambda \\ \beta + b\lambda \\ \gamma + c\lambda \end{pmatrix}$

The vector equation of first line,

$$\begin{pmatrix} 1 + 2\lambda \\ -1 + 3\lambda \\ 1 + 4\lambda \end{pmatrix} \quad (1)$$

The vector equation of Second line,

$$\begin{pmatrix} 3 + \lambda \\ k + 2\lambda \\ \lambda \end{pmatrix} \quad (2)$$

The lines $A + K_1m_1, B + K_2m_2$ will intersect if

$$\text{rank}(\mathbf{M} \quad \mathbf{B} - \mathbf{A}) = 2 \quad (3)$$

$$\mathbf{M} = \begin{pmatrix} m_1 & m_2 \end{pmatrix} \quad (4)$$

Here,

$$\mathbf{M} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ 4 & 1 \end{pmatrix} \quad (5)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 - 1 \\ k - (-1) \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ k + 1 \\ -1 \end{pmatrix} \quad (6)$$

$$\text{rank} \left(\begin{pmatrix} 2 & 1 & 2 \\ 3 & 2 & k + 1 \\ 4 & 1 & -1 \end{pmatrix} \right) = 2 \quad (7)$$

$$\begin{pmatrix} 2 & 1 & 2 \\ 3 & 2 & k + 1 \\ 4 & 1 & -1 \end{pmatrix} \xrightarrow[R_3 \rightarrow 2R_3 - 4R_1]{R_2 \rightarrow 2R_2 - 3R_1} \begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & 2k - 4 \\ 0 & -2 & -10 \end{pmatrix} \xrightarrow{R_3 \rightarrow R_3 + 2R_2} \begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & 2k - 4 \\ 0 & 0 & 4k - 18 \end{pmatrix}$$

For the rank($\mathbf{M} \quad \mathbf{B} - \mathbf{A}$) to be 2
the last row must be all zero implies

$$4k - 18 = 0 \quad (8)$$

$$k = \frac{9}{2} \quad (9)$$

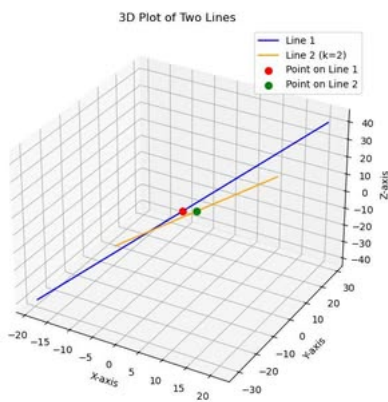


Fig. 0.1