12.235

Bhargav - EE25BTECH11013

September 30, 2025

Question

Question:
A system of equations represented as

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 4 \\ 1 & 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ y \\ 3 \end{pmatrix} is, \tag{1}$$

1. consistent and has a unique solution
2. inconsistent and has no solution
3. consistent and infinite solution
4. inconsistent and has unique solution

Solution

This can be represented as an augmented matrix and can be solved by using Gaussian elimination.

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 2 & 1 & 4 & y \\ 1 & 3 & 1 & 3 \end{array}\right) \xleftrightarrow[R_3 \leftarrow R_3 - R_1]{R_2 \leftarrow R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 3 & 0 & y-8 \\ 0 & 4 & -1 & -1 \end{array}\right) \tag{2}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - 4R_2]{R_2 \leftarrow \frac{R_2}{3}} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 1 & 0 & \frac{y-8}{3} \\ 0 & 0 & -1 & \frac{29-4y}{3} \end{array}\right) \xleftrightarrow[R_1 \leftarrow R_1 - 2R_3]{R_3 \leftarrow -R_3} \tag{3}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & \frac{70-8y}{3} \\ 0 & 1 & 0 & \frac{y-8}{3} \\ 0 & 0 & 1 & \frac{4y-29}{3} \end{array}\right) \xleftrightarrow{R_1 \leftarrow R_1 + R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{62-7y}{3} \\ 0 & 1 & 0 & \frac{y-8}{3} \\ 0 & 0 & 1 & \frac{4y-29}{3} \end{array}\right) \tag{4}$$

Solution

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{62-7y}{3} \\ \frac{y-8}{3} \\ \frac{4y-29}{3} \end{pmatrix} \tag{5}$$

Since $y \in \mathbf{R}$, we can conclude that there exists a unique solution and the system of equations is consistent.

Option (1) is the correct answer

Solution

This can be verified by finding the point of intersection of 3 planes:
As an example, take $y = 8$

$$x_1 - x_2 + 2x_3 = 4 \tag{6}$$

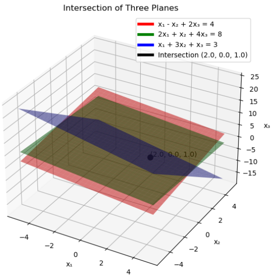
$$2x_1 + x_2 + 4x_3 = 8 \tag{7}$$

$$x_1 + 3x_2 + x_3 = 3 \tag{8}$$

The point of intersection of the planes from (5) is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \tag{9}$$

Plot



Intersection of Three Planes
x₁ - x₂ + 2x₃ = 4
2x₁ + x₂ + 4x₃ = 8
x₁ + 3x₂ + x₃ = 3
Intersection (2.0, 0.0, 1.0)
(2.0, 0.0, 1.0)
x₁
x₂
x₃

C Code

```
#include <stdio.h>

void solve_planes(double *A, double *b, double *x) {
    double detA, det1, det2, det3;

    detA = A[0]*(A[4]*A[8] - A[5]*A[7]) -
           A[1]*(A[3]*A[8] - A[5]*A[6]) +
           A[2]*(A[3]*A[7] - A[4]*A[6]);

    if(detA == 0) {
        x[0] = x[1] = x[2] = 0;
        return;
    }

    det1 = b[0]*(A[4]*A[8] - A[5]*A[7]) -
           A[1]*(b[1]*A[8] - A[5]*b[2]) +
           A[2]*(b[1]*A[7] - A[4]*b[2]);
```

C Code

```
    det2 = A[0]*(b[1]*A[8] - A[5]*b[2]) -
           b[0]*(A[3]*A[8] - A[5]*A[6]) +
           A[2]*(A[3]*b[2] - b[1]*A[6]);

    det3 = A[0]*(A[4]*b[2] - b[1]*A[7]) -
           A[1]*(A[3]*b[2] - b[1]*A[6]) +
           b[0]*(A[3]*A[7] - A[4]*A[6]);

    x[0] = det1 / detA;
    x[1] = det2 / detA;
    x[2] = det3 / detA;
}
```

Python + C Code

```
import numpy as np
import ctypes
import matplotlib.pyplot as plt

# Load shared C library
lib = ctypes.CDLL("./libcode.so")

# Define argument types
lib.solve_planes.argtypes = [
    np.ctypeslib.ndpointer(dtype=np.double, ndim=1, flags="C_CONTIGUOUS"),
    np.ctypeslib.ndpointer(dtype=np.double, ndim=1, flags="C_CONTIGUOUS"),
    np.ctypeslib.ndpointer(dtype=np.double, ndim=1, flags="C_CONTIGUOUS")
]
```

Python + C Code

```
# Coefficient matrix and constants
A = np.array([[1, -1, 2],
              [2, 1, 4],
              [1, 3, 1]], dtype=np.double)
b = np.array([4, 8, 3], dtype=np.double)
x = np.zeros(3, dtype=np.double)

# Call C function
lib.solve_planes(A.ravel(), b, x)
print("Point of intersection (x1, x2, x3):", x)

# --- Plotting ---
x_vals = np.linspace(-5, 5, 30)
y_vals = np.linspace(-5, 5, 30)
X, Y = np.meshgrid(x_vals, y_vals)

# Plane equations rearranged for z
Z1 = (4 - X + Y) / 2
Z2 = (8 - 2*X - Y) / 4
```

```
Z3 = 3 - X - 3*Y

fig = plt.figure(figsize=(9, 7))
ax = plt.axes(projection='3d')

# Plot each plane
ax.plot_surface(X, Y, Z1, alpha=0.5, color='red')
ax.plot_surface(X, Y, Z2, alpha=0.5, color='green')
ax.plot_surface(X, Y, Z3, alpha=0.5, color='blue')

# Plot intersection point
ax.scatter(x[0], x[1], x[2], color='black', s=60)
ax.text(x[0], x[1], x[2]+0.3, f'({x[0]:.1f}, {x[1]:.1f}, {x[2]:.1
    f})', color='black')

# Labels and title
ax.set_xlabel('x')
ax.set_ylabel('x')
ax.set_zlabel('x')
```

```python
colors = ['red', 'green', 'blue', 'black']
labels = [
    'x - x + 2x = 4',
    '2x + x + 4x = 8',
    'x + 3x + x = 3',
    'Intersection (2, 0, 1)'
]

handles = [plt.Line2D([0], [0], color=c, lw=4) for c in colors]
ax.legend(handles, labels)
plt.savefig("/mnt/c/Users/bharg/Documents/backupmatrix/
    ee25btech11013/matgeo/12.235/figs/Figure_1.png")
plt.show()
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
A = np.array([[1, -1, 2],
              [2, 1, 4],
              [1, 3, 1]], dtype=float)
b = np.array([4, 8, 3], dtype=float)
x = np.linalg.solve(A, b)
print("Point of intersection (x1, x2, x3):", x)
x_vals = np.linspace(-5, 5, 30)
y_vals = np.linspace(-5, 5, 30)
X, Y = np.meshgrid(x_vals, y_vals)
# Plane equations rearranged for z
Z1 = (4 - X + Y) / 2
Z2 = (8 - 2*X - Y) / 4
Z3 = 3 - X - 3*Y

fig = plt.figure(figsize=(9, 7))
ax = fig.add_subplot(111, projection='3d')
```

Python Code

```
# Plot each plane
surf1 = ax.plot_surface(X, Y, Z1, alpha=0.5, color='red')
surf2 = ax.plot_surface(X, Y, Z2, alpha=0.5, color='green')
surf3 = ax.plot_surface(X, Y, Z3, alpha=0.5, color='blue')

# Plot intersection point
ax.scatter(x[0], x[1], x[2], color='black', s=60)
ax.text(x[0], x[1], x[2]+0.3, f'({x[0]:.1f}, {x[1]:.1f}, {x[2]:.1
    f})', color='black')

# Labels and title
ax.set_xlabel('x')
ax.set_ylabel('x')
ax.set_zlabel('x')
ax.set_title('Intersection of Three Planes')
```

Python Code

```
colors = ['red', 'green', 'blue', 'black']
labels = [
    'x - x + 2x = 4',
    '2x + x + 4x = 8',
    'x + 3x + x = 3',
    f'Intersection ({x[0]:.1f}, {x[1]:.1f}, {x[2]:.1f})'
]
handles = [plt.Line2D([0], [0], color=c, lw=4) for c in colors]
ax.legend(handles, labels)


plt.savefig("/mnt/c/Users/bharg/Documents/backupmatrix/
    ee25btech11013/matgeo/12.235/figs/Figure_1.png")
plt.show()
```