

1.4.19

EE25BTECH11004 - Aditya Appana

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Question

Find the acute angle between the planes $\mathbf{r} \cdot (\hat{i} - 2\hat{j} - 2\hat{k})$ and $\mathbf{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k})$

Solution

Let vectors be

$$\mathbf{P} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \quad (1)$$

$$\mathbf{Q} = \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix} \quad (2)$$

The formula to calculate the angle between the two planes is

$$\begin{aligned} \theta &= \frac{\pi}{2} - \cos^{-1} \left(\frac{\mathbf{P}^T \mathbf{Q}}{|\mathbf{P}| |\mathbf{Q}|} \right) \\ &= \sin^{-1} \left(\frac{\mathbf{P}^T \mathbf{Q}}{|\mathbf{P}| |\mathbf{Q}|} \right) \end{aligned}$$

Substituting **P**, **Q** in this formula :

$$\begin{aligned}
 &= \sin^{-1} \left(\frac{\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}^T \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix}}{\left| \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \right| \left| \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix} \right|} \right) \\
 &= \sin^{-1} \left(\frac{19}{|3||7|} \right) \\
 &= \sin^{-1} \left(\frac{11}{21} \right)
 \end{aligned}$$

This is 31.58906757233914°

Plot of the planes

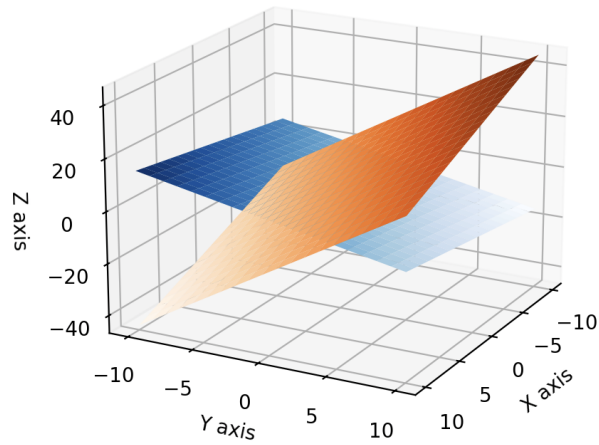


Figure 1: Plot

Plot of the planes

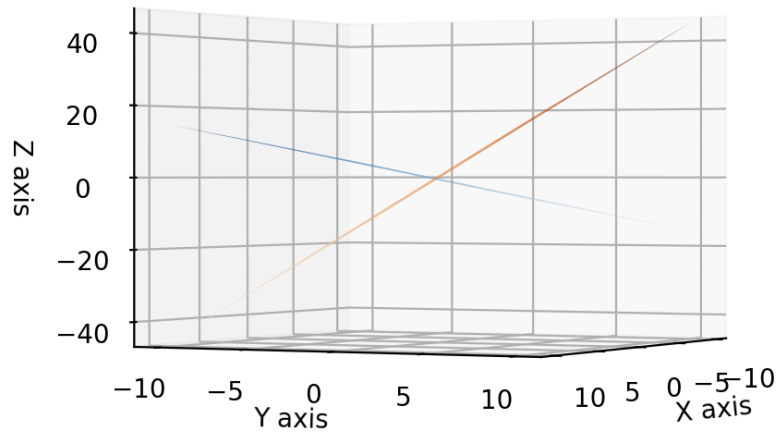


Figure 2: Plot