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Question

Given 3 vectors $\mathbf{A}$,$\mathbf{B}$,$\mathbf{C}$ are coplanar then show det($\mathbf{M}$) =0 where $\mathbf{M}$=($\mathbf{A}$ $\mathbf{B}$ $\mathbf{C}$)

Solution:

Equation of plane through 3 coplanar points is

$$\mathbf{n}^T\mathbf{x} = 0 \tag{1}$$
$$\implies \mathbf{n}^T\mathbf{A} = \mathbf{n}^T\mathbf{B} = \mathbf{n}^T\mathbf{C} = 0 \tag{2}$$
$$\mathbf{M} = \begin{pmatrix}\mathbf{A} & \mathbf{B} & \mathbf{C}\end{pmatrix} \tag{3}$$
$$\implies \mathbf{n}^T\mathbf{M} = \begin{pmatrix}\mathbf{n}^T\mathbf{A} & \mathbf{n}^T\mathbf{B} & \mathbf{n}^T\mathbf{C}\end{pmatrix} \tag{4}$$
$$\implies \mathbf{n}^T\mathbf{M} = \begin{pmatrix}0 & 0 & 0\end{pmatrix} \tag{5}$$
$$\implies \mathbf{n}^T\mathbf{M} = \mathbf{0} \tag{6}$$

From (6) it means $\mathbf{M}$ has a non trivial vector in it$'s$ null space

$$\implies \text{rank}(\mathbf{M}) < 3. \tag{7}$$

For a 3× 3 square matrix like $\mathbf{M}$ if det($\mathbf{M}$)≠0 means $\mathbf{M}$ is invertible which means $\mathbf{M}$ is a full rank matrix
$\implies$ rank($\mathbf{M}$)=3.(if det($\mathbf{M}$)≠0)
From (7) rank($\mathbf{M}$)<3
$\implies$ $\mathbf{M}$ is not invertible
$\implies$ det($\mathbf{M}$)=0

proof 2:

3 vectors $\mathbf{A}$,$\mathbf{B}$,$\mathbf{C}$ are coplanar means they are linearly dependent.
let′s assume

$$\mathbf{C} = \alpha\mathbf{A} + \beta\mathbf{B}. \tag{8}$$
$$\det(\mathbf{M}) = \det(\begin{pmatrix}\mathbf{A} & \mathbf{B} & \mathbf{C}\end{pmatrix} \tag{9}$$
$$= \det(\begin{pmatrix}\mathbf{A} & \mathbf{B} & \alpha\mathbf{A} + \beta\mathbf{B}\end{pmatrix} \tag{10}$$
$$= \alpha\det(\begin{pmatrix}\mathbf{A} & \mathbf{B} & \mathbf{A}\end{pmatrix} + \beta\det(\begin{pmatrix}\mathbf{A} & \mathbf{B} & \mathbf{B}\end{pmatrix} = 0 \tag{11}$$
$$\implies \det(\mathbf{M}) = 0 \tag{12}$$