1.2.21

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Question:

The centroid of a triangle ABC is at the point $(1, 1, 1)$. If the coordinates of A and B are $(3, -5, 7)$ and $(-1, 7, -6)$ respectively, find the coordinates of the point C.

Solution:

Let the position vectors of points A, B, and C be:

$$\mathbf{A} = \begin{pmatrix} 3 \\ -5 \\ 7 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -1 \\ 7 \\ -6 \end{pmatrix}, \quad \mathbf{C} = \mathbf{C} \tag{0.1}$$

The centroid $\mathbf{G}$ of triangle ABC is given by:

$$\mathbf{G} = \frac{1}{3}(\mathbf{A} + \mathbf{B} + \mathbf{C}) \tag{0.2}$$

Given:

$$\mathbf{G} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{0.3}$$

Substitute and solve:

$$\frac{1}{3}\left(\begin{pmatrix} 3 \\ -5 \\ 7 \end{pmatrix} + \begin{pmatrix} -1 \\ 7 \\ -6 \end{pmatrix} + \mathbf{C} \right) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{0.4}$$

Add vectors:

$$\frac{1}{3}\left(\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + \mathbf{C} \right) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{0.5}$$

Multiply both sides by 3:

$$\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} + \mathbf{C} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} \tag{0.6}$$

Subtract:

$$\mathbf{C} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \tag{0.7}$$

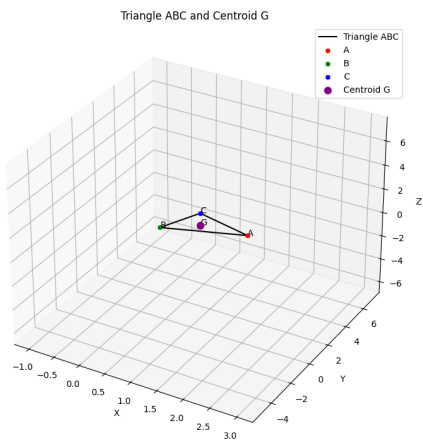**Therefore, the coordinates of point C are $(1, 1, 2)$.**

Fig. 0.1: 3D plot of triangle ABC and centroid G