# Matrices in Geometry 1.9.26'

EE25BTECH11037 - Divyansh

**Question:** Find the value of k, if the point $\mathbf{P}(2,4)$ is equidistant from point $\mathbf{A}(5,k)$ and $\mathbf{B}(k,7)$

**Given:** $\mathbf{P}\begin{pmatrix}2\\4\end{pmatrix}$, $\mathbf{A}\begin{pmatrix}5\\k\end{pmatrix}$ and a point $\mathbf{B}\begin{pmatrix}k\\7\end{pmatrix}$ such that $\mathbf{P}$ is equidistant from $\mathbf{A}$ and $\mathbf{B}$.

$$\therefore \|\mathbf{P} - \mathbf{A}\| = \|\mathbf{P} - \mathbf{B}\| \tag{1}$$

$$\text{On squaring both the sides, we get} \tag{2}$$

$$\|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{A}\|^2 \tag{3}$$

$$\left(\mathbf{P} - \mathbf{A}\right)^\top \left(\mathbf{P} - \mathbf{A}\right) = \left(\mathbf{P} - \mathbf{B}\right)^\top \left(\mathbf{P} - \mathbf{B}\right) \tag{4}$$

$$\mathbf{P}^\top\mathbf{P} - 2\mathbf{P}^\top\mathbf{A} + \mathbf{A}^\top\mathbf{A} = \mathbf{P}^\top\mathbf{P} - 2\mathbf{P}^\top\mathbf{B} + \mathbf{B}^\top\mathbf{B} \tag{5}$$

$$\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 = 2\mathbf{P}^\top\left(\mathbf{A} - \mathbf{B}\right) \tag{6}$$

$$\left\|\begin{pmatrix}5\\k\end{pmatrix}\right\| - \left\|\begin{pmatrix}k\\7\end{pmatrix}\right\| = 2\begin{pmatrix}2 & 4\end{pmatrix}\begin{pmatrix}5-k\\k-7\end{pmatrix} \tag{7}$$

$$25 + k^2 - 49 - k^2 = 2\left(10 - 2k + 4k - 28\right) \tag{8}$$

$$-24 = 2\left(2k - 18\right) \implies -12 = 2k - 18 \implies 2k = 6 \implies k = 3 \tag{9}$$
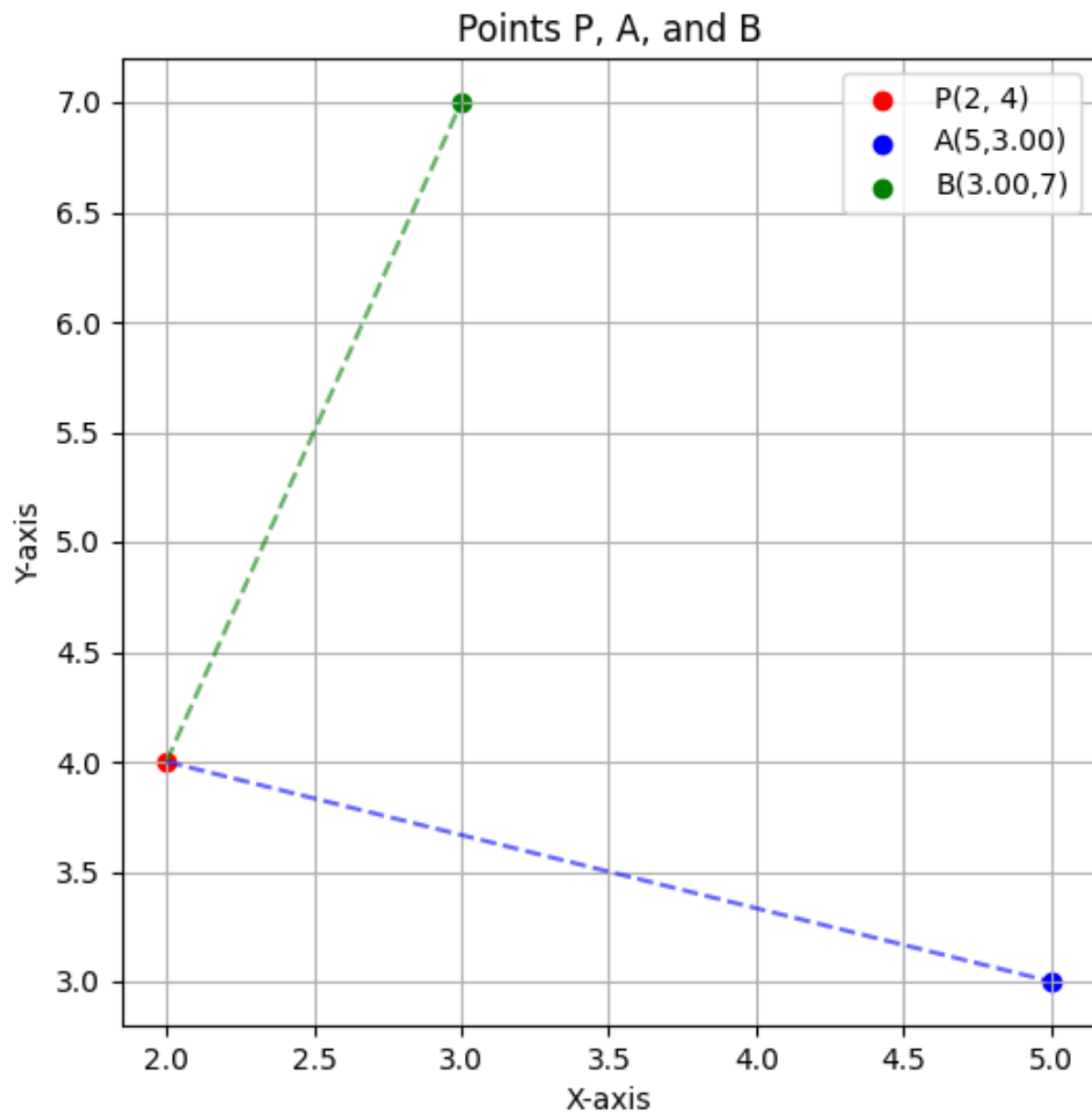
$$\text{Hence, the final answer is } \boxed{k = 3} \tag{10}$$

Fig. 1: Plot for 1.9.26