

1.4.13

AI25BTECH11038 – Tejas Uppala

Question:

Find the points on the X axis which are at a distance on $2\sqrt{5}$ from the point $(7, -4)$. How many such points are there?

Solution:

Given that the point $(7, -4)$ is at a distance $2\sqrt{5}$ from, assume a point P that lies on the X axis,

Let the given point be denoted A and its position vector \mathbf{a} and the position vector of P will be

$$\mathbf{p} = x \cdot \mathbf{e}_1 \quad (1)$$

The distance between the two given points will be,

$$\|\mathbf{a} - \mathbf{p}\| = 2\sqrt{5} \quad (2)$$

$$\|\mathbf{a} - x \cdot \mathbf{e}_1\| = 2\sqrt{5} \quad (3)$$

We know that,

$$\|H\|^2 = H \cdot H^T \quad (4)$$

So,

$$(\mathbf{a} - x \cdot \mathbf{e}_1) \cdot (\mathbf{a} - x \cdot \mathbf{e}_1)^T = (2\sqrt{5})^2 \quad (5)$$

$$(\mathbf{a} - x \cdot \mathbf{e}_1) \cdot (\mathbf{a}^T - x \cdot \mathbf{e}_1^T) = 20 \quad (6)$$

$$(\mathbf{a} \cdot \mathbf{a}^T) - (x \cdot \mathbf{a} \cdot \mathbf{e}_1^T) - (x \cdot \mathbf{a}^T \cdot \mathbf{e}_1) + (x^2 \cdot \mathbf{e}_1 \cdot \mathbf{e}_1^T) = 20 \quad (7)$$

$$(x^2 \cdot \mathbf{e}_1 \cdot \mathbf{e}_1^T) - ((\mathbf{a} \cdot \mathbf{e}_1^T + \mathbf{a}^T \cdot \mathbf{e}_1) \cdot x) + (\mathbf{a} \cdot \mathbf{a}^T) - 20 = 0 \quad (8)$$

$$(\mathbf{a} \cdot \mathbf{e}_1^T + \mathbf{a}^T \cdot \mathbf{e}_1) = 2 \cdot \mathbf{a} \cdot \mathbf{e}_1^T \quad (9)$$

On solving the quadratic for x,

$$x = \mathbf{a} \cdot \mathbf{e}_1^T \pm \sqrt{(\mathbf{a} \cdot \mathbf{e}_1^T)^2 - \|\mathbf{a}\|^2 + 20} \quad (10)$$

On substituting the values of \mathbf{a} and \mathbf{e}_1 ,

$$x = 7 \pm \sqrt{7^2 - 7^2 - 4^2 + 20} \quad (11)$$

$$x = 7 \pm 2 = 9, 5 \quad (12)$$

Hence, there exist two values if x i.e, there exist two points P on the X axis for the distance between the given point and P to be $2\sqrt{5}$

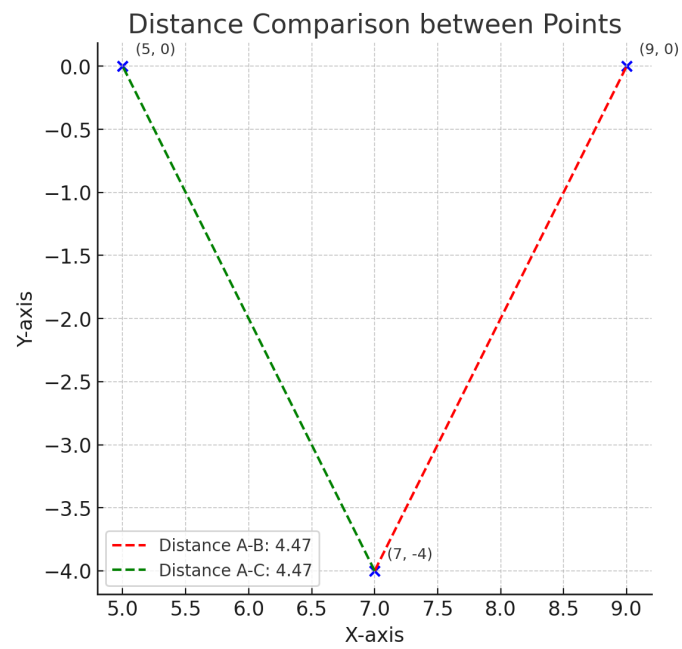


Fig. 1: The plot of the points A and the two points on the X axis