10.7.76

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Question.The number of common tangents to the circles $x^2+y^2 = 4$ and $x^2+y^2-6x-8y = 24$ is

1) 0
2) 1
3) 2
4) 3

Solution:
Let us solve the given equation theoretically and then verify the solution computationally.
Let the equation of 1st circle be:

$$\|\mathbf{x}\|^2 + 2\mathbf{u_1}^T\mathbf{x} + f_1 = 0 \tag{1}$$

Let the equation of 2nd circle be

$$\|\mathbf{x}\|^2 + 2\mathbf{u_2}^T\mathbf{x} + f_2 = 0 \tag{2}$$

Let $\mathbf{c_1}$ and $\mathbf{c_2}$ be the center of circles 1 and 2.
From the given information:

$$\mathbf{u_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \ , f_1 = -4 \ \ and \ \ \mathbf{c_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3}$$

$$\mathbf{u_2} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \ , f_2 = -24 \ \ and \ \ \mathbf{c_2} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{4}$$

$$r_1 = \sqrt{\|\mathbf{u_1}\|^2 - f_1} = \sqrt{0+4} = 2 \tag{5}$$

$$r_2 = \sqrt{\|\mathbf{u_2}\|^2 - f_2} = \sqrt{25+24} = 7 \tag{6}$$

Now calculating the distance between $\mathbf{c_1}$ and $\mathbf{c_2}$:

$$\|\mathbf{c_2} - \mathbf{c_1}\| = \left\| \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\| = 5 \tag{7}$$

We also have:

$$|r_2 - r_1| = 5 \tag{8}$$

Here we can observe that the distance between the centers of two circles is equal to the absolute difference of their radii . So the two circles touch each other internally at one

point
Therefore the number of common tangents to the given two circles is 1

From the figure it is clearly verified that the theoretical solution matches with the computational solution.