12.388

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Question:

For the matrix $\mathbf{A}$=$\begin{pmatrix} 5 & 3 \\ 1 & 3 \end{pmatrix}$,ONE of the normalized eigenvectors is given as (ME 2012)

1) $\begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \end{pmatrix}$ 2) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$ 3) $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \end{pmatrix}$ 4) $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$

Solution:
Given

$$\mathbf{A} = \begin{pmatrix} 5 & 3 \\ 1 & 3 \end{pmatrix} \tag{1}$$

For matrix $\mathbf{A}$ the characterstic polynomial is given by

$$|\mathbf{A} - \lambda\mathbf{I}| = 0 \tag{2}$$

$$\text{char}\mathbf{A} = \begin{vmatrix} 5-\lambda & 3 \\ 1 & 3-\lambda \end{vmatrix} = 0 \tag{3}$$

$$\implies (5-\lambda)(3-\lambda) - 3 = 0 \tag{4}$$

$$\implies \lambda^2 - 8\lambda + 12 = 0 \tag{5}$$

$$\implies (\lambda-2)(\lambda-6) = 0 \tag{6}$$

Thus,the eigen values are given by

$$\lambda_1 = 6 \text{ and } \lambda_2 = 2 \tag{7}$$

For λ_1, the augmented matrix formed from the eigenvalue-eigenvector equation is

$$\begin{pmatrix} -1 & 3 \\ 1 & -3 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 + R_1} \begin{pmatrix} -1 & 3 \\ 0 & 0 \end{pmatrix} \tag{8}$$

Hence,the normalized eigenvector is

$$\mathbf{v_1} = \frac{1}{\sqrt{10}} \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{9}$$

For λ_2, the augmented matrix formed from the eigenvalue-eigenvector equation is

$$\begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - \frac{1}{3} \times R_1} \begin{pmatrix} 3 & 3 \\ 0 & 0 \end{pmatrix} \tag{10}$$

Hence,the normalized eigenvector is

$$\mathbf{v_2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{11}$$

The normalized eigen vectors are

$$\mathbf{v_1} = \begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \end{pmatrix} \text{ and } \mathbf{v_2} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} \tag{12}$$