

4.3.13

EE25BTECH11026-Harsha

Question:

Equations of the diagonals of the square formed by the lines $x = 0$, $y = 0$, $x = 1$ and $y = 1$ are _____.

Solution:

Let us solve the given equation theoretically and then verify the solution computationally.

According to the question,

The vertices of the square are ,

$$\mathbf{a} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \mathbf{d} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

To compute the equation of the diagonals , we can use the normal form of the equation, which is given by

$$\mathbf{n}^T \mathbf{x} = 0 \text{ for the lines passing through the origin}$$

$$\mathbf{n}^T \mathbf{x} = 1 \text{ for the lines not passing through the origin}$$

where,

\mathbf{n} – vector orthogonal to the direction vector

$$\mathbf{x} = \begin{pmatrix} x & y \end{pmatrix}^T$$

For diagonal $\mathbf{c} - \mathbf{a}$, as it passes through the origin,

$$\therefore \mathbf{n}^T \mathbf{x} = 0$$

By substituting the vector through which it passes through,

$$\mathbf{n}^T \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0$$

$$\Rightarrow \mathbf{n} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

But, for diagonal $\mathbf{d} - \mathbf{b}$, as the diagonal doesn't pass through the origin,

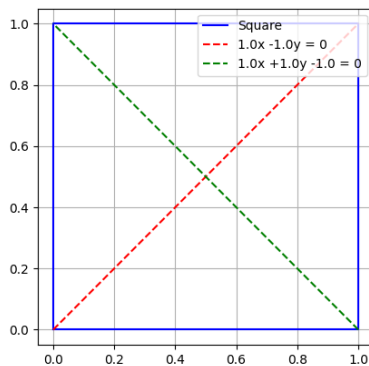
$$\mathbf{n}^\top \mathbf{x} = 1$$

$$\therefore \mathbf{n}^\top \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1$$

From the figure, it is clearly verified that the theoretical solution matches with the computational solution.



Plot of Square with diagonals