12.235

EE25BTECH11013 - Bhargav

Question:
A system of equations represented as

$$\begin{pmatrix}1 & -1 & 2\\ 2 & 1 & 4\\ 1 & 3 & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} = \begin{pmatrix}4\\ y\\ 3\end{pmatrix} is, \tag{0.1}$$

1) consistent and has a unique solution
2) inconsistent and has no solution
3) consistent and infinite solution
4) inconsistent and has unique solution

Solution:
This can be represented as an augmented matrix and can be solved by using Gaussian elimination.

$$\left(\begin{array}{ccc|c}1 & -1 & 2 & 4\\ 2 & 1 & 4 & y\\ 1 & 3 & 1 & 3\end{array}\right) \xleftrightarrow[R_3 \leftarrow R_3 - R_1]{R_2 \leftarrow R_2 - 2R_1} \left(\begin{array}{ccc|c}1 & -1 & 2 & 4\\ 0 & 3 & 0 & y-8\\ 0 & 4 & -1 & -1\end{array}\right) \tag{4.1}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - 4R_2]{R_2 \leftarrow \frac{R_2}{3}} \left(\begin{array}{ccc|c}1 & -1 & 2 & 4\\ 0 & 1 & 0 & \frac{y-8}{3}\\ 0 & 0 & -1 & \frac{29-4y}{3}\end{array}\right) \xleftrightarrow[R_1 \leftarrow R_1 - 2R_3]{R_3 \leftarrow -R_3} \tag{4.2}$$

$$\left(\begin{array}{ccc|c}1 & -1 & 0 & \frac{70-8y}{3}\\ 0 & 1 & 0 & \frac{y-8}{3}\\ 0 & 0 & 1 & \frac{4y-29}{3}\end{array}\right) \xleftrightarrow{R_1 \leftarrow R_1 + R_2} \left(\begin{array}{ccc|c}1 & 0 & 0 & \frac{62-7y}{3}\\ 0 & 1 & 0 & \frac{y-8}{3}\\ 0 & 0 & 1 & \frac{4y-29}{3}\end{array}\right) \tag{4.3}$$

$$\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix} = \begin{pmatrix}\frac{62-7y}{3}\\ \frac{y-8}{3}\\ \frac{4y-29}{3}\end{pmatrix} \tag{4.4}$$

Since $y \in \mathbf{R}$, we can conclude that there exists a unique solution and the system of equations is consistent.

Option (1) is the correct answer

This can be verified by finding the point of intersection of 3 planes: As an example, take $y = 8$

$$x_1 - x_2 + 2x_3 = 4 \tag{4.5}$$

$$2x_1 + x_2 + 4x_3 = 8 \tag{4.6}$$

$$x_1 + 3x_2 + x_3 = 3 \tag{4.7}$$

The point of intersection of the planes from (4.4) is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \tag{4.8}$$