12.495

AI25BTECH11001 - ABHISEK MOHAPATRA

October 4, 2025

Question: The matrix $\mathbf{P}$ is the inverse of a matrix $\mathbf{Q}$. If $\mathbf{I}$ denotes the identity matrix, which one of the following options is correct?

a) $\mathbf{PQ} = \mathbf{I}$ but $\mathbf{QP} \neq \mathbf{I}$

b) $\mathbf{QP} = \mathbf{I}$ but $\mathbf{PQ} \neq \mathbf{I}$

c) $\mathbf{PQ} = \mathbf{I}$ and $\mathbf{QP} = \mathbf{I}$

d) $\mathbf{PQ} - \mathbf{QP} = \mathbf{I}$

Solution: Let $\mathbf{P}$ is inverse of a matrix $\mathbf{Q}$ and $\mathbf{PQ} = \mathbf{I}$

Let there exist $\mathbf{C}$ such that $\mathbf{QC} = \mathbf{I}$

$$\mathbf{QC} = \mathbf{I} \tag{0.1}$$

$$\Rightarrow \mathbf{PQC} = \mathbf{P} \Rightarrow \mathbf{C} = \mathbf{P} \tag{0.2}$$

so $\mathbf{PQ} = \mathbf{QP} = \mathbf{I}$. So, option (c) is correct.