Question 2.10.29

Al25BTECH11040 - Vivaan Parashar

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Question:

The volume of the parallelopiped whose sides are given by $OA = 2\mathbf{i} - 2\mathbf{j}$, $OB = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $OC = 3\mathbf{i} - \mathbf{k}$, is

Solution:

To find the volume of the parallelopiped, we use the scalar triple product formula:

$$V = \det((\mathbf{OA}\,\mathbf{OB}\,\mathbf{OC}))$$
, where (1)

$$\mathbf{OA} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}, \quad \mathbf{OB} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \quad \mathbf{OC} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} \tag{2}$$

$$\therefore, \quad V = \det\begin{pmatrix} 2 & 1 & 3 \\ -2 & 1 & 0 \\ 0 & -1 & -1 \end{pmatrix}) \tag{3}$$

Therefore, $V = \sqrt{|\det(\mathbf{G})|} = \sqrt{4} = 2$.

Plot:

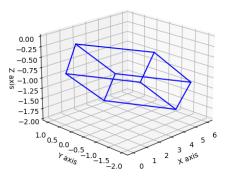


Figure: Parallelopiped formed by vectors OA, OB and OC