1.3.10

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Question (1.3.10)

Find the ratio in which the point $P = (8, y)$ divides the line segment joining $A = (1, 2)$ and $B = (2, 3)$. Also, find the value of y.

Collinearity Condition

Points **A**, **B**, and **P** are collinear: $\implies \text{rank}\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{P}-\mathbf{A}\end{pmatrix} = 1$

Calculate vectors:

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix}2-1\\3-2\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$$

$$\mathbf{P}-\mathbf{A} = \begin{pmatrix}8-1\\y-2\end{pmatrix} = \begin{pmatrix}7\\y-2\end{pmatrix}$$

Matrix Setup and Rank Reduction

Matrix: $\begin{pmatrix} 1 & 7 \\ 1 & y-2 \end{pmatrix}$

Row reduce: $R_1 \to R_1 - R_2 \implies \begin{pmatrix} 0 & 9-y \\ 1 & y-2 \end{pmatrix}$

For rank 1: $9 - y = 0 \implies y = 9$

$P = \begin{pmatrix} 8 \\ 9 \end{pmatrix}$

Ratio of Division

Using the section formula, $\mathbf{P} = \frac{k\mathbf{B}+\mathbf{A}}{k+1}$

The vector formula for ratio $k:1$: $k = \frac{(\left(A\right)-\left(P\right))^T(\left(P\right)-\left(B\right))}{\|\left(P\right)-\left(B\right)\|^2}$

Substitute

Substituting the values of **A**, **B** and **P**,

$$k = \frac{\begin{pmatrix} -7 & -7 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \end{pmatrix}}{\left\| \begin{pmatrix} 6 \\ 6 \end{pmatrix} \right\|^2} = \frac{-7}{6} \tag{1}$$

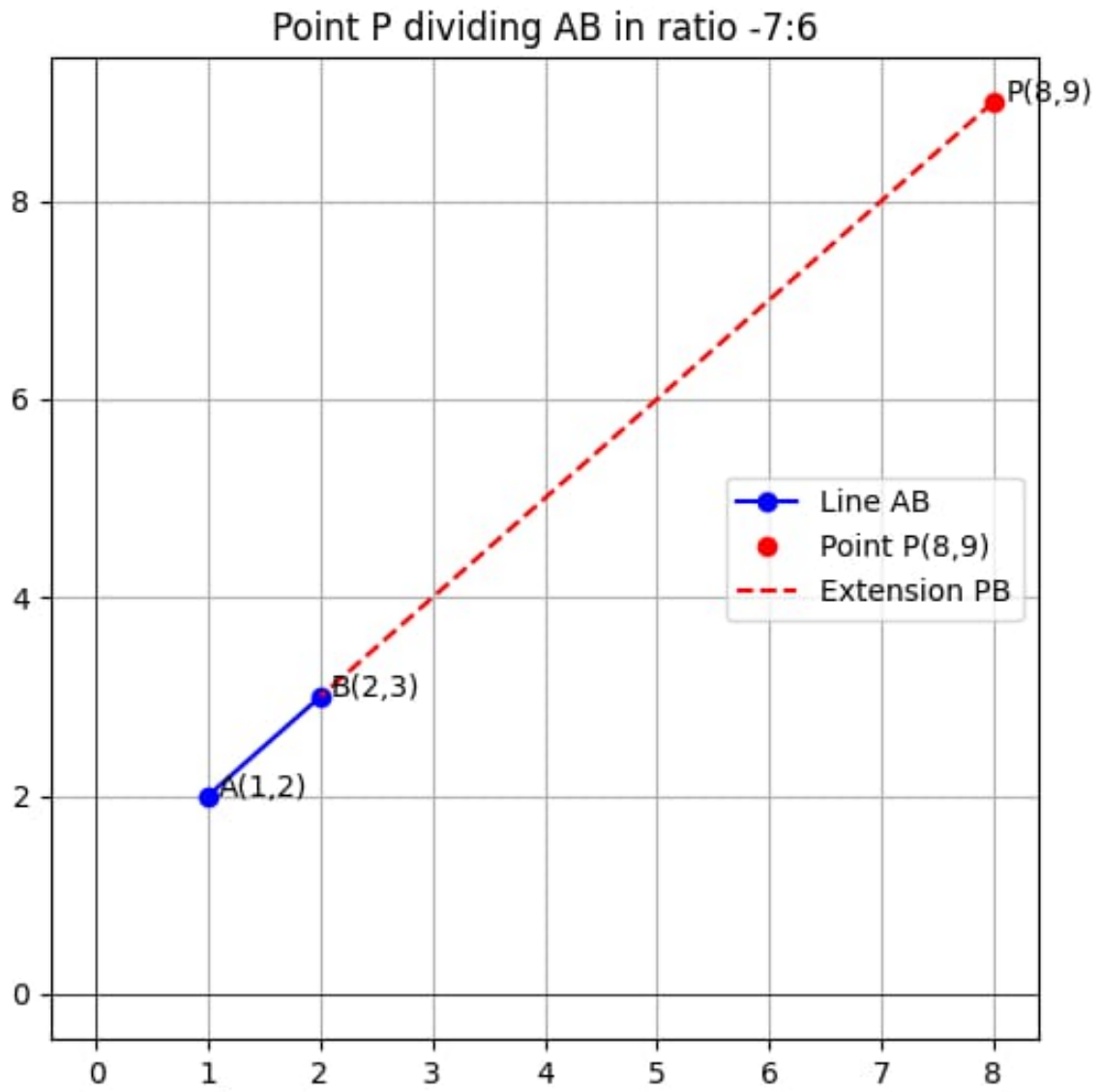
Result

$$P = \begin{pmatrix} 8 \\ 9 \end{pmatrix}$$

Thus, the ratio in which the point P divides the line segment AB is

$$\boxed{-7 : 6}$$

Plot


Point P dividing AB in ratio -7:6
P(8,9)
B(2,3)
A(1,2)
Line AB
Point P(8,9)
Extension PB
0
1
2
3
4
5
6
7
8
0
2
4
6
8

C Code

```c
#include <stdio.h>

int main() {
// Given points A and B
double Ax = 1, Ay = 2;
double Bx = 2, By = 3;

// Given x-coordinate of P
double Px = 8;
double Py;

// Solve for Py using rank condition:
// For matrix [[1, Px - Ax], [1, Py - Ay]] to have rank 1,
// second column must be proportional:
// => (Px - Ax) - (Py - Ay) = 0 => Py = Px - Ax + Ay
Py = Px - Ax + Ay;
```

C Code

```
printf("Calculated y-coordinate of P: %lf\n", Py);

// Calculate ratio k using section formula rearrangement
double numerator = (Ax - Px)*(Px - Bx) + (Ay - Py)*(Py - By);
double denominator = (Px - Bx)*(Px - Bx) + (Py - By)*(Py - By)

double k = numerator / denominator;

printf("Ratio k in which P divides AB: %lf\n", k);

return 0;
}
```

Python code

```
import numpy as np

# Given points A and B
A = np.array([1, 2])
B = np.array([2, 3])

# Given x-coordinate of P
Px = 8

# Solve for Py using collinearity rank condition:
# (Px - Ax) - (Py - Ay) = 0 => Py = Px - Ax + Ay
Py = Px - A[0] + A[1]
P = np.array([Px, Py])

print(f"Calculated y-coordinate of P: {Py}")
```

Python code

```
# Calculate ratio k using vector formula
numerator = np.dot(A - P, P - B)
denominator = np.linalg.norm(P - B) ** 2
k = numerator / denominator

print(f"Ratio k in which P divides AB: {k}")
```