4.3.20

EE25BTECH11031 - Sai Sreevallabh

Question:

Find the ratio in which the Y-axis divides the line segment joining the points $(5, -6)$ and $(-1, -4)$. Also find the point of intersection.

Solution:

Given points are

$$\mathbf{A} = \begin{pmatrix} 5 \\ -6 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} \tag{0.1}$$

Let $\mathbf{P}$ be a point on the Y-axis. We can assume it to be

$$\mathbf{P} = \begin{pmatrix} 0 \\ y \end{pmatrix} \tag{0.2}$$

$\mathbf{A}$, $\mathbf{B}$ and $\mathbf{P}$ are collinear.

$$\mathbf{P} - \mathbf{A} = \begin{pmatrix} -5 \\ y + 6 \end{pmatrix} \ , \ \mathbf{B} - \mathbf{A} = \begin{pmatrix} -6 \\ 2 \end{pmatrix} \tag{0.3}$$

$$\begin{pmatrix} \mathbf{P} - \mathbf{A} & \mathbf{B} - \mathbf{A} \end{pmatrix}^T = \begin{pmatrix} -5 & -6 \\ y + 6 & 2 \end{pmatrix}^T \tag{0.4}$$

$$= \begin{pmatrix} -5 & y + 6 \\ -6 & 2 \end{pmatrix} \tag{0.5}$$

Converting into echelon form using row operations

$$\begin{pmatrix} x - 1 & -3 \\ 3 & 2 \end{pmatrix} \xleftrightarrow{R_2 \to R_2 - \frac{6}{5} R_1} \begin{pmatrix} -5 & y + 6 \\ 0 & \frac{-6y - 26}{5} \end{pmatrix} \tag{0.6}$$

The points are collinear. Hence the rank of the above matrix must be 1. So,

$$\frac{6y + 26}{5} = 0 \tag{0.7}$$

$$\implies y = -\frac{13}{3} \tag{0.8}$$

Let $\mathbf{P}$ divide the line joining points $\mathbf{A}$ and $\mathbf{B}$ in the ratio k : 1.

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k+1} \tag{0.9}$$

$$k\,(\mathbf{P} - \mathbf{B}) = \mathbf{A} - \mathbf{P} \tag{0.10}$$

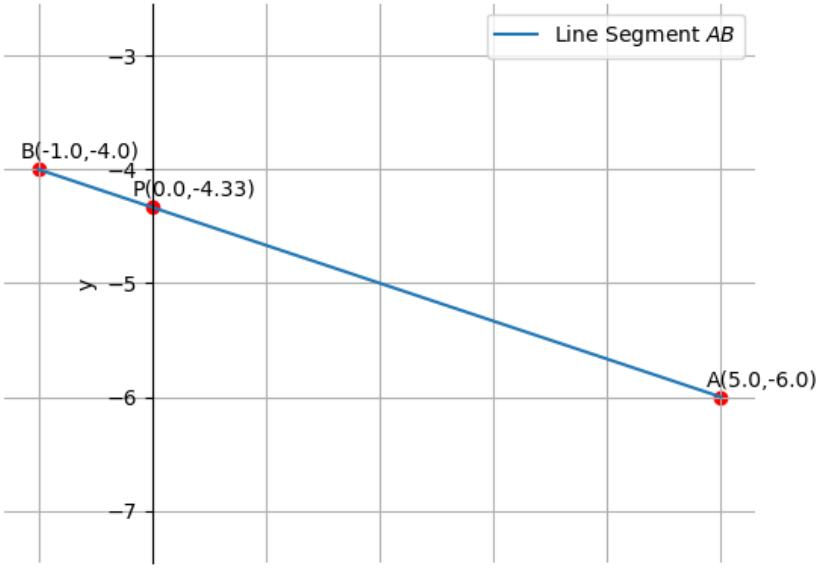
$$k = \frac{(\mathbf{P} - \mathbf{B})^T\,(\mathbf{A} - \mathbf{P})}{\|\,(\mathbf{P} - \mathbf{B})\,\|^2} \tag{0.11}$$

$$k = \frac{\begin{pmatrix}1 & y+4\end{pmatrix}\begin{pmatrix}5\\-y-6\end{pmatrix}}{\left\|\begin{pmatrix}1\\y+4\end{pmatrix}\right\|^2} \tag{0.12}$$

Substituting the value of y as $-\frac{13}{3}$, we get the value of k as

$$k = 5 \tag{0.13}$$

$\therefore$ The point $\mathbf{P}\begin{pmatrix}0\\-\frac{13}{3}\end{pmatrix}$ on the X-axis divides the line segment in the ratio 5 : 1.


Line Segment AB
B(-1.0,-4.0)
P(0.0,-4.33)
A(5.0,-6.0)
y
−3
−4
−5
−6
−7