2.6.14

AI25BTECH11008 - Chiruvella Harshith Sharan

Question: Find the area of the parallelogram whose diagonals are $\mathbf{d_1} = 2\hat{i} - \hat{j} + \hat{k}$ and $\mathbf{d_2} = \hat{i} + 3\hat{j} - \hat{k}$.

Solution:

The diagonals are

$$\mathbf{d_1} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \quad \mathbf{d_2} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}. \tag{1}$$

The area of the parallelogram is half the magnitude of the cross product of its diagonals,

$$A = \frac{1}{2} \|\mathbf{d_1} \times \mathbf{d_2}\|. \tag{2}$$

Now, using the identity

$$\|\mathbf{d_1} \times \mathbf{d_2}\|^2 = \|\mathbf{d_1}\|^2 \|\mathbf{d_2}\|^2 - (\mathbf{d_1} \cdot \mathbf{d_2})^2, \tag{3}$$

we compute

$$\|\mathbf{d_1}\|^2 = 2^2 + (-1)^2 + 1^2 = 6, \tag{4}$$

$$\|\mathbf{d_2}\|^2 = 1^2 + 3^2 + (-1)^2 = 11, \tag{5}$$

$$\mathbf{d_1} \cdot \mathbf{d_2} = (2)(1) + (-1)(3) + (1)(-1) = -2. \tag{6}$$

Hence,

$$\|\mathbf{d_1} \times \mathbf{d_2}\|^2 = (6)(11) - (-2)^2 \tag{7}$$

$$= 66 - 4 = 62. \tag{8}$$

$$\implies \|\mathbf{d_1} \times \mathbf{d_2}\| = \sqrt{62}.$$

Finally,

$$A = \frac{1}{2} \sqrt{62}.$$

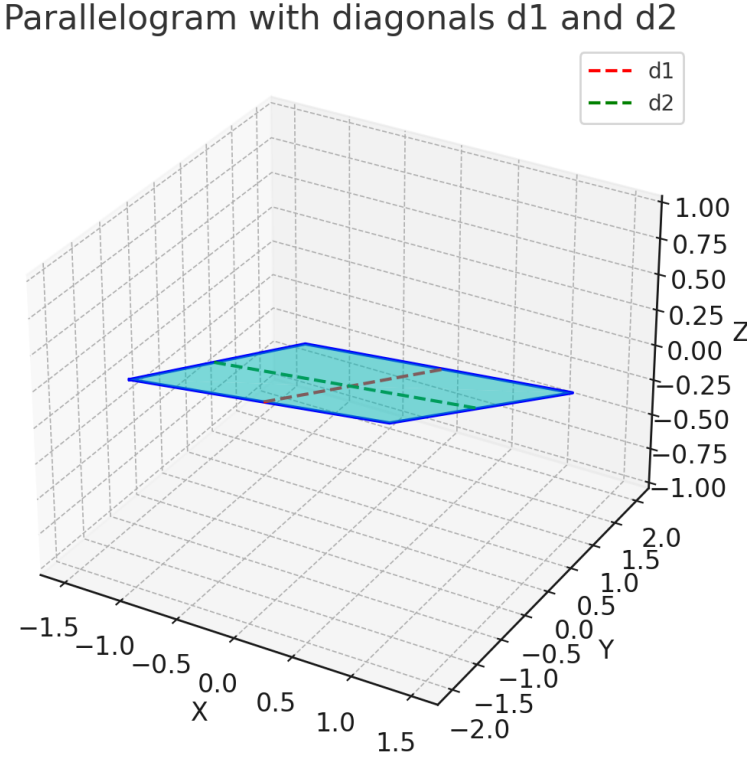
Parallelogram with diagonals d1 and d2
d1
d2
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Z
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
Y
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
X

Fig. 0