2.10.33

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Question

Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$, $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$

1. are collinear

2. form an equilateral triangle

3. form a scalene triangle

4. form a right angled triangle

Solution

Let $\mathbf{A}$ be $\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$, $\mathbf{B}$ be $\begin{pmatrix} \beta \\ \gamma \\ \alpha \end{pmatrix}$, and $\mathbf{C}$ be $\begin{pmatrix} \gamma \\ \alpha \\ \beta \end{pmatrix}$.

- $\mathbf{A} - \mathbf{B}$ is $\begin{pmatrix} \alpha - \beta \\ \beta - \gamma \\ \gamma - \alpha \end{pmatrix}$
- $\mathbf{B} - \mathbf{C}$ is $\begin{pmatrix} \beta - \gamma \\ \gamma - \alpha \\ \alpha - \beta \end{pmatrix}$
- $\mathbf{C} - \mathbf{A}$ is $\begin{pmatrix} \gamma - \alpha \\ \alpha - \beta \\ \beta - \gamma \end{pmatrix}$

Then norms of $\mathbf{A} - \mathbf{B}, \mathbf{B} - \mathbf{C}, \mathbf{C} - \mathbf{A}$ are all equal, and equal to

$$\sqrt{(\alpha - \beta)^2 + (\beta - \gamma)^2 + (\gamma - \alpha)^2}$$

The three points therefore form an equilateral triangle, so option (2) is correct.