4.3.32

EE25BTECH11042 - Nipun Dasari

Question:
Find the slope of a line which cuts off intercepts of equal length on the axes is. Solve using matrices.
Solution:

Consider normal form of a line:

$$\mathbf{n}^T\mathbf{x} = c, \text{where } \mathbf{n} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \tag{0.1}$$

Given that equal intercepts are cut off we get 2 cases: **Case 1: The intercepts are equal** $(b = a)$

$$\implies \mathbf{n}^T \begin{pmatrix} a \\ 0 \end{pmatrix} = c \implies a\cos\alpha = c \tag{0.2}$$

$$\implies \mathbf{n}^T \begin{pmatrix} 0 \\ a \end{pmatrix} = c \implies a\sin\alpha = c \tag{0.3}$$

$$(0.2)/(0.3) \implies \tan\alpha = 1 \tag{0.4}$$

Case 1: The intercepts are negatives of each other $(-b = a)$

$$\implies \mathbf{n}^T \begin{pmatrix} a \\ 0 \end{pmatrix} = c \implies a\cos\alpha = c \tag{0.5}$$

$$\implies \mathbf{n}^T \begin{pmatrix} 0 \\ -a \end{pmatrix} = c \implies -a\sin\alpha = c \tag{0.6}$$

$$(0.5)/(0.6) \implies \tan\alpha = -1 \tag{0.7}$$

This gives us two values of slope $m = \tan\alpha$

$$\therefore m = \pm 1 \tag{0.8}$$

Lines with Intercepts of Equal Length