4.10.17

EE25BTECH11015 - Bhoomika V

Question :-
Compute the area bounded by the lines $x + 2y = 2$, $y - x = 1$, and $2x + y = 7$.

Solution:
Equations of the lines

$$L_1 : x + 2y = 2, \quad L_2 : y - x = 1, \quad L_3 : 2x + y = 7$$

Finding intersection points using RREF

Intersection of L_1 and L_2:

$$\begin{cases} x + 2y = 2 \\ -x + y = 1 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ -1 & 1 & 1 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 1 \end{array}\right]$$

$$A = (0, 1)$$

Intersection of L_2 and L_3:

$$\begin{cases} -x + y = 1 \\ 2x + y = 7 \end{cases}$$

$$\left[\begin{array}{cc|c} -1 & 1 & 1 \\ 2 & 1 & 7 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \end{array}\right]$$

$$B = (2, 3)$$

Intersection of L_1 and L_3:

$$\begin{cases} x + 2y = 2 \\ 2x + y = 7 \end{cases}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 2 & 1 & 7 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -1 \end{array}\right]$$

$$C = (4, -1)$$

Area of the triangle

$$\Delta = \tfrac{1}{2} \left|(\mathbf{A} - \mathbf{B}) \times (\mathbf{B} - \mathbf{C})\right|$$

Compute:

$$\mathbf{A} - \mathbf{B} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}, \quad \mathbf{B} - \mathbf{C} = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$

$$(\mathbf{A}-\mathbf{B})\times(\mathbf{B}-\mathbf{C}) = \begin{vmatrix} -2 & -2 \\ -2 & 4 \end{vmatrix} = (-2)(4) - (-2)(-2) = -12$$

$$\Delta = \frac{1}{2}|-12| = 6$$

Final Answer:

$$\text{Area} = 6$$

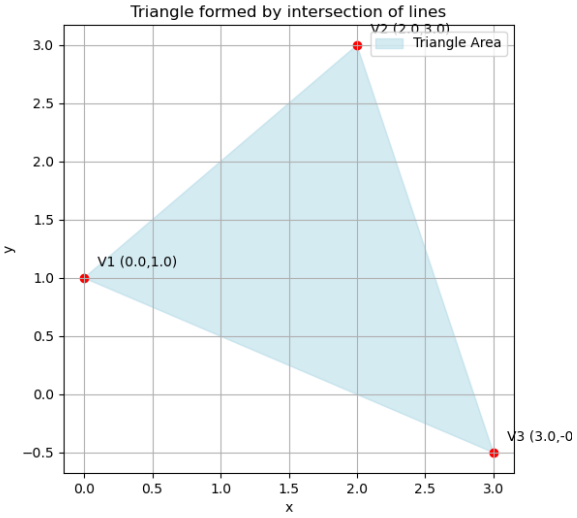

Fig. 0.1