5.8.8

EE25BTECH11010 - Arsh Dhoke

Question:
Places A and B are 100km apart on a highway. One car starts from A and another from B at the same time. If the car travel in the same direction at different speeds, they meet in 5hrs. If they travel towards each other, they meet in 1hr. What are the speeds of the two cars?

Solution:
Cars meet in 1 hr when moving towards each other:

$$v_1 + v_2 = \frac{100}{1} \tag{0.1}$$

Cars meet in 5 hr when moving in the same direction:

$$v_1 - v_2 = \frac{100}{5} \tag{0.2}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 100 \\ 20 \end{pmatrix} \tag{0.3}$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 100 \\ 20 \end{pmatrix} \tag{0.4}$$

The augmented matrix for the given matrix will be

$$\left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{array}\right) \xleftrightarrow[R_2 \to R_2 - R_1]{} \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & -2 & -1 & 1 \end{array}\right) \tag{0.5}$$

$$\xleftrightarrow[R_2 \to -\frac{1}{2}R_2]{} \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \end{array}\right) \tag{0.6}$$

$$\xleftrightarrow[R_1 \to R_1 - R_2]{} \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} \end{array}\right) \tag{0.7}$$

$$\therefore \quad \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \tag{0.8}$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 100 \\ 20 \end{pmatrix} \tag{0.9}$$

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 60 \\ 40 \end{pmatrix} \tag{0.10}$$

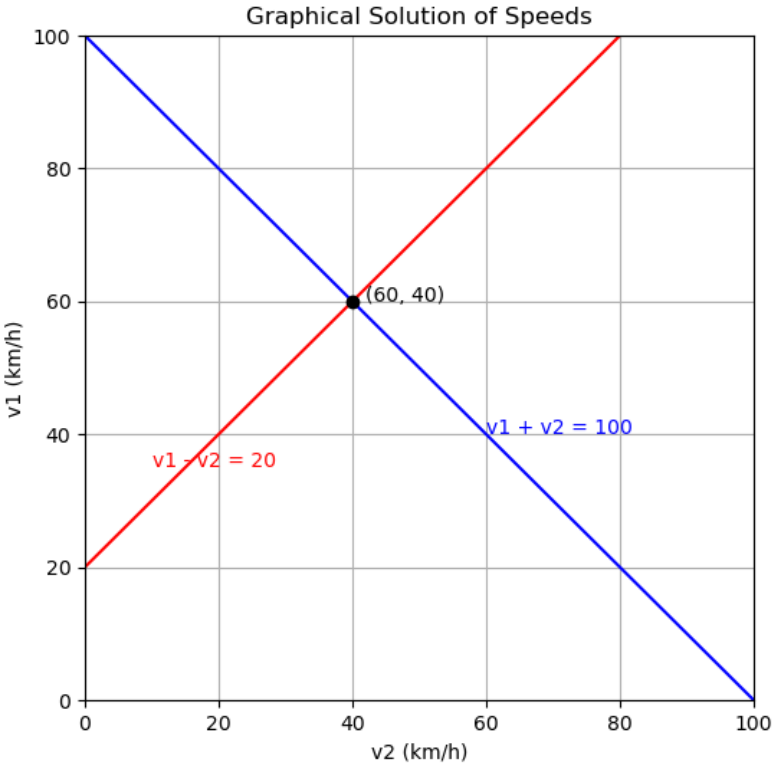
Graphical Solution of Speeds
v1 (km/h)
v2 (km/h)
(60, 40)
v1 + v2 = 100
v1 - v2 = 20
0
20
40
60
80
100

Fig. 0.1: Graph