

## 2.4.22

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**Question** Find the equation of a plane which bisects perpendicularly the line joining the points  $A(2, 3, 4)$  and  $B(4, 5, 8)$  at right angles.

**Solution:**

Let us solve the given equation theoretically and then verify the solution computationally. Let,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 4 \\ 5 \\ 8 \end{pmatrix} \quad (1)$$

Given that the plane is a perpendicular bisector to the line joining points A and B. Since it is a perpendicular bisector to the line joining points A and B, the midpoint of the line joining points A and B lies on the plane.

Let the midpoint of points A and B be X. Then

$$\|\mathbf{X} - \mathbf{A}\|^2 = \|\mathbf{X} - \mathbf{B}\|^2 \quad (2)$$

$$\|\mathbf{X}\|^2 + \|\mathbf{A}\|^2 - 2\|\mathbf{X}\| \|\mathbf{A}\| = \|\mathbf{X}\|^2 + \|\mathbf{B}\|^2 - 2\|\mathbf{X}\| \|\mathbf{B}\| \quad (3)$$

$$\|\mathbf{A}\|^2 - 2\|\mathbf{X}\| \|\mathbf{A}\| = \|\mathbf{B}\|^2 - 2\|\mathbf{X}\| \|\mathbf{B}\| \quad (4)$$

$$\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 = 2\mathbf{X}^T \mathbf{A} - 2\mathbf{X}^T \mathbf{B} \quad (5)$$

$$\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 = 2\mathbf{X}^T (\mathbf{A} - \mathbf{B}) \quad (6)$$

which can be written as :

$$\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 = 2(\mathbf{A} - \mathbf{B})^T \mathbf{X} \quad (7)$$

$$(\mathbf{A} - \mathbf{B})^T \mathbf{X} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2} \quad (8)$$

Now,

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \\ 8 \end{pmatrix} \quad (9)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} -2 \\ -2 \\ -4 \end{pmatrix} \quad (10)$$

And

$$\|\mathbf{A}\| = \sqrt{\mathbf{A}^T \mathbf{A}} \quad (11)$$

$$\|\mathbf{A}\| = \sqrt{2(2) + 3(3) + 4(4)} \quad (12)$$

$$\|\mathbf{A}\| = \sqrt{29} \quad (13)$$

$$\|\mathbf{B}\| = \sqrt{\mathbf{B}^T \mathbf{B}} \quad (14)$$

$$\|\mathbf{B}\| = \sqrt{4(4) + 5(5) + 8(8)} \quad (15)$$

$$\|\mathbf{B}\| = \sqrt{105} \quad (16)$$

Now substituting the respective value in Eq.8:

$$\begin{pmatrix} -2 \\ -2 \\ -4 \end{pmatrix}^T \mathbf{X} = \frac{29 - 105}{2} \quad (17)$$

$$\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}^T \mathbf{X} = 19 \quad (18)$$

Hence the equation of the plane is given by:

$$\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}^T \mathbf{X} = 19 \quad (19)$$

From the figure it is clearly verified that the theoretical solution matches with the computational solution.

