

5.13.7

AI25BTECH11021 - Abhiram Reddy N

QUESTION

Let A and B be two symmetric matrices of order 3.

Statement 1: $A(BA)$ and $(AB)A$ are symmetric matrices.

Statement 2: AB is symmetric matrix if matrix multiplication of A with B is commutative.

Step 1: Symmetry property of matrices

A matrix M is symmetric if

$$M^T = M \quad (0.1)$$

Step 2: Check $A(BA)$

Let

$$M = A \begin{bmatrix} BA \end{bmatrix}.$$

Now,

$$M^T = \left(A \begin{bmatrix} BA \end{bmatrix} \right)^T \quad (0.2)$$

$$= (BA)^T A^T \quad (0.3)$$

$$= A^T B^T A^T \quad (0.4)$$

$$= A \begin{bmatrix} BA \end{bmatrix} \quad (\text{since } A^T = A, B^T = B) \quad (0.5)$$

$$= M \quad (0.6)$$

Hence, $A(BA)$ is symmetric.

Step 3: Check $(AB)A$

Let

$$N = \begin{bmatrix} AB \end{bmatrix} A.$$

Now,

$$N^T = \left(\begin{bmatrix} AB \end{bmatrix} A \right)^T \quad (0.7)$$

$$= A^T (AB)^T \quad (0.8)$$

$$= A^T B^T A^T \quad (0.9)$$

$$= \begin{bmatrix} AB \end{bmatrix} A \quad (\text{since } A^T = A, B^T = B) \quad (0.10)$$

$$= N \quad (0.11)$$

Hence, $(AB)A$ is also symmetric.

Step 4: Condition for AB to be symmetric

For the product:

$$(AB)^T = B^T A^T$$

$$(AB)^T = BA \quad (\text{since } A^T = A, B^T = B) \quad (0.12)$$

Thus,

$$AB \text{ is symmetric} \iff AB = BA \quad (0.13)$$

Step 5: Conclusion

- Statement 1 is true, since $A(BA)$ and $(AB)A$ are symmetric.
- Statement 2 is true, since AB is symmetric when $AB = BA$.
- But Statement 2 is **not** the correct explanation for Statement 1.

Correct option: (a)