12.560

EE25BTECH11026-Harsha

Question:
A scalar function is given by $f(x, y) = x^2 + y^2$. Take $\hat{\imath}$ and $\hat{\jmath}$ as the unit vectors along the x and y axes, respectively. At $(x, y) = (3, 4)$, the direction along which f increases the fastest is

1) $\frac{1}{5}\left(4\hat{\imath} - 3\hat{\jmath}\right)$
2) $\frac{1}{5}\left(3\hat{\imath} - 4\hat{\jmath}\right)$
3) $\frac{1}{5}\left(3\hat{\imath} + 4\hat{\jmath}\right)$
4) $\frac{1}{5}\left(4\hat{\imath} + 3\hat{\jmath}\right)$

Solution:
Let us solve the given question theoretically and then verify the solution computationally.

Approach-1: The direction vector along which the function $f(x, y)$ is given by the gradient direction vector of the function, which is given by

$$\nabla f(x, y) = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix} \tag{4.1}$$

$$\therefore \nabla f(x, y) = \begin{pmatrix} 2x \\ 2y \end{pmatrix} \tag{4.2}$$

At $(x, y) = (3, 4)$,

$$\nabla f(3, 4) = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \tag{4.3}$$

$$\implies \text{Direction vector: } \frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{4.4}$$

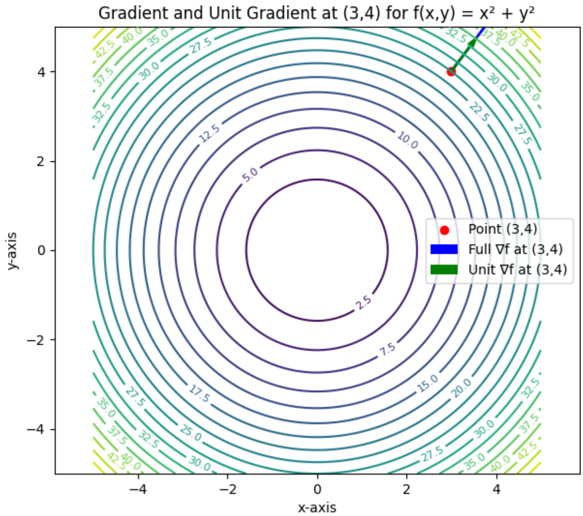

Fig. 4.1: Graph for approach-1

Approach-2: As the point is given to be $\begin{pmatrix}3\\4\end{pmatrix}$, it can be assumed that for the circle,

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} = 3^2 + 4^2 = 25 \tag{4.5}$$

where $\mathbf{V} = \mathbf{I}$.
We can infer that the function will increse along the direction vector of normal at that point. The direction vector of normal is given by

$$\mathbf{n} = (\mathbf{V}\mathbf{q} + \mathbf{u}) \tag{4.6}$$

where, $\mathbf{q}$ is the point of contact.

$$\therefore \mathbf{n} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}3\\4\end{pmatrix} + \begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}3\\4\end{pmatrix} \tag{4.7}$$

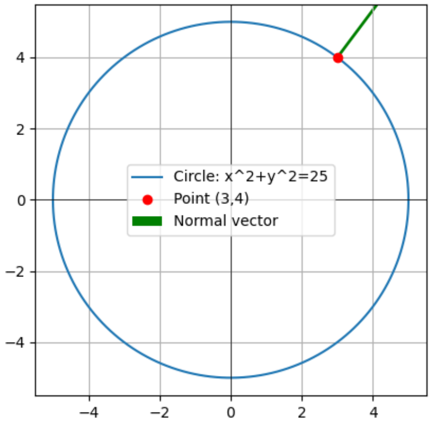

Fig. 4.2: Graph for approach-2