Assignment 3: 2.5.18

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Question:

Let $\mathbf{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\mathbf{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$. Show that the vectors $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ are perpendicular to each other.

Solution:

Given vectors:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \tag{1}$$

$$\mathbf{b} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \tag{2}$$

$\therefore$ We have:

$$\mathbf{C} = \mathbf{a} + \mathbf{b} = \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} \tag{3}$$

$$\mathbf{D} = \mathbf{a} - \mathbf{b} = \begin{pmatrix} -2 \\ 3 \\ -5 \end{pmatrix} \tag{4}$$

For two perpendicular vectors $\mathbf{P}$ and $\mathbf{Q}$:

$$\mathbf{A}^{\top}\mathbf{B} = 0 \tag{5}$$

For vectors $\mathbf{C}$ and $\mathbf{D}$:

$$\mathbf{C}^{\top}\mathbf{D} = \begin{pmatrix} 4 & 1 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ 3 \\ -5 \end{pmatrix} \tag{6}$$

$$= -8 + 3 + 5 = 0 \tag{7}$$

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