2.10.59

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Question Two adjacent sides of a parallelogram **ABCD** are given by $\mathbf{AB} = \begin{pmatrix} 2 \\ 10 \\ 11 \end{pmatrix}$ and $\mathbf{AD} = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$. The side **AD** is rotated by an acute angle α in the plane of the parallelogram so that **AD** becomes $\mathbf{AD}^1$. If $\mathbf{AD}^1$ makes a right angle with the side **AB** then the cosine of the angle α is given by

1) $\frac{8}{9}$
2) $\frac{\sqrt{17}}{9}$
3) $\frac{1}{9}$
4) $\frac{4\sqrt{5}}{9}$

Solution : Given details: **ABCD** is a parallelogram.

$$AB = \begin{pmatrix} 2 \\ 10 \\ 11 \end{pmatrix} \tag{1}$$

$$AD = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} \tag{2}$$

The side $\mathbf{AD^1}$ is perpendicular to **AB**.
Property: The cosine of the angle between vector 1 and vector 2 is given by $\dfrac{{n_1}^\top n_2}{\|n_1\|\,\|n_2\|}$.
Since AD^1 is perpendicular to AB,

Let the angle between the vectors be θ.

$$\alpha + \theta = \frac{\pi}{2}$$

$$\cos\theta = \frac{\mathbf{AB}^{\top}\mathbf{AD}}{||AB||\,||AD||} \tag{3}$$

$$\cos\theta = \frac{\begin{pmatrix}2 & 10 & 11\end{pmatrix}\begin{pmatrix}-1\\2\\2\end{pmatrix}}{\sqrt{225}\,\sqrt{9}} \tag{4}$$

$$\cos\theta = \frac{40}{45} = \frac{8}{9}\left(\because \sin\theta = \sqrt{1-\cos^2\theta}\right) \tag{5}$$

$$\sin\theta = \sqrt{1-\frac{64}{81}} \tag{6}$$

$$\sin\theta = \frac{\sqrt{17}}{9} \tag{7}$$

Since $\cos\alpha = \sin\theta = \frac{\sqrt{17}}{9}$
Ans. option 2

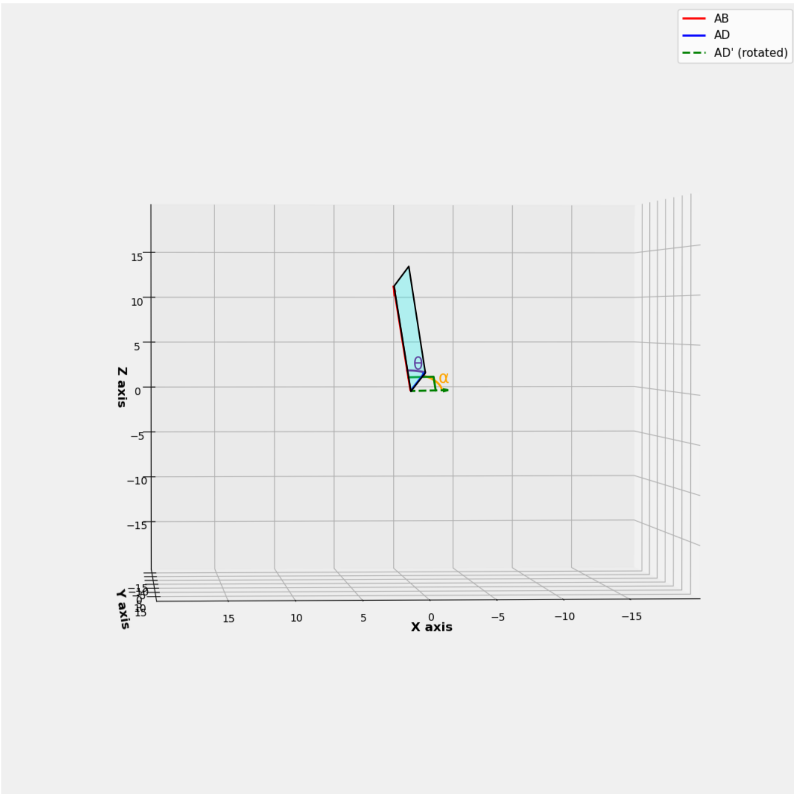

Fig. 4. Plot of the lines