Problem 7.3.5

If a circle passes through the points $(0,0)$, $(a,0)$ and $(0,b)$, then find the coordinates of its centre.

Input Variables

Variable	Description
$\mathbf{x}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	First point on circle
$\mathbf{x}_2 = \begin{pmatrix} a \\ 0 \end{pmatrix}$	Second point on circle
$\mathbf{x}_3 = \begin{pmatrix} 0 \\ b \end{pmatrix}$	Third point on circle

Solution

From (7.1.3.1), for three points $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ on a circle:

$$\begin{pmatrix} 2\mathbf{x}_1 & 2\mathbf{x}_2 & 2\mathbf{x}_3 \\ 1 & 1 & 1 \end{pmatrix}^\top \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = - \begin{pmatrix} \|\mathbf{x}_1\|^2 \\ \|\mathbf{x}_2\|^2 \\ \|\mathbf{x}_3\|^2 \end{pmatrix}, \quad \mathbf{c} = -\mathbf{u}. \tag{1}$$

Substituting the given points:

$$\begin{pmatrix} 0 & 2a & 0 \\ 0 & 0 & 2b \\ 1 & 1 & 1 \end{pmatrix}^\top \begin{pmatrix} u_1 \\ u_2 \\ f \end{pmatrix} = - \begin{pmatrix} 0 \\ a^2 \\ b^2 \end{pmatrix}. \tag{2}$$

This expands to:

$$f = 0, \tag{3}$$
$$2au_1 + a^2 = 0, \tag{4}$$
$$2bu_2 + b^2 = 0. \tag{5}$$

Solving:

$$u_1 = -\frac{a}{2}, \quad u_2 = -\frac{b}{2}, \quad f = 0. \tag{6}$$

Hence, the centre of the circle is

$$\mathbf{c} = -\mathbf{u} = \begin{pmatrix} \frac{a}{2} \\ \frac{b}{2} \end{pmatrix}. \tag{7}$$

$$\boxed{\text{Centre of the circle is } \left(\tfrac{a}{2}, \tfrac{b}{2}\right)} \tag{8}$$

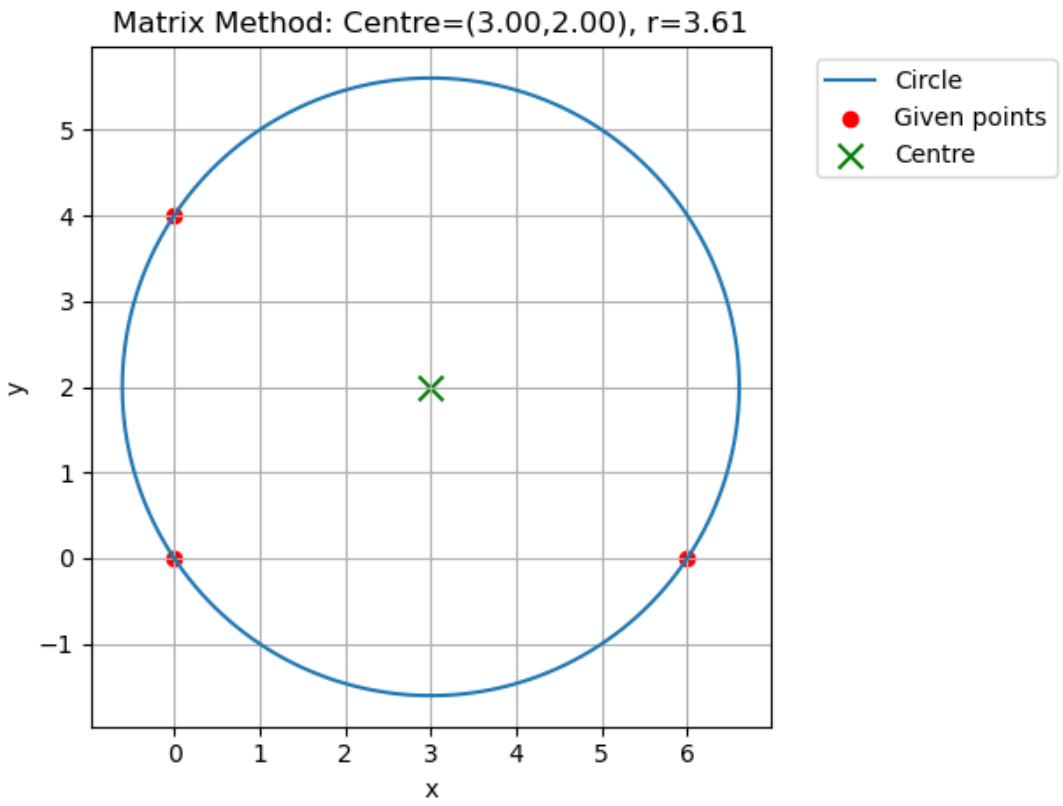
Matrix Method: Centre=(3.00,2.00), r=3.61
Circle
Given points
Centre
y
x
5
4
3
2
1
0
−1
0
1
2
3
4
5
6

Figure 1