1.6.21

AI25BTECH11006 - Nikhila

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Show that the three points **A** (a,b+c) , **B** (b,c+a) and **C** (c,a+b) are collinear.

The vector components of the given points are

$$\mathbf{A}\begin{pmatrix} a \\ b+c \end{pmatrix}, \mathbf{B}\begin{pmatrix} b \\ c+a \end{pmatrix}, \mathbf{C}\begin{pmatrix} c \\ b+a \end{pmatrix}.$$

We know that if the points **A B C** are collinear, the rank of the matrix

$$(\mathbf{B}-\mathbf{A} \quad \mathbf{C}-\mathbf{A}) = 1 \tag{1}$$

Solution

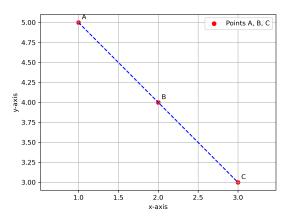$$(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^T = \begin{pmatrix} b - a & c - a \\ a - b & a - c \end{pmatrix} \tag{2}$$

$$R_2 = R_2 + R_1 \tag{3}$$

$$\begin{pmatrix} b - a & c - a \\ 0 & 0 \end{pmatrix} \tag{4}$$

Clearly the rank of the matrix is 1 and hence the points are collinear.

Graphical Representation

Let us take example points by taking a = 1, b = 2, c = 3 and plot the points A(1,5) B(2,4) C(3,3) on the graph