# 9.2.19

ee25btech11056 - Suraj.N

**Question :** Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$.
**Solution :**

| Name | Value |
|---|---|
| Circle | $\mathbf{x}^\top \mathbf{x} - a^2 = 0$ |
| Line | $\mathbf{x} = \begin{pmatrix} \frac{a}{\sqrt{2}} \\ 0 \end{pmatrix} + \kappa \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ |

Table : Circle

The parameters for the circle are :

$$\mathbf{V} = \mathbf{I} \qquad \mathbf{u} = \mathbf{0} \qquad f = -a^2 \tag{1}$$

The parameters for the line are :

$$\mathbf{h} = \begin{pmatrix} \frac{a}{\sqrt{2}} \\ 0 \end{pmatrix} \qquad \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2}$$

Substituting these in the below equation to find the intersection points :

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left( -\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{\left[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})\right]^2 - g(\mathbf{h}) \left(\mathbf{m}^\top \mathbf{V} \mathbf{m}\right)} \right) \tag{3}$$

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{x} - a^2 \tag{4}$$

$$g(\mathbf{h}) = \mathbf{h}^\top \mathbf{h} - a^2 \tag{5}$$

$$\kappa_i = \left( -\mathbf{m}^\top \mathbf{h} \pm \sqrt{a^2 - \mathbf{h}^\top \mathbf{h}} \right) \tag{6}$$

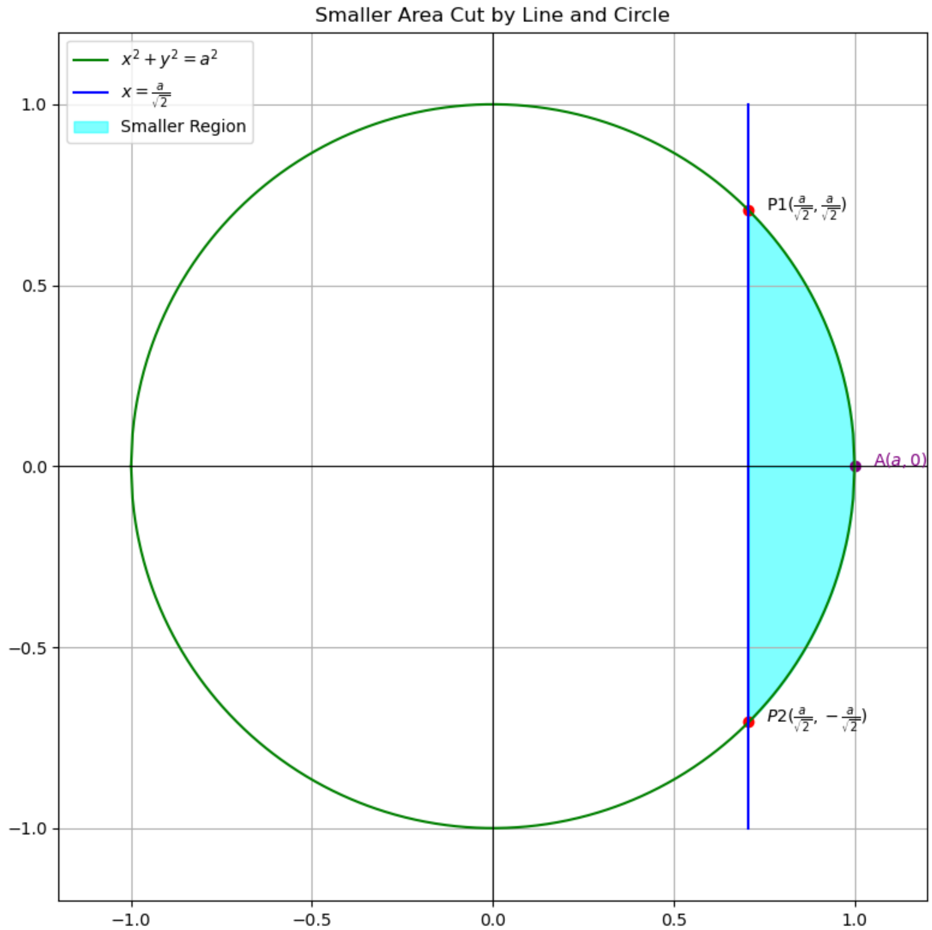
$$\kappa_i = \frac{a}{\sqrt{2}}, -\frac{a}{\sqrt{2}} \tag{7}$$

Therefore the points of intersection are :

$$\mathbf{P_1} = \begin{pmatrix} \frac{a}{\sqrt{2}} \\ \frac{a}{\sqrt{2}} \end{pmatrix} \qquad \mathbf{P_2} = \begin{pmatrix} \frac{a}{\sqrt{2}} \\ -\frac{a}{\sqrt{2}} \end{pmatrix} \tag{8}$$

Thus the area of the smaller part of the circle cut off by the line is :

$$2 \int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2 - x^2}\, dx = \frac{a^2}{2} \left( \frac{\pi}{2} - 1 \right) \tag{9}$$



Smaller Area Cut by Line and Circle
$x^2 + y^2 = a^2$
$x = \frac{a}{\sqrt{2}}$
Smaller Region
P1$(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}})$
P2$(\frac{a}{\sqrt{2}}, -\frac{a}{\sqrt{2}})$
A$(a, 0)$
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0


Fig : Circle