2.5.1

EE25BTECH11047 - RAVULA SHASHANK REDDY

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Question:

Check whether the points $(7, 10)$, $(-2, 5)$, $(3, 4)$ form an isosceles right triangle.

Solution:

Given:

$$\mathbf{A} = \begin{pmatrix} 7 \\ 10 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} \tag{2}$$

$$\mathbf{C} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{3}$$

Side vectors:

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 7 \\ 10 \end{pmatrix} - \begin{pmatrix} -2 \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \end{pmatrix} \tag{4}$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 7 \\ 10 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \tag{5}$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \end{pmatrix} \tag{6}$$

Isosceles check:

1. Altitude from **A**

$$\mathbf{D} = \frac{\mathbf{B}+\mathbf{C}}{2} = \frac{1}{2}\begin{pmatrix}-2+3\\5+4\end{pmatrix} = \begin{pmatrix}\frac{1}{2}\\\frac{9}{2}\end{pmatrix}. \tag{7}$$

$$\mathbf{A}-\mathbf{D} = \begin{pmatrix}\frac{13}{2}\\\frac{11}{2}\end{pmatrix}, \qquad \mathbf{B}-\mathbf{C} = \begin{pmatrix}-5\\1\end{pmatrix}. \tag{8}$$

$$(\mathbf{A}-\mathbf{D})^T(\mathbf{B}-\mathbf{C}) = \begin{pmatrix}\frac{13}{2} & \frac{11}{2}\end{pmatrix}\begin{pmatrix}-5\\1\end{pmatrix} = -\frac{65}{2}+\frac{11}{2} \neq 0. \tag{9}$$

2. Altitude from **B**

$$\mathbf{E} = \frac{\mathbf{C}+\mathbf{A}}{2} = \begin{pmatrix}5\\7\end{pmatrix}. \tag{10}$$

$$\mathbf{B}-\mathbf{E} = \begin{pmatrix}-7\\-2\end{pmatrix}, \qquad \mathbf{C}-\mathbf{A} = \begin{pmatrix}-4\\-6\end{pmatrix}. \tag{11}$$

$$(\mathbf{B}-\mathbf{E})^T(\mathbf{C}-\mathbf{A}) = \begin{pmatrix}-7 & -2\end{pmatrix}\begin{pmatrix}-4\\-6\end{pmatrix} = 28+12 = 40 \neq 0. \tag{12}$$

3.Altitude from **C**

$$\mathbf{F} = \frac{\mathbf{A}+\mathbf{B}}{2} = \begin{pmatrix}\frac{5}{2}\\\frac{15}{2}\end{pmatrix}. \tag{13}$$

$$\mathbf{C}-\mathbf{F} = \begin{pmatrix}\frac{1}{2}\\-\frac{7}{2}\end{pmatrix}, \qquad \mathbf{A}-\mathbf{B} = \begin{pmatrix}9\\5\end{pmatrix}. \tag{14}$$

$$(\mathbf{C}-\mathbf{F})^T(\mathbf{A}-\mathbf{B}) = \begin{pmatrix}\frac{1}{2} & -\frac{7}{2}\end{pmatrix}\begin{pmatrix}9\\5\end{pmatrix} = \frac{9}{2}-\frac{35}{2} = -13 \neq 0. \tag{15}$$

Hence it is not isosceles triangle.

Right angle check:

For a right angle, the dot product of two sides must be zero.

$$(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C}) = (9)(4)+(5)(6) = 66 \neq 0 \tag{16}$$

$$(\mathbf{A}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) = (9)(-5)+(5)(1) = -40 \neq 0 \tag{17}$$

$$(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = (4)(-5)+(6)(1) = -14 \neq 0 \tag{18}$$

Hence, the given points forms neither an isosceles nor a right-angled triangle.

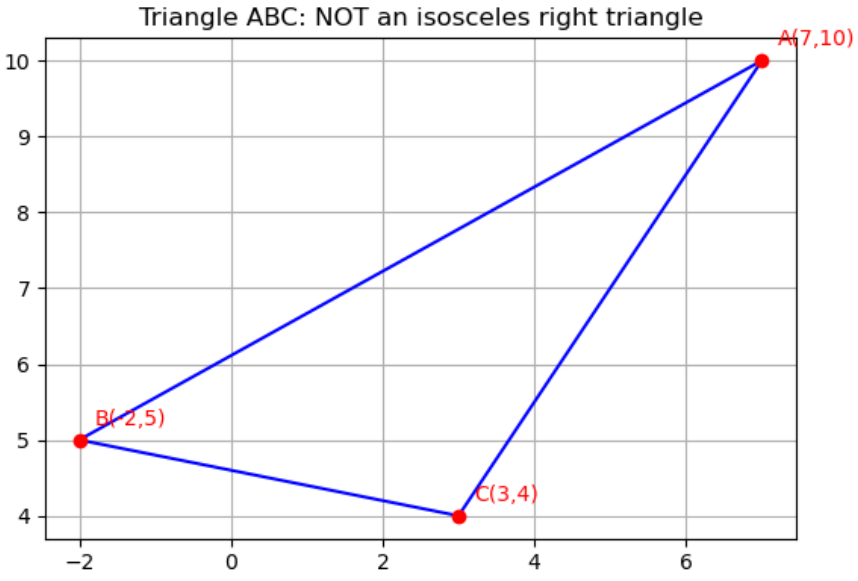
Triangle ABC: NOT an isosceles right triangle
A(7,10)
B(-2,5)
C(3,4)
10
9
8
7
6
5
4
−2
0
2
4
6

Figure 1