2.4.29

EE25BTECH11033 - Kavin

Question:
The points $\mathbf{A}(2,9), \mathbf{B}(a,5)$ and $\mathbf{C}(5,5)$ are the vertices of a triangle $\mathbf{ABC}$ right angled at $\mathbf{B}$. Find the values of a and hence the area of $\triangle\mathbf{ABC}$.

Solution:
Given the points,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 9 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} a \\ 5 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 5 \\ 5 \end{pmatrix} \tag{1}$$

Also it is given that the triangle $\mathbf{ABC}$ right angled at $\mathbf{B}$.

$\therefore$ The vectors $\mathbf{BA}$ and $\mathbf{BC}$ are perpendicular.

The angle θ between $\mathbf{BA}, \mathbf{BC}$, is given by

$$\cos\theta = \frac{(\mathbf{BA})^\top(\mathbf{BC})}{\|\mathbf{BA}\|\,\|\mathbf{BC}\|} \tag{2}$$

Here $\theta = 90°$.

$$\implies (\mathbf{BA})^\top(\mathbf{BC}) = 0 \tag{3}$$

$$\mathbf{BA} = \mathbf{A} - \mathbf{B} = \begin{pmatrix} 2-a \\ 4 \end{pmatrix}$$

$$\mathbf{BC} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} 5-a \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2-a \\ 4 \end{pmatrix}^\top \begin{pmatrix} 5-a \\ 0 \end{pmatrix} = 0 \tag{4}$$

$$\begin{pmatrix} 2-a & 4 \end{pmatrix} \begin{pmatrix} 5-a \\ 0 \end{pmatrix} = 0 \tag{5}$$

$$\implies (2-a)(5-a) + (4 \times 0) = 0 \tag{6}$$

$$\implies (2-a)(5-a) = 0 \tag{7}$$

$$\implies a = 2 \tag{8}$$

Here $a = 5$ is not considered because when $a = 5$, the points $\mathbf{B}$ and $\mathbf{C}$ will be the same and hence a triangle cannot be formed.

$$\mathbf{B} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

The area of $\triangle ABC$ is given by

$$Area = \frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C})\| \tag{9}$$

$$(\mathbf{A} - \mathbf{B}) = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$$

$$(\mathbf{A} - \mathbf{C}) = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

$$\implies Area = \frac{1}{2} \left\| \begin{pmatrix} 0 \\ 4 \end{pmatrix} \times \begin{pmatrix} -3 \\ 4 \end{pmatrix} \right\| \tag{10}$$

$$\implies Area = \frac{1}{2} \|0 + 12\| \tag{11}$$

$$\implies Area = 6 \tag{12}$$

Hence the area of $\triangle \mathbf{ABC}$ is 6 sq.units.
See Fig. 0 ,

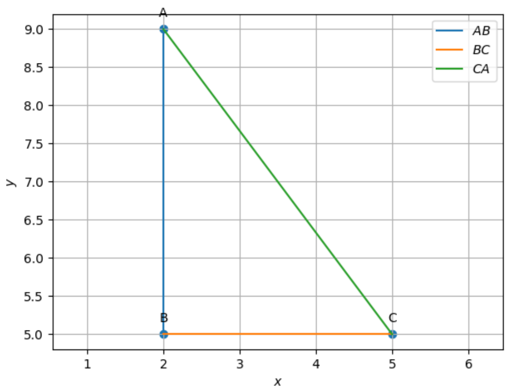

Fig. 0