Matrices in Geometry 1.5.23

EE25BTECH11035 - Kushal B N

Question: Show that the points $\mathbf{A}\left(-2\hat{i}+3\hat{j}+5\hat{k}\right)$, $\mathbf{B}\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and $\mathbf{C}\left(7\hat{i}-\hat{k}\right)$ are collinear.

Given: $\mathbf{A}=\begin{pmatrix}-2\\3\\5\end{pmatrix}$, $\mathbf{B}=\begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\mathbf{C}=\begin{pmatrix}7\\0\\-1\end{pmatrix}$ are three points.

They are defined to be collinear if rank of the collinearity matrix is 1.

$$\text{Collinearity matrix is } \begin{pmatrix}\mathbf{A}-\mathbf{C} & \mathbf{B}-\mathbf{C}\end{pmatrix}^{\top}$$

$$\mathbf{A}-\mathbf{C}=\begin{pmatrix}-9\\3\\6\end{pmatrix} \tag{1}$$

$$\mathbf{B}-\mathbf{C}=\begin{pmatrix}-6\\2\\4\end{pmatrix} \tag{2}$$

$$\implies \operatorname{rank}\begin{pmatrix}-9 & 3 & 6\\-6 & 2 & 4\end{pmatrix}=1. \tag{3}$$

$$\begin{pmatrix}-9 & 3 & 6\\-6 & 2 & 4\end{pmatrix}\overset{R_2\to R_2-\frac{2}{3}R_1}{\longrightarrow}\begin{pmatrix}-9 & 3 & 6\\0 & 0 & 0\end{pmatrix} \tag{4}$$

We know that for the rank of a matrix to be equal to 1, all the elements in the lower row of the matrix must be zero.
So it is proved that the given points are collinear.

Conclusion: Hence, as the rank of the collinearity matrix is 1, it is proved that the given three points are collinear.

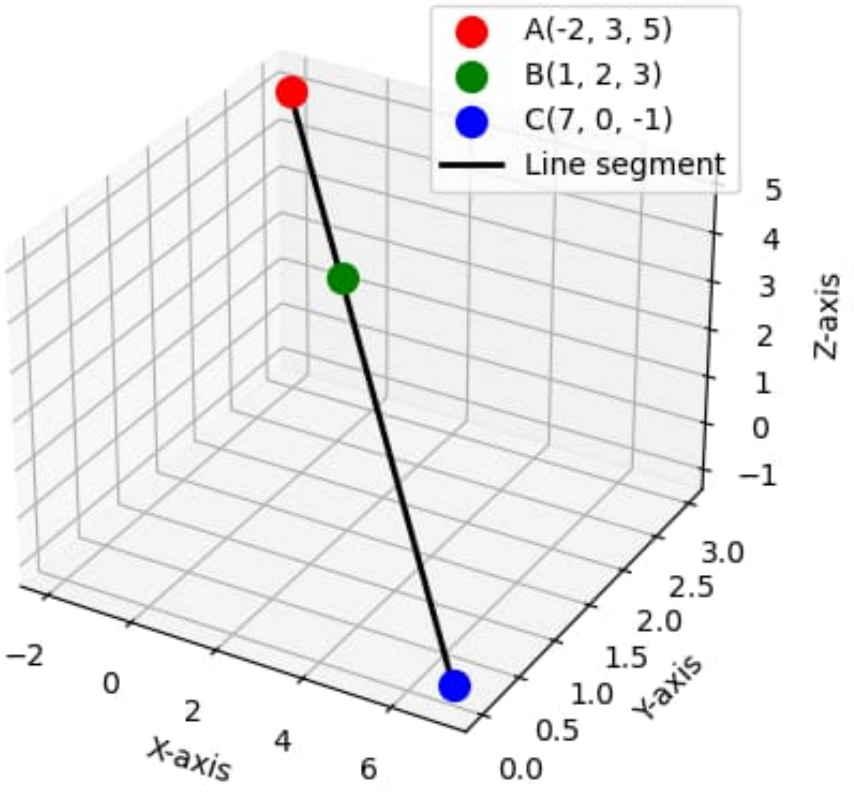

Fig. 1: Plot for 1.5.23