

## Question

If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentric

## Solution

### Matrix Representation of an Ellipse

The general quadratic form of a centered ellipse is:

$$\mathbf{x}^T A \mathbf{x} = 1 \quad \text{where } A = \begin{bmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{bmatrix} \quad (1)$$

Here,  $a$  and  $b$  are the semi-major and semi-minor axes respectively.

### Geometric Condition

The latus rectum  $L$  of an ellipse is given by:

$$L = \frac{2b^2}{a} \quad (2)$$

Given:

$$L = \frac{1}{2} \cdot 2b = b \quad \Rightarrow \quad \frac{2b^2}{a} = b \quad \Rightarrow \quad 2b = a \quad (3)$$

Thus, we have:

$$a = 2b \quad (4)$$

### Eccentricity Calculation

Eccentricity  $e$  of an ellipse is:

$$e = \sqrt{1 - \frac{b^2}{a^2}} \quad (5)$$

Substituting  $a = 2b$ :

$$e = \sqrt{1 - \frac{b^2}{(2b)^2}} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} \quad (6)$$

## Final Answer

$$e = \frac{\sqrt{3}}{2}$$

(7)

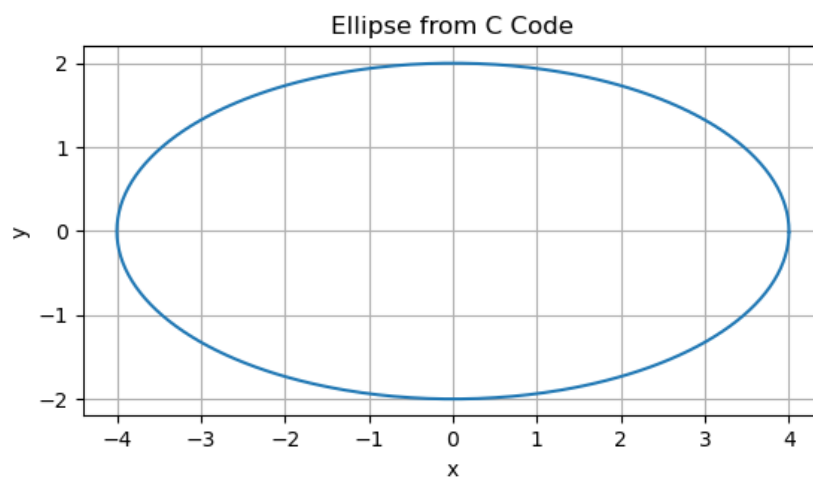


Figure 1