

5.13.46

EE25BTECH11020 - Darsh Pankaj Gajare

October 4, 2025

Question:

Consider the set  $A$  of all determinants of order 3 with entries 0 or 1 only. Let  $B$  be the subset of  $A$  consisting of all determinants with value 1. Let  $C$  be the subset of  $A$  consisting of all determinants with value  $-1$ . Then

- (A)  $C$  is empty  
(B)  $B$  has as many elements as  $C$   
(C)  $A = B \cup C$   
(D)  $B$  has twice as many elements as  $C$

**Solution:**

Let  $\mathbf{A}$  be

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad (0.1)$$

where  $a_{ij} \in \{0, 1\}$

$$\det(\mathbf{A}) \in \{-2, -1, 0, 1, 2\}. \quad (0.2)$$

## Cases

$$|\det| = 2 \Rightarrow \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}, \quad (0.3)$$

$$3 \text{ with } \det = 2, 3 \text{ with } \det = -2. \quad (0.4)$$

$$|\det| = 1 \Rightarrow (2^3 - 1)(2^3 - 2)(2^3 - 4) = 168 = 84(+1), 84(-1). \quad (0.5)$$

$$\det = 0 \Rightarrow 512 - (168 + 6) = 338. \quad (0.6)$$

## Distribution

$$-2 \implies 3 \quad (0.7)$$

$$-1 \implies 84 \quad (0.8)$$

$$0 \implies 338 \quad (0.9)$$

$$1 \implies 84 \quad (0.10)$$

$$2 \implies 3 \quad (0.11)$$

**Answer:** (b),

[Listing: C code](#)

```
#include <stdio.h>
int det3(int m[3][3]) {
    return m[0][0]*m[1][1]*m[2][2]
        + m[0][1]*m[1][2]*m[2][0]
        + m[0][2]*m[1][0]*m[2][1]
        - m[0][2]*m[1][1]*m[2][0]
        - m[0][0]*m[1][2]*m[2][1]
        - m[0][1]*m[1][0]*m[2][2];
}
```

```

}

void compute_counts(int counts[7]) {
    int mat[3][3];
    for (int i = 0; i < 7; i++) counts[i] = 0;

    for (int mask = 0; mask < (1<<9); mask++) {
        for (int i = 0; i < 9; i++) {
            mat[i/3][i%3] = (mask >> i) & 1;
        }
        int d = det3(mat);
        counts[d+3]++;
    }
}

```