3.2.29

AI25BTECH11013-Gautham

Question:
Construct a $\triangle ABC$ given

$$a = BC = 6 \text{ cm}, \qquad \angle B = 30°, \qquad AC - AB = 4 \text{ cm}. \tag{0.1}$$
$$\tag{0.2}$$

Solution:
In the usual notation $a = BC,\ b = CA,\ c = AB$. From the cosine formula in $\triangle ABC$

$$b^2 = a^2 + c^2 - 2ac\cos B. \tag{0.3}$$

Put $b = c + k$ where $k = 4$.

$$(c + k)^2 = a^2 + c^2 - 2ac\cos B. \tag{0.4}$$

Canceling c^2 and collecting terms in c:

$$2kc + k^2 = a^2 - 2ac\cos B \quad \implies \quad c(2k + 2a\cos B) = a^2 - k^2. \tag{0.5}$$

Hence the general expression for c when $b - c = k$ is

$$\boxed{c \ = \ \frac{a^2 - k^2}{2\,(k + a\cos B)}}. \tag{0.6}$$

Now substitute $a = 6,\ B = 30°,\ k = 4$:

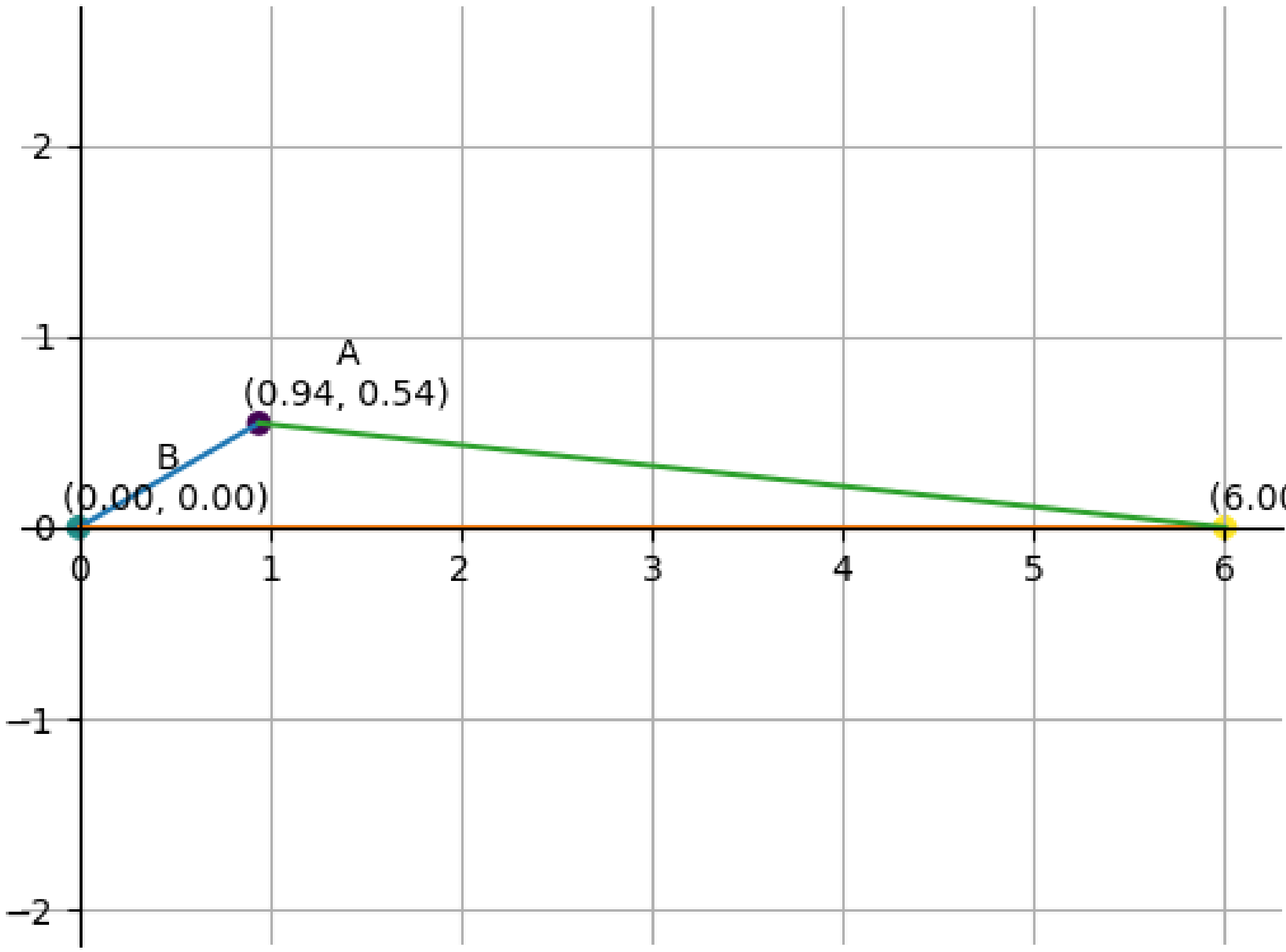
$$\cos 30° = \frac{\sqrt{3}}{2}, \qquad c = \frac{6^2 - 4^2}{2(4 + 6\cos 30°)} = \frac{36 - 16}{2(4 + 6 \cdot \frac{\sqrt{3}}{2})} = \frac{20}{2(4 + 3\sqrt{3})}. \tag{0.7}$$

Numerically,

$$c \approx 1.09 \text{ cm}, \qquad b = c + 4 \approx 5.09 \text{ cm}. \tag{0.8}$$

Place $\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$. Point A lies on this ray with $BA = c$, so

$$\mathbf{A} = c\begin{pmatrix} \cos B \\ \sin B \end{pmatrix} \approx (0.94, 0.54). \tag{0.9}$$


A
(0.94, 0.54)
B
(0.00, 0.00)
(6.00
2
1
0
−1
−2
0
1
2
3
4
5
6