6.4.1

AI25BTECH11001 - ABHISEK MOHAPATRA

Question: foci (3, 0), a = 4.
Solution: Given 2 foci exist for this conic,so it must be an ellipse or a hyperbola.

$$\therefore \mathbf{F_1} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \mathbf{F_2} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \tag{1}$$

$$\mathbf{u} = \frac{\mathbf{F_1} + \mathbf{F_2}}{2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{n} \equiv \mathbf{F_1} - \mathbf{F_2} \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3}$$

Using (3) in (8.1.2.2),

$$\mathbf{V} = \begin{pmatrix} 1 - e^2 & 0 \\ 0 & 1 \end{pmatrix} \tag{4}$$

Given, the length of the semi-major axis

$$4 = \sqrt{\frac{|f|}{|1 - e^2|}} \tag{5}$$

From (8.1.3.3), substituting from (1) and (3),

$$\pm ce^2 = 3 \tag{6}$$

Substituting all known values in (8.1.3.2),

$$c = \pm\frac{1}{e}\sqrt{\frac{|f|}{|e^2 - 1|}} \tag{7}$$

In (5)-(7), there are 3 unknowns, (c, f, e). Upon solving, we get

$$e = \frac{3}{4}, \quad c = \pm\frac{16}{3}, \quad |f| = 7. \tag{8}$$

Let

$$\mathbf{x} = \begin{pmatrix} \alpha \\ 0 \end{pmatrix} \tag{9}$$

be a vertex of the conic on the major axis. Substituting in (8.1.2.1),

$$\alpha^2 + f = 0 \quad \implies \quad f < 0 \tag{10}$$

or,

$$f = -7. \tag{11}$$

Thus, the desired equation of the conic is

$$\mathbf{x}^\top \begin{pmatrix} \frac{7}{16} & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - 7 = 0 \tag{12}$$

Graph: