Problem 5.3.8

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QUESTION

Solve the following system of linear equations using matrix row operations.

$$217a + 131b = 912$$

 $131a + 217b = 827$

MATRIX FORM

$$\mathbf{M} = \begin{bmatrix} 217 & 131 & 912 \\ 131 & 217 & 827 \end{bmatrix}$$

ROW OPERATIONS

Step 1: $R_1 \leftarrow R_1 \div 217$

$$R_1 = \begin{bmatrix} 1 & \frac{131}{217} & \frac{912}{217} \end{bmatrix}$$

Now M =

$$\left[\begin{array}{ccc}
1 & \frac{131}{217} & \frac{912}{217} \\
131 & 217 & 827
\end{array}\right]$$

Step 2: $R_2 \leftarrow R_2 - 131 \cdot R_1$

$$R_2 = \begin{bmatrix} 0 & 217 - 131 \cdot \frac{131}{217} & 827 - 131 \cdot \frac{912}{217} \end{bmatrix}$$

Simplify numerators:

$$217^2 = 47089$$
, $131^2 = 17161$
 $217 \cdot 827 = 179459$, $131 \cdot 912 = 119472$

Now M =

$$\begin{bmatrix} 1 & \frac{131}{217} & \frac{912}{217} \\ 0 & \frac{47089 - 17161}{217} & \frac{179459 - 119472}{217} \end{bmatrix} = \begin{bmatrix} 1 & \frac{131}{217} & \frac{912}{217} \\ 0 & \frac{29928}{217} & \frac{59987}{217} \end{bmatrix}$$

Step 3:
$$R_2 \leftarrow R_2 \div \frac{29928}{217}$$

$$R_2 = \begin{bmatrix} 0 & 1 & \frac{59987}{29928} \end{bmatrix}$$

Now M =

$$\begin{bmatrix}
1 & \frac{131}{217} & \frac{912}{217} \\
0 & 1 & \frac{59987}{29928}
\end{bmatrix}$$

Step 4:
$$R_1 \leftarrow R_1 - \frac{131}{217} \cdot R_2$$

$$R_1 = \begin{bmatrix} 1 & 0 & \frac{912}{217} - \frac{131}{217} \cdot \frac{59987}{29928} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{650}{217} \end{bmatrix}$$

Final matrix:

$$\left[\begin{array}{ccc} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right]$$

Solution

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

FIGURE

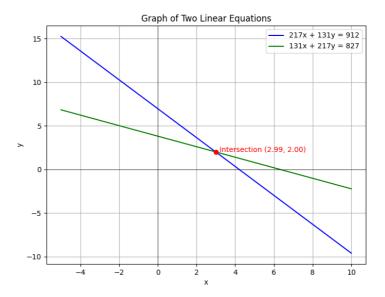


Fig. 1: System of equations from Problem 5.38