12.573

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Question:
a,**b**,**c** are three orthogonal vectors. Given that $\mathbf{a}=\hat{i}+2\hat{j}+5\hat{k}$ and $\mathbf{b}=\hat{i}+2\hat{j}-\hat{k}$, the vector **c** is parallel to (IN 2019)

(a) $\hat{i}+2\hat{j}+3\hat{k}$ (b) $2\hat{i}+\hat{j}$ (c) $2\hat{i}-\hat{j}$ (d) $4\hat{k}$

Solution:
Given

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} \tag{1}$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \tag{2}$$

$$\mathbf{a}^\top \mathbf{c} = 0 \tag{3}$$

$$\mathbf{b}^\top \mathbf{c} = 0 \tag{4}$$

(3) and (4) can be written as

$$\begin{pmatrix} \mathbf{a}^\top \\ \mathbf{b}^\top \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{5}$$

$$\implies \begin{pmatrix} 1 & 2 & 5 \\ 1 & 2 & -1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{6}$$

Forming the augmented matrix

$$\left(\begin{array}{ccc|c} 1 & 2 & 5 & 0 \\ 1 & 2 & -1 & 0 \end{array}\right) \xleftrightarrow{R_2 \to R_2 - R_1} \left(\begin{array}{ccc|c} 1 & 2 & 5 & 0 \\ 0 & 0 & -6 & 0 \end{array}\right) \tag{7}$$

$\implies$ vector **c** can be written in general as $\mathbf{c} = \begin{pmatrix} 2k \\ -k \\ 0 \end{pmatrix}$ (for some scalar k)

$\implies$ vector **c** is parallel to $2\hat{i}-\hat{j}$

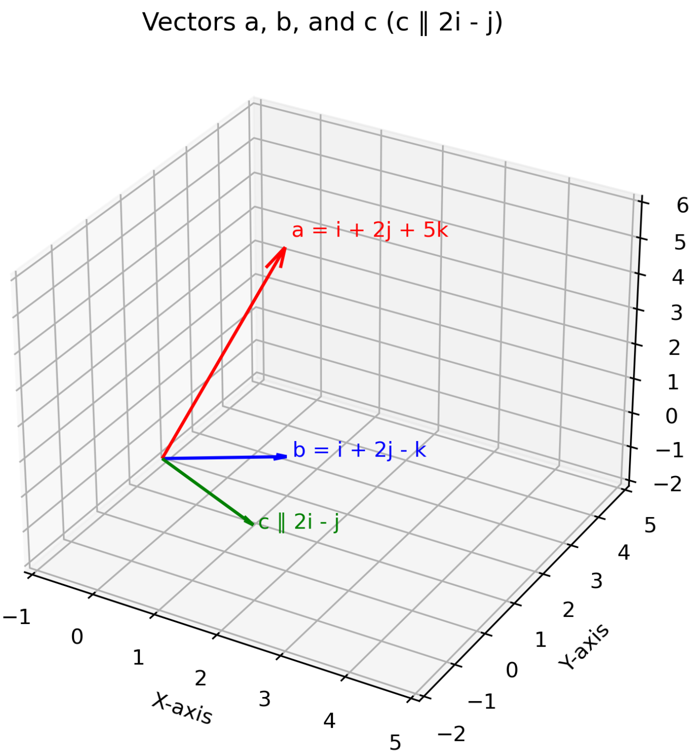

Fig. 4