2.9.1

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Question

Jagdish has a field which is in the shape of a right-angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining space for growing vegetables. In the field, there is a pole marked as O. Based on the above information, answer the following equations

- Taking O as the origin, P = (-200, 0) and Q = (200, 0). PQRS being a square, what are the coordinates of R and S?
-
  - What is the area of square PQRS ?
  - What is the length of diagonal PR in PQRS ?
- If S divides CA in the ratio K : 1, what is the value of K, where A = (200, 800)?

Theoretical Solution

Given that,
AQC is a right angled triangle at point Q and PQRS is a square inside the ΔAQC,
(a) We were given two points

$$P = (-200, 0), Q = (200, 0) \tag{1}$$

Let,
X be the vector along the side PQ,
Y be the vector along the side QR,
Z be the vector along the side PS then,

$$\mathbf{X} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} 200 \\ 0 \end{pmatrix} - \begin{pmatrix} -200 \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{X} = \begin{pmatrix} 400 \\ 0 \end{pmatrix} \tag{3}$$

Theoretical Solution

Rotation vector for 2x2 matrix is

$$\mathbf{R}_\theta = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \tag{4}$$

Rotate the vector $\mathbf{X}$ by 90° anticlockwise to get Y

$$\mathbf{Y} = \mathbf{R_{90}X} \tag{5}$$

$$\mathbf{Y} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 400 \\ 0 \end{pmatrix} \tag{6}$$

$$\mathbf{Y} = \begin{pmatrix} 0 \\ 400 \end{pmatrix} \tag{7}$$

So the vector along the side QR is $\mathbf{Y} = \begin{pmatrix} 0 \\ 400 \end{pmatrix}$ then,

$$\mathbf{Y} = \mathbf{R} - \mathbf{Q} \tag{8}$$

Theoretical Solution

$$\mathbf{R} = \mathbf{Y} + \mathbf{Q} \tag{9}$$

$$\mathbf{R} = \begin{pmatrix} 0 \\ 400 \end{pmatrix} + \begin{pmatrix} 200 \\ 0 \end{pmatrix} \tag{10}$$

$$\mathbf{R} = \begin{pmatrix} 200 \\ 400 \end{pmatrix} \tag{11}$$

Since the sides QR and PS are parallel, vectors $\mathbf{Y} = \mathbf{Z}$ then

$$\mathbf{Z} = \mathbf{S} - \mathbf{P} \tag{12}$$

$$\mathbf{S} = \mathbf{Z} + \mathbf{P} \tag{13}$$

$$\mathbf{S} = \begin{pmatrix} 0 \\ 400 \end{pmatrix} + \begin{pmatrix} -200 \\ 0 \end{pmatrix} \tag{14}$$

Theoretical Solution

$$\mathbf{S} = \begin{pmatrix} -200 \\ 400 \end{pmatrix} \tag{15}$$

Therefore the coordinates of the points R and S are (200,400) and (-200,400) (b)

- We know the points P(-200,0) and Q(200,0)
  Let length of the side of the square PQRS be x then,

$$x = \|\mathbf{Q} - \mathbf{P}\| \tag{16}$$

$$x = \left\| \begin{pmatrix} 400 \\ 0 \end{pmatrix} \right\| = 400 \tag{17}$$

  Area of the square $= x^2 = (400)^2 = 160000$ sq units
- Length of diagnol of the square $= x\sqrt{2} = 400\sqrt{2}$ units

(c) Given the point A=(200,800)
Since it was given that point S divides CA in the ratio K:1, this shows that points A,C and S are collinear. Since AQC is a right angled triangle, from this we can say that point C lies on X axis
Let point C be (t,0), Consider the matrix M

$$M = \begin{pmatrix} 200 & 800 & 1 \\ -200 & 400 & 1 \\ t & 0 & 1 \end{pmatrix} \tag{18}$$

$R_1 \to \frac{1}{200} R_1$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ -200 & 400 & 1 \\ t & 0 & 1 \end{pmatrix} \tag{19}$$

$R_2 \to R_2 + 200 R_1$ $\quad$ $R_3 \to R_3 - t R_1$

Theoretical Solution

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ 0 & 1200 & 2 \\ 0 & -4t & 1-\frac{t}{200} \end{pmatrix} \tag{20}$$

$R_2 \to \frac{1}{200} R_2$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ 0 & 1 & \frac{1}{600} \\ 0 & -4t & 1-\frac{t}{200} \end{pmatrix} \tag{21}$$

$R_3 \to R_3 + 4tR_2$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ 0 & 1 & \frac{1}{600} \\ 0 & 0 & 1-\frac{t}{200}+\frac{4t}{600} \end{pmatrix} \tag{22}$$

Since the three points A,S and C are collinear,
Rank of M $= 2$

Theoretical Solution

$$1 - \frac{t}{200} + \frac{4t}{600} = 0 \tag{23}$$

$$1 + \frac{t}{600} = 0 \tag{24}$$

$$\frac{t}{600} = -1 \tag{25}$$

$$t = -600 \tag{26}$$

Therefore point C=(-600,0), Now S divides CA in the ratio K:1,

$$S = \frac{KA + C}{K + 1} \tag{27}$$

Theoretical Solution

$$K = \frac{(S - A)^T (C - S)}{\|S - A\|^2} \tag{28}$$

$$K = \frac{1}{(400)^2 + (400)^2} \begin{pmatrix} -400 & -400 \end{pmatrix} \begin{pmatrix} -400 \\ -400 \end{pmatrix} \tag{29}$$

By solving ((c).12) we get K=1

C Code- Ploting the given vectors

```c
#include <stdio.h>

typedef struct {
    int x, y;
} Point;

void get_triangle(Point* pts) {
    pts[0].x = 200; pts[0].y = 800; // A
    pts[1].x = 200; pts[1].y = 0; // Q
    pts[2].x = -600; pts[2].y = 0; // C
}
```

C Code- Ploting the given vectors

```
void get_square(Point* pts) {
pts[0].x = -200; pts[0].y = 0; // P
pts[1].x = 200; pts[1].y = 0; // Q
pts[2].x = 200; pts[2].y = 400; // R
pts[3].x = -200; pts[3].y = 400; // S (shared corner, so Q
    again)
}
```

Python Code using shared output

```
                from ctypes import *
import matplotlib.pyplot as plt

# Load the C shared library
lib = CDLL(./2.9.1.so) # Ensure path matches your compiled output

class Point(Structure):
    _fields_ = [(x, c_int), (y, c_int)]

triangle = (Point * 3)()
square = (Point * 4)()

lib.get_triangle(triangle)
lib.get_square(square)
```

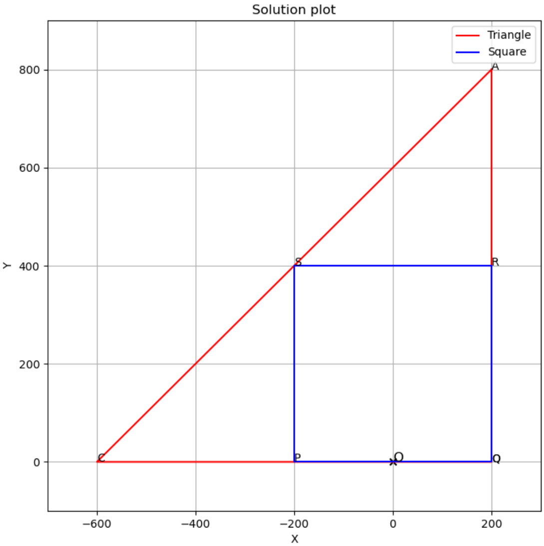
Python Code using shared output

```
    tri_x = [triangle[i].x for i in range(3)] + [triangle[0].x]
    tri_y = [triangle[i].y for i in range(3)] + [triangle[0].y]
     sqr_x = [square[i].x for i in range(4)] + [square[0].x]
sqr_y = [square[i].y for i in range(4)] + [square[0].y]

plt.figure(figsize=(8, 8))
plt.plot(tri_x, tri_y, 'r-', label='Triangle')
plt.plot(sqr_x, sqr_y, 'b-', label='Square')

# Annotate triangle points
labels_tri = ['A', 'Q', 'C']
for i in range(3):
    plt.text(triangle[i].x, triangle[i].y, labels_tri[i])
```

Python Code using shared output

```
# Annotate square points
labels_sqr = ['P', 'Q', 'R', 'S']
for i in range(4):
    plt.text(square[i].x, square[i].y, labels_sqr[i])

plt.scatter([0], [0], marker='x', color='black')
plt.text(0, 0, 'O', fontsize=12)
plt.xlim(-700, 300)
plt.ylim(-100, 900)
plt.xlabel('X')
plt.ylabel('Y')
plt.title('Solution plot')
plt.grid(True)
plt.legend()
plt.show()
```

Plot by python using shared output



Python code for the plot

```
    import matplotlib.pyplot as plt

# Triangle points
A = (200, 800)
Q = (200, 0)
C = (-600, 0)

# Square points
P = (-200, 0)
R = (200, 400)
S = (-200, 400)

# Lists for triangle
triangle_x = [A[0], Q[0], C[0], A[0]]
triangle_y = [A[1], Q[1], C[1], A[1]]
```

Python code for the plot

```
# Lists for square
square_x = [P[0], Q[0], R[0], S[0], P[0]]
square_y = [P[1], Q[1], R[1], S[1], P[1]]

plt.figure(figsize=(8, 8))

# Plot triangle
plt.plot(triangle_x, triangle_y, 'r-', label='Triangle')

# Plot square
plt.plot(square_x, square_y, 'b-', label='Square')

# Label triangle points
plt.text(A[0], A[1], 'A')
plt.text(Q[0], Q[1], 'Q')
plt.text(C[0], C[1], 'C')
```

Python code for plot

```
    # Label square points
plt.text(P[0], P[1], 'P')
plt.text(R[0], R[1], 'R')
plt.text(S[0], S[1], 'S')

# Mark and label the origin
plt.scatter([0], [0], marker='x', color='black')
plt.text(0, 0, 'O')

# To match axes and layout
plt.xlabel('X')
plt.ylabel('Y')
plt.title('Solution plot')
plt.xlim(-700, 300)
plt.ylim(-100, 900)
plt.grid(True)
plt.legend()
plt.show()
```

Plot of triangle and square