5.3.18

AI25BTECH11024 - Pratyush Panda

Question:

Using matrix method, solve the following system of equations

$$2x - 3y + 5z = 13 \tag{0.1}$$
$$3x + 2y - 4z = -2 \tag{0.2}$$
$$x + y - 2z = -2 \tag{0.3}$$

Solution:

The given system of equations can be written as;

$$\mathbf{AX = B} \quad where, \mathbf{A} = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 13 \\ -2 \\ -2 \end{pmatrix} and \mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{0.4}$$

Now, since $\mathbf{A}$ is not a singular matrix we can pre-multiply both sides with $\mathbf{A}^{-1}$
From that we get;

$$\mathbf{X = A^{-1}B} \tag{0.5}$$

On solving for $\mathbf{A}^{-1}$ we get $\mathbf{A}^{-1} = \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix}$

Thus,

$$\mathbf{X = A^{-1}B} = \begin{pmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{pmatrix} \begin{pmatrix} 13 \\ -2 \\ -2 \end{pmatrix} \tag{0.6}$$

$$or, \mathbf{X} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix} \tag{0.7}$$

Thus, the solution set is $(x, y, z) = (2, 2, 3)$.