1.9.17

EE25btech11028 - J.Navya sri

Question:
Write the coordinates of a point $\mathbf{P}$ on the x-axis which is equidistant from points $\mathbf{A}(-2, 0)$ and $\mathbf{B}(6, 0)$.

Solution: Let

$$\mathbf{A} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} b \\ 0 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} p \\ 0 \end{pmatrix} \tag{1}$$

Since $\mathbf{P}$ is equidistant from $\mathbf{A}$ and $\mathbf{B}$, their distances satisfy:

$$||\mathbf{P} - \mathbf{A}|| = ||\mathbf{P} - \mathbf{B}|| \tag{2}$$

Square both sides:

$$||\mathbf{P} - \mathbf{A}||^2 = ||\mathbf{P} - \mathbf{B}||^2 \tag{3}$$

Using the norm squared definition:

$$(\mathbf{P} - \mathbf{A})^\top (\mathbf{P} - \mathbf{A}) = (\mathbf{P} - \mathbf{B})^\top (\mathbf{P} - \mathbf{B}) \tag{4}$$

Expand both sides:

$$\mathbf{P}^\top \mathbf{P} - 2\mathbf{A}^\top \mathbf{P} + \mathbf{A}^\top \mathbf{A} = \mathbf{P}^\top \mathbf{P} - 2\mathbf{B}^\top \mathbf{P} + \mathbf{B}^\top \mathbf{B} \tag{5}$$

Cancel $\mathbf{P}^\top \mathbf{P}$ from both sides:

$$-2\mathbf{A}^\top \mathbf{P} + \mathbf{A}^\top \mathbf{A} = -2\mathbf{B}^\top \mathbf{P} + \mathbf{B}^\top \mathbf{B} \tag{6}$$

Rearranged:

$$2(\mathbf{B} - \mathbf{A})^\top \mathbf{P} = \mathbf{B}^\top \mathbf{B} - \mathbf{A}^\top \mathbf{A} \tag{7}$$

Substitute the vectors:

$$2(b - a)p = b^2 - a^2 \tag{8}$$

Rewrite right side as difference of squares:

$$2(b - a)p = (b - a)(b + a) \tag{9}$$

Since $b \neq a$, divide both sides by $(b - a)$:

$$2p = b + a \tag{10}$$

Solve for p:

$$p = \frac{a + b}{2} \tag{11}$$

Now substitute $a = -2$, $b = 6$:

$$p = \frac{-2 + 6}{2} = \frac{4}{2} = 2 \tag{12}$$

Hence, the coordinates of **P** are:

$$\boxed{\mathbf{P} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}} \tag{13}$$

Graphical Representation:

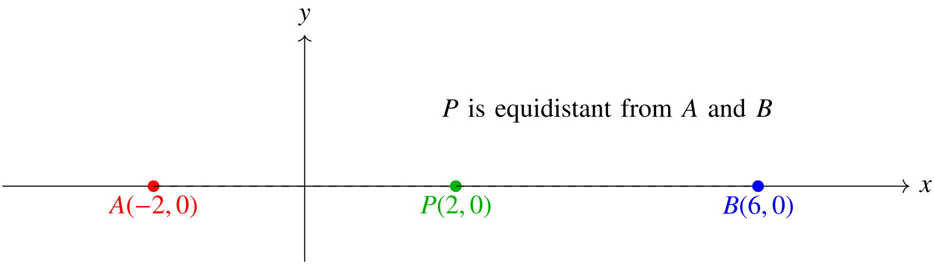

Fig. 0