1.5.33

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Question:
Find the ratio in which the Y-axis divides the line segment joining the points A$(5,-6)$ and B$(-1,-4)$. Also find the coordinates of the point of intersection.

Solution:
Let the given points be A and B

$$\mathbf{A} = \begin{pmatrix} 5 \\ -6 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$$

Let the Y-axis divide the line segment **AB** at point **P** in the ratio k : 1. Since **P** lies on Y-axis, let

$$\mathbf{P} = \begin{pmatrix} 0 \\ y \end{pmatrix}$$

The point **A**, **B**, **P** are collinear.

$$\implies \operatorname{rank}\left(\mathbf{B}-\mathbf{A}, \mathbf{P}-\mathbf{A}\right) = 1 \tag{1}$$

$$\implies \left\|\left(\mathbf{B}-\mathbf{A}, \mathbf{P}-\mathbf{A}\right)\right\| = 0 \tag{2}$$

$$\left\|\begin{pmatrix} -6, -5 \\ 2, y+6 \end{pmatrix}\right\| = 0 \tag{3}$$

$$-6 \times (y+6) - (-5) \times 2 = 0 \tag{4}$$

$$-36 - 6y + 10 = 0 \tag{5}$$

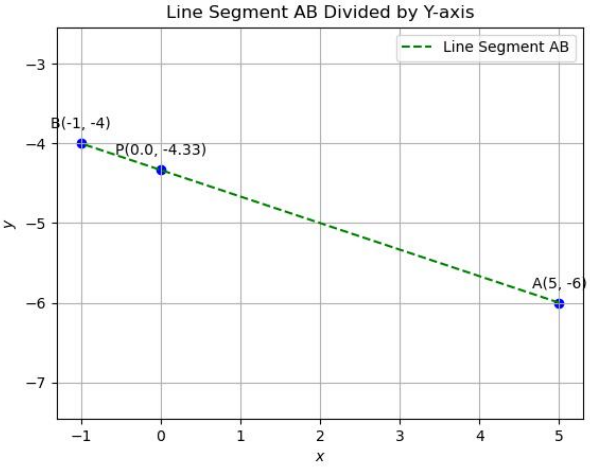
$$y = \frac{-13}{3} \tag{6}$$

$\therefore$ The coordinates of the point of intersection are

$$\mathbf{P} = \begin{pmatrix} 0 \\ -\frac{13}{3} \end{pmatrix}$$

The section formula is

$$\mathbf{P} \equiv \begin{pmatrix} x \\ y \end{pmatrix} = \frac{k\mathbf{B} + \mathbf{A}}{k+1} \tag{7}$$



Plot of Intersection of AB by Y-axis

Here, substituting the values,

$$\begin{pmatrix} 0 \\ \frac{-13}{3} \end{pmatrix} = \frac{1}{k+1}\left(\begin{pmatrix} 5 \\ -6 \end{pmatrix} + k\begin{pmatrix} -1 \\ -4 \end{pmatrix}\right) \tag{8}$$

$$\begin{pmatrix} 0 \\ \frac{-13}{3} \end{pmatrix} = \frac{1}{k+1}\left(\begin{pmatrix} 5-k \\ -6-4k \end{pmatrix}\right) \tag{9}$$

$$\tag{10}$$

$$0 = \frac{5-k}{k+1} \tag{11}$$

$$5 - k = 0 \tag{12}$$

$$\implies k = 5 \tag{13}$$

Thus, the ratio in which the point **P** divides the line segment **AB** is **5:1**.