4.3.32

EE25BTECH11042 - Nipun Dasari

Question:
Find the slope of a line which cuts off intercepts of equal length on the axes is. Solve using matrices.
Solution:
Consider normal form of a line:

$$\mathbf{n}^\top \mathbf{x} = 1 \tag{0.1}$$

Given that equal intercepts are cut off we get 2 cases:
On substituting the intercepts in place of $\mathbf{x}$:
Case 1: The intercepts are equal ($b = a$)

$$\implies \mathbf{n}^\top \begin{pmatrix} a \\ 0 \end{pmatrix} = 1 \tag{0.2}$$

$$\implies \mathbf{n}^\top \begin{pmatrix} 0 \\ a \end{pmatrix} = 1 \tag{0.3}$$

$$\tag{0.4}$$

from (0.2) and from (0.3)

$$\mathbf{n}^\top \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} = \begin{pmatrix} 1 & 1 \end{pmatrix} \tag{0.5}$$

$$\implies a\mathbf{n}^\top \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \end{pmatrix} \tag{0.6}$$

$$\implies a\mathbf{n}^\top \mathbf{I} = \begin{pmatrix} 1 & 1 \end{pmatrix} \implies a\mathbf{n}^\top = \begin{pmatrix} 1 & 1 \end{pmatrix} \implies \mathbf{n} = \frac{1}{a}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.7}$$

Thus, direction vector $\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Case 2: The intercepts are negatives of each other ($-b = a$)

$$\implies \mathbf{n}^\top \begin{pmatrix} a \\ 0 \end{pmatrix} = 1 \tag{0.8}$$

$$\implies \mathbf{n}^\top \begin{pmatrix} 0 \\ -a \end{pmatrix} = 1 \tag{0.9}$$

$$\tag{0.10}$$

By (0.8), (0.9):

$$\mathbf{n}^\top \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix} = \begin{pmatrix} 1 & 1 \end{pmatrix} \tag{0.11}$$

$$\implies a\mathbf{n}^\top \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \end{pmatrix} \tag{0.12}$$

Let $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

For this matrix $\mathbf{A} = \mathbf{A}^{-1}$

$$\implies a\mathbf{n}^\top = \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{A}^{-1} \implies a\mathbf{n}^\top = \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{A}^{-1} \implies \mathbf{n} = \frac{1}{a}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{0.13}$$

Thus, direction vector $\mathbf{n} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ The direction vector is given (in general) by:

$$\mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix} \text{ where } m \text{ is slope of given line} \tag{0.14}$$

On comparing with the obtained direction vectors

$$\therefore m = \pm 1 \tag{0.15}$$