1.9.14

EE25BTECH11025 - Ganachari Vishwambhar

Question:
If $\mathbf{P} = (2, 2)$, $\mathbf{Q} = (-4, -4)$, and $\mathbf{R} = (5, -8)$ are the vertices of a triangle ΔPQR, then find the length of the median through $\mathbf{R}$.
Solution:
Given position vectors of the points are:

$$\mathbf{P} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} -4 \\ -4 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 5 \\ -8 \end{pmatrix} \tag{1}$$

Let the position vectors of $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ be the columns of the 2×3 matrix:

$$V = \begin{pmatrix} \mathbf{R} & \mathbf{Q} & \mathbf{P} \end{pmatrix} \tag{2}$$

$$V = \begin{pmatrix} 5 & -4 & 2 \\ -8 & -4 & 2 \end{pmatrix} \tag{3}$$

The midpoint of PQ is:

$$\mathbf{M} = \frac{1}{2}\mathbf{P} + \frac{1}{2}\mathbf{Q} = V \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \tag{4}$$

$$\mathbf{RM} = \mathbf{M} - \mathbf{R} = V \begin{pmatrix} -1 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \tag{5}$$

Let

$$\mathbf{c_R} = \begin{pmatrix} -1 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \tag{6}$$

Let the gram matrix:

$$G = V^T V \tag{7}$$

$$G = \begin{pmatrix} 89 & 12 & -6 \\ 12 & 32 & -16 \\ -6 & -16 & 8 \end{pmatrix} \tag{8}$$

Then the squares length of the median from $\mathbf{R}$ is :

$$\|\mathbf{RM}\|^2 = (V\mathbf{c_R})^T (V\mathbf{c_R}) \tag{9}$$

$$= \mathbf{c_R}^T \left(V^T V\right) \mathbf{c_R} = \mathbf{c_R}^T G \mathbf{c_R} \tag{10}$$

$$\|\mathbf{RM}\| = \sqrt{85} \approx 9.2195 \tag{11}$$

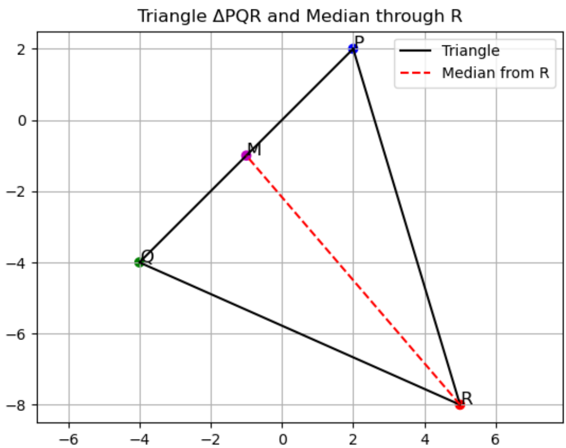

Fig. 1: Plot of triangle ΔPQR