

2.4.23

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Question:

Do the points $(3, 2)$, $(-2, -3)$, and $(2, 3)$ form a triangle? If so, name the type of triangle formed.

Solution:

Given points,

$$A = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \quad B = \begin{bmatrix} -2 \\ -3 \end{bmatrix}, \quad C = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad (1)$$

1. Collinearity check (using rank)

Form the matrix:

$$M = \begin{bmatrix} 3 & 2 & 1 \\ -2 & -3 & 1 \\ 2 & 3 & 1 \end{bmatrix} \quad (2)$$

Apply row operations:

$$R_2 \leftarrow R_2 + 2R_1, \quad R_3 \leftarrow 3R_3 - 2R_1 \Rightarrow \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 0 & 5 & 1 \end{bmatrix} \quad (3)$$

$$R_2 \leftarrow 3R_2 - 4R_1 \Rightarrow \begin{bmatrix} 3 & 2 & 1 \\ 0 & -5 & 5 \\ 0 & 5 & 1 \end{bmatrix} \quad (4)$$

$$R_3 \leftarrow R_3 + R_2 \Rightarrow \begin{bmatrix} 3 & 2 & 1 \\ 0 & -5 & 5 \\ 0 & 0 & 6 \end{bmatrix} \quad (5)$$

Since all three rows are nonzero:

$$\text{rank}(M) = 3 \quad (6)$$

\Rightarrow Points are not collinear, so they form a triangle.

2. Right-angle check

$$\overrightarrow{AB} = B - A = \begin{bmatrix} -5 \\ -5 \end{bmatrix}, \quad \overrightarrow{AC} = C - A = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad (7)$$

$$\overrightarrow{AB} \cdot \overrightarrow{AC} = (-5)(-1) + (-5)(1) = 0 \quad (8)$$

$$\Rightarrow \overrightarrow{AB} \perp \overrightarrow{AC}$$

So, the triangle is right-angled at

$$A = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad (9)$$

3. Final Answer

The given points form a triangle (rank = 3). (10)

The triangle is right-angled at $A = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. (11)

Graphical Representation:

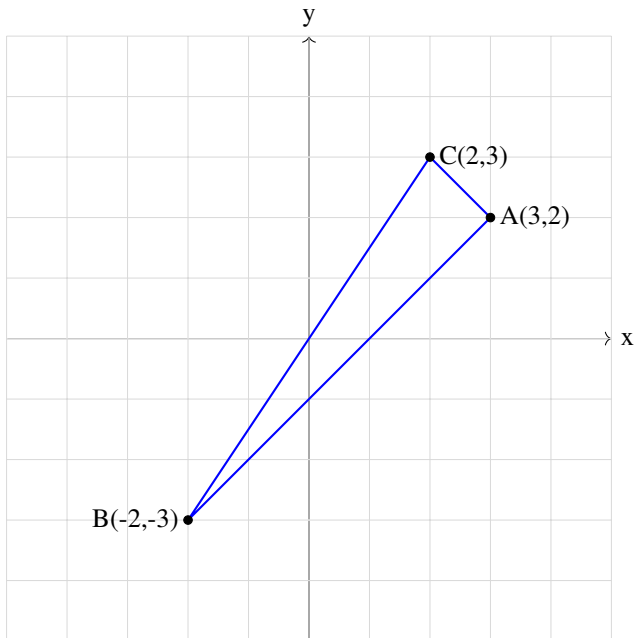


Fig. 0