2.10.10

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Question

Given that

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad \mathbf{a} \cdot \mathbf{b} = 3, \quad \mathbf{a} \times \mathbf{b} = \mathbf{c},$$

find **b**.

Solution - Step 1

Express vectors as column matrices:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}.$$

Solution - Step 2

Use the dot product condition:

$$\mathbf{a}^\top \mathbf{b} = x + y + z = 3.$$

Use the cross product condition:

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} z - y \\ x - z \\ y - x \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}.$$

Which gives:

$$\begin{cases} z - y = 0, \\ x - z = 1, \\ y - x = -1. \end{cases}$$

Solution - Step 3

Solve the system:
From $z - y = 0$:

$$z = y.$$

From $y - x = -1$:

$$y = x - 1.$$

Substitute in $x - z = 1$ (with $z = y$):

$$x - y = 1 \implies x - (x - 1) = 1 \implies 1 = 1,$$

which is consistent.

Solution - Step 4

Use the dot product:

$$x + y + z = x + (x-1) + (x-1) = 3x - 2 = 3 \implies 3x = 5 \implies x = \frac{5}{3}.$$

Then,

$$y = \frac{2}{3}, \quad z = \frac{2}{3}.$$

Final Answer

$$\boxed{\mathbf{b} = \begin{pmatrix} \frac{5}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{pmatrix}.}$$

C Code - Part 1

```
#include <stdio.h>

int main() {
    // Given vectors a and c
    double a[3] = {1, 1, 1};
    double c[3] = {0, 1, -1};

    // Variables for b components
    double x, y, z;
```

C Code - Part 2

```
    // From the solution:
    // z = y
    // y = x - 1
    // x + y + z = 3  =>  x + (x-1) + (x-1) = 3  => 3x - 2 = 3

    x = 5.0 / 3.0;
    y = x - 1.0;
    z = y;

    // Print result
    printf("Vector b is:\n");
    printf("b = [%.6f, %.6f, %.6f]\n", x, y, z);

    return 0;
}
```

Python Code - Part 1

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Define the vectors
a = np.array([1, 1, 1])
b = np.array([5/3, 2/3, 2/3])
c = np.array([0, 1, -1])

origin = np.array([0, 0, 0])

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
```

Python Code - Part 2

```
# Plot vectors a, b, and c
ax.quiver(*origin, *a, color='r', label='a', arrow_length_rati
ax.quiver(*origin, *b, color='g', label='b', arrow_length_rati
ax.quiver(*origin, *c, color='b', label='c', arrow_length_rati

ax.set_xlim([0, 2])
ax.set_ylim([0, 2])
ax.set_zlim([-2, 2])

ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')

ax.legend()
ax.set_title('Vectors a, b, and c in 3D')

plt.savefig('python_plot.png')
```

Plot

figs/python_plot.png