4.8.20

AI25BTECH11024 - Pratyush Panda

Question:

Find the distance between the point $(2, 3, 4)$ measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$

Solution:

Let the vector $\mathbf{A}$ be $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$, and the direction vector of the line $\mathbf{b} = \begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix}$.

The equation of the plane can be written as;

$$\mathbf{n}^T\mathbf{X} = 1 \qquad where, \mathbf{n} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} \tag{0.1}$$

The perpendicular distance between the point and the plane (x) can be written as;

$$x = \frac{\mathbf{n}^T\mathbf{A}}{||\mathbf{n}||} = \frac{20}{\sqrt{17}} \tag{0.2}$$

Now, the distance along the given line can be written as $\frac{x}{\cos\theta}$.

Where $\cos\theta$ is the angle between $\mathbf{b}$ (direction vector of the line) and $\mathbf{n}$ (normal vector of the plane).

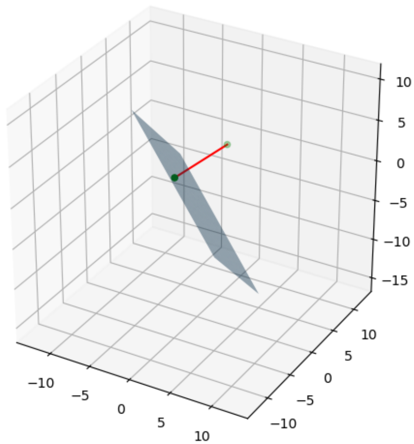
Thus $\cos\theta$ can be written as;

$$\cos\theta = \frac{\mathbf{n}^T\mathbf{b}}{||\mathbf{n}||.||\mathbf{b}||} = \frac{25}{7.\sqrt{17}} \tag{0.3}$$

Thus, the final distance along the line can be written as;

$$d = ||\mathbf{b}||.\frac{\mathbf{n}^T\mathbf{A}}{\mathbf{n}^T\mathbf{b}} = \frac{28}{5} \tag{0.4}$$

Thus, the distance between the point $(2, 3, 4)$ measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$ is 5.6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