1.3.10

AI25BTECH110031
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Question(1.3.10)

Find the ratio in which the point $P = (8, y)$ divides the line segment joining $A = (1, 2)$ and $B = (2, 3)$. Also, find the value of y.

Solution:

Let the given points be A and B

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

Let the point P divide the line segment $\overline{AB}$ in the ratio $k : 1$.

Given $P = \begin{pmatrix} 8 \\ y \end{pmatrix}$

The points A, B, P are collinear.

$$\implies \operatorname{rank}\begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{P} - \mathbf{A} \end{pmatrix} = 1 \tag{0.1}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 - 1 \\ 3 - 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.2}$$

$$\mathbf{P} - \mathbf{A} = \begin{pmatrix} 8 - 1 \\ y - 2 \end{pmatrix} = \begin{pmatrix} 7 \\ y - 2 \end{pmatrix} \tag{0.3}$$

Therefore, our matrix is:

$$\begin{pmatrix} 1 & 7 \\ 1 & y - 2 \end{pmatrix} \tag{0.4}$$

Row reducing:

$$R_1 \to R_1 - R_2 \implies \begin{pmatrix} 0 & 9 - y \\ 1 & y - 2 \end{pmatrix} \tag{0.5}$$

For the above matrix to be of rank 1,

$$9 - y = 0 \implies y = 9 \tag{0.6}$$

$\therefore$ The coordinates of the point of division are

$$P = \begin{pmatrix} 8 \\ 9 \end{pmatrix}$$

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k+1} \tag{0.7}$$

$$k = \frac{(\begin{pmatrix}A\end{pmatrix} - \begin{pmatrix}P\end{pmatrix})^T(\begin{pmatrix}P\end{pmatrix} - \begin{pmatrix}B\end{pmatrix})}{\|\begin{pmatrix}P\end{pmatrix} - \begin{pmatrix}B\end{pmatrix}\|^2} \tag{0.8}$$

Substituting the values of **A**, **B** and **P**,

$$k = \frac{\begin{pmatrix}-7 & -7\end{pmatrix}\begin{pmatrix}6\\6\end{pmatrix}}{\left\|\begin{pmatrix}6\\6\end{pmatrix}\right\|^2} = \frac{-7}{6} \tag{0.9}$$

Thus, the ratio in which the point P divides the line segment AB is $\boxed{-7:6}$.

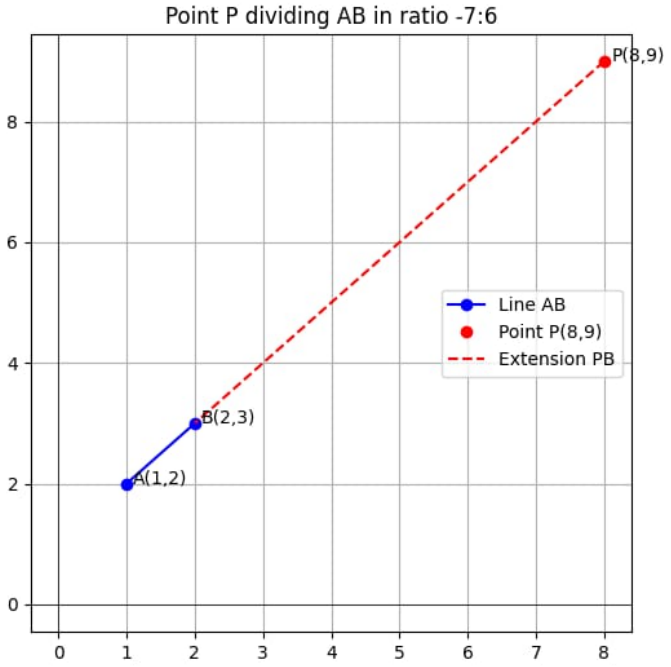

Fig. 0.1