

## 4.13.23

EE25BTECH11018 - Darisy Sreetej

**Question:**

Let  $a, b, c, d$  be non-zero numbers. If the point of intersection of the lines  $4ax + 2ay + c = 0$  and  $5bx + 2by + d = 0$  lies in the fourth quadrant and is equidistant from the two axes then

- 1)  $3bc - 2ad = 0$
- 2)  $2bc - 3ad = 0$
- 3)  $3bc + 2ad = 0$
- 4)  $2bc + 3ad = 0$

**Solution:**

The two lines are

$$4ax + 2ay + c = 0, \quad (1)$$

$$5bx + 2by + d = 0 \quad (2)$$

According to the condition, the intersection point is equidistant from the axes and lies in the fourth quadrant, so its coordinates satisfy  $y = -x$

$$x + y = 0 \quad (3)$$

This equation can be expressed in terms of matrices

$$\begin{pmatrix} 4a \\ 2a \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} = -c \quad (4)$$

$$\begin{pmatrix} 5b \\ 2b \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} = -d \quad (5)$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} = 0 \quad (6)$$

They can be represented as,

$$\begin{pmatrix} 4a & 5b \\ 2a & 2b \\ 1 & 1 \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -c \\ -d \\ 0 \end{pmatrix} \quad (7)$$

Using augmented matrix,

$$\left( \begin{array}{cc|c} 4a & 5b & -c \\ 2a & 2b & -d \\ 1 & 1 & 0 \end{array} \right) \quad (8)$$

$$R_3 = R_3 - 4aR_1$$

$$R_2 = R_2 - 5bR_1$$

$$\left( \begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -3b & -d \\ 0 & -2a & -c \end{array} \right) \quad (9)$$

$$R_3 = R_3 - \frac{2a}{3b}R_2$$

$$\left( \begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -3b & -d \\ 0 & 0 & -c + \frac{2ad}{3b} \end{array} \right) \quad (10)$$

$$\left( \begin{array}{cc|c} 1 & 1 & 0 \\ 0 & -3b & -d \\ 0 & 0 & \frac{2ad-3bc}{3b} \end{array} \right) \quad (11)$$

The last row of the matrix represents the equation

$$y0 + x0 = \frac{2ad - 3bc}{3b} \quad (12)$$

For the system to be consistent(i.e., to have a solution),the right-hand side must be zero.

$$\frac{2ad - 3bc}{3b} = 0 \quad (13)$$

$$2ad - 3bc = 0 \quad (14)$$

Therefore,

$$3bc = 2ad \quad (15)$$

For the point of intersection ,

$$-3by = -d \quad (16)$$

$$y = \frac{d}{3b} \quad (17)$$

$$x + y = 0 \quad (18)$$

$$x = \frac{-d}{3b} \quad (19)$$

The point of intersection is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{-d}{3b} \\ \frac{d}{3b} \end{pmatrix} \quad (20)$$

Also ,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{-c}{2a} \\ \frac{c}{2a} \end{pmatrix} \quad (from(15)) \quad (21)$$

Therefore, option(a) is correct

