

1.7.2

AI25BTECH11016-Varun

Question:

If $A(1, 2)$, $O(0, 0)$, and $C(a, 6)$ are collinear, then the value of a is

Solution:

The given points are

$$A = (1, 2) \quad O = (0, 0) \quad C = (a, 6) \quad (1)$$

$$\mathbf{A} - \mathbf{O} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2)$$

$$\mathbf{C} - \mathbf{O} = \begin{pmatrix} a \\ 6 \end{pmatrix} \quad (3)$$

Construct the matrix

$$M = \begin{pmatrix} 1 & a \\ 2 & 6 \end{pmatrix} \quad (4)$$

For the points to be collinear, the two vectors \mathbf{OA} and \mathbf{OC} must be linearly dependent. This means

$$\text{rank}(M) = 1 \quad \Leftrightarrow \quad \det(M) = 0 \quad (5)$$

For the rank to drop,

$$6 - 2a = 0 \quad (6)$$

$$a = 3 \quad (7)$$

When $a = 3$

$$\begin{pmatrix} 1 & 3 \\ 0 & 0 \end{pmatrix}$$

is the reduced row-echelon form (rank = 1)

The given points are collinear when

$$a = 3 \quad (8)$$

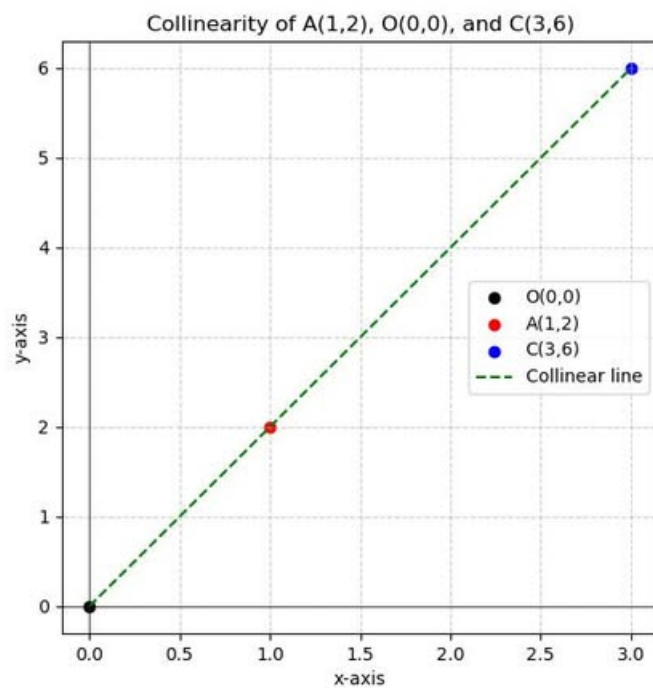


Fig. 0.1