

Matrices in Geometry - 2.4.32

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Problem Statement

The position vectors of the points **A**, **B**, **C** and **D** are $(3\hat{i} - 2\hat{j} - \hat{k})$, $(2\hat{i} + 3\hat{j} - 4\hat{k})$, $(-\hat{i} + \hat{j} + 2\hat{k})$ and $(4\hat{i} + 5\hat{j} + \lambda\hat{k})$ respectively. If the points **A**, **B**, **C** and **D** lie on a plane, find the value of λ .

Solution

Given,

$$\mathbf{A} \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}, \mathbf{B} \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}, \mathbf{C} \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} \text{ and } \mathbf{D} \begin{pmatrix} 4 \\ 5 \\ \lambda \end{pmatrix}.$$

The general equation for a plane with normal vector \mathbf{n} passing through point P is

$$\mathbf{n}^T \mathbf{P} = d \quad (1)$$

So,

$$\mathbf{n}^T \mathbf{A} = d \quad (2)$$

$$\mathbf{n}^T \mathbf{B} = d \quad (3)$$

$$\mathbf{n}^T \mathbf{C} = d \quad (4)$$

$$\mathbf{n}^T \mathbf{D} = d \quad (5)$$

Solution

Forming direction vectors in the plane,

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -1 \\ 5 \\ -3 \end{pmatrix} \quad (6)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -4 \\ 3 \\ 3 \end{pmatrix} \quad (7)$$

Now, the normal vector will be orthogonal to both of these direction vectors, so that

$$(\mathbf{B} - \mathbf{A})^\top \mathbf{n} = 0 \quad (8)$$

$$(\mathbf{C} - \mathbf{A})^\top \mathbf{n} = 0 \quad (9)$$

Solution

Combining the above two equations,

$$\begin{pmatrix} (\mathbf{B} - \mathbf{A})^\top \\ (\mathbf{C} - \mathbf{A})^\top \end{pmatrix} \mathbf{n} = 0 \quad (10)$$

$$\begin{pmatrix} -1 & 5 & -3 \\ -4 & 3 & 3 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (11)$$

Augmented matrix:

$$\Rightarrow \left(\begin{array}{ccc|c} -1 & 5 & -3 & 0 \\ -4 & 3 & 3 & 0 \end{array} \right) \quad (12)$$

$$\left(\begin{array}{ccc|c} -1 & 5 & -3 & 0 \\ -4 & 3 & 3 & 0 \end{array} \right) \xrightarrow{R_2 \rightarrow R_2 - 4R_1} \left(\begin{array}{ccc|c} -1 & 5 & -3 & 0 \\ 0 & -17 & 15 & 0 \end{array} \right) \quad (13)$$

Solution

$$\left(\begin{array}{ccc|c} -1 & 5 & -3 & 0 \\ 0 & -17 & 15 & 0 \end{array} \right) \xrightarrow{R_2 \rightarrow \frac{-1}{17} R_2} \left(\begin{array}{ccc|c} -1 & 5 & -3 & 0 \\ 0 & 1 & \frac{-15}{17} & 0 \end{array} \right) \quad (14)$$

$$\left(\begin{array}{ccc|c} -1 & 5 & -3 & 0 \\ 0 & 1 & \frac{-15}{17} & 0 \end{array} \right) \xrightarrow{R_1 \rightarrow R_1 - 5R_2} \left(\begin{array}{ccc|c} -1 & 0 & \frac{24}{17} & 0 \\ 0 & 1 & \frac{-15}{17} & 0 \end{array} \right) \quad (15)$$

$$\Rightarrow \mathbf{n} = \begin{pmatrix} \frac{24}{17} \\ \frac{15}{17} \\ 1 \end{pmatrix} t \quad (16)$$

So we can take $t = 17$ in order to get integer coefficients,

$$\mathbf{n} = \begin{pmatrix} 24 \\ 15 \\ 17 \end{pmatrix} \quad (17)$$

Solution

Substituting this in (2),

$$d = (24 \quad 15 \quad 17) \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} = 25 \quad (18)$$

So substituting for **D** and d in the equation (5), we have

$$(24 \quad 15 \quad 17) \begin{pmatrix} 4 \\ 5 \\ \lambda \end{pmatrix} = 25 \quad (19)$$

$$\implies \boxed{\lambda = \frac{-146}{17}} \quad (20)$$

Final Answer

The value of λ is $-\frac{146}{17}$.

3D Plot of Coplanar Points and Plane

