9.8.5

AI25BTECH11003 - Bhavesh Gaikwad

Question: Let $\mathbf{S}$ be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is

Solution:
Given:
Circle: $x^2 + y^2 - 2x - 4y = 0$
Parabola: $y^2 = 8x$

Parameters of the Circle:

$$\mathbf{V}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u}_1 = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, f_1 = 0 \tag{0.1}$$

Parameters of the Parabola:

$$\mathbf{V}_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, f_2 = 0, \mathbf{S} = \begin{pmatrix} 2e \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{0.2}$$

Points of Intersection of Circle and Parabola can be given as:

$$\mathbf{X}^\top(\mathbf{V}_1 + \mu\mathbf{V}_2)\mathbf{X} + 2(\mathbf{u}_1 + \mu\mathbf{u}_2)^\top\mathbf{X} + (f_1 + \mu f_2) \tag{0.3}$$

$$\mathbf{X}^\top \begin{pmatrix} 1 & 0 \\ 0 & 1 + \mu \end{pmatrix} \mathbf{X} - 2\begin{pmatrix} 1 + 4\mu & 2 \end{pmatrix} \mathbf{X} = 0 \tag{0.4}$$

From Equation 0.4, We get

$$\mathbf{X} = \begin{pmatrix} 1 + 4\mu \\ \dfrac{2}{1 + \mu} \end{pmatrix} \tag{0.5}$$

Putting Value of $\mathbf{X}$ in Equation 0.4, We get points of intersection as:

$$\mathbf{X}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \& \quad \mathbf{X}_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{0.6}$$

Therefore, Let $\mathbf{P} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

The Area of Triangle PQS is:

$$Area(\triangle PQS) = \frac{1}{2}\left\|\mathbf{SP}\times\mathbf{QP}\right\| = 4 \tag{0.7}$$

$$\boxed{\text{The Area of } \triangle PQS \text{ is 4 sq.units.}}$$

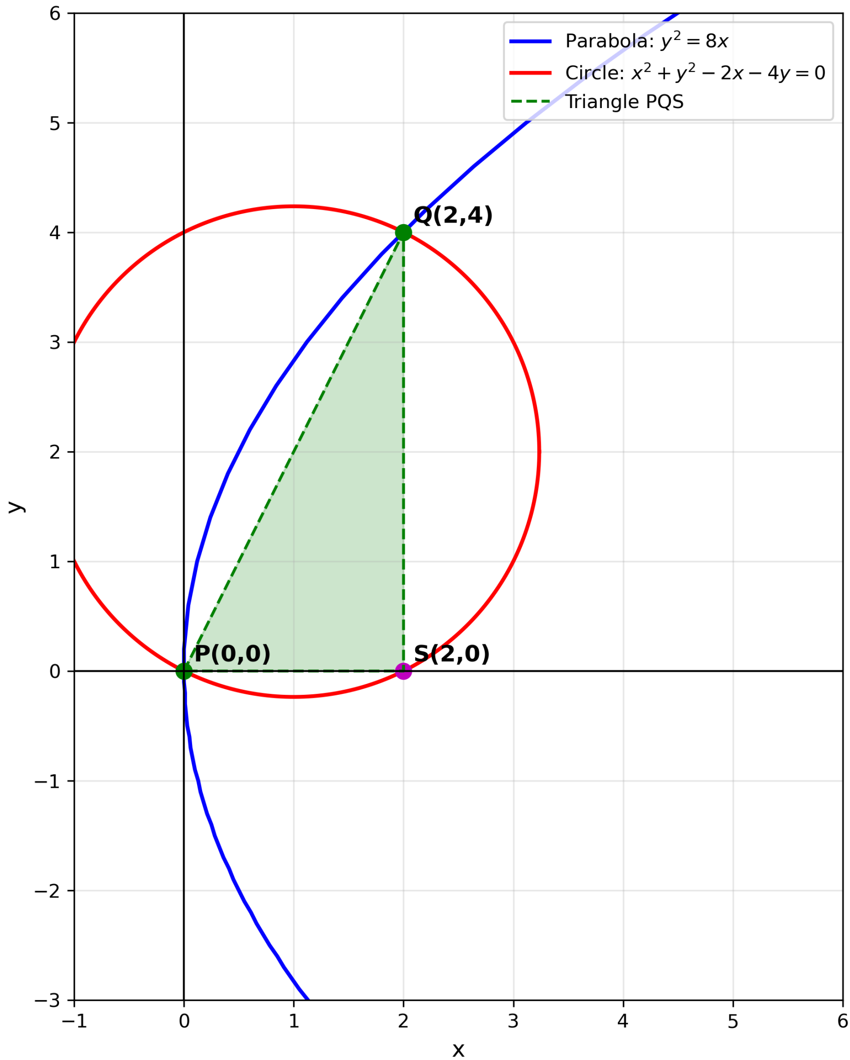

Fig. 0.1: Intersection of Two Conics and Triangle PQS