

5.2.15

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Question:

Solve the system of equations

$$\begin{aligned} 2x + y &= 5 \\ 3x + 2y &= 8 \end{aligned}$$

Solution: The equation of line:

$$n^T x = c \quad (0.1)$$

Line L:

$$\begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 5 \quad (0.2)$$

Line K:

$$\begin{pmatrix} 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 8 \quad (0.3)$$

Writing in matrix form:

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \quad (0.4)$$

The following augmented matrix can be solved by gaussian elimination

$$\left(\begin{array}{cc|c} 2 & 1 & 5 \\ 3 & 2 & 8 \end{array} \right) \xrightarrow{R_2 \rightarrow R_2 - \frac{3}{2}R_1} \left(\begin{array}{cc|c} 2 & 1 & 5 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{array} \right) \quad (0.5)$$

Since,

$$\text{rank}(A) = \text{rank}(A|b) = 2 \quad (0.6)$$

the system has a unique solution.

from 2nd row,

$$y = 1 \Rightarrow x = 2 \quad (0.7)$$

\therefore Solution of given system of equations is: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

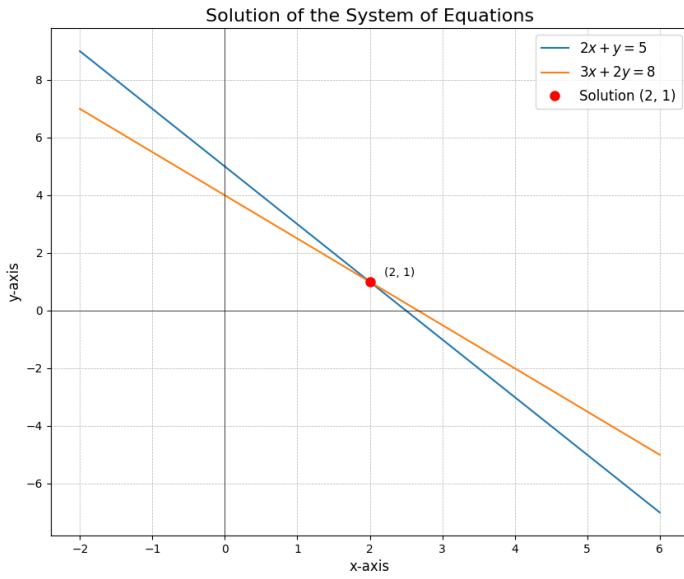


Fig. 0.1: 5.2.15