Matgeo Presentation - Problem 2.7.33

ee25btech11021 - Dhanush sagar

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Problem Statement

Find the value of p if

$$(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = 0.$$

solution

Solution:

The given vectors are

$$\mathbf{A} = \begin{pmatrix} 2 \\ 6 \\ 27 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 1 \\ 3 \\ p \end{pmatrix}. \tag{0.1}$$

Form the 2×3 matrix with these rows:

$$M = \begin{pmatrix} 2 & 6 & 27 \\ 1 & 3 & p \end{pmatrix}. \tag{0.2}$$

If $\mathbf{A} \times \mathbf{B} = \mathbf{0}$, then $\mathbf{A}$ and $\mathbf{B}$ are linearly dependent, so $\operatorname{rank}(M) < 2$. We perform row-reduction to find the rank and the condition on p.

Begin with M:

$$M = \begin{pmatrix} 2 & 6 & 27 \\ 1 & 3 & p \end{pmatrix}. \tag{0.3}$$

solution

Eliminate the leading entry of the second row by replacing R_2 with $R_2 - \frac{1}{2}R_1$:

$$R_2 \leftarrow R_2 - \tfrac{1}{2}R_1$$

$$\begin{pmatrix} 2 & 6 & 27 \\ 1 & 3 & p \end{pmatrix} \xrightarrow{R_2 - \frac{1}{2}R_1} \begin{pmatrix} 2 & 6 & 27 \\ 0 & 0 & p - \frac{27}{2} \end{pmatrix}. \tag{0.4}$$

Now scale the first row to make a leading 1: $R_1 \leftarrow \frac{1}{2}R_1$:

$$\begin{pmatrix} 2 & 6 & 27 \\ 0 & 0 & p - \frac{27}{2} \end{pmatrix} \xrightarrow{R_1 \div 2} \begin{pmatrix} 1 & 3 & \frac{27}{2} \\ 0 & 0 & p - \frac{27}{2} \end{pmatrix}. \tag{0.5}$$

This is the RREF form (up to the final optional normalization of the second row). The rank is the number of nonzero rows in RREF. Thus

$$\operatorname{rank}(M) = \begin{cases} 2, & \text{if } p - \frac{27}{2} \neq 0, \\ 1, & \text{if } p - \frac{27}{2} = 0. \end{cases}$$

solution

For $\mathbf{A} \times \mathbf{B} = \mathbf{0}$ we need $\operatorname{rank}(M) < 2$, hence

$$p - \tfrac{27}{2} = 0 \quad \Rightarrow \quad p = \tfrac{27}{2}. \tag{0.6}$$

Final answer:

$$\boxed{p = \frac{27}{2}} \tag{0.7}$$

C Source Code: cross.c

```
#include <stdio.h>
void cross_product(double a[3], double b[3], double result[3])
    result[0] = a[1]*b[2] - a[2]*b[1];
    result[1] = a[2]*b[0] - a[0]*b[2];
    result[2] = a[0]*b[1] - a[1]*b[0];
}
double find_p() {
    double a[3] = {2, 6, 27};
    double p = 27.0 / 2.0;
    double b[3] = {1, 3, p};
    double res[3];
    cross_product(a, b, res);
   if(res[0] == 0 && res[1] == 0 && res[2] == 0) {
        return p;
    }
    return -1;
}
```

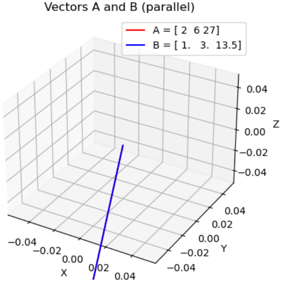
Python Script: vector solve.py

```
import ctypes
import numpy as np
# Load shared library
lib = ctypes.CDLL("./cross.so")
lib.find_p.restype = ctypes.c_double
# Call C function
p = lib.find_p()
print("Computed value of p:", p)
# Verify using numpy
A = np.array([2, 6, 27])
B = np.array([1, 3, p])
cross_prod = np.cross(A, B)
print("Cross product A × B =", cross_prod)
if np.allclose(cross_prod, [0, 0, 0]):
    print("✓ A and B are parallel. Solution verified.")
else:
    print("× Something went wrong.")
```

Python Script: plot vector.py

```
import numpy as np
import matplotlib.pyplot as plt
# Vectors
A = np.array([2, 6, 27])
p = 27/2
B = np.array([1, 3, p])
# Plot
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.quiver(0, 0, 0, A[0], A[1], A[2], color='r', label=f'A = {A
ax.quiver(0, 0, 0, B[0], B[1], B[2], color='b', label=f'B = {B
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.legend()
plt.title("Vectors A and B (parallel)")
plt.savefig("vectors.png")                plt.show()
```

Result Plot


Vectors A and B (parallel)
A = [2 6 27]
B = [1.   3. 13.5]
0.04
0.02
0.00
−0.02
−0.04
Z
0.04
0.02
0.00
−0.02
−0.04
Y
−0.04
−0.02
0.00
0.02
0.04
X