1.5.16

EE25btech11028 - J.Navya sri

Question:
Find the point A where AB is a diameter of a circle with center $(3,-1)$ and the point B is $(2,6)$

Point	Vector
B	$\begin{pmatrix}2\\6\end{pmatrix}$
C	$\begin{pmatrix}3\\-1\end{pmatrix}$

Using rank of matrix: Three points $\mathbf{A},\mathbf{C},\mathbf{B}$ are collinear if

$$\operatorname{rank}\begin{pmatrix}\mathbf{C}-\mathbf{A} & \mathbf{B}-\mathbf{A}\end{pmatrix} = 1$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix}3-x\\-1-y\end{pmatrix}, \quad \mathbf{B}-\mathbf{A} = \begin{pmatrix}2-x\\6-y\end{pmatrix}$$

Form matrix:

$$\begin{pmatrix}3-x & 2-x\\-1-y & 6-y\end{pmatrix}$$

Apply row operation:

$$R_2 \to (3-x)R_2 - (-1-y)R_1$$

$$(3-x)(6-y)+(1+y)(2-x)=0 \Rightarrow 20-y-7x=0 \Rightarrow \boxed{7x+y=20}$$

Using midpoint formula:

$$\mathbf{C} = \frac{\mathbf{A}+\mathbf{B}}{2} \Rightarrow \mathbf{A} = 2\mathbf{C}-\mathbf{B}$$

$$\mathbf{A} = 2\begin{pmatrix}3\\-1\end{pmatrix} - \begin{pmatrix}2\\6\end{pmatrix} = \begin{pmatrix}6\\-2\end{pmatrix} - \begin{pmatrix}2\\6\end{pmatrix} = \boxed{\mathbf{A} = \begin{pmatrix}4\\-8\end{pmatrix}}$$

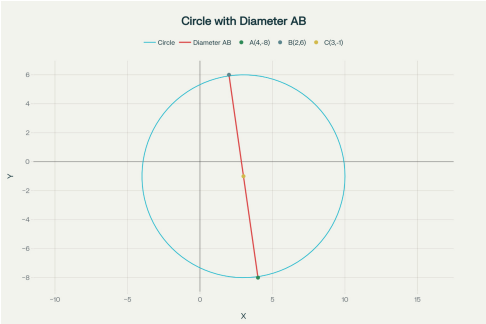
Circle with Diameter AB
Circle
Diameter AB
A(4,-8)
B(2,6)
C(3,-1)
6
4
2
0
-2
-4
-6
-8
Y
-10
-5
0
5
10
15
X

Fig. 0.1