

Matrices in Geometry 1.9.26‘

EE25BTECH11037 - Divyansh

Question: Find the value of k , if the point $\mathbf{P}(2, 4)$ is equidistant from point $\mathbf{A}(5, k)$ and $\mathbf{B}(k, 7)$

Given: $\mathbf{P}\begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $\mathbf{A}\begin{pmatrix} 5 \\ k \end{pmatrix}$ and a point $\mathbf{B}\begin{pmatrix} k \\ 7 \end{pmatrix}$ such that \mathbf{P} is equidistant from \mathbf{A} and \mathbf{B} .

$$\therefore \|\mathbf{A} - \mathbf{P}\| = \|\mathbf{B} - \mathbf{P}\| \quad (1)$$

$$\text{On squaring both the sides, we get} \quad (2)$$

$$\|\mathbf{A} - \mathbf{P}\|^2 = \|\mathbf{B} - \mathbf{P}\|^2 \quad (3)$$

$$(\mathbf{A} - \mathbf{P})^\top (\mathbf{A} - \mathbf{P}) = (\mathbf{B} - \mathbf{P})^\top (\mathbf{B} - \mathbf{P}) \quad (4)$$

$$(\mathbf{A} - \mathbf{P}) = \begin{pmatrix} 3 \\ k - 4 \end{pmatrix} \quad (5)$$

$$(\mathbf{B} - \mathbf{P}) = \begin{pmatrix} k - 2 \\ 3 \end{pmatrix} \quad (6)$$

$$(\mathbf{A} - \mathbf{P})^\top (\mathbf{A} - \mathbf{P}) = (3 \quad k - 4) \begin{pmatrix} 3 \\ k - 4 \end{pmatrix} \quad (7)$$

$$= 9 + (k - 4)^2 = 9 + k^2 - 8k + 16 = k^2 - 8k + 25 \quad (8)$$

$$(\mathbf{B} - \mathbf{P})^\top (\mathbf{B} - \mathbf{P}) = (k - 2 \quad 3) \begin{pmatrix} k - 2 \\ 3 \end{pmatrix} = \quad (9)$$

$$(k - 2)^2 + 9 = k^2 - 4k + 4 + 9 = k^2 - 4k + 13 \quad (10)$$

$$\text{From (8) and (10), } k^2 - 8k + 25 = k^2 - 4k + 13 \quad (11)$$

$$\implies -4k + 8k = 25 - 13 \implies 4k = 12 \implies k = 3 \quad (12)$$

$$\text{Hence, the final answer is } \boxed{k = 3} \quad (13)$$

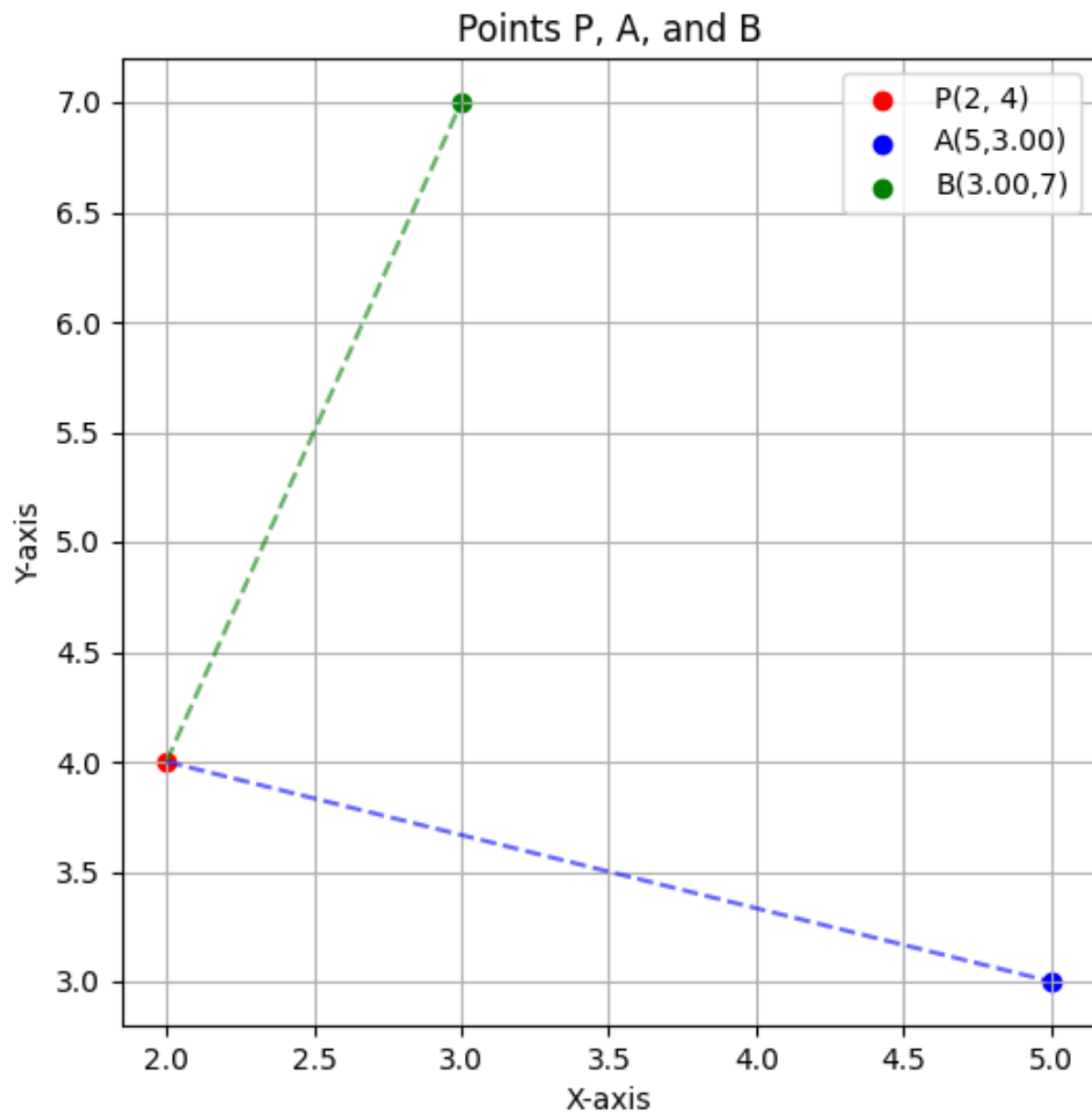


Fig. 1: Plot for 1.9.26