

1.7.7

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Problem Statement

Find the value of p such that the points:

$$A(2, 1), \quad B(p, -1), \quad C(-1, 3)$$

are **collinear**, using **matrices and echelon form**.

Step 1: Vectors

The points A, B, C are collinear if the vectors \overrightarrow{AB} and \overrightarrow{AC} are linearly dependent.

$$\overrightarrow{AB} = (p - 2, -2), \quad \overrightarrow{AC} = (-3, 2).$$

Step 2: Form the Matrix

We form a matrix M with these vectors as rows:

$$M = \begin{bmatrix} p-2 & -2 \\ -3 & 2 \end{bmatrix}.$$

For collinearity, $\text{rank}(M) = 1$.

Step 3: Row Reduction

Row operations to get echelon form:

$$R_2 \rightarrow R_2 + \frac{3}{p-2} R_1$$

$$R_2 = \begin{bmatrix} -3 & 2 \end{bmatrix} + \frac{3}{p-2} \begin{bmatrix} p-2 & -2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{2(p-2)-6}{p-2} \end{bmatrix}.$$

Step 4: Rank Condition

For $\text{rank}(M) = 1$, the second row must vanish:

$$\frac{2(p-2) - 6}{p-2} = 0$$

$$2(p-2) - 6 = 0 \implies 2p - 10 = 0$$

$$\boxed{p = 5}$$

Step 5: Verification

When $p = 5$, the points are

$$A = (2, 1), \quad B = (5, -1), \quad C = (-1, 3).$$

$$\text{Slope of } AB = \frac{-1 - 1}{5 - 2} = \frac{-2}{3}, \quad \text{Slope of } AC = \frac{3 - 1}{-1 - 2} = \frac{2}{-3} = \frac{-2}{3}.$$

Since slopes are equal, the points are collinear.

Final Answer

$$p = 5$$

C Code for echelon matrix

```
#include <stdio.h>

void echelonForm(double matrix[2][2]) {
    // Assuming matrix is 2x2
    double factor;

    // Make the first element of second row zero by row operation
    if (matrix[0][0] == 0) {
        printf("Cannot perform elimination as pivot is zero.\n");
        return;
    }

    factor = matrix[1][0] / matrix[0][0];

    // Subtract factor * first row from second row
    matrix[1][0] = matrix[1][0] - factor * matrix[0][0];
    matrix[1][1] = matrix[1][1] - factor * matrix[0][1];
```

C Code for echelon matrix

```
int main() {  
    double p;  
    printf(Enter value for p: );  
    scanf(%lf, &p);  
  
    // Create matrix with rows [p-2, -2] and [-3, 2]  
    double matrix[2][2] = {  
        {p - 2, -2},  
        {-3, 2}  
    };  
}
```

C Code for echelon matrix

```
printf(Original matrix:\n);
for(int i=0; i<2; i++) {
    for(int j=0; j<2; j++) {
        printf(%8.3f , matrix[i][j]);
    }
    printf(\n);
}

echelonForm(matrix);

printf(\nMatrix after echelon form operation:\n);
for(int i=0; i<2; i++) {
    for(int j=0; j<2; j++) {
        printf(%8.3f , matrix[i][j]);
    }
    printf(\n);
}
```

Python Code for plot

```
import matplotlib.pyplot as plt

# Points
A = (2, 1)
B = (5, -1)
C = (-1, 3)

# Plot points
plt.scatter(*A, color='red', label='A(2,1)')
plt.scatter(*B, color='blue', label='B(5,-1)')
plt.scatter(*C, color='green', label='C(-1,3)')

# Plot line through A and C
x_values = [A[0], C[0]]
y_values = [A[1], C[1]]
plt.plot(x_values, y_values, 'k--', label='Line through A and C')
```

Python Code for plot

```
plt.legend()
plt.grid(True)
plt.xlabel('x')
plt.ylabel('y')
plt.title('Collinear Points for p=5')

# Save the plot as an image file
plt.savefig('python_plot.png') # Saves to current directory

plt.show()
```

Plot

`figs/python_plot.png`