1.9.4

EE25BTECH11015 - Bhoomika V

Question :-
If $\|\mathbf{a}\| = 4$ and $-3 \le \lambda \le 2$, then $\|\lambda\mathbf{a}\|$ lies in

1) $[0, 12]$
2) $[2, 3]$
3) $[8, 12]$
4) $[-12, 8]$

Solution:

Using matrix definition of the norm:

$$\|\mathbf{a}\| = \sqrt{\mathbf{a}^T\mathbf{a}}, \qquad \text{hence} \qquad \mathbf{a}^T\mathbf{a} = \|\mathbf{a}\|^2 = 4^2 = 16. \tag{4.1}$$

The squared norm of $\lambda\mathbf{a}$ using matrix notation is:

$$\|\lambda\mathbf{a}\|^2 = (\lambda\mathbf{a})^T(\lambda\mathbf{a}) = \lambda^2(\mathbf{a}^T\mathbf{a}).$$

Substituting from Equation (4.1):

$$\|\lambda\mathbf{a}\|^2 = 16\lambda^2.$$

Taking square roots (norms are nonnegative) gives

$$\|\lambda\mathbf{a}\| = \sqrt{16\lambda^2} = 4\,|\lambda|.$$

The range of $|\lambda|$ given $-3 \le \lambda \le 2$.

$$0 \le |\lambda| \le \max\{|-3|, |2|\} = 3.$$

Multiplying by 4 yields

$$0 \le 4|\lambda| \le 12.$$

Therefore

$$\|\lambda\mathbf{a}\| = 4|\lambda| \in [0, 12].$$

$$\boxed{\|\lambda\mathbf{a}\| \in [0, 12]}$$

$$\implies k = 2$$

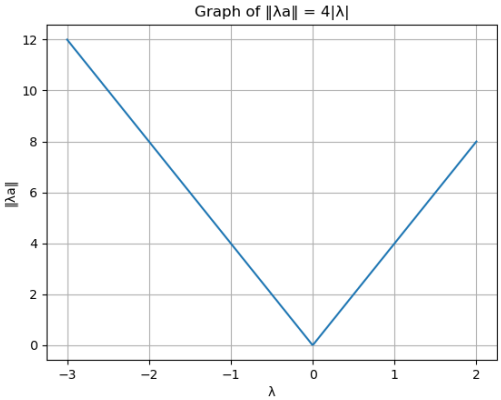

Fig. 4.1