

Question

For what value of k , will the following pair of equations have infinitely many solutions

$$2x + 3y = 7 \text{ and } (k + 2)x - 3(1 - k)y = 5k + 1$$

Solution

$$2x + 3y = 7 \quad (1)$$

$$(k + 2)x - 3(1 - k)y = 5k + 1 \quad (2)$$

Matrix Representation

Write the system as an augmented matrix:

$$\left(\begin{array}{cc|c} 2 & 3 & 7 \\ k+2 & -3(1-k) & 5k+1 \end{array} \right) = \left(\begin{array}{cc|c} 2 & 3 & 7 \\ k+2 & -3+3k & 5k+1 \end{array} \right) \quad (3)$$

Condition for Infinitely Many Solutions

For infinitely many solutions, the second row must be a scalar multiple of the first row. Let the scalar be λ . Then:

$$k + 2 = 2\lambda \quad (i) \quad (4)$$

$$-3 + 3k = 3\lambda \quad (ii) \quad (5)$$

$$5k + 1 = 7\lambda \quad (iii) \quad (6)$$

From (i), solve for λ :

$$\lambda = \frac{k + 2}{2} \quad (7)$$

Substitute into (ii):

$$-3 + 3k = 3\left(\frac{k + 2}{2}\right) \quad (8)$$

$$-3 + 3k = \frac{3k + 6}{2} \quad (9)$$

$$-6 + 6k = 3k + 6 \quad (10)$$

$$3k = 12 \quad (11)$$

$$k = 4 \quad (12)$$

Verification

Check with equation (iii):

$$\lambda = \frac{4+2}{2} = 3 \quad (13)$$

$$7\lambda = 21 \quad (14)$$

$$5k + 1 = 5(4) + 1 = 21 \quad (15)$$

Final Answer

$$\boxed{k = 4} \quad (16)$$