Question 4.13.5

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1 Question:

The set of lines $ax + by + c = 0$, where $3a + 2b + 4c = 0$ are concurrent at the point __________.

2 Solution:

We are given the fact that $3a + 2b + 4c = 0$. This can be written as:

$$\implies \begin{pmatrix} 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \tag{1}$$

Let the point of concurrency be $\mathbf{P}$, at coordinates $\begin{pmatrix} x \\ y \end{pmatrix}$. Because $\mathbf{P}$ lies on all lines $ax + by + c = 0$, we can write the following system of equations:

$$\begin{pmatrix} 3 & 2 & 4 \\ x & y & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2}$$

For this system to have a non-trivial solution in a, b and c, and for the two original equations to be linearly dependent (since equation 2 should be true whenever 1 is true), the rank of the coefficient matrix must be 1. Applying row reduction to get the reduced row echelon form:

$$\begin{pmatrix} 3 & 2 & 4 \\ x & y & 1 \end{pmatrix} \xleftrightarrow{R_1 \to \frac{R_1}{3}} \begin{pmatrix} 1 & \frac{2}{3} & \frac{4}{3} \\ x & y & 1 \end{pmatrix} \tag{3}$$

$$\xleftrightarrow{R_3 \to R_3 - \frac{3R_1}{4}} \begin{pmatrix} 1 & \frac{2}{3} & \frac{4}{3} \\ x - \frac{3}{4} & y - \frac{1}{2} & 0 \end{pmatrix} \tag{4}$$

Clearly, for the rank to be 1, the last row must be all zeros. Therefore the point of concurrency $\mathbf{P}$ is $\begin{pmatrix} \frac{3}{4} \\ \frac{1}{2} \end{pmatrix}$.

3 Plot:

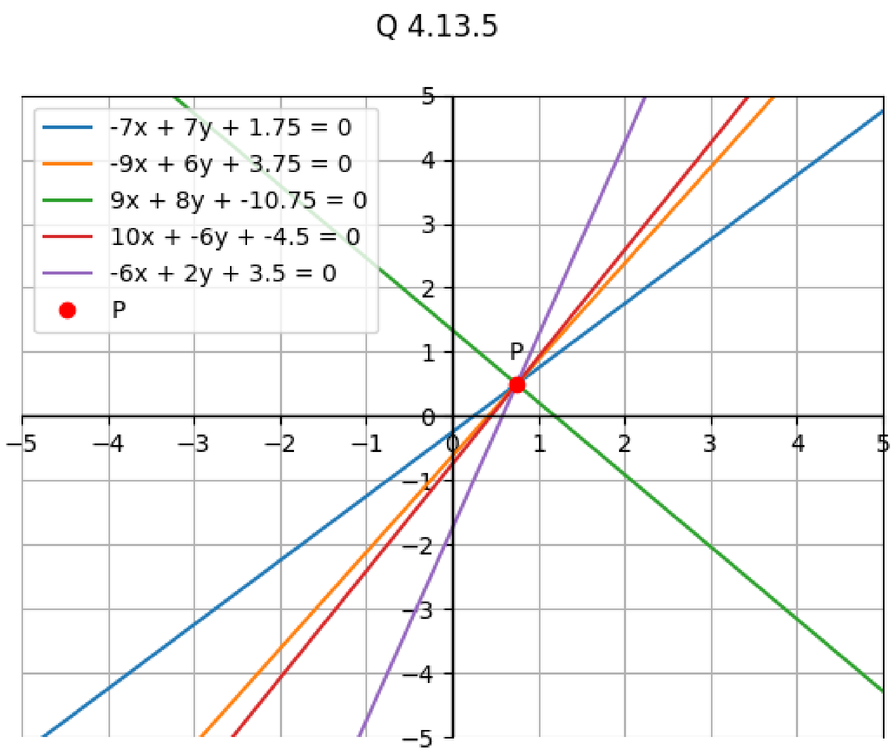

Figure 1: Graph of lines with randomly generated values of a and b satisfying $3a + 2b + 4c = 0$. All lines are concurrent at the point $\begin{pmatrix} \frac{3}{4} \\ \frac{1}{2} \end{pmatrix}$ (marked in red).