Matgeo Presentation - Problem 10.7.111

ee25btech11063 - Vejith

October 3, 2025

Question

Let the straight line $y = 2x$ touch a circle with center $(0, a)$, $a > 0$, and radius r at a point $\mathbf{A_1}$. Let $\mathbf{B_1}$ be the point on the circle such that the line segment $\mathbf{A_1B_1}$ is a diameter of the circle. Let $a + r = 5 + \sqrt{5}$. Match the following:

(A) a equals	(1) $(-2, 4)$
(B) r equals	2. $\sqrt{5}$
(C)$\mathbf{A_1}$ equals	(3)$(-2, 6)$
(D) $\mathbf{B_1}$ equals	(4) 5
	(5) $(2, 4)$

The correct option is (2024)

a) A−4, B−2, C−1, D−3

b) A−2, B−4, C−1, D−3

c) A−4, B−2, C−5, D−3

d) A−2, B−4, C−3, D−5

Solution

The equation of a Conic in Matrix form is

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.1}$$

For the given circle. let r be the radius of given circle

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ -a \end{pmatrix}, f = a^2 - r^2 \tag{0.2}$$

Equation of Tangent is given by

$$\mathbf{n}^\top \mathbf{x} = c \tag{0.3}$$

$$\implies \mathbf{n} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, c = 0 \tag{0.4}$$

$$\implies \begin{pmatrix} 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0 \tag{0.5}$$

For a circle, the points of contact are

$$\mathbf{q_j} = \left(\pm r \frac{\mathbf{n_j}}{\|\mathbf{n_j}\|} - \mathbf{u} \right), j = 1, 2 \tag{0.6}$$

Solution

let $\mathbf{A_1}$ be the point of contact

$$\mathbf{A_1} = \left(-r\frac{\mathbf{n}}{\|\mathbf{n}\|} - \mathbf{u}\right) \tag{0.7}$$

$$= \left(\frac{r}{\sqrt{5}}\begin{pmatrix}2\\-1\end{pmatrix} - \begin{pmatrix}0\\-a\end{pmatrix}\right) \quad \text{(given } a \text{ is positive)} \tag{0.8}$$

$$\implies \mathbf{A_1} = \begin{pmatrix}\frac{2r}{\sqrt{5}}\\ \frac{-r}{\sqrt{5}} + a\end{pmatrix} \tag{0.9}$$

$\mathbf{A_1}$ lies on (0.5)

$$\frac{-r}{\sqrt{5}} + a = \frac{4r}{\sqrt{5}} \tag{0.10}$$

$$\implies a = \sqrt{5}r \tag{0.11}$$

given

$$a + r = 5 + \sqrt{5} \tag{0.12}$$

Solution

substitute (0.11) in (0.12)

$$\sqrt{5}r + r = 5 + \sqrt{5} \tag{0.13}$$

$$\implies r = \sqrt{5} \tag{0.14}$$

From (0.11)

$$\implies a = 5 \tag{0.15}$$

$$\implies \mathbf{u} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \tag{0.16}$$

From (0.9) and (0.14)

$$\mathbf{A_1} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \tag{0.17}$$

Conclusion

Given $\mathbf{A_1}$ and $\mathbf{B_1}$ is the diameter of the circle

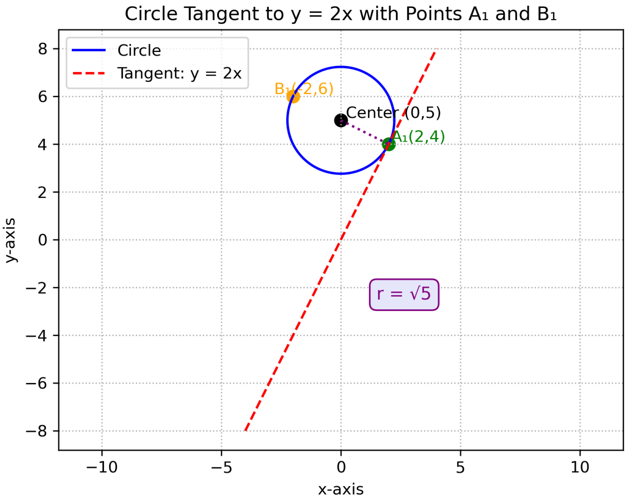
$$\frac{\mathbf{A_1}+\mathbf{B_1}}{2} = \mathbf{u} \tag{0.18}$$

$$\mathbf{B_1} = \begin{pmatrix}\mathbf{u} & \mathbf{A_1}\end{pmatrix}\begin{pmatrix}2\\-1\end{pmatrix} \tag{0.19}$$

$$\mathbf{B_1} = \begin{pmatrix}-2\\6\end{pmatrix} \tag{0.20}$$

Answer is c) A−4, B−2, C−5, D−3

Plot


Circle Tangent to y = 2x with Points A₁ and B₁
Circle
Tangent: y = 2x
B₁(-2,6)
Center (0,5)
A₁(2,4)
r = √5
y-axis
x-axis
8
6
4
2
0
−2
−4
−6
−8
−10
−5
0
5
10

Figure

C Code: circle.c

```
#include <stdio.h>
#include <math.h>

int main() {
    FILE *fp;
    double a, r;
    double A1x, A1y, B1x, B1y;

    // Given relation: a + r = 5 + sqrt(5)
    // and r = a / sqrt(5)
    // So we solve for a and r
    a = 5;
    r = sqrt(5);

    // Coordinates of A1 (point of tangency)
    A1x = 2;
    A1y = 4;

    // Coordinates of B1 (diametrically opposite point)
    B1x = -2;
    B1y = 6;

    // Open file to write output
    fp = fopen("circle.dat", "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }
```

C Code: circle.c

```c
    fprintf(fp, "Results␣of␣the␣circle␣problem:\n");
    fprintf(fp, "--------------------------------\n");
    fprintf(fp, "(A)␣a␣=␣%.2f\n", a);
    fprintf(fp, "(B)␣r␣=␣%.5f\n", r);
    fprintf(fp, "(C)␣A1␣=␣(%.2f,␣%.2f)\n", A1x, A1y);
    fprintf(fp, "(D)␣B1␣=␣(%.2f,␣%.2f)\n", B1x, B1y);
    fprintf(fp, "\nMatching␣options:\n(A)->(4),␣(B)->(2),␣(C)->(5),␣(D)->(3)\n");
    fprintf(fp, "Correct␣Option:␣(c)\n");

    fclose(fp);

    printf("Results␣written␣successfully␣to␣circle.dat\n");

    return 0;
}
```

Python: plot.py

```
import matplotlib.pyplot as plt
import numpy as np
import math

# Given values
a = 5 # y-coordinate of center
r = math.sqrt(5) # radius
center = (0, a)
A1 = (2, 4)
B1 = (-2, 6)

# Generate circle
theta = np.linspace(0, 2 * np.pi, 400)
x_circle = r * np.cos(theta)
y_circle = a + r * np.sin(theta)

# Tangent line y = 2x
x_line = np.linspace(-4, 4, 200)
y_line = 2 * x_line

# Plot circle and tangent
plt.plot(x_circle, y_circle, label='Circle', color='blue')
plt.plot(x_line, y_line, color='red', linestyle='--', label='Tangent: y = 2x')

# Plot key points
plt.scatter(*center, color='black', s=60)
plt.scatter(*A1, color='green', s=60)
plt.scatter(*B1, color='orange', s=60)

# Draw radius line
plt.plot([center[0], A1[0]], [center[1], A1[1]], color='purple', linestyle=':')
```

Python: plot.py

```
plt.text(2.1, 4.1, "A(2,4)", color='green', fontsize=10)
plt.text(-2.8, 6.1, "B(-2,6)", color='orange', fontsize=10)
plt.text(0.2, 5.1, "Center␣(0,5)", color='black', fontsize=10)

# Box below tangent equation showing radius
plt.text(1.5, -2.5, "r␣=␣5", fontsize=11, color='purple',
        bbox=dict(facecolor='lavender', edgecolor='purple', boxstyle='round,pad=0.4'))

# Make it neat
plt.axis('equal')
plt.grid(True, linestyle=':')
plt.xlabel('x-axis')
plt.ylabel('y-axis')
plt.title('Circle␣Tangent␣to␣y␣=␣2x␣with␣Points␣A␣and␣B')
plt.legend(loc='upper␣left')
plt.savefig('circle.png', dpi=300)
plt.show()
```