

MatGeo Assignment 2.6.13

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AI25BTECH11007

Question:

Using vectors, find the area of $\triangle ABC$ with vertices $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$.

Solution:

Compute vectors $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}, \quad (0.1)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix}. \quad (0.2)$$

Recall the identity:

$$\|\mathbf{B} - \mathbf{A} \times \mathbf{C} - \mathbf{A}\|^2 = \|\mathbf{B} - \mathbf{A}\|^2 \|\mathbf{C} - \mathbf{A}\|^2 - (\mathbf{B} - \mathbf{A}^T \mathbf{C} - \mathbf{A})^2. \quad (0.3)$$

Compute the inner products:

$$\|\mathbf{B} - \mathbf{A}\|^2 = (\mathbf{B} - \mathbf{A})^T (\mathbf{B} - \mathbf{A}) = 1^2 + (-3)^2 + 1^2 = 11, \quad (0.4)$$

$$\|\mathbf{C} - \mathbf{A}\|^2 = (\mathbf{C} - \mathbf{A})^T (\mathbf{C} - \mathbf{A}) = 3^2 + 3^2 + (-4)^2 = 34, \quad (0.5)$$

$$(\mathbf{B} - \mathbf{A})^T (\mathbf{C} - \mathbf{A}) = (1)(3) + (-3)(3) + (1)(-4) = -10. \quad (0.6)$$

Substitute into (??):

$$\begin{aligned} \|\mathbf{B} - \mathbf{A} \times \mathbf{C} - \mathbf{A}\|^2 &= (11)(34) - (-10)^2 \\ &= 374 - 100 \\ &= 274. \end{aligned} \quad (0.7)$$

Hence

$$\|\mathbf{B} - \mathbf{A} \times \mathbf{C} - \mathbf{A}\| = \sqrt{274}. \quad (0.8)$$

The area of $\triangle ABC$ is

$$\text{Area}(\triangle ABC) = \frac{1}{2} \|\mathbf{B} - \mathbf{A} \times \mathbf{C} - \mathbf{A}\| = \frac{\sqrt{274}}{2}. \quad (0.9)$$

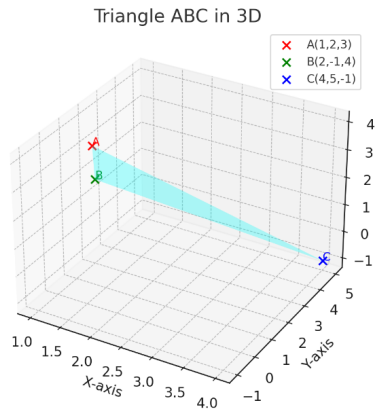


Fig. 0.1: Image Visual