5.13.81

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Question : Let $S = \{\mathbf{A} = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a, b, c, d, e \in \{0, 1\} \text{ and } |\mathbf{A}| \in \{-1, 1\}\}$. Find the number of elements in S.

Solution :

Name	**Matrix**
A	$\begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix}$ with $a, b, c, d, e \in \{0, 1\}$

Table : Matrix

Applyting row operation to **A**

$$\begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} \xleftrightarrow{R_2 \to R_2 - R_3} \begin{pmatrix} 0 & 1 & c \\ 0 & a-b & d-e \\ 1 & b & e \end{pmatrix} \tag{1}$$

Finding the determinant by the first column

$$\left|\mathbf{A}\right| = d - e - c(a - b) \tag{2}$$

Taking cases to find the possibilities of matrix **A**
Case 1 : $\left|\mathbf{A}\right| = 1$

if c = 0
the value of b and a can be 0 or 1.

$$d - e = 1 \tag{3}$$

So,

$$d = 1 \tag{4}$$
$$e = 0 \tag{5}$$

By permutation we get ,

$$2 \times 2 \times 1 \times 1 = 4 \tag{6}$$

if c = 1, we get 4 possibilities

$$d - e - (a - b) = 1 \tag{7}$$

So,

$$d = 1 \qquad e = 0 \tag{8}$$

$$b = a = 1 \qquad b = a = 0 \tag{9}$$

$$a = 0 \qquad b = 1 \tag{10}$$

$$d = e = 1 \qquad d = e = 0 \tag{11}$$

Case 2 : $\left|\mathbf{A}\right| = -1$

if c = 0
the value of b and a can be 0 or 1.

$$d - e = -1 \tag{12}$$

So,

$$d = 0 \tag{13}$$

$$e = 1 \tag{14}$$

By permutation we get ,

$$2 \times 2 \times 1 \times 1 = 4 \tag{15}$$

if c = 1, we get 4 possibilities

$$d - e - (a - b) = -1 \tag{16}$$

So,

$$d = 0 \qquad e = 1 \tag{17}$$

$$b = a = 1 \qquad b = a = 0 \tag{18}$$

$$a = 1 \qquad b = 0 \tag{19}$$

$$d = e = 1 \qquad d = e = 0 \tag{20}$$

By adding all the possibilities , we get

$$4 + 4 + 4 + 4 = 16 \tag{21}$$

Therefore, the number of elements in $S = 16$.