5.4.42

EE25BTECH11048 - Revanth Siva Kumar

Question:

Using elementary transformations, find the inverse of the following matrix.

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}$$

Solution:

We solve using Gauss-Jordan elimination.

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -3 & 0 & 1 & 0 \\ 3 & -2 & 4 & 0 & 0 & 1 \end{array}\right) \xleftrightarrow{R_3 \leftarrow R_3 - 3R_1} \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -3 & 0 & 1 & 0 \\ 0 & 1 & -2 & -3 & 0 & 1 \end{array}\right) \tag{0.1}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{1}{2}R_2} \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -3/2 & 0 & 1/2 & 0 \\ 0 & 1 & -2 & -3 & 0 & 1 \end{array}\right) \tag{0.2}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - R_2} \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -3/2 & 0 & 1/2 & 0 \\ 0 & 0 & -1/2 & -3 & -1/2 & 1 \end{array}\right) \tag{0.3}$$

$$\xleftrightarrow{R_3 \leftarrow -2R_3} \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -3/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 6 & 1 & -2 \end{array}\right) \tag{0.4}$$

$$\xleftrightarrow[R_1 \leftarrow R_1 - 2R_3]{R_2 \leftarrow R_2 + 3/2R_3} \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & -11 & -2 & 4 \\ 0 & 1 & 0 & 9 & 2 & -3 \\ 0 & 0 & 1 & 6 & 1 & -2 \end{array}\right) \tag{0.5}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & 1 \\ 0 & 1 & 0 & 9 & 2 & -3 \\ 0 & 0 & 1 & 6 & 1 & -2 \end{array}\right) \tag{0.6}$$

$$\therefore \text{Inverse of the given Matrix:} \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix} \tag{0.7}$$