7.4.27

EE25BTECH11020 - Darsh Pankaj Gajare

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Question:

The triangle PQR is inscribed in the circle $x^2+y^2=25$.If $\mathbf{Q}$ and $\mathbf{R}$ have co-ordinates $(3,4)$ and $(-4,3)$ respectively then $\angle QPR$ is equal to

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$

Solution:

Table

Q	$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$
R	$\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

$$\mathbf{x}^\top \mathbf{x} = 25 \tag{0.1}$$

The given points (position vectors) are

$$\mathbf{q} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \qquad \mathbf{r} = \begin{pmatrix} -4 \\ 3 \end{pmatrix} \tag{0.2}$$

Verify they lie on the circle:

$$\mathbf{q}^\top \mathbf{q} = 3^2 + 4^2 = 25, \tag{0.3}$$

$$\mathbf{r}^\top \mathbf{r} = (-4)^2 + 3^2 = 25. \tag{0.4}$$

Compute the inner product (matrix/dot product):

$$\mathbf{q}^\top \mathbf{r} = \begin{pmatrix} 3 & 4 \end{pmatrix} \begin{pmatrix} -4 \\ 3 \end{pmatrix} \tag{0.5}$$

$$= 3 \cdot (-4) + 4 \cdot 3 = -12 + 12 = 0. \tag{0.6}$$

Compute norms (using matrix notation) and the central angle θ:

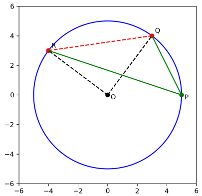
$$\|\mathbf{q}\| = \sqrt{\mathbf{q}^\top \mathbf{q}} = 5, \qquad \|\mathbf{r}\| = \sqrt{\mathbf{r}^\top \mathbf{r}} = 5, \tag{0.7}$$

$$\cos\theta = \frac{\mathbf{q}^\top \mathbf{r}}{\|\mathbf{q}\|\,\|\mathbf{r}\|} = \frac{0}{5 \cdot 5} = 0 \tag{0.8}$$

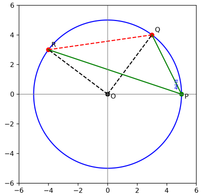
$$\implies \theta = \frac{\pi}{2}. \tag{0.9}$$

Since $\angle QPR$ is the angle subtended at the circumference by chord QR, it equals half the central angle:

$$\angle QPR = \frac{\theta}{2} = \frac{\pi}{4}. \tag{0.10}$$



Plot using C libraries



Plot using Python