4.3.13

EE25BTECH11026-Harsha

Question:
Equations of the diagonals of the square formed by the lines x = 0, y = 0, x = 1 and y = 1 are ___________.
Solution:
Let us solve the given equation theoretically and then verify the solution computationally.

According to the question,
The vertices of the square are ,

$$\mathbf{a} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \mathbf{d} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

To compute the equation of the diagnols , we can use the normal form of the equation,which is given by

$$\mathbf{n}^T\mathbf{x} = 0 \; for\ the\ lines\ passing\ through\ the\ origin$$

$$\mathbf{n}^T\mathbf{x} = 1 \; for\ the\ lines\ not\ passing\ through\ the\ origin$$

where,

$\mathbf{n}$-vector orthogonal to the direction vector

$\mathbf{x} = \begin{pmatrix} x & y \end{pmatrix}^T$

For diagonal $\mathbf{c} - \mathbf{a}$,

$$\mathbf{n} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{d}$$

where $\mathbf{d}$ is the direction vector of diagonal.

$$\therefore \mathbf{n} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \left(\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right)$$

$$\implies \mathbf{n} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

As we know that the diagonal $\mathbf{c} - \mathbf{a}$ passes through the origin,

$$\therefore \begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

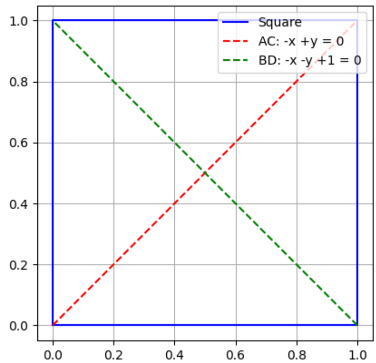
But, for diagonal $\mathbf{d}-\mathbf{b}$, as the diagonal doesn't pass through the origin,

$$\mathbf{n}^T\mathbf{x} = 1$$

As we know that the diagonal $\mathbf{d}-\mathbf{b}$ pass through the vectors $\begin{pmatrix}1 & 0\end{pmatrix}^T$ and $\begin{pmatrix}0 & 1\end{pmatrix}^T$, we can say that $\mathbf{n}$ would be $\begin{pmatrix}1 & 1\end{pmatrix}^T$

$$\therefore \begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}x \\ y\end{pmatrix} = 1$$

From the figure, it is clearly verified that the theoretical solution matches with the computational solution.



Plot of Square with diagonals