2.4.41

EE25BTECH11044 - Sai Hasini Pappula

Question:
Determine whether the points $A(3, 6, 9)$, $B(10, 20, 30)$, $C(24, -41, 5)$ are the vertices of a right-angled triangle using matrices.

Solution:

Let the position vectors of the points be

$$\mathbf{A} = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 24 \\ -41 \\ 5 \end{bmatrix}.$$

The vectors representing the sides of the triangle are

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = \begin{bmatrix} 7 \\ 14 \\ 21 \end{bmatrix}, \quad \mathbf{BC} = \mathbf{C} - \mathbf{B} = \begin{bmatrix} 14 \\ -61 \\ -25 \end{bmatrix}, \quad \mathbf{AC} = \mathbf{C} - \mathbf{A} = \begin{bmatrix} 21 \\ -47 \\ -4 \end{bmatrix}.$$

Check for right angles using the matrix multiplication form:

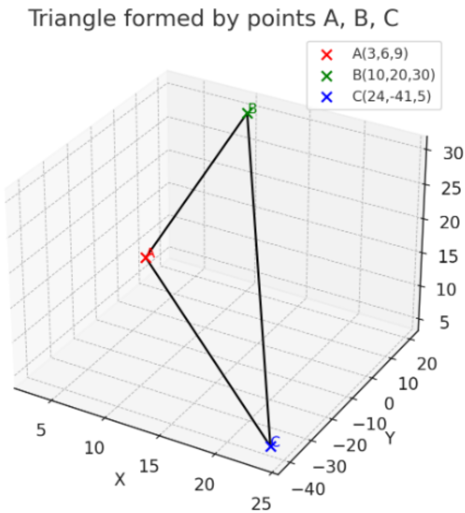
$$\mathbf{AB}^T\mathbf{AC} = \begin{bmatrix} 7 & 14 & 21 \end{bmatrix} \begin{bmatrix} 21 \\ -47 \\ -4 \end{bmatrix} = 7 \cdot 21 + 14 \cdot (-47) + 21 \cdot (-4) = -595 \neq 0$$

$$\mathbf{AB}^T\mathbf{BC} = \begin{bmatrix} 7 & 14 & 21 \end{bmatrix} \begin{bmatrix} 14 \\ -61 \\ -25 \end{bmatrix} = 7 \cdot 14 + 14 \cdot (-61) + 21 \cdot (-25) = -1281 \neq 0$$

$$\mathbf{AC}^T\mathbf{BC} = \begin{bmatrix} 21 & -47 & -4 \end{bmatrix} \begin{bmatrix} 14 \\ -61 \\ -25 \end{bmatrix} = 21 \cdot 14 + (-47) \cdot (-61) + (-4) \cdot (-25) = 3261 \neq 0$$

Since none of these products is zero, no angle of the triangle is $90°$.

Conclusion:
The points A, B, and C do **not** form a right-angled triangle.


Triangle formed by points A, B, C
A(3,6,9)
B(10,20,30)
C(24,-41,5)
A
B
C
X
Y
5
10
15
20
25
−40
−30
−20
−10
0
10
20
5
10
15
20
25
30