## 1

## Matrices in Geometry 10.7.86

## EE25BTECH11037 - Divyansh

**Question:** Let  $C_1$  and  $C_2$  be two circles with  $C_2$  lying inside  $C_1$ . A circle C lying inside  $C_1$  touches  $C_1$  internally and  $C_2$  externally. Identify the locus of center of C.

**Solution:** Let the center of C,  $C_1$  and  $C_2$  be O,  $O_1$  and  $O_2$ , respectively.

Let the radii of circles C,  $C_1$  and  $C_2$  be r,  $r_1$  and  $r_2$ 

It is given that C touches the circle  $C_1$  internally and  $C_2$  externally. Therefore,

$$\|\mathbf{O} - \mathbf{O}_1\| = r_1 - r \tag{1}$$

$$\|\mathbf{O} - \mathbf{O_2}\| = r_2 + r \tag{2}$$

Adding these two equations, we get

$$\|\mathbf{O} - \mathbf{O}_1\| + \|\mathbf{O} - \mathbf{O}_2\| = r_1 + r_2 \tag{3}$$

Substitute O as x

$$\|\mathbf{x} - \mathbf{O_1}\| + \|\mathbf{x} - \mathbf{O_2}\| = r_1 + r_2 \tag{4}$$

This is equation of an ellipse because it is of form

$$\|\mathbf{x} - \mathbf{S}_1\| + \|\mathbf{x} - \mathbf{S}_2\| = 2a \tag{5}$$

with focii as  $\mathbf{O_1}$  ,  $\mathbf{O_2}$  and length of the major axis as  $r_1 + r_2$ 

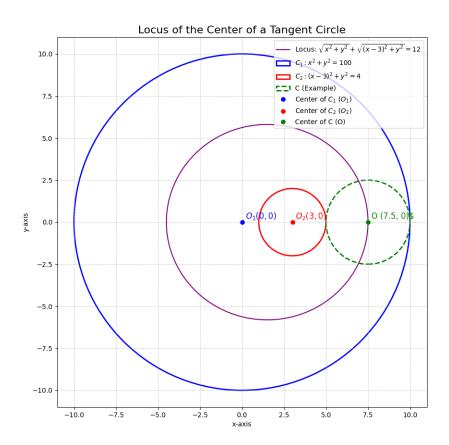


Fig. 1: Graph for 10.7.86