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AI25BTECH11027 - NAGA BHUVANA

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Question:

If the co-ordinates of the points **A**, **B**, **C**, **D** be (1,2,3), (4,5,7), (-4,3,-6) and (2,9,2) respectively, then find the angle between lines AB and CD.

Solution:

Let

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 5 \\ 7 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -4 \\ 3 \\ -6 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} 2 \\ 9 \\ 2 \end{pmatrix} \quad (0.1)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 4 \\ 5 \\ 7 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} \quad (0.2)$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} 2 \\ 9 \\ 2 \end{pmatrix} - \begin{pmatrix} -4 \\ 3 \\ -6 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \\ 8 \end{pmatrix} \quad (0.3)$$

Let the angle between $B - A$ and $D - C$ be θ

$$\cos \theta = \frac{(B - A)^T (D - C)}{\| \mathbf{B} - \mathbf{A} \| \| \mathbf{D} - \mathbf{C} \|} \quad (0.4)$$

$$\cos \theta = \frac{(3 \ 3 \ 4) \begin{pmatrix} 6 \\ 6 \\ 8 \end{pmatrix}}{\sqrt{34} \sqrt{136}} \quad (0.5)$$

$$\cos \theta = \frac{(3)(6) + (3)(6) + (4)(8)}{68} \quad (0.6)$$

$$\cos \theta = \frac{68}{68} \quad (0.7)$$

$$\cos \theta = 1 \quad (0.8)$$

$$\theta = 0^\circ \quad (0.9)$$

\therefore The angle between lines $(\mathbf{B} - \mathbf{A})$ and $(\mathbf{D} - \mathbf{C})$ is 0° (Collinear lines)

