1.2.16

AI25BTECH11010 - Dhanush Kumar

Question:
Show that $(-1, 2, 1)$, $(1, -2, 5)$, $(4, -7, 8)$ and $(2, -3, 4)$ are the vertices of a parallelogram.

Solution:
Let

$$\mathbf{A} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 4 \\ -7 \\ 8 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}. \tag{1}$$

Now,

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 - (-1) \\ -2 - 2 \\ 5 - 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}, \tag{2}$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} 4 - 2 \\ -7 - (-3) \\ 8 - 4 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}. \tag{3}$$

Hence,

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}. \tag{4}$$

Hence,

$$\mathbf{B} - \mathbf{A} \| \mathbf{C} - \mathbf{D}$$

Also,

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 4 - 1 \\ -7 - (-2) \\ 8 - 5 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \\ 3 \end{pmatrix}, \tag{5}$$

$$\mathbf{D} - \mathbf{A} = \begin{pmatrix} 2 - (-1) \\ -3 - 2 \\ 4 - 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -5 \\ 3 \end{pmatrix}. \tag{6}$$

Thus,

$$\mathbf{C} - \mathbf{B} = \mathbf{D} - \mathbf{A} = \begin{pmatrix} 3 \\ -5 \\ 3 \end{pmatrix}. \tag{7}$$

Hence,

$$\mathbf{C} - \mathbf{B} \| \mathbf{D} - \mathbf{A}$$

Therefore, A, B, C, D are the vertices of a parallelogram.

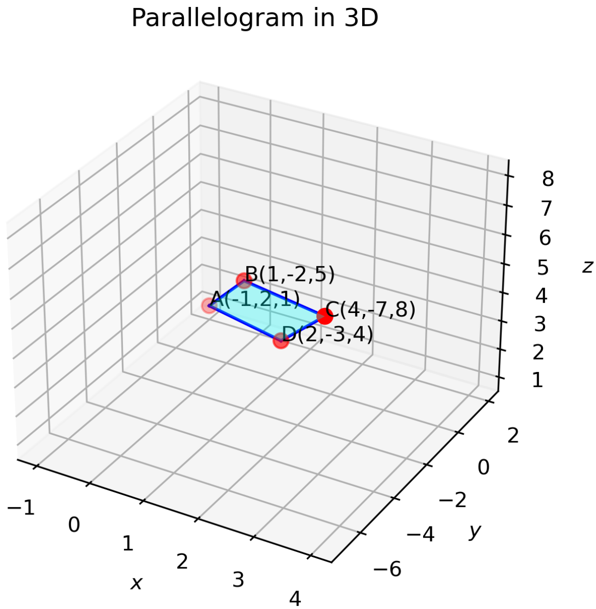

Fig. 0