5.10.4

AI25BTECH11001 - ABHISEK MOHAPATRA

Question: Write the balanced chemical equations for the following reaction.

$$BaCl_2 + K_2SO_4 \to BaSO_4 + KCl \tag{1}$$

Solution: Let the balanced version of given equation be

$$x_1BaCl_2 + x_2K_2SO_4 \to x_3BaSO_4 + x_4KCl \tag{2}$$

which results in the following equations:

$$(x_1 - x_3)\,Ba = 0 \tag{3}$$

$$(2x_1 - x_4)\,Cl = 0 \tag{4}$$

$$(2x_2 - x_4)\,K = 0 \tag{5}$$

$$(x_2 - x_3)\,S = 0 \tag{6}$$

$$(4x_2 - 4x_3)\,O = 0 \tag{7}$$

which can be expresseed as

$$x_1 + 0x_2 + (-1)x_3 + x_4 = 0 \tag{8}$$

$$2x_1 + 0x_2 + 0x_3 + (-1)x_4 = 0 \tag{9}$$

$$0x_1 + 2x_2 + 0x_3 + (-1)x_4 = 0 \tag{10}$$

$$0x_1 + x_2 + (-1)x_3 + 0x_4 = 0 \tag{11}$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 2 & 0 & 0 & -1 \\ 0 & 2 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix}\mathbf{X} = 0 \tag{12}$$

which can be reduced as follows

$$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 2 & 0 & 0 & -1 \\ 0 & 2 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 2 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \tag{13}$$

$$\xleftrightarrow{R_3 \leftrightarrow R_2} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \xleftrightarrow{R_4 \leftarrow R_4 + \frac{1}{2}R_2 - \frac{1}{2}R_1} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{14}$$

$$\xleftrightarrow{R_1 \leftrightarrow R_1 + \frac{1}{2}R_3} \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 2 & 0 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xleftrightarrow[R_3 \leftrightarrow \frac{1}{2}R_3]{R_2 \leftrightarrow \frac{1}{2}R_2} \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{15}$$

Thus,

$$x_1 = \frac{1}{2}x_4, x_2 = \frac{1}{2}x_4, x_3 = \frac{1}{2}x_4 \tag{16}$$

$$\Rightarrow \mathbf{X} = x_4 \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix} \tag{17}$$

by substituting $x_4 = 2$. Hence, The equation finally becomes

$$BaCl_2 + K_2SO_4 \rightarrow BaSO_4 + 2KCl \tag{18}$$