2.10.3

AI25BTECH11014 - Gooty Suhas

Problem

Find the unit vector perpendicular to the plane determined by the points:

$$\mathbf{P} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

Solution

Compute direction vectors:

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}, \quad \mathbf{R} - \mathbf{P} = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$$

Form the matrix system:

$$\begin{bmatrix} 1 & 1 & -3 \\ -1 & 3 & -1 \end{bmatrix} \mathbf{N} = \mathbf{0}$$

Apply row operations:

$$R_2 \leftarrow R_1 + R_2 \Rightarrow \begin{bmatrix} 1 & 1 & -3 \\ 0 & 4 & -4 \end{bmatrix} \Rightarrow \mathbf{N} = \begin{pmatrix} 8 \\ 2 \\ 4 \end{pmatrix}$$

Compute magnitude:

$$\|\mathbf{N}\| = \sqrt{8^2 + 2^2 + 4^2} = \sqrt{84}$$

Unit vector:

$$\hat{n} = \frac{1}{\sqrt{84}} \begin{pmatrix} 8 \\ 2 \\ 4 \end{pmatrix}$$

$$\boxed{\hat{n} = \begin{pmatrix} \frac{8}{\sqrt{84}} \\ \frac{2}{\sqrt{84}} \\ \frac{4}{\sqrt{84}} \end{pmatrix}}$$

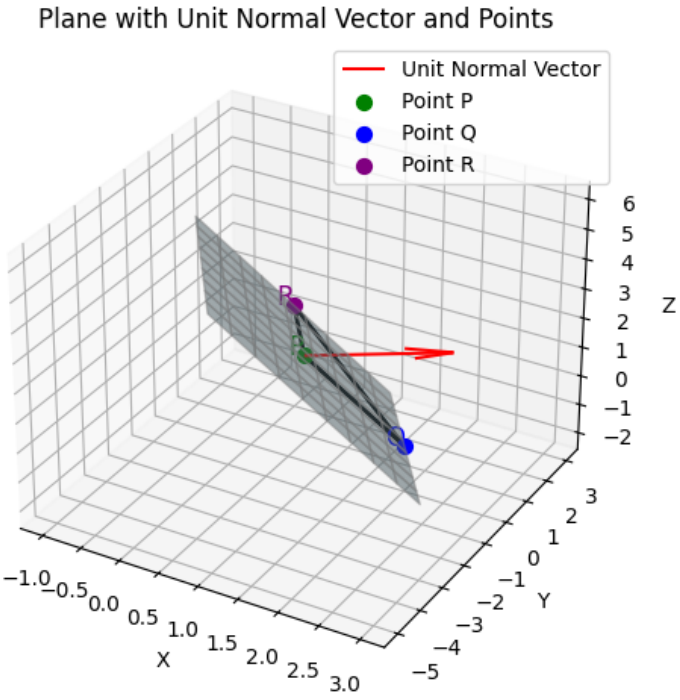

Fig. 0.1: Plane and its normal