4.7.13

INDHIRESH S- EE25BTECH11027

Question. Find the distance between the lines l_1 and l_2 given by

$$\overrightarrow{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$
$$\overrightarrow{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Solution:
Let us solve the given equation theoretically and then verify the solution computationally.
Given equation:

$$\overrightarrow{\mathbf{r}} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \tag{1}$$
$$\overrightarrow{\mathbf{r}} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k}) \tag{2}$$
$$\tag{3}$$

The given lines are in the form

$$\mathbf{r} = \mathbf{a_1} + \lambda\mathbf{b} \tag{4}$$
$$\mathbf{r} = \mathbf{a_2} + \mu\mathbf{b} \tag{5}$$

Where,

$$\mathbf{a_1} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} \quad \mathbf{a_2} = \begin{pmatrix} 3 \\ 3 \\ -5 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \tag{6}$$

The given two lines are parallel.The distance between two parallel lines is given by:

$$d = \frac{\|(\mathbf{a}_2 - \mathbf{a}_1) \times \mathbf{b}\|}{\|\mathbf{b}\|} \tag{7}$$

$$\mathbf{a_2} - \mathbf{a_1} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \quad and \quad \mathbf{b} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \tag{8}$$

Let,

$$\mathbf{a_2} - \mathbf{a_1} = \mathbf{a} \tag{9}$$

Now finding:

$$(\mathbf{a}_2 - \mathbf{a}_1) \times \mathbf{b} = \mathbf{a} \times \mathbf{b} \tag{10}$$

$$\begin{vmatrix}\mathbf{A_{23}} & \mathbf{B_{23}}\end{vmatrix} = \begin{vmatrix} 1 & 3 \\ -1 & 6 \end{vmatrix} = 9 \tag{11}$$

$$\left|\mathbf{A_{31}} \quad \mathbf{B_{31}}\right| = \begin{vmatrix} 2 & 2 \\ -1 & 6 \end{vmatrix} = 14 \tag{12}$$

$$\left|\mathbf{A_{12}} \quad \mathbf{B_{12}}\right| = \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} = 4 \tag{13}$$

$$\|\mathbf{a} \times \mathbf{b}\| = \left\| \begin{pmatrix} \left|\mathbf{A_{23}} \quad \mathbf{B_{23}}\right| \\ \\ \left|\mathbf{A_{31}} \quad \mathbf{B_{31}}\right| \\ \\ \left|\mathbf{A_{12}} \quad \mathbf{B_{12}}\right| \end{pmatrix} \right\| \tag{14}$$

$$\|\mathbf{a} \times \mathbf{b}\| = \left\| \begin{pmatrix} 9 \\ 14 \\ 4 \end{pmatrix} \right\| \tag{15}$$

$$\|\mathbf{a} \times \mathbf{b}\| = \sqrt{293} \tag{16}$$

$$\|(\mathbf{a}_2 - \mathbf{a}_1) \times \mathbf{b}\| = \sqrt{293} \tag{17}$$

$$\|\mathbf{b}\| = \sqrt{\mathbf{b^T b}} \tag{18}$$

$$\|\mathbf{b}\| = \sqrt{4 + 9 + 36} = \sqrt{49} \tag{19}$$

$$\|\mathbf{b}\| = 7 \tag{20}$$

Substituting the values in Eq.7:

$$d = \frac{\sqrt{293}}{7} \tag{21}$$

Therefore the distance between the lines l_1 and l_2 is $\frac{\sqrt{293}}{7}$

From the figure it is clearly verified that the theoretical solution matches with the computational solution.

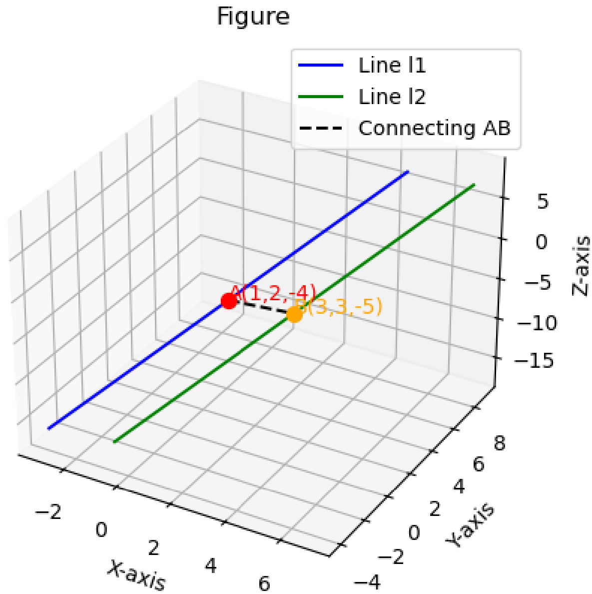

Figure
Line l1
Line l2
Connecting AB
A(1,2,-4)
B(3,3,-5)
X-axis
Y-axis
Z-axis
−2
0
2
4
6
−4
−2
0
2
4
6
8
5
0
−5
−10
−15