

# 7.2.18

AI25BTECH11023 - Pratik R

## QUESTION

Find the equation of the circle passing through  $(0, 0)$  and making intercepts  $a$  and  $b$  on the coordinate axes.

## Solution

Let:

$$\mathbf{x}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{x}_3 = \begin{pmatrix} 0 \\ b \end{pmatrix} \quad (0.1)$$

We use the general matrix form of a circle:

$$\begin{pmatrix} 2x_1 & 2x_2 & 2x_3 \\ 1 & 1 & 1 \end{pmatrix}^T \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = - \begin{pmatrix} \|\mathbf{x}_1\|^2 \\ \|\mathbf{x}_2\|^2 \\ \|\mathbf{x}_3\|^2 \end{pmatrix} \quad (0.2)$$

Substituting the values:

$$\begin{pmatrix} 0 & 0 & 1 \\ 2a & 0 & 1 \\ 0 & 2b & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ f \end{pmatrix} = - \begin{pmatrix} 0 \\ a^2 \\ b^2 \end{pmatrix} \quad (0.3)$$

Solving the system:

$$\begin{aligned} f &= 0 \\ 2au_1 + f &= -a^2 \Rightarrow u_1 = -\frac{a}{2} \\ 2bu_2 + f &= -b^2 \Rightarrow u_2 = -\frac{b}{2} \end{aligned}$$

we get

$$\mathbf{u} = \begin{pmatrix} -\frac{a}{2} \\ -\frac{b}{2} \end{pmatrix}, \quad f = 0 \quad (0.4)$$

So the equation of the circle becomes:

$$x^2 + y^2 + ax + by = 0 \quad (0.5)$$

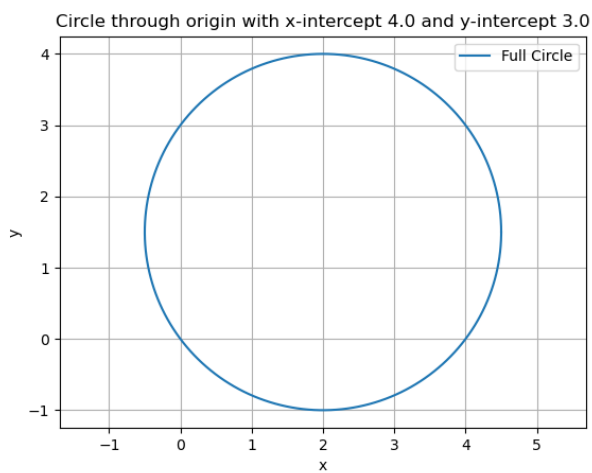


Fig. 0.1: plane