1.7.10

AI25BTECH11024 - Pratyush Panda

Question:
The value of $\hat{\mathbf{i}}.\left(\hat{\mathbf{j}}\times\hat{\mathbf{k}}\right)+\hat{\mathbf{j}}.\left(\hat{\mathbf{i}}\times\hat{\mathbf{k}}\right)+\hat{\mathbf{k}}.\left(\hat{\mathbf{i}}\times\hat{\mathbf{j}}\right)$ is ____________

Solution:
Given:

$$\hat{\mathbf{i}} = \mathbf{e_1} = \begin{pmatrix}1\\0\\0\end{pmatrix}, \hat{\mathbf{j}} = \mathbf{e_2} = \begin{pmatrix}0\\1\\0\end{pmatrix}, \hat{\mathbf{k}} = \mathbf{e_3} = \begin{pmatrix}0\\0\\1\end{pmatrix} \tag{0.1}$$

Each term of the expression in the question can be found using the scalar triple product determinant.
The first term can be written as:

$$\left(\hat{\mathbf{i}}.\left(\hat{\mathbf{j}}\times\hat{\mathbf{k}}\right)\right) = \begin{pmatrix}e_1 & e_2 & e_3\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix} \tag{0.2}$$

Determinant of this matrix is 1. Thus, the value of first term is 1.

The second term can be written as:

$$\left(\hat{\mathbf{j}}.\left(\hat{\mathbf{i}}\times\hat{\mathbf{k}}\right)\right) = \begin{pmatrix}e_2 & e_3 & e_1\end{pmatrix} = \begin{pmatrix}0 & 0 & 1\\1 & 0 & 0\\0 & 1 & 0\end{pmatrix} \tag{0.3}$$

Determinant of this matrix is -1. Thus, the value of first term is -1.

The third term can be written as:

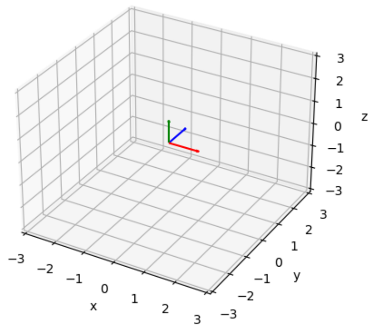
$$\left(\hat{\mathbf{k}}.\left(\hat{\mathbf{i}}\times\hat{\mathbf{j}}\right)\right) = \begin{pmatrix}e_3 & e_1 & e_2\end{pmatrix} = \begin{pmatrix}0 & 1 & 0\\0 & 0 & 1\\1 & 0 & 0\end{pmatrix} \tag{0.4}$$

Determinant of this matrix is 1. Thus, the value of first term is 1.

So, the sum of all the terms is;

$$\hat{\mathbf{i}}.\left(\hat{\mathbf{j}}\times\hat{\mathbf{k}}\right)+\hat{\mathbf{j}}.\left(\hat{\mathbf{i}}\times\hat{\mathbf{k}}\right)+\hat{\mathbf{k}}.\left(\hat{\mathbf{i}}\times\hat{\mathbf{j}}\right) = 1 + (-1) + 1 \tag{0.5}$$

$$or, \hat{\mathbf{i}}.\left(\hat{\mathbf{j}}\times\hat{\mathbf{k}}\right)+\hat{\mathbf{j}}.\left(\hat{\mathbf{i}}\times\hat{\mathbf{k}}\right)+\hat{\mathbf{k}}.\left(\hat{\mathbf{i}}\times\hat{\mathbf{j}}\right) = 1 \tag{0.6}$$



−3
−2
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