4.3.13

EE25BTECH11026-Harsha

Question:
Equations of the diagonals of the square formed by the lines x = 0, y = 0, x = 1 and y = 1 are ____________.
Solution:
Let us solve the given equation theoretically and then verify the solution computationally.

According to the question,
The vertices of the square are ,

$$\mathbf{a} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \mathbf{d} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{0.1}$$

To compute the equation of the diagnols , we can use the normal form of the equation,which is given by

$$\mathbf{n}^\top \mathbf{x} = 0 \; for\ the\ lines\ passing\ through\ the\ origin \tag{0.2}$$

$$\mathbf{n}^\top \mathbf{x} = 1 \; for\ the\ lines\ not\ passing\ through\ the\ origin \tag{0.3}$$

where,

$$\mathbf{n} - vector\ orthogonal\ to\ the\ direction\ vector \tag{0.4}$$

$$\mathbf{x} = \begin{pmatrix} x & y \end{pmatrix}^\top \tag{0.5}$$

For diagonal $\mathbf{c} - \mathbf{a}$, as it passes through the origin,

$$\therefore \mathbf{n}^\top \mathbf{x} = 0 \tag{0.6}$$

By substituting the vector through which it passes through,

$$\mathbf{n}^\top \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0 \tag{0.7}$$

$$\implies \mathbf{n} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{0.8}$$

$$\therefore \begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0 \tag{0.9}$$

But, for diagonal $\mathbf{d} - \mathbf{b}$, as the diagonal doesn't pass through the origin,

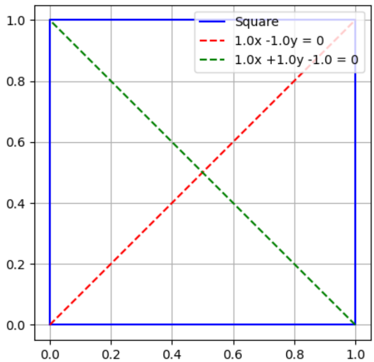
$$\mathbf{n}^\top \mathbf{x} = 1 \tag{0.10}$$

$$\therefore \mathbf{n}^\top \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.11}$$

$$\implies \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.12}$$

$$\therefore \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1 \tag{0.13}$$

From the figure, it is clearly verified that the theoretical solution matches with the computational solution.



Plot of Square with diagonals