2.4.28

EE25BTECH11032 - Kartik Lahoti

Question:

Find the coordinates of the point $\mathbf{Q}$ on the x-axis which lies on the perpendicular bisector of the line segment joining the points $\mathbf{A}(-5,-2)$ and $\mathbf{B}(4,-2)$. Name the type of triangle formed by points $\mathbf{Q}, \mathbf{A}$ and $\mathbf{B}$.

Solution:

Given :

$$\mathbf{A} = \begin{pmatrix} -5 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \tag{0.1}$$

Let $\mathbf{M}$ be the midpoint of $\mathbf{AB}$

$$\mathbf{M} = \frac{1}{2}(\mathbf{A} + \mathbf{B}) \tag{0.2}$$

$$= \frac{1}{2}\left(\begin{pmatrix} -5 \\ -2 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \end{pmatrix}\right) \tag{0.3}$$

$$= \begin{pmatrix} -0.5 \\ -2 \end{pmatrix} \tag{0.4}$$

To find the direction vector of perpendicular bisector , we can find the direction vector of $\mathbf{AB}$ and then rotate it by $90°$

Direction Vector of $\mathbf{AB}$ (represented by $\mathbf{V_{AB}}$):

$$\mathbf{B} - \mathbf{A} = \mathbf{V_{AB}} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} - \begin{pmatrix} -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 9 \\ 0 \end{pmatrix} \tag{0.5}$$

Rotation Matrix :

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{0.6}$$

Direction Vector for perpendicular bisector (represented by $\mathbf{V}$) :

$$\mathbf{V} = R(90°)\mathbf{V_{AB}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 9 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 9 \end{pmatrix} \tag{0.7}$$

Any arbitrary vector on perpendicular bisector can be given by :

$$\mathbf{Q} = \mathbf{M} + t\mathbf{V} \text{ where } t \in \mathbb{R} \tag{0.8}$$

Finding $\mathbf{Q}$,

$$\mathbf{Q} = \begin{pmatrix} -0.5 \\ -2 \end{pmatrix} + t \begin{pmatrix} 0 \\ 9 \end{pmatrix} \tag{0.9}$$

$$\mathbf{Q} = \begin{pmatrix} -0.5 \\ -2 + 9t \end{pmatrix} \tag{0.10}$$

Since y-coordinate of $\mathbf{Q}$ is zero

$$\mathbf{Q} = \begin{pmatrix} -0.5 \\ 0 \end{pmatrix} \tag{0.11}$$

Since $\mathbf{Q}$ lies on perpendicular bisector of $\mathbf{AB}$, it is equidistant from both $\mathbf{A}$ and $\mathbf{B}$

$$\|\mathbf{Q} - \mathbf{A}\| = \|\mathbf{Q} - \mathbf{B}\| \tag{0.12}$$

Hence $\triangle ABQ$ is an isosceles triangle.

Fig:2.4.28