

2.10.49

EE25BTECH11020 - Darsh Pankaj Gajare

Question:

The unit vector which is orthogonal to the vector $3\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar with vectors $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ is

1) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$

2) $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$

3) $\frac{3\hat{i}-\hat{k}}{\sqrt{10}}$

4) $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

Solution: Given:

TABLE I: Given data

Vector	matrix
A	$\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$
B	$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$
C	$\begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix}$

Assume Equation of plane through A, B .

$$\mathbf{n}^T \mathbf{x} = 1 \quad (1)$$

$$\mathbf{n}^T \mathbf{A} = 1 \quad (2)$$

$$\mathbf{n}^T \mathbf{B} = 1 \quad (3)$$

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} n = 1 \quad (4)$$

Augmented matrix,

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \end{array} \right). \quad (5)$$

$$R_1 = R_1 - R_2$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 1 & -1 & 1 & 1 \end{array} \right) \quad (6)$$

$$R_2 = R_2 - R_1$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & -3 & 1 & 1 \end{array} \right) \quad (7)$$

Let parametric constant be λ

$$n = \begin{pmatrix} -2\lambda \\ \lambda \\ 1 + 3\lambda \end{pmatrix} \quad (8)$$

$$\mathbf{n}^\top \mathbf{P} = 1 \quad (9)$$

$$\mathbf{P}^\top \mathbf{C} = 0 \quad (10)$$

$$\begin{pmatrix} -2\lambda & \lambda & 1+3\lambda \\ 3 & 2 & 6 \end{pmatrix} \mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (11)$$

Augmented matrix,

$$\left(\begin{array}{ccc|c} -2\lambda & \lambda & 1+3\lambda & 1 \\ 3 & 2 & 6 & 0 \end{array} \right). \quad (12)$$

Row operations: $R_1 = R_1 - \frac{\lambda}{2}R_2$

$$\left(\begin{array}{ccc|c} -3.5\lambda & 0 & 1 & 1 \\ 3 & 2 & 6 & 0 \end{array} \right). \quad (13)$$

$R_2 = R_2 - 6R_1$

$$\left(\begin{array}{ccc|c} -3.5\lambda & 0 & 1 & 1 \\ 3+21\lambda & 2 & 0 & -6 \end{array} \right). \quad (14)$$

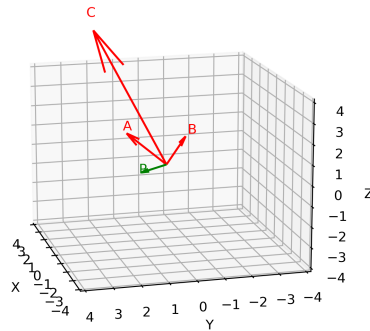
$$-3.5\lambda x + z = 1 \implies z = 1 + 3.5\lambda x, (3 + 21\lambda)x + 2y = -6 \implies y = -3 - \frac{x}{2}(3 + 21\lambda) \quad (15)$$

Let $x = \mu$ a parameter

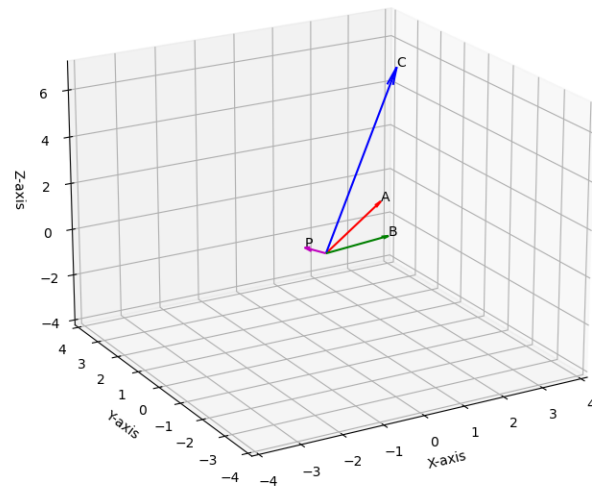
$$\mathbf{P} = \begin{pmatrix} -3 - \frac{\mu}{2}(3 + 21\lambda) \\ 1 + 3.5\lambda\mu \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} + \frac{\mu}{2} \begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix} + 7\lambda \frac{\mu}{2} \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}. \quad (16)$$

Normalizing,

$$\mathbf{P} = \pm \frac{1}{\sqrt{10}} \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} \quad (17)$$



Plot using C function



Plot using Python