1.9.20

EE25BTECH11031 - Sai Sreevallabh

Question:

Find the point on the Y-Axis which is equidistant from the points $(5, -2)$ and $(-3, 2)$

Solution:

Given points are

$$\mathbf{A} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -3 \\ 2 \end{pmatrix} \tag{0.1}$$

Let $\mathbf{P}$ be a point on the Y-Axis.

$$\mathbf{P} = \begin{pmatrix} 0 \\ y \end{pmatrix} \tag{0.2}$$

$\mathbf{P}$ is equidistant from both $\mathbf{A}$ and $\mathbf{B}$. Hence the norms of vectors $\mathbf{P} - \mathbf{B}$ and $\mathbf{P} - \mathbf{A}$ are equal.

$$\|\mathbf{P} - \mathbf{B}\| = \|\mathbf{P} - \mathbf{A}\| \tag{0.3}$$

$$\implies \|\mathbf{P} - \mathbf{B}\|^2 = \|\mathbf{P} - \mathbf{A}\|^2 \tag{0.4}$$

$$\implies \|\mathbf{P}\|^2 - 2\mathbf{P}^\top \mathbf{A} + \mathbf{A}^2 = \|\mathbf{P}\|^2 - 2\mathbf{P}^\top \mathbf{B} + \mathbf{B}^2 \tag{0.5}$$

Simplification of the above results in:

$$(\mathbf{A} - \mathbf{B})^\top \mathbf{P} = \frac{\|A\|^2 - \|B\|^2}{2} \tag{0.6}$$

$$\because \mathbf{P} = y\mathbf{e_2} \tag{0.7}$$

where, $\mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$$y = \frac{\|A\|^2 - \|B\|^2}{2\,(\mathbf{A} - \mathbf{B})^\top \mathbf{e_2}} \tag{0.8}$$

Substituting the values of $\mathbf{A}$ and $\mathbf{B}$:

$$y = \frac{\left\|\begin{pmatrix} 5 \\ -2 \end{pmatrix}\right\|^2 - \left\|\begin{pmatrix} -3 \\ 2 \end{pmatrix}\right\|^2}{2\begin{pmatrix} 8 & -4 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}} \tag{0.9}$$

$$y = -2 \tag{0.10}$$

$\therefore$ The point on the y-axis that is equidistant from the given two points is $\mathbf{P} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$.

plot(py).png