5.13.52

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Question

If the system of equations $x + ay = 0, az + y = 0$ and $ax + z = 0$ has infinite solutions, then the value of a is

1. -1
2. 1
3. 0
4. no real values

Theoretical Solution

From the given,

$$\begin{pmatrix} 1 & a & 0 \end{pmatrix} \mathbf{x} = 0 \tag{1}$$

$$\begin{pmatrix} 0 & 1 & a \end{pmatrix} \mathbf{x} = 0 \tag{2}$$

$$\begin{pmatrix} a & 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{3}$$

$$\therefore \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{4}$$

Theoretical Solution

To solve for a, we can use the fact that of rank of coefficient matrix should be less than 3.

$$\begin{pmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{pmatrix} \xleftrightarrow[R_3 \leftarrow R_3 + a^2 \times R_2]{R_3 \leftarrow R_3 - a \times R_1} \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & a \\ 0 & 0 & a^3 + 1 \end{pmatrix} \tag{5}$$

As the rank of the matrix should be less than 3, we require the last pivot to be zero.

$$\therefore a^3 + 1 = 0 \implies a = -1, -\omega, -\omega^2 \tag{6}$$

C Code -Finding the determinant of the matrix

```c
#include <stdio.h>

double det3x3(double a) {
    double det = 1 + a*a*a;
    return det;
}
```

Python+C code

```python
import ctypes
# Load the shared C library
lib = ctypes.CDLL("./libmatrix_solver.so")
lib.det3x3.argtypes = [ctypes.c_double]
lib.det3x3.restype = ctypes.c_double
# Real solution directly
a = -1.0
det_val = lib.det3x3(a)
tol = 1e-6

if abs(det_val) < tol:
    solutions = [a]
else:
    solutions = []

print("Real values of a for infinite solutions:")
print(solutions)
```

Python code

```python
import sympy as sp
a = sp.symbols('a')
A = sp.Matrix([
    [1, a, 0],
    [0, 1, a],
    [a, 0, 1]
])
# Solve det(A) = 0 for exact solution
solutions = sp.solve(A.det(), a)
print("Value(s) of a for infinite solutions:", solutions)
```