2.10.61

EE25BTECH11032 - Kartik Lahoti

Question:

If $\mathbf{a}$ and $\mathbf{b}$ are vectors such that $\left|\mathbf{a}+\mathbf{b}\right| = \sqrt{29}$ and

$$\mathbf{a}\times\left(2\hat{i}+3\hat{j}+4\hat{k}\right) = \left(2\hat{i}+3\hat{j}+4\hat{k}\right)\times\mathbf{b}$$

then a possible value of $(\mathbf{a}+\mathbf{b})\cdot\left(-7\hat{i}+2\hat{j}+3\hat{k}\right)$ is

1) 0 2) 3 3) 4 4) 8

Solution:

Given :

Symbol	Value	Description
$\mathbf{c}$	$\begin{pmatrix}2\\3\\4\end{pmatrix}$	Given Point
$\mathbf{d}$	$\begin{pmatrix}-7\\2\\3\end{pmatrix}$	Given Point
$\mathbf{a}+\mathbf{b}$	?	Desired Point

$$\mathbf{a}\times\mathbf{c} = \mathbf{c}\times\mathbf{b} \tag{4.1}$$

$$\mathbf{a}\times\mathbf{c} = -(\mathbf{b}\times\mathbf{c}) \tag{4.2}$$

$$(\mathbf{a}+\mathbf{b})\times\mathbf{c} = \mathbf{0} \tag{4.3}$$

If cross product of 2 vectors is zero , this implies both the vectors are parallel. Thus ,

$$(\mathbf{a}+\mathbf{b}) \parallel \mathbf{c} \tag{4.4}$$

$$\therefore (\mathbf{a}+\mathbf{b}) = \lambda\mathbf{c}, \text{ where } \lambda \in \mathbb{R} \tag{4.5}$$

Equating the magnitudes , we get

$$\|(\mathbf{a}+\mathbf{b})\|^2 = \lambda^2\|\mathbf{c}\|^2 \tag{4.6}$$

$$29 = \lambda^2 29 \tag{4.7}$$

$$\lambda = \pm 1 \tag{4.8}$$

Thus,

$$(\mathbf{a}+\mathbf{b}) = \begin{pmatrix}2\\3\\4\end{pmatrix} \text{ or } (\mathbf{a}+\mathbf{b}) = \begin{pmatrix}-2\\-3\\-4\end{pmatrix} \tag{4.9}$$

Hence,

$$(\mathbf{a}+\mathbf{b})^{\top}\,\mathbf{d} = 4 \text{ or } -4 \tag{4.10}$$

Answer : Option (3)

Fig:2.10.61