1.7.12

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Question

Find the value of k, if the points $P(5, 4)$, $Q(7, k)$ and $R(9, -2)$ are collinear.

Hint: Three points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ are collinear if the area of the triangle formed by them is zero.

Theoretical Solution

Solution:

$$\mathbf{P} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 7 \\ k \end{pmatrix}, \qquad \mathbf{R} = \begin{pmatrix} 9 \\ -2 \end{pmatrix} \tag{1}$$

Collinearity via rank Three points P, Q, R are collinear iff

$$\text{rank}\begin{pmatrix} \mathbf{Q}-\mathbf{P} & \mathbf{R}-\mathbf{P} \end{pmatrix} = 1. \tag{2}$$

Compute the direction columns:

$$\mathbf{Q}-\mathbf{P} = \begin{pmatrix} 7-5 \\ k-4 \end{pmatrix} = \begin{pmatrix} 2 \\ k-4 \end{pmatrix}, \qquad \mathbf{R}-\mathbf{P} = \begin{pmatrix} 9-5 \\ -2-4 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}. \tag{3}$$

Hence the collinearity matrix is

$$M = \begin{pmatrix} 2 & 4 \\ beginalign2pt]k-4 & -6 \end{pmatrix}. \tag{4}$$

Row reduction (rank = 1)

Theoretical Solution

$$\begin{pmatrix} 2 & 4 \\ k-4 & -6 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{1}{2}R_1} \begin{pmatrix} 1 & 2 \\ k-4 & -6 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2-(k-4)R_1} \begin{pmatrix} 1 & 2 \\ 0 & 2(1-k) \end{pmatrix}. \tag{5}$$

For $\text{rank}(M) = 1$, the second row must be the zero row:

$$2(1-k) = 0 \quad \Rightarrow \quad k = 1. \tag{6}$$

Conclusion For $k = \boxed{1}$, the three points $P(5,4)$, $Q(7,k)$, $R(9,-2)$ are collinear.

C Code

```c
#include <stdio.h>

int main() {
    int x1 = 5, y1 = 4;
    int x2 = 7, y2; // y2 = k
    int x3 = 9, y3 = -2;
    int k;

    // Equation: x1(y2 - y3) + x2(y3 - y1) + x3(y1 - y2) = 0
    // Substituting values
    // 5(k - (-2)) + 7((-2) - 4) + 9(4 - k) = 0
    // Solve manually inside program:

    // Simplified form: -4k + 4 = 0 => k = 1
    k = 1;
```

C Code

```
    printf("The value of k is: %d\n", k);

    return 0;
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Points
x1, y1 = 5, 4
x2, y2 = 7, 1 # k = 1 (solution)
x3, y3 = 9, -2

# Create figure
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
```

Python Code

```
# Plot the points in 3D (z = 0 for 2D points)
ax.scatter([x1, x2, x3], [y1, y2, y3], [0, 0, 0], c='r', s=100,
    label='Points')

# Draw line through the points
xs = np.array([x1, x2, x3])
ys = np.array([y1, y2, y3])
zs = np.array([0, 0, 0])
ax.plot(xs, ys, zs, label='Collinear Line')

# Labels and title
ax.set_xlabel('X axis')
ax.set_ylabel('Y axis')
ax.set_zlabel('Z axis')
ax.set_title('3D Visualization of Collinear Points')
ax.legend()
```

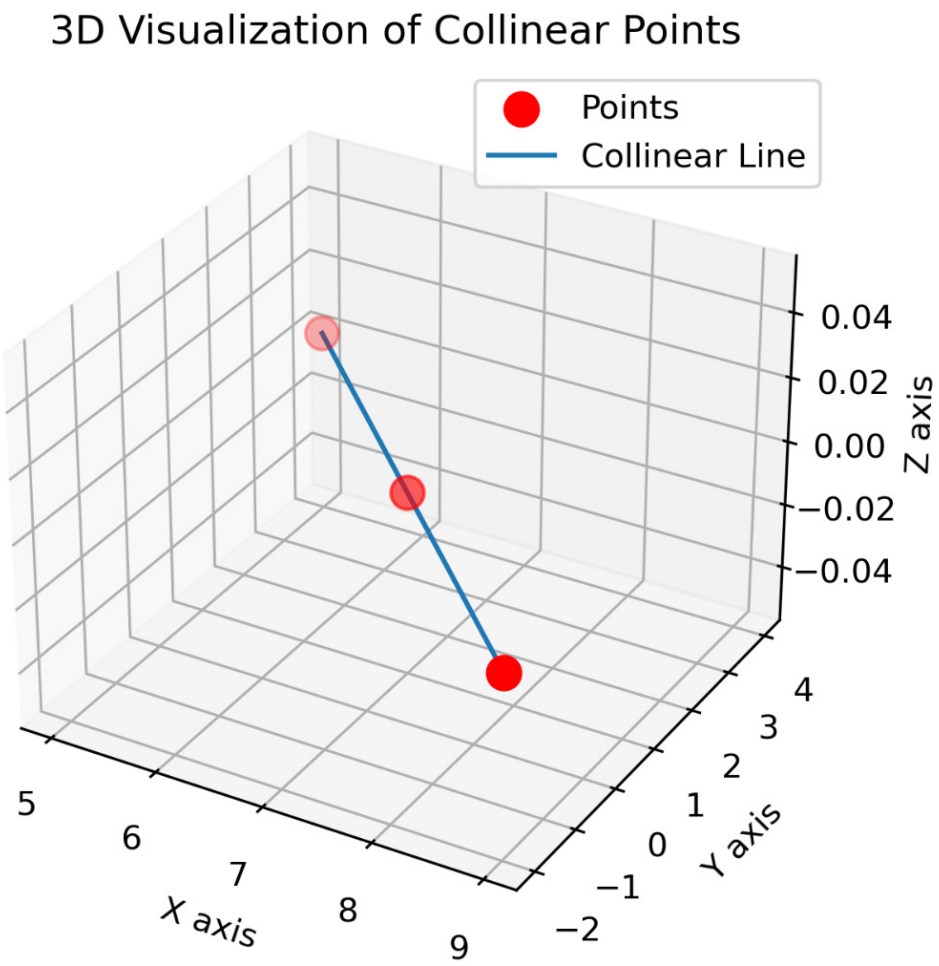
Python Code

```
# Save plot as picture
plt.savefig("collinear_points.png", dpi=300)

# Show the plot
plt.show()

print("Graph saved as collinear_points.png")
```

Plot


3D Visualization of Collinear Points
Points
Collinear Line
0.04
0.02
0.00
−0.02
−0.04
Z axis
5
6
7
8
9
X axis
−2
−1
0
1
2
3
4
Y axis