12.76

ee25btech11063-vejith

Question

Four points **P** $(0,1)$, **Q** $(0,-3)$, **R** $(-2,-1)$, **S** $(2,-1)$ represent the vertices of a quadrilateral. What is the area enclosed by the quadrilateral ? (ST 2022)

1) 4 2) $4\sqrt{2}$ 3) 8 4) $8\sqrt{2}$

Solution:

$$\mathbf{P} = \begin{pmatrix}0\\1\end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix}0\\-3\end{pmatrix} \quad \mathbf{R} = \begin{pmatrix}-2\\-1\end{pmatrix} \quad \mathbf{S} = \begin{pmatrix}2\\-1\end{pmatrix} \tag{1}$$

let PSQR be the quadrilateral then it's diagonals are $\mathbf{P}-\mathbf{Q}$ and $\mathbf{R}-\mathbf{S}$

$$\|\mathbf{P}-\mathbf{Q}\| = \left\|\begin{pmatrix}0\\4\end{pmatrix}\right\| = 4 \tag{2}$$

$$\|\mathbf{R}-\mathbf{S}\| = \left\|\begin{pmatrix}-4\\0\end{pmatrix}\right\| = 4 \tag{3}$$

$$(\mathbf{P}-\mathbf{Q})^{\top}(\mathbf{R}-\mathbf{S}) = \begin{pmatrix}0 & 4\end{pmatrix}\begin{pmatrix}-4\\0\end{pmatrix} \tag{4}$$

$$= 0 \tag{5}$$

$\implies$ diagonals of the quadrilateral are of equal length and they bisect each other perpendicularly
$\implies$ the given quadrilateral is a square

$$\text{area of the quadrilateral PSQR} = \frac{1}{2}\|\mathbf{P}-\mathbf{Q}\|^2 \tag{6}$$

$$= \frac{1}{2}\times 16 = 8 \tag{7}$$

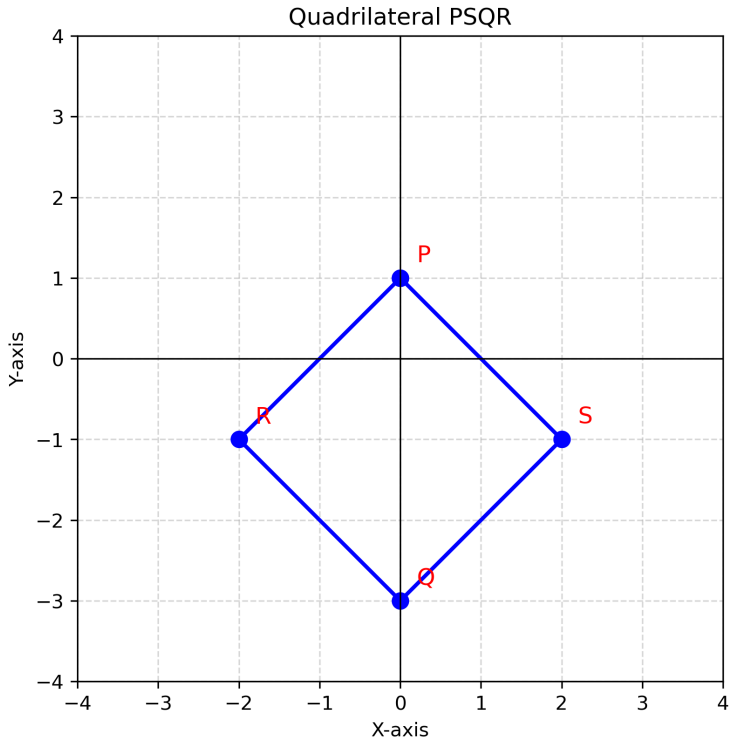

Fig. 4