Matgeo-q.4.13.1

AI25BTECH11036-SNEHAMRUDULA

September 17, 2025

Question

Consider the lines

$$L_1 : x + 3y - 5 = 0,$$
$$L_2 : 3x - ky - 1 = 0,$$
$$L_3 : 5x + 2y - 12 = 0.$$

Match the statements in Column I with Column II (choices refer to values of k or relations).

Column I

- L_1, L_2, L_3 are concurrent, if
- One of L_1, L_2, L_3 is parallel to at least one of the other two, if
- L_1, L_2, L_3 form a triangle, if
- L_1, L_2, L_3 do not form a triangle, if

Column II

- $k = 9$
- $k = \dfrac{-6}{5}$
- $k = \dfrac{5}{6}$
- $k = 5$

Solution — (A) Concurrency

Intersection of L_1 and L_3:

$$\begin{cases} x + 3y = 5, \\ 5x + 2y = 12, \end{cases} \quad \Rightarrow \quad 13y = 13, \ y = 1, \ x = 2.$$

So intersection point is

$$\mathbf{x}_0 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

This point lies on L_2 iff

$$\begin{pmatrix} 3 \\ -k \end{pmatrix}^{\top} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 1 \iff 6 - k = 1$$

$$\iff k = 5.$$

Hence the three lines are concurrent precisely when $k = 5$.

Solution — (B) Parallelism

Two lines are parallel iff their normals are proportional.
Normals of L_2 and L_3:

$$\mathbf{n}_2 = \begin{pmatrix} 3 \\ -k \end{pmatrix}, \qquad \mathbf{n}_3 = \begin{pmatrix} 5 \\ 2 \end{pmatrix}.$$

If $\mathbf{n}_2 = \lambda \mathbf{n}_3$ then

$$3 = 5\lambda, \quad -k = 2\lambda \Rightarrow \lambda = \tfrac{3}{5}, \ k = -2\lambda = -\tfrac{6}{5}.$$

So a parallel pair (between L_2 and L_3) occurs when $k = -\dfrac{6}{5}$. (Analogous checks show no other accidental proportionality with L_1 for these k.)

(C) Form a triangle: The three lines form a triangle iff no two are parallel and they are not concurrent. Therefore triangle occurs when

$$k \neq 5 \quad \text{and} \quad k \neq -\tfrac{6}{5}.$$

(So any explicit example values like $k = 9$ or $k = \tfrac{5}{6}$ satisfy the triangle condition.)

(D) Do not form a triangle: This happens when the lines are concurrent or when a parallel pair exists. Hence

$$\text{no triangle} \iff k = 5 \quad \text{or} \quad k = -\tfrac{6}{5}.$$

Final concise matching

- (A) Concurrency $\iff k = 5$.

- (B) One line parallel to another $\iff k = -\dfrac{6}{5}$.

- (C) They form a triangle $\iff k \neq 5$ and $k \neq -\dfrac{6}{5}$. (Examples: $k = 9, \ k = \frac{5}{6}$.)

- (D) They do *not* form a triangle $\iff k = 5$ or $k = -\dfrac{6}{5}$.

Graphical Representation