Question:

If $D\left(-\frac{1}{2}, \frac{5}{2}\right)$, $\quad E(7,3)$, $\quad F\left(\frac{7}{2}, \frac{7}{2}\right)$ are the midpoints of the sides of $\triangle ABC$, find the area of $\triangle ABC$.

Solution:

Let the position vectors of vertices A, B, C be $\mathbf{A}, \mathbf{B}, \mathbf{C}$.
Using midpoint relations:

$$\mathbf{D} = \frac{\mathbf{B}+\mathbf{C}}{2}, \quad \mathbf{E} = \frac{\mathbf{C}+\mathbf{A}}{2}, \quad \mathbf{F} = \frac{\mathbf{A}+\mathbf{B}}{2}$$

Rearranging,

$$\mathbf{A} - \mathbf{B} = 2(\mathbf{F} - \mathbf{D}), \quad \mathbf{A} - \mathbf{C} = 2(\mathbf{E} - \mathbf{D})$$

The area of $\triangle ABC$ is:

$$\text{Area} = \frac{1}{2}\,\|(\mathbf{A}-\mathbf{B}) \times (\mathbf{A}-\mathbf{C})\| = \frac{1}{2}\,\|2(\mathbf{F}-\mathbf{D}) \times 2(\mathbf{E}-\mathbf{D})\| = 2\,\|(\mathbf{F}-\mathbf{D}) \times (\mathbf{E}-\mathbf{D})\|$$

Calculate the difference vectors as matrices:

$$\mathbf{F} - \mathbf{D} = \begin{pmatrix} \frac{7}{2} \\ \frac{7}{2} \end{pmatrix} - \begin{pmatrix} -\frac{1}{2} \\ \frac{5}{2} \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

$$\mathbf{E} - \mathbf{D} = \begin{pmatrix} 7 \\ 3 \end{pmatrix} - \begin{pmatrix} -\frac{1}{2} \\ \frac{5}{2} \end{pmatrix} = \begin{pmatrix} \frac{15}{2} \\ \frac{1}{2} \end{pmatrix}$$

The magnitude of their cross product is the determinant:

$$\|(\mathbf{F}-\mathbf{D}) \times (\mathbf{E}-\mathbf{D})\| = \left\| \begin{matrix} 4 & \frac{15}{2} \\ 1 & \frac{1}{2} \end{matrix} \right\| = \left| 4 \times \frac{1}{2} - 1 \times \frac{15}{2} \right| = |2 - 7.5| = 5.5$$

the area of $\triangle ABC$ is:

$$\text{Area} = 2 \times 5.5 = 11$$

$$\boxed{11}$$

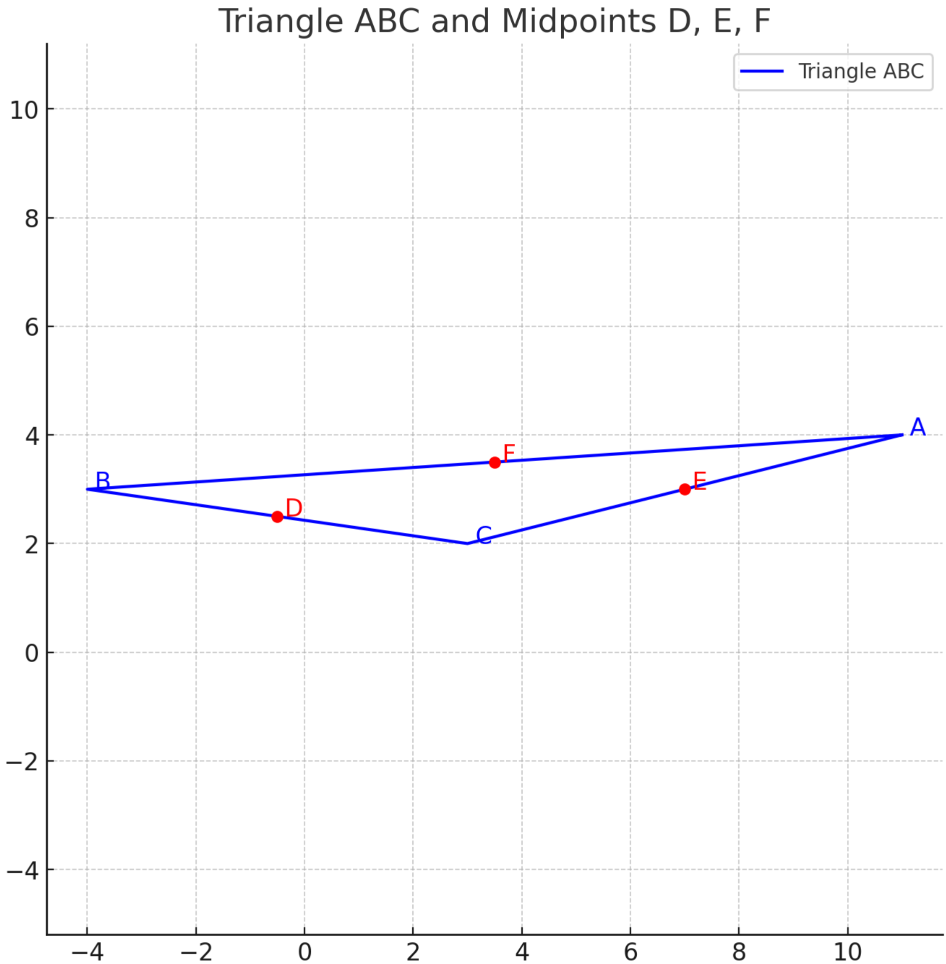
Triangle ABC and Midpoints D, E, F
Triangle ABC
A
B
C
D
E
F
10
8
6
4
2
0
−2
−4
−4
−2
0
2
4
6
8
10

Fig. 1: area