2.10.60

EE25BTECH11031 - Sai Sreevallabh

Question:

Let $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\mathbf{c} = \mathbf{i} - \mathbf{j} - \mathbf{k}$ be three vectors. A vector $\mathbf{v}$ in the plane of $\mathbf{a}$ and $\mathbf{b}$, whose projection on $\mathbf{c}$ is $\frac{1}{\sqrt{3}}$ is given by

1) $\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$
2) $-3\mathbf{i} - 3\mathbf{j} - \mathbf{k}$
3) $3\mathbf{i} - \mathbf{j} + 3\mathbf{k}$
4) $\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$

Solution:

Given vectors:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \tag{4.1}$$

Given that $\mathbf{v}$ is in the plane of $\mathbf{a}$ and $\mathbf{b}$, we can represent it as

$$\mathbf{v} = \alpha\mathbf{a} + \beta\mathbf{b} \tag{4.2}$$

Given that the projection of $\mathbf{v}$ on $\mathbf{c}$ is $\frac{1}{\sqrt{3}}$,

$$\frac{\mathbf{v}^\top \mathbf{c}}{\|\mathbf{c}\|} = \frac{1}{\sqrt{3}} \tag{4.3}$$

$\because \|c\| = \sqrt{3}$

$$\mathbf{v}^\top \mathbf{c} = 1 \tag{4.4}$$

$$\alpha\mathbf{a}^\top \mathbf{c} + \beta\mathbf{b}^\top \mathbf{c} = 1 \tag{4.5}$$

Substituting the values of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$, we get

$$\beta - \alpha = 1 \tag{4.6}$$

$$\beta = \alpha + 1 \tag{4.7}$$

Consequently,

$$\mathbf{v} = \alpha\mathbf{a} + (\alpha + 1)\,\mathbf{b} \tag{4.8}$$

$$\mathbf{v} = \alpha \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + (\alpha + 1) \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \tag{4.9}$$

$$\mathbf{v} = \begin{pmatrix} 2\alpha + 1 \\ -1 \\ 2\alpha + 1 \end{pmatrix} \tag{4.10}$$

This is the general expression for the vector $\mathbf{v}$. Out of the given options, only option 3 i.e. $3\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ satisfies the general expression (with $\alpha = 1$).

$\therefore\ \mathbf{v} = 3\mathbf{i} - \mathbf{j} + 3\mathbf{k}$