# 2.4.26

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**Question**Check whether (5,-2), (6,4) and (7,-2) are the vertices of an isosceles triangle.
**Solution:** Given details:

$$\mathbf{A} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \tag{1}$$

**Property:** In an isosceles triangle, the perpendicular bisector of a side passes through the opposite vertex.

$$\mathbf{A} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \tag{2}$$

Midpoint of side **AC**:

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{C}}{2} = \frac{\begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} 7 \\ -2 \end{pmatrix}}{2} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} \tag{3}$$

Direction vector of side **AC**:

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{4}$$

Vector from midpoint **M** to **B**:

$$\mathbf{B} - \mathbf{M} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \tag{5}$$

$$(\mathbf{C} - \mathbf{A})^{\top} (\mathbf{B} - \mathbf{M}) = \begin{pmatrix} 2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 6 \end{pmatrix} = 2(0) + 0(6) = 0 \tag{6}$$

**B** lies on the perpendicular bisector of side **AC**.

$$\therefore \mathbf{AB} = \mathbf{BC} \implies \triangle ABC \text{ is isosceles.}$$

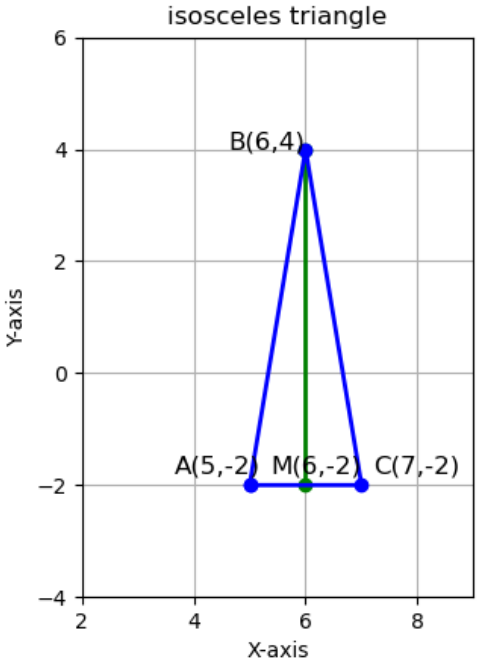


Fig. 0. isosceles triangle