

7.2.18

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Question

Find the equation of the circle passing through $(0, 0)$ and making intercepts a and b on the coordinate axes.

Let:

$$\mathbf{x}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{x}_2 = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{x}_3 = \begin{pmatrix} 0 \\ b \end{pmatrix} \quad (1)$$

Solution

We use the general matrix form of a circle:

$$\begin{pmatrix} 2x_1 & 2x_2 & 2x_3 \\ 1 & 1 & 1 \end{pmatrix}^T \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = - \begin{pmatrix} \|x_1\|^2 \\ \|x_2\|^2 \\ \|x_3\|^2 \end{pmatrix} \quad (2)$$

Substituting the values:

$$\begin{pmatrix} 0 & 0 & 1 \\ 2a & 0 & 1 \\ 0 & 2b & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ f \end{pmatrix} = - \begin{pmatrix} 0 \\ a^2 \\ b^2 \end{pmatrix} \quad (3)$$

Solution

Using augmented matrix and
applying $R_1 \leftrightarrow R_2$
and $R_2 \leftrightarrow R_3$

$$\left(\begin{array}{ccc|c} 2a & 0 & 1 & -a^2 \\ 0 & 2b & 1 & -b^2 \\ 0 & 0 & 1 & 0 \end{array} \right) \xleftrightarrow{R_1=R_1-R_3} \xleftrightarrow{R_2=R_2-R_3} \left(\begin{array}{ccc|c} 2a & 0 & 0 & -a^2 \\ 0 & 2b & 0 & -b^2 \\ 0 & 0 & 1 & 0 \end{array} \right) \quad (4)$$

we get

$$\mathbf{u} = \begin{pmatrix} -\frac{a}{2} \\ -\frac{b}{2} \end{pmatrix}, \quad f = 0 \quad (5)$$

So the equation of the circle becomes:

$$x^2 + y^2 + ax + by = 0 \quad (6)$$

