2.7.14

EE25BTECH11002 - Achat Parth Kalpesh

September 14,2025

Question

If θ is the angle between the two vectors $\mathbf{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\mathbf{b} = 3\hat{i} - 2\hat{j} + \hat{k}$, find $\sin\theta$.

Theoretical Solution

Let the given vectors be represented by column matrices **a** and **b**.

$$\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \tag{1}$$

The sine of the angle θ between two vectors is given by the formula:

$$\sin\theta = \frac{\|\mathbf{a} \times \mathbf{b}\|}{\|\mathbf{a}\| \cdot \|\mathbf{b}\|} \tag{2}$$

Theoretical Solution

For calculating cross product, $\mathbf{A} \times \mathbf{B}$

$$\begin{aligned} \mathbf{A}_{ij} &= \begin{pmatrix} a_i \\ a_j \end{pmatrix}, \\ \mathbf{B}_{ij} &= \begin{pmatrix} b_i \\ b_j \end{pmatrix} \end{aligned} \tag{3}$$

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} \begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \end{vmatrix} \\ \begin{vmatrix} \mathbf{A}_{31} & \mathbf{B}_{31} \end{vmatrix} \\ \begin{vmatrix} \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} \end{pmatrix} \tag{4}$$

Theoretical Solution

From (3) and (4), we calculate the components of the cross product $\mathbf{a} \times \mathbf{b}$:

$$\begin{vmatrix} A_{23} & B_{23} \end{vmatrix} = \begin{vmatrix} -2 & -2 \\ 3 & 1 \end{vmatrix} = (-2)(1) - (3)(-2) = 4 \tag{5}$$

$$\begin{vmatrix} A_{31} & B_{31} \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = (3)(3) - (1)(1) = 8 \tag{6}$$

$$\begin{vmatrix} A_{12} & B_{12} \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ -2 & -2 \end{vmatrix} = (1)(-2) - (-2)(3) = 4 \tag{7}$$

So,

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 4 \\ 8 \\ 4 \end{pmatrix} \tag{8}$$

Next, we calculate the magnitudes of the vectors:

$$\|\mathbf{a} \times \mathbf{b}\| = \sqrt{4^2 + 8^2 + 4^2} = \sqrt{16 + 64 + 16} = \sqrt{96} \tag{9}$$

$$\|\mathbf{a}\| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14} \tag{10}$$

$$\|\mathbf{b}\| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{9 + 4 + 1} = \sqrt{14} \tag{11}$$

Final Answer

Substituting these values into the formula for $\sin\theta$:

$$\sin\theta = \frac{\|\mathbf{a}\times\mathbf{b}\|}{\|\mathbf{a}\|\,\|\mathbf{b}\|} \tag{12}$$

$$= \frac{\sqrt{96}}{\sqrt{14}\cdot\sqrt{14}} = \frac{\sqrt{16\times 6}}{14} \tag{13}$$

$$= \frac{4\sqrt{6}}{14} = \frac{2\sqrt{6}}{7} \tag{14}$$

Therefore, the value of $\sin\theta$ is $\frac{2\sqrt{6}}{7}$.

C code

```
#include <stdio.h>
#include <math.h>
float formula(float *a,float *b)
{
    float c[3];
    c[0] = a[1]*b[2] - a[2]*b[1];
    c[1] = a[2]*b[0] - a[0]*b[2];
    c[2] = a[0]*b[1] - a[1]*b[0];

    return sqrt((c[0]*c[0] + c[1]*c[1] + c[2]*c[2]))/(sqrt(a[0]*a
        [0] + a[1]*a[1] + a[2]*a[2])*sqrt(b[0]*b[0] + b[1]*b[1] +
         b[2]*b[2]));
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
import ctypes
import os
import sys

# Load the shared C library
lib_path = ctypes.CDLL("./formula.so")

# Define the argument types for the C function
lib_path.formula.argtypes = [
    ctypes.POINTER(ctypes.c_float),
    ctypes.POINTER(ctypes.c_float)
]
# Define the return type for the C function
lib_path.formula.restype = ctypes.c_float
```

Python Code

```python
# Define the input vectors as numpy arrays
vecA = np.array([1, -2, 3], dtype=np.float32)
vecB = np.array([3, -2, 1], dtype=np.float32)

# Call the C function
sin_theta = lib_path.formula(
    vecA.ctypes.data_as(ctypes.POINTER(ctypes.c_float)),
    vecB.ctypes.data_as(ctypes.POINTER(ctypes.c_float))
)

print(f"The value of sin(theta) is: {sin_theta}")
# Theoretical value: 2*sqrt(6)/7 = 0.700
```

Python Code: Plotting

```
# Create a 3D plot
fig = plt.figure(figsize=(8, 8))
ax = fig.add_subplot(111, projection='3d')

# Define the origin point
origin = np.array([0, 0, 0])

# Plot the two vectors from the origin using ax.quiver
ax.quiver(*origin, *vecA, color='blue', label='Vector a = (1, -2,
    3)')
ax.quiver(*origin, *vecB, color='green', label='Vector b = (3,
    -2, 1)')

# Set the limits of the plot for better visualization
max_val = np.max(np.abs(np.concatenate((vecA, vecB)))) + 1
ax.set_xlim([-max_val, max_val])
ax.set_ylim([-max_val, max_val])
ax.set_zlim([-max_val, max_val])
```

Python Code: Finalizing Plot

```python
# Add labels and a title
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title('Two 3D Vectors')

# Add a legend
ax.legend()

# Add a grid for better visualization
ax.grid(True)

# Save and show the plot
plt.savefig("plot_c.jpg")
plt.show()
```

Plot

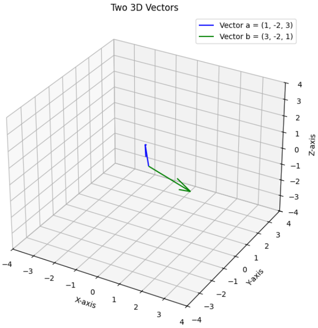

Figure: Visualization of the two vectors