

## Matgeo-q 2.3.2

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## Question

**Q.** Find the angle between unit vectors **a** and **b** such that  $\sqrt{3}\mathbf{a} - \mathbf{b}$  is also a unit vector.

## Solution

**Solution.** Let

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \quad \mathbf{a}^\top \mathbf{a} = \mathbf{b}^\top \mathbf{b} = 1.$$

If  $\sqrt{3}\mathbf{a} - \mathbf{b}$  is a unit vector, then

$$1 = \|\sqrt{3}\mathbf{a} - \mathbf{b}\|^2 = (\sqrt{3}\mathbf{a} - \mathbf{b})^\top (\sqrt{3}\mathbf{a} - \mathbf{b}) = 3\mathbf{a}^\top \mathbf{a} + \mathbf{b}^\top \mathbf{b} - 2\sqrt{3}\mathbf{a}^\top \mathbf{b}.$$

Hence

$$4 - 2\sqrt{3}(\mathbf{a}^\top \mathbf{b}) = 1 \implies \mathbf{a}^\top \mathbf{b} = \frac{\sqrt{3}}{2}.$$

Let  $\theta$  be the angle between  $\mathbf{a}$  and  $\mathbf{b}$ ;  $\cos \theta = \mathbf{a}^\top \mathbf{b}$ , so

$$\cos \theta = \frac{\sqrt{3}}{2} \implies \boxed{\theta = 30^\circ}.$$

# Plot

`figs/matgeo2.3.2.jpeg`