Matgeo Presentation - Problem 5.8.19

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Problem Statement

If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $1/2$ if we only add 1 to the denominator. What is the fraction?

solution

Given Let the unknown fraction be represented as:

$$\mathbf{v} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \tag{0.1}$$

so that the fraction equals:

$$\frac{x}{y}. \tag{0.2}$$

case 1 :Adding 1 to the numerator and subtracting 1 from the denominator:

$$\mathbf{T_1} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \tag{0.3}$$

$$\mathbf{T_1 v} = \begin{pmatrix} x+1 \\ y-1 \\ 1 \end{pmatrix} \tag{0.4}$$

solution

case 2 :Adding 1 to the denominator:

$$\mathbf{T_2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \tag{0.5}$$

$$\mathbf{T_2 v} = \begin{pmatrix} x \\ y+1 \\ 1 \end{pmatrix} \tag{0.6}$$

Condition for a fraction $\frac{a}{b} = k$:

$$\begin{pmatrix} 1 & -k & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ 1 \end{pmatrix} = 0 \tag{0.7}$$

Applying the first condition $(\mathbf{T_1} * \mathbf{v} = 1)$:

$$\mathbf{r_1} = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \mathbf{T_1} \tag{0.8}$$

$$= \begin{pmatrix} 1 & -1 & 2 \end{pmatrix} \tag{0.9}$$

$$\mathbf{r_1 v} = 0 \tag{0.10}$$

solution

Applying the second condition ($\mathbf{T_2} * \mathbf{v} = 1/2$):

$$\mathbf{r_2} = \begin{pmatrix} 2 & -1 & 0 \end{pmatrix} \mathbf{T_2} \tag{0.11}$$

$$= \begin{pmatrix} 2 & -1 & -1 \end{pmatrix} \tag{0.12}$$

$$\mathbf{r_2 v} = 0 \tag{0.13}$$

System of equations in matrix form:

$$\mathbf{Mv} = 0 \tag{0.14}$$

$$\mathbf{M} = \begin{pmatrix} 1 & -1 & 2 \\ 2 & -1 & -1 \end{pmatrix} \tag{0.15}$$

solution

Partitioning M into A and c, and vector v into u:

$$\mathbf{M} = \begin{pmatrix} \mathbf{A} & \mathbf{c} \end{pmatrix} \tag{0.16}$$

$$\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \tag{0.17}$$

$$\mathbf{v} = \begin{pmatrix} \mathbf{u} \\ 1 \end{pmatrix} \tag{0.18}$$

$$\mathbf{Au} + \mathbf{c} = 0 \implies \mathbf{Au} = -\mathbf{c} \tag{0.19}$$

Form the augmented matrix:

$$\left[\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \middle| \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right] \tag{0.20}$$

Eliminate below the pivot using $r_2 \leftarrow r_2 - 2r_1$:

$$\left[\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \middle| \begin{pmatrix} -2 \\ 5 \end{pmatrix} \right] \tag{0.21}$$

solution

Eliminate above the pivot using $r_1 \leftarrow r_1 + r_2$:

$$\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \middle| \begin{pmatrix} 3 \\ 5 \end{pmatrix}\right] \tag{0.22}$$

Reading off the solution for u:

$$\mathbf{u} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \tag{0.23}$$

Hence the homogeneous vector and fraction:

$$\mathbf{v} = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix} \tag{0.24}$$

$$\frac{x}{y} = \frac{3}{5} \tag{0.25}$$

C Source Code:fraction matrix.c

```
#include <stdio.h>

// Generate augmented matrix for fraction problem
void generate_matrix(double matrix[2][3]) {
    // Equation 1: x - y = -2
    matrix[0][0] = 1;
    matrix[0][1] = -1;
    matrix[0][2] = -2;

    // Equation 2: 2x - y = 1
    matrix[1][0] = 2;
    matrix[1][1] = -1;
    matrix[1][2] = 1;
}
```

Python Script:fraction matrix.py

```
import ctypes
import numpy as np

# Load the shared library
lib = ctypes.CDLL("./libfraction_matrix.so")

# Prepare 2x3 matrix
MatrixType = ctypes.c_double * 3 * 2
matrix = MatrixType()

# Call C function to generate matrix
lib.generate_matrix(matrix)
```

Python Script:fraction matrix.py

```
# Convert to numpy array
aug_matrix = np.array([[matrix[i][j] for j in range(3)] for i

# Solve system
A = aug_matrix[:, :2]
B = aug_matrix[:, 2]
solution = np.linalg.solve(A, B)
x, y = solution

print(f"The fraction is {x}/{y}")

# Optionally save fraction to file for plotting
with open("fraction.txt", "w") as f:
    f.write(f"{x} {y}")
```

Python Script: plot matrix.py

```
import numpy as np
import matplotlib.pyplot as plt

# Load fraction solution
with open("fraction.txt", "r") as f:
    x_sol, y_sol = map(float, f.read().split())

# Define the two lines from the equations:
# Equation 1: x - y = -2 → y = x + 2
# Equation 2: 2x - y = 1 → y = 2x - 1

x_vals = np.linspace(0, 5, 100)
y1 = x_vals + 2
y2 = 2*x_vals - 1
```

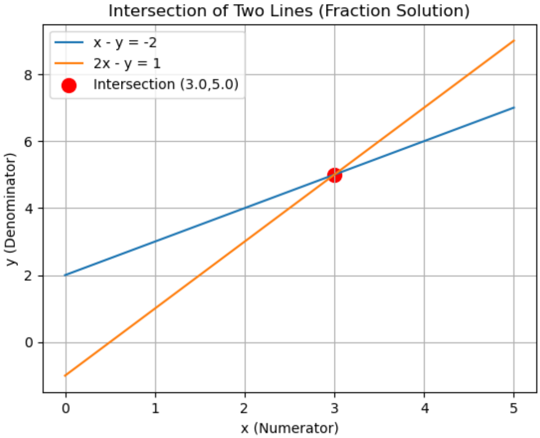
Python Script: plot matrix.py

```
# Plot the lines
plt.plot(x_vals, y1, label="x - y = -2")
plt.plot(x_vals, y2, label="2x - y = 1")

# Plot the intersection point
plt.scatter(x_sol, y_sol, color='red', s=100, label=f'Intersec

plt.xlabel("x (Numerator)")
plt.ylabel("y (Denominator)")
plt.title("Intersection of Two Lines (Fraction Solution)")
plt.grid(True)
plt.legend()
plt.show()
```

Result Plot



Intersection of Two Lines (Fraction Solution)
x - y = -2
2x - y = 1
Intersection (3.0,5.0)
y (Denominator)
x (Numerator)
0
1
2
3
4
5
0
2
4
6
8