

# 4.9.4

EE25BTECH11002 - Achat Parth Kalpesh

## Question:

The equations of the lines which pass through the point  $(3, -2)$  and are inclined at  $60^\circ$  to the line  $\sqrt{3}x + y = 1$  is

- 1)  $y + 2 = 0, \sqrt{3}x - y - 2 - 3\sqrt{3} = 0$
- 2)  $x - 2 = 0, \sqrt{3}x - y + 2 + 3\sqrt{3} = 0$
- 3)  $\sqrt{3}x - y - 2 - 3\sqrt{3} = 0$
- 4) None of these

## Solution:

The given line can be written in normal form as

$$\mathbf{n}_1^\top \mathbf{x} = 1, \quad (4.1)$$

where

$$\mathbf{n}_1 = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}. \quad (4.2)$$

Let the required line have normal vector  $\mathbf{n} = \begin{pmatrix} -m \\ 1 \end{pmatrix}$ , where  $m$  is the slope of line, then its equation is

$$\mathbf{n}^\top \mathbf{x} = c. \quad (4.3)$$

Since the line passes through  $\mathbf{P} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ ,

$$\mathbf{n}^\top \mathbf{P} = c \quad (4.4)$$

The angle  $\theta$  between two lines is given as;

$$\cos \theta = \frac{\mathbf{n}_1^\top \mathbf{n}}{\|\mathbf{n}_1\| \|\mathbf{n}\|}. \quad (4.5)$$

Here  $\theta = 60^\circ \implies \cos \theta = \frac{1}{2}$ , so

$$(\mathbf{n}_1^\top \mathbf{n})^2 = \frac{1}{4} \|\mathbf{n}_1\|^2 \|\mathbf{n}\|^2. \quad (4.6)$$

Substituting values:

$$(-\sqrt{3}m + 1)^2 = \frac{1}{4}(4)((-m)^2 + 1^2) \quad (4.7)$$

$$(-\sqrt{3}m + 1)^2 = m^2 + 1^2 \quad (4.8)$$

$$3m^2 - 2\sqrt{3}m + 1 = m^2 + 1, \quad (4.9)$$

$$2m^2 = 2\sqrt{3}m \quad (4.10)$$

$$m = 0 \text{ or } m = \sqrt{3} \quad (4.11)$$

For  $m = 0$  ;

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (4.12)$$

$$\mathbf{n}^\top \mathbf{P} = c \quad (4.13)$$

$$c = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix} = -2, \quad (4.14)$$

so the line is

$$y + 2 = 0 \quad (4.15)$$

For  $m = \sqrt{3}$  ;

$$\mathbf{n} = \begin{pmatrix} -\sqrt{3} \\ 1 \end{pmatrix} \quad (4.16)$$

$$\mathbf{n}^\top \mathbf{P} = c \quad (4.17)$$

$$c = \begin{pmatrix} -\sqrt{3} & 1 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix} = -3\sqrt{3} - 2 \quad (4.18)$$

so the line is

$$\sqrt{3}x - y - 3\sqrt{3} - 2 = 0 \quad (4.19)$$