EE25BTECH11059 - Vaishnavi Ramkrishna Anantheertha

Question:

if three points $\binom{h}{0}$, $\binom{a}{b}$, $\binom{0}{k}$ lie on a line, show that

$$\frac{a}{h} + \frac{b}{k} = 1$$

Solution:

Point	Name
(h, 0)	Point A
(0, k)	Point B
(a,b)	Point C

TABLE 0: Variables Used

If the rank of the Collinearity matrix is 1, then the points are collinear The Collinearity matrix is given by

$$\begin{pmatrix} \mathbf{C} - \mathbf{A} & \mathbf{B} - \mathbf{A} \end{pmatrix}^T = \begin{pmatrix} a - h & b \\ -h & k \end{pmatrix} \tag{0.1}$$

$$\stackrel{C_1 \to \frac{C_1}{-h}}{\longleftrightarrow} \begin{pmatrix} \frac{a-h}{-h} & b \\ 1 & k \end{pmatrix} \tag{0.2}$$

1

$$\stackrel{C_2 \to \frac{C_2}{k}}{\longleftrightarrow} \begin{pmatrix} \frac{a-h}{-h} & \frac{b}{k} \\ 1 & 1 \end{pmatrix} \tag{0.3}$$

$$\stackrel{R_1 \to C_1 - C_2}{\longleftrightarrow} \begin{pmatrix} \frac{a - h}{-h} - \frac{b}{k} & \frac{b}{k} \\ 0 & 1 \end{pmatrix} \tag{0.4}$$

since the rank of matrix=1

$$\frac{a-h}{-h} - \frac{b}{k} = 0 \implies \frac{a}{h} + \frac{b}{k} = 1$$

Refer to Fig. 0

Collinear Points (h,0,0), (0,k,0), and (a,b,c)

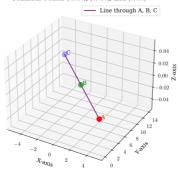


Fig. 0.1