1.9.25

EE25BTECH11036 - M Chanakya Srinivas

Question 1.9.25: If the point $P(x, y)$ is equidistant from $A(a+b, b-a)$ and $B(a-b, a+b)$, prove that $bx = ay$.

Solution:

1. Given Data
Define

$$\mathbf{z} = \begin{pmatrix} a \\ b \end{pmatrix}. \tag{1}$$

Then

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \mathbf{z}, \tag{2}$$

$$\mathbf{B} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \mathbf{z}, \tag{3}$$

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix}. \tag{4}$$

Notice that

$$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}^T \quad \Rightarrow \quad \mathbf{B} = \mathbf{A}^T \mathbf{z}. \tag{5}$$

2. Equidistant Condition

$$\|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2, \tag{6}$$

$$(\mathbf{P} - \mathbf{A})^T (\mathbf{P} - \mathbf{A}) = (\mathbf{P} - \mathbf{A}^T)^T (\mathbf{P} - \mathbf{A}^T). \tag{7}$$

3. Simplification using $B = A^T$

$$2(\mathbf{A}^T - \mathbf{A})^T \mathbf{P} = \mathbf{A}^T \mathbf{A}^T - \mathbf{A}^T \mathbf{A}. \tag{8}$$

But since $\mathbf{A}^T \mathbf{A}$ is symmetric,

$$\mathbf{A}^T \mathbf{A}^T - \mathbf{A}^T \mathbf{A} = 0. \tag{9}$$

Hence

$$(\mathbf{A}^T - \mathbf{A})^T \mathbf{P} = 0. \tag{10}$$

4. Final Result Expanding,

$$(\mathbf{B} - \mathbf{A})^T \mathbf{P} = \left(\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} \mathbf{z} \right)^T \mathbf{P}, \tag{11}$$

$$= -2bx + 2ay = 0, \tag{12}$$

$$\Rightarrow \quad bx = ay. \tag{13}$$

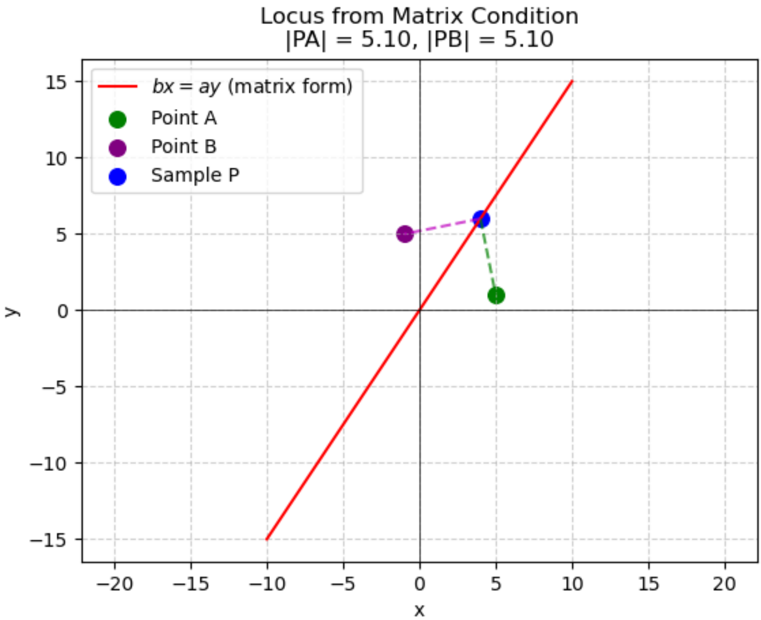
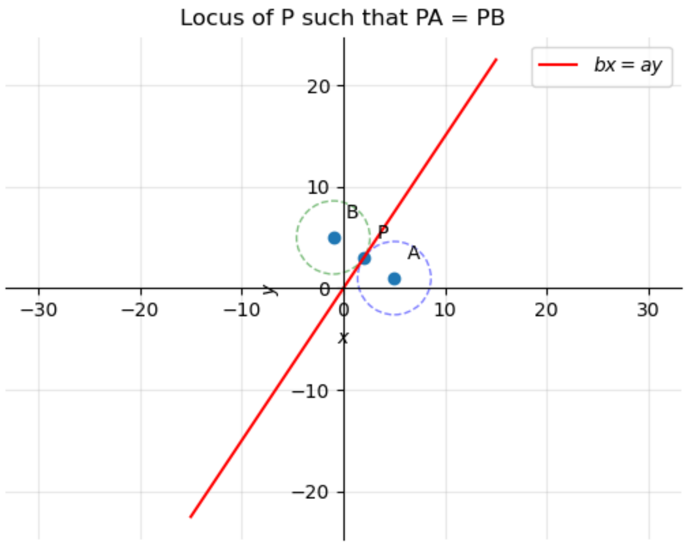


Fig. 1



Locus of P such that PA = PB
$bx = ay$
B
P
A
20
10
0
−10
−20
−30
−20
−10
0
10
20
30
x
y

Fig. 2