

4.13.50

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Question:

Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point $(1, -10)$. Determine the equation of the third side.

Solution: Let the two equal sides of the isosceles triangle be represented by

$$\mathbf{n}_1^\top \mathbf{x} = c_1$$

$$\mathbf{n}_2^\top \mathbf{x} = c_2$$

and the third side by the line

$$\mathbf{n}^\top \mathbf{x} = c$$

For a line passing through a given point \mathbf{p} ,

$$\mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{p} \quad (1)$$

$$\text{Here, } \mathbf{p} = \begin{pmatrix} 1 \\ -10 \end{pmatrix} \quad (2)$$

The third side of the isosceles, the base, is perpendicular to one of the angle bisector of the two equal sides and parallel to the other.

$$\mathbf{m}_1 = \frac{\mathbf{n}_1}{\|\mathbf{n}_1\|} + \frac{\mathbf{n}_2}{\|\mathbf{n}_2\|} \quad (3)$$

$$\mathbf{m}_2 = \frac{\mathbf{n}_1}{\|\mathbf{n}_1\|} - \frac{\mathbf{n}_2}{\|\mathbf{n}_2\|} \quad (4)$$

Hence,

$$\mathbf{n} = \mathbf{m}_2 \text{ or } \mathbf{n} = \mathbf{m}_1 \quad (5)$$

For the given question,

$$\mathbf{n}_1 = \begin{pmatrix} 7 \\ -1 \end{pmatrix} \text{ and } \mathbf{n}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (6)$$

$$\|\mathbf{n}_1\| = \sqrt{50} = 5\sqrt{2} \quad (7)$$

$$\|\mathbf{n}_2\| = \sqrt{2} \quad (8)$$

For the side parallel to \mathbf{m}_2 , using (3),

$$\mathbf{n} = \frac{1}{5\sqrt{2}} \left(\begin{pmatrix} 7 \\ -1 \end{pmatrix} + \begin{pmatrix} 5 \\ 5 \end{pmatrix} \right) = \frac{1}{5\sqrt{2}} \begin{pmatrix} 12 \\ 4 \end{pmatrix} \quad (9)$$

$$\mathbf{n} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad (10)$$

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -10 \end{pmatrix} \quad (11)$$

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = -7 \quad (12)$$

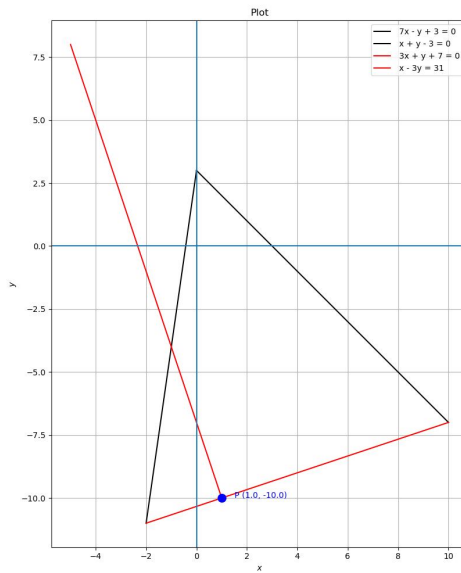
For the side parallel to \mathbf{m}_1 , using (4),

$$\mathbf{n} = \frac{1}{5\sqrt{2}} \left(\begin{pmatrix} 7 \\ -1 \end{pmatrix} - \begin{pmatrix} 5 \\ 5 \end{pmatrix} \right) = \frac{1}{5\sqrt{2}} \begin{pmatrix} 2 \\ -6 \end{pmatrix} \quad (13)$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \quad (14)$$

$$\begin{pmatrix} 1 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ -10 \end{pmatrix} \quad (15)$$

$$\begin{pmatrix} 1 & -3 \end{pmatrix} \mathbf{x} = 31 \quad (16)$$



Isosceles Triangle