2.8.9

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Question

Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three vectors such that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 4$, $|\mathbf{c}| = 5$, and each one of them is perpendicular to the sum of the other two. Find $|\mathbf{a} + \mathbf{b} + \mathbf{c}|$.

Solution

Let the Gram matrix G of the three vectors $(a), (b), (c)$ be

$$G = \begin{pmatrix} (a,a) & (a,b) & (a,c) \\ (b,a) & (b,b) & (b,c) \\ (c,a) & (c,b) & (c,c) \end{pmatrix} = \begin{pmatrix} 9 & x & z \\ x & 16 & y \\ z & y & 25 \end{pmatrix} \tag{1}$$

where

$$x = (a,b), \quad y = (b,c), \quad z = (c,a). \tag{2}$$

The conditions "each vector is perpendicular to the sum of the other two" give

$$(a,(b)+(c)) = 0, \tag{3}$$
$$(b,(c)+(a)) = 0, \tag{4}$$
$$(c,(a)+(b)) = 0. \tag{5}$$

Solution

In terms of x, y, z, equations (3)–(5) become

$$x + z = 0, \tag{6}$$
$$x + y = 0, \tag{7}$$
$$y + z = 0. \tag{8}$$

From (6) we get $z = -x$, and from (7) we get $y = -x$. Substituting into (8) gives

$$(-x) + (-x) = 0 \quad \Rightarrow \quad x = 0. \tag{9}$$

Hence

$$x = y = z = 0. \tag{10}$$

So $(a), (b), (c)$ are pairwise orthogonal.

Solution

Therefore

$$
\begin{aligned}
|(a)+(b)+(c)|^2 &= (a+b+c)\cdot(a+b+c) && (11)\\
&= (a,a)+(b,b)+(c,c) && (12)\\
&= |a|^2+|b|^2+|c|^2 && (13)\\
&= 9+16+25 && (14)\\
&= 50. && (15)
\end{aligned}
$$

Thus

$$
|(a)+(b)+(c)| = \sqrt{50} = 5\sqrt{2}. \tag{16}
$$

Python Code

```
import numpy as np
import matplotlib.pyplot as plt

# Define mutually perpendicular vectors
a = np.array([3, 0, 0])
b = np.array([0, 4, 0])
c = np.array([0, 0, 5])
s = a + b + c # resultant (3,4,5)

fig = plt.figure(figsize=(8, 6))
ax = fig.add_subplot(111, projection='3d')
```

Python Code

```
# Plot main vectors
ax.quiver(0, 0, 0, *a, color='r', linewidth=2,
          arrow_length_ratio=0.08, normalize=False, label='a (3)')
ax.quiver(0, 0, 0, *b, color='g', linewidth=2,
          arrow_length_ratio=0.08, normalize=False, label='b (4)')
ax.quiver(0, 0, 0, *c, color='b', linewidth=2,
          arrow_length_ratio=0.08, normalize=False, label='c (5)')

# Plot resultant
ax.quiver(0, 0, 0, *s, color='m', linewidth=2,
          arrow_length_ratio=0.05, normalize=False,
          label='a+b+c')
```

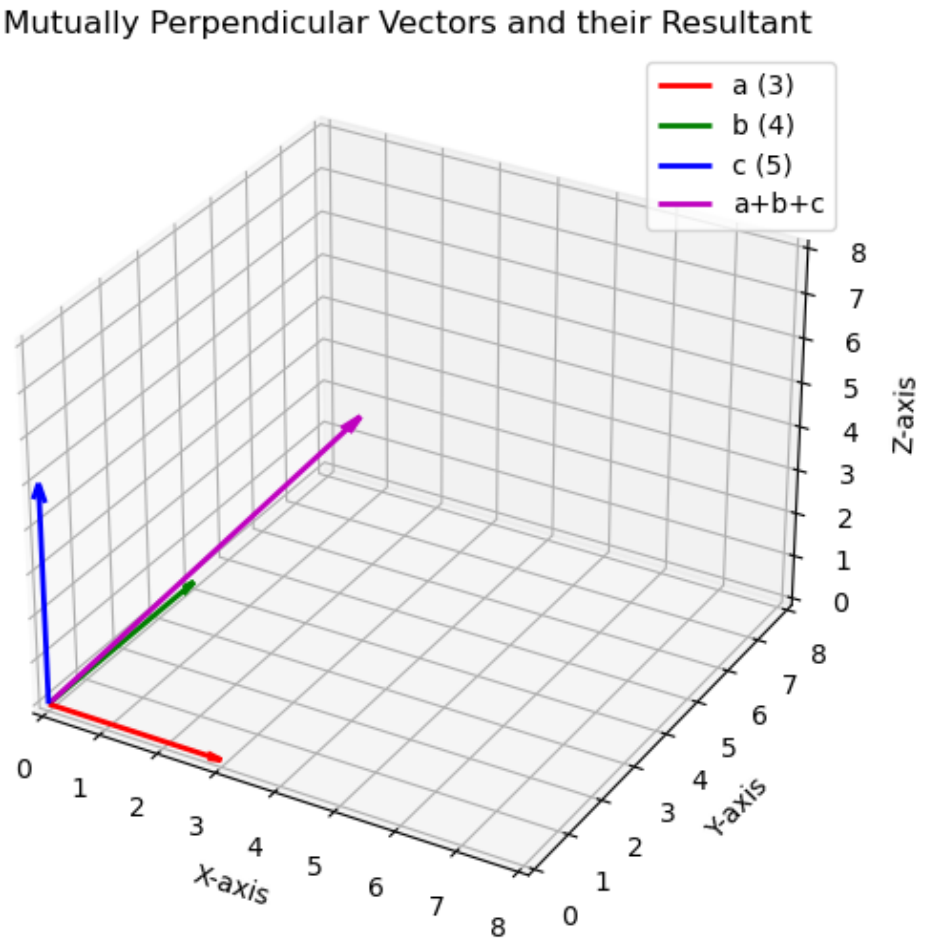
Python Code

```
# Axis limits
ax.set_xlim(0, 8)
ax.set_ylim(0, 8)
ax.set_zlim(0, 8)

# Axis labels
ax.set_xlabel("X-axis")
ax.set_ylabel("Y-axis")
ax.set_zlabel("Z-axis")
ax.set_title("Mutually Perpendicular Vectors and their Resultant"
    )

ax.legend()
plt.show()
```

C Code

```c
#include <stdio.h>
#include <math.h>

int main() {
    // Given magnitudes
    int a = 3, b = 4, c = 5;
    // Since a, b, c are mutually perpendicular (proved in
        solution),
    // |a + b + c|^2 = |a|^2 + |b|^2 + |c|^2
    int sum_sq = a*a + b*b + c*c;

    double magnitude = sqrt(sum_sq);

    // Print result
    printf("The magnitude |a + b + c| = %.2f\n", magnitude);

    return 0;
}
```

Plot-Using by Python



Python and C Code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load the compiled C library
lib = ctypes.CDLL("./vector_calc.so") # use "vector_calc.dll" on
    Windows

# Call the C function
lib.vector_magnitude.restype = ctypes.c_double
magnitude = lib.vector_magnitude()
print("Result from C code |a+b+c| =", magnitude)
```

Python and C Code

```
# ---- Plotting in Python ----
a = np.array([3, 0, 0])
b = np.array([0, 4, 0])
c = np.array([0, 0, 5])
resultant = a + b + c

fig = plt.figure(figsize=(8, 6))
ax = fig.add_subplot(111, projection="3d")

# plot vectors
origin = np.array([0, 0, 0])
ax.quiver(*origin, *a, color='r', label='a (3)')
ax.quiver(*origin, *b, color='g', label='b (4)')
ax.quiver(*origin, *c, color='b', label='c (5)')
ax.quiver(*origin, *resultant, color='m', label='a+b+c')
```

Python and C Code

```
ax.set_xlim([0, 8])
ax.set_ylim([0, 8])
ax.set_zlim([0, 8])

ax.set_xlabel("X axis")
ax.set_ylabel("Y axis")
ax.set_zlabel("Z axis")
ax.set_title("C code calculation + Python plot")

ax.legend()
plt.show()
```

Plot-Using by both C and Python