

4.8.26

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Question: Find the coordinates of the foot of the perpendicular drawn from the point

$$\mathbf{P} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$$

to the Y -axis.

Solution: The Y -axis has the direction vector

$$\mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

and passes through the origin. Its general point is

$$\mathbf{Q} = \begin{pmatrix} 0 \\ q \\ 0 \end{pmatrix}.$$

Any point \mathbf{Q} on the Y -axis satisfies $x = 0$ and $z = 0$.

$$\text{Let } \mathbf{P} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}.$$

The foot of the perpendicular \mathbf{Q} is given by projecting \mathbf{P} onto the Y -axis as

$$\mathbf{Q} = (\mathbf{e}_2^\top \mathbf{P}) \frac{\mathbf{e}_2}{\|\mathbf{e}_2\|^2}.$$

$$\mathbf{e}_2^\top \mathbf{P} = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}.$$

Since

$$\|\mathbf{e}_2\|^2 = 0^2 + 1^2 + 0^2 = 1,$$

the foot of the perpendicular is

$$\mathbf{Q} = \left(\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \right) \mathbf{e}_2.$$

$$\mathbf{Q} = (-3)\mathbf{e}_2 = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix}.$$

Final Answer: The coordinates of the foot of the perpendicular are

$$\begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix}.$$

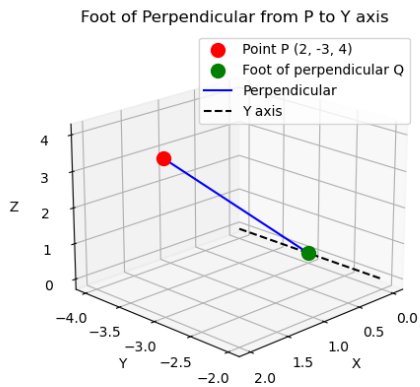


Fig. 0.1: Vector Representation