Vector Coplanarity and Perpendicularity Check

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Question

The vector(s) which is/are coplanar with the vectors

$$\hat{i} + \hat{j} + 2\hat{k}, \quad \hat{i} + 2\hat{j} + \hat{k}$$

and perpendicular to vector

$$\hat{i} + \hat{j} + \hat{k}$$

is/are:

a) $\hat{\mathbf{j}} - \hat{\mathbf{k}}$

b) $\hat{\mathbf{i}} + \hat{\mathbf{j}}$

c) $\hat{\mathbf{i}} - \hat{\mathbf{j}}$

d) $\hat{\mathbf{j}} + \hat{\mathbf{k}}$

Given Vectors

Variable	Vector
$\vec{A}$	$\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$
$\vec{B}$	$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$
$\vec{C}$	$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Options as vectors $\vec{D}_i$:

Input	Vector
$\vec{D}_1$	$\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$
$\vec{D}_2$	$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
$\vec{D}_3$	$\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$
$\vec{D}_4$	$\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

Checking Coplanarity

Condition:

$$(\vec{A} \times \vec{B})^T \vec{D}_i = 0$$

$$\vec{A} \times \vec{B} = \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix}, \quad (\vec{A} \times \vec{B})^T = \begin{pmatrix} -3 & 1 & 1 \end{pmatrix}$$

Vector	$(\vec{A} \times \vec{B})^T \vec{D}_i$	Coplanar?
$\vec{D}_1$	0	Yes
$\vec{D}_2$	-1	No
$\vec{D}_3$	-4	No
$\vec{D}_4$	2	No

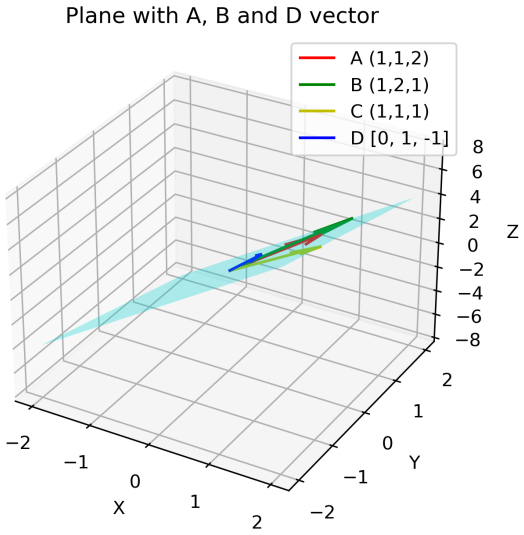
Checking Perpendicularity

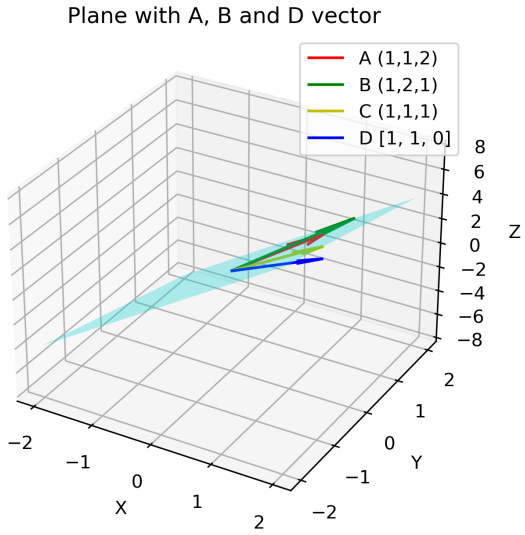
Condition:

$$\vec{C}^T \vec{D}_i = 0, \quad \vec{C}^T = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$

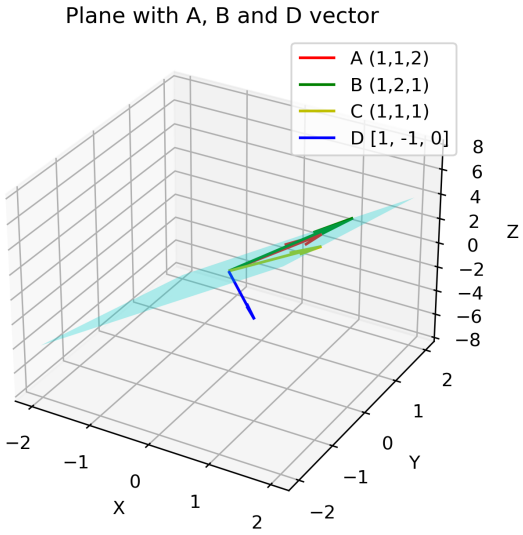
Vector	$\vec{C}^T \vec{D}_i$	Perpendicular?
$\vec{D}_1$	0	Yes
$\vec{D}_2$	3	No
$\vec{D}_3$	0	Yes
$\vec{D}_4$	2	No

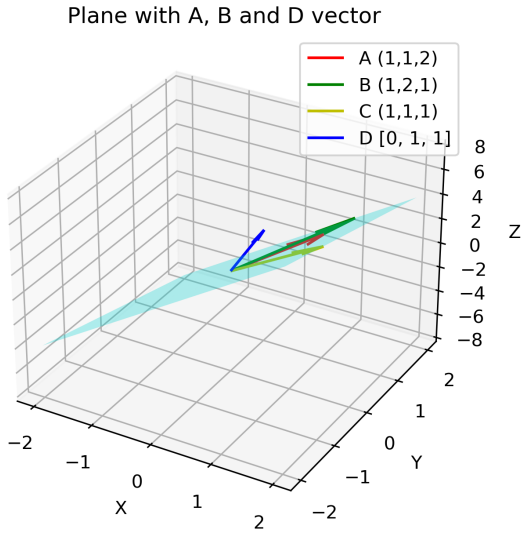
Figures (1/2)


Plane with A, B and D vector
A (1,1,2)
B (1,2,1)
C (1,1,1)
D [0, 1, -1]
X
Y
Z


Plane with A, B and D vector
A (1,1,2)
B (1,2,1)
C (1,1,1)
D [1, 1, 0]
X
Y
Z

Figures (2/2)


Plane with A, B and D vector
A (1,1,2)
B (1,2,1)
C (1,1,1)
D [1, -1, 0]
X
Y
Z


Plane with A, B and D vector
A (1,1,2)
B (1,2,1)
C (1,1,1)
D [0, 1, 1]
X
Y
Z

Final Answer

The vectors satisfying both conditions is:

$$\boxed{\vec{D_1}}$$