4.11.14

M Chanakya Srinivas- EE25BTECH11036

Problem

Find λ so that

$$\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}, \qquad \frac{x}{1} = \frac{y+\frac{1}{2}}{2\lambda} = \frac{z-1}{3}$$

are perpendicular; determine intersection condition.

Vector form & perpendicularity

Choose

$$\mathbf{A}_1 = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}, \ \mathbf{m}_1 = \begin{pmatrix} 5\lambda + 2 \\ -5 \\ 1 \end{pmatrix}, \qquad \mathbf{A}_2 = \begin{pmatrix} 0 \\ -\frac{1}{2} \\ 1 \end{pmatrix}, \ \mathbf{m}_2 = \begin{pmatrix} 1 \\ 2\lambda \\ 3 \end{pmatrix}.$$

Compute

$$\mathbf{m}_1^\top \mathbf{m}_2 = (5\lambda + 2) - 10\lambda + 3 = -5\lambda + 5.$$

Hence $\boxed{\lambda = 1}$ for perpendicular directions.

Set up augmented matrix

Intersection needs κ_1, κ_2 solving $\kappa_1 \mathbf{m}_1 - \kappa_2 \mathbf{m}_2 = \mathbf{A}_2 - \mathbf{A}_1$. Write the augmented matrix:

$$\mathcal{M}_0 = \left[\begin{array}{cc|c} 5\lambda + 2 & -1 & -5 \\ -5 & -2\lambda & -\frac{5}{2} \\ 1 & -3 & 0 \end{array}\right].$$

We reduce $\mathcal{M}_0$ to RREF using explicit row operations.

RREF — Step 1 (swap)

Swap $R_1 \leftrightarrow R_3$:

$$\mathcal{M}_1 = \left[\begin{array}{cc|c} 1 & -3 & 0 \\ -5 & -2\lambda & -\frac{5}{2} \\ 5\lambda+2 & -1 & -5 \end{array}\right].$$

RREF — Step 2 (eliminate col 1)

Eliminate first column using R_1:

$$R_2 \leftarrow R_2 + 5R_1, \qquad R_3 \leftarrow R_3 - (5\lambda + 2)R_1,$$

giving

$$\mathcal{M}_2 = \left[\begin{array}{cc|c} 1 & -3 & 0 \\ 0 & -2\lambda - 15 & -\frac{5}{2} \\ 0 & 15\lambda + 5 & -5 \end{array}\right].$$

RREF — Step 3 (scale R2 and eliminate)

Scale R_2 by $1/(-2\lambda - 15)$ (if nonzero) and eliminate column 2:

$$R_2 \leftarrow \frac{1}{-2\lambda - 15} R_2, \quad R_1 \leftarrow R_1 + 3R_2, \quad R_3 \leftarrow R_3 - (15\lambda + 5)R_2.$$

Resulting matrix has form

$$\mathcal{M}_4 = \left[\begin{array}{cc|c} 1 & 0 & * \\ 0 & 1 & * \\ 0 & 0 & r \end{array}\right],$$

where r is the residual determining consistency.

Consistency condition (RREF read-off)

From $\mathcal{M}_2$ the two scalar rows give

$$(-2\lambda - 15)\kappa_2 = -\tfrac{5}{2}, \qquad (15\lambda + 5)\kappa_2 = -5.$$

So

$$\kappa_2 = \frac{5}{30 + 4\lambda} \quad \text{and} \quad \kappa_2 = -\frac{1}{3\lambda + 1}.$$

Equate them:

$$-\frac{1}{3\lambda + 1} = \frac{5}{30 + 4\lambda}$$

which yields $\boxed{\lambda = -\tfrac{35}{19}}$ (intersection case).

Check $\lambda = 1$

At $\lambda = 1$ the two equations give

$$\kappa_2 = \frac{5}{34} \quad \text{and} \quad \kappa_2 = -\frac{1}{4},$$

a contradiction. Thus augmented system inconsistent $\Rightarrow$ lines are skew for $\lambda = 1$.

Summary

- $\lambda = 1$: direction vectors perpendicular; lines do not intersect (skew).
- $\lambda = -\dfrac{35}{19}$: RREF consistent; lines intersect (not perpendicular).

C code

```
    #include <math.h>
// Return lambda for which the two lines are perpendicular.
double perpendicular_lambda(void) {
    return 1.0;
}
// Return lambda for which the two lines intersect.
double intersection_lambda(void) {
    return -35.0/19.0;
}
// For a given lambda, check whether the two lines intersect.
// Writes s (parameter of line2) and t (parameter of line1) to
    the output pointers.
// Returns 1 if intersection exists (within tolerance), otherwise
    0.
int lines_intersection_params(double lambda, double *s_out,
    double *t_out) {
    const double EPS = 1e-9;
    double denom1 = 1.0 - 3.0*(5.0*lambda + 2.0);
    if (fabs(denom1) < EPS) return 0;
```

C code

```c
    double s1 = 5.0/denom1;
    double denom2 = 30.0 + 4.0*lambda;
    if (fabs(denom2) < EPS) return 0;
    double s2 = 5.0/denom2;
    if (fabs(s1 - s2) > 1e-6) return 0;
    double s = 0.5*(s1 + s2);
    double t = 3.0*s;
    if (s_out) *s_out = s;
    if (t_out) *t_out = t;
    return 1;
}
```

Python code through shared output

```python
import ctypes
from fractions import Fraction
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Load shared library
lib = ctypes.CDLL('./liblines.so')
lib.perpendicular_lambda.restype = ctypes.c_double
lib.intersection_lambda.restype = ctypes.c_double
lib.lines_intersection_params.restype = ctypes.c_int
lib.lines_intersection_params.argtypes = [
    ctypes.c_double,
    ctypes.POINTER(ctypes.c_double),
    ctypes.POINTER(ctypes.c_double)
]
```

Python code through shared output

```
# 1. Get lambdas
lam_perp = lib.perpendicular_lambda()
lam_inter = lib.intersection_lambda()

print(fLambda for perpendicular lines: {lam_perp} = {Fraction(
    lam_perp).limit_denominator()})
print(fLambda for intersection: {lam_inter} = {Fraction(lam_inter
    ).limit_denominator()})

# 2. Check intersection at perpendicular lambda
s1 = ctypes.c_double()
t1 = ctypes.c_double()
intersects_perp = lib.lines_intersection_params(lam_perp, ctypes.
    byref(s1), ctypes.byref(t1))

# 3. Check intersection at intersection lambda
s2 = ctypes.c_double()
t2 = ctypes.c_double()
intersects = lib.lines_intersection_params(lam_inter, ctypes.
```

Python code through shared output

```
if intersects_perp:
    print(Lines intersect when perpendicular.)
else:
    print( Lines do NOT intersect when they are perpendicular.)

if intersects:
    print( Lines intersect when  =, lam_inter)
    print(Intersection parameters: s =, s2.value, , t =, t2.value
        )
    inter_point = np.array([
        s2.value,
        -0.5 + 2*lam_inter*s2.value,
        1 + 3*s2.value
    ])
    print(Intersection point:, inter_point)
else:
    inter_point = None
    print( Lines do NOT intersect for intersection .)
```

Python code through shared output

```
# 4. Plot
t_vals = np.linspace(-2, 2, 100)
x1 = 5 + (5*lam_inter + 2)*t_vals
y1 = 2 - 5*t_vals
z1 = 1 + t_vals

s_vals = np.linspace(-2, 2, 100)
x2 = s_vals
y2 = -0.5 + 2*lam_inter*s_vals
z2 = 1 + 3*s_vals

fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')

# Plot lines
ax.plot(x1, y1, z1, label=Line 1, color='blue', linewidth=2)
ax.plot(x2, y2, z2, label=Line 2, color='red', linewidth=2)
```

Python code through shared output

```python
# Plot intersection
if inter_point is not None:
    ax.scatter(*inter_point, color='green', s=80, edgecolors='black', label=Intersection Point)
    ax.text(*inter_point + 0.3,
            f({inter_point[0]:.2f}, {inter_point[1]:.2f}, {inter_point[2]:.2f}),
            fontsize=10, color='green')
```

Python code through shared output

```
# Labels and legend
ax.set_xlabel(X Axis, fontsize=12)
ax.set_ylabel(Y Axis, fontsize=12)
ax.set_zlabel(Z Axis, fontsize=12)
ax.set_title(3D Plot: Intersection of Two Lines, fontsize=14)
ax.legend()
ax.grid(True)
ax.view_init(elev=25, azim=135)

plt.tight_layout()
plt.show()
```

Only Python code

```
import numpy as np
import matplotlib.pyplot as plt

# Local imports (assuming these scripts are available and in
    PYTHONPATH)
from line.funcs import *
from triangle.funcs import *
from conics.funcs import circ_gen

# Given intersection parameter lambda (from your C-types code or
    calculation)
lam = -35/19 # approx -1.8421

# Calculate intersection parameters s and t for the lines
# The two parametric lines from your problem:
```

Only Python code

```
# Line 1:
# x = 5 + (5*lam + 2)*t
# y = 2 - 5*t
# z = 1 + t

# Line 2:
# x = s
# y = -0.5 + 2*lam*s
# z = 1 + 3*s

# We want to find s, t such that both lines intersect

# From parametric equality:
# 5 + (5*lam + 2)*t = s ...(1)
# 2 - 5*t = -0.5 + 2*lam*s ...(2)
# 1 + t = 1 + 3*s ...(3)

# Solve (3):
# t = 3*s
```

Only Python code

```
# Substitute into (1):
# 5 + (5*lam + 2)*3*s = s
# 5 + 3*(5*lam + 2)*s = s
# 5 = s - 3*(5*lam + 2)*s = s*(1 - 3*(5*lam + 2))
# s = 5 / (1 - 3*(5*lam + 2))

denom = 1 - 3*(5*lam + 2)
s = 5 / denom

# Then t = 3*s
t = 3*s

# Calculate intersection point from line 2 (or line 1)
inter_point = np.array([
    s,
    -0.5 + 2*lam*s,
    1 + 3*s
])
```

Only Python code

```
print(fIntersection parameters: s = {s:.5f}, t = {t:.5f})
print(fIntersection Point: ({inter_point[0]:.5f}, {inter_point
    [1]:.5f}, {inter_point[2]:.5f}))

# Define parametric lines for plotting near intersection point

def line1(t_vals):
    x = 5 + (5*lam + 2)*t_vals
    y = 2 - 5*t_vals
    z = 1 + t_vals
    return x, y, z

def line2(s_vals):
    x = s_vals
    y = -0.5 + 2*lam*s_vals
    z = 1 + 3*s_vals
    return x, y, z
```

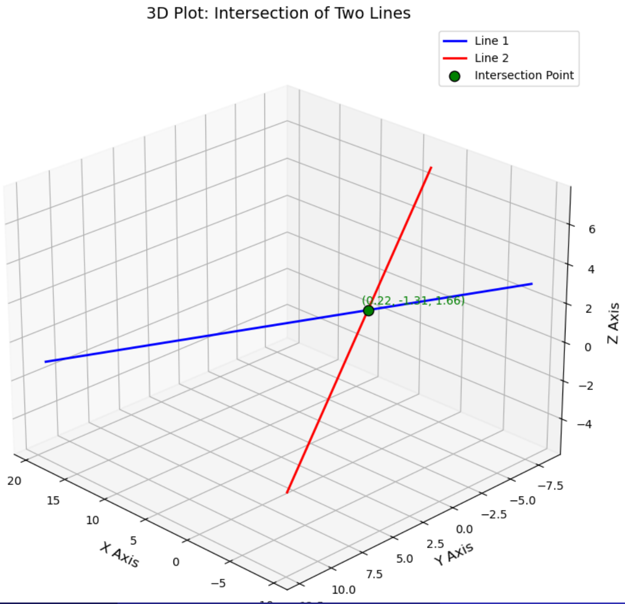
Only Python code

```
# Plot ranges close to intersection
t_vals = np.linspace(t - 1, t + 1, 100)
s_vals = np.linspace(s - 1, s + 1, 100)

x1, y1, z1 = line1(t_vals)
x2, y2, z2 = line2(s_vals)

# Plotting
fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')

ax.plot(x1, y1, z1, label='Line 1', color='blue', linewidth=2)
ax.plot(x2, y2, z2, label='Line 2', color='red', linewidth=2)

# Mark intersection
ax.scatter(*inter_point, color='green', s=80, label='Intersection
    Point')
```

Only Python code

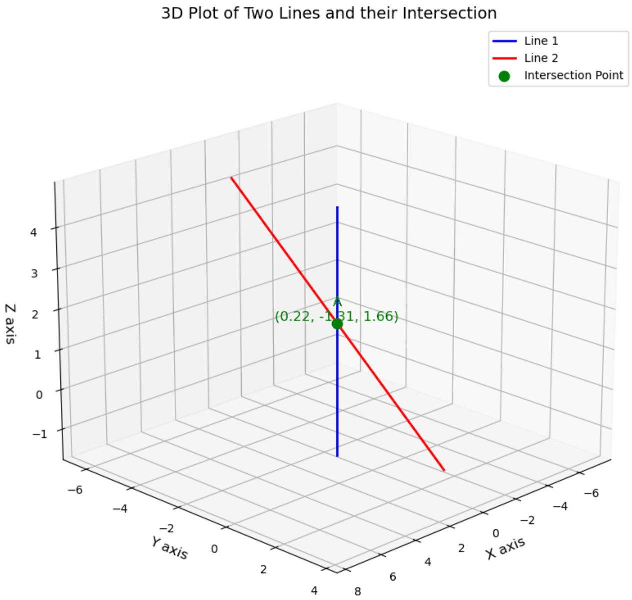
```
# Annotate intersection point
ax.text(inter_point[0], inter_point[1], inter_point[2],
        fA\n({inter_point[0]:.2f}, {inter_point[1]:.2f}, {
            inter_point[2]:.2f}),
        color='green', fontsize=12, ha='center', va='bottom')

# Axis labels and title
ax.set_xlabel(X axis, fontsize=12)
ax.set_ylabel(Y axis, fontsize=12)
ax.set_zlabel(Z axis, fontsize=12)
ax.set_title(3D Plot of Two Lines and their Intersection,
    fontsize=14)

ax.legend()
ax.grid(True)
ax.view_init(elev=20, azim=45)

plt.tight_layout()
plt.show()
```

PLOTS



3D Plot: Intersection of Two Lines
Line 1
Line 2
Intersection Point
(0.22, -1.31, 1.66)
X Axis
Y Axis
Z Axis
20
15
10
5
0
−5
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
6
4
2
0
−2
−4

PLOTS


3D Plot of Two Lines and their Intersection
Line 1
Line 2
Intersection Point
A
(0.22, -1.31, 1.66)
Z axis
Y axis
X axis
4
3
2
1
0
−1
−6
−4
−2
0
2
4
8
6
4
2
0
−2
−4
−6