

7.4.30

EE25BTECH11023 - Venkata Sai

Question:

A circle is given by $x^2 + (y - 1)^2 = 1$, another circle C touches it externally and also the X axis, then the locus of its centre is

Solution:

As the circle touches X-axis, Distance of a point from x-axis is given by

$$r = \mathbf{n}^T \mathbf{c} \quad (1)$$

where \mathbf{n} is the unit vector normal to x-axis

For the given circle with radius r_1 and center c_1

$$x^2 + (y - 1)^2 = 1 \quad (2)$$

$$\mathbf{p} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \mathbf{n} \text{ and } r_1 = 1 \quad (3)$$

Distance between their centers equal to sum of their radius

$$\|\mathbf{c} - \mathbf{p}\| = r + r_1 \quad (4)$$

$$\|\mathbf{c} - \mathbf{n}\| = \mathbf{n}^T \mathbf{c} + 1 \quad (5)$$

$$\|\mathbf{c} - \mathbf{n}\|^2 = (\mathbf{n}^T \mathbf{c} + 1)^2 \quad (6)$$

$$(\mathbf{c} - \mathbf{n})(\mathbf{c}^T - \mathbf{n}^T) = (\mathbf{n}^T \mathbf{c} + 1)^2 \quad (7)$$

$$\mathbf{c}^T \mathbf{c} + \mathbf{n} \mathbf{n}^T - \mathbf{c}^T \mathbf{n} - \mathbf{n}^T \mathbf{c} = (\mathbf{n}^T \mathbf{c})^T (\mathbf{n}^T \mathbf{c}) + 2\mathbf{n}^T \mathbf{c} + 1 \quad (8)$$

$$\mathbf{c}^T \mathbf{c} + \|\mathbf{n}\|^2 - 2\mathbf{n}^T \mathbf{c} = (\mathbf{c}^T \mathbf{n} \mathbf{n}^T \mathbf{c}) + 2\mathbf{n}^T \mathbf{c} + 1 \quad (9)$$

$$\mathbf{c}^T \mathbf{c} + 1 = (\mathbf{c}^T \mathbf{n} \mathbf{n}^T \mathbf{c}) + 4\mathbf{n}^T \mathbf{c} + 1 \quad (10)$$

$$\mathbf{c}^T \mathbf{c} - (\mathbf{c}^T \mathbf{n} \mathbf{n}^T \mathbf{c}) = 4\mathbf{n}^T \mathbf{c} \quad (11)$$

$$\mathbf{c}^T (\mathbf{I} - \mathbf{n} \mathbf{n}^T) \mathbf{c} = 4\mathbf{n}^T \mathbf{c} \quad (12)$$

$$\mathbf{c}^T \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \end{pmatrix} \right) \mathbf{c} = 4 \begin{pmatrix} 0 \\ 1 \end{pmatrix}^T \mathbf{c} \quad (13)$$

$$\mathbf{c}^T \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right) \mathbf{c} = 4 \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{c} \quad (14)$$

$$\begin{pmatrix} x & y \end{pmatrix} \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right) \begin{pmatrix} x \\ y \end{pmatrix} = 4 \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (15)$$

$$\begin{pmatrix} x & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 4y \quad (16)$$

$$x^2 = 4y \quad (17)$$

Hence $y \geq 0$. Then

$$\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \geq 0\} \quad (18)$$

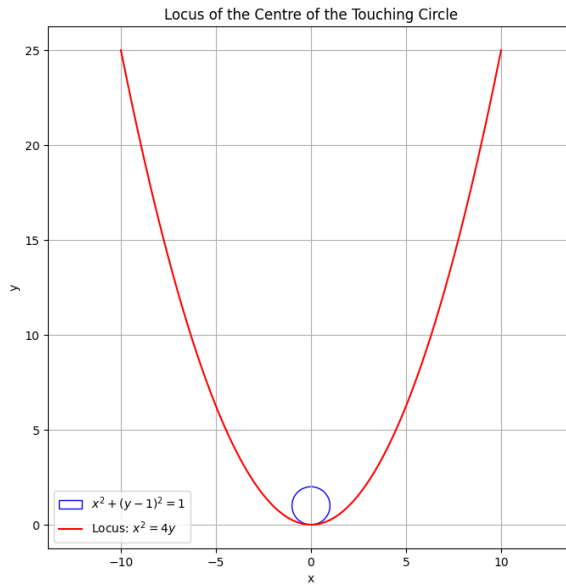


Fig. 0.1