5.4.10

EE25BTECH11015 - Bhoomika V

Question :-
Using elementary transformations, find the inverse of the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}.$$

Solution:

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}.$$

We form the augmented matrix $[A \mid I]$:

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 2 & 3 & 5 & 0 & 1 & 0 \\ -2 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 + 2R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 5 & 1 & -2 & 1 & 0 \\ 0 & -2 & 5 & 2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \to \frac{1}{5}R_2} \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & \frac{1}{5} & -\frac{2}{5} & \frac{1}{5} & 0 \\ 0 & -2 & 5 & 2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 + 2R_2]{R_1 \to R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{11}{5} & \frac{3}{5} & \frac{1}{5} & 0 \\ 0 & 1 & \frac{1}{5} & -\frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 0 & \frac{27}{5} & \frac{6}{5} & \frac{2}{5} & 1 \end{array}\right]$$

$$\xrightarrow{R_3 \to \frac{5}{27}R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{11}{5} & \frac{3}{5} & \frac{1}{5} & 0 \\ 0 & 1 & \frac{1}{5} & -\frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 & \frac{2}{9} & \frac{2}{27} & \frac{5}{27} \end{array}\right]$$

$$\xrightarrow[R_2 \to R_2 - \frac{1}{5}R_3]{R_1 \to R_1 - \frac{11}{5}R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{9} & \frac{1}{27} & -\frac{11}{27} \\ 0 & 1 & 0 & -\frac{4}{9} & \frac{5}{27} & -\frac{1}{27} \\ 0 & 0 & 1 & \frac{2}{9} & \frac{2}{27} & \frac{5}{27} \end{array}\right]$$

Thus the inverse is

$$A^{-1} = \begin{bmatrix} \frac{1}{9} & \frac{1}{27} & -\frac{11}{27} \\ -\frac{4}{9} & \frac{5}{27} & -\frac{1}{27} \\ \frac{2}{9} & \frac{2}{27} & \frac{5}{27} \end{bmatrix}.$$