Matgeo Presentation - Problem 12.180

ee25btech11063 - Vejith

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Question

The system of linear equations

$$4x+2y=7$$
$$2x+y=6 \text{ has}$$

a) a unique solution

b) no solution

c) infinite number of solutions

d) exactly two distinct solutions

Solution

Given linear equations are

$$\begin{pmatrix} 4 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 7 \tag{0.1}$$

$$\begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 6 \tag{0.2}$$

Equations (0.1) and (0.2) can be written as

$$\begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \end{pmatrix} \tag{0.3}$$

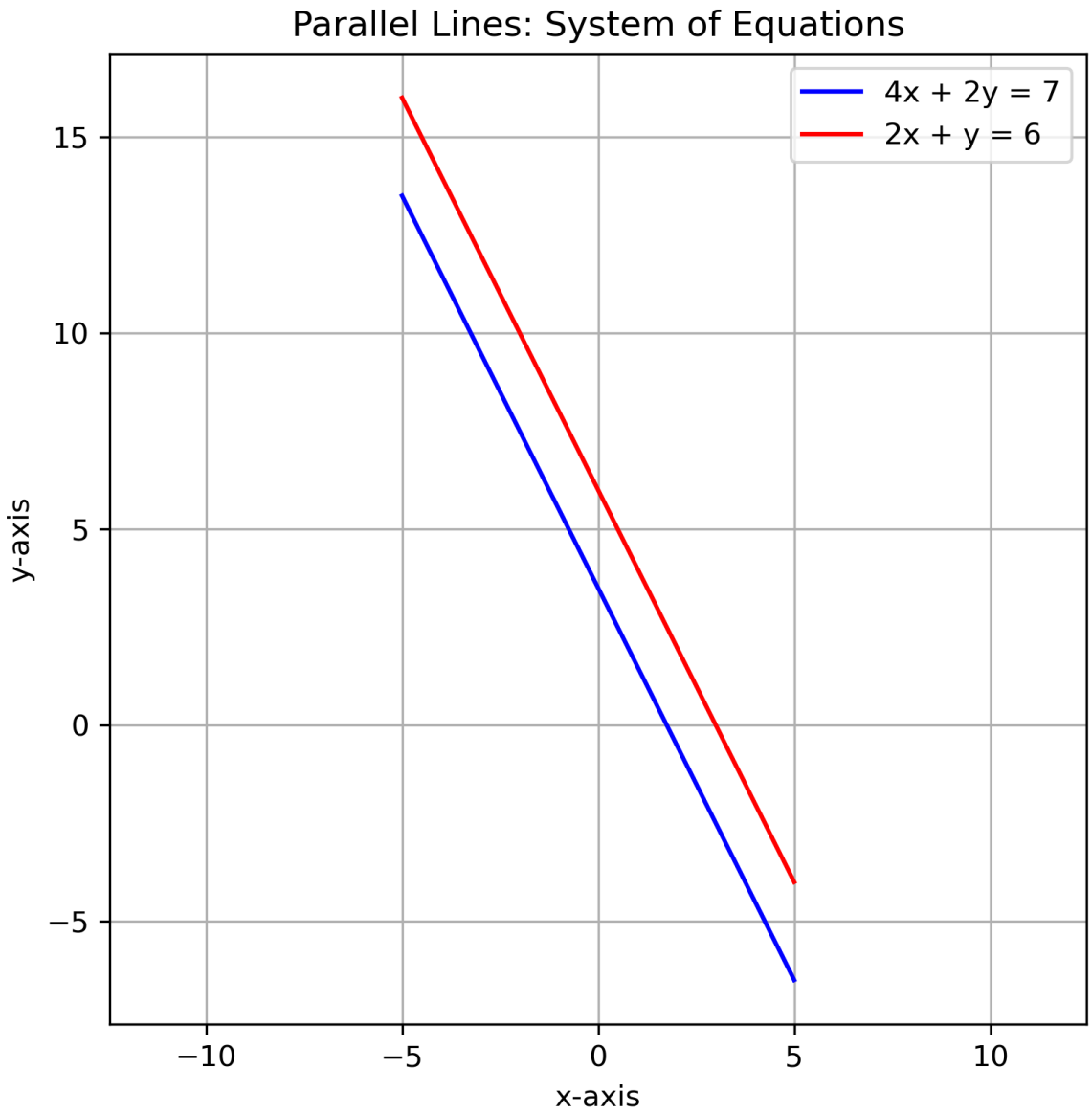
Forming the augmented matrix

$$\left(\begin{array}{cc|c} 4 & 2 & 7 \\ 2 & 1 & 6 \end{array}\right) \xleftrightarrow{R_1 \to R_1 - 2R_2} \left(\begin{array}{cc|c} 0 & 0 & -5 \\ 2 & 1 & 6 \end{array}\right) \tag{0.4}$$

As in the augmented matrix the entries of first row are 0 their linear combination should also give 0 but it is given as -5

$\Longrightarrow$ So,the given system of linear equations have no solution

Plot


Parallel Lines: System of Equations
4x + 2y = 7
2x + y = 6
15
10
5
0
−5
y-axis
−10
−5
0
5
10
x-axis

C Code: Solution.c

```c
#include <stdio.h>

int main() {
    // Coefficients of equations:
    // Equation 1: 4x + 2y = 7
    // Equation 2: 2x + y = 6
    int a1 = 4, b1 = 2, c1 = 7;
    int a2 = 2, b2 = 1, c2 = 6;

    // File pointer
    FILE *fp;
    fp = fopen("solution.dat", "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }

    // Calculate determinants
    int det = a1*b2 - a2*b1; // determinant of coefficients
    int detx = c1*b2 - c2*b1; // determinant replacing x-column
    int dety = a1*c2 - a2*c1; // determinant replacing y-column

    if (det != 0) {
        // Unique solution exists
        double x = (double)detx / det;
        double y = (double)dety / det;
        fprintf(fp, "The system has a unique solution: x = %.2f, y = %.2f\n", x, y);
    }
```

C Code: Solution.c

```
    else {
        if (detx == 0 && dety == 0) {
            // Infinite solutions
            fprintf(fp, "The system has infinite number of solutions.\n");
        } else {
            // No solution
            fprintf(fp, "The system has no solution.\n");
        }
    }

    fclose(fp);
    return 0;
}
```

Python: plot.py

```
import numpy as np
import matplotlib.pyplot as plt

# Define x range
x = np.linspace(-5, 5, 400)

# Line 1: 4x + 2y = 7 -> y = (7 - 4x) / 2
y1 = (7 - 4*x) / 2

# Line 2: 2x + y = 6 -> y = 6 - 2x
y2 = 6 - 2*x

# Plot the lines
plt.figure(figsize=(6,6))
plt.plot(x, y1, label="4x + 2y = 7", color="blue")
plt.plot(x, y2, label="2x + y = 6", color="red")

# Formatting
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.title("Parallel Lines: System of Equations")
plt.legend()
plt.grid(True)
plt.axis("equal")

# Save the figure
plt.savefig("parallel_lines.png", dpi=300)

# Show the plot
plt.show()
```