

4.4.22

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Question

Find the equation of a plane which passes through the point $(3, 2, 0)$ and contains the line

$$\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}. \quad (1)$$

Theoretical Solution

Solution:

Given: Find the plane using column vectors.

Let the normal be $\mathbf{n} = (a, b, c)^T$. We use the form

$$\mathbf{n}^T \mathbf{x} = 1. \quad (2)$$

The plane passes through the point $P = (3, 2, 0)$ and contains the line

$$\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}, \quad (3)$$

so take a point on the line $A = (3, 6, 4)$ and the direction vector $\mathbf{v} = (1, 5, 4)$.

The conditions are

$$\mathbf{n}^T P = 1, \quad \mathbf{n}^T A = 1, \quad \mathbf{n}^T \mathbf{v} = 0. \quad (4)$$

Theoretical Solution

Put these three column vectors together into a matrix (columns are the given points/vectors):

$$M = \begin{pmatrix} 3 & 3 & 1 \\ 2 & 6 & 5 \\ 0 & 4 & 4 \end{pmatrix} \quad (\text{columns are } P, A, \mathbf{v}). \quad (5)$$

Then the three scalar conditions above read compactly as

Theoretical Solution

$$\mathbf{n}^T M = \begin{pmatrix} 1 & 1 & 0 \end{pmatrix}. \quad (6)$$

Transposing both sides gives a standard linear system for \mathbf{n} :

$$M^T \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}. \quad (7)$$

Theoretical Solution

Write this out:

$$\begin{pmatrix} 3 & 2 & 0 \\ 3 & 6 & 4 \\ 1 & 5 & 4 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}. \quad (8)$$

Solving this 3×3 system (by Gaussian elimination or matrix inverse) yields

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}. \quad (9)$$

Thus a convenient normal vector is $\mathbf{n} = (1, -1, 1)^T$, and the plane equation in the requested form is

$$\boxed{\mathbf{n}^T \mathbf{x} = 1} \implies \boxed{x - y + z = 1}. \quad (10)$$

Plane and Line

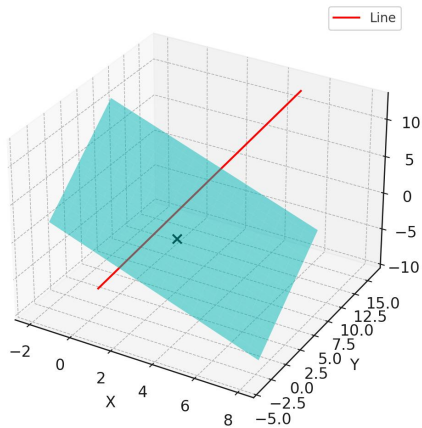


Figure: Caption