Question 2.6.37:
The vector from origin to the points A and B are

$$\mathbf{a} = 2\hat{i} - 3\hat{j} + 2\hat{k} \quad \text{and} \quad \mathbf{b} = 2\hat{i} + 3\hat{j} + \hat{k}, \tag{1}$$

respectively, then the area of $\triangle OAB$ is ___________.
Solution: Given

$$\mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}. \tag{2}$$

Using the triangle-area formula,

$$\operatorname{ar}(\triangle OAB) = \frac{1}{2}\|(A - O) \times (B - O)\| = \frac{1}{2}\|\mathbf{a} \times \mathbf{b}\|. \tag{3}$$

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -3 & 2 \\ 2 & 3 & 1 \end{pmatrix} = -9\,\hat{\imath} + 2\,\hat{\jmath} + 12\,\hat{k}, \tag{4}$$

hence

$$\|\mathbf{a} \times \mathbf{b}\| = \sqrt{(-9)^2 + 2^2 + 12^2} = \sqrt{229}. \tag{5}$$

Therefore,

$$\boxed{\operatorname{area}(\triangle OAB) = \frac{\sqrt{229}}{2}}. \tag{6}$$

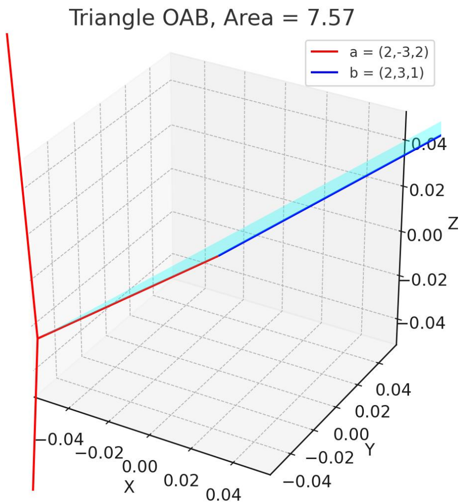
Triangle OAB, Area = 7.57
a = (2,-3,2)
b = (2,3,1)
0.04
0.02
0.00
−0.02
−0.04
Z
−0.04
−0.02
0.00
0.02
0.04
X
0.04
0.02
0.00
−0.02
−0.04
Y

Fig. 1