### 5.2.43

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# Question)

Solve the linear equation:

$$6x + 3y = 6xy \tag{1}$$

$$2x + 4y = 5xy \tag{2}$$

### Solution

Dividing both equations with xy

$$\frac{5}{y} + \frac{3}{x} = 6 \tag{3}$$

$$\frac{6}{y} + \frac{3}{x} = 6$$

$$\frac{2}{y} + \frac{4}{x} = 5$$
(3)

Let

$$\frac{1}{x} = a, \frac{1}{y} = b \tag{5}$$

### Solution

#### So, new equations

$$3a + 6b = 6 \tag{6}$$

$$4a + 2b = 5 \tag{7}$$

$$\begin{pmatrix} 3 & 6 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \begin{pmatrix} 6 \\ 5 \end{pmatrix}$$
 (8)

$$\mathbf{A}\mathbf{x} = \mathbf{c} \tag{9}$$

#### Gaussian elimination on A

$$\begin{pmatrix}
3 & 6 & | & 6 \\
4 & 2 & | & 5
\end{pmatrix} \xrightarrow{R_2 - \frac{4R_1}{3}} \begin{pmatrix}
3 & 6 & | & 6 \\
0 & -6 & | & -3
\end{pmatrix}$$
(10)

$$\xrightarrow{\frac{R_2}{-6}} \begin{pmatrix} 3 & 6 & | & 6 \\ 0 & 1 & | & \frac{1}{2} \end{pmatrix} \tag{11}$$

### Solution

Therefore, by putting values in (8)

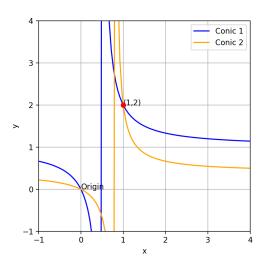
$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}$$
(12)

For x,y

$$\begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \tag{13}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{14}$$

# **Figure**



## Direct Python

```
import numpy as np
import matplotlib.pyplot as plt
plt.figure(figsize=(5,5), dpi=200)
plt.xlim(-1,4)
plt.ylim(-1,4)
A=np.array([[3,6],[4,2]])
c=np.array([6,5])
an=np.linalg.inv(A)
ans=np.dot(an,c)
Ans=np.linalg.solve(A,c)
```

## Direct Python

```
print("x=", Ans[0], "y=", Ans[1])
a=1/ans[0]
b=1/ans[1]
x = np.array([a,b]).reshape(-1,1)
x1 = np.linspace(-1, 4, 100)
11 = (6 \times x1)/(6 \times x1 - 3)
12 = (2 \times x1)/(5 \times x1 - 4)
plt.plot(x1,l1, color='blue', label="Line 1")
```

## Direct Python

```
plt.plot(x1,12, color='orange', label="Line 2")
 plt.scatter(1,2, c='r', zorder=5)
 plt.text(1,2,"(1,2)")
 plt.text(0,0,"Origin",)
 plt.xlabel("x")
 plt.ylabel("y")
plt.grid()
plt.legend()
 plt.savefig("figure.png", dpi=200)
 plt.show()
```

#### C code

```
#include <stdio.h>
typedef struct {
   double x;
   double y;
} Point;
typedef struct {
   Point sols[2];
    int count;
} SolutionSet;
// Solve 6x+3y=6xy, 2x+4y=5xy
SolutionSet solve_equations() {
   SolutionSet S:
   S.count = 0;
```

### C code

```
// Solution 1: (0,0)
S.sols[S.count].x = 0;
S.sols[S.count].y = 0;
S.count++;
// Solution 2: (1,2)
S.sols[S.count].x = 1;
S.sols[S.count].y = 2;
S.count++;
return S;
```

#### C code

## Python code with shared object

```
# main.py
import ctypes
from ctypes import Structure, c double, c int
import matplotlib.pyplot as plt
class Point(Structure):
   _fields_ = [("x", c_double), ("y", c_double)]
class SolutionSet(Structure):
   _fields_ = [("sols", Point * 2), ("count", c_int)]
# Load C lib
lib = ctypes.CDLL("./libsolver.so")
lib.solve_equations.restype = SolutionSet
```

# Python code with shared object

```
# Call function
solutions = lib.solve_equations()
print(f"Found {solutions.count} solutions:")
for i in range(solutions.count):
    x, y = solutions.sols[i].x, solutions.sols[i].y
    print(f"Solution {i+1}: ({x}, {y})")
# Plot equations and solutions
import numpy as np
|x \text{ vals} = \text{np.linspace}(-1, 3, 400)|
v1 = (6*x vals)/(6*x vals - 3) # from eqn (1)
v^2 = (2*x vals)/(5*x vals - 4) # from eqn (2)
```

## Python code with shared object

```
plt.figure(figsize=(6,6))
 plt.plot(x_vals, y1, label="6x+3y=6xy")
 plt.plot(x_vals, y2, label="2x+4y=5xy")
 for i in range(solutions.count):
     x, y = solutions.sols[i].x, solutions.sols[i].y
     plt.scatter(x, y, c='r', zorder=5)
     plt.text(x, y, f''(\{x:.0f\}, \{y:.0f\})'', fontsize=10)
 plt.ylim(-1,4)
 plt.grid(True)
 plt.legend()
plt.xlabel("x")
plt.ylabel("y")
 plt.title("Solutions of nonlinear system")
 plt.show()
```