1.9.2

EE25BTECH11013 - Bhargav

Question:
The point on the X axis which is equidistant from $(-4,0)$ and $(10,0)$ is
Solution:
Let the 2 points be **A** and **B** and let the desired point equidistant from both **A** and **B** be **O**:

$$\mathbf{A} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, \tag{0.1}$$

$$\mathbf{B} = \begin{pmatrix} 10 \\ 0 \end{pmatrix}, \tag{0.2}$$

$$\mathbf{O} = x\,\mathbf{e}_1 = \begin{pmatrix} \text{x} \\ 0 \end{pmatrix} \tag{0.3}$$

If **O** lies on X axis and is equidistant from **A** and **B**

$$\|\mathbf{O}-\mathbf{A}\| = \|\mathbf{O}-\mathbf{B}\| \tag{0.4}$$

$$\implies \|\mathbf{O}-\mathbf{A}\|^2 = \|vec{O}-\mathbf{B}\|^2 \tag{0.5}$$

$$\implies (\mathbf{O}-\mathbf{A})^\top(\mathbf{O}-\mathbf{A}) = (\mathbf{O}-\mathbf{B})^\top(\mathbf{O}-\mathbf{B}) \tag{0.6}$$

$$\implies \mathbf{O}^\top\mathbf{O} - 2\mathbf{O}^\top\mathbf{A} + \mathbf{A}^\top\mathbf{A} = \mathbf{O}^\top\mathbf{O} - 2\mathbf{O}^\top\mathbf{B} + \mathbf{B}^\top\mathbf{B} \tag{0.7}$$

$$\implies \|\mathbf{O}\|^2 - 2\mathbf{O}^\top\mathbf{A} + \|\mathbf{A}\|^2 = \|\mathbf{O}\|^2 - 2\mathbf{O}^\top\mathbf{B} + \|\mathbf{B}\|^2 \tag{0.8}$$

$$(\mathbf{A}-\mathbf{B})^\top\mathbf{O} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2}. \tag{0.9}$$

$$\mathbf{O} = x\mathbf{e}_1, \tag{0.10}$$

$$x = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2(\mathbf{A}-\mathbf{B})^\top \mathbf{e}_1}. \tag{0.11}$$

Solving for x, we get $x = 3$

$$\therefore \mathbf{O} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{0.12}$$