# Matgeo Presentation - 8.2.31

ee25btech11063 - Vejith

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# Question

Find the equation of conic if ends of the major axis are $(\pm 3, 0)$ and ends of the minor axis are $(0, \pm 2)$

# Solution

The equation of conic is represented as

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.1}$$

$$\mathbf{V} = \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n}\mathbf{n}^\top \tag{0.2}$$

As the major axis is along the X-axis

$$\mathbf{n} = \mathbf{e_1} \tag{0.3}$$

$$\implies \mathbf{V} = \begin{pmatrix} 1 - e^2 & 0 \\ 0 & 1 \end{pmatrix} \tag{0.4}$$

as the centre of ellipse is $\mathbf{c} = \mathbf{0}$

$$\implies \mathbf{u} = \mathbf{0} \tag{0.5}$$

let

$$\mathbf{P} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{0.6}$$

$\mathbf{P}$ satisfy (1)

# Solution

$$\mathbf{P}^\top \mathbf{V} \mathbf{P} + 2\mathbf{u}^\top \mathbf{P} + f = 0 \tag{0.7}$$

$$\begin{pmatrix} 0 & 2 \end{pmatrix} \begin{pmatrix} 1-e^2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} + f = 0 \tag{0.8}$$

$$4 + f = 0 \tag{0.9}$$

$$\implies f = -4 \tag{0.10}$$

End of the ellipse $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ also satisfy (1)

$$\begin{pmatrix} 3 & 0 \end{pmatrix} \begin{pmatrix} 1-e^2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix} + f = 0 \tag{0.11}$$

$$\implies 9\left(1-e^2\right) + f = 0 \tag{0.12}$$

from (10)

# Conclusion

$$1 - e^2 = \frac{4}{9} \tag{0.13}$$

$$\implies e^2 = \frac{5}{9} \tag{0.14}$$

$$\implies \mathbf{V} = \begin{pmatrix} \frac{4}{9} & 0 \\ 0 & 1 \end{pmatrix} \tag{0.15}$$

Equation of conic is

$$\mathbf{x}^\top \begin{pmatrix} \frac{4}{9} & 0 \\ 0 & 1 \end{pmatrix} \mathbf{X} - 4 = 0 \tag{0.16}$$

# Plot

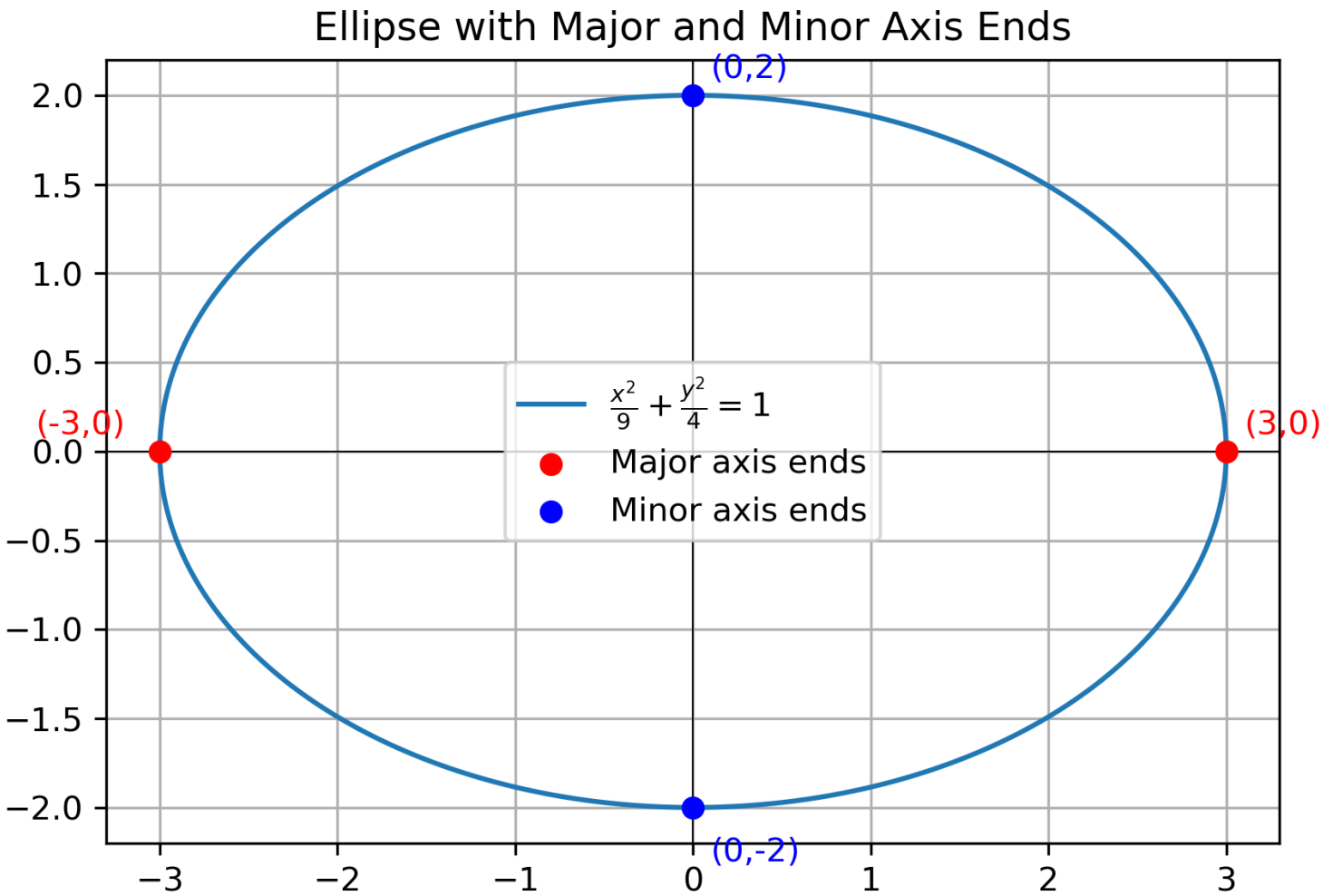


Figure: Caption

# C Code: ellipse.c

```c
#include <stdio.h>

int main() {
    FILE *fp;

    // Open file ellipse.dat for writing
    fp = fopen("ellipse.dat", "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }

    // Write the equation of ellipse into the file
    fprintf(fp, "Equation of the ellipse:\n");
    fprintf(fp, "(x^2)/9 + (y^2)/4 = 1\n");

    // Optionally write the matrix form as well
    fprintf(fp, "\nMatrix form:\n");
    fprintf(fp, "[x y] * [[1/9  0]\n");
    fprintf(fp, " [0    1/4]] * [x y]^T = 1\n");

    fclose(fp);
    printf("Equation successfully written to ellipse.dat\n");

    return 0;
}
```

# Python: plot.py

```
import numpy as np
import matplotlib.pyplot as plt

# Parameters of the ellipse
a = 3 # semi-major axis
b = 2 # semi-minor axis

# Generate theta values
theta = np.linspace(0, 2*np.pi, 400)

# Parametric equations of ellipse
x = a * np.cos(theta)
y = b * np.sin(theta)

# Plot ellipse
plt.plot(x, y, label=r"$\frac{x^2}{9}+\frac{y^2}{4}=1$")

# Mark ends of major axis (3,0)
plt.scatter([3, -3], [0, 0], color="red", zorder=5, label="Major␣axis␣ends")

# Mark ends of minor axis (0,2)
plt.scatter([0, 0], [2, -2], color="blue", zorder=5, label="Minor␣axis␣ends")

# Add annotations
plt.text(3.1, 0.1, "(3,0)", color="red")
plt.text(-3.7, 0.1, "(-3,0)", color="red")
plt.text(0.1, 2.1, "(0,2)", color="blue")
plt.text(0.1, -2.3, "(0,-2)", color="blue")

# Axes setup
plt.axhline(0, color="black", linewidth=0.5)
plt.axvline(0, color="black", linewidth=0.5)
plt.gca().set_aspect('equal') # keep aspect ratio equal
```

# Python: plot.py

```
plt.legend()
plt.title("Ellipse␣with␣Major␣and␣Minor␣Axis␣Ends")
plt.grid(True)
plt.savefig("ellipse.png", dpi=300, bbox_inches="tight")
plt.show()
```