4.12.44

AI25BTECH11030 -Sarvesh Tamgade

Question: Find the equation of the set of points which are equidistant from the points $\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$.

Solution: Let $\mathbf{X}$ be the position vector of any point equidistant from $\mathbf{A}$ and $\mathbf{B}$. The equidistance condition is

$$||\mathbf{X} - \mathbf{A}|| = ||\mathbf{X} - \mathbf{B}||. \tag{0.1}$$

Squaring both sides, we have

$$(\mathbf{X} - \mathbf{A})^\top (\mathbf{X} - \mathbf{A}) = (\mathbf{X} - \mathbf{B})^\top (\mathbf{X} - \mathbf{B}). \tag{0.2}$$

Expanding and simplifying,

$$\mathbf{X}^\top \mathbf{X} - 2\mathbf{A}^\top \mathbf{X} + \mathbf{A}^\top \mathbf{A} = \mathbf{X}^\top \mathbf{X} - 2\mathbf{B}^\top \mathbf{X} + \mathbf{B}^\top \mathbf{B}, \tag{0.3}$$

which reduces to

$$-2\mathbf{A}^\top \mathbf{X} + \mathbf{A}^\top \mathbf{A} = -2\mathbf{B}^\top \mathbf{X} + \mathbf{B}^\top \mathbf{B}. \tag{0.4}$$

Rearranging,

$$2(\mathbf{B} - \mathbf{A})^\top \mathbf{X} = \mathbf{B}^\top \mathbf{B} - \mathbf{A}^\top \mathbf{A}. \tag{0.5}$$

Calculate the vector difference:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 - 1 \\ 2 - 2 \\ -1 - 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}. \tag{0.6}$$

Calculate the scalar values:

$$\mathbf{B}^\top \mathbf{B} = 3^2 + 2^2 + (-1)^2 = 14, \quad \mathbf{A}^\top \mathbf{A} = 1^2 + 2^2 + 3^2 = 14, \tag{0.7}$$

so the right side is zero:

$$\mathbf{B}^\top \mathbf{B} - \mathbf{A}^\top \mathbf{A} = 0. \tag{0.8}$$

Thus, substituting the simplified difference vector, the plane equation becomes:

$$4 \begin{pmatrix} 1 & 0 & -2 \end{pmatrix} \mathbf{X} = 0, \tag{0.9}$$

or equivalently,

$$\begin{pmatrix} 1 & 0 & -2 \end{pmatrix} \mathbf{X} = 0. \tag{0.10}$$

Final Answer: The set of points equidistant from **A** and **B** lies on the plane defined by

$$\boxed{\begin{pmatrix} 1 & 0 & -2 \end{pmatrix} \mathbf{x} = 0}$$

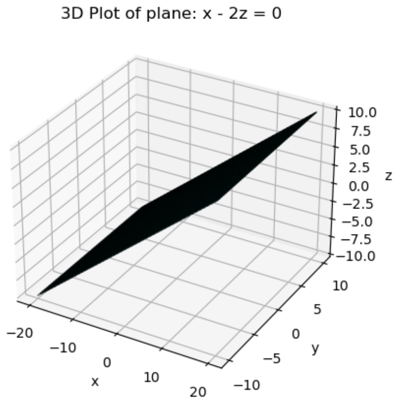

Fig. 0.1: Vector Representation