2.3.15

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Question

The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

given data

let **A**, **B** and **C** be the vectors, such that:

Variable	value
A	$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
B	$\begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}$
C	$\begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix}$

Table: Variables used

finding Scalar product of $\mathbf{A}$ with unit vector along $\mathbf{B}+\mathbf{C}$

The direction vector $\mathbf{B}+\mathbf{C} = \begin{pmatrix} 2+\lambda \\ 6 \\ -2 \end{pmatrix}$ — ①

The corresponding unit vector obtained is:

$$\frac{\mathbf{B}+\mathbf{C}}{\|\mathbf{B}+\mathbf{C}\|} = \frac{1}{\sqrt{\lambda^2+4\lambda+44}} \begin{pmatrix} 2+\lambda \\ 6 \\ -2 \end{pmatrix} \qquad -②$$

given,

$$\mathbf{A}^\top \cdot (B \hat{+} C) = 1 \qquad -③$$

$$\frac{1}{\sqrt{\lambda^2+4\lambda+44}} \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2+\lambda \\ 6 \\ -2 \end{pmatrix} = 1 \qquad -④$$

$$2 + \lambda + 6 - 2 = \sqrt{\lambda^2 + 4\lambda + 44} \qquad - ⑤$$

squaring on both sides:

$$\lambda^2 + 36 + 12\lambda = \lambda^2 + 4\lambda + 44 \qquad - ⑥$$

$$8\lambda = 8$$

$$\lambda = 1$$

Hence value of λ is 1.

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# --- 1. Define vectors with the calculated lambda = 1 ---
lambda_val = 1
A = np.array([1, 1, 1])
B = np.array([2, 4, -5])
C = np.array([lambda_val, 2, 3])
```

Python Code

```
# Calculate the resultant vectors ---
# Sum vector S = B + C
S = B + C
# Unit vector s_hat along S
s_hat = S / np.linalg.norm(S)
```

Python Code

```
# Set up the 3D plot ---
fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')
origin = [0, 0, 0]

# Plot all vectors from the origin ---
ax.quiver(*origin, *A, color='red', label=r'$\vec{A}$')
ax.quiver(*origin, *B, color='blue', label=r'$\vec{B}$')
ax.quiver(*origin, *C, color='green', label=r'$\vec{C}$ (with $\
    lambda=1$)')
ax.quiver(*origin, *S, color='purple', label=r'Sum Vector $\vec{S
    } = \vec{B} + \vec{C}$')
ax.quiver(*origin, *s_hat, color='orange', label=r'Unit Vector $\
    hat{s}$')
```

Python Code

```python
# Add text labels near the vector tips for clarity ---
ax.text(A[0], A[1], A[2], 'A', color='red', fontsize=12)
ax.text(B[0], B[1], B[2], 'B', color='blue', fontsize=12)
ax.text(C[0], C[1], C[2], 'C', color='green', fontsize=12)
ax.text(S[0], S[1], S[2], 'S', color='purple', fontsize=12)
ax.text(s_hat[0]*1.5, s_hat[1]*1.5, s_hat[2]*1.5, '', color='
    orange', fontsize=14)
```

Python Code

```
# Customize and display the plot ---
ax.set_title('Visualization of Vectors', fontsize=16)
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_xlim([-6, 6])
ax.set_ylim([-6, 6])
ax.set_zlim([-6, 6])
ax.legend()
ax.grid(True)
plt.show()
```

C Code

```
 #include <stdio.h>

int main() {
    // The problem statement is:
    // The scalar product of vector 'a' with the unit vector
        along the sum of vectors 'b' and 'c' is equal to one.
    // Vector a = i + j + k
   // Vector b = 2i + 4j - 5k
   // Vector c = i + 2j + 3k

    // Step 1: Find the sum vector, s = b + c
    // s = (2+)i + (4+2)j + (-5+3)k
    // s = (2+)i + 6j - 2k
   // The given condition is a  (s / |s|) = 1, which simplifies
        to a  s = |s|.
```

C Code

```
// Calculate the dot product a  s
// a  s = (1 * (2+)) + (1 * 6) + (1 * -2) = 2 +  + 6 - 2 =  + 6

// Find the magnitude squared, |s|^2
// |s|^2 = (2+)^2 + 6^2 + (-2)^2 = (2+)^2 + 36 + 4 = (2+)^2 + 40
// Set up the equation (a  s)^2 = |s|^2
// ( + 6)^2 = (2+)^2 + 40
// Step 6: Expand and simplify the equation
// ^2 + 12 + 36 = (^2 + 4 + 4) + 40
// ^2 + 12 + 36 = ^2 + 4 + 44
// 12 - 4 = 44 - 36
// 8 = 8
```

C Code

```c
    // Solve for
    float coefficient_of_lambda = 8.0;
    float constant = 8.0;
    float lambda = constant / coefficient_of_lambda;
    // Display the final result
    printf("The problem simplifies to the linear equation: 8 *
        lambda = 8\n");
    printf("Solving for lambda...\n");
    printf("The value of lambda is: %.0f\n", lambda);

  return 0;}
```

Python and C Code

```
import subprocess

# 1. Compile the C program
subprocess.run(["gcc", "code.c", "-o", "code"])

# 2. Run the compiled C program
result = subprocess.run(["./code"], capture_output=True, text=
    True)

# 3. Print the output from the C program
print(result.stdout)
```

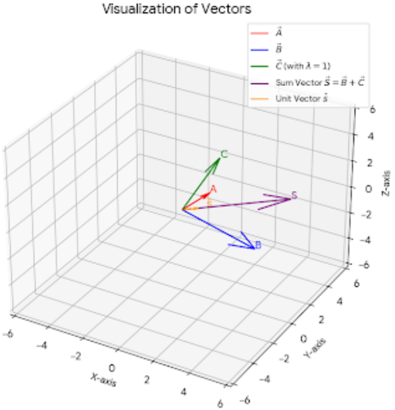
Visualization of Vectors
$\vec{A}$
$\vec{B}$
$\vec{C}$ (with $\lambda = 1$)
Sum Vector $\vec{S} = \vec{B} + \vec{C}$
Unit Vector $\hat{s}$
C
A
S
B
X-axis
Y-axis
Z-axis
-6
-4
-2
0
2
4
6

Figure: Plot