2.3.8

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Question

If $\hat{i}+\hat{j}+\hat{k},\ 2\hat{i}+5\hat{j},\ 3\hat{i}+2\hat{j}-3\hat{k},\ \hat{i}-6\hat{j}-\hat{k}$ respectively are the position vectors of points A, B, C, and D, then find the angle between the straight lines **AB** and **CD**. Also, determine whether **AB** and **CD** are collinear.

Given Information

$$\text{Let } \mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 1 \\ -6 \\ -1 \end{pmatrix}$$

Direction vectors:

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$$

$$\mathbf{CD} = \mathbf{D} - \mathbf{C} = \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix}$$

Formula

$$\cos\theta = \frac{\mathbf{AB}^T\mathbf{CD}}{|\mathbf{AB}|\,|\mathbf{CD}|} \tag{1}$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{AB}^T\mathbf{CD}}{|\mathbf{AB}|\,|\mathbf{CD}|}\right) \tag{2}$$

Vectors are collinear if $\theta = 0^\circ$ or 180°.

Solution

$$\mathbf{AB}^T\mathbf{CD} = \begin{pmatrix} 1 & 4 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix} = 1\times(-2)+4\times(-8)+(-1)\times 2 = -2-32-2 =$$

$$|\mathbf{AB}| = \sqrt{1^2 + 4^2 + (-1)^2} = \sqrt{18}$$

$$|\mathbf{CD}| = \sqrt{(-2)^2 + (-8)^2 + 2^2} = \sqrt{4 + 64 + 4} = \sqrt{72}$$

$$\cos\theta = \frac{-36}{\sqrt{18} \times \sqrt{72}} = \frac{-36}{36} = -1$$

$$\theta = \cos^{-1}(-1) = 180^\circ$$

Therefore, **AB** and **CD** are collinear but point in opposite directions.

Python Plot Code

```
import numpy as np
from mpl_toolkits.mplot3d import Axes3D
import matplotlib.pyplot as plt
A = np.array([1, 1, 1])
B = np.array([2, 5, 0])
C = np.array([3, 2, -3])
D = np.array([1, -6, -1])
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.quiver(*A, *(B-A), color='b', label='AB')
ax.quiver(*C, *(D-C), color='r', label='CD')
```

Python Plot Code

```
ax.scatter(*A, color='blue')
ax.scatter(*B, color='blue')
ax.scatter(*C, color='red')
ax.scatter(*D, color='red')
ax.text(*A, 'A')
ax.text(*B, 'B')
ax.text(*C, 'C')
ax.text(*D, 'D')
plt.show()
```

C Code

```c
#include<stdio.h>
#include<math.h>
double dot(double *a, double *b) {
double r=0;
for(int i=0;i<3;i++)
    r += a[i]*b[i];
return r;
}
double norm(double *a) {
return sqrt(a[0]*a[0] + a[1]*a[1] + a[2]*a[2]);}
```

C Code

```c
double angle_deg(double *a, double *b) {
double d = dot(a, b);
double n1 = norm(a);
double n2 = norm(b);
double ratio = d/(n1*n2);
if (ratio > 1) ratio=1;
if (ratio < -1) ratio=-1;
return acos(ratio)*180.0/M_PI;}
int collinear(double *a, double *b) {
    double ang = angle_deg(a, b);
    return (ang < 1e-8 || fabs(ang - 180) < 1e-8);
    }
```

Python and C Code

```
import numpy as np
import ctypes
lib = ctypes.CDLL('./mat3.so')
lib.angle_deg.argtypes = [ctypes.POINTER(ctypes.c_double),
    ctypes.POINTER(ctypes.c_double)]
lib.angle_deg.restype = ctypes.c_double
A = np.array([1., 1., 1.])
B = np.array([2., 5., 0.])
C = np.array([3., 2., -3.])
D = np.array([1., -6., -1.])
AB = B - A
CD = D - C
AB_c = np.ascontiguousarray(AB, dtype=np.double).ctypes.
    data_as(ctypes.POINTER(ctypes.c_double))
```

Python and C Code

```
CD_c = np.ascontiguousarray(CD, dtype=np.double).ctypes.
    data_as(ctypes.POINTER(ctypes.c_double))
angle = lib.angle_deg(AB_c, CD_c)
print("Angle between AB and CD (degrees):", angle)
lib.collinear.argtypes = [ctypes.POINTER(ctypes.c_double),
    ctypes.POINTER(ctypes.c_double)]

lib.collinear.restype = ctypes.c_int
is_collinear = lib.collinear(AB_c, CD_c)
print("Are AB and CD collinear?", "Yes" if is_collinear else
    "No")
```

Plot