2.7.14

EE25BTECH11002 - Achat Parth Kalpesh

Question:
If θ is the angle between the two vectors $\mathbf{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\mathbf{b} = 3\hat{i} - 2\hat{j} + \hat{k}$, find $\sin\theta$.
Solution:
Let the given vectors be represented by column matrices $\mathbf{a}$ and $\mathbf{b}$.

$$\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \tag{0.1}$$

The formula to calculate the angle $\sin\theta$ is given as,

$$\theta = \cos^{-1}\left(\frac{\left|\mathbf{a}^\top\mathbf{b}\right|}{\|\mathbf{b}\|\,\|\mathbf{a}\|}\right) \tag{0.2}$$

$$\sin\theta = \sin\left(\cos^{-1}\left(\frac{\left|\mathbf{a}^\top\mathbf{b}\right|}{\|\mathbf{a}\|\,\|\mathbf{b}\|}\right)\right) \tag{0.3}$$

$$= \sin\left(\cos^{-1}\left(\frac{\left|\begin{pmatrix}1 & -2 & 3\end{pmatrix}\begin{pmatrix}3\\-2\\1\end{pmatrix}\right|}{\left\|\begin{pmatrix}1\\-2\\3\end{pmatrix}\right\|\left\|\begin{pmatrix}3\\-2\\1\end{pmatrix}\right\|}\right)\right) \tag{0.4}$$

$$= \sin\left(\cos^{-1}\left(\frac{|(3)(1)+(-2)(-2)+(3)(1)|}{\sqrt{1^2+(-2)^2+3^2}\,\sqrt{3^2+(-2)^2+1^2}}\right)\right) \tag{0.5}$$

$$= \sin\left(\cos^{-1}\left(\frac{|3+4+3|}{\sqrt{14}\,\sqrt{14}}\right)\right) \tag{0.6}$$

$$= \sin\left(\cos^{-1}\left(\frac{10}{14}\right)\right) \tag{0.7}$$

$$= \frac{2\sqrt{6}}{7} \tag{0.8}$$

Therefore, the value of $\sin\theta$ is $\frac{2\sqrt{6}}{7}$.

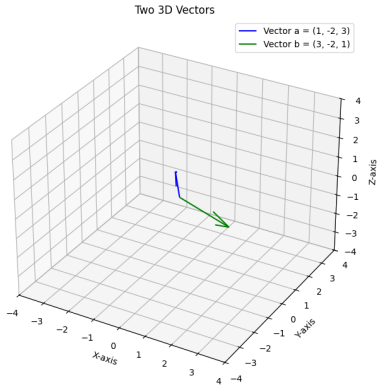

Fig. 0.1: Graph