1.2.17

AI25BTECH11011-VARUN

Question:
Three vertices of a parallelogram ABCD are A(3,-1,2),B(1,-2,4),C(-1,1,2). Find the coordinates of the fourth vertex.

Solution:
Let the vertices of parallelogram ABCD be $\mathbf{A}\begin{pmatrix}3\\-1\\2\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}1\\-2\\4\end{pmatrix}$, $\mathbf{C}\begin{pmatrix}-1\\1\\2\end{pmatrix}$. In any parallelogram, the diagonals bisect each other, so the midpoints of **AC** and **BD** are equal.

The midpoint of A and B is

$$\mathbf{M_{AB}} = \frac{\mathbf{A}+\mathbf{B}}{2} \tag{0.1}$$

Midpoint of **AC**:

$$\mathbf{M_{AC}} = \frac{\mathbf{A}+\mathbf{C}}{2} \tag{0.2}$$

Midpoint of **BD**:

$$\mathbf{M_{BD}} = \frac{\mathbf{B}+\mathbf{D}}{2} \tag{0.3}$$

As $\mathbf{M_{AC}} = \mathbf{M_{BD}}$:

$$\mathbf{A}+\mathbf{C} = \mathbf{B}+\mathbf{D} \tag{0.4}$$

$$\mathbf{D} = \mathbf{A}+\mathbf{C}-\mathbf{B} \tag{0.5}$$

$$\mathbf{D} = \begin{pmatrix}3\\-1\\2\end{pmatrix} + \begin{pmatrix}-1\\1\\2\end{pmatrix} - \begin{pmatrix}1\\-2\\4\end{pmatrix} \tag{0.6}$$

$$\mathbf{D} = \begin{pmatrix}3+(-1)-1\\-1+1-(-2)\\2+2-4\end{pmatrix} \tag{0.7}$$

$$\mathbf{D} = \begin{pmatrix}1\\2\\0\end{pmatrix} \tag{0.8}$$

The fourth vertex is $\mathbf{D}\begin{pmatrix}1\\2\\0\end{pmatrix}$.

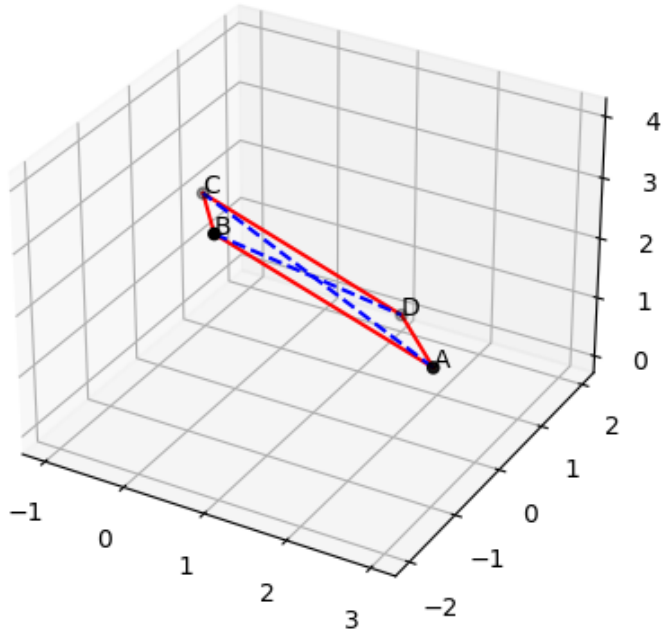

Fig. 0.1