2.8.8

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Question

If $\mathbf{a}$ is a unit vector and $(\mathbf{x}-\mathbf{a}).(\mathbf{x}+\mathbf{a})=8$, then find $|\mathbf{x}|$

Equation

Given equation:

$$(\mathbf{x}-\mathbf{a}).(\mathbf{x}+\mathbf{a}) = 8 \tag{1}$$

The given equation can be written as:

$$\left(\mathbf{x}-\mathbf{a}\right)^{T}\left(\mathbf{x}+\mathbf{a}\right) = 8 \tag{2}$$

Theoretical Solution

$$||\mathbf{x}||^2 - ||\mathbf{a}||^2 = 8 \tag{3}$$

Given that $\mathbf{a}$ is a unit vector . So,

$$||\mathbf{a}|| = 1 \tag{4}$$

Substituting the value of $||\mathbf{a}||$ in Eq.3.

$$||\mathbf{x}||^2 - 1 = 8 \tag{5}$$

Theoretical Solution

$$||\mathbf{x}||^2 = 9 \tag{6}$$

$$||\mathbf{x}|| = 3 \tag{7}$$

Hence, the final value of $|\mathbf{x}|$ is:

$$|\mathbf{x}| = 3 \tag{8}$$

Verification

For verification let us take:

$$\mathbf{x} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad \textit{and} \quad \mathbf{a} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{9}$$

Let's check whether Eq.2 is satisfied:

$$\left(\mathbf{x}-\mathbf{a}\right)^{T}\left(\mathbf{x}+\mathbf{a}\right) = \left(\begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right)^{T}\left(\begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) \tag{10}$$

$$\left(\mathbf{x}-\mathbf{a}\right)^{T}\left(\mathbf{x}+\mathbf{a}\right) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}^{T}\begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{11}$$

$$\left(\mathbf{x}-\mathbf{a}\right)^{T}\left(\mathbf{x}+\mathbf{a}\right) = 8 \tag{12}$$

C Code - Midpoint formula

```c
#include<stdio.h>

void get_x_coords(float ax, float ay, float *x_out, float *y_out)
     {
    float mag_x = 3.0f;
    *x_out = mag_x * ax;
    *y_out = mag_x * ay;
}
```

Python Code

```
import ctypes
import matplotlib.pyplot as plt
import numpy as np

# Load the shared library
lib = ctypes.CDLL('./magnitude.so')

# Specify argument and return types for the C function
lib.get_x_coords.argtypes = [
    ctypes.c_float, # ax
    ctypes.c_float, # ay
    ctypes.POINTER(ctypes.c_float), # x_out
    ctypes.POINTER(ctypes.c_float) # y_out
]
lib.get_x_coords.restype = None
```

Python Code

```
# Define the unit vector a direction
ax, ay = 1.0, 0.0 # (Choose your direction; this is along x-axis)

x_out = ctypes.c_float()
y_out = ctypes.c_float()

# Call the C function
lib.get_x_coords(ax, ay, ctypes.byref(x_out), ctypes.byref(y_out)
    )

# Prepare points for plotting
origin = np.array([0, 0])
a = np.array([ax, ay]) # Unit vector
x = np.array([x_out.value, y_out.value]) # Solution from C

plt.figure(figsize=(6, 6))
```

Python Code

```
# Draw vector a
plt.quiver(origin[0], origin[1], a[0], a[1], angles='xy',
    scale_units='xy', scale=1, color='green', label='a (unit)')
plt.text(a[0], a[1], f'a({a[0]:.1f}, {a[1]:.1f})', fontsize=10,
    ha='left', va='bottom')

# Draw vector x
plt.quiver(origin[0], origin[1], x[0], x[1], angles='xy',
    scale_units='xy', scale=1, color='blue', label='x = 3a')
plt.text(x[0], x[1], f'x({x[0]:.1f}, {x[1]:.1f})', fontsize=10,
    ha='left', va='bottom')

# Draw origin
plt.scatter(origin[0], origin[1], color='black', s=40)
plt.text(origin[0], origin[1], 'O(0,0)', fontsize=10, ha='left',
    va='top')
```

Python Code

```
plt.xlabel(X-axis)
plt.ylabel(Y-axis)
plt.title(Figure)
plt.legend()
plt.grid(True)
plt.xlim(-1, 4)
plt.ylim(-1, 2)
plt.savefig(/media/indhiresh-s/New Volume/Matrix/ee1030-2025/
    ee25btech11027/MATGEO/2.8.8/figs/figure1.png)
plt.show()
```

Plot

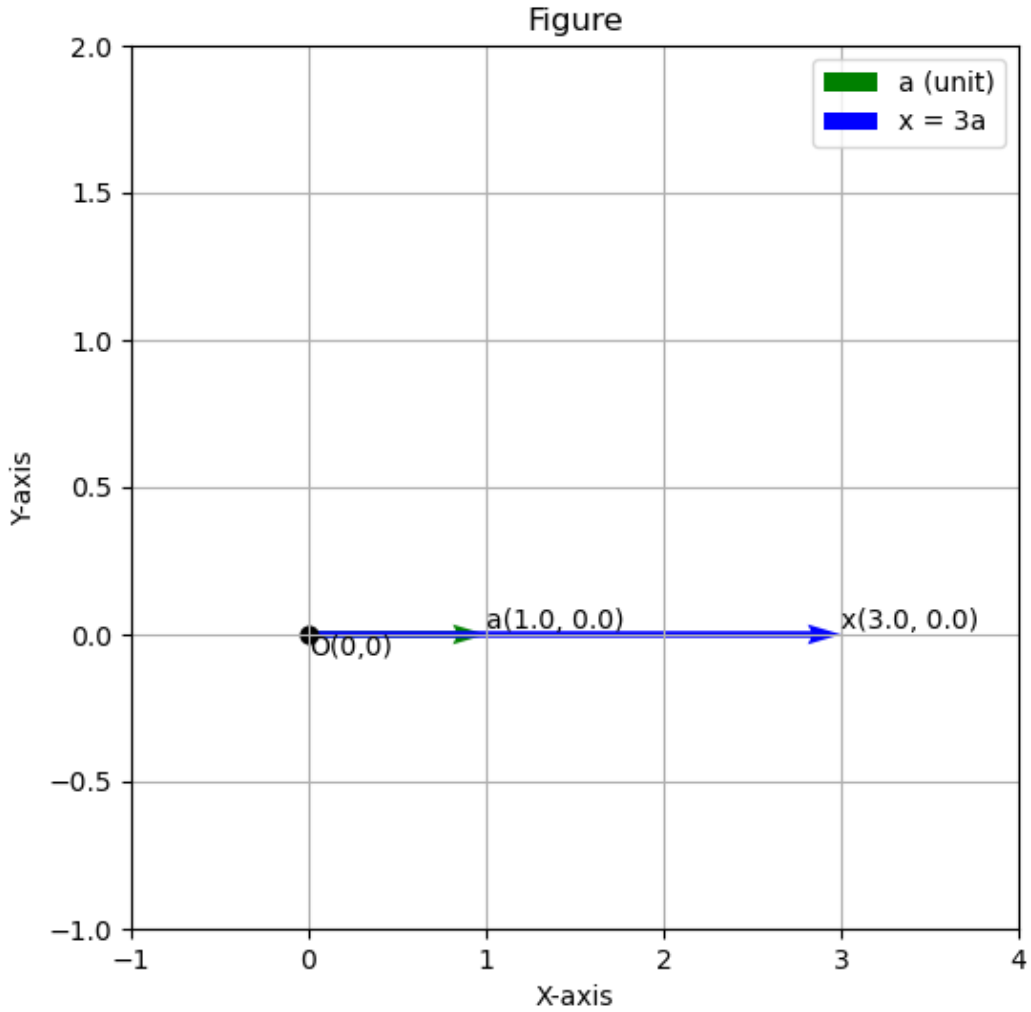
Figure
a (unit)
x = 3a
a(1.0, 0.0)
x(3.0, 0.0)
O(0,0)
Y-axis
X-axis
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1
0
1
2
3
4