10.3.11

EE25BTECH11012-BEERAM MADHURI

Question:
Find the normal at the point $(1, 1)$ on the curve

$$2y + x^2 = 3 \tag{0.1}$$

Solution:
Let $F = 2y + x^2 - 3 = 0$
gradient Vector is:

$$\begin{bmatrix} \frac{\partial F}{\partial x} \\ \frac{\partial F}{\partial y} \end{bmatrix} = \begin{bmatrix} 2x \\ 2 \end{bmatrix} \tag{0.2}$$

Normal Vector at $(1, 1)$ is:

$$n = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \tag{0.3}$$

let m be tangent vector,

$$\text{if } n = \begin{bmatrix} a \\ b \end{bmatrix} \tag{0.4}$$

$$\text{then } m = \begin{bmatrix} -b \\ a \end{bmatrix} \tag{0.5}$$

$$\therefore m = \begin{bmatrix} -2 \\ 2 \end{bmatrix} \tag{0.6}$$

let

$$p = \begin{bmatrix} x \\ y \end{bmatrix} \& p_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{0.7}$$

$$m^\top (P - P_0) = 0 \tag{0.8}$$

Substituting the values:-

$$\begin{bmatrix} -2 & 2 \end{bmatrix} \begin{bmatrix} x - 1 \\ y - 1 \end{bmatrix} = 0 \tag{0.9}$$

$$-2(x - 1) + 2(y - 1) = 0 \tag{0.10}$$

$$y = x \tag{0.11}$$

Hence equation of normal to $2y + x^2 - 3 = 0$ at $(1, 1)$ is $y = x$.
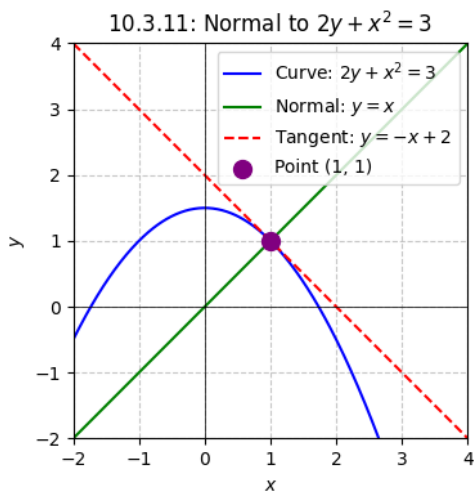
Graphs of Normal to the Curve

Fig. 0.1: 10.3.11