# 4.3.34

EE25BTECH11044 - Sai Hasini Pappula

**Question:** If the line

$$\frac{x}{a} + \frac{y}{b} = 1$$

passes through the points $(2, -3)$ and $(4, -5)$, then find $(a, b)$.

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**Solution**

The equation of a line can be expressed as

$$\mathbf{n}^T \mathbf{x} = c, \tag{0.1}$$

where $\mathbf{n}$ is the normal vector to the line.

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**Step 1: Direction vector of the line**

The line passes through

$$(x)_1 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}, \quad (x)_2 = \begin{pmatrix} 4 \\ -5 \end{pmatrix}. \tag{0.2}$$

Hence, its direction vector is

$$(m) = (x)_2 - (x)_1 = \begin{pmatrix} 2 \\ -2 \end{pmatrix}. \tag{0.3}$$

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**Step 2: Find the normal vector $(n)$**

The normal vector $(n) = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$ must satisfy

$$\mathbf{n}^T \mathbf{m} = 0. \tag{0.4}$$

That is,

$$\begin{pmatrix} n_1 & n_2 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \end{pmatrix} = 0, \tag{0.5}$$

$$2n_1 - 2n_2 = 0 \quad \Rightarrow \quad n_1 = n_2. \tag{0.6}$$

So, a valid choice is

$$(n) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{0.7}$$

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**Step 3: Find $c$**

Using the equation

$$\mathbf{n}^T \mathbf{x} = c, \tag{0.8}$$

substitute $(x)_1 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$:

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \end{pmatrix} = -1. \tag{0.9}$$

Thus,

$$c = -1. \tag{0.10}$$

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**Final Answer (matrix form):**

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -1 \quad \Rightarrow \quad x + y = -1. \tag{0.11}$$



Line $\frac{x}{a} + \frac{y}{b} = 1$ with (a,b)=(-1,-1)