5.5.31

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Problem Statement

Solve the following system using matrix row operations. Let $\mathbf{M} = \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{pmatrix}$, and find its value.

$$2 \cdot \frac{1}{x} + 3 \cdot \frac{1}{y} + 10 \cdot \frac{1}{z} = 4$$
$$4 \cdot \frac{1}{x} + 6 \cdot \frac{1}{y} + 5 \cdot \frac{1}{z} = 1$$
$$6 \cdot \frac{1}{x} + 9 \cdot \frac{1}{y} + 20 \cdot \frac{1}{z} = 2$$

Matrix Form

Coefficient matrix:

$$\mathbf{A} = \begin{bmatrix} \begin{pmatrix} 2 & 3 & 10 \end{pmatrix} \\ \begin{pmatrix} 4 & 6 & 5 \end{pmatrix} \\ \begin{pmatrix} 6 & 9 & 20 \end{pmatrix} \end{bmatrix}, \quad \mathbf{B} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$$

Augmented matrix:

$$\begin{bmatrix} \begin{pmatrix} 2 & 3 & 10 & 4 \end{pmatrix} \\ \begin{pmatrix} 4 & 6 & 5 & 1 \end{pmatrix} \\ \begin{pmatrix} 6 & 9 & 20 & 2 \end{pmatrix} \end{bmatrix}$$

Row Operations

Step 1: $R_1 \leftarrow R_1 \div 2$

$$\begin{bmatrix} \begin{pmatrix} 1 & \frac{3}{2} & 5 & 2 \end{pmatrix} \\ \begin{pmatrix} 4 & 6 & 5 & 1 \end{pmatrix} \\ \begin{pmatrix} 6 & 9 & 20 & 2 \end{pmatrix} \end{bmatrix}$$

Step 2: $R_2 \leftarrow R_2 - 4 \cdot R_1$

$$\begin{bmatrix} \begin{pmatrix} 1 & \frac{3}{2} & 5 & 2 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & -15 & -7 \end{pmatrix} \\ \begin{pmatrix} 6 & 9 & 20 & 2 \end{pmatrix} \end{bmatrix}$$

Step 3: $R_3 \leftarrow R_3 - 6 \cdot R_1$

$$\begin{bmatrix} \begin{pmatrix} 1 & \frac{3}{2} & 5 & 2 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & -15 & -7 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & -10 & -10 \end{pmatrix} \end{bmatrix}$$

Step 4: $R_3 \leftarrow R_3 - R_2$

$$\begin{bmatrix} \begin{pmatrix} 1 & \frac{3}{2} & 5 & 2 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & -15 & -7 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 5 & -3 \end{pmatrix} \end{bmatrix}$$

Step 5: $R_3 \leftarrow R_3 \div 5$

$$\begin{bmatrix} \begin{pmatrix} 1 & \frac{3}{2} & 5 & 2 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & -15 & -7 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 1 & -\frac{3}{5} \end{pmatrix} \end{bmatrix}$$

Step 6: $R_2 \leftarrow R_2 + 15 \cdot R_3$

$$\begin{bmatrix} \begin{pmatrix} 1 & \frac{3}{2} & 5 & 2 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 & \frac{4}{5} \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 1 & -\frac{3}{5} \end{pmatrix} \end{bmatrix}$$

Step 7: $R_1 \leftarrow R_1 - 5 \cdot R_3$

$$\begin{bmatrix} \begin{pmatrix} 1 & \frac{3}{2} & 0 & 5 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 & \frac{4}{5} \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 1 & -\frac{3}{5} \end{pmatrix} \end{bmatrix}$$

Final Matrix

After performing row operations, we arrive at:

$$\begin{bmatrix} \begin{pmatrix} 1 & \frac{3}{2} & 0 & 5 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 & \frac{4}{5} \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 1 & -\frac{3}{5} \end{pmatrix} \end{bmatrix}$$

This corresponds to:

$$\frac{1}{x} = 5$$
$$\frac{1}{z} = -\frac{3}{5}$$
$$0 = \frac{4}{5} \quad \text{(contradiction)}$$

Conclusion: The system is inconsistent and has no solution. The vector $\mathbf{M} = \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{pmatrix}$ is undefined.

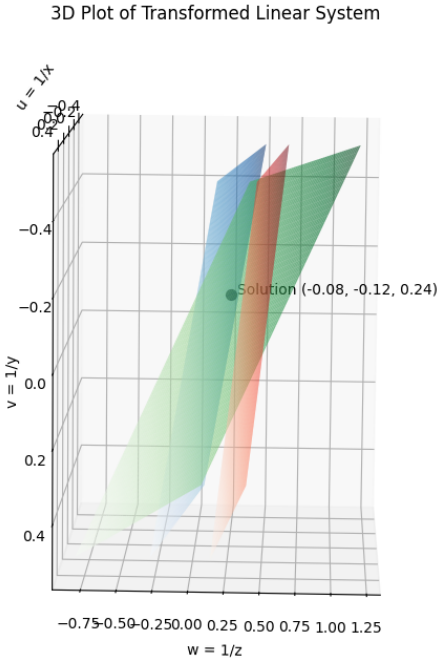

Fig. 1. Approximate solution