

2.5.34

AI25BTECH11001 - ABHISEK MOHAPATRA

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Question: Show that the lines

$$\frac{x - a + d}{\alpha - \delta} = \frac{y - a}{\alpha} = \frac{z - a - d}{\alpha + \delta} \quad (0.1)$$

$$\frac{x - b + c}{\beta - \delta} = \frac{y - b}{\beta} = \frac{z - b - c}{\beta + \delta} \quad (0.2)$$

are coplanar.

Solution: Given:

$$\mathbf{L}_1 = \mathbf{A} + \lambda \mathbf{m}_1 \quad (0.3)$$

$$\mathbf{L}_1 = \begin{pmatrix} a - d \\ a \\ a + d \end{pmatrix} + \lambda \begin{pmatrix} \alpha - \delta \\ \alpha \\ \alpha + \delta \end{pmatrix} \quad (0.4)$$

And,

$$\mathbf{L}_2 = \mathbf{B} + \lambda \mathbf{m}_2 \quad (0.5)$$

$$\mathbf{L}_2 = \begin{pmatrix} b-c \\ b \\ b+c \end{pmatrix} + \lambda \mathbf{m}_2 \begin{pmatrix} \beta-\delta \\ \beta \\ \beta+\delta \end{pmatrix} \quad (0.6)$$

If the lines lie in a plane, then they satisfy,

$$\text{nullity}(\mathbf{m}_1 \quad \mathbf{m}_2 \quad \mathbf{B} - \mathbf{A}) \geq 1 \quad (0.7)$$

$$\text{nullity} \begin{pmatrix} \alpha-\delta & \beta-\delta & a-b+c-d \\ \alpha & \beta & a-b \\ \alpha+\delta & \beta+\delta & a-b-c+d \end{pmatrix} \geq 1 \quad (0.8)$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} \alpha-\delta & \beta-\delta & a-b+c-d \\ \alpha+\delta & \beta+\delta & a-b-c+d \\ \alpha & \beta & a-b \end{pmatrix} \quad (0.9)$$

$$\xrightarrow{R_3 \rightarrow R_3 - \frac{R_1+R_2}{2}} \begin{pmatrix} \alpha-\delta & \beta-\delta & a-b+c-d \\ \alpha+\delta & \beta+\delta & a-b-c+d \\ 0 & 0 & 0 \end{pmatrix} \quad (0.10)$$

$$\xrightarrow{R_2 \rightarrow R_2 - R_1} \begin{pmatrix} \alpha - \delta & \beta - \delta & a - b + c - d \\ 2\delta & 2\delta & -2c + 2d \\ 0 & 0 & 0 \end{pmatrix} \quad (0.11)$$

$$\xrightarrow{C_1 \rightarrow C_1 - C_2} \begin{pmatrix} \alpha - \beta & \beta - \delta & a - b + c - d \\ 0 & 2\delta & -2c + 2d \\ 0 & 0 & 0 \end{pmatrix} \quad (0.12)$$

The matrix is in echelon form and the rank of the matrix is two. And, thus the lines are co-planer.

Graph(using some random values for the variables):

