

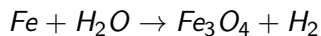
5.10.3

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September 28,2025

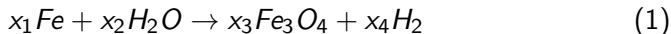
Question

Balance the following chemical equation.



Theoretical Solution

Let the balanced version of the equation be



which results in the following equations based on the conservation of each element:

$$\text{For Fe: } x_1 - 3x_3 = 0 \quad (2)$$

$$\text{For H: } 2x_2 - 2x_4 = 0 \implies x_2 - x_4 = 0 \quad (3)$$

$$\text{For O: } x_2 - 4x_3 = 0 \quad (4)$$

This can be expressed as a homogeneous system of linear equations:

$$x_1 + 0x_2 - 3x_3 + 0x_4 = 0 \quad (5)$$

$$0x_1 + x_2 + 0x_3 - x_4 = 0 \quad (6)$$

$$0x_1 + x_2 - 4x_3 + 0x_4 = 0 \quad (7)$$

Theoretical Solution

This results in the matrix equation $A\mathbf{x} = \mathbf{0}$, where:

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -4 & 0 \end{pmatrix} \mathbf{x} = \mathbf{0}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \quad (8)$$

The coefficient matrix can be reduced as follows using Gaussian elimination to find the null space:

Theoretical Solution

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -4 & 0 \end{pmatrix} \xrightarrow{R_3 \rightarrow R_3 - R_2} \quad (9)$$

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -4 & 1 \end{pmatrix} \xrightarrow{R_3 \rightarrow -\frac{1}{4}R_3} \quad (10)$$

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1/4 \end{pmatrix} \xrightarrow{R_1 \rightarrow R_1 + 3R_3} \quad (11)$$

$$\begin{pmatrix} 1 & 0 & 0 & -3/4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1/4 \end{pmatrix} \quad (12)$$

Theoretical Solution

From the reduced row echelon form, we get the solutions in terms of the free variable x_4 :

$$x_1 = \frac{3}{4}x_4, \quad x_2 = x_4, \quad x_3 = \frac{1}{4}x_4 \quad (13)$$

Theoretical Solution

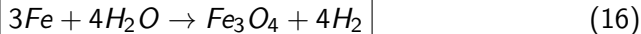
Thus,

$$\mathbf{x} = x_4 \begin{pmatrix} 3/4 \\ 1 \\ 1/4 \\ 1 \end{pmatrix} \quad (14)$$

By substituting $x_4 = 4$, the simplest integer solution is found. Hence,

$$\mathbf{x} = 4 \begin{pmatrix} 3/4 \\ 1 \\ 1/4 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \\ 4 \end{pmatrix} \quad (15)$$

This gives $x_1 = 3$, $x_2 = 4$, $x_3 = 1$, and $x_4 = 4$. Hence, the balanced equation finally becomes:



```
#include <stdio.h>

void solve_and_print_balance() {
    int x1, x2, x3, x4;
    int found = 0;

    printf(Searching for the smallest integer coefficients...\n\n
        );

    for (x1 = 1; x1 <= 100 && !found; x1++) {
        for (x2 = 1; x2 <= 100 && !found; x2++) {
            for (x3 = 1; x3 <= 100 && !found; x3++) {
                for (x4 = 1; x4 <= 100 && !found; x4++) {
                    if ((x1 == 3 * x3) && (x2 == x4) && (x2 == 4 *
                        x3)) {
                        printf(Solution found!\n);
                        printf(Coefficients are: x1=%d, x2=%d,
```



```
x3=%d, x4=%d\n\n, x1, x2, x3, x4);
    printf(The balanced chemical equation is:\n
        );
    printf(%dFe + %dH2O -> %dFe3O4 + %dH2\n, x1
        , x2, x3, x4);
    found = 1;
}
}
}
}
}
}
}

if (!found) {
    printf(No solution was found within the search range.\n);
}
}
```