4.11.6

EE25btech11028 - J.Navya sri

Question:

Find the equation of the plane passing through the intersection of the planes

$$\left(r\right) \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$$

and

$$\left(r\right) \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$$

and parallel to the X-axis. Hence, find the distance of the plane from the X-axis.

Solution:

Step1:Plane through Intersection and Distance from X-axis

Let the equations of the given planes be:

$$\left(r\right) \cdot (\hat{i} + \hat{j} + \hat{k}) = 1 \tag{1}$$

$$\left(r\right) \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 4 \tag{2}$$

Any plane passing through their intersection can be written as:

$$\left(r\right) \cdot (\hat{i} + \hat{j} + \hat{k}) - 1 + \lambda\left(\left(r\right) \cdot (2\hat{i} + 3\hat{j} - \hat{k}) - 4\right) = 0 \tag{3}$$

Expanding:

$$\left((\hat{i} + \hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})\right) \cdot \left(r\right) = 1 + 4\lambda \tag{4}$$

The normal vector of the plane is:

$$\left(N\right) = (1 + 2\lambda)\hat{i} + (1 + 3\lambda)\hat{j} + (1 - \lambda)\hat{k} \tag{5}$$

Step2:Parallel to X-Axis

Since the plane is parallel to the X-axis, its normal must be perpendicular to the X-axis:

$$1 + 2\lambda = 0 \implies \lambda = -\frac{1}{2} \tag{6}$$

Substitute $\lambda = -\frac{1}{2}$:

$$\left(N\right) = 0 \cdot \hat{i} + \left(1 + 3\left(-\frac{1}{2}\right)\right)\hat{j} + \left(1 - \left(-\frac{1}{2}\right)\right)\hat{k} = -\frac{1}{2}\hat{j} + \frac{3}{2}\hat{k} \tag{7}$$

Equation of the plane:

$$-\frac{1}{2}y + \frac{3}{2}z = 1 + 4\left(-\frac{1}{2}\right) = -1 \tag{8}$$

$$-\frac{1}{2}y + \frac{3}{2}z + 1 = 0 \quad \Rightarrow \quad -y + 3z + 2 = 0 \tag{9}$$

Step3:Distance from X-Axis

The X-axis is the line $y = 0, z = 0$.
Distance from the plane to the X-axis (taking point $(0, 0, 0)$) is:

$$D = \frac{|-0 + 3 \cdot 0 + 2|}{\sqrt{(-1)^2 + 3^2}} = \frac{2}{\sqrt{10}} \tag{10}$$

Final Answers:

- Required plane: $-y + 3z + 2 = 0$
- Distance from X-axis: $\dfrac{2}{\sqrt{10}}$

Graph presentation:

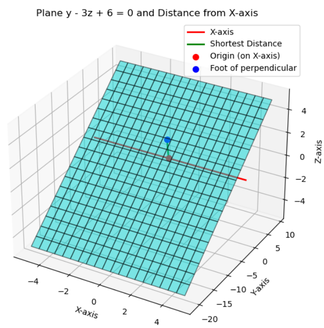

Fig. 1