Question 4.2.3

AI25BTECH11040 - Vivaan Parashar

September 28, 2025

Question:

Find the equation of the plane determined by the points **A**(3, -1, 2), **B**(5, 2, 4) and **C**(-1, -1, 6). Also find the distance of the point **P**(6, 5, 9) from the plane.

Solution:

A plane in 3D is represented by the equation $\mathbf{n}^{\mathrm{T}}\mathbf{x} = c$, where the vector $\mathbf{n}$ represents the normal to the plane. This vector $\mathbf{n}$ can be determined by using the cross-product of two vectors lying on the plane that aren't collinear, eg $\mathbf{A} - \mathbf{B}$ and $\mathbf{A} - \mathbf{C}$.

$$\therefore \mathbf{n} \equiv (\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C}) \tag{1}$$

$$\implies \mathbf{n} = \left[\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} - \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix} \right] \times \left[\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ -1 \\ 6 \end{pmatrix} \right] \tag{2}$$

$$\implies \mathbf{n} = \begin{pmatrix} 12 \\ -16 \\ 12 \end{pmatrix} \equiv \begin{pmatrix} 3 \\ -4 \\ 3 \end{pmatrix} \tag{3}$$

The constant c can be determined by substituting any of the three points in the plane into the plane equation.

$$\therefore c = \mathbf{n}^{\mathrm{T}}\mathbf{x}_{\mathbf{A}} = \begin{pmatrix} 3 & -4 & 3 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} = 19 \tag{4}$$

Thus, the equation of the plane is given by:

$$\begin{pmatrix} 3 & -4 & 3 \end{pmatrix} \mathbf{x} = 19 \tag{5}$$

The distance d of the point $\mathbf{P}$ from the plane is given by:

$$d = \frac{|\mathbf{n}^{\mathrm{T}} \mathbf{x}_{\mathbf{P}} - c|}{\|\mathbf{n}\|} \tag{6}$$

$$\implies d = \frac{|\begin{pmatrix} 3 & -4 & 3 \end{pmatrix} \begin{pmatrix} 6 \\ 5 \\ 9 \end{pmatrix} - 19|}{\sqrt{(3)^2 + (-4)^2 + (3)^2}} \tag{7}$$

$$\implies d = \frac{6}{\sqrt{34}} \tag{8}$$

Plot:

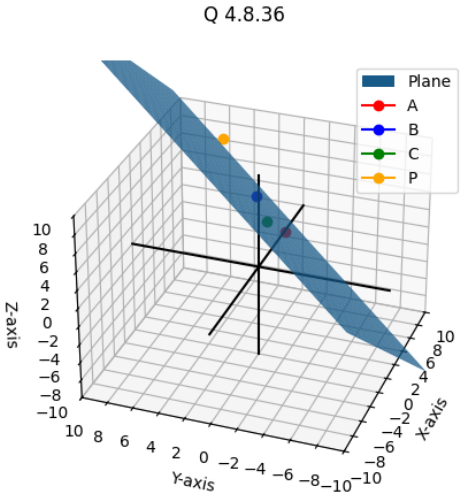

Figure: Graph of plane and points A, B, C and P