Matrices in Geometry - 2.4.32

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Problem Statement

The position vectors of the points **A**, **B**, **C** and **D** are $\left(3\hat{i} - 2\hat{j} - \hat{k}\right)$, $\left(2\hat{i} + 3\hat{j} - 4\hat{k}\right)$, $\left(-\hat{i} + \hat{j} + 2\hat{k}\right)$ and $\left(4\hat{i} + 5\hat{j} + \lambda\hat{k}\right)$ respectively. If the points **A**, **B**, **C** and **D** lie on a plane, find the value of λ.

Solution

Given,

$\mathbf{A}\begin{pmatrix}3\\-2\\-1\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}2\\3\\-4\end{pmatrix}$, $\mathbf{C}\begin{pmatrix}-1\\1\\2\end{pmatrix}$ and $\mathbf{D}\begin{pmatrix}4\\5\\\lambda\end{pmatrix}$.

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}-1\\5\\-3\end{pmatrix} \tag{1}$$

$$\mathbf{C}-\mathbf{A}=\begin{pmatrix}-4\\3\\3\end{pmatrix} \tag{2}$$

$$\mathbf{D}-\mathbf{A}=\begin{pmatrix}1\\7\\\lambda+1\end{pmatrix} \tag{3}$$

Solution

As the points **A**, **B**, **C** and **D** lie on a plane, this means that the vectors **B** − **A**, **C** − **A** and **D** − **A** are coplanar and hence, the determinant of the matrix

$$\implies \begin{pmatrix} \mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A} & \mathbf{D}-\mathbf{A} \end{pmatrix} = 0 \tag{4}$$

$$\begin{pmatrix} -1 & -4 & 1 \\ 5 & 3 & 7 \\ -3 & 3 & \lambda+1 \end{pmatrix} = 0 \tag{5}$$

Converting this matrix into row echelon form,

$$\begin{pmatrix} -1 & -4 & 1 \\ 5 & 3 & 7 \\ -3 & 3 & \lambda+1 \end{pmatrix} \xrightarrow{R_2 \to R_2 + 5R_1} \begin{pmatrix} -1 & -4 & 1 \\ 0 & -17 & 12 \\ -3 & 3 & \lambda+1 \end{pmatrix} \tag{6}$$

Solution

$$\begin{pmatrix}-1 & -4 & 1\\ 0 & -17 & 12\\ -3 & 3 & \lambda+1\end{pmatrix} \xrightarrow{R_3 \to R_3 - 3R_1} \begin{pmatrix}-1 & -4 & 1\\ 0 & -17 & 12\\ 0 & 15 & \lambda-2\end{pmatrix} \tag{7}$$

$$\begin{pmatrix}-1 & -4 & 1\\ 0 & -17 & 12\\ 0 & 15 & \lambda-2\end{pmatrix} \xrightarrow{R_2 \to -R_2} \begin{pmatrix}-1 & -4 & 1\\ 0 & 17 & -12\\ 0 & 15 & \lambda-2\end{pmatrix} \tag{8}$$

$$\begin{pmatrix}-1 & -4 & 1\\ 0 & 17 & -12\\ 0 & 15 & \lambda-2\end{pmatrix} \xrightarrow{R_3 \to R_3 - \frac{15}{17}R_2} \begin{pmatrix}-1 & -4 & 1\\ 0 & 17 & -12\\ 0 & 0 & \lambda+\frac{146}{17}\end{pmatrix} \tag{9}$$

Now for the determinant of this matrix to be zero, the complete row R_3 must be zero, so that

$$\implies \boxed{\lambda = \frac{-146}{17}} \tag{10}$$

Final Answer

The value of λ is $\frac{-146}{17}$.