4.4.26

AI25BTECH11030 -Sarvesh Tamgade

Question: Find the equation of the median through vertex $\mathbf{A}$ of the triangle ABC, having vertices

$$\mathbf{A}(2,5), \quad \mathbf{B}(-4,9), \quad \mathbf{C}(-2,-1).$$

Solution:
Using the section formula, the midpoint $\mathbf{M}$ of the side BC is

$$\mathbf{M} = \frac{\mathbf{B}+\mathbf{C}}{2} = \frac{1}{2}\begin{bmatrix}-4\\9\end{bmatrix} + \frac{1}{2}\begin{bmatrix}-2\\-1\end{bmatrix} = \begin{bmatrix}-3\\4\end{bmatrix}.$$

The median passes through points $\mathbf{A}(2,5)$ and $\mathbf{M}(-3,4)$. Let the required line have the equation

$$\mathbf{n}^\top \mathbf{x} = 1$$

where $\mathbf{n} = \begin{bmatrix}n_1\\n_2\end{bmatrix}$ is the direction vector.

Since both the points A and M lie on the median, they satisfy the line equation. That is,

$$\mathbf{n}^\top \mathbf{A} = 1, \quad \mathbf{n}^\top \mathbf{M} = 1$$

or, writing explicitly for the points $A(2,5)$, $M(-3,4)$:

$$\begin{pmatrix}2 & 5\\-3 & 4\end{pmatrix}\begin{pmatrix}n_1\\n_2\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}$$

We want to find the vector $\mathbf{n} = \begin{bmatrix}n_1\\n_2\end{bmatrix}$ satisfying the system:

$$\begin{pmatrix}2 & 5\\-3 & 4\end{pmatrix}\mathbf{n} = \mathbf{c}$$

Set up the augmented matrix with right-hand side 1:

$$\left(\begin{array}{cc|c}2 & 5 & 1\\-3 & 4 & 1\end{array}\right)$$

Perform row operations:

$$R_2 \to R_2 + \frac{3}{2}R_1 : \quad \left(\begin{array}{cc|c}2 & 5 & 1\\0 & \frac{23}{2} & \frac{5}{2}\end{array}\right)$$

$$R_1 \to R_1 - \frac{10}{23}R_2 : \quad \left(\begin{array}{cc|c}2 & 0 & 1-\frac{50}{46}\\0 & \frac{23}{2} & \frac{5}{2}\end{array}\right)$$

So the augmented matrix is:

$$\left(\begin{array}{cc|c} 2 & 0 & -\frac{2}{23} \\ 0 & \frac{23}{2} & \frac{5}{2} \end{array}\right)$$

Solve the system:

$$2n_1 = -\frac{2}{23} \quad \Rightarrow \quad n_1 = -\frac{1}{23}$$
$$\frac{23}{2}n_2 = \frac{5}{2} \quad \Rightarrow \quad n_2 = \frac{5}{23}$$

$$\mathbf{n} = \frac{1}{23}\begin{bmatrix} -1 \\ 5 \end{bmatrix}$$

$$\mathbf{n}^\top \mathbf{x} = 1$$

Substitute $\mathbf{n}$:

$$\left(\frac{1}{23}\begin{bmatrix} -1 \\ 5 \end{bmatrix}\right)^\top \mathbf{x} = 1$$

$$\begin{bmatrix} -1 & 5 \end{bmatrix}\mathbf{x} = 23$$

or equivalently,

$$5y - x = 23.$$

Therefore, equation of required line is :

$$\boxed{5y - x = 23.}$$

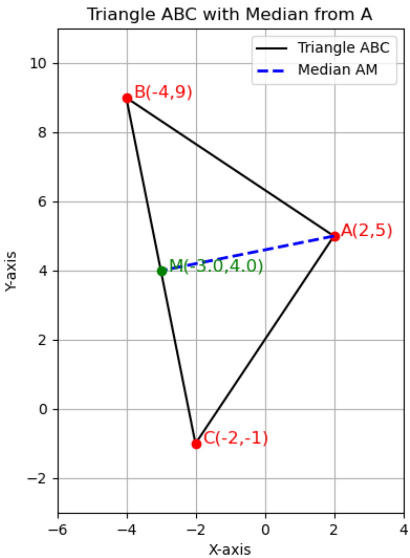

Fig. 0.1: Vector Representation