5.2.24

EE25BTECH11021 - Dhanush sagar

Question:
Solve the following system of linear equations

px + qy = p - q
qx - py = p + q

Solution:
Given

$$px + qy = p - q \tag{1}$$
$$qx - py = p + q \tag{2}$$

The matrix equation for a line is defined as

$$\mathbf{n}^\top \mathbf{x} = c \tag{3}$$

where $\mathbf{n}$ is the coefficient vector and $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$

Writing the two lines in matrix form:

$$\begin{pmatrix} p & q \end{pmatrix} \mathbf{x} = p - q \tag{4}$$
$$\begin{pmatrix} q & -p \end{pmatrix} \mathbf{x} = p + q \tag{5}$$

Combine into a single system:

$$\begin{pmatrix} p & q \\ q & -p \end{pmatrix} \mathbf{x} = \begin{pmatrix} p - q \\ p + q \end{pmatrix} \tag{6}$$

Observe that the right-hand side vector can itself be written as the coefficient matrix multiplied by a simple vector:

$$\begin{pmatrix} p & q \\ q & -p \end{pmatrix} \mathbf{x} = \begin{pmatrix} p - q \\ p + q \end{pmatrix} \tag{7}$$
$$= \begin{pmatrix} p & q \\ q & -p \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}. \tag{6}$$

Since the same coefficient matrix appears on both sides, and it is invertible whenever $p^2 + q^2 \neq 0$, we may cancel it to obtain

$$\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}. \tag{8}$$

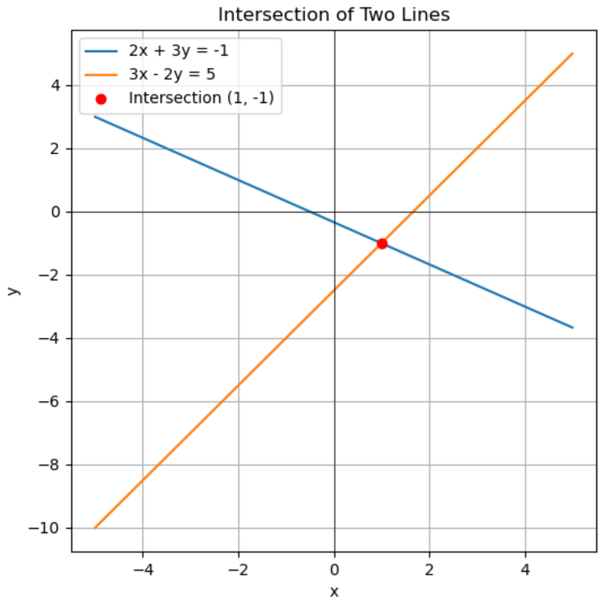
Intersection of Two Lines
2x + 3y = -1
3x - 2y = 5
Intersection (1, -1)
4
2
0
−2
−4
−6
−8
−10
y
−4
−2
0
2
4
x

Fig. 0.1