2.7.18

ADUDOTLA SRIVIDYA - EE25BTECH11006

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Question

Vertices of a $\triangle ABC$ are $\mathbf{A}(4,6)$, $\mathbf{B}(1,5)$ and $\mathbf{C}(7,2)$. A line segment DE is drawn intersecting AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of $\triangle ADE$ and compare it with the area of $\triangle ABC$.

Formula

- Section formula for a point P dividing A and B in ratio $m : n$:
  $\mathbf{P} = \frac{m\mathbf{B}+n\mathbf{A}}{m+n}$
- Area of triangle given 3 points P, Q, R:
  $\text{Area} = \frac{1}{2} \left|(Q_x - P_x)(R_y - P_y) - (Q_y - P_y)(R_x - P_x)\right|$

Solution

Let $A(4,6), B(1,5), C(7,2)$.

$$D = \frac{1\cdot B+2\cdot A}{1+2} = \frac{1}{3}\begin{bmatrix}1+8\\5+12\end{bmatrix} = \begin{pmatrix}3\\\frac{17}{3}\end{pmatrix}$$

$$E = \frac{1\cdot C+2\cdot A}{1+2} = \frac{1}{3}\begin{bmatrix}7+8\\2+12\end{bmatrix} = \begin{pmatrix}5\\\frac{14}{3}\end{pmatrix}$$

Areas

$$\text{ar}(\Delta ABC) = \frac{1}{2}\left|(1-4)(2-6)-(5-6)(7-4)\right| = \frac{15}{2}$$

$$\text{ar}(\Delta ADE) = \frac{1}{2}\left|(3-4)\left(\frac{14}{3}-6\right)-\left(\frac{17}{3}-6\right)(5-4)\right| = \frac{5}{6}$$

$$\frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta ABC)} = \frac{1}{9}$$

Pure Python Plot (Part 1)

```
import numpy as np
import matplotlib.pyplot as plt

A = np.array([4, 6])
B = np.array([1, 5])
C = np.array([7, 2])

# Points D and E using section formula (1:2)
D = (2*A + B)/3
E = (2*A + C)/3
```

Pure Python Plot (Part 2)

```
def area(P, Q, R):
    return 0.5 * abs(np.linalg.det(np.array([
        [Q[0] - P[0], R[0] - P[0]],
        [Q[1] - P[1], R[1] - P[1]]
    ])))

area_ABC = area(A, B, C)
area_ADE = area(A, D, E)
ratio = area_ADE / area_ABC
print("Ratio =", ratio)
```

Pure Python Plot (Part 3)

```
plt.plot([A[0], B[0], C[0], A[0]],
         [A[1], B[1], C[1], A[1]], 'k--', label='ABC')
plt.plot([A[0], D[0], E[0], A[0]],
         [A[1], D[1], E[1], A[1]], 'r--', label='ADE')

points = np.vstack([A,B,C,D,E])
labels = ['A(4,6)','B(1,5)','C(7,2)','D(3,17/3)','E(5,14/3)']
plt.scatter(points[:,0], points[:,1], color='black')
for i, txt in enumerate(labels):
    plt.annotate(txt, (points[i,0], points[i,1]),
                 textcoords="offset points", xytext=(0,10), ha='center')

plt.xlabel('$x$'); plt.ylabel('$y$')
plt.legend(); plt.grid(True); plt.axis('equal')
plt.savefig('figs/fig2_7_18.png')
plt.show()
```

C Code: formula.c

```c
#include <stdio.h>

// Section formula for point dividing AB in ratio m:n
void section_formula(float *P, float *A, float *B, int m, int n, int k){
    for (int i = 0; i < k ; i++) {
        P[i] = (m*B[i] + n*A[i])/(m+n);
    }
}

// Area of triangle given 3 points
float triangle_area(float *A, float *B, float *C){
    float det = (B[0]-A[0])*(C[1]-A[1]) - (B[1]-A[1])*(C[0]-A[0]);
    if(det < 0) det = -det;
    return 0.5f * det;
}
```

Python + Ctypes (cpython.py, Part 1)

```
import ctypes, numpy as np, matplotlib.pyplot as plt, os

c_lib = ctypes.CDLL('./formula.so')

c_lib.section_formula.argtypes = [
    ctypes.POINTER(ctypes.c_float),
    ctypes.POINTER(ctypes.c_float),
    ctypes.POINTER(ctypes.c_float),
    ctypes.c_int, ctypes.c_int, ctypes.c_int
]
c_lib.section_formula.restype = None

c_lib.triangle_area.argtypes = [
    ctypes.POINTER(ctypes.c_float),
    ctypes.POINTER(ctypes.c_float),
    ctypes.POINTER(ctypes.c_float)
]
```

Python + Ctypes (cpython.py, Part 2)

```
A = np.array([4, 6], dtype=np.float32)
B = np.array([1, 5], dtype=np.float32)
C = np.array([7, 2], dtype=np.float32)

D, E = np.zeros(2, dtype=np.float32), np.zeros(2, dtype=np.float32)

c_lib.section_formula(D.ctypes.data_as(ctypes.POINTER(ctypes.c_float)),
    A.ctypes.data_as(ctypes.POINTER(ctypes.c_float)),
    B.ctypes.data_as(ctypes.POINTER(ctypes.c_float)), 1, 2, 2)

c_lib.section_formula(E.ctypes.data_as(ctypes.POINTER(ctypes.c_float)),
    A.ctypes.data_as(ctypes.POINTER(ctypes.c_float)),
    C.ctypes.data_as(ctypes.POINTER(ctypes.c_float)), 1, 2, 2)

area_ABC = c_lib.triangle_area(A.ctypes.data_as(ctypes.POINTER(ctypes.
    c_float)),
    B.ctypes.data_as(ctypes.POINTER(ctypes.c_float))
```

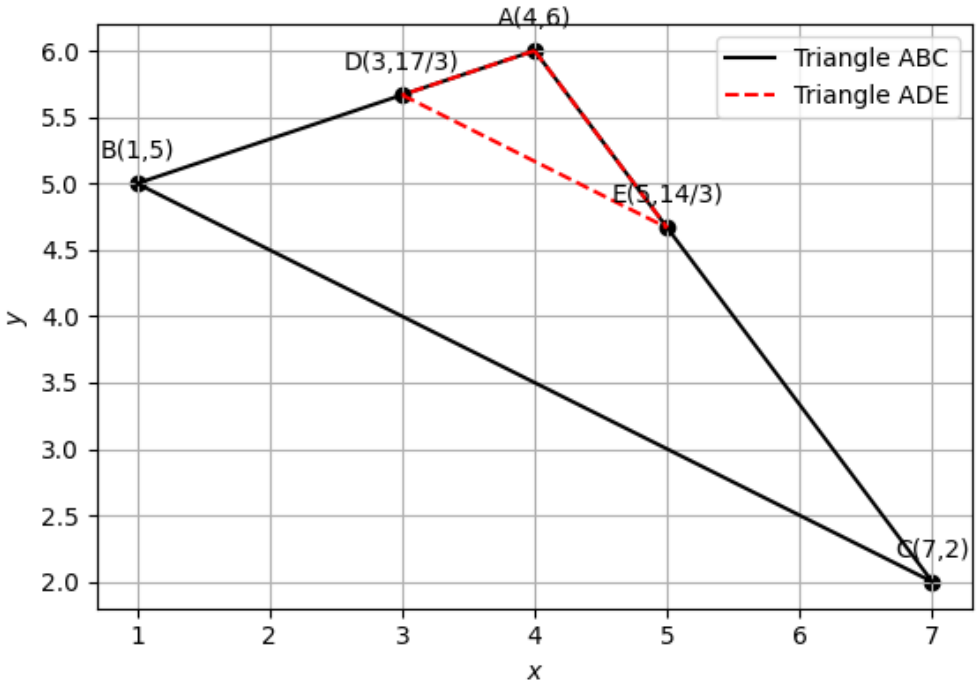
Python + Ctypes (cpython.py, Part 3)

```
ratio = area_ADE / area_ABC

print("A=", A, "B=", B, "C=", C)
print("D=", D, "E=", E)
print("Areas:ABC=", area_ABC, "ADE=", area_ADE)
print("Ratio=", ratio)

# Plot
os.makedirs("figs", exist_ok=True)
plt.plot([A[0], B[0], C[0], A[0]],[A[1], B[1], C[1], A[1]],'b-',label="ABC")
plt.plot([A[0], D[0], E[0], A[0]],[A[1], D[1], E[1], A[1]],'r--',label="ADE"
    )
plt.legend(); plt.grid(True); plt.axis('equal')
plt.savefig("figs/fig_cpython_2_7_18.png")
plt.show()
```

Plot



$\triangle ADE$ inside $\triangle ABC$