5.13.71

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Question:
If $\mathbf{M}$ is a 3×3 matrix, where

$$\left|\mathbf{M}\right| = 1 \text{ and } \mathbf{M}\mathbf{M}^\top = \mathbf{I}$$

. where $\mathbf{I}$ is an identity matrix, prove that

$$\left|\mathbf{M} - \mathbf{I}\right| = 0$$

Solution:

$$\mathbf{M}\mathbf{M}^\top = \mathbf{I}$$
$$\left|\mathbf{M}\right| = 1$$

$$\left|\mathbf{M} - \mathbf{I}\right| = \left|\mathbf{M} - \mathbf{M}\mathbf{M}^\top\right| \tag{1}$$
$$= \left|\mathbf{M}(\mathbf{I} - \mathbf{M}^\top)\right| \tag{2}$$
$$= \left|\mathbf{M}\right| \left|\mathbf{I} - \mathbf{M}^\top\right| \tag{3}$$
$$= 1 \cdot \left|(\mathbf{I} - \mathbf{M})^\top\right| \tag{4}$$
$$= \left|\mathbf{I} - \mathbf{M}\right| \tag{5}$$
$$= \left|-(\mathbf{M} - \mathbf{I})\right| \tag{6}$$
$$= (-1)^3 \left|\mathbf{M} - \mathbf{I}\right| \tag{7}$$
$$= -\left|\mathbf{M} - \mathbf{I}\right| \tag{8}$$

$$\left|\mathbf{M} - \mathbf{I}\right| = -\left|\mathbf{M} - \mathbf{I}\right| \tag{9}$$
$$2\left|\mathbf{M} - \mathbf{I}\right| = 0 \tag{10}$$
$$\left|\mathbf{M} - \mathbf{I}\right| = 0 \tag{11}$$