MatGeo Assignment 1.11.9

AI25BTECH11007

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If

$$\mathbf{a} = \hat{i} - 7\hat{j} + 7\hat{k} \quad \text{and} \quad \mathbf{b} = 3\hat{i} - 2\hat{j} + 2\hat{k},$$

find a unit vector perpendicular to both the vectors **a** and **b**.

Solution

We want $\mathbf{n}$ such that

$$\mathbf{a}^T\mathbf{n} = 0, \tag{1}$$

$$\mathbf{b}^T\mathbf{n} = 0. \tag{2}$$

This system can be written as

$$\begin{pmatrix} 1 & -7 & 7 \\ 3 & -2 & 2 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{3}$$

The solution is given by the **null space** of the coefficient matrix. Equivalently, $\mathbf{n}$ can be expressed as the cross product of $\mathbf{a}$ and $\mathbf{b}$:

$$\mathbf{n} = \mathbf{a} \times \mathbf{b}. \tag{4}$$

Using the **transpose method**, we write

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 1 \\ -7 \\ 7 \end{pmatrix} \times \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix} \tag{5}$$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \end{bmatrix} \begin{bmatrix} 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix}^T \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{pmatrix} 1 \\ -7 \\ 7 \end{pmatrix}. \tag{6}$$

Simplifying (6), we obtain

$$\mathbf{n} = \begin{pmatrix} 0 \\ 19 \\ 19 \end{pmatrix}. \tag{7}$$

Now, the unit vector is

$$\hat{n} = \frac{\mathbf{n}}{\|\mathbf{n}\|} \tag{8}$$

$$= \frac{1}{\sqrt{0^2 + 19^2 + 19^2}} \begin{pmatrix} 0 \\ 19 \\ 19 \end{pmatrix} \tag{9}$$

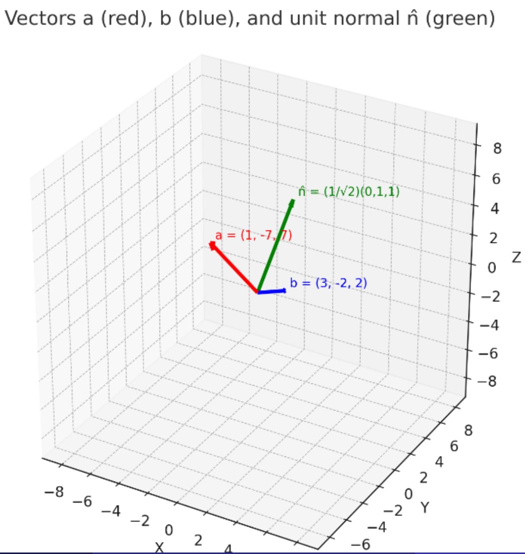
$$= \frac{1}{\sqrt{722}} \begin{pmatrix} 0 \\ 19 \\ 19 \end{pmatrix}. \tag{10}$$

Hence, the required unit vectors are

$$\hat{n} = \pm \frac{1}{\sqrt{722}} \begin{pmatrix} 0 \\ 19 \\ 19 \end{pmatrix}. \tag{11}$$

$$\hat{n} = \pm \frac{1}{\sqrt{2}} (\hat{j} + \hat{k}) \tag{12}$$

Plot


Vectors a (red), b (blue), and unit normal n̂ (green)
n̂ = (1/√2)(0,1,1)
a = (1, -7, 7)
b = (3, -2, 2)
X
Y
Z
−8
−6
−4
−2
0
2
4
6
8

Conclusion

Therefore, a unit vector perpendicular to both **a** and **b** is

$$\hat{n} = \frac{1}{\sqrt{2}}(\hat{j} + \hat{k}),$$

or its negative.