

Matrices in Geometry 1.5.25

EE25BTECH11037 - Divyansh

Question: In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points $\mathbf{P} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$? Also find the value of y .

Given: $\mathbf{P} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ and a point $\mathbf{R} = \begin{pmatrix} \frac{24}{11} \\ y \end{pmatrix}$ on PQ .

Let R divide PQ internally in the ratio $k : 1$.

Therefore, they are defined to be collinear if,

$$\text{rank}(\mathbf{R} - \mathbf{P} \quad \mathbf{Q} - \mathbf{R}) = 1$$

$$\mathbf{R} - \mathbf{P} = \begin{pmatrix} \frac{2}{11} \\ y + 2 \end{pmatrix}$$

$$\mathbf{Q} - \mathbf{R} = \begin{pmatrix} \frac{9}{11} \\ 7 - y \end{pmatrix}$$

$$\text{rank} \begin{pmatrix} \frac{2}{11} & \frac{9}{11} \\ y + 2 & 7 - y \end{pmatrix} = 1$$

$$\begin{pmatrix} \frac{2}{11} & \frac{9}{11} \\ y + 2 & 7 - y \end{pmatrix} \xrightarrow{R_2 \rightarrow 9R_2 - 11R_1(7-y)} \begin{pmatrix} \frac{2}{11} & \frac{9}{11} \\ 11y + 4 & 0 \end{pmatrix}$$

for the rank of this matrix to be 1, its 2nd row has to be 0

$$11y + 4 = 0$$

$$\therefore y = \frac{-4}{11}$$

We know that k is the ratio in which \mathbf{R} divides \mathbf{P} and \mathbf{Q} ,

$$\mathbf{R} = \frac{k\mathbf{Q} + \mathbf{P}}{1 + k}$$

$$k(\mathbf{R} - \mathbf{Q}) = \mathbf{P} - \mathbf{R}$$

$$\Rightarrow k = \frac{(\mathbf{P} - \mathbf{R})^\top (\mathbf{R} - \mathbf{Q})}{\|\mathbf{R} - \mathbf{Q}\|^2}$$

$$(\mathbf{P} - \mathbf{R})^\top = \begin{pmatrix} \frac{-2}{11} & \frac{-18}{11} \end{pmatrix}$$

$$(\mathbf{R} - \mathbf{Q}) = \begin{pmatrix} \frac{-9}{11} \\ \frac{-81}{11} \end{pmatrix}$$

$$\|\mathbf{R} - \mathbf{Q}\|^2 = \frac{81}{121} + \frac{6561}{121} = \frac{6642}{121}$$

$$\therefore k = \frac{\begin{pmatrix} \frac{-2}{11} & \frac{-18}{11} \end{pmatrix} \begin{pmatrix} \frac{-9}{11} \\ \frac{-81}{11} \end{pmatrix}}{\frac{6642}{121}}$$

$$\Rightarrow k = \frac{\frac{18}{121} + \frac{1458}{121}}{\frac{6642}{121}} \Rightarrow k = \frac{1476}{6624} = \frac{2}{9}$$

Hence, the final answer is $k = \frac{2}{9}$ and $y = \frac{-4}{11}$

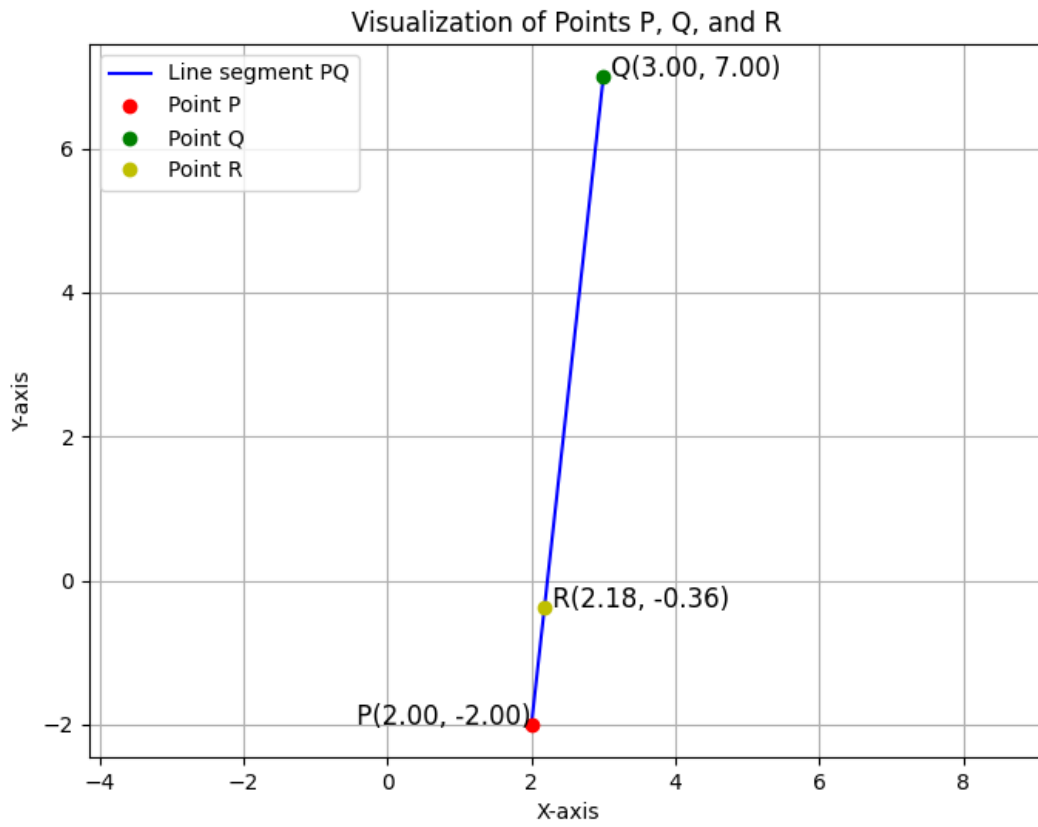


Fig. 1: Plot for 1.5.25