2.7.3

AI25BTECH110031
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Question(2.7.3) If $\mathbf{a}$ and $\mathbf{b}$ are two vectors such that $\mathbf{a} = \hat{i} - \hat{j} + \hat{k}$, $\mathbf{b} = 2\hat{i} - \hat{j} - 3\hat{k}$, then find the vector $\mathbf{c}$, given that $\mathbf{a} \times \mathbf{c} = \mathbf{b}$, $\mathbf{a} \cdot \mathbf{c} = 4$.

Solution:

$$\mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \tag{0.1}$$

$$\mathbf{a} \times \mathbf{c} = \mathbf{b} \tag{0.2}$$

$$\implies \mathbf{c} \perp \mathbf{b} \tag{0.3}$$

$$\therefore \mathbf{b}^\top \mathbf{c} = 0 \tag{0.4}$$

and $\mathbf{a}^\top \mathbf{c} = 4$ is given

$$\begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix}^\top \mathbf{c} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{0.5}$$

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & -3 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{0.6}$$

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & -1 & -3 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{0.7}$$

Let, $c_3 = \lambda$
Then

$$c_1 = -4 + 4\lambda \tag{0.8}$$

$$c_2 = -8 + 5\lambda \tag{0.9}$$

$$\mathbf{c} = \begin{pmatrix} -4 \\ -8 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 5 \\ 1 \end{pmatrix} \tag{0.10}$$

$$\tag{0.11}$$

This satisfies all the given conditions when $\lambda = 1$

Thus,

$$\mathbf{c} = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} \tag{0.12}$$

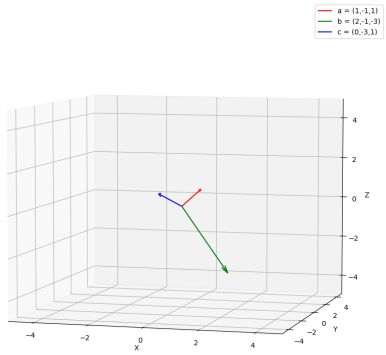

Fig. 0.1