

Matgeo Presentation - Problem 12.589

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Problem Statement

Solve the system of equations and find the condition for which the system has **no solution**:

$$x + y + z = 6$$

$$x + 4y + 6z = 20$$

$$x + 4y + \lambda z = \mu$$

Data

Name	Value
Equation 1	$x + y + z = 6 \iff \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{x} = 6$
Equation 2	$x + 4y + 6z = 20 \iff \begin{pmatrix} 1 & 4 & 6 \end{pmatrix} \mathbf{x} = 20$
Equation 3	$x + 4y + \lambda z = \mu \iff \begin{pmatrix} 1 & 4 & \lambda \end{pmatrix} \mathbf{x} = \mu$

Table : Equations

Solution

The system of equations in matrix form is :

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 4 & 6 \\ 1 & 4 & \lambda \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 20 \\ \mu \end{pmatrix} \quad (0.1)$$

Forming the augmented matrix,

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 1 & 4 & \lambda & \mu \end{array} \right) \quad (0.2)$$

Using Gaussian elimination,

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 1 & 4 & \lambda & \mu \end{array} \right) \xleftrightarrow[R_2 \rightarrow R_2 - R_1]{R_3 \rightarrow R_3 - R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & 5 & 14 \\ 0 & 3 & \lambda - 1 & \mu - 6 \end{array} \right) \quad (0.3)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & 5 & 14 \\ 0 & 3 & \lambda - 1 & \mu - 6 \end{array} \right) \xleftrightarrow{R_3 \rightarrow R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & 5 & 14 \\ 0 & 0 & \lambda - 6 & \mu - 20 \end{array} \right) \quad (0.4)$$

Solution

From back substitution we get:

$$(\lambda - 6)z = \mu - 20 \quad (0.5)$$

For the system to have **no solution**, we must have

$$\lambda = 6 \quad \mu \neq 20 \quad (0.6)$$

So, we get zero equal to a non-zero value which is not possible .

Therefore the system of equations has **no solution**.

Final Answer : The system has no solution when $\lambda = 6$ and $\mu \neq 20$.

Plot

No solution for $\lambda = 6$ and $\mu \neq 20$

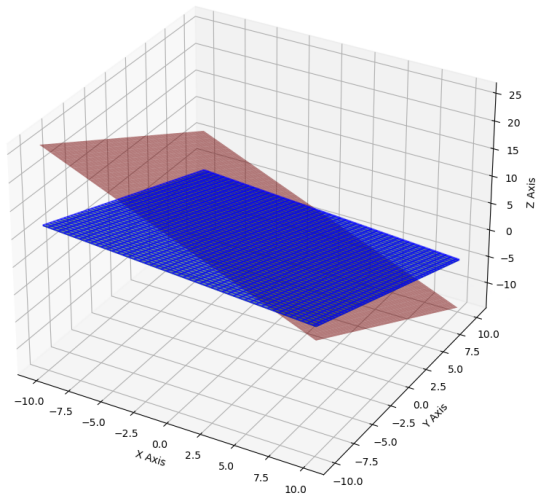


Fig : Planes