1.3.4

R Nikhil-AI25BTECH11025

September 8, 2025

Question

If $A(1,3)$, $B(4,2)$, $C(x,5)$, and $D(x,4)$ are the vertices of a parallelogram $ABCD$, then the value of x is ____________. (10, 2012)

Theoretical Solution

In a parallelogram, opposite sides are equal and parallel. Since *ABCD* is a parallelogram, vectors **AB** and **CD** must be equal.

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 4-1 \\ 2-3 \end{pmatrix} = \begin{pmatrix} 3, -1 \end{pmatrix} \tag{1}$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} x-x \\ 4-5 \end{pmatrix} = \begin{pmatrix} 0, -1 \end{pmatrix} \tag{2}$$

Clearly, $\mathbf{B} - \mathbf{A} \neq \mathbf{D} - \mathbf{C}$, so let's try using diagonals. In a parallelogram, the diagonals bisect each other.
Midpoint of diagonal *AC*:

$$\begin{pmatrix} \frac{1+x}{2} \\ \frac{3+5}{2} \end{pmatrix} = \begin{pmatrix} \frac{1+x}{2} \\ 4 \end{pmatrix} \tag{3}$$

Theoretical Solution

Midpoint of diagonal BD

$$\begin{pmatrix} \frac{4+x}{2} \\ \frac{2+4}{2} \end{pmatrix} = \begin{pmatrix} \frac{4+x}{2} \\ 3 \end{pmatrix} \tag{4}$$

Equating midpoints:

$$\frac{1+x}{2} = \frac{4+x}{2} \quad \text{and} \quad 4 = 3 \tag{5}$$

Theoretical Solution

The second equation is false, so diagonals do not bisect each other. Let's try using opposite sides again, but this time equating **AD** and **BC**:

$$\mathbf{D} - \mathbf{A} = \begin{pmatrix} x-1 \\ 4-3 \end{pmatrix} = \begin{pmatrix} x-1 \\ 1 \end{pmatrix} \tag{6}$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} x-4 \\ 5-2 \end{pmatrix} = \begin{pmatrix} x-4 \\ 3 \end{pmatrix} \tag{7}$$

Equating vectors:

$$x - 1 = x - 4 \quad \text{and} \quad 1 = 3 \tag{8}$$

Again, contradiction. So let's try using the property that opposite sides are equal in length.

Theoretical Solution

$$\text{Length of } \mathbf{D}-\mathbf{A}: \quad |AD| = \sqrt{(x-1)^2+(4-3)^2} = \sqrt{(x-1)^2+1} \tag{9}$$

$$\text{Length of } \mathbf{C}-\mathbf{B}: \quad |BC| = \sqrt{(x-4)^2+(5-2)^2} = \sqrt{(x-4)^2+9} \tag{10}$$

Equating lengths:

$$\sqrt{(x-1)^2+1} = \sqrt{(x-4)^2+9} \tag{11}$$

Squaring both sides:

$$(x-1)^2+1 = (x-4)^2+9 \tag{12}$$

$$x^2-2x+1+1 = x^2-8x+16+9 \tag{13}$$

$$x^2-2x+2 = x^2-8x+25 \tag{14}$$

Theoretical Solution

Subtract x^2 from both sides:

$$-2x + 2 = -8x + 25 \tag{15}$$

$$6x = 23 \Rightarrow x = \frac{23}{6} \tag{16}$$

Answer: $\boxed{\frac{23}{6}}$

Main C Code

```c
include <stdio.h>

// Function declarations (prototypes)
double dx_from_abc(double ax, double ay, double bx, double by,
    double cx, double cy);
double dy_from_abc(double ax, double ay, double bx, double by,
    double cx, double cy);
int write_points_file(const char *filepath,
                    double ax, double ay,
                    double bx, double by,
                    double cx, double cy);

   int main(void) {
```

Main C Code

```c
    // Given A(1,3), B(-1,2), C(2,5)
    double ax=1, ay=3, bx=-1, by=2, cx=2, cy=5;

    double dx = dx_from_abc(ax, ay, bx, by, cx, cy);
    double dy = dy_from_abc(ax, ay, bx, by, cx, cy);

    printf("Computed x for D: %.10g\n", dx);
    printf("Computed y for D (for consistency): %.10g\n", dy);

    // Write coordinates to a file
    if (write_points_file("points.dat", ax, ay, bx, by, cx, cy)
        != 0) {
        fprintf(stderr, "Failed to write points.dat\n");
        return 1;
    }
    printf("Wrote points to points.dat\n");

    return 0;
}
```

C Function

```c
   #include <stdio.h>

// Function to calculate Dx (x-coordinate of D)
double dx_from_abc(double ax, double ay, double bx, double by,
    double cx, double cy) {
    (void)ay; (void)by; (void)cy; // unused
    return ax + cx - bx;
}
```

C Function

```c

// Function to calculate Dy (y-coordinate of D)
double dy_from_abc(double ax, double ay, double bx, double by,
    double cx, double cy) {
    (void)ax; (void)bx; (void)cx; // unused
    return ay + cy - by;
}

// Function to write points into a file
int write_points_file(const char *filepath,
                    double ax, double ay,
                    double bx, double by,
                    double cx, double cy) {
    double dx = dx_from_abc(ax, ay, bx, by, cx, cy);
    double dy = dy_from_abc(ax, ay, bx, by, cx, cy);
```

C Function

```c

 FILE *fp = fopen(filepath, "w");
    if (!fp) return 1;
    fprintf(fp, "A %.10g %.10g\n", ax, ay);
    fprintf(fp, "B %.10g %.10g\n", bx, by);
    fprintf(fp, "C %.10g %.10g\n", cx, cy);
    fprintf(fp, "D %.10g %.10g\n", dx, dy);
    fclose(fp);
    return 0;
}
```

Python Code

```

   import ctypes
import pandas as pd
import matplotlib.pyplot as plt

# Load the shared library
lib = ctypes.CDLL("./libparallelogram.so")
lib.dx_from_abc.argtypes = [ctypes.c_double]*6
lib.dx_from_abc.restype = ctypes.c_double
lib.dy_from_abc.argtypes = [ctypes.c_double]*6
lib.dy_from_abc.restype = ctypes.c_double

# Given points
ax, ay = 1.0, 3.0
bx, by = -1.0, 2.0
cx, cy = 2.0, 5.0
```

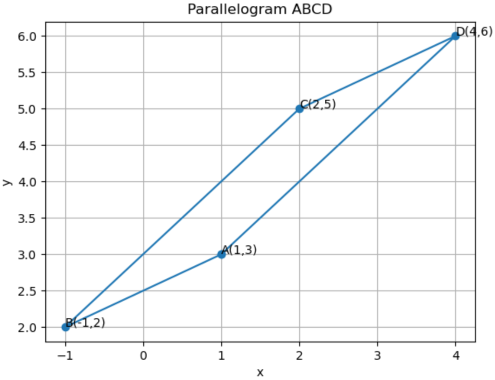
Python Code

```python
dx = lib.dx_from_abc(ax, ay, bx, by, cx, cy)
dy = lib.dy_from_abc(ax, ay, bx, by, cx, cy)

print("From Python via .so:")
print("D =", dx, dy)

# Read the points written by C main
df = pd.read_csv("points.dat", sep=r"\s+", header=None, names=["label","x","y"])

# Plot
order = ["A","B","C","D","A"]
xs = [df.loc[df["label"]==lbl,"x"].values[0] for lbl in order]
ys = [df.loc[df["label"]==lbl,"y"].values[0] for lbl in order]
```

Python Code

```python
plt.plot(xs, ys, marker="o")
for lbl in ["A","B","C","D"]:
    x = df.loc[df["label"]==lbl,"x"].values[0]
    y = df.loc[df["label"]==lbl,"y"].values[0]
    plt.text(x, y, f"{lbl}({x:.0f},{y:.0f})")

plt.title("Parallelogram ABCD")
plt.xlabel("x")
plt.ylabel("y")
plt.grid(True)
plt.savefig("/home/r-nikhil/ee1030-2025/ai25btech11025/matgeo/1.3.4/figs/plotc.png")
plt.show()
```

Plot


Parallelogram ABCD
D(4,6)
C(2,5)
A(1,3)
B(-1,2)
y
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
−1
0
1
2
3
4
x