2.10.3

AI25BTECH11014 - Gooty Suhas

Problem

Find the unit vector perpendicular to the plane determined by the points:

$$\mathbf{P} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

Solution

Let the points be:

$$\mathbf{P} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

Define direction vectors:

$$\mathbf{U} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}, \quad \mathbf{V} = \mathbf{R} - \mathbf{P} = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$$

We seek a normal vector $\mathbf{N}$ such that:

$$\mathbf{N}^T\mathbf{U} = 0, \quad \mathbf{N}^T\mathbf{V} = 0$$

Stacking the system:

$$\begin{bmatrix} \mathbf{U}^T \\ \mathbf{V}^T \end{bmatrix} \mathbf{N} = \mathbf{0} \Rightarrow \begin{bmatrix} 1 & 1 & -3 \\ -1 & 3 & -1 \end{bmatrix} \mathbf{N} = \mathbf{0}$$

Solving this homogeneous system yields:

$$\mathbf{N} = \begin{pmatrix} 8 \\ 2 \\ 4 \end{pmatrix}$$

Normalize:

$$||\mathbf{N}|| = \sqrt{8^2 + 2^2 + 4^2} = \sqrt{84} \Rightarrow \hat{n} = \frac{1}{\sqrt{84}} \begin{pmatrix} 8 \\ 2 \\ 4 \end{pmatrix}$$

Compute:

$$\hat{n}^T\mathbf{P} = \frac{1}{\sqrt{84}} \begin{pmatrix} 8 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \frac{1}{\sqrt{84}}(8 - 2 + 8) = \frac{14}{\sqrt{84}}$$

So the plane equation is:

$$\hat{n}^T \mathbf{x} = \frac{14}{\sqrt{84}}$$

Plot

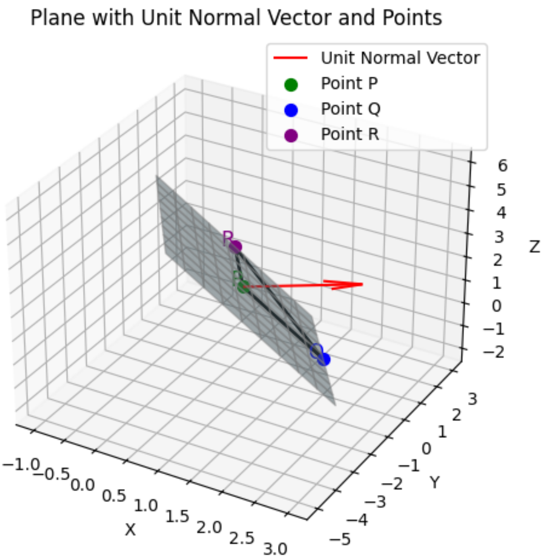

Fig. 0.1: Plane and its unit normal