

## 1.8.4

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September 4, 2025

**Question:**

Find the coordinates of a point on Y axis which is at a distance of  $5\sqrt{2}$  from the point  $P(3, -2, 5)$ .

**Solution:**

Given point:

$$\mathbf{P} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix} \quad (1)$$

We seek a point on the Y-axis:

$$\mathbf{Q} = y \mathbf{e}_2 = \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix} \quad (2)$$

Let the standard basis vectors be:

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (3)$$

The required distance:

$$\|\mathbf{P} - \mathbf{Q}\| = d = 5\sqrt{2} \quad (4)$$

So, equating squared norms:

$$\|\mathbf{P} - y\mathbf{e}_2\|^2 = d^2 \quad (5)$$

$$(\mathbf{P} - y\mathbf{e}_2)^T (\mathbf{P} - y\mathbf{e}_2) = d^2 \quad (6)$$

$$\mathbf{P}^T \mathbf{P} - 2y \mathbf{P}^T \mathbf{e}_2 + y^2 \mathbf{e}_2^T \mathbf{e}_2 = d^2 \quad (7)$$

$$\mathbf{P}^T \mathbf{P} - 2y(\mathbf{P} \cdot \mathbf{e}_2) + y^2 = d^2 \quad (8)$$

Rearrange:

$$y^2 - 2(\mathbf{P} \cdot \mathbf{e}_2)y + (\mathbf{P}^T \mathbf{P} - d^2) = 0 \quad (9)$$

With values:

$$\mathbf{P}^T \mathbf{P} = 3^2 + (-2)^2 + 5^2 = 38 \quad (10)$$

$$\mathbf{P} \cdot \mathbf{e}_2 = -2 \quad (11)$$

$$d^2 = (5\sqrt{2})^2 = 50 \quad (12)$$

So:

$$y^2 + 4y - 12 = 0 \quad (13)$$

Quadratic formula:

$$y = \frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot (-12)}}{2} \quad (14)$$

$$= \frac{-4 \pm \sqrt{16 + 48}}{2} \quad (15)$$

$$= \frac{-4 \pm 8}{2} \quad (16)$$

Thus,

$$y_1 = 2, \quad y_2 = -6 \quad (17)$$

**Answer:**

Therefore, the required points on the Y-axis are:

$$\mathbf{Q}_1 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \quad \mathbf{Q}_2 = \begin{pmatrix} 0 \\ -6 \\ 0 \end{pmatrix} \quad (18)$$

**Graph:**

### 3D Visualization of Point P and Points on Y-axis Q1, Q2

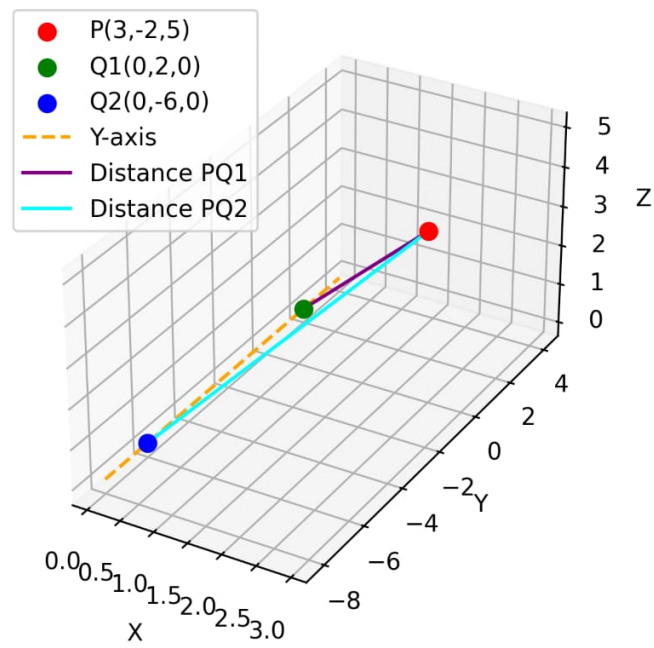


Figure 1: 3D Visualization of Point P and Points on Y-axis Q1,Q2