

Matrices in Geometry 5.8.33

EE25BTECH11035 - Kushal B N

Question: Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the co-ordinates of the vertices of the triangle formed by these lines and the y-axis.

Given:

The lines $(5 \ -1) \begin{pmatrix} x \\ y \end{pmatrix} = 5$ and $(3 \ -1) \begin{pmatrix} x \\ y \end{pmatrix} = 3$. The y-axis $(1 \ 0) \begin{pmatrix} x \\ y \end{pmatrix} = 0$.

Solution:

Solving for the intersection of the two lines

$$\begin{pmatrix} 5 & -1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \quad (1)$$

Forming the augmented matrix for solving this,

$$\left(\begin{array}{cc|c} 5 & -1 & 5 \\ 3 & -1 & 3 \end{array} \right) \quad (2)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \left(\begin{array}{cc|c} 2 & 0 & 2 \\ 3 & -1 & 3 \end{array} \right) \xleftrightarrow{R_2 \leftarrow -R_2} \left(\begin{array}{cc|c} 2 & 0 & 2 \\ -3 & 1 & -3 \end{array} \right) \quad (3)$$

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{2}} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ -3 & 1 & -3 \end{array} \right) \xleftrightarrow{R_2 \leftarrow R_2 + 3R_1} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \end{array} \right) \quad (4)$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (5)$$

Now, the intersection of the two lines with the y-axis,

First line:

$$\begin{pmatrix} 5 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad (6)$$

Forming augmented matrix and solving,

$$\left(\begin{array}{cc|c} 5 & -1 & 5 \\ 1 & 0 & 0 \end{array} \right) \xleftrightarrow{R_1 \leftarrow R_2} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 5 & -1 & 5 \end{array} \right) \quad (7)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 5R_1} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & -1 & 5 \end{array} \right) \xleftrightarrow{R_2 \leftarrow -R_2} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & -5 \end{array} \right) \quad (8)$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -5 \end{pmatrix} \quad (9)$$

Second Line:

$$\begin{pmatrix} 1 & 0 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad (10)$$

Forming the augmented Matrix,

$$\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 3 & -1 & 3 \end{array} \right) \xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & -1 & 3 \end{array} \right) \quad (11)$$

$$\xleftrightarrow{R_2 \leftarrow -R_2} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & -3 \end{array} \right) \quad (12)$$

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} \quad (13)$$

Conclusion:

\therefore The coordinates of the vertices of the triangle formed by these lines and the y-axis are $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$.

The graph of the system of equations:

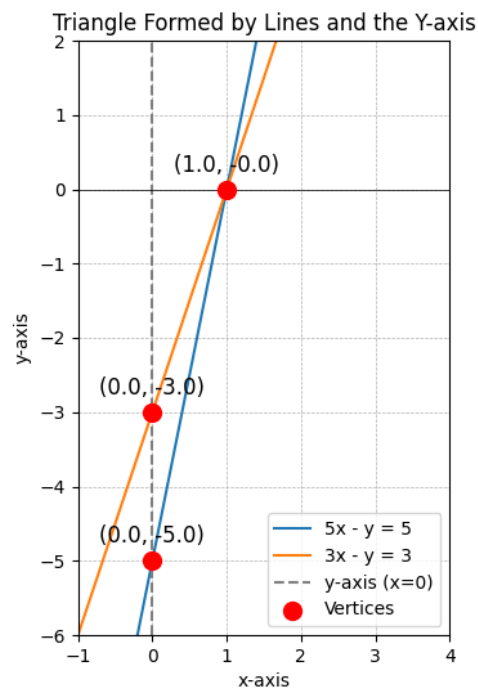


Fig. 1: Figure for 5.8.33