

1.3.6

AI25BTECH11027 - NAGA BHUVANA

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Question:

Show that the points **A** (6, 2), **B** (2, 1), **C** (1, 5) and **D** (5, 6) are vertices of a square.

Solution:

Given that

$$\mathbf{A} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} \quad (0.1)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 - 6 \\ 1 - 2 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \quad (0.2)$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} 1 - 5 \\ 5 - 6 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \quad (0.3)$$

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \quad (0.4)$$

By the above property we can say that **ABCD** is a parallelogram.
Consider the sides

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} 6 - 5 \\ 2 - 6 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} \quad (0.5)$$

$$(\mathbf{B} - \mathbf{A})^T = (-4 \quad -1) \quad (0.6)$$

Consider the angle θ between the sides $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{B}$ of the parallelogram

$$\cos \theta = \frac{(\mathbf{B} - \mathbf{A})^T (\mathbf{C} - \mathbf{B})}{\|\mathbf{B} - \mathbf{A}\| \|\mathbf{C} - \mathbf{B}\|} \quad (0.7)$$

$$\cos \theta = \frac{(-4 \quad -1) \begin{pmatrix} -1 \\ 4 \end{pmatrix}}{\sqrt{17}\sqrt{17}} \quad (0.8)$$

$$\cos \theta = \frac{(-4)(-1) + (-1)(4)}{17} \quad (0.9)$$

$$(0.10)$$

$$\cos \theta = 0 \quad (0.11)$$

$$\implies \theta = 90^\circ \quad (0.12)$$

Property:

A parallelogram with one angle 90° is a rectangle

Hence the parallelogram is a rectangle

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} \quad (0.13)$$

$$\implies (\mathbf{A} - \mathbf{C})^T = (5 \quad -3) \quad (0.14)$$

$$\mathbf{B} - \mathbf{D} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (0.15)$$

Let the angle between the diagonals of the rectangle be α

Now Consider the inner product of the diagonals of rectangle $\mathbf{A} - \mathbf{C}$ and $\mathbf{B} - \mathbf{D}$

$$\cos \alpha = \frac{(\mathbf{A} - \mathbf{C})^T (\mathbf{B} - \mathbf{D})}{\|\mathbf{A} - \mathbf{C}\| \|\mathbf{B} - \mathbf{D}\|} = \frac{\begin{pmatrix} 5 & -3 \end{pmatrix} \begin{pmatrix} -3 \\ -5 \end{pmatrix}}{\sqrt{34}\sqrt{34}} \quad (0.16)$$

$$\cos \alpha = 0 \quad (0.17)$$

$$\implies \alpha = 90^\circ \quad (0.18)$$

Property:

Rectangle with diagonals at right angle is a square

Hence given points forms a square

Graphical Representation

