1.7.13

AI25BTECH11027 - NAGA BHUVANA

Question:
Find the value of p for which the points $(-5, 1)$, $(1, p)$ and $(4, -2)$ are collinear.
solution:
Let the points be

$$A = \begin{pmatrix} -5 \\ 1 \end{pmatrix}, B = \begin{pmatrix} 1 \\ p \end{pmatrix} and C = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \tag{1}$$

$$\tag{2}$$

Given that the three points are collinear , That is Rank of the Augmented matrix of $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$ must be 1.

$$\implies rank \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix}^T = 1 \tag{3}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 - (-5) \\ p - 1 \end{pmatrix} = \begin{pmatrix} 6 \\ p - 1 \end{pmatrix} \tag{4}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 - (-5) \\ -2 - 1 \end{pmatrix} = \begin{pmatrix} 9 \\ -3 \end{pmatrix} \tag{5}$$

Now Consider the augmented matrix $\mathbf{M}$

$$\mathbf{M} = \begin{pmatrix} 6 & 9 \\ p - 1 & -3 \end{pmatrix}^T = \begin{pmatrix} 6 & p - 1 \\ 9 & -3 \end{pmatrix} \tag{6}$$

By doing Row operations $R_2 \longrightarrow R_2/3$ and $R_2 \longrightarrow 2R_2 - R_1$

$$\mathbf{M} = \begin{pmatrix} 6 & p - 1 \\ 0 & -p - 1 \end{pmatrix} \tag{7}$$

As the rank($\mathbf{M}$)=1

$$\implies -p - 1 = 0 \tag{8}$$

$$\tag{9}$$

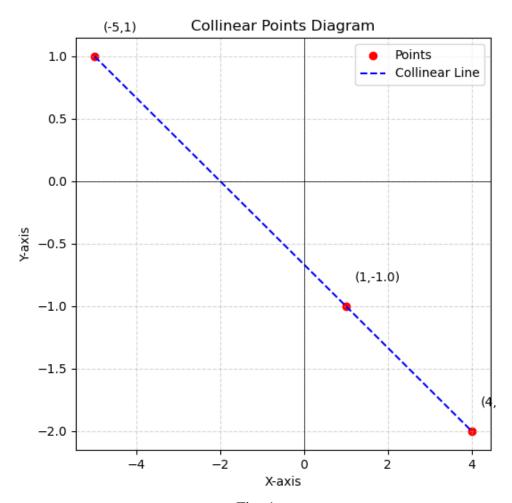$$\implies \boxed{p = -1} \tag{10}$$

$\therefore$ The value of p is -1

Fig. 1