4.11.5

INDHIRESH S - EE25BTECH11027

15 September, 2025

Question

Find the equation of the plane passing through the points $(2, 5, -3)$, $(-2, -3, 5)$, and $(5, 3, -3)$. Also, find the point of intersection of this plane with the line passing through points $(3, 1, 5)$ and $(-1, -3, -1)$.

Equation I

Let the given points be:

$$\mathbf{A} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ -3 \\ 5 \end{pmatrix} \textit{ and } \mathbf{C} = \begin{pmatrix} 5 \\ 3 \\ -3 \end{pmatrix} \tag{1}$$

Theoretical Solution

For equation of plane:

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}^T \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{2}$$

$$\begin{pmatrix} 2 & -2 & 5 \\ 5 & -3 & 3 \\ -3 & 5 & -3 \end{pmatrix}^T \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} 2 & 5 & -3 \\ -2 & -3 & 5 \\ 5 & 3 & -3 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c} 2 & 5 & -3 & 1 \\ -2 & -3 & 5 & 1 \\ 5 & 3 & -3 & 1 \end{array}\right) \tag{4}$$

Theoretical solution

$$\left(\begin{array}{ccc|c} 2 & 5 & -3 & 1 \\ -2 & -3 & 5 & 1 \\ 5 & 3 & -3 & 1 \end{array}\right) \xleftrightarrow[R_2 \leftarrow R_2 + R_1]{R_3 \leftarrow R_3 - \frac{5}{2}R_1} \left(\begin{array}{ccc|c} 2 & 5 & -3 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & \frac{-19}{2} & \frac{9}{2} & \frac{-3}{2} \end{array}\right) \tag{5}$$

$$\left(\begin{array}{ccc|c} 2 & 5 & -3 & 1 \\ 0 & 2 & 2 & 2 \\ 0 & \frac{-19}{2} & \frac{9}{2} & \frac{-3}{2} \end{array}\right) \xleftrightarrow{R_2 \leftarrow \frac{1}{2}R_2} \left(\begin{array}{ccc|c} 2 & 5 & -3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & \frac{-19}{2} & \frac{9}{2} & \frac{-3}{2} \end{array}\right) \tag{6}$$

$$\left(\begin{array}{ccc|c} 2 & 5 & -3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & \frac{-19}{2} & \frac{9}{2} & \frac{-3}{2} \end{array}\right) \xleftrightarrow{R_3 \leftarrow R_3 + \frac{19}{2}R_2} \left(\begin{array}{ccc|c} 2 & 5 & -3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 14 & 8 \end{array}\right) \tag{7}$$

Theoretical Solution

$$\left(\begin{array}{ccc|c} 2 & 5 & -3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 14 & 8 \end{array}\right) \xleftrightarrow{R_3 \longleftarrow \frac{R_3}{14}} \left(\begin{array}{ccc|c} 2 & 5 & -3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & \frac{4}{7} \end{array}\right) \tag{8}$$

$$\left(\begin{array}{ccc|c} 2 & 5 & -3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & \frac{4}{7} \end{array}\right) \xleftrightarrow[R_2 \longleftarrow R_2 - R_3]{R_1 \longleftarrow R_1 + 3R_3} \left(\begin{array}{ccc|c} 2 & 5 & 0 & \frac{19}{7} \\ 0 & 1 & 0 & \frac{3}{7} \\ 0 & 0 & 1 & \frac{4}{7} \end{array}\right) \tag{9}$$

$$\left(\begin{array}{ccc|c} 2 & 5 & 0 & \frac{19}{7} \\ 0 & 1 & 0 & \frac{3}{7} \\ 0 & 0 & 1 & \frac{4}{7} \end{array}\right) \xleftrightarrow{R_1 \longleftarrow R_1 - 5R_2} \left(\begin{array}{ccc|c} 2 & 0 & 0 & \frac{4}{7} \\ 0 & 1 & 0 & \frac{3}{7} \\ 0 & 0 & 1 & \frac{4}{7} \end{array}\right) \tag{10}$$

$$\left(\begin{array}{ccc|c} 2 & 0 & 0 & \frac{4}{7} \\ 0 & 1 & 0 & \frac{3}{7} \\ 0 & 0 & 1 & \frac{4}{7} \end{array}\right) \xleftrightarrow{R_1 \longleftarrow \frac{R_1}{2}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2}{7} \\ 0 & 1 & 0 & \frac{3}{7} \\ 0 & 0 & 1 & \frac{4}{7} \end{array}\right) \tag{11}$$

Theoretical Solution

The equation of plane can be given as:

$$\mathbf{n}^T\mathbf{x} = c \tag{12}$$

Therefore the equation of plane can be given as

$$\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}^T \mathbf{x} = 7 \tag{13}$$

Where

$$n = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad and \quad c = 7 \tag{14}$$

Theoretical Solution

The equation of line passing through the points $(3, 1, 5)$ and $(-1, -3, -1)$

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{15}$$

Multiply with $\mathbf{n}^\mathbf{T}$ on both sides:

$$\mathbf{n}^\mathbf{T}\mathbf{x} = \mathbf{n}^\mathbf{T}\mathbf{h} + k\mathbf{n}^\mathbf{T}\mathbf{m} \tag{16}$$

From Eq.12

$$c = \mathbf{n}^\mathbf{T}\mathbf{h} + k\mathbf{n}^\mathbf{T}\mathbf{m} \tag{17}$$

$$\frac{c - \mathbf{n}^\mathbf{T}\mathbf{h}}{\mathbf{n}^\mathbf{T}\mathbf{m}} = k \tag{18}$$

$$\mathbf{m} = \begin{pmatrix} 3 - (-1) \\ 1 - (-3) \\ 5 - (-1) \end{pmatrix} \tag{19}$$

Theoretical Solution

$$\mathbf{m} = \begin{pmatrix} 4 \\ 4 \\ 6 \end{pmatrix} \tag{20}$$

Now substitute the corresponding values in Eq.18

$$\frac{7 - \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}^T \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}}{\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}^T \begin{pmatrix} 4 \\ 4 \\ 6 \end{pmatrix}} = k \tag{21}$$

$$\frac{7 - 29}{44} = k \tag{22}$$

Theoretical Solution

Solving this we get

$$k = \frac{-1}{2} \tag{23}$$

Now substitute the value of k in Eq.15

$$\mathbf{x} = \begin{pmatrix} 3-2 \\ 1-2 \\ 5-3 \end{pmatrix} \tag{24}$$

Therefore the point of intersection is:

$$\mathbf{x} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \tag{25}$$

C Code

```
#include <stdio.h>

void solve(double *out) {
    int A[3] = {2, 5, -3};
    int B[3] = {-2, -3, 5};
    int C[3] = {5, 3, -3};
    int P[3] = {3, 1, 5};
    int Q[3] = {-1, -3, -1};

    // vectors
    int AB[3] = {B[0]-A[0], B[1]-A[1], B[2]-A[2]};
    int AC[3] = {C[0]-A[0], C[1]-A[1], C[2]-A[2]};

    // normal = AB x AC
    int n[3];
    n[0] = AB[1]*AC[2] - AB[2]*AC[1];
    n[1] = AB[2]*AC[0] - AB[0]*AC[2];
    n[2] = AB[0]*AC[1] - AB[1]*AC[0];
    int d = -(n[0]*A[0] + n[1]*A[1] + n[2]*A[2]);
```

C Code

```c
// direction of PQ
 int v[3] = {Q[0]-P[0], Q[1]-P[1], Q[2]-P[2]};
 int num = -(n[0]*P[0] + n[1]*P[1] + n[2]*P[2] + d);
 int den = n[0]*v[0] + n[1]*v[1] + n[2]*v[2];

 double t = (double)num / den;
 double X = P[0] + t*v[0];
 double Y = P[1] + t*v[1];
 double Z = P[2] + t*v[2];
```

C Code

```c
    // output: n0, n1, n2, d, X, Y, Z
    out[0] = n[0];
    out[1] = n[1];
    out[2] = n[2];
    out[3] = d;
    out[4] = X;
    out[5] = Y;
    out[6] = Z;
}
```

Python Code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Load the shared library
lib = ctypes.CDLL(./intersection.so) # use plane.dll on Windows

# Prepare output buffer (7 doubles)
out = (ctypes.c_double * 7)()
lib.solve(out)

# Extract results
n = [out[0], out[1], out[2], out[3]]
intersection = [out[4], out[5], out[6]]
```

Python Code

```
print(fEquation of plane: {n[0]}x + {n[1]}y + {n[2]}z + {n[3]} =
    0)
print(fIntersection point: {intersection})

# --- Plotting ---
A = np.array([2,5,-3])
B = np.array([-2,-3,5])
C = np.array([5,3,-3])
P = np.array([3,1,5])
Q = np.array([-1,-3,-1])
I = np.array(intersection)

fig = plt.figure()
ax = fig.add_subplot(111, projection=3d)

# Plot the plane
xx, yy = np.meshgrid(range(-2,7), range(-4,7))
zz = (-n[0]*xx - n[1]*yy - n[3]) / n[2]
```

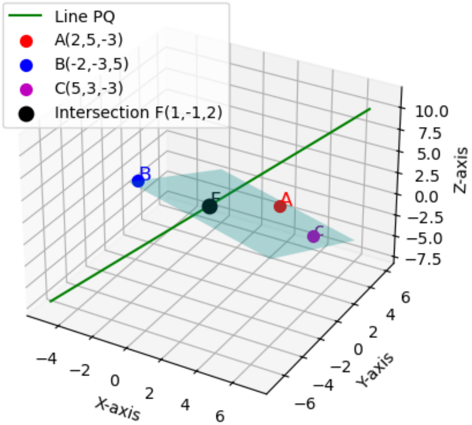
Python Code

```
ax.plot_surface(xx, yy, zz, alpha=0.3, color=cyan)

# Plot line PQ
t_vals = np.linspace(-1,2,20)
linePQ = np.array([P + t*(Q-P) for t in t_vals])
ax.plot(linePQ[:,0], linePQ[:,1], linePQ[:,2], g, label=Line PQ)

# Plot points
ax.scatter(*A, color=r, s=50, label=A(2,5,-3))
ax.scatter(*B, color=b, s=50, label=B(-2,-3,5))
ax.scatter(*C, color=m, s=50, label=C(5,3,-3))
ax.scatter(*I, color=k, s=80, marker=o, label=Intersection F
    (1,-1,2))
ax.text(2, 5, -3, A, color=red, fontsize=12)
ax.text(-2, -3, 5, B, color=blue, fontsize=12)
ax.text(5, 3, -3, C, color=purple, fontsize=12)
ax.text(1, -1, 2, F, color=black, fontsize=12)
```

Python Code

```
ax.set_xlabel(X-axis)
ax.set_ylabel(Y-axis)
ax.set_zlabel(Z-axis)
ax.legend()
plt.savefig(./figure1.png)
plt.show()
```

Plot



Line PQ
A(2,5,-3)
B(-2,-3,5)
C(5,3,-3)
Intersection F(1,-1,2)
B
F
A
C
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
Z-axis
−4
−2
0
2
4
6
X-axis
6
4
2
0
−2
−4
−6
Y-axis