10.3.12

EE25BTECH11013 - Bhargav

Question:
If the line $y = \sqrt{3}x + K$ touches the parabola $x^2 = 16y$, then find the value of K.

Solution:

The equation of the conic (*parabola*) can be written as

$$\mathbf{x}^{\mathbf{T}}\mathbf{V}\mathbf{x} + 2\mathbf{u}^{\mathbf{T}}\mathbf{x} + f = 0 \tag{0.1}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ -8 \end{pmatrix}, f = 0, \mathbf{m}^{\mathbf{T}} = \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} \tag{0.2}$$

Since the tangent is perpendicular to the normal of the conic at the point of contact($\mathbf{q}$), we can write:

$$\mathbf{m}^{\mathbf{T}} \left(\mathbf{V}\mathbf{q} + \mathbf{u}\right) = 0 \tag{0.3}$$

$$\begin{pmatrix} 1 & \sqrt{3} \end{pmatrix} \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{q} + \begin{pmatrix} 0 \\ -8 \end{pmatrix} \right) = 0 \tag{0.4}$$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{q} - 8\sqrt{3} = 0 \tag{0.5}$$

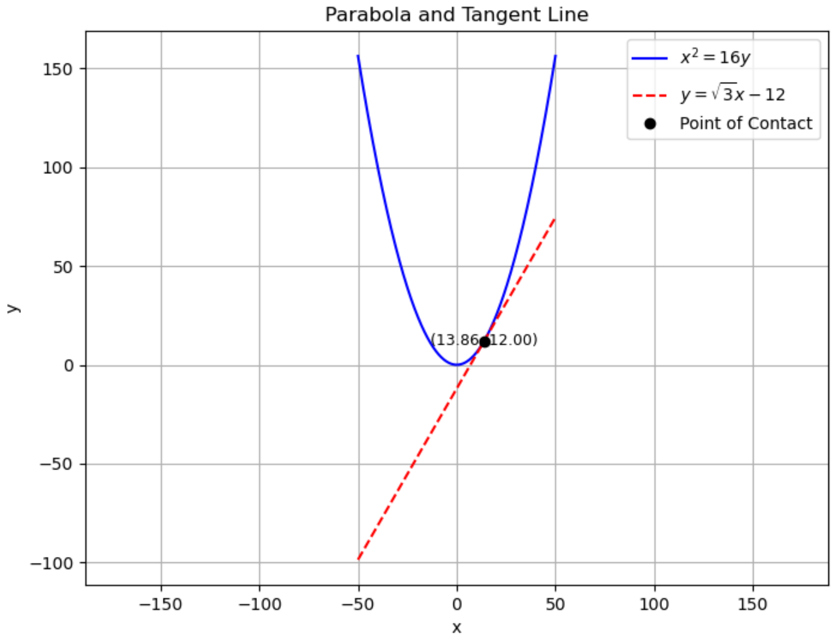
$$\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 8\sqrt{3} \tag{0.6}$$

$$\mathbf{x} = 8\sqrt{3} \tag{0.7}$$

Substituting the value of x in the parabola equation we get $y = 12$

$$\mathbf{q} = \begin{pmatrix} 8\sqrt{3} \\ 12 \end{pmatrix} \tag{0.8}$$

$$k = -12 \tag{0.9}$$



Parabola and Tangent Line
$x^2 = 16y$
$y = \sqrt{3}x - 12$
Point of Contact
(13.86, 12.00)
150
100
50
0
−50
−100
−150
−100
−50
0
50
100
150
x
y