1.6.28

AI25BTECH11013-Gautham

Question:
Show that the points $A(-2\hat{i}+3\hat{j}+5\hat{k}), B(\hat{i}+2\hat{j}+3\hat{k}) and C(7\hat{i}-\hat{k})$are collinear.
Solution:
Let the points are $\mathbf{A}\begin{pmatrix}-2\\3\\5\end{pmatrix}$,$\mathbf{B}\begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\mathbf{C}\begin{pmatrix}7\\0\\-1\end{pmatrix}$.

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}1\\2\\3\end{pmatrix}-\begin{pmatrix}-2\\3\\5\end{pmatrix} \tag{0.1}$$

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}1-(-2)\\2-3\\3-5\end{pmatrix} \tag{0.2}$$

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}3\\-1\\-2\end{pmatrix} \tag{0.3}$$

$$\mathbf{C}-\mathbf{A}=\begin{pmatrix}7\\0\\-1\end{pmatrix}-\begin{pmatrix}-2\\3\\5\end{pmatrix} \tag{0.4}$$

$$\mathbf{C}-\mathbf{A}=\begin{pmatrix}7-(-2)\\0-3\\-1-5\end{pmatrix} \tag{0.5}$$

$$\mathbf{C}-\mathbf{A}=\begin{pmatrix}9\\-3\\-6\end{pmatrix} \tag{0.6}$$

$$\tag{0.7}$$

If $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are collinear, then the Rank of matrix $(\mathbf{B}-\mathbf{A},\mathbf{C}-\mathbf{A})$ should be 1.

$$(\mathbf{B}-\mathbf{A},\mathbf{C}-\mathbf{A})=\begin{pmatrix}3&9\\-1&-3\\-2&-6\end{pmatrix} \tag{0.8}$$

$$R_3\to(\frac{R_1}{3}\times 2)+R_3 \tag{0.9}$$

$$R_2\to\frac{R_1}{3}+R_2 \tag{0.10}$$

$$=\begin{pmatrix}3&9\\0&0\\0&0\end{pmatrix} \tag{0.11}$$

$$\tag{0.12}$$

Since all elements of R_2 and R_3 are 0, The Rank of matrix $(\mathbf{B}-\mathbf{A},\mathbf{C}-\mathbf{A})$ is 1.
$\implies$ $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are collinear.

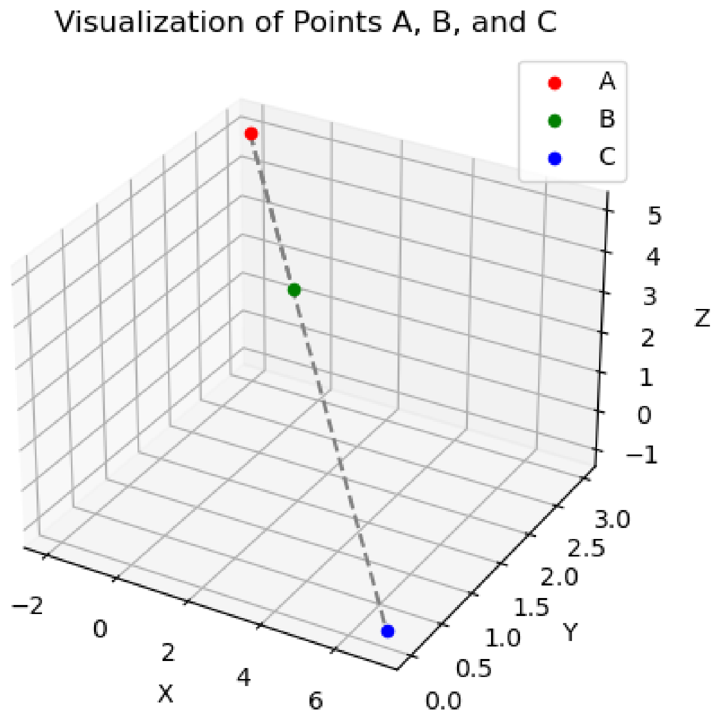
Visualization of Points A, B, and C
A
B
C
5
4
3
2
1
0
−1
Z
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Y
−2
0
2
4
6
X

Fig. 0.1