2.10.25

AI25BTECH11036–SNEHAMRUDULA

2.10.25. In $\triangle PQR$

, let $\mathbf{a} = \overrightarrow{QR}$, $\mathbf{b} = \overrightarrow{RP}$, $\mathbf{c} = \overrightarrow{PQ}$.

If $|\mathbf{a}| = 12$, $|\mathbf{b}| = 4\sqrt{3}$, $\mathbf{b} \cdot \mathbf{c} = 24$, then which of the following is (are) true?

(a) $\dfrac{|\mathbf{c}|^2}{2} - |\mathbf{a}| = 12$

(b) $\dfrac{|\mathbf{c}|^2}{2} + |\mathbf{a}| = 30$

(c) $|\mathbf{a} \times \mathbf{b} + \mathbf{c} \times \mathbf{a}| = 48\sqrt{3}$

(d) $\mathbf{a} \cdot \mathbf{b} = -72$

solution $\|\mathbf{a}\| = 12, \quad \|\mathbf{b}\| = 4\sqrt{3}, \quad \mathbf{b} \cdot \mathbf{c} = 24, \mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$ Thus, $\mathbf{c} = -(\mathbf{a} + \mathbf{b})$.

Find:

(a) $\|\mathbf{c}\|^2$

(b) Check $\frac{\|\mathbf{c}\|^2}{2} \pm \|\mathbf{a}\|$

(c) $\|\mathbf{a} \times \mathbf{b} + \mathbf{c} \times \mathbf{a}\|$

(d) $\mathbf{a} \cdot \mathbf{b}$

(d) Compute $\mathbf{a} \cdot \mathbf{b}$:

$$\mathbf{b} \cdot \mathbf{c} = -\mathbf{a} \cdot \mathbf{b} - \|\mathbf{b}\|^2 \quad \Rightarrow \quad 24 = -\mathbf{a} \cdot \mathbf{b} - 48, \tag{0.1}$$

$$\mathbf{a} \cdot \mathbf{b} = -72. \tag{0.2}$$

(a), (b) Compute $\|\mathbf{c}\|^2$:

$$\|\mathbf{c}\|^2 = \|\mathbf{a} + \mathbf{b}\|^2 = 144 + 48 + 2(-72) = 48, \tag{0.3}$$

$$\frac{48}{2} - 12 = 12 \text{ (True)}, \quad \frac{48}{2} + 12 = 36 \text{ (False)}. \tag{0.4}$$

(c) Compute $\|\mathbf{a} \times \mathbf{b} + \mathbf{c} \times \mathbf{a}\|$:

$$\mathbf{a} \times \mathbf{b} + \mathbf{c} \times \mathbf{a} = 2(\mathbf{a} \times \mathbf{b}), \tag{0.5}$$

$$\cos\theta = \frac{-72}{12 \cdot 4\sqrt{3}} = -\frac{\sqrt{3}}{2}, \quad \sin\theta = \frac{1}{2}, \|\cos\theta\| = 2 \cdot 12 \cdot 4\sqrt{3} \cdot \frac{1}{2} = 48\sqrt{3} \tag{0.6}$$

1) $\mathbf{a} \cdot \mathbf{b} = -72$
2) $\|\mathbf{c}\|^2 = 48$
3) $\frac{\|\mathbf{c}\|^2}{2} - \|\mathbf{a}\| = 12(True)$
4) $\frac{\|\mathbf{c}\|^2}{2} + \|\mathbf{a}\| = 36(False)$
5) $\|\mathbf{a} \times \mathbf{b} + \mathbf{c}\|$

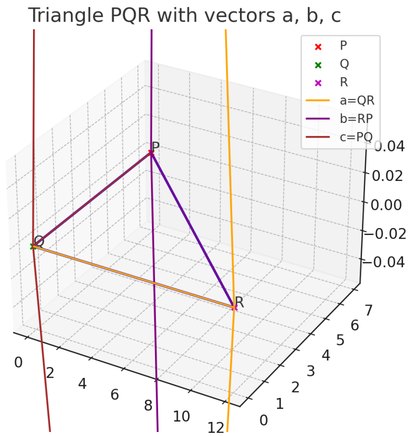
Triangle PQR with vectors a, b, c
P
Q
R
a=QR
b=RP
c=PQ
0.04
0.02
0.00
−0.02
−0.04
0
2
4
6
8
10
12
0
1
2
3
4
5
6
7

Fig. 5.1