2.10.48

EE25BTECH11019 - Darji Vivek M.

Question

If
$$\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$$
, $\mathbf{a} \cdot \mathbf{b} = 1$ and $\mathbf{a} \times \mathbf{b} = \mathbf{j} - \mathbf{k}$, then \mathbf{b} is 2

- **e**
- **●** 2i

Solution

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1}$$

Dot product condition:

$$x + y + z = 1$$

Solution

Cross product condition gives

$$z - y = 0$$
, $x - z = 1$, $y - x = -1$

The first equation can be expressed as

$$\begin{pmatrix} 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

Solution

Collecting all equations:

$$\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Solving,

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Final Answer

Hence,

$$\mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \mathbf{i}$$

Therefore, the correct option is **(c)**.

Answer

$$\mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \mathbf{i}$$

.: Correct option: (c)

C Code

```
1 // Function to compute b given a b and a b
2 #include <stdio.h>
3 #include <math.h>
 void find_b(double *b) {
     double a[3] = \{1, 1, 1\};
     double dot = 1;
7
     double cross [3] = \{0, 1, -1\};
8
     b[0] = 1; // x
     b[1] = 0; // y
11
     b[2] = 0; // z
```

C Code

```
int main() {
    double b[3];
    find_b(b);
    FILE *file = fopen("values.dat", "w");
    fprintf(file, "b_x\tb_y\tb_z\n");
    fprintf(file, "%.2lf\t%.2lf\t%.2lf\n", b[0], b[1],
        b[2]);
    fclose(file);
    printf("Vector b written: (%.21f, %.21f, %.21f)\n"
        , b[0], b[1], b[2]);
    return 0;
```

Python Plot

```
1 import ctypes
2 import numpy as np
3 import matplotlib.pyplot as plt
5 lib = ctypes.CDLL('./libb.so')
_{6} b = (ctypes.c_double * 3)()
7 lib.find_b(b)
9 | bx, by, bz = b[0], b[1], b[2]
print(f"Vector b = (\{bx\}, \{by\}, \{bz\})")
|a| = np.array([1, 1, 1])
|b| b_vec = np.array([bx, by, bz])
|0| = \text{np.array}([0, 0, 0])
```

Python plot

```
1 fig = plt.figure()
2 ax = fig.add_subplot(111, projection='3d')
ax.quiver(0,0,0, a[0], a[1], a[2], color='r', label='a=
     i+j+k')
4 ax.quiver(0,0,0, b_vec[0],b_vec[1],b_vec[2], color='b'
     , label='b')
5 points = {'0':0,'A (1,1,1)':a,f'B ({bx:.0f},{by:.0f},{
    bz:.0f})':b_vec}
6 for label, coord in points.items():
   ax.text(coord[0], coord[1], coord[2], f'{label}',
        fontsize=10, ha='center')
8 | ax.set_xlim([0, 2]); ax.set_ylim([0, 2]); ax.set_zlim
    ([0, 2])
9 | ax.set_xlabel('X'); ax.set_ylabel('Y'); ax.set_zlabel(
    77.1)
ax.legend(); plt.title("Vectors a and b with
    coordinates")
plt.show()
```

plot

Vectors a and b with coordinates

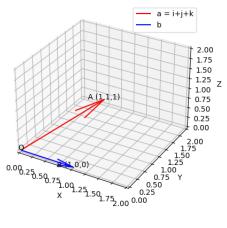


Figure: plot