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Matrix 2.6.24

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Question

Find the area of the parallelogram formed by the vectors

$$\mathbf{a} = 3\hat{i} + \hat{j} + 4\hat{k}, \qquad \mathbf{b} = \hat{i} - \hat{j} + \hat{k}.$$

Use the cross product definition.

Cross Product Definition

- Let

$$\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}. \tag{2}$$

- Define sub-vectors

$$\mathbf{A}_{ij} = \begin{pmatrix} a_i \\ a_j \end{pmatrix}, \quad \mathbf{B}_{ij} = \begin{pmatrix} b_i \\ b_j \end{pmatrix}. \tag{3}$$

- The cross product is

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} \begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \\ \mathbf{A}_{31} & \mathbf{B}_{31} \\ \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} \end{pmatrix}. \tag{4}$$

Applying the Formula

- For the given vectors

$$\mathbf{a} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}. \tag{5}$$

- Substituting:

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} \begin{vmatrix} \begin{pmatrix} 1 \\ 4 \end{pmatrix} & \begin{pmatrix} -1 \\ 1 \end{pmatrix} \end{vmatrix} \\ \begin{vmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} & \begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{vmatrix} \\ \begin{vmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix} & \begin{pmatrix} 1 \\ -1 \end{pmatrix} \end{vmatrix} \end{pmatrix}. \tag{6}$$

Simplification

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} (1)(1) - (4)(-1) \\ (4)(1) - (3)(1) \\ (3)(-1) - (1)(1) \end{pmatrix} \tag{7}$$

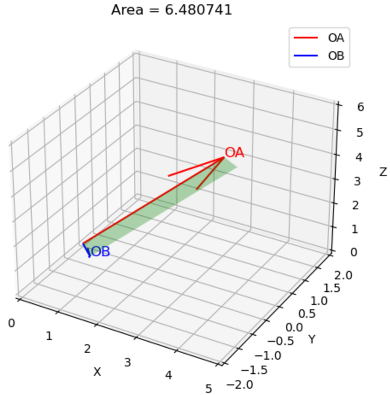
$$= \begin{pmatrix} 5 \\ 1 \\ -4 \end{pmatrix}. \tag{8}$$

Area of Parallelogram

$$
\begin{aligned}
\text{Area} &= \|\mathbf{a} \times \mathbf{b}\| && (9)\\
&= \sqrt{5^2 + 1^2 + (-4)^2} && (10)\\
&= \sqrt{42}. && (11)
\end{aligned}
$$

$$\boxed{\text{Area} = \sqrt{42}}$$

Figure



Parallelogram spanned by **a** and **b**.

C Code

```c
// C code to calculate area of parallelogram
#include <stdio.h>
#include "libs/matfun.h"
#include <math.h>

double main() {
    // create a b vectros
    double **a = createMat(3,1);
    a[0][0] = 3;
    a[1][0] = 1;
    a[2][0] = 4;

    double **b = createMat(3,1);
    b[0][0] = 1;
    b[1][0] = -1;
    b[2][0] = 1;


    double **a_cross_b = createMat(3,1);
    a_cross_b[0][0] = a[1][0] * b[2][0] - a[2][0] * b[1][0];
    a_cross_b[1][0] = a[2][0] * b[0][0] - a[0][0] * b[2][0];
    a_cross_b[2][0] = a[0][0] * b[1][0] - a[1][0] * b[0][0];

    double area = sqrt(Matdot(a_cross_b, a_cross_b, 3));
```

C Code

```c
    printf("Area of the parallelogram: %lf\n", area);

    FILE *fp = fopen("var.dat", "w");
    if (fp != NULL) {
        fprintf(fp, "%lf\n", area);
        fclose(fp);
    } else {
        printf("Error opening file for writing.\n");
    }
    return area;
}
```

Python Code

```
import numpy as np
from mpl_toolkits.mplot3d import Axes3D
from mpl_toolkits.mplot3d.art3d import Poly3DCollection
import ctypes
import matplotlib.pyplot as plt

# Load the shared object file
main_lib = ctypes.CDLL(’./main.so’)

# Define input and return types for the C function
# main_lib.main.argtypes = []
main_lib.main.restype = ctypes.c_double

# Call the C function to calculate the area
area_value = main_lib.main()
print(area_value)


with open(’var.dat’, ’r’) as f:
        area = f.read().strip()

a = np.array([3, 1, 4])
b = np.array([1, -1, 1])
O = np.array([0, 0, 0])
```

Python Code

```
fig = plt.figure()
ax = fig.add_subplot(111, projection=’3d’)

# Plot vectors OA and OB
ax.quiver(*O, *a, color=’r’, label=’OA’)
ax.quiver(*O, *b, color=’b’, label=’OB’)
ax.text(a[0], a[1], a[2], ’OA’, color=’r’, fontsize=12)
ax.text(b[0], b[1], b[2], ’OB’, color=’b’, fontsize=12)

verts = [ [O, a, a+b, b] ]
ax.add_collection3d(Poly3DCollection(verts, alpha=0.3, facecolor=’
    green’))

# Set limits and labels
ax.set_xlim([0, max(a[0], b[0], a[0]+b[0])+1])
ax.set_ylim([min(0, a[1], b[1], a[1]+b[1])-1, max(a[1], b[1], a[1]+
    b[1])+1])
ax.set_zlim([0, max(a[2], b[2], a[2]+b[2])+1])
ax.set_xlabel(’X’)
ax.set_ylabel(’Y’)
ax.set_zlabel(’Z’)
ax.legend()
```

Python Code

```python
plt.tight_layout()
plt.savefig('../figs/fig.png')
plt.close()
```