

2.9.1

EE25BTECH11050-Hema Havil

Question:

Jagdish has a field which is in the shape of a right-angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining space for growing vegetables. In the field, there is a pole marked as O. Based on the above information, answer the following equations

- Taking O as the origin, $P = (-200, 0)$ and $Q = (200, 0)$. PQRS being a square, what are the coordinates of R and S?
- i) What is the area of square PQRS ?
ii) What is the length of diagonal PR in PQRS ?
- If S divides CA in the ratio $K : 1$, what is the value of K, where $A = (200, 800)$?

Solution:

Given that,

AQC is a right angled triangle at point Q and PQRS is a square inside the ΔAQC ,

- (a) We were given two points

$$P = (-200, 0), Q = (200, 0) \quad ((a).1)$$

Let,

X be the vector along the side PQ,

Y be the vector along the side QR,

Z be the vector along the side PS then,

$$\mathbf{X} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} 200 \\ 0 \end{pmatrix} - \begin{pmatrix} -200 \\ 0 \end{pmatrix} \quad ((a).2)$$

$$\mathbf{X} = \begin{pmatrix} 400 \\ 0 \end{pmatrix} \quad ((a).3)$$

Rotation vector for 2x2 matrix is

$$\mathbf{R}_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \quad ((a).4)$$

Rotate the vector \mathbf{X} by 90° anticlockwise to get \mathbf{Y}

$$\mathbf{Y} = \mathbf{R}_{90} \mathbf{X} \quad ((a).5)$$

$$\mathbf{Y} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 400 \\ 0 \end{pmatrix} \quad ((a).6)$$

$$\mathbf{Y} = \begin{pmatrix} 0 \\ 400 \end{pmatrix} \quad ((a).7)$$

So the vector along the side QR is $\mathbf{Y} = \begin{pmatrix} 0 \\ 400 \end{pmatrix}$ then,

$$\mathbf{Y} = \mathbf{R} - \mathbf{Q} \quad ((a).8)$$

$$\mathbf{R} = \mathbf{Y} + \mathbf{Q} \quad ((a).9)$$

$$\mathbf{R} = \begin{pmatrix} 0 \\ 400 \end{pmatrix} + \begin{pmatrix} 200 \\ 0 \end{pmatrix} \quad ((a).10)$$

$$\mathbf{R} = \begin{pmatrix} 200 \\ 400 \end{pmatrix} \quad ((a).11)$$

Since the sides QR and PS are parallel, vectors $\mathbf{Y} = \mathbf{Z}$ then

$$\mathbf{Z} = \mathbf{S} - \mathbf{P} \quad ((a).12)$$

$$\mathbf{S} = \mathbf{Z} + \mathbf{P} \quad ((a).13)$$

$$\mathbf{S} = \begin{pmatrix} 0 \\ 400 \end{pmatrix} + \begin{pmatrix} -200 \\ 0 \end{pmatrix} \quad ((a).14)$$

$$\mathbf{S} = \begin{pmatrix} -200 \\ 400 \end{pmatrix} \quad ((a).15)$$

Therefore the coordinates of the points R and S are (200,400) and (-200,400)

(b)(i) We know the points P(-200,0) and Q(200,0)

Let length of the side of the square PQRS be x then,

$$x = \|\mathbf{Q} - \mathbf{P}\| \quad ((b).1)$$

$$x = \left\| \begin{pmatrix} 400 \\ 0 \end{pmatrix} \right\| = 400 \quad ((b).2)$$

$$\text{Area of the square} = x^2 = (400)^2 = 160000 \text{ sq units}$$

(ii) Length of diagonal of the square = $x\sqrt{2} = 400\sqrt{2}$ units

(c) Given the point A=(200,800)

Since it was given that point S divides CA in the ratio K:1, this shows that points A,C and S are collinear. Since AQC is a right angled triangle, from this we can say that point C lies on X axis

Let point C be (t,0), Consider the matrix M

$$M = \begin{pmatrix} 200 & 800 & 1 \\ -200 & 400 & 1 \\ t & 0 & 1 \end{pmatrix} \quad ((c).1)$$

$$R_1 \rightarrow \frac{1}{200}R_1$$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ -200 & 400 & 1 \\ t & 0 & 1 \end{pmatrix} \quad ((c).2)$$

$$R_2 \rightarrow R_2 + 200R_1$$

$$R_3 \rightarrow R_3 - tR_1$$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ 0 & 1200 & 2 \\ 0 & -4t & 1 - \frac{t}{200} \end{pmatrix} \quad ((c).3)$$

$$R_2 \rightarrow \frac{1}{200}R_2$$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ 0 & 1 & \frac{1}{600} \\ 0 & -4t & 1 - \frac{t}{200} \end{pmatrix} \quad ((c).4)$$

$$R_3 \rightarrow R_3 + 4tR_2$$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ 0 & 1 & \frac{1}{600} \\ 0 & 0 & 1 - \frac{t}{200} + \frac{4t}{600} \end{pmatrix} \quad ((c).5)$$

Since the three points A,S and C are collinear,
Rank of M = 2

$$1 - \frac{t}{200} + \frac{4t}{600} = 0 \quad ((c).6)$$

$$1 + \frac{t}{600} = 0 \quad ((c).7)$$

$$\frac{t}{600} = -1 \quad ((c).8)$$

$$t = -600 \quad ((c).9)$$

Therefore point C=(-600,0), Now S divides CA in the ratio K:1,

$$S = \frac{KA + C}{K + 1} \quad ((c).10)$$

$$K = \frac{(S - A)^T(C - S)}{\|S - A\|^2} \quad ((c).11)$$

$$K = \frac{1}{(400)^2 + (400)^2} (-400 - 400) \begin{pmatrix} -400 \\ -400 \end{pmatrix} \quad ((c).12)$$

By solving ((c).12) we get $K=1$

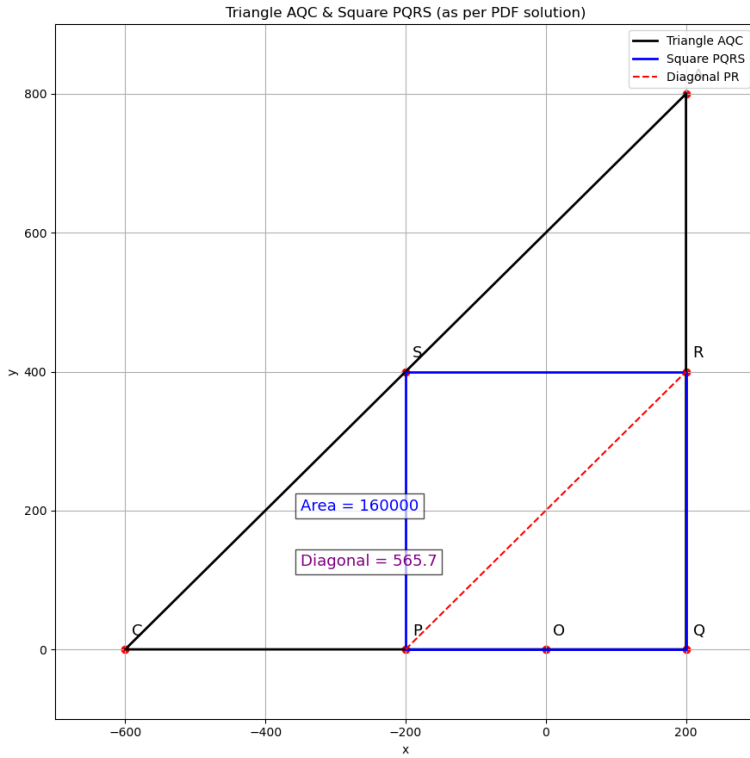


Fig. 3.1: Plot of the square and triangle