2.10.49

EE25BTECH11020 - Darsh Pankaj Gajare

Question:
The unit vector which is orthogonal to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is

1) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$
2) $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$
3) $\frac{3\hat{i}-\hat{k}}{\sqrt{10}}$
4) $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

Solution: Given:

TABLE I: Given data

Vector	matrix
A	$\begin{pmatrix}2\\1\\1\end{pmatrix}$
B	$\begin{pmatrix}1\\-1\\1\end{pmatrix}$
C	$\begin{pmatrix}3\\2\\6\end{pmatrix}$

Assume Equation of plane through A, B.

$$\mathbf{n}^\top \mathbf{x} = 1 \tag{1}$$

$$\mathbf{n}^\top \mathbf{A} = 1 \tag{2}$$

$$\mathbf{n}^\top \mathbf{B} = 1 \tag{3}$$

$$\begin{pmatrix}2 & 1 & 1\\1 & -1 & 1\end{pmatrix} n = 1 \tag{4}$$

Augmented matrix,

$$\left(\begin{array}{ccc|c}2 & 1 & 1 & 1\\1 & -1 & 1 & 1\end{array}\right). \tag{5}$$

$R_1 = R_1 - R_2$

$$\left(\begin{array}{ccc|c}1 & 2 & 0 & 0\\1 & -1 & 1 & 1\end{array}\right) \tag{6}$$

$R_2 = R_2 - R_1$

$$\left(\begin{array}{ccc|c}1 & 2 & 0 & 0\\0 & -3 & 1 & 1\end{array}\right) \tag{7}$$

Let parametric constant be λ

$$n = \begin{pmatrix}-2\lambda\\ \lambda\\ 1+3\lambda\end{pmatrix} \tag{8}$$

$$\mathbf{n}^\top \mathbf{P} = 1 \tag{9}$$

$$\mathbf{C}^\top \mathbf{P} = 0 \tag{10}$$

$$\begin{pmatrix} -2\lambda & \lambda & 1+3\lambda \\ 3 & 2 & 6 \end{pmatrix} \mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{11}$$

Augmented matrix,

$$\left(\begin{array}{ccc|c} -2\lambda & \lambda & 1+3\lambda & 1 \\ 3 & 2 & 6 & 0 \end{array}\right). \tag{12}$$

Row operations: $R_1 = R_1 - \frac{\lambda}{2} R_2$

$$\left(\begin{array}{ccc|c} -3.5\lambda & 0 & 1 & 1 \\ 3 & 2 & 6 & 0 \end{array}\right). \tag{13}$$

$R_2 = R_2 - 6R_1$

$$\left(\begin{array}{ccc|c} -3.5\lambda & 0 & 1 & 1 \\ 3+21\lambda & 2 & 0 & -6 \end{array}\right). \tag{14}$$

$$-3.5\lambda x + z = 1 \implies z = 1 + 3.5\lambda x \tag{15}$$

$$(3+21\lambda)\, x + 2y = -6 \implies y = -3 - \frac{x}{2}(3+21\lambda) \tag{16}$$

Let $x =\mu$ a parameter

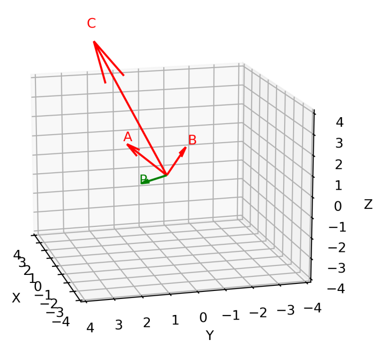
$$\mathbf{P} = \begin{pmatrix} \mu \\ -3 - \frac{\mu}{2}(3+21\lambda) \\ 1 + 3.5\lambda\mu \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} + \frac{\mu}{2}\begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix} + 7\lambda\frac{\mu}{2}\begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}. \tag{17}$$

Taking $\mu = 0$ we get,

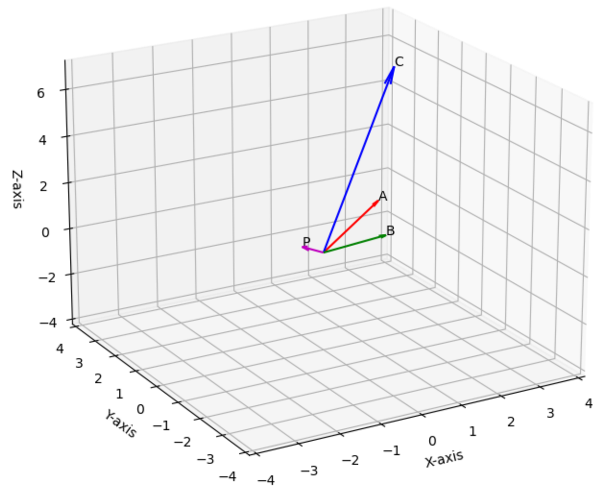
$$\mathbf{P} = \pm \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} \tag{18}$$

Normalizing,

$$\mathbf{P} = \pm \frac{1}{\sqrt{10}} \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} \tag{19}$$



Plot using C function



Plot using Python