Assignment : GATE 2014 MA

EE25BTECH11061 - Vankudoth Sainadh

1) A student is required to demonstrate a high level of <u>comprehension</u> of the subject, especially in the social sciences.
The word closest in meaning to <u>comprehension</u> is
(GATE MA 2014)
   a) understanding b) meaning c) concentration d) stability
2) Choose the most appropriate word from the options given below to complete the following sentence.
One of his biggest __________ was his ability to forgive.
(GATE MA 2014)
   a) vice b) virtues c) choices d) strength
3) Rajan was not happy that Sajan decided to do the project on his own. On observing his unhappiness, Sajan explained to Rajan that he preferred to work independently.
Which one of the statements below is logically valid and can be inferred from the above sentences?
(GATE MA 2014)
   a) Rajan has decided to work only in a group.
   b) Rajan and Sajan were formed into a group against their wishes.
   c) Sajan had decided to give in to Rajan's request to work with him.
   d) Rajan had believed that Sajan and he would be working together.
4) If $y = 5x^2 + 3$, then the tangent at $x = 0,\ y = 3$
(GATE MA 2014)
   a) passes through $x = 0,\ y = 0$
   b) has a slope of $+1$
   c) is parallel to the x-axis
   d) has a slope of -1
5) A foundry has a fixed daily cost of Rs 50,000 whenever it operates and a variable cost of Rs $800Q$, where Q is the daily production in tonnes. What is the cost of production in Rs per tonne for a daily production of 100 tonnes?
(GATE MA 2014)
6) Find the odd one in the following group: `ALRVX`, `EPVZB`, `ITZDF`, `OYEIK` (GATE MA 2014)
   a) ALRVX b) EPVZB c) ITZDF d) OYEIK
7) Anuj, Bhola, Chandan, Dilip, Eswar and Faisal live on different floors in a six-storeyed building (the ground floor is numbered 1, the floor above it 2, and so on). Anuj lives on an even-numbered floor. Bhola does not live on an odd numbered floor. Chandan does not live on any of the floors below Faisal's floor. Dilip does not live on floor number 2. Eswar does not live on a floor immediately above or immediately below Bhola. Faisal lives three floors above Dilip. Which of the following floor-person combinations is correct?
(GATE MA 2014)

	Anuj	Bhola	Chandan	Dilip	Eswar	Faisal
(A)	6	2	5	1	3	4
(B)	2	6	5	1	3	4
(C)	4	2	6	3	1	5
(D)	2	4	6	1	3	5

8) The smallest angle of a triangle is equal to two-thirds of the smallest angle of a quadrilateral. The ratio between the angles of the quadrilateral is $3 : 4 : 5 : 6$. The largest angle of the triangle is twice its smallest angle. What is the sum, in degrees, of the second largest angle of the triangle and the largest angle of the quadrilateral?

(GATE MA 2014)

9) One percent of the people of country X are taller than 6 ft. Two percent of the people of country Y are taller than 6 ft. There are thrice as many people in country X as in country Y. Taking both countries together, what is the percentage of people taller than 6 ft?

(GATE MA 2014)

a) 3.0 b) 2.5 c) 1.5 d) 1.25

10) The monthly rainfall chart based on 50 years of rainfall in Agra is shown in the following figure. Which of the following are true? (k) percentile is the value such that k percent of the data fall below that value

(GATE MA 2014)

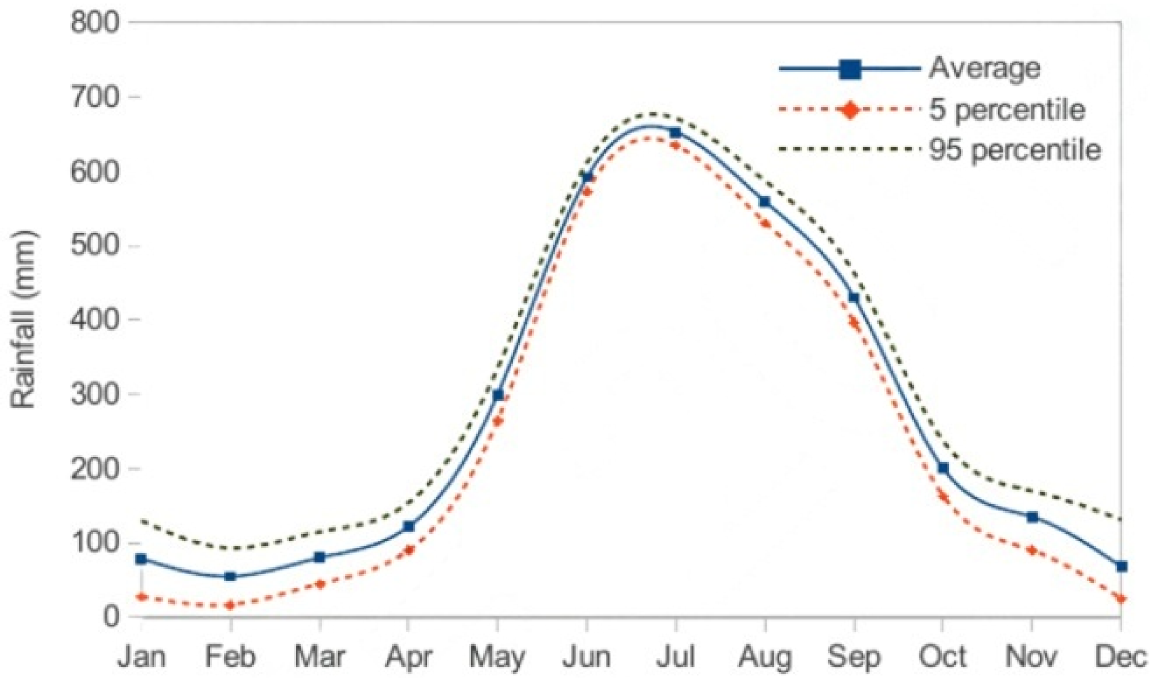

Fig. 10: *

a) On average, it rains more in July than in December
b) Every year, the amount of rainfall in August is more than that in January
c) July rainfall can be estimated with better confidence than February rainfall
d) In August, there is at least 500 mm of rainfall

a) (i) and (ii)
b) (i) and (iii)
c) (ii) and (iii)
d) (iii) and (iv)

11) The function $f(z) = \bar{z}^2 + i\,\bar{z} + 1$ is differentiable at

(GATE MA 2014)

a) i
b) 1
c) $-i$
d) no point in $\mathbb{C}$

12) The radius of convergence of the power series

$$\sum_{n=0}^{\infty} 4^{(-1)^n n}\, z^{2n}$$

is ____________.

(GATE MA 2014)

13) Let E_1 and E_2 be two non empty subsets of a normed linear space X, and let

$$E_1 + E_2 \;:=\; \{\, x + y \in X : x \in E_1 \text{ and } y \in E_2 \,\}.$$

Which of the following statements is **FALSE**?

(GATE MA 2014)

a) If E_1 and E_2 are convex, then $E_1 + E_2$ is convex
b) If either E_1 or E_2 is open, then $E_1 + E_2$ is open
c) $E_1 + E_2$ must be closed if E_1 and E_2 are closed
d) If E_1 is closed and E_2 is compact, then $E_1 + E_2$ is closed

14) Let $y(x)$ be the solution to the initial value problem

$$\frac{dy}{dx} = \sqrt{y} + 2x, \qquad y(1.2) = 2.$$

Using the Euler method with step size $h = 0.05$, the approximate value of $y(1.3)$, correct to two decimal places, is ______________.

(GATE MA 2014)

15) Let $\alpha \in \mathbb{R}$. If αx is the polynomial which interpolates the function $f(x) = \sin(\pi x)$ on $[-1, 1]$ at all the zeroes of the polynomial $4x^3 - 3x$, then α is _________.

(GATE MA 2014)

16) If $u(x,t)$ is the D'Alembert's solution to the wave equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \qquad x \in \mathbb{R},\ t > 0,$$

with the conditions $u(x,0) = 0$ and $\dfrac{\partial u}{\partial t}(x,0) = \cos x$, then $u\left(0, \dfrac{\pi}{4}\right)$ is __________.

(GATE MA 2014)

17) The solution of the integral equation

$$\phi(x) = x + \int_0^x \sin(x - \xi)\,\phi(\xi)\,d\xi$$

is

(GATE MA 2014)

a) $x^2 + \dfrac{x^3}{3}$
b) $x - \dfrac{x^3}{3!}$
c) $x + \dfrac{x^3}{3!}$
d) $x^2 - \dfrac{x^3}{3!}$

18) The general solution to the ordinary differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(4x^2-\frac{9}{25}\right)y=0$$

in terms of Bessel's functions $J_\nu(\cdot)$, is

(GATE MA 2014)

a) $y(x)=c_1\,J_{3/5}(2x)+c_2\,J_{-3/5}(2x)$
b) $y(x)=c_1\,J_{3/10}(x)+c_2\,J_{-3/10}(x)$
c) $y(x)=c_1\,J_{3/5}(x)+c_2\,J_{-3/5}(x)$
d) $y(x)=c_1\,J_{3/10}(2x)+c_2\,J_{-3/10}(2x)$

19) The inverse Laplace transform of

$$\frac{2s^2-4}{(s-3)\,(s^2-s-2)}$$

is

(GATE MA 2014)

a) $(1+t)\,e^{-t}+\frac{7}{2}\,e^{-3t}$
b) $\frac{e^t}{3}+t\,e^{-t}+2t$
c) $\frac{7}{2}\,e^{3t}-\frac{e^{-t}}{6}-\frac{4}{3}\,e^{2t}$
d) $\frac{7}{2}\,e^{-3t}-\frac{e^{t}}{6}-\frac{4}{3}\,e^{-2t}$

20) If X_1, X_2 is a random sample of size 2 from $\mathcal{N}(0,1)$ population, then

$$\frac{(X_1+X_2)^2}{(X_1-X_2)^2}\text{ follows}$$

follows

(GATE MA 2014)

a) $\chi^2(2)$
b) $F(2,2)$
c) $F(2,1)$
d) $F(1,1)$

21) Let $Z\sim\mathcal{N}(0,1)$ be a random variable. Then the value of $\mathbb{E}[\max\{Z,0\}]$ is

(GATE MA 2014)

a) $\frac{1}{\sqrt{\pi}}$
b) $\sqrt{\frac{2}{\pi}}$
c) $\frac{1}{\sqrt{2\pi}}$
d) $\frac{1}{\pi}$

22) The number of non-isomorphic groups of order 10 is ____________.

(GATE MA 2014)

23) Let $a,b,c,d\in\mathbb{R}$ with $a<c<d<b$. Consider the ring $C[a,b]$ with pointwise addition and multiplication. If

$$S=\{\,f\in C[a,b]:f(x)=0\text{ for all }x\in[c,d]\,\},$$

then

(GATE MA 2014)

a) S is NOT an ideal of $C[a,b]$
b) S is an ideal of $C[a,b]$ but NOT a prime ideal of $C[a,b]$
c) S is a prime ideal of $C[a,b]$ but NOT a maximal ideal of $C[a,b]$
d) S is a maximal ideal of $C[a,b]$

24) Let R be a ring. If $R[x]$ is a principal ideal domain, then R is necessarily a

(GATE MA 2014)

a) Unique Factorization Domain
b) Principal Ideal Domain
c) Euclidean Domain
d) Field

25) Consider the group homomorphism $\varphi : M_2(\mathbb{R}) \to \mathbb{R}$ given by $\varphi(A) = \text{trace}(A)$. The kernel of φ is isomorphic to

(GATE MA 2014)

a) $\{A \in M_2(\mathbb{R}) : \varphi(A) = 0\}$
b) $\mathbb{R}^2$
c) $\mathbb{R}^3$
d) $\text{GL}_2(\mathbb{R})$

26) Let X be a set with at least two elements. Let τ and τ' be two topologies on X with $\tau' \neq \{\emptyset, X\}$. Which of the following conditions is necessary for the identity function $\text{id} : (X, \tau) \to (X, \tau')$ to be continuous?

(GATE MA 2014)

a) $\tau \subseteq \tau'$
b) $\tau' \subseteq \tau$
c) no conditions on τ and τ'
d) $\tau \cap \tau' = \{\emptyset, X\}$

27) Let $A \in M_3(\mathbb{R})$ satisfy $\det(A - I) = 0$. If $\text{trace}(A) = 13$ and $\det(A) = 32$, then the sum of squares of the eigenvalues of A is ____________.

(GATE MA 2014)

28) Consider the group homomorphism $\varphi : M_2(\mathbb{R}) \to \mathbb{R}$ given by $\varphi(A) = \text{trace}(A)$. The kernel of φ is isomorphic to which of the following groups?

a) $M_2(\mathbb{R})/\{A \in M_2(\mathbb{R}) : \varphi(A) = 0\}$
b) $\mathbb{R}^2$
c) $\mathbb{R}^3$
d) $\text{GL}_2(\mathbb{R})$

29) Let V be a real inner product space of dimension 10. Let $x, y \in V$ be non-zero vectors such that $\langle x, y \rangle = 0$. Then the dimension of $\{x\}^1 \cap \{y\}^1$ is _______________.

(GATE MA 2014)

30) Consider the following linear programming problem
Minimize $\quad x_1 + x_2$
Subject to

$$2x_1 + x_2 \geq 8, \qquad 2x_1 + 5x_2 \geq 10, \qquad x_1, x_2 \geq 0.$$

The optimal value to this problem is _______________.

(GATE MA 2014)

31) Let

$$f(x) = \begin{cases} -3\pi, & -\pi < x \leq 0, \\ 3\pi, & 0 < x < \pi, \end{cases} \qquad \text{and extend } f \text{ to be } 2\pi\text{-periodic.}$$

The coefficient of $\sin(3x)$ in the Fourier series expansion of $f(x)$ on $[-\pi, \pi]$ is _______________.

(GATE MA 2014)

32) For the sequence of functions

$$f_n(x) = \frac{1}{x^2}\sin\left(\frac{1}{nx}\right), \qquad x \in [1,\infty),$$

consider the following quantities expressed in terms of Lebesgue integrals:

$$\text{I}: \lim_{n\to\infty}\int_1^{\infty} f_n(x)\,dx, \qquad \text{II}: \int_1^{\infty}\left(\lim_{n\to\infty} f_n(x)\right)dx.$$

Which of the following is **TRUE**?

(GATE MA 2014)

a) The limit in I does not exist
b) The integrand in II is not integrable on $[1,\infty)$
c) Quantities I and II are well-defined, but I $\neq$ II
d) Quantities I and II are well-defined and I = II

33) Which of the following statements about the spaces ℓ^p and $L^p[0,1]$ is **TRUE**?

(GATE MA 2014)

a) $\ell^3 \subset \ell^7$ and $L^6[0,1] \subset L^9[0,1]$
b) $\ell^3 \subset \ell^7$ and $L^9[0,1] \subset L^6[0,1]$
c) $\ell^7 \subset \ell^3$ and $L^6[0,1] \subset L^9[0,1]$
d) $\ell^7 \subset \ell^3$ and $L^9[0,1] \subset L^6[0,1]$

34) The maximum modulus of e^{z^2} on the set

$$S = \{\, z \in \mathbb{C} : 0 \le \Re(z) \le 1,\ 0 \le \Im(z) \le 1 \,\}$$

is

(GATE MA 2014)

a) $2/e$
b) e
c) $e+1$
d) e^2

35) Let d_1, d_2 and d_3 be metrics on a set X with at least two elements. Which of the following is *NOT* a metric on X?

(GATE MA 2014)

a) $\min\{d_1, 2\}$
b) $\max\{d_2, 2\}$
c) $\dfrac{d_3}{1+d_3}$
d) $\dfrac{d_1+d_2+d_3}{3}$

36) Let $\Omega = \{\, z \in \mathbb{C} : \Im(z) > 0 \,\}$ and let C be a smooth curve lying in Ω with initial point $-1+2i$ and final point $1+2i$. The value of $\displaystyle\int_C \frac{1+2z}{1+z}\,dz$ is

(GATE MA 2014)

a) $4 - \frac{1}{2}\ln 2 + i\frac{\pi}{4}$
b) $-4 + \frac{1}{2}\ln 2 + i\frac{\pi}{4}$
c) $4 + \frac{1}{2}\ln 2 - i\frac{\pi}{4}$
d) $4 - \frac{1}{2}\ln 2 + i\frac{\pi}{2}$

37) If $a \in \mathbb{C}$ with $|a| < 1$, then the value of

$$\frac{1-|a|^2}{\pi}\int_{\Gamma}\frac{|dz|}{|z+\overline{a}|^2}, \qquad \Gamma: \ |z| = 1 \text{ (positively oriented)},$$

is ______________.

(GATE MA 2014)

38) Consider $C[-1,1]$ equipped with the supremum norm

$$\|f\|_\infty = \sup\{\,|f(t)| : t \in [-1,1]\,\} \qquad \text{for } f \in C[-1,1].$$

Define a linear functional T on $C[-1,1]$ by

$$T(f) = \int_0^1 f(t)\,dt \;-\; \int_{-1}^0 f(t)\,dt \qquad \text{for all } f \in C[-1,1].$$

Then the value of $\|T\|$ is _______________.

(GATE MA 2014)

39) Consider the vector space $C[0,1]$ over $\mathbb{R}$. Consider the following statements:
P: If the set $\{\,tf_1,\ t^2 f_2,\ t^3 f_3\,\}$ is linearly independent, then the set $\{\,f_1, f_2, f_3\,\}$ is linearly independent, where $f_1, f_2, f_3 \in C[0,1]$ and t^n denotes the polynomial $t \mapsto t^n$, $n \in \mathbb{N}$.
Q: If $F : C[0,1] \to \mathbb{R}$ is given by

$$F(x) = \int_0^1 x(t^2)\,dt \qquad \text{for each } x \in C[0,1],$$

then F is a linear map.
Which of the above statements hold **TRUE**?

(GATE MA 2014)

a) Only **P**
b) Only **Q**
c) Both **P** and **Q**
d) Neither **P** nor **Q**

40) Using the Newton-Raphson method with the initial guess $x^{(0)} = 6$, the approximate value of the real root of $x\log_{10} x = 4.77$, after the second iteration, is ________________.

(GATE MA 2014)

41) Let the following discrete data be obtained from a curve $y = y(x)$:

x :	0	0.25	0.50	0.75	1.00
y :	1	0.9896	0.9589	0.9089	0.8415

Let S be the solid of revolution obtained by rotating the above curve about the x-axis between $x = 0$ and $x = 1$, and let V denote its volume. The approximate value of V, obtained using Simpson's $\frac{1}{3}$ rule, is ________________.

(GATE MA 2014)

42) The integral surface of the first-order partial differential equation

$$2y\,(z-3)\,\frac{\partial z}{\partial x} \;+\; (2x-z)\,\frac{\partial z}{\partial y} \;=\; y\,(2x-3),$$

passing through the curve $x^2 + y^2 = 2x,\ z = 0$, is

(GATE MA 2014)

A. $x^2 + y^2 - z^2 - 2x + 4z = 0$
B. $x^2 + y^2 - z^2 - 2x + 8z = 0$
C. $x^2 + y^2 + z^2 - 2x + 16z = 0$
D. $x^2 + y^2 + z^2 - 2x + 8z = 0$

43) The boundary value problem

$$\frac{d^2\phi}{dx^2} + \lambda\,\phi = x, \qquad \phi(0) = 0, \quad \frac{d\phi}{dx}(1) = 0,$$

is converted into the integral equation

$$\phi(x) = g(x) + \lambda \int_0^1 k(x,\xi)\,\phi(\xi)\,d\xi, \qquad k(x,\xi) = \begin{cases} \xi, & 0 < \xi < x, \\ x, & x < \xi < 1. \end{cases}$$

Then $g\left(\frac{2}{3}\right)$ is ______________. (GATE MA 2014)

44) If $y_1(x) = x$ is a solution to the differential equation

$$(1 - x^2)\,\frac{d^2y}{dx^2} - 2x\,\frac{dy}{dx} + 2y = 0,$$

then its general solution is

(GATE MA 2014)

a) $y(x) = c_1 x + c_2\left(x \ln\left|1 + x^2\right| - 1\right)$
b) $y(x) = c_1 x + c_2\left(\ln\left|\frac{1-x}{1+x}\right| + 1\right)$
c) $y(x) = c_1 x + c_2\left(\frac{x}{2}\ \ln\left|1 - x^2\right| + 1\right)$
d) $y(x) = c_1 x + c_2\left(\frac{x}{2}\ \ln\left|\frac{1+x}{1-x}\right| - 1\right)$

45) The solution to the initial value problem

$$\frac{d^2y}{dt^2} + 2\,\frac{dy}{dt} + 5y \;=\; 3e^{-t}\sin t, \qquad y(0) = 0, \quad y'(0) = 3,$$

is

(GATE MA 2014)

a) $y(t) = e^{t}(\sin t + \sin 2t)$
b) $y(t) = e^{-t}(\sin t + \sin 2t)$
c) $y(t) = 3e^{t}\sin t$
d) $y(t) = 3e^{-t}\sin t$

46) The time to failure, in months, of light bulbs manufactured at two plants A and B obey the exponential distributions with means 6 and 2 months, respectively. Plant B produces four times as many bulbs as plant A. Bulbs are indistinguishable, mixed, and sold together. Given that a randomly purchased bulb is working after 12 months, the probability that it was manufactured at plant A is ______________.
(GATE MA 2014)

47) Let X, Y be continuous random variables with joint density

$$f_{X,Y}(x,y) = \begin{cases} e^{-y}(1 - e^{-x}), & 0 < x < y < \infty, \\ e^{-x}(1 - e^{-y}), & 0 < y \le x < \infty, \\ 0, & \text{otherwise.} \end{cases}$$

The value of $E[X + Y]$ is ______________.

(GATE MA 2014)

48) Let $X = [0, 1) \cup (1, 2)$ be the subspace of $\mathbb{R}$ with the usual topology. Which of the following is **FALSE**?

(GATE MA 2014)

a) There exists a non-constant continuous function $f : X \to \mathbb{Q}$.
b) X is homeomorphic to $(-\infty, -3) \cup [0, \infty)$.
c) There exists an onto continuous function $f : \overline{X} \to [0, 1]$, where $\overline{X}$ is the closure of X in $\mathbb{R}$.
d) There exists an onto continuous function $f : X \to [0, 1]$.

49) Let X=$\begin{pmatrix} 2 & 0 & -3 \\ 3 & -1 & -3 \\ 0 & 0 & -1 \end{pmatrix}$. A matrix P such that $P^{-1}XP$ is diagonal is (GATE MA 2014)

a) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

b) $\begin{pmatrix} -1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

c) $\begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

d) $\begin{pmatrix} -1 & -1 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

50) Using the Gauss-Seidel iteration method with the initial guess $(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}) = (3.5, 2.25, 1.625)$, the second approximation $(x_1^{(2)}, x_2^{(2)}, x_3^{(2)})$ for the solution to the system

$$\begin{aligned} 2x_1 - x_2 &= 7, \\ -x_1 + 2x_2 - x_3 &= 1, \\ -x_2 + 2x_3 &= 1, \end{aligned}$$

is

(GATE MA 2014)

a) $x_1^{(2)} = 5.3125, \quad x_2^{(2)} = 4.4491, \quad x_3^{(2)} = 2.1563$
b) $x_1^{(2)} = 5.3125, \quad x_2^{(2)} = 4.3125, \quad x_3^{(2)} = 2.6563$
c) $x_1^{(2)} = 5.3125, \quad x_2^{(2)} = 4.4491, \quad x_3^{(2)} = 2.6563$
d) $x_1^{(2)} = 5.4991, \quad x_2^{(2)} = 4.4491, \quad x_3^{(2)} = 2.1563$

51) The fourth-order Runge-Kutta method

$$u_{j+1} = u_j + \frac{h}{6}(K_1 + 2K_2 + 2K_3 + K_4), \qquad j = 0, 1, 2, \ldots,$$

is used to solve the initial value problem $\frac{du}{dt} = u,\ u(0) = \alpha$. If $u(1) = 1$ is obtained by taking the step size $h = 1$, then the value of K_4 is _______________.

(GATE MA 2014)

52) A particle P of mass m moves along the cycloid $x = \theta - \sin\theta$ and $y = 1 + \cos\theta$, $0 \leq \theta \leq 2\pi$. Let g denote the acceleration due to gravity. Neglecting friction, the Lagrangian associated with the motion of the particle P is:

(GATE MA 2014)

A. $m(1 - \cos\theta)\,\dot{\theta}^2 \ - \ mg(1 + \cos\theta)$

B. $m(1 + \cos\theta)\,\dot{\theta}^2 \ + \ mg(1 + \cos\theta)$

C. $m(1 + \cos\theta)\,\dot{\theta}^2 \ + \ mg(1 - \cos\theta)$

D. $m(\theta - \sin\theta)\,\dot{\theta}^2 \ - \ mg(1 + \cos\theta)$

53) Suppose that X is a population random variable with probability density

$$f(x;\theta) = \begin{cases} \theta\, x^{\theta-1}, & 0 < x < 1, \\ 0, & \text{otherwise}, \end{cases}$$

where θ is a parameter. To test the null hypothesis $H_0 : \ \theta = 2$ against the alternative $H_1 : \ \theta = 3$, use the rule: reject H_0 if $X_1 \geq \frac{1}{2}$ and accept otherwise, where X_1 is a single observation drawn from the above population. Then the power of this test is _______________.

(GATE MA 2014)

54) Suppose that $X_1, X_2, \dots, X_n$ is a random sample of size n drawn from a population with probability density function

$$f(x;\theta) = \begin{cases} \dfrac{x}{\theta^2} e^{-x/\theta}, & x > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta > 0$. The maximum likelihood estimator of θ is

(GATE MA 2014)

a) $\frac{1}{n}\sum_{i=1}^{n} X_i$

b) $\frac{1}{n-1}\sum_{i=1}^{n} X_i$

c) $\frac{1}{2n}\sum_{i=1}^{n} X_i$

d) $\frac{2}{n}\sum_{i=1}^{n} X_i$

55) Let $\mathbf{F}$ be a vector field on $\mathbb{R}^2 \setminus \{(0,0)\}$ defined by

$$\mathbf{F}(x,y) = \left(\frac{y}{x^2+y^2},\ -\frac{x}{x^2+y^2}\right).$$

Let $\gamma, \alpha : [0,1] \to \mathbb{R}^2$ be given by

$$\gamma(t) = (8\cos(2\pi t),\ 17\sin(2\pi t)), \qquad \alpha(t) = (26\cos(2\pi t),\ -10\sin(2\pi t)).$$

If

$$3\oint_{\alpha} \mathbf{F}\cdot d\mathbf{r} \ - \ 4\oint_{\gamma} \mathbf{F}\cdot d\mathbf{r} \ = \ 2m\pi,$$

then m is _______.

(GATE MA 2014)

56) Let $g : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$g(x,y,z) = (3y+4z,\ 2x-3z,\ x+3y),$$

and let

$$S = \{(x,y,z) \in \mathbb{R}^3 :\ 0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le z \le 1\}.$$

If

$$\iiint_{g(S)} (2x+y-2z)\,dx\,dy\,dz \ = \ \alpha \iiint_{S} z\,dx\,dy\,dz,$$

then α is __________.

(GATE MA 2014)

57) Let $T_1, T_2 : \mathbb{R}^5 \to \mathbb{R}^3$ be linear transformations such that $\operatorname{rank}(T_1) = 3$ and $\operatorname{nullity}(T_2) = 3$. Let $T_3 : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T_3 \circ T_1 = T_2$. Then $\operatorname{rank}(T_3)$ is ____________.

(GATE MA 2014)

58) Let $\mathbb{F}_3$ be the field with 3 elements and let $\mathbb{F}_3 \times \mathbb{F}_3$ be the vector space over $\mathbb{F}_3$. The number of distinct linearly dependent sets of the form $\{u, v\}$, where $u, v \in \mathbb{F}_3 \times \mathbb{F}_3 \setminus \{(0,0)\}$ and $u \neq v$, is ____________.

(GATE MA 2014)

59) Let $\mathbb{F}_{125}$ be the field of 125 elements. The number of nonzero elements $\alpha \in \mathbb{F}_{125}$ such that $\alpha^5 = \alpha$ is ______________.

(GATE MA 2014)

60) The value of $\iint_R x\,y\,dx\,dy$, where R is the region in the first quadrant bounded by the curves $y = x^2$, $y + x = 2$ and $x = 0$, is ______________.

(GATE MA 2014)

61) Consider the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < \pi, \ t > 0,$$

with boundary conditions $u(0,t) = 0$, $u(\pi,t) = 0$ for $t > 0$, and initial condition $u(x,0) = \sin x$. Then $u\left(\frac{\pi}{2}, 1\right)$ is ____________.

(GATE MA 2014)

62) Consider the partial order on $\mathbb{R}^2$ given by

$$(x_1, y_1) < (x_2, y_2) \text{ either if } x_1 < x_2 \text{ or if } x_1 = x_2 \text{ and } y_1 < y_2.$$

Then, in the order topology on $\mathbb{R}^2$ defined by the above order, which of the following is **TRUE**?

(GATE MA 2014)

a) $[0,1] \times \{1\}$ is compact but $[0,1] \times [0,1]$ is *not* compact
b) $[0,1] \times [0,1]$ is compact but $[0,1] \times \{1\}$ is *not* compact
c) Both $[0,1] \times [0,1]$ and $[0,1] \times \{1\}$ are compact
d) Both $[0,1] \times [0,1]$ and $[0,1] \times \{1\}$ are *not* compact

63) Consider the following linear programming problem:

$$\begin{aligned} \text{Minimize} \quad & x_1 + x_2 + 2x_3 \\ \text{Subject to} \quad & x_1 + 2x_2 \ \geq \ 4, \\ & x_2 + 7x_3 \ \leq \ 5, \\ & x_1 - 3x_2 + 5x_3 \ = \ 6, \\ & x_1, \ x_2 \ \geq \ 0, \qquad x_3 \text{ is unrestricted.} \end{aligned}$$

The dual to this problem is:

$$\begin{aligned} \text{Maximize} \quad & 4y_1 + 5y_2 + 6y_3 \\ \text{Subject to} \quad & y_1 + y_3 \ \leq \ 1, \\ & 2y_1 + y_2 - 3y_3 \ \leq \ 1, \\ & 7y_2 + 5y_3 \ = \ 2. \end{aligned}$$

Which choice of signs on (y_1, y_2, y_3) is correct?

(GATE MA 2014)

a) $y_1 \geq 0$, $y_2 \leq 0$ and y_3 is unrestricted
b) $y_1 \geq 0$, $y_2 \geq 0$ and y_3 is unrestricted
c) $y_1 \geq 0$, $y_3 \leq 0$ and y_2 is unrestricted
d) $y_3 \geq 0$, $y_2 \leq 0$ and y_1 is unrestricted

64) Let $X = C^1[0,1]$. For each $f \in X$, define

$$p_1(f) := \sup\{|f(t)| : t \in [0,1]\}, \qquad p_2(f) := \sup\{|f'(t)| : t \in [0,1]\}, \qquad p_3(f) := p_1(f) + p_2(f).$$

Which of the following statements is **TRUE**? (GATE MA 2014)

a) (X, p_1) is a Banach space
b) (X, p_2) is a Banach space
c) (X, p_3) is *NOT* a Banach space
d) (X, p_3) does *NOT* have denumerable basis