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Matrix 2.6.24

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Question (2.6.24)

Find the area of the parallelogram formed by the vectors

$$\mathbf{a} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}, \qquad \mathbf{b} = \hat{\imath} - \hat{\jmath} + \hat{k}.$$

Use the cross product definition.

Solution

1. Let

$$\mathbf{A} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \tag{2}$$

   and define the sub-vectors

$$\mathbf{A}_{ij} = \begin{pmatrix} a_i \\ a_j \end{pmatrix}, \qquad \mathbf{B}_{ij} = \begin{pmatrix} b_i \\ b_j \end{pmatrix}. \tag{3}$$

2. The cross product of **A** and **B** is defined as

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} \begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \\ \mathbf{A}_{31} & \mathbf{B}_{31} \\ \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} \end{pmatrix}. \tag{4}$$

3. For the given vectors

$$\mathbf{a} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}. \tag{5}$$

4. Substituting into the definition:

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} \left| \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right| \\ \left| \begin{pmatrix} 4 \\ 3 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right| \\ \left| \begin{pmatrix} 3 \\ 1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right| \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} (1)(1) - (4)(-1) \\ (4)(1) - (3)(1) \\ (3)(-1) - (1)(1) \end{pmatrix} \tag{7}$$

$$= \begin{pmatrix} 5 \\ 1 \\ -4 \end{pmatrix}. \tag{8}$$

5. Area of the parallelogram is

$$\text{Area} = \|\mathbf{a} \times \mathbf{b}\| = \sqrt{5^2 + 1^2 + (-4)^2} = \sqrt{42}. \tag{9}$$

Final Answer

$$\boxed{\text{Area} = \sqrt{42}}$$

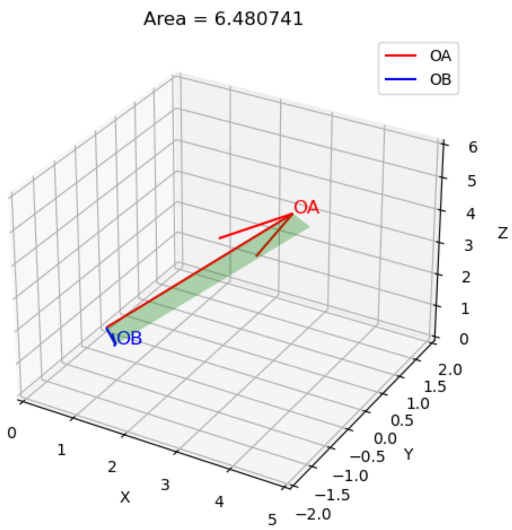

Figure 1: Parallelogram spanned by $\mathbf{a}$ and $\mathbf{b}$.