1.3.4

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If $A(1,3), B(-1,2), C(2,5)$ and $D(x,4)$ are the vertices of a parallelogram ABCD , then the value of x is _____(10, 2012)

Solution:
In a parallelogram, the opposite sides are equal. Therefore, the length of side AC equals to the length of side BD:

$$\mathbf{C} - \mathbf{A} = \mathbf{D} - \mathbf{B} \tag{0.1}$$
$$\mathbf{D} = \mathbf{B} + \mathbf{C} - \mathbf{A} \tag{0.2}$$

Substituting the coordinates:

$$\begin{pmatrix} x \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{0.3}$$
$$= \begin{pmatrix} -1+2-1 \\ 2+5-3 \end{pmatrix} \tag{0.4}$$
$$= \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{0.5}$$

This gives us the equations:

$$x = 0 \tag{0.6}$$
$$4 = 4 \tag{0.7}$$

Answer: x=0