# 2.4.27

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# Question

Three vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ satisfy the condition $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$. Evaluate the quantity $\mu = \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}$. If $|\mathbf{a}| = 3$, $|\mathbf{b}| = 4$ and $|\mathbf{c}| = 2$.

# Solution

Given:

$$\mathbf{a}+\mathbf{b}+\mathbf{c}=0 \text{ and } \|\mathbf{a}\|=3\ ,\ \|\mathbf{b}\|=4\ ,\ \|\mathbf{c}\|=2 \tag{1}$$

To find

$$\mu=\mathbf{a}^\top\mathbf{b}+\mathbf{b}^\top\mathbf{c}+\mathbf{c}^\top\mathbf{a} \tag{2}$$

# Solution

Multiplying $\mathbf{a}+\mathbf{b}+\mathbf{c}=0$ with $\mathbf{a}^\top$ on both sides

$$\mathbf{a}^\top\mathbf{a}+\mathbf{a}^\top\mathbf{b}+\mathbf{a}^\top\mathbf{c}=0 \tag{3}$$

Similarly, upon multiplying with $\mathbf{b}^\top$ and $\mathbf{c}^\top$, we get

$$\mathbf{b}^\top\mathbf{a}+\mathbf{b}^\top\mathbf{b}+\mathbf{b}^\top\mathbf{c}=0 \tag{4}$$

$$\mathbf{c}^\top\mathbf{a}+\mathbf{c}^\top\mathbf{b}+\mathbf{c}^\top\mathbf{c}=0 \tag{5}$$

# Solution

Adding the above three equations,

$$2\left(\mathbf{a}^\top\mathbf{b} + \mathbf{b}^\top\mathbf{c} + \mathbf{c}^\top\mathbf{a}\right) + \mathbf{a}^\top\mathbf{a} + \mathbf{b}^\top\mathbf{b} + \mathbf{c}^\top\mathbf{c} = 0 \tag{6}$$

$$\implies 2\mu + \mathbf{a}^\top\mathbf{a} + \mathbf{b}^\top\mathbf{b} + \mathbf{c}^\top\mathbf{c} = 0 \tag{7}$$

By using $\mathbf{x}^\top\mathbf{x} = \|\mathbf{x}\|^2$ we get

$$2\mu + \left(\|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + \|\mathbf{c}\|^2\right) = 0 \tag{8}$$

# Solution

Substituting the values of $\|\mathbf{a}\|, \|\mathbf{b}\|, \|\mathbf{c}\|$ we get

$$\mu = \frac{-29}{2} \tag{9}$$

$\therefore$ The value of $\mu$ is $\frac{-29}{2}$.