12.235

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Question

Question:
A system of equations represented as

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 4 \\ 1 & 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ y \\ 3 \end{pmatrix} \textit{is}, \tag{1}$$

1. consistent and has a unique solution
2. inconsistent and has no solution
3. consistent and infinite solution
4. inconsistent and has unique solution

Solution

This can be represented as an augmented matrix and can be solved by using Gaussian elimination.

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 2 & 1 & 4 & y \\ 1 & 3 & 1 & 3 \end{array}\right) \xleftrightarrow[R_3 \leftarrow R_3 - R_1]{R_2 \leftarrow R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 3 & 0 & y-8 \\ 0 & 4 & -1 & -1 \end{array}\right) \tag{2}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - 4R_2]{R_2 \leftarrow \frac{R_2}{3}} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 1 & 0 & \frac{y-8}{3} \\ 0 & 0 & -1 & \frac{29-4y}{3} \end{array}\right) \xleftrightarrow[R_1 \leftarrow R_1 - 2R_3]{R_3 \leftarrow -R_3} \tag{3}$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & \frac{41-8y}{3} \\ 0 & 1 & 0 & \frac{y-8}{3} \\ 0 & 0 & 1 & \frac{4y-29}{3} \end{array}\right) \xleftrightarrow{R_1 \leftarrow R_1 + R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{33-7y}{3} \\ 0 & 1 & 0 & \frac{y-8}{3} \\ 0 & 0 & 1 & \frac{4y-29}{3} \end{array}\right) \tag{4}$$

Solution

Since $y \in \mathbf{R}$, we can conclude that there exists a unique solution and the system of equations is consistent.

Option (1) is the correct answer

C Code

```c
#include <stdio.h>

int determinant(int mat[3][3]) {
    int det;
    det = mat[0][0]*(mat[1][1]*mat[2][2] - mat[1][2]*mat[2][1])
        - mat[0][1]*(mat[1][0]*mat[2][2] - mat[1][2]*mat[2][0])
        + mat[0][2]*(mat[1][0]*mat[2][1] - mat[1][1]*mat[2][0]);
    return det;
}
```

Python + C Code

```
import ctypes
import numpy as np

lib = ctypes.CDLL("./libcode.so")
lib.determinant.argtypes = [np.ctypeslib.ndpointer(dtype=np.int32
    , shape=(3,3))]
lib.determinant.restype = ctypes.c_int
A = np.array([[1, -1, 2],
              [2, 1, 4],
              [1, 3, 1]], dtype=np.int32)

det = lib.determinant(A)
if det != 0:
    print("Unique solution exists and consistent system of
        equations")
else:
    print("Inconsistent system of equations")
```

Python Code

```
import numpy as np

A = np.array([[1, -1, 2], [2, 1, 4], [1, 3, 1]])
det = np.linalg.det(A)

if(det!=0):
        print("Unique solution exists and consistent system of
            equations")
else:
        print("Inconsistent system of equations")
```