# Problem 12.349

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October 9, 2025

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# Problem

Let $T_1, T_2 : \mathbb{R}_5 \to \mathbb{R}_3$ be linear transformations such that $\operatorname{rank}(T_1) = 3$ and $\operatorname{nullity}(T_2) = 3$. Let $T_3 : \mathbb{R}_3 \to \mathbb{R}_3$ be a linear transformation such that $T_3 \circ T_1 = T_2$ . Then $\operatorname{rank}(T_3)$ is . . . (MA 2014)

# Given

According to Rank-Nullity theorem,
For a linear transformation $T : \mathbb{R}_m \to \mathbb{R}_n$

$$\text{rank}\,(T) + \text{nullity}\,(T) = \dim\,(\text{domain}) \tag{3.1}$$

where $\dim \mathbb{R}_m$ is the dimension of the domain i.e vector space $\mathbb{R}_m$

Given $T_2 : \mathbb{R}_5 \to \mathbb{R}_3$ and nullity($T_2$)=3

$$\text{rank}\,(T_2) + \text{nullity}\,(T_2) = \dim \mathbb{R}_5 \tag{3.2}$$
$$\text{rank}\,(T_2) + 3 = 5 \tag{3.3}$$
$$\text{rank}\,(T_2) = 2 \tag{3.4}$$

Given $T_1 : \mathbb{R}_5 \to \mathbb{R}_3$ and rank($T_1$)=3

$$\dim\,(\text{Co-domain}) = 3 \tag{3.5}$$
$$\text{rank}\,(T_1) = \dim\,(\text{Co-domain}) \tag{3.6}$$

## Conclusion

It is onto and hence

$$\dim\left(\mathrm{Im}\left(T_1\right)\right) = \dim\left(\text{Co-domain}\right) \tag{3.7}$$

$$\dim\left(\mathrm{Im}\left(T_1\right)\right) = 3 \implies \mathrm{Im}\left(T_1\right) = \mathbb{R}_3 \tag{3.8}$$

where $\mathrm{Im}\left(T\right)$ is the Image space of the linear transformation $T$

Given $T_3 : \mathbb{R}_3 \to \mathbb{R}_3$

$$T_3 \circ T_1 = T_2 \tag{3.9}$$

$$\left(T_3 \circ T_1\right)\left(\mathbb{R}_5\right) = \mathrm{Im}\left(T_2\right) \tag{3.10}$$

$$T_3\left(T_1\left(R_5\right)\right) = \mathrm{Im}\left(T_2\right) \tag{3.11}$$

$$T_3\left(\mathrm{Im}\, T_1\right) = \mathrm{Im}\left(T_2\right) \tag{3.12}$$

$$T_3\left(\mathbb{R}_3\right) = \mathrm{Im}\left(T_2\right) \tag{3.13}$$

$$\mathrm{Im}\left(T_3\right) = \mathrm{Im}\left(T_2\right) \tag{3.14}$$

$$\implies \mathrm{rank}(T_3) = \mathrm{rank}(T_2) \tag{3.15}$$

From (4)

$$\mathrm{rank}(T_3) = 2 \tag{3.16}$$