

Matrices in Geometry 1.5.25

EE25BTECH11037 - Divyansh

Question: In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points $\mathbf{P} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$? Also find the value of y .

Given: $\mathbf{P} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ and a point $\mathbf{R} = \begin{pmatrix} \frac{24}{11} \\ y \end{pmatrix}$ on PQ .

Let R divide PQ internally in the ratio $k : 1$.

Therefore, they are defined to be collinear if,

$$\begin{aligned} \text{rank}(\mathbf{R} - \mathbf{P} \quad \mathbf{Q} - \mathbf{R}) &= 1 \\ \mathbf{R} - \mathbf{P} &= \begin{pmatrix} \frac{2}{11} \\ y + 2 \end{pmatrix} \\ \mathbf{Q} - \mathbf{R} &= \begin{pmatrix} \frac{9}{11} \\ 7 - y \end{pmatrix} \\ \Rightarrow \text{rank} \begin{pmatrix} \frac{2}{11} & \frac{9}{11} \\ y + 2 & 7 - y \end{pmatrix} &= 1 \\ \Rightarrow \Delta &= 0 \\ \frac{2}{11}(7 - y) - \frac{9}{11}(y + 2) &= 0 \\ 14 - 2y - 18 - 9y &= 0 \\ \Rightarrow y &= \frac{-4}{11} \end{aligned}$$

We know that k is the ratio in which \mathbf{R} divides \mathbf{P} and \mathbf{Q} ,

$$\begin{aligned} \mathbf{R} &= \frac{k\mathbf{Q} + \mathbf{P}}{1 + k} \\ k(\mathbf{R} - \mathbf{Q}) &= \mathbf{P} - \mathbf{R} \\ \Rightarrow k &= \frac{(\mathbf{P} - \mathbf{R})^\top (\mathbf{R} - \mathbf{Q})}{\|\mathbf{R} - \mathbf{Q}\|^2} \\ (\mathbf{P} - \mathbf{R})^\top &= \begin{pmatrix} -\frac{2}{11} & -\frac{18}{11} \end{pmatrix} \\ (\mathbf{R} - \mathbf{Q}) &= \begin{pmatrix} \frac{-9}{11} \\ \frac{-81}{11} \end{pmatrix} \\ \|\mathbf{R} - \mathbf{Q}\|^2 &= \frac{81}{121} + \frac{6561}{121} = \frac{6642}{121} \\ \therefore k &= \frac{\begin{pmatrix} -\frac{2}{11} & -\frac{18}{11} \end{pmatrix} \begin{pmatrix} \frac{-9}{11} \\ \frac{-81}{11} \end{pmatrix}}{\frac{6642}{121}} \end{aligned}$$

$$\Rightarrow k = \frac{\frac{18}{121} + \frac{1458}{121}}{\frac{6642}{121}} \Rightarrow k = \frac{1476}{6624} = \frac{2}{9}$$

Hence, the final answer is $k = \frac{2}{9}$ and $y = \frac{-4}{11}$

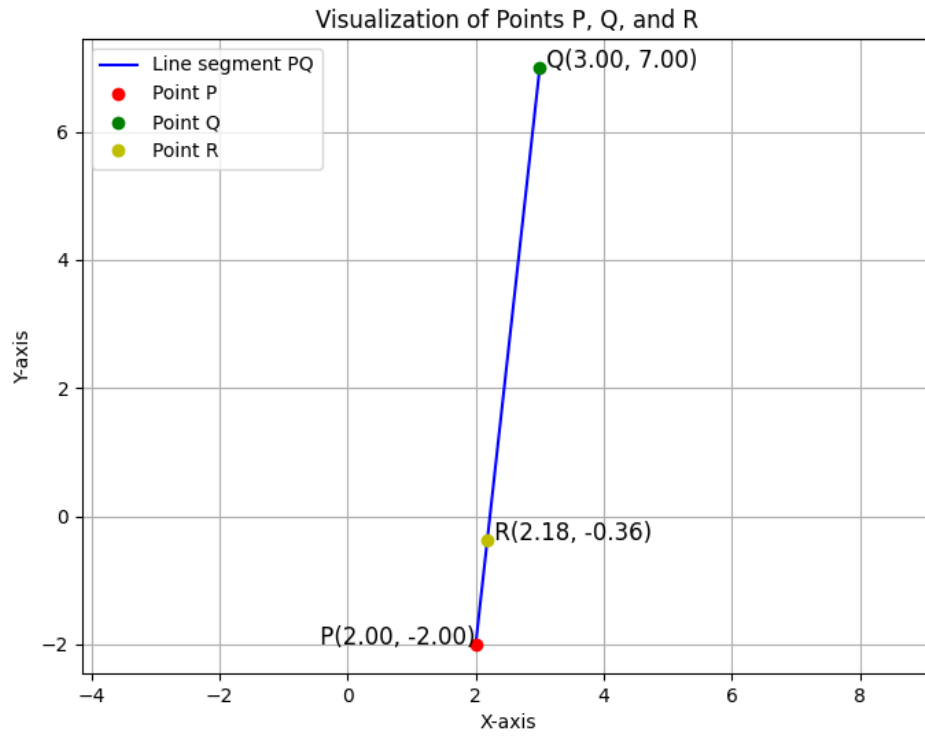


Fig. 1: Plot for 1.5.25