2.10.1

AI25BTECH11012 - GARIGE UNNATHI

Question:
Consider 3 points :

$$\mathbf{P} = (-\sin(\beta - \alpha), -\cos\beta), \mathbf{Q} = (\cos(\beta - \alpha), \sin\beta)$$

$$\mathbf{R} = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$$

where $\theta < \alpha, \beta, \theta < \frac{\pi}{4}$ Then,

1) P lies on the line segment RQ
2) Q lies on the line segment PR
3) R lies on the line segment QP
4) P,Q,R are non-collinear

Solution:
First we have to check if points can be collinear for the values satisfing the given conditions :

The eqution for coliinearity of the given points are :

$$Rank \begin{pmatrix} \mathbf{P} - \mathbf{Q} \\ \mathbf{R} - \mathbf{Q} \end{pmatrix} = 1 \tag{4.1}$$

$$Rank \begin{pmatrix} -\sin(\beta - \alpha) - \cos(\beta - \alpha) & -\cos\beta - \sin\beta \\ \cos(\beta - \alpha + \theta) - \cos(\beta - \alpha) & \sin(\beta - \theta) - \sin\beta \end{pmatrix} = 1 \tag{4.2}$$

$$\tag{4.3}$$

$$R_2 = R_2 - R_1 \tag{4.4}$$

$$Rank \begin{pmatrix} -\sin(\beta - \alpha) - \cos(\beta - \alpha) & -\cos\beta - \sin\beta \\ \cos(\beta - \alpha + \theta) - \sin(\beta - \alpha) & \sin(\beta - \theta) - \cos\beta \end{pmatrix} = 1 \tag{4.5}$$

For the rank to be 1 R_2 must be zero :

$$\cos(\beta - \alpha + \theta) - \sin(\beta - \alpha) = 0 \tag{4.6}$$

$$\tag{4.7}$$

This will only be satisfied if :

$$\theta = \frac{\pi}{2} + 2\pi K \quad or \quad \frac{\pi}{2} - 2\pi K \tag{4.8}$$

But ,given that :

$$0 < \theta < \frac{\pi}{4} \tag{4.9}$$

Which is contradictory :
Hence the points P,Q,R are not collinear .