

2.7.15

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Question

Find the volume of a parallelepiped whose edges are given by:

$$\mathbf{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}, \quad \mathbf{b} = -5\hat{i} + 7\hat{j} - 3\hat{k}, \quad \mathbf{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$$

Theoretical Solution

Let **a**, **b** and **c** be the vectors representing the edges of the parallelepiped:

$$\mathbf{a} = \begin{pmatrix} -3 \\ 7 \\ 5 \end{pmatrix},$$

$$\mathbf{b} = \begin{pmatrix} -5 \\ 7 \\ -3 \end{pmatrix},$$

$$\mathbf{c} = \begin{pmatrix} 7 \\ -5 \\ -3 \end{pmatrix}$$

Theoretical Solution

The volume is given by the scalar triple product:

$$V = |[\mathbf{a} \ \mathbf{b} \ \mathbf{c}]|$$

This is equivalent to the determinant:

$$V = \left| \begin{pmatrix} -3 & -5 & 7 \\ 7 & 7 & -5 \\ 5 & -3 & -3 \end{pmatrix} \right|$$

Expanding the determinant:

$$\begin{aligned}\det &= -3(-21 - 15) + 5(-21 + 25) + 7(-21 - 35) \\ &= -3(-36) + 5(4) + 7(-56) \\ &= 108 + 20 - 392 = -264\end{aligned}$$

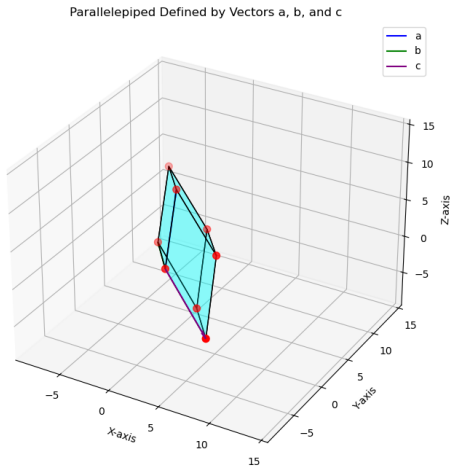
Theoretical Solution

Taking the absolute value of the determinant:

$$V = |-264| = 264 \text{ cubic units}$$

Therefore, the volume of the parallelepiped is 264 cubic units.

Plot



figureParallelepiped Defined by Vectors **a**, **b**, and **c**