9.5.4

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Question)

If one zero of the polynomial $6x^2 + 37x - (k - 2)$ is the reciprocal of the other, then what is the value of k?

Solution

General equation of conic

$$g(\mathbf{x}) = \mathbf{x}^{\mathbf{T}}\mathbf{V}\mathbf{x} + 2\mathbf{u}^{\mathbf{T}}\mathbf{x} + f \tag{1}$$

Equation of quadratic,

$$\mathbf{x}^{\mathbf{T}} \begin{pmatrix} 6 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} \frac{37}{2} \\ 0 \end{pmatrix}^{T} \mathbf{x} - (k-2) = 0 \tag{2}$$

Solution

Equation of line,

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{3}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{4}$$

Using following equation to find point of intersection of conic and line

$$k_i = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^T \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \pm \sqrt{\left[\mathbf{m}^T \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 - g(\mathbf{h})(\mathbf{m}^T \mathbf{V} \mathbf{m})} \right) \tag{5}$$

Solution

Solving for $g(\mathbf{h})$

$$g(\mathbf{h}) = \mathbf{h}^{\mathbf{T}} \begin{pmatrix} 6 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{h} + 2 \begin{pmatrix} \frac{37}{2} \\ 0 \end{pmatrix}^T \mathbf{h} - (k-2) \tag{6}$$

$$g(\mathbf{h}) = -(k-2) \tag{7}$$

Solving for $\mathbf{m}^T\mathbf{V}\mathbf{m}$

$$\mathbf{m}^T\mathbf{V}\mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \begin{pmatrix} 6 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{8}$$

$$= 6 \tag{9}$$

Solution

Solving for $\mathbf{m}^T (\mathbf{Vh} + \mathbf{u})$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \left(\begin{pmatrix} 6 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{37}{2} \\ 0 \end{pmatrix} \right) \tag{10}$$

$$= \frac{37}{2} \tag{11}$$

Solving (5)

$$k_i = \frac{1}{6} \left(-\frac{37}{2} \pm \sqrt{\frac{1369}{4} + (k-2) \times 6} \right) \tag{12}$$

Solution

Given condition

$$k_1 = \frac{1}{k_2} \tag{13}$$

Therefore

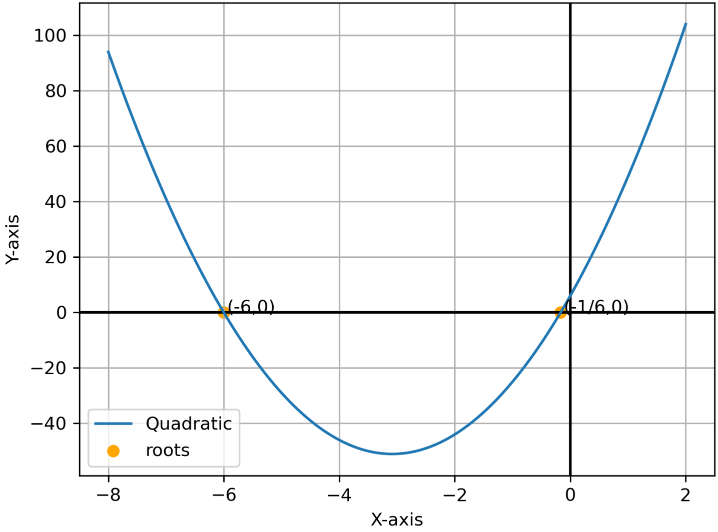
$$\frac{1}{6}\left(-\frac{37}{2} - \sqrt{\frac{1369}{4} + (k-2)\times 6}\right) = \frac{1}{\frac{1}{6}\left(-\frac{37}{2} + \sqrt{\frac{1369}{4} + (k-2)\times 6}\right)} \tag{14}$$

$$\frac{37}{2}^2 - \left(\frac{1369}{4} + 6(k-2)\right) = 36 \tag{15}$$

$$-6(k-2) = 36 \tag{16}$$

$$k = -4 \tag{17}$$

Figure



100
80
60
40
20
0
−20
−40
−8
−6
−4
−2
0
2
Y-axis
X-axis
(-6,0)
(-1/6,0)
Quadratic
roots

Direct Python

```
import numpy as np
import matplotlib.pyplot as plt

x=np.linspace(-8,2,300)
y=6*x*x+37*x+6

plt.xlabel("X-axis")
plt.ylabel("Y-axis")
xp = np.array([-6, -1/6])
yp=np.array([0,0])
```

Direct Python

```
plt.axhline(y=0, color='black')
plt.axvline(x=0, color='black')

plt.grid()
plt.plot(x,y, label='Quadratic')

plt.scatter(xp,yp, label='roots', color='orange')
plt.legend()
plt.text(xp[0]+0.05, yp[0]+0.05, "(-6,0)")
plt.text(xp[1]+0.05, yp[1]+0.05, "(-1/6,0)")

plt.savefig("figure.png", dpi=300)
plt.show()
```

C code

```
#include <stdio.h>

double find_k() {
    double a = 6, b = 37, c;
    double k;

    c = a;

    k = 2 - a;
```

C code

```c
    return k;
}

int main() {
    double k = find_k();
    printf("The value of k = %.2lf\n", k);
    return 0;
}
```

Python code with shared object

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load shared library
# Compile C code using: gcc -shared -fPIC -o main.so main.c
so = ctypes.CDLL('./main.so')
so.find_k.restype = ctypes.c_double

# Get value of k from C function
k = so.find_k()
print(f"The value of k = {k}")
```

Python code with shared object

```
# Define polynomial: 6x^2 + 37x - (k - 2)
a, b, c = 6, 37, -(k - 2)

# Generate x values
x = np.linspace(-10, 2, 400)
y = a * x**2 + b * x + c

# Plot
plt.figure(figsize=(8,6))
plt.plot(x, y, label=r'$6x^2 + 37x - (k - 2)$', color='blue')
```

Python code with shared object

```
# X and Y axis lines
plt.axhline(0, color='black', linewidth=1)
plt.axvline(0, color='black', linewidth=1)

# Roots of polynomial
roots = np.roots([a, b, c])
plt.scatter(roots, [0, 0], color='red', zorder=5, label='Roots')

# Labels and Title
plt.title(f"Graph of 6x + 37x - (k - 2), where k = {k:.2f}")
plt.xlabel('x-axis')
plt.ylabel('y-axis')
plt.legend()
plt.grid(True)
plt.show()
```