frame=single, breaklines=true, columns=fullflexible

5.3.9

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Question

If

$$M = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$$

is a decomposition into a symmetric matrix P and a skew-symmetric matrix Q, then find Q.

Solution

For any square matrix M,

$$P = \frac{M + M^\top}{2}, \qquad Q = \frac{M - M^\top}{2}.$$

Here,

$$M = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}, \quad M^\top = \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix}.$$

Thus,

$$Q = \frac{1}{2}(M - M^\top) = \frac{1}{2}\left(\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix}\right).$$

$$Q = \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}.$$

Answer

$$Q = \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}.$$