4.10.10

EE25BTECH11010 - Arsh Dhoke

Question:
Find the distance of the point (-1,-5,-10) from the point of intersection of the line $\mathbf{r} = 2\mathbf{i} - \mathbf{j} - 2\mathbf{k} + \lambda(3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k})$ and the plane $\mathbf{r} \cdot (\mathbf{i} - \mathbf{j} + \mathbf{k}) = 5$.
Solution:

On comparing equation of line with $\mathbf{x} = \mathbf{a} + \lambda\mathbf{b}$ and equation of plane with $\mathbf{n}^T\mathbf{x} = c$ we get:

Description	Vector
P	$\begin{pmatrix} -1 \\ -5 \\ -10 \end{pmatrix}$
a	$\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$
b	$\begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$
n	$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

$$\mathbf{n}^T\mathbf{x} = c \tag{0.1}$$

$$\mathbf{n}^T(\mathbf{a} + \lambda\mathbf{b}) = c \tag{0.2}$$

$$\lambda = \frac{c - \mathbf{n}^T\mathbf{a}}{\mathbf{n}^T\mathbf{b}} \tag{0.3}$$

$$\mathbf{x} = \mathbf{a} + \left(\frac{c - \mathbf{n}^T\mathbf{a}}{\mathbf{n}^T\mathbf{b}}\right)\mathbf{b} \tag{0.4}$$

$$\mathbf{n}^T\mathbf{a} = \begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} = 1 \tag{0.5}$$

$$\mathbf{n}^T\mathbf{b} = \begin{pmatrix}1 & -1 & 1\end{pmatrix} \cdot \begin{pmatrix}3\\4\\2\end{pmatrix} = 1 \tag{0.6}$$

Substituting given values in (0.4) we get:

$$\mathbf{x} = \begin{pmatrix}14\\15\\6\end{pmatrix} \tag{0.7}$$

Distance between $\mathbf{P}$ and $\mathbf{x}$:
$\|\mathbf{P} - \mathbf{x}\| = \sqrt{15^2 + 20^2 + 16^2} = \sqrt{225 + 400 + 256} = \sqrt{881}$

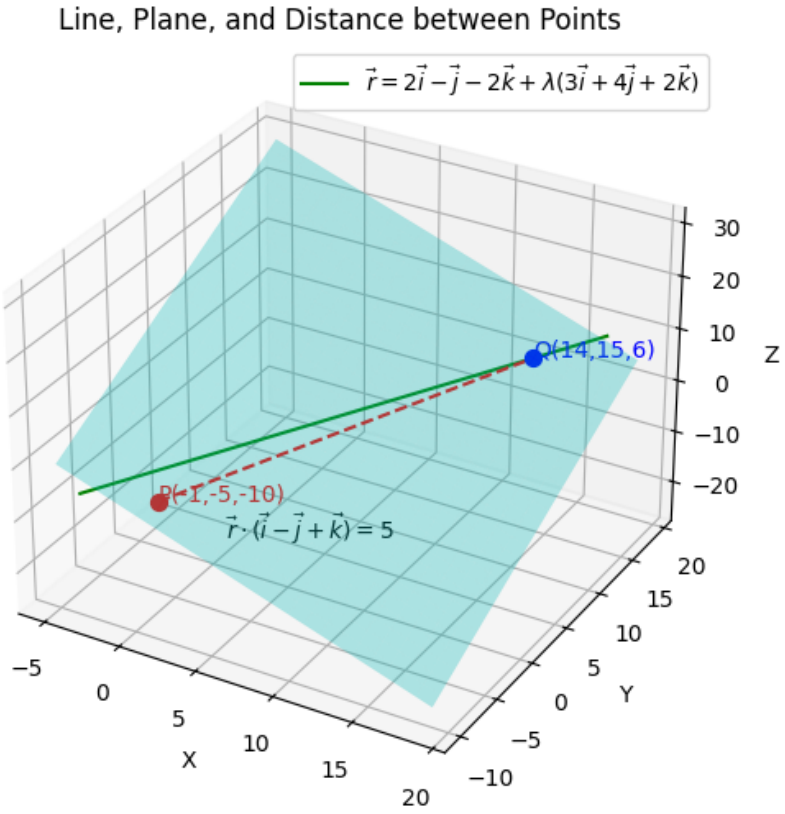

Fig. 0.1: Graph