2.10.33

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Question

Let α, β, γ be distinct real numbers. The points with position vectors $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$, $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$:

1. are collinear
2. form an equilateral triangle
3. form a scalene triangle
4. form a right angled triangle

Solution

To answer this question, we need to find the distance between each of these points.
Let **A** be $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$, **B** be $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$, and **C** be $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$.

Then

- **A** $-$ **B** is $(\alpha - \beta)\hat{i} + (\beta - \gamma)\hat{j} + (\gamma - \alpha)\hat{k}$
- **B** $-$ **C** is $(\beta - \gamma)\hat{i} + (\gamma - \alpha)\hat{j} + (\alpha - \beta)\hat{k}$
- **C** $-$ **A** is $(\gamma - \alpha)\hat{i} + (\alpha - \beta)\hat{j} + (\beta - \gamma)\hat{k}$

Solution

The norms of $\mathbf{A}-\mathbf{B}, \mathbf{B}-\mathbf{C}, \mathbf{C}-\mathbf{A}$ are all equal, and equal to

$$\sqrt{(\alpha-\beta)^2+(\beta-\gamma)^2+(\gamma-\alpha)^2}$$

The three points therefore form an equilateral triangle, so option (2) is correct.

Python Code

```
import numpy as np


vector = np.zeros(3)
vector[0] = input()
vector[1] = input()
vector[2] = input()

print(np.linalg.norm(vector))
```

C Code

```c
#include<stdio.h>
#include<math.h>

float norm(float a, float b, float c){

float answer;
answer = pow(a,2) + pow(b,2) + pow(c,2);
answer = sqrt(answer);

return answer;

}
```

Python and C Code

```
import numpy as np
import ctypes
c_lib=ctypes.CDLL('./5c.so')

c_lib.norm.argtypes = [ctypes.c_float, ctypes.c_float, ctypes.
    c_float]
c_lib.norm.restype = ctypes.c_float

vector = np.zeros(3)
vector[0] = input()
vector[1] = input()
vector[2] = input()

answer = c_lib.norm(
    ctypes.c_float(vector[0]),
    ctypes.c_float(vector[1]),
    ctypes.c_float(vector[2]))
print(answer)
```