4.10.23

EE25BTECH11021 - Dhanush Sagar

Question

Find the equation of the line passing through the point of intersection of 2x + y = 5 and x + 3y + 8 = 0 and parallel to the line 3x + 4y = 7.

Solution: The two given lines are written in matrix form as

$$l_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \mathbf{x} = 5, \tag{0.1}$$

$$l_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \mathbf{x} = -8. \tag{0.2}$$

$$l_3 = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \mathbf{x} = 7. \tag{0.3}$$

$$\tag{0.4}$$

Normals and constants for the given lines l_1 and l_2.

$$\mathbf{n}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad c_1 = 5 \tag{0.5}$$

$$\mathbf{n}_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \quad c_2 = -8 \tag{0.6}$$

General family of lines through the intersection, written as $l_1 + \lambda l_2$.

$$(\mathbf{n}_1^T \mathbf{x} - c_1) + \lambda(\mathbf{n}_2^T \mathbf{x} - c_2) = 0 \tag{0.7}$$

Explicit expanded form of the family of lines.

$$\begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} - 5 + \lambda(\begin{pmatrix} 1 & 3 \end{pmatrix} \mathbf{x} + 8) = 0 \tag{0.8}$$

$$\implies \begin{pmatrix} 2 + \lambda & 1 + 3\lambda \end{pmatrix} \mathbf{x} = 5 - 8\lambda. \tag{0.9}$$

Normal of the line l_3, to which our required line must be parallel.

$$\mathbf{m} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{0.10}$$

Normal vector of the family of lines.

$$\mathbf{N}(\lambda) = \mathbf{n}_1 + \lambda \mathbf{n}_2 = \begin{pmatrix} 2 + \lambda \\ 1 + 3\lambda \end{pmatrix} \tag{0.11}$$

Condition for parallelism with $\mathbf{m}$.

$$\mathbf{N}(\lambda) = \alpha\mathbf{m} \implies \begin{pmatrix} 2+\lambda \\ 1+3\lambda \end{pmatrix} = \alpha \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{0.12}$$

Solve these equations to determine λ.

$$3\alpha - \lambda = 2 \tag{0.13}$$
$$4\alpha - 3\lambda = 1 \tag{0.14}$$

$$\left(\begin{array}{cc|c} 3 & -1 & 2 \\ 4 & -3 & 1 \end{array}\right) \tag{0.15}$$

$$R_1 \to \frac{1}{3}R_1 \tag{0.16}$$
$$\left(\begin{array}{cc|c} 3 & -1 & 2 \\ 4 & -3 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{2}{3} \\ 4 & -3 & 1 \end{array}\right) \tag{0.17}$$

$$R_2 \to R_2 - 4R_1 \tag{0.18}$$
$$\left(\begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{2}{3} \\ 4 & -3 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & -\frac{5}{3} & -\frac{5}{3} \end{array}\right) \tag{0.19}$$

$$R_2 \to \left(-\frac{3}{5}\right)R_2 \tag{0.20}$$
$$\left(\begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & -\frac{5}{3} & -\frac{5}{3} \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 1 \end{array}\right) \tag{0.21}$$

$$R_1 \to R_1 + \frac{1}{3}R_2 \tag{0.22}$$
$$\left(\begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 1 \end{array}\right) \tag{0.23}$$

$$\lambda = 1 \tag{0.24}$$

substituting value of λ in eq(0.9) .Final equation of the required line in matrix form.

$$\begin{pmatrix} 3 & 4 \end{pmatrix}\mathbf{x} + 3 = 0 \tag{0.25}$$

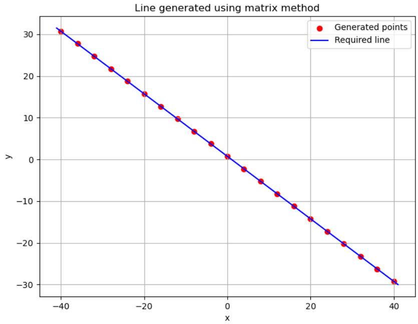
Line generated using matrix method
Generated points
Required line
30
20
10
0
−10
−20
−30
−40
−20
0
20
40
x
y

Fig. 0.1