2.9.2

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Question

If (-5, 3) and (5, 3) are two vertices of an equilateral triangle, then the coordinates of the third vertex, given that the origin lies inside the triangle (take $\sqrt{3} = 1.7$), are

Given Information

Let the two given points be represented as vectors, **A** and **B**, respectively

$$\mathbf{A} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \tag{1}$$

Let us assume the third point be **C**.

Solution

We have to first find the line equation of the line joining the points A and B.

$$\mathbf{x} = \mathbf{A} + t(\mathbf{B} - \mathbf{A}) \tag{2}$$

This gives,

$$\mathbf{x} = \begin{pmatrix} -5 \\ 3 \end{pmatrix} + t \begin{pmatrix} 10 \\ 0 \end{pmatrix} \tag{3}$$

Continuation

We have to find the lines aligned at 60° to this line at both $\mathbf{A}$ and $\mathbf{B}$. We can get this by multiplying a rotation vector to this vector, this is given by,

$$\mathbf{V}(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{4}$$

By multiplying this to 3 with $\theta = \pm 60^\circ$, we get the lines,

$$\mathbf{x} = \begin{pmatrix} -5 \\ 3 \end{pmatrix} + t(\mathbf{V}(\pm 60^\circ)) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{5}$$

The lines we get from this equation are,

$$\mathbf{x} = \begin{pmatrix} -5 \\ 3 \end{pmatrix} + t \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \tag{6}$$

Continuation

$$\mathbf{x} = \begin{pmatrix} -5 \\ 3 \end{pmatrix} + t \begin{pmatrix} \frac{1}{2} \\ \frac{-\sqrt{3}}{2} \end{pmatrix} \tag{7}$$

By doing the same thing taking point B,

$$\mathbf{x} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + t \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \tag{8}$$

Continuation

$$\mathbf{x} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + t \begin{pmatrix} \frac{1}{2} \\ \frac{-\sqrt{3}}{2} \end{pmatrix} \tag{9}$$

We can get two possible points that fit the given conditions for an equilateral triangle, let us assume these to be **C1** and **C2**

We can get **C1** by finding the point of intersection of 6 and 9

$$\begin{pmatrix} -5 \\ 3 \end{pmatrix} + t \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + t \begin{pmatrix} \frac{1}{2} \\ \frac{-\sqrt{3}}{2} \end{pmatrix} \tag{10}$$

Final Answer

On further solving, we get the point to be,

$$\mathbf{C1} = \begin{pmatrix} 0 \\ 3 + 5\sqrt{3} \end{pmatrix} = \begin{pmatrix} 0 \\ 11.5 \end{pmatrix} \tag{11}$$

Similarly, on solving for the other two lines, 7 and 8, we get,

$$\mathbf{C2} = \begin{pmatrix} 0 \\ 3 - 5\sqrt{3} \end{pmatrix} = \begin{pmatrix} 0 \\ -5.5 \end{pmatrix} \tag{12}$$

C code

```c
#include<stdio.h>
#include<math.h>

double norm(double *A, int m){
	double norm = 0;
	for(int i=0; i<m; i++){
		norm += A[i]*A[i];
	}
	norm = sqrt(norm);
	return norm;
}
```

Python code

```
import matplotlib.pyplot as plt
import numpy as np
import ctypes
import os
import sys

norm = ctypes.CDLL('./norm.so')
norm.norm.argtypes = [
        ctypes.POINTER(ctypes.c_double),
        ctypes.c_int
]
```

Python code

```
norm.norm.restype = ctypes.c_double

A=np.array([-5, 3], dtype=np.float64)
B=np.array([5, 3], dtype=np.float64)
m=len(A)

D=B-A

fig, ax=plt.subplots()
```

Python code

```
norm = norm.norm(
        D.ctypes.data_as(ctypes.POINTER(ctypes.c_double)),
        m
)

t=(1.7/2)*norm

C1=np.array([0, 3+t], dtype=np.float64)
C2=np.array([0, 3-t], dtype=np.float64)
```

Python code

```
def line_gen_num(A,B,num):
  dim = A.shape[0]
  x_AB = np.zeros((dim,num))
  lam_1 = np.linspace(0,1,num)
  for i in range(num):
    temp1 = A + lam_1[i]*(B-A)
    x_AB[:,i]= temp1.T
  return x_AB
```

Python code

```
x_AB = line_gen_num(A, B, 20)
x_BC1 = line_gen_num(C1, B, 20)
x_BC2 = line_gen_num(C2, B, 20)
x_AC1 = line_gen_num(A, C1, 20)
x_AC2 = line_gen_num(A, C2, 20)
```

Python code

```
plt.grid()
plt.title('2.9.2')
plt.plot(x_AB[0, :], x_AB[1, :], 'r--', label='Line from A to B')
plt.plot(x_BC1[0, :], x_BC1[1, :], 'r--')
plt.plot(x_BC2[0, :], x_BC2[1, :], 'r--')
plt.plot(x_AC1[0, :], x_AC1[1, :], 'r--')
plt.plot(x_AC2[0, :], x_AC2[1, :], 'r--')
```

Python code

```
plt.plot(A[0], A[1], 'go', label='Point A')
plt.annotate('(-5,3)', xy=(A[0],A[1]), fontsize=12)
plt.plot(B[0], B[1], 'go', label='Point B')
plt.annotate('(5,3)', xy=(B[0],B[1]), fontsize=12)
plt.plot(C1[0], C1[1], 'bo', label='Point C1')
plt.annotate('(0,11.5)', xy=(C1[0],C1[1]), fontsize=12)
plt.plot(C2[0], C2[1], 'bo', label='Point C2')
plt.annotate('(5,-5.5)', xy=(C2[0],C2[1]), fontsize=12)
```

Python code

```python

for axis in ['bottom', 'left']:
    ax.spines[axis].set_color('black')
    ax.spines[axis].set_linewidth(2)
```

Python code

```
plt.legend()
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.axis('equal')
plt.savefig('/home/shreyas/GVV_Assignments/matgeo/2.9.2/figs/fig1
    .png')

plt.show()
```

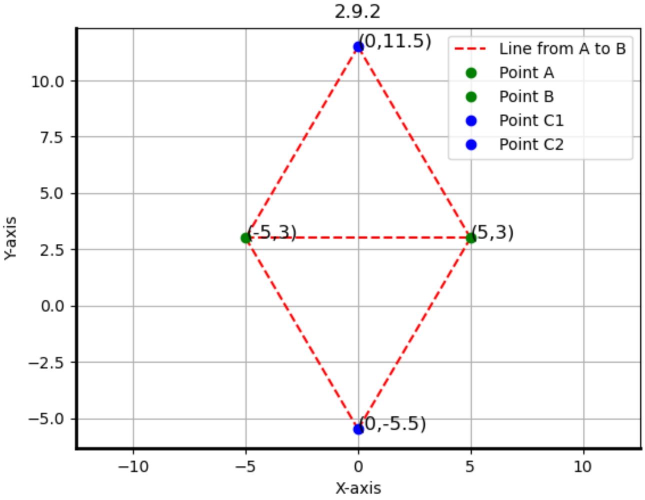

Figure: 2D Plot