9.4.45

EE25BTECH11032 - Kartik Lahoti

Question:

A motor boat whose speed is $18\,km/h$ in still water takes 1 hour more to go $24\,km$ upstream than to return downstream to the same spot. Find the speed of the stream.

Solution:

Let speed of boat be v and of stream be u.

Now, if $x - axis$ represents time t for downstream and $y - axis$ represents time t for upstream,

$$\mathbf{n}^\top \mathbf{x} = c \tag{0.1}$$

where c is distance (Given $= 24$) and

$$\mathbf{n} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} v \\ u \end{pmatrix} \tag{0.2}$$

If Line 0.1 cuts $x - axis$ at t_1 and $y - axis$ at t_2, then
Given,

$$t_1 = t_2 - 1 \tag{0.3}$$

To solve,

$$\begin{pmatrix} v & u \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} t_2 - 1 \\ 0 \end{pmatrix} = 24 \tag{0.4}$$

$$\begin{pmatrix} v & u \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ t_2 \end{pmatrix} = 24 \tag{0.5}$$

From 0.5

$$t_2 = \frac{24}{v - u} \tag{0.6}$$

Putting 0.6 in 0.4 , we get

$$u^2 + 48u - 324 = 0 \tag{0.7}$$

$$\implies y = x^2 + 48x - 324 = 0 \tag{0.8}$$

which can be expressed as the conic

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.9}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 24 \\ -\frac{1}{2} \end{pmatrix}, f = -324. \tag{0.10}$$

To find the roots of **??**, we find the points of intersection of the conic with the x-axis

$$\mathbf{x_i} + \mathbf{h} + k_i\mathbf{m} \tag{0.11}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.12}$$

Using ,

$$k_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \pm \sqrt{\left[\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 - g\left(\mathbf{h} \right) \left(\mathbf{m}^\top \mathbf{V}\mathbf{m} \right)} \right) \tag{0.13}$$

$$k_i = \frac{1}{1} \left(-24 \pm \sqrt{24^2 + 324} \right) \tag{0.14}$$

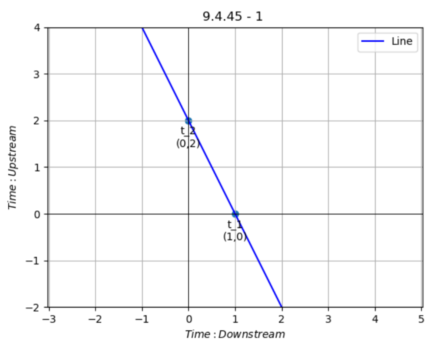
$$\implies k_1 = 6, k_2 = -54 \tag{0.15}$$

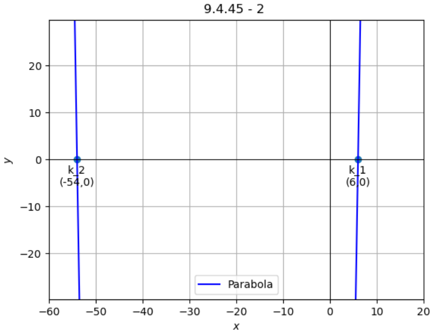
Hence the points of intersection are

$$\mathbf{h} + k\mathbf{m} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}, \begin{pmatrix} -54 \\ 0 \end{pmatrix} \tag{0.16}$$

Since u cannot be negative ,

$$u = 6\, km/h \tag{0.17}$$




9.4.45 - 2
Parabola
k_2
(-54,0)
k_1
(6,0)
x
y