5.3.35

EE25BTECH11001 - Aarush Dilawri

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Question

Question:
If the pair of equations

$$3x - y + 8 = 0 \tag{1}$$
$$6x - ry + 16 = 0 \tag{2}$$

represent coincident lines, then find the value of r.

Solution

The equation of line:

$$\mathbf{n}^\top \mathbf{x} = c \tag{3}$$

Line L:

$$\begin{pmatrix} 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -8 \tag{4}$$

Line K:

$$\begin{pmatrix} 6 & -r \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -16 \tag{5}$$

Matrix Form

These can be combined and written in matrix form:

$$\begin{pmatrix} 3 & -1 \\ 6 & -r \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ -16 \end{pmatrix} \tag{6}$$

The following augmented matrix can be solved by gaussian elimination

$$\left(\begin{array}{cc|c} 3 & -1 & -8 \\ 6 & -r & -16 \end{array}\right) \xleftrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{cc|c} 3 & -1 & -8 \\ 0 & -r+2 & 0 \end{array}\right) \tag{7}$$

Rouché–Capelli Application

Since the lines are coincident, they have infinitely many solutions.

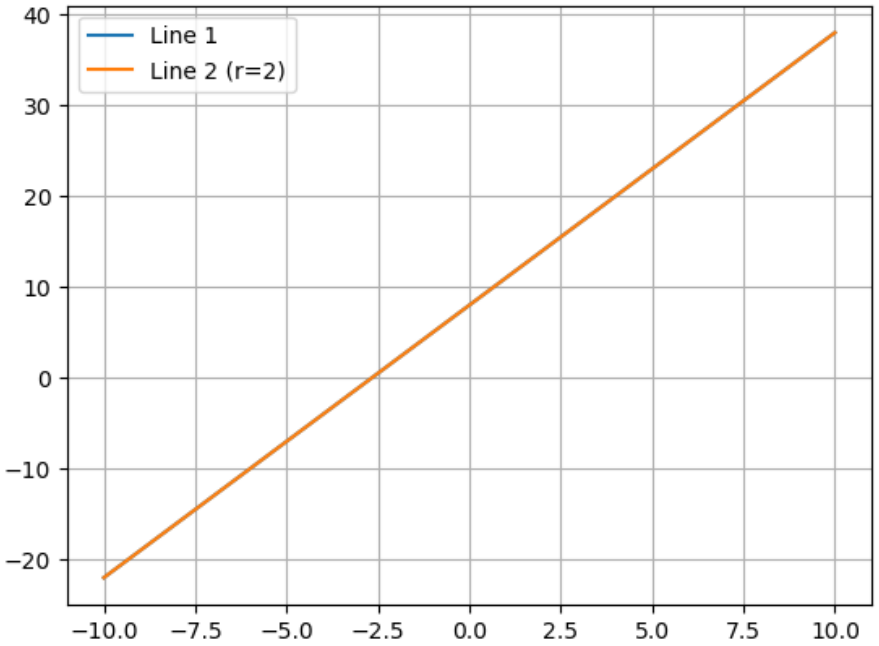
$$\text{Thus, } \operatorname{rank}(\mathbf{A}) = \operatorname{rank}([\mathbf{A} \mid b]) < n \quad \text{where } n \text{ is the number of variables.} \tag{8}$$

$$\implies -r + 2 = 0 \implies r = 2 \tag{9}$$

Hence, the value of r is 2.

Graphical Representation

See Figure,



C Code (code.c)

```c
#include <stdio.h>

// Function to find r such that the lines are coincident
// Lines: a1*x + b1*y + c1 = 0
// a2*x + (-r)*y + c2 = 0
double find_r(double a1, double b1, double c1, double a2, double c2) {
    // Since line2 normal = (a2, -r), it must be proportional to (a1, b1)
    // So, a2/a1 = (-r)/b1 AND c2/c1 = a2/a1
    double k = a2 / a1;
    double r = -k * b1;
    return r;
}
```

Python Code (code.py)

```
import numpy as np
import matplotlib.pyplot as plt

a1, b1, c1 = 3, -1, 8
a2, b2, c2 = 6, -2, 16 # since r = 2

def line1(x): return (a1*x + c1)/(-b1)
def line2(x): return (a2*x + c2)/(-b2)

x = np.linspace(-10, 10, 100)
plt.plot(x, line1(x), label="Line-1")
plt.plot(x, line2(x), label="Line-2-(r=2)")
plt.legend()
plt.grid(True)
plt.show()
```

Python Code (nativecode.py)

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load the shared library
code = ctypes.CDLL("./code.so")

# Declare function signature
code.find_r.restype = ctypes.c_double
code.find_r.argtypes = [ctypes.c_double, ctypes.c_double, ctypes.c_double,
    ctypes.c_double, ctypes.c_double]

# Given coefficients
a1, b1, c1 = 3, -1, 8
a2, c2 = 6, 16
```

Python Code (nativecode.py)

```
# Call C function
r = code.find_r(a1, b1, c1, a2, c2)
print("Value of r:", r)

# Define line equations
def line1(x): return (a1*x + c1)/(-b1)
def line2(x): return (a2*x + c2)/(-(-r))

# Plot
x = np.linspace(-10, 10, 100)
plt.plot(x, line1(x), label="Line 1")
plt.plot(x, line2(x), label="Line 2 (with r)")
plt.legend()
plt.grid(True)
plt.show()
```