4.12.23

Hemanth Reddy-AI25BTECH11018

September 30, 2025

Question

A point moves so that square of its distance from the point (3, -2) is numerically equal to its distance from the line $5x - 12y = 3$. The equation of its locus is

Solution:

$$\text{Let the position vector of point } \mathbf{P} \text{ is } = \begin{pmatrix} x \\ y \end{pmatrix} \tag{1}$$

$$\text{Let } \mathbf{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{2}$$

Square of distance of point $\mathbf{P}$ from $\mathbf{a}$ is $\|\mathbf{P}-\mathbf{a}\|^2 = (\mathbf{P}-\mathbf{a})^T(\mathbf{P}-\mathbf{a})$

$$(\mathbf{P}-\mathbf{a}) = \begin{pmatrix} x-3 \\ y+2 \end{pmatrix} \tag{3}$$

$$(\mathbf{P}-\mathbf{a})^T(\mathbf{P}-\mathbf{a}) = \begin{pmatrix} x-3 & y+2 \end{pmatrix} \begin{pmatrix} x-3 \\ y+2 \end{pmatrix} = (x-3)^2 + (y+2)^2 \tag{4}$$

Theoretical Solution

$$\text{Let} \quad \mathbf{n} = \begin{pmatrix} 5 \\ -12 \end{pmatrix} \tag{5}$$

$$|\mathbf{n}| = \sqrt{\mathbf{n}^T\mathbf{n}} = \sqrt{\begin{pmatrix} 5 & -12 \end{pmatrix} \begin{pmatrix} 5 \\ -12 \end{pmatrix}} = \sqrt{25 + 144} = 13 \tag{6}$$

$$d = \frac{|\mathbf{P}^T\mathbf{n} - 3|}{|\mathbf{n}|} = \frac{|5x - 12y - 3|}{13} \tag{7}$$

$$(x-3)^2 + (y+2)^2 = d = \frac{|5x - 12y - 3|}{13} \tag{8}$$

Theoretical Solution

$$(x-3)^2 + (y+2)^2 = \frac{(5x-12y-3)}{13} \tag{9}$$

$$13x^2 + 13y^2 - 83x + 64y + 172 = 0 \tag{10}$$

The locus of the point is a circle.

C Code

```c
#include <stdio.h>
#include <math.h>

int main() {
    // Fixed point coordinates
    double a_x = 3.0;
    double a_y = -2.0;

    // Line coefficients (Ax + By + C = 0)
    double line_A = 5.0;
    double line_B = -12.0;
    double line_C = -3.0;

    // The constant multiplier from the denominator of the
        distance formula
    // This is sqrt(A^2 + B^2)
    double constant_mult = sqrt(line_A * line_A + line_B * line_B
        );
```

C Code

```c
// Case 1: Positive side of the line
    // Equation: constant_mult * ((x - a_x)^2 + (y - a_y)^2) =
        line_A*x + line_B*y + line_C
    double coeff_x1 = -2 * a_x * constant_mult - line_A;
    double coeff_y1 = -2 * a_y * constant_mult - line_B;
    double const_term1 = constant_mult * (a_x * a_x + a_y * a_y)
        - line_C;

    // Case 2: Negative side of the line
    // Equation: constant_mult * ((x - a_x)^2 + (y - a_y)^2) = -(
        line_A*x + line_B*y + line_C)
    double coeff_x2 = -2 * a_x * constant_mult + line_A;
    double coeff_y2 = -2 * a_y * constant_mult + line_B;
    double const_term2 = constant_mult * (a_x * a_x + a_y * a_y)
        + line_C;
```

C Code

```
// Print the general form of the equations and the computed
     coefficients
    printf("The locus of the point is a pair of circles with the
        general equation:\n");
    printf("13x^2 + 13y^2 + (coeff_x)x + (coeff_y)y + (const_term
        ) = 0\n\n");

    printf("Equation for Case 1:\n");
    printf("13x^2 + 13y^2 + %.0fx + %.0fy + %.0f = 0\n\n",
        coeff_x1, coeff_y1, const_term1);

    printf("Equation for Case 2:\n");
    printf("13x^2 + 13y^2 + %.0fx + %.0fy + %.0f = 0\n", coeff_x2
        , coeff_y2, const_term2);

    return 0;
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt

def plot_locus():
    """
    Plots the two circle equations representing the locus of the
        point.
    """
    # Define the range for x and y
    x_range = np.linspace(-10, 10, 400)
    y_range = np.linspace(-10, 10, 400)
    X, Y = np.meshgrid(x_range, y_range)

    # Define the two implicit functions for the circles
    # Equation 1: 13x^2 + 13y^2 - 83x + 64y + 172 = 0
    F1 = 13*X**2 + 13*Y**2 - 83*X + 64*Y + 172
```

Python Code

```
# Equation 2: 13x^2 + 13y^2 - 73x + 40y + 166 = 0
    F2 = 13*X**2 + 13*Y**2 - 73*X + 40*Y + 166

    plt.figure(figsize=(8, 8))

    # Plot the first circle
    plt.contour(X, Y, F1, levels=[0], colors='blue', linewidths=2)

    # Plot the second circle
    plt.contour(X, Y, F2, levels=[0], colors='red', linewidths=2)

    # Plot the fixed point (3, -2)
    plt.plot(3, -2, 'o', color='green', markersize=8, label='Fixed Point (3, -2)')
```

Python Code

```
# Plot the line 5x - 12y = 3
    # Rearrange to y = (5x - 3) / 12
    x_line = np.linspace(-10, 10, 100)
    y_line = (5 * x_line - 3) / 12
    plt.plot(x_line, y_line, '--', color='gray', label='Line 5x -
        12y = 3')

    plt.title('Locus of the Point (Two Circles)')
    plt.xlabel('x')
    plt.ylabel('y')
    plt.axhline(0, color='black', linewidth=0.5)
    plt.axvline(0, color='black', linewidth=0.5)
    plt.grid(True, linestyle='--', alpha=0.6)
    plt.axis('equal') # Ensure circles appear as circles
    plt.legend()
```

Python Code

```
    plt.xlim(-5, 10) # Adjust x-axis limits for better visualization
    plt.ylim(-7, 5) # Adjust y-axis limits for better visualization

    plt.show()

# Run the plotting function
plot_locus()
```

Plot

Beamer/figs/circle.png