Matrices in Geometry 1.9.26

EE25BTECH11037 - Divyansh

Question: Find the value of k, if the point $\mathbf{P}(2,4)$ is equidistant from point $\mathbf{A}(5,k)$ and $\mathbf{B}(k,7)$

Given: $\mathbf{P}\begin{pmatrix}2\\4\end{pmatrix}$, $\mathbf{A}\begin{pmatrix}5\\k\end{pmatrix}$ and a point $\mathbf{B}\begin{pmatrix}k\\7\end{pmatrix}$ such that $\mathbf{P}$ is equidistant from $\mathbf{A}$ and $\mathbf{B}$.

$$\therefore \|\mathbf{A}-\mathbf{P}\| = \|\mathbf{B}-\mathbf{P}\| \tag{1}$$

On squaring both the sides, we get (2)

$$\|\mathbf{A}-\mathbf{P}\|^2 = \|\mathbf{B}-\mathbf{P}\|^2 \tag{3}$$

$$\left(\mathbf{A}-\mathbf{P}\right)^\top\left(\mathbf{A}-\mathbf{P}\right) = \left(\mathbf{B}-\mathbf{P}\right)^\top\left(\mathbf{B}-\mathbf{P}\right) \tag{4}$$

$$\left(\mathbf{A}-\mathbf{P}\right) = \begin{pmatrix}3\\k-4\end{pmatrix} \tag{5}$$

$$\left(\mathbf{B}-\mathbf{P}\right) = \begin{pmatrix}k-2\\3\end{pmatrix} \tag{6}$$

$$\left(\mathbf{A}-\mathbf{P}\right)^\top\left(\mathbf{A}-\mathbf{P}\right) = \begin{pmatrix}3 & k-4\end{pmatrix}\begin{pmatrix}3\\k-4\end{pmatrix} \tag{7}$$

$$= 9 + (k-4)^2 = 9 + k^2 - 8k + 16 = k^2 - 8k + 25 \tag{8}$$

$$\left(\mathbf{B}-\mathbf{P}\right)^\top\left(\mathbf{B}-\mathbf{P}\right) = \begin{pmatrix}k-2 & 3\end{pmatrix}\begin{pmatrix}k-2\\3\end{pmatrix} \tag{9}$$

$$= (k-2)^2 + 9 = k^2 - 4k + 4 + 9 = k^2 - 4k + 13 \tag{10}$$

From (8) and (10), $k^2 - 8k + 25 = k^2 - 4k + 13$ (11)

$$\implies -4k + 8k = 25 - 13 \implies 4k = 12 \implies k = 3 \tag{12}$$

Hence, the final answer is $\boxed{k = 3}$ (13)

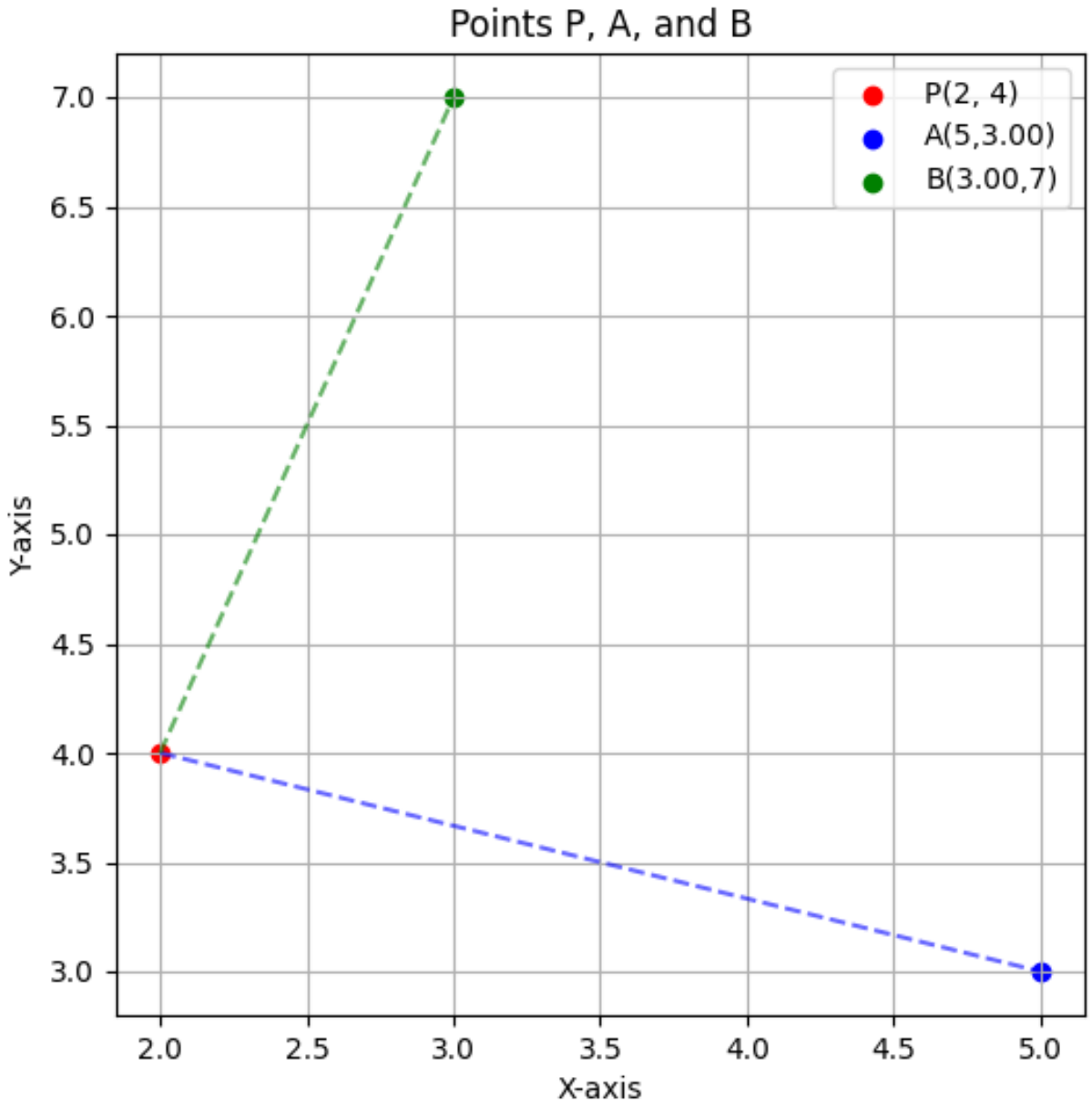

Fig. 1: Plot for 1.9.26