

## 5.13.30

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### Question

Let  $\mathbf{A}$  be a square matrix all of whose entries are integers. Then which of the following is true?

- A) If  $\det(\mathbf{A}) \neq \pm 1$ , then  $\mathbf{A}^{-1}$  exists but all its entries are not necessarily integers
- B) If  $\det(\mathbf{A}) \neq \pm 1$ , then  $\mathbf{A}^{-1}$  exists and all its entries are non-integers
- C) If  $\det(\mathbf{A}) = \pm 1$ , then  $\mathbf{A}^{-1}$  exists but all its entries are integers
- D) If  $\det(\mathbf{A}) = \pm 1$ , then  $\mathbf{A}^{-1}$  need not exist

### Solution

We will proceed by checking each option.

A) If  $\det(\mathbf{A}) \neq \pm 1$ , then  $\mathbf{A}^{-1}$  exists but all its entries are not necessarily integers

Let us take a square matrix  $\mathbf{A}$  having all integer entries. Let rows  $R_1$  and  $R_2$  be equal. By performing row operation  $R_1 \rightarrow R_1 - R_2$ , all elements in  $R_1$  become 0. Therefore,  $|\mathbf{A}| = 0$ . We know that if  $|\mathbf{A}| = 0$ ,  $\mathbf{A}^{-1}$  does not exist. Therefore, this option is wrong.

B) If  $\det(\mathbf{A}) \neq \pm 1$ , then  $\mathbf{A}^{-1}$  exists and all its entries are non-integers

This option is wrong according to the previous reasoning.

D) If  $\det(\mathbf{A}) = \pm 1$ , then  $\mathbf{A}^{-1}$  need not exist

We know that if  $|\mathbf{A}| \neq 0$ ,  $\mathbf{A}^{-1}$  exists. By this logic, this option is wrong.

Therefore, the correct answer is C).