

## 8.4.7

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# Question

Let **O** be the vertex and **Q** be any point on the parabola  $x^2 = 8y$ . If the point **P** divides the line segment **OQ** internally in the ratio (1:3), then the locus of **P** is:

①  $y^2 = 2x$

②  $x^2 = 2y$

③  $x^2 = y$

④  $y^2 = x$

# Solution

The equation of conic with directrix  $\mathbf{n}^T \mathbf{x} = c$  and focus at  $\mathbf{F}$ , and eccentricity  $e$  is

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0$$

where:

$$\mathbf{V} = \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n} \mathbf{n}^T$$

$$\mathbf{u} = c e^2 \mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F}$$

$$f = \|\mathbf{n}\|^2 \mathbf{F} - c^2 e^2$$

The directrix of the given parabola is  $y = -2$ , which expressed in the form  $\mathbf{n}^T \mathbf{x} = c$  is

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}^T \mathbf{x} = -2$$

The focus  $\mathbf{F} = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ . Since it is a parabola,  $e = 1$ .

# Solution

Therefore:

$$\mathbf{v} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$
$$u = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$$
$$f = 0$$

The parabola can be represented as

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} - \begin{pmatrix} 0 \\ 8 \end{pmatrix} \mathbf{x} = 0 \quad (1)$$

Since the point  $\mathbf{P}$  divides  $\mathbf{OQ}$  internally in the ratio 1:3,

$$\mathbf{P} = \frac{\mathbf{x}}{4} \quad (2)$$

# Solution

Substituting  $\mathbf{P}$  in (1),

$$4\mathbf{P}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{P} - \begin{pmatrix} 0 \\ 8 \end{pmatrix} \mathbf{P} = 0 \quad (3)$$

$$\mathbf{P}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{P} - \begin{pmatrix} 0 \\ 2 \end{pmatrix} \mathbf{P} = 0 \quad (4)$$

Expanding this equation, we get the locus of  $\mathbf{P}$  as  $x^2 = 2y$ .

The correct option is **B**.

# Python Code

```
import matplotlib.pyplot as plt
import numpy as np
fig = plt.figure(figsize = (6,6))
ax = fig.add_subplot(111)

X = np.linspace(-10,10,50)
p1 = X*X/8
p2 = X*X/2
l = np.zeros(50) - 2

ax.plot(X,p1, label = '$x^2 = 8y$', color = 'green')
ax.plot(X,p2, label = '$x^2 = 2y$', color = 'red')
ax.plot(X,l, label = '$y=-2$', color = 'orange')
ax.scatter(0,2, label = 'Focus')
ax.grid(True)
ax.legend()
plt.show()
```

# Figure

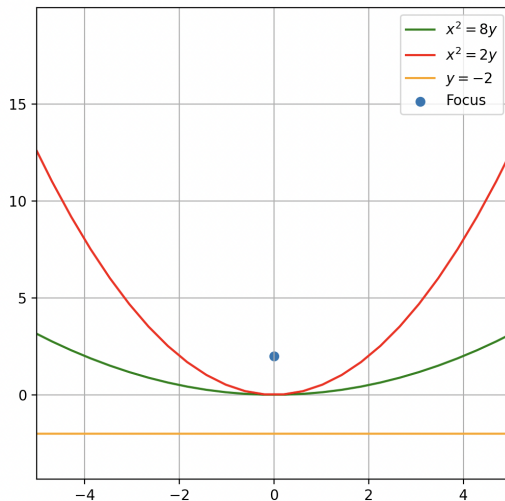


Figure: Plot