4.8.8 Matgeo

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Question

Find the equation of the plane passing through the point (-1,3,2) and perpendicular to the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

Solution

The equation of a plane can be given by the formula :

$$n^T \mathbf{x} = c \tag{1}$$

From the above formula we can write :

$$x + 2y + 3z = 5 = \mathbf{n_1}^T \mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}^T \mathbf{x} = 5 \tag{2}$$

$$3x + 3y + z = 0 = \mathbf{n_2}^T \mathbf{x} = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}^T \mathbf{x} = 0 \tag{3}$$

Solution

Let us assume the equation of the plane to be

$$\mathbf{n}^{\mathbf{T}}\mathbf{x} = 1 \quad or \quad \mathbf{x}^{\mathbf{T}}\mathbf{n} = 1 \tag{4}$$

As point **A** lies on the plane we can write :

$$\mathbf{A}^{\mathbf{T}}\mathbf{n} = 1 \tag{5}$$

If two planes are perpendicular then there normal vectors must also be perpendicular ,using this we can write :

$$\mathbf{n_1^T}\mathbf{n} = 0 \tag{6}$$

$$\mathbf{n_2^T}\mathbf{n} = 0 \tag{7}$$

Solution

Combining equations 5,6 and 7 ,we get :

$$\begin{pmatrix}\mathbf{A} & \mathbf{n_1} & \mathbf{n_2}\end{pmatrix}^{\mathbf{T}}\mathbf{n} = \begin{bmatrix}-1 & 3 & 2\\ 1 & 2 & 3\\ 3 & 3 & 1\end{bmatrix}\mathbf{n} = \begin{bmatrix}1\\0\\0\end{bmatrix} \tag{8}$$

Solving the above equation by row reduction we get :

$$\mathbf{n} = \begin{bmatrix}-\frac{7}{25}\\ \frac{8}{25}\\ -\frac{3}{25}\end{bmatrix} = \frac{1}{25}\begin{bmatrix}-7\\8\\-3\end{bmatrix} \tag{9}$$

Solution

From the equation 4 we can write the plane equation as :

$$\begin{bmatrix} -7 \\ 8 \\ -3 \end{bmatrix}^T \mathbf{x} = 25 \tag{10}$$

Graphical Representation