4.11.37

EE25BTECH11060 - V.Namaswi

Question

Show that the lines $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $\frac{x+4}{2} = \frac{y}{0} = \frac{z+1}{3}$ intersect.Find their point of intersection.

Solution

Let,

$$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda \tag{1}$$

$$\frac{x+4}{2} = \frac{y}{0} = \frac{z+1}{3} = \mu \tag{2}$$

To check point of intersection,

$$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \tag{3}$$

$$\implies \begin{pmatrix} 3 & -2 \\ -1 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \\ 0 \end{pmatrix} \tag{4}$$

$$\tag{5}$$

Using gaussian elimination
Argumented matrix

$$\begin{bmatrix} 3 & -2 & | & -5 \\ -1 & 0 & | & -1 \\ 0 & -3 & | & 0 \end{bmatrix} \tag{6}$$

Apply the row operation $R_2 \to 3R_2 + R_1$:

$$\begin{bmatrix} 3 & -2 & | & -5 \\ 0 & -2 & | & -8 \\ 0 & -3 & | & 0 \end{bmatrix} \tag{7}$$

Divide the second row by -2:

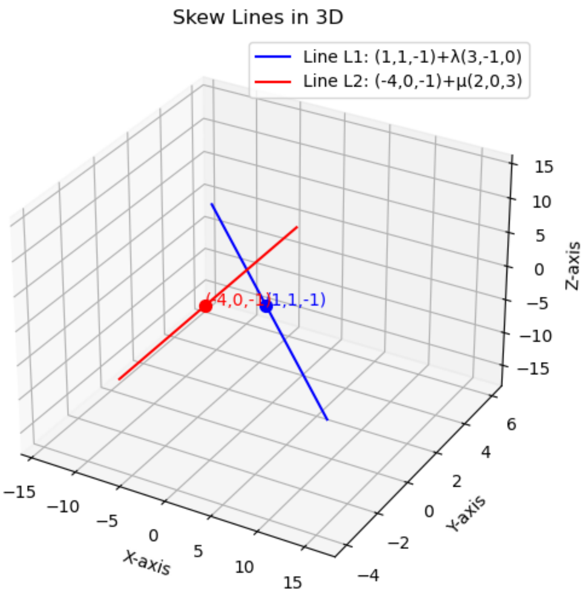
$$\begin{bmatrix} 3 & -2 & | & -5 \\ 0 & 1 & | & 4 \\ 0 & -3 & | & 0 \end{bmatrix} \tag{8}$$

Now eliminate using R_2:

$$R_1 \to R_1 + 2R_2, \quad R_3 \to R_3 + 3R_2$$

$$\begin{bmatrix} 3 & 0 & | & 3 \\ 0 & 1 & | & 4 \\ 0 & 0 & | & 12 \end{bmatrix} \tag{9}$$

The last row gives 0 = 12, which is a contradiction. Hence, the system is inconsistent and has **no solution**. Therefore, the two lines are skew and do not intersect.



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