

1.11.12

AI25BTECH11006 - Nikhila

Question: Find the sine of the angle between the vectors $\mathbf{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\mathbf{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$.

Solution:

The given vectors are $\mathbf{a} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$.

We know that

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}| |\mathbf{b}| \sin \theta \quad (0.1)$$

$$\sin \theta = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}||\mathbf{b}|}. \quad (0.2)$$

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{pmatrix} = 8 \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}. \quad (0.3)$$

$$|\mathbf{a} \times \mathbf{b}| = \left| 8 \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \right| = 8 \sqrt{3}. \quad (0.4)$$

$$|\mathbf{a}| = \sqrt{(3)^2 + (1)^2 + (2)^2} = \sqrt{14}, \quad (0.5)$$

$$|\mathbf{b}| = \sqrt{(2)^2 + (-2)^2 + (4)^2} = \sqrt{24}. \quad (0.6)$$

$$\sin \theta = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}||\mathbf{b}|} \quad (0.7)$$

$$= \frac{8 \sqrt{3}}{\sqrt{14} \cdot \sqrt{24}} \quad (0.8)$$

$$= \frac{2}{\sqrt{7}}. \quad (0.9)$$

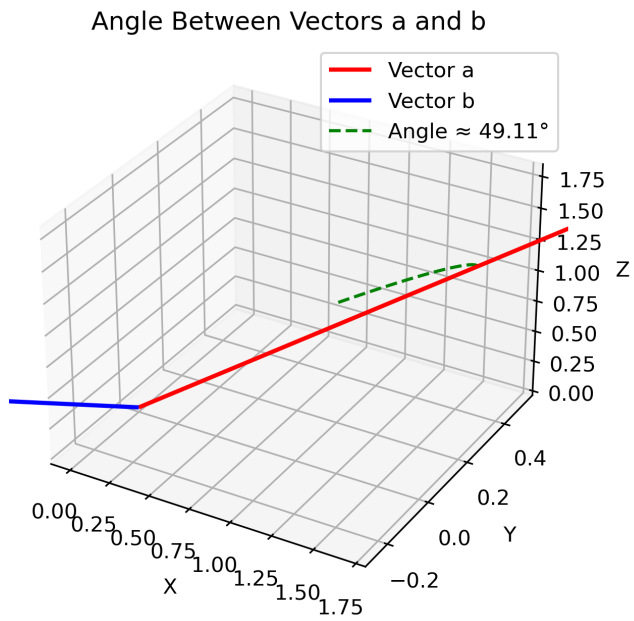


Fig. 0.1