1.8.19

EE25BTECH11002 - Achat Parth Kalpesh

Question:
If $\mathbf{Q}(0,1)$ is equidistant from $\mathbf{P}(5,-3)$ and $\mathbf{R}(x,6)$, find the values of x. Also find the distances **QR** and **PR**.

Solution:
Let the given points be represented by the column vectors **P**, **Q**, and **R**.

$$\mathbf{P} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} x \\ 6 \end{pmatrix} \tag{0.1}$$

According to given condition;

$$\|\mathbf{P} - \mathbf{Q}\|^2 = \|\mathbf{R} - \mathbf{Q}\|^2 \tag{0.2}$$

The squared norm of a vector $\mathbf{v}$ is given by the matrix product $\mathbf{v}^\top \mathbf{v}$.

$$(\mathbf{P} - \mathbf{Q})^\top (\mathbf{P} - \mathbf{Q}) = (\mathbf{R} - \mathbf{Q})^\top (\mathbf{R} - \mathbf{Q}) \tag{0.3}$$

$$\left(\mathbf{P}^\top - \mathbf{Q}^\top\right)(\mathbf{P} - \mathbf{Q}) = \left(\mathbf{R}^\top - \mathbf{Q}^\top\right)(\mathbf{R} - \mathbf{Q}) \tag{0.4}$$

$$\mathbf{P}^\top \mathbf{P} - 2\mathbf{Q}^\top \mathbf{P} = \mathbf{R}^\top \mathbf{R} - 2\mathbf{R}^\top \mathbf{Q} \tag{0.5}$$

$$\begin{pmatrix} 5 & -3 \end{pmatrix}\begin{pmatrix} 5 \\ -3 \end{pmatrix} - 2\begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ -3 \end{pmatrix} = \begin{pmatrix} x & 6 \end{pmatrix}\begin{pmatrix} x \\ 6 \end{pmatrix} - 2\begin{pmatrix} x & 6 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{0.6}$$

$$(5)(5) + (-3)(-3) - 2(0)(5) - 2(1)(-3) = (x)(x) + (6)(6) - 2(x)(0) - 2(6)(1) \tag{0.7}$$

$$25 + 9 + 6 = x^2 + 36 - 12 \tag{0.8}$$

$$x^2 = 16 \tag{0.9}$$

$$\implies x = \pm 4 \tag{0.10}$$

Therefore, the two possible vectors for **R** are:

$$\mathbf{R}_1 = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \tag{0.11}$$

$$\mathbf{R}_2 = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \tag{0.12}$$

$$\|\mathbf{Q} - \mathbf{R}\| = \|\mathbf{P} - \mathbf{Q}\| = \sqrt{5^2 + (-4)^2} = \sqrt{41} \approx 6.40 \tag{0.13}$$

- For $\mathbf{R}_1$:

$$\|\mathbf{R_1} - \mathbf{P}\| = \left\| \begin{pmatrix} 4-5 \\ 6-(-3) \end{pmatrix} \right\| = \left\| \begin{pmatrix} -1 \\ 9 \end{pmatrix} \right\| \tag{0.14}$$

$$= \sqrt{(-1)^2 + 9^2} = \sqrt{82} \approx 9.06 \tag{0.15}$$

- For $\mathbf{R}_2$:

$$\|\mathbf{R}_2 - \mathbf{P}\| = \left\| \begin{pmatrix} -4-5 \\ 6-(-3) \end{pmatrix} \right\| = \left\| \begin{pmatrix} -9 \\ 9 \end{pmatrix} \right\| \tag{0.16}$$

$$= \sqrt{(-9)^2 + 9^2} = \sqrt{162} = 9\sqrt{2} \approx 12.73 \tag{0.17}$$

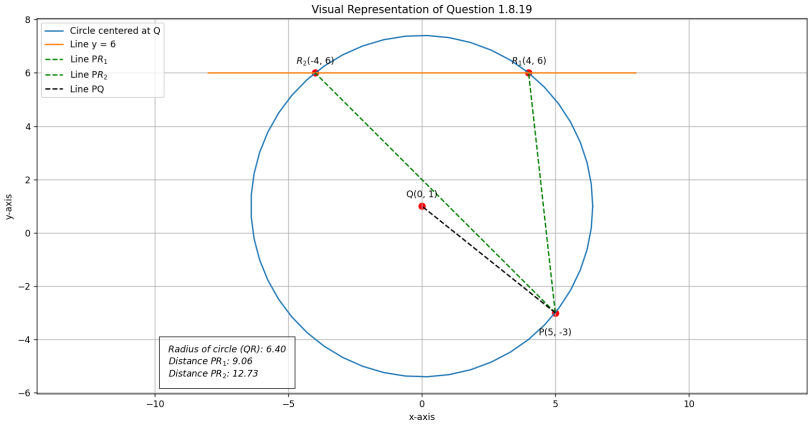

Fig. 0.1: Visual representation of the solution. The points R_1 and R_2 are the intersections of the circle centered at Q and the line $y = 6$.