4.8.14

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Question

Solve for x and y

$x + y = 6$, $2x - 3y = 4$

Theoretical Solution

Solution:

Let :

$$\mathbf{r_1} = \begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{x} = 6 \tag{1}$$

$$\mathbf{r_2} = \begin{pmatrix} 2 & -3 \end{pmatrix} \mathbf{x} = 4 \tag{2}$$

The augmented matrix of the above equations is given by,

$$\begin{pmatrix} 1 & 1 & 6 \\ 2 & -3 & 4 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & 1 & 6 \\ 0 & -5 & -8 \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} 1 & 1 & 6 \\ 0 & -5 & -8 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow 5R_1 + R_2} \begin{pmatrix} 5 & 0 & 22 \\ 0 & -5 & -8 \end{pmatrix} \tag{4}$$

Theoretical Solution

$$5x = 22 \qquad x = \frac{22}{5} \tag{5}$$

$$-5y = -8 \qquad y = \frac{8}{5} \tag{6}$$

C Code

```c
#include <stdio.h>

int main() {
    // Coefficients and constants for the system of linear
        equations
    // Equation 1: x + y = 6
    double a1 = 1.0;
    double b1 = 1.0;
    double c1 = 6.0;

    // Equation 2: 2x - 3y = 4
    double a2 = 2.0;
    double b2 = -3.0;
    double c2 = 4.0;

    // Use Cramer's Rule to solve for x and y
    // Determinant of the coefficient matrix
```

C Code

```
double determinant = a1 * b2 - a2 * b1;

  // Check if the determinant is close to zero, which means no
      unique solution exists
  if (determinant == 0) {
      printf("The system has no unique solution.\n");
      return 1;
  }

  // Determinant for x
  double determinant_x = c1 * b2 - c2 * b1;

  // Determinant for y
  double determinant_y = a1 * c2 - a2 * c1;
```

```c

    // Solve for x and y
    double x = determinant_x / determinant;
    double y = determinant_y / determinant;

    // Print the results
    printf("The solution to the system is:\n");
    printf("x = %.2f\n", x);
    printf("y = %.2f\n", y);

    return 0;
}
```

```python
import numpy as np
import matplotlib.pyplot as plt

def plot_solution():
    # Define the equations of the lines
    # Line 1: x + y = 6 => y = 6 - x
    # Line 2: 2x - 3y = 4 => y = (2x - 4) / 3

    # Generate x values to plot the lines
    x = np.linspace(-10, 10, 400)

    # Calculate corresponding y values for each line
    y1 = 6 - x
    y2 = (2 * x - 4) / 3

    # The solution is x = 22/5 = 4.4 and y = 8/5 = 1.6
    solution_x = 22 / 5
    solution_y = 8 / 5
```

Python Code

```python
# Set up the plot
plt.figure(figsize=(8, 8))
plt.title('Solution of the System of Linear Equations')
plt.xlabel('x-axis')
plt.ylabel('y-axis')
plt.grid(True, linestyle='--', alpha=0.6)
plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
plt.axis('equal') # Ensures correct aspect ratio

# Plot the lines
plt.plot(x, y1, label='x + y = 6', color='blue')
plt.plot(x, y2, label='2x - 3y = 4', color='red')
```

Python Code

```python

# Plot and annotate the intersection point
    plt.plot(solution_x, solution_y, 'o', color='green',
        markersize=10, label=f'Solution ({solution_x:.1f}, {
        solution_y:.1f})')
    plt.annotate(f'({solution_x:.1f}, {solution_y:.1f})',
                 (solution_x, solution_y),
                 textcoords="offset points",
                 xytext=(0,10),
                 ha='center')

    # Add a legend and display the plot
    plt.legend()
    plt.show()

# Run the plotting function
plot_solution()
```

Plot

Beamer/figs/plot.png

Figure: