3.3.7

AI25BTECH11023-Pratik R

Question:

Write the steps of construction for drawing a ΔABC in which $BC = 8cm$, $\angle B = 45$ and $\angle C = 30°$

solution:

Let $\begin{pmatrix}0\\0\end{pmatrix}$ be the position vector of point **B** and a,b and c be the sides opposite the vertices A,B and C, respectively in ΔABC.

Given a =8cm;

$$\mathbf{C} = \begin{pmatrix}8\\0\end{pmatrix} \tag{0.1}$$

$$\therefore \mathbf{A} = \begin{pmatrix}c\cos\angle B\\ c\sin\angle B\end{pmatrix} = \begin{pmatrix}c\times 1/\sqrt{2}\\ c\times 1/\sqrt{2}\end{pmatrix} \tag{0.2}$$

in ΔABC

$$b\cos\angle C + c\cos\angle B = 8 \tag{0.3}$$

$$b\sin\angle C - c\sin\angle B = 0 \tag{0.4}$$

Solving linear Equation in b and c:

$$\begin{pmatrix}\cos\angle C & \cos\angle B\\ \sin\angle C & -\sin\angle B\end{pmatrix}\begin{pmatrix}b\\c\end{pmatrix} = \begin{pmatrix}a\\0\end{pmatrix} \tag{0.5}$$

using augmented matrix

$$\left(\begin{array}{cc|c}\cos\angle C & \cos\angle B & a\\ \sin\angle C & -\sin\angle B & 0\end{array}\right) \tag{0.6}$$

(0.7)

putting $\angle C = 30°$ and $\angle B = 45°$

$$\left(\begin{array}{cc|c}\sqrt{3}/2 & 1/\sqrt{2} & 8\\ 1/2 & -1/\sqrt{2} & 0\end{array}\right) \tag{0.8}$$

Echelon form of the matrix is given by

$$\left(\begin{array}{cc|c}\sqrt{3}/2 & 1/\sqrt{2} & 8\\ 0 & (-\sqrt{3}-1)/\sqrt{2} & -8\end{array}\right) \tag{0.9}$$

$$\frac{(-\sqrt{3}-1)}{\sqrt{2}} \times c = -8 \tag{0.10}$$

$$\implies c = \frac{8\sqrt{2}}{(\sqrt{3}+1)} = \sqrt{3}-1 \tag{0.11}$$

$$\therefore \mathbf{A} = \begin{pmatrix} \sqrt{3}-1 \\ \sqrt{3}-1 \end{pmatrix} \tag{0.12}$$