1.5.15

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Agust, 2025

Question

The midpoint of the line segment joining $A(2a, 4)$ and $B(-2, 3b)$ is $(1, 2a + 1)$. Find the values of a and b.

$$\mathbf{A} = \begin{pmatrix} 2a \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 3b \end{pmatrix} \tag{1}$$

Let the midpoint of points A and B be C. where,

$$\mathbf{C} = \begin{pmatrix} 1 \\ 2a+1 \end{pmatrix} \tag{2}$$

Theoretical Solution

We know that the midpoint formula for the points A and B is

$$\mathbf{C} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{3}$$

Now we can find the values of a and b by using the midpoint formula(Eq.3):

$$\mathbf{C} = \frac{\begin{pmatrix} 2a \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 3b \end{pmatrix}}{2} \tag{4}$$

$$\begin{pmatrix} 1 \\ 2a+1 \end{pmatrix} = \frac{\begin{pmatrix} 2a \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 3b \end{pmatrix}}{2} \tag{5}$$

$$\begin{pmatrix} 1 \\ 2a+1 \end{pmatrix} = \frac{\begin{pmatrix} 2a-2 \\ 4+3b \end{pmatrix}}{2} \tag{6}$$

Theoretical Solution

$$\begin{pmatrix} 1 \\ 2a+1 \end{pmatrix} = \begin{pmatrix} a-1 \\ 2+\frac{3b}{2} \end{pmatrix} \tag{9}$$

As the two vector matrices are equal , their corresponding elements are also equal. By equating the corresponding elements , we get:

$$1 = a - 1 \ and \ 2a + 1 = 2 + \frac{3b}{2} \tag{10}$$

By solving the above equation we get

$$a = 2 \ and \ b = \frac{4a-2}{3} \tag{11}$$

Theoretical Solution

As the value of a=2, we get:

$$b = \frac{4(2) - 2}{3} \tag{12}$$

$$b = \frac{8 - 2}{3} \tag{13}$$

$$b = \frac{6}{3} \tag{14}$$

$$b = 2 \tag{15}$$

So the final answer is

$$a = 2 \text{ and } b = 2 \tag{16}$$

C Code - Midpoint formula

```c
#include <stdio.h>

// Function to calculate midpoint
void midpoint(float x1, float y1, float x2, float y2, float *mx,
    float *my) {
    *mx = (x1 + x2) / 2.0;
    *my = (y1 + y2) / 2.0;
}
```

Python Code

```python
import numpy as np
import ctypes
import matplotlib.pyplot as plt

# Load the shared library
lib = ctypes.CDLL(./midpoint.so) # use midpoint.dll on Windows

# Define function signature
lib.midpoint.argtypes = [
    ctypes.c_float, ctypes.c_float, # x1, y1
    ctypes.c_float, ctypes.c_float, # x2, y2
    ctypes.POINTER(ctypes.c_float), # mx
    ctypes.POINTER(ctypes.c_float) # my
]
```

Python Code

```python
# Given values from problem
a, b = 2, 2
A = (2*a, 4) # (4,4)
B = (-2, 3*b) # (-2,6)

# Prepare variables to hold midpoint
mx, my = ctypes.c_float(), ctypes.c_float()

# Call the C function
lib.midpoint(A[0], A[1], B[0], B[1], ctypes.byref(mx), ctypes.
    byref(my))
M = (mx.value, my.value)

print(fMidpoint from C: {M})
```

Python Code

```python
# --- Plot ---
plt.figure(figsize=(6,6))
plt.plot([A[0], B[0]], [A[1], B[1]], 'b-', linewidth=2, label='
    Line AB')

# Scatter points
plt.scatter(*A, color='red', s=100, label=fA{A})
plt.scatter(*B, color='green', s=100, label=fB{B})
plt.scatter(*M, color='purple', s=120, marker='*', label=fM{M})

# Annotate
plt.text(A[0]+0.2, A[1]+0.2, fA{A}, fontsize=10)
plt.text(B[0]+0.2, B[1]+0.2, fB{B}, fontsize=10)
plt.text(M[0]+0.2, M[1]+0.2, fM{M}, fontsize=10, color=purple)
```

Python Code

```python
# Axes formatting
plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
plt.grid(True, linestyle=--, alpha=0.6)
plt.legend()
plt.title(Midpoint using C + Python)
plt.xlabel(X-axis)
plt.ylabel(Y-axis)
plt.show()
```

Plot