4.13.100

AI25BTECH110031
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Question(4.13.100) Let **S** be the reflection of a point **Q** with respect to the plane given by $\mathbf{r} = -(t+p)\hat{i} + t\hat{j} + (1+p)\hat{k}$ where t, p are real parameters and $\hat{i}$, $\hat{j}$, $\hat{k}$ are the unit vectors along the three positive coordinate axes. If the position vectors of **Q** and **S** are $10\hat{i}+15\hat{j}+20\hat{k}$ and $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ respectively, then which of the following is/are TRUE ?

a $3(\alpha+\beta) = -101$
b $3(\beta+\gamma) = -71$
c $3(\gamma+\alpha) = -86$
d $3(\alpha+\beta+\gamma) = -121$

Solution:

The plane is given by

$$\mathbf{r} = t\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + p\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{0.1}$$

so two direction vectors are

$$\mathbf{u} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \qquad \mathbf{v} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}. \tag{0.2}$$

Hence the normal vector is

$$\mathbf{n} = \mathbf{u} \times \mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{0.3}$$

So the plane equation becomes

$$\mathbf{n}^\top \mathbf{x} = 1. \tag{0.4}$$

For a point $\mathbf{q} \in \mathbb{R}^3$, its reflection across the plane $\mathbf{n}^\top \mathbf{x} = 1$ is

$$\mathbf{S} = \mathbf{Q} - 2\frac{\mathbf{n}^\top \mathbf{Q} - 1}{\|n\|^2}\mathbf{n}, \tag{0.5}$$

Here

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 10 \\ 15 \\ 20 \end{pmatrix}. \tag{0.6}$$

$$\mathbf{n}^\top \mathbf{n} = 1^2 + 1^2 + 1^2 = 3. \tag{0.7}$$

$$\mathbf{S} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} \frac{58}{3} \\ -\frac{43}{3} \\ -\frac{28}{3} \end{pmatrix}. \tag{0.8}$$

$$3(\alpha+\beta) = -101, \quad 3(\beta+\gamma) = -71, \quad 3(\gamma+\alpha) = -86, \quad 3(\alpha+\beta+\gamma) = -129. \tag{0.9}$$

Hence, the correct options are

$$\boxed{(a), (b), (c)}. \tag{0.10}$$

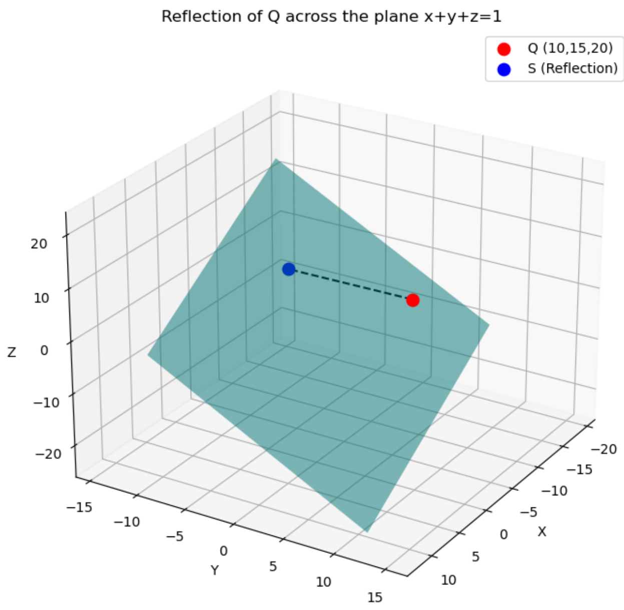

Fig. 4.1