Matgeo Presentation - Problem 2.5.2

Revanth Siva Kumar.D – EE25BTECH11048

September 29, 2025

Problem Statement

Verify the type of triangle formed by the points:A(-4,0),B(4,0),C(0,3).

Solution: Setup

$$\mathbf{A} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

Vectors:

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}, \quad \mathbf{AC} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \quad \mathbf{BC} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

Solution: Right-Angle Check

Step 2: Check for Right-Angled triangle (perpendicular sides)
Dot product should be 0.

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}8\\0\end{pmatrix},\quad \mathbf{C}-\mathbf{A}=\begin{pmatrix}4\\3\end{pmatrix},\qquad (\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A})=\begin{pmatrix}8 & 0\end{pmatrix}\begin{pmatrix}4\\3\end{pmatrix}=32 \tag{0.1}$$

$$\mathbf{A}-\mathbf{B}=\begin{pmatrix}-8\\0\end{pmatrix},\quad \mathbf{C}-\mathbf{B}=\begin{pmatrix}-4\\3\end{pmatrix},\qquad (\mathbf{A}-\mathbf{B})^\top(\mathbf{C}-\mathbf{B})=\begin{pmatrix}-8 & 0\end{pmatrix}\begin{pmatrix}-4\\3\end{pmatrix}=32 \tag{0.2}$$

$$\mathbf{A}-\mathbf{C}=\begin{pmatrix}-4\\-3\end{pmatrix},\quad \mathbf{B}-\mathbf{C}=\begin{pmatrix}4\\-3\end{pmatrix},\quad (\mathbf{A}-\mathbf{C})^\top(\mathbf{B}-\mathbf{C})=\begin{pmatrix}-4 & -3\end{pmatrix}\begin{pmatrix}4\\-3\end{pmatrix}=-7 \tag{0.3}$$

Since no pair of sides is perpendicular, the triangle is not right-angled.

Solution:Isosceles triangle check

$$\text{Midpoint of } AB: \quad \mathbf{M} = \frac{\mathbf{A}+\mathbf{B}}{2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.4}$$

$$\mathbf{C} - \mathbf{M} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{0.5}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \tag{0.6}$$

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{M}) = \begin{pmatrix} 8 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 3 \end{pmatrix} = 0 \tag{0.7}$$

Hence, C lies on the perpendicular bisector of AB.

$AC = BC = 5 \implies \triangle ABC$ is isosceles.

Equilateral and Scalene Triangle check

$$AB^2 = (\mathbf{B} - \mathbf{A})^\top(\mathbf{B} - \mathbf{A}) = \begin{pmatrix} 8 & 0 \end{pmatrix} \begin{pmatrix} 8 \\ 0 \end{pmatrix} = 64 \tag{0.8}$$

$$BC^2 = (\mathbf{C} - \mathbf{B})^\top(\mathbf{C} - \mathbf{B}) = \begin{pmatrix} -4 & 3 \end{pmatrix} \begin{pmatrix} -4 \\ 3 \end{pmatrix} = 16 + 9 = 25 \tag{0.9}$$

$$AC^2 = (\mathbf{C} - \mathbf{A})^\top(\mathbf{C} - \mathbf{A}) = \begin{pmatrix} 4 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = 16 + 9 = 25 \tag{0.10}$$

All sides are not equal (64, 25, 25), so the triangle is not equilateral.

Since two sides are equal, the triangle is not scalene.

Conclusion

Therefore, the triangle with vertices $(-4, 0), (4, 0), (0, 3)$ is an **isosceles triangle** with $AC = BC = 5$.

C Source Code: points.c

```
#include <math.h>

// Compute dot product of 2D vectors
double dot_product(double *u, double *v) {
return u[0]*v[0] + u[1]*v[1];
}

// Compute squared norm of 2D vector
double norm_squared(double *u) {
return u[0]*u[0] + u[1]*u[1];
}
```

Python Script: call c.py

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

lib = ctypes.CDLL("./points.so")

lib.dot_product.argtypes = [ctypes.POINTER(ctypes.c_double),
ctypes.POINTER(ctypes.c_double)]
lib.dot_product.restype = ctypes.c_double

lib.norm_squared.argtypes = [ctypes.POINTER(ctypes.c_double)]
lib.norm_squared.restype = ctypes.c_double

A = np.array([-4.0, 0.0])
B = np.array([4.0, 0.0])
C = np.array([0.0, 3.0])

AB = B - A
AC = C - A
BC = C - B

dp1 = lib.dot_product(AB.ctypes.data_as(ctypes.POINTER(ctypes.c_double)),
AC.ctypes.data_as(ctypes.POINTER(ctypes.c_double)))
dp2 = lib.dot_product((-AB).ctypes.data_as(ctypes.POINTER(ctypes.c_double)),
BC.ctypes.data_as(ctypes.POINTER(ctypes.c_double)))
dp3 = lib.dot_product((-AC).ctypes.data_as(ctypes.POINTER(ctypes.c_double)),
(-BC).ctypes.data_as(ctypes.POINTER(ctypes.c_double)))
```

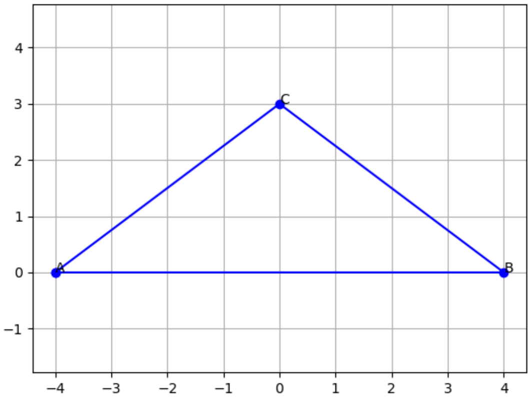
Python Script: call c.py

```
AB2 = lib.norm_squared(AB.ctypes.data_as(ctypes.POINTER(ctypes.c_double)))
AC2 = lib.norm_squared(AC.ctypes.data_as(ctypes.POINTER(ctypes.c_double)))
BC2 = lib.norm_squared(BC.ctypes.data_as(ctypes.POINTER(ctypes.c_double)))

print("Dot␣products:", dp1, dp2, dp3)
print("Squared␣lengths:␣AB^2␣=", AB2, "AC^2␣=", AC2, "BC^2␣=", BC2)

plt.plot([A[0], B[0]], [A[1], B[1]], 'r-', label='AB')
plt.plot([A[0], C[0]], [A[1], C[1]], 'g-', label='AC')
plt.plot([B[0], C[0]], [B[1], C[1]], 'b-', label='BC')

plt.scatter([A[0], B[0], C[0]], [A[1], B[1], C[1]], color='black')
plt.text(A[0], A[1], 'A(-4,0)', ha='right', va='top')
plt.text(B[0], B[1], 'B(4,0)', ha='left', va='top')
plt.text(C[0], C[1], 'C(0,3)', ha='center', va='bottom')

plt.axis('equal')
plt.legend()
plt.grid(True)
plt.show()
```

Python Script: plot.py

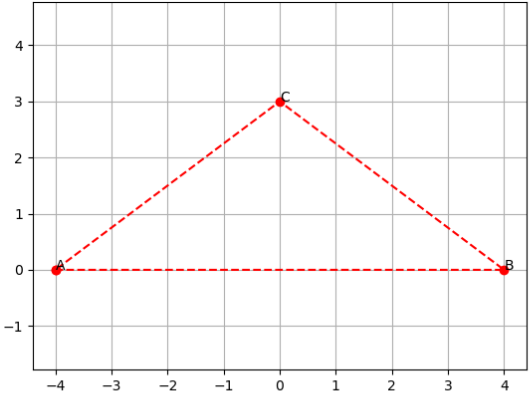
```
import matplotlib.pyplot as plt
import numpy as np

A = np.array([-4, 0])
B = np.array([4, 0])
C = np.array([0, 3])

plt.plot([A[0], B[0]], [A[1], B[1]], 'r-', label='AB')
plt.plot([A[0], C[0]], [A[1], C[1]], 'g-', label='AC')
plt.plot([B[0], C[0]], [B[1], C[1]], 'b-', label='BC')

plt.scatter([A[0], B[0], C[0]], [A[1], B[1], C[1]], color='black')
plt.text(A[0], A[1], 'A(-4,0)', ha='right', va='top')
plt.text(B[0], B[1], 'B(4,0)', ha='left', va='top')
plt.text(C[0], C[1], 'C(0,3)', ha='center', va='bottom')

plt.axis('equal')
plt.legend()
plt.grid(True)
plt.show()
```

Result Plot



Triangle ABC plotted using shared output

Result Plot



Triangle ABC plotted using direct python