Matrices in Geometry - 1.5.23

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Problem Statement

Show that the points **A** $\left(-2\hat{i}+3\hat{j}+5\hat{k}\right)$, **B** $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and **C** $\left(7\hat{i}-\hat{k}\right)$ are collinear.

Solution

Given $\mathbf{A} = \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 7 \\ 0 \\ -1 \end{pmatrix}$ are three points.

They are defined to be collinear if rank of the collinearity matrix is 1.

$$\text{Collinearity matrix is } \begin{pmatrix} \mathbf{A} - \mathbf{C} & \mathbf{B} - \mathbf{C} \end{pmatrix}^\top$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -9 \\ 3 \\ 6 \end{pmatrix} \tag{1}$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -6 \\ 2 \\ 4 \end{pmatrix} \tag{2}$$

Solution

$$\implies \text{rank}\begin{pmatrix} -9 & 3 & 6 \\ -6 & 2 & 4 \end{pmatrix} = 1 \tag{3}$$

$$\begin{pmatrix} -9 & 3 & 6 \\ -6 & 2 & 4 \end{pmatrix} \xrightarrow{R_2 \to R_2 - \frac{2}{3}R_1} \begin{pmatrix} -9 & 3 & 6 \\ 0 & 0 & 0 \end{pmatrix} \tag{4}$$

We know that for the rank of a matrix to be equal to 1, all the elements in the lower row of the matrix must be zero.
So it is proved that the given points are collinear

Conclusion

Hence, as the rank of the collinearity matrix is 1, it is proved that the given three points are collinear.