

4.8.17

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Question

The foot of a perpendicular drawn from the point $(-2, -1, -3)$ on a plane is $(1, -3, 3)$. Find the equation of the plane.

Step 1: Understanding the problem

Given points:

$$\mathbf{P} = \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix}$$

- **P**: Point from which perpendicular is drawn
- **F**: Foot of the perpendicular on the plane

Step 2: Vector along the perpendicular

Vector from **P** to **F**:

$$\mathbf{PF} = \mathbf{F} - \mathbf{P} = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$$

Step 3: Normal vector to the plane

Since **PF** is perpendicular to the plane, the normal vector **n** is:

$$\mathbf{n} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$$

Step 4: Equation of the plane in vector form

Equation of the plane:

$$\mathbf{n}^T(\mathbf{r} - \mathbf{F}) = 0$$

where $\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is any point on the plane.

Step 5: Substitute and expand

Substitute **n** and **F**:

$$(3 \quad -2 \quad 6) \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} \right) = 0$$

$$3(x - 1) - 2(y + 3) + 6(z - 3) = 0$$

Step 6: Simplify the equation

$$3x - 3 - 2y - 6 + 6z - 18 = 0 \implies 3x - 2y + 6z = 27$$

C Code to find plane equation

```
#include <stdio.h>

int main() {
    int a, b, c; // Normal vector components
    int x0, y0, z0; // Coordinates of foot of perpendicular

    printf(Enter normal vector (a b c): );
    scanf(%d %d %d, &a, &b, &c);

    printf(Enter foot coordinates (x0 y0 z0): );
    scanf(%d %d %d, &x0, &y0, &z0);

    int d = a*x0 + b*y0 + c*z0;

    printf(Plane equation: %dx + %dy + %dz = %d\n, a, b, c, d);

    return 0;
}
```

Python Code to plot plane and points (Part 1)

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Points
P = np.array([-2, -1, -3])
F = np.array([1, -3, 3])

# Normal vector
n = F - P

# Create grid
xx, yy = np.meshgrid(range(-5, 5), range(-5, 5))
a, b, c = n
x0, y0, z0 = F

if c != 0:
    zz = (-a * (xx - x0) - b * (yy - y0)) / c + z0
else:
```

Python Code to plot plane and points (Part 2)

```
fig = plt.figure(figsize=(10, 7))
ax = fig.add_subplot(111, projection='3d')

ax.plot_surface(xx, yy, zz, alpha=0.5, color='cyan')

ax.scatter(*P, color='red', s=100, label='Point P (-2,-1,-3)')
ax.scatter(*F, color='blue', s=100, label='Foot F (1,-3,3)')

ax.plot([P[0], F[0]], [P[1], F[1]], [P[2], F[2]],
        color='green', linestyle='--', label='Perpendicular')

ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('Plane and Perpendicular Foot')
ax.legend()

plt.show()
```

Plot

`figs/python_plot.png`