4.3.34

EE25BTECH11044 - Sai Hasini Pappula

Question: If the line

$$\frac{x}{a} + \frac{y}{b} = 1$$

passes through the points $(2, -3)$ and $(4, -5)$, then find (a, b)

Solution:

For $(2, -3)$:

$$\frac{2}{a} + \frac{-3}{b} = 1 \quad \Rightarrow \quad \frac{2}{a} - \frac{3}{b} = 1$$

For $(4, -5)$:

$$\frac{4}{a} + \frac{-5}{b} = 1 \quad \Rightarrow \quad \frac{4}{a} - \frac{5}{b} = 1$$

Let

$$u = \frac{1}{a}, \quad v = \frac{1}{b}.$$

Then the system becomes:

$$2u - 3v = 1, \qquad 4u - 5v = 1$$

or in matrix form:

$$\begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

$$A = \begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix}.$$

We compute

$$AA^T = \begin{bmatrix} 13 & 23 \\ 23 & 41 \end{bmatrix}.$$

Hence, the norm of A is

$$\|A\|_F = \sqrt{\text{trace}(AA^T)} = \sqrt{13 + 41} = \sqrt{54} \neq 0,$$

so the matrix is invertible.

$$A^{-1} = \frac{1}{2} \begin{bmatrix} -5 & 3 \\ -4 & 2 \end{bmatrix}.$$

Thus,

$$\begin{bmatrix} u \\ v \end{bmatrix} = A^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -5 & 3 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -2 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}.$$

Hence,

$$u = -1 \;\Rightarrow\; a = -1, \qquad v = -1 \;\Rightarrow\; b = -1.$$

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Verification: Equation of the line:

$$\frac{x}{-1} + \frac{y}{-1} = 1 \quad \Rightarrow \quad -x - y = 1.$$

For $(2, -3)$:

$$-(2) - (-3) = -2 + 3 = 1 \quad \checkmark$$

For $(4, -5)$:

$$-(4) - (-5) = -4 + 5 = 1 \quad \checkmark$$

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Final Answer:

$$(a, b) = (-1, -1).$$