MatGeo Assignment 1.2.14

AI25BTECH11008

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Question

The fourth vertex D of a parallelogram $ABCD$ whose three vertices are $A(-2, 3)$, $B(6, 7)$ and $C(8, 3)$ is

Theoretical Solution

Let us solve the given equation theoretically and then verify the solution computationally.
According to the question,

We are given three vertices of a parallelogram:

$$A(-2,3),\ B(6,7),\ C(8,3).$$

Property

In a parallelogram, the diagonals bisect each other. So, the midpoints of the diagonals are equal.

Theoretical Solution

Let $D(x, y)$ be the fourth vertex.

$$\text{Midpoint of } AC = \text{Midpoint of } BD$$

$$\frac{1}{2}\begin{pmatrix} -2+8 \\ 3+3 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 6+x \\ 7+y \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{6+x}{2} \\ \frac{7+y}{2} \end{pmatrix}$$

$$\frac{6+x}{2} = 3, \quad \frac{7+y}{2} = 3$$

$$x = 0, \quad y = -1$$

$$\therefore \; D(0, -1)$$

C-code

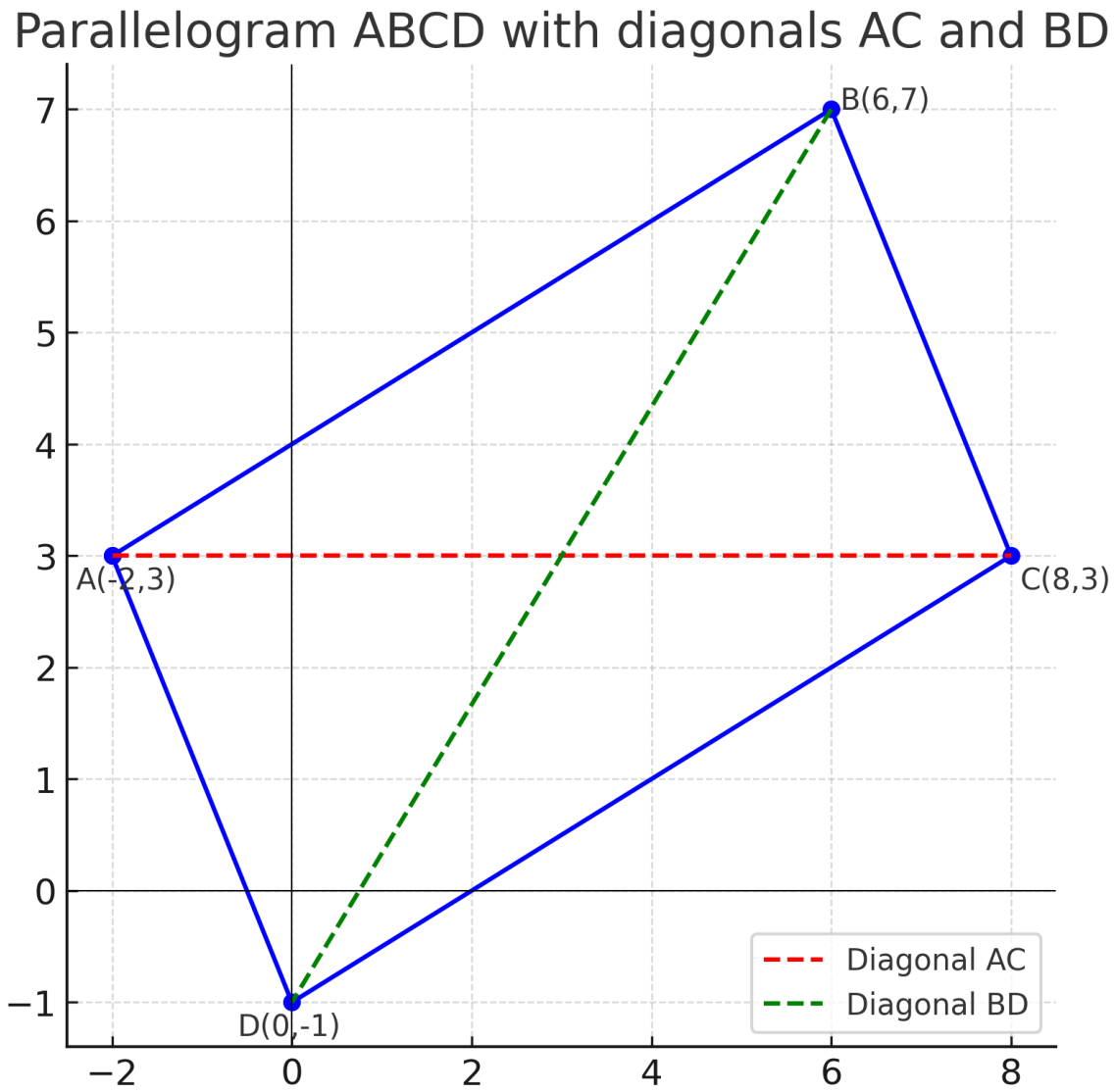
```c
#include <stdio.h>

int main() {
    // Given vertices
    int x1 , y1 ; // A
    int x2 , y2 ; // B
    int x3 , y3 ; // C
    int x, y; // D (to be calculated)

    // Using midpoint property: midpoint of AC = midpoint of BD
    x = x1 + x3 - x2; // Derived formula
    y = y1 + y3 - y2;

    return 0;
}
```

Plot


Parallelogram ABCD with diagonals AC and BD
B(6,7)
A(-2,3)
C(8,3)
D(0,-1)
Diagonal AC
Diagonal BD
7
6
5
4
3
2
1
0
−1
−2
0
2
4
6
8

Python code for plot

```
    import matplotlib.pyplot as plt

# Given points
A = (-2, 3)
B = (6, 7)
C = (8, 3)
D = (0, -1) # calculated fourth vertex

# Plotting the parallelogram
x_coords = [A[0], B[0], C[0], D[0], A[0]]
y_coords = [A[1], B[1], C[1], D[1], A[1]]

plt.figure(figsize=(6,6))
plt.plot(x_coords, y_coords, 'b-o')

# Plot diagonals
plt.plot([A[0], C[0]], [A[1], C[1]], 'r--', label='Diagonal AC')
plt.plot([B[0], D[0]], [B[1], D[1]], 'g--', label='Diagonal BD')
```

Python code for plot

```
# Label points
plt.text(A[0]-0.4, A[1]-0.3, 'A(-2,3)', fontsize=10)
plt.text(B[0]+0.1, B[1], 'B(6,7)', fontsize=10)
plt.text(C[0]+0.1, C[1]-0.3, 'C(8,3)', fontsize=10)
plt.text(D[0]-0.6, D[1]-0.3, 'D(0,-1)', fontsize=10)

# Axes and grid
plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
plt.grid(True, linestyle='--', alpha=0.5)

# Title and legend
plt.legend()
plt.title(Parallelogram ABCD with diagonals AC and BD)
```

Conclusion

From the figure it is clearly verified that the theoretical solution matches with the computational solution.