2.6.22

AI25BTECH11013-Gautham

Question:
Given vectors $\mathbf{a} = 2\vec{i} + \vec{j} + 3\vec{k}$ and $\mathbf{b} = 3\vec{i} + 5\vec{j} - 2\vec{k}$, find $|\mathbf{a} \times \mathbf{b}|$.

Solution:

The cross product or vector product of two vectors $\mathbf{A} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix}$ is defined as:

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} A_2B_3 - A_3B_2 \\ A_3B_1 - A_1B_3 \\ A_1B_2 - A_2B_1 \end{pmatrix} \tag{0.1}$$

Now, given

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix} \tag{0.2}$$

Using the formula for cross product,

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} 1 \times (-2) - 3 \times 5 \\ 3 \times 3 - 2 \times (-2) \\ 2 \times 5 - 1 \times 3 \end{pmatrix} \tag{0.3}$$

$$= \begin{pmatrix} -2 - 15 \\ 9 + 4 \\ 10 - 3 \end{pmatrix} \tag{0.4}$$

$$= \begin{pmatrix} -17 \\ 13 \\ 7 \end{pmatrix} \tag{0.5}$$

Finally, the magnitude of the cross product is:

$$\|\mathbf{a} \times \mathbf{b}\| = \sqrt{(-17)^2 + 13^2 + 7^2} \tag{0.6}$$

$$= \sqrt{289 + 169 + 49} \tag{0.7}$$

$$= \sqrt{507} \tag{0.8}$$

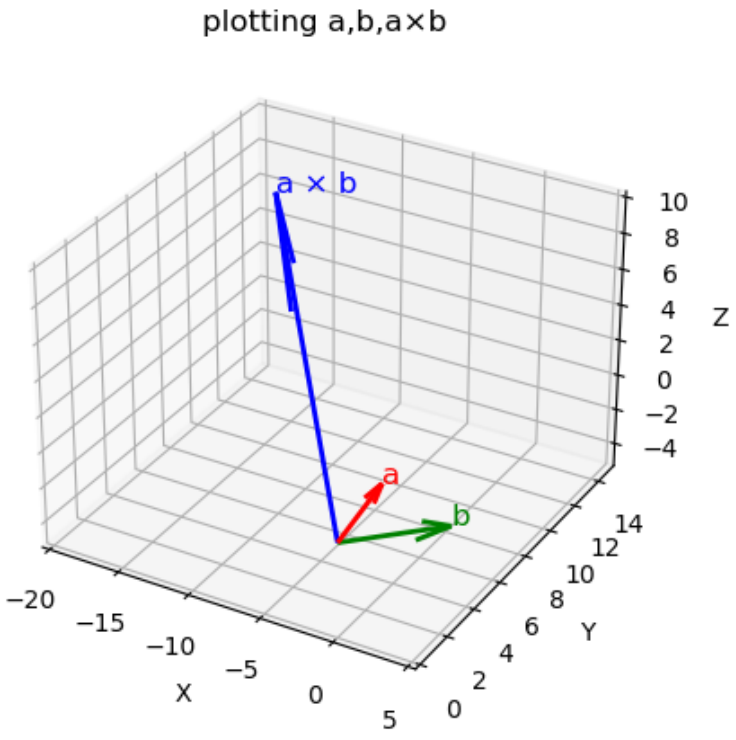

Fig. 0.1