Problem 4.8.26

Sarvesh Tamgade

September 14, 2025

Question: Find the coordinates of the foot of the perpendicular drawn from the point

$$\mathbf{P} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$$

to the Y-axis.

Solution

he Y-axis has the direction vector

$$\mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

and passes through the origin. Its general point is

$$\mathbf{Q} = \begin{pmatrix} 0 \\ q \\ 0 \end{pmatrix}.$$

Any point $\mathbf{Q}$ on the Y-axis satisfies $x = 0$ and $z = 0$.

Let $\mathbf{P} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$.

The foot of the perpendicular $\mathbf{Q}$ is given by projecting $\mathbf{P}$ onto the Y-axis as

$$\mathbf{Q} = \left(\mathbf{e_2}^\top \mathbf{P} \right) \frac{\mathbf{e_2}}{\|\mathbf{e_2}\|^2}.$$

Solution

$$\mathbf{e_2}^\top \mathbf{P} = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}.$$

Since

$$\|\mathbf{e_2}\|^2 = 0^2 + 1^2 + 0^2 = 1,$$

the foot of the perpendicular is

$$\mathbf{Q} = \left(\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} \right) \mathbf{e_2}.$$

$$\mathbf{Q} = (-3)\mathbf{e_2} = \begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix}.$$

Final Answer: The coordinates of the foot of the perpendicular are

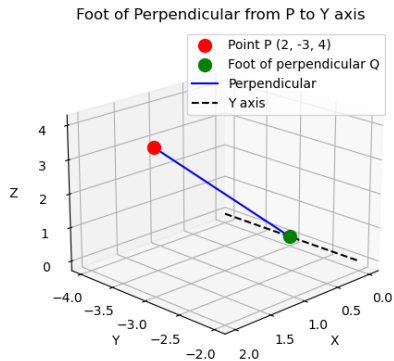$$\begin{pmatrix} 0 \\ -3 \\ 0 \end{pmatrix}.$$

Figure: Vector Representation

C Code

```c
#include <stdio.h>
#include "matfun.h"

int main() {
    double P[3] = {2.0, -3.0, 4.0};
    double Q[3];

    foot_of_perpendicular_to_Y_axis(P, Q);

    printf("Foot of the perpendicular from P(2, -3, 4) to Y-axis
        is: (%.2f, %.2f, %.2f)\n", Q[0], Q[1], Q[2]);
    return 0;
}
```

```python
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import numpy as np

# Points
P = np.array([2, -3, 4]) # Given point
Q = np.array([0, -3, 0]) # Foot of the perpendicular on Y axis

# Y-axis vector for reference
y_axis = np.array([[0, 0], [min(P[1], Q[1]) - 1, max(P[1], Q[1])
    + 1], [0, 0]])

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Plot point P
ax.scatter(P[0], P[1], P[2], color='r', s=100, label='Point P (2,
    -3, 4)')
```

```python
# Plot foot of perpendicular Q
ax.scatter(Q[0], Q[1], Q[2], color='g', s=100, label='Foot of
    perpendicular Q')

# Plot the perpendicular line from P to Q
ax.plot([P[0], Q[0]], [P[1], Q[1]], [P[2], Q[2]], color='b',
    label='Perpendicular')

# Plot Y axis
ax.plot(y_axis[0], y_axis[1], y_axis[2], color='k', linestyle='--
    ', label='Y axis')

# Labels and title
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('Foot of Perpendicular from P to Y axis')

ax.legend()
```

```python
# Adjust view angle
ax.view_init(elev=20, azim=45)

# Save the figure as  .png
plt.savefig('foot_of_perpendicular.png')
plt.show()
```