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EE25BTECH11032 - Kartik Lahoti

Question:

Rambha travels $300\,km$ to her home partly by train and partly by bus. She takes 4 hours if she travels $60\,km$ by train and the remaining by bus. If she travels $100\,km$ by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus seperately.

Solution:

Given,

Symbol	Value	Description
$\mathbf{n_1}$	$\begin{pmatrix} 60 \\ 240 \end{pmatrix}$	Normal Vector
$\mathbf{n_2}$	$\begin{pmatrix} 100 \\ 200 \end{pmatrix}$	Normal Vector
c_1	4	Constant 1
c_2	$\frac{25}{6}$	Constant 2
$\mathbf{P}$	?	Reciprocal Speed Vector

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Let the equations be,

$$\mathbf{n_1}^\top \mathbf{X} = c_1 \tag{0.1}$$
$$\mathbf{n_2}^\top \mathbf{X} = c_2 \tag{0.2}$$

Since $\mathbf{P}$ satisfies both the lines,

$$\mathbf{n_1}^\top \mathbf{P} = c_1 \tag{0.3}$$
$$\mathbf{n_2}^\top \mathbf{P} = c_2 \tag{0.4}$$

Solving for $\mathbf{P}$

$$\left(\begin{array}{cc|c} 60 & 240 & 4 \\ 100 & 200 & \frac{25}{6} \end{array} \right) \xleftrightarrow{R_1 \to \frac{R_1}{60}} \left(\begin{array}{cc|c} 1 & 4 & \frac{1}{15} \\ 100 & 200 & \frac{25}{6} \end{array} \right) \tag{0.5}$$

$$\begin{pmatrix} 1 & 4 \\ 100 & 200 \end{pmatrix} \begin{matrix} \frac{1}{15} \\ \frac{25}{6} \end{matrix} \xleftrightarrow{R_2 \to \frac{R_2}{100}} \begin{pmatrix} 1 & 4 \\ 1 & 2 \end{pmatrix} \begin{matrix} \frac{1}{15} \\ \frac{1}{24} \end{matrix} \qquad (0.6)$$

$$\begin{pmatrix} 1 & 4 \\ 1 & 2 \end{pmatrix} \begin{matrix} \frac{1}{15} \\ \frac{1}{24} \end{matrix} \xleftrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix} \begin{matrix} \frac{1}{15} \\ \frac{-1}{40} \end{matrix} \qquad (0.7)$$

$$\begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix} \begin{matrix} \frac{1}{15} \\ \frac{-1}{40} \end{matrix} \xleftrightarrow{R_2 \to \frac{R_2}{-2}} \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} \begin{matrix} \frac{1}{15} \\ \frac{1}{80} \end{matrix} \qquad (0.8)$$

$$\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} \begin{matrix} \frac{1}{15} \\ \frac{1}{80} \end{matrix} \xleftrightarrow{R_1 \to R_1 - 4R_2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{matrix} \frac{1}{60} \\ \frac{1}{80} \end{matrix} \qquad (0.9)$$

$$\therefore \mathbf{P} = \begin{pmatrix} \frac{1}{60} \\ \frac{1}{80} \end{pmatrix} \qquad (0.10)$$

Since $\mathbf{P}$ is the reciprocal of the speeds
The speed of train is $60\,km/h$ and bus is $80\,km/h$

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