

5.13.15

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Question

The area of a triangle formed by vertices **O**, **A** and **B**, where $\mathbf{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\mathbf{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$ is

Given:

$$\mathbf{A} = \begin{pmatrix} 5a & -b \\ 3 & 2 \end{pmatrix} \quad (1)$$

$$\text{Adj}(\mathbf{A}) = \begin{pmatrix} 2 & b \\ -3 & 5a \end{pmatrix} \quad (2)$$

Solution

Compute AA^T

First, compute the transpose:

$$A^T = \begin{pmatrix} 5a & 3 \\ -b & 2 \end{pmatrix} \quad (3)$$

Now multiply:

$$AA^T = \begin{pmatrix} 5a & -b \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 5a & 3 \\ -b & 2 \end{pmatrix} \quad (4)$$

Solution

Compute each entry:

$$AA^T = \begin{pmatrix} (5a)^2 + (-b)^2 & 5a \cdot 3 + (-b) \cdot 2 \\ 3 \cdot 5a + 2 \cdot (-b) & 3^2 + 2^2 \end{pmatrix} = \begin{pmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{pmatrix} \quad (5)$$

$$\begin{pmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{pmatrix} = \begin{pmatrix} 2 & b \\ -3 & 5a \end{pmatrix} \quad (6)$$

Solution

Compare bottom-right entries:

$$13 = 5a \Rightarrow a = \frac{13}{5} \quad (7)$$

Compare top-right entries:

$$15a - 2b = b \Rightarrow 15a = 3b \Rightarrow b = 13 \quad (8)$$

Final Step: Compute $5a + b$

$$5a + b = 5 \times \frac{13}{5} + 13 = 26 = \boxed{26} \quad (9)$$