2.9.11

EE25BTECH11060 - V.Namaswi

Question

If $\bar{a}$ and $\bar{b}$ are unit vectors and θ is angle between them then prove that $\sin\frac{\theta}{2} = \frac{1}{2}\,|\bar{a} - \bar{b}|$

Solution

Squaring on both sides,

$$\sin^2\frac{\theta}{2} = \frac{1}{4}|\bar{a} - \bar{b}|^2$$

consider RHS,

$$\implies \frac{1}{4}\|a - b\|^2 \tag{1}$$

$$= \frac{1}{4}(a - b)^\top(a - b) \tag{2}$$

$$= \frac{1}{4}\left(a^\top a - 2a^\top b + b^\top b\right) \tag{3}$$

$$= \frac{1}{4}\left(1 - 2a^\top b + 1\right) \tag{4}$$

$$= \frac{1}{2}(1 - a^\top b) \tag{5}$$

$$= \frac{1}{2}\left(1 - \cos\theta\right) \tag{6}$$

$$= \sin^2\frac{\theta}{2} \tag{7}$$

$$= LHS \tag{8}$$

Hence, $\sin\frac{\theta}{2} = \frac{1}{2}\,|\bar{a} - \bar{b}|$