1.3.6

AI25BTECH11027 - NAGA BHUVANA

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Question:
Show that the points **A** $(6,2)$,**B** $(2,1)$,**C** $(1,5)$ and **D** $(5,6)$ are vertices of a square.
Solution:
Given that

$$\mathbf{A} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} \tag{0.1}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2-6 \\ 1-2 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \tag{0.2}$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} 1-5 \\ 5-6 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \tag{0.3}$$

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \tag{0.4}$$

By the above property we can say that **ABCD** is a parallelogram. Consider the sides

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} 6-5 \\ 2-6 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} \tag{0.5}$$

$$(\mathbf{B} - \mathbf{A})^T = \begin{pmatrix} -4 & -1 \end{pmatrix} \tag{0.6}$$

$$\|\mathbf{B} - \mathbf{A}\| = \sqrt{17} \tag{0.7}$$

$$\|\mathbf{A} - \mathbf{D}\| = \sqrt{17} \tag{0.8}$$

$$\tag{0.9}$$

Consider the angle θ between the sides $\mathbf{B} - \mathbf{A}$ and $\mathbf{A} - \mathbf{D}$ of the parallelogram

$$\cos\theta = \frac{(B-A)^T (A-D)}{\|\mathbf{B} - \mathbf{A}\|\|\mathbf{A} - \mathbf{D}\|} \tag{0.10}$$

$$\cos\theta = \frac{\begin{pmatrix}-4 & -1\end{pmatrix}\begin{pmatrix}1 \\ -4\end{pmatrix}}{\sqrt{17}\sqrt{17}} \tag{0.11}$$

$$\cos\theta = \frac{(-4)(1)+(-1)(-4)}{17} \tag{0.12}$$

$$\tag{0.13}$$

$$\cos\theta = 0 \tag{0.14}$$

$$\implies \theta = 90^\circ \tag{0.15}$$

Property:
A parallelogram with one angle 90° is a rectangle
Hence the parallelogram is a rectangle

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix}5 \\ -3\end{pmatrix} \tag{0.16}$$

$$\implies (\mathbf{A} - \mathbf{C})^T = \begin{pmatrix}5 & -3\end{pmatrix} \tag{0.17}$$

$$\mathbf{B} - \mathbf{D} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \tag{0.18}$$

Let the angle between the diagonals of the rectangle be α
Now Consider the inner product of the diagonals of rectangle $\mathbf{A} - \mathbf{C}$ and $\mathbf{B} - \mathbf{D}$

$$\cos\alpha = \frac{\left(A - C\right)^T \left(B - D\right)}{\|\mathbf{A} - \mathbf{C}\|\|\mathbf{B} - \mathbf{D}\|} = \frac{\begin{pmatrix} 5 & -3 \end{pmatrix} \begin{pmatrix} -3 \\ -5 \end{pmatrix}}{\sqrt{34}\sqrt{34}} \tag{0.19}$$

$$\cos\alpha = 0 \tag{0.20}$$

$$\implies \alpha = 90^\circ \tag{0.21}$$

Property:
Rectangle with diagonals at right angle is a square
Hence given points forms a square

Graphical Representation