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EE25BTECH11044 - Sai Hasini Pappula

Question

Find the equation of the plane passing through the line of intersection of the planes

$$r.(2\hat{i} + 2\hat{j} - 3\hat{k}) = 7 \tag{0.1}$$

$$r.(2\hat{i} + 5\hat{j} + 3\hat{k}) = 9 \tag{0.2}$$

such that the intercepts made by the plane on the x-axis and z-axis are equal.

Solution

Step 1: Represent as a system

The general plane through the intersection of the planes can be written as:

$$(2 + 2\lambda)x + (2 + 5\lambda)y + (-3 + 3\lambda)z = 7 + 9\lambda, \tag{0.3}$$

where λ is a scalar.
We can represent the plane coefficients as a **row vector**:

$$\begin{bmatrix} a & b & c \end{bmatrix} = \begin{bmatrix} 2 + 2\lambda & 2 + 5\lambda & -3 + 3\lambda \end{bmatrix}, \quad d = 7 + 9\lambda.$$

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Step 2: Express intercept condition as a matrix equation

Let the plane intercepts on x and z axes be equal.

The x-intercept occurs when$y = 0, z = 0$ and the z-intercept occurs when $x = 0, y = 0$.
This gives the system

$$\begin{bmatrix} 2 + 2\lambda & 0 & 0 \\ 0 & 0 & -3 + 3\lambda \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \begin{bmatrix} 7 + 9\lambda \\ 7 + 9\lambda \end{bmatrix}. \tag{0.4}$$

we can write:

$$(2 + 2\lambda)x_0 = 7 + 9\lambda, \quad (-3 + 3\lambda)z_0 = 7 + 9\lambda. \tag{0.5}$$

Equal intercepts condition:

$$x_0 = z_0 \quad \Rightarrow \quad 2 + 2\lambda = -3 + 3\lambda \quad \Rightarrow \quad \lambda = 5. \tag{0.6}$$

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Step 3: Form the plane equation matrix

Substitute $\lambda = 5$

$$\begin{bmatrix} a & b & c \end{bmatrix} = \begin{bmatrix} 2+10 & 2+25 & -3+15 \end{bmatrix} = \begin{bmatrix} 12 & 27 & 12 \end{bmatrix}, \quad d = 7 + 45 = 52.$$

Hence, the plane equation is

$$12x + 27y + 12z = 52. \tag{0.7}$$