

1.4.16

EE25BTECH11001 - Aarush Dilawri

Question:

Find the coordinates of the points which trisect the line segment joining the points $P(4, 2, -6)$ and $Q(10, -16, 6)$.

Solution:

Let the vector \mathbf{P} be

$$\mathbf{P} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}$$

Let the vector \mathbf{Q} be

$$\mathbf{Q} = \begin{pmatrix} 10 \\ -16 \\ 6 \end{pmatrix}$$

Using Section formula, we have to find the coordinates of the points which divide the line segment PQ in the ratio $2 : 1$ and $1 : 2$.

Section formula for a vector \mathbf{P} which divides the line formed by vectors \mathbf{S} and \mathbf{R} in the ratio $k:1$ is given by

$$\mathbf{P} = \frac{k\mathbf{R} + \mathbf{S}}{k + 1}$$

Let the vector which divides PQ in the ratio $2 : 1$ be \mathbf{S} and the vector which divides PQ in the ratio $1 : 2$ be \mathbf{R} . Using section formula,

$$\begin{aligned} \mathbf{S} &= \frac{2 \begin{pmatrix} 10 \\ -16 \\ 6 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}}{3} \\ \Rightarrow \mathbf{S} &= \frac{\begin{pmatrix} 20 \\ -32 \\ 12 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}}{3} \\ \Rightarrow \mathbf{S} &= \frac{\begin{pmatrix} 24 \\ -30 \\ 6 \end{pmatrix}}{3} \Rightarrow \mathbf{S} = \begin{pmatrix} 8 \\ -10 \\ 2 \end{pmatrix} \end{aligned}$$

Similarly,

$$\begin{aligned}\mathbf{R} &= \frac{\begin{pmatrix} 10 \\ -16 \\ 6 \end{pmatrix} + 2 \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}}{3} \\ \Rightarrow \mathbf{R} &= \frac{\begin{pmatrix} 10 \\ -16 \\ 6 \end{pmatrix} + \begin{pmatrix} 8 \\ 4 \\ -12 \end{pmatrix}}{3} \\ \Rightarrow \mathbf{R} &= \frac{\begin{pmatrix} 18 \\ -12 \\ -6 \end{pmatrix}}{3} \Rightarrow \mathbf{R} = \begin{pmatrix} 6 \\ -4 \\ -2 \end{pmatrix}\end{aligned}$$

Therefore, the points of trisection of PQ are

$$\mathbf{S} = \begin{pmatrix} 8 \\ -10 \\ 2 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} 6 \\ -4 \\ -2 \end{pmatrix}$$

See Fig. 0 ,

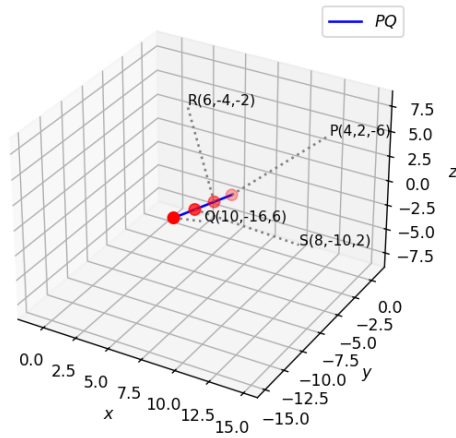


Fig. 0