4.12.17 Matgeo

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Question

Let P_1 : 2x + y - z = 3 and P_2 : x + 2y + z = 2 be two planes . Then, which of the following statements is/are TRUE ?

1. The line of intersection of P1 and P2 has direction ratios 1,2,-1
2. The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P1 and P2
3. The acute angle between P_1 and P_2 is 60°
4. If P_3 is the plane passing through the point (4,2,-2) and perpendicular to the line of intersection of P_1 and P_2 ,then the distance of the point (2,1,1) from the plane P_3 is $\frac{2}{\sqrt{3}}$

Solution

Let

$$P_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}^T \mathbf{X} = 3 \tag{1}$$

$$P_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}^T \mathbf{X} = 2 \tag{2}$$

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix} \mathbf{X} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \tag{3}$$

Combining both equations and solving by row reduction we get :

$$\mathbf{X} = \begin{bmatrix} 0 \\ \frac{5}{3} \\ -\frac{4}{3} \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \tag{4}$$

Hence , the direction ratios of the line of intersection are (1,-1,1) .

Solution

So option 1 is false
For option 2 :
simplifing the line equation we get the line equation to be :

$$\frac{x - \frac{4}{3}}{3} = \frac{y - \frac{1}{3}}{-3} = \frac{z}{3} \tag{5}$$

$$\mathbf{X} = \begin{bmatrix} -\frac{4}{3} \\ -\frac{1}{3} \\ 0 \end{bmatrix} + \mu \begin{bmatrix} 3 \\ -3 \\ 3 \end{bmatrix} \tag{6}$$

solving the equation by row reduction we get direction ratios of the line to be (3,-3,3)
For two lines to be perpendicular :

$$n_1^T n_2 = 0 \tag{7}$$

Solution

For the given lines :

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}^T \begin{bmatrix} 3 \\ -3 \\ 3 \end{bmatrix} = 9 \tag{8}$$

Hence the lines are not perpendicular . So option 2 is also false

Solution

We find the angle between two planes by the formula :

$$\cos\theta = \frac{|n_1^T n_2|}{\|n_1\|\|n_2\|} \tag{9}$$

By solving using above equation we get :

$$\cos\theta = \frac{1}{2} \tag{10}$$

Hence the angle $\theta = 60^\circ$. So option 3 is true

Solution

The plane perpendicular to a line has normal or direction ratios equal to the direction ratios of the line that is (1,-1,1)
Hence the plane equation can be written as :

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}^T \mathbf{X} = c \tag{11}$$

To find c we can substitute the point (4,2,-2) in the plane equation :

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}^T \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix} = 0 \tag{12}$$

Hence the plane equation is :

$$\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}^T \mathbf{X} = 0 \tag{13}$$

Solution

The distance of a point from a plane is given by the equation :

$$\frac{|n^T \mathbf{P} - c|}{\|n\|} \tag{14}$$

Solving using above equation for the point $\mathbf{P} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ we get :

$$\frac{|2-1+1|}{\sqrt{3}} = \frac{2}{\sqrt{3}} \tag{15}$$

Hence , option 4 is also true . Thus options 3 and 4 are true