

4.11.36

EE25BTECH11059 - Vaishnavi Ramkrishna Anantheertha

Question: Find the coordinates of the point where the line through the points $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane determined by the points $(1, 2, 3)$, $(4, 2, -3)$ and $(0, 4, 3)$.

Solution

Variable	Value
A	$(3, -4, -5)$
B	$(2, -3, 1)$
P	$(1, 2, 3)$
Q	$(4, 2, -3)$
R	$(0, 4, 3)$

TABLE 0: Variables Used

Let eq of plane be

$$\mathbf{n}^T \mathbf{x} = 1 \quad (0.1)$$

As **P, Q, R** lie on the plane

$$\mathbf{n}^T \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 1 \quad (0.2)$$

$$\mathbf{n}^T \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix} = 1 \quad (0.3)$$

$$\mathbf{n}^T \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix} = 1 \quad (0.4)$$

From eq (0.2), (0.3) and (0.4)

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 4 & 2 & -3 & 1 \\ 0 & 4 & 3 & 1 \end{array} \right) \xrightarrow{R_2 \rightarrow R_2 - 4R_1} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -6 & -15 & -3 \\ 0 & 4 & 3 & 1 \end{array} \right) \quad (0.5)$$

$$\xrightarrow{R_2 \rightarrow -\frac{1}{3}R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 2 & 5 & 1 \\ 0 & 4 & 3 & 1 \end{array} \right) \quad (0.6)$$

$$\xrightarrow{R_3 \rightarrow R_3 - 2R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & -7 & -1 \end{array} \right) \quad (0.7)$$

$$\xrightarrow{R_3 \rightarrow -\frac{1}{7}R_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & 1 & \frac{1}{7} \end{array} \right) \quad (0.8)$$

$$\xrightarrow{R_2 \rightarrow R_2 - 5R_3} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 2 & 0 & \frac{2}{7} \\ 0 & 0 & 1 & \frac{1}{7} \end{array} \right) \quad (0.9)$$

$$\xrightarrow{R_2 \rightarrow \frac{1}{2}R_2} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 0 & \frac{1}{7} \\ 0 & 0 & 1 & \frac{1}{7} \end{array} \right) \quad (0.10)$$

$$\xrightarrow{R_1 \rightarrow R_1 - 3R_3} \left(\begin{array}{ccc|c} 1 & 2 & 0 & \frac{4}{7} \\ 0 & 1 & 0 & \frac{1}{7} \\ 0 & 0 & 1 & \frac{1}{7} \end{array} \right) \quad (0.11)$$

$$\xrightarrow{R_1 \rightarrow R_1 - 2R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2}{7} \\ 0 & 1 & 0 & \frac{1}{7} \\ 0 & 0 & 1 & \frac{1}{7} \end{array} \right) \quad (0.12)$$

$$\mathbf{n} = \begin{pmatrix} \frac{2}{7} \\ \frac{1}{7} \\ \frac{1}{7} \end{pmatrix} \quad (0.13)$$

hence eq of plane is

$$\left(\frac{2}{7} \quad \frac{1}{7} \quad \frac{1}{7} \right) \mathbf{x} = 1 \quad (0.14)$$

let a point on line **AB** be

$$\mathbf{c} = k\mathbf{A} + (1 - k)\mathbf{B} \quad (0.15)$$

$$\mathbf{c} = \begin{pmatrix} 2 + k \\ -3 - k \\ 1 - 6k \end{pmatrix} \quad (0.16)$$

$$\begin{pmatrix} \frac{2}{7} & \frac{1}{7} & \frac{1}{7} \end{pmatrix} \begin{pmatrix} 2 + k \\ -3 - k \\ 1 - 6k \end{pmatrix} = 14 + 2k - 3 - k + 1 - 6k = 7 \quad (0.17)$$

$$2 - 5k = 7 \quad (0.18)$$

$$k = -1 \quad (0.19)$$

The point **c** is

$$\mathbf{c} = \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix} \quad (0.20)$$

Refer to Figure

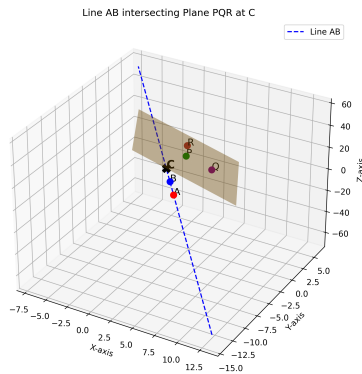


Fig. 0.1