4.13.20

EE25BTECH11015 - Bhoomika V

Question :-
A ray of light along $x + 3y = 3$ gets reflected upon reaching the X-axis. The equation of the reflected ray is:

$$\text{(a) } y = x + 3 \quad \text{(b) } 3y = x - 3$$

$$\text{(c) } y = 3x - 3 \quad \text{(d) } 3y = x - 1$$

Solution:

The given line in parametric (matrix) form

$$x + 3y = 3.$$

The normal vector is

$$\mathbf{n} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}.$$

A direction vector $\mathbf{d}$ satisfies $\mathbf{n}^\top \mathbf{d} = 0$.

$$\mathbf{d} = \begin{pmatrix} -3 \\ 1 \end{pmatrix},$$

A point on the line is

$$\mathbf{p} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (0 + 3 \cdot 1 = 3).$$

Hence, the parametric form is

$$\mathbf{r}(t) = \mathbf{p} + t\mathbf{d} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} -3 \\ 1 \end{pmatrix}.$$

Point of incidence (intersection with the x-axis)
For incidence with the x-axis, set $y = 0$. From the second component:

$$1 + t = 0 \quad \Rightarrow \quad t = -1.$$

Thus,

$$\mathbf{P} = \mathbf{r}(-1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} - 1 \begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}.$$

Reflection of the direction vector
Reflection in the x-axis is represented by the matrix

$$R = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

So,

$$\mathbf{d}' = R\mathbf{d} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}.$$

(Equivalently, we can take $\mathbf{d}' = (3, 1)$.)
Equation of the reflected ray
The reflected ray is

$$\mathbf{r}'(s) = \mathbf{P} + s\mathbf{d}' = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + s \begin{pmatrix} 3 \\ 1 \end{pmatrix}.$$

Coordinates:

$$x = 3 + 3s, \quad y = 0 + s.$$

Thus,

$$x - 3 = 3y \quad \Rightarrow \quad 3y = x - 3.$$

Equation of the reflected ray: 3y=x-3

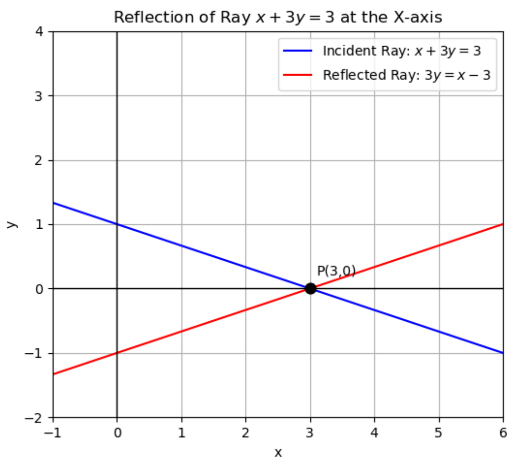

Fig. 0.1