Problem 12.214

The eigenvector pair of the matrix

$$A = \begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix} \tag{1}$$

is (PI 2008)

Options:

$$\text{(a)} \quad \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \end{pmatrix} \tag{2}$$

$$\text{(b)} \quad \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ -1 \end{pmatrix} \tag{3}$$

$$\text{(c)} \quad \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \begin{pmatrix} -2 \\ -1 \end{pmatrix} \tag{4}$$

$$\text{(d)} \quad \begin{pmatrix} 1 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{5}$$

Input Variables:

Symbol	Description
A	Given matrix $\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix}$
λ	Eigenvalue of A
$\mathbf{v}$	Corresponding eigenvector

Solution:

$$A = \begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix} \tag{6}$$

$$\det(A - \lambda I) = \det \begin{pmatrix} 3 - \lambda & 4 \\ 4 & -3 - \lambda \end{pmatrix} \tag{7}$$

$$= (3 - \lambda)(-3 - \lambda) - 16 \tag{8}$$

$$= \lambda^2 - 25 \tag{9}$$

$$\Rightarrow \lambda = \pm 5 \tag{10}$$

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For $\lambda = 5$**:**

$$(A - 5I)\mathbf{v} = 0 \tag{11}$$

$$\begin{pmatrix} -2 & 4 \\ 4 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{0} \tag{12}$$

$$-2x + 4y = 0 \Rightarrow x = 2y \tag{13}$$

$$\mathbf{v_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{14}$$

For $\lambda = -5$**:**

$$(A + 5I)\mathbf{v} = 0 \tag{15}$$

$$\begin{pmatrix} 8 & 4 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{0} \tag{16}$$

$$8x + 4y = 0 \Rightarrow y = -2x \tag{17}$$

$$\mathbf{v_2} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \tag{18}$$

Hence, the correct eigenvector pair is

$$\mathbf{v_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \mathbf{v_2} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}. \tag{19}$$
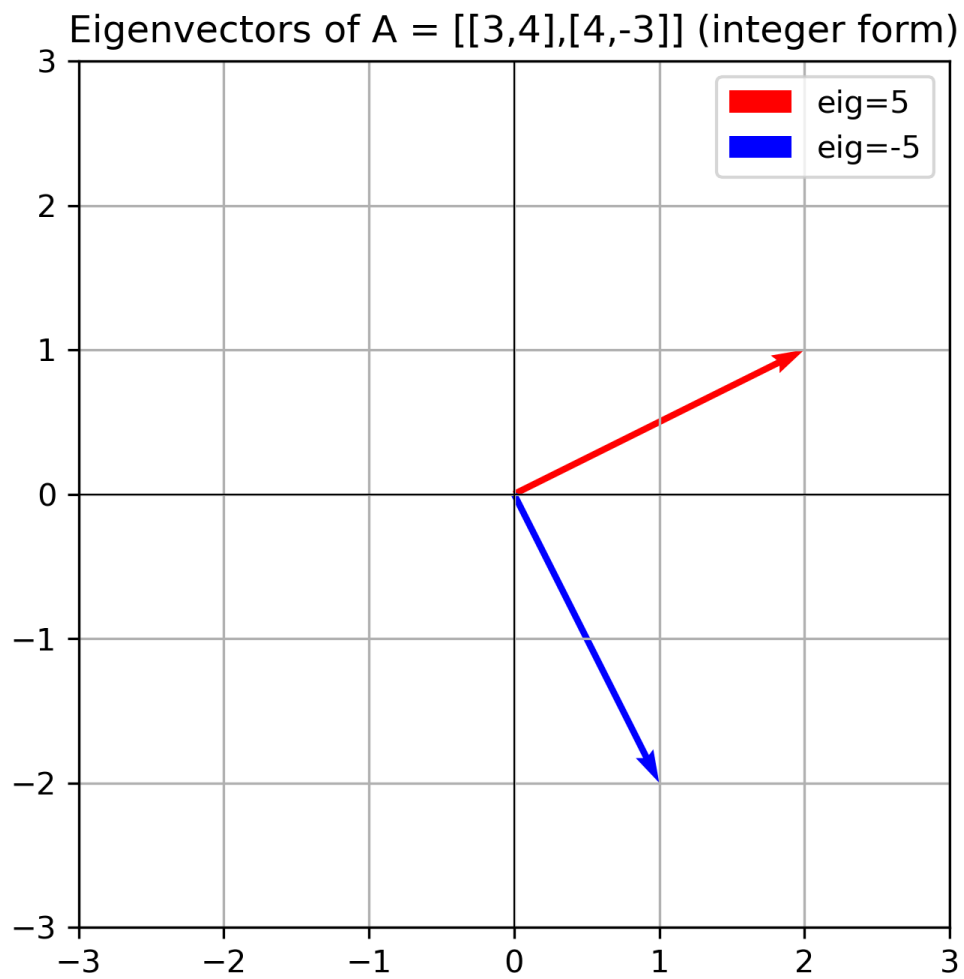
Answer: (a)

Figure 1