4.4.26

AI25BTECH11030 -Sarvesh Tamgade

Question: Find the equation of the median through vertex $\mathbf{A}$ of the triangle ABC, having vertices

$$\mathbf{A}(2,5), \quad \mathbf{B}(-4,9), \quad \mathbf{C}(-2,-1).$$

Solution:

Using the section formula, the midpoint $\mathbf{M}$ of the side BC is

$$\mathbf{M} = \frac{\mathbf{B}+\mathbf{C}}{2} = \frac{1}{2}\begin{pmatrix}-4\\9\end{pmatrix} + \frac{1}{2}\begin{pmatrix}-2\\-1\end{pmatrix} = \begin{pmatrix}-3\\4\end{pmatrix}. \tag{0.1}$$

The median passes through points $\mathbf{A} = \begin{pmatrix}2\\5\end{pmatrix}$ and $\mathbf{M} = \begin{pmatrix}-3\\4\end{pmatrix}$.

Let the required line have the equation

$$\mathbf{n}^\top \mathbf{x} = 1, \tag{0.2}$$

where

$$\mathbf{n} = \begin{pmatrix}n_1\\n_2\end{pmatrix} \tag{0.3}$$

is the column vector (normal vector).

Since both points $\mathbf{A}$ and $\mathbf{M}$ lie on the median, they satisfy the line equation:

$$\mathbf{n}^\top \mathbf{A} = 1, \quad \mathbf{n}^\top \mathbf{M} = 1, \tag{0.4}$$

or, explicitly,

$$\begin{pmatrix}2 & 5\\-3 & 4\end{pmatrix}\mathbf{n} = \begin{pmatrix}1\\1\end{pmatrix}. \tag{0.5}$$

We want to find $\mathbf{n}$ satisfying

$$\begin{pmatrix}2 & 5\\-3 & 4\end{pmatrix}\mathbf{n} = \mathbf{c}, \quad \text{where } \mathbf{c} = \begin{pmatrix}1\\1\end{pmatrix}. \tag{0.6}$$

Set up the augmented matrix with right-hand side 1:

$$\left(\begin{array}{cc|c}2 & 5 & 1\\-3 & 4 & 1\end{array}\right) \tag{0.7}$$

Perform row operation $R_2 \to R_2 + \frac{3}{2}R_1$:

$$\left(\begin{array}{cc|c}2 & 5 & 1\\0 & \frac{23}{2} & \frac{5}{2}\end{array}\right) \tag{0.8}$$

Perform row operation $R_1 \to R_1 - \frac{10}{23}R_2$:

$$\left(\begin{array}{cc|c} 2 & 0 & -\frac{2}{23} \\ 0 & \frac{23}{2} & \frac{5}{2} \end{array}\right) \tag{0.9}$$

The final augmented matrix is:

$$\left(\begin{array}{cc|c} 2 & 0 & -\frac{2}{23} \\ 0 & \frac{23}{2} & \frac{5}{2} \end{array}\right) \tag{0.10}$$

Solve the system:

$$2n_1 = -\frac{2}{23} \implies n_1 = -\frac{1}{23} \tag{0.11}$$

$$\frac{23}{2}n_2 = \frac{5}{2} \implies n_2 = \frac{5}{23} \tag{0.12}$$

$$\mathbf{n} = \frac{1}{23}\begin{pmatrix} -1 \\ 5 \end{pmatrix} \tag{0.13}$$

Therefore, equation of required line is:

$$\boxed{\begin{pmatrix} -1 & 5 \end{pmatrix}\mathbf{x} = 23}$$

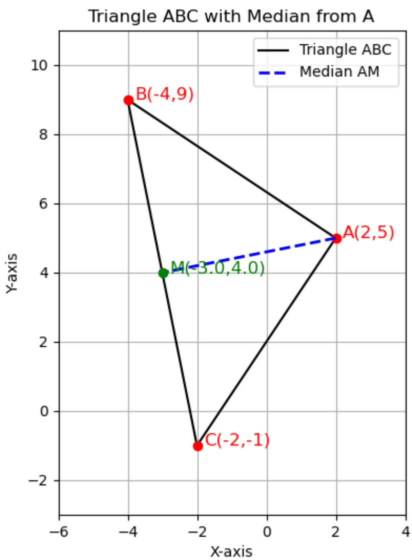

Fig. 0.1: Vector Representation