5.5.12

EE25BTECH11060 - V.Namaswi

Question

If $\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{pmatrix}$

find $\mathbf{A}^{-1}$. Hence solve the following system of equations

$$\begin{aligned} x + y + z &= 6 \\ x + 2z &= 7 \\ 3x + y + z &= 12 \end{aligned}$$

Solution

$$\mathbf{A}\mathbf{x} = \mathbf{I} \tag{1}$$

Forming Argumented Matrix

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 \\ 3 & 1 & 1 & 0 & 0 & 1 \end{array}\right) \tag{2}$$

Replace $R2 \to R2 - R1$ and $R3 \to R3 - 3R1$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 & 1 & 0 \\ 0 & -2 & -2 & -3 & 0 & 1 \end{array}\right) \tag{3}$$

Replace $R2 \to -R2$ and $R3 \to R3 + 2R2$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & -1 & 0 \\ 0 & 0 & -4 & -1 & -2 & 1 \end{array}\right) \tag{4}$$

Repalce $R_3 \leftarrow -\frac{1}{4}R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & 1 & -1 & 0 \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \end{array}\right) \tag{5}$$

Replace $R_1 \leftarrow R_1 - 2R_3,\ R_2 \leftarrow R_2 + R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{5}{4} & -\frac{1}{2} & -\frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \end{array}\right) \tag{6}$$

Thus

$$A^{-1} = \begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{4} & -\frac{1}{2} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \end{pmatrix}. \tag{7}$$

$$\mathbf{X} = \mathbf{A}^{-1}\mathbf{b} \tag{8}$$

$$\mathbf{X} = \begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{4} & -\frac{1}{2} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \end{pmatrix} \begin{pmatrix} 6 \\ 7 \\ 12 \end{pmatrix} \tag{9}$$

$$\mathbf{X} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \tag{10}$$