

**Question 2.6.37:**

The vector from origin to the points  $A$  and  $B$  are

$$\mathbf{a} = 2\hat{i} - 3\hat{j} + 2\hat{k} \quad \text{and} \quad \mathbf{b} = 2\hat{i} + 3\hat{j} + \hat{k}, \quad (1)$$

respectively, then the area of  $\triangle OAB$  is \_\_\_\_\_.

**Solution:** Given

The area of the triangle  $OAB$  is given by

$$\text{Area}(OAB) = \frac{1}{2} \|\mathbf{a} \times \mathbf{b}\|. \quad (2)$$

We have

$$\mathbf{a} = (2, -3, 2), \quad \mathbf{b} = (2, 3, 1). \quad (3)$$

Using the cross product definition,

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \\ a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \quad (4)$$

Substituting values:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \begin{pmatrix} -3 & 3 \\ 2 & 1 \end{pmatrix} \\ \begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix} \\ \begin{pmatrix} 2 & 2 \\ -3 & 3 \end{pmatrix} \end{vmatrix} = \begin{pmatrix} (-3)(1) - (3)(2) \\ (2)(2) - (1)(2) \\ (2)(3) - (2)(-3) \end{pmatrix}. \quad (5)$$

$$\mathbf{a} \times \mathbf{b} = (-9, 2, 12). \quad (6)$$

Now, its magnitude is

$$\|\mathbf{a} \times \mathbf{b}\| = \sqrt{(-9)^2 + (2)^2 + (12)^2} = \sqrt{81 + 4 + 144} = \sqrt{229}. \quad (7)$$

Therefore, the required area is

$$\text{Area}(OAB) = \frac{1}{2} \|\mathbf{a} \times \mathbf{b}\| = \frac{1}{2} \sqrt{229}. \quad (8)$$

$$\boxed{\text{Area}(OAB) = \frac{\sqrt{229}}{2}} \quad (9)$$