5.12.7

AI25BTECH11012 - GARIGE UNNATHI

Question:
Solve the following equations for x and y :

$$(ax - by) + (a + 4b) = 0$$
$$(bx + ay) + (b - 4a) = 0$$

Solution:
given two equations :

$$(ax - by) = -(a + 4b) \tag{0.1}$$
$$(bx + ay) = -(b - 4a) \tag{0.2}$$

these can be written as :

$$\begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -a - 4b \\ 4a - b \end{pmatrix} \tag{0.3}$$

Variable	**Formula**
A	$A = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$
B	$B = \begin{pmatrix} -a - 4b \\ 4a - b \end{pmatrix}$
X	$X = \begin{pmatrix} x \\ y \end{pmatrix}$

TABLE 0: Variables Used

From the equation 0.3 :

$$\mathbf{AX} = \mathbf{B} \tag{0.4}$$

To find X we need to multiply A^{-1} on both sides

$$\mathbf{X} = \mathbf{A^{-1}B} \tag{0.5}$$

Finding A^{-1} :

$$\begin{pmatrix} a - \lambda & -b \\ b & a - \lambda \end{pmatrix} = 0 \tag{0.6}$$
$$\lambda^2 - 2a\lambda + a^2 + b^2 = 0 \tag{0.7}$$

since :

$$|\mathbf{A} - \lambda \mathbf{I}| = 0 \tag{0.8}$$

$$\mathbf{A}^2 - 2a\mathbf{A} + (a^2 + b^2) = 0 \tag{0.9}$$

Multiply both sides by $\mathbf{A^{-1}}$:

$$\mathbf{A} - 2a\mathbf{I} + \mathbf{A^{-1}}(a^2 + b^2) = 0 \tag{0.10}$$

$$\mathbf{A^{-1}} = \frac{1}{a^2 + b^2}(2a\mathbf{I} - \mathbf{A}) \tag{0.11}$$

$$\mathbf{A^{-1}} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \tag{0.12}$$

from the equation 0.5 :

$$\mathbf{X} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} -a - 4b \\ 4a - b \end{pmatrix} \tag{0.13}$$

$$\mathbf{X} = \begin{pmatrix} -a^2 - b^2 \\ 4a^2 + 4b^2 \end{pmatrix} \tag{0.14}$$

Hence :

$$x = -a^2 - b^2$$

$$y = 4a^2 + 4b^2$$