4.11.6

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Question:
Find the equation of the plane passing through the intersection of the planes

$$\left(r\right) \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$$

and

$$\left(r\right) \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$$

and parallel to the X-axis. Hence, find the distance of the plane from the X-axis.

Solution:

Step 1: Plane through the intersection of the two given planes

The two given planes are

$$\Pi_1 : x + y + z = 1 \tag{1}$$

$$\Pi_2 : 2x + 3y - z + 4 = 0 \tag{2}$$

A plane passing through their intersection can be written as

$$\Pi : (x + y + z - 1) + \lambda(2x + 3y - z + 4) = 0 \tag{3}$$

Expanding:

$$x + y + z - 1 + \lambda(2x + 3y - z + 4) = 0 \tag{4}$$

$$x(1 + 2\lambda) + y(1 + 3\lambda) + z(1 - \lambda) + (-1 + 4\lambda) = 0 \tag{5}$$

Step 2: Condition for the plane to be parallel to the X-axis

A plane is parallel to the X-axis if the coefficient of x is zero:

$$1 + 2\lambda = 0 \quad \Rightarrow \quad \lambda = -\frac{1}{2} \tag{6}$$

Step 3: Substitute $\lambda = -\frac{1}{2}$

$$y\left(1 + 3\left(-\frac{1}{2}\right)\right) + z\left(1 - \left(-\frac{1}{2}\right)\right) + \left(-1 + 4\left(-\frac{1}{2}\right)\right) = 0 \tag{7}$$

$$y(-\tfrac{1}{2}) + z(\tfrac{3}{2}) - 3 = 0 \tag{8}$$

Multiply through by 2:

$$-y + 3z - 6 = 0 \quad \Rightarrow \quad y - 3z + 6 = 0 \tag{9}$$

So, the required plane is

$$\boxed{y - 3z + 6 = 0} \tag{10}$$

Step 4: Distance from the X-axis

The X-axis is the line $y = 0, z = 0$.

Distance from a plane $y - 3z + 6 = 0$ to a point $(0, 0, 0)$ on the X-axis is

$$d = \frac{|y_0 - 3z_0 + 6|}{\sqrt{1^2 + (-3)^2}} = \frac{|0 - 0 + 6|}{\sqrt{1 + 9}} = \frac{6}{\sqrt{10}} \tag{11}$$

$$\boxed{\text{Distance} = \frac{6}{\sqrt{10}}} \tag{12}$$

Graph presentation:

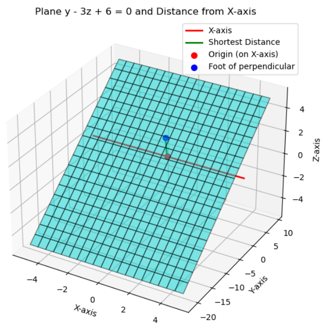

Fig. 1