

3.3.8

AI25BTECH11024 - Pratyush Panda

Question:

Construct a triangle ΔABC with side $BC = 7\text{cm}$, $\angle B = 45^\circ$, $\angle A = 105^\circ$.

Solution:

Let the position vector of \mathbf{B} be $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and the position vector of \mathbf{C} be $\begin{pmatrix} 7 \\ 0 \end{pmatrix}$ as $BC = 7\text{cm}$ is given.

Let a , b and c be the length of sides opposite to the vertex A , B and C respectively.

Now, we know that sum of all interior angles of a triangle is 180° , thus;

$$\angle A + \angle B + \angle C = 180^\circ \quad (0.1)$$

$$\text{or } 105^\circ + 45^\circ + \angle C = 180^\circ \quad (0.2)$$

$$\text{Thus, } \angle C = 30^\circ \quad (0.3)$$

We can form two equations to get the other two sides such as;

$$b \cos C + c \cos B = 8 \quad (0.4)$$

$$b \sin C - c \sin B = 0 \quad (0.5)$$

On writing this system of equation as a matrix equation, we get;

$$\begin{pmatrix} \cos C & \cos B \\ \sin C & -\sin B \end{pmatrix} \mathbf{X} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \text{ where } \mathbf{X} = \begin{pmatrix} b \\ c \end{pmatrix} \quad (0.6)$$

After putting the values of all the trigonometric values, we get;

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{\sqrt{2}} \end{pmatrix} \mathbf{X} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \quad (0.7)$$

Now we can do row operations to get the Echelon form of this matrix.

$$\begin{pmatrix} \sqrt{3}/2 & 1/\sqrt{2} \\ 0 & (-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}) \end{pmatrix} \mathbf{X} = \begin{pmatrix} 7 \\ \frac{7}{\sqrt{3}} \end{pmatrix} \quad (0.8)$$

On solving this equation we get;

$$\mathbf{X} = \begin{pmatrix} 7 \left(\frac{1-\sqrt{3}}{\sqrt{3}} \right) \\ \frac{7\sqrt{2}}{\sqrt{3}-1} \end{pmatrix} \quad (0.9)$$

From here we get, $c = \frac{7\sqrt{2}}{\sqrt{3}-1}$

Now, the coordinates of **A** can be written as;

$$\mathbf{A} = \begin{pmatrix} c \cos B \\ c \sin B \end{pmatrix} \quad (0.10)$$

$$\text{or, } \mathbf{A} = \begin{pmatrix} \frac{c}{\sqrt{2}} \\ \frac{c}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{7}{\sqrt{3}-1} \\ \frac{7}{\sqrt{3}-1} \end{pmatrix} \quad (0.11)$$

Now, we can plot the triangle using the three points.

