1.3.6

AI25BTECH11027 - NAGA BHUVANA

Question:
Show that the points $\mathbf{A}\,(6,2)$,$\mathbf{B}\,(2,1)$,$\mathbf{C}\,(1,5)$ and $\mathbf{D}\,(5,6)$ are vertices of a square.
Solution:
Given that

$$\mathbf{A} = \begin{pmatrix}6\\2\end{pmatrix}, \mathbf{B} = \begin{pmatrix}2\\1\end{pmatrix}, \mathbf{C} = \begin{pmatrix}1\\5\end{pmatrix}, \mathbf{D} = \begin{pmatrix}5\\6\end{pmatrix} \tag{1}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix}2-6\\1-2\end{pmatrix} = \begin{pmatrix}-4\\-1\end{pmatrix} \tag{2}$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix}1-5\\5-6\end{pmatrix} = \begin{pmatrix}-4\\-1\end{pmatrix} \tag{3}$$

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \tag{4}$$

By the above property we can say that **ABCD** is a parallelogram.
Consider the sides
Consider the inner product of the vectors $(\mathbf{B} - \mathbf{A})$ and $(\mathbf{C} - \mathbf{B})$ of the parallelogram

$$\implies (\mathbf{B} - \mathbf{A})^T(\mathbf{C} - \mathbf{B}) = (-4)(-1) + (-1)(4) = 0 \tag{5}$$

Hence the angle at vertex B is 90°
Property:
A parallelogram with one angle 90° is a rectangle
Hence the parallelogram is a rectangle
Now consider the inner product of the diagonals of the rectangle $(\mathbf{C} - \mathbf{A})$ and $(\mathbf{D} - \mathbf{B})$

$$\implies (\mathbf{C} - \mathbf{A})^T(\mathbf{D} - \mathbf{B}) = (-5)(3) + (3)(5) = 0 \tag{6}$$

Hence angle between the diagonals of rectangle is 90°
Property:
Rectangle with diagonals at right angle is a square
Hence given points forms a square

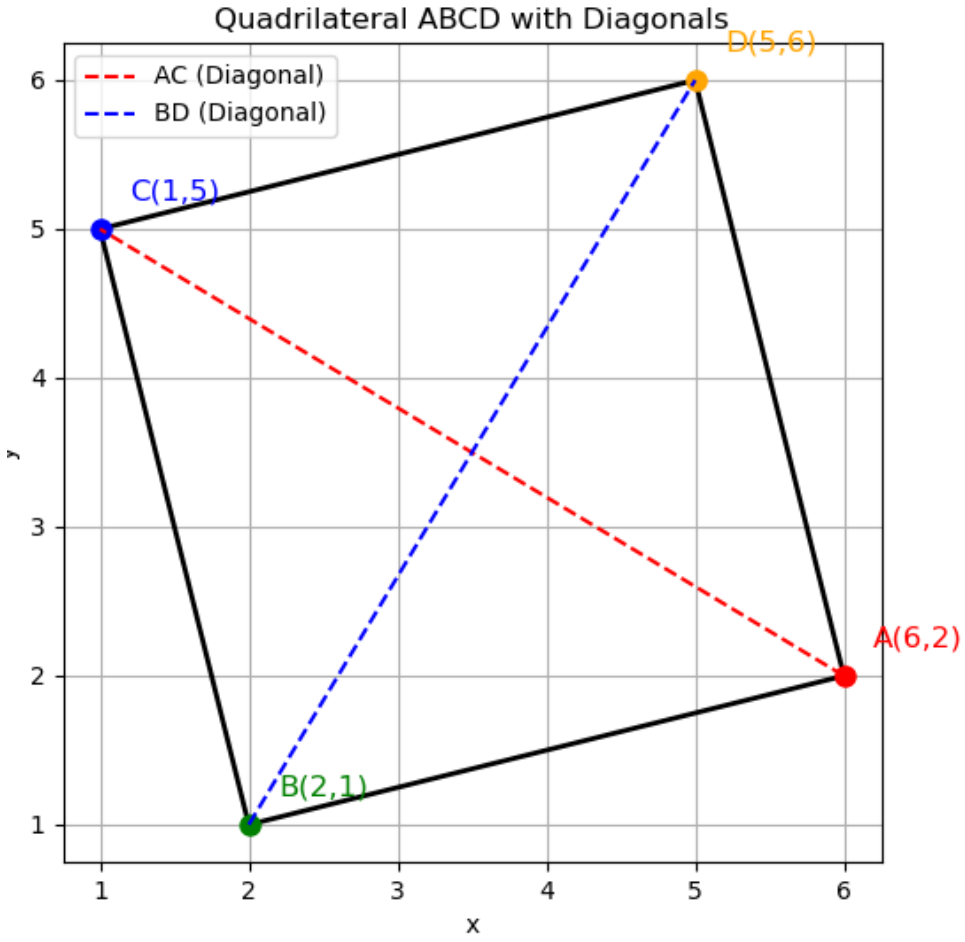
Quadrilateral ABCD with Diagonals
AC (Diagonal)
BD (Diagonal)
D(5,6)
C(1,5)
A(6,2)
B(2,1)
x
y
1
2
3
4
5
6

Fig. 1