Question 2.10.29

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The volume of the parallelopiped whose sides are given by
$OA = 2\mathbf{i} - 2\mathbf{j}$, $OB = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $OC = 3\mathbf{i} - \mathbf{k}$, is

Solution:

Let's define the position vectors **a**, **b** and **c** as follows:

$$\mathbf{a} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} \quad (1)$$

$$(2)$$

To find the volume of the parallelopiped, we use the scalar triple product formula:

$$V = |\det((\mathbf{a}\,\mathbf{b}\,\mathbf{c}))|, \text{ where} \quad (3)$$

$$\therefore V = \left| \det \left(\begin{pmatrix} 2 & 1 & 3 \\ -2 & 1 & 0 \\ 0 & -1 & -1 \end{pmatrix} \right) \right| \quad (4)$$
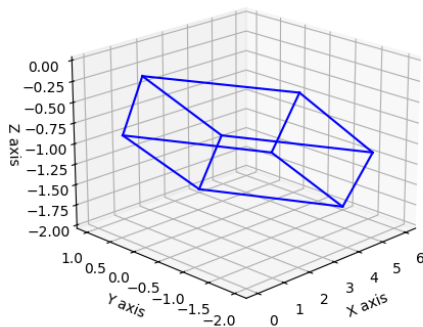
$$\therefore V = 2 \quad (5)$$

Plot:

Figure: Parallelopiped formed by vectors **OA**, **OB** and **OC**