

5.8.39

EE25BTECH11042 - Nipun Dasari

Question:

The cost of 2 pencils and 3 erasers is $9rs$ and the cost of 4 pencils and 6 erasers is $18rs$. Find the cost of each pencil and each eraser.

Solution: Let x and y denote the cost of each pencil and eraser respectively. By forming the equations we get

$$\mathbf{n}_1^\top \mathbf{x} = c_1 \quad (0.1)$$

$$\mathbf{n}_2^\top \mathbf{x} = c_2 \quad (0.2)$$

Stacking these gives:

$$\begin{pmatrix} \mathbf{n}_1^\top \\ \mathbf{n}_2^\top \end{pmatrix} \mathbf{x} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \quad (0.3)$$

$$\mathbf{n}_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \mathbf{n}_2 = \begin{pmatrix} 4 \\ 6 \end{pmatrix}, c_1 = 9, c_2 = 18 \quad (0.4)$$

Thus,

$$\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 9 \\ 18 \end{pmatrix} \quad (0.5)$$

The augmented matrix is

$$\left(\begin{array}{cc|c} 2 & 3 & 9 \\ 4 & 6 & 18 \end{array} \right) \quad (0.6)$$

By row transformations:

$$\left(\begin{array}{cc|c} 2 & 3 & 9 \\ 4 & 6 & 18 \end{array} \right) \xleftrightarrow{R_2 \leftarrow R_2 - 2R_1} \left(\begin{array}{cc|c} 2 & 3 & 9 \\ 0 & 0 & 0 \end{array} \right) \quad (0.7)$$

This implies that there exist infinitely many solutions as one row is a linear factor of the other. Both equations lead to the same line.

