

2.10.57

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Question:

If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ and \mathbf{d} are unit vectors such that

$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = 1 \quad \text{and} \quad \mathbf{a} \cdot \mathbf{c} = \frac{1}{2},$$

then

- (a) $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are non-coplanar
- (b) $\mathbf{b}, \mathbf{c}, \mathbf{d}$ are non-coplanar
- (c) \mathbf{b}, \mathbf{d} are non-parallel
- (d) \mathbf{a}, \mathbf{d} are parallel and \mathbf{b}, \mathbf{c} are parallel

Soultion: We are given that

$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = 1, \quad \mathbf{a} \cdot \mathbf{c} = \frac{1}{2}. \quad (1)$$

Step 1: Vector Identity

$$(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c}). \quad (2)$$

Step 2: Substitution Since $\mathbf{a} \cdot \mathbf{c} = \frac{1}{2}$,

$$1 = \frac{1}{2}(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c}). \quad (3)$$

Step 3: Assume $\mathbf{b} \parallel \mathbf{d}$ If $\mathbf{b} \cdot \mathbf{d} = 1$, then

$$1 = \frac{1}{2}(1) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c}). \quad (4)$$

$$1 = \frac{1}{2} - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c}). \quad (5)$$

$$(\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c}) = -\frac{1}{2}. \quad (6)$$

Step 4: Conclusion Thus, the condition is satisfied when

$$\mathbf{a} \parallel \mathbf{d}, \quad \mathbf{b} \parallel \mathbf{c}. \quad (7)$$

Option (D) is correct.

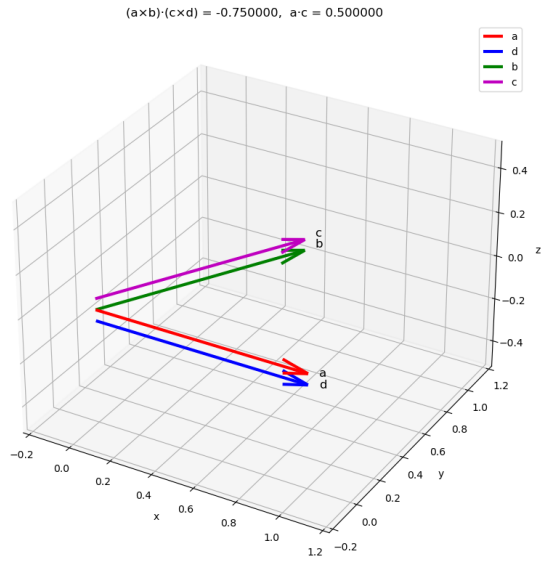


Fig. 1