

## 10.2.7

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**Question:**

At what points on the curve  $x^2 + y^2 - 2x - 4y + 1 = 0$ , the tangents are parallel to the y-axis?

## Solution:

The general equation of a conic is  $\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$  (1)

Comparing with  $x^2 + y^2 - 2x - 4y + 1 = 0$ , we get

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \quad f = 1 \quad (2)$$

# Solution

The centre and radius of the circle are given by

$$\mathbf{c} = -\mathbf{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad r = \sqrt{\|\mathbf{u}\|^2 - f} = \sqrt{(-1)^2 + (-2)^2 - 1} = \sqrt{4} = 2 \quad (3)$$

The points of contact of the tangent are given by

$$\mathbf{q}_{i,j} = \pm r \frac{\mathbf{n}_j}{\|\mathbf{n}_j\|} - \mathbf{u} \quad i, j = 1, 2 \quad (4)$$

# Solution

Since the tangents are parallel to the y-axis, the normal vector is

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (5)$$

Substituting in (0.4),

$$\mathbf{q}_{1,1} = \pm 2 \frac{\begin{pmatrix} 1 \\ 0 \end{pmatrix}}{\left\| \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\|} - \begin{pmatrix} -1 \\ -2 \end{pmatrix} \quad (6)$$

$$= \pm 2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (7)$$

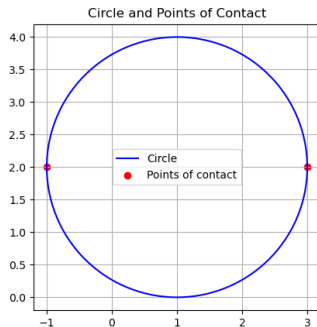
Therefore, the two points of contact are

$$\mathbf{q}_1 = \begin{pmatrix} 1+2 \\ 2+0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad (8)$$

$$\mathbf{q}_2 = \begin{pmatrix} 1-2 \\ 2+0 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad (9)$$

Hence, the required points are  $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$  and  $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

# Figure



`https://github.com/AarushDilawri/ee1030-2025/tree/main/ee25btech11001/MATGEO/10.2.7/codes`