4.8.14

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September 29, 2025

Question

Find the position vector of the foot of perpendicular and the perpendicular distance from the point **P** with position vector $2\hat{i} + 3\hat{j} + \hat{k}$ to the plane $\mathbf{r} \cdot (2\hat{i} + \hat{j} + 3\hat{k})$ -26 $= 0$. Also find image of **P** in the plane.

Theoretical Solution

Solution:

$$\text{The position vector of point } \mathbf{P} \text{ is } \mathbf{p} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \tag{1}$$

The normal vector of the plane is

$$\mathbf{n} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \tag{2}$$

The plane equation is

$$\mathbf{p}^T \cdot \mathbf{n} - 26 = 0 \tag{3}$$

1. Perpendicular Distance

The dot product $\mathbf{p} \cdot \mathbf{n}$ is given by the matrix multiplication $\mathbf{p}^T\mathbf{n}$.

$$\mathbf{p}^T\mathbf{n} = \begin{pmatrix} 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = (2)(2) + (3)(1) + (1)(3) = 10 \tag{4}$$

$$|\mathbf{n}| = \sqrt{\mathbf{n}^T\mathbf{n}} = \sqrt{\begin{pmatrix} 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}} = \sqrt{4+1+9} = \sqrt{14} \tag{5}$$

The perpendicular distance d is:

$$d = \frac{|\mathbf{p}^T\mathbf{n} - 26|}{|\mathbf{n}|} = \frac{|10-26|}{\sqrt{14}} = \frac{16}{\sqrt{14}} \tag{6}$$

Theoretical Solution

2. Foot of Perpendicular

The position vector of the foot of the perpendicular $\mathbf{q}$ is:

$$\mathbf{q} = \mathbf{p} - \frac{(\mathbf{p}^T\mathbf{n} - 26)}{|\mathbf{n}|^2}\mathbf{n} \tag{7}$$

$$\mathbf{q} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} - \frac{10-26}{14}\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \frac{16}{14}\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \tag{8}$$

$$\mathbf{q} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \frac{8}{7}\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2+16/7 \\ 3+8/7 \\ 1+24/7 \end{pmatrix} = \begin{pmatrix} 30/7 \\ 29/7 \\ 31/7 \end{pmatrix} \tag{9}$$

So the position vector of the foot of the perpendicular is $\frac{30}{7}\hat{i} + \frac{29}{7}\hat{j} + \frac{31}{7}\hat{k}$.

Theoretical Solution

3. Image of **P**

The position vector of the image **P'**, is:

$$\mathbf{P'} = 2\mathbf{q} - \mathbf{P} = 2\begin{pmatrix} 30/7 \\ 29/7 \\ 31/7 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 60/7 - 14/7 \\ 58/7 - 21/7 \\ 62/7 - 7/7 \end{pmatrix} = \begin{pmatrix} 46/7 \\ 37/7 \\ 55/7 \end{pmatrix} \quad (10)$$

So the position vector of the image is $\frac{46}{7}\hat{i} + \frac{37}{7}\hat{j} + \frac{55}{7}\hat{k}$.

C Code

```c

#include <stdio.h>
#include <math.h>

int main() {

    double p[3] = {2.0, 3.0, 1.0};
    double n[3] = {2.0, 1.0, 3.0};
    double d0 = 26.0;

    // --- 1. Perpendicular Distance ---


    double dot_product_pn = 0.0;
    for (int i = 0; i < 3; i++) {
        dot_product_pn += p[i] * n[i];
    }
```

C Code

```
double magnitude_n_sq = 0.0;
   for (int i = 0; i < 3; i++) {
       magnitude_n_sq += n[i] * n[i];
   }
   double magnitude_n = sqrt(magnitude_n_sq);


   double distance = fabs(dot_product_pn - d0) / magnitude_n;
   printf("Perpendicular Distance: %.2f / %.2f = %.4f\n\n", fabs
       (dot_product_pn - d0), magnitude_n, distance);

   // --- 2. Foot of Perpendicular ---
   double scalar_factor = (dot_product_pn - d0) / magnitude_n_sq
       ;
   double q[3];
```

```c
for (int i = 0; i < 3; i++) {
        q[i] = p[i] - scalar_factor * n[i];
    }
    printf("Position vector of Foot of Perpendicular (Q):\n");
    printf("(%.4f, %.4f, %.4f)\n\n", q[0], q[1], q[2]);

    // --- 3. Image of P ---
    double p_prime[3];
    for (int i = 0; i < 3; i++) {
        p_prime[i] = 2.0 * q[i] - p[i];
    }
    printf("Position vector of Image of P (P'):\n");
    printf("(%.4f, %.4f, %.4f)\n\n", p_prime[0], p_prime[1],
         p_prime[2]);
    return 0;
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

n = np.array([2, 1, 3]) # Normal vector to the plane
p = np.array([2, 3, 1]) # Position vector of point P
d = 26 # Plane constant

# Calculate the foot of perpendicular from P to the plane
t_foot = (d - np.dot(n, p)) / np.dot(n, n)
foot = p + t_foot * n

# Calculate image of P (reflection about the plane)
image = 2 * foot - p

# Create grid for the plane
xx, yy = np.meshgrid(np.linspace(0, 8, 8), np.linspace(0, 8, 8))
zz = (d - 2 * xx - yy) / 3
```

Python Code

```
# Plotting
fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')
ax.plot_surface(xx, yy, zz, alpha=0.6, color='cyan', rstride=1,
    cstride=1, edgecolor='none')

# Plot point P, foot of perpendicular, and image
ax.scatter(*p, color='blue', s=80, label='P (2,3,1)')
ax.scatter(*foot, color='red', s=80, label='Foot of Perpendicular
    ')
ax.scatter(*image, color='green', s=80, label='Image of P')

# Plot perpendicular line
ax.plot([p[0], foot[0]], [p[1], foot[1]], [p[2], foot[2]], color=
    'black', lw=2, linestyle='--', label='Perpendicular')
```

Python Code

```python

ax.set_xlim(0, 8)
ax.set_ylim(0, 8)
ax.set_zlim(0, 8)

ax.set_xlabel('X axis')
ax.set_ylabel('Y axis')
ax.set_zlabel('Z axis')
ax.set_title('3D Solution Graph: Foot, Distance, Image')
ax.legend()

plt.tight_layout()
plt.savefig('3d_plane_solution.png')
plt.show()
```

Plot

Beamer/figs/3d_plane_solution.png