2.10.3

AI25BTECH11014 - Gooty Suhas

Problem

Find the unit vector perpendicular to the plane determined by the points:

$$\mathbf{P} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$$

Solution

We solve using the plane equation:

$$\mathbf{n}^T \mathbf{x} = c \quad \text{where} \quad c = \mathbf{n}^T \mathbf{P}$$

Compute direction vectors:

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}, \quad \mathbf{R} - \mathbf{P} = \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$$

Let $\mathbf{n}$ be perpendicular to both:

$$\mathbf{n}^T (\mathbf{Q} - \mathbf{P}) = 0, \quad \mathbf{n}^T (\mathbf{R} - \mathbf{P}) = 0$$

Solving this system gives:

$$\mathbf{n} = \begin{pmatrix} 8 \\ 2 \\ 4 \end{pmatrix}$$

Then:

$$c = \mathbf{n}^T \mathbf{P} = 8 \cdot 1 + 2 \cdot (-1) + 4 \cdot 2 = 14$$

So the plane equation is:

$$\mathbf{n}^T \mathbf{x} = 14$$

Normalize:

$$\|\mathbf{n}\| = \sqrt{8^2 + 2^2 + 4^2} = \sqrt{84} \quad \Rightarrow \quad \hat{n} = \frac{1}{\sqrt{84}} \begin{pmatrix} 8 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{8}{\sqrt{84}} \\ \frac{2}{\sqrt{84}} \\ \frac{4}{\sqrt{84}} \end{pmatrix}$$

Then:

$$\hat{n}^T \mathbf{x} = \frac{1}{\sqrt{84}} \mathbf{n}^T \mathbf{x} = \frac{1}{\sqrt{84}} \cdot 14 = 1$$

Final Answer

$$\boxed{\hat{n} = \begin{pmatrix} \frac{8}{\sqrt{84}} \\ \frac{2}{\sqrt{84}} \\ \frac{4}{\sqrt{84}} \end{pmatrix} \quad \text{and} \quad \hat{n}^T \mathbf{x} = 1}$$

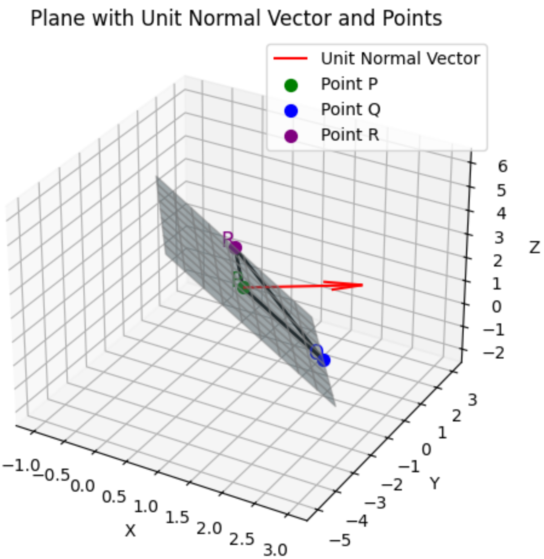

Fig. 0.1: Plane and its unit normal