MatGeo Assignment 3.2.23

AI25BTECH11007

Question:

Construct a triangle ABC in which

$$BC = 5\,\text{cm}, \quad \angle B = 60^\circ, \quad \text{and} \quad AC + AB = 7.5\,\text{cm}.$$

Solution:

Using the cosine formula in $\triangle ABC$,

$$\textbf{Cosine Formula : } b^2 = a^2 + c^2 - 2ac\cos B \tag{0.1}$$

$$\Rightarrow (7.5 - c)^2 = 5^2 + c^2 - 2 \cdot 5c\cos 60^\circ \tag{0.2}$$

$$\Rightarrow c = \frac{7.5^2 - 5^2}{2(7.5 - 5\cos 60^\circ)} \tag{0.3}$$

$$c = 3.125, \qquad b = 7.5 - 3.125 = 4.375.$$

The coordinates of $\triangle ABC$ can then be expressed as

$$A = \begin{pmatrix} \frac{3.125\cos 60^\circ}{\sin 60^\circ} \\ \frac{3.125}{\sin 60^\circ} \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} 5 \\ 0 \end{pmatrix}.$$

The coordinates of $\triangle ABC$ are

$$A = \begin{pmatrix} \frac{3.125}{\sqrt{3}} \\ \frac{6.25}{\sqrt{3}} \end{pmatrix} \approx \begin{pmatrix} 1.804 \\ 3.608 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} 5 \\ 0 \end{pmatrix}.$$

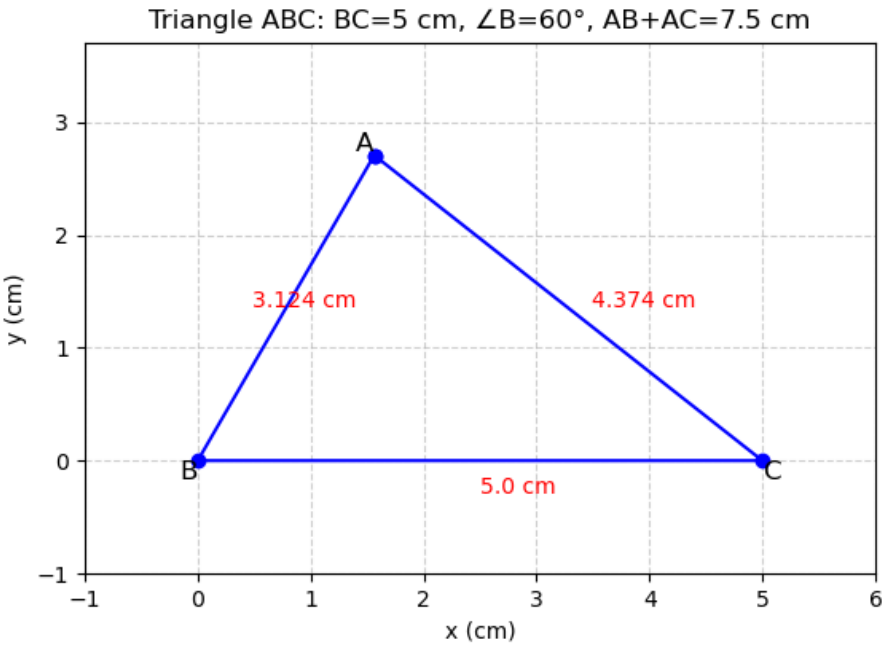

Fig. 0.1: Construction Plot