Matrices in Geometry - 1.9.26

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Problem Statement

Find the value of k, if the point $\mathbf{P}(2, 4)$ is equidistant from point $\mathbf{A}(5, k)$ and $\mathbf{B}(k, 7)$

Solution

Given: $\mathbf{P} \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $\mathbf{A} \begin{pmatrix} 5 \\ k \end{pmatrix}$ and a point $\mathbf{B} \begin{pmatrix} k \\ 7 \end{pmatrix}$ such that $\mathbf{P}$ is equidistant from $\mathbf{A}$ and $\mathbf{B}$.

$$\therefore \|\mathbf{A} - \mathbf{P}\| = \|\mathbf{B} - \mathbf{P}\|$$

On squaring both the sides, we get

$$\|\mathbf{A} - \mathbf{P}\|^2 = \|\mathbf{B} - \mathbf{P}\|^2$$

$$\left(\mathbf{A} - \mathbf{P}\right)^\top \left(\mathbf{A} - \mathbf{P}\right) = \left(\mathbf{B} - \mathbf{P}\right)^\top \left(\mathbf{B} - \mathbf{P}\right)$$

Solution

$$(\mathbf{A} - \mathbf{P}) = \begin{pmatrix} 3 \\ k - 4 \end{pmatrix}$$

$$(\mathbf{B} - \mathbf{P}) = \begin{pmatrix} k - 2 \\ 3 \end{pmatrix}$$

$$(\mathbf{A} - \mathbf{P})^\top (\mathbf{A} - \mathbf{P}) = \begin{pmatrix} 3 & k - 4 \end{pmatrix} \begin{pmatrix} 3 \\ k - 4 \end{pmatrix} = 9 + (k - 4)^2 = k^2 - 8k + 25$$

$$(\mathbf{B} - \mathbf{P})^\top (\mathbf{B} - \mathbf{P}) = \begin{pmatrix} k - 2 & 3 \end{pmatrix} \begin{pmatrix} k - 2 \\ 3 \end{pmatrix} = (k - 2)^2 + 9 = k^2 - 4k + 13$$

From above, $k^2 - 8k + 25 = k^2 - 4k + 13$

$$\implies 4k = 12 \implies k = 3$$

Final Answer

Hence, the final answer is $\boxed{k = 3}$

Point P is Equidistant from A and B for k = 3.00