4.7.64

AI25BTECH11003 - Bhavesh Gaikwad

Question: Find the distance between the point **P**(6, 5, 9) and the plane determined by the points **A**(3, -1, 2), **B**(5, 2, 4) and **C**(-1, -1, 6).

Solution:
Given:

$$P = \begin{pmatrix}6\\5\\9\end{pmatrix},\ A = \begin{pmatrix}3\\-1\\2\end{pmatrix},\ B = \begin{pmatrix}5\\2\\4\end{pmatrix},\ C = \begin{pmatrix}-1\\-1\\6\end{pmatrix}. \tag{0.1}$$

First, form two direction vectors on the plane using the given points.

$$LET\ \mathbf{u} = B - A = \begin{pmatrix}2\\3\\2\end{pmatrix}, \qquad \mathbf{v} = C - A = \begin{pmatrix}-4\\0\\4\end{pmatrix}. \tag{0.2}$$

Let $\mathbf{n} = \begin{pmatrix}n_1\\n_2\\n_3\end{pmatrix}$ be the perpendicular vector to the Plane.
Therefore, the equation of the Plane is $\mathbf{n}^\top\mathbf{x} = 1$
Let the equation of the Plane be $\begin{pmatrix}n_1 & n_2 & n_3\end{pmatrix}\mathbf{x} = 1$

Finding $\mathbf{n}$ which is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$ by solving the homogeneous system:

$$\begin{pmatrix}2 & 3 & 2\\-4 & 0 & 4\end{pmatrix}\begin{pmatrix}n_1\\n_2\\n_3\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}. \tag{0.3}$$

Row-reduce and solve for a convenient integer solution.

$$\begin{pmatrix}2 & 3 & 2\\-4 & 0 & 4\end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 + 2R_1} \begin{pmatrix}2 & 3 & 2\\0 & 6 & 8\end{pmatrix}, \tag{0.4}$$

$$6n_2 + 8n_3 = 0 \ \Rightarrow\ 3n_2 + 4n_3 = 0, \quad n_3 = 3,\ n_2 = -4,\ 2n_1 + 3n_2 + 2n_3 = 0 \Rightarrow n_1 = 3, \tag{0.5}$$

$$\therefore \mathbf{n} = \frac{1}{19}\begin{pmatrix}3\\-4\\3\end{pmatrix}. \tag{0.6}$$

Writing the plane as $\mathbf{n}^\top\mathbf{x} = 1$

Finally, applying the point-to-plane distance formula and simplify.

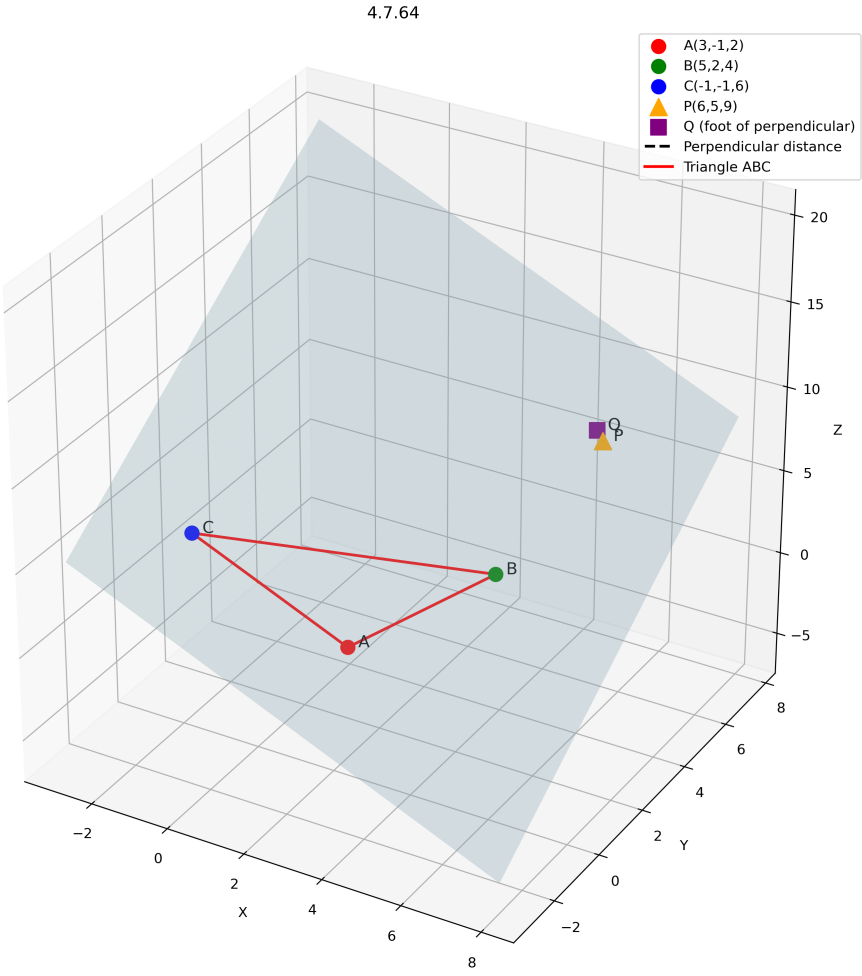
$$d = \frac{\left|\mathbf{n}^\top P - 1\right|}{\|\mathbf{n}\|} \tag{0.7}$$

$$= \frac{|3 \cdot 6 + (-4) \cdot 5 + 3 \cdot 9 - 19|}{\sqrt{3^2 + (-4)^2 + 3^2}} \tag{0.8}$$

$$= \frac{|25 - 19|}{\sqrt{34}} \tag{0.9}$$

$$= \frac{6}{\sqrt{34}} = \frac{3\sqrt{34}}{17}. \tag{0.10}$$

$$\boxed{\text{The Distance between the Plane and } \mathbf{P} \, is \, \frac{3\sqrt{34}}{17} \, units.} \tag{0.11}$$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A(3,-1,2)
B(5,2,4)
C(-1,-1,6)
P(6,5,9)
Q (foot of perpendicular)
Perpendicular distance
Triangle ABC
A
B
C
P
Q
X
Y
Z

Fig. 0.1: Plane