5.2.28

Vishwambhar - EE25BTECH11025

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Question

Solve the system of linear equations:

$$5x - 8y = -1 \tag{1}$$

$$3x - \frac{24}{5}y = \frac{-3}{5} \tag{2}$$

Given

Given

$$\begin{pmatrix} 5 & -8 \end{pmatrix} \mathbf{x} = -1; \begin{pmatrix} 3 & \left(\frac{-24}{5}\right) \end{pmatrix} \mathbf{x} = \frac{-3}{5} \tag{3}$$

$$A = \begin{pmatrix} 5 & -8 \\ 3 & \left(\frac{-24}{5}\right) \end{pmatrix}; \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}; \mathbf{b} = \begin{pmatrix} -1 \\ \left(\frac{-3}{5}\right) \end{pmatrix} \tag{4}$$

$$A\mathbf{x} = \mathbf{b} \tag{5}$$

Given

Let:
Rank of coefficient matrix $= r$
Rank of Augmented matrix $= r_a$
Order of coefficient matrix $= n$

Solving

Augmented Matrix:

$$\left(\begin{array}{cc|c} 5 & -8 & -1 \\ 3 & \left(\frac{-24}{5}\right) & \left(\frac{-3}{5}\right) \end{array}\right) \tag{6}$$

$$R_2 \to R_2 - \frac{3}{5}R_1 \tag{7}$$

$$\left(\begin{array}{cc|c} 5 & -8 & -1 \\ 0 & 0 & 0 \end{array}\right) \tag{8}$$

$$r = 1; r_a = 1; n = 2 \tag{9}$$

$$\because r = r_a < n \tag{10}$$

Infinite solutions exist for the given system of linear equations.

C Code

```c
#include<stdio.h>

double x = -24/5;
double y = -3/5;
double coefficient_mat[2][2] = {{5,-8},{3,-4.8}};
double constant[2][1] = {{-1}, {-0.6}};

double get_item(int i, int j){
    return coefficient_mat[i][j];
}
double get_constant(int i, int j){
    return constant[i][j];
}
```

Python Code 1

```
import ctypes
import sympy as sp
lib = ctypes.CDLL("./problem.so")
lib.get_item.argtypes = [ctypes.c_int, ctypes.c_int]
lib.get_item.restype = ctypes.c_double
lib.get_constant.argtypes = [ctypes.c_int, ctypes.c_int]
lib.get_constant.restype = ctypes.c_double
A = sp.Matrix([[0, 0, 0],
               [0, 0, 0]])
B = sp.Matrix([[0, 0],
               [0, 0]])
for i in range(0,2):
    for j in range(0,2):
        A[i, j] = lib.get_item(i, j)
        B[i, j] = lib.get_item(i, j)
```

Python Code 1

```
A[0, 2] = lib.get_constant(0, 0)
A[1, 2] = lib.get_constant(1, 0)
rA = A.rank()
rB = B.rank()
n = 2
if rA==rB and rB==n:
    print("Unique solution exist for the given system of linear
        equations.")
    rref_matrix, pivots = A.rref()
    print("The solution for the given system of linear equations
        is: x=", rref_matrix[0,2],", y=",rref_matrix[1,2])
elif rA==rB and rA!=n:
    print('Infinite solutions exist for the given system of
        linear equations in 2 variables.')
else:
    print("No solution exists for the given system of linear
        equations in 2 variables")
```

Python Code 2

```
import matplotlib.pyplot as plt
import numpy as np

x = np.linspace(-10, 10, 100)
y = (5/8)*x + (1/8)

X = np.linspace(-15, 15, 100)
Y = (5/8)*X + (1/8)

plt.plot(X, Y, '-k')
plt.plot(x, y, '-r')
```

Python Code 2

```
plt.text(-13.64, -8.96, r'$5x-8y=-1$', fontsize=10, color='black'
    )
plt.text(1.06, 1.08, r'$3x-\frac{24}{5}y=-\frac{3}{5}$', fontsize
    =10, color='black')

plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.axis('equal')
plt.grid(True)
plt.savefig("../figs/plot.png")
plt.show()
```

Plot

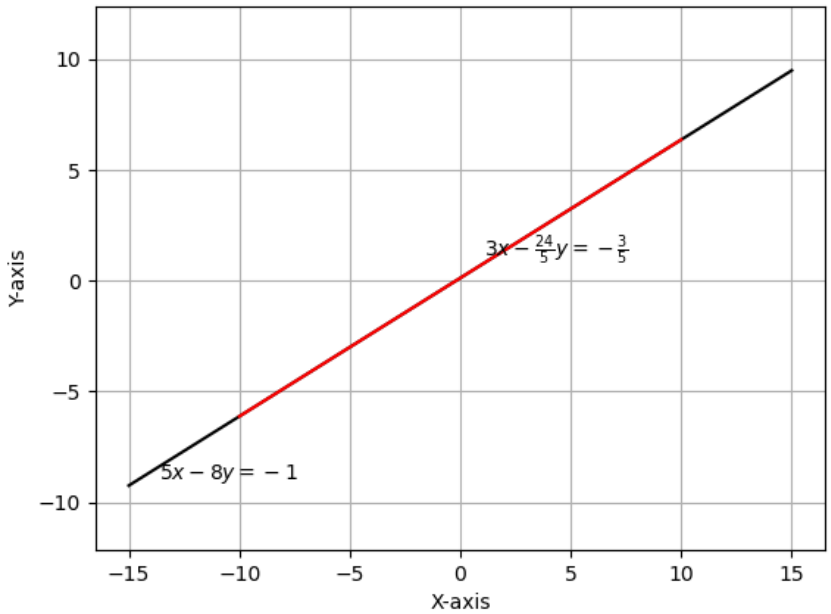

Figure: Plot of the given line equations