2.9.24

AI25BTECH11008
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Question

Find the co-ordinates of the point where the line

$$\mathbf{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 3\hat{k})$$

meets the plane which is perpendicular to the vector

$$\mathbf{n} = \hat{i} + \hat{j} + 3\hat{k}$$

and at a distance of $\frac{4}{\sqrt{11}}$ from origin.

Solution: Line and Plane Equations

Parametric form of the line:

$$\mathbf{r}(\lambda) = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 + 3\lambda \\ -2 + 4\lambda \\ -3 + 3\lambda \end{pmatrix} \tag{1}$$

Plane equation (distance d from origin, normal $\mathbf{n}$):

$$\mathbf{n}^T \mathbf{r} = \pm \|\mathbf{n}\| d \tag{2}$$

Computing Plane Constants

$$\|\mathbf{n}\| = \sqrt{1^2 + 1^2 + 3^2} = \sqrt{11}, \quad \pm\|\mathbf{n}\|d = \pm 4 \tag{3}$$

Plane equations:

$$\mathbf{n}^T\mathbf{r} = 4 \quad \text{or} \quad \mathbf{n}^T\mathbf{r} = -4 \tag{4}$$

Finding Intersection Points

Substitute line into plane:

$$\mathbf{n}^T\mathbf{r}(\lambda) = 1(-1+3\lambda) + 1(-2+4\lambda) + 3(-3+3\lambda) = -12+16\lambda \tag{5}$$

Case 1: $-12+16\lambda = 4 \Rightarrow \lambda = 1$
Case 2: $-12+16\lambda = -4 \Rightarrow \lambda = \frac{1}{2}$
Intersection points:

$$\mathbf{r}(1) = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}, \quad \mathbf{r}\left(\frac{1}{2}\right) = \begin{pmatrix} \frac{1}{2} \\ 0 \\ -\frac{3}{2} \end{pmatrix} \tag{6}$$

C Code

```c
#include <stdio.h>
#include <math.h>

int main() {
    // Step 1: Define points P and Q
    double P[3] = {4, 3, -5};
    double Q[3] = {-2, 1, 8};

    // Step 2: Direction vector PQ = Q - P
    double PQ[3];
    PQ[0] = Q[0] - P[0]; // -6
    PQ[1] = Q[1] - P[1]; // -2
    PQ[2] = Q[2] - P[2]; // 13

    // Step 3: Magnitude of PQ
    double mag = sqrt(PQ[0]*PQ[0] + PQ[1]*PQ[1] + PQ[2]*PQ[2]);
```

Python Code

```

    // Step 4: Direction cosines
    double cos_alpha = PQ[0] / mag;
    double cos_beta = PQ[1] / mag;
    double cos_gamma = PQ[2] / mag;

    // Output
    printf(Vector PQ = (%.0f, %.0f, %.0f)\n, PQ[0], PQ[1], PQ[2])
        ;
    printf(|PQ| = sqrt(209) = %.4f\n, mag);
    printf(Direction cosines:\n);
    printf(cos(alpha) = %.4f\n, cos_alpha);
    printf(cos(beta) = %.4f\n, cos_beta);
    printf(cos(gamma) = %.4f\n, cos_gamma);

    return 0;
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Define the points
P = np.array([4, 3, -5])
Q = np.array([-2, 1, 8])

# Generate line PQ
t = np.linspace(0, 1, 100)
line = np.outer(1-t, P) + np.outer(t, Q)

# Plot
fig = plt.figure(figsize=(8, 6))
ax = fig.add_subplot(111, projection='3d')

# Plot line PQ
ax.plot(line[:, 0], line[:, 1], line[:, 2], 'b-', label='$PQ$')
```

Python Code

```
# Plot points P and Q
ax.scatter(P[0], P[1], P[2], color='red', s=60, label='P(4,3,-5)')
ax.scatter(Q[0], Q[1], Q[2], color='green', s=60, label='Q(-2,1,8)')

# Annotate points
ax.text(P[0]+0.3, P[1]+0.3, P[2], 'P(4,3,-5)', fontsize=10, color='red')
ax.text(Q[0]+0.3, Q[1]+0.3, Q[2], 'Q(-2,1,8)', fontsize=10, color='green')

# Set labels
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
```

Python Code

```
# Title
ax.set_title(Line joining P(4,3,-5) and Q(-2,1,8))

# Grid and legend
ax.grid(True, linestyle='--', alpha=0.6)
ax.legend()

# Save and show
plt.savefig(fig1.png, dpi=300, bbox_inches=tight)
plt.show()
```

Plot

beamer/figs/fig1.jpg