# Matgeo Presentation - Problem 9.6.5

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# Problem Statement

Solve

$$\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}, \quad \frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=-\frac{1}{8}.$$

# Data

| Conic | Value |
|---|---|
| Hyperbola 1 | $\mathbf{x}^\top \begin{pmatrix} 9 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 0 \\ -4 \end{pmatrix}^\top \mathbf{x} = 0$ |
| Hyperbola 2 | $\mathbf{x}^\top \begin{pmatrix} 9 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -4 \\ 0 \end{pmatrix}^\top \mathbf{x} = 0$ |

Table : Hyperbola

# Solution

By rearranging the two equations we get the equation of two hyperbolas as :

$$9x^2 - y^2 - 8y = 0 \tag{0.1}$$
$$9x^2 - y^2 - 8x = 0 \tag{0.2}$$

The conic parameters for the two hyperbolas can be expressed as :

$$\mathbf{V_1} = \begin{pmatrix} 9 & 0 \\ 0 & -1 \end{pmatrix} \qquad \mathbf{u_1} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \qquad f1 = 0 \tag{0.3}$$
$$\mathbf{V_2} = \begin{pmatrix} 9 & 0 \\ 0 & -1 \end{pmatrix} \qquad \mathbf{u_2} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \qquad f2 = 0 \tag{0.4}$$

The intersection of two conics is defined as :

$$\mathbf{x}^\top(\mathbf{V_1} + \mu\mathbf{V_2})\mathbf{x} + 2(\mathbf{u_1} + \mu\mathbf{u_2})^\top\mathbf{x} + (f1 + \mu f2) = 0 \tag{0.5}$$

# Solution

The above equation represents a pair of straight lines if :

$$\begin{vmatrix} \mathbf{V_1} + \mu \mathbf{V_2} & \mathbf{u_1} + \mu \mathbf{u_2} \\ (\mathbf{u_1} + \mu \mathbf{u_2})^\top & f1 + \mu f2 \end{vmatrix} = 0 \tag{0.6}$$

Substituting the values in the above equation :

$$\begin{vmatrix} 9 + 9\mu & 0 & -4\mu \\ 0 & -1 - \mu & -4 \\ -4\mu & -4 & 0 \end{vmatrix} = 0 \tag{0.7}$$

# Solution

Applyint row reduction to find determinant:

$$\begin{vmatrix} 9+9\mu & 0 & -4\mu \\ 0 & -1-\mu & -4 \\ -4\mu & -4 & 0 \end{vmatrix} \xleftrightarrow{R_3 \to R_3 + \frac{4\mu}{9+9\mu} R_1} \begin{vmatrix} 9+9\mu & 0 & -4\mu \\ 0 & -1-\mu & -4 \\ 0 & -4 & -\frac{16\mu^2}{9+9\mu} \end{vmatrix} \tag{0.8}$$

$$\xleftrightarrow{R_3 \to R_3 - \frac{4}{1+\mu} R_2} \begin{vmatrix} 9+9\mu & 0 & -4\mu \\ 0 & -1-\mu & -4 \\ 0 & 0 & \frac{-16\mu^2+144}{9+9\mu} \end{vmatrix} \tag{0.9}$$

By finding the determinant we get

$$(1+\mu)(-16\mu^2+144) = 0 \tag{0.10}$$

$$\mu = -1, \mu = \pm 3 \tag{0.11}$$

# Solution

Substituting $\mu = -1$ in (0.5) we get equation of line as :

$$2\begin{pmatrix} 4 \\ -4 \end{pmatrix}^\top \mathbf{x} = 0 \tag{0.12}$$

$$\begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{0.13}$$

The parameters of the line are :

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \mathbf{m} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.14}$$

Substituting the parameters of line and the first hyperbola in the below equation :

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V_1} \mathbf{m}} \left( -\mathbf{m}^\top \left( \mathbf{V_1} \mathbf{h} + \mathbf{u_1} \right) \pm \sqrt{\left[ \mathbf{m}^\top \left( \mathbf{V_1} \mathbf{h} + \mathbf{u_1} \right) \right]^2 - g(\mathbf{h}) \left( \mathbf{m}^\top \mathbf{V_1} \mathbf{m} \right)} \right. \tag{0.15}$$

$$\kappa_i = 0, 1 \tag{0.16}$$

# Solution

Therefore the points of intersections of the line and first hyperbola are :

$$\mathbf{P_1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \mathbf{P_2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.17}$$

But if we substiture $\mathbf{P_2}$ in the original equation we get 0 in the denominator , which is undefined.
Therefore the solution for the given equations is :

$$\mathbf{P} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.18}$$

# Plot

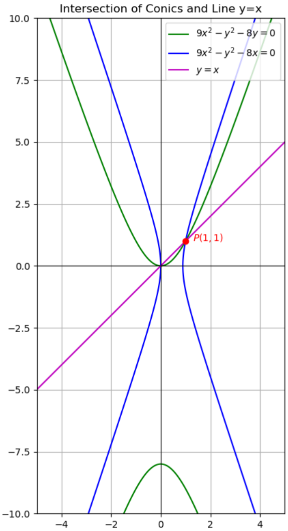


Fig : Hyperbola and Line