4.13.37

EE25BTECH11032 - Kartik Lahoti

Question:

If x_1, x_2, x_3 as well as y_1, y_2, y_3, are in G.P with the same common ratio then then points (x_1, y_1), (x_2, y_2) and (x_3, y_3)

1) lie on a straight line
2) lie on ellipse
3) lie on circle
4) are vertices of a triangle

Solution:

Given :

Symbol	Value	Description
A	$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$	Given Point
B	$\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$	Given Point
C	$\begin{pmatrix} x_3 \\ y_3 \end{pmatrix}$	Given Point

To check if **A** , **B** and **C** lie on a straight line ,

$$rank\begin{pmatrix} \mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{B} \end{pmatrix} = 1 \tag{4.1}$$

If r is the common ratio for the G.P , then vector **B** and **C** can also be written as

$$\mathbf{B} = r\mathbf{A} \qquad \mathbf{C} = r^2\mathbf{A} \tag{4.2}$$

$$rank\begin{pmatrix} r\mathbf{A}-\mathbf{A} & r^2\mathbf{A}-r\mathbf{A} \end{pmatrix} = 1 \tag{4.3}$$

Case 1: $x_1 \neq 0$

$$(r-1)\begin{pmatrix} x_1 & rx_1 \\ y_1 & ry_1 \end{pmatrix} \xleftrightarrow{R_2 \to R_2 - \frac{y_1}{x_1}R_1} \begin{pmatrix} x_1 & rx_1 \\ 0 & 0 \end{pmatrix} \tag{4.4}$$

Case 2: ($x_1 = 0$ and $y_1 \neq 0$) or ($x_1 \neq 0$ and $y_1 = 0$)

$$\begin{pmatrix} 0 & 0 \\ y_1 & ry_1 \end{pmatrix} \quad or \quad \begin{pmatrix} x_1 & rx_1 \\ 0 & 0 \end{pmatrix} \tag{4.5}$$

From Case 1 and Case 2 we can see *rank* = 1. Thus, the points lie on a straight line Hence, Answer : (1)

Taking an example as $\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $r = 3$, we get the following graph.