MatGeo Assignment 1.11.9

AI25BTECH11007

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Question

If

$$\mathbf{a} = \hat{i} - 7\hat{j} + 7\hat{k} \quad \text{and} \quad \mathbf{b} = 3\hat{i} - 2\hat{j} + 2\hat{k},$$

find a unit vector perpendicular to both the vectors **a** and **b**.

Solution

We need a vector $\mathbf{n}$ such that

$$\mathbf{n} \cdot \mathbf{a} = 0, \quad \mathbf{n} \cdot \mathbf{b} = 0 \tag{1}$$

where

$$\mathbf{a} = \begin{pmatrix} 1 \\ -7 \\ 7 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}. \tag{2}$$

Let

$$\mathbf{n} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \tag{3}$$

Orthogonality conditions,

$$\mathbf{n} \cdot \mathbf{a} = x - 7y + 7z = 0 \tag{4}$$

$$\mathbf{n} \cdot \mathbf{b} = 3x - 2y + 2z = 0 \tag{5}$$

Solve equations,
From (4),

$$x = 7y - 7z. \tag{6}$$

Substitute (6) into (5):

$$3(7y - 7z) - 2y + 2z = 0 \tag{7}$$
$$21y - 21z - 2y + 2z = 0 \tag{8}$$
$$19y - 19z = 0 \tag{9}$$
$$y = z. \tag{10}$$

From (6) and (10):

$$x = 7y - 7y = 0. \tag{11}$$

Thus,

$$\mathbf{n} = \begin{pmatrix} 0 \\ y \\ y \end{pmatrix} = y \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}. \tag{12}$$

Normalize,

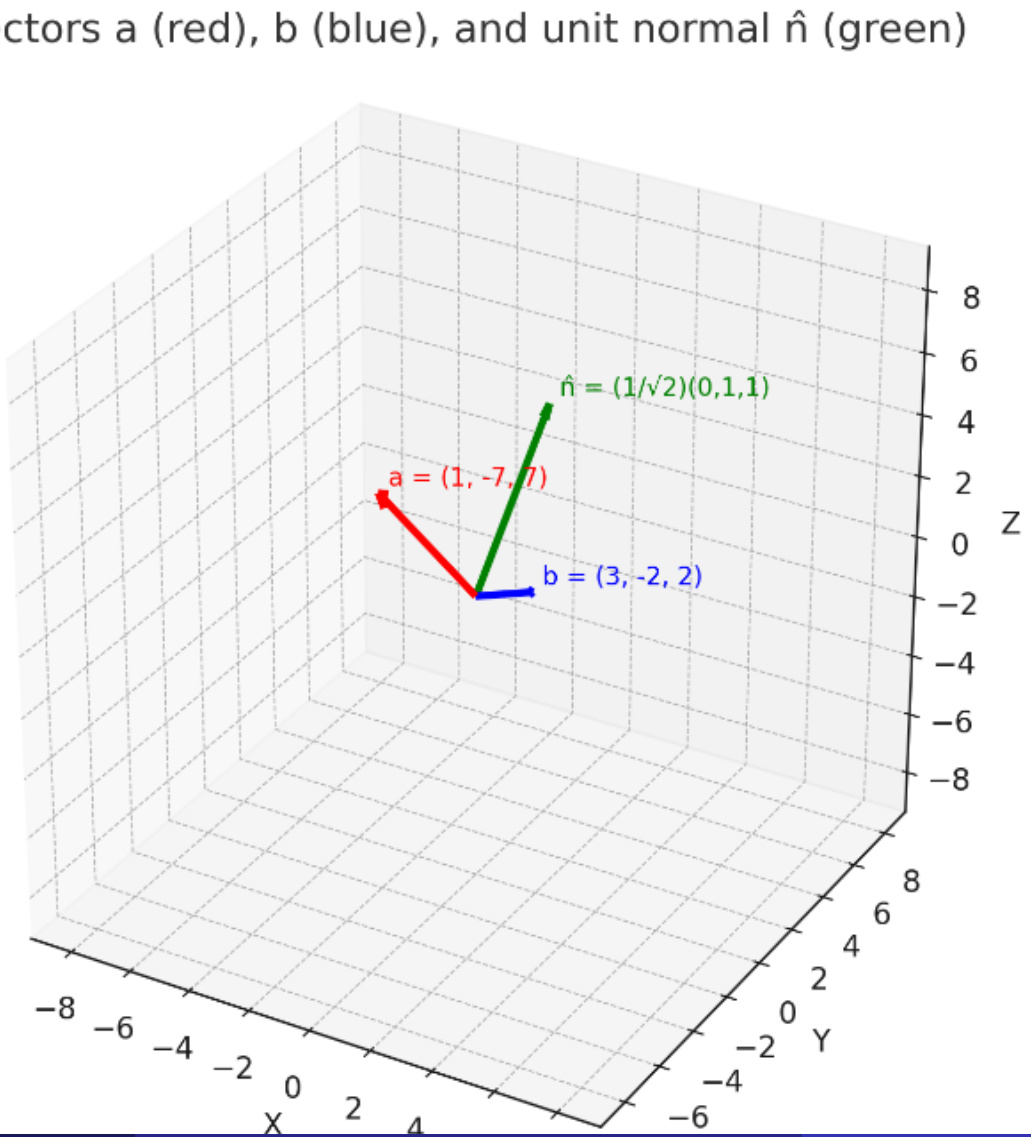
$$\hat{n} = \frac{\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}}{\sqrt{0^2 + 1^2 + 1^2}} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{13}$$

Hence, a unit vector perpendicular to both **a** and **b** is

$$\hat{n} = \frac{1}{\sqrt{2}}(\hat{j} + \hat{k}), \tag{14}$$

or its negative.

Plot


Vectors a (red), b (blue), and unit normal n̂ (green)
n̂ = (1/√2)(0,1,1)
a = (1, -7, 7)
b = (3, -2, 2)
X
Y
Z

Conclusion

Therefore, a unit vector perpendicular to both **a** and **b** is

$$\hat{n} = \frac{1}{\sqrt{2}}(\hat{j} + \hat{k}),$$

or its negative.