9.2.1

EE25BTECH11041 - Naman Kumar

Question:
Find the area bounded by the curve $y = \sqrt{x}, x = 2y + 3$, in the first quadrant and x-axis.
Solution:
General equation of conic

$$g(\mathbf{x}) = \mathbf{x^T V x} + 2\mathbf{u^T x} + f \tag{1}$$

Equation of parabola,

$$\mathbf{x^T} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix}^T \mathbf{x} = 0 \tag{2}$$

Equation of line,

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{3}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \tag{4}$$

Using following equation to find point of intersection of conic and line

$$k_i = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^T \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \pm \sqrt{\left[\mathbf{m}^T \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 - g(\mathbf{h})(\mathbf{m}^T \mathbf{V} \mathbf{m})} \right) \tag{5}$$

Solving for $g(\mathbf{h})$

$$g(\mathbf{h}) = \mathbf{h^T} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{h} + 2 \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix}^T \mathbf{h} \tag{6}$$

$$g(\mathbf{h}) = \frac{9}{4} \tag{7}$$

Solving for $\mathbf{m}^T \mathbf{V} \mathbf{m}$

$$\mathbf{m}^T \mathbf{V} \mathbf{m} = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \tag{8}$$

$$= \frac{1}{4} \tag{9}$$

Solving for $\mathbf{m}^T \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right)$

$$\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}^T \left(\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix} \right) \tag{10}$$

$$= -\frac{5}{4} \tag{11}$$

Solving (5)

$$k_i = \frac{1}{\frac{1}{4}}\left(\frac{5}{4} \pm \sqrt{\frac{25}{16} - \frac{9}{4} \times \frac{1}{4}}\right) \tag{12}$$

$$k_i = 4\left(\frac{5}{4} \pm 1\right) \tag{13}$$

$$k_1 = 9, k_2 = 1 \tag{14}$$

So with these values points are

$$\mathbf{x_1} = \begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix} + 9 \times \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \tag{15}$$

$$\mathbf{x_1} = \begin{pmatrix} 9 \\ 3 \end{pmatrix} \tag{16}$$

$$\mathbf{x_2} = \begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix} + 1 \times \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \tag{17}$$

$$\mathbf{x_2} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{18}$$

Area under curve in first quadrant between parabola and line

$$\int_0^3 \sqrt{x} + \int_3^9 \sqrt{x} - \left(\frac{x-3}{2}\right) \tag{19}$$

$$\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^9 - \left[\frac{x^2}{4} - \frac{3x}{2}\right]_3^9 \tag{20}$$

$$area = 9 \tag{21}$$

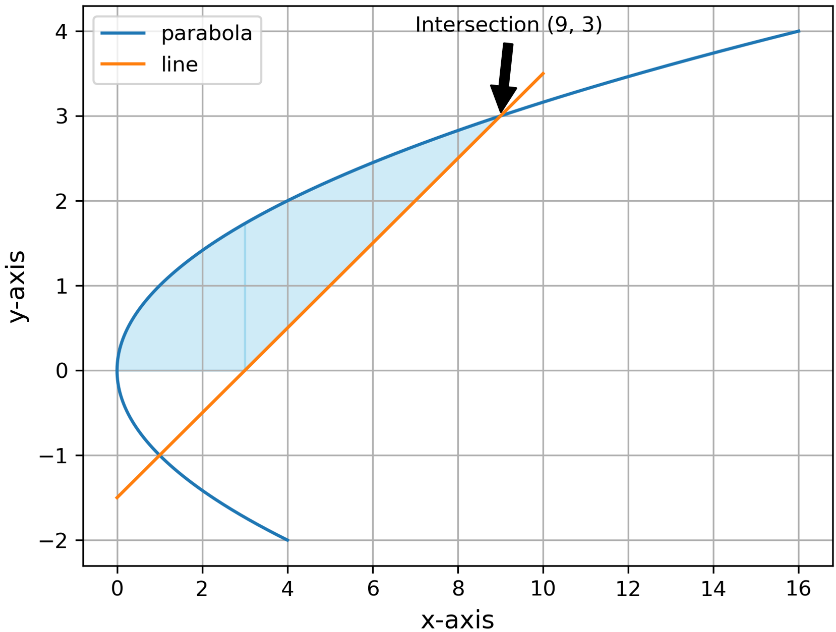
Intersection (9, 3)
parabola
line
y-axis
x-axis
4
3
2
1
0
−1
−2
0
2
4
6
8
10
12
14
16

Figure 1