2.8.19

EE25BTECH11036 - M Chanakya Srinivas

Problem Statement

Problem

If

$$\mathbf{r} \cdot \mathbf{a} = 0, \quad \mathbf{r} \cdot \mathbf{b} = 0, \quad \mathbf{r} \cdot \mathbf{c} = 0$$

for some non-zero vector $\mathbf{r}$, then find the value of

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}).$$

Solution

Since

$$\mathbf{r} \cdot \mathbf{a} = 0, \tag{1}$$

$$\mathbf{r} \cdot \mathbf{b} = 0, \tag{2}$$

$$\mathbf{r} \cdot \mathbf{c} = 0, \tag{3}$$

we conclude that the vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ all lie in the subspace orthogonal to $\mathbf{r}$:

$$\mathbf{a}, \mathbf{b}, \mathbf{c} \in \text{span}\{\mathbf{r}\}^{\perp}. \tag{4}$$

Since $\mathbf{r} \neq \mathbf{0}$, the orthogonal subspace $\text{span}\{\mathbf{r}\}^{\perp}$ is **at most 2-dimensional**:

$$\dim(\text{span}\{\mathbf{r}\}^{\perp}) = 2. \tag{5}$$

Thus, any three vectors lying in this at most 2-dimensional subspace are **linearly dependent**:

$$\mathbf{c} = \lambda_1 \mathbf{a} + \lambda_2 \mathbf{b} \quad \text{for some scalars } \lambda_1, \lambda_2. \tag{6}$$

The **scalar triple product** of linearly dependent vectors is zero:

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = \det \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix} \tag{7}$$

$$= 0. \tag{8}$$

Final Answer

$$\boxed{\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0} \tag{9}$$

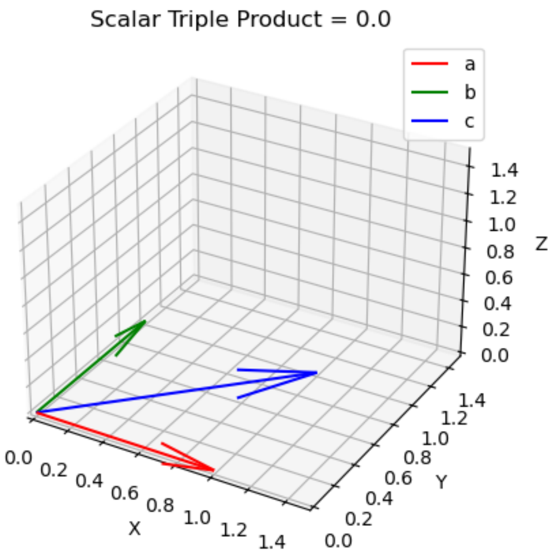

Fig. 1

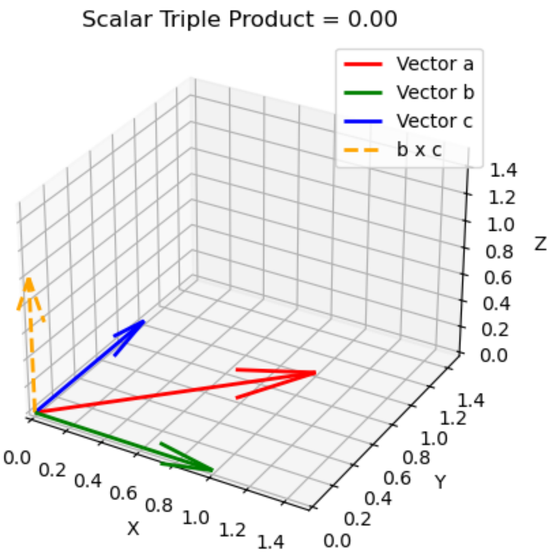
Scalar Triple Product = 0.00
Vector a
Vector b
Vector c
b x c
X
Y
Z
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Fig. 2