5.2.55

Shriyansh Chawda-EE25BTECH11052 September 23, 2025

Question

Solve the following system of linear equations.

$$\frac{2}{x} + \frac{3}{y} = 13 \quad \frac{5}{x} + \frac{4}{y} = -2$$

Solution

Let

$$u = \frac{1}{x}, \quad v = \frac{1}{y}. \tag{1}$$

The given system becomes

$$2u + 3v = 13 \tag{2}$$

$$5u + 4v = -2 \tag{3}$$

In matrix form:

$$\begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 13 \\ -2 \end{pmatrix}. \tag{4}$$

Solution

We solve using Gauss-Jordan elimination. We start with the augmented matrix $[A|\vec{b}]$ and reduce it to $[I|\vec{x}]$.

$$\left[\begin{array}{cc|c} 2 & 3 & 13 \\ 5 & 4 & -2 \end{array}\right] \xrightarrow{R_1 \to \frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{13}{2} \\ 5 & 4 & -2 \end{array}\right] \tag{5}$$

$$\xrightarrow{R_2 \to R_2 - 5R_1} \left[\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{13}{2} \\ 0 & -\frac{7}{2} & -\frac{69}{2} \end{array}\right] \tag{6}$$

Solution

Continuing the row reduction:

$$\xrightarrow{R_2 \to -\frac{2}{7}R_2} \left[\begin{array}{cc|c} 1 & \frac{3}{2} & \frac{13}{2} \\ 0 & 1 & \frac{69}{7} \end{array}\right] \tag{7}$$

$$\xrightarrow{R_1 \to R_1 - \frac{3}{2}R_2} \left[\begin{array}{cc|c} 1 & 0 & -\frac{58}{7} \\ 0 & 1 & \frac{69}{7} \end{array}\right] \tag{8}$$

The matrix is now in reduced row echelon form.

Solution

From the final matrix, we can directly read the solution:

$$u = -\frac{58}{7} \tag{9}$$

$$v = \frac{69}{7} \tag{10}$$

Back substituting to find x and y:

$$\frac{1}{x} = -\frac{58}{7} \implies x = -\frac{7}{58}, \tag{11}$$

$$\frac{1}{y} = \frac{69}{7} \implies y = \frac{7}{69}. \tag{12}$$

Thus, the solution is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{7}{58} \\ \frac{7}{69} \end{pmatrix}. \tag{13}$$

Plot