4.12.23

AI25BTECH11018-Hemanth Reddy

Question:

A point moves so that square of its distance from the point (3, -2) is numerically equal to its distance from the line 5x - 12y = 3. The equation of its locus is

Solution:

$$\text{Let the position vector of point } \mathbf{P} \text{ is } = \begin{pmatrix} x \\ y \end{pmatrix} \tag{0.1}$$

$$\text{Let } \mathbf{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{0.2}$$

Square of distance of point $\mathbf{P}$ from $\mathbf{a}$ is $\|\mathbf{P}-\mathbf{a}\|^2 = (\mathbf{P}-\mathbf{a})^T(\mathbf{P}-\mathbf{a})$

$$(\mathbf{P}-\mathbf{a}) = \begin{pmatrix} x-3 \\ y+2 \end{pmatrix} \tag{0.3}$$

$$(\mathbf{P}-\mathbf{a})^T(\mathbf{P}-\mathbf{a}) = \begin{pmatrix} x-3 & y+2 \end{pmatrix}\begin{pmatrix} x-3 \\ y+2 \end{pmatrix} = (x-3)^2 + (y+2)^2 \tag{0.4}$$

$$\text{Let} \quad \mathbf{n} = \begin{pmatrix} 5 \\ -12 \end{pmatrix} \tag{0.5}$$

$$|\mathbf{n}| = \sqrt{\mathbf{n}^T\mathbf{n}} = \sqrt{\begin{pmatrix} 5 & -12 \end{pmatrix}\begin{pmatrix} 5 \\ -12 \end{pmatrix}} = \sqrt{25+144} = 13 \tag{0.6}$$

$$d = \frac{|\mathbf{P}^T\mathbf{n} - 3|}{|\mathbf{n}|} = \frac{|5x-12y-3|}{13} \tag{0.7}$$

$$(x-3)^2 + (y+2)^2 = d = \frac{|5x-12y-3|}{13} \tag{0.8}$$

$$(x-3)^2 + (y+2)^2 = \frac{(5x-12y-3)}{13} \tag{0.9}$$

$$13x^2 + 13y^2 - 83x + 64y + 172 = 0 \tag{0.10}$$

The locus of the point is a circle.

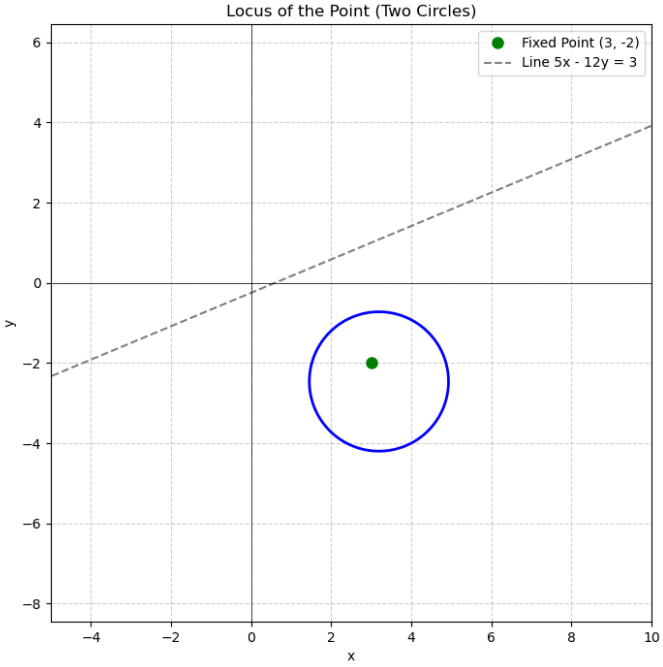

Fig. 0.1