Presentation - Matgeo

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AI25BTECH11032
EE1030 - Matrix Theory

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Problem Statement

Problem 7.3.5

If a circle passes through the points $(0, 0)$, $(a, 0)$ and $(0, b)$, then find the coordinates of its centre.

Description of Variables used

Variable	Description
$\mathbf{x}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$	First point on circle
$\mathbf{x}_2 = \begin{pmatrix} a \\ 0 \end{pmatrix}$	Second point on circle
$\mathbf{x}_3 = \begin{pmatrix} 0 \\ b \end{pmatrix}$	Third point on circle

Theoretical Solution

From (7.1.3.1), for three points $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ on a circle:

$$\begin{pmatrix} 2\mathbf{x}_1 & 2\mathbf{x}_2 & 2\mathbf{x}_3 \\ 1 & 1 & 1 \end{pmatrix}^\top \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = - \begin{pmatrix} \|\mathbf{x}_1\|^2 \\ \|\mathbf{x}_2\|^2 \\ \|\mathbf{x}_3\|^2 \end{pmatrix}, \quad \mathbf{c} = -\mathbf{u}. \tag{2.1}$$

Substituting the given points:

$$\begin{pmatrix} 0 & 2a & 0 \\ 0 & 0 & 2b \\ 1 & 1 & 1 \end{pmatrix}^\top \begin{pmatrix} u_1 \\ u_2 \\ f \end{pmatrix} = - \begin{pmatrix} 0 \\ a^2 \\ b^2 \end{pmatrix}. \tag{2.2}$$

Theoretical Solution

This expands to:

$$f = 0, \tag{2.3}$$

$$2au_1 + a^2 = 0, \tag{2.4}$$

$$2bu_2 + b^2 = 0. \tag{2.5}$$

Solving:

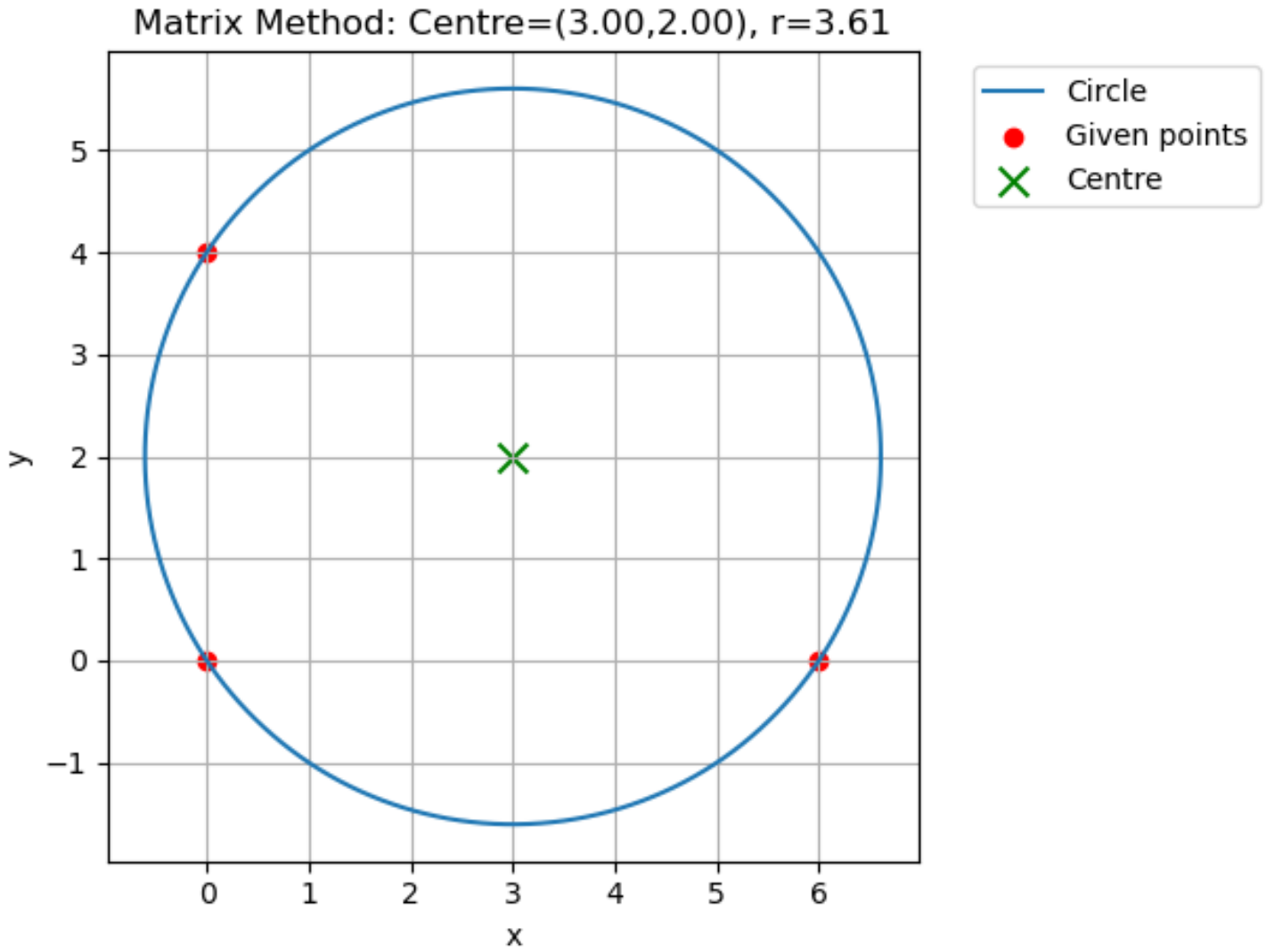
$$u_1 = -\frac{a}{2}, \quad u_2 = -\frac{b}{2}, \quad f = 0. \tag{2.6}$$

Hence, the centre of the circle is

$$\mathbf{c} = -\mathbf{u} = \begin{pmatrix} \frac{a}{2} \\ \frac{b}{2} \end{pmatrix}. \tag{2.7}$$

$$\boxed{\text{Centre of the circle is } \left(\tfrac{a}{2}, \tfrac{b}{2}\right)} \tag{2.8}$$

Plot


Matrix Method: Centre=(3.00,2.00), r=3.61
Circle
Given points
Centre
y
x
5
4
3
2
1
0
−1
0
1
2
3
4
5
6

Figure

Code - C

```c
#include <math.h>

// Circle through (0,0), (a,0), (0,b) using direct expansion of matrix
    equations
void circle_center_matrix(double a, double b,
                          double *cx, double *cy, double *r)
{
    // From equations:
    // f = 0
    // 2a u1 + f = -a^2 -> u1 = -a/2
    // 2b u2 + f = -b^2 -> u2 = -b/2
    double u1 = -a/2.0;
    double u2 = -b/2.0;

    *cx = -u1; // = a/2
    *cy = -u2; // = b/2
```

Code - C

```
    if (r) {
        *r = sqrt((*cx)*(*cx) + (*cy)*(*cy));
    }
}
```

Code - Python(with shared C code)

The code to obtain the required plot is

```
import ctypes as ct
import numpy as np
import matplotlib.pyplot as plt

# Load library
lib = ct.CDLL("./libcircle.so")

lib.circle_center_matrix.argtypes = [ct.c_double, ct.c_double,
                                     ct.POINTER(ct.c_double),
                                     ct.POINTER(ct.c_double),
                                     ct.POINTER(ct.c_double)]
lib.circle_center_matrix.restype = None

# Inputs
a, b = 6.0, 4.0
cx, cy, r = ct.c_double(), ct.c_double(), ct.c_double()
```

Code - Python(with shared C code)

```
lib.circle_center_matrix(a, b, ct.byref(cx), ct.byref(cy), ct.byref(r))
print(f"Centre=({cx.value},{cy.value}),Radius={r.value}")
# Plot
theta = np.linspace(0, 2*np.pi, 400)
X = cx.value + r.value*np.cos(theta)
Y = cy.value + r.value*np.sin(theta)

plt.plot(X, Y, label="Circle")
plt.scatter([0,a,0], [0,0,b], color="red", label="Given points")
plt.scatter([cx.value], [cy.value], color="green", marker="x", s=100, label
    ="Centre")
plt.gca().set_aspect("equal", adjustable="box")
plt.title(f"Direct Matrix Expansion: Centre=({cx.value:.2f},{cy.value:.2f}),
    r={r.value:.2f}")
plt.legend(); plt.grid(True)
plt.savefig("circle.png")
plt.show()
```

Code - Python only

```
# Circle through (0,0), (a,0), (0,b) using the matrix method
# Equation: ||x||^2 + 2u^T x + f = 0
# Centre = −u

import numpy as np
import matplotlib.pyplot as plt

# −−− Input values −−−
a = 6.0
b = 4.0

# −−− Build the 3x3 system A [u1, u2, f]^T = b_vec −−−
A = np.array([
      [0.0, 0.0, 1.0],
      [2.0*a, 0.0, 1.0],
      [0.0, 2.0*b, 1.0]
], dtype=float)
```

Code - Python only

```
b_vec = np.array([0.0, -a*a, -b*b], dtype=float)

# --- Solve for (u1, u2, f) ---
u1, u2, f = np.linalg.solve(A, b_vec)

# --- Centre and radius ---
cx, cy = -u1, -u2
r = np.hypot(cx, cy)

print(f"u = ({u1}, {u2}), f = {f}")
print(f"Centre = ({cx}, {cy})")
print(f"Radius = {r}")

# --- Plot the circle and points ---
theta = np.linspace(0, 2*np.pi, 400)
X = cx + r*np.cos(theta)
Y = cy + r*np.sin(theta)
```

Code - Python only

```
plt.figure()
plt.plot(X, Y, label="Circle")
plt.scatter([0, a, 0], [0, 0, b], color="red", label="Given points")
plt.scatter([cx], [cy], color="green", marker="x", s=100, label="Centre")
plt.gca().set_aspect("equal", adjustable="box")
plt.title(f"Matrix Method: Centre=({cx:.2f},{cy:.2f}), r={r:.2f}")
plt.xlabel("x"); plt.ylabel("y")
plt.grid(True)

# Place legend outside (right side)
plt.legend(loc="upper left", bbox_to_anchor=(1.05, 1.0))
plt.tight_layout()
plt.savefig("newcentre.png")
plt.show()
```