2.8.37

EE25BTECH11047 - RAVULA SHASHANK REDDY

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Question: If $|\mathbf{a} \times \mathbf{b}|^2 + (\mathbf{a}^T\mathbf{b})^2 = 144$ and $||\mathbf{a}|| = 4$, then $||\mathbf{b}||$ is equal to _____

Solution:

$$|\mathbf{a} \times \mathbf{b}|^2 + (\mathbf{a}^T\mathbf{b})^2 = (||\mathbf{a}||\,||\mathbf{b}||\,\sin\theta)^2 + (||\mathbf{a}||\,||\mathbf{b}||\,\cos\theta)^2 \tag{1}$$

$$= ||\mathbf{a}||^2||\mathbf{b}||^2(\sin^2\theta + \cos^2\theta) \tag{2}$$

$$= ||\mathbf{a}||^2||\mathbf{b}||^2. \tag{3}$$

Given :

$$|\mathbf{a} \times \mathbf{b}|^2 + (\mathbf{a}^T\mathbf{b})^2 = 144, \tag{4}$$

$$||\mathbf{a}|| = 4, \tag{5}$$

$$144 = ||\mathbf{a}||^2||\mathbf{b}||^2 \tag{6}$$

$$144 = 4^2||\mathbf{b}||^2 \tag{7}$$

$$144 = 16||\mathbf{b}||^2 \tag{8}$$

$$||\mathbf{b}||^2 = \frac{144}{16} = 9 \tag{9}$$

$$||\mathbf{b}|| = 3. \tag{10}$$