4.4.38

EE25BTECH11002 - Achat Parth Kalpesh

Question:
Find the equation of the line which bisects the line segment joining points $\mathbf{A}\,(2,3,4)$ and $\mathbf{B}\,(4,5,8)$ and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$
Solution:
Let the equation of the required line be

$$\mathbf{x} = \mathbf{h} + \kappa\mathbf{m} \tag{0.1}$$

where $\mathbf{h}$ is any point on the line and $\mathbf{m}$ is the direction vector of the line
Let the direction vectors of the given lines be $\mathbf{m_1}$ and $\mathbf{m_2}$

$$\mathbf{m_1} = \begin{pmatrix} 3 \\ -16 \\ 7 \end{pmatrix} \tag{0.2}$$

$$\mathbf{m_2} = \begin{pmatrix} 3 \\ 8 \\ -5 \end{pmatrix} \tag{0.3}$$

According to the given condition $\mathbf{h}$ is the midpoint of the line segment joining $\mathbf{A}$ and $\mathbf{B}$

$$\mathbf{h} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{0.4}$$

By the given condition,

$$\mathbf{m_1}^\top \mathbf{m} = \mathbf{0} \tag{0.5}$$

$$\mathbf{m_2}^\top \mathbf{m} = \mathbf{0} \tag{0.6}$$

$$\begin{pmatrix} \mathbf{m_1}^\top \\ \mathbf{m_2}^\top \end{pmatrix} \mathbf{m} = \mathbf{0} \tag{0.7}$$

$$\begin{pmatrix} 3 & -16 & 7 \\ 3 & 8 & -5 \end{pmatrix} \mathbf{m} = \mathbf{0} \xleftrightarrow{R_2 \to R_2 - R_1} \begin{pmatrix} 3 & -16 & 7 \\ 0 & 24 & -12 \end{pmatrix} \tag{0.8}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + \frac{2}{3}R_2} \begin{pmatrix} 3 & 0 & -1 \\ 0 & 24 & -12 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow \frac{R_2}{12}} \begin{pmatrix} 3 & 0 & -1 \\ 0 & 2 & -1 \end{pmatrix} \tag{0.9}$$

This yeilds

$$\mathbf{m} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \tag{0.10}$$

Hence, the vector equation of the line passing through $\mathbf{h}$ is

$$\mathbf{x} = \mathbf{h} + \kappa\mathbf{m} \tag{0.11}$$

$$\mathbf{x} = \left(\frac{\mathbf{A} + \mathbf{B}}{2}\right) + \kappa\mathbf{m} \tag{0.12}$$

$$\mathbf{x} = \left(\frac{\begin{pmatrix}2\\3\\4\end{pmatrix} + \begin{pmatrix}4\\5\\8\end{pmatrix}}{2}\right) + \kappa\mathbf{m} \tag{0.13}$$

$$\mathbf{x} = \begin{pmatrix}3\\4\\6\end{pmatrix} + \kappa\begin{pmatrix}2\\3\\6\end{pmatrix} \tag{0.14}$$

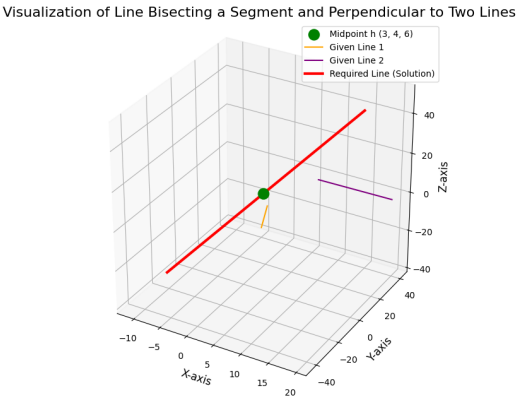

Fig. 0.1: Graph