12.163

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Question

The geometric transformation specified by

$$\begin{pmatrix} X' & Y' & 1 \end{pmatrix} = \begin{pmatrix} X & Y & 1 \end{pmatrix} \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.25 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

in a 2D CAD system represents

1. Scaling and Translation
2. Scaling and Rotation
3. Rotation and Translation
4. Rotation

Theoretical Solution

A 2D affine transformation is of the form $\mathbf{x}'^{\top} = \mathbf{x}^{\top}\mathbf{T}$, where the transformation matrix is

$$\mathbf{T} = \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{t}^{\top} & 1 \end{pmatrix}$$

The given transformation matrix is:

$$\mathbf{T} = \begin{pmatrix} 0.5 & 0 & 0 \\ 0 & 0.25 & 0 \\ 1 & 2 & 1 \end{pmatrix} \tag{1}$$

Theoretical Solution

From this, we identify the linear transformation matrix $\mathbf{A}$ and the translation vector $\mathbf{t}$.

$$\mathbf{A} = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.25 \end{pmatrix}, \quad \mathbf{t} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{2}$$

Since $\mathbf{A}$ is a diagonal matrix and not a multiple of the identity, it represents a non-uniform scaling. Since $\mathbf{t} \neq \mathbf{0}$, there is a translation.

Theoretical Solution

A pure rotation requires the linear part $\mathbf{A}$ to be an orthogonal matrix, where $\mathbf{A}\mathbf{A}^\top = \mathbf{I}$. We check this condition:

$$\mathbf{A}\mathbf{A}^\top = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.25 \end{pmatrix} \begin{pmatrix} 0.5 & 0 \\ 0 & 0.25 \end{pmatrix} = \begin{pmatrix} 0.25 & 0 \\ 0 & 0.0625 \end{pmatrix} \neq \mathbf{I} \tag{3}$$

Since $\mathbf{A}$ is not orthogonal, the transformation is not a rotation.

Example

Applying the transformation to the point $\begin{pmatrix}1\\1\end{pmatrix}$. In homogeneous coordinates, $\mathbf{x}^\top = \begin{pmatrix}1 & 1 & 1\end{pmatrix}$.

$$\mathbf{x}'^\top = \begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}0.5 & 0 & 0\\ 0 & 0.25 & 0\\ 1 & 2 & 1\end{pmatrix}$$

$$= \begin{pmatrix}1.5 & 2.25 & 1\end{pmatrix}$$

The point $\begin{pmatrix}1\\1\end{pmatrix}$ is transformed to $\begin{pmatrix}1.5\\2.25\end{pmatrix}$, demonstrating both scaling and translation.

The correct option is **1) Scaling and Translation**.

Plot

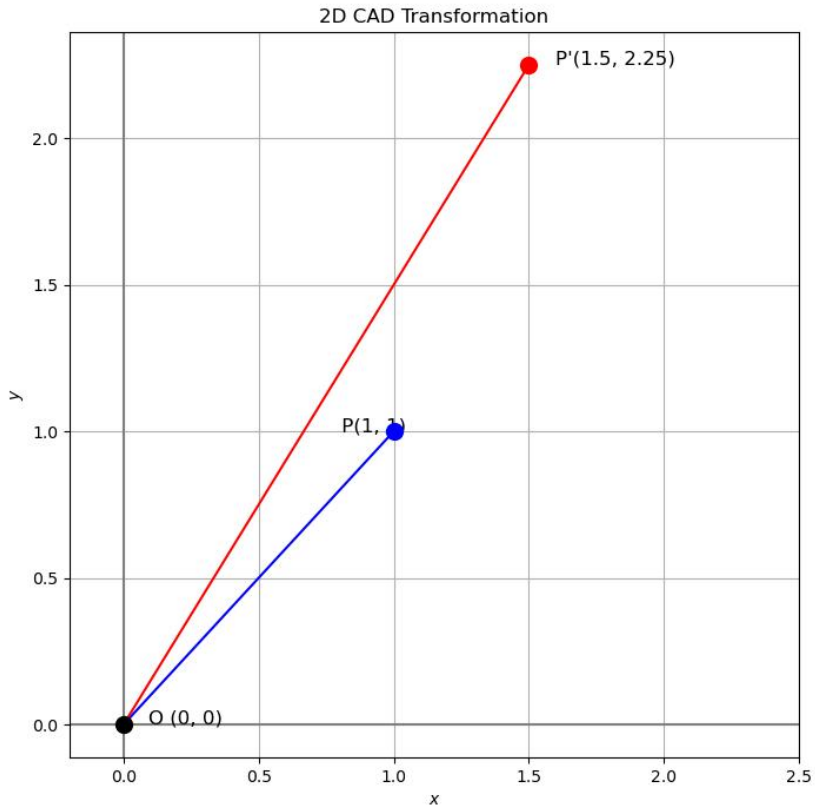

Figure: Plot