2.5.5

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Question

If (-5, 3) and (5, 3) are two vertices of an equilateral triangle, then the coordinates of the third vertex, given that the origin lies inside the triangle (take $\sqrt{3} = 1.7$), are

Given Information

Let the two given points be represented as vectors, **A** and **B**, respectively

$$\mathbf{A} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \tag{1}$$

Let us assume the third point be **C**.

Solution

C must be equidistant from both **A** and **B**, and it lies on the perpendicular bisector to both **A** and **B**.
The distance between **A** and **B**, is given by

$$\|\mathbf{A} - \mathbf{B}\| = \left\| \begin{pmatrix} -10 \\ 0 \end{pmatrix} \right\| \tag{2}$$

Continuation

We know that the norm of a vector is given by

$$\|\mathbf{A}-\mathbf{B}\|^2 = (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{B}) \implies \begin{pmatrix} -10 & 0 \end{pmatrix} . \begin{pmatrix} -10 \\ 0 \end{pmatrix} = 100 \tag{3}$$

As the norm of a vector is always greater than or equal to zero. From 3 we get

$$\|\mathbf{A}-\mathbf{B}\| = 10 \tag{4}$$

Continuation

The midpoint to the line segment **AB** is given by

$$\frac{\mathbf{A}+\mathbf{B}}{2} = \frac{\begin{pmatrix} -5 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ 3 \end{pmatrix}}{2} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{5}$$

Slope of line segment **AB** is given by

$$\mathbf{B}-\mathbf{A} = k\begin{pmatrix} 1 \\ m \end{pmatrix}, \text{ where m is the slope of the line segment} \tag{6}$$

Continuation

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 10 \\ 0 \end{pmatrix} \implies m = 0 \tag{7}$$

Therefore the perpendicular bisector for this line segment is a vertical line passing through the midpoint (0, 3).

Continuation

In parametric form

$$\mathbf{C} = \begin{pmatrix} 0 \\ t+3 \end{pmatrix}, \text{ where t is the distance between the point } \mathbf{C} \text{ and the line segm} \tag{8}$$

Final Answer

We know for an equilateral triangle, distance between a point and the opposite edge is $\frac{\sqrt{3}}{2}$ times the length of an edge of that triangle.

$$t = \pm\frac{\sqrt{3}}{2}\left\|\mathbf{A} - \mathbf{B}\right\| \implies t = \pm 5\sqrt{3} \tag{9}$$

Therefore the required points for $\mathbf{C}$ are given by

$$\mathbf{C} = \begin{pmatrix} 0 \\ \pm 5\sqrt{3} + 3 \end{pmatrix} \tag{10}$$

C code

```c
#include<stdio.h>
#include<math.h>

double norm(double *A, int m){
        double norm = 0;
        for(int i=0; i<m; i++){
                norm += A[i]*A[i];
        }
        norm = sqrt(norm);
        return norm;
}
```

Python code

```
import matplotlib.pyplot as plt
import numpy as np
import ctypes
import os
import sys

norm = ctypes.CDLL('./norm.so')
norm.norm.argtypes = [
        ctypes.POINTER(ctypes.c_double),
        ctypes.c_int
]
```

Python code

```
norm.norm.restype = ctypes.c_double

A=np.array([-5, 3], dtype=np.float64)
B=np.array([5, 3], dtype=np.float64)
m=len(A)

D=B-A

fig, ax=plt.subplots()
```

Python code

```
norm = norm.norm(
        D.ctypes.data_as(ctypes.POINTER(ctypes.c_double)),
        m
)

t=(1.7/2)*norm

C1=np.array([0, 3+t], dtype=np.float64)
C2=np.array([0, 3-t], dtype=np.float64)
```

Python code

```
def line_gen_num(A,B,num):
  dim = A.shape[0]
  x_AB = np.zeros((dim,num))
  lam_1 = np.linspace(0,1,num)
  for i in range(num):
    temp1 = A + lam_1[i]*(B-A)
    x_AB[:,i]= temp1.T
  return x_AB
```

Python code

```
x_AB = line_gen_num(A, B, 20)
x_BC1 = line_gen_num(C1, B, 20)
x_BC2 = line_gen_num(C2, B, 20)
x_AC1 = line_gen_num(A, C1, 20)
x_AC2 = line_gen_num(A, C2, 20)
```

```python

plt.grid()
plt.title('2.9.2')
plt.plot(x_AB[0, :], x_AB[1, :], 'r--', label='Line from A to B')
plt.plot(x_BC1[0, :], x_BC1[1, :], 'r--')
plt.plot(x_BC2[0, :], x_BC2[1, :], 'r--')
plt.plot(x_AC1[0, :], x_AC1[1, :], 'r--')
plt.plot(x_AC2[0, :], x_AC2[1, :], 'r--')
```

Python code

```
plt.plot(A[0], A[1], 'go', label='Point A')
plt.annotate('(-5,3)', xy=(A[0],A[1]), fontsize=12)
plt.plot(B[0], B[1], 'go', label='Point B')
plt.annotate('(5,3)', xy=(B[0],B[1]), fontsize=12)
plt.plot(C1[0], C1[1], 'bo', label='Point C1')
plt.annotate('(0,11.5)', xy=(C1[0],C1[1]), fontsize=12)
plt.plot(C2[0], C2[1], 'bo', label='Point C2')
plt.annotate('(5,-5.5)', xy=(C2[0],C2[1]), fontsize=12)
```

Python code

```python

for axis in ['bottom', 'left']:
    ax.spines[axis].set_color('black')
    ax.spines[axis].set_linewidth(2)
```

Python code

```
plt.legend()
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.axis('equal')
plt.savefig('/home/shreyas/GVV_Assignments/matgeo/2.9.2/figs/fig1
    .png')

plt.show()
```

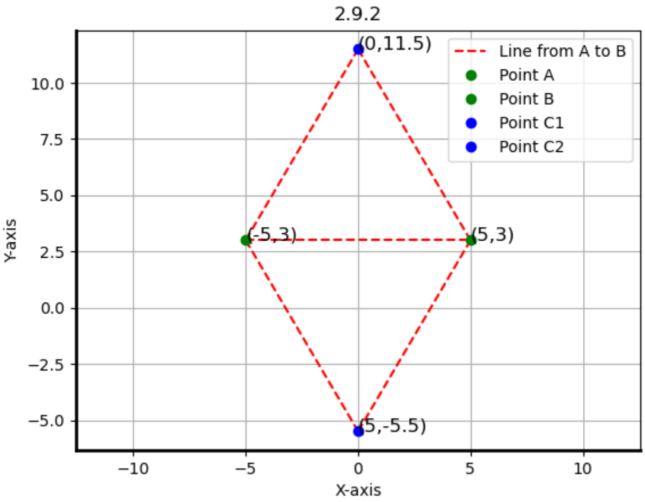

Figure: 2D Plot