

Matrices in Geometry - 5.4.30

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Problem Statement

Using elementary transformations, find the inverse of the following matrix.

$$\begin{pmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{pmatrix}$$

Solution

Let $\mathbf{A} = \begin{pmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{pmatrix}$

If \mathbf{B} is the inverse of the matrix i.e,

$$\mathbf{B} = \mathbf{A}^{-1} \quad (1)$$

$$\implies \mathbf{AB} = \mathbf{I} \quad (2)$$

Forming the augmented matrix for this in order to solve for \mathbf{B}

$$\left(\begin{array}{cc|cc} x^2 - x + 1 & x - 1 & 1 & 0 \\ x + 1 & x + 1 & 0 & 1 \end{array} \right) \quad (3)$$

Solution

$$\xleftrightarrow{R_2 \leftarrow R_2 - \frac{x+1}{x^2-x+1} R_1} \left(\begin{array}{cc|cc} x^2 - x + 1 & x - 1 & 1 & 0 \\ 0 & \frac{(x+1)(x^2-2x+2)}{x^2-x+1} & -\frac{x+1}{x^2-x+1} & 1 \end{array} \right) \quad (4)$$

$$\xleftrightarrow{R_2 \leftarrow \frac{x^2-x+1}{(x+1)(x^2-2x+2)} R_2} \left(\begin{array}{cc|cc} x^2 - x + 1 & x - 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{x^2-2x+2} & \frac{x^2-x+1}{(x+1)(x^2-2x+2)} \end{array} \right) \quad (5)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - (x-1)R_2} \left(\begin{array}{cc|cc} x^2 - x + 1 & 0 & \frac{x^2-x+1}{x^2-2x+2} & -\frac{(x-1)(x^2-x+1)}{(x+1)(x^2-2x+2)} \\ 0 & 1 & -\frac{1}{x^2-2x+2} & \frac{x^2-x+1}{(x+1)(x^2-2x+2)} \end{array} \right) \quad (6)$$

Solution

$$\xleftrightarrow{R_1 \leftarrow \frac{1}{x^2-x+1} R_1} \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{x^2-2x+2} & -\frac{x-1}{(x+1)(x^2-2x+2)} \\ 0 & 1 & -\frac{1}{x^2-2x+2} & \frac{x^2-x+1}{(x+1)(x^2-2x+2)} \end{array} \right) \quad (7)$$

$$\Rightarrow \mathbf{A}^{-1} = \begin{pmatrix} \frac{1}{x^2-2x+2} & -\frac{x-1}{(x+1)(x^2-2x+2)} \\ -\frac{1}{x^2-2x+2} & \frac{x^2-x+1}{(x+1)(x^2-2x+2)} \end{pmatrix} \quad (8)$$

$$\boxed{\mathbf{A}^{-1} = \begin{pmatrix} \frac{1}{x^2-2x+2} & \frac{1-x}{x^3-x^2+2} \\ -\frac{1}{x^2-2x+2} & \frac{x^2-x+1}{x^3-x^2+2} \end{pmatrix}} \quad (9)$$

final Answer

∴ The inverse of the given matrix is $\begin{pmatrix} \frac{1}{x^2-2x+2} & \frac{1-x}{x^3-x^2+2} \\ -\frac{1}{x^2-2x+2} & \frac{x^2-x+1}{x^3-x^2+2} \end{pmatrix}$