

12.235

EE25BTECH11013 - Bhargav

Question:

A system of equations represented as

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 4 \\ 1 & 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ y \\ 3 \end{pmatrix} \text{ is,} \quad (0.1)$$

- 1) consistent and has a unique solution
- 2) inconsistent and has no solution
- 3) consistent and infinite solution
- 4) inconsistent and has unique solution

Solution:

This can be represented as an augmented matrix and can be solved by using Gaussian elimination.

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 2 & 1 & 4 & y \\ 1 & 3 & 1 & 3 \end{array} \right) \xrightarrow[R_3 \leftarrow R_3 - R_1]{R_2 \leftarrow R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 3 & 0 & y-8 \\ 0 & 4 & -1 & -1 \end{array} \right) \quad (4.1)$$

$$\xrightarrow[R_3 \leftarrow R_3 - 4R_2]{R_2 \leftarrow \frac{R_2}{3}} \left(\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 1 & 0 & \frac{y-8}{3} \\ 0 & 0 & -1 & \frac{29-4y}{3} \end{array} \right) \xrightarrow[R_1 \leftarrow R_1 - 2R_3]{R_3 \leftarrow -R_3} \quad (4.2)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & \frac{41-8y}{3} \\ 0 & 1 & 0 & \frac{y-8}{3} \\ 0 & 0 & 1 & \frac{4y-29}{3} \end{array} \right) \xrightarrow{R_1 \leftarrow R_1 + R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{33-7y}{3} \\ 0 & 1 & 0 & \frac{y-8}{3} \\ 0 & 0 & 1 & \frac{4y-29}{3} \end{array} \right) \quad (4.3)$$

Since $y \in \mathbf{R}$, we can conclude that there exists a unique solution and the system of equations is consistent.

Option (1) is the correct answer