4.7.38

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14 September,2025

Question

P(0, 2) is the point of intersection of Y axis and perpendicular bisector of line segment joining the points A(-1, 1) and B(3, 3).

Solution

Given points,

$$\mathbf{A} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{1}$$

Mid point of **A** and **B**, Let it be **R**

$$\mathbf{R} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{2}$$

Solution

Slope, $\mathbf{m}$

$$\mathbf{m} = \mathbf{B} - \mathbf{A} \tag{3}$$

$$\tag{4}$$

Let $\mathbf{n}$ be the direction vector perpendicular to $\mathbf{m}$, If truly $\mathbf{P}$ is y-intercept of bisector

$$\mathbf{n} = \mathbf{P} - \mathbf{R} \tag{5}$$

Solution

Both $\mathbf{n}$ and $\mathbf{m}$ are perpendicular

$$\mathbf{n}^T\mathbf{m} = 0 \tag{6}$$

$$(\mathbf{P}-\mathbf{R})^T(\mathbf{B}-\mathbf{A}) = 0 \tag{7}$$

$$(\mathbf{P}^T - (\frac{\mathbf{A}+\mathbf{B}}{2})^T)(\mathbf{B}-\mathbf{A}) = 0 \tag{8}$$

$$\mathbf{P}^T(\mathbf{B}-\mathbf{A}) - \frac{(\mathbf{A}+\mathbf{B})^T(\mathbf{B}-\mathbf{A})}{2} = 0 \tag{9}$$

$$\begin{pmatrix} 0 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \end{pmatrix} - \frac{\begin{pmatrix} 2 & 4 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \end{pmatrix}}{2} = 0 \tag{10}$$

$$4 - \frac{16}{2} \neq 0 \tag{11}$$

Hence, $\mathbf{P}$ is not the y-intercept of perpendicular bisector of line $\mathbf{A}-\mathbf{B}$

Figure

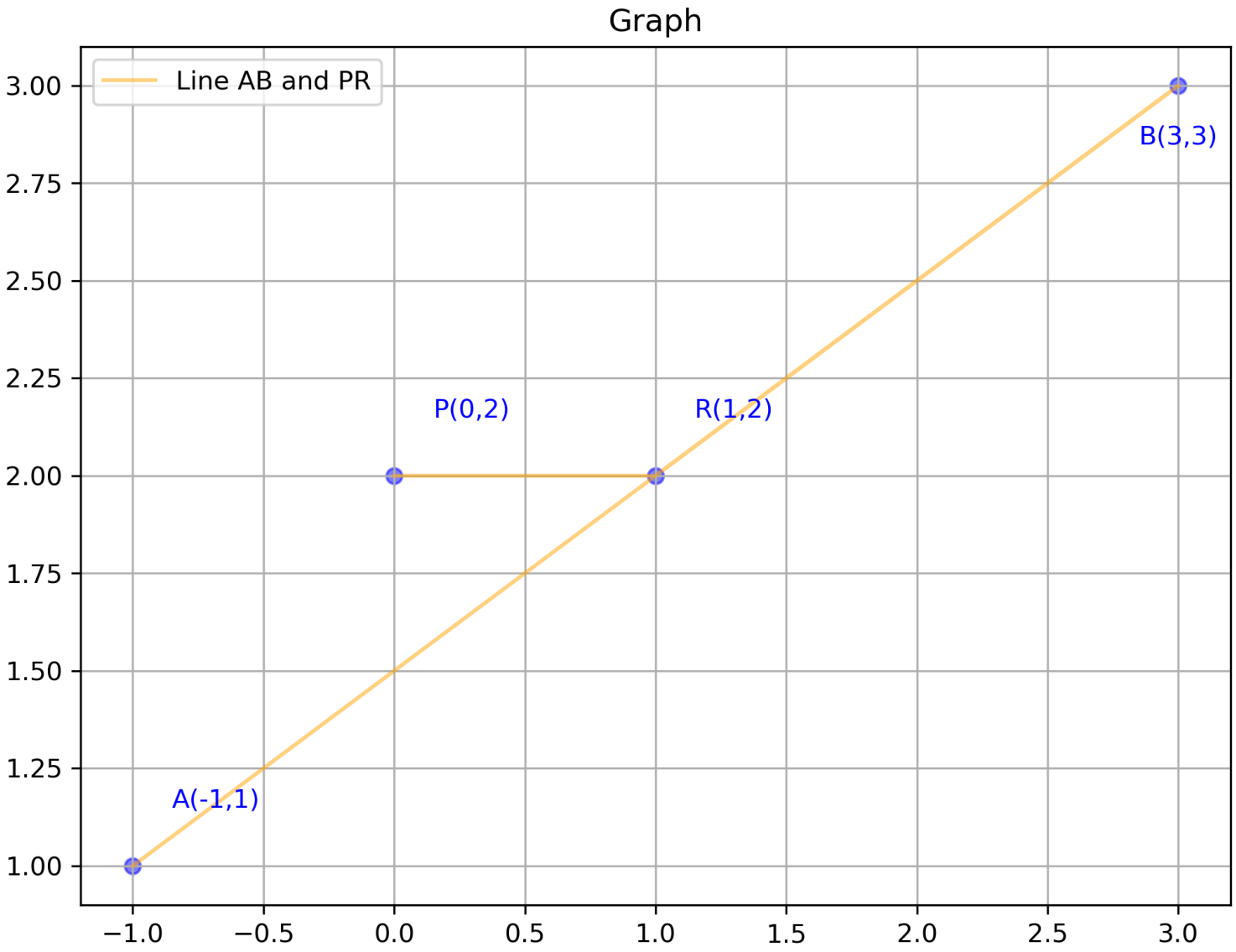

Figure: Caption

C code

```c
#include <stdio.h>

void midpoint(double x1, double y1, double x2, double y2, double
    *mx, double *my) {
    *mx = (x1 + x2) / 2.0;
    *my = (y1 + y2) / 2.0;
}
```

Python code

```
import ctypes
import matplotlib.pyplot as plt

# Load shared object
lib = ctypes.CDLL('./libmidpoint.so')

# Define argument and return types
lib.midpoint.argtypes = [ctypes.c_double, ctypes.c_double,
                         ctypes.c_double, ctypes.c_double,
                         ctypes.POINTER(ctypes.c_double),
                         ctypes.POINTER(ctypes.c_double)]

# Input points A and B
x1, y1 = -1, 1
x2, y2 = 3, 3
```

Python code

```
# Output variables
mx = ctypes.c_double()
my = ctypes.c_double()

# Call C function
lib.midpoint(x1, y1, x2, y2, ctypes.byref(mx), ctypes.byref(my))

print(f"Midpoint of A and B: ({mx.value}, {my.value})")

# Points
A = (x1, y1)
B = (x2, y2)
M = (mx.value, my.value)
P = (0, 2) # intersection with Y-axis
```

Python code

```
# Plot
plt.figure(figsize=(6,6))
plt.plot([A[0], B[0]], [A[1], B[1]], 'b-', label="Line AB")
plt.plot([M[0],P[0]], [M[1],P[1]], 'b-', label="Line MP")
plt.scatter(*A, color='red', label="A(-1,1)")
plt.scatter(*B, color='green', label="B(3,3)")
plt.scatter(*M, color='purple', label=f"M{M}")
plt.scatter(*P, color='orange', label="P(0,2)")
```

Python code

```
plt.text(A[0]+0.08,A[1]+0.1,'A', color='red')
plt.text(B[0]-0.08,B[1]-0.1,'B', color='green')
plt.text(M[0]+0.08,M[1]+0.1,'M', color='purple')
plt.text(0+0.08,2+0.1,'P', color='orange')


# Draw perpendicular bisector line
plt.axvline(x=0, color='gray', linestyle='--', label="Y-axis")
plt.legend()
plt.grid(True)
```

Python code

```
plt.xlabel("X-axis")
plt.ylabel("Y-axis")
plt.title("Midpoint and Perpendicular Bisector Intersection")
plt.savefig("figure.png", dpi=150)
plt.show()
```

Direct Python code

```
import numpy as np
import matplotlib.pyplot as plt

plt.figure(figsize=(8, 6), dpi=100)
a=0
b=0
x=np.array([-1,3,a,0])
y=np.array([1,3,b,2])
x[2]=(x[0]+x[1])/2
y[2]=(y[0]+y[1])/2
```

Direct Python code

```
plt.scatter(x,y, color='blue', alpha=0.5, )
plt.text(x[0]+0.15, y[0]+0.15, "A(-1,1)", color='blue')
plt.text(x[1]-0.15, y[1]-0.15, "B(3,3)", color='blue')
plt.text(x[2]+0.15, y[2]+0.15, "R(1,2)", color='blue')
plt.text(x[3]+0.15, y[3]+0.15, "P(0,2)", color='blue')
plt.title("Graph")

plt.grid()
plt.plot(x,y, 'o-', color='orange', mfc='blue', ms=0,alpha=0.5,
    label='Line AB and PR')
plt.legend()
plt.savefig('figure.png', dpi=300, bbox_inches='tight')
plt.show()
```