4.13.81

AI25BTECH11012 - GARIGE UNNATHI

Question:
Let P_1 : 2x + y - z = 3 and P_2 : x + 2y + z = 2 be two planes . Then, which of the following statements is/are TRUE ?

1) The line of intersection of P1 and P2 has direction ratios 1,2,-1
2) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P1 and P2
3) The acute angle between P_1 and P_2 is 60°
4) If P_3 is the plane passing through the point (4,2,-2) and perpendicular to the line of intersection of P_1 and P_2 ,then the distance of the point (2,1,1) from the plane P_3 is $\frac{2}{\sqrt{3}}$

Solution:
Let

$$P_1 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}^T \mathbf{X} = 3 \tag{4.1}$$

$$P_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}^T \mathbf{X} = 2 \tag{4.2}$$

$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \end{pmatrix} \mathbf{X} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{4.3}$$

Combining both equations and solving by row reduction we get :

$$\mathbf{X} = \begin{pmatrix} 0 \\ \frac{5}{3} \\ -\frac{4}{3} \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \tag{4.4}$$

Hence , the direction ratios of the line of intersection are (1,-1,1) . So option 1 is false

For option 2 :
simplifing the line equation we get the line equation to be :

$$\frac{x - \frac{4}{3}}{3} = \frac{y - \frac{1}{3}}{-3} = \frac{z}{3} \tag{4.5}$$

$$\mathbf{X} = \begin{pmatrix} -\frac{4}{3} \\ -\frac{1}{3} \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix} \tag{4.6}$$

solving the equation by row reduction we get direction ratios of the line to be (3,-3,3)
For two lines to be perpendicular :

$$n_1^T n_2 = 0 \tag{4.7}$$

For the given lines :

$$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}^T \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix} = 9 \tag{4.8}$$

Hence. the lines are not perpendicular . So option 2 is also false

We find the angle between two planes by the formula :

$$\cos\theta = \frac{|n_1^T n_2|}{||n_1||||n_2||} \tag{4.9}$$

By solving using above equation we get :

$$\cos\theta = \frac{1}{2} \tag{4.10}$$

Hence the angle $\theta = 60°$. So option 3 is true

The plane perpendicular to a line has normal or direction ratios equal to the direction ratios of the line that is (1,-1,1)

Hence the plane equation can be written as :

$$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}^T \mathbf{X} = c \tag{4.11}$$

To find c we can substitute the point (4,2,-2) in the plane equation :

$$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}^T \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} = 0 \tag{4.12}$$

Hence the plane equation is :

$$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}^T \mathbf{X} = 0 \tag{4.13}$$

The distance of a point from a plane is given by the equation :

$$\frac{|n^T \mathbf{P} - c|}{||n||} \tag{4.14}$$

Solving using above equation for the point $\mathbf{P}$= $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ we get :

$$\frac{|2 - 1 + 1|}{\sqrt{3}} = \frac{2}{\sqrt{3}} \tag{4.15}$$

Hence , option 4 is also true .

Thus options 3 and 4 are true