

5.5.11

EE25BTECH11059 - Vaishnavi Ramkrishna Anantheertha

Question: Find inverse of the following matrix,using elementary transformation

$$A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

Solution

Construct the augmented matrix of A and I

$$\left(\begin{array}{ccc|ccc} 2 & 0 & -1 & 1 & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array} \right) \xrightarrow{R_1 \leftarrow \frac{1}{2}R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array} \right) \quad (0.1)$$

$$\xrightarrow{R_2 \leftarrow R_2 - 5R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & -\frac{5}{2} & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array} \right) \quad (0.2)$$

$$\xrightarrow{R_3 \leftarrow R_3 - R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & -\frac{5}{2} & 1 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{5}{2} & -1 & 1 \end{array} \right) \quad (0.3)$$

$$\xrightarrow{R_3 \leftarrow 2R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{5}{2} & -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{array} \right) \quad (0.4)$$

$$\xrightarrow{R_2 \leftarrow R_2 - \frac{5}{2}R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & -15 & 6 & -5 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{array} \right) \quad (0.5)$$

$$\xrightarrow{R_1 \leftarrow R_1 + \frac{1}{2}R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & 1 \\ 0 & 1 & 0 & -15 & 6 & -5 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{array} \right) \quad (0.6)$$

$$A^{-1} = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix} \quad (0.7)$$