

Matgeo-2.2.20

AI25BTECH11027 - NAGA BHUVANA

Question:

If the co-ordinates of the points **A**, **B**, **C**, **D** be (1,2,3), (4,5,7), (-4,3,-6) and (2,9,2) respectively, then find the angle between lines AB and CD.

Solution:

Let

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 5 \\ 7 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -4 \\ 3 \\ -6 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} 2 \\ 9 \\ 2 \end{pmatrix} \quad (1)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 4 \\ 5 \\ 7 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix} \quad (2)$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} 2 \\ 9 \\ 2 \end{pmatrix} - \begin{pmatrix} -4 \\ 3 \\ -6 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \\ 8 \end{pmatrix} \quad (3)$$

Let the angle between $\mathbf{B} - \mathbf{A}$ and $\mathbf{D} - \mathbf{C}$ be θ

$$\cos \theta = \frac{(\mathbf{B} - \mathbf{A})^T (\mathbf{D} - \mathbf{C})}{\|\mathbf{B} - \mathbf{A}\| \|\mathbf{D} - \mathbf{C}\|} \quad (4)$$

$$\cos \theta = \frac{\begin{pmatrix} 3 & 3 & 4 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \\ 8 \end{pmatrix}}{\sqrt{34} \sqrt{136}} \quad (5)$$

$$\cos \theta = \frac{(3)(6) + (3)(6) + (4)(8)}{68} \quad (6)$$

$$\cos \theta = \frac{68}{68} \quad (7)$$

$$\cos \theta = 1 \quad (8)$$

$$\theta = 0^\circ \quad (9)$$

\therefore The angle between lines $(\mathbf{B} - \mathbf{A})$ and $(\mathbf{D} - \mathbf{C})$ is 0° (Collinear lines)

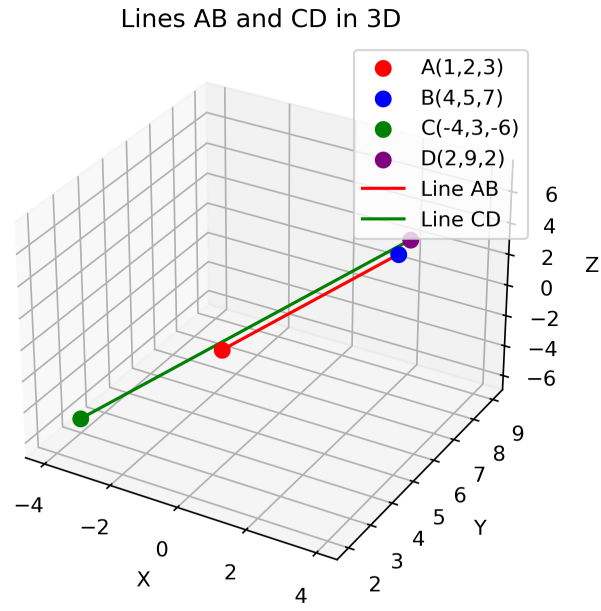


Fig. 1