2.3.5

EE25BTECH11002 - Achat Parth Kalpesh

Question:
Find the angle between the line $\mathbf{r} = \left(\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ and the plane $\mathbf{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 3$.
Solution:
From the given equations, we can identify the direction vector of the line, $\mathbf{d}$, and the normal vector of the plane, $\mathbf{n}$.

$$\mathbf{d} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \tag{0.1}$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{0.2}$$

If θ is the angle between the line and the plane, then the angle between the line's direction vector $\mathbf{d}$ and the plane's normal vector $\mathbf{n}$ is $90° - \theta$. The formula to calculate the angle θ is:

$$\theta = \sin^{-1}\left(\frac{\left|\mathbf{n}^\top \mathbf{d}\right|}{\|\mathbf{d}\| \, \|\mathbf{n}\|}\right) \tag{0.3}$$

Substituting the vectors $\mathbf{d}$ and $\mathbf{n}$ into this formula:

$$\theta = \sin^{-1}\left(\frac{\left|\begin{pmatrix} 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}\right|}{\left\|\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}\right\| \left\|\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}\right\|}\right) \tag{0.4}$$

$$= \sin^{-1}\left(\frac{|(3)(1) + (-1)(1) + (2)(1)|}{\sqrt{3^2 + (-1)^2 + 2^2}\sqrt{1^2 + 1^2 + 1^2}}\right) \tag{0.5}$$

$$= \sin^{-1}\left(\frac{|3 - 1 + 2|}{\sqrt{9 + 1 + 4}\sqrt{3}}\right) \tag{0.6}$$

$$= \sin^{-1}\left(\frac{|4|}{\sqrt{14}\sqrt{3}}\right) \tag{0.7}$$

$$= \sin^{-1}\left(\frac{4}{\sqrt{42}}\right) \tag{0.8}$$

So, the angle θ is:

$$\theta = \sin^{-1}\left(\frac{4}{\sqrt{42}}\right) \tag{0.9}$$

This is approximately 37.98°

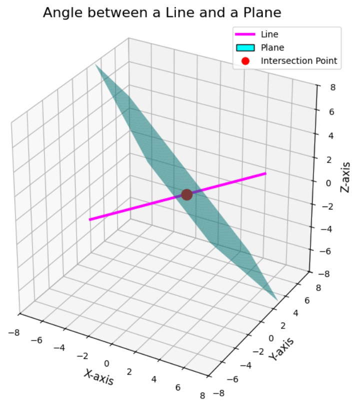

Fig. 0.1: Graph