

9.3.3

AI25BTECH11012 - GARIGE UNNATHI

Question:

Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$.

Solution: The points of intersection of the line :

$$L : \mathbf{x} = \mathbf{h} + \kappa \mathbf{m} \quad (0.1)$$

with the conic is given by

$$\mathbf{x}_i = \mathbf{h} + \kappa_i \mathbf{m} \quad (0.2)$$

where :

$$\kappa_i = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} (-\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^T \mathbf{V} \mathbf{m})})$$

For the parabola $3x^2 - 4y = 0$

$$\mathbf{V} = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \quad (0.3)$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \quad (0.4)$$

For the line $2y = 3x + 12$.

$$\mathbf{X} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} + \kappa \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix} \quad (0.5)$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \quad (0.6)$$

$$\mathbf{m} = \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix} \quad (0.7)$$

Substituting and solving we get :

$$\kappa = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad (0.8)$$

so the points of intersection after solving using the equation 0.2 are :

$$\mathbf{X} = \begin{pmatrix} 4 \\ 12 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -2 \\ 3 \end{pmatrix} \quad (0.9)$$

Calculating the area :

$$\int_{-2}^4 \frac{3}{2}x + 6 - \frac{3}{4}x^2 dx = 27 \quad (0.10)$$

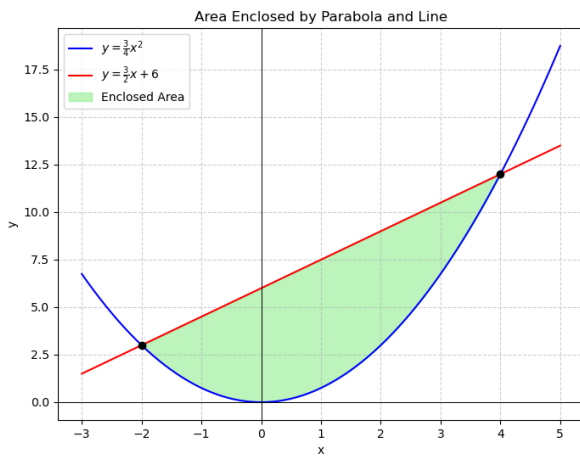


Fig. 0.1