5.7.14

EE25BTECH11001 - Aarush Dilawri

Question:
If $\mathbf{A} = \begin{pmatrix} -3 & 6 \\ -2 & 4 \end{pmatrix}$, then show that $\mathbf{A}^3 = \mathbf{A}$.
Solution:

The characteristic equation of $\mathbf{A}$ is given by:

$$f(\lambda) = |\mathbf{A} - \lambda\mathbf{I}| = 0 \tag{0.1}$$

Therefore,

$$f(\lambda) = \begin{vmatrix} -3-\lambda & 6 \\ -2 & 4-\lambda \end{vmatrix} = 0 \tag{0.2}$$

$$f(\lambda) = \lambda^2 - \lambda = 0 \tag{0.3}$$

By Cayley-Hamilton theorem,

$$f(\lambda) = f(\mathbf{A}) = 0 \tag{0.4}$$

Therefore,

$$\mathbf{A}^2 - \mathbf{A} = 0 \implies \mathbf{A}^2 = \mathbf{A} \tag{0.5}$$

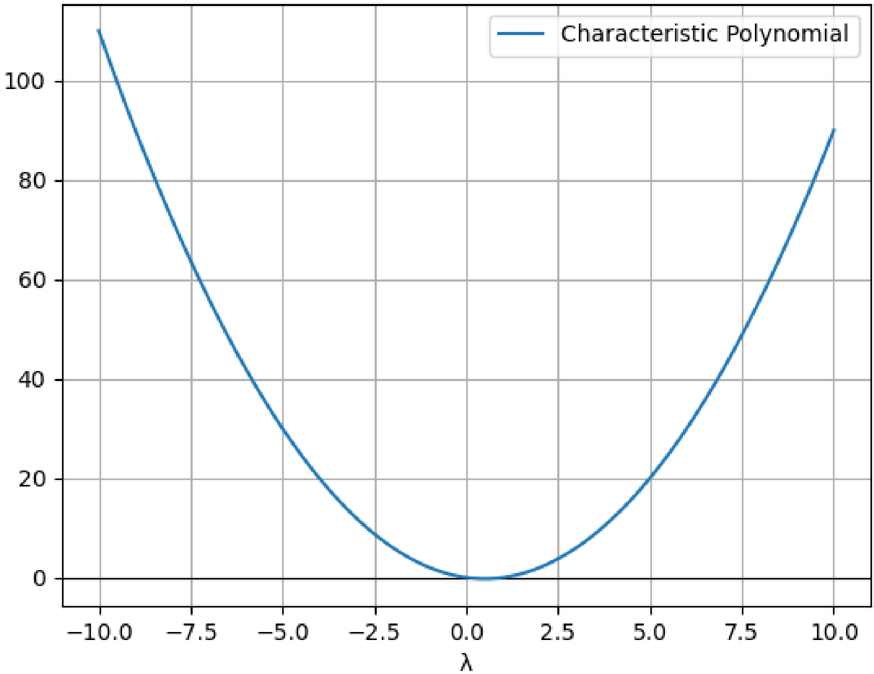
Pre-multiplying both sides by $\mathbf{A}$,

$$\mathbf{A}^3 = \mathbf{A}^2 \quad \text{but } \mathbf{A}^2 = \mathbf{A} \tag{0.6}$$

$$\implies \mathbf{A}^3 = \mathbf{A} \tag{0.7}$$

Hence proved.

See Figure,


Characteristic Polynomial
100
80
60
40
20
0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
λ