

MatGeo Assignment 1.11.9

AI25BTECH11007

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Question

If

$$\mathbf{a} = \hat{i} - 7\hat{j} + 7\hat{k} \quad \text{and} \quad \mathbf{b} = 3\hat{i} - 2\hat{j} + 2\hat{k},$$

find a unit vector perpendicular to both the vectors \mathbf{a} and \mathbf{b} .

Solution

We have

$$\mathbf{a} = \begin{pmatrix} 1 \\ -7 \\ 7 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}, \quad \mathbf{n} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

$$\mathbf{a}^T \mathbf{n} = 0, \tag{1}$$

$$\mathbf{b}^T \mathbf{n} = 0. \tag{2}$$

This gives the linear system

$$\begin{bmatrix} 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{3}$$

Step 1: Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & -7 & 7 & 0 \\ 3 & -2 & 2 & 0 \end{array} \right]. \tag{4}$$

Step 2: Row operations

$$R_2 \rightarrow R_2 - 3R_1 : \left[\begin{array}{ccc|c} 1 & -7 & 7 & 0 \\ 0 & 19 & -19 & 0 \end{array} \right], \quad (5)$$

$$R_2 \rightarrow \frac{1}{19}R_2 : \left[\begin{array}{ccc|c} 1 & -7 & 7 & 0 \\ 0 & 1 & -1 & 0 \end{array} \right], \quad (6)$$

$$R_1 \rightarrow R_1 + 7R_2 : \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{array} \right]. \quad (7)$$

Step 3: Solution

From RREF:

$$x = 0, \quad (8)$$

$$y - z = 0 \Rightarrow y = z. \quad (9)$$