# Matrices in Geometry - 2.4.34

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#### Problem Statement

What type of quadrilateral do the points  $\mathbf{A}(2,-2)$ ,  $\mathbf{B}(7,3)$ ,  $\mathbf{C}(11,-1)$  and  $\mathbf{D}(6,-6)$  taken in that order, form?

## Solution

Given: 
$$\mathbf{A} \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$
,  $\mathbf{B} \begin{pmatrix} 7 \\ 3 \end{pmatrix}$ ,  $\mathbf{C} \begin{pmatrix} 11 \\ -1 \end{pmatrix}$  and  $\mathbf{D} \begin{pmatrix} 6 \\ -6 \end{pmatrix}$ .

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 5 \\ 5 \end{pmatrix} \tag{1}$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 4 \\ -4 \end{pmatrix} \tag{2}$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} -5 \\ -5 \end{pmatrix} \tag{3}$$

$$\mathbf{A} - \mathbf{D} = \begin{pmatrix} -4\\4 \end{pmatrix} \tag{4}$$

#### Solution

Checking opposite sides, 
$$\mathbf{D} - \mathbf{C} = -(\mathbf{A} - \mathbf{B})$$
 and  $\mathbf{A} - \mathbf{D} = -(\mathbf{B} - \mathbf{C})$  (5)

Each pair of opposite sides are parallel and equal in length; this implies that the quadrilateral is a parallelogram.

Now, checking for right angle, we check for inner product.

$$(\mathbf{B} - \mathbf{A})^{\top} (\mathbf{C} - \mathbf{B}) = \begin{pmatrix} 5 & 5 \end{pmatrix} \begin{pmatrix} 4 \\ -4 \end{pmatrix} = 0$$
 (6)

#### Solution

This implies that the angle at  ${\bf B}$  is one right angle. A parallelogram with a right angle is a rectangle.

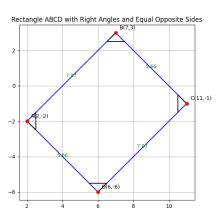
Checking for a square:

$$\|\mathbf{B} - \mathbf{A}\| = 5\sqrt{2} \text{ and } \|\mathbf{C} - \mathbf{B}\| = 4\sqrt{2}$$
 (7)

We can see that  $\|\mathbf{B} - \mathbf{A}\| \neq \|\mathbf{C} - \mathbf{B}\|$ , and therefore it is not a square. Therefore, the quadrilateral **ABCD** is a rectangle.

### Final Answer

#### The quadrilateral **ABCD** is a rectangle



(8)