4.13.23

EE25BTECH11018 - Darisy Sreetej

Question:
Let a, b, c, d be non-zero numbers. If the point of intersection of the lines $4a\mathbf{x} + 2a\mathbf{y} + c = 0$ and $5b\mathbf{x} + 2b\mathbf{y} + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then

1) $3bc - 2ad = 0$
2) $2bc - 3ad = 0$
3) $3bc + 2ad = 0$
4) $2bc + 3ad = 0$

Solution:
The two lines are

$$4a\mathbf{x} + 2a\mathbf{y} + c = 0, \tag{1}$$

$$5b\mathbf{x} + 2b\mathbf{y} + d = 0 \tag{2}$$

This equation can be expressed in terms of matrices

$$\begin{pmatrix} 4a \\ 2a \end{pmatrix}^\top \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = -c \tag{3}$$

$$\begin{pmatrix} 5b \\ 2b \end{pmatrix}^\top \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = -d \tag{4}$$

They can be represented as,

$$\begin{pmatrix} 4a & 5b \\ 2a & 2b \end{pmatrix}^\top \begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = \begin{pmatrix} -c \\ -d \end{pmatrix} \tag{5}$$

Using augmented matrix,

$$\left(\begin{array}{cc|c} 4a & 2a & -c \\ 5b & 2b & -d \end{array}\right) \tag{6}$$

$R_1 = \frac{R_1}{4a}$

$$\left(\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{-c}{4a} \\ 5b & 2b & -d \end{array}\right) \tag{7}$$

$R_2 = R_2 - 5bR_1$

$$\left(\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{-c}{4a} \\ 0 & \frac{-b}{2} & \frac{-4ad+5bc}{4a} \end{array}\right) \tag{8}$$

$$\mathbf{y}\frac{-b}{2} = \frac{-4ad + 5bc}{4a} \tag{9}$$

$$\mathbf{y} = \frac{4ad - 5bc}{2ab} \tag{10}$$

Also,

$$\mathbf{x} + \frac{1}{2}\mathbf{y} = \frac{-c}{4a} \tag{11}$$

$$\mathbf{x} = \frac{bc - ad}{ab} \tag{12}$$

$$\begin{pmatrix} \mathbf{x} \\ \mathbf{y} \end{pmatrix} = \begin{pmatrix} \frac{bc-ad}{ab} \\ \frac{4ad-5bc}{2ab} \end{pmatrix} \tag{13}$$

Therefore , the point of intersection is $\left(\frac{bc-ad}{ab}, \frac{4ad-5bc}{2ab}\right)$
According to the condition,the intersection point is equidistant from the axes and lies in the fourth quadrant, so its coordinates satisfy $y = -x$
Therefore,

$$\frac{4ad - 5bc}{2ab} = -\frac{bc - ad}{ab} \tag{14}$$

$$2ad - 2bc = 4ad - 5bc \tag{15}$$

$$3bc = 2ad \tag{16}$$

Therefore, option(a) is correct