

2.10.33

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Question

Let α, β, γ be distinct real numbers. The points with position vectors $\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}, \begin{pmatrix} \beta \\ \gamma \\ \alpha \end{pmatrix}, \begin{pmatrix} \gamma \\ \alpha \\ \beta \end{pmatrix}$:

1. are collinear
2. form an equilateral triangle
3. form a scalene triangle
4. form a right angled triangle

Solution

Let \mathbf{A} be $\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$, \mathbf{B} be $\begin{pmatrix} \beta \\ \gamma \\ \alpha \end{pmatrix}$, and \mathbf{C} be $\begin{pmatrix} \gamma \\ \alpha \\ \beta \end{pmatrix}$.

$$\bullet \mathbf{A} - \mathbf{B} \text{ is } \begin{pmatrix} \alpha - \beta \\ \beta - \gamma \\ \gamma - \alpha \end{pmatrix} \quad \bullet \mathbf{B} - \mathbf{C} \text{ is } \begin{pmatrix} \beta - \gamma \\ \gamma - \alpha \\ \alpha - \beta \end{pmatrix} \quad \bullet \mathbf{C} - \mathbf{A} \text{ is } \begin{pmatrix} \gamma - \alpha \\ \alpha - \beta \\ \beta - \gamma \end{pmatrix}$$

Then norms of $\mathbf{A} - \mathbf{B}$, $\mathbf{B} - \mathbf{C}$, $\mathbf{C} - \mathbf{A}$ are all equal, and equal to

$$\sqrt{(\alpha - \beta)^2 + (\beta - \gamma)^2 + (\gamma - \alpha)^2}$$

The three points therefore form an equilateral triangle, so option (2) is correct.