2.10.39

EE25BTECH11010 - Arsh Dhoke

Question:
Let $\mathbf{a} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and a unit vector $\mathbf{c}$ be coplanar. If $\mathbf{c}$ is perpendicular to $\mathbf{a}$, then $\mathbf{c}=$

1) $\frac{1}{\sqrt{2}}(-\mathbf{j} + \mathbf{k})$

2) $\frac{1}{\sqrt{3}}(-\mathbf{i} - \mathbf{j} - \mathbf{k})$

3) $\frac{1}{\sqrt{5}}(\mathbf{i} - 2\mathbf{j})$

4) $\frac{1}{\sqrt{3}}(\mathbf{i} - \mathbf{j} - \mathbf{k})$

Vector	Point
a	$\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$
b	$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

$$\mathbf{c} = \mathbf{a} + k\mathbf{b} \tag{4.1}$$

$$\mathbf{c} = \begin{pmatrix} 2 + k \\ 1 + 2k \\ 1 - k \end{pmatrix} \tag{4.2}$$

$$\mathbf{a}^T \mathbf{c} = 0 \tag{4.3}$$

$$2(2 + k) + 1(1 + 2k) + 1(1 - k) = 0 \tag{4.4}$$

$$6 + 3k = 0 \tag{4.5}$$

$$k = -2 \tag{4.6}$$

$$\mathbf{c} = \begin{pmatrix} 0 \\ -3 \\ 3 \end{pmatrix} \tag{4.7}$$

$$\|\mathbf{c}\| = \sqrt{0^2 + (-3)^2 + 3^2} = 3\sqrt{2} \tag{4.8}$$

$$\mathbf{c} = \frac{1}{3\sqrt{2}} \begin{pmatrix} 0 \\ -3 \\ 3 \end{pmatrix} \tag{4.9}$$

$$\mathbf{c} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \tag{4.10}$$
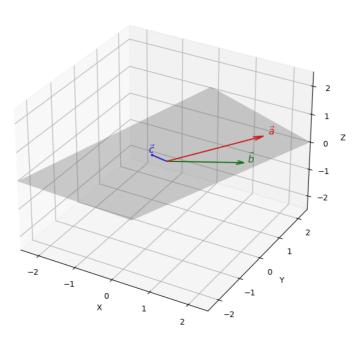
Thus option 1 is correct.

Fig. 4.1: Graph