

# 2.8.12

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**Question** Show that the tangent of an angle between the lines

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{a} - \frac{y}{b} = 1$$

is  $\frac{2ab}{a^2 - b^2}$

**Solution :** Given details:

$$\left(\frac{1}{a} \quad \frac{1}{b}\right) \mathbf{x} = 1 \quad (1)$$

$$\left(\frac{1}{a} \quad \frac{-1}{b}\right) \mathbf{x} = 1 \quad (2)$$

**Property:** The cosine of the angle between line 1 and line 2 is given by  $\frac{n_1^\top n_2}{\|n_1\| \|n_2\|}$ .

Let the angle between the lines be  $\theta$ .

$$\cos \theta = \frac{\left(\frac{1}{a} \quad \frac{1}{b}\right)^\top \left(\frac{1}{a} \quad \frac{-1}{b}\right)}{\left\|\left(\frac{1}{a} \quad \frac{1}{b}\right)\right\| \left\|\left(\frac{1}{a} \quad \frac{-1}{b}\right)\right\|} \quad (3)$$

$$\cos \theta = \frac{\frac{1}{a^2} - \frac{1}{b^2}}{\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{-1}{b}\right)^2}} \quad (4)$$

$$\cos \theta = \frac{b^2 - a^2}{a^2 + b^2} \left( \because \tan \theta = \frac{\sqrt{1 - \cos^2 \theta}}{\cos \theta} \right) \quad (5)$$

$$\tan \theta = \left| \frac{2ab}{b^2 - a^2} \right| \quad (6)$$

$\therefore$  The tan of the acute angle between the lines is  $\frac{2ab}{a^2 - b^2}$ .

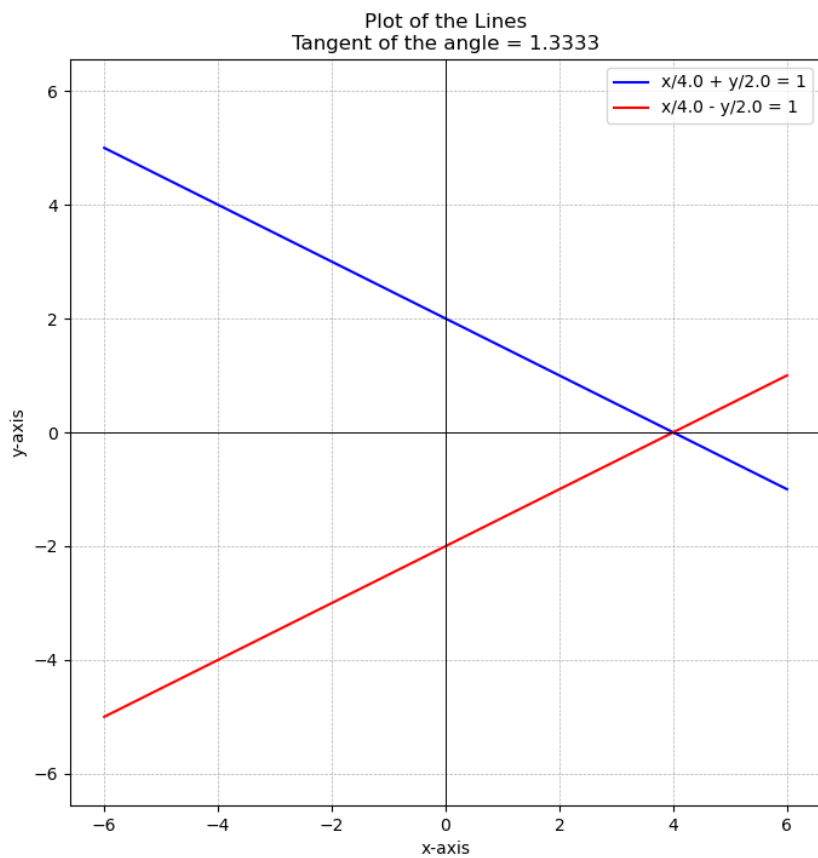


Fig. 0. Plot of the lines