Matrices in Geometry - 9.5.1

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Problem Statement

Find the roots of

$$x^2 + 3x - 10 = 0 \tag{1}$$

Solution

Expressing the given equation as parabola

$$y = x^2 + 3x - 10 \tag{2}$$

Representing this equation as a conic section

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0\ ,\ \mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\ ,\ \mathbf{u} = \begin{pmatrix} 3/2 \\ -1/2 \end{pmatrix}\ ,\ f = -10 \tag{3}$$

We need to find intersection points with $y = 0$, that is, the X-axis.

$$\mathbf{x} = \mathbf{h} + k\mathbf{m}\ ,\ \mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}\ ,\ \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{4}$$

Solution

Substituting $\mathbf{x} = k\mathbf{m}$

$$k^2\mathbf{m}^\top\mathbf{V}\mathbf{m} + 2k\mathbf{u}^\top\mathbf{m} + f = 0 \tag{5}$$

$$\implies k = \frac{1}{2}\left[-2\mathbf{u}^\top\mathbf{m} \pm \sqrt{4\left(\mathbf{u}^\top\mathbf{m}\right)^2 - 4f\mathbf{m}^\top\mathbf{V}\mathbf{m}}\right] \tag{6}$$

$$\implies k = -\mathbf{u}^\top\mathbf{m} \pm \sqrt{\left(\mathbf{u}^\top\mathbf{m}\right)^2 - f\mathbf{m}^\top\mathbf{V}\mathbf{m}} \tag{7}$$

$$\mathbf{u}^\top\mathbf{m} = \begin{pmatrix} 3/2 & -1/2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 3/2 \tag{8}$$

$$\mathbf{m}^\top\mathbf{V}\mathbf{m} = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1 \tag{9}$$

$$k = -\frac{3}{2} \pm \sqrt{\frac{9}{4} - (-10)\,1} = -\frac{3}{2} \pm \sqrt{\frac{49}{4}} \tag{10}$$

$$\implies k = -\frac{3}{2} \pm \frac{7}{2} \implies \boxed{k = 2 \text{ OR } k = -5} \tag{11}$$

Solution

Substituting k into $\mathbf{x}$, we get

$$\mathbf{x} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \text{ OR } \mathbf{x} = \begin{pmatrix} -5 \\ 0 \end{pmatrix} \tag{12}$$

This implies that the roots of $x^2 + 3x - 10 = 0$ are 2 and -5.

Solution

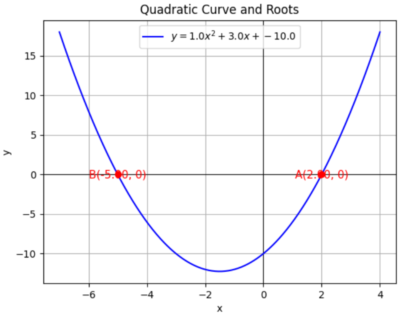

Figure: Graph for 9.5.1