4.9.4

EE25BTECH11002 - Achat Parth Kalpesh

Question:
The equations of the lines which pass through the point $(3, -2)$ and are inclined at $60°$ to the line $\sqrt{3}x + y = 1$ is

1) $y + 2 = 0, \ \sqrt{3}x - y - 2 - 3\sqrt{3} = 0$
2) $x - 2 = 0, \ \sqrt{3}x - y + 2 + 3\sqrt{3} = 0$
3) $\sqrt{3}x - y - 2 - 3\sqrt{3} = 0$
4) None of these

Solution:
The given line can be written in normal form as

$$\mathbf{n}_1^\top \mathbf{x} = 1, \tag{4.1}$$

where

$$\mathbf{n}_1 = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}. \tag{4.2}$$

Let the required line have normal vector $\mathbf{n} = \begin{pmatrix} -m \\ 1 \end{pmatrix}$, where m is the slope of line,then its equation is

$$\mathbf{n}^\top \mathbf{x} = c. \tag{4.3}$$

Since the line passes through $\mathbf{P} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$,

$$\mathbf{n}^\top \mathbf{P} = c \tag{4.4}$$

The angle θ between two lines is given as;

$$\cos\theta = \frac{\mathbf{n}_1^\top \mathbf{n}}{\|\mathbf{n}_1\| \, \|\mathbf{n}\|}. \tag{4.5}$$

Here θ =$60°$ $\implies$ $\cos\theta = \frac{1}{2}$, so

$$\left(\mathbf{n}_1^\top \mathbf{n}\right)^2 = \frac{1}{4} \|\mathbf{n}_1\|^2 \|\mathbf{n}\|^2 . \tag{4.6}$$

Substituting values:

$$\left(-\sqrt{3}m+1\right)^2 = \frac{1}{4}(4)\left((-m)^2+1^2\right) \tag{4.7}$$

$$\left(-\sqrt{3}m+1\right)^2 = m^2+1^2 \tag{4.8}$$

$$3m^2-2\sqrt{3}m+1 = m^2+1, \tag{4.9}$$

$$2m^2 = 2\sqrt{3}m \tag{4.10}$$

$$m = 0 \text{ or } m = \sqrt{3} \tag{4.11}$$

For m = 0 ;

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4.12}$$

$$\mathbf{n}^\top \mathbf{P} = c \tag{4.13}$$

$$c = \begin{pmatrix} 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix} = -2, \tag{4.14}$$

so the line is

$$y+2=0 \tag{4.15}$$

For m = $\sqrt{3}$;

$$\mathbf{n} = \begin{pmatrix} -\sqrt{3} \\ 1 \end{pmatrix} \tag{4.16}$$

$$\mathbf{n}^\top \mathbf{P} = c \tag{4.17}$$

$$c = \begin{pmatrix} -\sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix} = -3\sqrt{3}-2 \tag{4.18}$$

so the line is

$$\sqrt{3}x-y-3\sqrt{3}-2=0 \tag{4.19}$$

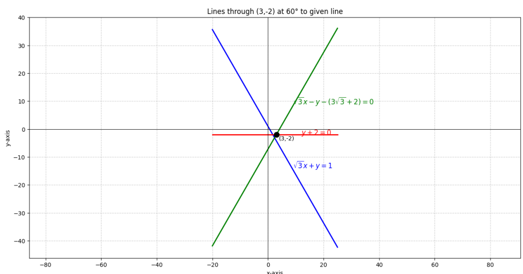

Fig. 4.1: Lines through $(3,-2)$ at 60°to given line