9.3.3

AI25BTECH11012 - GARIGE UNNATHI

Question:
Find the area enclosed by the parabola 4y = $3x^2$ and the line 2y = 3x + 12.

Solution: The points of intersection of the line :

$$L : \mathbf{x} = \mathbf{h} + \kappa \mathbf{m} \tag{0.1}$$

with the conic is given by

$$\mathbf{x}_i = \mathbf{h} + \kappa_i \mathbf{m} \tag{0.2}$$

where :

$$g(\mathbf{x}) = \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f$$

$$\kappa_i = \frac{1}{vecm^T \mathbf{V} \mathbf{m}} (-\mathbf{m}^T (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T (\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^T \mathbf{V} \mathbf{m})})$$

For the parabola $3x^2$ - 4y = 0

$$\mathbf{V} = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \tag{0.3}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{0.4}$$

For the line 2y = 3x + 12.

$$\mathbf{X} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} + \kappa \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix} \tag{0.5}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \tag{0.6}$$

$$\mathbf{m} = \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix} \tag{0.7}$$

Substituting and solving we get :

$$\kappa = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \tag{0.8}$$

so the points of intersection after solving using the equation 0.2 are :

$$\mathbf{X} = \begin{pmatrix} 4 \\ 12 \end{pmatrix} \quad and \quad \begin{pmatrix} -2 \\ 3 \end{pmatrix} \tag{0.9}$$

Calculating the area :

$$\int_{-2}^{4} \frac{3}{2}x + 6 - \frac{3}{4}x^2 \, dx = 27 \tag{0.10}$$

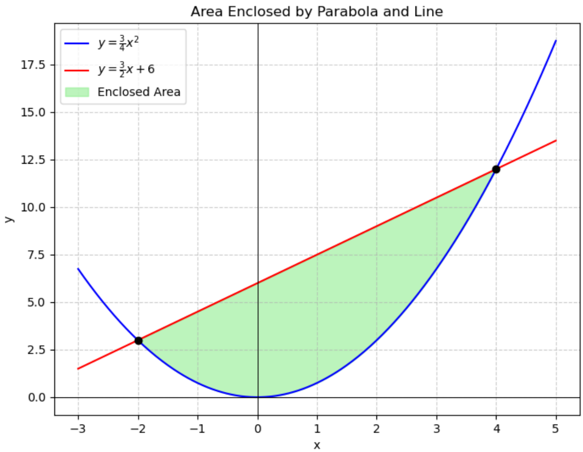

Fig. 0.1