

2.4.26

Josyula G S Avaneesh- EE25BTECH11030

Question Check whether (5,-2), (6,4) and (7,-2) are the vertices of an isosceles triangle.
Solution: Given details:

$$\mathbf{A} = \begin{pmatrix} 5 \\ -2 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \quad (1)$$

Property: In an isosceles triangle, the perpendicular bisector of a side passes through the opposite vertex.

$$\mathbf{A} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \quad (2)$$

Midpoint of side **AC**:

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{C}}{2} = \frac{\begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} 7 \\ -2 \end{pmatrix}}{2} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} \quad (3)$$

Direction vector of side **AC**:

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (4)$$

Vector from midpoint **M** to **B**:

$$\mathbf{B} - \mathbf{M} = \begin{pmatrix} 6 \\ 4 \end{pmatrix} - \begin{pmatrix} 6 \\ -2 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \quad (5)$$

$$(\mathbf{C} - \mathbf{A})^\top (\mathbf{B} - \mathbf{M}) = \begin{pmatrix} 2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 6 \end{pmatrix} = 2(0) + 0(6) = 0 \quad (6)$$

B lies on the perpendicular bisector of side **AC**.

$\therefore \mathbf{AB} = \mathbf{BC} \implies \triangle ABC$ is isosceles.

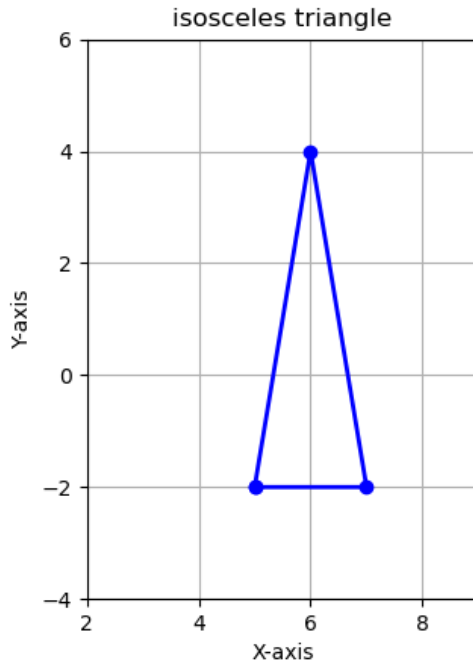


Fig. 0. isosceles triangle