2.7.11

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QUESTION

Q 2.7.11. Find the area of the triangle with vertices $A = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $B = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$, $C = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$.

SOLUTION

$$A = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad B = \begin{pmatrix} -4 \\ 6 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} -3 \\ -5 \\ 0 \end{pmatrix}. \tag{0.1}$$

$$\mathbf{AB} = B - A = \begin{pmatrix} -5 \\ 7 \\ 0 \end{pmatrix}, \qquad \mathbf{AC} = C - A = \begin{pmatrix} -4 \\ -4 \\ 0 \end{pmatrix}. \tag{0.2}$$

$$\mathbf{AB} \times \mathbf{AC} = \begin{pmatrix} 0 \\ 0 \\ (-5)(-4) - (7)(-4) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 48 \end{pmatrix}. \tag{0.3}$$

$$\therefore \ \text{Area} = \frac{1}{2} \ \|\mathbf{AB} \times \mathbf{AC}\| = \frac{1}{2} \cdot 48 = 24. \tag{0.4}$$
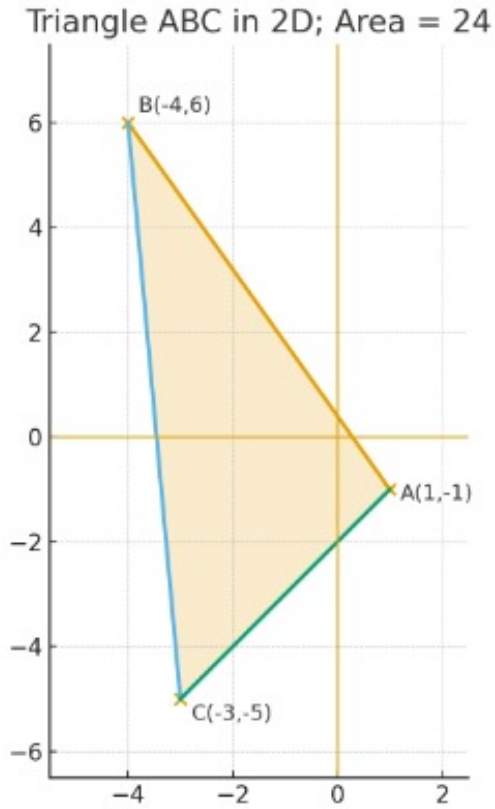
Fig. 0.1: Triangle *ABC* with $A(1, -1)$, $B(-4, 6)$, $C(-3, -5)$; area = 24.