4.3.10

EE25BTECH11007- Aniket

Question:
Find the direction and normal vectors of the line $x - y = 2$.

<center>SOLUTION</center>

A line can be expressed in two forms:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ c \end{pmatrix} + x \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{1}$$

where $\begin{pmatrix} 1 \\ m \end{pmatrix}$ is the direction vector and m is the slope.

$$\mathbf{n}^\top x = c \tag{2}$$

where $\mathbf{n}$ is the normal vector of the line.

$$\mathbf{n}^\top \begin{pmatrix} 1 \\ m \end{pmatrix} = 0 \tag{3}$$

From $x - y = 2$,the slope is $m = 1$. Hence, using (1),

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} + x \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{4}$$

Let $\begin{pmatrix} x \\ y \end{pmatrix}$ be a normal vector. Then, from (3),

$$\begin{pmatrix} x \\ y \end{pmatrix}^\top \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0 \implies x + y = 0 \implies \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{5}$$

Line in normal form using (2)

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix}^\top \begin{pmatrix} x \\ y \end{pmatrix} = 2$$

Hence, the direction vector is $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, and the normal vector is $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$.