4.8.27

AI25BTECH110031
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Question(4.8.27) Find the equation of the plane passing through $(-1, 3, 2)$ and perpendicular to the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

Solution:
Normals of the given planes are

$$\mathbf{n_1} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{n_2} = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}. \tag{0.1}$$

Let the required plane have normal vector $\mathbf{n}$
Since it is perpendicular to both given planes:

$$\mathbf{n_1}^\top \mathbf{n} = 0, \quad \mathbf{n_2}^\top \mathbf{n} = 0. \tag{0.2}$$

That is,

$$\begin{pmatrix} \mathbf{n_1} & \mathbf{n_2} \end{pmatrix}^\top \mathbf{n} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.3}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & 1 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{0.4}$$

Let

$$\mathbf{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \tag{0.5}$$

$$a + 2b + 3c = 0, \tag{0.6}$$

$$3a + 3b + c = 0. \tag{0.7}$$

From these, we get

$$\mathbf{n} = t \begin{pmatrix} 7 \\ -8 \\ 3 \end{pmatrix}, \quad t \in \mathbb{R}, \quad t \neq 0 \tag{0.8}$$

Equation of plain is

$$\mathbf{n}^\top \mathbf{x} = 1 \tag{0.9}$$

since point $\mathbf{p} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$ lies on the plain

$$\mathbf{n}^\top \mathbf{p} = 1 \tag{0.10}$$

Substituting,

$$\begin{pmatrix} 7 & -8 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 & -8 & 3 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}. \tag{0.11}$$

$$\begin{pmatrix} 7 & -8 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -25 \tag{0.12}$$

$$\frac{-1}{25} \begin{pmatrix} 7 & -8 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 1 \tag{0.13}$$

$$\mathbf{n} = \frac{-1}{25} \begin{pmatrix} 7 \\ -8 \\ 3 \end{pmatrix} \tag{0.14}$$

$$\mathbf{n}^\top \mathbf{x} = 1. \tag{0.15}$$

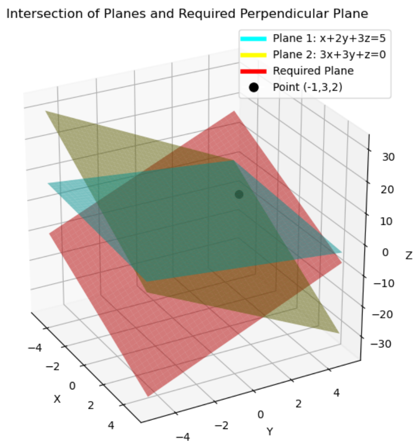

Fig. 0.1