

Question 1.7.12:

Find the value of k , if the points $P(5, 4)$, $Q(7, k)$ and $R(9, -2)$ are collinear.

Hint: Three points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ are collinear if the area of the triangle formed by them is zero.

Solution**QUESTION**

Find the value of a , if the distance between the points $A\begin{pmatrix} -3 \\ -14 \end{pmatrix}$ and $B\begin{pmatrix} a \\ -5 \end{pmatrix}$ is 9 units.

SOLUTION

$$\mathbf{P} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 7 \\ k \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} 9 \\ -2 \end{pmatrix} \quad (1)$$

Collinearity via rank Three points P, Q, R are collinear iff

$$\text{rank}(\mathbf{Q} - \mathbf{P} \quad \mathbf{R} - \mathbf{P}) = 1. \quad (2)$$

Compute the direction columns:

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} 7-5 \\ k-4 \end{pmatrix} = \begin{pmatrix} 2 \\ k-4 \end{pmatrix}, \quad \mathbf{R} - \mathbf{P} = \begin{pmatrix} 9-5 \\ -2-4 \end{pmatrix} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}. \quad (3)$$

Hence the collinearity matrix is

$$M = \begin{pmatrix} 2 & 4 \\ k-4 & -6 \end{pmatrix}. \quad (4)$$

Row reduction (rank = 1)

$$\begin{pmatrix} 2 & 4 \\ k-4 & -6 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{1}{2}R_1} \begin{pmatrix} 1 & 2 \\ k-4 & -6 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - (k-4)R_1} \begin{pmatrix} 1 & 2 \\ 0 & 2(1-k) \end{pmatrix}. \quad (5)$$

For $\text{rank}(M) = 1$, the second row must be the zero row:

$$2(1-k) = 0 \Rightarrow k = 1. \quad (6)$$

Conclusion For $k = \boxed{1}$, the three points $P(5, 4)$, $Q(7, k)$, $R(9, -2)$ are collinear.

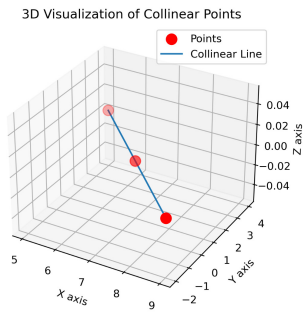


Fig. 1