2.3.11

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Question

Find the acute angle between the planes

$$x - 2y - 2z = 5$$
$$3x - 6y + 2z = 7$$

Solution

The angle between two planes is the angle between their normals. Let

$$\mathbf{n}_1 = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}, \quad \mathbf{n}_2 = \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix}.$$

The dot product is

$$\begin{aligned} \mathbf{n}_1^\top \mathbf{n}_2 &= 1 \cdot 3 + (-2)(-6) + (-2)(2) && (1) \\ &= 3 + 12 - 4 && (2) \\ &= 11. && (3) \end{aligned}$$

The norms are

$$\|\mathbf{n}_1\| = \sqrt{1^2 + (-2)^2 + (-2)^2} = \sqrt{9} = 3, \quad (4)$$
$$\|\mathbf{n}_2\| = \sqrt{3^2 + (-6)^2 + 2^2} = \sqrt{49} = 7. \quad (5)$$

Hence,

$$\cos\theta = \frac{\mathbf{n}_1^\top \mathbf{n}_2}{\|\mathbf{n}_1\|\|\mathbf{n}_2\|} \tag{6}$$

$$= \frac{11}{3 \cdot 7} \tag{7}$$

$$= \frac{11}{21}. \tag{8}$$

Therefore, the acute angle between the planes is

$$\boxed{\theta = \arccos\left(\tfrac{11}{21}\right) \approx 58.41^\circ}$$

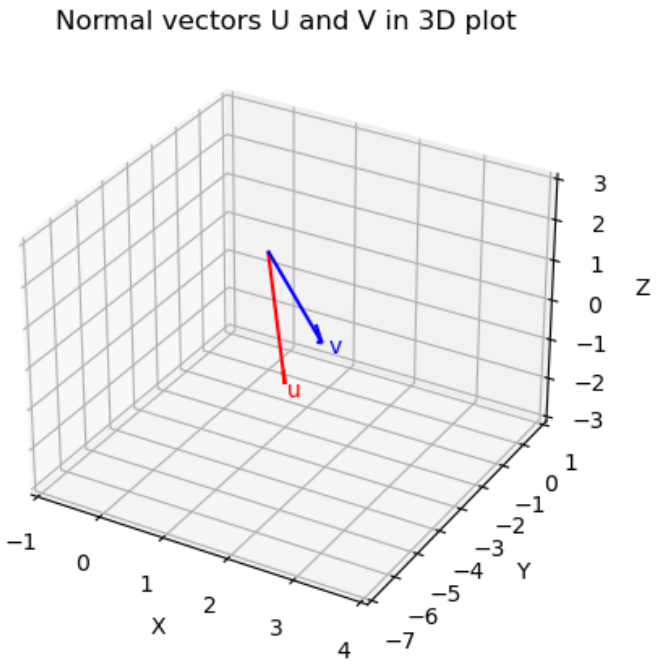

Figure 1: Normal vectors **U** and **V**