2.10.32

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Question

Let **p** and **q** be the position vectors of **P** and **Q** respectively, with respect to **O** and $|\mathbf{p}| = p, |\mathbf{q}| = q$.
The points **R** and **S** divide PQ internally and externally in the ratio $2 : 3$ respectively. If OR and OS are perpendicular, then

- $9p^2 = 4q^2$
- $4p^2 = 9q^2$
- $9p = 4q$
- $4p = 9q$

Theoretical Solution

Since R divides PQ internally in the ratio $2:3$,

$$\mathbf{R} = \frac{3\mathbf{p} + 2\mathbf{q}}{5}.$$

Since S divides PQ externally in the ratio $2:3$,

$$\mathbf{S} = 3\mathbf{p} - 2\mathbf{q}.$$

Theoretical Solution

Given $OR \perp OS$, we have

$$\mathbf{R}^T\mathbf{S} = 0.$$

Substitute $\mathbf{R}$ and $\mathbf{S}$:

$$\left(\frac{3\mathbf{p} + 2\mathbf{q}}{5}\right)^T (3\mathbf{p} - 2\mathbf{q}) = 0.$$

Theoretical Solution

Multiply through by 5:

$$(3\mathbf{p} + 2\mathbf{q})^T(3\mathbf{p} - 2\mathbf{q}) = 0.$$

Expanding:

$$9\mathbf{p}^T\mathbf{p} - 6\mathbf{p}^T\mathbf{q} + 6\mathbf{q}^T\mathbf{p} - 4\mathbf{q}^T\mathbf{q} = 0.$$

$$9\mathbf{p}^T\mathbf{p} - 4\mathbf{q}^T\mathbf{q} = 0.$$

Theoretical Solution

That is,

$$9\|\mathbf{p}\|^2 - 4\|\mathbf{q}\|^2 = 0 \quad \Longrightarrow \quad 9p^2 = 4q^2.$$

Answer: **(a)** $9p^2 = 4q^2$

Vectors OR and OS with $OR \perp OS$