# 4.13.93

AI25BTECH11024 - Pratyush Panda

**Question:**

Let $\mathbf{P}$ be the image of the point $(3, 1, 7)$ with respect to the plane $x - y + z = 3$. Then the equation of the plane passing through $\mathbf{P}$ and containing the straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ is

1) $x + y - 3z = 0$
2) $x - 4y + 7z = 0$
3) $x + 3z = 0$
4) $2x - y = 0$

**Solution:**

Let the vector $\mathbf{A} = \begin{pmatrix} 3 \\ 1 \\ 7 \end{pmatrix}$.

The given plane can be written as;

$$\mathbf{n_1}^T\mathbf{X} = 3 \qquad \textit{where}, \ \mathbf{n_1} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \tag{4.1}$$

And the line has the direction vector $\mathbf{b}$=$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

Now, the image of point $\mathbf{A}$ with respect to the plane can be found out by the formula;

$$\mathbf{P} = \mathbf{A} - \frac{2\left(\mathbf{n}^T\mathbf{A} - 3\right)}{||\mathbf{n}||}\mathbf{n} \tag{4.2}$$

From putting the values in the formula, we get the point $\mathbf{P}$ to be $\begin{pmatrix} -1 \\ 5 \\ 3 \end{pmatrix}$

Let the equation of the required plane be $\mathbf{X}^T\mathbf{n} = 0$ since the plane contains the line and the line passes through the origin.

From the given constraints we can get the following equations;

$$\mathbf{P}^T\mathbf{n} = 0 \qquad\qquad \mathbf{b}^T\mathbf{n} = 0 \tag{4.3}$$

If we combine the two equations we get;

$$\begin{pmatrix} P & b \end{pmatrix}^T \mathbf{n} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{4.4}$$

On solving this equation we get $\mathbf{n} = n.\begin{pmatrix} 1 \\ -4 \\ 7 \end{pmatrix}$. The parameter can be taken as 1.

Thus, the equation of the required plane is $\begin{pmatrix} 1 \\ -4 \\ 7 \end{pmatrix}^T \mathbf{X} = 0$ or $x - 4y + 7z = 0$.