

4.12.2

EE25BTECH11065 - Yoshita J

Question:

For which value of k will the following pair of linear equations have no solution?

$$\begin{aligned} 3x + y &= 1 \\ (2k - 1)x + (k - 1)y &= 2k + 1 \end{aligned}$$

Solution:

The given system of equations is

Line	Vector Form
l_1	$\begin{pmatrix} 3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1$
l_2	$\begin{pmatrix} 2k - 1 & k - 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 2k + 1$

TABLE 0: Answers

In matrix form:

$$\begin{pmatrix} 3 & 1 \\ 2k - 1 & k - 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2k + 1 \end{pmatrix} \quad (1)$$

Now, form the augmented matrix:

$$\left(\begin{array}{cc|c} 3 & 1 & 1 \\ 2k - 1 & k - 1 & 2k + 1 \end{array} \right) \quad (2)$$

Perform row reduction:

$$R_1 \rightarrow \frac{1}{3}R_1 \Rightarrow \left(\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{1}{3} \\ 2k-1 & k-1 & 2k+1 \end{array} \right) \quad (3)$$

$$R_2 \rightarrow R_2 - (2k-1)R_1 \Rightarrow \left(\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{k-2}{3} & \frac{4k+4}{3} \end{array} \right) \quad (4)$$

$$R_2 \rightarrow \frac{3}{k-2}R_2 \Rightarrow \left(\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & \frac{4k+4}{k-2} \end{array} \right) \quad (5)$$

$$R_1 \rightarrow R_1 - \frac{1}{3}R_2 \Rightarrow \left(\begin{array}{cc|c} 1 & 0 & \frac{1}{3} - \frac{1}{3} \cdot \frac{4k+4}{k-2} \\ 0 & 1 & \frac{4k+4}{k-2} \end{array} \right) \quad (6)$$

For inconsistency, we need:

$$k-2=0 \quad \text{and} \quad 4k+4 \neq 0 \quad (7)$$

So,

$$k=2, \quad 4(2)+4=12 \neq 0 \quad (8)$$

$$\boxed{k=2}$$

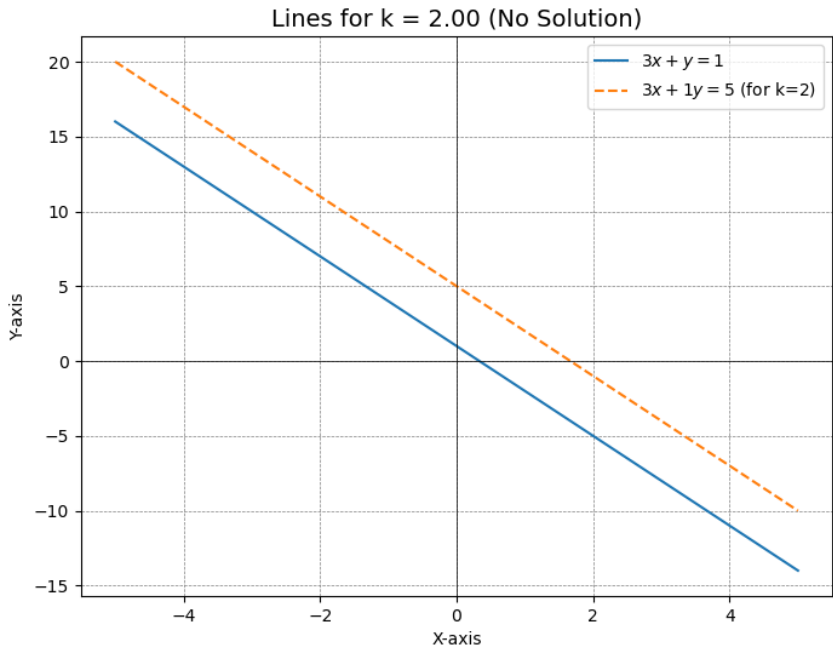


Fig. 0: A plane passing through point A with normal vector n .