4.3.46

ee25btech11056 - Suraj.N

Question : Find the coordinates of the point where the line through $(3,-4,-5)$ and $(2,-3,1)$ crosses the plane $2x+y+z=7$.

Solution :

Description	Value
Line	$\mathbf{x}=\begin{pmatrix}2\\-3\\1\end{pmatrix}+k\begin{pmatrix}-1\\1\\6\end{pmatrix}$
Plane	$\mathbf{n}^\top\mathbf{x}=7 \quad$ where $\quad \mathbf{n}=\begin{pmatrix}2\\1\\1\end{pmatrix}$

Table : Line and Plane

Let the point of intersection be $\mathbf{P}$.
The line is written as

$$\mathbf{x}=\mathbf{h}+k\,\mathbf{m} \tag{1}$$

$$\mathbf{h}=\begin{pmatrix}2\\-3\\1\end{pmatrix} \quad \mathbf{m}=\begin{pmatrix}-1\\1\\6\end{pmatrix} \tag{2}$$

So,

$$\mathbf{P}=\mathbf{h}+k\,\mathbf{m} \tag{3}$$

The point also lies on the plane , so

$$\mathbf{n}^\top\mathbf{P}=c \tag{4}$$

$$\mathbf{n}^\top(\mathbf{h}+k\mathbf{m})=c \tag{5}$$

$$\mathbf{n}^\top\mathbf{h}+k\mathbf{n}^\top\mathbf{m}=c \tag{6}$$

$$k=\frac{c-\mathbf{n}^\top\mathbf{h}}{\mathbf{n}^\top\mathbf{m}} \tag{7}$$

here c = 7 , by substituting the vectors for $\mathbf{n},\mathbf{h}$ and $\mathbf{m}$

$$k=\frac{7-\begin{pmatrix}2&1&1\end{pmatrix}\begin{pmatrix}2\\-3\\1\end{pmatrix}}{\begin{pmatrix}2&1&1\end{pmatrix}\begin{pmatrix}-1\\1\\6\end{pmatrix}} \tag{8}$$

$$k=\frac{7-2}{5}=1 \tag{9}$$

Substitute $k = 1$ in the line equation to obtain the point of intersection $\mathbf{P}$

$$\mathbf{P} = \mathbf{h} + \mathbf{m} \tag{10}$$

$$\mathbf{P} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} = \begin{pmatrix} 2-1 \\ -3+1 \\ 1+6 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix}. \tag{11}$$

Answer:

$$\mathbf{P} = \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix} \tag{12}$$

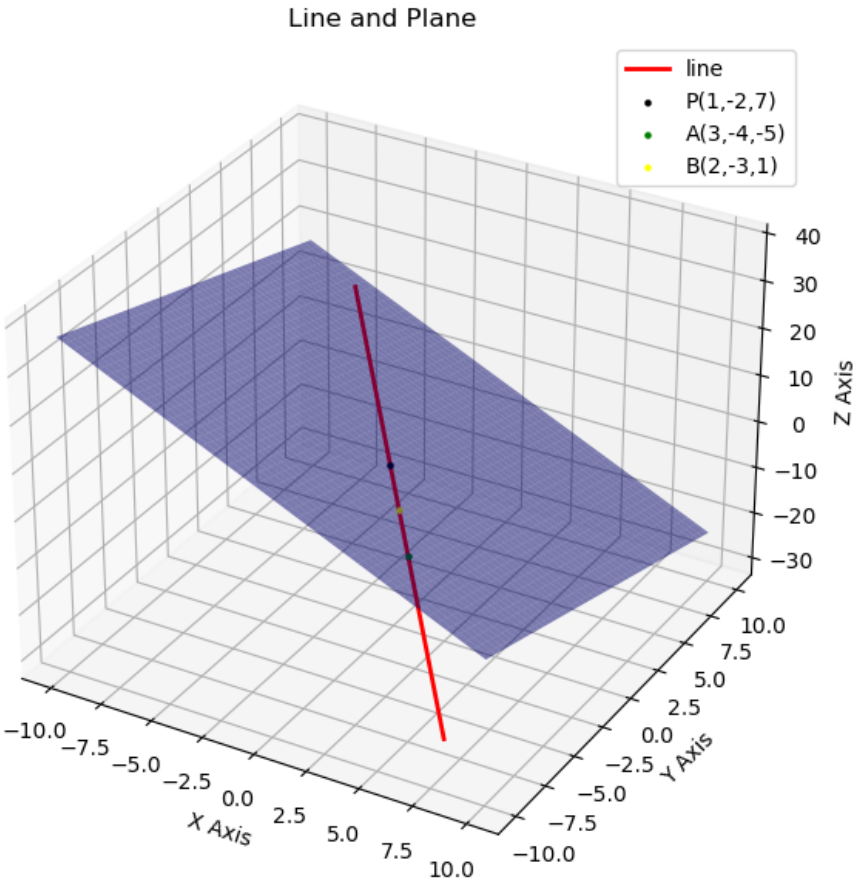

Fig : Line and Plane