2.10.34

EE25BTECH11013 - Bhargav

September 20, 2025

Question

Let $\mathbf{a} = \hat{i} - \hat{j},\ \mathbf{b} = \hat{j} - \hat{k},\ \mathbf{c} = \hat{k} - \hat{i}$. If $\mathbf{d}$ is a unit vector such that

$$\mathbf{a}^\top \mathbf{d} = 0 \quad \text{and} \quad [\mathbf{b}\ \mathbf{c}\ \mathbf{d}] = 0,$$

then $\mathbf{d}$ equals:

$$\text{a)}\ \pm \frac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}} \quad \text{b)}\ \pm \frac{\hat{i} + \hat{k} - \hat{j}}{\sqrt{3}} \quad \text{c)}\ \pm \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} \quad \text{d)}\ \pm \hat{k}$$

Vectors

$$\mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

Let

$$\mathbf{d} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Condition 1: Orthogonality

$$\mathbf{a}^\top \mathbf{d} = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x - y = 0$$

$$\Rightarrow x = y$$

Condition 2: Scalar Triple Product

$$[\mathbf{b}\ \mathbf{c}\ \mathbf{d}] = \det\begin{pmatrix} 0 & -1 & x \\ 1 & 0 & y \\ -1 & 1 & z \end{pmatrix} = 0$$

Expanding:

$$= (y + z) + x$$

$$\Rightarrow x + y + z = 0$$

Since $x = y$,

$$2x + z = 0 \quad \Rightarrow \quad z = -2x$$

Unit Vector Condition

$$\mathbf{d} = \begin{pmatrix} x \\ x \\ -2x \end{pmatrix}$$

$$\mathbf{d}^\top \mathbf{d} = x^2 + x^2 + (-2x)^2 = 6x^2 = 1$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{6}}$$

Thus,

$$\mathbf{d} = \pm \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

Final Answer

$$\mathbf{d} = \pm\frac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$$

Correct option: **a**