Matgeo-4.8.35

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Question

Q.4.8.35 Find the coordinates of the foot of the perpendicular drawn from the point $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ on the x-axis

Solution. Let the x-axis be represented as the intersection of the two planes

$$\mathbf{e}_2^T\mathbf{x} = 0, \quad \mathbf{e}_3^T\mathbf{x} = 0,$$

where

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \ \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \ \mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

The direction vector of the x-axis is $\mathbf{m} = \mathbf{e}_1$ and the given point is

$$\mathbf{P} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}.$$

By the foot-of-perpendicular relation,

$$\begin{pmatrix} \mathbf{m} & \mathbf{e}_2 & \mathbf{e}_3 \end{pmatrix}^T \mathbf{Q} = \begin{pmatrix} \mathbf{m}^T \mathbf{P} \\ 0 \\ 0 \end{pmatrix}. \tag{1}$$

Solution

Substituting
$$\mathbf{m} = \mathbf{e}_1$$
 and $\mathbf{P} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$,

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{Q} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \implies \mathbf{Q} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}. \tag{2}$$

Thus, the foot of the perpendicular from $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ to the x-axis is

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Plot

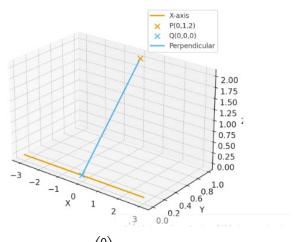


Figure: Perpendicular from
$$\mathbf{P} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$$
 to the x-axis with foot $\mathbf{Q} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.