4.10.22

EE25BTECH11020 - Darsh Pankaj Gajare

September 29, 2025

Question:
Find the equation of the plane through the intersection of the planes $\mathbf{r} \cdot \left(\hat{i} + 3\hat{j}\right) - 6 = 0$ and $\mathbf{r} \cdot \left(3\hat{i} - \hat{j} - 4\hat{k}\right) = 0$,whose perpendicular distance from origin is unity.

Solution:

Table

$\mathbf{n_1}$	$\begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$
$\mathbf{n_1}$	$\begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}$

Solution: The given planes are

$$\mathbf{x}^\top \mathbf{n}_1 - 6 = 0 \quad (0.1)$$

$$\mathbf{x}^\top \mathbf{n}_2 = 0 \quad (0.2)$$

Let the required plane be

$$\mathbf{x}^\top(\mathbf{n}_1 + \lambda\mathbf{n}_2) - 6 = 0 \tag{0.3}$$

the normal vector is

$$\mathbf{n} = \mathbf{n}_1 + \lambda\mathbf{n}_2 \tag{0.4}$$

$$\|\mathbf{n}\|^2 = \mathbf{n}^\top\mathbf{n} = \mathbf{n}_1^\top\mathbf{n}_1 + 2\lambda\mathbf{n}_1^\top\mathbf{n}_2 + \lambda^2\mathbf{n}_2^\top\mathbf{n}_2 \tag{0.5}$$

Perpendicular distance from origin is

$$\frac{|-6|}{\|\mathbf{n}\|} = 1 \tag{0.6}$$

$$\|\mathbf{n}\| = 6 \tag{0.7}$$

Hence,

$$\mathbf{n}_1^\top\mathbf{n}_1 + 2\lambda\mathbf{n}_1^\top\mathbf{n}_2 + \lambda^2\mathbf{n}_2^\top\mathbf{n}_2 = 36 \tag{0.8}$$

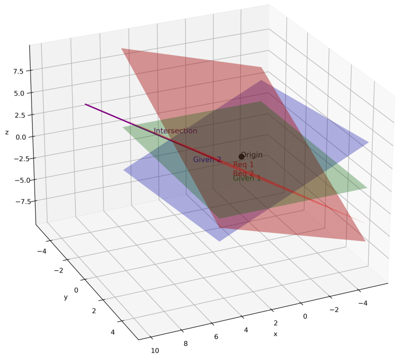
$$10 + 26\lambda^2 = 36 \tag{0.9}$$

$$\lambda = \pm 1 \tag{0.10}$$

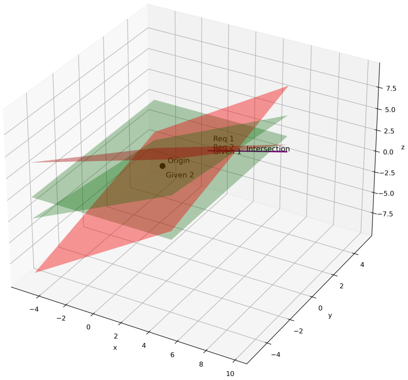
Thus, the required planes are

$$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}^\top \mathbf{x} = 3 \tag{0.11}$$

$$\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}^\top \mathbf{x} = -3 \tag{0.12}$$



Plot using C libraries



Plot using Python