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INDHIRESH S- EE25BTECH11027

Question. A plane contains the following three points: $\mathbf{P}(2, 1, 5), \mathbf{Q}(-1, 3, 4)$ and $\mathbf{R}(3, 0, 6)$. The vector perpendicular to the above plane can be represented as
Solution:
Let us solve the given equation theoretically and then verify the solution computationally.
Given points are:

$$\mathbf{P} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \ and \ \mathbf{R} = \begin{pmatrix} 3 \\ 0 \\ 6 \end{pmatrix} \tag{1}$$

Now finding two vectors in the plane:

$$\mathbf{PQ} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix} \tag{2}$$

$$\mathbf{PR} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \tag{3}$$

Now the required vector be $\mathbf{n}$

$$\mathbf{n} = \mathbf{PQ} \times \mathbf{PR} \tag{4}$$

Let

$$\mathbf{A} = \mathbf{PQ} \ and \ \mathbf{B} = \mathbf{PR} \tag{5}$$

Then

$$\mathbf{n} = \mathbf{A} \times \mathbf{B} \tag{6}$$

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} \left|\mathbf{A_{23}B_{23}}\right| \\ \\ \left|\mathbf{A_{31}B_{31}}\right| \\ \\ \left|\mathbf{A_{12}B_{12}}\right| \end{pmatrix} \tag{7}$$

$$\left|\mathbf{A_{23}B_{23}}\right| = \begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix} = 1 \tag{8}$$

$$\left|\mathbf{A_{31}B_{31}}\right| = -\begin{vmatrix} -3 & -1 \\ 1 & 1 \end{vmatrix} = 2 \tag{9}$$

$$\left|\mathbf{A_{12}B_{12}}\right| = \begin{vmatrix} -3 & 2 \\ 1 & -1 \end{vmatrix} = 1 \tag{10}$$

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \tag{11}$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \tag{12}$$

From the figure it is clearly verified that the theoretical solution matches with the computational solution.