2.6.37

RATHLAVATH JEEVAN -AI25BTECH11026

September 9, 2025

Question

The vector from origin to the points A and B are

$$\mathbf{a} = 2\hat{i} - 3\hat{j} + 2\hat{k} \quad \text{and} \quad \mathbf{b} = 2\hat{i} + 3\hat{j} + \hat{k}, \tag{1}$$

respectively, then the area of $\triangle OAB$ is _____.

Theoretical Solution

Solution:

Given

The area of the triangle OAB is given by

$$\text{Area}(OAB) = \frac{1}{2}\|\mathbf{a} \times \mathbf{b}\|. \tag{2}$$

We have

$$\mathbf{a} = (2, -3, 2), \quad \mathbf{b} = (2, 3, 1). \tag{3}$$

Using the cross product definition,

Theoretical Solution

Solution:

Using the cross product definition,

$$\left(\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_2 & b_2 \\ a_3 & b_3 \end{pmatrix} \quad \begin{pmatrix} a_3 & b_3 \\ a_1 & b_1 \end{pmatrix} \quad \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \right) \tag{4}$$

Theoretical Solution

Solution:

Substituting values:

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} \begin{pmatrix} -3 & 3 \\ 2 & 1 \end{pmatrix} \\ \begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix} \\ \begin{pmatrix} 2 & 2 \\ -3 & 3 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} (-3)(1) - (3)(2) \\ (2)(2) - (1)(2) \\ (2)(3) - (2)(-3) \end{pmatrix}. \tag{5}$$

Theoretical Solution

Solution:

$$\mathbf{a} \times \mathbf{b} = (-9,\ 2,\ 12). \tag{6}$$

Now, its magnitude is

$$\|\mathbf{a} \times \mathbf{b}\| = \sqrt{(-9)^2 + (2)^2 + (12)^2} = \sqrt{81 + 4 + 144} = \sqrt{229}. \tag{7}$$

Therefore, the required area is

$$\text{Area}(OAB) = \frac{1}{2}\|\mathbf{a} \times \mathbf{b}\| = \frac{1}{2}\sqrt{229}. \tag{8}$$

$$\boxed{\text{Area}(OAB) = \frac{\sqrt{229}}{2}} \tag{9}$$

C Code

```c
#include <stdio.h>
#include <math.h>

int main() {
    // Vectors a and b
    double ax = 2, ay = -3, az = 2;
    double bx = 2, by = 3, bz = 1;

    // Cross product a * b
    double cx = ay*bz - az*by;
    double cy = az*bx - ax*bz;
    double cz = ax*by - ay*bx;

    // Magnitude of cross product
    double magnitude = sqrt(cx*cx + cy*cy + cz*cz);
```

C Code

```c
// Area of triangle OAB
    double area = 0.5 * magnitude;

    printf("The area of triangle OAB is: %.2f\n", area);

    return 0;
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d.art3d import Poly3DCollection

# Vectors
a = np.array([2, -3, 2])
b = np.array([2, 3, 1])

# Cross product and area
cross = np.cross(a, b)
area = 0.5 * np.linalg.norm(cross)
print("Area of triangle OAB:", area)

# Points
origin = np.array([0, 0, 0])
A = a
B = b
```

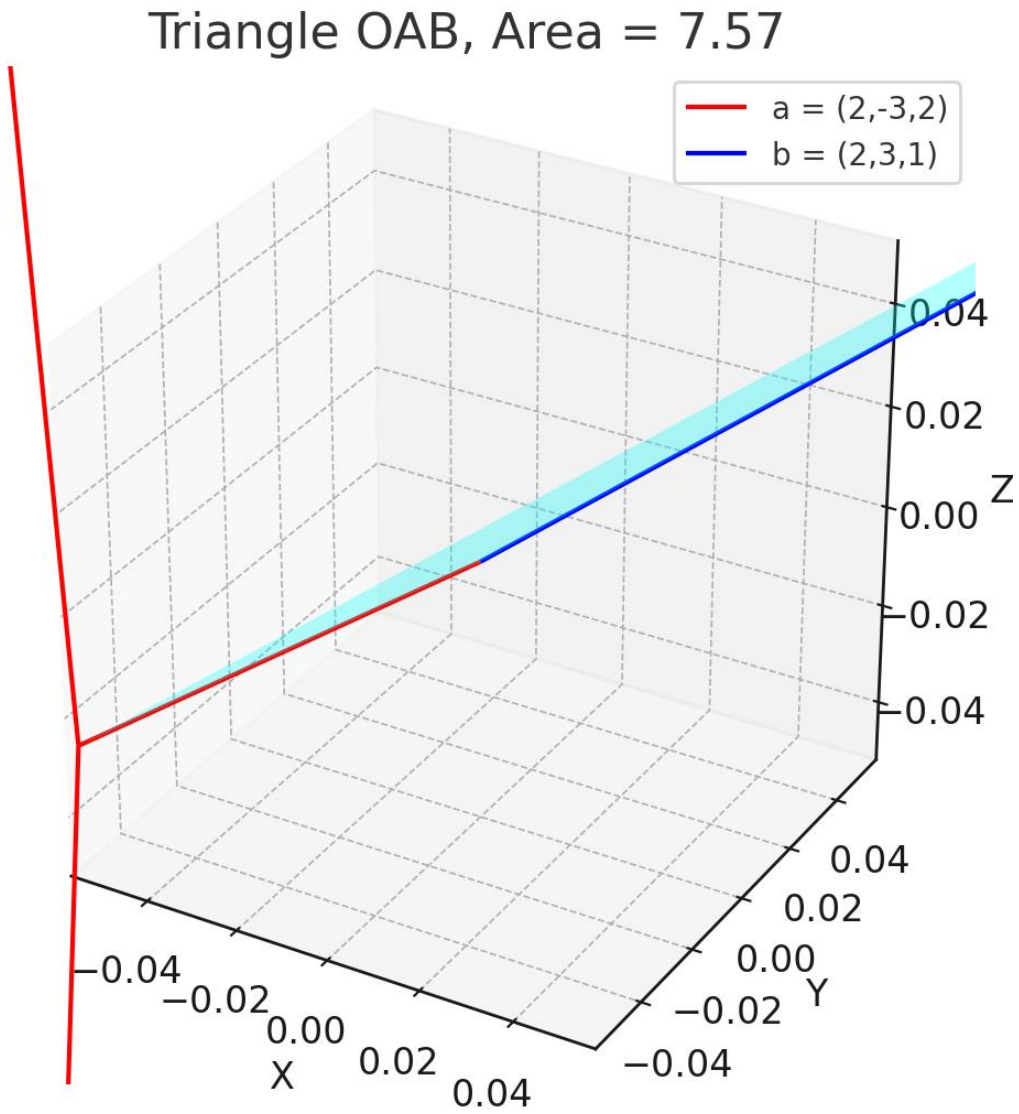
Python Code

```
    # Create 3D plot
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
# Plot vectors a and b
ax.quiver(0, 0, 0, a[0], a[1], a[2], color='r', label='a =
    (2,-3,2)')
ax.quiver(0, 0, 0, b[0], b[1], b[2], color='b', label='b =
    (2,3,1)')

# Draw triangle OAB
verts = [[origin, A, B]]
ax.add_collection3d(Poly3DCollection(verts, alpha=0.3, facecolor=
    'cyan'))

# Labels and legend
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
```

Python Code

```
    ax.legend()
# Set equal aspect ratio
ax.set_box_aspect([1,1,1])

# Save figure as image
plt.savefig("triangle_OAB.png", dpi=300)
plt.show()
```

Plot


Triangle OAB, Area = 7.57
a = (2,-3,2)
b = (2,3,1)
0.04
0.02
0.00
−0.02
−0.04
Z
−0.04
−0.02
0.00
0.02
0.04
X
0.04
0.02
0.00
−0.02
−0.04
Y