

4.7.54

EE25BTECH11057 - Rushil Shanmukha Srinivas

Problem: Find the vector equations of the line passing through the point (1,2,-4) and perpendicular to the two lines

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$

Solution: Given the line passes through the point

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} \quad (0.1)$$

The line is also perpendicular to

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}, \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}. \quad (0.2)$$

The direction vector of the line is given by

$$\begin{pmatrix} 3 & -16 & 7 \\ 3 & 8 & -5 \end{pmatrix} \mathbf{m} = 0 \xrightarrow{R_2 \rightarrow R_2 - R_1} \begin{pmatrix} 3 & -16 & 7 \\ 0 & 24 & -12 \end{pmatrix} \quad (0.3)$$

$$\begin{pmatrix} 3 & -16 & 7 \\ 0 & 24 & -12 \end{pmatrix} \xrightarrow{R_1 \rightarrow 3R_1 + 2R_2} \begin{pmatrix} 9 & 0 & -3 \\ 0 & 24 & -12 \end{pmatrix} \quad (0.4)$$

$$\begin{pmatrix} 9 & 0 & -3 \\ 0 & 24 & -12 \end{pmatrix} \xrightarrow{R_1 \rightarrow \frac{R_1}{9}} \begin{pmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 24 & -12 \end{pmatrix} \quad (0.5)$$

$$\begin{pmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 24 & -12 \end{pmatrix} \xrightarrow{R_2 \rightarrow \frac{R_2}{24}} \begin{pmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & -\frac{1}{2} \end{pmatrix} \quad (0.6)$$

So

$$\mathbf{m} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} = \text{Direction vector of line} \quad (0.7)$$

The vector equation of the line is

$$\mathbf{L}_1 = \mathbf{A} + k\mathbf{m} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} + k \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \quad (0.8)$$

Lines in 3D

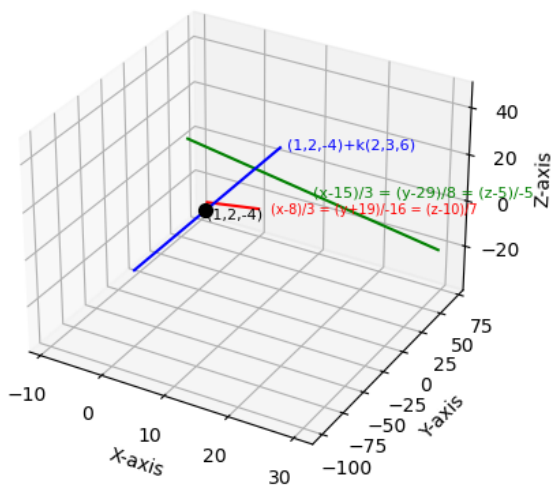


Fig: Representation of Lines and Point