Area of Triangle

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Problem Statement

Show that the area of the triangle formed by the lines $y = m_1 x + c_1$, $y = m_2 x + c_2$ and $x = 0$ is

$$\frac{(c_1 - c_2)^2}{2|m_1 - m_2|}.$$

Solution

Vertices:

$$\mathbf{A} = \begin{pmatrix} 0 \\ c_1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 \\ c_2 \end{pmatrix}.$$

Intersection of the two lines (RREF):

$$\left(\begin{array}{cc|c} m_1 & -1 & -c_1 \\ m_2 & -1 & -c_2 \end{array}\right) \xrightarrow{R_2 \leftarrow R_2 - R_1} \left(\begin{array}{cc|c} m_1 & -1 & -c_1 \\ m_2 - m_1 & 0 & -(c_2 - c_1) \end{array}\right) \tag{1}$$

$$\Rightarrow (m_2 - m_1)x = -(c_2 - c_1)$$

$$\Rightarrow x^* = \frac{c_2 - c_1}{m_1 - m_2}, \quad y^* = m_1 x^* + c_1. \tag{2}$$

Solution (cont..)

Vectors:

$$\mathbf{u} = \mathbf{AB} = \begin{pmatrix} 0 \\ c_2 - c_1 \end{pmatrix}, \quad \mathbf{v} = \mathbf{AC} = \begin{pmatrix} x^* \\ y^* - c_1 \end{pmatrix}.$$

Observe that $y^* - c_1 = m_1 x^*$, hence

$$\mathbf{v} = x^* \begin{pmatrix} 1 \\ m_1 \end{pmatrix}.$$

Compute norms and dot product:

$$\|\mathbf{u}\|^2 = (c_2 - c_1)^2, \tag{3}$$

$$\|\mathbf{v}\|^2 = x^{*2}(1 + m_1^2), \tag{4}$$

$$\mathbf{u} \cdot \mathbf{v} = (c_2 - c_1)(m_1 x^*). \tag{5}$$

Solution (cont..)

Using $\|\mathbf{u} \times \mathbf{v}\|^2 = \|\mathbf{u}\|^2\|\mathbf{v}\|^2 - (\mathbf{u} \cdot \mathbf{v})^2$:

$$\begin{aligned}
\|\mathbf{u} \times \mathbf{v}\|^2 &= (c_2 - c_1)^2 x^{*2}(1 + m_1^2) - (c_2 - c_1)^2 m_1^2 x^{*2} && (6)\\
&= (c_2 - c_1)^2 x^{*2}. && (7)
\end{aligned}$$

Thus,

$$\begin{aligned}
\|\mathbf{u} \times \mathbf{v}\| = |c_2 - c_1|\,|x^*| &= |c_2 - c_1| \left| \frac{c_2 - c_1}{m_1 - m_2} \right| && (8)\\
&= \frac{(c_2 - c_1)^2}{|m_1 - m_2|}. && (9)
\end{aligned}$$

Solution (cont..)

Area:

$$\text{Area} = \tfrac{1}{2}\|\mathbf{u} \times \mathbf{v}\| = \frac{(c_1 - c_2)^2}{2|m_1 - m_2|}. \tag{10}$$

$$\boxed{\frac{(c_1 - c_2)^2}{2|m_1 - m_2|}}$$

Python Code (Plotting Line and Vectors)

```
import numpy as np
import matplotlib.pyplot as plt

m1, c1 = 1.5, 2
m2, c2 = -0.5, 5
A = np.array([0, c1])
B = np.array([0, c2])
x_intersect = (c2 - c1) / (m1 - m2)
y_intersect = m1 * x_intersect + c1
C = np.array([x_intersect, y_intersect])
triangle = np.array([A, B, C, A])
```

Python Code (cont..)

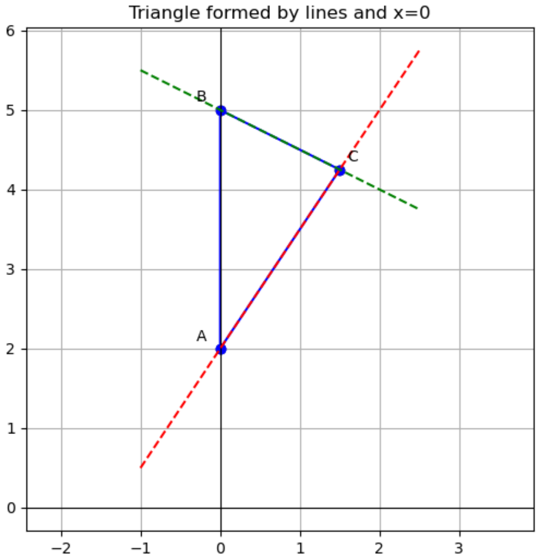
```
plt.figure(figsize=(6,6))
plt.plot(triangle[:,0], triangle[:,1], 'b-o', label='Triangle')
plt.axvline(0, color='k', linewidth=0.8)
plt.axhline(0, color='k', linewidth=0.8)
x_vals = np.linspace(min(0, C[0])-1, max(C[0]+1, 2), 100)
plt.plot(x_vals, m1*x_vals + c1, 'r--', label=f'y={m1}x+{c1}')
plt.plot(x_vals, m2*x_vals + c2, 'g--', label=f'y={m2}x+{c2}')
```

Python Code (cont..)

```
plt.scatter([A[0], B[0], C[0]], [A[1], B[1], C[1]], color='black')
plt.text(A[0]-0.3, A[1]+0.1, 'A')
plt.text(B[0]-0.3, B[1]+0.1, 'B')
plt.text(C[0]+0.1, C[1]+0.1, 'C')
plt.grid(True)
plt.axis('equal')
plt.title('Triangle formed by lines and x=0')

plt.show()
```

Plot



C Code (Computations)

```c
#include <math.h>

double dot(double* x, double* y, int l) {
    double ans = 0;
    for (int i=0; i<l; i++) {
        ans += x[i]*y[i];
    }
    return ans;
}
```

C Code (Cont..)

```
double triangle_area(double m1, double c1, double m2, double
    c2) {
    double A[2] = {0, c1};
    double B[2] = {0, c2};

    double x = (c2 - c1) / (m1 - m2);
    double y = m1 * x + c1;
    double C[2] = {x, y};

    double u[2] = {B[0] - A[0], B[1] - A[1]};
    double v[2] = {C[0] - A[0], C[1] - A[1]};

    double cross = pow(dot(u, u, 2)*dot(v, v, 2) - pow(dot(u, v
        , 2), 2), 0.5);

    return 0.5 * fabs(cross);
}
```

Python Code (Calling C)

```
import ctypes

lib = ctypes.CDLL("./area.so")

lib.triangle_area.argtypes = [ctypes.c_double, ctypes.c_double,
    ctypes.c_double, ctypes.c_double]
lib.triangle_area.restype = ctypes.c_double

m1 = float(input("Enter m1: "))
c1 = float(input("Enter c1: "))
m2 = float(input("Enter m2: "))
c2 = float(input("Enter c2: "))

area = lib.triangle_area(m1, c1, m2, c2)
print(f"Triangle area = {area}")
```