12.475

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Question

Consider a triangle PQR with initial coordinates of the vertices as P $\begin{pmatrix} 1 & 3 \end{pmatrix}^\top$, Q $\begin{pmatrix} 4 & 5 \end{pmatrix}^\top$ and R $\begin{pmatrix} 5 & 3.5 \end{pmatrix}^\top$. The triangle is rotated in the X-Y plane about the vertex P by angle θ in clockwise direction. If $\sin\theta = 0.6$ and $\cos\theta = 0.8$, the new coordinates of the vertex Q are

1. $\begin{pmatrix} 4.6 & 2.8 \end{pmatrix}^\top$
2. $\begin{pmatrix} 3.2 & 4.6 \end{pmatrix}^\top$
3. $\begin{pmatrix} 7.9 & 5.5 \end{pmatrix}^\top$
4. $\begin{pmatrix} 5.5 & 7.9 \end{pmatrix}^\top$

Theoretical Solution

Let the coordinates of the vertices be represented by vectors $\mathbf{p} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$. The rotation of a point $\mathbf{q}$ about a pivot point $\mathbf{p}$ is given by:

$$\mathbf{q}_{\text{new}} = \mathbf{R}\left(\mathbf{q} - \mathbf{p}\right) + \mathbf{p} \tag{1}$$

The matrix for a clockwise rotation by an angle θ is:

$$\mathbf{R} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{2}$$

Theoretical Solution

Substituting the given values and vectors into (1)

$$\mathbf{q}_{\text{new}} = \begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix} \left(\begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right) + \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{3}$$

$$= \begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{4}$$

$$= \begin{pmatrix} 0.8\,(3) + 0.6\,(2) \\ -0.6\,(3) + 0.8\,(2) \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{5}$$

$$= \begin{pmatrix} 2.4 + 1.2 \\ -1.8 + 1.6 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} 3.6 \\ -0.2 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4.6 \\ 2.8 \end{pmatrix} \tag{7}$$

Conclusion

The new coordinates of the vertex Q are $\begin{pmatrix} 4.6 \\ 2.8 \end{pmatrix}$.

The correct option is **1)**.

Plot

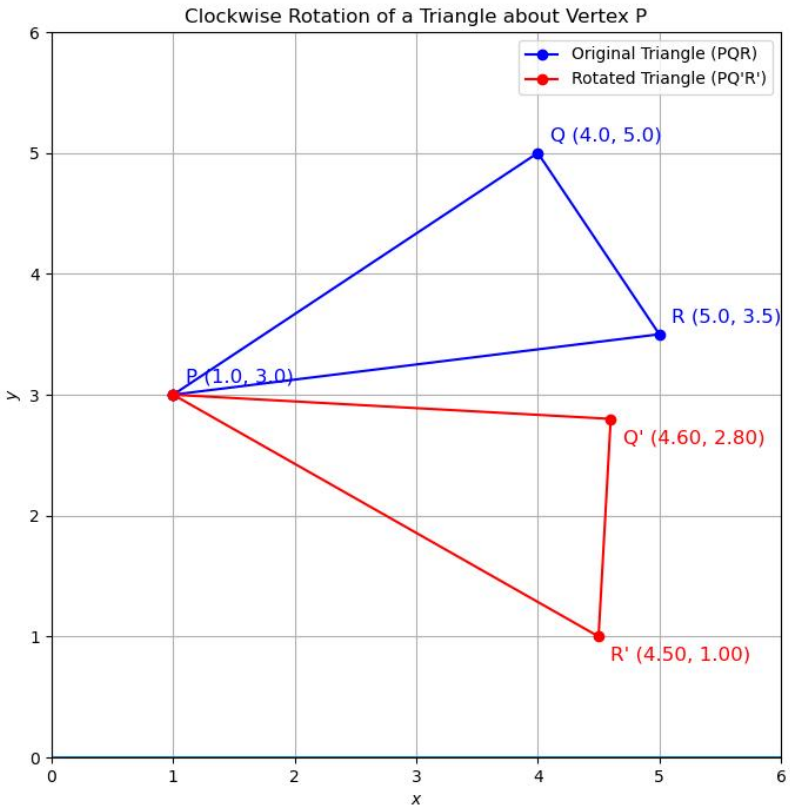

Figure: Plot