8.2.33

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Question

Find the equation of the conic with length of major axis 26, foci $(\pm 5, 0)$.

Theoretical Solution

The given foci are $\mathbf{F}_1 = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$ and $\mathbf{F}_2 = \begin{pmatrix} -5 \\ 0 \end{pmatrix}$.

The center of the conic is the midpoint of the foci:

$$\mathbf{u} = \frac{\mathbf{F}_1 + \mathbf{F}_2}{2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1}$$

The length of the major axis is given as $2a = 26$ So, The distance from the center to a focus is $c = 5$.

Eccentricity:

$$e = \frac{c}{a} = \frac{5}{13} \tag{2}$$

Theoretical Solution

The general equation of a conic is given by :

$$g(\mathbf{x}) = \mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{3}$$

where, x is a vertex on the major axis,
Since the center $\mathbf{u} = \mathbf{0}$, the equation simplifies to

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + f = 0. \tag{4}$$

where,

$$\mathbf{V} = ||\mathbf{n}||^2\mathbf{I} - e^2\mathbf{n}\mathbf{n}^T \tag{5}$$

This simplifies to:

$$\mathbf{V} = \begin{pmatrix} 1-e^2 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

Theoretical Solution

Substituting,

$$\mathbf{V} = \begin{pmatrix} 1-(5/13)^2 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 144/169 & 0 \\ 0 & 1 \end{pmatrix} \tag{7}$$

Simplifying equation (5) and (4),

$$\begin{pmatrix} 13 & 0 \end{pmatrix} \begin{pmatrix} 144/169 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 13 \\ 0 \end{pmatrix} + f = 0$$

$$144 + f = 0 \Rightarrow f = -144 \tag{8}$$

Final equation of the conic,

$$\mathbf{x}^T \begin{pmatrix} 144/169 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - 144 = 0.$$

C Code

```c
#include <stdio.h>
#include <math.h>

#define PI 3.1415926535

double calculate_circular_sector_area() {
    double radius = 2.0;
    double angle_in_radians = PI / 6.0;
    double area = 0.5 * radius * radius * angle_in_radians;

    return area;
}
```

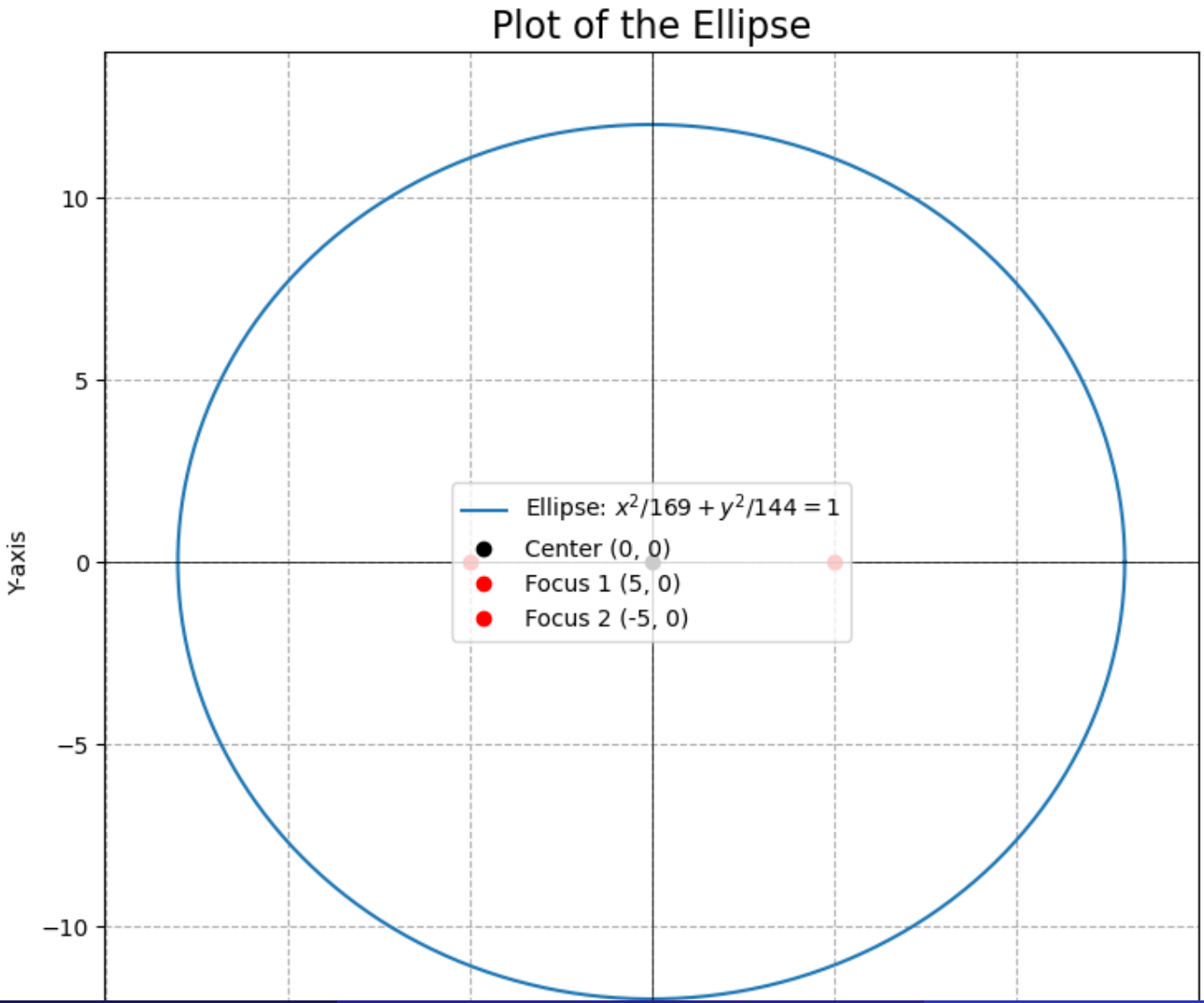
Python Code

```
import numpy as np
import matplotlib.pyplot as plt

a = 13
b = 12
c = 5
theta = np.linspace(0, 2 * np.pi, 200)

x = a * np.cos(theta)
y = b * np.sin(theta)

plt.figure(figsize=(10, 8))
ax = plt.gca()

ax.plot(x, y, label='Ellipse: $x^2/169 + y^2/144 = 1$')

ax.plot(0, 0, 'ko', label='Center (0, 0)')
ax.plot(c, 0, 'ro', label='Focus 1 (5, 0)')
ax.plot(-c, 0, 'ro', label='Focus 2 (-5, 0)')
```

Python Code

```
ax.set_title('Plot of the Ellipse', fontsize=16)
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')

ax.set_aspect('equal', adjustable='box')

ax.grid(True, linestyle='--')
ax.legend()

ax.set_xlim(-a - 2, a + 2)
ax.set_ylim(-b - 2, b + 2)

ax.axhline(0, color='black', linewidth=0.5)
ax.axvline(0, color='black', linewidth=0.5)

plt.show()
```

Plot


Plot of the Ellipse
Ellipse: $x^2/169 + y^2/144 = 1$
Center (0, 0)
Focus 1 (5, 0)
Focus 2 (-5, 0)
Y-axis
10
5
0
−5
−10