Matrices in Geometry - 1.5.25

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Problem Statement

In what ratio does the point $\vec{R} = \begin{pmatrix} \frac{24}{11} \\ y \end{pmatrix}$ divide the line segment joining the points $\vec{P} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ and $\vec{Q} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$? Also find the value of y. ;

Solution

$\vec{P} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $\vec{Q} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$ and a point $\vec{R} = \begin{pmatrix} \frac{24}{11} \\ y \end{pmatrix}$ on PQ.

Let R divide PQ internally in the ratio $k : 1$.

Therefore, they are defined to be collinear if,

$$\text{rank} \begin{pmatrix} \vec{R} - \vec{P} & \vec{Q} - \vec{R} \end{pmatrix} = 1$$

$$\vec{R} - \vec{P} = \begin{pmatrix} \frac{2}{11} \\ y + 2 \end{pmatrix}$$

$$\vec{Q} - \vec{R} = \begin{pmatrix} \frac{9}{11} \\ 7 - y \end{pmatrix}$$

$$\implies \text{rank} \begin{pmatrix} \frac{2}{11} & \frac{9}{11} \\ y + 2 & 7 - y \end{pmatrix} = 1$$

Solution

$$\begin{pmatrix} \frac{2}{11} & \frac{9}{11} \\ y+2 & 7-y \end{pmatrix} \xrightarrow{R_2 \to 9R_2 - 11R_1(7-y)} \begin{pmatrix} \frac{2}{11} & \frac{9}{11} \\ 11y+4 & 0 \end{pmatrix}$$

for the rank of this matrix to be 1, its 2^{nd} row has to be 0

$$11y + 4 = 0$$

$$\therefore y = \frac{-4}{11}$$

We know that k is the ratio in which $\vec{R}$ divides $\vec{P}$ and $\vec{Q}$,

$$\vec{R} = \frac{k\vec{Q} + \vec{P}}{1+k}$$

$$k\left(\vec{R} - \vec{Q}\right) = \vec{P} - \vec{R}$$

Solution

$$\implies k = \frac{\left(\vec{P} - \vec{R}\right)^{\top} \left(\vec{R} - \vec{Q}\right)}{\|\vec{R} - \vec{Q}\|^2}$$

$$\left(\vec{P} - \vec{R}\right)^{\top} = \begin{pmatrix} \frac{-2}{11} & \frac{-18}{11} \end{pmatrix}$$

$$\left(\vec{R} - \vec{Q}\right) = \begin{pmatrix} \frac{-9}{11} \\ \frac{-81}{11} \end{pmatrix}$$

$$\|\vec{R} - \vec{Q}\|^2 = \frac{81}{121} + \frac{6561}{121} = \frac{6642}{121}$$

$$\therefore k = \frac{\begin{pmatrix} \frac{-2}{11} & \frac{-18}{11} \end{pmatrix} \begin{pmatrix} \frac{-9}{11} \\ \frac{-81}{11} \end{pmatrix}}{\frac{6642}{121}}$$

Solution

$$\implies k = \frac{\frac{18}{121} + \frac{1458}{121}}{\frac{6642}{121}}$$

$$\implies k = \frac{1476}{6624} = \frac{2}{9}$$

Final Answer

Hence, the final answer is $\boxed{k = \frac{2}{9}}$ and $\boxed{y = \frac{-4}{11}}$