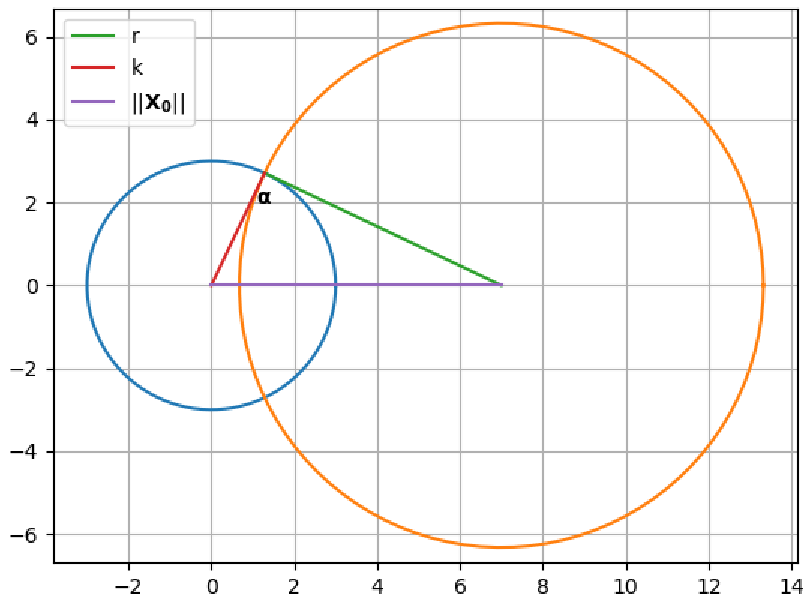
9.8.3

AI25BTECH11001 - ABHISEK MOHAPATRA

Question: If a circle is passing through the point (a, b) and it is cutting the circle $x^2 + y^2 = k^2$ orthogonally, then the equation of the locus of its centre is

Solution:

Graph:



Let center of the circle be $\mathbf{X_0}$ and radius of the circle be r. So, equation of the circle be

$$||\mathbf{X} - \mathbf{X_0}|| = r \tag{1}$$

$$||\mathbf{X} - \mathbf{X_0}||^2 = r^2 \tag{2}$$

$$(\mathbf{X} - \mathbf{X_0})^\top (\mathbf{X} - \mathbf{X_0}) = r^2 \tag{3}$$

$$||\mathbf{X}||^2 - 2\mathbf{X_0}^\top \mathbf{X} + ||\mathbf{X_0}||^2 - r^2 = 0 \tag{4}$$

And the other given circle be with center $\mathbf{0}$ and radius k.

As evident from the fig, for the circle to be orthogonal, $\angle\alpha = 90°$and

$$r^2 + k^2 = ||\mathbf{X_0} - 0||^2 = ||\mathbf{X_0}||^2 \tag{5}$$

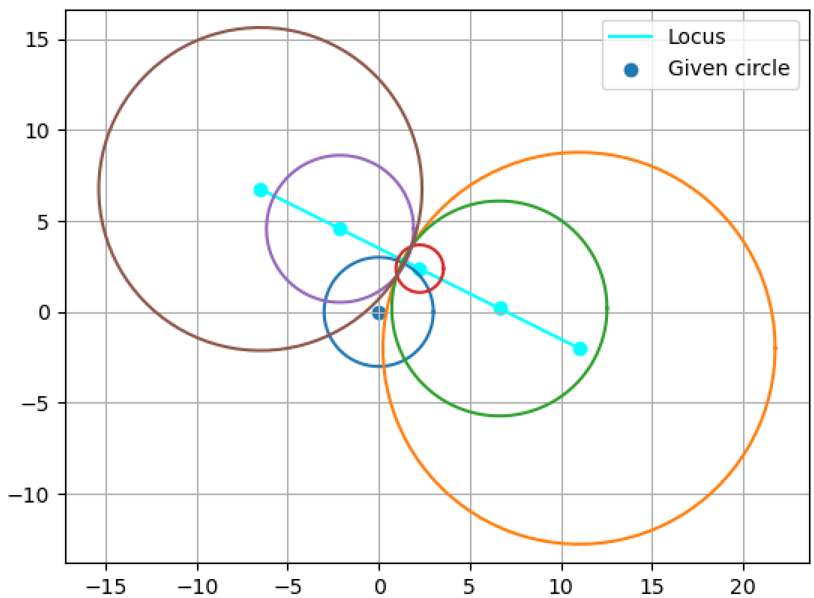
substituing in the equation,

$$||\mathbf{X}||^2 - 2\mathbf{X_0}^\top \mathbf{X} + k^2 = 0 \tag{6}$$

Putting th given point $\beta = \begin{pmatrix} a \\ b \end{pmatrix}$

$$||\beta||^2 - 2\mathbf{X_0}^\top \beta + k^2 = 0 \tag{7}$$

So, option (a) is correct. Graph:


Locus
Given circle
15
10
5
0
−5
−10
−15
−10
−5
0
5
10
15
20