

5.13.46

EE25BTECH11020 - Darsh Pankaj Gajare

Question:

Consider the set A of all determinants of order 3 with entries 0 or 1 only. Let B be the subset of A consisting of all determinants with value 1. Let C be the subset of A consisting of all determinants with value -1 .

Then

- 1) C is empty
- 2) B has as many elements as C
- 3) $A = B \cup C$
- 4) B has twice as many elements as C

Solution:

Let \mathbf{A} be

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad (1)$$

where $a_{ij} \in \{0, 1\}$

$$\det(\mathbf{A}) \in \{-2, -1, 0, 1, 2\}. \quad (2)$$

Cases

$$|\det| = 2 \Rightarrow \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}, \quad (3)$$

$$3 \text{ with } \det = 2, 3 \text{ with } \det = -2. \quad (4)$$

$$|\det| = 1 \Rightarrow (2^3 - 1)(2^3 - 2)(2^3 - 4) = 168 = 84(+1), 84(-1). \quad (5)$$

$$\det = 0 \Rightarrow 512 - (168 + 6) = 338. \quad (6)$$

Distribution

$$-2 \Rightarrow 3 \quad (7)$$

$$-1 \Rightarrow 84 \quad (8)$$

$$0 \Rightarrow 338 \quad (9)$$

$$1 \Rightarrow 84 \quad (10)$$

$$2 \Rightarrow 3 \quad (11)$$

Answer: (b),

Listing 1: C code

```
1 #include <stdio.h>
2 int det3(int m[3][3]) {
3     return m[0][0]*m[1][1]*m[2][2]
4         + m[0][1]*m[1][2]*m[2][0]
5         + m[0][2]*m[1][0]*m[2][1]
6         - m[0][2]*m[1][1]*m[2][0]
7         - m[0][0]*m[1][2]*m[2][1]
8         - m[0][1]*m[1][0]*m[2][2];
9 }
10
11 void compute_counts(int counts[7]) {
12     int mat[3][3];
13     for (int i = 0; i < 7; i++) counts[i] = 0;
14
15     for (int mask = 0; mask < (1<<9); mask++) {
16         for (int i = 0; i < 9; i++) {
17             mat[i/3][i%3] = (mask >> i) & 1;
18         }
19         int d = det3(mat);
20         counts[d+3]++;
21     }
22 }
```