1.8.5

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Question (1.8.5)

If **A** and **B** be the points $(3, 4, 5)$ and $(-1, 3, -7)$ respectively, find the equation of the set of a point **P** such that $\mathbf{PA}^2 + \mathbf{PB}^2 = k^2$

Given

$$\mathbf{A} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix} \tag{1}$$

According to the question,

$$\mathbf{PA}^2 + \mathbf{PB}^2 = k^2 \tag{2}$$

where, $\mathbf{PA} = \|P - A\|$ and $\mathbf{PB} = \|P - B\|$

Solution

The squared distances can be written as dot products:

$$\mathbf{PA}^2 = (\mathbf{P} - \mathbf{A}).(\mathbf{P} - \mathbf{A}) \tag{3}$$

$$\mathbf{PB}^2 = (\mathbf{P} - \mathbf{B}).(\mathbf{P} - \mathbf{B}) \tag{4}$$

Thus:

$$\mathbf{PA}^2 + \mathbf{PB}^2 = (\mathbf{P} - \mathbf{A}).(\mathbf{P} - \mathbf{A}) + (\mathbf{P} - \mathbf{B}).(\mathbf{P} - \mathbf{B}) \tag{5}$$

$$\mathbf{PA}^2 + \mathbf{PB}^2 = \mathbf{P}.\mathbf{P} - 2\mathbf{A}.\mathbf{P} + \mathbf{A}.\mathbf{A} + \mathbf{P}.\mathbf{P} - 2\mathbf{B}.\mathbf{P} + \mathbf{B}.\mathbf{B} \tag{6}$$

$$\tag{7}$$

Substitute the known values

$$\mathbf{A}.\mathbf{A} = 3^2 + 4^2 + 5^2 = 50 \tag{8}$$

$$\mathbf{B}.\mathbf{B} = (-1)^2 + 3^2 + (-7)^2 = 59 \tag{9}$$

$$\mathbf{A} + \mathbf{B} = \begin{pmatrix} 3 - 1 \\ 4 - 3 \\ 5 - 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 7 \\ -2 \end{pmatrix} \tag{10}$$

Result

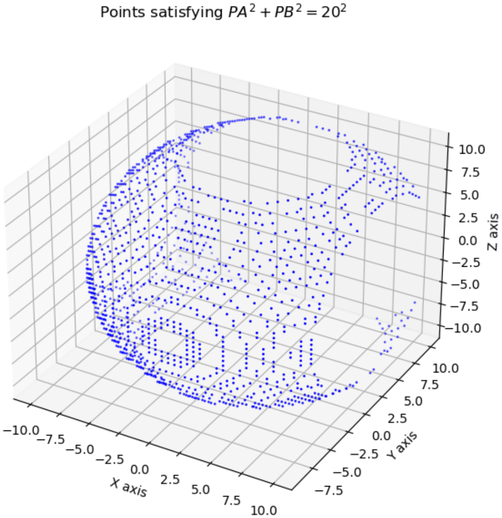
The equation of the locus is:

$$2\mathbf{P}.\mathbf{P} - 2\begin{pmatrix} 2 \\ 7 \\ -2 \end{pmatrix}.\mathbf{P} + 109 = K^2 \tag{11}$$

or equivalently,

$$2\mathbf{P}^T\mathbf{P} - 2\begin{pmatrix} 2 & 7 & -2 \end{pmatrix}.\mathbf{P} + 109 = K^2 \tag{12}$$

The plot show the locus for $k = 20$



C Code

```c
#include <stdio.h>

// Define a 3D vector struct
typedef struct {
    double x, y, z;
} Vector3;

// Dot product of two vectors
double dot(Vector3 v1, Vector3 v2) {
    return v1.x * v2.x + v1.y * v2.y + v1.z * v2.z;
}
int main() {
    Vector3 A = {3, 4, 5};
    Vector3 B = {-1, 3, -7};
```

C Code

```
    // Vector sum A+B
    Vector3 AplusB = {A.x + B.x, A.y + B.y, A.z + B.z};
    // Dot products |A|^2 and |B|^2
    double A_dot = dot(A, A);
    double B_dot = dot(B, B);
    // Equation in vector form:
    // 2 * (P·P) - 2 * (A+B)·P + |A|^2 + |B|^2 = K^2
    printf("Vector form equation of locus P satisfies:\n");
    printf("2 * (P · P) - 2 * (A+B) · P + |A|^2 + |B|^2 = K^2
    \n");
    printf("where\n");
    printf("A + B = (%.1f, %.1f, %.1f),\n", AplusB.x,
    AplusB.y,AplusB.z);
    printf("|A|^2 = %.1f, |B|^2 = %.1f\n", A_dot, B_dot);
    return 0;
}
```

Python Code

```
        import numpy as np

# Define points A and B
A = np.array([3, 4, 5])
B = np.array([-1, 3, -7])

# Compute vector sum A+B
AplusB = A + B

# Compute dot products |A|^2 and |B|^2
A_dot = np.dot(A, A)
B_dot = np.dot(B, B)
```

Python Code

```
print("Vector form equation of locus P satisfies:")
print("2 * (P · P) - 2 * (A+B) · P + |A|^2 + |B|^2 = K^2")
print("where")
print(f"A + B = ({AplusB[0]:.1f}, {AplusB[1]:.1f}, {AplusB[2]
print(f"|A|^2 = {A_dot:.1f}, |B|^2 = {B_dot:.1f}")
```