4.3.33

EE25BTECH11043 - Nishid Khandagre

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Question

If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$, then the equation of the line will be

Theoretical Solution

The equation of a line is

$$\mathbf{n}^\top \mathbf{x} = c \tag{1}$$

Where $\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$ is the normal vector and $\mathbf{x}$ is the position vector.

Theoretical Solution

X-axis intercept is at **A**

$$\mathbf{n}^\top \mathbf{A} = c \tag{2}$$

$$\begin{pmatrix} n_1 & n_2 \end{pmatrix} \begin{pmatrix} a \\ 0 \end{pmatrix} = c \tag{3}$$

$$n_1 a = c \tag{4}$$

$$\mathbf{A} = \begin{pmatrix} \frac{c}{n_1} \\ 0 \end{pmatrix} \tag{5}$$

Theoretical Solution

Y-axis intercept is at $\mathbf{B}$

$$\mathbf{n}^\top \mathbf{B} = c \tag{6}$$

$$\begin{pmatrix} n_1 & n_2 \end{pmatrix} \begin{pmatrix} 0 \\ b \end{pmatrix} = c \tag{7}$$

$$n_2 b = c \tag{8}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ \frac{c}{n_2} \end{pmatrix} \tag{9}$$

Theoretical Solution

The **M** is the midpoint of **A** and **B**

Given $\mathbf{M} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{10}$$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \frac{c}{n_1} \\ 0 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} 0 \\ \frac{c}{n_2} \end{pmatrix} \tag{11}$$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{c}{2n_1} \\ \frac{c}{2n_2} \end{pmatrix} \tag{12}$$

Theoretical Solution

$$\frac{c}{2n_1} = 3 \tag{13}$$

$$\frac{c}{2n_2} = 2 \tag{14}$$

$$\frac{n_1}{n_2} = \frac{2}{3} \tag{15}$$

Let $n_1 = 2$ and $n_2 = 3$. Then

$$c = 6 \times 2 = 12 \tag{16}$$

The final equation of the line is $\mathbf{n}^\top \mathbf{x} = c$

$$\begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} = 12 \tag{17}$$

C Code

```c
#include <stdio.h>

// Function to calculate 'a' (x-intercept) and 'b' (y-intercept)
// given the midpoint (xm, ym) of the line segment intercepted
    between the axes.
void findIntercepts(double xm, double ym, double *a, double *b) {
    // If the midpoint of (a, 0) and (0, b) is (xm, ym):
    // (a + 0) / 2 = xm -> a = 2 * xm
    // (0 + b) / 2 = ym -> b = 2 * ym
    *a = 2 * xm;
    *b = 2 * ym;
}
```

Python Code through shared output

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load the shared library
lib_line = ctypes.CDLL(./code6.so)

# Define the argument types and return type for the C function
lib_line.findIntercepts.argtypes = [
    ctypes.c_double, # xm (midpoint x-coordinate)
    ctypes.c_double, # ym (midpoint y-coordinate)
    ctypes.POINTER(ctypes.c_double), # a (x-intercept)
    ctypes.POINTER(ctypes.c_double) # b (y-intercept)
]
```

Python Code through shared output

```
lib_line.findIntercepts.restype = None

# Given midpoint coordinates
xm_given, ym_given = 3.0, 2.0 # Midpoint of the line segment

# Create ctypes doubles to hold the results for 'a' and 'b'
a_result = ctypes.c_double()
b_result = ctypes.c_double()

# Call the C function to find the intercepts
lib_line.findIntercepts(
    xm_given, ym_given,
    ctypes.byref(a_result),
    ctypes.byref(b_result)
)
```

Python Code through shared output

```
a_intercept = a_result.value
b_intercept = b_result.value

print(fGiven midpoint: ({xm_given:.2f}, {ym_given:.2f}))
print(fThe x-intercept (a) is: {a_intercept:.2f})
print(fThe y-intercept (b) is: {b_intercept:.2f})

# The equation of the line is x/a + y/b = 1
# Which can be rewritten as: b*x + a*y = a*b
# Or in standard form: b*x + a*y - a*b = 0

# For plotting, let's express y in terms of x:
# y = (-b/a) * x + b
slope = -b_intercept / a_intercept
y_intercept_calc = b_intercept
```

Python Code through shared output

```
print(fThe equation of the line is: {b_intercept:.0f}x + {
    a_intercept:.0f}y = {a_intercept * b_intercept:.0f})
# Plotting the line and intercepts
plt.figure(figsize=(8, 8))

# Generate points for the line
x_vals = np.linspace(-1, 7, 400)
y_vals = slope * x_vals + y_intercept_calc
plt.plot(x_vals, y_vals, 'b-', label=f'Line: {int(b_intercept)}x
    + {int(a_intercept)}y = {int(a_intercept * b_intercept)}')

# Plot the x-intercept
plt.scatter(a_intercept, 0, color='red', s=100, zorder=5, label=f
    'X-intercept ({a_intercept:.0f}, 0)')
plt.annotate(f'({a_intercept:.0f}, 0)', (a_intercept, 0),
    textcoords=offset points, xytext=(5,5), ha='left')
```

Python Code through shared output

```
# Plot the y-intercept
plt.scatter(0, b_intercept, color='green', s=100, zorder=5, label
    =f'Y-intercept (0, {b_intercept:.0f})')
plt.annotate(f'(0, {b_intercept:.0f})', (0, b_intercept),
    textcoords=offset points, xytext=(5,5), ha='left')
# Plot the midpoint
plt.scatter(xm_given, ym_given, color='purple', s=100, zorder=5,
    label=f'Midpoint ({xm_given:.0f}, {ym_given:.0f})')
plt.annotate(f'({xm_given:.0f}, {ym_given:.0f})', (xm_given,
    ym_given), textcoords=offset points, xytext=(5,5), ha='left')

plt.axhline(0, color='gray', linestyle='--', linewidth=0.5)
plt.axvline(0, color='gray', linestyle='--', linewidth=0.5)
```

Python Code through shared output

```
plt.gca().set_aspect('equal', adjustable='box')
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.title('Equation of a Line with Given Midpoint of Intercepts')
plt.grid(True)
plt.legend()
plt.xlim(min(0, a_intercept) - 1, max(0, a_intercept) + 1)
plt.ylim(min(0, b_intercept) - 1, max(0, b_intercept) + 1)
plt.show()
# plt.savefig(fig1.png)
```

Python Code (Direct)

```
import numpy as np
import matplotlib.pyplot as plt

def line_gen_num(A, B, num_points):
    A = A.flatten()
    B = B.flatten()
    t = np.linspace(0, 1, num_points)
    points = np.outer(A, (1 - t)) + np.outer(B, t)
    return points

def plot_line_with_intercepts(mid_x, mid_y):
    # Calculate the x-intercept (a) and y-intercept (b)
    # Since (mid_x, mid_y) is the midpoint of (a, 0) and (0, b)
    # mid_x = (a + 0) / 2 => a = 2 * mid_x
    # mid_y = (0 + b) / 2 => b = 2 * mid_y
    a = 2 * mid_x
    b = 2 * mid_y
```

Python Code (Direct)

```
# The equation of the line is x/a + y/b = 1
# Generate points for the line
x_intercept = np.array([a, 0]).reshape(-1, 1)
y_intercept = np.array([0, b]).reshape(-1, 1)
line_points = line_gen_num(x_intercept, y_intercept, 100)

# Plotting
plt.figure(figsize=(8, 6))
plt.plot(line_points[0, :], line_points[1, :], blue, label=f
    Line: x/{a} + y/{b} = 1)
# Plot the intercepts
plt.scatter([a, 0], [0, b], color='red', s=100, zorder=5)
plt.text(a + 0.5, 0, f'({a:.0f}, 0)', color='red')
plt.text(0.5, b + 0.5, f'(0, {b:.0f})', color='red')
# Plot the midpoint
plt.scatter([mid_x], [mid_y], color='green', s=100, zorder=5,
     label=fMidpoint: ({mid_x}, {mid_y}))
```

Python Code (Direct)

```
    plt.text(mid_x + 0.5, mid_y + 0.5, f'({mid_x:.0f}, {mid_y:.0f
        })', color='green')
    plt.xlabel('x')
    plt.ylabel('y')
    plt.legend(loc='best')
    plt.grid()
    plt.axhline(0, color='black', linewidth=0.5)
    plt.axvline(0, color='black', linewidth=0.5)
    plt.title(fLine Intercepted by Coordinate Axes with Midpoint
        ({mid_x}, {mid_y}))
    plt.axis('equal')
    plt.xlim(min(0, a, mid_x) - 2, max(0, a, mid_x) + 2)
    plt.ylim(min(0, b, mid_y) - 2, max(0, b, mid_y) + 2)
    plt.savefig(fig1.png)
    plt.show()
```

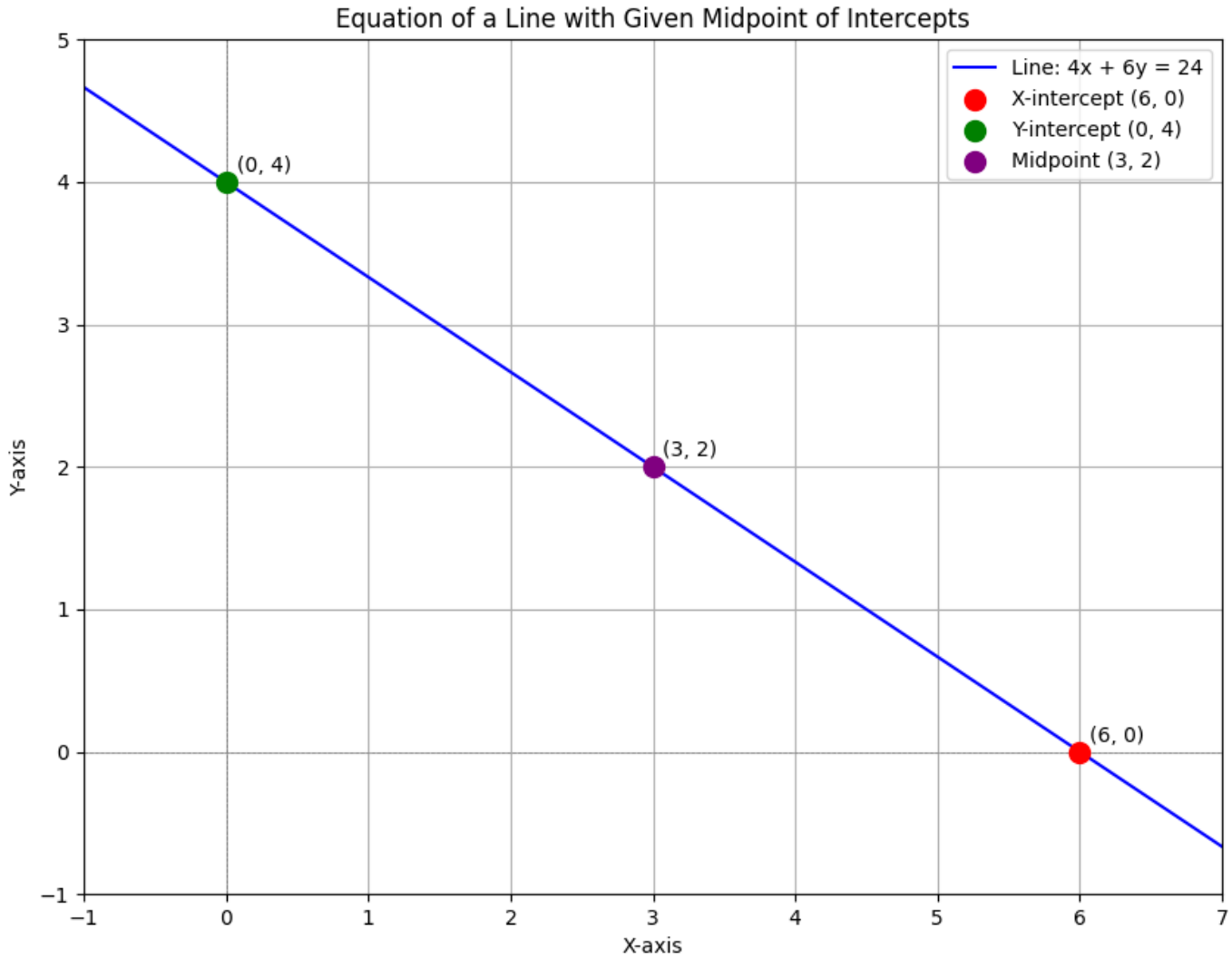
Python Code (Direct)

```
    print(fThe x-intercept is ({a}, 0))
    print(fThe y-intercept is (0, {b}))
    print(fThe equation of the line is x/{a} + y/{b} = 1)
    print(Figure saved as line_intercept_midpoint.png)

# Given midpoint coordinates
mid_x_coord = 3
mid_y_coord = 2

# Call the function to plot and calculate the equation
plot_line_with_intercepts(mid_x_coord, mid_y_coord)
```

Plot by Python using shared output from C



Plot by Python only