# 4.13.72

AI25BTECH11003 - Bhavesh Gaikwad

**Question**: A non-zero vector $\mathbf{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{\imath}$, $\hat{\imath}+\hat{\jmath}$ and the plane determined by the vectors $\hat{\imath}-\hat{\jmath}$, $\hat{\imath}+\hat{k}$. The angle between $\mathbf{a}$ and the vector $\hat{\imath}-2\hat{\jmath}+2\hat{k}$ is ______.

(1996)

**Solution:**

Let $\mathbf{A} = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $\mathbf{B} = \begin{pmatrix}1\\1\\0\end{pmatrix}$, $\mathbf{C} = \begin{pmatrix}1\\-1\\0\end{pmatrix}$, $\mathbf{D} = \begin{pmatrix}1\\0\\1\end{pmatrix}$

Let $\mathbf{n}_1$ be the perpendicular vector to Plane-1.

Let $\mathbf{n}_1 = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$

$$\therefore\ \mathbf{A}^\top\mathbf{n}_1 = 0 \,\&\, \mathbf{B}^\top\mathbf{n}_1 = 0 \tag{0.1}$$

From Equation 0.1,

$$x_1 = 0 \,\&\, x_2 + x_1 = 0 \quad \Rightarrow x_1 = x_2 = 0 \tag{0.2}$$

$$\mathbf{n}_1 = \begin{pmatrix}0\\0\\x_3\end{pmatrix} \quad OR \quad \mathbf{n}_1 = \begin{pmatrix}0\\0\\1\end{pmatrix} \tag{0.3}$$

Let $\mathbf{n}_2$ be the perpendicular vector to Plane-2.

Let $\mathbf{n}_2 = \begin{pmatrix}z_1\\z_2\\z_3\end{pmatrix}$

$$\therefore\ \mathbf{C}^\top\mathbf{n}_2 = 0 \,\&\, \mathbf{D}^\top\mathbf{n}_2 = 0 \tag{0.4}$$

From Equation 0.4,

$$z_1 - z_2 = 0 \,\&\, z_1 + z_3 = 0 \tag{0.5}$$

$$\mathbf{n}_2 = \begin{pmatrix}-z_3\\-z_3\\z_3\end{pmatrix} \quad OR \quad \mathbf{n}_2 = \begin{pmatrix}-1\\-1\\1\end{pmatrix} \tag{0.6}$$

Let $\mathbf{n}_3$ be the parallel vector to the line of intersection of planes.

Let $\mathbf{n}_3 = \begin{pmatrix}k_1\\k_2\\k_3\end{pmatrix}$

Since, $\mathbf{n}_3$ is perpendicular to both $\mathbf{n}_1$ and $\mathbf{n}_2$.

$$\therefore \mathbf{n}_3^\top \mathbf{n}_1 = 0 \,\&\, \mathbf{n}_3^\top \mathbf{n}_2 = 0 \tag{0.7}$$

$$\boxed{\text{The angle between } \mathbf{a}\, and\; 1\hat{i} - 2\hat{j} + 2\hat{k}\, is\, 45°.} \tag{0.8}$$

$$k_3 = 0 \,\&\, k_3 - k_1 - k_2 = 0 \quad \Rightarrow k_2 = -k_1 \tag{0.9}$$

$$\mathbf{n}_3 = \begin{pmatrix} k_1 \\ k_2 \\ 0 \end{pmatrix} \quad OR \quad \mathbf{n}_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \tag{0.10}$$

From Equation 0.10,

$$\therefore \mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \tag{0.11}$$

Let $\mathbf{u} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ [Already given in the Question]

We know,

$$\mathbf{a}^\top \mathbf{u} = \|\mathbf{a}\| \, \|\mathbf{u}\| \cos(\theta) \tag{0.12}$$

$$\|\mathbf{a}\| = \sqrt{\mathbf{a}^\top \mathbf{a}} = \sqrt{2}, \ \|\mathbf{u}\| = \sqrt{\mathbf{u}^\top \mathbf{u}} = 3 \tag{0.13}$$

From Equation 0.12 and 0.13,

$$\cos(\theta) = \frac{3}{3\sqrt{2}} \quad \Rightarrow \theta = 45° \tag{0.14}$$

$$\boxed{\text{The angle between } \mathbf{a}\, and\; 1\hat{i} - 2\hat{j} + 2\hat{k}\, is\, 45°.} \tag{0.15}$$

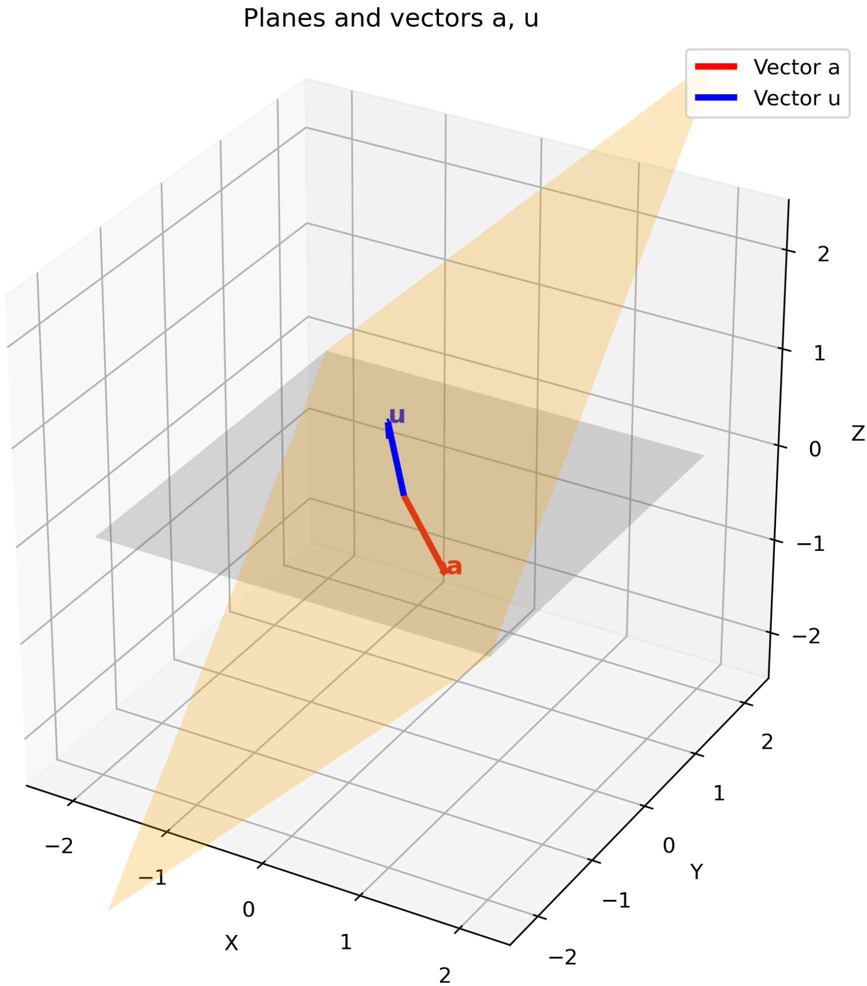


Fig. 0.1: Plane