EE25BTECH11009-Anshu kumar ram

Question:

Find a point on the Y axis which is equidistant from the points A(6,5) and B(-4,3).

Solution:

The input parameters for this problem are available in Table

| Symbol | Value | Description |
|--------|---|---------------|
| A | $\binom{6}{5}$ | First point |
| В | $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ | Second point |
| О | $\begin{pmatrix} 0 \\ y \end{pmatrix}$ | Desired point |

TABLE 0: Parameters for the problem

TABLE 0

If O lies on the y-axis and is equidistant from the points A and B,

$$\|\mathbf{O} - \mathbf{A}\| = \|\mathbf{O} - \mathbf{B}\| \tag{0.1}$$

$$\implies \|\mathbf{O} - \mathbf{A}\|^2 = \|\mathbf{O} - \mathbf{B}\|^2 \tag{0.2}$$

$$\implies \|\mathbf{O}\|^2 - 2\mathbf{O}^{\mathsf{T}}\mathbf{A} + \|\mathbf{A}\|^2 = \|\mathbf{O}\|^2 - 2\mathbf{O}^{\mathsf{T}}\mathbf{B} + \|\mathbf{B}\|^2, \tag{0.3}$$

which can be simplified to obtain

$$(\mathbf{A} - \mathbf{B})^{\mathsf{T}} \mathbf{O} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2}.$$
 (0.4)

$$\mathbf{O} = y\mathbf{e}_2, \tag{0.5}$$

$$y = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2(\mathbf{A} - \mathbf{B})^\top \mathbf{e}_2}.$$
 (0.6)

Substituting from table we get, y = 9. Thus,

$$\mathbf{O} = \begin{pmatrix} 0 \\ 9 \end{pmatrix}. \tag{0.7}$$

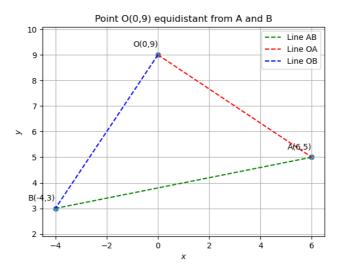


Fig. 0.1: Point O(0,9) on the y-axis is equidistant from A and B.