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SAMYAK GONDANE - AI25BTECH11029

Question

For what value of k, will the following pair of equations have infinitly many solutions

$2x + 3y = 7$ and $(k + 2)x - 3(1 - k)y = 5k + 1$

Solution

Given:

$$2x + 3y = 7 \tag{1}$$
$$(k+2)x - 3(1-k)y = 5k + 1 \tag{2}$$

Augmented Matrix

Convert the system to an augmented matrix:

$$\left(\begin{array}{cc|c} 2 & 3 & 7 \\ k+2 & -3+3k & 5k+1 \end{array}\right) \tag{3}$$

Solution

Let the second row be:

$$R_2 = \begin{pmatrix} a & b & | & c \end{pmatrix} \quad \text{where} \quad a = k+2, \quad b = -3+3k, \quad c = 5k+1 \tag{4}$$

Eliminate First Entry of Row 2

Apply row operation:

$$R_2 \to R_2 - \frac{a}{2} R_1 \tag{5}$$

Solution

Compute each entry:

$$\begin{aligned}\text{New second entry} &= b - \frac{a}{2}\cdot 3 \\ &= (-3+3k) - \frac{3(k+2)}{2} \\ &= \frac{-6+6k-3k-6}{2} = \frac{3k-12}{2}\end{aligned}$$

$$\begin{aligned}\text{New third entry} &= c - \frac{a}{2}\cdot 7 \\ &= (5k+1) - \frac{7(k+2)}{2} \\ &= \frac{10k+2-7k-14}{2} = \frac{3k-12}{2}\end{aligned}$$

Solution

So the matrix becomes:

$$\left(\begin{array}{cc|c} 2 & 3 & 7 \\ 0 & \frac{3k-12}{2} & \frac{3k-12}{2} \end{array}\right) \tag{6}$$

Condition for Infinitely Many Solutions

For infinitely many solutions, the second row must reduce to:

$$0x + 0y = 0 \Rightarrow \frac{3k-12}{2} = 0 \Rightarrow 3k - 12 = 0 \Rightarrow k = 4 \tag{7}$$

Final Answer

$$\boxed{k = 4} \tag{8}$$