1.9.17

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Question

Write the coordinates of a point **P** on the x-axis which is equidistant from the points $\mathbf{A}(-\mathbf{2}, \mathbf{0})$ and $\mathbf{B}(\mathbf{6}, \mathbf{0})$.

Theoretical solution

Let

$$\mathbf{A} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} b \\ 0 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} p \\ 0 \end{pmatrix} \tag{1}$$

Since $\mathbf{P}$ is equidistant from $\mathbf{A}$ and $\mathbf{B}$, their distances satisfy:

$$\|\mathbf{P} - \mathbf{A}\| = \|\mathbf{P} - \mathbf{B}\| \tag{2}$$

Square both sides:

$$\|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 \tag{3}$$

Using the norm squared definition:

$$(\mathbf{P} - \mathbf{A})^\top (\mathbf{P} - \mathbf{A}) = (\mathbf{P} - \mathbf{B})^\top (\mathbf{P} - \mathbf{B}) \tag{4}$$

Expand both sides:

$$\mathbf{P}^\top \mathbf{P} - 2\mathbf{A}^\top \mathbf{P} + \mathbf{A}^\top \mathbf{A} = \mathbf{P}^\top \mathbf{P} - 2\mathbf{B}^\top \mathbf{P} + \mathbf{B}^\top \mathbf{B} \tag{5}$$

Theoretical solution

Cancel $\mathbf{P}^\top\mathbf{P}$ from both sides:

$$-2\mathbf{A}^\top\mathbf{P} + \mathbf{A}^\top\mathbf{A} = -2\mathbf{B}^\top\mathbf{P} + \mathbf{B}^\top\mathbf{B} \tag{6}$$

Rearranged:

$$2(\mathbf{B} - \mathbf{A})^\top\mathbf{P} = \mathbf{B}^\top\mathbf{B} - \mathbf{A}^\top\mathbf{A} \tag{7}$$

Substitute the vectors:

$$2(b - a)p = b^2 - a^2 \tag{8}$$

Rewrite right side as difference of squares:

$$2(b - a)p = (b - a)(b + a) \tag{9}$$

Since $b \neq a$, divide both sides by $(b - a)$:

$$2p = b + a \tag{10}$$

Theoretical solution

Solve for x: Solve for p:

$$p = \frac{a+b}{2} \tag{11}$$

Now substitute $a = -2$, $b = 6$:

$$p = \frac{-2+6}{2} = \frac{4}{2} = 2 \tag{12}$$

Hence, the coordinates of **P** are:

$$\boxed{\mathbf{P} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}} \tag{13}$$

Python Code

```
import matplotlib.pyplot as plt
# Coordinates
A = (-2, 0)
B = (6, 0)
P = (2, 0)

# Plot points
plt.figure(figsize=(6,6))
plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
```

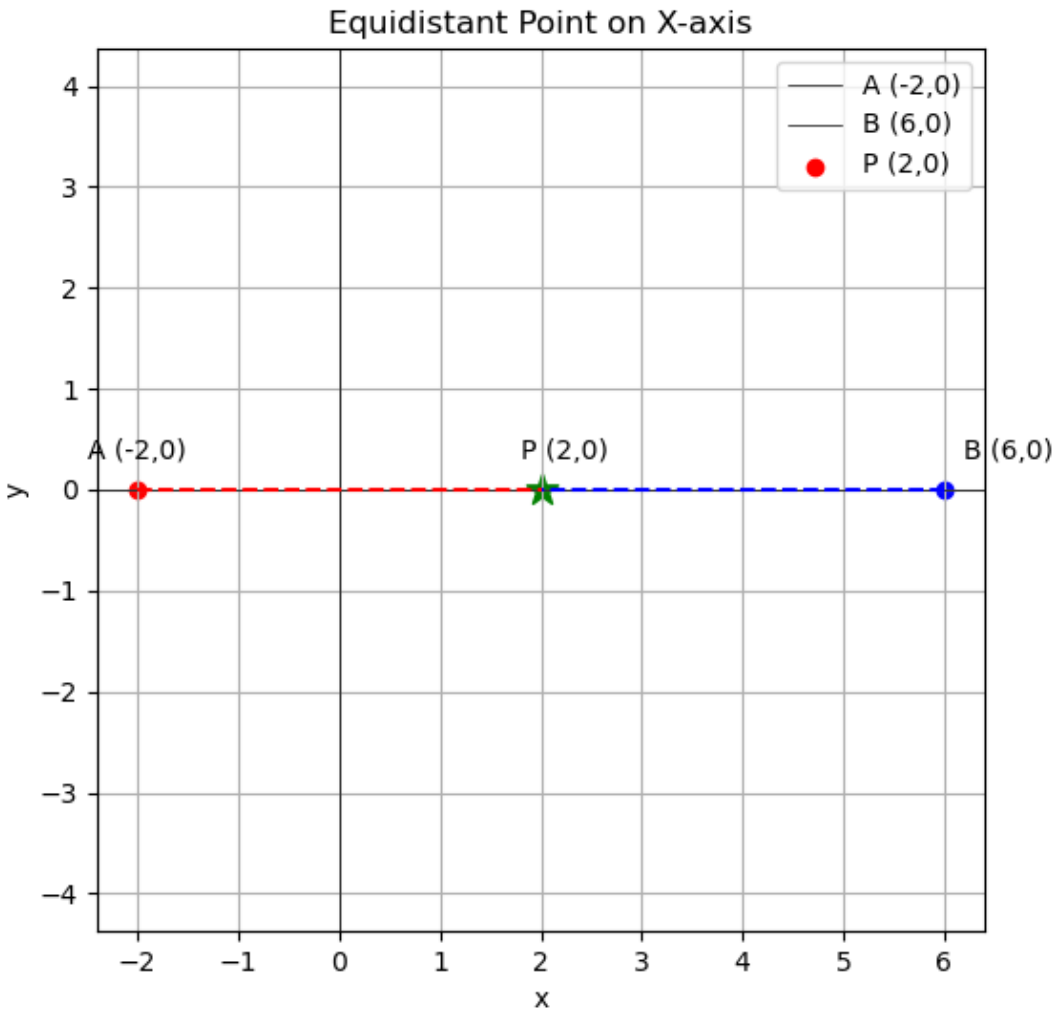
Python Code

```
# A point
plt.scatter(A[0], A[1], color="red")
plt.text(A[0]-0.5, A[1]+0.3, "A (-2,0)")
plt.plot([A[0], P[0]], [A[1], P[1]], "r--")

# B point
plt.scatter(B[0], B[1], color="blue")
plt.text(B[0]+0.2, B[1]+0.3, "B (6,0)")
plt.plot([B[0], P[0]], [B[1], P[1]], "b--")
```

Python Code

```
# P point (equidistant point)
plt.scatter(P[0], P[1], color="green", s=150, marker="*")
plt.text(P[0]-0.2, P[1]+0.3, "P (2,0)")

# Labels and grid
plt.title("Equidistant Point on X-axis")
plt.xlabel("x")
plt.ylabel("y")
plt.grid(True)
plt.legend(["A (-2,0)", "B (6,0)", "P (2,0)"])
plt.axis("equal")
plt.show()
```

Plot-Using by Python



C Code

```c
#include <stdio.h>
#include <math.h>

// Function to compute x-coordinate of equidistant point
double equidistant_point(double ax, double ay, double bx, double
    by) {
    // Norm squared of A and B
    double normA2 = ax*ax + ay*ay;
    double normB2 = bx*bx + by*by;

    double denom = 2 * (ax - bx);

    double x = (normA2 - normB2) / denom;
    return x;
}
```

Python and C Code

```
import ctypes
import matplotlib.pyplot as plt

lib = ctypes.CDLL("./libequidistant.so")
lib.compute_point.argtypes = [ctypes.c_double, ctypes.c_double,
                              ctypes.c_double, ctypes.c_double,
                              ctypes.POINTER(ctypes.c_double),
                                  ctypes.POINTER(ctypes.c_double)]


Ax, Ay = -2., 0.
Bx, By = 6., 0
```

Python and C Code

```
Px, Py = ctypes.c_double(), ctypes.c_double()

lib.compute_point(Ax, Ay, Bx, By, ctypes.byref(Px), ctypes.byref(
    Py))

Px_val, Py_val = Px.value, Py.value

print(f"A = ({Ax}, {Ay})")
print(f"B = ({Bx}, {By})")
print(f"Computed P = ({Px_val}, {Py_val})")
plt.figure()
plt.scatter([Ax, Bx, Px_val], [Ay, By, Py_val],
            color=["green", "red", "blue"], s=100)
```

Python and C Code

```
plt.text(Ax + 0.2, Ay, f"A({Ax:.2f},{Ay:.2f})", fontsize=12,
    color="green")
plt.text(Bx + 0.2, By, f"B({Bx:.2f},{By:.2f})", fontsize=12,
    color="red")
plt.text(Px_val + 0.2, Py_val, f"P({Px_val:.2f},{Py_val:.2f})",
    fontsize=12, color="blue")

plt.plot([Ax, Px_val, Bx], [Ay, Py_val, By],
         linestyle="--", color="gray")

plt.xlabel("X-axis")
plt.ylabel("Y-axis")
plt.title("Computed P = (A + B)/2")
plt.grid(True)
plt.axis("equal")
plt.savefig("fig2.1.png")
plt.show()
```

Plot-Using by both C and Python