2.7.20

EE25BTECH11008 - Anirudh M Abhilash

September 16, 2025

Question

The two adjacent sides of a parallelogram are

$$\mathbf{a} = \begin{pmatrix} 2 \\ -4 \\ -5 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix}$$

Find the two unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram.

Solution

Diagonals:

$$\mathbf{d}_1 = \mathbf{a} + \mathbf{b} \tag{1}$$

$$\mathbf{d}_2 = \mathbf{a} - \mathbf{b} \tag{2}$$

$$\mathbf{d}_1 = \begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix} \tag{3}$$

$$\mathbf{d}_2 = \begin{pmatrix} 0 \\ -6 \\ -8 \end{pmatrix} \tag{4}$$

Norms:

$$\|\mathbf{d}_1\|^2 = \mathbf{d}_1^\top \mathbf{d}_1 = 24 \tag{5}$$
$$\|\mathbf{d}_1\| = 2\sqrt{6} \tag{6}$$
$$\|\mathbf{d}_2\|^2 = \mathbf{d}_2^\top \mathbf{d}_2 = 100 \tag{7}$$
$$\|\mathbf{d}_2\| = 10 \tag{8}$$

Unit vectors along diagonals:

$$\mathbf{u}_1 = \frac{\mathbf{d}_1}{\|\mathbf{d}_1\|} = \frac{1}{2\sqrt{6}}\begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix} \tag{9}$$
$$\mathbf{u}_2 = \frac{\mathbf{d}_2}{\|\mathbf{d}_2\|} = \frac{1}{10}\begin{pmatrix} 0 \\ -6 \\ -8 \end{pmatrix}. \tag{10}$$

Area of the parallelogram:

$$\|\mathbf{a} \times \mathbf{b}\|^2 = (\mathbf{a}^\top \mathbf{a})(\mathbf{b}^\top \mathbf{b}) - (\mathbf{a}^\top \mathbf{b})^2. \tag{11}$$
$$\mathbf{a}^\top \mathbf{a} = 45, \quad \mathbf{b}^\top \mathbf{b} = 17, \quad \mathbf{a}^\top \mathbf{b} = -19. \tag{12}$$
$$\|\mathbf{a} \times \mathbf{b}\|^2 = 45 \cdot 17 - (-19)^2 = 404, \tag{13}$$
$$\text{Area} = \sqrt{404} = 2\sqrt{101}. \tag{14}$$

$$\boxed{\mathbf{u}_1 = \tfrac{1}{2\sqrt{6}}\begin{pmatrix} 4 \\ -2 \\ -2 \end{pmatrix}, \quad \mathbf{u}_2 = \tfrac{1}{10}\begin{pmatrix} 0 \\ -6 \\ -8 \end{pmatrix}, \quad \text{Area} = 2\sqrt{101}}$$

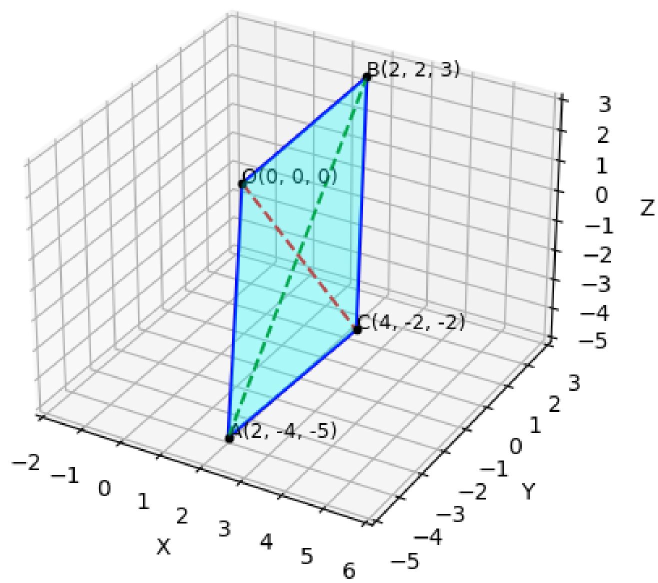

Figure 1: Parallelogram along with diagonal vectors