5.4.31

EE25BTECH11036 - M Chanakya Srinivas

Question 5.4.31: Using elementary row transformations, find the inverse of

$$A = \begin{pmatrix} 1 & 2 \\ 4 & 2 \end{pmatrix}.$$

Method: The inverse of a non-singular matrix A can be found using the augmented form

$$\left(nn | \mathrm{A} \quad \mathrm{I} \right) \xrightarrow{\text{row operations}} \left(nn | \mathrm{I} \quad \mathrm{A}^{-1} \right).$$

This is known as the **Gauss–Jordan elimination method**.

Solution:

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 4 & 2 & 0 & 1 \end{array}\right) \quad \text{Initial augmented matrix} \tag{1}$$

$$R_2 \leftarrow R_2 - 4R_1 : \quad (4 - 4 \cdot 1 = 0,\ 2 - 4 \cdot 2 = -6,\ 0 - 4 \cdot 1 = -4,\ 1 - 4 \cdot 0 = 1) \tag{2}$$

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & -6 & -4 & 1 \end{array}\right) \tag{3}$$

$$R_2 \leftarrow -\tfrac{1}{6} R_2 : \quad (0, -6, -4, 1) \cdot \left(-\tfrac{1}{6}\right) = (0, 1, \tfrac{2}{3}, -\tfrac{1}{6}) \tag{4}$$

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & \frac{2}{3} & -\frac{1}{6} \end{array}\right) \tag{5}$$

$$R_1 \leftarrow R_1 - 2R_2 : \quad (1, 2, 1, 0) - (0, 2, \tfrac{4}{3}, -\tfrac{1}{3}) = (1, 0, -\tfrac{1}{3}, \tfrac{1}{3}) \tag{6}$$

$$\left(\begin{array}{cc|cc} 1 & 0 & -\frac{1}{3} & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} & -\frac{1}{6} \end{array}\right) \tag{7}$$

From (**??**), the left block is I, hence the right block is the inverse:

$$A^{-1} = \begin{pmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{6} \end{pmatrix}.$$