1.5.16

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Question: Find the point A if AB is a diameter of the circle with center $C = (3, -1)$ and point $B = (2, 6)$.

Solution:

Point	Vector
B	$\begin{pmatrix}2\\6\end{pmatrix}$
C	$\begin{pmatrix}3\\-1\end{pmatrix}$

Section Formula:
If a point P divides the line joining A and B internally in the ratio $m : n$, then

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k+1} = \begin{pmatrix}\mathbf{A} & \mathbf{B}\end{pmatrix}\begin{pmatrix}\frac{1}{k+1}\\ \frac{k}{k+1}\end{pmatrix}$$

Here, C is the midpoint of AB, i.e. ratio $1 : 1$.

$$\mathbf{C} = \frac{\mathbf{A} + \mathbf{B}}{2} = \begin{pmatrix}\mathbf{A} & \mathbf{B}\end{pmatrix}\begin{pmatrix}\frac{1}{2}\\ \frac{1}{2}\end{pmatrix}.$$

Substitute values:

$$\begin{pmatrix}3\\-1\end{pmatrix} = \begin{pmatrix}\mathbf{A} & \begin{pmatrix}2\\6\end{pmatrix}\end{pmatrix}\begin{pmatrix}\frac{1}{2}\\ \frac{1}{2}\end{pmatrix}.$$

$$2\begin{pmatrix}3\\-1\end{pmatrix} = \mathbf{A} + \begin{pmatrix}2\\6\end{pmatrix} \quad \Rightarrow \quad \mathbf{A} = 2\begin{pmatrix}3\\-1\end{pmatrix} - \begin{pmatrix}2\\6\end{pmatrix} = \begin{pmatrix}4\\-8\end{pmatrix}.$$

Rank Verification:
Check collinearity of A, B, C:

$$\begin{pmatrix}\mathbf{C}-\mathbf{A} & \mathbf{B}-\mathbf{A}\end{pmatrix} = \begin{pmatrix}3-4 & 2-4\\ -1-(-8) & 6-(-8)\end{pmatrix} = \begin{pmatrix}-1 & -2\\ 7 & 14\end{pmatrix} = 0$$

Thus, rank = 1 and points are collinear.

$$\boxed{\mathbf{A} = \begin{pmatrix}4\\-8\end{pmatrix}}$$

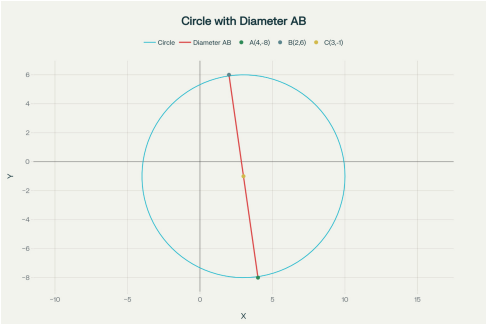
Circle with Diameter AB
Circle
Diameter AB
A(4,-8)
B(2,6)
C(3,-1)
6
4
2
0
-2
-4
-6
-8
Y
-10
-5
0
5
10
15
X

Fig. 0