2.4.40

EE25BTECH11043 - Nishid Khandagre

Question: Find the angle between the lines $-\sqrt{3}x + y - 5 = 0$ and $-x + \sqrt{3}y + 6 = 0$.
Solution: Given lines:

$$L_1 : -\sqrt{3}x + y - 5 = 0 \tag{0.1}$$

$$L_2 : -x + \sqrt{3}y + 6 = 0 \tag{0.2}$$

The matrix form of a line can be written as

$$\mathbf{n}^\top \mathbf{x} = C \tag{0.3}$$

Where $\mathbf{n}$ is the normal vector and $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$ is the position vector.

$$L_1 : \mathbf{n_1}^\top \mathbf{x} = c_1 \tag{0.4}$$

$$L_2 : \mathbf{n_2}^\top \mathbf{x} = c_2 \tag{0.5}$$

Where $\mathbf{n_1}$ and $\mathbf{n_2}$ are the normal vectors to the lines L_1 and L_2 respectively.

$$\mathbf{n_1} = \begin{pmatrix} -\sqrt{3} \\ 1 \end{pmatrix} \tag{0.6}$$

$$\mathbf{n_2} = \begin{pmatrix} -1 \\ \sqrt{3} \end{pmatrix} \tag{0.7}$$

The angle θ between the lines is the angle between their normal vectors.

$$\cos\theta = \frac{\mathbf{n_1}^\top \mathbf{n_2}}{\|\mathbf{n_1}\| \, \|\mathbf{n_2}\|} \tag{0.8}$$

$$\mathbf{n_1}^\top \mathbf{n_2} = \begin{pmatrix} -\sqrt{3} & 1 \end{pmatrix} \begin{pmatrix} -1 \\ \sqrt{3} \end{pmatrix} \tag{0.9}$$

$$= (-\sqrt{3})(-1) + (1)(\sqrt{3}) \tag{0.10}$$

$$= 2\sqrt{3} \tag{0.11}$$

$$\|\mathbf{n_1}\| = \sqrt{\mathbf{n_1}^\top \mathbf{n_1}} \tag{0.12}$$

$$= \sqrt{(-\sqrt{3})^2 + (1)^2} \tag{0.13}$$

$$= \sqrt{4} \tag{0.14}$$

$$= 2 \tag{0.15}$$

$$\|\mathbf{n_2}\| = \sqrt{\mathbf{n_2}^\top \mathbf{n_2}} \tag{0.16}$$
$$= \sqrt{(-1)^2 + (\sqrt{3})^2} \tag{0.17}$$
$$= \sqrt{4} \tag{0.18}$$
$$= 2 \tag{0.19}$$

Now, substitute these values into the formula (0.8)

$$\cos\theta = \frac{2\sqrt{3}}{(2)(2)} \tag{0.20}$$
$$= \frac{\sqrt{3}}{2} \tag{0.21}$$

$$\theta = \frac{\pi}{6} \text{ radians} \tag{0.22}$$

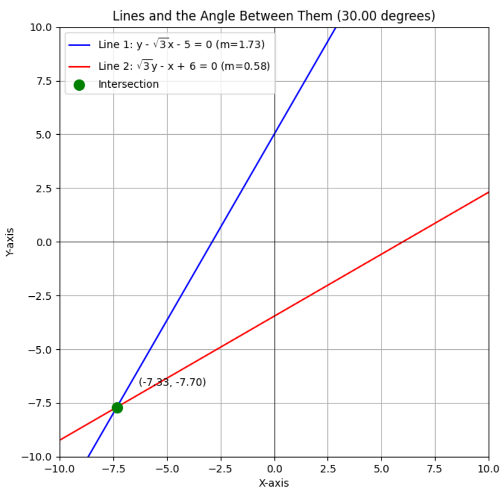

Fig. 0.1