

1.5.33

Puni Aditya - EE25BTECH11046

Question:

Find the ratio in which the Y-axis divides the line segment joining the points A(5, -6) and B(-1, -4). Also find the coordinates of the point of intersection.

Solution:

Let the given points be A and B

$$\mathbf{A} = \begin{pmatrix} 5 \\ -6 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$$

Let the Y-axis divide the line segment **AB** at point **P** in the ratio $k : 1$.

The coordinates of any point **P** on the line segment **AB** can be found using the section formula

$$\mathbf{P} \equiv \begin{pmatrix} x \\ y \end{pmatrix} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \quad (1)$$

Here, substituting the values, we get

$$\mathbf{P} \equiv \begin{pmatrix} x \\ y \end{pmatrix} = \frac{k \begin{pmatrix} -1 \\ -4 \end{pmatrix} + 1 \begin{pmatrix} 5 \\ -6 \end{pmatrix}}{k + 1} \quad (2)$$

$$\mathbf{P} = \begin{pmatrix} \frac{k(-1)+1(5)}{k+1} \\ \frac{k(-4)+1(-6)}{k+1} \end{pmatrix} \quad (3)$$

$$\mathbf{P} = \begin{pmatrix} \frac{-k+5}{k+1} \\ \frac{-4k-6}{k+1} \end{pmatrix} \quad (4)$$

Since the point **P** lies on the Y-axis only, its x component must be 0.

$$\frac{-k+5}{k+1} = 0 \quad (5)$$

$$-k+5=0 \implies k=5 \quad (6)$$

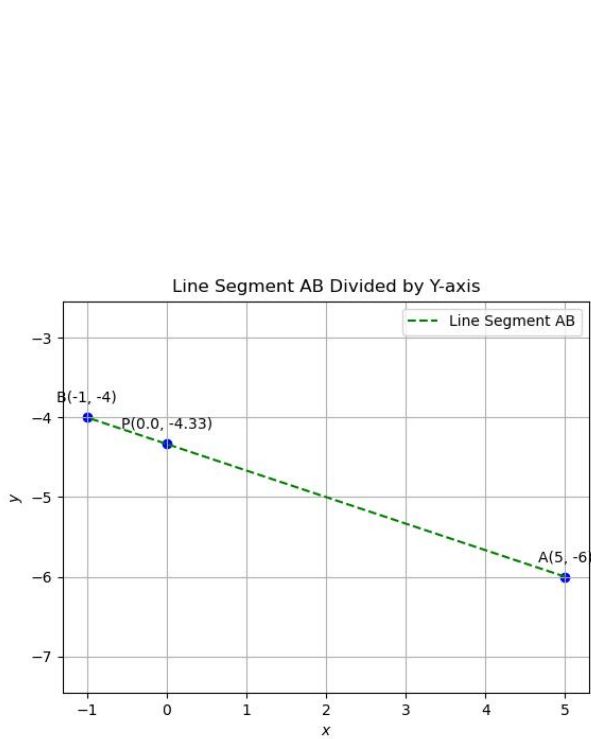
Thus, the ratio in which the Y-axis divides the line segment **AB** is **5:1**.

Now, we find the coordinates of the point of intersection, **P**, by substituting $k = 5$ into the equations for x and y components. The x component is 0. For the y component,

$$y = \frac{-4(5)-6}{5+1} = \frac{-20-6}{6} = \frac{-26}{6} = -\frac{13}{3} \quad (7)$$

\therefore The coordinates of the point of intersection are

$$\mathbf{P} = \begin{pmatrix} 0 \\ -\frac{13}{3} \end{pmatrix}$$



Plot of Intersection of AB by Y-axis