5.13.28

EE25BTECH11002 - Achat Parth Kalpesh

Question:
Let $\mathbf{A}$ be a 2×2 matrix with real entries. Let $\mathbf{I}$ be the 2×2 identity matrix. Denote by tr($\mathbf{A}$),the sum of diagonal entries of $\mathbf{A}$. Assume that $\mathbf{A}^2 = \mathbf{I}$.
Statement-1 : If $\mathbf{A} \neq \mathbf{I}$ and $\mathbf{A} \neq -\mathbf{I}$,then det($\mathbf{A}$)=-1
Statement-2 : If $\mathbf{A} \neq \mathbf{I}$ and $\mathbf{A} \neq -\mathbf{I}$,then $tr(\mathbf{A}) \neq 0$.

1) Statement-1 is false, Statement-2 is true
2) Statement-1 is true, Statement-2 is true;Statement-2 is a correct explanation for Statement-1
3) Statement-1 is true, Statement-2 is true;Statement-2 is not a correct explanation for Statement-1
4) Statement-1 is true, Statement-2 is false

Solution:
Let the eigenvalue of $\mathbf{A}$ be λ.

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{v} \tag{4.1}$$
$$\mathbf{A}^2\mathbf{v} = \lambda^2\mathbf{v} \tag{4.2}$$
$$\mathbf{A}^2 = \mathbf{I} \tag{4.3}$$
$$\lambda^2 = 1 \tag{4.4}$$
$$\lambda = \pm 1 \tag{4.5}$$

Thus, eigenvalues λ_1 , λ_2 of $\mathbf{A}$ are chosen from $\{1, -1\}$
As it is given as $\mathbf{A} \neq \mathbf{I}$ and $\mathbf{A} \neq -\mathbf{I}$, so the possible case is

$$\lambda_1 = 1, \lambda_2 = -1 \tag{4.6}$$

Thereby,

$$det(\mathbf{A}) = \lambda_1\lambda_2 \tag{4.7}$$
$$= -1 \tag{4.8}$$
$$tr(\mathbf{A}) = \lambda_1 + \lambda_2 \tag{4.9}$$
$$= 0 \tag{4.10}$$

Thus Statement-1 is true, Statement-2 is false