5.5.8

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Question : If

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{pmatrix}$$

find $\mathbf{A}^{-1}$. Hence, solve the system of equations

$$x + y + z = 6, \quad y + 3z = 11, \quad x - 2y + z = 0.$$

Solution :

Matrix	Value
$\mathbf{A}$	$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{pmatrix}$
$\mathbf{b}$	$\begin{pmatrix} 6 \\ 11 \\ 0 \end{pmatrix}$

Table : Equations

Forming the augmented matrix,

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 1 & -2 & 1 & 0 & 0 & 1 \end{array}\right) \tag{1}$$

Applying elementary row operations to find the inverse,

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 1 & -2 & 1 & 0 & 0 & 1 \end{array}\right) \xleftrightarrow{R_3 \to R_3 - R_1} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & -3 & 0 & -1 & 0 & 1 \end{array}\right) \xleftrightarrow{R_3 \to R_3 + 3R_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 9 & -1 & 3 & 1 \end{array}\right) \tag{2}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 9 & -1 & 3 & 1 \end{array}\right) \xleftrightarrow{R_3 \to \frac{R_3}{9}} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{array}\right) \xleftrightarrow[R_1 \to R_1 - R_3]{R_2 \to R_2 - 3R_3} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & \frac{10}{9} & -\frac{1}{3} & -\frac{1}{9} \\ 0 & 1 & 0 & \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{array}\right) \tag{3}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & \frac{10}{9} & -\frac{1}{3} & -\frac{1}{9} \\ 0 & 1 & 0 & \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{array}\right) \xleftrightarrow{R_1 \to R_1 - R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{7}{9} & -\frac{1}{3} & \frac{2}{9} \\ 0 & 1 & 0 & \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{array}\right) \tag{4}$$

The right side part of the augmented matrix is $\mathbf{A}^{-1}$

$$\mathbf{A}^{-1} = \begin{pmatrix} \frac{7}{9} & -\frac{1}{3} & \frac{2}{9} \\ \frac{1}{3} & 0 & -\frac{1}{3} \\ -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{pmatrix} \tag{5}$$

The solution for the system of equations is :

$$\mathbf{Ax} = \mathbf{b} \tag{6}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b} \tag{7}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{7}{9} & -\frac{1}{3} & \frac{2}{9} \\ \frac{1}{3} & 0 & -\frac{1}{3} \\ -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{pmatrix} \begin{pmatrix} 6 \\ 11 \\ 0 \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \tag{9}$$

Therefore the solution is :

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \tag{10}$$

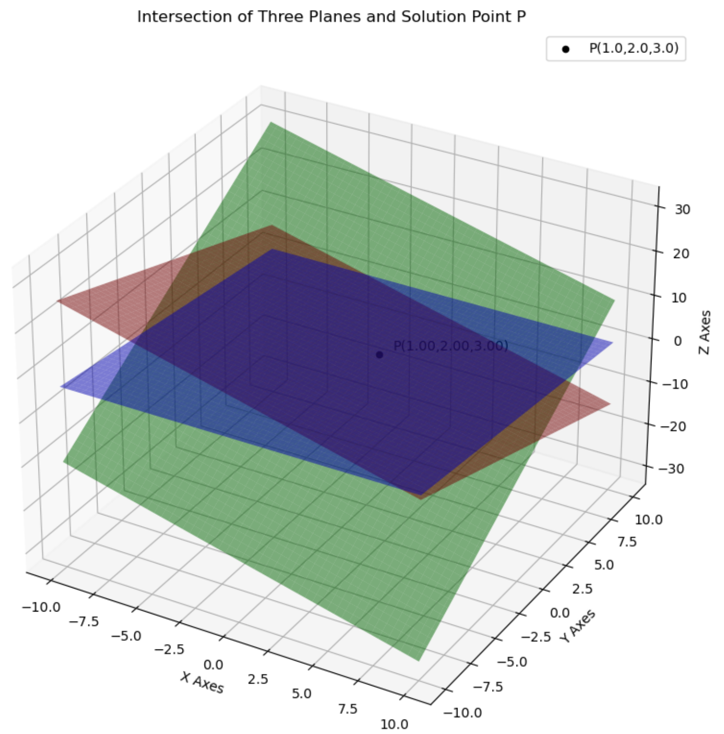
Intersection of Three Planes and Solution Point P
P(1.0,2.0,3.0)
P(1.00,2.00,3.00)
X Axes
Y Axes
Z Axes
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
30
20
10
0
−10
−20
−30

Fig : Planes