8.2.33

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September 19,2025

Question

Find the equation of the conic with length of major axis 26, foci $(\pm 5, 0)$.

Theoretical Solution

The equation of a conic is

$$\mathbf{x}^T V \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \tag{1}$$

where

$$V = \|\mathbf{n}\|^2 I - e^2 \mathbf{n}\mathbf{n}^T \tag{2}$$

The foci are

$$\mathbf{F}_1 = \begin{pmatrix} 5 \\ 0 \end{pmatrix}, \quad \mathbf{F}_2 = \begin{pmatrix} -5 \\ 0 \end{pmatrix} \tag{3}$$

The centre is

$$\mathbf{u} = \frac{\mathbf{F}_1 + \mathbf{F}_2}{2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{4}$$

The axis vector is

$$\mathbf{n} = \mathbf{F}_1 - \mathbf{F}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{5}$$

Theoretical Solution

Therefore, substituting $\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ in (2), we get

$$V = \begin{pmatrix} 1 - e^2 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

From the formula for the length of the major axis,

$$2\sqrt{\frac{|f|}{\lambda_1}} \tag{7}$$

where $\lambda_1 = 1 - e^2$. Hence

$$26 = 2\sqrt{\frac{|f|}{1 - e^2}} \tag{8}$$

The relation between focus and eccentricity is

$$\pm \mathbf{c} e^2 = 5 \tag{9}$$

Theoretical Solution

The distance $\mathbf{c}$ is

$$\mathbf{c} = \pm\frac{1}{e}\sqrt{\frac{|f|}{|e^2-1|}} \tag{10}$$

Thus from (8)–(10), solving the unknowns $(\mathbf{c}, e, f)$ we get

$$e = \tfrac{5}{13}, \quad \mathbf{c} = \pm 5, \quad |f| = 144. \tag{11}$$

Let $\mathbf{x} = \begin{pmatrix} 0 \\ \alpha \end{pmatrix}$ be a vertex on the minor axis. Substituting in (1):

$$\frac{12^2}{1} + f = 0 \implies f = -144. \tag{12}$$

Hence the equation of the conic is

$$\mathbf{x}^T \begin{pmatrix} \frac{144}{169} & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} - 144 = 0. \tag{13}$$

C Code

```c
#include <stdio.h>
#include <math.h>

#define PI 3.1415926535

double calculate_circular_sector_area() {
    double radius = 2.0;
    double angle_in_radians = PI / 6.0;
    double area = 0.5 * radius * radius * angle_in_radians;
    return area;
}
```

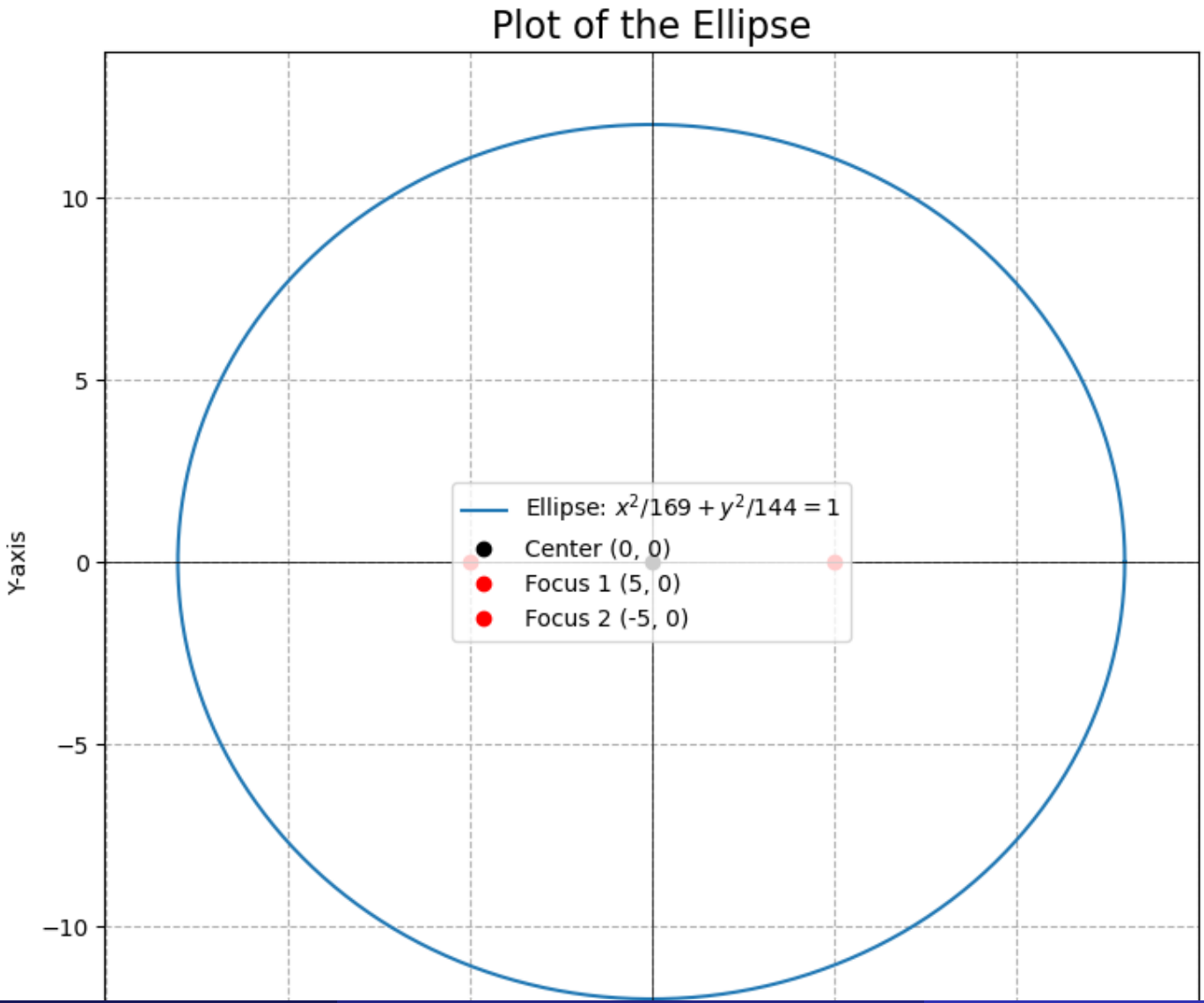
Python Code

```
import numpy as np
import matplotlib.pyplot as plt

a = 13
b = 12
c = 5
theta = np.linspace(0, 2 * np.pi, 200)

x = a * np.cos(theta)
y = b * np.sin(theta)

plt.figure(figsize=(10, 8))
ax = plt.gca()

ax.plot(x, y, label='Ellipse: $x^2/169 + y^2/144 = 1$')

ax.plot(0, 0, 'ko', label='Center (0, 0)')
ax.plot(c, 0, 'ro', label='Focus 1 (5, 0)')
ax.plot(-c, 0, 'ro', label='Focus 2 (-5, 0)')
```

Python Code

```
ax.set_title('Plot of the Ellipse', fontsize=16)
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')

ax.set_aspect('equal', adjustable='box')

ax.grid(True, linestyle='--')
ax.legend()

ax.set_xlim(-a - 2, a + 2)
ax.set_ylim(-b - 2, b + 2)

ax.axhline(0, color='black', linewidth=0.5)
ax.axvline(0, color='black', linewidth=0.5)

plt.show()
```

Plot


Plot of the Ellipse
Ellipse: $x^2/169 + y^2/144 = 1$
Center (0, 0)
Focus 1 (5, 0)
Focus 2 (-5, 0)
Y-axis
10
5
0
−5
−10