12.589

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Question : Solve the system of equations and find the condition for which the system has **no solution**:

$$x + y + z = 6$$
$$x + 4y + 6z = 20$$
$$x + 4y + \lambda z = \mu$$

Solution :

Name	Value
Equation 1	$x + y + z = 6 \iff \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{x} = 6$
Equation 2	$x + 4y + 6z = 20 \iff \begin{pmatrix} 1 & 4 & 6 \end{pmatrix} \mathbf{x} = 20$
Equation 3	$x + 4y + \lambda z = \mu \iff \begin{pmatrix} 1 & 4 & \lambda \end{pmatrix} \mathbf{x} = \mu$

Table : Equations

The system of equations in matrix form is :

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 4 & 6 \\ 1 & 4 & \lambda \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 20 \\ \mu \end{pmatrix} \tag{1}$$

Forming the augmented matrix,

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 1 & 4 & \lambda & \mu \end{array}\right) \tag{2}$$

Using Gaussian elimination,

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 1 & 4 & \lambda & \mu \end{array}\right) \xleftrightarrow[R_2 \to R_2 - R_1]{R_3 \to R_3 - R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & 5 & 14 \\ 0 & 3 & \lambda - 1 & \mu - 6 \end{array}\right) \tag{3}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & 5 & 14 \\ 0 & 3 & \lambda - 1 & \mu - 6 \end{array}\right) \xleftrightarrow{R_3 \to R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & 5 & 14 \\ 0 & 0 & \lambda - 6 & \mu - 20 \end{array}\right) \tag{4}$$

From back substitution we get:

$$(\lambda - 6)z = \mu - 20 \tag{5}$$

For the system to have **no solution**, we must have

$$\lambda = 6 \qquad \mu \neq 20 \tag{6}$$

So, we get zero equal to a non-zero value which is not possible .
Therefore the system of equations has **no solution**.
Final Answer : The system has no solution when $\lambda = 6$ and $\mu \neq 20$.

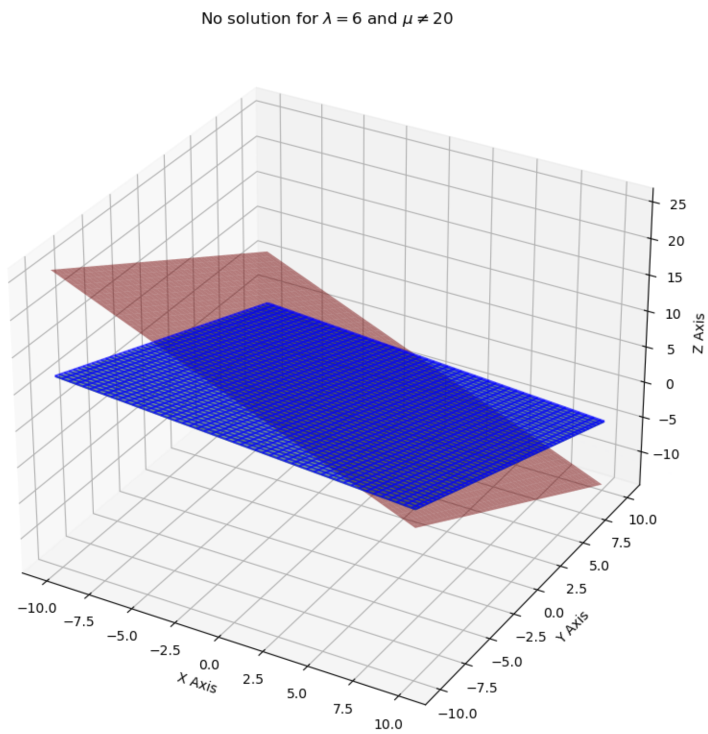
No solution for $\lambda = 6$ and $\mu \neq 20$
X Axis
Y Axis
Z Axis
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
−10
−5
0
5
10
15
20
25

Fig : Planes