

Matrices in Geometry - 1.9.24

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Aug, 2025

Problem Statement

The x-coordinate of a point **P** is twice its y-coordinate. If **P** is equidistant from the points **Q**(2, -5) and **R**(-3, 6), find the coordinates of **P**.

Solution

Given $\mathbf{P} \begin{pmatrix} 2k \\ k \end{pmatrix}$, $\mathbf{Q} \begin{pmatrix} 2 \\ -5 \end{pmatrix}$, $\mathbf{R} \begin{pmatrix} -3 \\ 6 \end{pmatrix}$.

Distances $PQ = PR$

So their norms must be equal and also the square of their norms.

$$\|\mathbf{PQ}\|^2 = \|\mathbf{PR}\|^2 \quad (1)$$

$$\|\mathbf{P}\|^2 - 2\mathbf{P}^\top \mathbf{Q} + \|\mathbf{Q}\|^2 = \|\mathbf{P}\|^2 - 2\mathbf{P}^\top \mathbf{R} + \|\mathbf{R}\|^2 \quad (2)$$

$$\implies \frac{\|\mathbf{Q}\|^2 - \|\mathbf{R}\|^2}{2} = \mathbf{P}^\top (\mathbf{Q} - \mathbf{R}) \quad (3)$$

Solution

$$\|\mathbf{Q}\|^2 = (2 \quad -5) \begin{pmatrix} 2 \\ -5 \end{pmatrix} = 4 + 25 = 29$$

$$\|\mathbf{R}\|^2 = (-3 \quad 6) \begin{pmatrix} -3 \\ 6 \end{pmatrix} = 9 + 36 = 45$$

$$\mathbf{Q} - \mathbf{R} = \begin{pmatrix} 5 \\ -11 \end{pmatrix}$$

$$\implies -8 = (2k \quad k) \begin{pmatrix} 5 \\ -11 \end{pmatrix} \tag{4}$$

$$-8 = 10k - 11k = -k \tag{5}$$

$$\implies \boxed{k = 8} \tag{6}$$

Final Answer

$$k = 8$$

The coordinates of point P are (16,8).

