Matgeo : 2.6.12

AI25BTECH11006 - Nikhila

Problem: Find the sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + -2\hat{j} + 4\hat{k}$

Solution:
We know that

$$\langle \boldsymbol{a}, \boldsymbol{b} \rangle = \|\boldsymbol{a}\|\|\boldsymbol{b}\| \cos\theta,$$

where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.
Thus,

$$\cos\theta = \frac{\langle \boldsymbol{a}, \boldsymbol{b} \rangle}{\|\boldsymbol{a}\|\|\boldsymbol{b}\|}.$$

Calculating each term:

$$\langle \boldsymbol{a}, \boldsymbol{b} \rangle = 3(2) + 1(-2) + 2(4) = 6 - 2 + 8 = 12, \tag{1}$$

$$\|\boldsymbol{a}\| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{14}, \tag{2}$$

$$\|\boldsymbol{b}\| = \sqrt{2^2 + (-2)^2 + 4^2} = \sqrt{24}, \tag{3}$$

$$\cos\theta = \frac{12}{\sqrt{14} \cdot \sqrt{24}} = \frac{12}{\sqrt{336}} = \frac{3}{\sqrt{21}}. \tag{4}$$

Now, to find $\sin\theta$:

$$\sin\theta = \sqrt{1 - \cos^2\theta} \tag{5}$$

$$= \sqrt{1 - \left(\frac{3}{\sqrt{21}}\right)^2} \tag{6}$$

$$= \sqrt{1 - \frac{9}{21}} \tag{7}$$

$$= \sqrt{\frac{12}{21}} \tag{8}$$

$$= \frac{2}{\sqrt{7}}. \tag{9}$$

The sine of the angle between the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is $\frac{2}{\sqrt{7}}$.

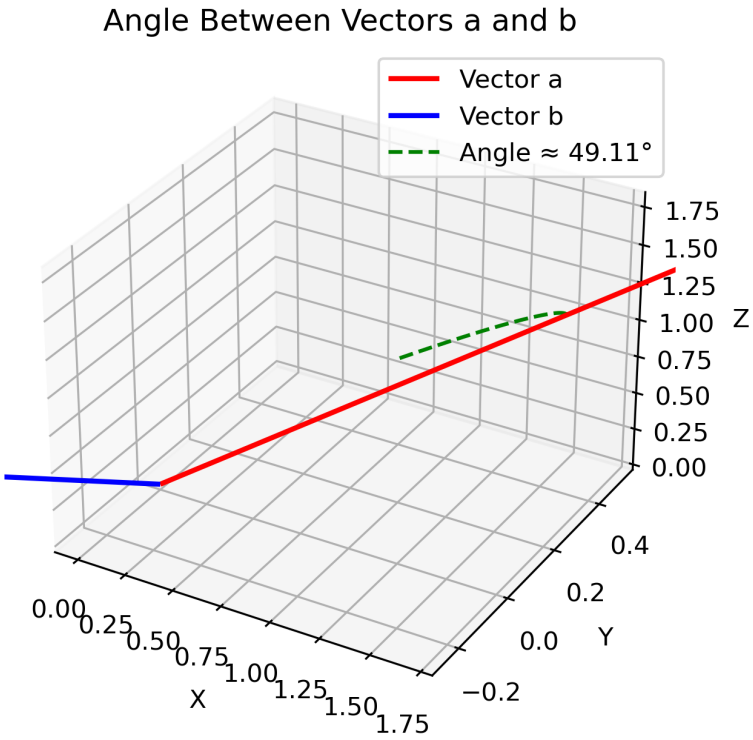
Angle Between Vectors a and b
Vector a
Vector b
Angle ≈ 49.11°
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
Z
0.4
0.2
0.0
−0.2
Y
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
X

Fig. 1.