

## 7.4.26

EE25BTECH11019 – Darji Vivek M.

# Question

## Question:

If two distinct chords, drawn from the point  $(p, q)$  on the circle

$$x^2 + y^2 = px + qy$$

(where  $pq \neq 0$ ) are bisected by the  $X$ -axis, then which of the following is true?

- ①  $p^2 = q^2$
- ②  $p^2 = 8q^2$
- ③  $p^2 < 8q^2$
- ④  $p^2 > 8q^2$

# Solution

Let

$$\mathbf{P} = \begin{pmatrix} p \\ q \end{pmatrix}, \quad \mathbf{c} = \frac{1}{2}\mathbf{P}, \quad r = \frac{1}{2}\sqrt{\mathbf{P}^\top \mathbf{P}}. \quad (1)$$

Hence, the circle in vector form is  $\|\mathbf{x} - \mathbf{c}\| = r$ .

Let the midpoint of a chord through  $\mathbf{P}$  lying on the  $x$ -axis be

$$\mathbf{M} = \begin{pmatrix} h \\ 0 \end{pmatrix}, \quad (2)$$

and the other end of the chord be

$$\mathbf{B} = 2\mathbf{M} - \mathbf{P}. \quad (3)$$

Since  $\mathbf{B}$  lies on the circle,

$$(\mathbf{B} - \mathbf{c})^\top (\mathbf{B} - \mathbf{c}) = r^2. \quad (4)$$

# Solution

Substitute  $\mathbf{B} = 2\mathbf{M} - \mathbf{P}$  and  $\mathbf{c} = \frac{1}{2}\mathbf{P}$ :

$$\left(2\mathbf{M} - \frac{3}{2}\mathbf{P}\right)^{\top} \left(2\mathbf{M} - \frac{3}{2}\mathbf{P}\right) = \frac{1}{4}\mathbf{P}^{\top}\mathbf{P}. \quad (5)$$

Expand and simplify:

$$4\mathbf{M}^{\top}\mathbf{M} - 6\mathbf{M}^{\top}\mathbf{P} + \frac{9}{4}\mathbf{P}^{\top}\mathbf{P} = \frac{1}{4}\mathbf{P}^{\top}\mathbf{P} \quad (6)$$

$$\implies 4\mathbf{M}^{\top}\mathbf{M} - 6\mathbf{M}^{\top}\mathbf{P} + 2\mathbf{P}^{\top}\mathbf{P} = 0. \quad (7)$$

With  $\mathbf{M} = \begin{pmatrix} h \\ 0 \end{pmatrix}$ ,

$$\mathbf{M}^{\top}\mathbf{M} = h^2, \quad \mathbf{M}^{\top}\mathbf{P} = ph.$$

Substitute:

$$4h^2 - 6ph + 2\mathbf{P}^{\top}\mathbf{P} = 0.$$

Divide by 2:

$$2h^2 - 3ph + \mathbf{P}^{\top}\mathbf{P} = 0.$$

# Solution

For two distinct chords, this quadratic in  $h$  must have two distinct real roots:

$$\Delta = 9p^2 - 8\mathbf{P}^\top \mathbf{P} > 0 \quad (8)$$

$$= 9p^2 - 8(p^2 + q^2) > 0 \quad (9)$$

$$\implies p^2 - 8q^2 > 0. \quad (10)$$

$$\boxed{p^2 > 8q^2}$$

Hence, option (d) is correct.

## C Code: parallel\_funcs.c

<https://github.com/vivekd03/ee1030-2025/blob/main/ee25btech11019/matgeo/7.4.26/codes/14.c>

# Python: Plotting Lines

```
https://github.com/vivekd03/ee1030-2025/blob/main/ee25btech11019/  
matgeo/7.4.26/codes/14.py
```

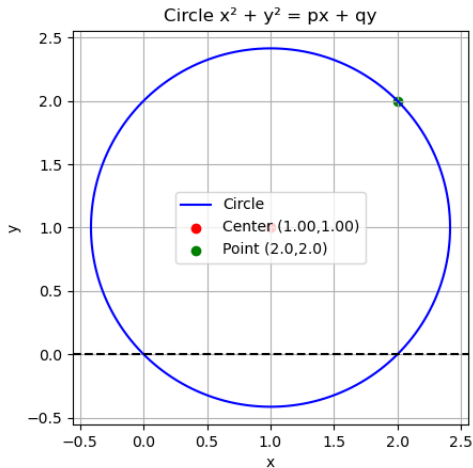


Figure: plot if  $p=2, q=2$