5.13.52

EE25BTECH11026-Harsha

Question:
If the system of equations $x + ay = 0, az + y = 0$ and $ax + z = 0$ has infinite solutions, then the value of a is

1) -1 2) 1 3) 0 4) no real values

Solution:
Let us solve the given question theoretically and then verify the solution computationally.

From the given,

$$\begin{pmatrix} 1 & a & 0 \end{pmatrix} \mathbf{x} = 0 \tag{4.1}$$

$$\begin{pmatrix} 0 & 1 & a \end{pmatrix} \mathbf{x} = 0 \tag{4.2}$$

$$\begin{pmatrix} a & 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{4.3}$$

$$\therefore \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{4.4}$$

To solve for a, we can use the fact that of rank of coefficient matrix should be less than 3.

$$\begin{pmatrix} 1 & a & 0 \\ 0 & 1 & a \\ a & 0 & 1 \end{pmatrix} \xleftrightarrow[R_3 \leftarrow R_3 + a^2 \times R_2]{R_3 \leftarrow R_3 - a \times R_1} \begin{pmatrix} 1 & a & 0 \\ 0 & 1 & a \\ 0 & 0 & a^3 + 1 \end{pmatrix} \tag{4.5}$$

As the rank of the matrix should be less than 3, we require the last pivot to be zero.

$$\therefore a^3 + 1 = 0 \implies a = -1, -\omega, -\omega^2 \tag{4.6}$$