AI25BTECH11034 - SUJAL CHAUHAN
2.10.23

Question:
The vector(s) which is/are coplanar with the vectors $\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}+\hat{k}$, and perpendicular to vector $\hat{i}+\hat{j}+\hat{k}$ is/are.

a) $\hat{\mathbf{j}}-\hat{\mathbf{k}}$

b) $\hat{\mathbf{i}}+\hat{\mathbf{j}}$

c) $\hat{\mathbf{i}}-\hat{\mathbf{j}}$

d) $\hat{\mathbf{j}}+\hat{\mathbf{k}}$

Variable	Vector
$\mathbf{A}$	$\begin{pmatrix}1\\1\\2\end{pmatrix}$
$\mathbf{B}$	$\begin{pmatrix}1\\2\\1\end{pmatrix}$
$\mathbf{C}$	$\begin{pmatrix}1\\1\\1\end{pmatrix}$

Listing options as vectors $\mathbf{D_i}$:

Input	Vector
$\mathbf{D_1}$	$\begin{pmatrix}0\\1\\-1\end{pmatrix}$
$\mathbf{D_2}$	$\begin{pmatrix}1\\1\\1\end{pmatrix}$
$\mathbf{D_3}$	$\begin{pmatrix}1\\-1\\0\end{pmatrix}$
$\mathbf{D_4}$	$\begin{pmatrix}0\\1\\1\end{pmatrix}$

Checking conditions

Let equation of plane be given by:

$$\mathbf{n}^\top \mathbf{X} = 1 \tag{1}$$

Let's find general solution $\mathbf{n}$ which is perpendicular to the plane

$$\begin{pmatrix} A & B \end{pmatrix}^\top \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2}$$

$$\text{Given:} \quad \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{3}$$

$$\tag{4}$$

$$\text{We want to solve:} \quad \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} \tag{5}$$

Form the augmented matrix:

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & 2 & 1 & 1 \end{array}\right) \tag{6}$$

$$\xrightarrow{R_2 \to R_2 - R_1} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -1 & 0 \end{array}\right) \tag{7}$$

$$\xrightarrow{R_1 \to R_1 - R_2} \left(\begin{array}{ccc|c} 1 & 0 & 3 & 1 \\ 0 & 1 & -1 & 0 \end{array}\right) \tag{8}$$

Thus the equations are:

$$n_1 + 3n_3 = 1, \quad n_2 - n_3 = 0 \tag{9}$$

$$\text{Let } n_3 = a \in \mathbb{R}, \quad n_2 = a, \quad n_1 = 1 - 3a \tag{10}$$

So the general solution is:

$$\mathbf{n} = \begin{pmatrix} 1 - 3a \\ a \\ a \end{pmatrix}, \quad a \in \mathbb{R} \tag{11}$$

General solution is

$$\mathbf{n} = \begin{pmatrix} 1-3a \\ a \\ a \end{pmatrix} \tag{12}$$

Now any vector following both condition will be solution of the equation:

$$\begin{pmatrix} n & C \end{pmatrix}^\top \mathbf{D_i} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{13}$$

Checking Values for all options:

$$\begin{pmatrix} 1-3a & a & a \\ 1 & 1 & 1 \end{pmatrix} \mathbf{D_i} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{14}$$

Vector	$\begin{pmatrix} 1-3a & a & a \\ 1 & 1 & 1 \end{pmatrix} \mathbf{D_i}$	Satisfies
$\mathbf{D_1}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Yes
$\mathbf{D_2}$	$\begin{pmatrix} 1-2a \\ 2 \end{pmatrix}$	No
$\mathbf{D_3}$	$\begin{pmatrix} 1-4a \\ 0 \end{pmatrix}$	No
$\mathbf{D_4}$	$\begin{pmatrix} 2a \\ 2 \end{pmatrix}$	No

So only $\mathbf{D_1}$ satisfies both conditions.

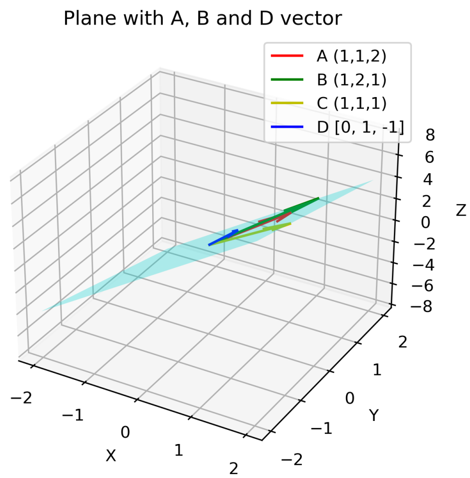

Figure 1: Vector $\mathbf{D_1}$ in plane

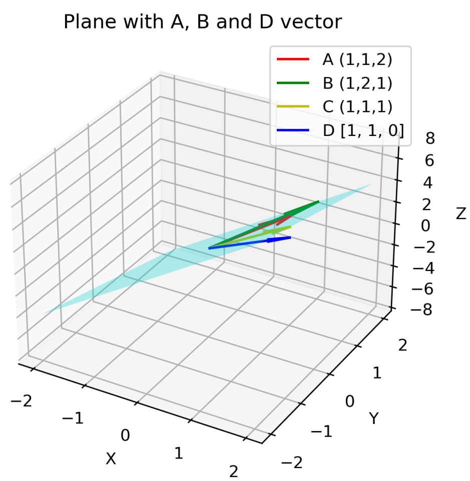

Figure 2: Vector $\mathbf{D_2}$ not coplanar

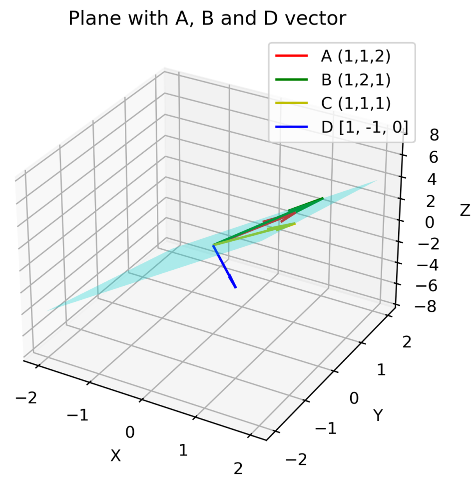

Figure 3: Vector $\mathbf{D_3}$ not coplanar

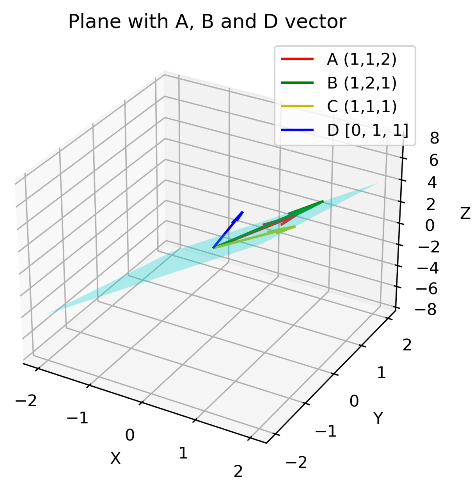

Figure 4: Vector $\mathbf{D_4}$ not coplanar