9.4.25

Bhargav - EE25BTECH11013

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Question

Question:
Find the roots of the quadratic equation graphically.

$$5x^2 - 6x - 2 = 0 \tag{1}$$

Solution

$$y = 5x^2 - 6x - 2 = 0 \tag{2}$$

This equation can be represented as the conic

$$\mathbf{x^T V x} + 2\mathbf{u^T x} + f = 0 \tag{3}$$

$$\mathbf{V} = \begin{pmatrix} 5 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}, f = -2 \tag{4}$$

Solution

To find the roots, we find the points of intersection of the conic with the x-axis.

$$\mathbf{x} = \mathbf{h} + k_i \mathbf{m} \tag{5}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{6}$$

Solution

The value of k_i can be found out by solving the line and conic equation

$$(\mathbf{h} + k_i\mathbf{m})^\top \mathbf{V}(\mathbf{h} + k_i\mathbf{m}) + 2\mathbf{u}^\top(\mathbf{h} + k_i\mathbf{m}) + f = 0 \tag{7}$$

$$\implies k_i^2\mathbf{m}^\top\mathbf{V}\mathbf{m} + 2k_i\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) + \mathbf{h}^\top\mathbf{V}\mathbf{h} + 2\mathbf{u}^\top\mathbf{h} + f = 0 \tag{8}$$

$$\text{or, } k_i^2\mathbf{m}^\top\mathbf{V}\mathbf{m} + 2k_i\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) + g(\mathbf{h}) = 0 \tag{9}$$

Solution

Solving the above quadratic gives the equation

$$k_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{\left[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})\right]^2 - g(\mathbf{h}) \left(\mathbf{m}^\top \mathbf{V}\mathbf{m}\right)} \right) \tag{10}$$

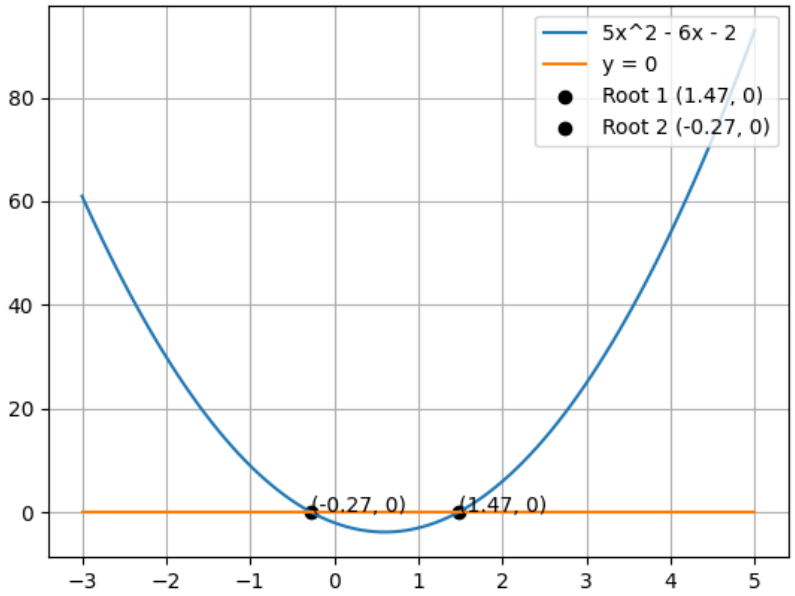
$$\therefore k_i = \frac{3}{5} \pm \frac{\sqrt{19}}{5} \tag{11}$$

Solution

$$\implies k_1 = \frac{3}{5} + \frac{\sqrt{19}}{5}, \quad k_2 = \frac{3}{5} - \frac{\sqrt{19}}{5} \tag{12}$$

$$\therefore \mathbf{x} = \mathbf{h} + k_i\mathbf{m} = \begin{pmatrix} \frac{3}{5} + \frac{\sqrt{19}}{5} \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{3}{5} - \frac{\sqrt{19}}{5} \\ 0 \end{pmatrix} \tag{13}$$

Plot


5x^2 - 6x - 2
y = 0
Root 1 (1.47, 0)
Root 2 (-0.27, 0)
(-0.27, 0)
(1.47, 0)
80
60
40
20
0
−3
−2
−1
0
1
2
3
4
5

C Code

```c
#include <stdio.h>
#include <math.h>

double root1(double a, double b, double c) {
    double d = b*b - 4*a*c;
    return (-b + sqrt(d)) / (2*a);
}

double root2(double a, double b, double c) {
    double d = b*b - 4*a*c;
    return (-b - sqrt(d)) / (2*a);
}
```

Python + C Code

```
import numpy as np
import matplotlib.pyplot as plt
import ctypes
lib = ctypes.CDLL(./libcode.so)
lib.root1.argtypes = [ctypes.c_double, ctypes.c_double, ctypes.
    c_double]
lib.root1.restype = ctypes.c_double
lib.root2.argtypes = [ctypes.c_double, ctypes.c_double, ctypes.
    c_double]
lib.root2.restype = ctypes.c_double
def quadratic(a, b, c):
    x1 = lib.root1(a, b, c)
    x2 = lib.root2(a, b, c)
    return x1, x2
def function(x):
    return 5*(x**2) - 6*x - 2
x = np.linspace(-3, 5, 100)
y = function(x)
```

Python + C Code

```
y1 = np.zeros(100)
x1, x2 = quadratic(5, -6, -2)
fig, ax = plt.subplots()
ax.plot(x, y, label='5x^2 - 6x - 2')
ax.plot(x, y1, label='y = 0')
ax.scatter(x1, 0, color=black, label=f'Root 1 ({x1:.2f}, 0)')
ax.text(x1, 0, f'({x1:.2f}, 0)')
ax.scatter(x2, 0, color=black, label=f'Root 2 ({x2:.2f}, 0)')
ax.text(x2, 0, f'({x2:.2f}, 0)')
ax.grid(True)
ax.legend(loc=upper right)
plt.savefig(/Users/bhargavkrish/Desktop/BackupMatrix/
    ee25btech11013/matgeo/9.4.25/figs/Figure_1.png)
plt.show()
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
def function(x):
  return 5*(x**2) - 6*x - 2;
x = np.linspace(-3, 5, 100)
y = function(x)
y1 = np.zeros(100)
def quadratic(a, b, c):
  d = b**2 - 4*a*c
  x1 = (-b + np.sqrt(d)) / (2*a)
  x2 = (-b - np.sqrt(d)) / (2*a)
  return x1, x2

x1, x2 = quadratic(5, -6, -2)
fig, ax = plt.subplots()
ax.plot(x, y, label='5x^2 - 6x - 2')
ax.plot(x, y1, label='y = 0')
```

Python Code

```
ax.scatter(x1, 0, color=black, label=f'Root 1 ({x1:.2f}, 0)')
ax.text(x1, 0, f'({x1:.2f}, 0)')
ax.scatter(x2, 0, color=black, label=f'Root 2 ({x2:.2f}, 0)')
ax.text(x2, 0, f'({x2:.2f}, 0)')
ax.grid(True)
ax.legend()
ax.legend(loc=upper right)
plt.savefig(/Users/bhargavkrish/Desktop/BackupMatrix/
    ee25btech11013/matgeo/9.4.25/figs/Figure_1.png)
plt.show()
```