4.8.6

AI25BTECH11010 - Dhanush Kumar

Question: Find the coordinates of the foot of the perpendicular **Q** drawn from $P(3, 2, 1)$ to the plane $2x - y + z + 1 = 0$. Also find the distance **PQ** and the image of the point **P** treating this plane as a mirror.

Solution:

The point and the plane normal are

$$\mathbf{P} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \qquad \mathbf{n} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \tag{1}$$

$$\mathbf{n}^T \mathbf{x} = -1. \tag{2}$$

Let **Q** be the foot of the perpendicular from **P** to the plane and let λ be the scalar such that

$$\mathbf{P} - \mathbf{Q} = \lambda \mathbf{n} \tag{3}$$

$$\mathbf{n}^T \mathbf{Q} = -1. \tag{4}$$

we have $\mathbf{Q} = \mathbf{P} - \lambda \mathbf{n}$. Therefore,

$$\mathbf{n}^T (\mathbf{P} - \lambda \mathbf{n}) = -1 \tag{5}$$

$$\mathbf{n}^T \mathbf{P} - \lambda \|\mathbf{n}\|^2 = -1. \tag{6}$$

Thus

$$5 - 6\lambda = -1 \quad \Rightarrow \quad \lambda = -1. \tag{7}$$

Therefore

$$\mathbf{Q} = \mathbf{P} - \lambda \mathbf{n} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} - (-1) \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}. \tag{8}$$

Distance:

$$PQ = \|\mathbf{P} - \mathbf{Q}\| = \|\lambda \mathbf{n}\| = |\lambda| \|\mathbf{n}\| = 1 \cdot \sqrt{6} = \sqrt{6}. \tag{9}$$

Image of **P** in the plane (reflection) is

$$\mathbf{R} = 2\mathbf{Q} - \mathbf{P} = 2 \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ -1 \end{pmatrix}. \tag{10}$$

Answer: $\mathbf{Q} = (1, 3, 0)$, $PQ = \sqrt{6}$, $\mathbf{R} = (-1, 4, -1)$.

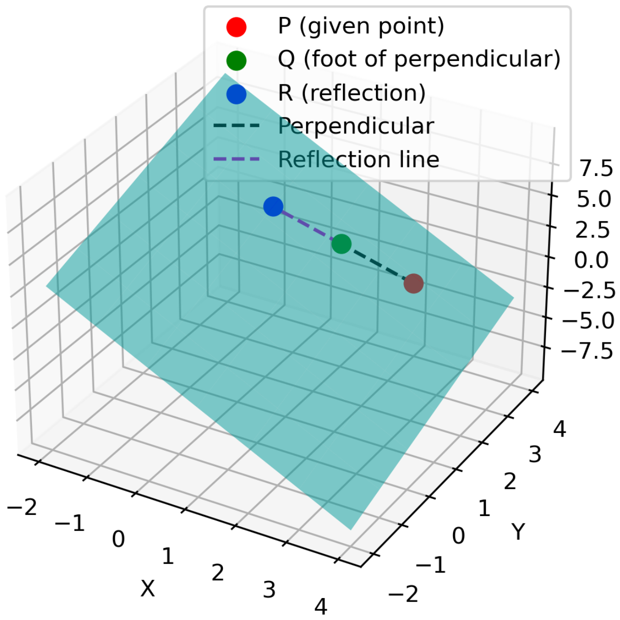
P (given point)
Q (foot of perpendicular)
R (reflection)
Perpendicular
Reflection line
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−2
−1
0
1
2
3
4
X
Y

Fig. 0