

2.5.2

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Question The points $(-4, 0), (4, 0), (0, 3)$ are the vertices of a:

- (a) right triangle
- (b) isosceles triangle
- (c) equilateral triangle
- (d) scalene triangle

Solution:

Step 1: Represent points as column vectors

$$\mathbf{A} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad (1)$$

Step 2: Check for right-angled triangle (perpendicular sides)

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}, \quad \mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \quad (\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} 8 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = 32 \neq 0 \quad (2)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} -8 \\ 0 \end{pmatrix}, \quad \mathbf{C} - \mathbf{B} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}, \quad (\mathbf{A} - \mathbf{B})^\top (\mathbf{C} - \mathbf{B}) = \begin{pmatrix} -8 & 0 \end{pmatrix} \begin{pmatrix} -4 \\ 3 \end{pmatrix} = 32 \neq 0 \quad (3)$$

$$\mathbf{A} - \mathbf{C} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}, \quad \mathbf{B} - \mathbf{C} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}, \quad (\mathbf{A} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C}) = \begin{pmatrix} -4 & -3 \end{pmatrix} \begin{pmatrix} 4 \\ -3 \end{pmatrix} = -7 \neq 0 \quad (4)$$

Since no pair of sides is perpendicular, the triangle is not right-angled.

Step 3: Check for isosceles triangle (perpendicular bisector method)

$$\text{Midpoint of } AB : \quad \mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (5)$$

$$\mathbf{C} - \mathbf{M} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad (6)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \quad (7)$$

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{M}) = \begin{pmatrix} 8 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 3 \end{pmatrix} = 0 \quad (8)$$

Hence, C lies on the perpendicular bisector of AB .

$$AC = BC = 5 \implies \triangle ABC \text{ is isosceles.}$$

Conclusion: Therefore, the triangle with vertices $(-4, 0), (4, 0), (0, 3)$ is an **isosceles triangle** with $AC = BC = 5$.

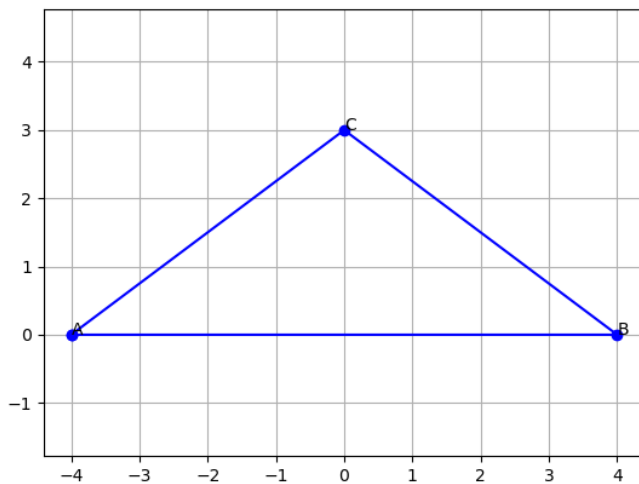


Fig. 1: Shared Output Plot

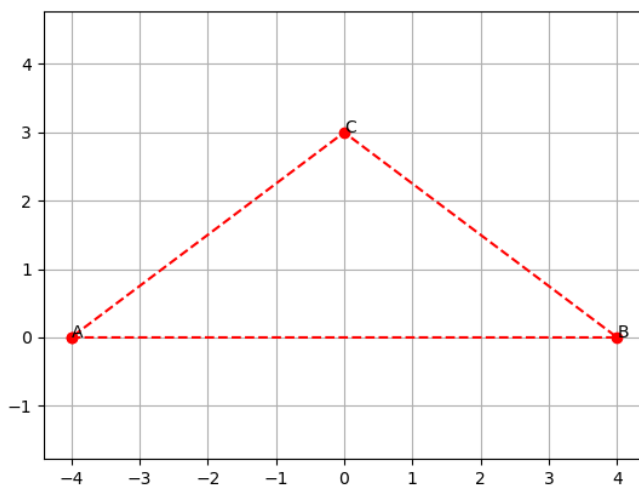


Fig. 2: Direct Python code plot