

1.8.3

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Question

Find the angle between two vectors **a** and **b** with magnitudes 1 and 2 respectively and when $\mathbf{a} \cdot \mathbf{b} = 1$.

Solution

We use the dot product formula:

$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \|\mathbf{b}\| \cos \theta \quad (1)$$

Substitute the given values:

$$1 = (1)(2) \cos \theta \Rightarrow \cos \theta = \frac{1}{2} \quad (2)$$

$$\theta = \cos^{-1} \left(\frac{1}{2} \right) = 60^\circ \quad (3)$$

Matrix Representation

Let:

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad (4)$$

Then:

$$\mathbf{a}^T \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = 1 \quad (5)$$

$$\|\mathbf{a}\| = \sqrt{a_1^2 + a_2^2 + a_3^2} = 1, \quad \|\mathbf{b}\| = \sqrt{b_1^2 + b_2^2 + b_3^2} = 2 \quad (6)$$

So the angle is:

$$\theta = \cos^{-1} \left(\frac{\mathbf{a}^T \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} \right) = \cos^{-1} \left(\frac{1}{2} \right) = 60^\circ \quad (7)$$

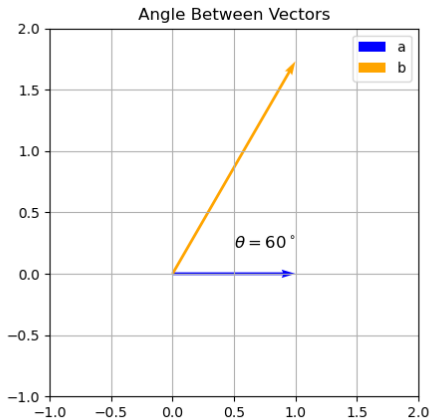


Figure: