1.8.15

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QUESTION
Find the value of a, if the distance between the points $A\begin{pmatrix} -3 \\ -14 \end{pmatrix}$ and $B\begin{pmatrix} a \\ -5 \end{pmatrix}$ is 9 units.

SOLUTION

$$\mathbf{A} = \begin{pmatrix} -3 \\ -14 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} a \\ -5 \end{pmatrix} \tag{0.1}$$

$$\|\mathbf{A} - \mathbf{B}\| = 9 \tag{0.2}$$

$$\implies \left\| \begin{pmatrix} -3 \\ -14 \end{pmatrix} - \begin{pmatrix} a \\ -5 \end{pmatrix} \right\| = 9 \tag{0.3}$$

$$\implies \left\| \begin{pmatrix} -3 - a \\ -9 \end{pmatrix} \right\| = 9 \tag{0.4}$$

$$\implies (-3 - a)^2 + (-9)^2 = 9^2 \tag{0.5}$$

$$(a + 3)^2 + 81 = 81 \tag{0.6}$$

$$(a + 3)^2 = 0 \tag{0.7}$$
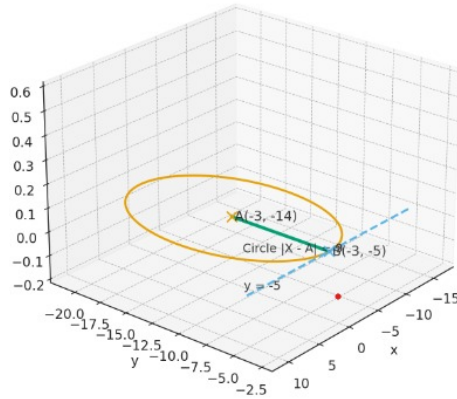
$$a = -3 \tag{0.8}$$

$$\boxed{a = -3}$$

Fig. 0.1: Circle centered at $A(-3, -14)$ with radius 9 intersecting the line $y = -5$ at $B(-3, -5)$.