# 5.5.17

EE25BTECH11065 - Yoshita J

**Question**

If

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{pmatrix},$$

find $\mathbf{A}^{-1}$ using elementary row transformations. Hence, solve the system:

$$\begin{aligned} x + y + z &= 6 \\ y + 3z &= 11 \\ x - 2y + z &= 0 \end{aligned}$$

**Solution:**

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{pmatrix} \tag{1}$$

The augmented matrix is:

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 1 & -2 & 1 & 0 & 0 & 1 \end{array}\right) \tag{2}$$

Row operations:

$$R_3 \to R_3 - R_1 \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & -3 & 0 & -1 & 0 & 1 \end{array}\right) \tag{3}$$

$$R_3 \to R_3 + 3R_2 \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 9 & -1 & 3 & 1 \end{array}\right) \tag{4}$$

$$R_3 \to \frac{1}{9}R_3 \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{array}\right) \tag{5}$$

$$R_1 \to R_1 - R_3, \quad R_2 \to R_2 - 3R_3 \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & \frac{10}{9} & -\frac{1}{3} & -\frac{1}{9} \\ 0 & 1 & 0 & \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{array}\right) \tag{6}$$

$$R_1 \to R_1 - R_2 \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{7}{9} & -\frac{1}{3} & \frac{2}{9} \\ 0 & 1 & 0 & \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{array}\right) \tag{7}$$

As the left block becomes identity, the right block is $\mathbf{A}^{-1}$:

$$\mathbf{A}^{-1} = \begin{pmatrix} \frac{7}{9} & -\frac{1}{3} & \frac{2}{9} \\ \frac{1}{3} & 0 & -\frac{1}{3} \\ -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{pmatrix} \tag{8}$$

Now solving: $\quad \mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$, where

$$\mathbf{b} = \begin{pmatrix} 6 \\ 11 \\ 0 \end{pmatrix}$$

$$\mathbf{x} = \begin{pmatrix} \frac{7}{9} & -\frac{1}{3} & \frac{2}{9} \\ \frac{1}{3} & 0 & -\frac{1}{3} \\ -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{pmatrix} \begin{pmatrix} 6 \\ 11 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \tag{9}$$

**Final Answer:**

$$\boxed{x = 1, \quad y = 2, \quad z = 3} \tag{10}$$

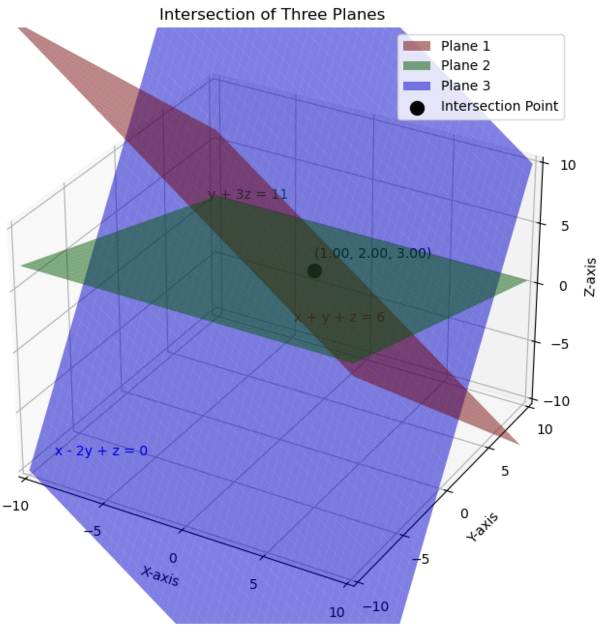


Fig. 0: A plane passing through point A with normal vector n.