2.8.36

Puni Aditya - EE25BTECH11046

9th September, 2025

Question

The value of the expression $\|\mathbf{a} \times \mathbf{b}\| + \left(\mathbf{a}^\top \mathbf{b}\right)$ is _____

Theoretical Solution

Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors, and let θ be the angle between them. The magnitude of the cross product is

$$\|\mathbf{a} \times \mathbf{b}\| = \|\mathbf{a}\| \, \|\mathbf{b}\| \sin(\theta) \tag{1}$$

The dot product or inner product is

$$\mathbf{a}^\top \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \cos(\theta) \tag{2}$$

Theoretical Solution

Now, we substitute these definitions into the given expression:

$$\|\mathbf{a} \times \mathbf{b}\| + \left(\mathbf{a}^\top \mathbf{b}\right) = \|\mathbf{a}\| \|\mathbf{b}\| \sin(\theta) + \|\mathbf{a}\| \|\mathbf{b}\| \cos(\theta) \tag{3}$$

$$\implies \|\mathbf{a} \times \mathbf{b}\| + \left(\mathbf{a}^\top \mathbf{b}\right) = \|\mathbf{a}\| \|\mathbf{b}\| (\sin(\theta) + \cos(\theta)) \tag{4}$$

$$\sin(\theta) + \cos(\theta) = \sqrt{2} \sin\left(\theta + \frac{\pi}{4}\right) \tag{5}$$

$$\therefore \|\mathbf{a} \times \mathbf{b}\| + \left(\mathbf{a}^\top \mathbf{b}\right) = \sqrt{2} \|\mathbf{a}\| \|\mathbf{b}\| \sin\left(\theta + \frac{\pi}{4}\right)$$

Example

Example: Let

$$\mathbf{a} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$\|\mathbf{a}\| = 1 \text{ and } \|\mathbf{b}\| = \sqrt{2}$$

$$\|\mathbf{a} \times \mathbf{b}\| + \left(\mathbf{a}^\top \mathbf{b}\right) = \left\| \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\| + \begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \tag{6}$$

$$\|\mathbf{a} \times \mathbf{b}\| + \left(\mathbf{a}^\top \mathbf{b}\right) = 1 + 1 = 2 \tag{7}$$

Example

$$\cos(\theta) = \frac{\begin{pmatrix} 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}}{\left\| \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\| \left\| \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\|} \tag{8}$$

$$\cos(\theta) = \frac{1}{\sqrt{2}} \tag{9}$$

$$\theta = \frac{\pi}{4} \tag{10}$$

From (10) (11)

$$\sqrt{2}\,\|\mathbf{a}\|\,\|\mathbf{b}\| \sin\left(\theta + \frac{\pi}{4}\right) = \sqrt{2} \times 1 \times \sqrt{2} \times \sin\left(\frac{\pi}{2}\right) \tag{12}$$

Example

$$\sqrt{2}\,\|\mathbf{a}\|\,\|\mathbf{b}\|\sin\left(\theta+\frac{\pi}{4}\right)=2 \tag{13}$$

From (7) and (13),

$$\|\mathbf{a}\times\mathbf{b}\|+\left(\mathbf{a}^{\top}\mathbf{b}\right)=\sqrt{2}\,\|\mathbf{a}\|\,\|\mathbf{b}\|\sin\left(\theta+\frac{\pi}{4}\right)$$

Theoretical Solution

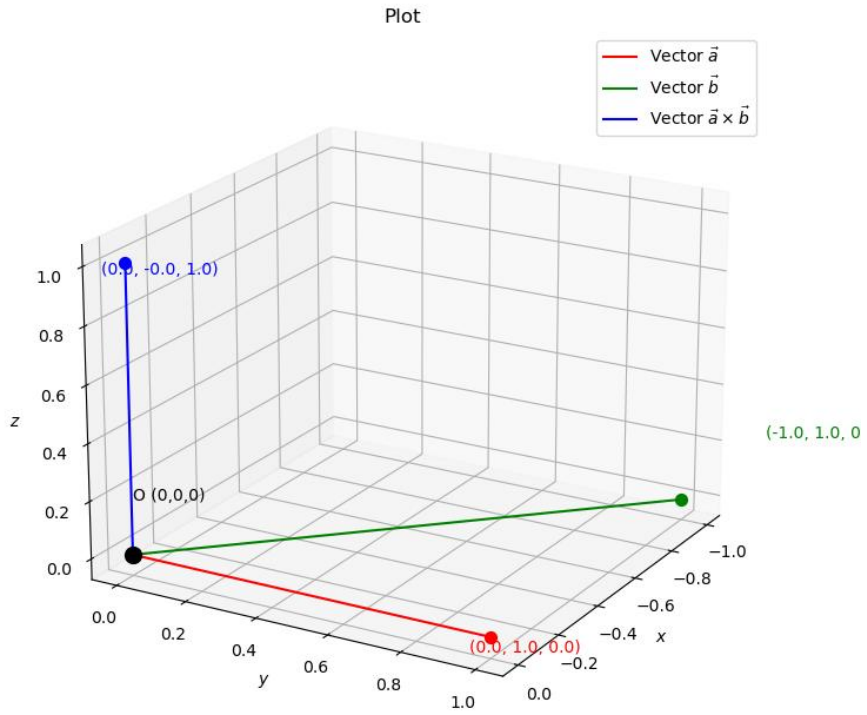

Figure: Example