

# 4.3.33

EE25BTECH11043 - Nishid Khandagre

**Question:** If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is  $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ , then the equation of the line will be?

**Solution:** The equation of line is

$$\mathbf{n}^\top \mathbf{x} = c \quad (0.1)$$

Where  $\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$  is the normal vector and  $\mathbf{x}$  is the position vector.

X-axis intercept is at **A**

$$\mathbf{n}^\top \mathbf{A} = c \quad (0.2)$$

$$\begin{pmatrix} n_1 & n_2 \end{pmatrix} \begin{pmatrix} a \\ 0 \end{pmatrix} = c \quad (0.3)$$

$$n_1 a = c \quad (0.4)$$

$$\mathbf{A} = \begin{pmatrix} \frac{c}{n_1} \\ 0 \end{pmatrix} \quad (0.5)$$

Y-axis intercept is at **B**

$$\mathbf{n}^\top \mathbf{B} = c \quad (0.6)$$

$$\begin{pmatrix} n_1 & n_2 \end{pmatrix} \begin{pmatrix} 0 \\ b \end{pmatrix} = c \quad (0.7)$$

$$n_2 b = c \quad (0.8)$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ \frac{c}{n_2} \end{pmatrix} \quad (0.9)$$

Let **M** is the midpoint of **A** and **B**

Given  $\mathbf{M} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ .

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (0.10)$$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \frac{c}{n_1} \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 \\ \frac{c}{n_2} \end{pmatrix} \quad (0.11)$$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{c}{2n_1} \\ \frac{c}{2n_2} \end{pmatrix} \quad (0.12)$$

$$\frac{c}{2n_1} = 3 \quad (0.13)$$

$$\frac{c}{2n_2} = 2 \quad (0.14)$$

$$\frac{n_1}{n_2} = \frac{2}{3} \quad (0.15)$$

Let  $n_1 = 2$  and  $n_2 = 3$ . Then

$$c = 6 \times 2 = 12 \quad (0.16)$$

The final equation of the line is  $\mathbf{n}^T \mathbf{x} = c$

$$\begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} = 12 \quad (0.17)$$

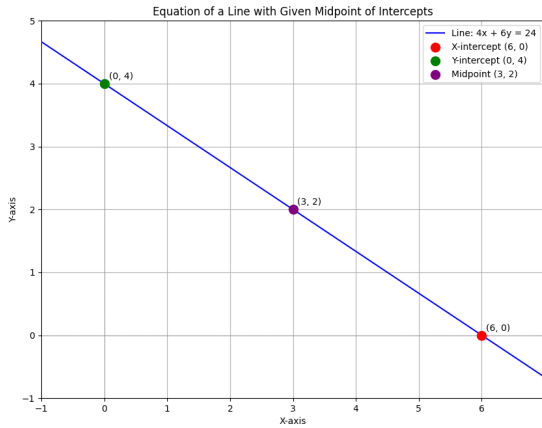


Fig. 0.1