4.4.17

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Problem Statement

A point **P** divides the line segment joining the points **A**$(3, -5)$ and **B**$(-4, 8)$ such that

$$\frac{AP}{PB} = \frac{K}{1}.$$

If **P** lies on the line $x + y = 0$, then find the value of K.

Answer: Step 1 - Represent points as vectors

$$\mathbf{A} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{bmatrix} 3 & -5 \end{bmatrix}^\top, \quad \mathbf{B} = \begin{pmatrix} -4 \\ 8 \end{pmatrix} = \begin{bmatrix} -4 & 8 \end{bmatrix}^\top.$$

$$\mathbf{P} = \frac{K\mathbf{B} + \mathbf{A}}{K+1} = \frac{K\begin{bmatrix} -4 \\ 8 \end{bmatrix} + \begin{bmatrix} 3 \\ -5 \end{bmatrix}}{K+1} = \frac{1}{K+1}\begin{bmatrix} 3-4K \\ -5+8K \end{bmatrix}.$$

Answer: Step 2 - Use the line condition

The point $\mathbf{P} = \begin{bmatrix} x \\ y \end{bmatrix}$ lies on the line $x + y = 0$:

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \mathbf{P} = 0,$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \frac{1}{K+1} \begin{bmatrix} 3 - 4K \\ -5 + 8K \end{bmatrix} = 0.$$

Answer: Step 3 - Solve for K

Multiply and simplify:

$$\frac{1}{K+1}\left((3-4K)+(-5+8K)\right)=0,$$

$$\Rightarrow \frac{1}{K+1}(-2+4K)=0.$$

Since $K+1 \neq 0$,

$$-2+4K=0 \implies K=\frac{1}{2}.$$

$$\boxed{K=\frac{1}{2}}.$$

C Code: Calculate Point P

```c
#include <stdio.h>

void calculateP(double A[2], double B[2], double K, double P[2])
    {
    P[0] = (K * B[0] + A[0]) / (K + 1);
    P[1] = (K * B[1] + A[1]) / (K + 1);
}

int main() {
    double A[2] = {3, -5};
    double B[2] = {-4, 8};
    double K = 0.5; // example value for K
    double P[2];

    calculateP(A, B, K, P);

    printf(Coordinates of P are: (%.2f, %.2f)\n, P[0], P[1]);

    return 0;
```

Python Plotting Code - Part 1

```python
import numpy as np
import matplotlib.pyplot as plt

# Given points A and B
A = np.array([3, -5])
B = np.array([-4, 8])

# Given ratio K
K = 0.5

# Calculate point P dividing AB in ratio K:1
P = (K * B + A) / (K + 1)
```

Python Plotting Code - Part 2

```
# Prepare line segment AB
line_AB_x = [A[0], B[0]]
line_AB_y = [A[1], B[1]]

# Prepare line x + y = 0 (y = -x)
x_vals = np.linspace(-10, 10, 400)
y_vals = -x_vals

# Plotting
plt.figure(figsize=(8, 8))
plt.plot(line_AB_x, line_AB_y, 'b-', label='Line segment AB')
plt.plot(x_vals, y_vals, 'g--', label='Line x + y = 0')
```

Python Plotting Code - Part 3

```
# Plot points
plt.plot(A[0], A[1], 'ro', label='Point A (3, -5)')
plt.plot(B[0], B[1], 'bo', label='Point B (-4, 8)')
plt.plot(P[0], P[1], 'mo', label=f'Point P (K={K})')

plt.xlabel('x')
plt.ylabel('y')
plt.title('Points A, B, P and line x + y = 0')
plt.legend()
plt.grid(True)
plt.axis('equal')

# Save plot
plt.savefig('python_plot.png')
plt.show()
```

Plot