Matgeo Presentation - Problem 12.388

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Question

For the matrix $\mathbf{A}=\begin{pmatrix} 5 & 3 \\ 1 & 3 \end{pmatrix}$,ONE of the normalized eigenvectors is given as (ME 2012)

a) $\begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \end{pmatrix}$

b) $\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix}$

c) $\begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \end{pmatrix}$

d) $\begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$

Solution

Given

$$\mathbf{A} = \begin{pmatrix} 5 & 3 \\ 1 & 3 \end{pmatrix} \tag{0.1}$$

For matrix $\mathbf{A}$ the characterstic polynomial is given by

$$|\mathbf{A} - \lambda\mathbf{I}| = 0 \tag{0.2}$$

$$\text{char}\mathbf{A} = \begin{vmatrix} 5-\lambda & 3 \\ \\ 1 & 3-\lambda \end{vmatrix} = 0 \tag{0.3}$$

$$\implies (5-\lambda)(3-\lambda) - 3 = 0 \tag{0.4}$$

$$\implies \lambda^2 - 8\lambda + 12 = 0 \tag{0.5}$$

$$\implies (\lambda - 2)(\lambda - 6) = 0 \tag{0.6}$$

Thus,the eigen values are given by

$$\lambda_1 = 6 \text{ and } \lambda_2 = 2 \tag{0.7}$$

Solution

For λ_1, the augmented matrix formed from the eigenvalue-eigenvector equation is

$$\begin{pmatrix} -1 & 3 \\ 1 & -3 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 + R_1} \begin{pmatrix} -1 & 3 \\ 0 & 0 \end{pmatrix} \tag{0.8}$$

Hence,the normalized eigenvector is

$$\mathbf{v_1} = \frac{1}{\sqrt{10}} \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{0.9}$$

For λ_2, the augmented matrix formed from the eigenvalue-eigenvector equation is

$$\begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - \frac{1}{3} \times R_1} \begin{pmatrix} 3 & 3 \\ 0 & 0 \end{pmatrix} \tag{0.10}$$

Hence,the normalized eigenvector is

$$\mathbf{v_2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{0.11}$$

Conclusion

The normalized eigen vectors are

$$\mathbf{v_1} = \begin{pmatrix} \frac{3}{\sqrt{10}} \\ \frac{1}{\sqrt{10}} \end{pmatrix} \text{ and } \mathbf{v_2} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} \tag{0.12}$$

C Code: eigen.c

```
#include <stdio.h>
#include <math.h>

int main() {
    FILE *fp;
    fp = fopen("eigen.dat", "w");
    if (fp == NULL) {
        printf("Error␣opening␣file!\n");
        return 1;
    }
    // Eigenvalues are already known: 1=6, 2=2
    // Well compute the corresponding eigenvectors.

    // Eigenvector for 1 = 6 --> [3, 1]
    double v1x = 3, v1y = 1;
    double norm1 = sqrt(v1x*v1x + v1y*v1y);
    v1x /= norm1;
    v1y /= norm1;

    // Eigenvector for 2 = 2 --> [1, -1]
    double v2x = 1, v2y = -1;
    double norm2 = sqrt(v2x*v2x + v2y*v2y);
    v2x /= norm2;
    v2y /= norm2;
    fprintf(fp, "Normalized␣Eigenvectors:\n");
    fprintf(fp, "For␣1␣=␣6␣:␣(␣%.6f␣,␣%.6f␣)\n", v1x, v1y);
    fprintf(fp, "For␣2␣=␣2␣:␣(␣%.6f␣,␣%.6f␣)\n", v2x, v2y);

    fclose(fp);

    printf("Eigenvectors␣written␣to␣eigen.dat\n");
    return 0;
}
```

Python: solution.py

```
import numpy as np
from sympy import Matrix

# Define the matrix
A = np.array([[5, 3],
              [1, 3]], dtype=float)

# --- Using NumPy ---
# Eigen decomposition
eigenvalues, eigenvectors = np.linalg.eig(A)

print("Using NumPy:")
for i in range(len(eigenvalues)):
    vec = eigenvectors[:, i]
    # Normalize vector
    norm_vec = vec / np.linalg.norm(vec)
    print(f"Eigenvalue: {eigenvalues[i]:.0f}  --> Normalized eigenvector: {norm_vec}")

# --- Using SymPy (symbolic check) ---
M = Matrix([[5, 3],
            [1, 3]])

eigs = M.eigenvects()

print("\nUsing SymPy:")
for val, mult, vecs in eigs:
    for v in vecs:
        # Normalize with sympy
        v_normalized = v.normalized()
        print(f"Eigenvalue: {val}  --> Normalized eigenvector: {v_normalized}")
```