4.2.7

EE25BTECH11004 - Aditya Appana

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Question

Find the direction and normal vectors of the line $y - 2 = 0$

Solution

A line can be expressed in two forms:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ c \end{pmatrix} + x \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{1}$$

where $\begin{pmatrix} 1 \\ m \end{pmatrix}$ is the direction vector of the line and m is the slope of the line.

$$\mathbf{n}^T x = c \tag{2}$$

where $\mathbf{n}$ is the normal vector of the line. $\mathbf{n}^T \begin{pmatrix} 1 \\ m \end{pmatrix} = 0$

The slope of the line $y - 2 = 0$ is 0, therefore it can be expressed in the first form as:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} + x \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3}$$

The vector orthogonal to $\begin{pmatrix}1\\0\end{pmatrix}$ in $\mathbb{R}^2$ is $\begin{pmatrix}0\\1\end{pmatrix}$. Therefore the line $y - 2 = 0$ can be expressed in the second form as:

$$\begin{pmatrix}0\\1\end{pmatrix}^T x = 2 \tag{4}$$

Therefore, the direction vector of $y - 2 = 0$ is $\begin{pmatrix}0\\1\end{pmatrix}$, and the normal vector is $\begin{pmatrix}1\\0\end{pmatrix}$.