5.2.44

EE25BTECH11042 - Nipun Dasari

Question:
Solve the following system of rational equations

$$\frac{10}{x+y} + \frac{2}{x-y} = 4 \tag{0.1}$$

$$\frac{15}{x+y} - \frac{5}{x-y} = -2 \tag{0.2}$$

Solution:
Introduce a and b as follows:

$$a = \frac{1}{x+y} \quad b = \frac{1}{x-y} \tag{0.3}$$

Also define

$$\mathbf{a} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{0.4}$$

This gives us simplified equations

$$\begin{pmatrix} 10 & 2 \end{pmatrix} \mathbf{a} = 4 \tag{0.5}$$

$$\begin{pmatrix} 15 & -5 \end{pmatrix} \mathbf{a} = -2 \tag{0.6}$$

Augmented matrix for the given system is

$$\left(\begin{array}{cc|c} 10 & 2 & 4 \\ 15 & -5 & -2 \end{array}\right) \tag{0.7}$$

By row reductions

$$\left(\begin{array}{cc|c} 10 & 2 & 4 \\ 15 & -5 & -2 \end{array}\right) \xleftrightarrow{R_2 \leftarrow R_2 - \frac{3}{2} \times R_1} \left(\begin{array}{cc|c} 10 & 2 & 4 \\ 0 & -8 & -8 \end{array}\right) \xleftrightarrow{R_1 \leftarrow R_1 + \frac{1}{4} \times R_2} \left(\begin{array}{cc|c} 10 & 0 & 2 \\ 0 & -8 & -8 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 10 & 0 & 2 \\ 0 & -8 & -8 \end{array}\right) \xleftrightarrow{R_1 \leftarrow \frac{1}{10} \times R_1} \left(\begin{array}{cc|c} 1 & 0 & \frac{1}{5} \\ 0 & -8 & -8 \end{array}\right) \xleftrightarrow{R_2 \leftarrow \frac{1}{-8} \times R_2} \left(\begin{array}{cc|c} 1 & 0 & \frac{1}{5} \\ 0 & 1 & 1 \end{array}\right)$$

$$\mathbf{a} = \begin{pmatrix} \frac{1}{5} \\ 1 \end{pmatrix} \tag{0.8}$$

Substituting value of a and b again we get

$$\begin{pmatrix} \frac{1}{x+y} \\ \frac{1}{x-y} \end{pmatrix} = \begin{pmatrix} \frac{1}{5} \\ 1 \end{pmatrix} \tag{0.9}$$

$$\implies \begin{pmatrix} x+y \\ x-y \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \tag{0.10}$$

Introduce

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{0.11}$$

This gives us the equation

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \tag{0.12}$$

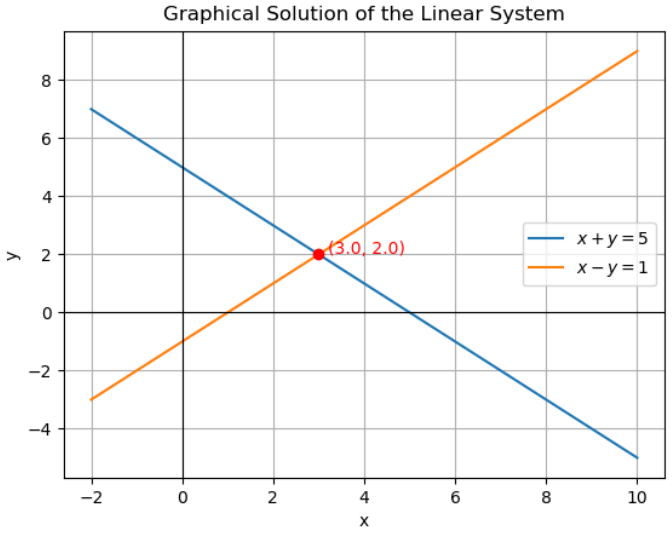
$$\implies \mathbf{x} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} \tag{0.13}$$

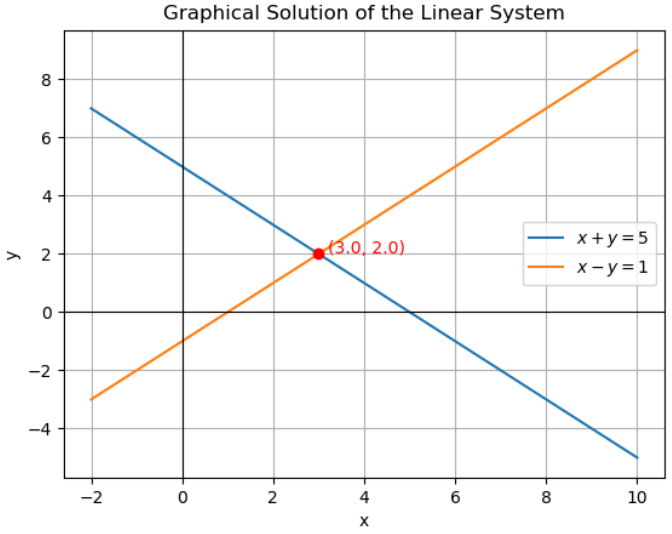
$$\implies \mathbf{x} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \tag{0.14}$$

$$\implies \mathbf{x} = \begin{pmatrix} \frac{5}{2} + \frac{1}{2} \\ \frac{5}{2} - \frac{1}{2} \end{pmatrix} \tag{0.15}$$

$$\implies \mathbf{x} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{0.16}$$

Thus $x = 3$ and $y = 2$


Graphical Solution of the Linear System
(3.0, 2.0)
$x+y=5$
$x-y=1$
x
y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