1.2.17

AI25BTECH11011-VARUN

Question:
Three vertices of a parallelogram ABCD are A(3,-1,2),B(1,-2,4),C(-1,1,2). Find the coordinates of the fourth vertex.

Solution:
Let the vertices of parallelogram ABCD be $\mathbf{A}\begin{pmatrix}3\\-1\\2\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}1\\-2\\4\end{pmatrix}$, $\mathbf{C}\begin{pmatrix}-1\\1\\2\end{pmatrix}$. In any parallelogram, the diagonals bisect each other, so the midpoints of **AC** and **BD** are equal.

The midpoint of A $\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix}$ **and B** $\begin{pmatrix}x_2\\y_2\\z_2\end{pmatrix}$ **is**

$$\mathbf{M_{AB}} = \begin{pmatrix}\frac{x_1+x_2}{2}\\\frac{y_1+y_2}{2}\\\frac{z_1+z_2}{2}\end{pmatrix} \tag{0.1}$$

Midpoint of **AC**:

$$\mathbf{M_{AC}} = \begin{pmatrix}\frac{3+(-1)}{2}\\\frac{-1+1}{2}\\\frac{2+2}{2}\end{pmatrix} = \begin{pmatrix}1\\0\\2\end{pmatrix}. \tag{0.2}$$

Let $\mathbf{D} = \begin{pmatrix}x\\y\\z\end{pmatrix}$. Midpoint of **BD**:

$$\mathbf{M_{BD}} = \begin{pmatrix}\frac{1+x}{2}\\\frac{-2+y}{2}\\\frac{4+z}{2}\end{pmatrix}. \tag{0.3}$$

Set $\mathbf{M_{AC}} = \mathbf{M_{BD}}$:

$$\frac{1+x}{2} = 1, \frac{-2+y}{2} = 0, \frac{4+z}{2} = 2. \tag{0.4}$$

Solving gives $x = 1, y = 2, z = 0$.

The fourth vertex is $\mathbf{D}\begin{pmatrix}1\\2\\0\end{pmatrix}$.

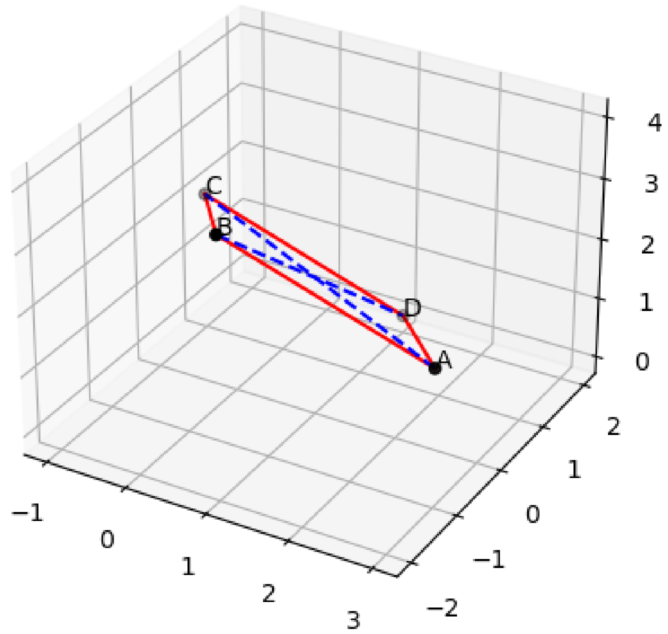

Fig. 0.1