

Presentation - Matgeo

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Problem Statement

Find the points on the X axis which are at a distance of $2\sqrt{5}$ from the point $(7, -4)$. How many such points are there?

Solution

Given that the point $(7, -4)$ is at a distance $2\sqrt{5}$ from, assume a point P that lies on the X axis,

Let the given point be denoted A and its position vector \mathbf{a} and the position vector of P will be

$$\mathbf{p} = x \cdot \mathbf{e}_1 \quad (1.1)$$

The distance between the two given points will be,

$$\|\mathbf{a} - \mathbf{p}\| = 2\sqrt{5} \quad (1.2)$$

$$\|\mathbf{a} - x \cdot \mathbf{e}_1\| = 2\sqrt{5} \quad (1.3)$$

We know that,

$$\|H\|^2 = H \cdot H^T \quad (1.4)$$

So,

$$(\mathbf{a} - x \cdot \mathbf{e}_1) \cdot (\mathbf{a} - x \cdot \mathbf{e}_1)^T = (2\sqrt{5})^2 \quad (1.5)$$

$$(\mathbf{a} - x \cdot \mathbf{e}_1) \cdot (\mathbf{a}^T - x \cdot \mathbf{e}_1^T) = 20 \quad (1.6)$$

$$(\mathbf{a} \cdot \mathbf{a}^T) - (x \cdot \mathbf{a} \cdot \mathbf{e}_1^T) - (x \cdot \mathbf{a}^T \cdot \mathbf{e}_1) + (x^2 \cdot \mathbf{e}_1 \cdot \mathbf{e}_1^T) = 20 \quad (1.7)$$

$$(x^2 \cdot \mathbf{e}_1 \cdot \mathbf{e}_1^T) - ((\mathbf{a} \cdot \mathbf{e}_1^T + \mathbf{a}^T \cdot \mathbf{e}_1) \cdot x) + (\mathbf{a} \cdot \mathbf{a}^T) - 20 = 0 \quad (1.8)$$

$$(\mathbf{a} \cdot \mathbf{e}_1^T + \mathbf{a}^T \cdot \mathbf{e}_1) = 2 \cdot \mathbf{a} \cdot \mathbf{e}_1^T \quad (1.9)$$

On solving the quadratic for x ,

$$x = \mathbf{a} \cdot \mathbf{e}_1^T \pm \sqrt{(\mathbf{a} \cdot \mathbf{e}_1^T)^2 - \|\mathbf{a}\|^2 + 20} \quad (1.10)$$

On substituting the values of \mathbf{a} and \mathbf{e}_1 ,

$$x = 7 \pm \sqrt{7^2 - 7^2 - 4^2 + 20} \quad (1.11)$$

$$x = 7 \pm 2 = 9, 5 \quad (1.12)$$

Hence, there exist two values of x i.e., there exist two points P on the X axis for the distance between the given point and P to be $2\sqrt{5}$

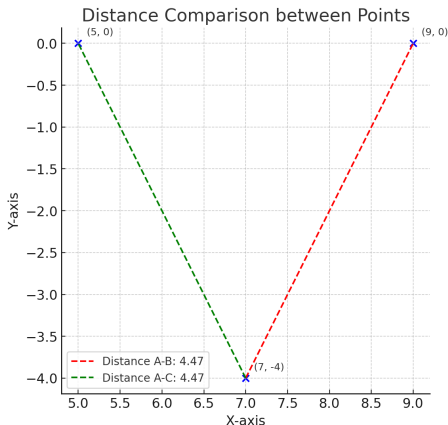


Figure: The plot of the points A and the two points on the X axis

Code - C

```
#include <math.h>
#include <stdio.h>

void find_x_axis_points(double x0, double y0, double d, double *x1,
    double *x2) {
    double rhs = d*d - y0*y0;
    if (rhs < 0) {
        *x1 = NAN;
        *x2 = NAN;
        return;
    }
    double root = sqrt(rhs);
    *x1 = x0 + root;
    *x2 = x0 - root;
}

int main() {
```