4.3.22

EE25BTECH11033 - Kavin

Question:
Find the ratio in which the line segment joining $\mathbf{A}(1, -5)$ and $\mathbf{B}(-4, 5)$ is divided by the X axis. Also find the coordinates of the point of division.

Solution:
Let the vector $\mathbf{P}$ be the point on x-axis

$$\mathbf{P} = \begin{pmatrix} x \\ 0 \end{pmatrix}, \tag{1}$$

Given the points,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -5 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -4 \\ 5 \end{pmatrix} \tag{2}$$

The points $\mathbf{A}$, $\mathbf{P}$, $\mathbf{B}$ are collinear.

Points $\mathbf{A}, \mathbf{P}, \mathbf{B}$ are defined to be collinear if

$$\text{rank}\begin{pmatrix} \mathbf{P} - \mathbf{A} & \mathbf{B} - \mathbf{A} \end{pmatrix} = 1 \tag{3}$$

$$\mathbf{P} - \mathbf{A} = \begin{pmatrix} x - 1 \\ 5 \end{pmatrix} \tag{4}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -5 \\ 10 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} \mathbf{P} - \mathbf{A} & \mathbf{B} - \mathbf{A} \end{pmatrix} = \begin{pmatrix} x - 1 & -5 \\ 5 & 10 \end{pmatrix} \tag{6}$$

$$R_1 \leftrightarrow R_2 \implies \begin{pmatrix} 5 & 10 \\ x - 1 & -5 \end{pmatrix} \tag{7}$$

$$R_2 \to 2R_2 + R_1 \implies \begin{pmatrix} 5 & 10 \\ 2x + 3 & 0 \end{pmatrix} \tag{8}$$

For rank 1, the second row must be zero:

$$2x + 3 = 0 \implies x = -3/2 \tag{9}$$

$$\therefore \mathbf{P} = \begin{pmatrix} -3/2 \\ 0 \end{pmatrix}$$

Section formula for a vector **P** which divides the line formed by vectors **A** and **B** in the ratio k:1 is given by

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k+1} \tag{10}$$

$$k(\mathbf{P} - \mathbf{B}) = \mathbf{A} - \mathbf{P} \tag{11}$$

$$\implies k = \frac{(\mathbf{A} - \mathbf{P})^\top (\mathbf{P} - \mathbf{B})}{\|\mathbf{P} - \mathbf{B}\|^2} \tag{12}$$

$$(\mathbf{A} - \mathbf{P})^\top (\mathbf{P} - \mathbf{B}) = \begin{pmatrix} 5/2 & -5 \end{pmatrix} \begin{pmatrix} 5/2 \\ -5 \end{pmatrix} = 125/4 \tag{13}$$

$$\|\mathbf{P} - \mathbf{B}\|^2 = \left(\sqrt{(5/2)^2 + (-5)^2}\right)^2 = 125/4 \tag{14}$$

$$\implies k = 1 \tag{15}$$

Therefore the ratio in which **P** divides the line segment joining the points **A** and **B** is $1:1$

See Fig. 0 ,

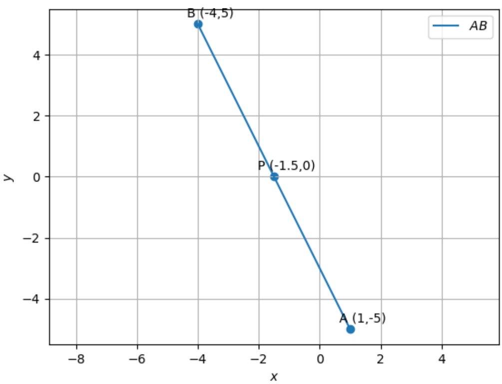

Fig. 0