9.2.27

EE25BTECH11060 - V.Namaswi

Question
Find the Area enclosed by the parabola $4y = 3x^2$ and the Line 2y=3x+12
Solution
Given Line

$$2y = 3x + 12 \tag{1}$$

$$\mathbf{x} = \mathbf{h} + k\mathbf{m}; k \in \mathbb{R} \tag{2}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \tag{3}$$

$$\mathbf{m} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{4}$$

Given curve

$$4y = 3x^2 \tag{5}$$

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{6}$$

$$\mathbf{V} = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \tag{7}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{8}$$

$$f = 0 \tag{9}$$

Points of Intersection

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h}) \cdot (\mathbf{m}^\top \mathbf{V}\mathbf{m})} \right) \tag{10}$$

where

$$g(\mathbf{h}) = \mathbf{h}^\top \mathbf{V}\mathbf{h} + 2\mathbf{u}^\top \mathbf{h} + f \tag{11}$$

$$\mathbf{V}\mathbf{h} = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{12}$$

$$\mathbf{V}\mathbf{h} + \mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{13}$$

$$\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) = \begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ -2 \end{pmatrix} = -6 \tag{14}$$

$$\mathbf{m}^\top \mathbf{V}\mathbf{m} = \begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} 6 \\ 0 \end{pmatrix} = 12 \tag{15}$$

$$\mathbf{u}^\top \mathbf{h} = \begin{pmatrix} 0 & -2 \end{pmatrix} \begin{pmatrix} 0 \\ 6 \end{pmatrix} = -12 \tag{16}$$

$$g(\mathbf{h}) = 0 + 2(-12) + 0 = -24 \tag{17}$$

$$\kappa_i = \frac{1}{12} \left(6 \pm \sqrt{(-6)^2 - (-24)(12)} \right) \tag{18}$$

$$= \frac{1}{12} \left(6 \pm \sqrt{36 + 288} \right) \tag{19}$$

$$= \frac{1}{12} \left(6 \pm \sqrt{324} \right) \tag{20}$$

$$= \frac{1}{12} (6 \pm 18) \tag{21}$$

$$= \kappa_1 = \frac{24}{12} = 2, \quad \kappa_2 = \frac{-12}{12} = -1 \tag{22}$$

The point of intersection are :

$$(4, 12) \quad \text{and} \quad (-2, 3) \tag{23}$$

Area Bounded by curves is given by

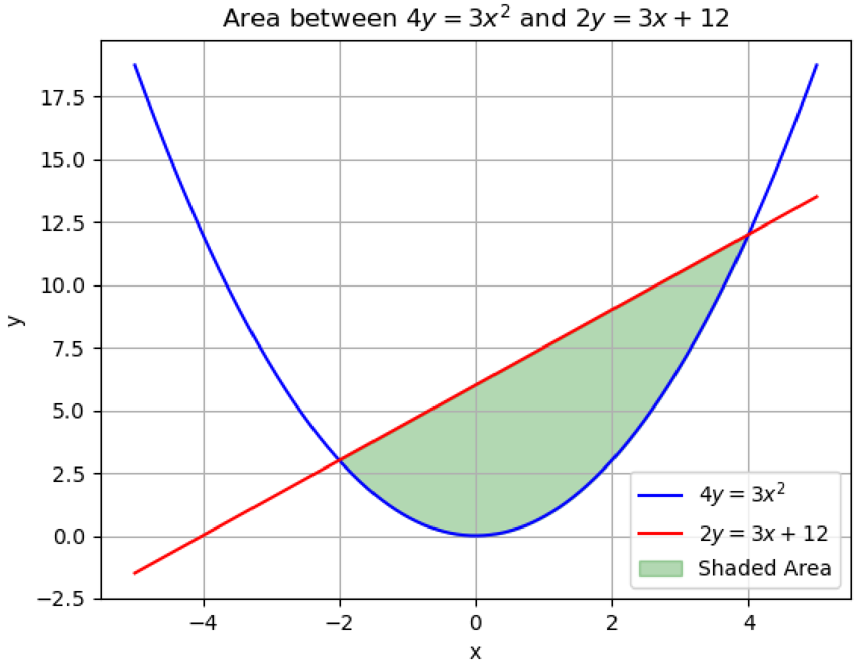
$$\left| \int_{-2}^{4} \frac{3x^2}{4} - \frac{3x + 12}{2} \right| \tag{24}$$

$$= \left| \frac{1}{4} \int_{-2}^{4} 3x^2 - 6x - 24 \right| \tag{25}$$

$$= \left| \frac{1}{4} \left(x^3 - 3x^2 - 24x \right)_{-2}^{4} \right| \tag{26}$$

$$= \left| \frac{1}{4} \left(4^3 - (-2)^3 - 3(4^2 - (-2)^2) - 24(4 - (-2)) \right) \right| \tag{27}$$

$$= 27 \tag{28}$$



Area between $4y = 3x^2$ and $2y = 3x + 12$
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
y
−4
−2
0
2
4
x
$4y = 3x^2$
$2y = 3x + 12$
Shaded Area

(29)