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Question

For what value of k, will the following pair of equations have infinitly many solutions

$2x + 3y = 7$ and $(k + 2)x - 3(1 - k)y = 5k + 1$

Solution

$$2x + 3y = 7 \quad (1) \tag{1}$$
$$(k+2)x - 3(1-k)y = 5k + 1 \quad (2) \tag{2}$$

Matrix Representation

Write the system as an augmented matrix:

$$\left(\begin{array}{cc|c} 2 & 3 & 7 \\ k+2 & -3(1-k) & 5k+1 \end{array}\right) = \left(\begin{array}{cc|c} 2 & 3 & 7 \\ k+2 & -3+3k & 5k+1 \end{array}\right) \tag{3}$$

Solution

Condition for Infinitely Many Solutions

For infinitely many solutions, the second row must be a scalar multiple of the first row. Let the scalar be λ. Then:

$$k + 2 = 2\lambda \quad \text{(i)} \tag{4}$$

$$-3 + 3k = 3\lambda \quad \text{(ii)} \tag{5}$$

$$5k + 1 = 7\lambda \quad \text{(iii)} \tag{6}$$

Solution

From (i), solve for λ:

$$\lambda = \frac{k+2}{2} \tag{7}$$

Substitute into (ii):

$$-3 + 3k = 3(\frac{k+2}{2}) \tag{8}$$

$$-3 + 3k = \frac{3k+6}{2} \tag{9}$$

$$-6 + 6k = 3k + 6 \tag{10}$$

$$3k = 12 \tag{11}$$

$$k = 4 \tag{12}$$

Solution

Verification

Check with equation (iii):

$$\lambda = \frac{4+2}{2} = 3 \tag{13}$$

$$7\lambda = 21 \tag{14}$$

$$5k + 1 = 5(4) + 1 = 21 \tag{15}$$

Final Answer

$$\boxed{k = 4} \tag{16}$$