1.4.6

AI25BTECH11033–SNEHAMRUDULA

If the point $P(2, 1)$ lies on the line segment joining points $A(4, 2)$ and $B(8, 4)$, then

1) $AP = \frac{1}{4}AB$
2) $AP = PE$
3) $PB = \frac{1}{3}AB$
4) $AP = \frac{3}{5}AB$

Solution:

$$\mathbf{A} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}, \qquad \mathbf{P} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}. \tag{4.1}$$

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \tag{4.2}$$
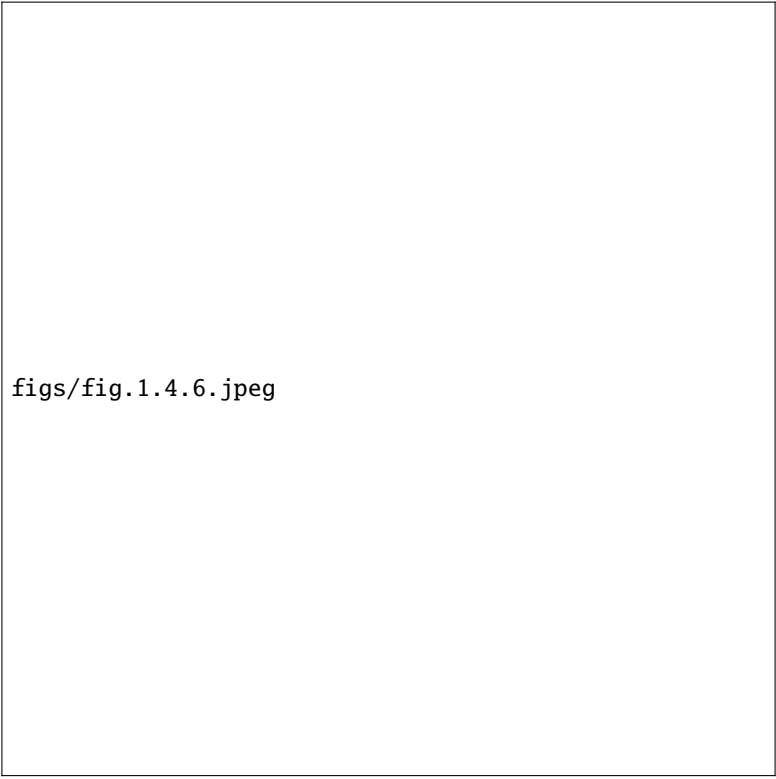
$$\mathbf{AP} = \mathbf{P} - \mathbf{A} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}. \tag{4.3}$$

$$\|\mathbf{AB}\|^2 = 4^2 + 2^2 = 20, \tag{4.4}$$

$$\|\mathbf{AP}\|^2 = (-2)^2 + (-1)^2 = 5. \tag{4.5}$$

$$\left(\frac{AP}{AB}\right)^2 = \frac{\|\mathbf{AP}\|^2}{\|\mathbf{AB}\|^2} = \frac{5}{20} = \frac{1}{4} \implies \frac{AP}{AB} = \frac{1}{2}. \tag{4.6}$$

$$\boxed{AP = \tfrac{1}{2} AB}$$

Fig. 4.1