1.4.20: Section Formula Problem

Aditya Mishra - ee25btech11005

August 27, 2025

Question

Find the coordinates of the point which divides the line segment joining

$$A(-2, 3, 5), \quad B(1, -4, 6)$$

in the ratio

(a) 2 : 3 internally

(b) 2 : 3 externally

Given Information

Given vector **A**:

$$\begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix}$$

Given vector **B**:

$$\begin{bmatrix} 1 \\ -4 \\ 6 \end{bmatrix}$$

Required Formulae

Internal division:

$$P = \frac{m\mathbf{B} + n\mathbf{A}}{m + n}$$

External division:

$$Q = \frac{m\mathbf{B} - n\mathbf{A}}{m - n}$$

Solution - Internal

$$P = \frac{2\begin{bmatrix}1\\-4\\6\end{bmatrix} + 3\begin{bmatrix}-2\\3\\5\end{bmatrix}}{5} = \frac{\begin{bmatrix}2-6\\-8+9\\12+15\end{bmatrix}}{5} = \begin{bmatrix}-\frac{4}{5}\\\frac{1}{5}\\\frac{27}{5}\end{bmatrix}$$

Solution - External

$$Q = \frac{2\begin{bmatrix}1\\-4\\6\end{bmatrix} - 3\begin{bmatrix}-2\\3\\5\end{bmatrix}}{2-3} = \frac{\begin{bmatrix}2+6\\-8-9\\12-15\end{bmatrix}}{-1} = \begin{bmatrix}-8\\17\\3\end{bmatrix}$$

Python Code-Plot

```
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

A = (-2, 3, 5)
B = (1, -4, 6)

P = (
    (2*B[0] + 3*A[0]) / 5,
    (2*B[1] + 3*A[1]) / 5,
    (2*B[2] + 3*A[2]) / 5
)

Q = (
    (2*B[0] - 3*A[0]) / (2-3),
    (2*B[1] - 3*A[1]) / (2-3),
    (2*B[2] - 3*A[2]) / (2-3)
)

print("Internal Division Point:", P)
print("External Division Point:", Q)
```

Python Code-Plot

```
fig = plt.figure(figsize=(8,8))
ax = fig.add_subplot(111, projection='3d')

ax.plot([A[0], B[0]], [A[1], B[1]], [A[2], B[2]], color='blue')

def plot_point(pt, label, color):
    ax.scatter(*pt, color=color, s=60)
    ax.text(pt[0], pt[1], pt[2], f"{label}{pt}", fontsize=10)

plot_point(A, "A", "red")
plot_point(B, "B", "red")
plot_point(P, "P", "green")
plot_point(Q, "Q", "purple")

ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title('3D Division of Line Segment')
```

Python Code-Plot

```
ax.set_xlim(-10, 5)
ax.set_ylim(-10, 20)
ax.set_zlim(0, 10)

plt.savefig("Figs/graph.png")
plt.show()
```

Python ctypes Call

```
import ctypes

lib = ctypes.CDLL('./mat1.so')

lib.sectionFormula.argtypes = [
    ctypes.POINTER(ctypes.c_float),
    ctypes.POINTER(ctypes.c_float),
    ctypes.c_float,
    ctypes.c_float,
    ctypes.POINTER(ctypes.c_float)
]
```

Python ctypes Call

```
lib.sectionFormula.restype = None

lib.sectionFormulaExternal.argtypes = [
    ctypes.POINTER(ctypes.c_float),
    ctypes.POINTER(ctypes.c_float),
    ctypes.c_float,
    ctypes.c_float,
    ctypes.POINTER(ctypes.c_float)
]

lib.sectionFormulaExternal.restype = None

p1 = (ctypes.c_float * 3)(-2.0, 3.0, 5.0)
p2 = (ctypes.c_float * 3)(1.0, -4.0, 6.0)

res_internal = (ctypes.c_float * 3)()
res_external = (ctypes.c_float * 3)()

m = 2.0
n = 3.0
```

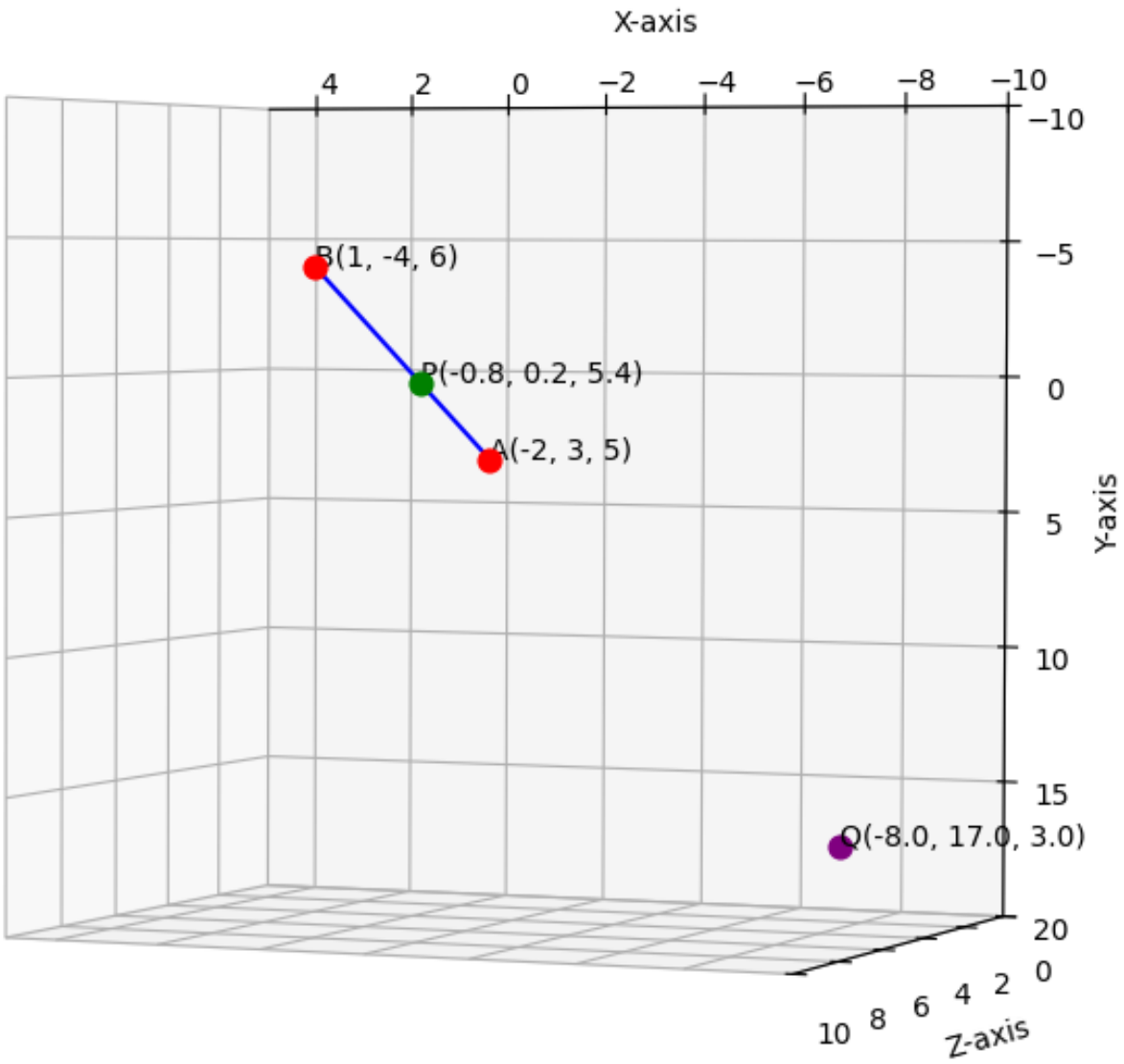
Python ctypes Call

```
lib.sectionFormula(p1, p2, m, n, res_internal)
lib.sectionFormulaExternal(p1, p2, m, n, res_external)

print("Internal division (2:3): [{:.2f}, {:.2f}, {:.2f}]".format(
    res_internal[0], res_internal[1], res_internal[2]
))

print("External division (2:3): [{:.2f}, {:.2f}, {:.2f}]".format(
    res_external[0], res_external[1], res_external[2]
))
```

C Code

```c
void sectionFormula(float p1[3], float p2[3], float m, float n,
float res[3]) {
    res[0] = (m * p2[0] + n * p1[0]) / (m + n);
    res[1] = (m * p2[1] + n * p1[1]) / (m + n);
    res[2] = (m * p2[2] + n * p1[2]) / (m + n);
}

void sectionFormulaExternal(float p1[3], float p2[3], float m,
float n, float res[3]) {
    res[0] = (m * p2[0] - n * p1[0]) / (m - n);
    res[1] = (m * p2[1] - n * p1[1]) / (m - n);
    res[2] = (m * p2[2] - n * p1[2]) / (m - n);
}
```

Plot


X-axis
4
2
0
−2
−4
−6
−8
−10
−10
−5
0
5
10
15
20
Y-axis
B(1, -4, 6)
P(-0.8, 0.2, 5.4)
A(-2, 3, 5)
Q(-8.0, 17.0, 3.0)
0
2
4
6
8
10
Z-axis