1.5.2

EE25BTECH11014 - Bhoomika Lokesh

Question: Find the ratio in which the Y axis divides the line segment joining the points $(6, -4)$ and $(-2, -7)$. Also find the point of intersection.

Solution: Given the points,

$$\mathbf{A} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \mathbf{B} = \begin{pmatrix} -2 \\ -7 \end{pmatrix} \tag{0.1}$$

Let the vector $\mathbf{P}$ be

$$\mathbf{P} = \begin{pmatrix} 0 \\ y \end{pmatrix}, \tag{0.2}$$

WKT points $\mathbf{A}$,$\mathbf{P}$,$\mathbf{B}$ are collinear.
The points to be collinear,

$$\text{rank}\begin{pmatrix} \mathbf{P} - \mathbf{A} & \mathbf{B} - \mathbf{A} \end{pmatrix} = 1 \tag{0.3}$$

$$\mathbf{P} - \mathbf{A} = \begin{pmatrix} -6 \\ y + 4 \end{pmatrix} \tag{0.4}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -8 \\ -3 \end{pmatrix} \tag{0.5}$$

$$\begin{pmatrix} \mathbf{P} - \mathbf{A} & \mathbf{B} - \mathbf{A} \end{pmatrix} = \begin{pmatrix} -6 & -8 \\ y + 4 & -3 \end{pmatrix} \tag{0.6}$$

Conversion to Row Echelon form, $R_2 \to R_2 + \frac{y+4}{6} R_1$:

$$\begin{pmatrix} -6 & -8 \\ 0 & -3 + \frac{y+4}{6}(-8) \end{pmatrix} \implies \begin{pmatrix} -6 & -8 \\ 0 & \frac{-4y-25}{3} \end{pmatrix} \tag{0.7}$$

$$\tag{0.8}$$

$$\frac{-4y - 25}{3} = 0 \implies y = -\frac{25}{4} \tag{0.9}$$

$$\therefore \mathbf{P} = \begin{pmatrix} 0 \\ -\frac{25}{4} \end{pmatrix}$$

Vector $\mathbf{P}$ divides the line joining vectors $\mathbf{A}$ and $\mathbf{B}$ in the ratio k:1

$$\textit{by using section formula}, \mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \tag{0.10}$$

$$k(\mathbf{P} - \mathbf{B}) = \mathbf{A} - \mathbf{P} \tag{0.11}$$

$$\implies k = \frac{(\mathbf{A} - \mathbf{P})^\top (\mathbf{P} - \mathbf{B})}{\|\mathbf{P} - \mathbf{B}\|^2} \tag{0.12}$$

$$(\mathbf{A} - \mathbf{P})^\top (\mathbf{P} - \mathbf{B}) = \begin{pmatrix} 6 & \frac{9}{4} \end{pmatrix} \begin{pmatrix} 2 \\ \frac{3}{4} \end{pmatrix} = \frac{219}{16} \tag{0.13}$$

$$\|\mathbf{P} - \mathbf{B}\|^2 = \left(\sqrt{2^2 + \left(\frac{3}{4}\right)^2} \right)^2 = \frac{73}{16} \tag{0.14}$$

$$k = \frac{\frac{219}{16}}{\frac{73}{16}} \tag{0.15}$$

$$\implies k = 3 \tag{0.16}$$

Therefore the ratio in which point **P** divides the line segment joining **A** and **B** is 3:1
See Fig.0.1,

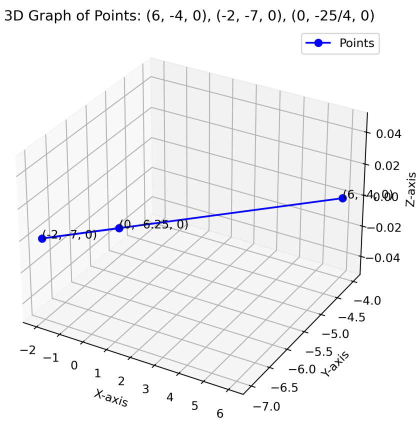

Fig. 0.1