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AI25BTECH11004-B.JASWANTH

Question

X and **Y** are two points with position vectors $3\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-3\mathbf{b}$, respectively. Write the position vector of a point **Z** which divides the line segment **XY** in the ratio 2:1 externally.

Solution:

Let the position vectors of the points *X* and *Y* be given by

$$\Rightarrow \mathbf{X} = \begin{pmatrix} 3 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} \quad (0.1)$$

$$(0.2)$$

$$\Rightarrow \mathbf{Y} = \begin{pmatrix} 1 & -3 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix}. \quad (0.3)$$

The formula for the point **Z** dividing the line segment joining **X** and **Y** **externally** in the ratio $k : 1$ is:

$$\mathbf{Z} = \frac{k\mathbf{Y} - \mathbf{X}}{k - 1}. \quad (0.4)$$

Substituting $k = 2$ and the above matrices:

$$\mathbf{Z} = \frac{2\mathbf{Y} - \mathbf{X}}{2 - 1} = 2\mathbf{Y} - \mathbf{X}. \quad (0.5)$$

Now compute $2\mathbf{Y} - \mathbf{X}$:

$$2\mathbf{Y} - \mathbf{X} = 2 \begin{pmatrix} 1 & -3 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} - \begin{pmatrix} 3 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} = \begin{pmatrix} -1 & -7 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix}. \quad (0.6)$$

Hence,

$$\mathbf{Z} = \begin{pmatrix} -1 & -7 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix}. \quad (0.7)$$

Therefore, the position vector of **Z** is

$$\boxed{\mathbf{Z} = -\mathbf{a} - 7\mathbf{b}} \quad (0.8)$$

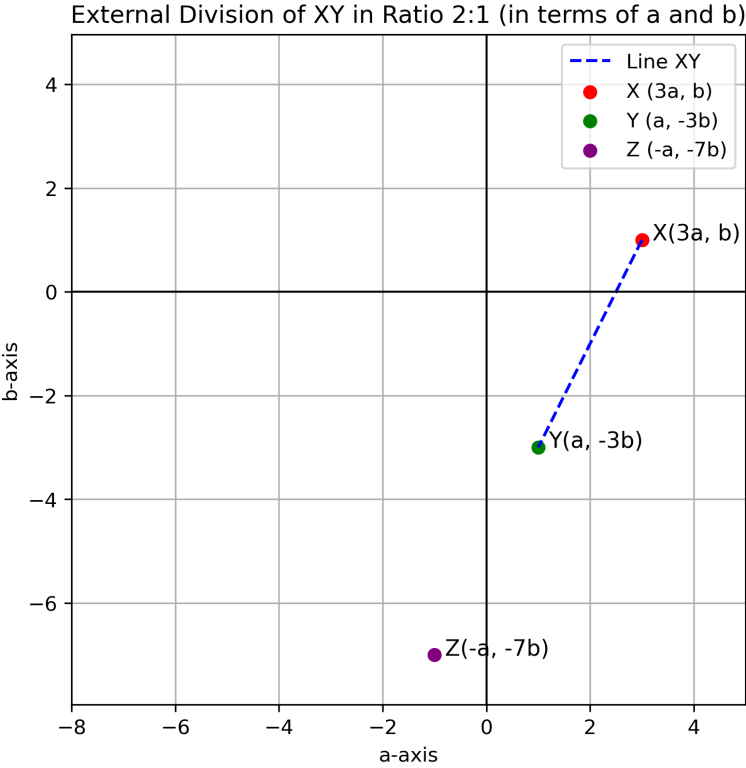


Fig. 0: Caption