AI25BTECH11006 - Nikhila

Question:

Show that the points A(a, b+c), B(b, c+a), C(c, b+a) are collinear.

Solution:

The vector components of the given points are $\mathbf{A} \begin{pmatrix} a \\ b+c \end{pmatrix}$, $\mathbf{B} \begin{pmatrix} b \\ c+a \end{pmatrix}$, $\mathbf{C} \begin{pmatrix} c \\ a+b \end{pmatrix}$.

We know that If the points A B C are collinear, the rank of the matrix

$$(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A}) = 1 \tag{0.1}$$

$$\begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix} = \begin{pmatrix} b - a & c - a \\ a - b & a - c \end{pmatrix} \tag{0.2}$$

$$\stackrel{R_2=R_2+R_1}{\longleftrightarrow} \begin{pmatrix} b-a & c-a \\ 0 & 0 \end{pmatrix} \tag{0.3}$$

Clearly the rank of the matrix is 1 and hence the points are colinear.

Let us take example points by taking a = 1, b = 2, c = 3 and plot the points A(1,5), B(2,4), C(3,3) on the graph.

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Graph:

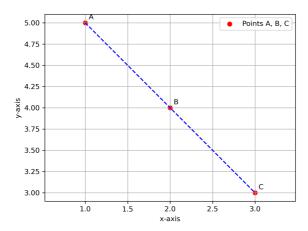


Fig. 0.1