

## 2.8.9

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### Question:

Let  $\mathbf{a}, \mathbf{b}, \mathbf{c}$  be three vectors such that  $|\mathbf{a}| = 3$ ,  $|\mathbf{b}| = 4$ ,  $|\mathbf{c}| = 5$ , and each one of them is perpendicular to the sum of the other two. Find  $|\mathbf{a} + \mathbf{b} + \mathbf{c}|$ .

### Solution:

Let the Gram matrix  $G$  of the three vectors  $(a), (b), (c)$  be

$$G = \begin{pmatrix} (a, a) & (a, b) & (a, c) \\ (b, a) & (b, b) & (b, c) \\ (c, a) & (c, b) & (c, c) \end{pmatrix} = \begin{pmatrix} 9 & x & z \\ x & 16 & y \\ z & y & 25 \end{pmatrix} \quad (1)$$

where

$$x = (a, b), \quad y = (b, c), \quad z = (c, a). \quad (2)$$

The conditions “each vector is perpendicular to the sum of the other two” give

$$(a, (b) + (c)) = 0, \quad (3)$$

$$(b, (c) + (a)) = 0, \quad (4)$$

$$(c, (a) + (b)) = 0. \quad (5)$$

In terms of  $x, y, z$ , equations (3)–(5) become

$$x + z = 0, \quad (6)$$

$$x + y = 0, \quad (7)$$

$$y + z = 0. \quad (8)$$

From (6) we get  $z = -x$ , and from (7) we get  $y = -x$ . Substituting into (8) gives

$$(-x) + (-x) = 0 \quad \Rightarrow \quad x = 0. \quad (9)$$

Hence

$$x = y = z = 0. \quad (10)$$

So  $(a), (b), (c)$  are pairwise orthogonal.

Therefore

$$|(a) + (b) + (c)|^2 = (a + b + c) \cdot (a + b + c) \quad (11)$$

$$= (a, a) + (b, b) + (c, c) \quad (12)$$

$$= |a|^2 + |b|^2 + |c|^2 \quad (13)$$

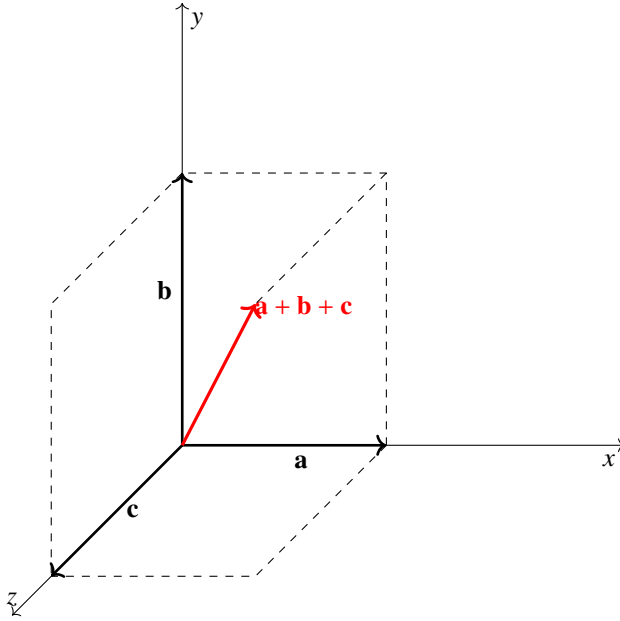
$$= 9 + 16 + 25 \quad (14)$$

$$= 50. \quad (15)$$

Thus

$$|(a) + (b) + (c)| = \sqrt{50} = 5\sqrt{2}. \quad (16)$$

**Graphical Representation:**



**Fig. 4**