5.3.15

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Question 5.3.15

What type of lines will you get by drawing the graph of the pair of equations:

$$x - 2y + 3 = 0 \quad \text{and} \quad 2x - 4y = 5?$$

Step 1: Convert to Standard Form

Write both equations in the standard form:

$$x - 2y = -3 \tag{1}$$

$$2x - 4y = 5 \tag{2}$$

We can represent them in matrix-vector form:

$$\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -3 \\ 5 \end{bmatrix} \tag{3}$$

Then the system becomes:

$$\mathbf{Ax} = \mathbf{b} \tag{4}$$

Step 2: Analyze the System

Observe the coefficient matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix}$$

We see:

$$\text{Row}_2 = 2 \times \text{Row}_1 \tag{5}$$

This implies the equations are **linearly dependent** in coefficients.
Now check the constants:

$$c_2 = 5 \neq 2 \times c_1 = 2 \times (-3) = -6 \tag{6}$$

So the constants are not in the same ratio as the coefficients.

Step 3: Matrix Rank and Conclusion

The augmented matrix is:

$$\left[\begin{array}{cc|c} 1 & -2 & -3 \\ 2 & -4 & 5 \end{array}\right]$$

Then:

$$\text{rank}(\mathbf{A}) = 1, \quad \text{rank}(\mathbf{A}|\mathbf{b}) = 2 \tag{7}$$

Therefore, the system is:

$$\boxed{\text{Inconsistent (no solution)}}$$

$$\Rightarrow \boxed{\text{Lines are parallel and distinct}}$$

C Code to Determine Line Relationship

```c
#include <stdio.h>
#include <stdbool.h>

bool areEqual(double a, double b, double epsilon) {
    return (a - b < epsilon) && (b - a < epsilon);
}

int main() {
    double a1 = 1, b1 = -2, c1 = -3;
    double a2 = 2, b2 = -4, c2 = 5;

    double ratio_a = a1 / a2;
    double ratio_b = b1 / b2;
    double ratio_c = c1 / c2;

    double epsilon = 1e-6;

    if (areEqual(ratio_a, ratio_b, epsilon) && !areEqual(ratio_b,
        ratio_c, epsilon)) {
```

Python Code: Plotting the Graphs

Visualization of the Locus and an Example Line

```
import matplotlib.pyplot as plt
import numpy as np

# 1. Define the range for x
x = np.linspace(-10, 10, 400)

# 2. Define the equations in slope-intercept form (y = mx + b)
# Equation 1: y = (1/2)x + 3/2
y1 = (1/2) * x + 3/2

# Equation 2: y = (1/2)x - 5/4
y2 = (1/2) * x - 5/4

# 3. Create the plot
plt.figure(figsize=(8, 6))

# Plot the lines
plt.plot(x, y1, label='$x - 2y + 3 = 0$ ($y = 0.5x + 1.5$)',
```

Plot