

# 5.13.12

AI25BTECH11036-SNEHAMRUDULA

**Question:**

The set of all values of  $\lambda$  for which the system of linear equations

$$2x_1 - 2x_2 + x_3 = \lambda x_1,$$

$$2x_1 - 3x_2 + 2x_3 = \lambda x_2,$$

$$-x_1 + 2x_2 = \lambda x_3,$$

has a non-trivial solution

- (a) contains two elements
- (b) contains more than two elements
- (c) is an empty set
- (d) is a singleton

**Solution:**

Bring all terms to one side:

$$(2 - \lambda)x_1 - 2x_2 + x_3 = 0$$

$$2x_1 + (-3 - \lambda)x_2 + 2x_3 = 0$$

$$-x_1 + 2x_2 - \lambda x_3 = 0$$

This system can be written in matrix form as:

$$\begin{pmatrix} 2 - \lambda & -2 & 1 \\ 2 & -3 - \lambda & 2 \\ -1 & 2 & -\lambda \end{pmatrix} \mathbf{x} = \mathbf{0} \quad \text{where} \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \quad \mathbf{0} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

To have a non-trivial solution, the system must be underdetermined, i.e.,

$$\text{rank}(\text{Coefficient Matrix}) < 3$$

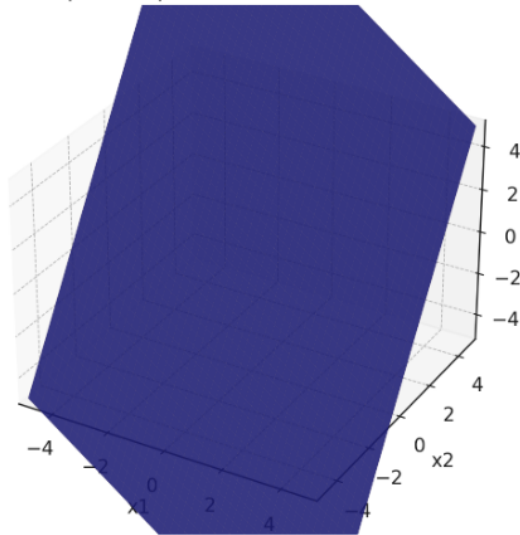
$$\text{rank} \left( \begin{pmatrix} 2 - \lambda & -2 & 1 \\ 2 & -3 - \lambda & 2 \\ -1 & 2 & -\lambda \end{pmatrix} \right) = 2 \quad \text{for exactly two values of } \lambda$$

So, the system has a non-trivial solution for exactly two values of  $\lambda$ .

Correct answer: (a) contains two elements

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3D plot of planes for  $\lambda \approx 1.000000$



3D plot of planes for  $\lambda \approx -3.000000$

