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Matrix 4.13.84

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Question (4.13.84)

Find the value of k such that the line

$$\frac{x-4}{1} = \frac{y-2}{1} = \frac{z-k}{2}$$

lies in the plane

$$2x - 4y + z = 7.$$

Parametric Form of Line

The line can be written as

$$\mathbf{r}(t) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{p} + t\,\mathbf{v},$$

where

$$\mathbf{p} = \begin{pmatrix} 4 \\ 2 \\ k \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}.$$

The plane can be written as

$$\mathbf{n}^T\mathbf{r} = 7, \quad \mathbf{n} = \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}.$$

Conditions for Line in Plane

Substituting, we get

$$\mathbf{n}^T(\mathbf{p} + t\mathbf{v}) = 7,$$
$$\mathbf{n}^T\mathbf{p} + t\,\mathbf{n}^T\mathbf{v} = 7.$$

For this to hold for all t:

$$\mathbf{n}^T\mathbf{v} = 0, \tag{1}$$
$$\mathbf{n}^T\mathbf{p} = 7. \tag{2}$$

Step 1: Direction Condition

$$\mathbf{n}^T\mathbf{v} = \begin{bmatrix} 2 & -4 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 2(1) + (-4)(1) + 1(2) = 0.$$

Thus, the condition is satisfied automatically.

Step 2: Point Condition

$$\mathbf{n}^T\mathbf{p} = \begin{bmatrix} 2 & -4 & 1 \end{bmatrix} \begin{pmatrix} 4 \\ 2 \\ k \end{pmatrix} = 8 - 8 + k = k.$$

Equating,

$$k = 7.$$

Final Answer

$$\boxed{k = 7}$$

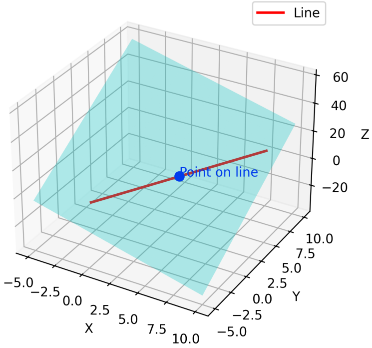

Figure: Line lying in the plane