4.13.72

AI25BTECH11003 - Bhavesh Gaikwad

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Question

A non-zero vector $\mathbf{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}$, $\hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}$, $\hat{i}+\hat{k}$. The angle between $\mathbf{a}$ and the vector $\hat{i}-2\hat{j}+2\hat{k}$ is?
(1996)

Theoretical Solution

First plane is determined by the vectors $\begin{pmatrix}1\\0\\0\end{pmatrix}$ and $\begin{pmatrix}1\\1\\0\end{pmatrix}$, so a normal is

$$\mathbf{n}_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} \times \begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix}. \tag{1}$$

Second plane is determined by $\begin{pmatrix}1\\-1\\0\end{pmatrix}$ and $\begin{pmatrix}1\\0\\1\end{pmatrix}$, so a normal is

$$\mathbf{n}_2 = \begin{pmatrix}1\\-1\\0\end{pmatrix} \times \begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\-1\\1\end{pmatrix}. \tag{2}$$

Theoretical Solution

Let vector $\mathbf{n}_3$ be the parallel vector of the intersection line.

$$\mathbf{n}_3 = \mathbf{n}_1 \times \mathbf{n}_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \tag{3}$$

Thus any vector $\mathbf{a}$ parallel to the intersection line is parallel to $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

$$\therefore \ \mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}. \tag{4}$$

Given vector in the question:

$$\text{Let } \mathbf{u} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}. \text{ (Already Given in the question)} \tag{5}$$

Theoretical Solution

Using the scalar product formula

$$\cos\theta = \frac{\mathbf{a}^\top \mathbf{u}}{\|\mathbf{a}\| \, \|\mathbf{u}\|}, \tag{6}$$

We compute

$$\mathbf{a}^\top \mathbf{u} = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} = 1 + 2 + 0 = 3, \tag{7}$$

$$\|\mathbf{a}\| = \sqrt{\mathbf{a}^\top \mathbf{a}} = \sqrt{1^2 + (-1)^2 + 0^2} = \sqrt{2}, \tag{8}$$

$$\|\mathbf{u}\| = \sqrt{\mathbf{u}^\top \mathbf{u}} = \sqrt{1^2 + (-2)^2 + 2^2} = 3. \tag{9}$$

Theoretical Solution

Substituting value from Equation 7 and 8 in Equation 5,

$$\cos\theta = \frac{3}{\sqrt{2}\cdot 3} = \frac{1}{\sqrt{2}} \implies \theta = 45^\circ. \tag{10}$$

$$\boxed{\text{The angle between } \mathbf{a} \textit{ and } 1\hat{i} - 2\hat{j} + 2\hat{k} \textit{ is } 45^\circ.} \tag{11}$$

Image