5.2.4

EE25BTECH11001 - Aarush Dilawri

Question:
Solve the system of equations

$$6x - 3y + 10 = 0 \tag{0.1}$$

$$2x - y + 9 = 0 \tag{0.2}$$

Solution:

The equation of line:

$$\mathbf{n}^\top \mathbf{x} = c \tag{0.3}$$

Line L:

$$\begin{pmatrix} 6 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -10 \tag{0.4}$$

Line K:

$$\begin{pmatrix} 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -9 \tag{0.5}$$

These can be combined and written in matrix form:

$$\begin{pmatrix} 6 & -3 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -10 \\ -9 \end{pmatrix} \tag{0.6}$$

The following augmented matrix can be solved by gaussian elimination

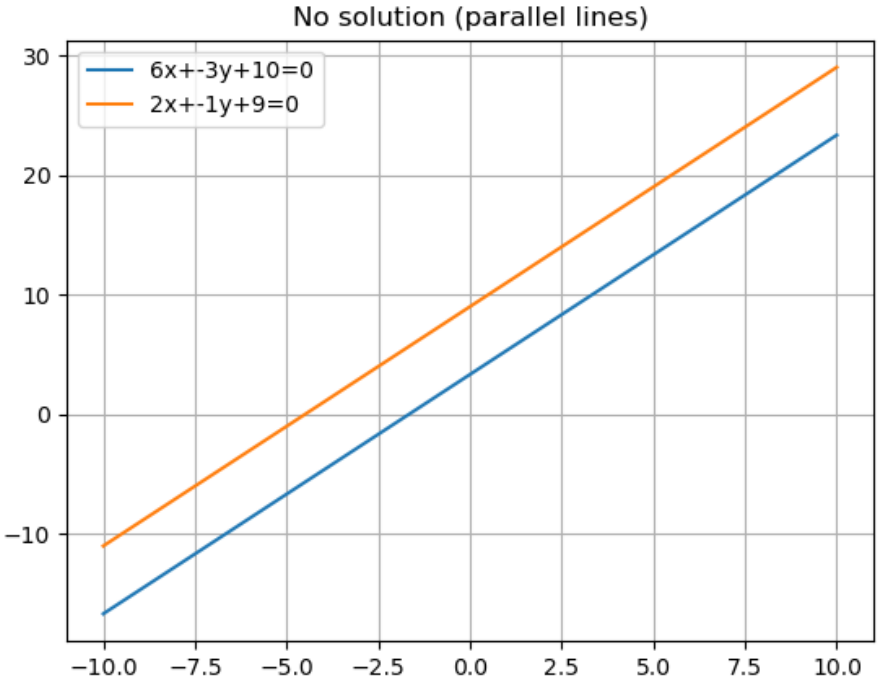
$$\left(\begin{array}{cc|c} 6 & -3 & -10 \\ 2 & -1 & -9 \end{array}\right) \xleftrightarrow{R_2 \to 3R_2 - R_1} \left(\begin{array}{cc|c} 6 & -3 & -10 \\ 0 & 0 & -17 \end{array}\right) \tag{0.7}$$

The rank of coefficient matrix is 1 whereas the rank of the Augmented matrix is 0. Thus, by Rouche Capelli theorem,

$$\text{If } \operatorname{rank}(\mathbf{A}) \neq \operatorname{rank}([\mathbf{A} \mid b]) \quad \text{, then the system has no solution.} \tag{0.8}$$

Hence, there are no solutions to this system of equations.

See Figure,


No solution (parallel lines)
6x+-3y+10=0
2x+-1y+9=0
30
20
10
0
−10
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0