Question

Consider two points P and Q with position vectors

$$\mathbf{P} = 3\mathbf{a} - 2\mathbf{b}, \qquad \mathbf{Q} = \mathbf{a} + \mathbf{b}.$$

Find the position vector of a point R which divides the line joining P and Q in the ratio $2:1$,
(a) internally, and
(b) externally.

Solution

We write the endpoints in matrix form:

$$\begin{pmatrix}\mathbf{P} & \mathbf{Q}\end{pmatrix}^T = \begin{pmatrix}3 & -2\\ 1 & 1\end{pmatrix}\begin{pmatrix}\mathbf{a}\\ \mathbf{b}\end{pmatrix}. \tag{1}$$

General section formulas (matrix form). For points $\mathbf{A}, \mathbf{B}$ and ratio $k:1$:

$$\mathbf{R}_{\text{int}} = \frac{k\,\mathbf{B} + \mathbf{A}}{k+1} = \frac{1}{k+1}\,[\,\mathbf{A}\;\;\mathbf{B}\,]\begin{pmatrix}1\\ k\end{pmatrix}. \tag{2}$$

$$\mathbf{R}_{\text{ext}} = \frac{k\,\mathbf{B} - \mathbf{A}}{k-1} = \frac{1}{k-1}\,[\,\mathbf{A}\;\;\mathbf{B}\,]\begin{pmatrix}-1\\ k\end{pmatrix}. \tag{3}$$

Internal division $2:1$**.** Using (2) with $\mathbf{A} = \mathbf{P},\ \mathbf{B} = \mathbf{Q},\ k = 2$,

$$\mathbf{R}_{\text{int}} = \frac{1}{3}\,[\,\mathbf{P}\;\;\mathbf{Q}\,]\begin{pmatrix}1\\ 2\end{pmatrix} = \begin{pmatrix}\tfrac{1}{3} & \tfrac{2}{3}\end{pmatrix}\begin{pmatrix}\mathbf{P} & \mathbf{Q}\end{pmatrix}. \tag{4}$$

Substitute (1) into (4):

$$\mathbf{R}_{\text{int}} = \begin{pmatrix}\tfrac{1}{3} & \tfrac{2}{3}\end{pmatrix}\begin{pmatrix}3 & -2\\ 1 & 1\end{pmatrix}\begin{pmatrix}\mathbf{a}\\ \mathbf{b}\end{pmatrix}. \tag{5}$$

$$\boxed{\mathbf{R}_{\text{int}} = \tfrac{5}{3}\,\mathbf{a}} \tag{6}$$

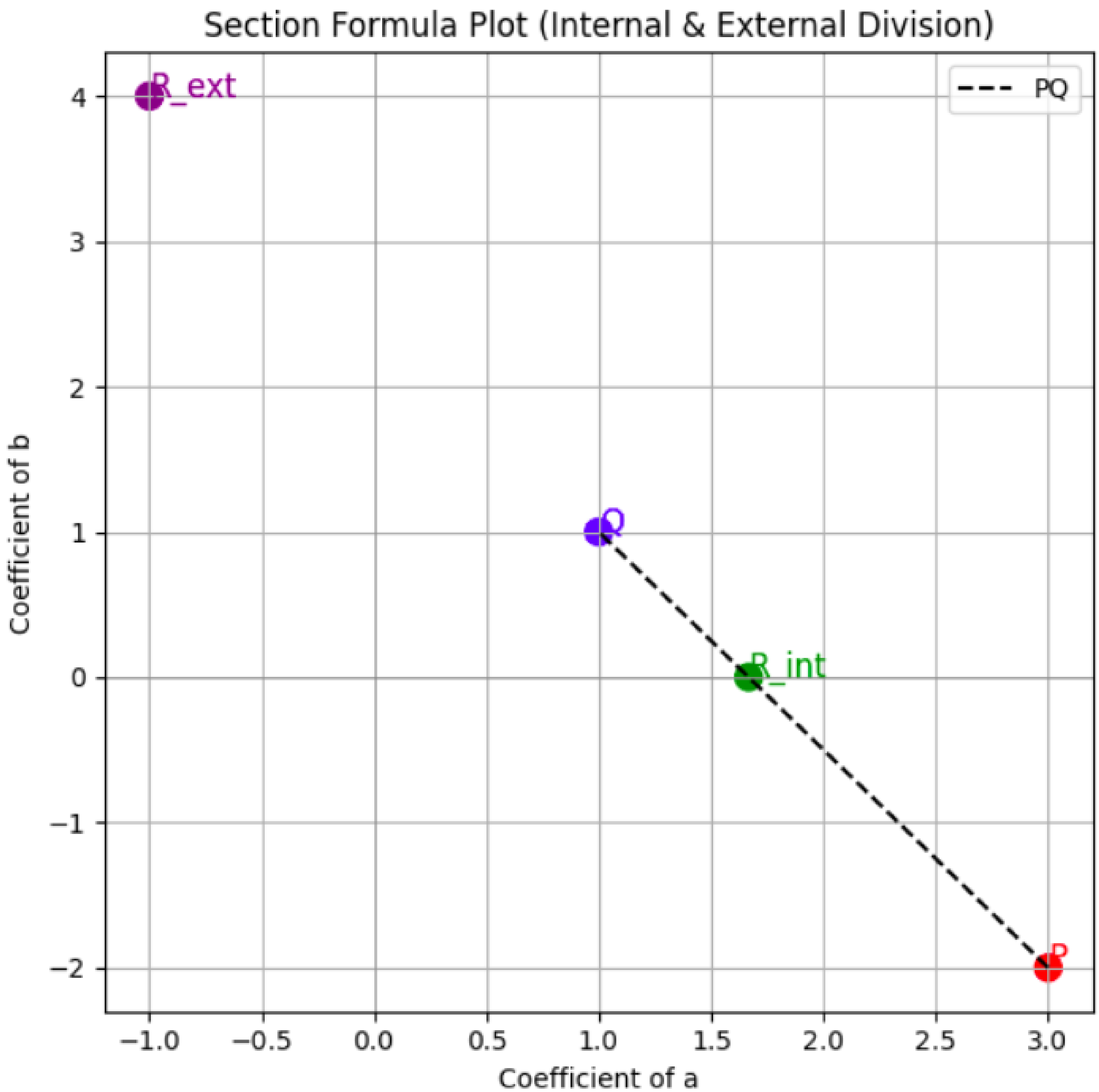
External division $2:1$**.** Using (3) with $\mathbf{A} = \mathbf{P},\ \mathbf{B} = \mathbf{Q},\ k = 2$,

$$\mathbf{R}_{\text{ext}} = \,[\,\mathbf{P}\;\;\mathbf{Q}\,]\begin{pmatrix}-1\\ 2\end{pmatrix} = \begin{pmatrix}-1 & 2\end{pmatrix}\begin{pmatrix}\mathbf{P} & \mathbf{Q}\end{pmatrix}. \tag{7}$$

Substitute (1) into (7):

$$\mathbf{R}_{\text{ext}} = \begin{pmatrix}-1 & 2\end{pmatrix}\begin{pmatrix}3 & -2\\ 1 & 1\end{pmatrix}\begin{pmatrix}\mathbf{a}\\ \mathbf{b}\end{pmatrix}. \tag{8}$$

$$\boxed{\mathbf{R}_{\text{ext}} = -\,\mathbf{a} + 4\,\mathbf{b}} \tag{9}$$


Section Formula Plot (Internal & External Division)
R_ext
Q
R_int
P
PQ
Coefficient of a
Coefficient of b
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
4
3
2
1
0
−1
−2