5.2.2

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Question. Solve the simultaneous linear equations

$$5u - 4v + 8 = 0, \qquad 7u + 6v - 9 = 0.$$

Solution.
Writing each line in normal form,

$$\begin{pmatrix} 5 & -4 \end{pmatrix} \mathbf{p} = -8, \qquad \begin{pmatrix} 7 & 6 \end{pmatrix} \mathbf{p} = 9, \tag{1}$$

where the unknown point vector is

$$\mathbf{p} = \begin{pmatrix} u \\ v \end{pmatrix}. \tag{2}$$

Equivalently,

$$\underbrace{\begin{pmatrix} 5 & -4 \\ 7 & 6 \end{pmatrix}}_{\mathbf{A}} \mathbf{p} = \underbrace{\begin{pmatrix} -8 \\ 9 \end{pmatrix}}_{\mathbf{b}}. \tag{3}$$

Since

$$\det(\mathbf{A}) = 5 \cdot 6 - (-4) \cdot 7 = 58 \neq 0, \tag{4}$$

the unique solution is

$$\mathbf{p} = \mathbf{A}^{-1}\mathbf{b} = \frac{1}{58}\,(\mathbf{A})\,\mathbf{b} = \frac{1}{58}\begin{pmatrix} 6 & 4 \\ -7 & 5 \end{pmatrix}\begin{pmatrix} -8 \\ 9 \end{pmatrix} = \frac{1}{58}\begin{pmatrix} -12 \\ 101 \end{pmatrix}. \tag{5}$$

Hence,

$$\boxed{u = -\frac{6}{29}, \quad v = \frac{101}{58}}. \tag{6}$$

(Here we used the matrix normal-form method $\mathbf{A}\mathbf{p} = \mathbf{b}$ and the $\mathbf{A}^{-1} = \frac{\text{adj}(\mathbf{A})}{\det(\mathbf{A})}$.)

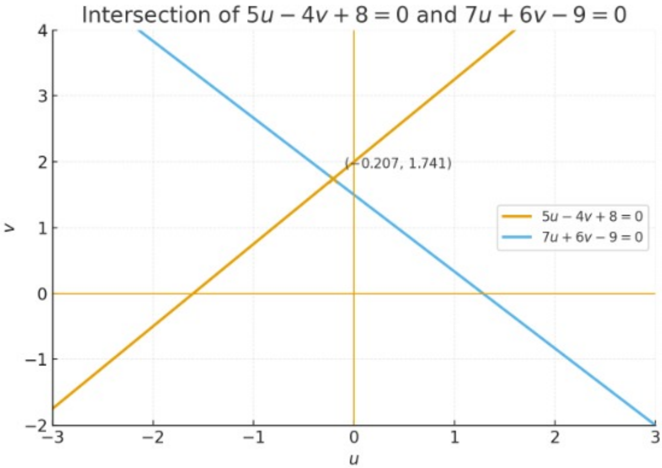

Fig. 0.1: Intersection of $5u - 4v + 8 = 0$ and $7u + 6v - 9 = 0$ at $\left(-\frac{6}{29},\ \frac{101}{58}\right)$.