Matrices in Geometry - 1.9.26

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Problem Statement

Find the value of k, if the point $\mathbf{P}\,(2, 4)$ is equidistant from point $\mathbf{A}\,(5, k)$ and $\mathbf{B}\,(k, 7)$

Solution

Given: $\mathbf{P}\begin{pmatrix}2\\4\end{pmatrix}$, $\mathbf{A}\begin{pmatrix}5\\k\end{pmatrix}$ and a point $\mathbf{B}\begin{pmatrix}k\\7\end{pmatrix}$ such that $\mathbf{P}$ is equidistant from $\mathbf{A}$ and $\mathbf{B}$.

$$\therefore \|\mathbf{P}-\mathbf{A}\| = \|\mathbf{P}-\mathbf{B}\| \tag{1}$$

On squaring both the sides, we get (2)

$$\|\mathbf{P}-\mathbf{A}\|^2 = \|\mathbf{P}-\mathbf{A}\|^2 \tag{3}$$

$$(\mathbf{P}-\mathbf{A})^\top(\mathbf{P}-\mathbf{A}) = (\mathbf{P}-\mathbf{B})^\top(\mathbf{P}-\mathbf{B}) \tag{4}$$

Solution

$$\mathbf{P}^\top\mathbf{P} - 2\mathbf{P}^\top\mathbf{A} + \mathbf{A}^\top\mathbf{A} = \mathbf{P}^\top\mathbf{P} - 2\mathbf{P}^\top\mathbf{B} + \mathbf{B}^\top\mathbf{B} \quad (5)$$

$$\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 = 2\mathbf{P}^\top(\mathbf{A} - \mathbf{B}) \quad (6)$$

$$\left\|\begin{pmatrix}5\\k\end{pmatrix}\right\| - \left\|\begin{pmatrix}k\\7\end{pmatrix}\right\| = 2\begin{pmatrix}2 & 4\end{pmatrix}\begin{pmatrix}5-k\\k-7\end{pmatrix} \quad (7)$$

$$25 + k^2 - 49 - k^2 = 2(10 - 2k + 4k - 28) \quad (8)$$

$$-24 = 2(2k - 18) \implies -12 = 2k - 18 \implies 2k = 6 \implies k = 3 \quad (9)$$

Final Answer

Hence, the final answer is $\boxed{k = 3}$ (10)