4.7.38

EE25BTECH11041 - Naman Kumar

Question:
P(0, 2) is the point of intersection of Y axis and perpendicular bisector of line segment joining the points A(-1, 1) and B(3, 3).

Solution:
Given points,

$$\mathbf{A} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{1}$$

Mid point of $\mathbf{A}$ and $\mathbf{B}$, Let it be $\mathbf{R}$

$$\mathbf{R} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{2}$$

Slope, $\mathbf{m}$

$$\mathbf{m} = \mathbf{B} - \mathbf{A} \tag{3}$$

(4)

Let $\mathbf{n}$ be the direction vector perpendicular to $\mathbf{m}$, If truly $\mathbf{P}$ is y-intercept of bisector

$$\mathbf{n} = \mathbf{P} - \mathbf{R} \tag{5}$$

Both $\mathbf{n}$ and $\mathbf{m}$ are perpendicular

$$\mathbf{n}^T\mathbf{m} = 0 \tag{6}$$

$$(\mathbf{P} - \mathbf{R})^T(\mathbf{B} - \mathbf{A}) = 0 \tag{7}$$

$$(\mathbf{P}^T - (\frac{\mathbf{A} + \mathbf{B}}{2})^T)(\mathbf{B} - \mathbf{A}) = 0 \tag{8}$$

$$\mathbf{P}^T(\mathbf{B} - \mathbf{A}) - \frac{(\mathbf{A} + \mathbf{B})^T(\mathbf{B} - \mathbf{A})}{2} = 0 \tag{9}$$

$$\begin{pmatrix} 0 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \end{pmatrix} - \frac{\begin{pmatrix} 2 & 4 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \end{pmatrix}}{2} = 0 \tag{10}$$

$$4 - \frac{16}{2} \neq 0 \tag{11}$$

Hence, $\mathbf{P}$ is not the y-intercept of perpendicular bisector of line $\mathbf{A} - \mathbf{B}$

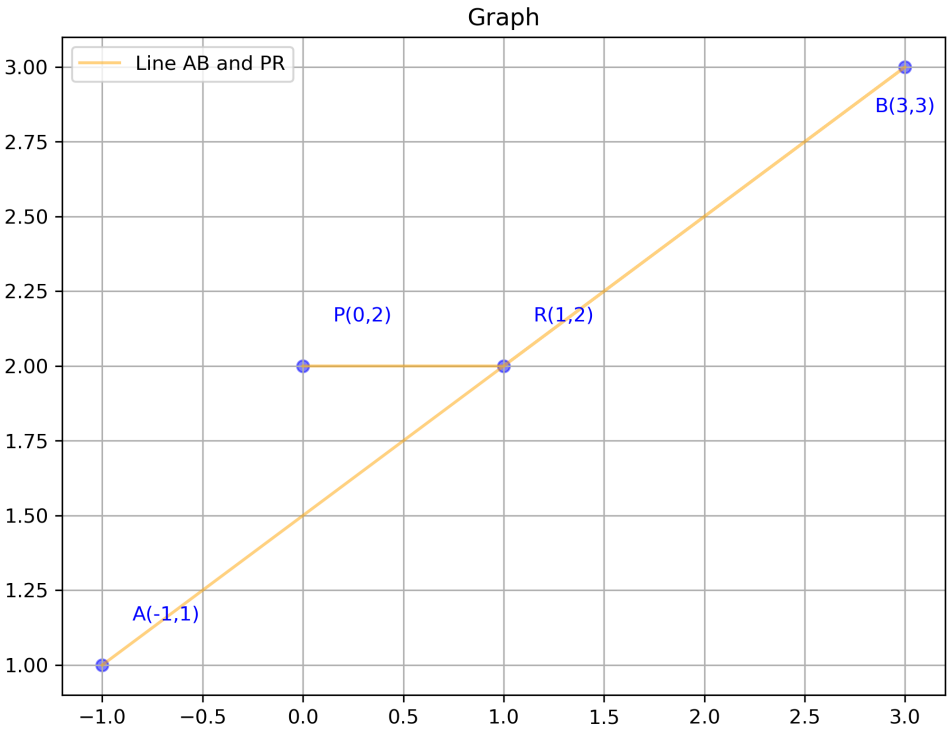
Graph
Line AB and PR
B(3,3)
P(0,2)
R(1,2)
A(-1,1)
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Fig. 1