

4.11.25

EE25BTECH11048 - Revanth Siva Kumar.D

Question

Find the distance of the point $(1, -2, 9)$ from the point of intersection of the line

$$\mathbf{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$$

and the plane

$$\mathbf{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10.$$

Solution:

Step 1: General setup

Let the line be

$$\mathbf{r} = \mathbf{r}_0 + \lambda \mathbf{d}, \quad (1)$$

and the plane be

$$\mathbf{n}^T \mathbf{r} = c, \quad (2)$$

where \mathbf{r}_0 is a point on the line, \mathbf{d} is its direction vector, \mathbf{n} is the plane normal, and c is a constant. The given external point is \mathbf{A} .

Step 2: Intersection point of line and plane

Substitute $\mathbf{r} = \mathbf{r}_0 + \lambda \mathbf{d}$ into the plane equation:

$$\mathbf{n}^T (\mathbf{r}_0 + \lambda \mathbf{d}) = c. \quad (3)$$

This gives

$$\lambda = \frac{c - \mathbf{n}^T \mathbf{r}_0}{\mathbf{n}^T \mathbf{d}}. \quad (4)$$

Hence, the intersection point is

$$\mathbf{P} = \mathbf{r}_0 + \frac{c - \mathbf{n}^T \mathbf{r}_0}{\mathbf{n}^T \mathbf{d}} \mathbf{d}. \quad (5)$$

Step 3: Distance formula

The displacement vector is

$$\mathbf{v} = \mathbf{P} - \mathbf{A}, \mathbf{v} = \mathbf{r}_0 + \frac{c - \mathbf{n}^T \mathbf{r}_0}{\mathbf{n}^T \mathbf{d}} \mathbf{d} - \mathbf{A} \quad (6)$$

and therefore the required distance is

$$d = \|\mathbf{v}\| = \sqrt{\mathbf{v}^T \mathbf{v}}. \quad (7)$$

Step 4: Substitution from the question

From the problem statement,

$$\mathbf{r}_0 = \begin{pmatrix} 4 \\ 2 \\ 7 \end{pmatrix}, \quad \mathbf{d} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}, \quad \mathbf{n} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \quad c = 10, \quad \mathbf{A} = \begin{pmatrix} 1 \\ -2 \\ 9 \end{pmatrix}. \quad (8)$$

Now compute:

$$\mathbf{n}^T \mathbf{d} = 1, \quad (9)$$

$$\mathbf{n}^T \mathbf{r}_0 = 9, \quad (10)$$

$$\lambda = \frac{10 - 9}{1} = 1. \quad (11)$$

So

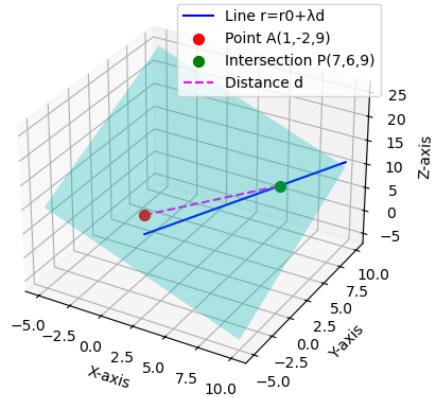
$$\mathbf{P} = \begin{pmatrix} 4 \\ 2 \\ 7 \end{pmatrix} + 1 \cdot \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \\ 9 \end{pmatrix}, \quad (12)$$

$$\mathbf{v} = \mathbf{P} - \mathbf{A} = \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix}, \quad (13)$$

$$d = \sqrt{6^2 + 8^2 + 0^2} = 10. \quad (14)$$

Final Answer:

Distance from Point to Line-Plane Intersection

Fig. 1: Intersection point P , given point A , and distance AP .