

1.4.12

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Problem:

In what ratio does the point $P(-4, 6)$ divide the line segment joining the points $A(-6, 0)$ and $C(3, -8)$?

Solution:

Given Points

$$\mathbf{A} = \begin{pmatrix} -6 \\ 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 3 \\ -8 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} -4 \\ 6 \end{pmatrix},$$

the ratio k in which P divides AC is

$$\begin{aligned} k &= \frac{(\mathbf{A} - \mathbf{P})^T (\mathbf{P} - \mathbf{C})}{\|\mathbf{P} - \mathbf{C}\|^2} \\ &= \frac{\begin{pmatrix} -6 + 4 \\ 0 - 6 \end{pmatrix}^T \begin{pmatrix} -4 - 3 \\ 6 + 8 \end{pmatrix}}{(-4 - 3)^2 + (6 + 8)^2} \\ &= \frac{\begin{pmatrix} -2 \\ -6 \end{pmatrix}^T \begin{pmatrix} -7 \\ 14 \end{pmatrix}}{(-7)^2 + 14^2} \end{aligned}$$

Therefore,

$$k = \frac{-70}{245} = -\frac{2}{7}.$$

Since $k = -\frac{2}{7}$, Point P divides the segment AC **externally** in the ratio $2 : 7$.

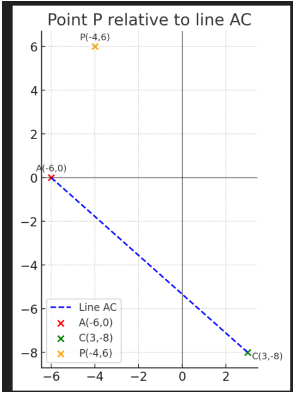


Fig. 0.1: Caption