2.8.9

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september 2025

Question

Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be three vectors with magnitudes given by

$$\|\mathbf{a}\| = 3, \qquad \|\mathbf{b}\| = 4, \qquad \|\mathbf{c}\| = 5 \tag{1}$$

Each vector is perpendicular to the sum of the other two, so

$$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = 0, \qquad \mathbf{b} \cdot (\mathbf{c} + \mathbf{a}) = 0, \qquad \mathbf{c} \cdot (\mathbf{a} + \mathbf{b}) = 0 \tag{2}$$

Introduce notation:

$$s = \mathbf{a} \cdot \mathbf{b}, \qquad t = \mathbf{b} \cdot \mathbf{c}, \qquad u = \mathbf{c} \cdot \mathbf{a} \tag{3}$$

From (2) the equations become

$$s + u = 0, \qquad t + s = 0, \qquad u + t = 0 \tag{4}$$

Theoretical solution

From the first equation,

$$u = -s \tag{5}$$

From the second equation,

$$t = -s \tag{6}$$

Substitute (5) and (6) into the third equation:

$$(-s) + (-s) = -2s = 0 \quad \Rightarrow \quad s = 0 \tag{7}$$

Hence,

$$s = t = u = 0 \tag{8}$$

Theoretical solution

Thus $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are mutually perpendicular Now compute the square of the magnitude:

$$\|\mathbf{a}+\mathbf{b}+\mathbf{c}\|^2 = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + \|\mathbf{c}\|^2 + 2(s+t+u) \tag{9}$$

Substitute values from (1) and (8):

$$= 3^2 + 4^2 + 5^2 + 2(0+0+0) \tag{10}$$

$$= 9 + 16 + 25 = 50 \tag{11}$$

Therefore,

$$\|\mathbf{a}+\mathbf{b}+\mathbf{c}\| = \sqrt{50} = 5\sqrt{2}. \tag{12}$$

Final Answer:

$$\boxed{5\sqrt{2}}$$

Python Code

```
import numpy as np
import matplotlib.pyplot as plt

# Define mutually perpendicular vectors
a = np.array([3, 0, 0])
b = np.array([0, 4, 0])
c = np.array([0, 0, 5])
s = a + b + c # resultant (3,4,5)

fig = plt.figure(figsize=(8, 6))
ax = fig.add_subplot(111, projection='3d')
```

Python Code

```
# Plot main vectors
ax.quiver(0, 0, 0, *a, color='r', linewidth=2,
          arrow_length_ratio=0.08, normalize=False, label='a (3)')
ax.quiver(0, 0, 0, *b, color='g', linewidth=2,
          arrow_length_ratio=0.08, normalize=False, label='b (4)')
ax.quiver(0, 0, 0, *c, color='b', linewidth=2,
          arrow_length_ratio=0.08, normalize=False, label='c (5)')

# Plot resultant
ax.quiver(0, 0, 0, *s, color='m', linewidth=2,
          arrow_length_ratio=0.05, normalize=False,
          label='a+b+c')
```

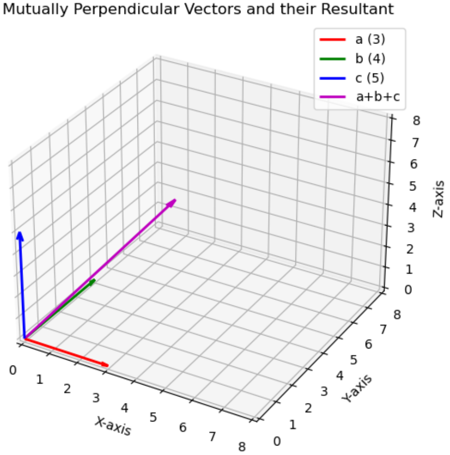
Python Code

```
# Axis limits
ax.set_xlim(0, 8)
ax.set_ylim(0, 8)
ax.set_zlim(0, 8)

# Axis labels
ax.set_xlabel("X-axis")
ax.set_ylabel("Y-axis")
ax.set_zlabel("Z-axis")
ax.set_title("Mutually Perpendicular Vectors and their Resultant"
    )

ax.legend()
plt.show()
```

C Code

```c
#include <stdio.h>
#include <math.h>

int main() {
    // Given magnitudes
    int a = 3, b = 4, c = 5;
    // Since a, b, c are mutually perpendicular (proved in
        solution),
    // |a + b + c|^2 = |a|^2 + |b|^2 + |c|^2
    int sum_sq = a*a + b*b + c*c;

    double magnitude = sqrt(sum_sq);

    // Print result
    printf("The magnitude |a + b + c| = %.2f\n", magnitude);

    return 0;
}
```

Plot-Using by Python



Python and C Code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load the compiled C library
lib = ctypes.CDLL("./vector_calc.so") # use "vector_calc.dll" on
    Windows

# Call the C function
lib.vector_magnitude.restype = ctypes.c_double
magnitude = lib.vector_magnitude()
print("Result from C code |a+b+c| =", magnitude)
```

Python and C Code

```
# ---- Plotting in Python ----
a = np.array([3, 0, 0])
b = np.array([0, 4, 0])
c = np.array([0, 0, 5])
resultant = a + b + c

fig = plt.figure(figsize=(8, 6))
ax = fig.add_subplot(111, projection="3d")

# plot vectors
origin = np.array([0, 0, 0])
ax.quiver(*origin, *a, color='r', label='a (3)')
ax.quiver(*origin, *b, color='g', label='b (4)')
ax.quiver(*origin, *c, color='b', label='c (5)')
ax.quiver(*origin, *resultant, color='m', label='a+b+c')
```

Python and C Code

```
ax.set_xlim([0, 8])
ax.set_ylim([0, 8])
ax.set_zlim([0, 8])

ax.set_xlabel("X axis")
ax.set_ylabel("Y axis")
ax.set_zlabel("Z axis")
ax.set_title("C code calculation + Python plot")

ax.legend()
plt.show()
```

Plot-Using by both C and Python