5.5.29

AI25BTECH11012 - GARIGE UNNATHI

Question:
If the inverse of the matrix $\begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ is the matrix $\begin{pmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{pmatrix}$, then find the value of λ .

Solution:
Let :

$$\mathbf{A} = \begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

The characteristic equation for a matrix $\mathbf{A}$ is

$$f(\lambda) = |\mathbf{A} - \lambda\mathbf{I}| = 0 \tag{0.1}$$

$$f(\lambda) = \begin{vmatrix} 7-\lambda & -3 & -3 \\ -1 & 1-\lambda & 0 \\ -1 & 0 & 1-\lambda \end{vmatrix} = 0 \tag{0.2}$$

Solving the above equation we get :

$$\lambda^3 - 9\lambda^2 + 9\lambda - 1 = 0 \tag{0.3}$$

By Cayley-Hamilton theorem :

$$f(\lambda) = f(\mathbf{A}) = 0 \tag{0.4}$$

$$\mathbf{A}^3 - 9\mathbf{A}^2 + 9\mathbf{A} - 1 = 0 \tag{0.5}$$

Multiplying the equation 0.5 by $\mathbf{A}^-1$ we get :

$$\mathbf{A}^2 - 9\mathbf{A} + 9\mathbf{I} - \mathbf{A}^{-1} = 0 \tag{0.6}$$

$$\mathbf{A}^{-1} = \mathbf{A}^2 - 9\mathbf{A} + 9\mathbf{I} \tag{0.7}$$

Solving the above equation we get :

$$\mathbf{A}^{-1} = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix} \tag{0.8}$$

Hence ,

$$\lambda = 4 \tag{0.9}$$