2.3.8

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Question

If $\hat{i}+\hat{j}+\hat{k},\ 2\hat{i}+5\hat{j},\ 3\hat{i}+2\hat{j}-3\hat{k},\ \hat{i}-6\hat{j}-\hat{k}$ respectively are the position vectors of points A, B, C, and D, then find the angle between the straight lines $(\mathbf{B}-\mathbf{A})$ and $(\mathbf{D}-\mathbf{C})$. Find whether $(\mathbf{B}-\mathbf{A})$ and $(\mathbf{D}-\mathbf{C})$ are collinear or not.

Given Information

$$\text{Let } \mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 1 \\ -6 \\ -1 \end{pmatrix}$$

Direction vectors:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix} \tag{2}$$

Formula

The angle θ between $(\mathbf{B}-\mathbf{A})$ and $(\mathbf{D}-\mathbf{C})$ is

$$\cos\theta = \frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{D}-\mathbf{C})}{|(\mathbf{B}-\mathbf{A})||(\mathbf{D}-\mathbf{C})|} \tag{3}$$

$$\theta = \cos^{-1}\left(\frac{(\mathbf{B}-\mathbf{A})^T(\mathbf{D}-\mathbf{C})}{|(\mathbf{B}-\mathbf{A})||(\mathbf{D}-\mathbf{C})|}\right) \tag{4}$$

Vectors are collinear if $\theta = 0^\circ$ or 180°.

Solution

$$(\mathbf{B}-\mathbf{A})^T(\mathbf{D}-\mathbf{C}) = \begin{pmatrix} 1 & 4 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix} = 1\times(-2)+4\times(-8)+(-1)\times 2 = -36$$

$$|(\mathbf{B}-\mathbf{A})| = \sqrt{1^2+4^2+(-1)^2} = \sqrt{18}$$

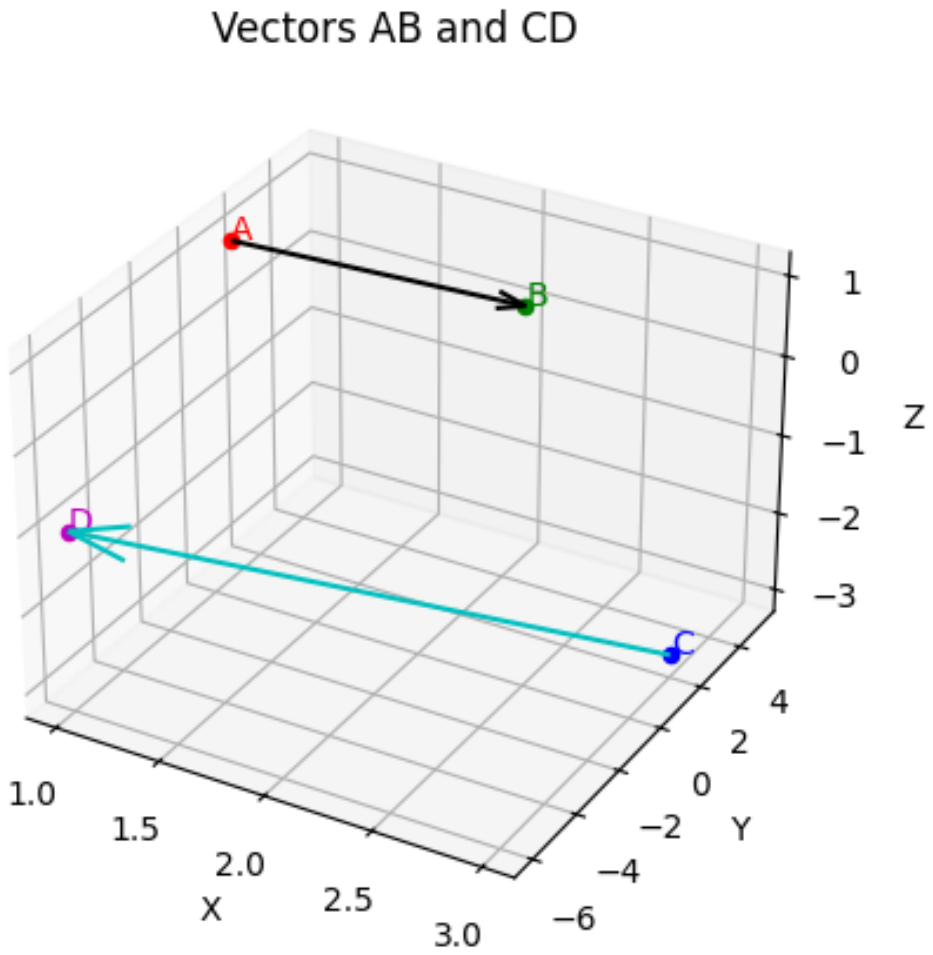
$$|(\mathbf{D}-\mathbf{C})| = \sqrt{(-2)^2+(-8)^2+2^2} = \sqrt{72}$$

$$\cos\theta = \frac{-36}{\sqrt{18}\times\sqrt{72}} = \frac{-36}{36} = -1$$

$$\theta = \cos^{-1}(-1) = 180^\circ$$

Therefore, $(\mathbf{B}-\mathbf{A})$ and $(\mathbf{D}-\mathbf{C})$ are collinear but in opposite directions.

Plot



Codes

For Codes, refer to the URL below:
https://github.com/Aditya-Mishra11005/ee1030-2025/tree/main/ee25btech11005/matgeo/2.3.8/Codes