12.371

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Question

Let T : $P_3[0,\ 1] \to P_2[0,\ 1]$ be defined by $(T\ \ p)(x) = p''(x) + p'(x)$. Then the matrix representation of T with respect to the bases $\{1, x, x^2, x^3\}$ and $\{1, x, x^2\}$ of $P_3[0,\ 1]$ and $P_2[0,\ 1]$ respectively is

1. $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 2 & 0 \\ 0 & 6 & 3 \end{pmatrix}$
2. $\begin{pmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 3 \end{pmatrix}$
3. $\begin{pmatrix} 0 & 2 & 1 & 0 \\ 6 & 2 & 0 & 0 \\ 3 & 0 & 0 & 0 \end{pmatrix}$
4. $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 2 \\ 3 & 6 & 0 \end{pmatrix}$

Theoretical Solution

The transformation is

$$T\left(p\right)\left(x\right) = p''\left(x\right) + p'\left(x\right) \tag{1}$$

The domain basis is

$$\mathcal{B} = \left\{1, x, x^2, x^3\right\} = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\} \tag{2}$$

The codomain basis is

$$\mathcal{C} = \left\{1, x, x^2\right\} = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3\} \tag{3}$$

Theoretical Solution

The matrix representation $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} [T(\mathbf{v}_1)]_{\mathcal{C}} & [T(\mathbf{v}_2)]_{\mathcal{C}} & [T(\mathbf{v}_3)]_{\mathcal{C}} & [T(\mathbf{v}_4)]_{\mathcal{C}} \end{pmatrix} \tag{4}$$

The columns of $\mathbf{A}$ are computed by applying the transformation (1) to each basis vector in $\mathcal{B}$.

$$T(\mathbf{v}_1) = T(1) = 0(1) + 0(x) + 0\left(x^2\right) \implies [T(\mathbf{v}_1)]_{\mathcal{C}} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

$$T(\mathbf{v}_2) = T(x) = 1(1) + 0(x) + 0\left(x^2\right) \implies [T(\mathbf{v}_2)]_{\mathcal{C}} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{6}$$

Theoretical Solution

$$T\left(\mathbf{v}_3\right) = T\left(x^2\right) = 2\left(1\right) + 2\left(x\right) + 0\left(x^2\right) \implies \left[T\left(\mathbf{v}_3\right)\right]_{\mathcal{C}} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \tag{7}$$

$$T\left(\mathbf{v}_4\right) = T\left(x^3\right) = 0\left(1\right) + 6\left(x\right) + 3\left(x^2\right) \implies \left[T\left(\mathbf{v}_4\right)\right]_{\mathcal{C}} = \begin{pmatrix} 0 \\ 6 \\ 3 \end{pmatrix} \tag{8}$$

This gives

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 3 \end{pmatrix} \tag{9}$$

The correct option is **2)**.