

5.2.48

Puni Aditya - EE25BTECH11046

Question:

Solve the following system of linear equations.

$$\begin{aligned}x + y + z &= 1 \\2x + 3y + 2z &= 2 \\ax + ay + 2az &= 4\end{aligned}$$

Solution:

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 3 & 2 & 2 \\ a & a & 2a & 4 \end{array} \right) \xleftrightarrow{R_3 \rightarrow \frac{1}{a}R_3, a \neq 0} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 3 & 2 & 2 \\ 1 & 1 & 2 & \frac{4}{a} \end{array} \right) \quad (1)$$

$$\xleftrightarrow[\begin{smallmatrix} R_3 \rightarrow R_3 - R_1 \\ R_2 \rightarrow R_2 - 2R_1 \end{smallmatrix}]{R_1 \rightarrow R_1 - R_2} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{4}{a} - 1 \end{array} \right) \xleftrightarrow{R_1 \rightarrow R_1 - R_2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{4}{a} - 1 \end{array} \right) \quad (2)$$

$$\xleftrightarrow{R_1 \rightarrow R_1 - R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 - \frac{4}{a} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{4}{a} - 1 \end{array} \right) \quad (3)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 - \frac{4}{a} \\ 0 \\ \frac{4}{a} - 1 \end{pmatrix}, \quad a \neq 0 \quad (4)$$

For $a = 0$, the third equation becomes $0 = 4$, which is inconsistent. Therefore, no solution exists for $a = 0$.

Example:

Let

$$a = 2$$

$$x + y + z = 1$$

$$2x + 3y + 2z = 2$$

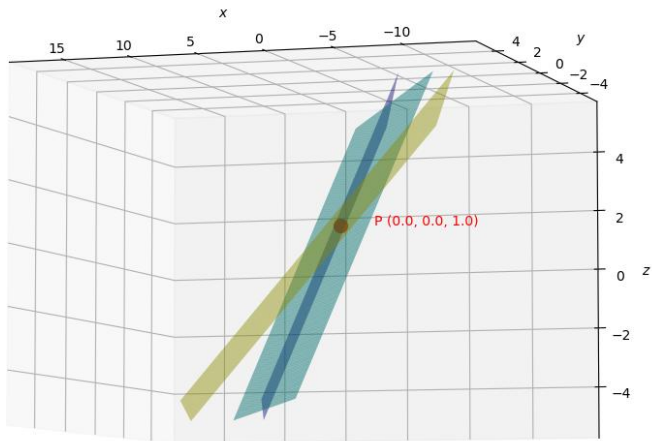
$$2x + 2y + 4z = 4$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

Using (4),

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (5)$$

The Planes and their Solution



Plot