# Matrices in Geometry 4.13.40

EE25BTECH11035 - Kushal B N

**Question:**

The number of integer values of $m$ for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is

1) 2 2) 0 3) 4 4) 1

**Given:**

The two lines

$\begin{pmatrix}3 & 4\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = 9$ and $\begin{pmatrix}m & -1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = -1$

**Solution:**

The given set of equations can be written as,

$$\begin{pmatrix}3 & 4\\ m & -1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}9\\-1\end{pmatrix} \tag{1}$$

Augmented Matrix:

$$\left(\begin{array}{cc|c}3 & 4 & 9\\ m & -1 & -1\end{array}\right) \tag{2}$$

$$\left(\begin{array}{cc|c}3 & 4 & 9\\ m & -1 & -1\end{array}\right) \overset{R_2 \to R_2 - \frac{m}{3}R_1}{\longrightarrow} \left(\begin{array}{cc|c}3 & 4 & 9\\ 0 & -1-\frac{4m}{3} & -1-3m\end{array}\right) \tag{3}$$

$$\implies y = 3\left(\frac{3m+1}{4m+3}\right) \tag{4}$$

$$\implies x = \frac{5}{4m+3} \tag{5}$$

Thus, for $x$ to be an integer, while keeping the denominator also an integer,

$$(4m+3) \in \{1, 5, -1, -5\} \tag{6}$$

$$\implies m \in \left\{\frac{-1}{2}, \frac{1}{2}, -1, -2\right\} \tag{7}$$

Hence, for m to be an integer value $m = -1$ or $m = -2$.

**Conclusion:**

$\therefore$ There are 2 integer values of $m$ for which the x-coordinate of the point of intersection of the given lines is also an integer.

Hence, the correct answer is option (1).

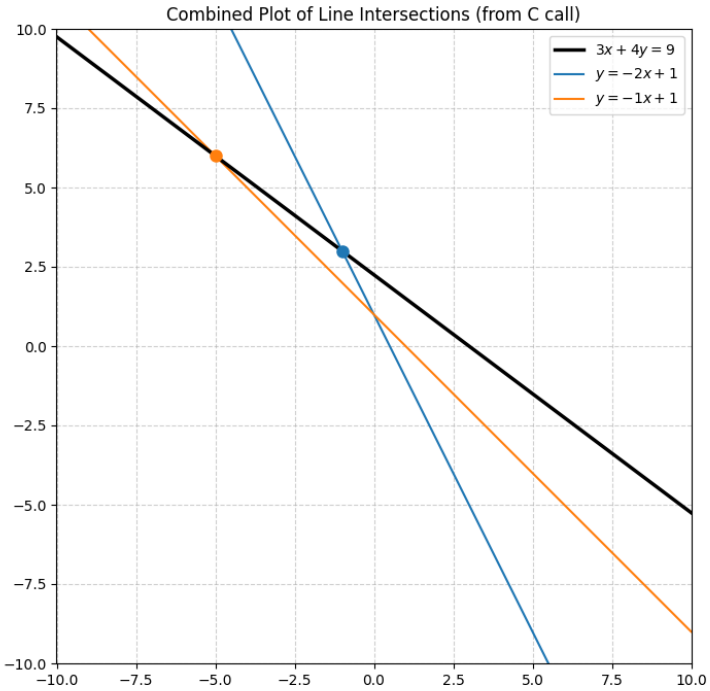
Combined Plot of Line Intersections (from C call)
3x + 4y = 9
y = −2x + 1
y = −1x + 1
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

Fig. 1