10.4.8

EE25BTECH11032 - Kartik Lahoti

Question:

For which value of m is the line

$$y = mx + 1 \tag{0.1}$$

a tangent to the curve

$$y^2 = 4x \tag{0.2}$$

Solution:

Given parabola

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.3}$$

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, f = 0. \tag{0.4}$$

Given Line equation

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{0.5}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ m \end{pmatrix}. \tag{0.6}$$

For the line to be a tangent , all the solution for k should be equal

$$k_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top \mathbf{V}\mathbf{m})} \right) \tag{0.7}$$

From 0.7

$$\left[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})\right]^2 = g(\mathbf{h})\left(\mathbf{m}^\top \mathbf{V}\mathbf{m}\right) \tag{0.8}$$

$$g(\mathbf{h}) = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} -2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + 0 = 1 \tag{0.9}$$

Using 0.9 in 0.8

$$\left[\begin{pmatrix} 1 & m \end{pmatrix} \left(\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix} \right) \right]^2 = 1 \left(\begin{pmatrix} 1 & m \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ m \end{pmatrix} \right) \tag{0.10}$$

$$\implies (m-2)^2 = m^2 \tag{0.11}$$

$$\implies 4 - 4m = 0 \tag{0.12}$$

$$\implies m = 1 \tag{0.13}$$