9.4.39

EE25BTECH11026-Harsha

Question:
The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.
Solution:
Let us solve the given question theoretically and then verify the solution computationally.

Let x and y be the 2 numbers such that $x > y$.
The given equations are,

$$x^2 - y^2 = 180 \tag{0.1}$$

$$y^2 = 8x \tag{0.2}$$

As the given equations are homogeneous, converting them into quadratic form,

$$\implies \mathbf{x}^\top \mathbf{F} \mathbf{x} = 0 \tag{0.3}$$

where $\mathbf{x}^\top = \begin{pmatrix} x & y & 1 \end{pmatrix}$ and $\mathbf{F} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -180 \end{pmatrix}$
And also,

$$\mathbf{x}^\top \mathbf{G} \mathbf{x} = 0 \tag{0.4}$$

where $\mathbf{x}^\top = \begin{pmatrix} x & y & 1 \end{pmatrix}^\top$ and $\mathbf{G} = \begin{pmatrix} 0 & 0 & -4 \\ 0 & 1 & 0 \\ -4 & 0 & 0 \end{pmatrix}$ To identify the intersection of conics, we can employ the approach of degenerating the conics.

This approach goes by the fact that as both the conics are 0 simultaneously, it's linear combination would also have the same solution.

$$\therefore \mathbf{x}^\top (\mathbf{F} + \lambda \mathbf{G}) \mathbf{x} = 0 \tag{0.5}$$

To degenerate the conic into a line, we can find the solutions of λ when $||\mathbf{F} + \lambda \mathbf{G}|| = 0$

$$\therefore ||\mathbf{F} + \lambda \mathbf{G}|| = 0 \tag{0.6}$$

$$\implies (\lambda - 1)\left(4\lambda^2 + 45\right) = 0 \tag{0.7}$$

$$\therefore \lambda = 1 \tag{0.8}$$

Substituting λ in the equation,

$$\mathbf{x}^\top (\mathbf{F} + \mathbf{G}) \mathbf{x} \tag{0.9}$$

$$\implies x^2 - 8x - 180 = 0 \tag{0.10}$$

$$\implies x = 18, -10 \tag{0.11}$$

for $x = -10$, there is no real solution of y,

$$\implies y = \pm 12 \tag{0.12}$$

$$\therefore \text{The two numbers are } (18, 12) \textit{ and } (18, -12) \tag{0.13}$$

From the figure, it is clearly verified that the theoretical solution matches with the computational solution.