12.601

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Question

The matrix $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$, one of the eigen values is 1. The eigen vectors corresponding to the eigne value 1 are: (CS 2016)

a) $\alpha \begin{pmatrix} 4 & -2 & 1 \end{pmatrix}, \alpha \neq 0, \ \alpha \epsilon \mathbb{R}$
b) $\alpha \begin{pmatrix} -4 & 2 & 1 \end{pmatrix}, \alpha \neq 0, \ \alpha \epsilon \mathbb{R}$
c) $\alpha \begin{pmatrix} -2 & 0 & 1 \end{pmatrix}, \alpha \neq 0, \ \alpha \epsilon \mathbb{R}$
d) $\alpha \begin{pmatrix} 2 & 0 & 1 \end{pmatrix}, \alpha \neq 0, \ \alpha \epsilon \mathbb{R}$

Theoretical Solution

Given: $\lambda = 1$, $Let\ \mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$

$$\mathbf{A} - \mathbf{I} = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{pmatrix} \tag{1}$$

Row Transformation-1: $R_1 \leftrightarrow R_3$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 2 \end{pmatrix} \tag{2}$$

Row Transformation-2: $R_2 \leftrightarrow R_3$

$$\mathbf{A} - \mathbf{I} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \tag{3}$$

Theoretical Solution

Let $\mathbf{v}$ be the corresponding eigenvector.

$$\Rightarrow \mathbf{A}\mathbf{v} = (1)\mathbf{v} \tag{4}$$

$$(\mathbf{A} - \mathbf{I})\mathbf{v} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{v} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \tag{6}$$

Let $\mathbf{v} = \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$

Substituting value of $\mathbf{v}$ in Equation 6,

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \tag{7}$$

Theoretical Solution

$$Row - 1 \rightarrow \; v_2 + 2v_2 = 0 \tag{8}$$
$$Row - 2 \rightarrow \; v_1 + 2v_3 = 0 \tag{9}$$
$$Row - 3 \rightarrow \; 0 + 0 + 0 = 0 \text{ (Always true)} \tag{10}$$

Let $v_3 = \alpha$ (Free parameter)
Substituting value of v_3 in Equations 8 and 9

$$\therefore \; v_2 = -2\alpha \,\&\, v_1 = 4\alpha \tag{11}$$

$$\therefore \; \mathbf{v} = \alpha \begin{pmatrix} 4 & -2 & 1 \end{pmatrix} \tag{12}$$

$$\boxed{\text{Thus, Option-A is correct.}}$$