2.4.18

EE25BTECH11022 - sankeertthan

Question: show that the line through the points $(1,-1,2)$, $(3,4,-2)$ is perpendicular to the line through the points $(0,3,2)$ and $(3,5,6)$

solution:

$$let\mathbf{A} = (1,-1,2)\,, \mathbf{B} = (3,4,-2)\,, \mathbf{C} = (0,3,2)\,, \mathbf{D} = (3,5,6) \tag{1}$$

Direction vector of line joining points **A** and **B** is

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3-1 \\ 4-(-1) \\ -2-2 \end{pmatrix} \tag{2}$$

$$= \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix} \tag{3}$$

$$\tag{4}$$

Direction vector of line joining points **C** and **D** is

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} 3-0 \\ 5-3 \\ 6-2 \end{pmatrix} \tag{5}$$

$$= \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} \tag{6}$$

$$\tag{7}$$

$$\left(\mathbf{B} - \mathbf{A}\right)^{\top} \left(\mathbf{D} - \mathbf{C}\right) = \begin{pmatrix} 2 & 5 & -4 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} \tag{8}$$

$$= 2\,(3) + 5\,(2) + (-4)\,(4) \tag{9}$$

$$= 6 + 10 - 16 \tag{10}$$

$$= 0 \tag{11}$$

$$\left(\mathbf{B} - \mathbf{A}\right)^{\top} \left(\mathbf{D} - \mathbf{C}\right) = 0 \tag{12}$$

Therefore, the lines joining points **A**,**B** and **C**,**D** are perpendicular

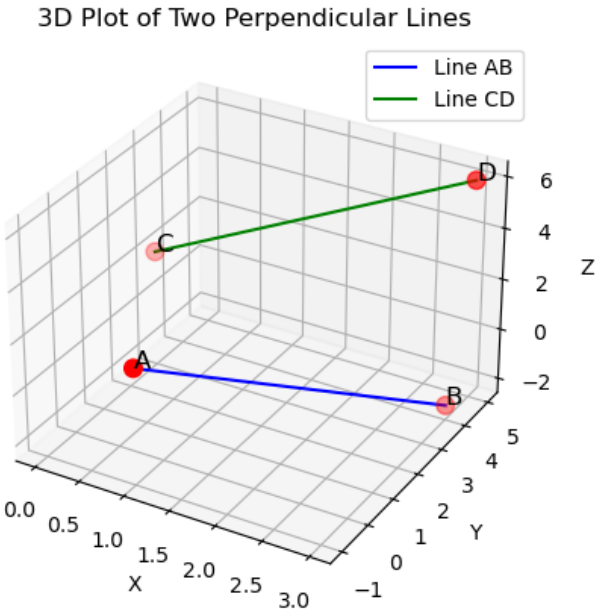
3D Plot of Two Perpendicular Lines
Line AB
Line CD
A
B
C
D
X
Y
Z
0.0
0.5
1.0
1.5
2.0
2.5
3.0
−1
0
1
2
3
4
5
−2
0
2
4
6

Fig. 0