5.7.15

EE25BTECH11002 - Achat Parth Kalpesh

Question:
If

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} \tag{0.1}$$

and $\mathbf{A}^3 - 6\mathbf{A}^2 + 7\mathbf{A} + k\mathbf{I} = 0$ find k.
Solution:
The characteristic equation for the matrix $\mathbf{A}$

$$f(\lambda) = |\mathbf{A} - \lambda\mathbf{I}| \tag{0.2}$$

From (0.2) the characteristic equation is

$$\begin{vmatrix} 1-\lambda & 0 & 2 \\ 0 & 2-\lambda & 1 \\ 2 & 0 & 3-\lambda \end{vmatrix} = 0 \tag{0.3}$$

which can be expanded to obtain

$$\lambda^3 - 6\lambda^2 + 7\lambda + 2 = 0 \tag{0.4}$$

Upon simplification, by using Cayley-Hamilton theorem,

$$\mathbf{A}^3 - 6\mathbf{A}^2 + 7\mathbf{A} + 2\mathbf{I} = 0 \tag{0.5}$$

Thereby, on comparing (0.5) with

$$\mathbf{A}^3 - 6\mathbf{A}^2 + 7\mathbf{A} + k\mathbf{I} = 0 \tag{0.6}$$

$$k = 2 \tag{0.7}$$