1.8.31

EE25BTECH11008 - Anirudh M Abhilash

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Question

Find the acute angle between the planes

$$x - 2y - 2z = 5$$
$$3x - 6y + 2z = 7$$

Solution

The angle between two planes is the angle between their normals. Let

$$\mathbf{n}_1 = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}, \quad \mathbf{n}_2 = \begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix}.$$

Form the matrix

$$A = \begin{pmatrix} 1 & 3 \\ -2 & -6 \\ -2 & 2 \end{pmatrix}, \tag{1}$$

whose columns are the normals. The Gram matrix is

$$G = A^\top A = \begin{pmatrix} \mathbf{n}_1^\top \mathbf{n}_1 & \mathbf{n}_1^\top \mathbf{n}_2 \\ \mathbf{n}_2^\top \mathbf{n}_1 & \mathbf{n}_2^\top \mathbf{n}_2 \end{pmatrix}. \tag{2}$$

Now,

$$\mathbf{n}_1^\top \mathbf{n}_1 = 9, \quad \mathbf{n}_2^\top \mathbf{n}_2 = 49, \quad \mathbf{n}_1^\top \mathbf{n}_2 = 11.$$

Thus,

$$G = \begin{pmatrix} 9 & 11 \\ 11 & 49 \end{pmatrix}. \tag{3}$$

Let

$$D = \begin{pmatrix} 9 & 0 \\ 0 & 49 \end{pmatrix}, \quad D^{-1/2} = \begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{7} \end{pmatrix}. \tag{4}$$

The normalized Gram matrix is

$$C = D^{-1/2} G D^{-1/2} = \begin{pmatrix} 1 & \frac{11}{21} \\ \frac{11}{21} & 1 \end{pmatrix}. \tag{5}$$

The off-diagonal entry gives

$$\cos\theta = \frac{11}{21}. \tag{6}$$

Hence, the acute angle between the planes is

$$\boxed{\theta = \arccos\left(\tfrac{11}{21}\right) \approx 58.41^\circ}$$

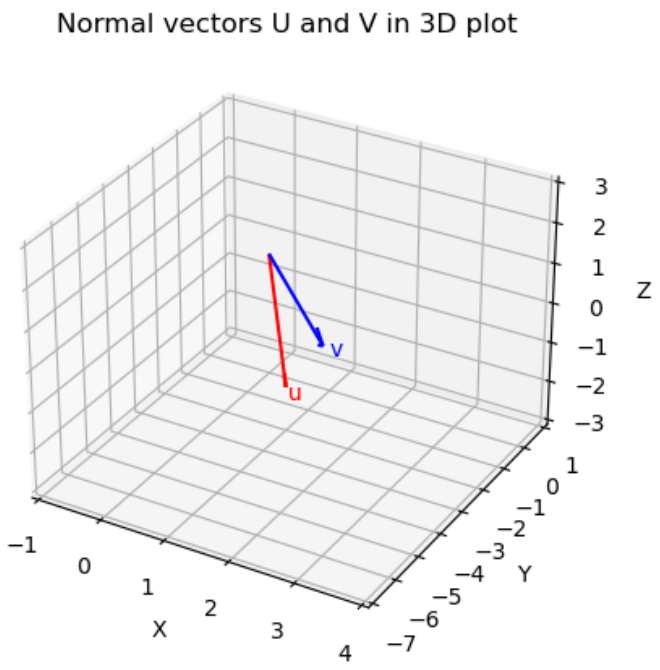

Figure 1: Normal vectors **U** and **V**