1.4.23

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Question

Consider two points P and Q with position vectors

$$\mathbf{P} = 3\mathbf{a} - 2\mathbf{b}, \quad \mathbf{Q} = \mathbf{a} + \mathbf{b}.$$

Find the position vector of a point R which divides the line joining P and Q in the ratio $2:1$,

1. internally, and
2. externally.

Theoretical Solution

We write the endpoints in matrix form:

$$\begin{pmatrix} \mathbf{P} & \mathbf{Q} \end{pmatrix}^T = \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix}. \tag{1}$$

General section formulas (matrix form). For points $\mathbf{A}, \mathbf{B}$ and ratio $k : 1$:

$$\mathbf{R}_{\text{int}} = \frac{k\,\mathbf{B} + \mathbf{A}}{k+1} = \frac{1}{k+1} \left[\mathbf{A} \;\; \mathbf{B} \right] \begin{pmatrix} 1 \\ k \end{pmatrix}. \tag{2}$$

$$\mathbf{R}_{\text{ext}} = \frac{k\,\mathbf{B} - \mathbf{A}}{k-1} = \frac{1}{k-1} \left[\mathbf{A} \;\; \mathbf{B} \right] \begin{pmatrix} -1 \\ k \end{pmatrix}. \tag{3}$$

Internal division $2:1$. Using (2) with $\mathbf{A} = \mathbf{P}$, $\mathbf{B} = \mathbf{Q}$, $k = 2$,

$$\mathbf{R}_{\text{int}} = \frac{1}{3}\left[\mathbf{P} \quad \mathbf{Q}\right]\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}\begin{pmatrix} \mathbf{P} & \mathbf{Q} \end{pmatrix}. \tag{4}$$

Substitute (1) into (4):

$$\mathbf{R}_{\text{int}} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix}. \tag{5}$$

$$\boxed{\mathbf{R}_{\text{int}} = \frac{5}{3}\,\mathbf{a}} \tag{6}$$

Theoretical Solution

External division $2:1$**.** Using (3) with $\mathbf{A} = \mathbf{P},\ \mathbf{B} = \mathbf{Q},\ k = 2,$

$$\mathbf{R}_{\text{ext}} = \begin{bmatrix} \mathbf{P} & \mathbf{Q} \end{bmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 & 2 \end{pmatrix} \begin{pmatrix} \mathbf{P} & \mathbf{Q} \end{pmatrix}. \tag{7}$$

Substitute (1) into (7):

$$\mathbf{R}_{\text{ext}} = \begin{pmatrix} -1 & 2 \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix}. \tag{8}$$

$$\boxed{\mathbf{R}_{\text{ext}} = -\mathbf{a} + 4\,\mathbf{b}} \tag{9}$$

C Code - Internal and External Division

```c
#include <stdio.h>

// Function to find point dividing line in ratio m:n
// flag = 1 for internal, -1 for external
void sectionFormula(float Px, float Py, float Qx, float Qy, int m
    , int n, int flag,
                  float *Rx, float *Ry) {
    if(flag == 1) { // internal
        *Rx = (m*Qx + n*Px) / (float)(m+n);
        *Ry = (m*Qy + n*Py) / (float)(m+n);
    }
    else if(flag == -1) { // external
        *Rx = (m*Qx - n*Px) / (float)(m-n);
        *Ry = (m*Qy - n*Py) / (float)(m-n);
    }
}
```

Python+C Code

```python
import ctypes
import matplotlib.pyplot as plt

# Load the shared library (make sure libsection.so is in the same
    folder)
lib = ctypes.CDLL("./mg1o.so")

# Define argument and return types
lib.sectionFormula.argtypes = [
    ctypes.c_float, ctypes.c_float, # Px, Py
    ctypes.c_float, ctypes.c_float, # Qx, Qy
    ctypes.c_int, ctypes.c_int, # m, n
    ctypes.c_int, # flag (1=internal, -1=external)
    ctypes.POINTER(ctypes.c_float), # Rx
    ctypes.POINTER(ctypes.c_float) # Ry
]
lib.sectionFormula.restype = None
```

Python+C Code

```python
# Example points (P = (3,-2), Q = (1,1)), ratio 2:1
Px, Py = 3.0, -2.0
Qx, Qy = 1.0, 1.0
m, n = 2, 1

# Prepare storage for results
Rx_int, Ry_int = ctypes.c_float(), ctypes.c_float()
Rx_ext, Ry_ext = ctypes.c_float(), ctypes.c_float()

# Call C function
lib.sectionFormula(Px, Py, Qx, Qy, m, n, 1, ctypes.byref(Rx_int),
    ctypes.byref(Ry_int)) # internal
lib.sectionFormula(Px, Py, Qx, Qy, m, n, -1, ctypes.byref(Rx_ext)
    , ctypes.byref(Ry_ext)) # external

print("Internal Division:", Rx_int.value, Ry_int.value)
print("External Division:", Rx_ext.value, Ry_ext.value)
```

Python+C Code

```python

# --- Plotting ---
plt.figure(figsize=(6,6))

# Plot P, Q, R_int, R_ext
plt.scatter([Px, Qx, Rx_int.value, Rx_ext.value],
            [Py, Qy, Ry_int.value, Ry_ext.value],
            color=["red","blue","green","purple"], s=100)

plt.text(Px+0.1, Py+0.1, "P", color="red")
plt.text(Qx+0.1, Qy+0.1, "Q", color="blue")
plt.text(Rx_int.value+0.1, Ry_int.value+0.1, "R_int", color="
    green")
plt.text(Rx_ext.value+0.1, Ry_ext.value+0.1, "R_ext", color="
    purple")
```

Python+C Code

```python
# Line PQ
plt.plot([Px, Qx], [Py, Qy], "k--")

plt.axhline(0, color="gray", linewidth=0.5)
plt.axvline(0, color="gray", linewidth=0.5)
plt.xlabel("Coefficient of a")
plt.ylabel("Coefficient of b")
plt.title("Section Formula using C + Python")
plt.grid(True)
plt.show()
```

Python

```
import numpy as np
import matplotlib.pyplot as plt

# Point coordinates (coefficients of a and b)
Px, Py = 3.0, -2.0 # P = 3a - 2b
Qx, Qy = 1.0, 1.0 # Q = 1a + 1b
m, n = 2, 1 # ratio 2:1

# Internal Division: (mQ + nP) / (m+n)
Rx_int = (m*Qx + n*Px) / (m+n)
Ry_int = (m*Qy + n*Py) / (m+n)

# External Division: (mQ - nP) / (m-n)
Rx_ext = (m*Qx - n*Px) / (m-n)
Ry_ext = (m*Qy - n*Py) / (m-n)
```

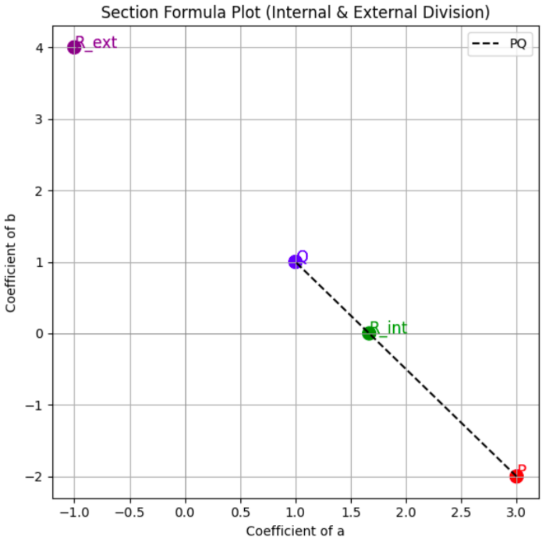
Python Code

```python
# Print results
print(f"Internal Division: R = {Rx_int:.2f}a + {Ry_int:.2f}b")
print(f"External Division: R = {Rx_ext:.2f}a + {Ry_ext:.2f}b")

# Plotting
plt.figure(figsize=(6,6))
plt.scatter([Px, Qx, Rx_int, Rx_ext],
            [Py, Qy, Ry_int, Ry_ext],
            color=["red","blue","green","purple"], s=100)

# Labels
plt.text(Px, Py, "P", fontsize=12, color="red")
plt.text(Qx, Qy, "Q", fontsize=12, color="blue")
plt.text(Rx_int, Ry_int, "R_int", fontsize=12, color="green")
plt.text(Rx_ext, Ry_ext, "R_ext", fontsize=12, color="purple")
```

Python Code

```python
# Draw line PQ
plt.plot([Px, Qx], [Py, Qy], "k--", label="PQ")

# Axes styling
plt.axhline(0, color="gray", linewidth=0.5)
plt.axvline(0, color="gray", linewidth=0.5)
plt.xlabel("Coefficient of a")
plt.ylabel("Coefficient of b")
plt.legend()
plt.title("Section Formula Plot (Internal & External Division)")
plt.grid(True)
plt.show()
```

Plot



Section Formula Plot (Internal & External Division)
PQ
R_ext
Q
R_int
P
Coefficient of b
Coefficient of a
4
3
2
1
0
−1
−2
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0