1.3.10

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Question:
Find the ratio in which the point $P(8, y)$ divides the line segment joining the points $A(1, 2)$ and $B(2, 3)$. Also, find the value of y.
Solution: Let

$$\mathbf{A} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} 8 \\ y \end{bmatrix}$$

According to the section formula, if a point divides the line segment in the ratio $k : 1$, then

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \tag{0.1}$$

Substituting the given values,

$$\begin{bmatrix} 8 \\ y \end{bmatrix} = \frac{1}{k + 1} \begin{bmatrix} 2k + 1 \\ 3k + 2 \end{bmatrix} \tag{0.2}$$

Comparing first coordinate,

$$8 = \frac{2k + 1}{k + 1} \tag{0.3}$$

$$8(k + 1) = 2k + 1 \quad \Rightarrow \quad 6k = -7 \tag{0.4}$$

$$k = -\frac{7}{6} \tag{0.5}$$

Now using second coordinate,

$$y = \frac{3k + 2}{k + 1} \tag{0.6}$$

$$y = 9 \tag{0.7}$$

Hence, the required ratio is

$$k : 1 = -\frac{7}{6} : 1 = -7 : 6$$

and

$$y = 9$$

From the figure it is clearly verified that the theoretical solution matches with the computational solution.

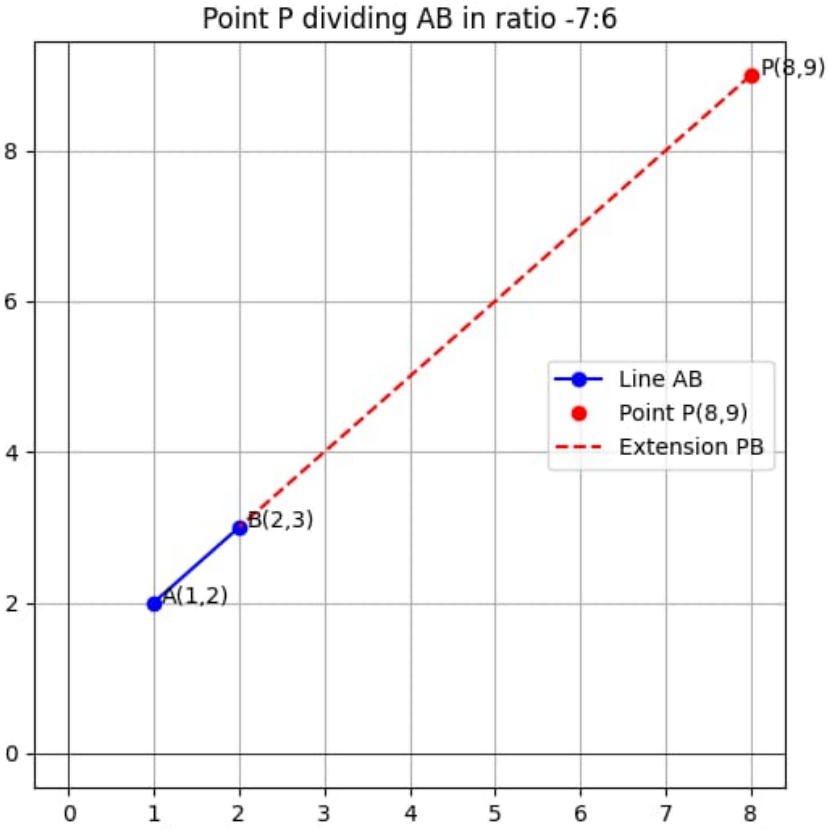
Point P dividing AB in ratio -7:6
P(8,9)
B(2,3)
A(1,2)
Line AB
Point P(8,9)
Extension PB
0
2
4
6
8
0
1
2
3
4
5
6
7
8

Fig. 0.1