2.7.15

EE25BTECH11003 - Adharvan Kshathriya Bommagani

Question:
Find the volume of a parallelepiped whose edges are given by $-3\hat{i}+7\hat{j}+5\hat{k}$, $-5\hat{i}+7\hat{j}-3\hat{k}$ and $7\hat{i}-5\hat{j}-3\hat{k}$.

Solution: Let **a**, **b** and **c** be the vectors representing the edges of the parallelepiped.

$$\mathbf{a} = \begin{pmatrix} -3 \\ 7 \\ 5 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -5 \\ 7 \\ -3 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 7 \\ -5 \\ -3 \end{pmatrix} \tag{1}$$

The volume of the parallelepiped is given by the absolute value of the scalar triple product:

$$V = |[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]|$$

which is equivalent to the determinant of the matrix formed by the components of **a**, **b**, and **c**:

$$V = \begin{Vmatrix} -3 & -5 & 7 \\ 7 & 7 & -5 \\ 5 & -3 & -3 \end{Vmatrix}$$

The determinant expression:

$$\det = -3(-36) + 5(4) + 7(-56) = 108 + 20 - 392 = -264$$

Thus, the volume is:

$$V = |-264| = 264 \text{ cubic units}$$

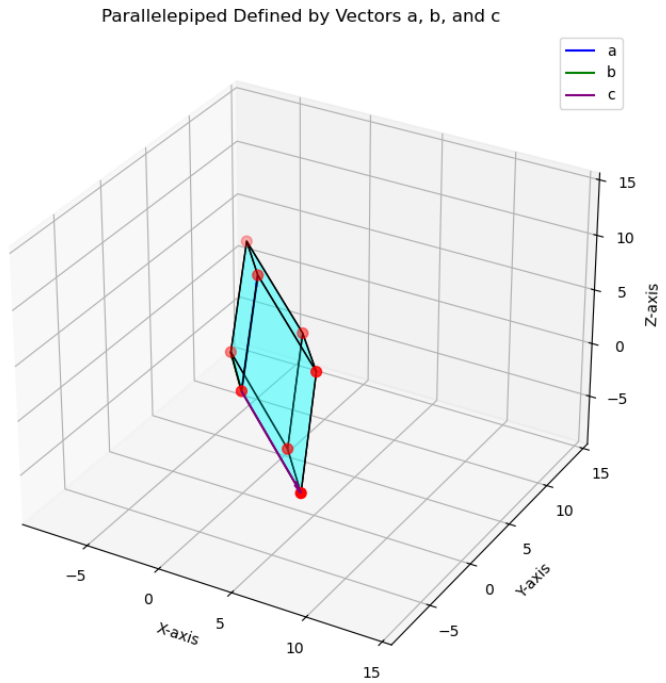Therefore, the volume of the parallelepiped is 264 cubic units.

Parellelopiped Defined by Vectors a, b and c:

Fig. 0: Figure for 2.7.15