

1.3.10

AI25BTECH110031
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Question(1.3.10)

Find the ratio in which the point $P = (8, y)$ divides the line segment joining $A = (1, 2)$ and $B = (2, 3)$. Also, find the value of y .

Solution:

Let the given points be A and B

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

Let the point P divide the line segment \overline{AB} in the ratio $k : 1$.

$$\text{Given } P = \begin{pmatrix} 8 \\ y \end{pmatrix}$$

The points A, B, P are collinear.

$$\implies \text{rank}(\mathbf{B} - \mathbf{A} \quad \mathbf{P} - \mathbf{A})^T = 1 \quad (0.1)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 - 1 \\ 3 - 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (0.2)$$

$$\mathbf{P} - \mathbf{A} = \begin{pmatrix} 8 - 1 \\ y - 2 \end{pmatrix} = \begin{pmatrix} 7 \\ y - 2 \end{pmatrix} \quad (0.3)$$

Therefore, our matrix is:

$$\begin{pmatrix} 1 & 1 \\ 7 & y - 2 \end{pmatrix} \quad (0.4)$$

Row reducing:

$$R_2 \rightarrow R_2 - 7R_1 \implies \begin{pmatrix} 1 & 1 \\ 0 & y - 9 \end{pmatrix} \quad (0.5)$$

For the above matrix to be of rank 1,

$$y - 9 = 0 \implies y = 9 \quad (0.6)$$

\therefore The coordinates of the point of division are

$$\mathbf{P} = \begin{pmatrix} 8 \\ 9 \end{pmatrix}$$

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \quad (0.7)$$

$$k = \frac{((\mathbf{A}) - (\mathbf{P}))^T((\mathbf{P}) - (\mathbf{B}))}{\|(\mathbf{P}) - (\mathbf{B})\|^2} \quad (0.8)$$

Substituting the values of \mathbf{A} , \mathbf{B} and \mathbf{P} ,

$$k = \frac{\begin{pmatrix} -7 & -7 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \end{pmatrix}}{\left\| \begin{pmatrix} 6 \\ 6 \end{pmatrix} \right\|^2} = \frac{-7}{6} \quad (0.9)$$

Thus, the ratio in which the point P divides the line segment AB is $\boxed{-7 : 6}$.

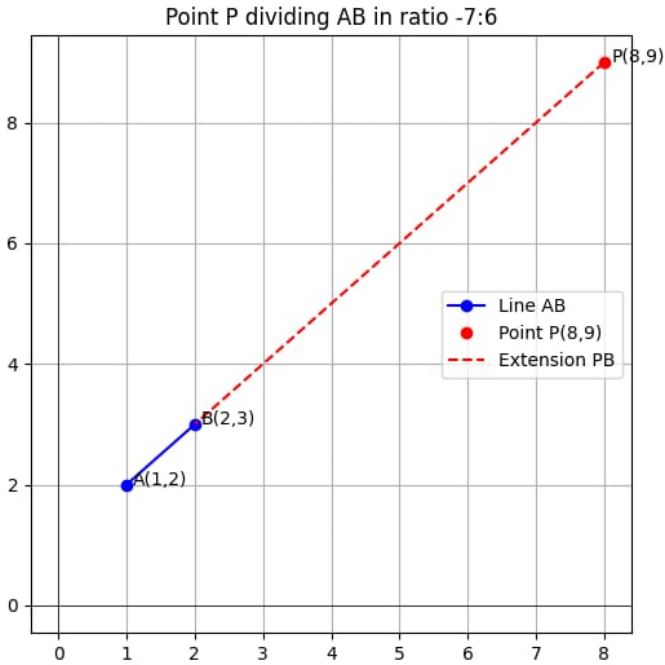


Fig. 0.1