4.3.13

EE25BTECH11026-Harsha

Question:
Find the distance of the line $4x - y = 0$ from the point P$(4, 1)$ measured along the line making an angle of 135° with the positive x-axis.

Solution:
Let us solve the given question theoretically and then verify the solution computationally.

According to the question,

$$\textit{Equation of target line} \ : \ \begin{pmatrix} 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

and

$$\mathbf{P} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$$

As the direction of line makes an angle of 135° with the $+x$ axis, the unit direction vector of the line is given by

$$\mathbf{m} = \begin{pmatrix} \cos 135^\circ \\ \sin 135^\circ \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

Parametrize the required line using $\mathbf{P}$,yielding

$$\mathbf{x} = \mathbf{P} + \kappa \mathbf{m}$$

Inserting the parametric form in the equation of target line,

$$\begin{pmatrix} 4 & -1 \end{pmatrix} (\mathbf{P} + \kappa \mathbf{m}) = 0$$

$$\therefore \kappa = \frac{-\begin{pmatrix} 4 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix}}{\begin{pmatrix} 4 & -1 \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}}$$

$$\implies \kappa = 3\sqrt{2}$$

Since $\mathbf{m}$ is a unit vector, the norm of vector $\mathbf{P}$ from the given line along the line with $\mathbf{m} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}^{\top}$ is

$$\kappa = 3\sqrt{2}\ units$$

From the figure, it is clearly verified that the theoretical solution matches with the computational solution.