2.7.33

EE25BTECH11021 - Dhanush Sagar

Question:
Find the value of p if

$$(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = 0.$$

Solution:
The given vectors are

$$\mathbf{A} = \begin{pmatrix} 2 \\ 6 \\ 27 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 3 \\ p \end{pmatrix}. \tag{0.1}$$

Since

$$\mathbf{A} \times \mathbf{B} = 0, \tag{0.2}$$

the vectors $\mathbf{A}$ and $\mathbf{B}$ are linearly dependent. Therefore, there exists a scalar t such that

$$\mathbf{B} = t\mathbf{A}. \tag{0.3}$$

Substituting the components,

$$\begin{pmatrix} 1 \\ 3 \\ p \end{pmatrix} = t \begin{pmatrix} 2 \\ 6 \\ 27 \end{pmatrix}. \tag{0.4}$$

From the first coordinate,

$$1 = 2t \Rightarrow t = \tfrac{1}{2}. \tag{0.5}$$

From the second coordinate,

$$3 = 6t \Rightarrow t = \tfrac{1}{2}, \tag{0.6}$$

which is consistent with the first.

Finally, from the third coordinate,

$$p = 27t = 27 \cdot \tfrac{1}{2} = \tfrac{27}{2}. \tag{0.7}$$

Final Answer:

$$\boxed{p = \tfrac{27}{2}} \tag{0.8}$$

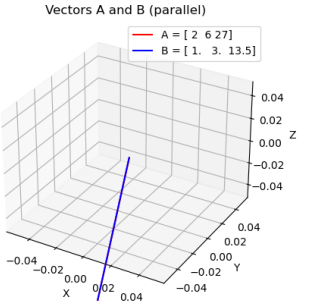
Vectors A and B (parallel)
A = [2 6 27]
B = [1.   3. 13.5]
0.04
0.02
0.00
−0.02
−0.04
Z
0.04
0.02
0.00
−0.02
−0.04
Y
−0.04
−0.02
0.00
0.02
0.04
X

Fig. 0.1