

Matrices in Geometry - 4.11.13

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Problem Statement

Find the equation of the plane passing through the points $(2,5,-3)$, $(-2,-3,5)$, and $(5,3,-3)$. Also find the point of intersection of this plane with the line passing through points $(3,1,5)$ and $(-1,-3,-1)$.

Solution

Given,

The points **A** $\begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}$, **B** $\begin{pmatrix} -2 \\ -3 \\ 5 \end{pmatrix}$ and **C** $\begin{pmatrix} 5 \\ 3 \\ -3 \end{pmatrix}$ which pass through a plane.

The points **D** $\begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$ and **E** $\begin{pmatrix} -1 \\ -3 \\ -1 \end{pmatrix}$ which pass through a line.

Solution

If \mathbf{n} is the normal vector to the plane, then by the plane equation we can write

$$\mathbf{n}^\top \mathbf{A} = c \quad (1)$$

$$\mathbf{n}^\top \mathbf{B} = c \quad (2)$$

$$\mathbf{n}^\top \mathbf{C} = c \quad (3)$$

which can be written as

$$(A \quad B \quad C)^\top \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (4)$$

Solution

Forming the augmented matrix for this

$$\left(\begin{array}{ccc|c} 2 & 5 & -3 & 1 \\ -2 & -3 & 5 & 1 \\ 5 & 3 & -3 & 1 \end{array} \right) \quad (5)$$

$$\left(\begin{array}{ccc|c} 2 & 5 & -3 & 1 \\ -2 & -3 & 5 & 1 \\ 5 & 3 & -3 & 1 \end{array} \right) \xleftrightarrow{R_1 \leftarrow \frac{R_1}{2}} \left(\begin{array}{ccc|c} 1 & \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \\ -2 & -3 & 5 & 1 \\ 5 & 3 & -3 & 1 \end{array} \right) \quad (6)$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{2}} \left(\begin{array}{ccc|c} 1 & \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \\ 0 & 1 & 1 & 1 \\ 0 & \frac{-19}{2} & \frac{9}{2} & \frac{-3}{2} \end{array} \right) \xleftrightarrow{\begin{array}{l} R_3 \leftarrow R_3 + \frac{19}{2} R_2 \\ R_1 \leftarrow R_1 - \frac{5}{2} R_2 \end{array}} \left(\begin{array}{ccc|c} 1 & 0 & -4 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 14 & 8 \end{array} \right) \quad (7)$$

Solution

$$\xleftrightarrow{R_3 \leftarrow \frac{1}{14} R_3} \left(\begin{array}{ccc|c} 1 & 0 & -4 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & \frac{4}{7} \end{array} \right) \xleftrightarrow{\begin{array}{l} R_1 \leftarrow R_1 + 4R_3 \\ R_2 \leftarrow R_2 - R_3 \end{array}} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2}{7} \\ 0 & 1 & 0 & \frac{3}{7} \\ 0 & 0 & 1 & \frac{4}{7} \end{array} \right) \quad (8)$$

Thus, multiplying by 7, the plane equation can be expressed as

$$(2 \quad 3 \quad 4) \mathbf{x} = 7 \quad (9)$$

Now, the line passing through the two given points in parametric form

$$\mathbf{x} = \mathbf{P} + \lambda \mathbf{m} \quad (10)$$

$$\text{where } \mathbf{P} = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} \text{ and } \mathbf{m} = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ -3 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \\ 6 \end{pmatrix} \equiv \begin{pmatrix} 2 \\ 2 \\ 3 \end{pmatrix},$$

Solution

Now substituting these in (11) to get x and then that in (10), we get

$$(2 \ 3 \ 4) \begin{pmatrix} 3 + 2\lambda \\ 1 + 2\lambda \\ 5 + 3\lambda \end{pmatrix} = 7 \quad (11)$$

$$\implies \boxed{\lambda = -1} \quad (12)$$

which gives the intersection point as $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

Conclusion

∴ The equation of the plane is $(2 \ 3 \ 4) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 7$ and the point of intersection of this plane with the line through the given two points is $\mathbf{P} \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

