2.8.19

EE25BTECH11036 - M Chanakya Srinivas

Problem

Suppose for some non-zero vector $\mathbf{r}$ we have

$$\mathbf{r}\cdot\mathbf{a} = 0, \quad \mathbf{r}\cdot\mathbf{b} = 0, \quad \mathbf{r}\cdot\mathbf{c} = 0.$$

Show that the scalar triple product $(\mathbf{a}\ \mathbf{b}\ \mathbf{c}) = 0$.

Solution

Step 1: Write as matrix equation

The three scalar equations can be written as

$$\mathbf{r}^\top\mathbf{a} = 0 \tag{1}$$

$$\mathbf{r}^\top\mathbf{b} = 0 \tag{2}$$

$$\mathbf{r}^\top\mathbf{c} = 0 \tag{3}$$

Stack them into a single matrix equation:

$$\begin{pmatrix}\mathbf{a}^\top\\ \mathbf{b}^\top\\ \mathbf{c}^\top\end{pmatrix}\mathbf{r} = \begin{pmatrix}0\\0\\0\end{pmatrix}. \tag{4}$$

Step 2: Define the matrix

Let

$$A = [\mathbf{a}\ \mathbf{b}\ \mathbf{c}] \tag{5}$$

be the 3×3 matrix with columns $\mathbf{a},\mathbf{b},\mathbf{c}$. Then the stacked equation becomes

$$A^\top\mathbf{r} = \begin{pmatrix}0\\0\\0\end{pmatrix}. \tag{6}$$

Step 3: Deduce singularity

Since $\mathbf{r}\neq\mathbf{0}$ and $A^\top\mathbf{r} = 0$, the matrix $A^\top$ is singular. Therefore,

$$\det(A^\top) = 0. \tag{7}$$

Step 4: Relate to scalar triple product

But $\det(A^\top) = \det(A)$, and the determinant of A is exactly the scalar triple product:

$$(\mathbf{a}\ \mathbf{b}\ \mathbf{c}) = \det\left[\mathbf{a}\ \mathbf{b}\ \mathbf{c}\right] = \det(A) = 0. \tag{8}$$

Conclusion

$$\boxed{(\mathbf{a}\ \mathbf{b}\ \mathbf{c}) = 0}.$$

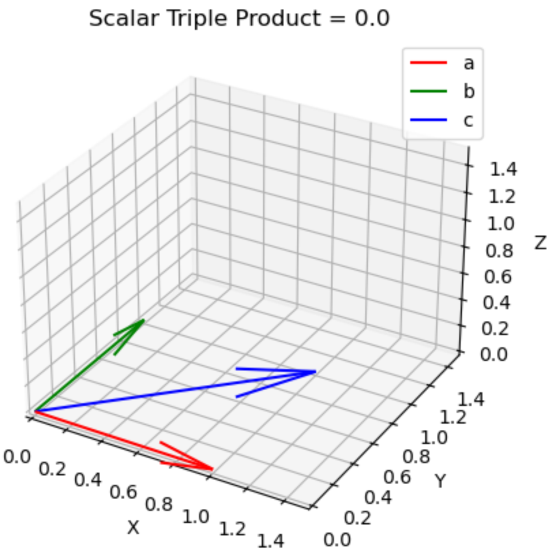
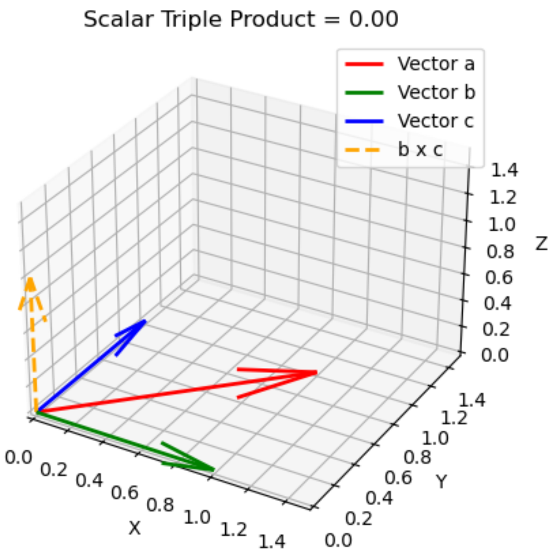


Fig. 1



Scalar Triple Product = 0.00
Vector a
Vector b
Vector c
b x c
X
Y
Z
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4

Fig. 2