

Matgeo Presentation - Problem 4.13.67

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Question

The area of the triangle formed by the intersection of line parallel to X axis and passing through $\mathbf{p}(h,k)$ with the lines $y=x$ and $x+y=2$ is $4h^2$. Find the locus of point \mathbf{p}

Solution

line parallel to X axis is of the form

$$y = c. \quad (0.1)$$

which can be expressed in the form of

$$\mathbf{n}^T \mathbf{x} = c. \quad (0.2)$$

$$\implies \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = c. \quad (0.3)$$

As the above line passes through $\mathbf{p}(h,k)$

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} h \\ k \end{pmatrix} = c. \implies c = k. \quad (0.4)$$

The three lines are as follows

$$y = k \implies \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = k. \quad (0.5)$$

$$-x + y = 0 \implies \begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0. \quad (0.6)$$

Solution

$$x + y = 2 \implies \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 2. \quad (0.8)$$

Let **A**, **B**, **C** be the point of intersection of above 3 lines
By solving equation (5) and (6) we get

$$\mathbf{A} = \begin{pmatrix} k \\ k \end{pmatrix}. \quad (0.9)$$

By solving equation (6) and (7) we get

$$\mathbf{B} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \quad (0.10)$$

By solving equation (5) and (7) we get

$$\mathbf{C} = \begin{pmatrix} 2 - k \\ k \end{pmatrix}. \quad (0.11)$$

Solution

$$\text{area of } \triangle ABC = \frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{B} - \mathbf{C})\| \quad (0.12)$$

$$= \frac{1}{2} \left\| \begin{pmatrix} k-1 \\ k-1 \end{pmatrix} \times \begin{pmatrix} k-1 \\ 1-k \end{pmatrix} \right\| \quad (0.13)$$

$$= \frac{1}{2} (2(k-1)^2) = (k-1)^2. \quad (0.14)$$

Given area of the triangle formed by the intersection of above 3 lines is $4h^2$.

$$\implies (k-1)^2 = 4h^2. \quad (0.15)$$

$$\implies (y-1)^2 = 4x^2 \quad (0.16)$$

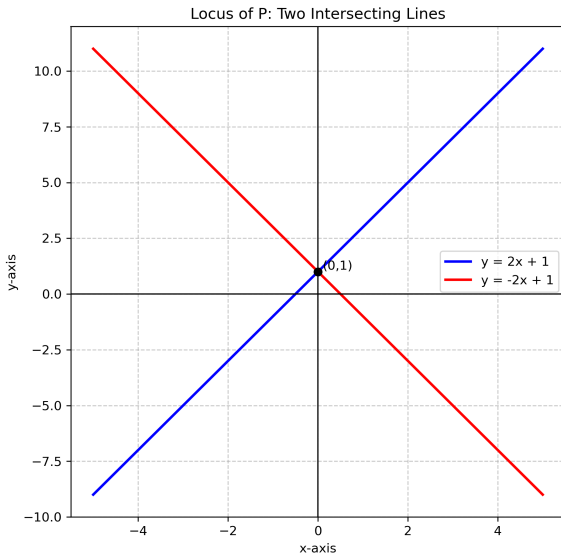
$$\implies (y-1-2x)(y-1+2x) = 0 \quad (0.17)$$

\implies The locus of \mathbf{p} is union of 2 straight lines

$$y-1-2x=0. \implies \begin{pmatrix} -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1. \quad (0.18)$$

$$y-1+2x=0. \implies \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1. \quad (0.19)$$

Plot



C Code: locus.c

```
#include <stdio.h>

int main() {
    FILE *fp;

    // Open the file locus.dat in write mode
    fp = fopen("locus.dat", "w");

    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }

    // Write the solution into the file
    fprintf(fp, "The locus of P is given by the equation:\n");
    fprintf(fp, " $(y-1)^2 = 4x^2$ \n");
    fprintf(fp, "Which represents two intersecting straight lines:\n");
    fprintf(fp, "1)  $y-1 = 2x \rightarrow y = 2x+1$ \n");
    fprintf(fp, "2)  $y-1 = -2x \rightarrow y = -2x+1$ \n");

    // Close the file
    fclose(fp);

    printf("Locus has been written successfully into locus.dat\n");

    return 0;
}
```

Python: plot.py

```
import numpy as np
import matplotlib.pyplot as plt

# Define range for x
x = np.linspace(-5, 5, 400)

# Equations of the two lines
y1 = 2*x + 1
y2 = -2*x + 1

# Create the plot
plt.figure(figsize=(7, 7))

# Plot both lines
plt.plot(x, y1, label="y=2x+1", color="blue", linewidth=2)
plt.plot(x, y2, label="y=-2x+1", color="red", linewidth=2)

# Mark the point of intersection (0,1)
plt.scatter(0, 1, color="black", zorder=5)
plt.text(0.1, 1.1, "(0,1)", fontsize=10)

# Axes setup
plt.axhline(0, color="black", linewidth=1)
plt.axvline(0, color="black", linewidth=1)

# Labels, grid and title
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.title("Locus of P: Two Intersecting Lines")
plt.grid(True, linestyle="--", alpha=0.7)
plt.legend()
plt.savefig("locus_plot.png", dpi=300)
plt.show()
```