

1.11.5

AI25BTECH11003 - Bhavesh Gaikwad

Question: The scalar product of vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

Solution:

Given: $\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix}$.

Let \mathbf{u} be the unit vector along $\mathbf{b} + \mathbf{c}$.

$$\mathbf{b} + \mathbf{c} = \begin{pmatrix} 2 + \lambda \\ 4 + 2 \\ -5 + 3 \end{pmatrix} = \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix}.$$

$$\|\mathbf{b} + \mathbf{c}\| = \sqrt{(2 + \lambda)^2 + 6^2 + (-2)^2} = \sqrt{\lambda^2 + 4\lambda + 44}.$$

$$\mathbf{u} = \frac{\mathbf{b} + \mathbf{c}}{\|\mathbf{b} + \mathbf{c}\|} = \frac{1}{\sqrt{\lambda^2 + 4\lambda + 44}} \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix}.$$

Given condition: $\mathbf{a} \cdot \mathbf{u} = 1$.

$$\mathbf{a} \cdot \mathbf{u} = \frac{\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})}{\|\mathbf{b} + \mathbf{c}\|} = \frac{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix}}{\sqrt{\lambda^2 + 4\lambda + 44}} = \frac{\lambda + 6}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1.$$

$$\Rightarrow (\lambda + 6)^2 = \lambda^2 + 4\lambda + 44 \Rightarrow \lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44 \Rightarrow 8\lambda = 8$$

$$\Rightarrow \boxed{\lambda = 1}$$

Now, with $\lambda = 1$: $\mathbf{b} + \mathbf{c} = \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix}$, $\|\mathbf{b} + \mathbf{c}\| = \sqrt{3^2 + 6^2 + (-2)^2} = \sqrt{49} = 7$.

Unit vector along, $\mathbf{b} + \mathbf{c}$ is: $\frac{1}{7} \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} = \frac{1}{7} \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix}.$

$\lambda = 1$

and

Unit vector along $\mathbf{b} + \mathbf{c} = \frac{1}{7} \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix}.$

(0.1)

Vectors a, b, c and unit vector along b+c

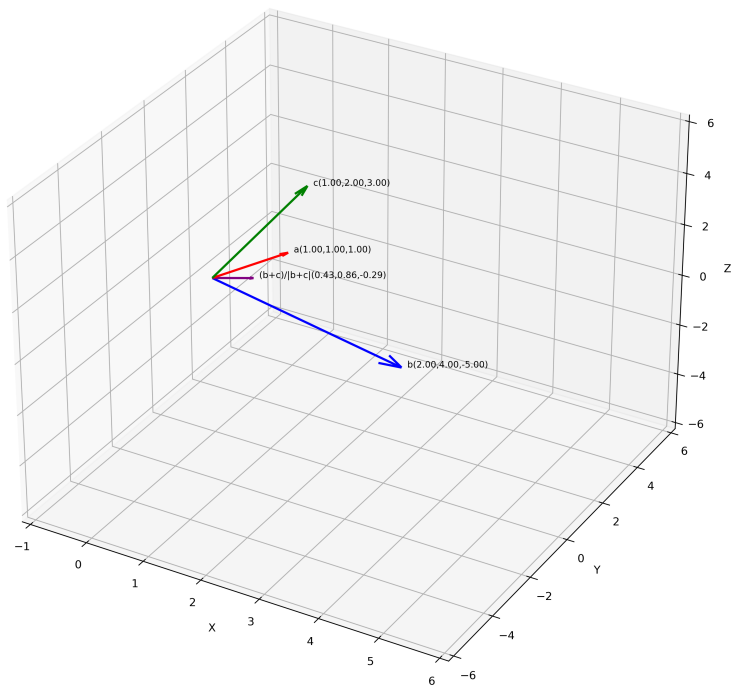


Fig. 0.1: Vector Representation