2.6.27

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Question:

If A (-5,7),B(-4,-5),C(-1,-6) and D(4,5) are the vertices of a quadrilateral, find the area of quadrilateral ABCD.

Solution:

Area of quadrilateral ABCD = The area of triangle ABC + The area of triangle ACD

Let $\mathbf{A} \begin{pmatrix} -5 \\ 7 \end{pmatrix}$, $\mathbf{B} \begin{pmatrix} -4 \\ -5 \end{pmatrix}$, $\mathbf{C} \begin{pmatrix} -1 \\ -6 \end{pmatrix}$, $\mathbf{D} \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ be vectors

$$\overrightarrow{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ -12 \end{pmatrix} \tag{0.1}$$

$$\overrightarrow{AC} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 \\ -13 \end{pmatrix} \tag{0.2}$$

$$\overrightarrow{AD} = \mathbf{D} - \mathbf{A} = \begin{pmatrix} 9 \\ -2 \end{pmatrix} \tag{0.3}$$

$$ar(ABC) = \frac{1}{2} \|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A})\| = 17.5 \tag{0.4}$$

$$ar(ACD) = \frac{1}{2} \|(\mathbf{C} - \mathbf{A}) \times (\mathbf{D} - \mathbf{A})\| = 54.5 \tag{0.5}$$

Therefore area of quadrilateral ABCD = 17.5+54.5 = 72 sq. units

OR

Area of quadrilateral ABCD = $\frac{1}{2}|d_1 \times d_2|$

where $d_1 = \overrightarrow{AC}$ and $d_2 = \overrightarrow{BD}$

$$\overrightarrow{AC} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 \\ -13 \end{pmatrix} \tag{0.6}$$

$$\overrightarrow{BD} = \mathbf{D} - \mathbf{B} = \begin{pmatrix} 8 \\ 10 \end{pmatrix} \tag{0.7}$$

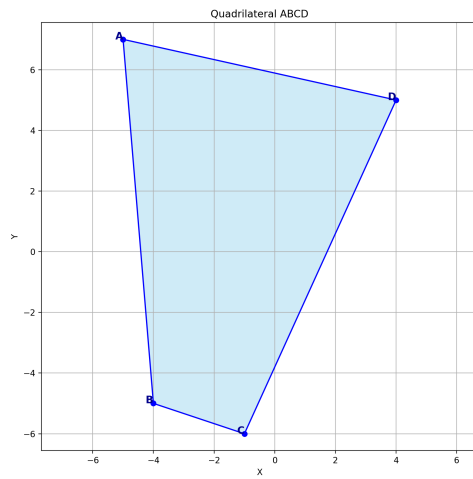$$ar(ABCD) = \frac{1}{2} \|(\mathbf{D} - \mathbf{B}) \times (\mathbf{C} - \mathbf{A})\| = 72 \, sq.units \tag{0.8}$$

Fig. 0.1