1.3.6

AI25BTECH11027 - NAGA BHUVANA

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Question:
Show that the points $\mathbf{A}\,(6,2)$,$\mathbf{B}\,(2,1)$,$\mathbf{C}\,(1,5)$ and $\mathbf{D}\,(5,6)$ are vertices of a square.

Solution:

Given that

$$\mathbf{A} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 5 \\ 6 \end{pmatrix} \tag{0.1}$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2-6 \\ 1-2 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \tag{0.2}$$

$$\mathbf{C} - \mathbf{D} = \begin{pmatrix} 1-5 \\ 5-6 \end{pmatrix} = \begin{pmatrix} -4 \\ -1 \end{pmatrix} \tag{0.3}$$

$$\mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \tag{0.4}$$

By the above property we can say that **ABCD** is a parallelogram.
Consider the inner product of the vectors $(\mathbf{B}-\mathbf{A})$ and $(\mathbf{C}-\mathbf{B})$

$$\implies (\mathbf{B}-\mathbf{A})\cdot(\mathbf{C}-\mathbf{B}) = (-4)(-1)+(-1)(4) = 0 \tag{0.5}$$

Hence the angle at the vertex B is 90°

Property:

A parallelogram with one angle 90° is a rectangle
Hence the parallelogram is a rectangle
Now consider the inner product of the diagonals of the rectangle $(\mathbf{C}-\mathbf{A})$ and $(\mathbf{D}-\mathbf{B})$

$$\implies (\mathbf{C}-\mathbf{A})\cdot(\mathbf{D}-\mathbf{B}) = (-5)(3)+(3)(5) = 0 \tag{0.6}$$

Hence the angle between the diagonals of the rectangle is 90°

Property:

Rectangle with diagonals at right angle is a square
Hence given points forms a square

Graphical Representation