2.3.15

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Question: The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to one. Find the value of λ.

Solution: let **A**, **B** and **C** be the vectors such that:

Variable	value
A	$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
B	$\begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix}$
C	$\begin{pmatrix} \lambda \\ 2 \\ 3 \end{pmatrix}$

TABLE 0: Variables used

The direction vector $\mathbf{B} + \mathbf{C} = \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix}$ — ①

The corresponding unit vector obtained is:

$$\frac{\mathbf{B} + \mathbf{C}}{\|\mathbf{B} + \mathbf{C}\|} = \frac{1}{\sqrt{\lambda^2 + 4\lambda + 44}} \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix} \quad - ②$$

given,

$$\mathbf{A}^\top \cdot (B \hat{+} C) = 1 \quad - ③$$

$$\frac{1}{\sqrt{\lambda^2 + 4\lambda + 44}} \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 + \lambda \\ 6 \\ -2 \end{pmatrix} = 1 \quad - ④$$

$$2 + \lambda + 6 - 2 = \sqrt{\lambda^2 + 4\lambda + 44} \quad - ⑤$$

squaring on both sides:

$$\lambda^2 + 36 + 12\lambda = \lambda^2 + 4\lambda + 44 \quad - ⑥$$
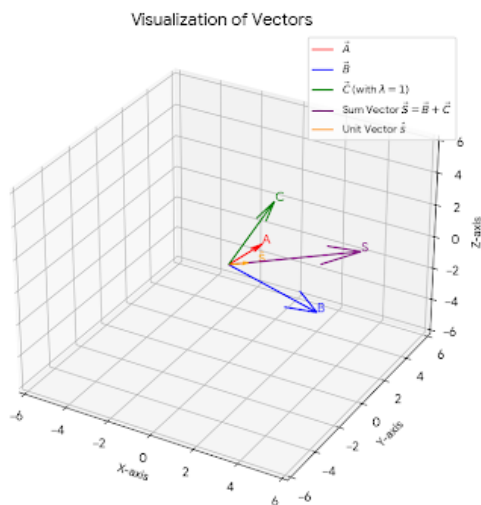
$$8\lambda = 8$$

$$\lambda = 1$$

Hence value of λ is 1.

Fig. 0.1