5.13.7

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Question 5.13.7

Let A and B be two symmetric matrices of order 3.
Statement 1: $A(BA)$ and $(AB)A$ are symmetric matrices.
Statement 2: AB is symmetric if A and B commute ($AB = BA$).
Determine the correct option.

Step 1: Symmetry property

A matrix M is symmetric if

$$M^T = M \tag{1}$$

Step 2: Check $A(BA)$

Let

$$M = A(BA)$$

Then,

$$\begin{aligned} M^T &= (A(BA))^T && (2)\\ &= (BA)^T A^T && (3)\\ &= A^T B^T A^T && (4)\\ &= ABA \quad (\text{since } A^T = A,\ B^T = B) && (5)\\ &= A(BA) = M && (6) \end{aligned}$$

Hence, $A(BA)$ is symmetric.

Step 3: Check $(AB)A$

Let

$$N = (AB)A$$

Then,

$$\begin{aligned} N^T &= ((AB)A)^T && (7)\\ &= A^T(AB)^T && (8)\\ &= A^T B^T A^T && (9)\\ &= ABA && (10)\\ &= (AB)A = N && (11) \end{aligned}$$

Hence, $(AB)A$ is symmetric.

Step 4: Condition for AB to be symmetric

$$(AB)^T = B^T A^T \tag{12}$$

$$= BA \quad (\text{since } A^T = A,\ B^T = B) \tag{13}$$

Thus,

$$AB \text{ is symmetric} \iff AB = BA \tag{14}$$

Conclusion: Statement 1 is true, Statement 2 is true but not the correct explanation for Statement 1.

Correct option: (a)

C Code (Part 1)

```c
#include <stdio.h>

int main() {
    int A[3][3] = {{1,2,3},{4,5,6},{7,8,9}};
    int B[3][3] = {{9,8,7},{6,5,4},{3,2,1}};
    int AB[3][3], ABt[3][3], BtAt[3][3], i, j, k;

    for(i=0;i<3;i++){
        for(j=0;j<3;j++){
            AB[i][j]=0;
            for(k=0;k<3;k++){
                AB[i][j]+=A[i][k]*B[k][j];
            }
        }
    }

    for(i=0;i<3;i++){
        for(j=0;j<3;j++){
            ABt[i][j]=AB[j][i];
```

C Code (Part 2)

```
    int At[3][3], Bt[3][3];
    for(i=0;i<3;i++){
        for(j=0;j<3;j++){
            At[i][j]=A[j][i];
            Bt[i][j]=B[j][i];
        }
    }

    for(i=0;i<3;i++){
        for(j=0;j<3;j++){
            BtAt[i][j]=0;
            for(k=0;k<3;k++){
                BtAt[i][j]+=Bt[i][k]*At[k][j];
            }
        }
    }

    printf(Transpose of AB:\n);
    for(i=0;i<3;i++){
```

Python Code

```
import numpy as np

A = np.array([[1,2,3],[4,5,6],[7,8,9]])
B = np.array([[9,8,7],[6,5,4],[3,2,1]])

AB = np.dot(A, B)
ABt = AB.T
BtAt = np.dot(B.T, A.T)

print(Transpose of AB:)
print(ABt)
print(Product B^T A^T:)
print(BtAt)
```