Matgeo-1.2.27

AI25BTECH11019-Menavath Sai Sanjana

Question

Rain is falling vertically with a speed of (30 , m/s). A woman rides a bicycle with a speed of (10 , m/s) in the north to south direction. What is the direction in which she should hold her umbrella?

Solution

Choose axes: x (south, +), y (downward, +).

$$\vec{v}_r = \begin{pmatrix} 0 \\ 30 \end{pmatrix} \quad \text{(rain velocity)} \tag{1}$$

$$\vec{v}_w = \begin{pmatrix} 10 \\ 0 \end{pmatrix} \quad \text{(woman velocity)} \tag{2}$$

$$\vec{v}_{r/w} = \vec{v}_r - \vec{v}_w \tag{3}$$

$$= \begin{pmatrix} -10 \\ 30 \end{pmatrix}. \tag{4}$$

Solution (using inner product)

Let the unit vector along the x-axis be $\hat{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

The angle θ between $\mathbf{v}_{r/w}$ and the x-axis is given by the inner product:

$$\cos\theta = \frac{\overrightarrow{v}_{r/w} \cdot \hat{i}}{\|\overrightarrow{v}_{r/w}\|\|\hat{i}\|} \tag{5}$$

$$= \frac{\begin{pmatrix} -10 \\ 30 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}}{\sqrt{(-10)^2 + 30^2} \cdot 1} \tag{6}$$

$$= \frac{-10}{\sqrt{100 + 900}} \tag{7}$$

$$= -\frac{1}{\sqrt{10}}. \tag{8}$$

Solution (conclusion)

$\theta = \cos^{-1}\left(-\frac{1}{\sqrt{10}}\right) \approx 108.43^\circ$
Since $90^\circ < \theta < 180^\circ$, the relative rain vector points *behind the x-axis*, i.e., slightly tilted from vertical.

Tilt of umbrella: From vertical, the tilt angle is
$180^\circ - 108.43^\circ = 71.57^\circ$ from the x-axis.
This corresponds to about 18.43° forward from vertical (towards south, the direction of motion).

Graphical Representation