2.8.12

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Question

Show that the tangent of an angle between the lines

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{a} - \frac{y}{b} = 1$$

is $\dfrac{2ab}{a^2 - b^2}$

Equation

Property: The cosine of the angle between line 1 and line 2 is given by $\frac{n_1^\top n_2}{\|n_1\| \|n_2\|}$.

Theoretical Solution

Given details:

$$\begin{pmatrix} \frac{1}{a} & \frac{1}{b} \end{pmatrix} \mathbf{x} = 1 \tag{1}$$

$$\begin{pmatrix} \frac{1}{a} & \frac{-1}{b} \end{pmatrix} \mathbf{x} = 1 \tag{2}$$

Theoretical Solution

Let the angle between the lines be θ.

$$\cos\theta = \frac{\begin{pmatrix}\frac{1}{a} & \frac{1}{b}\end{pmatrix}^{\top}\begin{pmatrix}\frac{1}{a} & \frac{-1}{b}\end{pmatrix}}{\left\|\begin{pmatrix}\frac{1}{a} & \frac{1}{b}\end{pmatrix}\right\|\left\|\begin{pmatrix}\frac{1}{a} & \frac{-1}{b}\end{pmatrix}\right\|} \tag{3}$$

$$\cos\theta = \frac{\frac{1}{a^2} - \frac{1}{b^2}}{\sqrt{\left(\frac{1}{a}^2\right) + \left(\frac{1}{b}\right)^2}\sqrt{\left(\frac{1}{a}^2\right) + \left(\frac{-1}{b}\right)^2}} \tag{4}$$

$$\cos\theta = \frac{b^2 - a^2}{a^2 + b^2} \tag{5}$$

Theoretical Solution

$$\cos\theta = \frac{b^2 - a^2}{a^2 + b^2} \left(\because \tan\theta = \frac{\sqrt{1 - \cos^2\theta}}{\cos\theta} \right) \tag{6}$$

$$\tan\theta = \left| \frac{2ab}{b^2 - a^2} \right| \tag{7}$$

$\therefore$ The tan of the acute angle between the lines is $\frac{2ab}{a^2 - b^2}$.

C Code (1) - Function to store the points

```c
#include <math.h>

double calculate_tangent(double a, double b) {
    // Calculate the denominator of the formula.
    double denominator = (a * a) - (b * b);

    // Check if the denominator is zero. This happens when the
        lines are
    // perpendicular, and the tangent would be undefined (
        division by zero).
    if (denominator == 0.0) {
        return NAN; // Return "Not a Number" to indicate an
            undefined result.
    }
```

C Code (1) - Function to store the points

```
    // Calculate the numerator.
    double numerator = 2.0 * a * b;

    // Return the final calculated tangent.
    return numerator / denominator;
}
```

Python Code - Using Shared Object

```
import ctypes
import math
import numpy as np
import matplotlib.pyplot as plt

# Load the shared object
tangent_lib = ctypes.CDLL("./func.so")

# Define the C function's argument and return types
tangent_lib.calculate_tangent.argtypes = [ctypes.c_double, ctypes
    .c_double]
tangent_lib.calculate_tangent.restype = ctypes.c_double

# Create a Python-callable function
calculate_tangent = tangent_lib.calculate_tangent
```

Python Code - Using Shared Object

```
def plot_lines(a, b, tangent_value):
    """
    Plots the two lines and displays the tangent of the angle
        between them.
    """
    # Create a range of x-values for the plot.
    x_range = np.linspace(-abs(a) * 1.5, abs(a) * 1.5, 400)

    # Calculate the corresponding y-values for each line's
        equation.
    # Line 1: y = b * (1 - x/a)
    y1 = b * (1 - x_range / a)
    # Line 2: y = b * (x/a - 1)
    y2 = b * (x_range / a - 1)
```

Python Code - Using Shared Object

```
# Create the plot figure.
    plt.figure(figsize=(8, 8))
    plt.plot(x_range, y1, label=f'x/{a} + y/{b} = 1', color='blue
        ')
    plt.plot(x_range, y2, label=f'x/{a} - y/{b} = 1', color='red'
        )


# Add plot enhancements for better visualization.
    plt.axhline(0, color='black', linewidth=0.7)
    plt.axvline(0, color='black', linewidth=0.7)
    plt.grid(True, which='both', linestyle='--', linewidth=0.5)
    plt.legend()
```

Python Code - Using Shared Object

```
    plt.xlabel("x-axis")
    plt.ylabel("y-axis")
    plt.axis('equal') # Use 'equal' scaling to ensure angles are
        not distorted.
# Set the plot title based on the tangent calculation result.
    if math.isnan(tangent_value):
        title = "Plot of the Lines\nTangent is undefined (lines
            are perpendicular)"
    else:
        title = f"Plot of the Lines\nTangent of the angle = {
            tangent_value:.4f}"
    plt.title(title)
    plt.savefig('figs/line.png')

    # Display the plot.
    plt.show()
```

Python Code - Using Shared Object

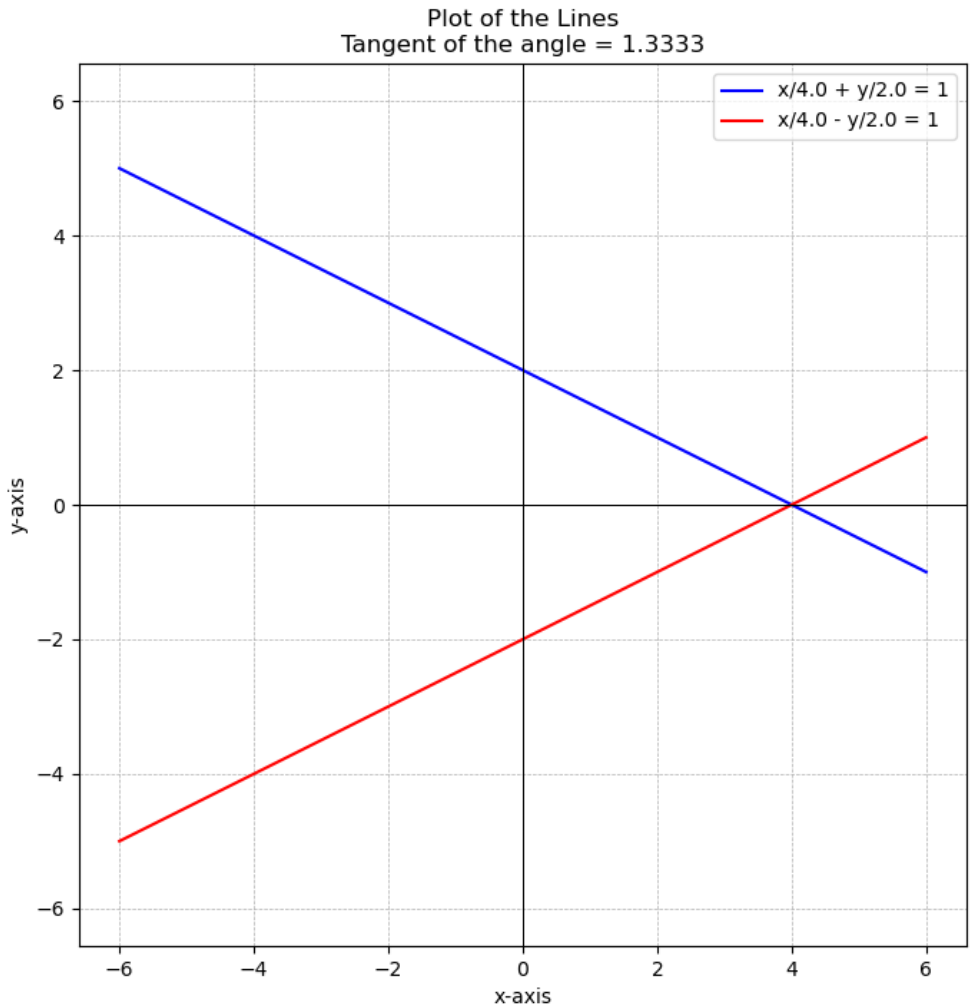
```
    def main():
    """
    Main function to define parameters, call the C function, and
        trigger the plot.
    """
    # Hardcoded values for 'a' and 'b'.
    a = 4.0
    b = 2.0

    # Call the C function with the hardcoded values.
    result = calculate_tangent(a, b)

    # Plot the lines using the results.
    plot_lines(a, b, result)


if __name__ == "__main__":
    main()
```

Plot-Using Both C and Python



Python Code

```
import math
import numpy as np
import matplotlib.pyplot as plt


def calculate_tangent_py(a, b):
    """
    Calculates the tangent of the angle between the two lines
        directly in Python.
    The formula is derived from the slopes of the lines x/a + y/b
         = 1 and x/a - y/b = 1.
    """
    # Calculate the denominator of the tangent formula.
    denominator = a**2 - b**2
```

Python Code

```
# Check for division by zero, which occurs when a = b or a = -b.
    # In this case, the lines are perpendicular, and the tangent
        is undefined.
    if denominator == 0:
        return math.nan # Return Not-a-Number for an undefined
            tangent.

    # Calculate the numerator.
    numerator = 2 * a * b

    return numerator / denominator
```

Python Code

```
def plot_lines(a, b, tangent_value):
    """
    Plots the two lines and displays the tangent of the angle
        between them.
    """
    # Create a range of x-values for the plot.
    x_range = np.linspace(-abs(a) * 1.5, abs(a) * 1.5, 400)

    # Calculate the corresponding y-values for each line's
        equation.
    # Line 1: y = b * (1 - x/a)
    y1 = b * (1 - x_range / a)
    # Line 2: y = b * (x/a - 1)
    y2 = b * (x_range / a - 1)
```

Python Code

```
# Create the plot figure.
    plt.figure(figsize=(8, 8))
    plt.plot(x_range, y1, label=f'x/{a} + y/{b} = 1', color='blue
        ')
    plt.plot(x_range, y2, label=f'x/{a} - y/{b} = 1', color='red'
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# Add plot enhancements for better visualization.
    plt.axhline(0, color='black', linewidth=0.7)
    plt.axvline(0, color='black', linewidth=0.7)
    plt.grid(True, which='both', linestyle='--', linewidth=0.5)
    plt.legend()
    plt.xlabel("x-axis")
    plt.ylabel("y-axis")
    plt.axis('equal') # Use 'equal' scaling to ensure angles are
        not distorted.
```

Python Code

```
# Set the plot title based on the tangent calculation result.
    if math.isnan(tangent_value):
        title = "Plot of the Lines\nTangent is undefined (lines
            are perpendicular)"
    else:
        title = f"Plot of the Lines\nTangent of the angle = {
            tangent_value:.4f}"
    plt.title(title)
    plt.savefig('figs/line2.png')

    # Display the plot.
    plt.show()
```

Python Code

```
def main():
    """
    Main function to define parameters, call the Python function,
        and trigger the plot.
    """
    # Hardcoded values for 'a' and 'b'.
    a = 4.0
    b = 2.0

    # Call the Python function with the hardcoded values.
    result = calculate_tangent_py(a, b)

    # Plot the lines using the results.
    plot_lines(a, b, result)


if __name__ == "__main__":
    main()
```

Plot-Using only Python