Matrices in Geometry 12.51

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Question: Let the eigenvalues of a square matrix $\mathbf{A}$ of order two be 1 and 2. The corresponding eigenvectors are $\begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix}$ and $\begin{pmatrix} 0.8 \\ -0.6 \end{pmatrix}$, respectively. Then, the element $\mathbf{A}(2,2)$ is

a) -0.48 b) 0.48 c) 1.36 d) 1.64

Solution:
The eigenvalues of $\mathbf{A}$ are $\lambda_1 = 1$ and $\lambda_2 = 2$.
Let the given eigenvectors be

$$\mathbf{v_1} = \begin{pmatrix} 0.6 \\ 0.8 \end{pmatrix}, \ \mathbf{v_2} = \begin{pmatrix} 0.8 \\ -0.6 \end{pmatrix} \tag{1}$$

Let

$$\mathbf{P} = \begin{pmatrix} \mathbf{v_1} & \mathbf{v_2} \end{pmatrix} = \begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix} \tag{2}$$

The given eigen vectors $\mathbf{v_1}$ and $\mathbf{v_2}$ are orthonormal, that is, they are unit vectors and their scalar product is zero.

$$\therefore \mathbf{P}^\top = \mathbf{P}^{-1} \tag{3}$$

Using spectral decomposition, we can find the matrix $\mathbf{A}$.

$$\mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^\top, \ \mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \tag{4}$$

$$\mathbf{A} = \begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{pmatrix} \tag{5}$$

$$\implies \mathbf{A} = \begin{pmatrix} 1.64 & -0.48 \\ -0.48 & 1.36 \end{pmatrix} \tag{6}$$

The element $\mathbf{A}(2,2) = 1.36$ which is option c)