2.10.78

EE25BTECH11050-Hema Havil

Question:

The point (4, 1) undergoes the following three transformations successively.

(a) Reflection about the line $y = x$.
(b) Translation through a distance 2 units along the positive direction of x-axis.
(c) Rotation through an angle $\frac{\pi}{4}$ about the origin in the counter clockwise direction.

Then the final position of the point is given by the coordinates.

(a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (b) $\left(-\sqrt{2}, 7\sqrt{2}\right)$ (c) $\left(\sqrt{2}, 7\sqrt{2}\right)$ (d) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$

Solution:

Let the given point be P=(4,1) and vector be $\mathbf{P} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$,

(a) Reflection matrix for $y = x$ is,

$$\mathbf{M} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{(a).1}$$

then the reflection of P about $y = x$ is,

$$\mathbf{P_1} = \mathbf{MP} \tag{(a).2}$$

$$\mathbf{P_1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix} \tag{(a).3}$$

$$\mathbf{P_1} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \tag{(a).4}$$

(b) Convert P’ into homogeneous form,

$$\mathbf{P_1^h} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} \tag{(b).1}$$

The translation matrix along the x direction is given as,

$$\mathbf{T} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{(b).2}$$

then the translated vector is,

$$\mathbf{P_2^h} = \mathbf{TP_1^h} \tag{(b).3}$$

$$\mathbf{P_2^h} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} \tag{(b).4}$$

$$\mathbf{P_2} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{(b).5}$$

(c) Rotation matrix is given as,

$$\mathbf{R} = \begin{pmatrix} cos\theta & -\ sin\theta \\ sin\theta & cos\theta \end{pmatrix} \tag{(c).1}$$

Rotation of P_2 by an angle $\frac{\pi}{4}$ about the origin in the counter clockwise direction,

$$\mathbf{P_3} = \mathbf{RP_2} \tag{(c).2}$$

$$\mathbf{P_3} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{(c).3}$$

$$\mathbf{P_3} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{7}{\sqrt{2}} \end{pmatrix} \tag{(c).4}$$

Therefore the final position of the point is $P_3 = (-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}})$, option (d) is correct

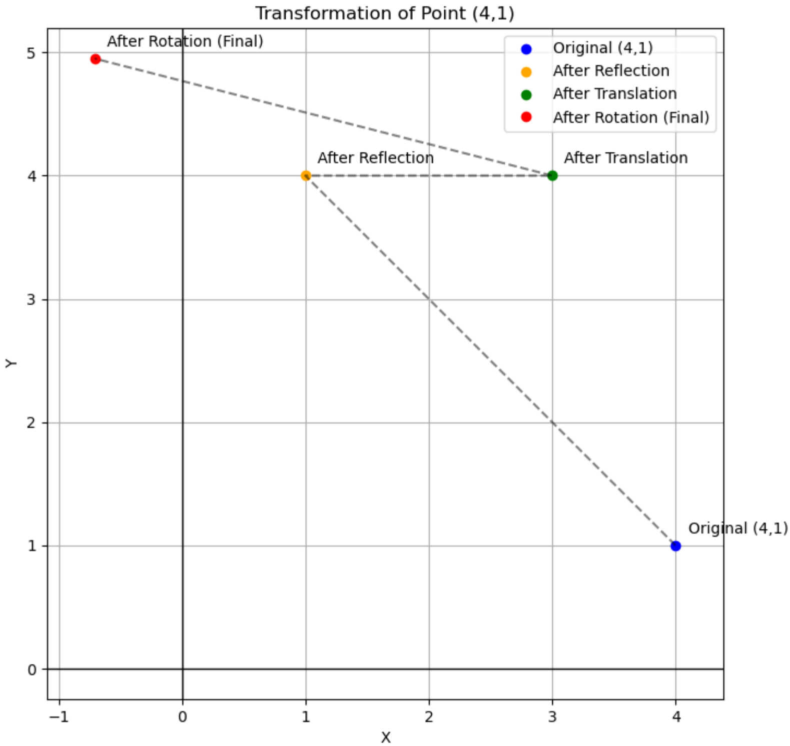

Fig. 3.1: Plot for the above Transformations