

1.2.12

AI25BTECH11006 - Nikhila

Question:

Show that the points A(a, b+c), B(b, c+a), C(c, b+a) are collinear.

Solution:

The vector components of the given points are $\mathbf{A} \begin{pmatrix} a \\ b+c \end{pmatrix}$, $\mathbf{B} \begin{pmatrix} b \\ c+a \end{pmatrix}$, $\mathbf{C} \begin{pmatrix} c \\ a+b \end{pmatrix}$.

We know that If the points $\mathbf{A} \mathbf{B} \mathbf{C}$ are collinear , the rank of the matrix

$$(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A}) = 1 \quad (0.1)$$

$$(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A}) = \begin{pmatrix} b-a & c-a \\ a-b & a-c \end{pmatrix} \quad (0.2)$$

$$\xleftrightarrow{R_2=R_2+R_1} \begin{pmatrix} b-a & c-a \\ 0 & 0 \end{pmatrix} \quad (0.3)$$

Clearly the rank of the matrix is 1 and hence the points are colinear.

Let us take example points by taking a = 1, b = 2, c = 3 and plot the points A(1,5), B(2,4), C(3,3) on the graph.

Graph:

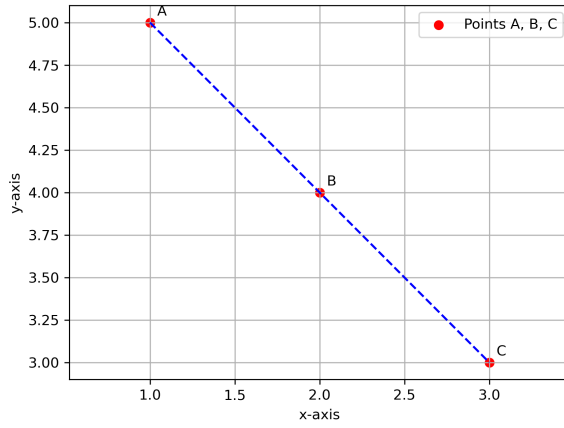


Fig. 0.1