5.2.48

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Question

Solve the following system of linear equations.

$$\begin{aligned} x+y+z &= 1 \\ 2x+3y+2z &= 2 \\ ax+ay+2az &= 4 \end{aligned}$$

Theoretical Solution

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 3 & 2 & 2 \\ a & a & 2a & 4 \end{array}\right) \xleftrightarrow{R_3 \to \frac{1}{a} R_3, a \neq 0} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 3 & 2 & 2 \\ 1 & 1 & 2 & \frac{4}{a} \end{array}\right) \tag{1}$$

$$\xleftrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - 2R_1} \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{4}{a} - 1 \end{array}\right) \xleftrightarrow{R_1 \to R_1 - R_2} \left(\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{4}{a} - 1 \end{array}\right) \tag{2}$$

Theoretical Solution

$$\xleftrightarrow{R_1 \to R_1 - R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 - \frac{4}{a} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{4}{a} - 1 \end{array}\right) \tag{3}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 - \frac{4}{a} \\ 0 \\ \frac{4}{a} - 1 \end{pmatrix}, \quad a \neq 0 \tag{4}$$

For $a = 0$, the third equation becomes $0 = 4$, which is inconsistent. Therefore, no solution exists for $a = 0$.

Example

Let

$$a = 2$$

$$\begin{aligned} x + y + z &= 1 \\ 2x + 3y + 2z &= 2 \\ 2x + 2y + 4z &= 4 \end{aligned}$$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 2 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$$

Using (4),

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{5}$$

Plot

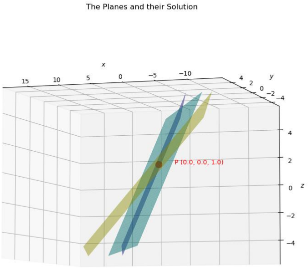

Figure: Plot