

Matrices in Geometry - 10.7.86

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Problem Statement

Let C_1 and C_2 be two circles with C_2 lying inside C_1 . A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of center of C .

Solution

Let the center of \mathbf{C} , \mathbf{C}_1 and \mathbf{C}_2 be \mathbf{O} , \mathbf{O}_1 and \mathbf{O}_2 , respectively.

Let the radii of circles \mathbf{C} , \mathbf{C}_1 and \mathbf{C}_2 be r , r_1 and r_2

It is given that \mathbf{C} touches the circle \mathbf{C}_1 internally and \mathbf{C}_2 externally.

Therefore,

$$\|\mathbf{O} - \mathbf{O}_1\| = r_1 - r \quad (1)$$

$$\|\mathbf{O} - \mathbf{O}_2\| = r_2 + r \quad (2)$$

Adding these two equations, we get

$$\|\mathbf{O} - \mathbf{O}_1\| + \|\mathbf{O} - \mathbf{O}_2\| = r_1 + r_2 \quad (3)$$

Solution

Substitute \mathbf{O} as \mathbf{x}

$$\|\mathbf{x} - \mathbf{O}_1\| + \|\mathbf{x} - \mathbf{O}_2\| = r_1 + r_2 \quad (4)$$

This is equation of an ellipse because it is of form

$$\|\mathbf{x} - \mathbf{S}_1\| + \|\mathbf{x} - \mathbf{S}_2\| = 2a \quad (5)$$

with focii as \mathbf{O}_1 , \mathbf{O}_2 and length of the major axis as $r_1 + r_2$

Solution

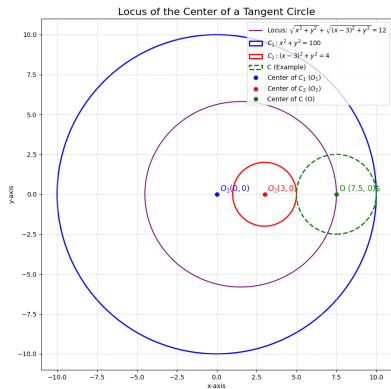


Figure: Graph for 10.7.86