2.5.31

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Question:
If two vertices of an equilateral triangle are $(3, 0)$ and $(6, 0)$,find the third vertex
Solution

vector	Name
$\begin{pmatrix}3\\0\end{pmatrix}$	Vector **A**
$\begin{pmatrix}6\\0\end{pmatrix}$	Vector **B**
$\begin{pmatrix}x\\y\end{pmatrix}$	Vector **C**

TABLE 0: Variables Used

$$\text{The vector joining from } \mathbf{A} \text{ to } \mathbf{B} \text{ is given by } \mathbf{B}-\mathbf{A} = \begin{pmatrix}6\\0\end{pmatrix} - \begin{pmatrix}3\\0\end{pmatrix} \tag{1}$$

$$\implies \mathbf{B}-\mathbf{A} = \begin{pmatrix}3\\0\end{pmatrix}. \tag{2}$$

(3)

An equilateral triangle can be obtained by rotating **B-A** by **A** about $+60°$ or $-60°$ .The rotation matrix p at angle θ is defined as

$$p(\theta) = \begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix} \tag{4}$$

(5)

$$p(60°) = \begin{pmatrix}\frac{1}{2} & \frac{-\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix} \qquad p(-60°) = \begin{pmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ \frac{-\sqrt{3}}{2} & \frac{1}{2}\end{pmatrix} \tag{6}$$

(7)

Apply p(60°) or p(−60°) to $\mathbf{B}-\mathbf{A}$ and add it to **A** to get **C**

$$\mathbf{C} = \mathbf{A} + \mathrm{p}(60°)(\mathbf{B}-\mathbf{A}) \quad \text{or} \quad \mathbf{C} = \mathbf{A} + \mathrm{p}(-60°)(\mathbf{B}-\mathbf{A}) \tag{8}$$

$$\mathrm{p}(60°)\begin{pmatrix}3\\0\end{pmatrix} = \begin{pmatrix}\frac{3}{2}\\ \frac{3\sqrt{3}}{2}\end{pmatrix} \quad \text{or} \quad \mathrm{p}(-60°)\begin{pmatrix}3\\0\end{pmatrix} = \begin{pmatrix}\frac{3}{2}\\ \frac{-3\sqrt{3}}{2}\end{pmatrix} \tag{9}$$

$$\mathbf{C} = \begin{pmatrix}\frac{3}{2}\\ \frac{3\sqrt{3}}{2}\end{pmatrix} \quad \text{or} \quad \mathbf{C} = \begin{pmatrix}\frac{3}{2}\\ \frac{-3\sqrt{3}}{2}\end{pmatrix} \tag{10}$$