1.5.26

EE25BTECH11038 - Gnanthik Lucky

Question:
Let **P** and **Q** be the points of trisection of the line segment that join the points **A** (2,-2) and **B** (-7,4) such that **P** is closer to **A**. Find the coordinates of **P** and **Q**.

Solution:

Let $\mathbf{A} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -7 \\ 4 \end{pmatrix}$

Point P (Nearer to A, Ratio 1 : 2):

$$\mathbf{P} = \frac{1}{3}\mathbf{B} + \frac{2}{3}\mathbf{A} = \frac{1}{3}\begin{pmatrix} -7 \\ 4 \end{pmatrix} + \frac{2}{3}\begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$$\mathbf{P} = \begin{pmatrix} \frac{1\times(-7)+2\times2}{3} \\ \frac{1\times4+2\times(-2)}{3} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

Point Q (Further from A, Ratio 2 : 1):

$$\mathbf{Q} = \frac{2}{3}\mathbf{B} + \frac{1}{3}\mathbf{A} = \frac{2}{3}\begin{pmatrix} -7 \\ 4 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

$$\mathbf{Q} = \begin{pmatrix} \frac{2\times(-7)+1\times2}{3} \\ \frac{2\times4+1\times(-2)}{3} \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

$$\boxed{\text{P} = (-1,\ 0) \qquad \text{Q} = (-4,\ 2)}$$
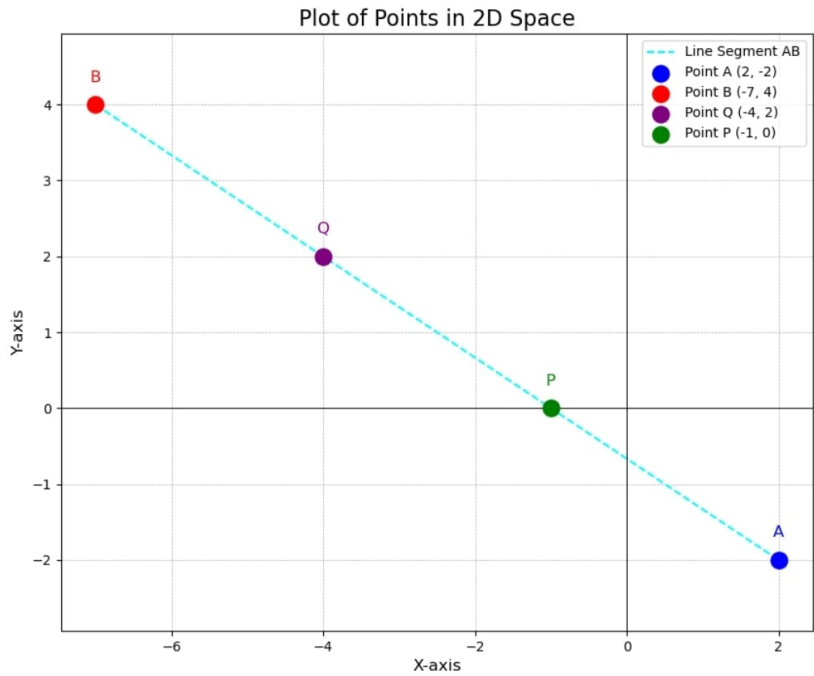
Graph of the line segment AB with trisection points P and Q

Fig. 0: Figure for 1.5.26