9.4.13

EE25BTECH11001 - Aarush Dilawri

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Question:

Find the roots of the following quadratic equation graphically

$$x^2 - 2x = (-2)(3 - x) \tag{1}$$

Solution

Solution:

$$y = x^2 - 2x - (-2)(3 - x) \tag{2}$$

$$y = x^2 - 4x + 6 = 0 \tag{3}$$

This quadratic can be represented as a conic in matrix form:

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{4}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, f = 6 \tag{5}$$

Solution

To find the roots, we find the points of intersection of the conic with the x-axis.

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{6}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{7}$$

Solution

The value of k can be found out by solving the line and conic equation

$$(\mathbf{h}+k\mathbf{m})^\top \mathbf{V}(\mathbf{h}+k\mathbf{m})+2\mathbf{u}^\top(\mathbf{h}+k\mathbf{m})+f=0 \tag{8}$$

$$\implies k^2\mathbf{m}^\top\mathbf{V}\mathbf{m}+2k\mathbf{m}^\top(\mathbf{V}\mathbf{h}+\mathbf{u})+\mathbf{h}^\top\mathbf{V}\mathbf{h}+2\mathbf{u}^\top\mathbf{h}+f=0 \tag{9}$$

$$\text{or, } k^2\mathbf{m}^\top\mathbf{V}\mathbf{m}+2k\mathbf{m}^\top(\mathbf{V}\mathbf{h}+\mathbf{u})+g(\mathbf{h})=0 \tag{10}$$

Solution

Solving the above quadratic gives the equation

$$k = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \pm \sqrt{\left[\mathbf{m}^\top \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 - g\left(\mathbf{h} \right) \left(\mathbf{m}^\top \mathbf{V}\mathbf{m} \right)} \right) \tag{11}$$

Substituting the values in the above equation gives

$$\therefore k = 2 \pm i\sqrt{2} \tag{12}$$

$$k_1 = 2 + i\sqrt{2} \tag{13}$$

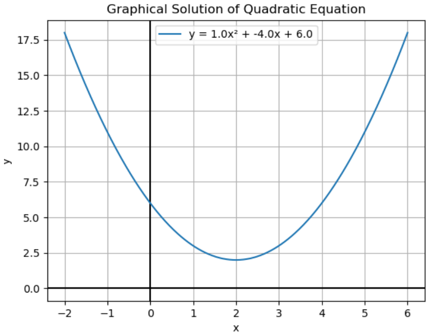
$$k_2 = 2 - i\sqrt{2} \tag{14}$$

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} = \begin{pmatrix} 2 + i\sqrt{2} \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 2 - i\sqrt{2} \\ 0 \end{pmatrix} \tag{15}$$

$\therefore$ The given quadratic equation has imaginary roots.

Graphical Representation

See Figure,



Codes

https://github.com/AarushDilawri/ee1030-2025/tree/main/ee25btech11001/MATGEO/9.4.13/codes