

Matrices in Geometry 5.2.38

EE25BTECH11035 - Kushal B N

Question:

Solve the following system of equations.

$$\begin{aligned}\frac{1}{2x} + \frac{1}{3y} &= 2 \\ \frac{1}{3x} + \frac{1}{2y} &= \frac{13}{6}\end{aligned}$$

Solution:

Let

$$\mathbf{x} = \begin{pmatrix} \frac{1}{2x} \\ \frac{1}{3y} \end{pmatrix} \quad (1)$$

So that the given equations, after multiplying by 6 on both sides, can be represented in the matrix form as

$$\begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 12 \\ 13 \end{pmatrix} \quad (2)$$

Which can be represented as the augmented matrix

$$\left(\begin{array}{cc|c} 3 & 2 & 12 \\ 2 & 3 & 13 \end{array} \right) \quad (3)$$

$$\left(\begin{array}{cc|c} 3 & 2 & 12 \\ 2 & 3 & 13 \end{array} \right) \xleftrightarrow{R_2 \leftarrow R_2 - \frac{2}{3}R_1} \left(\begin{array}{cc|c} 3 & 2 & 12 \\ 0 & \frac{5}{3} & 5 \end{array} \right) \quad (4)$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - \frac{6}{5}R_2} \left(\begin{array}{cc|c} 3 & 0 & 6 \\ 0 & \frac{5}{3} & 5 \end{array} \right) \quad (5)$$

So, by this, we get

$$\frac{1}{y} = 3 \implies y = \frac{1}{3} \quad (6)$$

$$\frac{1}{x} = 2 \implies x = \frac{1}{2} \quad (7)$$

Final Answer:

The solution for the given system of linear equations is $x = \frac{1}{2}$ and $y = \frac{1}{3}$.

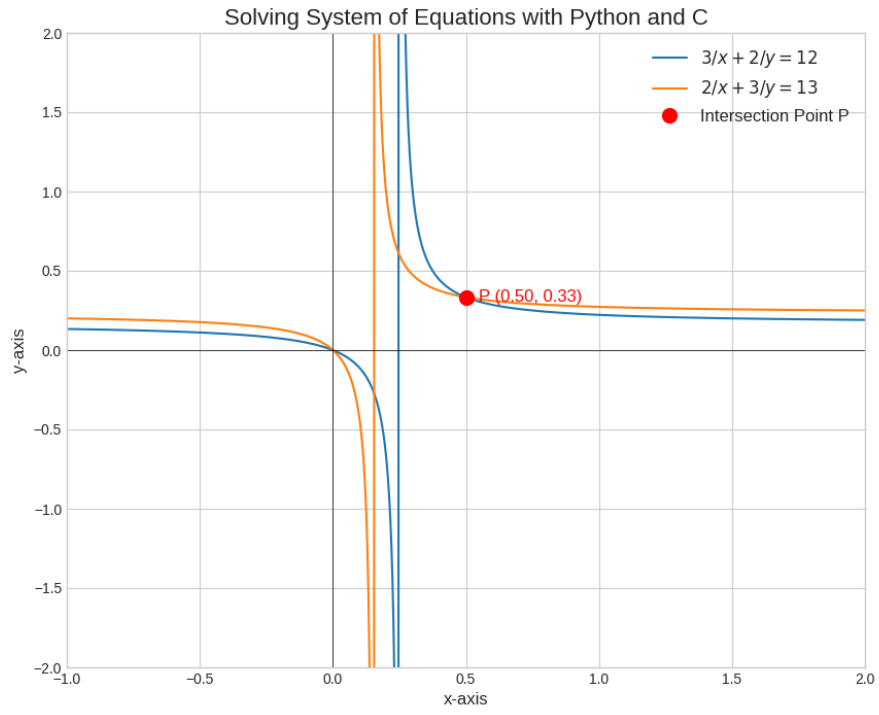


Fig. 1