1.11.5

AI25BTECH11003 - Bhavesh Gaikwad

Question: The scalar product of vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of the vectors $\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal to 1. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$.

Solution:

Given: $\vec{a} = \begin{pmatrix}1\\1\\1\end{pmatrix}$, $\vec{b} = \begin{pmatrix}2\\4\\-5\end{pmatrix}$, $\vec{c} = \begin{pmatrix}\lambda\\2\\3\end{pmatrix}$.

Let $\hat{u}$ be the unit vector along $\vec{b} + \vec{c}$.

$$\vec{b} + \vec{c} = \begin{pmatrix}2+\lambda\\4+2\\-5+3\end{pmatrix} = \begin{pmatrix}2+\lambda\\6\\-2\end{pmatrix}.$$

$$\|\vec{b} + \vec{c}\| = \sqrt{(2+\lambda)^2 + 6^2 + (-2)^2} = \sqrt{\lambda^2 + 4\lambda + 44}.$$

$$\hat{u} = \frac{\vec{b} + \vec{c}}{\|\vec{b} + \vec{c}\|} = \frac{1}{\sqrt{\lambda^2 + 4\lambda + 44}}\begin{pmatrix}2+\lambda\\6\\-2\end{pmatrix}.$$

Given condition: $\vec{a} \cdot \hat{u} = 1$.

$$\vec{a} \cdot \hat{u} = \frac{\vec{a} \cdot (\vec{b} + \vec{c})}{\|\vec{b} + \vec{c}\|} = \frac{\begin{pmatrix}1\\1\\1\end{pmatrix} \cdot \begin{pmatrix}2+\lambda\\6\\-2\end{pmatrix}}{\sqrt{\lambda^2 + 4\lambda + 44}} = \frac{\lambda + 6}{\sqrt{\lambda^2 + 4\lambda + 44}} = 1.$$

$$\Rightarrow (\lambda+6)^2 = \lambda^2 + 4\lambda + 44 \implies \lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44 \implies 8\lambda = 8 \implies \boxed{\lambda = 1}.$$

Now, with $\lambda = 1$: $\vec{b} + \vec{c} = \begin{pmatrix}3\\6\\-2\end{pmatrix}$, $\|\vec{b} + \vec{c}\| = \sqrt{3^2 + 6^2 + (-2)^2} = \sqrt{49} = 7$.

Unit vector along, $\vec{b} + \vec{c}$ is: $\frac{1}{7}\begin{pmatrix}3\\6\\-2\end{pmatrix} = \frac{1}{7}\begin{pmatrix}3\\6\\-2\end{pmatrix}$.

$\boxed{\lambda = 1}$ *and* $\boxed{\text{Unit vector along } \overrightarrow{b} + \overrightarrow{c} = \frac{1}{7}\begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix}}$. (0.1)

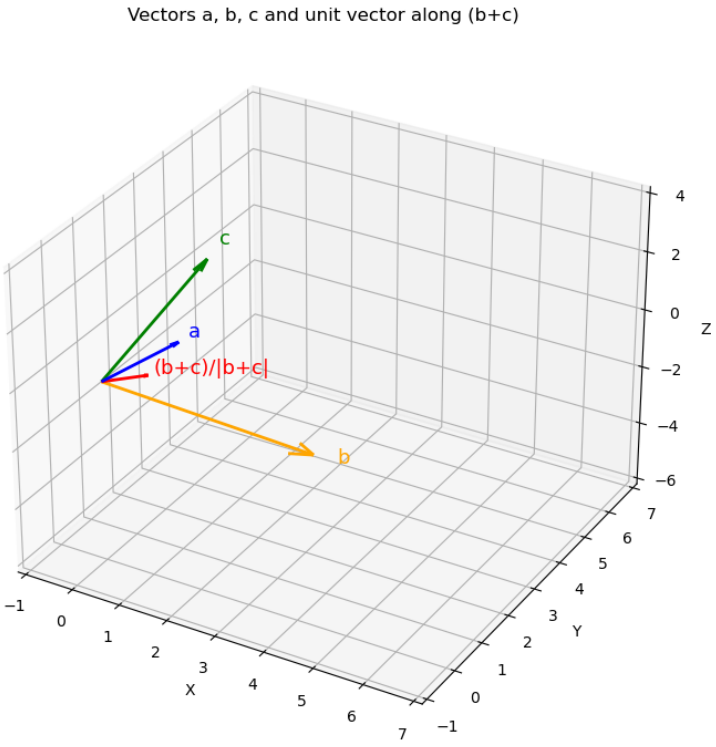

Fig. 0.1: Vector Representation