2.8.14

EE25BTECH11031 - Sai Sreevallabh

Question:

Three vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ satisfy the condition $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$. Evaluate the quantity $\mu = \mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{c} + \mathbf{c} \cdot \mathbf{a}$. If $|\mathbf{a}| = 3$, $|\mathbf{b}| = 4$ and $|\mathbf{c}| = 2$.

Solution:

Given:

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0 \text{ and } \|\mathbf{a}\| = 3 \ , \ \|\mathbf{b}\| = 4 \ , \ \|\mathbf{c}\| = 2 \tag{0.1}$$

To find

$$\mu = \mathbf{a}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{c} + \mathbf{c}^\top \mathbf{a} \tag{0.2}$$

Multiplying $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$ with $\mathbf{a}^\top$ on both sides

$$\mathbf{a}^\top \mathbf{a} + \mathbf{a}^\top \mathbf{b} + \mathbf{a}^\top \mathbf{c} = 0 \tag{0.3}$$

Similarly, upon multiplying with $\mathbf{b}^\top$ and $\mathbf{c}^\top$, we get

$$\mathbf{b}^\top \mathbf{a} + \mathbf{b}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{c} = 0 \tag{0.4}$$

$$\mathbf{c}^\top \mathbf{a} + \mathbf{c}^\top \mathbf{b} + \mathbf{c}^\top \mathbf{c} = 0 \tag{0.5}$$

Adding the above three equations,

$$2\left(\mathbf{a}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{c} + \mathbf{c}^\top \mathbf{a}\right) + \mathbf{a}^\top \mathbf{a} + \mathbf{b}^\top \mathbf{b} + \mathbf{c}^\top \mathbf{c} = 0 \tag{0.6}$$

$$\implies 2\mu + \mathbf{a}^\top \mathbf{a} + \mathbf{b}^\top \mathbf{b} + \mathbf{c}^\top \mathbf{c} = 0 \tag{0.7}$$

By using $\mathbf{x}^\top \mathbf{x} = \|\mathbf{x}\|^2$ we get

$$2\mu + \left(\|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + \|\mathbf{c}\|^2\right) = 0 \tag{0.8}$$

Substituting the values of $\|\mathbf{a}\|, \|\mathbf{b}\|, \|\mathbf{c}\|$ we get

$$\mu = \frac{-29}{2} \tag{0.9}$$

$\therefore$ The value of μ is $\frac{-29}{2}$.