5.8.43

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Question

The sum of three numbers is 6. If we multiply the third number by 3 and add the second number to it, we get 11. By adding the first and third numbers, we get double of the second number. Find the numbers.

Theoretical Solution

Given

$$x + y + z = 6 \tag{1}$$

$$0x + y + 3z = 11 (2)$$

$$x - 2y + z = 0 \tag{3}$$

$$\begin{pmatrix} 1 & 1 & 1 & | & 6 \\ 0 & 1 & 3 & | & 11 \\ 1 & -2 & 1 & | & 0 \end{pmatrix} \xrightarrow{\mathsf{R}_3 \to \mathsf{R}_3 - \mathsf{R}_1} \begin{pmatrix} 1 & 1 & 1 & | & 6 \\ 0 & 1 & 3 & | & 11 \\ 0 & -3 & 0 & | & -6 \end{pmatrix} \tag{4}$$

$$\stackrel{\mathsf{R}_3 \to -\frac{1}{3}\mathsf{R}_3}{\longleftrightarrow} \begin{pmatrix} 1 & 1 & 1 & | & 6 \\ 0 & 1 & 3 & | & 11 \\ 0 & 1 & 0 & | & 2 \end{pmatrix} \stackrel{\mathsf{R}_2 \leftrightarrow \mathsf{R}_3}{\longleftrightarrow} \begin{pmatrix} 1 & 1 & 1 & | & 6 \\ 0 & 1 & 0 & | & 2 \\ 0 & 1 & 3 & | & 11 \end{pmatrix} \tag{5}$$

Theoretical Solution

$$\frac{\stackrel{R_1 \to R_1 - R_2}{\to R_3 \to R_3 - R_2}}{\stackrel{R_3 \to R_3 - R_2}{\to 0}} \begin{pmatrix} 1 & 0 & 1 & | & 4 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 3 & | & 9 \end{pmatrix} \xrightarrow{\stackrel{R_3 \to \frac{1}{3}R_3}{\to 1}} \begin{pmatrix} 1 & 0 & 1 & | & 4 \\ 0 & 1 & 0 & | & 2 \\ 0 & 0 & 1 & | & 3 \end{pmatrix} \tag{6}$$

$$\stackrel{\mathsf{R}_1 \to \mathsf{R}_1 - \mathsf{R}_3}{\longleftrightarrow} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix} \tag{7}$$

Example

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

 \therefore The required numbers are 1,2 and 3.

Plot

The Planes and their Solution

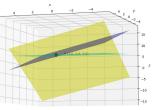


Figure: Plot