1.5.10

EE25BTECH11022 - sankeerthan

problem(1.5.10). Find the ratio in which the line segment joining the points

$$A = \begin{pmatrix} 1 \\ -5 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -4 \\ 5 \end{pmatrix}. \tag{0.1}$$

is divided by X-axis. Also,find the coordinates of the point of division

Solution:

Let the given points be A and B

$$\mathbf{A} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

Let the X-axis divide the line segment $\overline{\mathbf{AB}}$ at point **P** in the ratio $k : 1$. Since **P** lies on X-axis, let

$$\mathbf{P} = \begin{pmatrix} x \\ 0 \end{pmatrix}$$

The point **A**, **B**, **P** are collinear.

$$\implies \operatorname{rank}\begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{P} - \mathbf{A} \end{pmatrix} = 1 \tag{0.2}$$

$$\begin{pmatrix} -5 & x-1 \\ 10 & 5 \end{pmatrix} \xleftrightarrow{R_1 \to R_1 + \frac{1}{2}R_2} \begin{pmatrix} 0 & x - \frac{3}{2} \\ 10 & 5 \end{pmatrix} \xleftrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 10 & 5 \\ 0 & x - \frac{3}{2} \end{pmatrix} \tag{0.3}$$

The number of nonzero rows in the row reduced matrix (also known as *echelon form*) is defined as the rank. For above matrix to be of rank 1,

$$x + \frac{3}{2} = 0 \tag{0.4}$$

$$x = \frac{-3}{2} \tag{0.5}$$

$\therefore$ The coordinates of the point of intersection are

$$\mathbf{P} = \begin{pmatrix} \frac{-3}{2} \\ 0 \end{pmatrix}$$

Substituting the values of **A**, **B** and **P**,

$$k = \frac{\begin{pmatrix} \frac{5}{2} & -5 \end{pmatrix}\begin{pmatrix} \frac{5}{2} \\ -5 \end{pmatrix}}{\left\| \begin{pmatrix} \frac{5}{2} \\ -5 \end{pmatrix} \right\|^2} = 1 \tag{0.6}$$

Thus, the ratio in which the point **P** divides the line segment **AB** is **1:1**.

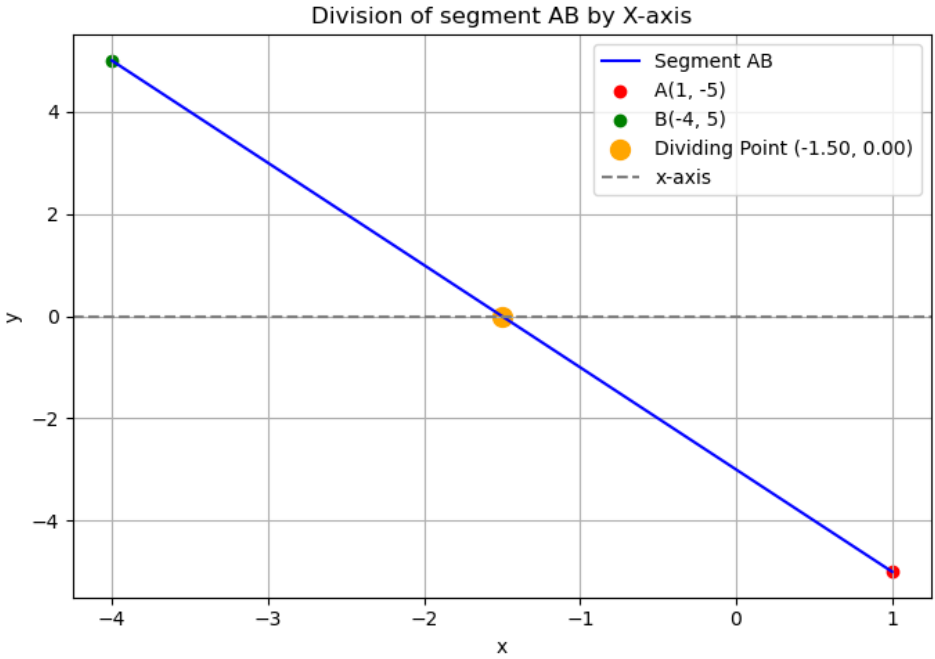

Fig. 0.1: Plot of line segment **AB**