4.11.3

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Question

Find the equation of the line passing through (2,-1,2) and (5,3,4) and the equation of the plane passing through (2,0,3), (1,1,5), and (3,2,4). Also, find their point of intersection.

Given

Let:

$$\mathbf{P}_1 = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}; \mathbf{P}_2 = \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix} \tag{1}$$

$$\mathbf{A} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}; \mathbf{B} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}; \mathbf{C} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} \tag{2}$$

Vector forms

Direction vector of the line:

$$\mathbf{m} = \mathbf{P}_2 - \mathbf{P}_1 = \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \tag{3}$$

Vector form of the line can be written as:

$$\mathbf{x} = \mathbf{P}_1 + \kappa \mathbf{m} \tag{4}$$

$$\mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \kappa \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \tag{5}$$

Vector form of the line can be written as:

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}^\top \mathbf{n} = \mathbf{1} \tag{6}$$

$$\begin{pmatrix} 2 & 0 & 3 \\ 1 & 1 & 5 \\ 3 & 2 & 4 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{7}$$

Augmented matrix

Augmented matrix can be written as:

$$\left(\begin{array}{ccc|c} 2 & 0 & 3 & 1 \\ 1 & 1 & 5 & 1 \\ 3 & 2 & 4 & 1 \end{array}\right) R_2 \leftrightarrow R_1 \left(\begin{array}{ccc|c} 1 & 1 & 5 & 1 \\ 2 & 0 & 3 & 1 \\ 3 & 2 & 4 & 1 \end{array}\right) \frac{R_2 \to R_2 - 2R_1}{R_3 \to R_3 - 3R_1} \tag{8}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 5 & 1 \\ 0 & -2 & -7 & -1 \\ 0 & -1 & -11 & -2 \end{array}\right) \frac{R_2 \leftrightarrow R_3}{R_2 \to -R_2} \left(\begin{array}{ccc|c} 1 & 1 & 5 & 1 \\ 0 & 1 & 11 & 2 \\ 0 & -2 & -7 & -1 \end{array}\right) \tag{9}$$

$$\tag{10}$$

Augmented matrix

$$\xrightarrow[R_3 \to R_3 + 2R_2]{R_1 \to R_1 - R_2} \left(\begin{array}{ccc|c} 1 & 0 & -6 & -1 \\ 0 & 1 & 11 & 2 \\ 0 & 0 & 15 & 3 \end{array}\right) R_3 \to \frac{1}{15} R_3 \tag{11}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -6 & -1 \\ 0 & 1 & 11 & 2 \\ 0 & 0 & 1 & \frac{1}{5} \end{array}\right) \xrightarrow[R_2 \to R_2 - 11R_3]{R_1 \to R_1 + 6R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{5} \\ 0 & 1 & 0 & \frac{-1}{5} \\ 0 & 0 & 1 & \frac{1}{5} \end{array}\right) \tag{12}$$

Therefore, the plane equation is:

$$\begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \mathbf{x} = 5 \tag{13}$$

General form of a point on the line can be written as:

$$\mathbf{x} = \begin{pmatrix} 2 + 3\kappa \\ -1 + 6\kappa \\ 2 + 2\kappa \end{pmatrix} \tag{14}$$

Point of intersection

Substituting (13) in (12), we get:

$$\begin{pmatrix} 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2+3\kappa \\ -1+6\kappa \\ 2+2\kappa \end{pmatrix} = 5 \tag{15}$$

$$\kappa = 0 \tag{16}$$

Hence the point of intersection of the line and the plane can be found by substituting (15) in (13):

$$\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \tag{17}$$

C Code

```c
#include <stdio.h>

int lpoint1[3] = {2, -1, 2};
int lpoint2[3] = {5, 3, 4};
int ppoint[3][4] = {{2, 0, 3, 1}, {1,1,5,1}, {3,2,4,1}};

int get_lpoint1(int index){
    return lpoint1[index];
}

int get_lpoint2(int index){
    return lpoint2[index];
}

int get_ppoint(int index1, int index2){
    return ppoint[index1][index2];
}
```

Python Code 1

```
import ctypes
import numpy as np
import sympy as sp

lib = ctypes.CDLL("./problem.so")

lpoint=[0, 0, 0]
lvec=[0, 0, 0]

for i in range(0,3):
    lpoint[i] = lib.get_lpoint1(i)

for i in range(0,3):
    lvec[i] = lib.get_lpoint1(i) - lib.get_lpoint2(i)
```

Python Code 1

```
print("Line equation is: ",lpoint,"+ k",lvec)

A = sp.Matrix([[2,0,3,1],
               [1,1,5,1],
               [3,2,4,1]])

rref, pivots = A.rref()

e = (rref[0,3],rref[1,3], rref[2,2])

print("Plane equation is: ", e[0],"x+",e[1],"y+",e[2],"/5z=",rref
    [0,0])
```

Python Code 2

```python
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D


a, b, c, d = 1, -1, 1, 5


xx, yy = np.meshgrid(np.linspace(-10, 10, 50), np.linspace(-10, 10, 50))


zz = (d - a*xx - b*yy) / c
```

Python Code 2

```
p0 = np.array([2, -1, 2])
v = np.array([3, 4, 2])

t = np.linspace(-5, 5, 50)
x_line = p0[0] + v[0]*t
y_line = p0[1] + v[1]*t
z_line = p0[2] + v[2]*t


fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')


ax.plot_surface(xx, yy, zz, alpha=0.5, color='cyan')
```

Python Code 2

```
ax.plot(x_line, y_line, z_line, color='red', linewidth=2)

ax.text(-12, -22, -8, r'$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z
    -2}{2}$', fontsize=12, color="black")
ax.text(-14,21,12, "x-y+z=5", fontsize=12, color="black")
ax.text(2.1, -0.9, 2.1, "(2,-1,2)", fontsize=12, color="black")

ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')

plt.savefig("../figs/plot.png")
plt.show()
```

Plot

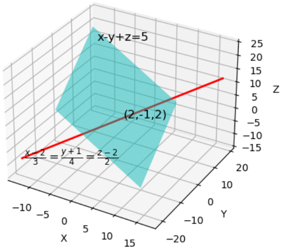

Figure: Plot of given plane and line