4.5.5

EE25BTECH11003 - Adharvan Kshathriya Bommagani

Question:

Find the vector equation of the line which is parallel to the vector $3\hat{i} - 2\hat{j} + 6\hat{k}$ and passes through the point $(1, -2, 3)$.

Solution:

The vector equation of a line is given by the formula:

$$\mathbf{x} = \mathbf{h} + t\mathbf{m}$$

where:

$\mathbf{x}$ is the position vector of any point on the line, represented as $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

$\mathbf{h}$ is the position vector of a known point on the line.
$\mathbf{m}$ is the direction vector of the line.
t is a scalar parameter.

The problem states that the line passes through the point $(1, -2, 3)$. The position vector for this point is:

$$\mathbf{h} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$$

The problem states that the line is parallel to the vector $3\hat{i} - 2\hat{j} + 6\hat{k}$. This is the direction vector of the line:

$$\mathbf{m} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$$

Now, substitute the vectors $\mathbf{h}$ and $\mathbf{m}$ into the general equation $\mathbf{x} = \mathbf{h} + t\mathbf{m}$. The vector equation of the line is:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$$

The vector equation of the line is:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$$

Plot of the Line:
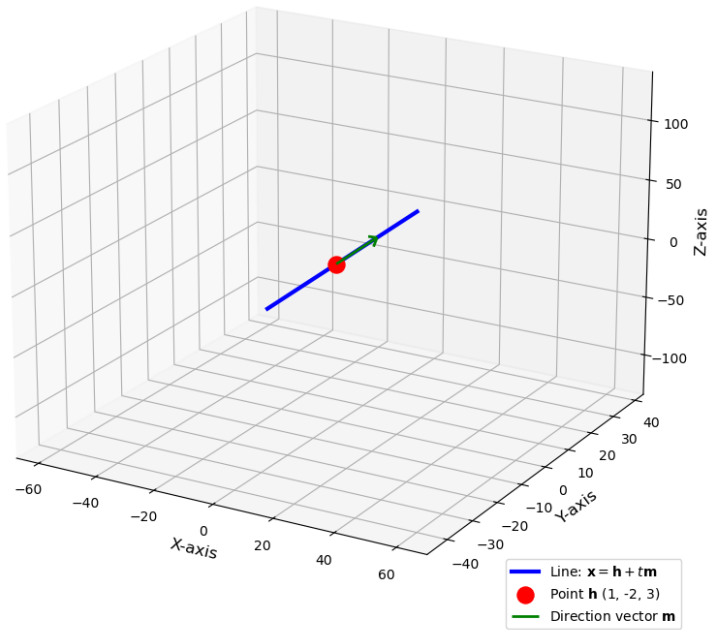
3D Plot of the Vector Equation of a Line (Extended)

Fig. 0: Figure for 4.5.5