

4.4.26

AI25BTECH11030 -Sarvesh Tamgade

Question: Find the equation of the median through vertex **A** of the triangle ABC , having vertices

$$\mathbf{A}(2, 5), \quad \mathbf{B}(-4, 9), \quad \mathbf{C}(-2, -1).$$

Solution: Using the section formula, the midpoint **M** of the side BC is

$$\mathbf{M} = \frac{\mathbf{B} + \mathbf{C}}{2} = \frac{1}{2} \begin{pmatrix} -4 & 9 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -2 & -1 \end{pmatrix} = \begin{pmatrix} -3 & 4 \end{pmatrix}.$$

The median passes through points $\mathbf{A}(2, 5)$ and $\mathbf{M}(-3, 4)$. Let the required line have the equation

$$\mathbf{n}^T \mathbf{x} = 1$$

where

$$\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

is the column vector (direction vector).

Since both points A and M lie on the median, they satisfy the line equation:

$$\mathbf{n}^T \mathbf{A} = 1, \quad \mathbf{n}^T \mathbf{M} = 1$$

or, explicitly,

$$\begin{pmatrix} 2 & 5 \\ -3 & 4 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

We want to find the vector \mathbf{n} satisfying

$$\begin{pmatrix} 2 & 5 \\ -3 & 4 \end{pmatrix} \mathbf{n} = \mathbf{c}.$$

Set up the augmented matrix with right-hand side 1:

$$\left(\begin{array}{cc|c} 2 & 5 & 1 \\ -3 & 4 & 1 \end{array} \right) \quad (0.1)$$

Perform row operation $R_2 \rightarrow R_2 + \frac{3}{2}R_1$:

$$\left(\begin{array}{cc|c} 2 & 5 & 1 \\ 0 & \frac{23}{2} & \frac{5}{2} \end{array} \right) \quad (0.2)$$

Perform row operation $R_1 \rightarrow R_1 - \frac{10}{23}R_2$:

$$\left(\begin{array}{cc|c} 2 & 0 & -\frac{2}{23} \\ 0 & \frac{23}{2} & \frac{5}{2} \end{array} \right) \quad (0.3)$$

The final augmented matrix is:

$$\left(\begin{array}{cc|c} 2 & 0 & -\frac{2}{23} \\ 0 & \frac{23}{2} & \frac{5}{2} \end{array} \right) \quad (0.4)$$

Solve the system:

$$2n_1 = -\frac{2}{23} \Rightarrow n_1 = -\frac{1}{23}$$

$$\frac{23}{2}n_2 = \frac{5}{2} \Rightarrow n_2 = \frac{5}{23}$$

$$\mathbf{n} = \frac{1}{23} \begin{pmatrix} -1 \\ 5 \end{pmatrix}$$

$$\mathbf{n}^\top \mathbf{x} = 1$$

Substitute \mathbf{n} :

$$\left(\frac{1}{23} \begin{pmatrix} -1 \\ 5 \end{pmatrix} \right)^\top \mathbf{x} = 1$$

$$\begin{pmatrix} -1 & 5 \end{pmatrix} \mathbf{x} = 23$$

or equivalently,

$$5y - x = 23.$$

Therefore, equation of required line is:

$5y - x = 23.$

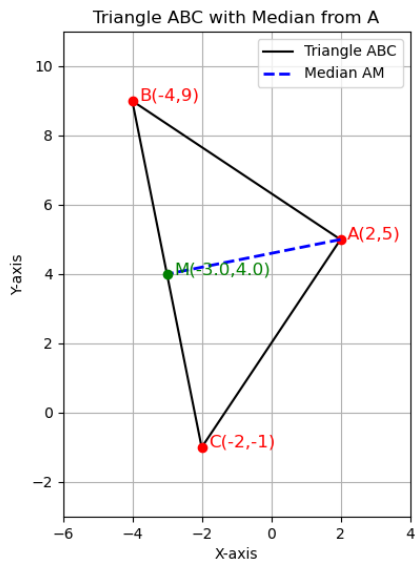


Fig. 0.1: Vector Representation