5.8.21

EE25BTECH11023 - Venkata Sai

Question:
The Sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number

Solution:
Let $\mathbf{x}$ be the matrix that contains the digits of the required number N

$$N = \begin{pmatrix}10 & 1\end{pmatrix}\mathbf{x} \tag{1}$$

Given Sum of the digits of a two-digit number is 9

$$\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x} = 9 \tag{2}$$

Nine times this number is twice the number obtained by reversing the order of the digits.

$$9\begin{pmatrix}10 & 1\end{pmatrix}\mathbf{x} = 2\begin{pmatrix}1 & 10\end{pmatrix}\mathbf{x} \tag{3}$$

$$\begin{pmatrix}90-2 & 9-20\end{pmatrix}\mathbf{x} = 0 \tag{4}$$

$$\begin{pmatrix}88 & -11\end{pmatrix}\mathbf{x} = 0 \tag{5}$$

$$11\begin{pmatrix}8 & -1\end{pmatrix}\mathbf{x} = 0 \implies \begin{pmatrix}8 & -1\end{pmatrix}\mathbf{x} = 0 \tag{6}$$

$$\begin{pmatrix}1 & 1\\ 8 & -1\end{pmatrix}\mathbf{x} = \begin{pmatrix}9\\ 0\end{pmatrix} \tag{7}$$

Augmented Matrix:

$$\left(\begin{array}{cc|c}1 & 1 & 9\\ 8 & -1 & 0\end{array}\right) \xrightarrow{R_2 \to R_2 - 8R_1} \left(\begin{array}{cc|c}1 & 1 & 9\\ 0 & -9 & -72\end{array}\right) \xrightarrow{R_1 \to R_1 + \frac{1}{9}R_2} \left(\begin{array}{cc|c}1 & 0 & 1\\ 0 & -1 & -8\end{array}\right) \tag{8}$$

$$\left(\begin{array}{cc|c}1 & 0 & 1\\ 0 & -1 & -8\end{array}\right) \xrightarrow{R_2 \to -R_2} \left(\begin{array}{cc|c}1 & 0 & 1\\ 0 & 1 & 8\end{array}\right) \tag{9}$$

$$\mathbf{x} = \begin{pmatrix}1\\ 8\end{pmatrix} \tag{10}$$

$$N = \begin{pmatrix}10 & 1\end{pmatrix}\begin{pmatrix}1\\ 8\end{pmatrix} = 10 + 8 = 18 \tag{11}$$

Hence the Required Number is 18