4.3.20

EE25BTECH11031 - Sai Sreevallabh

Question:

Find the area of the triangle $\triangle ABC$ bounded by the lines $4x - y + 5 = 0$, $x + y - 5 = 0$ and $x - 4y + 5 = 0$.

Solution:

Given lines can be written as:

$$\begin{pmatrix} 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -5 \tag{0.1}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 5 \tag{0.2}$$

$$\begin{pmatrix} 1 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -5 \tag{0.3}$$

Solving equations (0.1) and (0.2) to get the point of intersection $\mathbf{A}$:

$$\begin{pmatrix} 1 & 1 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \end{pmatrix} \tag{0.4}$$

Making the Augmented Matrix and converting to echelon form

$$\left(\begin{array}{cc|c} 1 & 1 & 5 \\ 4 & -1 & -5 \end{array}\right) \xleftrightarrow{R_2 \to R_2 - 4R_1} \left(\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & -5 & -25 \end{array}\right) \tag{0.5}$$

We get

$$\mathbf{A} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} \tag{0.6}$$

Solving equations (0.2) and (0.3) to get the point of intersection $\mathbf{B}$

$$\left(\begin{array}{cc|c} 1 & 1 & 5 \\ 1 & -4 & -5 \end{array}\right) \xleftrightarrow{R_2 \to R_2 - R_1} \left(\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & -5 & -10 \end{array}\right) \tag{0.7}$$

We get

$$\mathbf{B} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{0.8}$$

Solving equations (0.1) and (0.3) to get the point of intersection $\mathbf{C}$

$$\left(\begin{array}{cc|c} 1 & -4 & -5 \\ 4 & -1 & 5 \end{array}\right) \xleftrightarrow{R_2 \to R_2 - 4R_1} \left(\begin{array}{cc|c} 1 & -4 & -5 \\ 0 & 15 & 15 \end{array}\right) \tag{0.9}$$

We get

$$\mathbf{C} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{0.10}$$

The vertices of the triangle are

$$\mathbf{A} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}, \ \mathbf{B} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \ \mathbf{C} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{0.11}$$

Now

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ -3 \end{pmatrix} \text{ and } \mathbf{C} - \mathbf{A} = \begin{pmatrix} -1 \\ -4 \end{pmatrix} \tag{0.12}$$

Area of the triangle $\triangle ABC$ is given by

$$\frac{1}{2} \left\|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A})\right\| \tag{0.13}$$

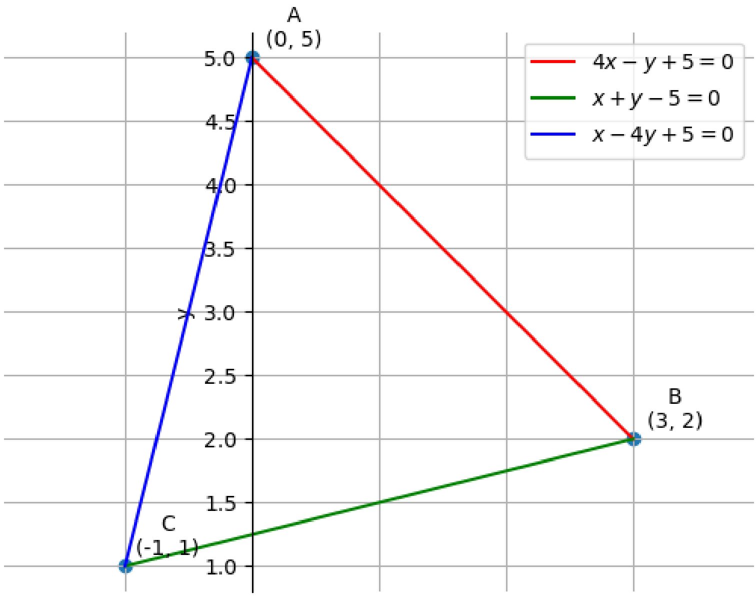
$$= \frac{1}{2} \left\| \begin{pmatrix} 3 \\ -3 \end{pmatrix} \times \begin{pmatrix} -1 \\ -4 \end{pmatrix} \right\| \tag{0.14}$$

$$= \frac{15}{2} \tag{0.15}$$

Hence,

$$ar\left(\triangle ABC\right) = \frac{15}{2} \tag{0.16}$$

$\therefore$ The area of the triangle formed by the given three lines is $\frac{15}{2}$ units.


A
(0, 5)
B
(3, 2)
C
(-1, 1)
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
y
$4x - y + 5 = 0$
$x + y - 5 = 0$
$x - 4y + 5 = 0$