

1.4.16

EE25BTECH11001 - Aarush Dilawri

August 29, 2025

Question

Find the coordinates of the points which trisect the line segment joining the points

$$\mathbf{P}(4, 2, -6) \quad \text{and} \quad \mathbf{Q}(10, -16, 6).$$

Let the vectors be

$$\mathbf{P} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}, \quad (1)$$

$$\mathbf{Q} = \begin{pmatrix} 10 \\ -16 \\ 6 \end{pmatrix}. \quad (2)$$

We want to find the points which divide PQ in the ratio $2 : 1$ and $1 : 2$.

Section formula: If a point divides the line joining **A** and **B** in the ratio $k : 1$, then

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1}.$$

First Trisection Point

Using section formula for ratio 2 : 1,

$$\mathbf{S} = \frac{2\mathbf{Q} + \mathbf{P}}{3} \quad (3)$$

$$= \frac{\begin{pmatrix} 20 \\ -32 \\ 12 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}}{3} \quad (4)$$

$$= \frac{\begin{pmatrix} 24 \\ -30 \\ 6 \end{pmatrix}}{3} \quad (5)$$

$$= \begin{pmatrix} 8 \\ -10 \\ 2 \end{pmatrix}. \quad (6)$$

Second Trisection Point

Using section formula for ratio 1 : 2,

$$\mathbf{R} = \frac{\mathbf{Q} + 2\mathbf{P}}{3} \quad (7)$$

$$= \frac{\begin{pmatrix} 10 \\ -16 \\ 6 \end{pmatrix} + \begin{pmatrix} 8 \\ 4 \\ -12 \end{pmatrix}}{3} \quad (8)$$

$$= \frac{\begin{pmatrix} 18 \\ -12 \\ -6 \end{pmatrix}}{3} \quad (9)$$

$$= \begin{pmatrix} 6 \\ -4 \\ -2 \end{pmatrix}. \quad (10)$$

Therefore, the points of trisection of PQ are

$$\mathbf{S} = \begin{pmatrix} 8 \\ -10 \\ 2 \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} 6 \\ -4 \\ -2 \end{pmatrix}.$$

Plot

