12.173

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Question : Consider the system

$$x + 10y = 5$$
$$y + 5z = 1$$
$$10x - y + z = 0$$

On applying **Gauss-Seidel** method , x correct up to 4 decimal places is
Solution :

Name	Value (normal form)
Equation 1	$x + 10y = 5$ $\begin{pmatrix} 1 & 10 & 0 \end{pmatrix} \mathbf{x} = 5$
Equation 2	y + 5z = 1 $\begin{pmatrix} 0 & 1 & 5 \end{pmatrix} \mathbf{x} = 1$
Equation 3	$10x - y + z = 0$ $\begin{pmatrix} 10 & -1 & 1 \end{pmatrix} \mathbf{x} = 0$

Table : Equations

Using **Gauss-Seidel** method

We reorder equations for diagonal dominance:

$$10x - y + z = 0 \tag{1}$$
$$x + 10y = 5 \tag{2}$$
$$y + 5z = 1 \tag{3}$$

$$\begin{pmatrix} 10 & -1 & 1 \\ 1 & 10 & 0 \\ 0 & 1 & 5 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ 5 \\ 1 \end{pmatrix} \tag{4}$$

Gauss–Seidel iteration formulas:

$$x^{(k+1)} = \frac{1}{10}(y^{(k)} - z^{(k)}) \tag{5}$$
$$y^{(k+1)} = \frac{1}{10}(5 - x^{(k+1)}) \tag{6}$$
$$z^{(k+1)} = \frac{1}{5}(1 - y^{(k+1)}) \tag{7}$$

Initial guess:

$$\mathbf{x}^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{8}$$

Iterations:

$$\mathbf{x}^{(1)} = \begin{pmatrix} 0 \\ 0.5 \\ 0.1 \end{pmatrix} \tag{9}$$

$$\mathbf{x}^{(2)} = \begin{pmatrix} 0.04 \\ 0.496 \\ 0.1008 \end{pmatrix} \tag{10}$$

$$\mathbf{x}^{(3)} = \begin{pmatrix} 0.03952 \\ 0.496048 \\ 0.1007904 \end{pmatrix} \tag{11}$$

$$\mathbf{x}^{(4)} = \begin{pmatrix} 0.03952576 \\ 0.49604742 \\ 0.10079052 \end{pmatrix} \tag{12}$$

$$\mathbf{x}^{(5)} = \begin{pmatrix} 0.03952569 \\ 0.49604743 \\ 0.10079051 \end{pmatrix} \tag{13}$$

Thus, the first component is

$$x \approx 0.03952569 \tag{14}$$

Correct to four decimal places:

$$x \approx 0.0395 \tag{15}$$

Answer: $x = 0.0395$

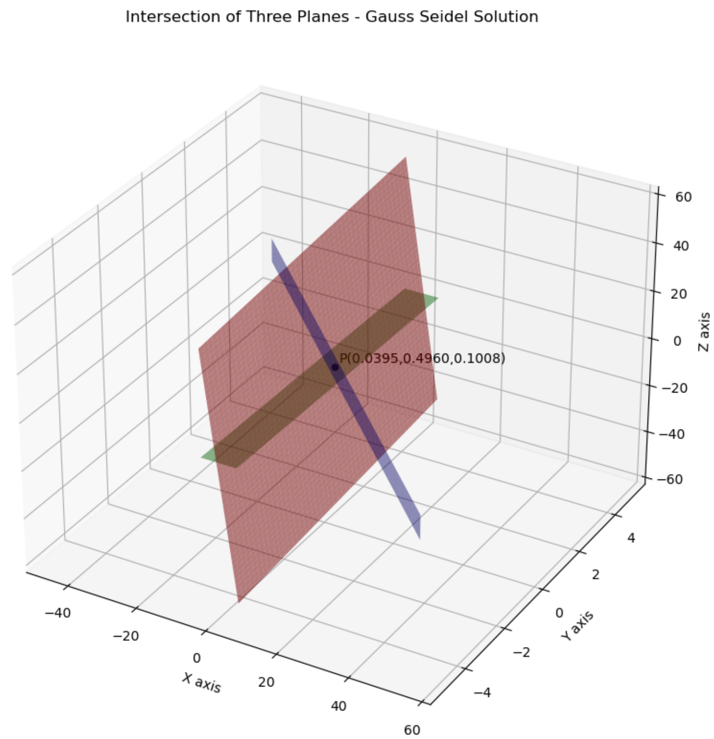
Intersection of Three Planes - Gauss Seidel Solution
P(0.0395,0.4960,0.1008)
X axis
Y axis
Z axis
−40
−20
0
20
40
60
−4
−2
0
2
4
−60
−40
−20
0
20
40
60

Fig : Planes