1.8.4

AI25BTECH110030 - SARVESH TAMGADE

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Question:
Find the coordinates of a point on Y axis which is at a distance of $5\sqrt{2}$ from the point $P(3, -2, 5)$.
Solution:
Let

$$\mathbf{P} \in \mathbb{R}^3, \quad \mathbf{Q} = y\,\mathbf{e}_2, \quad \text{where} \quad \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{1}$$

The required distance condition is

$$\|\mathbf{P} - \mathbf{Q}\| = d \tag{2}$$
$$\implies (\mathbf{P} - y\mathbf{e}_2)^T(\mathbf{P} - y\mathbf{e}_2) = d^2 \tag{3}$$

Expanding the quadratic form:

$$\mathbf{P}^T\mathbf{P} - 2y\,\mathbf{e}_2^T\mathbf{P} + y^2\mathbf{e}_2^T\mathbf{e}_2 = d^2 \tag{4}$$

Since $\mathbf{e}_2^T\mathbf{e}_2 = 1$, this leads to the quadratic equation in y:

$$y^2 - 2(\mathbf{e}_2^T\mathbf{P})y + \left(\mathbf{P}^T\mathbf{P} - d^2\right) = 0 \tag{5}$$

Applying the quadratic formula, the solution for y is:

$$y = \mathbf{e}_2^T\mathbf{P} \pm \sqrt{\left(\mathbf{e}_2^T\mathbf{P}\right)^2 - (\mathbf{P}^T\mathbf{P} - d^2)} \tag{6}$$

$$\mathbf{P} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix}, \quad d = 5\sqrt{2} \tag{7}$$

Calculate intermediate terms:

$$\mathbf{e}_2^T\mathbf{P} = -2 \tag{8}$$
$$\mathbf{P}^T\mathbf{P} = 3^2 + (-2)^2 + 5^2 = 38 \tag{9}$$
$$d^2 = (5\sqrt{2})^2 = 50 \tag{10}$$

Substitute into the general formula:

$$y = -2 \pm \sqrt{(-2)^2 - (38 - 50)} \tag{11}$$

$$= -2 \pm \sqrt{4 + 12} \tag{12}$$

$$= -2 \pm 4 \tag{13}$$

Solutions are:

$$y_1 = 2, \qquad y_2 = -6 \tag{14}$$

Answer:

Therefore, the required points on the Y-axis are:

$$\mathbf{Q}_1 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \qquad \mathbf{Q}_2 = \begin{pmatrix} 0 \\ -6 \\ 0 \end{pmatrix} \tag{15}$$
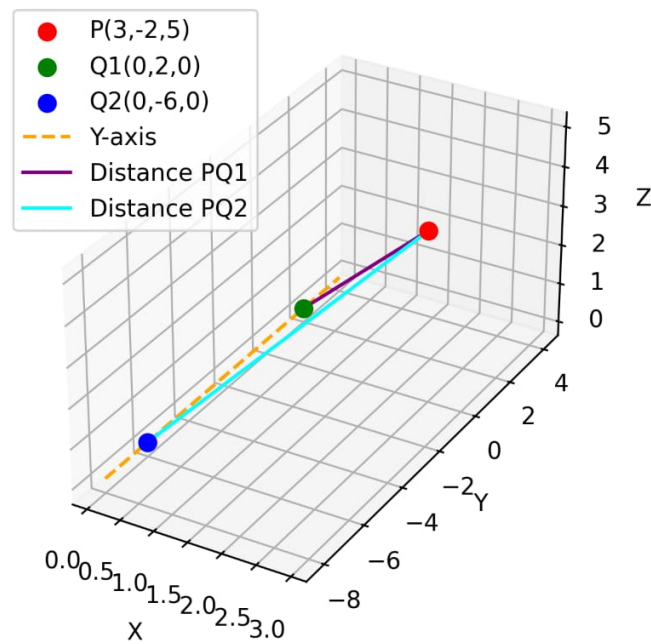
Graph:

Figure 1: 3D Visualization of Point P and Points on Y-axis Q1,Q2