

12.475

Puni Aditya - EE25BTECH11046

Question:

Consider a triangle PQR with initial coordinates of the vertices as P(1,3), Q(4,5) and R(5,3.5). The triangle is rotated in the X-Y plane about the vertex P by angle θ in clockwise direction. If $\sin \theta = 0.6$ and $\cos \theta = 0.8$, the new coordinates of the vertex Q are

- 1) (4.6, 2.8)
- 2) (3.2, 4.6)
- 3) (7.9, 5.5)
- 4) (5.5, 7.9)

Solution:

Let the coordinates of the vertices be represented by vectors $\mathbf{p} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$. The rotation of a point \mathbf{q} about a pivot point \mathbf{p} is given by:

$$\mathbf{q}_{\text{new}} = \mathbf{R}(\mathbf{q} - \mathbf{p}) + \mathbf{p} \quad (1)$$

The matrix for a clockwise rotation by an angle θ is:

$$\mathbf{R} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \quad (2)$$

Substituting the given values and vectors into (1)

$$\mathbf{q}_{\text{new}} = \begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix} \left(\begin{pmatrix} 4 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right) + \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (3)$$

$$= \begin{pmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (4)$$

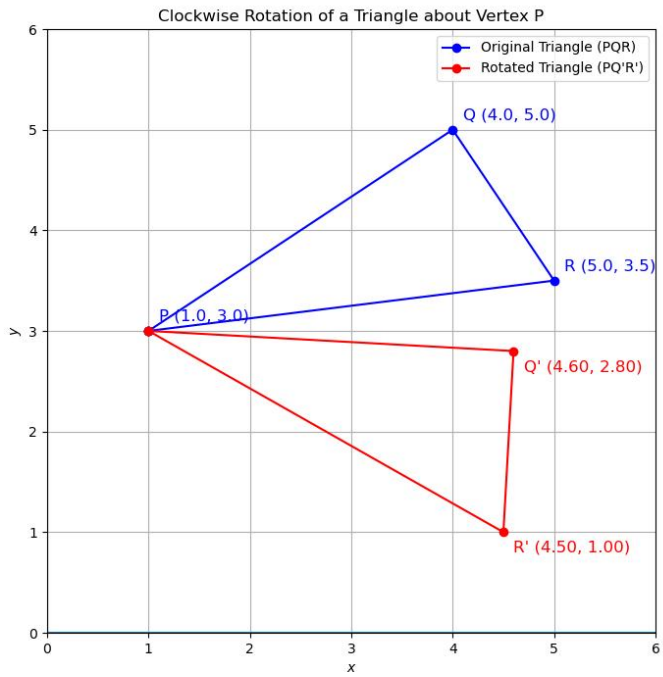
$$= \begin{pmatrix} 0.8(3) + 0.6(2) \\ -0.6(3) + 0.8(2) \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (5)$$

$$= \begin{pmatrix} 2.4 + 1.2 \\ -1.8 + 1.6 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} \quad (6)$$

$$= \begin{pmatrix} 3.6 \\ -0.2 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 4.6 \\ 2.8 \end{pmatrix} \quad (7)$$

The new coordinates of the vertex Q are $\begin{pmatrix} 4.6 \\ 2.8 \end{pmatrix}$.

The correct option is **1**).



Plot