2.4.29

EE25BTECH11042 - Nipun Dasari

Question:
The points $\mathbf{A}\,(2,9)$, $\mathbf{B}\,(a,5)$ and $\mathbf{C}\,(5,5)$ are the vertices of a triangle **ABC** right angled at **B**. Find the values of a and hence the area of Δ**ABC**.
Solution:

Given the points A, B and C, also consider **c** to be vector opposite to side AB and **b**, **a** similarly

$$\mathbf{A} = \begin{pmatrix} 2 \\ 9 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} a \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 5 \end{pmatrix} \tag{0.1}$$

Since the sides c and a are perpendicular their inner product will be 0
Take the inner product of **c** and **a**
Vector **c**:

$$\mathbf{c} = \mathbf{A} - \mathbf{B} = \begin{pmatrix} 2-a \\ 9-5 \end{pmatrix} = \begin{pmatrix} 2-a \\ 4 \end{pmatrix} \tag{0.2}$$

Vector **a**:

$$\mathbf{a} = \mathbf{B} - \mathbf{C} = \begin{pmatrix} a-5 \\ 5-5 \end{pmatrix} = \begin{pmatrix} a-5 \\ 0 \end{pmatrix} \tag{0.3}$$

Orthogonality $\implies$ matrix product is zero :

$$\mathbf{c}^T\mathbf{a} = \begin{pmatrix} 2-a & 4 \end{pmatrix} \begin{pmatrix} a-5 \\ 0 \end{pmatrix} = (2-a)(a-5) = 0 \tag{0.4}$$

So $(2-a)(5-a) = 0 \implies a = 2$ or $a = 5$.
$a = 5$ make **B**=**C**. $\therefore a = 2$
We can compute area using general formula since the vectors are perpendicular

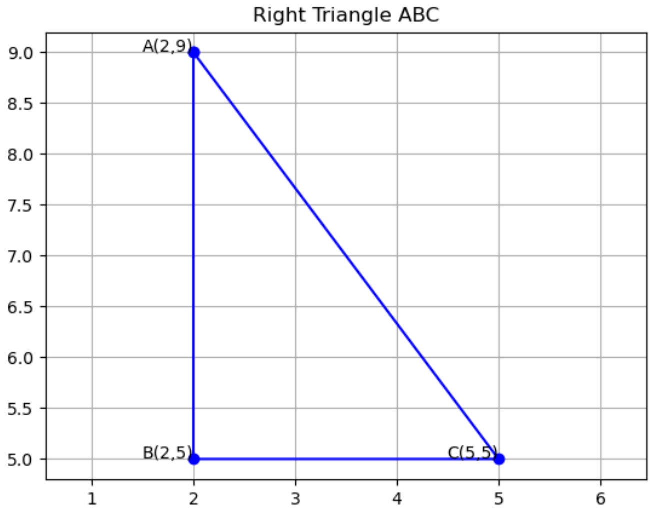
$$\text{AREA} = \frac{1}{2} \times \text{base} \times \text{height} \tag{0.5}$$

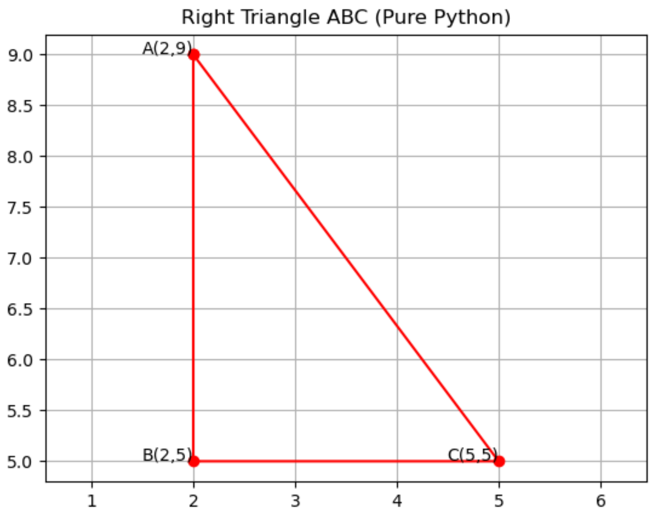
Using (0.5)

$$\Delta = \frac{1}{2} \times \|\mathbf{AB}\| \times \|\mathbf{BC}\| \tag{0.6}$$

$$\therefore \Delta = \frac{1}{2} \times 4 \times 3 = 6 \tag{0.7}$$

Thus area of triangle is 6


Right Triangle ABC
A(2,9)
B(2,5)
C(5,5)
9.0
8.5
8.0
7.5
7.0
6.5
6.0
5.5
5.0
1
2
3
4
5
6


Right Triangle ABC (Pure Python)
A(2,9)
B(2,5)
C(5,5)
9.0
8.5
8.0
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7.0
6.5
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5.5
5.0
1
2
3
4
5
6