AI25BTECH11025-R Nikhil

Question 1.3.4

If A(1,3), B(4,2), C(x,5), and D(x,4) are the vertices of a parallelogram ABCD, then the value of x is ______. (10, 2012)

Solution:

In a parallelogram, opposite sides are equal and parallel. Since ABCD is a parallelogram, vectors AB and CD must be equal.

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = (4 - 1, 2 - 3) = (3, -1) \tag{0.1}$$

$$\mathbf{CD} = \mathbf{D} - \mathbf{C} = (x - x, 4 - 5) = (0, -1)$$
(0.2)

Clearly, $AB \neq CD$, so let's try using diagonals. In a parallelogram, the diagonals bisect each other. Midpoint of diagonal AC:

$$\mathbf{M}_{\mathbf{AC}} = \begin{pmatrix} \frac{1+x}{2} \\ \frac{3+5}{2} \end{pmatrix} = \begin{pmatrix} \frac{1+x}{2} \\ 4 \end{pmatrix} \tag{0.3}$$

Midpoint of diagonal BD:

$$\mathbf{M_{BD}} = \begin{pmatrix} \frac{4+x}{2} \\ \frac{2+4}{2} \end{pmatrix} = \begin{pmatrix} \frac{4+x}{2} \\ 3 \end{pmatrix} \tag{0.4}$$

Equating midpoints:

$$\frac{1+x}{2} = \frac{4+x}{2} \quad \text{and} \quad 4 = 3 \tag{0.5}$$

The second equation is false, so diagonals do not bisect each other. Let's try using opposite sides again, but this time equating **AD** and **BC**:

$$\mathbf{AD} = \mathbf{D} - \mathbf{A} = \begin{pmatrix} x - 1 \\ 4 - 3 \end{pmatrix} = \begin{pmatrix} x - 1 \\ 1 \end{pmatrix} \tag{0.6}$$

$$\mathbf{BC} = \mathbf{C} - \mathbf{B} = \begin{pmatrix} x - 4 \\ 5 - 2 \end{pmatrix} = \begin{pmatrix} x - 4 \\ 3 \end{pmatrix} \tag{0.7}$$

Equating vectors:

$$x - 1 = x - 4$$
 and $1 = 3$ (0.8)

Again, contradiction. So let's try using the property that opposite sides are equal in length.

Length of **AD**:
$$|AD| = \sqrt{(x-1)^2 + (4-3)^2} = \sqrt{(x-1)^2 + 1}$$
 (0.9)

Length of **BC**:
$$|BC| = \sqrt{(x-4)^2 + (5-2)^2} = \sqrt{(x-4)^2 + 9}$$
 (0.10)

Equating lengths:

$$\sqrt{(x-1)^2 + 1} = \sqrt{(x-4)^2 + 9} \tag{0.11}$$

Squaring both sides:

$$(x-1)^2 + 1 = (x-4)^2 + 9 (0.12)$$

$$x^2 - 2x + 1 + 1 = x^2 - 8x + 16 + 9 ag{0.13}$$

$$x^2 - 2x + 2 = x^2 - 8x + 25 ag{0.14}$$

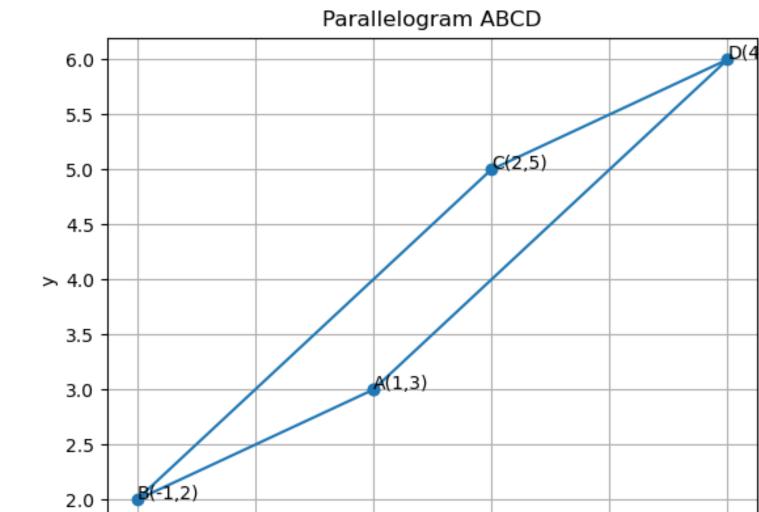
Subtract x^2 from both sides:

$$-2x + 2 = -8x + 25 \tag{0.15}$$

$$6x = 23 \Rightarrow x = \frac{23}{6} \tag{0.16}$$

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Answer: $\frac{23}{6}$



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