5.13.10

AI25BTECH11024 - Pratyush Panda

Question:
if

$$P = \begin{pmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{pmatrix} \tag{0.1}$$

is the adjoint of a 3×3 matrix $\mathbf{A}$ and $|\mathbf{A}| = 4$, then α is equal to

1) 4
2) 11
3) 5
4) 0

Solution:
Given

$$P = \begin{pmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{pmatrix} \tag{4.1}$$

is the adjoint of a 3×3 matrix $\mathbf{A}$ and $|A| = 4$.

We know that,

$$adj(\mathbf{A}) = |\mathbf{A}|\mathbf{A}^{-1}. \tag{4.2}$$

Hence,

$$\mathbf{P} = 4\mathbf{A}^{-1} \quad \Rightarrow \quad \mathbf{A} = 4\mathbf{P}^{-1}. \tag{4.3}$$

Taking determinants on both sides,

$$|\mathbf{A}| = |4\mathbf{P}^{-1}| = 4^3|\mathbf{P}^{-1}| = 64 \cdot \frac{1}{|\mathbf{P}|}. \tag{4.4}$$

Since $|\mathbf{A}| = 4$,

$$\frac{64}{\det(P)} = 4 \quad \Rightarrow \quad |\mathbf{P}| = 16. \tag{4.5}$$

Now compute $|\mathbf{P}|$:

$$|\mathbf{P}| = \begin{pmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{pmatrix} \tag{4.6}$$

Simplifying,

$$|\mathbf{P}| = 1(12-12) - \alpha(4-6) + 3(4-6) \tag{4.7}$$

$$|\mathbf{P}| = 0 + 2\alpha - 6 = 2(\alpha - 3). \tag{4.8}$$

Equating this with $|\mathbf{P}| = 16$,

$$2(\alpha - 3) = 16 \quad \Rightarrow \quad \alpha - 3 = 8 \quad \Rightarrow \alpha = 11. \tag{4.9}$$