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Matrix 5.3.9

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Question (5.3.9)

If

$$M = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = P + Q$$

is a decomposition into a symmetric matrix P and a skew-symmetric matrix Q , then find Q .

General Formula

For any square matrix M ,

$$P = \frac{M + M^T}{2}, \quad Q = \frac{M - M^T}{2}.$$

Step 1: Transpose of M

$$M = \begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix}, \quad M^T = \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix}.$$

Step 2: Skew-Symmetric Part

$$Q = \frac{1}{2}(M - M^T) = \frac{1}{2} \left(\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix} \right)$$

$$Q = \frac{1}{2} \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}.$$

$$Q = \begin{bmatrix} 0 & -\frac{5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}.$$