

# 4.8.17

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## QUESTION

The foot of a perpendicular drawn from the point  $(-2, -1, -3)$  on a plane is  $(1, -3, 3)$ . Find the equation of the plane.

## ANSWER

### *Step 1: Understanding the problem*

Given:

$$\mathbf{P} = \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix}$$

where  $\mathbf{P}$  is the point from which the perpendicular is dropped and  $\mathbf{F}$  is the foot of the perpendicular on the plane.

### *Step 2: Vector along the perpendicular*

The vector along the perpendicular from  $\mathbf{P}$  to  $\mathbf{F}$  is:

$$\mathbf{PF} = \mathbf{F} - \mathbf{P} = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$$

### *Step 3: Normal vector to the plane*

Since the perpendicular from  $\mathbf{P}$  to the plane hits the foot  $\mathbf{F}$ , the vector  $\mathbf{PF}$  is normal to the plane. Therefore, the normal vector to the plane  $\mathbf{n}$  is:

$$\mathbf{n} = \mathbf{PF} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$$

### *Step 4: Equation of the plane in vector form*

The vector form of the plane equation is:

$$\mathbf{n}^T(\mathbf{r} - \mathbf{F}) = 0$$

where  $\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$  is any point on the plane.

**Step 5:** Substitute values and expand

Substituting **n** and **F**:

$$\begin{pmatrix} 3 & -2 & 6 \end{pmatrix} \left( \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} \right) = 0$$

$$\Rightarrow \begin{pmatrix} 3 & -2 & 6 \end{pmatrix} \begin{pmatrix} x-1 \\ y+3 \\ z-3 \end{pmatrix} = 0$$

$$3(x-1) - 2(y+3) + 6(z-3) = 0$$

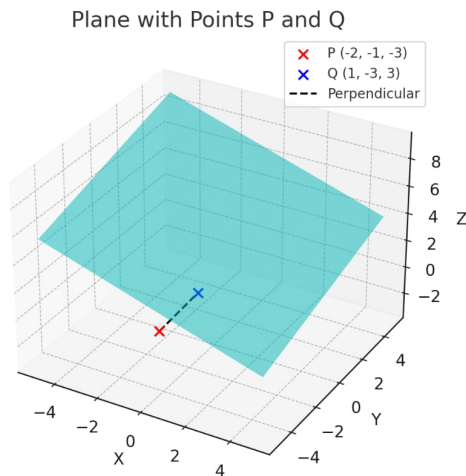
**Step 6:** Simplify the equation

$$3x - 3 - 2y - 6 + 6z - 18 = 0$$

$$3x - 2y + 6z - 27 = 0$$

**Final answer:**

$$3x - 2y + 6z = 27$$



Plot of the points and plane with perpendicular.  
fig:1