2.10.48

EE25BTECH11019 - Darji Vivek M.

Question:
If $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$, $\mathbf{a} \cdot \mathbf{b} = 1$ and $\mathbf{a} \times \mathbf{b} = \mathbf{j} - \mathbf{k}$, then $\mathbf{b}$ is

(a) $\mathbf{i} - \mathbf{j} + \mathbf{k}$
(b) $2\mathbf{j} - \mathbf{k}$
(c) $\mathbf{i}$
(d) $2\mathbf{i}$

Solution:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{1}$$

Dot product condition:

$$\mathbf{a} \cdot \mathbf{b} = x + y + z = 1 \tag{2}$$

Cross product:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} \tag{3}$$

$$= (z - y)\mathbf{i} + (x - z)\mathbf{j} + (y - x)\mathbf{k} \tag{4}$$

Comparing with $\mathbf{j} - \mathbf{k}$:

$$z - y = 0 \implies z = y \tag{5}$$

$$x - z = 1 \implies x = z + 1 \tag{6}$$

$$y - x = -1 \implies y = x - 1 \tag{7}$$

Hence $z = y = x - 1$. Substituting in dot product:

$$x + y + z = x + (x - 1) + (x - 1) = 3x - 2 = 1 \tag{8}$$

$$\implies x = 1, \; y = z = 0 \tag{9}$$

Thus,

$$\mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \mathbf{i} \tag{10}$$

Hence the answer is option (c).

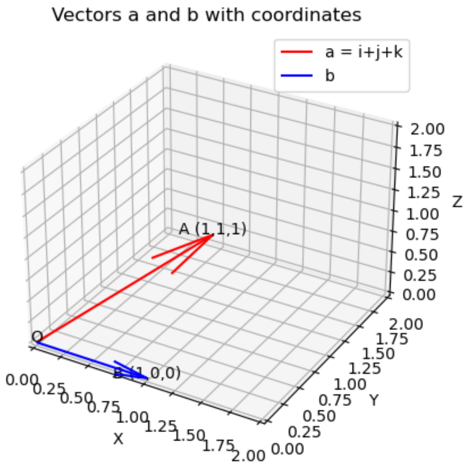
Vectors a and b with coordinates
a = i+j+k
b
A (1,1,1)
O
B (1,0,0)
X
Y
Z
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

Fig. 4.1: plot