Question 5.13.26

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Question:

If $\mathbf{A}$ and $\mathbf{B}$ are square matrices of size $n \times n$ such that $\mathbf{A}^2 - \mathbf{B}^2 = (\mathbf{A} - \mathbf{B})(\mathbf{A} + \mathbf{B})$, then which of the following will be always true?

(a) $\mathbf{A} = \mathbf{B}$

(b) $\mathbf{AB} = \mathbf{BA}$

(c) either of $\mathbf{A}$ or $\mathbf{B}$ is a zero matrix

(d) either $\mathbf{A}$ or $\mathbf{B}$ is an identity matrix

Solution:

We know that for any two square matrices $\mathbf{A}$ and $\mathbf{B}$ of size $n \times n$, the following is true:

$$(\mathbf{A} - \mathbf{B})(\mathbf{A} + \mathbf{B}) = (\mathbf{A} - \mathbf{B})\mathbf{A} + (\mathbf{A} - \mathbf{B})\mathbf{B} \tag{1}$$

$$= \mathbf{A}^2 - \mathbf{BA} + \mathbf{AB} - \mathbf{B}^2 \tag{2}$$

Given that $\mathbf{A}^2 - \mathbf{B}^2 = (\mathbf{A} - \mathbf{B})(\mathbf{A} + \mathbf{B})$, we can use equation 2:

$$\mathbf{A}^2 - \mathbf{B}^2 = \mathbf{A}^2 - \mathbf{BA} + \mathbf{AB} - \mathbf{B}^2 \tag{3}$$

$$\implies 0 = -\mathbf{BA} + \mathbf{AB} \tag{4}$$

$$\implies \mathbf{BA} = \mathbf{AB} \tag{5}$$

Thus, option (b) is always true.