4.13.7

EE25BTECH11002 - Achat Parth Kalpesh

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Question

If a, b and c are in A.P, then the straight line $ax + by + c = 0$ will always pass through a fixed point whose coordinates are _____.

Theoretical Solution

Let the equation $ax + by + c = 0$ be represented as

$$\mathbf{n}^\top \mathbf{x} = -c \tag{1}$$

$$\mathbf{n} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{2}$$

Let $\mathbf{p}$ be the fixed point.
The condition that a, b, c are in arithmetic progression is

$$2b = a + c \quad \Longrightarrow \quad a - 2b = -c, \tag{3}$$

Theoretical Solution

Thereby,

$$\begin{pmatrix} a \\ b \end{pmatrix}^{\top} \begin{pmatrix} 1 \\ -2 \end{pmatrix} = -c \tag{4}$$

Comparing it with (1) we get

$$\mathbf{p} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \tag{5}$$

C code

```c
#include <stdio.h>
void line_coefficients(int a, int d, int coeffs[3]) {
    coeffs[0] = a;
    coeffs[1] = a + d;
    coeffs[2] = a + 2*d;
}
```

Python Code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load shared library
mylib = ctypes.CDLL("./mylib.so")

# Define argument and return types
mylib.line_coefficients.argtypes = [ctypes.c_int, ctypes.c_int,
    ctypes.POINTER(ctypes.c_int)]
mylib.line_coefficients.restype = None

# Wrapper function to call C function
def get_line_coeffs(a, d):
    coeffs = (ctypes.c_int * 3)()
    mylib.line_coefficients(a, d, coeffs)
    return coeffs[0], coeffs[1], coeffs[2]
```

Python Code

```
# Fixed point
fixed_point = (1, -2)

# Define (a, d) pairs
lines = [(1, -1), (2, -1), (1, 0)]
colors = ['blue', 'green', 'purple']

x = np.linspace(-5, 5, 400)

plt.figure(figsize=(10,5))

for i, (a, d) in enumerate(lines):
    A, B, C = get_line_coeffs(a, d)
       if B != 0:
        y = (-A*x - C)/B
        plt.plot(x, y, color=colors[i], linewidth=1.5)
        if i == 1:
```

Python Code

```
            plt.text(-1, 4, f'2x + y = 0', fontsize=10, color=
                colors[i])
            plt.plot(x, y, color=colors[i], linewidth=1.5,
            label=f"(a={a}, d={d})") # <-- add label
        elif i == 2:
            plt.text(-4, 4, f'x + y = -1', fontsize=10, color=
                colors[i])
            plt.plot(x, y, color=colors[i], linewidth=1.5,
            label=f"(a={a}, d={d})") # <-- add label
    else:
        x_vert = -C / A
        plt.plot([x_vert]*len(x), x, color=colors[i], linewidth
            =1.5)
        plt.text(x_vert+0.2, 4.5, f'x = {x_vert}', fontsize=10,
            color=colors[i])
        plt.plot([x_vert]*len(x), x, color=colors[i], linewidth
            =1.5,
         label=f"(a={a}, d={d})")
```

Python Code

```
# Plot fixed point
plt.plot(fixed_point[0], fixed_point[1], 'ro', markersize=6)
plt.text(fixed_point[0]+0.5, fixed_point[1]-0.3, '(1, -2)',
    fontsize=10, color='red')

# Axes settings
plt.xlim(-5, 5)
plt.ylim(-5, 5)
plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
plt.title("Lines passing through fixed point (1, -2) for
    different values of a and common difference d")
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.legend(loc="upper right")
plt.grid(True)
plt.show()
```

Plot

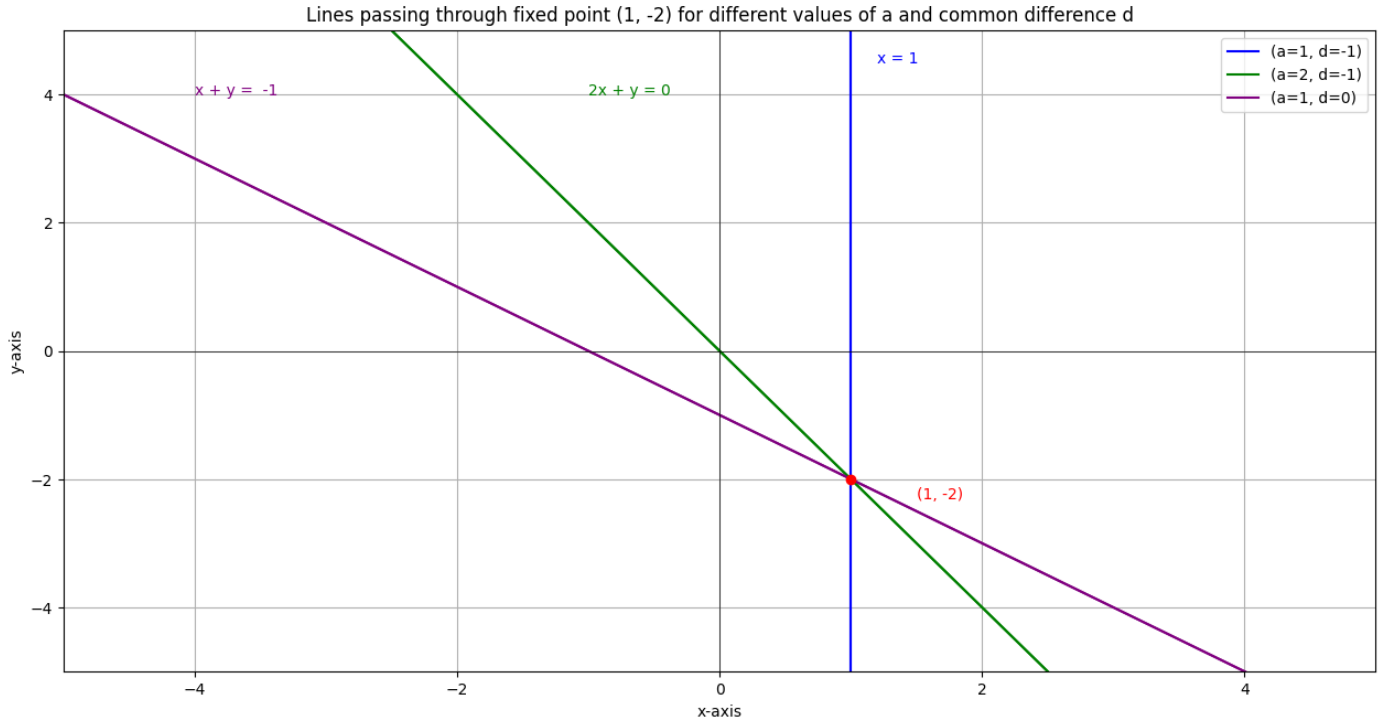

Figure: Line passing through a fixed point