2.3.8

EE25BTECH11005 - Aditya Mishra

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Question

If $\mathbf{A} = \hat{i} + \hat{j} + \hat{k},\ \mathbf{B} = 2\hat{i} + 5\hat{j},\ \mathbf{C} = 3\hat{i} + 2\hat{j} - 3\hat{k},\ \mathbf{D} = \hat{i} - 6\hat{j} - \hat{k}$ are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether $(\mathbf{B} - \mathbf{A})$ and $(\mathbf{D} - \mathbf{C})$ are collinear or not.

Solution

The direction vectors are

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} 1 \\ -6 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix} \tag{2}$$

The angle θ between $\mathbf{B} - \mathbf{A}$ and $\mathbf{D} - \mathbf{C}$ is

$$\cos\theta = \frac{(\mathbf{B} - \mathbf{A})^{\top}(\mathbf{D} - \mathbf{C})}{\|\mathbf{B} - \mathbf{A}\| \, \|\mathbf{D} - \mathbf{C}\|} \tag{3}$$

Substitute:

$$(\mathbf{B} - \mathbf{A})^{\top}(\mathbf{D} - \mathbf{C}) = \begin{pmatrix} 1 & 4 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix} \tag{4}$$

$$= -2 - 32 - 2 = -36 \tag{5}$$

Magnitudes:

$$\|\mathbf{B}-\mathbf{A}\| = \sqrt{1^2+4^2+(-1)^2} = \sqrt{18} \tag{6}$$

$$\|\mathbf{D}-\mathbf{C}\| = \sqrt{(-2)^2+(-8)^2+2^2} = \sqrt{72} \tag{7}$$

Thus,

$$\cos\theta = \frac{-36}{\sqrt{18}\sqrt{72}} = \frac{-36}{36} = -1 \tag{8}$$

$$\theta = \cos^{-1}(-1) = \pi \text{ radians} = 180^\circ \tag{9}$$

Collinearity using Rank

For collinearity, consider the matrix

$$M = \begin{pmatrix} 1 & 4 & -1 \\ -2 & -8 & 2 \end{pmatrix} \tag{10}$$

If rank$(M) = 1$, then the two vectors are linearly dependent (collinear).
Performing row reduction:

$$M = \begin{pmatrix} 1 & 4 & -1 \\ -2 & -8 & 2 \end{pmatrix} \tag{11}$$

$$R_2 \to R_2 + 2R_1 \implies \begin{pmatrix} 1 & 4 & -1 \\ 0 & 0 & 0 \end{pmatrix} \tag{12}$$

Hence,

$$\text{rank}(M) = 1 \tag{13}$$

Therefore, $(\mathbf{B}-\mathbf{A})$ and $(\mathbf{D}-\mathbf{C})$ are collinear. Since $\theta = 180^\circ$, they are anti-parallel.

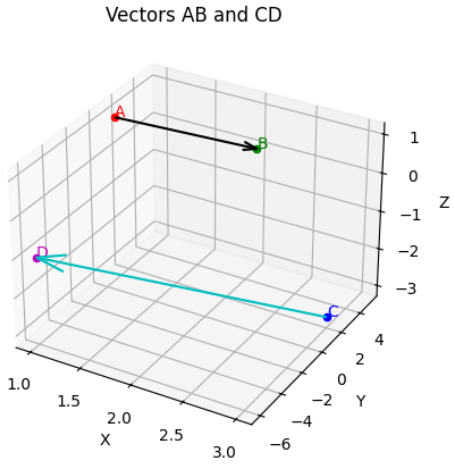

Figure 1: Line directions