

5.13.71

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# Question

If  $\mathbf{M}$  is a  $3 \times 3$  matrix, where

$$|\mathbf{M}| = 1 \text{ and } \mathbf{M}\mathbf{M}^T = \mathbf{I}$$

. where  $\mathbf{I}$  is an identity matrix, prove that

$$|\mathbf{M} - \mathbf{I}| = 0$$

# Theoretical Solution

$$\mathbf{M}\mathbf{M}^T = \mathbf{I}$$

$$|\mathbf{M}| = 1$$

$$|\mathbf{M} - \mathbf{I}| = |\mathbf{M} - \mathbf{M}\mathbf{M}^T| \quad (1)$$

$$= |\mathbf{M}(\mathbf{I} - \mathbf{M}^T)| \quad (2)$$

$$= |\mathbf{M}| |\mathbf{I} - \mathbf{M}^T| \quad (3)$$

$$= 1 \cdot |(\mathbf{I} - \mathbf{M})^T| \quad (4)$$

$$= |\mathbf{I} - \mathbf{M}| \quad (5)$$

$$= |-(\mathbf{M} - \mathbf{I})| \quad (6)$$

# Theoretical Solution

$$|\mathbf{M} - \mathbf{I}| = (-1)^3 |\mathbf{M} - \mathbf{I}| \quad (7)$$

$$= -|\mathbf{M} - \mathbf{I}| \quad (8)$$

$$|\mathbf{M} - \mathbf{I}| = -|\mathbf{M} - \mathbf{I}| \quad (9)$$

$$2|\mathbf{M} - \mathbf{I}| = 0 \quad (10)$$

$$|\mathbf{M} - \mathbf{I}| = 0 \quad (11)$$