1.5.36

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Question

The point P (x, 4) lies in the line segment that joins the points A (5, 8) and B (4, 10). Find the ratio in which point P divides the line segment AB. Also, find the value of x

Theoretical Solution

Let

$$\mathbf{A} = \begin{pmatrix} -5 \\ 8 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ -10 \end{pmatrix}$$

Let $\mathbf{P} = \lambda\mathbf{A} + \mu\mathbf{B}$ with $\lambda + \mu = 1$. Using the y-coordinates:

$$\begin{pmatrix} 8 & -10 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \tag{1}$$

Hence the internal division ratio

$$AP : PB = \mu : \lambda = 2 : 7 \tag{2}$$

and

$$x = \lambda(-5) + \mu(4) = -\frac{35}{9} + \frac{8}{9} = -3 \tag{3}$$

So, $\mathbf{P} = (-3, 4)$

Solution by Laplace Transform

$$\frac{dy}{dx} = x + y - 5 \tag{4}$$

Applying the Laplace transform to both sides:

$$\mathcal{L}\left\{\frac{dy}{dx}\right\} = \mathcal{L}\left\{x + (y - 5)\right\} \tag{5}$$

$$sY(s) - y(0) = \frac{1}{s^2} + Y(s) - \frac{5}{s} \tag{6}$$

$$Y(s) = \frac{\frac{1}{s^2} - \frac{5}{s} + y(0)}{s - 1} \tag{7}$$

$$Y(s) = \frac{1}{s^2(s-1)} - \frac{1}{s(s-1)} \cdot 5 + \frac{y(0)}{s-1} \tag{8}$$

Solution by Laplace Transform

Inverse Laplace Transform of Each Term

$$\frac{1}{s^2(s-1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-1} \tag{9}$$

$$1 = A\ s\ (s-1) + B\ (s-1) + C\ s^2 \tag{10}$$

we get

$$\frac{1}{s^2(s-1)} = -\frac{1}{s} - \frac{1}{s^2} + \frac{1}{s-1} \tag{11}$$

Taking the inverse Laplace transform

$$\mathcal{L}^{-1}\left\{-\frac{1}{s} - \frac{1}{s^2} + \frac{1}{s-1}\right\} = -1 - x + e^x \tag{12}$$

Solution by Laplace Transform

Inverse Laplace of $\frac{5}{s(s-1)}$

$$\frac{5}{s(s-1)} = \frac{A}{s} + \frac{B}{s-1} \tag{13}$$

$$5 = A(s-1) + Bs \tag{14}$$

Solving gives $A = 5$ and $B = -5$

$$\frac{5}{s(s-1)} = \frac{5}{s} - \frac{5}{s-1} \tag{15}$$

Taking the inverse Laplace transform

$$\mathcal{L}^{-1}\left\{\frac{5}{s} - \frac{5}{s-1}\right\} = 5 - 5e^{x} \tag{16}$$

Solution by Laplace Transform

Inverse Laplace of $\frac{y(0)}{s-1}$

$$\mathcal{L}^{-1}\left\{\frac{y(0)}{s-1}\right\} = y(0)e^{x} \tag{17}$$

combining the results from all parts, we have the solution for $y(x)$ The general solution too this differential equation is

$$y(x) = 4 - x + (y(0) - 4)e^{x} \tag{18}$$

$$y(x) = 4 - x + ce^{x} \tag{19}$$

To find c, put x=0 and y=2 in **??**

$$c = -2 \tag{20}$$

The curve is

$$x + y = 4 - 2e^{x} \tag{21}$$

Verification

Now lets verify the solution computationally from the definition of $\frac{dy}{dx}$

$$y_{n+1} = y_n + \frac{dy}{dx} \cdot h \tag{22}$$

From the differential equation given,

$$\frac{dy}{dx} = x + y - 5 \tag{23}$$

Substituting **??** in **??**

$$y_{n+1} = y_n + (x_n + y_n - 5) \cdot h \tag{24}$$

C Code - Eulers Method

```c
#include <stdio.h>
#include <stdio.h>
#include <math.h>
#include <stdlib.h>
// Function to calculate dy/dx for the differential equation
float dy_dx(float x, float y) {
    return x + y - 5; // Differential equation dy/dx = x + y - 5
}
// Function to calculate points using Euler's method
void points(float x_0, float y_0, float x_end, float h, float *
    x_points, float *y_points, int steps) {
    float x_n = x_0;
    float y_n = y_0;

    for (int i = 0; i < steps; i++) {
        x_points[i] = x_n; // Store current x value
        y_points[i] = y_n; // Store current y value
}
```

```
        // Calculate the next y using Euler's method
        y_n = y_n + h * dy_dx(x_n, y_n);
        x_n = x_n + h; // Move to the next x value
    }
    // Main function
int main() {
    float x_0 = 0.0; // Initial condition for x
    float y_0 = 2.0; // Initial condition for y
    float x_end = 1.0; // Final value of x
    float step_size = 0.001; // Step size for Euler's method
    int steps = (int)((x_end - x_0) / step_size) + 1;
    // Allocate memory for arrays to store points
    float *x_points = (float *)malloc(steps * sizeof(float));
    float *y_points = (float *)malloc(steps * sizeof(float));
    if (x_points == NULL || y_points == NULL) {
        printf(Memory allocation failed.\n);
        return 1;
    }
}
```

C Code - Eulers Method

```c
// Call the points function
    points(x_0, y_0, x_end, step_size, x_points, y_points, steps);

    // Print the calculated points (optional, for debugging purposes)
    printf(x\t\ty\n);
    for (int i = 0; i < steps; i++) {
        printf(%f\t%f\n, x_points[i], y_points[i]);
    }

    // Free allocated memory
    free(x_points);
    free(y_points);

    return 0;
}
```

Python Code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load the shared library
lib = ctypes.CDLL(./c.so)

# Define the function signature for points
lib.points.argtypes = [
    ctypes.c_float, # x_0
    ctypes.c_float, # y_0
    ctypes.c_float, # x_end
    ctypes.c_float, # h
    np.ctypeslib.ndpointer(dtype=np.float32, ndim=1), # x_points
    np.ctypeslib.ndpointer(dtype=np.float32, ndim=1), # y_points
    ctypes.c_int # stepsclass 12 differential equations
]
```

Python Code

```python
# Parameters for simulation
x_0 = 0.0 # Initial condition for x
y_0 = 2.0 # Initial condition for y
x_end = 1.0 # Final value of x
step_size = 0.001 # Reduced step size for higher accuracy
steps = int((x_end - x_0) / step_size) + 1

# Create numpy arrays to hold the points
x_points = np.zeros(steps, dtype=np.float32)
y_points = np.zeros(steps, dtype=np.float32)

# Call the points function from the C shared library
lib.points(x_0, y_0, x_end, step_size, x_points, y_points, steps)

# Define the theoretical solution with C = -2
def theoretical_solution(x):
    return (-x + 4 - 2* np.exp(x)) # C = -2
```

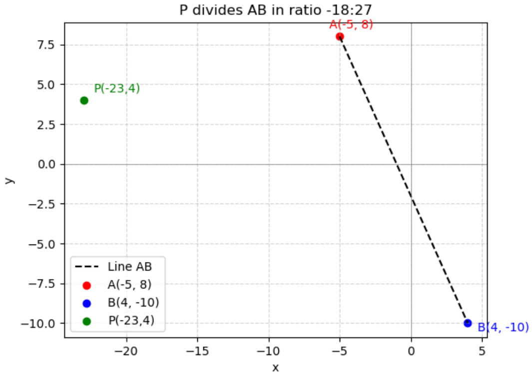
Python Code

```python
# Generate theoretical values for y
x_theory = np.linspace(x_0, x_end, 1000)
y_theory = theoretical_solution(x_theory)

# Plot the results
plt.figure(figsize=(10, 6))

# Plot Euler's method results
plt.plot(x_points, y_points, 'ro-', markersize=2, linewidth=4,
    label=sim)

# Plot the theoretical solution
plt.plot(x_theory, y_theory, 'b-', linewidth=2, label=theory)
```

Python Code

```
  # Add labels, title, grid, and legend
plt.xlabel(x) 1
plt.ylabel(y)
plt.grid(True, linestyle=--)
plt.legend()

# Display the plot
plt.show()
```

Plot



P divides AB in ratio -18:27
A(-5, 8)
P(-23,4)
B(4, -10)
Line AB
A(-5, 8)
B(4, -10)
P(-23,4)
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−20
−15
−10
−5
0
5
x
y