4.9.4

EE25BTECH11002 - Achat Parth Kalpesh

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Question

The equations of the lines which pass through the point $(3, -2)$ and are inclined at $60°$ to the line $\sqrt{3}x + y = 1$ is

1. $y + 2 = 0, \ \sqrt{3}x - y - 2 - 3\sqrt{3} = 0$
2. $x - 2 = 0, \ \sqrt{3}x - y + 2 + 3\sqrt{3} = 0$
3. $\sqrt{3}x - y - 2 - 3\sqrt{3} = 0$
4. None of these

Solution

The given line can be written in normal form as

$$\mathbf{n}_1^\top \mathbf{x} = 1, \tag{1}$$

where

$$\mathbf{n}_1 = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}. \tag{2}$$

Let the required line have normal vector $\mathbf{n} = \begin{pmatrix} -m \\ 1 \end{pmatrix}$, where m is the slope of line,then its equation is

$$\mathbf{n}^\top \mathbf{x} = c. \tag{3}$$

Since the line passes through $\mathbf{P} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$,

$$\mathbf{n}^\top \mathbf{P} = c \tag{4}$$

Solution

The angle θ between two lines is given as;

$$\cos\theta = \frac{\mathbf{n}_1^\top \mathbf{n}}{\left\|\mathbf{n}_1\right\| \left\|\mathbf{n}\right\|}. \tag{5}$$

Here $\theta = 60^\circ \implies \cos\theta = \frac{1}{2}$, so

$$\left(\mathbf{n}_1^\top \mathbf{n}\right)^2 = \frac{1}{4} \left\|\mathbf{n}_1\right\|^2 \left\|\mathbf{n}\right\|^2. \tag{6}$$

Substituting values:

$$\left(-\sqrt{3}m + 1\right)^2 = \frac{1}{4}(4)\left((-m)^2 + 1^2\right) \tag{7}$$

$$\left(-\sqrt{3}m + 1\right)^2 = m^2 + 1^2 \tag{8}$$

$$3m^2 - 2\sqrt{3}m + 1 = m^2 + 1, \tag{9}$$

$$2m^2 = 2\sqrt{3}m \tag{10}$$

$$m = 0 \text{ or } m = \sqrt{3} \tag{11}$$

Solution

For m = 0 ;

$$\mathbf{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{12}$$

$$\mathbf{n}^\top \mathbf{P} = c \tag{13}$$

$$c = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix} = -2, \tag{14}$$

so the line is

$$y + 2 = 0 \tag{15}$$

Solution

For m = $\sqrt{3}$;

$$\mathbf{n} = \begin{pmatrix} -\sqrt{3} \\ 1 \end{pmatrix} \tag{16}$$

$$\mathbf{n}^\top \mathbf{P} = c \tag{17}$$

$$c = \begin{pmatrix} -\sqrt{3} & 1 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix} = -3\sqrt{3} - 2 \tag{18}$$

so the line is

$$\sqrt{3}x - y - 3\sqrt{3} - 2 = 0 \tag{19}$$

C code

```c
#include <stdio.h>
#include <math.h>
void line_equation(double m, double px, double py, double *a,
    double *b, double *c) {
    if (isinf(m)) {
        *a = 1;
        *b = 0;
        *c = -px;
    } else {
        *a = -m;
        *b = 1;
        *c = -( (*a)*px + (*b)*py );
    }
}
```

Python Code

```python
import ctypes
import numpy as np
import matplotlib.pyplot as plt

lib = ctypes.CDLL(r"D:\Matgeo\4.9.4\codes\lines.so")

lib.line_equation.argtypes = [
    ctypes.c_double, ctypes.c_double, ctypes.c_double,
    ctypes.POINTER(ctypes.c_double),
    ctypes.POINTER(ctypes.c_double),
    ctypes.POINTER(ctypes.c_double)
]

# --- Python wrapper for the C function ---
def line_equation(m, px, py):
    a = ctypes.c_double()
    b = ctypes.c_double()
    c = ctypes.c_double()
```

```
    lib.line_equation(m, px, py,
                      ctypes.byref(a), ctypes.byref(b), ctypes.
                          byref(c))
    return a.value, b.value, c.value
    # --- Given point ---
P = np.array([3, -2])

# --- Define functions for lines ---
def line_y_given_x(a, b, c, x_vals):
    # Line: ax + by + c = 0 => y = (-a*x - c)/b
    return (-a * x_vals - c) / b

# --- Given line: sqrt(3)x + y - 1 = 0 ---
a1, b1, c1 = np.sqrt(3), 1, -1

# --- Required lines ---
# Line 1: y + 2 = 0 => 0*x + 1*y + 2 = 0
a2, b2, c2 = 0, 1, 2
```

Python Code

```python
# Line 2: sqrt(3)x - y - (3sqrt(3)+2) = 0
a3, b3, c3 = np.sqrt(3), -1, -(3*np.sqrt(3) + 2)

# --- Generate x values over a much wider range ---
x_vals = np.linspace(-20, 25, 400)

# --- Compute y values for each line ---
y1 = line_y_given_x(a1, b1, c1, x_vals)
y2 = line_y_given_x(a2, b2, c2, x_vals)
y3 = line_y_given_x(a3, b3, c3, x_vals)
# --- Plot ---
plt.figure(figsize=(12,8))

plt.plot(x_vals, y1, 'b-', linewidth=2)
plt.plot(x_vals, y2, 'r-', linewidth=2)
plt.plot(x_vals, y3, 'g-', linewidth=2)
```

Python Code

```
# Plot the common point
plt.scatter(P[0], P[1], color='k', s=80, zorder=5)
plt.text(P[0]+0.9, P[1]-2.0, "(3,-2)", fontsize=10, color='k')

# --- Place labels on the lines directly in non-overlapping
    positions ---
# For the blue line, place the label in the bottom-right quadrant
    .
# A small vertical offset (+ 0.5) is added to prevent it from
    sitting directly on the line.
plt.text(9, line_y_given_x(a1, b1, c1, 9) + 0.6, r'$\sqrt{3}x+y=1
    $', color='b', fontsize=12)

# For the red line, place the label to the right of the
    intersection point.
plt.text(12, line_y_given_x(a2, b2, c2, 5), r'$y+2=0$', color='r'
    , fontsize=12)
```

Python Code

```
# For the green line, place the label in the upper-right quadrant
    , away from the blue line's label.
plt.text(9, line_y_given_x(a3, b3, c3, 9) + 0.5, r'$\sqrt{3}x - y
     -(3\sqrt{3}+2)=0$', color='g', fontsize=12)

# Axes settings
plt.axhline(0, color='black', linewidth=0.7)
plt.axvline(0, color='black', linewidth=0.7)
plt.grid(True, linestyle='--', alpha=0.6)
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.title("Lines through (3,-2) at 60 to given line")

# --- Set plot boundaries to ensure lines touch the edge ---
plt.xlim(-200, 250)
plt.ylim(-100, 100)
plt.axis("equal")
plt.show()
```

Python Plot

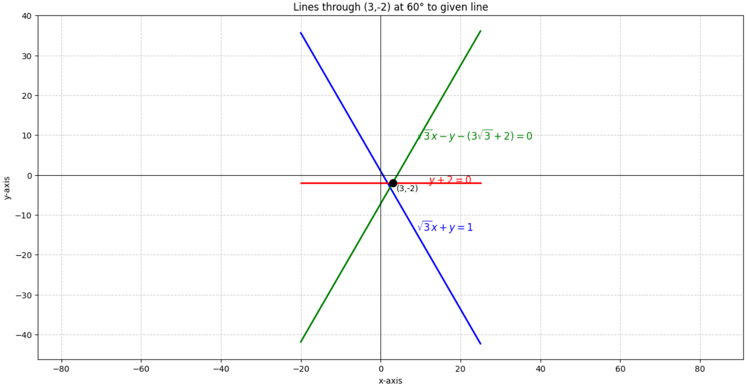

Figure: Lines through $(3, -2)$ at $60°$to given line