MatGeo Assignment 1.11.9

AI25BTECH11007

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Question

If

$$\mathbf{a} = \hat{i} - 7\hat{j} + 7\hat{k} \quad \text{and} \quad \mathbf{b} = 3\hat{i} - 2\hat{j} + 2\hat{k},$$

find a unit vector perpendicular to both the vectors **a** and **b**.

Solution

We want $\mathbf{n} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ such that

$$\mathbf{a}^T\mathbf{n} = 0, \tag{1}$$
$$\mathbf{b}^T\mathbf{n} = 0. \tag{2}$$

This gives the linear system

$$\begin{bmatrix} 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{3}$$

Step 1: Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & -7 & 7 & 0 \\ 3 & -2 & 2 & 0 \end{array}\right]. \tag{4}$$

Step 2: Row operations

$$R_2 \to R_2 - 3R_1 : \left[\begin{array}{ccc|c} 1 & -7 & 7 & 0 \\ 0 & 19 & -19 & 0 \end{array}\right], \tag{5}$$

$$R_2 \to \frac{1}{19}R_2 : \left[\begin{array}{ccc|c} 1 & -7 & 7 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right], \tag{6}$$

$$R_1 \to R_1 + 7R_2 : \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right]. \tag{7}$$

Step 3: Solution
From RREF:

$$x = 0, \tag{8}$$

$$y - z = 0 \quad \Rightarrow \quad y = z. \tag{9}$$

Thus the general solution is

$$\mathbf{n} = t \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad t \in \mathbb{R}. \tag{10}$$

Step 4: Unit vector
Since

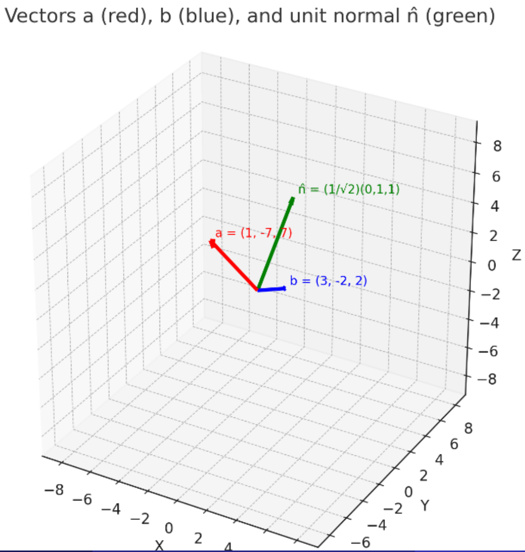
$$\|(0, 1, 1)\| = \sqrt{2}, \tag{11}$$

the unit vectors are

$$\hat{n} = \pm \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \tag{12}$$

$$= \pm \frac{1}{\sqrt{2}} (\hat{j} + \hat{k}). \tag{13}$$

Plot


Vectors a (red), b (blue), and unit normal n̂ (green)
n̂ = (1/√2)(0,1,1)
a = (1, -7, 7)
b = (3, -2, 2)
X
Y
Z

Conclusion

Therefore, a unit vector perpendicular to both **a** and **b** is

$$\hat{n} = \frac{1}{\sqrt{2}}(\hat{j} + \hat{k}),$$

or its negative.