10.5.8

EE25BTECH11041 - Naman Kumar

Question:
Draw two concentric circles of radii 3 cm and 5 cm. Taking a point on outer circle construct the pair of tangents to the other. Measure the length of a tangent and verify it by actual calculation.

Solution:
General equation of conic

$$g(\mathbf{x}) = \mathbf{x}^{\mathbf{T}}\mathbf{V}\mathbf{x} + 2\mathbf{u}^{\mathbf{T}}\mathbf{x} + f \tag{1}$$

Equation of circle,

$$\mathbf{x}^{\mathbf{T}}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{x} + 2\begin{pmatrix}0\\0\end{pmatrix}^T \mathbf{x} - r^2 = 0, r = \text{radius od circle} \tag{2}$$

$$r_1 = 3cm, r_2 = 5cm \tag{3}$$

A point lies on the tangent to the conic if it satisfies the following equation

$$\mathbf{m}^T\left[(\mathbf{V}\mathbf{h}+\mathbf{u})(\mathbf{V}\mathbf{h}+\mathbf{u})^T - \mathbf{V}g(\mathbf{h})\right]\mathbf{m} = 0 \tag{4}$$

Assuming a point on outer circle as $\mathbf{A}(5, 0)$
putting $\mathbf{A}$ in (2) for inner circle

$$\mathbf{A}^{\mathbf{T}}\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\mathbf{A} + 2\begin{pmatrix}0\\0\end{pmatrix}^T \mathbf{A} - (r_1)^2 \tag{5}$$

$$25 - 9 = 16 \tag{6}$$

$$g(\mathbf{A})_1 = 16 \tag{7}$$

Calculating $(\mathbf{VA} + \mathbf{u})$

$$\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}\begin{pmatrix}5\\0\end{pmatrix} + \begin{pmatrix}0\\0\end{pmatrix} \tag{8}$$

$$\begin{pmatrix}5\\0\end{pmatrix} \tag{9}$$

putting in (4)

$$\mathbf{m}^T \left[(\mathbf{VA}+\mathbf{u})(\mathbf{VA}+\mathbf{u})^T - \mathbf{V}g(\mathbf{A})_1 \right] \mathbf{m} = 0 \tag{10}$$

$$\mathbf{m}^T \left[\begin{pmatrix}5\\0\end{pmatrix}\begin{pmatrix}5\\0\end{pmatrix}^T - \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} \times 16 \right] \mathbf{m} = 0 \tag{11}$$

$$\mathbf{m}^T \left[\begin{pmatrix}9 & 0\\0 & -16\end{pmatrix} \right] \mathbf{m} = 0 \tag{12}$$

$$\begin{pmatrix}1\\m\end{pmatrix}^T \begin{pmatrix}9 & 0\\0 & -16\end{pmatrix}\begin{pmatrix}1\\m\end{pmatrix} = 0 \tag{13}$$

$$9 - 16m^2 = 0 \tag{14}$$

$$m = \pm\frac{3}{4} \tag{15}$$

$$\mathbf{m} = \begin{pmatrix}1\\ \pm\frac{3}{4}\end{pmatrix} \tag{16}$$

Using following formula to find point of contact of tangent

$$\mathbf{q}_j = \left(\pm r \frac{\mathbf{n}_j}{||\mathbf{n}_j||} - \mathbf{u} \right), j = 1, 2 \tag{17}$$

$$\mathbf{q_1} = \left(\pm 3 \frac{\begin{pmatrix}\frac{3}{4}\\1\end{pmatrix}}{\sqrt{\left(\frac{3}{4}\right)^2 + 1}} \right) \tag{18}$$

$$\mathbf{q}_1 = \pm \begin{pmatrix}\frac{9}{5}\\ \frac{12}{5}\end{pmatrix} \tag{19}$$

$$Similarly, \mathbf{q_2} = \pm \begin{pmatrix}\frac{9}{5}\\ \frac{-12}{5}\end{pmatrix} \tag{20}$$

To take the ones passing through $\mathbf{A}$ taking $\mathbf{q}_1$ and $\mathbf{q}_2$ as

$$\mathbf{q}_1 = \begin{pmatrix}\frac{9}{5}\\ \frac{12}{5}\end{pmatrix} \tag{21}$$

$$\mathbf{q}_2 = \begin{pmatrix}\frac{9}{5}\\ \frac{-12}{5}\end{pmatrix} \tag{22}$$

Length of both tangent will be equal and will be

$$||\mathbf{q_1} - \mathbf{A}|| \tag{23}$$

$$||\begin{pmatrix}\frac{9}{5}\\ \frac{12}{5}\end{pmatrix} - \begin{pmatrix}5\\0\end{pmatrix}|| \tag{24}$$

$$||\begin{pmatrix}\frac{-16}{5}\\ \frac{12}{5}\end{pmatrix}|| \tag{25}$$

$$= 4 \tag{26}$$

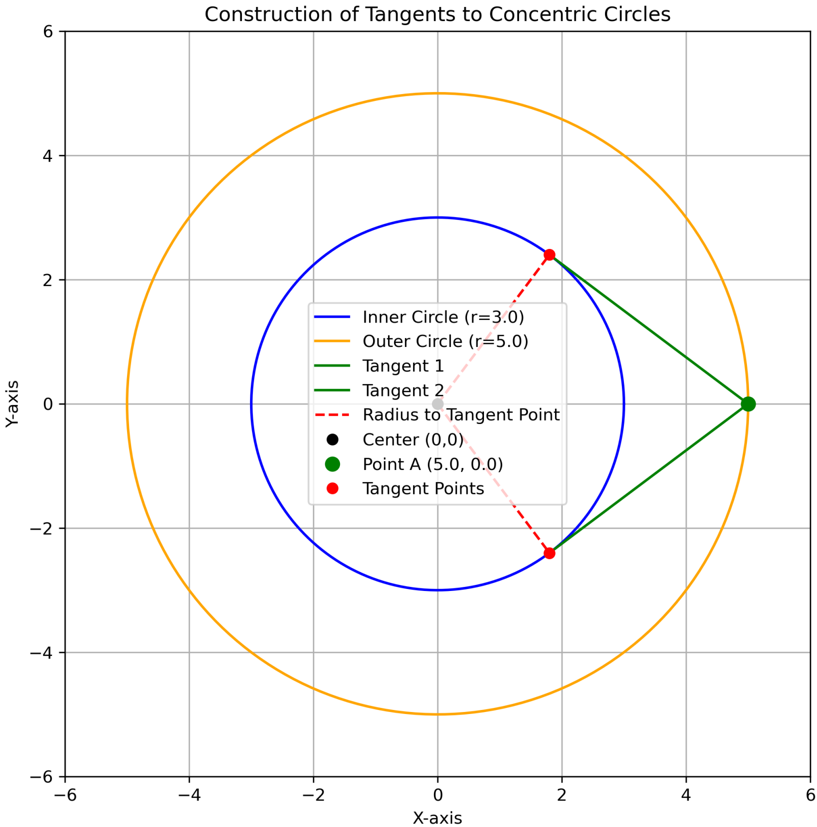
Construction of Tangents to Concentric Circles
Inner Circle (r=3.0)
Outer Circle (r=5.0)
Tangent 1
Tangent 2
Radius to Tangent Point
Center (0,0)
Point A (5.0, 0.0)
Tangent Points
Y-axis
X-axis
6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6

Figure 1