2.10.42

EE25BTECH11012 - BEERAM MADHURI

Question:
If $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are unit coplanar vectors, then the scalar triple product

$$\begin{bmatrix} 2\mathbf{a}-\mathbf{b} & 2\mathbf{b}-\mathbf{c} & 2\mathbf{c}-\mathbf{a} \end{bmatrix} =$$

Solution:

$$\mathbf{B} = \begin{pmatrix} 2\mathbf{a}-\mathbf{b} & 2\mathbf{b}-\mathbf{c} & 2\mathbf{c}-\mathbf{a} \end{pmatrix} = \begin{pmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{pmatrix} \begin{pmatrix} 2 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{pmatrix} \tag{0.1}$$

Since $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are coplanar,

$$\begin{vmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{vmatrix} = 0 \tag{0.2}$$

$$\begin{vmatrix} 2\mathbf{a}-\mathbf{b} & 2\mathbf{b}-\mathbf{c} & 2\mathbf{c}-\mathbf{a} \end{vmatrix} = \begin{vmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{vmatrix} \begin{vmatrix} 2 & 0 & -1 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \end{vmatrix} = 0 \tag{0.3}$$

Hence, the value of $\begin{bmatrix} 2\mathbf{a}-\mathbf{b} & 2\mathbf{b}-\mathbf{c} & 2\mathbf{c}-\mathbf{a} \end{bmatrix} = 0$.

Proof of $\begin{bmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{bmatrix}$ is singular:

Given $\mathbf{a}$,$\mathbf{b}$,$\mathbf{c}$ are coplanar
plane equation of the plane through $\mathbf{a}$,$\mathbf{b}$,$\mathbf{c}$ be

$$\mathbf{n}^\top \mathbf{r} = 0 \tag{0.4}$$

where $\mathbf{n}$ is normal to plane

$$\mathbf{n}^\top \mathbf{a} = 0 \tag{0.5}$$

$$\mathbf{n}^\top \mathbf{b} = 0 \tag{0.6}$$

$$\mathbf{n}^\top \mathbf{c} = 0 \tag{0.7}$$

let $M = \begin{bmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{bmatrix}$

$$\mathbf{n}^\top M = 0^\top \tag{0.8}$$

For a homogeneous linear system

$$\mathbf{n}^\top M = 0^\top, \quad \mathbf{n} \neq 0 \tag{0.9}$$

M must be singular.

$$\therefore \begin{bmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} \end{bmatrix} \text{ is singular}$$

Hence proved.

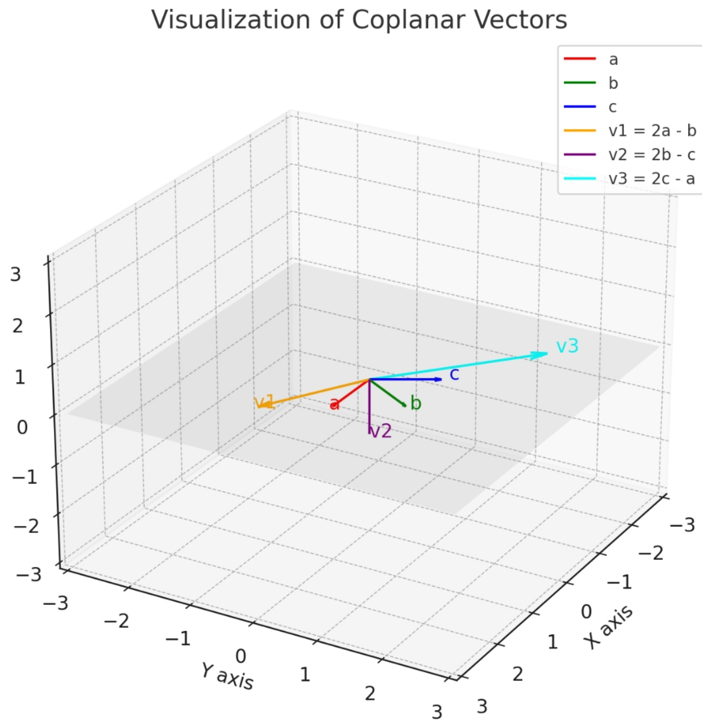

Fig. 0.1: Plot