

9.4.45

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Question

A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Theoretical Solution

Let speed of boat be v and of stream be u .

Now, if x – axis represents time t for downstream and y – axis represents time t for upstream,

$$\mathbf{n}^T \mathbf{x} = c \quad (1)$$

where c is distance (Given = 24) and

$$\mathbf{n} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} v \\ u \end{pmatrix} \quad (2)$$

Theoretical Solution

If Line 1 cuts x - *axis* at t_1 and y - *axis* at t_2 , then
Given,

$$t_1 = t_2 - 1 \quad (3)$$

To solve,

$$\begin{pmatrix} v & u \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} t_2 - 1 \\ 0 \end{pmatrix} = 24 \quad (4)$$

$$\begin{pmatrix} v & u \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ t_2 \end{pmatrix} = 24 \quad (5)$$

Theoretical Solution

From 5

$$t_2 = \frac{24}{v - u} \quad (6)$$

Putting 6 in 4 , we get

$$u^2 + 48u - 324 = 0 \quad (7)$$

$$\implies y = x^2 + 48x - 324 = 0 \quad (8)$$

which can be expressed as the conic

$$\mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \quad (9)$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 24 \\ -\frac{1}{2} \end{pmatrix}, f = -324. \quad (10)$$

Theoretical Solution

To find the roots of 9, we find the points of intersection of the conic with the x -axis

$$\mathbf{x}_i = \mathbf{h} + k_i \mathbf{m} \quad (11)$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (12)$$

Using ,

$$k_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^\top \mathbf{V} \mathbf{m})} \right) \quad (13)$$

$$k_i = \frac{1}{1} \left(-24 \pm \sqrt{24^2 + 324} \right) \quad (14)$$

$$\implies k_1 = 6, k_2 = -54 \quad (15)$$

Theoretical Solution

the points of intersection are

$$\mathbf{h} + k\mathbf{m} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}, \begin{pmatrix} -54 \\ 0 \end{pmatrix} \quad (16)$$

Since u cannot be negative ,

$$u = 6 \text{ km/h} \quad (17)$$



