Question 4.2.3

AI25BTECH11040 - Vivaan Parashar

September 22, 2025

1 Question:

Find the direction and normal vectors of each of the following line: $-2x + 3y = 6$

2 Solution:

Assuming points on the line are represented by $\mathbf{x}$, we can express the line equation as:

$$\left\| \begin{pmatrix} -2 \\ 3 \end{pmatrix} \mathbf{x} \right\| = 6 \tag{1}$$

In such a form, the normal vector $\mathbf{n}$ is given by:

$$\mathbf{n} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \equiv \frac{1}{\sqrt{13}} \begin{pmatrix} 2 \\ -3 \end{pmatrix} \tag{2}$$

The direction vector $\mathbf{d}$ can be derived from the normal vector by rotating it by 90 degrees clockwise (or anticlockwise). To do that, we multiply with the transformation matrix $\mathbf{r}$:

$$\mathbf{r} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{3}$$

$$\therefore \mathbf{d} = \mathbf{rn} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix} \equiv \frac{1}{\sqrt{13}} \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{4}$$

3 Plot:

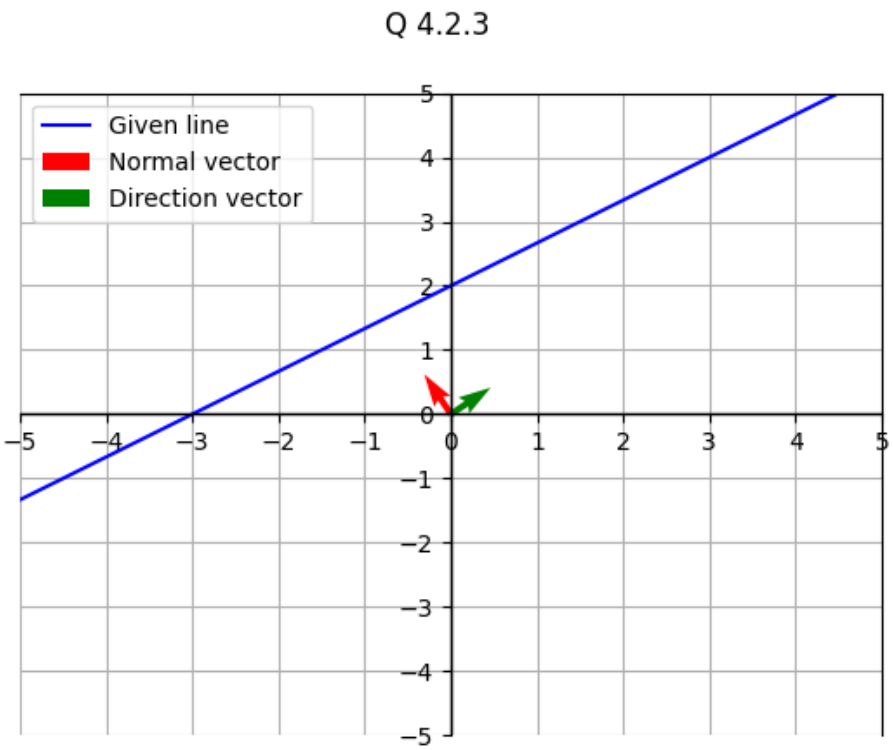

Figure 1: Graph of line with direction and normal vectors