9.2.1

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Question)

Find the area bounded by the curve $y = \sqrt{x}, x = 2y + 3$, in the first quadrant and x-axis.

Solution

General equation of conic

$$g(\mathbf{x}) = \mathbf{x}^{\mathbf{T}}\mathbf{V}\mathbf{x} + 2\mathbf{u}^{\mathbf{T}}\mathbf{x} + f \tag{1}$$

Equation of parabola,

$$\mathbf{x}^{\mathbf{T}}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{x} + 2\begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix}^{T}\mathbf{x} = 0 \tag{2}$$

Solution

Equation of line,

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{3}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \tag{4}$$

Using following equation to find point of intersection of conic and line

$$k_i = \frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}} \left(-\mathbf{m}^T \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \pm \sqrt{\left[\mathbf{m}^T \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \right]^2 - g(\mathbf{h})(\mathbf{m}^T\mathbf{V}\mathbf{m})} \right) \tag{5}$$

Solution

Solving for $g(\mathbf{h})$

$$g(\mathbf{h}) = \mathbf{h}^{\mathbf{T}} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{h} + 2 \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix}^T \mathbf{h} \tag{6}$$

$$g(\mathbf{h}) = \frac{9}{4} \tag{7}$$

Solving for $\mathbf{m}^T\mathbf{V}\mathbf{m}$

$$\mathbf{m}^T\mathbf{V}\mathbf{m} = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \tag{8}$$

$$= \frac{1}{4} \tag{9}$$

Solution

Solving for $\mathbf{m}^T \left(\mathbf{V}\mathbf{h} + \mathbf{u}\right)$

$$\begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}^T \left(\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix} + \begin{pmatrix} -\frac{1}{2} \\ 0 \end{pmatrix} \right) \tag{10}$$

$$= -\frac{5}{4} \tag{11}$$

Solving (5)

$$k_i = \frac{1}{\frac{1}{4}} \left(\frac{5}{4} \pm \sqrt{\frac{25}{16} - \frac{9}{4} \times \frac{1}{4}} \right) \tag{12}$$

$$k_i = 4 \left(\frac{5}{4} \pm 1 \right) \tag{13}$$

$$k_1 = 9, k_2 = 1 \tag{14}$$

Solution

So with these values points are

$$\mathbf{x_1} = \begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix} + 9 \times \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \tag{15}$$

$$\mathbf{x_1} = \begin{pmatrix} 9 \\ 3 \end{pmatrix} \tag{16}$$

$$\mathbf{x_2} = \begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix} + 1 \times \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \tag{17}$$

$$\mathbf{x_2} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{18}$$

Solution

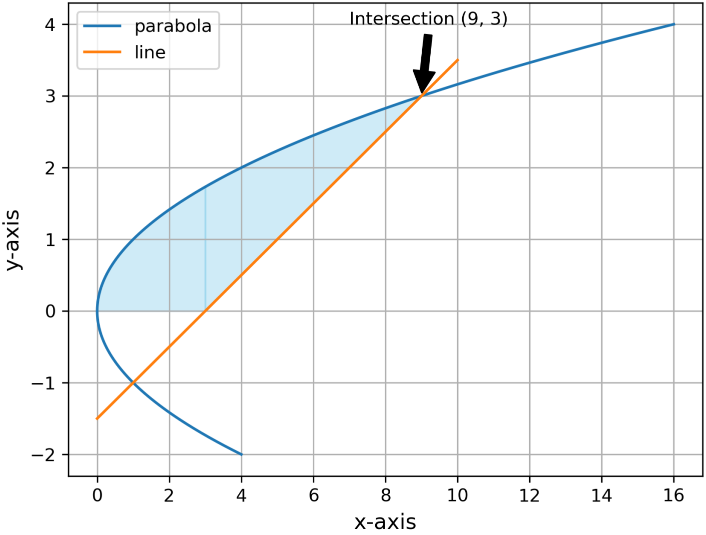
Area under curve in first quadrant between parabola and line

$$\int_0^3 \sqrt{x} + \int_3^9 \sqrt{x} - \left(\frac{x-3}{2}\right) \tag{19}$$

$$\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^9 - \left[\frac{x^2}{4} - \frac{3x}{2}\right]_3^9 \tag{20}$$

$$area = 9 \tag{21}$$

Figure



parabola
line
Intersection (9, 3)
y-axis
x-axis
4
3
2
1
0
−1
−2
0
2
4
6
8
10
12
14
16

Direct Python

```
import numpy as np
import matplotlib.pyplot as plt
import math

y = np.linspace(-2,4,300)
x = y*y
xl = np.linspace(0,10,300)
yl = (xl-3)/2

x1 = np.linspace(0,3, 200)
y1 = np.sqrt(x1)
x2 = np.linspace(3,9, 200)
y2 = np.sqrt(x2)
yl2 = (x2-3)/2
```

Direct Python

```
plt.fill_between(x1, y1, 0, color='skyblue', alpha=0.4)
plt.fill_between(x2,y2, yl2, color='skyblue', alpha=0.4)
plt.annotate('Intersection (9, 3)', xy=(9, 3), xytext=(7, 4),
             arrowprops=dict(facecolor='black', shrink=0.05))
plt.xlabel('x-axis', fontsize=12)
plt.ylabel('y-axis', fontsize=12)
plt.plot(x,y, label="parabola")
plt.plot(xl,yl, label="line")
plt.grid()
plt.legend()

plt.savefig("figure.png", dpi=300)

plt.show()
```

C code

```c
#include <stdio.h>

double area_bounded() {
    double y1 = 0, y2 = 3;
    double area;
```

C code

```c
    area = (y2 * y2 + 3 * y2 - (y2 * y2 * y2) / 3.0) -
           (y1 * y1 + 3 * y1 - (y1 * y1 * y1) / 3.0);
    return area;
}
int main() {
    printf("Area bounded = %.2f\n", area_bounded());
    return 0;
}
```

Python code with shared object

```
import ctypes
import matplotlib.pyplot as plt
import numpy as np

lib = ctypes.CDLL('./area.so')
lib.area_bounded.restype = ctypes.c_double

area = lib.area_bounded()
print("Area bounded =", area)
```

```
y = np.linspace(0, 3, 100)
x1 = y**2 # x = y^2 (y = sqrt(x))
x2 = 2*y + 3 # x = 2y + 3

plt.plot(x1, y, label='y = sqrt(x)  x = y')
plt.plot(x2, y, label='x = 2y + 3')

plt.fill_betweenx(y, x1, x2, color='lightblue', alpha=0.5)
plt.xlabel("x")
plt.ylabel("y")
plt.title(f"Area bounded by curves = {area:.2f}")
plt.legend()
plt.grid(True)
plt.show()
```