

2.7.2

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Question: Which of the following expressions are meaningful?

- | | |
|---|---|
| (a) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$ | (c) $(\mathbf{u} \cdot \mathbf{v}) \mathbf{w}$ |
| (b) $(\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w}$ | (d) $\mathbf{u} \times (\mathbf{v} \cdot \mathbf{w})$ |

Solution:

Let \mathbf{u} , \mathbf{v} , \mathbf{w} be vectors in \mathbb{R}^3 .

- (a) $\mathbf{u}(\mathbf{v} \times \mathbf{w})$:

The expression $\mathbf{v} \times \mathbf{w}$ is a vector (cross product), and the expression $\mathbf{u}(\mathbf{v} \times \mathbf{w})$ denotes the scalar triple product (sometimes written as the inner product of \mathbf{u} and the vector $\mathbf{v} \times \mathbf{w}$).

Meaningful.

- (b) $(\mathbf{u}^\top \mathbf{v})\mathbf{w}$:

Here, $(\mathbf{u}^\top \mathbf{v})$ represents the inner (dot) product, which is a scalar. Multiplying a scalar by a vector \mathbf{w} is valid. However, if it is interpreted as $(\mathbf{u}^\top \mathbf{v})^\top \mathbf{w}$ having a dot between scalar and vector, that is not defined.

Not meaningful if interpreted as scalar dot vector.

- (c) $\langle \mathbf{u}^\top \mathbf{v} \rangle^\top \mathbf{w}$:

$\langle \mathbf{u}, \mathbf{v} \rangle$ denotes the inner product (a scalar) and multiplying this scalar by vector \mathbf{w} is valid scalar multiplication of a vector.

Meaningful.

- (d) $\mathbf{u} \times (\mathbf{v}^\top \mathbf{w})$:

$\mathbf{v}^\top \mathbf{w}$ inside parentheses denotes the inner product (scalar), and cross product between a vector and scalar is undefined.

Not meaningful.

Answer: Only (a) and (c) are meaningful

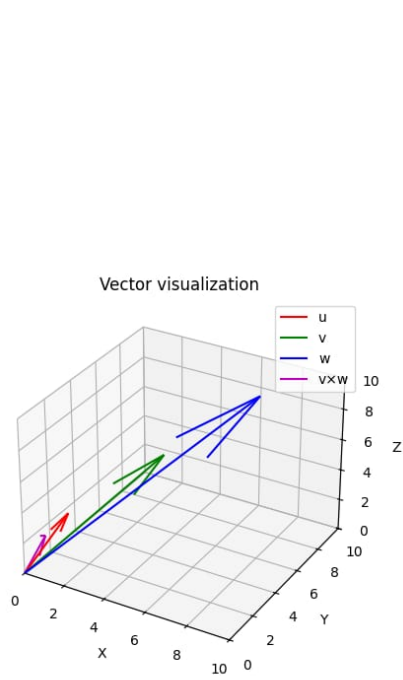


Fig. 4.1: Vector Representation