1.10.20

Rushil Shanmukha Srinivas
EE25BTECH11057
Electrical Enggineering ,
IIT Hyderabad.

August 31, 2025

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Problem Statement

Find the direction cosines of the line passing through the two points (-2,4,-5) and (1,2,3).

Variable	**Description**	**Values**
A	Point	$(-2, 4, -5)$
B	Point	$(1, 2, 3)$

Table: Variables Used

Echelon Form and Column Operations

Let

$$\mathbf{A} = \begin{pmatrix} -2 \\ 4 \\ -5 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}. \tag{3.1}$$

Form the 3×2 matrix with these as columns:

$$\mathbf{M} = \begin{pmatrix} -2 & 1 \\ 4 & 2 \\ -5 & 3 \end{pmatrix}. \tag{3.2}$$

Apply the column operation $C_2 \leftarrow C_2 - C_1$ to extract the difference vector as the second column:

$$\mathbf{M} \xrightarrow{C_2 \leftarrow C_2 - C_1} \begin{pmatrix} -2 & 3 \\ 4 & -2 \\ -5 & 8 \end{pmatrix}. \tag{3.3}$$

Thus the direction (difference) vector of the line is

$$\mathbf{v} = \mathbf{AB} = \begin{pmatrix} 3 \\ -2 \\ 8 \end{pmatrix}. \tag{3.4}$$

Norm and Direction Cosines

The length of $\mathbf{v}$ is

$$\begin{aligned}\mathbf{v}^\top \mathbf{v} &= \begin{pmatrix} 3 & -2 & 8 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \\ 8 \end{pmatrix} \\ &= 3^3 + (-2)^2 + (8)^2 \\ &= 9 + 4 + 64 = 77\end{aligned}$$

Therefore, the norm of $\mathbf{v}$ is

$$\|\mathbf{v}\| \triangleq \sqrt{\mathbf{v}^\top \mathbf{v}} = \sqrt{77}$$

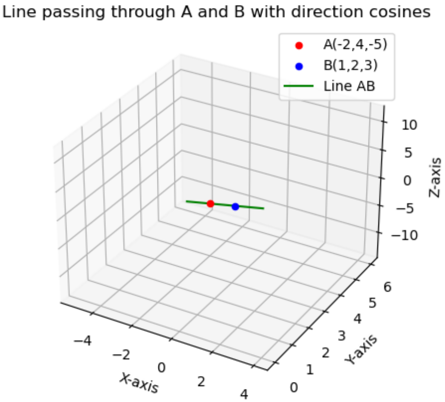
The unit vector in the direction of $\mathbf{v}$ is

$$\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\sqrt{77}} \begin{pmatrix} 3 \\ -2 \\ 8 \end{pmatrix}$$

Let α, β, γ be the angles made by the line with the x, y, z axes respectively.Then, the direction cosines are the elements of the above direction vector

$$\cos\alpha = \frac{3}{\sqrt{77}}, \quad \cos\beta = -\frac{2}{\sqrt{77}}, \quad \cos\gamma = \frac{8}{\sqrt{77}}$$

Plots



Figure

C Code

```
#include <stdio.h>
#include <math.h>

// Function to compute direction cosines between two 3D points
void direction_cosines(double x1, double y1, double z1,
double x2, double y2, double z2,
double *l, double *m, double *n) {
double dx = x2 - x1;
double dy = y2 - y1;
double dz = z2 - z1;
double mag = sqrt(dx*dx + dy*dy + dz*dz);

*l = dx / mag;
*m = dy / mag;
*n = dz / mag;
}
```

```
// For testing in C directly
int main() {
double l, m, n;
direction_cosines(-2,4,-5, 1,2,3, &l,&m,&n);
printf("Direction cosines: (%lf, %lf, %lf)\n", l, m, n);
return 0;
}
```

Python: call_c.py

```
import ctypes

# Load the shared object
lib = ctypes.CDLL("./direction_cosines.so")

# Define argument and return types
lib.direction_cosines.argtypes = [
ctypes.c_double, ctypes.c_double, ctypes.c_double,
ctypes.c_double, ctypes.c_double, ctypes.c_double,
ctypes.POINTER(ctypes.c_double),
ctypes.POINTER(ctypes.c_double),
ctypes.POINTER(ctypes.c_double)
]

# Prepare variables
l = ctypes.c_double()
m = ctypes.c_double()
```

```
n = ctypes.c_double()

# Call the function
lib.direction_cosines(-2, 4, -5, 1, 2, 3, ctypes.byref(l), ctypes.byref(m),
    ctypes.byref(n))

print("Direction cosines (C via .so):", (l.value, m.value, n.value))
```

Python Code for Plotting

```
import numpy as np
import matplotlib.pyplot as plt

# Given points
A = np.array([-2, 4, -5])
B = np.array([1, 2, 3])

# Direction ratios
AB = B - A
print("Direction ratios:", AB)

# Direction cosines
magnitude = np.linalg.norm(AB)
direction_cosines = AB / magnitude
print("Direction cosines:", direction_cosines)
```

```
# Plotting
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Plot points A and B
ax.scatter(*A, color='red', label='A(-2,4,-5)')
ax.scatter(*B, color='blue', label='B(1,2,3)')

# Plot line passing through A and B
t = np.linspace(-1, 2, 100) # parameter for line
line = A.reshape(3,1) + np.outer(AB, t)
ax.plot(line[0], line[1], line[2], color='green', label='Line AB')



# Labels
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
```

```
ax.legend()
ax.set_title("Line passing through A and B with direction cosines")

plt.savefig("../figs/fig_vector.png")

plt.show()
```