1.3.4

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If $A(1,3), B(-1,2), C(2,5)$ and $D(x,4)$ are the vertices of a parallelogram ABCD , then the value of x is _____(10, 2012)

Solution:

In a parallelogram, the diagonals bisect each other. Therefore, the midpoint of diagonal AC equals the midpoint of diagonal BD:

$$\frac{\mathbf{A}+\mathbf{C}}{2} = \frac{\mathbf{B}+\mathbf{D}}{2} \tag{0.1}$$

$$\mathbf{A}+\mathbf{C} = \mathbf{B}+\mathbf{D} \tag{0.2}$$

$$\mathbf{D} = \mathbf{A}+\mathbf{C}-\mathbf{B} \tag{0.3}$$

Substituting the coordinates:

$$\begin{pmatrix} x \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix} \tag{0.4}$$

$$= \begin{pmatrix} 1+2-(-1) \\ 3+5-2 \end{pmatrix} \tag{0.5}$$

$$= \begin{pmatrix} 4 \\ 6 \end{pmatrix} \tag{0.6}$$

This gives us the equations:

$$x = 4 \tag{0.7}$$

$$4 = 6 \tag{0.8}$$

this indicates that the assumption ABCD is a parallelogram with the given order might be incorrect. Perhaps the vertices are not in order A,B,C,DA,B,C,D. Let's try a different pairing.

$$\frac{\mathbf{A}+\mathbf{D}}{2} = \frac{\mathbf{B}+\mathbf{C}}{2} \tag{0.9}$$

$$\mathbf{A}+\mathbf{D} = \mathbf{B}+\mathbf{C} \tag{0.10}$$

$$\mathbf{D} = \mathbf{B}+\mathbf{C}-\mathbf{A} \tag{0.11}$$

Substituting the coordinates:

$$\begin{pmatrix} x \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{0.12}$$

$$= \begin{pmatrix} -1+2-(-1) \\ 2+5-3 \end{pmatrix} \tag{0.13}$$

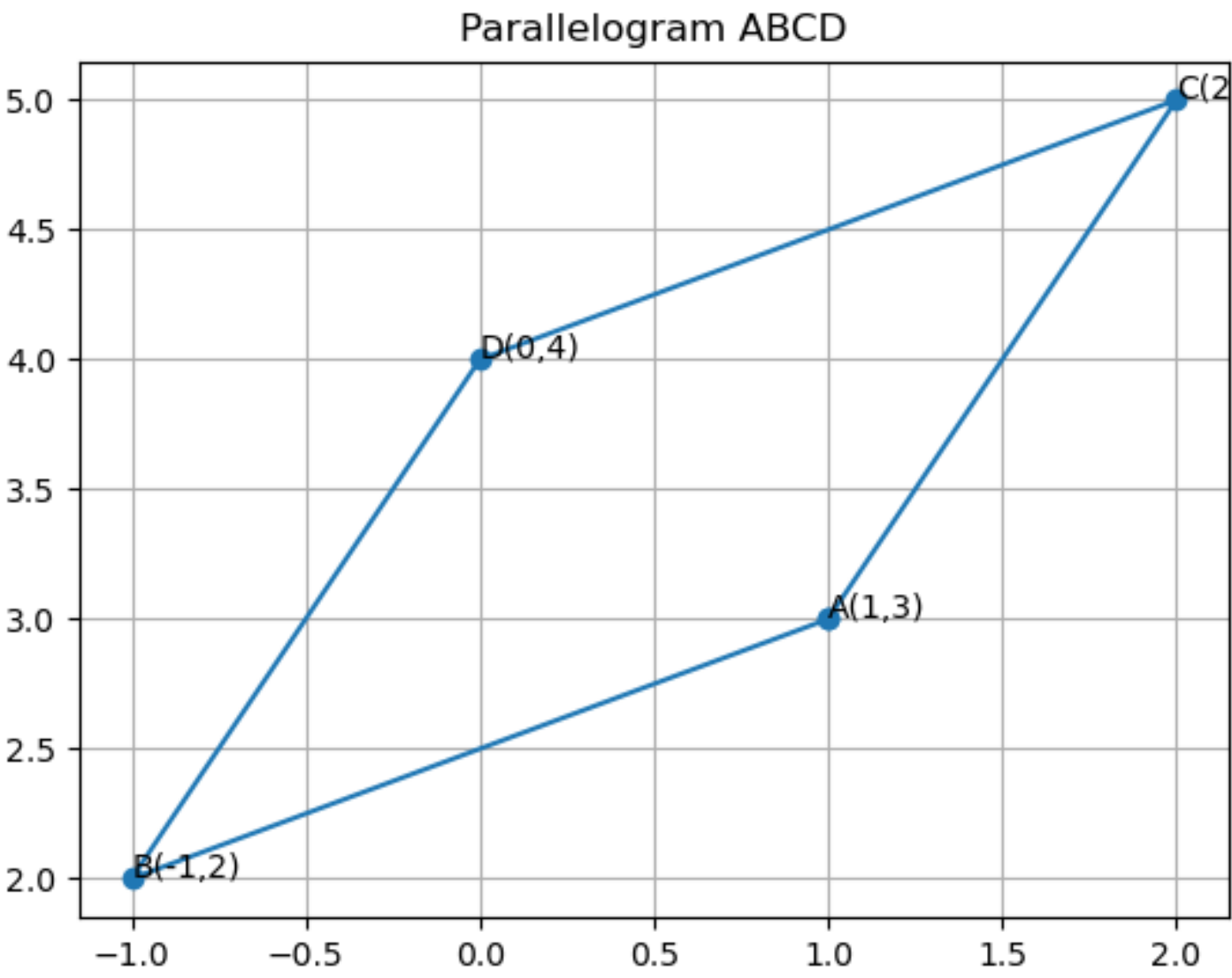
$$= \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{0.14}$$

This gives us the equations:

$$x = 0 \tag{0.15}$$

$$4 = 4 \tag{0.16}$$

Answer: x=0


Parallelogram ABCD
C(2
D(0,4)
A(1,3)
B(−1,2)
5.0
4.5
4.0
3.5
3.0
2.5
2.0
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0