Question 5.2.3

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1 Question:

Solve the following system of linear equations:

$$9x + 3y + 12 = 018x + 6y + 24 = 0$$

2 Solution:

The given equations can be rewritten as:

$$9x + 3y = -12 \tag{1}$$
$$18x + 6y = -24 \tag{2}$$

We can represent this system of equations in matrix form as:

$$\mathbf{MX} = \mathbf{D} \tag{3}$$
$$\implies \begin{pmatrix} 9 & 3 \\ 18 & 6 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -12 \\ -24 \end{pmatrix} \tag{4}$$

To obtain values of x and y, we can multiply both sides by the inverse of the coefficient matrix on the left side, but in this case, the coefficient matrix has a rank of 1 ($R_2 = 2R_1$). This implies that either the system has no solutions or infinite solutions.
If the system has infinite solution, then the rank of the augmented matrix $\begin{pmatrix} \mathbf{M} & \mathbf{D} \end{pmatrix}$ should also have a rank of 1.

$$\text{rank}(\begin{pmatrix} \mathbf{M} & \mathbf{D} \end{pmatrix}) = \text{rank}(\begin{pmatrix} 9 & 3 & -12 \\ 18 & 6 & -24 \end{pmatrix}) \tag{5}$$
$$\begin{pmatrix} 9 & 3 & -12 \\ 18 & 6 & -24 \end{pmatrix} \xleftrightarrow{R_2 = R_2 - 2R_1} \begin{pmatrix} 9 & 3 & -12 \\ 0 & 0 & 0 \end{pmatrix} \tag{6}$$
$$\therefore \text{rank}(\begin{pmatrix} \mathbf{M} & \mathbf{D} \end{pmatrix}) = 1 \tag{7}$$

Therefore, this system of equations has infinite solutions, which are all values of $(x, y) \in \mathbb{R}^2$ that satisfy the equation $3x + y = -4$.

3 Plot:

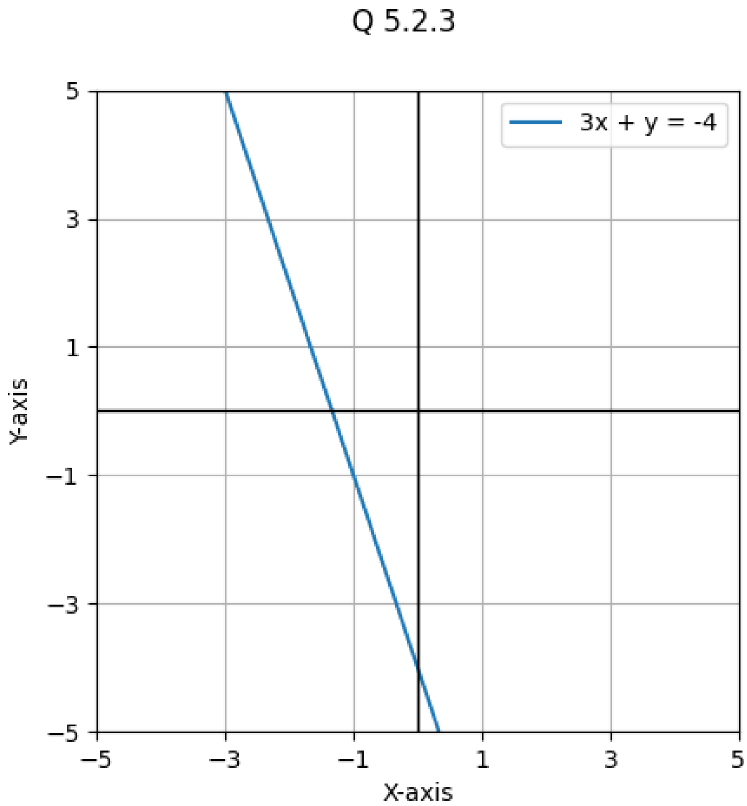

Figure 1: Graph of line representing all possible solutions