Problem 2.10.20.

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Question

Question: Which of the following expressions are meaningful?

(a) $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$

(b) $(\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{w}$

(c) $(\mathbf{u} \cdot \mathbf{v})\ \mathbf{w}$

(d) $\mathbf{u} \times (\mathbf{v} \cdot \mathbf{w})$

Solution

Let $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ be vectors in $\mathbb{R}^3$.
Let

$$\mathbf{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}. \tag{2.1}$$

$$\tag{2.2}$$

a) $\mathbf{u}^\top(\mathbf{v} \times \mathbf{w})$

$$\mathbf{v} \times \mathbf{w} = \begin{pmatrix} v_{23} & w_{23} \\ v_{31} & w_{31} \\ v_{12} & w_{12} \end{pmatrix} = \begin{pmatrix} v_2 w_3 - v_3 w_2 \\ v_3 w_1 - v_1 w_3 \\ v_1 w_2 - v_2 w_1 \end{pmatrix} = \begin{pmatrix} 1 \times 0 - 0 \times 5 \\ 0 \times 0 - 4 \times 0 \\ 4 \times 5 - 1 \times 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 20 \end{pmatrix}$$

Solution

$$\mathbf{u}^\top(\mathbf{v}\times\mathbf{w}) = \begin{bmatrix}2 & 3 & 0\end{bmatrix}\begin{bmatrix}0\\0\\20\end{bmatrix} = 0$$

Since the scalar (dot) product of two vectors is defined, the expression $\mathbf{u}^\top(\mathbf{v}\times\mathbf{w})$ is meaningful.$(\mathbf{u}^\top\mathbf{v})^\top\mathbf{w}$

$$\mathbf{u}^\top\mathbf{v} = \begin{bmatrix}2 & 3\end{bmatrix}\begin{bmatrix}4\\1\end{bmatrix} = 2\times4+3\times1 = 11,$$

$$(\mathbf{u}^\top\mathbf{v})^\top\mathbf{w} = 11^\top\mathbf{w} \quad \text{(scalar dot vector)} \quad \text{undefined.}$$

$(\mathbf{u}^\top\mathbf{v})\mathbf{w}$

$$(\mathbf{u}^\top\mathbf{v})\mathbf{w} = 11\times\begin{bmatrix}0\\5\end{bmatrix} = \begin{bmatrix}0\\55\end{bmatrix}.$$

Solution

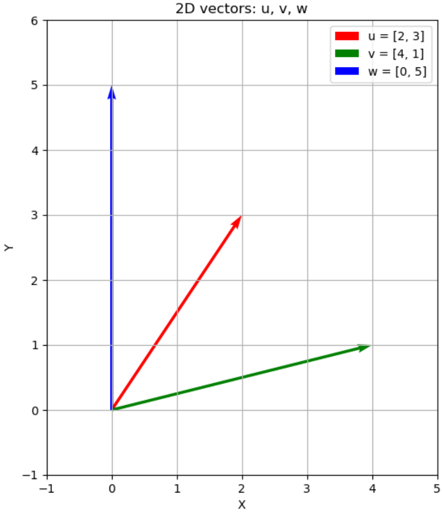
This is meaningful scalar multiplication.$\mathbf{u} \times (\mathbf{v}^\top \mathbf{w})$

$$\mathbf{v}^\top \mathbf{w} = \begin{bmatrix} 4 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \end{bmatrix} = 0 + 5 = 5,$$

$$\mathbf{u} \times 5 = \text{cross product of vector and scalar – undefined.}$$

Answer: $\boxed{\text{Only (a) and (c) are meaningful}}$

Graph



d)

Figure: Vector Representation

C Code

```c
#include <stdio.h>
#include "matfun.h"

void print_vector(const char* name, const double v[3]) {
    printf("%s = (%.2f, %.2f, %.2f)\n", name, v[0], v[1], v[2]);
}

int main() {
    double u[3] = {1, 2, 3};
    double v[3] = {4, 5, 6};
    double w[3] = {7, 8, 9};

    double cross_vw[3];
    cross_product(v, w, cross_vw);

    double dot_u_crossvw = dot_product(u, cross_vw);
    printf("u  (v  w) = %.2f\n", dot_u_crossvw);
```

C Code

```
double dot_uv = dot_product(u, v);
   printf("(u  v) = %.2f\n", dot_uv);

   printf("(u  v)  w is NOT meaningful as dot product of scalar and vector.\n");

   printf("(u  v) * w (scalar multiplication) = (%.2f, %.2f, %.2f)\n",
          dot_uv * w[0], dot_uv * w[1], dot_uv * w[2]);

   printf("v  w = %.2f\n", dot_product(v, w));
   printf("u  (v  w) is NOT meaningful as cross product of vector and scalar.\n");

   return 0;
}
```

Python Code for Plotting

```python
import matplotlib.pyplot as plt
import numpy as np

# Vectors u, v, w in 2D (using first two components)
u = np.array([2, 3])
v = np.array([4, 1])
w = np.array([0, 5])

# Origin point
origin = np.array([0, 0])

# Plotting the vectors
plt.figure(figsize=(7, 7))
plt.quiver(*origin, *u, angles='xy', scale_units='xy', scale=1,
    color='red', label='u = [2, 3]')
plt.quiver(*origin, *v, angles='xy', scale_units='xy', scale=1,
    color='green', label='v = [4, 1]')
plt.quiver(*origin, *w, angles='xy', scale_units='xy', scale=1,
    color='blue', label='w = [0, 5]')
```

Python Code for Plotting

```python
# Setting the limits
plt.xlim(-1, 5)
plt.ylim(-1, 6)

# Adding labels and title
plt.xlabel('X')
plt.ylabel('Y')
plt.title('2D vectors: u, v, w')
plt.grid()
plt.legend()
plt.gca().set_aspect('equal')

# Save the figure as a PNG file
plt.savefig('2D_vectors.png')
plt.close()
```