2.10.61

EE25BTECH11032 - Kartik Lahoti

Question:

If **a** and **b** are vectors such that $\left|\mathbf{a} + \mathbf{b}\right| = \sqrt{29}$ and

$$\mathbf{a} \times \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right) \times \mathbf{b}$$

then a possible value of $(\mathbf{a} + \mathbf{b}) \cdot \left(-7\hat{i} + 2\hat{j} + 3\hat{k}\right)$ is

1) 0   2) 3   3) 4   4) 8

Solution:

Given :

Symbol	Value	Description
c	$\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$	Given Point
d	$\begin{pmatrix} -7 \\ 2 \\ 3 \end{pmatrix}$	Given Point
a + b	?	Desired Point

$$\mathbf{a} \times \mathbf{c} = \mathbf{c} \times \mathbf{b} \tag{4.1}$$

$$\mathbf{a} \times \mathbf{c} = -(\mathbf{b} \times \mathbf{c}) \tag{4.2}$$

$$(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = 0 \tag{4.3}$$

If cross product of 2 vectors is zero , this implies both the vectors are parallel. Thus ,

$$(\mathbf{a} + \mathbf{b}) \parallel \mathbf{c} \tag{4.4}$$

$$\therefore (\mathbf{a} + \mathbf{b}) = \lambda\mathbf{c}, \text{ where } \lambda \in \mathbb{R} \tag{4.5}$$

Equating the magnitudes , we get

$$\|(\mathbf{a} + \mathbf{b})\|^2 = \lambda^2 \|c\|^2 \tag{4.6}$$

$$29 = \lambda^2 29 \tag{4.7}$$

$$\lambda = \pm 1 \tag{4.8}$$

Thus,

$$(\mathbf{a} + \mathbf{b}) = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \text{ or } (\mathbf{a} + \mathbf{b}) = \begin{pmatrix} -2 \\ -3 \\ -4 \end{pmatrix} \tag{4.9}$$

Hence,

$$(\mathbf{a} + \mathbf{b})^\top \mathbf{d} = 4 \text{ or } -4 \tag{4.10}$$

Answer : Option (3)

Fig:2.10.61