Assignment 3: 2.5.18

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Question:
Let $\mathbf{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\mathbf{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$. Show that the vectors $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ are perpendicular to each other.

Solution:
Given vectors:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \tag{1}$$

$$\mathbf{b} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \tag{2}$$

∴ We have:

$$\mathbf{C} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{3}$$

$$\mathbf{D} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{4}$$

For two perpendicular vectors $\mathbf{P}$ and $\mathbf{Q}$:

$$\mathbf{P}^\top \mathbf{Q} = 0 \tag{5}$$

For vectors $\mathbf{C}$ and $\mathbf{D}$:

$$\mathbf{C}^\top \mathbf{D} = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix}^\top \begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{6}$$

$$= \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} \|\mathbf{a}\|^2 & \mathbf{a}^\top\mathbf{b} \\ \mathbf{a}^\top\mathbf{b} & \|\mathbf{b}\|^2 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{7}$$

$$= \|\mathbf{a}\|^2 - \mathbf{a}^\top\mathbf{b} + \mathbf{a}^\top\mathbf{b} - \|\mathbf{b}\|^2 \tag{8}$$

$$= 14 - 14 = 0 \tag{9}$$

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