4.8.17

AI25BTECH11021 - Abhiram Reddy N

Question

The foot of a perpendicular drawn from the point $(-2,-1,-3)$ on a plane is $(1,-3,3)$. Find the equation of the plane.

Answer

***Step 1**: Understanding the problem*

Given:

$$\mathbf{P} = \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix}$$

where $\mathbf{P}$ is the point from which the perpendicular is dropped and $\mathbf{F}$ is the foot of the perpendicular on the plane.

***Step 2**: Vector along the perpendicular*

The vector along the perpendicular from $\mathbf{P}$ to $\mathbf{F}$ is:

$$\mathbf{PF} = \mathbf{F} - \mathbf{P} = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ -1 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$$

***Step 3**: Normal vector to the plane*

Since the perpendicular from $\mathbf{P}$ to the plane hits the foot $\mathbf{F}$, the vector $\mathbf{PF}$ is normal to the plane. Therefore, the normal vector to the plane $\mathbf{n}$ is:

$$\mathbf{n} = \mathbf{PF} = \begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}$$

***Step 4**: Equation of the plane in vector form*

The vector form of the plane equation is:

$$\mathbf{n}^T(\mathbf{r} - \mathbf{F}) = 0$$

where $\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ is any point on the plane.

***Step 5**: Substitute values and expand*

Substituting **n** and **F**:

$$\begin{pmatrix}3 & -2 & 6\end{pmatrix}\left(\begin{pmatrix}x\\y\\z\end{pmatrix}-\begin{pmatrix}1\\-3\\3\end{pmatrix}\right)=0$$

$$\Rightarrow\begin{pmatrix}3 & -2 & 6\end{pmatrix}\begin{pmatrix}x-1\\y+3\\z-3\end{pmatrix}=0$$

$$3(x-1)-2(y+3)+6(z-3)=0$$

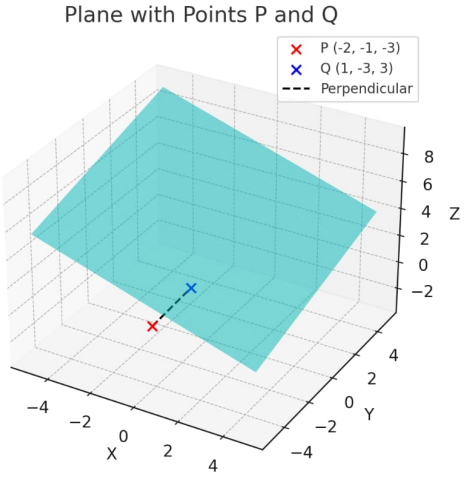
***Step 6**: Simplify the equation*

$$3x-3-2y-6+6z-18=0$$

$$3x-2y+6z-27=0$$

Final answer:

$$\boxed{3x-2y+6z=27}$$



Plot of the points and plane with perpendicular.
fig:1