4.7.34

EE25BTECH11036 - M Chanakya Srinivas

Problem

Find the equation of a plane at distance $3\sqrt{3}$ from the origin, whose normal is equally inclined to the coordinate axes.

Solution

Step 1: Normal vector

If the normal is equally inclined to all coordinate axes,

$$\mathbf{n} = \lambda \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \lambda \neq 0 \tag{1}$$

Step 2: General plane equation

The equation of a plane is

$$\mathbf{n}^\top \mathbf{x} = p \tag{2}$$

Step 3: Distance condition

The distance from origin to plane (**??**) is

$$d = \frac{|p|}{\|\mathbf{n}\|} \tag{3}$$

$$\|\mathbf{n}\| = |\lambda| \sqrt{1^2 + 1^2 + 1^2} = |\lambda| \sqrt{3} \tag{4}$$

So

$$3\sqrt{3} = \frac{|p|}{|\lambda| \sqrt{3}} \tag{5}$$

$$|p| = 9|\lambda| \tag{6}$$

Step 4: Simplification

Divide (**??**) by λ:

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{x} = \frac{p}{\lambda} \tag{7}$$

Since $\frac{p}{\lambda} = \pm 9$,

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{x} = 9 \tag{8}$$

$$\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{x} = -9 \tag{9}$$

Final Answer

Thus, the required planes are:

$$\mathbf{n}^\top \mathbf{x} = \pm 9, \quad \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{10}$$

Algebraic Form

Equivalently,

$$x + y + z = 9 \quad \text{or} \quad x + y + z = -9 \tag{11}$$

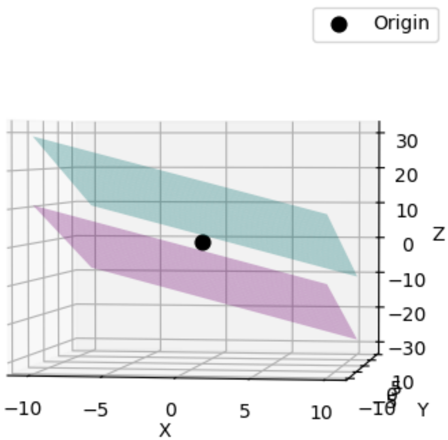

Fig. 1

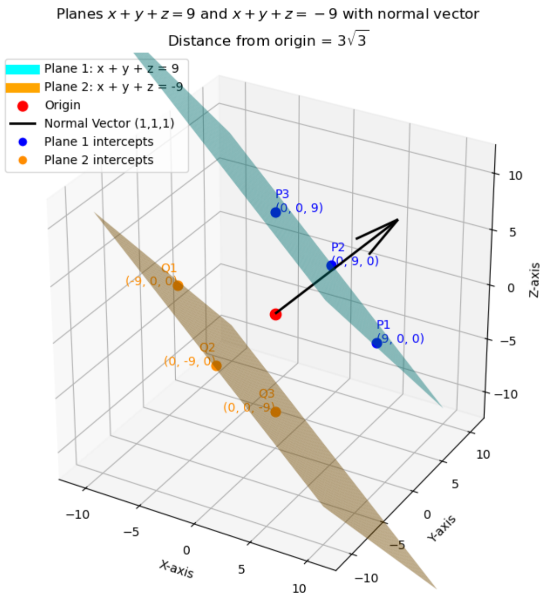
Planes $x + y + z = 9$ and $x + y + z = -9$ with normal vector
Distance from origin = $3\sqrt{3}$
Plane 1: x + y + z = 9
Plane 2: x + y + z = -9
Origin
Normal Vector (1,1,1)
Plane 1 intercepts
Plane 2 intercepts
P3
(0, 0, 9)
P2
(0, 9, 0)
P1
(9, 0, 0)
Q1
(-9, 0, 0)
Q2
(0, -9, 0)
Q3
(0, 0, -9)
X-axis
Y-axis
Z-axis
−10
−5
0
5
10

Fig. 2