12.560

EE25BTECH11026-Harsha

Question:
A scalar function is given by $f(x, y) = x^2 + y^2$. Take $\hat{i}$ and $\hat{j}$ as the unit vectors along the x and y axes, respectively. At $(x, y) = (3, 4)$, the direction along which f increases the fastest is

1) $\frac{1}{5}\left(4\hat{i} - 3\hat{j}\right)$ 2) $\frac{1}{5}\left(3\hat{i} - 4\hat{j}\right)$ 3) $\frac{1}{5}\left(3\hat{i} + 4\hat{j}\right)$ 4) $\frac{1}{5}\left(4\hat{i} + 3\hat{j}\right)$

Solution:
Let us solve the given question theoretically and then verify the solution computationally.

The direction vector along which the function $f(x, y)$ is given by the gradient direction vector of the function, which is given by

$$\nabla f(x, y) = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix} \tag{4.1}$$

$$\therefore \nabla f(x, y) = \begin{pmatrix} 2x \\ 2y \end{pmatrix} \tag{4.2}$$

At $(x, y) = (3, 4)$,

$$\nabla f(3, 4) = \begin{pmatrix} 6 \\ 8 \end{pmatrix} \tag{4.3}$$

$$\implies \text{Direction vector: } \frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{4.4}$$

Gradient and Unit Gradient at (3,4) for f(x,y) = x² + y²