2.9.19

AI25BTECH11003 - Bhavesh Gaikwad

Question: Let $\overrightarrow{a}$, $\overrightarrow{b}$, and $\overrightarrow{c}$ be three vectors such that $|\overrightarrow{a}| = 1$, $|\overrightarrow{b}| = 2$, and $|\overrightarrow{c}| = 3$. If the projection of $\overrightarrow{b}$ along $\overrightarrow{a}$ is equal to the projection of $\overrightarrow{c}$ along $\overrightarrow{a}$, and $\overrightarrow{b}$ and $\overrightarrow{c}$ are perpendicular to each other, then find $|3\overrightarrow{a} - 2\overrightarrow{b} + 2\overrightarrow{c}|$.

Solution:

Given: $\|\mathbf{a}\| = 1$, $\|\mathbf{b}\| = 2$, $\|\mathbf{c}\| = 3$

$\mathbf{b}^T \dfrac{\mathbf{a}}{\|\mathbf{a}\|} = \mathbf{c}^T \dfrac{\mathbf{a}}{\|\mathbf{a}\|}$

Since $\mathbf{b}$ and $\mathbf{c}$ are perpendicular: $\mathbf{b}^T\mathbf{c} = 0$

Let $\mathbf{v} = 3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c}$
$\|\mathbf{v}\|^2 = (3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c})^T(3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c})$
$= 9\|\mathbf{a}\|^2 + 4\|\mathbf{b}\|^2 + 4\|\mathbf{c}\|^2 - 12(\mathbf{a}^T\mathbf{b}) + -12(\mathbf{a}^T\mathbf{b}) - 8(\mathbf{b}^T\mathbf{c}) = 9 + 16 + 36$

$$\|\mathbf{v}\|^2 = 61 \quad \Rightarrow \quad \|\mathbf{v}\| = \sqrt{61} \tag{0.1}$$

$$\boxed{\|3\mathbf{a} - 2\mathbf{b} + 2\mathbf{c}\| = \sqrt{61}} \tag{0.2}$$

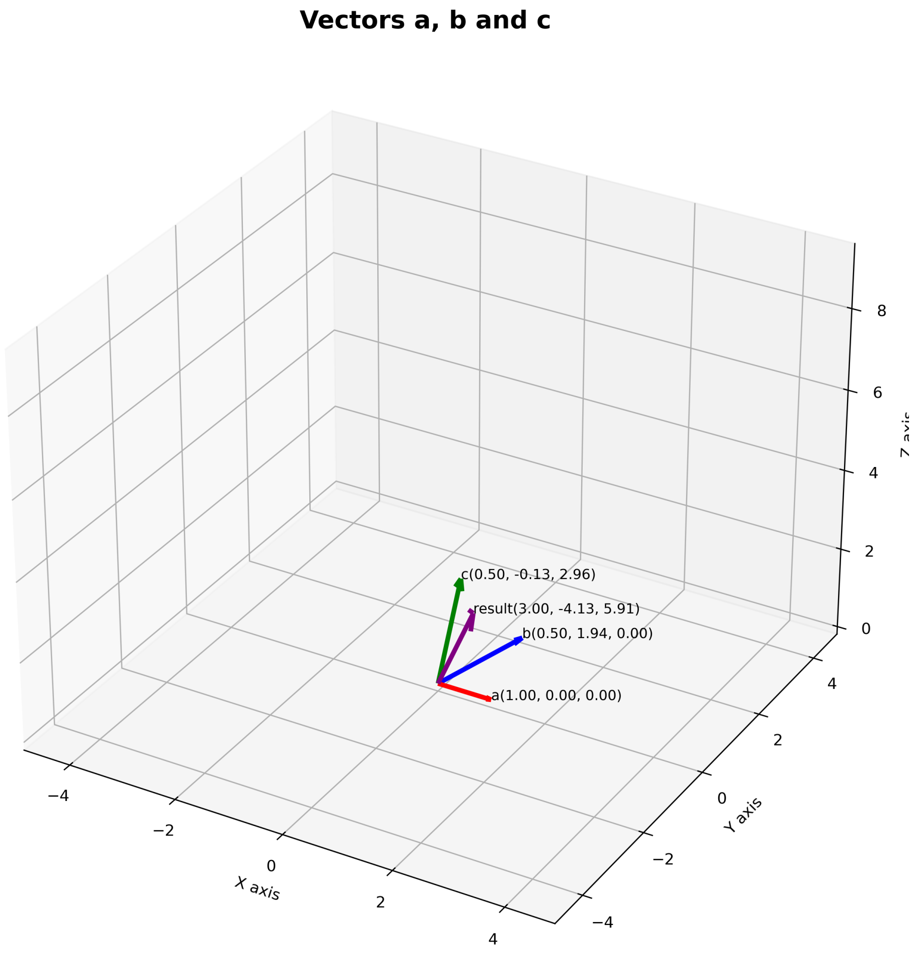

Fig. 0.1: Vector Representation