1.2.17

AI25BTECH11011-VARUN

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Question

Three vertices of a parallelogram ABCD are A(3,-1,2),B(1,-2,4),C(-1,1,2). Find the coordinates of the fourth vertex.

Theoretical Solution

Let the vertices of parallelogram ABCD be $\mathbf{A}\begin{pmatrix}3\\-1\\2\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}1\\-2\\4\end{pmatrix}$, $\mathbf{C}\begin{pmatrix}-1\\1\\2\end{pmatrix}$.

In any parallelogram, the diagonals bisect each other, so the midpoints of **AC** and **BD** are equal.

Equation

The midpoint of A $\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ **and B** $\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$ **is**

$$\mathbf{M_{AB}} = \begin{pmatrix} \frac{x_1+x_2}{2} \\ \frac{y_1+y_2}{2} \\ \frac{z_1+z_2}{2} \end{pmatrix} \tag{1}$$

Theoretical Solution

Midpoint of **AC**:

$$\mathbf{M_{AC}} = \begin{pmatrix} \frac{3+(-1)}{2} \\ \frac{-1+1}{2} \\ \frac{2+2}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}. \tag{2}$$

Let $\mathbf{D} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. Midpoint of **BD**:

$$\mathbf{M_{BD}} = \begin{pmatrix} \frac{1+x}{2} \\ \frac{-2+y}{2} \\ \frac{4+z}{2} \end{pmatrix}. \tag{3}$$

Theoretical Solution

Set $\mathbf{M_{AC}} = \mathbf{M_{BD}}$:

$$\frac{1+x}{2} = 1, \frac{-2+y}{2} = 0, \frac{4+z}{2} = 2. \tag{4}$$

Solving gives $x = 1, y = 2, z = 0$.

The fourth vertex is $\mathbf{D} \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$.

main C Code

```c
#include <stdio.h>

int main() {
    double A[3] = {3, -1, 2};
    double B[3] = {1, -2, 4};
    double C[3] = {-1, 1, 2};
    double D[3];

    D[0] = A[0] + C[0] - B[0];
    D[1] = A[1] + C[1] - B[1];
    D[2] = A[2] + C[2] - B[2];

    FILE *fp = fopen("coords.dat", "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }
```

main C Code

```
    fprintf(fp, "%lf %lf %lf\n", A[0], A[1], A[2]);
    fprintf(fp, "%lf %lf %lf\n", B[0], B[1], B[2]);
    fprintf(fp, "%lf %lf %lf\n", C[0], C[1], C[2]);
    fprintf(fp, "%lf %lf %lf\n", D[0], D[1], D[2]);

    fclose(fp);

    printf("Fourth vertex D: (%.2lf, %.2lf, %.2lf)\n", D[0], D
        [1], D[2]);

    return 0;
}
```

C Code

```c
#include <stdio.h>

void find_fourth_vertex(double A[3], double B[3], double C[3],
    double D[3]) {
    D[0] = A[0] + C[0] - B[0];
    D[1] = A[1] + C[1] - B[1];
    D[2] = A[2] + C[2] - B[2];
}
```

Python Code

```
from ctypes import CDLL, c_double, POINTER
import numpy as np
import matplotlib.pyplot as plt

# Load the shared library
lib = CDLL("./libvertex.so")

# Define argument and return types
lib.find_fourth_vertex.argtypes = [POINTER(c_double), POINTER(
    c_double), POINTER(c_double), POINTER(c_double)]

# Define points
A = (c_double * 3)(3, -1, 2)
B = (c_double * 3)(1, -2, 4)
C = (c_double * 3)(-1, 1, 2)
D = (c_double * 3)()
```

Python Code

```
# Call the C function
lib.find_fourth_vertex(A, B, C, D)

# Convert to Python list
D_point = [D[i] for i in range(3)]
print(f"Fourth vertex D: {D_point}")

# Read coordinates from .dat file (generated by main C code)
coords = np.loadtxt("coords.dat")

# Plot the parallelogram
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Points
x = coords[:, 0]
y = coords[:, 1]
z = coords[:, 2]
```

Python Code

```
# Plot parallelogram edges
ax.plot([x[0], x[1]], [y[0], y[1]], [z[0], z[1]], 'r-')
ax.plot([x[1], x[2]], [y[1], y[2]], [z[1], z[2]], 'r-')
ax.plot([x[2], x[3]], [y[2], y[3]], [z[2], z[3]], 'r-')
ax.plot([x[3], x[0]], [y[3], y[0]], [z[3], z[0]], 'r-')

# Plot diagonals
ax.plot([x[0], x[2]], [y[0], y[2]], [z[0], z[2]], 'b--')
ax.plot([x[1], x[3]], [y[1], y[3]], [z[1], z[3]], 'b--')

# Labels
ax.scatter(x, y, z, color='black')
for i, txt in enumerate(['A', 'B', 'C', 'D']):
    ax.text(x[i], y[i], z[i], txt)
plt.savefig("/home/gara-varun-kumar/ee1030-2025/ai25btech11011/
    matgeo/1.2.17/figs/Fig 1.png")
plt.show()
```

Plot