2.10.17

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Question

The vector

$$\frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k})$$

is

1. a unit vector
2. parallel to the vector $(-\hat{i} + \hat{j} - \frac{1}{2}\hat{k})$
3. perpendicular to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

.

Solution

Given

$$\mathbf{v} = \frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k}) = \begin{pmatrix} \frac{2}{3} \\ -\frac{2}{3} \\ \frac{1}{3} \end{pmatrix}.$$

$$\begin{aligned} \|\mathbf{v}\| &= \sqrt{\mathbf{v}^T\mathbf{v}} \\ &= \sqrt{\left(\frac{2}{3}\right)^2 + \left(-\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2} \\ &= 1 \end{aligned}$$

Hence, $\mathbf{v}$ is a unit vector.

Solution

Let

$$\mathbf{u} = \begin{pmatrix} -1 \\ 1 \\ -\frac{1}{2} \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}.$$

$$\mathbf{v} = -\frac{2}{3}\mathbf{u} \Rightarrow \mathbf{v} \parallel \mathbf{u},$$

$$\begin{aligned} \mathbf{v}^T\mathbf{w} &= \begin{pmatrix} \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \\ &= \frac{2}{3} \times 3 + \left(-\frac{2}{3}\right) \times 2 + \frac{1}{3} \times (-2) \\ &= 0 \Rightarrow \mathbf{v} \perp \mathbf{w}. \end{aligned}$$

C Code

```c
#include <math.h>

// Analyze vector v with respect to u and w
void analyze_vector(double vx, double vy, double vz,
                    double ux, double uy, double uz,
                    double wx, double wy, double wz,
                    double *mag, // magnitude of v
                    double *par_t, // scalar t if parallel, else
                        NAN
                    double *dot_vw) // dot product v.w
{
    // magnitude
    *mag = sqrt(vx*vx + vy*vy + vz*vz);

    // dot product
    *dot_vw = vx*wx + vy*wy + vz*wz;
```

C Code

```
    // check parallel using cross product
    double cx = vy*uz - vz*uy;
    double cy = vz*ux - vx*uz;
    double cz = vx*uy - vy*ux;

    if (cx == 0 && cy == 0 && cz == 0) {
        // parallel --> find scalar t
        if (ux != 0) *par_t = vx / ux;
        else if (uy != 0) *par_t = vy / uy;
        else if (uz != 0) *par_t = vz / uz;
        else *par_t = NAN; // u is zero vector
    } else {
        *par_t = NAN; // not parallel
    }
}
```

Python+C Code

```
   import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load shared library (relative path)
vec = ctypes.CDLL("./libvec.so")

# Define C function signature
vec.analyze_vector.argtypes = (
    ctypes.c_double, ctypes.c_double, ctypes.c_double, # v
    ctypes.c_double, ctypes.c_double, ctypes.c_double, # u
    ctypes.c_double, ctypes.c_double, ctypes.c_double, # w
    ctypes.POINTER(ctypes.c_double), # mag
    ctypes.POINTER(ctypes.c_double), # par_t
    ctypes.POINTER(ctypes.c_double) # dot_vw
)
```

Python+C Code

```
 vec.analyze_vector.restype = None

def analyze_vector(v: np.ndarray, u: np.ndarray, w: np.ndarray):
    """Call the C function via ctypes."""
    mag = ctypes.c_double()
    par_t = ctypes.c_double()
    dot_vw = ctypes.c_double()

    vec.analyze_vector(
        v[0], v[1], v[2],
        u[0], u[1], u[2],
        w[0], w[1], w[2],
        ctypes.byref(mag),
        ctypes.byref(par_t),
        ctypes.byref(dot_vw)
    )
    return mag.value, par_t.value, dot_vw.value
```

Python+C Code

```
if __name__ == "__main__":
    v = np.array([2/3, -2/3, 1/3])
    u = np.array([-1, 1, -0.5])
    w = np.array([3, 2, -2])

    mag, par_t, dot_vw = analyze_vector(v, u, w)

    print("||v|| =", mag)
    if np.isnan(par_t):
        print("v is NOT parallel to u")
    else:
        print(f"v is parallel to u, scalar t = {par_t}")
    print("v^T w =", dot_vw)

    # === Plotting ===
    fig = plt.figure()
    ax = fig.add_subplot(111, projection="3d")
```

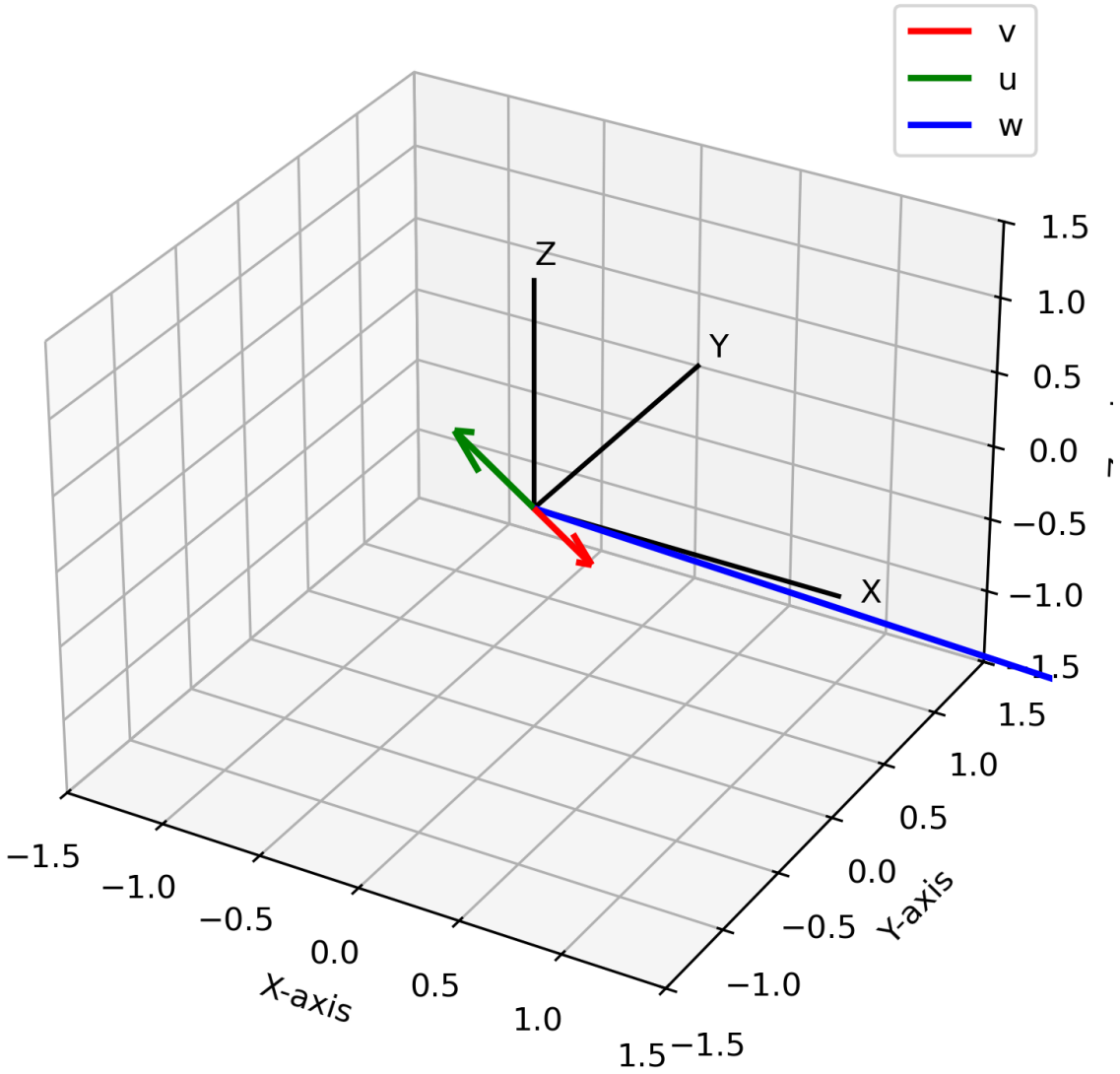
Python+C Code

```
origin = np.array([0, 0, 0])
    ax.quiver(*origin, *v, color="r", label="v")
    ax.quiver(*origin, *u, color="g", label="u")
    ax.quiver(*origin, *w, color="b", label="w")

    ax.set_xlabel("X")
    ax.set_ylabel("Y")
    ax.set_zlabel("Z")
    ax.legend()


# Save before show
plt.savefig("/storage/emulated/0/matrix/Matgeo/2.10.17/figs/
    Figure_1.png", dpi=300, bbox_inches='tight')
plt.show()
```

Plot



Python Code

```
import numpy as np
import matplotlib.pyplot as plt

# === Vector analysis functions ===
def magnitude(v):
    return np.sqrt(v.T @ v) # sqrt(v^T v)

def is_parallel(v, u, tol=1e-9):
    """Check if v is parallel to u. Return scalar t if v = t u,
        else None."""
    if np.allclose(u, 0): # u is zero vector
        return None
    ratios = []
    for vi, ui in zip(v, u):
        if abs(ui) > tol:
            ratios.append(vi / ui)
        elif abs(vi) > tol:
            return None # ui = 0 but vi != 0 --> not parallel
```

Python Code

```
if len(ratios) == 0:
        return None
    t = ratios[0]
    if all(abs(r - t) < tol for r in ratios):
        return t
    return None

def dot_product(v, w):
    return v.T @ w # v^T w

# === Example usage ===
v = np.array([2/3, -2/3, 1/3])
u = np.array([-1, 1, -0.5])
w = np.array([3, 2, -2])

# Analysis
mag_v = magnitude(v)
t = is_parallel(v, u)
dot_vw = dot_product(v, w)
```

Python Code

```
    print("||v|| =", mag_v)
if t is None:
    print("v is NOT parallel to u")
else:
    print(f"v is parallel to u, scalar t = {t}")
print("v^T w =", dot_vw)

# === Plotting ===
fig = plt.figure(figsize=(6,6))
ax = fig.add_subplot(111, projection="3d")

origin = np.array([0,0,0])

# Draw vectors
ax.quiver(*origin, *v, color="r", linewidth=2, label="v")
ax.quiver(*origin, *u, color="g", linewidth=2, label="u")
ax.quiver(*origin, *w, color="b", linewidth=2, label="w")
```

Python Code

```
    # Axes
ax.plot([0,1.5],[0,0],[0,0], color="k")
ax.plot([0,0],[0,1.5],[0,0], color="k")
ax.plot([0,0],[0,0],[0,1.5], color="k")
ax.text(1.6,0,0,"X",color="k")
ax.text(0,1.6,0,"Y",color="k")
ax.text(0,0,1.6,"Z",color="k")
ax.set_xlim([-1.5,1.5])
ax.set_ylim([-1.5,1.5])
ax.set_zlim([-1.5,1.5])
ax.set_xlabel("X-axis")
ax.set_ylabel("Y-axis")
ax.set_zlabel("Z-axis")
ax.legend()

    # Save before show
plt.savefig("/storage/emulated/0/matrix/Matgeo/2.10.17/figs/
    Figure_1.png", dpi=300, bbox_inches='tight')
plt.show()
```