4.13.90

AI25BTECH11021 - Abhiram Reddy N

Question

If the distance of the point $\mathbf{P}(1, -2, 1)$ from the plane

$$x + 2y - 2z = \alpha, \quad \text{where } \alpha > 0,$$

is 5, then the foot of the perpendicular from $\mathbf{P}$ to the plane is:

1) $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$
2) $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$
3) $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$
4) $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

Solution

Step 1: Let the plane be $\mathbf{n}^T\mathbf{x} = \alpha$

Let the normal vector to the plane be

$$\mathbf{n} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

The distance D from point $\mathbf{P}$ to the plane is given by:

$$D = \frac{|\mathbf{n}^T\mathbf{P} - \alpha|}{\|\mathbf{n}\|} \tag{4.1}$$

Given that $D = 5$, we calculate:

$$\mathbf{n}^T\mathbf{P} = \begin{bmatrix} 1 & 2 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = 1 - 4 - 2 = -5 \tag{4.2}$$

$$\|\mathbf{n}\| = \sqrt{1^2 + 2^2 + (-2)^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3 \tag{4.3}$$

Substituting (**??**) and (**??**) into (**??**):

$$5 = \frac{|-5 - \alpha|}{3} \Rightarrow |-5 - \alpha| = 15 \Rightarrow -5 - \alpha = \pm 15 \tag{4.4}$$

Solving:

$$\text{Case 1: } -5 - \alpha = 15 \Rightarrow \alpha = -20 \quad (\text{Invalid since } \alpha > 0)$$

$$\text{Case 2: } -5 - \alpha = -15 \Rightarrow \alpha = 10$$

So the plane becomes:

$$x + 2y - 2z = 10 \tag{4.5}$$

Step 2: Find foot of perpendicular

Let the foot of the perpendicular be:

$$\mathbf{Q} = \mathbf{P} + \lambda\mathbf{n} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 1+\lambda \\ -2+2\lambda \\ 1-2\lambda \end{bmatrix} \tag{4.6}$$

Substitute into the plane equation (**??**):

$$(1+\lambda) + 2(-2+2\lambda) - 2(1-2\lambda) = 10$$

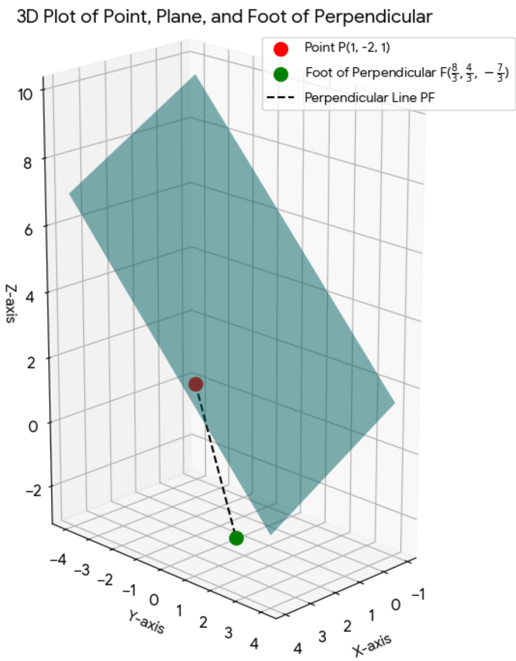
$$1 + \lambda - 4 + 4\lambda - 2 + 4\lambda = 10 \Rightarrow -5 + 9\lambda = 10 \Rightarrow \lambda = \frac{15}{9} = \frac{5}{3}$$

Substitute $\lambda = \frac{5}{3}$ into equation (**??**):

$$\mathbf{Q} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \frac{5}{3} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} \frac{8}{3} \\ \frac{4}{3} \\ -\frac{7}{3} \end{bmatrix} \tag{4.7}$$

Answer

$$\boxed{\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)} \quad \text{Option 1}$$



Plot of the curves
Fig1