MatGeo Assignment 1.6.22

AI25BTECH11007

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Question

Show that the points A(2, -3, 4), B(-1, 2, 1) and C(0, 1/3, 2) are collinear.

Solution

Let us solve the given equation theoretically and then verify the solution computationally According to the question,
Show that the points

$$A(2, -3, 4), B(-1, 2, 1), C(0, 1/3, 2)$$

are collinear (rank method).

Theoretical Solution

$$A = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \quad B = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 \\ \frac{1}{3} \\ 2 \end{pmatrix} \tag{1}$$

$$B - A = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} -3 \\ 5 \\ -3 \end{pmatrix} \tag{2}$$

$$C - A = \begin{pmatrix} 0 \\ \frac{1}{3} \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} -2 \\ \frac{10}{3} \\ -2 \end{pmatrix} \tag{3}$$

Form the 3×2 matrix whose columns are $B - A$ and $C - A$:

$$M = \begin{bmatrix} B - A & C - A \end{bmatrix} = \begin{pmatrix} -3 & -2 \\ 5 & \frac{10}{3} \\ -3 & -2 \end{pmatrix} \tag{4}$$

The vectors $B - A$ and $C - A$ are linearly dependent (hence the three points are collinear) iff the rank of M is 1. For a 3×2 matrix this is equivalent to all 2×2 minors of M vanishing. Compute the minors:
Minor from rows 1 and 2:

$$\begin{vmatrix} -3 & -2 \\ 5 & \frac{10}{3} \end{vmatrix} = (-3) \cdot \left(\frac{10}{3}\right) - (-2) \cdot 5 = -10 - (-10) = 0 \tag{5}$$

Minor from rows 1 and 3:

$$\begin{vmatrix} -3 & -2 \\ -3 & -2 \end{vmatrix} = (-3)(-2) - (-2)(-3) = 6 - 6 = 0 \tag{6}$$

Minor from rows 2 and 3:

$$\begin{vmatrix} 5 & \frac{10}{3} \\ -3 & -2 \end{vmatrix} = 5 \cdot (-2) - \left(\frac{10}{3}\right) \cdot (-3) = -10 - (-10) = 0 \tag{7}$$

All 2×2 minors are zero, so $\text{rank}(M) = 1$. Therefore the columns are linearly dependent, i.e,

$$\text{rank}\begin{bmatrix} B - A & C - A \end{bmatrix} = 1. \tag{8}$$

Hence the vectors $B - A$ and $C - A$ are proportional and the points.

Plot

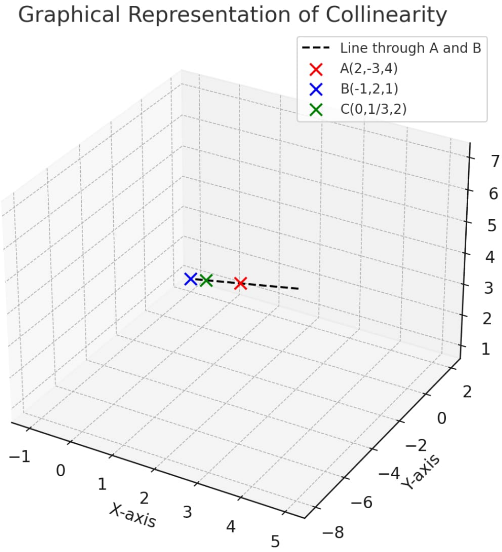

Figure: Image Visual

Conclusion

As the rank of the matrix M is 1, the three given points are collinear. From the figure it is clearly verified that theoritical solution matches with the computational solution.