12.339

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Question

Question:
If

$$\mathbf{A} = \begin{pmatrix} 3 & -3 \\ -3 & 4 \end{pmatrix} \tag{1}$$

then

$$\det\left(-\mathbf{A}^2 + 7\mathbf{A} - 3\mathbf{I}\right) \tag{2}$$

is

Solution

The characteristic equation of matrix $\mathbf{A}$ is

$$f(\lambda) = |\mathbf{A} - \lambda\mathbf{I}| = 0 \tag{3}$$

$$\left| \begin{pmatrix} 3-\lambda & -3 \\ -3 & 4-\lambda \end{pmatrix} \right| = 0 \tag{4}$$

$$\implies (3-\lambda)(4-\lambda) - 9 = 0 \tag{5}$$

$$-\lambda^2 + 7\lambda - 3 = 0 \tag{6}$$

Solution

According to Cayley-Hamilton Theorem:

$$f(\lambda) = f(\mathbf{A}) \tag{7}$$

$$\therefore -\mathbf{A}^2 + 7\mathbf{A} - 3\mathbf{I} = 0 \tag{8}$$

$$\therefore \left|-\mathbf{A}^2 + 7\mathbf{A} - 3\mathbf{I}\right| = 0 \tag{9}$$

C Code

```c
#include <stdio.h>

double det(double *mat) {
    return mat[0]*mat[3] - mat[1]*mat[2];
}
```

Python + C Code

```
import numpy as np
import ctypes
lib = ctypes.CDLL("./libcode.so")
lib.det.argtypes = [ctypes.POINTER(ctypes.c_double)]
lib.det.restype = ctypes.c_double

A = np.array([[3, -3],
              [-3, 4]], dtype=np.float64)

I = np.eye(2, dtype=np.float64)
expr = -A @ A + 7*A - 3*I
mat_flat = expr.flatten()
det_value = lib.det(mat_flat.ctypes.data_as(ctypes.POINTER(ctypes
    .c_double)))
print("Matrix (-A^2 + 7A - 3I):\n", expr)
print("Determinant =", det_value)
```

Python Code

```
import numpy as np
A = np.array([[3, -3],
              [-3, 4]])
I = np.eye(2)

expr = -A @ A + 7*A - 3*I

det_value = np.linalg.det(expr)

print("Matrix A:\n", A)
print("Matrix (-A^2 + 7A - 3I):\n", expr)
print("Determinant =", det_value)
```