question

Find the values of k for which the points $A(k+1, 2k)$, $B(3k, 2k+3)$, $C(5k-1, 5k)$ are collinear.

Theoritical solution

First, form the difference vectors:

$$B - A = \begin{pmatrix} 3k - (k+1) \\ (2k+3) - 2k \end{pmatrix} = \begin{pmatrix} 2k-1 \\ 3 \end{pmatrix}$$

$$C - A = \begin{pmatrix} (5k-1) - (k+1) \\ 5k - 2k \end{pmatrix} = \begin{pmatrix} 4k-2 \\ 3k \end{pmatrix}$$

Form the matrix:

$$M = \begin{pmatrix} 2k-1 & 3 \\ 4k-2 & 3k \end{pmatrix}$$

For the points to be collinear, the rank of M must be 1. Perform the row operation:

$$R_2 \to -\frac{4k-2}{2k-1} R_1 + R_2 \qquad (\text{for } 2k-1 \neq 0)$$

Which gives:

$$\begin{pmatrix} 2k-1 & 3 \\ 0 & 3k - \frac{3(4k-2)}{2k-1} \end{pmatrix}$$

Theoritical solution

Set the second row entry to zero for rank 1:

$$3k - \frac{3(4k-2)}{2k-1} = 0$$

$$3k = \frac{3(4k-2)}{2k-1}$$

$$3k(2k-1) = 3(4k-2)$$

$$6k^2 - 3k = 12k - 6$$

$$6k^2 - 15k + 6 = 0$$

$$2k^2 - 5k + 2 = 0$$

Solving for k:

$$k = \frac{5 \pm 3}{4}$$

$$k = 2 \quad \text{or} \quad k = \frac{1}{2}$$

C code

```c
int areCollinear(double Ax, double Ay, double Bx, double By,
    double Cx, double Cy) {
    double BA_x = Bx - Ax;
    double BA_y = By - Ay;
    double CA_x = Cx - Ax;
    double CA_y = Cy - Ay;

    if (BA_x * CA_y == BA_y * CA_x) {
        return 1; // Points are collinear
    } else {
        return 0; // Points are not collinear
    }
}
```

Python Plotting Code - Part 1

```
import matplotlib.pyplot as plt
import numpy as np
# Part 1: Define points for each k and plot points
k_values = [0.5, 2] # values of k for collinearity
plt.figure(figsize=(8,6))
for k in k_values:
    A = np.array([k+1, 2*k])
    B = np.array([3*k, 2*k+3])
    C = np.array([5*k-1, 5*k])
     plt.scatter(*A, color='red')
    plt.scatter(*B, color='green')
    plt.scatter(*C, color='blue')
     plt.text(A[0], A[1], 'A', fontsize=12, color='red', ha='
         right')
    plt.text(B[0], B[1], 'B', fontsize=12, color='green', ha='
        right')
    plt.text(C[0], C[1], 'C', fontsize=12, color='blue', ha='
        right')
```

Python Plotting Code - Part 2

```
    x_line = np.linspace(min(A[0], B[0], C[0])-1, max(A[0], B[0],
        C[0])+1, 100)
    y_line = ((B[1]-A[1])/(B[0]-A[0])) * (x_line - A[0]) + A[1]
    plt.plot(x_line, y_line, label=f'Line through A and B for k={
        k}')

plt.title('Collinearity of Points A, B, C for values of k')
plt.xlabel('x')
plt.ylabel('y')
plt.legend()
plt.grid(True)
plt.show()
```

plot

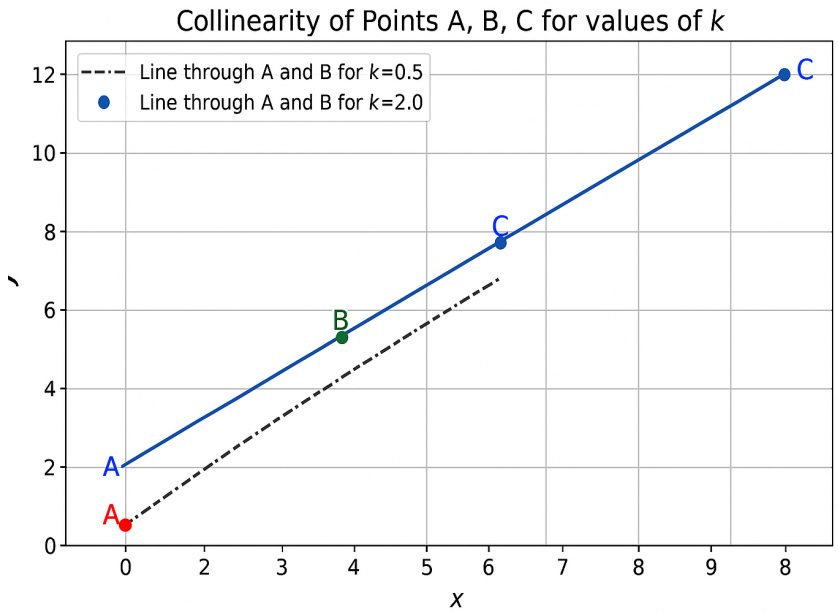

Figure: collinear