2.7.18

EE25BTECH11006 - ADUDOTLA SRIVIDYA

Question:
Vertices of a $\triangle ABC$ are $\mathbf{A}(4,6)$, $\mathbf{B}(1,5)$ and $\mathbf{C}(7,2)$. A line segment DE is drawn intersecting AB and AC at D and E respectively such that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Calculate the area of $\triangle ADE$ and compare it with the area of $\triangle ABC$.

Solution:
Let

$$\mathbf{A} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$$

Point $\mathbf{D}$ divides AB in ratio 1 : 2:

$$\mathbf{D} = \frac{2\mathbf{A} + 1\mathbf{B}}{3} = \frac{2\begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \end{pmatrix}}{3} = \begin{pmatrix} 3 \\ \frac{17}{3} \end{pmatrix}$$

Point $\mathbf{E}$ divides AC in ratio 1 : 2:

$$\mathbf{E} = \frac{2\mathbf{A} + 1\mathbf{C}}{3} = \frac{2\begin{pmatrix} 4 \\ 6 \end{pmatrix} + \begin{pmatrix} 7 \\ 2 \end{pmatrix}}{3} = \begin{pmatrix} 5 \\ \frac{14}{3} \end{pmatrix}$$

Area of a triangle with vertices $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ is

$$\Delta = \frac{1}{2}\left| \det\begin{pmatrix} x_Q - x_P & x_R - x_P \\ y_Q - y_P & y_R - y_P \end{pmatrix} \right|$$

So,

$$\begin{aligned}\Delta_{ABC} &= \tfrac{1}{2}\left|\det\begin{pmatrix}1-4 & 7-4\\ 5-6 & 2-6\end{pmatrix}\right|\\ &= \tfrac{1}{2}\left|\det\begin{pmatrix}-3 & 3\\ -1 & -4\end{pmatrix}\right|\\ &= \tfrac{1}{2}(12+3)\\ &= \tfrac{15}{2}\end{aligned}$$

Similarly,

$$\begin{aligned}\Delta_{ADE} &= \tfrac{1}{2}\left|\det\begin{pmatrix}3-4 & 5-4\\ \frac{17}{3}-6 & \frac{14}{3}-6\end{pmatrix}\right|\\ &= \tfrac{1}{2}\left|\det\begin{pmatrix}-1 & 1\\ -\frac{1}{3} & -\frac{4}{3}\end{pmatrix}\right|\\ &= \tfrac{1}{2}\left(\tfrac{4}{3}+\tfrac{1}{3}\right)\\ &= \tfrac{5}{6}\end{aligned}$$

Thus,

$$\frac{\Delta_{ADE}}{\Delta_{ABC}} = \frac{\frac{5}{6}}{\frac{15}{2}} = \frac{1}{9}$$

Therefore,

$$\text{Area of } \triangle ADE = \tfrac{1}{9} \text{ of area of } \triangle ABC.$$

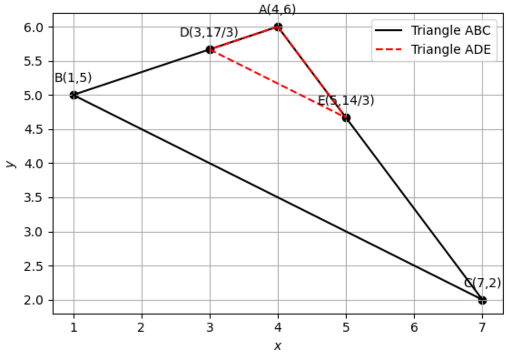

Fig. 0.1: Triangle ABC with inner triangle ADE