

2.4.31

EE25BTECH11034 - Kishora Karthik

Question:

Check if the point $\mathbf{A}(2, 7)$ lies on the perpendicular bisector of line segment joining the points $\mathbf{P}(6, 5)$ and $\mathbf{Q}(0, -4)$.

Formulae:

The equation of the perpendicular bisector of PQ is

$$\left(\mathbf{A} - \frac{\mathbf{P} + \mathbf{Q}}{2} \right)^T (\mathbf{P} - \mathbf{Q}) = 0 \quad (1)$$

Solution: The given points are,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 7 \end{pmatrix} \quad (2)$$

$$\mathbf{P} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} \quad (3)$$

$$\mathbf{Q} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \quad (4)$$

$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix} 6 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ -4 \end{pmatrix} \quad (5)$$

$$\mathbf{P} - \mathbf{Q} = \begin{pmatrix} 6 \\ 9 \end{pmatrix} \quad (6)$$

$$\frac{\mathbf{P} + \mathbf{Q}}{2} = \frac{\begin{pmatrix} 6 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix}}{2} \quad (7)$$

$$\frac{\mathbf{P} + \mathbf{Q}}{2} = \frac{\begin{pmatrix} 6 \\ 1 \end{pmatrix}}{2} \quad (8)$$

$$\frac{\mathbf{P} + \mathbf{Q}}{2} = \begin{pmatrix} 3 \\ 0.5 \end{pmatrix} \quad (9)$$

$$\mathbf{A} - \frac{\mathbf{P} + \mathbf{Q}}{2} = \begin{pmatrix} 2 \\ 7 \end{pmatrix} - \begin{pmatrix} 3 \\ 0.5 \end{pmatrix} \quad (10)$$

$$\mathbf{A} - \frac{\mathbf{P} + \mathbf{Q}}{2} = \begin{pmatrix} -1 \\ 6.5 \end{pmatrix} \quad (11)$$

$$\left(\mathbf{A} - \frac{\mathbf{P} + \mathbf{Q}}{2} \right)^\top (\mathbf{P} - \mathbf{Q}) = \begin{pmatrix} -1 & 6.5 \end{pmatrix} \begin{pmatrix} 6 \\ 9 \end{pmatrix} \quad (12)$$

$$\left(\mathbf{A} - \frac{\mathbf{P} + \mathbf{Q}}{2} \right)^\top (\mathbf{P} - \mathbf{Q}) = (-1)(6) + (6.5)(9) \quad (13)$$

$$\left(\mathbf{A} - \frac{\mathbf{P} + \mathbf{Q}}{2} \right)^\top (\mathbf{P} - \mathbf{Q}) = 52.5 \neq 0 \quad (14)$$

The equation of perpendicular bisector is not satisfied.

Therefore, point \mathbf{A} does not lie on the perpendicular bisector of line segment joining the points \mathbf{P} and \mathbf{Q} .