

1.6.9

EE25BTECH11059 - Vaishnavi Ramkrishna Anantheertha

Question:

if three points $\begin{pmatrix} h \\ 0 \end{pmatrix}, \begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} 0 \\ k \end{pmatrix}$ lie on a line, show that

$$\frac{a}{h} + \frac{b}{k} = 1$$

Solution:

Point	Name
$(h, 0)$	Point A
$(0, k)$	Point B
(a, b)	Point C

TABLE 0: Variables Used

If the rank of the Collinearity matrix is 1, then the points are collinear
The Collinearity matrix is given by

$$(\mathbf{C} - \mathbf{A} \quad \mathbf{B} - \mathbf{A})^T = \begin{pmatrix} a-h & b \\ -h & k \end{pmatrix} \quad (0.1)$$

$$\xleftrightarrow{R_1 \rightarrow \frac{R_1}{a-h}} \begin{pmatrix} 1 & \frac{b}{a-h} \\ -h & k \end{pmatrix} \quad (0.2)$$

$$\xleftrightarrow{R_2 \rightarrow \frac{R_2}{-h}} \begin{pmatrix} 1 & \frac{b}{a-h} \\ 1 & \frac{-k}{h} \end{pmatrix} \quad (0.3)$$

$$\xleftrightarrow{R_2 \rightarrow R_2 - R_1} \begin{pmatrix} 1 & \frac{b}{a-h} \\ 0 & \frac{-k}{h} - \frac{b}{a-h} \end{pmatrix} \quad (0.4)$$

since the rank of matrix=1

$$\frac{-k}{h} - \frac{b}{a-h} = 0 \quad (0.5)$$

$$\Rightarrow -bh - ka + kh = 0 \quad (0.6)$$

$$(\text{Dividing Eq.(0.6) by } kh) \Rightarrow \frac{a}{h} + \frac{b}{k} = 1 \quad (0.7)$$

Refer to Fig

Collinear Points $(h, 0, 0)$, $(0, k, 0)$, and (a, b, c)

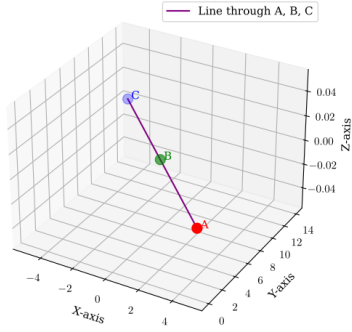


Fig. 0.1