12.18

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Question

X is 1 km northeast of **Y**. **Y** is 1 km southeast of **Z**. **W** is 1 km west of **Z**. **P** is 1 km south of **W**. **Q** is 1 km east of **P**. What is the distance between **X** and **Q** in km?

1. 1
2. $\sqrt{2}$
3. $\sqrt{3}$
4. 2

Solution

Let $\mathbf{X}$ be $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Every subsequent vector can be expressed as a rotation by a particular angle θ with respect to the previous vector.
The matrix to rotate a vector by angle θ about the origin is:

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{1}$$

Solution

X is 1km north-east of **Y**, so **Y** is 1km south-west of **X**. Therefore:

$$\mathbf{Y}-\mathbf{X}=\begin{pmatrix}\cos 225 & -\sin 225\\ \sin 225 & \cos 225\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{2}$$

$$\mathbf{Y}-\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}\frac{-1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{pmatrix} \tag{3}$$

$$\mathbf{Y}=\begin{pmatrix}\frac{-1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{pmatrix} \tag{4}$$

Solution

Y is 1km south-east of **Z**, so **Z** is 1km north-west of **Y**. Therefore:

$$\mathbf{Z}-\mathbf{Y}=\begin{pmatrix}\cos 135 & -\sin 135\\ \sin 135 & \cos 135\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{5}$$

$$\mathbf{Y}-\begin{pmatrix}\frac{-1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{pmatrix}=\begin{pmatrix}\frac{-1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix} \tag{6}$$

$$\mathbf{Z}=\begin{pmatrix}-\sqrt{2}\\ 0\end{pmatrix} \tag{7}$$

Solution

W is 1km west of **Z**. Therefore:

$$\mathbf{W} - \mathbf{Z} = \begin{pmatrix} \cos 180 & -\sin 180 \\ \sin 180 & \cos 180 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{8}$$

$$\mathbf{Y} - \begin{pmatrix} -\sqrt{2} \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \tag{9}$$

$$\mathbf{W} = \begin{pmatrix} -1 - \sqrt{2} \\ 0 \end{pmatrix} \tag{10}$$

Solution

P is 1km south of **W**. Therefore:

$$\mathbf{P}-\mathbf{W}=\begin{pmatrix}\cos 270 & -\sin 270\\ \sin 270 & \cos 270\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{11}$$

$$\mathbf{P}-\begin{pmatrix}-1-\sqrt{2}\\0\end{pmatrix}=\begin{pmatrix}0\\-1\end{pmatrix} \tag{12}$$

$$\mathbf{P}=\begin{pmatrix}-1-\sqrt{2}\\-1\end{pmatrix} \tag{13}$$

Solution

Q is 1km east of **P**. Therefore:

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} \cos 0 & -\sin 0 \\ \sin 0 & \cos 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{14}$$

$$\mathbf{Q} - \begin{pmatrix} -1-\sqrt{2} \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{15}$$

$$\mathbf{Q} = \begin{pmatrix} -\sqrt{2} \\ -1 \end{pmatrix} \tag{16}$$

The distance between $\mathbf{X} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} -\sqrt{2} \\ -1 \end{pmatrix}$ is:

$$\|\mathbf{X} - \mathbf{Q}\| = \left\| \begin{pmatrix} \sqrt{2} \\ 1 \end{pmatrix} \right\| \tag{17}$$

$$\sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3} \tag{18}$$

Code

Codes Permalink

Figure

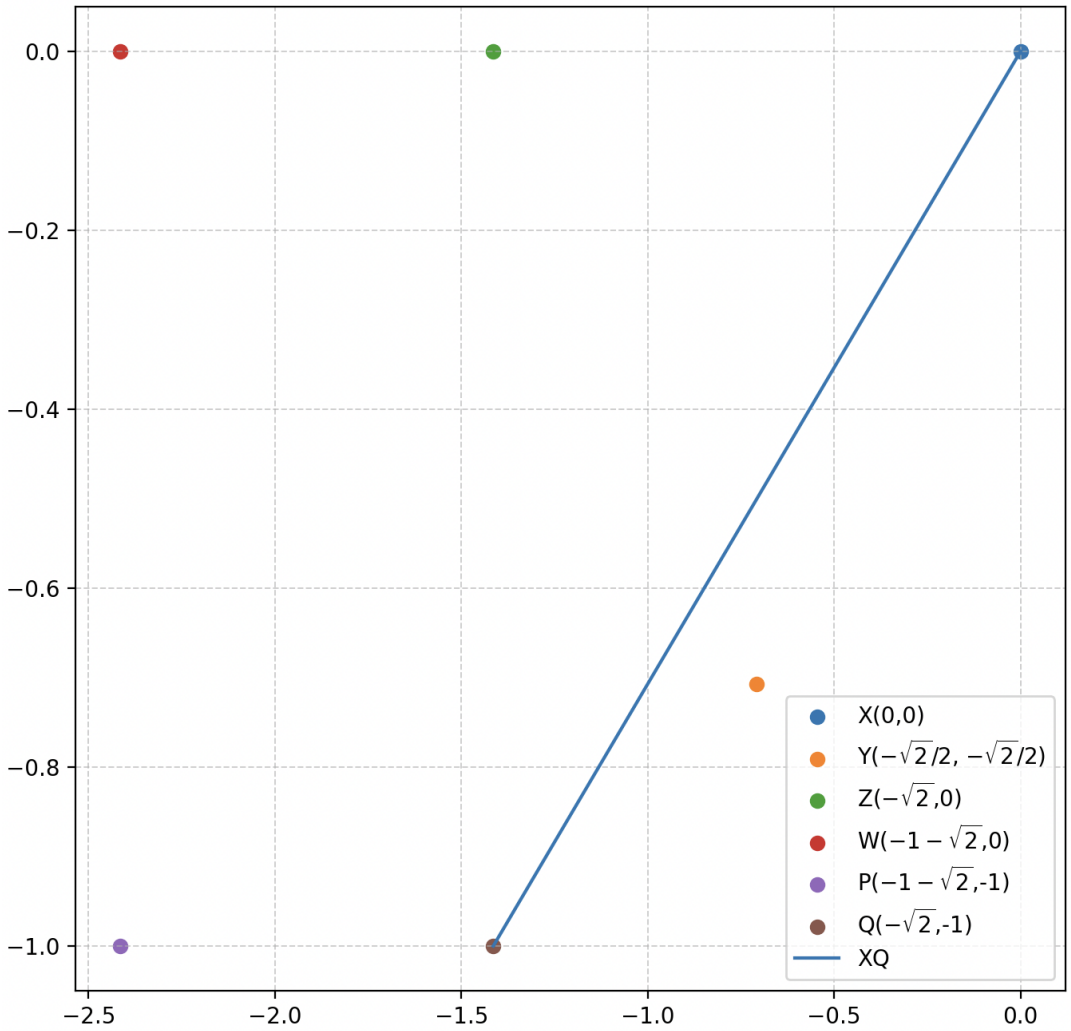

Figure: Plot