5.2.55

EE25BTECH11052 - Shriyansh Kalpesh Chawda

Question:
Solve the following system of linear equations.

$$\frac{2}{x}+\frac{3}{y}=13 \quad \frac{5}{x}+\frac{4}{y}=-2$$

Solution:
Let

$$u=\frac{1}{x}, \quad v=\frac{1}{y}. \tag{1}$$

The given system becomes

$$2u+3v=13 \tag{2}$$
$$5u+4v=-2 \tag{3}$$

In matrix form:

$$\begin{pmatrix}2 & 3\\5 & 4\end{pmatrix}\begin{pmatrix}u\\v\end{pmatrix}=\begin{pmatrix}13\\-2\end{pmatrix}. \tag{4}$$

Let

$$A=\begin{pmatrix}2 & 3\\5 & 4\end{pmatrix}, \quad \mathbf{b}=\begin{pmatrix}13\\-2\end{pmatrix}. \tag{5}$$

We solve using Gauss-Jordan elimination by reducing the augmented matrix $[A|\mathbf{b}]$ to $[I|\mathbf{x}]$.

$$\left[\begin{array}{cc|c}2 & 3 & 13\\5 & 4 & -2\end{array}\right] \xrightarrow{R_1\to\frac{1}{2}R_1} \left[\begin{array}{cc|c}1 & \frac{3}{2} & \frac{13}{2}\\5 & 4 & -2\end{array}\right] \tag{6}$$
$$\xrightarrow{R_2\to R_2-5R_1} \left[\begin{array}{cc|c}1 & \frac{3}{2} & \frac{13}{2}\\0 & -\frac{7}{2} & -\frac{69}{2}\end{array}\right] \tag{7}$$
$$\xrightarrow{R_2\to -\frac{2}{7}R_2} \left[\begin{array}{cc|c}1 & \frac{3}{2} & \frac{13}{2}\\0 & 1 & \frac{69}{7}\end{array}\right] \tag{8}$$
$$\xrightarrow{R_1\to R_1-\frac{3}{2}R_2} \left[\begin{array}{cc|c}1 & 0 & -\frac{58}{7}\\0 & 1 & \frac{69}{7}\end{array}\right] \tag{9}$$

From the reduced row echelon form, we have the solution:

$$\begin{pmatrix}u\\v\end{pmatrix}=\begin{pmatrix}-\frac{58}{7}\\\frac{69}{7}\end{pmatrix} \tag{10}$$

Back substituting:

$$u = \frac{1}{x} = -\frac{58}{7} \implies x = -\frac{7}{58}, \tag{11}$$

$$v = \frac{1}{y} = \frac{69}{7} \implies y = \frac{7}{69}. \tag{12}$$

Thus, the solution is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{7}{58} \\ \frac{7}{69} \end{pmatrix}. \tag{13}$$