# 5.12.7 Matgeo

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# Question

Solve the following equations for x and y :

$$(ax - by) + (a + 4b) = 0$$
$$(bx + ay) + (b - 4a) = 0$$

# Solution

given two equations :

$$(ax - by) = -(a + 4b) \tag{1}$$
$$(bx + ay) = -(b - 4a) \tag{2}$$

these can be written as :

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -a - 4b \\ 4a - b \end{bmatrix} \tag{3}$$

# Solution

$$\mathbf{AX} = \mathbf{B} \tag{4}$$

To find X we need to multiply $A^{-1}$ on both sides

$$X = A^{-1}B \tag{5}$$

Finding $A^{-1}$ :

$$\begin{bmatrix} a-\lambda & -b \\ b & a-\lambda \end{bmatrix} = 0 \tag{6}$$

$$\lambda^2 - 2a\lambda + a^2 + b^2 = 0 \tag{7}$$

since :

$$|\mathbf{A} - \lambda\mathbf{I}| = 0 \tag{8}$$

$$\mathbf{A}^2 - 2a\mathbf{A} + (a^2 + b^2) = 0 \tag{9}$$

Multiply both sides by $A^{-1}$:

$$\mathbf{A} - 2a\mathbf{I} + A^{-1}(a^2 + b^2) = 0 \tag{10}$$

$$A^{-1} = \frac{1}{a^2 + b^2}(2a\mathbf{I} - \mathbf{A}) \tag{11}$$

$$A^{-1} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \tag{12}$$

from the equation 0.5 :

$$\mathbf{X} = \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} -a - 4b \\ 4a - b \end{bmatrix} \tag{13}$$

$$\mathbf{X} = \begin{bmatrix} -a^2 - b^2 \\ 4a^2 + 4b^2 \end{bmatrix} \tag{14}$$

# Solution

Hence :

$$x = -a^2 - b^2$$
$$y = 4a^2 + 4b^2$$