

9.4.36

EE25BTECH11023 - Venkata Sai

Question:

The sum of the reciprocals of Ram's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age

Solution:

The input parameters are available in Table 1.

Variable	Description
x	Present Age of Ram
$x - 3$	Age of Ram 3 years ago
$x + 5$	Age of Ram 5 years from now

Given sum of reciprocal of Ram's ages 3 years ago and 5 years from now is $\frac{1}{3}$

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3} \quad (1)$$

$$\frac{(x+5) + (x-3)}{(x-3)(x+5)} = \frac{1}{3} \quad (2)$$

$$\frac{2x+2}{x^2+5x-3x-15} = \frac{1}{3} \quad (3)$$

$$\frac{2x+2}{x^2+2x-15} = \frac{1}{3} \quad (4)$$

$$(2x+2) = x^2+2x-15 \quad (5)$$

$$6x+6 = x^2+2x-15 \quad (6)$$

$$x^2+2x-15-6x-6=0 \quad (7)$$

$$x^2-4x-21=0 \quad (8)$$

$$\Rightarrow y = x^2 - 4x - 21 \quad (9)$$

$$\Rightarrow x^2 - 4x - y - 21 = 0 \quad (10)$$

$$x^2 + 2(-2x - \frac{1}{2}y) - 21 = 0 \quad (11)$$

which can be expressed as the conic

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (12)$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -2 \\ -\frac{1}{2} \end{pmatrix}, f = -21 \quad (13)$$

To find the roots of (9), we find the points of intersection of the conic with the x-axis

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \quad (14)$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (15)$$

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{(\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u}))^2 - g(\mathbf{h})(\mathbf{m}^\top \mathbf{V} \mathbf{m})} \right) \quad (16)$$

where

$$g(\mathbf{h}) = \mathbf{h}^\top \mathbf{V} \mathbf{h} + 2\mathbf{u}^\top \mathbf{h} + f \quad (17)$$

$$g(\mathbf{h}) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} -2 \\ -\frac{1}{2} \end{pmatrix}^\top \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 21 \quad (18)$$

$$g(\mathbf{h}) = \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} -2 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 21 \quad (19)$$

$$g(\mathbf{h}) = \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2(0) - 21 \quad (20)$$

$$g(\mathbf{h}) = 0 + 0 - 21 = -21 \quad (21)$$

$$\mathbf{m}^\top \mathbf{V} \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1 \quad (22)$$

$$\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^\top \left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ -\frac{1}{2} \end{pmatrix} \right) = \begin{pmatrix} 1 & 0 \end{pmatrix} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ -\frac{1}{2} \end{pmatrix} \right) = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ -\frac{1}{2} \end{pmatrix} = -2 \quad (23)$$

From equation (14)

$$\kappa_i = \frac{1}{1} \left(-(-2) \pm \sqrt{(-2)^2 + 21} \right) \quad (24)$$

$$= 2 \pm \sqrt{25} = 2 \pm 5 \quad (25)$$

$$= 7, -3 \quad (26)$$

Hence the points of intersection are

$$\mathbf{h} + k\mathbf{m} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad (27)$$

Hence the solutions are $x = -3$ and $x = 7$. We reject $x = -3$ as the Age cannot be negative. Hence, the present age of Ram will be 7 years

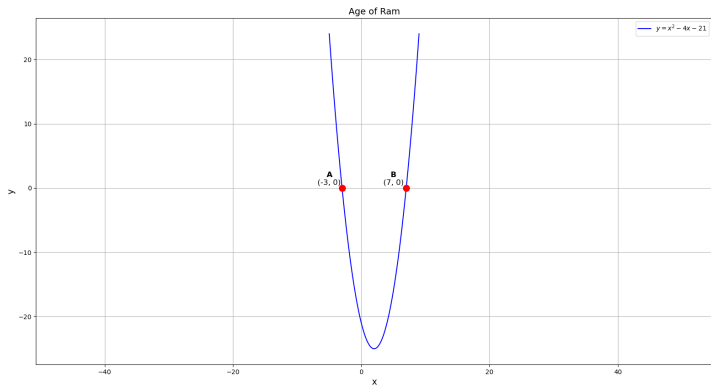


Fig. 0.1