

# AI25BTECH11034 - SUJAL CHAUHAN

4.12.48

**Question:**

Prove that if a plane has the intercept a,b,c and is at a distance of p units from the origin, then  $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$

**solution** Equation of plane in vector form is given by

Point	Positon vector	
a	$\begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}$	(1)
b	$\begin{pmatrix} 0 \\ b \\ 0 \end{pmatrix}$	
c	$\begin{pmatrix} 0 \\ 0 \\ c \end{pmatrix}$	

$$\mathbf{n}^T \mathbf{P} = d \quad (2)$$

where  $\mathbf{n}$  is normal unit vector to the plane and  $d$  is the distance of the plane from the origin.

Now given plane satisfys the vectors

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \mathbf{n} = \begin{pmatrix} p \\ p \\ p \end{pmatrix} \quad (3)$$

Now multiply both side with inverse of the matrix

$$\mathbf{n} = \frac{1}{abc} \begin{pmatrix} bc & 0 & 0 \\ 0 & ac & 0 \\ 0 & 0 & ab \end{pmatrix} \begin{pmatrix} p \\ p \\ p \end{pmatrix} \quad (4)$$

$$\mathbf{n} = \begin{pmatrix} \frac{p}{a} \\ \frac{p}{b} \\ \frac{p}{c} \end{pmatrix} \quad (5)$$

Now since  $\mathbf{n}$  is a unit vector then

$$\mathbf{n}^T \mathbf{n} = 1 \quad (6)$$

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2} \quad (7)$$

Checking and Ploting using numerical values of a,b,c

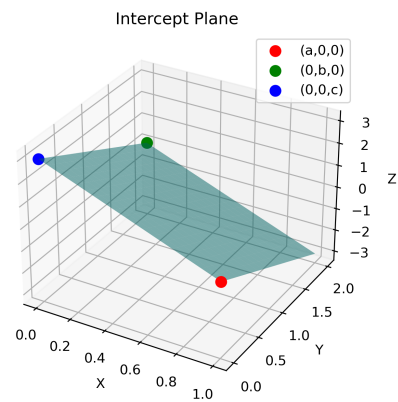


Figure 1: plotting