1.6.18

AI25BTECH11003 - Bhavesh Gaikwad

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Question

Prove that points A(2,1), B(0,5) and C(-1,2) are collinear.

Theoretical Solution

$$\text{B-A}=\begin{pmatrix}0-2\\5-1\end{pmatrix}=\begin{pmatrix}-2\\4\end{pmatrix} \qquad C-A=\begin{pmatrix}-1-2\\2-1\end{pmatrix}=\begin{pmatrix}-3\\1\end{pmatrix}$$

$$\text{M}=\begin{pmatrix}B-A & C-A\end{pmatrix}=\begin{pmatrix}-2 & -3\\4 & 1\end{pmatrix}$$

Row-reduce to compute the rank:

$$\begin{pmatrix}-2 & -3\\4 & 1\end{pmatrix}\xrightarrow{R_2\leftarrow R_2+2R_1}\begin{pmatrix}-2 & -3\\0 & -5\end{pmatrix}$$

The echelon form has two nonzero rows, hence rank(M)=2≠1

$$\boxed{\textit{Therefore, The points } A(2,1),\ B(0,5) \textit{ and } C(-1,2) \textit{ are not collinear.}} \tag{1}$$

C Code

```c
#include <stdio.h>
int main(void) {
int Ax = 2, Ay = 1;
int Bx = 0, By = 5;
int Cx = -1, Cy = 2;

// Columns of M: B-A and C-A
int m11 = Bx - Ax;
int m21 = By - Ay;
int m12 = Cx - Ax;
int m22 = Cy - Ay;

int det = m11 * m22 - m12 * m21;
```

C Code

```
  printf("Matrix M = [[%d, %d],[%d, %d]]\n", m11, m12, m21, m22);
printf("det(M) = %d\n", det);

if (det != 0) {
    printf("rank(M) = 2 -> Points are NOT collinear.\n");
} else if (m11 != 0 || m21 != 0 || m12 != 0 || m22 != 0) {
    printf("rank(M) = 1 -> Points are collinear.\n");
} else {
    printf("rank(M) = 0 (degenerate).\n");
}
return 0;
}
```

Python Code

```
import numpy as np
import sympy as sp
import matplotlib.pyplot as plt

A = np.array([2, 1])
B = np.array([0, 5])
C = np.array([-1, 2])

# Collinearity matrix [B-A C-A]
M = sp.Matrix.hstack(sp.Matrix(B - A), sp.Matrix(C - A))
print("Collinearity matrix M =\n", np.array(M, dtype=int))
print("rank(M) =", M.rank())
print("Conclusion:", "Collinear" if M.rank() == 1 else "NOT
    collinear")
```

Python Code

```
plt.figure(figsize=(4, 4))
pts = np.vstack([A, B, C])
plt.scatter(pts[:, 0], pts[:, 1], color=['tab:blue', 'tab:orange'
    , 'tab:green'], s=80)
for (x, y), label in zip(pts, ['A(2,1)', 'B(0,5)', 'C(-1,2)']):
    plt.annotate(label, (x + 0.05, y + 0.05)) # <-- This must be
        indented

plt.plot([A[0], B[0]], [A[1], B[1]], 'k--', alpha=0.6)
plt.plot([A[0], C[0]], [A[1], C[1]], 'k--', alpha=0.6)
plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
plt.grid(True, linestyle=':')
plt.xlim(-2, 4)
plt.ylim(0, 7)
plt.title('Points A, B, C (rank test)')
plt.tight_layout()
plt.savefig('fig1.png', dpi=150)
```

Graph