Question: Find the values of k for which the points $A(k+1, 2k)$, $B(3k, 2k+3)$, $C(5k-1, 5k)$ are collinear.

Solution:
First, form the difference vectors:

$$B - A = \begin{pmatrix} 3k - (k+1) \\ (2k+3) - 2k \end{pmatrix} = \begin{pmatrix} 2k - 1 \\ 3 \end{pmatrix}$$

$$C - A = \begin{pmatrix} (5k-1) - (k+1) \\ 5k - 2k \end{pmatrix} = \begin{pmatrix} 4k - 2 \\ 3k \end{pmatrix}$$

Form the matrix:

$$M = \begin{pmatrix} 2k-1 & 3 \\ 4k-2 & 3k \end{pmatrix}$$

For the points to be collinear, the rank of M must be 1. Perform the row operation:

$$R_2 \to -\frac{4k-2}{2k-1} R_1 + R_2 \qquad (\text{for } 2k-1 \neq 0)$$

Which gives:

$$\begin{pmatrix} 2k-1 & 3 \\ 0 & 3k - \frac{3(4k-2)}{2k-1} \end{pmatrix}$$

Set the second row entry to zero for rank 1:

$$3k - \frac{3(4k-2)}{2k-1} = 0$$

$$3k = \frac{3(4k-2)}{2k-1}$$

$$3k(2k-1) = 3(4k-2)$$

$$6k^2 - 3k = 12k - 6$$

$$6k^2 - 15k + 6 = 0$$

$$2k^2 - 5k + 2 = 0$$

Solving for k:

$$k = \frac{5 \pm 3}{4}$$

$$k = 2 \quad \text{or} \quad k = \frac{1}{2}$$