

1.5.21

EE25BTECH11033 - Kavın

Question:

Find the ratio in which $\mathbf{P}(4, m)$ divides the line segment joining the points $\mathbf{A}(2, 3)$ and $\mathbf{B}(6, -3)$. Hence, find m .

Solution:

Let the vector \mathbf{P} be

$$\mathbf{P} = \begin{pmatrix} 4 \\ m \end{pmatrix}, \quad (1)$$

Given the points,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 6 \\ -3 \end{pmatrix} \quad (2)$$

The points A, P, B are collinear.

Points $\mathbf{A}, \mathbf{P}, \mathbf{B}$ are defined to be collinear if

$$\text{rank}(\mathbf{P} - \mathbf{A} \quad \mathbf{B} - \mathbf{A}) = 1 \quad (3)$$

$$\mathbf{P} - \mathbf{A} = \begin{pmatrix} 2 \\ m - 3 \end{pmatrix} \quad (4)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} \quad (5)$$

$$(\mathbf{P} - \mathbf{A} \quad \mathbf{B} - \mathbf{A}) = \begin{pmatrix} 2 & 4 \\ m - 3 & -6 \end{pmatrix} \quad (6)$$

$$R_2 \rightarrow 2R_2 + 3R_1 \implies \begin{pmatrix} 2 & 4 \\ 2m & 0 \end{pmatrix}$$

For rank 1, the second row must be zero:

$$2m = 0 \implies m = 0 \quad (7)$$

$$\therefore \mathbf{P} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

Section formula for a vector \mathbf{P} which divides the line formed by vectors \mathbf{A} and \mathbf{B} in the ratio $k:1$ is given by

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \quad (8)$$

Using section formula,

$$\begin{pmatrix} 4 \\ 0 \end{pmatrix} = \frac{\begin{pmatrix} 2 \\ 3 \end{pmatrix} + k \begin{pmatrix} 6 \\ -3 \end{pmatrix}}{1 + k} \quad (9)$$

$$\Rightarrow \begin{pmatrix} 4 \\ 0 \end{pmatrix} + k \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + k \begin{pmatrix} 6 \\ -3 \end{pmatrix} \quad (10)$$

$$\Rightarrow k \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad (11)$$

$$\Rightarrow k = \frac{1}{1}. \quad (12)$$

Therefore the ratio in which **P** divides the line segment joining the points **A** and **B** is 1 : 1

See Fig. 0 ,

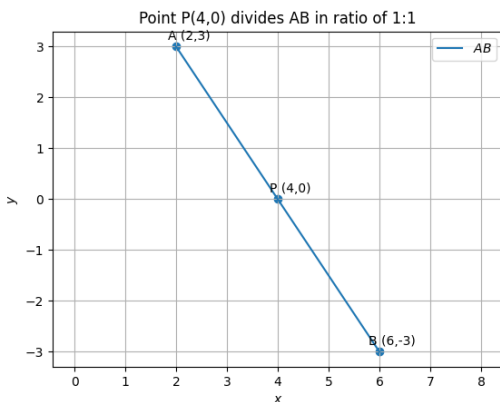


Fig. 0