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Question

The two vectors $[1, 1, 1]$ and $[1, a, a^2]$, where $a = \left(\frac{-1}{2} + j\frac{\sqrt{3}}{2}\right)$

- ① orthonormal
- ② orthogonal
- ③ parallel
- ④ collinear

Theoretical Solution

Given ,

$$\mathbf{P} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (1)$$

$$\mathbf{Q} = \begin{pmatrix} 1 \\ a \\ a^2 \end{pmatrix} \quad (2)$$

Theoretical Solution

Let,

$$\mathbf{z}_1 = x_1 + jy_1 \longrightarrow \begin{pmatrix} x_1 & -y_1 \\ y_1 & x_1 \end{pmatrix} \quad (3)$$

$$\mathbf{z}_2 = x_2 + jy_2 \longrightarrow \begin{pmatrix} x_2 & -y_2 \\ y_2 & x_2 \end{pmatrix} \quad (4)$$

Look at

$$\mathbf{z}_1 + \mathbf{z}_2 = (x_1 + x_2) + j(y_1 + y_2) \quad (5)$$

Theoretical Solution

Which is equivalent to

$$\begin{pmatrix} x_1 & -y_1 \\ y_1 & x_1 \end{pmatrix} + \begin{pmatrix} x_2 & -y_2 \\ y_2 & x_2 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 & -y_1 - y_2 \\ y_1 + y_2 & x_1 + x_1 \end{pmatrix} \quad (6)$$

Also,

$$\mathbf{z_1 z_2} = (x_1 x_2 - y_1 y_2) + j(x_1 y_2 + x_2 y_1) \quad (7)$$

This is equivalent to

$$\begin{pmatrix} x_1 & -y_1 \\ y_1 & x_1 \end{pmatrix} \begin{pmatrix} x_2 & -y_2 \\ y_2 & x_2 \end{pmatrix} = \begin{pmatrix} (x_1 x_2 - y_1 y_2) & -(x_1 y_2 + x_2 y_1) \\ (x_1 y_2 + x_2 y_1) & (x_1 x_2 - y_1 y_2) \end{pmatrix} \quad (8)$$

Theoretical Solution

∴ Complex Numbers can be represented as this matrix form since it satisfies Addition and Multiplication properties.

$$x + jy \longrightarrow \begin{pmatrix} x & -y \\ y & x \end{pmatrix} \quad (9)$$

$$a = \left(\frac{-1}{2} + j\frac{\sqrt{3}}{2} \right) \longrightarrow \mathbf{A} = \begin{pmatrix} \frac{-1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \quad (10)$$

Similarly

$$a^2 = \left(\frac{-1}{2} - j\frac{\sqrt{3}}{2} \right) \longrightarrow \mathbf{A}^2 = \begin{pmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \quad (11)$$

$$1 \longrightarrow \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (12)$$

Theoretical Solution

Now,

$$\mathbf{I} + \mathbf{A} + \mathbf{A}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \frac{-1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} + \begin{pmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \quad (13)$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad (14)$$

$$\implies 1 + a + a^2 = 0 \quad (15)$$

Theoretical Solution

Now, Look At ,

$$\mathbf{P}^T \mathbf{Q} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ a \\ a^2 \end{pmatrix} = 1 + a + a^2 = 0 \quad (16)$$

Hence \mathbf{P} and \mathbf{Q} are orthogonal.

Answer : Option (2)