

Question 4.4.36

AI25BTECH11040 - Vivaan Parashar

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1 Question:

The area of the triangle formed by the lines $\frac{x}{a} + \frac{y}{b} = 1$ and the coordinate axes is _____.

2 Solution:

Let the origin be \mathbf{O} , the x-intercept be \mathbf{A} , and the y-intercept be \mathbf{B} . We then need the area of triangle OAB . The x-intercept is found by setting $y = 0$ in the equation of the line, written as $\mathbf{m}^T \mathbf{x} = 1$, where $\mathbf{m} = \begin{pmatrix} \frac{1}{a} \\ \frac{1}{b} \end{pmatrix}$ and \mathbf{x} represents a point on the line. Similarly, the y-intercept is found by setting $x = 0$.

$$\therefore \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 \\ b \end{pmatrix} \quad (1)$$

Putting \mathbf{A} and \mathbf{B} in the equation of the line, we get

$$x = a, \quad y = b \quad (2)$$

Clearly, the triangle formed is a right angled triangle, with the right angle at \mathbf{O} (due to the axes being perpendicular). The area of the triangle is given by

$$\Delta OAB = \frac{1}{2} \times \text{base} \times \text{height}, \text{ in this case} \quad (3)$$

$$\Delta OAB = \frac{1}{2} \times OA \times OB \quad (4)$$

$$\Delta OAB = \frac{1}{2} \times |a| \times |b| \quad (5)$$

$$\therefore \Delta OAB = \left| \frac{ab}{2} \right| \quad (6)$$

3 Plot:

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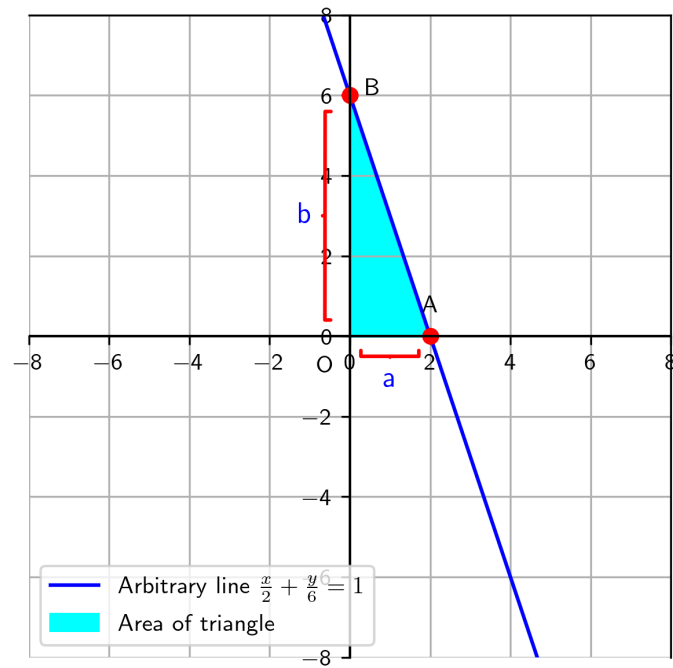


Figure 1: Graph of line and triangle formed by intercepts with axes