2.6.39

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Question: The area of the quadrilateral ABCD, where $\mathbf{A}(0,4,1), \mathbf{B}(2,3,-1), \mathbf{C}(4,5,0)$ and $\mathbf{D}(2,6,2)$, is equal to

Solution: The area of a quadrilateral is given by half the magnitude of the cross product of its diagonals.

First, we find the vectors for the diagonals

$$\mathbf{P} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 \\ 5 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$$

$$\mathbf{Q} = \mathbf{D} - \mathbf{B} = \begin{pmatrix} 2 \\ 6 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 3 \end{pmatrix}$$

Now, we compute the cross product $\mathbf{P} \times \mathbf{Q}$ using the determinant expansion:

$$\mathbf{P} \times \mathbf{Q} = \begin{pmatrix} \left|\begin{pmatrix} 1 & 3 \\ -1 & 3 \end{pmatrix}\right| \\ -\left|\begin{pmatrix} 4 & 3 \\ -1 & 3 \end{pmatrix}\right| \\ \left|\begin{pmatrix} 4 & 1 \\ 0 & 3 \end{pmatrix}\right| \end{pmatrix}$$

$$= \begin{pmatrix} (1)(3) - (3)(-1) \\ -((4)(3) - (3)(-1)) \\ (4)(3) - (1)(0) \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ -12 \\ 12 \end{pmatrix}$$

The area is half the magnitude of this vector:

$$
\begin{aligned}
\text{Area} &= \frac{1}{2}||\mathbf{P} \times \mathbf{Q}|| \\
&= \frac{1}{2}\sqrt{6^2 + (-12)^2 + 12^2} \\
&= \frac{1}{2}\sqrt{36 + 144 + 144} \\
&= \frac{1}{2}\sqrt{324} \\
&= \frac{1}{2}(18) \\
&= 9
\end{aligned}
$$

Thus, the area of the quadrilateral is 9 square units.