1.6.9

EE25BTECH11059 - Vaishnavi Ramkrishna Anantheertha

Question:
if three points $\begin{pmatrix} h \\ 0 \end{pmatrix}, \begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} 0 \\ k \end{pmatrix}$ lie on a line,show that

$$\frac{a}{h} + \frac{b}{k} = 1$$

Solution:

Point	**Name**
$(h, 0)$	Point A
$(0, k)$	Point B
(a, b)	Point C

TABLE 0: Variables Used

If the rank of the Collinearity matrix is 1, then the points are collinear
The Collinearity matrix is given by

$$\begin{pmatrix} \mathbf{C} - \mathbf{A} & \mathbf{B} - \mathbf{A} \end{pmatrix}^T = \begin{pmatrix} a - h & b \\ -h & k \end{pmatrix} \tag{0.1}$$

$$\xleftrightarrow{R_1 \to \frac{R_1}{a-h}} \begin{pmatrix} 1 & \frac{b}{a-h} \\ -h & k \end{pmatrix} \tag{0.2}$$

$$\xleftrightarrow{R_2 \to \frac{R_2}{-h}} \begin{pmatrix} 1 & \frac{b}{a-h} \\ 1 & \frac{-k}{h} \end{pmatrix} \tag{0.3}$$

$$\xleftrightarrow{R_1 \to R_1 - R_2} \begin{pmatrix} 0 & \frac{b}{a-h} + \frac{k}{h} \\ 1 & \frac{-k}{h} \end{pmatrix} \tag{0.4}$$

since the rank of matrix=1

$$\frac{b}{a-h} + \frac{k}{h} = 0$$
$$\implies bh + ka - kh = 0 \text{ (dividing the eq with } kh) \implies \frac{a}{h} + \frac{b}{k} = 1$$

Refer to Fig

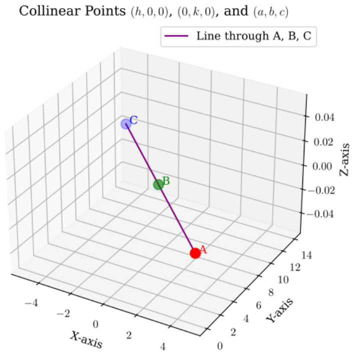
Collinear Points $(h,0,0)$, $(0,k,0)$, and (a,b,c)
Line through A, B, C
C
B
A
X-axis
Y-axis
Z-axis
−4
−2
0
2
4
0
2
4
6
8
10
12
14
0.04
0.02
0.00
−0.02
−0.04

Fig. 0.1