5.10.4

AI25BTECH11001 - ABHISEK MOHAPATRA

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Question: Write the balanced chemical equations for the following reaction.

$$BaCl_2 + K_2SO_4 \rightarrow BaSO_4 + KCl \tag{0.1}$$

Solution: Let the balanced version of given equation be

$$x_1 BaCl_2 + x_2 K_2SO_4 \rightarrow x_3 BaSO_4 + x_4 KCl \tag{0.2}$$

which results in the following equations:

$$(x_1 - x_3)\, Ba = 0 \tag{0.3}$$

$$(2x_1 - x_4)\, Cl = 0 \tag{0.4}$$

$$(2x_2 - x_4)\, K = 0 \tag{0.5}$$

$$(x_2 - x_3)\, S = 0 \tag{0.6}$$

$$(4x_2 - 4x_3)\,O = 0 \tag{0.7}$$

which can be expresseed as

$$x_1 + 0x_2 + (-1)x_3 + x_4 = 0 \tag{0.8}$$

$$2x_1 + 0x_2 + 0x_3 + (-1)x_4 = 0 \tag{0.9}$$

$$0x_1 + 2x_2 + 0x_3 + (-1)x_4 = 0 \tag{0.10}$$

$$0x_1 + x_2 + (-1)x_3 + 0x_4 = 0 \tag{0.11}$$

resulting in the matrix equation

$$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 2 & 0 & 0 & -1 \\ 0 & 2 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \mathbf{X} = 0 \tag{0.12}$$

which can be reduced as follows

$$\begin{pmatrix} 1 & 0 & -1 & 0 \\ 2 & 0 & 0 & -1 \\ 0 & 2 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 2 & -1 \\ 0 & 2 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \tag{0.13}$$

$$\xleftrightarrow{R_3 \leftrightarrow R_2} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \xleftrightarrow{R_4 \leftarrow R_4 + \frac{1}{2}R_2 - \frac{1}{2}R_1} \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{0.14}$$

$$\xleftrightarrow{R_1 \leftrightarrow R_1 + \frac{1}{2}R_3} \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 2 & 0 & -1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \xleftrightarrow[R_3 \leftrightarrow \frac{1}{2}R_3]{R_2 \leftrightarrow \frac{1}{2}R_2} \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{0.15}$$

Thus,

$$x_1 = \frac{1}{2}x_4, x_2 = \frac{1}{2}x_4, x_3 = \frac{1}{2}x_4 \tag{0.16}$$

$$\Rightarrow \mathbf{X} = x_4 \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix} \tag{0.17}$$

by substituting $x_4 = 2$. Hence, The equation finally becomes

$$BaCl_2 + K_2SO_4 \rightarrow BaSO_4 + 2KCl \tag{0.18}$$