4.11.37

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Question

Show that the lines $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $\frac{x+4}{2} = \frac{y}{0} = \frac{z+1}{3}$ intersect.Find their point of intersection

Lines

Let,

$$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda \tag{1}$$

$$\frac{x+4}{2} = \frac{y}{0} = \frac{z+1}{3} = \mu \tag{2}$$

Solution

$$\begin{pmatrix}1 & 1 & -1\end{pmatrix} + \lambda\begin{pmatrix}3 & -1 & 0\end{pmatrix} = \begin{pmatrix}-4 & 0 & -1\end{pmatrix} + \mu\begin{pmatrix}2 & 0 & 3\end{pmatrix} \tag{3}$$

$$\implies \begin{pmatrix}3 & -2\\ -1 & 0\\ 0 & -3\end{pmatrix}\begin{pmatrix}\lambda\\ \mu\end{pmatrix} = \begin{pmatrix}-5\\ -1\\ 0\end{pmatrix} \tag{4}$$

(5)

Using gaussian elimination
Argumented matrix

$$\left[\begin{array}{cc|c} 3 & -2 & -5 \\ -1 & 0 & -1 \\ 0 & -3 & 0 \end{array}\right] \tag{6}$$

Apply the row operation $R_2 \to 3R_2 + R_1$:

$$\left[\begin{array}{cc|c} 3 & -2 & -5 \\ 0 & -2 & -8 \\ 0 & -3 & 0 \end{array}\right] \tag{7}$$

Divide the second row by -2:

$$\left[\begin{array}{cc|c} 3 & -2 & -5 \\ 0 & 1 & 4 \\ 0 & -3 & 0 \end{array}\right] \tag{8}$$

solution

Now eliminate using R_2:

$$R_1 \to R_1 + 2R_2, \quad R_3 \to R_3 + 3R_2$$

$$\begin{bmatrix} 3 & 0 & | & 3 \\ 0 & 1 & | & 4 \\ 0 & 0 & | & 12 \end{bmatrix} \tag{9}$$

The last row gives $0 = 12$, which is a contradiction. Hence, the system is inconsistent and has **no solution**. Therefore, the two lines are skew and do not intersect.

C Code

```c
#include <stdio.h>
int main() {
    // Augmented matrix for the system:
    // 3 - 2 = -5
    // - = -1
    // -3 = 0

    double A[3][3] = {
        {3, -2, -5},
        {-1, 0, -1},
        {0, -3, 0}
    };
```

C Code

```
    // Perform Gaussian elimination
   for (int i = 0; i < 3; i++) {
       // Normalize row i if pivot nonzero
       if (A[i][i] != 0) {
           double pivot = A[i][i];
           for (int j = i; j < 3; j++) {
               A[i][j] /= pivot;
           }
       }
  // Eliminate column i from other rows
       for (int k = 0; k < 3; k++) {
           if (k != i) {
               double factor = A[k][i];
               for (int j = i; j < 3; j++) {
                   A[k][j] -= factor * A[i][j];
               }
           }
       }
   }
```

C Code

```
// After elimination, check last row
    if (A[2][0] == 0 && A[2][1] == 0 && A[2][2] != 0) {
        printf("System is inconsistent -> No solution. Lines are
            skew.\n");
    } else {
        double lambda = A[0][2];
        double mu = A[1][2];
        printf("Solution: lambda = %.2f, mu = %.2f\n", lambda, mu
            );
    }

    return 0;
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt

# Parameter range
t = np.linspace(-5, 5, 100)

# Line L1: (x,y,z) = (1,1,-1) + (3,-1,0)
x1 = 1 + 3*t
y1 = 1 - t
z1 = -1 * np.ones_like(t)

# Line L2: (x,y,z) = (-4,0,-1) + (2,0,3)
x2 = -4 + 2*t
y2 = np.zeros_like(t)
z2 = -1 + 3*t
```

Python Code

```
# Create 3D plot
fig = plt.figure(figsize=(8,6))
ax = fig.add_subplot(111, projection='3d')

# Plot the two lines
ax.plot(x1, y1, z1, label="Line L1: (1,1,-1)+(3,-1,0)", color='
    blue')
ax.plot(x2, y2, z2, label="Line L2: (-4,0,-1)+(2,0,3)", color='
    red')

# Mark given points
ax.scatter(1, 1, -1, color='blue', s=50, marker='o')
ax.text(1, 1, -1, "(1,1,-1)", color='blue')
```

Python Code

```
ax.scatter(-4, 0, -1, color='red', s=50, marker='o')
ax.text(-4, 0, -1, "(-4,0,-1)", color='red')

# Labels and title
ax.set_xlabel("X-axis")
ax.set_ylabel("Y-axis")
ax.set_zlabel("Z-axis")
ax.set_title("Skew Lines in 3D")
ax.legend()

plt.show()
```

C and Python Code

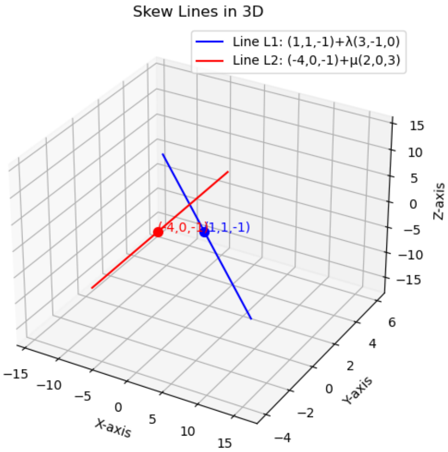
```python
import ctypes

# Load the shared object file
lib = ctypes.CDLL('./skew.so')

# Call the function
result = lib.check_intersection()

if result == 0:
    print("System is inconsistent -> Lines are skew (no
        intersection).")
else:
    print("Lines intersect.")
```

Plot



Skew Lines in 3D
Line L1: (1,1,-1)+λ(3,-1,0)
Line L2: (-4,0,-1)+μ(2,0,3)
(-4,0,-1)
(1,1,-1)
X-axis
Y-axis
Z-axis