Matgeo Presentation - Problem 3.2.4

ee25btech11063 - Vejith

September 2, 2025

Question

Construct the triangle BD′C′ similar to △BDC with scale factor $\frac{4}{3}$.Draw the line segment D′A′. parallel to DA where A^p prime lies on extended side BA.Is A′BC′D′ a parallelogram?

Description

Solution:

Vector	Name
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Vector **A**
$\begin{pmatrix} 4 \\ 0 \end{pmatrix}$	Vector **B**
$\begin{pmatrix} 4 \\ 3 \end{pmatrix}$	Vector **C**
$\begin{pmatrix} 0 \\ 3 \end{pmatrix}$	Vector **D**

Table: Variables Used

Solution

consider $\triangle$BDC.constructs a $\triangle$BD$'C'$ with scale factor $\frac{4}{3}$.
This means

$$\triangle BD'C' \sim \triangle BDC. \tag{0.1}$$

$$\frac{\mathbf{D}' - \mathbf{B}}{\mathbf{D} - \mathbf{B}} = \frac{\mathbf{C}' - \mathbf{B}}{\mathbf{C} - \mathbf{B}} = \frac{\mathbf{C}' - \mathbf{D}'}{\mathbf{C} - \mathbf{D}} = \frac{4}{3}. \tag{0.2}$$

$$\mathbf{D}' = \mathbf{B} + \frac{4}{3}(\mathbf{D} - \mathbf{B}) \tag{0.3}$$

$$\mathbf{D}' = \begin{pmatrix} -4/3 \\ 4 \end{pmatrix} \tag{0.4}$$

$$\mathbf{C}' = \mathbf{B} + \frac{4}{3}(\mathbf{C} - \mathbf{B}) \tag{0.5}$$

$$\mathbf{C}' = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{0.6}$$

Construct $\mathbf{A}'$

Mark $\mathbf{D}'$ and $\mathbf{A}'$ parallel to $\mathbf{D} - \mathbf{A}$ with $\mathbf{A}'$ along the direction of $\mathbf{B} - \mathbf{A}$.

Solution

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}4\\0\end{pmatrix}-\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}4\\0\end{pmatrix} \tag{0.7}$$

$$\mathbf{C}-\mathbf{D}=\begin{pmatrix}4\\3\end{pmatrix}-\begin{pmatrix}0\\3\end{pmatrix}=\begin{pmatrix}4\\0\end{pmatrix} \tag{0.8}$$

$$\implies \mathbf{B}-\mathbf{A}=\mathbf{C}-\mathbf{D} \tag{0.9}$$

$$\mathbf{D}-\mathbf{A}=\begin{pmatrix}0\\3\end{pmatrix}-\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}0\\3\end{pmatrix} \tag{0.10}$$

$$\mathbf{C}-\mathbf{B}=\begin{pmatrix}4\\3\end{pmatrix}-\begin{pmatrix}4\\0\end{pmatrix}=\begin{pmatrix}0\\3\end{pmatrix} \tag{0.11}$$

$$\implies \mathbf{D}-\mathbf{A}=\mathbf{C}-\mathbf{B} \tag{0.12}$$

$\implies$ ABCD is a Parallelogram

Solution

Check the parallelogram property of $\mathbf{A'BC'D'}$

$$\mathbf{B} - \mathbf{A'} = -t(\mathbf{A} - \mathbf{B}) \tag{0.13}$$

$$\mathbf{D'} - \mathbf{C'} = k(\mathbf{C} - \mathbf{D}) \tag{0.14}$$

$$\text{From 9 } \mathbf{B} - \mathbf{A} = \mathbf{C} - \mathbf{D} \tag{0.15}$$

$$\implies \mathbf{B} - \mathbf{A'} = -t(\mathbf{A} - \mathbf{B}) = t(\mathbf{C} - \mathbf{D}) = \frac{t}{k}\mathbf{D'} - \mathbf{C'} \tag{0.16}$$

$$\implies \mathbf{B} - \mathbf{A'} \parallel \mathbf{D'} - \mathbf{C'} \tag{0.17}$$

By construction of $\mathrm{A'}$

$$\mathbf{D'} - \mathbf{A'} \parallel \mathbf{D} - \mathbf{A} \tag{0.18}$$

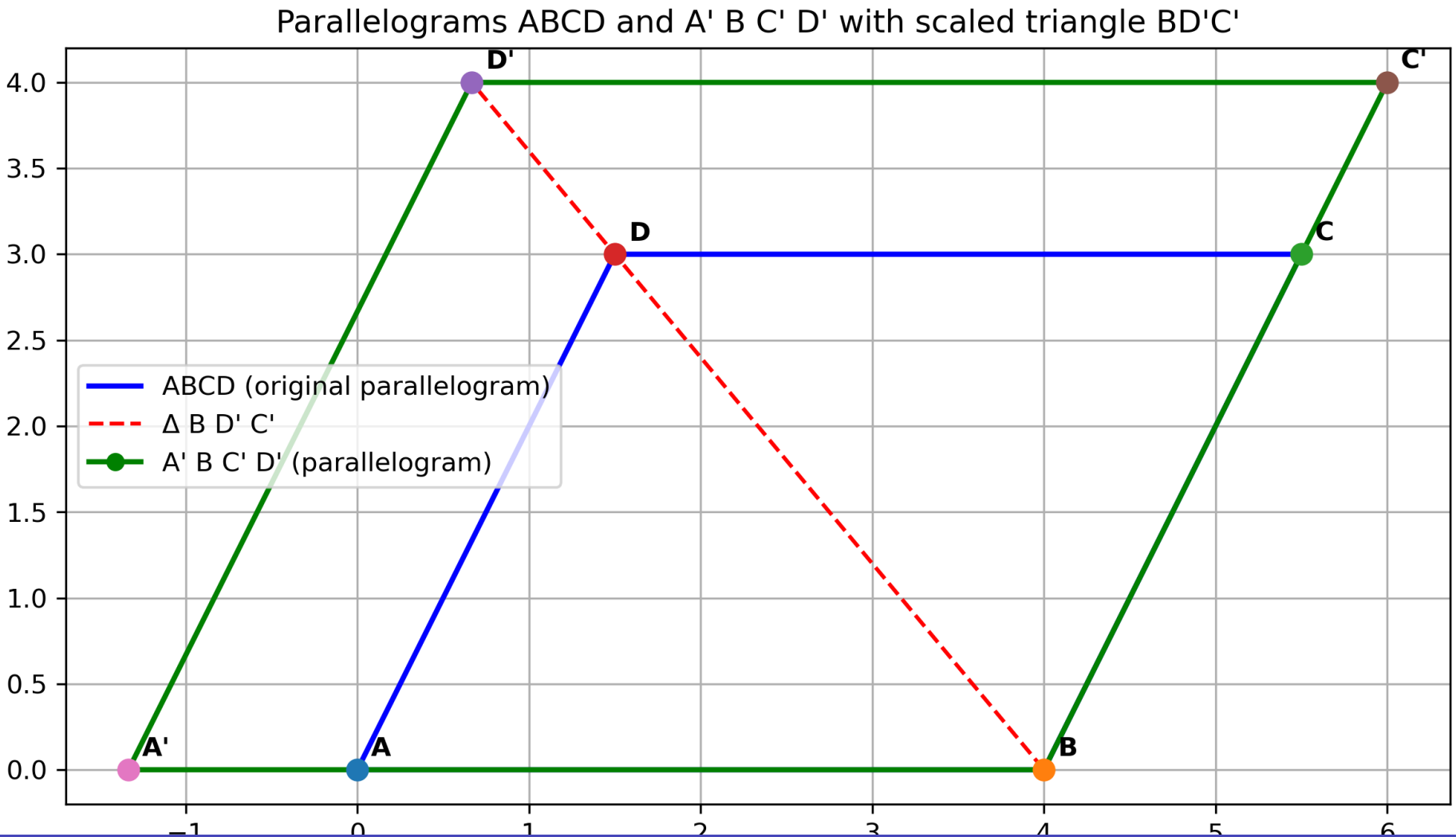
$$\mathbf{D} - \mathbf{A} \parallel \mathbf{C} - \mathbf{B} \tag{0.19}$$

$$\mathbf{C} - \mathbf{B} \parallel \mathbf{C'} - \mathbf{B} \tag{0.20}$$

$$\implies \mathbf{D'} - \mathbf{A'} \parallel \mathbf{C'} - \mathbf{B} \tag{0.21}$$

Conclusion and plot

$\Longrightarrow$ $A'BC'D'$ a parallelogram



C Code: triangle.c

```c
/* triangle.c
   - writes points to "triangle.dat"
   - computes A' so that D'A' || DA and A' lies on extended BA
   - checks whether A' B C' D' is a parallelogram
*/
#include <stdio.h>
#include <math.h>

typedef struct { double x, y; } Point;

int areParallel(Point p1, Point p2, Point q1, Point q2) {
    double dx1 = p2.x - p1.x, dy1 = p2.y - p1.y;
    double dx2 = q2.x - q1.x, dy2 = q2.y - q1.y;
    return fabs(dy1 * dx2 - dy2 * dx1) < 1e-8;
}

int main(void) {
    FILE *fp = fopen("triangle.dat", "w");
    if (!fp) {
        perror("fopen");
        return 1;
    }

    /* Choose ABCD to be a parallelogram (so final shape will be a parallelogram).
       Example: rectangle/parallelogram with A=(0,0), B=(4,0), D=(0,3).
       Then C = B + D - A = (4,3).
    */
    Point A = {0.0, 0.0};
    Point B = {4.0, 0.0};
    Point D = {0.0, 3.0};
    Point C = { B.x + D.x - A.x, B.y + D.y - A.y }; /* ensures ABCD is parallelogram */

    double k = 4.0 / 3.0;
```

C Code: triangle.c

```
/* BD'C' similar to BDC with scale factor k: D' = B + k*(D - B), C' = B + k*(C - B) */
Point Dp = { B.x + k * (D.x - B.x), B.y + k * (D.y - B.y) };
Point Cp = { B.x + k * (C.x - B.x), B.y + k * (C.y - B.y) };

/* Solve for t where A' = B + t*(A - B) and D'A' || DA.
   Derivation:
     Let v = A - D.
     Let u(t) = (B - D') + t*(A - B). (u = A' - D')
     Parallel condition: u.x * v.y - u.y * v.x = 0
   => t = [ (B.y - D'.y)*v.x - (B.x - D'.x)*v.y ]
          / [ (A.x - B.x)*v.y - (A.y - B.y)*v.x ]
*/
double vx = A.x - D.x;
double vy = A.y - D.y;
double numerator = (B.y - Dp.y) * vx - (B.x - Dp.x) * vy;
double denominator = (A.x - B.x) * vy - (A.y - B.y) * vx;

if (fabs(denominator) < 1e-12) {
    fprintf(stderr, "Denominator ~ 0: can't find unique A' (degenerate configuration)\n");
    fclose(fp);
    return 1;
}

double t = numerator / denominator;
Point Ap = { B.x + t * (A.x - B.x), B.y + t * (A.y - B.y) };

/* Write coordinates */
fprintf(fp, "A   = (%.6f, %.6f)\n", A.x, A.y);
fprintf(fp, "B   = (%.6f, %.6f)\n", B.x, B.y);
fprintf(fp, "C   = (%.6f, %.6f)\n", C.x, C.y);
fprintf(fp, "D   = (%.6f, %.6f)\n", D.x, D.y);
fprintf(fp, "D' = (%.6f, %.6f)\n", Dp.x, Dp.y);
fprintf(fp, "C' = (%.6f, %.6f)\n", Cp.x, Cp.y);
```

C Code: triangle.c

```
    fprintf(fp, "A'␣=␣(%.6f,␣%.6f)\n", Ap.x, Ap.y);

    /* Check parallelogram: opposite sides parallel */
    int cond1 = areParallel(Ap, B, Dp, Cp); /* A'B || D'C' */
    int cond2 = areParallel(Ap, Dp, B, Cp); /* A'D' || B C' */

    if (cond1 && cond2) {
        fprintf(fp, "\nA'BC'D'␣is␣a␣parallelogram.\n");
        printf("A'BC'D'␣is␣a␣parallelogram.\n");
    } else {
        fprintf(fp, "\nA'BC'D'␣is␣NOT␣a␣parallelogram.\n");
        printf("A'BC'D'␣is␣NOT␣a␣parallelogram.\n");
    }

    fclose(fp);
    return 0;
}
```

Python: plot.py

```
   import numpy as np
import matplotlib.pyplot as plt

# --- Define a slanted parallelogram ABCD
A = np.array([0.0, 0.0])
B = np.array([4.0, 0.0])
D = np.array([1.5, 3.0]) # slanted
C = B + D - A # ensures ABCD is a parallelogram

k = 4.0 / 3.0 # scale factor for BD'C' ~ BDC

# --- Compute D' and C' by homothety about B
Dp = B + k * (D - B) # D' = B + k*(D - B)
Cp = B + k * (C - B) # C' = B + k*(C - B)

# --- Build A' so that A'BC'D' is parallelogram
Ap = B + Dp - Cp

# --- Plotting
plt.figure(figsize=(8,6))

# original ABCD (parallelogram)
poly_ABCD = np.vstack([A,B,C,D,A])
plt.plot(poly_ABCD[:,0], poly_ABCD[:,1], 'b-', linewidth=2, label="ABCD␣(original␣parallelogram)")

# scaled triangle B-D'-C'
plt.plot([B[0], Dp[0], Cp[0], B[0]], [B[1], Dp[1], Cp[1], B[1]],
        'r--', linewidth=1.5, label="␣B␣D'␣C'")

# parallelogram A'-B-C'-D'
poly_par = np.vstack([Ap, B, Cp, Dp, Ap])
plt.plot(poly_par[:,0], poly_par[:,1], 'g-o', linewidth=2, label="A'␣B␣C'␣D'␣(parallelogram)")
```

Python: plot.py

```
# plot points and annotate (only names, no coordinates)
pts = {"A":A,"B":B,"C":C,"D":D,"D'":Dp,"C'":Cp,"A'":Ap}
for name,p in pts.items():
    plt.scatter(p[0], p[1], s=60, zorder=5)
    plt.text(p[0]+0.08, p[1]+0.08, f"{name}", fontsize=10, fontweight="bold")

plt.gca().set_aspect('equal', adjustable='box')
plt.grid(True)
plt.legend()
plt.title("Parallelograms␣ABCD␣and␣A'␣B␣C'␣D'␣with␣scaled␣triangle␣BD'C'")
plt.tight_layout()

# --- Save the figure ---
plt.savefig("parallelogram_with_scaling.png", dpi=300)

plt.show()
```