4.8.3

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Question

Find the equation of the plane passing through the points $A(2, 5, -3), B(-2, -3, 5)$ and $C(5, 3, -3)$.

Solution

$$\mathbf{A} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} -2 \\ -3 \\ 5 \end{pmatrix}, \qquad \mathbf{C} = \begin{pmatrix} 5 \\ 3 \\ -3 \end{pmatrix}. \tag{1}$$

Let the equation of the plane be

$$\mathbf{n}^T \mathbf{x} = 1. \tag{2}$$

Since $\mathbf{A}, \mathbf{B}, \mathbf{C}$ lie in the plane:

$$\mathbf{n}^T \mathbf{A} = 1, \qquad \mathbf{n}^T \mathbf{B} = 1, \qquad \mathbf{n}^T \mathbf{C} = 1, \tag{3}$$

or equivalently

Solution

$$\mathbf{A}^T\mathbf{n} = 1, \qquad \mathbf{B}^T\mathbf{n} = 1, \qquad \mathbf{C}^T\mathbf{n} = 1. \tag{4}$$

Hence,

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}^T \mathbf{n} = 1. \tag{5}$$

$$\begin{pmatrix} 2 & 5 & -3 \\ -2 & -3 & 5 \\ 5 & 3 & -3 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}. \tag{6}$$

Solution

Performing row operations:

$$R_2 \leftarrow R_2 + R_1, \tag{7}$$

$$\begin{pmatrix} 2 & 5 & -3 \\ 0 & 2 & 2 \\ 5 & 3 & -3 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \tag{8}$$

$$R_3 \leftarrow 2R_3 - 5R_1, \tag{9}$$

$$\begin{pmatrix} 2 & 5 & -3 \\ 0 & 2 & 2 \\ 0 & -19 & 9 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \tag{10}$$

$$R_3 \leftarrow 19R_2 + 2R_3, \tag{11}$$

$$\begin{pmatrix} 2 & 5 & -3 \\ 0 & 2 & 2 \\ 0 & 0 & 56 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ 32 \end{pmatrix}. \tag{12}$$

Solution

Thus, solving we get

$$\mathbf{n} = \begin{pmatrix} \frac{2}{7} \\ \frac{3}{7} \\ \frac{4}{7} \end{pmatrix}. \tag{13}$$

Therfore, The equation of plane is

$$\begin{pmatrix} \frac{2}{7} \\ \frac{3}{7} \\ \frac{4}{7} \end{pmatrix}^T \mathbf{x} = 1. \tag{14}$$

Python code

```python
import numpy as np
from fractions import Fraction
import matplotlib.pyplot as plt
import os

# Create figs folder if it doesn't exist
os.makedirs(figs, exist_ok=True)

# Define points
A = np.array([2, 5, -3])
B = np.array([-2, -3, 5])
C = np.array([5, 3, -3])

# Coefficient matrix
M = np.array([A, B, C])
b = np.array([1, 1, 1])
```

Python code

```
# Solve for normal vector n (float)
n_float = np.linalg.solve(M, b)

# Convert to fractions
n_frac = [Fraction(x).limit_denominator() for x in n_float]

# Display normal vector as column matrix
print(Normal vector n (column matrix in fractions):)
for val in n_frac:
    print(f| {val} |)

# Plane equation in fraction form
x, y, z = 'x', 'y', 'z'
eq_terms = [f{val}*{var} for val, var in zip(n_frac, [x, y, z])]
plane_eq =  + .join(eq_terms) +  = 1
print(\nEquation of the plane (n^T x = 1) in fractions:)
print(plane_eq)
```

Python code

```
# ---------------- Plotting ----------------
n1, n2, n3 = n_float # Use float for plotting

# Create grid
xx = np.linspace(-5, 5, 20)
yy = np.linspace(-5, 5, 20)
X, Y = np.meshgrid(xx, yy)

# Solve for Z from plane equation
Z = (1 - n1*X - n2*Y) / n3
```

Python code

```

# Plotting
fig = plt.figure(figsize=(8,6))
ax = fig.add_subplot(111, projection='3d')
ax.plot_surface(X, Y, Z, alpha=0.5, color='cyan', rstride=1,
    cstride=1)

# Plot points
points = {'A': A, 'B': B, 'C': C}
colors = {'A': 'red', 'B': 'green', 'C': 'blue'}

for label, point in points.items():
    ax.scatter(*point, color=colors[label], s=50, label=label)
    # Annotate with coordinates
    ax.text(point[0], point[1], point[2], f'{label}{tuple(point)}
        ', color=colors[label])
```

Python code

```

ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.legend()
plt.title(Plane passing through points A, B, C)

# Save figure in figs folder
plt.savefig(figs/fig1.png)
plt.show()
```

C code

```c
#include <stdio.h>

typedef struct {
    double x, y, z;
} Point;

int main() {
    Point A = {2, 5, -3};
    Point B = {-2, -3, 5};
    Point C = {5, 3, -3};

    // Compute vectors AB and AC
    double AB[3] = {B.x - A.x, B.y - A.y, B.z - A.z};
    double AC[3] = {C.x - A.x, C.y - A.y, C.z - A.z};
```

C code

```
// Normal vector n = AB x AC
double n[3];
n[0] = AB[1]*AC[2] - AB[2]*AC[1];
n[1] = AB[2]*AC[0] - AB[0]*AC[2];
n[2] = AB[0]*AC[1] - AB[1]*AC[0];

// Plane equation: nX = d
double d = n[0]*A.x + n[1]*A.y + n[2]*A.z;

// Save points and plane to file
FILE *fp = fopen(plane_points.dat, w);
fprintf(fp, # Plane: %lf*x + %lf*y + %lf*z = %lf\n, n[0], n
    [1], n[2], d);
fprintf(fp, %lf %lf %lf\n, A.x, A.y, A.z);
fprintf(fp, %lf %lf %lf\n, B.x, B.y, B.z);
fprintf(fp, %lf %lf %lf\n, C.x, C.y, C.z);
fclose(fp);
```

C code

```c

    // Print normal and d for Python
    printf(%lf %lf %lf %lf\n, n[0], n[1], n[2], d);

    return 0;
}
```

Plot

beamer/figs/fig1.png