2.4.5

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Question

Show that the points $\mathbf{A}(2\hat{i} - \hat{j} + \hat{k}),\ \mathbf{B}(\hat{i} - 3\hat{j} - 5\hat{k}),\ \mathbf{C}(3\hat{i} - 4\hat{j} - 4\hat{k})$ are the vertices of a right angled triangle.

The vertices of a triangle

Point	Vector
A	$\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$
B	$\begin{bmatrix} 1 \\ -3 \\ -5 \end{bmatrix}$
C	$\begin{bmatrix} 3 \\ -4 \\ -4 \end{bmatrix}$

Table: Vectors

Sides of Triangle

The sides of the triangle will be

$$\mathbf{B}-\mathbf{A}=\begin{bmatrix}-1\\-2\\-6\end{bmatrix},\quad \mathbf{C}-\mathbf{B}=\begin{bmatrix}2\\-1\\1\end{bmatrix},\quad \mathbf{A}-\mathbf{C}=\begin{bmatrix}-1\\3\\5\end{bmatrix} \tag{1}$$

Condition for right angled triabgle

In a right angled triangle

$$(\mathbf{C}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})=0 \tag{2}$$

theoretical solution

from Equation (1)

$$(\mathbf{C}-\mathbf{B})^T(\mathbf{C}-\mathbf{A}) = \begin{bmatrix} 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix} = 0 \tag{3}$$

C Code - A function to find if triangle is right angled

```c
#include <stdio.h>

// Function to compute dot product of two 3D vectors
float dot_product(float v1[3], float v2[3]) {
    return v1[0]*v2[0] + v1[1]*v2[1] + v1[2]*v2[2];
}

// Function to check if A, B, C form a right-angled triangle
// Returns 1 if true, 0 otherwise
int is_right_triangle(float A[3], float B[3], float C[3]) {
    float AB[3], BC[3], AC[3];

    // Compute vectors
    AB[0] = B[0] - A[0]; AB[1] = B[1] - A[1]; AB[2] = B[2] - A
        [2];
    BC[0] = C[0] - B[0]; BC[1] = C[1] - B[1]; BC[2] = C[2] - B
        [2];
    AC[0] = C[0] - A[0]; AC[1] = C[1] - A[1]; AC[2] = C[2] - A
        [2];
```

C Code - A function to find if the triangle is right angled

```c

    // Check dot products
    if (dot_product(AB, AC) == 0) {
        printf("Angle A is 90 degrees.\n");
        return 1;
    }
    if (dot_product(AB, BC) == 0) {
        printf("Angle B is 90 degrees.\n");
        return 1;
    }
    if (dot_product(AC, BC) == 0) {
     printf("Angle C is 90 degrees.\n");
        return 1;
    }
    return 0;
}
```

Python Code

```python
    import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d.art3d import Poly3DCollection
import ctypes
import os

# --- Load the C library ---
try:
    c_lib = ctypes.CDLL('./code.so')
except OSError:
    print("Error: 'code.so' not found. Compile using: gcc -shared -o code.so -fPIC triangle.c")
    exit()
```

Python Code

```
    # Define argument and return types
c_lib.is_right_triangle.argtypes = [ctypes.c_float, ctypes.
    c_float, ctypes.c_float,
                                 ctypes.c_float, ctypes.c_float,
                                     ctypes.c_float,
                                 ctypes.c_float, ctypes.c_float,
                                     ctypes.c_float]
c_lib.is_right_triangle.restype = ctypes.c_int

# --- Given points ---
A = np.array([2, -1, 1], dtype=np.float32)
B = np.array([1, -3, -5], dtype=np.float32)
C = np.array([3, -4, -4], dtype=np.float32)

# --- Call C function ---
result = c_lib.is_right_triangle(A[0], A[1], A[2],
                              B[0], B[1], B[2],
                              C[0], C[1], C[2])
```

Python Code

```
    print(" The points form a right-angled triangle.")
else:
    print(" The points do not form a right-angled triangle.")

# --- Plotting ---
fig = plt.figure(figsize=(8,6))
ax = fig.add_subplot(111, projection='3d')

# Points
ax.scatter(*A, color="red", s=50)
ax.scatter(*B, color="blue", s=50)
ax.scatter(*C, color="green", s=50)

# Triangle surface
triangle = np.array([A, B, C])
ax.add_collection3d(Poly3DCollection([triangle], alpha=0.2,
    facecolor='cyan'))
```

Python Code

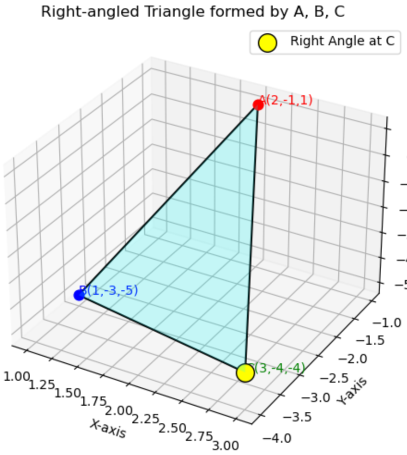
```python
    # Edges
ax.plot(*zip(A,B), color="black")
ax.plot(*zip(B,C), color="black")
ax.plot(*zip(C,A), color="black")

# Labels
ax.text(*A, "A(2,-1,1)", color="red")
ax.text(*B, "B(1,-3,-5)", color="blue")
ax.text(*C, "C(3,-4,-4)", color="green")

# Axes labels
ax.set_xlabel("X-axis")
ax.set_ylabel("Y-axis")
ax.set_zlabel("Z-axis")
ax.set_title("Triangle formed by A, B, C")

plt.show()
```

Plot


Right-angled Triangle formed by A, B, C
Right Angle at C
A(2,-1,1)
B(1,-3,-5)
C(3,-4,-4)
X-axis
Y-axis
Z-axis
1.00
1.25
1.50
1.75
2.00
2.25
2.50
2.75
3.00
−1.0
−1.5
−2.0
−2.5
−3.0
−3.5
−4.0
1
0
−1
−2
−3
−4
−5