4.7.12

EE25BTECH11026-Harsha

Question:
Find the distance of the line $4x - y = 0$ from the point P$(4, 1)$ measured along the line making an angle of 135° with the positive x-axis.
Solution:
Let us solve the given question theoretically and then verify the solution computationally.

According to the question,

$$\text{Equation of target line: } \begin{pmatrix} 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0 \tag{0.1}$$

and

$$\mathbf{P} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \tag{0.2}$$

As the direction of line makes an angle of 135° with the $+x$ axis, the unit direction vector of the line is given by

$$\mathbf{m_0} = \begin{pmatrix} \cos 135^\circ \\ \sin 135^\circ \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \tag{0.3}$$

To calculate the distance κ of a vector $\mathbf{P}$ from the target line $\mathbf{n}^\top \mathbf{x} = c$ along a line with direction vector $\mathbf{m_0}$,

$$\text{Parametric form: } \mathbf{x} = \mathbf{P} + \kappa \mathbf{m_0} \tag{0.4}$$

$$\implies \mathbf{n}^\top (\mathbf{P} + \kappa \mathbf{m_0}) = c \tag{0.5}$$

$$\therefore \kappa = \frac{c - \mathbf{n}^\top \mathbf{P}}{\mathbf{n}^\top \mathbf{m_0}} \tag{0.6}$$

$$\tag{0.7}$$

$$\implies \kappa = \frac{-\begin{pmatrix} 4 & -1 \end{pmatrix} \begin{pmatrix} 4 \\ 1 \end{pmatrix}}{\begin{pmatrix} 4 & -1 \end{pmatrix} \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}} \tag{0.8}$$

$$\kappa = 3\sqrt{2} \text{ units} \tag{0.9}$$

From the figure, it is clearly verified that the theoretical solution matches with the computational solution.