

# MatGeo Assignment 2.6.13

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AI25BTECH11007

## Question:

Given that vectors **a**, **b**, **c** form a triangle such that

$$\mathbf{a} = \mathbf{b} + \mathbf{c},$$

find  $p, q, r, s$  given that

$$\mathbf{a} = p\hat{i} + q\hat{j} + r\hat{k}, \quad \mathbf{b} = s\hat{i} + 3\hat{j} + 4\hat{k}, \quad \mathbf{c} = 3\hat{i} + 1\hat{j} - 2\hat{k},$$

and the area of the triangle is  $5\sqrt{6}$ .

## Solution:

We are given:

$$\mathbf{a} = \mathbf{b} + \mathbf{c} \quad (0.1)$$

$$\mathbf{a} = p\hat{i} + q\hat{j} + r\hat{k}, \quad \mathbf{b} = s\hat{i} + 3\hat{j} + 4\hat{k}, \quad \mathbf{c} = 3\hat{i} + 1\hat{j} - 2\hat{k} \quad (0.2)$$

and the area of the triangle formed by these vectors is:

$$\text{Area} = 5\sqrt{6} \quad (0.3)$$

**Observation:** For three vectors to form a triangle, they must sum to zero:

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0} \quad (0.4)$$

However, we are told:

$$\mathbf{a} = \mathbf{b} + \mathbf{c} \Rightarrow \mathbf{a} - \mathbf{b} - \mathbf{c} = \mathbf{0} \quad (0.5)$$

This implies:

$$\mathbf{a} + (-\mathbf{b}) + (-\mathbf{c}) = \mathbf{0} \quad (0.6)$$

So, the triangle is formed by the vectors **a**, **-b**, **-c**. For these to form a triangle, they must not lie along the same line (i.e., must not be collinear).

Now, if we assume:

$$\mathbf{a} = \mathbf{0} \Rightarrow \mathbf{b} + \mathbf{c} = \mathbf{0} \Rightarrow \mathbf{b} = -\mathbf{c} \quad (0.7)$$

Given:

$$\mathbf{c} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} \Rightarrow \mathbf{b} = -\mathbf{c} = -\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix} \quad (0.8)$$

Then:

$$\mathbf{a} = \mathbf{b} + \mathbf{c} = \mathbf{0} \Rightarrow p = 0, \quad q = 0, \quad r = 0 \quad (0.9)$$

We now compute the area of the triangle using:

$$\text{Area} = \frac{1}{2} \|\mathbf{b} \times \mathbf{c}\| \quad (0.10)$$

Compute the cross product:

$$\mathbf{b} \times \mathbf{c} = \mathbf{0} \Rightarrow \text{Area} = 0 \quad (0.11)$$

If we assume  $\mathbf{a} = \mathbf{0}$ , then  $\mathbf{b} = -\mathbf{c}$ , and the triangle is degenerate (i.e., the vectors lie on a straight line). Therefore, the area is zero:

$$\boxed{\text{Area} = 0} \quad (0.12)$$

This contradicts the given area of  $5\sqrt{6}$ . Therefore, no solution exists such that:

$$\mathbf{a} = \mathbf{b} + \mathbf{c} \quad \text{and} \quad \text{Area} = 5\sqrt{6} \quad (0.13)$$