2.5.2

Revanth Siva Kumar D - EE25BTECH11048

Question The points $(-4,0),(4,0),(0,3)$ are the vertices of a:

(a) right triangle
(b) isosceles triangle
(c) equilateral triangle
(d) scalene triangle

Solution:

Step 1: Represent points as column vectors

$$\mathbf{A} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{1}$$

Step 2: Check for right-angled triangle (perpendicular sides)

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}, \quad \mathbf{C}-\mathbf{A} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \quad (\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A}) = \begin{pmatrix} 8 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = 32 \neq 0 \tag{2}$$

$$\mathbf{A}-\mathbf{B} = \begin{pmatrix} -8 \\ 0 \end{pmatrix}, \quad \mathbf{C}-\mathbf{B} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}, \quad (\mathbf{A}-\mathbf{B})^\top(\mathbf{C}-\mathbf{B}) = \begin{pmatrix} -8 & 0 \end{pmatrix}\begin{pmatrix} -4 \\ 3 \end{pmatrix} = 32 \neq 0 \tag{3}$$

$$\mathbf{A}-\mathbf{C} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}, \quad \mathbf{B}-\mathbf{C} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}, \quad (\mathbf{A}-\mathbf{C})^\top(\mathbf{B}-\mathbf{C}) = \begin{pmatrix} -4 & -3 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \end{pmatrix} = -7 \neq 0 \tag{4}$$

Since no pair of sides is perpendicular, the triangle is not right-angled.

Step 3: Check for isosceles triangle (perpendicular bisector method)

$$\text{Midpoint of } AB: \quad \mathbf{M} = \frac{\mathbf{A}+\mathbf{B}}{2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{5}$$

$$\mathbf{C}-\mathbf{M} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{6}$$

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \tag{7}$$

$$(\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{M}) = \begin{pmatrix} 8 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 3 \end{pmatrix} = 0 \tag{8}$$

Hence, C lies on the perpendicular bisector of AB.

$$AC = BC = 5 \implies \triangle ABC \text{ is isosceles.}$$

Step 4: Check for equilateral triangle (using norm squared)

$$\|AB\|^2 = (\mathbf{B}-\mathbf{A})^\top(\mathbf{B}-\mathbf{A}) = \begin{pmatrix} 8 & 0 \end{pmatrix}\begin{pmatrix} 8 \\ 0 \end{pmatrix} = 64 \tag{9}$$

$$\|BC\|^2 = (\mathbf{C}-\mathbf{B})^\top(\mathbf{C}-\mathbf{B}) = \begin{pmatrix} -4 & 3 \end{pmatrix}\begin{pmatrix} -4 \\ 3 \end{pmatrix} = 16+9 = 25 \tag{10}$$

$$\|AC\|^2 = (\mathbf{C}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A}) = \begin{pmatrix} 4 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = 16+9 = 25 \tag{11}$$

All sides are not equal (64, 25, 25), so the triangle is not equilateral.

Step 5: Check for scalene triangle

Since two sides are equal, the triangle is not scalene.

Conclusion: Therefore, the triangle with vertices $(-4,0),(4,0),(0,3)$ is an **isosceles triangle** with $AC = BC = 5$.

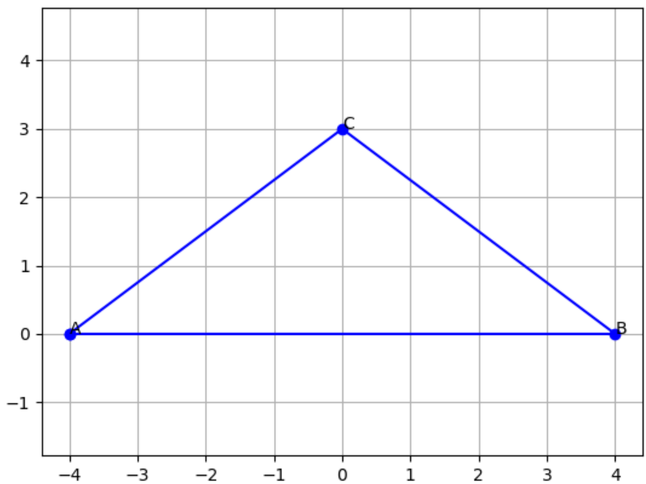

Fig. 1: Shared Output Plot

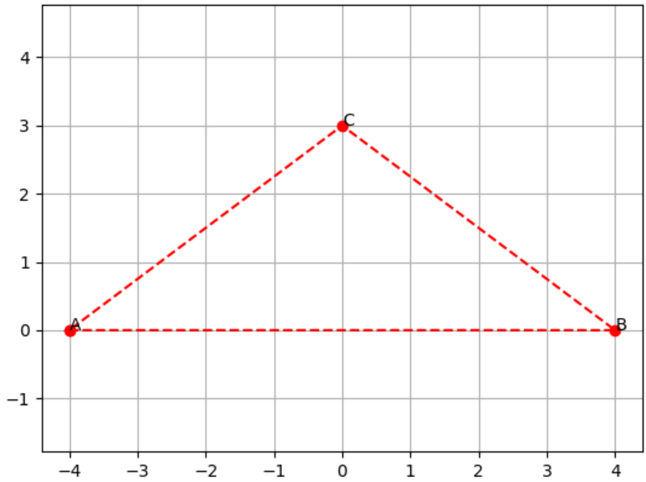

Fig. 2: Direct Python code plot