12.150

EE25BTECH11032 - Kartik Lahoti

Question:

Two sides of a triangle are represented by vectors $\mathbf{a} = \hat{i} + \hat{j} + \hat{k}$ and $\mathbf{b} = -\hat{i} + -\hat{j} + \hat{k}$. The area (magnitude) of the triangle is

1) $\frac{1}{\sqrt{2}}$ 2) 1 3) $\sqrt{2}$ 4) $2\sqrt{2}$

Solution:

Given ,

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \tag{4.1}$$

Area of Trianle

$$\frac{1}{2} \|\mathbf{a} \times \mathbf{b}\| \tag{4.2}$$

Also,

$$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} \begin{vmatrix} \mathbf{a}_{23} & \mathbf{b}_{23} \\ \mathbf{a}_{31} & \mathbf{b}_{31} \end{vmatrix} \\ \begin{vmatrix} \mathbf{a}_{12} & \mathbf{b}_{12} \end{vmatrix} \end{pmatrix} \tag{4.3}$$

$$= \begin{pmatrix} 1 \cdot 1 - 1 \cdot (-1) \\ 1 \cdot (-1) - 1 \cdot 1 \\ 1 \cdot (-1) - 1 \cdot (-1) \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} \tag{4.4}$$

$$Ar(\Delta) = \frac{1}{2} \left\| \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} \right\| \tag{4.5}$$

$$= \frac{1}{2} 2\sqrt{2} \tag{4.6}$$

$$= \sqrt{2} \tag{4.7}$$

Hence, Answer : Option 3

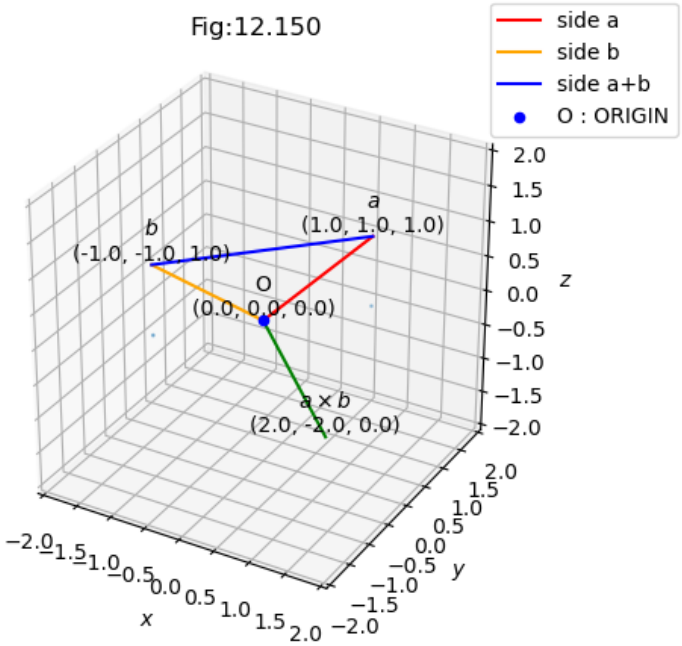
Fig:12.150
side a
side b
side a+b
O : ORIGIN
a
(1.0, 1.0, 1.0)
b
(-1.0, -1.0, 1.0)
O
(0.0, 0.0, 0.0)
a × b
(2.0, -2.0, 0.0)
x
y
z