4.3.34

EE25BTECH11044 - Sai Hasini Pappula

Question: If the line

$$\frac{x}{a} + \frac{y}{b} = 1$$

passes through the points $(2, -3)$ and $(4, -5)$, then find (a, b).

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Solution

The line in intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1. \tag{0.1}$$

Substituting the points $\mathbf{x}_1 = (2, -3)^T$ and $\mathbf{x}_2 = (4, -5)^T$ yields

$$\frac{2}{a} - \frac{3}{b} = 1, \qquad \frac{4}{a} - \frac{5}{b} = 1. \tag{0.2}$$

Introduce unknowns

$$u = \frac{1}{a}, \quad v = \frac{1}{b}, \tag{0.3}$$

so the system becomes the linear matrix equation

$$\begin{bmatrix} 2 & -3 \\ 4 & -5 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{0.4}$$

Write the augmented matrix and perform Gauss–Jordan elimination:

$$\left[\begin{array}{cc|c} 2 & -3 & 1 \\ 4 & -5 & 1 \end{array}\right] \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \left[\begin{array}{cc|c} 2 & -3 & 1 \\ 0 & 1 & -1 \end{array}\right] \tag{0.5}$$

$$\xrightarrow{R_1 \leftarrow R_1 + 3R_2} \left[\begin{array}{cc|c} 2 & 0 & -2 \\ 0 & 1 & -1 \end{array}\right] \xrightarrow{R_1 \leftarrow \frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & -1 \end{array}\right]. \tag{0.6}$$

Thus the solution for the unknown vector is

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}.$$

Back-substitute $u = \frac{1}{a}$, $v = \frac{1}{b}$:

$$\frac{1}{a} = -1 \Rightarrow a = -1, \qquad \frac{1}{b} = -1 \Rightarrow b = -1. \tag{0.7}$$

Final Answer:

$$(a, b) = (-1, -1), \tag{0.8}$$

and the line becomes

$$\frac{x}{-1} + \frac{y}{-1} = 1 \quad \Longrightarrow \quad x + y = -1.$$