

4.4.8 Matgeo

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Question

Find the value of x such that the four points $A(x, 5, -1)$, $B(3, 2, 1)$, $C(4, 5, 5)$, and $D(4, 2, -2)$ are coplanar.

Solution

The equation of a plane can be given by the formula :

$$n^T \mathbf{x} = 1 \quad (1)$$

$$\text{or} \quad (2)$$

$$\mathbf{x}^T \mathbf{n} = 1 \quad (3)$$

Since all the points A,B,C,D are on the plane :

$$A^T n = 1 \quad B^T n = 1 \quad C^T n = 1 \quad D^T n = 1 \quad (4)$$

Solution

To find **D** we find **n** :

Combining the above equation we get :

$$\begin{bmatrix} B \\ C \\ D \end{bmatrix}^T \mathbf{n} = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 5 & 5 \\ 4 & 2 & -2 \end{bmatrix} \mathbf{n} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad (5)$$

solving the equation by row reduction we get

$$\mathbf{n} = \begin{bmatrix} \frac{9}{16} \\ \frac{7}{16} \\ \frac{3}{16} \end{bmatrix} = \frac{1}{16} \begin{bmatrix} 9 \\ 7 \\ 3 \end{bmatrix} \quad (6)$$

Solution

substituting in the equation $A^T n = 1$ we get:

$$\begin{bmatrix} x & 5 & -1 \end{bmatrix} \begin{bmatrix} 9 \\ -7 \\ 3 \end{bmatrix} = 16 \quad (7)$$

$$9x - 35 - 3 = 16 \quad (8)$$

$$9x = 54 \quad (9)$$

$$x = 6 \quad (10)$$

Graphical Representation

