

4.3.35

EE25BTECH11045 - P.Navya Priya

Question:

Find the length and the foot of perpendicular from the point $\left(1, \frac{3}{2}, 2\right)$ to the plane $2x - 2y + 4z + 5 = 0$.

Solution:

Given plane equation $2x - 2y + 4z + 5 = 0$ can be written as

$$\mathbf{n}^T \mathbf{x} = c \quad (1)$$

Where

$$\mathbf{n} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} \text{ and } c = -5$$

Let the point be $\mathbf{p} \begin{pmatrix} 1 \\ \frac{3}{2} \\ 2 \end{pmatrix}$ and the point on the plane be \mathbf{x}_o . The equation of the line joining \mathbf{p} and \mathbf{x}_o is

$$\mathbf{x}_o = \mathbf{p} + \lambda \mathbf{n} \quad (2)$$

Multiply equation(2) on both sides by \mathbf{n}^T

$$\mathbf{n}^T (\mathbf{x}_o) = \mathbf{n}^T (\mathbf{p} + \lambda \mathbf{n}) \quad (3)$$

$$\lambda = \frac{\mathbf{n}^T \mathbf{x}_o}{\mathbf{n}^T \mathbf{p} + \lambda \mathbf{n}^T \mathbf{n}} \quad (4)$$

$$-5 = (2 \ -2 \ 4) \begin{pmatrix} 1 \\ \frac{3}{2} \\ 2 \end{pmatrix} + \lambda (2 \ -2 \ 4) \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} \quad (5)$$

$$\lambda = -\frac{1}{2} \quad (6)$$

Substitute the value of λ in equation(2) to get \mathbf{x}_o

$$\mathbf{x}_o = \begin{pmatrix} 0 \\ \frac{5}{2} \\ 0 \end{pmatrix} \quad (7)$$

$$\therefore \text{Foot of perpendicular is } \mathbf{x}_o = \begin{pmatrix} 0 \\ \frac{5}{2} \\ 0 \end{pmatrix}$$

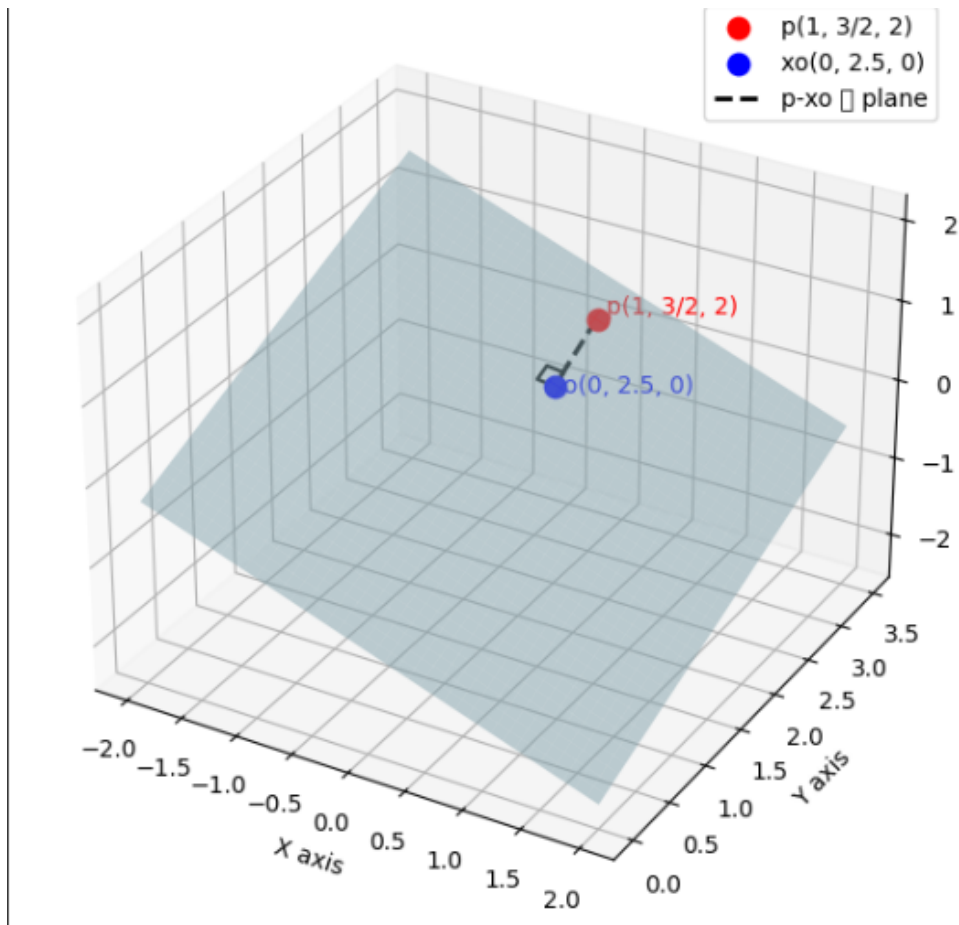
The length of point $\begin{pmatrix} 1 \\ 3/2 \\ 2 \end{pmatrix}$ to the plane $2x - 2y + 4z + 5 = 0$ is

$$\|x_0 - p\| = \sqrt{(x_0 - p)^T (x_0 - p)} \quad (8)$$

$$= \sqrt{6} \quad (9)$$

$$\therefore \|x_0 - p\| = \sqrt{6}$$

From the graph, theoretical solution matches with the computational solution.



Perpendicular to the plane (length= $\sqrt{6}$)