4.4.17

AI25BTECH11021 - Abhiram Reddy N

QUESTION

A point **P** divides the line segment joining the points $\mathbf{A}(3, -5)$ and $\mathbf{B}(-4, 8)$ such that $\frac{AP}{PB} = \frac{K}{1}$. If **P** lies on the line $x + y = 0$, then find the value of K.

ANSWER

Step 1: Represent points as column vectors

$$\mathbf{A} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} = \begin{bmatrix} 3 & -5 \end{bmatrix}^\top, \quad \mathbf{B} = \begin{pmatrix} -4 \\ 8 \end{pmatrix} = \begin{bmatrix} -4 & 8 \end{bmatrix}^\top.$$

*Step 2: Express **P** using section formula in vector form*

Since **P** divides **AB** in the ratio $K : 1$,

$$\mathbf{P} = \frac{K\mathbf{B} + \mathbf{A}}{K + 1} = \frac{K\begin{bmatrix} -4 \\ 8 \end{bmatrix} + \begin{bmatrix} 3 \\ -5 \end{bmatrix}}{K + 1} = \frac{1}{K + 1}\begin{bmatrix} 3 - 4K \\ -5 + 8K \end{bmatrix}.$$

Step 3: Use the line equation condition

The point $\mathbf{P} = \begin{bmatrix} x \\ y \end{bmatrix}$ lies on the line $x + y = 0$, which can be written as

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \mathbf{P} = 0.$$

Substitute **P**:

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \frac{1}{K + 1}\begin{bmatrix} 3 - 4K \\ -5 + 8K \end{bmatrix} = 0.$$

Step 4: Simplify and solve for K

Multiply:

$$\frac{1}{K + 1}\left((3 - 4K) + (-5 + 8K)\right) = 0,$$

$$\frac{1}{K + 1}(3 - 4K - 5 + 8K) = 0,$$

$$\frac{1}{K + 1}(-2 + 4K) = 0.$$

Since $K + 1 \neq 0$,

$$-2 + 4K = 0,$$

$$4K = 2,$$

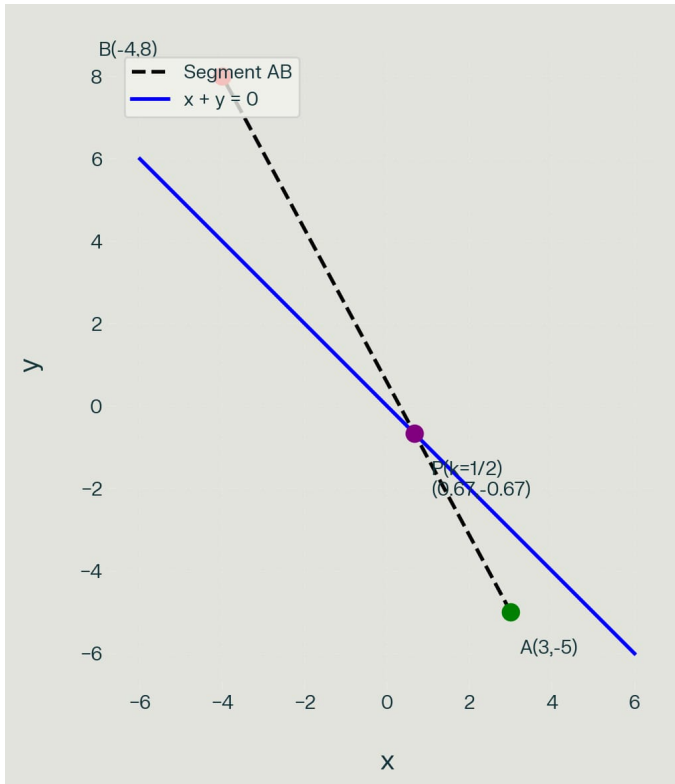$$K = \frac{1}{2}.$$

Final answer

$$\boxed{K = \frac{1}{2}}.$$

Fig. 0.1: plot