5.13.10

AI25BTECH11024 - Pratyush Panda

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Question:

if

$$P = \begin{pmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{pmatrix} \tag{0.1}$$

is the adjoint of a 3×3 matrix $\mathbf{A}$ and $|\mathbf{A}| = 4$, then α is equal to

a 4

b 11

c 5

d 0

Solution:
Given

$$P = \begin{pmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{pmatrix} \tag{0.2}$$

is the adjoint of a 3×3 matrix $\mathbf{A}$ and $|A| = 4$.
We know that,

$$adj(\mathbf{A}) = |\mathbf{A}|\mathbf{A}^{-1}. \tag{0.3}$$

Hence,

$$\mathbf{P} = 4\mathbf{A}^{-1} \quad \Rightarrow \quad \mathbf{A} = 4\mathbf{P}^{-1}. \tag{0.4}$$

Taking determinants on both sides,

$$|\mathbf{A}| = |4\mathbf{P}^{-1}| = 4^3|\mathbf{P}^{-1}| = 64 \cdot \frac{1}{|\mathbf{P}|}. \tag{0.5}$$

Since $|\mathbf{A}| = 4$,

$$\frac{64}{\det(P)} = 4 \quad \Rightarrow \quad |\mathbf{P}| = 16. \tag{0.6}$$

Now compute $|\mathbf{P}|$:

$$|\mathbf{P}| = \begin{pmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{pmatrix} \tag{0.7}$$

Simplifying,

$$|\mathbf{P}| = 1(12 - 12) - \alpha(4 - 6) + 3(4 - 6) \tag{0.8}$$

$$|\mathbf{P}| = 0 + 2\alpha - 6 = 2(\alpha - 3). \tag{0.9}$$

Equating this with $|\mathbf{P}| = 16$,

$$2(\alpha - 3) = 16 \quad \Rightarrow \quad \alpha - 3 = 8 \quad \Rightarrow \alpha = 11. \tag{0.10}$$