## 7.4.27

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Question:

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The triangle PQR is inscribed in the circle  $x^2 + y^2 = 25.$ If **Q** and **R** have co-ordinates (3,4) and (-4,3) respectively then  $\angle QPR$  is equal to

(A)  $\frac{\pi}{2}$ 

(B)  $\frac{\pi}{3}$ 

(C)  $\frac{\pi}{4}$ 

(D)  $\frac{7}{6}$ 

Solution:

**Table** 

$$\begin{array}{|c|c|}
\hline
\mathbf{Q} & \begin{pmatrix} 3\\4 \end{pmatrix} \\
\hline
\mathbf{R} & \begin{pmatrix} -4\\3 \end{pmatrix}
\end{array}$$

$$\mathbf{x}^{\mathsf{T}}\mathbf{x} = 25$$

(0.1)

The given points (position vectors) are

$$\mathbf{q} = \begin{pmatrix} 3 \\ 4 \end{pmatrix},$$

$$\mathbf{r} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

Verify they lie on the circle:

$$\mathbf{q}^{\top}\mathbf{q} = 3^2 + 4^2 = 25,$$
  
 $\mathbf{r}^{\top}\mathbf{r} = (-4)^2 + 3^2 = 25.$ 

Compute the inner product (matrix/dot product):

$$\mathbf{q}^{ op}$$
r

$$\mathbf{q}^{\top}\mathbf{r} = \begin{pmatrix} 3 & 4 \end{pmatrix} \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$
$$= 3 \cdot (-4) + 4 \cdot 3 = -12 + 12 = 0.$$

(0.2)

Compute norms (using matrix notation) and the central angle  $\theta$ :

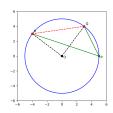
$$\|\mathbf{q}\| = \sqrt{\mathbf{q}^{\mathsf{T}}\mathbf{q}} = 5,$$
  $\|\mathbf{r}\| = \sqrt{\mathbf{r}^{\mathsf{T}}\mathbf{r}} = 5,$  (0.7)

$$\cos \theta = \frac{\mathbf{q}^{\top} \mathbf{r}}{\|\mathbf{q}\| \|\mathbf{r}\|} = \frac{0}{5 \cdot 5} = 0 \tag{0.8}$$

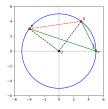
$$\implies \theta = \frac{\pi}{2}.\tag{0.9}$$

Since  $\angle QPR$  is the angle subtended at the circumference by chord QR, it equals half the central angle:

$$\angle QPR = \frac{\theta}{2} = \frac{\pi}{4}.\tag{0.10}$$



## Plot using C libraries



Plot using Python