2.4.19

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Problem Statement

If $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are the points with position vectors $\hat{i} + \hat{j} - \hat{k}$, $2\hat{i} - \hat{j} + 3\hat{k}$, $2\hat{i} - 3\hat{k}$, $3\hat{i} - 2\hat{j} + \hat{k}$ respectively, find the projection of AB along CD.

Solution:

Symbol	Value	Description
A	$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$	First Point
B	$\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$	Second Point
C	$\begin{pmatrix} 2 \\ 0 \\ -3 \end{pmatrix}$	Third Point
D	$\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$	Fourth Point

From the the given points we find AB and CD

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}, \mathbf{D} - \mathbf{C} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} \tag{2.4.19.1}$$

Let P be the projection of AB along CD. We know that

$$\mathbf{P} = \left(\frac{(\mathbf{B} - \mathbf{A})^\top (\mathbf{D} - \mathbf{C})}{\|\mathbf{D} - \mathbf{C}\|^2}\right)(\mathbf{D} - \mathbf{C}) \tag{2.4.19.2}$$

Substituting (**??**) in (**??**) we get

$$P = (\mathbf{D} - \mathbf{C}) = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} \tag{2.4.19.3}$$

Thus, the projection of AB along CD = $\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix}$

See Figure **??**.

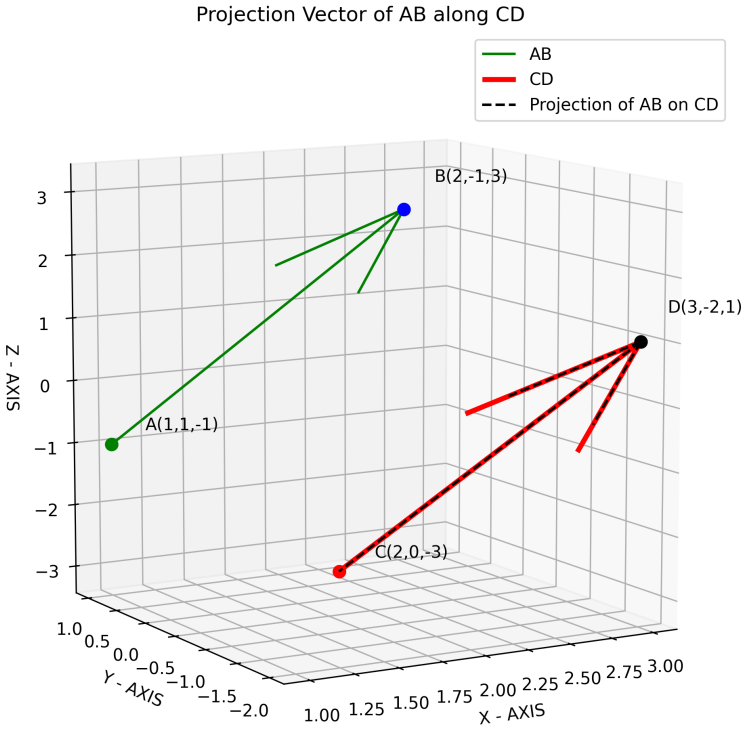
Projection Vector of AB along CD
AB
CD
Projection of AB on CD
B(2,-1,3)
D(3,-2,1)
A(1,1,-1)
C(2,0,-3)
Z - AXIS
3
2
1
0
−1
−2
−3
Y - AXIS
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
X - AXIS
1.00
1.25
1.50
1.75
2.00
2.25
2.50
2.75
3.00

Fig. 2.4.19.1