2.7.11

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Question

Q 2.7.11. Find the area of the triangle with vertices $A = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $B = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$, $C = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$.

Solution

Solution

Let the vertices be

$$\mathbf{A} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} -3 \\ -5 \end{bmatrix}.$$

The area of the triangle is

$$\Delta = \frac{1}{2} \left| \det\Big((\mathbf{B} - \mathbf{A}), (\mathbf{C} - \mathbf{A})\Big) \right|.$$

Now,

$$\mathbf{B} - \mathbf{A} = \begin{bmatrix} -4 \\ 6 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -5 \\ 7 \end{bmatrix},$$

$$\mathbf{C} - \mathbf{A} = \begin{bmatrix} -3 \\ -5 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ -4 \end{bmatrix}.$$

So,

$$\det\Big((\mathbf{B} - \mathbf{A}), (\mathbf{C} - \mathbf{A})\Big) = \begin{vmatrix} -5 & -4 \\ 7 & -4 \end{vmatrix} = (-5)(-4) - (7)(-4) = 20 + 28 = 48.$$

Hence,

$$\Delta = \frac{1}{2} \times |48| = 24.$$

Final Answer: The area of the triangle is

$$\Delta = 24.$$

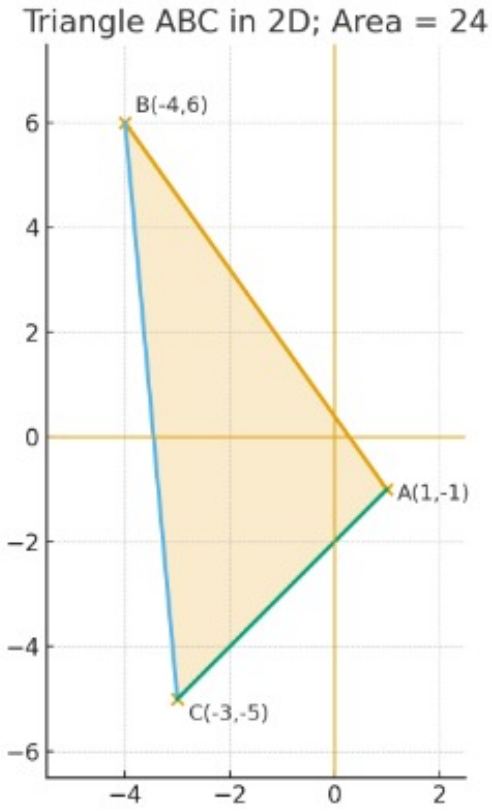

Fig. 0.1: Triangle ABC with $A(1,-1)$, $B(-4,6)$, $C(-3,-5)$; area = 24.