4.7.25

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Question

Find the points on the line $x + y = 4$ which lie at a unit distance from the line $4x + 3y = 10$.

Theoretical Solution

According to the question,

$$\text{Equation of line } L_1\text{: } \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 4 \tag{1}$$

and

$$\text{Equation of line } L_2\text{: } \begin{pmatrix} 4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 10 \tag{2}$$

Any point $\mathbf{P}$ on line L_1 is given by ,

$$\mathbf{P} = \begin{pmatrix} k \\ 4-k \end{pmatrix} \tag{3}$$

Formulae

The distance λ of a vector $\mathbf{P}$ from the line $\mathbf{n}^\top \mathbf{x} = c$ is given by ,

$$\lambda = \frac{\left|\mathbf{n}^\top \mathbf{P} - c\right|}{\|\mathbf{n}\|} \tag{4}$$

Theoretical Solution

where,

$$\mathbf{n}^\top = \begin{pmatrix} 4 & 3 \end{pmatrix} \ , c = 10 \text{ and } \lambda = 1$$

$$\implies \lambda \left\| \mathbf{n} \right\| = \left| \mathbf{n}^\top \mathbf{P} - c \right| \tag{5}$$

Also,

$$\left\| \mathbf{n} \right\| = \sqrt{\mathbf{n}^\top \mathbf{n}} = \sqrt{25} = 5 \tag{6}$$

$$\mathbf{n}^\top \mathbf{P} = k + 12 \tag{7}$$

Theoretical Solution

$$\implies 5 = |k + 12 - 10| \tag{8}$$

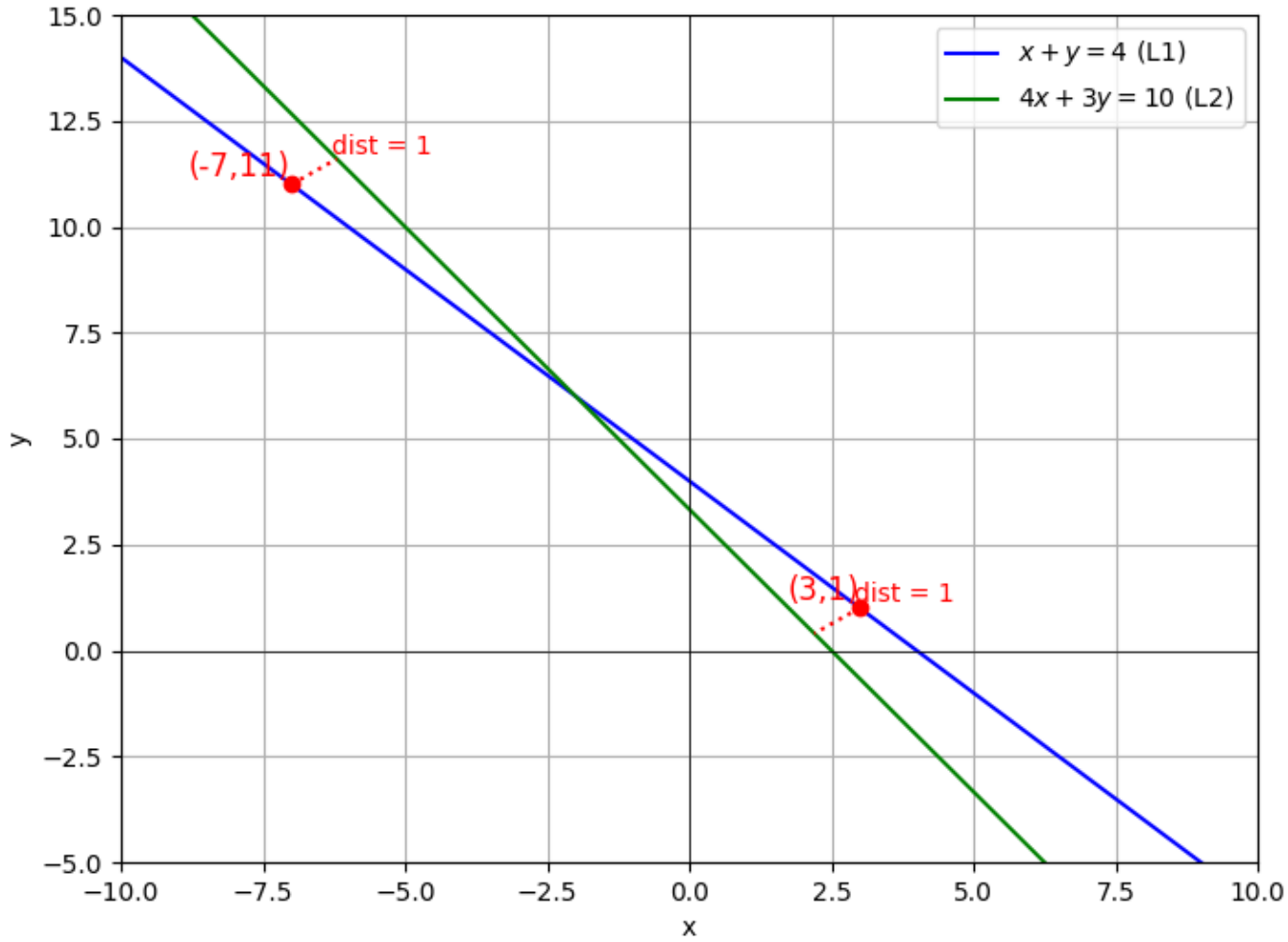
$$\implies 5 = |k + 2| \tag{9}$$

$$\implies k = 3, -7 \tag{10}$$

Therefore the points on L_1 which lie at a unit distance from the line L_2 are ,

$$\begin{pmatrix} 3 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} -7 \\ 11 \end{pmatrix}$$

Plot


15.0
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
y
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
x
$x + y = 4$ (L1)
$4x + 3y = 10$ (L2)
(-7,11)
dist = 1
(3,1)
dist = 1