4.8.20

AI25BTECH11024 - Pratyush Panda

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Question:

Find the distance between the point $(2, 3, 4)$ measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$

Solution:

Let the vector $\mathbf{A}$ be $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$, and the direction vector of the line $\mathbf{b} = \begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix}$.

The equation of the plane can be written as;

$$\mathbf{n}^T \mathbf{X} = 1 \qquad \textit{where}, \mathbf{n} = \begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} \textit{ and } \mathbf{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{0.1}$$

The equation of the line passing through $\mathbf{A}$ and with the direction vector $\mathbf{b}$ is;

$$\mathbf{x} = \mathbf{A} + \lambda \mathbf{b} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 6 \\ 2 \end{pmatrix} \tag{0.2}$$

The point on the plane lying on this line can be found out by substituting the parametric point in the equation of the plane and find out the value of λ.

$$3(2+3\lambda)+2(3+6\lambda)+2(4+2\lambda)+5=0 \tag{0.3}$$
$$\lambda=\frac{-25}{25} \tag{0.4}$$
$$or,\ \lambda=-1 \tag{0.5}$$

After solving for λ we got $\lambda=-1$. Thus, the point is **B** would be $\begin{pmatrix}-1\\-3\\2\end{pmatrix}$.

Thus, the final distance along the line can be written as;

$$d = \mathbf{A}^T.\mathbf{B} = 7 \tag{0.6}$$

Thus, the distance between the point $(2, 3, 4)$ measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$ is 7