2.2.21

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Question

If the angle between two lines is $\pi/4$ and slope of one of the lines is $1/2$,find the slope of the other line.

Equation

From the given Information,
Angle between the lines is $\pi/4$
The angle θ between $\mathbf{a}, \mathbf{b}$, is given by

$$\cos\theta = \frac{\mathbf{a}^\top \mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|} \tag{1}$$

Solution

let vector **A** be the line slope $1/2$ and **B** be the line with slope m_2

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1/2 \end{pmatrix} \tag{2}$$

$$\mathbf{B} = \begin{pmatrix} 1 \\ m_2 \end{pmatrix} \tag{3}$$

$$\cos \pi/4 = \frac{\mathbf{A}^\top \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|} \tag{4}$$

Solution

Now,

$$\mathbf{A}^\top \mathbf{B} = \begin{pmatrix} 1 \\ 1/2 \end{pmatrix}^\top \begin{pmatrix} 1 \\ m_2 \end{pmatrix} = 1^2 + m_2/2 \tag{5}$$

$$\|\mathbf{A}\| = \sqrt{1^2 + \left(\frac{1}{2}\right)^2} = \sqrt{1 + \frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2} \tag{6}$$

$$\|\mathbf{B}\| = \sqrt{1^2 + m_2^2} = \sqrt{1 + m_2^2} \tag{7}$$

From this,

$$\frac{1}{\sqrt{2}} * \frac{\sqrt{5}}{2} * \sqrt{1 + m_2^2} = 1 + \frac{m_2}{2} \tag{8}$$

$$\sqrt{5} * \sqrt{1 + m_2^2} = \sqrt{2}(2 + m_2) \tag{9}$$

Solution

Now squaring on both sides;

$$5 + 5m_2^2 = 2(4 + m_2^2 + 4m_2) \tag{10}$$

$$3m_2^2 - 8m_2 - 3 = 0 \tag{11}$$

Therefore,

$$m_2 = 3 or -1/3 \tag{12}$$

C Code

```c
#include <stdio.h>
#include <math.h>

// Function to find slopes of second line given m1 and angle
int find_other_slopes(double m1, double theta, double *slopes) {
    // Equation derived: tan(theta) = |(m2 - m1) / (1 + m1*m2)|
    // Rearranged: (m2 - m1) = +/- tan(theta) * (1 + m1*m2)

    double t = tan(theta);
```

C Code

```
    // Case 1: (m2 - m1) = +t(1 + m1*m2)
// => m2 - t*m1*m2 = m1 + t
// => m2(1 - t*m1) = m1 + t
slopes[0] = (m1 + t) / (1 - t*m1);

// Case 2: (m2 - m1) = -t(1 + m1*m2)
// => m2 + t*m1*m2 = m1 - t
// => m2(1 + t*m1) = m1 - t
slopes[1] = (m1 - t) / (1 + t*m1);

return 2; // number of solutions
}
```

Python+C Code

```
  import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load shared library
lib = ctypes.CDLL("./libline_angle.so")

# Define function signature
lib.find_other_slopes.argtypes = [ctypes.c_double, ctypes.
    c_double, ctypes.POINTER(ctypes.c_double)]
lib.find_other_slopes.restype = ctypes.c_int
```

Python+C Code

```
 def other_slopes_c(m1, theta):
    result = (ctypes.c_double * 2)()
    n = lib.find_other_slopes(m1, theta, result)
    return [result[i] for i in range(n)]

m1 = 0.5
theta = np.pi/4

slopes = other_slopes_c(m1, theta)
print("Slopes from C library:", slopes)
```

Python+C Code

```
# --- Plotting ---
x = np.linspace(-10, 10, 100)

plt.axhline(0, color='k')
plt.axvline(0, color='k')

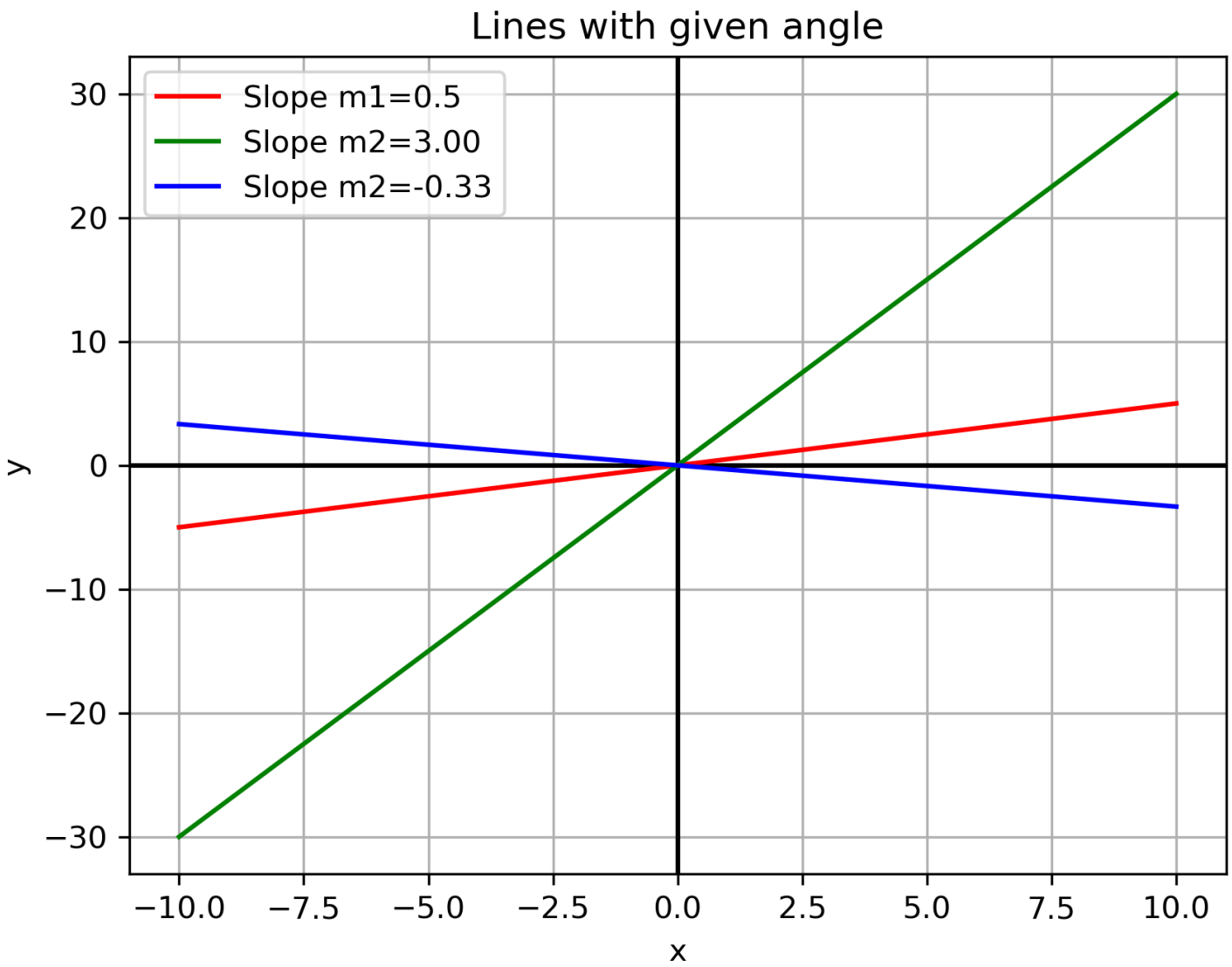
plt.plot(x, m1*x, 'r', label=f"Slope m1={m1}")
plt.plot(x, slopes[0]*x, 'g', label=f"Slope m2={slopes[0]:.2f}")
plt.plot(x, slopes[1]*x, 'b', label=f"Slope m2={slopes[1]:.2f}")

plt.legend()
plt.grid(True)
plt.xlabel("x")
plt.ylabel("y")
plt.title("Lines with given angle")
```

Python+C Code

```
# Save before show
plt.savefig("/storage/emulated/0/matrix/Matgeo/2.2.21/figs/
    Figure_1.png", dpi=300, bbox_inches='tight')
plt.show()
```

Plot


Lines with given angle
Slope m1=0.5
Slope m2=3.00
Slope m2=-0.33
30
20
10
0
−10
−20
−30
y
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
x

Python Code

```
        import numpy as np

def other_slopes(m1, theta):
    t = np.tan(theta)
    m2_1 = (m1 + t) / (1 - t*m1)
    m2_2 = (m1 - t) / (1 + t*m1)
    return m2_1, m2_2

m1 = 0.5
theta = np.pi/4

slopes = other_slopes(m1, theta)
print("Slopes of other line(s):", slopes)
```

Python Code

```
    # --- Plotting ---
import matplotlib.pyplot as plt

x = np.linspace(-10, 10, 100)

plt.axhline(0, color='k')
plt.axvline(0, color='k')

plt.plot(x, m1*x, 'r', label=f"Slope m1={m1}")
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Python Code

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