7.4.39

EE25BTECH11032 - Kartik Lahoti

Question:

If $\left(m_i, \frac{1}{m_i}\right)$, $m_i > 0, i = 1, 2, 3, 4$ are four distinct points on a circle, then show that $m_1 m_2 m_3 m_4 = 1$

Solution:

Let the circle equation be

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.1}$$

where, $\mathbf{u} = \begin{pmatrix} a \\ b \end{pmatrix}$ with a and b as constants.

Let $\mathbf{P} = \begin{pmatrix} m \\ \frac{1}{m} \end{pmatrix}$ be a arbitrary vector in space.

Putting $\mathbf{P}$ in the circle , we get

$$\|\mathbf{P}\|^2 + 2\mathbf{u}^\top \mathbf{P} + f = 0 \tag{0.2}$$

$$m^2 + \frac{1}{m^2} + 2am + \frac{2b}{m} + f = 0 \tag{0.3}$$

$$m^4 + 2am^3 + fm^2 + 2bm + 1 = 0 \tag{0.4}$$

For a general polynomial of degree n

$$a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} \cdots + a_n x^0 = 0 \tag{0.5}$$

Product of roots is given by

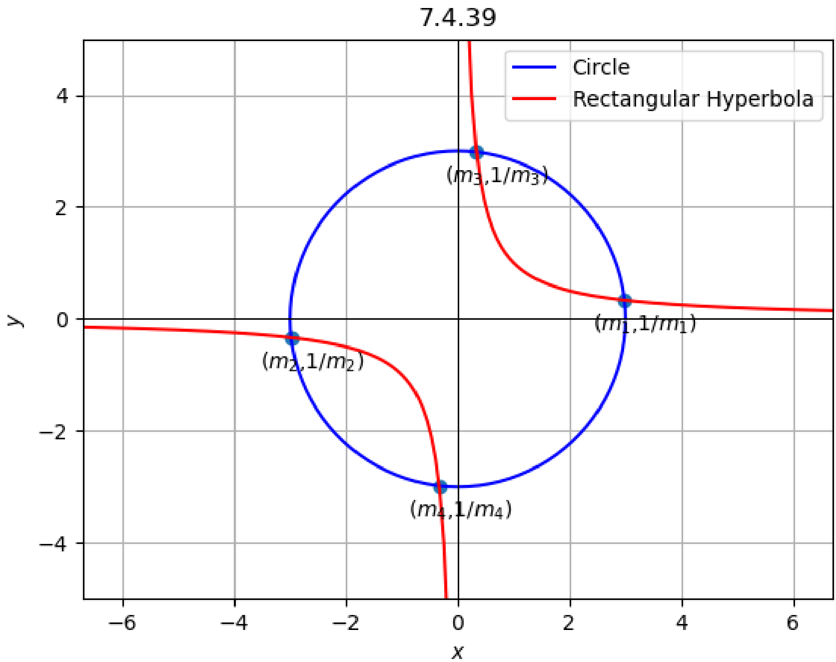
$$(-1)^n \frac{a_n}{a_0} \tag{0.6}$$

Since , m_i, where $i \in \{1, 2, 3, 4\}$ satisfies the equation 0.4.

we can say

$$m_1 m_2 m_3 m_4 = 1 \tag{0.7}$$

Hence Proved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Circle
Rectangular Hyperbola
$(m_3, 1/m_3)$
$(m_1, 1/m_1)$
$(m_2, 1/m_2)$
$(m_4, 1/m_4)$
x
y
−6
−4
−2
0
2
4
6