Matgeo Presentation - Problem 9.2.31

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Question

Find the area of the region bounded by the curve $y^2 = 4x$ and $x^2 = 4y$

variables

Variable	**Name**
$\mathbf{x_1},\mathbf{x_2}$	points of intersection
A	vector Area of the desired region

Table: Variables Used

Solution

The equation of a parabola in Matrix form is

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.1}$$

For $y^2=4x$

$$\mathbf{V_1} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{0.2}$$

$$\mathbf{u_1} = -2\mathbf{e_1} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{0.3}$$

$$f_1 = 0 \tag{0.4}$$

For $x^2=4y$

$$\mathbf{V_2} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{0.5}$$

$$\mathbf{u_2} = -2\mathbf{e_2} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{0.6}$$

$$f_2 = 0 \tag{0.7}$$

Solution

The intersection of two conics with parameters $\mathbf{v_i}$,$\mathbf{u_i}$,f_i is defined as

$$\mathbf{X}^T\left(\mathbf{V}_1+\mu\mathbf{V}_2\right)\mathbf{X}+2(\mathbf{u_1}+\mu\mathbf{u_2})^T\mathbf{X} \;+\; (f_1+\mu f_2) \;=\; 0 \tag{0.8}$$

$$\implies \begin{vmatrix} \mathbf{V}_1+\mu\mathbf{V}_2 & \mathbf{u}_1+\mu\mathbf{u}_2 \\ (\mathbf{u}_1+\mu\mathbf{u}_2)^{\mathrm{T}} & f_1+\mu f_2 \end{vmatrix} = 0 \tag{0.9}$$

$$\implies \begin{vmatrix} \mu & 0 & -2 \\ 0 & 1 & -2\mu \\ -2 & -2\mu & 0 \end{vmatrix} = 0 \tag{0.10}$$

$$\implies \begin{vmatrix} \mu & 0 & -2 \\ 0 & 1 & -2\mu \\ -2 & -2\mu & 0 \end{vmatrix} \xleftrightarrow{R_3 \leftrightarrow R_3+\frac{2}{\mu}\times R_1} \begin{vmatrix} \mu & 0 & -2 \\ 0 & 1 & -2\mu \\ 0 & -2\mu & -\frac{4}{\mu} \end{vmatrix} \tag{0.11}$$

Solution

$$\xleftrightarrow{R_3 \leftrightarrow R_3 + 2\mu \times R_2} \begin{vmatrix} \mu & 0 & -2 \\ 0 & 1 & -2\mu \\ 0 & 0 & -(\frac{4}{\mu} + 4\mu^2) \end{vmatrix} = 0 \tag{0.12}$$

$$\implies -(4 + +4\mu^3) = 0 \tag{0.13}$$

$$\implies \mu = -1 \tag{0.14}$$

substituting the value of μ=-1 in (8) we get points of intersection as

$$\mathbf{x_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.15}$$

$$\mathbf{X_2} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} \tag{0.16}$$

Conclusion

Area of the desired region is given by

$$A = \int_0^4 2\sqrt{x} - \frac{x^2}{4} dx \tag{0.17}$$

$$A = \frac{32}{3} - \frac{16}{3} \tag{0.18}$$

$$A = \frac{16}{3} \tag{0.19}$$

Plot

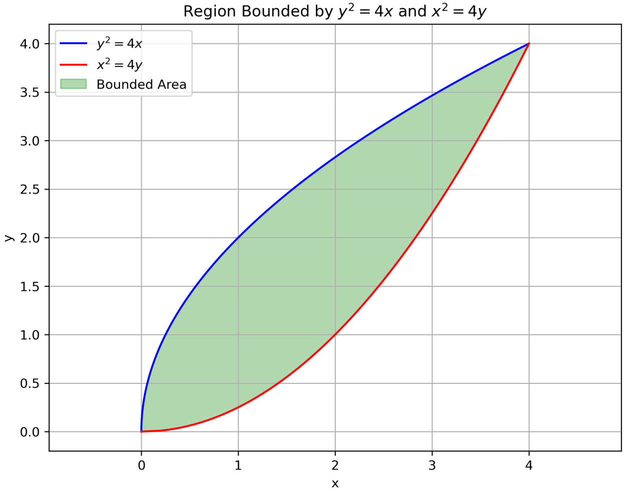

Figure: Area

C Code: area.c

```
#include <stdio.h>
#include <math.h>

double f(double y) {
    return 2 * sqrt(y) - (pow(y, 2) / 4.0);
}

int main() {
    double a = 0.0, b = 4.0; // Limits of integration
    int n = 100000; // Number of intervals
    double h = (b - a) / n;
    double area = 0.0;

    for (int i = 1; i < n; i++) {
        double y = a + i * h;
        area += f(y);
    }

    area += (f(a) + f(b)) / 2.0;
    area *= h;

    // Write the result to area.dat
    FILE *fp = fopen("area.dat", "w");
    if (fp == NULL) {
        printf("Error␣opening␣file.\n");
        return 1;
    }

    fprintf(fp, "The␣area␣bounded␣by␣the␣curves␣is:␣%.6lf\n", area);
    fclose(fp);

    return 0;}
```

Python: plot.py

```
import numpy as np
import matplotlib.pyplot as plt

# Define y range (limits from earlier analysis)
y = np.linspace(0, 4, 400)

# Curve 1: y^2 = 4x  x = y^2 / 4
x1 = y**2 / 4

# Curve 2: x^2 = 4y  x = 2 * sqrt(y)
x2 = 2 * np.sqrt(y)

# Create the plot
plt.figure(figsize=(8, 6))

# Plot the two curves
plt.plot(x1, y, label=r'$y^2␣=␣4x$', color='blue')
plt.plot(x2, y, label=r'$x^2␣=␣4y$', color='red')

# Fill the region between the curves
plt.fill_betweenx(y, x1, x2, where=(x2 > x1), color='green', alpha=0.3, label='Bounded␣Area')

# Add labels, grid, legend
plt.xlabel("x")
plt.ylabel("y")
plt.title("Region␣Bounded␣by␣$y^2␣=␣4x$␣and␣$x^2␣=␣4y$")
plt.legend()
plt.grid(True)
plt.axis('equal')

# Save the figure
plt.savefig("bounded_region.png", dpi=300, bbox_inches='tight')
print("Plot␣saved␣as␣'bounded_region.png'")
```