12.381

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Question : Consider the matrix

$$\mathbf{A} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$$

Which one of the following vectors is **NOT** a valid eigenvector of the above matrix?

1) $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ 2) $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ 3) $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$ 4) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Solution :

Name	Value
$\mathbf{A}$	$\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$

Table : Matrix

Since $\mathbf{A}$ is diagonal, its eigenvalues are the diagonal entries.

$$\lambda_1 = 4, \quad \lambda_2 = 4 \tag{1}$$

To find eigenvectors, we solve:

$$(\mathbf{A} - \lambda\mathbf{I})\mathbf{v} = \mathbf{0} \tag{2}$$

For $\lambda = 4$:

$$\mathbf{A} - 4\mathbf{I} = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \tag{3}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{4}$$

So:

$$(\mathbf{A} - 4\mathbf{I})\mathbf{v} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\mathbf{v} = \mathbf{0} \tag{5}$$

Thus, every $\mathbf{v} \in \mathbb{R}^2$ satisfies this equation. But an eigenvector must be nonzero.

Conclusion: Any nonzero vector is an eigenvector corresponding to $\lambda = 4$. The zero vector $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is **not** a valid eigenvector.

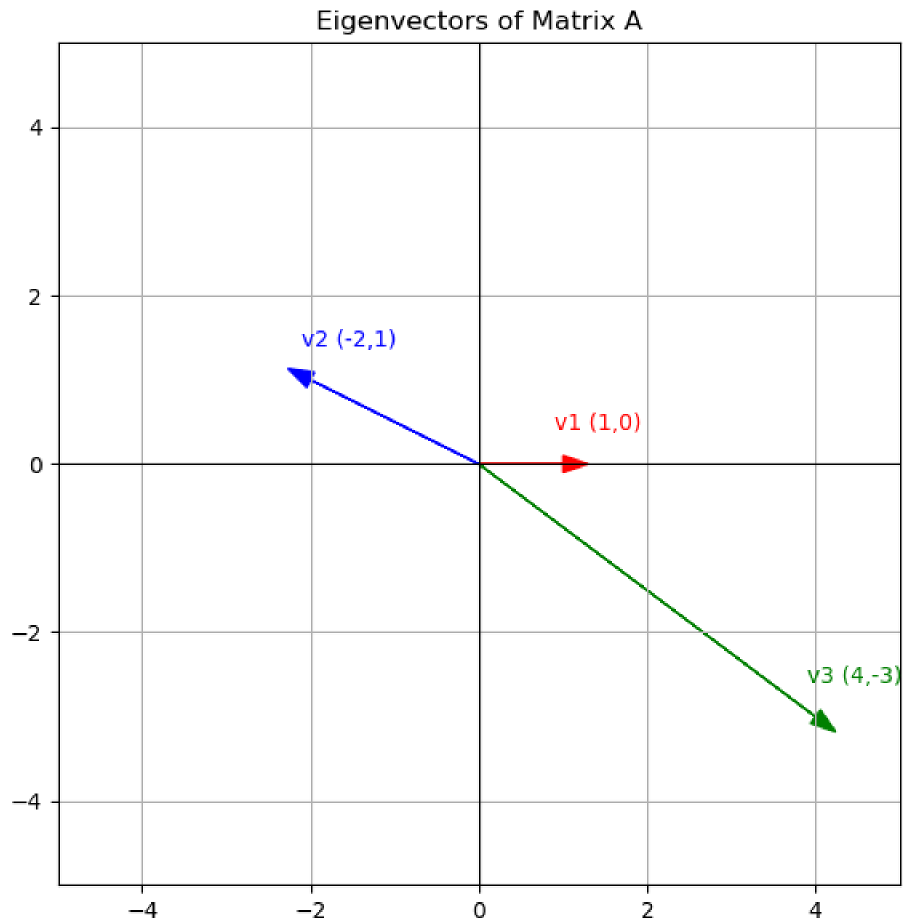

Fig : Eigen Vectors