## EE25BTECH11050-Hema Havil

## **Question:**

Jagdish has a field which is in the shape of a right-angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining space for growing vegetables. In the field, there is a pole marked as O. Based on the above information, answer the following equations

- a) Taking O as the origin, P = (-200, 0) and Q = (200, 0). PQRS being a square, what are the coordinates of R and S?
- b) i) What is the area of square PQRS?
  - ii) What is the length of diagonal PR in PQRS?
- c) If S divides CA in the ratio K: 1, what is the value of K, where A = (200, 800)?

## **Solution:**

Given that,

AQC is a right angled triangle at point Q and PQRS is a square inside the  $\Delta$ AQC,

(a) We were given two points

$$P = (-200, 0), Q = (200, 0)$$
 ((a).1)

Let,

X be the vector along the side PQ,

Y be the vector along the side QR,

Z be the vector along the side PS then,

$$\mathbf{X} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} 200 \\ 0 \end{pmatrix} - \begin{pmatrix} -200 \\ 0 \end{pmatrix} \tag{(a).2}$$

$$\mathbf{X} = \begin{pmatrix} 400\\0 \end{pmatrix} \tag{(a).3}$$

Rotation vector for 2x2 matrix is

$$\mathbf{R}_{\theta} = \begin{pmatrix} \cos\theta - \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{(a).4}$$

Rotate the vector  $\boldsymbol{X}$  by  $90^{\circ}$  anticlockwise to get Y

$$\mathbf{Y} = \mathbf{R}_{90}\mathbf{X} \tag{(a).5}$$

$$\mathbf{Y} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 400 \\ 0 \end{pmatrix} \tag{(a).6}$$

$$\mathbf{Y} = \begin{pmatrix} 0 \\ 400 \end{pmatrix} \tag{(a).7}$$

So the vector along the side QR is  $\mathbf{Y} = \begin{pmatrix} 0 \\ 400 \end{pmatrix}$  then,

$$\mathbf{Y} = \mathbf{R} - \mathbf{Q} \tag{(a).8}$$

$$\mathbf{R} = \mathbf{Y} + \mathbf{Q} \tag{(a).9}$$

$$\mathbf{R} = \begin{pmatrix} 0 \\ 400 \end{pmatrix} + \begin{pmatrix} 200 \\ 0 \end{pmatrix} \tag{(a).10}$$

$$\mathbf{R} = \begin{pmatrix} 200\\ 400 \end{pmatrix} \tag{(a).11}$$

Since the sides QR and PS are parallel, vectors  $\mathbf{Y} = \mathbf{Z}$  then

$$\mathbf{Z} = \mathbf{S} - \mathbf{P} \tag{(a).12}$$

$$\mathbf{S} = \mathbf{Z} + \mathbf{P} \tag{(a).13}$$

$$\mathbf{S} = \begin{pmatrix} 0 \\ 400 \end{pmatrix} + \begin{pmatrix} -200 \\ 0 \end{pmatrix} \tag{(a).14}$$

$$\mathbf{S} = \begin{pmatrix} -200\\400 \end{pmatrix} \tag{(a).15}$$

Therefore the coordinates of the points R and S are (200,400) and (-200,400)

(b)(i) We know the points P(-200,0) and Q(200,0)

Let length of the side of the square PQRS be x then,

$$x = \|\mathbf{Q} - \mathbf{P}\| \tag{(b).1}$$

$$x = \left\| \begin{pmatrix} 400 \\ 0 \end{pmatrix} \right\| = 400 \tag{(b).2}$$

Area of the square =  $x^2 = (400)^2 = 160000$  sq units

- (ii) Length of diagnol of the square =  $x\sqrt{2} = 400\sqrt{2}$  units
- (c) Given the point A=(200,800)

Since it was given that point S divides CA in the ratio K:1, this shows that points A,C and S are collinear. Since AQC is a right angled triangle, from this we can say that point C lies on X axis

Let point C be (t,0), Consider the matrix M

$$M = \begin{pmatrix} 200\ 800\ 1\\ -200\ 400\ 1\\ t\ 0\ 1 \end{pmatrix} \tag{(c).1}$$

$$R_1 \rightarrow \frac{1}{200} R_1$$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ -200 & 400 & 1 \\ t & 0 & 1 \end{pmatrix} \tag{(c).2}$$

$$R_2 \rightarrow R_2 + 200R_1$$

$$R_3 \rightarrow R_3 - tR_1$$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ 0 & 1200 & 2 \\ 0 & -4t & 1 - \frac{t}{200} \end{pmatrix}$$
 ((c).3)

$$R_2 \rightarrow \frac{1}{200}R_2$$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ 0 & 1 & \frac{1}{600} \\ 0 & -4t & 1 - \frac{t}{200} \end{pmatrix}$$
 ((c).4)

 $R_3 \rightarrow R_3 + 4tR_2$ 

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ 0 & 1 & \frac{1}{600} \\ 0 & 0 & 1 - \frac{t}{200} + \frac{4t}{600} \end{pmatrix}$$
 ((c).5)

Since the three points A,S and C are collinear,

Rank of M = 2

$$1 - \frac{t}{200} + \frac{4t}{600} = 0 \tag{(c).6}$$

$$1 + \frac{t}{600} = 0 \tag{(c).7}$$

$$\frac{t}{600} = -1\tag{(c).8}$$

$$t = -600 ((c).9)$$

Therefore point C=(-600,0), Now S divides CA in the ratio K:1,

$$S = \frac{KA + C}{K + 1} \tag{(c).10}$$

$$K = \frac{(S - A)^{T}(C - S)}{\|S - A\|^{2}}$$
 ((c).11)

$$K = \frac{1}{(400)^2 + (400)^2} \left( -400 - 400 \right) \begin{pmatrix} -400 \\ -400 \end{pmatrix} \tag{(c).12}$$

By solving ((c).12) we get K=1

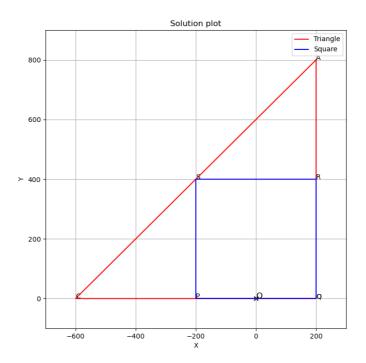


Fig. 3.1: Plot of the square and triangle