Matgeo Presentation - Problem 5.8.19

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Problem Statement

If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $1/2$ if we only add 1 to the denominator. What is the fraction?

solution

Let the unknown fraction be represented as

$$\mathbf{u} = \begin{pmatrix} x \\ y \end{pmatrix}, \qquad \frac{x}{y}. \tag{0.1}$$

Affine transformations are written as a vector plus a translation.
case 1:add 1 to numerator, subtract 1 from denominator

$$T_1(\mathbf{u}) = \mathbf{u} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \tag{0.2}$$

case 2:add 1 to denominator

$$T_2(\mathbf{u}) = \mathbf{u} + \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{0.3}$$

Condition for a fraction: For any vector $\begin{pmatrix} a \\ b \end{pmatrix}$, the requirement $\frac{a}{b} = k$ is equivalent to the linear equation

$$\mathbf{r}_k \begin{pmatrix} a \\ b \end{pmatrix} = 0, \qquad \mathbf{r}_k = \begin{pmatrix} 1 & -k \end{pmatrix}, \tag{0.4}$$

solution

since $\mathbf{r}_k \begin{pmatrix} a \\ b \end{pmatrix} = a - kb = 0 \iff \frac{a}{b} = k.$

Case 1: $T_1(\mathbf{u})$ must yield fraction 1.use $\mathbf{r}_1 = \begin{pmatrix} 1 & -1 \end{pmatrix}$

$$\mathbf{r}_1\big(T_1(\mathbf{u})\big) = 0 \tag{0.5}$$

$$\implies \mathbf{r}_1\mathbf{u} + \mathbf{r}_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = 0 \tag{0.6}$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix} \mathbf{u} + 2 = 0 \tag{0.7}$$

This gives the first equation.

Case 2: $T_2(\mathbf{u})$ must yield fraction $\frac{1}{2}$. To avoid fractions, multiply the functional by 2, i.e., use $\mathbf{r}_2 = \begin{pmatrix} 2 & -1 \end{pmatrix}$.

$$\mathbf{r}_2\big(T_2(\mathbf{u})\big) = 0 \tag{0.8}$$

$$\implies \mathbf{r}_2\mathbf{u} + \mathbf{r}_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0 \tag{0.9}$$

$$\begin{pmatrix} 2 & -1 \end{pmatrix} \mathbf{u} - 1 = 0 \tag{0.10}$$

This gives the second equation.

solution

System of equations: Both conditions together form the system

$$\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \mathbf{u} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}. \tag{0.11}$$

Gaussian elimination: Form the augmented matrix

$$\left[\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \middle| \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right]. \tag{0.12}$$

Eliminate the entry below the pivot:

$$R_2 \leftarrow R_2 - 2R_1 \;\; \Rightarrow \;\; \left[\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \middle| \begin{pmatrix} -2 \\ 5 \end{pmatrix} \right]. \tag{0.13}$$

Now eliminate above the pivot:

$$R_1 \leftarrow R_1 + R_2 \;\; \Rightarrow \;\; \left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \middle| \begin{pmatrix} 3 \\ 5 \end{pmatrix} \right]. \tag{0.14}$$

Final solution:

$$\mathbf{u} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \frac{x}{y} = \frac{3}{5}. \tag{0.15}$$

C Source Code:fraction matrix.c

```
#include <stdio.h>

// Generate augmented matrix for fraction problem
void generate_matrix(double matrix[2][3]) {
    // Equation 1: x - y = -2
    matrix[0][0] = 1;
    matrix[0][1] = -1;
    matrix[0][2] = -2;

    // Equation 2: 2x - y = 1
    matrix[1][0] = 2;
    matrix[1][1] = -1;
    matrix[1][2] = 1;
}
```

Python Script:fraction matrix.py

```
import ctypes
import numpy as np

# Load the shared library
lib = ctypes.CDLL("./libfraction_matrix.so")

# Prepare 2x3 matrix
MatrixType = ctypes.c_double * 3 * 2
matrix = MatrixType()

# Call C function to generate matrix
lib.generate_matrix(matrix)
```

Python Script:fraction matrix.py

```
# Convert to numpy array
aug_matrix = np.array([[matrix[i][j] for j in range(3)] for i

# Solve system
A = aug_matrix[:, :2]
B = aug_matrix[:, 2]
solution = np.linalg.solve(A, B)
x, y = solution

print(f"The fraction is {x}/{y}")

# Optionally save fraction to file for plotting
with open("fraction.txt", "w") as f:
    f.write(f"{x} {y}")
```

Python Script: plot matrix.py

```
import numpy as np
import matplotlib.pyplot as plt

# Load fraction solution
with open("fraction.txt", "r") as f:
    x_sol, y_sol = map(float, f.read().split())

# Define the two lines from the equations:
# Equation 1: x - y = -2 → y = x + 2
# Equation 2: 2x - y = 1 → y = 2x - 1

x_vals = np.linspace(0, 5, 100)
y1 = x_vals + 2
y2 = 2*x_vals - 1
```

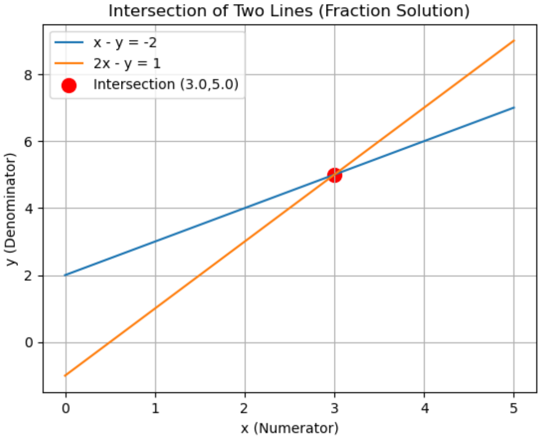
Python Script: plot matrix.py

```
# Plot the lines
plt.plot(x_vals, y1, label="x - y = -2")
plt.plot(x_vals, y2, label="2x - y = 1")

# Plot the intersection point
plt.scatter(x_sol, y_sol, color='red', s=100, label=f'Intersec

plt.xlabel("x (Numerator)")
plt.ylabel("y (Denominator)")
plt.title("Intersection of Two Lines (Fraction Solution)")
plt.grid(True)
plt.legend()
plt.show()
```

Result Plot



Intersection of Two Lines (Fraction Solution)
x - y = -2
2x - y = 1
Intersection (3.0,5.0)
y (Denominator)
x (Numerator)
0
2
4
6
8
0
1
2
3
4
5