5.2.43

EE25BTECH11041 - Naman Kumar

Question:
Solve the linear equation:

$$6x + 3y = 6xy \tag{1}$$
$$2x + 4y = 5xy \tag{2}$$

Solution:
Dividing both equations with xy

$$\frac{6}{y} + \frac{3}{x} = 6 \tag{3}$$
$$\frac{2}{y} + \frac{4}{x} = 5 \tag{4}$$

Let

$$\frac{1}{x} = a, \frac{1}{y} = b \tag{5}$$

So, new equations

$$3a + 6b = 6 \tag{6}$$
$$4a + 2b = 5 \tag{7}$$
$$\begin{pmatrix} 3 & 6 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix}\begin{pmatrix} 6 \\ 5 \end{pmatrix} \tag{8}$$
$$\mathbf{Ax} = \mathbf{c} \tag{9}$$

Gaussian elimination on $\mathbf{A}$

$$\left(\begin{array}{cc|c} 3 & 6 & 6 \\ 4 & 2 & 5 \end{array}\right) \xrightarrow{R_2 - \frac{4R_1}{3}} \left(\begin{array}{cc|c} 3 & 6 & 6 \\ 0 & -6 & -3 \end{array}\right) \tag{10}$$
$$\xrightarrow{\frac{R_2}{-6}} \left(\begin{array}{cc|c} 3 & 6 & 6 \\ 0 & 1 & \frac{1}{2} \end{array}\right) \tag{11}$$

Therefore,by putting values in (8)

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \tag{12}$$

For x,y

$$\begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} \tag{13}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{14}$$

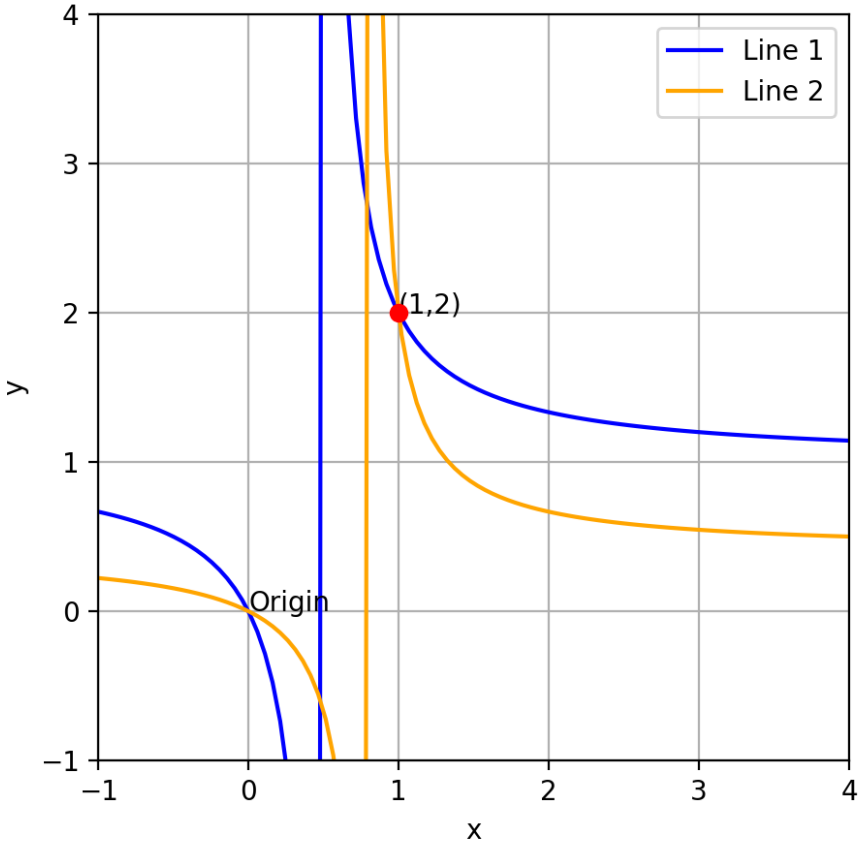

Fig. 1