7.4.9

EE25BTECH11002 - Achat Parth Kalpesh

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Question

The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \leq 6$ into two parts.
If S is $\{(2, 3/4), (5/2, 3/4), (1/4, -1/4), (1/8, 1/4)\}$ then the number of point(s) in S lying inside the smaller part is _____.

Solution:

Let the points be

$$\mathbf{p_1} = \begin{pmatrix} 2 \\ \frac{3}{4} \end{pmatrix} \quad \mathbf{p_2} = \begin{pmatrix} \frac{5}{2} \\ \frac{3}{4} \end{pmatrix} \quad \mathbf{p_3} = \begin{pmatrix} \frac{1}{4} \\ -\frac{1}{4} \end{pmatrix} \quad \mathbf{p_4} = \begin{pmatrix} \frac{1}{8} \\ \frac{1}{4} \end{pmatrix} \tag{1}$$

The circular region is

$$\mathbf{x}^\top \mathbf{x} \leq 6 \tag{2}$$

The line is

$$\mathbf{n}^\top \mathbf{x} = 1 \tag{3}$$

$$\mathbf{n} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \tag{4}$$

Solution:

Since the origin $\mathbf{0}$ lies inside the circle, checking which side of the line it belongs to:

$$\mathbf{n}^\top \mathbf{0} - 1 = -1 < 0 \tag{5}$$

Thus, the smaller part of the circle is the region

$$R = \left\{ \mathbf{x} : \mathbf{x}^\top \mathbf{x} \leq 6, \mathbf{n}^\top \mathbf{x} - 1 > 0 \right\} \tag{6}$$

Solution:

For $\mathbf{p_1}$

$$\mathbf{p_1}^\top \mathbf{p_1} = 4 + \frac{9}{16} = \frac{73}{16} \leq 6 \tag{7}$$

$$\mathbf{n}^\top \mathbf{p_1} - 1 = 4 - \frac{9}{4} - 1 = \frac{3}{4} > 0 \tag{8}$$

For $\mathbf{p_2}$

$$\mathbf{p_2}^\top \mathbf{p_2} = \frac{109}{16} > 6 \tag{9}$$

$$\mathbf{n}^\top \mathbf{p_2} - 1 = 5 - \frac{9}{4} - 1 = \frac{7}{4} > 0 \tag{10}$$

For $\mathbf{p_3}$

$$\mathbf{p_3}^\top \mathbf{p_3} = \frac{1}{8} \leq 6 \tag{11}$$

$$\mathbf{n}^\top \mathbf{p_3} - 1 = \frac{1}{2} + \frac{3}{4} - 1 = \frac{1}{4} > 0 \tag{12}$$

Solution:

For $\mathbf{p_4}$

$$\mathbf{p_4}^\top \mathbf{p_4} = \frac{5}{64} \leq 6 \tag{13}$$

$$\mathbf{n}^\top \mathbf{p_4} - 1 = \frac{1}{4} - \frac{3}{4} - 1 = -\frac{3}{2} < 0 \tag{14}$$

Thus, the points lying in the smaller part of the circle are

$$\mathbf{p_1} = \begin{pmatrix} 2 \\ \frac{3}{4} \end{pmatrix} \quad \mathbf{p_3} = \begin{pmatrix} \frac{1}{4} \\ -\frac{1}{4} \end{pmatrix} \tag{15}$$

C Code

```c
#include <stdio.h>
double get_circle_radius() {
    return 2.449489742783178; // sqrt(6)
}
double line_equation(double x) {
    return (2*x - 1)/3;
}
int is_inside_circle(double x, double y) {
    return (x*x + y*y <= 6) ? 1 : 0;
}
int is_on_positive_side(double x, double y) {
    return (2*x - 3*y - 1 > 0) ? 1 : 0;
}
int is_inside_smaller_region(double x, double y) {
    return is_inside_circle(x, y) && is_on_positive_side(x, y);
}
```

Python Code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt
formula = ctypes.CDLL("./formula.so")

# Declare argument & return types for functions (same as in C)
formula.get_circle_radius.restype = ctypes.c_double
formula.line_equation.argtypes = [ctypes.c_double]
formula.line_equation.restype = ctypes.c_double
formula.is_inside_circle.argtypes = [ctypes.c_double, ctypes.
    c_double]
formula.is_inside_circle.restype = ctypes.c_int
formula.is_on_positive_side.argtypes = [ctypes.c_double, ctypes.
    c_double]
formula.is_on_positive_side.restype = ctypes.c_int
formula.is_inside_smaller_region.argtypes = [ctypes.c_double,
    ctypes.c_double]
formula.is_inside_smaller_region.restype = ctypes.c_int
```

Python Code

```
# Circle parameters: x^2 + y^2 = 6
r = np.sqrt(6)

# Line: 2x - 3y = 1 => y = (2x - 1)/3
x_line = np.linspace(-3, 3, 400)
y_line = (2*x_line - 1) / 3

# Points
points = {
    "$p_1$(2, 3/4)": (2, 0.75),
    "$p_3$(1/4, -1/4)": (0.25, -0.25),
    "$p_4$(1/8, 1/4)": (0.125, 0.25)
}
point = {
    "$p_2$(5/2 ,3/4)": (2.5, 0.75),
}
```

Python Code

```
# Circle boundary
theta = np.linspace(0, 2*np.pi, 500)
x_circle = r * np.cos(theta)
y_circle = r * np.sin(theta)
plt.figure(figsize=(7,7))
plt.plot(x_circle, y_circle, 'b')
plt.text(r*np.cos(np.pi/4), r*np.sin(np.pi/4), "x + y = 6", color
    ='b', fontsize=10)
# Line
plt.plot(x_line, y_line, 'r')
plt.text(-2.5, -2.5, "2x - 3y = 1", color='r', fontsize=10)

# Shaded region (smaller part)
xx, yy = np.meshgrid(np.linspace(-r, r, 400), np.linspace(-r, r,
    400))
mask_circle = xx**2 + yy**2 <= 6
mask_line = 2*xx - 3*yy - 1 > 0
mask = mask_circle & mask_line
```

Python Code

```
plt.contourf(xx, yy, mask, levels=[0.5, 1], colors=['#ffcccc'],
    alpha=0.5)
plt.text(0, -2, "Smaller region", color='darkred', fontsize=10)

# Plot points
for label, (x, y) in points.items():
    plt.scatter(x, y, s=70)
    plt.text(x-0.5, y-0.35, label, fontsize=9)

for label, (x, y) in point.items():
    plt.scatter(x, y, s=70)
    plt.text(x-0.3, y-0.3, label, fontsize=9)
```

Python Code

```
# Formatting
plt.gca().set_aspect('equal', adjustable='box')
plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
plt.xlim(-3,3)
plt.ylim(-3,3)
plt.grid(True)
plt.title("Circle cut by the Line")
plt.show()
```

Plot

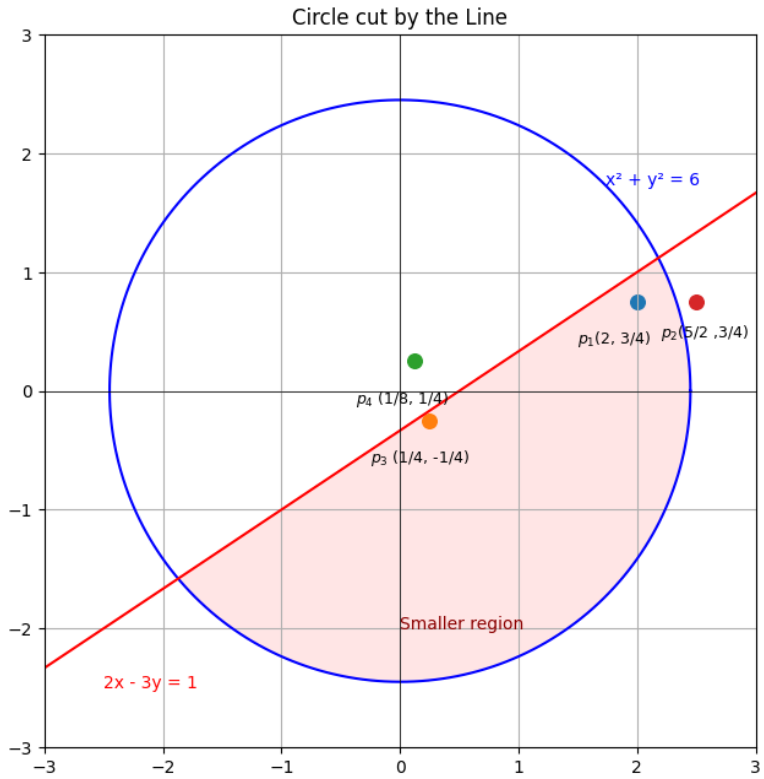

Figure: Visualization of the solution.