4.13.17

EE25BTECH11012-BEERAM MADHURI

Question:
Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point $(1, 0)$ to the distance from the point $(-1, 0)$ is equal to $\frac{1}{3}$. Then the circumcentre of the triangle ABC is at the point:

1) $\left(\frac{5}{4}, 0\right)$ 2) $\left(\frac{5}{2}, 0\right)$ 3) $\left(\frac{5}{3}, 0\right)$ 4) $(0, 0)$

Solution: let $\mathbf{F_1}$, $\mathbf{F_2}$ be the vectors such that:

Point	Vector
$\left(F_1\right)$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\left(F_2\right)$	$\begin{pmatrix} -1 \\ 0 \end{pmatrix}$

TABLE 4: Variables used

Let P $\begin{pmatrix} x \\ y \end{pmatrix}$ be any point in the plane of A,B,C.
given,

$$\frac{\|PF_1\|}{\|PF_2\|} = \frac{1}{3} \tag{4.1}$$

$$\frac{\sqrt{(P - F_1)^T (P - F_1)}}{\sqrt{(P - F_2)^T (P - F_2)}} = \frac{1}{3} \tag{4.2}$$

By Substituting Values and Simplifying:

$$2x^2 + 2y^2 - 5x + 2 = 0 \tag{4.3}$$

$$\left\| \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} \frac{5}{4} \\ 0 \end{pmatrix} \right\| = \sqrt{\frac{21}{4}} \tag{4.4}$$

On comparing the equation with general form:-

$$\|P - C\| = r \tag{4.5}$$

$$\text{where , P = any point on the circle} \tag{4.6}$$

$$\text{C = Center of circle} \tag{4.7}$$

$$\text{r = radius of circle} \tag{4.8}$$

$$\text{Center of circle} = (\frac{5}{4}, 0) \tag{4.9}$$

Hence, the circumcenter of the triangle is $(\frac{5}{4}, 0)$

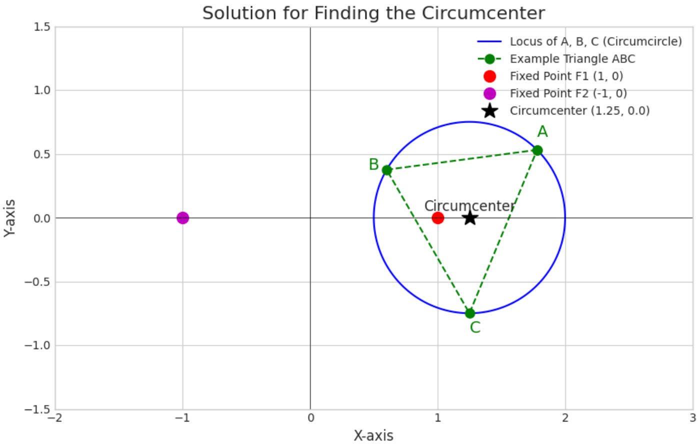

Fig. 4.1: 4.13.17