9.4.14

EE25BTECH11002 - Achat Parth Kalpesh

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Question

Find the roots of the following quadratic equations graphically

$$(x - 3)(2x - 1) = x(x + 5) \tag{1}$$

$$y = (x-3)(2x-1) - x(x+5) = 0 \tag{2}$$

$$y = x^2 - 12x + 3 = 0 \tag{3}$$

This quadratic can be represented as a conic in matrix form:

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{4}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -6 \\ 0 \end{pmatrix}, f = 3 \tag{5}$$

To find the roots, we find the points of intersection of the conic with the x-axis.

$$\mathbf{x} = \mathbf{h} + k_i \mathbf{m} \tag{6}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{7}$$

Theoretical Solution

The value of k_i can be found out by solving the line and conic equation

$$(\mathbf{h}+k_i\mathbf{m})^\top \mathbf{V}(\mathbf{h}+k_i\mathbf{m})+2\mathbf{u}^\top(\mathbf{h}+k_i\mathbf{m})+f=0 \tag{8}$$

$$\implies k_i^2\mathbf{m}^\top\mathbf{V}\mathbf{m}+2k_i\mathbf{m}^\top(\mathbf{V}\mathbf{h}+\mathbf{u})+\mathbf{h}^\top\mathbf{V}\mathbf{h}+2\mathbf{u}^\top\mathbf{h}+f=0 \tag{9}$$

$$\text{or, } k_i^2\mathbf{m}^\top\mathbf{V}\mathbf{m}+2k_i\mathbf{m}^\top(\mathbf{V}\mathbf{h}+\mathbf{u})+g(\mathbf{h})=0 \tag{10}$$

Solving the above quadratic gives the equation

$$k_i=\frac{1}{\mathbf{m}^\top\mathbf{V}\mathbf{m}}\left(-\mathbf{m}^\top(\mathbf{V}\mathbf{h}+\mathbf{u})\pm\sqrt{\left[\mathbf{m}^\top(\mathbf{V}\mathbf{h}+\mathbf{u})\right]^2-g(\mathbf{h})\ (\mathbf{m}^\top\mathbf{V}\mathbf{m})}\right) \tag{11}$$

Theoretical Solution

Substituting the values in the above equation gives

$$\therefore k_i = 6 \pm \sqrt{33} \tag{12}$$

$$k_1 = 6 + \sqrt{33} \tag{13}$$

$$k_2 = 6 - \sqrt{33} \tag{14}$$

$$\mathbf{x} = \mathbf{h} + k_i\mathbf{m} = \begin{pmatrix} 6 + \sqrt{33} \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 6 - \sqrt{33} \\ 0 \end{pmatrix} \tag{15}$$

C code

```c
#include <stdio.h>
#include <math.h>

double root1(double a, double b, double c) {
    double d = b*b - 4*a*c;
    return (-b + sqrt(d)) / (2*a);
}

double root2(double a, double b, double c) {
    double d = b*b - 4*a*c;
    return (-b - sqrt(d)) / (2*a);
}
```

```python
import numpy as np
import matplotlib.pyplot as plt
import ctypes

lib = ctypes.CDLL("./formula.so")

lib.root1.argtypes = [ctypes.c_double, ctypes.c_double, ctypes.
    c_double]
lib.root1.restype = ctypes.c_double

lib.root2.argtypes = [ctypes.c_double, ctypes.c_double, ctypes.
    c_double]
lib.root2.restype = ctypes.c_double

def quadratic(a, b, c):
    x1 = lib.root1(a, b, c)
    x2 = lib.root2(a, b, c)
    return x1, x2
```

Python Code

```
def function(x):
    return x**2 - 12*x + 3

x = np.linspace(-3, 15, 100)
y = function(x)
y1 = np.zeros(100)
x1, x2 = quadratic(1, -12, 3)
fig, ax = plt.subplots()
ax.plot(x, y, label='x^2 - 12x + 3')
ax.plot(x, y1, label='y = 0')
ax.scatter(x1, 0, color="black", label=f'Root 1 ({x1:.2f}, 0)')
ax.text(x1, 0, f'({x1:.2f}, 0)')
ax.scatter(x2, 0, color="black", label=f'Root 2 ({x2:.2f}, 0)')
ax.text(x2, 0, f'({x2:.2f}, 0)')
ax.grid(True)
ax.legend(loc="lower right")
plt.show()
```

Plot

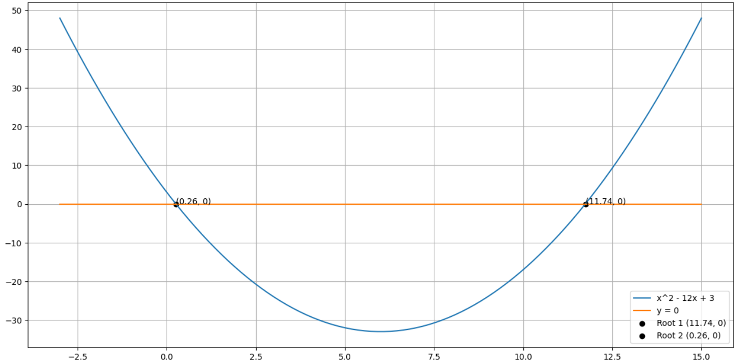

Figure: Graphical Representation of quadratic equation