2.3.15

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Question: The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ with the unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

Solution: let **A**, **B** and **C** be the vectors such that:

Variable	value
A	$\begin{pmatrix}1\\1\\1\end{pmatrix}$
B	$\begin{pmatrix}2\\4\\-5\end{pmatrix}$
C	$\begin{pmatrix}\lambda\\2\\3\end{pmatrix}$

TABLE 0: Variables used

The direction vector $\mathbf{B}+\mathbf{C}=\begin{pmatrix}2+\lambda\\6\\-2\end{pmatrix}$

$$B \hat{+} C = \frac{\mathbf{B}+\mathbf{C}}{\|\mathbf{B}+\mathbf{C}\|}$$

The corresponding unit vector obtained is:

$$\begin{pmatrix}\frac{2+\lambda}{\sqrt{\lambda^2+4\lambda+44}}\\ \frac{6}{\sqrt{\lambda^2+4\lambda+44}}\\ \frac{-2}{\sqrt{\lambda^2+4\lambda+44}}\end{pmatrix}$$

given,

$$\mathbf{A}^\top \cdot (B \hat{+} C) = 1$$

$$\frac{1}{\sqrt{\lambda^2+4\lambda+44}}\begin{pmatrix}1 & 1 & 1\end{pmatrix}\cdot\begin{pmatrix}2+\lambda\\6\\-2\end{pmatrix}=1$$

$$2+\lambda+6-2=\sqrt{\lambda^2+4\lambda+44}$$

squaring on both sides:

$$\lambda^2+36+12\lambda=\lambda^2+4\lambda+44$$

$$8\lambda=8$$

$$\lambda=1$$

Hence value of λ is 1.

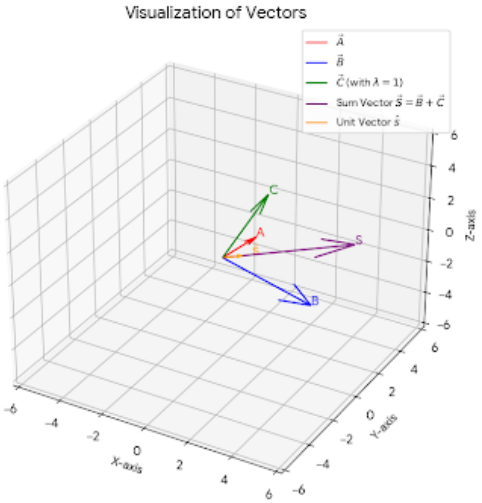

Fig. 0.1