5.5.31

AI25BTECH11014 - Suhas

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Question

Solve the following system using matrix row operations. Let $\mathbf{M} = \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{pmatrix}$,

and find its value.

$$2 \cdot \frac{1}{x} + 3 \cdot \frac{1}{y} + 10 \cdot \frac{1}{z} = 4$$
$$4 \cdot \frac{1}{x} + 6 \cdot \frac{1}{y} + 5 \cdot \frac{1}{z} = 1$$
$$6 \cdot \frac{1}{x} + 9 \cdot \frac{1}{y} + 20 \cdot \frac{1}{z} = 2$$

Matrix Form

Augmented matrix:

$$\left[\begin{array}{cccc} \begin{pmatrix}2 & 3 & 10 & 4\end{pmatrix} \\ \begin{pmatrix}4 & 6 & 5 & 1\end{pmatrix} \\ \begin{pmatrix}6 & 9 & 20 & 2\end{pmatrix} \end{array}\right]$$

Row Operations

Step 1: $R_1 \leftarrow R_1 \div 2$

$$\begin{bmatrix} \begin{pmatrix} 1 & \frac{3}{2} & 5 & 2 \end{pmatrix} \\ \begin{pmatrix} 4 & 6 & 5 & 1 \end{pmatrix} \\ \begin{pmatrix} 6 & 9 & 20 & 2 \end{pmatrix} \end{bmatrix}$$

Step 2: $R_2 \leftarrow R_2 - 4 \cdot R_1$

$$\begin{bmatrix} \begin{pmatrix} 1 & \frac{3}{2} & 5 & 2 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & -15 & -7 \end{pmatrix} \\ \begin{pmatrix} 6 & 9 & 20 & 2 \end{pmatrix} \end{bmatrix}$$

Row Operations (contd.)

Step 3: $R_3 \leftarrow R_3 - 6 \cdot R_1$

$$\begin{bmatrix} \begin{pmatrix} 1 & \frac{3}{2} & 5 & 2 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & -15 & -7 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & -10 & -10 \end{pmatrix} \end{bmatrix}$$

Step 4: $R_3 \leftarrow R_3 - R_2$

$$\begin{bmatrix} \begin{pmatrix} 1 & \frac{3}{2} & 5 & 2 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & -15 & -7 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 5 & -3 \end{pmatrix} \end{bmatrix}$$

Row Operations (contd.)

Step 5: $R_3 \leftarrow R_3 \div 5$

$$\begin{bmatrix} \begin{pmatrix} 1 & \frac{3}{2} & 5 & 2 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & -15 & -7 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 1 & -\frac{3}{5} \end{pmatrix} \end{bmatrix}$$

Step 6: $R_2 \leftarrow R_2 + 15 \cdot R_3$

$$\begin{bmatrix} \begin{pmatrix} 1 & \frac{3}{2} & 5 & 2 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 & \frac{4}{5} \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 1 & -\frac{3}{5} \end{pmatrix} \end{bmatrix}$$

Row Operations (final)

Step 7: $R_1 \leftarrow R_1 - 5 \cdot R_3$

$$\begin{bmatrix} \begin{pmatrix} 1 & \frac{3}{2} & 0 & 5 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 0 & \frac{4}{5} \end{pmatrix} \\ \begin{pmatrix} 0 & 0 & 1 & -\frac{3}{5} \end{pmatrix} \end{bmatrix}$$

Conclusion

From the final matrix:

$$\begin{bmatrix} 1 & \frac{3}{2} & 0 & 5 \\ 0 & 0 & 0 & \frac{4}{5} \\ 0 & 0 & 1 & -\frac{3}{5} \end{bmatrix}$$

This corresponds to:

$$\frac{1}{x} = 5$$
$$\frac{1}{z} = -\frac{3}{5}$$
$$0 = \frac{4}{5} \quad \text{(contradiction)}$$

Since the second row implies a false statement,

The system is inconsistent and has no solution.

Python Code (Part 1)

```
import numpy as np

A = np.array([
    [2, 3, 10],
    [4, 6, 5],
    [6, 9, 20]
], dtype=np.float32)

B = np.array([4, 1, 2], dtype=np.float32)

U, residuals, rank, s = np.linalg.lstsq(A, B, rcond=None)
```

Python Code (Part 2)

```
if residuals.size > 0 and residuals[0] > 1e-6:
    print("System is inconsistent. No exact solution exists.")
else:
    u, v, w = U
    x, y, z = 1/u, 1/v, 1/w
    print(f"x = {x:.3f}, y = {y:.3f}, z = {z:.3f}")
```

Python Code Using .so (Part 3)

```
# After calling solve_system(...)
if abs(U[0]*2 + U[1]*3 + U[2]*10 - 4) > 1e-3 or \
   abs(U[0]*4 + U[1]*6 + U[2]*5 - 1) > 1e-3 or \
   abs(U[0]*6 + U[1]*9 + U[2]*20 - 2) > 1e-3:
    print("System is inconsistent. No exact solution exists.")
else:
    x, y, z = 1/U[0], 1/U[1], 1/U[2]
    print(f"x = {x}, y = {y}, z = {z}")
```

Python Code Using .so (Part 1)

```
import ctypes
import numpy as np

lib = ctypes.CDLL('./libsystem.so')
lib.solve_system.argtypes = [
    ctypes.POINTER(ctypes.c_float),
    ctypes.POINTER(ctypes.c_float),
    ctypes.POINTER(ctypes.c_float)
]

lib.solve_system.restype = None
```

Python Code Using .so (Part 2)

```
A = np.array([
    [2, 3, 10],
    [4, 6, 5],
    [6, 9, 20]
], dtype=np.float32).flatten()

B = np.array([4, 1, 2], dtype=np.float32)
U = np.zeros(3, dtype=np.float32)

lib.solve_system(
    A.ctypes.data_as(ctypes.POINTER(ctypes.c_float)),
    B.ctypes.data_as(ctypes.POINTER(ctypes.c_float)),
    U.ctypes.data_as(ctypes.POINTER(ctypes.c_float))
)
```

Python Code Using .so (Part 3)

```
def check_inconsistency(A, B, U):
    A = A.reshape(3, 3)
    residuals = A @ U - B
    return np.any(np.abs(residuals) > 1e-3)

if check_inconsistency(A, B, U):
    print("System is inconsistent. No exact solution exists.")
else:
    x, y, z = 1/U[0], 1/U[1], 1/U[2]
    print(f"x = {x:.3f}, y = {y:.3f}, z = {z:.3f}")
```

Plot

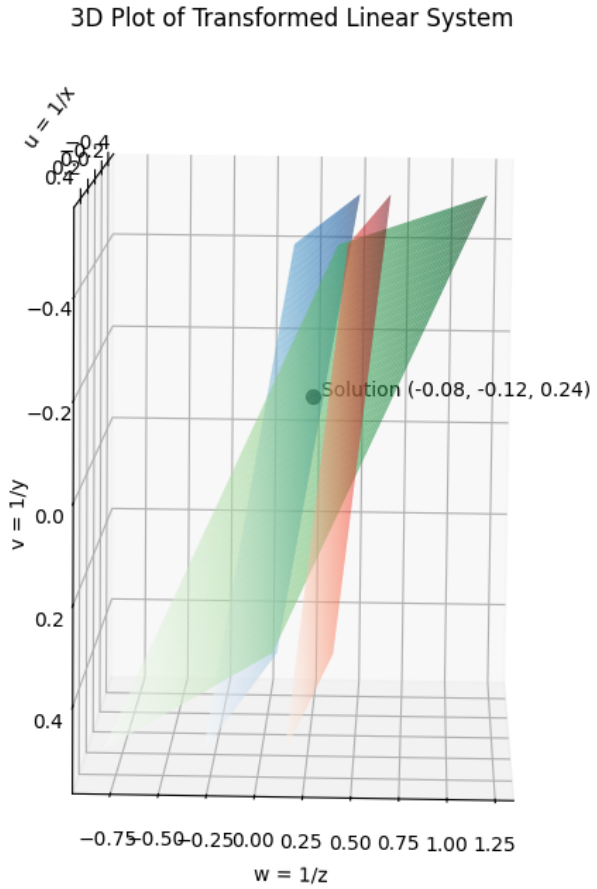

Figure: Approximate solution