

## 2.10.33

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### Question

Let  $\alpha, \beta, \gamma$  be distinct real numbers. The points with position vectors  $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ ,  $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$ ,  $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$

1. are collinear
2. form an equilateral triangle
3. form a scalene triangle
4. form a right angled triangle

### Solution

Let  $\mathbf{A}$  be  $\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$ ,  $\mathbf{B}$  be  $\begin{pmatrix} \beta \\ \gamma \\ \alpha \end{pmatrix}$ , and  $\mathbf{C}$  be  $\begin{pmatrix} \gamma \\ \alpha \\ \beta \end{pmatrix}$ .

$$\mathbf{A} - \mathbf{B} \text{ is } \begin{pmatrix} \alpha - \beta \\ \beta - \gamma \\ \gamma - \alpha \end{pmatrix} \quad (1)$$

$$\mathbf{B} - \mathbf{C} \text{ is } \begin{pmatrix} \beta - \gamma \\ \gamma - \alpha \\ \alpha - \beta \end{pmatrix} \quad (2)$$

$$\mathbf{C} - \mathbf{A} \text{ is } \begin{pmatrix} \gamma - \alpha \\ \alpha - \beta \\ \beta - \gamma \end{pmatrix} \quad (3)$$

Then  $\|\mathbf{A} - \mathbf{B}\|$ ,  $\|\mathbf{B} - \mathbf{C}\|$ ,  $\|\mathbf{C} - \mathbf{A}\|$  are all equal, and equal to

$$\sqrt{(\alpha - \beta)^2 + (\beta - \gamma)^2 + (\gamma - \alpha)^2}$$

The three points therefore form an equilateral triangle, so option (2) is correct.