

AI25BTECH11034 - SUJAL CHAUHAN
2.2.27

question

Find the angle between the pair of lines:

$$\frac{x+3}{3} = \frac{y-1}{5} = \frac{z+3}{4}$$

$$\frac{x+1}{1} = \frac{y-4}{1} = \frac{z+5}{2}$$

solution

Given lines can be represented as

$$\mathbf{X} = \begin{pmatrix} -3 \\ 1 \\ -3 \end{pmatrix} + k_1 \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix} \quad (1)$$

$$\mathbf{Y} = \begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix} + k_2 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad (2)$$

where k_1 and k_2 are any real numbers.

clearly direction vector of lines are \mathbf{m}_1 and \mathbf{m}_2

$$\mathbf{m}_1 = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix} \quad (3)$$

and

$$\mathbf{m}_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad (4)$$

angle between them is

$$\cos \theta = \frac{\mathbf{m}_1 \mathbf{m}_2^T}{|\mathbf{m}_1| |\mathbf{m}_2|} \quad (5)$$

so

$$\theta = \cos^{-1} \frac{\mathbf{m}_1 \mathbf{m}_2^T}{|\mathbf{m}_1| |\mathbf{m}_2|} \quad (6)$$

$$\mathbf{m}_1 \mathbf{m}_2^T = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \end{pmatrix} = 16 \quad (7)$$

$$|m_1| = \sqrt{50}, |m_2| = \sqrt{6} \quad (8)$$

$$\theta = \cos^{-1} \frac{16}{\sqrt{300}} = 22.52^\circ \quad (9)$$

Angle between two given line is 22.52°

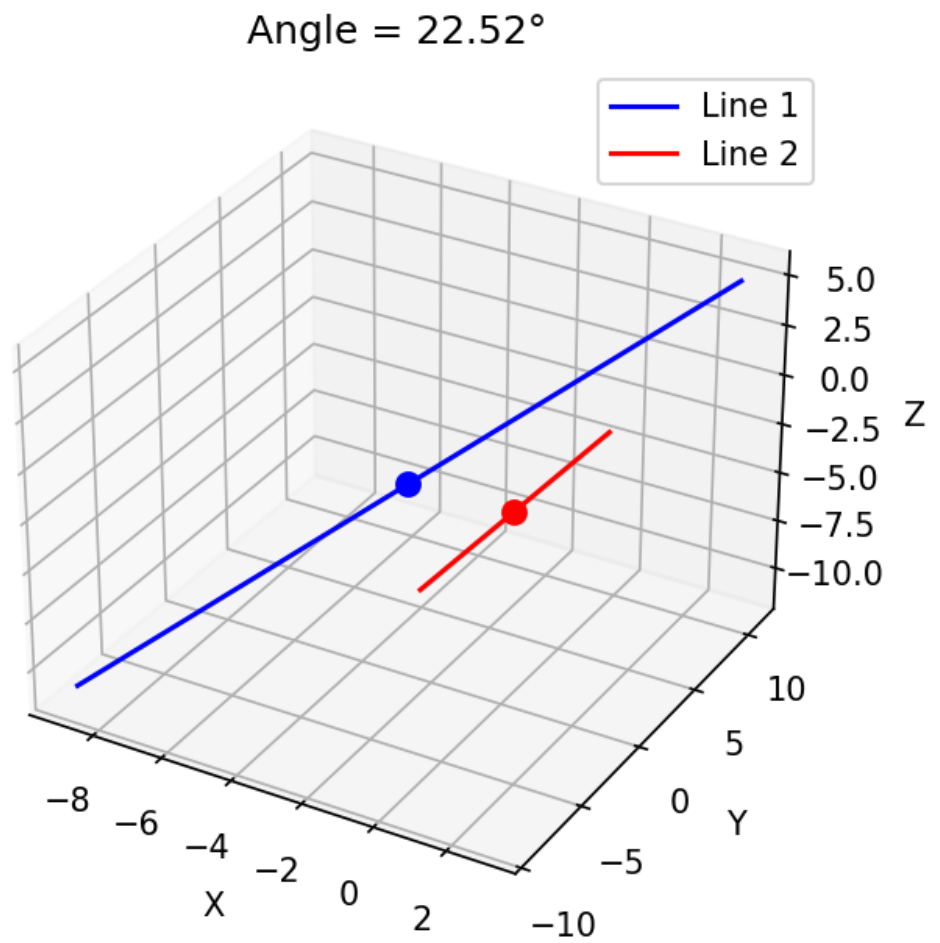


Figure 1