2.9.24

AI25BTECH11008 - Chiruvella Harshith Sharan

Question: 2.9.24 Find the co-ordinates of the point where the line

$$\mathbf{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 3\hat{k})$$

meets the plane which is perpendicular to the vector

$$\mathbf{n} = \hat{i} + \hat{j} + 3\hat{k}$$

and at a distance of $\frac{4}{\sqrt{11}}$ from origin.

Solution:

The parametric form of the line is

$$\mathbf{r}(\lambda) = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 + 3\lambda \\ -2 + 4\lambda \\ -3 + 3\lambda \end{pmatrix}. \tag{1}$$

The equation of the plane with normal $\mathbf{n}$ and distance $d = \frac{4}{\sqrt{11}}$ from origin is

$$\mathbf{n}^T \mathbf{r} = \pm \|\mathbf{n}\| d. \tag{2}$$

Now,

$$\|\mathbf{n}\| = \sqrt{1^2 + 1^2 + 3^2} = \sqrt{11}, \quad \pm\|\mathbf{n}\|d = \pm 4. \tag{3}$$

So the plane equations are

$$\mathbf{n}^T \mathbf{r} = 4 \quad \text{or} \quad \mathbf{n}^T \mathbf{r} = -4. \tag{4}$$

Substitute $\mathbf{r}(\lambda)$:

$$\mathbf{n}^T \mathbf{r}(\lambda) = (1)(-1 + 3\lambda) + (1)(-2 + 4\lambda) + (3)(-3 + 3\lambda). \tag{5}$$

Simplifying,

$$\mathbf{n}^T \mathbf{r}(\lambda) = -1 + 3\lambda - 2 + 4\lambda - 9 + 9\lambda = -12 + 16\lambda. \tag{6}$$

Case 1:

$$-12 + 16\lambda = 4 \quad \Rightarrow \quad \lambda = 1. \tag{7}$$

Case 2:

$$-12 + 16\lambda = -4 \quad \Rightarrow \quad \lambda = \tfrac{1}{2}. \tag{8}$$

Hence, the intersection points are:

$$\mathbf{r}(1) = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}, \quad \mathbf{r}\left(\frac{1}{2}\right) = \begin{pmatrix} \frac{1}{2} \\ 0 \\ -\frac{3}{2} \end{pmatrix}. \tag{9}$$

Final Answer: The required points are

$$(2, 2, 0) \quad \text{and} \quad \left(\frac{1}{2}, 0, -\frac{3}{2}\right). \tag{10}$$

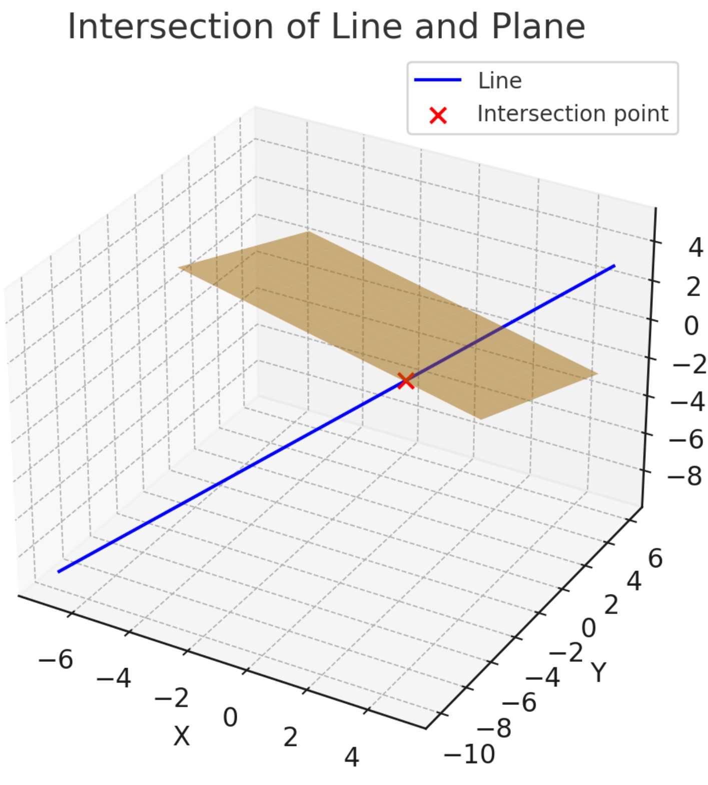

Fig. 1: Intersection of the line with the plane