2.3.5

EE25BTECH11002 - Achat Parth Kalpesh

September 13,2025

Question

Find the angle between the line $\mathbf{r} = \left(\hat{i} - \hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)$ and the plane $\mathbf{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 3$.

Theoretical Solution

Let the direction vector of the line be **d** and the normal vector to the plane be **n**.

$$\mathbf{d} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \tag{1}$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{2}$$

The angle θ between a line and a plane is the complement of the angle ϕ between the line's direction vector **d** and the plane's normal vector **n**.

Equation

The formula to calculate the angle θ between the line and the plane is given by:

$$\theta = \sin^{-1}\left(\frac{\left|\mathbf{n}^\top \mathbf{d}\right|}{\left\|\mathbf{d}\right\| \left\|\mathbf{n}\right\|}\right) \tag{3}$$

For the given vectors:

$$\mathbf{n}^{\top}\mathbf{d} = (3)(1) + (-1)(1) + (2)(1) = 4 \tag{4}$$

$$\|\mathbf{d}\| = \sqrt{3^2 + (-1)^2 + 2^2} = \sqrt{14} \tag{5}$$

$$\|\mathbf{n}\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} \tag{6}$$

Substituting these values into the formula:

$$\theta = \sin^{-1}\left(\frac{4}{\sqrt{14}\sqrt{3}}\right) = \frac{4}{\sqrt{42}} \tag{7}$$

So, the angle θ is $sin^{-1}\left(\frac{4}{\sqrt{42}}\right) \approx 37.98$.

C code

```
#include <stdio.h>
#include <math.h>
float formula(float *d,float *n)
{
    return asin((d[0]*n[0] + d[1]*n[1] + d[2]*n[2])/(sqrt(d[0]*d
        [0] + d[1]*d[1] + d[2]*d[2])*sqrt(n[0]*n[0] + n[1]*n[1] +
         n[2]*n[2])));
}
```

Python Code

```python
import ctypes
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Define the absolute path to the compiled C library.
lib_path = ctypes.CDLL('./f.so')

# Define the argument and return types for the C function
#c_float_p = ctypes.POINTER(ctypes.c_float)
lib_path.formula.argtypes = [
    ctypes.POINTER(ctypes.c_float),
    ctypes.POINTER(ctypes.c_float)
]
lib_path.formula.restype = ctypes.c_float
```

Python Code

```python
# Prepare the input vectors for problem 2.3.5
d_vec = np.array([3, -1, 2], dtype=np.float32)
n_vec = np.array([1, 1, 1], dtype=np.float32)

#d_p = d_vec.ctypes.data_as(ctypes.POINTER(ctypes.c_float))
#n_p = n_vec.ctypes.data_as(ctypes.POINTER(ctypes.c_float))

# Call the C function to calculate the angle
angle = lib_path.formula(
    d_vec.ctypes.data_as(ctypes.POINTER(ctypes.c_float)),
    n_vec.ctypes.data_as(ctypes.POINTER(ctypes.c_float))
)

print(f"Angle between line and plane: {angle:.4f} degrees")
```

Python Code

```python
# Plane and Line definitions
a, b, c, d_plane = 1, 1, 1, 3
line_point = np.array([1, -1, 1])
line_direction = d_vec
intersection_point = line_point + 0.5 * line_direction

# Plotting setup
fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')
plot_lim = 8

x_plane = np.linspace(-plot_lim, plot_lim, 50)
y_plane = np.linspace(-plot_lim, plot_lim, 50)
X, Y = np.meshgrid(x_plane, y_plane)
Z = (d_plane - a*X - b*Y) / c
Z[(Z > plot_lim) | (Z < -plot_lim)] = np.nan # Masking
```

Python Code

```
# Plotting the geometry
ax.plot_surface(X, Y, Z, alpha=0.6, color='cyan')
t = np.linspace(-3, 3, 100)
ax.plot(line_point[0] + t * line_direction[0],
        line_point[1] + t * line_direction[1],
        line_point[2] + t * line_direction[2],
        color='magenta', linewidth=3)
ax.scatter(intersection_point[0], intersection_point[1],
    intersection_point[2],
           color='red', s=150, zorder=10)
```

Python Code

```python

# Formatting the plot
ax.set_xlabel('X-axis'); ax.set_ylabel('Y-axis'); ax.set_zlabel('
    Z-axis')
ax.set_title('Intersection Plot (Angle calculated in C)',
    fontsize=16)
ax.set_xlim([-plot_lim, plot_lim]); ax.set_ylim([-plot_lim,
    plot_lim]); ax.set_zlim([-plot_lim, plot_lim])
ax.set_box_aspect([1,1,1])
plt.grid(True)
plt.legend()

# Save and show the final plot
plt.savefig('plot_from_c_and_python_absolute_path.pdf')
plt.show()
```

Plot

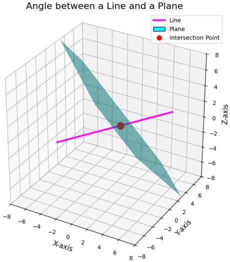

Figure: Angle between the line and plane