8.2.3

EE25BTECH11041 - Naman Kumar

Question:

$$y^2 = -8x \tag{1}$$

Solution:
Since it is a parabola we have

Eccentricity	e=1
Eigenvalue	$\lambda_1 = 0$
Determinant	$\|V\| = 0$

General equation of conic

$$\mathbf{x^T V x} + 2\mathbf{u^T x} + f = 0 \tag{2}$$

For the equation (1), we can write

$$\mathbf{x^T} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 4 \\ 0 \end{pmatrix}^T \mathbf{x} = 0 \tag{3}$$

$\mathbf{V}$	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
$\mathbf{u}$	$4e_1$
f	0

using general equations we know for any conic

$$\mathbf{V} = \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n}\mathbf{n}^T \tag{4}$$

$$\mathbf{u} = ce^2 \mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F} \tag{5}$$

$$f = \|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 e^2 \tag{6}$$

Let

$\mathbf{n}$	$\begin{pmatrix} a \\ b \end{pmatrix}$	normal vector to directrix
$\mathbf{F}$	$\begin{pmatrix} g \\ h \end{pmatrix}$	focus
c		be constant of directrix

Firstly in (4)

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{n}^T \mathbf{n}\mathbf{I} - (1)^2 \mathbf{n}\mathbf{n}^T \tag{7}$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{pmatrix} - \begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix} \tag{8}$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \mathbf{e_1} \tag{9}$$

In (5)

$$4\mathbf{e_1} = c(1)^2 \mathbf{e_1} - (1)^2 \begin{pmatrix} g \\ h \end{pmatrix} \tag{10}$$

$$\begin{pmatrix} g \\ h \end{pmatrix} = (c-4)\mathbf{e_1} = (c-4) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{11}$$

$$\mathbf{F} = \begin{pmatrix} c-4 \\ 0 \end{pmatrix} \tag{12}$$

In (6)

$$0 = (1)^2 \begin{pmatrix} c-4 \\ 0 \end{pmatrix}^T \begin{pmatrix} c-4 \\ 0 \end{pmatrix} - c^2(1) \tag{13}$$

$$c^2 + 16 - 8c - c^2 = 0 \tag{14}$$

$$c = 2 \tag{15}$$

$$\mathbf{F} = -2\mathbf{e_1} \tag{16}$$

Directrix is

$$\mathbf{n^T x} = c \tag{17}$$

$$\mathbf{e_1}^T \mathbf{x} = 2 \tag{18}$$

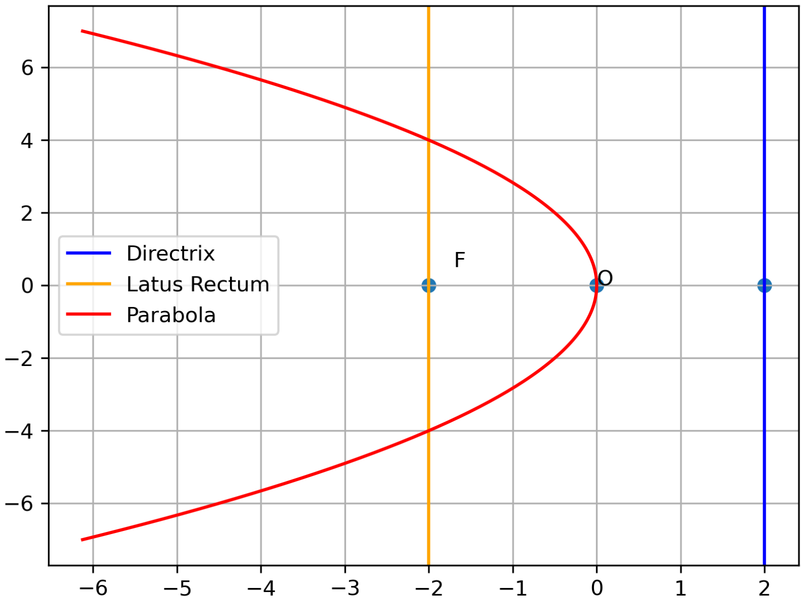

Figure 1