2.10.5

AI25BTECH11016-Varun

Question:

A, B, C and D, are four points in a plane respectively such that $(A - D) \cdot (B - C) = (B - D) \cdot (C - A) = 0$. The point D, then, is the _____ of $\triangle ABC$.

Solution:

Consider the equation,

$$\left(A - D\right) \cdot \left(B - C\right) = 0 \tag{1}$$

This implies line joining A and D is perpendicular to line joining B and C

Consider the equation,

$$\left(B - D\right) \cdot \left(C - A\right) = 0 \tag{2}$$

This implies line joining B and D is perpendicular to line joining A and C

In $\triangle ABC$,

side BC is perpendicular to AD

side AC is perpendicular to BD

We know that,

The altitudes(The perpendiculars drawn from a vertex to opposite sides) are concurrent at Orthocentre.

Therefore,
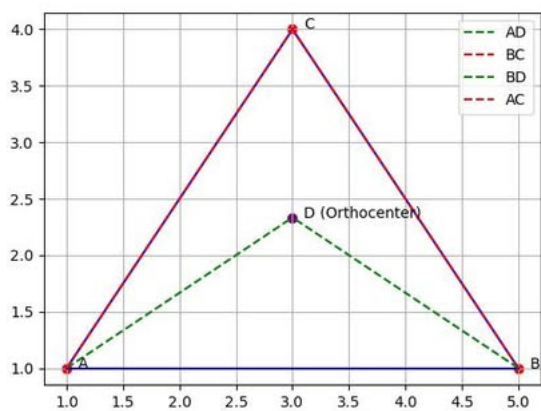
D must be Orthocentre of $\triangle ABC$

Fig. 0.1