1.7.2

AI25BTECH11016-Varun

Question:
If A(1, 2), O(0, 0), and C(a, 6) are collinear, then the value of a is
Solution:
The given points are

$$A = (1, 2), \quad O = (0, 0), \quad C = (a, 6). \tag{1}$$

$$\mathbf{A} - \mathbf{O} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \tag{2}$$

$$\mathbf{C} - \mathbf{O} = \begin{pmatrix} a \\ 6 \end{pmatrix}. \tag{3}$$

Construct the matrix

$$M = \begin{pmatrix} 1 & a \\ 2 & 6 \end{pmatrix}. \tag{4}$$

For the points to be collinear, the two vectors **OA** and **OC** must be linearly dependent. This means

$$\text{rank}(M) = 1 \quad \Leftrightarrow \quad \det(M) = 0. \tag{5}$$

$$\begin{pmatrix} 1 & a \\ 2 & 6 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - 2R_1} \begin{pmatrix} 1 & a \\ 0 & 6 - 2a \end{pmatrix}. \tag{6}$$

For the rank to drop,

$$6 - 2a = 0 \tag{7}$$

$$a = 3. \tag{8}$$

When $a = 3$,

$$\begin{pmatrix} 1 & 3 \\ 0 & 0 \end{pmatrix}$$

is the reduced row-echelon form (rank = 1).

The given points are collinear when

$$a = 3. \tag{9}$$

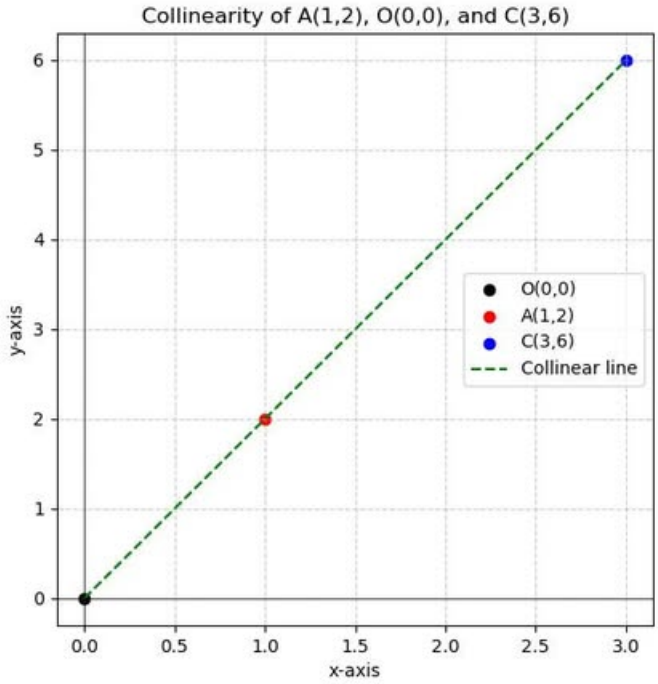
Collinearity of A(1,2), O(0,0), and C(3,6)
y-axis
x-axis
0
1
2
3
4
5
6
0.0
0.5
1.0
1.5
2.0
2.5
3.0
O(0,0)
A(1,2)
C(3,6)
Collinear line

Fig. 0.1