1.4.26

EE25BTECH11010 - Arsh Dhoke

Question:
The position vector of the point which divides the join of points $2\mathbf{a} - 3\mathbf{b}$ and $\mathbf{a} + \mathbf{b}$ in the ratio 3 : 1 is _____.

Solution:

Let $\mathbf{a} = \begin{pmatrix}1\\0\end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix}0\\1\end{pmatrix}$.

Then,

$$\mathbf{P} = 2\mathbf{a} - 3\mathbf{b} \tag{0.1}$$

$$\mathbf{Q} = \mathbf{a} + \mathbf{b} \tag{0.2}$$

$$\mathbf{P} = \begin{pmatrix}\mathbf{a} & \mathbf{b}\end{pmatrix}\begin{pmatrix}2\\-3\end{pmatrix} \mathbf{Q} = \begin{pmatrix}\mathbf{a} & \mathbf{b}\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} \tag{0.3}$$

Using section formula, the point $\mathbf{R}$ dividing $\mathbf{PQ}$ in ratio 3 : 1 is:

$$\mathbf{R} = \frac{3\mathbf{Q} + 1\mathbf{P}}{3+1}. \tag{0.4}$$

$$\mathbf{R} = \begin{pmatrix}\mathbf{Q} & \mathbf{P}\end{pmatrix}\begin{pmatrix}\frac{3}{4}\\\frac{1}{4}\end{pmatrix} \tag{0.5}$$

$$\mathbf{R} = \begin{pmatrix}\begin{pmatrix}\mathbf{a}\\\mathbf{b}\end{pmatrix} & \begin{pmatrix}2\mathbf{a}\\-3\mathbf{b}\end{pmatrix}\end{pmatrix}\begin{pmatrix}\frac{3}{4}\\\frac{1}{4}\end{pmatrix} \tag{0.6}$$

$$\mathbf{R} = \frac{1}{4}\left(3\begin{pmatrix}\mathbf{a}\\\mathbf{b}\end{pmatrix} + \begin{pmatrix}2\mathbf{a}\\-3\mathbf{b}\end{pmatrix}\right) \tag{0.7}$$

$$= \frac{1}{4}\begin{pmatrix}3\mathbf{a} + 2\mathbf{a}\\3\mathbf{b} - 3\mathbf{b}\end{pmatrix} \tag{0.8}$$

$$= \frac{1}{4}\begin{pmatrix}5\mathbf{a}\\0\end{pmatrix} \tag{0.9}$$

$$= \begin{pmatrix}\frac{5\mathbf{a}}{4}\\0\end{pmatrix}. \tag{0.10}$$

$$\boxed{\mathbf{R} = \begin{pmatrix} \frac{5\mathbf{a}}{4} \\ 0 \end{pmatrix}} \tag{0.11}$$

Let **a**=1 and **b**=0.

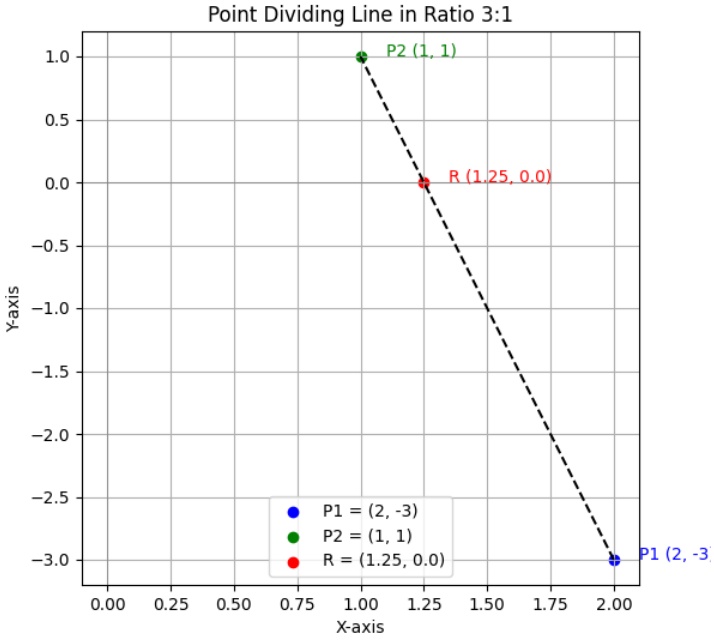

Fig. 0.1. Graph for question 1