

1.10.9

EE25BTECH11050-Hema Havil

Question:

Find the unit vector in the direction of the vector PQ, where P and Q are the points (1, 2, 3) and (4, 5, 6), respectively.

Solution:

Given,

The points:

$$\mathbf{P} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} \quad (0.1)$$

Let the required unit vector be \mathbf{x} , then

The formula for unit vector along a line joining two points

$$\mathbf{x} = \frac{\mathbf{X}}{\|\mathbf{X}\|} \quad (0.2)$$

The vector along \mathbf{P} and \mathbf{Q} is

$$\mathbf{X} = \mathbf{Q} - \mathbf{P} \quad (0.3)$$

$$\mathbf{X} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad (0.4)$$

$$\mathbf{X} = \begin{pmatrix} 4 - 1 \\ 5 - 2 \\ 6 - 3 \end{pmatrix} \quad (0.5)$$

$$\mathbf{X} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} \quad (0.6)$$

Magnitude of the vector \mathbf{X} is

$$\|\mathbf{X}\| = \sqrt{\mathbf{X}^T \mathbf{X}} \quad (0.7)$$

$$\|\mathbf{X}\| = \sqrt{(3, 3, 3) \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}} \quad (0.8)$$

$$\|\mathbf{X}\| = \sqrt{(3)^2 + (3)^2 + (3)^2} \quad (0.9)$$

$$\|\mathbf{X}\| = \sqrt{3(3)^2} \quad (0.10)$$

$$\|\mathbf{X}\| = 3\sqrt{3} \quad (0.11)$$

Then the unit vector,

$$\mathbf{x} = \frac{1}{3\sqrt{3}}\mathbf{X} = \mathbf{x} = \frac{1}{3\sqrt{3}} \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} \quad (0.12)$$

$$\mathbf{x} = \frac{3}{3\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (0.13)$$

$$\mathbf{x} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right) \quad (0.14)$$

Therefore, the required unit vector is

$$\mathbf{x} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right) \quad (0.15)$$

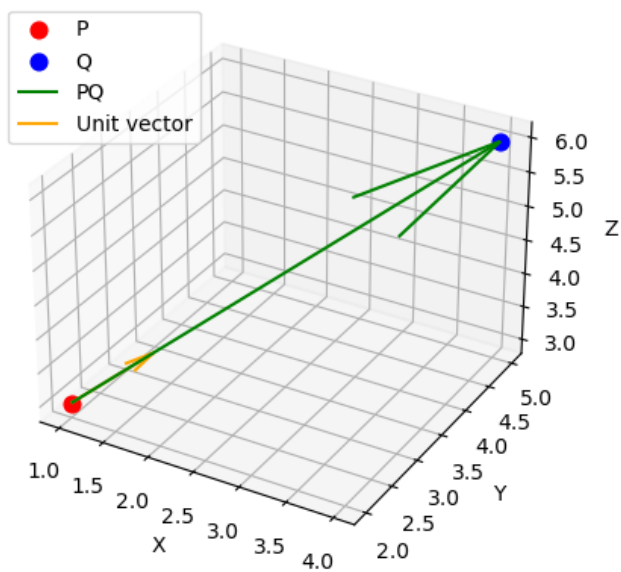


Fig. 0.1: Plot for the unit vector along PQ