

2.10.49

EE25BTECH11020 - Darsh Pankaj Gajare

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Question:

The unit vector which is orthogonal to the vector  $3\hat{i} + 2\hat{j} + 6\hat{k}$  and is coplanar with vectors  $2\hat{i} + \hat{j} + \hat{k}$  and  $\hat{i} - \hat{j} + \hat{k}$  is

(A)  $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$

(B)  $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$

(C)  $\frac{3\hat{i}-\hat{k}}{\sqrt{10}}$

(D)  $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

**Solution:** Given:

Table: Given data

Vector	matrix
<b>A</b>	$\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$
<b>B</b>	$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$
<b>C</b>	$\begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix}$

Assume Equation of plane through  $A, B$ .

$$\mathbf{n}^\top \mathbf{x} = 1 \quad (0.1)$$

$$\mathbf{n}^\top \mathbf{A} = 1 \quad (0.2)$$

$$\mathbf{n}^\top \mathbf{B} = 1 \quad (0.3)$$

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} n = 1 \quad (0.4)$$

Augmented matrix,

$$\left( \begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \end{array} \right). \quad (0.5)$$

$$R_1 = R_1 - R_2$$

$$\left( \begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 1 & -1 & 1 & 1 \end{array} \right) \quad (0.6)$$

$$R_2 = R_2 - R_1$$

$$\left( \begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & -3 & 1 & 1 \end{array} \right) \quad (0.7)$$

Let parametric constant be  $\lambda$

$$n = \begin{pmatrix} -2\lambda \\ \lambda \\ 1 + 3\lambda \end{pmatrix} \quad (0.8)$$

$$\mathbf{n}^\top \mathbf{P} = 1 \quad (0.9)$$

$$\mathbf{P}^\top \mathbf{C} = 0 \quad (0.10)$$

$$\begin{pmatrix} -2\lambda & \lambda & 1 + 3\lambda \\ 3 & 2 & 6 \end{pmatrix} \mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (0.11)$$

Augmented matrix,

$$\left( \begin{array}{ccc|c} -2\lambda & \lambda & 1+3\lambda & 1 \\ 3 & 2 & 6 & 0 \end{array} \right). \quad (0.12)$$

Row operations:  $R_1 = R_1 - \frac{\lambda}{2}R_2$

$$\left( \begin{array}{ccc|c} -3.5\lambda & 0 & 1 & 1 \\ 3 & 2 & 6 & 0 \end{array} \right). \quad (0.13)$$

$R_2 = R_2 - 6R_1$

$$\left( \begin{array}{ccc|c} -3.5\lambda & 0 & 1 & 1 \\ 3+21\lambda & 2 & 0 & -6 \end{array} \right). \quad (0.14)$$

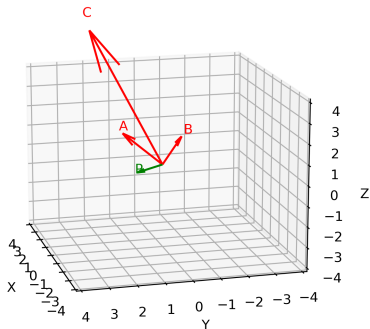
$$-3.5\lambda x + z = 1 \implies z = 1 + 3.5\lambda x, (3 + 21\lambda)x + 2y = -6 \implies y = -3 - (0.15)$$

Let  $x = \mu$  a parameter

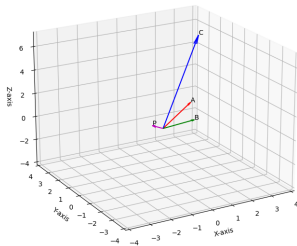
$$\mathbf{P} = \begin{pmatrix} \mu \\ -3 - \frac{\mu}{2}(3 + 21\lambda) \\ 1 + 3.5\lambda\mu \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} + \frac{\mu}{2} \begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix} + 7\lambda \frac{\mu}{2} \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}. \quad (0.16)$$

Normalizing,

$$\mathbf{P} = \pm \frac{1}{\sqrt{10}} \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} \quad (0.17)$$



## Plot using C functions



## Plot using Python