

4.4.20

AI25BTECH11024 - Pratyush Panda

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**Question:**

The pair of linear equations  $2x = 5y + 6$  and  $15y = 6x - 18$  represents two lines which are:

- (a) intersecting
- (b) parallel
- (c) coincident
- (d) either intersecting or parallel

**Solution:**

First, let us rewrite the equation of lines as;

$$ax + by = c \quad (0.1)$$

$$2x - 5y = 6 \quad (0.2)$$

$$6x - 15y = 18 \text{ or } 2x - 5y = 6 \quad (0.3)$$

Normal vectors of the given lines can be written as:

$$\mathbf{n} = \begin{pmatrix} a \\ b \end{pmatrix} \quad (0.4)$$

$$\mathbf{n}_1 = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \quad \mathbf{n}_2 = \begin{pmatrix} 2 \\ -5 \end{pmatrix} \quad (0.5)$$

Let the matrix  $\mathbf{M}$  be;

$$\mathbf{M} = \begin{pmatrix} n_1 & n_2 \end{pmatrix}^T = \begin{pmatrix} 2 & -5 \\ 2 & -5 \end{pmatrix} \quad (0.6)$$

After reducing it to its Echelon form;

$$\begin{pmatrix} 2 & -5 \\ 2 & -5 \end{pmatrix} \xrightarrow{R_2 \longleftrightarrow R_2 - R_1} \begin{pmatrix} 2 & -5 \\ 0 & 0 \end{pmatrix} \quad (0.7)$$

We can see that the rank of  $\mathbf{M}$  is 1. Therefore, the given lines can be either parallel or coincident.

Now, consider the matrices  $\mathbf{P}$  and  $\mathbf{Q}$ ;

$$\mathbf{P} = \begin{pmatrix} a_1 & c_1 \\ a_2 & c_2 \end{pmatrix} \qquad \mathbf{Q} = \begin{pmatrix} b_1 & c_1 \\ b_2 & c_2 \end{pmatrix} \qquad (0.8)$$

Since, the rank of both the matrices is 1. Thus, we can conclude that the the given lines are coincident.

