4.3.33

EE25BTECH11043 - Nishid Khandagre

Question: If the coordinates of the middle point of the portion of a line intercepted between the coordinate axes is $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$, then the equation of the line will be?

Solution: The equation of line is

$$\mathbf{n}^\top \mathbf{x} = c \tag{0.1}$$

Where $\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$ is the normal vector and $\mathbf{x}$ is the position vector.

X-axis intercept is at $\mathbf{A}$

$$\mathbf{n}^\top \mathbf{A} = c \tag{0.2}$$

$$\begin{pmatrix} n_1 & n_2 \end{pmatrix} \begin{pmatrix} a \\ 0 \end{pmatrix} = c \tag{0.3}$$

$$n_1 a = c \tag{0.4}$$

$$\mathbf{A} = \begin{pmatrix} \frac{c}{n_1} \\ 0 \end{pmatrix} \tag{0.5}$$

Y-axis intercept is at $\mathbf{B}$

$$\mathbf{n}^\top \mathbf{B} = c \tag{0.6}$$

$$\begin{pmatrix} n_1 & n_2 \end{pmatrix} \begin{pmatrix} 0 \\ b \end{pmatrix} = c \tag{0.7}$$

$$n_2 b = c \tag{0.8}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ \frac{c}{n_2} \end{pmatrix} \tag{0.9}$$

Thus, $\mathbf{B}$ is $\begin{pmatrix} 0 \\ \frac{c}{n_2} \end{pmatrix}$

Let $\mathbf{M}$ is the midpoint of $\mathbf{A}$ and $\mathbf{B}$

Given $\mathbf{M} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{0.10}$$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \frac{c}{n_1} \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 \\ \frac{c}{n_2} \end{pmatrix} \tag{0.11}$$

$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{c}{2n_1} \\ \frac{c}{2n_2} \end{pmatrix} \tag{0.12}$$

$$\frac{c}{2n_1} = 3 \tag{0.13}$$

$$\frac{c}{2n_2} = 2 \tag{0.14}$$

$$\frac{n_1}{n_2} = \frac{2}{3} \tag{0.15}$$

Let $n_1 = 2$ and $n_2 = 3$. Then

$$c = 6 \times 2 = 12 \tag{0.16}$$

The final equation of the line is $\mathbf{n}^\top \mathbf{x} = c$

$$\begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{x} = 12 \tag{0.17}$$

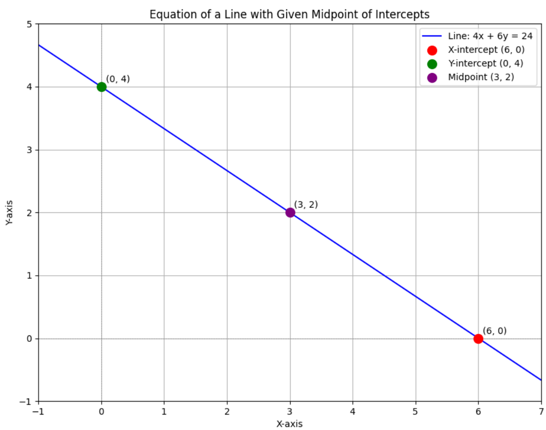

Fig. 0.1