7.4.11

EE25BTECH11004 - Aditya Appana

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Question

If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure 45° at the major segment of the circle then the value of m is

a) $2 \pm \sqrt{2}$ b) $-2 \pm \sqrt{2}$ c) $-1 \pm \sqrt{2}$ d) none of these

Solution

The given line subtends an angle 45° at the major segment of the circle. Therefore, it will subtend $2 \times 45^\circ = 90^\circ$ at the centre of the circle.

The line $y = mx + 1$ can be expressed as:

$$\begin{pmatrix} m \\ -1 \end{pmatrix}^T \mathbf{x} + 1 = 0 \tag{1}$$

This line always passes through $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, which lies on the circle $x^2 + y^2 = 1$ (since $0^2 + 1^2 = 1$). Therefore one point of intersection is $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Let the other point of intersection be **P**. **P** will be a $\pm 90^\circ$ rotation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ about the origin.

The rotation matrix is:

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{2}$$

Therefore:

$$\mathbf{P} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} \cos(\pm 90^\circ) & \sin(\pm 90^\circ) \\ -\sin(\pm 90^\circ) & \cos(\pm 90^\circ) \end{pmatrix} \tag{3}$$

$$\mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ or } \begin{pmatrix} -1 \\ 0 \end{pmatrix} \tag{4}$$

$$\text{If } \mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, m = \frac{0-1}{1-0} = -1 \tag{5}$$

$$\text{If } \mathbf{P} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, m = \frac{0-1}{-1-0} = 1 \tag{6}$$

$$m = \pm 1 \tag{7}$$

Therefore, **d)** is the correct answer.

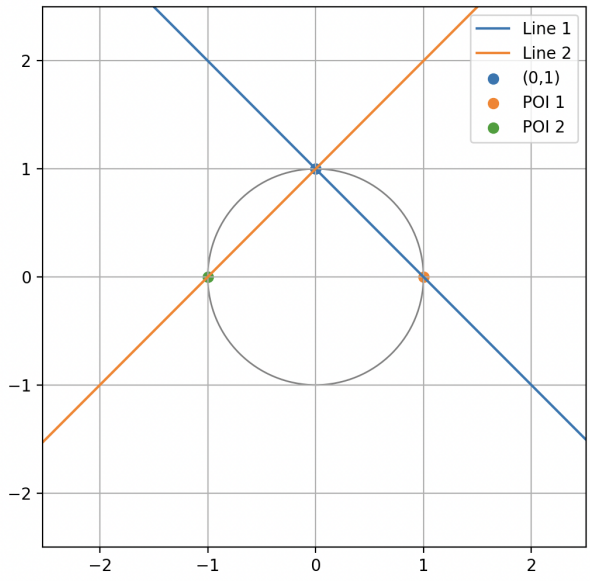

Figure 1: Plot