2.7.11

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Question

Q 2.7.11. Find the area of the triangle with vertices $A = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $B = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$, $C = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$.

Solution

Given: The vertices of the triangle are

$$A = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad B = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \quad C = \begin{pmatrix} -3 \\ -5 \end{pmatrix}.$$

The area of a triangle with vertices A, B, C is given by

$$\Delta = \frac{1}{2} \left| \det\left(\begin{pmatrix} B - A & C - A \end{pmatrix} \right) \right| = \frac{1}{2} \left\| (B - A) \times (C - A) \right\|,$$

where the cross product in 2D is interpreted as the determinant of a 2×2 matrix.

$$B - A = \begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}, \quad C - A = \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix}.$$

Now,

$$\det \begin{pmatrix} -5 & -4 \\ 7 & -4 \end{pmatrix} = (-5)(-4) - (7)(-4) = 20 + 28 = 48.$$

Hence,

$$\Delta = \frac{1}{2} |48| = 24.$$

Final Answer:

$$\boxed{24}$$

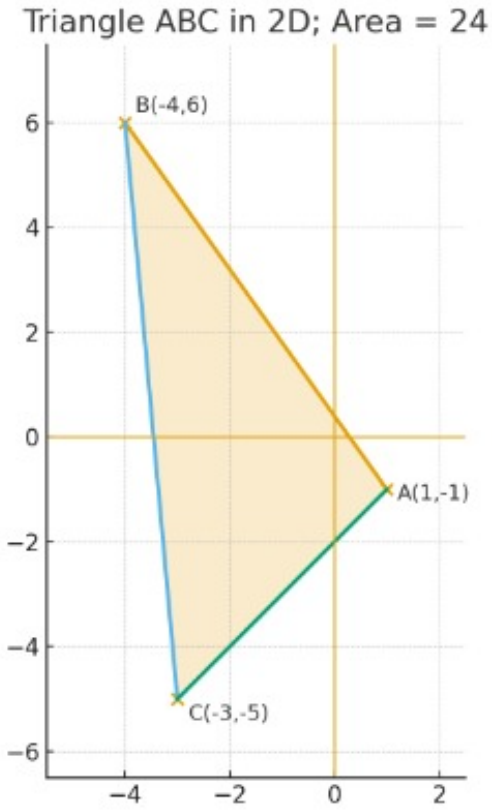

Fig. 0.1: Triangle ABC with $A(1,-1)$, $B(-4,6)$, $C(-3,-5)$; area = 24.