# 9.2.27

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# Question

Find the Area enclosed by the parabola $4y = 3x^2$ and the Line 2y=3x+12

# Solution

Given Line

$$2y = 3x + 12 \tag{1}$$

$$\mathbf{x} = \mathbf{h} + k\mathbf{m};\ k \in \mathbb{R} \tag{2}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 6 \end{pmatrix} \tag{3}$$

$$\mathbf{m} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{4}$$

# Solution

Given curve

$$4y = 3x^2 \tag{5}$$

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{6}$$

$$\mathbf{V} = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \tag{7}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{8}$$

$$f = 0 \tag{9}$$

# Solution

Points of Intersection

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left( -\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{\left[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})\right]^2 - g(\mathbf{h}) \cdot (\mathbf{m}^\top \mathbf{V}\mathbf{m})} \right) \tag{10}$$

where

$$g(\mathbf{h}) = \mathbf{h}^\top \mathbf{V}\mathbf{h} + 2\mathbf{u}^\top \mathbf{h} + f \tag{11}$$

$$\tag{12}$$

# Solution

$$\mathbf{V}\mathbf{h} = \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{13}$$

$$\mathbf{V}\mathbf{h} + \mathbf{u} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \tag{14}$$

$$\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) = \begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ -2 \end{pmatrix} = -6 \tag{15}$$

$$\mathbf{m}^\top\mathbf{V}\mathbf{m} = \begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} 6 \\ 0 \end{pmatrix} = 12 \tag{16}$$

$$\mathbf{u}^\top\mathbf{h} = \begin{pmatrix} 0 & -2 \end{pmatrix} \begin{pmatrix} 0 \\ 6 \end{pmatrix} = -12 \tag{17}$$

$$g(\mathbf{h}) = 0 + 2(-12) + 0 = -24 \tag{18}$$

# Solution

$$\kappa_i = \frac{1}{12}\left(6 \pm \sqrt{(-6)^2 - (-24)(12)}\right) \tag{19}$$

$$= \frac{1}{12}\left(6 \pm \sqrt{36 + 288}\right) \tag{20}$$

$$= \frac{1}{12}\left(6 \pm \sqrt{324}\right) \tag{21}$$

$$= \frac{1}{12}(6 \pm 18) \tag{22}$$

$$= \kappa_1 = \frac{24}{12} = 2, \quad \kappa_2 = \frac{-12}{12} = -1 \tag{23}$$

The point of intersection are :

$$(4, 12) \quad \text{and} \quad (-2, 3) \tag{24}$$

# Solution

Area Bounded by curves is given by

$$\left|\int_{-2}^{4} \frac{3x^2}{4} - \frac{3x+12}{2}\right| \tag{25}$$

$$= \left|\frac{1}{4}\int_{-2}^{4} 3x^2 - 6x - 24\right| \tag{26}$$

$$= \left|\frac{1}{4}\left(x^3 - 3x^2 - 24x\right)_{-2}^{4}\right| \tag{27}$$

$$= \left|\frac{1}{4}\left(4^3 - (-2)^3 - 3(4^2 - (-2)^2) - 24(4 - (-2)\right)\right| \tag{28}$$

$$= 27 \tag{29}$$

# C Code

```
        #include <stdio.h>
#include <math.h>
double* compute_points_and_area() {
    static double results[5]; // results[0,1]=P1, [2,3]=P2, [4]=
        area

    double h[2] = {0, 6};
    double m[2] = {2, 3};
    double V[2][2] = {{3,0},{0,0}};
    double u[2] = {0, -2};
    double f = 0;
```

# C Code

```c
	// Step 1: V*h + u
    double Vh[2];
    Vh[0] = V[0][0]*h[0] + V[0][1]*h[1];
    Vh[1] = V[1][0]*h[0] + V[1][1]*h[1];

    double Vh_plus_u[2] = {Vh[0]+u[0], Vh[1]+u[1]};

    // Step 2: m^T*(Vh+u)
    double mT_Vh_plus_u = m[0]*Vh_plus_u[0] + m[1]*Vh_plus_u[1];
```

# C Code

```c
    // Step 3: m^T * V * m
double Vm[2] = { V[0][0]*m[0] + V[0][1]*m[1], V[1][0]*m[0] + V[1][1]*m[1] };
double mT_V_m = m[0]*Vm[0] + m[1]*Vm[1];

// Step 4: g(h)
double hT_V_h = h[0]*(V[0][0]*h[0]+V[0][1]*h[1]) + h[1]*(V[1][0]*h[0]+V[1][1]*h[1]);
double uT_h = u[0]*h[0] + u[1]*h[1];
double g_h = hT_V_h + 2*uT_h + f;
```

# C Code

```c
        // Step 5: kappa values
    double sqrt_term = sqrt(mT_Vh_plus_u*mT_Vh_plus_u - g_h*
        mT_V_m);
    double kappa1 = (-mT_Vh_plus_u + sqrt_term)/mT_V_m;
    double kappa2 = (-mT_Vh_plus_u - sqrt_term)/mT_V_m;

    // Step 6: Intersection points
    results[0] = h[0] + kappa1*m[0]; // P1 x
    results[1] = h[1] + kappa1*m[1]; // P1 y
    results[2] = h[0] + kappa2*m[0]; // P2 x
    results[3] = h[1] + kappa2*m[1]; // P2 y
```

# C Code

```
        // Step 7: Area
    double x1 = results[2]; // -2
    double x2 = results[0]; // 4
    results[4] = fabs((1.0/4)*(pow(x2,3)-pow(x1,3)-3*(pow(x2,2)-
        pow(x1,2))-24*(x2-x1)));

    return results;
}
```

# Python Code

```
import numpy as np
import matplotlib.pyplot as plt

# Define the x range
x = np.linspace(-5, 5, 500)

# Define the curves
y_curve = (3/4) * x**2 # 4y = 3x^2 => y = 3/4 x^2
y_line = (3/2) * x + 6 # 2y = 3x+12 => y = 3/2 x + 6
```

# Python Code

```
# Plot the curves
plt.plot(x, y_curve, label=r'$4y=3x^2$', color='blue')
plt.plot(x, y_line, label=r'$2y=3x+12$', color='red')

# Find intersection points for shading
# Solve (3/4)x^2 = (3/2)x + 6 => 3x^2/4 - 3x/2 - 6 = 0
# Multiply by 4: 3x^2 - 6x - 24 = 0 => x^2 - 2x - 8 = 0
# Using quadratic formula: x = 1  3
x1 = -2
x2 = 4
```

# Python Code

```python

# Shade the area between the curves
x_fill = np.linspace(x1, x2, 500)
plt.fill_between(x_fill, (3/4)*x_fill**2, (3/2)*x_fill + 6, color
    ='green', alpha=0.3, label='Shaded Area')

# Add labels, grid, and legend
plt.xlabel('x')
plt.ylabel('y')
plt.title('Area between $4y=3x^2$ and $2y=3x+12$')
plt.grid(True)
plt.legend()
plt.show()
```

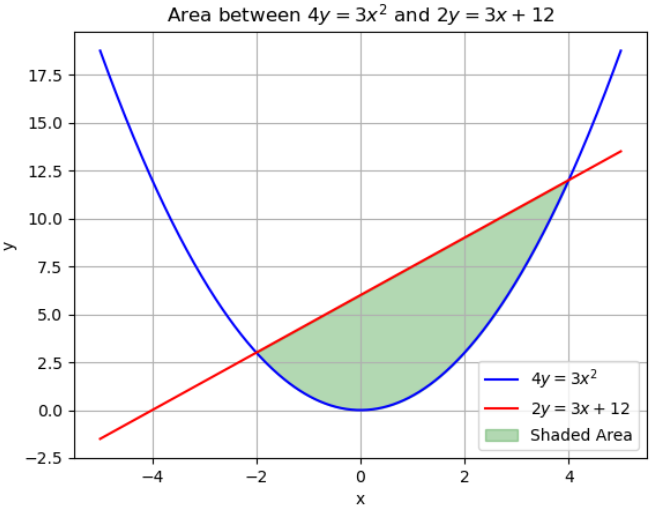
# C and Python Code

```
        import ctypes
# Load the shared object
lib = ctypes.CDLL("./curves.so")

# Specify return type
lib.compute_points_and_area.restype = ctypes.POINTER(ctypes.
    c_double)

# Call the function
result_ptr = lib.compute_points_and_area()
results = [result_ptr[i] for i in range(5)]
```

# C and Python Code

```
        P1 = (results[0], results[1])
P2 = (results[2], results[3])
area = results[4]

print("Intersection points:")
print("P1:", P1)
print("P2:", P2)
print("Area bounded by curves:", area)
```

# plot


Area between $4y = 3x^2$ and $2y = 3x + 12$
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
y
−4
−2
0
2
4
x
$4y = 3x^2$
$2y = 3x + 12$
Shaded Area