4.10.13

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Question:
Find the equation of the plane passing through the line of intersection of the planes $\mathbf{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\mathbf{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to the X axis.
Solution:
let P1 and P2 be the plane equations whose normals are:

Plane	**Normal vector**
P1	$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$
P2	$\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$

TABLE 0: 4.10.13

Given equations of planes are:-

$$P_1 : \quad \mathbf{r}^\top \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 1 \tag{0.1}$$

$$P_2 : \quad \mathbf{r}^\top \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} = -4 \tag{0.2}$$

expressing the plane equations in matrix form:-

$$\begin{bmatrix} 1 & 1 & 1 & | & 1 \\ 2 & 3 & -4 & | & -4 \end{bmatrix} \tag{0.3}$$

Using row reductions:

$$R_2 \rightarrow R_2 - 2R_1 \tag{0.4}$$

$$\begin{bmatrix} 1 & 1 & 1 & | & 1 \\ 0 & 1 & -3 & | & -6 \end{bmatrix} \tag{0.5}$$

$$\therefore \text{equation of planes:} \tag{0.6}$$

$$\mathbf{r}^\top \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 1 \tag{0.7}$$

$$\mathbf{r}^\top \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix} = -6 \tag{0.8}$$

Solving the equations to find the line of intersection of planes

$$\mathbf{r}(\lambda) = \begin{pmatrix} 0 \\ -\frac{3}{4} \\ \frac{7}{6} \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3/4 \\ -1/6 \end{pmatrix} \tag{0.9}$$

normal to plane is orthogonal to the line and x-axis

$$\mathbf{n}^\top \mathbf{e}_1 = 0 \tag{0.10}$$

$$\mathbf{n}^\top \mathbf{n}_1 = 0 \tag{0.11}$$

$$where,$$

$$\mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{0.12}$$

$$\mathbf{n}_1 = \begin{pmatrix} 1 \\ -3/4 \\ -1/6 \end{pmatrix} \tag{0.13}$$

Solving using row reductions:-

$$\begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 4 & -3 & -1 & | & 0 \end{bmatrix} \tag{0.14}$$

$$R_2 \rightarrow R_2 - 4R_1 \tag{0.15}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & -3 & -1 & | & 0 \end{bmatrix} \tag{0.16}$$

plane equation using normal and a point on the line:

$$\mathbf{n}^\top(\mathbf{r}-\mathbf{r}_0)=0 \tag{0.17}$$

$$\mathbf{r}_0=\begin{pmatrix}0\\-3/4\\-7/4\end{pmatrix} \tag{0.18}$$

$$\mathbf{n}=\begin{pmatrix}0\\1\\-3\end{pmatrix} \tag{0.19}$$

$$\mathbf{r}=\begin{pmatrix}x\\y\\z\end{pmatrix} \tag{0.20}$$

Hence, equation of the plane is: $y-3z+6=0$

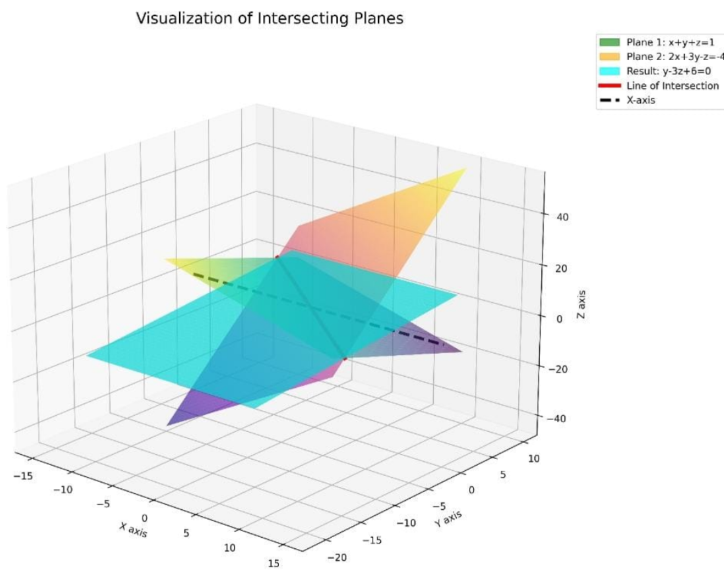

Fig. 0.1: 4.10.13