

MatGeo Assignment 1.2.13

AI25BTECH11007

September 1, 2025

Question

Using vectors, find the area of $\triangle ABC$ with vertices $A(1, 2, 3)$, $B(2, -1, 4)$ and $C(4, 5, -1)$.

Solution

Compute vectors $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 - 1 \\ -1 - 2 \\ 4 - 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \quad (1)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 - 1 \\ 5 - 2 \\ -1 - 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix} \quad (2)$$

Compute the cross product $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$:

$$\begin{aligned}
 [\mathbf{B} - \mathbf{A} \times \mathbf{C} - \mathbf{A}] &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix} \\
 &= \hat{i}((-3)(-4) - (1)(3)) - \hat{j}((1)(-4) - (1)(3)) + \hat{k}((1)(3) - (1)(-9)) \\
 &= \hat{i}(12 - 3) - \hat{j}(-4 - 3) + \hat{k}(3 + 9) \\
 &= \hat{i}(9) - \hat{j}(-7) + \hat{k}(12) \\
 &= \begin{pmatrix} 9 \\ 7 \\ 12 \end{pmatrix}
 \end{aligned} \tag{3}$$

Compute the magnitude of $[\mathbf{B} - \mathbf{A} \times \mathbf{C} - \mathbf{A}]$:

$$\begin{aligned}
 |[\mathbf{B} - \mathbf{A} \times \mathbf{C} - \mathbf{A}]| &= \sqrt{9^2 + 7^2 + 12^2} \\
 &= \sqrt{81 + 49 + 144} \\
 &= \sqrt{274}
 \end{aligned} \tag{4}$$

Area of $\triangle ABC$ is half the magnitude of the cross product:

$$\begin{aligned}\text{Area}(\triangle ABC) &= \frac{1}{2} \|[\mathbf{B} - \mathbf{A} \times \mathbf{C} - \mathbf{A}]\| \\ &= \frac{1}{2} \sqrt{274} \\ &= \frac{\sqrt{274}}{2}\end{aligned}\tag{5}$$

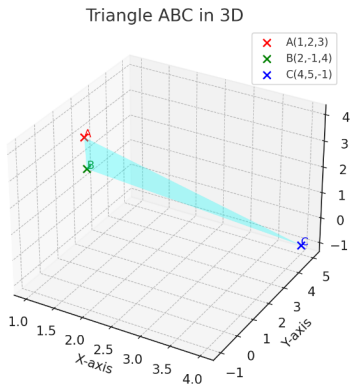


Figure: Image Visual