2.7.1

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Question

The area of a triangle formed by vertices **O**, **A** and **B**, where $\mathbf{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\mathbf{OB} = -3\hat{i} - 2\hat{j} + \hat{k}$ is

Solution

Represent the vectors in matrix form:

$$\mathbf{OA} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{OB} = \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} \tag{1}$$

The area of triangle OAB is given by:

$$\text{Area} = \frac{1}{2} \|\mathbf{OA} \times \mathbf{OB}\| \tag{2}$$

Compute the cross product using determinant:

$$\mathbf{OA} \times \mathbf{OB} = \begin{pmatrix} (2)(1) - (3)(1) \\ (3)(-3) - (1)(1) \\ (1)(1) - (2)(-3) \end{pmatrix} = \begin{pmatrix} 2 - 3 \\ -9 - 1 \\ 1 + 6 \end{pmatrix} = \begin{pmatrix} -1 \\ -10 \\ 7 \end{pmatrix} \tag{3}$$

Solution

Simplifying:

$$= -\hat{i} - 10\hat{j} + 7\hat{k} \Rightarrow \mathbf{OA} \times \mathbf{OB} = \begin{pmatrix} -1 \\ -10 \\ 7 \end{pmatrix} \tag{4}$$

Magnitude of the cross product:

$$\|\mathbf{OA} \times \mathbf{OB}\| = \sqrt{(-1)^2 + (-10)^2 + 7^2} = \sqrt{1 + 100 + 49} = \sqrt{150} \tag{5}$$

Final area:

$$\text{Area} = \frac{1}{2}\sqrt{150} \tag{6}$$

Plot

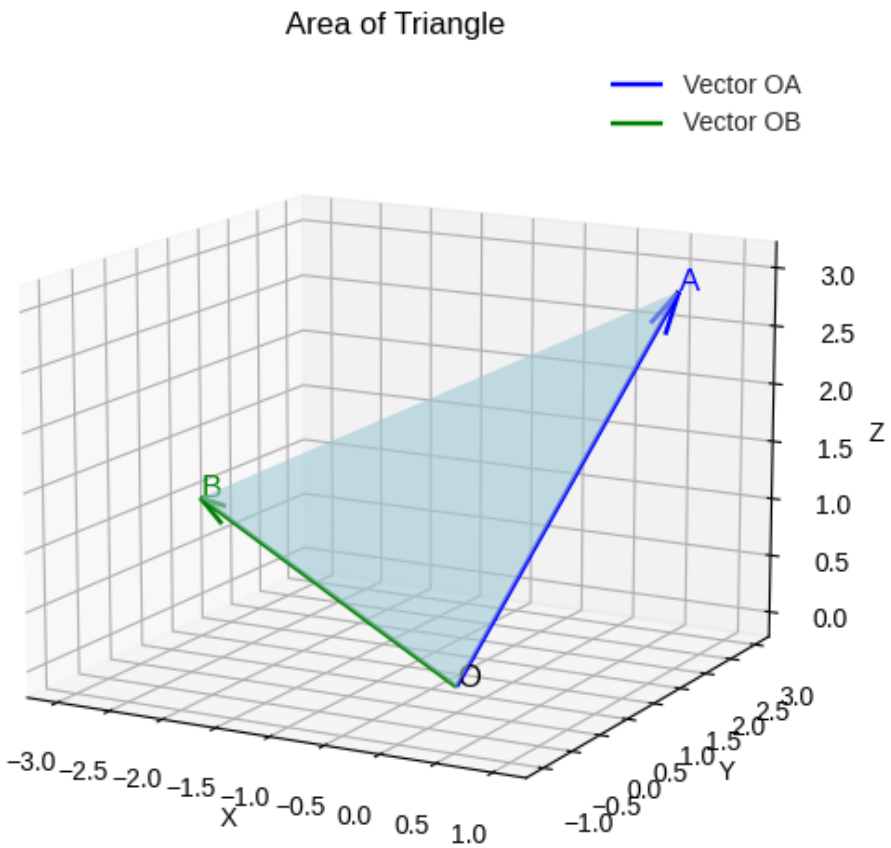

Figure: