2.9.1

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Question

Jagdish has a field which is in the shape of a right-angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining space for growing vegetables. In the field, there is a pole marked as O. Based on the above information, answer the following equations

(a) Taking O as the origin, P = (-200, 0) and Q = (200, 0). PQRS being a square, what are the coordinates of R and S?

(b)

(i) What is the area of square PQRS ?

(ii) What is the length of diagonal PR in PQRS ?

(c) If S divides CA in the ratio K : 1, what is the value of K, where A = (200, 800)?

Theoretical Solution

Given that,
AQC is a right angled triangle at point Q and PQRS is a square inside the ΔAQC,
(a) We were given two points

$$P = (-200, 0), Q = (200, 0) \tag{1}$$

Let,
X be the vector along the side PQ,
Y be the vector along the side QR,
Z be the vector along the side PS then,

$$\mathbf{X} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} 200 \\ 0 \end{pmatrix} - \begin{pmatrix} -200 \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{X} = \begin{pmatrix} 400 \\ 0 \end{pmatrix} \tag{3}$$

Theoretical Solution

Rotation vector for 2x2 matrix is

$$\mathbf{R}_\theta = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \tag{4}$$

Rotate the vector $\mathbf{X}$ by 90° anticlockwise to get Y

$$\mathbf{Y} = \mathbf{R_{90}X} \tag{5}$$

$$\mathbf{Y} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 400 \\ 0 \end{pmatrix} \tag{6}$$

$$\mathbf{Y} = \begin{pmatrix} 0 \\ 400 \end{pmatrix} \tag{7}$$

So the vector along the side QR is $\mathbf{Y} = \begin{pmatrix} 0 \\ 400 \end{pmatrix}$ then,

$$\mathbf{Y} = \mathbf{R} - \mathbf{Q} \tag{8}$$

$$\mathbf{R} = \mathbf{Y} + \mathbf{Q} \tag{9}$$

$$\mathbf{R} = \begin{pmatrix} 0 \\ 400 \end{pmatrix} + \begin{pmatrix} 200 \\ 0 \end{pmatrix} \tag{10}$$

$$\mathbf{R} = \begin{pmatrix} 200 \\ 400 \end{pmatrix} \tag{11}$$

Since the sides QR and PS are parallel, vectors $\mathbf{Y} = \mathbf{Z}$ then

$$\mathbf{Z} = \mathbf{S} - \mathbf{P} \tag{12}$$

$$\mathbf{S} = \mathbf{Z} + \mathbf{P} \tag{13}$$

$$\mathbf{S} = \begin{pmatrix} 0 \\ 400 \end{pmatrix} + \begin{pmatrix} -200 \\ 0 \end{pmatrix} \tag{14}$$

$$\mathbf{S} = \begin{pmatrix} -200 \\ 400 \end{pmatrix} \tag{15}$$

Therefore the coordinates of the points R and S are (200,400) and (-200,400) (b)
(i)) We know the points P(-200,0) and Q(200,0)
Let length of the side of the square PQRS be x then,

$$x = \|\mathbf{Q} - \mathbf{P}\| \tag{16}$$

$$x = \left\| \begin{pmatrix} 400 \\ 0 \end{pmatrix} \right\| = 400 \tag{17}$$

Area of the square $= x^2 = (400)^2 = 160000$ sq units

(ii) Length of diagnol of the square $= x\sqrt{2} = 400\sqrt{2}$ units

(c) Given the point A=(200,800)
Since it was given that point S divides CA in the ratio K:1, this shows that points A,C and S are collinear. Since AQC is a right angled triangle, from this we can say that point C lies on X axis
Let point C be (t,0), Consider the matrix M

$$M = \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} \tag{18}$$

Where the points in the matrix are A(200,800),S(-200,400) and C(t,0) then,
substitute in equation 18

$$M = \begin{pmatrix} 200\ 800 & 1 \\ -200\ 400 & 1 \\ t \quad 0 & 1 \end{pmatrix} \tag{19}$$

$R_1 \to \frac{1}{200} R_1$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ -200 & 400 & 1 \\ t & 0 & 1 \end{pmatrix} \tag{20}$$

$R_2 \to R_2 + 200 R_1$ $\quad$ $R_3 \to R_3 - t R_1$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ 0 & 1200 & 2 \\ 0 & -4t & 1 - \frac{t}{200} \end{pmatrix} \tag{21}$$

$R_2 \to \frac{1}{200} R_2$

Theoretical Solution

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ 0 & 1 & \frac{1}{600} \\ 0 & -4t & 1 - \frac{t}{200} \end{pmatrix} \tag{22}$$

$R_3 \to R_3 + 4tR_2$

$$M = \begin{pmatrix} 1 & 4 & \frac{1}{200} \\ 0 & 1 & \frac{1}{600} \\ 0 & 0 & 1 - \frac{t}{200} + \frac{4t}{600} \end{pmatrix} \tag{23}$$

Since the rank of matrix M is 2,

$$1 - \frac{t}{200} + \frac{4t}{600} = 0 \tag{24}$$

$$1 + \frac{t}{600} = 0 \tag{25}$$

$$\frac{t}{600} = -1 \tag{26}$$

$$t = -600 \tag{27}$$

Therefore point C=(-600,0), Now S divides CA in the ratio K:1,

$$S = \frac{KA + C}{K + 1} \tag{28}$$

Theoretical Solution

$$K = \frac{(S-A)^T(C-S)}{\left\|S-A\right\|^2} \tag{29}$$

$$K = \frac{1}{(400)^2+(400)^2}\begin{pmatrix}-400 & -400\end{pmatrix}\begin{pmatrix}-400\\-400\end{pmatrix} \tag{30}$$

By solving (30) we get K=1

C Code- Ploting the given vectors

```c
#include <stdio.h>
#include <math.h>

// Output: out[0]=Rx, out[1]=Ry, out[2]=Sx, out[3]=Sy, out[4]=
    area, out[5]=diagonal, out[6]=Cx, out[7]=Cy, out[8]=K
void solve_from_pdf(double* out) {
    // Points from PDF
    double P[2] = {-200, 0};
    double Q[2] = {200, 0};
    double A[2] = {200, 800};

    // Side vector PQ
    double X[2] = {Q[0] - P[0], Q[1] - P[1]}; // [400, 0]
    // Rotate X by 90 deg anticlockwise to get Y (QR)
    double Y[2] = {0 - X[1], X[0]}; // [0, 400]

    // R = Q + Y = [200 + 0, 0 + 400] = [200, 400]
```

```c
double R[2] = {Q[0] + Y[0], Q[1] + Y[1]};

// Z = Y, PS parallel to QR, so Z = [0, 400]
// S = P + Z = [-200, 0 + 400] = [-200, 400]
double S[2] = {P[0] + Y[0], P[1] + Y[1]};

// Area and diagonal
double x = sqrt(X[0]*X[0] + X[1]*X[1]); // 400
double area = x * x;
double diag = x * sqrt(2);

// Point C from PDF, lying on x-axis, with collinearity
double C[2] = {-600, 0}; // from matrix rank/collinearity in
    PDF
```

C Code- Ploting the given vectors

```
        // K for S dividing CA in K:1
    // S = (K*A + C)/(K+1) --> solve for K using x or y (use y)
    double K = (S[1] - C[1]) / (A[1] - S[1]);

    out[0] = R[0];
    out[1] = R[1];
    out[2] = S[0];
    out[3] = S[1];
    out[4] = area;
    out[5] = diag;
    out[6] = C[0];
    out[7] = C[1];
    out[8] = K;
}
```

Python Code using shared output

```
                    import ctypes
import numpy as np
import matplotlib.pyplot as plt

# Load the compiled shared C lib (use correct path if needed)
lib = ctypes.CDLL('./2.9.1.so')

# Set up output variables
outputs = (ctypes.c_double * 9)()
lib.solve_from_pdf(outputs)

# Extract variables (variable names as in C and PDF)
R = (outputs[0], outputs[1])
S = (outputs[2], outputs[3])
area = outputs[4]
diagonal = outputs[5]
C = (outputs[6], outputs[7])
K = outputs[8]
```

Python Code using shared output

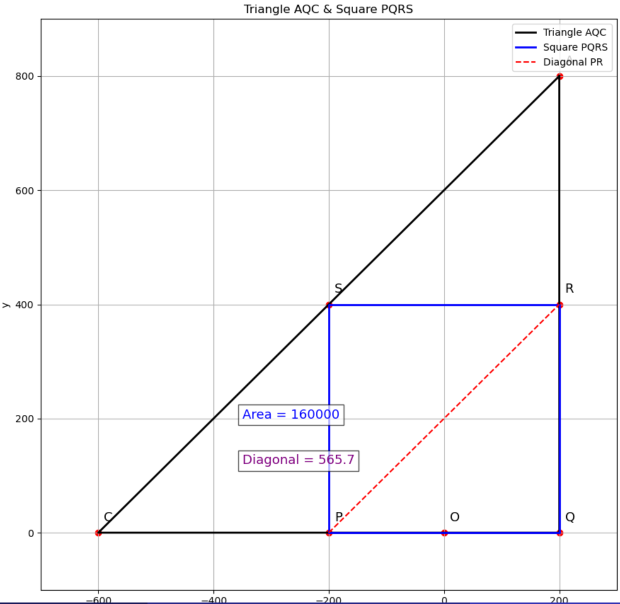
```
    P = (-200, 0)
Q = (200, 0)
A = (200, 800)
O = (0, 0)

# --- Print for verification ---
print(Coordinates of R:, R)
print(Coordinates of S:, S)
print(Area of PQRS:, area)
print(Length of diagonal PR:, diagonal)
print(Coordinates of C:, C)
print(Value of K:, K)

# --- Plot as in the PDF ---
plt.figure(figsize=(9,9))
# Triangle AQC (A,Q,C)
triangle_x = [A[0], Q[0], C[0], A[0]]
triangle_y = [A[1], Q[1], C[1], A[1]]
```

Python Code using shared output

```
plt.plot(triangle_x, triangle_y, 'k-', label='Triangle AQC',
    linewidth=2)

# Square PQRS
square_x = [P[0], Q[0], R[0], S[0], P[0]]
square_y = [P[1], Q[1], R[1], S[1], P[1]]
plt.plot(square_x, square_y, 'b-', label='Square PQRS', linewidth
    =2)

plt.scatter([O[0], P[0], Q[0], A[0], R[0], S[0], C[0]],
            [O[1], P[1], Q[1], A[1], R[1], S[1], C[1]], color='red
                ')
for pt, name in zip([O, P, Q, A, R, S, C], ['O', 'P', 'Q', 'A', '
    R', 'S', 'C']):
    plt.text(pt[0]+10, pt[1]+20, name, fontsize=13)

# Diagonal PR in square
plt.plot([P[0], R[0]], [P[1], R[1]], 'r--', label='Diagonal PR')
```

```
        # Annotate area & diagonal
plt.text(-350, 200, f'Area = {area:.0f}', fontsize=13, color='
    blue', bbox=dict(facecolor='white', alpha=0.7))
plt.text(-350, 120, f'Diagonal = {diagonal:.1f}', fontsize=13,
    color='purple', bbox=dict(facecolor='white', alpha=0.7))

plt.xlabel('x')
plt.ylabel('y')
plt.xlim(-700, 300)
plt.ylim(-100, 900)
plt.grid(True)
plt.legend()
plt.title('Triangle AQC & Square PQRS (as per PDF solution)')
plt.tight_layout()
plt.show()
```

Plot by python using shared output



Python code for the plot

```
   import numpy as np
import matplotlib.pyplot as plt

# Given points (from solution and problem statement)
P = np.array([-200, 0])
Q = np.array([200, 0])
A = np.array([200, 800])
O = np.array([0, 0])

# Step 1: Vector from P to Q
X = Q - P # [400, 0]

# Step 2: Rotate X by 90 degrees anticlockwise to get Y (QR)
Y = np.array([0 - X[1], X[0]]) # [0, 400]

# Step 3: Find R and S using vector addition as in PDF
R = Q + Y # [200, 400]
S = P + Y # [-200, 400]
```

Python code for the plot

```python
# Lists for square
square_x = [P[0], Q[0], R[0], S[0], P[0]]
square_y = [P[1], Q[1], R[1], S[1], P[1]]

plt.figure(figsize=(8, 8))

# Plot triangle
plt.plot(triangle_x, triangle_y, 'r-', label='Triangle')

# Plot square
plt.plot(square_x, square_y, 'b-', label='Square')

# Label triangle points
plt.text(A[0], A[1], 'A')
plt.text(Q[0], Q[1], 'Q')
plt.text(C[0], C[1], 'C')
```

Python code for the plot

```
    # Step 4: Area and diagonal of the square
side = np.linalg.norm(X) # 400
area = side ** 2 # 160000
diagonal = side * np.sqrt(2) # 400*sqrt(2)  565.69

# Step 5: Find C as in PDF by solving with collinearity (x-axis, so y=0)
# Using determinant as in PDF: |A-Q| |C-Q| = 0 for being collinear with Q as origin
# But given in PDF as C = (-600, 0)
C = np.array([-600, 0])

# Step 6: Find K for S dividing CA in K:1, S = (K*A + C)/(K+1)
# Solve for K using y-coordinates
S_y = S[1]
K = (S_y - C[1]) / (A[1] - S_y)
```

Python code for the plot

```
        # ---- Plot as in the PDF ----
plt.figure(figsize=(9,9))
# Triangle AQC (A->Q->C->A)
triangle_x = [A[0], Q[0], C[0], A[0]]
triangle_y = [A[1], Q[1], C[1], A[1]]
plt.plot(triangle_x, triangle_y, 'k-', label='Triangle AQC',
    linewidth=2)

# Square PQRS
square_x = [P[0], Q[0], R[0], S[0], P[0]]
square_y = [P[1], Q[1], R[1], S[1], P[1]]
plt.plot(square_x, square_y, 'b-', label='Square PQRS', linewidth
    =2)

# Points O, P, Q, A, R, S, C
pts = [O, P, Q, A, R, S, C]
lbls = ['O', 'P', 'Q', 'A', 'R', 'S', 'C']
for pt, name in zip(pts, lbls):
```

Python code for the plot

```
        plt.scatter(pt[0], pt[1], color='red')
    plt.text(pt[0]+10, pt[1]+20, name, fontsize=13)

# Diagonal PR
plt.plot([P[0], R[0]], [P[1], R[1]], 'r--', label='Diagonal PR')

# Area, Diagonal annotations
plt.text(-350, 200, f'Area = {area:.0f}', fontsize=13, color='blue', bbox=dict(facecolor='white', alpha=0.7))
plt.text(-350, 120, f'Diagonal = {diagonal:.2f}', fontsize=13, color='purple', bbox=dict(facecolor='white', alpha=0.7))

plt.xlabel('x')
plt.ylabel('y')
plt.xlim(-700, 300)
plt.ylim(-100, 900)
plt.grid(True)
```

Python code for the plot

```
        plt.legend()
plt.title('Triangle AQC & Square PQRS (PDF method, pure Python)')
plt.tight_layout()
plt.show()

# For verification
print(Coordinates of R:, tuple(R))
print(Coordinates of S:, tuple(S))
print(Area of PQRS:, area)
print(Length of diagonal PR:, diagonal)
print(Coordinates of C:, tuple(C))
print(Value of K:, K)
```

Plot of triangle and square