2.10.49

EE25BTECH11020 - Darsh Pankaj Gajare

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Question:
The unit vector which is orthogonal to the vector $3\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar with vectors $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ is

(A) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$ (B) $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$ (C) $\frac{3\hat{i}-\hat{k}}{\sqrt{10}}$ (D) $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

Solution: Given:

Table: Given data

Vector	**matrix**
A	$\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$
B	$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$
C	$\begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix}$

Assume Equation of plane through A, B.

$$\mathbf{n}^\top \mathbf{x} = 1 \tag{0.1}$$

$$\mathbf{n}^\top \mathbf{A} = 1 \tag{0.2}$$

$$\mathbf{n}^\top \mathbf{B} = 1 \tag{0.3}$$

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} n = 1 \tag{0.4}$$

Augmented matrix,

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \end{array}\right). \tag{0.5}$$

$R_1 = R_1 - R_2$

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 1 & -1 & 1 & 1 \end{array}\right) \tag{0.6}$$

$R_2 = R_2 - R_1$

$$\left(\begin{array}{ccc|c} 1 & 2 & 0 & 0 \\ 0 & -3 & 1 & 1 \end{array}\right) \tag{0.7}$$

Let parametric constant be λ

$$n = \begin{pmatrix} -2\lambda \\ \lambda \\ 1 + 3\lambda \end{pmatrix} \tag{0.8}$$

$$\mathbf{n}^\top \mathbf{P} = 1 \tag{0.9}$$

$$\mathbf{C}^\top \mathbf{P} = 0 \tag{0.10}$$

$$\begin{pmatrix} -2\lambda & \lambda & 1 + 3\lambda \\ 3 & 2 & 6 \end{pmatrix} \mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{0.11}$$

Augmented matrix,

$$\left(\begin{array}{ccc|c} -2\lambda & \lambda & 1+3\lambda & 1 \\ 3 & 2 & 6 & 0 \end{array}\right). \tag{0.12}$$

Row operations: $R_1 = R_1 - \frac{\lambda}{2} R_2$

$$\left(\begin{array}{ccc|c} -3.5\lambda & 0 & 1 & 1 \\ 3 & 2 & 6 & 0 \end{array}\right). \tag{0.13}$$

$R_2 = R_2 - 6R_1$

$$\left(\begin{array}{ccc|c} -3.5\lambda & 0 & 1 & 1 \\ 3+21\lambda & 2 & 0 & -6 \end{array}\right). \tag{0.14}$$

$$-3.5\lambda x + z = 1 \implies z = 1 + 3.5\lambda x \tag{0.15}$$

$$(3+21\lambda)\, x + 2y = -6 \implies y = -3 - \frac{x}{2}(3+21\lambda) \tag{0.16}$$

Let $x = \mu$ a parameter

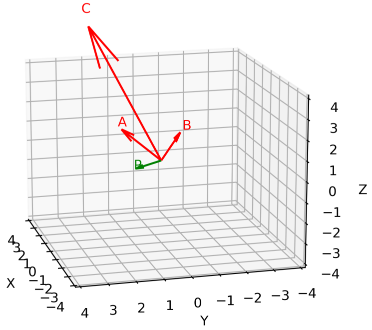
$$\mathbf{P} = \begin{pmatrix} \mu \\ -3 - \frac{\mu}{2}(3 + 21\lambda) \\ 1 + 3.5\lambda\mu \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} + \frac{\mu}{2}\begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix} + 7\lambda\frac{\mu}{2}\begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}. \quad (0.17)$$

Taking $\mu = 0$ we get,

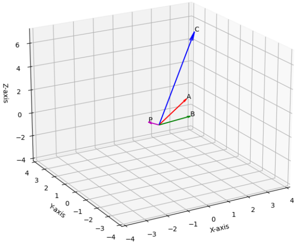
$$\mathbf{P} = \pm\begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} \quad (0.18)$$

Normalizing,

$$\mathbf{P} = \pm\frac{1}{\sqrt{10}}\begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} \quad (0.19)$$



Plot using C functions



Plot using Python