

## 1.4.6

snehamrudula-AI25BTECH11036

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# Question

If the point  $P(2, 1)$  lies on the line segment joining points  $A(4, 2)$  and  $B(8, 4)$ , then which of the following is true?

$$AP = \frac{1}{3}AB$$

$$AP = PB$$

$$PB = \frac{1}{3}AB$$

$$AP = \frac{1}{2}AB$$

# Theoretical Solution

## Solution:

Let the points be represented as vectors:

$$\mathbf{A} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 8 \\ 4 \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Since  $P$  lies on the line joining  $A$  and  $B$ , we have:

$$\mathbf{P} = \mathbf{A} + t(\mathbf{B} - \mathbf{A})$$

## Solution (continued)

Compute:

$$\mathbf{B} - \mathbf{A} = \begin{bmatrix} 8 - 4 \\ 4 - 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

So,

$$\mathbf{P} = \begin{bmatrix} 4 \\ 2 \end{bmatrix} + t \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 + 4t \\ 2 + 2t \end{bmatrix}$$

But  $\mathbf{P} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ , so:

$$4 + 4t = 2 \quad \text{and} \quad 2 + 2t = 1$$

## Solution (continued)

Solving:

$$4t = -2 \implies t = -0.5$$

$$2t = -1 \implies t = -0.5$$

Thus,  $t = -0.5$ .

The distance  $AP = |t| \cdot AB = 0.5 \cdot AB = \frac{1}{2}AB$ .

Therefore, the correct option is **d)**  $AP = \frac{1}{2}AB$ .