4.3.36

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Question

The line $\mathbf{r} = \left(2\hat{i} - 3\hat{j} - \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + 2\hat{k}\right)$ lies in the plane $\mathbf{r} \cdot \left(3\hat{i} + \hat{j} - \hat{k}\right) + 2 = 0$.

Theoretical Solution

Let the line L be $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ and the plane P be $\mathbf{r}^\top\mathbf{n} = c$ where

$$\mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \text{c} = \text{-2}$$

$$\mathbf{b}^\top\mathbf{n} = \begin{pmatrix} 1 & -1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \tag{1}$$

$$= (1)(3) + (-1)(1) + (2)(-1) \tag{2}$$

$$= 3 - 1 - 2 \tag{3}$$

$$\mathbf{b}^\top\mathbf{n} = 0 \tag{4}$$

$\because$ $\mathbf{b}^\top\mathbf{n} = 0$, the line L is parallel to plane P.

$$\mathbf{a}^\top \mathbf{n} = \begin{pmatrix} 2 & -3 & -1 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \tag{5}$$

$$= (2)(3) + (-3)(1) + (-1)(-1) \tag{6}$$

$$= 6 - 3 + 1 \tag{7}$$

$$= 4 \neq \text{ c} \tag{8}$$

$\because \mathbf{a}^\top \mathbf{n} \neq c$, the point **a** doesn't line in the plane P. Hence, the line L containing **a** also doesn't lie in the plane.
The given statement is **false**.

Plot

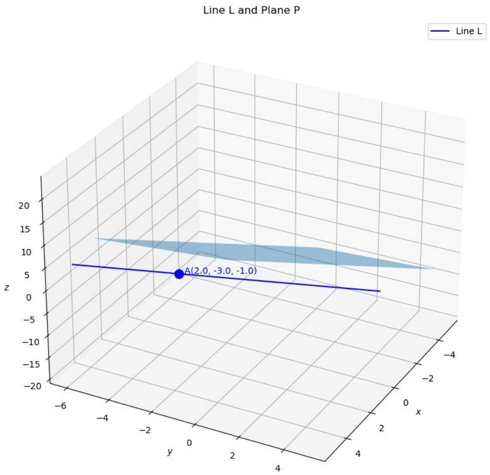

Figure: Plot