4.13.30

EE25BTECH11025 - Ganachari Vishwambhar

Question:
If $\mathbf{P} = (1, 0)$, $\mathbf{Q} = (-1, 0)$ and $\mathbf{R} = (2, 0)$ are three given points, then the locus of point $\mathbf{S}$ satisfying the relation $(SQ)^2 + (SR)^2 = 2(SP)^2$, is:

1) a straight line parallel to X axis
2) a circle passing through the origin
3) a circle with the center at the origin
4) a straight line parallel to Y axis

Solution:
Given:

$$\mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \mathbf{Q} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}; \mathbf{R} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{1}$$

$$\mathbf{S} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{2}$$

Solving:

$$\|\mathbf{Q} - \mathbf{S}\|^2 + \|\mathbf{R} - \mathbf{S}\|^2 = 2\|\mathbf{P} - \mathbf{S}\|^2 \tag{3}$$

$$\left(\mathbf{Q} - \mathbf{S}\right)^\top \left(\mathbf{Q} - \mathbf{S}\right) + \left(\mathbf{R} - \mathbf{S}\right)^\top \left(\mathbf{R} - \mathbf{S}\right) = 2\left(\mathbf{P} - \mathbf{S}\right)^\top \left(\mathbf{P} - \mathbf{S}\right) \tag{4}$$

$$\|\mathbf{Q}\|^2 + \|\mathbf{R}\|^2 - 2\|\mathbf{P}\|^2 = \mathbf{S}^\top + \mathbf{Q}^\top\mathbf{S} + \mathbf{S}^\top\mathbf{R} + \mathbf{R}^\top\mathbf{S} - 2\mathbf{S}^\top\mathbf{P} - 2\mathbf{P}^\top\mathbf{S} \tag{5}$$

$$\|\mathbf{Q}\|^2 + \|\mathbf{R}\|^2 - 2\|\mathbf{P}\|^2 = \mathbf{S}^\top\left(\mathbf{Q} + \mathbf{R} - 2\mathbf{P}\right) + \mathbf{S}\left(\mathbf{Q} + \mathbf{R} - 2\mathbf{P}\right)^\top \tag{6}$$

$$\|\mathbf{Q}\|^2 + \|\mathbf{R}\|^2 - 2\|\mathbf{P}\|^2 = 2\left(\mathbf{Q} + \mathbf{R} - 2\mathbf{P}\right)^\top \mathbf{S} \tag{7}$$

Equation (7) is of the form:

$$\mathbf{n}^\top\mathbf{x} = c \tag{8}$$

$$\left(\mathbf{Q} + \mathbf{R} - 2\mathbf{P}\right)^\top \mathbf{S} = \frac{\|\mathbf{Q}\|^2 + \|\mathbf{R}\|^2 - 2\|\mathbf{P}\|^2}{2} \tag{9}$$

Substituting values:

$$\left(\begin{pmatrix} -1 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \end{pmatrix} - 2\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right)^\top \mathbf{S} = \frac{\left((-1)^2 + 0^2\right) + \left(2^2 + 0^2\right) - 2\left(1^2 + 0^2\right)}{2} \tag{10}$$

$$\begin{pmatrix} -1 \\ 0 \end{pmatrix}^\top \mathbf{S} = \frac{3}{2} \tag{11}$$

Hence the locus of $\mathbf{s}$ is a line parallel to Y-axis.

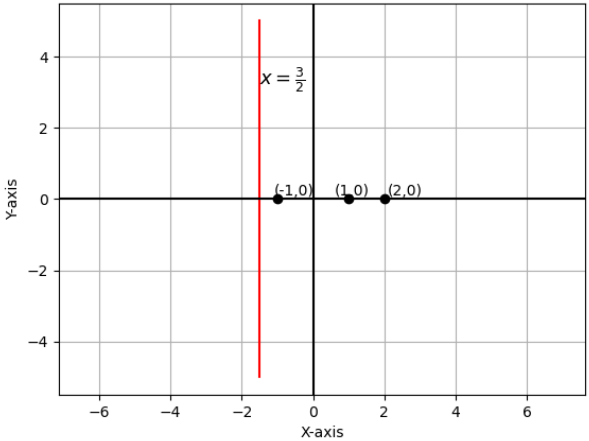

Fig. 1: Plot of the given points and locus of **S**