4.13.72

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Question

A non-zero vector **a** is parallel to the line of intersection of the plane determined by the vectors $\hat{i}$, $\hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}$, $\hat{i}+\hat{k}$. The angle between **a** and the vector $\hat{i}-2\hat{j}+2\hat{k}$ is?
(1996)

Theoretical Solution

Let $\mathbf{A} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

Let $\mathbf{n}_1$ be the perpendicular vector to Plane-1.

Let $\mathbf{n}_1 = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$$\therefore \mathbf{A}^\top \mathbf{n}_1 = 0 \,\&\, \mathbf{B}^\top \mathbf{n}_1 = 0 \tag{1}$$

From Equation 1,

$$x_1 = 0 \,\&\, x_2 + x_1 = 0 \quad \Rightarrow x_1 = x_2 = 0 \tag{2}$$

$$\mathbf{n}_1 = \begin{pmatrix} 0 \\ 0 \\ x_3 \end{pmatrix} \quad OR \quad \mathbf{n}_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{3}$$

Let $\mathbf{n}_2$ be the perpendicular vector to Plane-2.

Let $\mathbf{n}_2 = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}$

$$\therefore \ \mathbf{C}^\top \mathbf{n}_2 = 0 \,\&\, \mathbf{D}^\top \mathbf{n}_2 = 0 \tag{4}$$

From Equation 4,

$$z_1 - z_2 = 0 \,\&\, z_1 + z_3 = 0 \tag{5}$$

$$\mathbf{n}_2 = \begin{pmatrix} -z_3 \\ -z_3 \\ z_3 \end{pmatrix} \quad OR \quad \mathbf{n}_2 = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} \tag{6}$$

Let $\mathbf{n}_3$ be the parallel vector to the line of intersection of planes.

Let $\mathbf{n}_3 = \begin{pmatrix} k_1 \\ k_2 \\ k_3 \end{pmatrix}$

Theoretical Solution

Since, $\mathbf{n}_3$ is perpendicular to both $\mathbf{n}_1$ and $\mathbf{n}_2$.

$$\therefore\ \mathbf{n}_3^\top \mathbf{n}_1 = 0 \,\&\, \mathbf{n}_3^\top \mathbf{n}_2 = 0 \tag{7}$$

$$\boxed{\text{The angle between } \mathbf{a} \textit{ and } 1\hat{i} - 2\hat{j} + 2\hat{k} \textit{ is } 45^\circ.} \tag{8}$$

$$k_3 = 0 \,\&\, k_3 - k_1 - k_2 = 0 \quad \Rightarrow k_2 = -k_1 \tag{9}$$

$$\mathbf{n}_3 = \begin{pmatrix} k_1 \\ k_2 \\ 0 \end{pmatrix} \quad OR \quad \mathbf{n}_3 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \tag{10}$$

From Equation 10,

$$\therefore\ \mathbf{a} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \tag{11}$$

Theoretical Solution

Let $\mathbf{u} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ [Already given in the Question]

We know,

$$\mathbf{a}^\top \mathbf{u} = \|\mathbf{a}\| \, \|\mathbf{u}\| \cos(\theta) \tag{12}$$

$$\|\mathbf{a}\| = \sqrt{\mathbf{a}^\top \mathbf{a}} = \sqrt{2}, \; \|\mathbf{u}\| = \sqrt{\mathbf{u}^\top \mathbf{u}} = 3 \tag{13}$$

From Equation 12 and 13,

$$\cos(\theta) = \frac{3}{3\sqrt{2}} \quad \Rightarrow \theta = 45^\circ \tag{14}$$

$$\boxed{\text{The angle between } \mathbf{a} \; and \; 1\hat{i} - 2\hat{j} + 2\hat{k} \; is \; 45^\circ.} \tag{15}$$

Image