4.13.38

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Question

Let PS be the median of the triangle with vertices $\mathbf{P}(2,2)$, $\mathbf{Q}(6,-1)$ and $\mathbf{R}(7,3)$. The equation of the line passing through $(1,-1)$ and parallel to PS is 2

1. $4x + 7y + 3 = 0$
2. $2x - 9y - 11 = 0$
3. $4x - 7y - 11 = 0$
4. $2x + 9y + 7 = 0$

Theoretical Solution

Given the points,

$$\mathbf{P} = \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad \mathbf{Q} = \begin{pmatrix} 6 \\ -1 \end{pmatrix} \quad \mathbf{R} = \begin{pmatrix} 7 \\ 3 \end{pmatrix} \tag{1}$$

S is the midpoint of the line segment joining points **Q** and **R**.
If **S** divides QR in the ratio $k : 1$,

Formulae

Section formula for a vector $\mathbf{S}$ which divides the line formed by vectors $\mathbf{Q}$ and $\mathbf{R}$ in the ratio $k : 1$ is given by

$$\mathbf{S} = \frac{k\mathbf{R} + \mathbf{Q}}{k+1} \tag{2}$$

Theoretical Solution

where,

$$k = 1 \tag{3}$$

$$\mathbf{S} = \frac{\mathbf{R} + \mathbf{Q}}{2} \tag{4}$$

$$\implies \mathbf{S} = \begin{pmatrix} 13/2 \\ 1 \end{pmatrix} \tag{5}$$

As **P** and **S** are collinear,

$$\mathbf{n}^\top \mathbf{P} = c \tag{6}$$

$$\mathbf{n}^\top \mathbf{S} = c \tag{7}$$

Theoretical Solution

which can be expressed as

$$\begin{pmatrix} \mathbf{P} & \mathbf{S} \end{pmatrix}^\top \mathbf{n} = c \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{8}$$

$$\equiv \begin{pmatrix} \mathbf{P} & \mathbf{S} \end{pmatrix}^\top \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{9}$$

$$\implies \begin{pmatrix} 2 & 2 \\ 13/2 & 1 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{10}$$

$$\implies \left(\begin{array}{cc|c} 2 & 2 & 1 \\ 13/2 & 1 & 1 \end{array}\right) \tag{11}$$

$$R_2 \to 2R_2 \implies \left(\begin{array}{cc|c} 2 & 2 & 1 \\ 13 & 2 & 2 \end{array}\right) \tag{12}$$

Theoretical Solution

$$R_2 \to 2R_2 - 13R_1 \implies \left(\begin{array}{cc|c} 2 & 2 & 1 \\ 0 & -22 & -9 \end{array}\right) \tag{13}$$

$$R_1 \to 1/2R_1 \implies \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -22 & -9 \end{array}\right) \tag{14}$$

$$R_2 \to -1/22R_1 \left(\begin{array}{cc|c} 1 & 1 & 1/2 \\ 0 & 1 & 9/22 \end{array}\right) \tag{15}$$

$$R_1 \to R_1 - R_2 \implies \left(\begin{array}{cc|c} 1 & 0 & 1/11 \\ 0 & 1 & 9/22 \end{array}\right) \tag{16}$$

$$\implies n = \begin{pmatrix} 1/11 \\ 9/22 \end{pmatrix} \tag{17}$$

Theoretical Solution

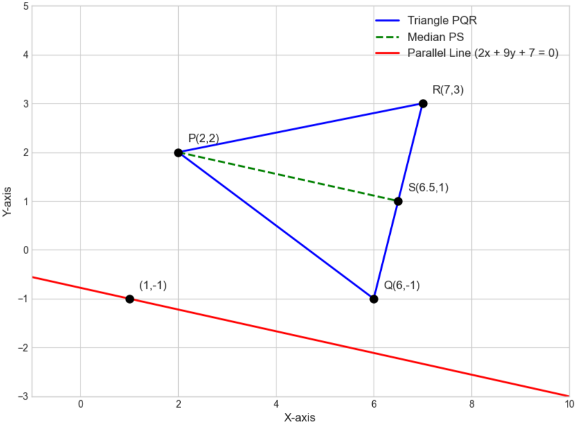
$\therefore$ The equation of the line passing through $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and parallel to PS is given by

$$\mathbf{n}^\top \left(\mathbf{x} - \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right) = 0 \tag{18}$$

$$\begin{pmatrix} 1/11 & 9/22 \end{pmatrix} \begin{pmatrix} x - 1 \\ y + 1 \end{pmatrix} = 0 \tag{19}$$

$$\implies 2x + 9y + 7 = 0 \tag{20}$$

Plot


Triangle PQR
Median PS
Parallel Line (2x + 9y + 7 = 0)
R(7,3)
P(2,2)
S(6.5,1)
(1,-1)
Q(6,-1)
X-axis
Y-axis