

1.6.16

AI25BTECH11001 - ABHISEK MOHAPATRA

August 21, 2025

Question:

Find the values of k if the points $A(k + 1, 2k)$, $B(3k, 2k + 3)$ and $C(5k - 1, 5k)$ are collinear.

Solution: From the given information,

$$A = \begin{pmatrix} k + 1 \\ 2k \end{pmatrix}, B = \begin{pmatrix} 3k \\ 2k + 3 \end{pmatrix}, C = \begin{pmatrix} 5k - 1 \\ 5k \end{pmatrix} \quad (0.1)$$

To check if the points are collinear, we can use

$$\text{rank} (B - A \quad C - A) = 1 \quad (0.2)$$

So,

$$(B - A \quad C - A) = \begin{pmatrix} 2k - 1 & 4k - 2 \\ 3 & 3k \end{pmatrix} \quad (0.3)$$

$$\xleftrightarrow{C_2 = C_2 - 2C_1} \begin{pmatrix} 2k - 1 & 0 \\ 3 & 3k - 6 \end{pmatrix} \quad (0.4)$$

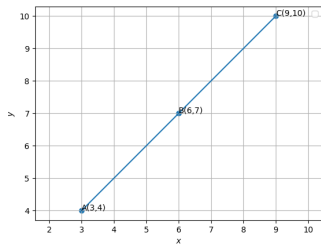
The rank of the matrix will be 1 when

$$3k - 6 = 0 \quad (0.5)$$

$$\Rightarrow k = 2$$

(0.6)

Graph:



Therefore, $k = 2$.