

## 4.3.23

EE25BTECH11034 - Kishora Karthik

### Question:

The line segment joining the points  $\mathbf{A}(3, 2)$  and  $\mathbf{B}(5, 1)$  is divided at the point  $\mathbf{P}$  in the ratio  $1 : 2$  which lies on  $3x - 18y + k = 0$ . Find the value of  $k$ .

### Solution:

Given the points,

$$\mathbf{A} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad (1)$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \quad (2)$$

and the line  $L_1$ ,

$$L_1 : \begin{pmatrix} 3 & -18 \end{pmatrix} \mathbf{x} = -k \quad (3)$$

$$\Rightarrow \mathbf{n}^\top \mathbf{x} = 0 \quad (4)$$

Where,

$$\mathbf{n} = \begin{pmatrix} 3 \\ -18 \end{pmatrix} \quad (5)$$

Let the vector  $\mathbf{P}$  be a point on the line  $3x - 18y + k = 0$  which divides the line segment joining the points  $\mathbf{A}$  and  $\mathbf{B}$ .

Section formula for a vector  $\mathbf{P}$  which divides the line formed by vectors  $\mathbf{A}$  and  $\mathbf{B}$  in the ratio  $k : 1$  is given by

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \quad (6)$$

$$\mathbf{P} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \frac{1}{k+1} \\ \frac{k}{k+1} \end{pmatrix} \quad (7)$$

Here,  $k = 1/2$ .

$$\Rightarrow \mathbf{P} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} \quad (8)$$

Since  $\mathbf{P}$  lies on line  $L_1$ ,

$$\mathbf{n}^T \mathbf{P} = 0 \quad (9)$$

$$\Rightarrow \begin{pmatrix} 3 & -18 \end{pmatrix} \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} = -k \quad (10)$$

$$\Rightarrow \begin{pmatrix} 3 & -18 \end{pmatrix} \begin{pmatrix} 3 & 5 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} = -k \quad (11)$$

$$\Rightarrow \begin{pmatrix} 3 \cdot 3 + (-18) \cdot 2 & 3 \cdot 5 + (-18) \cdot 1 \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} = -k \quad (12)$$

$$\Rightarrow \begin{pmatrix} -27 & -3 \end{pmatrix} \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} = -k \quad (13)$$

$$\Rightarrow \left( (-27) \cdot \frac{2}{3} + (-3) \cdot \frac{1}{3} \right) = -k \quad (14)$$

$$\Rightarrow k = 19 \quad (15)$$

$\therefore$  The value of  $k$  is 19 and the equation of the line is  $3x - 18y + 19 = 0$ .

