MatGeo Assignment 2.6.13

AI25BTECH11007

Question:

Using vectors, find the area of $\triangle ABC$ with vertices A(1, 2, 3),B(2, -1, 4) and C(4, 5, -1).

Solution:

Compute vectors $\mathbf{B}-\mathbf{A}$ and $\mathbf{C}-\mathbf{A}$:

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix} 2-1 \\ -1-2 \\ 4-3 \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \tag{0.1}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix} 4-1 \\ 5-2 \\ -1-3 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix} \tag{0.2}$$

Compute the cross product $\mathbf{B}-\mathbf{A}$ and $\mathbf{C}-\mathbf{A}$:

$$\begin{aligned}
[\mathbf{B}-\mathbf{A} \times \mathbf{C}-\mathbf{A}] &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix} \\
&= \hat{i}((-3)(-4)-(1)(3)) - \hat{j}((1)(-4)-(1)(3)) + \hat{k}((1)(3)-(-3)(3)) \\
&= \hat{i}(12-3) - \hat{j}(-4-3) + \hat{k}(3+9) \\
&= \hat{i}(9) - \hat{j}(-7) + \hat{k}(12) \\
&= \begin{pmatrix} 9 \\ 7 \\ 12 \end{pmatrix}
\end{aligned} \tag{0.3}$$

Compute the magnitude of $[\mathbf{B}-\mathbf{A} \times \mathbf{C}-\mathbf{A}]$:

$$\begin{aligned}
\|[\mathbf{B}-\mathbf{A} \times \mathbf{C}-\mathbf{A}]\| &= \sqrt{9^2+7^2+12^2} \\
&= \sqrt{81+49+144} \\
&= \sqrt{274}
\end{aligned} \tag{0.4}$$

Area of $\triangle ABC$ is half the magnitude of the cross product:

$$\begin{aligned}
\text{Area}(\triangle ABC) &= \frac{1}{2}\|[\mathbf{B}-\mathbf{A} \times \mathbf{C}-\mathbf{A}]\| \\
&= \frac{1}{2}\sqrt{274} \\
&= \frac{\sqrt{274}}{2}
\end{aligned} \tag{0.5}$$

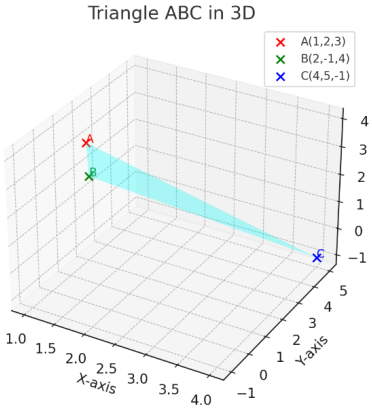

Fig. 0.1: Image Visual