2.4.28

EE25BTECH11032 - Kartik Lahoti

Question:

Find the coordinates of the point $\mathbf{Q}$ on the x-axis which lies on the perpendicular bisector of the line segment joining the points $\mathbf{A}(-5,-2)$ and $\mathbf{B}(4,-2)$. Name the type of triangle formed by points $\mathbf{Q},\mathbf{A}$ and $\mathbf{B}$.

Solution:

Symbol	Value	Description
$\mathbf{A}$	$\begin{pmatrix}-5\\-2\end{pmatrix}$	First Point
$\mathbf{B}$	$\begin{pmatrix}4\\-2\end{pmatrix}$	Second Point
$\mathbf{Q}$	?	Desired Point

Table:2.4.28

If $\mathbf{Q}$ lies on the x-axis and on the perpendicular bisector of the points $\mathbf{A}$ and $\mathbf{B}$, i.e $\mathbf{Q}$ is equidistant from points $\mathbf{A}$ and $\mathbf{B}$

$$\|\mathbf{Q}-\mathbf{A}\| = \|\mathbf{Q}-\mathbf{B}\| \tag{0.1}$$

$$\implies \|\mathbf{Q}-\mathbf{A}\|^2 = \|\mathbf{Q}-\mathbf{B}\|^2 \tag{0.2}$$

$$\implies \|\mathbf{Q}\|^2 - 2\mathbf{Q}^\top\mathbf{A} + \|\mathbf{A}\|^2 = \|\mathbf{Q}\|^2 - 2\mathbf{Q}^\top\mathbf{B} + \|\mathbf{B}\|^2, \tag{0.3}$$

which can be simplified to obtain,

$$(\mathbf{A}-\mathbf{B})^\top \mathbf{Q} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2}. \tag{0.4}$$

$$\because \mathbf{Q} = x\mathbf{e}_1, \tag{0.5}$$

$$x = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{2(\mathbf{A}-\mathbf{B})^\top \mathbf{e}_1.} \tag{0.6}$$

$$\|\mathbf{A}\|^2 = 29, \|\mathbf{B}\|^2 = 20 \tag{0.7}$$

$$(\mathbf{A}-\mathbf{B})^\top = \begin{pmatrix}-9 & 0\end{pmatrix}, \mathbf{e}_1 = \begin{pmatrix}1\\0\end{pmatrix} \tag{0.8}$$

Substituting from (0.7) and (0.8), $x = -0.5$. Thus,

$$\mathbf{Q} = \begin{pmatrix} -0.5 \\ 0 \end{pmatrix}. \tag{0.9}$$

Since $\mathbf{Q}$ lies on perpendicular bisector of $\mathbf{AB}$, it is equidistant from both $\mathbf{A}$ and $\mathbf{B}$

$$\|\mathbf{Q} - \mathbf{A}\| = \|\mathbf{Q} - \mathbf{B}\| \tag{0.10}$$

Hence $\triangle ABQ$ is an isosceles triangle.

See Fig. 0.1.

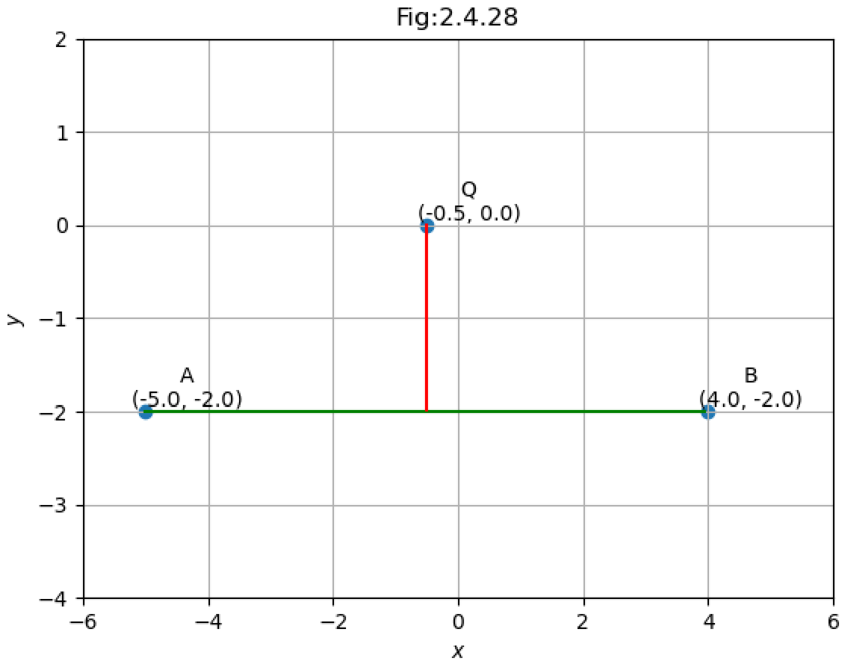

Fig. 0.1