

5.2.35

EE25BTECH11032 - Kartik Lahoti

Question:

Solve the following system of linear equations.

$$3x + 4y = 10$$

$$2x - 2y = 2$$

Solution:

The equation of line L_1 is,

$$\begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} = 10 \quad (1)$$

The equation of line L_2 is,

$$\begin{pmatrix} 2 & -2 \end{pmatrix} \mathbf{x} = 2 \quad (2)$$

On putting the equations in a matrix, we will get

$$\Rightarrow \begin{pmatrix} 3 & 4 \\ 2 & -2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 10 \\ 2 \end{pmatrix} \quad (3)$$

So the augmented matrix is,

$$\left(\begin{array}{cc|c} 3 & 4 & 10 \\ 2 & -2 & 2 \end{array} \right) \quad (4)$$

$$\left(\begin{array}{cc|c} 3 & 4 & 10 \\ 2 & -2 & 2 \end{array} \right) \xleftrightarrow{R_2 \rightarrow R_2 - \frac{2}{3}R_1} \left(\begin{array}{cc|c} 3 & 4 & 10 \\ 0 & -\frac{14}{3} & -\frac{14}{3} \end{array} \right) \quad (5)$$

$$\left(\begin{array}{cc|c} 3 & 4 & 10 \\ 0 & -\frac{14}{3} & -\frac{14}{3} \end{array} \right) \xleftrightarrow{R_2 \rightarrow -\frac{3}{14}R_2} \left(\begin{array}{cc|c} 3 & 4 & 10 \\ 0 & 1 & 1 \end{array} \right) \quad (6)$$

$$\left(\begin{array}{cc|c} 3 & 4 & 10 \\ 0 & 1 & 1 \end{array} \right) \xleftrightarrow{R_1 \rightarrow R_1 - 4R_2} \left(\begin{array}{cc|c} 3 & 0 & 6 \\ 0 & 1 & 1 \end{array} \right) \quad (7)$$

$$\left(\begin{array}{cc|c} 3 & 0 & 6 \\ 0 & 1 & 1 \end{array} \right) \xleftrightarrow{R_1 \rightarrow \frac{1}{3}R_1} \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array} \right) \quad (8)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (9)$$

Therefore the two lines will intersect at $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

