9.4.13

EE25BTECH11001 - Aarush Dilawri

Question:
Find the roots of the following quadratic equation graphically

$$x^2 - 2x = (-2)(3 - x) \tag{1}$$

Solution:

$$y = x^2 - 2x - (-2)(3 - x) \tag{2}$$

$$y = x^2 - 4x + 6 = 0 \tag{3}$$

This quadratic can be represented as a conic in matrix form:

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{4}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}, f = 6 \tag{5}$$

To find the roots, we find the points of intersection of the conic with the x-axis.

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{6}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{7}$$

The value of k can be found out by solving the line and conic equation

$$(\mathbf{h} + k\mathbf{m})^\top \mathbf{V}(\mathbf{h} + k\mathbf{m}) + 2\mathbf{u}^\top (\mathbf{h} + k\mathbf{m}) + f = 0 \tag{8}$$

$$\implies k^2\mathbf{m}^\top \mathbf{V}\mathbf{m} + 2k\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) + \mathbf{h}^\top \mathbf{V}\mathbf{h} + 2\mathbf{u}^\top \mathbf{h} + f = 0 \tag{9}$$

$$\text{or, } k^2\mathbf{m}^\top \mathbf{V}\mathbf{m} + 2k\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) + g(\mathbf{h}) = 0 \tag{10}$$

Solving the above quadratic gives the equation

$$k = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}}\left(-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})\ (\mathbf{m}^\top \mathbf{V}\mathbf{m})}\right) \tag{11}$$

Substituting the values in the above equation gives

$$\therefore k = 2 \pm i\sqrt{2} \tag{12}$$

$$k_1 = 2 + i\sqrt{2} \tag{13}$$

$$k_2 = 2 - i\sqrt{2} \tag{14}$$

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} = \begin{pmatrix} 2 + i\sqrt{2} \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 2 - i\sqrt{2} \\ 0 \end{pmatrix} \tag{15}$$

$\therefore$ The given quadratic equation has imaginary roots.

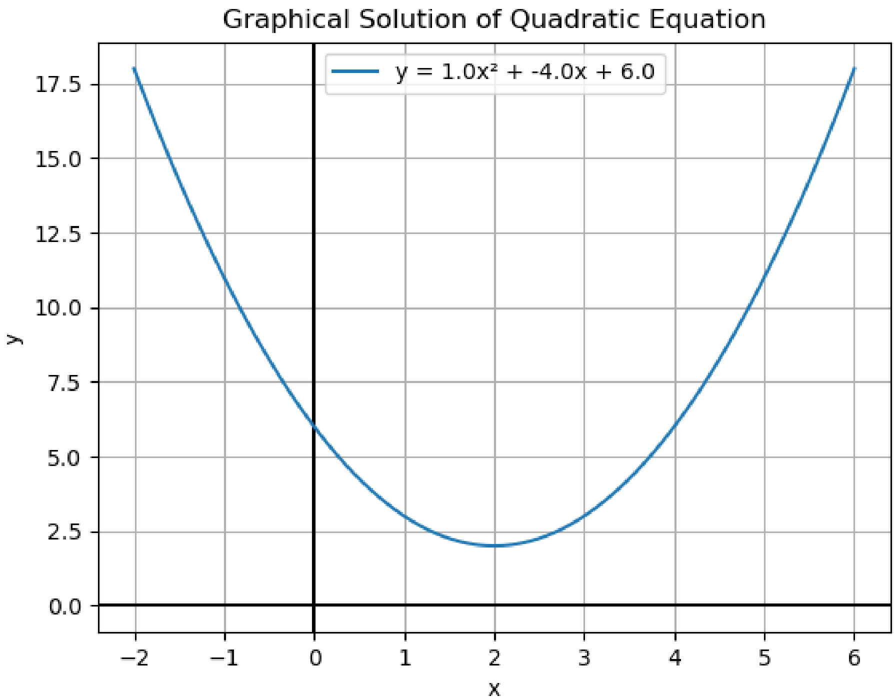

Fig. 0: Graph