

# Matgeo-q.4.13.1

AI25BTECH11036-SNEHAMRUDULA

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# Question

Consider the lines

$$L_1 : x + 3y - 5 = 0,$$

$$L_2 : 3x - ky - 1 = 0,$$

$$L_3 : 5x + 2y - 12 = 0.$$

Match the statements in Column I with Column II (choices refer to values of  $k$  or relations).

## Column I

- ☐  $L_1, L_2, L_3$  are concurrent, if
- ☐ One of  $L_1, L_2, L_3$  is parallel to at least one of the other two, if
- ☐  $L_1, L_2, L_3$  form a triangle, if
- ☐  $L_1, L_2, L_3$  do not form a triangle, if

## Column II

- ☐  $k = 9$
- ☐  $k = \frac{-6}{5}$
- ☐  $k = \frac{5}{6}$
- ☐  $k = 5$

# Solution — (A) Concurrency

**(A) Concurrency.** Intersection of  $L_1$  and  $L_3$ :

$$\begin{pmatrix} 1 & 3 \\ 5 & 2 \end{pmatrix} \mathbf{x}_0 = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \quad (1)$$

From row reduction  $\mathbf{x}_0 = (2, 1)^\top$  lies on  $L_2$  if

$$\mathbf{n}_2^\top \mathbf{x}_0 = p_2 \iff 1 \iff k = 5.$$

Thus concurrency  $\iff k = 5$ .

## Solution — (B) Parallelism

Two lines are parallel iff their normals are proportional.

Normals of  $L_2$  and  $L_3$ :

$$\mathbf{n}_2 = \begin{pmatrix} 3 \\ -k \end{pmatrix}, \quad \mathbf{n}_3 = \begin{pmatrix} 5 \\ 2 \end{pmatrix}.$$

If  $\mathbf{n}_2 = \lambda \mathbf{n}_3$  then

$$3 = 5\lambda, \quad -k = 2\lambda \Rightarrow \lambda = \frac{3}{5}, \quad k = -2\lambda = -\frac{6}{5}.$$

So a parallel pair (between  $L_2$  and  $L_3$ ) occurs when  $k = -\frac{6}{5}$ . (Analogous checks show no other accidental proportionality with  $L_1$  for these  $k$ .)

## Solution — (C) Triangle & (D) Not a triangle

**(C) Form a triangle:** The three lines form a triangle iff no two are parallel and they are not concurrent. Therefore triangle occurs when

$$k \neq 5 \quad \text{and} \quad k \neq -\frac{6}{5}.$$

(So any explicit example values like  $k = 9$  or  $k = \frac{5}{6}$  satisfy the triangle condition.)

**(D) Do not form a triangle:** This happens when the lines are concurrent or when a parallel pair exists. Hence

$$\text{no triangle} \iff k = 5 \quad \text{or} \quad k = -\frac{6}{5}.$$

# Final concise matching

- (A) Concurrency  $\iff k = 5$ .
- (B) One line parallel to another  $\iff k = -\frac{6}{5}$ .
- (C) They form a triangle  $\iff k \neq 5$  and  $k \neq -\frac{6}{5}$ . (Examples:  $k = 9$ ,  $k = \frac{5}{6}$ .)
- (D) They do *not* form a triangle  $\iff k = 5$  or  $k = -\frac{6}{5}$ .

# Graphical Representation

