8.3.9

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If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentric

Matrix Representation of an Ellipse

The general quadratic form of a centered ellipse is:

$$\mathbf{x}^T A \mathbf{x} = 1 \quad \text{where } A = \begin{bmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{bmatrix} \tag{1}$$

Here, a and b are the semi-major and semi-minor axes respectively.

SOlution

Geometric Condition

The latus rectum L of an ellipse is given by:

$$L = \frac{2b^2}{a} \tag{2}$$

Given:

$$L = \frac{1}{2} \cdot 2b = b \quad \Rightarrow \quad \frac{2b^2}{a} = b \quad \Rightarrow \quad 2b = a \tag{3}$$

Thus, we have:

$$a = 2b \tag{4}$$

Solution

Eccentricity Calculation

Eccentricity e of an ellipse is:

$$e = \sqrt{1 - \frac{b^2}{a^2}} \tag{5}$$

Substituting $a = 2b$:

$$e = \sqrt{1 - \frac{b^2}{(2b)^2}} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} \tag{6}$$
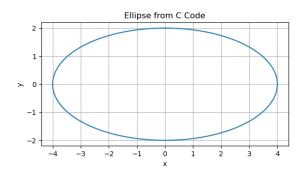
Solution

Final Answer

$$\boxed{e = \frac{\sqrt{3}}{2}} \tag{7}$$

Plot

Figure: