9.4.40

INDHIRESH S- EE25BTECH11027

Question.A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would have taken 1 hour less for the same journey. Find the speed of the train.
Solution:
Let us solve the given equation theoretically and then verify the solution computationally.
Let the uniform speed of the train be s km/hr.
Let the time taken for the journey be t hours.
From 1st journey:

$$360 = s \times t \tag{1}$$

$$t = \frac{360}{s} \tag{2}$$

For the second scenario:

$$360 = (s+5)(t-1) \tag{3}$$

Now substitute Eq.2 in Eq.3

$$360 = (s+5)(\frac{360}{s} - 1) \tag{4}$$

$$s^2 + 5s - 1800 = 0 \tag{5}$$

Let

$$u = s^2 + 5s - 1800 \tag{6}$$

This can be expressed as:

$$\mathbf{x^T V x} + 2\mathbf{u^T x} + f = 0 \tag{7}$$

Where,

$$\mathbf{x} = \begin{pmatrix} s \\ u \end{pmatrix} \ , \mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \ , \mathbf{u} = \begin{pmatrix} 2.5 \\ -0.5 \end{pmatrix} \ \textit{and} \ \ f = -1800 \tag{8}$$

Now finding the point of intersection of parabola with s-axis:

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{9}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \ \textit{and} \ \ \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{10}$$

$$\mathbf{x} = k\begin{pmatrix}1\\0\end{pmatrix} \tag{11}$$

Now substitute Eq.11 in Eq.7

$$k^2\begin{pmatrix}1\\0\end{pmatrix}^T\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} + 2\begin{pmatrix}2.5\\-0.5\end{pmatrix}^T k\begin{pmatrix}1\\0\end{pmatrix} - 1800 = 0 \tag{12}$$

$$k^2 + 5k - 1800 = 0 \tag{13}$$

$$k = \frac{-5 \pm \sqrt{25 - 4(-1800)}}{4} \tag{14}$$

$$k = 40 \;\; and \;\; k = -45 \tag{15}$$

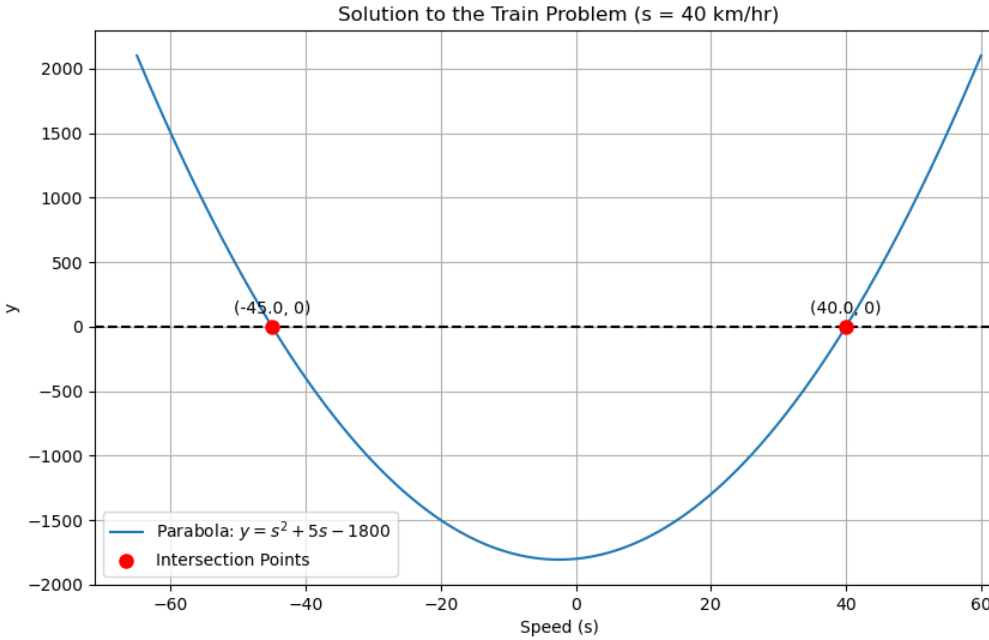
Speed cannot be negative. So,

$$k = 40 \tag{16}$$

Substitute in Eq.11

$$s = 40\ km/hr \tag{17}$$

From the figure it is clearly verified that the theoretical solution matches with the computational solution.


Solution to the Train Problem (s = 40 km/hr)
2000
1500
1000
500
0
−500
−1000
−1500
−2000
y
(-45.0, 0)
(40.0, 0)
Parabola: $y = s^2 + 5s - 1800$
Intersection Points
−60
−40
−20
0
20
40
60
Speed (s)