# 4.3.53

EE25BTECH11062 - Vivek K Kumar

**Question**:
The Fahrenheit temperature $F$ and absolute temperature $K$ satisfy a linear equation. Given that $K = 273$ when $F = 32$ and that $K = 373$ when $F = 212$. Express K in terms of $F$ and find the value of $F$, when $K = 0$.

**Solution:**

| Name | Point |
|---|---|
| **x** | $\begin{pmatrix} K \\ F \end{pmatrix}$ |
| **A** | $\begin{pmatrix} 273 \\ 32 \end{pmatrix}$ |
| **B** | $\begin{pmatrix} 373 \\ 212 \end{pmatrix}$ |
| **C** | $\begin{pmatrix} 0 \\ F \end{pmatrix}$ |

TABLE 0: Variables used

Since there is a linear relation, the equation of the straight line can be expressed as

$$\mathbf{n}^\top \mathbf{x} = c \tag{0.1}$$

$$\mathbf{A}^\top \mathbf{n} = c \tag{0.2}$$

$$\mathbf{B}^\top \mathbf{n} = c \tag{0.3}$$

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix}^\top \mathbf{n} = c \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.4}$$

$$\begin{pmatrix} 273 & 32 \\ 373 & 212 \end{pmatrix} \mathbf{n} = c \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.5}$$

As $\operatorname{rank}\begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix}^\top \neq 1$ from above equation, $c \neq 0$.
Taking $c = 1$,

$$\begin{pmatrix} 273 & 32 \\ 373 & 212 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.6}$$

$$\implies \left(\begin{array}{cc|c} 273 & 32 & 1 \\ 373 & 212 & 1 \end{array}\right) \xleftrightarrow{R_2 \leftarrow R_2 - 373/273 R_1} \left(\begin{array}{cc|c} 273 & 32 & 1 \\ 0 & 45940/273 & -100/273 \end{array}\right) \tag{0.7}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - 8736/45940 R_2} \left(\begin{array}{cc|c} 273 & 0 & 2457/2297 \\ 0 & 45940/273 & -100/273 \end{array}\right) \tag{0.8}$$

$$\mathbf{n} = \frac{1}{2297} \begin{pmatrix} 9 \\ -5 \end{pmatrix} \tag{0.9}$$

Substituting in line equation

$$\mathbf{n}^\top \mathbf{x} = 1 \tag{0.10}$$

$$\begin{pmatrix} 9 & -5 \end{pmatrix} \begin{pmatrix} K \\ F \end{pmatrix} = 2297 \tag{0.11}$$

Solving for point $\mathbf{C}$, $\begin{pmatrix} 0 \\ F \end{pmatrix}$

We have,

$$\begin{pmatrix} 9 & -5 \end{pmatrix} \begin{pmatrix} 0 \\ F \end{pmatrix} = 2297 \tag{0.12}$$

$$F = -\frac{2297}{5} \tag{0.13}$$

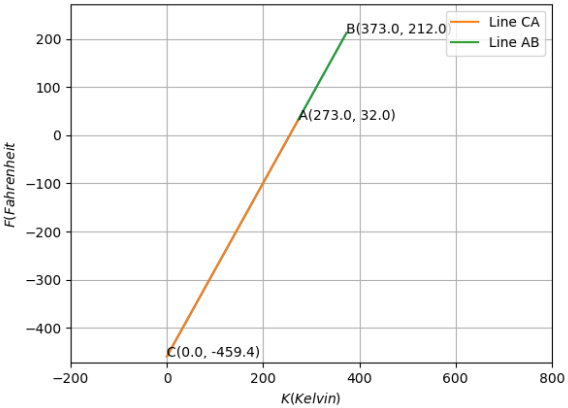


Fig. 0.1: Given points on a line