4.11.20

EE25BTECH11043 - Nishid Khandagre

Question: Find the coordinates of the point where the line through the points $\mathbf{A}\begin{pmatrix}3\\4\\1\end{pmatrix}$ and $\mathbf{B}\begin{pmatrix}5\\1\\6\end{pmatrix}$ crosses the XZ plane. Also find the angle which this line makes with the XZ plane.

Solution: Direction vector

$$\mathbf{d} = \mathbf{B} - \mathbf{A} \tag{0.1}$$

$$= \begin{pmatrix}5\\1\\6\end{pmatrix} - \begin{pmatrix}3\\4\\1\end{pmatrix} = \begin{pmatrix}2\\-3\\5\end{pmatrix} \tag{0.2}$$

The normal vector $\mathbf{n}$ to the XZ-plane is:

$$\mathbf{n} = \begin{pmatrix}0\\1\\0\end{pmatrix} \tag{0.3}$$

General point $\mathbf{P}$ on the line

$$\mathbf{P} = \mathbf{A} + t\mathbf{d} \tag{0.4}$$

For the line to intersect the XZ-plane, the point $\mathbf{P}$ must lie on the plane. Therefore

$$\mathbf{n}^\top\mathbf{P} = 0 \tag{0.5}$$

$$\mathbf{n}^\top(\mathbf{A} + t\mathbf{d}) = 0 \tag{0.6}$$

$$\mathbf{n}^\top\mathbf{A} + t\,\mathbf{n}^\top\mathbf{d} = 0 \tag{0.7}$$

$$t = -\frac{\mathbf{n}^\top\mathbf{A}}{\mathbf{n}^\top\mathbf{d}} \tag{0.8}$$

$$\mathbf{n}^\top\mathbf{A} = \begin{pmatrix}0 & 1 & 0\end{pmatrix}\begin{pmatrix}3\\4\\1\end{pmatrix} = 4 \tag{0.9}$$

$$\mathbf{n}^\top\mathbf{d} = \begin{pmatrix}0 & 1 & 0\end{pmatrix}\begin{pmatrix}2\\-3\\5\end{pmatrix} = -3 \tag{0.10}$$

$$t = -\frac{4}{-3} = \frac{4}{3} \tag{0.11}$$

Intersection point:

$$\mathbf{P} = \mathbf{A} + \frac{4}{3}\mathbf{d} \tag{0.12}$$

$$= \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \frac{4}{3}\begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} \tag{0.13}$$

$$= \begin{pmatrix} \frac{17}{3} \\ 0 \\ \frac{23}{3} \end{pmatrix} \tag{0.14}$$

The angle θ between a line with direction vector $\mathbf{d}$ and a plane with normal vector $\mathbf{n}$ is given by:

$$\sin\theta = \frac{|\mathbf{n}^\top \mathbf{d}|}{||\mathbf{n}||\,||\mathbf{d}||} \tag{0.15}$$

$$||\mathbf{n}|| = \sqrt{\mathbf{n}^\top \mathbf{n}} \tag{0.16}$$

$$||\mathbf{n}|| = \sqrt{0^2 + 1^2 + 0^2} \tag{0.17}$$

$$||\mathbf{n}|| = 1 \tag{0.18}$$

$$||\mathbf{d}|| = \sqrt{\mathbf{d}^\top \mathbf{d}} \tag{0.19}$$

$$||\mathbf{d}|| = \sqrt{2^2 + (-3)^2 + 5^2} \tag{0.20}$$

$$||\mathbf{d}|| = \sqrt{38} \tag{0.21}$$

$$\sin\theta = \frac{|-3|}{1 \cdot \sqrt{38}} = \frac{3}{\sqrt{38}} \tag{0.22}$$

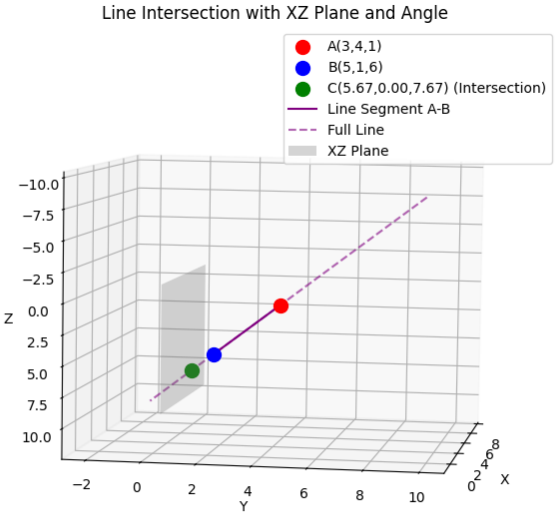
Line Intersection with XZ Plane and Angle
A(3,4,1)
B(5,1,6)
C(5.67,0.00,7.67) (Intersection)
Line Segment A-B
Full Line
XZ Plane
Z
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
−2
0
2
4
6
8
10
Y
0
2
4
6
8
X

Fig. 0.1