Assignment 8: 4.11.32

EE25BTECH11055 - Subhodeep Chakraborty

Question:
Find the equation of the line passing through $(2, -1, 2)$ and $(5, 3, 4)$ and of the plane passing through $(2, 0, 3)$, $(1, 1, 5)$ and $(3, 2, 4)$. Also, find their point of intersection. $(12, 2018)$
Solution:
Given:

$$\mathbf{A} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix} \tag{2}$$

$$\mathbf{P} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \tag{3}$$

$$\mathbf{Q} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} \tag{4}$$

$$\mathbf{R} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} \tag{5}$$

We know, for line $\mathbf{x} = \mathbf{h} + k\mathbf{m}$ and plane $\mathbf{n}^\top \mathbf{y} = 1$,

$$\mathbf{h} = \mathbf{A} \tag{6}$$

$$\mathbf{m} = \mathbf{B} - \mathbf{A} \tag{7}$$

$$\begin{pmatrix} P & Q & R \end{pmatrix}^\top \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{8}$$

$$\text{Also, for intersection} \tag{9}$$

$$\mathbf{x} = \mathbf{y} \implies \mathbf{n}^\top (\mathbf{h} + k\mathbf{m}) = 1 \tag{10}$$

$$k = \frac{1 - \mathbf{n}^\top \mathbf{h}}{\mathbf{n}^\top \mathbf{m}} \tag{11}$$

$$\mathbf{x} = \mathbf{h} + \left(\frac{1 - \mathbf{n}^\top \mathbf{h}}{\mathbf{n}^\top \mathbf{m}} \right) \mathbf{m} \tag{12}$$

Thus

$$\begin{pmatrix} P & Q & R \end{pmatrix}^\top \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{13}$$

$$\begin{pmatrix} 2 & 0 & 3 \\ 1 & 1 & 5 \\ 3 & 2 & 4 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{14}$$

$$\left(\begin{array}{ccc|c} 2 & 0 & 3 & 1 \\ 1 & 1 & 5 & 1 \\ 3 & 2 & 4 & 1 \end{array}\right) \xleftrightarrow{R_2=2R_2-R_1;R_3=2R_3-3R_1} \left(\begin{array}{ccc|c} 2 & 0 & 3 & 1 \\ 0 & 2 & 7 & 1 \\ 0 & 4 & 2 & -1 \end{array}\right) \tag{15}$$

$$\xleftrightarrow{R_3=R_3-2R_2} \left(\begin{array}{ccc|c} 2 & 0 & 3 & 1 \\ 0 & 2 & 7 & 1 \\ 0 & 0 & -12 & -3 \end{array}\right) \xleftrightarrow{R_1=4R_1+R_3;R_2=12R_2+7R_3} \left(\begin{array}{ccc|c} 8 & 0 & 0 & 1 \\ 0 & 24 & 0 & -9 \\ 0 & 0 & -12 & -3 \end{array}\right) \tag{16}$$

$$\xleftrightarrow{R_1=R_1/8;R_2=R_2/24;R_3=-R_3/12} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1/8 \\ 0 & 1 & 0 & -3/8 \\ 0 & 0 & 1 & 1/4 \end{array}\right) \tag{17}$$

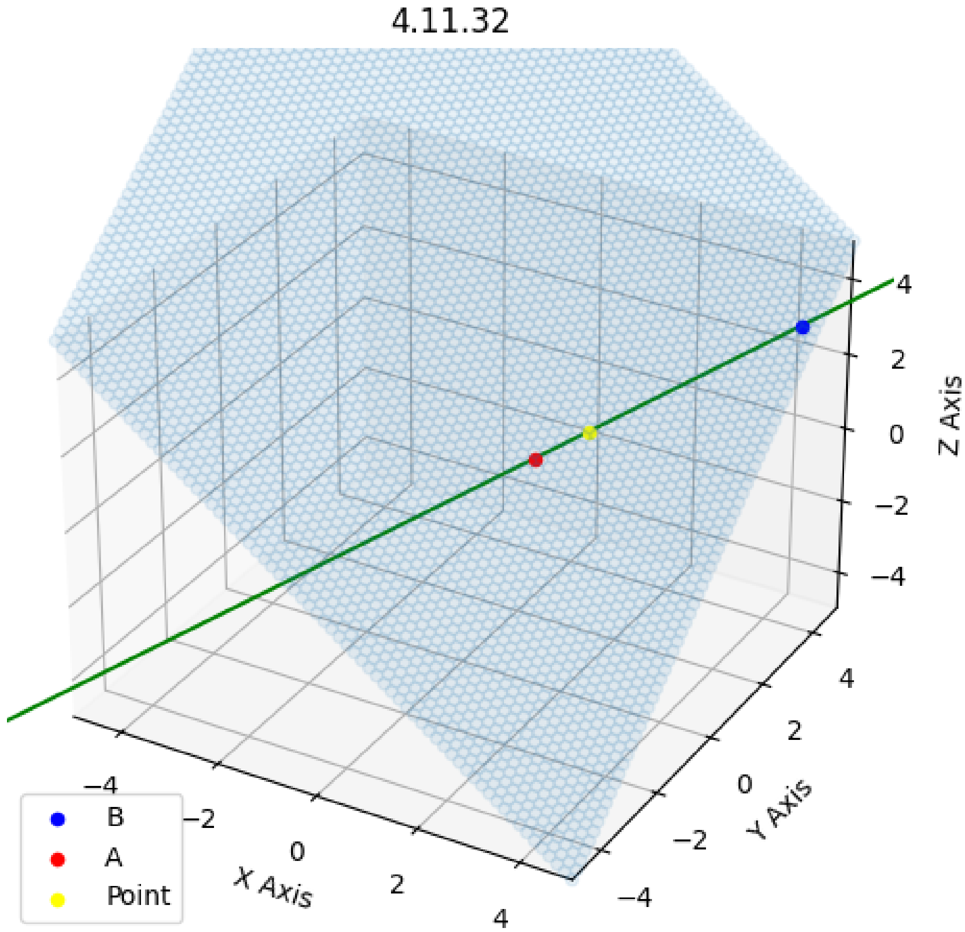
$$\mathbf{n} = \frac{1}{8}\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \tag{18}$$

So we have:

$$\mathbf{n}^\top \mathbf{h} = \frac{1}{8}\begin{pmatrix} 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \frac{9}{8} \tag{19}$$

$$\mathbf{n}^\top \mathbf{m} = \frac{1}{8}\begin{pmatrix} 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} = -\frac{5}{8} \tag{20}$$

$$\mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + \left(\frac{1-9/8}{-5/8}\right)\begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 13/5 \\ -1/5 \\ 12/5 \end{pmatrix} \tag{21}$$


4.11.32
B
A
Point
X Axis
Y Axis
Z Axis
−4
−2
0
2
4