1.6.6

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In each of the following, find the value of k for which the points are collinear:

1) $(7, -2)\,,\ (5, 1)\,,\ (3, k)$
2) $(8, 1)\,,\ (k, -4)\,,\ (2, -5)$

Solution: Three points A, B, C are collinear iff the vectors $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$ are linearly dependent, i.e., the collinearity matrix $M = \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{C} - \mathbf{A} \end{pmatrix}^\top$ has $\text{rank}\,(M) = 1$.

(a) Let $A = \begin{pmatrix} 7 \\ -2 \end{pmatrix},\ B = \begin{pmatrix} 5 \\ 1 \end{pmatrix},\ C = \begin{pmatrix} 3 \\ k \end{pmatrix}$.

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 5 - 7 \\ 1 - (-2) \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}, \qquad \mathbf{C} - \mathbf{A} = \begin{pmatrix} 3 - 7 \\ k - (-2) \end{pmatrix} = \begin{pmatrix} -4 \\ k + 2 \end{pmatrix}$$

$$M = \begin{pmatrix} -2 & 3 \\ -4 & k + 2 \end{pmatrix}$$

Apply row transformations:

$$R_2 \leftarrow R_2 - 2R_1 \quad \implies \quad \begin{pmatrix} -2 & 3 \\ 0 & k - 4 \end{pmatrix}$$

For collinearity, $\text{rank}\,(M) = 1 \iff k - 4 = 0 \implies \boxed{k = 4}$.

(b) Let $A = \begin{pmatrix} 8 \\ 1 \end{pmatrix},\ B = \begin{pmatrix} k \\ -4 \end{pmatrix},\ C = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$.

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} k - 8 \\ -5 \end{pmatrix}, \qquad \mathbf{C} - \mathbf{A} = \begin{pmatrix} 2 - 8 \\ -5 - 1 \end{pmatrix} = \begin{pmatrix} -6 \\ -6 \end{pmatrix}$$

$$M = \begin{pmatrix} k - 8 & -5 \\ -6 & -6 \end{pmatrix}$$

Use a combination of scaling and addition to eliminate the first entry of R_2:

$$R_2 \leftarrow (k - 8)\ R_2 + 6\,R_1 \quad \implies \quad \begin{pmatrix} k - 8 & -5 \\ 0 & 18 - 6k \end{pmatrix}$$

For collinearity, $\text{rank}\,(M) = 1 \iff 18 - 6k = 0 \implies \boxed{k = 3}$.

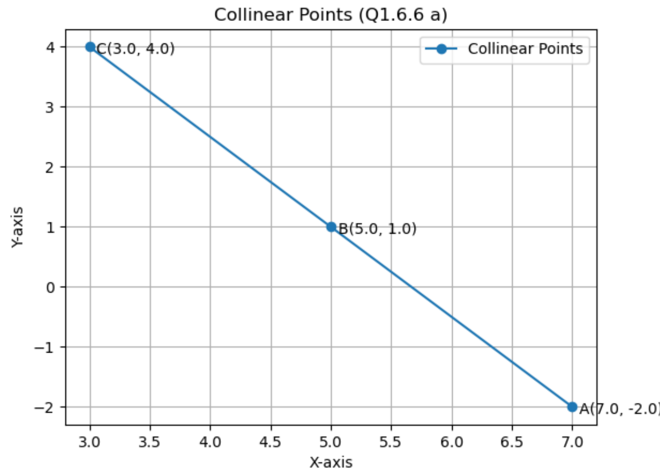

Fig 1 : Line through the given points

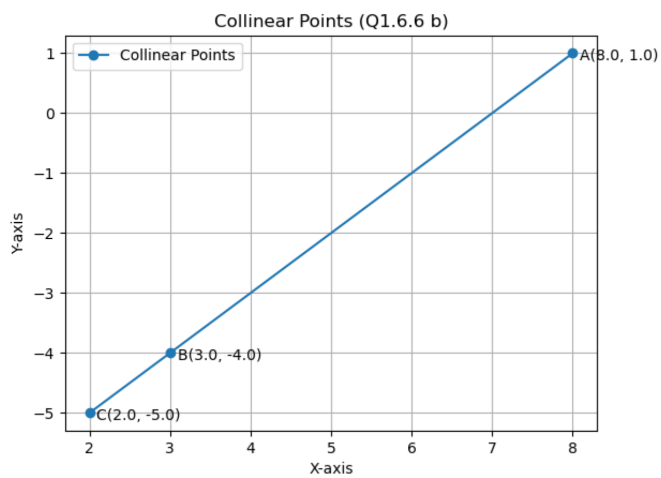

Fig 2 : Line through the given points