1.6.19

AI25BTECH11004-B.JASWANTH

Question

The vectors $\mathbf{A} = \lambda\hat{i} + \lambda\hat{j} + 2\hat{k}$,$\mathbf{B} = \hat{i} + \lambda\hat{j} - \hat{k}$ and $\mathbf{C} = 2\hat{i} - \hat{j} + \lambda\hat{k}$ are coplanar if $\lambda =$

Solution:

The vectors are coplanar $\iff$ they are linearly dependent. Form the matrix with these vectors as columns:

$$A = \begin{bmatrix} \lambda & 1 & 2 \\ \lambda & \lambda & -1 \\ 2 & -1 & \lambda \end{bmatrix}. \tag{0.1}$$

The three vectors are linearly dependent $\iff$ $\det(A) = 0$. We compute $\det(A)$ using row reduction.

$$A = \begin{bmatrix} \lambda & 1 & 2 \\ \lambda & \lambda & -1 \\ 2 & -1 & \lambda \end{bmatrix} \tag{0.2}$$

$$R_2 \to R_2 - R_1 \quad \Rightarrow \quad \begin{bmatrix} \lambda & 1 & 2 \\ 0 & \lambda - 1 & -3 \\ 2 & -1 & \lambda \end{bmatrix} \tag{0.3}$$

$$R_3 \to R_3 - \tfrac{2}{\lambda}R_1 \quad \Rightarrow \quad \begin{bmatrix} \lambda & 1 & 2 \\ 0 & \lambda - 1 & -3 \\ 0 & -1 - \frac{2}{\lambda} & \lambda - \frac{4}{\lambda} \end{bmatrix} \tag{0.4}$$

$$R_3 \to R_3 - \frac{-1 - \frac{2}{\lambda}}{\lambda - 1}R_2 \quad \Rightarrow \quad \begin{bmatrix} \lambda & 1 & 2 \\ 0 & \lambda - 1 & -3 \\ 0 & 0 & \dfrac{\lambda^3 - \lambda^2 - 7\lambda - 2}{\lambda(\lambda - 1)} \end{bmatrix} \tag{0.5}$$

Now the matrix is upper triangular, so

Solve the cubic

$$\lambda^3 - \lambda^2 - 7\lambda - 2 = 0. \tag{0.6}$$

Factor:

$$(\lambda + 2)(\lambda^2 - 3\lambda - 1) = 0. \tag{0.7}$$

So the solutions are

$$\lambda = -2, \qquad \lambda = \frac{3+\sqrt{13}}{2}, \qquad \lambda = \frac{3-\sqrt{13}}{2}. \tag{0.8}$$

Conclusion:

- For these values of λ, $\det(A) = 0 \implies \operatorname{rank}(A) < 3$, so the vectors are **linearly dependent** (coplanar).

$\boxed{\text{The vectors are coplanar for } \lambda = -2,\ \frac{3+\sqrt{13}}{2},\ \frac{3-\sqrt{13}}{2}.}$