

2.10.48

EE25BTECH11019 - Darji Vivek M.

Question

If

$$\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}, \quad \mathbf{a} \cdot \mathbf{b} = 1, \quad \mathbf{a} \times \mathbf{b} = \mathbf{j} - \mathbf{k},$$

then \mathbf{b} is 2

☐ $\mathbf{i} - \mathbf{j} + \mathbf{k}$

☐ $2\mathbf{j} - \mathbf{k}$

☐ \mathbf{i}

☐ $2\mathbf{i}$

(GATE XL 2025)

Setup

$$\mathbf{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Dot product condition:

$$\mathbf{a} \cdot \mathbf{b} = x + y + z = 1$$

Cross product:

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = (z - y)\mathbf{i} + (x - z)\mathbf{j} + (y - x)\mathbf{k}$$

Solving Conditions

Compare with $\mathbf{j} - \mathbf{k}$:

$$z - y = 0 \Rightarrow z = y$$

$$x - z = 1 \Rightarrow x = z + 1$$

$$y - x = -1 \Rightarrow y = x - 1$$

So,

$$z = y = x - 1$$

Dot product:

$$x + y + z = x + (x - 1) + (x - 1) = 3x - 2 = 1$$

$$\Rightarrow x = 1, \quad y = z = 0$$

$$\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \mathbf{i}$$

∴ Correct option: (c)

C Code

```
1 // Function to compute b given a b and a b
2 #include <stdio.h>
3 #include <math.h>
4
5 void find_b(double *b) {
6     double a[3] = {1, 1, 1};
7     double dot = 1;
8     double cross[3] = {0, 1, -1};
9
10    b[0] = 1; // x
11    b[1] = 0; // y
12    b[2] = 0; // z
13 }
```

```
1 int main() {  
2     double b[3];  
3     find_b(b);  
4     FILE *file = fopen("values.dat", "w");  
5     fprintf(file, "b_x\tb_y\tb_z\n");  
6     fprintf(file, "%.21f\t%.21f\t%.21f\n", b[0], b[1],  
7         b[2]);  
8     fclose(file);  
9     printf("Vector b written: (%.21f, %.21f, %.21f)\n",  
10         b[0], b[1], b[2]);  
11     return 0;  
12 }
```

Python Plot

```
1 import ctypes
2 import numpy as np
3 import matplotlib.pyplot as plt
4
5 lib = ctypes.CDLL('./libb.so')
6 b = (ctypes.c_double * 3)()
7 lib.find_b(b)
8
9 bx, by, bz = b[0], b[1], b[2]
10 print(f"Vector b = ({bx}, {by}, {bz})")
11
12 a = np.array([1, 1, 1])
13 b_vec = np.array([bx, by, bz])
14 0 = np.array([0, 0, 0])
```


Python plot

```
1 fig = plt.figure()
2 ax = fig.add_subplot(111, projection='3d')
3 ax.quiver(0,0,0, a[0],a[1],a[2], color='r', label='a =
    i+j+k')
4 ax.quiver(0,0,0, b_vec[0],b_vec[1],b_vec[2], color='b',
    , label='b')
5 points = {'0':0,'A (1,1,1)':a,f'B ({bx:.0f},{by:.0f},{
    bz:.0f})':b_vec}
6 for label, coord in points.items():
7     ax.text(coord[0], coord[1], coord[2], f'{label}',
8         fontsize=10, ha='center')
9 ax.set_xlim([0, 2]); ax.set_ylim([0, 2]); ax.set_zlim
    ([0, 2])
10 ax.set_xlabel('X'); ax.set_ylabel('Y'); ax.set_zlabel(
    'Z')
11 ax.legend(); plt.title("Vectors a and b with
    coordinates")
plt.show()
```

Vectors a and b with coordinates

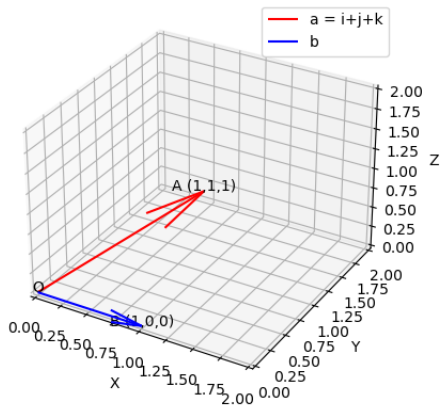


Figure: plot