

Question

Draw a triangle ABC with $BC = 7 \text{ cm}$, $\angle B = 45^\circ$ and $\angle C = 60^\circ$.

Solution

Given

- $BC = a = 7 \text{ cm}$
- $\angle B = 45^\circ$
- $\angle C = 60^\circ$

Let \mathbf{B} be the origin

$$\angle A = 180^\circ - (45^\circ + 60^\circ) = 75^\circ \quad (1)$$

$$K = \frac{a \sin C}{\sin A} = \frac{7 \sin 60^\circ}{\sin 75^\circ} = \frac{7 \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}+1}{2}} = \frac{7\sqrt{6}}{\sqrt{3}+1} \quad (2)$$

$$c = \frac{K^2 - a^2}{2(K - a \cos B)} = \frac{(\frac{7\sqrt{6}}{\sqrt{3}+1})^2 - 49}{2(\frac{7\sqrt{6}}{\sqrt{3}+1} - 7 \times \frac{1}{\sqrt{2}})} = -3.64 \quad (3)$$

Let:

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \quad (4)$$

Direction of \mathbf{A} is along angle $B = 45^\circ$:

$$\mathbf{A} = c \begin{pmatrix} \cos B \\ \sin B \end{pmatrix} = c \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (5)$$

$$\mathbf{A} = c \begin{pmatrix} \cos B \\ \sin B \end{pmatrix} = -3.64 \begin{pmatrix} 0.7071 \\ 0.7071 \end{pmatrix} \approx \begin{pmatrix} -2.574 \\ -2.574 \end{pmatrix} \quad (6)$$

Final Coordinates

$$\mathbf{A} = \begin{pmatrix} -2.574 \\ -2.574 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \quad (7)$$

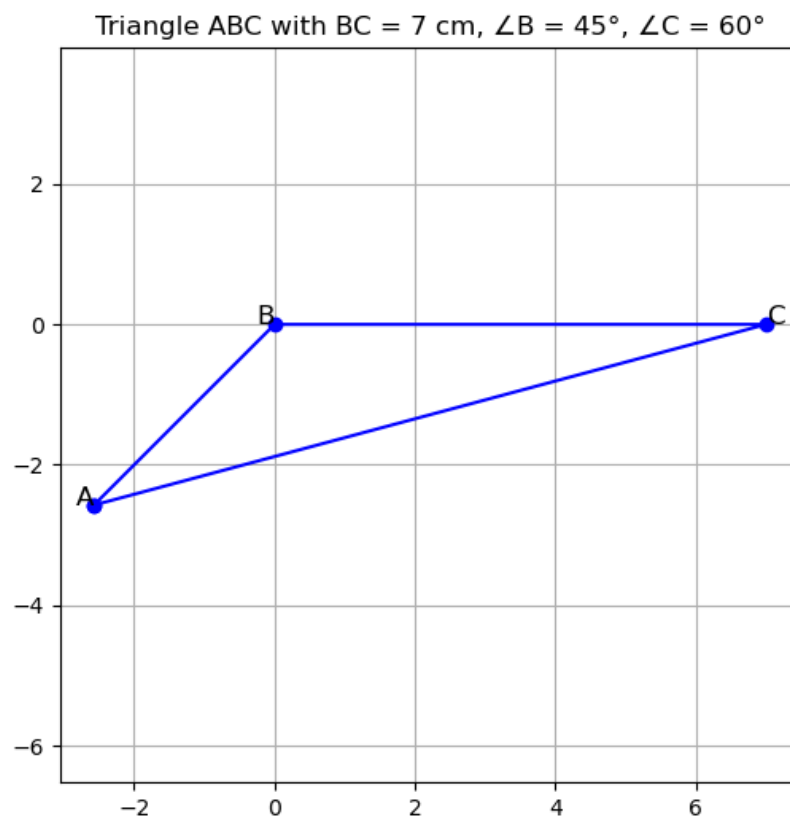


Figure 1