Matrices in Geometry - 2.10.64

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Problem Statement

The position vectors of the points **A**, **B**, **C** and **D** are $\left(3\hat{i} - 2\hat{j} - \hat{k}\right)$, $\left(2\hat{i} + 3\hat{j} - 4\hat{k}\right)$, $\left(-\hat{i} + \hat{j} + 2\hat{k}\right)$ and $\left(4\hat{i} + 5\hat{j} + \lambda\hat{k}\right)$ respectively. If the points **A**, **B**, **C** and **D** lie on a plane, find the value of λ.

Solution

Given,

$\mathbf{A}\begin{pmatrix}3\\-2\\-1\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}2\\3\\-4\end{pmatrix}$, $\mathbf{C}\begin{pmatrix}-1\\1\\2\end{pmatrix}$ and $\mathbf{D}\begin{pmatrix}4\\5\\\lambda\end{pmatrix}$.

The general equation for a plane with normal vector $\mathbf{n}$ passing through point P is

$$\mathbf{n}^\top \mathbf{P} = d \tag{1}$$

So,

$$\mathbf{n}^\top \mathbf{A} = d \tag{2}$$

$$\mathbf{n}^\top \mathbf{B} = d \tag{3}$$

$$\mathbf{n}^\top \mathbf{C} = d \tag{4}$$

$$\mathbf{n}^\top \mathbf{D} = d \tag{5}$$

Solution

Forming direction vectors in the plane,

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -1 \\ 5 \\ -3 \end{pmatrix} \tag{6}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -4 \\ 3 \\ 3 \end{pmatrix} \tag{7}$$

Now, the normal vector will be orthogonal to both of these direction vectors, so that

$$(\mathbf{B} - \mathbf{A})^\top \mathbf{n} = 0 \tag{8}$$

$$(\mathbf{C} - \mathbf{A})^\top \mathbf{n} = 0 \tag{9}$$

Solution

Combining the above two equations,

$$\begin{pmatrix} (\mathbf{B}-\mathbf{A})^\top \\ (\mathbf{C}-\mathbf{A})^\top \end{pmatrix} \mathbf{n} = 0 \tag{10}$$

$$\begin{pmatrix} -1 & 5 & -3 \\ -4 & 3 & 3 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{11}$$

Augmented matrix:

$$\implies \left(\begin{array}{ccc|c} -1 & 5 & -3 & 0 \\ -4 & 3 & 3 & 0 \end{array}\right) \tag{12}$$

$$\left(\begin{array}{ccc|c} -1 & 5 & -3 & 0 \\ -4 & 3 & 3 & 0 \end{array}\right) \xrightarrow{R_2 \to R_2 - 4R_1} \left(\begin{array}{ccc|c} -1 & 5 & -3 & 0 \\ 0 & -17 & 15 & 0 \end{array}\right) \tag{13}$$

Solution

$$\left(\begin{array}{ccc|c} -1 & 5 & -3 & 0 \\ 0 & -17 & 15 & 0 \end{array}\right) \xrightarrow{R_2 \to \frac{-1}{17} R_2} \left(\begin{array}{ccc|c} -1 & 5 & -3 & 0 \\ 0 & 1 & \frac{-15}{17} & 0 \end{array}\right) \tag{14}$$

$$\left(\begin{array}{ccc|c} -1 & 5 & -3 & 0 \\ 0 & 1 & \frac{-15}{17} & 0 \end{array}\right) \xrightarrow{R_1 \to R_1 - 5R_2} \left(\begin{array}{ccc|c} -1 & 0 & \frac{24}{17} & 0 \\ 0 & 1 & \frac{-15}{17} & 0 \end{array}\right) \tag{15}$$

$$\implies \mathbf{n} = \begin{pmatrix} \frac{24}{17} \\ \frac{15}{17} \\ 1 \end{pmatrix} t \tag{16}$$

So we can take $t = 17$ in order to get integer coefficients,

$$\mathbf{n} = \begin{pmatrix} 24 \\ 15 \\ 17 \end{pmatrix} \tag{17}$$

Solution

Substituting this in (2),

$$d = \begin{pmatrix} 24 & 15 & 17 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} = 25 \tag{18}$$

So substituting for **D** and d in the equation (5), we have

$$\begin{pmatrix} 24 & 15 & 17 \end{pmatrix} \begin{pmatrix} 4 \\ 5 \\ \lambda \end{pmatrix} = 25 \tag{19}$$

$$\implies \boxed{\lambda = \frac{-146}{17}} \tag{20}$$

Final Answer

The value of λ is $\frac{-146}{17}$.