

1.8.24

EE25BTECH11007- Aniket

Question:

If (a, b) is the mid-point of the line segment joining the point **A** $(10, -6)$ and **B** $(k, 4)$ and $a - 2b = 18$, find the value of a, b and the distance **AB**.

Solution:

Let $\mathbf{A} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$. By the (matrix) section formula, the point dividing **AB** in the ratio $k : 1$ is

$$\mathbf{R}_{\text{int}} = \frac{1}{k+1} [\mathbf{A} \ \mathbf{B}] \begin{pmatrix} 1 \\ k \end{pmatrix}. \quad (1)$$

With $\mathbf{A} = \begin{pmatrix} 10 \\ -6 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} k \\ 4 \end{pmatrix}$ and $\mathbf{O} = \begin{pmatrix} a \\ b \end{pmatrix}$ the midpoint ($k = 1$) is

$$\mathbf{O} = \frac{1}{2} [\mathbf{A} \ \mathbf{B}] \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 10 & k \\ -6 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 10+k \\ -2 \end{pmatrix} = \begin{pmatrix} \frac{10+k}{2} \\ -1 \end{pmatrix}. \quad (2)$$

Thus,

$$a = \frac{10+k}{2}, \quad b = -1 \quad (3)$$

Using the given condition $a - 2b = 18$:

$$\frac{10+k}{2} - 2(-1) = 18 \quad (4)$$

$$\frac{10+k}{2} = 16 \quad (5)$$

$$k = 22 \quad (6)$$

So,

$$a = \frac{10+22}{2} = \boxed{16}, \quad b = \boxed{-1} \quad (7)$$

Distance Between AB:

$$D = \|\mathbf{A} - \mathbf{B}\| \quad (8)$$

Where,

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{(\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{B})}. \quad (9)$$

Given,

$$\mathbf{A} = \begin{pmatrix} 10 \\ -6 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 22 \\ 4 \end{pmatrix}. \quad (10)$$

Now using the values of **A** and **B**,

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 10 - 22 \\ -6 - 4 \end{pmatrix} = \begin{pmatrix} -12 \\ -10 \end{pmatrix}. \quad (11)$$

Next,

$$(\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{B}) = \begin{pmatrix} -12 & -10 \end{pmatrix} \begin{pmatrix} -12 \\ -10 \end{pmatrix}. \quad (12)$$

$$\|\mathbf{A} - \mathbf{B}\| = \sqrt{(-12)^2 + (-10)^2} \quad (13)$$

$$= \sqrt{144 + 100} \quad (14)$$

$$= \sqrt{244} \quad (15)$$

$$= \boxed{2\sqrt{61}}. \quad (16)$$

