5.4.35

EE25BTECH11041 - Naman Kumar

Question:
Find inverse with elementary transformations of matrix

$$\begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{pmatrix} \tag{1}$$

Solution:
For elementary transformation, matrix can be written in form

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ -1 & 0 & -3 & 0 & 1 & 0 \\ -2 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \tag{2}$$

Here, it is in form

$$[\mathbf{A}|\mathbf{I}] \tag{3}$$

With elementary transformation, we get

$$\left[\mathbf{I}|\mathbf{A}^{-1}\right] \tag{4}$$

So now in (2)

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 2 & 1 & 0 & 0 \\ -1 & 0 & -3 & 0 & 1 & 0 \\ -2 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \tag{5}$$

$$\xrightarrow{R_1 \to R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 5 & 1 & -1 & 0 \\ -1 & 0 & -3 & 0 & 1 & 0 \\ -2 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \tag{6}$$

$$\xrightarrow{R_2 \to R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 5 & 1 & -1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ -2 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \tag{7}$$

$$\xrightarrow{R_1 \to R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ -2 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \tag{8}$$

$$\xrightarrow{R_3 \to R_3 + 2R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 3 & 6 & 0 & -2 & 1 \end{array}\right] \tag{9}$$

$$\xrightarrow{R_3 \to R_3 - 3R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 & -2 & 1 \end{array}\right] \tag{10}$$

At this point, we have obtained a row of all zeros on the left side of the augmented matrix. It's now impossible to continue the process to form the identity matrix.

Because the RREF of the original matrix is not the identity matrix, the matrix is singular and its inverse does not exist.