4.7.64

AI25BTECH11003 - Bhavesh Gaikwad

September 4,2025

Question

Find the distance between the point **P**(6, 5, 9) and the plane determined by the points **A**(3, -1, 2), **B**(5, 2, 4) and **C**(-1, -1, 6).

Theoretical Solution

Let the Equation of the Plane be $\mathbf{n}^\top \mathbf{x} = 1$.

Since, **A**, **B**, **C** lie on the Plane.

$$\therefore \mathbf{n}^\top \mathbf{A} = 1, \mathbf{n}^\top \mathbf{B} = 1, \mathbf{n}^\top \mathbf{C} = 1 \tag{1}$$

OR

$$\mathbf{A}^\top \mathbf{n} = 1, \mathbf{B}^\top \mathbf{n} = 1, \mathbf{C}^\top \mathbf{n} = 1 \tag{2}$$

Let $\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$

From Equation 2,

$$(\mathbf{A}\,\mathbf{B}\,\mathbf{C})^\top n = 1 \tag{3}$$

Theoretical Solution

$$\begin{pmatrix} 3 & -1 & 2 \\ 5 & 2 & 4 \\ -1 & -1 & 6 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = 1 \tag{4}$$

From Equation 4,

$$n_1 = 3/19,\ n_2 = -4/19,\ n_3 = 3/19 \tag{5}$$

$$\therefore \text{ The Equation of the Plane is: } \frac{1}{19} \begin{pmatrix} 3 & -4 & 3 \end{pmatrix} \mathbf{x} = 1 \tag{6}$$

The Distance of Point from a Plane is given by the formula:

$$d = \frac{\left\| \mathbf{n}^\top \mathbf{Q} - 1 \right\|}{\left\| \mathbf{n} \right\|} \tag{7}$$

Theoretical Solution

Let d be the distance between Point $\mathbf{P}$ and the Plane.

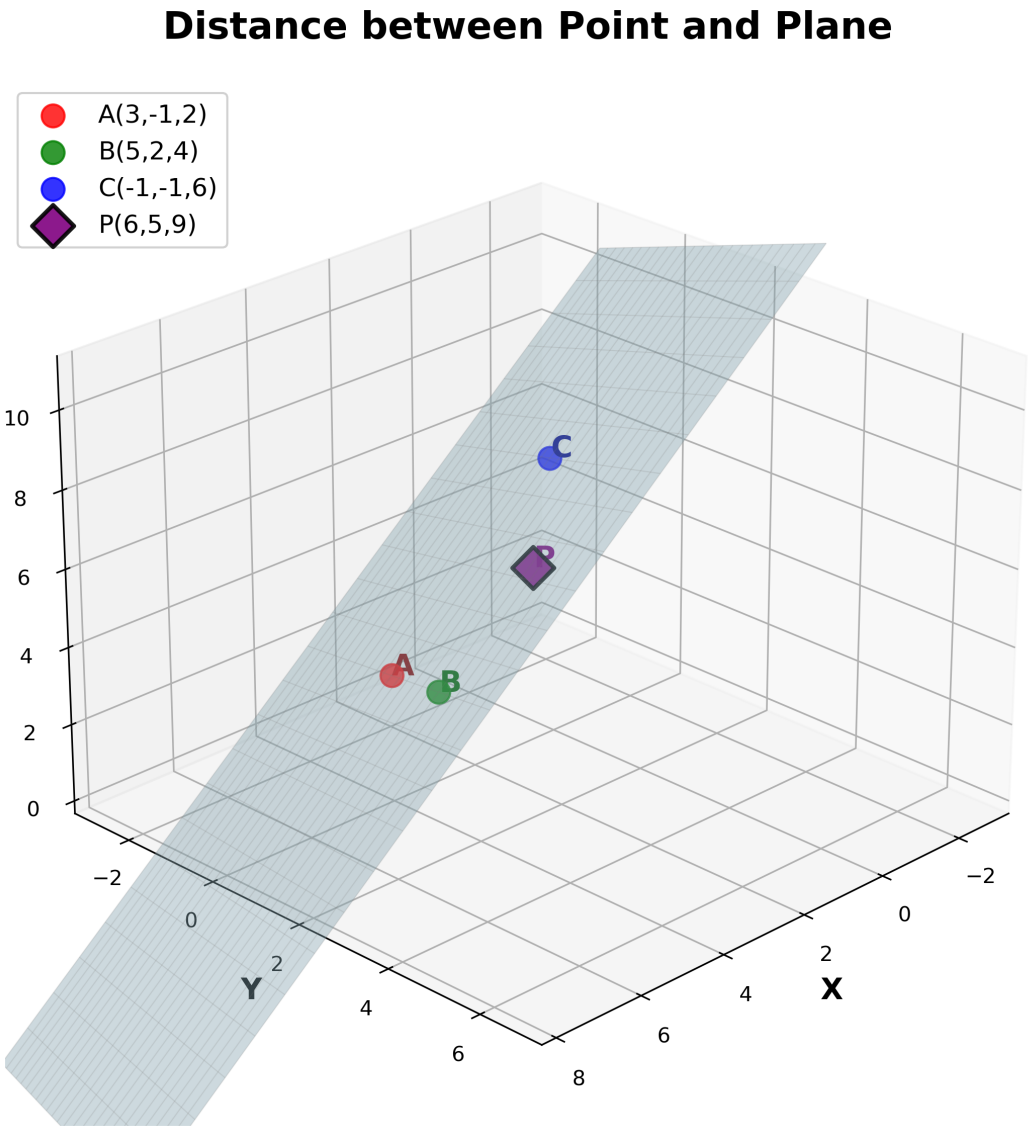
$$Then,\ d = \frac{\left\|\mathbf{n}^\top \mathbf{P} - 1\right\|}{\|\mathbf{n}\|} \tag{8}$$

$$d = \frac{\left\|\frac{18-20+27}{19} - 1\right\|}{\left(\frac{\sqrt{34}}{19}\right)} \tag{9}$$

$$\therefore\ d = \frac{3\sqrt{34}}{17} \tag{10}$$

$$\boxed{\text{The Distance between the Plane and } \mathbf{P} \textit{ is } \frac{3\sqrt{34}}{17} \textit{ units}.} \tag{11}$$

Plane


Distance between Point and Plane
A(3,-1,2)
B(5,2,4)
C(-1,-1,6)
P(6,5,9)
10
8
6
4
2
0
−2
0
2
4
6
8
Y
−2
0
2
4
6
8
X
A
B
C
P