

Matrices in Geometry 12.675

EE25BTECH11037 - Divyansh

Question: The ratio of the product of eigenvalues to the sum of the eigenvalues of the given matrix

$$\begin{pmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{pmatrix}$$

Solution: Let

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{pmatrix} \quad (1)$$

The eigenvalues are the values of λ that satisfy $|\mathbf{A} - \lambda\mathbf{I}| = 0$

$$\Rightarrow \left| \begin{pmatrix} 3-\lambda & 1 & 2 \\ 2 & -3-\lambda & -1 \\ 1 & 2 & 1-\lambda \end{pmatrix} \right| = 0 \quad (2)$$

$$\Rightarrow (3-\lambda)((-3-\lambda)(1-\lambda)+2)-1(2-2\lambda+1)+2(4+3+\lambda)=0 \quad (3)$$

$$\Rightarrow \lambda^3 - \lambda^2 - 11\lambda - 8 = 0 \quad (4)$$

Let the eigenvalues be $\lambda_1, \lambda_2, \lambda_3$, then

$$\lambda_1 + \lambda_2 + \lambda_3 = -\frac{-1}{1} = 1 \quad (5)$$

$$\lambda_1 \lambda_2 \lambda_3 = -\frac{-8}{1} \quad (6)$$

Thus the ratio of product of eigenvalues to sum of eigenvalues of \mathbf{A} is r

$$r = \frac{8}{1} = 8 \quad (7)$$