4.6.8

EE25BTECH11020 - Darsh Pankaj Gajare

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Question:

Find the equation of the plane passing through the points $(2, 2, -1)$,$(3, 4, 2)$ and $(7, 0, 6)$. Also find the equation of the plane passing through $(4, 3, 1)$ and parallel to the plane obtained above.

Solution:

Table

A	$\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$
B	$\begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$
C	$\begin{pmatrix} 7 \\ 0 \\ 6 \end{pmatrix}$
D	$\begin{pmatrix} 4 \\ 3 \\ 1) \end{pmatrix}$

Let the equation of plane be

$$\mathbf{n}^\top \mathbf{x} = C_1 \tag{0.1}$$

A,B,C satisfies this equation,

$$\mathbf{n}^\top \mathbf{A} = C_1, \mathbf{n}^\top \mathbf{B} = C_1, \mathbf{n}^\top \mathbf{C} = C_1 \tag{0.2}$$

$$\begin{pmatrix} \mathbf{A} \\ \mathbf{B} \\ \mathbf{C} \end{pmatrix}^\top \mathbf{n} = \begin{pmatrix} C_1 \\ C_1 \\ C_1 \end{pmatrix} \tag{0.3}$$

Using augmented matrix,

$$\left(\begin{array}{ccc|c} 2 & 2 & -1 & C_1 \\ 3 & 4 & 2 & C_1 \\ 7 & 0 & 6 & C_1 \end{array}\right) \tag{0.4}$$

$R_1 = R_1/2$

$$\left(\begin{array}{ccc|c} 1 & 1 & -0.5 & \frac{C_1}{2} \\ 3 & 4 & 2 & C_1 \\ 7 & 0 & 6 & C_1 \end{array}\right) \tag{0.5}$$

$R_2 = R_2 - 3R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & -0.5 & \frac{C_1}{2} \\ 0 & 1 & 3.5 & \frac{-C_1}{2} \\ 7 & 0 & 6 & C_1 \end{array}\right) \tag{0.6}$$

$R_3 = R_3 - 7R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & -0.5 & \frac{C_1}{2} \\ 0 & 1 & 3.5 & \frac{-C_1}{2} \\ 0 & -7 & 9.5 & \frac{-5C_1}{2} \end{array}\right) \tag{0.7}$$

$R_3 = R_3 + 7R_2$

$$\left(\begin{array}{ccc|c} 1 & 1 & -0.5 & \frac{C_1}{2} \\ 0 & 1 & 3.5 & \frac{-C_1}{2} \\ 0 & 0 & 34 & -6C_1 \end{array}\right) \tag{0.8}$$

$$34z + 6C_1 = 0 \implies z = \frac{-3C_1}{17} \tag{0.9}$$

$$y + 3.5z + 0.5C_1 = 0 \implies y = \frac{2C_1}{17} \tag{0.10}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -0.5 & \frac{C_1}{2} \\ 0 & 1 & 3.5 & \frac{-C_1}{2} \\ 0 & -7 & 9.5 & \frac{-5C_1}{2} \end{array}\right) \tag{0.11}$$

$$x + y - 0.5z = 0.5C_1 \implies x = \frac{5C_1}{17} \tag{0.12}$$

Let $C_1 = 17$

$$\mathbf{n} = \begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}, C_1 = 17 \tag{0.13}$$

Equation of plane parallel to given plane passing through D,

$$\mathbf{n}^\top \mathbf{x} = C_2 \tag{0.14}$$

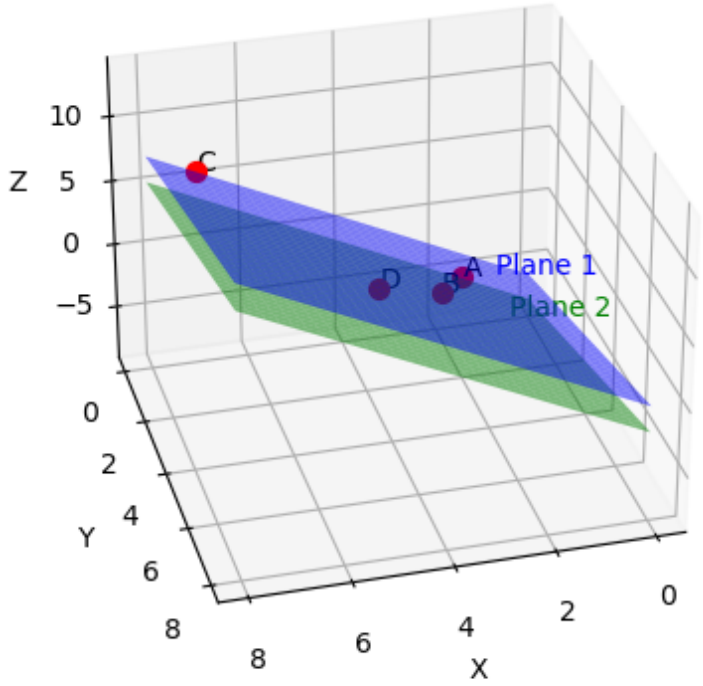
$$\mathbf{n}^\top \mathbf{D} = C_2 \tag{0.15}$$

$$C_2 = \begin{pmatrix} 5 & 2 & -3 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} \tag{0.16}$$

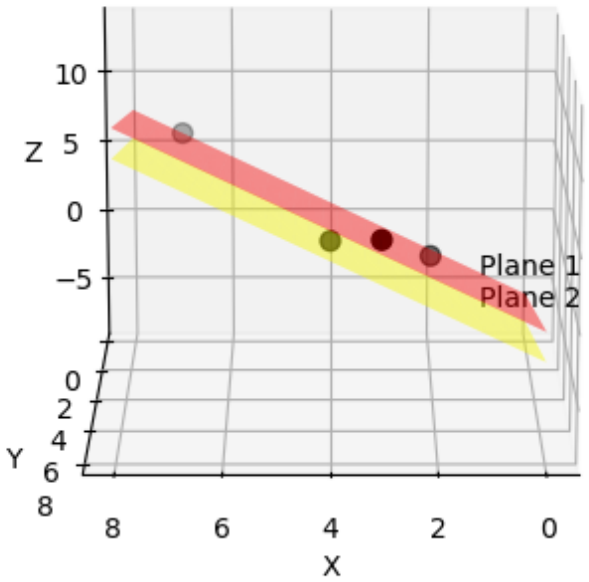
$$C_2 = 23 \tag{0.17}$$

Equation of plane

$$\begin{pmatrix} 5 \\ 2 \\ -3 \end{pmatrix}^\top \mathbf{x} = 23 \tag{0.18}$$



Plot using Python:


10
Z 5
0
−5
0
2
4
Y 6
8
8
6
4
2
0
X
Plane 1
Plane 2