4.8.14

AI25BTECH11018-Hemanth Reddy

Question:

Find the position vector of the foot of perpendicular and the perpendicular distance from the point **P** with position vector $2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ to the plane $\mathbf{r} \cdot (2\hat{\imath} + \hat{\jmath} + 3\hat{k})$ -26 = 0. Also find image of **P** in the plane.
Solution:

$$\text{The position vector of point } \mathbf{P} \text{ is } \mathbf{p} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \tag{0.1}$$

The normal vector of the plane is

$$\mathbf{n} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \tag{0.2}$$

The plane equation is

$$\mathbf{p}^T \cdot \mathbf{n} - 26 = 0 \tag{0.3}$$

1. Perpendicular Distance

The dot product $\mathbf{p} \cdot \mathbf{n}$ is given by the matrix multiplication $\mathbf{p}^T \mathbf{n}$.

$$\mathbf{p}^T \mathbf{n} = \begin{pmatrix} 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = (2)(2) + (3)(1) + (1)(3) = 10 \tag{0.4}$$

$$|\mathbf{n}| = \sqrt{\mathbf{n}^T \mathbf{n}} = \sqrt{\begin{pmatrix} 2 & 1 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}} = \sqrt{4 + 1 + 9} = \sqrt{14} \tag{0.5}$$

The perpendicular distance d is:

$$d = \frac{|\mathbf{p}^T \mathbf{n} - 26|}{|\mathbf{n}|} = \frac{|10 - 26|}{\sqrt{14}} = \frac{16}{\sqrt{14}} \tag{0.6}$$

2. Foot of Perpendicular

The position vector of the foot of the perpendicular $\mathbf{q}$ is:

$$\mathbf{q} = \mathbf{p} - \frac{(\mathbf{p}^T\mathbf{n} - 26)}{|\mathbf{n}|^2}\mathbf{n} \tag{0.7}$$

$$\mathbf{q} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} - \frac{10 - 26}{14}\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \frac{16}{14}\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \tag{0.8}$$

$$\mathbf{q} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \frac{8}{7}\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 + 16/7 \\ 3 + 8/7 \\ 1 + 24/7 \end{pmatrix} = \begin{pmatrix} 30/7 \\ 29/7 \\ 31/7 \end{pmatrix} \tag{0.9}$$

So the position vector of the foot of the perpendicular is $\frac{30}{7}\hat{\imath} + \frac{29}{7}\hat{\jmath} + \frac{31}{7}\hat{k}$.

3. Image of **P**

The position vector of the image **P'**, is:

$$\mathbf{P'} = 2\mathbf{q} - \mathbf{P} = 2\begin{pmatrix} 30/7 \\ 29/7 \\ 31/7 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 60/7 - 14/7 \\ 58/7 - 21/7 \\ 62/7 - 7/7 \end{pmatrix} = \begin{pmatrix} 46/7 \\ 37/7 \\ 55/7 \end{pmatrix} \tag{0.10}$$

So the position vector of the image is $\frac{46}{7}\hat{\imath} + \frac{37}{7}\hat{\jmath} + \frac{55}{7}\hat{k}$.

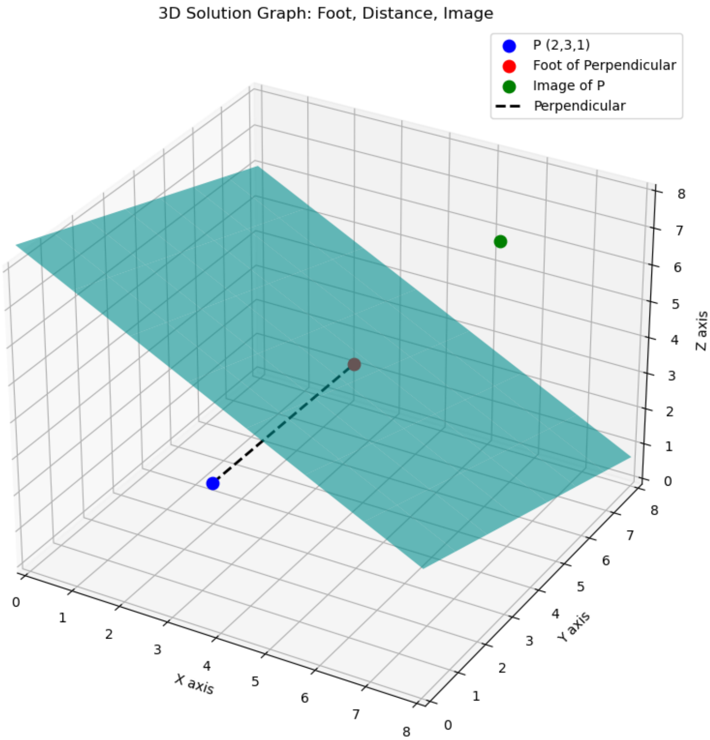
3D Solution Graph: Foot, Distance, Image
P (2,3,1)
Foot of Perpendicular
Image of P
Perpendicular
X axis
Y axis
Z axis
0
1
2
3
4
5
6
7
8

Fig. 0.1