12.339

EE25BTECH11013 - Bhargav

Question:
If

$$\mathbf{A} = \begin{pmatrix} 3 & -3 \\ -3 & 4 \end{pmatrix} \tag{0.1}$$

then

$$\det\left(-\mathbf{A}^2 + 7\mathbf{A} - 3\mathbf{I}\right) \tag{0.2}$$

is

Solution:

$$\det(\mathbf{A}) = 3, \mathrm{Tr}(\mathbf{A}) = 3 + 4 = 7 \tag{0.3}$$

(where $\mathrm{Tr}(\mathbf{A})$ represents the trace of matrix $\mathbf{A}$, i.e. the sum of the diagonal entries of $\mathbf{A}$)

The determinant of the expression can be found out by using the Cayley-Hamilton theorem.

$$\mathbf{A}^2 - (\mathrm{Tr}(\mathbf{A}))\,\mathbf{A} + \det(\mathbf{A})\mathbf{I} = 0 \tag{0.4}$$

$$\mathbf{A}^2 - 7\mathbf{A} + 3\mathbf{I} = 0 \implies -\mathbf{A}^2 + 7\mathbf{A} - 3\mathbf{I} = 0 \tag{0.5}$$

$$\therefore \det\left(-\mathbf{A}^2 + 7\mathbf{A} - 3\mathbf{I}\right) = 0 \tag{0.6}$$