Matgeo-q.2.7.8

AI25BTECH11036-SNEHAMRUDULA

September 8, 2025

Question

2.7.8 Find $|\mathbf{a} \times \mathbf{b}|$, if $\mathbf{a} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$.

solution

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 3 \\ 5 \\ -2 \end{pmatrix}. \tag{1}$$

Using the triangle-area formula,

$$\text{ar}(\triangle OAB) = \frac{1}{2}\|(A - O) \times (B - O)\| = \frac{1}{2}\|\mathbf{a} \times \mathbf{b}\|. \tag{2}$$

from the vector cross product definition $\mathbf{a} \times \mathbf{b} = \begin{pmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{pmatrix}$ (3)

$$= -17\,\hat{\imath} + 13\,\hat{\jmath} + 7\,\hat{k}. \tag{4}$$

$$\|\mathbf{a} \times \mathbf{b}\| = \sqrt{(-17)^2 + 13^2 + 7^2} = \sqrt{507} = 13\sqrt{3}. \tag{5}$$

$$\boxed{|\mathbf{a} \times \mathbf{b}| = 13\sqrt{3}}. \tag{6}$$

Graphical Representation