12.768

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Question

In the figure, the vectors $\mathbf{u}$ and $\mathbf{v}$ are related as $\mathbf{A}\mathbf{u} = \mathbf{v}$ by a transformation matrix $\mathbf{A}$. The correct choice of $\mathbf{A}$ is

1. $\begin{pmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & -\frac{4}{5} \end{pmatrix}$
2. $\begin{pmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix}$
3. $\begin{pmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{pmatrix}$
4. $\begin{pmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & -\frac{4}{5} \end{pmatrix}$

Question

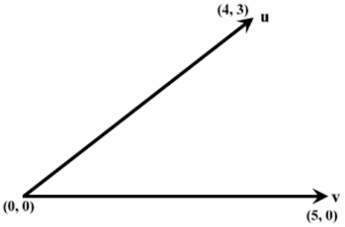

Figure: Figure-1

Theoretical solution

Given,

$$\mathbf{u} = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \qquad \mathbf{v} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{1}$$

From 1,

$$\|\mathbf{u}\| = \|\mathbf{v}\| = 5 \textit{ units} \tag{2}$$

This implies, **A** is a rotation matrix.
Rotation matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{3}$$

where θ is the angle between the vectors in counter-clockwise sense.

Theoretical solution

$$\therefore \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} 4\cos\theta - 3\sin\theta \\ 4\sin\theta + 3\cos\theta \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{5}$$

The above equation can be re-arranged as,

$$\begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{6}$$

We need to solve for $\cos\theta$ and $\sin\theta$ to get the transformation matrix $\mathbf{A}$.

Theoretical solution

We can see that in (6), the columns of the coefficient matrix are orthogonal to each other and also the column vectors have the same norm.

$$\therefore \frac{1}{5}\begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{7}$$

$$\implies \begin{pmatrix} \frac{4}{5} & -\frac{3}{5} \\ \frac{3}{5} & \frac{4}{5} \end{pmatrix}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{8}$$

In equation (8), the coefficient matrix is an orthogonal matrix.

$$\implies \mathbf{Ax} = \mathbf{b} \Rightarrow \mathbf{A}^\top\mathbf{Ax} = \mathbf{A}^\top\mathbf{b} \Rightarrow \mathbf{x} = \mathbf{A}^\top\mathbf{b} \qquad \left(\because \mathbf{A}^\top\mathbf{A} = \mathbf{I}\right) \tag{9}$$

Theoretical solution

$$\therefore \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \begin{pmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{10}$$

$$\therefore \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \begin{pmatrix} \frac{4}{5} \\ -\frac{3}{5} \end{pmatrix} \tag{11}$$

$$\implies \mathbf{A} = \begin{pmatrix} \frac{4}{5} & \frac{3}{5} \\ -\frac{3}{5} & \frac{4}{5} \end{pmatrix} \tag{12}$$