2.3.4

EE25BTECH11001 - Aarush Dilawri

Question:
a and **b** are two unit vectors such that

$$|2\mathbf{a}+3\mathbf{b}| = |3\mathbf{a}-2\mathbf{b}| \tag{1}$$

Find the angle between **a** and **b**.
Solution:
Let $\mathbf{a}, \mathbf{b}$ be unit vectors with angle θ between them. We know that $\mathbf{a}\cdot\mathbf{a} = 1,\ \mathbf{b}\cdot\mathbf{b} = 1,\ \mathbf{a}\cdot\mathbf{b} = \cos\theta$.

The Gram matrix is

$$\mathbf{G} = \begin{pmatrix} 1 & \cos\theta \\ \cos\theta & 1 \end{pmatrix}.$$

$$\left\|2\mathbf{a}+3\mathbf{b}\right\|^2 = \begin{pmatrix} 2 & 3 \end{pmatrix} \mathbf{G} \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{2}$$

$$= \begin{pmatrix} 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & \cos\theta \\ \cos\theta & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{3}$$

$$= 13 + 12\cos\theta \tag{4}$$

$$\left\|3\mathbf{a}-2\mathbf{b}\right\|^2 = \begin{pmatrix} 3 & -2 \end{pmatrix} \mathbf{G} \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{5}$$

$$= \begin{pmatrix} 3 & -2 \end{pmatrix} \begin{pmatrix} 1 & \cos\theta \\ \cos\theta & 1 \end{pmatrix} \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{6}$$

$$= 13 - 12\cos\theta \tag{7}$$

$$13 + 12\cos\theta = 13 - 12\cos\theta \tag{8}$$

$$24\cos\theta = 0 \tag{9}$$

$$\cos\theta = 0 \tag{10}$$

Therefore, the angle between **a** and **b** is

$$\theta = \frac{\pi}{2} \quad (90^\circ).$$

See Fig. 0 ,

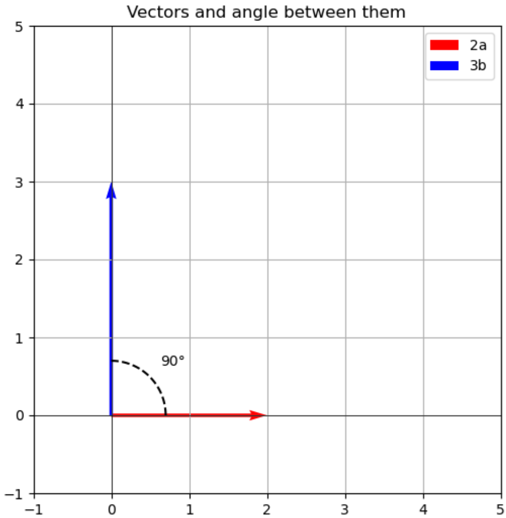

Fig. 0