Matgeo Presentation - Problem 8.4.24

ee25btech11021 - Dhanush sagar

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Problem Statement

If the line $x - 1 = 0$ is the directrix of the parabola

$$y^2 - kx + 8 = 0,$$

then one of the values of k is:

1) 18 2) 8 3) 4 4) 14

solution

We are given the parabola

$$y^2 - kx + 8 = 0 \tag{0.1}$$

with directrix $x - 1 = 0$. Represent the parabola in matrix form:

$$\mathbf{x}^\top V\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{0.2}$$

For a conic with directrix $\mathbf{n}^\top\mathbf{x} = c$, eccentricity e and focus $\mathbf{F}$, the matrix formulas are:

$$\mathbf{V} = \|\mathbf{n}\|^2 I - e^2\mathbf{n}\mathbf{n}^\top \tag{0.3}$$

$$\mathbf{u} = ce^2\mathbf{n} - \|\mathbf{n}\|^2\mathbf{F} \tag{0.4}$$

$$f = \|\mathbf{n}\|^2\|\mathbf{F}\|^2 - c^2e^2 \tag{0.5}$$

For the parabola $y^2 - kx + 8 = 0$, we write the matrices as

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \mathbf{u} = \begin{pmatrix} -k/2 \\ 0 \end{pmatrix}, \qquad f = 8 \tag{0.6}$$

solution

The directrix is $\mathbf{n}^\top \mathbf{x} = c \implies \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, c = 1$, and for a parabola $e = 1$. Then

$$\mathbf{V} = \|\mathbf{n}\|^2 I - e^2 \mathbf{n}\mathbf{n}^\top = 1 \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 1 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{0.7}$$

The vector $\mathbf{u}$ gives the focus:

$$\mathbf{u} = ce^2\mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F} \implies \mathbf{F} = c\mathbf{n} - \mathbf{u} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} -k/2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 + k/2 \\ 0 \end{pmatrix} \tag{0.8}$$

The constant term is

$$f = \|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 e^2 = 1 \cdot \left(\begin{pmatrix} 1 + k/2 \\ 0 \end{pmatrix}^\top \begin{pmatrix} 1 + k/2 \\ 0 \end{pmatrix} \right) - 1 = (1 + k/2)^2 - \tag{0.9}$$

solution

Equating with the given $f = 8$:

$$(1 + k/2)^2 - 1 = 8 \implies (1 + k/2)^2 = 9 \tag{0.10}$$

Solving the matrix equation:

$$1 + k/2 = 3 \implies k = 4 \tag{0.11}$$

$$1 + k/2 = -3 \implies k = -8 \tag{0.12}$$

Hence, one of the values of k is

$$\boxed{4} \tag{0.13}$$

C Source Code:

```
#include <stdio.h>
#include <math.h>

void generate_parabola_points(double k, double x_start,
double x_end, int n_points,double *X, double *Y) {
    double step = (x_end - x_start) / (n_points - 1);
    for(int i = 0; i < n_points; i++){
        double x = x_start + i*step;
        double y_sq = k*x - 8;  // y^2 = kx - 8
        if (y_sq >= 0) {
            X[i] = x; Y[i] = sqrt(y_sq);           // positive
            X[i + n_points] = x; Y[i + n_points] = -sqrt(y_sq
        } else {
            X[i] = X[i + n_points] = x;
            Y[i] = Y[i + n_points] = 0; // skip imaginary
        }
    }
```

C Source Code:

```
}

// Solve for k using matrix method formulas
double solve_k_matrix() {
    double f_target = 8;
    double temp = f_target + 1;  // f = ||F||^2 - 1 => (1 + k/
    double k1 = 2*(sqrt(temp) - 1);
    double k2 = 2*(-sqrt(temp) - 1);
    printf("Possible values of k: %f , %f\n", k1, k2);
    return k1; // return first value
}
```

Python Script:solve

```
import numpy as np
import ctypes
lib = ctypes.CDLL('./libparabola.so')
n = np.array([[1], [0]])  # directrix vector
c = 1
e = 1
f_target = 8  # from parabola y^2 - kx + 8
temp = f_target + 1
k1 = 2*(np.sqrt(temp) - 1)
k2 = 2*(-np.sqrt(temp) - 1)

print("Possible k values:", k1, k2)

# Save k1 for plotting
with open("k_value.txt", "w") as f:
    f.write(str(k1))
```

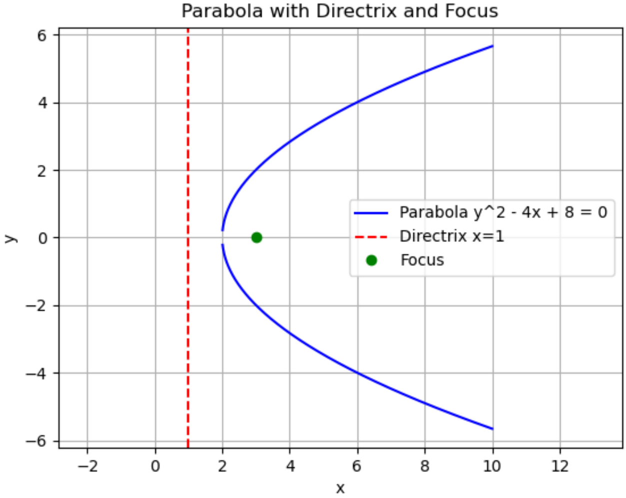
Python Script: plot

```
import numpy as np
import matplotlib.pyplot as plt

k = 4  # choose k1
x = np.linspace(-5, 10, 1000)
y_square = k*x - 8

mask = y_square >= 0
x_real = x[mask]
y_real = np.sqrt(y_square[mask])
plt.plot(x_real, y_real, 'b', label=f"Parabola y^2 - {k}x + 8
plt.plot(x_real, -y_real, 'b')
plt.axvline(x=1, color='r', linestyle='--', label='Directrix x
F_x = 1 + k/2
F_y = 0
```

Python Script: plot

```
plt.plot(F_x, F_y, 'go', label='Focus')
plt.xlabel("x")
plt.ylabel("y")
plt.title("Parabola with Directrix and Focus")
plt.legend()
plt.grid(True)
plt.axis('equal')
plt.show()
```

Result Plot



Parabola with Directrix and Focus
Parabola y^2 - 4x + 8 = 0
Directrix x=1
Focus
x
y