12.339 [1]

EE25BTECH11013 - Bhargav

Question:
If

$$\mathbf{A} = \begin{pmatrix} 3 & -3 \\ -3 & 4 \end{pmatrix} \tag{0.1}$$

then

$$\det\left(-\mathbf{A}^2 + 7\mathbf{A} - 3\mathbf{I}\right) \tag{0.2}$$

is

Solution:

The characteristic equation of matrix $\mathbf{A}$ is

$$f(\lambda) = |\mathbf{A} - \lambda\mathbf{I}| = 0 \tag{0.3}$$

$$\left| \begin{pmatrix} 3 - \lambda & -3 \\ -3 & 4 - \lambda \end{pmatrix} \right| = 0 \tag{0.4}$$

$$\implies (3 - \lambda)(4 - \lambda) - 9 = 0 \tag{0.5}$$

$$-\lambda^2 + 7\lambda - 3 = 0 \tag{0.6}$$

According to Cayley-Hamilton Theorem:

$$f(\lambda) = f(\mathbf{A}) \tag{0.7}$$

$$\therefore -\mathbf{A}^2 + 7\mathbf{A} - 3\mathbf{I} = 0 \tag{0.8}$$

$$\therefore \left| -\mathbf{A}^2 + 7\mathbf{A} - 3\mathbf{I} \right| = 0 \tag{0.9}$$