4.3.32

EE25BTECH11042 - Nipun Dasari

Question:
Find the slope of a line which cuts off intercepts of equal length on the axes is. Solve using matrices.
Solution:

Let the line cut the x-axis at an intercept 'a' and the y-axis at an intercept 'b'. The points where the line intersects the axes can be represented by position vectors (column matrices).

The point of x-intercept is $P_1 = (a, 0)$. Its position vector is:

$$\mathbf{p_1} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{0.1}$$

The point of y-intercept is $P_2 = (0, b)$. Its position vector is:

$$\mathbf{p_2} = \begin{pmatrix} 0 \\ b \end{pmatrix} \tag{0.2}$$

A direction vector for the line can be found by taking the difference between the two position vectors:

$$\mathbf{v} = \mathbf{p_2} - \mathbf{p_1} = \begin{pmatrix} 0 \\ b \end{pmatrix} - \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} -a \\ b \end{pmatrix} \tag{0.3}$$

The slope, m, is defined as the ratio of the change in y to the change in x.

$$m = \frac{\Delta y}{\Delta x} = \frac{b}{-a} \tag{0.4}$$

The problem states that the intercepts have equal length, which means their magnitudes are equal:

$$|a| = |b| \tag{0.5}$$

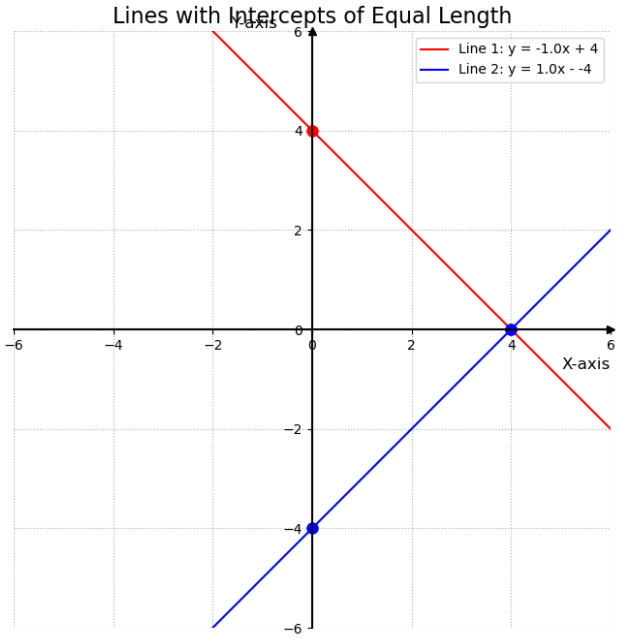
Case 1: The intercepts are equal ($b = a$)
Substituting $b = a$ into the slope equation (assuming $a \neq 0$): By (0.4)

$$m_1 = \frac{a}{-a} = -1 \tag{0.6}$$

Case 2: The intercepts are opposite ($b = -a$)
Substituting $b = -a$ into the slope equation (assuming $a \neq 0$): By (0.4)

$$m_2 = \frac{-a}{-a} = 1 \tag{0.7}$$

Thus, using a matrix representation for the points, we find that the two possible slopes are -1 and 1.


Lines with Intercepts of Equal Length
Y-axis
X-axis
Line 1: y = -1.0x + 4
Line 2: y = 1.0x - -4
6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6