1.8.23

EE25BTECH11006 - ADUDOTLA SRIVIDYA

Question:
If the point $\mathbf{A}(2,-4)$ is equidistant from $\mathbf{P}(3,8)$ and $\mathbf{Q}(-10,y)$, find the values of y. Also find distance **PQ**.
Solution: :
The input parameters for this problem are available in Table

Symbol	Value	Description
A	$\begin{pmatrix}2\\-4\end{pmatrix}$	equidistant point
P	$\begin{pmatrix}3\\8\end{pmatrix}$	First point
Q	$\begin{pmatrix}-10\\y\end{pmatrix}$	Secong point

TABLE I: Parameters for the problem

Since **A** is equidistant from **P** and **Q**,

$$\left\|\left(\mathbf{A}-\mathbf{P}\right)\right\| = \left\|\left(\mathbf{A}-\mathbf{Q}\right)\right\| \tag{1}$$

$$\left\|\left(\mathbf{A}-\mathbf{P}\right)\right\|^2 = \left\|\left(\mathbf{A}-\mathbf{Q}\right)\right\|^2 \tag{2}$$

$$\|\mathbf{A}\|^2 - 2\mathbf{A}^\top\mathbf{P} + \|\mathbf{P}\|^2 = \|\mathbf{A}\|^2 - 2\mathbf{A}^\top\mathbf{Q} + \|\mathbf{Q}\|^2 \tag{3}$$

$$(\mathbf{P}-\mathbf{Q})^\top\mathbf{A} = \frac{\|\mathbf{P}\|^2 - \|\mathbf{Q}\|^2}{2} \tag{4}$$

After substituting the values,

$$\begin{pmatrix}3-(-10)\\8-y\end{pmatrix}^\top\begin{pmatrix}2\\-4\end{pmatrix} = \frac{73-(-10)^2-y^2}{2} \tag{5}$$

$$y^2 + 8y + 15 = 0 \tag{6}$$

Therefore,

$$y = -5, -3 \tag{7}$$

$$\mathbf{Q_1} = \begin{pmatrix} -10 \\ -5 \end{pmatrix}, \quad \mathbf{Q_2} = \begin{pmatrix} -10 \\ -3 \end{pmatrix} \tag{8}$$

$$\left\| \left(\mathbf{P} - \mathbf{Q_1} \right) \right\| = \left\| \begin{pmatrix} 3 \\ 8 \end{pmatrix} - \begin{pmatrix} -10 \\ -5 \end{pmatrix} \right\| \tag{9}$$

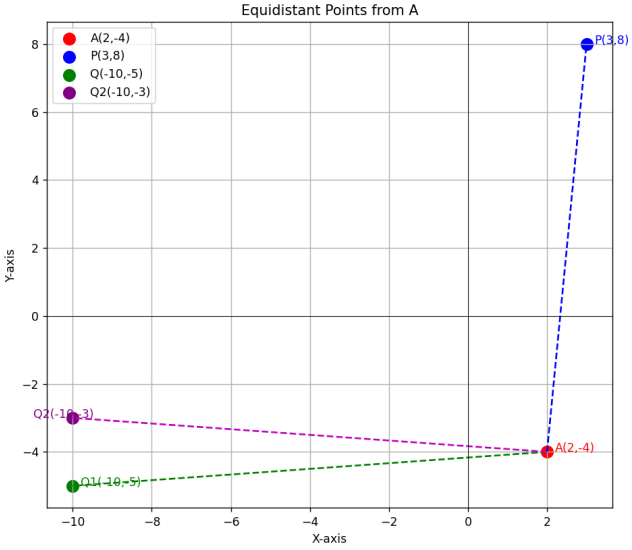
$$= \left\| \begin{pmatrix} 13 \\ 13 \end{pmatrix} \right\| \tag{10}$$

$$= 13\sqrt{2} \tag{11}$$

$$\left\| \left(\mathbf{P} - \mathbf{Q_2} \right) \right\| = \left\| \begin{pmatrix} 3 \\ 8 \end{pmatrix} - \begin{pmatrix} -10 \\ -3 \end{pmatrix} \right\| \tag{12}$$

$$= \left\| \begin{pmatrix} 13 \\ 11 \end{pmatrix} \right\| \tag{13}$$

$$= \sqrt{290} \tag{14}$$



Equidistant Points from **A** with Distances