4.11.23

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Question

Find the co-ordinates of the point where the line $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$ crosses the plane passing through the points $\left(\frac{7}{2}, 0, 0\right)$, $(0, 7, 0)$, and $(0, 0, 7)$.

Theoretical Solution

For the intersection of a line

$$\mathbf{x} = \mathbf{p} + \lambda \mathbf{m}$$

with the plane

$$\mathbf{n}^\top \mathbf{x} = c$$

$$\mathbf{n}^\top (\mathbf{p} + \lambda \mathbf{m}) = c \tag{1}$$

$$\mathbf{n}^\top \mathbf{p} + \lambda \mathbf{n}^\top \mathbf{m} = c \tag{2}$$

$$\lambda = \frac{c - \mathbf{n}^\top \mathbf{p}}{\mathbf{n}^\top \mathbf{m}} \tag{3}$$

$$\mathbf{x} = \mathbf{p} + \left(\frac{c - \mathbf{n}^\top \mathbf{p}}{\mathbf{n}^\top \mathbf{m}} \right) \mathbf{m} \tag{4}$$

Let the equation of the plane be

$$\begin{pmatrix} n_1 & n_2 & n_3 \end{pmatrix} \mathbf{x} = c \tag{5}$$

The three points

$$\mathbf{P_1} = \begin{pmatrix} \frac{7}{2} \\ 0 \\ 0 \end{pmatrix}, \mathbf{P_2} = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix}, \mathbf{P_3} = \begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix}$$

satisfy this equation, giving the system:

$$\frac{7}{2} n_1 + 0 n_2 + 0 n_3 = c \tag{6}$$

$$0 n_1 + 7 n_2 + 0 n_3 = c \tag{7}$$

$$0 n_1 + 0 n_2 + 7 n_3 = c \tag{8}$$

Theoretical Solution

This system of equations gives the augmented matrix

$$\left(\begin{array}{ccc|c} \frac{7}{2} & 0 & 0 & c \\ 0 & 7 & 0 & c \\ 0 & 0 & 7 & c \end{array}\right) \xleftrightarrow[R_3 \to \frac{1}{7}R_3]{R_1 \to \frac{2}{7}R_1, R_2 \to \frac{1}{7}R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{2c}{7} \\ 0 & 1 & 0 & \frac{c}{7} \\ 0 & 0 & 1 & \frac{c}{7} \end{array}\right) \tag{9}$$

From the row-reduced echelon form,

$$n_1 = \frac{2c}{7}, n_2 = \frac{c}{7}, n_3 = \frac{c}{7} \tag{10}$$

Substituting (10) in (5),

$$\begin{pmatrix} \frac{2c}{7} & \frac{c}{7} & \frac{c}{7} \end{pmatrix} \mathbf{x} = c \tag{11}$$

Assuming $c \neq 0$, the equation simplifies to the normal form of the plane, $\mathbf{n}^\top \mathbf{x} = c$, which is

$$\begin{pmatrix} 2 & 1 & 1 \end{pmatrix} \mathbf{x} = 7 \tag{12}$$

The vector equation of the line passing through

$$\mathbf{p} = \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}$$

with direction vector

$$\mathbf{m} = \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix}$$

is

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \tag{13}$$

Theoretical Solution

Using (4),

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} + \left(\frac{7 - \begin{pmatrix} 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix}}{\begin{pmatrix} 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix}} \right) \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \tag{14}$$

$$\mathbf{x} = \begin{pmatrix} 3 \\ -4 \\ -5 \end{pmatrix} + 2 \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix} \tag{15}$$

The co-ordinates of the point of intersection are $\begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix}$.

Plot

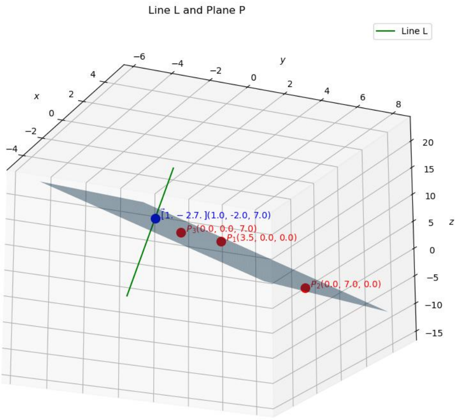

Figure: Plot