5.5.17

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Question

If

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{pmatrix},$$

find $\mathbf{A}^{-1}$ using elementary row transformations. Hence, solve the system:

$$\begin{aligned} x + y + z &= 6 \\ y + 3z &= 11 \\ x - 2y + z &= 0 \end{aligned}$$

Theoretical Solution

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{pmatrix} \tag{1}$$

The augmented matrix is:

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 1 & -2 & 1 & 0 & 0 & 1 \end{array}\right) \tag{2}$$

Theoretical Solution

Row operations:

$$R_3 \to R_3 - R_1 \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & -3 & 0 & -1 & 0 & 1 \end{array}\right) \tag{3}$$

$$R_3 \to R_3 + 3R_2 \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 9 & -1 & 3 & 1 \end{array}\right) \tag{4}$$

$$R_3 \to \frac{1}{9} R_3 \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 0 & 1 & 0 \\ 0 & 0 & 1 & -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{array}\right) \tag{5}$$

Theoretical Solution

$$R_1 \to R_1 - R_3, \quad R_2 \to R_2 - 3R_3 \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & \frac{10}{9} & -\frac{1}{3} & -\frac{1}{9} \\ 0 & 1 & 0 & \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{array}\right) \tag{6}$$

$$R_1 \to R_1 - R_2 \Rightarrow \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{7}{9} & -\frac{1}{3} & \frac{2}{9} \\ 0 & 1 & 0 & \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{array}\right) \tag{7}$$

As the left block becomes identity, the right block is $\mathbf{A}^{-1}$:

$$\mathbf{A}^{-1} = \begin{pmatrix} \frac{7}{9} & -\frac{1}{3} & \frac{2}{9} \\ \frac{1}{3} & 0 & -\frac{1}{3} \\ -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{pmatrix} \tag{8}$$

Now solving: $\quad \mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$, where

$$\mathbf{b} = \begin{pmatrix} 6 \\ 11 \\ 0 \end{pmatrix}$$

Theoretical Solution

$$\mathbf{x} = \begin{pmatrix} \frac{7}{9} & -\frac{1}{3} & \frac{2}{9} \\ \frac{1}{3} & 0 & -\frac{1}{3} \\ -\frac{1}{9} & \frac{1}{3} & \frac{1}{9} \end{pmatrix} \begin{pmatrix} 6 \\ 11 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Final Answer:

$$\boxed{x = 1, \quad y = 2, \quad z = 3}$$

C Code

```c
#include <stdio.h>

void inverse3x3(double A[3][3], double inv[3][3]);
void multiply3x3_3x1(double A[3][3], double B[3], double result[3]);
void printVector(double v[3]);

int main() {
    double A[3][3] = {
        {1, 1, 1},
        {0, 1, 3},
        {1, -2, 1}
    };
    double b[3] = {6, 11, 0};
    double A_inv[3][3];
    double x[3];
```

C Code

```
inverse3x3(A, A_inv);
    multiply3x3_3x1(A_inv, b, x);

    printf(Solution:\n);
    printf(x = %.6lf\n, x[0]);
    printf(y = %.6lf\n, x[1]);
    printf(z = %.6lf\n, x[2]);

    return 0;
}
void inverse3x3(double A[3][3], double inv[3][3]) {
    double det =
          A[0][0]*(A[1][1]*A[2][2] - A[1][2]*A[2][1])
        - A[0][1]*(A[1][0]*A[2][2] - A[1][2]*A[2][0])
        + A[0][2]*(A[1][0]*A[2][1] - A[1][1]*A[2][0]);

    if(det == 0) {
        printf(Matrix is singular, no inverse.\n);
        return;
```

C Code

```
    inv[0][0] = (A[1][1]*A[2][2] - A[1][2]*A[2][1]) * invDet;
    inv[0][1] = -(A[0][1]*A[2][2] - A[0][2]*A[2][1]) * invDet;
    inv[0][2] = (A[0][1]*A[1][2] - A[0][2]*A[1][1]) * invDet;

    inv[1][0] = -(A[1][0]*A[2][2] - A[1][2]*A[2][0]) * invDet;
    inv[1][1] = (A[0][0]*A[2][2] - A[0][2]*A[2][0]) * invDet;
    inv[1][2] = -(A[0][0]*A[1][2] - A[0][2]*A[1][0]) * invDet;

    inv[2][0] = (A[1][0]*A[2][1] - A[1][1]*A[2][0]) * invDet;
    inv[2][1] = -(A[0][0]*A[2][1] - A[0][1]*A[2][0]) * invDet;
    inv[2][2] = (A[0][0]*A[1][1] - A[0][1]*A[1][0]) * invDet;
}
```

C Code

```
void multiply3x3_3x1(double A[3][3], double B[3], double result
    [3]) {
    for(int i = 0; i < 3; i++) {
        result[i] = 0;
        for(int j = 0; j < 3; j++) {
            result[i] += A[i][j] * B[j];
        }
    }
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

x_vals = np.linspace(-10, 10, 50)
y_vals = np.linspace(-10, 10, 50)
x, y = np.meshgrid(x_vals, y_vals)

z1 = 6 - x - y
z2 = (11 - y) / 3
z3 = 2*y - x

A = np.array([
    [1, 1, 1],
    [0, 1, 3],
    [1, -2, 1]
])
b = np.array([6, 11, 0])
```

Python Code

```
intersection_point = np.linalg.solve(A, b)
px, py, pz = intersection_point

fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')

ax.plot_surface(x, y, z1, alpha=0.5, color='red', label='Plane 1')
ax.plot_surface(x, y, z2, alpha=0.5, color='green', label='Plane 2')
ax.plot_surface(x, y, z3, alpha=0.5, color='blue', label='Plane 3')

ax.scatter(px, py, pz, color='black', s=100, label='Intersection Point')
ax.text(px, py, pz + 1, f({px:.2f}, {py:.2f}, {pz:.2f}), color='black')
```

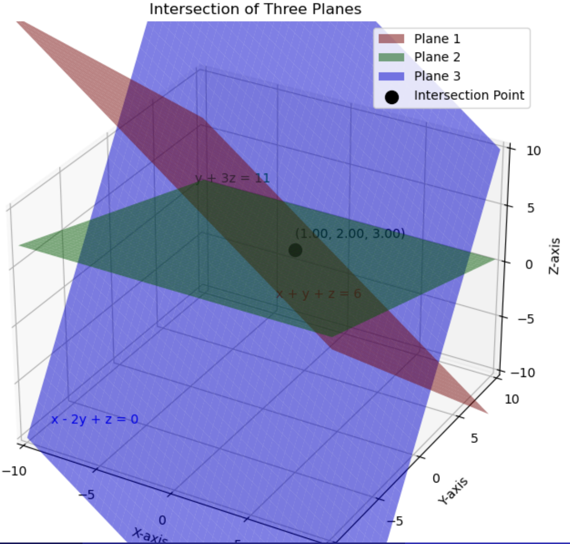
Python Code

```
ax.text(6, -9, 8, x + y + z = 6, color='red')
ax.text(-10, 9, (11 - 9)/3, y + 3z = 11, color='green')
ax.text(-9, -9, 2*(-9) - (-9), x - 2y + z = 0, color='blue')
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_xlim(-10, 10)
ax.set_ylim(-10, 10)
ax.set_zlim(-10, 10)
ax.set_xticks(np.linspace(-10, 10, 5))
ax.set_yticks(np.linspace(-10, 10, 5))
ax.set_zticks(np.linspace(-10, 10, 5))

ax.grid(True)
ax.set_title('Intersection of Three Planes')
ax.legend()

plt.show()
```

Plot


Intersection of Three Planes
Plane 1
Plane 2
Plane 3
Intersection Point
y + 3z = 11
(1.00, 2.00, 3.00)
x + y + z = 6
x - 2y + z = 0
10
5
0
−5
−10
Z-axis
10
5
0
−5
Y-axis
−10
−5
0
X-axis