Problem 2.10.20.

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Question

Question: Which of the following expressions are meaningful?

(a) $\vec{u} \cdot (\vec{v} \times \vec{w})$

(b) $(\vec{u} \cdot \vec{v}) \cdot \vec{w}$

(c) $(\vec{u} \cdot \vec{v})\ \vec{w}$

(d) $\vec{u} \times (\vec{v} \cdot \vec{w})$

Solution

Let $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ be vectors in $\mathbb{R}^3$.

(a) $\mathbf{u}\big(\mathbf{v} \times \mathbf{w}\big)$:
The expression $\mathbf{v} \times \mathbf{w}$ is a vector (cross product), and the expression $\mathbf{u}\big(\mathbf{v} \times \mathbf{w}\big)$ denotes the scalar triple product (sometimes written as the inner product of $\mathbf{u}$ and the vector $\mathbf{v} \times \mathbf{w}$).
Meaningful.

(b) $\big(\mathbf{u}^\top \mathbf{v}\big)\mathbf{w}$:
Here, $\big(\mathbf{u}^\top \mathbf{v}\big)$ represents the inner (dot) product, which is a scalar. Multiplying a scalar by a vector $\mathbf{w}$ is valid. However, if it is interpreted as $(\mathbf{u}^\top \mathbf{v})^\top \mathbf{w}$ having a dot between scalar and vector, that is not defined.
Not meaningful if interpreted as scalar dot vector.

(c) $\langle \mathbf{u}^\top \mathbf{v} \rangle^\top \mathbf{w}$:
$\langle \mathbf{u}, \mathbf{v} \rangle$ denotes the inner product (a scalar) and multiplying this scalar by vector $\mathbf{w}$ is valid scalar multiplication of a vector.
Meaningful.

Solution

(d) $\mathbf{u} \times (\mathbf{v}^\top \mathbf{w})$:
$\mathbf{v}^\top \mathbf{w}$ inside parentheses denotes the inner product (scalar), and cross product between a vector and scalar is undefined.
Not meaningful.

Answer: Only (a) and (c) are meaningful

Graph

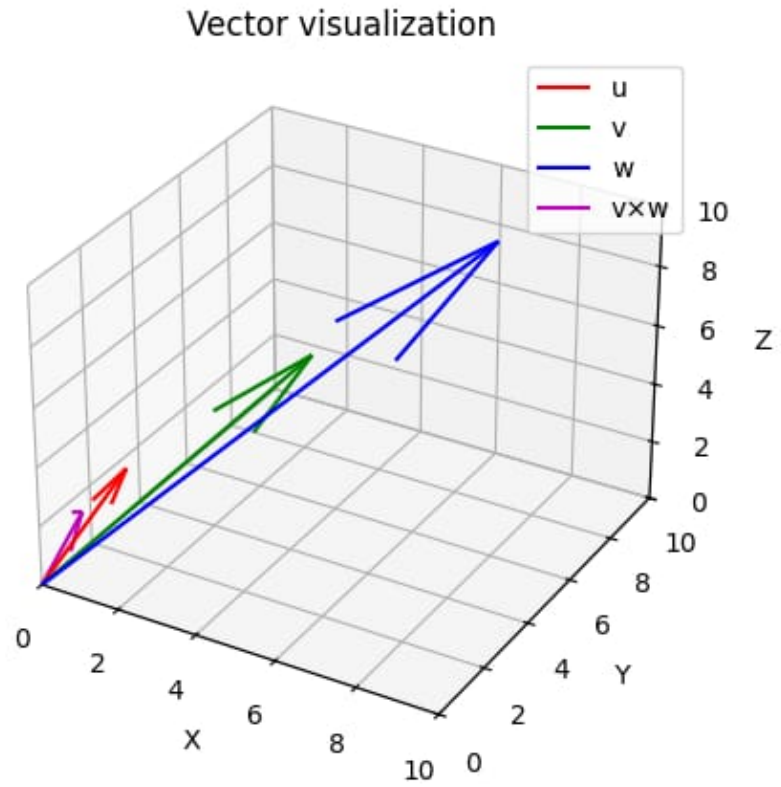

Figure: Vector Representation

C Code

```c
#include <stdio.h>
#include "matfun.h"

void print_vector(const char* name, const double v[3]) {
    printf("%s = (%.2f, %.2f, %.2f)\n", name, v[0], v[1], v[2]);
}

int main() {
    double u[3] = {1, 2, 3};
    double v[3] = {4, 5, 6};
    double w[3] = {7, 8, 9};

    double cross_vw[3];
    cross_product(v, w, cross_vw);

    double dot_u_crossvw = dot_product(u, cross_vw);
    printf("u  (v  w) = %.2f\n", dot_u_crossvw);
```

C Code

```
 double dot_uv = dot_product(u, v);
    printf("(u  v) = %.2f\n", dot_uv);

    printf("(u  v)  w is NOT meaningful as dot product of scalar
        and vector.\n");

    printf("(u  v) * w (scalar multiplication) = (%.2f, %.2f, %.2
        f)\n",
           dot_uv * w[0], dot_uv * w[1], dot_uv * w[2]);

    printf("v  w = %.2f\n", dot_product(v, w));
    printf("u  (v  w) is NOT meaningful as cross product of
        vector and scalar.\n");

    return 0;
}
```

Python Code for Plotting

```python
import matplotlib.pyplot as plt
import numpy as np

# Define the three points
points = np.array([[1, -1], [0, 5], [3, 2]])

# Extract x and y coordinates
x = points[:, 0]
y = points[:, 1]

# Plot the points
plt.plot(x, y, 'ro')

# Annotate the points
for i, (xi, yi) in enumerate(points):
    plt.text(xi + 0.1, yi, f'({xi},{yi})')
```

Python Code for Plotting

```
# Draw the triangle by connecting points and closing the loop
triangle = plt.Polygon(points, closed=True, fill=True, color='
    cyan', alpha=0.3)
plt.gca().add_patch(triangle)

# Set limits
plt.xlim(min(x)-1, max(x)+1)
plt.ylim(min(y)-1, max(y)+1)

# Title and labels
plt.title('Triangle formed by points')
plt.xlabel('X-axis')
plt.ylabel('Y-axis')

# Save the figure
plt.savefig('triangle_area.png')

plt.show()
```