8.4.28

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Question

The axis of the parabola is along the line $y = x$ and trhe distance of its vertex and focus from origin are $\sqrt{2}$ and $2\sqrt{2}$ respectively. If the vertex and focus both lie in the first quadrant, then the equation of the parabola is

$$2(x+y)^2 = (x-y-2)(x-y)^2 = (x+y-2)(x-y)^2 = 4(x+y-2)(x-y)^2 = 8(x+y-2)$$

Let:
Focus of the parabola be $\mathbf{F}$
Vertex of the parabola be $\mathbf{V}$
Normal vector to the directrix be $\mathbf{n}$
The point of intersection of directrix and axis be $\mathbf{P}$
Direction vector and slope of axis be $\mathbf{m}_1$ and m_1
Direction vector and slope of directrix be $\mathbf{m}_2$ and m_2
Equation of axis be $\mathbf{x} = \lambda \mathbf{m}_1$

Given

Given:

$$\|\mathbf{F}\| = 2\sqrt{2} \tag{1}$$

$$\|\mathbf{V}\| = \sqrt{2} \tag{2}$$

$$\mathbf{m}_1 = \begin{pmatrix} 1 \\ m_1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{3}$$

Focus, Vertex

Finding focus($\mathbf{F}$):

$$\lambda \mathbf{m}_1 = \mathbf{F} \tag{4}$$

$$\lambda = \pm \frac{\|\mathbf{F}\|}{\|\mathbf{m}_1\|} \tag{5}$$

$$\mathbf{F} = \pm \frac{\|\mathbf{F}\|}{\|\mathbf{m}_1\|} \mathbf{m}_1 \tag{6}$$

Finding vertex($\mathbf{V}$)

$$\lambda \mathbf{m}_1 = \mathbf{V} \tag{7}$$

$$\lambda = \pm \frac{\|\mathbf{V}\|}{\|\mathbf{m}_1\|} \tag{8}$$

$$\mathbf{V} = \pm \frac{\|\mathbf{V}\|}{\|\mathbf{m}_1\|} \mathbf{m}_1 \tag{9}$$

Directrix

Since directrix will be perpendicular to axis $m_1 m_2 = -1$

$$\mathbf{m_2} = \begin{pmatrix} 1 \\ m_2 \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{-1}{m_1} \end{pmatrix} \tag{10}$$

$$\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{m_2} \tag{11}$$

Since **V** will be midpoint of **P** and **F** and also **P**:

$$\frac{\mathbf{P} + \mathbf{F}}{2} = \mathbf{V} \tag{12}$$

$$\mathbf{P} = 2\mathbf{V} - \mathbf{F} \tag{13}$$

Parabola

Now, finding the directrix equation in normal form;

$$\mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{P} \tag{14}$$

From equation of conic $\mathbf{x}^\top V\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$
For parabola:

$$V = \|\mathbf{n}\|^2 I - \mathbf{n}\mathbf{n}^\top \tag{15}$$

$$\mathbf{u} = c\mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F} = \mathbf{n}^2 \mathbf{P}\mathbf{n} - \|\mathbf{n}\|^2 \frac{\|\mathbf{F}\|}{\|\mathbf{m}_1\|}\mathbf{m}_1 \tag{16}$$

$$f = \|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 = \|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - \left(\mathbf{n}^\top \mathbf{P}\right)^2 \tag{17}$$

Conclusion

Substituting values given in question we get:

$$V = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \tag{18}$$

$$\mathbf{u} = -4\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{19}$$

$$f = 16 \tag{20}$$

Substituting (18), (19) and (20) in conic equation we get:

$$\begin{pmatrix} x & y \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + (-4)\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + 16 = 0 \tag{21}$$

$$(x+y)^2 = 8\left(x+y-2\right) \tag{22}$$

Hnece, option(4) is correct.

C Code

```
#include<stdio.h>
#include<math.h>

double normf;
double normv;
double dvectora[2] = {1,1};
double m1 = 1;

double norm(double arr1[2]){
    double sum = 0;
    for(int i = 0; i<2; i++){
        sum+=(sqrt(arr1[i]*arr1[i]));
    }
    return sum;
}
```

C code

```c
double vdotv(double arr1[2], double arr2[2]){
    double sum = 0;
    for(int i = 0; i<2; i++){
        sum+=(arr1[i]*arr2[i]);
    }
    return sum;
}
void vplusv(double t, double v1[2], double v2[2], double P[2]){
    if(t==1){
        for(int i = 0; i<2; i++){
            P[i]=v1[i]+v2[i];
        }
    }
    if(t==-1){
        for(int i = 0; i<2; i++){
            P[i]=v1[i]-v2[i];
        }
    }
}
```

C code

```
void give_data(double *points){

    normf = 2*sqrt(2); normv = sqrt(2);
    double lambda;
    lambda =(normf/norm(dvectora));
    double vectorF[2];
    for(int i = 0; i<2; i++){
        vectorF[i] = lambda*dvectora[i];
    }
    lambda = (normv/norm(dvectora));
    double vectorV[2];
    for(int i = 0; i<2; i++){
        vectorV[i] = lambda*dvectora[i];
    }
```

C code

```
double m2 = -1/m1;
double dvectorb[2] = {1, m2};
double X[2] = {2*vectorV[0], 2*vectorV[1]};
double P[2];
vplusv(1, X, vectorF, P);
double c;
double n[2]={0};
double imp[2][2] = {{0, 1},{-1, 0}};
for(int i = 0; i<2; i++){
    for(int j = 0; j<2; j++){
        n[i]+=imp[i][j]*dvectora[j];}}
c = vdotv(n, P);
 points[0] = -8;
 points[1] = -2;
 points[2] = 16;}
```

Python code 1

```python
import ctypes as ct

lib = ct.CDLL("./problem.so")

lib.give_data.argtypes = [ct.POINTER(ct.c_double)]

points = ct.c_double*3
data = points()
lib.give_data(data)

def send_data():
    return data[0], data[1], data[2]
```

Python code 2

```
import matplotlib.pyplot as plt
import numpy as np
from call import send_data
a, b, c = send_data()
def parabola(x,y):
    return x**2+y**2+b*x*y+a*x+a*y+c
X, Y = np.meshgrid((np.linspace(-15, 15, 400)), (np.linspace(-15,
     15, 400)))
Z = parabola(X,Y)
p = np.linspace(-10, 10, 200)
q = p
r = p = np.linspace(-10, 10, 200)
s = -r
```

Python code 2

```
plt.plot(p,q,"-r")
plt.plot(r,s,"-g")
plt.plot(2,2,'ko')
plt.plot(1,1,'ko')
plt.contour(X,Y,Z,levels=[0], colors = "black", linewidths = 1)
plt.text(-6.5,-5.7,"y=x",color='black',fontsize = 12)
plt.text(-7.14,7.18,"y=-x", color = 'black', fontsize = 12)
plt.text(2.52, 11.64, r'$(x+y)^2=8(x+y-2)$', color = 'black',
    fontsize = 12)
plt.text(2.4,2,"(2,2)", color = 'black', fontsize = 10)
plt.text(1.05,0.65, "(1,1)", color = 'black', fontsize = 10)
```

Python code 2

```
plt.xlabel("X-axis")
plt.ylabel("Y-axis")
plt.axvline(0, color='black', linewidth=2)
plt.axhline(0,color='black',linewidth=2)
plt.axis("equal")
plt.grid(True)
plt.show()
```

Plot

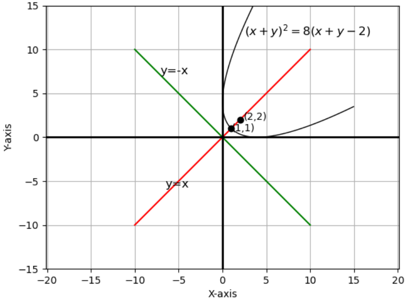

Figure: Plot of the parabola