4.2.22

EE25BTECH11019 - Darji Vivek M.

Question:
Show that the two lines

$$a_1x + b_1y + c_1 = 0, \qquad a_2x + b_2y + c_2 = 0$$

where $b_1b_2 \neq 0$ are parallel if $\frac{a_1}{b_1} = \frac{a_2}{b_2}$.
Solution:

$$\text{Normal vector of first line: } \mathbf{n}_1 = \begin{pmatrix} a_1 \\ b_1 \end{pmatrix}, \tag{1}$$

$$\text{Normal vector of second line: } \mathbf{n}_2 = \begin{pmatrix} a_2 \\ b_2 \end{pmatrix} \tag{2}$$

If $\frac{a_1}{b_1} = \frac{a_2}{b_2}$ then

$$a_1b_2 - a_2b_1 = 0 \tag{3}$$

Noting that

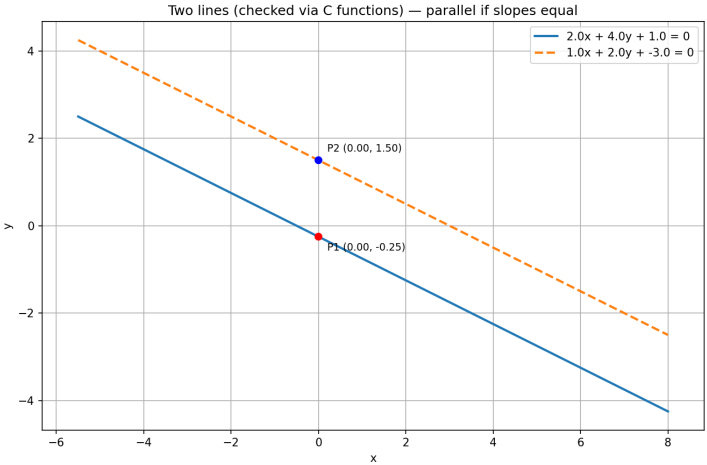
$$a_1b_2 - a_2b_1 = \det\begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} \tag{4}$$

we have $\det[\mathbf{n}_1, \mathbf{n}_2] = 0$, which means the two normal vectors are collinear, i.e.

$$\mathbf{n}_1 = \lambda\,\mathbf{n}_2 \quad \text{for some } \lambda \in \mathbb{R} \tag{5}$$

Since normals are scalar multiples, the direction vectors of the two lines are also scalar multiples, hence the two lines have identical slopes and are parallel.

Therefore, the two lines are parallel whenever $\frac{a_1}{b_1} = \frac{a_2}{b_2}$ (with $b_1b_2 \neq 0$).



Two lines (checked via C functions) — parallel if slopes equal
2.0x + 4.0y + 1.0 = 0
1.0x + 2.0y + -3.0 = 0
P2 (0.00, 1.50)
P1 (0.00, -0.25)
4
2
0
−2
−4
y
−6
−4
−2
0
2
4
6
8
x

Fig. 0.1: plot