2.4.29

EE25BTECH11042 - Nipun Dasari

Question:
The points $\mathbf{A}(2, 9)$, $\mathbf{B}(a, 5)$ and $\mathbf{C}(5, 5)$ are the vertices of a triangle **ABC** right angled at **B**. Find the values of a and hence the area of Δ**ABC**.
Solution:

Given the points A, B and C, also consider **c** to be vector opposite to side AB and **b**, **a** similarly

$$\mathbf{A} = \begin{pmatrix} 2 \\ 9 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} a \\ 5 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 5 \\ 5 \end{pmatrix} \tag{0.1}$$

Since the sides c and a are perpendicular their inner product will be 0
Take the inner product of **c** and **a**
Vector **c**:

$$\mathbf{c} = \mathbf{A} - \mathbf{B} = \begin{pmatrix} 2 - a \\ 9 - 5 \end{pmatrix} = \begin{pmatrix} 2 - a \\ 4 \end{pmatrix} \tag{0.2}$$

Vector **a**:

$$\mathbf{a} = \mathbf{B} - \mathbf{C} = \begin{pmatrix} a - 5 \\ 5 - 5 \end{pmatrix} = \begin{pmatrix} a - 5 \\ 0 \end{pmatrix} \tag{0.3}$$

Orthogonality $\implies$ matrix product is zero :

$$\mathbf{c}^T\mathbf{a} = \begin{pmatrix} 2 - a & 4 \end{pmatrix} \begin{pmatrix} a - 5 \\ 0 \end{pmatrix} = (2 - a)(a - 5) = 0 \tag{0.4}$$

So $(2 - a)(5 - a) = 0 \implies a = 2$ or $a = 5$.
$a = 5$ make **B**=**C**. $\therefore a = 2$
We can compute area using cross product formula

$$\Delta = \frac{1}{2} \|\mathbf{c} \times \mathbf{a}\| \tag{0.5}$$

The general cross product of two vectors is defined as:

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} |\mathbf{A}_{23} & \mathbf{B}_{23}| \\ |\mathbf{A}_{31} & \mathbf{B}_{31}| \\ |\mathbf{A}_{12} & \mathbf{B}_{12}| \end{pmatrix} \tag{0.6}$$

The vectors in 3-D space look like

$$\mathbf{c} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \tag{0.7}$$

$$\mathbf{a} = \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix} \tag{0.8}$$

$$\mathbf{c}_{31} \ \mathbf{a}_{31}| = | \begin{vmatrix} 0 & 0 \\ 0 & -3 \end{vmatrix} = 0 \tag{0.9}$$

$$|\mathbf{c}_{23} \ \mathbf{a}_{23}| = \begin{vmatrix} 4 & 0 \\ 0 & 0 \end{vmatrix} = 0 \tag{0.10}$$

$$|\mathbf{c}_{12} \ \mathbf{a}_{12}| = \begin{vmatrix} 0 & 4 \\ -3 & 0 \end{vmatrix} = 12 \tag{0.11}$$

By (0.6):

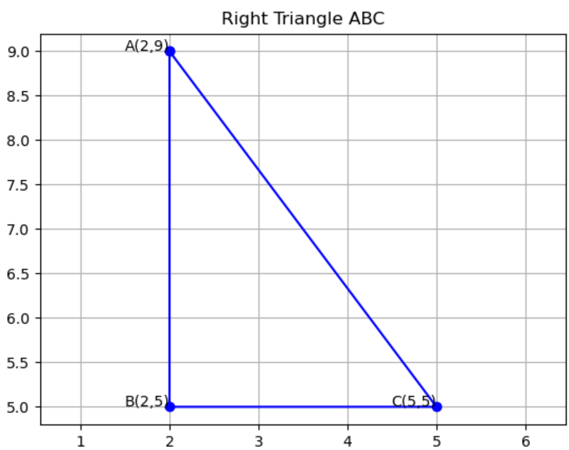
$$\mathbf{c} \times \mathbf{a} = \begin{pmatrix} 0 \\ 0 \\ 12 \end{pmatrix} \tag{0.12}$$

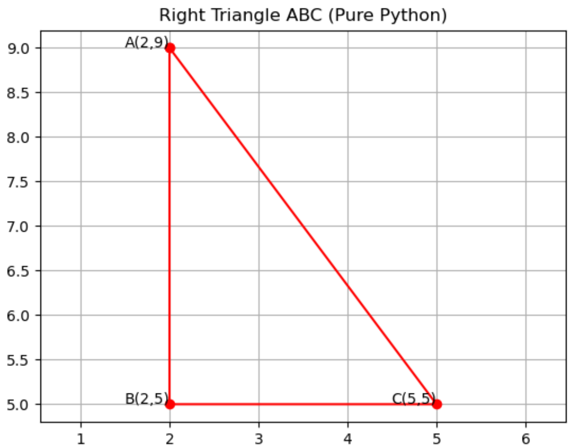
$$\|\mathbf{c} \times \mathbf{a}\| = 12 \tag{0.13}$$

Using (0.5)

$$\therefore \Delta = \frac{1}{2} \|\mathbf{c} \times \mathbf{a}\| = 6 \tag{0.14}$$

Thus area of triangle is 6




Right Triangle ABC (Pure Python)
A(2,9)
B(2,5)
C(5,5)
9.0
8.5
8.0
7.5
7.0
6.5
6.0
5.5
5.0
1
2
3
4
5
6