

2.2.14

AI25btech11022 - Narshitha

Question:

The angle between the line

$$\mathbf{r} = (5\hat{i} - \hat{j} - 4\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k}) \quad (1)$$

and the plane

$$\mathbf{r} \cdot (3\hat{i} - 4\hat{j} - \hat{k}) + 5 = 0 \quad (2)$$

is

Solution: The given line can be expressed in the form as

$$\mathbf{r} = \begin{pmatrix} 5 \\ -1 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \quad (3)$$

Hence the vector direction of this line is

$$\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \quad (4)$$

the normal vector of the given plane is

$$\begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix} \quad (5)$$

by the formula

$$\cos \theta = \frac{\mathbf{a}^T \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} \quad (6)$$

Thus the cosine of the angle between the two is

$$\frac{3\sqrt{3}}{2\sqrt{13}} \quad (7)$$

which is sine of the angle between the plane and the line.

\therefore The angle between the line and plane is $\sin^{-1} \frac{3\sqrt{3}}{2\sqrt{13}}$

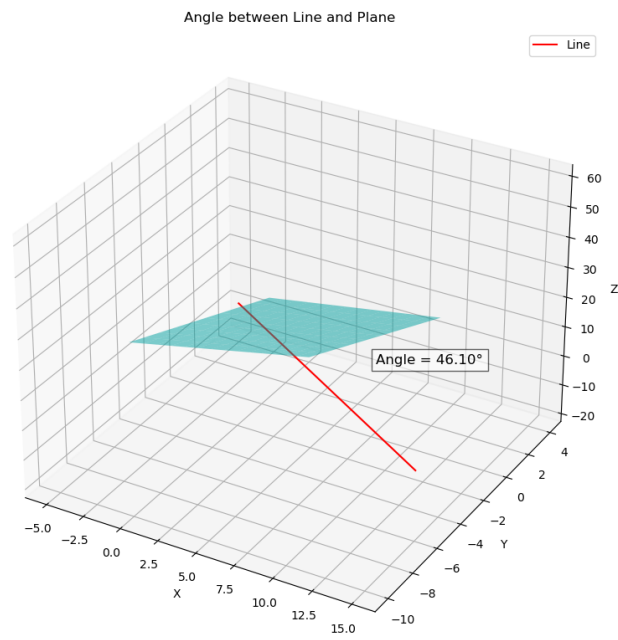


Fig. 0. Caption