4.13.90

Abhiram Reddy-AI25BTECH11021

September 25,2025

Question 4.13.90

If the distance of the point $\mathbf{P}(1, -2, 1)$ from the plane

$$x + 2y - 2z = \alpha, \quad \text{where } \alpha > 0,$$

is 5, then the foot of the perpendicular from $\mathbf{P}$ to the plane is:
4

1. $\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)$
2. $\left(\frac{4}{3}, -\frac{4}{3}, \frac{1}{3}\right)$
3. $\left(\frac{1}{3}, \frac{2}{3}, \frac{10}{3}\right)$
4. $\left(\frac{2}{3}, -\frac{1}{3}, \frac{5}{2}\right)$

Step 1: Use Distance Formula

Let

$$\mathbf{n} = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

The distance from point $\mathbf{P}$ to the plane is:

$$D = \frac{|\mathbf{n}^T \mathbf{P} - \alpha|}{\|\mathbf{n}\|} \tag{1}$$

Compute:

$$\mathbf{n}^T \mathbf{P} = 1 - 4 - 2 = -5 \tag{2}$$

$$\|\mathbf{n}\| = \sqrt{1^2 + 2^2 + (-2)^2} = \sqrt{9} = 3 \tag{3}$$

Given $D = 5$, solve:

$$5 = \frac{|-5 - \alpha|}{3} \Rightarrow |-5 - \alpha| = 15 \tag{4}$$

Step 2: Solve for α

From:

$$|-5-\alpha| = 15$$

Case 1:

$$-5-\alpha = 15 \Rightarrow \alpha = -20 \quad \text{(invalid)} \tag{5}$$

Case 2:

$$-5-\alpha = -15 \Rightarrow \alpha = 10 \tag{6}$$

So, the plane becomes:

$$x + 2y - 2z = 10 \tag{7}$$

Step 3: Foot of Perpendicular

Let the foot be $\mathbf{Q} = \mathbf{P} + \lambda \mathbf{n}$

$$\mathbf{Q} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 1+\lambda \\ -2+2\lambda \\ 1-2\lambda \end{bmatrix} \tag{8}$$

Substitute into plane:

$$(1+\lambda) + 2(-2+2\lambda) - 2(1-2\lambda) = 10$$

$$1 + \lambda - 4 + 4\lambda - 2 + 4\lambda = 10 \Rightarrow -5 + 9\lambda = 10 \Rightarrow \lambda = \frac{15}{9} = \frac{5}{3}$$

Step 4: Final Answer

Substitute $\lambda = \frac{5}{3}$ into $\mathbf{Q}$:

$$\mathbf{Q} = \begin{bmatrix} 1 + \frac{5}{3} \\ -2 + \frac{10}{3} \\ 1 - \frac{10}{3} \end{bmatrix} = \begin{bmatrix} \frac{8}{3} \\ \frac{4}{3} \\ -\frac{7}{3} \end{bmatrix} \tag{9}$$

$$\boxed{\left(\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}\right)} \quad \text{Option 1}$$

C Code – Function

```c
#include <math.h>

double distanceFromPointToPlane(double A, double B, double C,
    double D_plane,
                                double x0, double y0, double z0) {
    double numerator = fabs(A * x0 + B * y0 + C * z0 - D_plane);
    double denominator = sqrt(A * A + B * B + C * C);
    return numerator / denominator;
}
```

C Code – Main Function

```c
#include <stdio.h>

int main() {
    double A = 1, B = 2, C = -2;
    double alpha = 10;
    double x0 = 1, y0 = -2, z0 = 1;
    double distance = distanceFromPointToPlane(A, B, C, alpha, x0
        , y0, z0);
    printf(Distance from point to plane = %.2f\n, distance);
    return 0;
}
```

Python Code: Setup and Locus Data

Generating Data for the Plot

We use $c = 5$ and an example line with intercepts $a = 25/3$ and $b = 25/4$.

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

P = np.array([1, -2, 1])
F = np.array([8/3, 4/3, -7/3])
A, B, C, D_plane = 1, 2, -2, 10

fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')

ax.scatter(*P, color='r', s=100, label=r'Point P(1, -2, 1)')
ax.scatter(*F, color='g', s=100, label=r'Foot of Perpendicular F($\frac{8}{3}, \frac{4}{3}, -\frac{7}{3}$)')

ax.plot([P[0], F[0]], [P[1], F[1]], [P[2], F[2]], 'k--', label='Perpendicular Line PF')
```

Python Code: Plotting the Graphs

Visualization of the Locus and an Example Line

```
zz = (A * xx + B * yy - D_plane) / C

ax.plot_surface(xx, yy, zz, alpha=0.5, rstride=100, cstride=100,
    color='c')

ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title(r'3D Plot of Point, Plane, and Foot of Perpendicular
    ')

ax.legend()
ax.view_init(elev=20, azim=45)
ax.set_box_aspect([np.ptp(a) for a in [ax.get_xlim(), ax.get_ylim
    (), ax.get_zlim()]])

plt.savefig('python_plot.png')
```

Plot

figs/python_plot.png