

4.4.33

AI25BTECH11037-stalin

Question:

Find the value of x such that the four points with position vectors $\mathbf{A}(3\hat{i}+2\hat{j}+\hat{k})$, $\mathbf{B}(4\hat{i}+x\hat{j}+5\hat{k})$, $\mathbf{C}(4\hat{i}+2\hat{j}-2\hat{k})$, and $\mathbf{D}(6\hat{i}+5\hat{j}-\hat{k})$ are coplanar. (12, 2018)

Solution:

Let us solve the given equation theoretically and then verify the solution computationally

According to the question,

Given four position vectors

$$\mathbf{A} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 4 \\ x \\ 5 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 6 \\ 5 \\ -1 \end{pmatrix} \quad (0.1)$$

$$\mathbf{A}^T \mathbf{n} = 1 \quad (0.2)$$

$$\mathbf{B}^T \mathbf{n} = 1 \quad (0.3)$$

$$\mathbf{C}^T \mathbf{n} = 1 \quad (0.4)$$

$$\mathbf{D}^T \mathbf{n} = 1 \quad (0.5)$$

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \end{pmatrix}^T \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad (0.6)$$

Let

$$\mathbf{i} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad \mathbf{z} = \begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \end{pmatrix}^T \quad (0.7)$$

condition is Rank of $\begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \end{pmatrix}^T = 3$ and $\begin{pmatrix} \mathbf{z} & \mathbf{i} \end{pmatrix} = 3$

From solving we get $x=5$.

3D Graph of Points A, B, C, D

