Problem 4.12.44

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Question: Find the equation of the set of points which are equidistant from the points $\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$.

Solution

Solution: Let $\mathbf{X}$ be the position vector of any point equidistant from $\mathbf{A}$ and $\mathbf{B}$. The equidistance condition is

$$\|\mathbf{X} - \mathbf{A}\| = \|\mathbf{X} - \mathbf{B}\|. \tag{2.1}$$

Squaring both sides, we have

$$(\mathbf{X} - \mathbf{A})^\top (\mathbf{X} - \mathbf{A}) = (\mathbf{X} - \mathbf{B})^\top (\mathbf{X} - \mathbf{B}). \tag{2.2}$$

Expanding and simplifying,

$$\mathbf{X}^\top \mathbf{X} - 2\mathbf{A}^\top \mathbf{X} + \mathbf{A}^\top \mathbf{A} = \mathbf{X}^\top \mathbf{X} - 2\mathbf{B}^\top \mathbf{X} + \mathbf{B}^\top \mathbf{B}, \tag{2.3}$$

which reduces to

$$-2\mathbf{A}^\top \mathbf{X} + \mathbf{A}^\top \mathbf{A} = -2\mathbf{B}^\top \mathbf{X} + \mathbf{B}^\top \mathbf{B}. \tag{2.4}$$

Rearranging,

$$2(\mathbf{B} - \mathbf{A})^\top \mathbf{X} = \mathbf{B}^\top \mathbf{B} - \mathbf{A}^\top \mathbf{A}. \tag{2.5}$$

Calculate the vector difference:

Solution

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3-1 \\ 2-2 \\ -1-3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}. \tag{2.6}$$

Calculate the scalar values:

$$\mathbf{B}^\top\mathbf{B} = 3^2 + 2^2 + (-1)^2 = 14, \quad \mathbf{A}^\top\mathbf{A} = 1^2 + 2^2 + 3^2 = 14, \tag{2.7}$$

so the right side is zero:

$$\mathbf{B}^\top\mathbf{B} - \mathbf{A}^\top\mathbf{A} = 0. \tag{2.8}$$

Thus, substituting the simplified difference vector, the plane equation becomes:

$$4\begin{pmatrix} 1 & 0 & -2 \end{pmatrix}\mathbf{X} = 0, \tag{2.9}$$

or equivalently,

$$\begin{pmatrix} 1 & 0 & -2 \end{pmatrix}\mathbf{X} = 0. \tag{2.10}$$

Solution

Final Answer: The set of points equidistant from **A** and **B** lies on the plane defined by

$$\boxed{\begin{pmatrix} 1 & 0 & -2 \end{pmatrix} \mathbf{x} = 0}$$

Graph

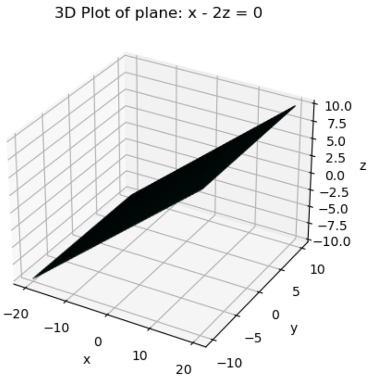

Figure: Vector Representation

C Code

```c
#include <stdio.h>
#include "matfun.h"

int main() {
    double A[3] = {1, 2, 3};
    double B[3] = {3, 2, -1};
    double BA[3];
    double rhs;

    // Compute B - A
    vector_subtract(B, A, BA, 3);

    // Compute dot products B.B and A.A
    double BTB = dot_product(B, B, 3);
    double ATA = dot_product(A, A, 3);

    rhs = BTB - ATA;
```

C Code

```c
    // Multiply BA by 2
    scalar_multiply(BA, BA, 2, 3);

    printf("Vector (2(B - A)) is: [%.2f, %.2f, %.2f]\n", BA[0],
        BA[1], BA[2]);
    printf("Right-hand side value (B.B - A.A): %.2f\n", rhs);

    printf("Equation of the plane in vector form: [%.2f %.2f %.2f
        ] \\cdot X = %.2f\n", BA[0], BA[1], BA[2], rhs/2);

    return 0;
}
```

Python Code for Plotting

```python
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Create grid for y and z
y_vals = np.linspace(-10, 10, 100)
z_vals = np.linspace(-10, 10, 100)
Y, Z = np.meshgrid(y_vals, z_vals)

# Calculate corresponding x using the plane equation: x = 2z
X = 2 * Z

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
```

Python Code for Plotting

```
# Plot the plane
ax.plot_surface(X, Y, Z, alpha=0.5, color='cyan', edgecolor='k')

# Set axis labels with x, y, z
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')
ax.set_title('3D Plot of plane: x - 2z = 0')
plt.savefig("fig1.png")
plt.show()
```

Python Code - Using Shared Object

```python
import ctypes
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Load the shared library
matfun = ctypes.CDLL('./matfun.so')

# Define argument and return types
matfun.vector_subtract.argtypes = [
    np.ctypeslib.ndpointer(dtype=np.float64),
    np.ctypeslib.ndpointer(dtype=np.float64),
    np.ctypeslib.ndpointer(dtype=np.float64),
    ctypes.c_int
]
matfun.dot_product.argtypes = [
    np.ctypeslib.ndpointer(dtype=np.float64),
    np.ctypeslib.ndpointer(dtype=np.float64),
```

Python Code - Using Shared Object

```
    ctypes.c_int
]
matfun.dot_product.restype = ctypes.c_double
matfun.scalar_multiply.argtypes = [
    np.ctypeslib.ndpointer(dtype=np.float64),
    np.ctypeslib.ndpointer(dtype=np.float64),
    ctypes.c_double,
    ctypes.c_int
]

# Define points A and B
A = np.array([1.0, 2.0, 3.0])
B = np.array([3.0, 2.0, -1.0])

BA = np.zeros(3)
rhs = 0.0
```

Python Code - Using Shared Object

```python
# Compute B - A using shared lib
matfun.vector_subtract(B, A, BA, 3)

# Compute dot products B.B and A.A using shared lib
rhs = matfun.dot_product(B, B, 3) - matfun.dot_product(A, A, 3)

# Calculate 2*(B - A)
BA2 = np.zeros(3)
matfun.scalar_multiply(BA, BA2, 2, 3)

print(f"Vector 2(B - A): {BA2}")
print(f"RHS (B.B - A.A): {rhs}")

# Create grid for y and z
y_vals = np.linspace(-10, 10, 100)
z_vals = np.linspace(-10, 10, 100)
Y, Z = np.meshgrid(y_vals, z_vals)
```

Python Code - Using Shared Object

```python
# From plane equation 2(B - A)^T X = rhs
# Which is BA2^T * X = rhs
# BA2 = [4, 0, -8], so the equation is 4x - 8z = 0
# => x = 2z

# Calculate corresponding x using x = 2z
X = 2 * Z

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Plot the plane surface
ax.plot_surface(X, Y, Z, alpha=0.5, color='cyan', edgecolor='k')
```

Python Code - Using Shared Object

```
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_zlabel('z')
ax.set_title('3D Plot of plane: x - 2z = 0 (from shared library)'
    )
plt.savefig("fig2.png")
plt.show()
```

Plot-Using Both C and Python