Matgeo Presentation - Problem 4.13.67

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September 10, 2025

Question

The area of the triangle formed by the intersection of line parallel to X axis and passing through **p**(h,k) with the lines y=x and x+y=2 is $4h^2$.Find the locus of point **p**

Solution

line parallel to X axis is of the form

$$y = c. \tag{0.1}$$

which can be expressed in the form of

$$\mathbf{n}^T \mathbf{x} = c. \tag{0.2}$$

$$\implies \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = c. \tag{0.3}$$

As the above line passes through $\mathbf{p}$(h,k)

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} h \\ k \end{pmatrix} = c. \implies c = k. \tag{0.4}$$

The three lines are as follows

$$y = k \implies \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = k. \tag{0.5}$$

$$-x + y = 0 \implies \begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0. \tag{0.6}$$

(0.7)

Solution

$$x + y = 2 \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 2. \tag{0.8}$$

Let **A**,**B**,**C** be the point of intersection of above 3 lines
By solving equation (5) and (6) we get

$$\mathbf{A} = \begin{pmatrix} k \\ k \end{pmatrix}. \tag{0.9}$$

By solving equation (6) and (7) we get

$$\mathbf{B} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{0.10}$$

By solving equation (5) and (7) we get

$$\mathbf{C} = \begin{pmatrix} 2 - k \\ k \end{pmatrix}. \tag{0.11}$$

Solution

$$\text{area of } \triangle \text{ABc} = \frac{1}{2}\|(\mathbf{A}-\mathbf{B})\times(\mathbf{B}-\mathbf{C})\| \tag{0.12}$$
$$= \frac{1}{2}\|\begin{pmatrix} k-1 \\ k-1 \end{pmatrix} \times \begin{pmatrix} k-1 \\ 1-k \end{pmatrix}\| \tag{0.13}$$
$$= \frac{1}{2}(2(k-1)^2) = (k-1)^2. \tag{0.14}$$

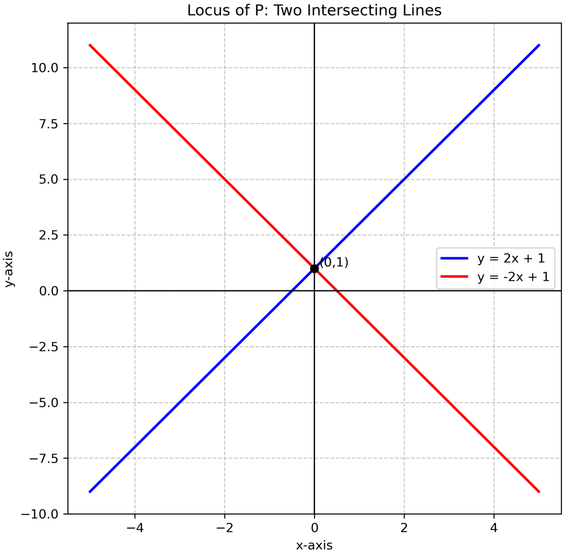
Given area of the triangle formed by the intersection of above 3 lines is 4h^2.

$$\implies (k-1)^2 = 4h^2. \tag{0.15}$$
$$\implies (y-1)^2 = 4x^2 \tag{0.16}$$
$$\implies (y-1-2x)(y-1+2x) = 0 \tag{0.17}$$

$\implies$ The locus of $\mathbf{p}$ is union of 2 straight lines

$$y-1-2x = 0. \implies \begin{pmatrix} -2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1. \tag{0.18}$$
$$y-1+2x = 0. \implies \begin{pmatrix} 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1. \tag{0.19}$$

Plot


Locus of P: Two Intersecting Lines
y-axis
x-axis
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−4
−2
0
2
4
(0,1)
y = 2x + 1
y = -2x + 1

C Code: locus.c

```c
#include <stdio.h>

int main() {
    FILE *fp;

    // Open the file locus.dat in write mode
    fp = fopen("locus.dat", "w");

    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }

    // Write the solution into the file
    fprintf(fp, "The locus of P is given by the equation:\n");
    fprintf(fp, "(y - 1)^2 = 4x^2\n");
    fprintf(fp, "Which represents two intersecting straight lines:\n");
    fprintf(fp, "1) y - 1 = 2x  -->  y = 2x + 1\n");
    fprintf(fp, "2) y - 1 = -2x -->  y = -2x + 1\n");

    // Close the file
    fclose(fp);

    printf("Locus has been written successfully into locus.dat\n");

    return 0;
}
```

Python: plot.py

```
 import numpy as np
import matplotlib.pyplot as plt

# Define range for x
x = np.linspace(-5, 5, 400)

# Equations of the two lines
y1 = 2*x + 1
y2 = -2*x + 1

# Create the plot
plt.figure(figsize=(7, 7))

# Plot both lines
plt.plot(x, y1, label="y␣=␣2x␣+␣1", color="blue", linewidth=2)
plt.plot(x, y2, label="y␣=␣-2x␣+␣1", color="red", linewidth=2)

# Mark the point of intersection (0,1)
plt.scatter(0, 1, color="black", zorder=5)
plt.text(0.1, 1.1, "(0,1)", fontsize=10)

# Axes setup
plt.axhline(0, color="black", linewidth=1)
plt.axvline(0, color="black", linewidth=1)

# Labels, grid and title
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.title("Locus␣of␣P:␣Two␣Intersecting␣Lines")
plt.grid(True, linestyle="--", alpha=0.7)
plt.legend()
plt.savefig("locus_plot.png", dpi=300)
plt.show()
```