

Matgeo Presentation - Problem 10.7.111

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Question

Let the straight line $y = 2x$ touch a circle with center $(0, a)$, $a > 0$, and radius r at a point \mathbf{A}_1 . Let \mathbf{B}_1 be the point on the circle such that the line segment $\mathbf{A}_1\mathbf{B}_1$ is a diameter of the circle. Let $a + r = 5 + \sqrt{5}$. Match the following:

- | | |
|---------------------------|---------------|
| (A) a equals | (1) $(-2, 4)$ |
| (B) r equals | 2. $\sqrt{5}$ |
| (C) \mathbf{A}_1 equals | (3) $(-2, 6)$ |
| (D) \mathbf{B}_1 equals | (4) 5 |
| | (5) $(2, 4)$ |

The correct option is

(2024)

- a) A-4, B-2, C-1, D-3
- b) A-2, B-4, C-1, D-3
- c) A-4, B-2, C-5, D-3
- d) A-2, B-4, C-3, D-5

Solution

The equation of a Conic in Matrix form is

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (0.1)$$

For the given circle. let r be the radius of given circle

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ -a \end{pmatrix}, f = a^2 - r^2 \quad (0.2)$$

Equation of Tangent is given by

$$\mathbf{n}^T \mathbf{x} = c \quad (0.3)$$

$$\Rightarrow \mathbf{n} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}, c = 0 \quad (0.4)$$

$$\Rightarrow (2 \quad -1) \begin{pmatrix} x \\ y \end{pmatrix} = 0 \quad (0.5)$$

Solution

The distance of point \mathbf{P} to the line $\mathbf{n}^\top \mathbf{x} = c$ is given by

$$d = \frac{|\mathbf{n}^\top \mathbf{P} - c|}{\|\mathbf{n}\|} \quad (0.6)$$

$$\Rightarrow r = \frac{\left| (2 \ 1) \begin{pmatrix} 0 \\ -a \end{pmatrix} - 0 \right|}{\sqrt{5}} \quad (0.7)$$

$$\Rightarrow r = \frac{a}{\sqrt{5}} \quad (0.8)$$

$$a + r = 5 + \sqrt{5} \quad (0.9)$$

substitute (8) in (9)

$$\sqrt{5}r + r = 5 + \sqrt{5} \quad (0.10)$$

$$\Rightarrow r = \sqrt{5} \quad (0.11)$$

$$\Rightarrow a = 5 \quad (0.12)$$

Solution

For a circle, the points of contact are

$$\mathbf{q}_j = \left(\pm r \frac{\mathbf{n}_j}{\|\mathbf{n}_j\|} - \mathbf{u} \right), j = 1, 2 \quad (0.13)$$

let \mathbf{A}_1 be the point of contact

$$\mathbf{A}_1 = \left(-r \frac{\mathbf{n}}{\|\mathbf{n}\|} - \mathbf{u} \right) \quad (0.14)$$

$$= \left(\frac{r}{\sqrt{5}} \begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 0 \\ -a \end{pmatrix} \right) \quad (0.15)$$

$$\Rightarrow \mathbf{A}_1 = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad (0.16)$$

Conclusion

Given \mathbf{A}_1 and \mathbf{B}_1 is the diameter of the circle

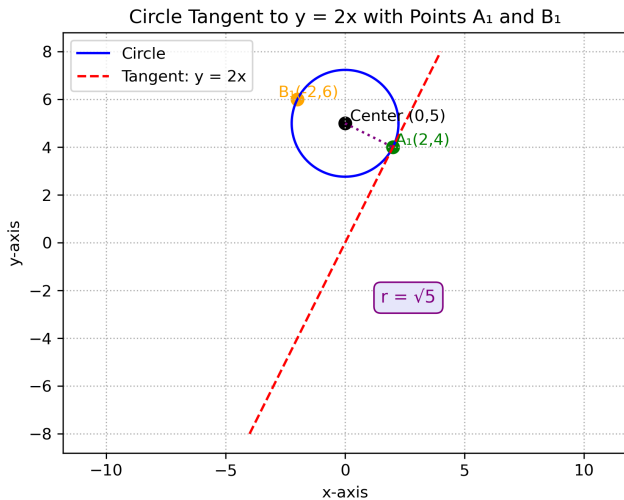
$$\frac{\mathbf{A}_1 + \mathbf{B}_1}{2} = \mathbf{u} \quad (0.17)$$

$$\mathbf{B}_1 = (\mathbf{u} \quad \mathbf{A}_1) \begin{pmatrix} 2 \\ -1 \end{pmatrix} \quad (0.18)$$

$$\mathbf{B}_1 = \begin{pmatrix} -2 \\ 6 \end{pmatrix} \quad (0.19)$$

Answer is c) A-4, B-2, C-5, D-3

Plot



Figure

C Code: circle.c

```
#include <stdio.h>
#include <math.h>

int main() {
    FILE *fp;
    double a, r;
    double A1x, A1y, B1x, B1y;

    // Given relation:  $a + r = 5 + \sqrt{5}$ 
    // and  $r = a / \sqrt{5}$ 
    // So we solve for a and r
    a = 5;
    r = sqrt(5);

    // Coordinates of A1 (point of tangency)
    A1x = 2;
    A1y = 4;

    // Coordinates of B1 (diametrically opposite point)
    B1x = -2;
    B1y = 6;

    // Open file to write output
    fp = fopen("circle.dat", "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }
}
```


C Code: circle.c

```
fprintf(fp, "Results of the circle problem:\n");
fprintf(fp, "-----\n");
fprintf(fp, "(A) a=%.2f\n", a);
fprintf(fp, "(B) r=%.5f\n", r);
fprintf(fp, "(C) A1=%.2f, %.2f\n", A1x, A1y);
fprintf(fp, "(D) B1=%.2f, %.2f\n", B1x, B1y);
fprintf(fp, "\nMatching options:\n(A)->(4), (B)->(2), (C)->(5), (D)->(3)\n");
fprintf(fp, "Correct Option: (c)\n");

fclose(fp);

printf("Results written successfully to circle.dat\n");

return 0;
}
```

Python: plot.py

```
import matplotlib.pyplot as plt
import numpy as np
import math

# Given values
a = 5 # y-coordinate of center
r = math.sqrt(5) # radius
center = (0, a)
A1 = (2, 4)
B1 = (-2, 6)

# Generate circle
theta = np.linspace(0, 2 * np.pi, 400)
x_circle = r * np.cos(theta)
y_circle = a + r * np.sin(theta)

# Tangent line  $y = 2x$ 
x_line = np.linspace(-4, 4, 200)
y_line = 2 * x_line

# Plot circle and tangent
plt.plot(x_circle, y_circle, label='Circle', color='blue')
plt.plot(x_line, y_line, color='red', linestyle='--', label='Tangent:  $y = 2x$ ')

# Plot key points
plt.scatter(*center, color='black', s=60)
plt.scatter(*A1, color='green', s=60)
plt.scatter(*B1, color='orange', s=60)

# Draw radius line
plt.plot([center[0], A1[0]], [center[1], A1[1]], color='purple', linestyle=':')
```

Python: plot.py

```
plt.text(2.1, 4.1, "A(2,4)", color='green', fontsize=10)
plt.text(-2.8, 6.1, "B(-2,6)", color='orange', fontsize=10)
plt.text(0.2, 5.1, "Center(0,5)", color='black', fontsize=10)

# Box below tangent equation showing radius
plt.text(1.5, -2.5, "r=5", fontsize=11, color='purple',
        bbox=dict(facecolor='lavender', edgecolor='purple', boxstyle='round,pad=0.4'))

# Make it neat
plt.axis('equal')
plt.grid(True, linestyle=':')
plt.xlabel('x-axis')
plt.ylabel('y-axis')
plt.title('Circle Tangent to y=2x with Points A and B')
plt.legend(loc='upper left')
plt.savefig('circle.png', dpi=300)
plt.show()
```