1.4.19

Namaswi - EE25BTECH11060

August 2025

Question

Show that the points $\mathbf{A}(1,-2,-8)$,$\mathbf{B}(5,0,-2)$ and $\mathbf{C}(11,3,7)$ are collinear and find the ratio in which B divides AC.

given data

Point	x	y	z
A	1	-2	-8
B	5	0	-2
C	11	3	7

Formula

collinearity matrix can be expressed as

$$\begin{pmatrix} B-A & C-A \end{pmatrix} = \begin{pmatrix} 4 & 10 \\ 2 & 5 \\ 6 & 15 \end{pmatrix}$$

Row reduction

$$\begin{pmatrix} 4 & 10 \\ 2 & 5 \\ 6 & 15 \end{pmatrix} R_3 \leftarrow R_3 - (R_1 + R_2) \implies \begin{pmatrix} 4 & 10 \\ 2 & 5 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 10 \\ 2 & 5 \\ 0 & 0 \end{pmatrix} R_2 \leftarrow R_2 - (R_1/2) \implies \begin{pmatrix} 4 & 10 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 10 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} R_1 \leftarrow R_1/4 \implies \begin{pmatrix} 1 & 2.5 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Which is a Rank 1 matrix , Hence $\mathbf{A}(1,-2,-8)$,$\mathbf{B}(5,0,-2)$ and $\mathbf{C}(11,3,7)$ are collinear.

Section formula for a vector $\mathbf{B}$ which divides the line formed by vectors $\mathbf{A}$ and $\mathbf{C}$ in the ratio k:1 is given by

$$\mathbf{B} = \frac{k\mathbf{C} + \mathbf{A}}{k+1} \tag{1}$$

$$\tag{2}$$

$$k\left(\mathbf{B} - \mathbf{C}\right) = \mathbf{A} - \mathbf{B} \tag{3}$$

Taking dot product on both sides with $(\mathbf{B} - \mathbf{C})$

$$k = \frac{\left(\mathbf{A} - \mathbf{B}\right)^{\top}\left(\mathbf{B} - \mathbf{C}\right)}{\left\|\mathbf{B} - \mathbf{C}\right\|^{2}} \tag{4}$$

$$\left(\mathbf{A} - \mathbf{B}\right)^{\top}\left(\mathbf{B} - \mathbf{C}\right) = \begin{pmatrix} -4 & -2 & -6 \end{pmatrix}\begin{pmatrix} -6 \\ -3 \\ -9 \end{pmatrix} = 84 \tag{5}$$

$$\left\|\mathbf{B} - \mathbf{C}\right\|^{2} = \left(\sqrt{(-6^2) + (-3)^2 + (-9)^2}\right)^{2} = 126 \tag{6}$$

B which divides **AC** in the ratio 2:3

Python Code

```
 # Plotting points A(1, -2, -8), B(5, 0, -2), and C(11, 3, 7)

import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Define the points as numpy arrays
A = np.array([1, -2, -8])
B = np.array([5, 0, -2])
C = np.array([11, 3, 7])
```

Python Code

```
 # Create a 3D plot
fig = plt.figure(figsize=(8, 6))
ax = fig.add_subplot(111, projection='3d')

# Plot the points
ax.scatter(*A, color='red', s=100, label='A(1, -2, -8)')
ax.scatter(*B, color='green', s=100, label='B(5, 0, -2)')
ax.scatter(*C, color='blue', s=100, label='C(11, 3, 7)')
```

Python Code

```
 # Plot line AC
ax.plot([A[0], C[0]], [A[1], C[1]], [A[2], C[2]], color='purple',
     label='Line AC')

# Annotate points
ax.text(*A, ' A', color='red', fontsize=10)
ax.text(*B, ' B', color='green', fontsize=10)
ax.text(*C, ' C', color='blue', fontsize=10)
```

Python Code

```
 # Set axes labels
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title('3D Plot of Points A, B, C and Line AC')
ax.legend()
ax.grid(True)

# Show the plot
plt.show()
```

C Code

```c
#include <stdio.h>

int main(){
    double Ax = 1, Ay = -2, Az = -8;
    double Bx = 5, By = 0, Bz = -2;
    double Cx = 11, Cy = 3, Cz = 7;

    double kx = (Bx - Ax) / (Cx - Bx);
    double ky = (By - Ay) / (Cy - By);
    double kz = (Bz - Az) / (Cz - Bz);

    printf("%lf",kx);

    return 0;

}
```

Python and C Code

```
 import subprocess

# Compile the C program
subprocess.run(["gcc", "points.c", "-o", "points"])

# Run the compiled C program
result = subprocess.run(["./points"], capture_output=True, text=
    True)

# Print the output from the C program (solution steps for k=2/3)
print(result.stdout)
```

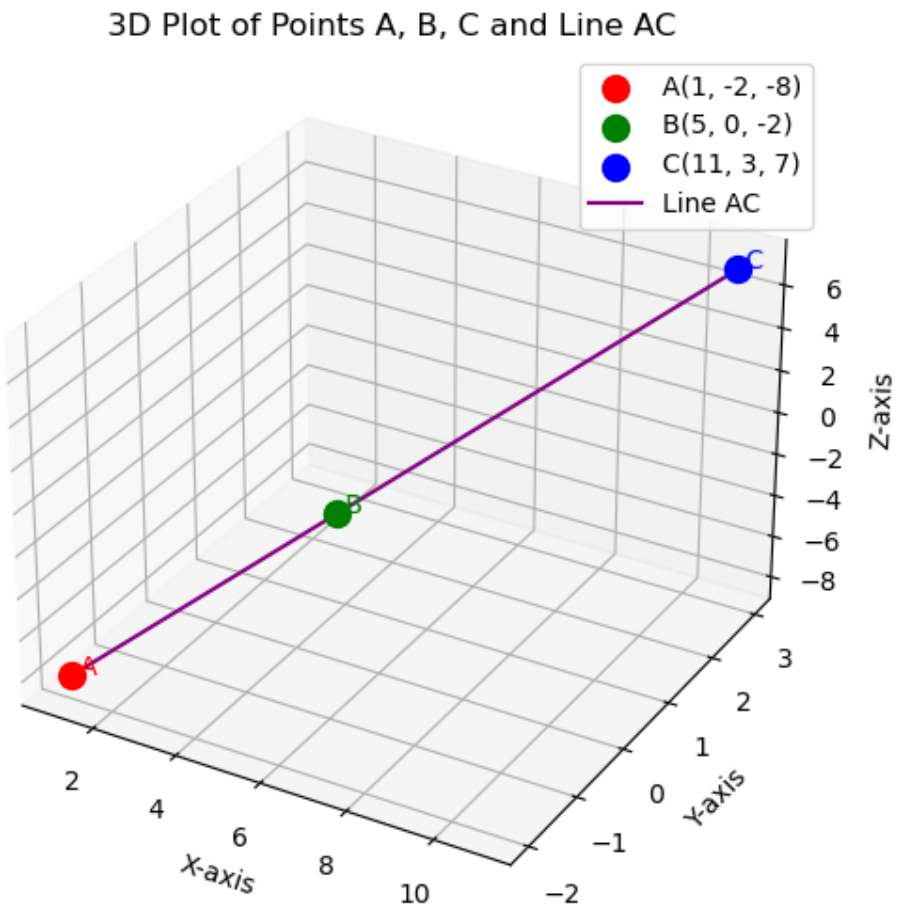
3D Plot of Points A, B, C and Line AC
A(1, -2, -8)
B(5, 0, -2)
C(11, 3, 7)
Line AC
A
B
C
X-axis
Y-axis
Z-axis
2
4
6
8
10
−2
−1
0
1
2
3
6
4
2
0
−2
−4
−6
−8

Figure: Plot