1.8.22: Equidistant Points Problem

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September 9, 2025

Question

Find all points that are equidistant from the points

$$\mathbf{A} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ 6 \end{pmatrix}.$$

How many such points exist?

Solution

Let the desired point be

$$\mathbf{O} = \begin{pmatrix} x \\ y \end{pmatrix}.$$

The equidistance condition is:

$$\|\mathbf{O} - \mathbf{A}\| = \|\mathbf{O} - \mathbf{B}\|.$$

Squaring both sides:

$$\|\mathbf{O} - \mathbf{A}\|^2 = \|\mathbf{O} - \mathbf{B}\|^2.$$

Using vector dot product,

$$(\mathbf{O} - \mathbf{A})^\top(\mathbf{O} - \mathbf{A}) = (\mathbf{O} - \mathbf{B})^\top(\mathbf{O} - \mathbf{B}).$$

Expanding,

$$\mathbf{O}^\top\mathbf{O} - 2\mathbf{A}^\top\mathbf{O} + \mathbf{A}^\top\mathbf{A} = \mathbf{O}^\top\mathbf{O} - 2\mathbf{B}^\top\mathbf{O} + \mathbf{B}^\top\mathbf{B}.$$

Canceling terms,

$$-2\mathbf{A}^\top\mathbf{O} + \mathbf{A}^\top\mathbf{A} = -2\mathbf{B}^\top\mathbf{O} + \mathbf{B}^\top\mathbf{B}.$$

Rearranged,

$$2(\mathbf{B} - \mathbf{A})^\top\mathbf{O} = \mathbf{B}^\top\mathbf{B} - \mathbf{A}^\top\mathbf{A}.$$

Substituting values,

$$\begin{pmatrix}4 & 2\end{pmatrix}\begin{pmatrix}x \\ y\end{pmatrix} = \frac{37 - 41}{2} = -2.$$

Simplified line equation:

$$2x + y = -1.$$

Number of such points: Infinitely many points lying on the above line.

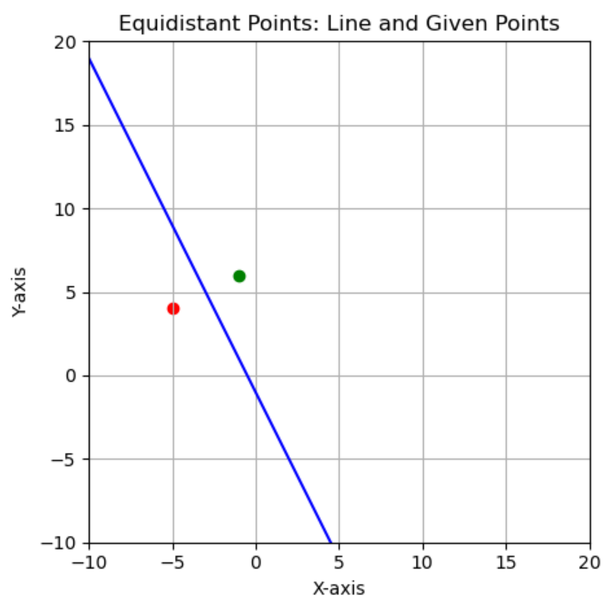

Figure 1: Points A, B and equidistant line $2x + y = -1$.