4.11.25

EE25BTECH11048 - Revanth Siva Kumar.D

Question

Find the distance of the point $\left(1, -2, 9\right)$ from the point of intersection of the line

$$\mathbf{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$$

and the plane

$$\mathbf{r} \cdot \left(\hat{i} - \hat{j} + \hat{k}\right) = 10.$$

Solution:

The line is

$$\mathbf{r} = \mathbf{r}_0 + \lambda\mathbf{d}, \tag{1}$$

$$\mathbf{r}_0 = \begin{pmatrix} 4 \\ 2 \\ 7 \end{pmatrix}, \quad \mathbf{d} = \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}. \tag{2}$$

The plane has normal

$$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \quad \mathbf{r}^T\mathbf{n} = 10. \tag{3}$$

Substitute $\mathbf{r} = \mathbf{r}_0 + \lambda\mathbf{d}$ into the plane equation:

$$\mathbf{n}^T\left(\mathbf{r}_0 + \lambda\mathbf{d}\right) = 10 \tag{4}$$

$$\implies \mathbf{n}^T\mathbf{d}\,\lambda = 10 - \mathbf{n}^T\mathbf{r}_0. \tag{5}$$

Now,

$$\mathbf{n}^T\mathbf{d} = \begin{pmatrix} 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} = 1, \tag{6}$$

$$\mathbf{n}^T\mathbf{r}_0 = \begin{pmatrix} 1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 4 \\ 2 \\ 7 \end{pmatrix} = 9. \tag{7}$$

Thus,

$$\lambda = \frac{10 - 9}{1} = 1. \tag{8}$$

Hence, the intersection point is

$$\mathbf{P} = \mathbf{r}_0 + \lambda \mathbf{d} \tag{9}$$

$$= \begin{pmatrix} 4 \\ 2 \\ 7 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} \tag{10}$$

$$= \begin{pmatrix} 7 \\ 6 \\ 9 \end{pmatrix}. \tag{11}$$

Given point is

$$\mathbf{A} = \begin{pmatrix} 1 \\ -2 \\ 9 \end{pmatrix}. \tag{12}$$

The displacement vector is

$$\mathbf{v} = \mathbf{P} - \mathbf{A} = \begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix}. \tag{13}$$

Therefore, the distance is

$$d = ||\mathbf{v}|| = \sqrt{\mathbf{v}^T \mathbf{v}} \tag{14}$$

$$= \sqrt{6^2 + 8^2 + 0^2} \tag{15}$$

$$= \sqrt{100} = 10. \tag{16}$$

Final Answer: The required distance is

$$\boxed{10}$$

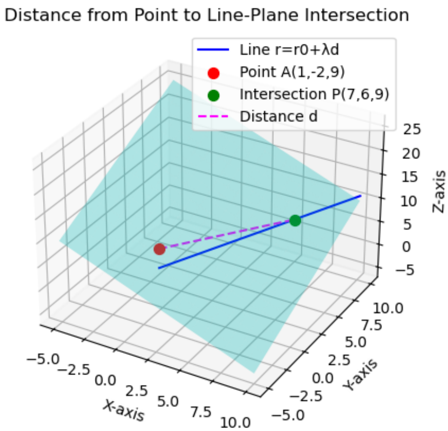
Distance from Point to Line-Plane Intersection
Line r=r0+λd
Point A(1,-2,9)
Intersection P(7,6,9)
Distance d
X-axis
Y-axis
Z-axis
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
−5
0
5
10
15
20
25

Fig. 1: PLOT