

9.3.9

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Question

Find the area of the region

$$\{(x, y) : 0 \leq y \leq x^2, 0 \leq y \leq x + 2, -1 \leq x \leq 3\}.$$

Theoretical Solution

Solution:

The parabola $y = x^2$ can be written as

$$y - x^2 = 0$$

or, in conic matrix form:

$$\mathbf{x}^T V \mathbf{x} + 2 \mathbf{u}^T \mathbf{x} + f = 0, \quad (1)$$

$$\mathbf{V} = \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} 0 \\ \frac{1}{2} \end{pmatrix}, \quad f = 0. \quad (2)$$

The line $y = x + 2$ can be written as

$$x - y + 2 = 0,$$

Theoretical Solution

The General Equation of a Line:

$$\mathbf{x} = k\mathbf{m} + \mathbf{h} \quad (3)$$

On comparing, we get:

$$\mathbf{m} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \mathbf{h} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (4)$$

The Intersection of the given conic with the given line can be written as:

$$\mathbf{x}_i = \mathbf{h} + k_i\mathbf{m} \quad (5)$$

$$\text{where, } k_i = \left(\frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \right) \left(-\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(h)(\mathbf{m}^\top \mathbf{V} \mathbf{m})} \right) \quad (6)$$

$$\text{Let } \mathbf{K} = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$$

Theoretical Solution

The Solution Matrix can be expressed as:

$$\mathbf{X} = \begin{pmatrix} \mathbf{h} & \mathbf{m} \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{k} \end{pmatrix}^T \quad (7)$$

Therefore, The points of intersection are:

$$\mathbf{x}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad \& \quad \mathbf{x}_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad (8)$$

From Fig.0.1, the required area is given by:

$$\int_{-1}^2 [(x+2) - (x^2)] dx = \int_{-1}^2 [2 + x - x^2] dx \quad (9)$$

$$\int_{-1}^2 [x^2] dx + \int_2^3 [x+2] dx = \frac{15}{2} = 7.5 \text{ sq.units} \quad (10)$$

Therefore, the required area is 7.5 sq.units.

Plot

Beamer/figs/plot.png