

Matrices in Geometry - 2.8.20

EE25BTECH11037 Divyansh

Sept, 2025

Problem Statement

If $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$, then prove that \mathbf{a} and \mathbf{b} are orthogonal.

Solution

Given: $\mathbf{A} \begin{pmatrix} 2 \\ -2 \end{pmatrix}$, $\mathbf{B} \begin{pmatrix} 7 \\ 3 \end{pmatrix}$, $\mathbf{C} \begin{pmatrix} 11 \\ -1 \end{pmatrix}$ and $\mathbf{D} \begin{pmatrix} 6 \\ -6 \end{pmatrix}$.

$$\|\mathbf{a} + \mathbf{b}\| = \|\mathbf{a} - \mathbf{b}\| \quad (1)$$

Two vectors \mathbf{a} and \mathbf{b} are orthogonal if, $\mathbf{a}^\top \mathbf{b} = 0$ (2)

$$\|\mathbf{a} + \mathbf{b}\|^2 = \|\mathbf{a} - \mathbf{b}\|^2 \quad (3)$$

Solution

We know that $\|\mathbf{a}\|^2 = \mathbf{a}^\top \mathbf{a}$

$$\implies (\mathbf{a} + \mathbf{b})^\top (\mathbf{a} + \mathbf{b}) = (\mathbf{a} - \mathbf{b})^\top (\mathbf{a} - \mathbf{b}) \quad (4)$$

$$\implies \mathbf{a}^\top \mathbf{a} + 2\mathbf{a}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{b} = \mathbf{a}^\top \mathbf{a} - 2\mathbf{a}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{b} \quad (5)$$

$$\implies 4\mathbf{a}^\top \mathbf{b} = 0 \quad (6)$$

$$\implies \mathbf{a}^\top \mathbf{b} = 0 \quad (7)$$

This shows that \mathbf{a} and \mathbf{b} are orthogonal.

Hence, proved