12.18

EE25BTECH11004 - Aditya Appana

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Question

X is 1 km northeast of **Y**. **Y** is 1 km southeast of **Z**. **W** is 1 km west of **Z**. **P** is 1 km south of **W**. **Q** is 1 km east of **P**. What is the distance between **X** and **Q** in km?

A) 1 B) $\sqrt{2}$ C) $\sqrt{3}$ D) 2

Solution

Let **X** be $\begin{pmatrix}0\\0\end{pmatrix}$. Every subsequent vector can be expressed as a rotation by a particular angle θ with respect to the previous vector.
The matrix to rotate a vector by angle θ about the origin is:

$$\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix} \tag{1}$$

X is 1km north-east of **Y**, so **Y** is 1km south-west of **X**. Therefore:

$$\mathbf{Y}-\mathbf{X} = \begin{pmatrix}\cos 225^\circ & -\sin 225^\circ\\ \sin 225^\circ & \cos 225^\circ\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} \tag{2}$$

$$\mathbf{Y}-\begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}\frac{-1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{pmatrix} \tag{3}$$

$$\mathbf{Y} = \begin{pmatrix}\frac{-1}{\sqrt{2}}\\ \frac{-1}{\sqrt{2}}\end{pmatrix} \tag{4}$$

Y is 1km south-east of **Z**, so **Z** is 1km north-west of **Y**. Therefore:

$$\mathbf{Z} - \mathbf{Y} = \begin{pmatrix} \cos 135° & -\sin 135° \\ \sin 135° & \cos 135° \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{5}$$

$$\mathbf{Y} - \begin{pmatrix} \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \tag{6}$$

$$\mathbf{Z} = \begin{pmatrix} -\sqrt{2} \\ 0 \end{pmatrix} \tag{7}$$

W is 1km west of **Z**. Therefore:

$$\mathbf{W} - \mathbf{Z} = \begin{pmatrix} \cos 180° & -\sin 180° \\ \sin 180° & \cos 180° \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{8}$$

$$\mathbf{Y} - \begin{pmatrix} -\sqrt{2} \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \tag{9}$$

$$\mathbf{W} = \begin{pmatrix} -1 - \sqrt{2} \\ 0 \end{pmatrix} \tag{10}$$

P is 1km south of **W**. Therefore:

$$\mathbf{P} - \mathbf{W} = \begin{pmatrix} \cos 270° & -\sin 270° \\ \sin 270° & \cos 270° \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{11}$$

$$\mathbf{P} - \begin{pmatrix} -1 - \sqrt{2} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{12}$$

$$\mathbf{P} = \begin{pmatrix} -1 - \sqrt{2} \\ -1 \end{pmatrix} \tag{13}$$

Q is 1km east of **P**. Therefore:

$$\mathbf{Q} - \mathbf{P} = \begin{pmatrix} \cos 0° & -\sin 0° \\ \sin 0° & \cos 0° \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{14}$$

$$\mathbf{Q} - \begin{pmatrix} -1 - \sqrt{2} \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{15}$$

$$\mathbf{Q} = \begin{pmatrix} -\sqrt{2} \\ -1 \end{pmatrix} \tag{16}$$

The distance between $\mathbf{X} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} -\sqrt{2} \\ -1 \end{pmatrix}$ is:

$$\|\mathbf{X} - \mathbf{Q}\| = \left\| \begin{pmatrix} \sqrt{2} \\ 1 \end{pmatrix} \right\| \tag{17}$$

$$\sqrt{(\sqrt{2})^2 + 1^2} = \sqrt{3} \tag{18}$$

Therefore the correct option is **C**.

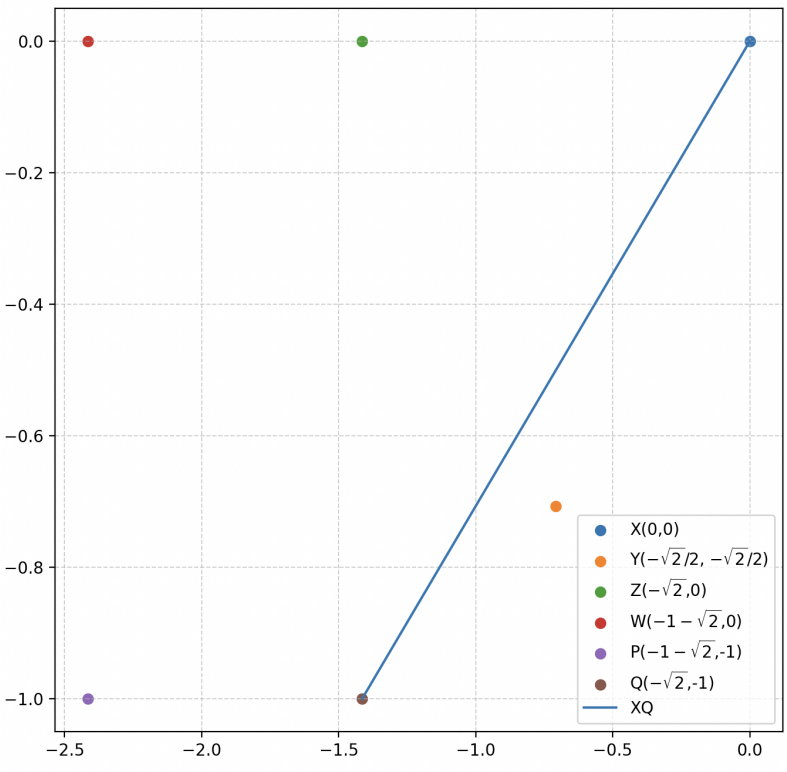

Figure 1: Plot