4.13.67

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Question

The area of the triangle formed by the intersection of line parallel to X axis and passing through $\mathbf{p}$(h,k) with the lines y=x and x+y=2 is $4h^2$.Find the locus of point $\mathbf{p}$

Solution:

line parallel to X axis is of the form

$$y = c. \tag{1}$$

which can be expressed in the form of

$$\mathbf{n}^T\mathbf{x} = c. \tag{2}$$

$$\implies \begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = c. \tag{3}$$

As the above line passes through $\mathbf{p}$(h,k)

$$\begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}h\\k\end{pmatrix} = c. \implies c = k. \tag{4}$$

The three lines are as follows

$$y = k \implies \begin{pmatrix}0 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = k. \tag{5}$$

$$-x + y = 0 \implies \begin{pmatrix}-1 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = 0. \tag{6}$$

$$x + y = 2 \implies \begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = 2. \tag{7}$$

Let $\mathbf{A}$,$\mathbf{B}$,$\mathbf{C}$ be the point of intersection of above 3 lines
By solving equation (5) and (6) we get

$$\mathbf{A} = \begin{pmatrix}k\\k\end{pmatrix}. \tag{8}$$

By solving equation (6) and (7) we get

$$\mathbf{B} = \begin{pmatrix}1\\1\end{pmatrix}. \tag{9}$$

By solving equation (5) and (7) we get

$$\mathbf{C} = \begin{pmatrix}2-k\\k\end{pmatrix}. \tag{10}$$

$$\text{area of } \triangle\text{ABc} = \frac{1}{2}\left\|(\mathbf{A}-\mathbf{B})\times(\mathbf{B}-\mathbf{C})\right\| \tag{11}$$

$$= \frac{1}{2}\left\|\begin{pmatrix}k-1\\k-1\end{pmatrix}\times\begin{pmatrix}k-1\\1-k\end{pmatrix}\right\| \tag{12}$$

$$= \frac{1}{2}(2(k-1)^2) = (k-1)^2. \tag{13}$$

Given area of the triangle formed by the intersection of above 3 lines is $4h^2$.

$$\implies (k-1)^2 = 4h^2. \tag{14}$$

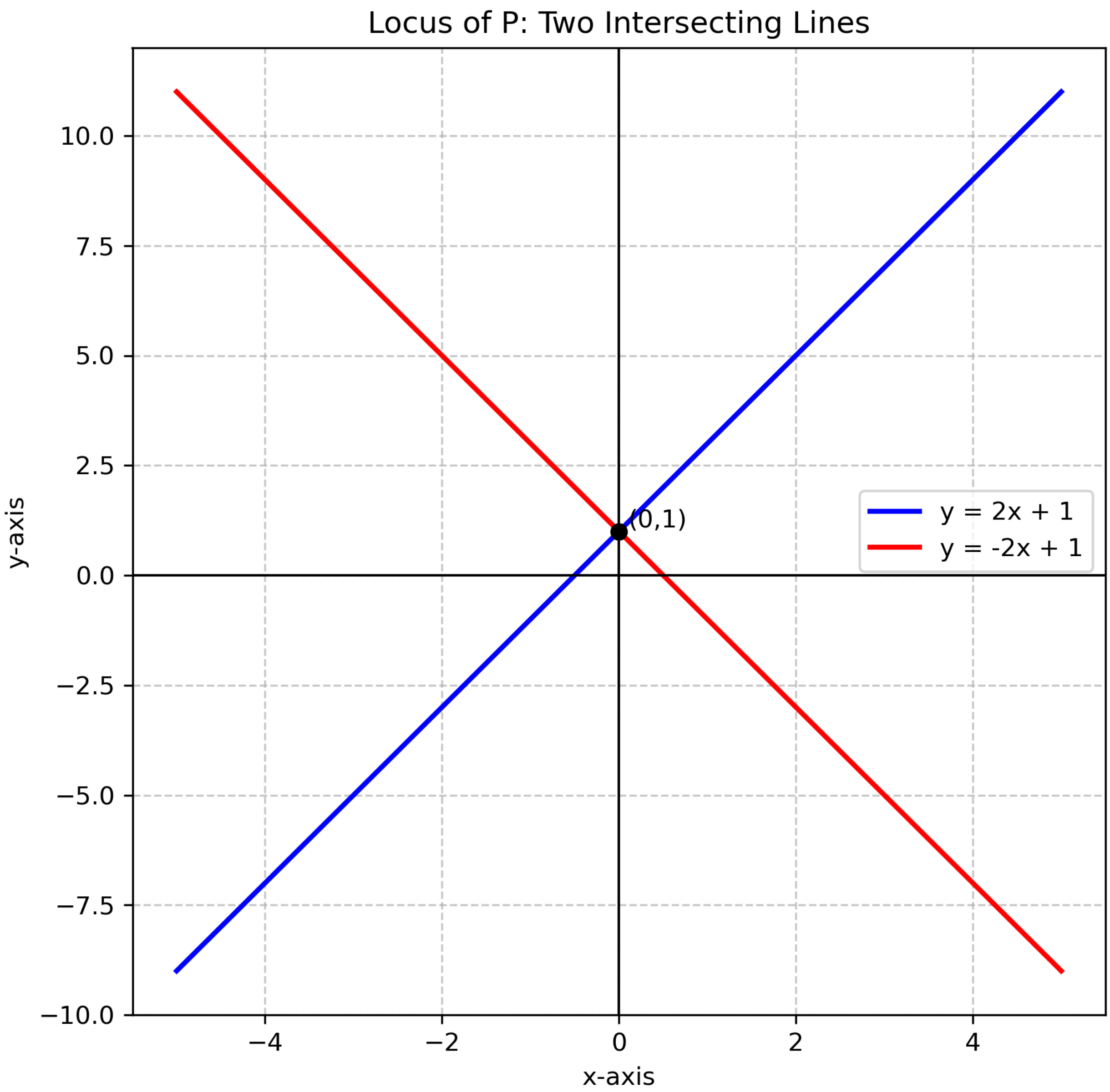
$$\implies (y-1)^2 = 4x^2 \tag{15}$$

$$\implies (y-1-2x)(y-1+2x) = 0 \tag{16}$$

$\implies$ The locus of $\mathbf{p}$ is union of 2 straight lines

$$y - 1 - 2x = 0. \implies \begin{pmatrix}-2 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = 1. \tag{17}$$

$$y - 1 + 2x = 0. \implies \begin{pmatrix}2 & 1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = 1. \tag{18}$$


Locus of P: Two Intersecting Lines
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
y-axis
−4
−2
0
2
4
x-axis
(0,1)
y = 2x + 1
y = -2x + 1