Question 2.10.29

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1 Question:

The volume of the parallelopiped whose sides are given by $OA = 2\mathbf{i} - 2\mathbf{j}$, $OB = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $OC = 3\mathbf{i} - \mathbf{k}$, is

2 Solution:

To find the volume of the parallelopiped, we can use the scalar triple product formula:

$$V = \left\|\mathbf{OA}^{\mathrm{T}}(\mathbf{OB} \times \mathbf{OC})\right\| \tag{1}$$

$$\mathbf{OA} = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix}, \quad \mathbf{OB} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \quad \mathbf{OC} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} \tag{2}$$

$$\tag{3}$$

To find the cross product between two vectors $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$, we can use the matrix multiplication method, by first defining a new matrix $[\mathbf{a}]_\times$ as follows:

$$[\mathbf{a}]_\times = \begin{pmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{pmatrix} \tag{4}$$

$$\text{Now, } \mathbf{a} \times \mathbf{b} = [\mathbf{a}]_\times \mathbf{b} \tag{5}$$

Using this method, we can find the cross product $\mathbf{OB} \times \mathbf{OC}$ as follows:

$$[\mathbf{OB}]_\times = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \tag{6}$$

$$\mathbf{OB} \times \mathbf{OC} = [\mathbf{OB}]_\times \mathbf{OC} = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix} \tag{7}$$

Now, we can find the scalar triple product $\mathbf{OA}^\mathrm{T}(\mathbf{OB} \times \mathbf{OC})$ as follows:

$$\mathbf{OA}^\mathrm{T}(\mathbf{OB} \times \mathbf{OC}) = \begin{pmatrix} 2 & -2 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 + 4 + 0 \end{pmatrix} = \begin{pmatrix} 2 \end{pmatrix} \tag{8}$$

Finally, we can find the volume of the parallelopiped as follows:

$$V = \left\| \mathbf{OA}^\mathrm{T}(\mathbf{OB} \times \mathbf{OC}) \right\| = \left\| \begin{pmatrix} 2 \end{pmatrix} \right\| = 2 \tag{9}$$