

2.3.8

EE25BTECH11005 - Aditya Mishra

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Question

If $\mathbf{A} = \hat{i} + \hat{j} + \hat{k}$, $\mathbf{B} = 2\hat{i} + 5\hat{j}$, $\mathbf{C} = 3\hat{i} + 2\hat{j} - 3\hat{k}$, $\mathbf{D} = \hat{i} - 6\hat{j} - \hat{k}$ are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD . Find whether $(\mathbf{B} - \mathbf{A})$ and $(\mathbf{D} - \mathbf{C})$ are collinear or not.

Solution

Let the direction vectors be

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} \quad (1)$$

$$\mathbf{D} - \mathbf{C} = \begin{pmatrix} 1 \\ -6 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix} \quad (2)$$

The angle θ between $(\mathbf{B} - \mathbf{A})$ and $(\mathbf{D} - \mathbf{C})$ is given by

$$\cos \theta = \frac{(\mathbf{B} - \mathbf{A})^\top (\mathbf{D} - \mathbf{C})}{|(\mathbf{B} - \mathbf{A})| |(\mathbf{D} - \mathbf{C})|} \quad (3)$$

Substitute the values:

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{D} - \mathbf{C}) = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}^\top \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix} \quad (4)$$

$$= (1)(-2) + (4)(-8) + (-1)(2) \quad (5)$$

$$= -2 - 32 - 2 = -36 \quad (6)$$

Magnitudes:

$$|(\mathbf{B} - \mathbf{A})| = \sqrt{1^2 + 4^2 + (-1)^2} = \sqrt{1 + 16 + 1} = \sqrt{18} \quad (7)$$

$$|(\mathbf{D} - \mathbf{C})| = \sqrt{(-2)^2 + (-8)^2 + 2^2} = \sqrt{4 + 64 + 4} = \sqrt{72} \quad (8)$$

Thus,

$$\cos \theta = \frac{-36}{\sqrt{18} \sqrt{72}} = \frac{-36}{\sqrt{1296}} = \frac{-36}{36} = -1 \quad (9)$$

$$\theta = \cos^{-1}(-1) = \pi \text{ radians} = 180^\circ \quad (10)$$

So, $(\mathbf{B} - \mathbf{A})$ and $(\mathbf{D} - \mathbf{C})$ are collinear but point in opposite directions, i.e., they are anti-parallel.

The lines are collinear (anti-parallel).

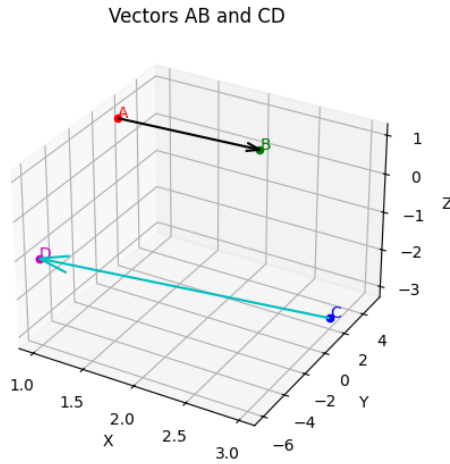


Figure 1: Line directions