2.2.21

AI25BTECH11028 - RAYIDI MANOHAR

Question: If the angle between two lines is $\pi/4$ and slope of one of the lines is $1/2$,find the slope of the other line.

Solution:

We know that, The angle θ between $\mathbf{a}, \mathbf{b}$, is given by

$$\cos\theta = \frac{\mathbf{a}^\top\mathbf{b}}{\|\mathbf{a}\|\|\mathbf{b}\|} \tag{0.1}$$

From the given Information, Angle between the lines is $\pi/4$ let vector $\mathbf{A}$ be the line slope $1/2$ and $\mathbf{B}$ be the line with slope m_2

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1/2 \end{pmatrix} \tag{0.2}$$

$$\mathbf{B} = \begin{pmatrix} 1 \\ m_2 \end{pmatrix} \tag{0.3}$$

$$\cos\pi/4 = \frac{\mathbf{A}^\top\mathbf{B}}{\|\mathbf{A}\|\|\mathbf{B}\|} \tag{0.4}$$

Now,

$$\mathbf{A}^\top\mathbf{B} = \begin{pmatrix} 1 \\ 1/2 \end{pmatrix}^\top \begin{pmatrix} 1 \\ m_2 \end{pmatrix} = 1^2 + m_2/2 \tag{0.5}$$

$$\|\mathbf{A}\| = \sqrt{1^2 + \left(\frac{1}{2}\right)^2} = \sqrt{1 + \frac{1}{4}} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2} \tag{0.6}$$

$$\|\mathbf{B}\| = \sqrt{1^2 + m_2^2} = \sqrt{1 + m_2^2} \tag{0.7}$$

From this,

$$\frac{1}{\sqrt{2}} * \frac{\sqrt{5}}{2} * \sqrt{1 + m_2^2} = 1 + \frac{m_2}{2} \tag{0.8}$$

$$\sqrt{5} * \sqrt{1 + m_2^2} = \sqrt{2}(2 + m_2) \tag{0.9}$$

Now squaring on both sides;

$$5 + 5m_2^2 = 2(4 + m_2^2 + 4m_2) \tag{0.10}$$

$$3m_2^2 - 8m_2 - 3 = 0 \tag{0.11}$$

Therefore,

$$m_2 = 3 \, or -1/3 \tag{0.12}$$