10.7.76

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Question

The number of common tangents to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y = 24$ is

1. 0
2. 1
3. 2
4. 3

Equation I

Let the equation of 1st circle be:

$$\|\mathbf{x}\|^2 + 2\mathbf{u_1}^T\mathbf{x} + f_1 = 0 \tag{1}$$

Let the equation of 2nd circle be

$$\|\mathbf{x}\|^2 + 2\mathbf{u_2}^T\mathbf{x} + f_2 = 0 \tag{2}$$

Theoretical Solution

From the given information:

$$\mathbf{u_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \textit{and} \quad f_1 = -4 \tag{3}$$

$$\mathbf{u_2} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \quad \textit{and} \quad f_2 = -24 \tag{4}$$

The intersection of two curves can be given as :

$$\mathbf{x}^T(\mathbf{V_1} + \mu\mathbf{V_2})\mathbf{x} + 2(\mathbf{u_1} + \mu\mathbf{u_2})^T\mathbf{x} + (f_1 + \mu f_2) = 0 \tag{5}$$

Given conic is a circle. So,

$$\mathbf{V_1} = \mathbf{V_2} = \mathbf{I} \tag{6}$$

Theoretical solution

Now subdtituitng the given values:

$$(\mu+1)\mathbf{x}^T\mathbf{x}+2\mu\begin{pmatrix}-3\\-4\end{pmatrix}^T\mathbf{x}+(-4-24\mu)=0 \tag{7}$$

$$(\mu+1)\left\|\mathbf{x}\right\|^2-2\mu\begin{pmatrix}3\\4\end{pmatrix}^T\mathbf{x}-4(1+6\mu)=0 \tag{8}$$

$\mathbf{x}$ lies on the circle 1. So,

$$\left\|\mathbf{x}\right\|^2=4 \tag{9}$$

$$4(\mu+1)-2\mu\begin{pmatrix}3\\4\end{pmatrix}^T\mathbf{x}-4(1+6\mu)=0 \tag{10}$$

Theoretical solution

$$4\mu - 2\mu \begin{pmatrix} 3 \\ 4 \end{pmatrix}^T \mathbf{x} - 24\mu = 0; \tag{11}$$

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix}^T \mathbf{x} = -10 \tag{12}$$

Which is the equation of a single line
So the number of common tangents is 1

C Code

```c
#include <math.h>
int find_common_tangents(double x1, double y1, double r1, double
    x2, double y2, double r2) {
    // Calculate the distance between the centers
    double d = sqrt(pow(x2 - x1, 2) + pow(y2 - y1, 2));

    // Calculate the sum and difference of the radii
    double r_sum = r1 + r2;
    double r_diff = fabs(r1 - r2);

    // Determine the relationship between the circles
    if (d > r_sum) {
        return 4; // Circles are separate and do not intersect
    } else if (d == r_sum) {
        return 3; // Circles touch externally
    } else if (d > r_diff && d < r_sum) {
        return 2; // Circles intersect at two points
    }
```

C Code

```c
    else if (d == r_diff) {
        return 1; // Circles touch internally
    }
    else if (d < r_diff) {
        return 0; // One circle is completely inside the other
    } else if (d == 0 && r1 == r2) {
        return -1; // Concentric and identical
    }

    return 0; // Default case, including d=0 and r1!=r2
}
```

Python Code

```
import ctypes
import platform
import numpy as np
import matplotlib.pyplot as plt

# --- 1. Load the C shared library ---
lib_name = 'circle.so'
if platform.system() == 'Windows':
    lib_name = 'circle.dll'

try:
    c_lib = ctypes.CDLL(f'./{lib_name}')
except OSError as e:
    print(fError loading shared library: {e})
    print(fPlease compile circle.c into {lib_name} first.)
    exit()
```

Python Code

```
# --- 2. Define the C function signature for Python ---
c_lib.find_common_tangents.argtypes = [
    ctypes.c_double, ctypes.c_double, ctypes.c_double,
    ctypes.c_double, ctypes.c_double, ctypes.c_double
]
c_lib.find_common_tangents.restype = ctypes.c_int

def solve_with_c(c1, r1, c2, r2):
    A Python wrapper that calls the C function.
    return c_lib.find_common_tangents(c1[0], c1[1], r1, c2[0], c2
        [1], r2)

def plot_circles(c1, r1, c2, r2, tangency_point):
    Plots the two circles, their centers, and the point of
        tangency.
    fig, ax = plt.subplots(figsize=(10, 8))
```

Python Code

```
 # Create circle patches
circle1 = plt.Circle(c1, r1, color='blue', fill=False,
    linewidth=2, label=f'Circle 1: $r_1={r1}$')
circle2 = plt.Circle(c2, r2, color='red', fill=False,
    linewidth=2, label=f'Circle 2: $r_2={r2}$')

ax.add_patch(circle1)
ax.add_patch(circle2)

# Plot centers and label them with coordinates
ax.plot(c1[0], c1[1], 'bo', markersize=8, label='Center $C_1$
    ')
ax.text(c1[0] + 0.3, c1[1] + 0.3, f'$C_1$ ({c1[0]:.1f}, {c1
    [1]:.1f})', fontsize=12, color='blue')
```

Python Code

```
    ax.plot(c2[0], c2[1], 'ro', markersize=8, label='Center $C_2$')
    ax.text(c2[0] + 0.3, c2[1] + 0.3, f'$C_2$ ({c2[0]:.1f}, {c2[1]:.1f})', fontsize=12, color='red')

    # Plot tangency point and label it with coordinates
    ax.plot(tangency_point[0], tangency_point[1], 'go', markersize=8, label='Tangency Point T')
    ax.text(tangency_point[0] + 0.3, tangency_point[1] - 0.5, f'T ({tangency_point[0]}, {tangency_point[1]})', fontsize=12, color='green')

    # Set plot properties
    ax.set_aspect('equal', adjustable='box')
    plt.title('Relationship Between Two Circles (Internal Tangency)')
```

Python Code

```python
    plt.xlabel('x-axis')
    plt.ylabel('y-axis')
    plt.grid(True)
    plt.legend(loc='upper right')
    plt.savefig(/media/indhiresh-s/New Volume/Matrix/ee1030-2025/
        ee25btech11027/MATGEO/10.7.76/figs/figure1.png)
    plt.show()

# --- Main execution ---
if __name__ == __main__:
    # Parameters for the circles from the problem
    C1 = (0.0, 0.0)
    r1 = 2.0
    C2 = (3.0, 4.0)
    r2 = 7.0
```

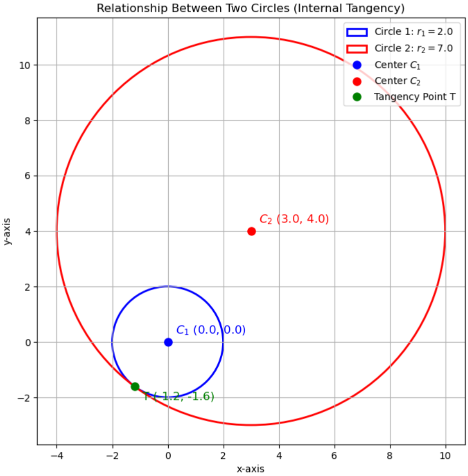
Python Code

```
# Calculate the number of tangents using the C function
num_tangents = solve_with_c(C1, r1, C2, r2)
print(fNumber of common tangents (via C function): {
    num_tangents})

# Calculate the point of tangency for plotting
v = np.array(C1) - np.array(C2)
u = v / np.linalg.norm(v)
T = tuple(np.array(C2) + r2 * u)
T_rounded = (round(T[0], 2), round(T[1], 2))

# Plot the result
plot_circles(C1, r1, C2, r2, T_rounded)
```

Plot



Relationship Between Two Circles (Internal Tangency)
Circle 1: $r_1 = 2.0$
Circle 2: $r_2 = 7.0$
Center C_1
Center C_2
Tangency Point T
C_2 (3.0, 4.0)
C_1 (0.0, 0.0)
T(-1.2, -1.6)
y-axis
x-axis