5.4.42

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Question

Using elementary transformations, find the inverse of the following matrix:

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}$$

Theoretical Solution

We solve using Gauss-Jordan elimination.

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -3 & 0 & 1 & 0 \\ 3 & -2 & 4 & 0 & 0 & 1 \end{array}\right) \tag{1}$$

$$R_3 \leftarrow R_3 - 3R_1 \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -3 & 0 & 1 & 0 \\ 0 & 1 & -2 & -3 & 0 & 1 \end{array}\right) \tag{2}$$

$$R_2 \leftarrow \frac{1}{2}R_2 \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -3/2 & 0 & 1/2 & 0 \\ 0 & 1 & -2 & -3 & 0 & 1 \end{array}\right) \tag{3}$$

Theoretical Solution

$$R_3 \leftarrow R_3 - R_2 \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -3/2 & 0 & 1/2 & 0 \\ 0 & 0 & -1/2 & -3 & -1/2 & 1 \end{array}\right) \tag{4}$$

$$R_3 \leftarrow -2R_3 \left(\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -3/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 6 & 1 & -2 \end{array}\right) \tag{5}$$

$$[\,R_1 \leftarrow R_1 - 2R_3]\, R_2 \leftarrow R_2 + 3/2R_3 \left(\begin{array}{ccc|ccc} 1 & -1 & 0 & -11 & -2 & 4 \\ 0 & 1 & 0 & 9 & 2 & -3 \\ 0 & 0 & 1 & 6 & 1 & -2 \end{array}\right) \tag{6}$$

$$R_1 \leftarrow R_1 + R_2 \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 0 & 1 \\ 0 & 1 & 0 & 9 & 2 & -3 \\ 0 & 0 & 1 & 6 & 1 & -2 \end{array}\right) \tag{7}$$

Conclusion

$$\therefore \text{Inverse of the given Matrix: } \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix}$$

C Code

```c
#include <stdio.h>
#define N 3
void inverse(double A[N][N], double inv[N][N]) {
        double aug[N][2*N];
        for (int i = 0; i < N; i++) {
                for (int j = 0; j < N; j++) {
                        aug[i][j] = A[i][j];
                        aug[i][j+N] = (i==j)?1:0;
                }
        }
```

C Code

```
// Gauss-Jordan elimination
for (int i=0;i<N;i++){
        double pivot = aug[i][i];
        for(int j=0;j<2*N;j++) aug[i][j]/=pivot;
        for(int k=0;k<N;k++){
                if(k!=i){
                        double factor = aug[k][i];
                        for(int j=0;j<2*N;j++)
                                aug[k][j]-=factor*aug[i][j];
                }
        }
}
// Extract inverse
for(int i=0;i<N;i++)
        for(int j=0;j<N;j++)
                inv[i][j]=aug[i][j+N];
}
```

Python Code using shared output

```
import sympy as sp

A = sp.Matrix([[1, -1, 2], [0, 2, -3], [3, -2, 4]])
A_inv = A.inv()
sp.pprint(A_inv)
```

Python:plot.py

```python
import matplotlib.pyplot as plt
import numpy as np

# Inverse matrix from 5.4.42
A_inv = np.array([
    [-2, 0, 1],
    [9, 2, -3],
    [6, 1, -2]
])

fig, ax = plt.subplots()
cax = ax.matshow(A_inv, cmap='coolwarm')
fig.colorbar(cax)
for (i, j), val in np.ndenumerate(A_inv):
    ax.text(j, i, f'{val}', ha='center', va='center', color='black')
ax.set_title('Inverse of Matrix 5.4.42')
plt.show()
```