Matgeo-q.1.4.6

AI25BTECH11036-SNEHAMRUDULA

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Question

If the point $P(2,1)$ lies on the line segment joining points $A(4,2)$ and $B(8,4)$, then which of the following is true?

$AP = \frac{1}{3}AB$

$AP = PB$

$PB = \frac{1}{3}AB$

$AP = \frac{1}{2}AB$

Solution

Given:

$$\mathbf{A} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}, \qquad \mathbf{P} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}. \tag{1}$$

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, \tag{2}$$

$$\mathbf{AP} = \mathbf{P} - \mathbf{A} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}. \tag{3}$$

$$\|\mathbf{AB}\|^2 = 4^2 + 2^2 = 20, \tag{4}$$

$$\|\mathbf{AP}\|^2 = (-2)^2 + (-1)^2 = 5. \tag{5}$$

$$\left(\frac{AP}{AB}\right)^2 = \frac{\|\mathbf{AP}\|^2}{\|\mathbf{AB}\|^2} = \frac{5}{20} = \frac{1}{4} \Rightarrow \frac{AP}{AB} = \frac{1}{2}. \tag{6}$$

$$\boxed{AP = \tfrac{1}{2}\, AB}$$

Graphical Representation