# 5.10.3

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# Question

Balance the following chemical equation.

$$Fe + H_2O \rightarrow Fe_3O_4 + H_2$$

# Theoretical Solution

Let the balanced version of the equation be

$$x_1 Fe + x_2 H_2 O \to x_3 Fe_3 O_4 + x_4 H_2 \tag{1}$$

which results in the following equations based on the conservation of each element:

$$\text{For Fe: } x_1 - 3x_3 = 0 \tag{2}$$

$$\text{For H: } 2x_2 - 2x_4 = 0 \implies x_2 - x_4 = 0 \tag{3}$$

$$\text{For O: } x_2 - 4x_3 = 0 \tag{4}$$

This can be expressed as a homogeneous system of linear equations:

$$x_1 + 0x_2 - 3x_3 + 0x_4 = 0 \tag{5}$$

$$0x_1 + x_2 + 0x_3 - x_4 = 0 \tag{6}$$

$$0x_1 + x_2 - 4x_3 + 0x_4 = 0 \tag{7}$$

# Theoretical Solution

This results in the matrix equation $A\mathbf{x} = \mathbf{0}$, where:

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -4 & 0 \end{pmatrix} \mathbf{x} = \mathbf{0}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} \tag{8}$$

The coefficient matrix can be reduced as follows using Gaussian elimination to find the null space:

# Theoretical Solution

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -4 & 0 \end{pmatrix} \xrightarrow{R_3 \to R_3 - R_2} \tag{9}$$

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -4 & 1 \end{pmatrix} \xrightarrow{R_3 \to -\frac{1}{4} R_3} \tag{10}$$

$$\begin{pmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1/4 \end{pmatrix} \xrightarrow{R_1 \to R_1 + 3R_3} \tag{11}$$

$$\begin{pmatrix} 1 & 0 & 0 & -3/4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1/4 \end{pmatrix} \tag{12}$$

# Theoretical Solution

From the reduced row echelon form, we get the solutions in terms of the free variable $x_4$:

$$x_1 = \frac{3}{4}x_4, \quad x_2 = x_4, \quad x_3 = \frac{1}{4}x_4 \tag{13}$$

# Theoretical Solution

Thus,

$$\mathbf{x} = x_4 \begin{pmatrix} 3/4 \\ 1 \\ 1/4 \\ 1 \end{pmatrix} \tag{14}$$

By substituting $x_4 = 4$, the simplest integer solution is found. Hence,

$$\mathbf{x} = 4 \begin{pmatrix} 3/4 \\ 1 \\ 1/4 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \\ 4 \end{pmatrix} \tag{15}$$

This gives $x_1 = 3, x_2 = 4, x_3 = 1,$ and $x_4 = 4.$ Hence, the balanced

equation finally becomes:

$$\boxed{3Fe + 4H_2O \rightarrow Fe_3O_4 + 4H_2} \tag{16}$$

# C Code

```
#include <stdio.h>

void solve_and_print_balance() {
    int x1, x2, x3, x4;
    int found = 0;

    printf(Searching for the smallest integer coefficients...\n\n
        );

    for (x1 = 1; x1 <= 100 && !found; x1++) {
        for (x2 = 1; x2 <= 100 && !found; x2++) {
            for (x3 = 1; x3 <= 100 && !found; x3++) {
                for (x4 = 1; x4 <= 100 && !found; x4++) {
                    if ((x1 == 3 * x3) && (x2 == x4) && (x2 == 4 *
                        x3)) {
                        printf(Solution found!\n);
                        printf(Coefficients are: x1=%d, x2=%d,
```

# C Code

```
x3=%d, x4=%d\n\n, x1, x2, x3, x4);
                        printf(The balanced chemical equation is:\n);
                        printf(%dFe + %dH2O -> %dFe3O4 + %dH2\n, x1, x2, x3, x4);
                        found = 1;
                    }
                }
            }
        }
    }

    if (!found) {
        printf(No solution was found within the search range.\n);
    }
}
```