5.8.19

EE25BTECH11021 - Dhanush sagar

Question:
If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes 1/2 if we only add 1 to the denominator. What is the fraction?

Solution: Given Let the unknown fraction be represented as:

$$\mathbf{v} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \tag{1}$$

so that the fraction equals:

$$\frac{x}{y}. \tag{2}$$

case 1 :Adding 1 to the numerator and subtracting 1 from the denominator:

$$\mathbf{T_1} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

$$\mathbf{T_1 v} = \begin{pmatrix} x+1 \\ y-1 \\ 1 \end{pmatrix} \tag{4}$$

case 2 :Adding 1 to the denominator:

$$\mathbf{T_2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \tag{5}$$

$$\mathbf{T_2 v} = \begin{pmatrix} x \\ y+1 \\ 1 \end{pmatrix} \tag{6}$$

Condition for a fraction $\frac{a}{b} = k$:

$$\begin{pmatrix} 1 & -k & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ 1 \end{pmatrix} = 0 \tag{7}$$

Applying the first condition ($\mathbf{T_1} * \mathbf{v} = 1$):

$$\mathbf{r_1} = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \mathbf{T_1} \tag{8}$$

$$= \begin{pmatrix} 1 & -1 & 2 \end{pmatrix} \tag{9}$$

$$\mathbf{r_1 v} = 0 \tag{10}$$

Applying the second condition ($\mathbf{T_2} * \mathbf{v} = 1/2$):

$$\mathbf{r_2} = \begin{pmatrix} 2 & -1 & 0 \end{pmatrix} \mathbf{T_2} \tag{11}$$
$$= \begin{pmatrix} 2 & -1 & -1 \end{pmatrix} \tag{12}$$
$$\mathbf{r_2 v} = 0 \tag{13}$$

System of equations in matrix form:

$$\mathbf{Mv} = 0 \tag{14}$$
$$\mathbf{M} = \begin{pmatrix} 1 & -1 & 2 \\ 2 & -1 & -1 \end{pmatrix} \tag{15}$$

Partitioning M into A and c, and vector v into u:

$$\mathbf{M} = \begin{pmatrix} \mathbf{A} & \mathbf{c} \end{pmatrix} \tag{16}$$
$$\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} \tag{17}$$
$$\mathbf{v} = \begin{pmatrix} \mathbf{u} \\ 1 \end{pmatrix} \tag{18}$$
$$\mathbf{Au} + \mathbf{c} = 0 \implies \mathbf{Au} = -\mathbf{c} \tag{19}$$

Form the augmented matrix:

$$\left[\begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \middle| \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right] \tag{20}$$

Eliminate below the pivot using $r_2 \leftarrow r_2 - 2r_1$:

$$\left[\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \middle| \begin{pmatrix} -2 \\ 5 \end{pmatrix} \right] \tag{21}$$

Eliminate above the pivot using $r_1 \leftarrow r_1 + r_2$:

$$\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \middle| \begin{pmatrix} 3 \\ 5 \end{pmatrix} \right] \tag{22}$$

Reading off the solution for u:

$$\mathbf{u} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \tag{23}$$

Hence the homogeneous vector and fraction:

$$\mathbf{v} = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix} \tag{24}$$
$$\frac{x}{y} = \frac{3}{5} \tag{25}$$

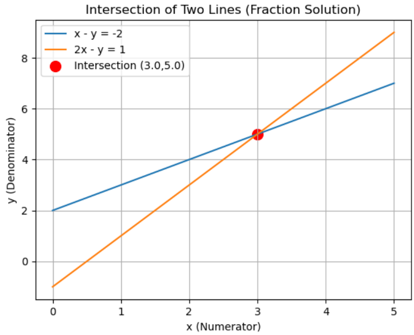
Intersection of Two Lines (Fraction Solution)
x - y = -2
2x - y = 1
Intersection (3.0,5.0)
y (Denominator)
x (Numerator)
0
1
2
3
4
5
0
2
4
6
8

Fig. 0.1