1.2.1.1

AI25BTECH11001 - ABHISEK MOHAPATRA

Question:
Find the values of k if the points A(k +1,2k), B(3k, 2k +3) and C(5k-1,5k) are collinear.

Solution: From the given information,

$$A = \begin{pmatrix} k+1 \\ 2k \end{pmatrix}, B = \begin{pmatrix} 3k \\ 2k+3 \end{pmatrix}, C = \begin{pmatrix} 5k-1 \\ 5k \end{pmatrix} \tag{0.1}$$

To check if the points are collinear, we can use

$$rank\begin{pmatrix} B-A & C-A \end{pmatrix} = 1 \tag{0.2}$$

So,

$$\begin{pmatrix} B-A & C-A \end{pmatrix} = \begin{pmatrix} 2k-1 & 4k-2 \\ 3 & 3k \end{pmatrix} \tag{0.3}$$

$$\xleftrightarrow{C_2=C_2-2C_1} \begin{pmatrix} 2k-1 & 0 \\ 3 & 3k-6 \end{pmatrix} \tag{0.4}$$

The rank of the matrix will be 1 when

$$3k - 6 = 0 \tag{0.5}$$

$$\Rightarrow k = 2 \tag{0.6}$$

Therefore, k = 2.