4.4.12

AI25BTECH11016-Varun

Question:

Find the equation of the plane passing through the points $(2, 5, -3)$, $(-2, -3, 5)$ and $(5, 3, -3)$. Also find the point of intersection of this plane with the line passing through points $(3, 1, 5)$ and $(-1, -3, -1)$. .

Solution:

Let the vectors be

$$\mathbf{A} = \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} -2 \\ -3 \\ 5 \end{pmatrix}, \qquad \mathbf{C} = \begin{pmatrix} 5 \\ 3 \\ -3 \end{pmatrix}. \tag{1}$$

The vectors lying on the plane are

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -2 \\ -3 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} = \begin{pmatrix} -4 \\ -8 \\ 8 \end{pmatrix}, \tag{2}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 5 \\ 3 \\ -3 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}. \tag{3}$$

The normal vector to the plane is given by the cross product

$$\mathbf{n} = (\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A}) \tag{4}$$

$$= \begin{pmatrix} -4 \\ -8 \\ 8 \end{pmatrix} \times \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 16 \\ 24 \\ 32 \end{pmatrix}$$

Equation of the plane passing through $\mathbf{A}$ is

$$\mathbf{n}^T (\mathbf{x} - \mathbf{A}) = 0, \tag{5}$$

$$\begin{pmatrix} 16 & 24 & 32 \end{pmatrix} \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \\ -3 \end{pmatrix} \right) = 0.$$

Hence, the equation of the plane is

$$2x + 3y + 4z = 7. \tag{6}$$

Now, the line passes through

$$\mathbf{P} = \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} -1 \\ -3 \\ -1 \end{pmatrix}. \tag{7}$$

The direction vector is

$$\mathbf{d} = \mathbf{Q} - \mathbf{P} = \begin{pmatrix} -1 \\ -3 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \\ -6 \end{pmatrix}. \tag{8}$$

Thus, the parametric equation of the line is

$$\mathbf{r} = \mathbf{P} + \lambda \mathbf{d}, \tag{9}$$

$$\mathbf{r} = \begin{pmatrix} 3 - 4\lambda \\ 1 - 4\lambda \\ 5 - 6\lambda \end{pmatrix}. \tag{10}$$

Substitute into the plane equation:

$$2x + 3y + 4z = 7 \tag{11}$$

$$2(3 - 4\lambda) + 3(1 - 4\lambda) + 4(5 - 6\lambda) = 7, \tag{12}$$

$$\lambda = \tfrac{1}{2}. \tag{13}$$

Thus, the point of intersection is

$$\mathbf{r} = \begin{pmatrix} 3 - 4\left(\frac{1}{2}\right) \\ 1 - 4\left(\frac{1}{2}\right) \\ 5 - 6\left(\frac{1}{2}\right) \end{pmatrix} \tag{14}$$

$$= \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}. \tag{15}$$

Final Answer:

The plane equation is $2x + 3y + 4z = 7$, and the point of intersection is $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

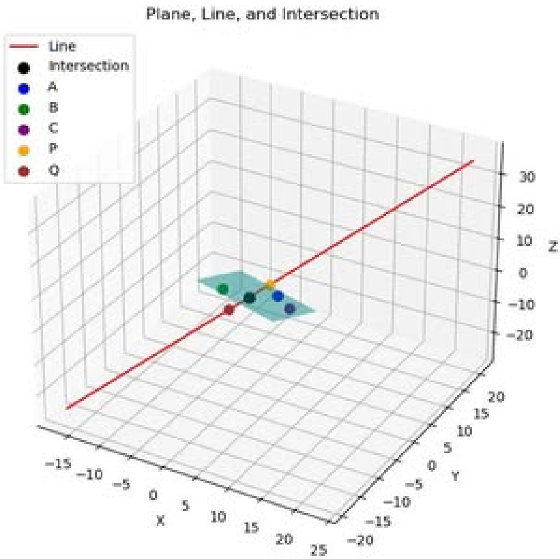
Plane, Line, and Intersection
Line
Intersection
A
B
C
P
Q
X
Y
Z
−15
−10
−5
0
5
10
15
20
25
−20
30

Fig. 0.1