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Matrix 3.2.31

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Question (3.2.31)

A triangle ABC can be constructed in which

$$\angle B = 60°, \qquad \angle C = 45°,$$

and

$$AB + BC + AC = 12 \text{ cm}.$$

Solution

Side $a = BC,\ b = CA,\ c = AB$, and angles A, B, C opposite to a, b, c respectively. Given $B = 60°,\ C = 45°$.

The three linear equations in the unknowns a, b, c are

$$a + b + c = 12, \tag{1}$$
$$-a + (\cos C)\, b + (\cos B)\, c = 0, \tag{2}$$
$$0 \cdot a + (\sin C)\, b - (\sin B)\, c = 0. \tag{3}$$

Write the augmented matrix corresponding to (**??**)–(**??**):

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ -1 & \frac{\sqrt{2}}{2} & \frac{1}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{3}}{2} & 0 \end{array}\right]. \tag{4}$$

Perform RREF on the augmented matrix (**??**). One convenient path is:

1. Add row 2 to row 1 (to eliminate the -1 in row 2):

$$R_1 \leftarrow R_1 + R_2 : \left[\begin{array}{ccc|c} 0 & 1 + \frac{\sqrt{2}}{2} & 1 + \frac{1}{2} & 12 \\ -1 & \frac{\sqrt{2}}{2} & \frac{1}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{3}}{2} & 0 \end{array}\right].$$

2. On Solving with RREF we get

$$a = c \cdot \frac{\sqrt{3}+1}{2}, \qquad b = c \cdot \frac{\sqrt{6}}{2}, \tag{5}$$

and from the sum $a + b + c = 12$ we get

$$c\left(\frac{\sqrt{3}+1}{2} + \frac{\sqrt{6}}{2} + 1\right) = 12. \tag{6}$$

Solving (**??**) for c gives

$$c \;=\; \frac{24}{\sqrt{3}+\sqrt{6}+3}. \tag{7}$$

Substituting back, we obtain

$$b = \frac{\sqrt{6}}{2}\,c \;=\; \frac{12\sqrt{6}}{\sqrt{3}+\sqrt{6}+3}, \tag{8}$$

$$a = \frac{\sqrt{3}+1}{2}\,c \;=\; \frac{12(\sqrt{3}+1)}{\sqrt{3}+\sqrt{6}+3}. \tag{9}$$

Numerically (for quick checking):

$$a \approx 4.565, \quad b \approx 4.093, \quad c \approx 3.342, \tag{10}$$

which indeed satisfy $a + b + c = 12$.

Plot

Place $B = (0, 0)$ and $C = (a, 0)$ and A $= (\text{c}\cos B,\; c\sin B)$

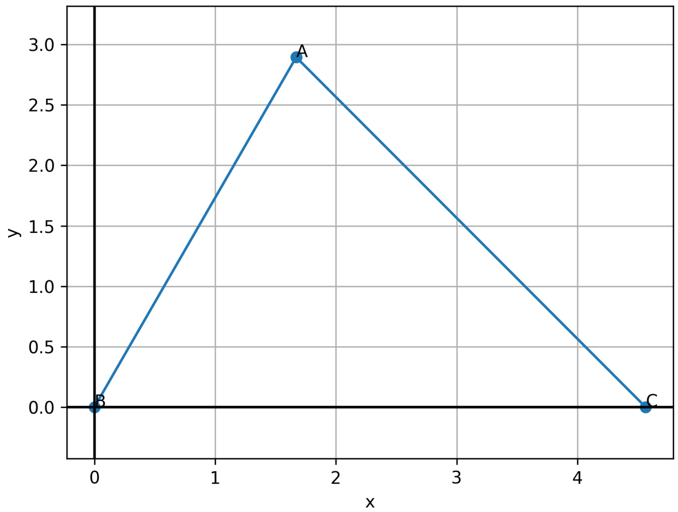

Figure 1: Triangle formed by points A, B, and C.