5.4.5

EE25BTECH11010 - Arsh Dhoke

Question:
Using elementary transformations, find the inverse of the following matrix:
$\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.

Solution:
We know

$$\mathbf{A}^{-1}\mathbf{A} = \mathbf{I} \tag{0.1}$$

where $\mathbf{I}$ is the identity matrix $\mathbf{I}_2$

The augmented matrix for the given matrix will be

$$\left(\begin{array}{cc|cc} 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right) \xleftrightarrow[R_2 \to R_2 - 2R_1]{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|cc} 1 & 2 & 0 & 1 \\ 0 & -3 & 1 & -2 \end{array}\right) \tag{0.2}$$

$$\xleftrightarrow[R_1 \to R_1 - 2R_2]{R_2 \to -\frac{1}{3}R_2} \left(\begin{array}{cc|cc} 1 & 0 & \frac{2}{3} & -\frac{1}{3} \\ 0 & 1 & -\frac{1}{3} & \frac{2}{3} \end{array}\right) \tag{0.3}$$

$$\therefore \quad \mathbf{A}^{-1} = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \tag{0.4}$$

$$\mathbf{A}^{-1} = \frac{1}{3}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \tag{0.5}$$

We can verify the computed inverse using python code by showing $\mathbf{A}^{-1}\mathbf{A} = \mathbf{I}$.