1.8.2

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Question

Find the distance between the following pairs of points:

1. (2,3,5) and (4,3,1)
2. (-3,7,2) and (2,4,-1)
3. (-1,3,-4) and (1,-3,4)
4. (2,-1,3) and (-2,1,3)

Theoretical Solution

We know that,
The length of a vector is defined as

$$\|\mathbf{x}\| = \sqrt{\mathbf{x}^\top \mathbf{x}} \tag{1}$$

Therefore,
distance between $\mathbf{P}$ and $\mathbf{Q}$ is

$$d(\mathbf{P}, \mathbf{Q}) = \|\mathbf{P} - \mathbf{Q}\| = \sqrt{(\mathbf{P} - \mathbf{Q})^\top (\mathbf{P} - \mathbf{Q})}. \tag{2}$$

Theoretical Solution

Let,

$$\mathbf{A} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$$

$$d(\mathbf{A}, \mathbf{B}) = \left\| \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} \right\| = \left\| \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix} \right\|$$

$$= \sqrt{\begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}^{\top} \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}} = \sqrt{(-2)^2 + 0^2 + 4^2} = \sqrt{20} = 2\sqrt{5}.$$

Theoretical Solution

$$\mathbf{C} = \begin{pmatrix} -3 \\ 7 \\ 2 \end{pmatrix}, \ \mathbf{D} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$$

$$d(\mathbf{C}, \mathbf{D}) = \| \begin{pmatrix} -3 \\ 7 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} \| = \| \begin{pmatrix} -5 \\ 3 \\ 3 \end{pmatrix} \|$$

$$= \sqrt{\begin{pmatrix} -5 \\ 3 \\ 3 \end{pmatrix}^{\top} \begin{pmatrix} -5 \\ 3 \\ 3 \end{pmatrix}} = \sqrt{(-5)^2 + 3^2 + 3^2} = \sqrt{43}.$$

Theoretical Solution

$$\mathbf{E} = \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$$

$$d(\mathbf{E}, \mathbf{F}) = \| \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} \| = \| \begin{pmatrix} -2 \\ 6 \\ -8 \end{pmatrix} \|$$

$$= \sqrt{\begin{pmatrix} -2 \\ 6 \\ -8 \end{pmatrix}^{\top} \begin{pmatrix} -2 \\ 6 \\ -8 \end{pmatrix}} = \sqrt{(-2)^2 + 6^2 + (-8)^2} = \sqrt{104} = 2\sqrt{26}.$$

Theoritical Solution

$$\mathbf{G} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \ \mathbf{H} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$$

$$d(\mathbf{G}, \mathbf{H}) = \| \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \| = \| \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix} \|$$

$$= \sqrt{\begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}^{\top} \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}} = \sqrt{4^2 + (-2)^2 + 0^2} = \sqrt{20} = 2\sqrt{5}.$$

C Code - Sum of vectors and Magnitude of vectors

```c
#include<stdio.h>
#include<math.h>

#define DIST(p,q) sqrt( ((p[0]-q[0])*(p[0]-q[0])) + \
                        ((p[1]-q[1])*(p[1]-q[1])) + \
                        ((p[2]-q[2])*(p[2]-q[2])) )
int main()
{
    int A[3] = {2,3,5}, B[3] = {4,3,1};
    int C[3] = {-3,7,2}, D[3] = {2,4,-1};
    int E[3] = {-1,3,-4}, F[3] = {1,-3,4};
    int G[3] = {2,-1,3}, H[3] = {-2,1,3};
    printf("AB = %.3f\n", DIST(A,B));
    printf("CD = %.3f\n", DIST(C,D));
    printf("EF = %.3f\n", DIST(E,F));
    printf("GH = %.3f\n", DIST(G,H));

    return 0;
}
```

Python Code - Distance calculation

```
import numpy as np
import matplotlib.pyplot as plt

def distance3D(p1, p2):
    return np.linalg.norm(np.array(p1) - np.array(p2))

# Points
A, B = (2,3,5), (4,3,1)
C, D = (-3,7,2), (2,4,-1)
E, F = (-1,3,-4), (1,-3,4)
G, H = (2,-1,3), (-2,1,3)

print("AB =", distance3D(A,B))
print("CD =", distance3D(C,D))
print("EF =", distance3D(E,F))
print("GH =", distance3D(G,H))
```

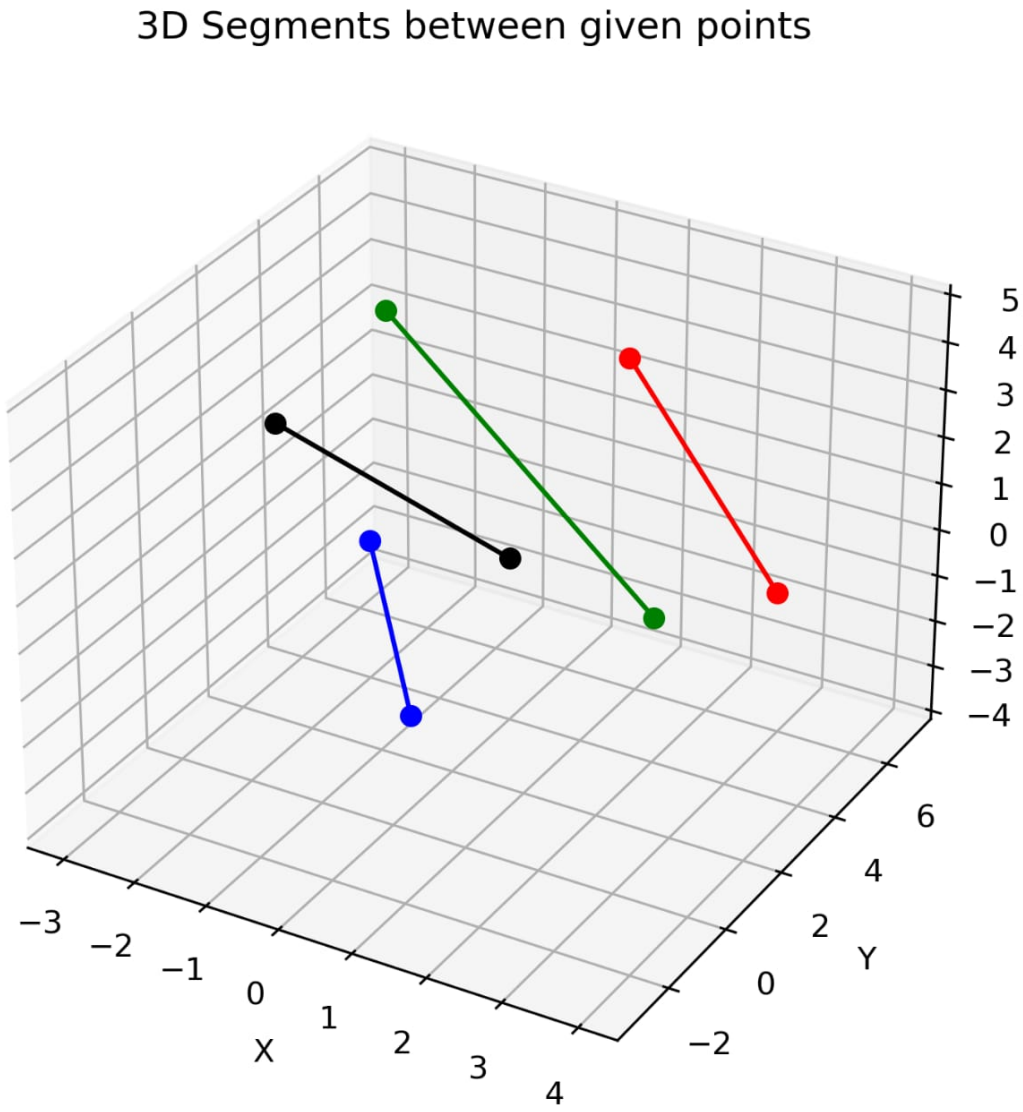
Python Code - 3D Plot

```
fig = plt.figure(figsize=(8,6))
ax = fig.add_subplot(111, projection='3d')

pairs = [(A,B,'r'),(C,D,'g'),(E,F,'b'),(G,H,'k')]
for P, Q, c in pairs:
    ax.plot([P[0],Q[0]], [P[1],Q[1]], [P[2],Q[2]], c+"-o")

ax.set_xlabel("X")
ax.set_ylabel("Y")
ax.set_zlabel("Z")
ax.set_title("3D Segments between given points")
plt.savefig("/home/user/Matrix/Matgeo_assignments/1.9.15/figs/
    Figure_1.png", dpi=300, bbox_inches='tight')
plt.show()
```

Plot


3D Segments between given points
X
Y
−3
−2
−1
0
1
2
3
4
−2
0
2
4
6
5
4
3
2
1
0
−1
−2
−3
−4