

3.3.7

Pratik R-AI25BTECH11023

September 16, 2025

Question

A unit vector perpendicular to the plane determined by the points $P(1, -1, 2)$, $Q(2, 0, -1)$ and $R(0, 2, 1)$ is

Solution

Let $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ be the position vector of point **B** and a,b and c be the sides opposite the vertices A,B and C, respectively in ΔABC .

Given $a = 8\text{cm}$;

$$\mathbf{c} = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \quad (1)$$

$$\therefore \mathbf{A} = \begin{pmatrix} c \cos \angle B \\ c \sin \angle B \end{pmatrix} = \begin{pmatrix} c \times 1/\sqrt{2} \\ c \times 1/\sqrt{2} \end{pmatrix} \quad (2)$$

Solution

in $\triangle ABC$

$$b \cos \angle C + c \cos \angle B = 8 \quad (3)$$

$$b \sin \angle C - c \sin \angle B = 0 \quad (4)$$

Solving linear Equation in b and c:

$$\begin{pmatrix} \cos \angle C & \cos \angle B \\ \sin \angle C & -\sin \angle B \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} a \\ 0 \end{pmatrix} \quad (5)$$

Solution

using augmented matrix

$$\left(\begin{array}{cc|c} \cos \angle C & \cos \angle B & a \\ \sin \angle C & -\sin \angle B & 0 \end{array} \right) \quad (6)$$

putting $\angle C = 30^\circ$ and $\angle B = 45^\circ$

$$\left(\begin{array}{cc|c} \sqrt{3}/2 & 1/\sqrt{2} & 8 \\ 1/2 & -1/\sqrt{2} & 0 \end{array} \right) \quad (7)$$

Echelon form of the matrix is given by

$$\left(\begin{array}{cc|c} \sqrt{3}/2 & 1/\sqrt{2} & 8 \\ 0 & (-\sqrt{3}-1)/\sqrt{2} & -8 \end{array} \right) \quad (8)$$

$$\frac{(-\sqrt{3}-1)}{\sqrt{2}} \times c = -8 \quad (9)$$

$$\Rightarrow c = \frac{8\sqrt{2}}{(\sqrt{3}+1)} = \sqrt{3}-1 \quad (10)$$

$$\therefore \mathbf{A} = \begin{pmatrix} \sqrt{3}-1 \\ \sqrt{3}-1 \end{pmatrix} \quad (11)$$

