8.3.9

SAMYAK GONDANE - AI25BTECH11029

Question

If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentric

Solution

Parametric Vector Form of Ellipse
An ellipse centered at the origin is defined by the vector function:

$$\mathbf{r}(\theta) = \begin{pmatrix} x(\theta) \\ y(\theta) \end{pmatrix} = \begin{pmatrix} a\cos\theta \\ b\sin\theta \end{pmatrix} \quad \text{where } \theta \in [0, 2\pi] \tag{1}$$

Here, a and b are the semi-major and semi-minor axes respectively.

Solution

Focus and Latus Rectum

The focus of the ellipse lies at:

$$\mathbf{F} = \begin{pmatrix} ae \\ 0 \end{pmatrix} \quad \text{where } e = \text{eccentricity} \tag{2}$$

The latus rectum L is the chord perpendicular to the major axis passing through the focus:

$$L = \frac{2b^2}{a} \tag{3}$$

Solution

Given Condition

We are given:

$$L = \frac{1}{2} \cdot 2b = b \quad \Rightarrow \quad \frac{2b^2}{a} = b \quad \Rightarrow \quad 2b = a \tag{4}$$

Thus:

$$a = 2b \tag{5}$$

Solution

Eccentricity via Vector Displacement

Eccentricity is defined as:

$$e = \frac{c}{a} \quad \text{where } c = \text{distance from center to focus} = \sqrt{a^2 - b^2} \tag{6}$$

Substitute $a = 2b$:

$$c = \sqrt{(2b)^2 - b^2} = \sqrt{4b^2 - b^2} = \sqrt{3b^2} = b\sqrt{3} \tag{7}$$

$$e = \frac{b\sqrt{3}}{2b} = \frac{\sqrt{3}}{2} \tag{8}$$

Final Answer

$$\boxed{e = \frac{\sqrt{3}}{2}} \tag{9}$$

Plot

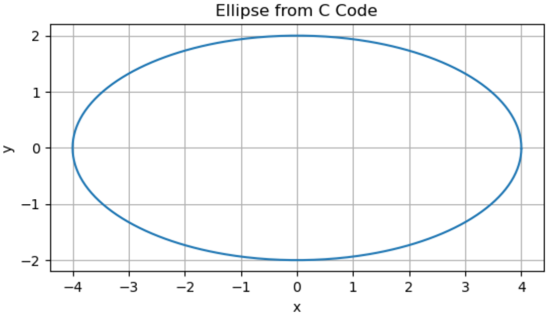

Figure: