4.12.44

AI25BTECH11030 -Sarvesh Tamgade

Question: Find the equation of the set of points which are equidistant from the points $\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$.

Solution: Let $\mathbf{X} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ be the position vector of any point equidistant from $\mathbf{A}$ and $\mathbf{B}$.

The condition for $\mathbf{X}$ to be equidistant is:

$$\|\mathbf{X} - \mathbf{A}\| = \|\mathbf{X} - \mathbf{B}\| \tag{0.1}$$

Squaring both sides we get:

$$(\mathbf{X} - \mathbf{A})^\top (\mathbf{X} - \mathbf{A}) = (\mathbf{X} - \mathbf{B})^\top (\mathbf{X} - \mathbf{B}) \tag{0.2}$$

Expanding,

$$\mathbf{X}^\top \mathbf{X} - 2\mathbf{A}^\top \mathbf{X} + \mathbf{A}^\top \mathbf{A} = \mathbf{X}^\top \mathbf{X} - 2\mathbf{B}^\top \mathbf{X} + \mathbf{B}^\top \mathbf{B} \tag{0.3}$$

Simplifying,

$$-2\mathbf{A}^\top \mathbf{X} + \mathbf{A}^\top \mathbf{A} = -2\mathbf{B}^\top \mathbf{X} + \mathbf{B}^\top \mathbf{B} \tag{0.4}$$

Rearranging,

$$2(\mathbf{B} - \mathbf{A})^\top \mathbf{X} = \mathbf{B}^\top \mathbf{B} - \mathbf{A}^\top \mathbf{A} \tag{0.5}$$

Calculate $\mathbf{B} - \mathbf{A}$:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3-1 \\ 2-2 \\ -1-3 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix} \tag{0.6}$$

Calculate $\mathbf{B}^\top \mathbf{B}$ and $\mathbf{A}^\top \mathbf{A}$:

$$\mathbf{B}^\top \mathbf{B} = 3^2 + 2^2 + (-1)^2 = 14, \quad \mathbf{A}^\top \mathbf{A} = 1^2 + 2^2 + 3^2 = 14 \tag{0.7}$$

Thus,

$$2\begin{pmatrix} 2 & 0 & -4 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 14 - 14 = 0 \tag{0.8}$$

Simplifying,

$$\begin{pmatrix} 4 & 0 & -8 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \tag{0.9}$$

This matrix equation represents the plane:

$$4a - 8c = 0 \tag{0.10}$$

or equivalently,

$$a - 2c = 0 \tag{0.11}$$

Final Answer: The set of points equidistant from **A** and **B** lies on the plane defined by

$$\boxed{\begin{pmatrix} 4 & 0 & -8 \end{pmatrix} \mathbf{x} = 0}$$

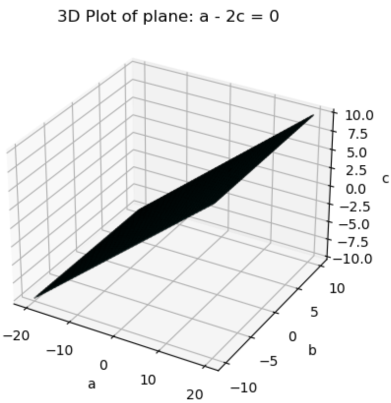

Fig. 0.1: Vector Representation