2.4.33

EE25BTECH11036 - M Chanakya Srinivas

Q 2.4.33.

Name the type of triangle formed by the points

$$A(-5,6),\ B(-4,-2),\ C(7,5). \tag{1}$$

Solution

Vertices

$$\mathbf{A} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}, \tag{2}$$

$$\mathbf{B} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}, \tag{3}$$

$$\mathbf{C} = \begin{pmatrix} 7 \\ 5 \end{pmatrix}. \tag{4}$$

Difference vectors

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ -8 \end{pmatrix}, \tag{5}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} 12 \\ -1 \end{pmatrix}, \tag{6}$$

$$\mathbf{C} - \mathbf{B} = \begin{pmatrix} 11 \\ 7 \end{pmatrix}. \tag{7}$$

Angle checks using dot products

At vertex A:

$$(\mathbf{B} - \mathbf{A})^{\top}(\mathbf{C} - \mathbf{A}) = \begin{pmatrix} 1 & -8 \end{pmatrix}\begin{pmatrix} 12 \\ -1 \end{pmatrix} \tag{8}$$

$$= 20 > 0. \tag{9}$$

Hence, $\angle A$ is acute.

At vertex B:

$$(\mathbf{A} - \mathbf{B})^{\top}(\mathbf{C} - \mathbf{B}) = \begin{pmatrix} -1 & 8 \end{pmatrix}\begin{pmatrix} 11 \\ 7 \end{pmatrix} \tag{10}$$

$$= 45 > 0. \tag{11}$$

Hence, $\angle B$ is acute.

At vertex C:

$$(\mathbf{A}-\mathbf{C})^{\top}(\mathbf{B}-\mathbf{C}) = \begin{pmatrix}-12 & 1\end{pmatrix}\begin{pmatrix}-11\\-7\end{pmatrix} \tag{12}$$

$$= 125 > 0. \tag{13}$$

Hence, $\angle C$ is acute.

Conclusion

All three angles are acute. Since the vectors in (**??**), (**??**), (**??**) are not multiples of each other and none of the angles is right or obtuse, the triangle is

an acute scalene triangle.

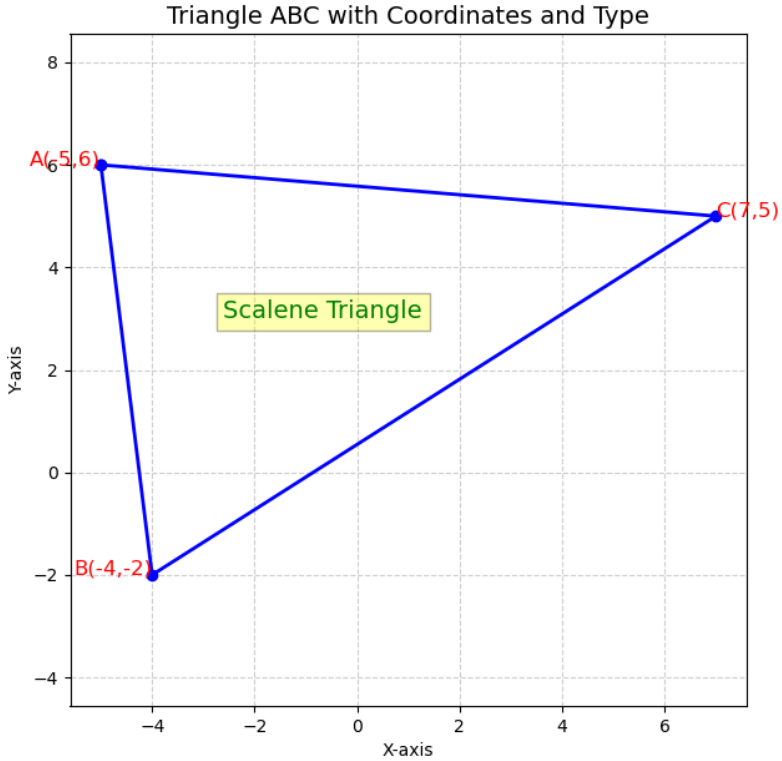

Fig. 1

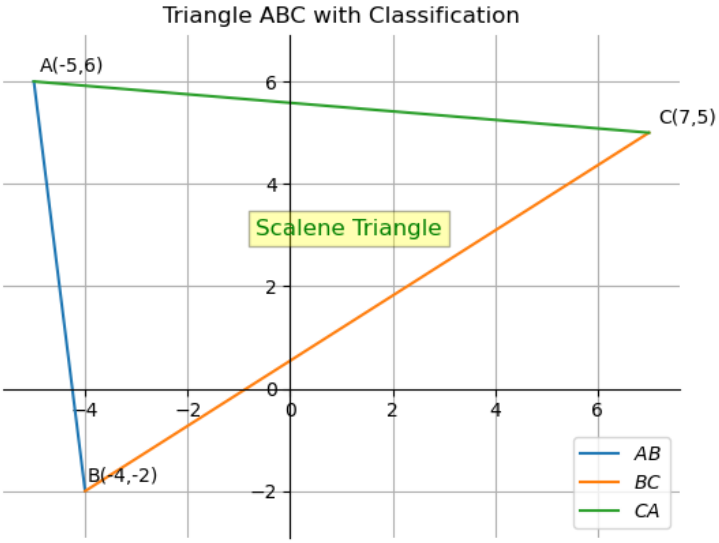
Triangle ABC with Classification
A(-5,6)
C(7,5)
Scalene Triangle
B(-4,-2)
AB
BC
CA

Fig. 2