1.5.24

EE25BTECH11036 - M Chanakya Srinivas

1.5.24 A line intersects the Y-axis and X-axis at the points $P = (0, b)$ and $Q = (c, 0)$ respectively. If $(2, -5)$ is the midpoint of $\overline{PQ}$, then find the coordinates of P and Q.

$$\mathbf{P} = \begin{pmatrix} 0 \\ b \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} c \\ 0 \end{pmatrix}, \quad \mathbf{M} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}.$$

(i) Rank/collinearity:

$$\text{Since } \mathbf{P}, \mathbf{Q}, \mathbf{M} \text{ are collinear, } \operatorname{rank}\begin{pmatrix} \mathbf{P}-\mathbf{M} & \mathbf{Q}-\mathbf{M} \end{pmatrix}^{\top} = 1.$$

$$\begin{pmatrix} \mathbf{P}-\mathbf{M} & \mathbf{Q}-\mathbf{M} \end{pmatrix}^{\top} = \begin{pmatrix} -2 & c-2 \\ b+5 & 5 \end{pmatrix} \xrightarrow{R_2 \leftarrow -2R_2 - (b+5)R_1} \begin{pmatrix} -2 & c-2 \\ 0 & -10-(b+5)(c-2) \end{pmatrix}.$$

For rank = 1, the last entry must be 0:

$$-10 - (b+5)(c-2) = 0 \implies (b+5)(c-2) = -10. \tag{*}$$

(ii) Midpoint:

$$\mathbf{M} = \frac{\mathbf{P}+\mathbf{Q}}{2} \implies \begin{pmatrix} 2 \\ -5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 \\ b \end{pmatrix} + \frac{1}{2}\begin{pmatrix} c \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} c \\ b \end{pmatrix} \implies \begin{pmatrix} c \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ -10 \end{pmatrix}.$$

Thus $c = 4$, $b = -10$, and these satisfy $(*)$.

Answer:

$$\mathbf{P} = \begin{pmatrix} 0 \\ -10 \end{pmatrix}, \qquad \mathbf{Q} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}.$$

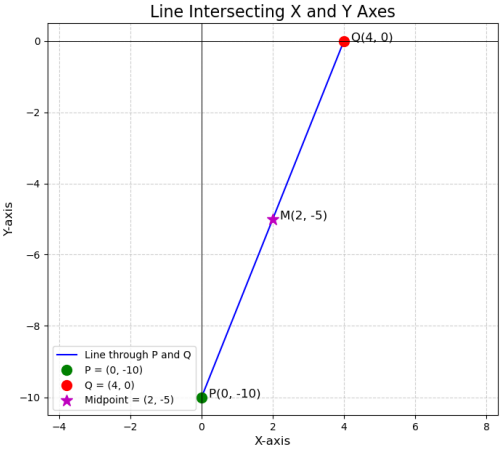

Fig. 0.1: Plot using Shared output

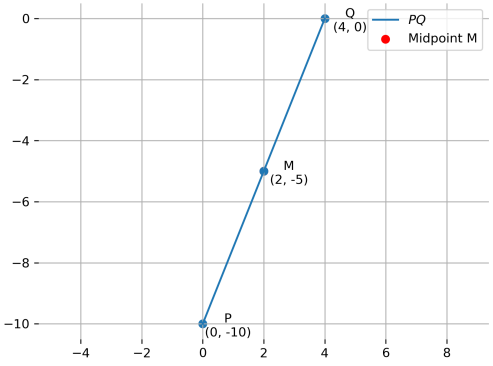

Fig. 0.2: Plot using Python