

Question 5.13.26

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1 Question:

If \mathbf{A} and \mathbf{B} are square matrices of size $n \times n$ such that $\mathbf{A}^2 - \mathbf{B}^2 = (\mathbf{A} - \mathbf{B})(\mathbf{A} + \mathbf{B})$, then which of the following will be always true?

- (a) $\mathbf{A} = \mathbf{B}$
- (b) $\mathbf{AB} = \mathbf{BA}$
- (c) either of \mathbf{A} or \mathbf{B} is a zero matrix
- (d) either \mathbf{A} or \mathbf{B} is an identity matrix

2 Solution:

We know that for any two square matrices \mathbf{A} and \mathbf{B} of size $n \times n$, the following is true:

$$(\mathbf{A} - \mathbf{B})(\mathbf{A} + \mathbf{B}) = (\mathbf{A} - \mathbf{B})\mathbf{A} + (\mathbf{A} - \mathbf{B})\mathbf{B} \quad (1)$$

$$= \mathbf{A}^2 - \mathbf{BA} + \mathbf{AB} - \mathbf{B}^2 \quad (2)$$

Given that $\mathbf{A}^2 - \mathbf{B}^2 = (\mathbf{A} - \mathbf{B})(\mathbf{A} + \mathbf{B})$, we can use equation 2:

$$\mathbf{A}^2 - \mathbf{B}^2 = \mathbf{A}^2 - \mathbf{BA} + \mathbf{AB} - \mathbf{B}^2 \quad (3)$$

$$\implies 0 = -\mathbf{BA} + \mathbf{AB} \quad (4)$$

$$\implies \mathbf{BA} = \mathbf{AB} \quad (5)$$

Thus, option (b) is always true.