10.6.11

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Question : Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.

Solution :

Name	**Value**
Circle	$\mathbf{x}^\top\mathbf{x} - 16 = 0$
P	$\begin{pmatrix}8\\0\end{pmatrix}$

Table : Circle

The parameters of the circle with center $\mathbf{0}$ are :

$$\mathbf{V} = \mathbf{I} \qquad \mathbf{u} = \mathbf{0} \qquad f = -16 \tag{1}$$

Let the point from which tangent is being drawn be $\mathbf{p}$.

Let the point of contact be $\mathbf{q}$ and

$$\mathbf{q}^\top\mathbf{q} = 16 \tag{2}$$

From the condition of tangency we get

$$\mathbf{q}^\top(\mathbf{q} - \mathbf{p}) = 0 \tag{3}$$

$$\mathbf{p}^\top\mathbf{q} = \mathbf{q}^\top\mathbf{q} \tag{4}$$

$$\mathbf{p}^\top\mathbf{q} = 16 \tag{5}$$

If the angle between the tangents is 60° then the angle betweent the normals at the points of contact is 120°.

Therefore,

$$\cos(\tfrac{120°}{2}) = \frac{\mathbf{p}^\top\mathbf{q}}{\|\mathbf{p}\|\,\|\mathbf{q}\|} \tag{6}$$

$$\|\mathbf{p}\| = 8 \tag{7}$$

$$\mathbf{p}^\top\mathbf{p} - 64 = 0 \tag{8}$$

Therefore the locus of point $\mathbf{p}$ is a circle with center $\mathbf{0}$ and radius 8 cm.

Consider point $\mathbf{P} = \begin{pmatrix}8\\0\end{pmatrix}$ from which tangents are drawn.

Let the slope of tangent be m and the tangent equationn is given as :

$$\mathbf{n}^\top\mathbf{x} = \mathbf{n}^\top\mathbf{P} \qquad \mathbf{n} = \begin{pmatrix}-m\\1\end{pmatrix} \tag{9}$$

The length of perpendicular from the center of the circle to the tangent is equal to the radius and is given by :

$$4 = \frac{|\mathbf{n}^\top \mathbf{0} - \mathbf{n}^\top \mathbf{P}|}{||\mathbf{n}||} \tag{10}$$

$$|\mathbf{n}^\top \mathbf{P}| = 4\,||\mathbf{n}|| \tag{11}$$

$$|-8m| = 4\sqrt{m^2+1} \tag{12}$$

$$m = \pm\frac{1}{\sqrt{3}} \tag{13}$$

The normal vectors for the tangents are given as :

$$\mathbf{n_1} = \begin{pmatrix} -\frac{1}{\sqrt{3}} \\ 1 \end{pmatrix} \qquad \mathbf{n_2} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ 1 \end{pmatrix} \tag{14}$$

The points of contacts are given as :

$$\mathbf{q_i} = \pm r\frac{\mathbf{n_i}}{||\mathbf{n_i}||} \tag{15}$$

From (5) , $\mathbf{P}^\top \mathbf{q} = 16$, so the points of contact are :

$$\mathbf{q_1} = \begin{pmatrix} 2 \\ 2\sqrt{3} \end{pmatrix} \qquad \mathbf{q_2} = \begin{pmatrix} 2 \\ -2\sqrt{3} \end{pmatrix} \tag{16}$$

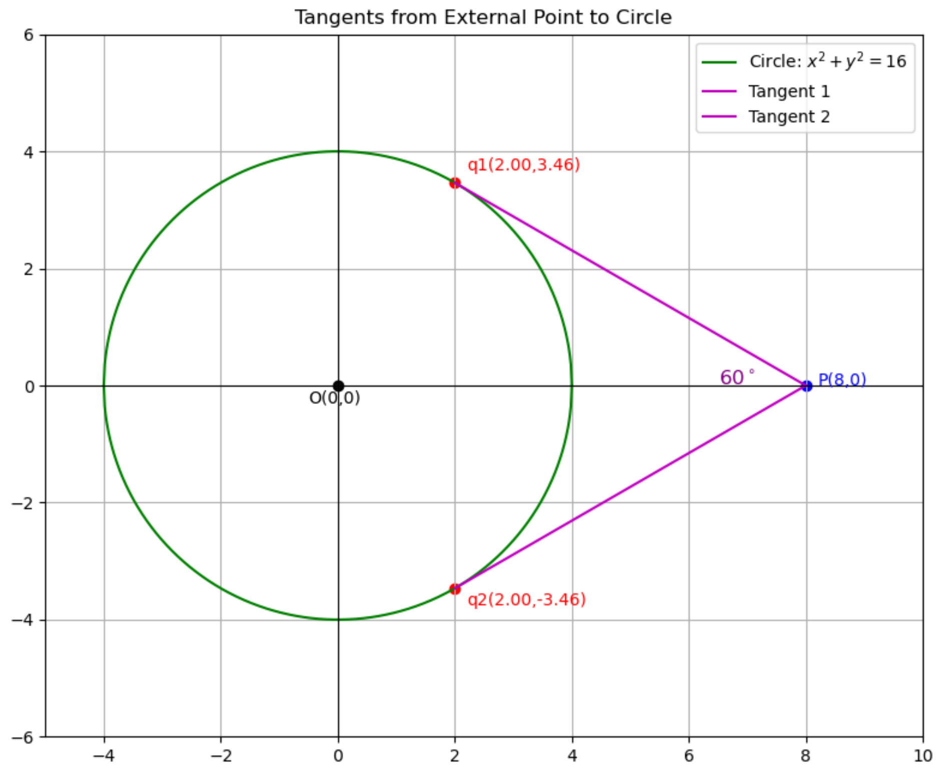

Fig : Circle and Tangents