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Question

For what value of k, will the following pain of equations have infinitly many solutions

$2x + 3y = 7$ and $(k + 2)x - 3(1 - k)y = 5k + 1$

Solution

Given:

$$2x + 3y = 7 \tag{1}$$
$$(k + 2)x - 3(1 - k)y = 5k + 1 \tag{2}$$

Augmented Matrix

Convert the system to an augmented matrix:

$$\begin{pmatrix} 2 & 3 & | & 7 \\ k+2 & -3+3k & | & 5k+1 \end{pmatrix} \tag{3}$$

Let the second row be:

$$R_2 = \begin{pmatrix} a & b & | & c \end{pmatrix} \quad \text{where} \quad a = k + 2, \quad b = -3 + 3k, \quad c = 5k + 1 \tag{4}$$

Eliminate First Entry of Row 2

Apply row operation:

$$R_2 \to R_2 - \frac{a}{2}R_1 \tag{5}$$

Compute each entry:

$$\text{New second entry} = b - \frac{a}{2} \cdot 3 \tag{6}$$
$$= (-3 + 3k) - \frac{3(k + 2)}{2} \tag{7}$$
$$= \frac{-6 + 6k - 3k - 6}{2} = \frac{3k - 12}{2} \tag{8}$$

$$\text{New third entry} = c - \frac{a}{2} \cdot 7 \tag{9}$$
$$= (5k + 1) - \frac{7(k + 2)}{2} \tag{10}$$
$$= \frac{10k + 2 - 7k - 14}{2} = \frac{3k - 12}{2} \tag{11}$$

So the matrix becomes:

$$\begin{pmatrix} 2 & 3 & | & 7 \\ 0 & \frac{3k-12}{2} & | & \frac{3k-12}{2} \end{pmatrix} \tag{12}$$

Condition for Infinitely Many Solutions

For infinitely many solutions, the second row must reduce to:

$$0x + 0y = 0 \Rightarrow \frac{3k-12}{2} = 0 \Rightarrow 3k - 12 = 0 \Rightarrow k = 4 \tag{13}$$

Final Answer

$$\boxed{k = 4} \tag{14}$$