10.7.81

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Question

Two circles, each of radius 5 units, touch each other at $(1, 2)$. If the equation of common tangent is $4x + 3y = 10$, find the equations of circles.

Theoretical Solution

Let ,

$$\mathbf{P} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{1}$$

Given Line,

$$\mathbf{n}^{\top}\mathbf{x} = 10 \tag{2}$$

Normal Vector $\mathbf{n} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

Theoretical Solution

Unit vector $\mathbf{u}$ in direction of $\mathbf{n}$

$$\mathbf{u} = \frac{\mathbf{n}}{\|\mathbf{n}\|} = \begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix} \tag{3}$$

Let $\mathbf{O_i}$ be the center of Circles, then

$$\mathbf{O_i} = \mathbf{P} \pm 5\mathbf{u} \tag{4}$$

$$\mathbf{O_i} = \begin{pmatrix} 1 \pm 4 \\ 2 \pm 3 \end{pmatrix} \tag{5}$$

$$\therefore \mathbf{O_1} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}, \mathbf{O_2} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} \tag{6}$$

Theoretical Solution

Equation of Circles are :

$$O_1 : g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f \tag{7}$$

$\mathbf{V}$	$\mathbf{u}$	f
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 5 \\ 5 \end{pmatrix}$	25

Table: 1

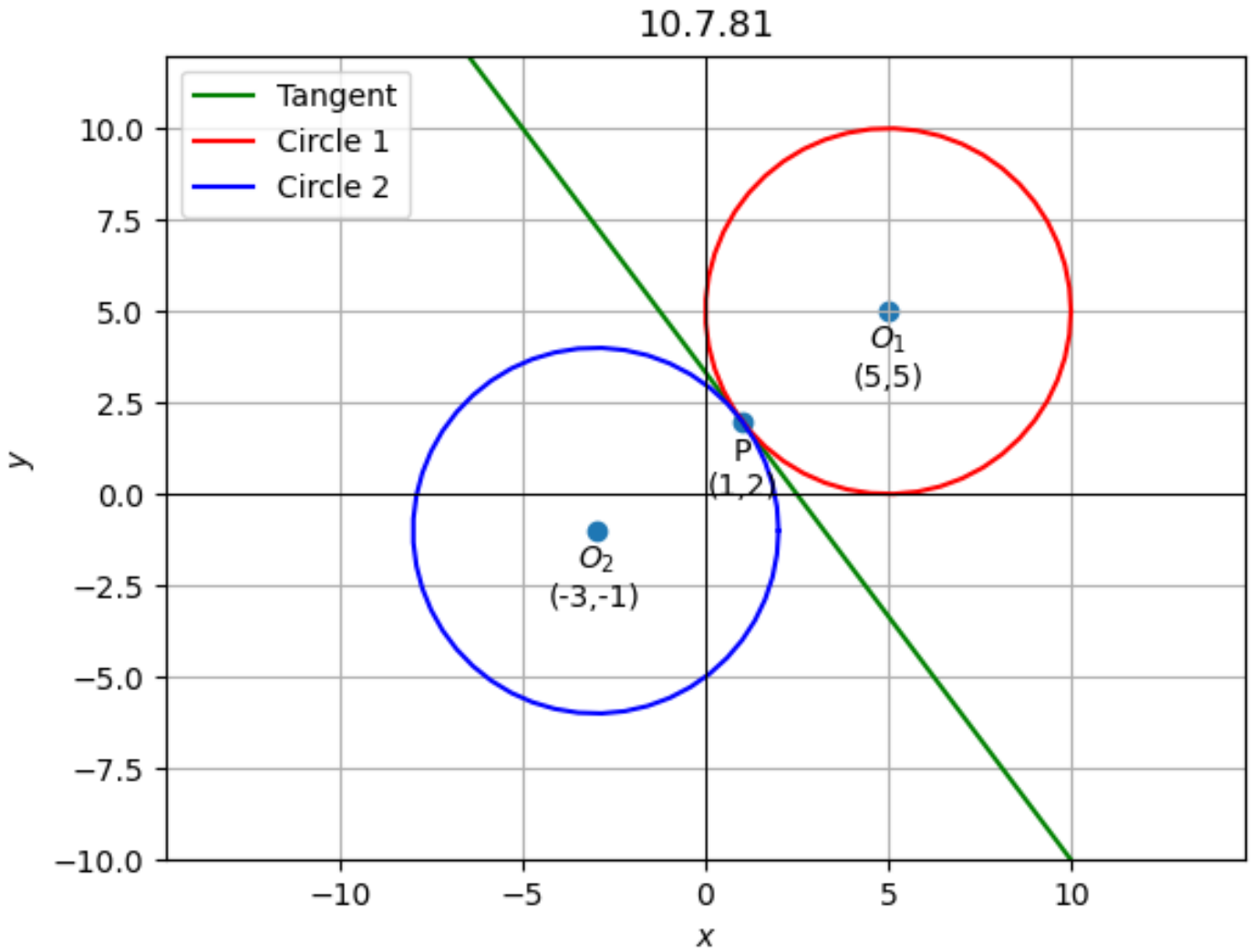
Theoretical Solution

$$O_2 : g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f \tag{8}$$

$\mathbf{V}$	$\mathbf{u}$	f
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -3 \\ -1 \end{pmatrix}$	-15

Table: 2

Plot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Tangent
Circle 1
Circle 2
O_1
(5,5)
P
(1,2)
O_2
(-3,-1)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−10
−5
0
5
10
x
y