4.13.28

EE25BTECH11023 - Venkata Sai

Question:
Slope of a line passing through $\mathbf{P}(2,3)$ and intersecting the line $x+y=7$ at a distance of 4 units from $\mathbf{P}$, is

Solution: Given

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{1}$$

Equation of a line through $\mathbf{P}$ and having slope m is

$$\mathbf{r} = \mathbf{p} + t\mathbf{b} \tag{2}$$

$$\mathbf{b} = \begin{pmatrix} 1 \\ m \end{pmatrix} \tag{3}$$

$$x + y = 7 \implies \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 7 \tag{4}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix}(\mathbf{p} + t\mathbf{b}) = 7 \tag{5}$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{p} + t\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{b} = 7 \tag{6}$$

$$t\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{b} = 7 - \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{p} \tag{7}$$

$$t = \frac{7 - \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{p}}{\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{b}} \tag{8}$$

$\mathbf{Q}$ be the point of intersection

$$\mathbf{q} = \mathbf{p} + t\mathbf{b} \tag{9}$$

$$\mathbf{q} - \mathbf{p} = t\mathbf{b} \tag{10}$$

$$\|\mathbf{q} - \mathbf{p}\| = |t|\,\|\mathbf{b}\| \implies |t| = \frac{\|\mathbf{q} - \mathbf{p}\|}{\|\mathbf{b}\|} \tag{11}$$

$$\left|\frac{7 - \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{p}}{\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{b}}\right| = \frac{\|\mathbf{q} - \mathbf{p}\|}{\|\mathbf{b}\|} \tag{12}$$

Given the point is at a distance of 4 units from point $\mathbf{P}$

$$\left|\frac{7 - \begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix}}{\begin{pmatrix} 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ m \end{pmatrix}}\right| = \frac{4}{\sqrt{1 + m^2}} \tag{13}$$

$$\left|\frac{7-5}{1+m}\right| = \frac{4}{\sqrt{1+m^2}} \tag{14}$$

$$\left(\frac{7-5}{1+m}\right)^2 = \frac{16}{1+m^2} \implies \frac{4}{(1+m)^2} = \frac{16}{1+m^2} \tag{15}$$

$$4\,(1+m)^2 = 1+m^2 \tag{16}$$

$$4\left(m^2+2m+1\right) = 1+m^2 \tag{17}$$

$$4m^2+8m+4 = 1+m^2 \implies 3m^2+8m+3 = 0 \tag{18}$$

$$m^2+\frac{8m}{3}+1 = 0 \tag{19}$$

$$m^2+\frac{8m}{3}+1+\left(\frac{4}{3}\right)^2 = \frac{16}{9} \tag{20}$$

$$\left(m+\frac{4}{3}\right)^2 = \frac{16-9}{9} = \frac{7}{9} \tag{21}$$

$$m+\frac{4}{3} = \pm\frac{\sqrt{7}}{3} \tag{22}$$

$$m = \frac{-4-\sqrt{7}}{3} \text{ (or) } \frac{-4+\sqrt{7}}{3} \tag{23}$$

According to options

$$m = \frac{-4+\sqrt{7}}{3} = \frac{8-2\sqrt{7}}{-6} = \frac{\left(1-\sqrt{7}\right)^2}{\left(1+\sqrt{7}\right)\left(1-\sqrt{7}\right)} = \frac{1-\sqrt{7}}{1+\sqrt{7}} \tag{24}$$

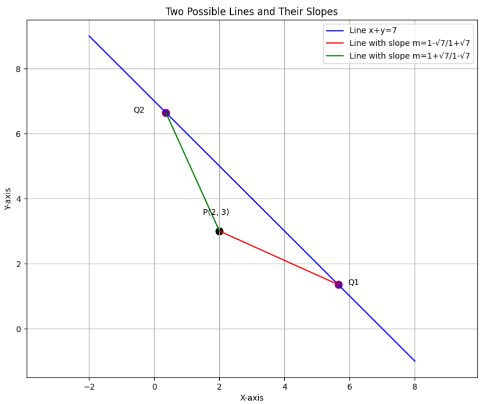

Fig. 0.1