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Matrix 3.2.31

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Question (3.2.31)

A triangle ABC can be constructed in which

$$\angle B = 60^\circ, \qquad \angle C = 45^\circ,$$

and

$$AB + BC + AC = 12 \text{ cm}.$$

Setup

Let the sides be

$$a = BC, \quad b = CA, \quad c = AB,$$

with opposite angles A, B, C.
The equations are:

$$a + b + c = 12, \tag{1}$$

$$-a + (\cos C)b + (\cos B)c = 0, \tag{2}$$

$$(\sin C)b - (\sin B)c = 0. \tag{3}$$

Augmented Matrix

Substituting $\cos 60 = \frac{1}{2},\ \sin 60 = \frac{\sqrt{3}}{2},\ \cos 45 = \sin 45 = \frac{\sqrt{2}}{2}$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 12 \\ -1 & \frac{\sqrt{2}}{2} & \frac{1}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{3}}{2} & 0 \end{array}\right].$$

RREF Step

Add row 2 to row 1:

$$R_1 \leftarrow R_1 + R_2$$

$$\left[\begin{array}{ccc|c} 0 & 1+\frac{\sqrt{2}}{2} & 1+\frac{1}{2} & 12 \\ -1 & \frac{\sqrt{2}}{2} & \frac{1}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{3}}{2} & 0 \end{array}\right].$$

Relations

From RREF we get:

$$a = c \cdot \frac{\sqrt{3}+1}{2}, \qquad b = c \cdot \frac{\sqrt{6}}{2}.$$

Using $a+b+c=12$:

$$c\left(\frac{\sqrt{3}+1}{2} + \frac{\sqrt{6}}{2} + 1\right) = 12,$$

so

$$c = \frac{24}{\sqrt{3}+\sqrt{6}+3}.$$

Final Values

Substituting:

$$b = \frac{12\sqrt{6}}{\sqrt{3}+\sqrt{6}+3}, \qquad a = \frac{12(\sqrt{3}+1)}{\sqrt{3}+\sqrt{6}+3}.$$

Numerically:

$$a \approx 4.565, \quad b \approx 4.093, \quad c \approx 3.342.$$

Check: $a + b + c = 12.$

Plot

Place

$$B = (0, 0), \quad C = (a, 0).$$

Then

$$A = (c \cos B,\ c \sin B).$$

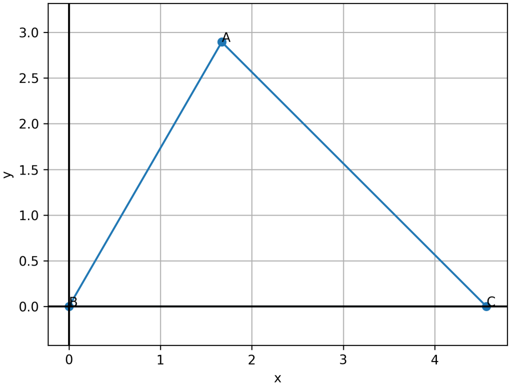

Figure: Triangle formed by points A, B, and C.