Question 4.2.3

AI25BTECH11040 - Vivaan Parashar

September 30, 2025

Question:

Find the equation of the plane determined by the points **A**(3, -1, 2), **B**(5, 2, 4) and **C**(-1, -1, 6). Also find the distance of the point **P**(6, 5, 9) from the plane.

Solution:

A plane in 3D is represented by the equation $\mathbf{n}^{\mathrm{T}}\mathbf{x} = c$, where the vector $\mathbf{n}$ represents the normal to the plane, and c is an arbitrary constant, that can be set to 1 for simplicity. We have three points that lie on the plane, $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$. We therefore have the following equations:

$$\mathbf{n}^{\mathrm{T}}\mathbf{A} = 1 \tag{1}$$

$$\mathbf{n}^{\mathrm{T}}\mathbf{B} = 1 \tag{2}$$

$$\mathbf{n}^{\mathrm{T}}\mathbf{C} = 1 \tag{3}$$

$$\implies \mathbf{n}^{\mathrm{T}} \begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \tag{4}$$

$$\tag{5}$$

Let's call the matrix $\begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix}$ $\mathbf{M}$. Multiply both sides by $\mathbf{M}^{-1}$ on the right:

$$\therefore \mathrm{n}^{\mathrm{T}} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathrm{M}^{-1} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3 & 5 & -1 \\ -1 & 2 & -1 \\ 2 & 4 & 6 \end{pmatrix}^{-1} \tag{6}$$

$$\implies \mathbf{n}^{\mathrm{T}} = \frac{1}{19} \begin{pmatrix} 3 & -4 & 3 \end{pmatrix} \tag{7}$$

Thus, the equation of the plane is given by:

$$\begin{pmatrix} 3 & -4 & 3 \end{pmatrix} \mathbf{x} = 19 \tag{8}$$

The distance d of the point $\mathbf{P}$ from the plane is given by:

$$d = \frac{|\mathbf{n}^{\mathrm{T}} \mathbf{x_P} - c|}{\|\mathbf{n}\|} \tag{9}$$

$$\implies d = \frac{|\begin{pmatrix} 3 & -4 & 3 \end{pmatrix} \begin{pmatrix} 6 \\ 5 \\ 9 \end{pmatrix} - 19|}{\sqrt{(3)^2 + (-4)^2 + (3)^2}} \tag{10}$$

$$\implies d = \frac{6}{\sqrt{34}} \tag{11}$$

Plot:

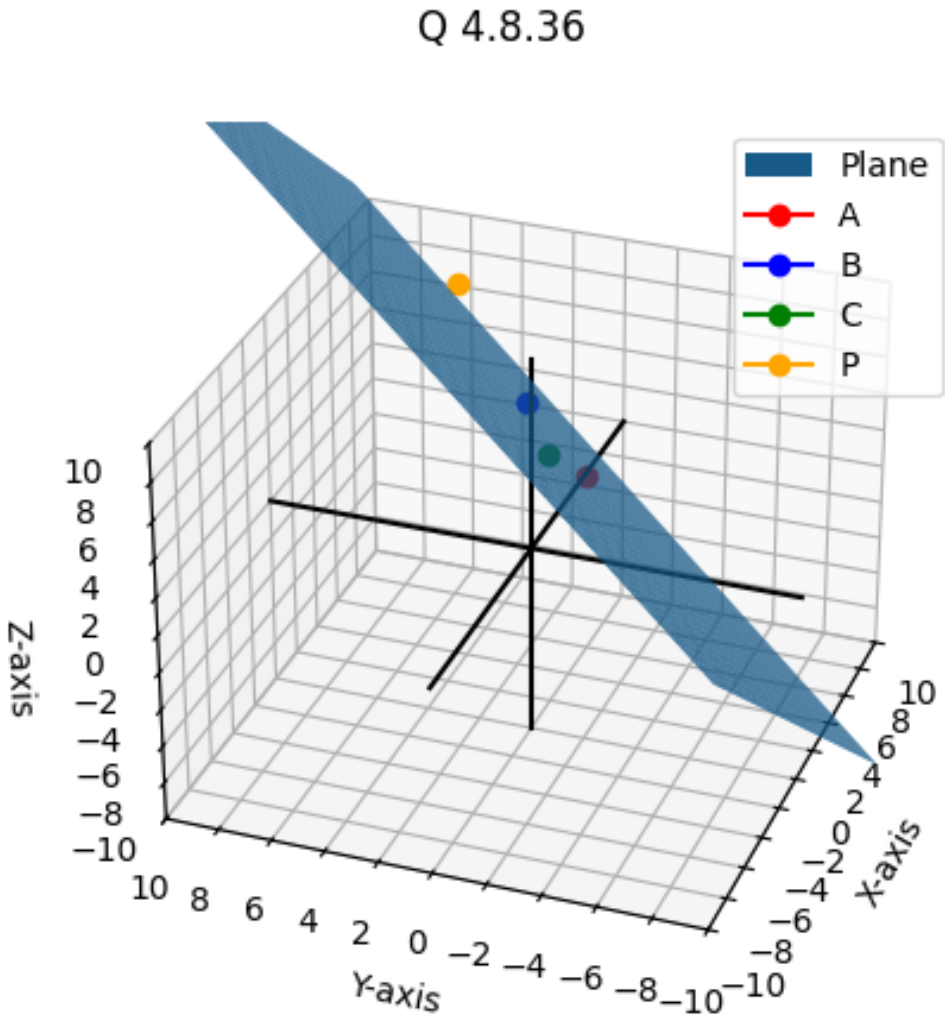

Figure: Graph of plane and points A, B, C and P