1.7.7

AI25BTECH11021 - Abhiram Reddy N

Question:
For what value of p are the points (2, 1), (p, -1), and (-1, 3) collinear?

Solution using Rank and RREF

We are given the points

$$A = (2, 1), \quad B = (p, -1), \quad C = (-1, 3).$$

Step 1: Vectors for Collinearity The points A, B, C are collinear if the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are linearly dependent.

$$\overrightarrow{AB} = (p - 2, -2), \qquad \overrightarrow{AC} = (-3, 2).$$

Step 2: Form Matrix We form a matrix M with these vectors as rows:

$$M = \begin{bmatrix} p-2 & -2 \\ -3 & 2 \end{bmatrix}.$$

For collinearity, the rank of M must be 1 (since the two rows are linearly dependent).

Step 3: Row Reduction (Echelon Form) Perform row operations to reduce M to echelon form.

$$R_1 = \begin{bmatrix} p-2 & -2 \end{bmatrix}, \quad R_2 = \begin{bmatrix} -3 & 2 \end{bmatrix}.$$

Eliminate the first element of R_2:

$$R_2 \to R_2 + \frac{3}{p-2} R_1.$$

$$R_2 = \begin{bmatrix} -3 & 2 \end{bmatrix} + \frac{3}{p-2} \begin{bmatrix} p-2 & -2 \end{bmatrix} = \begin{bmatrix} -3+3 & 2 - \frac{6}{p-2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{2(p-2)-6}{p-2} \end{bmatrix}.$$

Step 4: Condition for Rank = 1 For rank to be 1, the second row must vanish:

$$\frac{2(p-2) - 6}{p-2} = 0.$$

$$2(p-2) - 6 = 0 \quad \Longrightarrow \quad 2p - 10 = 0.$$

$$\boxed{p = 5}$$

Step 5: Verification When $p = 5$, the points are

$$A = (2, 1), \quad B = (5, -1), \quad C = (-1, 3).$$

The slope of AB is

$$\frac{-1-1}{5-2} = \frac{-2}{3},$$

and the slope of AC is

$$\frac{3-1}{-1-2} = \frac{2}{-3} = \frac{-2}{3}.$$

Since slopes are equal, A, B, C are collinear.

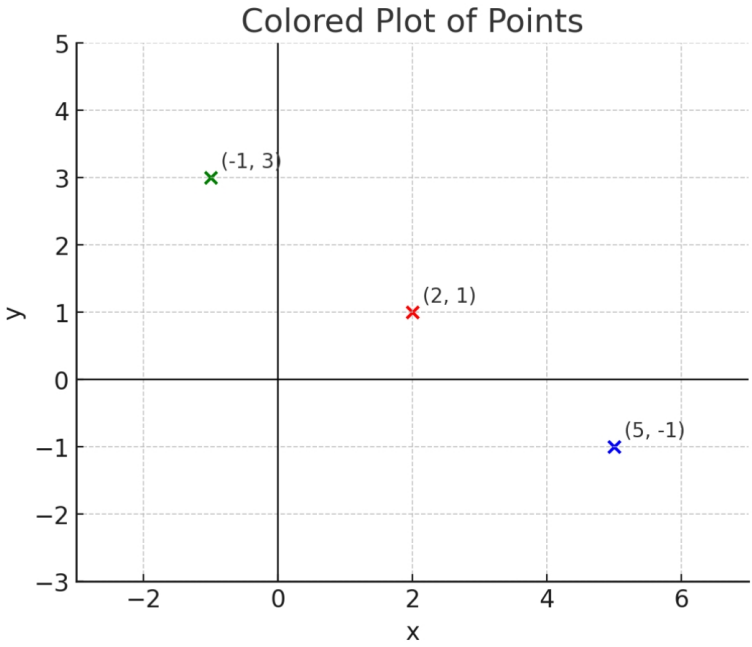

Fig. 0.1: plot