

## Matgeo-q 2.3.2

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## Question

**Q.** Find the angle between unit vectors **a** and **b** such that  $\sqrt{3}\mathbf{a} - \mathbf{b}$  is also a unit vector.

## Solution

**Solution. Given:**  $\| (a) \| = \| (b) \| = 1$  and  $\| \sqrt{3} (a) - (b) \| = 1$ .

Use the length definition  $\|x\|^2 = x^\top x$  and the scalar-product relation  $(a)^\top (b) = \| (a) \| \| (b) \| \cos \theta$ .

$$\begin{aligned}\| \sqrt{3} (a) - (b) \|^2 &= \left( \sqrt{3} (a) - (b) \right)^\top \left( \sqrt{3} (a) - (b) \right) \\&= 3 (a)^\top (a) + (b)^\top (b) - 2\sqrt{3} (a)^\top (b) \\&= 3\| (a) \|^2 + \| (b) \|^2 - 2\sqrt{3} \| (a) \| \| (b) \| \cos \theta \\&= 3 + 1 - 2\sqrt{3} \cos \theta.\end{aligned}$$

Since  $\| \sqrt{3} (a) - (b) \| = 1$ , we get

$$1 = 4 - 2\sqrt{3} \cos \theta \implies 3 = 2\sqrt{3} \cos \theta \implies \cos \theta = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}.$$

Hence,

$$\boxed{\theta = 30^\circ}.$$

# Plot

2D Illustration (xy-projection): Parallelogram spanned by  $\vec{a}$  and  $\vec{b}$

$$|\vec{a} \times \vec{b}| = 22.517 \quad (= 13\sqrt{3})$$

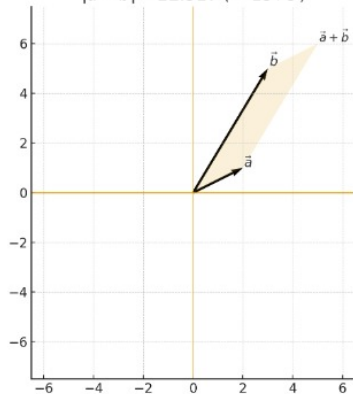


Figure: xy-projection of  $\mathbf{a}$  and  $\mathbf{b}$ ;  $|\mathbf{a} \times \mathbf{b}| = 13\sqrt{3}$ .