

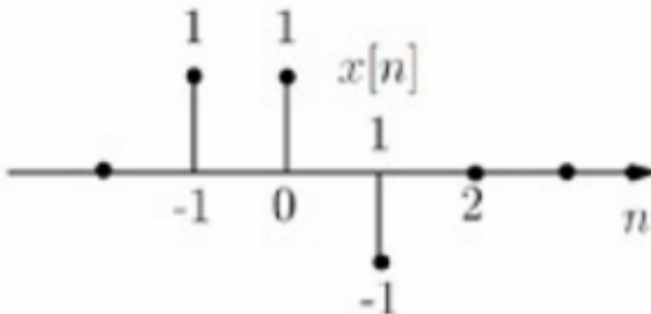
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# Question

The signal  $x[n]$  shown is convolved with itself to get  $y[n]$ . The value of  $y[-1]$  is



# Theoretical Solution

The Operation

$$x[n] * x[n] = y[n] \quad (1)$$

Can be written as

$$\mathbf{y} = \mathbf{M}\mathbf{x} \quad (2)$$

Where ,  $\mathbf{M}$  is a special kind of matrix called a Toeplitz matrix formed from the signal  $x[n]$

# Theoretical Solution

Given ,

$$\mathbf{x} = \begin{pmatrix} x[-1] \\ x[0] \\ x[1] \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}. \quad (3)$$

$$x[n] = 0, \text{ where } n \notin \{-1, 0, 1\} \quad (4)$$

$$\mathbf{M} = \begin{pmatrix} x[-1] & 0 & 0 \\ x[0] & x[-1] & 0 \\ x[1] & x[0] & x[-1] \\ 0 & x[1] & x[0] \\ 0 & 0 & x[1] \end{pmatrix} \quad (5)$$

# Theoretical Solution

$$\mathbf{y} = \mathbf{M} \begin{pmatrix} x[-1] \\ x[0] \\ x[1] \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \quad (6)$$

To find  $y[-1]$ , we perform matrix multiplication for the second row.

$$y[-1] = (1 \quad 1 \quad 0) \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \quad (7)$$

$$= 2 \quad (8)$$

Hence,  $y[-1] = 2$