2.7.33

EE25BTECH11021 - Dhanush Sagar

Question:
Find the value of p if

$$(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = 0.$$

Solution:
The given vectors are

$$\mathbf{A} = \begin{pmatrix} 2 \\ 6 \\ 27 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 1 \\ 3 \\ p \end{pmatrix}. \tag{0.1}$$

Construct the matrix

$$M = \begin{pmatrix} 2 & 6 & 27 \\ 1 & 3 & p \end{pmatrix}. \tag{0.2}$$

If $\mathbf{A} \times \mathbf{B} = 0$, then $\mathbf{A}$ and $\mathbf{B}$ are linearly dependent. Thus,

$$\text{rank}(M) < 2. \tag{0.3}$$

For a 2×3 matrix, this happens exactly when all 2×2 minors vanish.
First minor:

$$\det \begin{pmatrix} 2 & 6 \\ 1 & 3 \end{pmatrix} = 6 - 6 = 0. \tag{0.4}$$

Second minor:

$$\det \begin{pmatrix} 2 & 27 \\ 1 & p \end{pmatrix} = 2p - 27. \tag{0.5}$$

Third minor:

$$\det \begin{pmatrix} 6 & 27 \\ 3 & p \end{pmatrix} = 6p - 81. \tag{0.6}$$

For $\text{rank}(M) < 2$, all three determinants must vanish. The first is already zero. From the second,

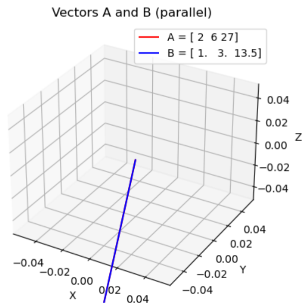
$$2p - 27 = 0 \Rightarrow p = \tfrac{27}{2}. \tag{0.7}$$

From the third,

$$6p - 81 = 0 \Rightarrow p = \tfrac{27}{2}. \tag{0.8}$$

Thus, the required value is

$$\boxed{p = \tfrac{27}{2}} \tag{0.9}$$



Vectors A and B (parallel)
A = [2 6 27]
B = [1.   3. 13.5]
0.04
0.02
0.00
−0.02
−0.04
Z
−0.04
−0.02
0.00
0.02
0.04
X
0.04
0.02
0.00
−0.02
−0.04
Y

Fig. 0.1