Assignment 1: 1.6.5

EE25BTECH11055 - Subhodeep Chakraborty

Question:
Show that the points $(2,3,4)$, $(-1,-2,1)$, $(5,8,7)$ are collinear.
Solution:
For three points $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ to be collinear:

$$\text{rank}\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix} = \text{rank}\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix}^{\top} = 1 \tag{1}$$

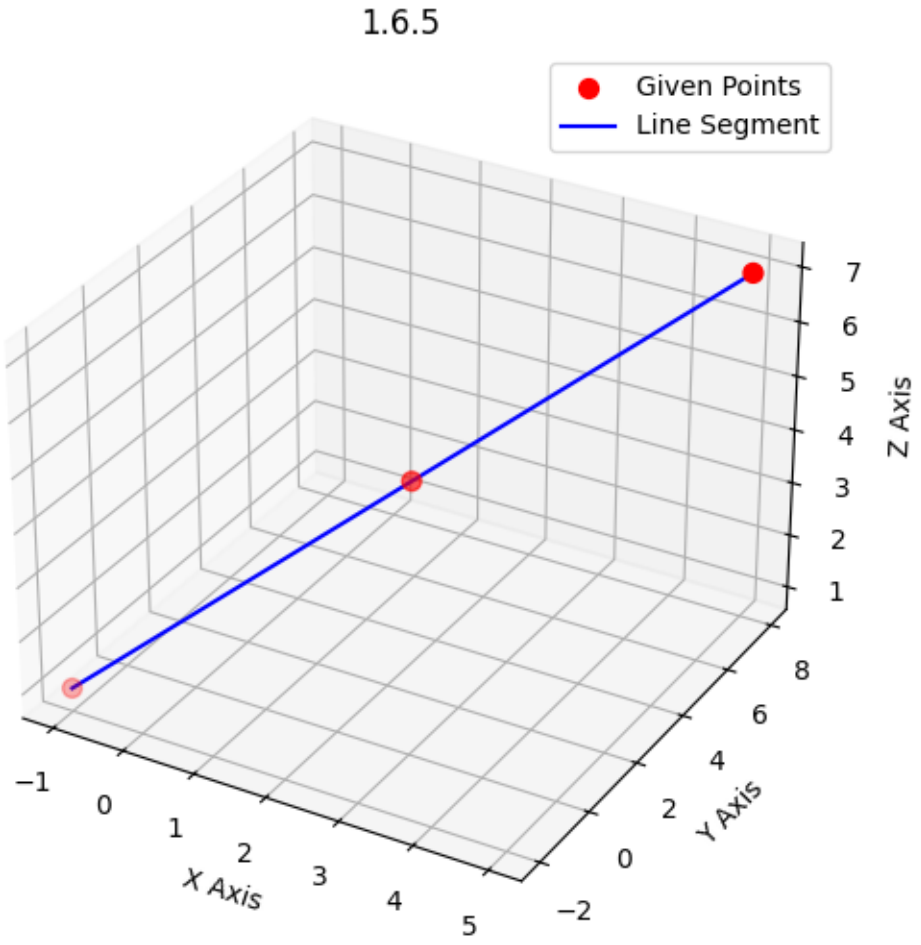
Given:

$$\mathbf{A} \equiv (2,3,4) \tag{2}$$
$$\mathbf{B} \equiv (-1,-2,1) \tag{3}$$
$$\mathbf{C} \equiv (5,8,7) \tag{4}$$

The transpose of the collinearity matrix can be expressed as:

$$\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix}^{\top} = \begin{pmatrix}-3 & -5 & -3\\ 3 & 5 & 3\end{pmatrix} \tag{5}$$
$$\xleftrightarrow{R_2=R_2+R_1} \begin{pmatrix}-3 & -5 & -3\\ 0 & 0 & 0\end{pmatrix} \tag{6}$$

which has rank 1. Using 1, we conclude that the given points are collinear.


1.6.5
Given Points
Line Segment
X Axis
Y Axis
Z Axis
−1
0
1
2
3
4
5
−2
0
2
4
6
8
1
2
3
4
5
6
7