

1.6.16.1

AI25BTECH11001 - ABHISEK MOHAPATRA

Question:

Find the values of k if the points $A(k+1, 2k)$, $B(3k, 2k+3)$ and $C(5k-1, 5k)$ are collinear.

Solution: From the given information,

$$A = \begin{pmatrix} k+1 \\ 2k \end{pmatrix}, B = \begin{pmatrix} 3k \\ 2k+3 \end{pmatrix}, C = \begin{pmatrix} 5k-1 \\ 5k \end{pmatrix} \quad (0.1)$$

To check if the points are collinear, we can use

$$\text{rank}(B-A \quad C-A) = 1 \quad (0.2)$$

So,

$$(B-A \quad C-A) = \begin{pmatrix} 2k-1 & 4k-2 \\ 3 & 3k \end{pmatrix} \quad (0.3)$$

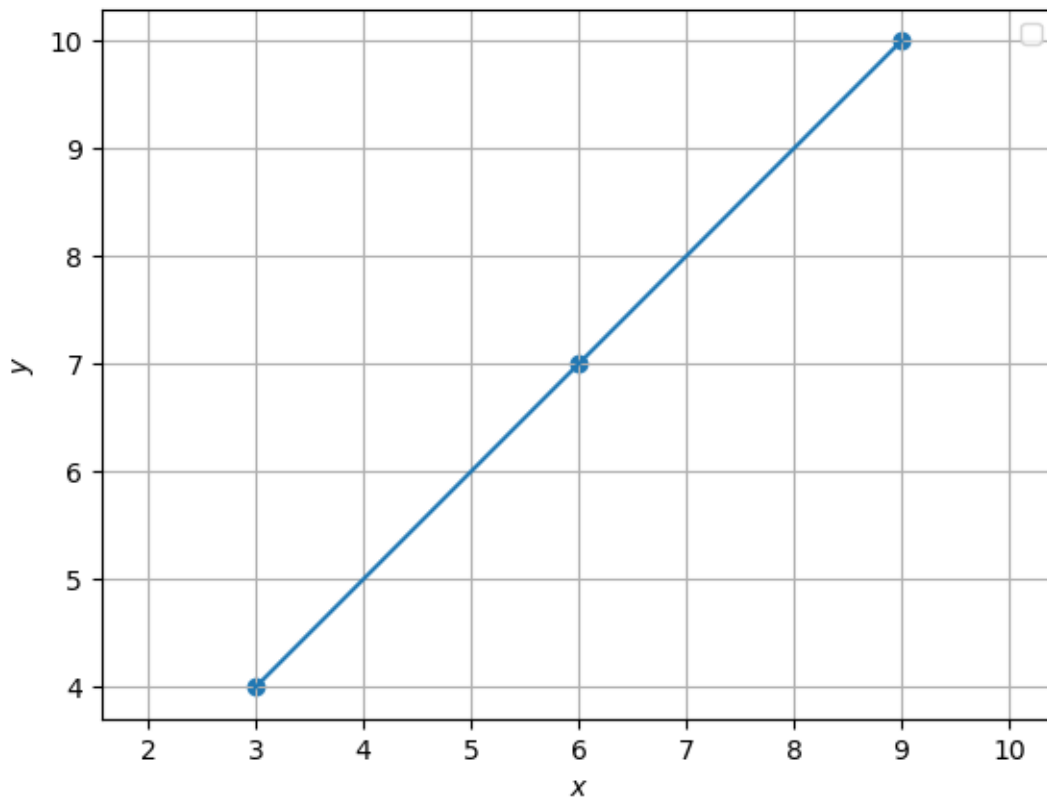
$$\xrightarrow{C_2 = C_2 - 2C_1} \begin{pmatrix} 2k-1 & 0 \\ 3 & 3k-6 \end{pmatrix} \quad (0.4)$$

The rank of the matrix will be 1 when

$$3k - 6 = 0 \quad (0.5)$$

$$\Rightarrow k = 2 \quad (0.6)$$

Graph:



Therefore, $k = 2$.