12.443

EE25BTECH11013 - Bhargav

Question:

The positive eigenvalue of $\begin{pmatrix} 2 & 1 \\ 5 & 2 \end{pmatrix}$ is

Solution:

The eigenvalue of matrix **A** can be found out by

$$\mathbf{A x} = \lambda \mathbf{x} \implies (\mathbf{A} - \lambda \mathbf{I})\,\mathbf{x} = 0 \tag{0.1}$$

$$|\mathbf{A} - \lambda \mathbf{I}| = 0 \tag{0.2}$$

where λ is the eigenvalue, **x** is the eigenvector, **I** is the identity matrix

$$\left| \begin{pmatrix} 2 - \lambda & 1 \\ 5 & 2 - \lambda \end{pmatrix} \right| = 0 \tag{0.3}$$

$$(2 - \lambda)^2 - 5 = 0 \implies \lambda^2 - 4\lambda - 1 = 0 \tag{0.4}$$

Using the quadratic formula,

$$\lambda = \frac{4 \pm \sqrt{16 + 4}}{2} \tag{0.5}$$

$$\lambda = 2 \pm \sqrt{5} \tag{0.6}$$

The positive eigenvalue of $\begin{pmatrix} 2 & 1 \\ 5 & 2 \end{pmatrix}$ is $2 + \sqrt{5}$