# Matrices in Geometry - 1.9.26

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## Problem Statement

Find the value of k, if the point P(2,4) is equidistant from point A(5,k) and B(k,7)

## Solution

**Given:**  $P \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ ,  $A \begin{pmatrix} 5 \\ k \end{pmatrix}$  and a point  $B \begin{pmatrix} k \\ 7 \end{pmatrix}$  such that P is equidistant from A and B.

$$\therefore \|\mathbf{A} - \mathbf{P}\| = \|\mathbf{B} - \mathbf{P}\| \tag{1}$$

$$\|\mathbf{A} - \mathbf{P}\|^2 = \|\mathbf{B} - \mathbf{P}\|^2$$
 (3)

$$(\mathbf{A} - \mathbf{P})^{\top} (\mathbf{A} - \mathbf{P}) = (\mathbf{B} - \mathbf{P})^{\top} (\mathbf{B} - \mathbf{P})$$
(4)

## Solution

$$(\mathbf{A} - \mathbf{P}) = \begin{pmatrix} 3 \\ k - 4 \end{pmatrix} \tag{5}$$

$$(\mathbf{B} - \mathbf{P}) = \binom{k-2}{3} \tag{6}$$

$$(\mathbf{A} - \mathbf{P})^{\top} (\mathbf{A} - \mathbf{P}) = (3 \quad k - 4) \begin{pmatrix} 3 \\ k - 4 \end{pmatrix}$$
 (7)

$$= 9 + (k-4)^2 = 9 + k^2 - 8k + 16 = k^2 - 8k + 25$$
 (8)

## Solution

$$(\mathbf{B} - \mathbf{P})^{\top} (\mathbf{B} - \mathbf{P}) = (k - 2 \quad 3) \begin{pmatrix} k - 2 \\ 3 \end{pmatrix}$$
(9)

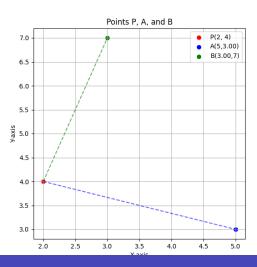
$$= (k-2)^2 + 9 = k^2 - 4k + 4 + 9 = k^2 - 4k + 13$$
 (10)

From (8) and (10), 
$$k^2 - 8k + 25 = k^2 - 4k + 13$$
 (11)

$$\implies -4k + 8k = 25 - 13 \implies 4k = 12 \implies k = 3 \tag{12}$$

## Final Answer

Hence, the final answer is k = 3



(13)