4.13.30

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Question

If $\mathbf{P} = (1, 0)$, $\mathbf{Q} = (-1, 0)$ and $\mathbf{R} = (2, 0)$ are three given points, then the locus of point $\mathbf{S}$ satisfying the relation $(SQ)^2 + (SR)^2 = 2(SP)^2$, is:

1. a straight line parallel to X axis
2. a circle passing through the origin
3. a circle with the center at the origin
4. a straight line parallel to Y axis

Given

Given

$$\mathbf{P} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \mathbf{Q} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}; \mathbf{R} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \tag{1}$$

$$\mathbf{S} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{2}$$

$$||\mathbf{Q}-\mathbf{S}||^2+||\mathbf{R}-\mathbf{S}||^2=2||\mathbf{P}-\mathbf{S}||^2 \tag{3}$$

$$\left(\mathbf{Q}-\mathbf{S}\right)^\top\left(\mathbf{Q}-\mathbf{S}\right)+\left(\mathbf{R}-\mathbf{S}\right)^\top\left(\mathbf{R}-\mathbf{S}\right)=2\left(\mathbf{P}-\mathbf{S}\right)^\top\left(\mathbf{P}-\mathbf{S}\right) \tag{4}$$

$$||\mathbf{Q}||^2+||\mathbf{R}||^2-2||\mathbf{P}||^2=\mathbf{S}^\top+\mathbf{Q}^\top\mathbf{S}+\mathbf{S}^\top\mathbf{R}+\mathbf{R}^\top\mathbf{S}-2\mathbf{S}^\top\mathbf{P}-2\mathbf{P}^\top\mathbf{S} \tag{5}$$

$$||\mathbf{Q}||^2+||\mathbf{R}||^2-2||\mathbf{P}||^2=\mathbf{S}^\top\left(\mathbf{Q}+\mathbf{R}-2\mathbf{P}\right)+\mathbf{S}\left(\mathbf{Q}+\mathbf{R}-2\mathbf{P}\right)^\top \tag{6}$$

$$||\mathbf{Q}||^2+||\mathbf{R}||^2-2||\mathbf{P}||^2=2\left(\mathbf{Q}+\mathbf{R}-2\mathbf{P}\right)^\top\mathbf{S} \tag{7}$$

Solving

Equation (7) is of the form:

$$\mathbf{n}^\top \mathbf{x} = c \tag{8}$$

$$\left(\mathbf{Q} + \mathbf{R} - 2\mathbf{P}\right)^\top \mathbf{S} = \frac{||\mathbf{Q}||^2 + ||\mathbf{R}||^2 - 2||\mathbf{P}||^2}{2} \tag{9}$$

Substituting

Substituting values:

$$\left(\begin{pmatrix}-1\\0\end{pmatrix}+\begin{pmatrix}2\\0\end{pmatrix}-2\begin{pmatrix}1\\0\end{pmatrix}\right)^\top \mathbf{s} = \frac{\left((-1)^2+0^2\right)+\left(2^2+0^2\right)-2\left(1^2+0^2\right)}{2} \tag{10}$$

$$\begin{pmatrix}-1\\0\end{pmatrix}^\top \mathbf{s} = \frac{3}{2} \tag{11}$$

Hence the locus of **s** is a line parallel to Y-axis.

C Code

```c
#include<stdio.h>
#include<math.h>
double arrP[2] = {1,0};
double arrQ[2] = {-1,0};
double arrR[2] = {2,0};

void give_data(double *points){
    double normal[2];
    for(int i = 0; i<2; i++){
        normal[i] = arrQ[i]+arrR[i]-(2*arrP[i]);
    }
```

C Code

```
  double k=0;
    for(int i = 0; i<2; i++){
        k+=(pow(arrQ[i],2)+pow(arrR[i],2)-(2*pow(arrP[i],2)))/2;

    }
    points[0] = arrP[0]; points[1] = arrP[1];
    points[2] = arrQ[0]; points[3] = arrQ[1];
    points[4] = arrR[0]; points[5] = arrR[1];
    points[6] = normal[0]; points[7] = normal[1];
    points[8] = k;
}
```

Python Code 1

```
import ctypes as ct

lib = ct.CDLL("./problem.so")

lib.give_data.argtypes = [ct.POINTER(ct.c_double)]

points = ct.c_double*9

data = points()

lib.give_data(data)

def send_data():
    return data
```

```python
import matplotlib.pyplot as plt
import numpy as np
from call import send_data

data = send_data()

y = np.linspace(-5, 5, 100)
x = ((data[7]*y)+data[8])/data[6]

X = [data[0], data[2], data[4]]
Y = [data[1], data[3], data[5]]

plt.plot(x, y, '-r')
plt.plot(X, Y, 'ko')
```

Python Code 2

```
plt.text(0.6, 0.1, "(1,0)", fontsize=10, color="black")
plt.text(-1.1, 0.1, "(-1,0)", fontsize=10, color="black")
plt.text(2.1, 0.1, "(2,0)", fontsize=10, color="black")
plt.text(-1.51, 3.20, r"$x=\frac{3}{2}$", fontsize=13, color="
    black")

plt.axvline(x=0, color='k', linewidth=1.5)
plt.axhline(y=0, color='k', linewidth=1.5)

plt.xlabel("X-axis")
plt.ylabel("Y-axis")
plt.grid(True)
plt.axis("equal")
plt.savefig("../figs/plot.png")
plt.show()
```

Plot

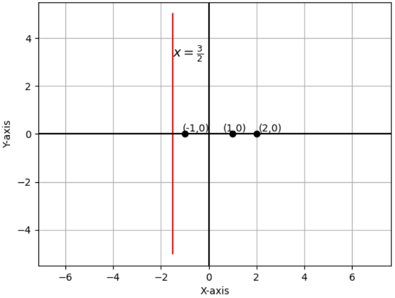

Figure: Plot of the given points and locus of **S**