2.2.10

AI25BTECH11018-Hemanth Reddy

Question:
The vectors $\mathbf{A} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\mathbf{B} = \hat{i} - 2\hat{k}$ are the adjancent sides of a parallelogram. The acute angle between its diagonals is ___________.

Solution:
The diagonals of the parallelogram are given by

$$\mathbf{A} + \mathbf{B} = \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}, \mathbf{A} - \mathbf{B} = \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$$

The angle θ between them satisfies $\cos\theta = \frac{\mathbf{d}_1^T \mathbf{d}_2}{\|\mathbf{d}_1\| \, \|\mathbf{d}_2\|} = \frac{(\mathbf{A}+\mathbf{B})^T(\mathbf{A}-\mathbf{B})}{\|\mathbf{A}+\mathbf{B}\| \, \|\mathbf{A}-\mathbf{B}\|} = \frac{\|\mathbf{A}\|^2 - \|\mathbf{B}\|^2}{\|\mathbf{A}+\mathbf{B}\| \, \|\mathbf{A}-\mathbf{B}\|}$.

Now compute: $\|\mathbf{A}\|^2 = 3^2 + (-2)^2 + 2^2 = 17, \qquad \|\mathbf{B}\|^2 = 1^2 + 0^2 + (-2)^2 = 5,$

$$\mathbf{A} + \mathbf{B} = \langle 4, -2, 0 \rangle, \quad \|\mathbf{A} + \mathbf{B}\| = \sqrt{20} = 2\sqrt{5},$$

$$\mathbf{A} - \mathbf{B} = \langle 2, -2, 4 \rangle, \quad \|\mathbf{A} - \mathbf{B}\| = \sqrt{24} = 2\sqrt{6}.$$

Hence $\cos\theta = \frac{17-5}{(2\sqrt{5})(2\sqrt{6})} = \frac{12}{4\sqrt{30}} = \frac{3}{\sqrt{30}}$.

Therefore, the acute angle between the diagonals is $\theta = \cos^{-1}\left(\frac{3}{\sqrt{30}}\right) \approx 56.7^\circ$.

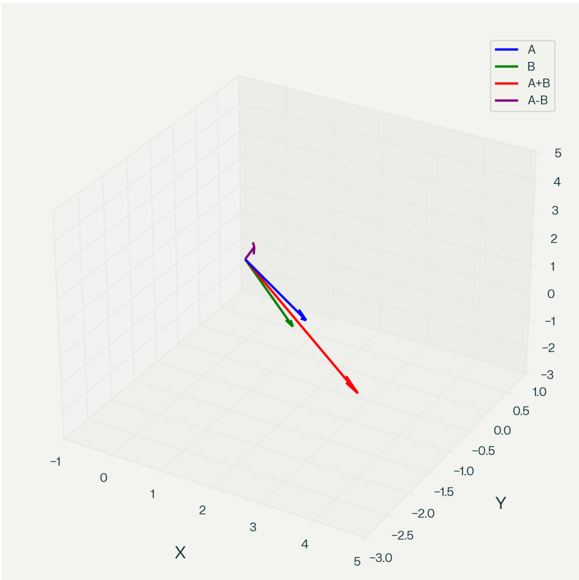

Fig. 0.1