4.4.37

EE25BTECH11001 - Aarush Dilawri

Question:
Find the vector equation of the line passing through the point $(2, 3, -5)$ and making equal angles with the coordinate axes.
Solution:

Let the line be

$$\mathbf{x} = \mathbf{h} + \kappa\mathbf{m} \tag{0.1}$$

where $\mathbf{m}$ is the direction unit vector of the line , $\mathbf{h}$ is any given point on the line and $|\kappa|$ is the distance of $\mathbf{x}$ from $\mathbf{h}$ along the line.
Here,

$$\mathbf{h} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix} \tag{0.2}$$

We are given that the line makes equal angles with the coordinate axes. Therefore,

$$\mathbf{m}^\top \mathbf{e_1} = \mathbf{m}^\top \mathbf{e_2} = \mathbf{m}^\top \mathbf{e_3} = \lambda \tag{0.3}$$

where,

$$\mathbf{e_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{e_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{e_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{0.4}$$

from (0.3),

$$\mathbf{m}^\top \begin{pmatrix} \mathbf{e_1} & \mathbf{e_2} & \mathbf{e_3} \end{pmatrix} = \lambda \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \tag{0.5}$$

$$\mathbf{m}^\top \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \lambda \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \tag{0.6}$$

$$\mathbf{m}^\top \mathbf{I} = \lambda \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \tag{0.7}$$

$$\mathbf{m}^\top = \lambda \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \tag{0.8}$$

Taking transpose on both sides,

$$\mathbf{m} = \begin{pmatrix} \lambda \\ \lambda \\ \lambda \end{pmatrix} \tag{0.9}$$

Since,

$$\left\|\mathbf{m}\right\| = 1 \tag{0.10}$$

$$\lambda = \pm\frac{1}{\sqrt{3}} \tag{0.11}$$

Therefore the equation of the line is

$$\mathbf{x} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix} + \kappa \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} \tag{0.12}$$

From the figure, it is clearly verified that the theoretical solution matches with the computational solution.