Matrices in Geometry - 12.675

EE25BTECH11037 Divyansh

Sept, 2025

Problem Statement

The ratio of the product of eigenvalues to the sum of the eigenvalues of the given matrix

$$\begin{pmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{pmatrix}$$

Solution

Let

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{pmatrix} \tag{1}$$

The eigenvalues are the values of λ that satisfy $|\mathbf{A} - \lambda \mathbf{I}| = 0$

$$\implies \left| \begin{pmatrix} 3-\lambda & 1 & 2 \\ 2 & -3-\lambda & -1 \\ 1 & 2 & 1-\lambda \end{pmatrix} \right| = 0 \tag{2}$$

$$(3-\lambda)\left((-3-\lambda)(1-\lambda)+2\right) - 1\left(2-2\lambda+1\right) + 2\left(4+3+\lambda\right) = 0 \tag{3}$$

Solution

$$\implies \lambda^3 - \lambda^2 - 11\lambda - 8 = 0 \tag{4}$$

Let the eigenvalues be $\lambda_1, \lambda_2, \lambda_3$, then

$$\lambda_1 + \lambda_2 + \lambda_3 = -\frac{-1}{1} = 1 \tag{5}$$

$$\lambda_1 \lambda_2 \lambda_3 = -\frac{-8}{1} \tag{6}$$

Thus the ratio of product of eigenvalues to sum of eigenvalues of $\mathbf{A}$ is r

$$r = \frac{8}{1} = 8 \tag{7}$$