Problem 4.6.11

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Problem

Find the equation of the line passing through the point $(1, -3, 2)$ and parallel to the line

$$\mathbf{r} = (2 + \lambda)\,\hat{i} + \lambda\hat{j} + (2\lambda - 1)\,\hat{k} \tag{1.1}$$

Finding Direction Vector

Given line is

$$\mathbf{r} = \begin{pmatrix} 2+\lambda \\ \lambda \\ 2\lambda - 1 \end{pmatrix} \tag{2.1}$$

The vector equation of given line is given by

$$\mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \tag{2.2}$$

The direction vectors of given line are

$$\mathbf{m} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \tag{2.3}$$

Conclusion

The lines with direction vectors $\mathbf{m}$ and $\mathbf{n}$ are parallel if

$$\mathbf{m} = \mathbf{n} \implies \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \tag{2.4}$$

The equation of a line is given by

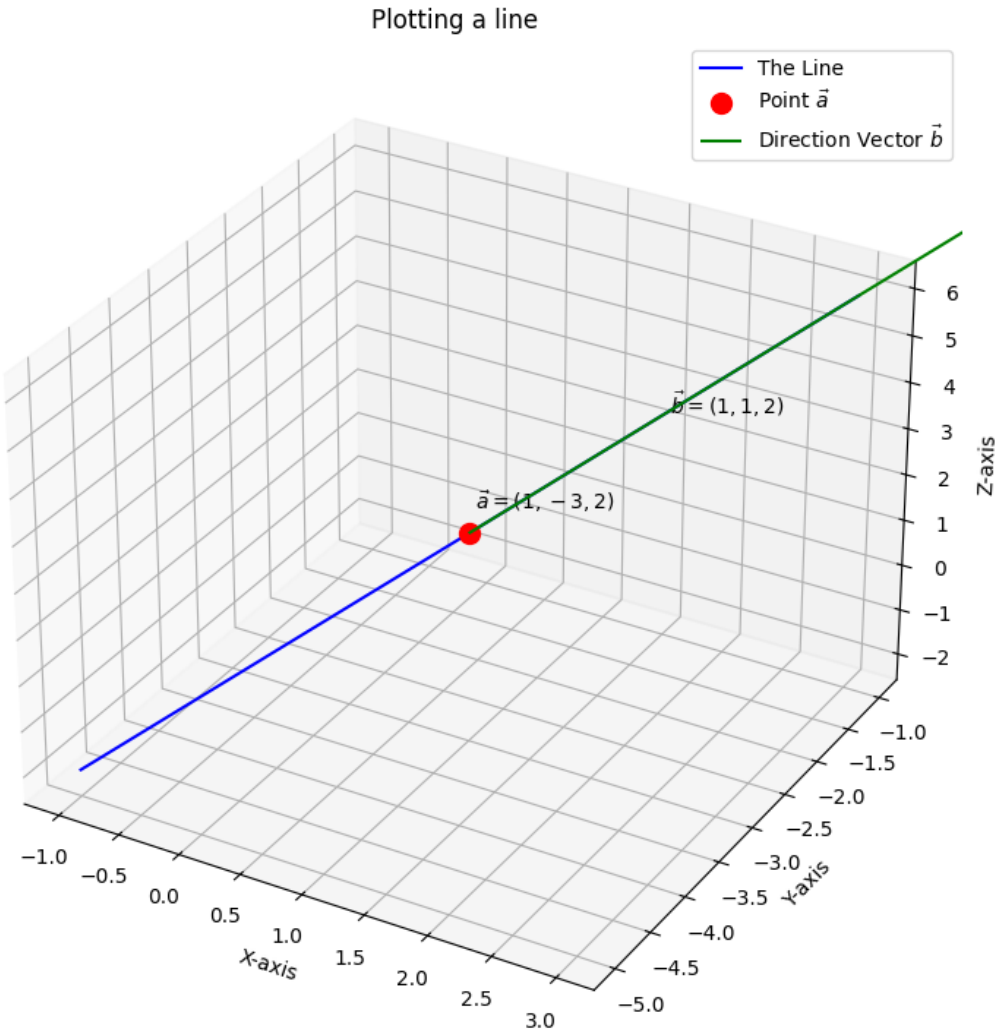
$$\mathbf{n}^\top (\mathbf{x} - \mathbf{a}) = 0 \tag{2.5}$$

$$\begin{pmatrix} 1 & 1 & 2 \end{pmatrix} \left(\mathbf{x} - \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \right) = 0 \tag{2.6}$$

$$\begin{pmatrix} 1 & 1 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \tag{2.7}$$

$$\begin{pmatrix} 1 & 1 & 2 \end{pmatrix} \mathbf{x} = 2 \tag{2.8}$$

Plot


Plotting a line
The Line
Point $\vec{a}$
Direction Vector $\vec{b}$
$\vec{b} = (1, 1, 2)$
$\vec{a} = (1, -3, 2)$
X-axis
Y-axis
Z-axis

C Code

```
void get_line_vectors(double* out_data) {

    double point_a[3] = {1.0, -3.0, 2.0};
    double dir_b[3] = {1.0, 1.0, 2.0};

    out_data[0] = point_a[0];
    out_data[1] = point_a[1];
    out_data[2] = point_a[2];

    out_data[3] = dir_b[0];
    out_data[4] = dir_b[1];
    out_data[5] = dir_b[2];
}
```

Calling C Function

```
import ctypes
import numpy as np

def get_vectors_from_c():
    lib = ctypes.CDLL('./line_eq.so')

    # The C function expects a pointer to a C double array of
        size 6
    double_array_6 = ctypes.c_double * 6
    lib.get_line_vectors.argtypes = [ctypes.POINTER(ctypes.
        c_double)]

    # Create the C-style array to receive the output data
    out_data_c = double_array_6()

    # Call the C function, which will fill the array
    lib.get_line_vectors(out_data_c)
```

Calling C Function

```
# Convert the C array back into a NumPy array
    all_data = np.array(out_data_c)

    # Split the data into the point vector and direction vector
    point_a = all_data[:3]
    dir_b = all_data[3:]

    return point_a, dir_b
```

Python Code for Plotting

```
#Code by GVV Sharma
#September 12, 2023
#Revised July 21, 2024
#released under GNU GPL

import sys
import matplotlib.pyplot as plt
import numpy as np
from mpl_toolkits.mplot3d import Axes3D
sys.path.insert(0, '/workspaces/urban-potato/matgeo/codes/
    CoordGeo/')
from call import get_vectors_from_c
hat_symbol = '\u0302'
from line.funcs import *
from triangle.funcs import *
from conics.funcs import circ_gen

point_a, dir_b = get_vectors_from_c()
```

Python Code for Plotting

```
from call import get_vectors_from_c

point_a, dir_b = get_vectors_from_c()

lambda_vals = np.array([-2, 2])
line_points = point_a + lambda_vals[:, np.newaxis] * dir_b

# --- Plotting ---
fig = plt.figure(figsize=(9, 9))
ax = fig.add_subplot(111, projection='3d')

# Plot the line segment itself
ax.plot(line_points[:, 0], line_points[:, 1], line_points[:, 2],
    color='blue', label='The Line')

# Plot the point 'a' on the line
ax.scatter(point_a[0], point_a[1], point_a[2], color='red', s
    =100, label='Point $\\vec{a}$')
```

Python Code for Plotting

```
  # Plot the direction vector 'd' starting from point 'a'
ax.quiver(point_a[0], point_a[1], point_a[2],
          dir_b[0], dir_b[1], dir_b[2],
          color='green', label='Direction Vector $\\vec{b}$',
              length=5, arrow_length_ratio=0.3)

# Add text labels for the point and vectors
ax.text(point_a[0], point_a[1], point_a[2] + 0.5, f' $\\vec{{a}}
    = ({point_a[0]:.0f}, {point_a[1]:.0f}, {point_a[2]:.0f})$')
ax.text(point_a[0] + dir_b[0], point_a[1] + dir_b[1], point_a[2]
    + dir_b[2], ' $\\vec{b}$')
```

Python Code for Plotting

```
# --- Formatting ---
ax.set_title('Plotting a line')
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')

ax.grid(True)
ax.legend()
plt.show()
plt.savefig('../figs/fig.png')
```