Problem 2.2.23

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Question

Question:
Find angle θ between the vectors $\mathbf{a} = \hat{i} + \hat{j} - \hat{k}$ and $\mathbf{b} = \hat{i} - \hat{j} + \hat{k}$.

Solution

Solution:

Express vectors in column form:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \qquad \mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

The cosine of the angle θ is given by:

$$\cos\theta = \frac{\mathbf{ab}}{\|\mathbf{a}\|\,\|\mathbf{b}\|}$$

Compute dot product:

$$\mathbf{ab} = (1)(1) + (1)(-1) + (-1)(1) = 1 - 1 - 1 = -1$$

Compute magnitudes:

$$\|\mathbf{a}\| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$$

$$\|\mathbf{b}\| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$$

Solution

Substitute:

$$\cos\theta = \frac{-1}{\sqrt{3}\sqrt{3}} = -\frac{1}{3}$$

$$\theta = \cos^{-1}\left(-\frac{1}{3}\right)$$

Answer:

The required angle is:

$$\boxed{\theta = \cos^{-1}\left(-\frac{1}{3}\right)}$$

Graph

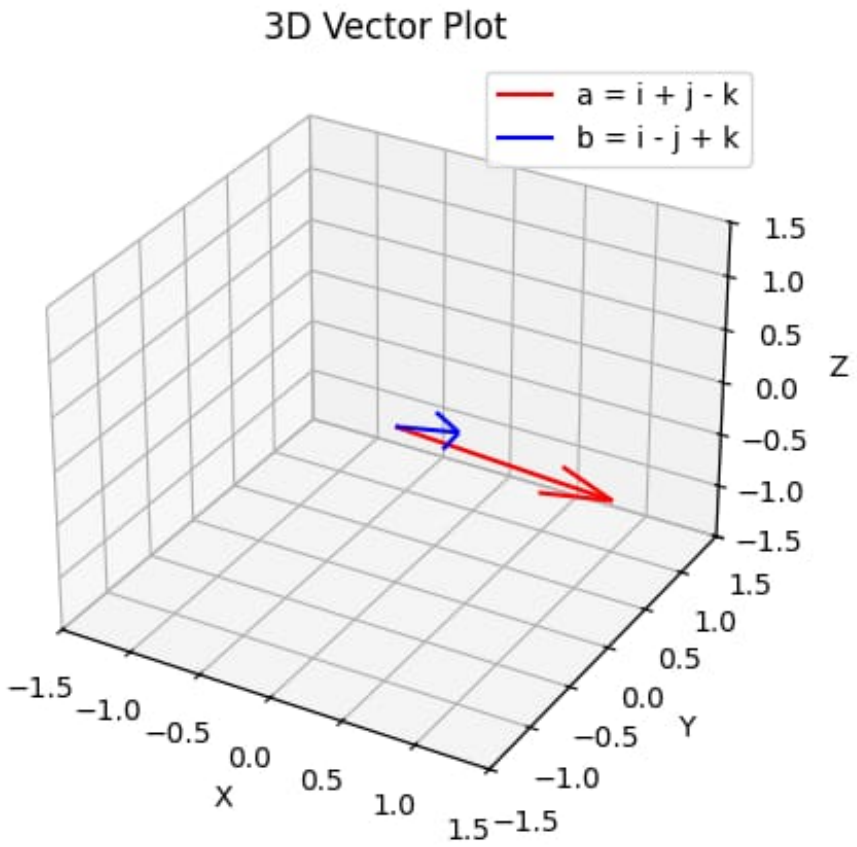

Figure: 3D Visualisation of two vectors and angle between them

C Code

```c
#include <stdio.h>
#include <math.h>
#include "VectorLib.h"

int main() {
    Vector3D a = createVector(1, 1, -1); // vector a = i + j - k
    Vector3D b = createVector(1, -1, 1); // vector b = i - j + k

    double theta = angleBetween(a, b); // radians

    printf("Angle between vectors a and b is %.6f radians\n",
        theta);
    printf("Angle between vectors a and b is %.6f degrees\n",
        theta * (180.0 / M_PI));

    return 0;
}
```

Python Code for Plotting

```
import numpy as np
import matplotlib.pyplot as plt

from mpl_toolkits.mplot3d import Axes3D

a = np.array([1, 1, -1])
b = np.array([1, -1, 1])
origin = np.array([0, 0, 0])

fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

ax.quiver(*origin, *a, color='r', label='a = i + j - k')
ax.quiver(*origin, *b, color='b', label='b = i - j + k')

limit = 1.5
ax.set_xlim([-limit, limit])
ax.set_ylim([-limit, limit])
ax.set_zlim([-limit, limit])
```

Python Code for Plotting

```
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.legend()
ax.set_title('3D Vector Plot')

plt.savefig("3d_vector_plot.png") # Save plot to file instead of
    plt.show()
```