Problem 2.10.52

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Problem

Let $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{c} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$. A vector in the plane of $\mathbf{a}$ and $\mathbf{b}$ whose projection on $\mathbf{c}$ is $\frac{1}{\sqrt{3}}$, is

1. $4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$
2. $3\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}}$
3. $2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}}$
4. $4\hat{\mathbf{i}} + \hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

Coplanarity

Given $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

Let $\mathbf{r}$ be coplanar to $\mathbf{a}$ and $\mathbf{b}$

$$\mathbf{r} = \mathbf{a} + t\mathbf{b} \tag{1.1}$$

Simplify

Given the projection of $\mathbf{r}$ on $\mathbf{c}$ is $\frac{1}{\sqrt{3}}$

$$\frac{\left|\mathbf{r}^\top \mathbf{c}\right|}{\|\mathbf{c}\|} = \frac{1}{\sqrt{3}} \tag{1.2}$$

$$\begin{aligned}
\mathbf{r}^\top \mathbf{c} &= (\mathbf{a} + t\mathbf{b})^\top \mathbf{c} && (1.3)\\
&= \left(\mathbf{a}^\top + t\mathbf{b}^\top\right) c && (1.4)\\
&= \mathbf{a}^\top \mathbf{c} + t\left(\mathbf{b}^\top \mathbf{c}\right) && (1.5)
\end{aligned}$$

$$\begin{aligned}
\mathbf{r}^\top \mathbf{c} - \mathbf{a}^\top \mathbf{c} &= t\left(\mathbf{b}^\top \mathbf{c}\right) && (1.6)\\
\implies t &= \frac{\mathbf{r}^\top \mathbf{c} - \mathbf{a}^\top \mathbf{c}}{\mathbf{b}^\top \mathbf{c}} && (1.7)
\end{aligned}$$

Finding Values

$$\mathbf{r} = \mathbf{a} + \left(\frac{\mathbf{r}^\top \mathbf{c} - \mathbf{a}^\top \mathbf{c}}{\mathbf{b}^\top \mathbf{c}} \right) \mathbf{b} \tag{1.8}$$

$$\mathbf{r} = \mathbf{a} + \left(\frac{\pm \frac{\|\mathbf{c}\|}{\sqrt{3}} - \mathbf{a}^\top \mathbf{c}}{\mathbf{b}^\top \mathbf{c}} \right) \mathbf{b} \tag{1.9}$$

$$\|\mathbf{c}\|^2 = \mathbf{c}^\top \mathbf{c} = \begin{pmatrix} 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \tag{1.10}$$

$$= 1 + 1 + 1 = 3 \implies \|\mathbf{c}\| = \sqrt{3} \tag{1.11}$$

$$\mathbf{a}^\top \mathbf{c} = \begin{pmatrix} 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = 1 + 2 - 1 = 2 \tag{1.12}$$

Conclusion

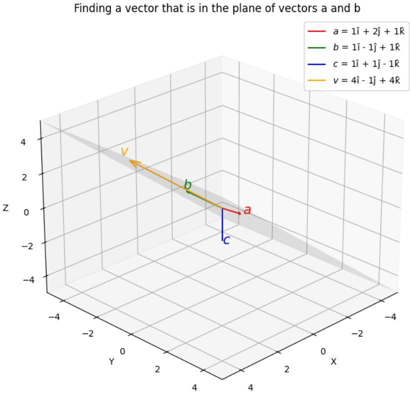
$$\mathbf{b}^\top \mathbf{c} = \begin{pmatrix}1 & -1 & 1\end{pmatrix} \begin{pmatrix}1 \\ 1 \\ -1\end{pmatrix} = 1 - 1 - 1 = -1 \tag{1.13}$$

$$\mathbf{r} = \begin{pmatrix}1 \\ 2 \\ 1\end{pmatrix} + \left(\frac{\pm 1 - 2}{-1}\right) \begin{pmatrix}1 \\ -1 \\ 1\end{pmatrix} \tag{1.14}$$

$$\implies \mathbf{r} = \begin{pmatrix}1 \\ 2 \\ 1\end{pmatrix} + \begin{pmatrix}1 \\ -1 \\ 1\end{pmatrix} \text{ or } \begin{pmatrix}1 \\ 2 \\ 1\end{pmatrix} + 3\begin{pmatrix}1 \\ -1 \\ 1\end{pmatrix} \implies \mathbf{r} = \begin{pmatrix}2 \\ 1 \\ 2\end{pmatrix} \text{ or } \begin{pmatrix}4 \\ -1 \\ 4\end{pmatrix} \tag{1.15}$$

Hence Option(1) is the correct answer

Plot



Finding a vector that is in the plane of vectors a and b
a = 1î + 2ĵ + 1k̂
b = 1î - 1ĵ + 1k̂
c = 1î + 1ĵ - 1k̂
v = 4î - 1ĵ + 4k̂
v
b
a
c
Z
X
Y

Figure:

C Code

```c
void get_vectors(double* vector_data) {
    vector_data[0] = 1.0; vector_data[1] = 2.0; vector_data[2] =
        1.0;
    vector_data[3] = 1.0; vector_data[4] = -1.0; vector_data[5] =
         1.0;
    vector_data[6] = 1.0; vector_data[7] = 1.0; vector_data[8] =
        -1.0;
    vector_data[9] = 4.0; vector_data[10] = -1.0; vector_data[11]
         = 4.0;
}
```

Calling C Function

```
import ctypes
import numpy as np

def load_vectors_from_c():

    # Load the shared C library. Assumes the file exists.
    lib = ctypes.CDLL('./plane.so')

    double_array_12=ctypes.c_double*12
    out_vectors=double_array_12()
    lib.get_vectors.argtypes=[ctypes.POINTER(ctypes.c_double)]
    lib.get_vectors(out_vectors)
    # Reshape the data and return the individual vectors
    out_vector = np.array(out_vectors).reshape(4, 3)
    a, b, c, v = out_vector[0], out_vector[1], out_vector[2],
        out_vector[3]
    return a, b, c, v
```

Python Code for Plotting

```
#Code by GVV Sharma
#September 12, 2023
#Revised July 21, 2024
#released under GNU GPL

import sys
import matplotlib.pyplot as plt
import numpy as np
sys.path.insert(0, '/workspaces/urban-potato/matgeo/codes/
    CoordGeo/')
from call import load_vectors_from_c
hat_symbol = '\u0302'
from line.funcs import *
from triangle.funcs import *
from conics.funcs import circ_gen

a, b, c, v = load_vectors_from_c()
```

Python Code for Plotting

```
vectors = [a, b, c, v]
colors = ['r', 'g', 'b', 'orange']
labels = ['a', 'b', 'c', 'v']
def format_original_label(name, vec):
    x, y, z = int(vec[0]), int(vec[1]), int(vec[2])
    y_part = f+ {y} if y >= 0 else f- {abs(y)}
    z_part = f+ {z}k if z >= 0 else f- {abs(z)}k
    return f'${name}$ = {x} {y_part} {z_part}'

# Create a 3D plot
fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')
origin = [0, 0, 0]

# Plot vectors
 for vec, color, name in zip(vectors, colors, labels):
    legend_label = format_original_label(name, vec)
    ax.quiver(*origin, *vec, color=color,arrow_length_ratio=0.1,
        label=legend_label)
```

Python Code for Plotting

```
    ax.text(*(vec * 1.1), f'${name}$', color=color, fontsize=15)
x_plane = np.linspace(-5, 5, 10)
y_plane = np.linspace(-5, 5, 10)
X, Y = np.meshgrid(x_plane, y_plane)
Z = X
ax.plot_surface(X, Y, Z, alpha=0.2, color='gray')
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('Finding a vector that is in the plane of vectors a
    and b')
ax.legend()
ax.grid(True)
plt.show()
plt.savefig('fig.png')
```