

1.5.2

EE25BTECH11014 - Bhoomika Lokesh

Question: Find the ratio in which the Y axis divides the line segment joining the points $(6, -4)$ and $(-2, -7)$. Also find the point of intersection.

Solution: Given the points,

$$\mathbf{A} = \begin{pmatrix} 6 \\ -4 \end{pmatrix} \mathbf{B} = \begin{pmatrix} -2 \\ -7 \end{pmatrix} \quad (0.1)$$

Let the vector \mathbf{P} be

$$\mathbf{P} = \begin{pmatrix} 0 \\ y \end{pmatrix}, \quad (0.2)$$

WKT points $\mathbf{A}, \mathbf{P}, \mathbf{B}$ are collinear.

The points to be collinear,

$$\text{rank}(\mathbf{P} - \mathbf{A} \quad \mathbf{B} - \mathbf{A}) = 1 \quad (0.3)$$

$$\mathbf{P} - \mathbf{A} = \begin{pmatrix} -6 \\ y + 4 \end{pmatrix} \quad (0.4)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -8 \\ -3 \end{pmatrix} \quad (0.5)$$

$$(\mathbf{P} - \mathbf{A} \quad \mathbf{B} - \mathbf{A}) = \begin{pmatrix} -6 & -8 \\ y + 4 & -3 \end{pmatrix} \quad (0.6)$$

$$\text{From Row Echelon form, } R_2 \rightarrow 3R_1 - 8R_2 : \begin{pmatrix} -6 & -8 \\ -8y - 50 & 0 \end{pmatrix}$$

$$-8y - 50 = 0 \implies y = -\frac{25}{4} \quad (0.7)$$

$$\therefore \mathbf{P} = \begin{pmatrix} 0 \\ -\frac{25}{4} \end{pmatrix}$$

Vector \mathbf{P} divides the line joining vectors \mathbf{A} and \mathbf{B} in the ratio $k:1$

$$\text{by using section formula, } \mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \quad (0.8)$$

$$k(\mathbf{P} - \mathbf{B}) = \mathbf{A} - \mathbf{P} \quad (0.9)$$

$$\implies k = \frac{(\mathbf{A} - \mathbf{P})^\top (\mathbf{P} - \mathbf{B})}{\|\mathbf{P} - \mathbf{B}\|^2} \quad (0.10)$$

$$(\mathbf{A} - \mathbf{P})^\top (\mathbf{P} - \mathbf{B}) = \left(6 \quad \frac{9}{4}\right) \begin{pmatrix} 2 \\ \frac{3}{4} \end{pmatrix} = \frac{219}{16} \quad (0.11)$$

$$\|\mathbf{P} - \mathbf{B}\|^2 = \left(\sqrt{2^2 + \left(\frac{3}{4}\right)^2} \right)^2 = \frac{73}{16} \quad (0.12)$$

$$k = \frac{\frac{219}{16}}{\frac{73}{16}} \quad (0.13)$$

$$\implies k = 3 \quad (0.14)$$

Therefore the ratio in which point \mathbf{P} divides the line segment joining \mathbf{A} and \mathbf{B} is 3:1

See Fig.0.1,

3D Graph of Points: (6, -4, 0), (-2, -7, 0), (0, -25/4, 0)

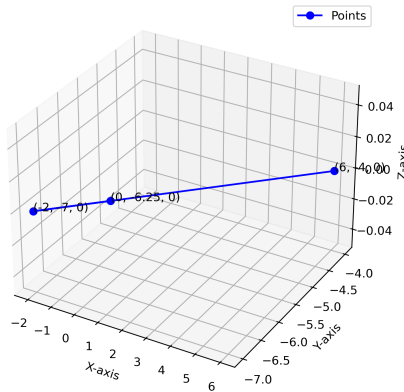


Fig. 0.1