

1.6.25

AI25BTECH11010 - Dhanush Kumar

Question:

If the sum of two unit vectors is a unit vector, prove that the magnitude of their difference is $\sqrt{3}$.

Solution:

Let \mathbf{u} and \mathbf{v} be unit vectors.
Hence,

$$\mathbf{u}^T \mathbf{u} = 1, \quad \mathbf{v}^T \mathbf{v} = 1. \quad (1)$$

$$\|\mathbf{u} + \mathbf{v}\|^2 = (\mathbf{u} + \mathbf{v})^T (\mathbf{u} + \mathbf{v}) \quad (2)$$

$$= \mathbf{u}^T \mathbf{u} + \mathbf{v}^T \mathbf{v} + 2\mathbf{u}^T \mathbf{v} \quad (3)$$

$$1^2 = 1 + 1 + 2\mathbf{u}^T \mathbf{v} \quad (4)$$

$$1 = 2 + 2\mathbf{u}^T \mathbf{v} \quad (5)$$

$$\Rightarrow \mathbf{u}^T \mathbf{v} = -\frac{1}{2}. \quad (6)$$

Now,

$$\|\mathbf{u} - \mathbf{v}\|^2 = (\mathbf{u} - \mathbf{v})^T (\mathbf{u} - \mathbf{v}) \quad (7)$$

$$= \mathbf{u}^T \mathbf{u} + \mathbf{v}^T \mathbf{v} - 2\mathbf{u}^T \mathbf{v} \quad (8)$$

$$= 1 + 1 - 2\left(-\frac{1}{2}\right) \quad (9)$$

$$= 2 + 1 = 3. \quad (10)$$

Therefore,

$$\|\mathbf{u} - \mathbf{v}\| = \sqrt{3}. \quad (11)$$

\therefore The required result is proved.

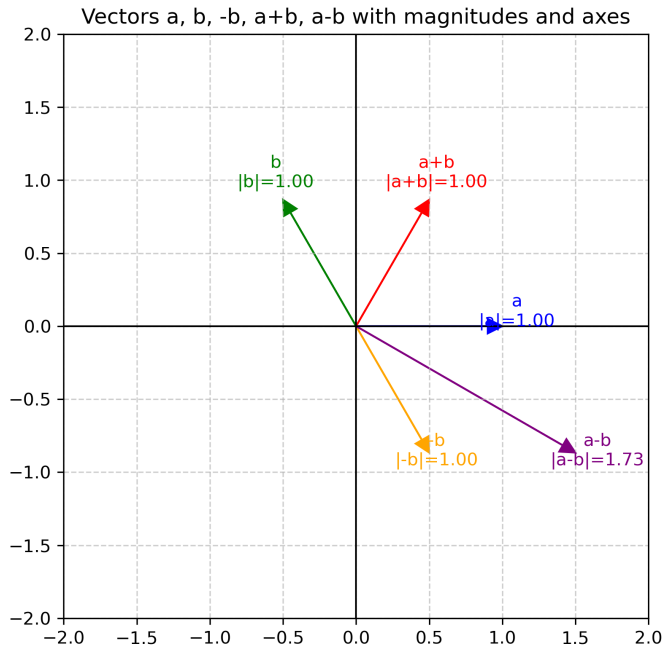


Fig. 0