

Matrices in Geometry 4.3.26

EE25BTECH11037 - Divyansh

Question: Find the ratio in which the line segment joining the points $\mathbf{A} = (4, 8, 10)$ and $\mathbf{B} = (6, 10, -8)$ is divided by the YZ plane.

Solution: We have two points $\mathbf{A} = \begin{pmatrix} 4 \\ 8 \\ 10 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 6 \\ 10 \\ -8 \end{pmatrix}$

Let \mathbf{P} be the point on the Y-Z plane. Since it is collinear to \mathbf{A} and \mathbf{B} ,

Since \mathbf{P} lies on Y-Z plane, $\mathbf{P} = \begin{pmatrix} 0 \\ P_y \\ P_z \end{pmatrix}$.

From (4.1.2.5)

$$(\mathbf{B} \quad \mathbf{A} \quad \mathbf{P})^T \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} 6 & 10 & -8 \\ 4 & 8 & 10 \\ 0 & P_y & P_z \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c} 6 & 10 & -8 & 1 \\ 4 & 8 & 10 & 1 \\ 0 & P_y & P_z & 1 \end{array} \right) \xrightarrow{R_1 \rightarrow \frac{(R_1 - R_2)}{2}} \left(\begin{array}{ccc|c} 1 & 1 & -9 & 0 \\ 4 & 8 & 10 & 1 \\ 0 & P_y & P_z & 1 \end{array} \right) \quad (2)$$

$$\xrightarrow{R_2 \rightarrow R_2 - 4R_1} \left(\begin{array}{ccc|c} 1 & 1 & -9 & 0 \\ 0 & 4 & 46 & 1 \\ 0 & P_y & P_z & 1 \end{array} \right) \xrightarrow{R_3 \rightarrow R_3 - R_2} \left(\begin{array}{ccc|c} 1 & 1 & -9 & 0 \\ 0 & 4 & 46 & 1 \\ 0 & P_y - 4 & P_z - 46 & 0 \end{array} \right) \quad (3)$$

Since \mathbf{P} , \mathbf{A} and \mathbf{B} are collinear, the rank of this matrix must be less than or equal to 2. Therefore, the

third row should be a zero row, and therefore, $\mathbf{P} = \begin{pmatrix} 0 \\ 4 \\ 46 \end{pmatrix}$

Let \mathbf{P} divide \mathbf{A} and \mathbf{B} in the ratio $k : 1$

Using the formula (1.1.5.2)

$$k = \frac{(\mathbf{A} - \mathbf{P})^T (\mathbf{P} - \mathbf{B})}{\|\mathbf{P} - \mathbf{B}\|^2} \quad (4)$$

$$\Rightarrow k = \frac{\begin{pmatrix} 4 & 4 & -36 \end{pmatrix} \begin{pmatrix} -6 \\ -6 \\ 54 \end{pmatrix}}{\left\| \begin{pmatrix} -6 \\ -6 \\ 54 \end{pmatrix} \right\|^2} \quad (5)$$

$$\Rightarrow k = \frac{-24 - 24 - 1944}{36 + 36 + 2916} = \frac{-1992}{2988} \quad (6)$$

$$\Rightarrow k = \frac{-2}{3} \quad (7)$$

Hence, the Y-Z plane divides the line segment that joins the points \mathbf{A} and \mathbf{B} in the external ratio 2 : 3.

3D Plot: Points A, B, P, Line AB, YZ-Plane, and Connections

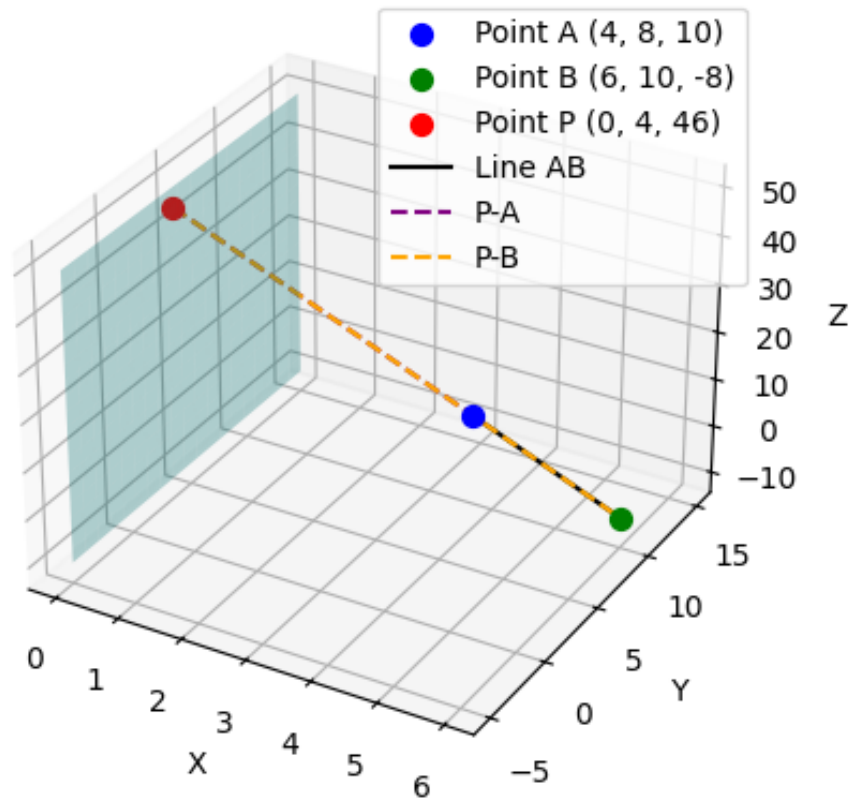


Fig. 1: Graph for 4.3.26