1.4.23

Aniket-EE25BTECH11007

September 29, 2025

Question

Consider two points P and Q with position vectors

$$\mathbf{OP} = 3\mathbf{a} - 2\mathbf{b}, \quad \mathbf{OQ} = \mathbf{a} + \mathbf{b}.$$

Find the position vector of a point R which divides the line joining P and Q in the ratio $2 : 1$,

1. internally, and
2. externally.

Theoretical Solution

In the basis $\{\mathbf{a}, \mathbf{b}\}$, we can write

$$\mathbf{A} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{1}$$

(a) Internal Division. If R divides AB in the ratio $k : 1$ internally, then

$$\mathbf{R} = \frac{k\mathbf{B} + \mathbf{A}}{k+1}. \tag{2}$$

Theoretical Solution

With $k = 2$

$$\mathbf{R} = \frac{2\begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}3\\-2\end{pmatrix}}{2+1} \tag{3}$$

$$= \frac{\begin{pmatrix}2\\2\end{pmatrix} + \begin{pmatrix}3\\-2\end{pmatrix}}{3} \tag{4}$$

$$= \frac{\begin{pmatrix}5\\0\end{pmatrix}}{3} \tag{5}$$

$$= \begin{pmatrix}\frac{5}{3}\\0\end{pmatrix}. \tag{6}$$

$$\boxed{\mathbf{R}_{\text{internal}} = \begin{pmatrix}\frac{5}{3}\\0\end{pmatrix}}$$

(b) External Division. If R divides AB in the ratio $k : 1$ externally, then

$$\mathbf{R} = \frac{k\mathbf{B} - \mathbf{A}}{k - 1}. \tag{7}$$

With $k = 2$

$$\mathbf{R} = \frac{2\begin{pmatrix}1\\1\end{pmatrix} - \begin{pmatrix}3\\-2\end{pmatrix}}{2-1} \tag{8}$$

$$= \begin{pmatrix}2\\2\end{pmatrix} - \begin{pmatrix}3\\-2\end{pmatrix} \tag{9}$$

$$= \begin{pmatrix}-1\\4\end{pmatrix} \tag{10}$$

$$\boxed{\mathbf{R}_{\text{external}} = \begin{pmatrix}-1\\4\end{pmatrix}}$$

Theoretical Solution

From the calculations above, we obtain:

$$\boxed{\mathbf{R}_{\text{internal}} = \frac{5}{3}\mathbf{a}}$$

$$\boxed{\mathbf{R}_{\text{external}} = -\mathbf{a} + 4\mathbf{b}}$$

C Code - Internal and External Division

```c
#include <stdio.h>

// Function to find point dividing line in ratio m:n
// flag = 1 for internal, -1 for external
void sectionFormula(float Px, float Py, float Qx, float Qy, int m
    , int n, int flag,
                    float *Rx, float *Ry) {
    if(flag == 1) { // internal
        *Rx = (m*Qx + n*Px) / (float)(m+n);
        *Ry = (m*Qy + n*Py) / (float)(m+n);
    }
    else if(flag == -1) { // external
        *Rx = (m*Qx - n*Px) / (float)(m-n);
        *Ry = (m*Qy - n*Py) / (float)(m-n);
    }
}
```

Python+C Code

```
import ctypes
import matplotlib.pyplot as plt

# Load the shared library (make sure libsection.so is in the same
    folder)
lib = ctypes.CDLL("./mg1o.so")

# Define argument and return types
lib.sectionFormula.argtypes = [
    ctypes.c_float, ctypes.c_float, # Px, Py
    ctypes.c_float, ctypes.c_float, # Qx, Qy
    ctypes.c_int, ctypes.c_int, # m, n
    ctypes.c_int, # flag (1=internal, -1=external)
    ctypes.POINTER(ctypes.c_float), # Rx
    ctypes.POINTER(ctypes.c_float) # Ry
]
lib.sectionFormula.restype = None
```

Python+C Code

```python
# Example points (P = (3,-2), Q = (1,1)), ratio 2:1
Px, Py = 3.0, -2.0
Qx, Qy = 1.0, 1.0
m, n = 2, 1

# Prepare storage for results
Rx_int, Ry_int = ctypes.c_float(), ctypes.c_float()
Rx_ext, Ry_ext = ctypes.c_float(), ctypes.c_float()

# Call C function
lib.sectionFormula(Px, Py, Qx, Qy, m, n, 1, ctypes.byref(Rx_int),
    ctypes.byref(Ry_int)) # internal
lib.sectionFormula(Px, Py, Qx, Qy, m, n, -1, ctypes.byref(Rx_ext)
    , ctypes.byref(Ry_ext)) # external

print("Internal Division:", Rx_int.value, Ry_int.value)
print("External Division:", Rx_ext.value, Ry_ext.value)
```

Python+C Code

```
# --- Plotting ---
plt.figure(figsize=(6,6))

# Plot P, Q, R_int, R_ext
plt.scatter([Px, Qx, Rx_int.value, Rx_ext.value],
            [Py, Qy, Ry_int.value, Ry_ext.value],
            color=["red","blue","green","purple"], s=100)

plt.text(Px+0.1, Py+0.1, "P", color="red")
plt.text(Qx+0.1, Qy+0.1, "Q", color="blue")
plt.text(Rx_int.value+0.1, Ry_int.value+0.1, "R_int", color="
    green")
plt.text(Rx_ext.value+0.1, Ry_ext.value+0.1, "R_ext", color="
    purple")
```

Python+C Code

```python
# Line PQ
plt.plot([Px, Qx], [Py, Qy], "k--")

plt.axhline(0, color="gray", linewidth=0.5)
plt.axvline(0, color="gray", linewidth=0.5)
plt.xlabel("Coefficient of a")
plt.ylabel("Coefficient of b")
plt.title("Section Formula using C + Python")
plt.grid(True)
plt.show()
```

Python

```
import numpy as np
import matplotlib.pyplot as plt

# Point coordinates (coefficients of a and b)
Px, Py = 3.0, -2.0 # P = 3a - 2b
Qx, Qy = 1.0, 1.0 # Q = 1a + 1b
m, n = 2, 1 # ratio 2:1

# Internal Division: (mQ + nP) / (m+n)
Rx_int = (m*Qx + n*Px) / (m+n)
Ry_int = (m*Qy + n*Py) / (m+n)

# External Division: (mQ - nP) / (m-n)
Rx_ext = (m*Qx - n*Px) / (m-n)
Ry_ext = (m*Qy - n*Py) / (m-n)
```

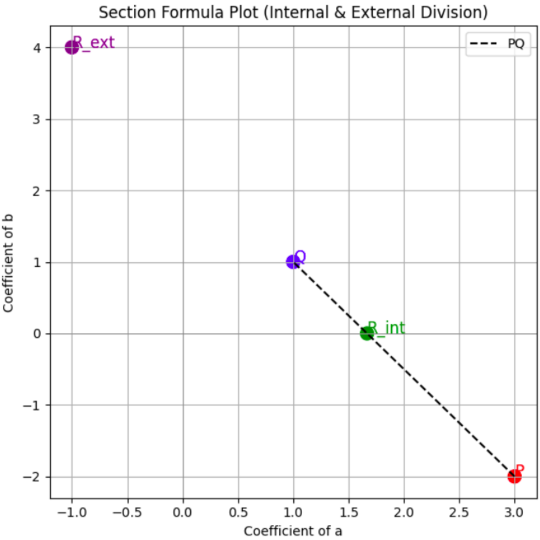
Python Code

```
# Print results
print(f"Internal Division: R = {Rx_int:.2f}a + {Ry_int:.2f}b")
print(f"External Division: R = {Rx_ext:.2f}a + {Ry_ext:.2f}b")

# Plotting
plt.figure(figsize=(6,6))
plt.scatter([Px, Qx, Rx_int, Rx_ext],
            [Py, Qy, Ry_int, Ry_ext],
            color=["red","blue","green","purple"], s=100)

# Labels
plt.text(Px, Py, "P", fontsize=12, color="red")
plt.text(Qx, Qy, "Q", fontsize=12, color="blue")
plt.text(Rx_int, Ry_int, "R_int", fontsize=12, color="green")
plt.text(Rx_ext, Ry_ext, "R_ext", fontsize=12, color="purple")
```

Python Code

```python
# Draw line PQ
plt.plot([Px, Qx], [Py, Qy], "k--", label="PQ")

# Axes styling
plt.axhline(0, color="gray", linewidth=0.5)
plt.axvline(0, color="gray", linewidth=0.5)
plt.xlabel("Coefficient of a")
plt.ylabel("Coefficient of b")
plt.legend()
plt.title("Section Formula Plot (Internal & External Division)")
plt.grid(True)
plt.show()
```

Plot


Section Formula Plot (Internal & External Division)
R_ext
Q
R_int
P
PQ
Coefficient of b
Coefficient of a
4
3
2
1
0
−1
−2
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0