

1.9.17

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Question:

Write the coordinates of a point \mathbf{P} on the x -axis which is equidistant from points $\mathbf{A}(-2, 0)$ and $\mathbf{B}(6, 0)$.

Solution: Let

$$\mathbf{A} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} b \\ 0 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} x \\ 0 \end{pmatrix} \quad (1)$$

Since \mathbf{P} is equidistant from \mathbf{A} and \mathbf{B} , their distances satisfy:

$$\|\mathbf{P} - \mathbf{A}\| = \|\mathbf{P} - \mathbf{B}\| \quad (2)$$

Square both sides:

$$\|\mathbf{P} - \mathbf{A}\|^2 = \|\mathbf{P} - \mathbf{B}\|^2 \quad (3)$$

Using the norm squared definition:

$$(\mathbf{P} - \mathbf{A}) \cdot (\mathbf{P} - \mathbf{A}) = (\mathbf{P} - \mathbf{B}) \cdot (\mathbf{P} - \mathbf{B}) \quad (4)$$

Substitute vectors:

$$(x - a)^2 + 0^2 = (x - b)^2 + 0^2 \quad (5)$$

Simplify:

$$(x - a)^2 = (x - b)^2 \quad (6)$$

Rewrite as difference of squares:

$$(x - a)^2 - (x - b)^2 = 0 \quad (7)$$

Factorize:

$$[(x - a) - (x - b)] \cdot [(x - a) + (x - b)] = 0 \quad (8)$$

Simplify:

$$(b - a) \cdot (2x - (a + b)) = 0 \quad (9)$$

Since $b \neq a$:

$$2x - (a + b) = 0 \quad (10)$$

Solve for x :

$$x = \frac{a + b}{2} \quad (11)$$

Now substitute $a = -2$, $b = 6$:

$$x = \frac{-2 + 6}{2} = \frac{4}{2} = 2 \quad (12)$$

Hence, the coordinates of \mathbf{P} are

$$\mathbf{P} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad (13)$$

Graphical Representation:

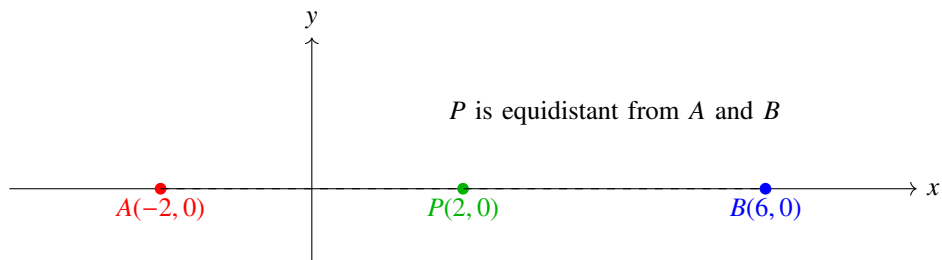


Fig. 0