Question 4.13.5

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1 Question:

The set of lines $ax + by + c = 0$, where $3a + 2b + 4c = 0$ are concurrent at the point _____.

2 Solution:

We are given the fact that $3a + 2b + 4c = 0$. This can be written as:

$$\implies \begin{pmatrix} 3 & 2 & 4 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \tag{1}$$

Any arbitrary point on the line $ax + by + c = 0$ satisfies the equation:

$$\begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \tag{2}$$

Let the point of concurrency be $\mathbf{P}$, at coordinates $\begin{pmatrix} p_x \\ p_y \end{pmatrix}$. We can therefore write the following system of equations:

$$\begin{pmatrix} 3 & 2 & 4 \\ x & y & 1 \\ p_x & p_y & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{3}$$

For any (x, y) on the line $ax+by+c = 0$, the above equation must not have a unique solution as there are infinite values of a, b, c satisfying the equation. Therefore, the

rank of the coefficient matrix must be less than 3.
Applying row reduction:

$$\begin{pmatrix} 3 & 2 & 4 \\ x & y & 1 \\ p_x & p_y & 1 \end{pmatrix} \xleftrightarrow{R_3 \to R_3 - \frac{R_1}{4}} \begin{pmatrix} 3 & 2 & 4 \\ x & y & 1 \\ p_x - \frac{3}{4} & p_y - \frac{1}{2} & 0 \end{pmatrix} \tag{4}$$

Clearly, for the rank to be less than 3, the last row must be all zeros. Therefore the point of concurrency **P** is $\begin{pmatrix} \frac{3}{4} \\ \frac{1}{2} \end{pmatrix}$.

3 Plot:

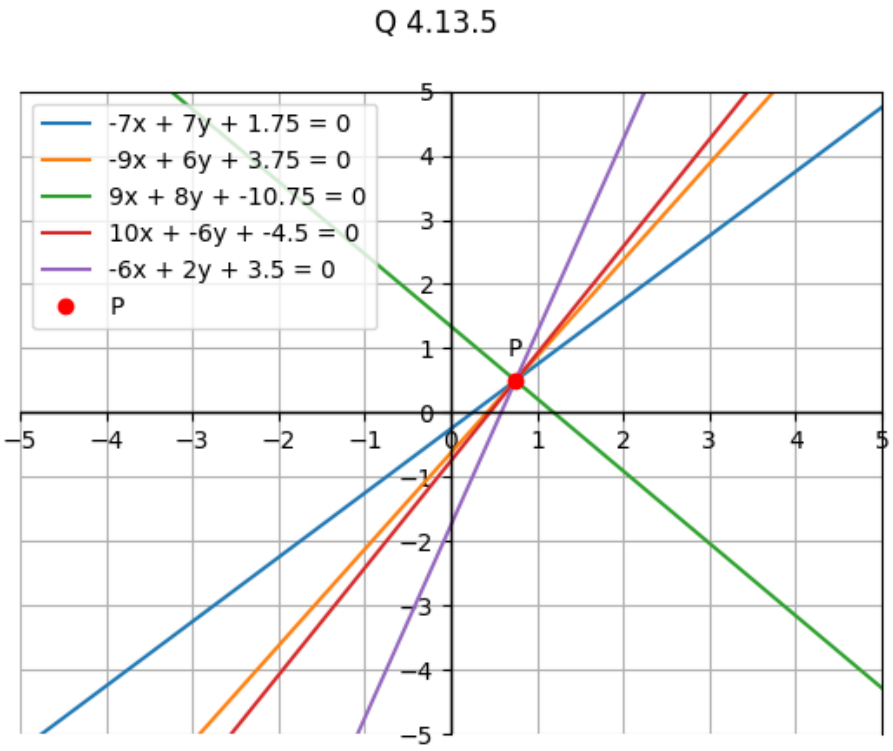

Figure 1: Graph of lines with randomly generated values of a and b satisfying $3a + 2b + 4c = 0$. All lines are concurrent at the point $\begin{pmatrix} \frac{3}{4} \\ \frac{1}{2} \end{pmatrix}$ (marked in red).