4.3.56

EE25BTECH11065 - Yoshita

Question:
Find the equation of the plane with intercepts 2, 3 and 4 on the x, y and z - axis respectively.

Solution:
The intercepts define three points on the plane, which we can label A, B, and C.

Point	Vector
A	$\begin{pmatrix}2\\0\\0\end{pmatrix}$
B	$\begin{pmatrix}0\\3\\0\end{pmatrix}$
C	$\begin{pmatrix}0\\0\\4\end{pmatrix}$

TABLE 0: Answers

The equation of the plane can also be written in the form

$$\mathbf{n}^T\mathbf{x} = 1$$

where $\mathbf{n}$ is the normal vector.

Substituting the three intercept points $A = \begin{pmatrix}2\\0\\0\end{pmatrix}$, $B = \begin{pmatrix}0\\3\\0\end{pmatrix}$, and $C = \begin{pmatrix}0\\0\\4\end{pmatrix}$ into this equation:

$$\mathbf{n}^T\mathbf{A} = 1 \implies \mathbf{n}^T\begin{pmatrix}2\\0\\0\end{pmatrix} = 1 \tag{1}$$

$$\mathbf{n}^T\mathbf{B} = 1 \implies \mathbf{n}^T\begin{pmatrix}0\\3\\0\end{pmatrix} = 1 \tag{2}$$

$$\mathbf{n}^T\mathbf{C} = 1 \implies \mathbf{n}^T\begin{pmatrix}0\\0\\4\end{pmatrix} = 1 \tag{3}$$

Combining these into a single matrix equation:

$$\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

Solving, we obtain

$$\mathbf{n} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{4} \end{pmatrix}$$

Hence, the plane equation

$$\mathbf{n}^T \mathbf{x} = 1$$

$$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{4} \end{pmatrix} \mathbf{x} = 1$$

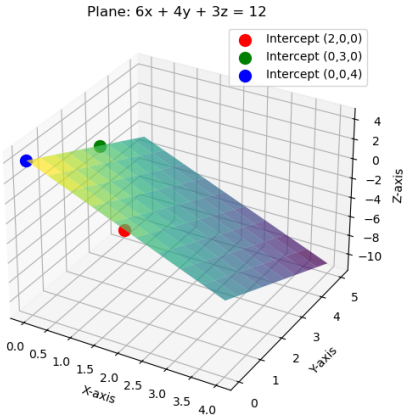

Fig. 0: A plane intersecting the x, y, and z axes.