Question 2.3.3

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Question:

If $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are three non-zero unequal vectors such that $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, then find the angle between $\mathbf{a}$ and $\mathbf{b} - \mathbf{c}$.

Solution:

Given that $\mathbf{a}^{\mathrm{T}}\mathbf{b} = \mathbf{a}^{\mathrm{T}}\mathbf{c}$, we can rewrite this as:

$$\mathbf{a}^{\mathrm{T}}\mathbf{b} - \mathbf{a}^{\mathrm{T}}\mathbf{c} = 0 \tag{1}$$

$$\mathbf{a}^{\mathrm{T}}(\mathbf{b} - \mathbf{c}) = 0 \tag{2}$$

This implies that the dot product of $\mathbf{a}$ and $\mathbf{b} - \mathbf{c}$ is zero, ie these are orthogonal matrices. Therefore, the angle between $\mathbf{a}$ and $\mathbf{b} - \mathbf{c}$ is 90°.