4.3.21

EE25BTECH11032 - Kartik Lahoti

Question:

In what ratio does the X-axis divide the line segment joining the points $(-4, -6)$ and $(-1, 7)$? Find the coordinates of the point of division.

Solution:

Given :

Symbol	Value	Description
A	$\begin{pmatrix} -4 \\ -6 \end{pmatrix}$	Given Point
B	$\begin{pmatrix} -1 \\ 7 \end{pmatrix}$	Given Point
P	$\begin{pmatrix} x \\ 0 \end{pmatrix}$	Desired Point

Let , **P** be the point on x-axis dividing the line **AB** in the ratio say $k : 1$.

Now, **A** , **B** and **P** are collinear ,

$$\therefore rank \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{P} - \mathbf{B} \end{pmatrix} = 1 \tag{0.1}$$

$$\begin{pmatrix} 3 & x+1 \\ 13 & -7 \end{pmatrix} \xleftarrow{R_2 \to \frac{-13}{3} R_1 + R_2} \begin{pmatrix} 3 & x+1 \\ 0 & -\frac{13x}{3} - \frac{34}{3} \end{pmatrix} \tag{0.2}$$

The number of nonzero rows in the echelon form is defined as the rank. For the above matrix to be of rank 1 ,

$$-\frac{13x}{3} - \frac{34}{3} = 0 \tag{0.3}$$

$$x = -\frac{34}{13} \tag{0.4}$$

$\therefore$ The coordinates of the point of intersection is

$$\mathbf{P} = \begin{pmatrix} -\frac{34}{13} \\ 0 \end{pmatrix} \tag{0.5}$$

Now,

$$k = \frac{(\mathbf{A} - \mathbf{P})^\top (\mathbf{P} - \mathbf{B})}{\|\mathbf{P} - \mathbf{B}\|^2} \tag{0.6}$$

Substituting the values of **A** , **B** and **P** , we get,

$$k = \frac{\left(-\frac{18}{13} \quad -6\right)\left(\begin{matrix} -\frac{21}{13} \\ -7 \end{matrix}\right)}{\left\|\left(\begin{matrix} -\frac{21}{13} \\ -7 \end{matrix}\right)\right\|^2} \tag{0.7}$$

$$k = \frac{6}{7} \tag{0.8}$$

Thus, the ratio in which the point **P** divides the line segment **AB** is $6\colon 7$