Matrices in Geometry - 1.9.26

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Problem Statement

Find the value of k, if the point $\mathbf{P}\,(2,4)$ is equidistant from point $\mathbf{A}\,(5,k)$ and $\mathbf{B}\,(k,7)$

Solution

Given: $\mathbf{P}\begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $\mathbf{A}\begin{pmatrix} 5 \\ k \end{pmatrix}$ and a point $\mathbf{B}\begin{pmatrix} k \\ 7 \end{pmatrix}$ such that $\mathbf{P}$ is equidistant from $\mathbf{A}$ and $\mathbf{B}$.

$$\therefore \|\mathbf{A} - \mathbf{P}\| = \|\mathbf{B} - \mathbf{P}\| \tag{1}$$

$$\text{On squaring both the sides, we get} \tag{2}$$

$$\|\mathbf{A} - \mathbf{P}\|^2 = \|\mathbf{B} - \mathbf{P}\|^2 \tag{3}$$

$$\left(\mathbf{A} - \mathbf{P}\right)^\top \left(\mathbf{A} - \mathbf{P}\right) = \left(\mathbf{B} - \mathbf{P}\right)^\top \left(\mathbf{B} - \mathbf{P}\right) \tag{4}$$

Solution

$$(\mathbf{A} - \mathbf{P}) = \begin{pmatrix} 3 \\ k - 4 \end{pmatrix} \tag{5}$$

$$(\mathbf{B} - \mathbf{P}) = \begin{pmatrix} k - 2 \\ 3 \end{pmatrix} \tag{6}$$

$$(\mathbf{A} - \mathbf{P})^{\top} (\mathbf{A} - \mathbf{P}) = \begin{pmatrix} 3 & k - 4 \end{pmatrix} \begin{pmatrix} 3 \\ k - 4 \end{pmatrix} \tag{7}$$

$$= 9 + (k - 4)^2 = 9 + k^2 - 8k + 16 = k^2 - 8k + 25 \tag{8}$$

Solution

$$\left(\mathbf{B} - \mathbf{P}\right)^{\top}\left(\mathbf{B} - \mathbf{P}\right) = \begin{pmatrix} k - 2 & 3 \end{pmatrix} \begin{pmatrix} k - 2 \\ 3 \end{pmatrix} \tag{9}$$

$$= (k-2)^2 + 9 = k^2 - 4k + 4 + 9 = k^2 - 4k + 13 \tag{10}$$

$$\text{From (8) and (10), } k^2 - 8k + 25 = k^2 - 4k + 13 \tag{11}$$

$$\implies -4k + 8k = 25 - 13 \implies 4k = 12 \implies k = 3 \tag{12}$$

Final Answer

Hence, the final answer is $\boxed{k = 3}$ (13)

Points P, A, and B