4.4.8

AI25BTECH11012 - GARIGE UNNATHI

Question:
Find the value of x such that the four points A(x,5,-1), B(3,2,1), C(4,5,5), and D(4,2,-2) are coplanar.

Solution:

Variable	Value
A	$\begin{pmatrix} x \\ 5 \\ -1 \end{pmatrix}$
B	$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$
C	$\begin{pmatrix} 4 \\ 5 \\ 5 \end{pmatrix}$
D	$\begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$

TABLE 0: Variables Used

The equation of a plane can be given by the formula :

$$n^T\mathbf{x} = 1 \tag{0.1}$$
$$or \tag{0.2}$$
$$x^T n = 1 \tag{0.3}$$

Since all the points A,B,C,D are on the plane :

$$A^T n = 1 \quad B^T n = 1 \quad C^T n = 1 \quad D^T n = 1 \tag{0.4}$$

To find **D** we find **n** :
Combining the above equation we get :

$$\begin{pmatrix} B \\ C \\ D \end{pmatrix}^T n = \begin{pmatrix} 3 & 2 & 1 \\ 4 & 5 & 5 \\ 4 & 2 & -2 \end{pmatrix} n = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{0.5}$$

solving the equation by row reduction we get

$$\mathbf{n} = \begin{pmatrix} \frac{9}{16} \\ -\frac{7}{16} \\ \frac{3}{16} \end{pmatrix} = \frac{1}{16}\begin{pmatrix} 9 \\ -7 \\ 3 \end{pmatrix} \tag{0.6}$$

substituting in the equation $A^T n = 1$ we get:

$$\begin{pmatrix} x & 5 & -1 \end{pmatrix}\begin{pmatrix} 9 \\ -7 \\ 3 \end{pmatrix} = 16 \tag{0.7}$$

$$9x - 35 - 3 = 16 \tag{0.8}$$

$$9x = 54 \tag{0.9}$$

$$x = 6 \tag{0.10}$$

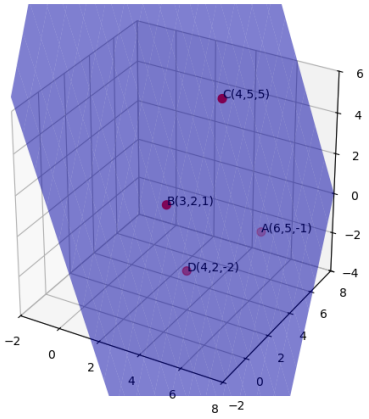

Fig. 0.1