5.2.43

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Question)

Solve the linear equation:

$$6x + 3y = 6xy \tag{1}$$
$$2x + 4y = 5xy \tag{2}$$

Solution

General equation of conic

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f \tag{3}$$

Given set of equations in the form of general conic can be written as

$$\mathbf{x}^T\begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}\mathbf{x} + 2\begin{pmatrix} -3 \\ -1.5 \end{pmatrix}^T\mathbf{x} = 0 \tag{4}$$

$$\mathbf{x}^T\mathbf{V_1}\mathbf{x} + 2\mathbf{u_1}^T\mathbf{x} = 0 \tag{5}$$

Similarly

$$\mathbf{x}^T\begin{pmatrix} 0 & 2.5 \\ 2.5 & 0 \end{pmatrix}\mathbf{x} + 2\begin{pmatrix} -1 \\ -2 \end{pmatrix}^T\mathbf{x} = 0 \tag{6}$$

$$\mathbf{x}^T\mathbf{V_2}\mathbf{x} + 2\mathbf{u_2}^T\mathbf{x} = 0 \tag{7}$$

Solution

Intersection of two conic

$$\mathbf{x}^T(\mathbf{V_1} + \mu\mathbf{V_2})\mathbf{x} + 2(\mathbf{u_1} + \mu\mathbf{u_2})^T\mathbf{x} = 0 \tag{8}$$

General equation of conic represent pair of lines if

$$\begin{vmatrix} \mathbf{V} & \mathbf{u} \\ \mathbf{u}^T & f \end{vmatrix} = 0 \tag{9}$$

From (8)

$$\begin{vmatrix} \mathbf{V_1} + \mu\mathbf{V_2} & \mathbf{u_1} + \mu\mathbf{u_2} \\ (\mathbf{u_1} + \mu\mathbf{u_2})^T & 0 \end{vmatrix} = 0 \tag{10}$$

Solution

Here

$$\mathbf{A} = \mathbf{V_1} + \mu \mathbf{V_2} = \begin{pmatrix} 0 & 3+2.5\mu \\ 3+2.5\mu & 0 \end{pmatrix} \tag{11}$$

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \tag{12}$$

$$\tag{13}$$

$$\mathbf{B} = \mathbf{u_1} + \mu \mathbf{u_2} = \begin{pmatrix} -3+\mu(-1) \\ -1.5+\mu(-2) \end{pmatrix} \tag{14}$$

$$\mathbf{B} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} \tag{15}$$

Solution

Putting values in (10)

$$\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ b_1 & b_2 & 0 \end{vmatrix} \tag{16}$$

$$-b_2(b_2a_{11} - b_1a_{21}) + b_1(b_2a_{12} - b_1a_{22}) \tag{17}$$

Putting values from (11) (14)

$$(-3 - 2.5\mu)(3 + \mu)(1.5 + 2\mu) \tag{18}$$

$$\mu = \frac{-6}{5}, -3, \frac{-3}{4} \tag{19}$$

Solution

Case 1:$\mu = -3$ in (8)

$$\mathbf{x}^T \begin{pmatrix} 0 & 4.5 \\ 4.5 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 0 \\ -4.5 \end{pmatrix}^T \mathbf{x} = 0 \tag{20}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} 0 & 4.5 \\ 4.5 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} 0 \\ -4.5 \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} = 0 \tag{21}$$

$$2 \times 4.5xy + 2(0 - 4.5y) \tag{22}$$

$$= 9xy - 9y = 9y(x-1) = 0 \tag{23}$$

$$y = 0, x = 1 \tag{24}$$

Solution

Case 2: $\mu = \frac{-6}{5}$ in (8)

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 1.8 \\ -0.9 \end{pmatrix}^T \mathbf{x} = 0 \tag{25}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} 1.8 \\ -0.9 \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} = 0 \tag{26}$$

$$2x - y = 0 \tag{27}$$

Solution

Case 3: $\mu = \frac{-3}{4}$ in (8)

$$\mathbf{x}^T \begin{pmatrix} 0 & -1.125 \\ -1.125 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 2.25 \\ 0 \end{pmatrix}^T \mathbf{x} = 0 \tag{28}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} 0 & -1.125 \\ -1.125 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} 2.25 \\ 0 \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} = 0 \tag{29}$$

$$-2.25xy + 4.5 = 2.25x(2 - y) = 0 \tag{30}$$

$$x = 0, y = 2 \tag{31}$$

Solution

Now checking point of intersection with conic

from $\mu = -3$ factors y=0 and x=1

- y=0 in (1) 6x=6x.0 $\implies$ x=0 and in (2) 2x=0, so point (0,0)
- x=1 in (1) 6+3y=6y $\implies$ y=2, so (1,2)

similarly for $\mu = \frac{-3}{4}$ factors are x=0 and y=2

- x=0 gives (0,0)
- y=2 gives (1,2)

And for $\mu = \frac{-6}{5}$ line y=2x

- put y=2x in (1) $12x = 12x^2 \implies x = 0, 1$ so (0,0),(1,2)

All three cases have same points ,

so Points are (0,0) and (1,2)

Figure

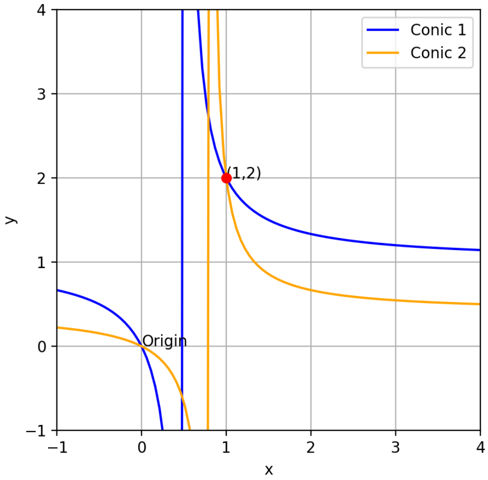

Figure:

Direct Python

```
import numpy as np
import matplotlib.pyplot as plt

plt.figure(figsize=(5,5), dpi=200)
plt.xlim(-1,4)
plt.ylim(-1,4)
A=np.array([[3,6],[4,2]])
c=np.array([6,5])

an=np.linalg.inv(A)
ans=np.dot(an,c)

Ans=np.linalg.solve(A,c)
```

Direct Python

```
print("x=",Ans[0],"y=",Ans[1])
a=1/ans[0]
b=1/ans[1]

x = np.array([a,b]).reshape(-1,1)

x1= np.linspace(-1,4,100)
l1= (6*x1)/(6*x1-3)
l2= (2*x1)/(5*x1-4)

plt.plot(x1,l1, color='blue', label="Line 1")
```

Direct Python

```
plt.plot(x1,l2, color='orange', label="Line 2")
plt.scatter(1,2, c='r', zorder=5)
plt.text(1,2,"(1,2)")
plt.text(0,0,"Origin",)

plt.xlabel("x")
plt.ylabel("y")
plt.grid()
plt.legend()
plt.savefig("figure.png", dpi=200)
plt.show()
```

C code

```c
#include <stdio.h>

typedef struct {
    double x;
    double y;
} Point;

typedef struct {
    Point sols[2];
    int count;
} SolutionSet;

// Solve 6x+3y=6xy, 2x+4y=5xy
SolutionSet solve_equations() {
    SolutionSet S;
    S.count = 0;
```

C code

```c
    // Solution 1: (0,0)
    S.sols[S.count].x = 0;
    S.sols[S.count].y = 0;
    S.count++;

    // Solution 2: (1,2)
    S.sols[S.count].x = 1;
    S.sols[S.count].y = 2;
    S.count++;

    return S;
}
```

C code

```c
#ifdef TEST_C
int main(){
    SolutionSet S = solve_equations();
    for(int i=0; i<S.count; i++){
        printf("Solution %d: (%.2f, %.2f)\n", i+1, S.sols[i].x, S
            .sols[i].y);
    }
    return 0;
}
#endif
```

Python code with shared object

```
# main.py
import ctypes
from ctypes import Structure, c_double, c_int
import matplotlib.pyplot as plt

class Point(Structure):
    _fields_ = [("x", c_double), ("y", c_double)]

class SolutionSet(Structure):
    _fields_ = [("sols", Point * 2), ("count", c_int)]

# Load C lib
lib = ctypes.CDLL("./libsolver.so")
lib.solve_equations.restype = SolutionSet
```

Python code with shared object

```python
# Call function
solutions = lib.solve_equations()
print(f"Found {solutions.count} solutions:")
for i in range(solutions.count):
    x, y = solutions.sols[i].x, solutions.sols[i].y
    print(f"Solution {i+1}: ({x}, {y})")

# Plot equations and solutions
import numpy as np
x_vals = np.linspace(-1, 3, 400)
y1 = (6*x_vals)/(6*x_vals - 3) # from eqn (1)
y2 = (2*x_vals)/(5*x_vals - 4) # from eqn (2)
```

Python code with shared object

```
plt.figure(figsize=(6,6))
plt.plot(x_vals, y1, label="6x+3y=6xy")
plt.plot(x_vals, y2, label="2x+4y=5xy")
for i in range(solutions.count):
    x, y = solutions.sols[i].x, solutions.sols[i].y
    plt.scatter(x, y, c='r', zorder=5)
    plt.text(x, y, f"({x:.0f},{y:.0f})", fontsize=10)

plt.ylim(-1,4)
plt.grid(True)
plt.legend()
plt.xlabel("x")
plt.ylabel("y")
plt.title("Solutions of nonlinear system")
plt.show()
```