

## 8.2.3

EE25BTECH11041 - Naman Kumar

Question:

$$y^2 = -8x \quad (1)$$

**Solution:**

Since it is a parabola we have

Eccentricity	$e=1$
Eigenvalue	$\lambda_1 = 0$
Determinant	$ V  = 0$

General equation of conic

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2)$$

For the equation (1), we can write

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 4 \\ 0 \end{pmatrix}^T \mathbf{x} = 0 \quad (3)$$

$\mathbf{V}$	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
$\mathbf{u}$	$4e_1$
$f$	$0$

using general equations we know for any conic

$$\mathbf{V} = \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n} \mathbf{n}^T \quad (4)$$

$$\mathbf{u} = ce^2 \mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F} \quad (5)$$

$$f = \|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 e^2 \quad (6)$$

Let

$\mathbf{n}$	$\begin{pmatrix} a \\ b \end{pmatrix}$	normal vector to directrix
$\mathbf{F}$	$\begin{pmatrix} g \\ h \end{pmatrix}$	focus
$c$		be constant of directrix

Firstly in (4)

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{n}^T \mathbf{n} \mathbf{I} - (1)^2 \mathbf{n} \mathbf{n}^T \quad (7)$$

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{pmatrix} - \begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix} \quad (8)$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = e_1 \quad (9)$$

In (5)

$$4e_1 = c(1)^2 e_1 - (1)^2 \begin{pmatrix} g \\ h \end{pmatrix} \quad (10)$$

$$\begin{pmatrix} g \\ h \end{pmatrix} = (c - 4)e_1 = (c - 4) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (11)$$

$$\mathbf{F} = \begin{pmatrix} c - 4 \\ 0 \end{pmatrix} \quad (12)$$

In (6)

$$0 = (1)^2 \begin{pmatrix} c - 4 \\ 0 \end{pmatrix}^T \begin{pmatrix} c - 4 \\ 0 \end{pmatrix} - c^2 (1) \quad (13)$$

$$c^2 + 16 - 8c - c^2 = 0 \quad (14)$$

$$c = 2 \quad (15)$$

$$\mathbf{F} = -2e_1 \quad (16)$$

Directrix is

$$\mathbf{n}^T \mathbf{x} = c \quad (17)$$

$$\mathbf{e}_1^T \mathbf{x} = 2 \quad (18)$$

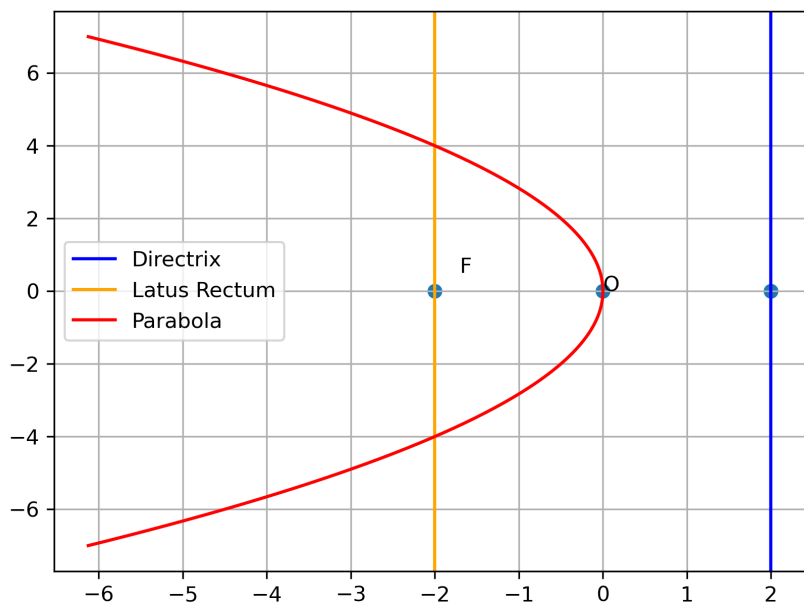


Figure 1