2.9.20

AI25BTECH11004-B.JASWANTH

Question

X and **Y** are two points with position vectors 3**a**+**b** and **a**-3**b**, respectively. Write the position vector of a point Z which divides the line segment **XY** in the ratio 2:1 externally.

Solution:

Let the position vectors of the points X and Y be given by

$$\mathbf{X} = \begin{pmatrix} 3\mathbf{a} \\ \\ \mathbf{b} \end{pmatrix} \tag{0.1}$$

$$\implies \mathbf{X} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} \tag{0.2}$$

$$\mathbf{Y} = \begin{pmatrix} \mathbf{a} \\ \\ -3\mathbf{b} \end{pmatrix} \tag{0.3}$$

$$\implies \mathbf{Y} = \begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix}. \tag{0.4}$$

The formula for the point **Z** dividing the line segment joining **X** and **Y** **externally** in the ratio $k:1$ is:

$$\mathbf{Z} = \frac{k\mathbf{Y} - \mathbf{X}}{k-1}. \tag{0.5}$$

Substituting $k = 2$ and the above matrices:

$$\mathbf{Z} = \frac{2\mathbf{Y} - \mathbf{X}}{2-1} = 2\mathbf{Y} - \mathbf{X}. \tag{0.6}$$

Now compute $2\mathbf{Y} - \mathbf{X}$:

$$2\mathbf{Y} - \mathbf{X} = 2\begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} - \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -7 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix}. \tag{0.7}$$

Hence,

$$\mathbf{Z} = \begin{pmatrix} -1 & 0 \\ 0 & -7 \end{pmatrix} \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} = \begin{pmatrix} -a \\ -7b \end{pmatrix}. \tag{0.8}$$

Therefore, the position vector of Z is

$$\boxed{\mathbf{Z} = \begin{pmatrix} -\mathbf{a} \\ -7\mathbf{b} \end{pmatrix}.} \tag{0.9}$$

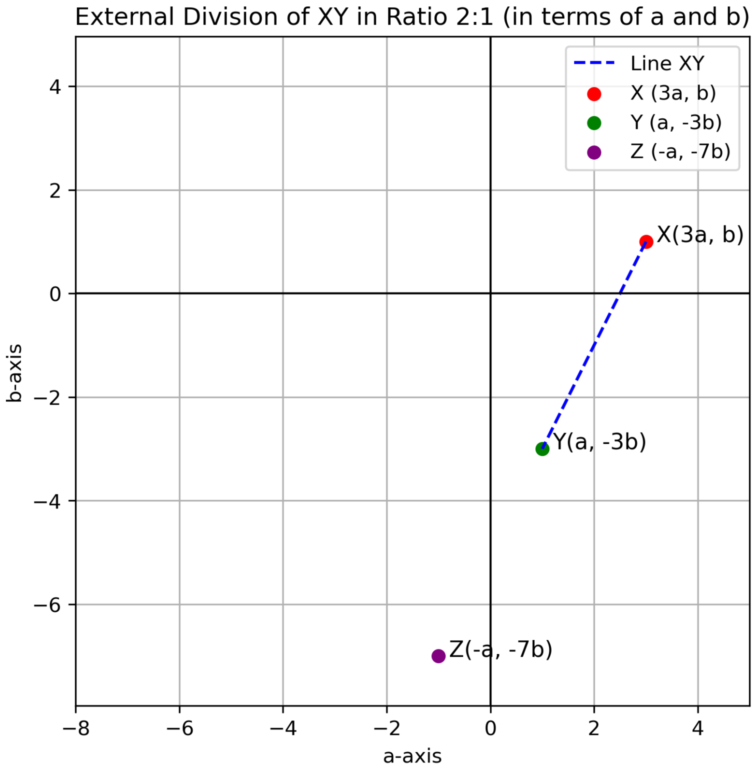

Fig. 0: Caption