

# Assignment 11: 5.5.7

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**Question:**

Find the inverse of the following matrix, using elementary transformations

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$$\begin{pmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{pmatrix}$$

**Solution:**

Given:

$$\mathbf{A} = \begin{pmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{pmatrix} \quad (1)$$

Let  $\mathbf{A}^{-1}$  be inverse of  $\mathbf{A}$  We know

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I} \quad (2)$$

Thus augmented matrix to find  $\mathbf{A}^{-1}$  is given by:  $(\mathbf{A} \mid \mathbf{I})$

$$\left( \begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ 3 & 7 & 2 & 0 & 0 & 1 \end{array} \right) \xleftarrow{R1 \leftrightarrow R3; R1 = R1/3} \quad (3)$$

$$\left( \begin{array}{ccc|ccc} 1 & 7/3 & 2/3 & 0 & 0 & 1/3 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ 2 & 3 & 1 & 1 & 0 & 0 \end{array} \right) \xleftarrow{R2 = R2 - 2R1; R3 = R3 - 2R1} \quad (4)$$

$$\left( \begin{array}{ccc|ccc} 1 & 7/3 & 2/3 & 0 & 0 & 1/3 \\ 0 & -2/3 & -1/3 & 0 & 1 & -2/3 \\ 0 & -5/3 & -1/3 & 1 & 0 & -2/3 \end{array} \right) \xleftarrow{R3 = -5/3 R3} \quad (5)$$

$$\left( \begin{array}{ccc|ccc} 1 & 7/3 & 2/3 & 0 & 0 & 1/3 \\ 0 & -2/3 & -1/3 & 0 & 1 & -2/3 \\ 0 & 1 & 1/5 & -3/5 & 0 & 2/5 \end{array} \right) \xleftarrow{R1 = R1 - 7/3 R3; R2 = 2/3 R3} \quad (6)$$

$$\left( \begin{array}{ccc|ccc} 1 & 0 & 1/5 & 7/5 & 0 & -3/5 \\ 0 & 0 & -1/5 & -2/5 & 1 & -2/5 \\ 0 & 1 & 1/5 & -3/5 & 0 & 2/5 \end{array} \right) \xleftarrow{R2 \leftrightarrow R3} \quad (7)$$

$$\left( \begin{array}{ccc|ccc} 1 & 0 & 1/5 & 7/5 & 0 & -3/5 \\ 0 & 1 & 1/5 & -3/5 & 0 & 2/5 \\ 0 & 0 & -1/5 & -2/5 & 1 & -2/5 \end{array} \right) \xleftarrow{R1 = R1 + R3; R2 = R2 - R3} \quad (8)$$

$$\left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -5 & 2 \end{array} \right) \quad (9)$$

So we have:

$$\mathbf{A}^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{pmatrix} \quad (10)$$

