4.7.64

AI25BTECH11003 - Bhavesh Gaikwad

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Question

Find the distance between the point **P**(6, 5, 9) and the plane determined by the points **A**(3, -1, 2), **B**(5, 2, 4) and **C**(-1, -1, 6).

Theoretical Solution

Given:

$$P = \begin{pmatrix} 6 \\ 5 \\ 9 \end{pmatrix}, A = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, B = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}, C = \begin{pmatrix} -1 \\ -1 \\ 6 \end{pmatrix}. \tag{1}$$

First, form two direction vectors on the plane using the given points.

$$LET\ \mathbf{u} = B - A = \begin{pmatrix} 2 \\ 3 \\ 2 \end{pmatrix}, \qquad \mathbf{v} = C - A = \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix}. \tag{2}$$

Let $\mathbf{n} = \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix}$ be the perpendicular vector to the Plane.

Therefore, the equation of the Plane is $\mathbf{n}^\top \mathbf{x} = 1$

Let the equation of the Plane be $\begin{pmatrix} n_1 & n_2 & n_3 \end{pmatrix} \mathbf{x} = 1$

Theoretical Solution

Finding $\mathbf{n}$ which is orthogonal to both $\mathbf{u}$ and $\mathbf{v}$ by solving the homogeneous system:

$$\begin{pmatrix} 2 & 3 & 2 \\ -4 & 0 & 4 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{3}$$

Row-reduce and solve for a convenient integer solution.

$$\begin{pmatrix} 2 & 3 & 2 \\ -4 & 0 & 4 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 + 2R_1} \begin{pmatrix} 2 & 3 & 2 \\ 0 & 6 & 8 \end{pmatrix}, \tag{4}$$

$$6n_2 + 8n_3 = 0 \;\Rightarrow\; 3n_2 + 4n_3 = 0, \quad n_3 = 3,\; n_2 = -4,\; 2n_1 + 3n_2 + 2n_3 = \tag{5}$$

$$\therefore \mathbf{n} = \frac{1}{19} \begin{pmatrix} 3 \\ -4 \\ 3 \end{pmatrix}. \tag{6}$$

Writing the plane as $\mathbf{n}^\top \mathbf{x} = 1$

Finally, applying the point-to-plane distance formula and simplify.

$$d = \frac{\left|\mathbf{n}^\top P - 1\right|}{\|\mathbf{n}\|} \tag{7}$$

$$= \frac{|3 \cdot 6 + (-4) \cdot 5 + 3 \cdot 9 - 19|}{\sqrt{3^2 + (-4)^2 + 3^2}} \tag{8}$$

$$= \frac{|25 - 19|}{\sqrt{34}} \tag{9}$$

$$= \frac{6}{\sqrt{34}} = \frac{3\sqrt{34}}{17}. \tag{10}$$

$$\boxed{\text{The Distance between the Plane and } \mathbf{P} \textit{ is } \frac{3\sqrt{34}}{17} \textit{ units}.} \tag{11}$$

Theoretical Solution

C Code

```c
#include <stdio.h>
#include <math.h>

/* 3D point and vector types */
typedef struct { double x, y, z; } Point3D;
typedef struct { double x, y, z; } Vector3D;

/* Subtract two points (p1 - p2) -> vector */
Vector3D subtract_points(Point3D p1, Point3D p2) {
    Vector3D r;
    r.x = p1.x - p2.x;
    r.y = p1.y - p2.y;
    r.z = p1.z - p2.z;
    return r;
}
```

C Code

```c

/* Cross product v1 x v2 */
Vector3D cross_product(Vector3D v1, Vector3D v2) {
    Vector3D r;
    r.x = v1.y * v2.z - v1.z * v2.y;
    r.y = v1.z * v2.x - v1.x * v2.z;
    r.z = v1.x * v2.y - v1.y * v2.x;
    return r;
}

/* Vector magnitude */
double magnitude(Vector3D v) {
    return sqrt(v.x * v.x + v.y * v.y + v.z * v.z);
}
```

C Code

```c
/* Plane normal from three points: n = (B - A) x (C - A) */
Vector3D find_plane_normal(Point3D A, Point3D B, Point3D C) {
    Vector3D AB = subtract_points(B, A);
    Vector3D AC = subtract_points(C, A);
    return cross_product(AB, AC);
}

/* Plane constant k in: n.x*x + n.y*y + n.z*z = k, using a point
    on plane */
double find_plane_constant(Point3D A, Vector3D n) {
    return n.x * A.x + n.y * A.y + n.z * A.z;
}
```

C Code

```c
/* Foot of perpendicular from P to plane with normal n and
    constant k */
Point3D find_foot_of_perpendicular(Point3D P, Vector3D n, double
    k) {
    /* Line: L(t) = P + t n; impose n * L(t) = k to solve for t
        */
    double num = k - (n.x * P.x + n.y * P.y + n.z * P.z);
    double den = n.x * n.x + n.y * n.y + n.z * n.z;
    double t = num / den;

    Point3D Q;
    Q.x = P.x + t * n.x;
    Q.y = P.y + t * n.y;
    Q.z = P.z + t * n.z;
    return Q;
}
```

C Code

```c
/* Distance between two points */
double distance_between_points(Point3D p1, Point3D p2) {
    Vector3D d = subtract_points(p1, p2);
    return magnitude(d);
}

/* Save points to points.dat in text format */
void save_points_to_file(Point3D A, Point3D B, Point3D C, Point3D
     P, Point3D Q) {
    FILE *fp = fopen("points.dat", "w");
    if (!fp) return;
    fprintf(fp, "A %.10f %.10f %.10f\n", A.x, A.y, A.z);
    fprintf(fp, "B %.10f %.10f %.10f\n", B.x, B.y, B.z);
    fprintf(fp, "C %.10f %.10f %.10f\n", C.x, C.y, C.z);
    fprintf(fp, "P %.10f %.10f %.10f\n", P.x, P.y, P.z);
    fprintf(fp, "Q %.10f %.10f %.10f\n", Q.x, Q.y, Q.z);
    fclose(fp);
}
```

C Code

```
/* Solve distance from P to plane ABC, return distance, write Q
    via pointer, and save A,B,C,P,Q to points.dat */
double solve_point_to_plane_distance(Point3D A, Point3D B,
    Point3D C, Point3D P, Point3D *Q_out) {
    Vector3D n = find_plane_normal(A, B, C);
    double k = find_plane_constant(A, n);
    Point3D Q = find_foot_of_perpendicular(P, n, k);
    if (Q_out) *Q_out = Q;
    save_points_to_file(A, B, C, P, Q);
    return distance_between_points(P, Q);
}
```

C Code

```c
/* Convenience function with the exact problem data from the PDF:
   A(3,-1,2), B(5,2,4), C(-1,-1,6), P(6,5,9).
   Calls the solver, writes points.dat, and returns the distance.
       */
double generate_points_and_save(void) {
    Point3D A = { 3.0, -1.0, 2.0 };
    Point3D B = { 5.0, 2.0, 4.0 };
    Point3D C = {-1.0, -1.0, 6.0 };
    Point3D P = { 6.0, 5.0, 9.0 };
    Point3D Q;
    return solve_point_to_plane_distance(A, B, C, P, &Q);
}
```

Python Code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import os

# Load the shared library
lib = ctypes.CDLL('./plane.so')

# Define the Point3D structure for ctypes
class Point3D(ctypes.Structure):
    _fields_ = [("x", ctypes.c_double),
                ("y", ctypes.c_double),
                ("z", ctypes.c_double)]
```

Python Code

```
class Vector3D(ctypes.Structure):
    _fields_ = [("x", ctypes.c_double),
                ("y", ctypes.c_double),
                ("z", ctypes.c_double)]

# Define function signatures
lib.solve_point_to_plane_distance.argtypes = [Point3D, Point3D,
    Point3D, Point3D, ctypes.POINTER(Point3D)]
lib.solve_point_to_plane_distance.restype = ctypes.c_double

def read_points_from_file(filename):
    """Read points from the data file"""
    points = {}
```

Python Code

```
if not os.path.exists(filename):
        print(f"File {filename} not found. Running C program
            first...")
        # If points.dat doesn't exist, we need to run the C
            program
        os.system('make main')

    with open(filename, 'r') as f:
        for line in f:
            parts = line.strip().split()
            if len(parts) == 4:
                label = parts[0]
                x, y, z = map(float, parts[1:])
                points[label] = (x, y, z)

    return points
```

Python Code

```
def create_visualization(points):
    """Create 3D visualization of points and plane"""
    fig = plt.figure(figsize=(12, 10))
    ax = fig.add_subplot(111, projection='3d')

    # Extract point coordinates
    A = np.array(points['A'])
    B = np.array(points['B'])
    C = np.array(points['C'])
    P = np.array(points['P'])
    Q = np.array(points['Q'])
```

Python Code

```
# Plot the points
ax.scatter(*A, color='red', s=100, label='A(3,-1,2)')
ax.scatter(*B, color='green', s=100, label='B(5,2,4)')
ax.scatter(*C, color='blue', s=100, label='C(-1,-1,6)')
ax.scatter(*P, color='orange', s=150, label='P(6,5,9)',
    marker='^')
ax.scatter(*Q, color='purple', s=120, label='Q (foot of
    perpendicular)', marker='s')

# Add text labels for points
ax.text(A[0], A[1], A[2], ' A', fontsize=12)
ax.text(B[0], B[1], B[2], ' B', fontsize=12)
ax.text(C[0], C[1], C[2], ' C', fontsize=12)
ax.text(P[0], P[1], P[2], ' P', fontsize=12)
ax.text(Q[0], Q[1], Q[2], ' Q', fontsize=12)
```

Python Code

```
# Draw line from P to Q (perpendicular distance)
    ax.plot([P[0], Q[0]], [P[1], Q[1]], [P[2], Q[2]], 'k--',
        linewidth=2, label='Perpendicular distance')

    # Create a grid around the points
    all_points = np.array([A, B, C, P, Q])
    x_min, x_max = all_points[:, 0].min() - 2, all_points[:, 0].
        max() + 2
    y_min, y_max = all_points[:, 1].min() - 2, all_points[:, 1].
        max() + 2

    xx, yy = np.meshgrid(np.linspace(x_min, x_max, 20), np.
        linspace(y_min, y_max, 20))
    zz = (19 - 3*xx + 4*yy) / 3

    # Plot the plane
    ax.plot_surface(xx, yy, zz, alpha=0.3, color='lightblue')
```

Python Code

```
# Draw triangle ABC on the plane to show the plane boundary
    triangle_x = [A[0], B[0], C[0], A[0]]
    triangle_y = [A[1], B[1], C[1], A[1]]
    triangle_z = [A[2], B[2], C[2], A[2]]
    ax.plot(triangle_x, triangle_y, triangle_z, 'r-', linewidth
        =2, label='Triangle ABC')

    # Set labels and title
    ax.set_xlabel('X')
    ax.set_ylabel('Y')
    ax.set_zlabel('Z')
    ax.set_title('4.7.64')
    ax.legend()

    # Set equal aspect ratio
    ax.set_box_aspect([1,1,1])
```

Python Code

```
# Save the figure
    plt.tight_layout()
    plt.savefig('fig1.png', dpi=300, bbox_inches='tight')
    plt.close()

    print("Visualization saved as fig1.png")

def main():
    """Main function"""
    # Define the points from the problem
    A = Point3D(3.0, -1.0, 2.0)
    B = Point3D(5.0, 2.0, 4.0)
    C = Point3D(-1.0, -1.0, 6.0)
    P = Point3D(6.0, 5.0, 9.0)
    Q = Point3D()
```

Python Code

```
print("Using shared library to solve the problem...")
   print("Points:")
   print(f"A = ({A.x}, {A.y}, {A.z})")
   print(f"B = ({B.x}, {B.y}, {B.z})")
   print(f"C = ({C.x}, {C.y}, {C.z})")
   print(f"P = ({P.x}, {P.y}, {P.z})")

   try:
       # Call the C function
       distance = lib.solve_point_to_plane_distance(A, B, C, P,
           ctypes.byref(Q))
       print(f"\nCalculated distance: {distance:.6f} units")
       print(f"Foot of perpendicular Q: ({Q.x:.6f}, {Q.y:.6f}, {
           Q.z:.6f})")

       # Read points from file and create visualization
       points_dict = read_points_from_file('points.dat')
       print(f"\nPoints read from file: {points_dict}")
```

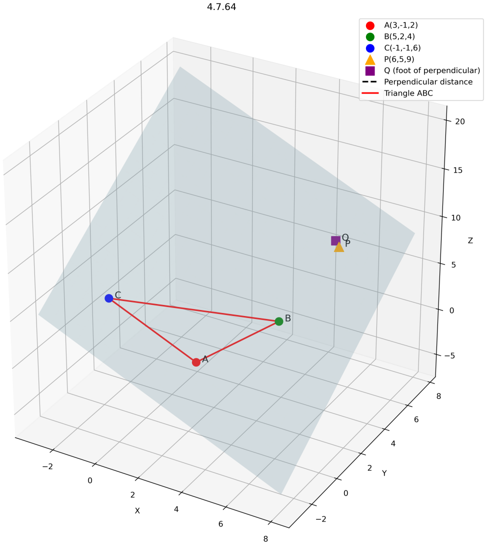
Python Code

```
create_visualization(points_dict)

    except Exception as e:
        print(f"Error: {e}")
        print("Make sure to compile the shared library first with
            : make plane.so")

if __name__ == "__main__":
    main()
```

Plane


4.7.64
A(3,-1,2)
B(5,2,4)
C(-1,-1,6)
P(6,5,9)
Q (foot of perpendicular)
Perpendicular distance
Triangle ABC
C
B
A
Q
P
X
Y
Z
−2
0
2
4
6
8
−2
0
2
4
6
8
−5
0
5
10
15
20