12.27

Bhargav - EE25BTECH11013

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Question:
1200 men and 500 women can build a bridge in 2 weeks. 900 men and 250 women will take 3 weeks to build the same bridge. How many men will be needed to build the bridge in one week?

Solution

Let one man complete work in X weeks and one woman complete work in Y weeks
In one week a man can complete $\frac{1}{X}$ work and woman can complete $\frac{1}{Y}$

$$\frac{1200}{X} + \frac{500}{Y} = \frac{1}{2} \implies XY - 1000X - 2400Y = 0 \tag{1}$$

$$\frac{900}{X} + \frac{250}{Y} = \frac{1}{3} \implies XY - 750X - 2700Y = 0 \tag{2}$$

Solution

Rotate the axis by 45° to remove the XY term in the equations

$$\mathbf{X} = \mathbf{Q}\mathbf{x} \tag{3}$$

(where $\mathbf{Q}$ is the rotation matrix)

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{4}$$

$$\implies XY = \frac{x^2 - y^2}{2} \tag{5}$$

The conic equations become:

$$x^2 - y^2 - 3400\sqrt{2}x + 1400\sqrt{2}y = 0 \tag{6}$$

$$x^2 - y^2 - 3450\sqrt{2}x + 1950\sqrt{2}y = 0 \tag{7}$$

Solution

These can be represented as general conic equations:

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{8}$$

For the conics: $\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $\mathbf{u_1} = \begin{pmatrix} -1700\sqrt{2} \\ 700\sqrt{2} \end{pmatrix}$, $\mathbf{u_2} = \begin{pmatrix} -1725\sqrt{2} \\ 975\sqrt{2} \end{pmatrix}$,
$f = 0$
In homogeneous coordinates, using the form $\mathbf{x}^\top C\mathbf{x} = 0$, where
$\mathbf{x} = \begin{pmatrix} x & y & 1 \end{pmatrix}^T$, the matrices for the conics are:

$$\mathbf{C} = \begin{pmatrix} \mathbf{V} & \mathbf{u^T} \\ \mathbf{u} & f \end{pmatrix} \tag{9}$$

Solution

$$\implies \mathbf{C_1} = \begin{pmatrix} 1 & 0 & -1700\sqrt{2} \\ 0 & -1 & 700\sqrt{2} \\ -1700\sqrt{2} & 700\sqrt{2} & 0 \end{pmatrix} \tag{10}$$

$$\implies \mathbf{C_2} = \begin{pmatrix} 1 & 0 & -1725\sqrt{2} \\ 0 & -1 & 975\sqrt{2} \\ -1725\sqrt{2} & 975\sqrt{2} & 0 \end{pmatrix} \tag{11}$$

The intersection point of both the conics lies on the conic formed by their individual linear combination $\mathbf{C}(\mu) = \mathbf{C_1} + \mu\mathbf{C_2}$. We must find the value of μ that makes the determinant of the conic's matrix as 0.

Solution

$$\mathbf{C}(\mu) = \begin{pmatrix} 1+\mu & 0 & -1700\sqrt{2} - 1725\sqrt{2 \\ 0 & -\mu - 1 & 700\sqrt{2} + 975\sqrt{2}\mu \\ -1700\sqrt{2} - 1725\sqrt{2}\mu & 700\sqrt{2} + 975\sqrt{2}\mu & 0 \end{pmatrix} \tag{12}$$

On solving $\det\left(\mathbf{C}(\mu)\right) = 0$, the simplest value of $\mu = -1$

$$\left(\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u_1}^\top \mathbf{x} + f\right) + (-1)\left(\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u_2}^\top \mathbf{x} + f\right) = 0 \tag{13}$$

The chord of intersection of the 2 hyperbolas is:

$$\left(\mathbf{u_1^T} - \mathbf{u_2^T}\right)\mathbf{x} = 0 \implies \begin{pmatrix} 1 & -11 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0 \tag{14}$$

The point of intersection of the common chord and the first hyperbola can be found out by solving them

$$\mathbf{x} = \mathbf{h} + k_i\mathbf{m}, \mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 11 \\ 1 \end{pmatrix} \tag{15}$$

$$(\mathbf{h} + k_i\mathbf{m})^\top \mathbf{V}(\mathbf{h} + k_i\mathbf{m}) + 2\mathbf{u}^\top(\mathbf{h} + k_i\mathbf{m}) + f = 0 \tag{16}$$

$$\implies k_i^2\mathbf{m}^\top\mathbf{V}\mathbf{m} + 2k_i\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) + \mathbf{h}^\top\mathbf{V}\mathbf{h} + 2\mathbf{u}^\top\mathbf{h} + f = 0 \tag{17}$$

$$\text{or, } k_i^2\mathbf{m}^\top\mathbf{V}\mathbf{m} + 2k_i\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) + g(\mathbf{h}) = 0 \tag{18}$$

Solution

Solving the above quadratic gives the equation

$$k_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{\left[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})\right]^2 - g(\mathbf{h}) \left(\mathbf{m}^\top \mathbf{V}\mathbf{m}\right)} \right) \tag{19}$$

Solving we get:

$$k_1 = 0, k_2 = 300\sqrt{2} \tag{20}$$

The point of intersection:

$$\mathbf{x_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 3300\sqrt{2} \\ 300\sqrt{2} \end{pmatrix} \tag{21}$$

The point $\mathbf{x_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is not possible because it causes division by 0.

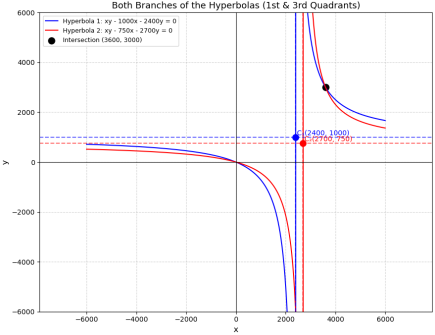
Solution

Substituting $\mathbf{x_2}$ in the rotation matrix equation:

$$\mathbf{X} = \begin{pmatrix} 3600 \\ 3000 \end{pmatrix} \tag{22}$$

A man can complete the work in 3600 weeks, a woman can complete the work in 3000 weeks

Plot


Both Branches of the Hyperbolas (1st & 3rd Quadrants)
Hyperbola 1: xy - 1000x - 2400y = 0
Hyperbola 2: xy - 750x - 2700y = 0
Intersection (3600, 3000)
C (2400, 1000)
C (2700, 750)
6000
4000
2000
0
−2000
−4000
−6000
−6000
−4000
−2000
0
2000
4000
6000
x
y

C Code

```
#include <stdio.h>

double determinant3(double A[3][3]) {
    double det =
        A[0][0]*(A[1][1]*A[2][2] - A[1][2]*A[2][1]) -
        A[0][1]*(A[1][0]*A[2][2] - A[1][2]*A[2][0]) +
        A[0][2]*(A[1][0]*A[2][1] - A[1][1]*A[2][0]);
    return det;
}
```

Python + C Code

```
import numpy as np
import matplotlib.pyplot as plt
import ctypes

lib = ctypes.CDLL("./libcode.so")
lib.determinant3.restype = ctypes.c_double

# Example use of determinant from C
A = ((ctypes.c_double * 3) * 3)()
data = [
    [1, 2, 3],
    [0, 1, 4],
    [5, 6, 0]
]
for i in range(3):
    for j in range(3):
        A[i][j] = data[i][j]

det_val = lib.determinant3(A)
```

```
print("Determinant (computed in C):", det_val)
def hyperbola1(x):
    return (1000 * x) / (x - 2400)

def hyperbola2(x):
    return (750 * x) / (x - 2700)

center1 = (2400, 1000)
center2 = (2700, 750)
intersection = (3600, 3000)

# Plot both branches
x_vals = np.linspace(-6000, 6000, 2000)
x_vals1 = x_vals[x_vals != 2400]
x_vals2 = x_vals[x_vals != 2700]

y1 = hyperbola1(x_vals1)
y2 = hyperbola2(x_vals2)
```

Python + C Code

```
plt.figure(figsize=(9, 7))

plt.plot(x_vals1, y1, 'b', label='Hyperbola 1: xy - 1000x - 2400y
     = 0')
plt.plot(x_vals2, y2, 'r', label='Hyperbola 2: xy - 750x - 2700y
    = 0')

# Centers and asymptotes
plt.scatter(*center1, color='blue', s=70)
plt.scatter(*center2, color='red', s=70)
plt.axvline(x=2400, color='b', linestyle='--', alpha=0.6)
plt.axhline(y=1000, color='b', linestyle='--', alpha=0.6)
plt.axvline(x=2700, color='r', linestyle='--', alpha=0.6)
plt.axhline(y=750, color='r', linestyle='--', alpha=0.6)

# Intersection
plt.scatter(*intersection, color='black', s=90, label='
    Intersection (3600,3000)')
plt.text(intersection[0]+80, intersection[1]+80, f"({intersection
```

```
# Formatting
plt.title("Hyperbolas in Original Coordinates (1st & 3rd
    Quadrants)", fontsize=13)
plt.xlabel("x")
plt.ylabel("y")
plt.grid(True, linestyle='--', alpha=0.6)
plt.axhline(0, color='k', linewidth=0.8)
plt.axvline(0, color='k', linewidth=0.8)
plt.axis('equal')
plt.xlim(-6000, 6000)
plt.ylim(-6000, 6000)
plt.legend(fontsize=9, loc='upper left')
plt.tight_layout()
plt.savefig("/mnt/c/Users/bharg/Documents/backupmatrix/
    ee25btech11013/matgeo/12.27/figs/Figure_1.png")

plt.show()
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt

def hyperbola1(x):
    # From: xy - 1000x - 2400y = 0 => y = 1000x / (x - 2400)
    return (1000 * x) / (x - 2400)

def hyperbola2(x):
    # From: xy - 750x - 2700y = 0 => y = 750x / (x - 2700)
    return (750 * x) / (x - 2700)

# Centers (from completing the product form)
center1 = (2400, 1000)
center2 = (2700, 750)

x1 = np.linspace(-6000, 6000, 2000)

# Avoid division by zero at vertical asymptotes
x1 = x1[x1 != 2400]
```

Python Code

```
y1 = hyperbola1(x1)
y2 = hyperbola2(x1)

plt.figure(figsize=(9, 7))

# Plot both hyperbolas
plt.plot(x1, y1, 'b', label='Hyperbola 1: xy - 1000x - 2400y = 0'
    )
plt.plot(x1, y2, 'r', label='Hyperbola 2: xy - 750x - 2700y = 0')

# Plot asymptotes for each hyperbola
plt.axvline(x=2400, color='b', linestyle='--', alpha=0.6)
plt.axhline(y=1000, color='b', linestyle='--', alpha=0.6)
plt.axvline(x=2700, color='r', linestyle='--', alpha=0.6)
plt.axhline(y=750, color='r', linestyle='--', alpha=0.6)

# Plot centers
plt.scatter(*center1, color='blue', s=80, zorder=10)
plt.text(center1[0]+50, center1[1]+80, f"C({center1[0]}, {center1
```

Python Code

```
plt.scatter(*center2, color='red', s=80, zorder=10)
plt.text(center2[0]+50, center2[1]+80, f"C({center2[0]}, {center2
    [1]})", fontsize=10, color='red')
intersection = (3600, 3000)
plt.scatter(*intersection, color='black', s=90, label=f'
    Intersection {intersection}')
plt.title("Both Branches of the Hyperbolas (1st & 3rd Quadrants)"
    , fontsize=13)
plt.xlabel("x", fontsize=12)
plt.ylabel("y", fontsize=12)
plt.grid(True, linestyle='--', alpha=0.6)
plt.axhline(0, color='k', linewidth=0.8)
plt.axvline(0, color='k', linewidth=0.8)
plt.legend(fontsize=9, loc='upper left')
plt.axis('equal')
plt.xlim(-6000, 6000)
plt.ylim(-6000, 6000)
plt.tight_layout()
plt.savefig("/mnt/c/Users/bharg/Documents/backupmatrix/
```