Matrices in Geometry 2.10.64

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Question: The position vectors of the points **A**, **B**, **C** and **D** are $\left(3\hat{i}-2\hat{j}-\hat{k}\right)$, $\left(2\hat{i}+3\hat{j}-4\hat{k}\right)$, $\left(-\hat{i}+\hat{j}+2\hat{k}\right)$ and $\left(4\hat{i}+5\hat{j}+\lambda\hat{k}\right)$ respectively. If the points **A**, **B**, **C** and **D** lie on a plane, find the value of λ.

Given:

$\mathbf{A}\begin{pmatrix}3\\-2\\-1\end{pmatrix}$, $\mathbf{B}\begin{pmatrix}2\\3\\-4\end{pmatrix}$, $\mathbf{C}\begin{pmatrix}-1\\1\\2\end{pmatrix}$ and $\mathbf{D}\begin{pmatrix}4\\5\\\lambda\end{pmatrix}$.

Solution:

$$\mathbf{B}-\mathbf{A}=\begin{pmatrix}-1\\5\\-3\end{pmatrix} \tag{1}$$

$$\mathbf{C}-\mathbf{A}=\begin{pmatrix}-4\\3\\3\end{pmatrix} \tag{2}$$

$$\mathbf{D}-\mathbf{A}=\begin{pmatrix}1\\7\\\lambda+1\end{pmatrix} \tag{3}$$

As the points **A**, **B**, **C** and **D** lie on a plane, this means that the vectors $\mathbf{B}-\mathbf{A}$, $\mathbf{C}-\mathbf{A}$ and $\mathbf{D}-\mathbf{A}$ are coplanar and hence,

$$\implies \left|\begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A} & \mathbf{D}-\mathbf{A}\end{pmatrix}\right|=0 \tag{4}$$

$$\left|\begin{pmatrix}-1 & -4 & 1\\5 & 3 & 7\\-3 & 3 & \lambda+1\end{pmatrix}\right|=0 \tag{5}$$

Converting this matrix into row echelon form,

$$\begin{pmatrix}-1 & -4 & 1\\5 & 3 & 7\\-3 & 3 & \lambda+1\end{pmatrix}\overset{R_2\to R_2+5R_1}{\longrightarrow}\begin{pmatrix}-1 & -4 & 1\\0 & -17 & 12\\-3 & 3 & \lambda+1\end{pmatrix} \tag{6}$$

$$\begin{pmatrix}-1 & -4 & 1\\0 & -17 & 12\\-3 & 3 & \lambda+1\end{pmatrix}\overset{R_3\to R_3-3R_1}{\longrightarrow}\begin{pmatrix}-1 & -4 & 1\\0 & -17 & 12\\0 & 15 & \lambda-2\end{pmatrix} \tag{7}$$

$$\begin{pmatrix}-1 & -4 & 1\\0 & -17 & 12\\0 & 15 & \lambda-2\end{pmatrix}\overset{R_2\to -R_2}{\longrightarrow}\begin{pmatrix}-1 & -4 & 1\\0 & 17 & -12\\0 & 15 & \lambda-2\end{pmatrix} \tag{8}$$

$$\begin{pmatrix}-1 & -4 & 1\\0 & 17 & -12\\0 & 15 & \lambda-2\end{pmatrix}\overset{R_3\to R_3-\frac{15}{17}R_2}{\longrightarrow}\begin{pmatrix}-1 & -4 & 1\\0 & 17 & -12\\0 & 0 & \lambda+\frac{146}{17}\end{pmatrix} \tag{9}$$

Now for the determinant of this matrix to be zero, the complete row R_3 must be zero, so that

$$\implies \boxed{\lambda = \frac{-146}{17}} \tag{10}$$

Final Answer:
The value of λ is $\frac{-146}{17}$.