5.3.35

EE25BTECH11001 - Aarush Dilawri

Question:
If the pair of equations

$$3x - y + 8 = 0 \tag{0.1}$$
$$6x - ry + 16 = 0 \tag{0.2}$$

represent coincident lines, then find the value of r.

Solution:
The equation of line:

$$\mathbf{n}^\top \mathbf{x} = c \tag{0.3}$$

Line L:

$$\begin{pmatrix} 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -8 \tag{0.4}$$

Line K:

$$\begin{pmatrix} 6 & -r \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -16 \tag{0.5}$$

These can be combined and written in matrix form:

$$\begin{pmatrix} 3 & -1 \\ 6 & -r \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -8 \\ -16 \end{pmatrix} \tag{0.6}$$

The following augmented matrix can be solved by gaussian elimination

$$\left(\begin{array}{cc|c} 3 & -1 & -8 \\ 6 & -r & -16 \end{array} \right) \xleftarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{cc|c} 3 & -1 & -8 \\ 0 & -r+2 & 0 \end{array} \right) \tag{0.7}$$

Since the lines are coincident, they have infinitely many solutions.

Thus, $\operatorname{rank}(\mathbf{A}) = \operatorname{rank}([\mathbf{A} \mid b]) < n$ where n is the number of variables. (0.8)

$$\implies -r + 2 = 0 \implies r = 2 \tag{0.9}$$

Hence, the value of r is 2.

See Figure,