Matgeo Presentation - Problem 9.7.7

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September 21, 2025

Question

Find the solution of the pair of equations:
$\frac{3}{x} + \frac{8}{y} = -1, \qquad \frac{1}{x} - \frac{2}{y} = 2, \qquad x, y \neq 0.$

Solution

let

$$\frac{1}{x} = u \tag{0.1}$$

$$\frac{1}{y} = v \tag{0.2}$$

$$\implies 3u + 8v = -1 \tag{0.3}$$

$$u - 2v = 2 \tag{0.4}$$

Equations (0.3) and (0.4) cann be written as

$$\begin{pmatrix} 3 & 8 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \tag{0.5}$$

Forming the augmented matrix

$$\implies \left(\begin{array}{cc|c} 3 & 8 & -1 \\ 1 & -2 & 2 \end{array} \right) \xleftrightarrow{R_2 \to R_2 - \frac{1}{3} \times R_1} \left(\begin{array}{cc|c} 3 & 8 & -1 \\ 0 & -\frac{14}{3} & \frac{7}{3} \end{array} \right) \tag{0.6}$$

Conclusion

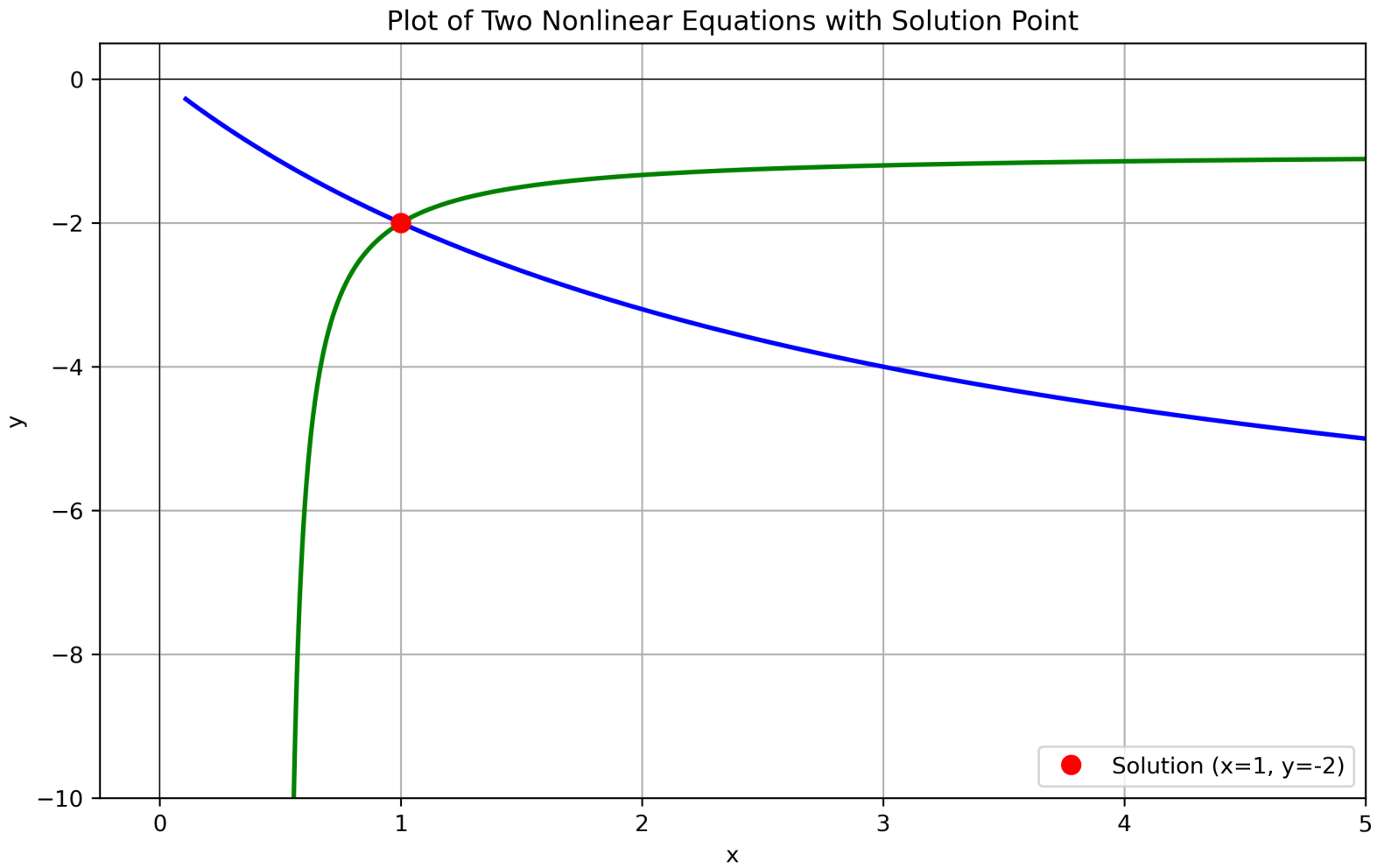
on back substitution we get

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{1}{2} \end{pmatrix} \tag{0.7}$$

From (0.1) and (0.2) we get

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \tag{0.8}$$

Plot


Plot of Two Nonlinear Equations with Solution Point
0
−2
−4
−6
−8
−10
y
0
1
2
3
4
5
x
Solution (x=1, y=-2)

Figure

C Code: area.c

```c
#include <stdio.h>

int main() {
    FILE *fp;
    double u, v, x, y;

    // Open file for writing (including status and solution)
    fp = fopen("answer.dat", "w");

    if (fp == NULL) {
        return 1; // Cannot proceed if file can't be opened
    }

    // Check for division by zero
    if (u == 0.0 || v == 0.0) {
        fprintf(fp, "Error: Division by zero encountered while computing x or y.\n");
        fclose(fp);
        return 1;
    }

    // Compute x and y
    x = 1.0 / u; // x = 1.0
    y = 1.0 / v; // y = -2.0

    // Write the solution to file
    fprintf(fp, "The solution is:\n");
    fprintf(fp, "x = %.2lf\n", x);
    fprintf(fp, "y = %.2lf\n", y);

    fclose(fp);
    return 0;
}
```

Python: plot.py

```
import numpy as np
import matplotlib.pyplot as plt

# Solution point from algebra
x_sol = 1
y_sol = -2

# Define ranges, avoiding x=0 and y=0
x_vals = np.linspace(0.1, 5, 400)
y_vals = np.linspace(-10, -0.1, 400)

# Create meshgrid
X, Y = np.meshgrid(x_vals, y_vals)

# Define the equations:
# 1. (3/x + 8/y + 1 = 0)
# 2. (1/x - 2/y - 2 = 0)
eq1 = (3 / X) + (8 / Y) + 1
eq2 = (1 / X) - (2 / Y) - 2

# Plot
plt.figure(figsize=(10, 6))

# Contour where each equation is zero
plt.contour(X, Y, eq1, levels=[0], colors='blue', linewidths=2, linestyles='solid')
plt.contour(X, Y, eq2, levels=[0], colors='green', linewidths=2, linestyles='solid')

# Plot point of intersection
plt.plot(x_sol, y_sol, 'ro', markersize=8, label=f'Solution (x={x_sol}, y={y_sol})')

# Labels and styling
plt.title('Plot of Two Nonlinear Equations with Solution Point')
plt.xlabel('x')
```

Python: plot.py

```
plt.ylabel('y')
plt.axhline(0, color='black', lw=0.5)
plt.axvline(0, color='black', lw=0.5)
plt.grid(True)
plt.legend()

# Save the plot
plt.savefig("equation_plot.png", dpi=300, bbox_inches='tight')
plt.close() # Close the figure to avoid showing it if not needed

print("Plot␣saved␣as␣'equation_plot.png'")
```