Matgeo Presentation - Problem 4.3.46

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Problem Statement

Find the coordinates of the point where the line through $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane $2x + y + z = 7$.

Data

Description	**Value**
Line	$\mathbf{x} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + k \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix}$
Plane	$\mathbf{n}^\top \mathbf{x} = 7 \quad \text{where} \quad \mathbf{n} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

Table : Line and Plane

Solution

Let the point of intersection be $\mathbf{P}$.
The line is written as

$$\mathbf{x} = \mathbf{h} + k\,\mathbf{m} \tag{0.1}$$

$$\mathbf{h} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \quad \mathbf{m} = \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} \tag{0.2}$$

So,

$$\mathbf{P} = \mathbf{h} + k\,\mathbf{m} \tag{0.3}$$

The point also lies on the plane , so

$$\mathbf{n}^\top \mathbf{P} = c \tag{0.4}$$

$$\mathbf{n}^\top (\mathbf{h} + k\mathbf{m}) = c \tag{0.5}$$

$$\mathbf{n}^\top \mathbf{h} + k\mathbf{n}^\top \mathbf{m} = c \tag{0.6}$$

$$k = \frac{c - \mathbf{n}^\top \mathbf{h}}{\mathbf{n}^\top \mathbf{m}} \tag{0.7}$$

here c $= 7$, by substituting the vectors for $\mathbf{n}, \mathbf{h}$ and $\mathbf{m}$

$$k = \frac{7 - \begin{pmatrix} 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}}{\begin{pmatrix} 2 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix}} \tag{0.8}$$

$$k = \frac{7 - 2}{5} = 1 \tag{0.9}$$

Substitute $k = 1$ in the line equation to obtain the point of intersection $\mathbf{P}$

$$\mathbf{P} = \mathbf{h} + \mathbf{m} \tag{0.10}$$

$$\mathbf{P} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ 6 \end{pmatrix} = \begin{pmatrix} 2-1 \\ -3+1 \\ 1+6 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix}. \tag{0.11}$$

Answer:

$$\mathbf{P} = \begin{pmatrix} 1 \\ -2 \\ 7 \end{pmatrix} \tag{0.12}$$

Plot

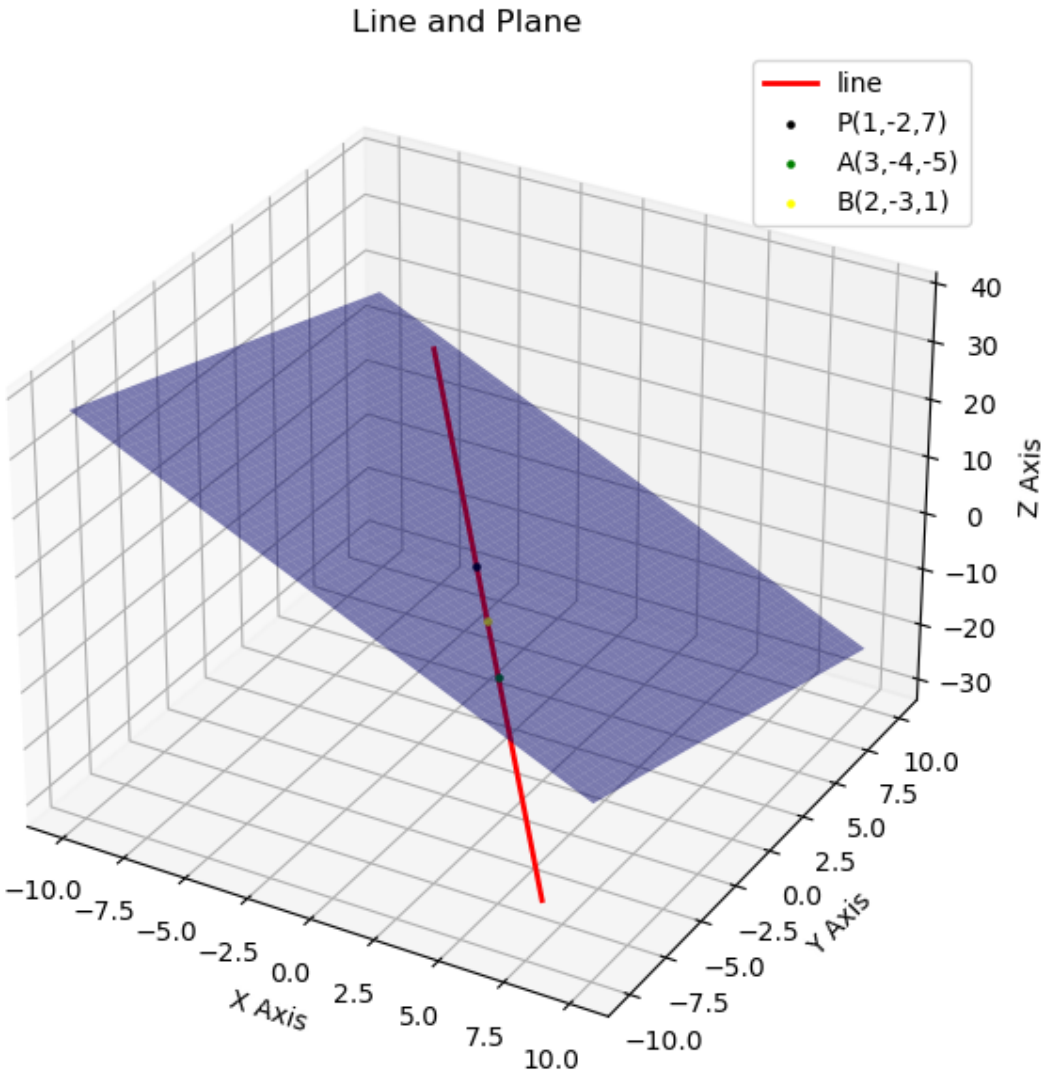

Fig : Line and Plane

C Code: points.c

```c
#include <stdio.h>

double intersection() {

  double k, final = -1;

  double a[3] = {2, 1, 1};

  double dot;

  for (k = -100; k <= 100; k++) {

    double x[3] = {2 - k, -3 + k, 1 + 6 * k};

    dot = a[0] * x[0] + a[1] * x[1] + a[2] * x[2];

    if (dot == 7) {
      final = k;
      break;
    }
  }

  return final;
}
```

Python: call_c.py

```
import sys
import ctypes
import numpy as np
import matplotlib.pyplot as plt
import os

#for generating figure in figs folder
figs_folder= os.path.join("..","figs")


#loading shared object , load the file into lib which is an objedt

lib = ctypes.CDLL("./points.so") #ctypes constructor to load a shared c library
lib.intersection.restype = ctypes.c_double #to tell the return type is a c double
lib.intersection.argtypes=[] # to tell that function takes no arguments

#call the c function , by using attributes for lib
k_val = lib.intersection()

if (k_val == -1) :
    print("no solution found")
    sys.exit(0)

print(f"solution found k = {k_val}")

#writing points in the form of array for line
```

Python: call_c.py

```
#parametric form of the given line
t = np.linspace(-5,5,200)

p = 2 + (-1)*t
q = (-3) + t
r = 1 + 6*t

#for plane

#coefficients for plane equation
a,b,c,d=2,1,1,7


l = np.linspace(-10,10,100)
m = np.linspace(-10,10,100)
l,m = np.meshgrid(l,m)

n = (d - a*l -b*m)/c

#plot

fig = plt.figure(figsize=(8,6))

ax = fig.add_subplot(111,projection="3d")

#line
ax.plot(p,q,r,label='line',color='red',linewidth=2)

ax.scatter(1,-2,7,color='black',s=5,label='I(1,-2,7)')
ax.scatter(3,-4,-5,color='green',s=5,label='A(3,-4,-5)')
ax.scatter(2,-3,1,color='yellow',s=5,label='B(2,-3,1)')
```

Python: call_c.py

```
#plane
ax.plot_surface(l,m,n,alpha=0.5,color='blue',edgecolor='none') #edgecolor='none' means edges are not drawn
      for the mesh and hence surface appears smooth without grid lines.

ax.set_xlabel("X Axis")
ax.set_ylabel("Y Axis")
ax.set_zlabel("Z Axis")
ax.set_title("Line and Plane")
ax.grid(True)
ax.legend()

plt.tight_layout()
fig.savefig(os.path.join(figs_folder,"intersection.png")) #savingfig using figure object
plt.show()
```

Python: plot.py

```
import numpy as np
import matplotlib.pyplot as plt
import os

#for generating figure in figs folder
figs_folder= os.path.join("..","figs")

# fixed value of k
k_val = 1

# line points
#parametric form of the given line
t = np.linspace(-5,5,200)

p = 2 + (-1)*t
q = (-3) + t
r = 1 + 6*t

# plane coefficients: 2x + y + z = 7
a, b, c, d = 2, 1, 1, 7

l = np.linspace(-10, 10, 100)
m = np.linspace(-10, 10, 100)
l, m = np.meshgrid(l, m)

n = (d - a * l - b * m) / c

# plot
fig = plt.figure(figsize=(8, 6))
ax = fig.add_subplot(111, projection="3d")
```

Python: plot.py

```
# line
ax.plot(p, q, r, label="line", color="red",linewidth=2)

# points
ax.scatter(1, -2, 7, color="black", s=5, label="I(1,-2,7)")
ax.scatter(3, -4, -5, color="green", s=5, label="A(3,-4,-5)")
ax.scatter(2, -3, 1, color="yellow", s=5, label="B(2,-3,1)")

# plane
ax.plot_surface(l, m, n, alpha=0.5, color="blue", edgecolor="none")

ax.set_xlabel("X␣Axis")
ax.set_ylabel("Y␣Axis")
ax.set_zlabel("Z␣Axis")
ax.set_title("Line␣and␣Plane")
ax.grid(True)
ax.legend()

plt.tight_layout()
fig.savefig(os.path.join(figs_folder,"intersection.png"))
plt.show()
```