Problem 9.4.36

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Problem

The sum of the reciprocals of Ram's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age

Input

Variable	Description
x	Present Age of Ram
$x-3$	Age of Ram 3 years ago
$x+5$	Age of Ram 5 years from now

Equation

Given sum of reciprocal of Ram's ages 3 years ago and 5 years from now is $\frac{1}{3}$

$$\frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3} \tag{1.1}$$

$$\frac{(x+5)+(x-3)}{(x-3)(x+5)}=\frac{1}{3} \tag{1.2}$$

$$\frac{2x+2}{x^2+5x-3x-15}=\frac{1}{3} \tag{1.3}$$

$$\frac{2x+2}{x^2+2x-15}=\frac{1}{3} \tag{1.4}$$

$$(2x+2)=x^2+2x-15 \tag{1.5}$$

$$6x+6=x^2+2x-15 \tag{1.6}$$

$$x^2+2x-15-6x-6=0 \tag{1.7}$$

$$x^2-4x-21=0 \tag{1.8}$$

$$\implies y=x^2-4x-21 \tag{1.9}$$

Equation

$$\implies x^2 - 4x - y - 21 = 0 \tag{1.10}$$

$$x^2 + 2(-2x - \frac{1}{2}y) - 21 = 0 \tag{1.11}$$

which can be expressed as the conic

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{1.12}$$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -2 \\ -\frac{1}{2} \end{pmatrix}, f = -21 \tag{1.13}$$

To find the roots of (9), we find the points of intersection of the conic with the x-axis

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \tag{1.14}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.15}$$

Formula

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{\{\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})\}^2 - g(\mathbf{h}) (\mathbf{m}^\top \mathbf{V}\mathbf{m})} \right) \tag{1.16}$$

where

$$g(\mathbf{h}) = \mathbf{h}^\top \mathbf{V}\mathbf{h} + 2\mathbf{u}^\top \mathbf{h} + f \tag{1.17}$$

$$g(\mathbf{h}) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} -2 \\ -\frac{1}{2} \end{pmatrix}^\top \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 21 \tag{1.18}$$

$$g(\mathbf{h}) = \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2 \begin{pmatrix} -2 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 21 \tag{1.19}$$

$$g(\mathbf{h}) = \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + 2(0) - 21 \tag{1.20}$$

$$g(\mathbf{h}) = 0 + 0 - 21 = -21 \tag{1.21}$$

Finding the solutions

$$\mathbf{m}^\top \mathbf{V}\mathbf{m} = \begin{pmatrix}1\\0\end{pmatrix}^\top \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = 1 \tag{1.22}$$

$$\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) = \begin{pmatrix}1\\0\end{pmatrix}^\top \left(\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\begin{pmatrix}0\\0\end{pmatrix} + \begin{pmatrix}-2\\-\frac{1}{2}\end{pmatrix}\right)$$

$$= \begin{pmatrix}1 & 0\end{pmatrix}\left(\begin{pmatrix}0\\0\end{pmatrix} + \begin{pmatrix}-2\\-\frac{1}{2}\end{pmatrix}\right) = \begin{pmatrix}1 & 0\end{pmatrix}\begin{pmatrix}-2\\-\frac{1}{2}\end{pmatrix} = -2 \tag{1.23}$$

From equation (1.14)

$$\kappa_i = \frac{1}{1}\left(-(-2) \pm \sqrt{(-2)^2 + 21}\right) \tag{1.24}$$

$$= 2 \pm \sqrt{25} = 2 \pm 5 \tag{1.25}$$

$$= 7, -3 \tag{1.26}$$

Conclusion

Hence the points of intersection are

$$\mathbf{h} + k\mathbf{m} = \begin{pmatrix} 7 \\ 0 \end{pmatrix}, \begin{pmatrix} -3 \\ 0 \end{pmatrix} \tag{1.27}$$

Hence the solutions are x = -3 and x = 7. We reject x = -3 as the Age cannot be negative. Hence, the present age of Ram willl be 7 years

Plot

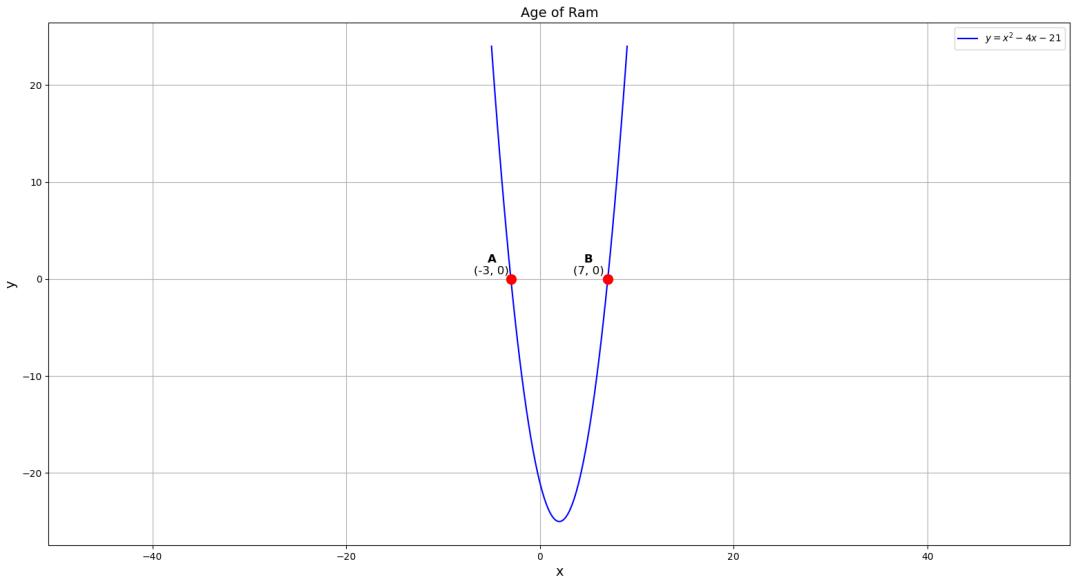

Figure:

C Code

```c
#include <math.h>
void get_parabola_data(double* out_data) {
      double a = 1.0,b = -4.0,c = -21.0;
        double discriminant = sqrt(b*b - 4*a*c);
    double root1 = (-b + discriminant) / (2 * a);
    double root2 = (-b - discriminant) / (2 * a);
    out_data[0] = root1;
    out_data[1] = root2;
    int num_points = 101;
    out_data[2] = (double)num_points;
    int index = 3;
    for (int i = 0; i < num_points; i++) {
        double x = -5.0 + (14.0 * i) / (num_points - 1);
        double y = a*x*x + b*x + c;
        out_data[index] = x;
        out_data[index + 1] = y;
        index += 2;
    }
}
```

Python Code for Solving

```
import ctypes
import numpy as np

def get_data_from_c():

    lib = ctypes.CDLL('./code.so')

    data_size = 3 + 101 * 2
    double_array = ctypes.c_double * data_size
    lib.get_parabola_data.argtypes = [ctypes.POINTER(ctypes.
        c_double)]

    out_data_c = double_array()
    lib.get_parabola_data(out_data_c)

    return np.array(out_data_c)
```

Python Code for Plotting

```
# Code by /sdcard/github/matgeo/codes/CoordGeoVV Sharma
# September 12, 2023
# Revised July 21, 2024
# Released under GNU GPL
# Section Formula
import sys
sys.path.insert(0, '/workspaces/urban-potato/matgeo/codes/
    CoordGeo/')
import numpy as np
import matplotlib.pyplot as plt

from call import get_data_from_c
all_data = get_data_from_c()
num_points = int(all_data[2])
roots = all_data[0:2]
parabola_points = all_data[3:].reshape((num_points, 2))

positive_root = max(roots)
fig, ax = plt.subplots(figsize=(8, 8))
```

Python Code for Plotting

```
ax.plot(parabola_points[:, 0], parabola_points[:, 1], 'b-', label
    ='$y = x^2 - 4x - 21$')
ax.scatter(roots, [0, 0], color='red', s=100, zorder=5)
pointA = np.array([min(roots), 0])
pointB = np.array([max(roots), 0])

label_A = f$\\mathbf{{A}}$\n({pointA[0]:.0f}, {pointA[1]:.0f})
ax.annotate(label_A,
            xy=pointA,
            xytext=(-20, 5),
            textcoords='offset points',
            ha='center',
            fontsize=12)
label_B = f$\\mathbf{{B}}$\n({pointB[0]:.0f}, {pointB[1]:.0f})
ax.annotate(label_B,
            xy=pointB,
            xytext=(-20, 5),
            textcoords='offset points',
            ha='center',
```

Python Code for Plotting

```
 fontsize=12)
 x.set_title(Age of Ram,fontsize=12)
ax.set_xlabel(x,fontsize=12)
ax.set_ylabel('y'.fontsize=12)

ax.grid(True)
ax.axis('equal')
ax.legend(loc='best')
plt.show()
```