4.11.18

EE25BTECH11041 - Naman Kumar

Question:
Find the equation of the plane which contains the line of intersection of the planes $\mathbf{r} \cdot (\imath - 2\jmath + 3\hat{k}) - 4 = 0$ and $\mathbf{r} \cdot (-2\imath + \jmath + \hat{k}) + 5 = 0$ and whose intercept on X axis is equal to that of on Y axis.

Solution:
Given Planes,

$$n_1^T x = c_1, n_2^T x = c_2 \tag{1}$$

Where

$$n_1 = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, n_2 = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, c_1 = 4, c_2 = -5 \tag{2}$$

Let Required equation of plane

$$n_3^T x = c_3 \tag{3}$$

Since we can write,

$$P_3 = P_1 - \lambda P_2 \text{ (Where } P_1, P_2, P_3 \text{ are equation of planes)} \tag{4}$$

Because All three planes intersect at same line,Therefore

$$(n_1 - \lambda n_2)^T x = c_1 - \lambda c_2 \tag{5}$$

$$\tag{6}$$

Given,

$$X - intercept = Y - intercept \tag{7}$$

$$\tag{8}$$

for X-intercept

$$(n_1 - \lambda n_2)^T \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} = c_1 - \lambda c_2 \tag{9}$$

$$(n_1 - \lambda n_2)^T x e_1 = c_1 - \lambda c_2 \tag{10}$$

Therefore,

$$X - intercept = \frac{c_1 - \lambda c_2}{(n_1 - \lambda n_2)^T e_1} \tag{11}$$

Similarly

$$Y - intercept = \frac{c_1 - \lambda c_2}{(n_1 - \lambda n_2)^T e_2} \tag{12}$$

Comparing equations (11) and (12)

$$\frac{c_1 - \lambda c_2}{(n_1 - \lambda n_2)^T e_1} = \frac{c_1 - \lambda c_2}{(n_1 - \lambda n_2)^T e_2} \tag{13}$$

$$(n_1 - \lambda n_2)^T e_1 = (n_1 - \lambda n_2)^T e_2 \tag{14}$$

$$\begin{pmatrix} 1 + 2\lambda \\ -2 - 1\lambda \\ 3 - 1\lambda \end{pmatrix}^T e_1 = \begin{pmatrix} 1 + 2\lambda \\ -2 - 1\lambda \\ 3 - 1\lambda \end{pmatrix}^T e_2 \tag{15}$$

$$1 + 2\lambda = -2 - 1\lambda \tag{16}$$

$$\lambda = -1 \tag{17}$$

Therefore equation of required plane is

$$\begin{pmatrix} 1 + 2(-1) \\ -2 - 1(-1) \\ 3 - 1(-1) \end{pmatrix}^T x = 4 + 5(-1) \tag{18}$$

$$\begin{pmatrix} -1 \\ -1 \\ 4 \end{pmatrix}^T x = -1 \tag{19}$$

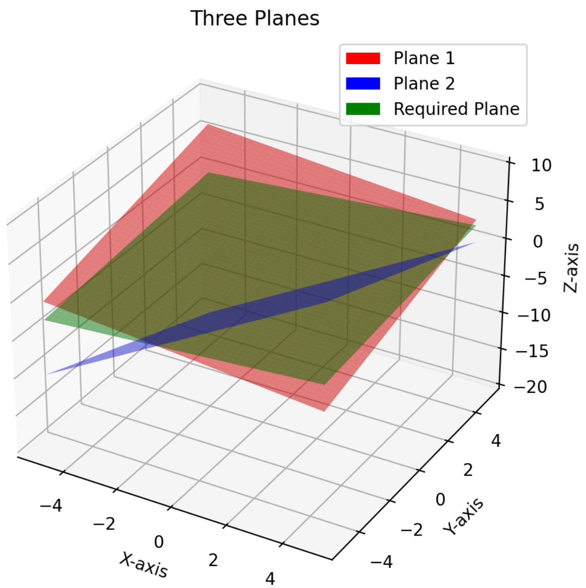
Three Planes
Plane 1
Plane 2
Required Plane
X-axis
Y-axis
Z-axis
−4
−2
0
2
4
10
5
0
−5
−10
−15
−20

Fig. 1