4.7.13

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Question

Find the distance between the lines l_1 and l_2 given by

$$\overrightarrow{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$
$$\overrightarrow{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Equation

Given equation:

$$\overrightarrow{\mathbf{r}} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k}) \tag{1}$$

$$\overrightarrow{\mathbf{r}} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k}) \tag{2}$$

$$\tag{3}$$

The given lines are in the form

$$\mathbf{r} = \mathbf{a_1} + \lambda \mathbf{b} \tag{4}$$

$$\mathbf{r} = \mathbf{a_2} + \mu \mathbf{b} \tag{5}$$

Where,

$$\mathbf{a_1} = \begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} \quad \mathbf{a_2} = \begin{pmatrix} 3 \\ 3 \\ -5 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \tag{6}$$

The given two lines are parallel.The distance between two parallel lines is given by:

$$d = \frac{\left\|(\mathbf{a}_2 - \mathbf{a}_1) \times \mathbf{b}\right\|}{\|\mathbf{b}\|} \tag{7}$$

$$\mathbf{a_2} - \mathbf{a_1} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \quad and \quad \mathbf{b} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \tag{8}$$

Theoretical solution

Let,

$$\mathbf{a_2} - \mathbf{a_1} = \mathbf{a} \tag{9}$$

Now finding:

$$(\mathbf{a}_2 - \mathbf{a}_1) \times \mathbf{b} = \mathbf{a} \times \mathbf{b} \tag{10}$$

$$\begin{vmatrix} \mathbf{A_{23}} & \mathbf{B_{23}} \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ -1 & 6 \end{vmatrix} = 9 \tag{11}$$

$$\begin{vmatrix} \mathbf{A_{31}} & \mathbf{B_{31}} \end{vmatrix} = \begin{vmatrix} 2 & 2 \\ -1 & 6 \end{vmatrix} = 14 \tag{12}$$

$$\begin{vmatrix} \mathbf{A_{12}} & \mathbf{B_{12}} \end{vmatrix} = \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} = 4 \tag{13}$$

$$\|\mathbf{a} \times \mathbf{b}\| = \left\| \begin{pmatrix} \begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \end{vmatrix} \\ \\ \begin{vmatrix} \mathbf{A}_{31} & \mathbf{B}_{31} \end{vmatrix} \\ \\ \begin{vmatrix} \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} \end{pmatrix} \right\| \tag{14}$$

$$\|\mathbf{a} \times \mathbf{b}\| = \left\| \begin{pmatrix} 9 \\ 14 \\ 4 \end{pmatrix} \right\| \tag{15}$$

$$\|\mathbf{a} \times \mathbf{b}\| = \sqrt{293} \tag{16}$$

$$\|(\mathbf{a}_2 - \mathbf{a}_1) \times \mathbf{b}\| = \sqrt{293} \tag{17}$$

Theoretical Solution

$$\|\mathbf{b}\| = \sqrt{\mathbf{b}^{\mathsf{T}}\mathbf{b}} \tag{18}$$

$$\|\mathbf{b}\| = \sqrt{4+9+36} = \sqrt{49} \tag{19}$$

$$\|\mathbf{b}\| = 7 \tag{20}$$

Substituting the values in Eq.7:

$$d = \frac{\sqrt{293}}{7} \tag{21}$$

Therefore the distance between the lines l_1 and l_2 is $\frac{\sqrt{293}}{7}$

C Code

```c
#include <stdio.h>
#include <math.h>

int main() {
    double A[3] = {1,2,-4};
    double B[3] = {3,3,-5};
    double d[3] = {2,3,6};
    double AB[3], cross[3];
    double dist;

    // Compute B - A
    for(int i=0;i<3;i++) AB[i] = B[i] - A[i];

    // Cross product (AB x d)
    cross[0] = AB[1]*d[2] - AB[2]*d[1];
    cross[1] = AB[2]*d[0] - AB[0]*d[2];
    cross[2] = AB[0]*d[1] - AB[1]*d[0];
```

C Code

```
    // Norms
    double num = sqrt(cross[0]*cross[0] + cross[1]*cross[1] +
        cross[2]*cross[2]);
    double den = sqrt(d[0]*d[0] + d[1]*d[1] + d[2]*d[2]);

    dist = num / den;
    printf(Distance between lines = %lf\n, dist);

    // Output A, B, direction vector for plotting
    printf(A: %lf %lf %lf\n, A[0], A[1], A[2]);
    printf(B: %lf %lf %lf\n, B[0], B[1], B[2]);
    printf(d: %lf %lf %lf\n, d[0], d[1], d[2]);

    return 0;
}
```

Python Code

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d.art3d import Poly3DCollection

# Load shared C library
lib = ctypes.CDLL(./distance.so)
lib.main()

# Define points and direction vector (same as in C)
A = np.array([1,2,-4])
B = np.array([3,3,-5])
d = np.array([2,3,6])

# Generate points for both lines
t = np.linspace(-2,2,10)
line1 = A[:,None] + d[:,None]*t
line2 = B[:,None] + d[:,None]*t
```

Python Code

```
# Plot
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Lines
ax.plot(line1[0], line1[1], line1[2], 'b-', label='Line l1')
ax.plot(line2[0], line2[1], line2[2], 'g-', label='Line l2')

# Points
ax.scatter(*A,color='r',s=50)
ax.text(*A,A(1,2,-4),color='red')
ax.scatter(*B,color='orange',s=50)
ax.text(*B,B(3,3,-5),color='orange')

# Dotted line AB
ax.plot([A[0],B[0]], [A[1],B[1]], [A[2],B[2]], 'k--', label='
    Connecting AB')
```

Python Code

```
ax.set_title(Figure)
ax.set_xlabel(X-axis)
ax.set_ylabel(Y-axis)
ax.set_zlabel(Z-axis)
ax.legend()
plt.savefig(/media/indhiresh-s/New Volume/Matrix/ee1030-2025/
    ee25btech11027/MATGEO/4.7.13/figs/figure1.png)
plt.show()
```

Plot

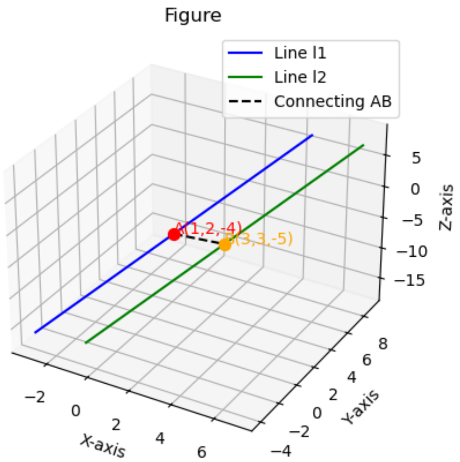

Figure
Line l1
Line l2
Connecting AB
A(1,2,-4)
B(3,3,-5)
X-axis
Y-axis
Z-axis
5
0
−5
−10
−15
−2
0
2
4
6
8
−4