

2.2.23

AI25BTECH110030 - SARVESH TAMGADE

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Question:

Find angle θ between the vectors $\mathbf{a} = \hat{i} + \hat{j} - \hat{k}$ and $\mathbf{b} = \hat{i} - \hat{j} + \hat{k}$.

Solution:

Express vectors in column form:

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

The cosine of the angle θ is given by:

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|}$$

Compute dot product:

$$\mathbf{a} \cdot \mathbf{b} = (1)(1) + (1)(-1) + (-1)(1) = 1 - 1 - 1 = -1$$

Compute magnitudes:

$$\|\mathbf{a}\| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$$

$$\|\mathbf{b}\| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$$

Substitute:

$$\cos \theta = \frac{-1}{\sqrt{3} \sqrt{3}} = -\frac{1}{3}$$

$$\theta = \cos^{-1} \left(-\frac{1}{3} \right)$$

Answer:

The required angle is:

$$\theta = \cos^{-1} \left(-\frac{1}{3} \right)$$

Graph:

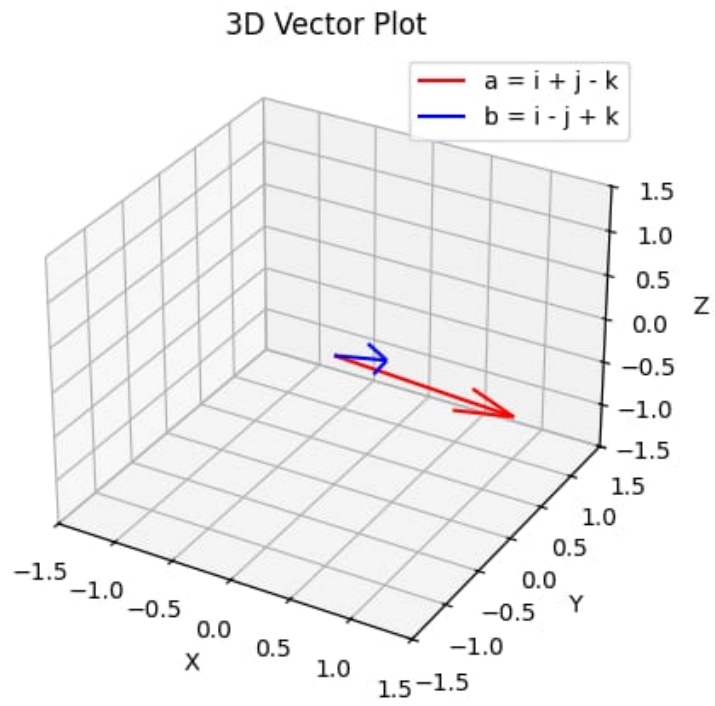


Figure 1: 3D Visualisation of two vectors and angle between them