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Question

Find the equation of the circle passing through $(0, 0)$ and making intercepts a and b on the coordinate axes.

Solution

Let:

$$\mathbf{x_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \ \mathbf{x_2} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \ \mathbf{x_3} = \begin{pmatrix} 0 \\ b \end{pmatrix} \tag{1}$$

We use the general matrix form of a circle:

$$\begin{pmatrix} 2x_1 & 2x_2 & 2x_3 \\ 1 & 1 & 1 \end{pmatrix}^T \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = -\begin{pmatrix} \|x_1\|^2 \\ \|x_2\|^2 \\ \|x_3\|^2 \end{pmatrix} \tag{2}$$

$$\begin{pmatrix} 2x_1^T & 1 \\ 2x_2^T & 1 \\ 2x_3^T & 1 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ f \end{pmatrix} = -\begin{pmatrix} \|x_1\|^2 \\ \|x_2\|^2 \\ \|x_3\|^2 \end{pmatrix} \tag{3}$$

Substituting the values:

$$\begin{pmatrix} 0 & 0 & 1 \\ 2a & 0 & 1 \\ 0 & 2b & 1 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ f \end{pmatrix} = -\begin{pmatrix} 0 \\ a^2 \\ b^2 \end{pmatrix} \tag{4}$$

Solving the system:

$$f = 0$$
$$2au_1 + f = -a^2 \Rightarrow u_1 = -\frac{a}{2}$$
$$2bu_2 + f = -b^2 \Rightarrow u_2 = -\frac{b}{2}$$

Final Result

$$\mathbf{u} = \begin{pmatrix} -\frac{a}{2} \\ -\frac{b}{2} \end{pmatrix}, \quad f = 0 \tag{5}$$

So the equation of the circle becomes:

$$x^2 + y^2 + ax + by = 0 \tag{6}$$

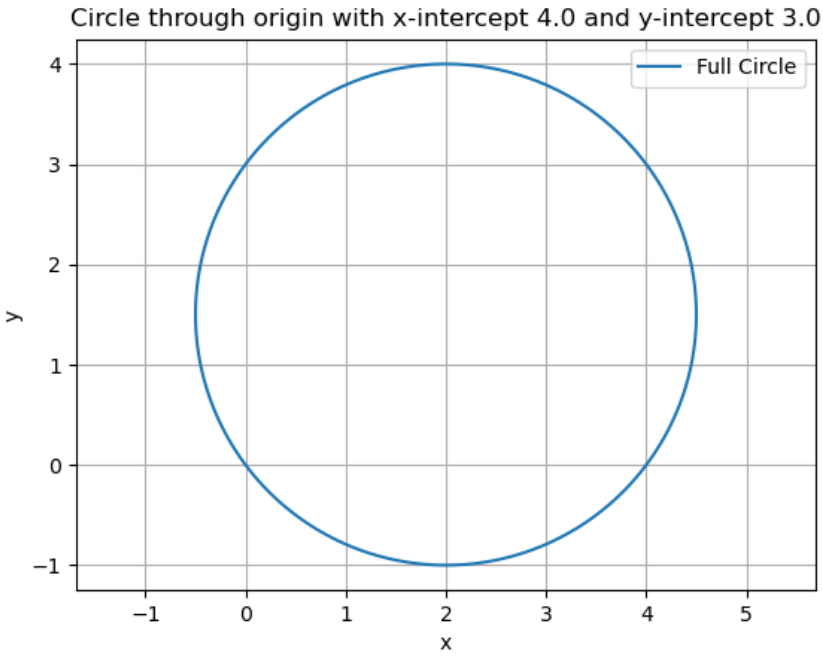
Circle through origin with x-intercept 4.0 and y-intercept 3.0
Full Circle
4
3
2
1
0
−1
y
−1
0
1
2
3
4
5
x

Figure 1