

Matgeo : 2.6.12

AI25BTECH11006 - Nikhila

Problem: Find the sine of the angle between the vectors $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + -2\hat{j} + 4\hat{k}$

Solution:

We know that

$$\langle \mathbf{a}, \mathbf{b} \rangle = \|\mathbf{a}\| \|\mathbf{b}\| \cos \theta,$$

where θ is the angle between \mathbf{a} and \mathbf{b} .

Thus,

$$\cos \theta = \frac{\langle \mathbf{a}, \mathbf{b} \rangle}{\|\mathbf{a}\| \|\mathbf{b}\|}.$$

Calculating each term:

$$\langle \mathbf{a}, \mathbf{b} \rangle = 3(2) + 1(-2) + 2(4) = 6 - 2 + 8 = 12, \quad (1)$$

$$\|\mathbf{a}\| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{14}, \quad (2)$$

$$\|\mathbf{b}\| = \sqrt{2^2 + (-2)^2 + 4^2} = \sqrt{24}, \quad (3)$$

$$\cos \theta = \frac{12}{\sqrt{14} \cdot \sqrt{24}} = \frac{12}{\sqrt{336}} = \frac{3}{\sqrt{21}}. \quad (4)$$

Now, to find $\sin \theta$:

$$\sin \theta = \sqrt{1 - \cos^2 \theta} \quad (5)$$

$$= \sqrt{1 - \left(\frac{3}{\sqrt{21}} \right)^2} \quad (6)$$

$$= \sqrt{1 - \frac{9}{21}} \quad (7)$$

$$= \sqrt{\frac{12}{21}} \quad (8)$$

$$= \frac{2}{\sqrt{7}}. \quad (9)$$

The sine of the angle between the vectors \mathbf{a} and \mathbf{b} is $\frac{2}{\sqrt{7}}$.

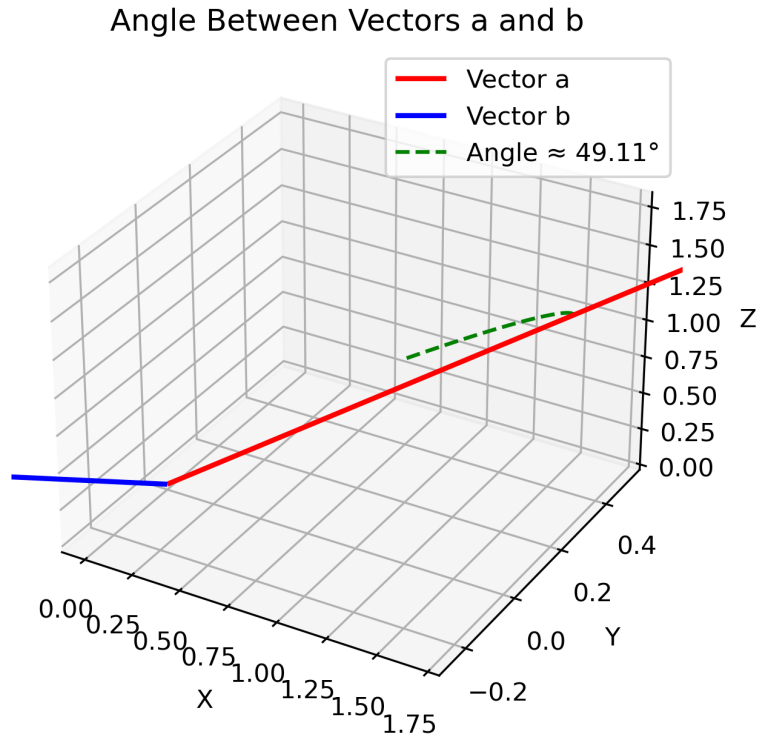


Fig. 1.