

# 1.5.32

EE25BTECH11045 - P.Navya Priya

Find the ratio in which the line segment joining the points  $(1, -3)$  and  $(4, 5)$  is divided by X axis.

**Solution:** Let  $\mathbf{A} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$  and  $\mathbf{C} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

Consider a point  $\mathbf{B} = \begin{pmatrix} x \\ 0 \end{pmatrix}$  on the X-axis. As the points  $\mathbf{A}, \mathbf{B}, \mathbf{C}$  are collinear The matrix  $(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^\top$  has rank 1.

$$(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^\top = \begin{pmatrix} 3 & x-1 \\ 8 & 3 \end{pmatrix}^\top \quad (1)$$

$$(\mathbf{B} - \mathbf{A} \quad \mathbf{C} - \mathbf{A})^\top = \begin{pmatrix} 3 & 8 \\ x-1 & 3 \end{pmatrix} \quad (2)$$

$$\begin{pmatrix} 3 & 8 \\ x-1 & 3 \end{pmatrix} \xrightarrow{R_2=8R_2-3R_1} \begin{pmatrix} 3 & 8 \\ 8(x-1)-9 & 0 \end{pmatrix} \quad (3)$$

$$\begin{pmatrix} 3 & 8 \\ 8(x-1)-9 & 0 \end{pmatrix} \xrightarrow{R_1 \rightarrow \frac{R_1}{3}} \begin{pmatrix} 1 & \frac{8}{3} \\ 8(x-1)-9 & 0 \end{pmatrix} \quad (4)$$

The matrix is in Row Reduced Echelon Form(RREF). To satisfy collinearity condition, the rank of matrix should be 1. Hence,

$$8(x-1)-9=0 \quad (5)$$

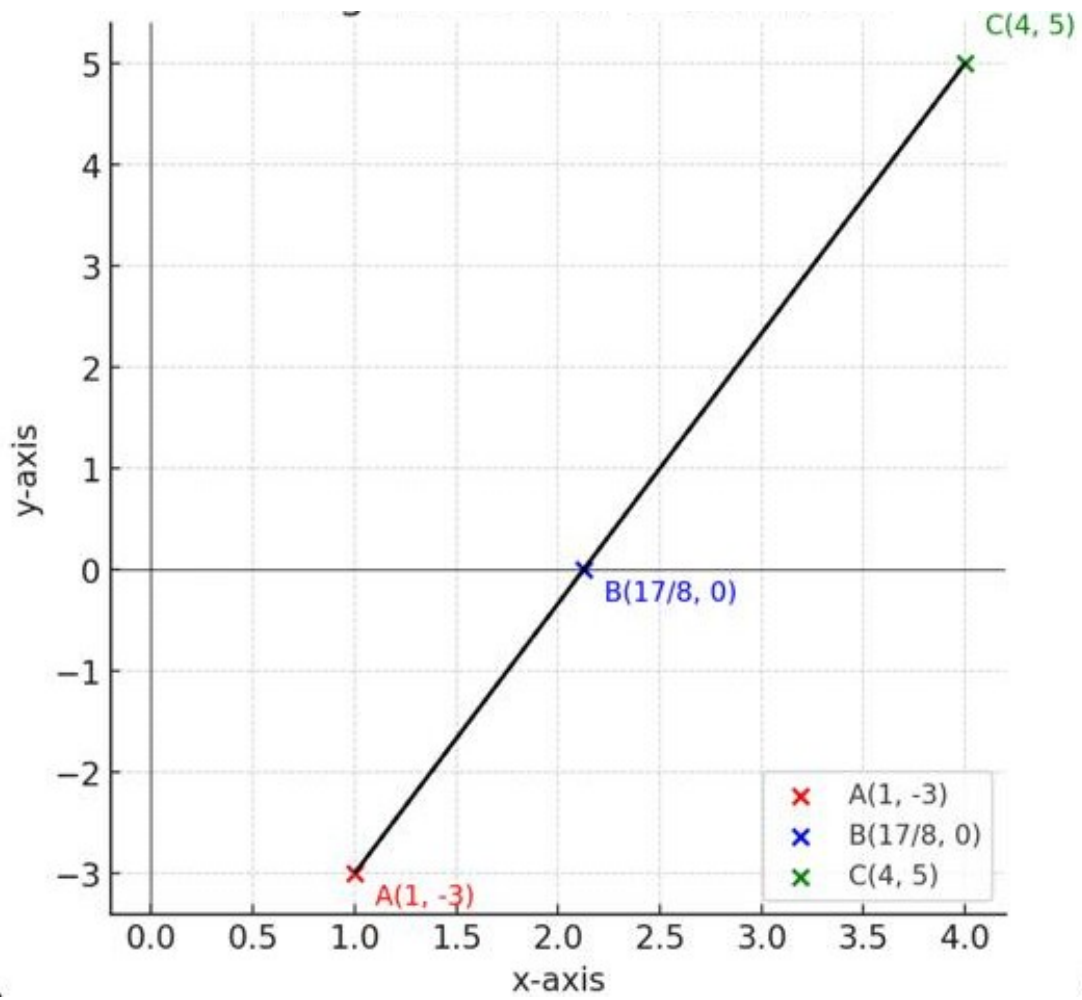
Assume the ratio  $\mathbf{B}$  divides  $\mathbf{A}$  and  $\mathbf{C}$  be k:1

$$k = \frac{(\mathbf{A} - \mathbf{B})^\top (\mathbf{B} - \mathbf{C})}{\|(\mathbf{B} - \mathbf{C})\|^2} \quad (6)$$

$$k = \frac{1095}{1825} \quad (7)$$

$$k = \frac{3}{5} \quad (8)$$

Hence the ratio is 3:5.



Plot of Intersection of AB by X-axis