Question 4.4.36

AI25BTECH11040 - Vivaan Parashar

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Question:

The area of the triangle formed by the lines $\frac{x}{a} + \frac{y}{b} = 1$ and the coordinate axes is ____________.

Solution:

Let the origin be $\mathbf{O}$, the x-intercept be $\mathbf{A}$, and the y-intercept be $\mathbf{B}$. We then need the area of triangle OAB. The x-intercept is some multiple of the basis vector $\mathbf{e_1}$, and the y-intercept is some multiple of the basis vector $\mathbf{e_2}$. Thus,

$$\therefore \mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{A} = \lambda \mathbf{e_1}, \quad \mathbf{B} = \mu \mathbf{e_2} \tag{1}$$

The equation of the line can be written as

$$\mathbf{n}^{\mathrm{T}} \mathbf{x} = 1 \tag{2}$$

Where $\mathbf{n} = \begin{pmatrix} \frac{1}{a} \\ \frac{1}{b} \end{pmatrix}$, and $\mathbf{x}$ is any point on the line. By putting the points $\mathbf{A}$ and $\mathbf{B}$ in the equation of the line, we get

$$\lambda \mathbf{n}^{\mathrm{T}} \mathbf{e_1} = 1, \quad \mu \mathbf{n}^{\mathrm{T}} \mathbf{e_2} = 1 \tag{3}$$

$$\implies \lambda = \frac{1}{\mathbf{n}^{\mathrm{T}} \mathbf{e_1}} = a, \quad \mu = \frac{1}{\mathbf{n}^{\mathrm{T}} \mathbf{e_2}} = b \tag{4}$$

Now,

$$\Delta \mathrm{OAB} = \frac{1}{2} \left| \mathbf{A} \times \mathbf{B} \right| \tag{5}$$

$$\implies \Delta \mathrm{OAB} = \frac{1}{2} \lambda \mu \left| \mathbf{e_1} \times \mathbf{e_2} \right| \tag{6}$$

$$\therefore \Delta \mathrm{OAB} = \left| \frac{ab}{2} \right| \tag{7}$$

Plot:

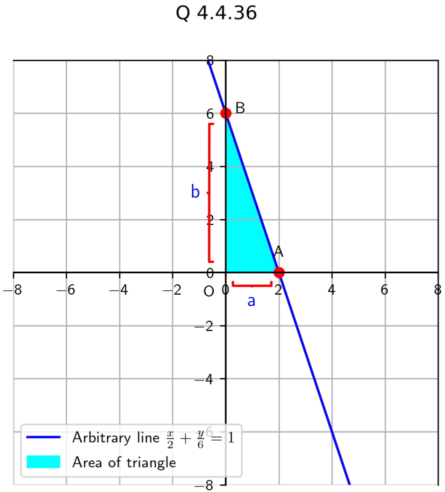

Figure: Graph of line and triangle formed by intercepts with axes