

5.7.15

EE25BTECH11002 - Achat Parth Kalpesh

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Question

If

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} \quad (1)$$

and $\mathbf{A}^3 - 6\mathbf{A}^2 + 7\mathbf{A} + k\mathbf{I} = 0$ find k .

Solution

The characteristic equation for the matrix **A**

$$f(\lambda) = |\mathbf{A} - \lambda \mathbf{I}| \quad (2)$$

From (??) the characteristic equation is

$$\begin{vmatrix} 1 - \lambda & 0 & 2 \\ 0 & 2 - \lambda & 1 \\ 2 & 0 & 3 - \lambda \end{vmatrix} = 0 \quad (3)$$

Solution

which can be expanded to obtain

$$\lambda^3 - 6\lambda^2 + 7\lambda + 2 = 0 \quad (4)$$

Upon simplification, by using Cayley-Hamilton theorem,

$$\mathbf{A}^3 - 6\mathbf{A}^2 + 7\mathbf{A} + 2\mathbf{I} = 0 \quad (5)$$

Thereby, on comparing (??) with

$$\mathbf{A}^3 - 6\mathbf{A}^2 + 7\mathbf{A} + k\mathbf{I} = 0 \quad (6)$$

$$k = 2 \quad (7)$$