5.4.36

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Question:
Using elementary transformations, find the inverse of the following matrix.

$$\begin{pmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{pmatrix}$$

Solution:
Let us solve the given question theoretically and then verify the solution computationally.

To solve for the inverse of a matrix, we can employ the Gauss-Jordan approach.

$$\left(\begin{array}{ccc|ccc} 2 & -1 & -2 & 1 & 0 & 0 \\ 0 & 2 & -1 & 0 & 1 & 0 \\ 3 & -5 & 0 & 0 & 0 & 1 \end{array}\right) \xleftrightarrow{R_1 \leftarrow \frac{1}{2}R_1} \left(\begin{array}{ccc|ccc} 1 & -1/2 & -1 & 1/2 & 0 & 0 \\ 0 & 2 & -1 & 0 & 1 & 0 \\ 3 & -5 & 0 & 0 & 0 & 1 \end{array}\right) \tag{0.1}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - 3R_1} \left(\begin{array}{ccc|ccc} 1 & -1/2 & -1 & 1/2 & 0 & 0 \\ 0 & 2 & -1 & 0 & 1 & 0 \\ 0 & -7/2 & 3 & -3/2 & 0 & 1 \end{array}\right) \tag{0.2}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{1}{2}R_2} \left(\begin{array}{ccc|ccc} 1 & -1/2 & -1 & 1/2 & 0 & 0 \\ 0 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & -7/2 & 3 & -3/2 & 0 & 1 \end{array}\right) \tag{0.3}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 + \frac{7}{2}R_2]{R_1 \leftarrow R_1 + \frac{1}{2}R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & -5/4 & 1/2 & 1/4 & 0 \\ 0 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 5/4 & -3/2 & 7/4 & 1 \end{array}\right) \tag{0.4}$$

$$\xleftrightarrow{R_3 \leftarrow \frac{4}{5}R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & -5/4 & 1/2 & 1/4 & 0 \\ 0 & 1 & -1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & -6/5 & 7/5 & 4/5 \end{array}\right) \tag{0.5}$$

$$\xleftrightarrow[R_2 \leftarrow R_2 + \frac{1}{2}R_3]{R_1 \leftarrow R_1 + \frac{5}{4}R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 2 & 1 \\ 0 & 1 & 0 & -3/5 & 6/5 & 2/5 \\ 0 & 0 & 1 & -6/5 & 7/5 & 4/5 \end{array}\right) \tag{0.6}$$

$$\therefore \text{Inverse of the given Matrix:} \begin{pmatrix} -1 & 2 & 1 \\ -3/5 & 6/5 & 2/5 \\ -6/5 & 7/5 & 4/5 \end{pmatrix} \tag{0.7}$$