

# 1.5.33

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## Question:

Find the ratio in which the Y-axis divides the line segment joining the points A(5, -6) and B(-1, -4). Also find the coordinates of the point of intersection.

## Solution:

Let the given points be A and B

$$\mathbf{A} = \begin{pmatrix} 5 \\ -6 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -1 \\ -4 \end{pmatrix}$$

Let the Y-axis divide the line segment  $\mathbf{AB}$  at point  $\mathbf{P}$  in the ratio  $k : 1$ . Since  $\mathbf{P}$  lies on Y-axis, let

$$\mathbf{P} = \begin{pmatrix} 0 \\ y \end{pmatrix}$$

The point  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\mathbf{P}$  are collinear.

$$\Rightarrow \text{rank}(\mathbf{B} - \mathbf{A} \quad \mathbf{P} - \mathbf{A}) = 1 \quad (1)$$

$$\Rightarrow |\mathbf{B} - \mathbf{A} \quad \mathbf{P} - \mathbf{A}| = 0 \quad (2)$$

$$\begin{vmatrix} -6 & -5 \\ 2 & y+6 \end{vmatrix} = 0 \quad (3)$$

$$-6 \times (y+6) - (-5) \times 2 = 0 \quad (4)$$

$$-36 - 6y + 10 = 0 \quad (5)$$

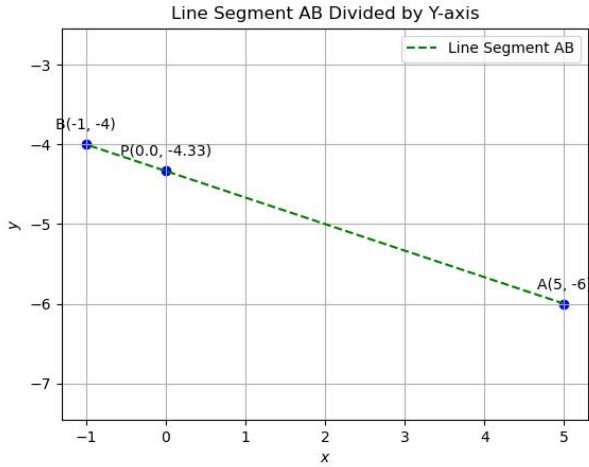
$$y = \frac{-13}{3} \quad (6)$$

$\therefore$  The coordinates of the point of intersection are

$$\mathbf{P} = \begin{pmatrix} 0 \\ -\frac{13}{3} \end{pmatrix}$$

The section formula is

$$\mathbf{P} \equiv \begin{pmatrix} x \\ y \end{pmatrix} = \frac{k\mathbf{B} + \mathbf{A}}{k+1} \quad (7)$$



Plot of Intersection of AB by Y-axis

Here, substituting the values,

$$\begin{pmatrix} 0 \\ -\frac{13}{3} \end{pmatrix} = \frac{1}{k+1} \left( \begin{pmatrix} 5 \\ -6 \end{pmatrix} + k \begin{pmatrix} -1 \\ -4 \end{pmatrix} \right) \quad (8)$$

$$\begin{pmatrix} 0 \\ -\frac{13}{3} \end{pmatrix} = \frac{1}{k+1} \begin{pmatrix} 5-k \\ -6-4k \end{pmatrix} \quad (9)$$

(10)

$$0 = \frac{5-k}{k+1} \quad (11)$$

$$5-k=0 \quad (12)$$

$$\Rightarrow k=5 \quad (13)$$

Thus, the ratio in which the point **P** divides the line segment **AB** is **5:1**.