5.3.15

AI25BTECH11021 - Abhiram Reddy N

QUESTION

What type of lines will you get by drawing the graph of the pair of equations:

$$x - 2y + 3 = 0 \quad \text{and} \quad 2x - 4y = 5 \text{ ?}$$

SOLUTION

We will analyze the system using matrix and vector notation.

Step 1: Write both equations in standard form

First, express both equations in the form:

$$Ax + By = C$$

Equation 1:

$$x - 2y = -3 \tag{0.1}$$

Equation 2:

$$2x - 4y = 5 \tag{0.2}$$

Step 2: Represent as matrices

Let us write both equations in matrix form:

$$\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -3 \\ 5 \end{bmatrix} \tag{0.3}$$

Then the system is:

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{0.4}$$

Step 3: Check for consistency and dependency

We analyze the coefficient matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 2 & -4 \end{bmatrix}$$

Observe that:

$$\text{Row } 2 = 2 \times \text{Row } 1 \tag{0.5}$$

So, the equations are **linearly dependent** in coefficients. However, check the constants:

$$\text{Equation 2 constant} = 5 \neq 2 \times (-3) = -6$$

So the augmented matrix is:

$$\left[\begin{array}{cc|c} 1 & -2 & -3 \\ 2 & -4 & 5 \end{array} \right]$$

Now,

$$\text{Rank of coefficient matrix } \mathbf{A} = 1, \quad \text{Rank of augmented matrix} = 2 \qquad (0.6)$$
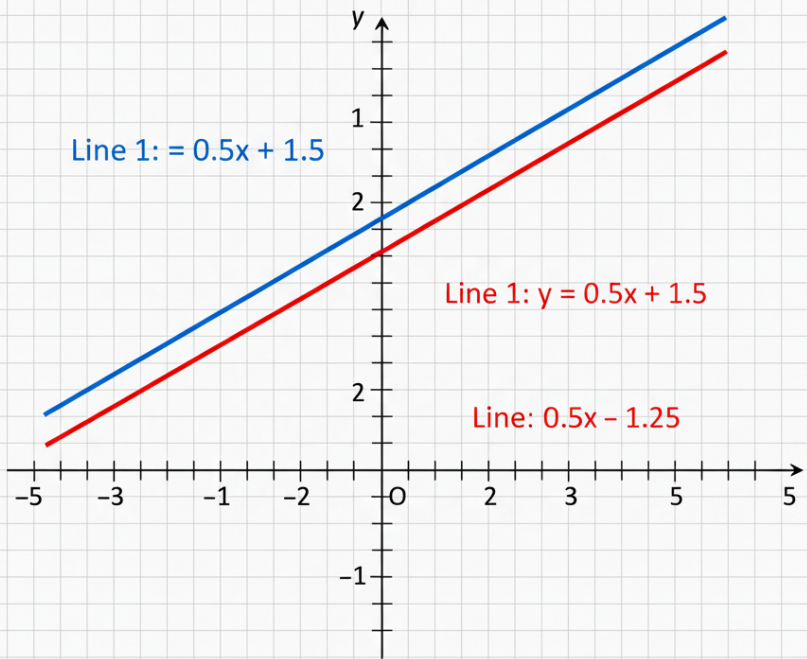
Step 4: Conclusion

Since

$$\text{rank}(\mathbf{A}) \neq \text{rank}(\mathbf{A}|\mathbf{b}),$$

the system is **inconsistent**. Therefore, the lines are:

Parallel and distinct (no solution)

Plot of Parallel Lines

Equations: $x - 2y + 3 = 0$ and $2x - 4y = 5$

Line 1: $= 0.5x + 1.5$

Line 1: $y = 0.5x + 1.5$

Line: $0.5x - 1.25$

Plot of the curves
Fig1