

# 1.6.7

EE25BTECH11057 - Rushil Shanmukha Srinivas

**Question:** Find a relation between  $x$  and  $y$  if the points  $(x,y)$ ,  $(1,2)$  and  $(7,0)$  are collinear.

**Solution:** Let the three points be  $A = \begin{pmatrix} x \\ y \end{pmatrix}$   $B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$   $C = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$ .

For collinearity,  $\text{rank}\left(\begin{pmatrix} B - A & C - A \end{pmatrix}^T\right) = 1$ .

Now,  $B - A = \begin{pmatrix} 1 - x \\ 2 - y \end{pmatrix}$   $C - A = \begin{pmatrix} 7 - x \\ -y \end{pmatrix}$ .

So the matrix is  $M = \begin{pmatrix} B - A & C - A \end{pmatrix}^T = \begin{pmatrix} 1 - x & 2 - y \\ 7 - x & -y \end{pmatrix}$ .

*Row Reduction*

Step 1: Start with  $M = \begin{pmatrix} 1 - x & 2 - y \\ 7 - x & -y \end{pmatrix}$ .

Step 2: Eliminate the first entry of the second row:

$$R_2 \longrightarrow R_2 - \frac{7-x}{1-x}R_1 \quad (\text{assuming } x \neq 1).$$

$$\begin{pmatrix} 1 - x & 2 - y \\ 7 - x & -y \end{pmatrix} \longrightarrow \begin{pmatrix} 1 - x & 2 - y \\ 0 & -y - \frac{7-x}{1-x}(2 - y) \end{pmatrix}.$$

*Rank Condition*

For  $\text{rank}(M) = 1$ , the second row must vanish:

$$-y - \frac{7-x}{1-x}(2 - y) = 0.$$

Multiply through by  $(1 - x)$ :

$$-y(1 - x) - (7 - x)(2 - y) = 0.$$

Expand:

$$-y + xy - (14 - 2x - 7y + xy) = 0.$$

$$-y + xy - 14 + 2x + 7y - xy = 0.$$

$$2x + 6y - 14 = 0.$$

Thus, the condition for collinearity is  $\boxed{x + 3y = 7}$ .

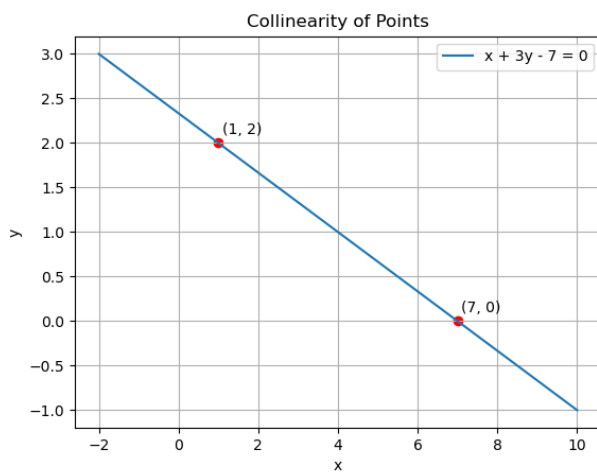


Fig. 0.1