1.5.33

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Find the ratio in which the Y-axis divides the line segment joining the points A(5, −6) and B(−1, −4). Also find the coordinates of the point of intersection.

Equation

Let the Y-axis divide the line segment **AB** at point **P** in the ratio $k : 1$.
Since **P** lies on Y-axis, let

$$\mathbf{P} = \begin{pmatrix} 0 \\ y \end{pmatrix}$$

The point **A**, **B**, **P** are collinear.

$$\implies \operatorname{rank}\left(\mathbf{B} - \mathbf{A}, \mathbf{P} - \mathbf{A}\right) = 1 \tag{1}$$

$$\implies \left\| \left(\mathbf{B} - \mathbf{A}, \mathbf{P} - \mathbf{A} \right) \right\| = 0 \tag{2}$$

$$\left\| \begin{pmatrix} -6, -5 \\ 2, y+6 \end{pmatrix} \right\| = 0 \tag{3}$$

$$-6 \times (y+6) - (-5) \times 2 = 0 \tag{4}$$

$$-36 - 6y + 10 = 0 \tag{5}$$

$$y = \frac{-13}{3} \tag{6}$$

$\therefore$ The coordinates of the point of intersection are

$$\mathbf{P} = \begin{pmatrix} 0 \\ -\frac{13}{3} \end{pmatrix}$$

The section formula is

$$\mathbf{P} \equiv \begin{pmatrix} x \\ y \end{pmatrix} = \frac{k\mathbf{B} + \mathbf{A}}{k+1} \tag{7}$$

Theoretical Solution

Here, substituting the values,

$$\begin{pmatrix} 0 \\ \frac{-13}{3} \end{pmatrix} = \frac{1}{k+1} \left(\begin{pmatrix} 5 \\ -6 \end{pmatrix} + k \begin{pmatrix} -1 \\ -4 \end{pmatrix} \right) \tag{8}$$

$$\begin{pmatrix} 0 \\ \frac{-13}{3} \end{pmatrix} = \frac{1}{k+1} \left(\begin{pmatrix} 5-k \\ -6-4k \end{pmatrix} \right) \tag{9}$$

$$\tag{10}$$

$$0 = \frac{5-k}{k+1} \tag{11}$$

$$5 - k = 0 \tag{12}$$

$$\implies k = 5 \tag{13}$$

Thus, the ratio in which the point **P** divides the line segment **AB** is **5:1**.

C Code

```c
#include <stdio.h>
#include <math.h>
void function(double *P, double *B, double *A , int m, int k) {
    for ( int i = 0 ; i < m ; i++ ) {
        P[i] = (1*A[i] + k*B[i])/(k+1) ;
    }
}
```

Python Code

```python
import sys
import math
import numpy as np
import matplotlib.pyplot as plt
import ctypes

section_formula = ctypes.CDLL('/home/puniaditya/GitHub/ee1030
    -2025/ee25btech11046/matgeo/1.5.33/codes/section_formula.so')
```

Python Code

```
section_formula.argtypes = [
    ctypes.POINTER(ctypes.c_double),
    ctypes.POINTER(ctypes.c_double),
    ctypes.POINTER(ctypes.c_double),
    ctypes.c_int,
    ctypes.c_int,
]
section_formula.restype = None # void function

m = 2
k = 5

A = np.array([[5, -6]], dtype=np.float64)
B = np.array([[-1, -4]], dtype=np.float64)
P = np.zeros(m, dtype=np.float64)
```

Python Code

```python
section_formula.function(
    P.ctypes.data_as(ctypes.POINTER(ctypes.c_double)),
    B.ctypes.data_as(ctypes.POINTER(ctypes.c_double)),
    A.ctypes.data_as(ctypes.POINTER(ctypes.c_double)),
    m, #len(P) alternate
    k
)

A = np.array([5, -6]).reshape(-1,1)
B = np.array([-1, -4]).reshape(-1,1)
P = P.reshape(-1,1)
```

```python
plt.plot([A[0,0], B[0,0]], [A[1,0], B[1,0]], 'g--', label="Line
    Segment AB")

plot_coords = np.block([[A, B, P]])
plt.scatter(plot_coords[0,:], plot_coords[1,:], color='blue')

vert_labels = [
    f'A({A[0,0]}, {A[1,0]})',
    f'B({B[0,0]}, {B[1,0]})',
    f'P({P[0,0]}, {P[1,0]:.2f})'
]
```

Python Code

```python
for i, txt in enumerate(vert_labels):
    plt.annotate(txt,
            (plot_coords[0,i],plot_coords[1,i]),
            textcoords="offset points",
            xytext=(0,10),
            ha='center')

plt.xlabel('$x$')
plt.ylabel('$y$')
plt.title("Line Segment AB Divided by Y-axis")
plt.legend(loc='best')
plt.grid()
plt.axis('equal')

plt.savefig("../figs/plot_c.jpg")
plt.show()
```
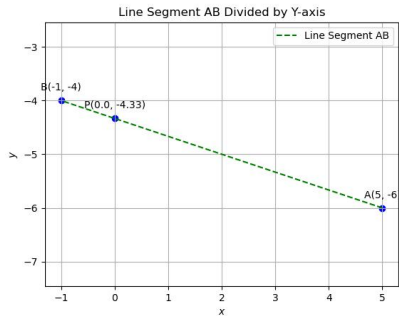
Plot

Figure: Plot of Intersection of AB by Y-axis