4.3.36

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Question

Show that the points $\left(\hat{i} - \hat{j} + 3\hat{k}\right)$ and $3\left(\hat{i} + \hat{j} + \hat{k}\right)$ are equidistant from the plane $\mathbf{r} \cdot \left(5\hat{i} + 2\hat{j} - 7\hat{k}\right) + 9 = 0$ and lie on opposite sides of it.

Theoretical Solution

Let the given points be $\mathbf{P_1} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{P_2} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$. The equation of the given plane is

$$\begin{pmatrix} 5 & 2 & -7 \end{pmatrix} \mathbf{x} + 9 = 0 \tag{1}$$

This can be written in the standard form $\mathbf{n}^\top \mathbf{x} = k$. Here, $\mathbf{n} = \begin{pmatrix} 5 \\ 2 \\ -7 \end{pmatrix}$ and $k = -9$.

$$\begin{pmatrix} 5 & 2 & -7 \end{pmatrix} \mathbf{x} = -9 \tag{2}$$

Theoretical Solution

The reflection of point $\mathbf{Q}$ with respect to the plane $\mathbf{n}^\top \mathbf{x} = k$ is given by

$$\mathbf{R} = \mathbf{Q} - \frac{2\left(\mathbf{n}^\top \mathbf{Q} - k\right)}{\|\mathbf{n}\|^2}\mathbf{n} \tag{3}$$

Let the reflection of point $\mathbf{P_1}$ with respect to the plane be $\mathbf{Q}$.

$$\mathbf{Q} = \mathbf{P_1} - \frac{2\left(\mathbf{n}^\top \mathbf{P_1} - k\right)}{\|\mathbf{n}\|^2}\mathbf{n} \tag{4}$$

$$= \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} - \frac{-18}{78}\begin{pmatrix} 5 \\ 2 \\ -7 \end{pmatrix} \tag{5}$$

$$= \begin{pmatrix} \frac{28}{13} \\ -\frac{7}{13} \\ \frac{18}{13} \end{pmatrix} \tag{6}$$

Theoretical Solution

Let a plane parallel to given plane pass through $\mathbf{P_1}$. Let this be $\mathbf{n}^\top \mathbf{x} = c$

$$\mathbf{n}^\top \mathbf{Q} = c \tag{7}$$

$$\begin{pmatrix} 5 & 2 & -7 \end{pmatrix} \begin{pmatrix} \frac{28}{13} \\ -\frac{7}{13} \\ \frac{18}{13} \end{pmatrix} = c \tag{8}$$

$$c = \frac{140}{13} - \frac{14}{13} - \frac{126}{13} \tag{9}$$

$$c = 0 \tag{10}$$

Theoretical Solution

$$\mathbf{n}^\top \mathbf{P_2} = \begin{pmatrix} 5 & 2 & -7 \end{pmatrix} \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix} \tag{11}$$
$$= 15 + 6 - 21 \tag{12}$$
$$= 0 = c \tag{13}$$

$\because$ $\mathbf{P_2}$ lies in the plane $\mathbf{n}^\top \mathbf{x} = c$, the point $\mathbf{P_2}$ and $\mathbf{P_1}$ are equidistant from the plane $\mathbf{n}^\top \mathbf{x} = k$ and lie on the opposite sides of the plane.

Plot

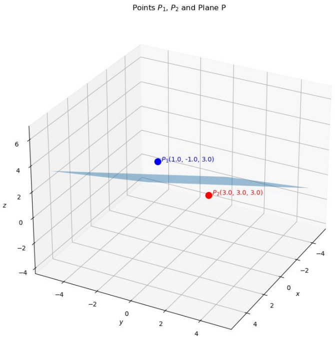

Figure: Plot