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Question

If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentric

Solution

Parametric Vector Form of Ellipse

An ellipse centered at the origin is defined by the vector function:

$$\mathbf{r}(\theta) = \begin{pmatrix} x(\theta) \\ y(\theta) \end{pmatrix} = \begin{pmatrix} a\cos\theta \\ b\sin\theta \end{pmatrix} \quad \text{where } \theta \in [0, 2\pi] \tag{1}$$

Here, a and b are the semi-major and semi-minor axes respectively.

Focus and Latus Rectum

The focus of the ellipse lies at:

$$\mathbf{F} = \begin{pmatrix} ae \\ 0 \end{pmatrix} \quad \text{where } e = \text{eccentricity} \tag{2}$$

The latus rectum L is the chord perpendicular to the major axis passing through the focus:

$$L = \frac{2b^2}{a} \tag{3}$$

Given Condition

We are given:

$$L = \frac{1}{2} \cdot 2b = b \quad \Rightarrow \quad \frac{2b^2}{a} = b \quad \Rightarrow \quad 2b = a \tag{4}$$

Thus:

$$a = 2b \tag{5}$$

Eccentricity via Vector Displacement

Eccentricity is defined as:

$$e = \frac{c}{a} \quad \text{where } c = \text{distance from center to focus} = \sqrt{a^2 - b^2} \tag{6}$$

Substitute $a = 2b$:

$$c = \sqrt{(2b)^2 - b^2} = \sqrt{4b^2 - b^2} = \sqrt{3b^2} = b\sqrt{3} \tag{7}$$

$$e = \frac{b\sqrt{3}}{2b} = \frac{\sqrt{3}}{2} \tag{8}$$

Final Answer

$$\boxed{e = \frac{\sqrt{3}}{2}} \tag{9}$$

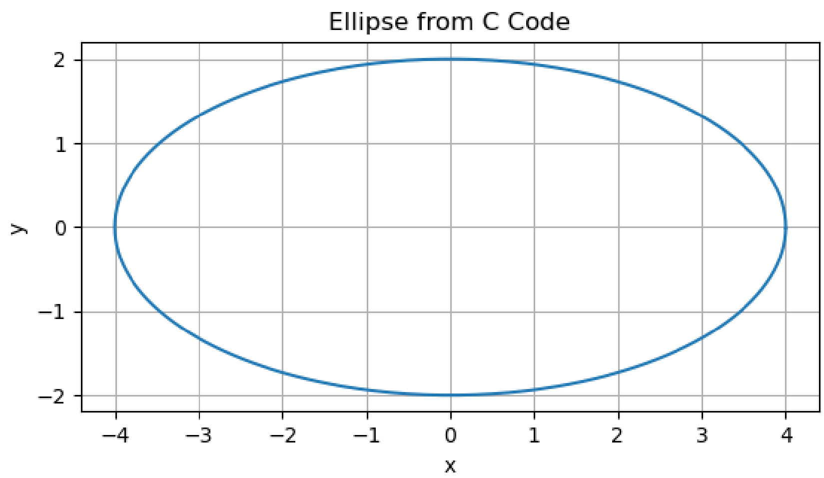

Figure 1