Assignment 8: 4.11.32

EE25BTECH11055 - Subhodeep Chakraborty

Question:

Find the equation of the line passing through $(2,-1,2)$ and $(5,3,4)$ and of the plane passing through $(2,0,3)$, $(1,1,5)$ and $(3,2,4)$. Also, find their point of intersection. (12, 2018)

Solution:

Given:

$$\mathbf{A} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \tag{1}$$

$$\mathbf{B} = \begin{pmatrix} 5 \\ 3 \\ 4 \end{pmatrix} \tag{2}$$

$$\mathbf{P} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} \tag{3}$$

$$\mathbf{Q} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix} \tag{4}$$

$$\mathbf{R} = \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} \tag{5}$$

We know, for line $\mathbf{x} = \mathbf{h} + k\mathbf{m}$ and plane $\mathbf{n}^\top \mathbf{y} = 1$,

$$\mathbf{h} = \mathbf{A} \tag{6}$$

$$\mathbf{m} = \mathbf{B} - \mathbf{A} \tag{7}$$

$$\begin{pmatrix} P & Q & R \end{pmatrix}^\top \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{8}$$

Thus

$$\begin{pmatrix} P & Q & R \end{pmatrix}^\top \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{9}$$

$$\begin{pmatrix} 2 & 0 & 3 \\ 1 & 1 & 5 \\ 3 & 2 & 4 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{10}$$

$$\left(\begin{array}{ccc|c} 2 & 0 & 3 & 1 \\ 1 & 1 & 5 & 1 \\ 3 & 2 & 4 & 1 \end{array}\right) \xleftrightarrow{R_2=2R_2-R_1;R_3=2R_3-3R_1} \left(\begin{array}{ccc|c} 2 & 0 & 3 & 1 \\ 0 & 2 & 7 & 1 \\ 0 & 4 & 2 & -1 \end{array}\right) \tag{11}$$

$$\xleftrightarrow{R_3=R_3-2R_2} \left(\begin{array}{ccc|c} 2 & 0 & 3 & 1 \\ 0 & 2 & 7 & 1 \\ 0 & 0 & -12 & -3 \end{array}\right) \xleftrightarrow{R_1=4R_1+R_3;R_2=12R_2+7R_3} \left(\begin{array}{ccc|c} 8 & 0 & 0 & 1 \\ 0 & 24 & 0 & -9 \\ 0 & 0 & -12 & -3 \end{array}\right) \tag{12}$$

$$\xleftrightarrow{R_1=R_1/8;R_2=R_2/24;R_3=-R_3/12} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1/8 \\ 0 & 1 & 0 & -3/8 \\ 0 & 0 & 1 & 1/4 \end{array}\right) \tag{13}$$

$$\mathbf{n} = \frac{1}{8}\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \tag{14}$$

So we have:

$$\mathbf{x} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} + k\begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 2+3k \\ -1+4k \\ 2+2k \end{pmatrix} \tag{15}$$

$$\begin{pmatrix} 1 & -3 & 2 \end{pmatrix}\mathbf{y} = 8 \tag{16}$$

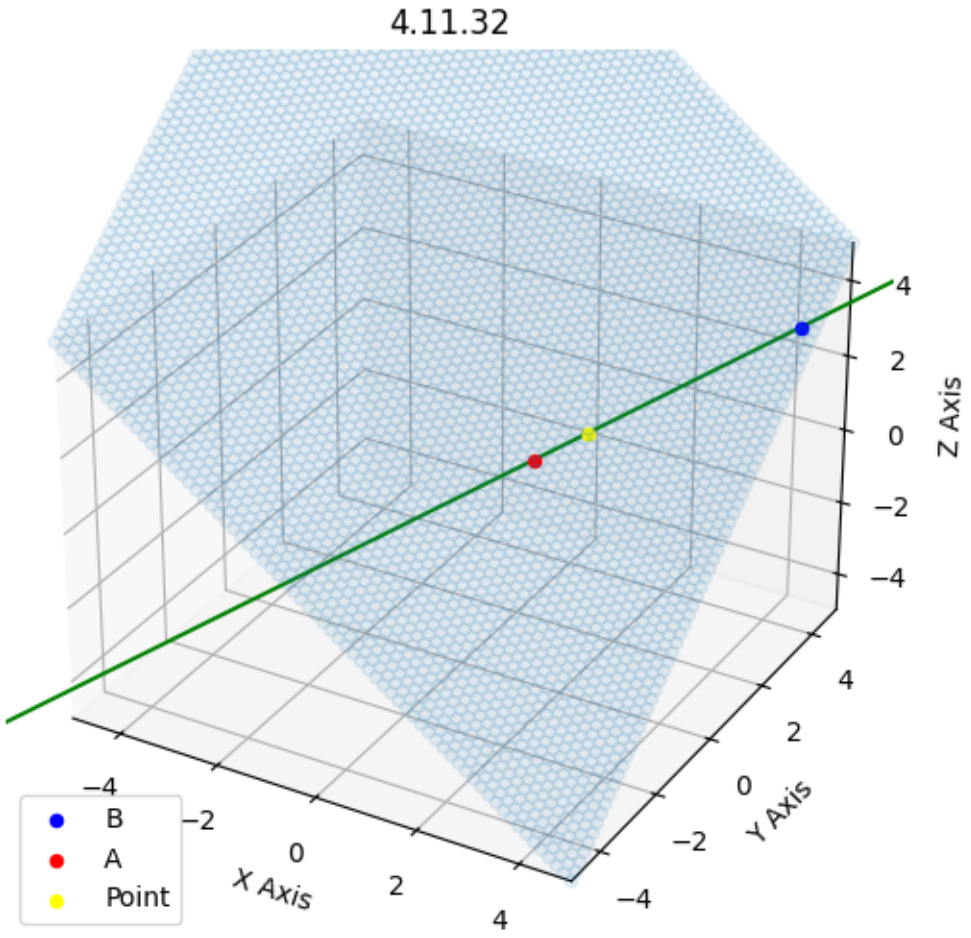
At point of intersection,

$$\mathbf{x} = \mathbf{y} = \mathbf{S} \tag{17}$$

$$\begin{pmatrix} 1 & -3 & 2 \end{pmatrix}\begin{pmatrix} 2+3k \\ -1+4k \\ 2+2k \end{pmatrix} = 8 \tag{18}$$

$$k = 1/5 \tag{19}$$

$$\mathbf{S} = \begin{pmatrix} 13/5 \\ -1/5 \\ 12/5 \end{pmatrix} \tag{20}$$


4.11.32
B
A
Point
X Axis
Y Axis
Z Axis
−4
−2
0
2
4