question

If $D\left(-\frac{1}{2}, \frac{5}{2}\right)$, $E(7,3)$, $F\left(\frac{7}{2}, \frac{7}{2}\right)$ are the midpoints of the sides of $\triangle ABC$, find the area of $\triangle ABC$.

theoritical solution

Solution:

Let the position vectors of vertices A, B, C be $\mathbf{A}, \mathbf{B}, \mathbf{C}$.

Using midpoint relations:

$$\mathbf{D} = \frac{\mathbf{B}+\mathbf{C}}{2}, \quad \mathbf{E} = \frac{\mathbf{C}+\mathbf{A}}{2}, \quad \mathbf{F} = \frac{\mathbf{A}+\mathbf{B}}{2}$$

Rearranging,

$$\mathbf{A}-\mathbf{B} = 2(\mathbf{F}-\mathbf{D}), \quad \mathbf{A}-\mathbf{C} = 2(\mathbf{E}-\mathbf{D})$$

The area of $\triangle ABC$ is:

$$\text{Area} = \frac{1}{2}\left\|(\mathbf{A}-\mathbf{B})\times(\mathbf{A}-\mathbf{C})\right\| = \frac{1}{2}\left\|2(\mathbf{F}-\mathbf{D})\times 2(\mathbf{E}-\mathbf{D})\right\| = 2\left\|(\mathbf{F}-\mathbf{D})\times\right.$$

theoritical solution

Calculate the difference vectors as matrices:

$$\mathbf{F}-\mathbf{D}=\begin{pmatrix}\frac{7}{2}\\ \frac{7}{2}\end{pmatrix}-\begin{pmatrix}-\frac{1}{2}\\ \frac{5}{2}\end{pmatrix}=\begin{pmatrix}4\\ 1\end{pmatrix}$$

$$\mathbf{E}-\mathbf{D}=\begin{pmatrix}7\\ 3\end{pmatrix}-\begin{pmatrix}-\frac{1}{2}\\ \frac{5}{2}\end{pmatrix}=\begin{pmatrix}\frac{15}{2}\\ \frac{1}{2}\end{pmatrix}$$

The magnitude of their cross product is the determinant:

$$\|(\mathbf{F}-\mathbf{D})\times(\mathbf{E}-\mathbf{D})\|=\left\|\begin{matrix}4 & \frac{15}{2}\\ 1 & \frac{1}{2}\end{matrix}\right\|=\left|4\times\frac{1}{2}-1\times\frac{15}{2}\right|=|2-7.5|=5.5$$

the area of $\triangle ABC$ is:

$$\text{Area}=2\times 5.5=11$$

$$\boxed{11}$$

equation

$$\text{Area} = \frac{1}{2} \left\| (\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C}) \right\|$$

C code

```c
#include <stdio.h>
#include <math.h>

// Function to compute the area of a triangle given coordinates
    of points
double triangleArea(double Ax, double Ay, double Bx, double By,
    double Cx, double Cy) {
    double cross_product = (Ax - Bx) * (Ay - Cy) - (Ay - By) * (
        Ax - Cx);
    return 0.5 * fabs(cross_product);
}
```

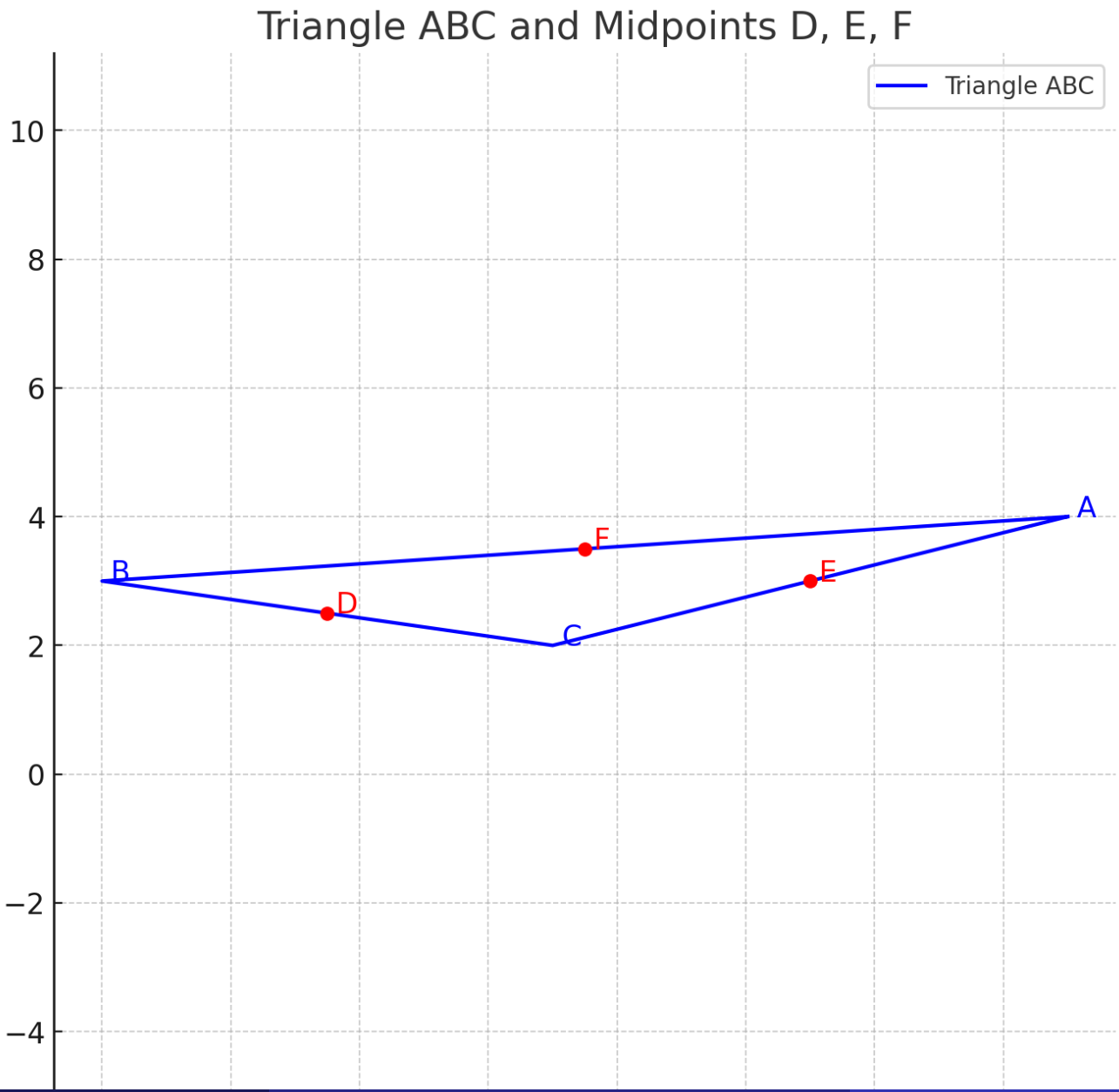
Python Plotting Code - Part 1

```
import matplotlib.pyplot as plt
# Given midpoints D, E, F
D = (-0.5, 2.5)
E = (7, 3)
F = (3.5, 3.5)
# Reconstruct vertices A, B, C using midpoint formulas
A = (F[0] + E[0] - D[0], F[1] + E[1] - D[1])
B = (D[0] + F[0] - E[0], D[1] + F[1] - E[1])
C = (D[0] + E[0] - F[0], D[1] + E[1] - F[1])
plt.figure(figsize=(8,8))
# Plot triangle ABC
triangle_x = [A[0], B[0], C[0], A[0]]
triangle_y = [A[1], B[1], C[1], A[1]]
plt.plot(triangle_x, triangle_y, 'b-', label='Triangle ABC')
# Mark midpoints D, E, F as red dots
plt.plot(D[0], D[1], 'ro')
plt.plot(E[0], E[1], 'ro')
plt.plot(F[0], F[1], 'ro')
```

Python Plotting Code - Part 2

```
    # Annotate vertices
plt.text(A[0], A[1], ' A', fontsize=12, color='blue')
plt.text(B[0], B[1], ' B', fontsize=12, color='blue')
plt.text(C[0], C[1], ' C', fontsize=12, color='blue')
# Annotate midpoints
plt.text(D[0], D[1], ' D', fontsize=12, color='red')
plt.text(E[0], E[1], ' E', fontsize=12, color='red')
plt.text(F[0], F[1], ' F', fontsize=12, color='red')
plt.title('Triangle ABC and Midpoints D, E, F')
plt.grid(True)
plt.axis('equal')
plt.legend()
plt.show()
```

plot


Triangle ABC and Midpoints D, E, F
Triangle ABC
10
8
6
4
2
0
−2
−4
A
B
C
D
E
F