4.7.60

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Question:
Reduce the equation $\sqrt{3}x+y-8=0$ into normal form.Find the values of p and ω.

Solution:
Given line equation is

$$\sqrt{3}x + y - 8 = 0 \tag{1}$$

which can be written as

$$\mathbf{n}^T\mathbf{x} = c \tag{2}$$

$$\implies \begin{pmatrix} \sqrt{3} & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 8 \tag{3}$$

$$\mathbf{n} = \begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix} \text{ and } \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \text{ and } c = 8 \tag{4}$$

Length (norm) of $\mathbf{n}$ is given as

$$\|\mathbf{n}\| = \sqrt{\mathbf{n}^T\mathbf{n}} = 2. \tag{5}$$

The unit normal is given by

$$\hat{\mathbf{n}} = \frac{\mathbf{n}}{\|\mathbf{n}\|} = \begin{pmatrix} \sqrt{3}/2 \\ 1/2 \end{pmatrix} \tag{6}$$

Divide the line equation by $\|\mathbf{n}\|$ to get the normal form

$$\implies \mathbf{n}^T\mathbf{x} = 4. \tag{7}$$

$$\implies \begin{pmatrix} \dfrac{\sqrt{3}}{2} & \dfrac{1}{2} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 4. \tag{8}$$

The standard form of line in normal form is given by

$$\begin{pmatrix} \cos\omega & \sin\omega \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \text{p}. \tag{9}$$

On comparing equations (8) and (9) we get

$$\text{p} = 4 \text{ and } \omega = \frac{\pi}{6} \tag{10}$$

Line in Normal Form