4.13.28

EE25BTECH11023 - Venkata Sai

Question:
Slope of a line passing through $\mathbf{P}(2,3)$ and intersecting the line $x+y=7$ at a distance of 4 units from $\mathbf{P}$, is

Solution: Given

$$\mathbf{P} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{1}$$

Equation of a line through $\mathbf{P}$ and having slope m is

$$\begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} x-2 \\ y-3 \end{pmatrix} = 0 \tag{2}$$

$$\begin{pmatrix} -m & 1 \end{pmatrix} \left(\begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right) \implies \begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} -m & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 3-2m \tag{4}$$

$$x+y=7 \implies \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 7 \tag{5}$$

$$\begin{pmatrix} -m & 1 & 3-2m \\ 1 & 1 & 7 \end{pmatrix} \xleftrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 1 & 7 \\ -m & 1 & 3-2m \end{pmatrix} \xleftrightarrow{R_2 \to R_2 + mR_1} \begin{pmatrix} 1 & 1 & 7 \\ 0 & 1+m & 3+5m \end{pmatrix} \tag{6}$$

$$y = \frac{3+5m}{1+m} \tag{7}$$

$$x+y=7 \implies x = 7-y \implies x = 7 - \frac{3+5m}{1+m} \tag{8}$$

$$x = \frac{7+7m-3-5m}{1+m} = \frac{4+2m}{1+m} \tag{9}$$

Given the point is at a distance of 4 units from point $\mathbf{P}$

$$\left\| \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix} \right\| = 4 \implies \left\| \begin{pmatrix} \frac{4+2m}{1+m} - 2 \\ \frac{3+5m}{1+m} - 3 \end{pmatrix} \right\| = 4 \tag{10}$$

$$\left\| \begin{pmatrix} \frac{4+2m-2-2m}{1+m} \\ \frac{3+5m-3-3m}{1+m} \end{pmatrix} \right\| = \left\| \begin{pmatrix} \frac{2}{1+m} \\ \frac{2m}{1+m} \end{pmatrix} \right\| = 4 \tag{11}$$

$$\sqrt{\left(\frac{2}{1+m} \right)^2 + \left(\frac{2m}{1+m} \right)^2} = 4 \tag{12}$$

$$\frac{4+4m^2}{(1+m)^2} = 4^2 = 16 \tag{13}$$

$$4\left(1+m^2\right) = 16\left(1+m^2+2m\right) \implies \left(1+m^2\right) = 4\left(1+m^2+2m\right) \tag{14}$$

$$4+4m^2+8m = 1+m^2 \implies 3m^2+8m+3 = 0 \tag{15}$$

$$m^2+\frac{8m}{3}+1 = 0 \tag{16}$$

$$m^2+\frac{8m}{3}+1+\left(\frac{4}{3}\right)^2 = \left(\frac{4}{3}\right)^2 \tag{17}$$

$$\left(m+\frac{4}{3}\right)^2 = \frac{16}{9}-1 = \frac{7}{9} \tag{18}$$

$$m+\frac{4}{3} = \pm\frac{\sqrt{7}}{3} \tag{19}$$

$$m = \frac{-4+\sqrt{7}}{3} \text{ or } \frac{-4-\sqrt{7}}{3} \tag{20}$$

According to options

$$\frac{-4+\sqrt{7}}{3} = \frac{-8+2\sqrt{7}}{6} = \frac{1-\sqrt{7}}{1+\sqrt{7}} \tag{21}$$

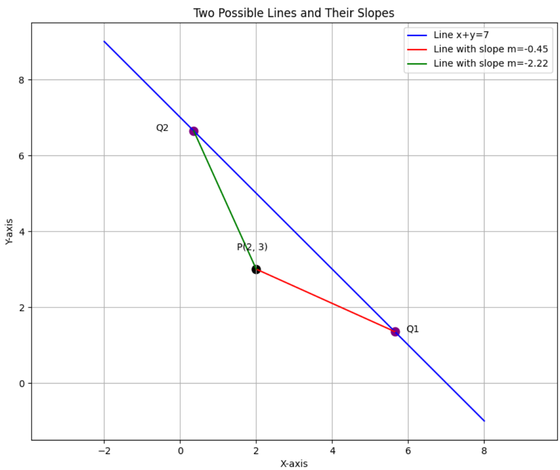

Fig. 0.1