2.10.49

EE25BTECH11020 - Darsh Pankaj Gajare

Question:
The unit vector which is orthogonal to the vector $3\hat{i}+2\hat{j}+6\hat{k}$ and is coplanar with vectors $2\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is

1) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$ 2) $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$ 3) $\frac{3\hat{i}-\hat{k}}{\sqrt{10}}$ 4) $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

Solution: Given:

TABLE I: Given data

Vector	matrix
A	$\begin{pmatrix}2\\1\\1\end{pmatrix}$
B	$\begin{pmatrix}1\\-1\\1\end{pmatrix}$
C	$\begin{pmatrix}3\\2\\6\end{pmatrix}$

Equation of plane through $\mathbf{A},\mathbf{B}$.

$$\begin{pmatrix}2 & 1 & 1\\ 1 & -1 & 1\end{pmatrix} n = 0. \tag{1}$$

As augmented matrix,

$$\left(\begin{array}{ccc|c}2 & 1 & 1 & 0\\ 1 & -1 & 1 & 0\end{array}\right). \tag{2}$$

Using Row operations: $R_1 = R_1 - 2R_2$, $R_2 = R_2 + R_1$

$$\left(\begin{array}{ccc|c}0 & 3 & -1 & 0\\ 1 & 2 & 0 & 0\end{array}\right). \tag{3}$$

$$\mathbf{n} = \begin{pmatrix}-2\\1\\3\end{pmatrix} \tag{4}$$

condition with $\mathbf{C}$.

$$\mathbf{C}^{\top}\mathbf{P} = 0 \tag{5}$$

So $\mathbf{P}$ satisfies

$$\begin{pmatrix}-2 & 1 & 3\\ 3 & 2 & 6\end{pmatrix}\mathbf{x} = 0. \tag{6}$$

$$\left(\begin{array}{ccc|c}-2 & 1 & 3 & 0\\ 3 & 2 & 6 & 0\end{array}\right). \tag{7}$$

Row operations: $R_2 = 2R_2 - 3R_1$, $R_2 = R_2/7$, $R_1 = R_1 + R_2$

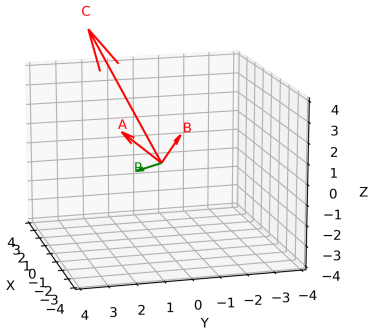
$$\left(\begin{array}{ccc|c} 2 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 \end{array}\right). \tag{8}$$

From first row: $2x = 0 \implies x = 0$. From second: $y + 3z = 0 \implies y = -3z$. So

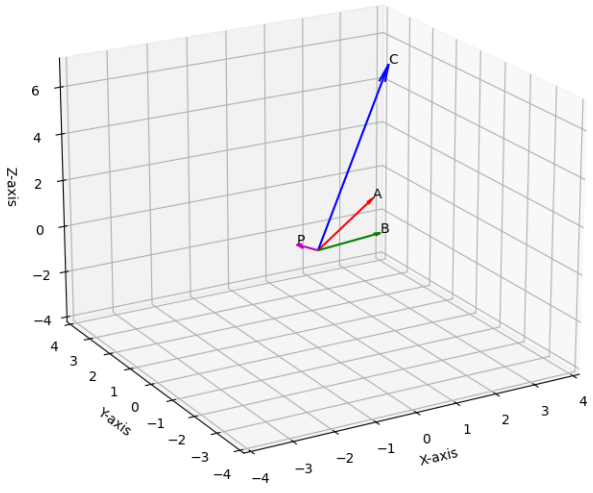
$$\mathbf{P} = \begin{pmatrix} 0 \\ -3z \\ z \end{pmatrix} = z \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}. \tag{9}$$

Normalizing,

$$\mathbf{P} = \pm \frac{1}{\sqrt{10}} \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} \tag{10}$$



Plot using C function



Plot using Python