1.9.30

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Question

Solve the following system of rational equations

$$\frac{10}{x+y} + \frac{2}{x-y} = 4 \tag{1}$$

$$\frac{15}{x+y} - \frac{5}{x-y} = -2 \tag{2}$$

Theoretical Solution

Introduce a and b as follows:

$$a = \frac{1}{x+y} \;\; b = \frac{1}{x-y} \tag{3}$$

Also define

$$\mathbf{a} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{4}$$

This gives us simplified equations

$$\begin{pmatrix} 10 & 2 \end{pmatrix} \mathbf{a} = 4 \tag{5}$$

$$\begin{pmatrix} 15 & -5 \end{pmatrix} \mathbf{a} = -2 \tag{6}$$

Augmented matrix for the given system is

$$\left(\begin{array}{cc|c} 10 & 2 & 4 \\ 15 & -5 & -2 \end{array}\right) \tag{7}$$

Theoretical Solution

By row reductions

$$\left(\begin{array}{cc|c} 10 & 2 & 4 \\ 15 & -5 & -2 \end{array}\right)$$
$$\mathrm{R}_2 \leftarrow R_2 - \frac{3}{2} \times R_1 \left(\begin{array}{cc|c} 10 & 2 & 4 \\ 0 & -8 & -8 \end{array}\right) R_1 \leftarrow R_1 + \frac{1}{4} \times R_2 \left(\begin{array}{cc|c} 10 & 0 & 2 \\ 0 & -8 & -8 \end{array}\right)$$
$$\left(\begin{array}{cc|c} 10 & 0 & 2 \\ 0 & -8 & -8 \end{array}\right) R_1 \leftarrow \frac{1}{10} \times R_1 \left(\begin{array}{cc|c} 1 & 0 & \frac{1}{5} \\ 0 & -8 & -8 \end{array}\right) R_2 \leftarrow \frac{1}{-8} \times R_2 \left(\begin{array}{cc|c} 1 & 0 & \frac{1}{5} \\ 0 & 1 & 1 \end{array}\right)$$

$$\mathbf{a} = \begin{pmatrix} \frac{1}{5} \\ 1 \end{pmatrix} \tag{8}$$

Substituting value of a and b again we get

$$\begin{pmatrix} \frac{1}{x+y} \\ \frac{1}{x-y} \end{pmatrix} = \begin{pmatrix} \frac{1}{5} \\ 1 \end{pmatrix} \tag{9}$$

Theoretical Solution

$$\implies \begin{pmatrix} x+y \\ x-y \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \tag{10}$$

Introduce

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{11}$$

This gives us the equation

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \tag{12}$$

$$\implies \mathbf{x} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1} \tag{13}$$

Theoretical Solution

$$\implies \mathbf{x} = \begin{pmatrix} 5 \\ 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \tag{14}$$

$$\implies \mathbf{x} = \begin{pmatrix} \frac{5}{2} + \frac{1}{2} \\ \frac{5}{2} - \frac{1}{2} \end{pmatrix} \tag{15}$$

$$\implies \mathbf{x} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{16}$$

Thus $x = 3$ and $y = 2$

C Code- equidistant check function

```c
#include <stdio.h>

void rref_solver(double aug[2][3], double solution[2]) {
    // Normalize first row (pivot = aug[0][0])
    double pivot = aug[0][0];
    for (int j = 0; j < 3; j++) {
        aug[0][j] /= pivot;
    }

    // Eliminate below pivot
    double factor = aug[1][0];
    for (int j = 0; j < 3; j++) {
        aug[1][j] -= factor * aug[0][j];
    }
```

```c
 pivot = aug[1][1];
for (int j = 0; j < 3; j++) {
        aug[1][j] /= pivot;
}

// Eliminate above pivot
factor = aug[0][1];
for (int j = 0; j < 3; j++) {
        aug[0][j] -= factor * aug[1][j];
}

// Extract solution
solution[0] = aug[0][2]; // x
solution[1] = aug[1][2]; // y
```

Python Code using shared output

```
import ctypes
import numpy as np
import matplotlib.pyplot as plt
import matplotlib as mp
# Load the shared C library
lib = ctypes.CDLL(./5.2.44.so)
# Define argument and return types
lib.rref_solver.argtypes = [ctypes.c_double
    * 6, ctypes.c_double * 2]
# Create augmented matrix for system:
aug = (ctypes.c_double * 6)(1, 1, 5, 1, -1,
    1) # Flattened 2x3
solution = (ctypes.c_double * 2)()
```

Python Code using shared output

```
# Call C function
lib.rref_solver(aug, solution)
# Convert result to numpy vector (ensure
    flat)
x_sol = np.array([solution[0], solution
    [1]], dtype=float).flatten()
print(Solution vector from C:, x_sol) #
    This correctly prints [3. 2.]
# plot
x_vals = np.linspace(-2, 10, 400)
y1 = 5 - x_vals # Correct for x + y = 5
y2 = x_vals - 1 # CORRECTED for x - y = 1
plt.plot(x_vals, y1, label=r$x+y=5$)
plt.plot(x_vals, y2, label=r$x-y=1$)
plt.scatter(x_sol[0], x_sol[1], color=red,
    zorder=5)
plt.text(float(x_sol[0])+0.2, float(x_sol
    [1]), f({x_sol[0]:.1f}, {x_sol[1]:.1f})
    , color=red)
```

Python Code using shared output

```
plt.plot(x_vals, y1, label=r$x+y=5$)
plt.plot(x_vals, y2, label=r$x-y=1$)
plt.scatter(x_sol[0], x_sol[1], color=red, zorder=5)
plt.text(float(x_sol[0])+0.2, float(x_sol[1]), f({x_sol[0]:.1f}, {x_sol[1]:.1f}), color=red)
plt.xlabel(x)
plt.ylabel(y)
plt.title(Graphical Solution of the Linear System)
plt.axhline(0, color=black, linewidth=0.8)
plt.axvline(0, color=black, linewidth=0.8)
plt.legend()
plt.grid(True)
plt.savefig(Figure_1_Corrected.png)
plt.show()
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib as mp
mp.use(TkAgg)
A=np.array([[1,1],[1,-1]],dtype=float)
b=np.array([5,1], dtype=float)
x=np.linalg.solve(A,b)
print(Solution vector for the system of equations:,x)
```

Python Code

```
# Making a plot
x_vals = np.linspace(-2, 10, 400)
# Rearranged equations to express y in terms of x
y1 = (5 - x_vals) # from x + 3y = 6
y2 = (x_vals-1) # from 2x - 3y = 12
# Plot lines
plt.plot(x_vals, y1, label=r$x + y = 5$)
plt.plot(x_vals, y2, label=r$x - y = 1$)
# Mark solution
plt.scatter(x[0], x[1], color=red, zorder=5)
plt.text(x[0]+0.2, x[1], f({x[0]:.1f}, {x[1]:.1f})
    , color=red)
```

Python Code

```
# Formatting
plt.xlabel(x)
plt.ylabel(y)
plt.title(Graphical Solution of the Linear System)
plt.axhline(0, color='black', linewidth=0.8)
plt.axvline(0, color='black', linewidth=0.8)
plt.legend()
plt.grid(True)
plt.savefig(Figure_2)
plt.show()
```

Plot by python using shared output from c

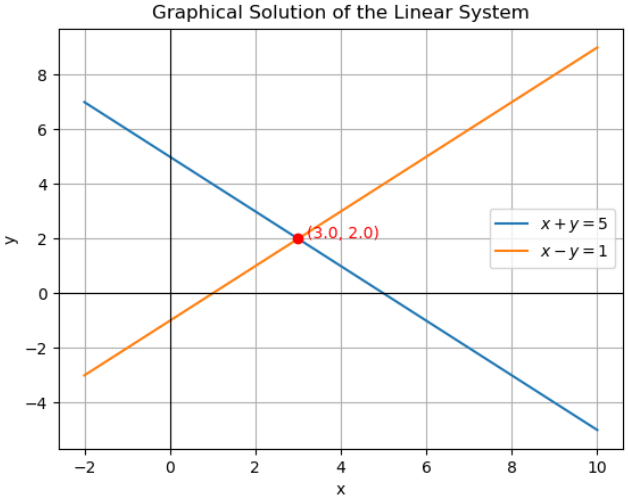

Figure: *

Plot by python

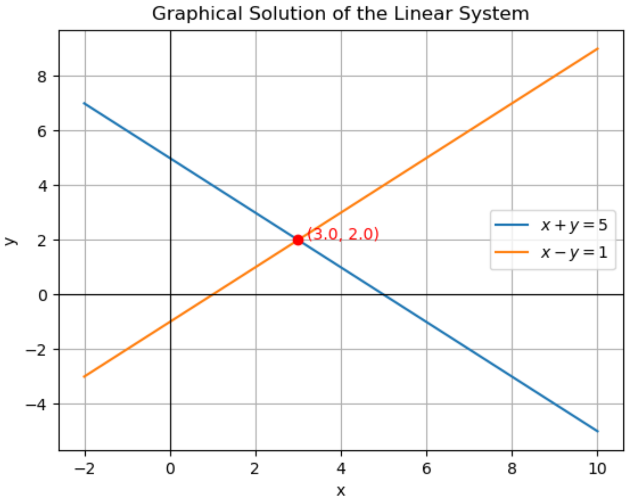

Figure: *