8.2.52

AI25BTECH11014 - Gooty Suhas

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Problem

Given:

- Eccentricity $e = \frac{2}{3}$
- Latus rectum $l = 5$
- Centre at origin $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

Find the equation of the conic in matrix form using matrix algebra.

Ellipse Setup

Let the conic be:

$$\mathbf{x}^T\mathbf{V}\mathbf{x} = 1 \quad \text{where} \quad \mathbf{V} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

Choose two points:

$$\mathbf{P}_1 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \quad \mathbf{P}_2 = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

Substitute:

$$\mathbf{P}_1^T\mathbf{V}\mathbf{P}_1 = 1, \quad \mathbf{P}_2^T\mathbf{V}\mathbf{P}_2 = 1$$

Matrix System

From substitution:

$$16a + 9b = 1, \quad 36a + 4b = 1$$

Write as matrix equation:

$$\begin{pmatrix} 16 & 9 \\ 36 & 4 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Row Reduction

Augmented matrix:

$$\left(\begin{array}{cc|c} 16 & 9 & 1 \\ 36 & 4 & 1 \end{array}\right) \xrightarrow{R_2 \to R_2 - \frac{9}{4}R_1} \left(\begin{array}{cc|c} 16 & 9 & 1 \\ 0 & -\frac{61}{4} & -\frac{5}{4} \end{array}\right)$$

Solution

Choose $a = \frac{4}{81} \Rightarrow 16a = \frac{64}{81}$

Then:

$$\frac{64}{81} + 9b = 1 \Rightarrow b = \frac{17}{81}$$

So:

$$\mathbf{V} = \begin{pmatrix} \frac{4}{81} & 0 \\ 0 & \frac{17}{81} \end{pmatrix}$$

Final Answer

$$\boxed{\mathbf{x}^T \begin{pmatrix} \frac{4}{81} & 0 \\ 0 & \frac{17}{81} \end{pmatrix} \mathbf{x} = 1}$$ is the equation of the ellipse

Python Code

```
from sympy import Rational, symbols, solve, Eq

e = Rational(2, 3)
l = 5
a = symbols('a')

b2 = a**2 * (1 - e**2)
eq = Eq(2 * b2 / a, l)
a_val = solve(eq, a)[0]
b2_val = a_val**2 * (1 - e**2)

print("a = ", a_val)
print("b = ", b2_val)
print("V = [[{:.6f}, 0], [0, {:.6f}]]".format(
    1 / a_val**2, 1 / b2_val))
```

C Code

```c
#include <math.h>
#include <stdio.h>

void solve_matrix(float* V) {
  float e = 2.0 / 3.0;
  float l = 5.0;

  float a = 9.0 / 2.0;
  float b2 = a * a * (1 - e * e);

  V[0] = 1 / (a * a);
  V[1] = 1 / b2;
}
```

Python Wrapper Code

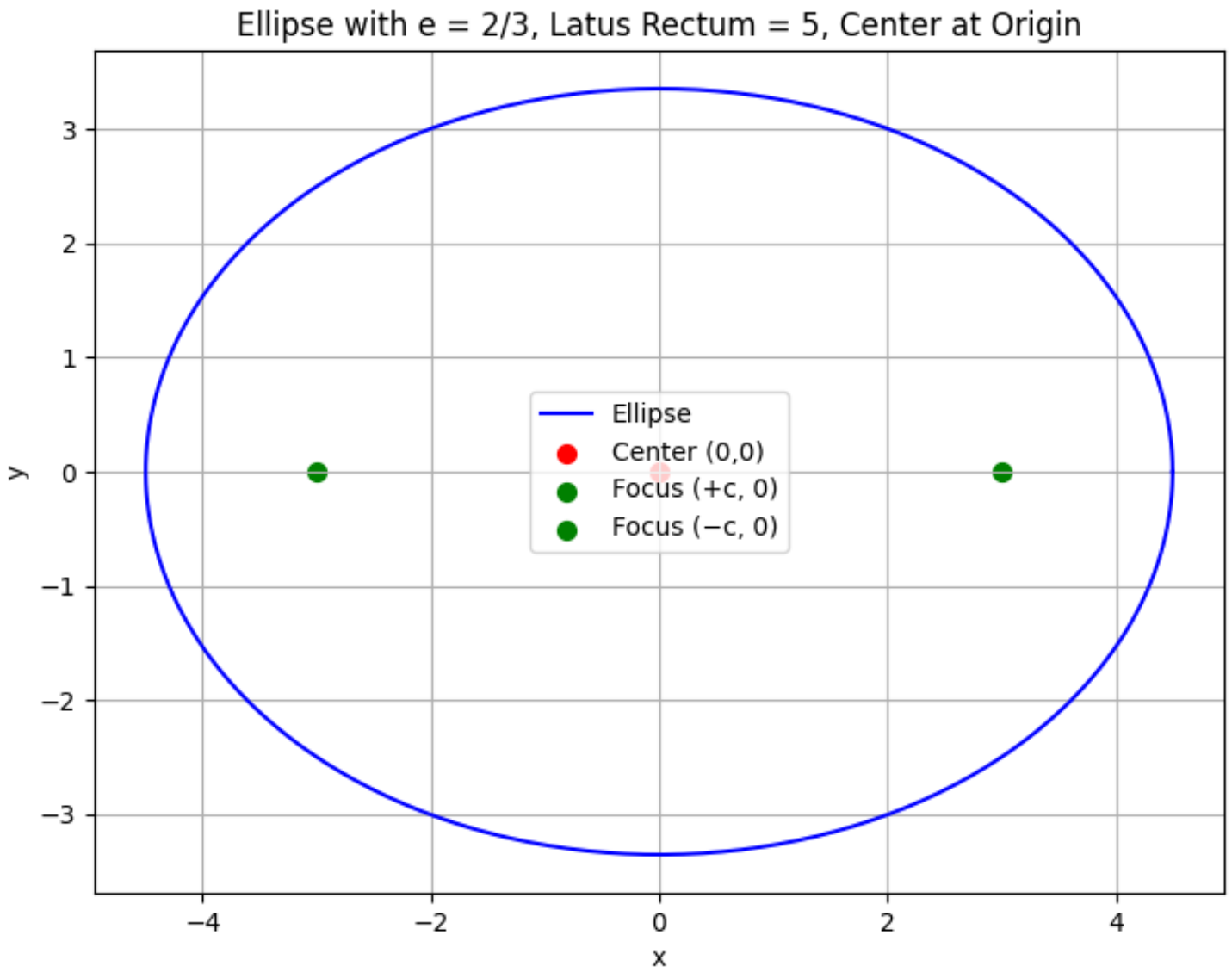
```
import ctypes

lib = ctypes.CDLL('./libellipse.so')
lib.solve_matrix.argtypes = [ctypes.POINTER(ctypes.c_float)]
lib.solve_matrix.restype = None

V = (ctypes.c_float * 2)()
lib.solve_matrix(V)

print("Ellipse equation: x V x = 1")
print(f"V = [[{V[0]:.6f}, 0], [0, {V[1]:.6f}]]")
```

Diagram


Ellipse with e = 2/3, Latus Rectum = 5, Center at Origin
Ellipse
Center (0,0)
Focus (+c, 0)
Focus (−c, 0)
x
y
−4
−2
0
2
4
3
2
1
0
−1
−2
−3