1.5.22

EE25BTECH11034 - Kishora Karthik

Question: $\mathbf{X}$ and $\mathbf{Y}$ are two points with position vectors $3\mathbf{a}+\mathbf{b}$ and $\mathbf{a}-3\mathbf{b}$ respectively. Write the position vector of a point $\mathbf{V}$ which divides the line segment XY in the ratio $2:1$ externally.

Solution:
Vectors $\mathbf{A}$ and $\mathbf{B}$ are given. Let $\mathbf{A} = \begin{pmatrix}1\\0\end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix}0\\1\end{pmatrix}$.
Then,

$$\mathbf{X} = 3\mathbf{A} + \mathbf{B} \tag{1}$$

$$\mathbf{Y} = \mathbf{A} - 3\mathbf{B} \tag{2}$$

Or,

$$\mathbf{X} = \begin{pmatrix}\mathbf{A} & \mathbf{B}\end{pmatrix}\begin{pmatrix}3\\1\end{pmatrix} \tag{3}$$

$$\mathbf{Y} = \begin{pmatrix}\mathbf{A} & \mathbf{B}\end{pmatrix}\begin{pmatrix}1\\-3\end{pmatrix} \tag{4}$$

Formula: Section formula for a vector $\mathbf{P}$ which divides the line formed by vectors $\mathbf{A}$ and $\mathbf{B}$ in the ratio k:1 externally is given by,

$$\mathbf{P} = \frac{k\mathbf{B} - \mathbf{A}}{k-1} \tag{5}$$

It is given that k=2.

$$\mathbf{V} = \frac{k\mathbf{Y} - \mathbf{X}}{k-1}$$

$$\implies \mathbf{V} = \frac{2\mathbf{Y} - \mathbf{X}}{1}$$

$$\implies \mathbf{V} = \frac{-2\begin{pmatrix}\mathbf{A} & \mathbf{B}\end{pmatrix}\begin{pmatrix}1\\-3\end{pmatrix} - \begin{pmatrix}\mathbf{A} & \mathbf{B}\end{pmatrix}\begin{pmatrix}3\\1\end{pmatrix}}{1}$$

$$\implies \mathbf{V} = \frac{\begin{pmatrix}\mathbf{A} & \mathbf{B}\end{pmatrix}\begin{pmatrix}2\\-6\end{pmatrix} - \begin{pmatrix}\mathbf{A} & \mathbf{B}\end{pmatrix}\begin{pmatrix}3\\1\end{pmatrix}}{1} \tag{6}$$

$$\tag{7}$$

$$\implies \mathbf{V} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \end{pmatrix} \begin{pmatrix} -1 \\ -7 \end{pmatrix} \tag{8}$$

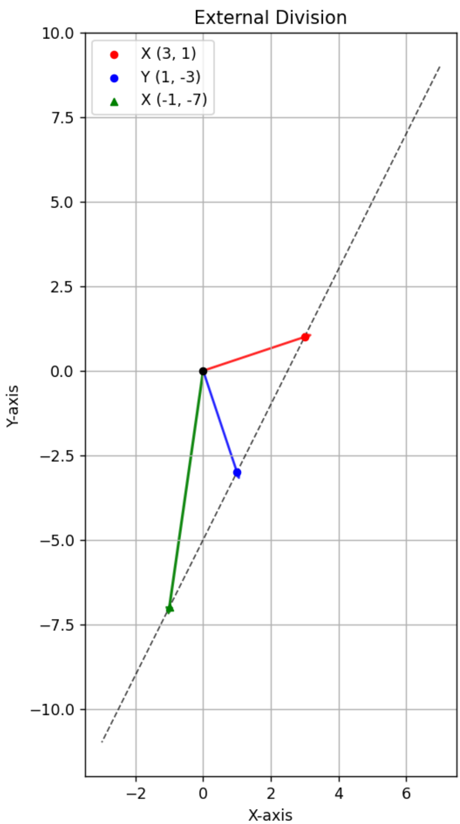
$$\tag{9}$$

$$\implies \mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ -7 \end{pmatrix} \tag{10}$$

$$\tag{11}$$

$$\implies \mathbf{V} = \begin{pmatrix} -1 \\ -7 \end{pmatrix} \tag{12}$$

See Fig. 1 ,



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