

2.2.24

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## Question (2.2.24)

If  $\hat{i} + \hat{j} + \hat{k}$ ,  $2\hat{i} + 5\hat{j}$ ,  $3\hat{i} + 2\hat{j} - 3\hat{k}$  and  $\hat{i} - 6\hat{j} - \hat{k}$  are the position vectors of the point **A**, **B**, **C** and **D** respectively, then find the angle between **AB** and **CD**. Deduce that **AB** and **CD** are collinear.

# Solution

Given points are

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 1 \\ -6 \\ -1 \end{pmatrix}. \quad (1)$$

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}, \quad \mathbf{CD} = \mathbf{D} - \mathbf{C} = \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix}. \quad (2)$$

# Solution

The angle  $\theta$  between **AB** and **CD** is given by

$$\cos \theta = \frac{\mathbf{AB}^T \mathbf{CD}}{\|\mathbf{AB}\| \|\mathbf{CD}\|}. \quad (3)$$

$$\mathbf{AB}^T \mathbf{CD} = \begin{pmatrix} 1 & 4 & -1 \end{pmatrix} \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix} = (1)(-2) + (4)(-8) + (-1)(2) = -36. \quad (4)$$

$$\|\mathbf{AB}\| = 3\sqrt{2}, \quad \|\mathbf{CD}\| = 6\sqrt{2}. \quad (5)$$

# Solution

$$\cos \theta = \frac{-36}{(3\sqrt{2})(6\sqrt{2})} = \frac{-36}{36} = -1. \quad (6)$$

Hence,

$$\theta = \cos^{-1}(-1) = \pi \text{ } (180^\circ). \quad (7)$$

Since, angle between vectors is  $180^\circ$  the given vectors are collinear

# Solution

Proof of collinearity by rank method

Let,

$$\mathbf{P} = (B - A \quad D - C) \quad (8)$$

$$\mathbf{P} = \begin{pmatrix} 1 & -2 \\ 4 & -8 \\ -1 & 2 \end{pmatrix} \quad (9)$$

$$\mathbf{P}^T = \begin{pmatrix} 1 & 4 & -1 \\ -2 & -8 & 2 \end{pmatrix} \quad (10)$$

$$R_2 \rightarrow R_2 - 2R_1 \quad (11)$$

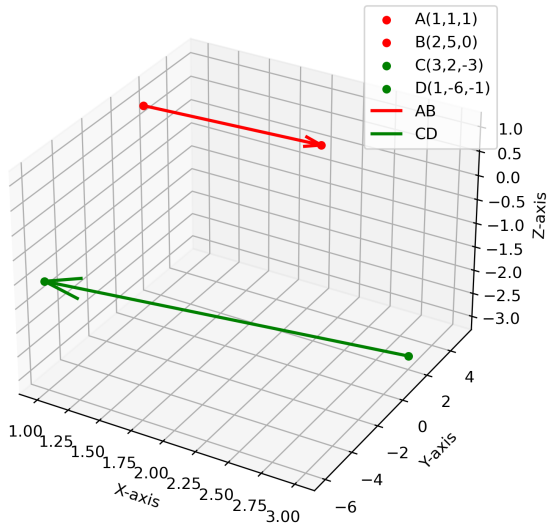
$$\mathbf{P}^T = \begin{pmatrix} 1 & 4 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad (12)$$

$$\text{rank} \mathbf{P} = \text{rank} \mathbf{P}^T = 1 \quad (13)$$

$$(14)$$

Thus the given vectors are collinear.

Vectors AB and CD (Collinear)



```
#include <stdio.h>
#include <math.h>

// Function to compute dot product of two vectors
double dot_product(double v1[3], double v2[3]) {
    return v1[0]*v2[0] + v1[1]*v2[1] + v1[2]*v2[2];
}

// Function to compute norm of a vector
double norm(double v[3]) {
    return sqrt(v[0]*v[0] + v[1]*v[1] + v[2]*v[2]);
}
```



```
int main() {  
    double A[3], B[3], C[3], D[3];  
    double AB[3], CD[3];  
    int i;  
    // Input points  
    printf("Enter coordinates of A (x y z): ");  
    scanf("%lf %lf %lf", &A[0], &A[1], &A[2]);  
  
    printf("Enter coordinates of B (x y z): ");  
    scanf("%lf %lf %lf", &B[0], &B[1], &B[2]);  
  
    printf("Enter coordinates of C (x y z): ");  
    scanf("%lf %lf %lf", &C[0], &C[1], &C[2]);  
  
    printf("Enter coordinates of D (x y z): ");  
    scanf("%lf %lf %lf", &D[0], &D[1], &D[2]);  
}
```

```
// Compute vectors AB and CD
for (i = 0; i < 3; i++) {
    AB[i] = B[i] - A[i];
    CD[i] = D[i] - C[i];
}

// Print vectors
printf("\nVector AB = (%.21f, %.21f, %.21f)\n", AB[0], AB[1], AB[2]);
printf("Vector CD = (%.21f, %.21f, %.21f)\n", CD[0], CD[1], CD[2]);

// Compute angle
double dot = dot_product(AB, CD);
double cos_theta = dot / (norm(AB) * norm(CD));
int x = (cos_theta * 100);
double y = x/100;
double theta = acos(y)*180/M_PI;
```

```
printf("\nDot product = %.2lf\n", dot);  
printf("cos(theta) = %.2lf\n", cos_theta);  
printf("Angle between AB and CD = %.2lf degrees\n", th  
  
if (cos_theta == 1 || -1){  
    printf("\n AB and CD are collinear.\n");  
} else {  
    printf("\n AB and CD are not collinear.\n");  
}  
  
return 0;  
}
```

# Python Code

```
import numpy as np

# Function to read a point from user
def read_point(name):
    coords = input(f"Enter coordinates of {name} (x y z):")
    return np.array(list(map(float, coords)))

# Input points
A = read_point("A")
B = read_point("B")
C = read_point("C")
D = read_point("D")

# Vectors
AB = B - A
CD = D - C
```

```
# Step 1: Angle between AB and CD
dot_product = np.dot(AB, CD)
norms = np.linalg.norm(AB) * np.linalg.norm(CD)
cos_theta = dot_product / norms
x = cos_theta*100
y = int(x)/100
theta_deg = np.degrees(np.arccos(y))

# Step 2: Rank method for collinearity
M = np.column_stack((AB, CD))    # Matrix with AB and CD as
rank = np.linalg.matrix_rank(M)
```

```
# Print results
print("\n--- Results ---")
print("Vector AB:", AB)
print("Vector CD:", CD)
print("Dot product =", dot_product)
print("cos(theta) =", y)
print("Angle between AB and CD =", theta_deg, "degrees")
print("Matrix M:\n", M)
print("Rank of M =", rank)

if rank == 1:
    print("Vectors AB and CD are collinear.")
else:
    print("Vectors AB and CD are not collinear.")
```