4.13.7

EE25BTECH11002 - Achat Parth Kalpesh

Question:
If a, b and c are in A.P, then the straight line $ax + by + c = 0$ will always pass through a fixed point whose coordinates are _____.

Solution:
Let the equation $ax + by + c = 0$ be represented as

$$\mathbf{n}^\top \mathbf{x} = -c \tag{0.1}$$

$$\mathbf{n} = \begin{pmatrix} a \\ b \end{pmatrix} \tag{0.2}$$

Let $\mathbf{p}$ be the fixed point.
The condition that a, b, c are in arithmetic progression is

$$2b = a + c \quad \implies \quad a - 2b = -c, \tag{0.3}$$

Thereby,

$$\begin{pmatrix} a \\ b \end{pmatrix}^\top \begin{pmatrix} 1 \\ -2 \end{pmatrix} = -c \tag{0.4}$$

Comparing it with (0.1) we get

$$\mathbf{p} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \tag{0.5}$$

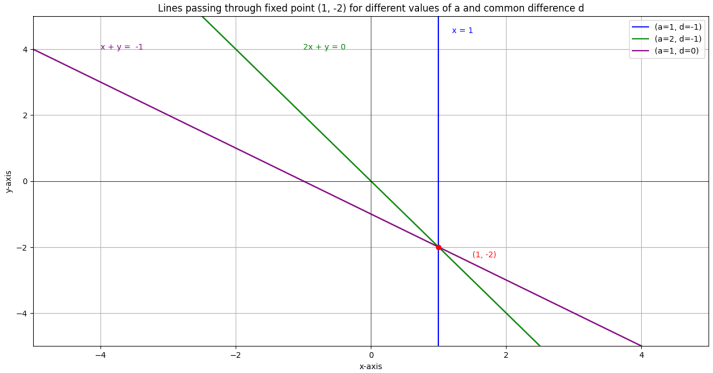

Fig. 0.1: Graph