12.601

AI25BTECH11003 - Bhavesh Gaikwad

September 28, 2025

Question

The matrix $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$, one of the eigen values is 1. The eigen vectors corresponding to the eigne value 1 are: (CS 2016)

a) $\alpha \begin{pmatrix} 4 & -2 & 1 \end{pmatrix}, \alpha \neq 0, \ \alpha \epsilon \mathbb{R}$

b) $\alpha \begin{pmatrix} -4 & 2 & 1 \end{pmatrix}, \alpha \neq 0, \ \alpha \epsilon \mathbb{R}$

c) $\alpha \begin{pmatrix} -2 & 0 & 1 \end{pmatrix}, \alpha \neq 0, \ \alpha \epsilon \mathbb{R}$

d) $\alpha \begin{pmatrix} 2 & 0 & 1 \end{pmatrix}, \alpha \neq 0, \ \alpha \epsilon \mathbb{R}$

Theoretical Solution

Given: $\lambda = 1$, $Let\ \mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$

Let $\mathbf{v}$ be the corresponding eigenvector.

$$\Rightarrow \mathbf{Av} = (1)\mathbf{v} \tag{1}$$

$$(\mathbf{A} - \mathbf{I})\mathbf{v} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \tag{2}$$

$$\begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{pmatrix} \mathbf{v} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \tag{3}$$

$$\therefore \mathbf{v} = \alpha \begin{pmatrix} 4 & -2 & 1 \end{pmatrix} \tag{4}$$

$\boxed{\text{Thus, Option-A is correct.}}$