4.7.47

EE25BTECH11050-Hema Havil

Question:

The foot of perpendiculars from the point (2, 3) on the line $y = 3x + 4$ is given by

Solution:

Let the given point be P=(2,3) and let the foot of perpendicular be Q and let the given line be written as,

$$\mathbf{n}^T\mathbf{x} = c \tag{0.1}$$

where

$$\mathbf{n} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$
$$c = -4$$

then Q is a point on the line, Hence it satisfies the line equation

$$\mathbf{n}^T\mathbf{Q} = c \tag{0.2}$$

Let $\mathbf{m} = (a, b)$ be the direction vector of the line

$$\mathbf{m}^T\mathbf{n} = 0 \tag{0.3}$$

$$\begin{pmatrix} a & b \end{pmatrix}\begin{pmatrix} 3 \\ -1 \end{pmatrix} = 0 \tag{0.4}$$

$$3a = b \tag{0.5}$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{0.6}$$

Then m is perpendicular to direction vector along PQ

$$\mathbf{m}^T(\mathbf{Q} - \mathbf{P}) = 0 \tag{0.7}$$

$$\mathbf{m}^T\mathbf{Q} = \mathbf{m}^T\mathbf{P} \tag{0.8}$$

from equation (0.2) and (0.8) we can write

$$\begin{pmatrix} \mathbf{m} & \mathbf{n} \end{pmatrix}^T \mathbf{Q} = \begin{pmatrix} \mathbf{m}^T\mathbf{P} \\ c \end{pmatrix} \tag{0.9}$$

We can find the value of $\mathbf{m}^T\mathbf{P}$

$$\mathbf{m}^T\mathbf{P} = \begin{pmatrix}1 & 3\end{pmatrix}\begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}2+9\end{pmatrix} = \begin{pmatrix}11\end{pmatrix} \tag{0.10}$$

From this we can find Q, substitute values in (0.9)

$$\begin{pmatrix}1 & 3\\3 & -1\end{pmatrix}^T \mathbf{Q} = \begin{pmatrix}11\\-4\end{pmatrix} \tag{0.11}$$

$$\begin{pmatrix}1 & 3\\3 & -1\end{pmatrix}\mathbf{Q} = \begin{pmatrix}11\\-4\end{pmatrix} \tag{0.12}$$

This can be solved using augmented matrix and let the augmented matrix be A

$$\mathbf{A} = \begin{pmatrix}1 & 3 & 11\\3 & -1 & -4\end{pmatrix} \tag{0.13}$$

$R_2 \rightarrow R_2 - 3R_1$

$$\mathbf{A} = \begin{pmatrix}1 & 3 & 11\\0 & -10 & -37\end{pmatrix} \tag{0.14}$$

$R_1 \rightarrow R_1 + \frac{3}{10}R_2 \qquad R_2 \rightarrow \frac{-1}{10}R_2$

$$\mathbf{A} = \begin{pmatrix}1 & 0 & \frac{-1}{10}\\0 & 1 & \frac{-37}{10}\end{pmatrix} \tag{0.15}$$

Therefore from (0.15) the value of Q is

$$\mathbf{Q} = \begin{pmatrix}\frac{-1}{10}\\\frac{-37}{10}\end{pmatrix} \tag{0.16}$$

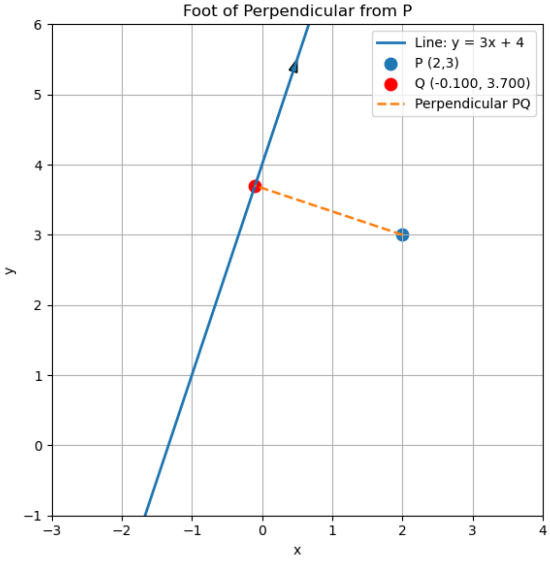

Fig. 0.1: Plot for foot of perpendicular of P