2.10.1 Matgeo

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Question

Consider 3 points :

$$\mathbf{P} = (-\sin(\beta - \alpha), -\cos\beta), \mathbf{Q} = (\cos(\beta - \alpha), \sin\beta)$$

$$\mathbf{R} = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$$

where $\theta < \alpha, \beta, \theta < \frac{\pi}{4}$ Then,

1. P lies on the line segment RQ
2. Q lies on the line segment PR
3. R lies on the line segment QP
4. P,Q,R are non-collinear

Solution

First we have to check if points can be collinear for the values satisfing the given conditions :
The eqution for coliinearity of the given points are :

$$Rank \begin{bmatrix} \mathbf{P} - \mathbf{Q} \\ \mathbf{R} - \mathbf{Q} \end{bmatrix} = 1 \tag{1}$$

$$Rank\begin{bmatrix} -\sin(\beta-\alpha)-\cos(\beta-\alpha) & -\cos\beta-\sin\beta \\ \cos(\beta-\alpha+\theta)-\cos(\beta-\alpha) & \sin(\beta-\theta)-\sin\beta \end{bmatrix}=1 \tag{2}$$

$$R_2 = R_2 - R_1 \tag{3}$$

$$Rank\begin{bmatrix} -\sin(\beta-\alpha)-\cos(\beta-\alpha) & -\cos\beta-\sin\beta \\ \cos(\beta-\alpha+\theta)-\sin(\beta-\alpha) & \sin(\beta-\theta)-\cos\beta \end{bmatrix}=1 \tag{4}$$

Solution

For the rank to be 1 R_2 must be zero :

$$\cos(\beta - \alpha + \theta) - \sin(\beta - \alpha) = 0 \tag{5}$$

$$\tag{6}$$

This will only be satisfied if :

$$\theta = \frac{\pi}{2} + 2\pi K \quad or \quad \frac{\pi}{2} - 2\pi K \tag{7}$$

But ,given that :

$$0 < \theta < \frac{\pi}{4} \tag{8}$$

Which is contradictory :

Hence the points P,Q,R are not collinear .