Direction and Normal Vectors

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Problem Statement

Find the equation of the line passing through the point $(1, 2, 3)$ and parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

Solution

Let the point $\mathbf{h}$ and direction vector $\mathbf{m}$ be

$$\mathbf{h} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix}.$$

The vector equation of the line is given by

$$\mathbf{x} = \mathbf{h} + \kappa \mathbf{m}, \quad \kappa \in \mathbb{R}.$$

Solution (cont..)

Expanding,

$$\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \kappa \begin{pmatrix} 3 \\ 2 \\ -2 \end{pmatrix} \tag{1}$$

$$= \begin{pmatrix} 1 + 3\kappa \\ 2 + 2\kappa \\ 3 - 2\kappa \end{pmatrix}. \tag{2}$$

Hence the parametric equations of the line are

$$x = 1 + 3\kappa, \tag{3}$$

$$y = 2 + 2\kappa, \tag{4}$$

$$z = 3 - 2\kappa, \quad \kappa \in \mathbb{R}. \tag{5}$$

$$\boxed{\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{-2}}$$

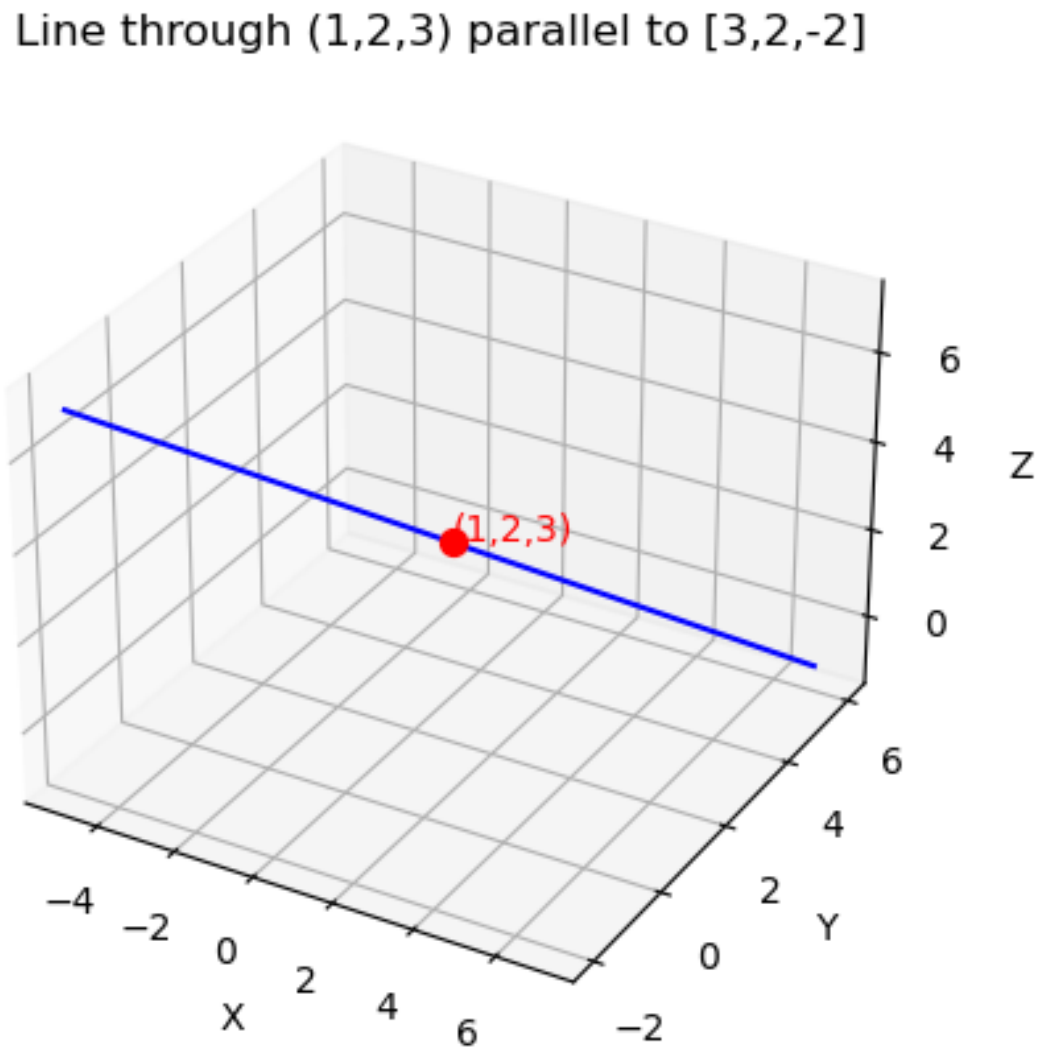
Python Code (Plotting Line and Vectors)

```
import numpy as np
import matplotlib.pyplot as plt

h = np.array([1, 2, 3])
m = np.array([3, 2, -2])
kappa = np.linspace(-2, 2, 100)

x = h[0] + kappa*m[0]
y = h[1] + kappa*m[1]
z = h[2] + kappa*m[2]
```

Python Code (cont..)

```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.plot(x, y, z, color='blue')
ax.scatter(h[0], h[1], h[2], color='red', s=50)
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('Line through (1,2,3) parallel to [3,2,−2]')
ax.text(h[0], h[1], h[2], '(1,2,3)', color='red')

plt.show()
```

Plot



C Code (Computations)

```
#include <stdio.h>

void line_point(double kappa, double h[3], double m[3], double
    out[3]) {
    out[0] = h[0] + kappa * m[0];
    out[1] = h[1] + kappa * m[1];
    out[2] = h[2] + kappa * m[2];
}
```

Python Code (Calling C)

```
import numpy as np
import ctypes
import matplotlib.pyplot as plt

lib = ctypes.CDLL("./pts.so")

lib.line_point.argtypes = [ctypes.c_double,
                           ctypes.POINTER(ctypes.c_double),
                           ctypes.POINTER(ctypes.c_double),
                           ctypes.POINTER(ctypes.c_double)]

h = np.array([1.0, 2.0, 3.0], dtype=np.float64)
m = np.array([3.0, 2.0, -2.0], dtype=np.float64)
```

Python Code (cont..)

```
kappa_values = np.linspace(−2, 2, 100)

points = np.zeros((len(kappa_values), 3), dtype=np.float64)

for i, k in enumerate(kappa_values):
    lib.line_point(ctypes.c_double(k),
                   h.ctypes.data_as(ctypes.POINTER(ctypes.
                       c_double)),
                   m.ctypes.data_as(ctypes.POINTER(ctypes.
                       c_double)),
                   points[i].ctypes.data_as(ctypes.POINTER(
                       ctypes.c_double)))
```

Python Code (cont..)

```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.plot(points[:,0], points[:,1], points[:,2], color='blue')
ax.scatter(h[0], h[1], h[2], color='red', s=50)
ax.text(h[0], h[1], h[2], '(1,2,3)', color='red')
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('3D-Line-through-(1,2,3)-parallel-to-[3,2,−2]')

plt.show()
```