

# 1.5.36

EE25BTECH11049 - Sai Krishna Bakki

## Question:

Point  $P(x, 4)$  lies on the line segment joining the points  $A(-5, 8)$  and  $B(4, -10)$ . Find the ratio in which point  $P$  divides the line segment  $AB$ . Also, find the value of  $x$ .

## Solution:

Let

$$\mathbf{A} = \begin{pmatrix} -5 \\ 8 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ -10 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} x \\ 4 \end{pmatrix} \quad (0.1)$$

Since  $\mathbf{P}$  lies on  $\mathbf{A}$  and  $\mathbf{B}$ , they must be collinear

$$\therefore \text{rank} \begin{pmatrix} \mathbf{B} - \mathbf{A} & \mathbf{P} - \mathbf{A} \end{pmatrix} = 1 \quad (0.2)$$

$$\text{rank} \begin{pmatrix} 9 & x+5 \\ -18 & -4 \end{pmatrix} = 1 \quad (0.3)$$

$$\begin{pmatrix} 9 & x+5 \\ -18 & -4 \end{pmatrix} \xrightarrow{R_2 \rightarrow 2R_1 + R_2} \begin{pmatrix} 9 & x+5 \\ 0 & 2x+6 \end{pmatrix} \quad (0.4)$$

The number of non zero rows in the row reduced matrix (also known as *echelon form*) is defined as the rank. For above matrix to be of rank 1,

$$2x + 6 = 0 \quad (0.5)$$

$$\therefore x = -3 \quad (0.6)$$

Thus  $\mathbf{P}$  is :

$$\mathbf{P} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \quad (0.7)$$

let  $\mathbf{P}$  divide the line joining points  $\mathbf{A}$  and  $\mathbf{B}$  in the ratio  $k : 1$ .

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \quad (0.8)$$

$$k(\mathbf{P} - \mathbf{B}) = \mathbf{A} - \mathbf{P}k = \frac{(\mathbf{P} - \mathbf{B})^T (\mathbf{A} - \mathbf{P})}{\|(\mathbf{P} - \mathbf{B})\|^2} \quad (0.9)$$

$$k = \frac{\begin{pmatrix} x-4 \\ -14 \end{pmatrix} \cdot \begin{pmatrix} -5-x \\ 4 \end{pmatrix}}{\left\| \begin{pmatrix} x-4 \\ -14 \end{pmatrix} \right\|^2} \quad (0.10)$$

substituting the value of  $x$  as , we get the value of  $k$  as

$$k = 2/7 \quad (0.11)$$

