

10.7.81

EE25BTECH11032 - Kartik Lahoti

Question:

Two circles, each of radius 5 units, touch each other at $(1, 2)$. If the equation of common tangent is $4x + 3y = 10$, find the equations of circles.

Solution:

Let ,

$$\mathbf{P} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (0.1)$$

Given Line,

$$\mathbf{n}^\top \mathbf{x} = 10 \quad (0.2)$$

Normal Vector $\mathbf{n} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

Unit vector \mathbf{u} in direction of \mathbf{n}

$$\mathbf{u} = \frac{\mathbf{n}}{\|\mathbf{n}\|} = \begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix} \quad (0.3)$$

Let \mathbf{O}_i be the center of Circles, then

$$\mathbf{O}_i = \mathbf{P} \pm 5\mathbf{u} \quad (0.4)$$

$$\mathbf{O}_i = \begin{pmatrix} 1 \pm 4 \\ 2 \pm 3 \end{pmatrix} \quad (0.5)$$

$$\therefore \mathbf{O}_1 = \begin{pmatrix} 5 \\ 5 \end{pmatrix}, \mathbf{O}_2 = \begin{pmatrix} -3 \\ -1 \end{pmatrix} \quad (0.6)$$

Equation of Circles are :

$$O_1 : g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V} \mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f \quad (0.7)$$

\mathbf{V}	\mathbf{u}	f
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 5 \\ 5 \end{pmatrix}$	25

$O_2\colon g(\mathbf{x}) = \mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f$

(0.8)

\mathbf{V}	\mathbf{u}	f
$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -3 \\ -1 \end{pmatrix}$	-15

