# Assignment 10: 5.2.57

EE25BTECH11055 - Subhodeep Chakraborty

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**Question:**
Solve the following system of linear equations.

$$2x + y + z = 1$$
$$x - 2y - z = \frac{3}{2}$$
$$3y - 5z = 9$$

**Solution:**
Given:

$$\mathbf{n_1}^\top \mathbf{x} = c_1 \qquad \mathbf{n_1} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} c_1 = 1 \tag{1}$$

$$\mathbf{n_2}^\top \mathbf{x} = c_2 \qquad \mathbf{n_2} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} c_2 = 3/2 \tag{2}$$

$$\mathbf{n_3}^\top \mathbf{x} = c_3 \qquad \mathbf{n_3} = \begin{pmatrix} 0 \\ 3 \\ -5 \end{pmatrix} c_3 = 9 \tag{3}$$

Thus

$$\begin{pmatrix} \mathbf{n_1} & \mathbf{n_2} & \mathbf{n_3} \end{pmatrix}^\top \mathbf{x} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \tag{4}$$

On forming augmented matrix and applying Gaussian elimination, we can solve for $\mathbf{x}$

$$\implies \left(\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 1 & -2 & -1 & 3/2 \\ 0 & 3 & -5 & 9 \end{array}\right) \xleftrightarrow{R_2 = 2R_2 - R_1} \tag{5}$$

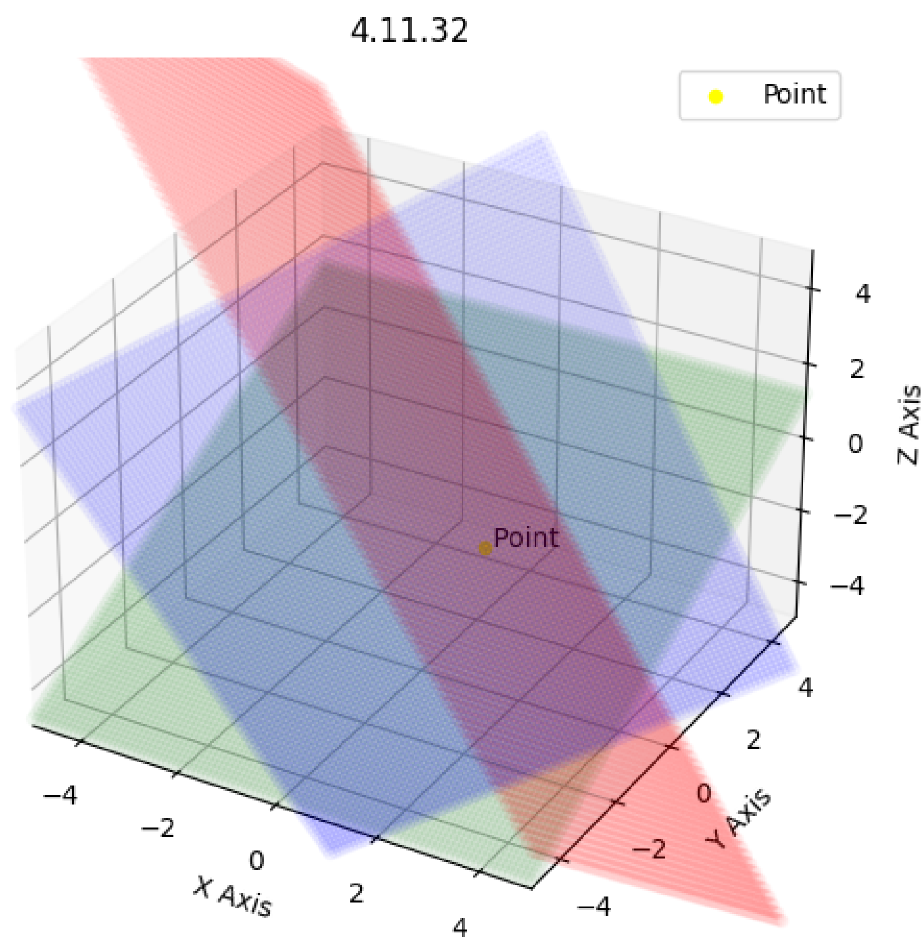
$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & -5 & -3 & 2 \\ 0 & 3 & -5 & 9 \end{array}\right) \xleftrightarrow{R_3 = 5R_3 + 3R_2} \left(\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & -5 & -3 & 2 \\ 0 & 0 & -34 & 51 \end{array}\right) \tag{6}$$

$$\xleftrightarrow{R_3 = -R_3/34; R_2 = R_2 + 3R_3} \left(\begin{array}{ccc|c} 2 & 1 & 1 & 1 \\ 0 & -5 & 0 & -5/2 \\ 0 & 0 & 1 & -3/2 \end{array}\right) \xleftrightarrow{R_2 = -R_2/5; R_1 = R_1 - R_2 - R_3} \left(\begin{array}{ccc|c} 2 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & -3/2 \end{array}\right) \tag{7}$$

$$\xleftrightarrow{R_1 = R_1/2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1/2 \\ 0 & 0 & 1 & -3/2 \end{array}\right) \tag{8}$$

So we have:

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1/2 \\ -3/2 \end{pmatrix} \tag{9}$$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