4.4.33

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Question

Find the value of x such that the four points with position vectors $\mathbf{A}(3\hat{i}+2\hat{j}+\hat{k}), \mathbf{B}(4\hat{i}+x\hat{j}+5\hat{k}), \mathbf{C}(4\hat{i}+2\hat{j}-2\hat{k}),$ and $\mathbf{D}(6\hat{i}+5\hat{j}-\hat{k})$ are coplanar. (12, 2018)

Theoretical Solution

Let us solve the given equation theoretically and then verify the solution computationally
According to the question,
Given four position vectors

$$\mathbf{A} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \ \mathbf{B} = \begin{pmatrix} 4 \\ x \\ 5 \end{pmatrix} \ \mathbf{C} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \ \mathbf{D} = \begin{pmatrix} 6 \\ 5 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{A}^T \mathbf{n} = 1 \tag{2}$$

$$\mathbf{B}^T \mathbf{n} = 1 \tag{3}$$

$$\mathbf{C}^T \mathbf{n} = 1 \tag{4}$$

Theoretical Solution

$$\mathbf{D}^T\mathbf{n} = 1 \tag{5}$$

$$\begin{pmatrix} A & B & C & D \end{pmatrix}^T \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \tag{6}$$

Let

$$\mathbf{i} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \mathbf{z} = \begin{pmatrix} A & B & C & D \end{pmatrix}^T \tag{7}$$

condition is Rank of $\begin{pmatrix} A & B & C & D \end{pmatrix}^T$=3 and $\begin{pmatrix} z & i \end{pmatrix}$=3
From solving we get x=5.

C Code

```
#include <stdio.h>

// Function to calculate the scalar triple product condition for
    coplanarity
double scalar_triple_product_condition(double x) {
    // Components of vectors AB, AC, AD based on x
    // A = (3, 2, 1)
    // B = (4, x, 5)
    // C = (4, 2, -2)
    // D = (6, 5, -1)

    // AB = B - A = (1, x-2, 4)
    double AB_x = 1;
    double AB_y = x - 2;
    double AB_z = 4;

    // AC = C - A = (1, 0, -3)
    double AC_x = 1;
    double AC_y = 0;
```

C Code

```c
double AC_z = -3;

 // AD = D - A = (3, 3, -2)
 double AD_x = 3;
 double AD_y = 3;
 double AD_z = -2;

 // Cross product AC x AD
 double cross_x = AC_y * AD_z - AC_z * AD_y; // 0*(-2) - (-3)*3 = 9
 double cross_y = AC_z * AD_x - AC_x * AD_z; // (-3)*3 - 1*(-2) = -7
 double cross_z = AC_x * AD_y - AC_y * AD_x; // 1*3 - 0*3 = 3

 // Dot product AB . (AC x AD)
 double scalar_triple = AB_x * cross_x + AB_y * cross_y + AB_z * cross_z;
```

C Code

```c

    return scalar_triple;
}

int main() {
    // From the math, we have linear equation 35 - 7x = 0 => x = 5
    // But let's also verify numerically:

    double x = 5.0;
    double result = scalar_triple_product_condition(x);

    printf("Value of x for coplanarity: %.2f\n", x);
    printf("Scalar triple product at x=%.2f: %.2f (should be close to 0)\n", x, result);

    return 0;
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Given vectors (with x unknown)
# A = 3i + 2j + k
A = np.array([3, 2, 1])

# Solve for x such that points are coplanar

# Let x be the unknown coordinate in B's j component and k
    component
# B = 4i + xj + 5k
# C = 4i + 2j - 2k
# D = 6i + 5j - k

# Set up vectors AB, AC, AD
def scalar_triple_product(x):
    B = np.array([4, x, 5])
```

Python Code

```
    C = np.array([4, 2, -2])
   D = np.array([6, 5, -1])
 AB = B - A
   AC = C - A
   AD = D - A

   return np.dot(AB, np.cross(AC, AD))

# Solve for x using the derived formula or by root finding
from scipy.optimize import fsolve

x_solution = fsolve(scalar_triple_product, 0)[0]

# Recalculate B with the found x
B = np.array([4, x_solution, 5])
C = np.array([4, 2, -2])
D = np.array([6, 5, -1])

# 3D plot
```

Python Code

```
fig = plt.figure(figsize=(8,6))
ax = fig.add_subplot(111, projection='3d')
# Plot points
ax.scatter(*A, color='r', label='A')
ax.scatter(*B, color='g', label=f'B (x={x_solution:.2f})')
ax.scatter(*C, color='b', label='C')
ax.scatter(*D, color='purple', label='D')

# Draw vectors from origin for clarity
for vec, name, color in zip([A, B, C, D], ['A', 'B', 'C', 'D'], [
    'r', 'g', 'b', 'purple']):
    ax.text(vec[0], vec[1], vec[2], f'{name}', size=12, color=
        color)

# Plot the plane defined by A, C, D (since points are coplanar)
# Plane normal vector
normal = np.cross(C - A, D - A)
```

Python Code

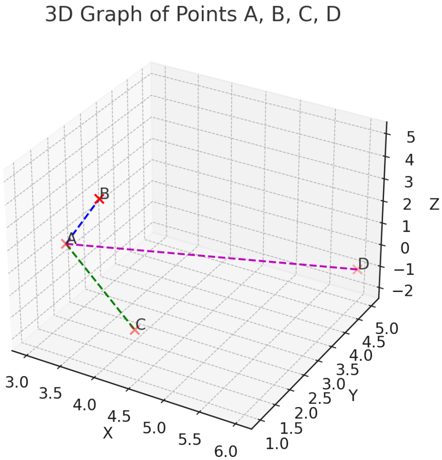
```
   # Create a grid of points on the plane
d = -np.dot(normal, A)
xx, yy = np.meshgrid(np.linspace(2, 7, 10), np.linspace(0, 6, 10)
    )
zz = (-normal[0] * xx - normal[1] * yy - d) / normal[2]

ax.plot_surface(xx, yy, zz, alpha=0.3, color='cyan')

ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('Coplanar Points A, B, C, D')

ax.legend()
plt.savefig('coplanar_points.png')
plt.show()
```

Plot



3D Graph of Points A, B, C, D
A
B
C
D
X
Y
Z
3.0
3.5
4.0
4.5
5.0
5.5
6.0
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
−2
−1
0
1
2
3
4
5