Question

Consider two points P and Q with position vectors

$$\mathbf{OP} = 3\mathbf{a} - 2\mathbf{b}, \qquad \mathbf{OQ} = \mathbf{a} + \mathbf{b}.$$

Find the position vector of a point R which divides the line joining P and Q in the ratio $2:1$,

(a) internally, and

(b) externally.

Solution

In the basis $\{\mathbf{a}, \mathbf{b}\}$, we can write

$$\mathbf{A} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}. \tag{1}$$

(a) Internal Division. If R divides AB in the ratio $k:1$ internally, then

$$\mathbf{R} = \frac{k\mathbf{B} + \mathbf{A}}{k+1}. \tag{2}$$

With $k = 2$

$$\mathbf{R} = \frac{2\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix}}{2+1} \tag{3}$$

$$= \frac{\begin{pmatrix} 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ -2 \end{pmatrix}}{3} \tag{4}$$

$$= \frac{\begin{pmatrix} 5 \\ 0 \end{pmatrix}}{3} \tag{5}$$

$$= \begin{pmatrix} \frac{5}{3} \\ 0 \end{pmatrix}. \tag{6}$$

$$\boxed{\mathbf{R}_{\text{internal}} = \begin{pmatrix} \frac{5}{3} \\ 0 \end{pmatrix}}$$

(b) External Division. If R divides AB in the ratio $k:1$ externally, then

$$\mathbf{R} = \frac{k\mathbf{B} - \mathbf{A}}{k-1}. \tag{7}$$

With $k = 2$

$$\mathbf{R} = \frac{2\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix}}{2-1} \tag{8}$$

$$= \begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix} \tag{9}$$

$$= \begin{pmatrix} -1 \\ 4 \end{pmatrix} \tag{10}$$

$$\boxed{\mathbf{R}_{\text{external}} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}}$$

From the calculations above, we obtain:

$$\boxed{\mathbf{R}_{\text{internal}} = \tfrac{5}{3}\mathbf{a}}$$

$$\boxed{\mathbf{R}_{\text{external}} = -\mathbf{a} + 4\mathbf{b}}$$