Matgeo Presentation - Problem 4.10.23

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Problem Statement

Find the equation of the line passing through the point of intersection of $2x + y = 5$ and $x + 3y + 8 = 0$ and parallel to the line $3x + 4y = 7$.

solution

The two given lines are written in matrix form as

$$l_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \mathbf{x} = 5, \tag{0.1}$$

$$l_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \mathbf{x} = -8. \tag{0.2}$$

$$l_3 = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \mathbf{x} = 7. \tag{0.3}$$

$$\tag{0.4}$$

Normals and constants for the given lines l_1 and l_2.

$$\mathbf{n}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad c_1 = 5 \tag{0.5}$$

$$\mathbf{n}_2 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \quad c_2 = -8 \tag{0.6}$$

General family of lines through the intersection, written as $l_1 + \lambda l_2$.

solution

$$(\mathbf{n}_1^T\mathbf{x} - c_1) + \lambda(\mathbf{n}_2^T\mathbf{x} - c_2) = 0 \tag{0.7}$$

Explicit expanded form of the family of lines.

$$\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{x} - 5 + \lambda\left(\begin{pmatrix}1 & 3\end{pmatrix}\mathbf{x} + 8\right) = 0 \tag{0.8}$$

$$\implies \begin{pmatrix}2+\lambda & 1+3\lambda\end{pmatrix}\mathbf{x} = 5 - 8\lambda. \tag{0.9}$$

Normal of the line l_3, to which our required line must be parallel.

$$\mathbf{m} = \begin{pmatrix}3\\4\end{pmatrix} \tag{0.10}$$

Normal vector of the family of lines.

$$\mathbf{N}(\lambda) = \mathbf{n}_1 + \lambda\mathbf{n}_2 = \begin{pmatrix}2+\lambda\\1+3\lambda\end{pmatrix} \tag{0.11}$$

solution

Condition for parallelism with $\mathbf{m}$.

$$\mathbf{N}(\lambda) = \alpha\mathbf{m} \implies \begin{pmatrix} 2+\lambda \\ 1+3\lambda \end{pmatrix} = \alpha \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{0.12}$$

Solve these equations to determine λ.

$$3\alpha - \lambda = 2 \tag{0.13}$$
$$4\alpha - 3\lambda = 1 \tag{0.14}$$

augmented matrix

$$\left(\begin{array}{cc|c} 3 & -1 & 2 \\ 4 & -3 & 1 \end{array}\right) \tag{0.15}$$

$$R_1 \to \tfrac{1}{3} R_1 \tag{0.16}$$
$$\left(\begin{array}{cc|c} 3 & -1 & 2 \\ 4 & -3 & 1 \end{array}\right) \Rightarrow \left(\begin{array}{cc|c} 1 & -\frac{1}{3} & \frac{2}{3} \\ 4 & -3 & 1 \end{array}\right) \tag{0.17}$$

solution

$$\lambda = 1 \tag{0.24}$$

substituting value of λ in eq(0.9) .Final equation of the required line in matrix form.

$$\begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} + 3 = 0 \tag{0.25}$$

C Source Code:line generator.c

```
/* line_generator.c */
#include <stdio.h>
/* Generate points along line N^T x + c = 0
   Nx, Ny = components of normal vector
   c = constant
   tmin, tmax = steps along direction perpendicular to N
*/
void generate_line_points(double Nx, double Ny, double c, int
    double dx = -Ny;  // direction vector perpendicular to no
    double dy = Nx;
// Pick a point on the line: x0 = 0, y0 = -c/Ny
    double x0 = 0.0;
    double y0 = -c / Ny;
    for (int t = tmin; t <= tmax; t++) {
        double x = x0 + t*dx;
        double y = y0 + t*dy;
        printf("%.6f %.6f\n", x, y);
```

Python Script:solve matrix line.py

```
import ctypes
import numpy as np

# --- Step 0: Load C library ---
lib = ctypes.CDLL("./line_generator.so")
lib.generate_line_points.argtypes = [ctypes.c_double, ctypes.c
                                     ctypes.c_int, ctypes.c_in

# --- Step 1: Matrix method to find required line ---
n1 = np.array([2, 1])
c1 = 5
n2 = np.array([1, 3])
c2 = -8
n3 = np.array([3, 4])  # line to be parallel
```

Python Script:solve matrix line.py

```
# Family of lines: N(lambda) = n1 + lambda*n2
# Parallel condition: N(lambda) = alpha * n3
# Solve for lambda:
lambda_val = 1  # from calculation
# Normal vector and constant of required line
N = n1 + lambda_val * n2
c = c1 + lambda_val * c2

print("Normal vector of required line:", N)
print("Constant term:", c)
print(f"Equation of line: {N[0]}x + {N[1]}y + {c} = 0\n")

# --- Step 2: Call C function to generate points ---
print("Points on the line:")
lib.generate_line_points(float(N[0]), float(N[1]), float(c),
-5, 5)
```

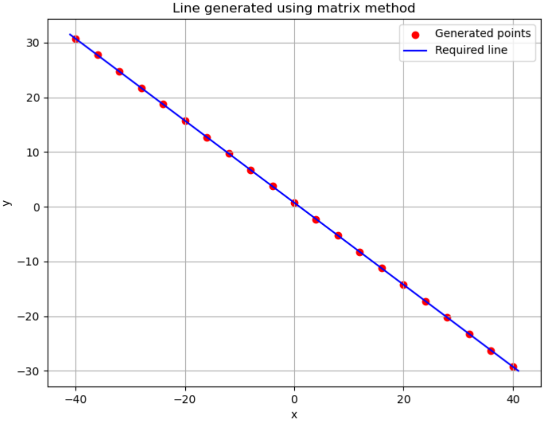
Python Script: plot matrix line.py

```
import numpy as np
import matplotlib.pyplot as plt
# --- Step 1: Matrix method to find required line ---
n1 = np.array([2, 1])
c1 = 5
n2 = np.array([1, 3])
c2 = -8
n3 = np.array([3, 4])  # line to be parallel
lambda_val = 1  # from calculation
# Normal vector and constant of required line
N = n1 + lambda_val * n2
c = c1 + lambda_val * c2
print("Normal vector of required line:", N)
print("Constant term:", c)
print(f"Equation of line: {N[0]}x + {N[1]}y + {c} = 0\n")
# --- Step 2: Generate points along the line in Python ---
d = np.array([-N[1], N[0]])
```

Python Script: plot lines plane.py

```
t_values = np.linspace(-10, 10, 21)  # 21 points
# Pick a point on the line: x0 = 0, y0 = -c/N[1]
x0 = 0
y0 = -c / N[1]
points = np.array([ [x0 + t*d[0], y0 + t*d[1]] for t in t_valu
# --- Step 3: Plot ---
plt.figure(figsize=(8,6))
plt.scatter(points[:,0], points[:,1], color='red', label='Gene
# Plot the exact line
x_vals = np.linspace(points[:,0].min()-1, points[:,0].max()+1,
y_vals = -(N[0]*x_vals + c)/N[1]
plt.plot(x_vals, y_vals, label='Required line', color='blue')
plt.xlabel('x')
plt.ylabel('y')
plt.title('Line generated using matrix method')
plt.grid(True)
plt.legend() plt.axis('equal') plt.show()
```

Result Plot


Line generated using matrix method
Generated points
Required line
30
20
10
0
−10
−20
−30
y
−40
−20
0
20
40
x