4.13.85

AI25BTECH11016-Varun

Question:
If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then the value of k is
Solution:
The vector equation of a line $\frac{x-\alpha}{a} = \frac{y-\beta}{b} = \frac{z-\gamma}{c} is \begin{pmatrix} \alpha + a\lambda \\ \beta + b\lambda \\ \gamma + c\lambda \end{pmatrix}$
The vector equation of first line,

$$\begin{pmatrix} 1+2\lambda \\ -1+3\lambda \\ 1+4\lambda \end{pmatrix} \tag{1}$$

The vector equation of Second line,

$$\begin{pmatrix} 3+\lambda \\ k+2\lambda \\ \lambda \end{pmatrix} \tag{2}$$

The lines $A + K_1 m_1, B + K_2 m_2$ will intersect if

$$\text{rank}(\mathbf{M} \quad \mathbf{B}-\mathbf{A}) = 2 \tag{3}$$

$$\mathbf{M} = \begin{pmatrix} m_1 & m_2 \end{pmatrix} \tag{4}$$

Here,

$$\mathbf{M} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ 4 & 1 \end{pmatrix} \tag{5}$$

$$\mathbf{B}-\mathbf{A} = \begin{pmatrix} 3-1 \\ k-(-1) \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ k+1 \\ -1 \end{pmatrix} \tag{6}$$

$$\text{rank}\left(\begin{pmatrix} 2 & 1 & 2 \\ 3 & 2 & k+1 \\ 4 & 1 & -1 \end{pmatrix}\right) = 2 \tag{7}$$

$$\begin{pmatrix} 2 & 1 & 2 \\ 3 & 2 & k+1 \\ 4 & 1 & -1 \end{pmatrix} \xrightarrow[R_3 \to 2R_3 - 4R_1]{R_2 \to 2R_2 - 3R_1} \begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & 2k-4 \\ 0 & -2 & -10 \end{pmatrix} \xrightarrow{R_3 \to R_3 + 2R_2} \begin{pmatrix} 2 & 1 & 2 \\ 0 & 1 & 2k-4 \\ 0 & 0 & 4k-18 \end{pmatrix}$$

For the rank$(\mathbf{M} \quad \mathbf{B}-\mathbf{A})$ to be 2
the last row must be all zero implies

$$4k - 18 = 0 \tag{8}$$

$$k = \frac{9}{2} \tag{9}$$

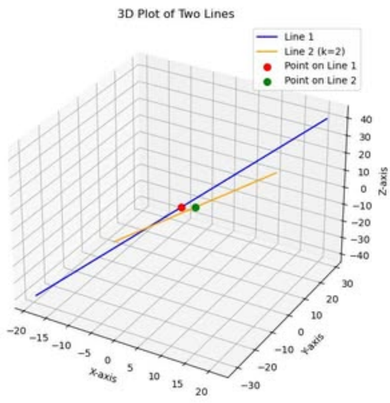

Fig. 0.1