

12.601

AI25BTECH11003 - Bhavesh Gaikwad

Question: The matrix $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$, one of the eigen values is 1. The eigen vectors corresponding to the eigen value 1 are: (CS 2016)

a) $\alpha \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix}, \alpha \neq 0, \alpha \in \mathbb{R}$

b) $\alpha \begin{pmatrix} -4 \\ 2 \\ 1 \end{pmatrix}, \alpha \neq 0, \alpha \in \mathbb{R}$

c) $\alpha \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}, \alpha \neq 0, \alpha \in \mathbb{R}$

d) $\alpha \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \alpha \neq 0, \alpha \in \mathbb{R}$

Solution:

Given: $\lambda = 1$, Let $\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$

$$\mathbf{A} - \mathbf{I} = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{pmatrix} \quad (0.1)$$

Row Transformation-1: $R_1 \leftrightarrow R_3$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 2 \end{pmatrix} \quad (0.2)$$

Row Transformation-2: $R_2 \leftrightarrow R_3$

$$\mathbf{A} - \mathbf{I} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \quad (0.3)$$

Let \mathbf{v} be the corresponding eigenvector.

$$\Rightarrow \mathbf{A}\mathbf{v} = (1)\mathbf{v} \quad (0.4)$$

$$(\mathbf{A} - \mathbf{I})\mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (0.5)$$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{v} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (0.6)$$

$$\text{Let } \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

Substituting value of \mathbf{v} in Equation 0.6,

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (0.7)$$

$$\text{Row} - 1 \rightarrow v_2 + 2v_2 = 0 \quad (0.8)$$

$$\text{Row} - 2 \rightarrow v_1 + 2v_3 = 0 \quad (0.9)$$

$$\text{Row} - 3 \rightarrow 0 + 0 + 0 = 0 \text{ (Always true)} \quad (0.10)$$

Let $v_3 = \alpha$ (Free parameter)

Substituting value of v_3 in Equations 0.8 and 0.9

$$\therefore v_2 = -2\alpha \text{ \& } v_1 = 4\alpha \quad (0.11)$$

$$\therefore \mathbf{v} = \alpha \begin{pmatrix} 4 \\ -2 \\ 1 \end{pmatrix} \quad (0.12)$$

Thus, Option-A is correct.