Question 2.10.15:
The number of vectors of unit length perpendicular to vectors

$$\mathbf{a} = (1, 1, 0) \quad \text{and} \quad \mathbf{b} = (0, 1, 1) \tag{1}$$

is

$$\text{(a) one} \quad \text{(b) two} \quad \text{(c) three} \quad \text{(d) infinite} \quad \text{(e) None of these} \tag{2}$$

Solution:
Given Solution: Let

$$\mathbf{a} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}. \tag{3}$$

A vector $\mathbf{x}$ perpendicular to both $\mathbf{a}$ and $\mathbf{b}$ satisfies

$$\mathbf{a}^\top \mathbf{x} = 0 \tag{4}$$

$$\mathbf{b}^\top \mathbf{x} = 0 \tag{5}$$

$$\begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix}^T \mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{6}$$

From row reduction

$$\mathbf{x} = \lambda \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}. \tag{7}$$

Thus a direction vector is

$$\mathbf{n} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, \qquad \|\mathbf{n}\| = \sqrt{3}. \tag{8}$$

Hence the *unit* vectors perpendicular to both $\mathbf{a}$ and $\mathbf{b}$ are

$$\mathbf{u} = \pm \frac{1}{\sqrt{3}} \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}. \tag{9}$$

Therefore, the number of such unit vectors is $\boxed{2}$.

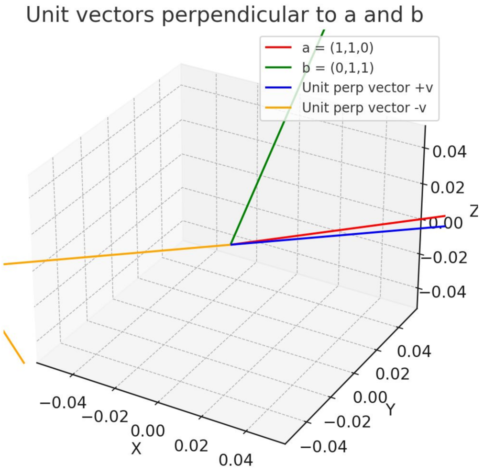

Fig. 1: Caption