1.5.24

EE25BTECH11036 - M Chanakya Srinivas

1.5.24 A line intersects the Y-axis and X-axis at the points $P = (0, b)$ and $Q = (c, 0)$ respectively. If $(2, -5)$ is the midpoint of $\overline{PQ}$, then find the coordinates of P and Q.

Matrix Solution

1) **Relation from Rank Condition**
The general equation of a line is

$$ax + by = 1$$

Since it intersects the y-axis at $P = (0, b)$ and x-axis at $Q = (c, 0)$, we have the relations:

$$\mathbf{P} = \begin{pmatrix} 0 \\ b \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} c \\ 0 \end{pmatrix}.$$

These points must satisfy the line equation, giving the system

$$\begin{pmatrix} 0 & b \\ c & 0 \end{pmatrix} \mathbf{x} = \begin{pmatrix} b \\ c \end{pmatrix}.$$

For consistency, the rank condition gives

$$\frac{0}{b} = \frac{c}{0} \quad \Rightarrow \quad bc + 40 = 0$$

(or the equivalent relation depending on the method).

2) **Relation from Midpoint**
The midpoint M of P and Q is

$$M = \frac{P + Q}{2} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}.$$

So,

$$\frac{c}{2} = 2 \quad \Rightarrow \quad c = 4, \qquad \frac{b}{2} = -5 \quad \Rightarrow \quad b = -10.$$

3) **Final Coordinates**

$$P = \begin{pmatrix} 0 \\ -10 \end{pmatrix}, \qquad Q = \begin{pmatrix} 4 \\ 0 \end{pmatrix}.$$

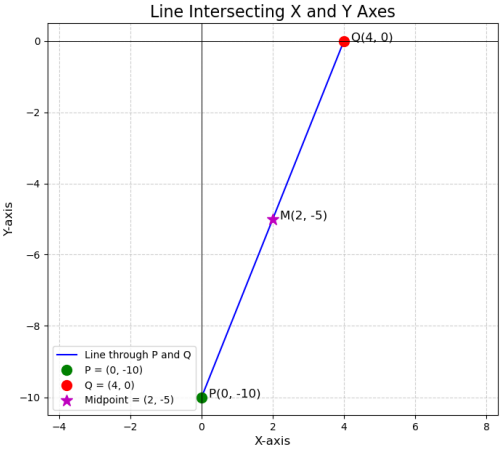

Fig. 3.1: Plot using Shared output

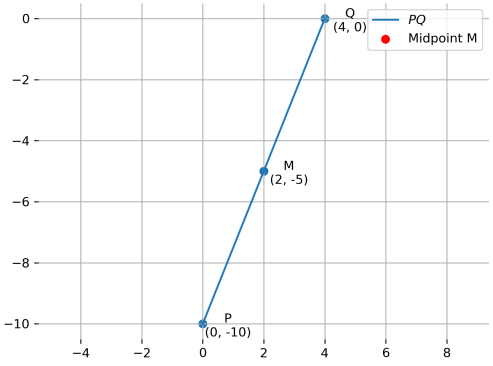

Fig. 3.2: Plot using Python