4.12.2

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Question

For which value of k will the following pair of linear equations have no solution?

$$3x + y = 1$$
$$(2k-1)x + (k-1)y = 2k+1$$

Theoretical Solution

The given system of equations is

Line	Vector Form
l_1	$\begin{pmatrix} 3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1$
l_2	$\begin{pmatrix} 2k-1 & k-1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 2k+1$

Table: Answers

In matrix form:

$$\begin{pmatrix} 3 & 1 \\ 2k-1 & k-1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2k+1 \end{pmatrix} \tag{1}$$

Now, form the augmented matrix:

$$\left(\begin{array}{cc|c} 3 & 1 & 1 \\ 2k-1 & k-1 & 2k+1 \end{array}\right) \tag{2}$$

Theoretical Solution

Perform row reduction:

$$R_1 \to \tfrac{1}{3}R_1 \quad \Rightarrow \quad \left(\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{1}{3} \\ 2k-1 & k-1 & 2k+1 \end{array}\right) \tag{3}$$

$$R_2 \to R_2 - (2k-1)R_1 \quad \Rightarrow \quad \left(\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{k-2}{3} & \frac{4k+4}{3} \end{array}\right) \tag{4}$$

$$R_2 \to \tfrac{3}{k-2}R_2 \quad \Rightarrow \quad \left(\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & \frac{4k+4}{k-2} \end{array}\right) \tag{5}$$

$$R_1 \to R_1 - \tfrac{1}{3}R_2 \quad \Rightarrow \quad \left(\begin{array}{cc|c} 1 & 0 & \frac{1}{3} - \frac{1}{3}\cdot\frac{4k+4}{k-2} \\ 0 & 1 & \frac{4k+4}{k-2} \end{array}\right) \tag{6}$$

Theoretical Solution

For inconsistency, we need:

$$k - 2 = 0 \quad \text{and} \quad 4k + 4 \neq 0 \tag{7}$$

So,

$$k = 2, \quad 4(2) + 4 = 12 \neq 0 \tag{8}$$

$$\boxed{k = 2}$$

C Code

```c
#include<stdio.h>
double solve_for_k(void) {
    double a1 = 3.0;
    double b1 = 1.0;
    double c1 = 1.0;
    double a2_k_coeff = 2.0;
    double a2_const = -1.0;
    double b2_k_coeff = 1.0;
    double b2_const = -1.0;
    double c2_k_coeff = 2.0;
    double c2_const = 1.0;
    double k_coeff_det = a1 * b2_k_coeff - b1 * a2_k_coeff;
    double const_det = a1 * b2_const - b1 * a2_const;
    double k = -const_det / k_coeff_det;

    return k;
}
```

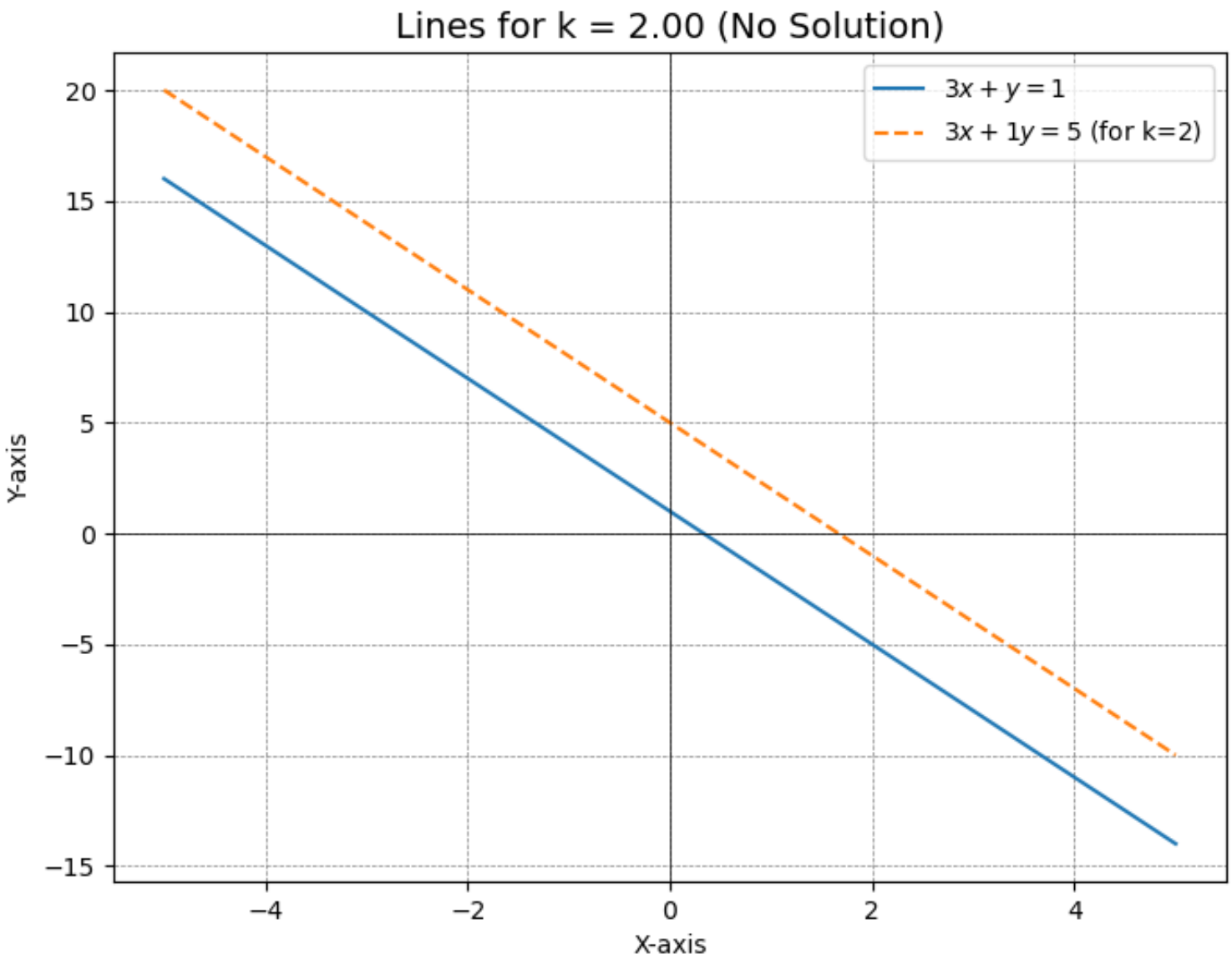
Python Code

```
import numpy as np
import matplotlib.pyplot as plt

k_value = 2.0
print(fThe value of k for which there is no solution is:)
print(f k = {k_value:.2f})

x = np.linspace(-5, 5, 100)

y1 = 1 - 3 * x

y2 = (2 * k_value + 1 - (2 * k_value - 1) * x) / (k_value - 1)

plt.figure(figsize=(8, 6))
plt.plot(x, y1, label=r'$3x + y = 1$')
plt.plot(x, y2, label=f'${{{(2*k_value-1):.0f}}}x + {{{(k_value
    -1):.0f}}}y = {{{(2*k_value+1):.0f}}}$ (for k={k_value:.0f})'
    , linestyle='--')
```

Python Code

```
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.title(f'Lines for k = {k_value:.2f} (No Solution)', fontsize
    =14)
plt.axhline(0, color='black',linewidth=0.5)
plt.axvline(0, color='black',linewidth=0.5)
plt.grid(color = 'gray', linestyle = '--', linewidth = 0.5)
plt.legend()
plt.show()
```

Plot


Lines for k = 2.00 (No Solution)
3x + y = 1
3x + 1y = 5 (for k=2)
Y-axis
X-axis
20
15
10
5
0
−5
−10
−15
−4
−2
0
2
4