1.11.9

AI25BTECH11007

Question:
If
$$\mathbf{a} = \hat{\imath} - 7\hat{\jmath} + 7\hat{k} \quad \text{and} \quad \mathbf{b} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k},$$
find a unit vector perpendicular to both the vectors **a** and **b**.
Solution:
We need a vector **n** such that
$$\mathbf{n} \cdot \mathbf{a} = 0, \quad \mathbf{n} \cdot \mathbf{b} = 0 \tag{0.1}$$
where
$$\mathbf{a} = \begin{pmatrix} 1 \\ -7 \\ 7 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}. \tag{0.2}$$
Let
$$\mathbf{n} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \tag{0.3}$$
Orthogonality conditions
$$\mathbf{n} \cdot \mathbf{a} = x - 7y + 7z = 0 \tag{0.4}$$
$$\mathbf{n} \cdot \mathbf{b} = 3x - 2y + 2z = 0 \tag{0.5}$$
Solve equations
From (**??**),
$$x = 7y - 7z. \tag{0.6}$$
Substitute (**??**) into (**??**):
$$3(7y - 7z) - 2y + 2z = 0 \tag{0.7}$$
$$21y - 21z - 2y + 2z = 0 \tag{0.8}$$
$$19y - 19z = 0 \tag{0.9}$$
$$y = z. \tag{0.10}$$
From (**??**) and (**??**):
$$x = 7y - 7y = 0. \tag{0.11}$$
Thus,
$$\mathbf{n} = \begin{pmatrix} 0 \\ y \\ y \end{pmatrix} = y \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}. \tag{0.12}$$

Normalize

$$\hat{n} = \frac{\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}}{\sqrt{0^2 + 1^2 + 1^2}} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{0.13}$$

Hence, a unit vector perpendicular to both **a** and **b** is

$$\hat{n} = \frac{1}{\sqrt{2}}(\hat{j} + \hat{k}), \tag{0.14}$$

or its negative.

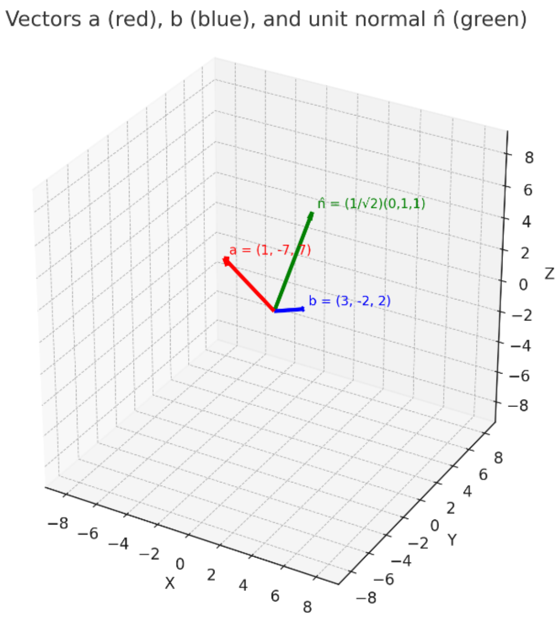

Fig. 0.1: Image Visual