9.4.14

EE25BTECH11002 - Achat Parth Kalpesh

Question:
Find the roots of the following quadratic equations grapically

$$(x-3)(2x-1) = x(x+5) \tag{0.1}$$

Solution:

$$y = (x-3)(2x-1) - x(x+5) = 0 \tag{0.2}$$
$$y = x^2 - 12x + 3 = 0 \tag{0.3}$$

This quadratic can be represented as a conic in matrix form:

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.4}$$
$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -6 \\ 0 \end{pmatrix}, f = 3 \tag{0.5}$$

To find the roots, we find the points of intersection of the conic with the x-axis.

$$\mathbf{x} = \mathbf{h} + k_i \mathbf{m} \tag{0.6}$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.7}$$

The value of k_i can be found out by solving the line and conic equation

$$(\mathbf{h} + k_i\mathbf{m})^\top \mathbf{V}(\mathbf{h} + k_i\mathbf{m}) + 2\mathbf{u}^\top (\mathbf{h} + k_i\mathbf{m}) + f = 0 \tag{0.8}$$
$$\implies k_i^2 \mathbf{m}^\top \mathbf{V}\mathbf{m} + 2k_i\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) + \mathbf{h}^\top \mathbf{V}\mathbf{h} + 2\mathbf{u}^\top \mathbf{h} + f = 0 \tag{0.9}$$
$$\text{or, } k_i^2 \mathbf{m}^\top \mathbf{V}\mathbf{m} + 2k_i\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) + g(\mathbf{h}) = 0 \tag{0.10}$$

Solving the above quadratic gives the equation

$$k_i = \frac{1}{\mathbf{m}^\top \mathbf{V}\mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{\left[\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u})\right]^2 - g(\mathbf{h})\ (\mathbf{m}^\top \mathbf{V}\mathbf{m})} \right) \tag{0.11}$$

Substituting the values in the above equation gives

$$\therefore k_i = 6 \pm \sqrt{33} \tag{0.12}$$

$$k_1 = 6 + \sqrt{33} \tag{0.13}$$
$$k_2 = 6 - \sqrt{33} \tag{0.14}$$

$$\mathbf{x} = \mathbf{h} + k_i\mathbf{m} = \begin{pmatrix} 6 + \sqrt{33} \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 6 - \sqrt{33} \\ 0 \end{pmatrix} \tag{0.15}$$

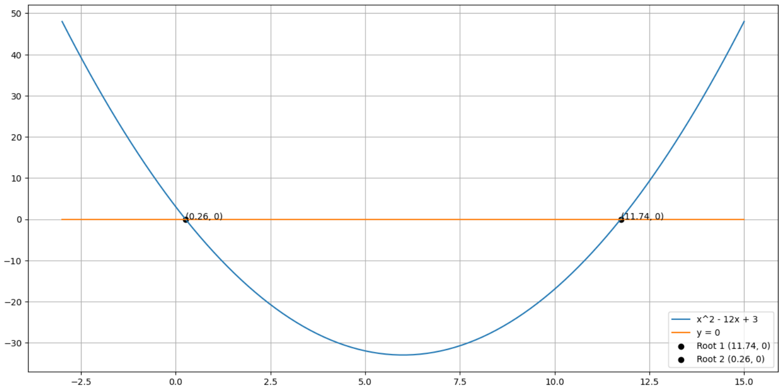

Fig. 0.1: Graph