2.10.63

EE25BTECH11034 - Kishora Karthik

Question:
A vector $\mathbf{A}$ has components A_1, A_2, A_3 in a right-handed rectangular Cartesian coordinate system $oxyz$. The coordinate system is rotated about the x-axis through an angle $\frac{\pi}{2}$. Find the components of $\mathbf{A}$ in the new coordinate system in terms of A_1, A_2, A_3.
Solution:
In the original coordinate system S,

$$\mathbf{A_S} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} \tag{1}$$

Let the new coordinate system be S', obtained by rotating S about the x-axis by an angle $\theta = \frac{\pi}{2}$. The components of the same vector $\mathbf{A}$ in the new system are,

$$\begin{pmatrix} A_1' \\ A_2' \\ A_3' \end{pmatrix} = R \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} \tag{2}$$

where R is the rotation matrix. For a rotation of the coordinate system by an angle θ about the x-axis,

$$R_x(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \tag{3}$$

Given, $\theta = \frac{\pi}{2}$.So,

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\frac{\pi}{2}) & \sin(\frac{\pi}{2}) \\ 0 & -\sin(\frac{\pi}{2}) & \cos(\frac{\pi}{2}) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} A_1' \\ A_2' \\ A_3' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} \tag{5}$$

$$A_1' = (1 \cdot A_1) + (0 \cdot A_2) + (0 \cdot A_3) = A_1 \tag{6}$$

$$A_2' = (0 \cdot A_1) + (0 \cdot A_2) + (1 \cdot A_3) = A_3 \tag{7}$$

$$A_3' = (0 \cdot A_1) + (-1 \cdot A_2) + (0 \cdot A_3) = -A_2 \tag{8}$$

$$\implies \begin{pmatrix} A'_1 \\ A'_2 \\ A'_3 \end{pmatrix} = \begin{pmatrix} A_1 \\ A_3 \\ -A_2 \end{pmatrix} \tag{9}$$

$\therefore$ The components of the vector $\mathbf{A}$ in the new coordinate system are: $A'_1 = A_1$, $A'_2 = A_3$ and $A'_3 = -A_2$.