1.4.13

AI25BTECH11038 – Tejas Uppala

Question:
Given that $\mathbf{P}(3,2,-4)$, $\mathbf{Q}(5,4,-6)$ and $\mathbf{R}(9,8,-10)$ are collinear. Find the ratio in which $\mathbf{Q}$ divides PR.
Solution:

$$P = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}, \qquad Q = \begin{pmatrix} 5 \\ 4 \\ -6 \end{pmatrix}, \qquad R = \begin{pmatrix} 9 \\ 8 \\ -10 \end{pmatrix} \tag{1}$$

From the section formula,

$$\mathbf{Q} = \frac{k\mathbf{P} + \mathbf{R}}{k+1} \tag{2}$$

for some scalar k. Where $\mathbf{Q}$ divides PR in the ratio $k:1$.
From equation (1):

$$(\mathbf{R} - \mathbf{P})\, t = (\mathbf{Q} - \mathbf{P}) \tag{3}$$

$$k = \frac{(\mathbf{Q} - \mathbf{P})\,(\mathbf{R} - \mathbf{P})^T}{\|\mathbf{R} - \mathbf{P}\|^2} \tag{4}$$

$$(\mathbf{R} - \mathbf{P}) = \begin{pmatrix} 6 \\ 6 \\ -6 \end{pmatrix}, \qquad (\mathbf{Q} - \mathbf{P}) = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix}. \tag{5}$$

$$\mathbf{R} - \mathbf{P} = \begin{pmatrix} 6 \\ 6 \\ -6 \end{pmatrix}, \qquad \mathbf{Q} - \mathbf{P} = \begin{pmatrix} 2 \\ 2 \\ -2 \end{pmatrix} \tag{6}$$

$$(\mathbf{Q} - \mathbf{P})\,(\mathbf{R} - \mathbf{P})^T = \begin{pmatrix} 2 & 2 & -2 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \\ -6 \end{pmatrix} = 2 \cdot 6 + 2 \cdot 6 + (-2)(-6) = 36 \tag{7}$$

$$(\mathbf{R} - \mathbf{P})\,(\mathbf{R} - \mathbf{P})^T = \begin{pmatrix} 6 & 6 & -6 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \\ -6 \end{pmatrix} = 6^2 + 6^2 + (-6)^2 = 108 \tag{8}$$

$$\therefore \quad k = \frac{36}{108} = \frac{1}{3} \tag{9}$$

$$\text{Thus, } PQ : QR = k : (1-k) = \tfrac{1}{3} : \tfrac{2}{3} = 1 : 2 \tag{10}$$

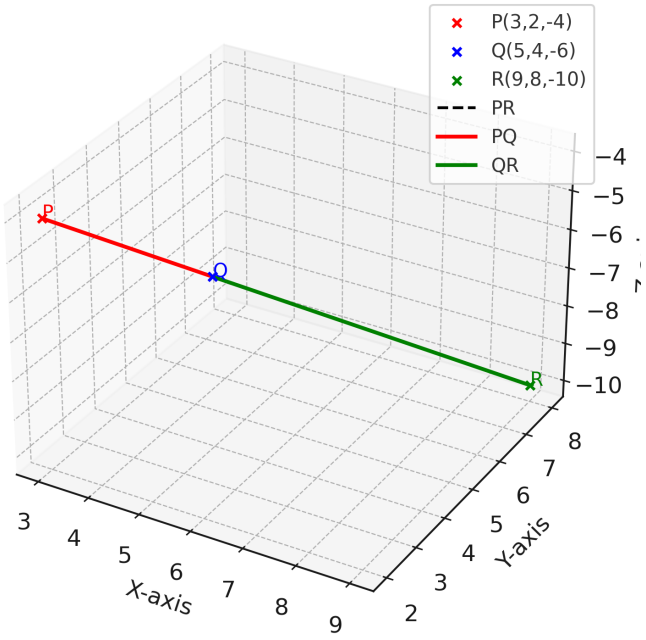

Fig. 1: Plot of the points P, Q and R