2.6.12

AI25BTECH11006 - Nikhila

31-08-2025

Question

Find the sine of the angle between the vectors $\mathbf{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\mathbf{b} = 2\hat{i} + -2\hat{j} + 4\hat{k}$.

Solution

The given vectors are $\vec{a} \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ and $\vec{b} \begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix}$

We know that

$$\cos \theta = \frac{\vec{a}^T \vec{b}}{\|\vec{a}\| \, \|\vec{b}\|} \tag{1}$$

and

$$\sin \theta = \sqrt{1 - \cos^2 \theta} \tag{2}$$

$$\vec{a}^T \vec{b} = 3(2) + 1(-2) + 2(4) = 6 - 2 + 8 = 12 \tag{3}$$

Solution

$$\|\vec{a}\| = \sqrt{(3)^2 + (1)^2 + (2)^2} = \sqrt{14}, \tag{4}$$

$$\|\vec{b}\| = \sqrt{(2)^2 + (-2)^2 + (4)^2} = \sqrt{24}. \tag{5}$$

$$\cos\theta = \frac{12}{\sqrt{14} \cdot \sqrt{24}} \tag{6}$$

$$= \frac{12}{\sqrt{336}} \tag{7}$$

$$= \frac{\sqrt{14}}{2\sqrt{6}} \tag{8}$$

Solution

$$\sin\theta = \sqrt{1 - \left(\frac{\sqrt{14}}{2\sqrt{6}}\right)^2} \tag{9}$$

$$= \sqrt{1 - \frac{14}{4 \cdot 6}} \tag{10}$$

$$= \sqrt{1 - \frac{14}{24}} \tag{11}$$

$$= \sqrt{\frac{10}{24}} \tag{12}$$

$$= \sqrt{\frac{5}{12}} \tag{13}$$

Angle Between Vectors a and b