4.7.11

EE25BTECH11025 - Ganachari Vishwambhar

Question:

Show that the path of a moving point such that its distance from two lines 3x - 2y = 5 and 3x + 2y = 5 are equal is a straight line.

Solution:

Given line equations can be written as:

$$\mathbf{n}_1^{\mathsf{T}} \mathbf{x} = c_1 \tag{1}$$

1

$$\mathbf{n}_1 = \begin{pmatrix} 3 \\ -2 \end{pmatrix}; c_1 = 5 \tag{2}$$

$$\mathbf{n}_2^{\mathsf{T}}\mathbf{x} = c_2 \tag{3}$$

$$\mathbf{n}_2 = \begin{pmatrix} 3\\2 \end{pmatrix}; c_2 = 5 \tag{4}$$

let the point equidistant from the given lines be:

$$\mathbf{P} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{5}$$

Proof:

From distance formula:

$$d_1 = \frac{|\mathbf{n}_1^{\mathsf{T}} \mathbf{P} - c_1|}{\|\mathbf{n}_1\|} \tag{6}$$

$$d_2 = \frac{|\mathbf{n}_2^{\mathsf{T}} \mathbf{P} - c_2|}{\|\mathbf{n}_2\|} \tag{7}$$

$$\therefore d_1 = d_2 \tag{8}$$

$$\frac{|\mathbf{n}_1^{\mathsf{T}} \mathbf{P} - c_1|}{\|\mathbf{n}_1\|} = \frac{|\mathbf{n}_2^{\mathsf{T}} \mathbf{P} - c_2|}{\|\mathbf{n}_2\|}$$
(9)

$$\mathbf{n}_1^{\mathsf{T}} \mathbf{P} - c_1 = \pm \left(\mathbf{n}_2^{\mathsf{T}} \mathbf{P} - c_2 \right) \tag{11}$$

First, by taking +:

$$\mathbf{n}_1^{\mathsf{T}} \mathbf{P} - c_1 = + \left(\mathbf{n}_2^{\mathsf{T}} \mathbf{P} - c_2 \right) \tag{12}$$

$$\mathbf{n}_1^{\mathsf{T}} \mathbf{P} - \mathbf{n}_2^{\mathsf{T}} \mathbf{P} = c_1 - c_2 \tag{13}$$

$$(\mathbf{n}_1 - \mathbf{n}_2)^{\mathsf{T}} \mathbf{P} = c_1 - c_2 \tag{14}$$

$$\begin{pmatrix} 0 & -4 \end{pmatrix} \mathbf{P} = 0 \tag{15}$$

Now by taking -:

$$\mathbf{n}_1^{\mathsf{T}} \mathbf{P} - c_1 = -\left(\mathbf{n}_2^{\mathsf{T}} \mathbf{P} - c_2\right) \tag{16}$$

$$\mathbf{n}_1^{\mathsf{T}} \mathbf{P} + \mathbf{n}_2^{\mathsf{T}} \mathbf{P} = c_1 + c_2 \tag{17}$$

$$(\mathbf{n}_1 + \mathbf{n}_2)^{\mathsf{T}} \mathbf{P} = c_1 + c_2 \tag{18}$$

$$\begin{pmatrix} 6 & 0 \end{pmatrix} \mathbf{P} = 10 \tag{19}$$

Since equations (15) and (19) are in the form of line equation $\mathbf{n}^{\mathsf{T}}\mathbf{x} = c$, the given path of the moving point is a line.

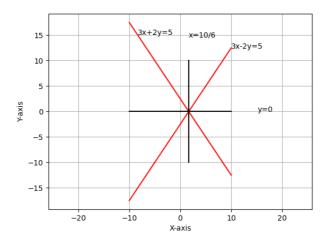


Fig. 1: Plot of the given lines and path of the moving point