5.6.4

AI25BTECH11018-Hemanth Reddy

Question:
If $\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find k so that $\mathbf{A}^2 = k\mathbf{A} - 2\mathbf{I}$.
Solution:

The characteristic equation for a matrix **A** is f(λ) = **A** - λ **I** = 0

$$\mathbf{A} - \lambda\mathbf{I} = \begin{vmatrix} 3-\lambda & -2 \\ 4 & -2-\lambda \end{vmatrix} = 0 \tag{0.1}$$

Upon expanding we get λ^2- λ+2=0

$$\lambda^2 = \lambda - 2 \tag{0.2}$$

Using the Cayley-Hamilton theorem f(λ) = f(**A**)=0

$$\mathbf{A}^2 = \mathbf{A} - 2\mathbf{I} \tag{0.3}$$

Value of k=1