2.7.14

EE25BTECH11002 - Achat Parth Kalpesh

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Question

If θ is the angle between the two vectors $\mathbf{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\mathbf{b} = 3\hat{i} - 2\hat{j} + \hat{k}$, find $\sin\theta$.

Theoretical Solution

Let the given vectors be represented by column matrices $\mathbf{a}$ and $\mathbf{b}$.

$$\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} \tag{1}$$

The sine of the angle θ between two vectors is given by the formula:

$$\sin\theta = \sin\left(\cos^{-1}\left(\frac{\left|\mathbf{a}^\top \mathbf{b}\right|}{\|\mathbf{a}\|\,\|\mathbf{b}\|}\right)\right) \tag{2}$$

Theoretical Solution

$$\sin\theta = \sin\left(\cos^{-1}\left(\frac{\left|\begin{pmatrix}1 & -2 & 3\end{pmatrix}\begin{pmatrix}3\\-2\\1\end{pmatrix}\right|}{\left\|\begin{pmatrix}1\\-2\\3\end{pmatrix}\right\|\left\|\begin{pmatrix}3\\-2\\1\end{pmatrix}\right\|}\right)\right) \tag{3}$$

$$= \sin\left(\cos^{-1}\left(\frac{|(3)(1)+(-2)(-2)+(3)(1)|}{\sqrt{1^2+(-2)^2+3^2}\sqrt{3^2+(-2)^2+1^2}}\right)\right) \tag{4}$$

Theoretical Solution

$$= \sin\left(\cos^{-1}\left(\frac{|3+4+3|}{\sqrt{14}\sqrt{14}}\right)\right) \tag{5}$$

$$= \sin\left(\cos^{-1}\left(\frac{10}{14}\right)\right) \tag{6}$$

$$= \frac{2\sqrt{6}}{7} \tag{7}$$

Therefore, the value of $\sin\theta$ is $\frac{2\sqrt{6}}{7}$.

C code

```c
#include <stdio.h>
#include <math.h>
float formula(float *a,float *b)
{
    float c[3];
    c[0] = a[1]*b[2] - a[2]*b[1];
    c[1] = a[2]*b[0] - a[0]*b[2];
    c[2] = a[0]*b[1] - a[1]*b[0];

    return sqrt((c[0]*c[0] + c[1]*c[1] + c[2]*c[2]))/(sqrt(a[0]*a
        [0] + a[1]*a[1] + a[2]*a[2])*sqrt(b[0]*b[0] + b[1]*b[1] +
         b[2]*b[2]));
}
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt
import ctypes
import os
import sys

# Load the shared C library
lib_path = ctypes.CDLL("./formula.so")

# Define the argument types for the C function
lib_path.formula.argtypes = [
    ctypes.POINTER(ctypes.c_float),
    ctypes.POINTER(ctypes.c_float)
]
# Define the return type for the C function
lib_path.formula.restype = ctypes.c_float
```

Python Code

```
# Define the input vectors as numpy arrays
vecA = np.array([1, -2, 3], dtype=np.float32)
vecB = np.array([3, -2, 1], dtype=np.float32)

# Call the C function
sin_theta = lib_path.formula(
    vecA.ctypes.data_as(ctypes.POINTER(ctypes.c_float)),
    vecB.ctypes.data_as(ctypes.POINTER(ctypes.c_float))
)

print(f"The value of sin(theta) is: {sin_theta}")
# Theoretical value: 2*sqrt(6)/7 = 0.700
```

Python Code: Plotting

```
# Create a 3D plot
fig = plt.figure(figsize=(8, 8))
ax = fig.add_subplot(111, projection='3d')

# Define the origin point
origin = np.array([0, 0, 0])

# Plot the two vectors from the origin using ax.quiver
ax.quiver(*origin, *vecA, color='blue', label='Vector a = (1, -2,
    3)')
ax.quiver(*origin, *vecB, color='green', label='Vector b = (3,
   -2, 1)')

# Set the limits of the plot for better visualization
max_val = np.max(np.abs(np.concatenate((vecA, vecB)))) + 1
ax.set_xlim([-max_val, max_val])
ax.set_ylim([-max_val, max_val])
ax.set_zlim([-max_val, max_val])
```

Python Code: Finalizing Plot

```
# Add labels and a title
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Z-axis')
ax.set_title('Two 3D Vectors')

# Add a legend
ax.legend()

# Add a grid for better visualization
ax.grid(True)

# Save and show the plot
plt.savefig("plot_c.jpg")
plt.show()
```

Plot

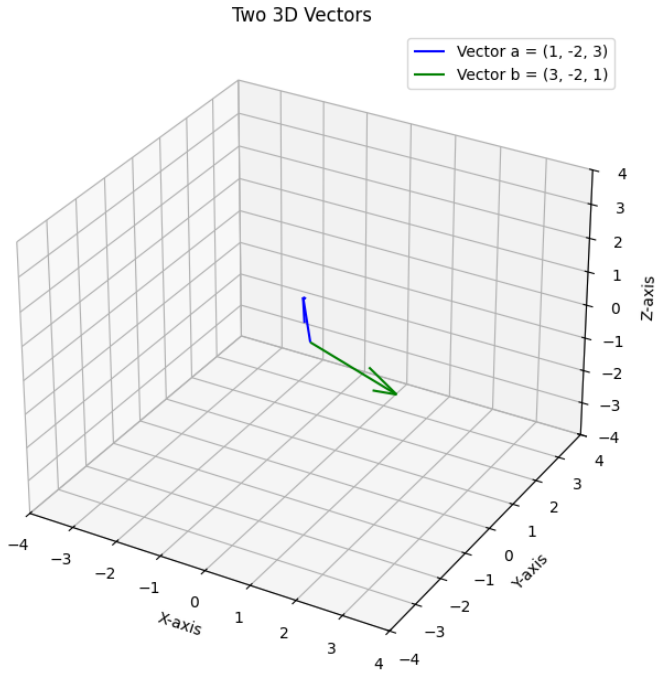

Figure: Visualization of the two vectors