

# Matrices in Geometry 1.5.25

EE25BTECH11037 - Divyansh

**Question:** In what ratio does the point  $\left(\frac{24}{11}, \frac{y}{y}\right)$  divide the line segment joining the points  $\mathbf{P}=\begin{pmatrix} 2 \\ -2 \end{pmatrix}$  and  $\mathbf{Q}=\begin{pmatrix} 3 \\ 7 \end{pmatrix}$ ? Also find the value of  $y$ .

**Given:**  $\mathbf{P} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ ,  $\mathbf{Q} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}$  and a point  $\mathbf{R} = \begin{pmatrix} \frac{24}{11} \\ y \end{pmatrix}$  on  $PQ$ .

Let  $R$  divide  $PQ$  internally in the ratio  $k : 1$ .

Therefore, they are defined to be collinear if,

$$\begin{aligned} \text{rank}(\mathbf{R} - \mathbf{P} \quad \mathbf{Q} - \mathbf{R}) &= 1 \\ \mathbf{R} - \mathbf{P} &= \begin{pmatrix} \frac{2}{11} \\ y+2 \end{pmatrix} \\ \mathbf{Q} - \mathbf{R} &= \begin{pmatrix} \frac{9}{11} \\ 7-y \end{pmatrix} \\ \Rightarrow \text{rank} \begin{pmatrix} \frac{2}{11} & \frac{9}{11} \\ y+2 & 7-y \end{pmatrix} &= 1 \\ \Rightarrow \Delta &= 0 \\ \frac{2}{11}(7-y) - \frac{9}{11}(y+2) &= 0 \\ 14 - 2y - 18 - 9y &= 0 \\ \Rightarrow y &= \frac{-4}{11} \end{aligned}$$

We know that  $k$  is the ratio in which  $\mathbf{R}$  divides  $\mathbf{P}$  and  $\mathbf{Q}$ ,

$$\begin{aligned} k &= \frac{\|\overline{PR}\|}{\|\overline{RQ}\|} \\ \overline{PR} &= \begin{pmatrix} -2/11 \\ -18/11 \end{pmatrix} \\ \Rightarrow \|\overline{PR}\| &= \sqrt{4/121 + 324/121} = \sqrt{328/121} \Rightarrow \|\overline{PR}\| = 2\sqrt{82}/11 \\ \overline{QR} &= \begin{pmatrix} 9/11 \\ 81/11 \end{pmatrix} \\ \Rightarrow \|\overline{QR}\| &= \sqrt{81/121 + 6561/121} = \sqrt{6642/121} \Rightarrow \|\overline{QR}\| = 9\sqrt{82}/11 \\ \therefore k &= \frac{\|\overline{PR}\|}{\|\overline{RQ}\|} = \frac{2}{9} \end{aligned}$$

Hence, the final answer is  $\boxed{k = \frac{2}{9}}$  and  $\boxed{y = \frac{-4}{11}}$

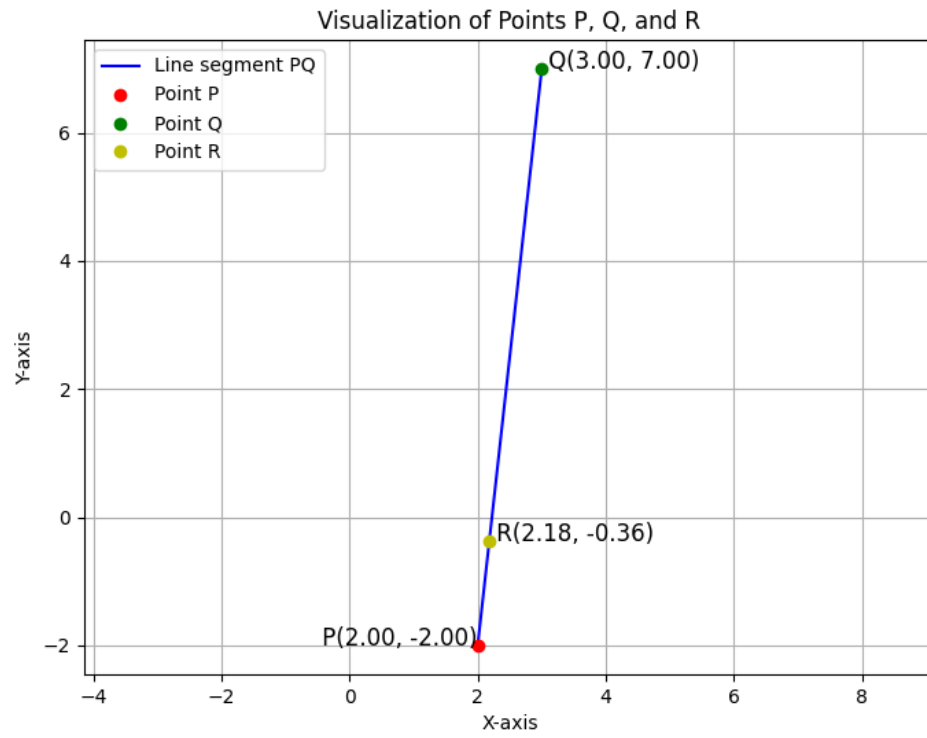


Fig. 1: Plot for 1.5.25