5.8.24

EE25BTECH11026-Harsha

Question:
The ages of two friends Ani and Bijoya differ by 3 years. Ani's father Dharam is twice as old as Ani and Bijoya is twice as old as sister Kanta. The ages of Kanta and Dharam differ by 30 years. Find the ages of Ani and Bijoya.

Solution:
Let us solve the given question theoretically and then verify the solution computationally.

Let the ages of Ani,Bijoya,Dharam and Kanta form a age vector $\mathbf{A}$ of the form,

$$\mathbf{A} = \begin{pmatrix} \mathbf{a} & \mathbf{b} & \mathbf{c} & \mathbf{d} \end{pmatrix}^{\top} \tag{0.1}$$

According to the data given,

$$\mathbf{a} - \mathbf{b} = 3 \tag{0.2}$$

$$\mathbf{c} = 2\mathbf{a} \tag{0.3}$$

$$\mathbf{b} = 2\mathbf{d} \tag{0.4}$$

$$\mathbf{c} - \mathbf{d} = 30 \tag{0.5}$$

$$\therefore \begin{pmatrix} 1 & -1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \end{pmatrix} \mathbf{A} = \begin{pmatrix} 3 \\ 0 \\ 0 \\ 30 \end{pmatrix} \tag{0.6}$$

We can find the solution of the matrix by doing Gaussian elimination,

$$\left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 3 \\ -2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -1 & 30 \end{array}\right) \xleftrightarrow{R_2 \leftarrow R_2 + 2\times R_1} \left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 3 \\ 0 & -2 & 1 & 0 & 6 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -1 & 30 \end{array}\right) \xleftrightarrow{R_2 \leftrightarrow R_3} \tag{0.7}$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & -2 & 1 & 0 & 6 \\ 0 & 0 & 1 & -1 & 30 \end{array}\right) \xleftrightarrow{R_3 \leftarrow 2\times R_2 + R_3} \left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -4 & 6 \\ 0 & 0 & 1 & -1 & 30 \end{array}\right) \xleftrightarrow{R_4 \leftarrow R_4 - R_3} \tag{0.8}$$

$$\left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -4 & 6 \\ 0 & 0 & 0 & 3 & 24 \end{array}\right) \xleftrightarrow[R_3 \leftarrow R_3 + 4\times R_4]{R_4 \leftarrow \frac{R_4}{3}} \left(\begin{array}{cccc|c} 1 & -1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & 38 \\ 0 & 0 & 0 & 1 & 8 \end{array}\right) \xleftrightarrow[R_1 \leftarrow R_1 + R_2]{R_2 \leftarrow R_2 + 2\times R_4} \tag{0.9}$$

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 19 \\ 0 & 1 & 0 & 0 & 16 \\ 0 & 0 & 1 & 0 & 38 \\ 0 & 0 & 0 & 1 & 8 \end{array}\right) \tag{0.10}$$

$\therefore$ Ages of Ani and Bijoya is 19 and 16 respectively.