4.7.40

EE25BTECH11043 - Nishid Khandagre

Question: Find the foot of the perpendicular and the perpendicular distance from the point $\begin{pmatrix} 2 \\ 3 \\ -8 \end{pmatrix}$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$.

Solution: Given line:

$$\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3} = t \tag{0.1}$$

$$x = 4 - 2t \tag{0.2}$$

$$y = 6t \tag{0.3}$$

$$z = 1 - 3t \tag{0.4}$$

Line in vector form:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + t \begin{pmatrix} -2 \\ 6 \\ -3 \end{pmatrix} \tag{0.5}$$

$$\mathbf{r} = \mathbf{a} + t\mathbf{m} \tag{0.6}$$

$$\mathbf{a} = \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} \tag{0.7}$$

$$\mathbf{m} = \begin{pmatrix} -2 \\ 6 \\ -3 \end{pmatrix} \tag{0.8}$$

Given point: $\mathbf{p} = \begin{pmatrix} 2 \\ 3 \\ -8 \end{pmatrix}$.

Let the foot of the perpendicular be $\mathbf{f}$. Since $\mathbf{f}$ lies on the line, we can write:

$$\mathbf{f} = \mathbf{a} + \alpha\mathbf{m} \tag{0.9}$$

$\left(\mathbf{p} - \mathbf{f}\right)$ must be orthogonal to the direction vector of the line $\mathbf{m}$.

Therefore

$$\left(\mathbf{p} - \mathbf{f}\right)^{\top} \mathbf{m} = 0 \tag{0.10}$$

$$\left(\mathbf{p} - \left(\mathbf{a} + \alpha\mathbf{m}\right)\right)^{\top} \mathbf{m} = 0 \tag{0.11}$$

$$\left(\mathbf{p} - \mathbf{a} - \alpha\mathbf{m}\right)^{\top} \mathbf{m} = 0 \tag{0.12}$$

$$\left(\mathbf{p} - \mathbf{a}\right)^{\top} \mathbf{m} - \alpha\left(\mathbf{m}^{\top}\mathbf{m}\right) = 0 \tag{0.13}$$

$$\alpha = \frac{\left(\mathbf{p} - \mathbf{a}\right)^{\top} \mathbf{m}}{\mathbf{m}^{\top}\mathbf{m}} \tag{0.14}$$

$$\mathbf{p} - \mathbf{a} = \begin{pmatrix} 2 \\ 3 \\ -8 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ -9 \end{pmatrix} \tag{0.15}$$

$$\left(\mathbf{p} - \mathbf{a}\right)^{\top} \mathbf{m} = \begin{pmatrix} -2 & 3 & -9 \end{pmatrix} \begin{pmatrix} -2 \\ 6 \\ -3 \end{pmatrix} \tag{0.16}$$

$$= (-2)(-2) + (3)(6) + (-9)(-3) \tag{0.17}$$

$$= 4 + 18 + 27 = 49 \tag{0.18}$$

$$\mathbf{m}^{\top}\mathbf{m} = (-2)^2 + 6^2 + (-3)^2 \tag{0.19}$$

$$= 4 + 36 + 9 = 49 \tag{0.20}$$

Therefore

$$\alpha = \frac{49}{49} = 1 \tag{0.21}$$

foot of the perpendicular **f**:

$$\mathbf{f} = \mathbf{a} + \alpha\mathbf{m} \tag{0.22}$$

$$= \begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} + 1 \begin{pmatrix} -2 \\ 6 \\ -3 \end{pmatrix} \tag{0.23}$$

$$= \begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix} \tag{0.24}$$

$$\text{Perpendicular Distance} = \|\mathbf{p} - \mathbf{f}\| \tag{0.25}$$

$$\mathbf{p}-\mathbf{f} = \begin{pmatrix} 2 \\ 3 \\ -8 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix} \tag{0.26}$$

$$= \begin{pmatrix} 0 \\ -3 \\ -6 \end{pmatrix} \tag{0.27}$$

$$\|\mathbf{p}-\mathbf{f}\| = \sqrt{\left(\mathbf{p}-\mathbf{f}\right)^{\top}\left(\mathbf{p}-\mathbf{f}\right)} \tag{0.28}$$

$$= \sqrt{0^2 + (-3)^2 + (-6)^2} \tag{0.29}$$

$$= \sqrt{0 + 9 + 36} \tag{0.30}$$

$$= \sqrt{45} \tag{0.31}$$

$$= 3\sqrt{5} \tag{0.32}$$

The perpendicular distance is $3\sqrt{5}$.

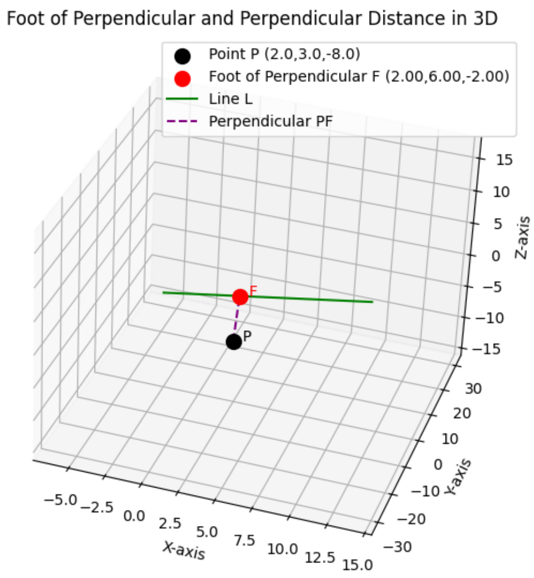

Fig. 0.1