Matrices in Geometry - 5.8.33

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Problem Statement

Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$.
Determine the co-ordinates of the vertices of the triangle formed by these lines and the y-axis.

Solution

Given,

The lines $\begin{pmatrix} 5 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 5$ and $\begin{pmatrix} 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 3$. The y-axis $\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$.

Solving for the intersection of the two lines

$$\begin{pmatrix} 5 & -1 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \tag{1}$$

Forming the augmented matrix for solving this,

$$\left(\begin{array}{cc|c} 5 & -1 & 5 \\ 3 & -1 & 3 \end{array}\right) \tag{2}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \left(\begin{array}{cc|c} 2 & 0 & 2 \\ 3 & -1 & 3 \end{array}\right) \xleftrightarrow{R_2 \leftarrow -R_2} \left(\begin{array}{cc|c} 2 & 0 & 2 \\ -3 & 1 & -3 \end{array}\right) \tag{3}$$

Solution

$$\xleftrightarrow{R_1 \leftarrow \frac{R_1}{2}} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ -3 & 1 & -3 \end{array}\right) \xleftrightarrow{R_2 \leftarrow R_2 + 3R_1} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \end{array}\right) \tag{4}$$

$$\implies \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{5}$$

Now, the intersection of the two lines with the y-axis,
First line:

$$\begin{pmatrix} 5 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{6}$$

Forming augmented matrix and solving,

$$\left(\begin{array}{cc|c} 5 & -1 & 5 \\ 1 & 0 & 0 \end{array}\right) \xleftrightarrow{R_1 \leftarrow R_2} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 5 & -1 & 5 \end{array}\right) \tag{7}$$

Solution

$$\xleftrightarrow{R_2 \leftarrow R_2 - 5R_1} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & -1 & 5 \end{array}\right) \xleftrightarrow{R_2 \leftarrow -R_2} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & -5 \end{array}\right) \tag{8}$$

$$\implies \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -5 \end{pmatrix} \tag{9}$$

Second Line:

$$\begin{pmatrix} 1 & 0 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \tag{10}$$

Forming the augmented Matrix,

$$\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 3 & -1 & 3 \end{array}\right) \xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & -1 & 3 \end{array}\right) \tag{11}$$

$$\xleftrightarrow{R_2 \leftarrow -R_2} \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & -3 \end{array}\right) \tag{12}$$

$$\implies \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -3 \end{pmatrix} \tag{13}$$

Conclusion

$\therefore$ The coordinates of the vertices of the triangle formed by these lines anad the y-axis are $\begin{pmatrix}1\\0\end{pmatrix}$, $\begin{pmatrix}0\\-5\end{pmatrix}$ and $\begin{pmatrix}0\\-3\end{pmatrix}$.

The graph of the system of equations:

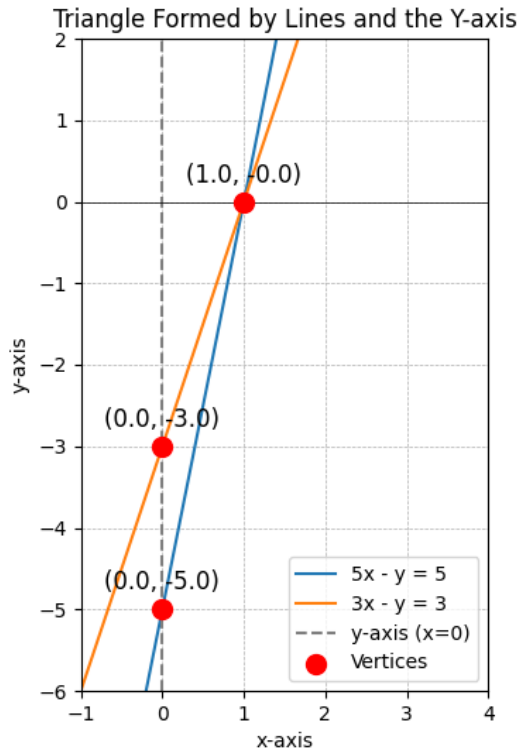

Figure: Figure for 5.8.33