1.8.23

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Question

If the point $\mathbf{A}(2, -4)$ is equidistant from $\mathbf{P}(3, 8)$ and $\mathbf{Q}(-10, y)$, find the values of y. Also find distance $\mathbf{PQ}$.

Theoretical Solution

Since **A** is equidistant from **P** and **Q**,

$$\left\|\left(\mathbf{A}-\mathbf{P}\right)\right\| = \left\|\left(\mathbf{A}-\mathbf{Q}\right)\right\| \tag{1}$$

$$\left\|\left(\mathbf{A}-\mathbf{P}\right)\right\|^2 = \left\|\left(\mathbf{A}-\mathbf{Q}\right)\right\|^2 \tag{2}$$

$$||\mathbf{A}||^2 - 2\mathbf{A}^\top\mathbf{P} + ||\mathbf{P}||^2 = ||\mathbf{A}||^2 - 2\mathbf{A}^\top\mathbf{Q} + ||\mathbf{Q}||^2 \tag{3}$$

$$(\mathbf{P}-\mathbf{Q})^\top\mathbf{A} = \frac{||\mathbf{P}||^2 - ||\mathbf{Q}||^2}{2} \tag{4}$$

$$\begin{pmatrix} 3-(-10) \\ 8-y \end{pmatrix}^{\top} \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \frac{73-(-10)^2-y^2}{2} \tag{5}$$

$$y^2+8y+15=0 \tag{6}$$

Therefore,

$$y = -5, -3 \tag{7}$$

$$\mathbf{Q_1} = \begin{pmatrix} -10 \\ -5 \end{pmatrix}, \quad \mathbf{Q_2} = \begin{pmatrix} -10 \\ -3 \end{pmatrix} \tag{8}$$

$$\left\|\left(\mathbf{P}-\mathbf{Q_1}\right)\right\| = \left\|\begin{pmatrix}3\\8\end{pmatrix}-\begin{pmatrix}-10\\-5\end{pmatrix}\right\| \tag{9}$$

$$= \left\|\begin{pmatrix}13\\13\end{pmatrix}\right\| \tag{10}$$

$$= 13\sqrt{2} \tag{11}$$

$$\left\|\left(\mathbf{P}-\mathbf{Q_2}\right)\right\| = \left\|\begin{pmatrix}3\\8\end{pmatrix}-\begin{pmatrix}-10\\-3\end{pmatrix}\right\| \tag{12}$$

$$= \left\|\begin{pmatrix}13\\11\end{pmatrix}\right\| \tag{13}$$

$$= \sqrt{290} \tag{14}$$

C code

```c
 #include <stdio.h>
#include <math.h>
int main() {
    int Ax = 2, Ay = -4;
    int Px = 3, Py = 8;
    int Qx = -10;
     // Quadratic from ||A-P||^2 = ||A-Q||^2
     // (Ax - Px)^2 + (Ay - Py)^2 = (Ax - Qx)^2 + (Ay - y)^2
    int a = 1, b = 8, c = 15;
    int discrim = b*b - 4*a*c;
    int y1 = (-b + (int)sqrt(discrim)) / (2*a);
    int y2 = (-b - (int)sqrt(discrim)) / (2*a);
    // Distance ||P-Q|| for each y
    double d1 = sqrt((Px - Qx)*(Px - Qx) + (Py - y1)*(Py - y1));
    double d2 = sqrt((Px - Qx)*(Px - Qx) + (Py - y2)*(Py - y2));
    return 0;
}
```

Call C.py

```
import subprocess

# Compile the C program (only once)
subprocess.run(["gcc", "equidistant.c", "-o", "equidistant", "-lm
    "])

# Run the compiled program and capture output
result = subprocess.run(["./equidistant"], capture_output=True,
    text=True)

print("Output from C program:")
print(result.stdout)
```

Plot.py

```
import matplotlib.pyplot as plt
# Points
A = (2, -4)
P = (3, 8)
Q1 = (-10, -5)
Q2 = (-10, -3)
# Plot points with markers
plt.scatter(*A, color='red', s=100, marker='o', label='A(2,-4)')
plt.scatter(*P, color='blue', s=100, marker='o', label='P(3,8)')
plt.scatter(*Q1, color='green', s=100, marker='o', label='Q
    (-10,-5)')
plt.scatter(*Q2, color='purple', s=100, marker='o', label='Q2
    (-10,-3)')
# Draw lines AP, AQ1, AQ2
plt.plot([A[0], P[0]], [A[1], P[1]], 'b--')
plt.plot([A[0], Q1[0]], [A[1], Q1[1]], 'g--')
plt.plot([A[0], Q2[0]], [A[1], Q2[1]], 'm--')
```

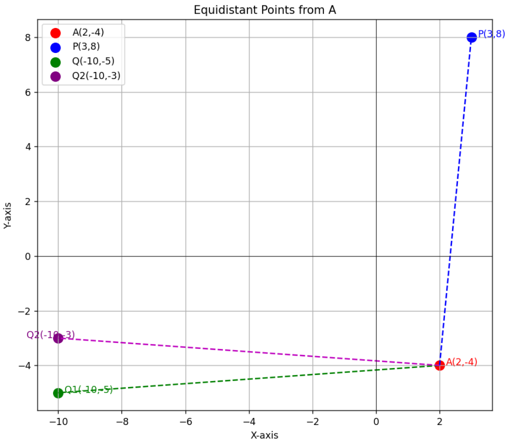
Plot.py

```
# Annotate points
plt.text(A[0]+0.2, A[1], "A(2,-4)", fontsize=10, color='red')
plt.text(P[0]+0.2, P[1], "P(3,8)", fontsize=10, color='blue')
plt.text(Q1[0]+0.2, Q1[1], "Q1(-10,-5)", fontsize=10, color='
    green')
plt.text(Q2[0]-1, Q2[1], "Q2(-10,-3)", fontsize=10, color='purple
    ')

# Labels and grid
plt.xlabel('X-axis')
plt.ylabel('Y-axis')
plt.title('Equidistant Points from A')
plt.legend()
plt.grid(True)
plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)

plt.show()
```

Plot



Equidistant Points from **A**