5.2.24

EE25BTECH11021 - Dhanush sagar

Question:
Solve the following system of linear equations

px + qy = p - q
qx - py = p + q

Solution:
Given

$$px + qy = p - q \tag{1}$$
$$qx - py = p + q \tag{2}$$

The matrix equation for a line is defined as

$$\mathbf{n}^\top \mathbf{x} = c \tag{3}$$

where $\mathbf{n}$ is the coefficient vector and $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$

Writing the two lines in matrix form:

$$\begin{pmatrix} p & q \end{pmatrix} \mathbf{x} = p - q \tag{4}$$
$$\begin{pmatrix} q & -p \end{pmatrix} \mathbf{x} = p + q \tag{5}$$

Combine into a single system:

$$\begin{pmatrix} p & q \\ q & -p \end{pmatrix} \mathbf{x} = \begin{pmatrix} p - q \\ p + q \end{pmatrix} \tag{6}$$

Multiply both sides by the transpose of the coefficient matrix:

$$\begin{pmatrix} p & q \\ q & -p \end{pmatrix}^\top \begin{pmatrix} p & q \\ q & -p \end{pmatrix} \mathbf{x} = \begin{pmatrix} p & q \\ q & -p \end{pmatrix}^\top \begin{pmatrix} p - q \\ p + q \end{pmatrix} \tag{7}$$

Compute the products:

$$\begin{pmatrix} p^2 + q^2 & 0 \\ 0 & p^2 + q^2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} p^2 + q^2 \\ -(p^2 + q^2) \end{pmatrix} \tag{8}$$

Factor out $(p^2 + q^2)$ as a scalar multiplying the identity matrix:

$$(p^2 + q^2)\mathbf{I}\mathbf{x} = \begin{pmatrix} p^2 + q^2 \\ -(p^2 + q^2) \end{pmatrix} \tag{9}$$

Divide both sides by $p^2 + q^2$:

$$\mathbf{I}\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{10}$$

$$\mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{11}$$

Hence the solution is:

$$x = 1, \quad y = -1 \tag{12}$$

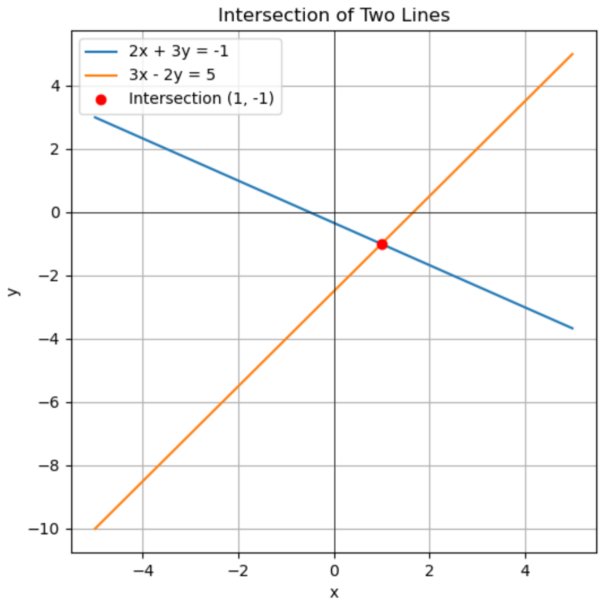

Fig. 0.1