2.8.37

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Question

If $|\mathbf{a} \times \mathbf{b}|^2 + (\mathbf{a}^T\mathbf{b})^2 = 144$ and $||\mathbf{a}|| = 4$, then $||\mathbf{b}||$ is equal to _____

Equation

$$|\mathbf{a} \times \mathbf{b}| = ||\mathbf{a}||\,||\mathbf{b}||\,\sin\theta$$
$$\mathbf{a}^T\mathbf{b} = ||\mathbf{a}||\,||\mathbf{b}||\,\cos\theta$$

Theoretical Solution

$$|\mathbf{a} \times \mathbf{b}|^2 + (\mathbf{a}^T\mathbf{b})^2 = \left(||\mathbf{a}||\,||\mathbf{b}||\,\sin\theta\right)^2 + \left(||\mathbf{a}||\,||\mathbf{b}||\,\cos\theta\right)^2 \tag{1}$$

$$= ||\mathbf{a}||^2||\mathbf{b}||^2\left(\sin^2\theta + \cos^2\theta\right) \tag{2}$$

$$= ||\mathbf{a}||^2||\mathbf{b}||^2. \tag{3}$$

Given :

$$|\mathbf{a} \times \mathbf{b}|^2 + (\mathbf{a}^T\mathbf{b})^2 = 144, \tag{4}$$

$$||\mathbf{a}|| = 4, \tag{5}$$

Theoretical Solution

$$144 = ||\mathbf{a}||^2||\mathbf{b}||^2 \tag{6}$$

$$144 = 4^2||\mathbf{b}||^2 \tag{7}$$

$$144 = 16||\mathbf{b}||^2 \tag{8}$$

$$||\mathbf{b}||^2 = \frac{144}{16} = 9 \tag{9}$$

$$||\mathbf{b}|| = 3. \tag{10}$$

C Code

```c
#include <stdio.h>
#include <math.h>

int main() {
    // Given values
    double a = 4.0; // |a| = 4
    double lhs = 144; // |a  b|^2 + (a  b)^2 = 144

    // From identity: |a  b|^2 + (a  b)^2 = |a|^2 * |b|^2
    // => |b|^2 = lhs / (|a|^2)
    double b_squared = lhs / (a * a);
    double b = sqrt(b_squared);

    printf(|b| = %.2f\n, b);

    return 0;
}
```

Python Shared Output

```
      import numpy as np
import ctypes
# Local imports
from libs.line.funcs import *
from libs.triangle.funcs import *
from libs.conics.funcs import circ_gen

# Load C math library for sqrt
libm = ctypes.CDLL(libm.so.6)
libm.sqrt.restype = ctypes.c_double
libm.sqrt.argtypes = [ctypes.c_double]
```

Python Shared Output

```
# Given values
a = 4.0 # |a|
lhs = 144.0 # |a  b|^2 + (a  b)^2

# Compute b^2 using NumPy
b_squared = lhs / np.power(a, 2)

# Compute b using C's sqrt via ctypes
b = libm.sqrt(ctypes.c_double(b_squared))

print(f|b| = {b})
```

Python Direct

```
import numpy as np

# Given values
a = 4.0 # |a|
lhs = 144.0 # |a  b|^2 + (a  b)^2

# Using numpy
b_squared = lhs / np.power(a, 2)
b = np.sqrt(b_squared)

print(f|b| = {b})
```