4.13.100

Shivam Sawarkar
AI25BTECH11031

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Question

Let **S** be the reflection of a point **Q** with respect to the plane given by $\mathbf{r} = -(t+p)\hat{i} + t\hat{j} + (1+p)\hat{k}$ where t, p are real parameters and $\hat{i}$, $\hat{j}$, $\hat{k}$ are the unit vectors along the three positive coordinate axes. If the position vectors of **Q** and **S** are $10\hat{i} + 15\hat{j} + 20\hat{k}$ and $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ respectively, then which of the following is/are TRUE ?

a. $3(\alpha + \beta) = -101$
b. $3(\beta + \gamma) = -71$
c. $3(\gamma + \alpha) = -86$
d. $3(\alpha + \beta + \gamma) = -121$

Soluion

The plane is given by

$$\mathbf{r} = t\begin{pmatrix}-1\\1\\0\end{pmatrix} + p\begin{pmatrix}-1\\0\\1\end{pmatrix} + \begin{pmatrix}0\\0\\1\end{pmatrix} \tag{1}$$

so two direction vectors are

$$\mathbf{u} = \begin{pmatrix}-1\\1\\0\end{pmatrix}, \qquad \mathbf{v} = \begin{pmatrix}-1\\0\\1\end{pmatrix}. \tag{2}$$

Hence the normal vector is

$$\mathbf{n} = \mathbf{u} \times \mathbf{v} = \begin{pmatrix}1\\1\\1\end{pmatrix}. \tag{3}$$

So the plane equation becomes

$$\mathbf{n}^{\top}\mathbf{x} = 1. \tag{4}$$

Solution

For a point $\mathbf{q} \in \mathbb{R}^3$, its reflection across the plane $\mathbf{n}^\top \mathbf{x} = 1$ is

$$\mathbf{S} = \mathbf{Q} - 2\frac{\mathbf{n}^\top \mathbf{Q} - 1}{\|n\|^2}\mathbf{n}, \tag{5}$$

Here

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 10 \\ 15 \\ 20 \end{pmatrix}. \tag{6}$$

$$\mathbf{n}^\top \mathbf{n} = 1^2 + 1^2 + 1^2 = 3. \tag{7}$$

$$\mathbf{S} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} \frac{58}{3} \\ -\frac{43}{3} \\ -\frac{28}{3} \end{pmatrix}. \tag{8}$$

Solution

$$3(\alpha+\beta)=-101,\quad 3(\beta+\gamma)=-71,\quad 3(\gamma+\alpha)=-86,\quad 3(\alpha+\beta+\gamma)= \tag{9}$$

Hence, the correct options are

$$\boxed{(a),(b),(c)}. \tag{10}$$

Plot

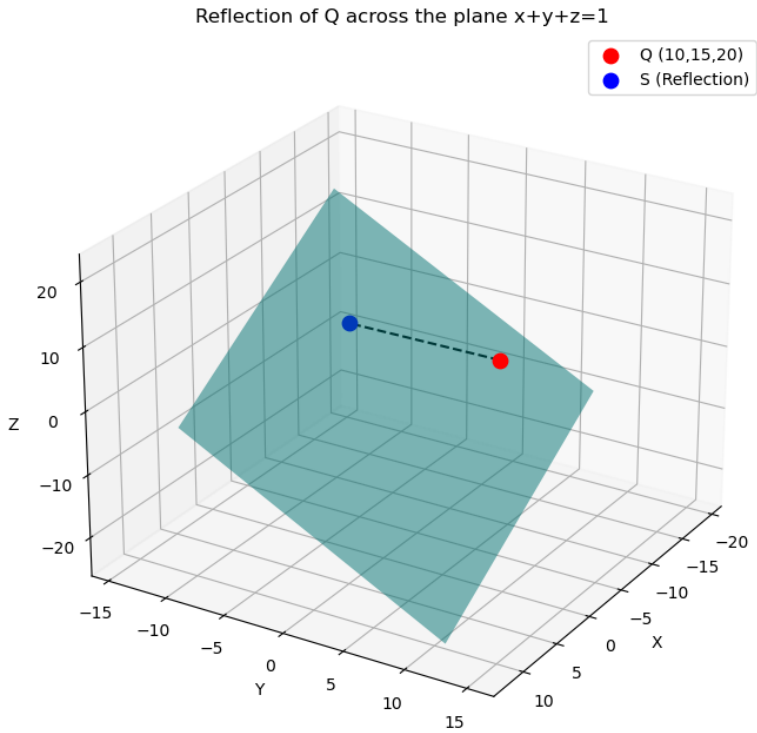
Reflection of Q across the plane x+y+z=1
Q (10,15,20)
S (Reflection)
Z
20
10
0
−10
−20
−15
−10
−5
0
5
10
15
Y
−20
−15
−10
−5
0
5
10
X

Figure:

C Code

```
#ifndef REFLECTION_H
#define REFLECTION_H

#include <stdio.h>

// Function to compute reflection of a point (x,y,z)
// across the plane x+y+z=1.
// The result is stored in (alpha, beta, gamma).
void reflect_point(double x, double y, double z,
                   double *alpha, double *beta, double *gamma) {
    double n[3] = {1.0, 1.0, 1.0};
    double d = 1.0;
    double norm_sq = 3.0;
```

C Code

```
    // Dot product n.q
    double dot = n[0]*x + n[1]*y + n[2]*z;

    // Reflection formula: s = q - 2((n.q - d)/||n||^2) * n
    *alpha = x - 2.0*(dot - d)/norm_sq * n[0];
    *beta  = y - 2.0*(dot - d)/norm_sq * n[1];
    *gamma = z - 2.0*(dot - d)/norm_sq * n[2];
}

#endif
```

C Code

```
#include "solution.h"

int main() {
    double x, y, z;
    double alpha, beta, gamma;

    printf("Enter coordinates of Q (x y z): ");
    scanf("%lf %lf %lf", &x, &y, &z);
    reflect_point(x, y, z, &alpha, &beta, &gamma);
    printf("Reflected point S = (%.6lf, %.6lf, %.6lf)\n", alph

    return 0;
}
```

Python Code

```
import numpy as np

def reflect_point(x, y, z):
    n = np.array([1.0, 1.0, 1.0])
    d = 1.0  # plane constant
    norm_sq = np.dot(n, n)  # = 3
    q = np.array([x, y, z])
    dot = np.dot(n, q)
    s = q - 2 * (dot - d) / norm_sq * n
    return s

    x, y, z = map(float, input("Enter coordinates of Q (x y z)
    : ").split())
    alpha, beta, gamma = reflect_point(x, y, z)
    print(f"Reflected point S = ({alpha:.6f}, {beta:.6f},
    {gamma:.6f})")
```

Python + C Code

```
import ctypes

# Load the shared library
lib = ctypes.CDLL("./solution.so")

# Define argument and return types for the function
lib.reflect_point.argtypes = [ctypes.c_double, ctypes.c_double
ctypes.c_double,
                              ctypes.POINTER(ctypes.c_double),
                             ctypes.POINTER(ctypes.c_double),
                             ctypes.POINTER(ctypes.c_double)]

def reflect_point(x, y, z):
    alpha = ctypes.c_double()
    beta  = ctypes.c_double()
    gamma = ctypes.c_double()
```

Python + C Code

```
    lib.reflect_point(x, y, z,
                      ctypes.byref(alpha),
                      ctypes.byref(beta),
                      ctypes.byref(gamma))
    return alpha.value, beta.value, gamma.value


# --- Main ---
if __name__ == "__main__":
    x, y, z = map(float, input("Enter coordinates of Q (x y z)
    ").split())
    alpha, beta, gamma = reflect_point(x, y, z)
    print(f"Reflected point S = ({alpha:.6f}, {beta:.6f},
    {gamma:.6f})")
```