

# 9.5.4

EE25BTECH11041 - Naman Kumar

Question:

If one zero of the polynomial  $6x^2 + 37x - (k - 2)$  is the reciprocal of the other, then what is the value of  $k$ ?

**Solution:**

General equation of conic

$$g(\mathbf{x}) = \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f \quad (1)$$

Equation of quadratic,

$$\mathbf{x}^T \begin{pmatrix} 6 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \left( \frac{37}{2} \right)^T \mathbf{x} - (k - 2) = 0 \quad (2)$$

Equation of line,

$$\mathbf{x} = \mathbf{h} + k\mathbf{m} \quad (3)$$

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (4)$$

Using following equation to find point of intersection of conic and line

$$k_i = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left( -\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})(\mathbf{m}^T \mathbf{V} \mathbf{m})} \right) \quad (5)$$

Solving for  $g(\mathbf{h})$

$$g(\mathbf{h}) = \mathbf{h}^T \begin{pmatrix} 6 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{h} + 2 \left( \frac{37}{2} \right)^T \mathbf{h} - (k - 2) \quad (6)$$

$$g(\mathbf{h}) = -(k - 2) \quad (7)$$

Solving for  $\mathbf{m}^T \mathbf{V} \mathbf{m}$

$$\mathbf{m}^T \mathbf{V} \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \begin{pmatrix} 6 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (8)$$

$$= 6 \quad (9)$$

Solving for  $\mathbf{m}^T (\mathbf{V} \mathbf{h} + \mathbf{u})$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}^T \left( \begin{pmatrix} 6 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 37 \\ 2 \end{pmatrix} \right) \quad (10)$$

$$= \frac{37}{2} \quad (11)$$

Solving (5)

$$k_i = \frac{1}{6} \left( -\frac{37}{2} \pm \sqrt{\frac{1369}{4} + (k-2) \times 6} \right) \quad (12)$$

Given condition

$$k_1 = \frac{1}{k_2} \quad (13)$$

Therefore

$$\frac{1}{6} \left( -\frac{37}{2} - \sqrt{\frac{1369}{4} + (k-2) \times 6} \right) = \frac{1}{\frac{1}{6} \left( -\frac{37}{2} + \sqrt{\frac{1369}{4} + (k-2) \times 6} \right)} \quad (14)$$

$$\frac{37^2}{2} - \left( \frac{1369}{4} + 6(k-2) \right) = 36 \quad (15)$$

$$-6(k-2) = 36 \quad (16)$$

$$k = -4 \quad (17)$$

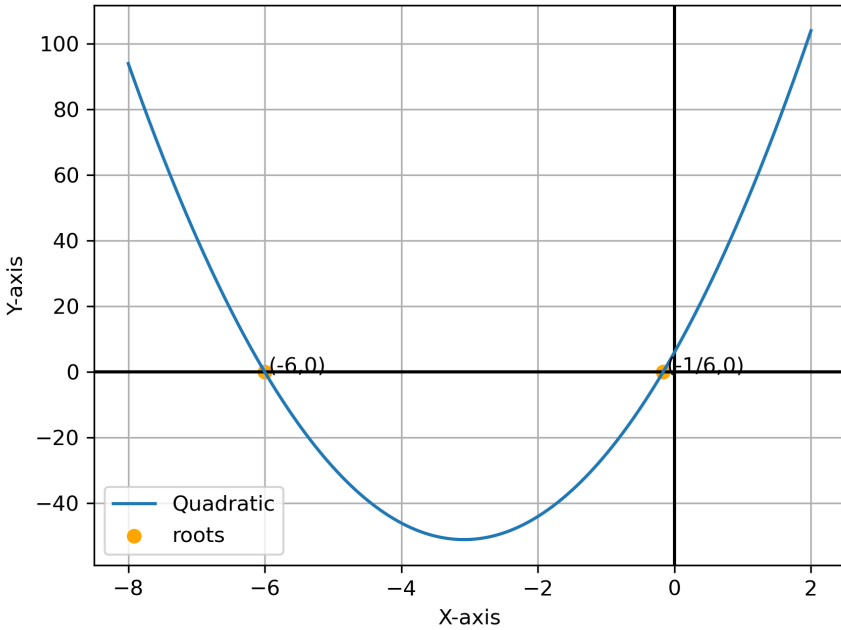


Figure 1