

4.3.57

AI25BTECH11001 - ABHISEK MOHAPATRA

Question: Show that the lines

$$\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha-\delta} \quad (1)$$

$$\frac{x-b+c}{\beta-\delta} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta-\delta} \quad (2)$$

are coplanar.

Solution: Given:

$$\mathbf{L}_1 = \mathbf{A} + \lambda \mathbf{m}_1 \quad (3)$$

$$\mathbf{L}_1 = \begin{pmatrix} a-d \\ a \\ a+d \end{pmatrix} + \lambda \begin{pmatrix} \alpha-\delta \\ \alpha \\ \alpha+\delta \end{pmatrix} \quad (4)$$

And,

$$\mathbf{L}_2 = \mathbf{B} + \lambda \mathbf{m}_2 \quad (5)$$

$$\mathbf{L}_2 = \begin{pmatrix} b-c \\ b \\ b+c \end{pmatrix} + \lambda \mathbf{m}_2 \begin{pmatrix} \beta-\delta \\ \beta \\ \beta+\delta \end{pmatrix} \quad (6)$$

If the lines lie in a plane, then they satisfy,

$$\text{nullity}(\mathbf{m}_1 \quad \mathbf{m}_2 \quad \mathbf{B} - \mathbf{A}) \geq 1 \quad (7)$$

$$\text{nullity} \begin{pmatrix} \alpha-\delta & \beta-\delta & a-b+c-d \\ \alpha & \beta & a-b \\ \alpha+\delta & \beta+\delta & a-b-c+d \end{pmatrix} \geq 1 \quad (8)$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} \alpha-\delta & \beta-\delta & a-b+c-d \\ \alpha+\delta & \beta+\delta & a-b-c+d \\ \alpha & \beta & a-b \end{pmatrix} \quad (9)$$

$$\xrightarrow{R_3 \rightarrow R_3 - \frac{R_1+R_2}{2}} \begin{pmatrix} \alpha-\delta & \beta-\delta & a-b+c-d \\ \alpha+\delta & \beta+\delta & a-b-c+d \\ 0 & 0 & 0 \end{pmatrix} \quad (10)$$

$$\xrightarrow{R_2 \rightarrow R_2 - R_1} \begin{pmatrix} \alpha-\delta & \beta-\delta & a-b+c-d \\ 2\delta & 2\delta & -2c+2d \\ 0 & 0 & 0 \end{pmatrix} \quad (11)$$

$$\xrightarrow{C_1 \rightarrow C_1 - C_2} \begin{pmatrix} \alpha-\beta & \beta-\delta & a-b+c-d \\ 0 & 2\delta & -2c+2d \\ 0 & 0 & 0 \end{pmatrix} \quad (12)$$

The matrix is in echelon form and the rank of the matrix is two. And, thus the lines are co-planar.

Graph:

