1.8.24

Aniket-EE25BTECH11007

October 2, 2025

Question

If **a** and **b** are unit vectors, find the angle θ between **a** and **b** such that $\mathbf{a} - \sqrt{2}\,\mathbf{b}$ is a unit vector.

Theoretical Solution

Since **a** and **b** are unit vectors,

$$\|\mathbf{a}\| = 1 \tag{1}$$

$$\|\mathbf{b}\| = 1 \tag{2}$$

The condition that $\mathbf{a} - \sqrt{2}\,\mathbf{b}$ is also a unit vector gives

$$\|\mathbf{a} - \sqrt{2}\,\mathbf{b}\| = 1. \tag{3}$$

Squaring both sides:

$$\|\mathbf{a} - \sqrt{2}\,\mathbf{b}\|^2 = (\mathbf{a} - \sqrt{2}\,\mathbf{b})^\top(\mathbf{a} - \sqrt{2}\,\mathbf{b}) = 1. \tag{4}$$

Theoretical Solution

$$(\mathbf{a} - \sqrt{2}\,\mathbf{b})^\top(\mathbf{a} - \sqrt{2}\,\mathbf{b}) = \mathbf{a}^\top(\mathbf{a} - \sqrt{2}\,\mathbf{b}) - \sqrt{2}\,\mathbf{b}^\top(\mathbf{a} - \sqrt{2}\,\mathbf{b}). \tag{5}$$

Using (1) and (2) and $\mathbf{a}^\top\mathbf{b} = \mathbf{b}^\top\mathbf{a}$:

$$1 = (\mathbf{a} - \sqrt{2}\,\mathbf{b})^\top(\mathbf{a} - \sqrt{2}\,\mathbf{b}) = 1 - \sqrt{2}\,\mathbf{a}^\top\mathbf{b} - \sqrt{2}\,\mathbf{a}^\top\mathbf{b} + 2 = 3 - 2\sqrt{2}\,(\mathbf{a}^\top\mathbf{b}). \tag{6}$$

$$2\sqrt{2}\,(\mathbf{a}^\top\mathbf{b}) = 2 \implies \mathbf{a}^\top\mathbf{b} = \frac{1}{\sqrt{2}}. \tag{7}$$

Theoretical Solution

Using the angle formula from dot product,

$$\cos\theta = \frac{\mathbf{a}^\top \mathbf{b}}{\|\mathbf{a}\| \, \|\mathbf{b}\|} = \frac{1/\sqrt{2}}{1 \cdot 1} = \frac{1}{\sqrt{2}}, \tag{8}$$

$$\boxed{\theta = \frac{\pi}{4} = 45^\circ} \tag{9}$$

C Code

```
#include <math.h>
#include <stdio.h>

int solve_angle_simple(const double *a, const double *b, int n,
    double *theta_deg) {
    const double SQRT2 = 1.4142135623730950488;
    const double PI = 3.14159265358979323846;
    const double TOL = 1e-9;

    double aa = 0.0, bb = 0.0, lhs_sq = 0.0;
    for (int i = 0; i < n; ++i) {
        aa += a[i] * a[i];
        bb += b[i] * b[i];
        double t = a[i] - SQRT2 * b[i];
        lhs_sq += t * t;
    }
    double na = sqrt(aa);
    double nb = sqrt(bb);
    double lhs = sqrt(lhs_sq);
```

C Code

```
    double ab = 0.0;
    for (int i = 0; i < n; ++i) ab += a[i] * b[i];

    double denom = na * nb;
    double cos_theta = (denom > 0.0) ? (ab / denom) : 1.0;
    if (cos_theta > 1.0) cos_theta = 1.0;
    if (cos_theta < -1.0) cos_theta = -1.0;
    *theta_deg = acos(cos_theta) * (180.0 / PI);

    if (fabs(na - 1.0) <= TOL && fabs(nb - 1.0) <= TOL && fabs(
        lhs - 1.0) <= TOL) {
        *theta_deg = 45.0;
        return 0;
    }
    return 1;
}
```

Python+C Code

```python
import ctypes
from ctypes import c_double, c_int
import numpy as np
import matplotlib.pyplot as plt

# --- load the shared lib ---
# Change name below if you're on macOS (.dylib) or Windows (.dll)
lib = ctypes.CDLL("./mg3.so")

# C signature:
# int solve_angle_simple(const double *a, const double *b, int n,
     double *theta_deg)
lib.solve_angle_simple.argtypes = [
    ctypes.POINTER(c_double),
    ctypes.POINTER(c_double),
    c_int,
    ctypes.POINTER(c_double),
]
lib.solve_angle_simple.restype = c_int
```

Python+C Code

```python
def solve_angle_np(a: np.ndarray, b: np.ndarray):
    """
    Call the C function using NumPy arrays a, b (1D, same length)
        .
    Returns (theta_deg, status) where status==0 if the unit
        checks passed.
    """
    a = np.asarray(a, dtype=np.float64).ravel()
    b = np.asarray(b, dtype=np.float64).ravel()
    if a.shape != b.shape:
        raise ValueError("a and b must have the same shape")

    theta = c_double(0.0)
    pa = a.ctypes.data_as(ctypes.POINTER(c_double))
    pb = b.ctypes.data_as(ctypes.POINTER(c_double))
    status = lib.solve_angle_simple(pa, pb, a.size, ctypes.byref(
        theta))
    return theta.value, status
```

Python+C Code

```
def plot_vectors(a: np.ndarray, b: np.ndarray, theta_deg: float):
    """Plot a, b, and a - 2 b on the unit circle for context."""
    a = np.asarray(a, dtype=np.float64).ravel()
    b = np.asarray(b, dtype=np.float64).ravel()
    c = a - np.sqrt(2.0) * b

    # 2D only for plotting
    if a.size != 2:
        raise ValueError("Plot expects 2D vectors (length 2).")

    # unit circle
    t = np.linspace(0, 2*np.pi, 400)
    plt.figure(figsize=(6,6))
    plt.plot(np.cos(t), np.sin(t), label="Unit circle")
```

Python+C Code

```
# vectors
plt.quiver(0,0, a[0], a[1], angles='xy', scale_units='xy',
    scale=1, label="a")
plt.quiver(0,0, b[0], b[1], angles='xy', scale_units='xy',
    scale=1, label="b")
plt.quiver(0,0, c[0], c[1], angles='xy', scale_units='xy',
    scale=1, label="a  2 b")

# angle arc (between a and b) if both unit and 2D
theta = np.deg2rad(theta_deg)
arc = np.linspace(0, theta, 100)
plt.plot(0.25*np.cos(arc), 0.25*np.sin(arc))
plt.text(0.32*np.cos(theta/2), 0.32*np.sin(theta/2), rf"$\
    theta={theta_deg:.0f}^\circ$")

plt.gca().set_aspect('equal', adjustable='box')
plt.grid(True, linewidth=0.4, alpha=0.5)
plt.xlim(-1.6, 1.6); plt.ylim(-1.6, 1.6)
plt.legend()
```

Python+C Code

```
    plt.title("Vectors a, b, and a  2 b")
    plt.tight_layout()
    plt.show()

if __name__ == "__main__":
    # Example: a is x-axis unit; b is unit at 45
    a = np.array([1.0, 0.0])
    b = np.array([np.cos(np.pi/4), np.sin(np.pi/4)])

    theta_deg, status = solve_angle_np(a, b)
    print(f"theta from C = {theta_deg:.6f}, status={status} (0
        means unit checks passed)")
    print(f"||a||={np.linalg.norm(a):.3f}, ||b||={np.linalg.norm(
        b):.3f}, "
          f"||a-2 b||={np.linalg.norm(a - np.sqrt(2)*b):.3f}")

    plot_vectors(a, b, theta_deg)
```

Python Code

```
import numpy as np
import matplotlib.pyplot as plt

# --- choose 2D unit vectors (you can edit these) ---
theta_true = np.deg2rad(45) # 45
a = np.array([1.0, 0.0]) # unit along +x
b = np.array([np.cos(theta_true), np.sin(theta_true)]) # unit at
    45

# --- compute angle  (in degrees) ---
dot = float(np.dot(a, b))
na = float(np.linalg.norm(a))
nb = float(np.linalg.norm(b))
cos_theta = dot / (na * nb)
cos_theta = max(-1.0, min(1.0, cos_theta)) # clamp for safety
theta_deg = float(np.degrees(np.arccos(cos_theta)))
```

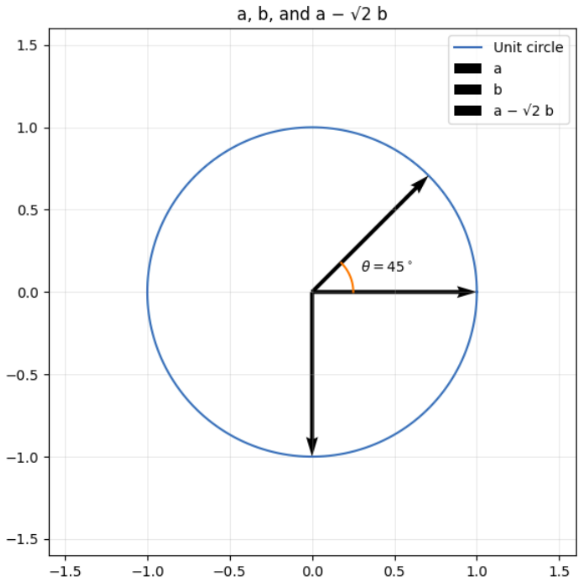
Python Code

```
# --- check the condition ||a||=||b||=||a-2 b||=1 ---
c = a - np.sqrt(2.0) * b
print(f" = {theta_deg:.2f}, ||a||={na:.2f}, ||b||={nb:.2f}, ||a-2
    b||={np.linalg.norm(c):.2f}")

# --- plot unit circle + vectors ---
t = np.linspace(0, 2*np.pi, 400)
plt.figure(figsize=(6,6))
plt.plot(np.cos(t), np.sin(t), label="Unit circle")
plt.quiver(0,0, a[0], a[1], angles='xy', scale_units='xy', scale
    =1, label="a")
plt.quiver(0,0, b[0], b[1], angles='xy', scale_units='xy', scale
    =1, label="b")
plt.quiver(0,0, c[0], c[1], angles='xy', scale_units='xy', scale
    =1, label="a  2 b")
```

Python Code

```
# angle arc (purely for illustration)
arc = np.linspace(0, np.deg2rad(theta_deg), 100)
plt.plot(0.25*np.cos(arc), 0.25*np.sin(arc))
plt.text(0.32*np.cos(np.deg2rad(theta_deg)/2),
         0.32*np.sin(np.deg2rad(theta_deg)/2),
         rf"$\theta={theta_deg:.0f}^\circ$")

plt.gca().set_aspect('equal', adjustable='box')
plt.grid(True, linewidth=0.4, alpha=0.5)
plt.xlim(-1.6, 1.6); plt.ylim(-1.6, 1.6)
plt.legend(); plt.title("a, b, and a  2 b"); plt.tight_layout()
plt.show()
```

Plot


a, b, and a − √2 b
Unit circle
a
b
a − √2 b
θ = 45°
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5