# 4.11.34

EE25BTECH11057 - Rushil Shanmukha Srinivas

**Problem:** Find the area of the region bounded by the lines 3x-2y+1=0,2x+3y-21=0 and x-5y+9=0 .

**Solution:** Given three lines are

$$\begin{pmatrix}3 & -2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = -1 \implies \mathbf{n}^\top\mathbf{x} = -1 \tag{0.1}$$

$$\begin{pmatrix}2 & 3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = 21 \implies \mathbf{m}^\top\mathbf{x} = 21 \tag{0.2}$$

$$\begin{pmatrix}1 & -5\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} = -9 \implies \mathbf{p}^\top\mathbf{x} = -9 \tag{0.3}$$

The three lines form a triangle. The vertices of triangle are obtained by

**Intersection of :**

$$\mathbf{n}^\top\mathbf{x} = -1 \; and \; \mathbf{m}^\top\mathbf{x} = 21 \tag{0.4}$$

The augmented system in matrix form is

$$\left(\begin{array}{cc|c}3 & -2 & -1\\2 & 3 & 21\end{array}\right) \xrightarrow{R_2 \longrightarrow 3R_2 - 2R_1} \left(\begin{array}{cc|c}3 & -2 & -1\\0 & 13 & 65\end{array}\right) \tag{0.5}$$

*From the second row we get* $y = 5$ *so* $x = 3 \implies \mathbf{A} = \begin{pmatrix}3\\5\end{pmatrix}$

$$\mathbf{m}^\top\mathbf{x} = 21 \; and \; \mathbf{p}^\top\mathbf{x} = -9 \tag{0.6}$$

The augmented matrix is

$$\left(\begin{array}{cc|c}2 & 3 & 21\\1 & -5 & -9\end{array}\right) \xrightarrow{R_2 \longrightarrow 2R_2 - R_1} \left(\begin{array}{cc|c}2 & 3 & 21\\0 & -13 & -39\end{array}\right) \tag{0.7}$$

*From the second row we get* $y = 3$ *so* $x = 6 \implies \mathbf{B} = \begin{pmatrix}6\\3\end{pmatrix}$

$$\mathbf{p}^\top\mathbf{x} = -9 \; and \; \mathbf{n}^\top\mathbf{x} = -1 \tag{0.8}$$

The augmented matrix is

$$\left(\begin{array}{cc|c} 1 & -5 & -9 \\ 3 & -2 & -1 \end{array}\right) \xrightarrow{R_2 \longrightarrow R_2 - 3R_1} \left(\begin{array}{cc|c} 1 & -5 & -9 \\ 0 & 13 & 26 \end{array}\right) \tag{0.9}$$

*From the second row we get* $y = 2$ *so* $x = 1 \implies \mathbf{C} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}, \mathbf{A} - \mathbf{C} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \tag{0.10}$$

$$||(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C})|| = |\begin{vmatrix} -3 & 2 \\ 2 & 3 \end{vmatrix}| = |-9-4| = |-13| = 13 \tag{0.11}$$

$$\textit{Area of the triangle} = \frac{1}{2}||(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} - \mathbf{C}|| = \frac{13}{2} \tag{0.12}$$

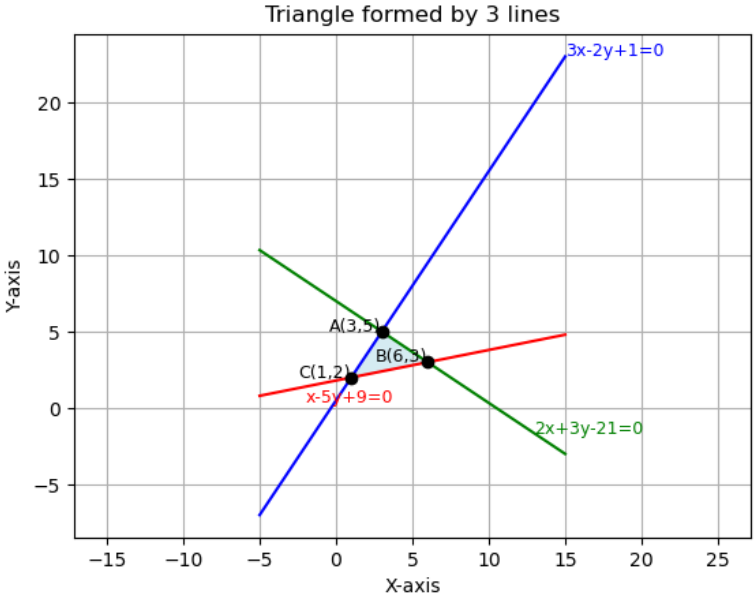


Fig: Representation of Triangle