

# 1.5.36

EE25BTECH11049 - Sai Krishna Bakki

## Question:

Point  $P(x, 4)$  lies on the line segment joining the points  $A(-5, 8)$  and  $B(4, -10)$ . Find the ratio in which point  $P$  divides the line segment  $AB$ . Also, find the value of  $x$ .

## Solution:

Let

$$\mathbf{A} = \begin{pmatrix} -5 \\ 8 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ -10 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} x \\ 4 \end{pmatrix}$$

Since  $\mathbf{P}$  lies on  $\mathbf{A}$  and  $\mathbf{B}$ , they must be collinear

$$\therefore \text{rank}(\mathbf{P} - \mathbf{A} \quad \mathbf{B} - \mathbf{A}) = 1$$

$$\text{rank} \begin{pmatrix} x+5 & 9 \\ -4 & -18 \end{pmatrix} = 1$$

By transformation  $R_2 \rightarrow R_2 + 2R_1$

$$\text{rank} \begin{pmatrix} x+5 & 9 \\ 2x+6 & 0 \end{pmatrix} = 1$$

Thus for rank to be 1

$$2x+6=0$$

$$\therefore x = -3$$

Thus  $\mathbf{P}$  is :

$$\mathbf{P} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$$

let  $\mathbf{P}$  divide the line joining points  $\mathbf{A}$  and  $\mathbf{B}$  in the ratio  $k : 1$ .

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k+1}$$

$$k(\mathbf{P} - \mathbf{B}) = \mathbf{A} - \mathbf{P}k = \frac{(\mathbf{P} - \mathbf{B})^T (\mathbf{A} - \mathbf{P})}{\|(\mathbf{P} - \mathbf{B})\|^2}$$

$$k = \frac{\begin{pmatrix} x-4 \\ -14 \end{pmatrix} \cdot \begin{pmatrix} -5-x \\ 4 \end{pmatrix}}{\left\| \begin{pmatrix} x-4 \\ -14 \end{pmatrix} \right\|^2}$$

substituting the value of  $x$  as , we get the value of  $k$  as

$$k = 2/7$$

