

2.6.35

AI25BTECH11024 - Pratyush Panda

Question:

The value of $\hat{\mathbf{i}} \cdot (\hat{\mathbf{j}} \times \hat{\mathbf{k}}) + \hat{\mathbf{j}} \cdot (\hat{\mathbf{i}} \times \hat{\mathbf{k}}) + \hat{\mathbf{k}} \cdot (\hat{\mathbf{i}} \times \hat{\mathbf{j}})$ is _____

Solution:

Given:

$$\hat{\mathbf{i}} = \mathbf{e}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \hat{\mathbf{j}} = \mathbf{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \hat{\mathbf{k}} = \mathbf{e}_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (0.1)$$

Each term of the expression in the question can be found using the scalar triple product determinant.

The first term can be written as:

$$(\hat{\mathbf{i}} \cdot (\hat{\mathbf{j}} \times \hat{\mathbf{k}})) = \begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (0.2)$$

Determinant of this matrix is 1. Thus, the value of first term is 1.

The second term can be written as:

$$(\hat{\mathbf{j}} \cdot (\hat{\mathbf{i}} \times \hat{\mathbf{k}})) = \begin{pmatrix} e_2 & e_3 & e_1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad (0.3)$$

Determinant of this matrix is -1. Thus, the value of first term is -1.

The third term can be written as:

$$(\hat{\mathbf{k}} \cdot (\hat{\mathbf{i}} \times \hat{\mathbf{j}})) = \begin{pmatrix} e_3 & e_1 & e_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad (0.4)$$

Determinant of this matrix is 1. Thus, the value of first term is 1.

So, the sum of all the terms is;

$$\hat{\mathbf{i}} \cdot (\hat{\mathbf{j}} \times \hat{\mathbf{k}}) + \hat{\mathbf{j}} \cdot (\hat{\mathbf{i}} \times \hat{\mathbf{k}}) + \hat{\mathbf{k}} \cdot (\hat{\mathbf{i}} \times \hat{\mathbf{j}}) = 1 + (-1) + 1 \quad (0.5)$$

$$\text{or, } \hat{\mathbf{i}} \cdot (\hat{\mathbf{j}} \times \hat{\mathbf{k}}) + \hat{\mathbf{j}} \cdot (\hat{\mathbf{i}} \times \hat{\mathbf{k}}) + \hat{\mathbf{k}} \cdot (\hat{\mathbf{i}} \times \hat{\mathbf{j}}) = 1 \quad (0.6)$$

