5.2.55

EE25BTECH11052 - Shriyansh Kalpesh Chawda

Question:
Solve the following system of linear equations.

$$\frac{2}{x} + \frac{3}{y} = 13 \quad \frac{5}{x} + \frac{4}{y} = -2$$

Solution:
Let

$$u = \frac{1}{x}, \quad v = \frac{1}{y}. \tag{1}$$

The given system becomes

$$2u + 3v = 13 \tag{2}$$
$$5u + 4v = -2 \tag{3}$$

In matrix form:

$$\begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} 13 \\ -2 \end{pmatrix}. \tag{4}$$

Let

$$A = \begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 13 \\ -2 \end{pmatrix}. \tag{5}$$

We solve using the inverse:

$$\begin{pmatrix} u \\ v \end{pmatrix} = A^{-1}\mathbf{b}. \tag{6}$$

We start with the augmented matrix $[A|I]$.

$$\left[\begin{array}{cc|cc} 2 & 3 & 1 & 0 \\ 5 & 4 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \to \frac{1}{2}R_1} \left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 5 & 4 & 0 & 1 \end{array}\right] \tag{7}$$

$$\xrightarrow{R_2 \to R_2 - 5R_1} \left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & -\frac{7}{2} & -\frac{5}{2} & 1 \end{array}\right] \tag{8}$$

$$\xrightarrow{R_2 \to -\frac{2}{7}R_2} \left[\begin{array}{cc|cc} 1 & \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 1 & \frac{5}{7} & -\frac{2}{7} \end{array}\right] \tag{9}$$

$$\xrightarrow{R_1 \to R_1 - \frac{3}{2}R_2} \left[\begin{array}{cc|cc} 1 & 0 & -\frac{4}{7} & \frac{3}{7} \\ 0 & 1 & \frac{5}{7} & -\frac{2}{7} \end{array}\right] \tag{10}$$

The inverse matrix is:

$$A^{-1} = \begin{pmatrix} -\frac{4}{7} & \frac{3}{7} \\ \frac{5}{7} & -\frac{2}{7} \end{pmatrix} = -\frac{1}{7}\begin{pmatrix} 4 & -3 \\ -5 & 2 \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \end{pmatrix} = A^{-1}\mathbf{b} = -\frac{1}{7}\begin{pmatrix} 4 & -3 \\ -5 & 2 \end{pmatrix}\begin{pmatrix} 13 \\ -2 \end{pmatrix} \tag{11}$$

$$= -\frac{1}{7}\begin{pmatrix} 52+6 \\ -65-4 \end{pmatrix} \tag{12}$$

$$= -\frac{1}{7}\begin{pmatrix} 58 \\ -69 \end{pmatrix} \tag{13}$$

$$= \begin{pmatrix} -\frac{58}{7} \\ \frac{69}{7} \end{pmatrix} \tag{14}$$

Back substituting:

$$u = \frac{1}{x} = -\frac{58}{7} \implies x = -\frac{7}{58}, \tag{15}$$

$$v = \frac{1}{y} = \frac{69}{7} \implies y = \frac{7}{69}. \tag{16}$$

Thus, the solution is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -\frac{7}{58} \\ \frac{7}{69} \end{pmatrix}. \tag{17}$$

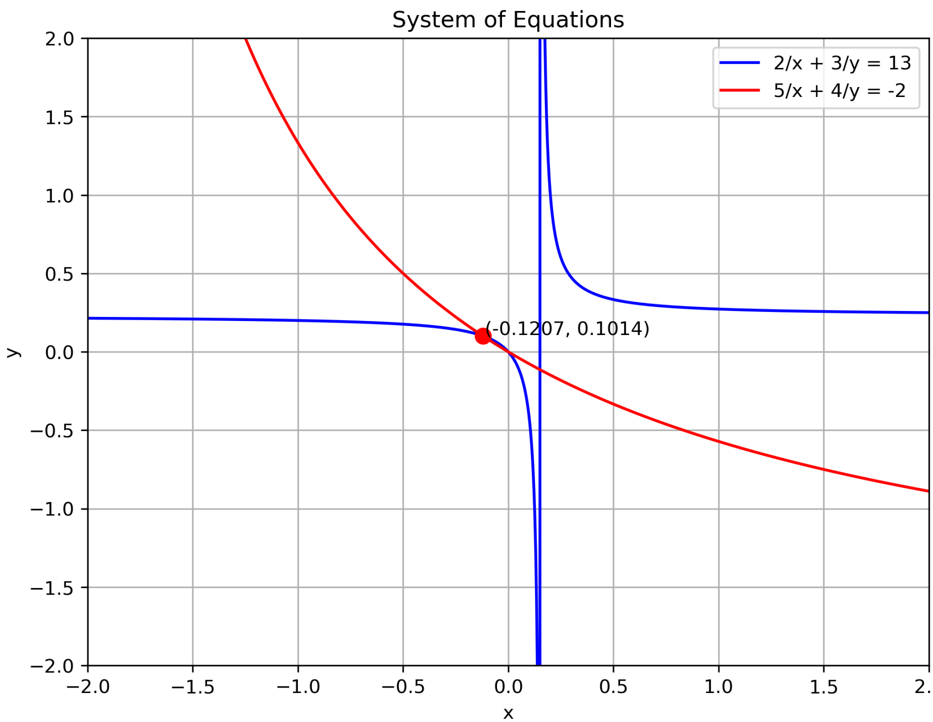

Fig. 1