Question:

Find the coordinates of a point A where AB is the diameter of the circle with center is $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and B is the point $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$.

Solution:

Given data:

Point	Vector
В	$\begin{pmatrix} 2 \\ 6 \end{pmatrix}$
P	$\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

Theory: Center of a circle is the mid-point of the diameter.

Let P be the center of the given circle, with AB as the diameter.

Let A be the Vector to be found

Given:

$$B \equiv \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \quad P \equiv \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

Center of a circle is the mid point of the diameter. For a circle with center P and ends of diameters represented by vectors A and B

$$\mathbf{P} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{0.1}$$

Rearranging, we get:

$$\mathbf{A} = 2\mathbf{P} - \mathbf{B} \tag{0.2}$$

Substituting the given vectors, we get:

$$\mathbf{A} = 2 \begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \end{pmatrix} \tag{0.3}$$

$$\mathbf{A} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \end{pmatrix} \tag{0.4}$$

$$\therefore \mathbf{A} \equiv \begin{pmatrix} 4 \\ -8 \end{pmatrix}$$

Hence, Coordinates of A are

$$\begin{pmatrix} 4 \\ -8 \end{pmatrix}$$

Graph representation:

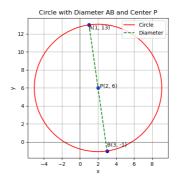


Fig. 0.1