5.4.40

Puni Aditya - EE25BTECH11046

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Question

Using elementary transformations, find the inverse of the following matrix.

$$\begin{pmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & 1 \end{pmatrix}$$

Theoretical Solution

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & 1 \end{pmatrix} \tag{1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 3 & 0 & 0 & 1 & 0 \\ 5 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \xleftrightarrow[R_3 \to R_3 - 5R_1]{R_2 \to R_2 - 3R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 3 & 0 & -3 & 1 & 0 \\ 0 & 2 & 1 & -5 & 0 & 1 \end{array}\right) \tag{2}$$

Theoretical Solution

$$\xleftrightarrow{R_2 \to \frac{1}{3}R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & \frac{1}{3} & 0 \\ 0 & 2 & 1 & -5 & 0 & 1 \end{array}\right) \xleftrightarrow{R_3 \to R_3 - 2R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & -3 & -\frac{2}{3} & 1 \end{array}\right) \tag{3}$$

$$\mathbf{A}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & \frac{1}{3} & 0 \\ -3 & -\frac{2}{3} & 1 \end{pmatrix} \tag{4}$$