4.13.50

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Question

Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point $(1, -10)$. Determine the equation of the third side.

Theoretical Solution

Let the two equal sides of the isosceles triangle be represented by

$$\mathbf{n_1}^\top \mathbf{x} = c_1$$
$$\mathbf{n_2}^\top \mathbf{x} = c_2$$

and the third side by the line

$$\mathbf{n}^\top \mathbf{x} = c$$

The third side of the isosceles, the base, is perpendicular to the angle bisector of the two equal sides.

Theoretical Solution

$$\frac{\left|\mathbf{n}^\top \mathbf{n_1}\right|}{\left\|\mathbf{n}\right\| \left\|\mathbf{n_1}\right\|} = \frac{\left|\mathbf{n}^\top \mathbf{n_2}\right|}{\left\|\mathbf{n}\right\| \left\|\mathbf{n_2}\right\|} \tag{1}$$

$$\frac{\left|\mathbf{n}^\top \mathbf{u_1}\right|}{\left\|\mathbf{n}\right\|} = \frac{\left|\mathbf{n}^\top \mathbf{u_2}\right|}{\left\|\mathbf{n}\right\|} \tag{2}$$

$$\left|\mathbf{n}^\top \mathbf{u_1}\right| = \left|\mathbf{n}^\top \mathbf{u_2}\right| \tag{3}$$

$$\mathbf{n}^\top \mathbf{u_1} = \pm \mathbf{n}^\top \mathbf{u_2} \tag{4}$$

$$\mathbf{n}^\top (\mathbf{u_1} \mp \mathbf{u_2}) = 0 \tag{5}$$

Here, $\mathbf{u_1}$ and $\mathbf{u_2}$ represent the unit vectors of $\mathbf{n_1}$ and $\mathbf{n_2}$ respectively.

Theoretical Solution

A vector perpendicular to given vector $\begin{pmatrix} 1 \\ m \end{pmatrix}$ is

$$\mathbf{n} = \begin{pmatrix} -m \\ 1 \end{pmatrix} \tag{6}$$

For the given question,

$$\mathbf{n_1} = \begin{pmatrix} 7 \\ -1 \end{pmatrix} \text{ and } \mathbf{n_2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{7}$$

$$\|\mathbf{n_1}\| = \sqrt{50} = 5\sqrt{2} \tag{8}$$

$$\|\mathbf{n_2}\| = \sqrt{2} \tag{9}$$

$$\mathbf{u_1} = \frac{1}{5\sqrt{2}} \begin{pmatrix} 7 \\ -1 \end{pmatrix}, \mathbf{u_2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{5}{5\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{10}$$

Theoretical Solution

$$\mathbf{u_1} - \mathbf{u_2} = \frac{1}{5\sqrt{2}}\left(\begin{pmatrix} 7 \\ -1 \end{pmatrix} - \begin{pmatrix} 5 \\ 5 \end{pmatrix}\right) = \frac{1}{5\sqrt{2}}\begin{pmatrix} 2 \\ -6 \end{pmatrix} \tag{11}$$

$$\mathbf{n_a} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \tag{12}$$

$$\mathbf{u_1} + \mathbf{u_2} = \frac{1}{5\sqrt{2}}\left(\begin{pmatrix} 7 \\ -1 \end{pmatrix} + \begin{pmatrix} 5 \\ 5 \end{pmatrix}\right) = \frac{1}{5\sqrt{2}}\begin{pmatrix} 12 \\ 4 \end{pmatrix} \tag{13}$$

$$\mathbf{n_b} = \begin{pmatrix} 1 \\ \frac{1}{3} \end{pmatrix} \tag{14}$$

For the bisector parallel to $\mathbf{n_a}$, using (6),

$$\mathbf{n_p} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{15}$$

For the bisector parallel to $\mathbf{n_b}$, using (6),

$$\mathbf{n_q} = \begin{pmatrix} -\frac{1}{3} \\ 1 \end{pmatrix} \tag{16}$$

For a line passing through a given point $\mathbf{p}$,

$$\mathbf{p} = \begin{pmatrix} 1 \\ -10 \end{pmatrix} \tag{17}$$

$$\mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{p} \tag{18}$$

Theoretical Solution

For $\mathbf{n_p}$,

$$\mathbf{n} = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \tag{19}$$

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -10 \end{pmatrix} \tag{20}$$

$$\begin{pmatrix} 3 & 1 \end{pmatrix} \mathbf{x} = -7 \tag{21}$$

For $\mathbf{n_q}$,

$$\mathbf{n} = \begin{pmatrix} 1 \\ -3 \end{pmatrix} \tag{22}$$

$$\begin{pmatrix} 1 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ -10 \end{pmatrix} \tag{23}$$

$$\begin{pmatrix} 1 & -3 \end{pmatrix} \mathbf{x} = 31 \tag{24}$$

Plot