2.10.49

EE25BTECH11020 - Darsh Pankaj Gajare

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Question:
The unit vector which is orthogonal to the vector $3\hat{i} + 2\hat{j} + 6\hat{k}$ and is coplanar with vectors $2\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} - \hat{j} + \hat{k}$ is

(A) $\frac{2\hat{i}-6\hat{j}+\hat{k}}{\sqrt{41}}$ (B) $\frac{2\hat{i}-3\hat{j}}{\sqrt{13}}$ (C) $\frac{3\hat{i}-\hat{k}}{\sqrt{10}}$ (D) $\frac{4\hat{i}+3\hat{j}-3\hat{k}}{\sqrt{34}}$

Solution: Given:

Table: Given data

Vector	**matrix**
A	$\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$
B	$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$
C	$\begin{pmatrix} 3 \\ 2 \\ 6 \end{pmatrix}$

Equation of plane through $\mathbf{A}, \mathbf{B}$.

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix} \mathbf{n} = 0. \tag{0.1}$$

As augmented matrix,

$$\left(\begin{array}{ccc|c} 2 & 1 & 1 & 0 \\ 1 & -1 & 1 & 0 \end{array}\right). \tag{0.2}$$

Using Row operations: $R_1 = R_1 - 2R_2$, $R_2 = R_2 + R_1$

$$\left(\begin{array}{ccc|c} 0 & 3 & -1 & 0 \\ 1 & 2 & 0 & 0 \end{array}\right). \tag{0.3}$$

$$\mathbf{n} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \tag{0.4}$$

condition with $\mathbf{C}$.

$$\mathbf{C}^\top \mathbf{P} = 0 \tag{0.5}$$

So **P** satisfies

$$\begin{pmatrix} -2 & 1 & 3 \\ 3 & 2 & 6 \end{pmatrix} \mathbf{x} = 0. \tag{0.6}$$

$$\left(\begin{array}{ccc|c} -2 & 1 & 3 & 0 \\ 3 & 2 & 6 & 0 \end{array}\right). \tag{0.7}$$

Row operations: $R_2 = 2R_2 - 3R_1$, $R_2 = R_2/7$, $R_1 = R_1 + R_2$

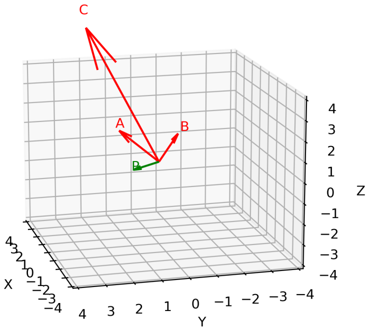
$$\left(\begin{array}{ccc|c} 2 & 0 & 0 & 0 \\ 0 & 1 & 3 & 0 \end{array}\right). \tag{0.8}$$

From first row: $2x = 0 \implies x = 0$. From second:
$y + 3z = 0 \implies y = -3z$. So

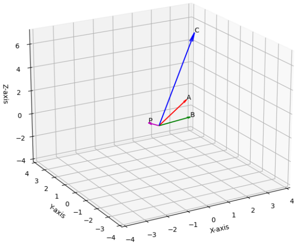
$$\mathbf{P} = \begin{pmatrix} 0 \\ -3z \\ z \end{pmatrix} = z \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}. \tag{0.9}$$

Normalizing,

$$\mathbf{P} = \pm \frac{1}{\sqrt{10}} \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} \tag{0.10}$$



Plot using C functions



Plot using Python