4.2.6

EE25BTECH11003 - Adharvan Kshathriya Bommagani

Question:
Find the direction and normal vectors of each of the following lines.
4.2.6 $3x + 2 = 0$

Solution:

To find the normal and direction vectors for the line $3x + 2 = 0$, we can express the equation in a standard vector form.
The equation of a line can be written in the form $\mathbf{n}^T\mathbf{x} = c$, where $\mathbf{n}$ is the normal vector and $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$.
The given equation can be rewritten to include the y term explicitly:

$$3x + 0y = -2$$

This can be expressed in vector form:

$$\begin{pmatrix} 3 \\ 0 \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} = -2$$

By comparing this to the general form $\mathbf{n}^T\mathbf{x} = c$, we can directly identify the **normal vector** as:

$$\mathbf{n} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$

The direction vector $\mathbf{m}$ is orthogonal (perpendicular) to the normal vector $\mathbf{n}$. This means their dot product is zero:

$$\mathbf{m}^T\mathbf{n} = 0$$

Let the direction vector be $\mathbf{m} = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}$. We can set up the equation:

$$\begin{aligned} \begin{pmatrix} m_1 & m_2 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix} &= 0 \\ (m_1)(3) + (m_2)(0) &= 0 \\ 3m_1 &= 0 \\ m_1 &= 0 \end{aligned}$$

Since $m_1 = 0$, the direction vector is of the form $\begin{pmatrix} 0 \\ m_2 \end{pmatrix}$. The vector must be non-zero, so we can choose any non-zero value for m_2. The simplest choice is $m_2 = 1$.

Therefore, the **direction vector** is:

$$\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Answer:

Normal Vector: $\mathbf{n} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$

Direction Vector: $\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Illustration of the Line and Vectors:

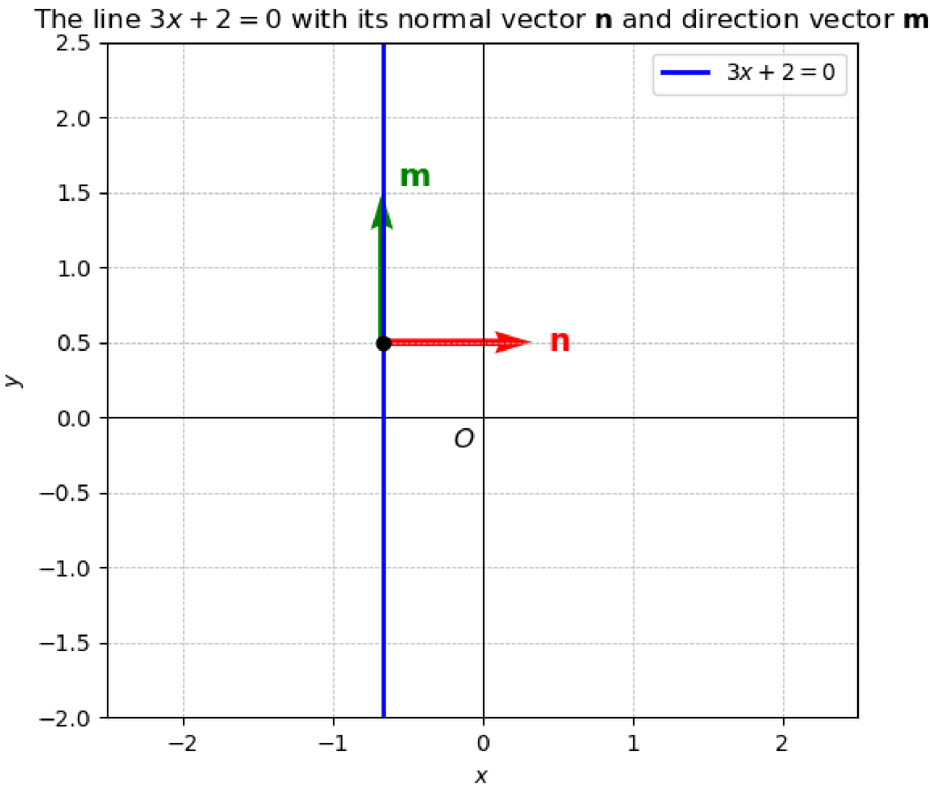

Fig. 6: Figure for 4.2.6