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Kartik Lahoti - EE25BTECH11032

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Question

The two vectors $[1, 1, 1]$ and $\left[1, a, a^2\right]$, where $a = \left(\frac{-1}{2} + j\frac{\sqrt{3}}{2}\right)$

1. orthonormal
2. orthogonal
3. paralle
4. collinear

Theoretical Solution

Given ,

$$\mathbf{P} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (1)$$

$$\mathbf{Q} = \begin{pmatrix} 1 \\ a \\ a^2 \end{pmatrix} \quad (2)$$

Theoretical Solution

we know,

$$x + jy \longrightarrow \begin{pmatrix} x & -y \\ y & x \end{pmatrix} \tag{3}$$

$$a = \left(\frac{-1}{2} + j\frac{\sqrt{3}}{2}\right) \longrightarrow \mathbf{A} = \begin{pmatrix} \frac{-1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \tag{4}$$

Similarly

$$a^2 = \left(\frac{-1}{2} - j\frac{\sqrt{3}}{2}\right) \longrightarrow \mathbf{A}^2 = \begin{pmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \tag{5}$$

$$1 \longrightarrow \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

Theoretical Solution

Now,

$$\mathbf{I} + \mathbf{A} + \mathbf{A}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \frac{-1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{-1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} + \begin{pmatrix} \frac{-1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \tag{7}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{8}$$

$$\implies 1 + a + a^2 = 0 \tag{9}$$

Theoretical Solution

Now, Look At ,

$$\mathbf{P}^\top \mathbf{Q} = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ a \\ a^2 \end{pmatrix} = 1 + a + a^2 = 0 \tag{10}$$

Hence **P** and **Q** are orthogonal.
Answer : Option (2)