

4.2.22

EE25BTECH11019 - Darji Vivek M.

Question:

Show that the two lines

$$a_1x + b_1y + c_1 = 0, \quad a_2x + b_2y + c_2 = 0$$

with $b_1b_2 \neq 0$ are parallel iff $\frac{a_1}{b_1} = \frac{a_2}{b_2}$.

Solution:

$$\text{Form the } 2 \times 2 \text{ coefficient matrix of normals: } \mathbf{M} = \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix}. \quad (1)$$

Assume $\frac{a_1}{b_1} = \frac{a_2}{b_2}$. Then there exists $k \in \mathbb{R}$ such that

$$a_2 = k a_1, \quad b_2 = k b_1. \quad (2)$$

Write the rows of \mathbf{M} as row vectors:

$$\mathbf{R}_1 = (a_1 \quad a_2) = (a_1 \quad k a_1) = a_1 (1 \quad k). \quad (3)$$

$$\mathbf{R}_2 = (b_1 \quad b_2) = (b_1 \quad k b_1) = b_1 (1 \quad k). \quad (4)$$

$$\mathbf{R}_2 \mapsto \mathbf{R}_2 - \frac{b_1}{a_1} \mathbf{R}_1, \quad \mathbf{R}_2 = (0, 0). \quad (5)$$

Thus the row-echelon form of \mathbf{M} has exactly one nonzero row, so

$$\text{rank}(\mathbf{M}) = 1. \quad (6)$$

if $\text{rank}(\mathbf{M}) = 1$ then the two rows (or columns) are proportional, therefore $\frac{a_1}{b_1} = \frac{a_2}{b_2}$.

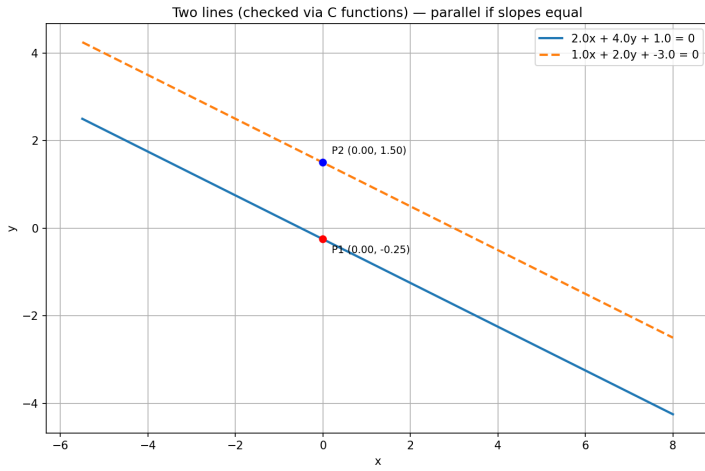


Fig. 0.1: plot