

5.5.18

AI25BTECH11001 - ABHISEK MOHAPATRA

Question: Find the inverse of the following matrix, using elementary transformations

$$\begin{pmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{pmatrix}$$

Solution: Given:

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{I} \quad (1)$$

$$\begin{pmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{pmatrix} \mathbf{A}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2)$$

Augumented Matrix:

$$\left(\begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ 3 & 7 & 2 & 0 & 0 & 1 \end{array} \right) \quad (3)$$

$$\xrightarrow{R_2 \rightarrow R_2 - R_1} \left(\begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 3 & 7 & 2 & 0 & 0 & 1 \end{array} \right) \quad (4)$$

$$\xrightarrow{R_3 \rightarrow R_3 - \frac{3}{2}R_1} \left(\begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & \frac{5}{2} & \frac{1}{2} & -\frac{3}{2} & 0 & 1 \end{array} \right) \quad (5)$$

$$\xrightarrow{R_3 \rightarrow R_3 - \frac{5}{2}R_2} \left(\begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & \frac{1}{2} & 1 & -\frac{5}{2} & 1 \end{array} \right) \quad (6)$$

$$\xrightarrow{R_1 \rightarrow R_1 - 3R_2 - 2R_3} \left(\begin{array}{ccc|ccc} 2 & 0 & 0 & 2 & 2 & -2 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & \frac{1}{2} & 1 & -\frac{5}{2} & 1 \end{array} \right) \quad (7)$$

$$\xrightarrow{R_1 \rightarrow R_1 - 3R_2 - 2R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -5 & 2 \end{array} \right) \quad (8)$$

So,

$$\mathbf{A}^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{pmatrix} \quad (9)$$