

Matgeo Presentation - Problem 10.6.11

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Problem Statement

Draw a circle of radius 4 cm. Draw two tangents to the circle inclined at an angle of 60° to each other.

Data

Name	Value
Circle	$\mathbf{x}^\top \mathbf{x} - 16 = 0$
P	$\begin{pmatrix} 8 \\ 0 \end{pmatrix}$

Table : Circle

Solution

The parameters of the circle with center $\mathbf{0}$ are :

$$\mathbf{V} = \mathbf{I} \qquad \mathbf{u} = \mathbf{0} \qquad f = -16 \qquad (0.1)$$

Let the point from which tangent is being drawn be \mathbf{p} .

Let the point of contact be \mathbf{q} and

$$\mathbf{q}^\top \mathbf{q} = 16 \qquad (0.2)$$

From the condition of tangency we get

$$\mathbf{q}^\top (\mathbf{q} - \mathbf{p}) = 0 \qquad (0.3)$$

$$\mathbf{p}^\top \mathbf{q} = \mathbf{q}^\top \mathbf{q} \qquad (0.4)$$

$$\mathbf{p}^\top \mathbf{q} = 16 \qquad (0.5)$$

If the angle between the tangents is 60° then the angle between the normals at the points of contact is 120° .

Solution

Therefore,

$$\cos\left(\frac{120^\circ}{2}\right) = \frac{\mathbf{p}^\top \mathbf{q}}{\|\mathbf{p}\| \|\mathbf{q}\|} \quad (0.6)$$

$$\|\mathbf{p}\| = 8 \quad (0.7)$$

$$\mathbf{p}^\top \mathbf{p} - 64 = 0 \quad (0.8)$$

Therefore the locus of point \mathbf{p} is a circle with center $\mathbf{0}$ and radius 8 cm.

Consider point $\mathbf{P} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$ from which tangents are drawn.

Let the slope of tangent be m and the tangent equationn is given as :

$$\mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{P} \quad \mathbf{n} = \begin{pmatrix} -m \\ 1 \end{pmatrix} \quad (0.9)$$

Solution

The length of perpendicular from the center of the circle to the tangent is equal to the radius and is given by :

$$4 = \frac{|\mathbf{n}^T \mathbf{0} - \mathbf{n}^T \mathbf{P}|}{\|\mathbf{n}\|} \quad (0.10)$$

$$|\mathbf{n}^T \mathbf{P}| = 4\|\mathbf{n}\| \quad (0.11)$$

$$|-8m| = 4\sqrt{m^2 + 1} \quad (0.12)$$

$$m = \pm \frac{1}{\sqrt{3}} \quad (0.13)$$

The normal vectors for the tangents are given as :

$$\mathbf{n}_1 = \begin{pmatrix} -\frac{1}{\sqrt{3}} \\ 1 \end{pmatrix} \quad \mathbf{n}_2 = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ 1 \end{pmatrix} \quad (0.14)$$

Solution

The points of contacts are given as :

$$\mathbf{q}_i = \pm r \frac{\mathbf{n}_i}{\|\mathbf{n}_i\|} \quad (0.15)$$

From (0.5) , $\mathbf{P}^\top \mathbf{q} = 16$, so the points of contact are :

$$\mathbf{q}_1 = \begin{pmatrix} 2 \\ 2\sqrt{3} \end{pmatrix} \quad \mathbf{q}_2 = \begin{pmatrix} 2 \\ -2\sqrt{3} \end{pmatrix} \quad (0.16)$$

Plot

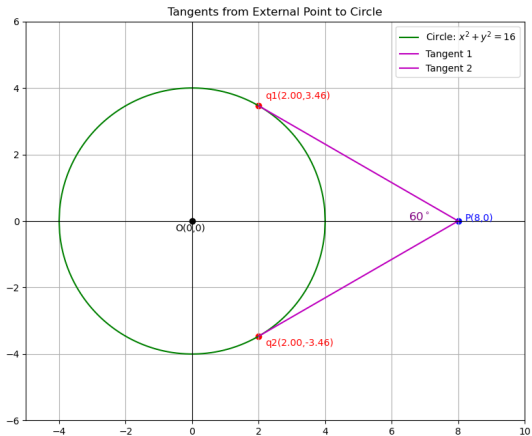


Fig : Circle and Tangents