Matrices in Geometry - 5.4.32

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Problem Statement

Find inverse of the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{pmatrix}$$

Solution

Let **B** be the inverse of **A**, then

$$\mathbf{AB} = \mathbf{I} \tag{1}$$

forming the augmented matrix,

$$\begin{pmatrix}
1 & 0 & 1 & | & 1 & 0 & 0 \\
0 & 1 & 2 & | & 0 & 1 & 0 \\
0 & 0 & 4 & | & 0 & 0 & 1
\end{pmatrix}
\xrightarrow{R_3 \to R_3/4}
\begin{pmatrix}
1 & 0 & 1 & | & 1 & 0 & 0 \\
0 & 1 & 2 & | & 0 & 1 & 0 \\
0 & 0 & 1 & | & 0 & 0 & \frac{1}{4}
\end{pmatrix}
\xrightarrow{R_2 \to R_2 - 2R_3}$$

$$\begin{pmatrix}
1 & 0 & 1 & | & 1 & 0 & 0 \\
0 & 1 & 0 & | & 0 & 1 & \frac{-1}{4} \\
0 & 1 & 0 & | & 0 & 1 & \frac{-1}{2} \\
0 & 0 & 1 & | & 0 & 0 & \frac{1}{4}
\end{pmatrix}
\xrightarrow{R_1 \to R_1 - R_3}
\begin{pmatrix}
1 & 0 & 0 & | & 1 & 0 & \frac{-1}{4} \\
0 & 1 & 0 & | & 0 & 1 & \frac{-1}{2} \\
0 & 0 & 1 & | & 0 & 0 & \frac{1}{4}
\end{pmatrix}$$
(3)

Solution

Thus,

$$\mathbf{B} = \mathbf{A}^{-1} = \begin{pmatrix} 1 & 0 & \frac{-1}{4} \\ 1 & 0 & \frac{-1}{2} \\ 1 & 0 & \frac{1}{4} \end{pmatrix} \tag{4}$$