Matrices in Geometry - 1.4.24

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Problem Statement

If $P(9a-2,-b)$ divides line segment joining $A(3a+1,-3)$ and $B(8a,5)$ in the ratio 3:1, find a and b.

Solution

Section Formula: If P divides AB in $m : n$, then

$$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right).$$

By applying the section formula:

$$9a - 2 = \frac{27a + 1}{4}, \qquad -b = 3.$$

$$\Rightarrow \quad a = 1, \quad b = -3.$$

We will now compute (a, b) in Python and verify them using C.

Python Code (Computation + Plot)

```
import numpy as np
import numpy.linalg as la
import matplotlib.pyplot as plt

# Solving eqns
coeff = np.array([[9-27/4, 0], [0, -1]])
rhs = np.array([2+1/4, 3])
soln = la.solve(coeff, rhs)
a = soln[0]
b = soln[1]

# Coordinates
A = (3*a + 1, -3)
B = (8*a, 5)
P = (9*a - 2, -b)

# Plotting points
plt.figure(figsize=(6,6))
plt.plot([A[0], B[0]], [A[1], B[1]], 'k--', label='Line␣AB')
# Line AB
plt.scatter(*A, color='blue', s=100, label='A␣(3a+1,␣-3)')
plt.scatter(*B, color='green', s=100, label='B␣(8a,␣5)')
plt.scatter(*P, color='red', s=100, label='P␣(9a-2,␣-b)')
```

Python Code (Cont..)

```
# Annotating points
plt.text(A[0]+0.2, A[1], 'A', fontsize=12)
plt.text(B[0]+0.2, B[1], 'B', fontsize=12)
plt.text(P[0]+0.2, P[1], 'P', fontsize=12)
```

```
# Axis labels and grid
plt.xlabel('x')
plt.ylabel('y')
plt.title('Plot of Points A, B and P dividing AB in 3:1')
plt.grid(True)
plt.legend()
plt.axis('equal')
plt.show()
```

C Code for verification

```
#include<stdio.h>

int solve(float a, float b) {
    float A[2] = {3*a + 1, -3};
    float B[2] = {8*a, 5};
    float P[2] = {9*a - 2, -b};
    int ratio = 3;

    if (((A[0]+ratio*B[0])/(ratio+1) == P[0]) && ((A[1]+ratio*B[1]
        return 1;
    }
    return 0;
}
```

This function is compiled as a shared library and called from Python using ctypes.

Using the C code in Python

```
import ctypes
import numpy as np
import numpy.linalg as la
import matplotlib.pyplot as plt

check = ctypes.CDLL("./verify.so")
check.solve.argtypes = [ctypes.c_float, ctypes.c_float]
check.solve.restype = ctypes.c_int

coeff = np.array([[9-27/4, 0], [0, -1]])
rhs = np.array([2+1/4, 3])
soln = la.solve(coeff, rhs)
a = soln[0]
b = soln[1]

correct = check.solve(a,b)
A = (3*a + 1, -3)
B = (8*a, 5)
P = (9*a - 2, -b)
```

Using the C code in Python (Cont..)

```
if correct:
    # Plotting points
    plt.figure(figsize=(6,6))
    plt.plot([A[0], B[0]], [A[1], B[1]], 'k--', label='Line AB')
    plt.scatter(*A, color='blue', s=100, label='A (3a+1, -3)')
    plt.scatter(*B, color='green', s=100, label='B (8a, 5)')
    plt.scatter(*P, color='red', s=100, label='P (9a-2, -b)')

    # Annotating points
    plt.text(A[0]+0.2, A[1], 'A', fontsize=12)
    plt.text(B[0]+0.2, B[1], 'B', fontsize=12)
    plt.text(P[0]+0.2, P[1], 'P', fontsize=12)

    # Axis labels and grid
    plt.xlabel('x')
    plt.ylabel('y')
    plt.title('Plot of Points A, B and P dividing AB in 3:1')
    plt.grid(True)
    plt.legend()
    plt.axis('equal')
    plt.show()
```

Resulting Plot