

Matrices in Geometry 10.5.5

EE25BTECH11037 - Divyansh

Question: Construct a tangent to a circle of radius $4cm$ from a point on the concentric circle of radius $6cm$ and measure its length. Also verify the measurement by actual calculation.

Solution: Consider two concentric circles of radii $4cm$ and $6cm$, respectively. Let the center be \mathbf{O} .

$$\mathbf{C}_1 : \|\mathbf{x} - \mathbf{O}\| = 4 \quad (1)$$

$$\mathbf{C}_2 : \|\mathbf{x} - \mathbf{O}\| = 6 \quad (2)$$

Let \mathbf{P} be a point on the \mathbf{C}_2 . From point \mathbf{P} a tangent is drawn to the \mathbf{C}_1 that intersects \mathbf{C}_1 at \mathbf{T}

$$(\mathbf{P} - \mathbf{T})^\top (\mathbf{T} - \mathbf{O}) = 0 \quad (\because \mathbf{P} - \mathbf{T} \text{ is a tangent to } \mathbf{C}_1) \quad (3)$$

Thus, $\triangle \mathbf{PTO}$ is a right-angled triangle. Using Pythagorean theorem,

$$\|\mathbf{P} - \mathbf{T}\|^2 + \|\mathbf{T} - \mathbf{O}\|^2 = \|\mathbf{P} - \mathbf{O}\|^2 \quad (4)$$

$$\|\mathbf{T} - \mathbf{O}\| = 4, \quad \|\mathbf{P} - \mathbf{O}\| = 6 \quad (5)$$

$$\|\mathbf{P} - \mathbf{T}\|^2 = 36 - 16 = 20 \implies \|\mathbf{P} - \mathbf{T}\| = 2\sqrt{5} \quad (6)$$

Thus, the length of the tangent is $2\sqrt{5} \text{ cm}$

Let us show this in graph using center $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

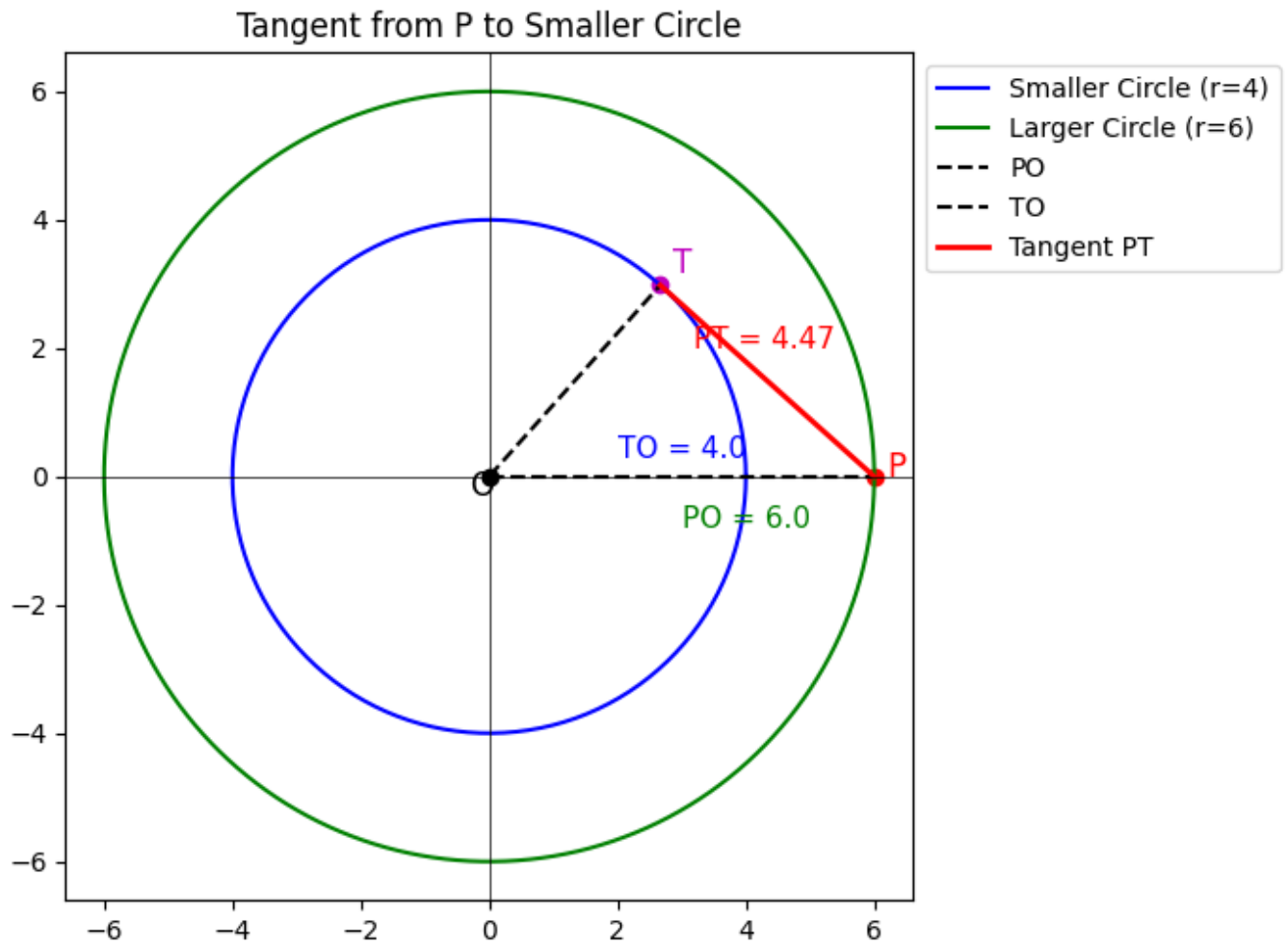


Fig. 1: Graph for 10.5.5