10.7.94

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Question

A circle touches the X axis and also touches the circle with centre at $(0, 3)$ and radius 2. The locus of the centre of the circle is

1. an ellipse
2. a circle
3. a hyperbola
4. a parabola

Theoretical Solution

Let the center of the moving circle be

$$\mathbf{c} = \begin{pmatrix} x \\ y \end{pmatrix}$$

and its radius be r. The circle touches the X-axis, so its radius is the y-coordinate of its center.

$$r = y = \mathbf{e_2}^\top \mathbf{c} \ \ (y > 0) \tag{1}$$

Theoretical Solution

The fixed circle has center

$$\mathbf{c}_f = 3\mathbf{e_2} \tag{2}$$

and radius

$$r_f = 2 \tag{3}$$

The distance between the centers of two touching circles is the sum of their radii (for external tangency).

$$\|\mathbf{c} - \mathbf{c}_f\| = r + r_f \tag{4}$$

$$\|\mathbf{c} - 3\mathbf{e_2}\| = \mathbf{e_2}^\top \mathbf{c} + 2 \tag{5}$$

Theoretical Solution

Squaring both sides,

$$(\mathbf{c} - 3\mathbf{e_2})^\top (\mathbf{c} - 3\mathbf{e_2}) = \left(\mathbf{e_2}^\top \mathbf{c} + 2\right)^2 \tag{6}$$

$$\mathbf{c}^\top \mathbf{c} - 6\mathbf{e_2}^\top \mathbf{c} + 9 = \left(\mathbf{e_2}^\top \mathbf{c}\right)^2 + 4\mathbf{e_2}^\top \mathbf{c} + 4 \tag{7}$$

$$x^2 + y^2 - 6y + 9 = y^2 + 4y + 4 \tag{8}$$

$$x^2 - 10y + 5 = 0 \tag{9}$$

Theoretical Solution

The locus in the standard form of the conic is

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 2 \begin{pmatrix} 0 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + 5 = 0 \tag{10}$$

The matrix of the quadratic part is

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{11}$$

The type of conic section is determined by the eigenvalues of $\mathbf{V}$. For a diagonal matrix, the eigenvalues are the diagonal entries.

$$\lambda_1 = 1, \ \lambda_2 = 0 \tag{12}$$

$$\left|\mathbf{V}\right| = \lambda_1 \lambda_2 = 1 \cdot 0 = 0 \tag{13}$$

Since one of the eigenvalues is zero, the locus is a parabola.
The correct option is **4) a parabola**.

Plot

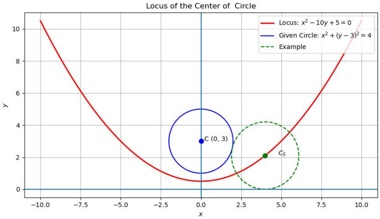

Figure: Plot