Matgeo Presentation - Problem 4.11.40

ee25btech11063 - Vejith

September 8, 2025

Question

Find the area of the region bounded by line y=3x+2, the X axis and the ordinates x=-2 and x=1.

Solution

let

$$\mathbf{A} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \tag{0.1}$$

$$\mathbf{C} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.2}$$

$$\tag{0.3}$$

let **D** and **E** be the vectors on the line corresponding to x= -2 and x=1
Given line equation is

$$-3x + y = 2 \tag{0.4}$$

which can be expressed as

$$\mathbf{n}^T \mathbf{x} = c \tag{0.5}$$

$$\implies \begin{pmatrix} -3 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 2 \tag{0.6}$$

$$\mathbf{n} = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \text{ and } \mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \text{ and } c = 2 \tag{0.7}$$

Solution

let us find the vector **D**

$$\begin{pmatrix} -3 & 1 \end{pmatrix} \begin{pmatrix} -2 \\ y \end{pmatrix} = 2 \quad \implies y = -4 \tag{0.8}$$

$$\implies \mathbf{D} = \begin{pmatrix} -2 \\ -4 \end{pmatrix} \tag{0.9}$$

as y<0 we should find the **B** where the line meets the x axis

$$\begin{pmatrix} -3 & 1 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} = 2 \quad \implies 3x = -2 \tag{0.10}$$

$$\implies \mathbf{B} = \begin{pmatrix} \frac{-2}{3} \\ 0 \end{pmatrix} \tag{0.11}$$

let us find the vector **E**

$$\begin{pmatrix} -3 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ y \end{pmatrix} = 2 \quad \implies y = 5 \tag{0.12}$$

$$\implies \mathbf{E} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \tag{0.13}$$

Conclusion

The area to be computed is area of $\triangle$EBC+area of $\triangle$ABD

$$ar(\triangle ABD) = \frac{1}{2}\|(A-B)\times(A-D)\| \tag{0.14}$$

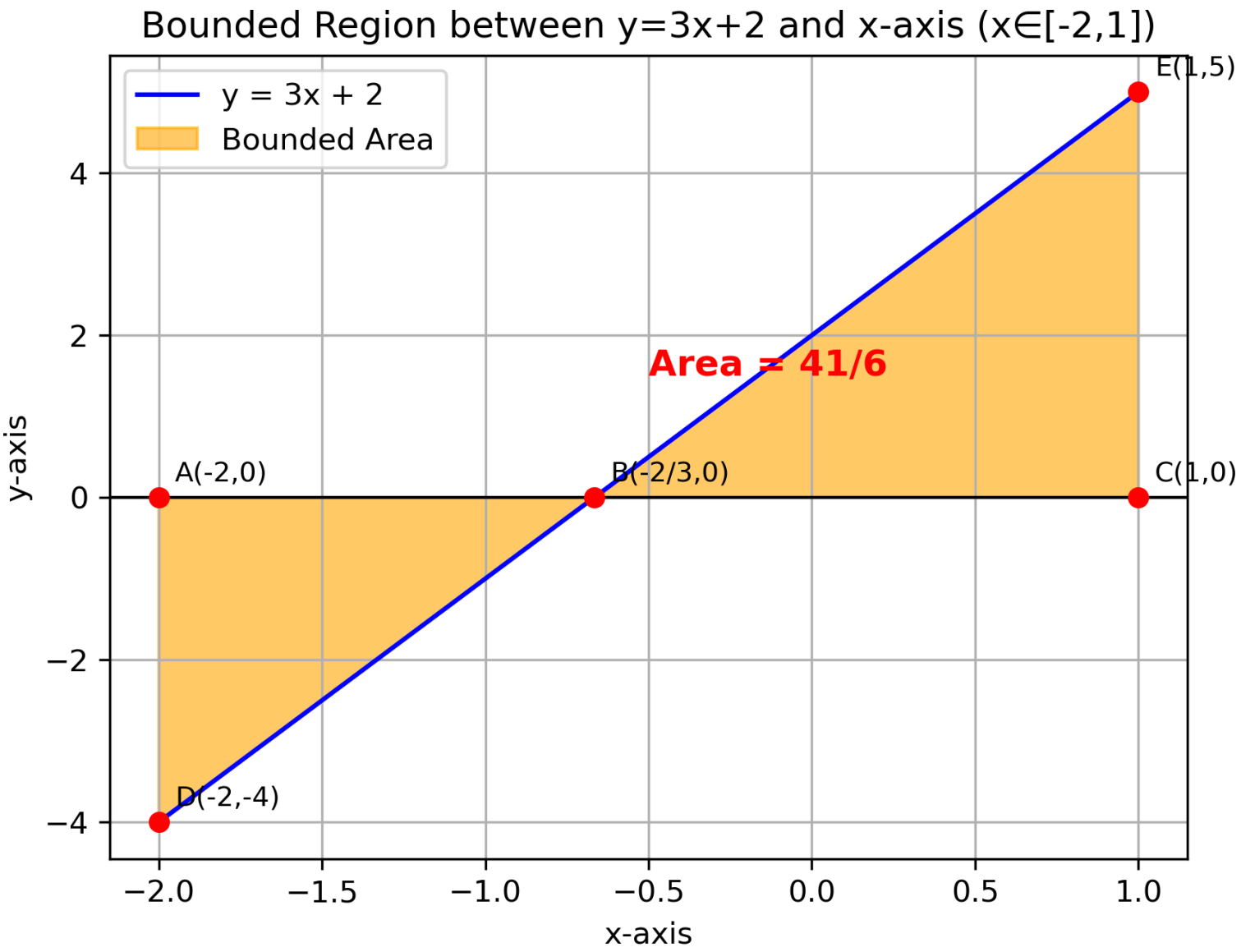
$$= \frac{1}{2}\|\left(\tfrac{4}{3}\right)\times\begin{pmatrix}0\\4\end{pmatrix}\| = \frac{8}{3} \tag{0.15}$$

$$ar(\triangle EBC) = \frac{1}{2}\|(E-c)\times(B-c)\| \tag{0.16}$$

$$= \frac{1}{2}\|\begin{pmatrix}0\\5\end{pmatrix}\times\begin{pmatrix}-5/3\\0\end{pmatrix}\| = \frac{25}{6} \tag{0.17}$$

$$\implies \text{area of the region is } = \frac{8}{3}+\frac{25}{6} = \frac{41}{6} \tag{0.18}$$

Plot


Bounded Region between y=3x+2 and x-axis (x∈[-2,1])
y = 3x + 2
Bounded Area
Area = 41/6
A(-2,0)
B(-2/3,0)
C(1,0)
D(-2,-4)
E(1,5)
y-axis
x-axis
4
2
0
−2
−4
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0

C Code: area.c

```c
#include <stdio.h>
#include <stdlib.h>
#include <math.h> // for fabs()

int main() {
    FILE *fp;
    double area;

    // Compute area =  from -2 to 1 |3x+2| dx
    // Split into two parts: [-2, -2/3] and [-2/3, 1]

    double x0 = -2.0/3.0;

    // Antiderivative F(x) = (3/2)x^2 + 2x
    double F_at_x0 = (1.5 * x0 * x0) + (2.0 * x0);
    double F_at_neg2 = (1.5 * (-2) * (-2)) + (2.0 * (-2));
    double F_at_1 = (1.5 * 1 * 1) + (2.0 * 1);

    // Left interval [-2, -2/3] -> integrand is negative
    double left = -(F_at_x0 - F_at_neg2);

    // Right interval [-2/3, 1] -> integrand is positive
    double right = (F_at_1 - F_at_x0);

    area = left + right; // total geometric area

    // Open file area.dat
    fp = fopen("area.dat", "w");
    if (fp == NULL) {
        printf("Error opening file!\n");
        return 1;
    }
```

C Code: area.c

```
    // Write output
    fprintf(fp, "Line␣equation␣in␣matrix␣form:␣[3␣␣-1]␣[x␣y]^T␣=␣-2\n");
    fprintf(fp, "Area␣of␣the␣region␣=␣%.6f␣(exact␣=␣41/6)\n", area);

    fclose(fp);

    printf("Area␣successfully␣written␣to␣area.dat\n");
    return 0;
}
```

Python: plot.py

```
 import matplotlib.pyplot as plt
import numpy as np

# Define line
x = np.linspace(-2, 1, 400)
y = 3*x + 2

# Vertices
A = (-2, 0)
B = (-2/3, 0) # x-intercept
C = (1, 0)
D = (-2, -4)
E = (1, 5)

# Plot line
plt.plot(x, y, 'b-', label="y␣=␣3x␣+␣2")

# Plot x-axis
plt.axhline(0, color='black', linewidth=1)

# Shade the bounded area (polygon A-B-C-E-D-A)
polygon_x = [A[0], B[0], C[0], E[0], D[0], A[0]]
polygon_y = [A[1], B[1], C[1], E[1], D[1], A[1]]
plt.fill(polygon_x, polygon_y, color='orange', alpha=0.6, label="Bounded␣Area")

# Mark and label vertices (B shown symbolically)
vertices = {
    "A(-2,0)": A,
    "B(-2/3,0)": B, # keep symbolic form
    "C(1,0)": C,
    "D(-2,-4)": D,
    "E(1,5)": E
}
```

Python: plot.py

```
for name, (px, py) in vertices.items():
    plt.plot(px, py, 'ro')
    plt.text(px+0.05, py+0.2, name, fontsize=9)

# Annotate area
plt.text(-0.5, 1.5, "Area = 41/6", fontsize=12, color="red", weight="bold")

# Labels and formatting
plt.xlabel("x-axis")
plt.ylabel("y-axis")
plt.title("Bounded Region between y=3x+2 and x-axis (x[-2,1])")
plt.legend()
plt.grid(True)

# Save figure
plt.savefig("bounded_area_vertices.png", dpi=300)
plt.show()
```