

1.8.2

AI25BTECH11028 -R.Manohar

Question: Find the distance between the following pairs of points:

- 1) (2,3,5) and (4,3,1)
- 2) (-3,7,2) and (2,4,-1)
- 3) (-1,3,-4) and (1,-3,4)
- 4) (2,-1,3) and (-2,1,3)

Solution: We know that,
The length of a vector is defined as

$$\|\mathbf{x}\| = \sqrt{\mathbf{x}^\top \mathbf{x}} \quad (1)$$

Therefore,
distance between \mathbf{P} and \mathbf{Q} is

$$d(\mathbf{P}, \mathbf{Q}) = \|\mathbf{P} - \mathbf{Q}\| = \sqrt{(\mathbf{P} - \mathbf{Q})^\top (\mathbf{P} - \mathbf{Q})}. \quad (2)$$

Let,

$$1) \mathbf{A} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$$

$$d(\mathbf{A}, \mathbf{B}) = \left\| \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} \right\| = \left\| \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix} \right\| = \sqrt{\begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}^\top \begin{pmatrix} -2 \\ 0 \\ 4 \end{pmatrix}} = \sqrt{(-2)^2 + 0^2 + 4^2} = \sqrt{20} = 2\sqrt{5}.$$

$$2) \mathbf{C} = \begin{pmatrix} -3 \\ 7 \\ 2 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$$

$$d(\mathbf{C}, \mathbf{D}) = \left\| \begin{pmatrix} -3 \\ 7 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} \right\| = \left\| \begin{pmatrix} -5 \\ 3 \\ 3 \end{pmatrix} \right\| = \sqrt{\begin{pmatrix} -5 \\ 3 \\ 3 \end{pmatrix}^\top \begin{pmatrix} -5 \\ 3 \\ 3 \end{pmatrix}} = \sqrt{(-5)^2 + 3^2 + 3^2} = \sqrt{43}.$$

$$3) \mathbf{E} = \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}, \mathbf{F} = \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$$

$$d(\mathbf{E}, \mathbf{F}) = \left\| \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} \right\| = \left\| \begin{pmatrix} -2 \\ 6 \\ -8 \end{pmatrix} \right\| = \sqrt{\begin{pmatrix} -2 \\ 6 \\ -8 \end{pmatrix}^\top \begin{pmatrix} -2 \\ 6 \\ -8 \end{pmatrix}} = \sqrt{(-2)^2 + 6^2 + (-8)^2} = \sqrt{104} = 2\sqrt{26}.$$

$$4) \mathbf{G} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}, \mathbf{H} = \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$$

$$d(\mathbf{G}, \mathbf{H}) = \left\| \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix} \right\| = \sqrt{\begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}^T \begin{pmatrix} 4 \\ -2 \\ 0 \end{pmatrix}} = \sqrt{4^2 + (-2)^2 + 0^2} = \sqrt{20} = 2\sqrt{5}.$$

FIGURE

3D Segments between given points

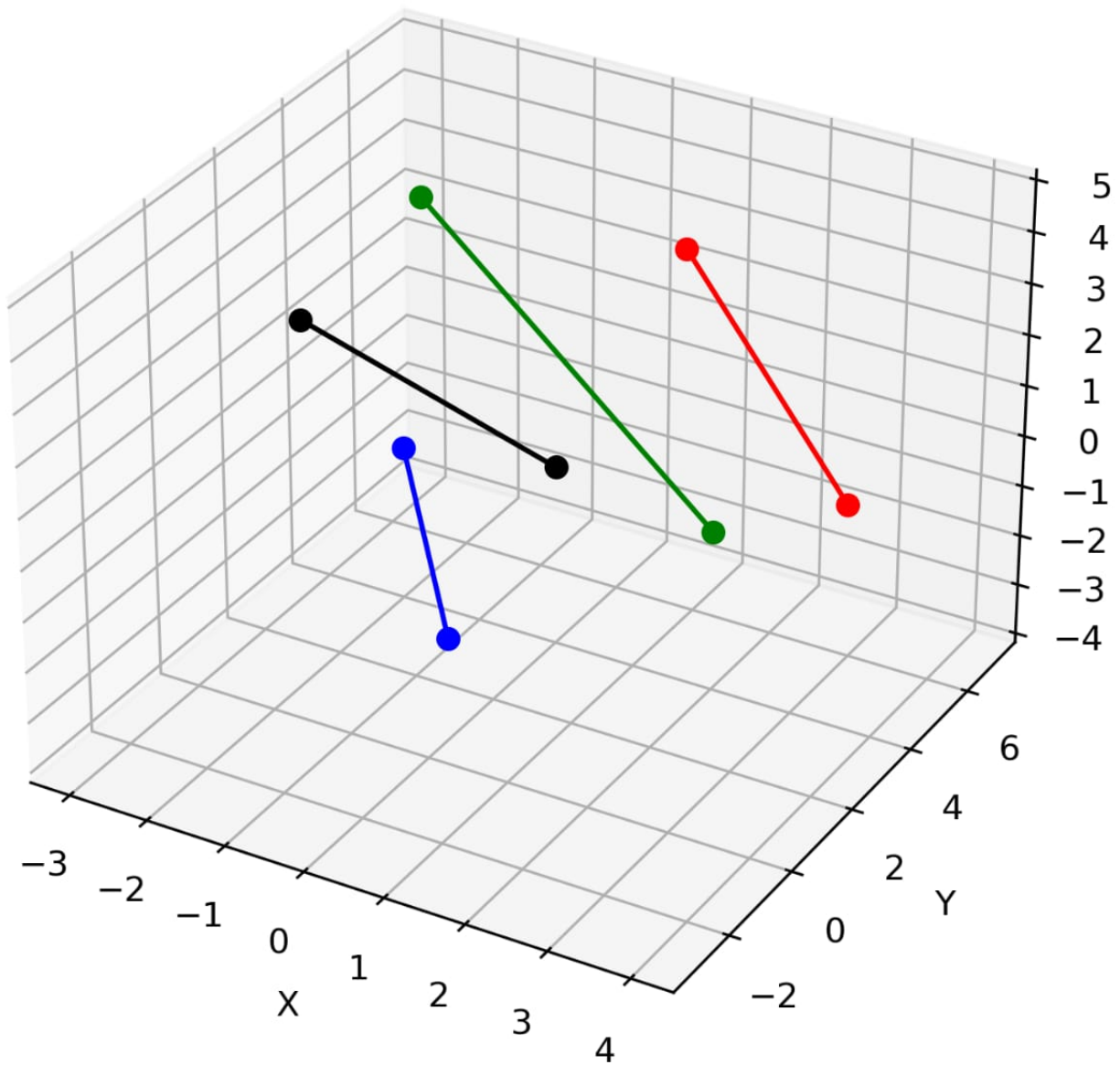


Fig. 4