

8.2.48

AI25BTECH11012 - GARIGE UNNATHI

Question:

Find the equation of the conic, that satisfies the given conditions.
focus $(-1, -2)$ and directrix $x - 2y + 3 = 0$.

Solution: Let :

$$\mathbf{F} = \begin{pmatrix} -1 \\ -2 \end{pmatrix} \quad (0.1)$$

$$\text{directrix equation is : } \begin{pmatrix} 1 \\ -2 \end{pmatrix}^T \mathbf{x} = -3 \quad (0.2)$$

The equation of a conic with directrix $\mathbf{n}^T \mathbf{x} = c$, eccentricity e and focus \mathbf{F} is given by:

$$g(\mathbf{x}) = \mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (0.3)$$

where :

$$\mathbf{V} = \|\mathbf{n}\|^2 \mathbf{I} - e^2 \mathbf{n} \mathbf{n}^T,$$

$$\mathbf{u} = ce^2 \mathbf{n} - \|\mathbf{n}\|^2 \mathbf{F},$$

$$f = \|\mathbf{n}\|^2 \|\mathbf{F}\|^2 - c^2 e^2$$

From the question we can say that the conic is a parabola that is $e = 1$;
Calculating \mathbf{V} , \mathbf{u} and f by using the above equations we get :

$$\mathbf{V} = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \quad (0.4)$$

$$\mathbf{u} = \begin{pmatrix} 2 \\ 16 \end{pmatrix} \quad (0.5)$$

$$f = 16 \quad (0.6)$$

Substituting in the equation 0.3 we get :

$$\mathbf{x}^T \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 2 & 16 \end{pmatrix} \mathbf{x} + 16 = 0 \quad (0.7)$$

Solving it we get :

$$4x^2 + 4xy + y^2 + 4x + 32y + 16 = 0 \quad (0.8)$$

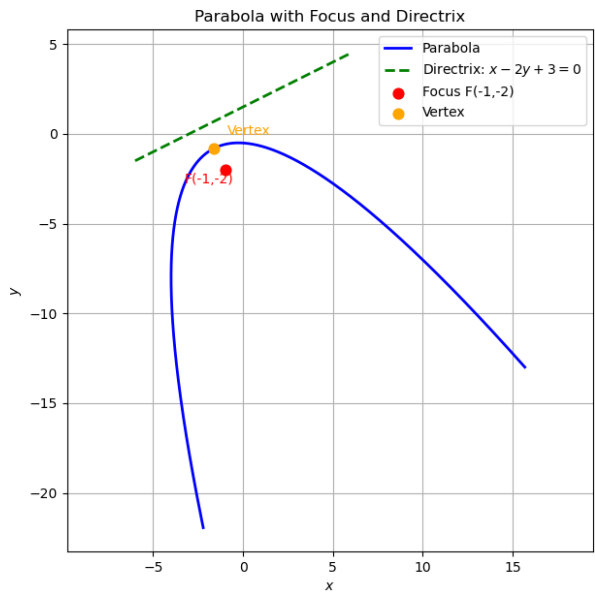


Fig. 0.1