10.7.75

EE25BTECH11026-Harsha

Question:
Find the equations of tangents drawn fron origin to the circle $x^2+y^2-2rx-2hy+h^2 = 0$,are

1) $x = 0$
2) $y = 0$
3) $\left(h^2 - r^2\right)x - 2rhy = 0$
4) $\left(h^2 - r^2\right)x + 2rhy = 0$

Solution:
Let us solve the given question theoretically and then verify the solution computationally.

Given the equation of circle,

$$\|\mathbf{x}\|^2 - 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{4.1}$$

where, $\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix}$, centre of circle $\mathbf{u} = \begin{pmatrix} r \\ h \end{pmatrix}$ and $f = h^2$.
It is given that the tangents pass through the origin.

$$\therefore \mathbf{n}^\top\mathbf{x} = 0 \tag{4.2}$$

where $\mathbf{n}$ is the direction vector of the tangent. From (4.1),

$$\therefore \frac{|n^\top\mathbf{u}|}{\|\mathbf{n}\|} = r_c \tag{4.3}$$

where r_c is the radius of the circle. For the given equation of circle,

$$\therefore r_c = r \tag{4.4}$$

$$\implies \frac{|\mathbf{n}^\top\mathbf{u}|}{\|\mathbf{n}\|} = r \tag{4.5}$$

Let $\mathbf{n} = \begin{pmatrix} a \\ b \end{pmatrix}$, substituting in the equation (4.5),

$$\therefore \frac{|ar + bh|}{\sqrt{a^2 + b^2}} = r \tag{4.6}$$

Squaring on both sides,

$$a^2r^2 + b^2h^2 + 2arbh = r^2a^2 + r^2b^2 \implies b\left(h^2b + 2arh - br^2\right) = 0 \tag{4.7}$$

From (4.7),

$$b = 0 \tag{4.8}$$

$$a = \frac{r^2 - h^2}{2rh} b \tag{4.9}$$

$\therefore$ The required equations of tangents are,

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{4.10}$$

$$\begin{pmatrix} h^2 - r^2 & -2rh \end{pmatrix} \mathbf{x} = 0 \tag{4.11}$$

From the figure, it is clearly verified that the theoretical solution matches with the computational solution.