

# 1.5.19

EE25BTECH11031 - Sai Sreevallabh

## Question:

Find the ratio in which the segment joining the points (1, 3) and (4, 5) is divided by the X-axis. Also find the coordinates of this point on the X-axis.

## Solution:

Given points are

$$\mathbf{A} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 4 \\ 5 \end{pmatrix} \quad (0.1)$$

Let  $\mathbf{P}$  be a point on the x-axis. We can assume it to be

$$\mathbf{P} = \begin{pmatrix} x \\ 0 \end{pmatrix} \quad (0.2)$$

$\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{P}$  are collinear.

$$\mathbf{P} - \mathbf{A} = \begin{pmatrix} x-1 \\ -3 \end{pmatrix}, \quad \mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad (0.3)$$

$$\begin{pmatrix} \mathbf{P} - \mathbf{A} & \mathbf{B} - \mathbf{A} \end{pmatrix}^T = \begin{pmatrix} x-1 & 3 \\ -3 & 2 \end{pmatrix}^T \quad (0.4)$$

$$= \begin{pmatrix} x-1 & -3 \\ 3 & 2 \end{pmatrix} \quad (0.5)$$

Converting into echelon form using row operations

$$\begin{pmatrix} x-1 & -3 \\ 3 & 2 \end{pmatrix} \xleftrightarrow{R_2 \rightarrow R_2 - \frac{3}{x-1} R_1} \begin{pmatrix} x-1 & -3 \\ 0 & \frac{2x+7}{x-1} \end{pmatrix} \quad (0.6)$$

Since the points are collinear, we can say that the rank of the matrix is 1 i.e.

$$\frac{2x+7}{x-1} = 0 \quad (0.7)$$

$$\Rightarrow x = -\frac{7}{2} \quad (0.8)$$

Let  $\mathbf{P}$  divide the line joining points  $\mathbf{A}$  and  $\mathbf{B}$  in the ratio  $k : 1$ .

$$\mathbf{P} = \frac{k\mathbf{B} + \mathbf{A}}{k + 1} \quad (0.9)$$

$$k(\mathbf{P} - \mathbf{B}) = \mathbf{A} - \mathbf{P} \quad (0.10)$$

$$k = \frac{(\mathbf{P} - \mathbf{B})^T (\mathbf{A} - \mathbf{P})}{\|(\mathbf{P} - \mathbf{B})\|^2} \quad (0.11)$$

$$k = \frac{(x-4 \quad -5) \begin{pmatrix} 1-x \\ 3 \end{pmatrix}}{\left\| \begin{pmatrix} x-4 \\ -5 \end{pmatrix} \right\|^2} \quad (0.12)$$

Substituting the value of  $x$  as  $-\frac{7}{2}$ , we get the value of  $k$  as

$$k = -\frac{3}{5} \quad (0.13)$$

Therefore

The point  $\mathbf{P} \begin{pmatrix} -\frac{7}{2} \\ 0 \end{pmatrix}$  on the X-axis divides the line segment in the ratio  $-3 : 5$  i.e. externally in the ratio  $3 : 5$ .

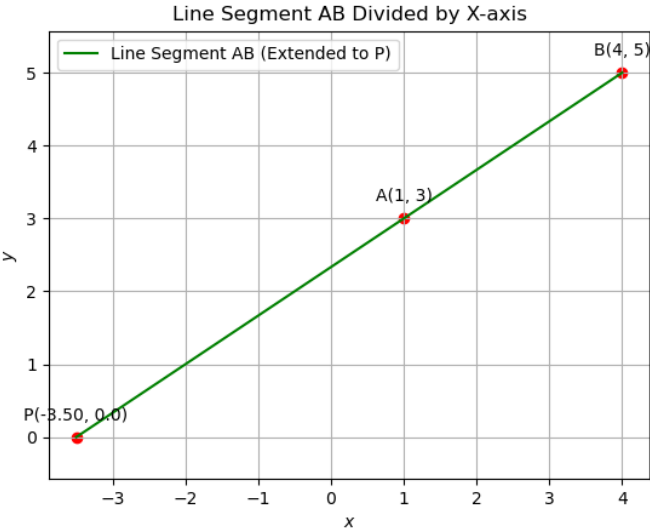


Fig. 0.1: Caption