

Question 4.2.3

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Question:

Find the direction and normal vectors of each of the following line:

$$-2x + 3y = 6$$

Solution:

Assuming points on the line are represented by \mathbf{x} , we can express the line equation as:

$$\left\| \begin{pmatrix} -2 \\ 3 \end{pmatrix}^T \mathbf{x} \right\| = 6 \quad (1)$$

In such a form, the normal vector \mathbf{n} is given by:

$$\mathbf{n} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \equiv \frac{1}{\sqrt{13}} \begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad (2)$$

The direction vector \mathbf{d} can be derived from the normal vector by rotating it by 90 degrees clockwise (or anticlockwise). To do that, we multiply with the transformation matrix \mathbf{r} :

$$\mathbf{r} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (3)$$

$$\therefore \mathbf{d} = \mathbf{r}\mathbf{n} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix} \equiv \frac{1}{\sqrt{13}} \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad (4)$$

Plot:

Q 4.2.3

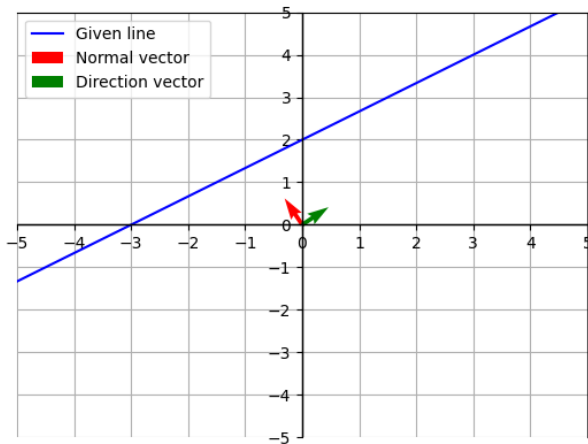


Figure: Graph of line with direction and normal vectors