4.13.8

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Question

The orthocentre of the triangle formed by the lines $x + y = 1$, $2x + 3y = 6$ and $4x - y + 4 = 0$ lies in the quadrant number.

Theoretical Solution

The three lines, written in the vector normal form $\mathbf{n}^\top \mathbf{x} = c$, are:

$$L_1 : \begin{pmatrix} 1 \\ 1 \end{pmatrix}^\top \begin{pmatrix} x \\ y \end{pmatrix} = 1 \tag{1}$$

$$L_2 : \begin{pmatrix} 2 \\ 3 \end{pmatrix}^\top \begin{pmatrix} x \\ y \end{pmatrix} = 6 \tag{2}$$

$$L_3 : \begin{pmatrix} 4 \\ -1 \end{pmatrix}^\top \begin{pmatrix} x \\ y \end{pmatrix} = -4 \tag{3}$$

The vertices of the triangle, $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$, are the intersection points of these lines.

Theoretical Solution

Vertex A : Solving $x + y = 1$ and $2x + 3y = 6$.

$$\left(\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & 3 & 6 \end{array}\right) \xrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & 4 \end{array}\right) \implies y = 4, x = -3. \quad \mathbf{A} = \begin{pmatrix} -3 \\ 4 \end{pmatrix} \tag{4}$$

Vertex B : Solving $2x + 3y = 6$ and $4x - y = -4$.

$$\left(\begin{array}{cc|c} 2 & 3 & 6 \\ 4 & -1 & -4 \end{array}\right) \xrightarrow{R_2 \to R_2 - 2R_1} \left(\begin{array}{cc|c} 2 & 3 & 6 \\ 0 & -7 & -16 \end{array}\right) \implies y = \frac{16}{7}, x = -\frac{3}{7} \tag{5}$$

$$\mathbf{B} = \begin{pmatrix} -3/7 \\ 16/7 \end{pmatrix} \tag{6}$$

Theoretical Solution

Vertex C : Solving $x + y = 1$ and $4x - y = -4$.

$$\left(\begin{array}{cc|c} 1 & 1 & 1 \\ 4 & -1 & -4 \end{array}\right) \xrightarrow{R_2 \to R_2 - 4R_1} \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -5 & -8 \end{array}\right) \implies y = \frac{8}{5}, x = -\frac{3}{5} \tag{7}$$

$$\mathbf{C} = \begin{pmatrix} -3/5 \\ 8/5 \end{pmatrix} \tag{8}$$

Theoretical Solution

The altitude from **A** is perpendicular to the opposite side, which lies on line $L_3 : 4x - y = -4$.

- The direction of the altitude is parallel to the normal of L_3, which is $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$.
- The normal to the altitude is thus perpendicular to this direction, e.g., $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

So, the equation of the altitude is $x + 4y = k$. Since it passes through

$\mathbf{A} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$:

$$(-3) + 4(4) = k \implies k = 13. \tag{9}$$

Equation of Altitude AD: $x + 4y = 13$.

Theoretical Solution

The altitude from **C** is perpendicular to the opposite side, which lies on line $L_2 : 2x + 3y = 6$.

The direction of the altitude is parallel to the normal of L_2, which is $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

The normal to the altitude is thus perpendicular to this direction, e.g., $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$.

So, the equation of the altitude is $3x - 2y = k$. Since it passes through

$\mathbf{C} = \begin{pmatrix} -3/5 \\ 8/5 \end{pmatrix}$:

$$3(-\tfrac{3}{5}) - 2(\tfrac{8}{5}) = k \implies k = -\frac{9}{5} - \frac{16}{5} = -5 \tag{10}$$

Equation of Altitude CE: $3x - 2y = -5$.

Theoretical Solution

The orthocentre ($\mathbf{H}$) is the intersection of the altitudes. We solve the system:

$$x + 4y = 13 \tag{11}$$

$$3x - 2y = -5 \tag{12}$$

Using Gaussian elimination:

$$\left(\begin{array}{cc|c} 1 & 4 & 13 \\ 3 & -2 & -5 \end{array}\right) \xrightarrow{R_2 \to R_2 - 3R_1} \left(\begin{array}{cc|c} 1 & 4 & 13 \\ 0 & -14 & -44 \end{array}\right) \tag{13}$$

From the second row: $-14y = -44 \implies y = \frac{22}{7}$.
Substituting into the first row: $x + 4(\frac{22}{7}) = 13 \implies x = \frac{3}{7}$.

Theoretical Solution

The coordinates of the orthocentre are:

$$\mathbf{H} = \begin{pmatrix} \frac{3}{7} \\ \frac{22}{7} \end{pmatrix} \tag{14}$$

Since the x-coordinate $(\frac{3}{7})$ and the y-coordinate $(\frac{22}{7})$ are both positive, the orthocentre lies in the **first quadrant**.

Plot of the Lines and Orthocentre: