12.755

EE25BTECH11013 - Bhargav

Question:
Which one of the following vectors is an eigenvector corresponding to the eigenvalue $\lambda = 1$ for the matrix $\mathbf{A} = \begin{pmatrix} 1 & -1 & 0 \\ 1 & -1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$ is

Solution:

The eigenvalue of matrix $\mathbf{A}$ can be found out by (where λ is the eigenvalue, $\mathbf{x}$ is the eigenvector, $\mathbf{I}$ is the identity matrix)

$$\mathbf{A}\mathbf{x} = \lambda\mathbf{x} \implies (\mathbf{A} - \lambda\mathbf{I})\,\mathbf{x} = \mathbf{0} \tag{0.1}$$

$$(\mathbf{A} - \mathbf{I})\,\mathbf{x} = \mathbf{0} \tag{0.2}$$

$$\implies \begin{pmatrix} 0 & -1 & 0 \\ 1 & -2 & 1 \\ -1 & 0 & 0 \end{pmatrix} \mathbf{x} = \mathbf{0} \tag{0.3}$$

This can be solved by representing it as an augmented matrix and using row elimination

$$\left(\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{array}\right) \xleftrightarrow{R_1 \leftarrow R_1 + R_2} \left(\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{array}\right) \xleftrightarrow[R_3 \leftarrow R_3 + R_1]{R_2 \leftarrow R_2 - R_1} \tag{0.4}$$

$$\left(\begin{array}{ccc|c} 1 & -3 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \end{array}\right) \xleftrightarrow[R_3 \leftarrow R_3 + 3R_2]{R_1 \leftarrow R_1 + 3R_2} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right) \tag{0.5}$$

Thus we get $\mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ which is the eigenvector of the matrix $\mathbf{A}$ corresponding to the eigenvalue $\lambda = 1$

This can be further verified by the intersection of planes

$$-y = 0 \tag{0.6}$$

$$x - 2y + z = 0 \tag{0.7}$$

$$-x = 0 \tag{0.8}$$

The intersection point is $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$