5.8.40

EE25BTECH11043 - Nishid Khandagre

Question: The ratio of incomes of two persons is 9:7 and the ratio of their expenditures is 4:3. If each of them manages to save rupees 2000 per month, find their monthly incomes.

Solution: Let the monthly incomes be x and y.

Given income ratio:

$$\frac{x}{y} = \frac{9}{7} \tag{0.1}$$

$$7x - 9y = 0 \tag{0.2}$$

Expenditures are Income − Savings. Expenditures are $x - 2000$ and $y - 2000$. Given expenditure ratio:

$$\frac{x - 2000}{y - 2000} = \frac{4}{3} \tag{0.3}$$

$$3x - 6000 = 4y - 8000 \tag{0.4}$$

$$3x - 4y = -2000 \tag{0.5}$$

$$7x - 9y = 0 \tag{0.6}$$

$$3x - 4y = -2000 \tag{0.7}$$

Matrix form:

$$\begin{pmatrix} 7 & -9 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ -2000 \end{pmatrix} \tag{0.8}$$

Augmented matrix:

$$\left(\begin{array}{cc|c} 7 & -9 & 0 \\ 3 & -4 & -2000 \end{array}\right) \tag{0.9}$$

Then $R_2 \to 7R_2 - 3R_1$

$$\left(\begin{array}{cc|c} 7 & -9 & 0 \\ 0 & -1 & -14000 \end{array}\right) \tag{0.10}$$

Then $R_2 \to -R_2$

$$\left(\begin{array}{cc|c} 7 & -9 & 0 \\ 0 & 1 & 14000 \end{array}\right) \tag{0.11}$$

Then $R_1 \to R_1 + 9R_2$

$$\left(\begin{array}{cc|c} 7 & 0 & 126000 \\ 0 & 1 & 14000 \end{array}\right) \tag{0.12}$$

Then $R_1 \to \frac{1}{7}R_1$

$$\left(\begin{array}{cc|c} 1 & 0 & 18000 \\ 0 & 1 & 14000 \end{array}\right) \tag{0.13}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 18000 \\ 14000 \end{pmatrix} \tag{0.14}$$

$$x = 18000 \tag{0.15}$$

$$y = 14000 \tag{0.16}$$

Thus, the monthly incomes are ₹18000 and ₹14000.

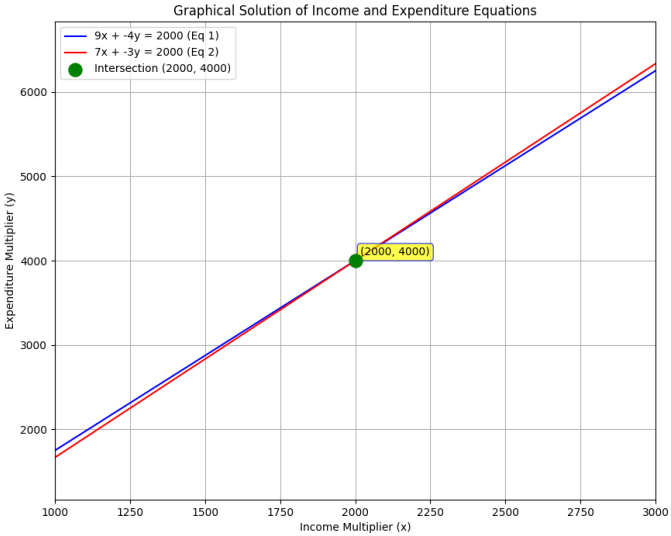

Fig. 0.1