

PROBLEM 7.2.5

Check whether the point $\mathbf{P} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$ lies on a circle of radius 6 centered at $\mathbf{C} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$.

MATRIX FORM

The general equation of a circle with center \mathbf{C} and radius r is:

$$\|\mathbf{x} - \mathbf{C}\|^2 = r^2$$

Substituting $\mathbf{C} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$, $r = 6$:

$$\|\mathbf{x} - \begin{pmatrix} 3 \\ 5 \end{pmatrix}\|^2 = 36$$

Expanding the norm:

$$(\mathbf{x} - \begin{pmatrix} 3 \\ 5 \end{pmatrix})^T (\mathbf{x} - \begin{pmatrix} 3 \\ 5 \end{pmatrix}) = 36$$

SUBSTITUTION

Let $\mathbf{x} = \mathbf{P} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$. Then:

$$\left(\begin{pmatrix} -2 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix}\right)^T \left(\begin{pmatrix} -2 \\ 4 \end{pmatrix} - \begin{pmatrix} 3 \\ 5 \end{pmatrix}\right) = \begin{pmatrix} -5 \\ -1 \end{pmatrix}^T \begin{pmatrix} -5 \\ -1 \end{pmatrix} = (-5)^2 + (-1)^2 = 25 + 1 = 26$$

COMPARISON

$$\text{LHS} = 26, \quad \text{RHS} = 36 \Rightarrow 26 \neq 36$$

CONCLUSION

The point $\mathbf{P} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$ does not satisfy the equation of the circle. Hence,

\mathbf{P} does not lie on the circle

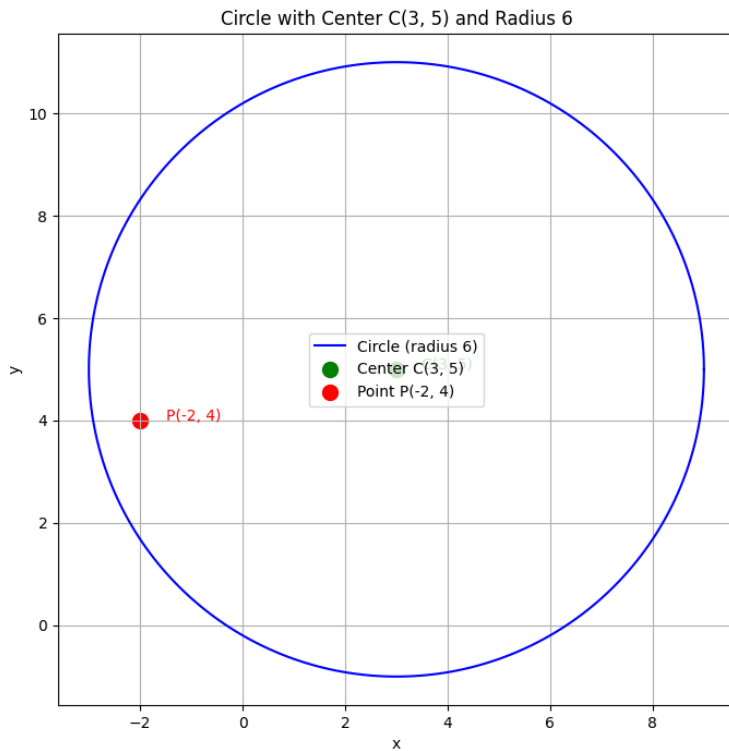


Fig. 1: Circle and the point