4.4.4

AI25BTECH11008 - Chiruvella Harshith Sharan

Question: A line passes through the point with position vector

$$\mathbf{A} = 2\hat{i} - \hat{j} + 4\hat{k}$$

and is in the direction of the vector

$$\mathbf{d} = \hat{i} + \hat{j} - 2\hat{k}.$$

Find the equation of the line.

Solution:

The line passes through point

$$\mathbf{A} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \tag{0.1}$$

and has direction vector

$$\mathbf{d} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}. \tag{0.2}$$

The vector equation of a line is

$$\mathbf{r} = \mathbf{A} + \lambda\mathbf{d}, \quad \lambda \in \mathbb{R}. \tag{0.3}$$

Substituting the given values,

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}. \tag{0.4}$$

Thus, the equation of the line is

$$\mathbf{r} = \begin{pmatrix} 2 + \lambda \\ -1 + \lambda \\ 4 - 2\lambda \end{pmatrix}, \quad \lambda \in \mathbb{R}.$$

Or equivalently, in symmetric form,

$$\frac{x-2}{1} = \frac{y+1}{1} = \frac{z-4}{-2}.$$

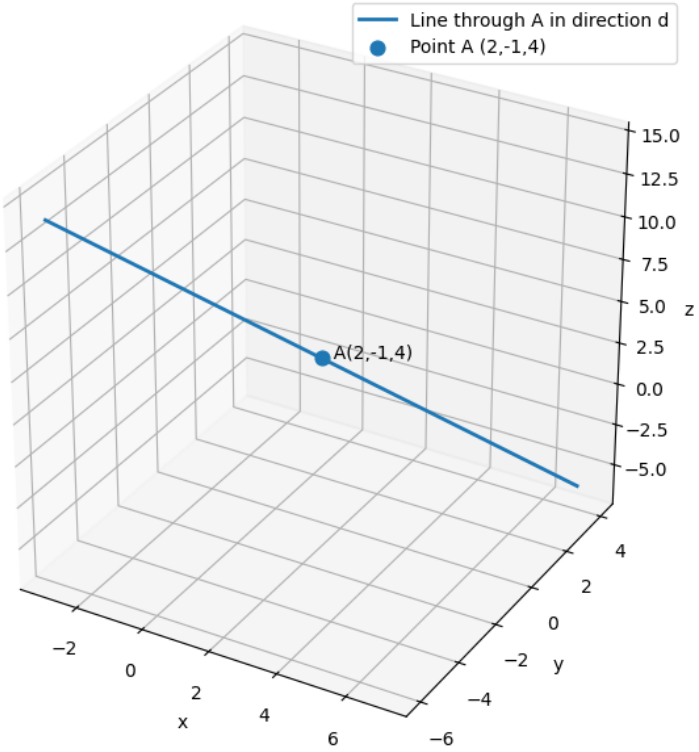

Fig. 0.1: Graph showing the line through point **A** in direction **d**