4.13.50

Puni Aditya - EE25BTECH11046

27th September, 2025

Question

Two equal sides of an isosceles triangle are given by the equations $7x - y + 3 = 0$ and $x + y - 3 = 0$ and its third side passes through the point $(1, -10)$. Determine the equation of the third side.

Theoretical Solution

Let the two equal sides of the isosceles triangle be represented by

$$\mathbf{n_1}^\top \mathbf{x} = c_1$$
$$\mathbf{n_2}^\top \mathbf{x} = c_2$$

and the third side by the line

$$\mathbf{n}^\top \mathbf{x} = c$$

The third side of the isosceles, the base, is perpendicular to the angle bisector of the two equal sides.

Theoretical Solution

For a line passing through a given point $\mathbf{p}$,

$$\mathbf{n}^\top \mathbf{x} = \mathbf{n}^\top \mathbf{p} \tag{1}$$

$$\text{Here, } \mathbf{p} = \begin{pmatrix} 1 \\ -10 \end{pmatrix} \tag{2}$$

The third side of the isosceles, the base, is perpendicular to one of the angle bisector of the two equal sides and parallel to the other.

$$\mathbf{m_1} = \frac{\mathbf{n_1}}{\|\mathbf{n_1}\|} + \frac{\mathbf{n_2}}{\|\mathbf{n_2}\|} \tag{3}$$

$$\mathbf{m_2} = \frac{\mathbf{n_1}}{\|\mathbf{n_1}\|} - \frac{\mathbf{n_2}}{\|\mathbf{n_2}\|} \tag{4}$$

Theoretical Solution

Hence,

$$\mathbf{n} = \mathbf{m_2} \text{ or } \mathbf{n} = \mathbf{m_1} \tag{5}$$

For the given question,

$$\mathbf{n_1} = \begin{pmatrix} 7 \\ -1 \end{pmatrix} \text{ and } \mathbf{n_2} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{6}$$

$$\|\mathbf{n_1}\| = \sqrt{50} = 5\sqrt{2} \tag{7}$$

$$\|\mathbf{n_2}\| = \sqrt{2} \tag{8}$$

Theoretical Solution

For the side parallel to $\mathbf{m_2}$, using (3),

$$\mathbf{n} = \frac{1}{5\sqrt{2}}\left(\begin{pmatrix}7\\-1\end{pmatrix}+\begin{pmatrix}5\\5\end{pmatrix}\right) = \frac{1}{5\sqrt{2}}\begin{pmatrix}12\\4\end{pmatrix} \tag{9}$$

$$\mathbf{n} = \begin{pmatrix}3\\1\end{pmatrix} \tag{10}$$

$$\begin{pmatrix}3 & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}3 & 1\end{pmatrix}\begin{pmatrix}1\\-10\end{pmatrix} \tag{11}$$

$$\begin{pmatrix}3 & 1\end{pmatrix}\mathbf{x} = -7 \tag{12}$$

Theoretical Solution

For the side parallel to $\mathbf{m_1}$, using (4),

$$\mathbf{n} = \frac{1}{5\sqrt{2}}\left(\begin{pmatrix}7\\-1\end{pmatrix} - \begin{pmatrix}5\\5\end{pmatrix}\right) = \frac{1}{5\sqrt{2}}\begin{pmatrix}2\\-6\end{pmatrix} \tag{13}$$

$$\mathbf{n} = \begin{pmatrix}1\\-3\end{pmatrix} \tag{14}$$

$$\begin{pmatrix}1 & -3\end{pmatrix}\mathbf{x} = \begin{pmatrix}1 & -3\end{pmatrix}\begin{pmatrix}1\\-10\end{pmatrix} \tag{15}$$

$$\begin{pmatrix}1 & -3\end{pmatrix}\mathbf{x} = 31 \tag{16}$$

Plot

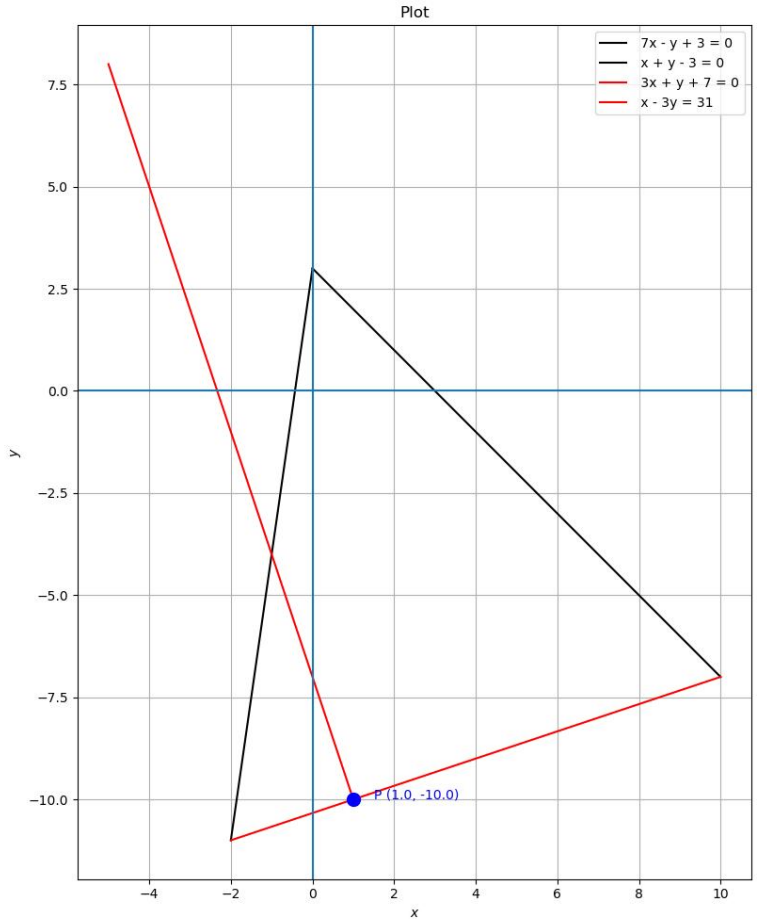

Figure: Isosceles Triangle