Question 2.7.12

AI25BTECH11040 - Vivaan Parashar

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1 Question:

Find the area of the triangle formed by joining the midpoints of the sides of the triangle ABC, whose vertices are A$(0, -1)$, B$(2, 1)$, and C$(0, 3)$

2 Solution:

Let us start by finding the midpoints, let's call them D, E and F. The midpoint formula is: (Here the vectors represent position vectors of the points from the origin)

$$\mathbf{D} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{1}$$

$$\mathbf{E} = \frac{\mathbf{B} + \mathbf{C}}{2} \tag{2}$$

$$\mathbf{F} = \frac{\mathbf{C} + \mathbf{A}}{2} \tag{3}$$

$$\therefore \mathbf{D} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{F} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4}$$

Now the area formula for a triangle with vertices at $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$ is given by:

$$\text{Area} = \frac{1}{2} \|(\mathbf{P} - \mathbf{Q}) \times (\mathbf{P} - \mathbf{R})\| \tag{5}$$

$$\therefore \text{Area of } \triangle DEF = \frac{1}{2} \|(\mathbf{D} - \mathbf{E}) \times (\mathbf{D} - \mathbf{F})\| \tag{6}$$

$$= \frac{1}{2} \left\| \left(\frac{\mathbf{A} + \mathbf{B}}{2} - \frac{\mathbf{B} + \mathbf{C}}{2} \right) \times \left(\frac{\mathbf{A} + \mathbf{B}}{2} - \frac{\mathbf{C} + \mathbf{A}}{2} \right) \right\| \tag{7}$$

$$= \frac{1}{2} \left\| \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right) \times \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) \right\| \tag{8}$$

$$= \frac{1}{2} \left\| \begin{pmatrix} 0 \\ -2 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\| \tag{9}$$

$$= \frac{1}{2} |0 - 2| = 1 \tag{10}$$

3 Diagram:

The diagram showing the triangle ABC and the triangle DEF is shown below:

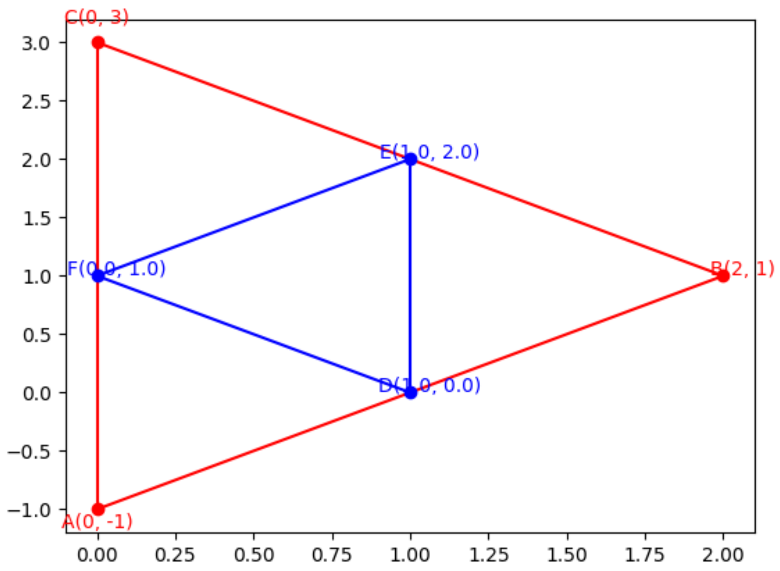

Figure 1: Diagram showing the triangle ABC and the triangle DEF.