

2.8.8

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Question If \mathbf{a} is a unit vector and $(\mathbf{x} - \mathbf{a}) \cdot (\mathbf{x} + \mathbf{a}) = 8$, then find $|\mathbf{x}|$

Solution:

Let us solve the given equation theoretically and then verify the solution computationally.

Given equation:

$$(\mathbf{x} - \mathbf{a}) \cdot (\mathbf{x} + \mathbf{a}) = 8 \quad (1)$$

The given equation can be written as:

$$(\mathbf{x} - \mathbf{a})^T (\mathbf{x} + \mathbf{a}) = 8 \quad (2)$$

$$\|\mathbf{x}\|^2 - \|\mathbf{a}\|^2 = 8 \quad (3)$$

Given that \mathbf{a} is a unit vector . So,

$$\|\mathbf{a}\| = 1 \quad (4)$$

Substituting the value of $\|\mathbf{a}\|$ in Eq.3.

$$\|\mathbf{x}\|^2 - 1 = 8 \quad (5)$$

$$\|\mathbf{x}\|^2 = 9 \quad (6)$$

$$\|\mathbf{x}\| = 3 \quad (7)$$

For verification let us take:

$$\mathbf{x} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \text{ and } \mathbf{a} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (8)$$

Let's check whether Eq.2 is satisfied:

$$(\mathbf{x} - \mathbf{a})^T (\mathbf{x} + \mathbf{a}) = \left(\begin{pmatrix} 3 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right)^T \left(\begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) \quad (9)$$

$$(\mathbf{x} - \mathbf{a})^T (\mathbf{x} + \mathbf{a}) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}^T \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (10)$$

$$(\mathbf{x} - \mathbf{a})^T (\mathbf{x} + \mathbf{a}) = 8 \quad (11)$$

From the figure it is clearly verified that the theoretical solution matches with the computational solution.

