Matgeo Presentation - Problem 9.4.34

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Problem Statement

Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

solution

Let the present ages be represented as the vector:

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} \tag{0.1}$$

where x and y denote Rohan's and his mother's present ages respectively.
given,
eq 1 :Since the mother is 26 years older than Rohan,

$$y = x + 26 \tag{0.2}$$

eq 2 :The product of their ages three years from now is given as

$$(x + 3)(y + 3) = 360 \tag{0.3}$$

Expanding the above equation:

$$xy + 3x + 3y - 351 = 0 \tag{0.4}$$

This can be written in quadratic (matrix) form as

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.5}$$

solution

where

$$\mathbf{V} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} \frac{3}{2} \\ \frac{3}{2} \end{pmatrix}, \quad f = -351 \tag{0.6}$$

The line $y = x + 26$ can be expressed parametrically as

$$\mathbf{x} = \mathbf{h} + \kappa \mathbf{m}, \quad \kappa \in \mathbb{R} \tag{0.7}$$

where

$$\mathbf{h} = \begin{pmatrix} 0 \\ 26 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.8}$$

Substituting $\mathbf{x} = \mathbf{h} + \kappa \mathbf{m}$ in the conic equation:

$$(\mathbf{h} + \kappa \mathbf{m})^\top \mathbf{V} (\mathbf{h} + \kappa \mathbf{m}) + 2\mathbf{u}^\top (\mathbf{h} + \kappa \mathbf{m}) + f = 0 \tag{0.9}$$

Grouping powers of κ, we get:

$$\kappa^2 (\mathbf{m}^\top \mathbf{V} \mathbf{m}) + 2\kappa\, \mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) + g(\mathbf{h}) = 0 \tag{0.10}$$

solution

where

$$g(\mathbf{h}) = \mathbf{h}^\top \mathbf{V}\mathbf{h} + 2\mathbf{u}^\top \mathbf{h} + f \tag{0.11}$$

Now compute each term:

$$\mathbf{m}^\top \mathbf{V}\mathbf{m} = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1 \tag{0.12}$$

$$\mathbf{V}\mathbf{h} + \mathbf{u} = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 26 \end{pmatrix} + \begin{pmatrix} \frac{3}{2} \\ \frac{3}{2} \end{pmatrix} = \begin{pmatrix} 14.5 \\ 1.5 \end{pmatrix} \tag{0.13}$$

$$\mathbf{m}^\top (\mathbf{V}\mathbf{h} + \mathbf{u}) = \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 14.5 \\ 1.5 \end{pmatrix} = 16 \tag{0.14}$$

$$g(\mathbf{h}) = \mathbf{h}^\top \mathbf{V}\mathbf{h} + 2\mathbf{u}^\top \mathbf{h} + f = 0 + 2(\tfrac{3}{2} \times 26) - 351 = -273 \tag{0.15}$$

solution

Substituting these results gives:

$$\kappa^2 + 32\kappa - 273 = 0 \tag{0.16}$$

The general quadratic solution is

$$\kappa = \frac{-\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top(\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})\,(\mathbf{m}^\top\mathbf{V}\mathbf{m})}}{\mathbf{m}^\top\mathbf{V}\mathbf{m}} \tag{0.17}$$

Substituting numerical values:

$$\kappa = \frac{-16 \pm \sqrt{16^2 - (-273)}}{1} = -16 \pm \sqrt{529} = -16 \pm 23 \tag{0.18}$$

Thus,

$$\kappa_1 = -39, \quad \kappa_2 = 7 \tag{0.19}$$

The intersection points are

$$\mathbf{x}_i = \mathbf{h} + \kappa_i\mathbf{m}, \quad i = 1, 2 \tag{0.20}$$

solution

Hence,

$$\mathbf{x}_1 = \begin{pmatrix} 0 \\ 26 \end{pmatrix} - 39 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -39 \\ -13 \end{pmatrix}, \quad \mathbf{x}_2 = \begin{pmatrix} 0 \\ 26 \end{pmatrix} + 7 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 33 \end{pmatrix} \quad (0.21)$$

The physically meaningful (non-negative) intersection corresponds to

$$\boxed{x = 7, \quad y = 33} \quad (0.22)$$

Therefore, Rohan's present age is 7 yearsand his mother's present age is 33years.

C Source Code:

```
#include <stdio.h>
#define N 100
void generate_line(double *x, double *y, int n) {
    for (int i = 0; i < n; i++) {
        x[i] = i;          // x values
        y[i] = x[i] + 26; // y = x + 26
    }
}
void generate_conic(double *x, double *y, int n) {
    for (int i = 0; i < n; i++) {
        x[i] = i + 0.1;       // avoid division by zero
        y[i] = 360.0 / (x[i] + 3.0) - 3.0;
    }
}
```

Python Script:solve

```
import ctypes
import numpy as np
lib = ctypes.CDLL('./libpoints.so')
N = 100
x_line = np.zeros(N, dtype=np.double)
y_line = np.zeros(N, dtype=np.double)
lib.generate_line.argtypes = [ctypes.POINTER(ctypes.c_double),
                              ctypes.POINTER(ctypes.c_double),
                              ctypes.c_int]
lib.generate_line.restype = None
lib.generate_line(x_line.ctypes.data_as(ctypes.POINTER(ctypes.
                  y_line.ctypes.data_as(ctypes.POINTER(ctypes.
                  N)
print("Sample points from C library (line):")
for i in range(5):
    print(f"x={x_line[i]}, y={y_line[i]}")
```

Python Script:solve

```
V = np.array([[0, 0.5],
              [0.5, 0]])
u = np.array([[1.5],
              [1.5]])
f = -351
h = np.array([[0],
              [26]])
m = np.array([[1],
              [1]])
mT_V_m = (m.T @ V @ m)[0,0]
Vh_plus_u = V @ h + u
mT_Vh_plus_u = (m.T @ Vh_plus_u)[0,0]
g_h = (h.T @ V @ h + 2*(u.T @ h) + f)[0,0]
a = mT_V_m
b = 2 * mT_Vh_plus_u
c = g_h
kappa = np.roots([a, b, c])
```

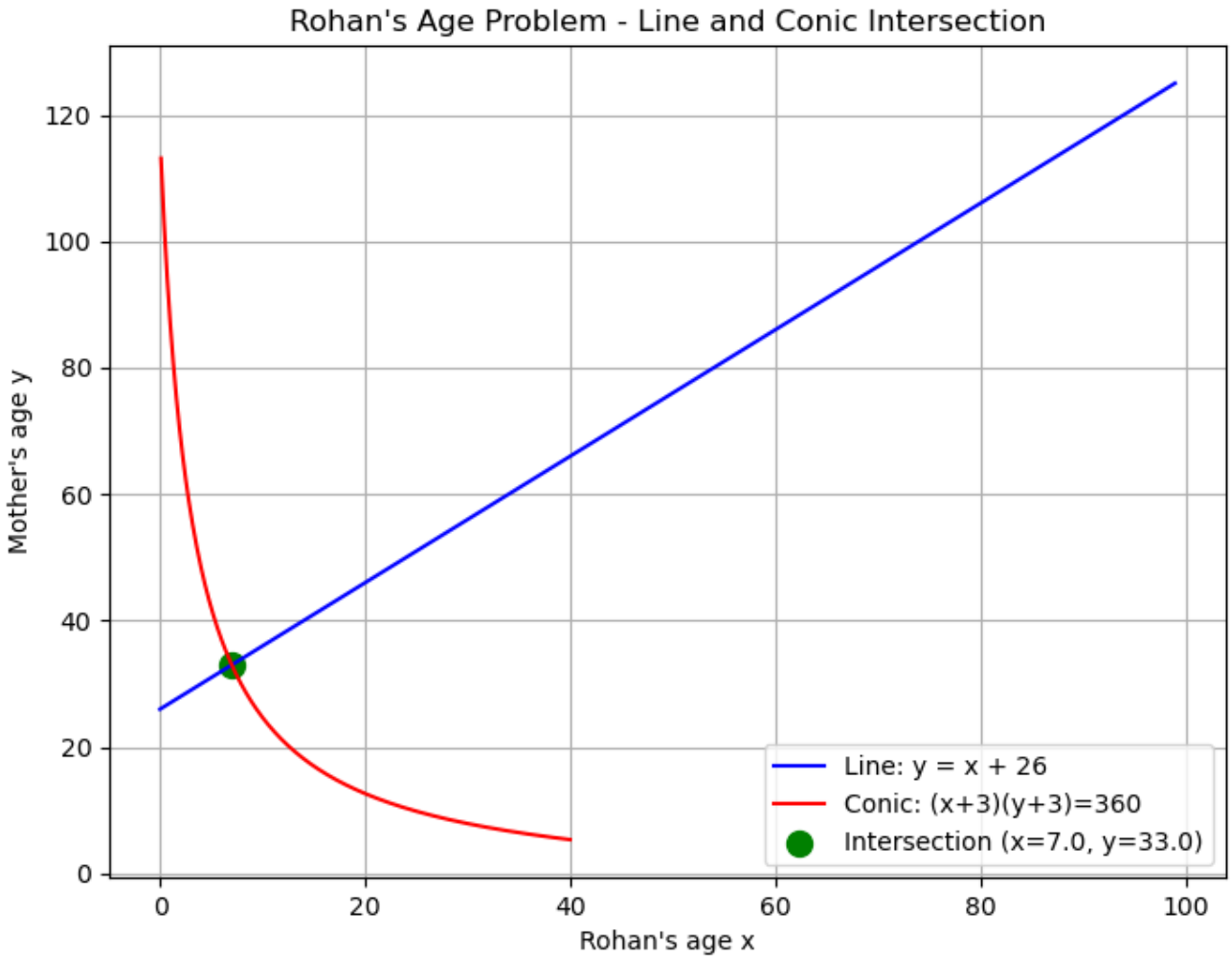
Python Script:solve

```
points = [h + k*m for k in kappa]
for pt in points:
    x_val = pt[0,0]
    y_val = pt[1,0]
    if x_val >= 0 and y_val >= 0:
        rohan_age = x_val
        mother_age = y_val
print(f"\nRohan's present age: {rohan_age} years")
print(f"Mother's present age: {mother_age} years")
np.savez("rohan_points.npz",
         x_line=x_line, y_line=y_line,
         rohan_age=rohan_age, mother_age=mother_age)
```

Python Script: plot

```
import numpy as np
import matplotlib.pyplot as plt
data = np.load("rohan_points.npz")
x_line = data['x_line']
y_line = data['y_line']
rohan_age = data['rohan_age']
mother_age = data['mother_age']
x_conic = np.linspace(0.1, 40, 400)  # avoid x=-3
y_conic = 360 / (x_conic + 3) - 3
plt.figure(figsize=(8,6))
plt.plot(x_line, y_line, label="Line: y = x + 26", color='blue
plt.plot(x_conic, y_conic, label="Conic: (x+3)(y+3)=360", colo
plt.scatter(rohan_age, mother_age, color='green', s=100, label
plt.xlabel("Rohan's age x")
plt.ylabel("Mother's age y")
plt.title("Rohan's Age Problem - Line and Conic Intersection")
plt.grid(True) plt.legend() plt.show()
```

Result Plot


Rohan's Age Problem - Line and Conic Intersection
Mother's age y
Rohan's age x
0
20
40
60
80
100
120
Line: y = x + 26
Conic: (x+3)(y+3)=360
Intersection (x=7.0, y=33.0)