12.596

ee25btech11063-vejith

Question:
Consider the system of linear equations:

$$x - 2y + 3z = -1,$$
$$x - 3y + 4z = 1,$$
$$-2x + 4y - 6z = k$$

The value of k for which the system has infinitely many solutions is _____ (EC 2015)

Solution:
Given equations are

$$(1 \quad -2 \quad 3)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = -1 \tag{1}$$

$$(1 \quad -3 \quad 4)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 1 \tag{2}$$

$$(-2 \quad 4 \quad -6)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = k \tag{3}$$

These equations can be written in matrix form as

$$\begin{pmatrix} 1 & -2 & 3 \\ 1 & -3 & 4 \\ -2 & 4 & -6 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ k \end{pmatrix} \tag{4}$$

Forming the augmented matrix

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 1 & -3 & 4 & 1 \\ -2 & 4 & -6 & k \end{array}\right) \xleftarrow{R_2 \leftarrow R_2 - R_1} \left(\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & -1 & 1 & 2 \\ -2 & 4 & -6 & k \end{array}\right) \tag{5}$$

$$\xleftarrow{R_3 \leftarrow R_3 + 2R_1} \left(\begin{array}{ccc|c} 1 & -2 & 3 & -1 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 0 & k-2 \end{array}\right) \tag{6}$$

As in the augmented matrix the entries of third row are 0 their linear combination should also give 0

$$k - 2 = 0 \tag{7}$$
$$\implies k = 2 \tag{8}$$

Now the system has 2 equations and 3 variables which has infinite solutions
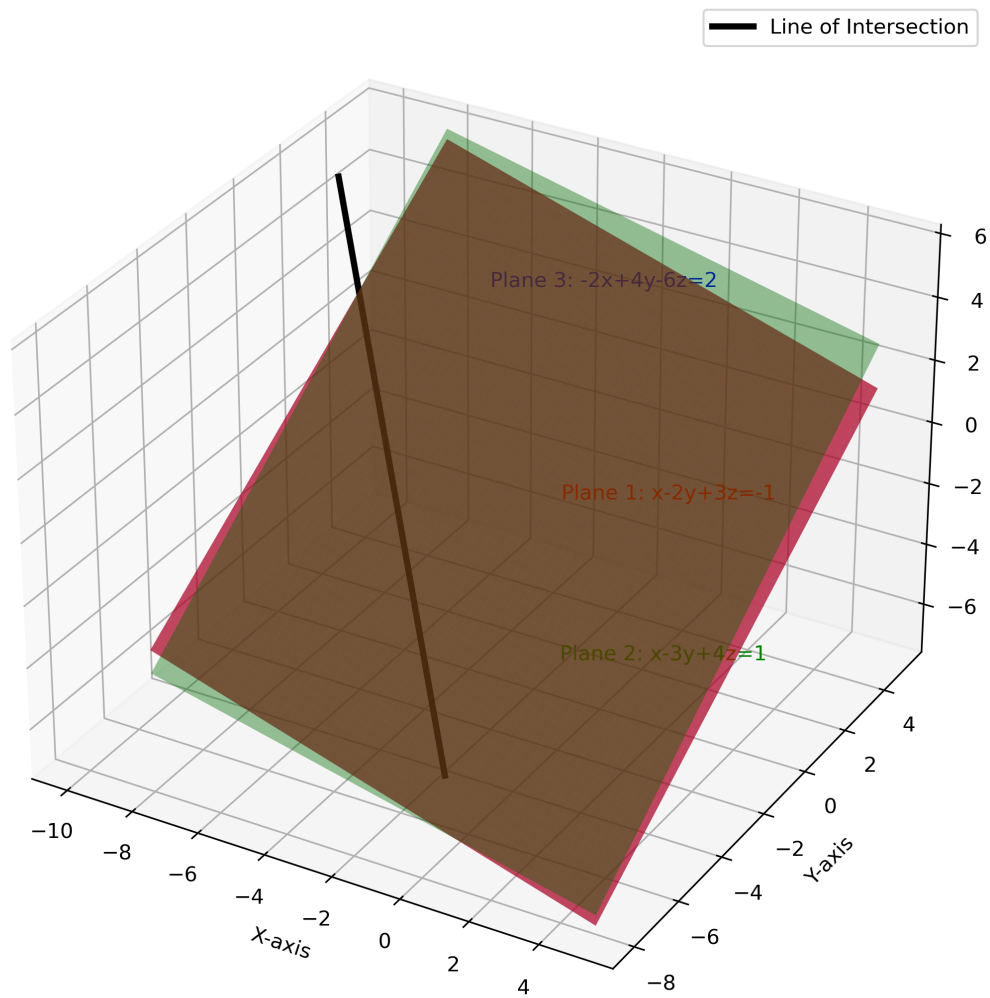
Intersection of 3 Planes (k=2): Infinite Solutions

Line of Intersection

Plane 3: -2x+4y-6z=2

Plane 1: x-2y+3z=-1

Plane 2: x-3y+4z=1

Fig. 0