Samyak Gondane-AI25BTECH11029

Question

Draw a triangle ABC with $BC = 7\ cm$, $\angle B = 45°$ and $\angle C = 60°$.

Solution

Given

- $BC = a = 7\ cm$
- $\angle B = 45°$
- $\angle C = 60°$

Let **B** be the origin

$$\angle A = 180° - (45° + 60°) = 75° \tag{1}$$

Let:

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} a \\ 0 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \tag{2}$$

Direction of **A** is along angle $B = 45°$:

$$\mathbf{A} = c \begin{pmatrix} \cos B \\ \sin B \end{pmatrix} = c \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{3}$$

in $\triangle ABC$

$$b \cos \angle C + c \cos \angle B = 7 \tag{4}$$

$$b \sin \angle C - c \sin \angle B = 0 \tag{5}$$

Solving linear Equation in b and c:

$$\begin{pmatrix} \cos \angle C & \cos \angle B \\ \sin \angle C & -\sin \angle B \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} \tag{6}$$

Using augmented matrix

$$\left(\begin{array}{cc|c} \cos \angle C & \cos \angle B & 7 \\ \sin \angle C & -\sin \angle B & 0 \end{array}\right) \tag{7}$$

putting $\angle C = 60°$ and $\angle B = 45°$

$$\left(\begin{array}{cc|c} 1/2 & 1/\sqrt{2} & 7 \\ \sqrt{3}/2 & -1/\sqrt{2} & 0 \end{array}\right) \tag{8}$$

Echelon form of the matrix is given by

$$\left(\begin{array}{cc|c} 1 & 2/\sqrt{2} & 14 \\ \sqrt{3}/2 & -1/\sqrt{2} & 0 \end{array}\right) \tag{9}$$

$$\left(\begin{array}{cc|c} 1 & 2/\sqrt{2} & 14 \\ 0 & (-1+\sqrt{3})/\sqrt{2} & -7\sqrt{3} \end{array}\right) \tag{10}$$

$$\frac{(-1+\sqrt{3})}{\sqrt{2}} \times c = -7\sqrt{3} \tag{11}$$

$$c = \frac{-7\sqrt{3}\cdot\sqrt{2}}{-1+\sqrt{3}} = \frac{-7\sqrt{6}}{-1+\sqrt{3}} \tag{12}$$

$$\mathbf{A} = c\begin{pmatrix}\cos\angle B \\ \sin\angle B\end{pmatrix} = -23.42\begin{pmatrix}0.7071 \\ 0.7071\end{pmatrix} \approx \begin{pmatrix}-16.56 \\ -16.56\end{pmatrix} \tag{13}$$

Final Coordinates

$$\mathbf{A} = \begin{pmatrix}-16.56 \\ -16.56\end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix}0 \\ 0\end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix}7 \\ 0\end{pmatrix} \tag{14}$$

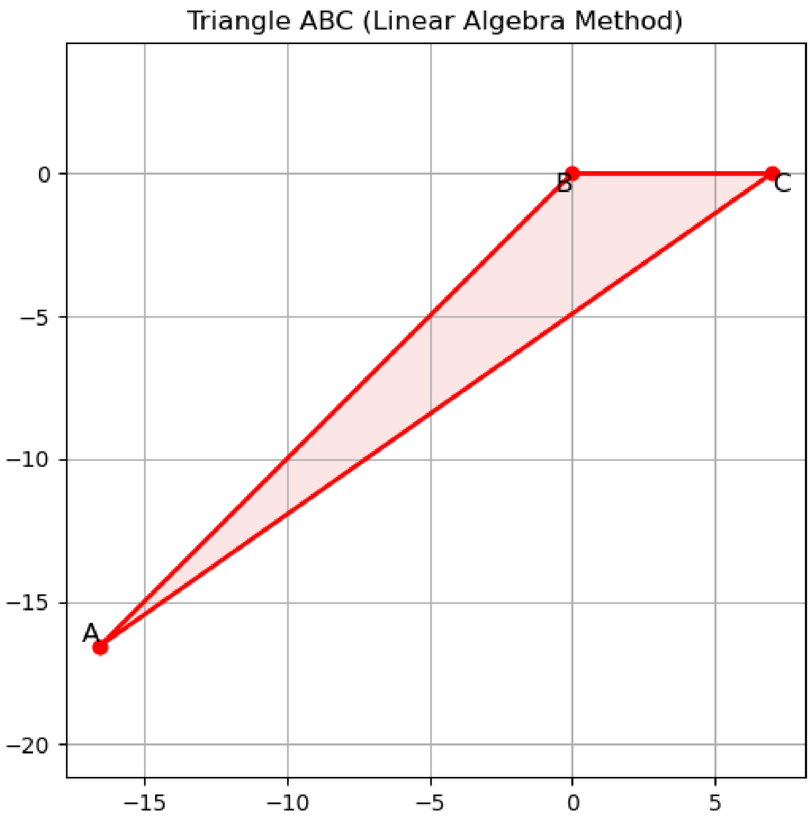

Figure 1