

Matgeo Presentation - Problem 9.2.19

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Problem Statement

Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$.

Data

Name	Value
Circle	$\mathbf{x}^\top \mathbf{x} - a^2 = 0$
Line	$\mathbf{x} = \begin{pmatrix} \frac{a}{\sqrt{2}} \\ 0 \end{pmatrix} + \kappa \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Table : Circle

Solution

The parameters for the circle are :

$$\mathbf{V} = \mathbf{I} \qquad \mathbf{u} = \mathbf{0} \qquad f = -a^2 \qquad (0.1)$$

The parameters for the line are :

$$\mathbf{h} = \begin{pmatrix} \frac{a}{\sqrt{2}} \\ 0 \end{pmatrix} \qquad \mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (0.2)$$

Substituting these in the below equation to find the intersection points :

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^\top (\mathbf{V} \mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h}) (\mathbf{m}^\top \mathbf{V} \mathbf{m})} \right) \qquad (0.3)$$

$$g(\mathbf{x}) = \mathbf{x}^\top \mathbf{x} - a^2 \qquad (0.4)$$

$$g(\mathbf{h}) = \mathbf{h}^\top \mathbf{h} - a^2 \qquad (0.5)$$

Solution

$$\kappa_i = \left(-\mathbf{m}^\top \mathbf{h} \pm \sqrt{a^2 - \mathbf{h}^\top \mathbf{h}} \right) \quad (0.6)$$

$$\kappa_i = \frac{a}{\sqrt{2}}, -\frac{a}{\sqrt{2}} \quad (0.7)$$

Therefore the points of intersection are :

$$\mathbf{P}_1 = \begin{pmatrix} \frac{a}{\sqrt{2}} \\ \frac{a}{\sqrt{2}} \end{pmatrix} \quad \mathbf{P}_2 = \begin{pmatrix} \frac{a}{\sqrt{2}} \\ -\frac{a}{\sqrt{2}} \end{pmatrix} \quad (0.8)$$

Thus the area of the smaller part of the circle cut off by the line is :

$$2 \int_{\frac{a}{\sqrt{2}}}^a \sqrt{a^2 - x^2} \, dx = \frac{a^2}{2} \left(\frac{\pi}{2} - 1 \right) \quad (0.9)$$

Plot

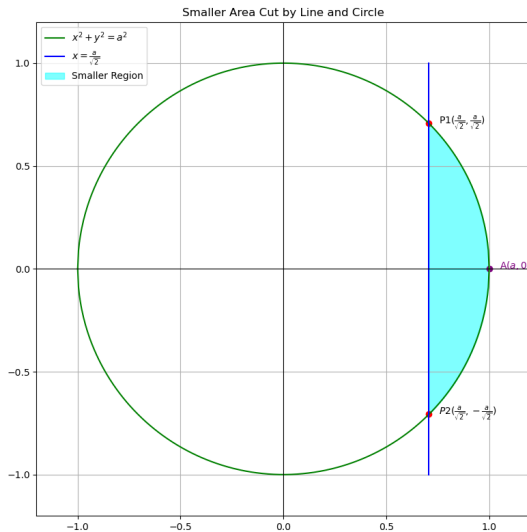


Fig : Circle