4.12.17 Matgeo

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Question

P_1,P_2 are points on either of the two lines y - $\sqrt{3}|x| = 2$ at a distance of 5 units from their point of intersection. Find the coordinates of the foot of the perpendiculars drawn from P_1,P_2 on the bisector of the angle between the given lines.

Solution

The equation of the lines is :

$$y - \sqrt{3}x = \begin{bmatrix} -\sqrt{3} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \tag{1}$$

$$y + \sqrt{3}x = \begin{bmatrix} \sqrt{3} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \tag{2}$$

Combining both the equations 0.1 and 0.2 ,we get :

$$\begin{bmatrix} -\sqrt{3} & 1 \\ \sqrt{3} & 1 \end{bmatrix} \mathbf{X} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \tag{3}$$

Solving by row reduction we get :

$$\mathbf{Q} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \tag{4}$$

Solution

The equation for the point $\mathbf{P_1}$ are:

$$\begin{bmatrix}-\sqrt{3} & 1\end{bmatrix}\mathbf{P_1} = 2 \tag{5}$$

$$\|P_1 - Q\| = 5 \tag{6}$$

The equation for the point $\mathbf{P_2}$ are:

$$\begin{bmatrix}\sqrt{3} & 1\end{bmatrix}\mathbf{P_2} = 2 \tag{7}$$

$$\|P_2 - Q\| = 5 \tag{8}$$

Solving the equations we get :

$$\mathbf{P_1} = \begin{bmatrix}\frac{5}{2} \\ 2 + \frac{5\sqrt{3}}{2}\end{bmatrix} \tag{9}$$

$$\mathbf{P_2} = \begin{bmatrix}-\frac{5}{2} \\ 2 - \frac{5\sqrt{3}}{2}\end{bmatrix} \tag{10}$$

Solution

The equation of the angle bisector is given by
Let us take a point $\mathbf{P}$ on the angle bisector , substitution it in the line equtions and equating the angles we get the equation :

$$\frac{|n_1\mathbf{P}-2|}{\|n_1\|} = \frac{|n_2\mathbf{P}-2|}{\|n_1\|} \tag{11}$$

$$\frac{n_1\mathbf{P}-2}{\|n_1\|} \pm \frac{n_2\mathbf{P}-2}{\|n_1\|} = 0 \tag{12}$$

solving the above equation we get locus of $\vec{P}$ as two lines which are the angle bisectors :

$$\begin{bmatrix}1\\0\end{bmatrix}^T \mathbf{x} = 0 \tag{13}$$

$$\begin{bmatrix}0\\1\end{bmatrix}^T \mathbf{x} = 2 \tag{14}$$

Solution

Let Q be the foot of the perpendicular from P to the line

$$\mathbf{n}^T\mathbf{x} = c \tag{15}$$

Then :

$$\begin{bmatrix}\mathbf{m} & \mathbf{n}\end{bmatrix}^T \mathbf{Q} = \begin{bmatrix}\mathbf{m}^T\mathbf{P} \\ c\end{bmatrix} \tag{16}$$

solving this equation for the line $\begin{bmatrix}1 \\ 0\end{bmatrix}^T \mathbf{x} = 0$,we get :

$$\begin{bmatrix}0 \\ 2 + \frac{5\sqrt{3}}{2}\end{bmatrix} \quad and \quad \begin{bmatrix}0 \\ 2 - \frac{5\sqrt{3}}{2}\end{bmatrix} \tag{17}$$

and solving it for the line $\begin{bmatrix}0 \\ 1\end{bmatrix}^T \mathbf{x} = 2$, we get :

$$\begin{bmatrix}\frac{5}{2} \\ 2\end{bmatrix} \quad and \quad \begin{bmatrix}-\frac{5}{2} \\ 2\end{bmatrix} \tag{18}$$

Graphical Representation