2.3.8

EE25BTECH11005 - Aditya Mishra

September 12, 2025

Question

If $\mathbf{A} = \hat{\imath} + \hat{\jmath} + \hat{k},\ \mathbf{B} = 2\hat{\imath} + 5\hat{\jmath},\ \mathbf{C} = 3\hat{\imath} + 2\hat{\jmath} - 3\hat{k},\ \mathbf{D} = \hat{\imath} - 6\hat{\jmath} - \hat{k}$ are the position vectors of points A, B, C and D, then find the angle between the straight lines AB and CD. Find whether $\mathbf{AB}$ and $\mathbf{CD}$ are collinear or not.

Solution

Let the direction vectors be

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix} \tag{1}$$

$$\mathbf{CD} = \mathbf{D} - \mathbf{C} = \begin{pmatrix} 1 \\ -6 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix} \tag{2}$$

The angle θ between $\mathbf{AB}$ and $\mathbf{CD}$ is given by

$$\cos\theta = \frac{\mathbf{AB}^\top \mathbf{CD}}{|\mathbf{AB}|\,|\mathbf{CD}|} \tag{3}$$

Substitute the values:

$$\mathbf{AB}^T\mathbf{CD} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}^\top \begin{pmatrix} -2 \\ -8 \\ 2 \end{pmatrix} \tag{4}$$

$$= (1)(-2) + (4)(-8) + (-1)(2) \tag{5}$$

$$= -2 - 32 - 2 = -36 \tag{6}$$

Magnitudes:

$$|\mathbf{AB}| = \sqrt{1^2 + 4^2 + (-1)^2} = \sqrt{1 + 16 + 1} = \sqrt{18} \tag{7}$$

$$|\mathbf{CD}| = \sqrt{(-2)^2 + (-8)^2 + 2^2} = \sqrt{4 + 64 + 4} = \sqrt{72} \tag{8}$$

Thus,

$$\cos\theta = \frac{-36}{\sqrt{18}\sqrt{72}} = \frac{-36}{\sqrt{1296}} = \frac{-36}{36} = -1 \tag{9}$$

$$\theta = \cos^{-1}(-1) = \pi \text{ radians } = 180^\circ \tag{10}$$

So, **AB** and **CD** are collinear but point in opposite directions, i.e., they are anti-parallel.

The lines are collinear (anti-parallel).

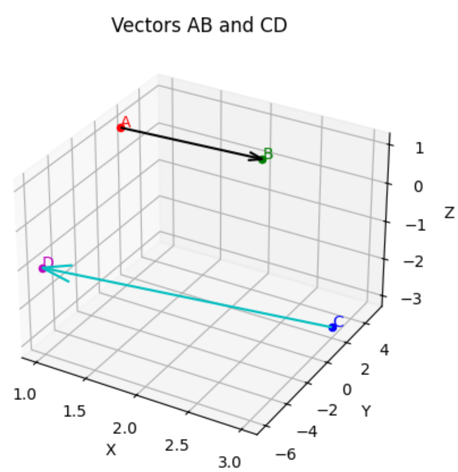

Figure 1: Line directions