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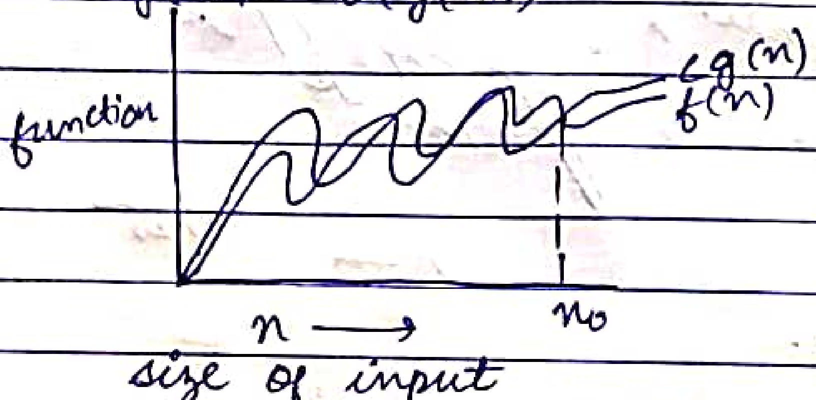
Tutorial-1

1. Asymptotic Notations - They are the mathematical notations used to describe the running time of an algorithm when the input tends towards a particular value or a limiting value.

Different asymptotic notations -

i) Big O(n)

$$f(n) = O(g(n))$$



$$f(n) = O(g(n))$$

iff $f(n) \le c\,g(n)$

$\forall\ n \ge n_0$

for some constant, $c > 0$

$g(n)$ is "tight" upper bound of $f(n)$.

ex. $f(n) = n^2 + n$

$g(n) = n^3$

$n^2 + n \le cn^3$

$n^2 + n = O(n^3)$

ii) Big omega (Ω)

$$f(n) = \Omega(g(n))$$

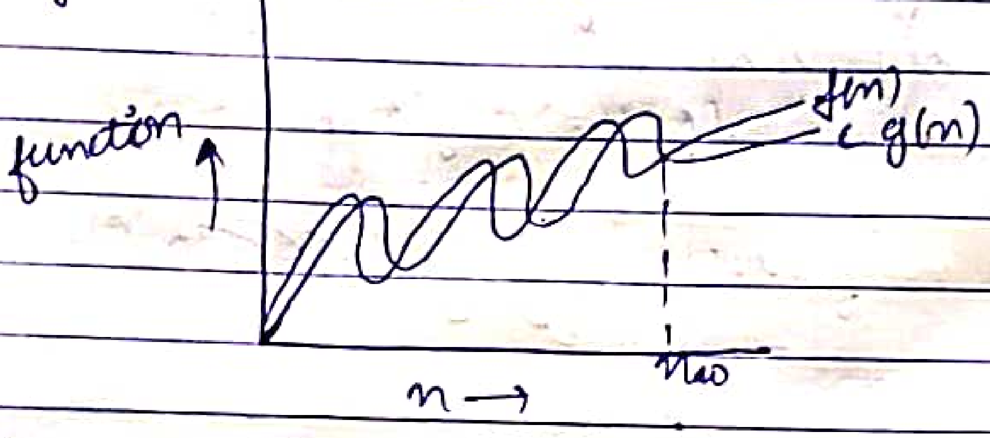
g(n) is "tight" lower bound of function f(n).

$$f(n) = \Omega(g(n))$$

iff $f(n) \geq c\,g(n)$

$\forall \; n \geq n_0$

for some constant $c > 0$



ex.

$$f(n) = n^5 + 4n^2$$

$$g(n) = n^2$$

$$n^3 + 4n^2 = \Omega(n^2)$$

iii) Big Theta (θ)

$$f(n) = \theta(g(n))$$

g(n) is both "tight" upper and "lower" bound of function f(n).

$$f(n) = \theta(g(n))$$

iff

$$c_1\, g(n) \leq f(n) \leq c_2\, g(n)$$

$$\forall \; n \geq \max(n_1, n_2)$$

for some constant $c_1 > 0$ and $c_2 > 0$

Ex-

$3n+2 = \theta(n)$ as $3n+2 \geq 3n$ and
$3n+2 \leq 4(n)$ for n, $k_1 = 3$, $k_2 = 4$ & $n_0 = 2$

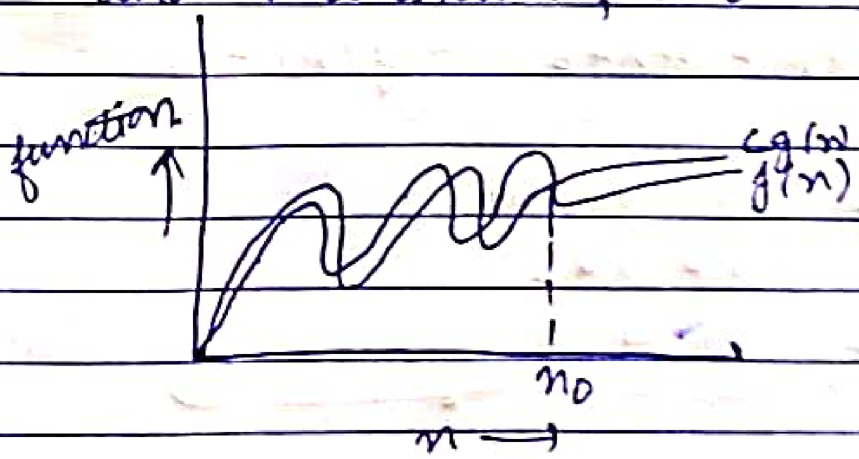
iv) Small $o(o)$ -

$$f(n) = o(g(n))$$

$g(n)$ is upper bound of function $f(n)$.

$$f(n) = o(g(n))$$

when $f(n) < c\,g(n)$
$\forall\ n > n_0$
and $\forall$ constants, $c > 0$



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ex - $f(n) = n^2$

$g(n) = n^3$

$n^2 = o(n^3)$

v. Small omega (ω)

$f(n) \geq \omega(g(n))$

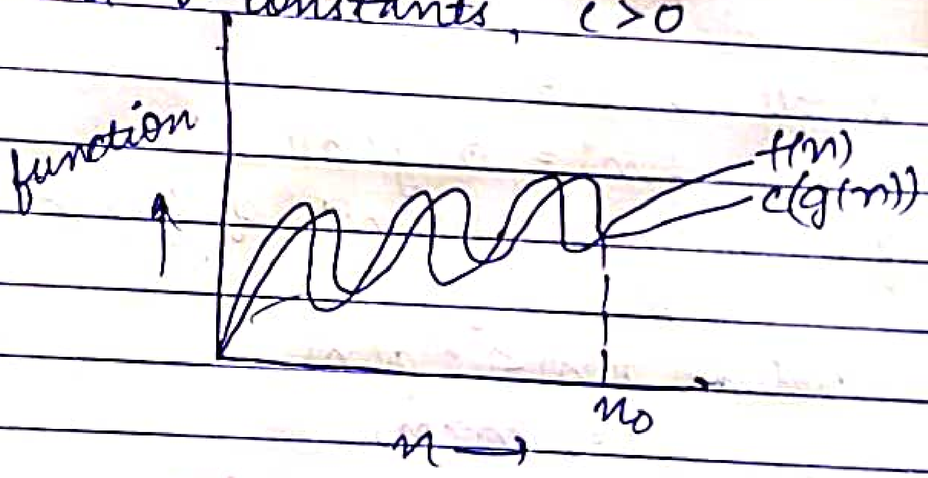
$g(n)$ is lower bound of $f(n)$.

$f(n) = \omega(g(n))$

when $f(n) > c\, g(n)$

$\forall\ n > n_0$

and $\forall$ constants, $c > 0$



$f(n) = 4n+6 \qquad g(n) = (1)$

2. for ($i = 1$ to n)

{ $i = i * 2$ }

→ $i = \underbrace{1, 2, 4, 8, 16, \text{——}, n}_{k}$ (G.P.)

— $O(k)$

$a = 1, \; r = \frac{2}{1} = 2.$

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GP kth value = $t_k = ar^{k-1}$

$$n = 1 \times 2^{k-1}$$

$$n = \frac{2^k}{2}$$

$$2n = 2^k$$

$$\log(2n) = k \log 2$$

$$k = \log_2 2n$$

$$k = \log_2 2 + \log_2 n$$

$$k = 1 + \log n$$

Time comp = $O(1 + \log_2 n)$

$= O(\log_2 n)$

3. $T(n) = 3T(n-1)$ — ①

Let $n = n-1$

$T(n-1) = 3T(n-2)$ — ②

Put ② in ①

$T(n) = 3 \times 3T(n-2)$ — ③

Put $n = n-2$

$T(n-2) = 3T(n-3)$ — ④

Put ④ in ③

$T(n) = 3 \times 3 \times 3T(n-3)$ — ⑤

$T(n) = 3^n T(n-n)$

$= 3^n T(0)$

$= 3^n$

$= O(3^n)$

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4. $T(n) = 2T(n-1)-1$

$= 2(2T(n-2)-1)-1$

$= 2^2(T(n-2))-2-1$

$= 2^3T(n-3)-2^2-2^1-2^0$

$---$

$= 2^nT(n-n)-2^{n-1}-2^{n-2}-2^{n-3}--$

$--2^2-2^1-2^0$

$= 2^n-2^{n-1}-2^{n-2}-2^{n-3}---2^2-2^1-2^0$

$= 2^n-(2^n-1)$

$T(n) = 1$

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$= 2^n T(n-n) - 2^{n-1} - 2^{n-2} - 2^{n-3}$
$\quad ..... 2^2 - 2^1 - 2^0$
$= 2^n - 2^{n-1} - 2^{n-2} - 2^{n-3} ... 2^2 - 2^1 - 2^0$
$= 2^n - (2^n - 1)$
$T(n) = 1$

5.
```
int i=1, s=1;
while (s<=n) {
    i++; s=s+i;
    printf("#");
}
```

$s_i = s_{i-1} + i$

i is incrementing by one step
s in incrementing by value of i
Following will be values after few iterations -
⇒ $i = 2$, $s = 3$ 1st iteration
⇒ $i = 3$, $s = 6$ 2nd iteration
⇒ $i = 4$, $s = 10$ 3rd iteration
Let the value of n be k.
Values of s ⇒ 1, 3, 6, 10 ---
S represents a series of sum of first n natural numbers
for $i = k$, $s = \frac{k(k+1)}{2}$

for stopping loop.

$$\frac{k(k+1)}{2} > n \Rightarrow \frac{k^2+k}{2} > n$$

$$T(n) = O(\sqrt{n})$$

6.
```
void function(int n){
    int i, count = 0;
    for(i=1; i*i<=n; i++)
        count++;
}
```

$i = 1, 2, 3 \ldots n$

$i^2 = 1, 4, 9 \ldots n$

So $i^2 <= n$ or $i <= \sqrt{n}$

$a_k = a + (k-1)d$

$a = 1 \quad d = 1$

$a_k <= \sqrt{n}$

$\sqrt{n} = 1 + (k-1) \cdot 1$

$\sqrt{n} = k$

$T(n) = O(\sqrt{n})$

7.
```
void function(int n){
    int i, j, k, count = 0;
    for(i=n/2; i<=n; i++)
        for(j=1; j<=n; j=j*2)
            for(k=1; k<=n; k=k*2)
                count++;
            }
    }   }
}
```

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$i = n/2$ $\quad j = \log_2 n$ $\quad k = \log_2 n$

$\left(\frac{n}{2}+1\right)$ times $\quad \log_2 n \quad \log_2 n$

$O(i * j * k) = O\left(\left(\frac{n}{2}+1\right) * \log_2 n * \log_2 n\right)$

$= O\left(\left(\frac{n}{2}+1\right) \times (\log n)^2\right)$

$T(n) = O(n(\log n)^2)$

8. function (int n) {
    if (n==1) return;
    for (i=1 to n){
        for (j=1 to n) {
            print("*");
        }
        function(n-3);
    }

$T(n) = T(n-3) + n^2$ —— ①

$T(1) = 1$ —— ②

put $n = n-3$ in ①

$T(n-3) = T(n-6) + (n-3)^2$ —— ③

Put ③ in ①

$T(n) = T(n-6) + (n-3)^2 + n^2$ —— ④

put $n = n-6$ in ①

$T(n-6) = T(n-9) + (n-6)^2$ —— ⑤

Put (5) in (4)

$T(n) = T(n-9) + (n-6)^2 + (n-3)^2 + n$

Generalizing

$T(n) = T(n-3k) + (n-3(k-1))^2 + (n-3(k-2))^2 + \dots + n^2$

Let $n-3k = 1$

$\frac{n-1}{3} = k$

$T(n) = T(1) + \left(n-3\left(\frac{n-1}{3} - 1\right)\right)^2 + \left(n-3\left(n-\frac{1}{3}\right)\right)^2 + \dots n^2$

$T(n) = T(1) + (n-((n-1)-3])^2 + (n-(n-1-6))^2 + (n-(n-1-9))^2 + \dots n^2$

$T(n) = 1 + (3+1)^2 + (6+1)^2 + \dots n^2$

$T(n) = 1^2 + 4^2 + 7^2 \dots n^2$

$T(n) = n^2 + \dots 1$

$\boxed{T_n = O(n^2)}$

9.
```
void function(int n){
    for(i=1 to n){
        for(j=1; j<=n; j=j+i){
            printf("*");
        }
    }
}
```

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for $i = 1$, $j \to n$ times

for $i = 2$, $j = 1 + 3 + 5 \cdots + n$

$a_n = a + (k-1)d$

$a = 1 \qquad d = 2$

$n = 1 + (k-1) \times 2$

$\frac{n-1}{2} = k - 1$

$k = \frac{n-1}{2} + 1$

$\boxed{k = \frac{n+1}{2}}$ No. of terms

for $i = 2$, $j \to \frac{n+1}{2}$ times

for $i = 3$, $j = 1 + 4 + 7 + \cdots n$

$n = 1 + (k-1) \times 3$

$\boxed{\frac{n-1}{3} + 1 = k}$

for $i = 3$, $j = \frac{n+2}{3}$ times

Generalising

for $i = n$, $j = \frac{n+k-1}{k}$ times

Time complexity is

$$\underbrace{n + \frac{n+1}{2} + \frac{n+2}{3} + \cdots + \frac{n+k-1}{k}}_{n \text{ terms}}$$

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General term = $\frac{n+k-1}{k}$

$$\sum_{k=1}^{n} \frac{n+k-1}{k} = \frac{\sum_{1}^{n} n + \sum_{1}^{n} k - \sum 1}{k}$$

$$\Rightarrow \frac{\frac{n(n+1)}{2} + nk - n}{k}$$

$$\Rightarrow \frac{n^2 + \frac{n}{2} + nk - n}{k}$$

$$T(n) = \frac{n^2 + \frac{n}{2} + nk - n}{k}$$

Neglecting constant terms.

$$\boxed{T(n) = O(n^2)}$$

10. as given n^k & c^n
relation b/w n^k & c^n is
$n^k = O(c^n)$
as $n^k \leq ac^n$
$\forall\ n \geq n_0$ & some constant $a>0$
for $n_0 = 1$
$c = 2$
$\Rightarrow 1^k \leq a2$
$n_0 = 1$ & $c = 2$