1

Modelling Options Prices with Mandelbrotian Movement of Prices

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***Abstract*—Option pricing models are important to the financial markets. They not only help in trading strategies through finding arbitrage but are also a core part of risk assessment and valuation of OTC products. There have been many option pricing models which have tried to accurately estimate option price. The Black Scholes model, which has been the industry standard, has flaws in the basic underlying assumptions leading to inaccuracies. The purpose of the paper is to investigate drawbacks of conventional option pricing models and show that Mandelbrotian Multifractal Model can help in reducing some of the inaccuracies.**

I. Introduction

Traditional Option Pricing Models:

The Black Scholes model was first introduced in 1973. Since then, the BSM has gained significant traction in option pricing and financial markets. The main reason behind its rise was easy of use and computation and its comprehensible results. But as Shinde [1] pointed out, the assumptions which the Black Scholes take are not in sync with market dynamics. For example, Black Scholes assumes log normal return distribution and a constant volatility which are not correct. The market in reality has much fatter tails. Furthermore, there are further jumps and abrupt price changes which are common in several asset classes which are not accounted for by BSM.

Owing to the shortcomings of Black Scholes model, researchers have come up with the jump diffusion model [2] and the binomial model [3]. The jump diffusion model is better suited for pricing options on illiquid assets or assets with large fluctuations in value since it takes jumps or discontinuous price movements into account. However, the jump diffusion model still struggles to capture long term memory of the market. This model finds it difficult to explain fractal features and long-term memory found in market dynamics. Conversely, the binomial model provides flexibility in accommodating different assumptions and parameters; but, its computing intensity and discrete-time approximation have drawbacks, particularly for complicated market dynamics or long-dated options.

II. Theoretical Framework

Mandelbrotian Multifractal Model:

Owing to the shortcomings of traditional models, Mandelbrot suggested the Mandelbrotian Multifractal Model for pricing options. This model better captures the market dynamics while addressing some of the concerns with the traditional models. The model is based on the observation that price movements do not follow the normal distribution that is supposed by modern finance, as McMillan (2020) [4] explains and that there are probabilities of extreme fluctuations, displaying traits of both short- and long-term memory.

The consequences of Mandelbrot's model for option pricing are pointed out by Markham (2020) [5]. According to him, it can more accurately represent the fractal nature, high volatility and fat-tailed distributions which are seen in financial markets. The Mandelbrotian model helps in creating a more realistic depiction of market dynamics by adding time and Brownian motion, especially for long-dated options and higher-risk scenarios.

Evaluation and Implications:

The Mandelbrot model for option pricing stands as an alternative to established models. There are reasons though this approach is not widely accepted or successful in its usage, with the first being its infancy; it has not attained the same popularity level as the more traditional models such as Black-Scholes.

One more problem emerges from the complexity of implementing, calibrating, and estimating parameters in the Mandelbrotian system - which hinders its adoption. Despite these challenges, the Mandelbrotian model can address certain issues that other methodologies cannot: such as not being able to properly deal with intricate market dynamics. This means that by combining it with alternative models such as machine learning or agent based modeling its accuracy and applicability could be enhanced even further. Additionally, given industry shifts towards accounting for extreme events and market risk in light of changing regulatory requirements; The ability of this particular models to reflect wild swings alongside thick-tailed distributions will find more usefulness going forward

The Mandelbrotian Multifractal Model(MMM) emerges as an alternative to the traditional option pricing models that fail to reflect the real market dynamics, thus overcoming their limitations. Despite acceptance issues, this model is an important one for those dealing with option pricing because it can capture wild price movements and fat-tailed distributions along with fractal features. We see here that the accurate

1

pricing of derivative instruments could be further enhanced by new approaches or integration of the Mandelbrotian model with other techniques given the evolution of financial markets as well as computer resources.

Real World Applications:

Mandelbrotian option pricing models have various applications and are highly used by fintech, hedge funds and retail traders. Some of the major use-cases are listed below -

- To capture leptokurtic distributions
- To find market anomalies
- Risk Management and Position Sizing in Extreme Events
- Detecting long-term trends and events
- To Investigate Non-Stationary Processes

The financial markets are dynamic in nature and always changing, with trends and variations that might alter over time. The Mandelbrotian model is a good choice for analyzing option pricing in dynamic market environments because of its capacity to represent non-stationary processes and adjust to shifting market conditions.

The ability of the Mandelbrotian model signifies the true essence of market dynamics such as market volatility, capturing rare and extreme events and the same get's reflected in option prices as well.
Thus, in terms of Non-Gaussian distributions of returns and the presence of heavy tails (aka leptokurtic) capturing spikes, the Madelbrotian approach can help improve risk assessment and portfolio diversification. Algotraders, Quants, Financial Analysts and various other financial institutions use this technique to boost their decision making skills in terms of arbitrage opportunities, hedging strategies and finding overbought and oversold zones.

III. Methodology

a) *Equations*

Black-Scholes Model - This model provides theoretical estimates of how option prices are calculated using vanilla European-style options. It assumes that the price of the underlying asset has a log-normal distribution.

We know that Black Scholes equation does not have a closed-form solution, however, we can express it form of a partial differential equation:

$$\partial v/\partial t + 0.5 * (\sigma^2) * (S^2) * (\partial^2 v/\partial S^2) + (r * S * \partial v/\partial S) - (r * V) = 0$$

where,

- **v** is option price,
- **t** is time to maturity
- **σ** is the volatility of the underlying asset
- **S** is the current price of the underlying asset
- **r** is the risk-free interest rate

- Binomial Model - Binomial models leverages discrete time option pricing strategy using a binomial distribution in order to simulate possible price changes of the underlying asset over time. These models are most popular in pricing both American and European styles options.

  We already know that binomial model does not have a closed-form solution, however, it can be expressed as a on-loop recursive equation:

  $$C[i] = \max(0, (p * C[i]+1 + (1-p) * C[i]-1) / (1 + r))$$
  where,
  - C[i] is the option price at given time i
  - p is the probability of an up move in given asset
  - C[i]+1 is the option value if the asset price moves up
  - C[i]-1 is the option value if the asset price moves down
  - r is the risk-free interest rate

- Jump Diffusion Model: This model took motivation from Black Scholes model which incorporates the chances of unusual, big price jumps or spikes in the underlying asset's prices. It is useful for pricing options on trading assets where we observe discontinuous price movements, such as stocks, bonds, cryto-currency, commodities, currencies markets.

  As we know, the jump diffusion model does not provide a closed-form solution, because of jump processes and the integration of the resulting option price. The model can be also be expressed in terms of partial differential equation below:

  $$\partial v/\partial t + 0.5 * (\sigma^2) * (S^2)* (\partial^2 v/\partial S^2) + r * S * (\partial v/\partial S) - (r * V) + \lambda * \int(V(S*Y,t) - V(S,t)) * f(Y)\, dY = 0$$

  where,
  - v is the option value
  - t is the time to maturity
  - σ is the volatility of the underlying asset
  - S is the current price of the underlying asset
  - r is the risk-free interest rate
  - λ is the jump intensity
  - f(Y) is the jump size distribution

- Mandelbrotian Model: The model is also known as the Fractal Market Hypothesis (FMH), is an alternative option pricing framework that leverages the ideas of fractal geometry and self-similarity of the asset prices. In regards to managing the asset returns' specifically non-normal distribution , this model aims to overcome the shortcomings of the conventional Black-Scholes model.

Despite knowing the fact that the Mandelbrotian model uses scaling laws and fractal dimensions to represent the prices and market dynamics of the underlying asset, we know that it lacks a closed-form solution. The approach here is to find those fractal patterns, power-law distributions, and long-range dependence over characteristics of discrete time series.

As we already know that this model accounts for non-linearity and incorporates complexity of global financial markets, it has the potential to outperform other alternative pricing models such as Black Scholes, Binomial and Jump Diffusion. Typically models, like Black-Scholes, have a shortfall in explaining the long-range dependence, fat-tailed distributions, and volatility clustering that are seen in financial data. Thus, considering various studies on fractals, this technique can effectively improve option pricing by combining fractal concepts.

Mandelbrot explains in his framework that the price of a financial asset is viewed as a long memory multiscaling process making it a multistep process. The steps included are based on the work of Rostislav Sibirtsev from his 2019 essay.[7]

Here's a summary of the key steps in simulating price paths using MMM, in point form:

1. Obtain and format daily closing price data for the asset, ensuring the number of data points is a highly composite value.
2. Calculate the log returns between each observed price which helps in measuring all variables in a comparable metric,thus easing evaluation of analytical relationships despite unequal values.
3. Define the stochastic process $X(t)$ to represent the total growth of the asset.

   $$X(t) = \ln P(t) - \ln P(0),$$

   for $0 < t < T$,where t is an instance of time since time zero.

4. Select the values of q (raw statistical moments) which will be used for empirical estimations .
5. Calculate the partition function to describe the statistical properties of the system.This is done by splitting the data into N non-overlapping subintervals, whose length Δt is less than our total time T.
6. Determine the Hurst exponent H to find how much the financial time series is deviating from a random walk..
7. Estimate the multifractal spectrum $f(\alpha)$ using a Legendre transformation.
8. Find the most probable Hölder exponent $\alpha 0$ which denotes the maximum value of the multifractal spectrum.
9. Estimate the log-normal distribution parameters λ and σ^2 for the multiplicative cascade using H and $\alpha 0$.
10. Determine the number of data points to simulate and generate a log-normal multiplicative cascade.
11. Find the trading time function $\theta(t)$ by converting values of multiplicative cascade into a cumulative distribution function.
12. Simulate a fractional Brownian motion BH(.).
13. Merge the trading time CDF with the fractional Brownian motion to obtain the compound process as a simulation of price growth over time.

IV. Results

We have implemented the following option pricing models (for code, please see the github repo here: https://github.com/Aarvin01/Modelling-Options-Prices-with-Mandelbrotian-Movement-of-Prices). The underlying asset used is Nifty 50. The spot price at the time of writing is 22256 and the strike price we are using is 22000 and the time to expiry is 15 days. Using the above parameters we have calculated the prices of options from different option pricing models.

Below are the details about the derivatives used for option price estimation :

Symbol	**Values**
Derivative Asset	Nifty50 Index
Spot Price	21730
Strike Price	22200
Time to expiry	128 days

Table 1 : Details about the derivative asset, strike prices and spot price

a) Binomial option pricing

Below is the summary of steps, which we followed to performed as part of Binomial option pricing:

1. We have calculated the change in timesteps as (Δt) as the maturity divided by the number of periods.
2. We calculated the up factor (u) and down factor (d) using the volatility and (Δt).
3. Then. determined the probability of an up move (p) using the risk-free rate, (u), and (d).
4. Set the initial call price to zero.
5. For each possible number of up moves (k) from 0 to the number of periods (N):
- Calculate the binomial coefficient for (N) choose (k).
- Estimate probability of k up moves and N-k down moves and simulate for each period over k up moves.

- Then, calculate weighted call option payoff.

6. Discount the factor using formula [e^(-annual_interest_rate * maturity)].
7. Compute the put price using the relationship between call and put prices, the strike price, and the initial stock price.

Following is the result of binomial option pricing for the above mentioned parameters. As stated above, the time step (Δt) was computed by dividing the maturity with the number of periods. After calculating (u) and (d), we calculated the binomial coefficient and calculated the stock price at the end of the period (k).

Option	Price
Call	64.80
Put	148.90

Table 2 : Estimated option prices using Binomial Option Pricing

b) Black scholes option pricing

Here are the steps used for Black scholes option pricing:

1) We defined parameters such as mean, standard deviation, number of paths, number of time steps, number of paths to plot, time horizon, time step, and initial stock price.
2) Created a time grid using numpy.linspace from 0 to the time horizon, with a number of steps equal to the number of time steps plus one.
3) Calculated the increments for the arithmetic Brownian motion using the formula $(\mu - 0.5\times\sigma^2)\times\Delta t + \sigma \times \mathrm{sqrt}(\Delta t) \times$ normal random variables
4) Accumulated these increments starting from zero to simulate the paths of the stock price
5) Transformed the accumulated increments to geometric Brownian motion by exponentiating and multiplying by the initial stock price.
6) Plotted a sample of the simulated stock price paths, selecting a subset of paths and using matplotlib to visualize them.
7) Plotted the average path of the stock prices over time using matplotlib. Created a histogram of the stock prices at the final time step using matplotlib.
8) Fit a log-normal distribution to the final stock prices and overlay this fitted distribution on the histogram.
9) Defined market parameters such as maturity, spot price, strike price, risk-free interest rate, dividend rate, and volatility.
10) Calculated the drift under the risk-neutral measure using the formula $r - q - 0.5\times \sigma^2$
11) Initialized arrays to store call and put option values for the Monte Carlo simulation.
12) For each block in the Monte Carlo simulation, simulate stock prices at maturity using the risk-neutral drift and volatility.
13) Calculate the call option value for each block by taking the mean of the maximum of the simulated stock prices minus the strike price, discounted by the risk-free rate.
14) Averaged the call option values across all blocks to get the final call option price
15) Averaged the put option values across all blocks to get the final put option price.
16) Printed the calculated prices for the call and put options.

Below are the results from the Black Scholes option pricing:

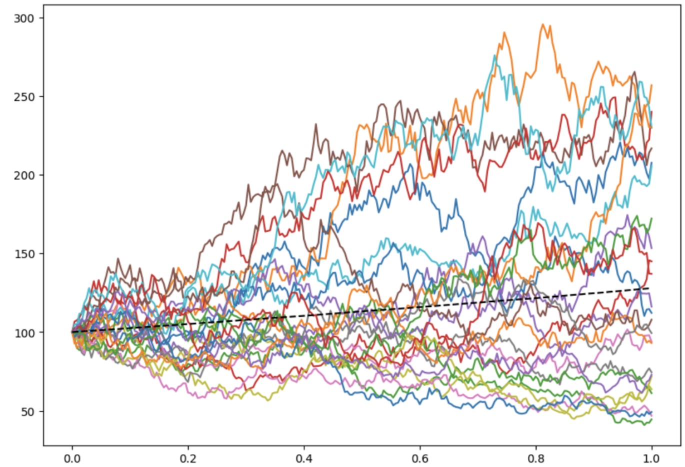


Fig 1 : Monte carlo simulation using Black Scholes Pricing Method over Nifty50 Index

The above graph represents simulated stock paths obtained from increments from Geometric Brownian motion. The Geometric brownian motion was generated from the increments from the normal random variable and then that was multiplied with initial stock price to get different paths.

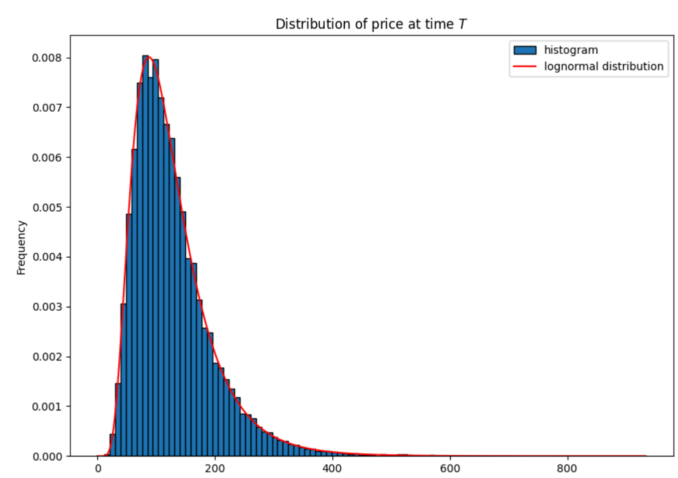


Fig 2 : Distribution of final stock price exhibiting log normal distribution

The above graph represents the distribution of final stock price after generating various price paths from Geometric Brownian motion. After defining the parameters (stock price, strike, maturite, risk free rate), drift under the risk neutral measure was calculated. Then, for each block in Monte Carlo simulation, call option and put option price was calculated. After averaging all the price paths, below price of call and put options was obtained.

Option	Price
Call	66.20
Put	150.40

Table 3 : Estimated option prices using Black Scholes method

c) Merton Jump diffusion option pricing

Here are the steps used for Merton Jump diffusion option pricing:

1. Defined market and model parameters including maturity, spot price, strike price, risk-free interest rate, dividend rate, volatility, jump intensity, expected jump size, and standard deviation of jump size.
2. Initialized Monte Carlo parameters with the number of blocks and samples per block.
3. Created arrays to store calls and put option values.
4. Run a Monte Carlo simulation for each block:
   - Generated normal random variables for stock price paths.
   - Generated Poisson-distributed random variables for the number of jumps.
   - Generated normal random variables for jump sizes.
   - Calculate the stock price paths incorporating jumps.
   - Calculate and store the call and put option values for each block.
5. Computed the final call and put option values by averaging over all blocks.
6. Printed the calculated call and put option prices.
7. Defined parameters for simulating stock price paths including the number of paths, number of time steps, and number of paths to plot.
8. Created a time grid for the simulation.
9. Simulated the increments of the arithmetic Brownian motion for stock prices.
10. Incorporated jumps into the increments.
11. Accumulated the increments to simulate stock price paths.
12. Transformed the simulated paths to geometric Brownian motion.
13. Plotted sample stock price paths and the mean path.
14. Plotted the distribution of stock prices at maturity.
15. Fit and plot a log-normal distribution curve on the histogram of final stock prices.

Below are the results from the Black Scholes option pricing:

Firstly, parameters such as maturity, spot price, strike price, risk-free interest rate, dividend rate, volatility, jump intensity, expected jump size, and standard deviation of jump size are defined. Then Monte Carlo simulation was run for each block and stock price paths were generated incorporating jump sizes.

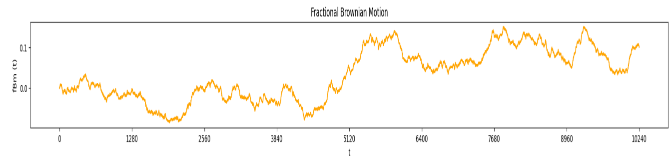


Fig 3 : Brownian Motion time series for Nifty50 Index

The above graph represents a path generated from the Brownian time motions series.

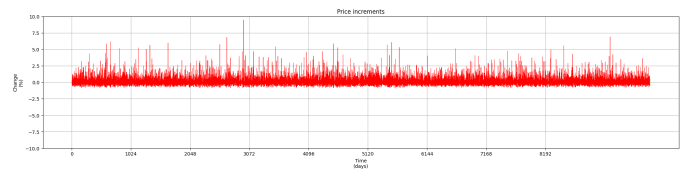


Fig 4 : Price Fluctuation for Brownian Motion time series over Nifty50 Index

The above graph represents the returns generated from various brownian motion paths.

Option	Price
Call	66.46
Put	151.28

Table 4 : Estimated option prices using Jump diffusion method

The above table represents the call and put option price after running the Jump diffusion model.

d) MMM (Mandelbrotian Multifractal Model) generated paths.

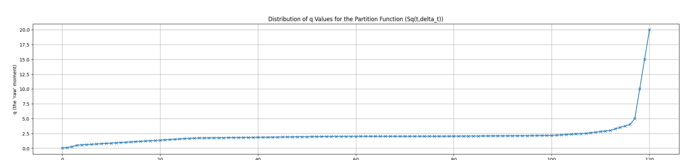
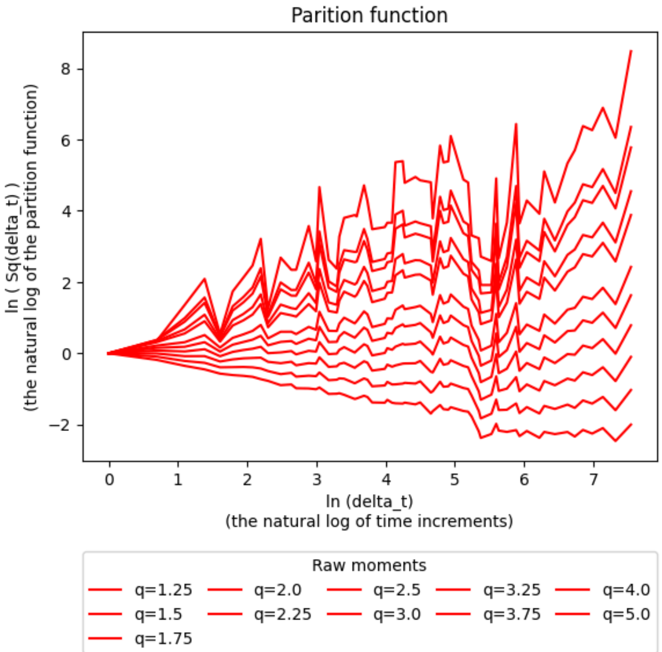


Fig 5 : Distribution of q values of partition function for MMM



The partition function seems to show scaling behaviour.This suggests that the price movements are fractal.The graph has also been normalized to start at zero to make the scaling behavior easier to see.

Fig 6 : Graph for Partition function applied using Mandelbrotian techniques based option price estimation

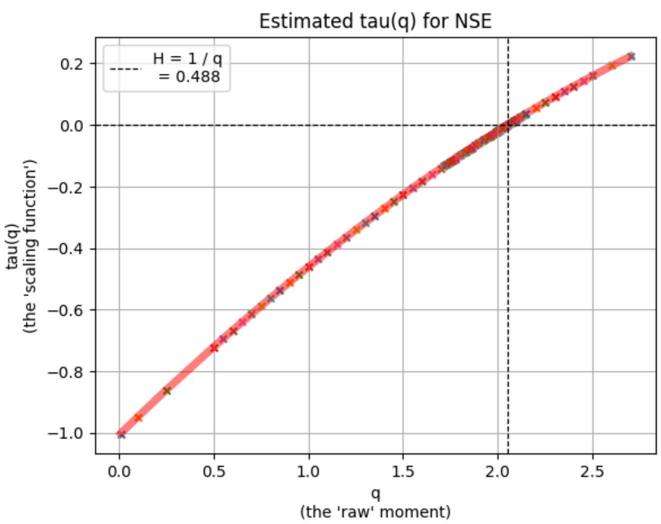


Fig 7 :Estimated Tau(q) for NSE for Mandelbrotian based option price estimation

We re-estimate the scaling function to have it in its continuous form and use it to determine H which is a key parameter in the Mandelbrotian techniques.

For NSE, we estimate

Params	Estimated Value
λ	1.1158713625139898
σ^2	0.3343340801599607

Table 4 : Estimated statistics using Mendalbrotian (MMM) method

Option	Price
Call	69.92
Put	104.02

Table 5 : Estimated option prices using Mendalbrotian (MMM) method

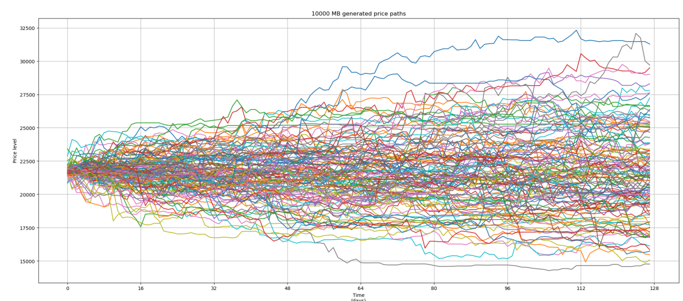


Fig 8 : Average Price over 100 generated price paths using MMM method

Simulating 10000 MMM price paths for NSE data.

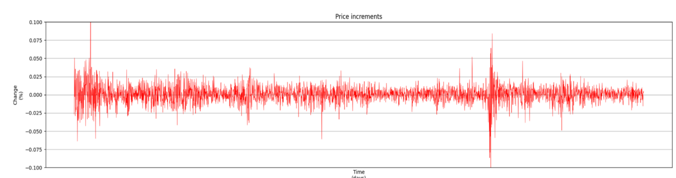


Fig 9: Price increments for NSE data in time period 2008-2024

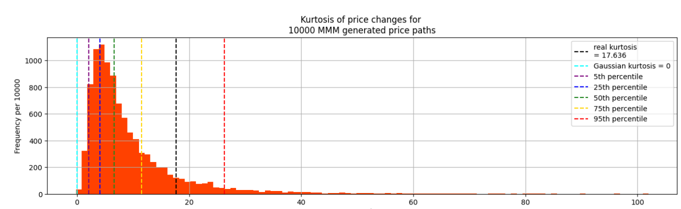


Fig 10: Kurtosis of price changes for 10000 paths generated for MMM based option price estimation.

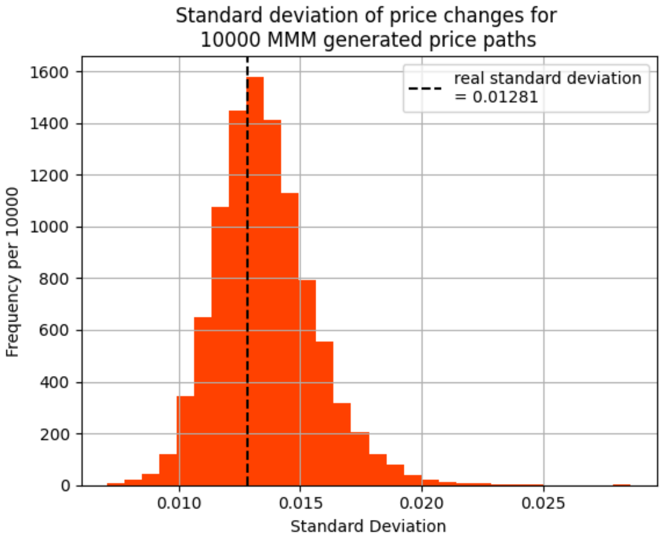


Fig 11: Standard Deviation (Volatility) of Price fluctuations using 10000 generated price paths using Mandelbrotian methods.

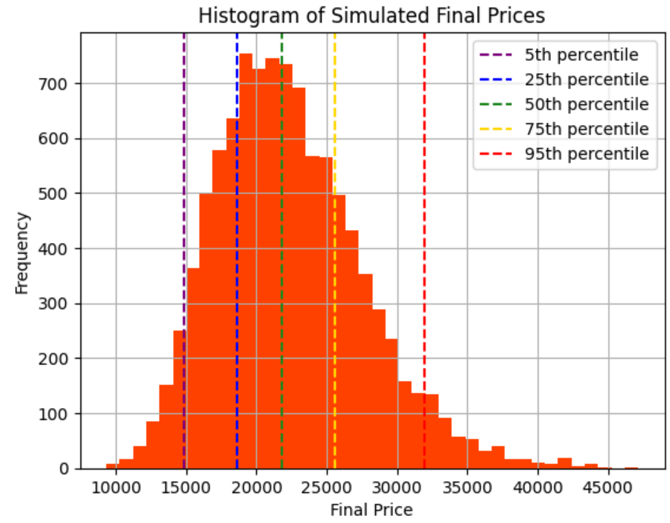


Fig 13: The histogram of the standard deviations appears to be converging to a clear, smooth bell curve.

We can see that,in general MMM tends to produce kurtosis levels that are smaller than that of real data. However, practically it never gets close to zero. Data with higher kurtosis produces a wider variety of simulations.

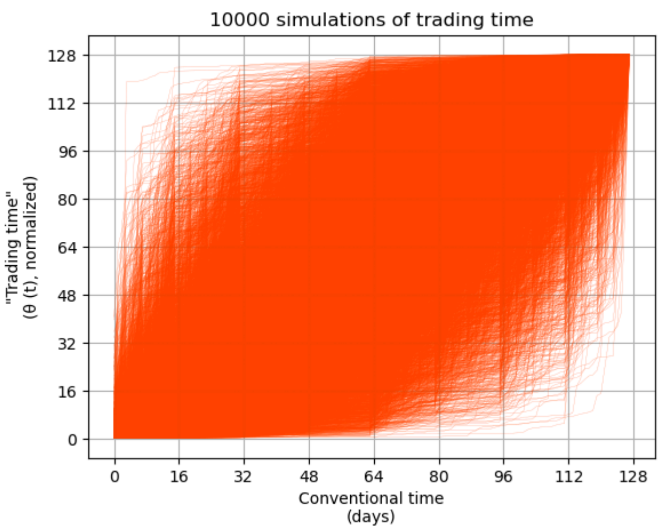


Fig 12: 10000 simulations of trading time over MMM based option pricing method

Stats	Estimated Values
count	3781.000000
mean	0.000549
std	0.011737
min	-0.139038
25%	-0.005112
50%	0.000741
75%	0.006534
max	0.163343

Table 6 : Estimated statistics for price change (Nifty50 Index) using Mendalbrotian method

Based on above, the fractional Brownian motion graph [Fig-10] shows the long-dependence properties observed in asset price movements. The MMM-generated paths explain how multifractality influences market behavior, capturing the complex correlations between various scales of fluctuations.

Finally, the price increment graph[Fig-3] based on MMM shows the evolution of asset prices over time, highlighting the multifractal nature of market dynamics.

V. Discussion

Model of option Pricing	Call Price	Put Price
Binomial Option Pricing	64.80	148.90
Black Scholes Model	66.20	150.40
Merton Jump Diffusion Model	66.46	151.28
Mandelbrotian Multifractal Model	69.92	104.02

We have compared four methods comparing option pricing techniques majorly used to find arbitrage opportunities. We observed that the mandelbrotian approach (MMM) differs from the other three approaches in terms of estimating option prices due to its fractal component of generating price estimates. As observed, we have used Nifty50 Index data of the last 16 years [2008-2024] and mapped the spot prices with the at-the-money strike prices to estimate how these four approaches are different over the real market prices.

While applying the approach, binomial option prices had wider price variations and high kurtosis of 4.5 compared to other three methods making it. Binomial and Black-Scholes are more computationally intensive compared to MMM, making it inefficient to find arbitrage opportunities and exploit market prices. As the results show, Black Scholes method has higher variation in call-put option prices due to its log-normal and constant volatility assumption, Thus, it fails to capture prices accurately.

The jump diffusion model addresses one of the shortcomings of the Black Scholes model by taking into account jumps or sudden price movements. This makes it more suitable for pricing options on illiquid assets or assets with high variations. However, it still struggles to capture the long-term memory and fractal features present in market dynamics.

Finally, we observed MMM statistics which uses fractals and self similarity to simulate option price for Nifty50 data. Based on the given results, it better captured high volatility or higher risk scenarios and regime changes especially for long-dated option expiries which other listed methods cannot estimate.MMR thus proved a better way of calculating option prices considering the nature of the market.

VI. Conclusion

As seen in the research paper, there are multiple challenges that the traditional option models face. Models such as Black scholes and Binomial Option pricing generally don't capture the complex nature of financial markets and fail to take into account factors such as fat tailed distributions of returns and long-term memory.

One of the most significant ways in which the Mandelbrotian Multifractal Model (MMM) differs from other models is that it offers a complete framework for simulating and modeling the complex dynamics observed in financial markets. Instead of neglecting important characteristics such as fat-tailed distributions, volatility clustering and long-term memory as many traditional models do, MMM attempts to achieve a more realistic representation of market behavior by utilizing fractal geometry and self-similarity.

To effectively simulate non-random walks and accommodate multifractality in asset price movements, MMM introduces such building blocks as log-normal multiplicative cascade, trading time function and fractional Brownian motion. This will improve option pricing, risk management, and trading strategies by better catching the actual market dynamics.

The financial sector is changing day by day and the regulators are also pushing for stringent risk measures and a robust risk assessment. Thus MMR can become increasingly important for a robust risk assessment as it can better model options by considering fat tailed distributions and long term memory of the market. The Mandelbrotian fractal model will also help traders in getting a more accurate picture of the market. MMR can be further refined by combining with the Machine Learning models. This would further enhance the accuracy of the model.

The Mandelbrotian Multifractal Model of Asset Returns is a significant improvement in pricing of options. Our research paper has used MMR along with Black scholes, Merton Jump Diffusion and Binomial model on Nifty 50 and MMR is promising in calculating option prices. It promises to offer a better alternative to traditional models thus opening up new areas for further research as well as practical applications. Thus, combining traditional methods with MMR methods can boost model performance and efficacy in real market scenarios.

VII. References

[1] Shinde, A. (2013). Study of Black-Scholes Model and its applications ResearchGate. https://www.researchgate.net/publication/257725235_Study_of_Black-Scholes_Model_and_its_Applications

[2] Kou, S. G. (2002). A Jump-diffusion model for option pricing. ResearchGate. https://www.researchgate.net/publication/299116401_A_Jump-diffusion_model_for_option_pricing

[3] Loffe, Mark (2014). BINOMIAL OPTIONS PRICING MODEL. ResearchGate. https://www.researchgate.net/publication/266376142_BINOMIAL_OPTIONS_PRICING_MODEL

[4] Moses Todd (2021). Mandelbrot's Multifractal Model of Asset Returns Explained. Medium.

https://medium.datadriveninvestor.com/mandelbrots-multifractal-model-of-asset-returns-explained-bde7b5121be0

[5] Moses Todd (2021). Implications of a Multifractal Model of Stocks on Options Pricing. Medium. https://medium.datadriveninvestor.com/implications-of-a-multifractal-model-of-stocks-on-options-pricing-741ca1a9eb8e

[6] Wengert, C., 2010. Multifractal Model of Asset Returns (MMAR). https://www.mathworks.com/matlabcentral/fileexchange/29686-multifractal-model-of-asset-returns-mmar

[7] Rostislav Sibirtsev from his 2019 essay

"How to Avoid Bankruptcy: Monte Carlo Simulation of Three Financial Markets, using the Multifractal Model of Asset Returns." -

[8] Mandelbrot, B. B. (1997). Fractals and scaling in finance: Discontinuity, concentration, risk. Springer Science & Business Media.

[9] Mandelbrot, B. B., Fisher, A., & Calvet, L. (1997). A multifractal model of asset returns. Cowles Foundation Discussion Paper, 1164.

[10] Calvet, L., & Fisher, A. (2002). Multifractality in asset returns: theory and evidence. Review of Economics and Statistics, 84(3), 381-406.

[11] Muzy, J. F., Delour, J., & Bacry, E. (2000). Modeling fluctuations of financial time series: from cascade process to stochastic volatility model. The European Physical Journal B-Condensed Matter and Complex Systems, 17(3), 537-548.

[12] Lux, T. (2008). The Markov-switching multifractal model of asset returns: GMM estimation and linear forecasting of volatility. Journal of Business & Economic Statistics, 26(2), 194-210.

[13] Bianchi, S. and Pianese, A., 2007. Modeling stock price movements: multifractality or multifractionality?. Quantitative Finance, 7(3), pp.301-319.

[14] Kantelhardt, J. W., Zschiegner, S. A., Koscielny-Bunde, E., Havlin, S., Bunde, A., & Stanley, H. E. (2002). Multifractal detrended fluctuation analysis of nonstationary time series. Physica A: Statistical Mechanics and its Applications, 316(1-4), 87-114.

[15] Ghashghaie, S., Breymann, W., Peinke, J., Talkner, P., & Dodge, Y. (1996). Turbulent cascades in foreign exchange markets. Nature, 381(6585), 767-770.

VIII. Appendix

Code : https://github.com/Aarvin01/GWP/tree/main