Assignment -04

On Non Linear Ordinary Differential Equations

PH1050 Computational Physics

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Introduction

Non Linear differential Equations are often non integrable by hand. In such cases we numerically solve the equations. Our goal is to analyse a particular Potential function and see the variance of Time period with total energy. We later see the case of a driven and damped oscillator and plot it’s position with respect to time. Using graph we can clearly see the change from transient state to steady state.

Aim

1) Plot the Dependence of Time period on total energy
2) Deriving the equation of motion with respec to time of a damped and driven oscillator

Code

In[114]:=

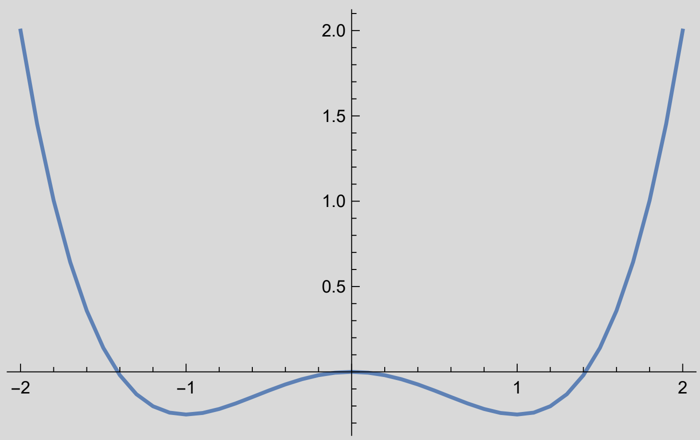
```
Clear["Global`*"]
Potential[x_] = x^4 /4 - x^2 /2
PotentialPlotData = Table[{x,Potential[x]}, {x,-2,2,0.1}];
Print["The plot of Potential energy"]
ListLinePlot[PotentialPlotData]
```

Out[115]=

$$-\frac{x^2}{2}+\frac{x^4}{4}$$

The plot of Potential energy

Out[118]=



In[∘]:=

```
totEnergy[x_] := m / 2 (dx / dt) ^2 + Potential[x] == energy
totEnergy[x];
dTsol = Solve[totEnergy[x], dt];
dTsol1 = dTsol /. {{{x0_}, {y0_}} → {x0, y0}};
dTime = dTsol1[[2]] /. {m → 1};
Print["The integral for time is"]
dt /. dTime
```

The integral for time is

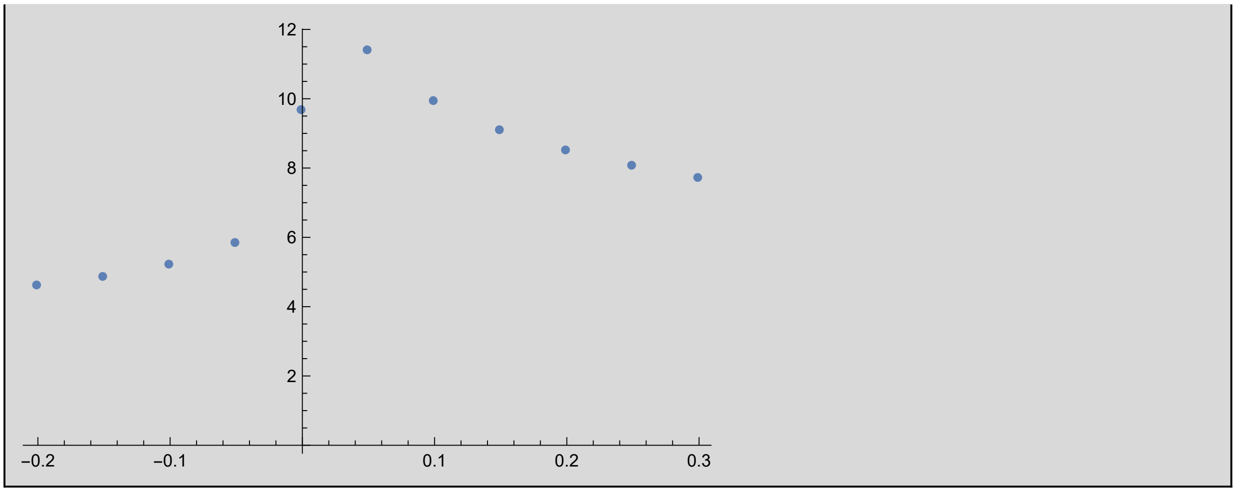
Out[∘]=

$$\frac{\sqrt{2}\ dx}{\sqrt{4\ energy+2\ x^2-x^4}}$$

In[119]:=

```
data = {};
For[i = -0.201, i ≤ 0.301, i = i + 0.05,
 ExtreamasNegative = SolveValues[Potential[x] == i, x, Reals][[{1, 2}]];
 TimePeriod2 =
  2 NIntegrate[√2 / √(4 i + 2 x^2 - x^4), {x, ExtreamasNegative[[1]], ExtreamasNegative[[2]]}];
 AppendTo[data, {i, TimePeriod2}]]
ListPlot[data, PlotRange → Automatic]
```

Out[121]=



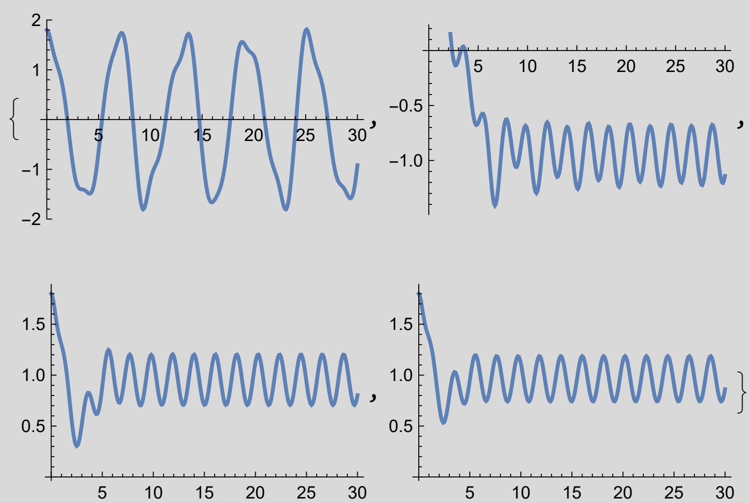
In[122]:=

```
Force[x_] = - D[Potential[x], {x, 1}]
plts = {};
For[i = 0, i ≤ 3, i++,
 eqn := {x''[t] == Force[x[t]] + A Sin[2 ω t] - γ x'[t] ,
    x[0] == 18 / 10, x'[0] == 0 } /. {A → 2, ω → 1.5 , γ → 6 * i / 10};
 soln = NDSolve[eqn, x[t], {t, 0, 100}];
 data = Table[{t, x[t] /. Flatten[soln]} , {t, 0, 30, 0.1}];
 plt = ListLinePlot[data];
 AppendTo[plts, plt];
]
plts
```

Out[122]=

$x - x^3$

Out[125]=



Conclusions

-For Negative total energy, the time period increases, then it starts decreasing
-It would take infinite time for 0 energy as the object would just reach origin
We can also say that near 0, x^2 term will dominate and hence no attractive force towards minima

-The damping helps the oscillator reach steady state, hence for greater γ it reaches steady state much early
- When gamma is not too large, the particle crosses the origin and oscillates below X-axis, but for large gamma, the partical can never reach the origin