Assignment 09

On Probability Distribution and Monte Carlo Method

PH 1050 Computational Physics

Aarya Gosar (EP23B025)
Engineering Physics
25th Oct 2023

Introduction

Monty Carlo is a way to numerically solve the problem using random numbers and exploiting the central limit theorem to get a precise approximation of the solution. In this assignment we first study how to create random numbers and their various distributions. In the second part we use the Monte Carlo method to integrate and check validity of CLT.

Aim

Part 1

1) Creating pseudorandom numbers and seeing their distribution.
2) computing statistical values of two different distribution.
3) Verifying CLT for a non-normal distribution.

Part 2

1) Computing the value of Pi
2) Integrating single variable function
3) integrating multivariable function

Code

Part1 Probability Distribution

Creating PseudoRandom numbers

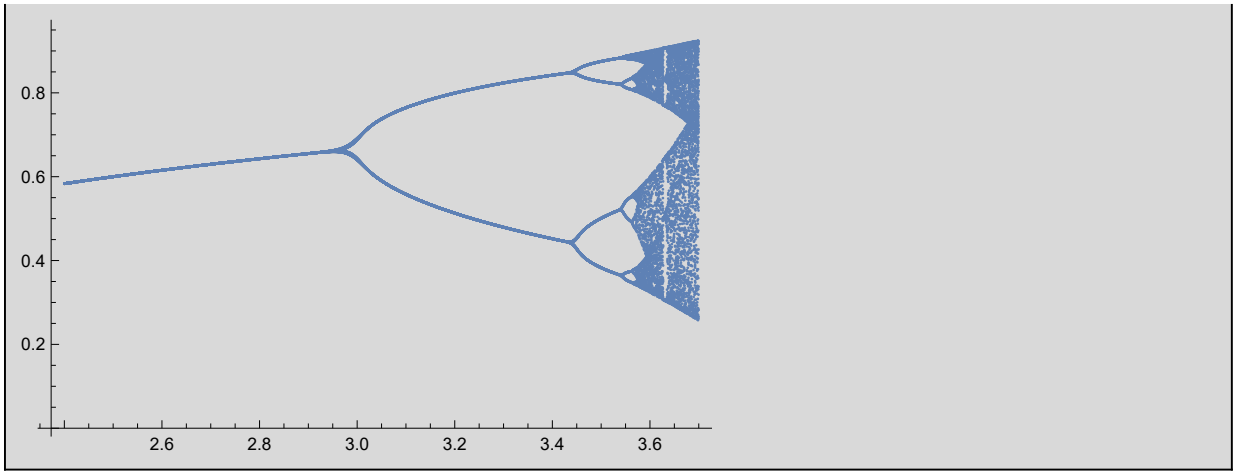
We can see that for a bifurcation model, we get pseudorandom numbers for large R

In[1]:=
```
P={x0};
x0=0.1;
n=100;
maxBi=30;
model[x_]=R (x-x^2);

Data={}
For[R=2.4,R<3.7,R=R+0.0005,P={x0};
For[i=2,i≤n,i++,AppendTo[P,model[P[[i-1]]]];
For[i=0,i<maxBi,i++,AppendTo[Data,{R,P[[n-i]]}]];]
ListPlot[Data]
```

Out[6]=
```
{}
```

Out[8]=

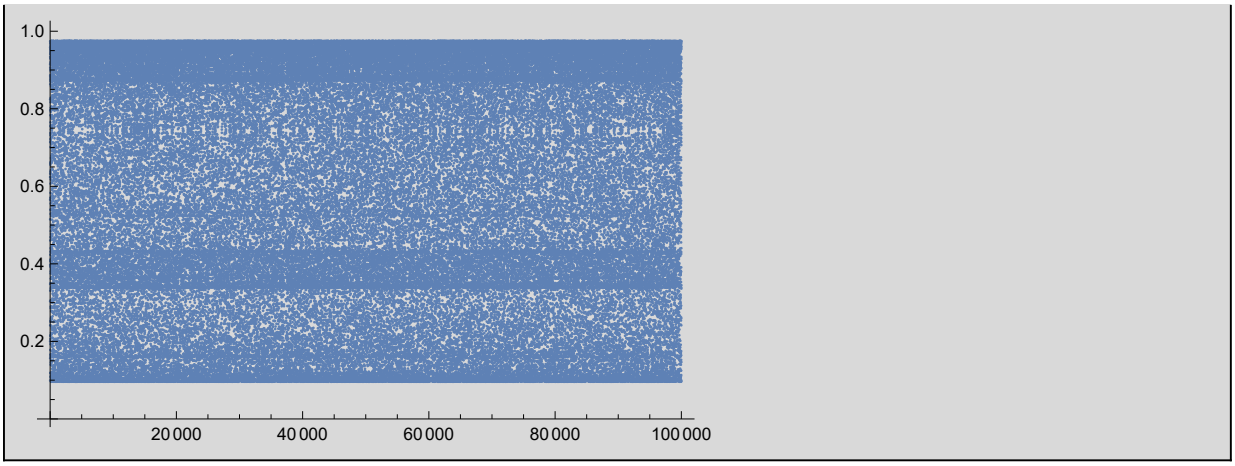


Using this chaotic behaviour at a large R

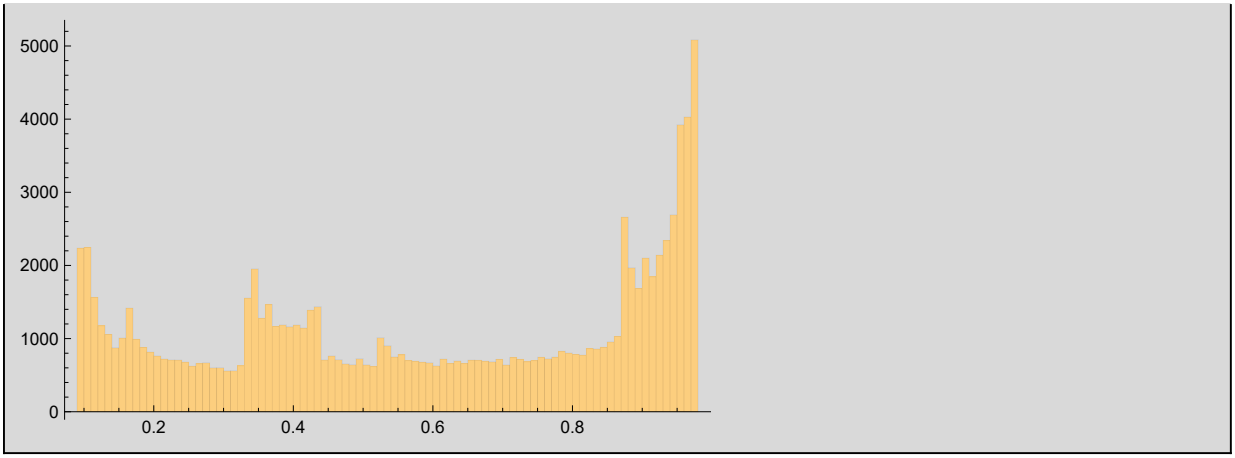
In[15]:=
```
R = 3.899;
model[x_] = R (x - x^2);
x0 = 0.2;
P = {x0};
n = 100 000;
For[i = 2, i ≤ n, i++, AppendTo[P, model[P[[i - 1]]]];
```

In[21]:=
```
ListPlot[P]
Histogram[P, 100]
```

Out[21]=



Out[22]=

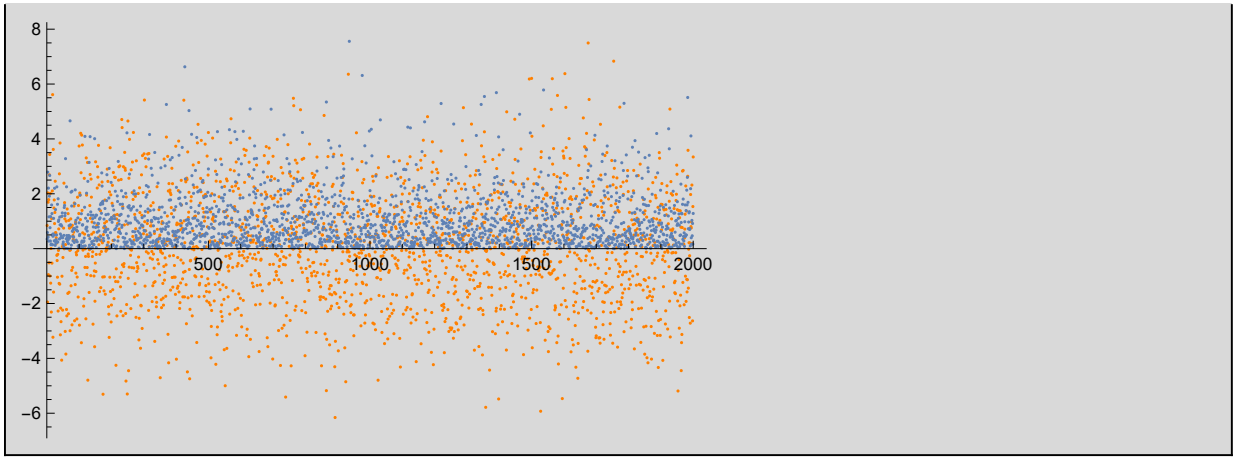


This is not a Uniform distribution, this model has tendencies to go towards certain values, but they are still pseudorandom
(as we can see there are bands forming in the listplot, it would be interesting to see why those bands exist)

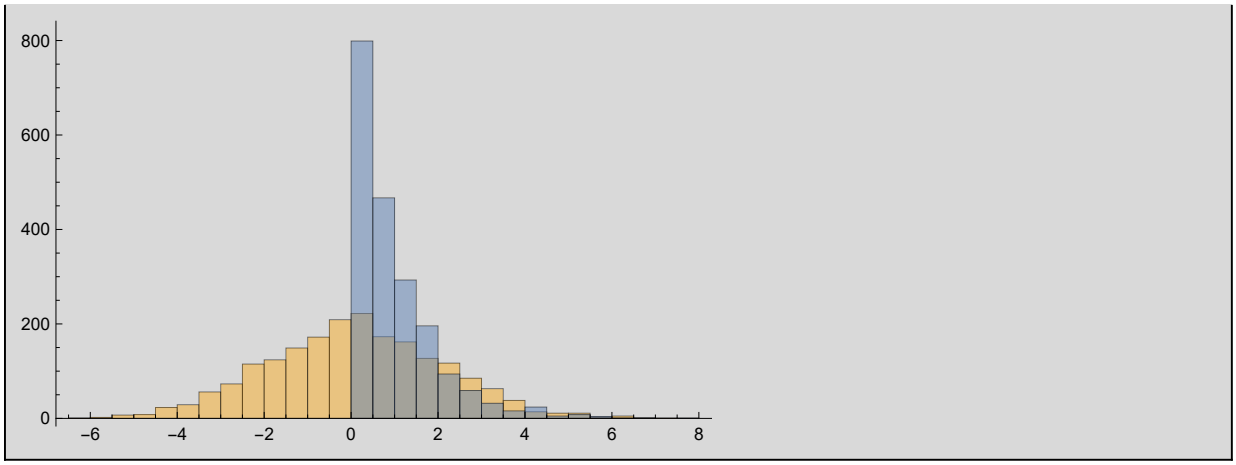
Statistical measures for two different distribution

In[53]:=
```
datExp = RandomVariate[NormalDistribution[0,2],n];
datExp2 = RandomVariate[ExponentialDistribution[1],n];
Show[{ListPlot[datExp, PlotStyle→Orange],ListPlot[datExp2]}]
Histogram[{datExp,datExp2}]
```

Out[55]=



Out[56]=



Means

In[57]:=
```
Mean[datExp]
Mean[datExp2]
```

Out[57]=
```
0.0802406
```

Out[58]=
```
0.983997
```

Variance

In[59]:=
```
Variance[datExp]
Variance[datExp2]
```

Out[59]=
```
4.03955
```

Out[60]=
```
0.947836
```

In[61]:=
```
Covariance[datExp, datExp2]
```

Out[61]=
```
-0.0015145
```

Skewness

In[62]:=
```
Skewness[datExp]
Skewness[datExp2]
```

Out[62]=
```
0.0542331
```

Out[63]=
```
1.94367
```

kurtosis

In[64]:=
```
Kurtosis[datExp]
Kurtosis[datExp2]
```

Out[64]=
```
3.01648
```

Out[65]=
```
8.12456
```

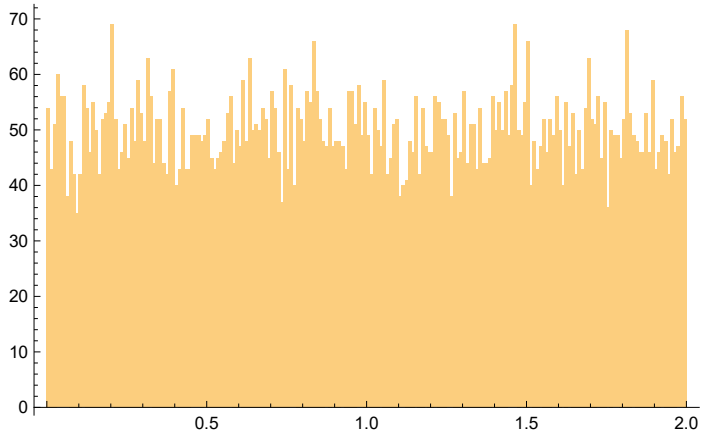
Verifying Central Limit theorem

We will use uniform distribution.

In[27]:=
```
For[ i = 1 , i ≤ 3, i++,
 avgs = {};
 For[j = 1, j ≤ 10000, j++,
  AppendTo[avgs, Mean[RandomReal[{0, 2} , i]]]
 ];
 Print["i = "];
 Print[i];
 Print[Histogram[avgs, {0, 2, 0.01}]]
]
```

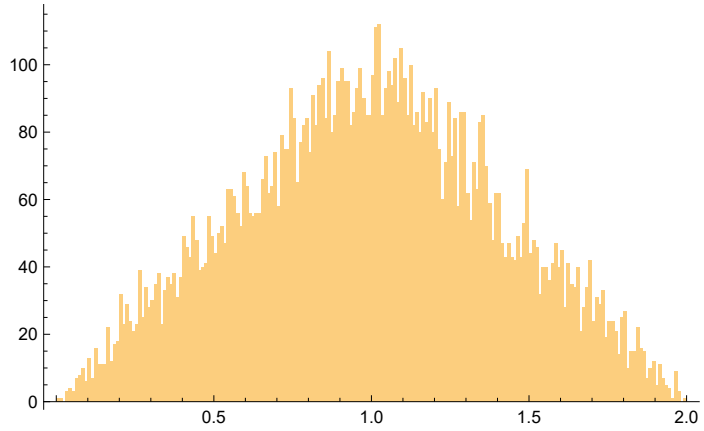
i =

1


70
60
50
40
30
20
10
0
0.5
1.0
1.5
2.0

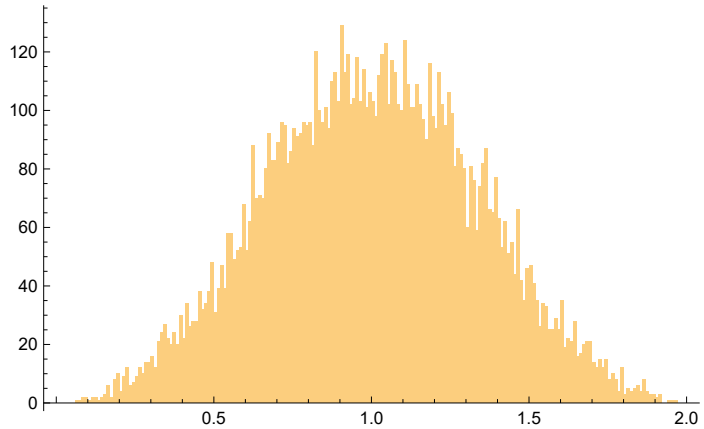
i =

2


100
80
60
40
20
0
0.5
1.0
1.5
2.0

i =

3


120
100
80
60
40
20
0
0.5
1.0
1.5
2.0

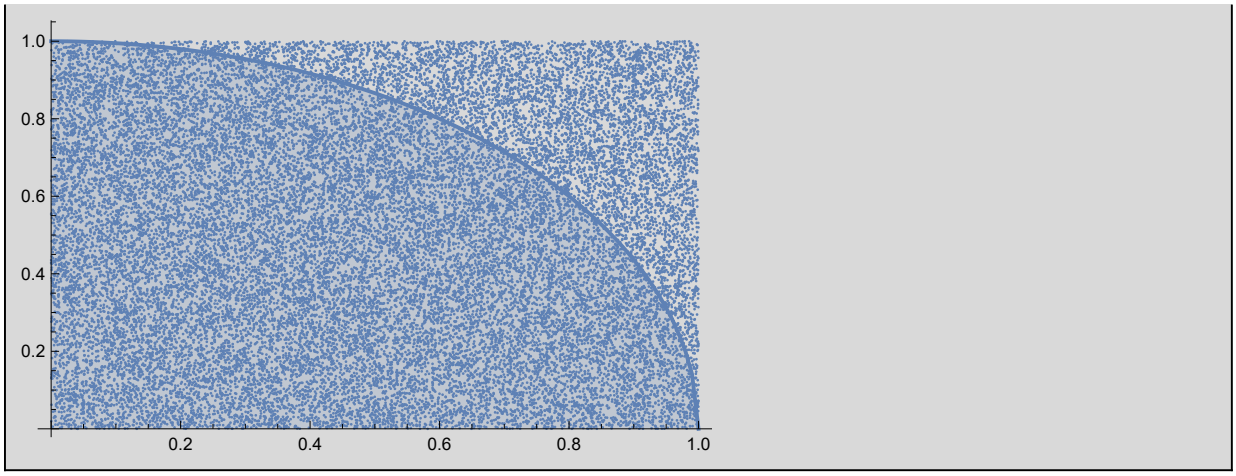
Part 2 Using Monte Carlo Method

For Computing π

In[28]:=
```
Clear["Global`*"]
pltCircle = ListLinePlot[Table[{x,Sqrt[1-x^2]} , {x,0,1,0.01}], Filling→Bottom];
pts = {};
a = 0;
b = 0;
For[i = 1 , i<30000, i++ ,

x = RandomReal[{0,1}];
y = RandomReal[{0,1}];
b = b + 1;
If[x^2 + y^2 ≤ 1, a = a + 1;];
AppendTo[pts,{x,y}];
]
pltPoints = ListPlot[pts];
Show[{pltPoints , pltCircle}]
4 a / b // N
```

Out[35]=



Out[36]=
```
3.14437
```

This value is close enough to 3.141

For Integrating single variable function

The function I have used is

In[37]:=
```
Clear["Global`*"]
a = 1;
b = 3;
n = 2000;
f[x_] =Exp[-3 x ] Cos[x]/(1+Sinh[2*x]*(Log[x])^2)
```

Out[41]=
$$\frac{e^{-3x}\,\text{Cos}[x]}{1+\text{Log}[x]^2\,\text{Sinh}[2x]}$$

The actual value of integration is

In[42]:=
```
NIntegrate[f[x] , {x,a,b}]
```

Out[42]=
```
0.00415512
```

Using Monte Carlo

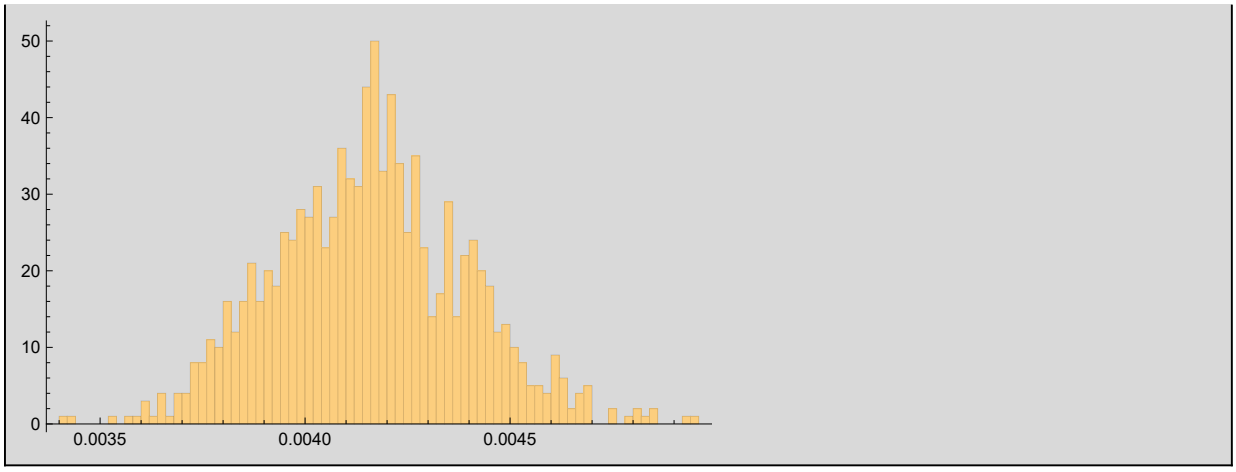
In[43]:=
```
sum = 0;
For[j = 1, j ≤ n, j++,
 sum = sum + (b-a) * f[RandomReal[{a,b}]]/ n
];
sum
```

Out[45]=
```
0.0042621
```

Verifying CLT For this by repeating it 1000 times

In[ ]:=
```
avgs = {};
For[i=1, i ≤ 1000, i ++ ,
sum = 0;
For[j = 1, j ≤ n, j++,
sum = sum + (b-a) * f[RandomReal[{a,b}]]/ n
];
AppendTo[avgs, sum]
]
Histogram[avgs, 100]
```

Out[ ]=



The peak should give predicted value

In[ ]:=
```
h = HistogramList[avgs, 100];
h[[1]][[Flatten[Position[h[[2]],Max[h[[2]]]]][[1]]] // N
```

Out[ ]=
```
0.00416
```

This is a more accurate value

Doing the same for Multivariable

The Function is

In[46]:=
```
Clear["Global`*"]
f[x_, y_] = Log[1 + x^2 + y^2] Exp[π/(1 + x)] Sin[y]
a = 2;
b = π;
n = 2000;
```

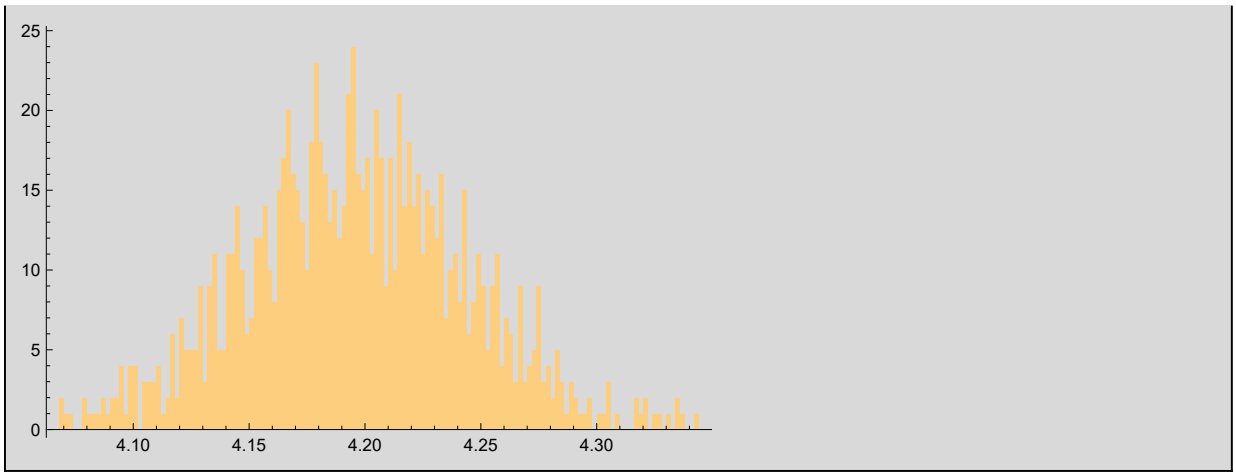
Out[47]=

$$e^{\frac{\pi}{1+x}} \operatorname{Log}\left[1 + x^2 + y^2\right] \operatorname{Sin}[y]$$

In[•]:=
```
avgs = {};
For[i = 1, i ≤ 1000, i ++ ,
 sum = 0;
 For[j = 1, j ≤ n, j++,
  sum = sum + (b - a)^2 * f[RandomReal[{a, b}] , RandomReal[{a, b}]] / n
 ];
 AppendTo[avgs, sum]]
```

In[•]:=
```
Histogram[avgs, 100]
```

Out[•]=



In[•]:=
```
h = HistogramList[avgs, 100 ];
h[[1]][[Flatten[Position[h[[2]], Max[h[[2]]]]][[1]]] // N
```

Out[•]=
```
4.194
```

The actual value of integration is

In[•]:=
```
NIntegrate[f[x, y] , {x, a, b}, {y, a, b}]
```

Out[•]=
```
4.195
```

Conclusion/Comments

1) CLT is valid for these MonteCarlo integrations

2) The Equation we used is similar to that of Rienmann Sum. But taking Random values helps us to exploit CLT

3) In Rienmann sum, we take steps at regular interval. This is why we need to take random values in Uniform Distribution for MonteCarlo