Assignment 08:

On Response of Linear systems to arbitrary inputs

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Engineering Physics

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Introduction

Circuits are often used to transmit signals. In this assignment we look at Output signal of various input signals in RC and RLC Circuits. NDSolve gives us ability to analyse piecewise inputs without having to compute them by hand.

RC circuit

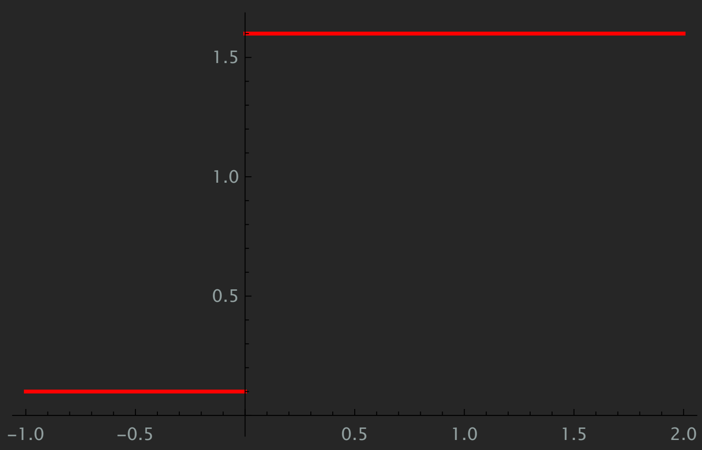
Step input

In[110]:=

```
Clear["Global`*"]
Vin[t_] := 1.5 HeavisideTheta[2 t] + 0.1
Print["Input"]
pltin = Plot[Vin[t], {t, -1, 2}, PlotStyle → Red]
r = 100;
c = 10^(-3);
τ = r * c;
eqn = v'[t] * τ + v[t] == Vin[t];
Vo = NDSolve[{eqn, v[0.001] == 0}, v, {t, 0, 2}]
plto = Plot[Evaluate[v[t] /. Vo], {t, 0.3, 2}]
```

Input

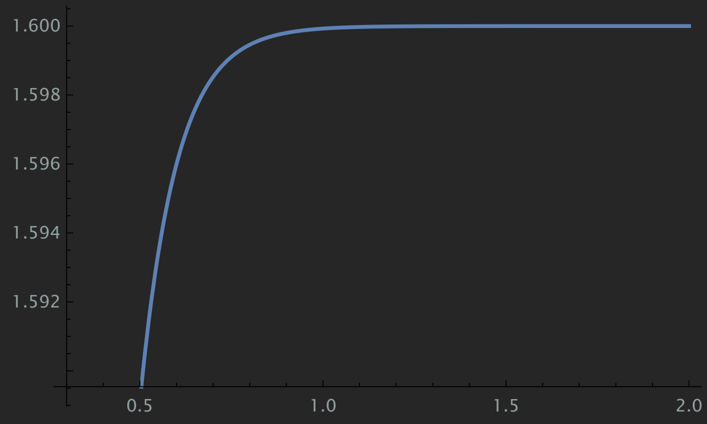
Out[113]=



Out[118]=

{{v → InterpolatingFunction[Domain: {{2.84×10^{-17}, 2.}} Output: scalar]}}

Out[119]=



Piecewise input

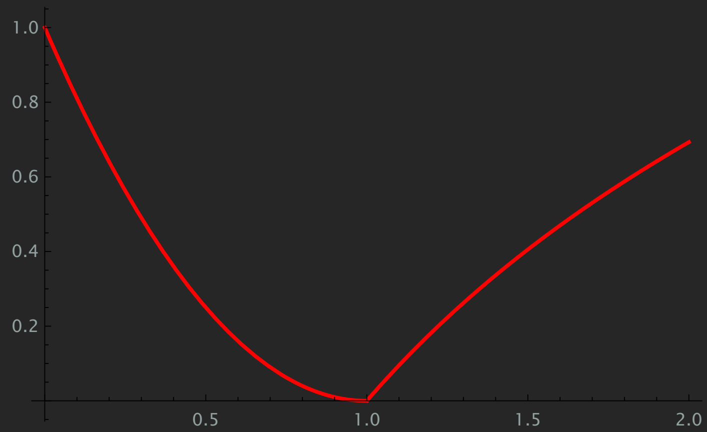
In[142]:=

```
Clear["Global`*"]
Vin[t_] = Piecewise[{{(t - 1)^2, 0 < t < 1 }, {Log[t], t ≥ 1}}]
pltin = Plot[Vin[t], {t, 0, 2}, PlotStyle → Red]
eqn = v'[t] * 0.1 + v[t] == Vin[t]; /. τ → 0.1
Vo = NDSolve[{eqn, v[0] == 0}, v, {t, 0, 2, 0.1 }]
plto = Plot[Evaluate[v[t] /. Vo], {t, 0, 2}];
Show[{pltin, plto}]
```

Out[143]=

$$\begin{cases} (-1+t)^2 & 0 < t < 1 \\ \text{Log}[t] & t \geq 1 \\ 0 & \text{True} \end{cases}$$

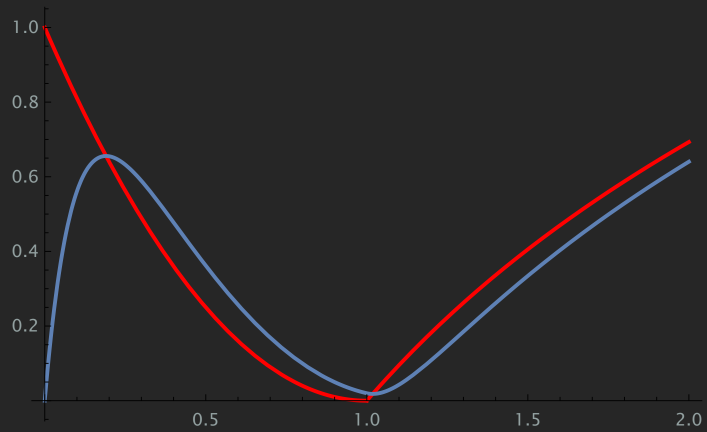
Out[144]=



Out[146]=

{{v → InterpolatingFunction[Domain: {{0., 2.}} Output: scalar]}}

Out[148]=



Square Pulse

In[154]:=

```
Clear["Global`*"]
Vin[t_] = { 0.6 * SquareWave[t]   t < 0.003`
          { 0                     t > 0.003
eqn = v'[t] * 0.1 + v[t] == Vin[t];
Vo = NDSolve[{eqn, v[0] == 0}, v, {t, 0, 2, 0.001 }]
Plot[Evaluate[v[t] /. Vo], {t, 0, 2}, PlotRange → All, PlotStyle → Green]
```

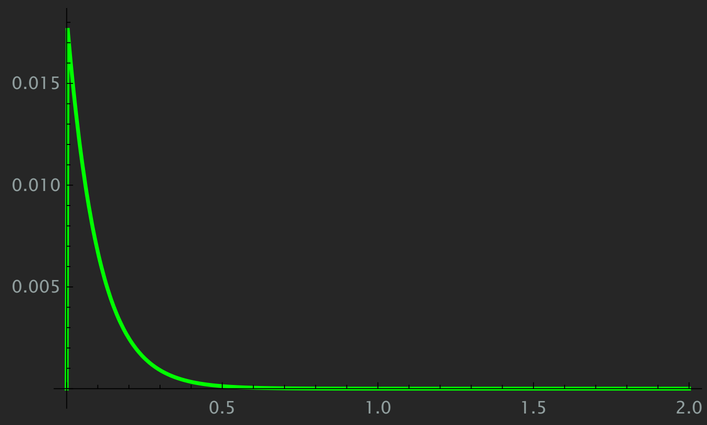
Out[155]=

```
{ 0.6 SquareWave[t]   t < 0.003
{ 0                   True
```

Out[157]=

```
{{v → InterpolatingFunction[ Domain: {{0., 2.}} Output: scalar ]}}
```

Out[158]=



Transfer Function

In[170]:=

```
r = 100;
c = 10^(-3);
H[ω_] = 1/(1 + I ω r c)
RealH[ω_] = 1/(1 + (ω r c)^2)
ImH[ω_] = - ω r c / (1 + (ω r c)^2)
phase[ω_] = -ArcTan[ImH[ω] / RealH[ω]]
Len[ω_] = Sqrt[ImH[ω]^2 + RealH[ω]^2]
Print["Real Part"]
Plot[RealH[ω] , {ω, 0, 30}, PlotRange → All]
Print["Imaginary part"]
Plot[ImH[ω] , {ω, 0, 30}]
Print["Phase"]
Plot[phase[ω] , {ω, 0, 30}]
Print["Absolute value"]
Plot[Len[ω] , {ω, 0, 30}]
```

Out[172]=

$$\frac{1}{1+\frac{i\,\omega}{10}}$$

Out[173]=

$$\frac{1}{1+\frac{\omega^2}{100}}$$

Out[174]=

$$-\frac{\omega}{10\left(1+\frac{\omega^2}{100}\right)}$$

Out[175]=

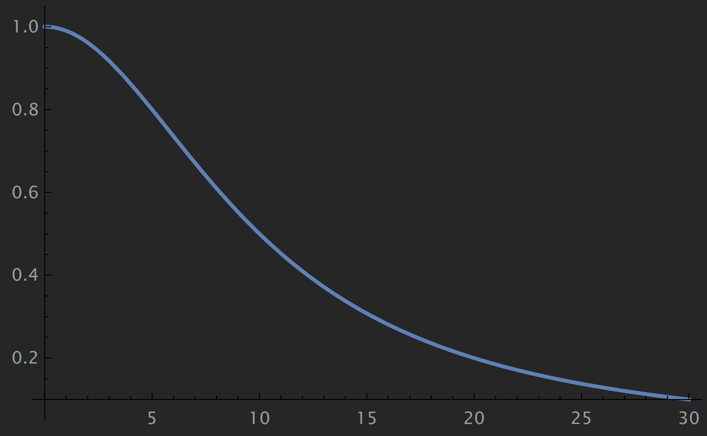
$$\text{ArcTan}\left[\frac{\omega}{10}\right]$$

Out[176]=

$$\sqrt{\frac{1}{\left(1+\frac{\omega^2}{100}\right)^2}+\frac{\omega^2}{100\left(1+\frac{\omega^2}{100}\right)^2}}$$

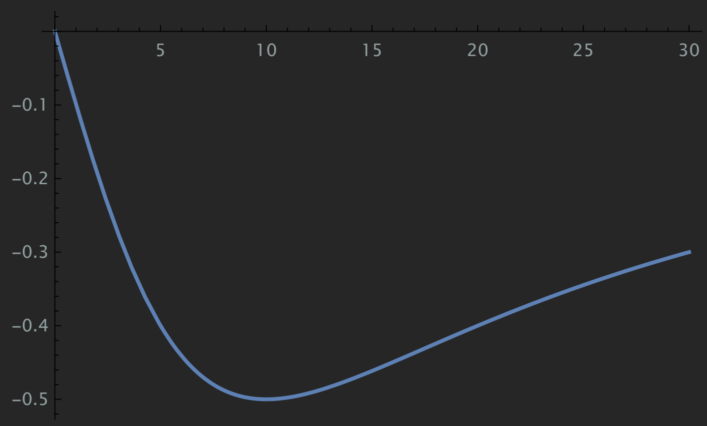
Real Part

Out[178]=



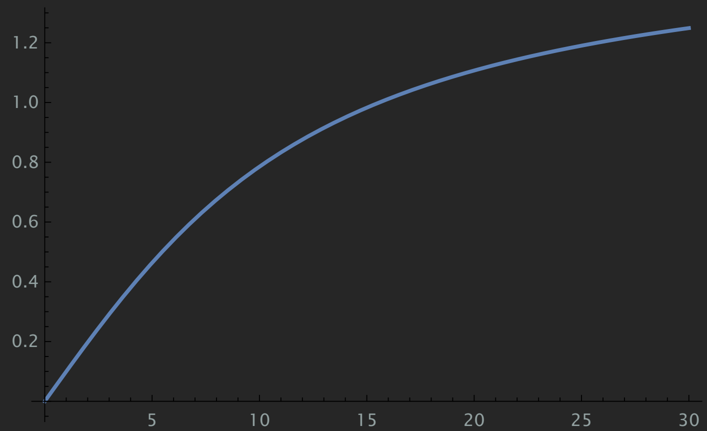
Imaginary part

Out[180]=



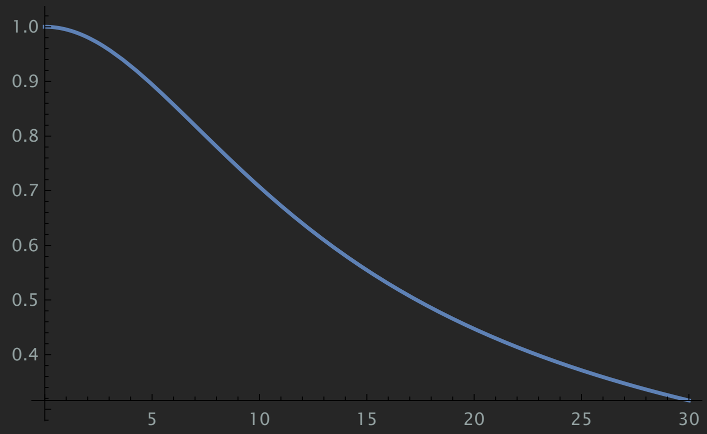
Phase

Out[182]=



Absolute value

Out[184]=



LCR Circuit

Impulse Response

In[191]:=

```
Clear["Global`*"]

Vin[t_] = { 0.6 * SquareWave[t]   t < 0.003`
          { 0                      t > 0.003
eqn = v''[t] + 6 v'[t] + 50 v[t] == 100 Vin[t];
Vo = NDSolve[{eqn, v[0] == 0, v'[0] == 0}, v, {t, 0, 2, 0.001 }]
Plot[Evaluate[v[t] /. Vo], {t, 0, 2}, PlotRange → All]
```

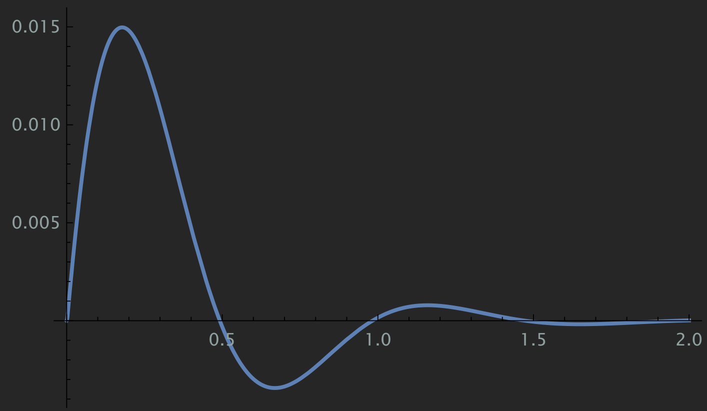
Out[192]=

```
{ 0.6 SquareWave[t]   t < 0.003
{ 0                   True
```

Out[194]=

```
{{v → InterpolatingFunction[ Domain: {{0., 2.}} Output: scalar ]}}
```

Out[195]=



Response to square wave

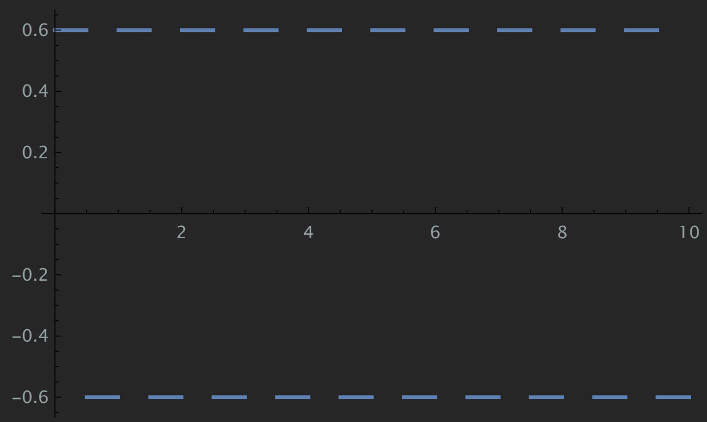
In[196]:=

```
Vin2[t_] = 0.6 SquareWave[t]
Plot[Vin2[t], {t, 0, 10}]
eqn2 = b''[t] + 6 b'[t] + 50 b[t] == 100 Vin2[t]
Vo2 = NDSolve[{eqn2, b[0] == 0, b'[0] == 0}, b, {t, 0, 10, 0.1 }]
Plot[Evaluate[b[t] /. Vo2], {t, 0, 10}]
```

Out[196]=

```
0.6 SquareWave[t]
```

Out[197]=



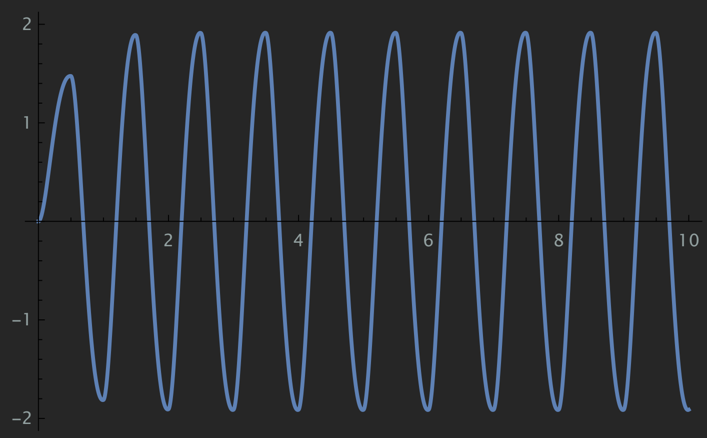
Out[198]=

$50\, b[t] + 6\, b'[t] + b''[t] == 60.\ \mathrm{SquareWave}[t]$

Out[199]=

{{b → InterpolatingFunction[Domain: {{0., 10.}} Output: scalar]}}

Out[200]=



Arbitrary input

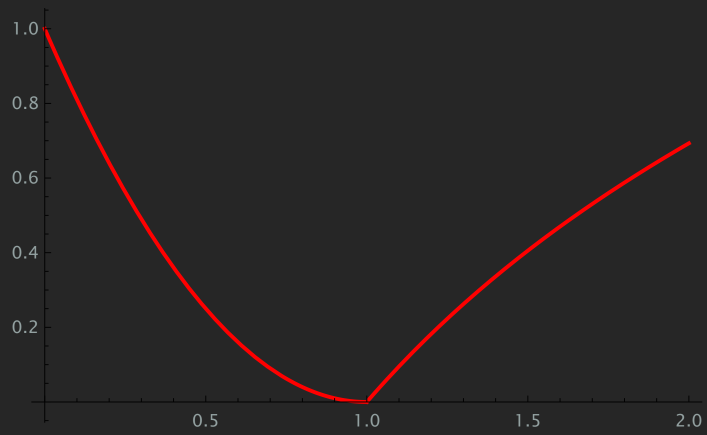
In[206]:=

```
Vin[t_] = Piecewise[{{(t - 1)^2, 0 < t < 1 }, {Log[t], t ≥ 1}}]
pltin = Plot[Vin[t], {t, 0, 2}, PlotStyle → Red]
eqn = v''[t] + 6 v'[t] + 50 v[t] == 100 Vin[t];
Vo = NDSolve[{eqn, v[0] == 0, v'[0] == 0}, v, {t, 0, 2, 0.001 }]
plto = Plot[Evaluate[v[t] /. Vo], {t, 0, 2}, PlotRange → All];
Show[{pltin, plto}]
```

Out[206]=

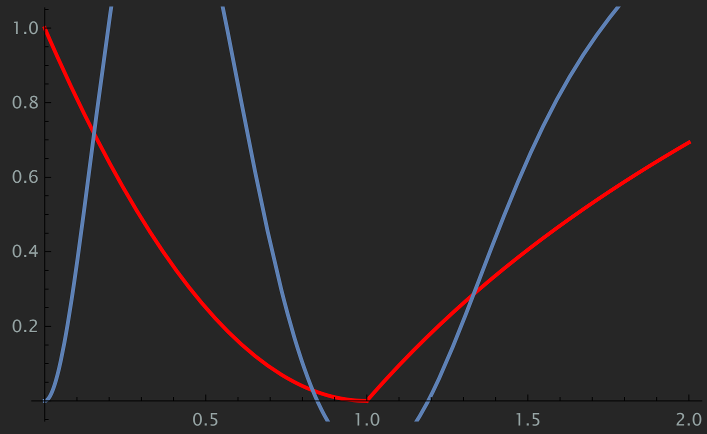
$$\begin{cases} (-1+t)^2 & 0<t<1 \\ \text{Log}[t] & t \geq 1 \\ 0 & \text{True} \end{cases}$$

Out[207]=



Out[209]=

Out[211]=



Comments

I am curious on why the outputs are the way they are. I still don’t get the intuition behind the Impulse Response & Transfer Function. This Assignment motivates me to look into more in depth explanation of digital signals.