On Phase trajectories

Assignment -6

PH1050 - Computational Physics

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Engineering Physics
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Note : I submitted late because I had 20hr train Journey to Pune

Introduction

Graphs comparing velocity and position, also known as Phase Trajectories, are known to give additional information regarding the stability of a dynamic system. In this assignment, we analyse the phase space of a system whose Potential energy is given by a modified gaussian function.

Aim

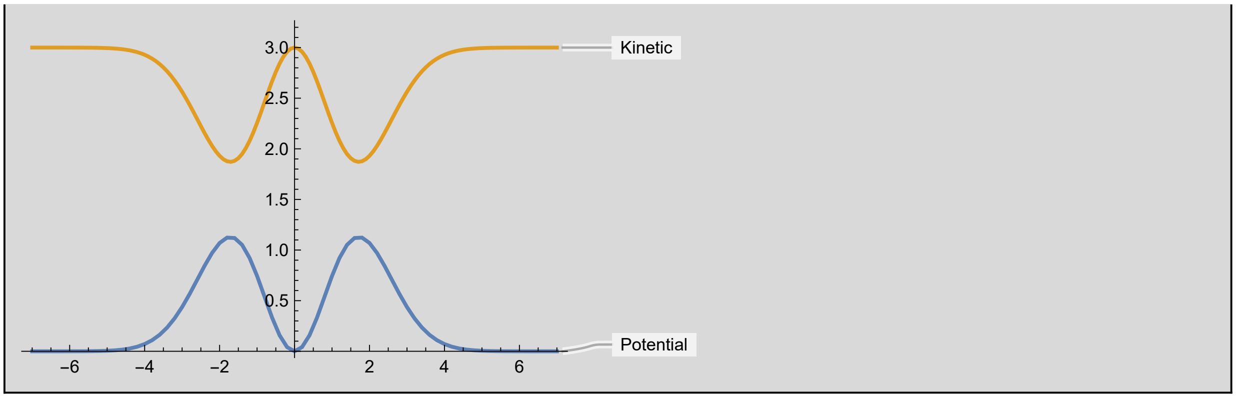
- look at the plot of the potential function
- Draw phase trajectory near critical points using Taylor expansion
- Making the same for entire region without using approximations
- Solving the equation of motion by brute force
- Solving the coupled differential equation

Code

In[432]:=

```
Clear["Global`*"]
U[x_] := a x^2 Exp[-b x^2] /. {a→ π/3, b → 2/(E + π)}
range = 7;
PotData  = Table[{x,U[x]} , {x,-range,range,0.2}];
H [p_,x_] = p^2 / (2 m) + U[x] /. (m→ 1);
KinData = Table[{x,(SolveValues[H[p,x]==3,p][[2]])^2 /2} , {x,-7,7,0.1}];
pltEnergy = ListLinePlot[{PotData,KinData} , PlotLabels→{"Potential"  , "Kinetic"}]
```

Out[438]=



In[439]:=

```
Print["Critical points: "]
Critpoints = SolveValues[D[U[x] , x]  == 0 , x , Reals]
Print[" The taylor expansion at these 3 critical points "]
Taylorseries[x_] = Normal[Series[U[x], {x, #, 2}]] & /@ Critpoints
Taylordata1 = Table[{x, Taylorseries[x][[2]]} , {x, -2.5, -1, 0.1}];
Taylordata2 = Table[{x, Taylorseries[x][[1]]} , {x, -0.5, 0.5, 0.1}];
Taylordata3 = Table[{x, Taylorseries[x][[3]]} , {x, 1, 2.5, 0.1}];
Print["we see that the parabolas can
   approximate the potential energy close to the critical points"]
ListLinePlot[{PotData, Taylordata1 , Taylordata2 , Taylordata3} ,
 PlotRange → {{-7, 7}, {0, 1.5}}]
```

Critical points:

Out[440]=

$$\left\{0, -\sqrt{\frac{e+\pi}{2}}, \sqrt{\frac{e+\pi}{2}}\right\}$$

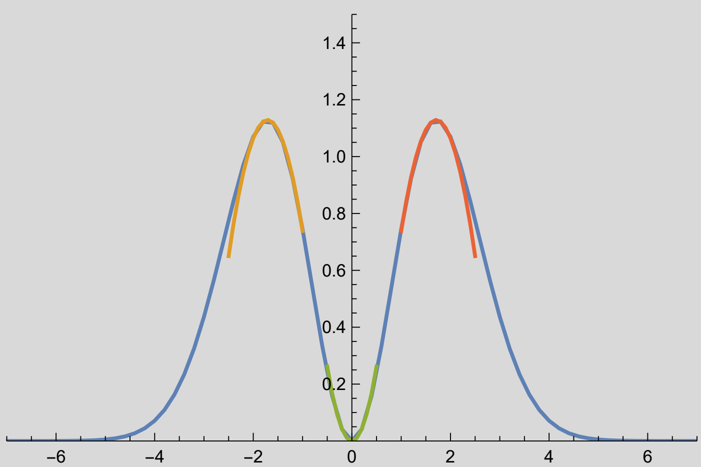
The taylor expansion at these 3 critical points

Out[442]=

$$\left\{\frac{\pi x^2}{3}, \frac{\pi (e+\pi)}{6 e} - \frac{2\pi\left(\sqrt{\frac{e+\pi}{2}}+x\right)^2}{3 e}, \frac{\pi (e+\pi)}{6 e} - \frac{2\pi\left(-\sqrt{\frac{e+\pi}{2}}+x\right)^2}{3 e}\right\}$$

we see that the parabolas can approximate the potential energy close to the critical points

Out[447]=



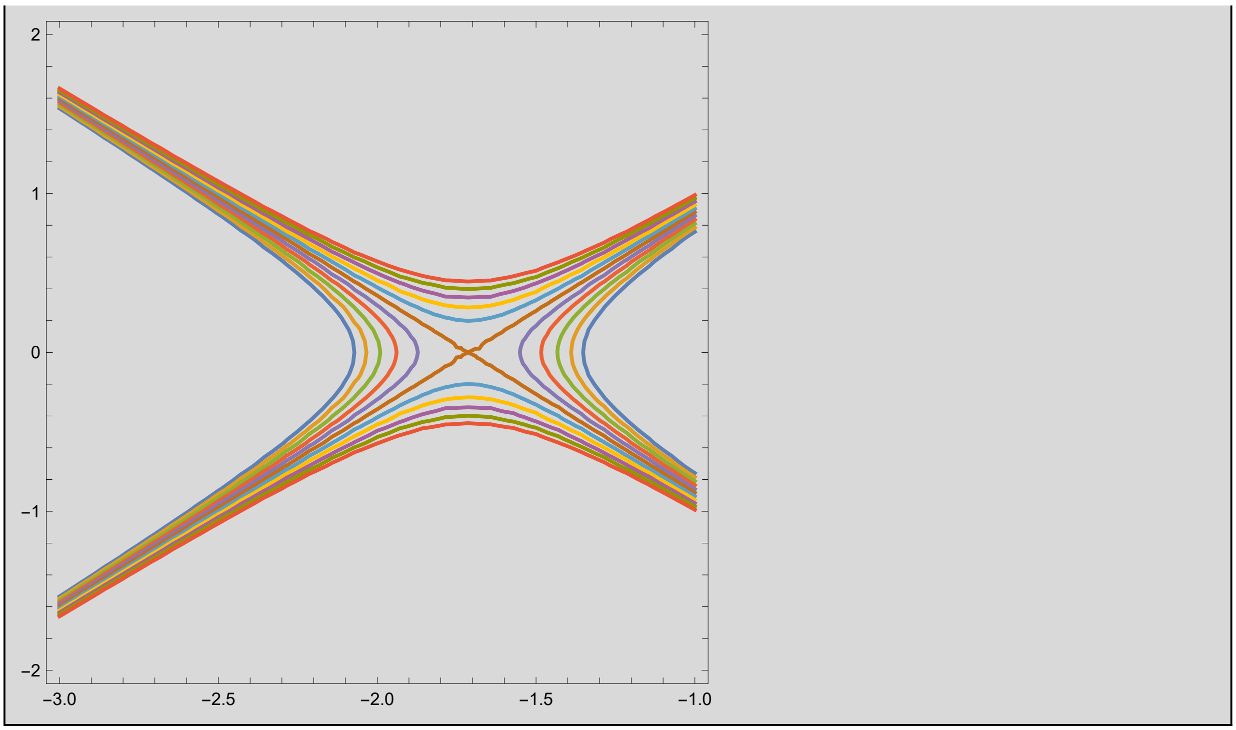
Phase trajectory close to maxima and minima:

In[448]:=

```
maxEnergy = U[Critpoints[[2]]];
energy1[p_,x_] = Taylorseries[x][[2]] + p^2 /2;
phase1 = Table[energy1[p,x]== e1, {e1,maxEnergy- 0.1  , maxEnergy + 0.1, 0.02}];
Print["Hyperbolic near maxima"]
plt1 = ContourPlot[Evaluate[phase1] , {x,-3,-1} , {p,-2,2}]
energy2[p_,x_] = Taylorseries[x][[1]] + p^2 /2;
phase2 = Table[energy2[p,x] == e1, { e1, 0, 0.2 , 0.02}];
Print["Elliptical near minima"]
plt2 = ContourPlot[Evaluate[phase2] , {x,-0.5,0.5},{p,-2,2}]
```

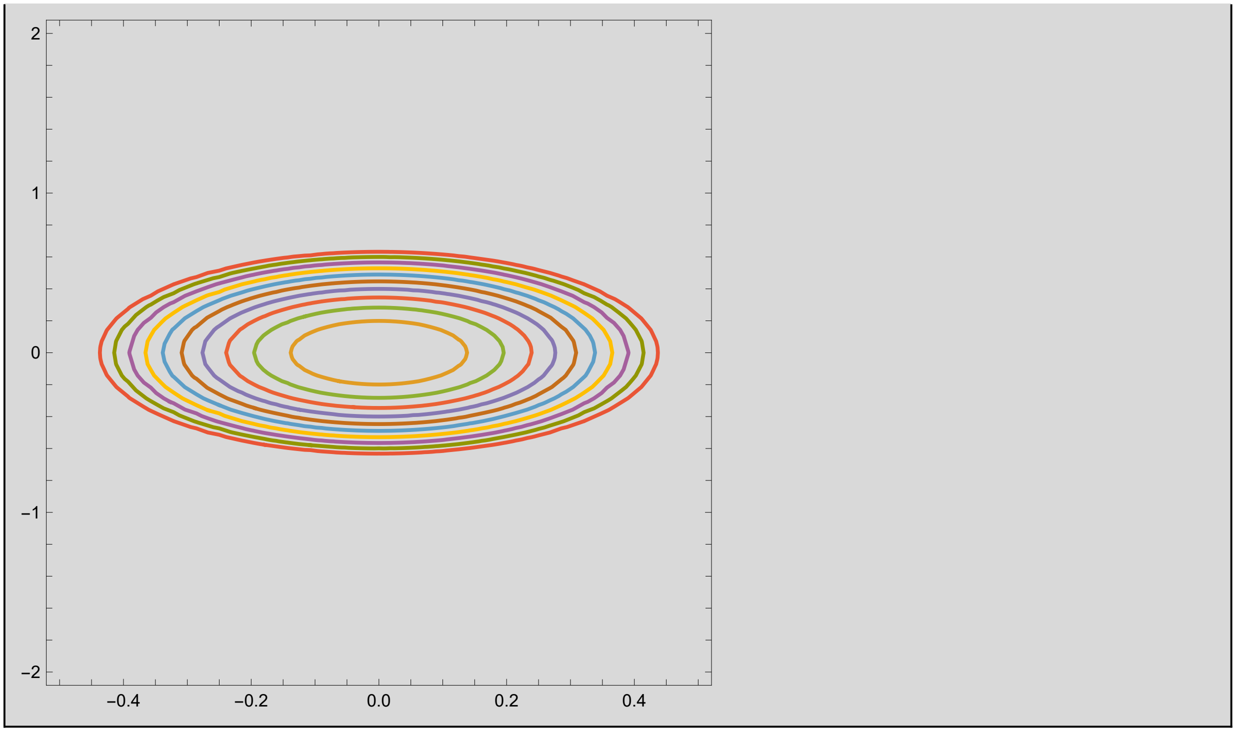
Hyperbolic near maxima

Out[452]=



Elliptical near minima

Out[456]=



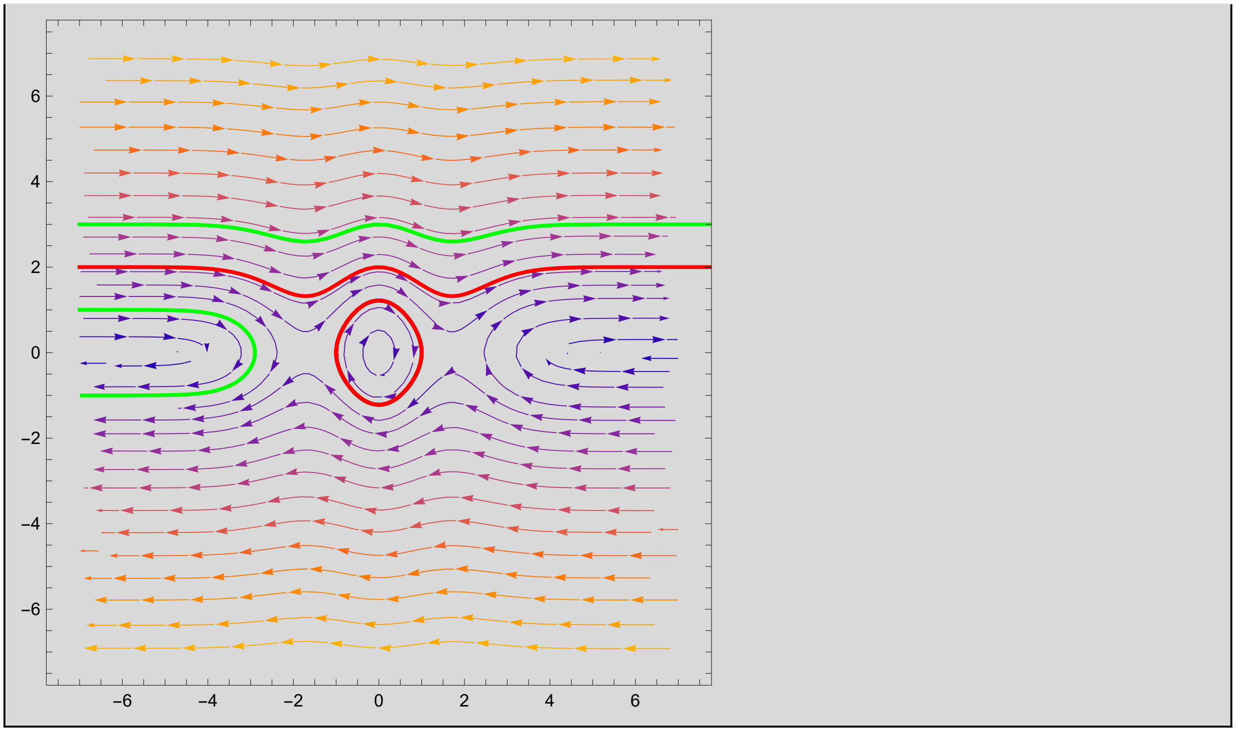
In[457]:=

```
Hamiltonian[p_, x_] = p^2 / 2 + U[x];
vctr[x_] = -D[Hamiltonian[p, x], x];
vctrPlot = StreamPlot[{p, vctr[x] }, {x, -range, range}, {p, -range, range}];
EqnMotion1 := {y''[t] == -D[U[y[t]], y[t]], y'[0] == 0 , y[0] == 1};
Solution1 = NDSolveValue[EqnMotion1 , {y[t], y'[t]}, {t, 0, 20}];
x1[t_] = Solution1[[1]];
v1[t_] = Solution1[[2]];
EqnData1 = Table[{x1[t], v1[t]} , {t, 0, 20, 0.1}];

EqnMotion2 := {z''[t] == -D[U[z[t]], z[t]], z'[0] == 2 , z[0] == -7};
Solution2 = NDSolveValue[EqnMotion2 , {z[t], z'[t]}, {t, 0, 20}];
x2[t_] = Solution2[[1]];
v2[t_] = Solution2[[2]];
EqnData2 = Table[{x2[t], v2[t]}, {t, 0, 10, 0.1}];
EqnPlot =
  ListLinePlot[{EqnData1, EqnData2} , PlotStyle → {{Red, Thick}, {Red, Thick}}];
CoupledEquation1 :=
  {v3'[t] == -D[U[x3[t]], x3[t]] , x3'[t] == v3[t], x3[0] == -7, v3[0] == 3 };
{x3[t_], v3[t_]} = NDSolveValue[CoupledEquation1, {x3[t], v3[t]}, {t, 0, 20}];
EqnData3 = Table[{x3[t], v3[t]}, {t, 0, 10, 0.1}];

CoupledEquation2 :=
  {v4'[t] == -D[U[x4[t]], x4[t]] , x4'[t] == v4[t], x4[0] == -7, v4[0] == 1 };
{x4[t_], v4[t_]} = NDSolveValue[CoupledEquation2, {x4[t], v4[t]}, {t, 0, 20}];
EqnData4 = Table[{x4[t], v4[t]}, {t, 0, 10, 0.1}];
CoupledPlot = ListLinePlot[{EqnData3, EqnData4},
   PlotRange → {{-7, 7} , {-1.5, 3.2}}, PlotStyle → Green];
```

In[478]:=

```
Show[vctrPlot, EqnPlot, CoupledPlot]
```

Out[478]=



Conclusion

-We can see that Plots made from various methods are the same

-There are 3 possible states for the object,

1) oscillate in the middle

2)turn back and return to infinity

3) cross both the potential hills and go to infinity again