Q1: vector $\Rightarrow$ $\boxed{\begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix} = \vec{v}}$

Order: ① rotation by $-\pi/6$ about Y

② rotat —— $+\pi/4$ —— X

③ reflectⁿ across XZ-plane.

④ translation by $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$

overall coord. Transformatⁿ Matrix (T)

(I) Rotation about Y axis by $-\pi/6$.

$$R_y(\theta) = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

For $\theta = -\pi/6$; $\cos(-\pi/6) = \cos(\pi/6) = \sqrt{3}/2$; $\sin(-\pi/6) = -\sin(\pi/6) = -1/2$

$$\therefore R_y(-\pi/6) = \begin{bmatrix} \sqrt{3}/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & \sqrt{3}/2 \end{bmatrix}$$

(II) Rotation about X-axis by $\pi/4$.

$$R_x(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{pmatrix}$$

For $\phi = \pi/4$;

$\cos(\pi/4) = 1/\sqrt{2}$, $\sin(\pi/4) = 1/\sqrt{2}$

$$\therefore R_x(\pi/4) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}$$

(C)(iii) Reflection across XZ-plane.

$$R_{ref} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(#) Combined (Linear) Transformation :-

$\because$ Ops. applied in sequence, overall 3×3 transformation matrix (w/o translation) is :-

$$\boxed{T = R_{ref} \cdot \underbrace{R_x(\pi/4) \cdot R_y(-\pi/6)}_{M}}$$

(S1) $M = R_x(\pi/4) \cdot R_y(-\pi/6)$

$$A = R_y(-\pi/6) = \begin{pmatrix} \sqrt{3}/2 & 0 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & \sqrt{3}/2 \end{pmatrix}$$

$$B = R_x(\pi/4) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix}$$

$$\Rightarrow M_{ij} = \sum_{k=1}^{3} B_{ij} A_{kj}$$

After multiplying;

$\cancel{}$ $M = R_x(\pi/4) \cdot R_y(-\pi/6)$

$$= \begin{pmatrix} \sqrt{3}/2 & 0 & -1/2 \\ -\sqrt{2}/4 & \sqrt{2}/2 & -\sqrt{6}/4 \\ \sqrt{2}/4 & \sqrt{2}/2 & \sqrt{6}/4 \end{pmatrix}$$

(S2) $\boxed{T = R_{ref} \cdot M}$ (where $R_{ref} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$)

$$\boxed{\therefore T = \begin{pmatrix} \sqrt{3}/2 & 0 & -1/2 \\ \sqrt{2}/4 & -\sqrt{2}/2 & \sqrt{6}/4 \\ \sqrt{2}/4 & \sqrt{2}/2 & \sqrt{6}/4 \end{pmatrix}}$$

// ~~Ans~~

Transformatⁿ of the Given vector and the Origin

a) Transforming the vector

$\vec{t} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$; $\vec{v} \Rightarrow$ original coordinate

$\Rightarrow \boxed{\vec{v}' = T\vec{v} + \vec{t}}$

[Note: considering small letters to be vectors unless stated
$\Rightarrow \vec{t} \sim t$ (for easier readability)]

$$v = \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}$$

Calculating v';

$$v_1' = \sqrt{3}/2 \cdot 3 + 0\cdot(-1) + \left(-\tfrac{1}{2}\right)\cdot 4 = \frac{3\sqrt{3}}{2} - 2$$

$$v_2' = \sqrt{2}/4 \cdot 3 + \left(\frac{-\sqrt{2}}{2}\right)\cdot(-1) + \sqrt{6}/4 \cdot 4 = \frac{5\sqrt{2}}{4} + \sqrt{6}$$

$$v_3' = \frac{\sqrt{2}}{4}\cdot 3 + \frac{\sqrt{2}}{2}\cdot(-1) + \frac{\sqrt{6}}{4}\cdot 4$$

$$= \sqrt{2}/4 + \sqrt{6}$$

Adding translation t; and presenting final transformed vector:-

$$v' = \begin{pmatrix} 3\sqrt{3}/2 - 1 \\ 5\sqrt{2}/4 + \sqrt{6} \\ \sqrt{2}/4 + \sqrt{6} - 2 \end{pmatrix}$$

b) Mapping of the origin:-

$\because$ Rotation and reflection work about the origin, hence the only effect on the origin is translation.

$\Rightarrow$ Original origin $(0,0,0)^T$ maps to $(1,0,-2)^T$.

Axis of the combined Rotat^n & Angle (w/o reflect^n)

$$R = R_x(\pi/4)\cdot R_y(-\pi/6) = M$$

where;
$$R = \begin{pmatrix} \sqrt{3}/2 & 0 & -1/2 \\ -\sqrt{2}/4 & \sqrt{2}/2 & -\sqrt{6}/4 \\ \sqrt{2}/4 & \sqrt{2}/2 & \sqrt{6}/4 \end{pmatrix}$$

a) Finding the angle of Rotation :-

We know; $\boxed{\cos\theta = \frac{tr(R)-1}{2}}$

trace: $tr(R) = \sqrt{3}/2 + \sqrt{2}/2 + \sqrt{6}/4$

$\Rightarrow \cos\theta = \left(\frac{\sqrt{3}/2 + \sqrt{2}/2 + \sqrt{6}/4 - 1}{2}\right)$

$\Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{2\sqrt{3}+2\sqrt{2}+\sqrt{6}-4}{8}\right)}$ // Ans

b) Finding the ~~Axis~~ Axis of Rotation.

We know, If R is a rotation matrix, than the axis is the (normalised) eigen vector corresponding to the eigen value 1.

Also; $K = (K_x, K_y, K_z)^T$

$$\boxed{\therefore K_x = \frac{R_{32}-R_{23}}{2\sin\theta},\ K_y = \frac{R_{13}-R_{31}}{2\sin\theta},\ K_z = \frac{R_{21}-R_{12}}{2\sin\theta}}$$

After putting values;
$$K \propto \begin{pmatrix} 2\sqrt{2}+\sqrt{6} \\ -(2+\sqrt{2}) \\ -\sqrt{2} \end{pmatrix}$$ // Ans

$\therefore \hat{K} = \frac{1}{\lambda}\begin{pmatrix} 2\sqrt{2}+\sqrt{6} \\ -(2+\sqrt{2}) \\ -\sqrt{2} \end{pmatrix}$; $\cancel{\lambda = \sqrt{(2\sqrt{2}+\sqrt{6})^2 + (2+\sqrt{2})^2}}$

$\lambda = \sqrt{(2\sqrt{2}+\sqrt{6})^2 + (2+\sqrt{2})^2 + (\sqrt{2})^2}$

Verification via Rodrigues' Formula

We know;

$$\boxed{R = I + \sin\theta . \mathbf{K} + (1-\cos\theta) . K^2}$$

(Rotation about an angle θ w.r.t. unit axis $\hat{k}$; where $\hat{k} = (k_x, k_y, k_z)^T$)

$$\mathbf{K} = \begin{pmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{pmatrix}$$

Verifying :-

① Identify θ and $\hat{k}$ (mentioned above)

$$\theta = \cos^{-1}\left(\frac{2\sqrt{3} + 2\sqrt{2} + \sqrt{6} - 4}{8}\right)$$

② have to calculate $\mathbf{K}^2$

$$K^2 = \begin{pmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{pmatrix}\begin{pmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{pmatrix}$$

$$= \begin{pmatrix} -(k_z^2+k_y^2) & k_x k_y & k_x k_z \\ k_x k_y & -(k_x^2+k_z^2) & k_z k_y \\ k_x k_z & k_y k_z & -(k_y^2+k_x^2) \end{pmatrix}$$

③ $R = I + \sin\theta . \mathbf{K} + (1-\cos\theta) . \mathbf{K}^2$

$$\Rightarrow R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \sin\theta . \begin{pmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{pmatrix}$$

$$+ (1-\cos\theta)\begin{pmatrix} -(k_z^2+k_y^2) & k_x k_y & k_x k_z \\ k_y . k_x & -(k_x^2+k_z^2) & k_y k_z \\ k_x k_z & k_y k_z & -(k_y^2+k_x^2) \end{pmatrix}$$

After substituting all values and calculating;

$$R = \begin{pmatrix} \sqrt{3}/2 & 0 & -1/2 \\ -\sqrt{2}/4 & \sqrt{2}/2 & -\sqrt{6}/4 \\ \sqrt{2}/4 & \sqrt{2}/2 & \sqrt{6}/4 \end{pmatrix}$$

$\Rightarrow$ Matches with what we have calculated as:

$$R = R_x(\pi/4) \cdot R_y(-\pi/6)$$

—— END ——

Q2 Image formation model in homogenous coordinates:-

$$x = K \cdot [R \mid t] \cdot X$$

For a given pt. (3D) (say X); its projections in the two cameras are given by:-

① Camera 1: Using world coord. frame as the camera frame:

$$x_1 \sim K_1 [I \mid 0] X \Rightarrow \boxed{x_1 \sim K_1 \cdot X}$$

② Camera 2: Assuming there's no translation (i.e. $t=0$); hence orientation is a result of pure rotation.

$$x_2 \sim K_2 [R \mid 0] X \Rightarrow \boxed{x_2 \sim K_2 \cdot R \cdot X}$$

Since K_1 is invertible $\Rightarrow X \sim K_1^{-1} x_1$

Substituting this expression:

$$x_2 \sim K_2 \cdot R \cdot (K_1^{-1} \cdot x_1)$$
$$= (K_2 \cdot R \cdot K_1^{-1}) \cdot x_1$$

Let $(H' = K_2 \cdot R \cdot K_1^{-1})$;

$$\boxed{\therefore x_2 \sim H' \cdot x_1}$$

Since H' is invertible $\Rightarrow \boxed{x_1 \sim (H')^{-1} \cdot x_2}$

$$\therefore H = (H')^{-1} = K_1 \cdot R^{-1} \cdot K_2^{-1}$$

($\because$ R is a rotation matrix $\Rightarrow R^{-1} = R^T$)

$$\boxed{\therefore H = K_1 \cdot R^T \cdot K_2^{-1}}$$

∴ The homogenous representations of the img pts.
x_1 and x_2 are related by :-

$$\boxed{x_1 = H \cdot x_2} \quad \boxed{H = K_1 \cdot R^T \cdot K_2^{-1}}$$