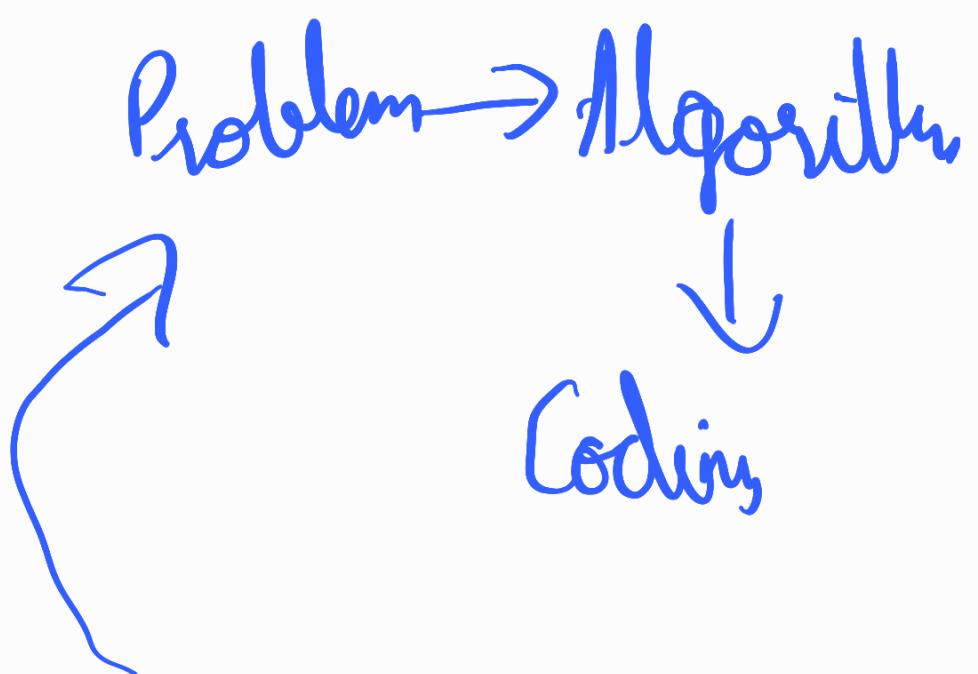


17/07/23



TOC (To check whether  
Problem is Solvable)

Finite Automata → (Machine)  
Push down Automata  
Turing Machine

## Language

1) Symbols 'a' 'z' 'y' '^' 'Δ'

Alphabet  $\rightarrow$  set of symbols in a language  $a, b \in \{a, b\}$

- 2) string  $\rightarrow$  word or group of symbols
- 3) Language  $\rightarrow$  All the words put together in a language.

L is a language

$L = \{ \text{string only start with } a \}$

$L = \{ a, aa, ab, aaa, aab, aba, aaaa \dots \}$

b-a

acceptable string not  
acceptable by this language

$\gamma = \{ \text{set of strings having length } 2 \}$

$\gamma = \{ aa, ab, ba, bb \}$

$\gamma_3 = \{ \text{string having length 3} \}$

$\delta = \{ aaa, aab, aba, abb, baa, bab, bba, bbb \}$

If the machine stops at  
the final state then the word  
or string is acceptable

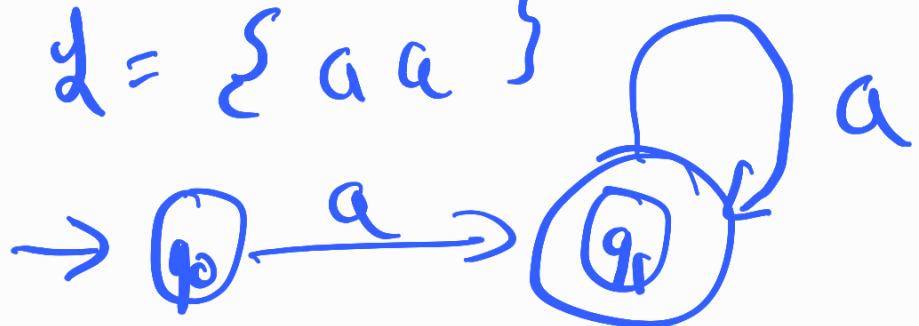
$$L = \{ \text{string start with } a \}$$



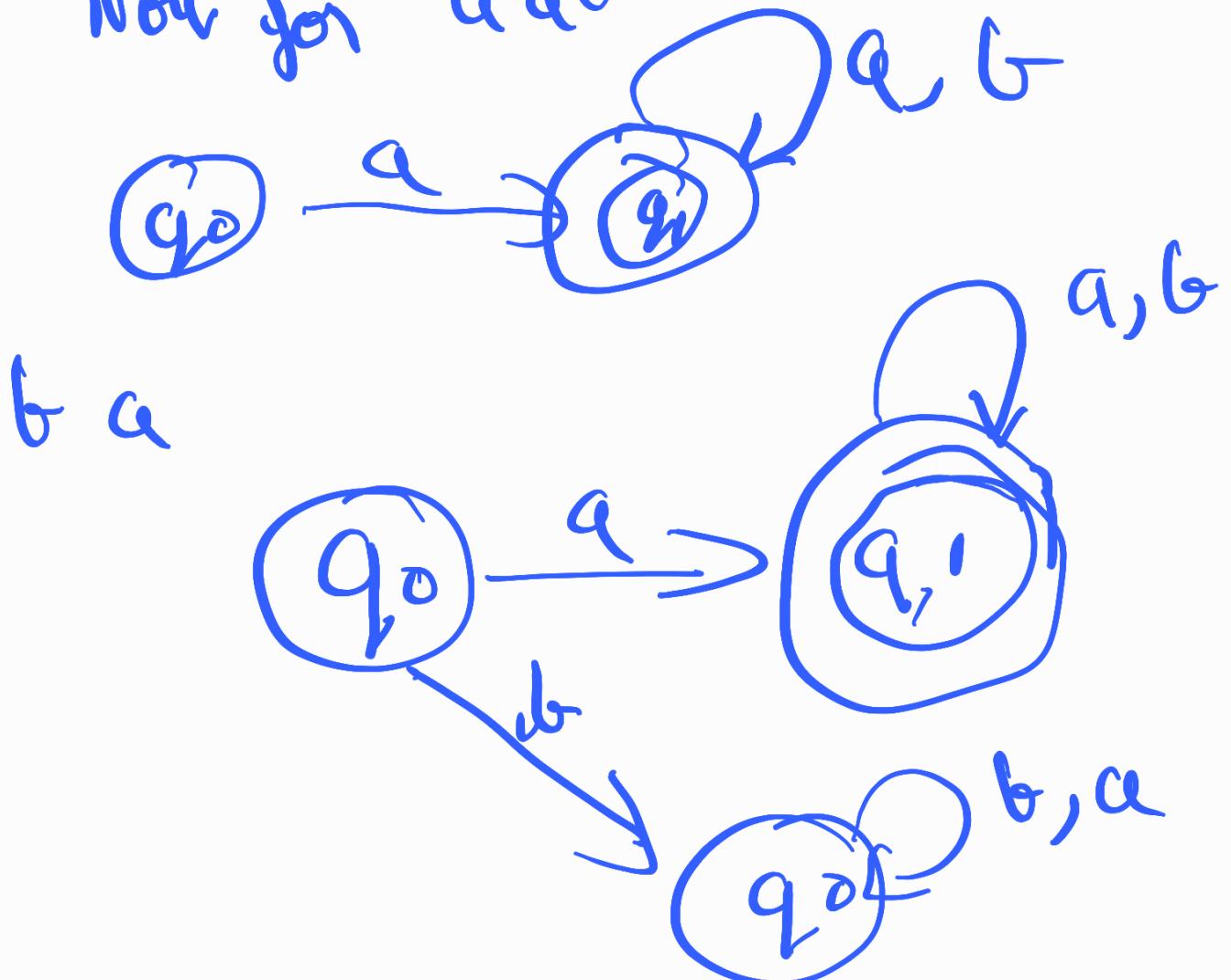
$$L = \{ a \}$$



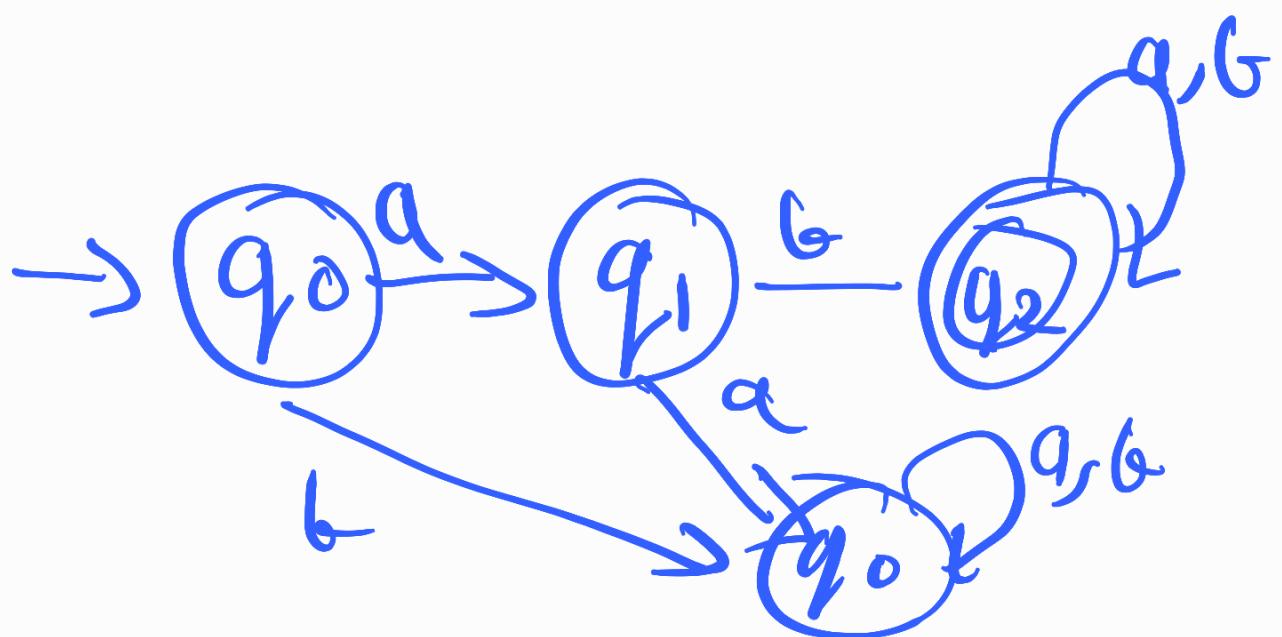
$$L = \{ aa \}$$



Now for aab

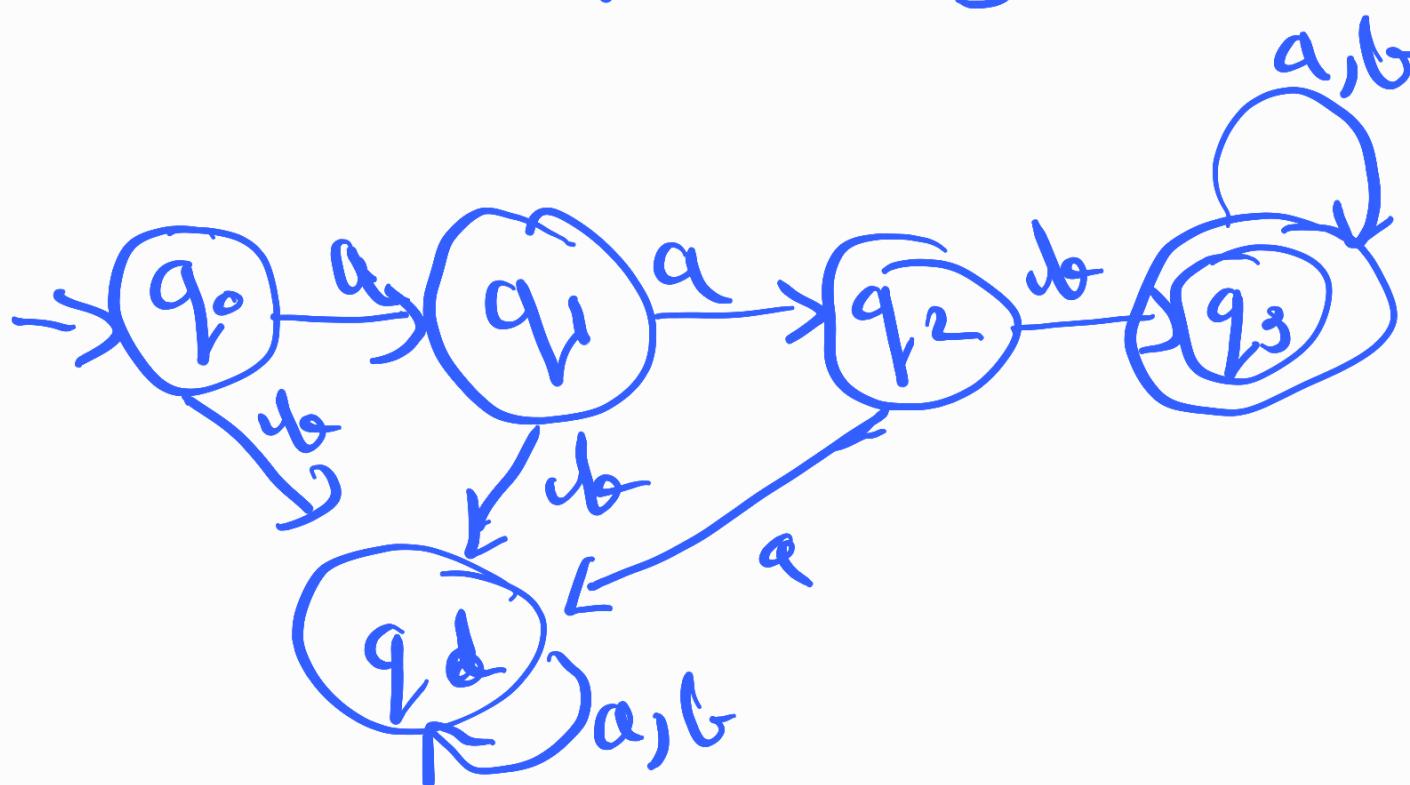


$Y_2 = \{ \text{string must start with } ab \}$

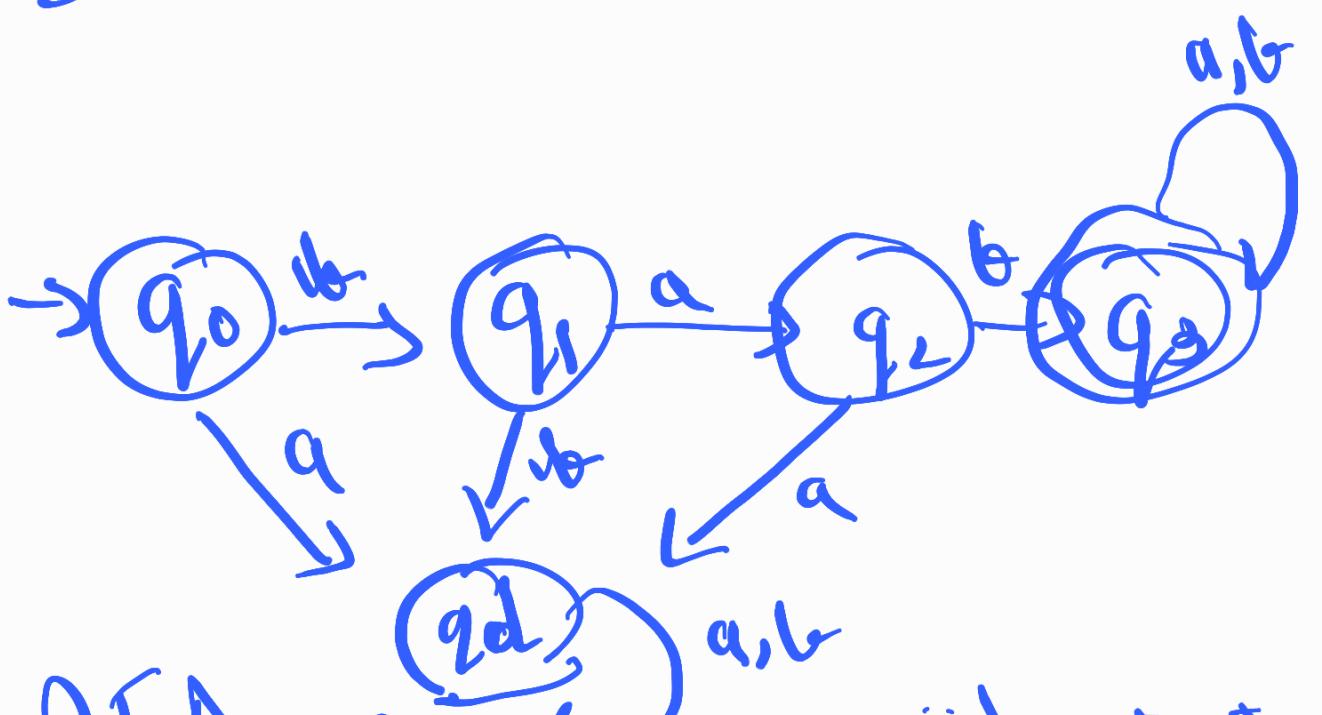


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$q = \{ q_1 \}$  it should accept string starting from  $aabb^3$



$q_2 = \{ q_1 \}$  starts with  $babb^3$



DFA  $\rightarrow$  Deterministic finite automata

DFA  $\rightarrow$  has transition for all symbols.

$$\Theta, \Sigma, \delta, q_0, \delta$$

$\Theta \rightarrow$  set represents the number of states

$$\Theta = \{q_0, q_1, q_2, q_3, q_d\}$$

$\Sigma \rightarrow$  set of symbols in input

$$\Sigma = \{\alpha, \beta\}$$

$\delta \rightarrow$  transition function

$$\delta \rightarrow \Theta \times \Sigma \rightarrow \Theta$$

↑                      ↑                      ↑  
current state    new input    new state

$q_0 \rightarrow$  initial state (only init)

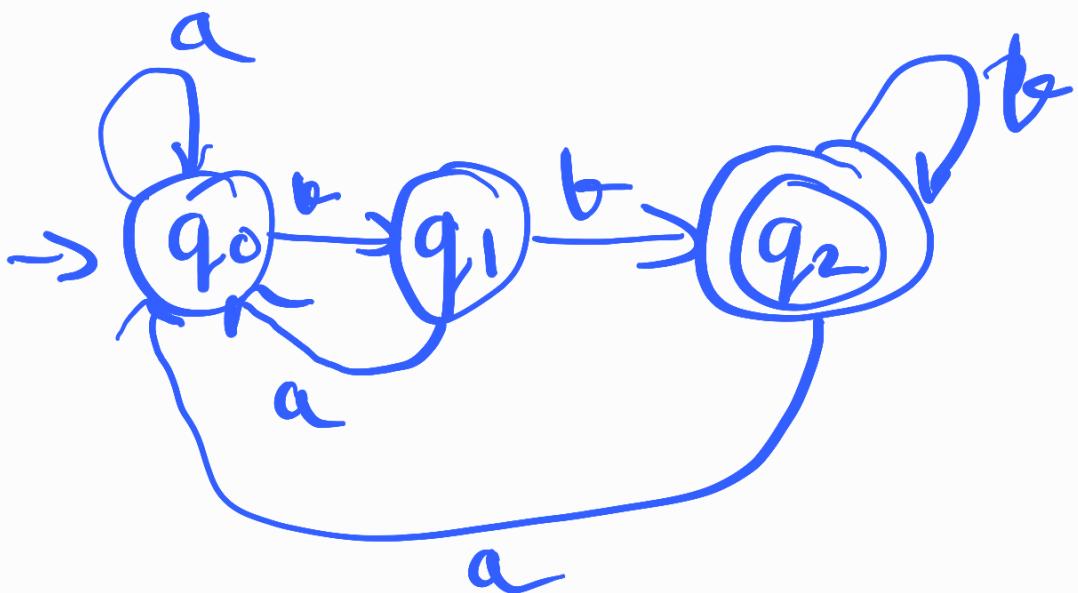
$F \rightarrow \{q_3\}$

final state

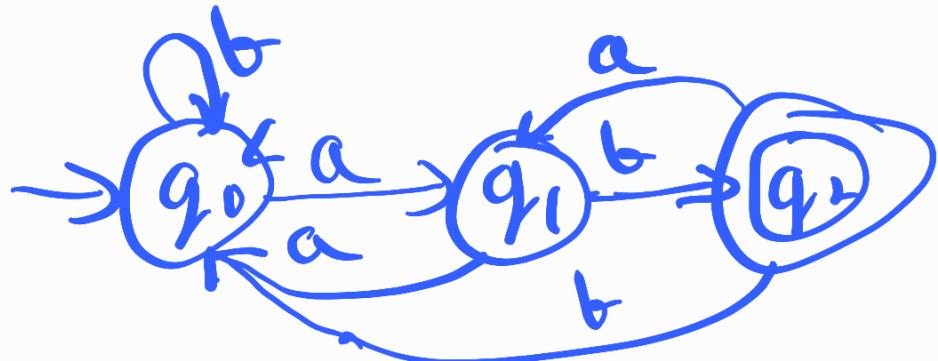
Staples

makes it the definition  
of a machine

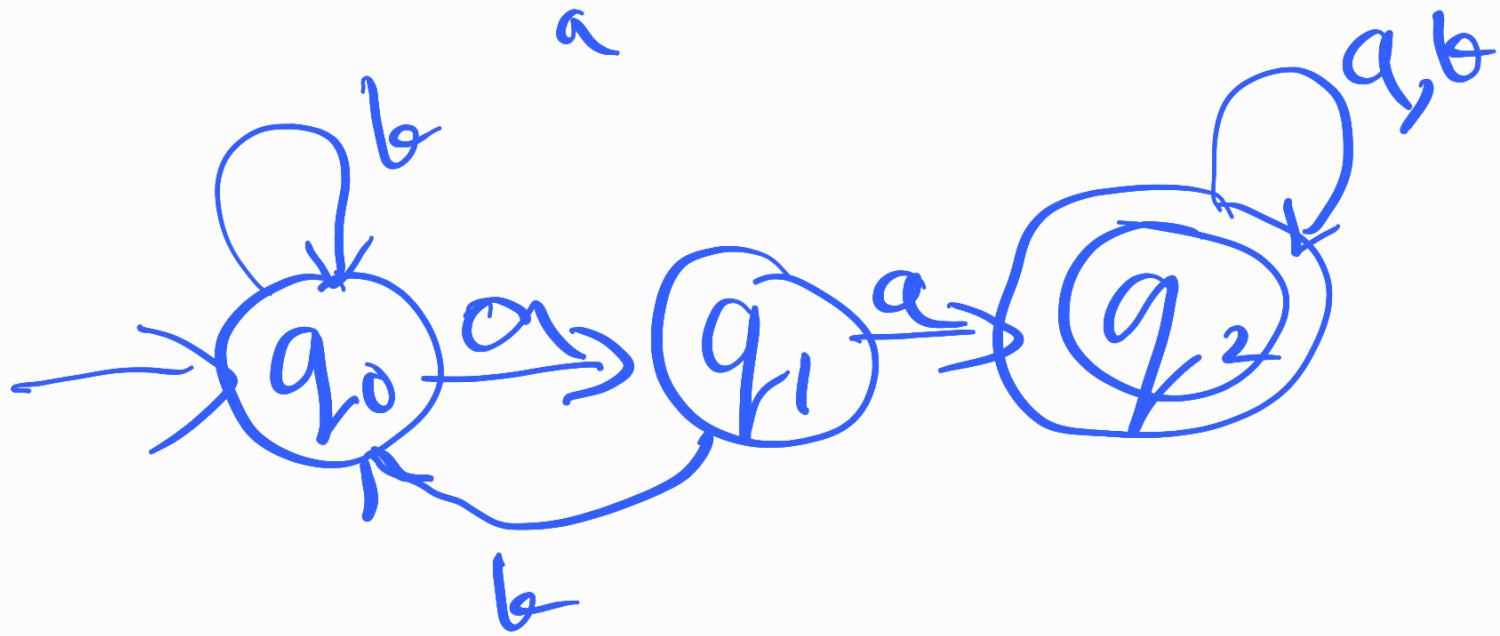
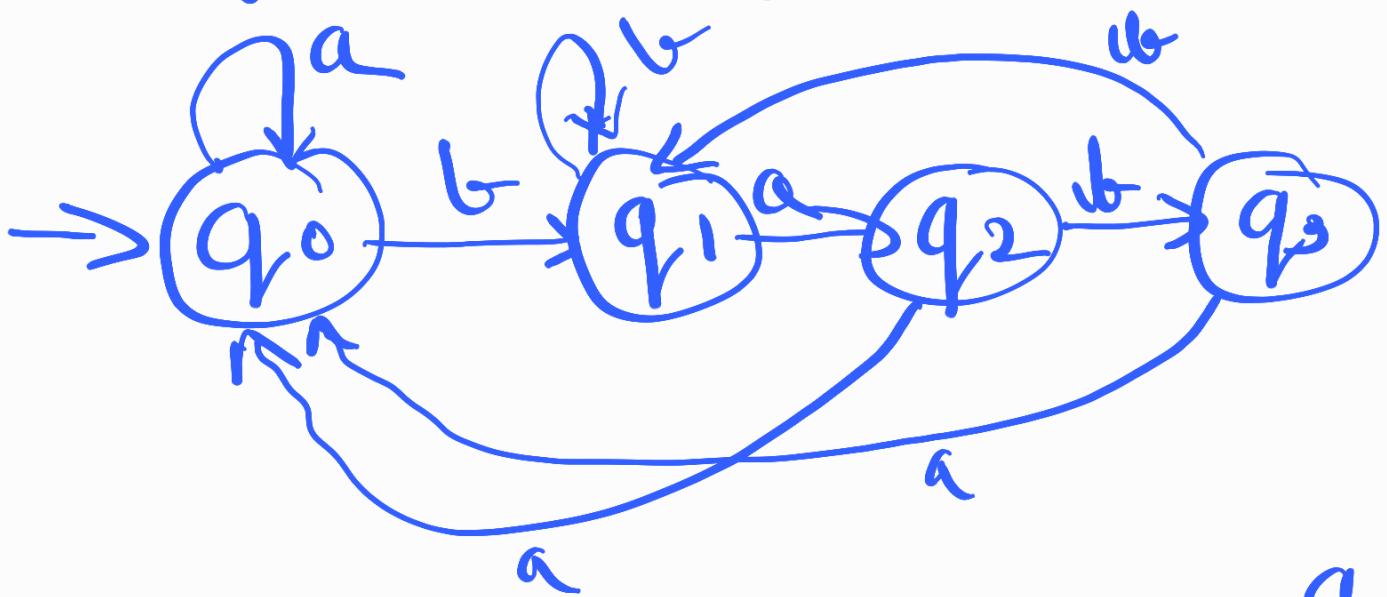
$L = \{ \text{string ending with } bb \}$



$L = \{ \text{string ends with } ab \}$

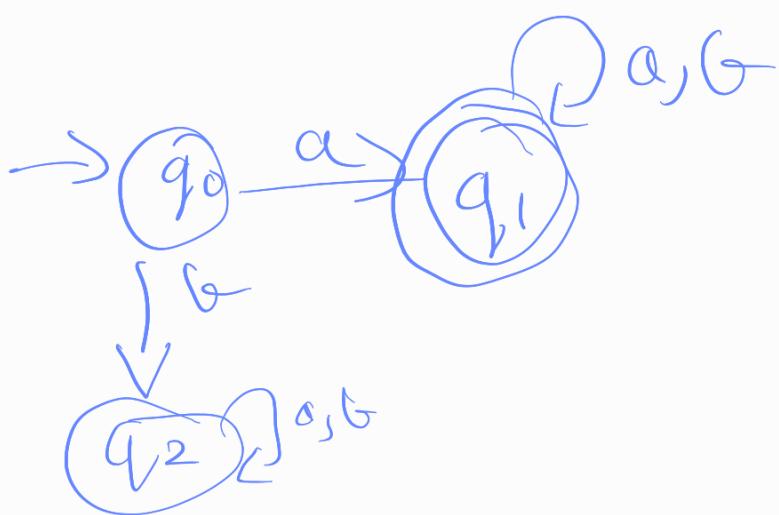


$L = \Sigma$  ends with  $bab^3$



# String starting with a

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$q_2 \setminus \Sigma$	a	b
$q_0$	$q_1$	$q_2$
$q_1$	$q_1$	$q_1$
$q_2$	$q_2$	$q_2$

$S \rightarrow \Sigma^*$

$Q \rightarrow \text{set of states}$

$\Sigma \rightarrow \text{Set of symbols}$

$\delta \rightarrow \text{transition function } \delta : Q \times \Sigma \rightarrow Q$

$q_0 \rightarrow \text{initial state}$

$F \rightarrow \text{set of final states}$

when it stops at final state

write

'aa ba' accepted by language

$\uparrow$

example string

L

String  $\rightarrow$  L for

off  $\rightarrow$  state diagram  $\rightarrow$  transition

String acceptable  $\leftarrow$  table

or not  $\rightarrow$  required

$Q$  = Set of states

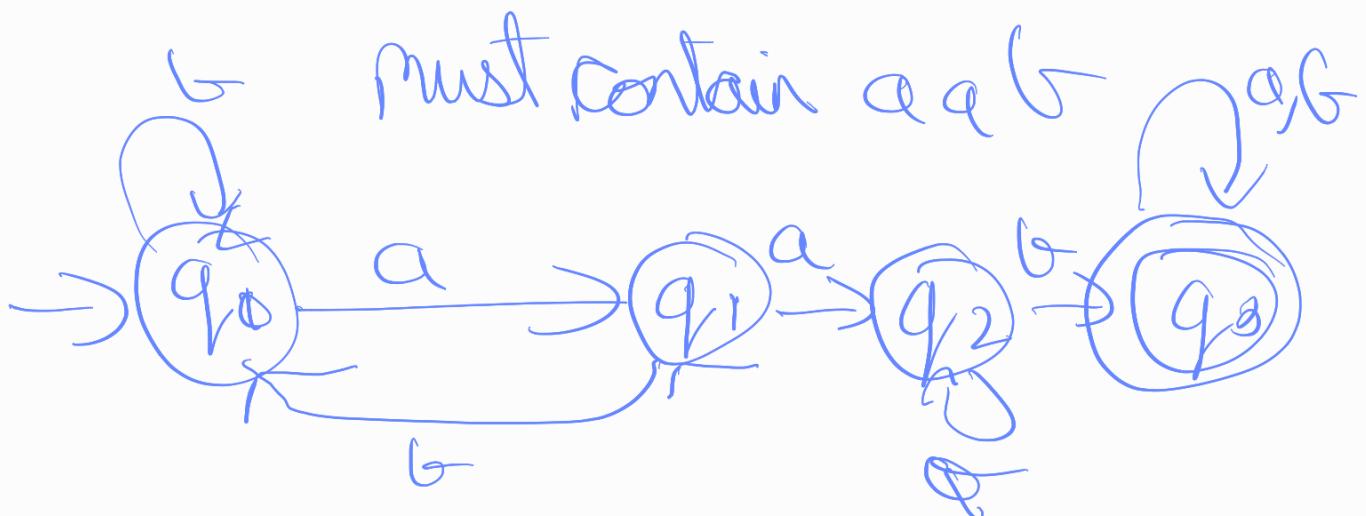
$\Sigma$  = Alphabetic set of tuples

$S$  = Transition function

$Q \times \Sigma \rightarrow Q$  used to find next state

$q_0$ : Initial state

$F$ : Final state



$\Sigma$		
$a$	$a$	$b$
$q_0$	$q_1$	$q_0$
$q_1$	$q_2$	$q_3$
$q_2$	$q_2$	$q_3$
$q_3$	$q_3$	$q_3$

bababab

$$\delta(a, q_0) = q_1$$

$$\delta(b, q_1) = q_0$$

$$\delta(a, q_0) = q_1$$

$$\delta(b, q_1) = q_0$$

$$\delta(a, q_0) = q^*$$

$$\delta(a, q^*) = q_2$$

$$\delta(b, q_2) = q_3$$

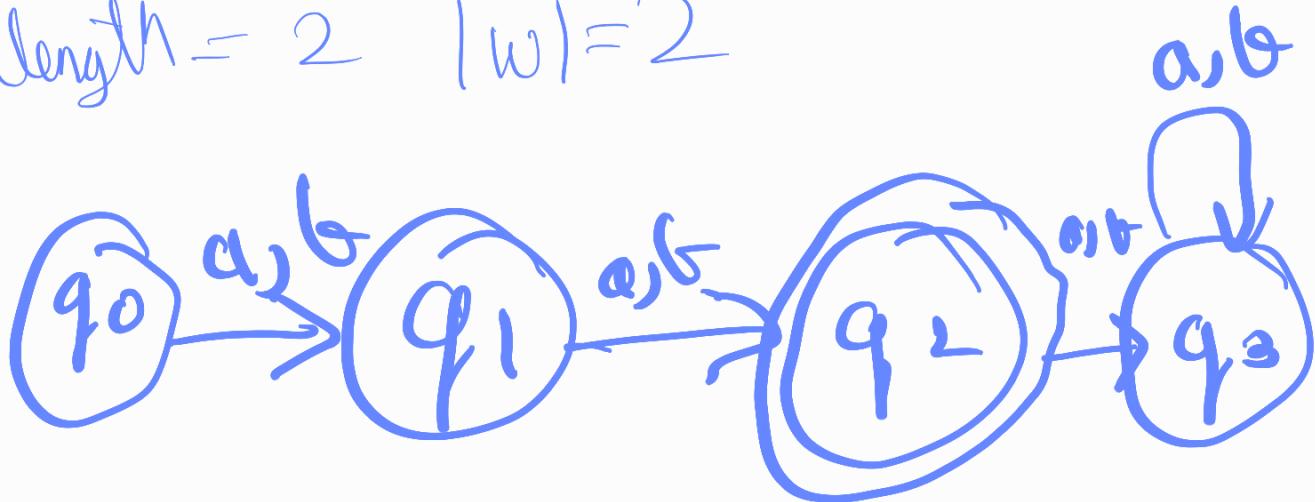
; the string ends on a final state

it is accepted by the language

Design a DFA using alphabet set

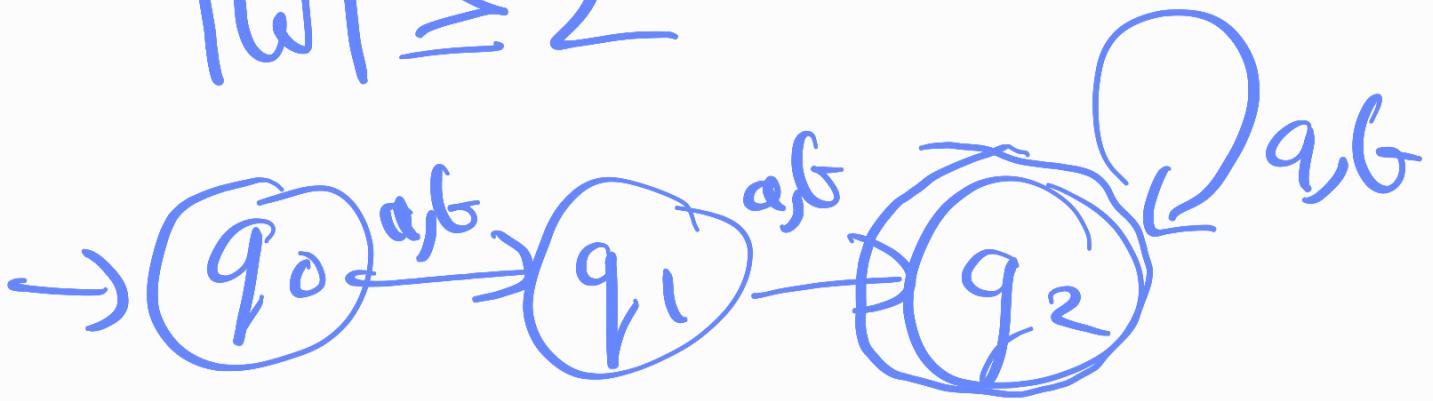
$$\Sigma = \{a, b\}$$

every string accepted must have  
length = 2  $|w|=2$

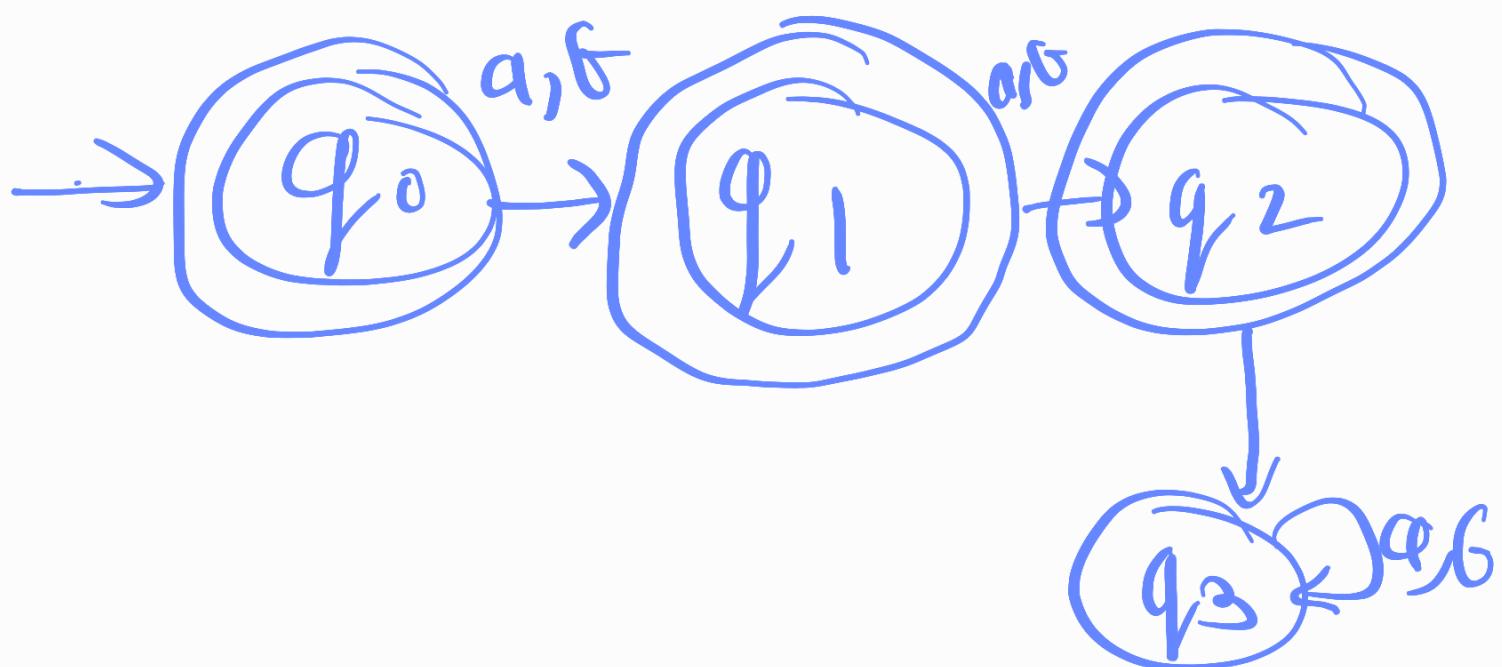


$q_i$	a	b
$q_0$	$q_1$	$q_1$
$q_1$	$q_2$	$q_2$
$q_2$	$q_3$	$q_3$
$q_3$	$q_3$	$q_3$

$$|w| \geq 2$$



$$|w| \leq 2$$



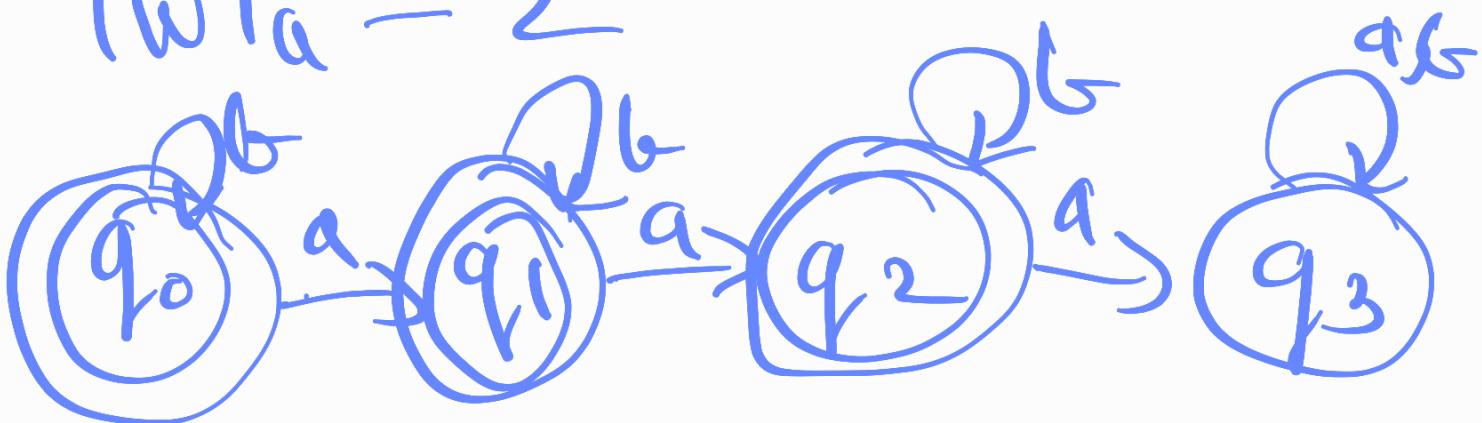
$$|w|_a = 2$$



$$|w|_a \geq 2$$

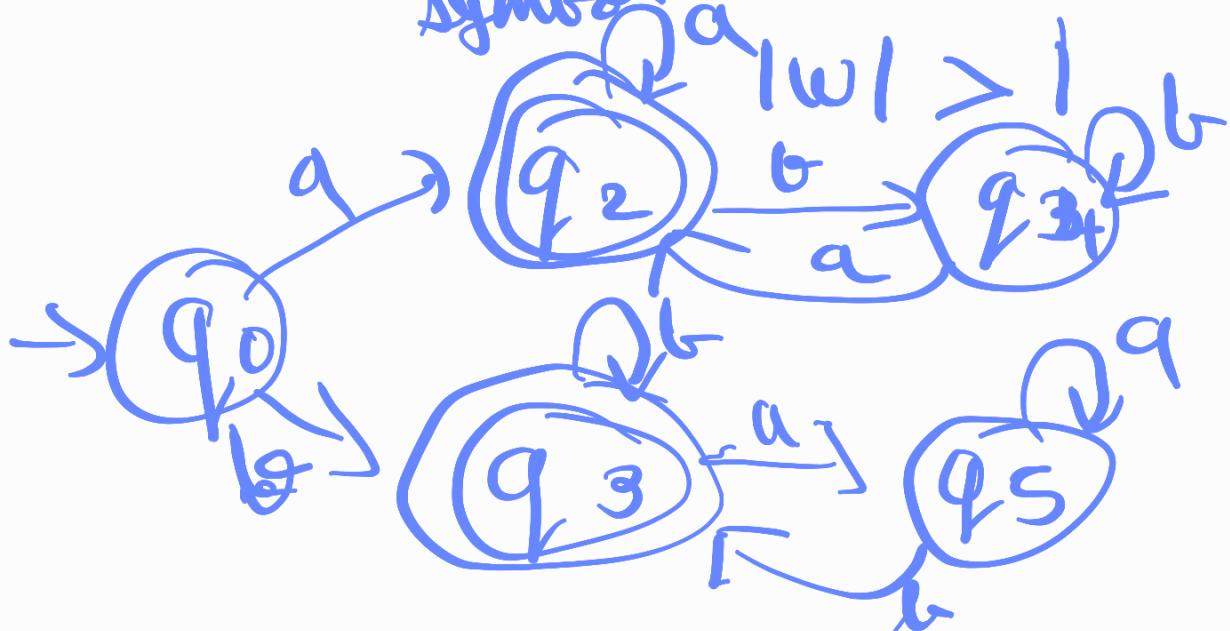


$$|w|_a \leq 2$$

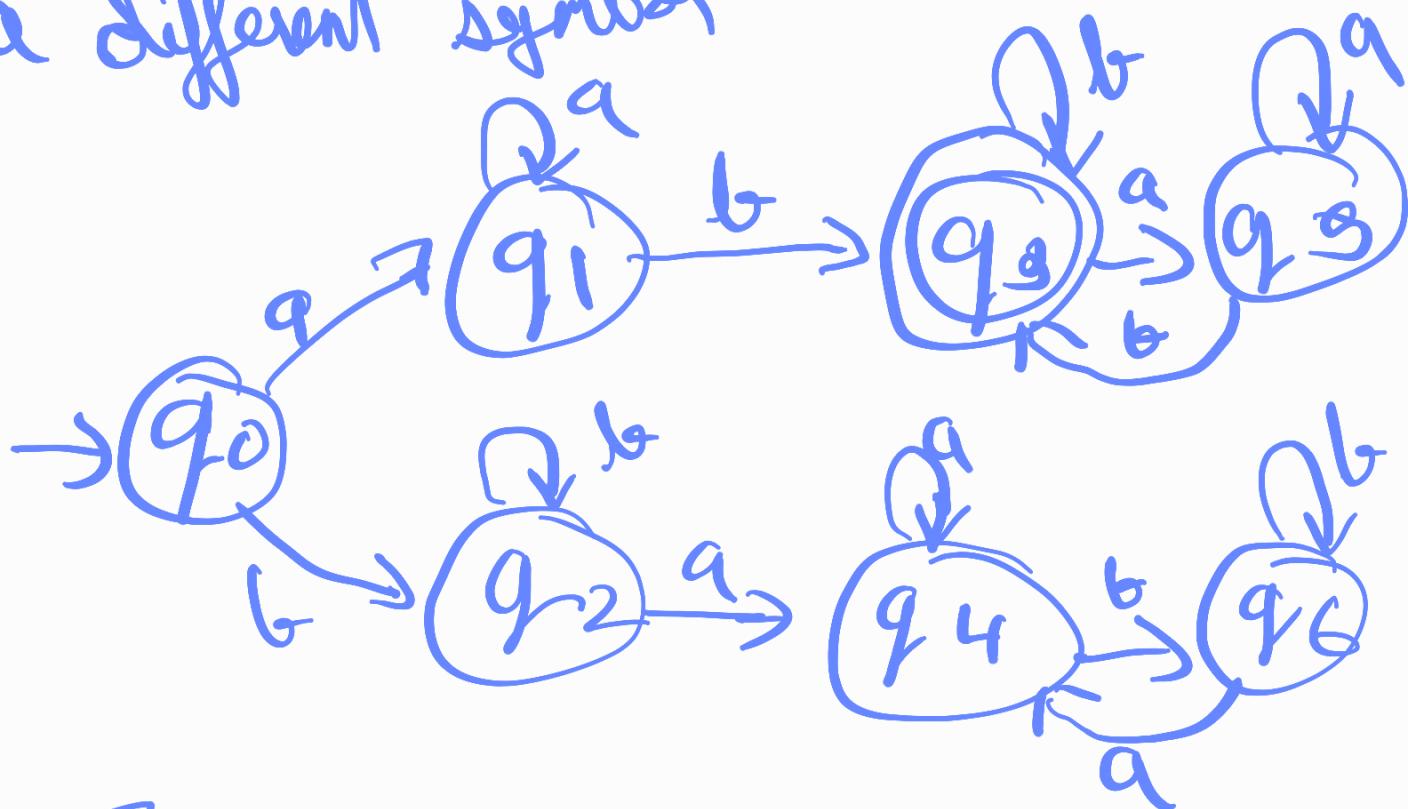


Design a DFA over  $\Sigma = \{a, b\}$

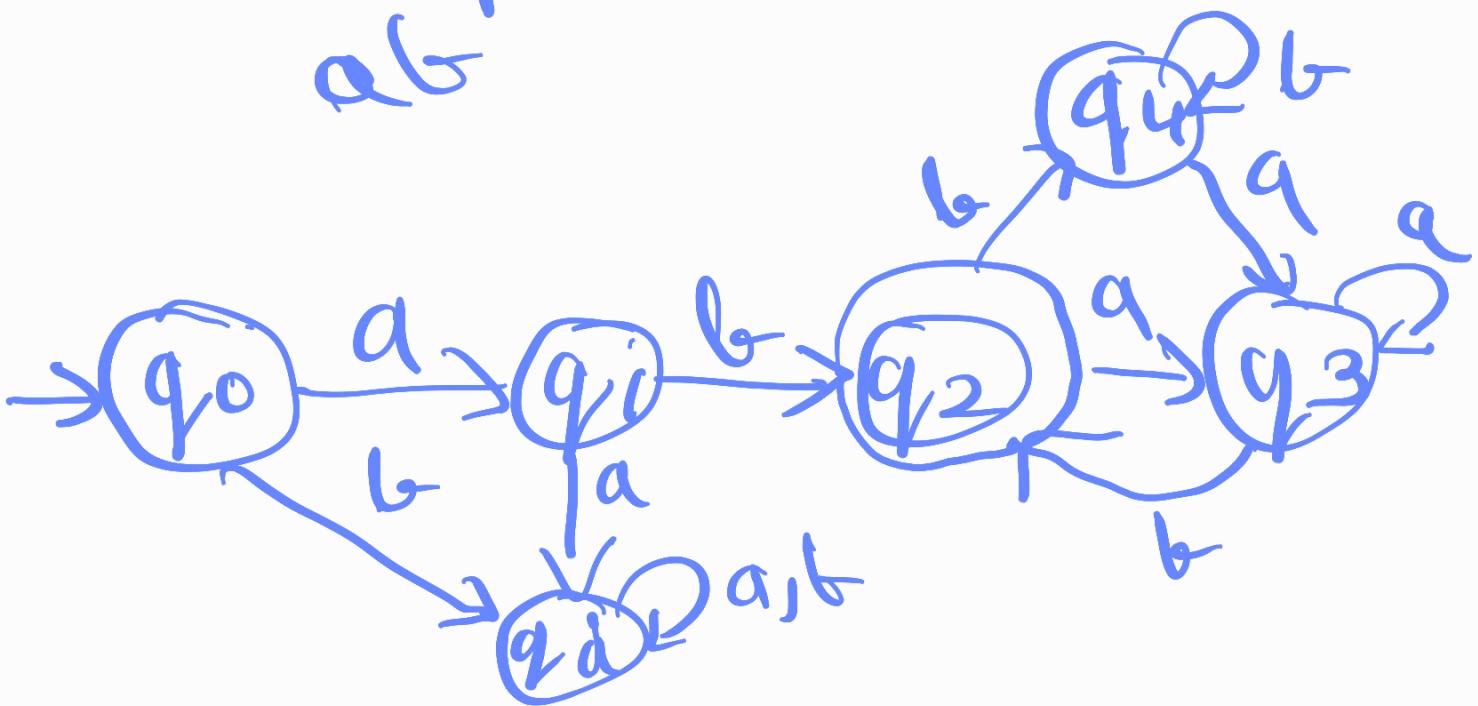
The string accepted must start and end with the same symbol.



Design a DFA over  
 $\Sigma = \{a, b\}$  such that every  
 string accepted must end with  
 a different symbol



DFA over  $a, b$  every string  
 accepted must start and end  
 $ab$

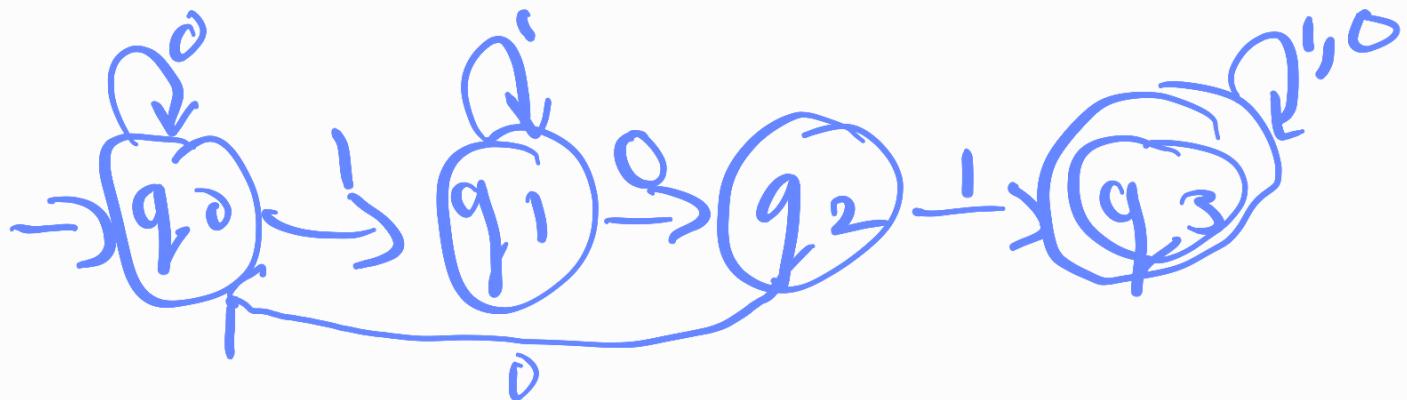


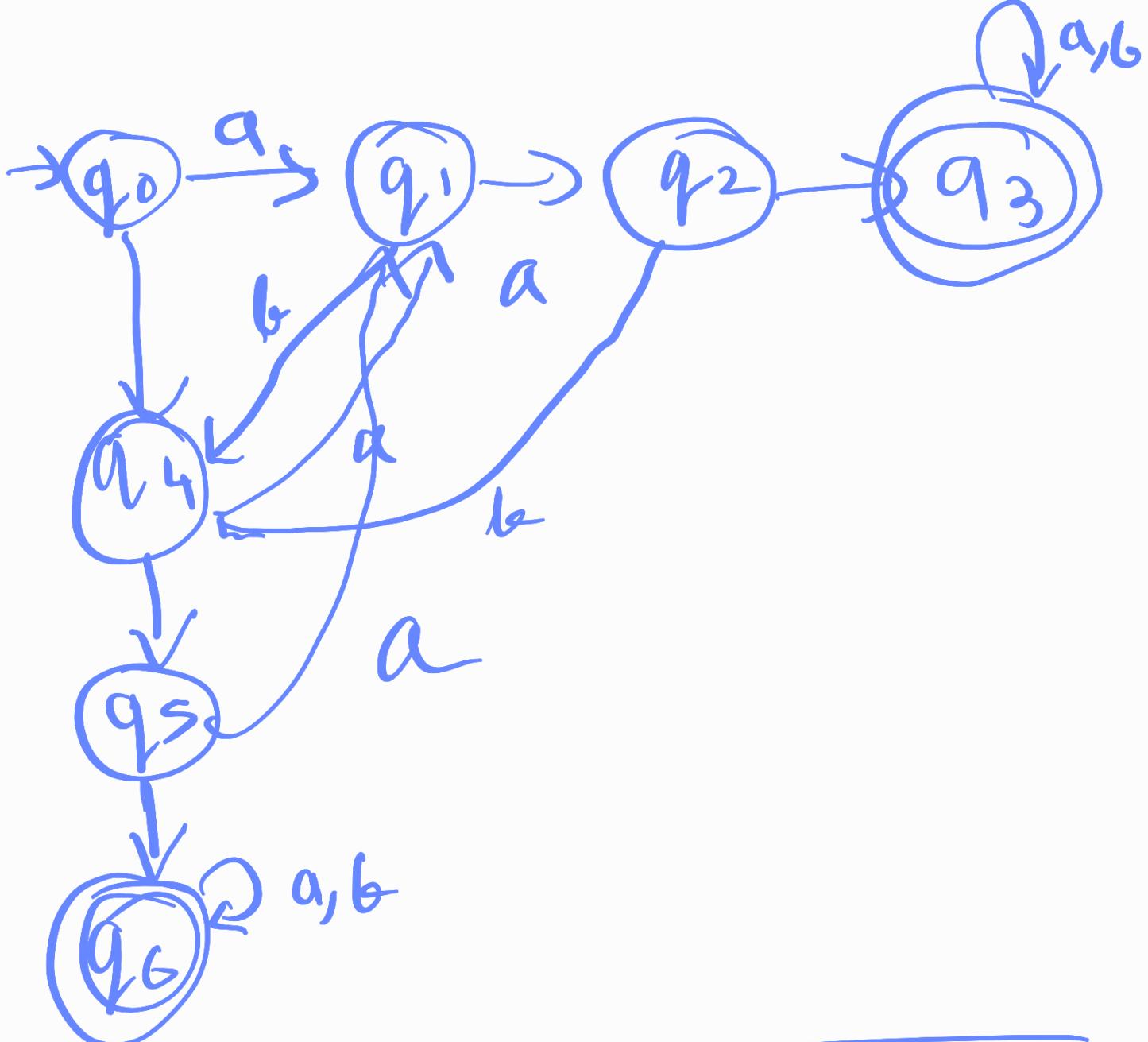
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- Q1. Design a DFA for the alphabet set  $\Sigma = \{0, 1\}$  which accepts a string which contains 101.
- Q2. Design a DFA that accepts set of all string over  $\Sigma = \{a, b\}$  contains either subsequence aaa or bbb.
- Q3. Design a DFA  $\Sigma = \{0, 1\}$  which accepts all string that contains substring 11 and does not contain 00.

$$\Sigma = \{0, 1\}$$

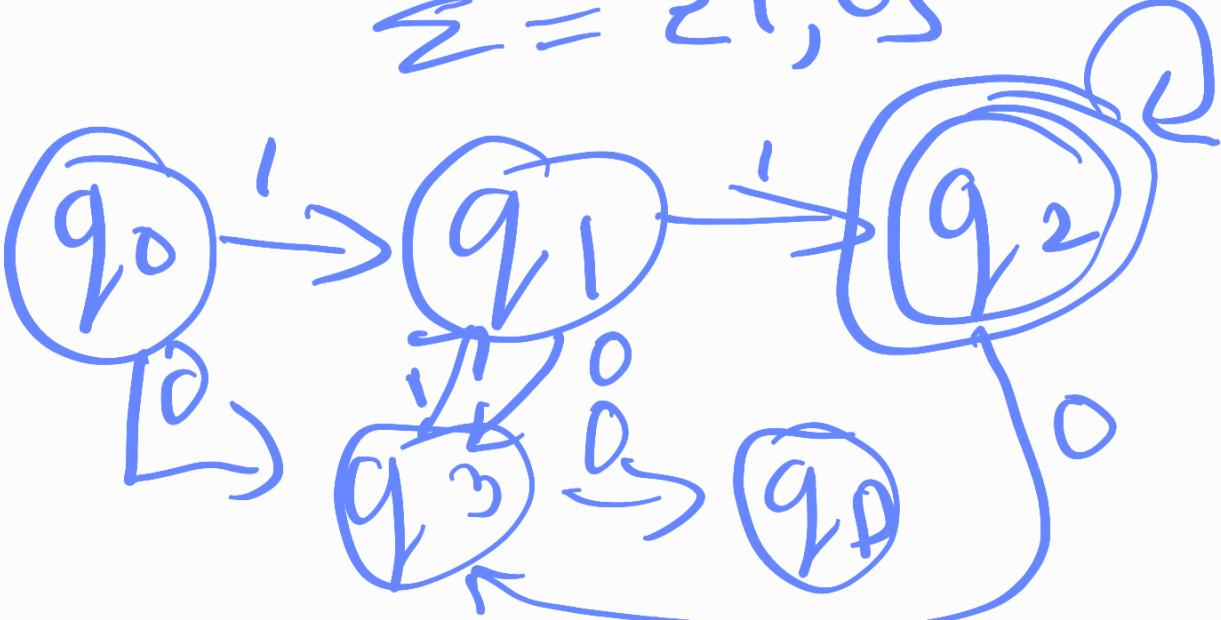
must contain 101

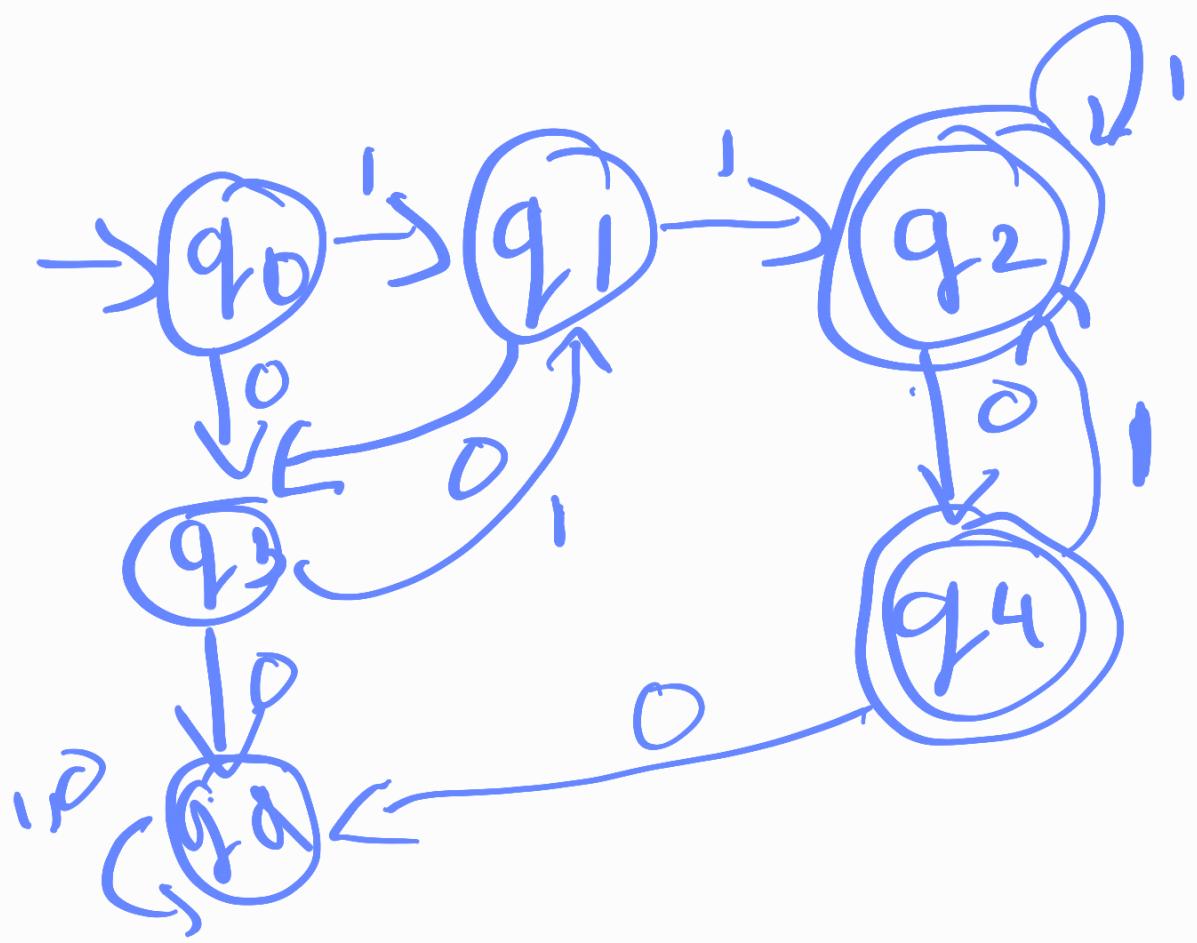





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Contains 11 and not 00  
 $\Sigma = \{1, 0\}$



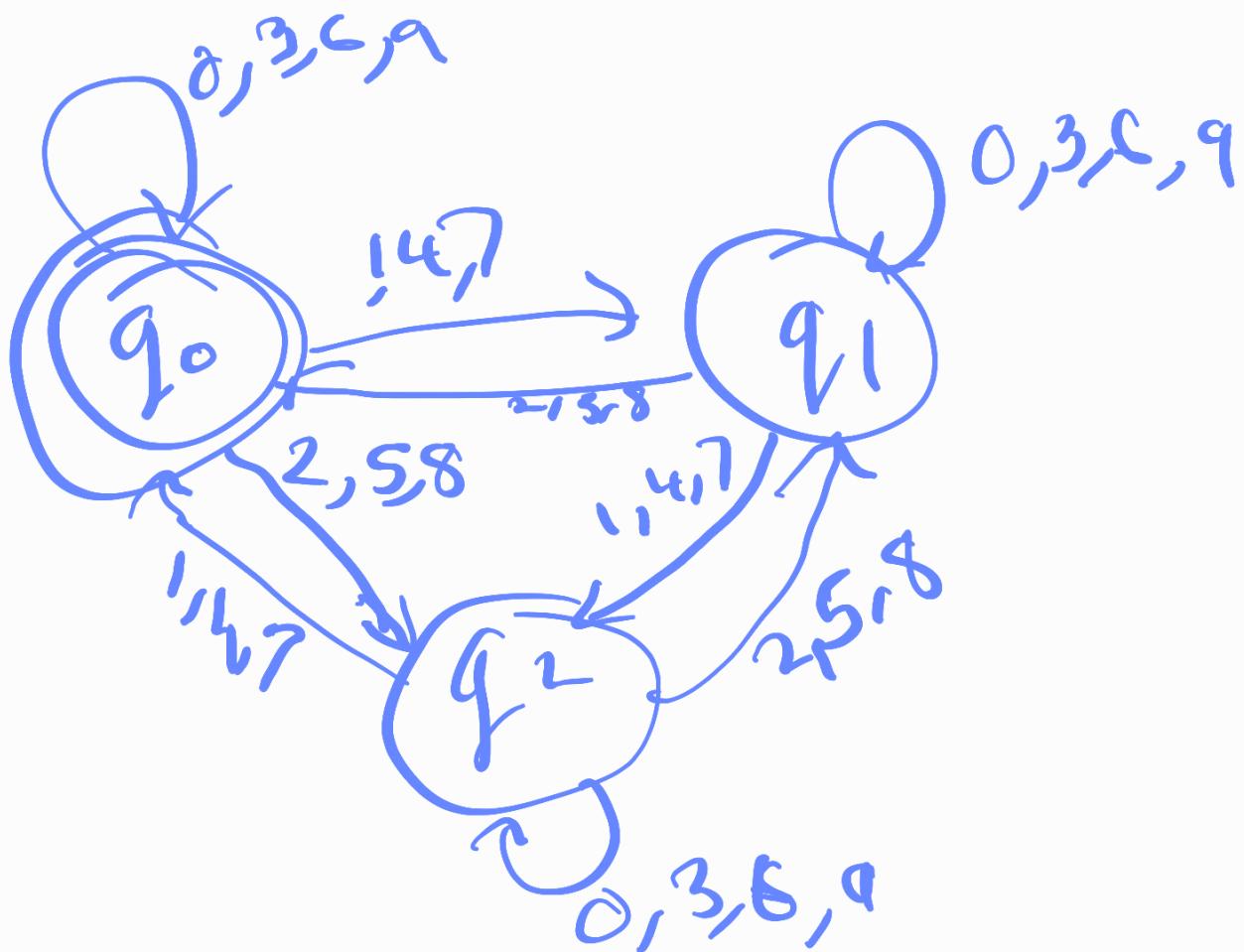


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FA  $\rightarrow$  FSM

$$I = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$O = \Sigma Y_N \}$$



$$0/3 = 3/3 = 6/3 = 0 \text{ remainder } \rightarrow q_0$$

$$1/3 = 4/3 = 7/3 = 1 \text{ remainder } \rightarrow q_1$$

$$2/3 = 5/3 = 8/3 = 2 \text{ remainder}$$

$\varphi \setminus \Sigma$	0, 3, 6, 9	1, 4, 7	2, 5, 8
rem 0	$q_0$	$q_1$	$q_2$
rem 1	$q_1$	$q_2$	$q_0$
rem 2	$q_2$	$q_0$	$q_1$

8869135

$$\delta(q_0, 6) = q_0 \quad \left| \begin{array}{l} \delta(q_0, 3) = q_0 \\ \delta(q_0, 5) = q_2 \end{array} \right.$$

$$\delta(q_0, 8) = q_2$$

$$\delta(q_2, 6) = q_1$$

$$\delta(q_2, 9) = q_2$$

$$\delta(q_2, 1) = q_0$$

$\therefore$  The string ended on a non final state the string is not accepted in the language.

Design a P.F.A whether a number is divisible by 3 or not -

a) 4

b) 2

c) 5

$$10 = \frac{4}{4} = \frac{8}{4} = 0 \rightarrow \text{remainder}$$

$$\frac{1}{4} = \frac{5}{4} = \frac{9}{4} = 1 \rightarrow \text{remainder}$$

$$\frac{2}{4} = \frac{6}{4} = 2 \rightarrow \text{remainder}$$

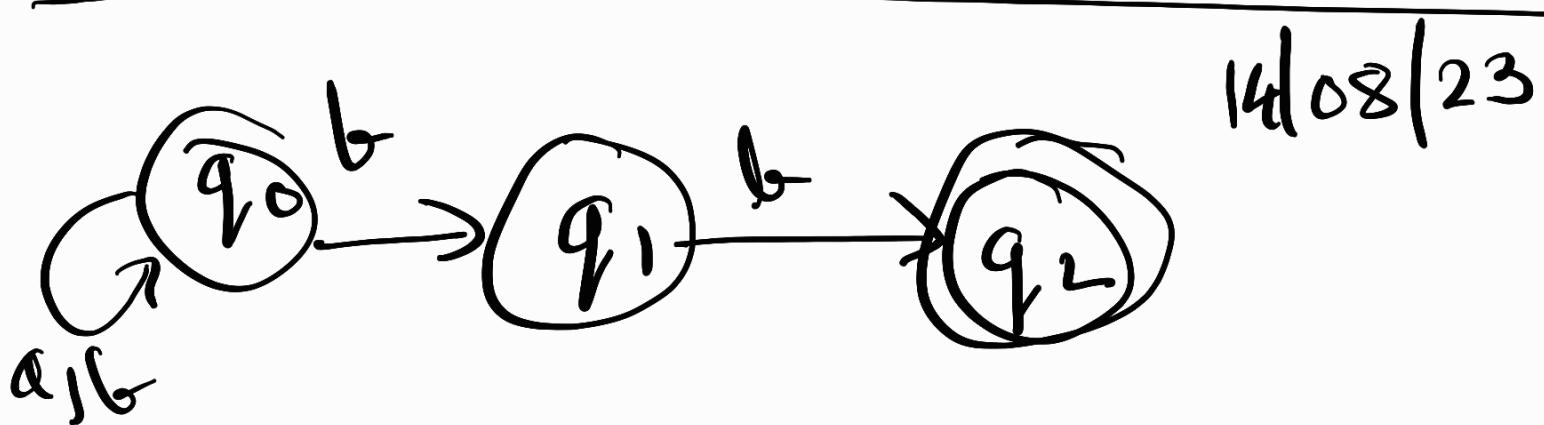
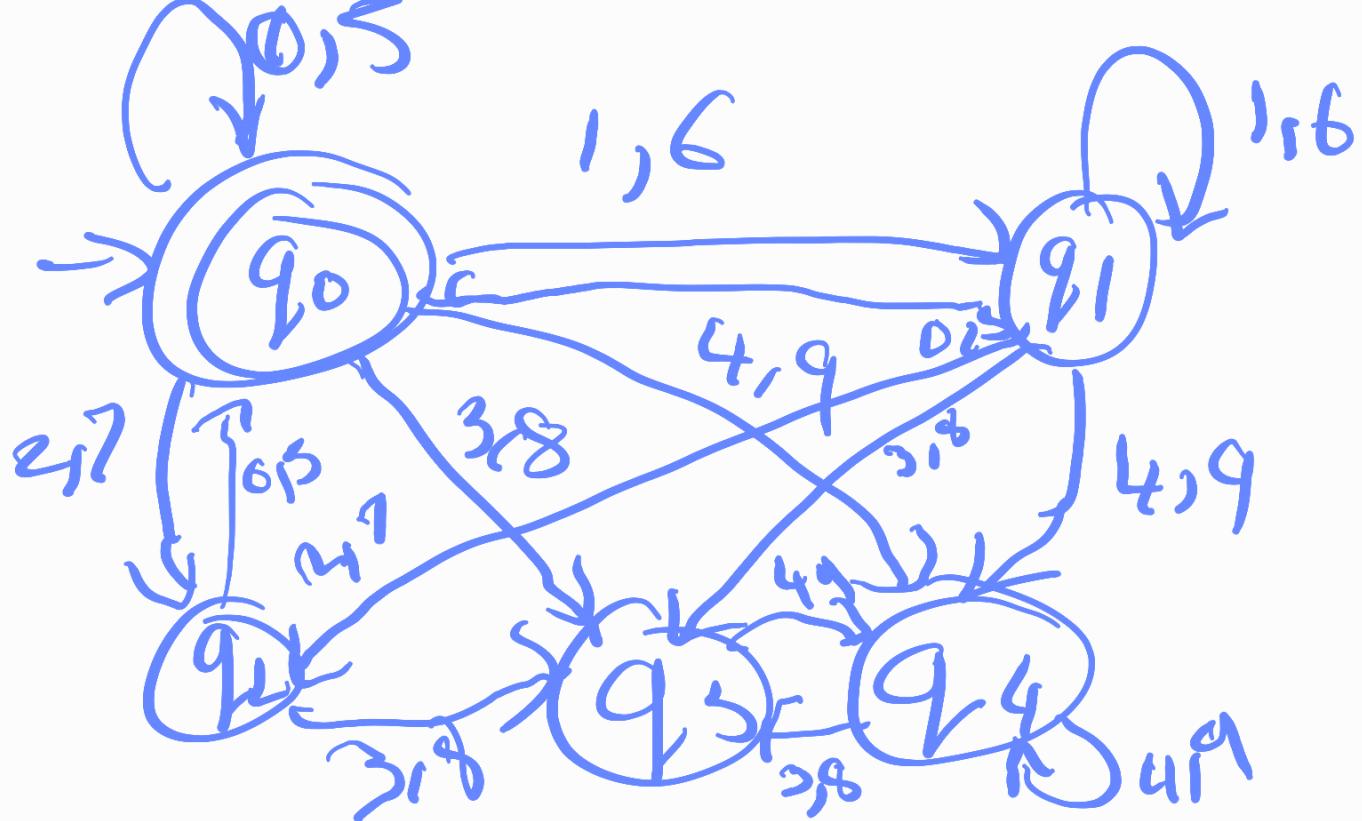
$$\frac{3}{4} = \frac{7}{4} = 3$$

	$\Sigma$	$0, 4, 8$	$1, 5, 9$	$2, 6$	$3, 7$
$rem^0 q_F^0$	$q_0$	$q_1$	$q_2$	$q_3$	
$rem^1 q_1$	$q_2$	$q_3$	$q_0$	$q_1$	
$rem^2 q_2$	$q_0$	$q_1$	$q_2$	$q_3$	
$rem^3 q_3$	$q_2$	$q_3$	$q_0$	$q_1$	
	$3, 7$	$1, 5, 9$	$2, 6$	$1, 5, 9$	

Diagram illustrating state transitions between states  $q_0, q_1, q_2, q_3$ :

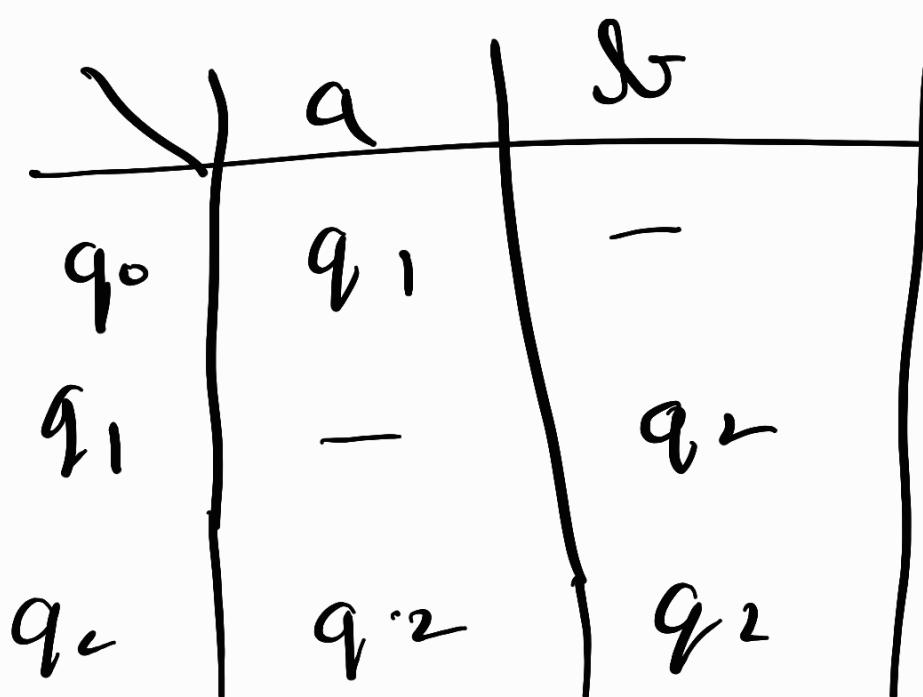
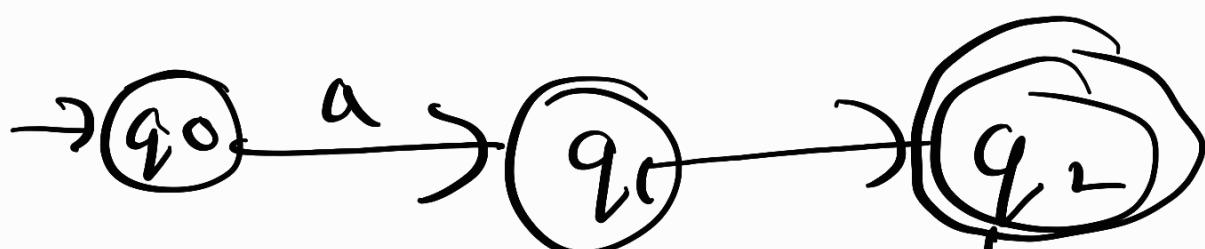
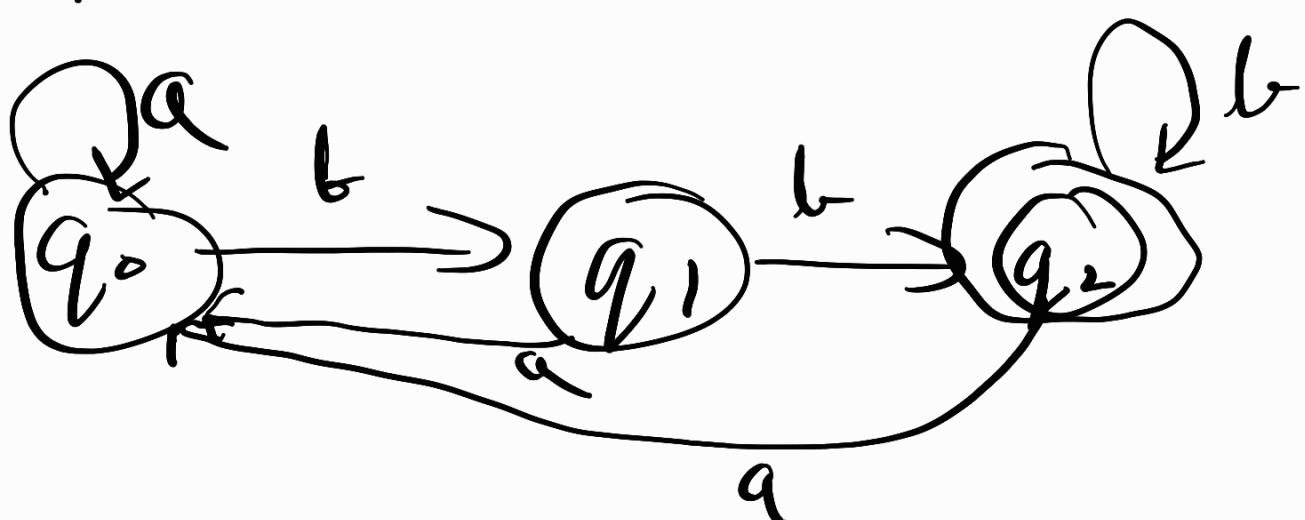
- From  $q_0$  to  $q_1$ : labeled  $1, 5, 9$
- From  $q_1$  to  $q_2$ : labeled  $0, 4, 8$
- From  $q_2$  to  $q_3$ : labeled  $2, 6$
- From  $q_3$  to  $q_0$ : labeled  $2, 6$
- From  $q_0$  to  $q_2$ : labeled  $2, 6$
- From  $q_1$  to  $q_3$ : labeled  $2, 6$
- From  $q_2$  to  $q_1$ : labeled  $2, 6$
- From  $q_3$  to  $q_0$ : labeled  $2, 6$

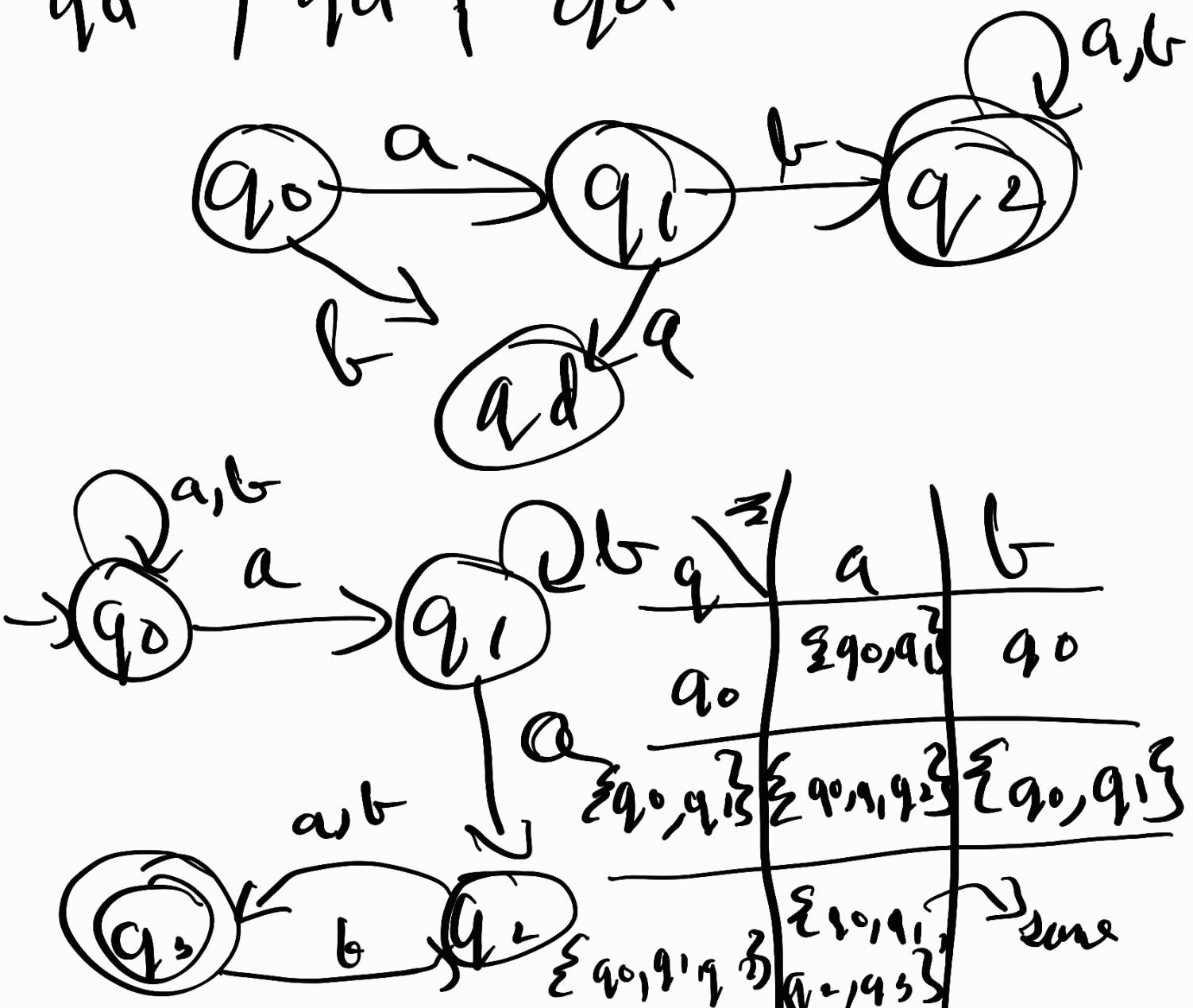
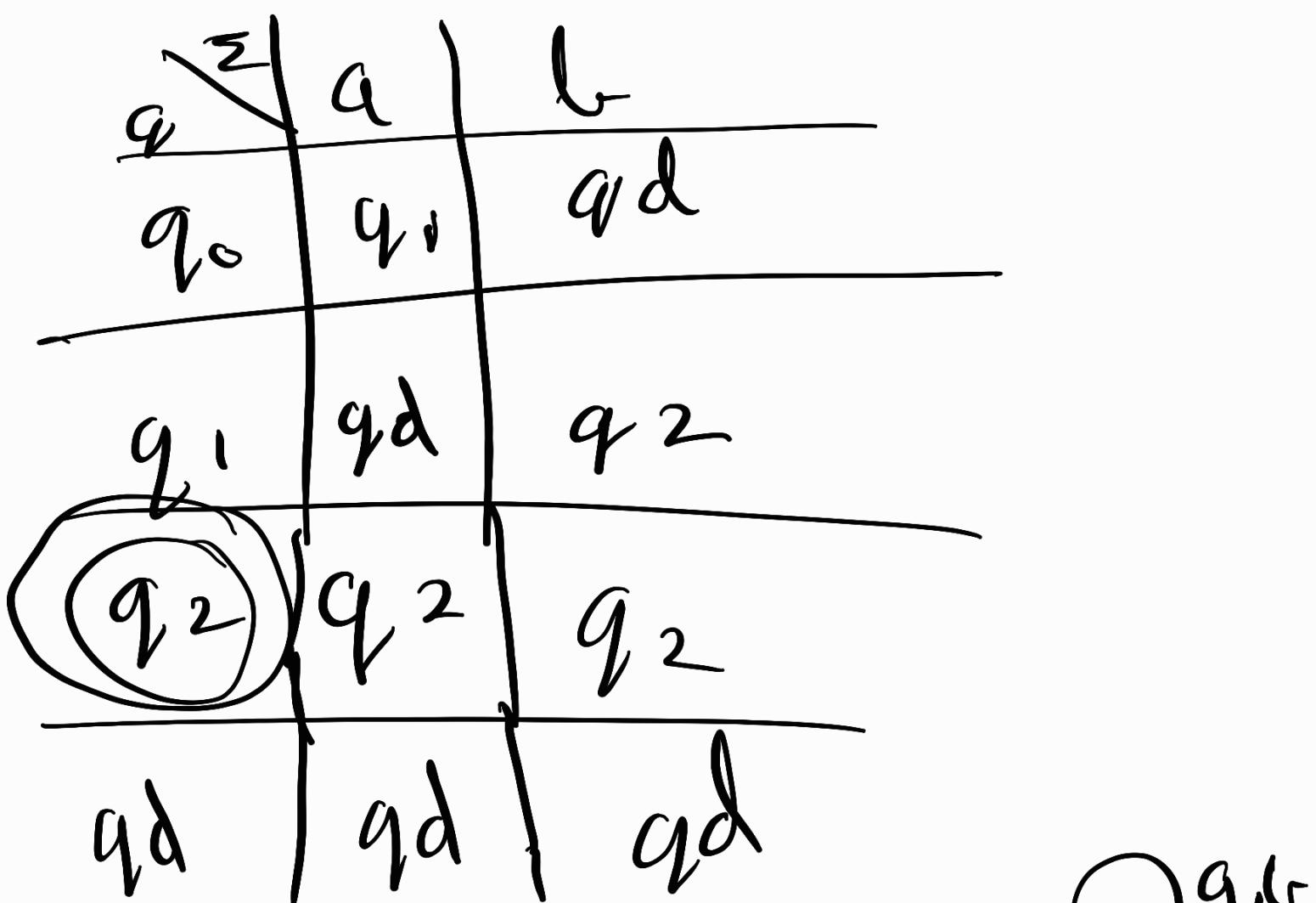


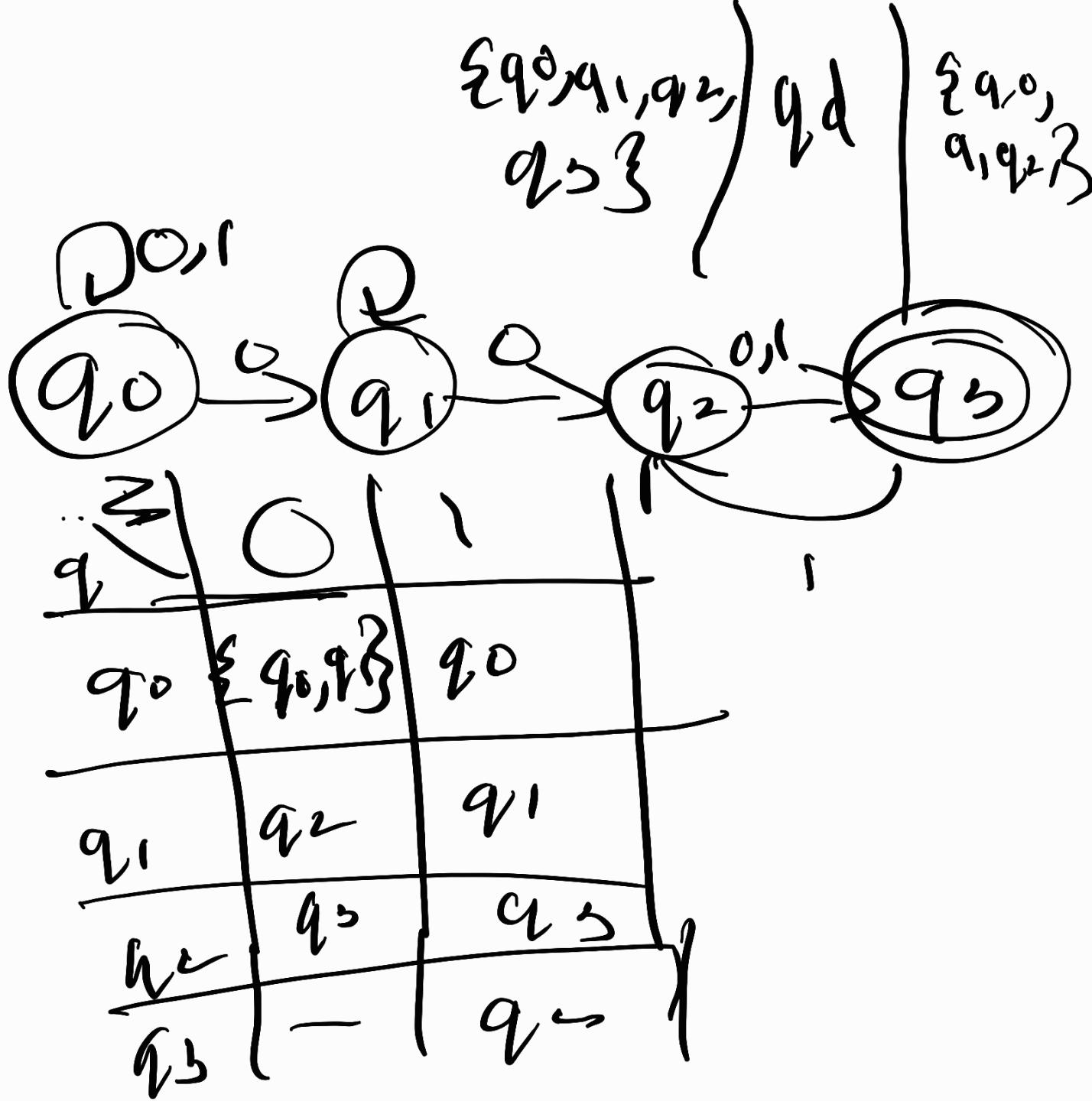


$a$	$\Sigma$	$a$	$b$
$q_0$	$q_0$	$\Sigma_{q_0, q_1, 3}$	
$q_1$	-	$q_2$	
$q_2$	-	-	-

$q_0$	$q_0$	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$q_0$	$\{q_0, q_1, q_2\}$
$\{q_0, q_1, q_2\}$	$q_0$	$\{q_0, q_1, q_2\}$



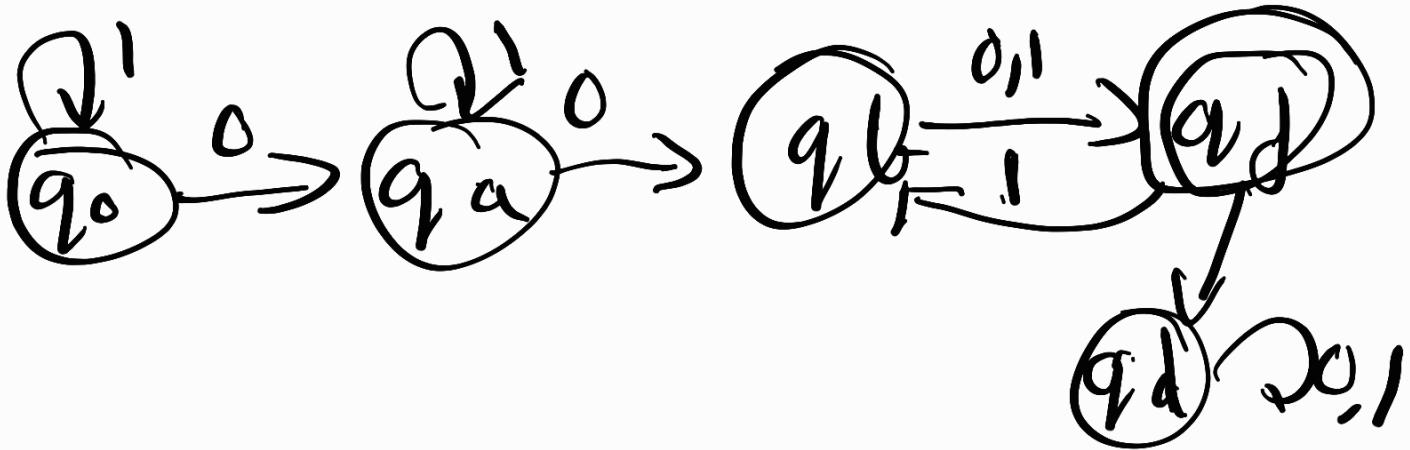




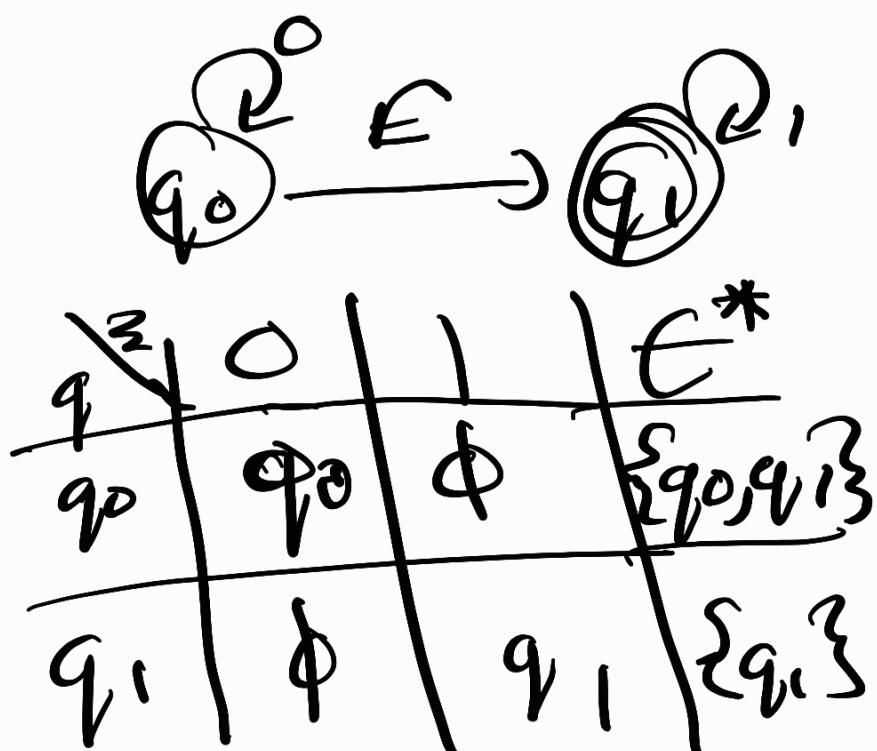
<del>q</del>	0	1
$q_0$	$\{q_0, q_1\}$	$q_0$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_3\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_3\}$	

Bottom row labels:

- $q_0, q_1, q_2, q_3$
- $q_0, q_1, q_3$
- $q_0, q_1, q_2$



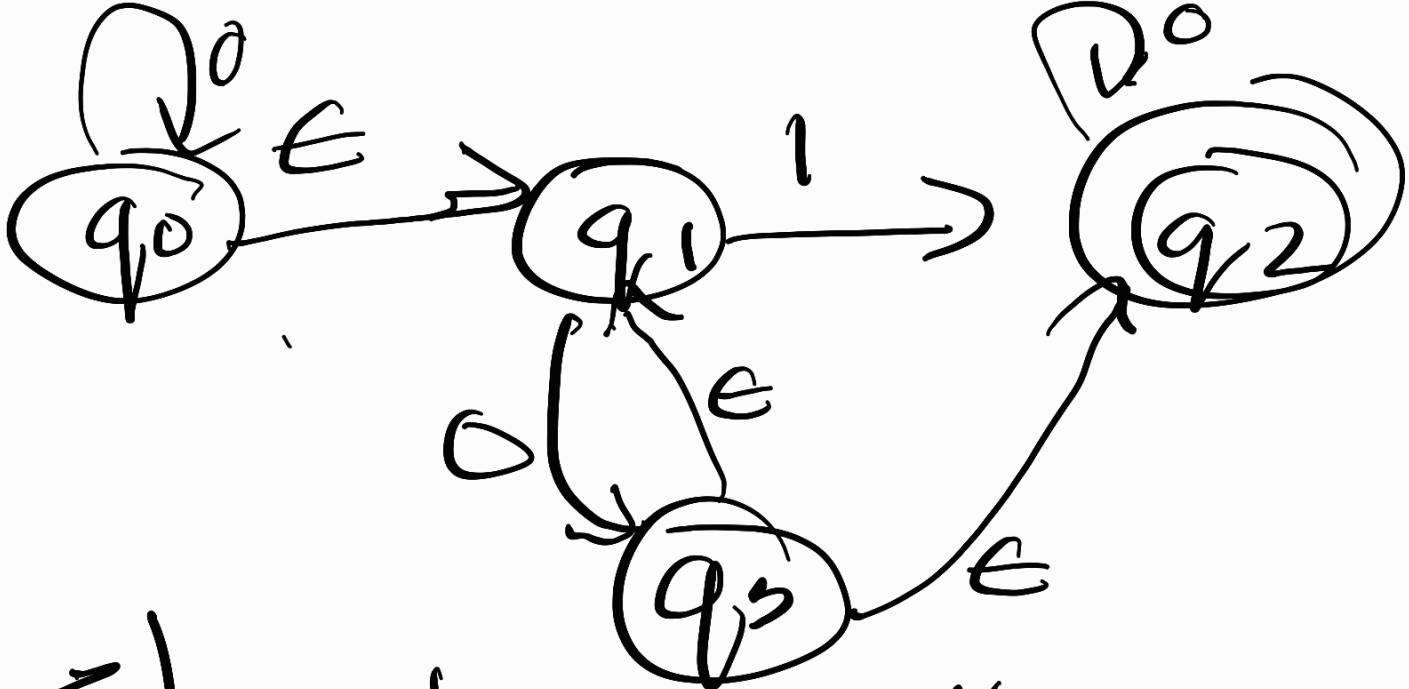
$$S' = \epsilon\text{-close}(\delta(\epsilon\text{-close}(q_0), 2))$$



$$\delta'(q_0, 0) = \epsilon\text{-close}(\delta(\epsilon\text{-close}(q_0), 0))$$

$$= \epsilon\text{-close}(\delta(q_0, q_1, 0))$$

$$= \epsilon\text{-close}(q_0) = \{q_0, q_1\}$$



$q_1$	$0$	$1$	$\Sigma^*$
$q_0$	$q_0$	$\emptyset$	$\{q_0, q_1\}$
$q_1$	$q_2$	$q_2$	$\{q_1\}$
$q_2$	$q_2$	$\emptyset$	$\{q_2\}$
$q_3$	$\emptyset$	$\emptyset$	$\{q_1, q_2, q_3\}$

$$S'(q_0, 0) = \{q_0, q_1\} \rightarrow S_0 \\ - \rightarrow \{q_0, q_3\}$$

$$\delta'(q_0, 1) = \{q_0, q_1\} \rightarrow 1$$

$\rightarrow q_2$

$$\delta'(q_1, 0) = \{q_1\} \rightarrow SD$$

$\rightarrow q_3 \Rightarrow \{q_1, q_2, q_3\}$

$$S(q_1, 1) = \{q_1\} \rightarrow S1$$

$q_2 \rightarrow \{q_2\}$

$$S(q_2, 0) = \{q_2\} \rightarrow SO$$

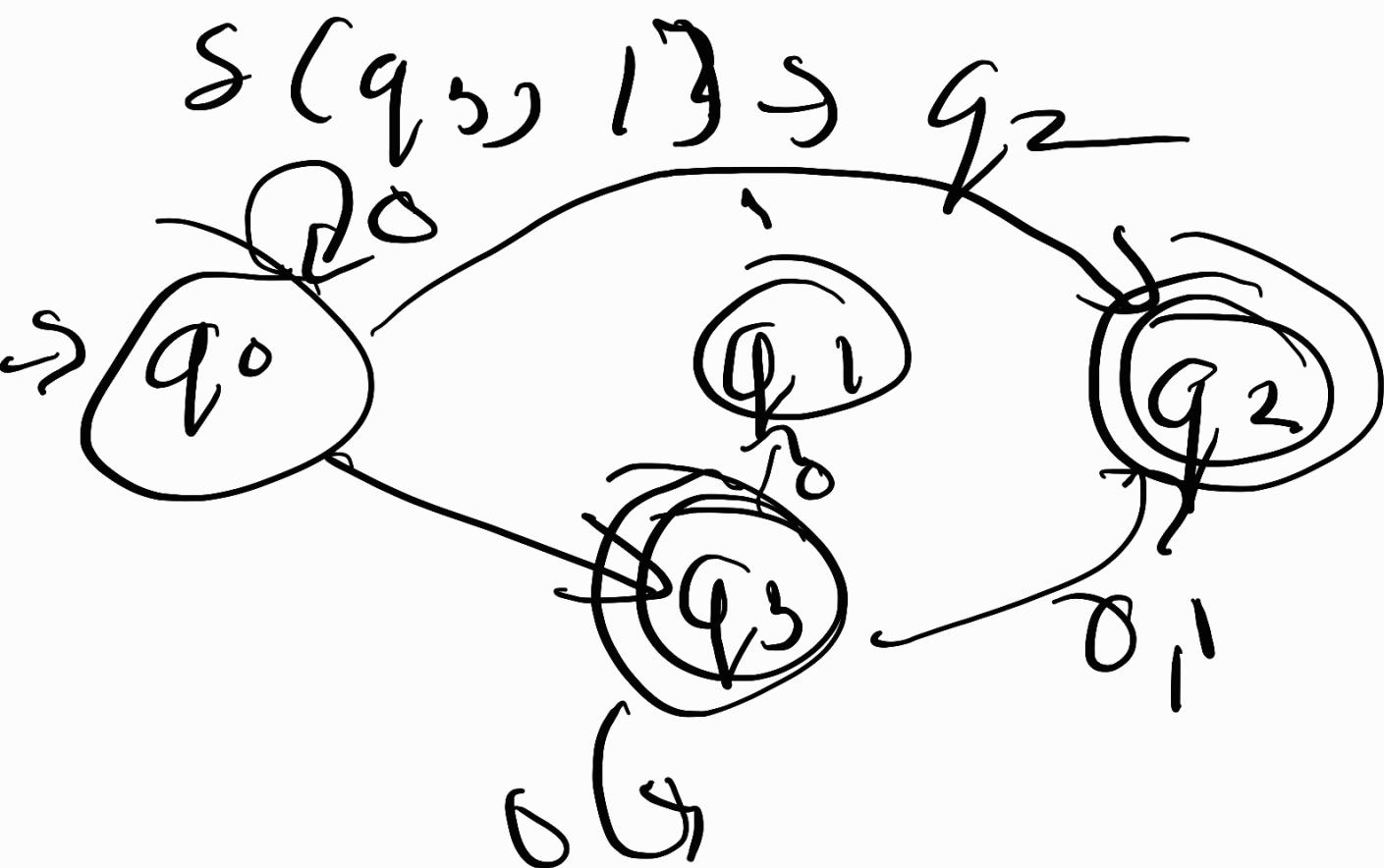
$\rightarrow q_2$

$$S(q_2, 1) = \{q_2\} \rightarrow S1$$

$\rightarrow \emptyset$

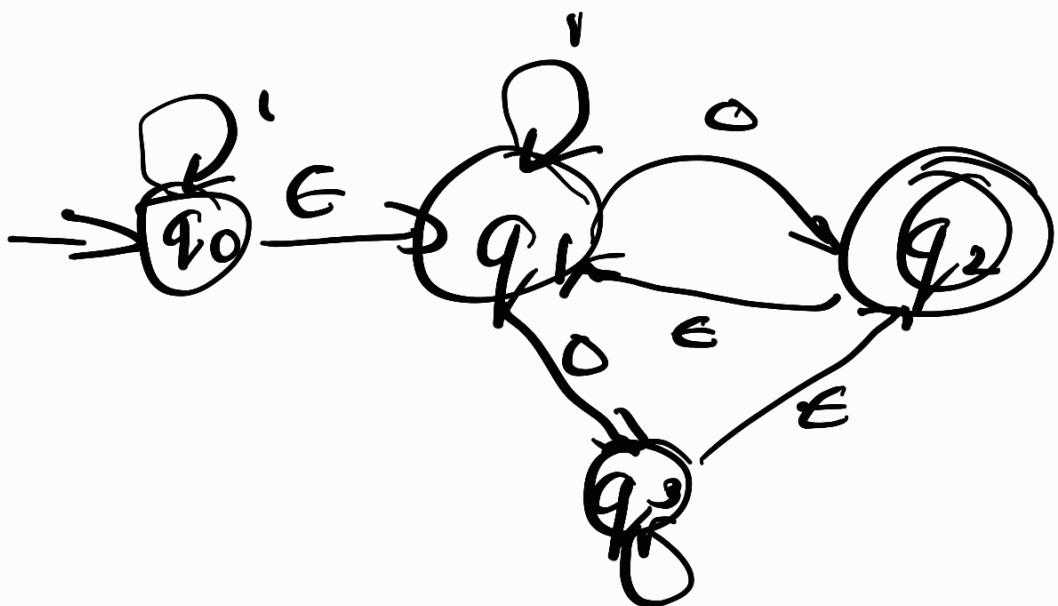
$$S(q_3, 0) = q_1 q_2 q_3 \rightarrow 80$$

$$\begin{aligned} & q_3 q_2 \\ & = \{q_1 q_2 q_3\} \end{aligned}$$



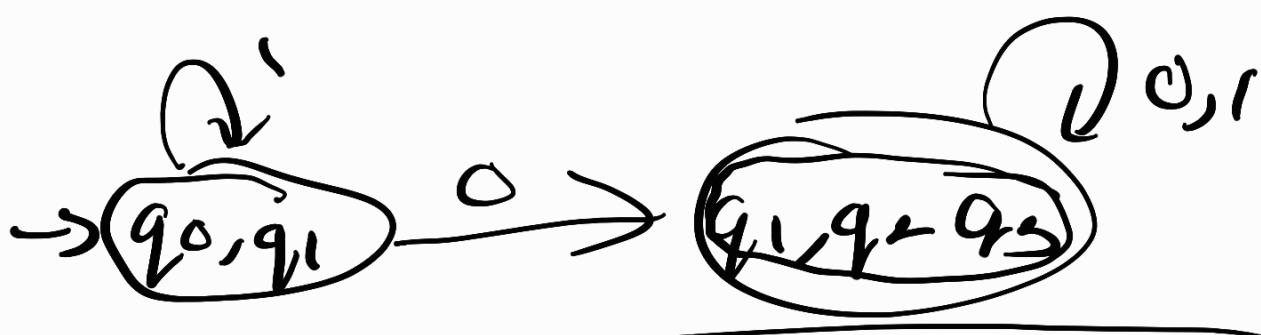
ENFA  $\rightarrow$  DFA

$S^*(q_0, \alpha) = \epsilon\text{-closure}$   
 $(S(q_0, \alpha))$



$a/\epsilon$	$\emptyset$	$I$	$\epsilon^*$
$q_0$	$\emptyset$	$q_0$	$q_0, q_1$
$q_1$	$\{q_2, q_3\}$	$q_1$	$q_1$
$q_2$	$\emptyset$	$\emptyset$	$\Sigma_{q_2, q_3}$

$$q_0 \left\{ \notin \left\{ q_0 \right\} \right\} \{q_2, q_1, 3$$



Kleen  
closure

$$L = \{ \epsilon, a, aa, a^2 \}$$

↳ for any symbol you can add that symbol any number of times.

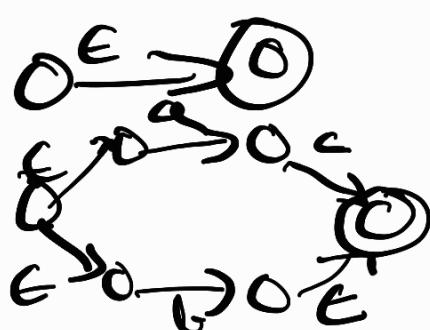
$a^+$  → positive closure

$$L = \{ a, aaaa \}$$

same as above but does not include  $\epsilon$

$$\lambda = \epsilon$$

$$\lambda = a+b$$



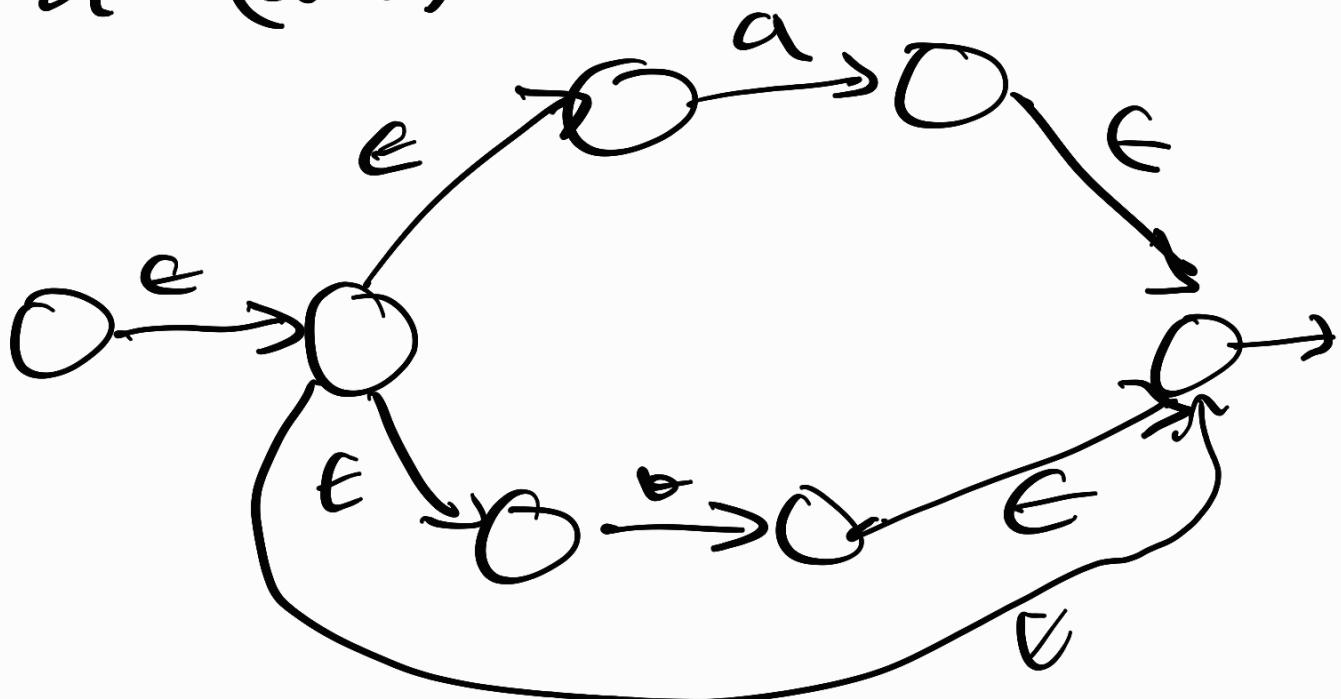
$$g = a \cdot b$$



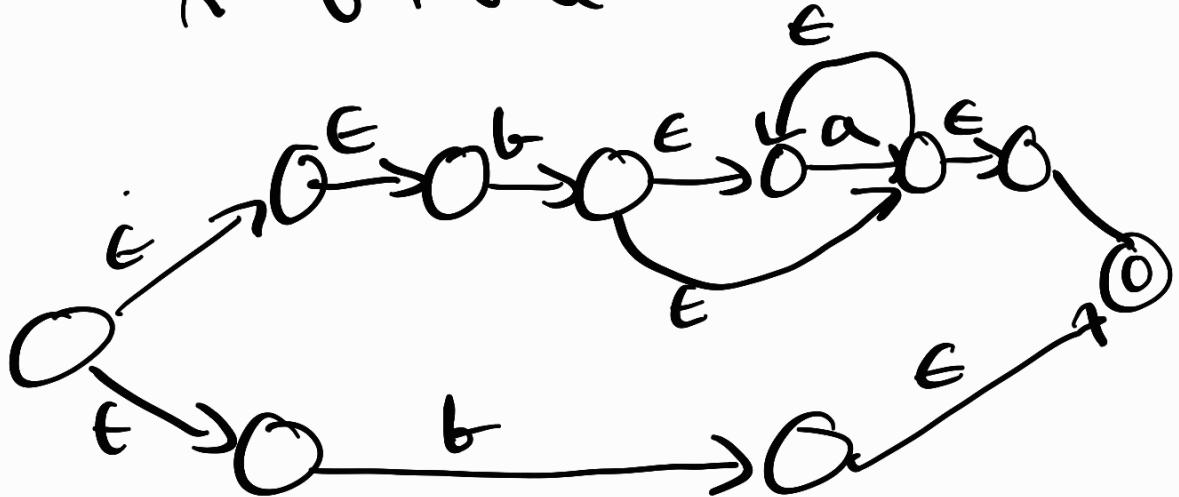
$$g = a^* \rightarrow \{ \epsilon, a, aa, aaaa \}$$



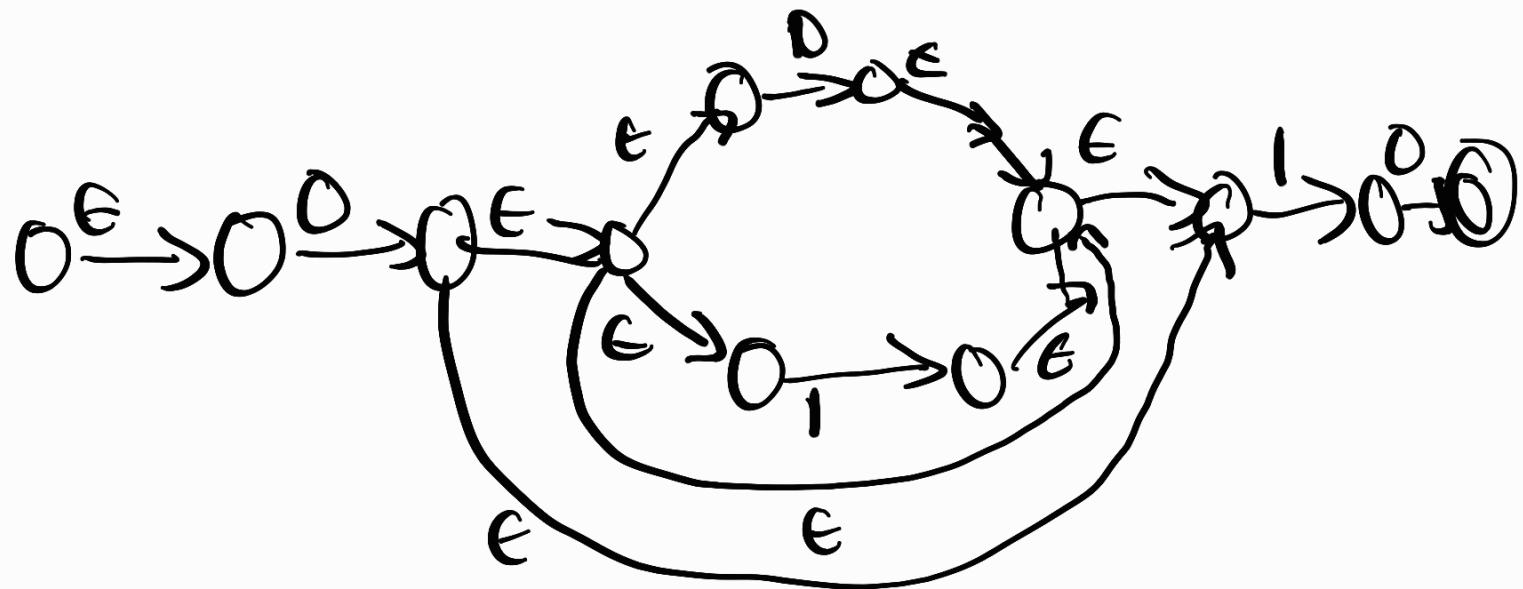
$$d = (a+b)^*$$



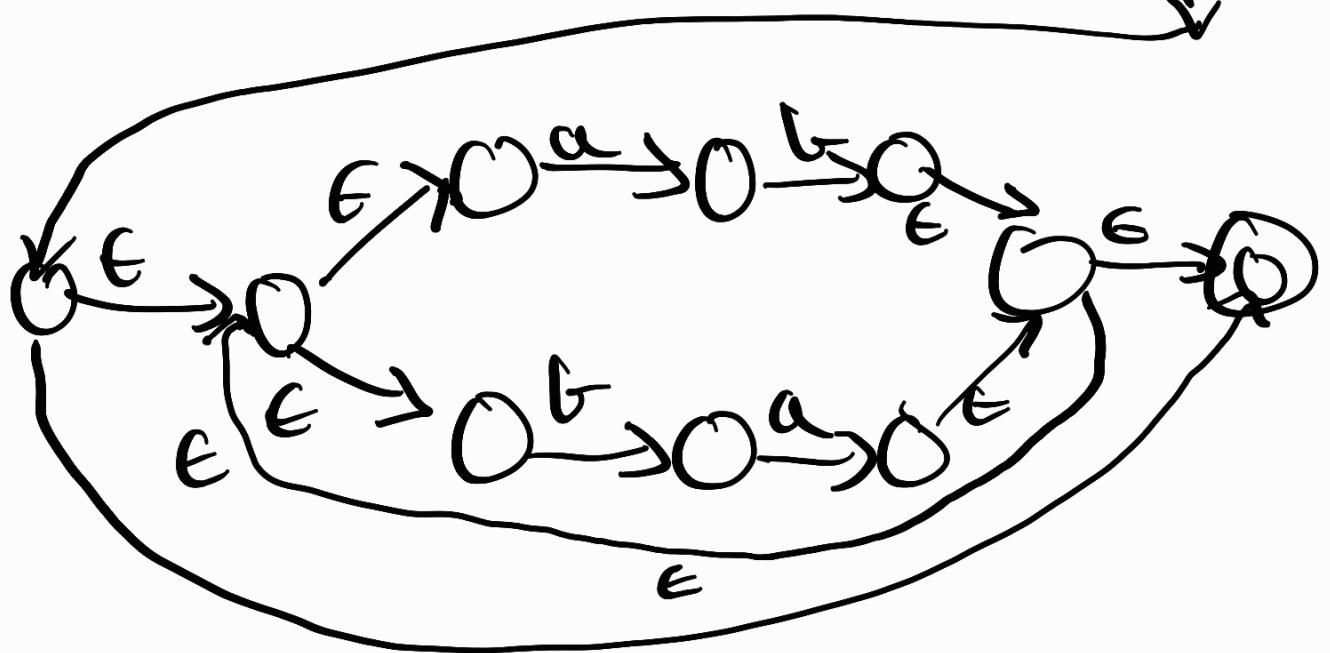
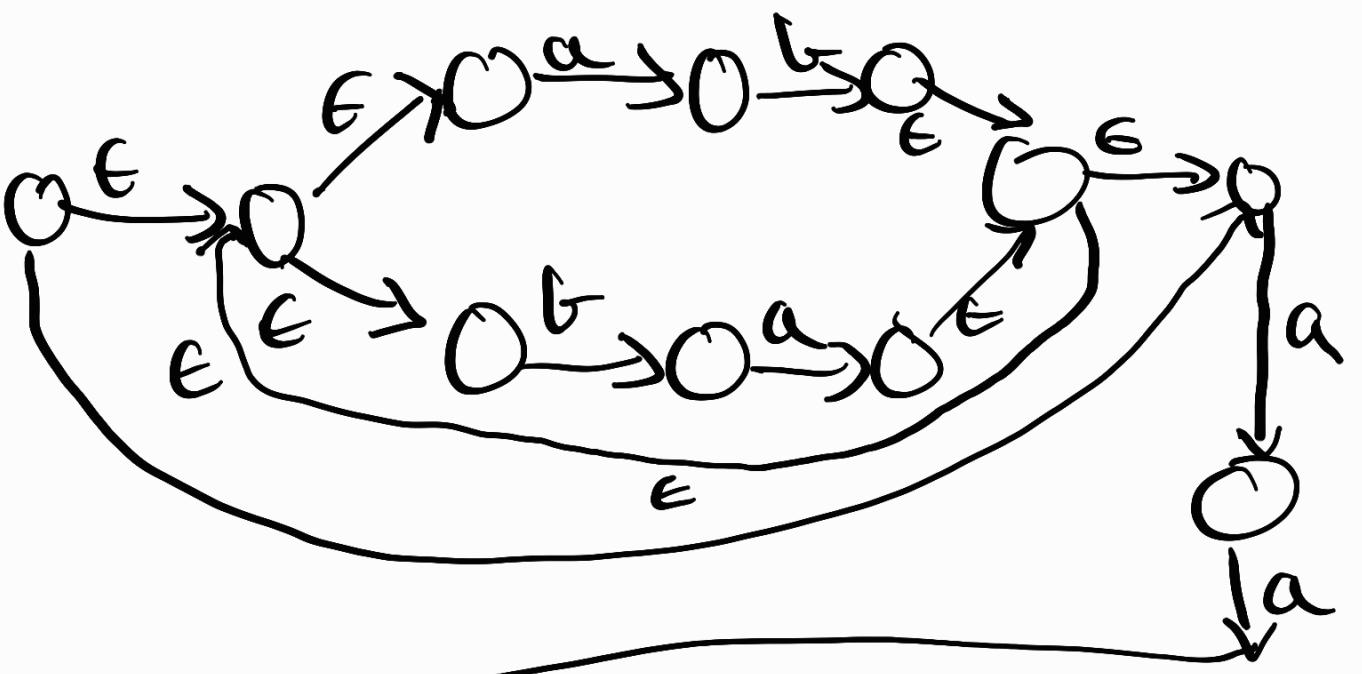
$$r = b + ba^*$$



$$r = \emptyset \cdot (\emptyset + 1)^* \cdot 1 \cdot \emptyset$$



$$r = (ab + ba)^* \cdot a \cdot a \cdot (ab + ba)^*$$



Starting with  $ab$

$$ab(a+b)^t$$

starts with bb a

$$L = b \cdot b \cdot a \cdot (a+b)^*$$

long ends with abb

$$L = (a+b)^* \cdot a \cdot b \cdot b$$

long string that contained

$$\underline{(a+b)^* a a b (a+b)^*}$$

starts and ends with a

$$L = a \cdot (a+b)^* \cdot a + a$$

Starts and ends with  
same symbol

$$\begin{aligned} L &\in \left( a \cdot (a+b)^* \cdot a + b \right) \\ &+ \left( b \cdot (a+b)^* \cdot b + a \right) \\ &= (a \cdot (a+b)^* \cdot a) + (b \cdot (a+b)^* \\ &\quad + b) + (a+b) \end{aligned}$$

starts and ends with different symbols

$$\delta = (a \cdot (a+b)^* b) + (b \cdot (a+b)^* a)$$

---

$$|w| = 3 \text{ for } \Delta w$$

$$\delta = (a+b) \cdot (a+b) \cdot (a+b)$$

$$|w| \geq 3$$

$$\delta = (a+b) \cdot (a+b) \cdot (a+b) \cdot (a+b)^*$$

$$|w| \geq 3$$

$$\delta = (a+b) + (a+b)^2 + (a+b)^3 + \dots$$

$$(\Delta w)_a = 2$$

$$|\Delta w|_a \geq 2$$

$$|\Delta w|_a \leq 2$$

3rd symbol from the left  
is 'b'

28th symbol from right is 'a'