

## Finite Automata to RE

Method 1  $\Rightarrow$  Arden's Theorem

If  $P$  &  $Q$  are two RE over  $\Sigma$  and if  $P$  does not contain  $\epsilon$  then the following equation in  $R$  given by

$R = Q + RP$  has unique solution  $R = QP^*$

$$\Rightarrow R = Q + (Q + RP)P$$

$$= Q + QP + RP^2$$

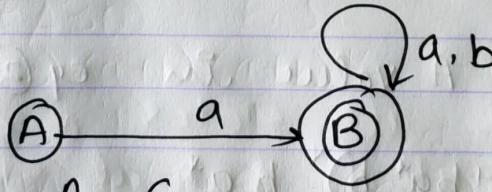
$$= Q + QP + (Q + RP)P^2$$

$$= Q + QP + QP^2 + RP^3$$

$$= Q(P^0 + P^1 + P^2 + P^3 + \dots)$$

$$\therefore \boxed{R = QP^*}$$

eg



$$A = \epsilon$$

$$B = Aa + Ba + Bb$$

As we know

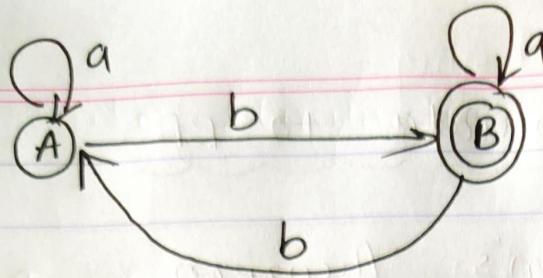
$$= \epsilon a + B(a+b)$$

$$B = a + B(a+b)$$

$$R = Q + RP$$

$$\Rightarrow \boxed{R = QP^*}$$

$$B = a + (a+b)^*$$



Eq 1

$$A = \epsilon + Aa + Bb \quad - \textcircled{1}$$

$$B = \underbrace{Ab + Ba}_{R} \quad - \textcircled{2} \quad R = Q + RP$$

$$B = Ab a^* \quad - \textcircled{3}$$

Substitute B in eq ①

$$A = \epsilon + Aa + Ab a^* b$$

$$A = \epsilon + A(a + ba^* b)$$

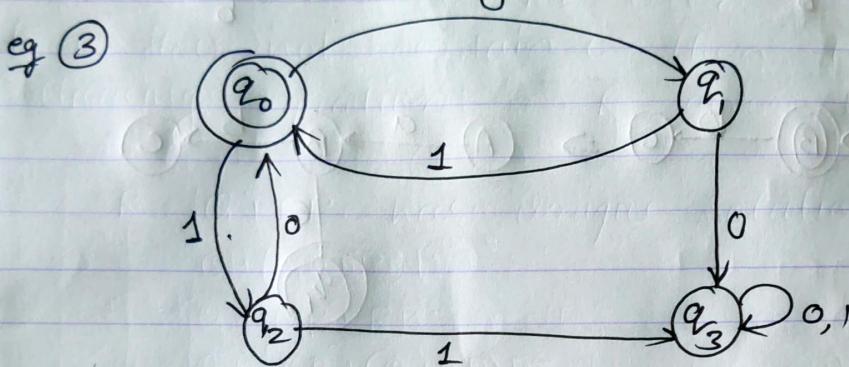
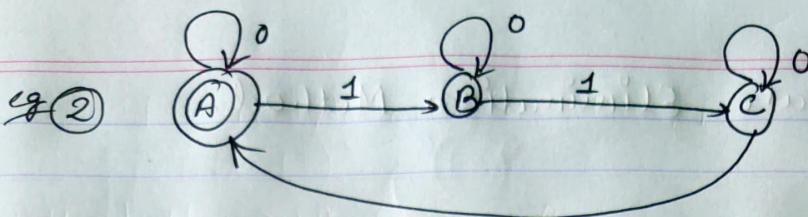
$$R = Q + RP$$

~~∴ RECURS~~

$$\therefore AR = \epsilon (a + ba^* b)^* \quad - \textcircled{4}$$

Substitute A value in eq ③

$$\therefore B = (a + ba^* b)^* b a^*$$



$$q_0 = q_1 \cdot 1 + q_2 \cdot 0 + e \quad - \textcircled{1}$$

$$q_1 = q_0 \cdot 0 \quad - \textcircled{2}$$

$$- \textcircled{3}$$

$$q_2 = q_0 \cdot 1$$

$$q_3 = q_1 \cdot 0 + q_2 \cdot 1 + q_3 \cdot 0 + q_3 \cdot 1$$

$$R = Q + RP$$

$$R = QP$$

Put the values of  $q_1$  &  $q_2$  in eq ①

$$q_0 = q_0 \cdot 0 \cdot 1 + q_0 \cdot 1 \cdot 0 + e$$

$$q_0 = q_0(01 + 10) + e$$

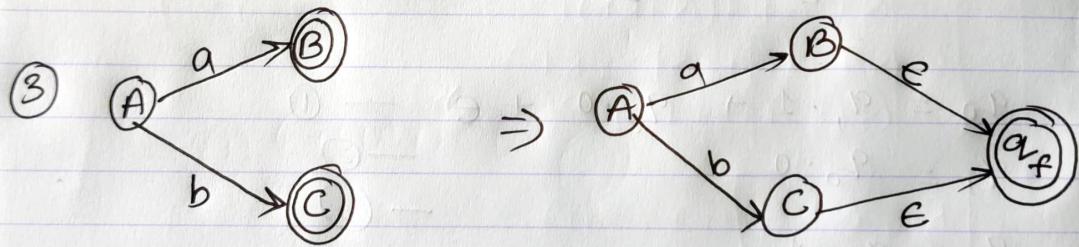
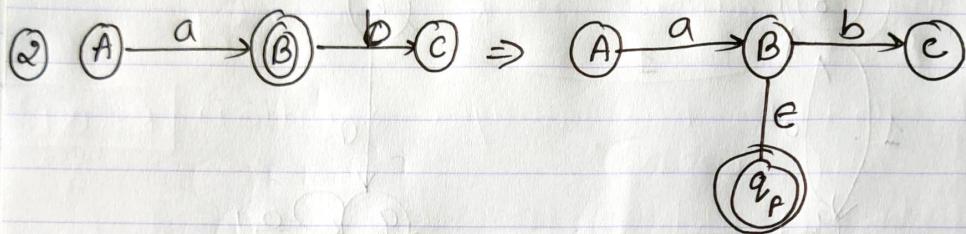
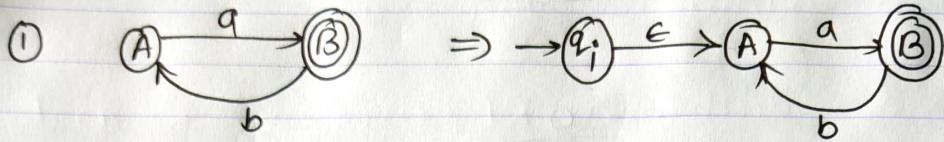
$$R = R + P + Q$$

$$\Rightarrow q_0 = e(01 + 10)^*$$

$$q_0 = (01 + 10)^*$$

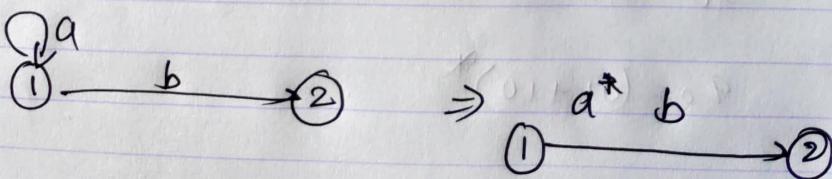
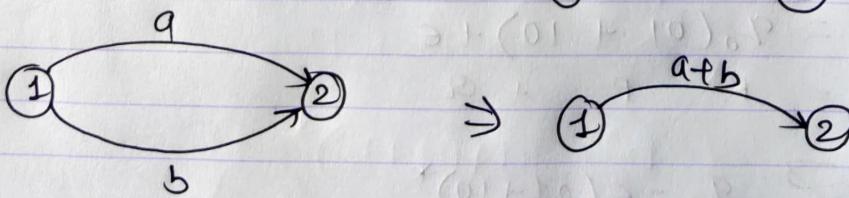
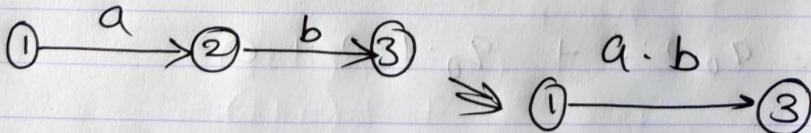
## Method-2 State Elimination Method

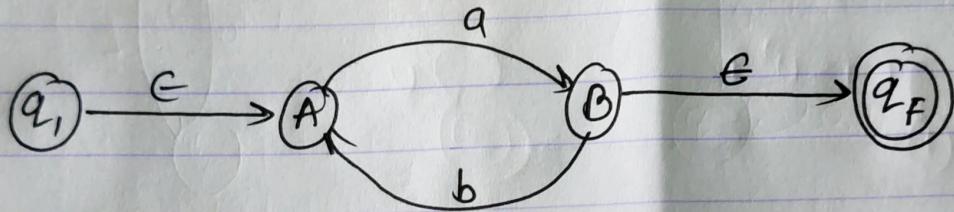
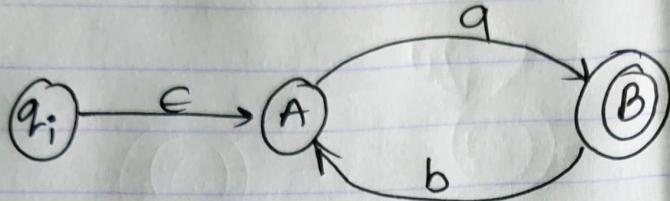
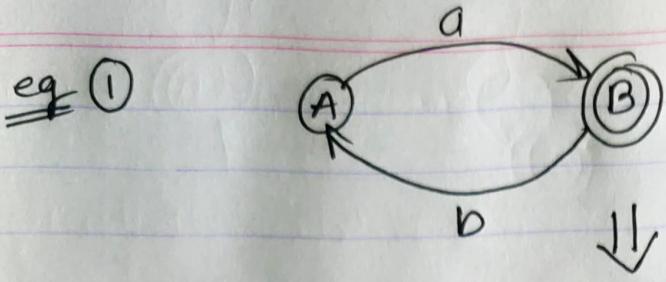
### Rules



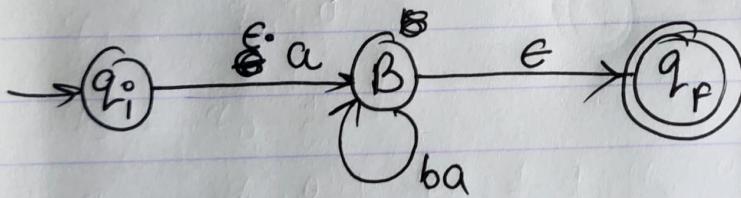
④ Eliminate all states one by one except initial & final state.

### Rules

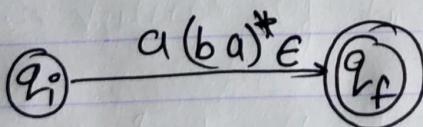




Eliminate ①

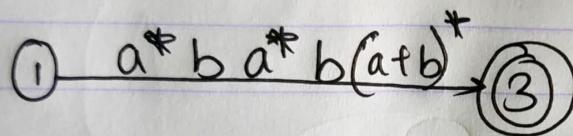
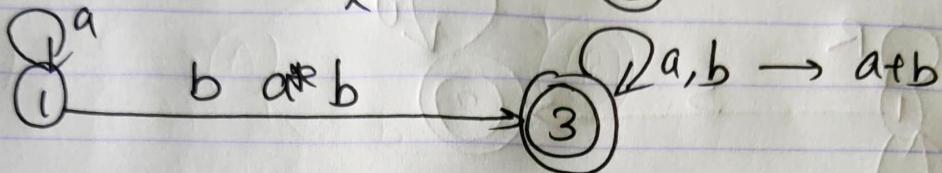
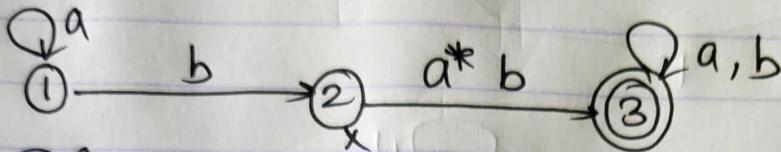
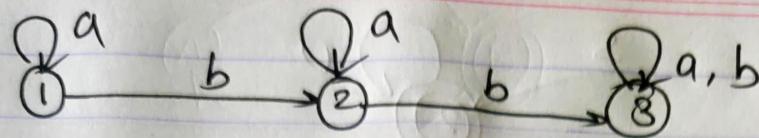


Eliminate ②

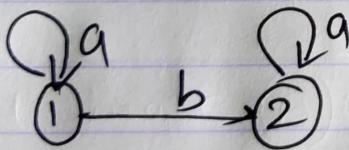
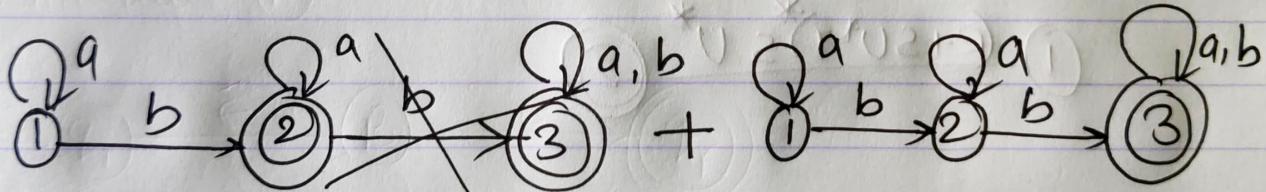
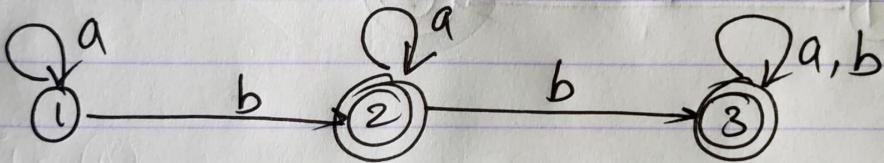


a(ba)\*

eg ①

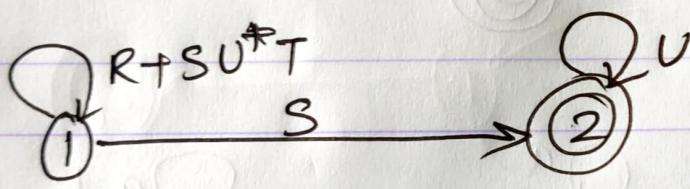
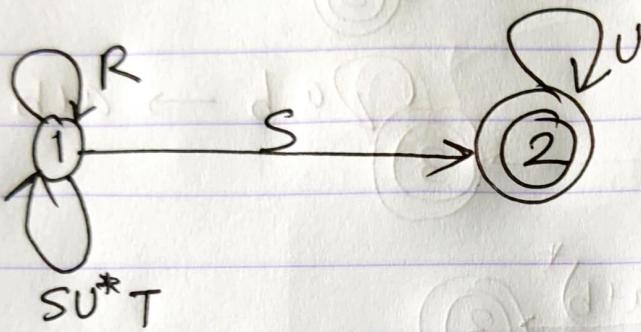
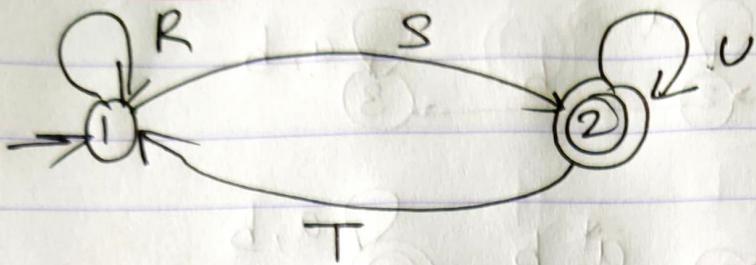


eg ②



$$a^* b a^* + a^* b a^* b (a+b)^*$$

eg 3



$$① \underline{(R + SU^* T)^* S} \quad \underline{U^*} \quad ②$$

