Formulae Queuing Theory

Performance measures of Queuing System

W_q- average time an arriving customer has to wait in a queue before being served

W_s- average time an arriving customer spends in the system including waiting in queue and being served

L_q- average number of customers has to wait in a queue before being served

L_s- average number of customers in the system including waiting in queue and being served

Other Notations

n-number of customers in the system including waiting in queue and being served
P_n- probability of n customers in the system
P_0- probability of no customers in the system (idle time)
$1 - P_0$ - probability that an customer has to wait in the system (system is busy)
λ- average number of arrivals per unit time in the system
μ - average number of customers served per unit time in the system

$$\rho = \frac{average\ service\ completion\ time(1/\mu)}{average\ interarrival\ time(1/\lambda)} = \frac{\lambda}{\mu}$$

= traffic intensity or service utilization factor
s – number of service channels (servers)
N – maximum number of customers allowed in the system

Model-1
Infinite Queuing Model
(M/M/s):(∞,FCFS)

Formulae for (M/M/s):(∞,FCFS) model

- $\rho = \frac{\lambda}{\mu}$ = traffic intensity or service utilization factor
- $P_0 = 1 - \rho$ =probability of no customers in the system (idle time)
- $P_n = \rho^n P_0 = \rho^n(1-\rho)$
- $P(n \geq k) = \rho^k$ and $P(n > k) = \rho^{k+1}$
- $L_s = \frac{\rho}{1-\rho}$
- $L_q = L_s - \rho = \frac{\rho^2}{1-\rho}$
- $W_s = \frac{L_s}{\lambda}$
- $W_q = \frac{L_q}{\lambda}$
- $P(W_s > t) = e^{-\mu(1-\rho)t}$
- $P(W_q > t) = \rho\, e^{-\mu(1-\rho)t}$
- We can derive that $W_s = W_q + \frac{1}{\mu}$ and $L_s = L_q + \frac{\lambda}{\mu}$

Model II
finite Queuing Model
(M/M/1):(N,FCFS)

Formulae for (M/M/s):(N,FCFS) model

- $\rho = \frac{\lambda}{\mu}$ = traffic intensity or service utilization factor
- $P_0 = \begin{cases} \frac{1-\rho}{1-\rho^{N+1}} & if\, \rho \neq 1 \\ \frac{1}{N+1} & if\, \rho = 1 \end{cases}$ =probability of no customers in the system (idle time)
- $P_n = \rho^n P_0$
- $L_s = \begin{cases} \frac{\rho}{1-\rho} - \frac{(N+1)\rho^{N+1}}{1-\rho^{N+1}} & if\, \rho \neq 1 \\ \frac{N}{2} & if\, \rho = 1 \end{cases}$
- $L_q = L_s - \rho$
- $W_s = \frac{L_s}{\lambda(1-P_N)}$
- $W_q = \frac{L_q}{\lambda(1-P_N)}$
- λ_{eff}=effective arrival rate= $\lambda/(1 - P_N)$
- $\rho_{eff} = \frac{\lambda_{eff}}{\mu}$