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C04-IA-2

Q1 Find (i) All basic solutions (ii) All feasible solution (iii) All Degenerate solution hence ~~also~~ decide the optimal feasible basis for the following L.P.P.

maximize $z = 2x_1 + 3x_2 + x_3 + x_4$

Subjected to $x_1 + 2x_2 - x_3 + x_4 = 5$

$2x_1 + x_2 + 2x_3 - 2x_4 = 3$

where $x_1, x_2, x_3, x_4 \geq 0$

→ Here $m=2$ & $n=4$
 $\xrightarrow{\text{No. of eq.}} \quad \xrightarrow{\text{No. of variable}}$

no. of basic ~~sol~~ solutions $= {}^nC_m = {}^4C_2 = 6$

To get basic feasible solution we put $n-m$ Variable $= 0$

No. of basic sol.	Non basic variable	Basic Variable	eq. & value of basic variable	BFS	Deg	Value of z	optimal or
1	$x_3=0$ $x_4=0$	x_1 x_2	$x_1 + 2x_2 = 5$ $2x_1 + x_2 = 3$ } $x_1 = 1/3$ $x_2 = 7/3$	Yes	Non Deg	9.6	No
2	$x_2=0$ $x_4=0$	x_1 x_3	$x_1 - x_3 = 5$ $2x_1 + 2x_3 = 3$ } $x_1 = 3.25$ $x_3 = -1.75$	Non	-	-	-
3	$x_1=0$ $x_4=0$	x_2 x_3	$2x_2 - x_3 = 5$ $x_2 + 2x_3 = 3$ } $x_2 = 2.6$ $x_3 = 0.2$	Yes	Non Deg	8	No
4	$x_2=0$ $x_3=0$	x_1 x_4	$x_1 + x_4 = 5$ $2x_1 - 2x_4 = 3$ } $x_1 = 3.25$ $x_4 = 1.75$	Yes	Non Deg	8.25	Yes
5	$x_1=0$ $x_3=0$	x_2 x_4	$2x_2 + x_4 = 5$ $x_2 - 2x_4 = 3$ } $x_2 = 2.6$ $x_4 = -0.2$	No	-	-	-
6	$x_1=0$ $x_2=0$	x_3 x_4	$-x_3 + x_4 = 5$ $2x_3 - 2x_4 = 3$ } No solution	-	-	-	-

$$i) \quad x_1 = 1/3, \quad x_2 = 7/3, \quad x_3 = x_4 = 0$$

$$x_1 = 3.25, \quad x_2 = 1.75, \quad x_3 = x_4 = 0$$

$$x_2 = 2.6, \quad x_3 = 0.2, \quad x_1 = x_4 = 0$$

$$x_1 = 3.25, \quad x_4 = 1.75, \quad x_2 = x_3 = 0$$

$$x_2 = 2.6, \quad x_4 = -0.2, \quad x_1 = x_3 = 0$$

$x_1 = x_2 = 0, \quad x_3 = x_4 \rightarrow$ No solution. There are basic solution

ii) Feasible basic solution are.

$$x_1 = 1/3, \quad x_2 = 7/3, \quad x_3 = x_4 = 0$$

$$x_2 = 2.6, \quad x_3 = 0.2, \quad x_1 = x_4 = 0$$

$$x_1 = 3.25, \quad x_4 = 1.75, \quad x_2 = x_3 = 0$$

iii) There are no degenerate solution

iv) Optimal feasible basic ~~for~~ is $z = 8.25$

$$x_1 = 3.25, \quad x_2 = 1.75, \quad x_3 = x_4 = 0$$

Q2 Convert the given LPP into standard form
 minimize $Z = 7x_1 - 48x_2 + 23x_3$
~~max~~ Subject to $x_1 + 2x_2 - x_3 + x_4 = 5$
 $2x_1 + x_2 + 2x_3 - 2x_4 = 3$
 where x_i

minimize $Z = 7x_1 - 48x_2 + 23x_3$

Subject to

$$61x_1 - 29x_2 + 12x_3 \leq 93$$

$$3x_1 - 61x_2 + 81x_3 \geq 9$$

$$x_1 - 33x_2 + 53x_3 \leq -5$$

where $x_1, x_2 \geq 0$ & x_3 is unrestricted in sign.

→ Maximize $Z = -7x_1 + 48x_2 - 23x_3$
 (constraints)

$$61x_1 - 29x_2 + 12x_3 \leq 93$$

$$3x_1 - 61x_2 + 81x_3 \geq 9$$

$$x_1 - 33x_2 + 53x_3 \leq -5 \quad (\times -1) \Rightarrow \geq 5$$

$x_1, x_2 \geq 0$; x_3 is unrestricted.

Thus, while observing the problem we notice that x_3 is unrestricted.

$$x_1, x_2, x_3 = x_3' - x_3''$$

$$\text{where } x_1, x_2, x_3', x_3'' \geq 0$$

Such that

$$\rightarrow -x_1 + 33x_2 - 53x_3 \geq 5$$

$$61x_1 - 29x_2 + 12(x_3' - x_3'') + s_1 + 0s_2 + 0s_3 = 93$$

$$3x_1 - 61x_2 + 81(x_3' - x_3'') + 0s_1 - s_2 + 0s_3 = 9$$

$$-x_1 + 33x_2 - 53(x_3' - x_3'') + 0s_1 + 0s_2 - s_3 = +5$$

$$Z = -7x_1 + 48x_2 - 23(x_3' - x_3'') + 0s_1 + 0s_2 + 0s_3$$

$$x_1, x_2, x_3', x_3'', s_1, s_2, s_3 \geq 0$$

[Standard form]

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CO4-IA2

Q3 Solve the given LPP by simplex method
minimize $z = 3x_1 + 5x_2 + 4x_3$

Subject to

$$2x_1 + 3x_2 \leq 8$$

$$2x_2 + 5x_3 \leq 10$$

$$3x_1 + 2x_2 + 4x_3 \leq 15$$

$$\text{where } x_1, x_2, x_3 \geq 0$$

→ we first express the given problem in standard form

$$z - 3x_1 - 5x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3 = 0$$

$$2x_1 + 3x_2 + 0x_3 + S_1 + 0S_2 + 0S_3 = 8$$

$$0x_1 + 2x_2 + 5x_3 + 0S_1 + S_2 + 0S_3 = 10$$

$$3x_1 + 2x_2 + 4x_3 + 0S_1 + 0S_2 + S_3 = 15$$

We put this information in tabular form as follows.

Iteration number	Basic Variable	Coefficients							RHS Value	Ratio
0	S1 leave, x_1 enter	x_1	x_2	x_3	S_1	S_2	S_3			
R_0	2	-3	-5	-4	0	0	0		0	
R_1	S_1	2	3	0	1	0	0		8	$8/3 = 2.67$
R_2	S_2	0	2	5	0	1	0		10	$10/2 = 5$
R_3	S_3	3	2	4	0	0	1		15	$15/2 = 7.5$
1	S_2 leave, x_2 enter	x_1	x_2	x_3	S_1	S_2	S_3			
$R_0' = R_0 - 5R_2$	2	1/3	0	-4	5/3	0	0		40/3	
$R_1' = R_1 - 3R_2$	x_2	2/3	1	0	1/3	0	0		8/3	-
$R_2' = R_2 - 2R_1'$	S_2	-4/3	0	5	-2/3	1	0		14/3	$14/5$
$R_3' = R_3 - 2R_1'$	S_3	5/3	0	4	-2/3	0	1		29/3	$29/12$
2	S_3 leave, x_3 enter	x_1	x_2	x_3	S_1	S_2	S_3			
$R_0'' = R_0' + 4R_2'$	2	-11/15	0	0	17/15	4/3	0		256/15	
$R_1'' = R_1'$	x_2	2/3	1	0	1/3	0	0		8/3	4
$R_2'' = R_2'/5$	x_3	4/15	0	1	-2/15	1/5	0		14/15	-
$R_3'' = R_3' - 4R_2''$	S_3	11/15	0	0	-2/15	-4/5	1		89/15	$89/41$
3		x_1	x_2	x_3	S_1	S_2	S_3			
$R_0''' = R_0'' + 11/15 R_3''$	2	0	0	0	49/41	24/41	11/41		765/41	
$R_1''' = R_1'' - 2/3 R_3''$	x_2	0	1	0	5/41	8/41	10/41		50/41	
$R_2''' = R_2'' + 4/15 R_3''$	x_3	0	0	1	6/41	5/41	4/41		62/41	
$R_3''' = (11/15) R_3''$	x_1	1	0	0	-2/41	12/41	815/41		89/41	

$$\therefore x_1 = 89/41$$

$$x_2 = 50/41$$

$$x_3 = 62/41$$

$$Z_{max} = 765/41$$