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Batch: B1 Roll No.: 16010421073
Experiment / assignment / tutorial No. 2
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ITC Tutorial 2

Q.1 Encode the message "CAFFEINE" using Shanon- Fano

Q.2 Compress the message "HEARTBEAT" using 1) haffmaan encoding
2) Shanon Fano coding
Calculate and compare the code efficiency in both cases

Q.3 Define Kraft's inequality theoram. Check wether it is satisfying in question 1 and 2

Q.1 x_i Px_i

word -"CAFFEINE"

Now $P(C) = \frac{1}{8}$

$P(A) = \frac{1}{8}$

$P(F) = \frac{1}{4}$

$P(E) = \frac{1}{4}$

$P(I) = \frac{1}{8}$

$P(N) = \frac{1}{8}$

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x_i	Px_i	col1	col2	col3	code	codeword length
E	0.25	0	0		00	2
F	0.25	0	1		01	2
A	0.125	1	0	0	100	3
C	0.125	1	0	1	101	3
N	0.125	1	1	0	110	3
I	0.125	1	1	1	111	3

$\bar{L} = P \times n$

$= 0.25 \times 2 + 0.25 \times 2 + 0.125 \times 3 + 0.125 \times 3 + 0.125 \times 3 + 0.125 \times 3$

$0.5 + 0.5 + 1.5$

$\bar{L} = 2.5$ bits / symbol

$H(x) = 0.25 \log(4) + 0.25 \log 4 + 0.125 \left(\log\left(\frac{1}{0.125}\right)\right) \times 4$

$H(x) = 1 \times 25$

$= 2.5$ bits symbol

$\eta = \frac{H(S)}{L}$

$= \frac{2.5}{2.5}$

$\eta = 1$

$\eta\% = 100\%$

$\therefore$ The efficiency is 100%.

Q.2 b) word = "HEARTBEAT" - By method shannon fano coding

$P(H) = \frac{1}{9}$ $P(R) = \frac{1}{9}$

$P(E) = \frac{2}{9}$ $P(B) = \frac{1}{9}$

$P(A) = \frac{2}{9}$ $P(T) = \frac{2}{9}$

x_i	Px_i	col1	col2	col3	code	codeword length
A	0.22	0	0	0	00	2
E	0.22	0	1	1	01	2
T	0.22	1	0	0	100	3
B	0.11	1	0	1	101	3
H	0.11	1	1	0	110	3
R	0.11	1	1	1	111	3

$\bar{L} = P \times n$

$0.22 \times 2 + 0.22 \times 2 + 0.22 \times 3 + 0.11 \times 3 + 0.11 \times 3 + 0.11 \times 3$

$= 0.44 + 0.44 + 0.66 + 0.33 + 0.33 + 0.33$

$\bar{L} = 2.53 \sim 2.55$

$$H = \sum_{i=1}^{6} Px_i \log\left(\frac{1}{Px_i}\right)$$

$$0.22 \log\left(\frac{1}{0.22}\right) + 0.22 \log\left(\frac{1}{0.22}\right) + 0.22 \log\left(\frac{1}{0.22}\right)$$

$$+ 0.11 \log\left(\frac{1}{0.11}\right) + 0.11 \log\left(\frac{1}{0.11}\right) + 0.11 \log\left(\frac{1}{0.11}\right)$$

$$= 2.49$$

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$$\eta = \frac{H}{L}$$

$$\eta = \frac{2.49}{2.55}$$

$$= 0.976$$

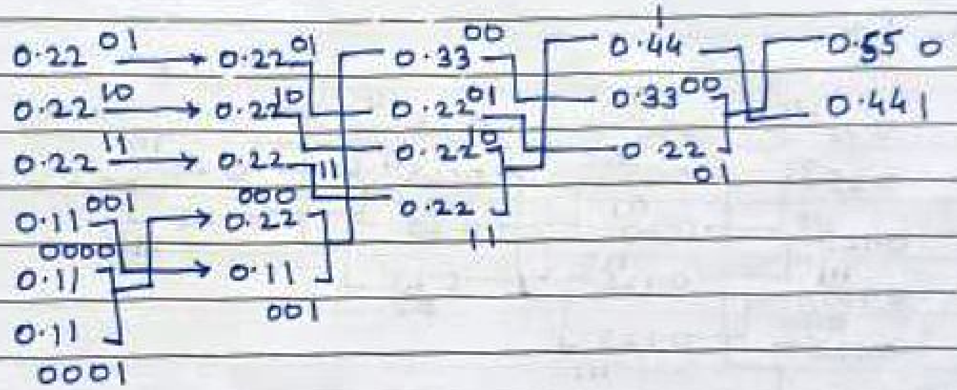
$\eta\%$ = 97.6% efficiency

Q.2 a) By haffman encoding

x_i	P_{x_i}	code	codeword length
E	0.22		
A	0.22		
T	0.22		
B	0.11		
H	0.11		
R	0.11		

x_i $\quad$ P_{x_i}

E
A
T
B
H
R



x_i	P_{x_i}	code	codeword length
E	0.22	01	2
A	0.22	10	2
T	0.22	11	2
B	0.11	001	3
H	0.11	0000	4
R	0.11	0001	4

$H(x) = \sum_{i=1}^{6} P_{x_i} \log_2\left(\frac{1}{P_{x_i}}\right)$

$0.22\times2 + 0.22\times2 + 0.22$

$0.22\log\left(\frac{1}{0.22}\right) + 0.22\log\left(\frac{1}{0.22}\right) + 0.22\log\left(\frac{1}{0.22}\right)$

$+ 0.11\log\left(\frac{1}{0.11}\right) + 0.11\log\left(\frac{1}{0.11}\right) + 0.11\log\left(\frac{1}{0.11}\right)$

$= 2.49$

$\bar{L} = 0.22\times2 + 0.22\times2 + 0.22\times2 + 0.11\times3 + 0.11\times4$
0.11×4

$0.44\times3 + 0.33 + 0.44 + 0.44$

$1.32 + 0.77 + 0.44$

$\bar{L} = 2.53$

$\therefore \eta = \frac{2.49}{2.53}$

0.984

$\eta\% = 98.4\%$

$\therefore$ The efficiency is 98.4%.

3) Kraft's inequality condition

Kraft's inequality is a necessary and sufficient condition to prove existence of prefix code (uniquely decodable code) given n symbol and l_i are no of bits used to represent a symbol for all $i = 1$ to n

$$L = \sum_{i=1}^{n} 2^{-l_i} \leq 1$$

l_i = codeword length

For Q1 - (Question 1)

x_i	P_{x_i}	codeword length
E	0.25	2
F	0.25	2
A	0.125	3
C	0.125	3
N	0.125	3
I	0.125	3

$$\text{formula} = \sum_{i=1}^{n} 2^{-l_i} \leq 1$$

$$\therefore \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^3} + \frac{1}{2^3} + \frac{1}{2^3}$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

$$\therefore \sum_{i=1}^{n} 2^{-l_i} = 1$$

$\therefore$ condition satisfied

$\therefore$ Q.1 follows Krafts inequality condition

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for Q.2a)

x_i	Px_i	codeword length
E	0.22	2
A	0.22	2
T	0.22	2
B	0.11	3
H	0.11	4
R	0.11	4

formula - $\sum_{i=1}^{n} 2^{-l_i} \leq 1$

$\frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^4}$

$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16}$

$= \frac{3}{4} + \frac{1}{8} + \frac{1}{8}$

$= \frac{3}{4} + \frac{1}{4}$

$= 1$

$\therefore \sum_{i=1}^{n} 2^{-l_i} = 1$

$\therefore$ Q.2a) follows Kraft's inequality condition

For Q.2 b)

x_i	$P x_i$	codeword Length
A	0.22	2
E	0.22	2
T	0.22	3
B	0.11	3
H	0.11	3
R	0.11	3

$\therefore$ formula $= \sum_{i=1}^{n} 2^{-l_i} \leq 1$

$$\therefore \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1$$

$$\therefore \sum_{i=1}^{n} 2^{-l_i} = 1$$

$\therefore$ Q.2 b) follows kraft's inequality condition