Somaiya TRUST

Batch: B2 Roll No.: 16010421119
Experiment / assignment / tutorial No.
Grade: AA/ AB / BB / BC / CC / CD / DD

Signature of the Faculty In-Charge with date

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Tutorial #1

Q1) What is the entropy of the image below.

Image

99	99	99	99	99	99	99	99
20	20	20	20	20	20	20	20
0	0	0	0	0	0	0	0
0	0	50	50	50	50	0	0
0	0	50	50	50	50	0	0
0	0	50	50	50	50	0	0
0	0	50	50	50	50	0	0
0	0	50	50	50	50	0	0

{0, 20, 50, 99}
↳ Intensity levels.

Q2) $S = \{ s_1, s_2, s_3, s_4 \}$

$P = \{ \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \}$

Find Self-Information of each symbol. and Entropy.

Q3) An event has 4 possible outcomes. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}$. Determine entropy. Also obtain rate of info. if there are 16 outcomes/s.

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Ans 1) $P(99) = 8/64 = 1/8 = 0.125$

$P(50) = 16/64 = 1/4 = 0.25$

$P(0) = 32/64 = 1/2 = 0.5$

$P(20) = 8/64 = 1/8 = 0.125.$

∴ Formula for Entropy.

$$H = \sum_{i=1}^{n} P_i \log\left(\frac{1}{P_i}\right)$$

$$H = \frac{1}{8}\log_2(8) + \frac{1}{8}\log_2(8) + \frac{1}{2}\log_2(2) + \frac{1}{4}\log_2\left(\frac{4}{1}\right)$$

$\log_2(8) = 3 \rightarrow \log_2(2)^3 = 3\log_2(2) = 3$

$\log_2(4) = 2 \rightarrow \log_2(2)^2 = 2\log_2(2) = 2$

$\log_2(2) = 1 \Rightarrow 1.$

$$H = \therefore \frac{1}{8} \times 3 + \frac{1}{8} \times 3 + \frac{1}{2} + \frac{1}{4} \times 2$$

$$H = 1 + \frac{6}{8} = 1 + \frac{3}{4} = 1.75 \text{ bits/symb.}$$

H = 1.75 bits/pixels.

Ans 2) $P_1 = \frac{1}{2}$ $P_3 = \frac{1}{4}$

$P_2 = \frac{1}{4}$

Self - Information $\rightarrow \log_2\left(\frac{1}{P}\right)$

$\therefore I_1 = \log_2(2) = 1$ bit

$I_2 = \log_2(4) = 2\log_2(2) = 2$ bits.

$I_3 = \log_2(4) = 2\log_2(2) = 2$ bits.

$$\text{Entropy} = \sum_{i=1}^{n} P_i \log\left(\frac{1}{P_i}\right) = \sum_{i=1}^{n} P_i \times I_i$$

$$H = \frac{1}{2} \times 1 + \frac{1}{4} \times 2 + \frac{1}{4} \times 2$$

$$H = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1.5 \text{ bits/symb}$$

$\therefore$ **H = 1.5 bits/symb**

Ans 3) $P_1 = \frac{1}{2}$ $P_2 = \frac{1}{4}$, $P_3 = \frac{1}{8}$, $P_4 = \frac{1}{8}$

$$\therefore \text{Entropy } (H) = \sum_{i=1}^{n} P_i \log\left(\frac{1}{P_i}\right)$$

$$\therefore H = \frac{1}{2} \times \log_2(2) + \frac{1}{4} \log_2(4) + \frac{1}{8} \log_2(8) + \frac{1}{8} \log_2(8)$$

$$= \frac{1}{2} \times 1 + \frac{1}{4} \times 2 + \frac{1}{8} \times 3 + \frac{1}{8} \times 3.$$

$$\frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{3}{8} = 1 + \frac{6}{8} = \boxed{1.75 \text{ bits/outcome}}$$

$\boxed{H = 1.75 \text{ bits/outcome}}$

Rate of info = Rate of outcome transfer x Entropy.

$\therefore \boxed{R = r \times H.}$

Rate of info = 16 outcomes/sec x 1.75 bits/outcome

$\therefore$ Rate of Info = $16 \times 1.75 = 28$

$\boxed{\text{Rate of Info} = 28 \text{ bits/sec.}}$