Batch: B2 Roll No.: 16010421119
Experiment / assignment / tutorial No. 2
Grade: AA/ AB / BB / BC / CC / CD / DD

Signature of the Faculty In-Charge with date

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Batch - B2

Q1) Encode the message 'CAFFEINE' using Shannon - Fano coding and calculate the efficiency.

Q2) Compress the message using 'HEARTBEAT'

Using - a) Huffmann Coding
b) Shannon - Fano coding

Q3) Define Kraft- Inequality Theorem. - Check whether it is satisfied in Q1 and Q2.

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Ans(i) Word - CAFFEINE

Where $P(C) = \frac{1}{8}$ $P(A) = \frac{1}{8}$ $P(E) = \frac{1}{4}$

$P(F) = \frac{1}{4}$ $P(I) = \frac{1}{8}$ $P(N) = \frac{1}{8}$

n_i	P_{n_i}	col_1	col_2	col_3	code
A	0.125	0	0	0	3
C	0.125	0	0	1	3
N	0.125	0	1	0	3
I	0.125	0	1	1	3
F	0.25	1	0		2
E	0.25	1	1		2

P.T.O for Table.

$$\boxed{\bar{L} = \sum P \times n}$$

$$\bar{L} = 0.25\times2 + 0.25\times2 + 0.125\times3 + 0.125\times3 + 0.125\times3 + 0.125\times3$$

$$\bar{L} = 0.5 + 0.5 + 0.375 + 0.375 + 0.375 + 0.375$$

$$\bar{L} = 1 + 1.5$$

$$\boxed{\bar{L} = 2.5 \text{ bits/symbol}}$$

Table. →

Entropy $(H) = \sum P \log\left(\frac{1}{P}\right)$

$\therefore \quad H = 0.25 \log_2(4) + 0.25 \log_2(4)$

$+ 0.125 \log(8) \times 4$

$\therefore \quad = 0.5 + 0.5 + (0.375 \times 4)$

$= 2.5$ bits/message

η (Efficiency) $= \frac{H(s)}{L}$

$= \frac{2.5}{2.5}$

$\therefore$

$\therefore$ Efficiency $= 1$

% Efficiency $= \frac{1}{\cancel{100}} \times 100 = 100\%$

$\boxed{\text{Efficiency} = 100\%}$

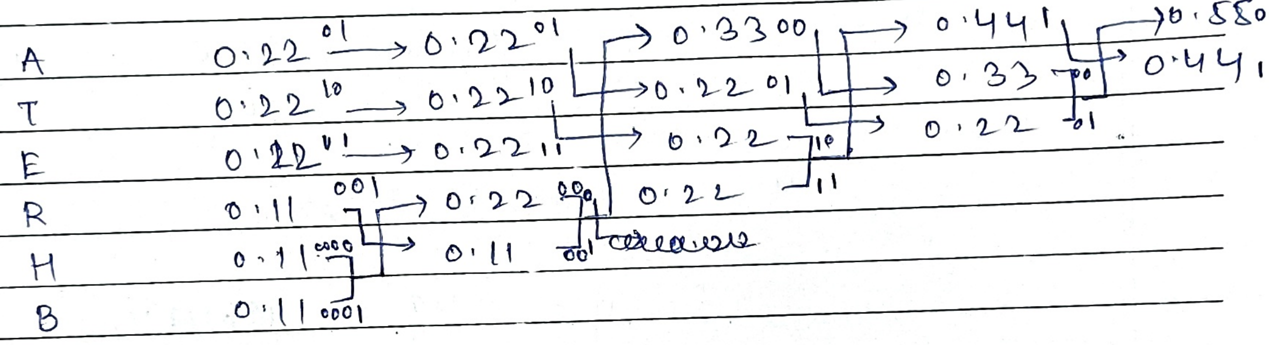
Table – (Redrawn)

ni	P(ni)	1	2	3	code	length
F	0.25	0	0		00	2
F	0.25	0	1		01	2
C	0.125	1	0	0	100	3
A	0.125	1	0	1	101	3
N	0.125	1	1	0	110	3
T	0.125	1	1	1	111	3

Q2)

a) By Huffmann Coding.

x_i $P(x_i)$



Final Table.

		code	codeword len.
A	0.22	01	2
T	0.22	10	2
E	0.22	11	2
R	0.11	001	3
H	0.11	0000	4
B	0.11	0001	4

$\bar{L} = \sum P \times n.$

$\bar{L} = 3(0.22 \times 2) + 2(0.11 \times 4) + (0.11 \times 3)$

$\bar{L} = 1.32 + 0.88 + 0.33$

$\bar{L} = 2.53$

$\therefore \boxed{\bar{L} = 2.53 \text{ bits/symb.}}$

$$H = \Sigma P \log\left(\frac{1}{P}\right)$$

$$H = 3\left[0.22 \log_2\left(\frac{100}{22}\right)\right] + 3\left[0.11 \log_2\left(\frac{100}{11}\right)\right]$$

$$H = 3\left[0.22 \log_2(4.547)\right] + 3\left[0.11 \log_2(9.09)\right]$$

$$H = 2.49 \text{ bits/symb.}$$

$$\therefore \eta = \frac{2.49}{2.53} = 98.41\%.$$

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Q2) Word - 'HEARTBEAT'

B) By Shannon-Fano coding.

$P(H) = \frac{1}{9}$ $P(R) = \frac{1}{9}$ $P(E) = \frac{2}{9}$

$P(B) = \frac{1}{9}$ $P(A) = \frac{2}{9}$ $P(T) = \frac{2}{9}$

x_i	P_{x_i}	1	2	3	code	codeword
A	0.22	0	0		00	2
T	0.22	0	1		01	2
E	0.22	1	0	0	100	3
R	0.11	1	0	1	101	3
H	0.11	1	1	0	110	3
B	0.11	1	1	1	111	3

$$\boxed{\bar{L} = P \times n}$$

$2(0.22 \times 2) + 3(0.11 \times 3) + (0.22 \times 3)$

$\therefore (0.88) + (0.99) + (0.66)$

$$\boxed{\bar{L} = 2.53 \text{ bits/symb.}}$$

$$\boxed{\text{Entropy } (H) = \Sigma P \log\left(\frac{1}{P}\right)}$$

$$2\left[0.22\log_2\left(\frac{100}{22}\right)\right] + 3\left[0.11\log_2\left(\frac{100}{11}\right)\right]$$

$$= 3\left[0.22\log_2(4.54) + 0.11\log_2(9.09)\right]$$

$$= 3\left[0.22 \times 2.182 + 0.11 \times 3.184\right]$$

$$= 2.49 \text{ bits/symb.}$$

$$\therefore \eta = \frac{2.49}{2.53} = 0.9841$$

$$\boxed{\eta = 98.41\%} \text{ efficiency.}$$

3) Krafts inequality condition.

Krafts inequality is a necessary and sufficient condition to prove existence of prefix code (uniquely decodeable code) given n symbol and l_i are no. of bits used to represent a symbol for all $i = 1$ to n

$$L = \sum_{i=1}^{n} 2^{-l_i} \leq 1$$

l_i is the codeword length

x_i	$P x_i$	code word
E	0.25	2
F	0.25	2
A	0.125	3
C	0.125	3
N	0.125	3
I	0.125	3

$$\therefore \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^3} + \frac{1}{2^3} + \frac{1}{2^3}$$

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$\frac{1}{2} + \frac{1}{2}$$

$= 1$ $\therefore$ condition satisfied

For Q2)

			codeword len.
a)	A	0.22	2
	T	0.22	2
	E	0.22	2
	R	0.11	3
	H	0.11	4
	B	0.11	4

bco L $\sum 2^{-l_i} \leq 1$

$$\therefore = \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^4} \leq 1$$

$$= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16}$$

$$= 0.75 + 0.125 + 0.125$$

$= 1 \leq 1$ condition satisfied.

b) same as a)

A	0.22	2
T	0.22	2
E	0.22	2
R	0.11	3
H	0.11	4
B	0.11	4

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{16}$$

$$0.75 + 0.125 + 0.125$$

$1 \leq 1$ condition satisfied.

so Kraft's equality is satisfied in both cases.