classmate

Date

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# Probability, Statistics and Optimization Techniques.

## Module 1. - Probability and Probability Distribution.

### Random Experiment

Any action which gives one or more outcomes is called a random experiment. Each result of the experiment is called an outcome.

→ Sample space - set of all possible outcomes.

### * Algebra of Events

#### (1) Union of two Events.

Let A and B be two events. The union of A & B is defined as the occurrence of atleast one event out of A or B

$\therefore$ $A \cup B = \{x \mid x \in A, B \text{ or } x \in B\}$

#### (2) Intersection of Events.

Let A and B be two events. The intersection of two events is when both events occur simultaneously.

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

* Conditional Probability.

→ Conditional Probability is the probability of an event A given that another event B has already occurred.

Denoted by $P(A|B)$

Formula – $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

→ Multiplication Theorem – $P(A \cap B) = P(A) \times P(B|A)$

* Bayes Theorem.

→ If $A_1, A_2, A_3 \cdots A_n$ form a partition of a sample space and B is any other event of S then.

$$P(A_1|B) = \frac{P(B|A_1) \times P(A_1)}{P(A_1)\,P(B|A_1) * P(A_2)\,P(B|A_2) + P(A_3)\,P(B|}$$

## * Discrete Random Variable

A discrete random variable is a type of random variable that can only take on a countable number of values.

Say x is a discrete random variable, the range of values X can take is called image set of X(S).

## * Probability Density Function (P.d.f)

A probability density function (p.d.f) is a mathematical function that describes the likelyhood of a continuous random variable.

Unlike discrete random variable, which can take only a countable no. of values, p.d.f has continuous random variables which can take any value within a certain range.

It satisfies 2 properties.

① $0 \leq P(x) \leq 1 \quad \forall \; x \in X(S)$

② $\Sigma p(x) = 1$.

* Cumulative distribution function (c.d.f)

$F(x) = P(X \leq x)$

c.d.f is a mathematical function that describes the probability that a continuous or discrete random variable takes on a value less than or equal to the given value.

Example of p.d.f and c.d.f.

* Conversion of random variable to p.d.f

Let X be a continuous random variable.

Then the p.d.f of X must satisfy these two conditions.

(1) $0 \leq f(x) \leq 1$

(2) $\int_{-\infty}^{+\infty} f(t) \cdot dt = 1$

Also for any two numbers a and b with $a \leq b$.

$$P(a \leq X \leq b) = \int_a^b f(t) \cdot dt.$$

If $F(x)$ is the cumulative distribution function of a continuous random variable $X$, the p.d.f of $X$ is.

$$f(x) = \frac{d}{dx}\{F(x)\}$$

Here :-

① $F(x) = P(X \leq x)$

② $1 - F(x) = P(X \geq x)$

③ $P(a < X < b) = F(b) - F(a)$

④ $P(X = x) = 0.$

* Expectation of discrete random variable

$$E(x) = (x_1 \cdot p_1) + (x_2 \cdot p_2) + (x_3 \cdot p_3) \cdots (x_n \cdot p_n)$$

$$= \sum_{i=1}^{n} x_i \cdot p_i = \sum_{i=1}^{n} x_i \cdot P(X = x_i)$$

$$\mu = E(X) = \Sigma x \cdot p(x).$$

* Variance

$$Var(x) = E(x-\bar{x})^2$$

$$= E[x - E(x)]^2$$

$$= E[x^2 - 2x E(x) + \{E(x)\}^2]$$

$$\therefore V(x) = E(x^2) - [E(x)]^2.$$

* Expectation of continuous variable

$$E(x^n) = \int_{-\infty}^{\infty} x^n \cdot f(x) \cdot dx.$$

* Binomial Distribution.

→ The experiment consists of n repeated trials in which there are only two outcomes possible - success or failure.

p.d.f of x in binomial distribution.

$$P(x) = {}^nC_x \, p^x q^{n-x}$$

$n$ = no. of trials

$p$ = probability of success.

$q = 1-p$ : probability of failure

$x$ = no. of success out of n trials.

→ Mean of Binomial Distribution - $n \times p$.

→ Variance of Binomial Distribution - $n \times p \times q$

* Additive Property in Binomial

If $x$ is an r.v $= B(n_1, p)$
If $y$ is a r.v $= B(n_2, p)$

Then $x+y$ is $B(n_1 + n_2, p)$

* Poisson Distribution.

When the no. of trials which are independent increase indefinitely and probability of getting success 'p' in each trial is very small so that 'np' remains constant.

p.d.f of Poisson distribution is given by :-

$$f(x) = \frac{e^{-m} \cdot m^x}{x!} \quad X = 0, 1, 2.$$

Mean = Variance = m.

* Uniform Distribution.

A random variable x is in uniform distribution in an interval [a, b], if its p.d.f is of the form.

$$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$$

Mean of Uniform Dist. $= \frac{b+a}{2}$

Variance of Uniform Dist $= \frac{(b-a)^2}{12}$

* Exponential Distribution.

When p.d.f of x is

$$f(x) = \lambda e^{-\lambda x}, \quad 0 \leq x \text{ and } 0 \leq \lambda$$
$$= 0, \quad \text{otherwise}$$

c.d.f ↓

$$F(X=x) = P(X \leq x) = 1 - e^{-\lambda x}, \quad 0 \leq x$$

Mean $= \frac{1}{\lambda}$

Variance $= \frac{1}{\lambda^2}$