ITC Tutorial 5 Even 2021-2022

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Q1. Show how the parity matrix for a (7,4) Hamming code is generated.

soln The (7,4) Hamming code is represented as

$D_7\ D_6\ D_5\ P_4\ D_3\ P_2\ P_1$ where

D_7, D_6, D_5, D_4 = 4 data bits

P_4, P_2, P_1 = 3 parity bits.

$P_1 = 1$ for all 4 bit patterns with a '1' in 2^0 positi

$P_2 = 1$ " " .... 2^1 position.

$P_4 =$ " " " 2^2 position.

	P_4	P_2	P_1
D_0	0	0	0
D_1	0	0	1
D_2	0	1	0
D_3	0	1	1
D_4	1	0	0
D_5	1	0	1
D_6	1	1	0
D_7	1	1	1

$$P_1 = D_3 \oplus D_5 \oplus D_7$$

$$P_2 = D_3 \oplus D_6 \oplus D_7$$

$$P_4 = D_5 \oplus D_6 \oplus P_7$$

$$\therefore \quad P = \begin{array}{c|c} & \begin{matrix} P_1 & P_2 & P_4 \end{matrix} \\ \begin{matrix} D_3 \\ D_5 \\ D_6 \\ D_7 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

Q2 Consider a (6,3) linear block code whose generator matrix is

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

(a) Find all code vectors.

(b) Find the minimum Hamming distance d_{min}.

(c) Check if the received vector $r = [110111]$ contains any error using the syndrome method.

Soln:

(a) Codeword = $[D][G]$

$$= [d_1 \; d_2 \; d_3] \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

$$[C] = [d_1, d_2, d_3, d_1 \oplus d_2, d_2 \oplus d_3, d_1 \oplus d_3]$$

message	Codeword	weight
000	000000	0
001	001011	3
010	010110	3
011	011101	4
100	100101	3
101	101110	4
110	110011	4
111	111000.	3

(b) From the previous table $d_{min} = 3$.

Error ~~corre~~ detection capability.

$$d_{min} \geq t+1$$

$$3 \geq t+1$$

$$t \leq 2.$$

$\therefore$ This code can detect upto 2 bit errors

Error correction capability.

$$d_{min} \geq 2t+1$$

$$3 \geq 2t+1$$

$$t \leq 1$$

$\therefore$ This code can correct 1 bit errors.

(c) To calculate the syndrome we need to construct the parity check matrix.

$$H = [P^T \mid I_{n-k}]$$

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad \therefore P^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$H = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

~~K~~ If the received vector r has no error then $r \cdot H^T = 0$.

$$\cancel{r \cdot H^T} = \left[\cancel{1\,1\,0\,1\,1\,1}\right] \begin{bmatrix} \cancel{1\,1\,0\,1\,0\,0} \\ 0\,1\,1\quad 0\,1\,0 \\ 1\,0\,1\quad 0\,0\,1 \end{bmatrix}$$

$$=$$

$$r \cdot H^T = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

~~as~~ $r \cdot H^T \neq 0$, the syndrome vector $[1\,0\,0]$ matches row no 4 from the top.

$\therefore\ e = [0\,0\,0\,1\,0\,0]$ $r = [1\,1\,0\,1\,1\,1]$

$$\therefore\ e \oplus r = \begin{array}{r} 0\,0\,0\,1\,0\,0 \\ \oplus\ 1\,1\,0\,1\,1\,1 \\ \hline 1\,1\,0\,\underline{0}\,1\,1 \end{array}$$

$\therefore$ The original message sent was $[1\,1\,0\,0\,1\,1]$