Somaiya TRUST

Batch: B2 Roll No: 16010421119
Experiment / assignment / tutorial No.
Grade: AA / AB / BB / BC / CC / CD / DD
Signature of the Faculty In-Charge with date

ITC Tut 4

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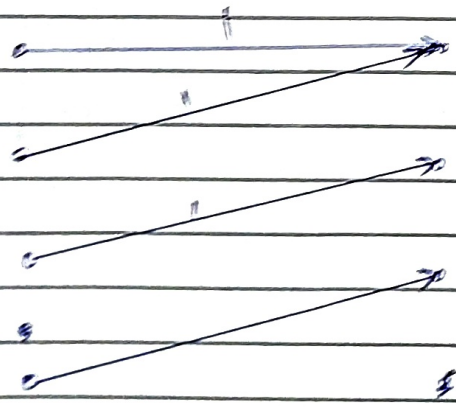
Roll no. - 16010421119
Batch - B2
Date - 23/02/23

a) For the following channel transition matrix.

$$\begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \end{bmatrix}$$

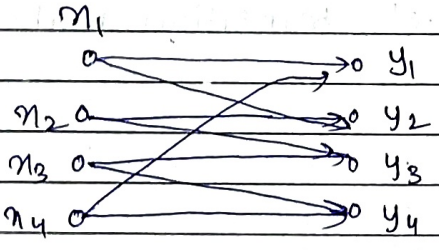
b) Write the probability transition matrix for the following channel diagram and write comment about it.

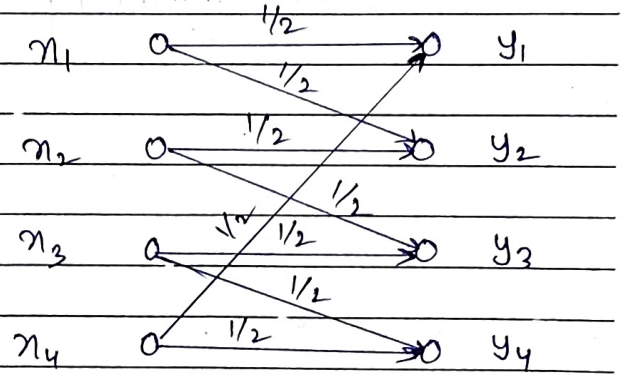


Ans:

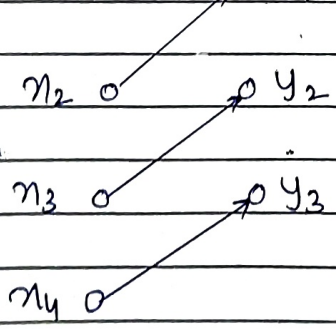
Q1 a)

	y_1	y_2	y_3	y_4
x_1	1/2	1/2	0	0
x_2	0	1/2	1/2	0
x_3	0	0	1/2	1/2
x_4	1/2	0	0	1/2





Q1 b)



$P(Y|X) =$

	y_1	y_2	y_3
x_1	1	0	0
x_2	1	0	0
x_3	0	1	0
x_4	0	0	1

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Q2)

i) $H(x) = -\sum_{j=1}^{m} P(x_j) \cdot \log_2(P(x_j))$ Finding H(K)

↳ Formula for Entropy

$= -\sum_{j=1}^{2} P(x_j) \log_2(P(x_j))$

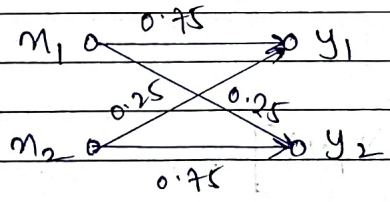
$= -\left(P(x_1) \log_2(P(x_1)) + P(x_2) \cdot \log_2(P(x_2))\right)$

$= -\left(\frac{2}{3} \log_2\left(\frac{2}{3}\right) + \frac{1}{3} \log_2\left(\frac{1}{3}\right)\right)$

$= -(0.389 + 0.528)$

$= -0.91829$

ii)



Channel Diagram

$P(x_1) = 2/3 \qquad P(x_2) = 1/3$

$P(Y) = P(X)\ P(Y|X)$

$$P(Y) = \begin{bmatrix} 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$

65	67
66	68
67	64
68	72
69	76
70	64
71	70
67	68

$\bar{x} = 69.125$

$\therefore \bar{x} \neq 70$

$$P(Y) = \begin{bmatrix} \frac{2}{3}(0.75) + \frac{1}{3}(0.25) & \frac{2}{3}[0.25) + \frac{1}{3}(0.75)] \end{bmatrix}$$

$$P(Y) = \begin{bmatrix} 0.583 & 0.416 \end{bmatrix}$$

iii) $[P(X,Y)] = [P(X)] [P(Y/X)]$

$$[P(X,Y)] = \begin{bmatrix} 2/3 & 0 \\ 0 & 1/3 \end{bmatrix} \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$

$$\therefore [P(X,Y)] = \begin{bmatrix} 1/2 & 1/6 \\ 1/2 & 1/4 \end{bmatrix}$$

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$$H(Y|X) = -\sum_{j=1}^{m}\sum_{K=1}^{n} P(y_K, x_j) \log_2 P(y_K | x_j)$$

$$P(X,Y) = \begin{bmatrix} 1/2 & 1/6 \\ 1/12 & 1/4 \end{bmatrix}$$

$$P(Y|X) = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$

$$H(Y|X) = \frac{1}{2}\log_2\left(\frac{4}{3}\right) + \frac{1}{6}\left(\log_2(4)\right) + \frac{1}{12}\left(\log_2(4)\right) + \frac{1}{4}\left(\log_2\left(\frac{4}{3}\right)\right)$$

$$= 0.7386 \text{ bits/msg}$$

Q3) Match the following.

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Q1)

	A	C	B
i	$H(X,Y)$	IV) Joint Entropy of X and Y	2) $-\sum_{j=1}^{m}\sum_{k=1}^{n} P(x_j, y_k) \times \log_2(x_j, y_k)$
ii)	$H(x)$	III) Priori Entropy	1) $-\sum_{j=1}^{m} P(x_j)\log_2(P x_j)$
iii)	$H(X/Y)$	II) Describes how well one can recover the transmitted symbols from the received symbols.	4) $-\sum_{j=1}^{m}\sum_{k=1}^{n} P(x_j, y_k)\log_2(P(x_j/y_k))$
iv)	$I(X,Y)$	I) Mutual Information	3 $H(X) - H(X/Y)$