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# ITC Tutorial 5.

Q1) Show how the parity matrix for a (7,4) Hamming code is generated

Q2) Consider a (6,3) linear block code whose generator matrix is

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

a) Find all code vectors

b) Find the minimum Hamming distance (dmin)

c) Check if the recieved vector r = [110111] contains any error using the syndrome method.

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Q1) The (7,4) Hamming code is represented as

$D_7, D_6, D_5, ~~D_4~~ P_4, D_3, P_2, P_1$

Where $D_7, D_6, D_5, D_3 \longrightarrow$ 4 data bits

$P_4, P_2, P_1 \longrightarrow$ 3 parity bits

$P_1 = 1$ for all 4 bit patterns with a '1' at $2^0$

$P_2 = 1$ for all 4 bit patterns with a '1' at $2^1$

$P_4 = 1$ for all 4 bit patterns with a '1' at $2^2$

| $P_4$ | $P_2$ | $P_1$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |
| 1 | 1 | 1 |

$P_1 = D_3 \oplus D_5 \oplus D_7$

$P_2 = D_3 \oplus D_6 \oplus D_7$

$P_4 = D_5 \oplus D_6 \oplus D_7.$

$$P = \begin{array}{c c} & \begin{array}{ccc} P_1 & P_2 & P_4 \end{array} \\ \begin{array}{c} D_3 \\ D_5 \\ D_6 \\ D_7 \end{array} & \left[\begin{array}{ccc} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{array}\right] \end{array}$$

Q2) codeword = [D][G]

a) $= [d_1 \; d_2 \; d_3] \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$

$[e] = [d_1, d_2, d_3, d_1 \oplus d_2, d_2 \oplus d_3, d_1 \oplus d_3]$

| Message | Codeword | Weight |
|---|---|---|
| 000 | 000000 | 0 |
| 001 | 001011 | 3 |
| 010 | 010110 | 3 |
| 011 | 011101 | 4 |
| 100 | 100101 | 3 |
| 101 | 101110 | 4 |
| 110 | 110011 | 4 |
| 111 | 111000 | 3 |

(b) From the previous table $d_{min} = 3$

Error detection capability.

$d_{min} \geq t+1$

$3 \geq t+1$

$t \leq 2$

$\therefore$ This code can detect upto 2 bits errors.

Error correction capability.

$d_{min} \geq 2t+1$

$3 \geq 2t+1$

$t \leq 1$

$\therefore$ This code can correct 1 bit errors.

(c) To calculate the syndrome we need to construct the parity check matrix.

$H = [P^T \mid I_{n-k}]$

$P = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad \therefore \quad P^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

classmate

Date

Page

$$\therefore H = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

If the recieved vector $r$ has no error then $r \cdot H^T = 0$

$$r \cdot H^T = [1\ 1\ 0\ 1\ 1\ 1] \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$s = [1\ 0\ 0]$$

$r \cdot H^T \neq 0$ The syndrome vector $[100]$ matches row no. 4 from the top.

$$\therefore e = [0\ 0\ 0\ 1\ 0\ 0] \qquad r = [1\ 1\ 0\ 1\ 1\ 1]$$

$$\begin{array}{rcccccc} e \oplus r = & 0 & 0 & 0 & 1 & 0 & 0 \\ \oplus & 1 & 1 & 0 & 1 & 1 & 1 \\ \hline & 1 & 1 & 0 & 0 & 1 & 1 \end{array}$$

$\therefore$ The orignal message sent was $[1\ 1\ 0\ 0\ 1\ 1]$