QUESTION: **<u>PROBABILITY DISTRIBUTION</u>**

1. Write down the probability distribution of the maximum of numbers appearing on the toss of two unbiased dice. Hence find mean of the distribution

2. Write down the probability distribution of the sum of numbers appearing on the toss of two unbiased dice. Hence find mean of the distribution

3. Find probability distribution and cumulative distribution function of X. Determine $P(X<3), P(1<X\le 2), P(0<X\le 2)$ Also find mean and variance of X if the random variable X takes the values 1,2,3&4 such that

   2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)

4. Find C, mean and variance of X, find K (where K is a +ve integer) if $P(X\le K)>1/2$, $P(1.5<X<4.5/X>2)$

   Where probability function of a discrete random variable X is

$X=x_i$	0	1	2	3	4	5	6	7
$P(x_i)$	0	C	2C	2C	3C	C^2	$2C^2$	$7C^2+C$

5. A shipment of 8 microcomputers contains 3 that are defective. If a college makes a random purchase of 2 of these computers, find the probability distribution of the defective computers.

6. 7. If X_1 has mean 5 and variance 5, X_2 has mean -2 and variance 3. If X_1 & X_2 are independent random variables find : i) $E(X_1+X_2)$, $V(X_1+X_2)$ ii) $E(2X_1+3X_2-5)$, $V(2X_1+3X_2-5)$

8. An urn contains 4 white and 3 black balls. Find the probability distribution of the number of black balls in three draws made successively with replacement from the urn.

9. A random variable x has the following probability function

X	1	2	3	4	5	6	7
P(x)	k	2k	3k	k^2	k^2+k	$2k^2$	$4k^2$

   Find i) k ii) P(x<5) iii) P(x>5) iv) $P(0\le X\le 5)$ v) mean

10. Verify that $P(X=x)$ probability function of random variable X where $P(X=x)=\frac{1}{2^x}$, $x=1,2,3,\dots\infty$ also find mean

11. If the following distribution of a discrete random variable X has mean =16 then find m, n and the variance of X.

X	8	12	16	20	24
P(x)	1/8	m	n	1/4	1/12

12. A continuous random variable X has the probability density function $f(x)=k\,x^2e^{-x}, x\ge 0$. Find k, mean and variance

13. A continuous random variable X has the probability density function defined by $f(x)=A+Bx, 0\le x\le 1$. If the mean of the distribution is 1/3, find A and B

14. Let X be a continuous random variable with probability density function $f(x)=kx^2(1-x), 0\le x\le 1$ Find k , mean,mode

15. Let X be a continuous random variable with probability density function $f(x)=k\,x(1-x), 0\le x\le 1$. Find k, mean and determine a number b such that $P(x\le b)=P(x\ge b)$.

16. Verify that the function given below is a distribution function

   $F(x)=\begin{cases} 0, & x<0 \\ 1-e^{-x/4}, & x\ge 0\end{cases}$ Also find the probabilities $P(x\le 4), P(x\ge 8)$ $P(4\le x\le 8)$

17. Find k, $P(1\le x\le 3)$, $\bar{X}$. if probability density function of a random variable is

$$f(x)=kx,\quad 0\le x\le 2$$
$$=2k,\quad 2\le x\le 4$$
$$=6k-kx,\quad 4\le x\le 6$$

18	A random variable x has the p.d.f. $f(x)=\frac{k}{1+x^2}, \quad -\infty < x < \infty$. Determine k, mean, variance, & the distribution function. Also evaluate $P(x \geq 0)$.
19	The probability density function of a random variable x is given by $f(x)=k\,e^{-x/6}, 0 < x < \infty$. Find the mean & standard deviation of x
20	The daily consumption of electric power (in million kwh) is a random variable X with probability distribution function $f(x)=\begin{cases} k\,x e^{-x/3} & for \quad x \geq 0 \\ 0 & for \quad x < 0 \end{cases}$ find the value of k and the probability that on a given day the electric consumption is more than expected value
21	Determine the constant 'a' and findmean , $P(4 \leq x \leq 7)$ if the distribution function of a continuous random variable is defined as $f(x) = \frac{a}{x^5}$ $, 2 \leq x \leq 10$
22	If the distribution function of a continuous random variable is defined as follows ,find the value of 'a' ,mean , var , c.d.f. and p $(1 \leq x \leq 2)$ $f(x) = \begin{cases} ax, 0 \leq x \leq 1 \\ a, 1 \leq x \leq 2 \\ 3a - ax, 2 \leq x \leq 3 \\ 0, else\ where \end{cases}$
23	The time a person has to wait for a bus at a bus stop is a random variable has distribution function $$\begin{aligned} F(x) &= 0, \quad x \leq 0 \\ &= x/3, \quad 0 \leq x \leq 1 \\ &= 1/3, \quad 1 \leq x \leq 3 \\ &= x/9, \quad 3 \leq x \leq 9 \\ &= 1, \quad x \geq 9 \end{aligned}$$ Find the probability density function and verify that the given function is a distribution function. Find mean and variance
24	The length of time (in minutes) a leady speaks on telephone is found to be a random variable with probability density function $f(x)=\begin{cases} A\,e^{-x/5} & for\ x \geq 0 \\ 0 & elsewhere \end{cases}$ find A and the probability that she will speak for i)more than 10 minutes, ii) less than 5 minutes, iii) between 5 and 10 minutes.
25	For the probability density function $$\begin{aligned} f(x) &= \frac{2(b+x)}{b(a+b)} \quad -b \leq x < 0 \\ &= \frac{2(a-x)}{a(a+b)} \quad 0 \leq x \leq a \end{aligned}$$ check that above is a p.d.f. and find the mean
26	If a fair coin is tossed till a head appears then what is expectation of number of tosses required ?
27	A fair coin is tossed 3 times. A person received Rs. X^2 if he get X heads. Find his expectation
28	A and B throw a fair die for a stake of Rs.44, which is won by player who throws 6 first. If A starts first, find their expectations
29	A & B toss fair coin alternately. One who gets a head first, wins Rs 12. A starts. Find their expectations
30	If a coin is tossed by a player two times , he wins Rs 3 for each head and Rs 2 for each tail. Find the probability distribution table and his expectation

31 If a player wins Rs 3 if he draws one white ball and wins Rs 2 if he draws one black ball from a bag containing 5 white and 4 black balls ,then find his expectation

32 Three fair coins are tossed. Find the expectation and the variance of number of heads

33 Find mean and variance of a binomial variate if if $n = 6,\ 9P(x = 4) = P(x = 2)$

34 Find the Binomial distribution if the mean is 5 & variance is 10/3. Find $P(x = 2)$, $P(x \le 4)$

35 The ratio of the probability of 3 successes in 5 independent trials to the probability of 2 successes in 5 independent trials is 1/4.what is the probability of 4 successes in 6 independent trials?

36 If a probability of a defective bulb is 0.2, find the mean & the standard deviation for the distribution of defective bulbs in a lot of 1000 bulbs. What is the expectation of defective bulbs in the lot?

37 The probability that a man aged 60 will live up to 70 is 0.65. What is the probability that out of 10 such men now at 60 (i)at least 7 will live up to 70(ii) at most 8 will live up to 70?

38 In a precision bombing attack there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give at least 99% chance of destroying the target

39 If 10% 0f the rivets produced by a machine are defective, find the probability that out of 5 randomly chosen rivets i) none will be defective ii) at the most two will be defective.

40 Out of 800 families with 5 children each how many would you expect to have i) 3 boys & 2 girls, ii) 5 girls iii) 5 boys?

41 Seven dice are thrown 729 times. How many times do you expect at least four dice to show three or five?

42 Let X, Y be two independent binomial variates with parameters $(n_1 = 6, p = 1/2)$ & $(n_2 = 4, p = 1/2)$
respectively. Evaluate $P(X + Y) = 3$ & $P(X + Y) \ge 3$.

43 In a multiple choice examination there are 20 questions. Each question has 4 alternative answers following it and the student must select one correct answer. 4 marks are given for correct answer and 1 mark is deducted for wrong answer. A student must secure at least 50% of maximum possible marks to pass the examination. Suppose a student has not studied at all, so that he answers the question by guessing only. What is the probability that he will pass the examination?

44 Assume that 50% of all engineering students are good in mathematics. Determine the probabilities that among 18 engineering students (i) at least 10, (ii) at least 2 and at most 9 are good in mathematics

45 Five fair coins are tossed 3200 times; Find the frequency distribution of number of heads obtained. Also find mean and standard deviation

46 Five dice are thrown together 96 times. The number of times 4, 5 or 6 was obtained is given below.

No.of imes 4, 5 or 6 is obtained	0	1	2	3	4	5
Freq.	1	10	24	35	18	8

47 Fit a Binomial distribution to the following data.

x:	0	1	2	3	4	5	6
f:	5	18	28	12	7	6	4

48 Seven coins are tossed and the number of heads obtained noted. The experiment is repeated 128 times and the following distribution is obtained. Fit a Binomial distribution

0	1	2	3	4	5	6	7	total
7	6	19	35	30	23	7	1	128

49 The probability that at any moment one telephone line out of 10 will be busy is 0.2.
(i)What is the probability that 5 lines are busy?(ii)Find the expected number of busy lines and also find the probability of this number.(iii)What is the probability that all lines are busy

50 If a random variable x follow Poisson distribution such that $P(x = 1) = 2P(x = 2)$, Find the mean

	and the variance of the distribution. Also find $P(x=3)$.
51	A variable x follows a Poisson distribution with variance 3. Calculate i) $P(x=2)$, ii) $P(x \geq 4)$.
52	If X, Y are independent Poisson variates such that $P(x=1)=P(x=2)$ & $P(y=2)=P(y=3)$ find the variance of $2X-3Y$.
53	An insurance company found that only 0.01% of the population is involved in a certain type of accident each year. If its 1000 policyholders were randomly selected from the population, what is the probability that no more than two of its clients are involved in such accident next year?
54	Find the probability that (i)at most 4 defective bulbs(ii) no defective bulbs will be found in a box of 200 bulbs if it is known that 2 percent of the bulbs are defective
55	Between the hours of 2 & 4 P.M. the average number of phone calls per minute coming in to the switchboard of a company is 2.5, find the probability that during a particular minute there will be i) no phone calls at all ii) more than 6 calls.
56	It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced item are send to the market in packets of 20, find the number of packets containing i)at least ii) exactly & iii)at most 2 defective items in a consignment of 1000 packets using Binomial distribution & Poisson approximation to the Binomial distribution
57	A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as Poisson variate with mean 1.5 Calculate the proportion of days on which i) neither car is used, ii) some demand is refused
58	Accidents occur on a particular stretch of highway at an average rate 3 per week. What is the probability that there will be(i) exactly two accidents (ii) atmost two accidents in a given week?
59	A firm produces articles, 0.1 percent of which one defective. It packs them in cases containing 500 articles. If a wholesaler purchases 100 such cases how many cases can be expected i) to be free from defective ii) to have one defective?
60	A manufacturer finds that the average demand per day for the mechanic to repair his new production is 1.5 over a period of one year & the demand per day is distributed as Poisson distribution. If he employs two mechanics on how many days in a year i) both mechanics would be free ii) some demand is refused?
61	In a certain factory producing certain articles the probability that an article is defective is 1/500. The articles are supplied in packets of 20. Find approximately the number of packets containing no defective, one defective, two defective in a consignment of 20000 packets
62	If the mean of the Poisson distribution is 4, find $P(m-2\sigma < x < m+2\sigma)$
63	If 2 percent bulbs are known to be defective bulbs, find the probability that in a lot of 20 bulbs, there will be 2 or 3 defective bulbs using i) Binomial distribution, ii) Poisson distribution.
64	If X_1, X_2, X_3 are three independent Poisson variates with parameters $m_1=1, m_2=2, m_3=3$ respectively, find $P[(X_1+X_2+X_3)\geq 3]$
65	Fit a Poisson distribution If the following mistakes per page were observed in a book
66	Fit a Poisson distribution to the following data.
67	Fit the data to Poisson distribution
68	Three factories A,B,C produce 40%,40% & 20% of the total production of an item .Out of their production 15%,10% & 2% are defective.An item is chosen at random and found to be defective . Using Bayes theorem find the probability that it was produced by the factory A.
69	A bag contains 8 red balls and 4 black balls and another bag contains 6 red balls and 5 black balls.

Question 65:

No. of mistakes	0	1	2	3	4	Total
No. of pages	211	90	19	5	0	325

Question 66:

x:	0	1	2	3	4	5	Total
f:	142	156	69	27	5	1	400

Question 67:

No. of mistakes	0	1	2	3	4	Total
No. of pages	123	59	14	3	1	200

	One ball is transferred from the first bag to the second bag then a ball is drawn from the second bag . If this ball happens to be red ,find the probability that a black ball was transferred
70	A lot of IC chips is known to contain 4% defective chips, each chip is tested before delivery but the test is not reliable .It is known that p(Tester says the chip is defective / the chip is actually defective) = 0.97 and p(Tester says the chip is good / the chip is actually good) = 0.98 If a tested chip is declared defective by the tester . find the probability that it is actually defective
71	An urn contains 5 white balls and 4 black balls and another urn contains 6 white balls and 4 black balls. One ball is transferred from the first urn to the second urn then a ball is drawn from the second urn . If this ball happens to be white ,find the probability that a black ball was transferred
72	In a certain college 4% of the boys and 1% of the girls are taller than 1.8 m. Furthermore 60% of the students are girls Now if a student is selected at random and taller than 1.8 m what is probability that the student is girl ?
73	A box contains 3 coins , first coin is fair , second coin is two headed , third coin is weighted so that the probability of a head appearing is 1/3 . A coin is selected at random from the box and tossed (i) find the probability that head appears (ii) If head appears what is probability that it comes on first coin?
74	Box A contains 9 cards numbered from 1 to 9 Box B contains 5 cards numbered from 1 to 5. A box is selected at random and a card is drawn. If the number is even what is probability that the card comes from box A ?
75	For a certain binary communication channel , the probability that a transmitted ‘0’ is received as a ‘0’ is 0.95 and the probability that a transmitted ‘1’ is received as a ‘1’ is 0.90 If the probability that a ‘0’ is transmitted is 0.4 what is probability that ‘1’ was transmitted given that ‘1’ was received ?