

Important definitions.

- ① Graph - A graph is an ordered pair $G = (V, E)$ where V is a set of vertices and E is a set of edges.
- ② A connected graph - A graph is called a connected graph if there is a path from any vertex to any other vertex.
- ③ Walk - When we traverse a graph in any way we want to, its a walk.
- ④ Trail - A walk where no edge is repeated but vertices can be repeated.
- ⑤ Circuit - A trail where start and end vertex is same. No edge is repeated.
- ⑥ Path - A walk where edges and vertices are not repeated.
- ⑦ Cycle - A path where start and end vertices are same.
- ⑧ Eulerian Graph - A graph where all the edges are covered and no edge is repeated.

T1 - G is a Eulerian graph if and only if all vertices of G are even degree.

~~if all vertices have even degree then graph is Eulerian~~

T2 - If G is a connected graph having exactly 2 vertices of degree 3, then G contains a Eulerian path b/w those two vertices.

(1) Hamiltonian Graph.

→ A connected graph is Hamiltonian if there is a cycle which includes every vertex of G and this cycle is called Hamiltonian.

→ T1 - If G is an a graph with n vertices where $\deg(x) + \deg(y) \geq n$ for each pair of non adjacent vertices x and y then the graph is Hamiltonian.

(2) Isomorphic graphs.

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$

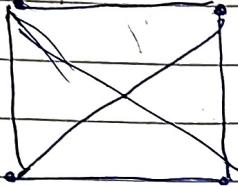
If there exists a bijective function $f: V_1 \rightarrow V_2$ with $\{a, b\} \in E_1$ iff $\{f(a), f(b)\} \in E_2$

$\{a, b\}$ is an edge of G_1

$\{f(a), f(b)\}$ is an edge of G_2

Denoted by $G_1 \cong G_2$

- ⑩ Planar Graph - If it is possible to draw the diagram of a given graph in such a way that 2 edges don't intersect each other then its a planar graph.



- ⑪ Plane Graph - The diagram of a graph drawn when no 2 edges intersect each other.

- ⑫ A Tree - A connected graph which doesn't have a cycle.

- ⑬ Terminal node - A vertex of degree 1 in tree is called as Terminal node or leaf.

- ⑭ Internal node - A vertex of degree greater than 1.

→ T₁ - In a tree there is only one path for every pair of vertices.

→ T₂ - A tree with n vertices has (n-1) edges.

→ A binary tree is a tree which has no more than 2 branches is called binary tree.

→ Has exactly one vertex of degree 2, rest of degree 1 and 3.

→ All vertices with degree 1 are called pendant vertices.

→ All vertices of degree 2 or 3 are called non-pendant vertices.

→ No. of pendant vertices = $(n+1)/2$

→ No. of non-pendant vertices = $\lceil (n+1)/2 \rceil - 1$

→ Max. and min height

$$\text{Max} = \frac{(n-1)}{2}$$

$$\text{Min} \log_2 \lceil n+1 \rceil - 1$$

Q. If $n = 11$ what is the maximum height of the tree?

$$\text{Max H} = \frac{n-1}{2} = \frac{11-1}{2} = 5$$

$$\text{Min Height} = \log_2(n+1) - 1$$

$$\text{Min Height} = \lceil \log_2(11+1) - 1 \rceil = \lceil 2.58 \rceil = 3$$

→ A sequence is an ordered set using 0s and 1s to create or decode a message. Also used in forming binary trees.

Eg: $e = \{00, 001, 01, 010, 11, 111\}$

