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**06EC65**

## Sixth Semester B.E. Degree Examination, December 2011
# Information Theory and Coding

Time: 3 hrs. Max. Marks:100

**Note: *Answer any FIVE full questions, selecting at least TWO questions from each part.***



### PART – A

1 a. The international morse code uses a sequence of dots and dashes to transmit letters of English alphabets. The dash is represented by a current pulse that has a duration of 3 units and the dot has a duration of 1 unit. The probability of occurrence of dash is 1/3 of the probability of occurrence of a dot.
i) Calculate the information content of a dot and a dash.
ii) Calculate the average information in the dot dash code.
iii) Assume that the dot lasts 1 msec which is the same interval as the pause between symbols. Find the average rate of information transmission. **(08 Marks)**

b. For the source model shown in Fig.Q1(b), find the source entropy and the average information content per symbol in messages containing one, two and three symbols.

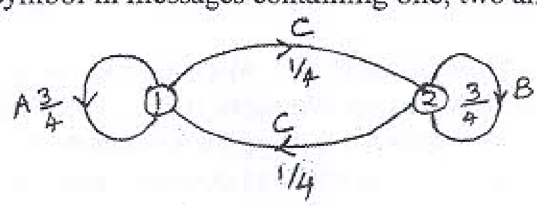
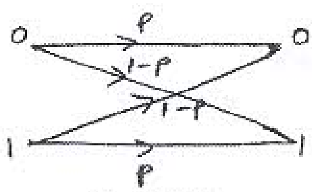



Fig.Q1(b) **(12 Marks)**



2 a. For the binary symmetric channel shown in Fig.Q2(a), find the rate of information transmission over the channel when p = 0.9, 0.8 and 0.6, given that the symbols rate is 1000/sec. **(06 Marks)**




$P[x = 0] = 1/2$

$P[x = 1] = 1/2$

Fig.Q2(a)

b. A source emits an independent sequence of symbols A, B, C, D and E with the probabilities 1/4, 1/8, 1/8, 3/16 and 5/16 respectively. Find the Shannon code and efficiency. **(08 Marks)**

c. A binary source emits an independent sequence of 0's and 1's, with probabilities p and 1 – p. Prove that the entropy is maximum at p = ½. Plot the entropy. **(06 Marks)**

3 a. Explain the prefix coding and decision tree with examples. **(08 Marks)**

b. A discrete memoryless source has an alphabet of five symbols, with their probabilities as given below.

| Symbol | $S_0$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ |
|---|---|---|---|---|---|
| Probability | 0.55 | 0.15 | 0.15 | 0.10 | 0.05 |

Compute two different Huffman codes for this source. For each of the two codes, find
i) The average code word length ii) The variance of the average code word length over the ensemble of source symbols. **(12 Marks)**

4 a. Derive an equation for the capacity 'C' of a channel of Bandwidth B Hz effected by additive white Gaussian noise of power spectral density of $N_o/2$. **(10 Marks)**

b. An analog signal has a 4 kHz bandwidth and is sampled at 2.5 times the Nyquist rate and each sample is quantized into one of 256 equally likely levels.
   i) What is the information rate of the source?
   ii) Can the output of this source be transmitted without errors over a Gaussian channel with a Bandwidth of 50 kHz and S/N of 20 db?
   iii) What will be the bandwidth requirements of an analog channel for transmitting the output of the source without errors, if the S/N ratio is 10 db? **(10 Marks)**

## PART – B

5 a. Write a note on encoding and decoding of linear block code. **(06 Marks)**

b. The parity clock bits of a (8, 4) block code are generated by

$$C_5 = d_1 + d_2 + d_4 \qquad C_6 = d_1 + d_2 + d_3 \qquad C_7 = d_1 + d_3 + d_4 \qquad C_8 = d_2 + d_3 + d_4$$

where $d_1$, $d_2$, $d_3$ and $d_4$ are message bits.
   i) Find the generator matrix ad parity check matrix.
   ii) Find the minimum weight of this code.
   iii) Show through an example that this code can detect and correct errors. **(08 Marks)**

c. Design a single error correcting code with a message block size of 8-bits. **(06 Marks)**

6 a. The generator polynomial of a (7, 4) cyclic code is $g(x) = 1 + x + x^3$.
   i) Find the codewords for messages 1010, 1110, 1100, 1111.
   ii) Find the codewords for i) using the systematic form. **(08 Marks)**

b. Discuss the features of encoder and decoder, used for cyclic codes, with examples. **(12 Marks)**



7 Write short notes on any four:
i) RS codes ii) BCH codes iii) Golay codes
iv) Shortened cyclic codes v) Burst error correcting codes. **(20 Marks)**

8 Consider a (3, 1, 2) convolutional code with

$$g^{(1)} = 110\,; \qquad g^{(2)} = 101\,; \qquad g^{(3)} = 111$$

   i) Draw the encoder block diagram
   ii) Find the generator matrix
   iii) Find the codeward corresponding to the information sequence 11101, using the time domain and transform domain approach. **(20 Marks)**

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2002 SCHEME

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# Sixth Semester B.E. Degree Examination, December 2011
## Information Theory and Coding

Time: 3 hrs. Max. Marks:100

**Note:** ***Answer any FIVE full questions.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

1 a. The output of a DMS consists of the possible letters $x_1, x_2, \ldots, x_n$ which occur with probabilities $p_1, p_2 \ldots p_n$ respectively. Prove that the entropy H(x) of the source is at the most $\log_2 n$. **(05 Marks)**

b. A discrete source emits one of the five symbols once every symbols once every msec. The symbol probabilities are $\frac{2}{5}, \frac{1}{5}, \frac{3}{20}, \frac{3}{20}, \frac{1}{10}$ respectively. Find the source entropy and information rate. **(03 Marks)**

c. For the first order Markov model shown in the fig. 1(c), find the entropy of each state, entropy of the source and show that $G_1 \geq G_2 \geq H(s)$. **(12 Marks)**

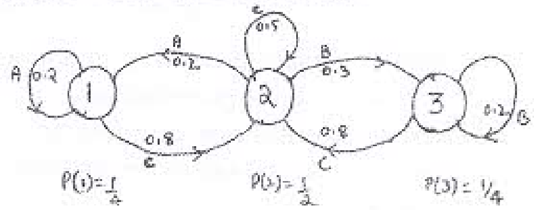

Fig.Q1(c)

2 a. i) What is a source encoder? **(02 Marks)**



ii) List the properties of a source encoder. **(03 Marks)**

iii) Explain prefix code and kraft McMillan inequality. Give an example and draw the decision tree for source symbols $S_0, S_1, S_2, S_3, S_4$. **(03 Marks)**

b. Consider an information source modeled by a discrete ergodic Markov random process whose graph is shown in fig. 2(b). Design a source encoder for this information source and compute the average bit rate and efficiency of the coder for N = 1 and 2 where N is the number of symbols in the message. **(12 Marks)**

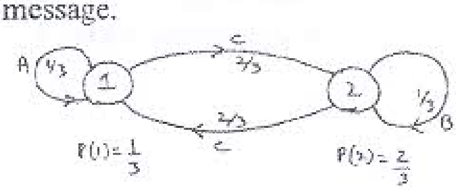

Fig.Q2(b)

3 a. A source produces two symbols $S_1$ and $S_2$ with probabilities $\frac{5}{6}$ and $\frac{1}{6}$ respectively. Device a coding scheme using Shannon – Fano encoding procedure to get a coding efficiency of at least 95%. **(12 Marks)**

b. A DMS with seven possible symbols $x_i$, i = 1, 2…7 has corresponding probabilities $P_1 = 0.46$, $P_2 = 0.26$, $P_3 = 0.12$, $P_4 = 0.06$, $P_5 = 0.03$, $P_6 = 0.02$, $P_7 = 0.01$. Show that Huffman coding is not unique. Find the coding efficiency and variance in both cases. Which coding is preferred? **(08 Marks)**

4 a. Prove the following relationships :

i) $I(x\,y) = H(x) - H(x/y)$ ii) $I(x\,y) = H(y) - H(y/x)$. **(05 Marks)**

b. Prove that mutual information is always positive i.e. $I(x\ y) \geq 0$. **(05 Marks)**

c. For the binary symmetric channel shown in the fig. Q4(c).

i) Calculate $H(x)$, $H(y)$, $H(y/x)$, $H(x/y)$, $I(x,y)$ and channel capacity 'c'.

ii) If $P(x_0) = 0.4$ and $P(x_1) = 0.6$, calculate the efficiency of the source and hence its redundancy.

iii) If the symbols are received on an average with 4 in every 100 symbols in error, calculate the transmission rate of the system. Take $r_s = 100$ symbols / sec. **(10 Marks)**

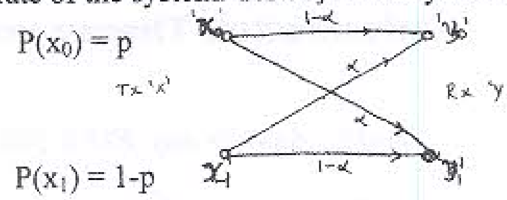


Fig.Q4(c)

5 a. State and prove the Shannon – Hartley law. Derive an expression for upper limit on channel capacity as the bandwidth tends to '∞'. **(08 Marks)**

b. Find the capacity of the channel shown in fig. Q5(b). Take $r_s = 1000$ symbols / sec. **(04 Marks)**

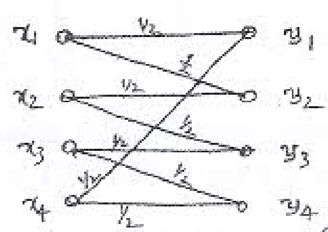

Fig.Q5(b)

c. A black and white TV picture consists of about $2 \times 10^6$ picture elements with 16 different brightness levels with equal probabilities. If pictures are repeated at the rate of 32 per second, calculate the average rate of information conveyed by this TV picture source. If the SNR is 30dB, what is the maximum bandwidth required to support the transmission of the resultant video signal. **(08 Marks)**



6 a. Explain the following terms : i) Code rate ii) Hamming distance iii) Minimum distance iv) Systematic code v) Linear code. **(05 Marks)**

b. For a linear block code with generator matrix 'G' and parity check matrix 'H', in a systematic form, prove that $GH^T = 0$. **(05 Marks)**

c. For a systematic linear block code, the parity matrix is given by $[p] = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

Find i) All possible code words ii) Minimum weight of this code iii) Parity check matrix iv) For a received code vector R = 1 1 1 1 0 1 detect and correct an error that has occurred due to noise. **(10 Marks)**

7 a. Define cycle code. Explain how cyclic codes are generated from the generating polynomials. **(06 Marks)**

b. The generator polynomial for a (7, 4) binary cyclic code is $g(x) = 1 + x + x^3$.

i) Find the code vector in systematic form for a message vector 1 1 0 1.

ii) Design an encoder for the above and verify its operation for message vector 1 1 0 1. Write the output code word in binary form and also in polynomial form. **(14 Marks)**

8 Consider a (3, 1, 2) convolutional code with $g^{(1)} = 110$, $g^{(2)} = 101$ and $g^{(3)} = 111$.

a. Draw the encoder block diagram b. Find the generator matrix.

c. Find the code word corresponding to the information sequence (1 1 1 0 0 ), using time domain and transform domain approach.

d. Draw the state table e. State diagram f. Code tree. **(20 Marks)**

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