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06EC65

Sixth Semester B.E. Degree Examination, December 2012
Information Theory and Coding

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. An analog signal is band limited to 500Hz and is sampled at "Nyquist rate". The samples are quantized into 4 levels and each level represent one message. The quantization levels are assumed to be independent. The probabilities of occurrence of 4 levels are $P_1 = P_4 = 1/8$ and $P_2 = P_3 = 3/8$. Find the information rate of the source. **(04 Marks)**

b. Design a system to report the heading of a collection of 400 cars. The heading is to be quantized into three levels: heading straight (s), turning left (L), and turning right ®. This information is to be transmitted every second. Based on the data given below, construct a model for the source and calculate: i) the entropy of each state; ii) entropy of the source; iii) the rate of transmission.

I. On the average, during a given reporting interval, 200 cars were heading straight, 100 were turning left and 100 cars were turning right.

II. Out of 200 cars that reported heading straight during a reporting period, 100 of them (on the average) going straight during the next reporting period, 50 of them reported turning left during the next period, and 50 of them reported turning right during the next period.

III. On the average, out of 100 cars that reported as turning during a signaling period, 50 of them continued their turn during the next period and the remaining headed straight during next reporting period.

IV. The dynamic of the cars did not allow them to change their heading from left to right or right to left during subsequent reporting periods. **(10 Marks)**

c. Define the following:
i) Entropy
ii) Self information
iii) Information rate. **(06 Marks)**

2 a. Using Shannon's encoding algorithm find the binary code for the symbol of length two (ie. N = 2) generated by the information source given in Fig.Q.2(a). Also compute the average number of bits/symbols and efficiency of the codes. **(12 Marks)**

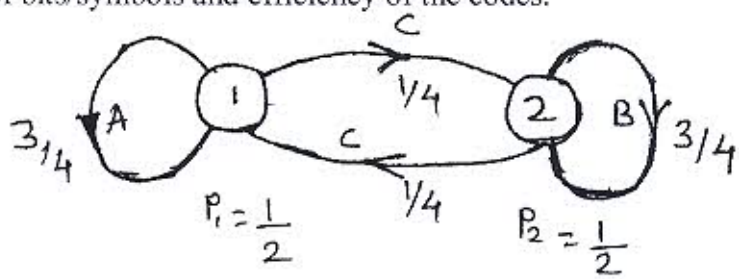

Fig.Q.2(a)

b. A Gaussian channel has a 10 MHz bandwidth and S/N = 100. Calculate the capacity of a channel and the maximum information rate. **(04 Marks)**

c. Determine the capacity of the channel shown in Fig.Q.2(c). **(04 Marks)**

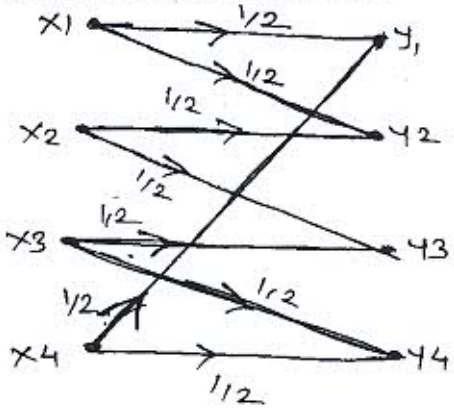

Fig.Q.2(c)

3 a. A source emits an independent sequence of symbols from an alphabet consisting of a 6 symbols A, B, C , D, E and F with probabilities $P = \left\{\frac{1}{3}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{12}, \frac{1}{12}\right\}$. Determine the Huffman code by shifting the combined symbols as high as possible. Also find the coding efficiency of the code. **(08 Marks)**

b. Explain the properties of mutual information and prove that the mutual information of the channel is symmetric. **(12 Marks)**

4 a. Prove that the maximum value of the differential entropy of white Gaussian noise with variance σ^2 is given by $h(x) = 1/2 \log_2 (2\pi e\sigma^2)$. **(10 Marks)**

b. A channel has the following characteristics :

$$P\left(\frac{Y}{X}\right) = \begin{array}{c} \\ X_1 \\ X_2 \end{array} \begin{array}{c} \begin{array}{cccc} Y_1 & Y_2 & Y_3 & Y_4 \end{array} \\ \begin{bmatrix} 1/3 & 1/3 & 1/6 & 1/6 \\ 1/6 & 1/6 & 1/3 & 1/3 \end{bmatrix} \end{array}$$

Find H(X), H(Y), H(X, Y) and channel capacity if r = 1000 symbols/sec. Assume $P(X_1) = P(X_2) = 1/2$. **(10 Marks)**

PART – B

5 a. In a linear block code the syndrome is given by :

$S_1 = r_1 + r_2 + r_3 + r_5$

$S_2 = r_1 + r_2 + r_4 + r_6$

$S_3 = r_1 + r_3 + r_4 + r_7$

Find:

i) Generator matrix [G]

ii) Parity check matrix [H]

iii) Find the code word for all the messages

iv) A single error has occurred in the received vector 1011011. Detect and correct this error. **(10 Marks)**

b. Explain syndrome properties. **(10 Marks)**

6 a. The generator polynomial for (15, 5) cyclic code is $g(x) = 1 + x^4 + x^6 + x^7 + x^8$. Find the code-vector in systematic form for the message vector $D(x) = x^2 + x^3 + x^4$. **(06 Marks)**

b. The generator polynomial of a (15, 7) cyclic code is $g(x) = 1 + x + x^4$. Illustrate the encoding procedure by listing the state of the register with the message vector 10010110111. **(08 Marks)**

c. Explain the error correction procedure for cyclic codes. **(06 Marks)**

7 a. Explain the interlacing technique for the correction of burst and random errors. Consider a (15, 7) BCH code generated by $g(x) = x^8 + x^4 + x^2 x + 1$; construct a interleaved code with $\lambda = 5$, with a burst error correcting ability of 10. **(08 Marks)**

b. Write short notes on any three:
i) RS codes
ii) Shortened cyclic codes
iii) BCH codes
iv) Golay codes. **(12 Marks)**

8 a. Consider a (2, 1, 2) convolution code with the impulse response $g^{(1)} = (1, 1, 1)$ and $g^{(2)} = (1, 0, 1)$ and the incoming message sequence is 10011.
i) Draw the encoder block diagram.
ii) Find the generator matrix.
iii) Find the code vector if the encoder generates.
The two output sequences by convoluing the message sequence with the impulse response of the path using time domain approach. **(10 Marks)**

b. For the (3, 2, 1) convolution encoder shown in Fig.Q.8(b), find the codeword for the input sequence 110110 using i) Time domain approach (using generator matrix); ii) Transfer domain approach by constructing transfer function matrix. **(10 Marks)**

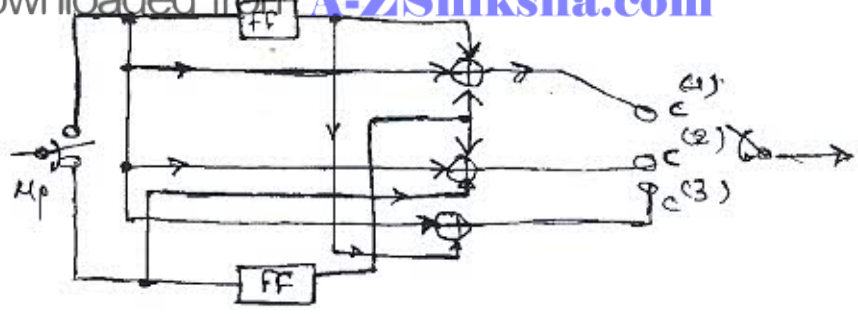

Fig.Q.8(b)

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06EC65

~~Sixth~~ V Semester B.E. Degree Examination, June 2012
Information Theory and Coding

Changed

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. A binary source is emitting independent sequence of 0's and 1's with probabilities p and (1 - p) respectively. Plot the entropy of this sources versus probability (0 < p < 1). Write the conclusion. **(04 Marks)**

b. Find the interrelationships between hartleys, hats and bits. **(06 Marks)**

c. For the 1st order Markov sources shown in the Fig.Q1(c)
i) Find the stationary distribution ii) Entropy of each state and hence entropy of source
iii) Entropy of adjacent source and verify whether $H(s) < H(\bar{s})$ **(10 Marks)**

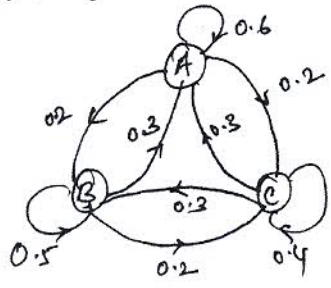

Fig.Q1(c)

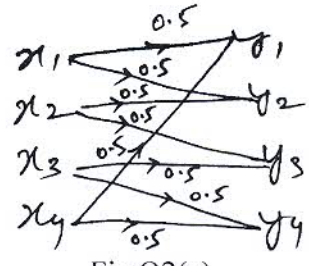

Fig.Q2(c)

2 a. Explain the important properties of codes to be considered while encoding a source. **(05 Marks)**

b. Using Shannon's binary encoding procedure, construct a code for the following discrete source.
$S = \{ s_1, s_2, s_3, s_4, s_5 \}$
$P = \{ 0.4, 0.25, 0.15, 0.12, 0.08 \}$ **(10 Marks)**

c. Determine the channel capacity of the discrete channel depicted in the Fig.Q2(c). **(05 Marks)**

3 a. A discrete memoryless source with alphabets A to H has respective probabilities 0.22, 0.20, 0.18, 0.15, 0.10, 0.08, 0.05 and 0.02. Construct binary and ternary codes for the same using Huffman's encoding algorithm. Determine code efficiency in each case. **(12 Marks)**

b. Noise matrix of a binary symmetric channel is illustrated below which has the following source symbol probabilities:

$$P(x_1) = 2/3 ; \quad P(x_2) = 1/3 ; \quad P(y/x) = \begin{bmatrix} 3/4 & 1/4 \\ 1/4 & 3/4 \end{bmatrix}$$

i) Determine H(x), H(y), H(x, y), H(x/y), H(y/x) and I(x, y)
ii) Also determine channel capacity. **(08 Marks)**

4 a. State and explain the Shannon-Hartley law. Obtain an expression for the maximum capacity of a continuous channel. **(10 Marks)**

b. A b/w TV picture may be viewed as consisting of approximately 3×10^5 elements, each one of which may occupy 10 distinct brightness levels with equal probability. Assuming the rate of transmission as 30 picture frames/sec and an SNR of 30 db, calculate the minimum bandwidth required to support the transmission of the resultant video signal. **(10 Marks)**

PART – B

5 a. Parity matrix for a systematic (6, 3) linear block code is given as

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$, find all the possible code vectors. **(10 Marks)**

b. Define hamming weight, hamming distance and minimum distance of linear block codes. **(06 Marks)**

c. If 'C' is a valid code vector such as C = DG, then prove that $CH^T = 0$, where H is the parity check matrix. **(04 Marks)**

6 a. A (15, 5) cyclic code has a generator polynomial $g(x) = 1 + x + x^2 + x^4 + x^5 + x^8 + x^{10}$.

i) Draw the block diagram of an encoder and syndrome calculator of this code.

ii) Find code polynomial for $D(x) = 1 + x^2 + x^4$ in systematic form.

iii) If $V(x) = 1 + x^4 + x^6 + x^8 + x^{14}$ check whether it is a valid code polynomial or not. **(12 Marks)**

b. Consider a (15, 11) cyclic code generated using $g(x) = 1 + x + x^4$.

i) Design a feedback resister encoder for the same.

ii) Illustrate the encoding procedure with the message vector [11001101011] by listing the states of the register assuming right most bit as the earliest bit. **(08 Marks)**

7 Write short notes on:

a. RS codes

b. Shortened cyclic codes

c. Golay codes

d. Burst error correcting codes. **(20 Marks)**

8 a. For the convolutional encoder shown in Fig.Q8(a), if information sequence D = 10011, find the output sequence using

i) time domain approach ii) transform domain approach. **(12 Marks)**

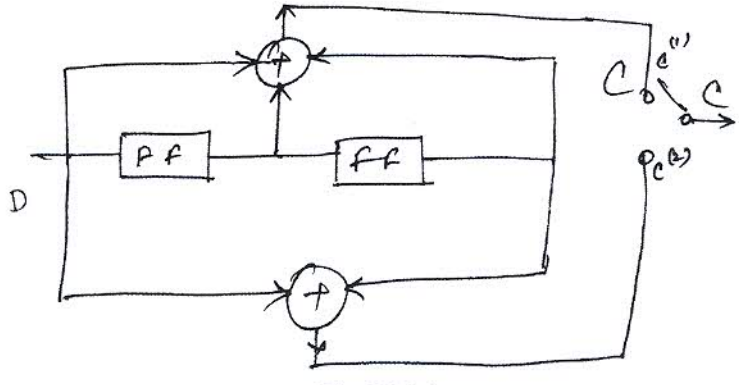

Fig.Q8(a)

b. For a (3, 1, 2) convolutional code with $g^{(1)} = (110)$, $g^{(2)} = (101)$ and $g^{(3)} = (111)$. Draw the encoder block diagram and also find the generator matrix. **(08 Marks)**

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