

Q1) What is the entropy of the image below (0, 20, 50, 99) denotes the grey level intensities

99	99	99	99	99	99	99	99
20	20	20	20	20	20	20	20
0	0	0	0	0	0	0	0
0	0	50	50	50	50	0	0
0	0	50	50	50	50	0	0
0	0	50	50	50	50	0	0
0	0	50	50	50	50	0	0
0	0	0	0	0	0	0	0

Q2) Consider a source  $S = \{s_1, s_2, s_3, s_4\}$  with probabilities  $P = \{1/2, 1/4, 1/4\}$ . Find self information of each image & entropy of  $S$

Q3) An event has four possible outcomes with probabilities  $P_1 = 1/2$ ,  $P_2 = 1/4$ ,  $P_3 = 1/8$ ,  $P_4 = 1/8$ . Determine self information in each outcome, entropy of the system. Also obtain rate of information if there are 16 outcomes / second.

Q4) A code is composed of dots & dashes. Assuming dash is 3 times dot & has  $1/3$ rd probability. Calculate

- ① information in dots & dash
- ② entropy of dot-dash code
- ③ Avg rate of info if a dot lasts for 10 milisec and this same interval is allowed between symbols.

1)  $8 \times 8$  image: 64 values

$$P(0) = \frac{1}{64} = \frac{1}{2^6}$$

$$P(20) = \frac{8}{64} = \frac{1}{8}$$

$$P(50) = \frac{16}{64} = \frac{1}{4}$$

$$P(40) = \frac{8}{64} = \frac{1}{8}$$

$$H(s) = \sum_{i=1}^n P_i \times \log_2(1/P_i)$$

$$H(s) = \frac{1}{2} \times \log_2(2) + \frac{1}{8} \times \log_2(8) + \frac{1}{4} \times \log_2(4) + \frac{1}{8} \times \log_2(8)$$

$$= \frac{7}{4} = 1.75 \text{ bits/symbol}$$

2)  $S = \{s_1, s_2, s_3\}$   $P = \{P_1, P_2, P_3\} = \{1/2, 1/4, 1/4\}$

Self information,  $I = \log_2(1/P)$

$$I_1 = 1 \text{ bit}$$

$$I_2 = 2 \text{ bits}$$

$$I_3 = 2 \text{ bits}$$

$$\text{Entropy } (H(s)) = \frac{1}{2} \times \log_2(2) + \frac{1}{4} \log_2(4) + \frac{1}{4} \log_2(4)$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2} = 1.5 \text{ bits/symbol}$$

$$3) I_1 = \log_2(2) = 1 \text{ bit}$$

$$I_2 = \log_2(4) = 2 \text{ bits}$$

$$I_3 = \log_2(8) = 3 \text{ bits}$$

$$I_4 = \log_2(8) = 3 \text{ bits}$$

$$H(s) = \frac{1}{2} I_1 + \frac{1}{4} I_2 + \frac{1}{8} I_3 + \frac{1}{8} I_4$$

$$= \frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{3}{8} = \frac{7}{4} = 1.75 \text{ bits/symbol}$$

$$R \cdot O \cdot I = 16 \times H(s) \\ = 16 \times 1.75 = \underline{\underline{28}}$$

4)  $P(\text{dot}) + P(\text{dash}) = 1 \dots \dots \textcircled{1}$   
 $P(\text{dash}) = \frac{1}{3} \times P(\text{dot})$

$$P(\text{dot}) = 3 P(\text{dash}) \dots \textcircled{2}$$

Put ② in ①

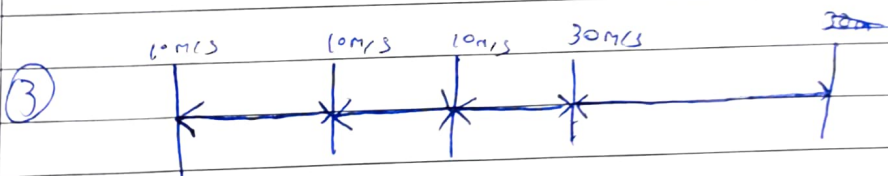
$$3 P(\text{dash}) + P(\text{dash}) \\ P(\text{dash}) = \frac{1}{4} \quad \therefore \quad P(\text{dot}) = \frac{3}{4}$$

①

$$I(\text{dash}) = \log_2(4) = 2 \text{ bits}$$

$$I(\text{dot}) = \log_2\left(\frac{4}{3}\right) = 0.41 \text{ bits}$$

$$\textcircled{2} \quad H(s) = \frac{1}{4} \log_2(4) + \frac{3}{4} \log_2\left(\frac{4}{3}\right) \\ = \frac{1}{2} + 0.311 = 0.81 \text{ bits/symbol}$$



$$f_s = \frac{\text{msg symbol}}{\text{time duration}} = \frac{4}{100} = 0.04 \text{ symbol/millisecond}$$

$$I_{\text{rate}} = f_s \times H(s) \\ = 0.04 \times 0.811 \\ = 0.034 \text{ bits/second}$$