

# 2D TRANSFORMATIONS

## COMPUTER GRAPHICS

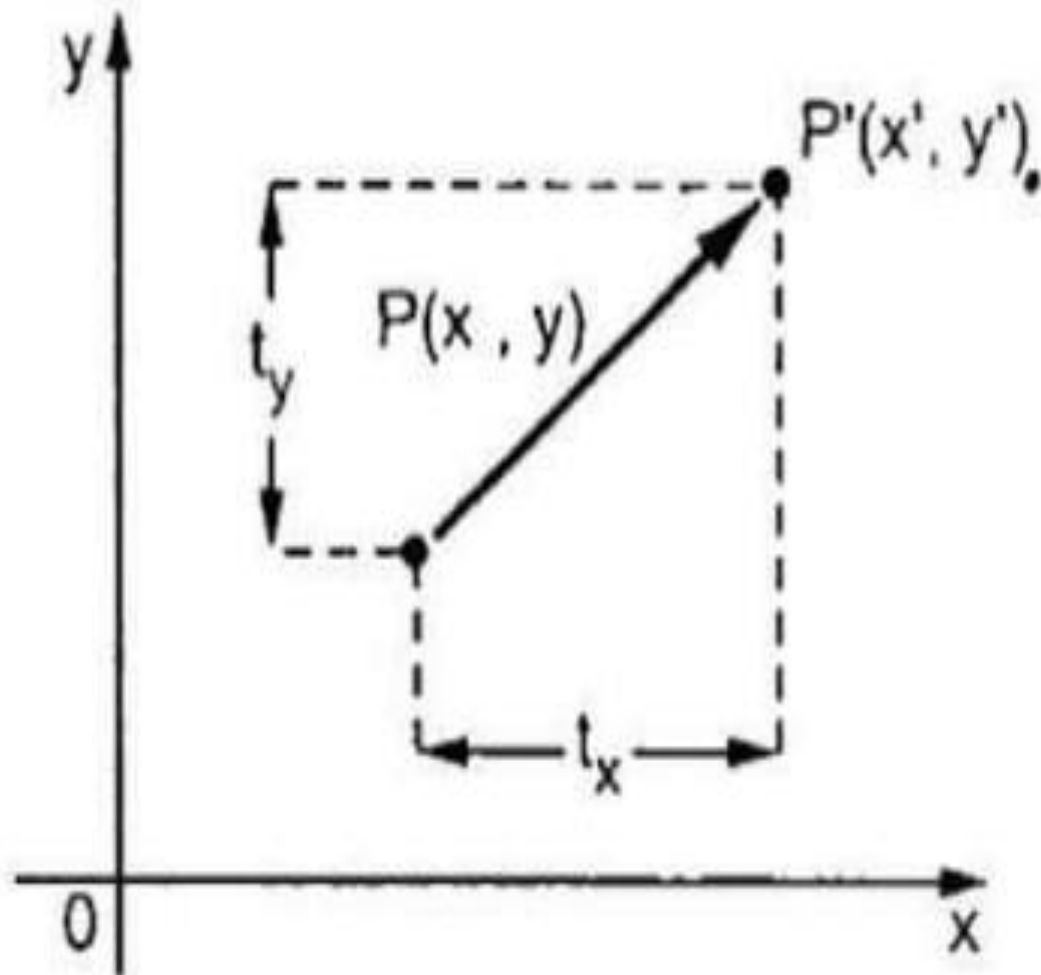
## 2D Transformations

“Transformations are the operations applied to geometrical description of an object to change its position, orientation, or size are called geometric transformations”.

# Translation

- ❖ Translation is a process of changing the position of an object in a straight-line path from one co-ordinate location to another.
- ❖ We can translate a two dimensional point by adding translation distances,  $t_x$  and  $t_y$ .
- ❖ Suppose the original position is  $(x, y)$  then new position is  $(x', y')$ .
- ❖ Here  $x' = x + t_x$  and  $y' = y + t_y$ .

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# Translation



- 1) Translate the point  $(2,4)$  where  $T(-1,14)$  Find  $P'$
- 2) Translate the polygon with  $A(2,7)$   $B(7,10)$   $C(10,2)$  by 3 units in X direction and 4 units in Y direction

- ❖ Matrix form of the equations:

$$X' = X + tx \quad \text{and} \quad Y' = Y + ty \quad \text{is}$$

$$P = \begin{pmatrix} x \\ y \end{pmatrix} \quad P' = \begin{pmatrix} x' \\ y' \end{pmatrix} \quad T = \begin{pmatrix} tx \\ ty \end{pmatrix}$$

- ❖ we can write it,

$$P' = P + T$$

✧ Translate a polygon with co-ordinates A(2,5) B(7,10) and C(10,2) by 3 units in X direction and 4 units in Y direction.

✧  $A' = A + T$

$$= \begin{pmatrix} 2 \\ 5 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \end{pmatrix}$$

✧  $B' = B + T$

$$= \begin{pmatrix} 7 \\ 10 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 10 \\ 14 \end{pmatrix}$$

✧  $C' = C + T$

$$= \begin{pmatrix} 10 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 13 \\ 7 \end{pmatrix}$$



# Rotation

- ❖ A two dimensional rotation is applied to an object by repositioning it along a circular path in the xy plane.
- ❖ Using standard trigonometric equations , we can express the transformed co-ordinates in terms of  $\theta$  and  $\phi$  as

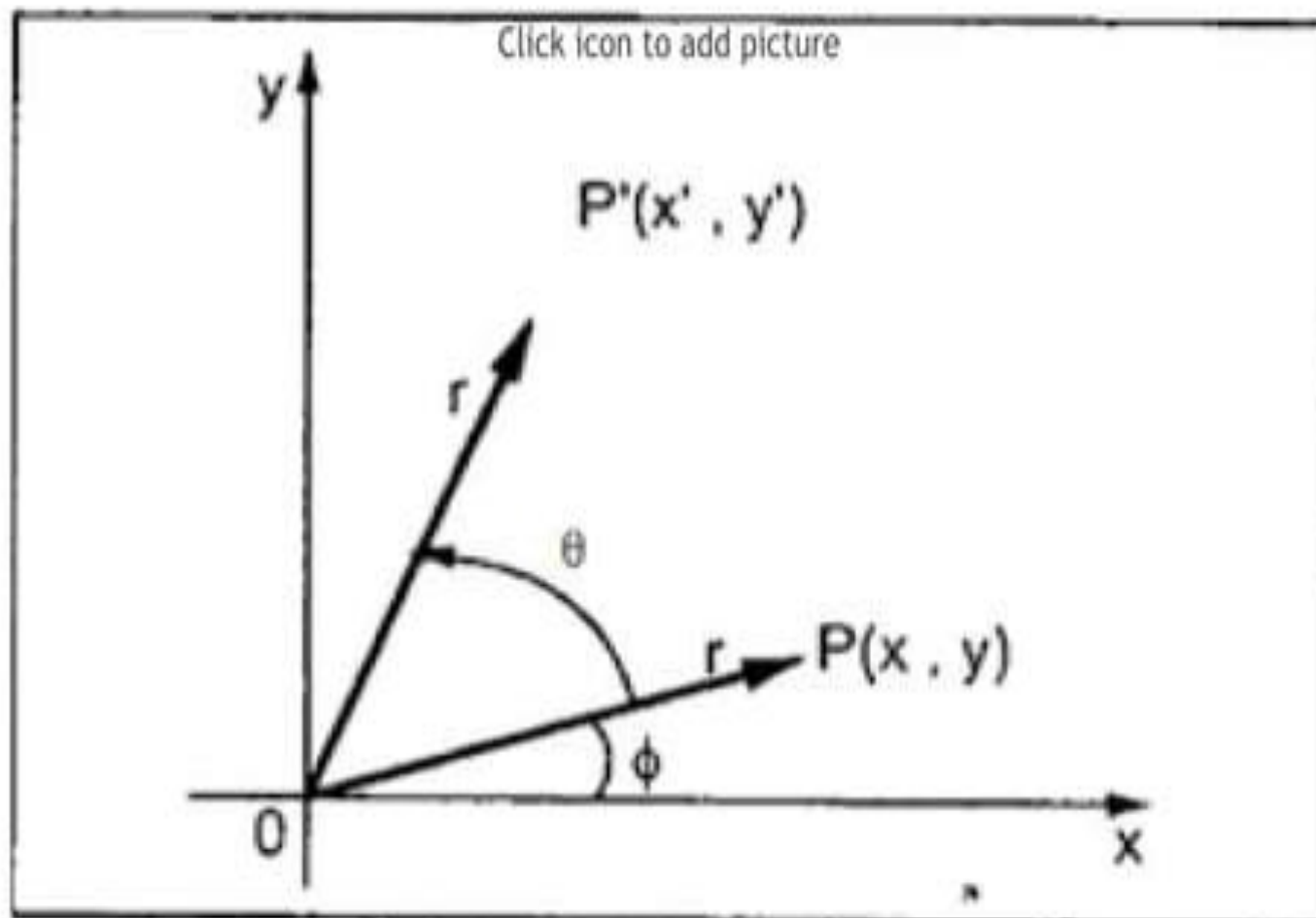
$$x' = r \cos(\phi + \theta) = r \cos\phi \cos\theta - r \sin\phi \sin\theta$$

$$y' = r \sin(\phi + \theta) = r \cos\phi \sin\theta + r \sin\phi \cos\theta$$

- ❖ The original co-ordinates of the point is

$$x = r \cos \phi$$

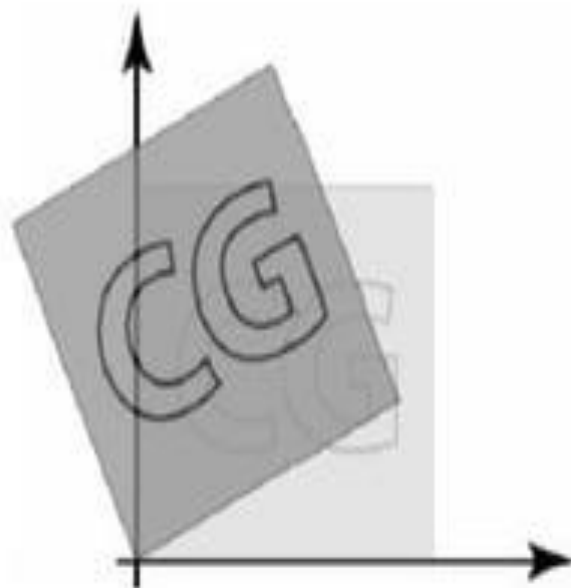
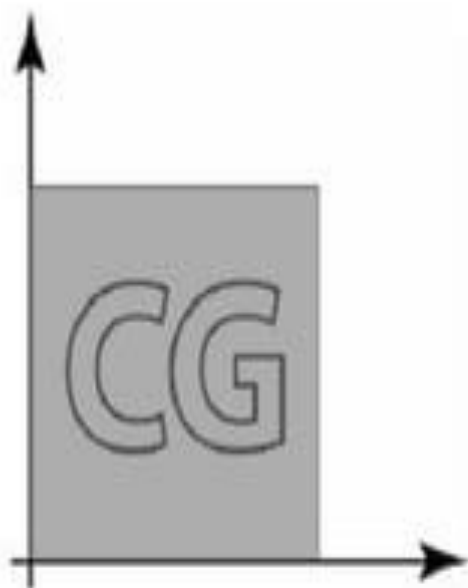
$$y = r \sin \phi$$



After substituting equation 2 in equation 1 we get

$$x' = x \cos \theta - y \sin \theta$$

# Rotation



- ❖ That equation can be represented in matrix form

$$\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

- ❖ we can write this equation as,

$$P' = P \cdot R$$

- ❖ Where R is a rotation matrix and it is given as

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

- ❖ A point (4,3) is rotated counterclockwise by angle of 45.  
find the rotation matrix and the resultant point.

$$\begin{aligned}\text{❖ } R &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \cos 45 & \sin 45 \\ -\sin 45 & \cos 45 \end{pmatrix} \\ &= \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}\end{aligned}$$

$$\begin{aligned}P' &= \begin{bmatrix} 4 & 3 \end{bmatrix} \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \\ &= \begin{bmatrix} 4/\sqrt{2} - 3/\sqrt{2} & 4/\sqrt{2} + 3/\sqrt{2} \end{bmatrix} \\ &= \begin{bmatrix} 1/\sqrt{2} & 7/\sqrt{2} \end{bmatrix}\end{aligned}$$

# Scaling

- ❖ A scaling transformation changes the size of an object.
- ❖ This operation can be carried out for polygons by multiplying the co-ordinates values  $(x, y)$  of each vertex by scaling factors  $S_x$  and  $S_y$  to produce the transformed co-ordinates  $(x', y')$ .

$$x' = x \cdot S_x$$

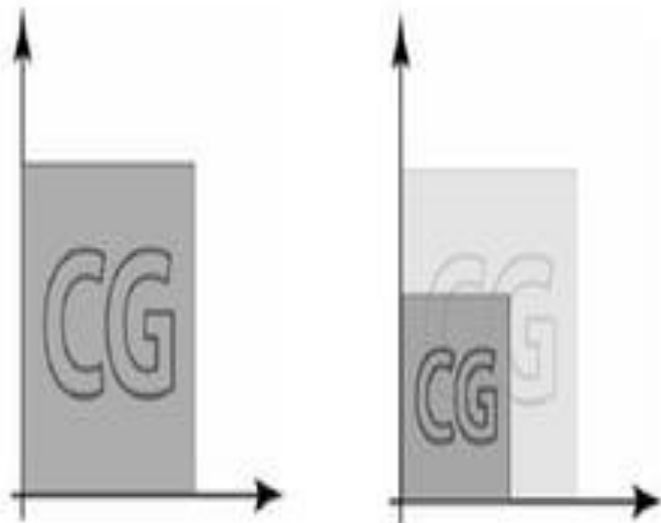
$$y' = y \cdot S_y$$

- ❖ In the matrix form

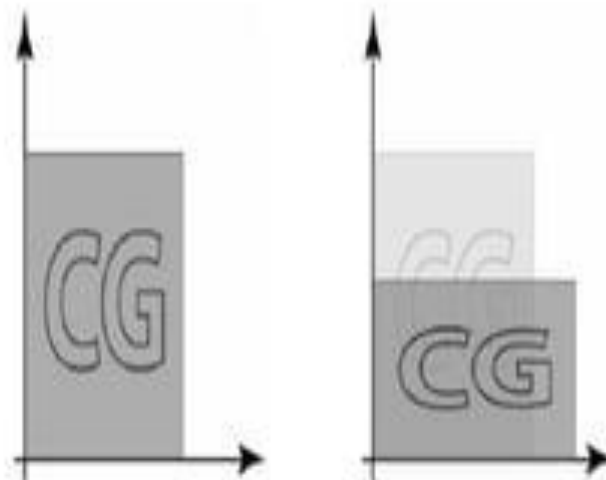
$$\begin{bmatrix} x' & y' \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

# Scaling

## • Uniform Scaling



## Un-uniform Scaling



- Ex. Scale a polygon with co-ordinates A(2,5), B(7,10) and C(10,2) by 2 units in X direction and 3 units in Y direction



- Translate a square ABCD A(0,0), B(3,0), C(3,3) and D(0,3) by 2 units in both direction then scale it by 1:5 units in X-direction and 0.5 in Y direction. Determine the resultant co-ordinated of polygon

# Answer

- Translation:

$A'(2,2)$     $B'(5,2)$     $C'(5,5)$     $D'(2,5)$

- Scaling:

$A''(3,1)$     $B''(7.5,1)$     $C''(7.5,2.5)$     $D''(3,2.5)$

Ex. Magnify the triangle with vertices  $(0,0)$ ,  $(1,1)$ ,  $(5,2)$  to twice its size while keeping  $(5,2)$  fixed

- Scaling:  $A'(0,0)$        $B'(2,2)$        $C'(10,4)$
- Translation:  $A''(-5,-2)$     $B''(-3,0)$     $C''(5,2)$

- Rotate a Triangle  $A(0,0)$   $B(2,2)$   $C(4,2)$   
about the origin by an angle of  $45^\circ$

# Transformation in 2D: Translation|Scaling|Rotation|Reflection|Shearing with nu

angle of  $45^\circ$ .

$$A' = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$B' = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4\sqrt{2} \end{bmatrix}$$

$$C' = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 2\sqrt{2} \\ 3\sqrt{2} \end{bmatrix}$$

## Transformation in 2D: Translation|Scaling|Rotation|Reflection|Shearing with num

→ A homogenous coordinate system is an abstract system in which we represent a 2-D point  $P(x, y)$  with a vector  $P_h(x_h, y_h, h)$  with the relationship

$$x = \frac{x_h}{h} \quad , \quad y = \frac{y_h}{h}$$

The term  $h$  is the homogeneous factor & can take any value.

For eg: point  $(3, 4)$  has homogenous coordinates  $(6, 8, 2)$

★ जब हम किसी coordinate को Translate, Rotate, Scale process में Time लगता है तो इस process को करने के लिए instead of  $2 \times 2$  matrix we use  $3 \times 3$  matrix

★ इसी के लिए अपने को  $h$  (dummy coordinate) को add करना

★ In this system, we can represent all the transformations

# Homogeneous co-ordinates for Translation

- ❖ The homogeneous co-ordinates for translation are given as

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{pmatrix}$$

- ❖ Therefore, we have

$$\begin{aligned} \begin{pmatrix} x' & y' & 1 \end{pmatrix} &= \begin{pmatrix} x & y & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ t_x & t_y & 1 \end{pmatrix} \\ &= \begin{pmatrix} x + t_x & y + t_y & 1 \end{pmatrix} \end{aligned}$$

# Homogeneous co-ordinates for rotation

- ❖ The homogeneous co-ordinates for rotation are given as

$$R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- ❖ Therefore , we have

$$\begin{aligned} \begin{bmatrix} x' & y' & 1 \end{bmatrix} &= \begin{bmatrix} x & y & 1 \end{bmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{bmatrix} x \cos\theta - y \sin\theta & x \sin\theta + y \cos\theta & 1 \end{bmatrix} \end{aligned}$$



## Homogeneous co-ordinates for scaling

- ❖ The homogeneous co-ordinate for scaling are given as

$$S = \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- ❖ Therefore , we have

$$\begin{aligned} [x' \ y' \ 1] &= [x \ y \ 1] \begin{pmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= [x \cdot S_x \ y \cdot S_y \ 1] \end{aligned}$$

# Composite Transformations

## (A) Translations

If two successive translation vectors  $(t_{x1}, t_{y1})$  and  $(t_{x2}, t_{y2})$  are applied to a coordinate position  $P$ , the final transformed location  $P'$  is calculated as: -

$$\begin{aligned} P' &= T(t_{x2}, t_{y2}) \cdot \{T(t_{x1}, t_{y1}) \cdot P\} \\ &= \{T(t_{x2}, t_{y2}) \cdot T(t_{x1}, t_{y1})\} \cdot P \end{aligned}$$

Where  $P$  and  $P'$  are represented as homogeneous-coordinate column vectors. We can verify this result by calculating the matrix product for the two associative groupings. Also, the composite transformation matrix for this sequence of transformations is: -

$$\begin{vmatrix} 1 & 0 & t_{x2} \\ 0 & 1 & t_{y2} \\ 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & t_{x1} \\ 0 & 1 & t_{y1} \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & t_{x1}+t_{x2} \\ 0 & 1 & t_{y1}+t_{y2} \\ 0 & 0 & 1 \end{vmatrix}$$

## (B) Rotations

Two successive rotations applied to point P produce the transformed position: -

$$\begin{aligned} P' &= R(\Theta_2) \cdot \{R(\Theta_1) \cdot P\} \\ &= \{R(\Theta_2) \cdot R(\Theta_1)\} \cdot P \end{aligned}$$

By multiplication the two rotation matrices, we can verify that two successive rotations are additive:

$$R(\Theta_2) \cdot R(\Theta_1) = R(\Theta_1 + \Theta_2)$$

So that the final rotated coordinates can be calculated with the composite rotation matrix as: -

$$P' = R(\Theta_1 + \Theta_2) \cdot P$$

## (C) Scaling

Concatenating transformation matrices for two successive scaling operations produces the following composite scaling matrix: -

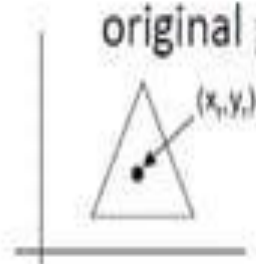
$$\begin{vmatrix} S_{x2} & 0 & 0 \\ 0 & S_{y2} & 0 \\ 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} S_{x1} & 0 & 0 \\ 0 & S_{y1} & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} S_{x1} \cdot S_{x2} & 0 & 0 \\ 0 & S_{y1} \cdot S_{y2} & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Or, 
$$S(S_{x2}, S_{y2}) \cdot S(S_{x1}, S_{y1}) = S(S_{x1} \cdot S_{x2}, S_{y1} \cdot S_{y2})$$

The resulting matrix in this case indicates that successive scaling operations are multiplicative.

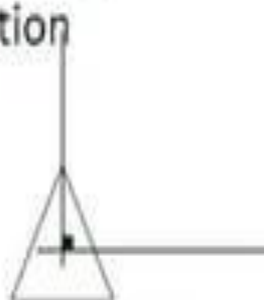
# General pivot point rotation

- Translate the object so that pivot-position is moved to the coordinate origin
- Rotate the object about the coordinate origin
- Translate the object so that the pivot point is returned to its original position



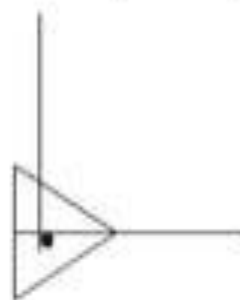
(a)

Original Position  
of Object and



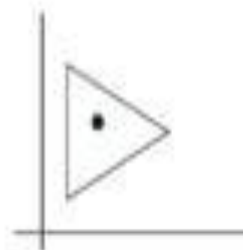
(b)

Translation of  
object so that  
pivot point  $(x_p, y_p)$



(c)

Rotation was  
about origin



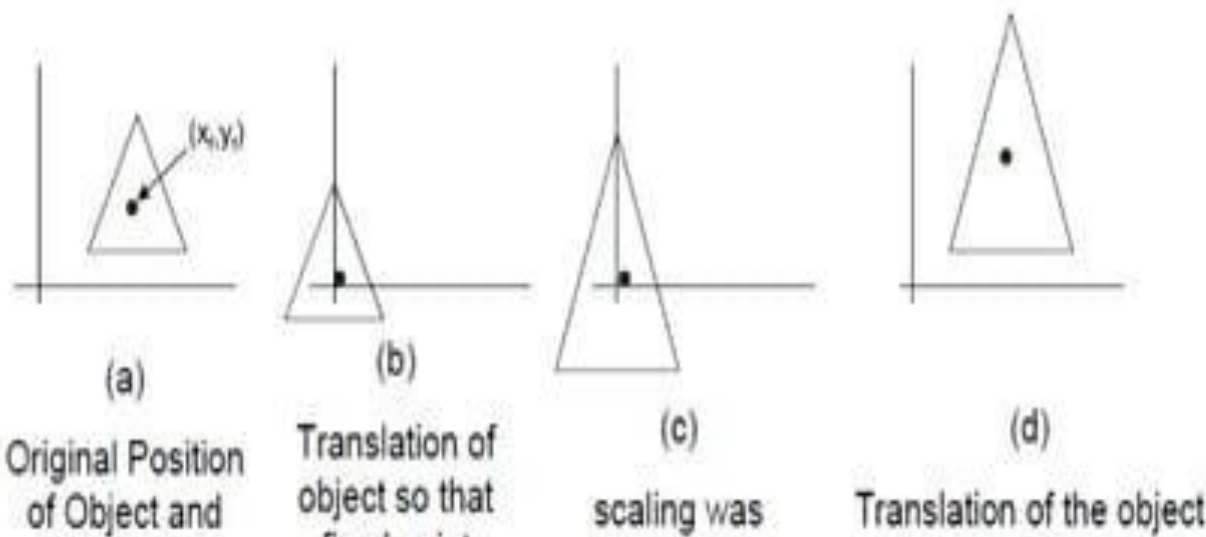
(d)

Translation of the object  
so that the pivot point is  
returned to position



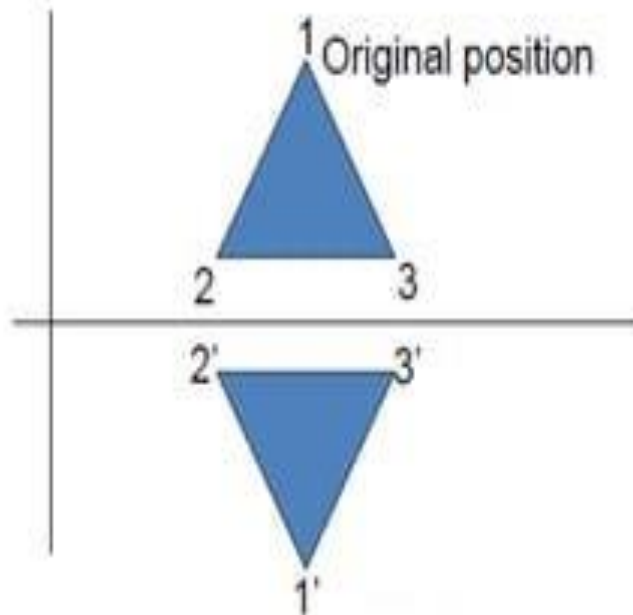
# General fixed point scaling

- Translate object so that the fixed point coincides with the coordinate origin
- Scale the object with respect to the coordinate origin
- Use the inverse translation of step 1 to return the object to its original position



## Other transformations

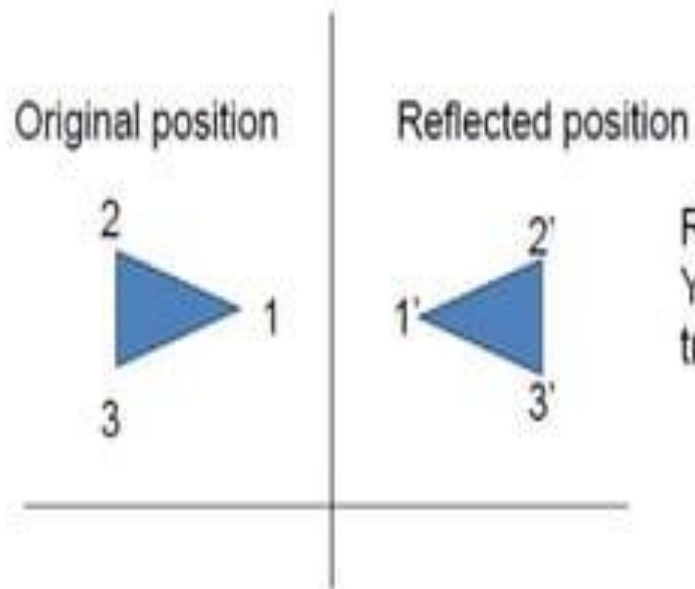
- **Reflection** is a transformation that produces a mirror image of an object. It is obtained by rotating the object by 180 deg about the reflection axis



Reflection about the line  $y=0$ , the X- axis , is accomplished with the transformation matrix

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

# Reflection

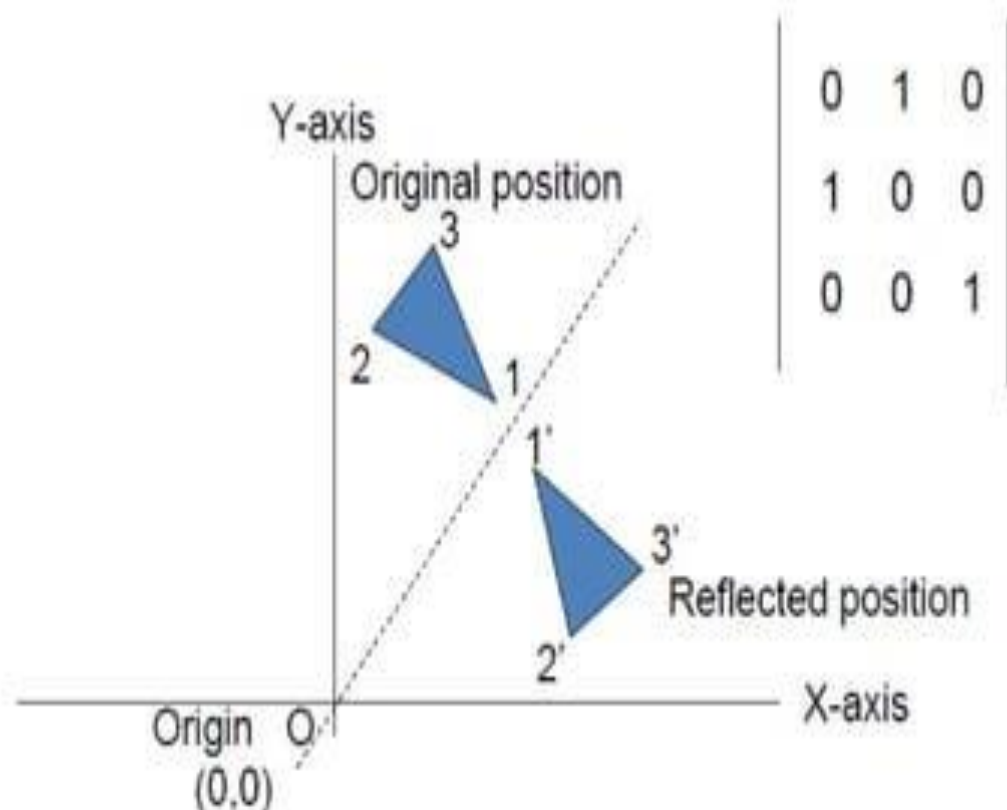


Reflection about the line  $x=0$ , the Y- axis , is accomplished with the transformation matrix

$$\begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$



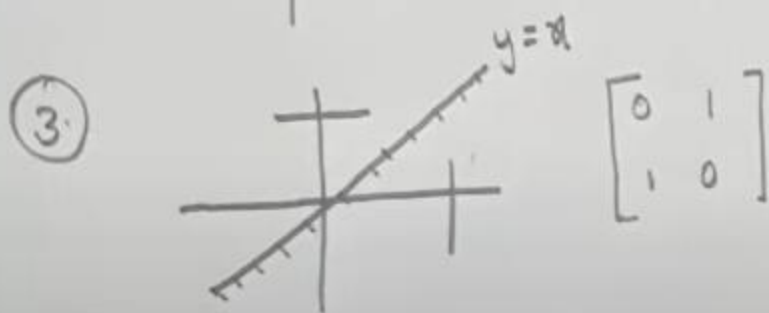
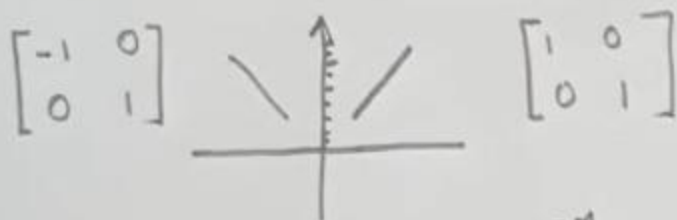
## Reflection of an object w.r.t the straight line $y=x$



# Transformation in 2D: Translation|Scaling|Rotation|Reflection|Shearing with num



② About X-axis:

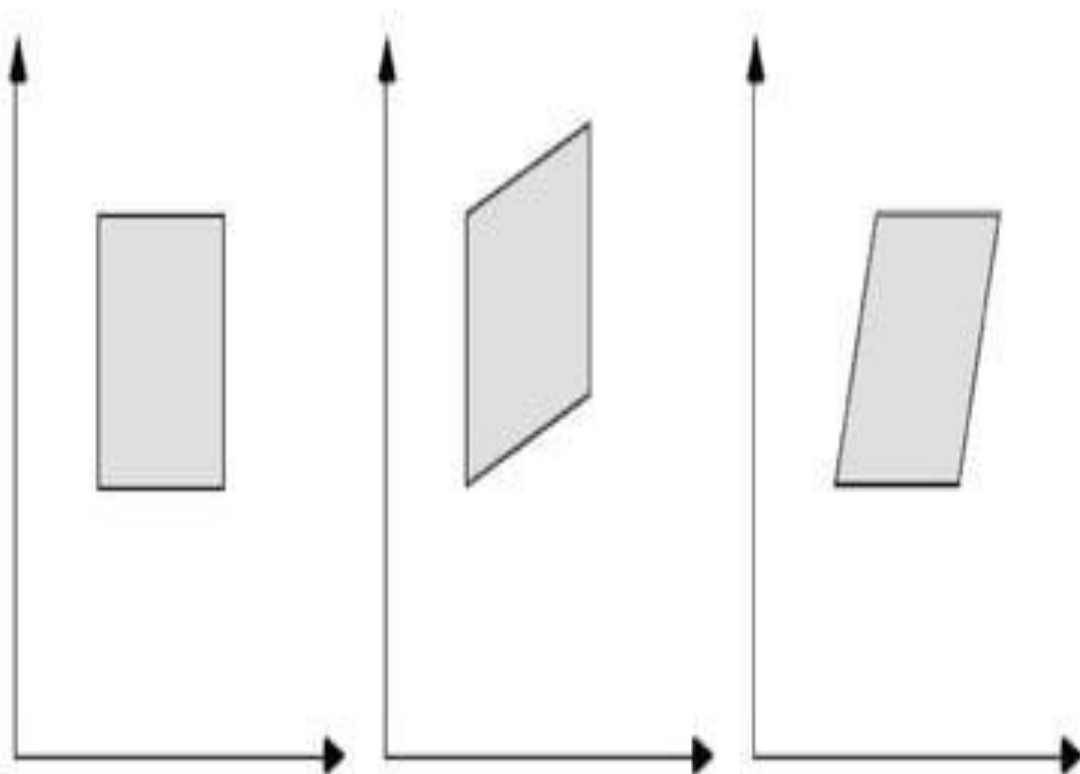


- Determine the transformation matrix of a triangle  $A(4,1)$   $B(5,2)$   $C(4,3)$  about the line  $x=0$  and determine the resultant co-ordinates.

# Shear Transformations

- Shear is a transformation that distorts the shape of an object such that the transformed shape appears as if the object were composed of internal layers that had been caused to slide over each other
- Two common shearing transformations are those that shift coordinate  $x$  values and those that shift  $y$  values

# Shears



## CONCLUSION

- ▶ To manipulate the initially created object and to display the modified object without having to redraw it, we use Transformations.

THANK YOU