AE 353 DesignProblem02

November 04 2019

1 Requirements and Verification's

1.1 Requirements

Since we must make our glider fly the furthest possible I would try to get a mean distance travelled of 20 meters

1.2 Verification

We will verify our requirements by making sure our mean of 1000 attempts is X.

We will also make sure our standard deviation does not exceed 5 meters(subject to change as per the performance of the controller)

Finally we will plot a histogram to study the data more easily.

2 Model

We want to maximize: C_L/C_D

$$\frac{C_L}{C_D} = \frac{2sin\alpha cos\alpha}{2sin\alpha sin\alpha} = \frac{1}{tan\alpha}$$

This is maximized when $\alpha = 0$

To find our state space model I solved the following equation using the Matlab function fsolve and found equilibrium points to linearize around. The equilibrium point is as follows.

$$f(\theta, \phi, \dot{x}, \dot{y}, 0, 0) = 0$$

$$\begin{bmatrix} \theta_e \\ \phi_e \\ \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} 0.0052 \\ -0.0824 \\ 6.0976 \\ -0.5499 \\ 0 \end{bmatrix}$$

Next I found the A and B matrices by finding the Jacobian (will explain in detail along with code in final report adding them to make sure they are correct)

$$\text{A} = \begin{bmatrix} -0.0475 & -0.5270 & 0.2767 & -6.5704 & 3.1240 \\ -23.8267 & 3.0641 & 1.0688 & 144.6979 & 48.3275 \\ 2.3034 & 25.5413 & -13.4106 & -157.0078 & -181.2602 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0.0611 \\ 0.7888 \\ -2.9589 \\ 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \end{bmatrix}$$

Checking for controllability , length(A) - rank(ctrb(A,B)) = 0 which is true. controllability matrix is as follows

$$\text{W} = 10^5 \text{ x} \begin{bmatrix} 0 & 0 & -0.0003 & 0.0220 & -0.4201 \\ 0 & 0.0002 & -0.0129 & 0.2112 & -2.7186 \\ 0 & -0.0012 & 0.0262 & -0.4902 & 7.9043 \\ 0 & 0 & -0.0012 & 0.0262 & -0.4902 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

To check if system if observable, I checked if the rank of the observable matrix is the same as length of A. The observable matrix(O) which is as follows had the rank 5 and thus the system is observable

$$\text{O} = 10^5 \text{ x} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0003 & -0.0001 & -0.0016 & -0.0018 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0095 & -0.0010 & 0.0579 & 0.0367 \\ 0 & 0 & 0 & 0 & 0 \\ 0.0078 & 0.2529 & 0.0150 & -1.5375 & -0.6447 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3 Controller

I designed my controller by finding the K matrix using the lqr function in matlab $K = lqr(A, b, Q_c, R_c)$ I used the following as my Q_c and R_c matrices

$$Q_c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \; R_c = [1]$$

similarly for L, L = lqr(A',C',$inv(R_0), inv(Q_0)$)

$$\text{where } Q_o = 3 \text{ x} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; R_o = 2 \text{ x} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Values for K and L are as follows

$$\text{L} = \begin{bmatrix} -3.671 & 7.1389 \\ 18.9889 & -3.5338 \\ 6.3420 & -4.1910 \\ 3.6449 & -0.9482 \\ -0.9482 & 0.7752 \end{bmatrix}, \; \text{K} = \begin{bmatrix} 0.7379 & -0.6037 & -0.6487 & -4.7614 & 11.6997 \end{bmatrix}$$

checking for asymptotical stability, found eigenvalues for (A-BK) and since they have all negative values the system is asymptotically stable

$$\text{eig(A-BK)} = \begin{bmatrix} -18.3066 + 8.0777i \\ -18.3066 - 8.0777i \\ -1.6781 \\ -6.0908 + 2.8444i \\ -6.0908 - 2.8444i \end{bmatrix}$$

4 Implementing and Testing

To Test the controller, I ran the simulation 1000 times and recorded the data using a loop in the test.m file. The following figures portray the result.

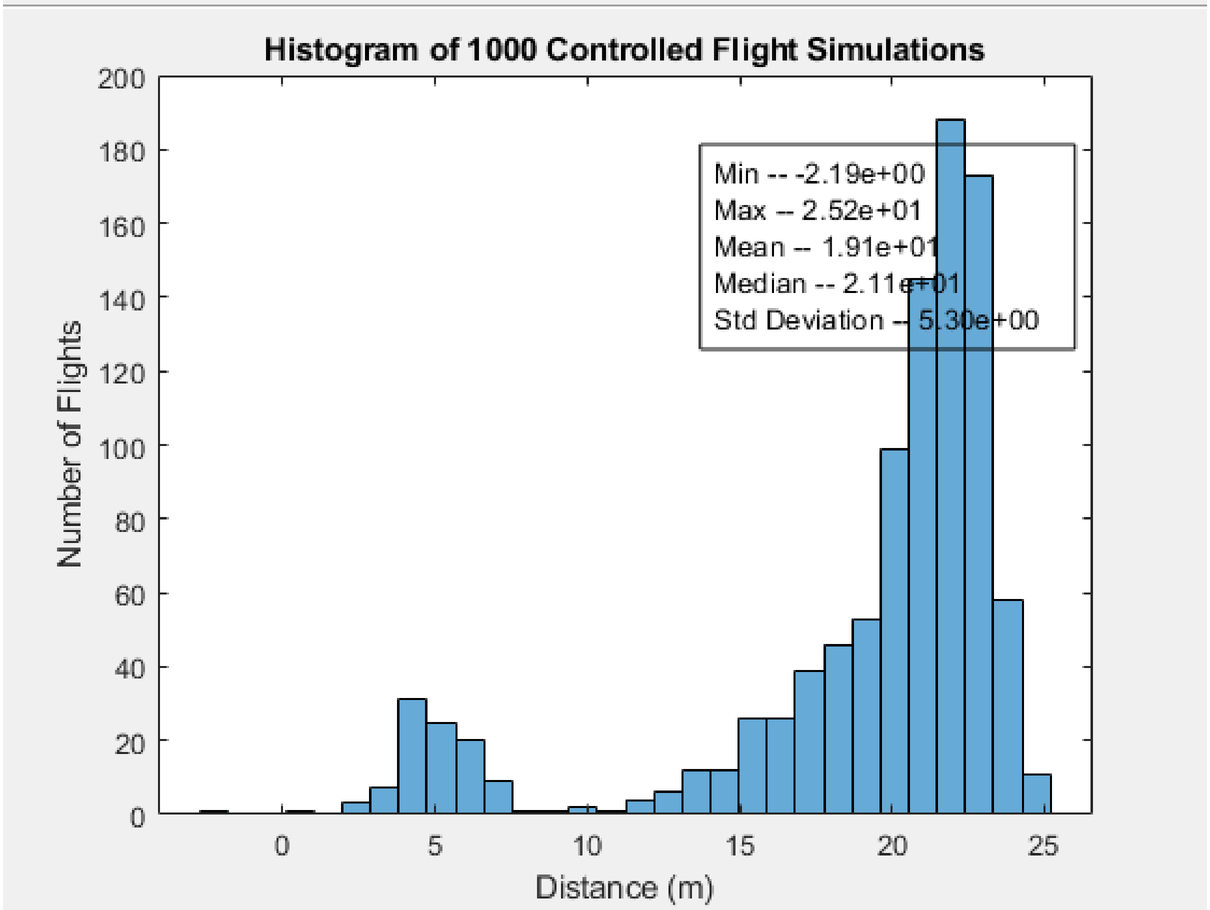

Figure 1: The results of 1000 iterations of the Simulation

The statistics are as follows

$$x_{max} = -25.2m$$
$$\bar{x} = 19.1m$$
$$x_{median} = 21.1m$$
$$\sigma = 5.3$$

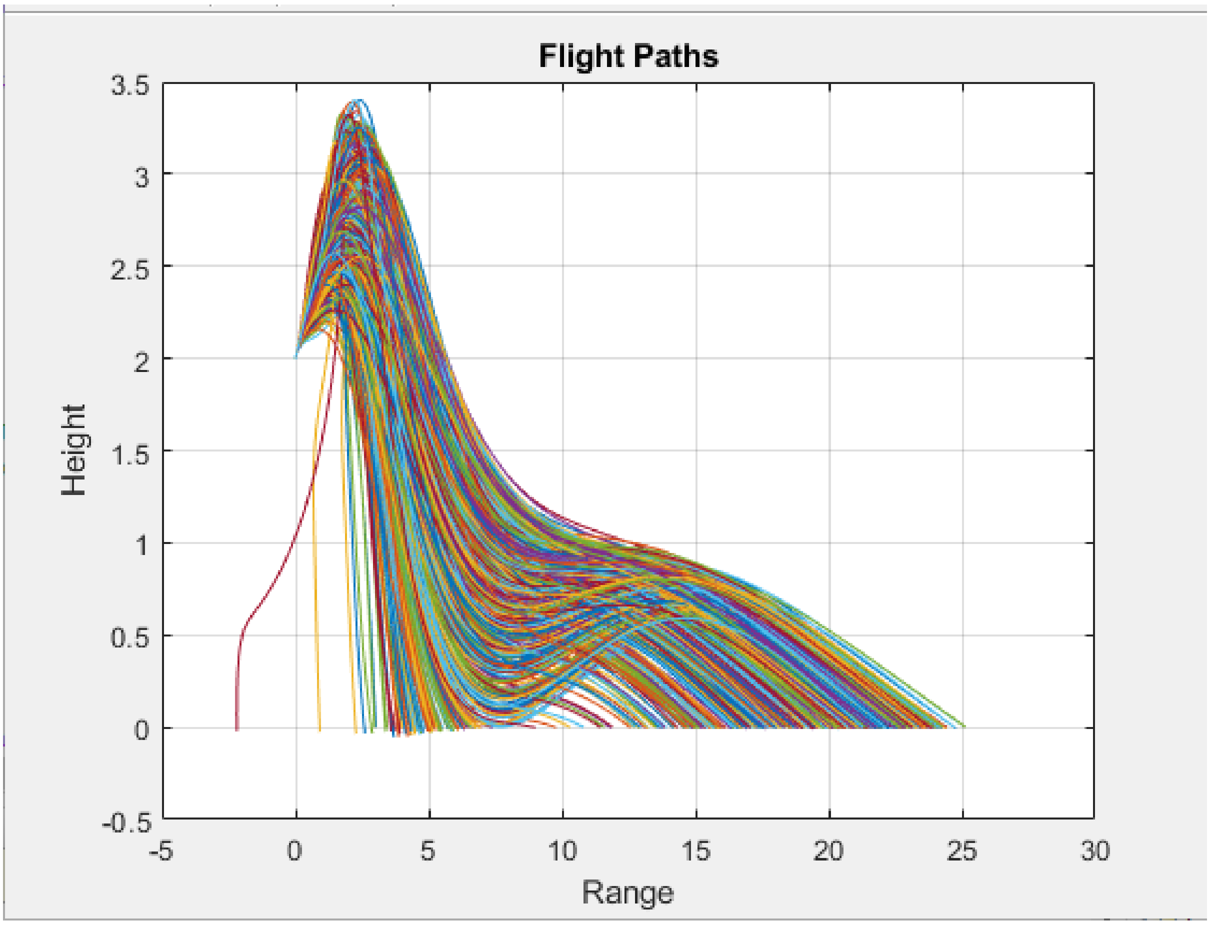

Figure 2: The results of 1000 iterations of the Simulation

5 Conclusion

My results still contain plenty of nose dives which causes my standard deviation to be so high. I changed values for $Q_0 and R_o$ to get a better result but could not get a result much higher than 25 meters. Unfortunately I could not get the standard deviation to be lower than what I achieved. A little more tweaking of the values could help me achieve the standard deviation required to verify