

Gaussian Processes

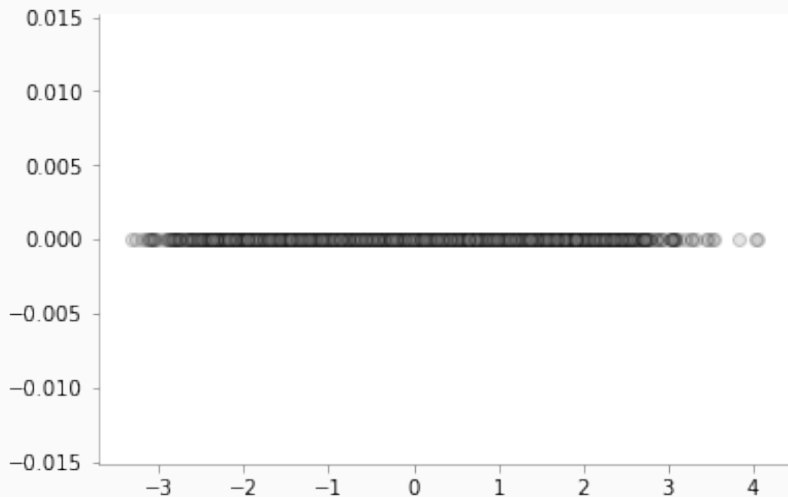
Nipun Batra

July 2, 2019

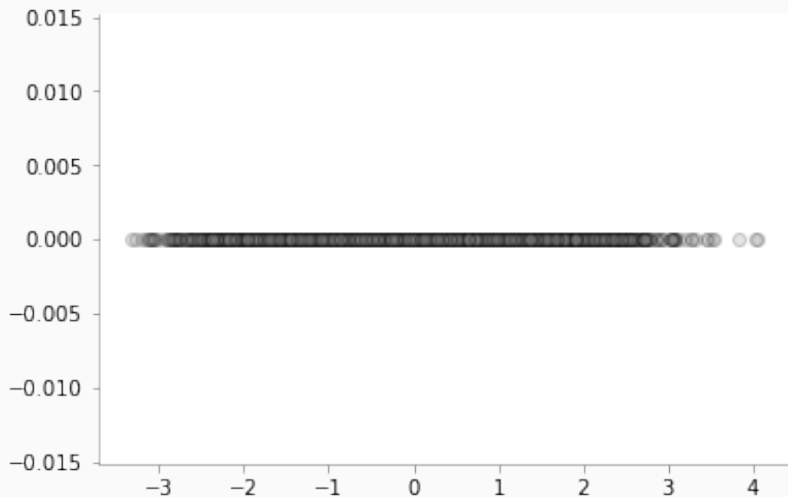
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Gaussian Distribution

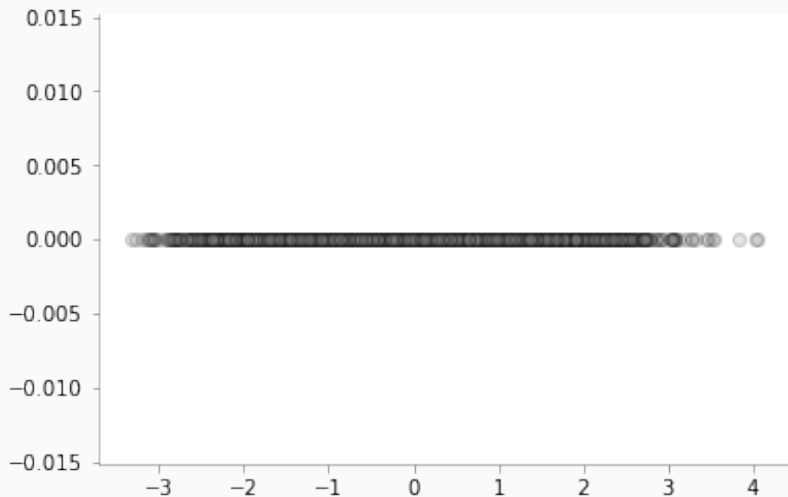
1d Gaussian Scatter Plot



1d Gaussian Histogram



Varying 1d Gaussian Variance



$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} a & \rho \\ \rho & b \end{pmatrix} \right)$$

Cholesky Decomposition I

$$\mathbf{A} = \mathbf{L}\mathbf{L}^T$$

where \mathbf{L} is a real lower triangular matrix.

We can thus re-write the posterior mean and covariance as:

$$p(y_* | X_*, X, y) \sim \mathcal{N}(\mu', \Sigma')$$
$$K = \mathbf{L}\mathbf{L}^T$$

Cholesky Decomposition II

$$\alpha = K^{-1}(x - \mu)$$

$$\text{or, } \alpha = LL^T^{-1}(x - \mu)$$

$$\text{or, } \alpha = L^{-T}L^{-1}(x - \mu)$$

$$\text{Let, } K^{-1}(x - \mu) = \beta$$

$$\text{Thus, } L^{-T}L^{-1}(x - \mu) = \beta$$

$$\text{Let, } L^{-1}(x - \mu) = \gamma$$

$$\text{Thus, } L\gamma = x - \mu$$

$$\text{Thus, } \gamma = L^{-1}(x - \mu)$$

$$\text{Thus, } \alpha = L^T \setminus (L \setminus (x - \mu))$$