Convention, Accuracy metrics, Classification, Regression

Nipun Batra December 25, 2023

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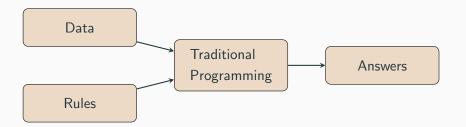
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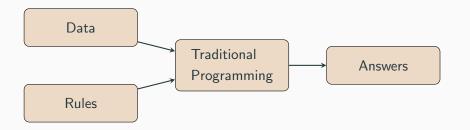
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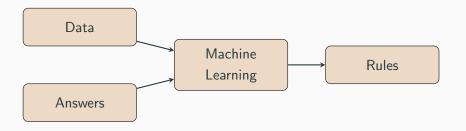
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"A computer program is said to learn from experience E with respect to some class of tasks T and performance measure P if its performance at tasks in T, as measured by P, improves with experience E." - Tom Mitchell

First ML Task: Grocery store tomatoes quality prediction

Problem statement: You want to predict the quality/condition of a tomato given its visual features.

Imagine you have some past data on quality of tomatoes. What visual features do you think will be useful?

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Size

Imagine you have some past data on quality of tomatoes. What visual features do you think will be useful?

- Size
- Colour

Imagine you have some past data on quality of tomatoes. What visual features do you think will be useful?

- Size
- Colour
- Texture

Imagine you have some past data on quality of tomatoes.

Sample	Colour	Size	Texture	Condition
1	Orange	Small	Smooth	Good
2	Red	Small	Rough	Good
3	Orange	Medium	Smooth	Bad
4	Yellow	Large	Smooth	Bad

Useful Features

Is the sample number a useful feature for predicting quality of a tomato?

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Let us modify our data table for now.

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The training set consists of two parts:

Colour	Size	Texture	Condition
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1. Features, Attributes or Covariates

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The training set consists of two parts:

- 1. Features, Attributes or Covariates
- 2. Output or Response Variable

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We call this matrix as \mathcal{D} , containing:

Colour	Size	Texture	Condition
Orange	Small	Smooth	Good
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Orange	Medium	Smooth	Bad
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We call this matrix as \mathcal{D} , containing:

1. Feature matrix $(\mathbf{X} \in \mathcal{R}^{\mathbf{N} \times \mathbf{P}})$ containing data of N samples each of which is P dimensional.

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• Example $x_1 = \begin{bmatrix} Orange \\ Small \\ Smooth \end{bmatrix}$

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2. Output Vector $(y \in \mathcal{R}^N)$ containing output variable for N samples.

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- Example $x_1 = \begin{bmatrix} Orange \\ Small \\ Smooth \end{bmatrix}$
- 2. Output Vector $(y \in \mathcal{R}^N)$ containing output variable for N samples.
- 3. Thus, we can also write $\mathcal{D} = \{(x_i^T, y_i)\}_{i=1}^N$

Prediction Task

Estimate condition for unseen tomatoes (#5, 6) based on data set.

Colour	Size	Texture	Condition
Orange	Small	Smooth	Good
Red	Small	Rough	Good
Orange	Medium	Smooth	Bad
Yellow	Large	Smooth	Bad
Red	Large	Rough	?
Orange	Large	Rough	?

Testing Set

Testing set is similar to training set, but, does not contain labels for output variable.

Colour	Size	Texture	Condition
Orange	Small	Smooth	Good
Red	Small	Rough	Good
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Yellow	Large	Smooth	Bad
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Orange	Large	Rough	?

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1. Learn f: Condition = f(colour, size, texture)

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We hope to:

- 1. Learn f: Condition = f(colour, size, texture)
- 2. From Training Dataset
- 3. To Predict the condition for the Testing set

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Orange	Small	Smooth	Good
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- Q: Is predicting on test set enough to say our model generalises?
- A: Ideally, no!
- Ideally we want to predict "well" on all possible inputs. But, can we test that?
- No! Since, the test set is only a sample from all possible inputs.

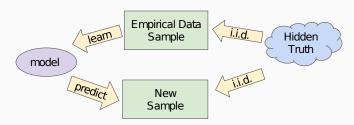


Image courtesy Google ML crash course

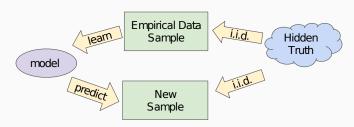


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Both the training set and the test set are samples drawn from the hidden true distribution (also sometimes called population)

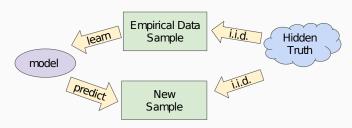


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More discussion later once we study bias and variance

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# People	Temp (C)	Energy (kWh)
4000	30	30
4200	30	32
4200	35	40
3000	20	?
1000	45	?

Classification

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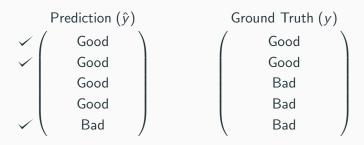
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 - Examples Predicting:
 - How much energy will campus consume?
 - How much rainfall will fall?

Metrics for Classification

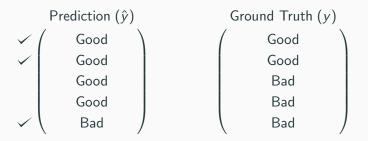
Ground Truth: From the actual training set

Prediction: Made by the model

Accuracy



Accuracy



$$\begin{aligned} \mathsf{Accuracy} &= \frac{||y = \hat{y}||}{||y||} \\ &= \frac{3}{5} = 0.6 \end{aligned}$$

Types of Data: Imbalanced Classes

$$\begin{array}{c} 1 \; \mathsf{sample} \; \{ \; \left(\begin{array}{c} \mathsf{Bad} \\ \mathsf{Good} \\ \mathsf{Good} \\ \dots \\ \mathsf{Good} \end{array} \right) \\ & \mathsf{Imbalanced} \; \mathsf{Classes} \end{array}$$

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Cases for this:

- Cancer Screening
- Planet Detection

Accuracy Metrics: Precision

Precision =
$$\frac{||y = \hat{y} = Good||}{||\hat{y} = Good||} = \frac{2}{4} = 0.5$$

"the fraction of relevant instances among the retrieved instances", i.e. "out of the number of times we predict Good, how many times is the condition actually Good"

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Accuracy Metrics: Recall

Recall =
$$\frac{||y = \hat{y} = \text{Good}||}{||y = \text{Good}||} = \frac{2}{3} = 0.67$$

"the fraction of the total amount of relevant instances that were actually retrieved"

Types of Data: Imbalanced Classes

Given predictions of whether a tissue is cancerous or not (n = 100).

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Given predictions of whether a tissue is cancerous or not (n = 100).

$$\mbox{Accuracy} = \frac{98}{100} = 0.98 \qquad \qquad \mbox{Recall} = \frac{0}{1} = 0$$

$$\mbox{Precision} = \frac{0}{1} = 0$$

		Ground Truth	
		Yes	No
ted	Yes	0	1
redicted	No	1	98
Д			

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		Yes	No
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$$Precision = \frac{T.P.}{T.P.+F.P.}$$

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		Yes	No	
redicted	Yes	True Positive	False Positive	
redi	No	False Negative	True Negative	
Д				

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Accuracy Metrics: F-Score

		Ground Truth	
		Yes	No
ted	Yes	True Positive	False Positive
Predicted	No	False Negative	True Negative
Д			

$$F-$$
 Score = $\frac{2 \times Precision \times Recall}{Precision + Recall}$

Accuracy Metrics: Matthew's Correlation Coefficient

		Ground Truth	
		Yes	No
cted	Yes	True Positive	False Positive
redicted	No	False Negative	True Negative
Д			

$$\frac{\text{TP} \times \text{TN} - \text{FP} \times \text{FN}}{\sqrt{(\text{TP} + \text{FP})(\text{TP} + \text{FN})(\text{TN} + \text{FP})(\text{TN} + \text{FN})}}$$

Accuracy Metrics: Example

For the data given below, calculate:

$$\begin{array}{cccc} & & G.T. \ Positive & G.T. \ Negative \\ Pred \ Positive & & 90 & 4 \\ Pred \ Negative & & 1 & 1 \end{array}$$

Precision = ?

Recall = ?

F-Score = ?

Matthew's Coeff. =?

Accuracy Metrics: Answer

For the same data

G.T. Positive G.T. Negative Pred Positive
$$\begin{pmatrix} 90 & 4 \\ 1 & 1 \end{pmatrix}$$

Precision =
$$\frac{90}{94}$$

Recall = $\frac{90}{91}$
F-Score = 0.9524
Matthew's Coeff. = 0.14

Metrics for Regression MSE & MAE

Prediction
$$(\hat{y})$$
 Ground Truth (y)

$$\begin{pmatrix}
10 \\
20 \\
30 \\
40 \\
50 \\
60
\end{pmatrix}$$

Mean Squared Error (MSE) =
$$\frac{\sum_{i=1}^{N} (\hat{y}_i - y_i)^2}{N}$$
Root Mean Square Error (RMSE) = $\sqrt{\text{MSE}}$

Accuracy Metrics: MAE & ME

Prediction
$$(\hat{y})$$
 Ground Truth

 $\begin{pmatrix}
10 & & & \\
20 & & & \\
30 & & & \\
40 & & & \\
50 & & & & \\
60 & & & & \\
\end{pmatrix}$

Mean Absolute Error (ME) =
$$\frac{\sum_{i=1}^{N} |\hat{y}_i - y_i|}{N}$$
Mean Error =
$$\frac{\sum_{i=1}^{N} \hat{y}_i - y_i}{N}$$

Accuracy Metrics: MAE & ME

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Is there any downside with using mean error?

Accuracy Metrics: MAE & ME

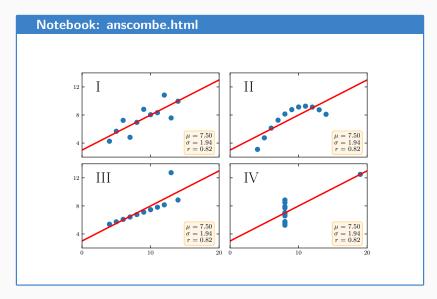
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Is there any downside with using mean error? Errors can get cancelled out

The Importance of Plotting



The Importance of Plotting

Property	Value	Accross datasets
mean(X)	9	exact
mean(Y)	7.5	upto 3 decimal places
Linear regression line	y = 3.00 + 0.500x	upto 2 decimal places