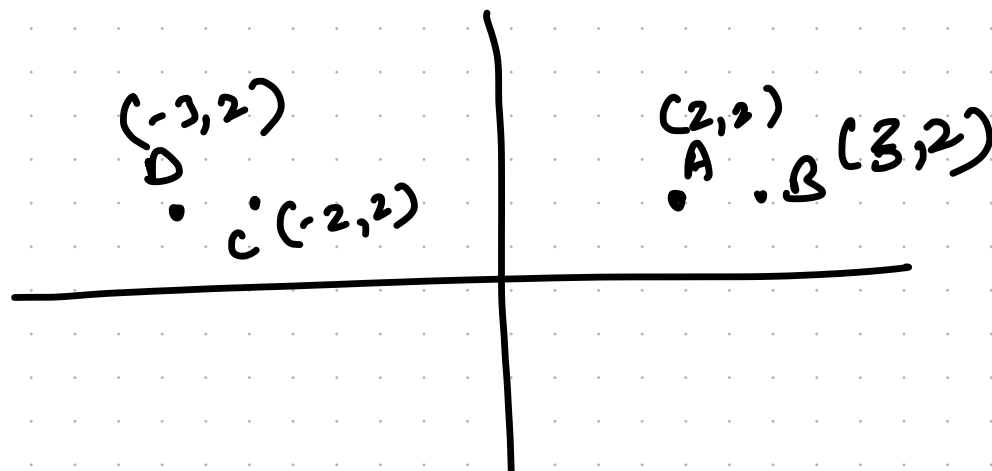


$$X = \begin{bmatrix} 2 & 2 \\ 2 & 3 \\ -2 & 3 \\ -2 & 2 \end{bmatrix} \in \mathbb{R}^{N \times D}$$

$$N = 4$$

$$D = 2$$



$$X = \begin{bmatrix} 2 & 2 \\ 3 & 2 \\ -2 & 2 \\ -3 & 2 \end{bmatrix} \in \mathbb{R}^{N \times D}$$

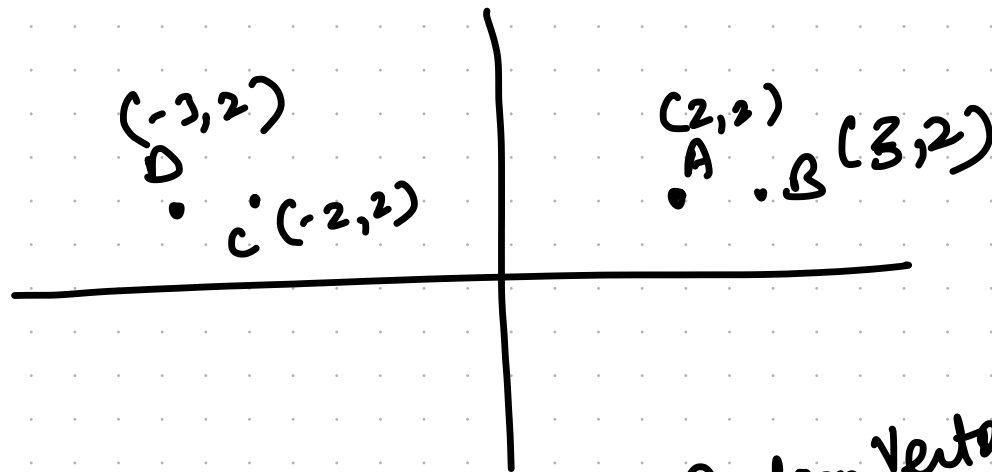
$$N = 4$$

$$D = 2$$

$$R = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \in \mathbb{R}^{D \times K}$$

$$K = 2$$

Usually  $K \ll D$   
 (Here for illustration  
 $K = D = 2$ )



$$X = \begin{bmatrix} 2 & 2 \\ 3 & 2 \\ -2 & 2 \\ -3 & 2 \end{bmatrix} \in \mathbb{R}^{N \times D}$$

Random Vector 1 (R.V.1)

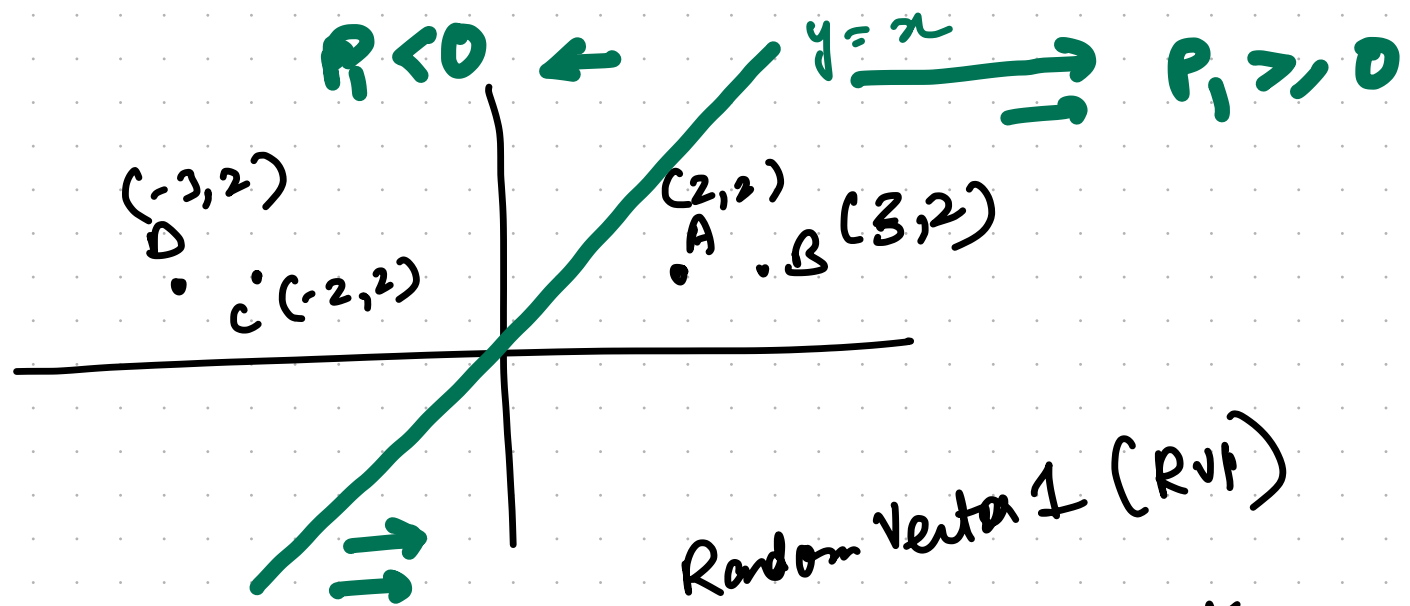
$$R = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \in \mathbb{R}^{D \times K}$$

$$N = 4$$

$$D = 2$$

Proj<sup>2</sup> of (x,y) using R.V.1. is:

$$p_1 = [x \ y] \begin{bmatrix} 1 \\ -1 \end{bmatrix} = x - y$$



$$X = \begin{bmatrix} 2 & 2 \\ 3 & 2 \\ -2 & 2 \\ -3 & 2 \end{bmatrix} \in \mathbb{R}^{N \times D}$$

Random Vector 1 (R.V.1)

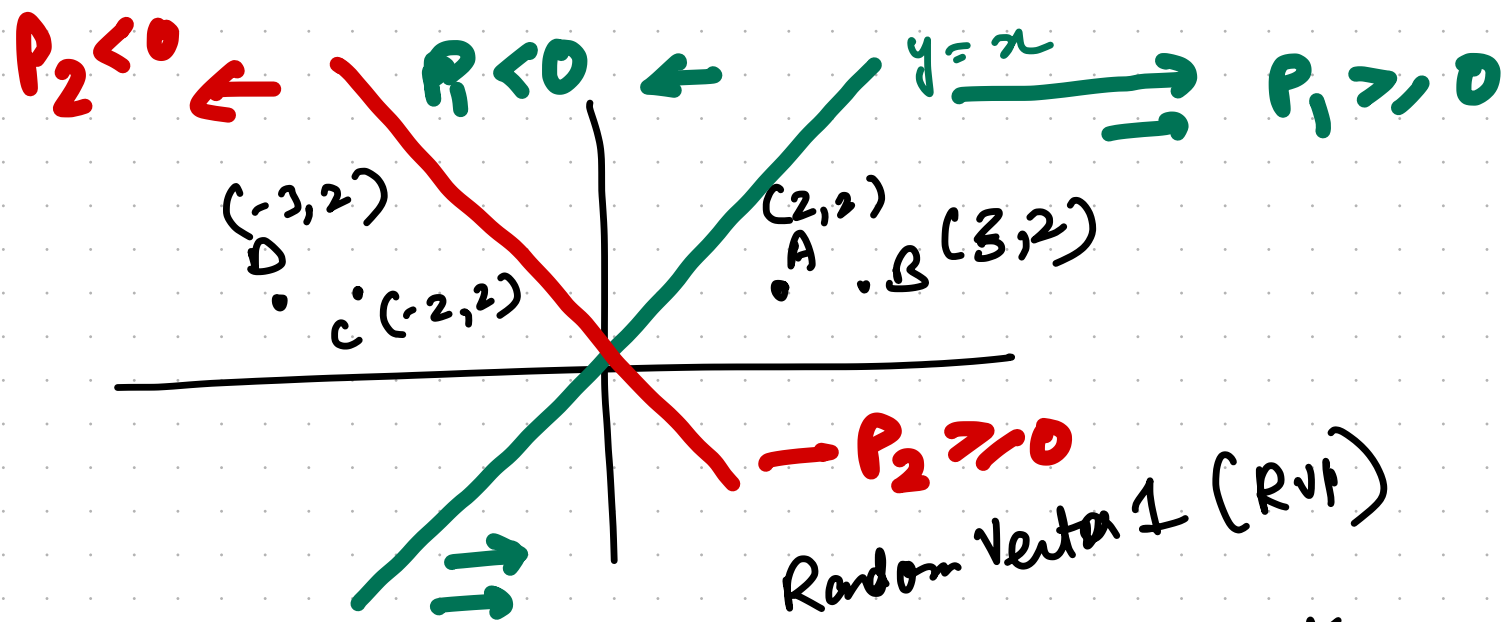
$$R = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \in \mathbb{R}^{D \times K}$$

$$N = 4$$

$$D = 2$$

Proj<sup>2</sup> of  $(x, y)$  using R.V.1. is:

$$p_1 = [x \ y] \begin{bmatrix} 1 \\ -1 \end{bmatrix} = x - y$$



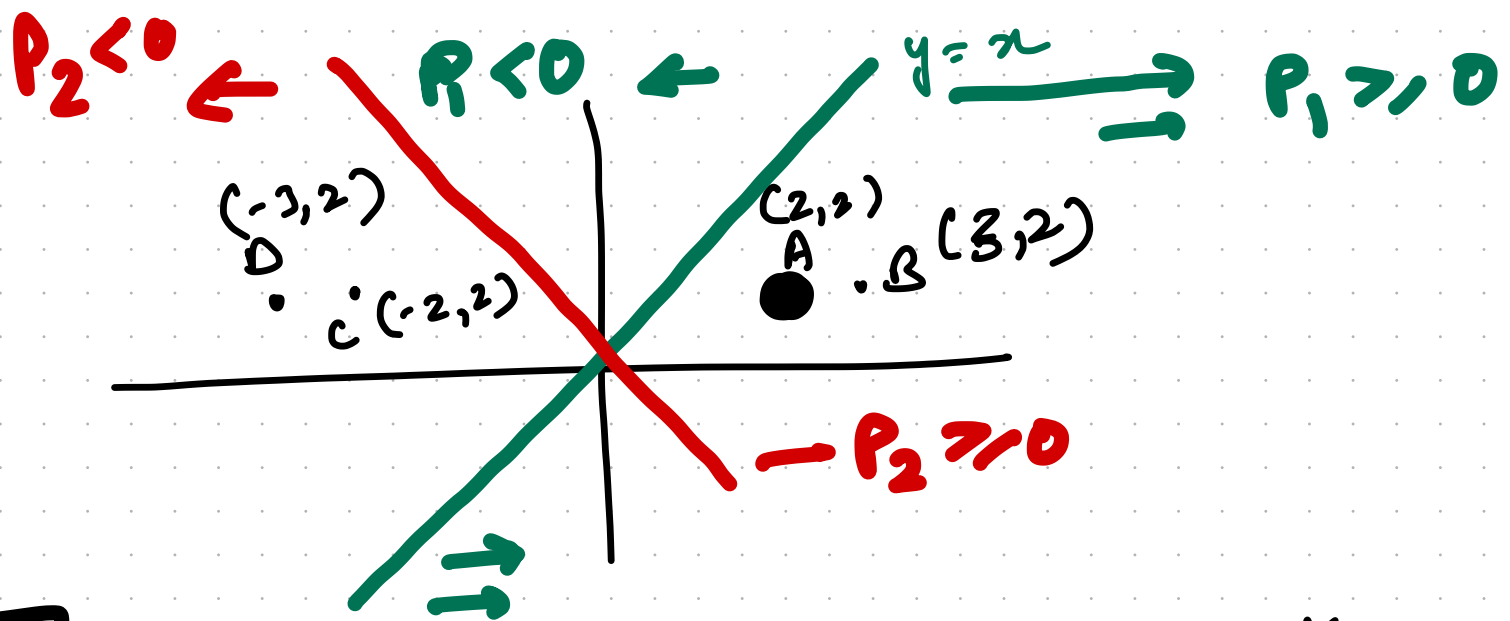
$$X = \begin{bmatrix} 2 & 2 \\ 3 & 2 \\ -2 & 2 \\ -3 & 2 \end{bmatrix} \in \mathbb{R}^{N \times D}$$

$$N = 4$$

$$D = 2$$

Project  $P_2$  of  $(x, y)$  using

$$p = [x \ y] \begin{bmatrix} 1 \\ 1 \end{bmatrix} = x + y$$



$$X = \begin{bmatrix} 2 & 2 \\ 3 & 2 \\ -2 & 2 \\ -3 & 2 \end{bmatrix} \in \mathbb{R}^{N \times D}$$

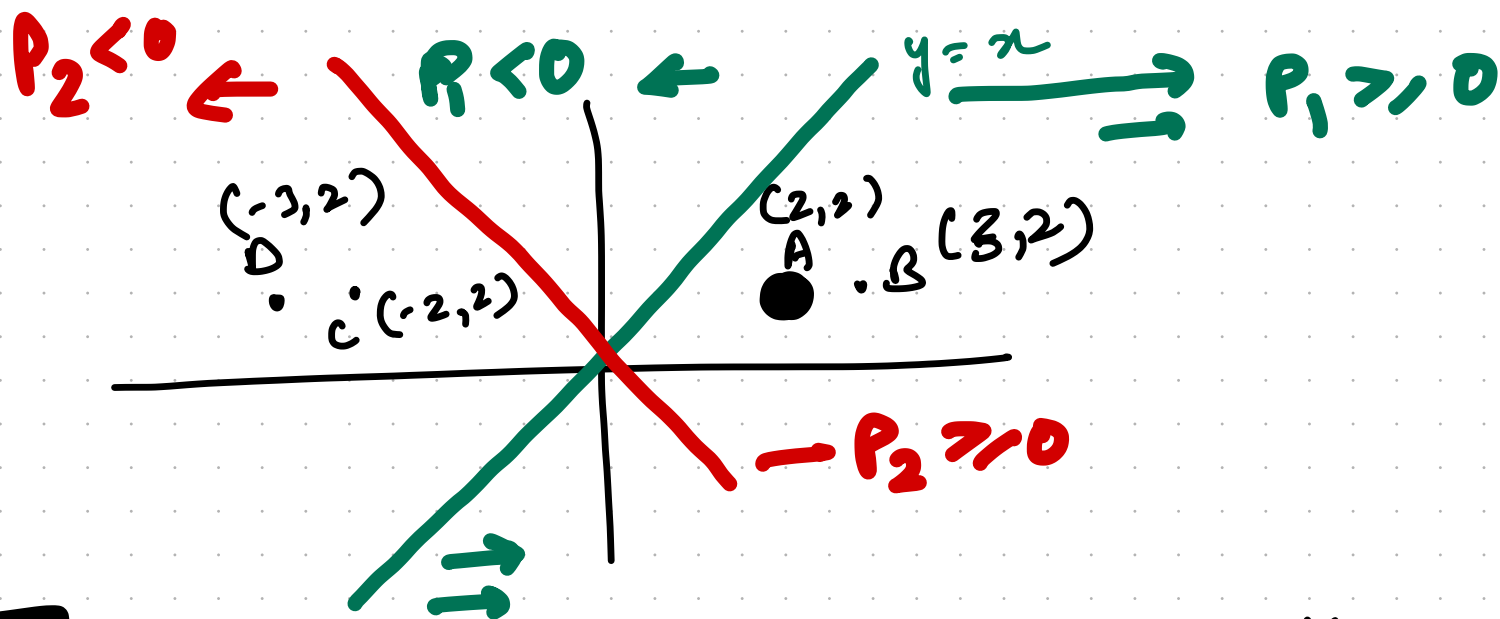
$$R = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \in \mathbb{R}^{D \times K}$$

$$N = 4$$

$$D = 2$$

Focus on  $A(2,2)$

$$\begin{bmatrix} 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}_{(R)} = \begin{bmatrix} 0 & 4 \end{bmatrix} = P_A$$



$$X = \begin{bmatrix} 2 & 2 \\ 3 & 2 \\ -2 & 2 \\ -3 & 2 \end{bmatrix} \in \mathbb{R}^{N \times D}$$

$$N = 4$$

$$D = 2$$

$$R = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \in \mathbb{R}^{D \times K}$$

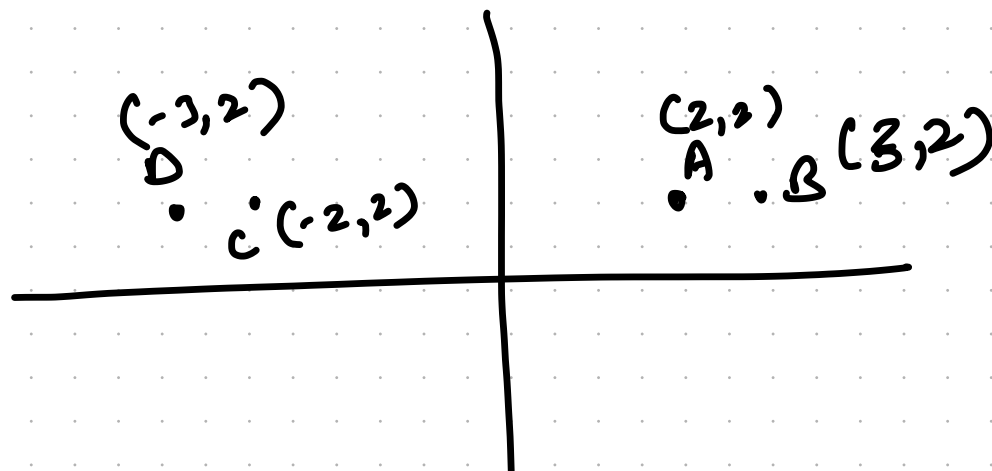
FOCUS ON  $A(2,2)$

$$\begin{bmatrix} 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 4 \end{bmatrix}$$

$$(R) = P_A$$

THRESHOLD PROJECTION (BINARIZE)

$$T(P_A) = P_A \geq 0 = \begin{bmatrix} 1 & 1 \end{bmatrix}$$



$$X = \begin{bmatrix} 2 & 2 \\ 3 & 2 \\ -2 & 2 \\ -3 & 2 \end{bmatrix} \in \mathbb{R}^{N \times D}$$

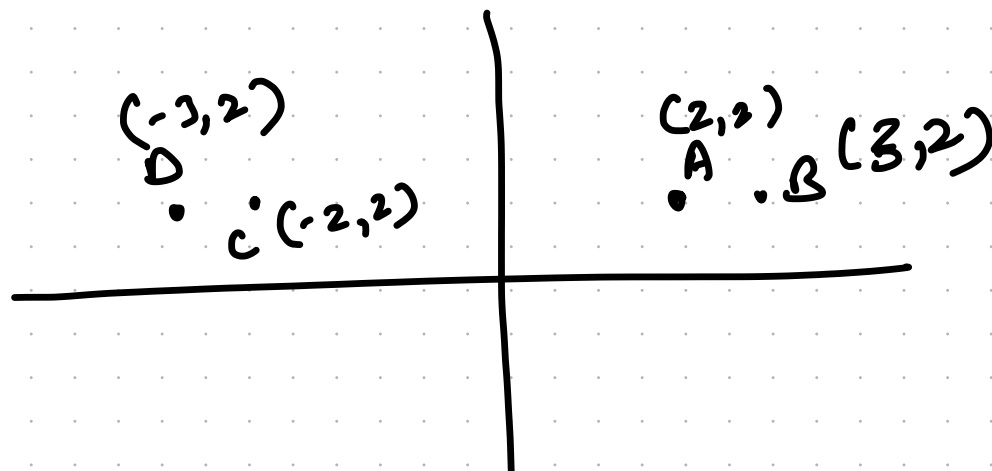
$$N = 4$$

$$D = 2$$

$$T(XR) = T \left( \begin{bmatrix} 2 & 2 \\ 3 & 2 \\ -2 & 2 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \right)$$

$$T(XR) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$





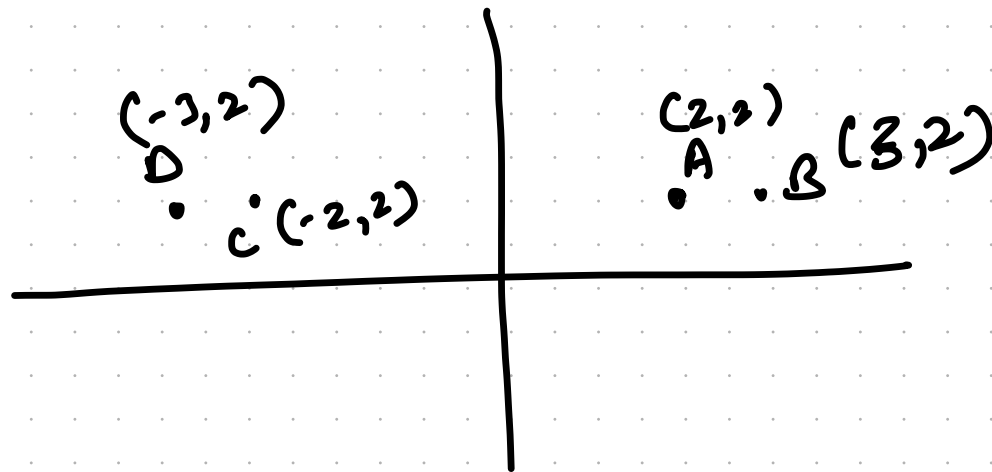
$$X = \begin{bmatrix} 2 & 2 \\ 3 & 2 \\ -2 & 2 \\ -3 & 2 \end{bmatrix} \in \mathbb{R}^{N \times D}$$

$$N = 4$$

$$D = 2$$

$$T(XR) = T \left( \begin{bmatrix} 2 & 2 \\ 3 & 2 \\ -2 & 2 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \right)$$

$$T(XR) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \left. \vphantom{\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}} \right\} \begin{array}{l} A \text{ \& } B \\ \text{have same} \\ \text{"hash"} \\ \text{value} \end{array}$$



$$X = \begin{bmatrix} 2 & 2 \\ 3 & 2 \\ -2 & 2 \\ -3 & 2 \end{bmatrix} \in \mathbb{R}^{N \times D}$$

$$N = 4$$

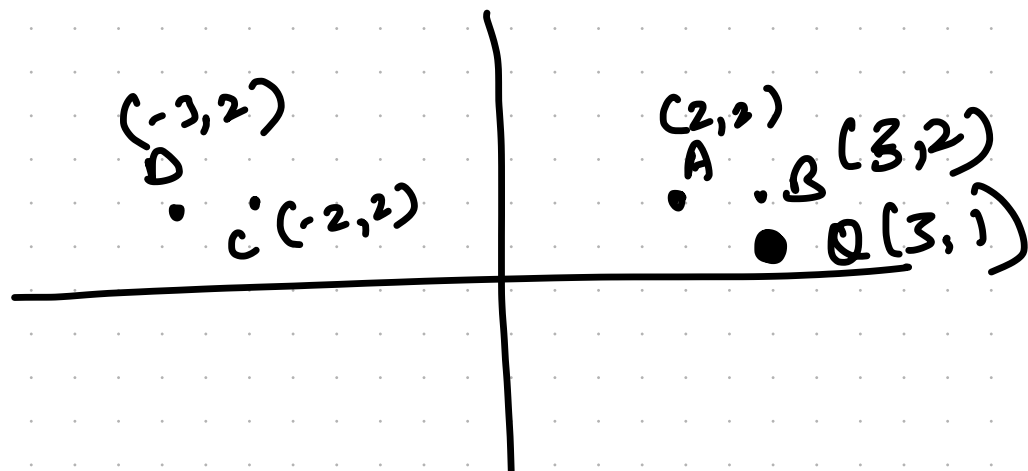
$$D = 2$$

Hash Table

11  $\rightarrow$  (A, B)

01  $\rightarrow$  C

00  $\rightarrow$  D



$$X = \begin{bmatrix} 2 & 2 \\ 3 & 2 \\ -2 & 2 \\ -3 & 2 \end{bmatrix} \in \mathbb{R}^{N \times D}$$

$$N = 4$$

$$D = 2$$

Hash Table

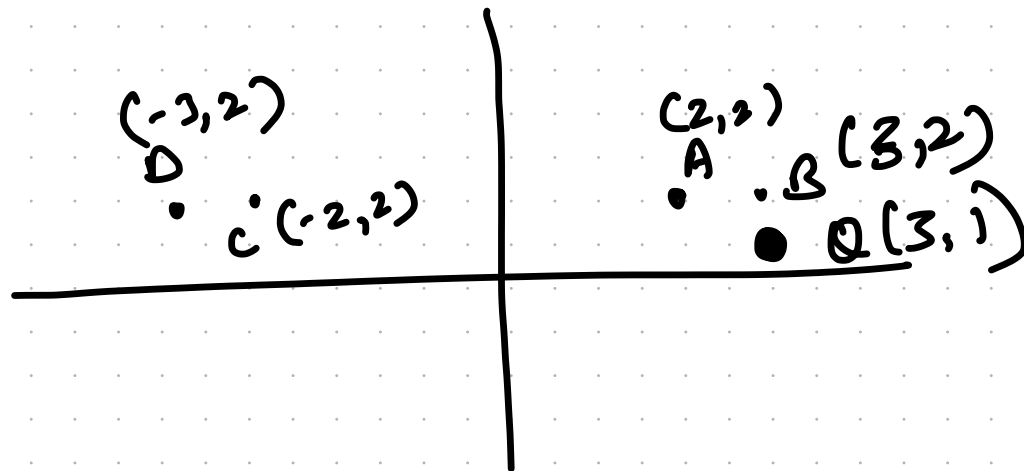
11  $\rightarrow$  (A, B)

01  $\rightarrow$  C

00  $\rightarrow$  D

TEST POINT Q (3, 1)

$$T(QR) = T\left(\begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = 11$$



Sublinear  
Algo

$$X = \begin{bmatrix} 2 & 2 \\ 3 & 2 \\ -2 & 2 \\ -3 & 2 \end{bmatrix} \in \mathbb{R}^{N \times D}$$

$$N = 4$$

$$D = 2$$

Hash Table

11  $\rightarrow$  (A, B)

01  $\rightarrow$  C

00  $\rightarrow$  D

Need to  
compare only  
in this  
bucket

TEST POINT Q (3, 1)

$$T(QR) = T\left[\begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right] = 11$$