

$$\text{MINIMIZE } x^2 + y^2$$

s.t.

$$x^2 + y^2 \leq 5$$

$$x + 2y = 4$$

$$x, y \geq 0$$

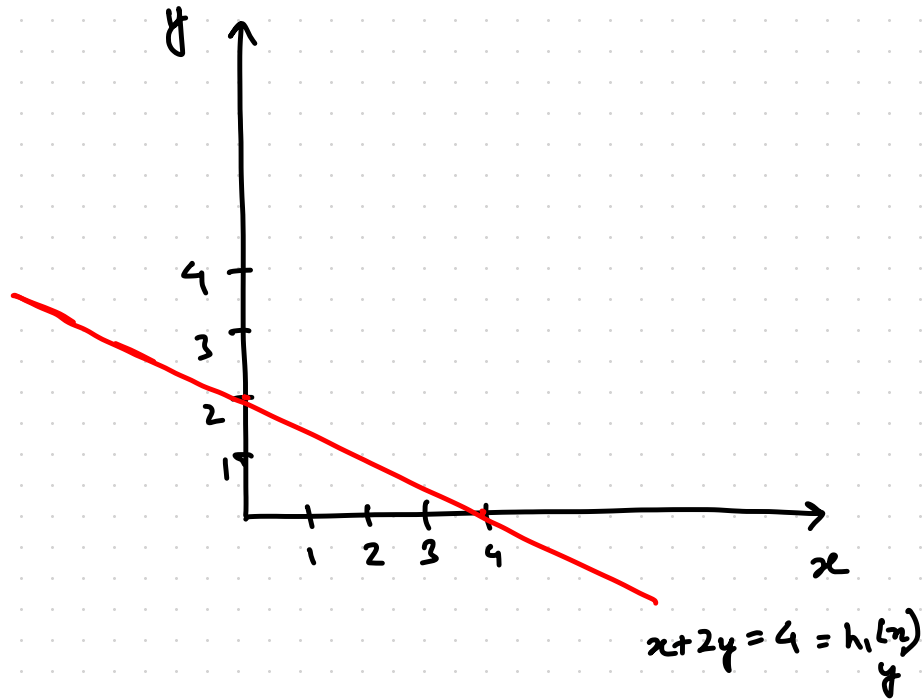
$$\text{or } g_1(x, y) = x^2 + y^2 - 5 \leq 0 \quad (\mu_1)$$

$$\text{or } h(x, y) = x + 2y - 4 = 0$$

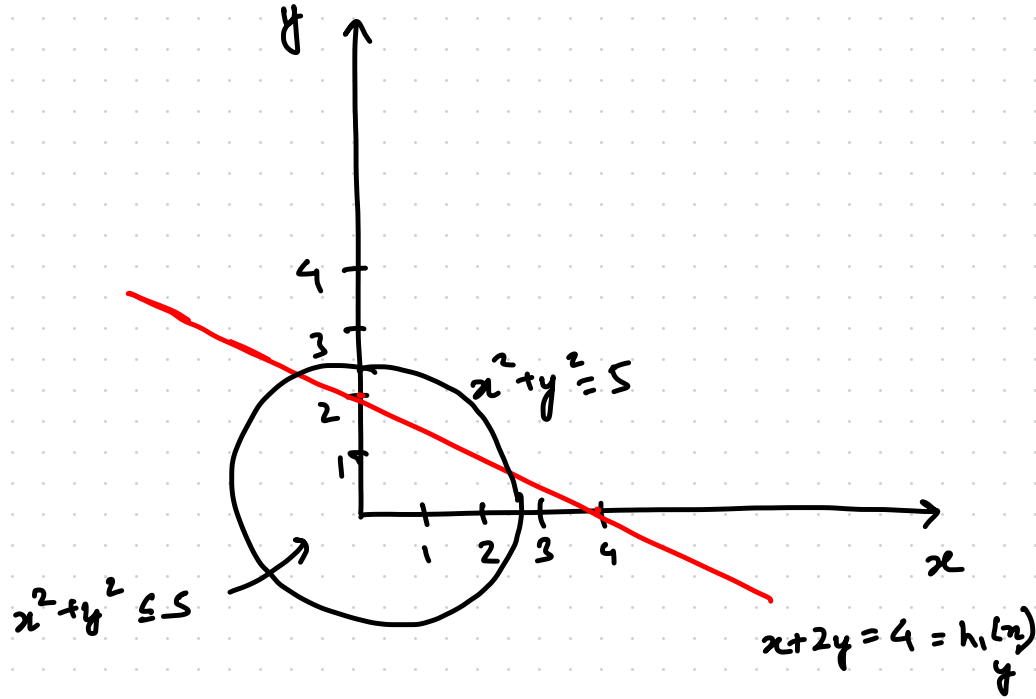
$$\text{or } g_2(x, y) = -x \leq 0 \quad (\mu_2)$$

$$g_3(x, y) = -y \leq 0 \quad (\text{multiplier: } \mu_3)$$

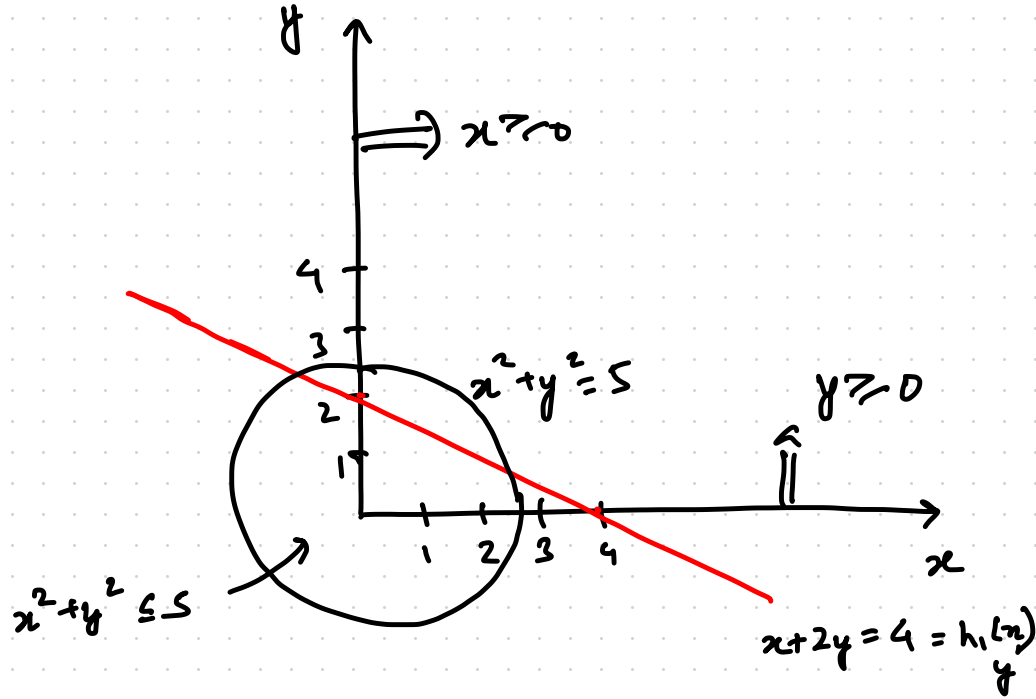
MINIMIZE  $x^2 + y^2$



MINIMIZE  $x^2 + y^2$

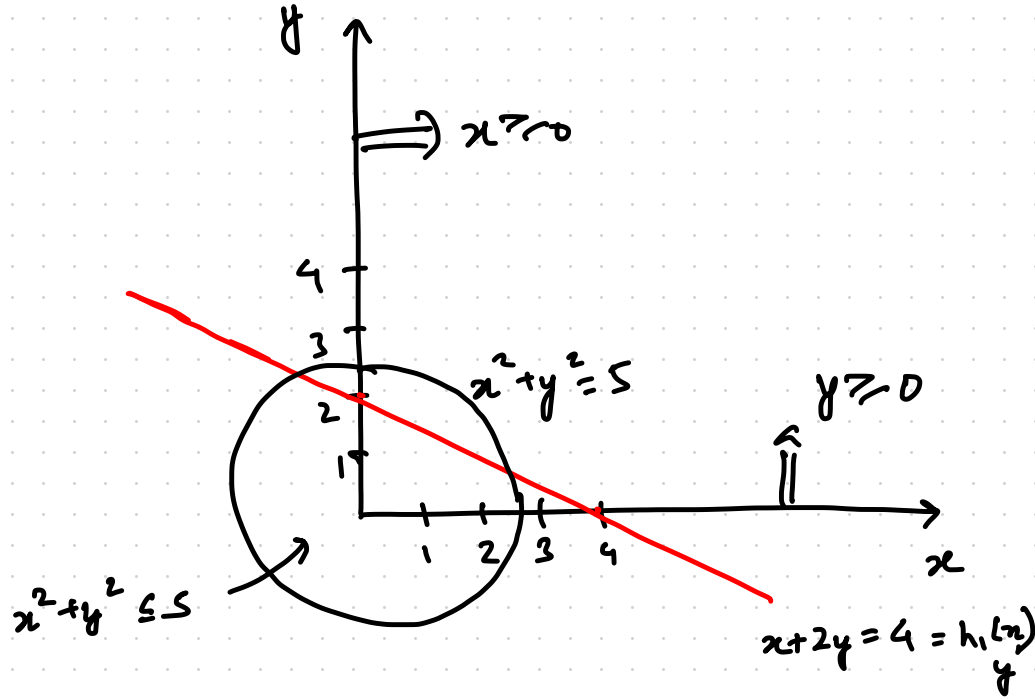


MINIMIZE  $x^2 + y^2$



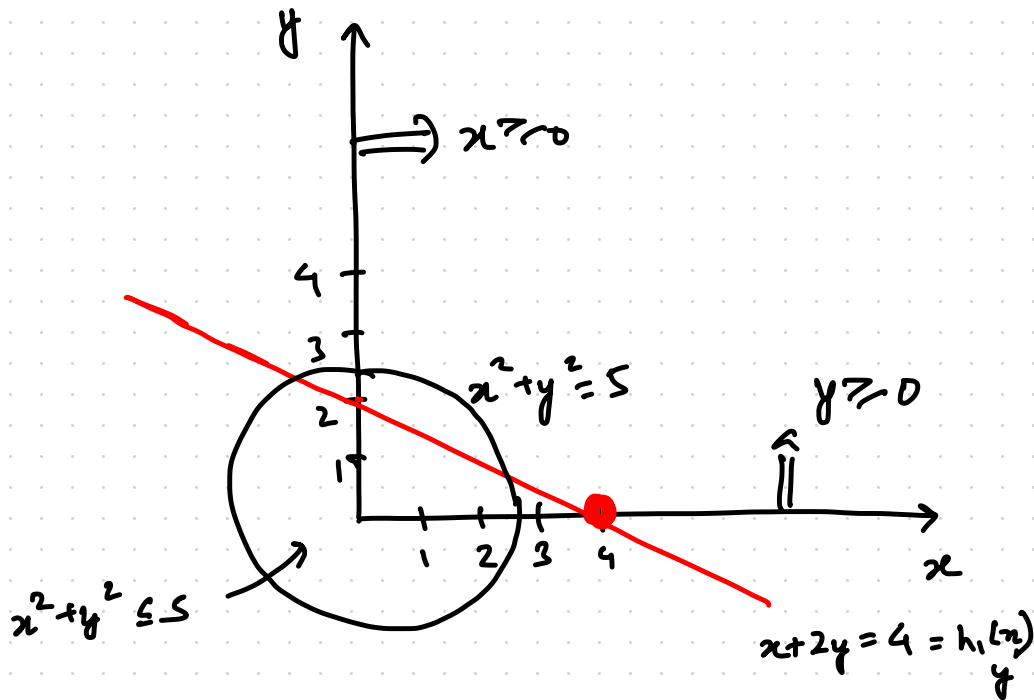
MINIMIZE  $x^2 + y^2$

$\mu_3 = 0$   $\lambda > 0$



MINIMIZE

$$x^2 + y^2$$



$$\mu_3 = 0 \quad \text{or } > 0$$

$$\mu_3 > 0$$

$$\Rightarrow y = 0$$

$$\Rightarrow x = 4$$

But doesn't  
satisfy

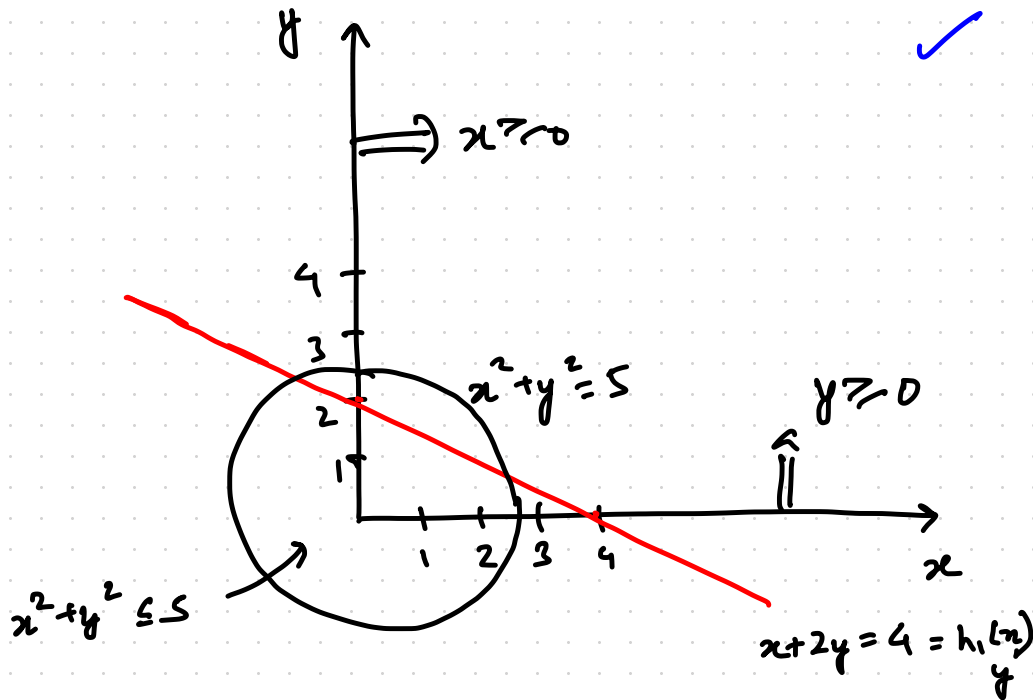
$$x^2 + y^2 \leq 5$$

$$\therefore \mu_3 \neq 0$$

MINIMIZE

$$x^2 + y^2$$

$$\mu_3 = 0$$



$$\mu_3 = 0 \quad \checkmark$$

$$\mu_3 > 0$$

$$\Rightarrow y = 0$$

$$\Rightarrow x = 4$$

But doesn't satisfy

$$x^2 + y^2 \leq 5$$

$$\therefore \mu_3 \neq 0$$

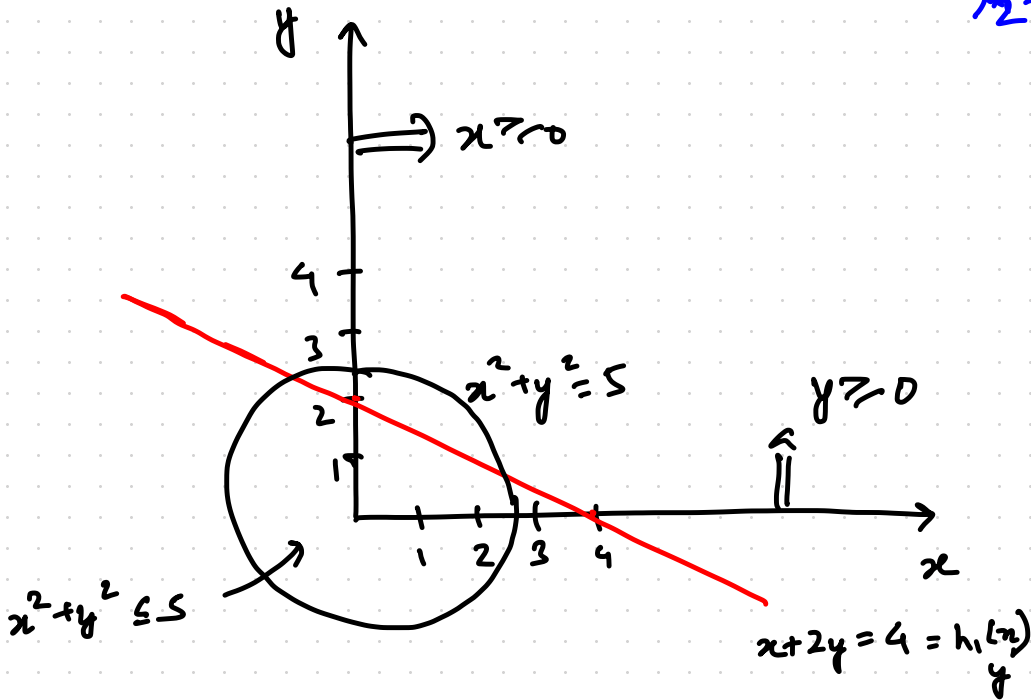
MINIMIZE

$$x^2 + y^2$$

$$\mu_2 = 0$$

$$\mu_2 > 0$$

$$\mu_3 = 0$$

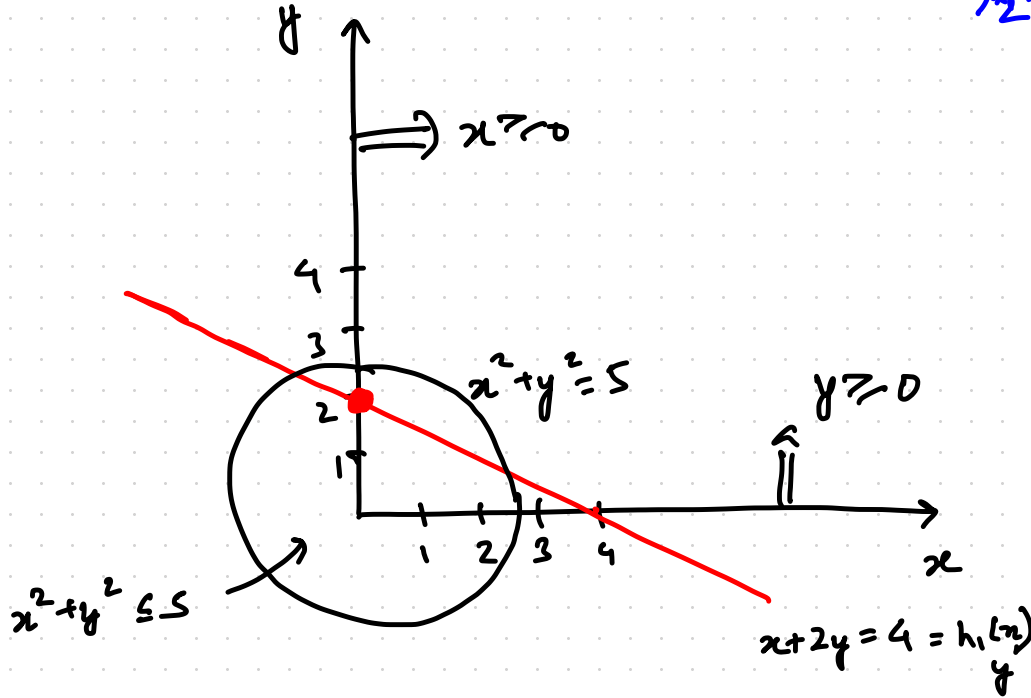




MINIMIZE

$$x^2 + y^2$$

$$\mu_3 = 0$$



$$\mu_2 = 0$$

$$\mu_2 > 0$$

$$\Rightarrow x = 0$$

$$\Rightarrow y = 2$$

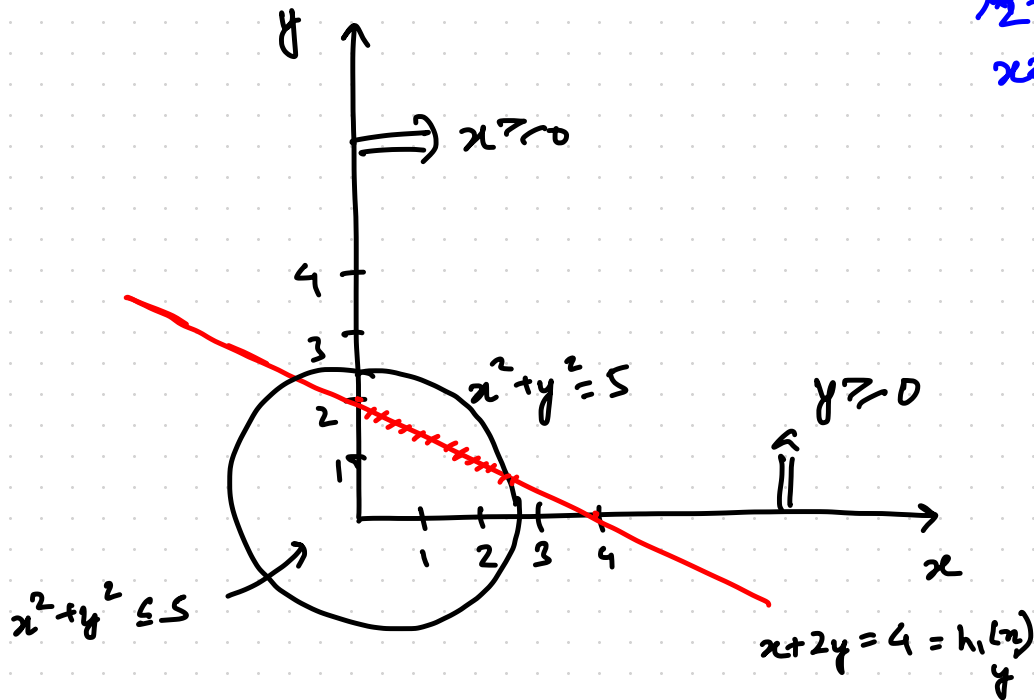
Feasible  
Sol<sup>n</sup>.

$$x^2 + y^2 = 4$$

MINIMIZE

$$x^2 + y^2$$

$$\mu_3 = 0$$



$$\mu_2 = 0$$

$$x \geq 0$$

$$\mu_2 > 0$$

$$\Rightarrow x = 0$$

$$\Rightarrow y = 2$$

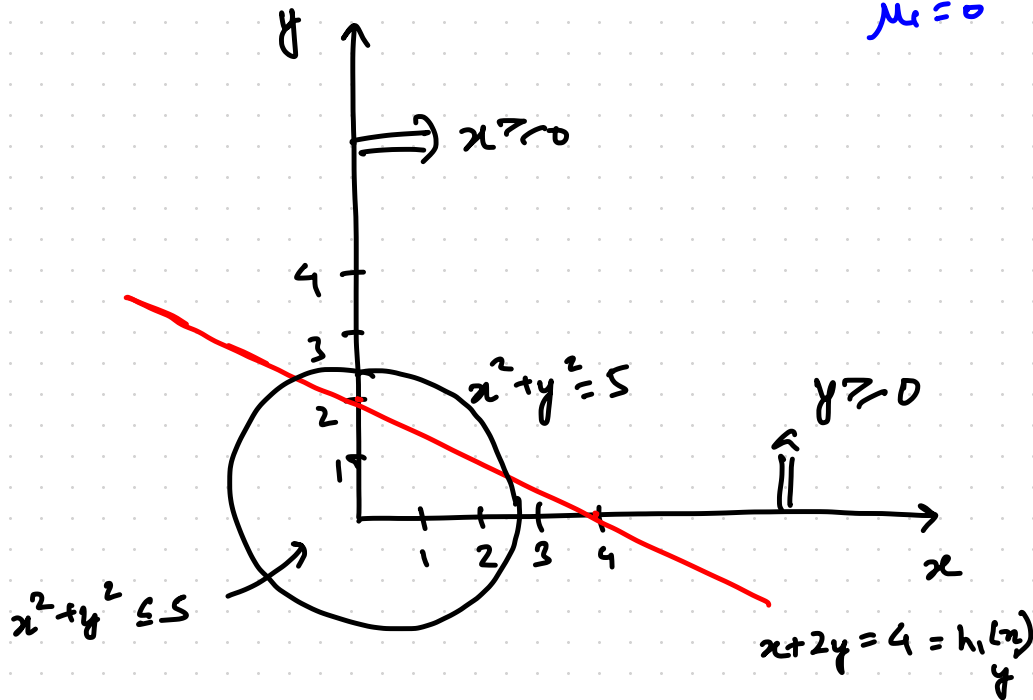
Feasible  
Sol<sup>n</sup>.

$$x^2 + y^2 = 4$$

MINIMIZE

$$x^2 + y^2$$

$$\mu_3 = 0$$



$$\mu_2 = 0$$

$$\mu_1 > 0$$

$$\Rightarrow x^2 + y^2 = 5$$

But we have  
seen  
better  
sol<sup>n</sup>

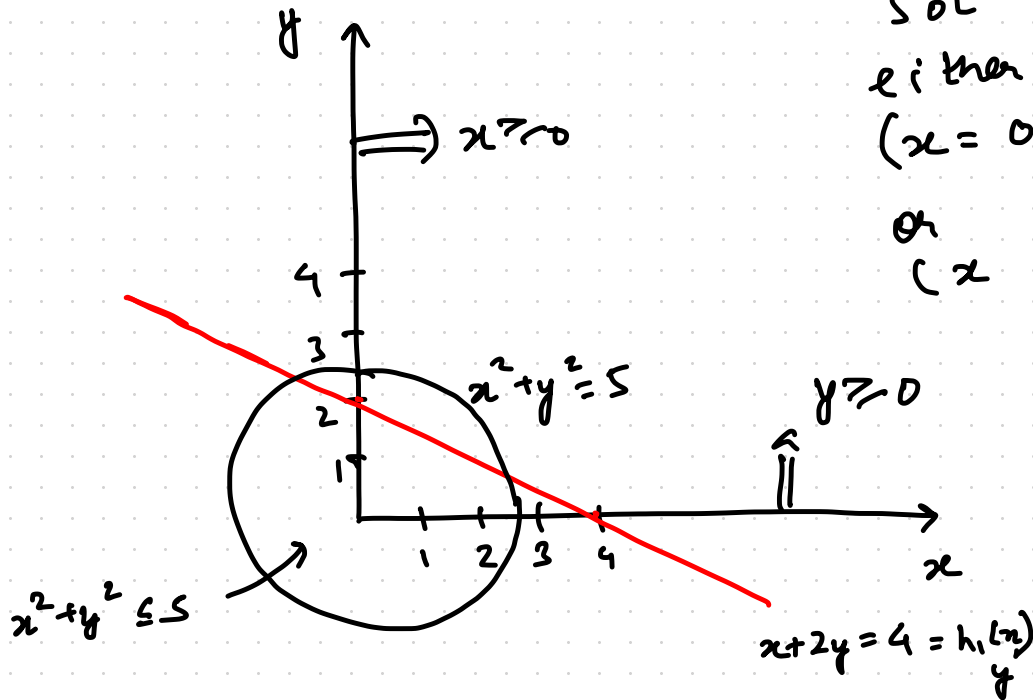
$$\therefore \mu_1 = 0$$

MINIMIZE

$$x^2 + y^2$$

$$\mu_3 = 0$$

$$\mu_1 = 0$$



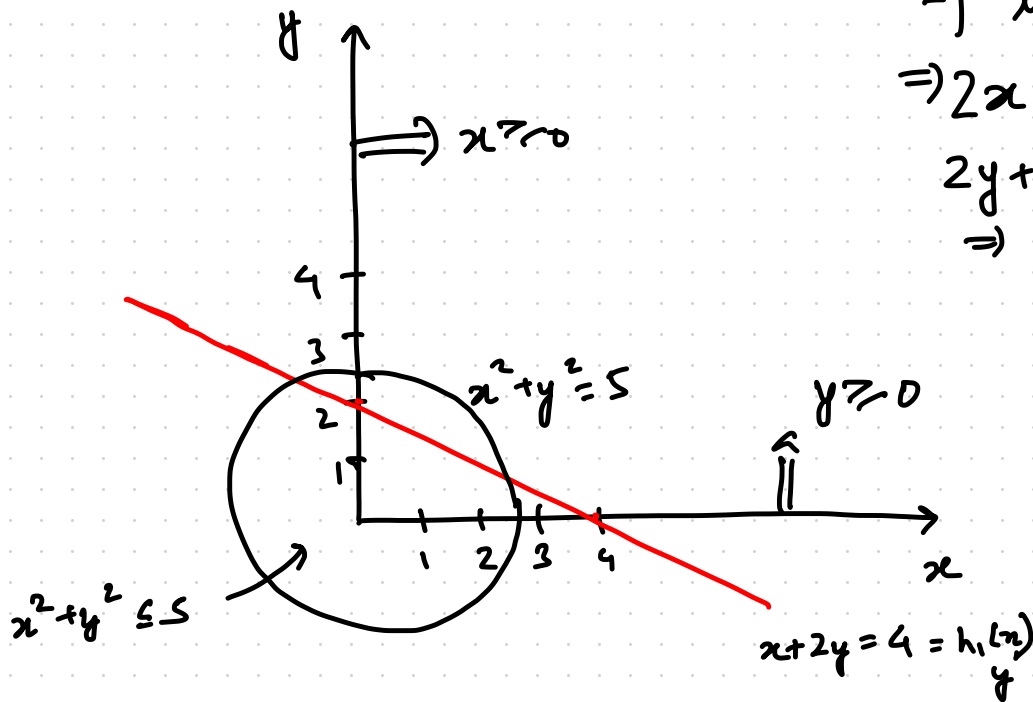
Sol<sup>n</sup> is  
either  
( $x = 0, y = 2$ )

or  
( $x$  on  $x + 2y = 4$   
line  
till it  
intersects  
circle)

MINIMIZE

$$x^2 + y^2$$

$$\begin{aligned} \mu_3 &= 0 \\ \mu_1 &= 0 \\ \mu_2 &= 0 \end{aligned}$$



If  $\mu_2 = 0$

$$\Rightarrow 2x + \lambda = 0$$

$$2y + 2\lambda = 0$$

$$\Rightarrow x = -\frac{\lambda}{2}$$

$$y = -\lambda$$

$$\therefore y = 2x$$

$$\text{or}$$

$$x + 2y = 4$$

$$\Rightarrow 5x = 4$$

$$\text{or } x = 0.8$$

$$y = 1.6$$

