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Batch: A1

TUT5

LAPLACE TRANSFORM:

Q.1 Find the Laplace Transform of the following functions:

In [3]:
```
t, s = var('t s')
f = t^3 * cos(2*t)
laplace_transform = laplace(f, t, s)
show(laplace_transform)
```

$$\frac{48\,s^4}{\left(s^2+4\right)^4} - \frac{48\,s^2}{\left(s^2+4\right)^3} + \frac{6}{\left(s^2+4\right)^2}$$

In [4]:
```
t, s = var('t', 's')
f = (e**(2*t) - e**(3*t))/t
laplace_transform = laplace(f, t, s)
show(laplace_transform)
```

$$\log\left(\frac{s-3}{s-2}\right)$$

In [6]:
```
t,s =var('t s')
f= exp(-5 * t) * sin(3*t)
show(f.laplace(t,s))
```

$$\frac{3}{s^2+10\,s+34}$$

Q.2 Find the Inverse Laplace Transform of the following Functions:

In [7]:
```
s = var('s')
F = 1 / (s^4 + 13*s^2 + 36)
inverse_laplace(F(s),s,t)
show(inverse_laplace(F(s),s,t))
```

$$-\frac{1}{15}\sin(3\,t) + \frac{1}{10}\sin(2\,t)$$

In [8]:
```
s = var('s')
F = (s + s^2) / ((s^2 + 1) * (s^2 + 2*s + 2))
show(inverse_laplace(F(s),s,t))
```

$$-\frac{1}{5}\left(3\,\cos(t) - \sin(t)\right)e^{(-t)} + \frac{3}{5}\cos(t) + \frac{1}{5}\sin(t)$$

Q.3 Solve the following differential equation using Laplace Transform:

In [9]:
```
s,t = var('s t')
x=function('x')(t)
de=diff(x,t,t)-diff(x,t)-2*x== 20*sin(2*t)
show(desolve_laplace(de,x,ics=[0,1,2]))
```

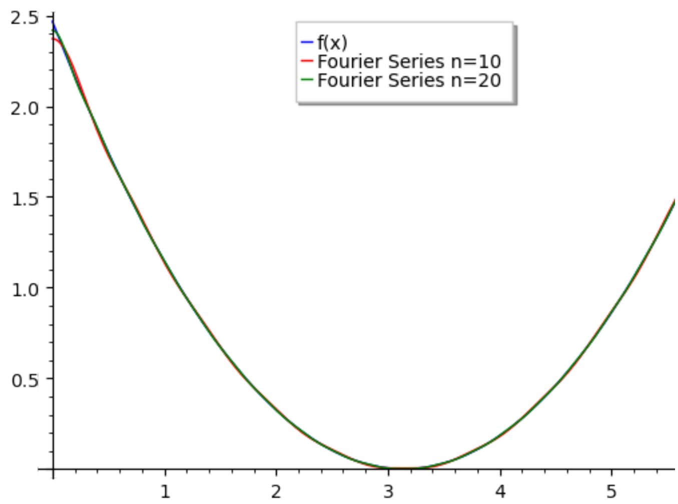
$$\cos(2\,t) + \frac{8}{3}e^{(2\,t)} - \frac{8}{3}e^{(-t)} - 3\,\sin(2\,t)$$

FOURIER SERIES:

Q.1 Find all the Fourier Coefficients and Fourier Series for the following func series:

(i) f(x)= ((π-x)/2)^2 in (0,2π) for n=10 and n=20

In [11]:
```
var('x n')
L = pi
f(x) = ((pi - x)/2)^2
a0 = (1/L) * integrate(f(x), x, 0, 2*pi)
an = (1/L) * integrate(f(x) * cos(n*pi*x/L), x, 0, 2*pi)
bn = (1/L) * integrate(f(x) * sin(n*pi*x/L), x, 0, 2*pi)
s1=a0/2 + sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,10)
s2=a0/2 + sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,20)
p1 = plot(f(x), (x, 0, 2*L), color="blue", legend_label="f(x)")
p2 = plot(s1, (x, 0, 2*L), color="red", legend_label="Fourier Seri
p3 = plot(s2, (x, 0, 2*L), color="green", legend_label="Fourier Se
(p1 + p2 + p3).show()
show(a0)
show(an)
show(bn)
print("Fourier series for n=10 is \n")
show(s1)
print("Fourier series for n=20 is \n")
show(s2)
```



$$\frac{1}{6}\pi^2$$

$$\frac{1}{n^2}$$

$$0$$

Fourier series for n=10 is

$$\frac{1}{12}\pi^2 + \frac{1}{100}\cos(10\,x) + \frac{1}{81}\cos(9\,x) + \frac{1}{64}\cos(8\,x) + \frac{1}{49}\cos(7\,x) + \frac{1}{36}\text{c}$$

$$(x)$$

Fourier series for n=20 is

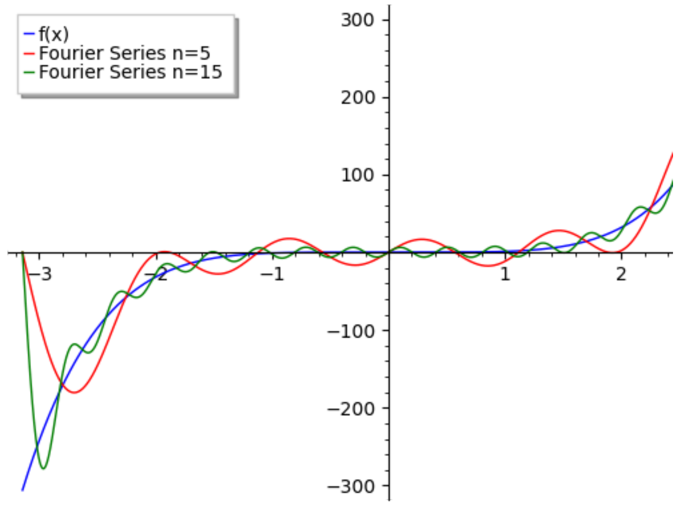
$$\frac{1}{12}\pi^2 + \frac{1}{400}\cos(20\,x) + \frac{1}{361}\cos(19\,x) + \frac{1}{324}\cos(18\,x) + \frac{1}{289}\cos(17\,x)$$

$$+ \frac{1}{144}\cos(12\,x) + \frac{1}{121}\cos(11\,x) + \frac{1}{100}\cos(10\,x) + \frac{1}{81}\cos(9\,x) + \frac{1}{64}\cos$$

$$\cos(3\,x) + \frac{1}{4}\cos(2$$

(ii) f(x)=x^5 in (-π,π) for n=5 and n=15

In [12]:
```
var('x n')
L = pi
f(x) = x^5
a0 = (1/L) * integrate(f(x), x, -L, L)
an = (1/L) * integrate(f(x) * cos(n*pi*x/L), x, -L, L)
bn = (1/L) * integrate(f(x) * sin(n*pi*x/L), x, -L, L)
s1= a0/2+sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,5)
s2= a0/2+sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,15)
p1 = plot(f(x), (x, -L, L), color="blue", legend_label="f(x)")
p2 = plot(s1, (x, -L, L), color="red", legend_label="Fourier Serie
p3 = plot(s2, (x, -L, L), color="green", legend_label="Fourier Ser
(p1 + p2 + p3).show()
show(a0)
show(an)
show(bn)
print("Fourier series for n=5 is \n")
show(s1)
print("Fourier series for n=15 is \n")
show(s2)
```



0

0

$$-\frac{2\left(120\,\pi + \pi^5 n^4 - 20\,\pi^3 n^2\right)(-1)^n}{\pi n^5}$$

```
Fourier series for n=5 is
```

$$\frac{2}{625}\left(125\,\pi^4 - 100\,\pi^2 + 24\right)\sin(5\,x) - \frac{1}{64}\left(32\,\pi^4 - 40\,\pi^2 + 15\right)\sin(4\,x) +$$
$$+ 2\left(\pi^4 - 20\,\pi^2 +\right.$$

```
Fourier series for n=15 is
```

$$\frac{2}{50625}\left(3375\,\pi^4 - 300\,\pi^2 + 8\right)\sin(15\,x) - \frac{1}{33614}\left(4802\,\pi^4 - 490\,\pi^2 + 15\right.$$
$$\left(864\,\pi^4 - 120\,\pi^2 + 5\right)\sin(12\,x) + \frac{2}{161051}\left(14641\,\pi^4 - 2420\,\pi^2 + 1\right.$$
$$\left(2187\,\pi^4 - 540\,\pi^2 + 40\right)\sin(9\,x) - \frac{1}{2048}\left(512\,\pi^4 - 160\,\pi^2 + 15\right)$$
$$\left(54\,\pi^4 - 30\,\pi^2 + 5\right)\sin(6\,x) + \frac{2}{625}\left(125\,\pi^4 - 100\,\pi^2 + 24\right)\sin(5\,x) - \frac{1}{64}$$
$$-\frac{1}{2}\left(2\,\pi^4 - 10\,\pi^2 + 15\right)\sin(2\,x) +$$

Q.2 Find the Half range cosine series for f(x)= x 0<x<2 for n=20 . Also plot the

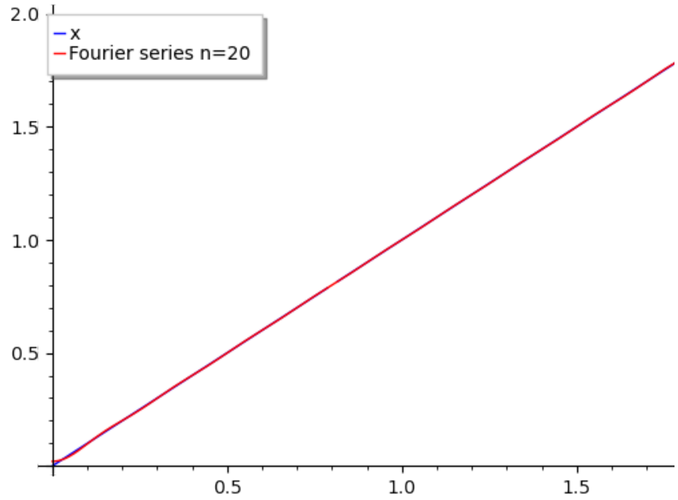
In [13]:
```
var('x')
var('n')
assume(n,'integer')
L = 2
f(x) = x
a0 = (2/L) * integrate(f(x), x, 0, 2)
an = (2/L) * integrate(f(x) * cos(n * pi * x / L), x, 0, 2)
S=a0/2 + sum(an*cos(n*pi*x/L),n,1,20)
show(a0)
show(an)
show(S)
plot(f(x),0,L,legend_label="x") + plot(S,0,L,color = "red",legend_
```

$$2$$

$$\frac{4\,(-1)^n}{\pi^2 n^2} - \frac{4}{\pi^2 n^2}$$

$$-\frac{8\left(586396035225\,\cos\left(\frac{19}{2}\,\pi x\right) + 732487781025\,\cos\left(\frac{17}{2}\,\pi x\right) + 94083986\ldots \left(\frac{11}{2}\,\pi x\right) + 2613444058225\,\cos\left(\frac{9}{2}\,\pi x\right) + 4320183035025\,\cos\left(\frac{7}{2}\,\pi x\right) + \ldots + 211688968716225\,\cos\left(\frac{1}{2}\,\pi x\right)\right)}{211688968\ldots} + 1$$

Out[13]:



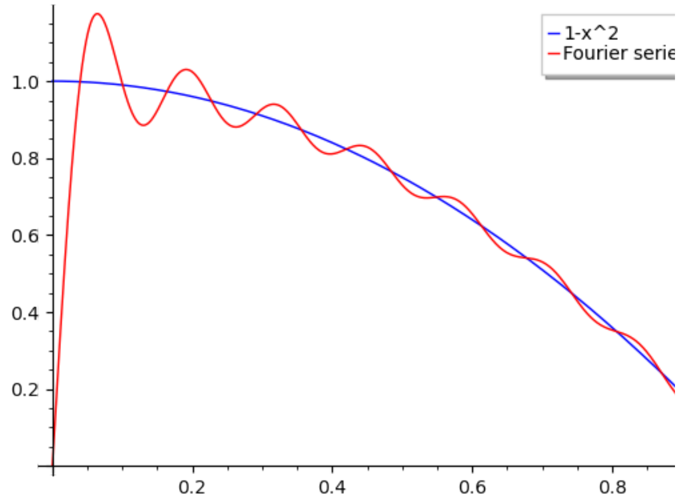
Q.3 Find the Half range sine series for f(x)= 1-x^2 in (0,1) for n=15 . Also plot

In [14]:
```
var('x')
var('n')
assume(n,'integer')
L = 1
f(x)= 1-x^2
bn=(2/L)*integrate(f(x)*sin(n*pi*x/L),x,0,L)
s= sum(bn*sin(n*pi*x/L),n,1,15)
show(bn)
show(s)
plot(f(x),0,L,legend_label="1-x^2") + plot(s,0,L,color = "red",leg
```

$$\frac{2\left(\pi^2 n^2 + 2\right)}{\pi^3 n^3} - \frac{4\,(-1)^n}{\pi^3 n^3}$$

$$52227799123500\,\pi^2 \sin(14\,\pi x) + 60932432310750\,\pi^2 \sin(12\,\pi x) + 731189$$
$$+ 121864864621500\,\pi^2 \sin(6\,\pi x) + 182797296932250\,\pi^2 \sin(4\,\pi x) + 3655$$
$$+ 332812557000\left(169\,\pi^2 + 4\right)\sin(13\,\pi x) + 549353259000\left(121\,\pi^2 + 4\right) \mathrm{si}$$
$$\left(49\,\pi^2 + 4\right)\sin(7\,\pi x) + 5849513501832\left(25\,\pi^2 + 4\right)\sin(5\,\pi x) + 27081081$$
$$36559459386$$

Out[14]:



Q.4 Find the Fourier series (n=15) , a10 and b15 for f(x)=x(π-x) in (-π,π).

In [15]:
```
var('x n')
assume(n,'integer')
f(x)=x*(pi-x)
L=pi
a0=1/L*integrate(f,x,-L,L)
an=1/L*integrate(f*cos(n*pi*x/L),x,-L,L)
bn=1/L*integrate(f*sin(n*pi*x/L),x,-L,L)
s=a0/2+sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,15)
f1=an.substitute(n=10)
f2=bn.substitute(n=15)
show("value of a0: ",a0)
show("value of an: ",an)
show("value of bn: ",bn)
show("sum of fourier series upto n=15:  ",s)
show("value of a10: ",f1)
show("value of b15: ",f2)
```

value of a0: $-\frac{2}{3}\pi^2$

value of an: $-\frac{4(-1)^n}{n^2}$

value of bn: $-\frac{2\left(\frac{(\pi^2 n^2-1)(-1)^n}{n^3}+\frac{(-1)^n}{n^3}\right)}{\pi}$

sum of fourier series upto n=15: $-\frac{1}{3}\pi^2+\frac{2}{15}\pi\sin(15x)-\frac{1}{7}\pi$ si

$(10x)+\frac{2}{9}\pi\sin(9x)-\frac{1}{4}\pi\sin(8x)+\frac{2}{7}\pi\sin(7x)-\frac{1}{3}\pi\sin(6x)+\frac{2}{5}\pi$ sin(

$(15x)-\frac{1}{49}\cos(14x)+\frac{4}{169}\cos(13x)-\frac{1}{36}\cos(12x)+\frac{4}{121}\cos(11x)-$

$+\frac{4}{25}\cos(5x)-\frac{1}{4}\cos(4x)+\frac{4}{9}$ cc

value of a10: $-\frac{1}{25}$

value of b15: $\frac{2}{15}\pi$