(Autonomous College Affiliated to Univ...)

Candidate Roll No. ____________ (In figures)

**Test Exam.**

Name : ______________________

Date : ____________ 20

Examination : ____________ Branch/Semester ____________

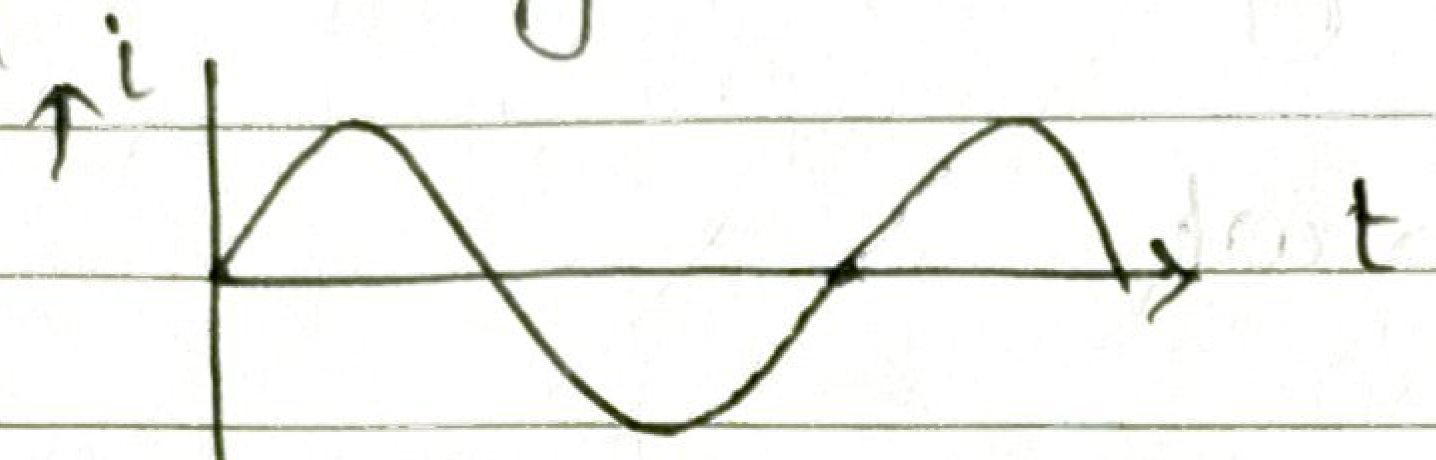
Subject : ______________________

| Junior Supervisor's full Signature with Date |
| --- |
| |

| Question No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Total |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Marks Obtained | | | | | | | | | | | | | |

# AC Circuits

When current flowing in the circuit varies in magnitude as well as direction periodically is called alternating current.



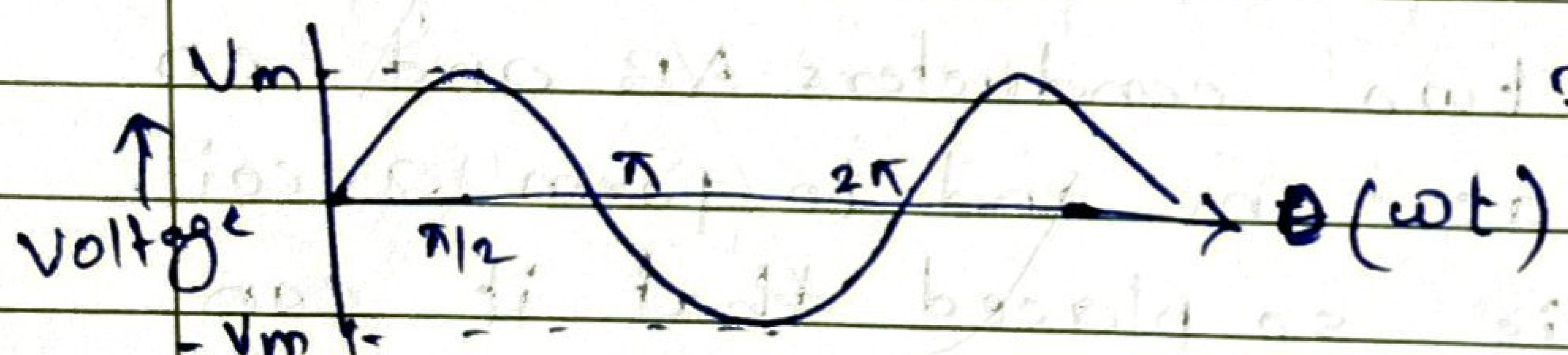

A volg. that changes its polarity at regular intervals of time is called as alternating volg.

## Advantgges of AC

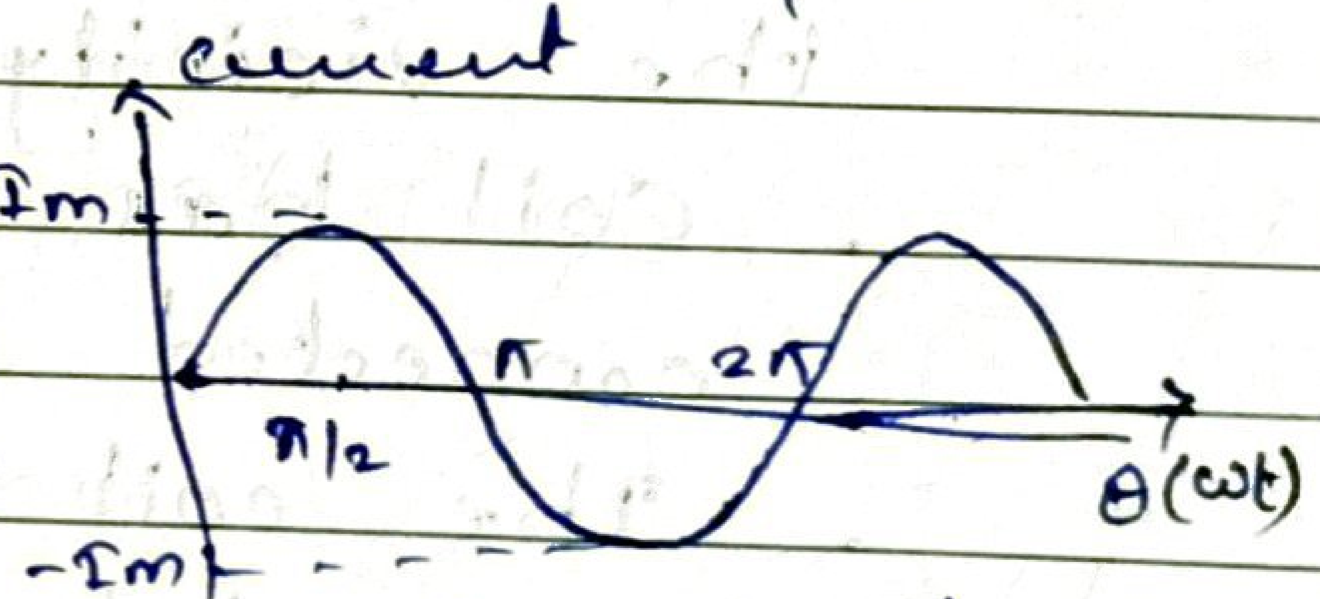
- Voltages in this system can be raised or Lowered with the help of a transformer.
- High-voltage ac transmission is possible and economical through the use of transformers.
- AC motors are simple, cheap and require less attention from maintenance point of view.
- AC supply can be easily converted into a dc supply.

## Sinusoidal Alternating Voltage and Current

- A sinusoidal alternating quantity is one whose instantaneous value varies according to the sine function of time. i.e. it produces a sine wave.




Voltage




Current

Mathematical eqⁿs

$v = V_m \sin\omega t$

Where $v$ = Instantaneous value of alternating voltage

$V_m$ = Max. value of alternating voltage

$\omega$ = Angular velocity of coil

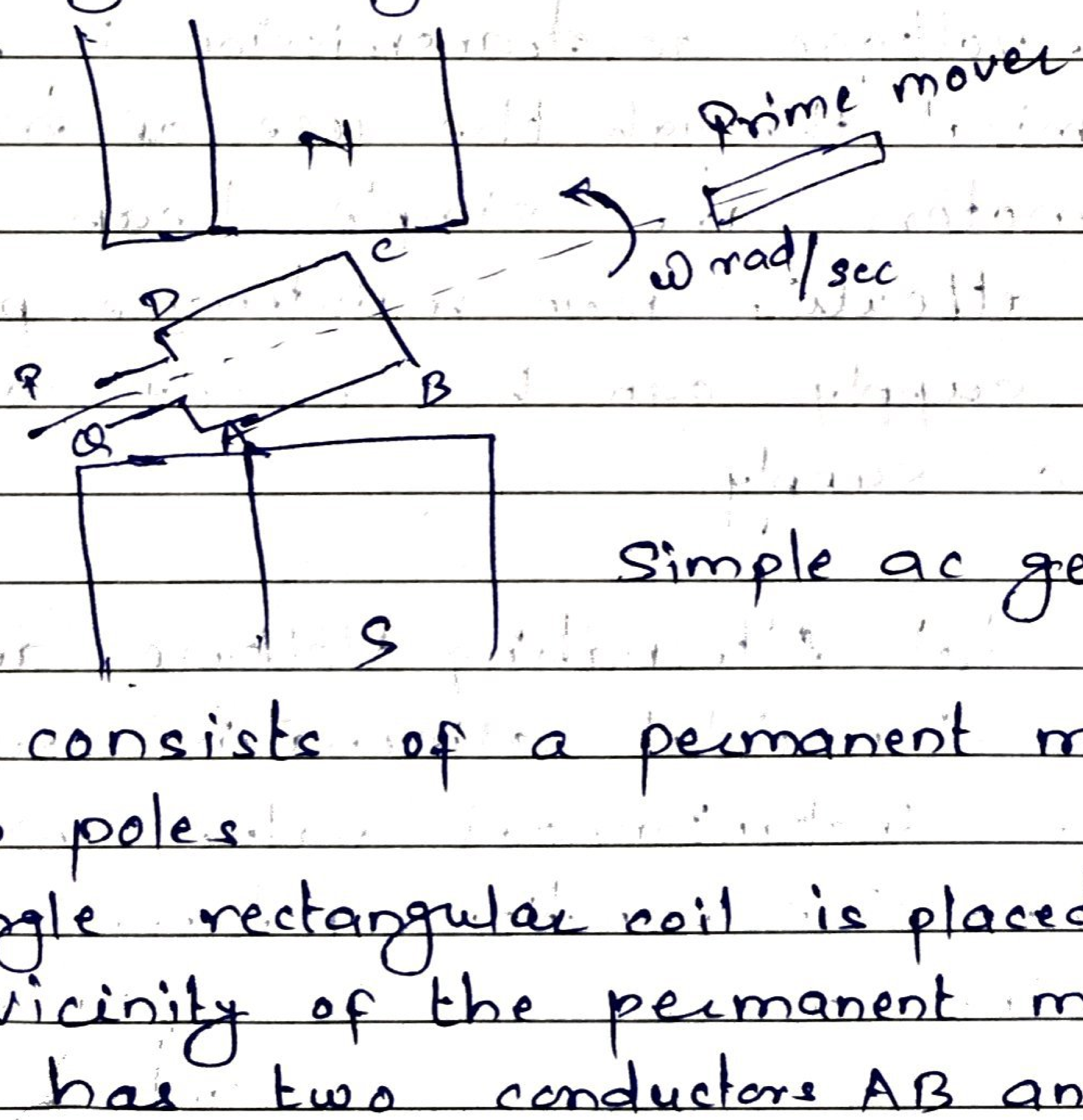
$V_m$ and $\omega$ are constant.

Similarly alternating ~~voltage~~ current varying sinusoidally can be expressed as

$i = I_m \sin\omega t$

AC Generator :-

Machine used to generate a sinusoidal alternating voltage.




Simple ac generator.

- It consists of a permanent magnet of two poles.
- A single rectangular coil is placed in the vicinity of the permanent magnet.
- Coil has two conductors AB and CD connected at one end to form a coil.
- The coil is so placed that it can be rotated about its own axis in clockwise or anticlockwise direction.

∴ P & Q are the ends of the coil.
∴ When the coil is rotated in anticlockwise direction with constant angular velocity (by means of prime mover) in a uniform magnetic field, its conductors AB and CD cut the magnetic lines of flux and according to Faraday's laws of electromagnetic induction, emf gets induced in them.

emf induced in one conductor = $BLV \sin\theta$

∴ Assuming two conductors identical

∴ Total emf induced in a coil

$$v = 2BlV\sin\theta \quad \text{—} (1)$$

$\omega$ = angular velocity rad/sec

L = active length of each conductor

B = flux density of mag. field.

V = linear velocity of conductors.

$\theta$ = angle betn plane of the coil & x-axis

When $\theta = 90^\circ$, induced emf is max. ∴ Vm

∴ Vm = max. value of induced emf

∴ $V_m = 2BlV\sin 90^\circ$

$V_m = 2BlV$

∴ Eqn (1) becomes

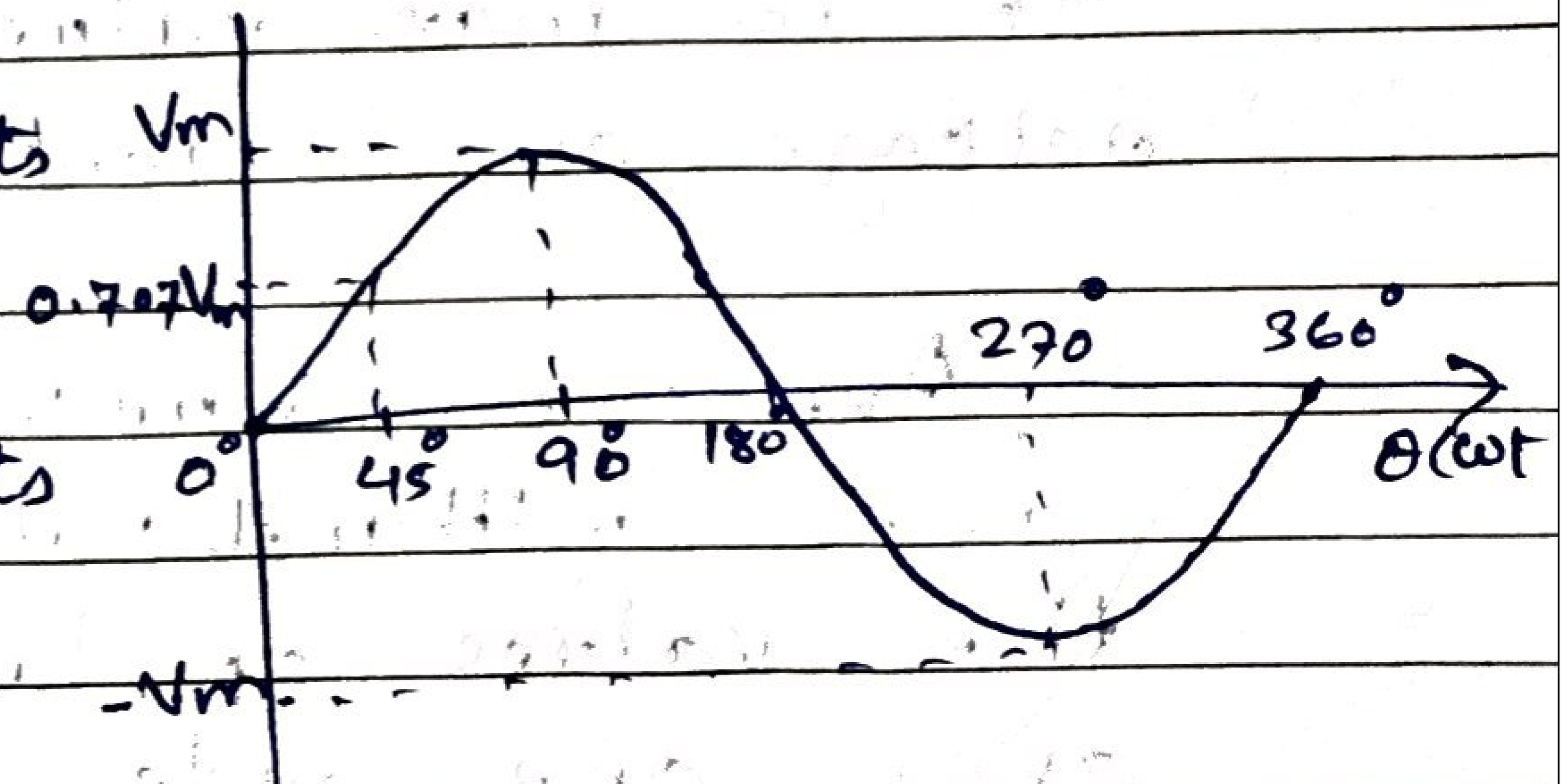
$$v = V_m \sin\theta \quad \text{—} (2)$$

This is the instantaneous value of volg. induced in a coil.

$\theta$ continuously varies from $0^\circ$ to $360^\circ$

∴ the magnitude of induced emf in coil continuously varies.

| | |
|---|---|
| $\theta = 0^\circ$ | $v = 0$ volt |
| $\theta = 45^\circ$ | $v = 0.707\,V_m$ volts |
| $\theta = 90^\circ$ | $v = V_m$ Volts |
| $\theta = 180^\circ$ | $v = 0$ volts |
| $\theta = 270^\circ$ | $v = -V_m$ volts |
| $\theta = 360^\circ$ | $v = 0$ volts |



Standard forms of alternating quantity
$\omega$ = angular velocity $\omega$ rad / sec.
Angle turned by the coil
$\theta = \omega t$ rad
Time taken to complete one cycle is the time period T of the alternating volg.

Angular velocity $\omega = \frac{\text{Angle turned}}{\text{Time taken}} = \frac{2\pi}{T}$

or $\omega = 2\pi f$ $\left(f = \frac{1}{T}\right)$

Standard forms of sinusoidal alternating volg are given by

$v = V_m \sin\theta$

$v = V_m \sin\omega t$

$v = V_m \sin 2\pi f t$

$v = V_m \sin \frac{2\pi}{T} t$

Values of alternating voltage and current :-

Magnitude of alternating voltage or current is expressed by 3 ways:
1. Peak value
2. Average value
3. rms value or effective value.

1. Peak value :-

It is max. value attained by alternating voltage or current. It is $V_m$ or $I_m$.

2. Average value :-

The arithmetical average of all the values of alternating quantity over one cycle is called its average value.

Average value = $\frac{\text{Area under the curve}}{\text{Base}}$

For symmetrical waveforms (+ve half = -ve half)

$$\text{Average value} = \frac{\text{Area of half cycle}}{\text{Base Length of half cycle}}$$

For unsymmetrical waveforms (+ve half ≠ -ve half)

$$\text{Average value} = \frac{\text{Area of full cycle}}{\text{Base length of full cycle.}}$$

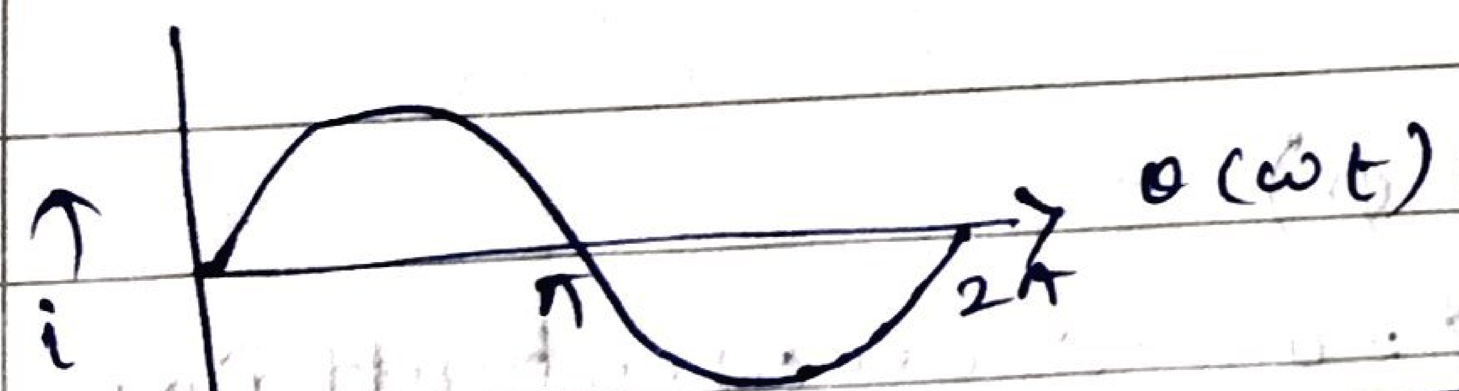
3. rms value or effective value:-

Rms value of current is equal to that direct current which when flowing thr' a given resistance for a given time produces the same amount of heat as produced by ac when flowing thr' the same resistance for the same time.

For symmetrical waveform

$$\text{Rms value} = \sqrt{(\text{Average value})^2}$$

$$= \sqrt{\frac{\text{Area of half/full cycle of squared wave}}{\text{Base length of half/full cycle}}}$$



$$I_{rms} = \sqrt{\frac{\int_0^{\pi} i^2 d\theta}{\pi}}$$

or $$I^2_{rms} = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta$$

$$\boxed{I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707 I_m}$$

Similarly $$\boxed{V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707 V_m}$$

## Form factor and Peak factor

Relationship among peak value, average value and the rms value of alternating quantity.

1) **Form factor** :- It gives peakiness of the waveform.

$$\text{Form factor} = \frac{\text{rms value}}{\text{Average value}}$$

e.g. for sinusoidal volg. or current

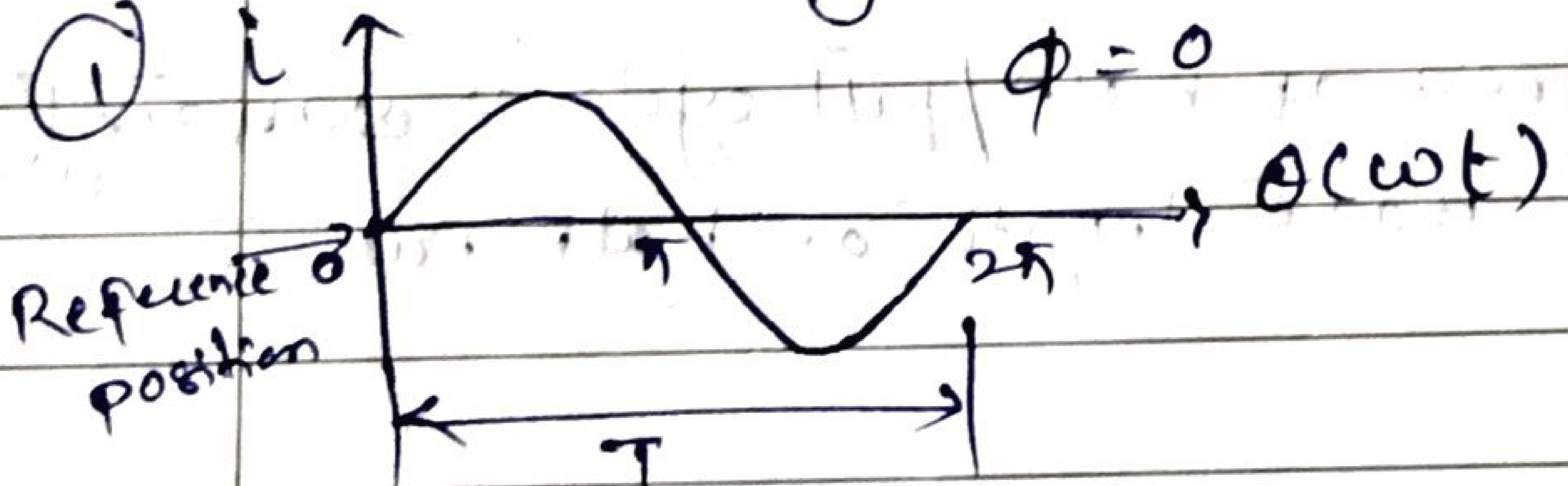
$$\text{Form factor} = \frac{0.707 \times \text{Max value}}{0.637 \times \text{Max value}} = 1.11$$

2) $$\text{Peak factor} = \frac{\text{Max. value}}{\text{rms value}}$$

e.g. for sinusoidal volg. or current

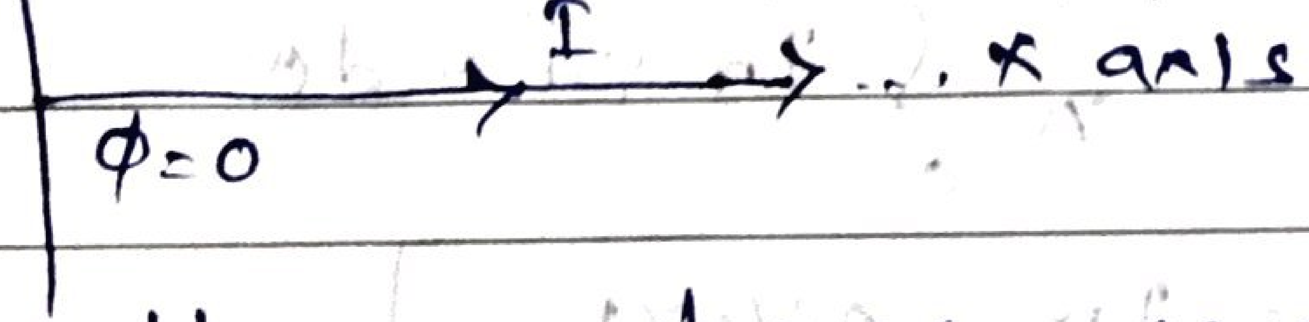
$$\text{Peak factor} = \frac{\text{Max. value}}{0.707 \times \text{Max value}} = 1.414$$

## Phase Angle and Phasor :-

1) 

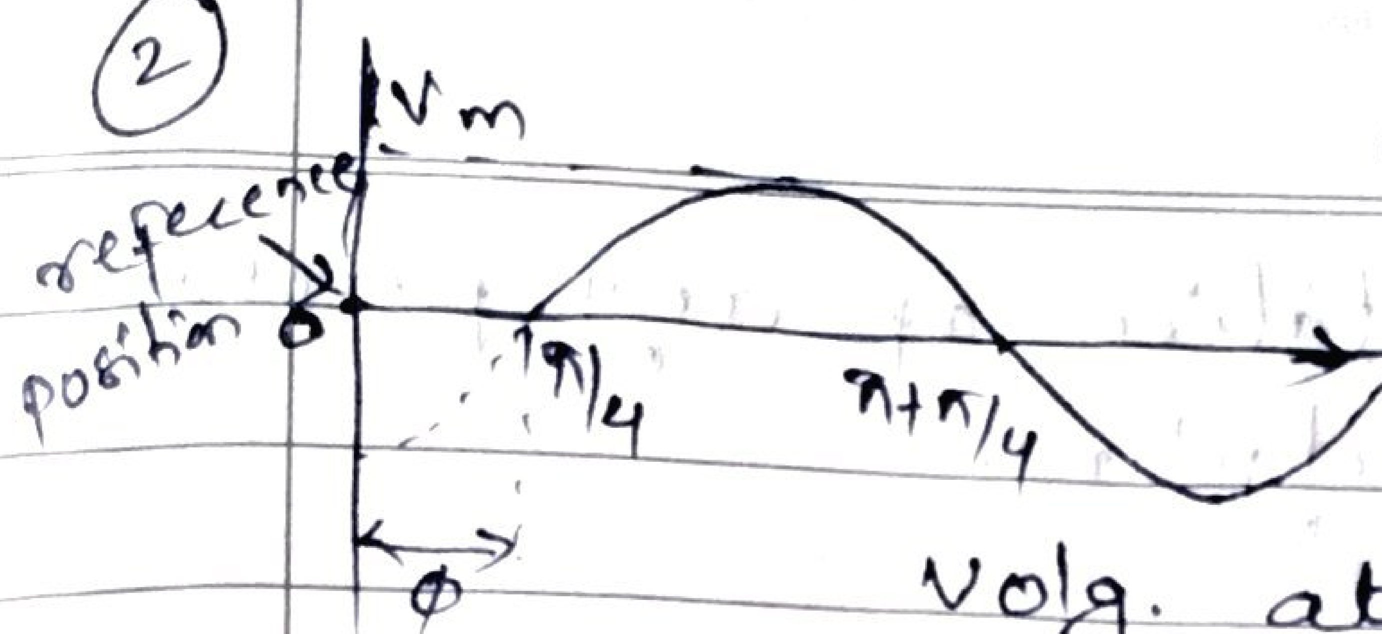

$$i = I_m \sin(\omega t \pm \phi)$$

In above case $\phi = 0$ ∴ current attains its zero value exactly at reference position.
∴ phase angle of current is zero.
Alternating current/volg. can be represented by Phasor.




- Length of phasor represents magnitude i.e. rms value.
- Inclination w.r.t. reference axis is equal to the phase angle of that quantity.

(2)

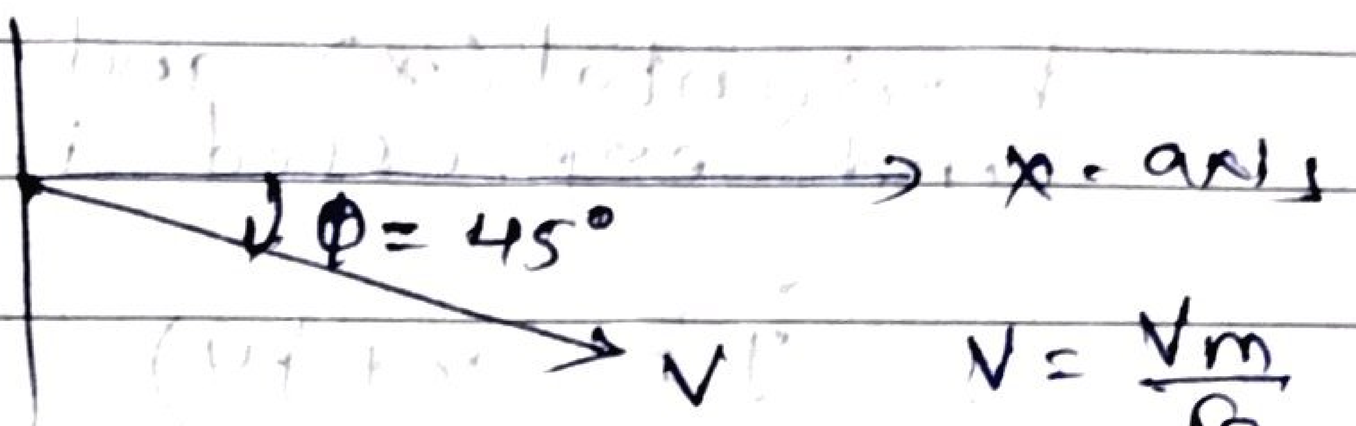



Phase angle $= -\frac{\pi}{4}$

Volg. attains its zero value (first time) after a reference position by angle $\pi/4$ rad or $45^\circ$ means it lags behind reference so phase angle $\phi = -\frac{\pi}{4}$ rad or $-45^\circ$

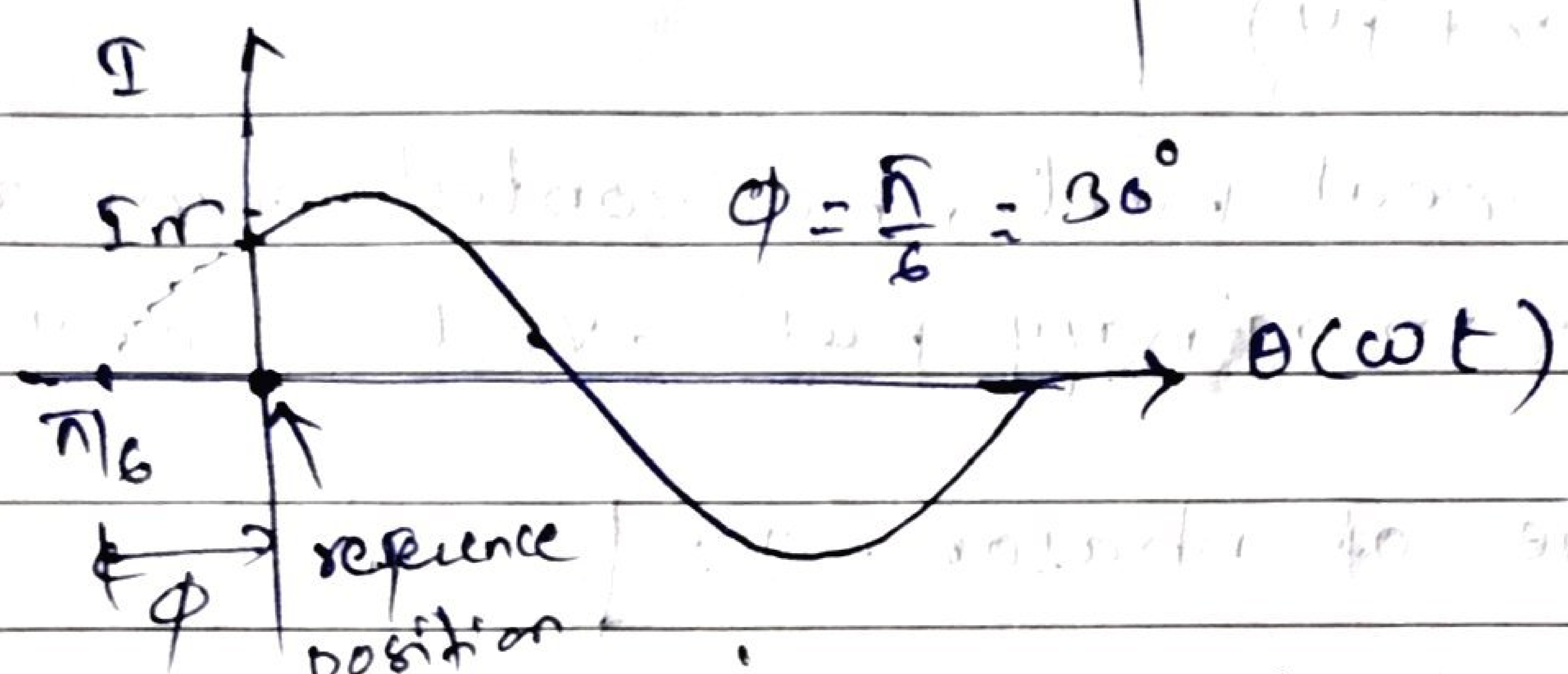
$\therefore\ v = V_m \sin\left(\omega t - \frac{\pi}{4}\right)$ or $v = V_m \sin(\omega t - 45^\circ)$

Voltage phasor is



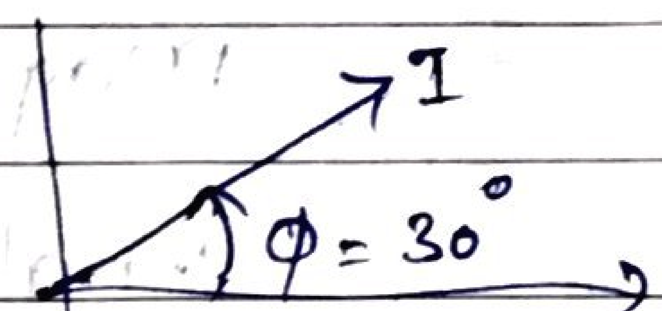

$V = \frac{V_m}{\sqrt{2}}$

(3)




$\phi = \frac{\pi}{6} = 30^\circ$

phasor

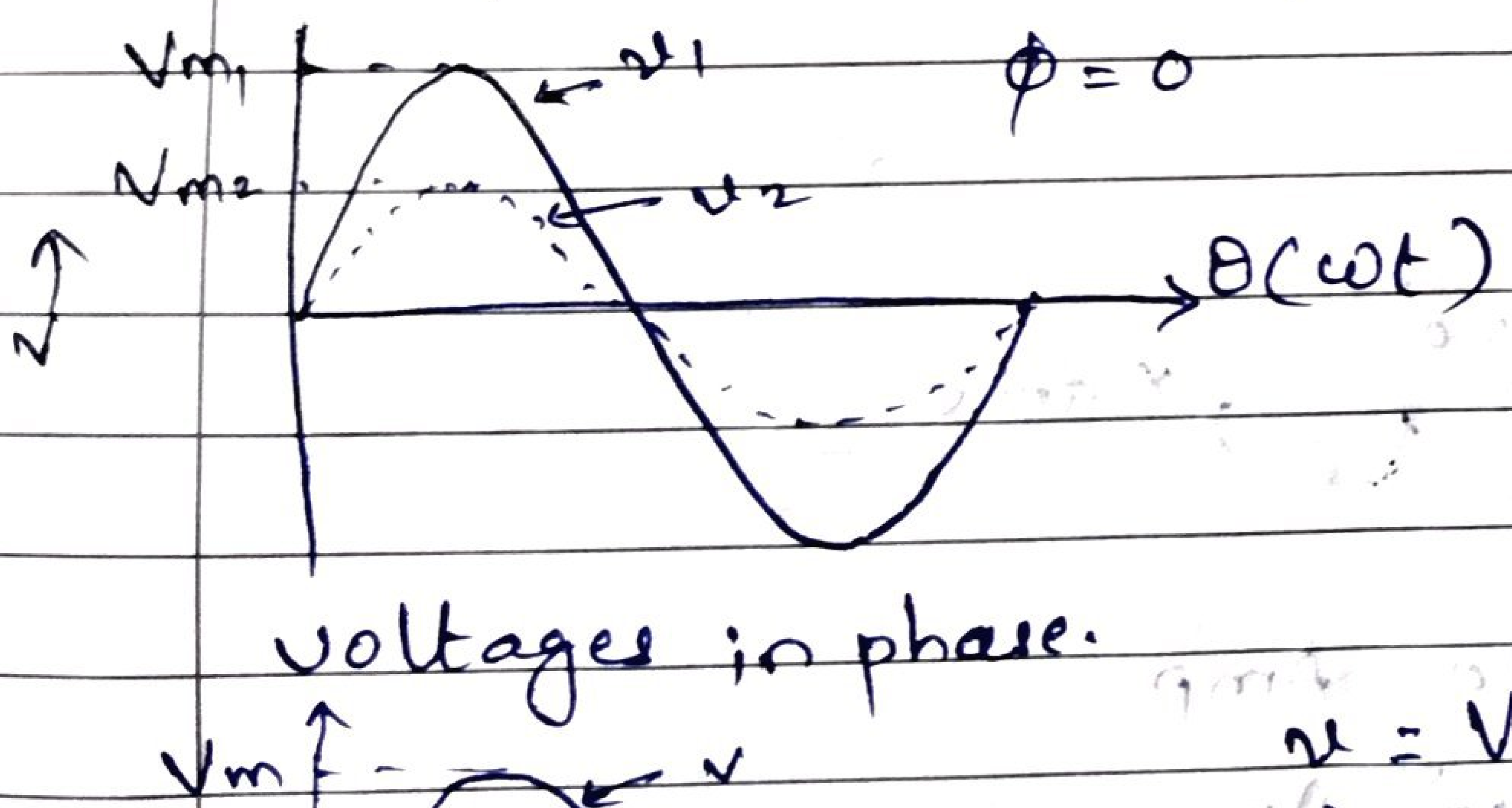



$i = I_m \sin\left(\omega t + \frac{\pi}{6}\right)$

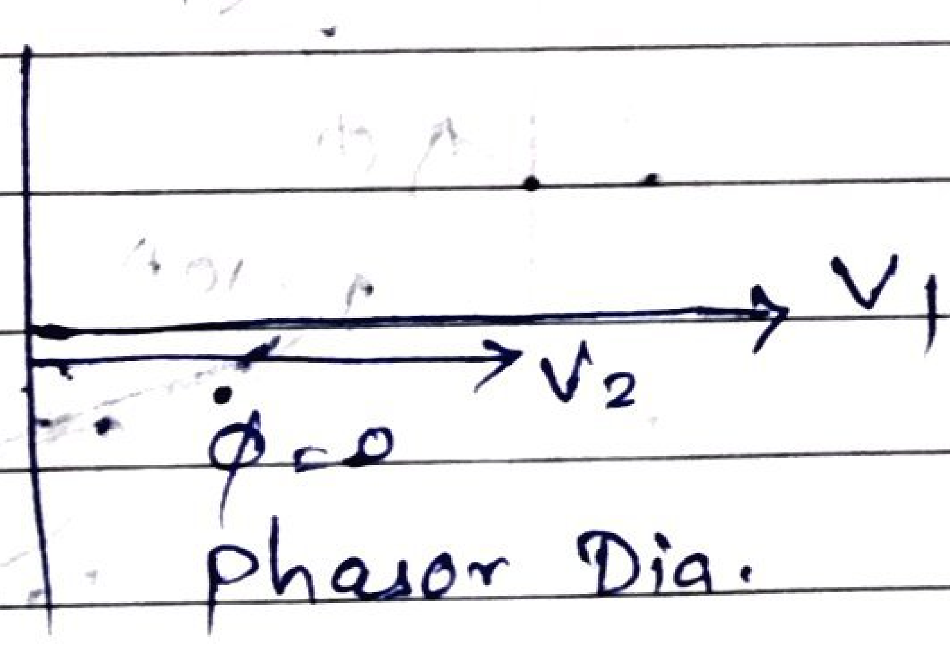
or $i = I_m \sin(\omega t + 30^\circ)$

$I = \frac{I_m}{\sqrt{2}}$

In phase, Out of phase and phase difference

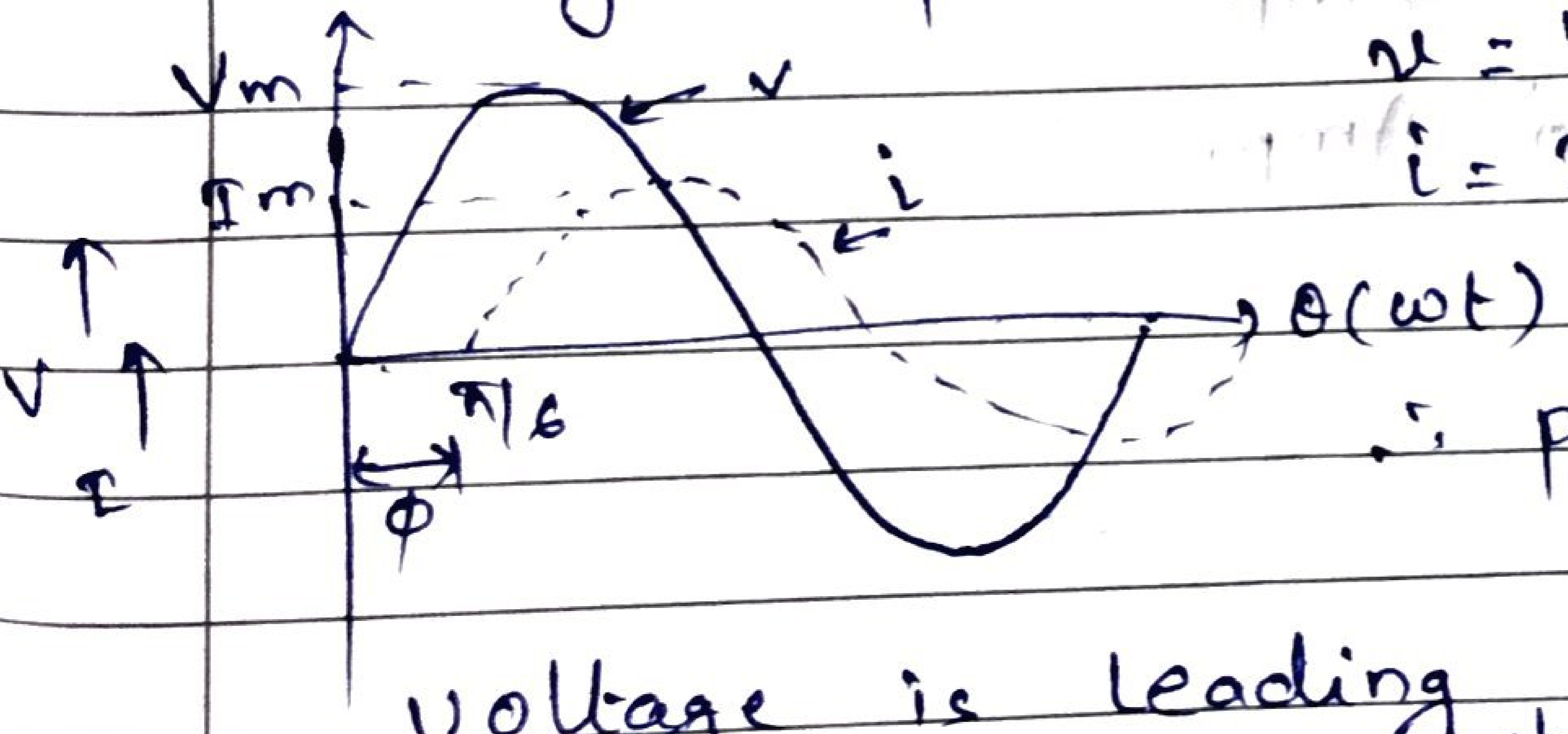



$\phi = 0$



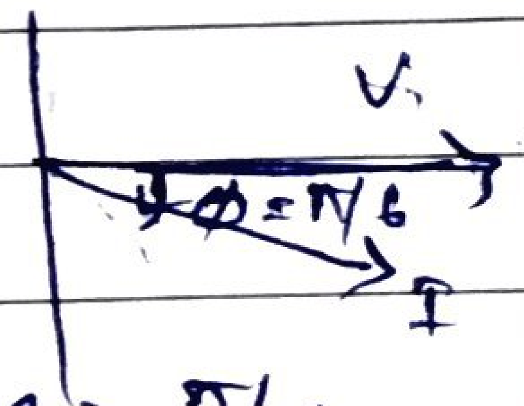

Phasor Dia.

Voltages in phase.




$v = V_m \sin \omega t$

$i = I_m \sin(\omega t - \pi/6)$




$\therefore$ phase difference $= \pi/6$

Voltage is leading current or current is lagging behind volg. by $\phi = 30^\circ$

Phasor Algebra

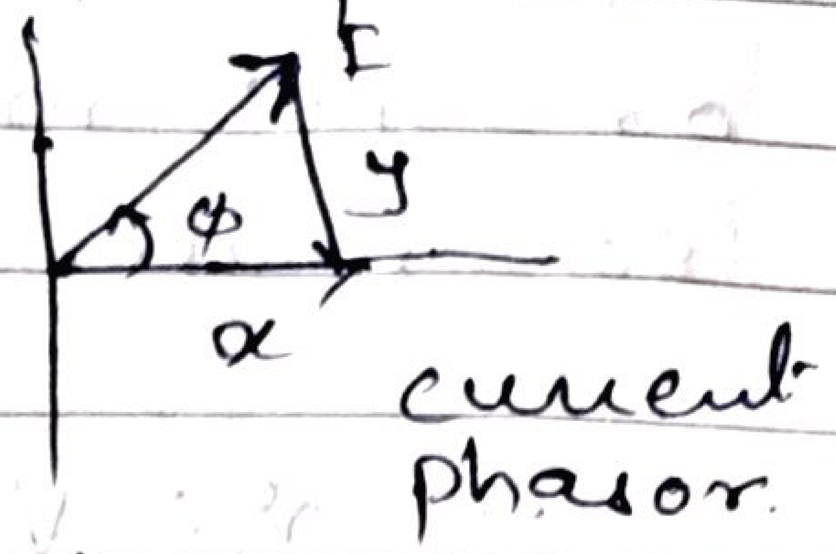
- Mathematical representation of any phasor is known as phasor algebra.

Two ways:

1) Rectangular form  2) Polar form.

1) Rectangular form

It is a complex form.




In this form, the phasor is resolved into two components: horizontal (x) and vertical (y) components. and expressed in complex form.

$$\bar{I} = (x + jy)$$

x = real part, horizontal component, x-co

y = Imaginary part, vertical or y - compone

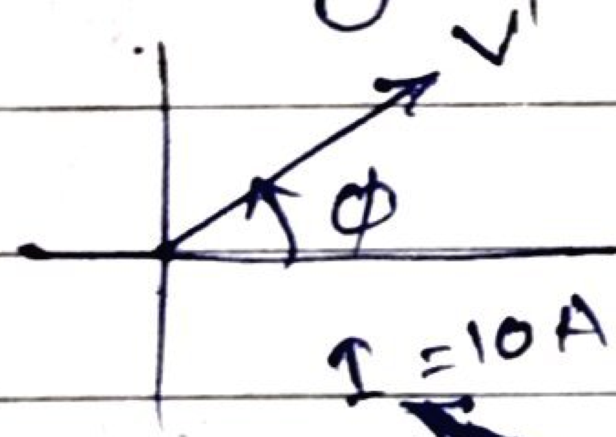
Magnitude of phasor $= I = \sqrt{x^2+y^2}$

Angle w.r.t. x-axis $= \phi = \tan^{-1}\dfrac{y}{x}$

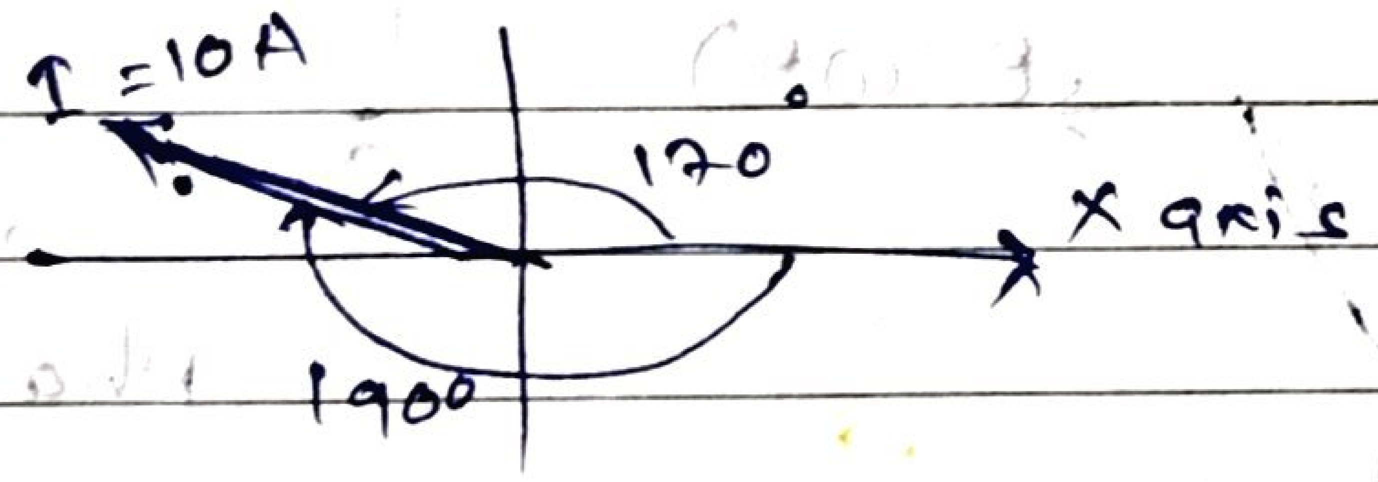
2) Polar form.

Current phasor is represented as $I \angle \pm \phi$

voltage phasor —11— as $V \angle \pm \phi$




e.g.




$$\bar{I} = 10 \angle 170^\circ \text{ Amp}$$

or $\bar{I} = 10 \angle -190^\circ$ Amp.

Candidate Roll No. ____________ (In figures)

**Test Exam.**

Name : ____________

Date : ____________ 20

Examination : ____________ Branch/Semester ____________

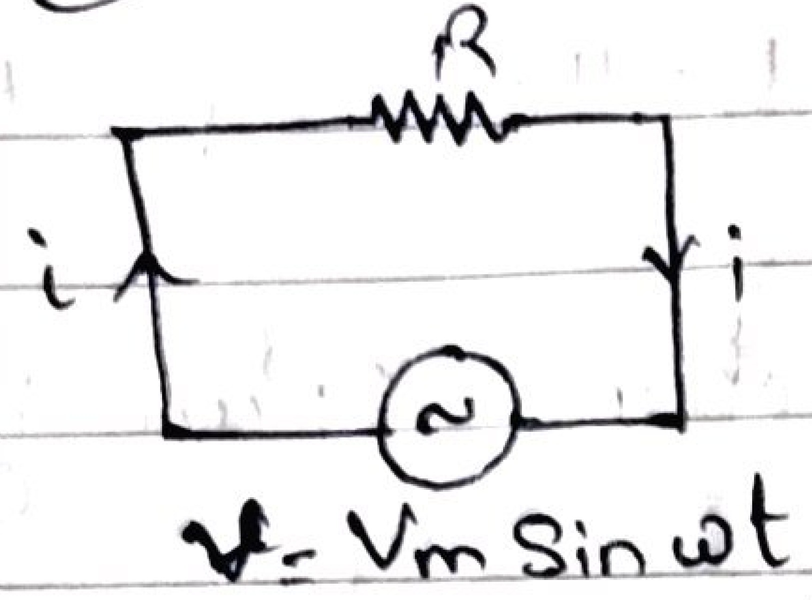
Subject : ____________

| Junior Supervisor's full Signature with Date |
|---|
| |

| Question No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Marks Obtained | | | | | | | | | | | | | |

## Fundamental AC circuits :-

① AC circuit with Resistance




$v = V_m \sin \omega t$ — ①

$i = \frac{v}{R}$

$= \frac{V_m \sin \omega t}{R}$ — ②

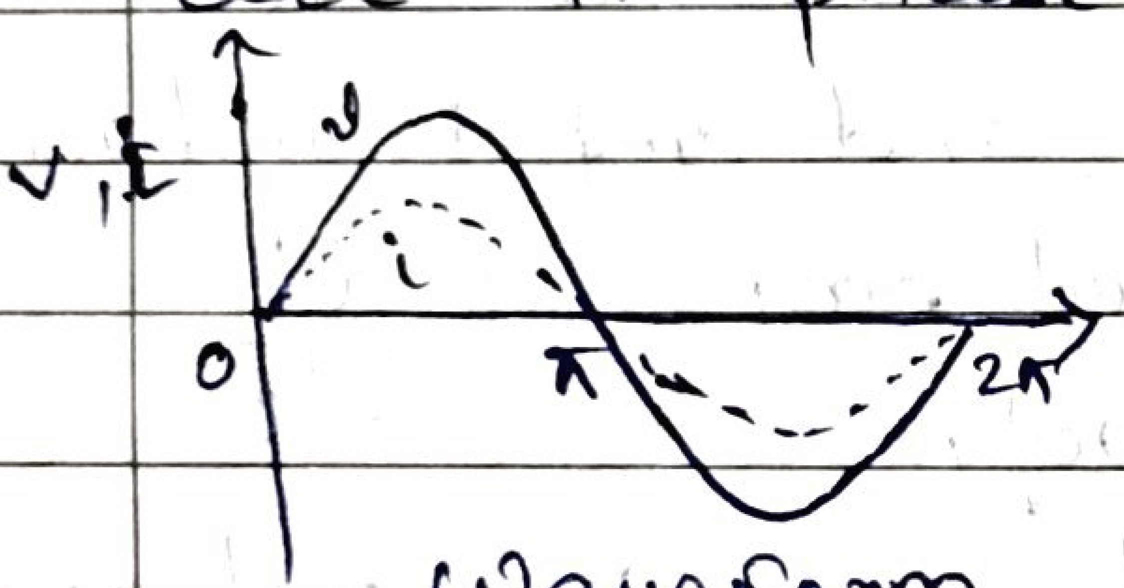
i will be max. when $\sin \omega t = 1$

$\therefore I_m = \frac{V_m}{R}$

$\therefore i = I_m \sin \omega t$ — ③

From eq$^n$s ① & ③ it can be found that the applied voltage and the circuit current are in phase with each other. i.e. $\phi = 0$

$\phi = 0$ (phase angle of the ckt)




Waveform


phasor diagram

Impedance (Z) :- Opposition offered by ckt element to the current flow.

$\frac{V}{I} = Z$

for resistive ckt $Z = R$

**Power (P)** :- $p = v.i$

instantaneous power $= (V_m \sin \omega t) \cdot (I_m \sin \omega t)$

$= V_m I_m \sin^2 \omega t$

$= V_m I_m \left( \frac{1 - \cos 2\omega t}{2} \right)$

$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$

Power is a scalar quantity, average power over a complete cycle is to be considered.

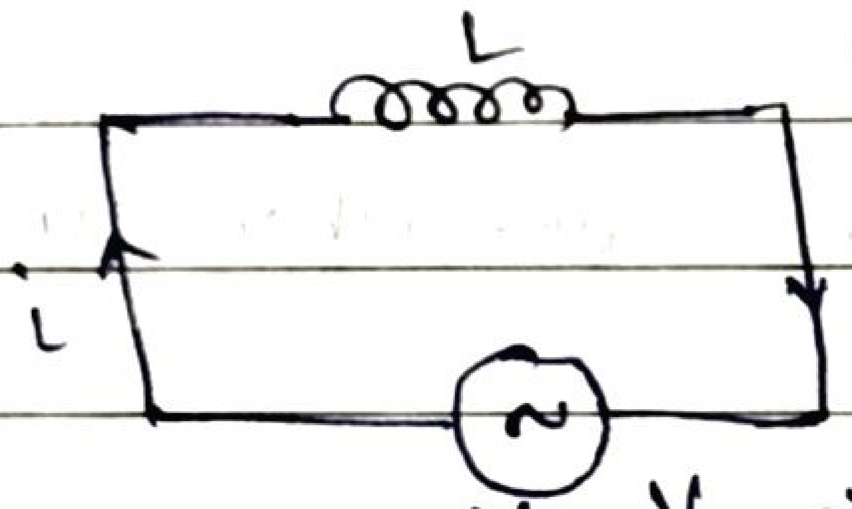
$\therefore$ Power consumed $P = \frac{1}{2\pi}\int_0^{2\pi} p\,d\omega t$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t\right] d$$

$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

$\therefore P = VI$    $V$ - rms value of applied v
$I$ - " - " - of " - curr

AC circuit with pure inductance (Pure inductive ckt)



$v = V_m \sin\omega t$

Applied voltage
$v = V_m \sin\omega t$ — (1)

As a result of this voltage alternating current $i$ flows through the inductance $L$. Alternating current sets up alternating magnetic field around the inductance. This changing flux links the coil and emf is induced in it is called self-induced emf $= L\frac{di}{dt}$
This emf opposes applied volg.
At any instant self induced emf is equal and opposite to the applied volg.

$$\therefore +v = L\frac{di}{dt}$$

$$L\,di = +v\,dt$$

$$L\,di = V_m \sin\omega t\,dt$$

$$di = \frac{V_m}{L}\sin\omega t\,dt$$

Integrating both sides

$$i = \frac{V_m}{L}\int \sin\omega t\,dt$$

Power factor has magnitude and nature. Magnitude equals to $\cos\phi$. Nature is same as nature of current.

$$i = \frac{V_m}{\omega L}(-\cos\omega t)$$

$$\therefore i = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{——} \quad (2)$$

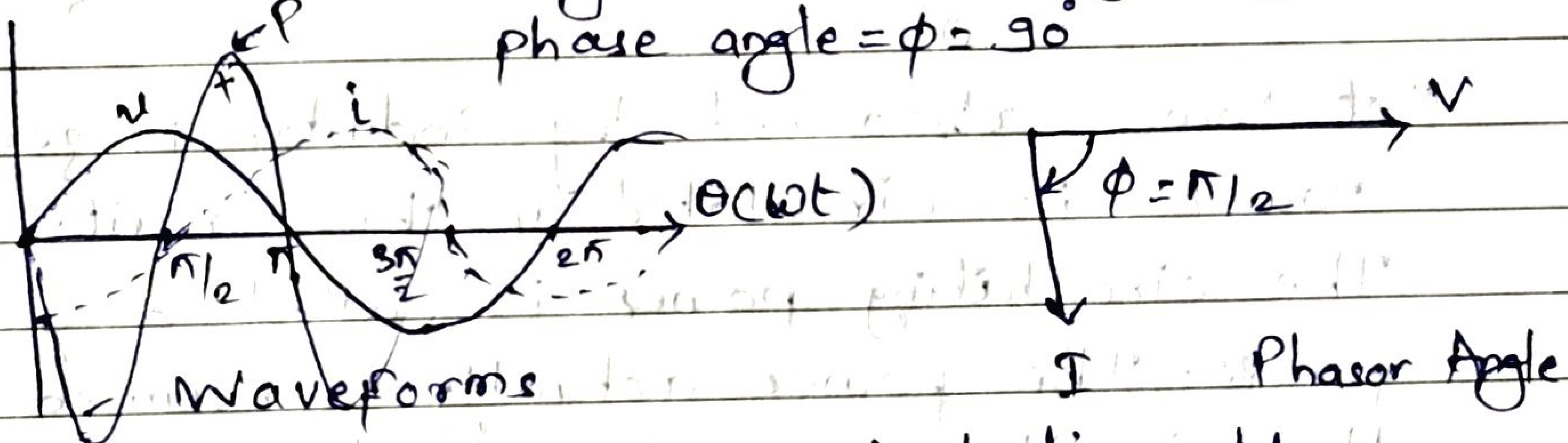
$i$ is max when $\sin\left(\omega t - \frac{\pi}{2}\right) = 1$

$$\therefore I_m = \frac{V_m}{\omega L}$$

$$\therefore i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) \quad \text{——} \quad (3)$$

Phase angle and power factor:-
From eq$^n$'s (1), (2) and (3) it is observed that the current lags behind voltage by $\pi/2$ or $90^\circ$

phase angle $= \phi = 90^\circ$




Power factor of pure inductive ckt

$pf = \cos\phi = \cos 90^\circ = 0$ Lagging

Impedance (Z)

$I_m = \frac{V_m}{\omega L}$ or $\omega L = \frac{V_m}{I_m}$

$$\frac{\frac{V_m}{\sqrt{2}}}{\frac{I_m}{\sqrt{2}}} = \omega L$$

$$\therefore \frac{V}{I} = \omega L$$

$$\therefore \frac{V}{I} = Z = \omega L$$

$\omega L$ is called as inductive reactance $X_L$ of coil. $\therefore X_L = \omega L \ \Omega$

$X_L = 2\pi f L \ \Omega$

Power :- Instantaneous power,

$$p = vi$$
$$= V_m \sin\omega t \times I_m \sin(\omega t - \frac{\pi}{2})$$
$$= -V_m I_m \sin\omega t \cos\omega t$$
$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

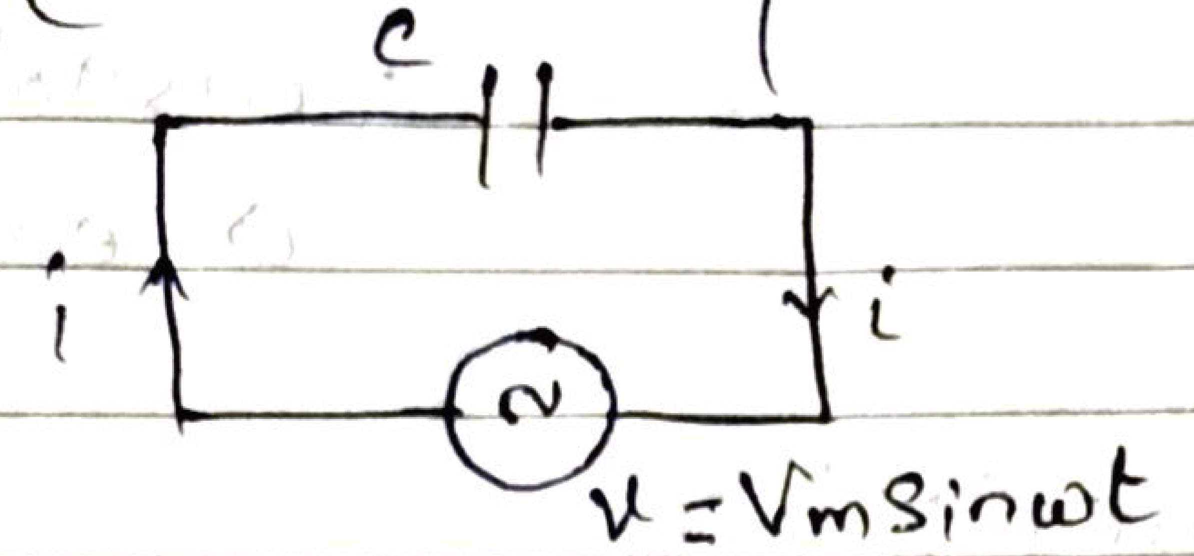
Since power is a scalar quantity, avg. power over a complete cycle to be considered.

$\therefore$ Average power $= P =$ Average of p over one cycle.

$$P = \frac{1}{2\pi}\int_0^{2\pi} p \, d\omega t$$
$$= \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t \, d\omega t$$
$$= 0$$

Net Power absorbed by inductor is zero.
Power is circulated in the circuit.
The circulating power is called as reactive power. The power actually consumed in the circuit is called as active power.

## AC Circuit Containing Pure Capacitance only (Pure Capacitive Circuit)




- Applied voltage
$v = V_m \sin\omega t$ — (1)
- Due to this alternating voltage alternating current i will flow through the circuit.

- charge on capacitor $q = Cv$
Circuit current $i = \frac{dq}{dt}$ — rate of change of charge

$= \frac{dCv}{dt}$

$= \frac{d}{dt}(CV_m \sin\omega t)$

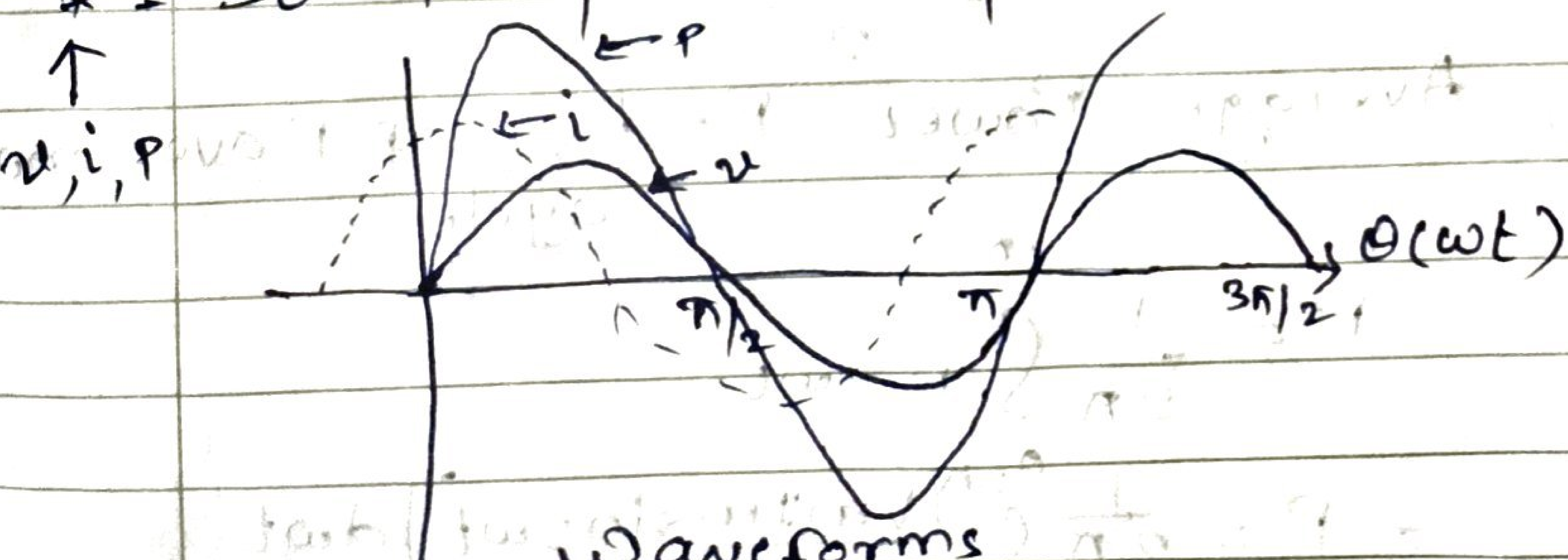
$= \omega C V_m \cos\omega t$

or $i = \omega C V_m \sin(\omega t + \frac{\pi}{2})$

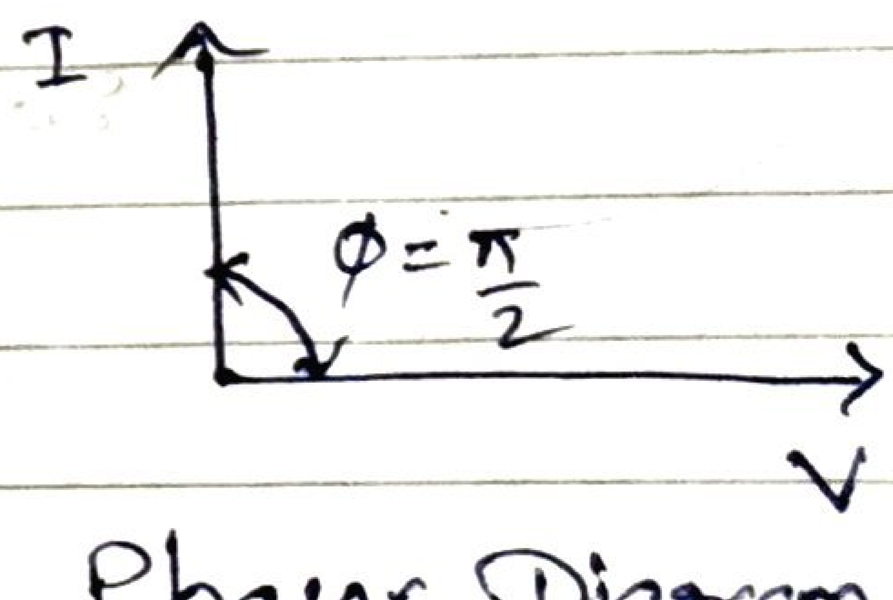
value of i will be max. i.e. $I_m$ when
$\sin(\omega t + \frac{\pi}{2}) = 1$
so $I_m = \omega C V_m$

$\therefore \boxed{i = I_m \sin(\omega t + \frac{\pi}{2})}$ — (2)

Phase angle and power factor:-
- From eq^n's (1) and (2) we get the current leads the voltage by $\pi/2$ or $90°$.
* - So in pure capacitor current leads volg. by $90°$




Waveforms




Phasor Diagram

- phase angle of the circuit $\phi = 90°$
- Power factor of the circuit, $pf = \cos\phi$

$$= \cos 90°$$

$$= 0 \text{ leading.}$$

Opposition to current (Z)

we have

$$I_m = \omega C V_m$$

$$\frac{V_m}{I_m} = \frac{1}{\omega C}$$

Dividing both numerator and denominator by $\sqrt{2}$ for rms value

$$\frac{V_m/\sqrt{2}}{I_m/\sqrt{2}} = \frac{1}{\omega C}$$

$$\frac{V}{I} = \frac{1}{\omega C}$$

$$\therefore \boxed{Z = \frac{1}{\omega C}\ \Omega}$$

This is called as capacitive reactance $X_C$ of the capacitor.

$$\therefore \boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}\ \Omega}$$

Power (P) :- Instantaneous power $p = vi$

$$= V_m \sin\omega t \times I_m \sin(\omega t + \frac{\pi}{2})$$

$$= V_m I_m \sin\omega t \cos\omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

So, Average Power P = Avg. of P over one cycle

$$P = \frac{1}{2\pi}\int_0^{2\pi} p\, d\omega t$$

$$= P = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m \sin 2\omega t}{2}\right] d\omega t$$

* Power absorbed in pure capacitance is zero.

Sums on pure L and C

# ① A 10mH inductor has current of $i = 5\cos(2000t)$A. Obtain the volg. $V_L$ across it.

Sol$^n$. $L = 10mH = 10\times10^{-3}$ H , $i = 5\cos(2000t)$

we can write i eq$^n$ as foll.

$i = 5\sin(2000t + \frac{\pi}{2})$

$\omega = 2000$ rad/sec. & $\phi = \frac{\pi}{2}$ rad $= 90°$

rms value of current $I = I_m/\sqrt{2} = \frac{5}{\sqrt{2}} = 3.54A$

$X_L = \omega L = 2000\times10\times10^{-3} = 20\Omega$

$\therefore V_L = I X_L = 3.54\times20 = 70.8V$

or

$v_L = L\,di/dt$

$= 10\times10^{-3}\frac{d}{dt}\left[5\cos(2000t)\right]$

$= -10\times10^{-3}\times5\times2000\sin 2000t$

or $v_L = 100\sin(2000t + 180°)$

$\therefore V_m = 100V.$

$\therefore V_L = \frac{100}{\sqrt{2}} = 70.71V$

# 2 A 318μF cap$^r$ is connected across 230V, 50Hz system. Determine ① capacitive reactance ⅱ rms value of current and ⅲ eq$^n$s for volg & current.

Sol$^n$. ① Capacitive reactance $= X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi\times50\times318\times10^{-6}}$

$X_C = 10\Omega$

ⅱ Rms value of current $= I = \frac{V}{Z} = \frac{V}{X_C} = \frac{230}{10} = 23$ Amp

(iii) $V_m = \sqrt{2} \times V$

$= \sqrt{2} \times 230$

$= 325.27\ V$

$I_m = \sqrt{2} \times I$

$= \sqrt{2} \times 23$

$= 32.53\ A$

$\therefore$ volt & current eq's are

$v = 325.27 \sin(2\pi \times 50)t$

$= 325.27 \sin 314t$

$i = 32.53 \sin\left(314t + \frac{\pi}{2}\right)$

SOMAIYA COLLEGE OF ENGINEERING

(Autonomous College Affiliated to University of Mumbai)

Candidate Roll No. ____________ (In figures)

Name : ____________ Test Exam.

Date : ____________ 20

Examination : ____________ Branch/Semester ____________

Subject : ____________

Junior Supervisor's full Signature with Date

| Question No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Marks Obtained | | | | | | | | | | | | | |

# Ac Series Circuits

**Series ac circuit :-**

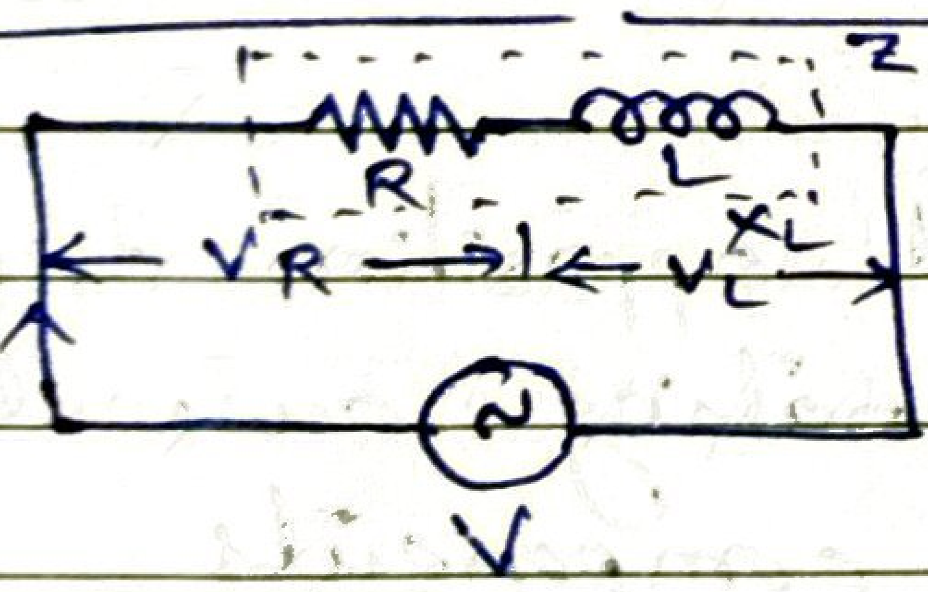
A ckt in which the same alternating current flows through all the elements R, L, C. Different types

1) R-L series ckt
2) R-C ,, ,,
3) R-L-C ,, ,,

For these types study of foll is important:

1) phase angle $\phi$ bet$^n$ applied volg & current
2) nature of the circuit (whether resistive, inductive or capacitive)
3) Circuit impedance
4) Power consumed

**1) R-L series circuit :-**




V = rms value of applied volg.

I = rms value of current

Z = impedance of the circuit

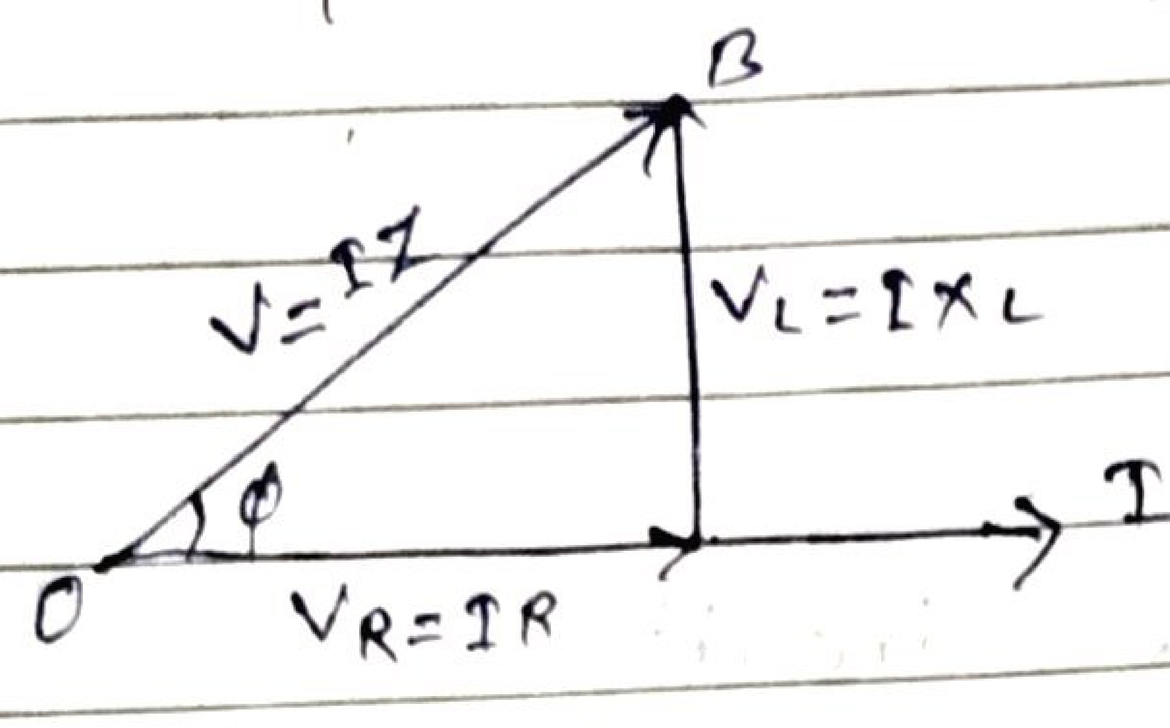
As per Ohm's law

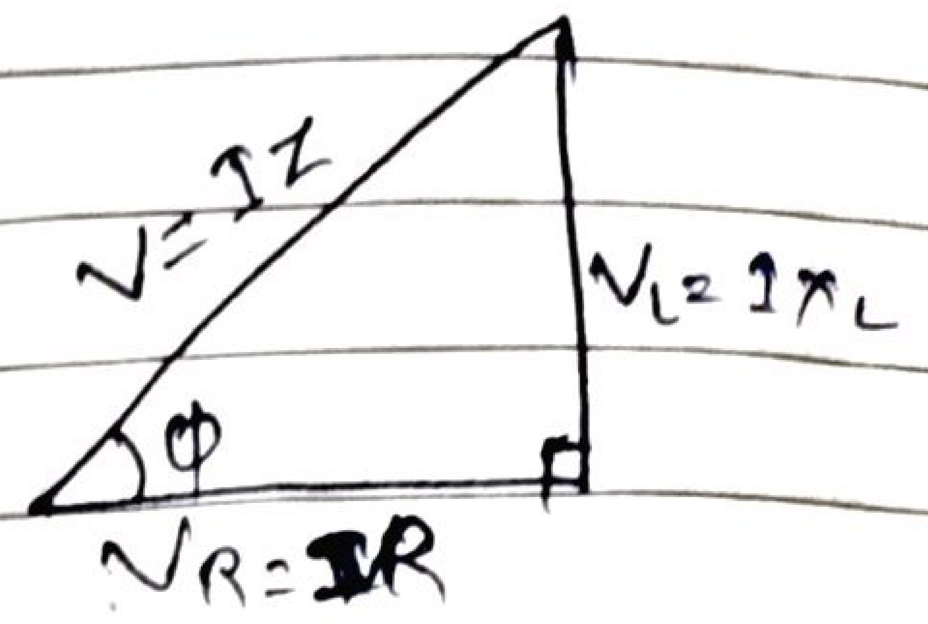
$V_R = IR$ — $V_R$ in phase with I

$V_L = IX_L$ — $V_L$ leads I by $90°$

$V = IZ$

Phasor diagram :-
Taking current as the reference phasor, the phasor dia. of the ckt will be




Phasor dia. voltage triangle

w.r.t. the ckt dia. volg. eqⁿ

$\bar{V} = \bar{V}_R + \bar{V}_L$

phase angle of ckt $= \phi = \tan^{-1}\frac{V_L}{V_R} = \tan^{-1}\frac{X_L}{R}$

Power factor of ckt $Pf = \cos\phi = \frac{V_R}{V} = \frac{IR}{IZ}$

$$\boxed{Pf = \frac{R}{Z} \text{ lagging}}$$

Ckt current I lags behind applied volg. nature of power factor is lagging.

Voltage triangle :- The voltage along with its components can be represented by a right-angled triangle is known as voltage triangle

Impedance Z :- Total opposition offered to the flow of alternating current.

Impedance has two components

Resistance (R) Inductive reactance ($X_L$)

Impedance along with its components can be represented by right-angled triangle known as impedance triangle.


Impedance Triangle

From impedance triangle,

$\boxed{\text{Impedance } Z = \sqrt{R^2 + X_L^2}\ \Omega}$

In rectangular and polar form Z is expressed as:

$\bar{Z} = (R + jX_L)\ \Omega$ — rect. form

In polar form

$\bar{Z} = (Z\angle\phi)\ \Omega$

Power P

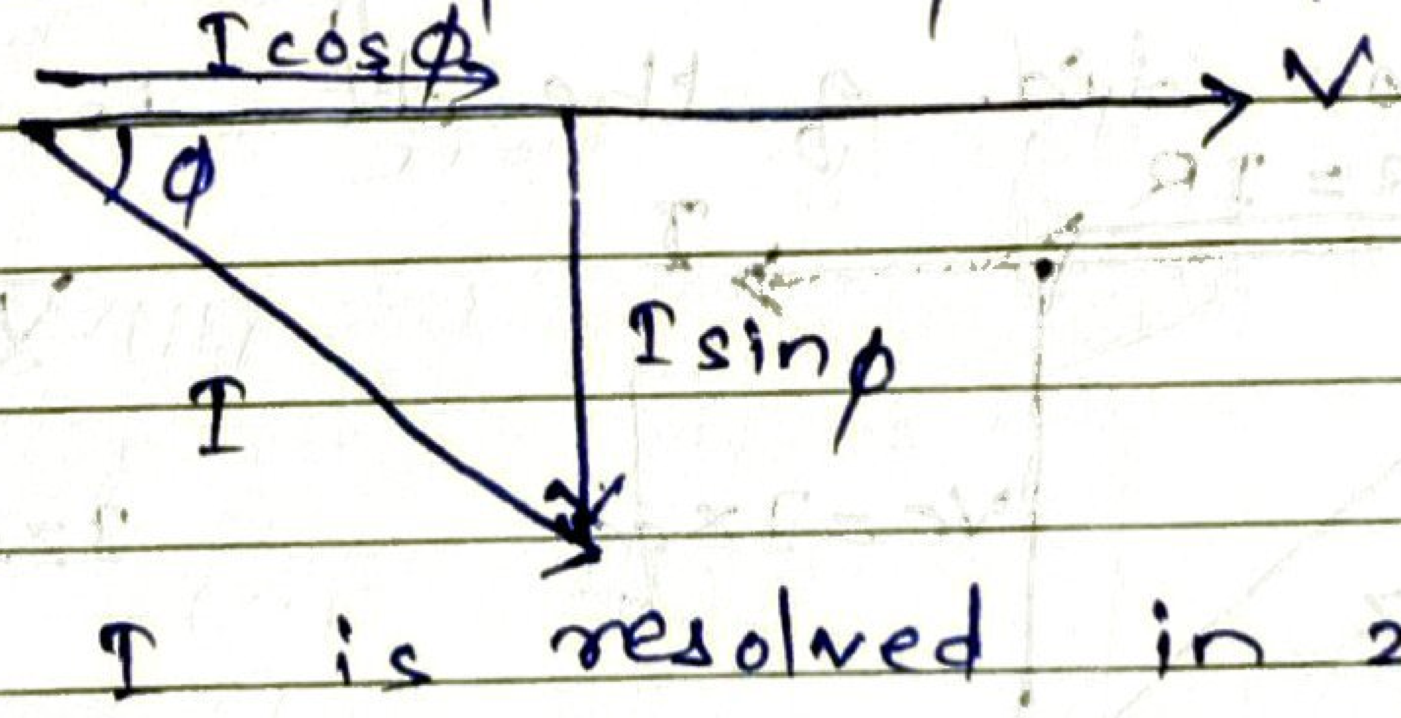
- Power consumed in the ckt is called active or true power (in case of R)
- Circulating power is called reactive power (in case of L or C).

Active power = P = voltage × current in phase with voltage

Reactive power = Q = voltage × current 90° out of phase with volt.

For R-L ckt

Taking V as reference phasor,




Phasor dia.

Current I is resolved in 2 components

i) $I\cos\phi$ — in phase with voltage

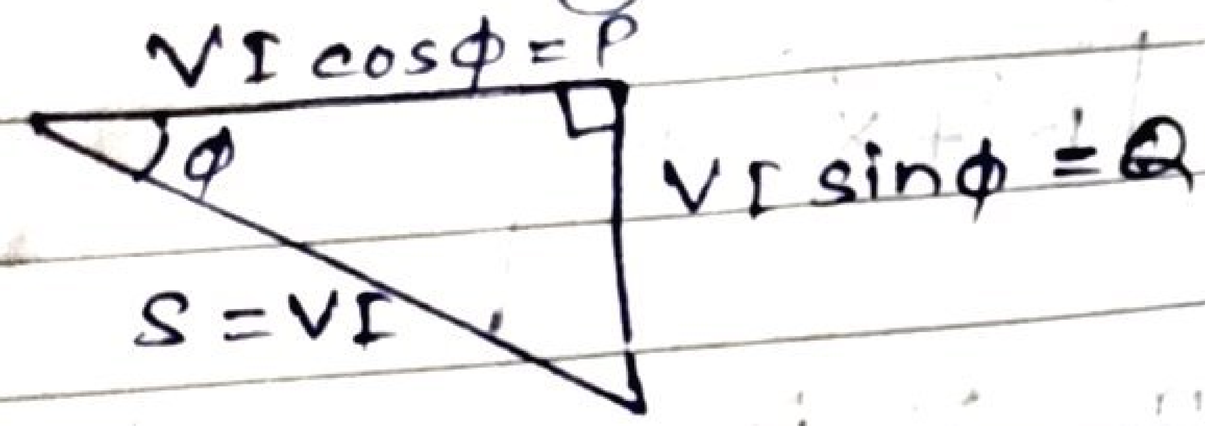
ii) $I\sin\phi$ — 90° out of phase with voltage

Active power $P = VI\cos\phi$ Watts or Killowatt

Reactive Power $Q = VI\sin\phi$ VAR or kVAR

Apparent Power $S = VI$ VA or kVA

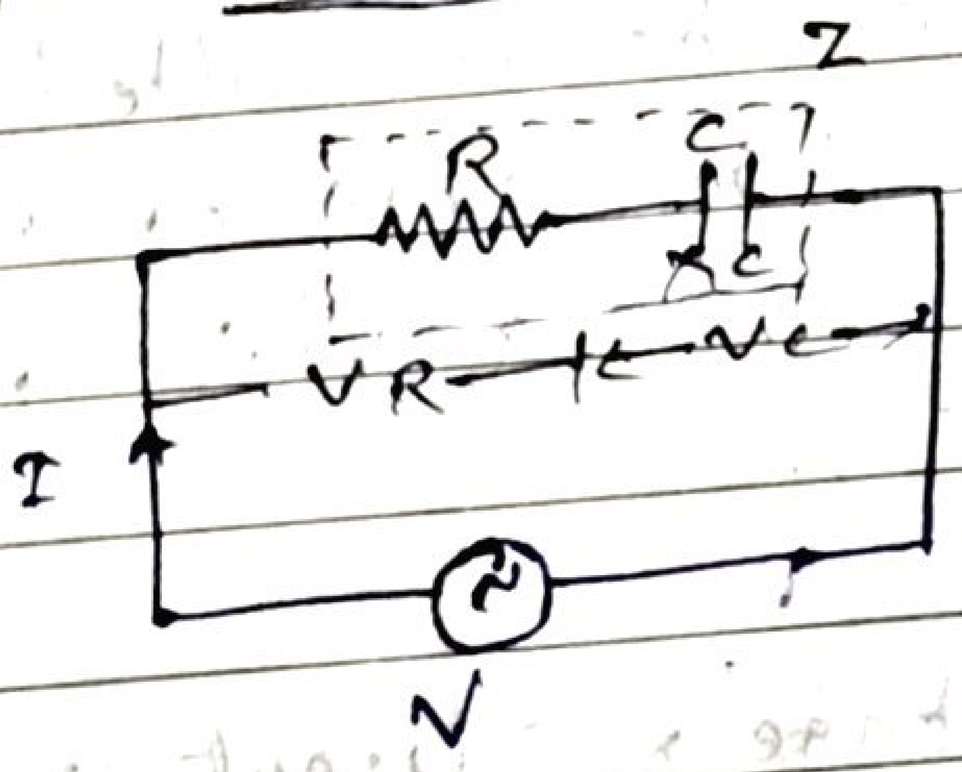
Power triangle:-




from power triangle
(Active power)$^2$ + (Reactive power)$^2$ = (Apparent power)$^2$

$$P^2 + Q^2 = S^2$$

## R - C series Circuit




V = rms value of applied volg.
I = rms value of ckt current
Z = impedance of the ckt

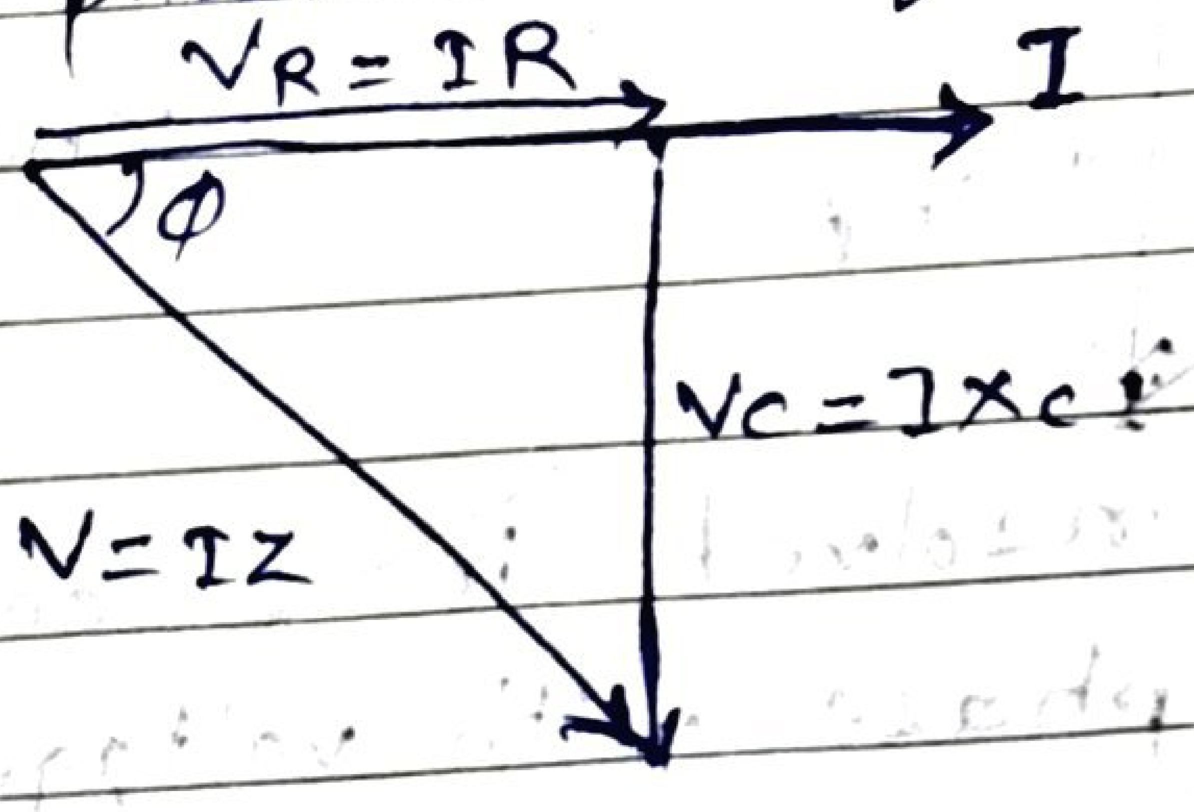
By Ohm's law

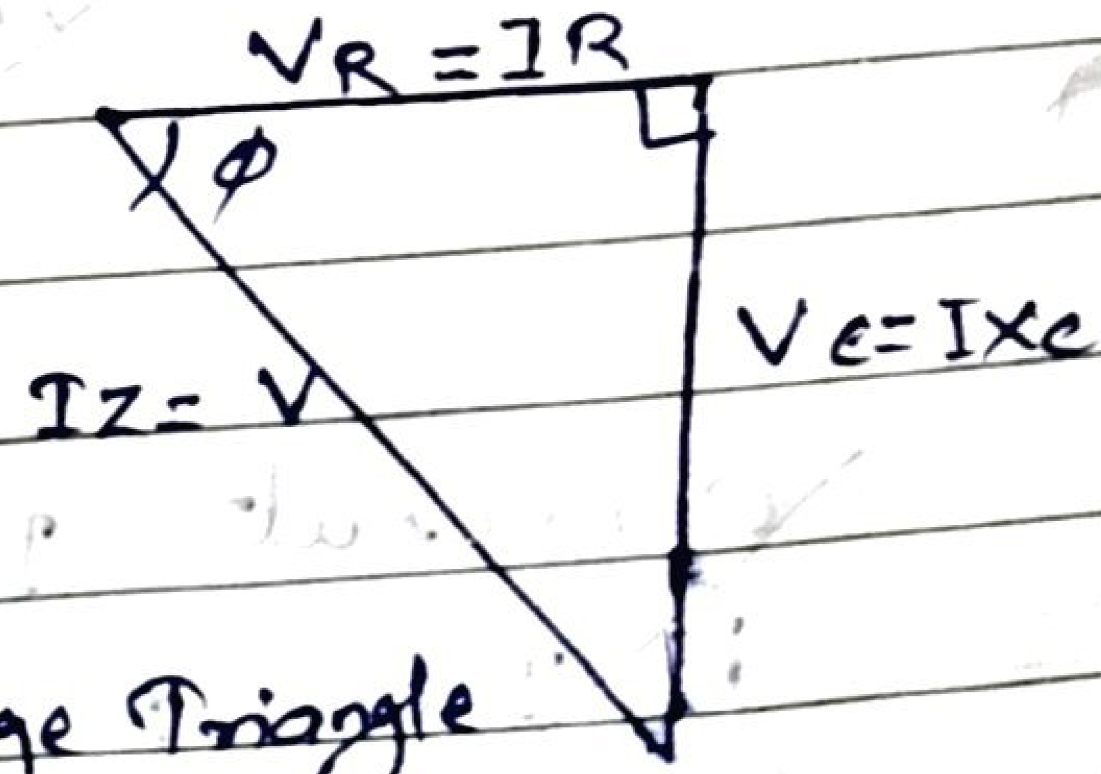
$V_R = IR \rightarrow V_R$ in phase I
$V_C = IX_C \rightarrow V_C$ lags behind I by 90°

$$V = IZ$$

Phasor Dia.

Taking current as the reference phasor, the phasor dia. of the ckt is.




Voltage Triangle

$$\overline{V} = \overline{V_R} + \overline{V_C}$$

Current I leads the applied volg. V by ϕ° ($\phi < 90°$) ∴ Nature of ckt is capacitive.

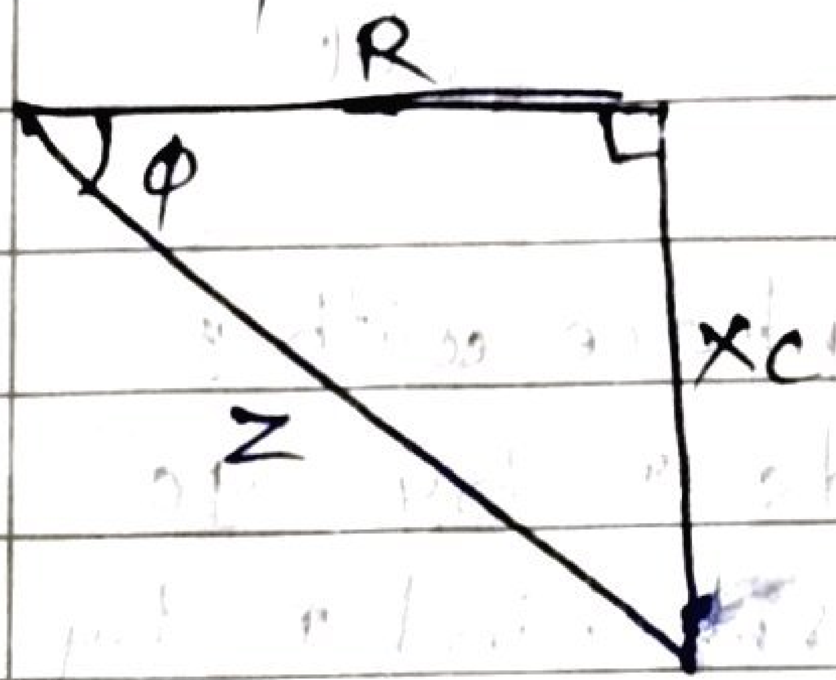
$$\phi = \tan^{-1}\frac{V_C}{V_R} = \tan^{-1}\frac{X_C}{R}$$

Power factor of the ckt $pf = \cos\phi = \frac{V_R}{V} = \frac{IR}{IZ}$

$= \frac{R}{Z}$ leading

As current I leads applied volg. nature of power factor is leading.

Impedance (Z)




Dividing each of volg. phasor by I in a volg. triangle we get the impedance triangle.

Impedance Triangle

Circuit impedance

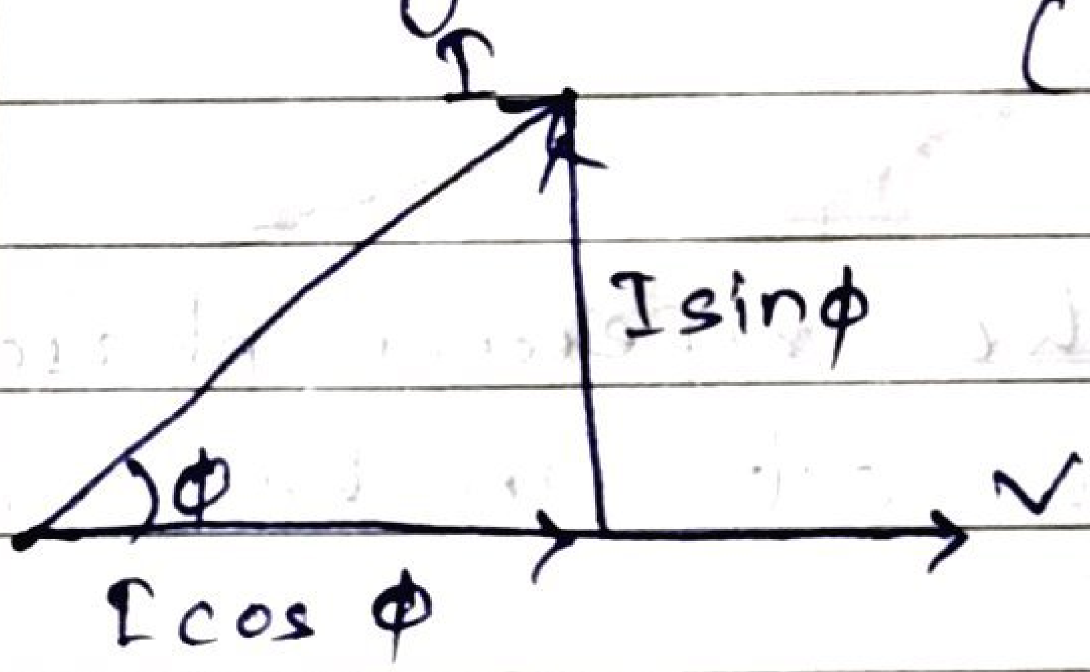
$Z = \sqrt{R^2 + X_c^2}\ \Omega$

$\bar{Z} = (R - jXc)\ \Omega$

$\bar{Z} = (Z\angle -\phi)\ \Omega$

Power (P)

Taking V as reference phasor dia. will be (only volg + current phasor are drawn)



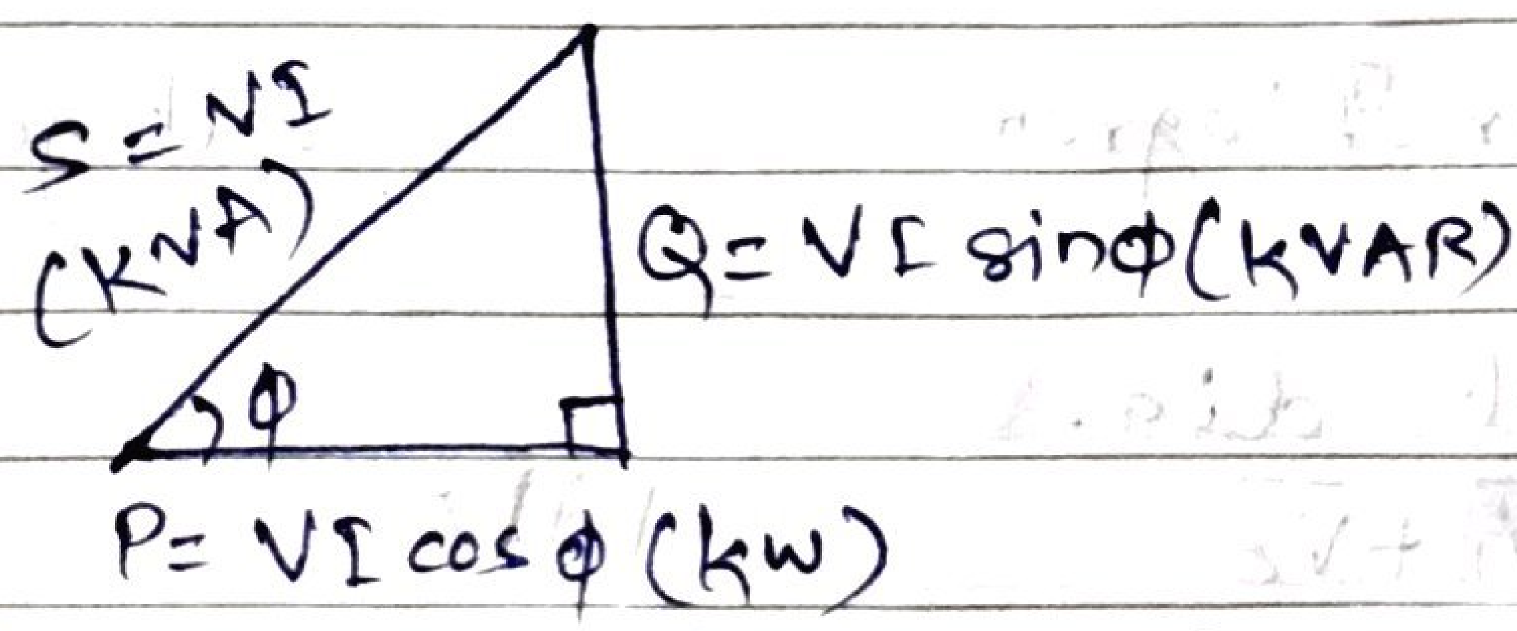

Phasor diagram

Circuit current can be resolved into two components

i) $I\cos\phi$ — in phase with V

ii) $I\sin\phi$, — 90° out of phase with Voltage

Multiplying each of current phasor by V in above dia. we get power triangle as:



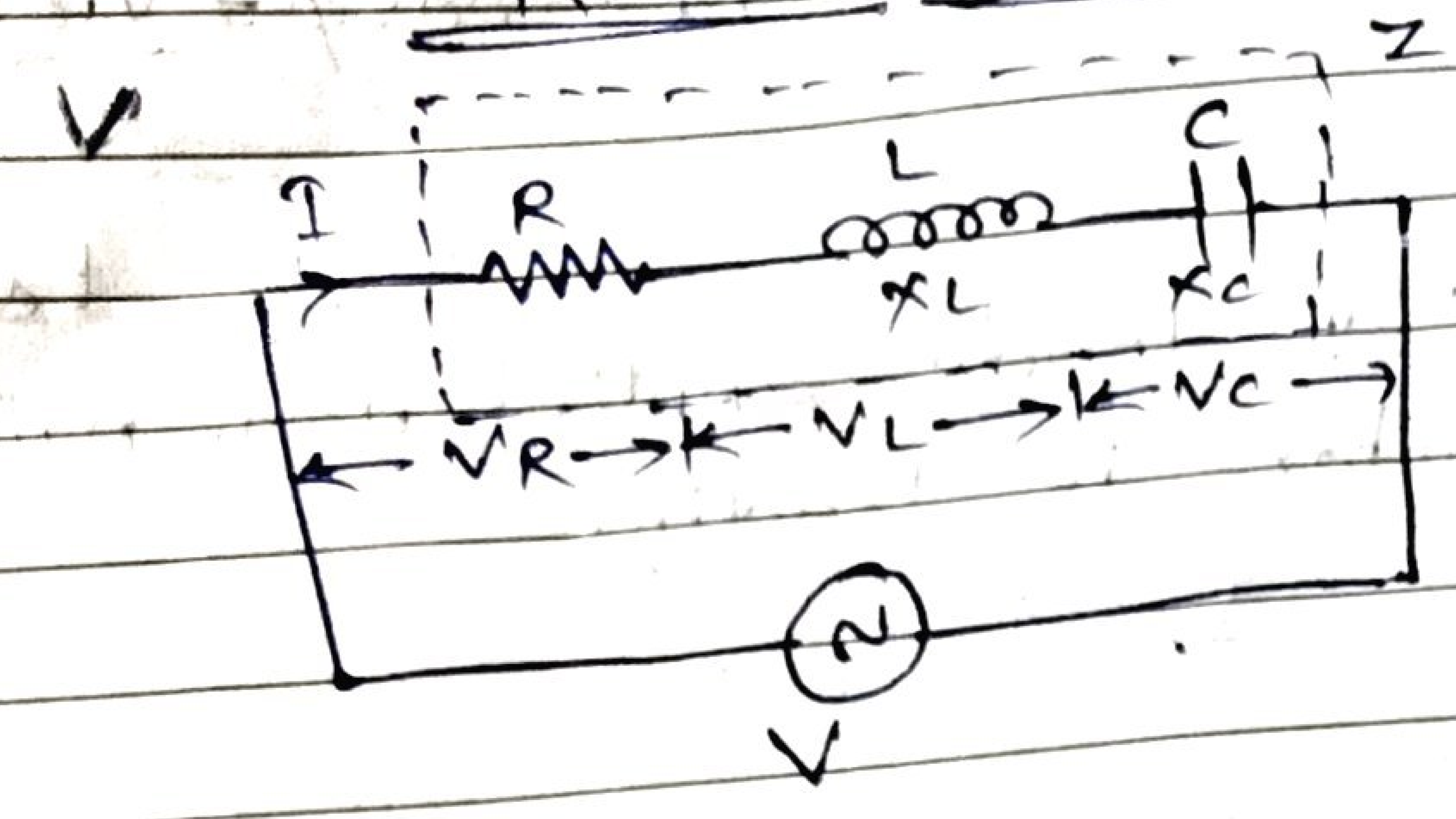

Power triangle

$(\text{Apparent power})^2 = (\text{Active power})^2 + (\text{Reactive power})^2$

$S^2 = P^2 + Q^2$

So power factor $pf = \cos\phi = \frac{P}{S}$

# R-L-C Series Circuit




V = rms value of applied volg.
I = rms value of circuit current
Z = Impedance of ckt

By Ohm's law

$V_R = I R$ — $V_R$ in phase with I
$V_L = I X_L$ — $V_L$ leads I by $90^\circ$
$V_C = I X_C$ — $V_C$ lags behind I by $90^\circ$

$V = IZ$

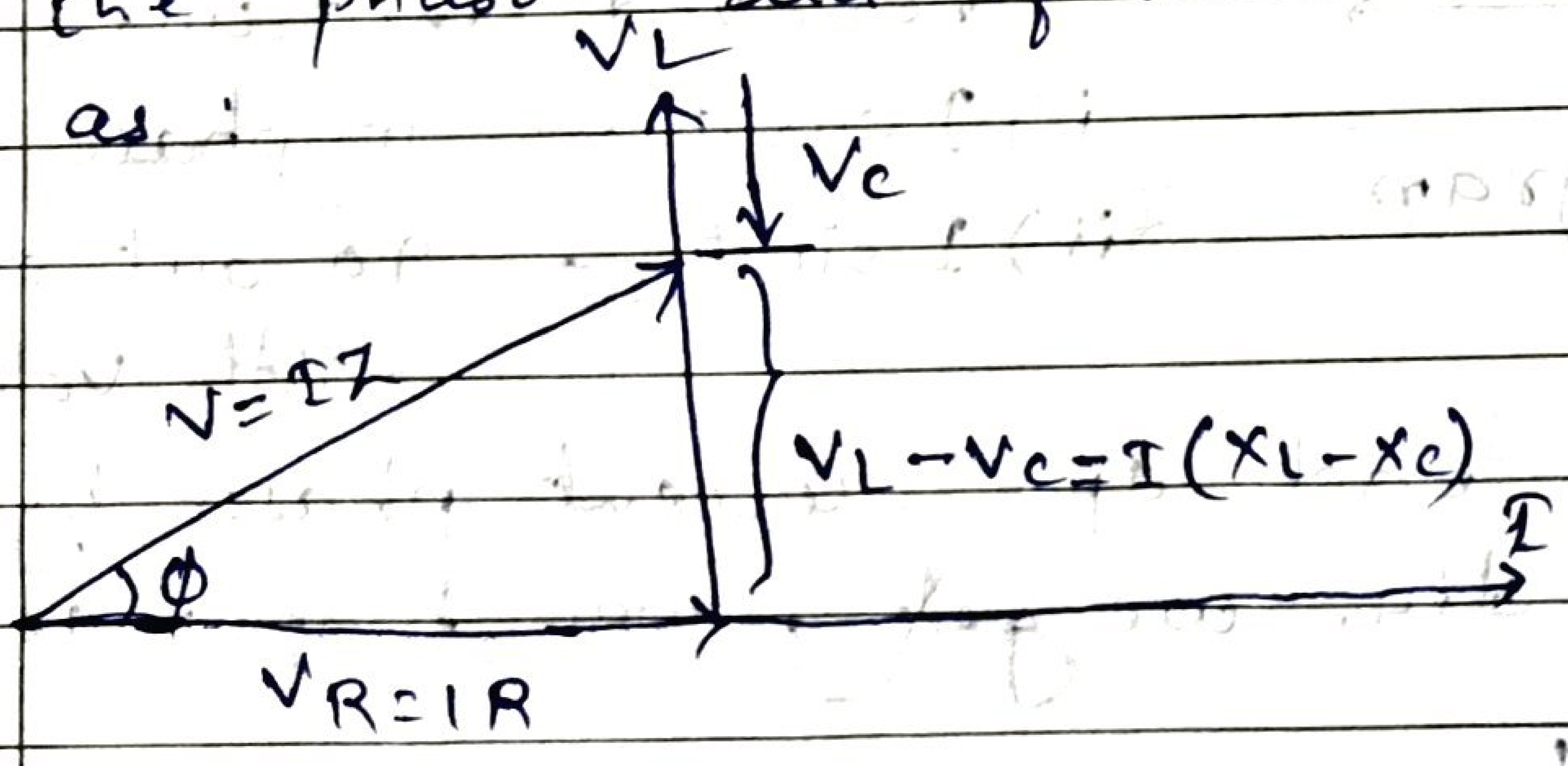
R-L-C series ckt can be effectively inductive, capacitive or resistive that depends on values of $X_L$ and $X_C$.

Case ① $X_L > X_C$

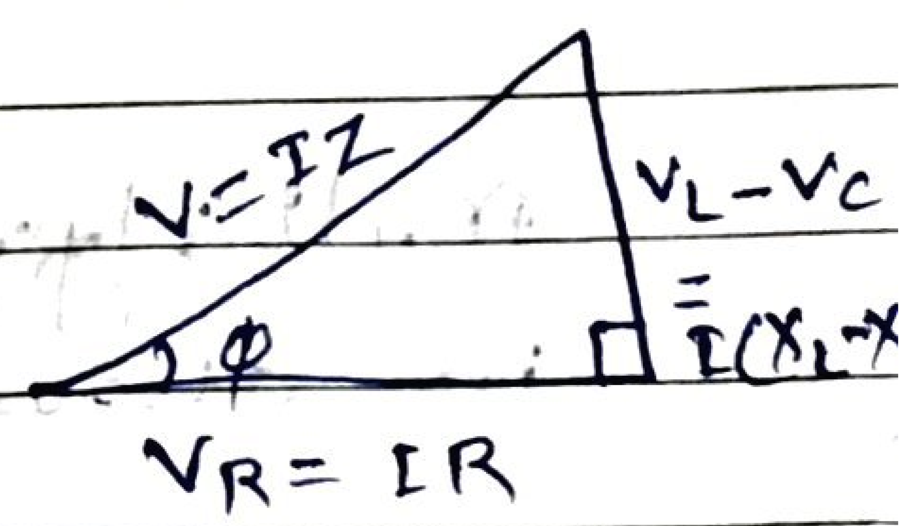
If $X_L > X_C$ then $V_L > V_C$, capacitive effect gets neutralized and the circuit behaves like R-L ckt.

Phasor diagram :-

Taking current as the reference phasor, the phasor dia. of the ckt can be drawn as :




Phasor Diagram




Voltage triangle

From the ckt dia.

$\bar{V} = \bar{V_R} + \bar{V_L} + \bar{V_C}$

- From phasor dia. it is clear that the current I lags behind the applied voltage V by $\phi^\circ$ ($\phi^\circ < 90$).
- So, the nature of the circuit is inductive.

- Phase angle of the circuit

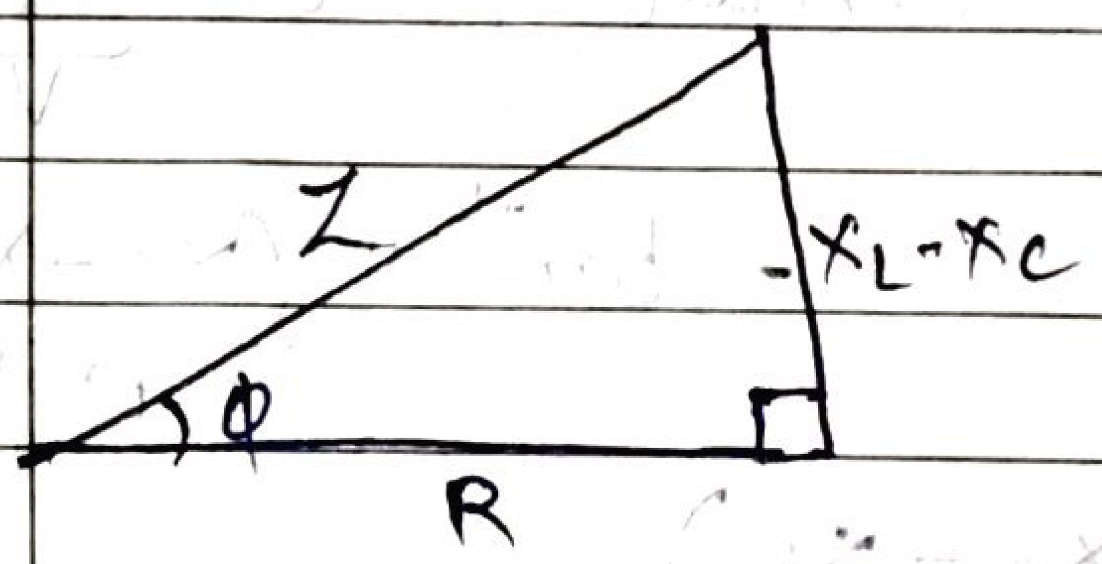
$$\phi = \tan^{-1}\frac{V_L - V_C}{V_R} = \tan^{-1}\frac{I(X_L - X_C)}{IR}$$

$$= \tan^{-1}\left[\frac{(X_L - X_C)}{R}\right]$$

- So power factor of the circuit

$$pf = \cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \text{ lagging}$$

## Impedance (Z)




Impedance Triangle (when $X_L > X_C$)

- Dividing each of voltage phasor by I in voltage triangle we get impedance triangle as shown.

From impedance triangle

$$Z = \sqrt{R^2 + (X_L - X_C)^2}\ \Omega$$

Impedance in rectangular form

$$\bar{Z} = R + j(X_L - X_C)\ \Omega$$

Impedance in polar form

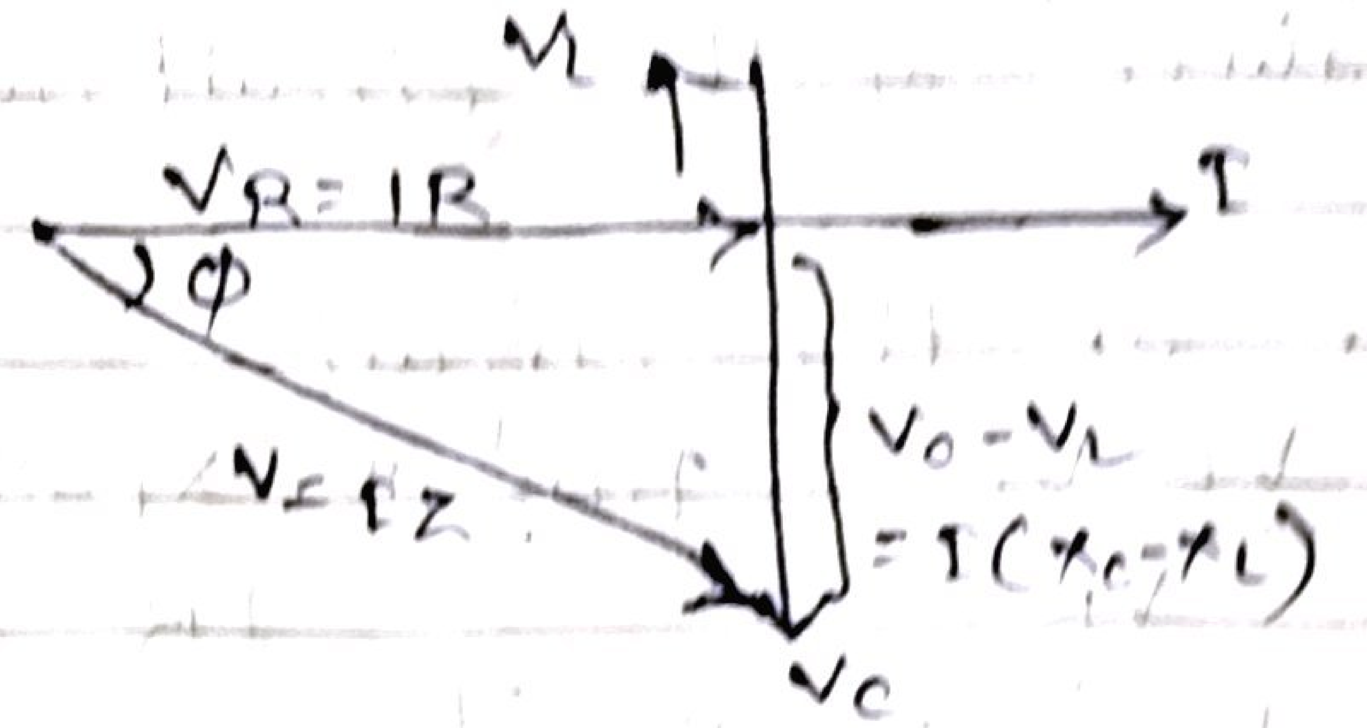
$$\bar{Z} = (Z\angle\phi)\ \Omega$$

Case (2) $X_C > X_L$

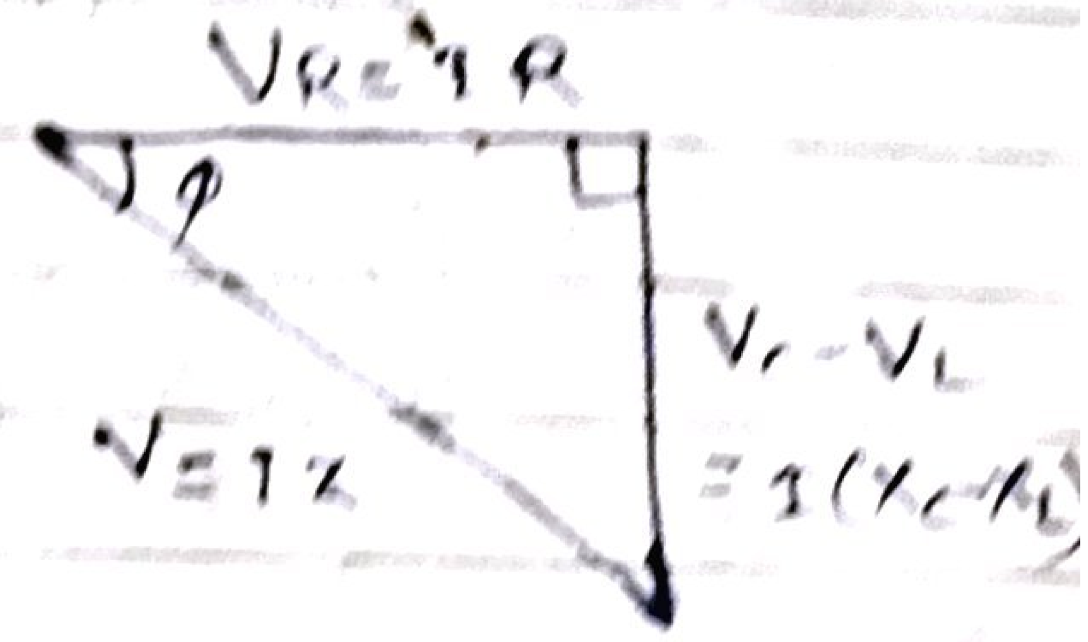
- If $X_C > X_L$, then inductive effect get neutralized and the circuit behaves like a R-C circuit.

Phasor Diagram:-

Taking current as reference phasor, phasor dia. will be




Phasor diagram




Voltage Triangle

- Current I leads the applied voltage V by $\phi^\circ$ ($\phi^\circ < 90^\circ$), so, nature of the ckt is capacitive

- Phase angle of the circuit $\phi = \tan^{-1} \frac{V_C - V_L}{V_R}$

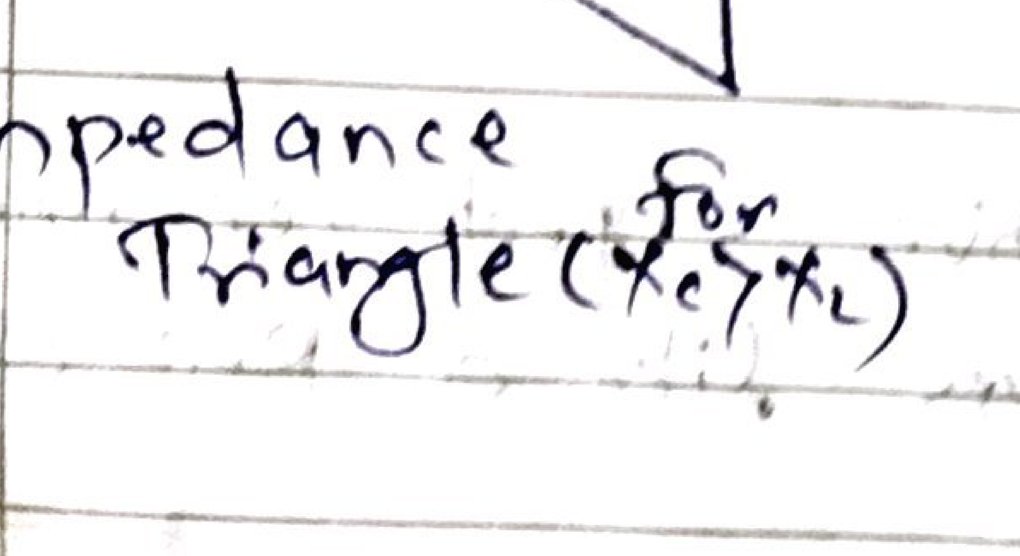
$$= \tan^{-1} \frac{I(X_C - X_L)}{IR}$$

$$= \tan^{-1} \left( \frac{X_C - X_L}{R} \right)$$

- Power factor of the circuit

$$PF = \cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z} \text{ leading}$$

Impedance (Z)




Impedance Triangle (for $X_C > X_L$)

$$Z = \sqrt{R^2 + (X_C - X_L)^2} \ \Omega$$

$$\bar{Z} = R - j(X_C - X_L) \ \Omega$$

or $\bar{Z} = (Z \angle -\phi) \ \Omega$

K. J. SOMAIYA COLLEGE OF ENGINEERING

AC Circuits

(4)

(Autonomous College Affiliated to University of Mumbai)

Candidate Roll No. ____________ (In figures)

Test Exam.

Name: ____________

Date: ____________ 20

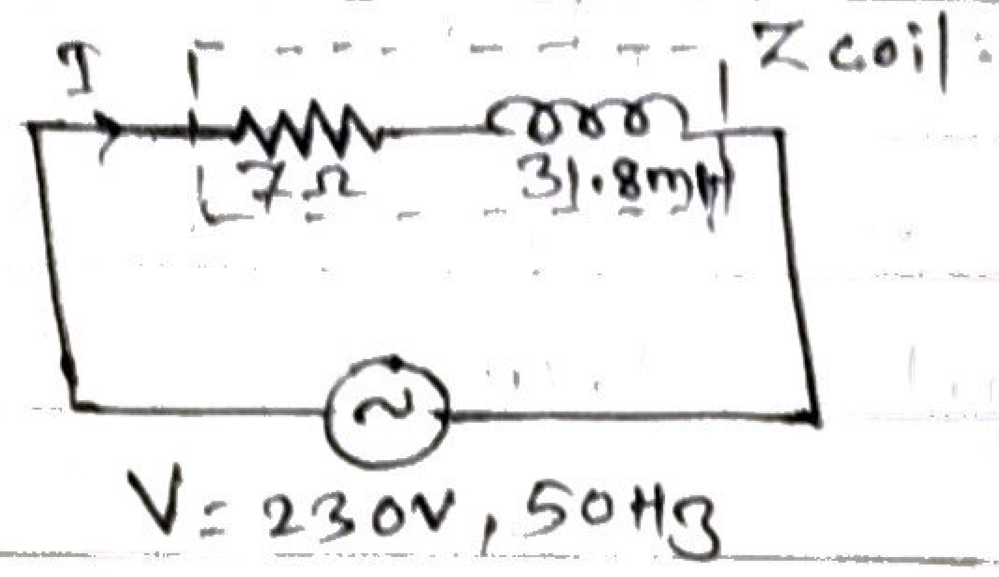
Examination: ____________ Branch/Semester ____________

Subject: ____________

| Junior Supervisor's full Signature with Date |
|---|
| |

| Question No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Marks Obtained | | | | | | | | | | | | | |

# 1




A coil having a resistance of $7\Omega$ and inductance of $31.8$ mH is connected to 230V, 50 Hz supply.
Calculate: (1) Circuit current (2) phase angle (3) power factor (4) power consumed

Sol$^n$.

Inductive reactance $X_L = 2\pi f L$

$= 2\pi \times 50 \times 31.8 \times 10^{-3}$

$= 10\Omega$

Impedance of coil, $Z_{coil} = \sqrt{R^2 + X_L^2}$

$= \sqrt{(7)^2 + (10)^2}$

$Z_{coil} = 12.21\,\Omega$

(1) Using Ohm's law

Circuit current $I = \dfrac{V}{Z_{coil}} = \dfrac{230}{12.21}$

$\boxed{I = 18.84 \text{ Amp}}$

(2) Phase angle $\phi = \tan^{-1}\dfrac{X_L}{R} = \tan^{-1}\dfrac{10}{7}$

$\boxed{\phi = 55^\circ}$

(3) Power Factor, $PF = \cos\phi = \cos 55^\circ$

$= 0.574$ lagging

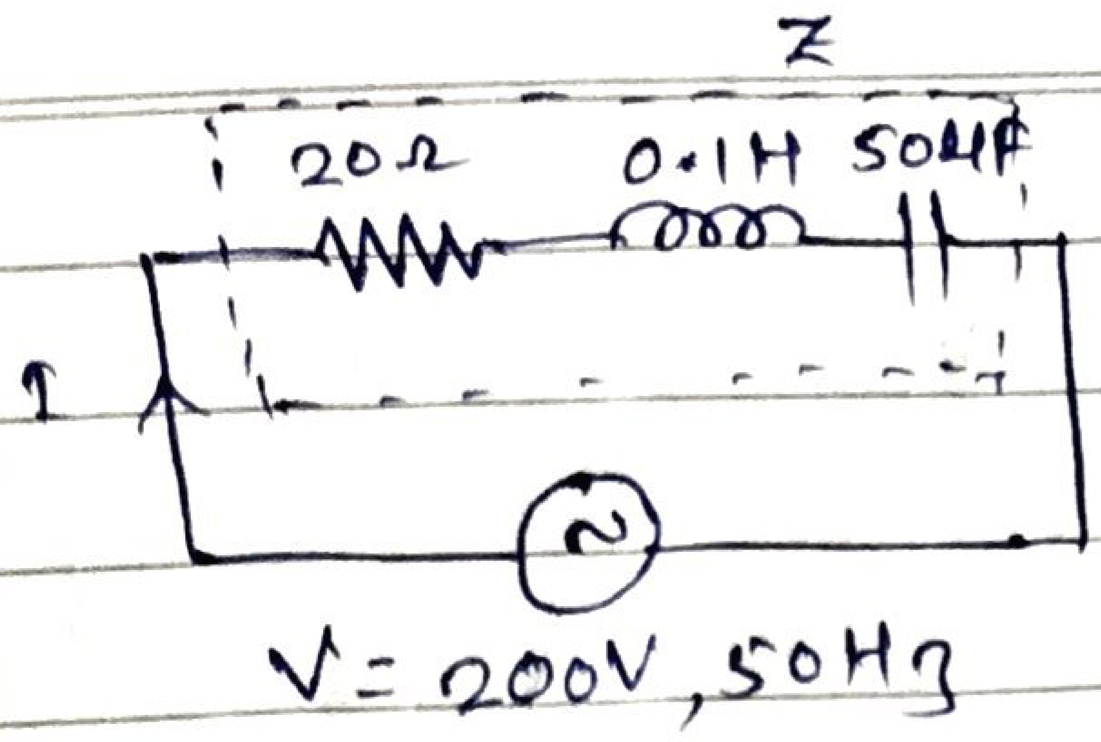
(ckt is inductive)

(4) Power consumed $P = VI\cos\phi$

$= 230 \times 18.84 \times 0.574$

$\boxed{P = 2487.26\,W}$

# 2




For the given circuit determine
1. Circuit impedance
2. Circuit current
3. Power factor
4. Active Power
5. Reactive Power
6. Apparent Power

Soln. Total reactance $= X = X_C - X_L \quad (X_C > X_L)$

$X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.42\,\Omega$

$X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = 63.66\,\Omega$

As $X_C > X_L$ circuit is capacitive

& Total reactance $X = X_C - X_L$

$= (63.66) - (31.42)$

$\boxed{X = 32.24\,\Omega}$

Applied volg $\bar{V} = 200\angle 0$ V

$\bar{Z} = R - jX$

$= (20 - j32.24)\,\Omega$

or $\bar{Z} = Z\angle\phi \qquad \phi = \tan^{-1}\left(\dfrac{X_C - X_L}{R}\right)$

(1) $= 37.94\angle -58.19)\,\Omega \qquad = -58.19^\circ$

$Z = \sqrt{R^2 + (X_C - X_L)^2} \qquad \therefore Z = 37.94\,\Omega$

(2) Circuit current $\bar{I} = \dfrac{\bar{V}}{\bar{Z}} = \dfrac{200\angle 0}{(37.94\angle -58.19)}$

$= 5.27\angle 58.19$ Amp

(3) Power factor $PF = \cos\phi$

$= \cos(-58.19^\circ)$

$= 0.527$ Leading

As $X_C > X_L$ ckt is capacitive $\therefore$ leading PF.

(4) Active Power $= P = VI\cos\phi$

$= 200 \times 5.27 \times \cos(-58.19^\circ)$

$\boxed{P = 555.46\,W}$

(5) Reactive power = $Q = VI \sin\phi$

$= 200 \times 5.27 \times \sin(-58.19^\circ)$

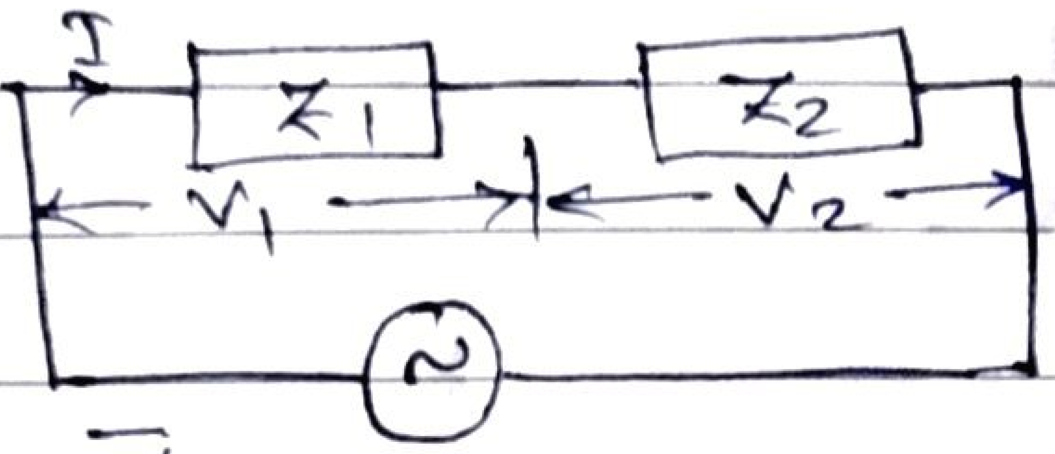
$$\boxed{Q = -895.69 \text{ VAR}}$$

(6) Apparent Power = $S = VI$

$= 200 \times 5.27$

$$\boxed{S = 1054 \text{ VA}}$$

\# 3




$\bar{V} = 100\angle 0$

Rms volg. of $100\angle 0$ volts is applied to series combination of $Z_1$ and $Z_2$ where $\bar{Z}_1 = 20\angle 30^\circ\ \Omega$. The effective volg. drop across $Z_1$ is known to be $(40\angle -30^\circ)$ volts.

Find the reactive component of $Z_2$.

Sol$^n$.

$$\bar{Z}_2 = \frac{\bar{V}_2}{\bar{I}}$$

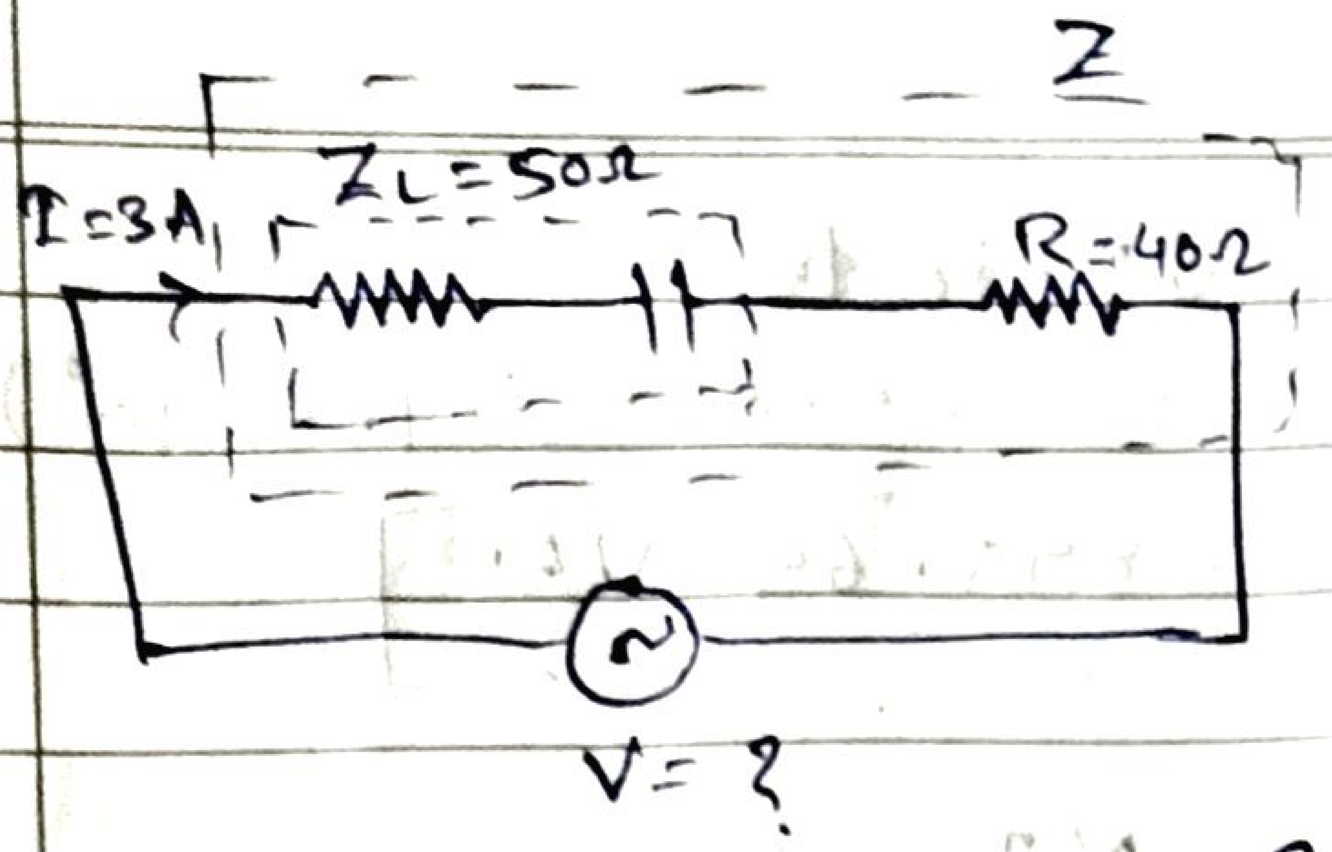
$$\bar{V}_2 = \bar{V} - \bar{V}_1 = (100\angle 0) - (40 - \angle -30)$$
$$= (100 + j0) - (34.64 - j20)$$
$$= (65.36 + j20) \text{ volts}$$
$$= (68.35\angle 17.01) \text{ volts}$$

$$\bar{I} = \frac{\bar{V}_1}{\bar{Z}_1} = \frac{(40\angle -30)}{(20\angle 30)} = (2\angle -60) \text{ Amp}$$

$$\bar{Z}_2 = \frac{\bar{V}_2}{\bar{I}} = \frac{68.35\angle 17.01}{2\angle -60}$$
$$= (34.175\angle 77.01)\ \Omega$$
$$= (7.68 + j33.3)\ \Omega$$

$$\boxed{\text{Reactive component of } Z_2 = 33.3\ \Omega}$$

# 4




A load consisting of a capacitor in series with a resistor has impedance of 50Ω and pf 0.707 leading. The load is connected in series with 40Ω res^n. across the supply and the resulting current is 3A. Determine the supply voltage and the overall phase angle.

Sol^n.

pf = 0.707 Leading

$\therefore \phi = 45^\circ$

$\bar{Z}_L = 50\angle -45\ \Omega$

$\bar{Z}_L = (35.36 - j35.36)\ \Omega$

R of Load = 35.36 Ω

Capacitive reactance of load = 35.36 Ω

Total impedance of the circuit

$\bar{Z} = [(35.36 + 40) - j35.36]\ \Omega$

$= (75.36 - j35.36)\ \Omega$

$\therefore \bar{Z} = (83.24\angle -25.14)\ \Omega$

The applied voltage is given by

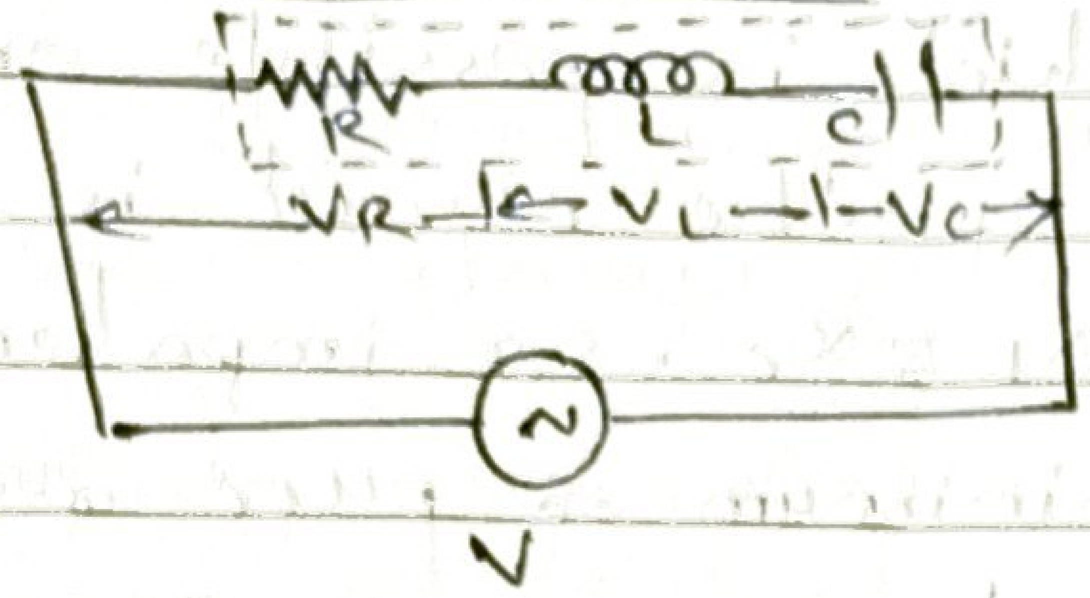
$\bar{V} = IZ = 3 \times 83.24$

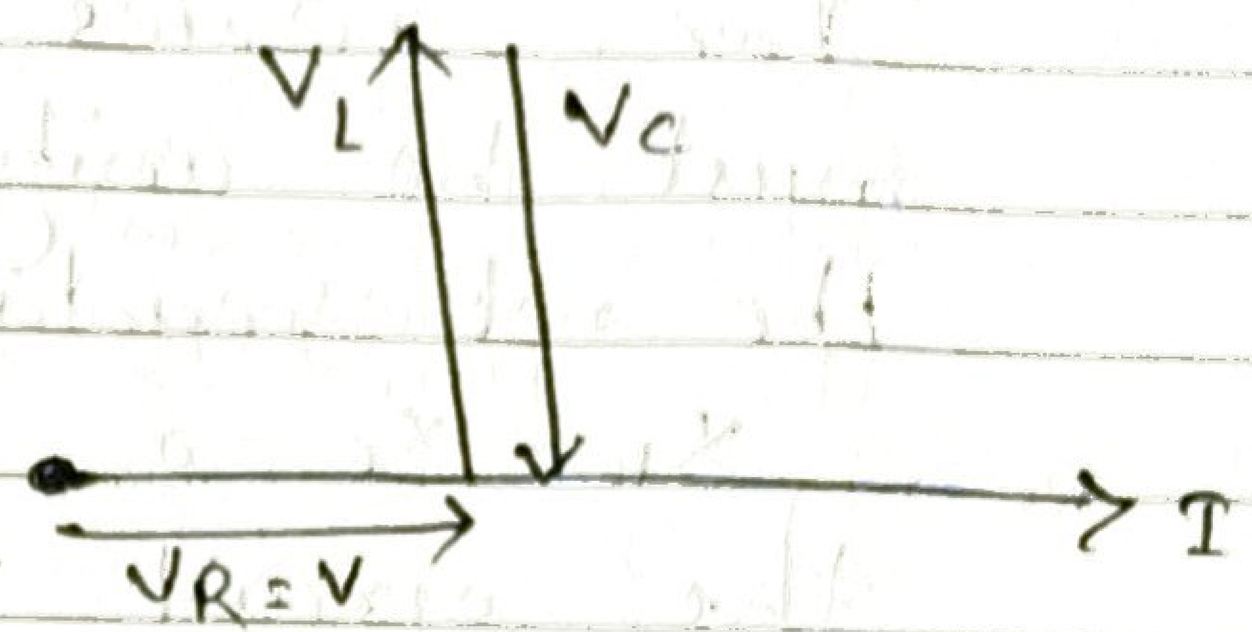
$= 249.72\ V$

overall phase angle = $25.14^\circ$

Resonance in AC circuits is the state of the ckt in which the inductive reactance of the ckt is equal to capacitive reactance.

## Series Resonance :-




$\phi = 0$

pf = unity

Phasor Diagram

- In series RLC ckt
  - When $X_L > X_C$, ckt is predominantly inductive total current lags behind applied volg.
  - When $X_C > X_L$, ckt is predominantly capacitive total current leads applied volg.

Considering series R-L-C ckt as shown in above dia.

Net reactance of the ckt is

$$X = (X_L - X_C) \ \Omega$$

where $X_L = 2\pi f L \ \Omega$

& $X_C = \dfrac{1}{2\pi f C} \ \Omega$

At a certain supply freq. inductive reactance becomes equal to capacitive reactance, this freq. is called as resonant freq. fr.

Net reactance becomes zero. So impedance Z of the circuit behaves purely resistive ($Z = R$)

The whole circuit behaves as a purely resistive circuit and current remains in phase with the applied voltage (pf = 1). This condition is said to be the condition for electrical resonance.

Phasor diagram is shown for resonance condition.

ckt containing reactive elements (L and C) is resonant when ckt power factor is unity. the applied voltage and current are in phase. when this condition occurs in series R-L-C ckt, it is termed as Series resonance.

For series resonance, the circuit power must be unity which is possible only if the net reactance of the circuit is zero.
$X_L - X_C = 0$ or $X_L = X_C$. So impedance of the circuit is minimum & pure resistive.
Current is max. at resonance. $I_r = \frac{V}{Z_r} = \frac{V}{R}$
As current is max. power absorbed by the circuit will also be at its max. value.
volg. drop across L & C are very large. They are greater than the applied volg. But volg. drop across L-C combination will be zero because drops are equal in magnitude but $180^\circ$ out of phase with each other.

Resonant Frequency :- ($f_r$)
- Series resonance at $X_L = X_C$ is achieved by changing supply freq. because $X_L = 2\pi f L$ & $X_C = \frac{1}{2\pi f C}$ are freq. dependent. The freq. at which $X_L = X_C$ in series R-L-C ckt is called the resonant freq. $f_r$.

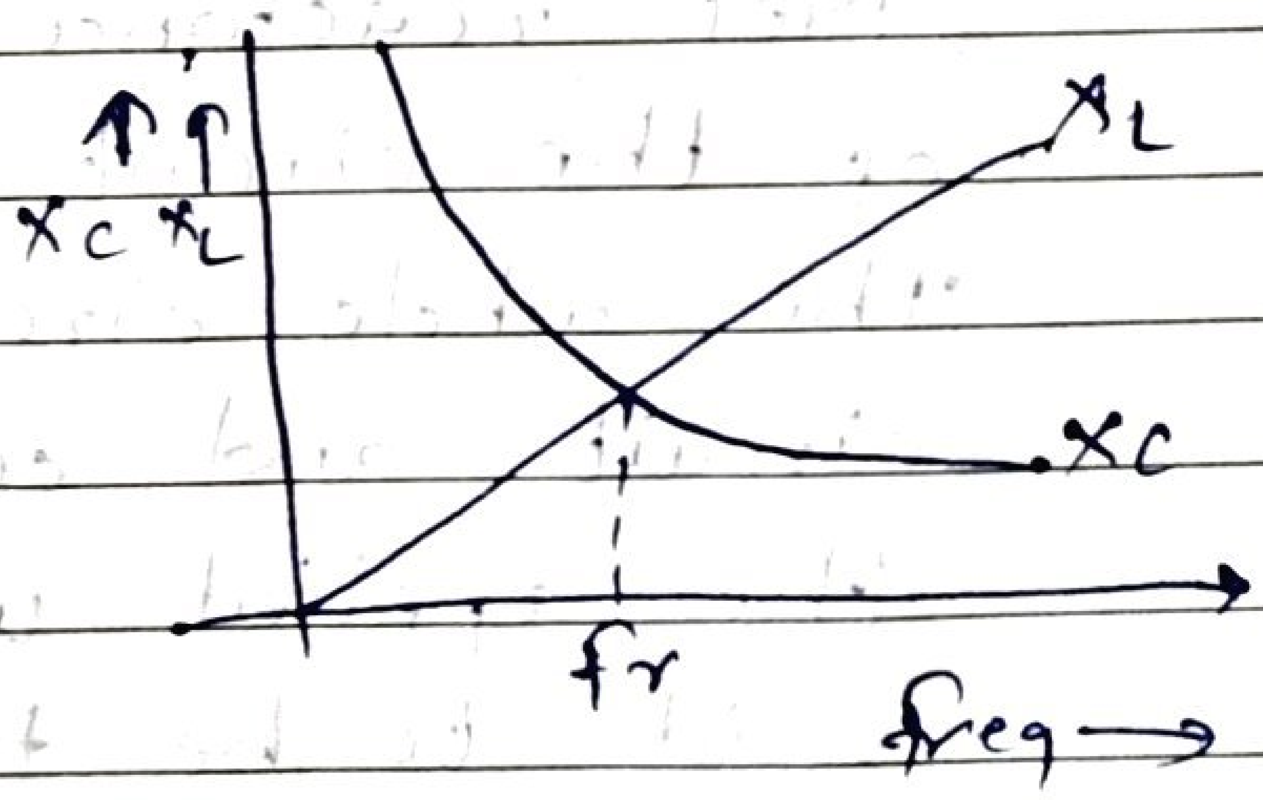
At series Resonance $X_L = X_C$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

$$\therefore \boxed{f_r = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}}$$

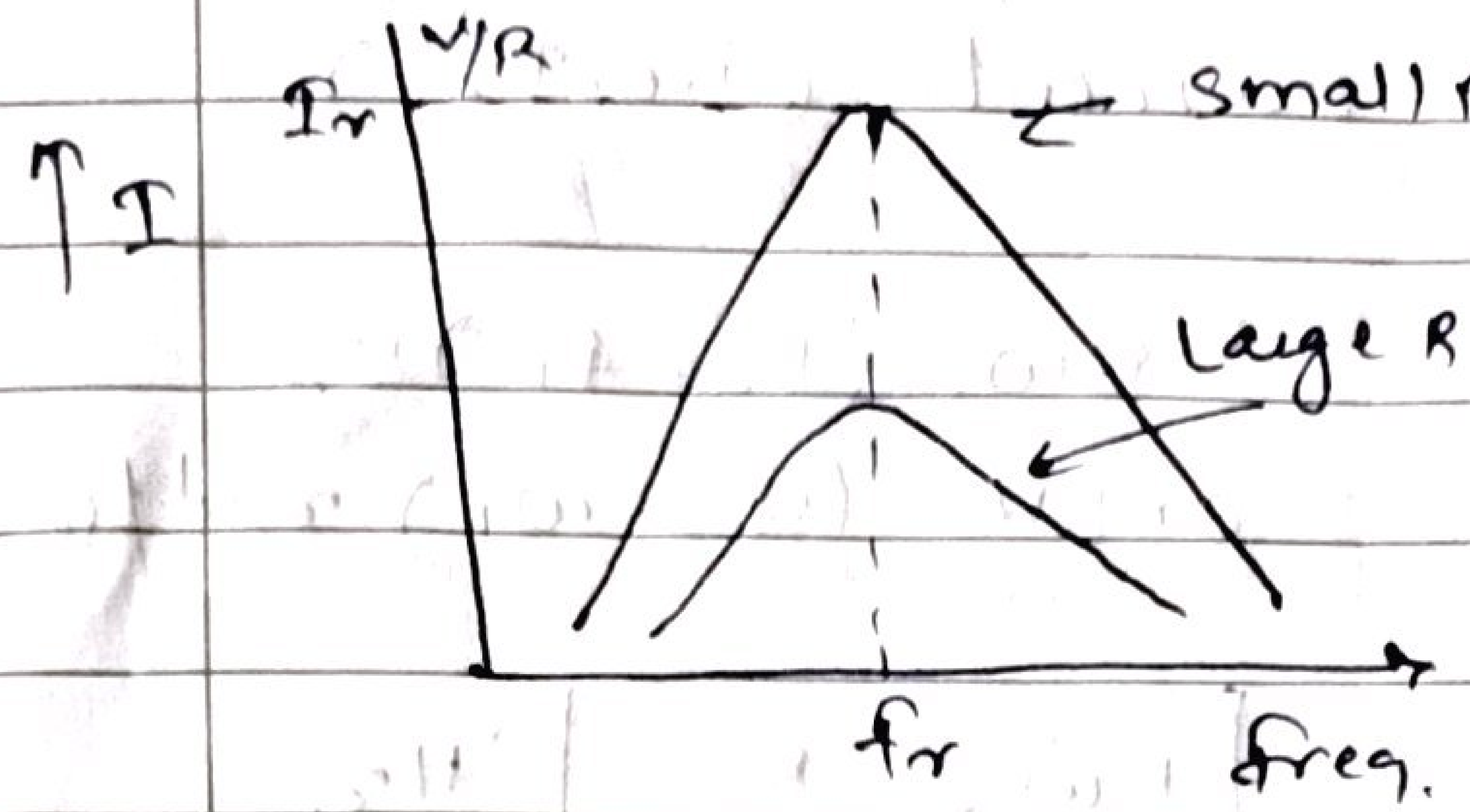
$$\boxed{\omega_r = \frac{1}{\sqrt{LC}} \text{ rad/sec}}$$



Variation of reactances with freq.

## Resonance Curve :-

The curve betⁿ current and freq. is known as resonance curve.




Below fr

$X_C > X_L$ & reactance is not zero.

Above fr

$X_L > X_C$ & net reactance is not zero

So for both cases circuit impedance will be greater than impedance $Z_r = R$ at resonance.

Shape of resonance curve depends upon value of resi.

## Q factor of series resonance ckt :-

$$Q\ \text{factor} = \frac{\text{voltage across L or C}}{\text{Applied voltage}}$$

$$= \frac{V_L \text{ or } V_C}{V}$$

$$= \frac{V_L}{V_R} \quad (\text{At resonance } V = V_R)$$

$$= \frac{I_r X_L}{I_r R}$$

$$= \frac{X_L}{R}$$

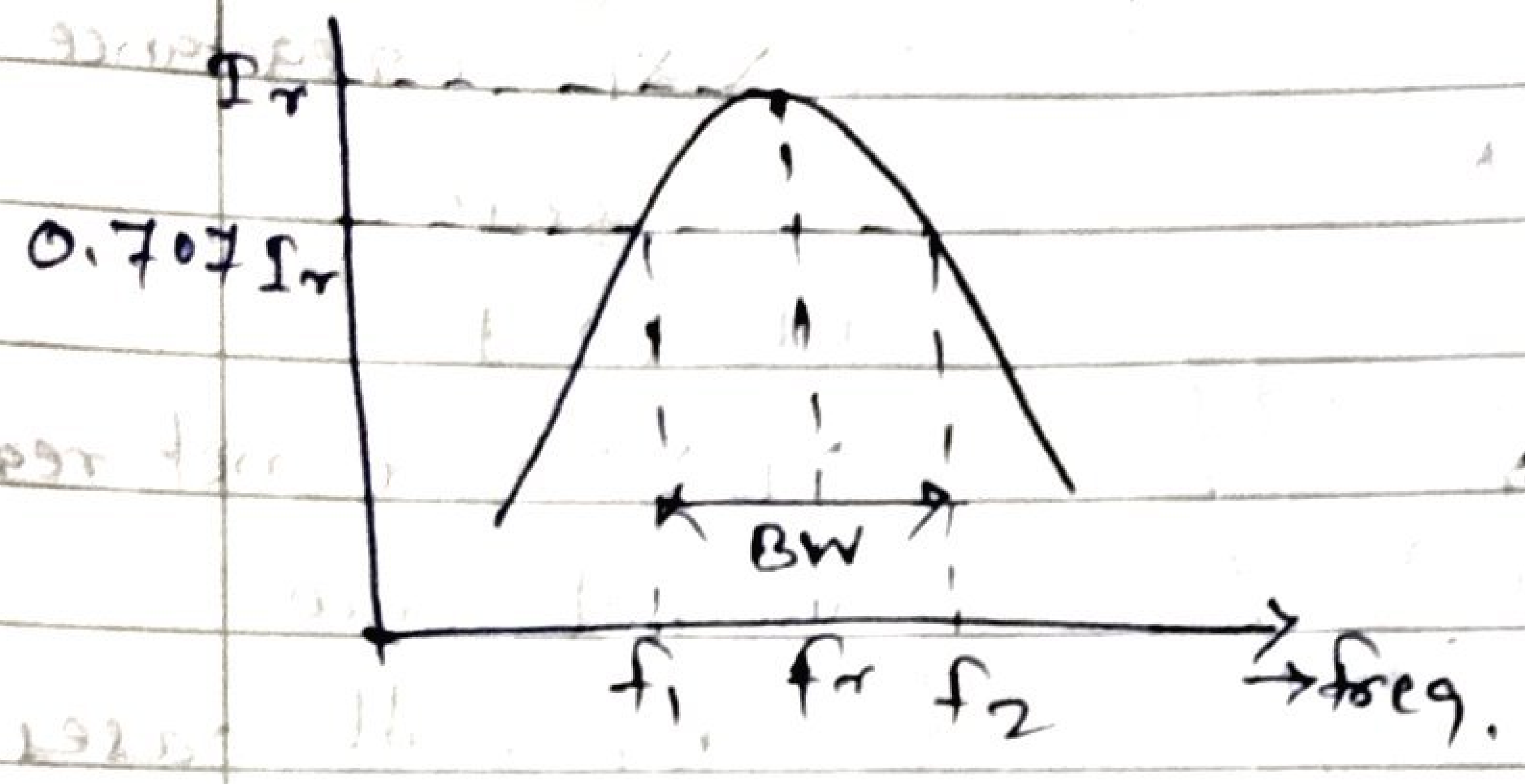
$$= \frac{2\pi f_r L}{R}$$

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore \boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

## Bandwidth of series resonance circuit:-

Bandwidth:- The range of frequency over which circuit current is equal to or greater than 70.7% of max. current. (i.e. $I_r$).




$BW = (f_2 - f_1)$ Hz

or $BW = (\omega_2 - \omega_1)$ rad/sec.

$$\boxed{BW = \frac{R}{2\pi L}} \text{ Hz.}$$

or $\boxed{BW = \frac{R}{L} \text{ rad/sec.}}$

$f_1$ - lower cut-off freq.

$f_2$ - upper cut-off freq.

$$\boxed{\begin{aligned} f_2 &= f_r + \frac{BW}{2} \\ f_1 &= f_r - \frac{BW}{2} \end{aligned}}$$

Relation bet^n Q factor & BW

$$Q \text{ factor} = \frac{2\pi f_r L}{R} \qquad \left(\frac{2\pi L}{R} = \frac{1}{BW}\right)$$

$$\therefore \boxed{Q \text{ factor} = \frac{f_r}{BW}} \quad \text{or} \quad \boxed{f_r = Q \text{ factor} \times BW}$$

Voltage drops at resonance:

$$V_R = I_r R = \frac{V}{R} \times R = V$$

$$V_L = I_r X_L$$

$$= \frac{V}{R} 2\pi f_r L \qquad \left(f_r = \frac{1}{2\pi\sqrt{LC}}\right)$$

$$\therefore V_L = \frac{V}{R} 2\pi \frac{1}{2\pi\sqrt{LC}} L$$

$$\boxed{V_L = \frac{V}{R}\sqrt{\frac{L}{C}}}$$

$$V_C = I_r X_C$$

$$= \frac{V}{R} \cdot \frac{1}{2\pi f_r C} \qquad \left(f_r = \frac{1}{2\pi\sqrt{LC}}\right)$$

$$= \frac{V}{R} \cdot \frac{1}{2\pi \cdot \frac{1}{2\pi\sqrt{LC}} C}$$

$$\therefore \boxed{V_C = \frac{V}{R}\sqrt{\frac{L}{C}}}$$

# K. J. SOMAIYA COLLEGE OF ENGINEERING

(Autonomous College Affiliated to University of Mumbai)

Candidate Roll No. ____________
(In figures)

Name : ____________ **Test Exam.**

Date : ____________ 20

Examination : ____________ Branch/Semester ____________

Subject : ____________

Junior Supervisor's full Signature with Date

| Question No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | Total |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Marks Obtained | | | | | | | | | | | | | |

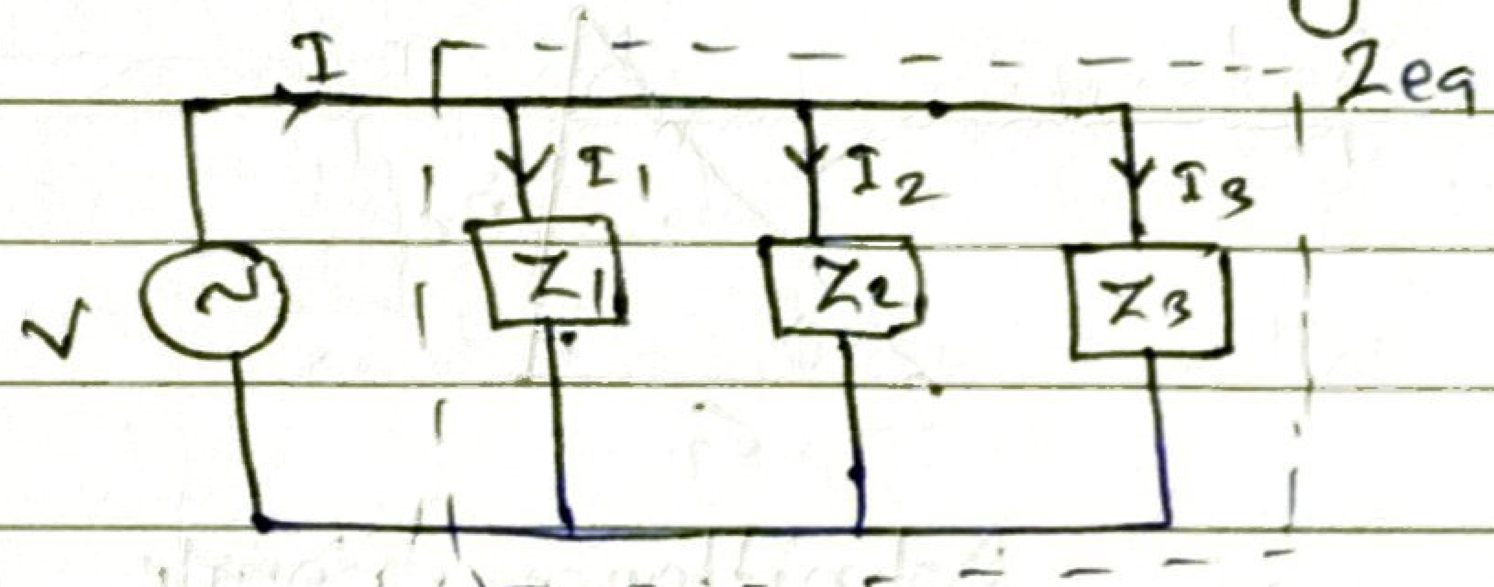
## Admittance (Y)

Admittance = $Y = \frac{1}{Z} = \frac{I}{V}$

or $\bar{Y} = \frac{1}{\bar{Z}} = \frac{\bar{I}}{\bar{V}}$

Unit is mho (℧)

- Circuit with higher value of admittance will have a higher value of current.




Parallel circuit

By law of parallel ckt

$$\frac{1}{\bar{Z}_{eq}} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3}$$

$$\therefore \bar{Y}_{eq} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3$$

Line current $\bar{I} = \frac{\bar{V}}{\bar{Z}_{eq}} = \bar{V}\bar{Y}_{eq}$

In complex form admittance $\bar{Y}$ can be expressed as

$\bar{Y} = (G - jB_L)$ ℧ or $\bar{Y} = (G + jB_C)$ ℧

$G$ = conductance ; in-phase component of $\bar{Y}$

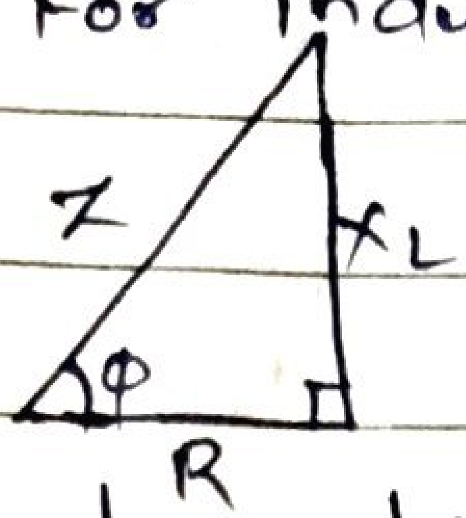
$B$ = Susceptance ; quadrature -11- -11-

$B_L$ = inductive suceptance.

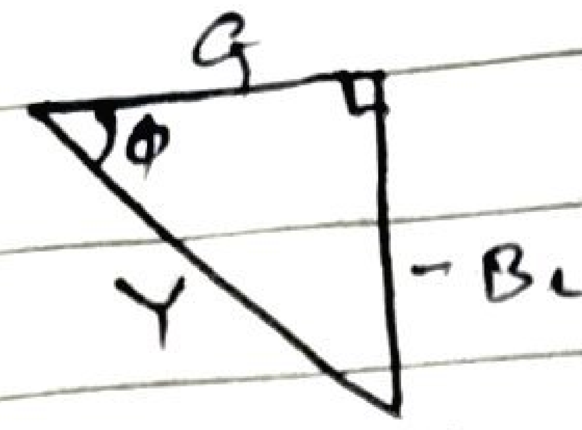
$B_C$ = capacitive -11-

Admittance triangle:

① For inductive ckt




Impedance triangle




Admittance triangle

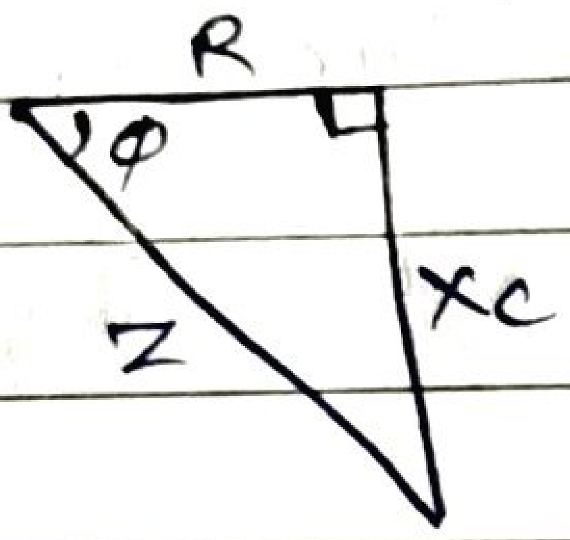
Conductance $G = Y\cos\phi = \frac{1}{Z} \times \frac{R}{Z}$

$$G = \frac{R}{Z^2} = \frac{R}{R^2 + X_L^2} \ \mho$$

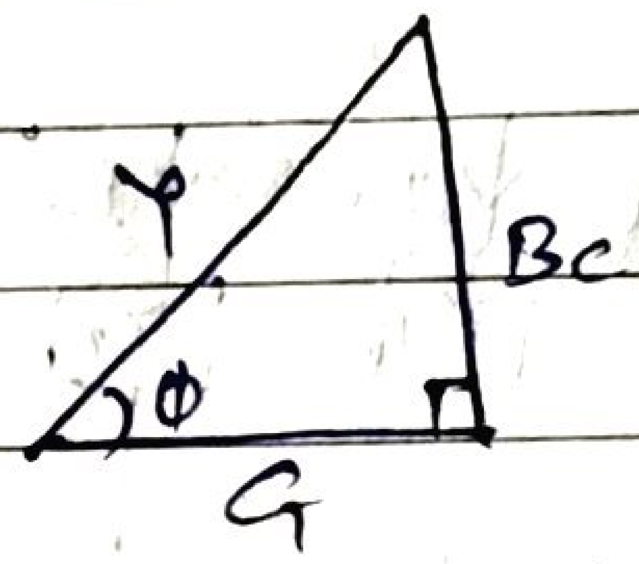
Susceptance $B_L = Y\sin\phi = \frac{1}{Z} \times \frac{X_L}{Z}$

$$B_L = \frac{X_L}{Z^2} = \frac{X_L}{R^2 + X_L^2} \ \mho$$

② For capacitive circuit




Impedance triangle




Admittance triangle

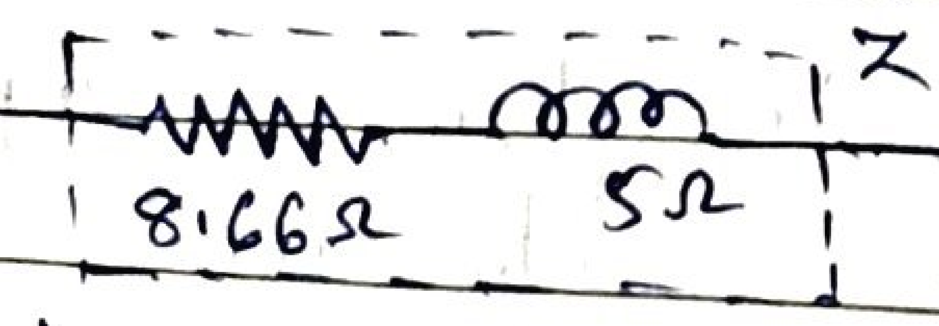
Conductance $G = Y\cos\phi = \frac{1}{Z} \times \frac{R}{Z}$

$$\therefore G = \frac{R}{Z^2} = \frac{R}{R^2 + X_C^2} \ \mho$$

Susceptance $B_C = Y\sin\phi = \frac{1}{Z} \cdot \frac{X_C}{Z}$

$$\therefore B_C = \frac{X_C}{Z^2} = \frac{X_C}{R^2 + X_C^2} \ \mho$$

#1 Calculate the admittance $\bar{Y}$ and draw the admittance triangle of following ckt.




ckt impedance $\bar{Z} = (8.66 + j5)\,\Omega$

$$= 10\angle 30 \ \Omega$$

ckt admittance $\bar{Y} = \frac{1}{\bar{Z}} = \frac{1}{10\angle 30}$

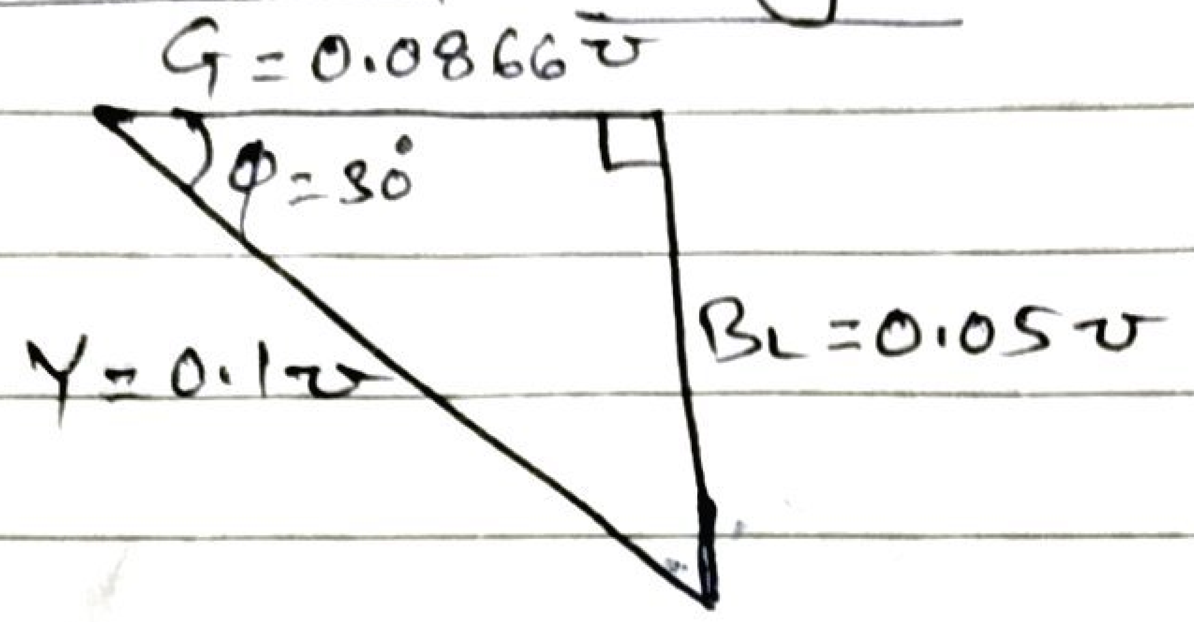
$= (0.1\angle -30)\,℧$

$= (0.0866 - j0.05)\,℧$

$G = 0.0866\,℧$

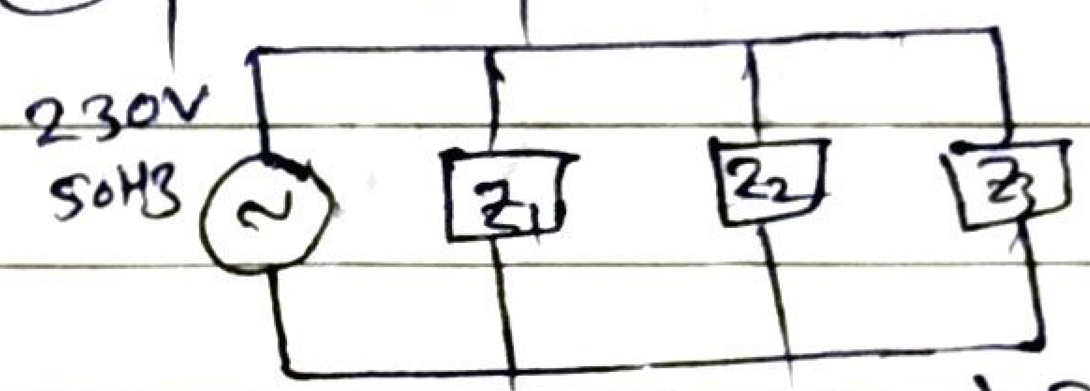
$B_L = 0.05\,℧$

Admittance triangle




#2 For foll. ckt, determine (1) admittance (2) equivalent impedance (3) power consumed (4) power factor

Solution:-




$Z_1 = (10\angle -30)\,\Omega$

$Z_2 = (20\angle 60)\,\Omega$

$Z_3 = (40\angle 0)\,\Omega$

$\bar{Y} = \frac{1}{\bar{Z}_1} = \frac{1}{(10\angle -30)} = (0.1\angle 30)\,℧$

$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{(20\angle 60)} = (0.05\angle -60)\,℧$

$\bar{Y}_3 = \frac{1}{\bar{Z}_3} = \frac{1}{(40\angle 0)} = (0.025\angle 0)\,℧$

(1) Total admittance of the ckt

$\bar{Y} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3$

$= (0.1\angle 30) + (0.05\angle -60) + (0.025\angle 0)$

$= (0.0866 + j0.05) + (0.025 - j0.0433) + (0.025 + j0)$

$= (0.1366 + j0.0067)\,℧$

$= (0.137\angle 2.81)\,℧$

(2) Equivalent impedance of the ckt

$\bar{Z} = \frac{1}{\bar{Y}} = \frac{1}{(0.137\angle 2.81)} = (7.3\angle -2.81)\,\Omega$

(3) Total current $= I = \frac{V}{Z} = \frac{230}{7.3} = 31.51$ Amp

(4) Power factor $= pf = \cos\phi = \cos(-2.81)$

$= 0.998$ leading