Show that fermi-level for intrinsic semiconductor lies half way between valence band and conduction band

For intrinsic semiconductor,

$\Rightarrow$ $N_C\, e^{(-E_C-E_F)/KT} = N_V\, {}^{(-E_F-E_V)/KT}$

$\Rightarrow$ $\dfrac{e^{(-E_C-E_F)}}{e^{(-E_F-E_V)}} = \dfrac{N_V}{N_C}$ — (1)

Now,

$\because$ $N_V = 2\left(\dfrac{2\pi m^*_h KT}{h^2}\right)^{3/2}$

and $N_e = 2\left(\dfrac{2\pi m^*_e KT}{h^2}\right)^{3/2}$

$\therefore$ $\dfrac{N_V}{N_e} = \dfrac{m^*_h}{m^*_e}$ m^*_h — mass of e-holes, m^*_e — mass of e^-

$\because$ $m^*_h \approx m^*_e$

then, $\boxed{\dfrac{N_V}{N_e} = 1}$ — (2)

then, on solving (1) further

$\dfrac{e^{-(E_C-E_F)+(E_F-E_V)}}{KT} = 1$ (from (1) and (2))

on applying log $\rightarrow$

$\dfrac{-E_C+2E_F-E_V}{KT} = \ln 1$

$-E_C+2E_F-E_V = \ln 1\,(KT)$

$\because \ln 1 = 0$

$\therefore$ $-E_C+2E_F-E_V = 0$

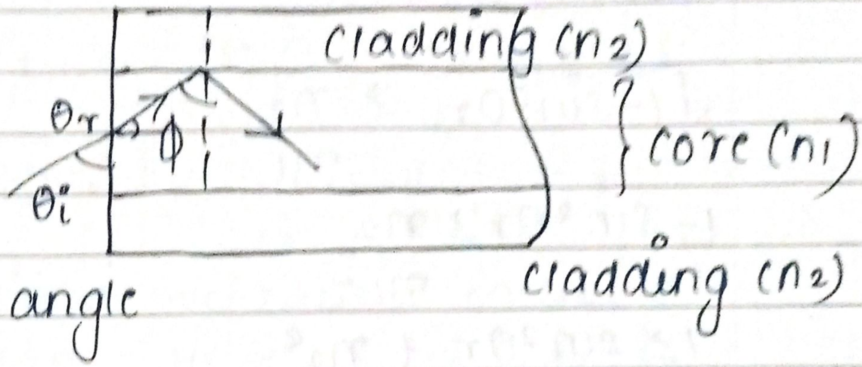
$\therefore$ $\boxed{E_F = \dfrac{E_C+E_V}{2}}$

2. Optical fibre (continuation)

Expression for numerical aperture (NA):-

1) Let θ_m = acceptance angle (i.e max angle of incidence for which TIR takes place)

$NA = \sin\theta_m$ - ①



2) Let ϕ_c = critical angle

$\sin\phi_c = \frac{n_2}{n_1} \rightarrow \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

for TIR $\phi \geq \phi_c$

$\phi \geq \sin^{-1}\left(\frac{n_2}{n_1}\right)$

$\sin\phi \geq \frac{n_2}{n_1}$ — ②

3) from figure, $\theta_r + \phi = 90^\circ$

$\phi = (90 - \theta_r)$

$\sin\phi = \sin(90 - \theta_r)$

$\sin\phi = \cos\theta_r \rightarrow$ ③

from ① and ②

$\cos\theta_r = \frac{n_2}{n_1}$ - ④

4) Snell's law,

$\frac{\sin\theta_i}{\sin\theta_r} = \frac{n_1}{n_0}$ note ($n_1/n_0 \neq n_1/n$ refractive indices

$(\sin\theta_i) n_0 = (n_1) \sin\theta_r$

$\sin\theta_r = \dfrac{(\sin\theta_i)(n_0)}{n_1}$ — (5)

(5) $\because \sin^2\theta + \cos^2\theta - 1$

$\therefore \cos\theta_r = \sqrt{1-\sin^2\theta_r}$ — (6)

from (4) and (6)

$\sqrt{1-\sin^2\theta_r} \geq \dfrac{n_2}{n_1}$

$1-\sin^2\theta_r \geq \dfrac{n_2^2}{n_1^2}$

$1 \geq \sin^2\theta_r + \dfrac{n_2^2}{n_1^2}$

$1 - \dfrac{n_2^2}{n_1^2} \geq \sin^2\theta_r$

$\sin^2\theta_r \leq \dfrac{n_1^2-n_2^2}{n_1^2}$

$\sin\theta_r \leq \sqrt{\dfrac{n_1^2-n_2^2}{n_1^2}}$

$\sin\theta_r \leq \sqrt{\dfrac{n_1^2-n_2^2}{n_1^2}}$ — (7)

from (5) and (7)

$\dfrac{(\sin\theta_i)(n_0)}{\cancel{n_1}} \leq \dfrac{\sqrt{n_1^2-n_2^2}}{\cancel{n_1}}$

$$\sin \theta_1 \leq \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

Let θm = max angle for which TIR takes place

$$\sin \theta m = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

$$NA = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

for air $(n_0) = 1$

$$\boxed{\therefore NA = \sqrt{n_1^2 - n_2^2}}$$

Note:- (1) fractional refractive index (Δ)

$$\Delta = \frac{\Delta_1 - \Delta_2}{\Delta_1}$$

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$= \sqrt{(n_1 + n_2)\frac{(n_1 - n_2) n_1}{n_1}}$$

(1) $n_1 \approx n_2$

$n_1 + n_2 \approx 2n_1$

$$NA = \sqrt{(2n_1)(\Delta) n_1}$$

$$\boxed{NA = (n_1)(\sqrt{2\Delta})}$$

(2) $\phi c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

For conductor/conducting material with drift velocity (V_d) and mobility (μ)

the average velocity when potential is applied,

$V_d \propto E$

$V_d = \mu E$

$\hookrightarrow$ mobility

(unit of mobility

$$\frac{m/s}{V/m} = \frac{\cancel{m}/s}{V/\cancel{m}} \boxed{\frac{m^2}{Vs}}$$

$J = I/A$

$\hookrightarrow$ current density

Let $n \to$ number of electrons / unit volume

$\left(\frac{A}{m^2}\right)$ unit

$$I = \frac{Q}{t} = \frac{Ne}{t}$$

$$I = \frac{n(A)l(e)}{(t)}$$

$\because \frac{l}{t} = V_d \quad \therefore \boxed{I = neAV_d}$

Ohm's law:-

$$\therefore \sigma = \frac{l}{RA}$$

$$R\sigma = \frac{V}{I}$$

$$\sigma = \left.\frac{l}{V}\frac{(I)}{(A)}\right\} \quad \because \frac{I}{A} = J \text{ and } \frac{V}{l} = E$$

$$\therefore \sigma = \frac{1}{E} \cdot J$$

$\rightarrow \boxed{J = \sigma E}$

Page No.
Date / /

Relation b/w conductivity (σ) and mobility (μ)

$\because J = \sigma E$

$J = n V_d \cancel{E} e \qquad \because V_d = \frac{n}{\sigma}$

$\sigma \cancel{E} = (n)\left(\frac{V_d}{E}\right)(e)\cancel{(E)}$

$\sigma = (n)(\mu)(e)$

$\boxed{J = \sigma E = (n)(\mu)}$

Conductivity for semiconductors

$\boxed{\sigma_n = (n)(\mu_e)(e)}$ — n-type

$\boxed{\sigma_p = (p)(\mu_h)(e)}$ $\qquad [e = 1.6 \times 10^{-19}\ C]$
(charge of an e^-)

→ for intrinsic semiconductor

$\sigma = \sigma_n + \sigma_p$

$= n\mu_e e + p\mu_h e$

$\boxed{\sigma = e(n\mu_e + p\mu_h)}$

$\frac{\sigma}{e} = n\mu_e + p\mu_h$

Current in semiconductor

$(J_{drift})_{electron} = \cancel{n(e)(\mu_e)}\ q\left(n\mu_e E + D_e \frac{dn}{dx}\right)$

$(J_{drift})_{hole} = \cancel{p(e)(\mu_h)}\ q\left(p\mu_h E + D_h \frac{dp}{dx}\right)$

Page No.
Date / /

$$J_{total} = \left[(n)(e)(\mu_e)(E) + P(e)(\mu_n) E + eD_e \frac{dn}{dx} - p\, D_h \frac{dP}{dx} \right]$$

Fermi Dirac formula

① $F(E) = \dfrac{1}{1 + \exp\left(\dfrac{E - E_f}{KT}\right)}$

② probability of not finding an e^- :- $\boxed{P = 1 - f(E)}$

③ % probability → $\boxed{P \times 100 = P\%}$

Expression for induced or polarized surface charge density (σ_p) :-

$E = E_0 - E_i$

$$\frac{\sigma}{\epsilon_0 \epsilon_r} = \frac{\sigma}{\epsilon_0 \epsilon_r} - \frac{\sigma_p}{\epsilon_0 \epsilon_r}$$

$$\frac{\sigma}{\epsilon_0 \epsilon_r} = \frac{\sigma - \sigma_p}{\epsilon_0 \epsilon_r}$$

$$\boxed{\sigma = \sigma - \sigma_p}$$

→ Electric dipole moment :-

$\mu = (q)(x)$

Polarizability / Susceptibility

$P \propto \epsilon_0 E$

$P = \chi \epsilon_0 E$

P — Polarization

Polarizability (α)

$\mu \propto E$

$\mu = \alpha E$

and :- $P = N\mu$

$\boxed{P = N\alpha E}$ ⟶ relation b/w polarization and polarizability

DOMS Page No.
Date / /

(Einstein's Coefficients)

1) for Absorption:- $\boxed{N_{ab} = (B_{12})(N_1)(Q)(\Delta t)}$

2) for stimulated emission :- $\boxed{N_{st} = (B_{21})(N_2)(Q)(\Delta t)}$

3) for Spontaneous Emission :- $\boxed{N_{sp} = (A_{21})(N_2)(\Delta t)}$

4) for equilibrium :- $\boxed{N_{ab} = N_{sp} + N_{st}}$

Derivation :-

$$N_{ab} = N_{sp} + N_{st}$$

$$(B_{12})(N_1)(Q)(\Delta t) = (A_{21})(N_2)(\Delta t) + (B_{21})(N_2)(Q)(\Delta t)$$

$$(B_{12})(N_1)(Q) = (A_{21})(N_2) + (B_{21})(N_2)(Q)$$

$$Q\left[(B_{12})(N_1) - (B_{21})(N_2)\right] = (A_{21})(N_2)$$

$$Q = \frac{(A_{21})(N_2)}{(B_{12})(N_1) - (B_{21})(N_2)}$$

$$Q = \frac{\dfrac{(A_{21})(N_2)}{(B_{21})(N_2)}}{\left[\dfrac{(B_{12})(N_1)}{(B_{21})(N_2)} - 1\right]}$$

$$Q = \frac{\dfrac{(A_{21})}{(B_{21})}}{\left[\dfrac{(B_{12})}{(B_{21})}\left(\dfrac{N_1}{N_2}\right)\right] - 1} \quad \text{—— (1)}$$

Date / /

$$\left(\frac{N_1}{N_2}\right) = \frac{e^{-E_1/KT}}{e^{-E_2/KT}} = e^{(E_2-E_1)/KT} \quad \text{(2)}$$

$\therefore (E_2 - E_1) = h\nu$

which also equals,

$$\left(\frac{N_1}{N_2}\right) = e^{(h\nu)/KT}$$

from (1) and (2)

$$Q = \frac{(A_{21})}{(B_{21})} \left[\left(\frac{B_{12}}{B_{21}}\right)\left(e^{h\nu/KT}\right)\right] - 1 \quad \text{(3)}$$

$$Q = \frac{(A_{21})}{(B_{21})} \left[\frac{1}{\frac{(B_{12})}{(B_{21})}\left[\left(e^{h\nu/KT}\right)\right] - 1}\right] \quad \text{(4)}$$

from (3) and (4)

$$\frac{(A_{21})}{(B_{21})} = \frac{(8\pi h\nu^3)}{(c^3)} \qquad \left[\frac{(B_{12})}{(B_{21})} = 1\right] \quad (B_{12} = B_{21})$$

$$\boxed{\frac{8\pi h}{\lambda} = (A_{21})(B_{12})}$$

Primary Relations in terms of unity

1) $\frac{N_{st}}{N_{ab}} = \left(\frac{N_2}{N_1}\right)\left(\frac{B_{21}}{B_{12}}\right)$

less than 1 — 1

$$\frac{N_{st}}{N_{ab}} < 1$$

$$\boxed{N_{st} < N_{ab}}$$

2) $$\frac{N_{sp}}{N_{st}} = \left[\frac{(A_{21})}{(B_{21})} \frac{1}{Q} \right]$$

$$= \frac{(A_{21})\,(N_2)(\Delta t)}{(B_{21})\,(N_2)\,(Q)\,(\Delta t)}$$

$$\left[\frac{N_{sp}}{N_{st}} = \frac{(A_{21})}{(B_{21})} \frac{(1)}{(Q)} \right]$$

i.e $\cancel{N_{st} <}$ $[N_{st} > N_{sp}]$

for $N_{sp} > 1$

$N_{st} = 1$

Date / /

- Internodal dispersion

for SI (step index) $\tau_i = \frac{(n_1)(L)(\Delta)}{c}$

for GI (graded index) $\tau_i = \frac{(n_2)(L)(\Delta)^2}{2c}$

- Total dispersion

$$\tau_{total} = \sqrt{\tau_i^2 + \tau_m^2}$$

(τ_m: material dispersion)

- $$\boxed{\text{Bitrate} = \frac{0.7}{\tau_{total}}}$$