TM

Turing Machine

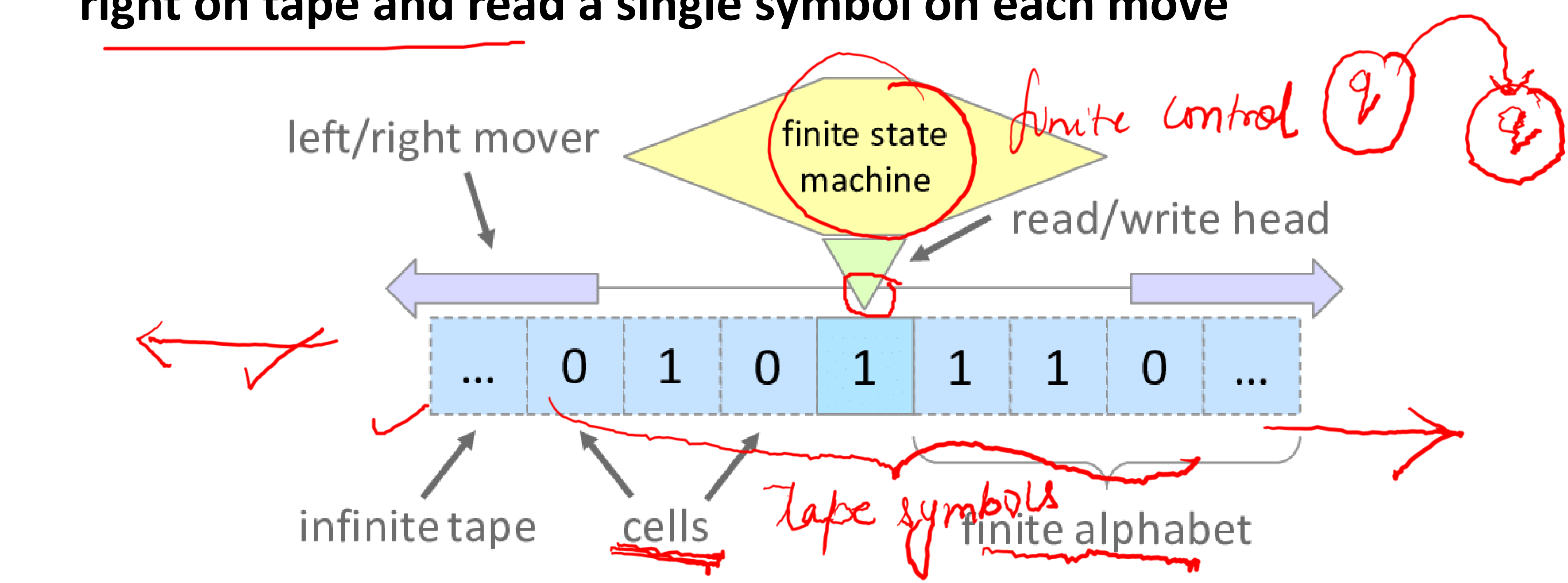
Consists of:-

- A Tape

Tape is infinite extending on either sides, divided into cells, each cell capable of holding one symbol

- Head-

Associated with tape is read-write head that can travel left or right on tape and read a single symbol on each move



Turing Machine

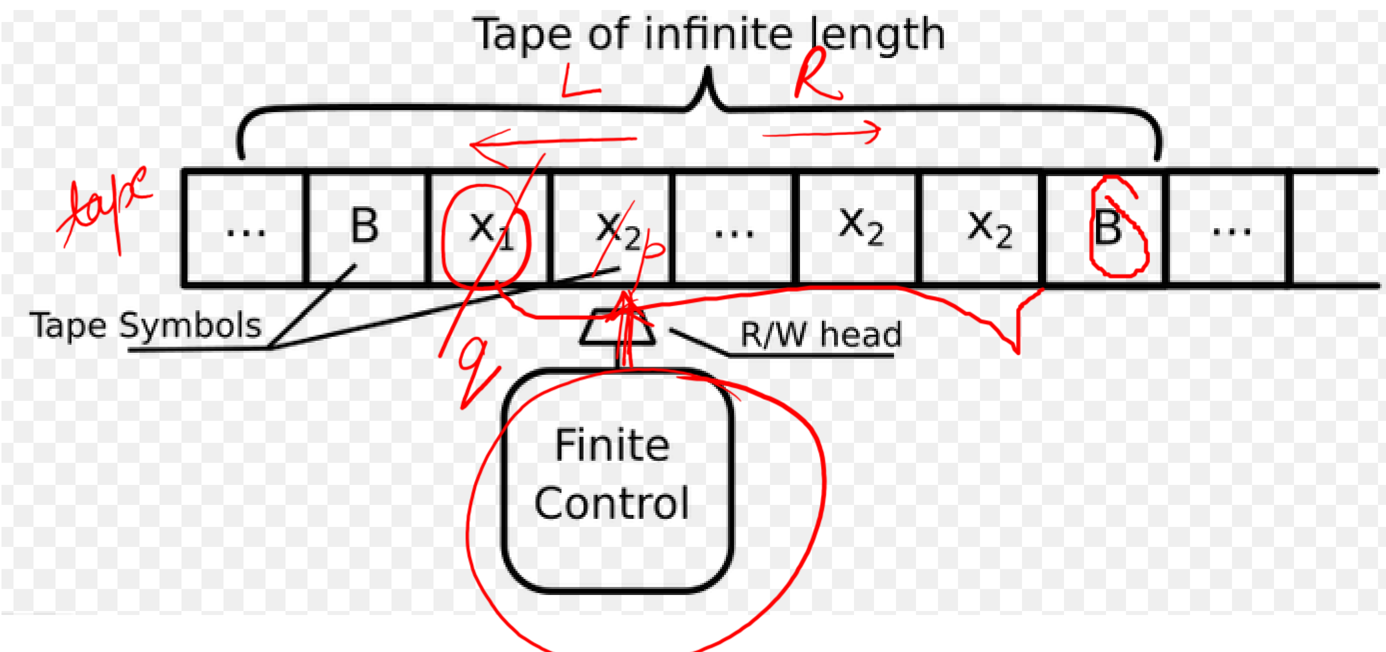
Consists of:-

- **Tape Symbols-**

Finite set of symbols, consists of lowercase letters, digits, usual punctuation marks and the Blank B

- **States-**

Set of states in which the machine can reside



Turing Machine

A Turing machine is a mathematical model of computation that defines an abstract machine[1] that manipulates symbols on a strip of tape according to a table of rules

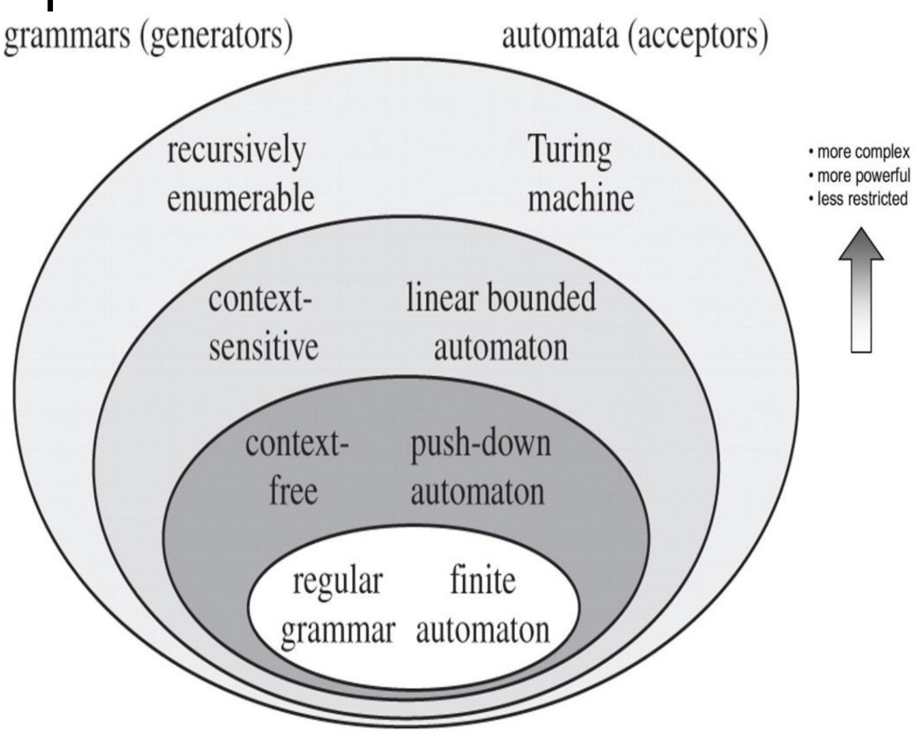
-Wikipedia

Turing Machine

- Turing Machine was invented by Alan Turing in 1936
- Accepts Recursive Enumerable Languages (generated by Type-0 Grammar).

The Hierarchy

Class	Grammars	Languages	Automaton
Type-0	Unrestricted	Recursive Enumerable	Turing Machine
Type-1	Context Sensitive	Context Sensitive	Linear-Bound
Type-2	Context Free	Context Free	Pushdown
Type-3	Regular	Regular	Finite



TM: Formal Definition

PDA is denoted by 7 Tuple: $M=(Q, \Sigma, \Gamma, \delta, q_0, B, F)$ where

PDA $(Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

- Q:Finite set of Internal states of the Control Unit
- Σ:Finite input alphabet
- Γ: Finite Set of Tape Symbols

/ PDA Pushdown Stack Symbols

- $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L,R,N\}$: Transition function(TF)
- q_0: Start state of Control Unit, $q_0 \in Q$
- B : Blank Symbol
- F: Set of Final States/ Accept State, $F \subseteq Q$
- $\Sigma \subseteq \Gamma - \{B\}$

δ Function

δ Function:-

- $\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L,R,N\}$: Transition function(TF)

If $\delta(q,X)=(p,Y,L)$

- If present state is q and the next symbol on the tape is X then the Turing Machine changes the state to p, replaces the symbol X by symbol Y and moves the tape head one position left.

$\delta(q,x)=(p,y,N)$

If $\delta(q,X)=(p,Y,R)$

- If present state is q and the next symbol on the tape is X then the Turing Machine changes the state to p, replaces the symbol X by symbol Y and moves the tape head one position right.

TM Example 1

Transition Rules

Construct a TM to replace each occurrence of 'a' by 'b' for the strings over ∑={a,b}

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$

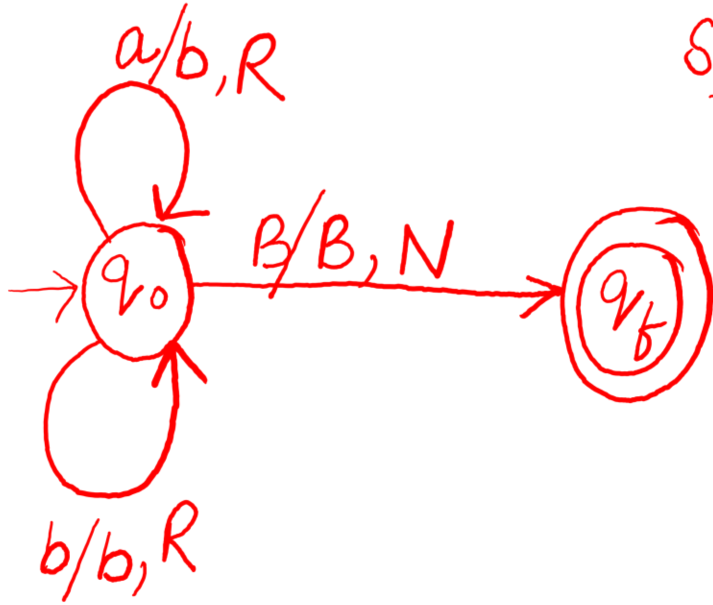
~~δ Rules~~

$\delta(q_0, a) = (q_0, b, R)$ Move Right

$\delta(q_0, b) = (q_0, b, R)$ Move Right

$\delta(q_0, B) = (q_f, B, N)$ No Movement

δ diag



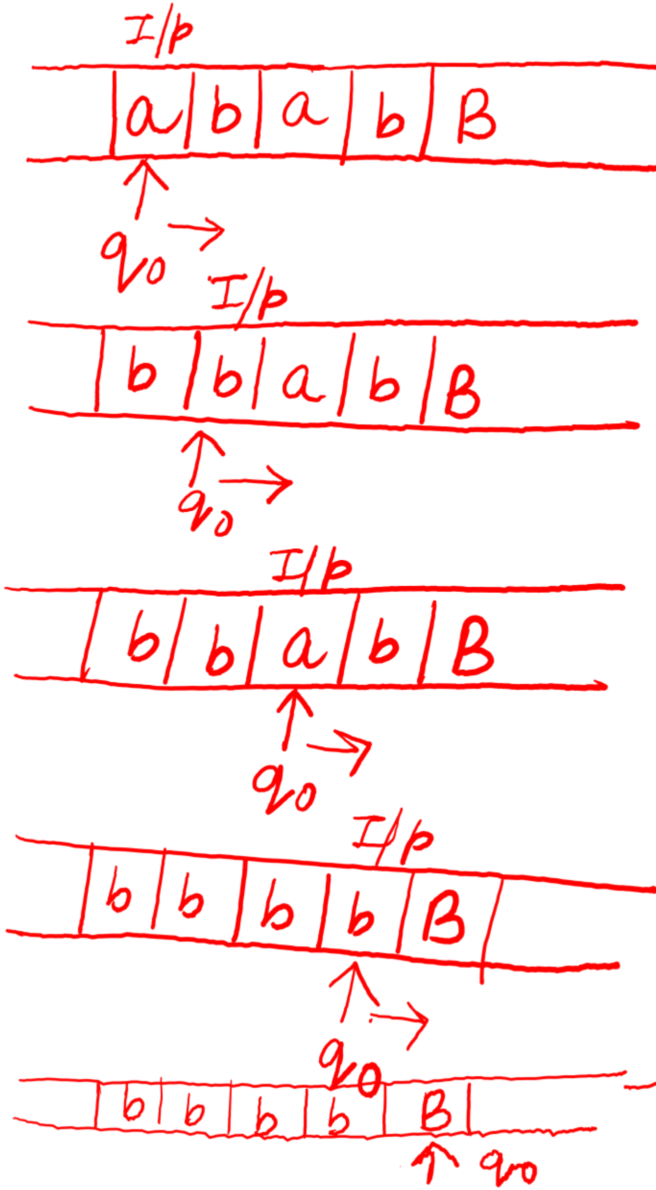
δ table

	Tape Symbols		
States	a	b	B
q_0	(q_0, b, R)	(q_0, b, R)	(q_f, B, N)
q_f	—	—	—

TM Example 1

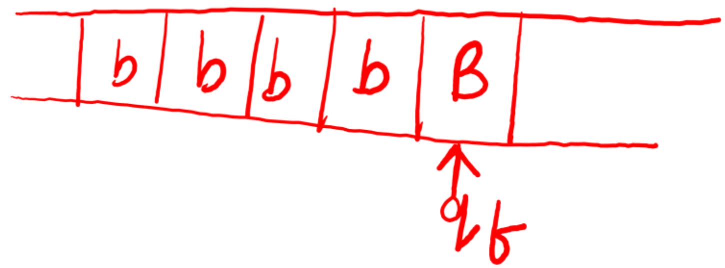
Construct a TM to replace each occurrence of 'a' by 'b' for the strings over ∑={a,b}

String = "abab"



$\delta(q_0, a) = (q_0, b, R)$

$\delta(q_0, b) = (q_0, b, R)$



$\delta(q_0, a) = (q_0, b, R)$

$\delta(q_0, b) = (q_0, b, R)$

$\delta(q_0, B) = (q_f, \underline{B}, N)$

TM Example 2

Design a TM to replace all occurrences of '111' by '101' over ∑={0,1}

$\delta(q_0, 0) = (q_0, 0, R)$

$\delta(q_0, 1) = (q_1, 1, R)$

$\delta(q_0, B) = (q_f, B, N)$

$\delta(q_1, 0) = (q_0, 0, R)$

$\delta(q_1, 1) = (q_2, 1, R)$

$\delta(q_1, B) = (q_f, B, N)$

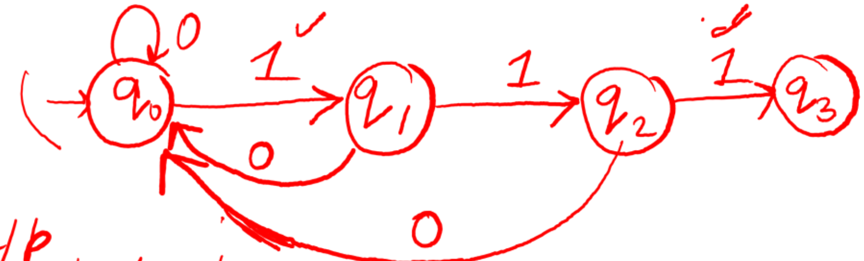
$\delta(q_2, 0) = (q_0, 0, R)$

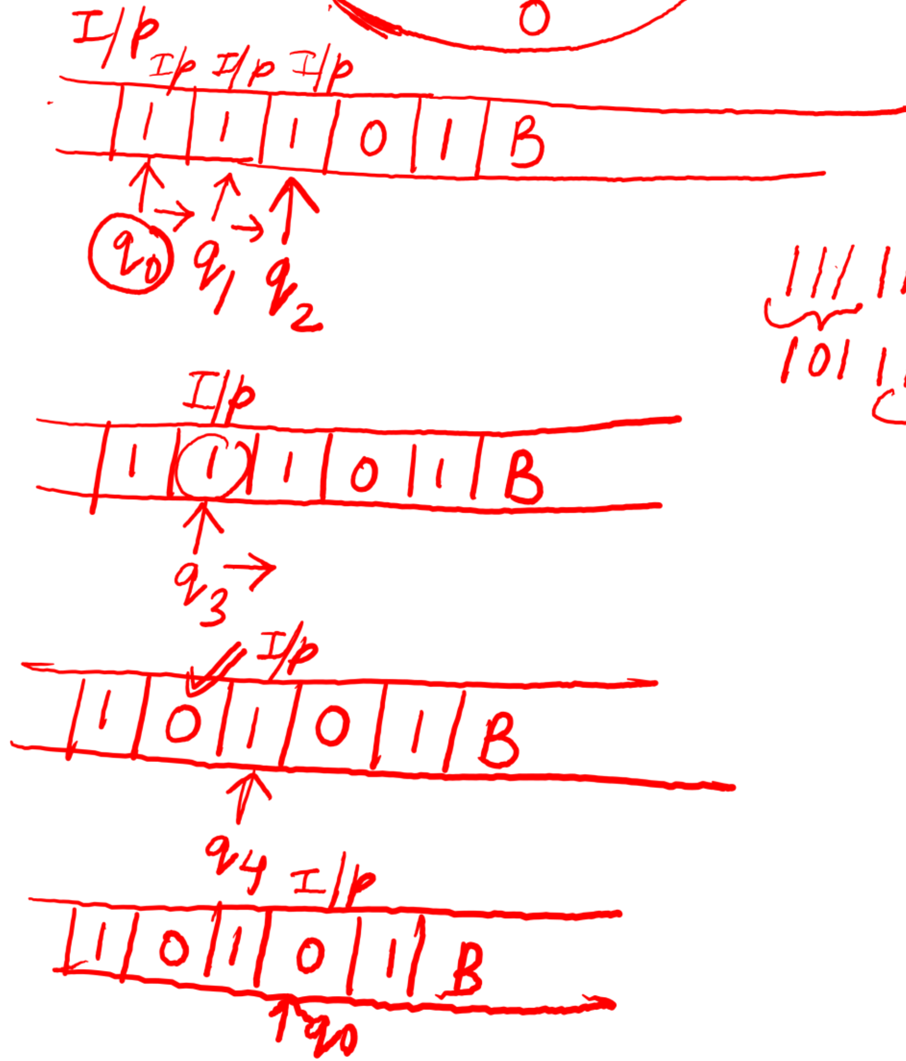
$\delta(q_2, 1) = (q_3, 1, L)$

$\delta(q_2, B) = (q_f, B, N)$

$\delta(q_3, 1) = (q_4, 0, R)$

$\delta(q_4, 1) = (q_0, 1, R)$





111 111

101 111

TM Example 2

Transition Diagram ,Table

Design a TM to replace all occurrences of '111' by '101' over $\Sigma=\{0,1\}$

TM Example 3

Design a TM accept the language containing substring "aba" over $\sum=\{a,b\}$

Logic-

During the process if TM discovers "aba" on the tape, it halts and thereby accepts entire input string

$M=(Q, \sum, \Gamma, \delta, q_0, B, F)$

TM Example 3

Design a TM accept the language containing substring "aba" over $\sum=\{a,b\}$

$\delta(q0,a)=(q1,a,R)$

$\delta(q0,b)=(q0,b,R)$ self loop

$\delta(q1,a)=(q1,a,R)$ self loop

$\delta(q1,b)=(q2,b,R)$

$\delta(q2,b)=(q0,b,R)$ go back

$\delta(q2,a)=(qf,a,N)$

TM Example 4

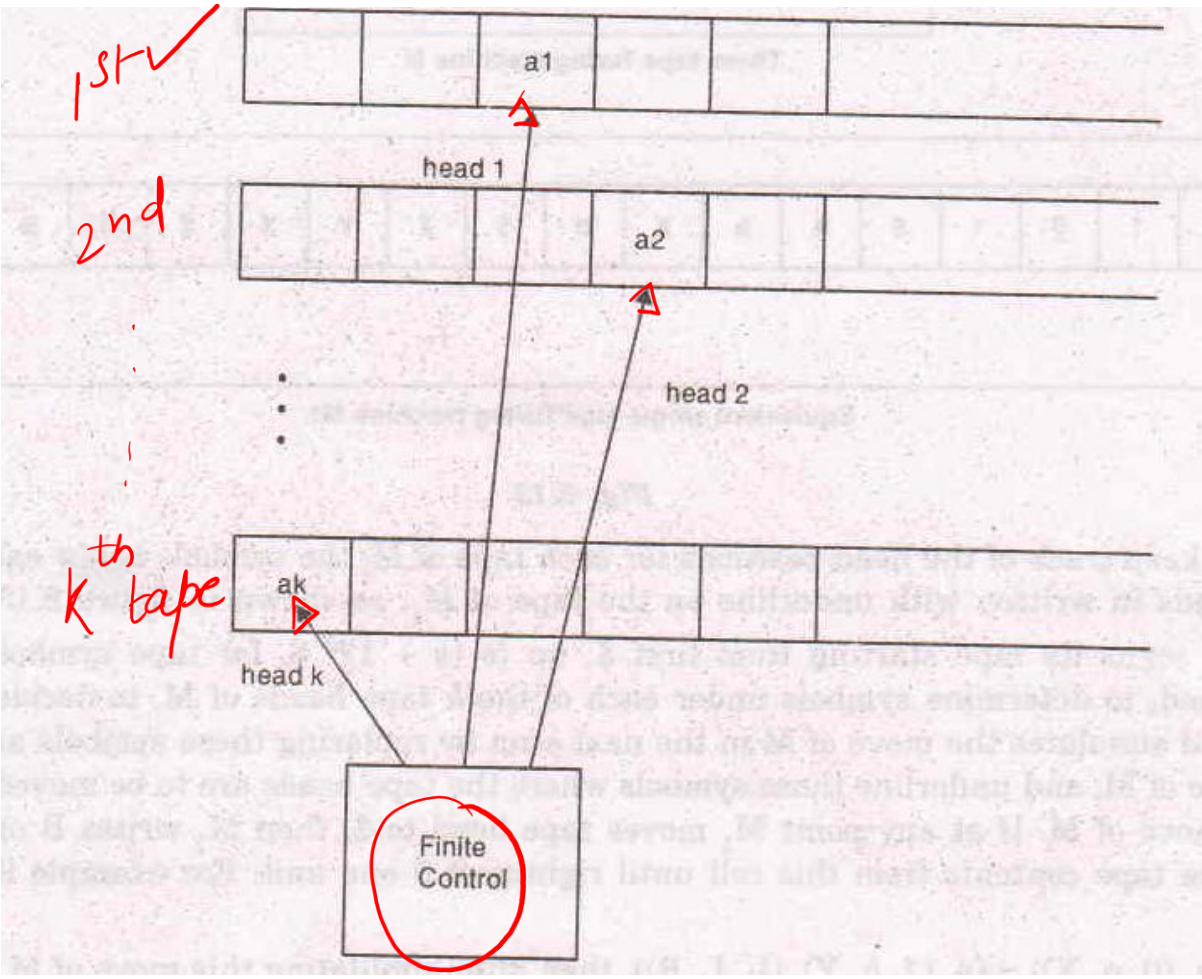
Design a TM to replace all occurrences of '101' by '011' over ∑={0,1}

Modifications of Turing Machines or Variants of Turing Machines

- **Multi Tape Turing Machine**
- **Non-Deterministic Turing Machine**

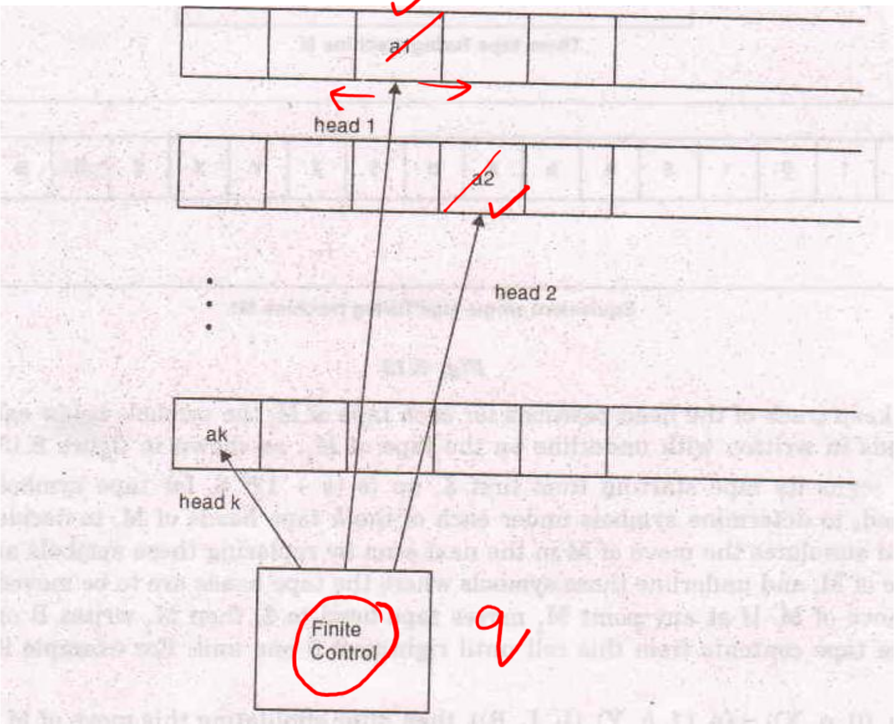
Multi Tape Turing Machine

- **A Multitape Turing Machine-**



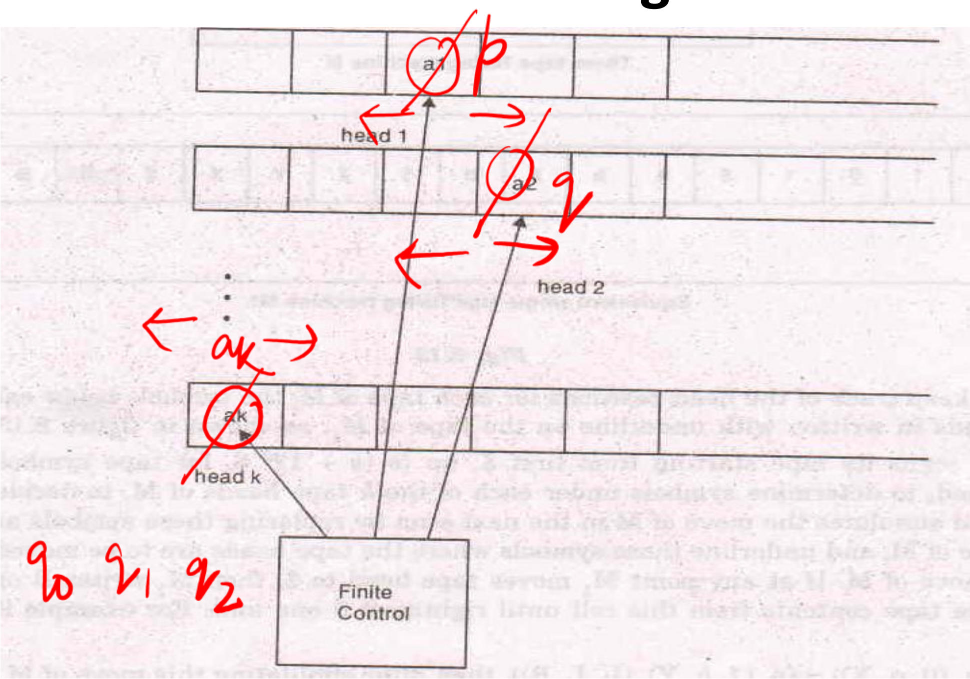
Multi Tape Turing Machine

- It is a Turing Machine with several tapes , each has its own tape heads
- It consists of a finite control with k tape heads and k tapes (k>1)
- The move of this Turing machine depends on the present state of the finite control and the symbol scanned by each of the tape heads.



Multi Tape Turing Machine

- In each move, depending on the present state and the symbols scanned by each of the tape heads , the machine can
  - change state of the finite control
  - print a new symbol on each of cells scanned by each of the tape heads
  - move each tape one cell left or one cell right independently



Multi Tape Turing Machine

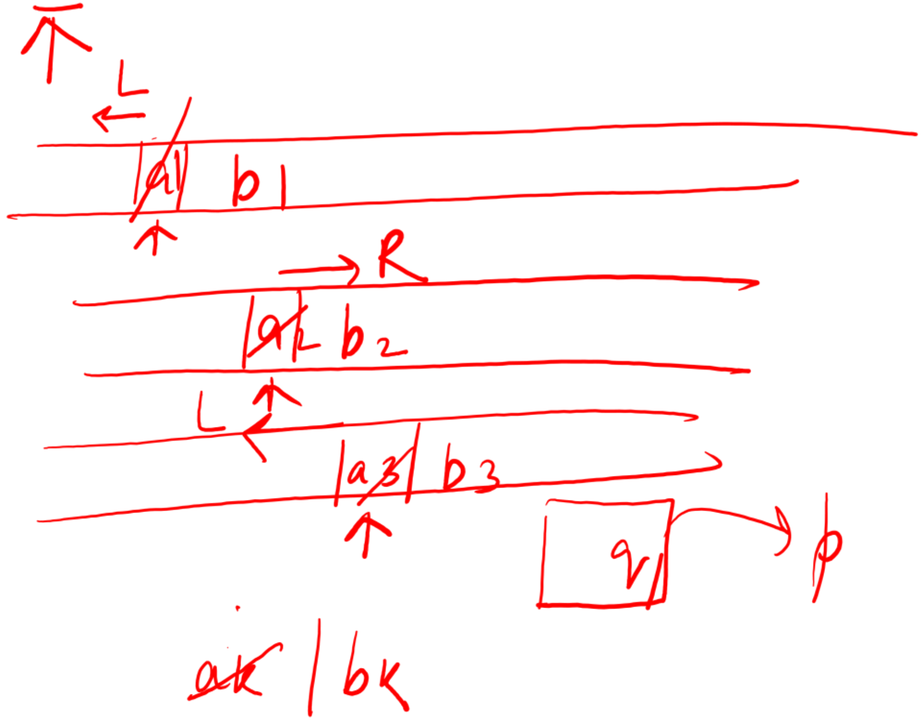
$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, N\}$

i/p from tapes

- The transition function of Multitape turing machine is :
- δ:Q X {Γ1 X Γ2 X .... Γk} ->Q X {Γ1 X Γ2 X .... Γk} X { {L,R,N} X {L,R,N} ....{L,R,N} k times}

tape 1 tape 2 tapek

- δ(q1, (a1,a2,......ak))=(p,(b1,b2,.......bk),(L,R,L,L......k times))

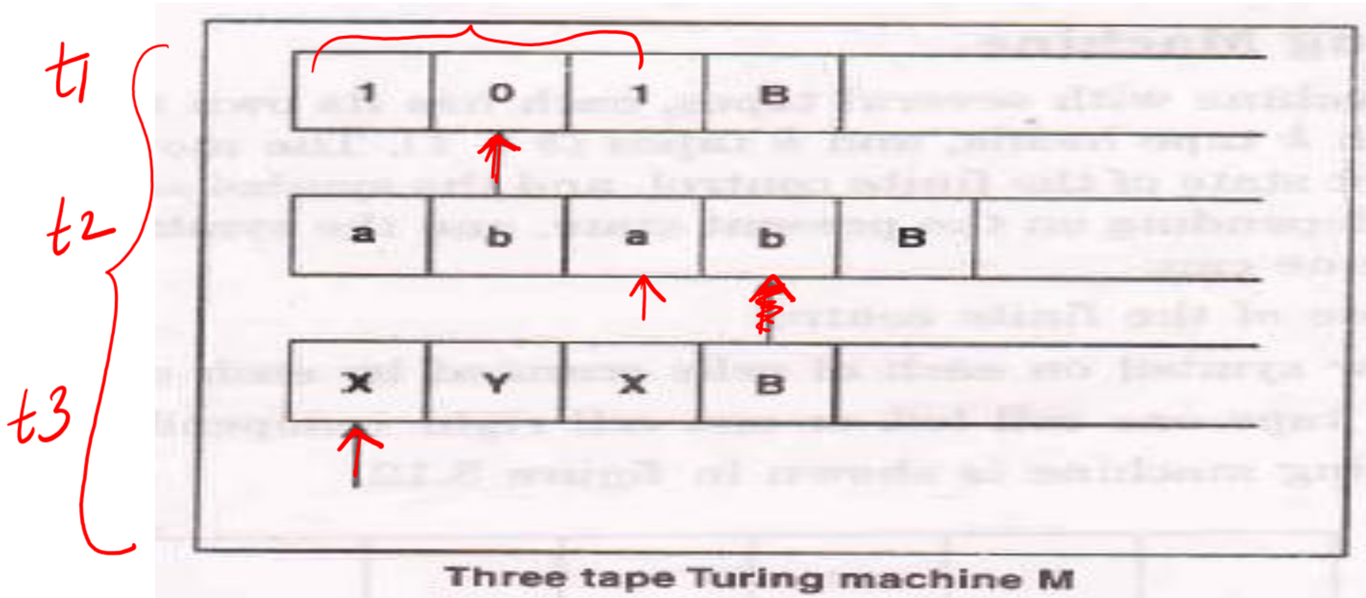


Multi Tape Turing Machine

- **All Variants have same power, every multitape turing machine has an equivalent single tape Turing machine**

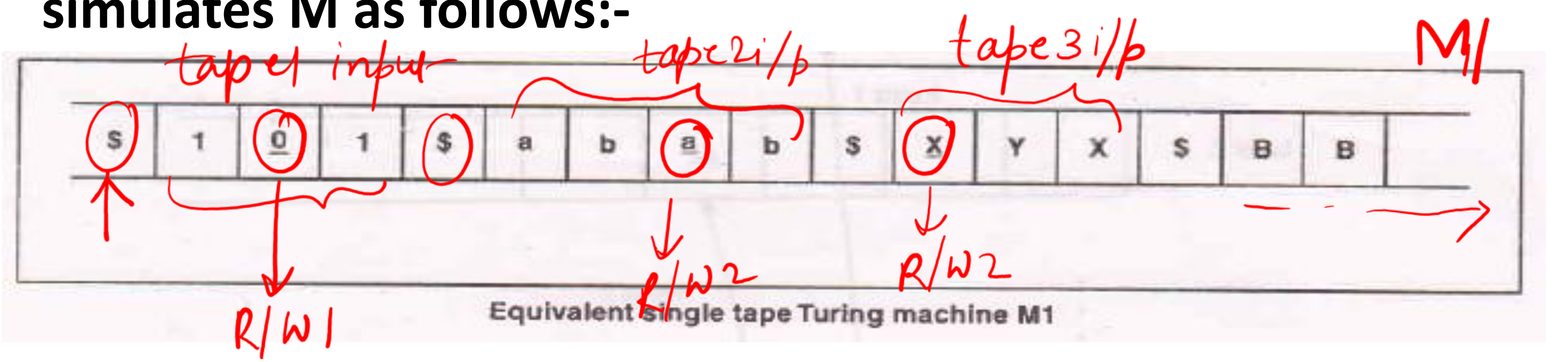
Multi Tape Turing Machine

- **Let M be a k tape Turing Machine(k>1)**



Three tape Turing machine M

- **Let us construct a single tape turing machine M1, which simulates M as follows:-**



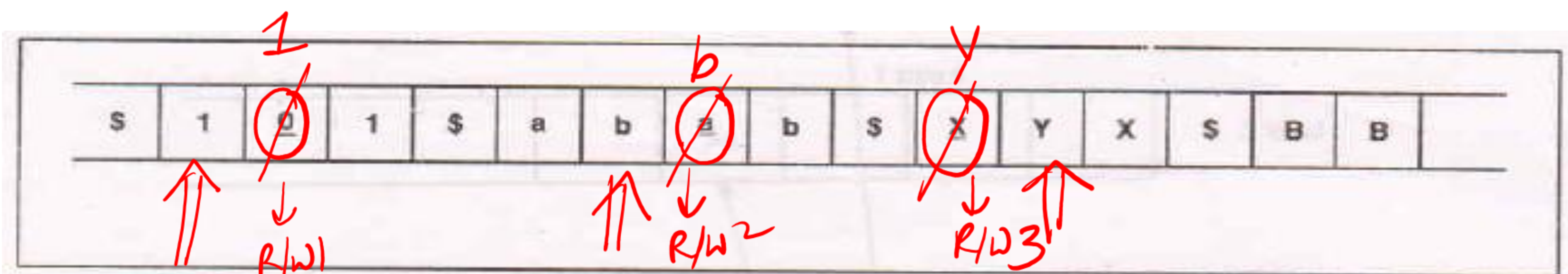
Equivalent single tape Turing machine M1

Multi Tape Turing Machine

$$\delta(q,(0,a,X)) = (p,(1,b,Y),(L,L,R))$$

- **M1 stores the information available on the k tapes of M on its tape, using a symbol $ as delimiter**

- **To keep a track of the head positions for each tape of M, the symbols below each of these tape heads are written with underline on the tape of M1**

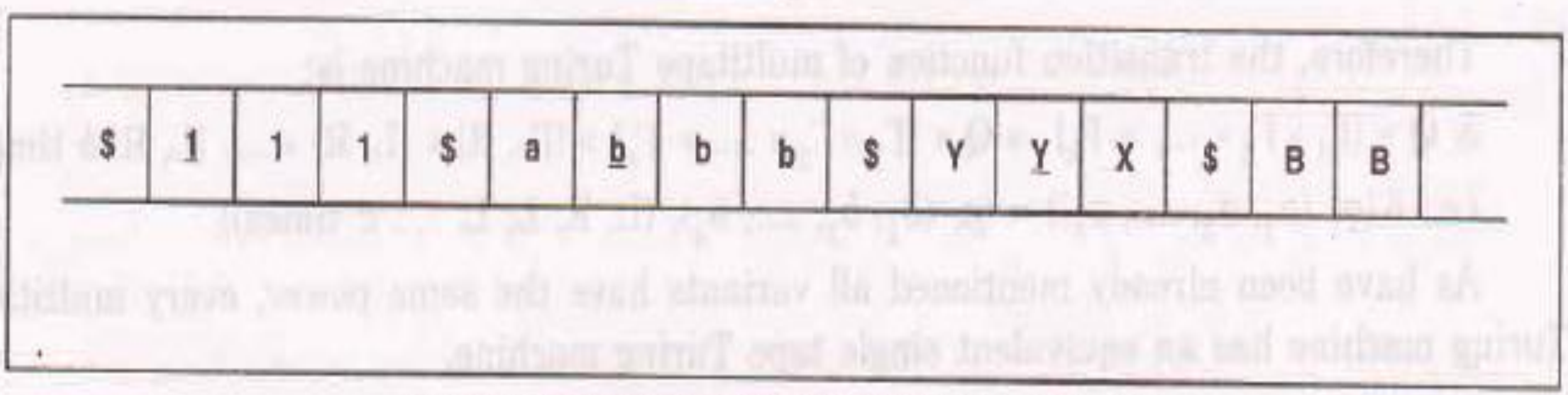


Equivalent single tape Turing machine M1

Multi Tape Turing Machine

- M1 scans its tape starting from first \$ up to (k+1)st \$ for tape symbols that are underlined to simulate the move by replacing these symbols according to the move of M

- $\delta(q, (0,a,X)) = (p,(1,b,Y),(L,L,R))$
- After this move the contents of the tape are-



Contents of tape of M1 after simulating the move of M

Non-Deterministic Turing Machine

- Turing Machine which at any point while carrying out computation , may proceed according to several possibilities
- For a tape symbol scanned, The machine has a finite number of choices of next move
- The transition function defines mapping from
- Q X Γ to a finite subset of Q X Γ X {L,R,N}
- It accepts input, if some sequence of choices of moves lead to an accepting state

$\delta(q_0, a) = (q_f, a, N)$

$\delta(q_0, a) = (q_1, b, R)$

$\delta(q_0, a) = (q_2, a, L)$

$(Q, \Gamma) \rightarrow (Q, \Gamma, \{L, R, N\})$

3 choices ⇒ out of these 3 choices, for 1 choice

multiple moves

non-determinism

we reach final state

acceptance by final

Non-Deterministic Turing Machine

- **All variants have the same power, thus Every Non-Deterministic Turing Machine has an equivalent deterministic Turing machine**

$$P(DTM) = P(NDTM)$$

Non-Deterministic Turing Machine

- For a Non-Deterministic Turing machine M,
- It is always possible to construct a turing machine M1, which tries all possible sequences of computations or branches of computations of M
- If M enters into an accept state on any of these branches, M1 accepts otherwise continues to simulate Mwithout termination
- Thus, M1 is accepting the same language as M

$\Rightarrow$ NDTM = M

DTM = M_1

either it enters into q_f $\Rightarrow$ string is accepted

else, goes on simulating without termination

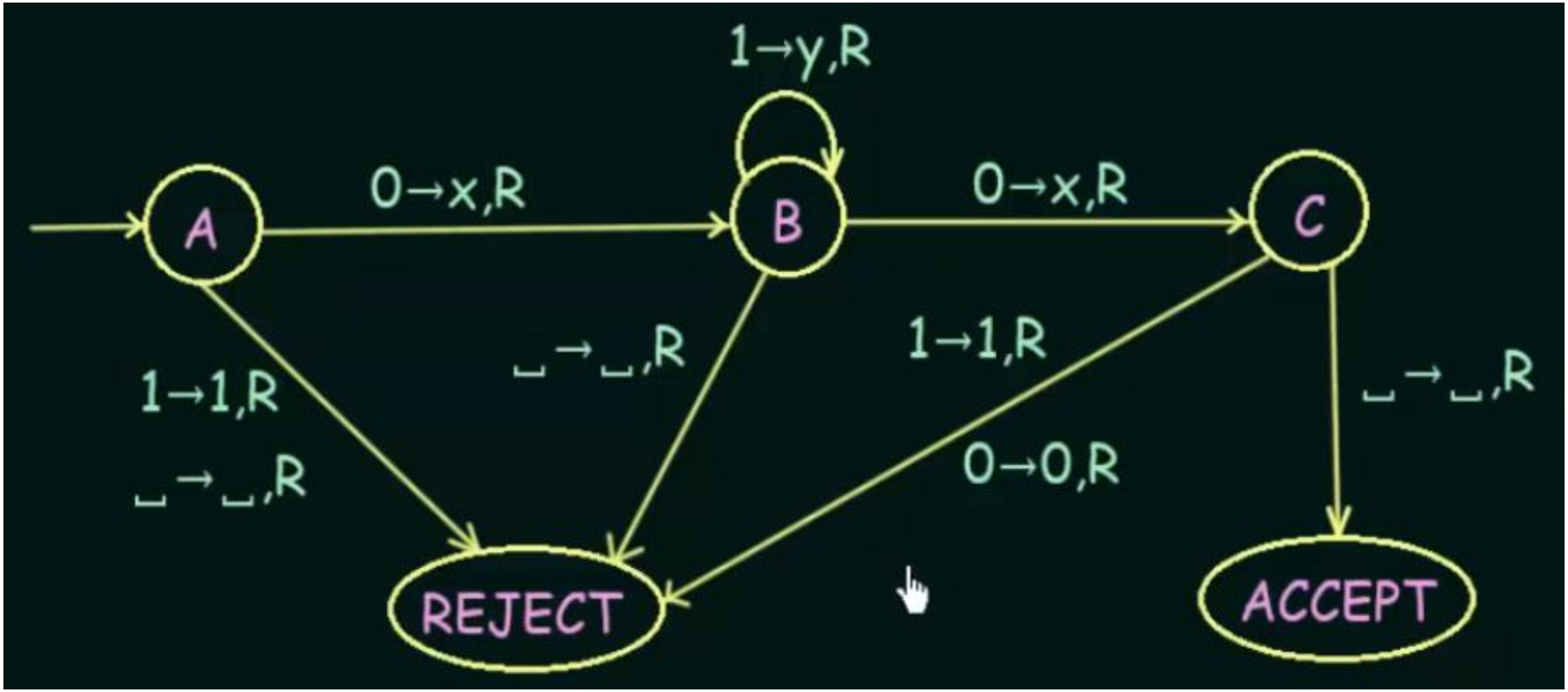
TM Example 5

Design a TM which recognizes the language L=01*0 for ∑={0,1}

Lets take string 0110

TM Example 5

Design a TM which recognizes the language L=01*0 for ∑={0,1}



0	1	1	0	B
X	Y	Y	X	B

TM Example 5

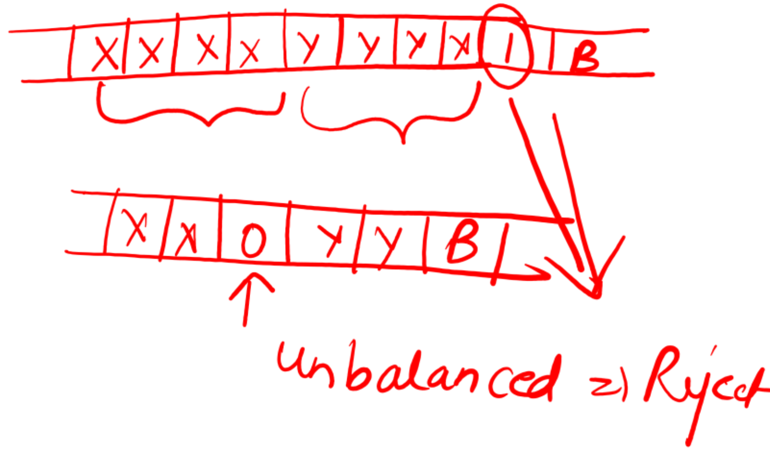
Design a TM which recognizes the language $L=0^N1^N$ for $\sum=\{0,1\}$

TM Example 5

Design a TM which recognizes the language $L=0^N1^N$ for $\sum=\{0,1\}$

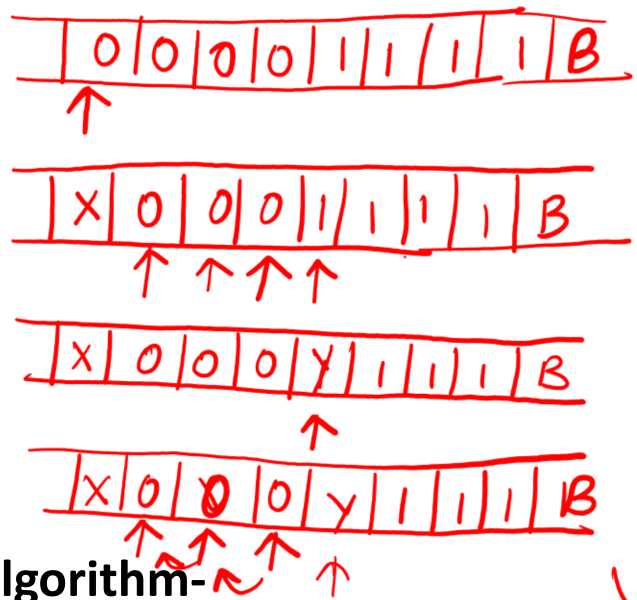
Algorithm-

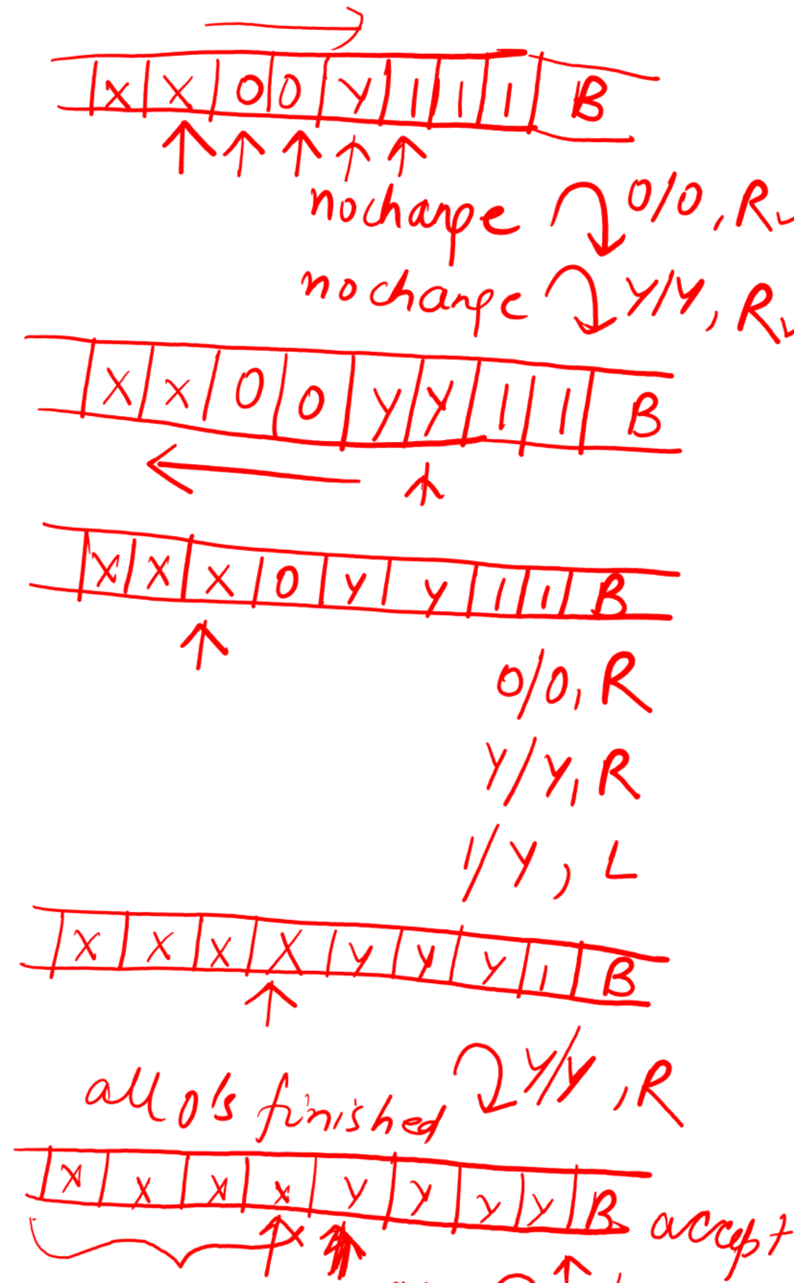
- Change 0 to X
- Move Right to first "1"
- If None:Reject
- Change "1" to "y"
- Move LEFT to Leftmost "0"
- Repeat the above steps until no more "0"s
- Make sure no more "1"s remain



TM Example 5

Simulation for $L=0^N1^N$, String 00001111





Algorithm-

- Change 0 to X
- Move Right to first "1"
- If None:Reject
- Change "1" to "y"
- Move LEFT to Leftmost "0"
- Repeat the above steps until no more "0"s
- Make sure no more "1"s remain

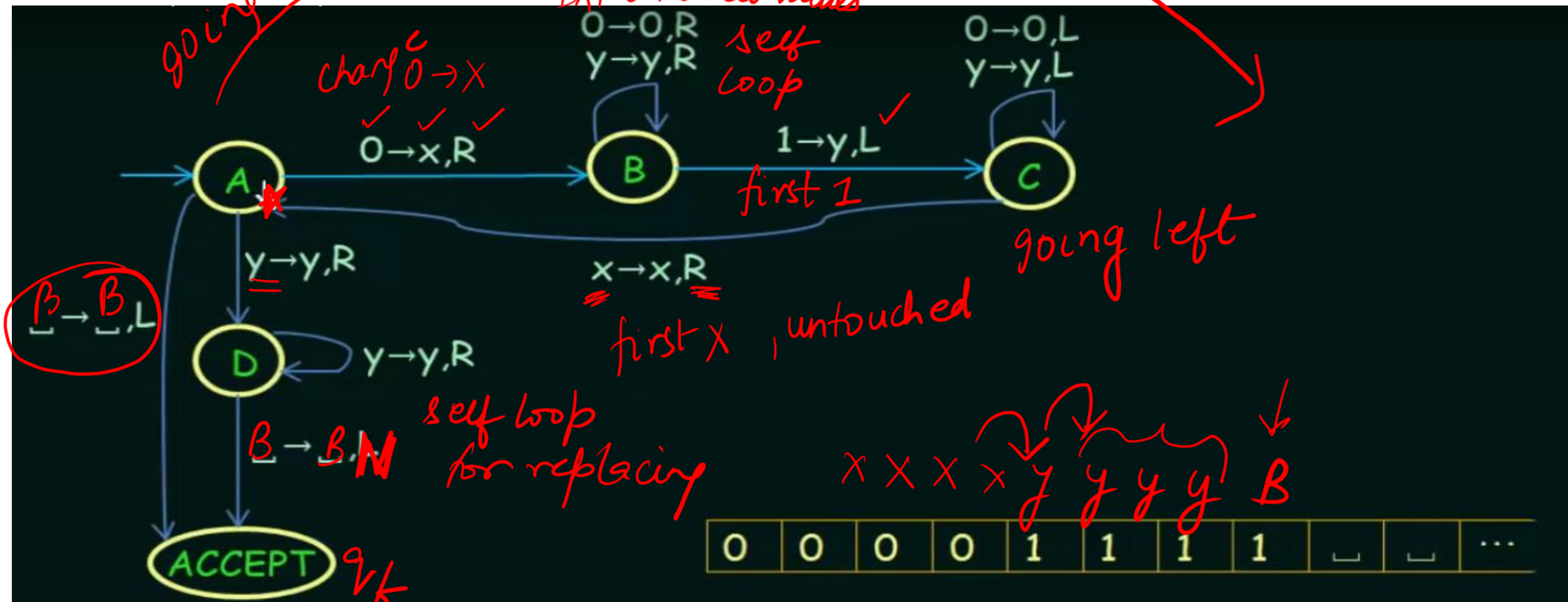
TM Example 5

going right

untounched values

Design a TM which recognizes the language $L=0^N1^N$ for $\sum=\{0,1\}$

Intermediate values



Algorithm-

Change 0 to X

Move Right to first “1”

If None:Reject

Change “1” to “y”

Move LEFT to Leftmost “0”

Repeat the above steps until no more “0”s

Make sure no more “1”s remain

$a^n b^n \Rightarrow n=0$

$\epsilon \Rightarrow$ empty string