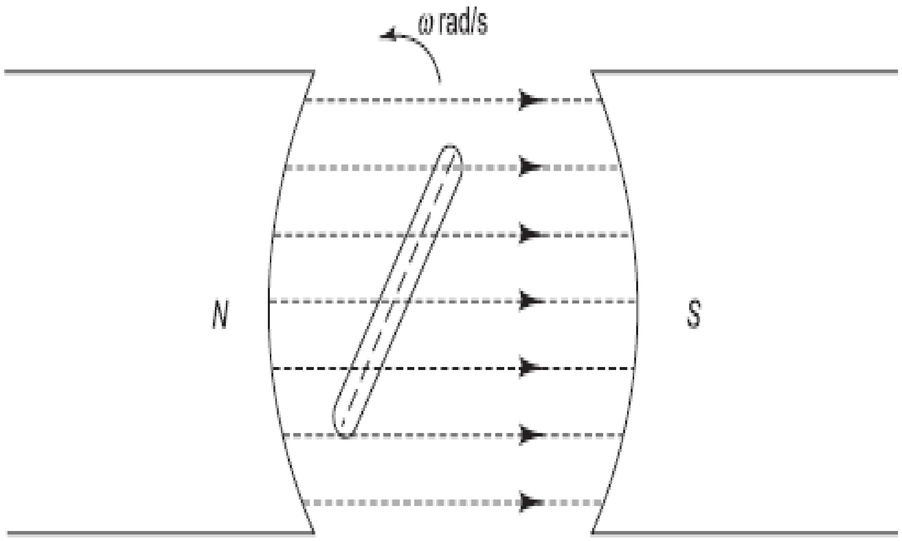
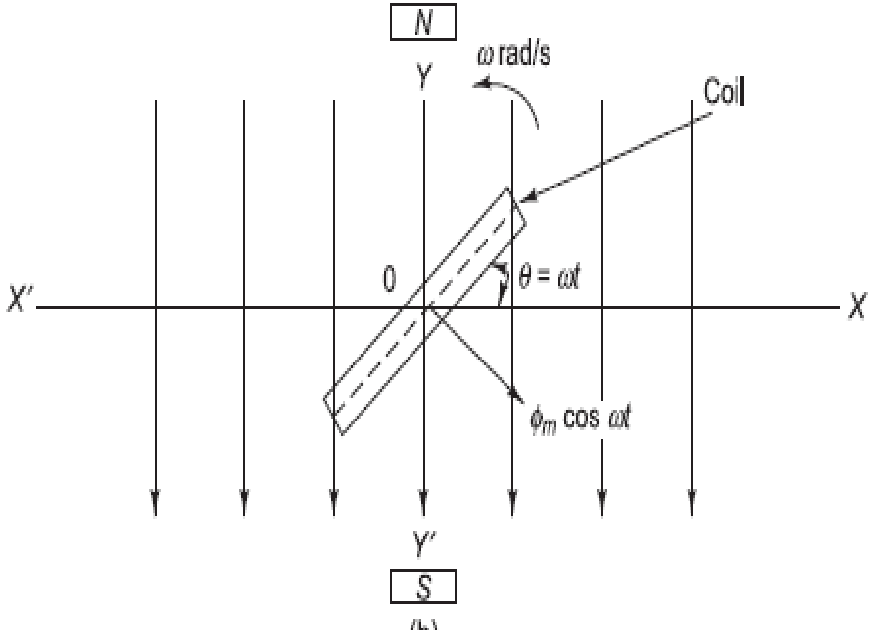
A.C. Fundamentals

Generation Of Alternating Voltages





(b)

Consider a rectangular coil of N turns of area A m2 and rotating in anti-clockwise direction with angular velocity of w radians per second in uniform magnetic field.

Let ϕm be the maximum flux cutting the coil when its axis coincides with the XX' axis (reference position of the coil). Thus when the coil is along XX' the flux linking with it is maximum, i.e., ϕm. When the coil is along YY' i.e., parallel to the lines of flux, the flux linking with it is zero.

The coil rotates through an angle Θ = w t at any instant t.

At this instant, the flux linking with the coil is ϕ= ϕm cos wt

Generation Of Alternating Voltages

According to Faraday's laws of electromagnetic induction,

$$e = -N\frac{d\phi}{dt}$$

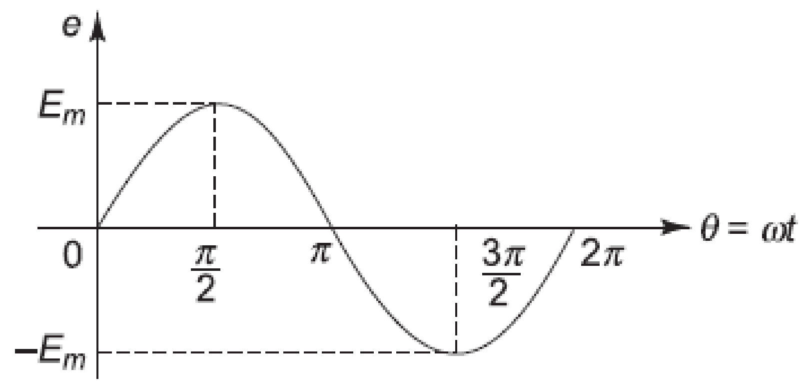
$$= -N\frac{d}{dt}(\phi_m \cos \omega t)$$

$$= N\,\phi_m\,\omega \sin \omega t$$

$$= E_m \sin \omega t$$

where $\quad E_m = N\,\phi_m\omega$

= maximum value of induced emf

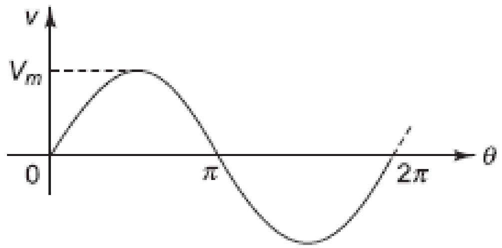


When $\quad \omega t = 0, \quad \sin \omega t = 0, \quad e = 0$

When $\quad \omega t = \frac{\pi}{2}, \quad \sin \frac{\pi}{2} = 1 \quad e = E_m$

If the induced emf is plotted against time, a sinusoidal waveform is obtained.

RMS value of sinusoidal waveform



$$v = V_m \sin\theta \qquad 0 < \theta < 2\pi$$

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\theta d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi} \sin^2\theta d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_0^{2\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{2\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{2\pi}{2}-0-0+0\right]}$$

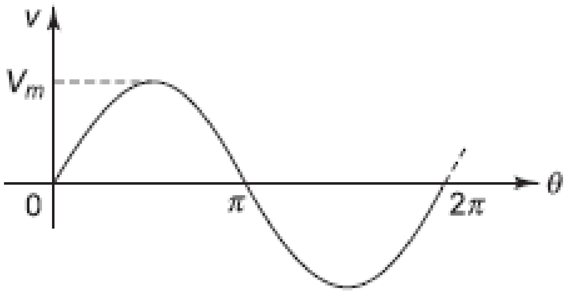
$$= \sqrt{\frac{V_m^2}{2}}$$

$$= \frac{V_m}{\sqrt{2}}$$

$$= 0.707\ V_m$$

$$\text{Peak factor } (k_p) = \frac{\text{Maximum value}}{\text{rms value}}$$

Average value of sinusoidal waveform



$$v = V_m \sin\theta \qquad 0 < \theta < 2\pi$$

$$V_{\text{avg}} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\; d\theta$$

$$= \frac{V_m}{\pi}\int_0^{\pi} \sin\theta\; d\theta$$

$$= \frac{V_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{V_m}{\pi}[1+1]$$

$$= \frac{2V_m}{\pi}$$

$$= 0.637\, V_m$$

$$\text{Form factor } (k_f) = \frac{\text{rms value}}{\text{Average value}}$$

Find the following parameters of a voltage

v = 200 sin 314 t, i) frequency, ii) form factor, iii) Crest factor.

Frequency,

$$v = V_m \sin 2\pi f t$$

$$f = \frac{314}{2\pi} = 50 \text{ Hz}$$

For a sinusoidal waveform,

$$V_{avg} = \frac{2V_m}{\pi}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Form Factor,

$$k_f = \frac{V_{rms}}{V_{avg}} = \frac{\frac{V_m}{\sqrt{2}}}{\frac{2V_m}{\pi}} = 1.11$$

Crest Factor,

$$k_p = \frac{V_m}{V_{rms}} = \frac{V_m}{\frac{V_m}{\sqrt{2}}} = 1.414$$

The waveform of a voltage has a form factor of 1.15 and peak factor of 1.5. If the maximum value of the voltage is 4500 V, calculate the average value and rms value of the voltage.

$$k_f = 1.15$$

$$k_p = 1.5$$

$$V_m = 4500 \text{ V}$$

rms value of the voltage

$$k_p = \frac{V_m}{V_{\text{rms}}}$$

$$1.5 = \frac{4500}{V_{\text{rms}}}$$

$$V_{\text{rms}} = 3000 \text{ V}$$

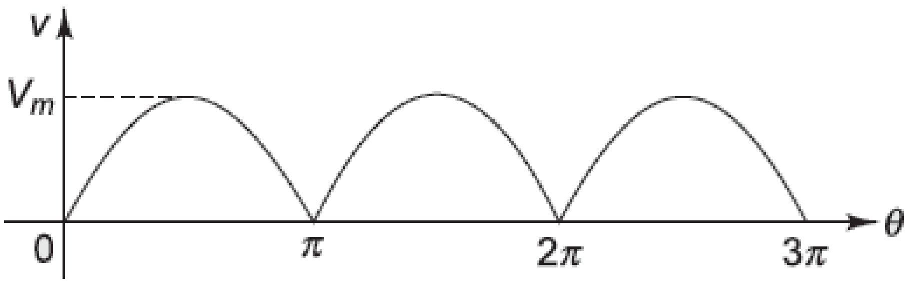
Average value of the voltage

$$k_f = \frac{V_{\text{rms}}}{V_{\text{avg}}}$$

$$1.15 = \frac{3000}{V_{\text{avg}}}$$

$$V_{\text{avg}} = 2608.7 \text{ V}$$

Find the average value and rms value of the waveform shown in figure



$$v = V_m \sin\theta \qquad 0 < \theta < \pi$$

Average value of the waveform

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\, d\theta$$

$$= \frac{V_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{V_m}{\pi}[1+1] \quad = \frac{2V_m}{\pi} \quad = 0.637\, V_m$$

rms value of the waveform

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2(\theta)\, d\theta}$$

$$= \sqrt{\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta}$$

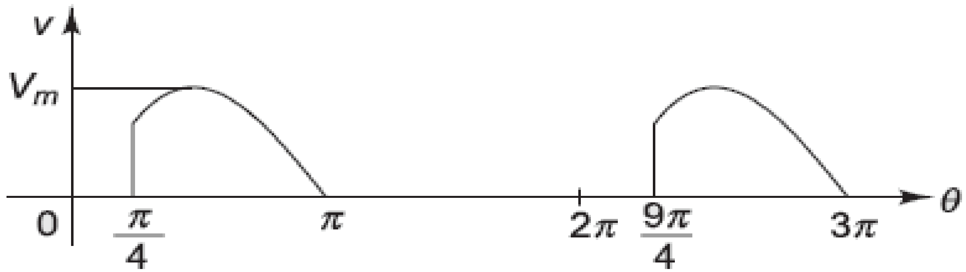
$$= \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi} \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{V_m^2}{\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - 0 + \frac{\sin 0}{4}\right]}$$

$$= 0.707\, V_m$$

Find the average value and rms value of the waveform shown in fig.



$v = 0 \qquad 0 < \theta < \pi/4$

$= V_m \sin\theta \qquad \pi/4 < \theta < \pi$

$= 0 \qquad \pi < \theta < 2\pi$

Average value of the waveform

$$V_{\text{avg}} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\, d\theta$$

$$= \frac{1}{2\pi}\int_{\pi/4}^{\pi} V_m \sin\theta\, d\theta$$

$$= \frac{V_m}{2\pi}[-\cos\theta]_{\pi/4}^{\pi}$$

$$= \frac{V_m}{2\pi}[1 + 0.707] \quad = 0.272\, V_m$$

rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\, d\theta}$$

$$= \sqrt{\frac{1}{2\pi}\int_{\pi/4}^{\pi} V_m^2 \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\int_{\pi/4}^{\pi} \sin^2\theta\, d\theta}$$

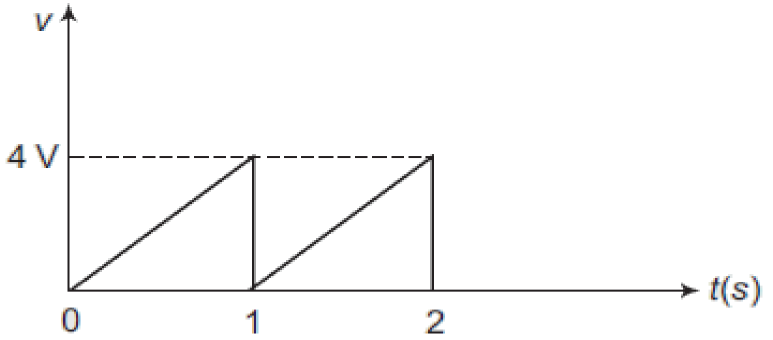
$$= \sqrt{\frac{V_m^2}{2\pi}\int_{\pi/4}^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_{\pi/4}^{\pi}}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - \frac{\pi}{8} + \frac{\sin \pi/2}{4}\right]}$$

$$= \sqrt{0.227\, V_m^2} \quad = 0.476\, V_m$$

Find the average value of the waveform shown in figure



$$v = 4t$$

$$V_{\text{avg}} = \frac{1}{T}\int_0^T v(t)\,\mathrm{d}t$$

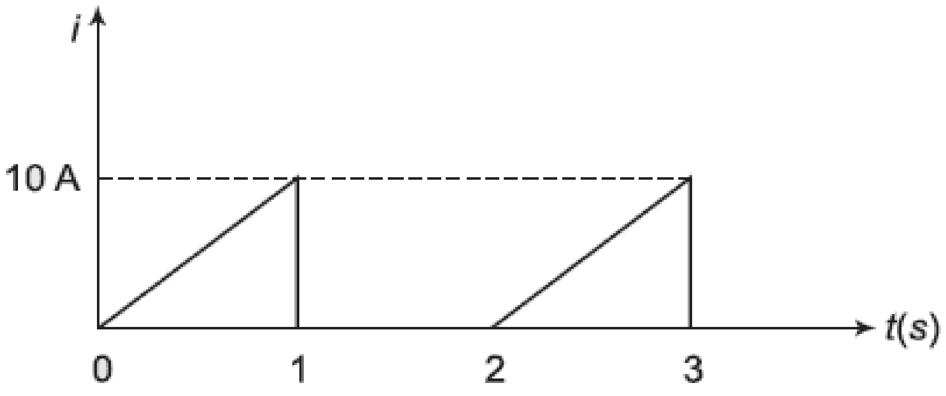
$$= \frac{1}{1}\int_0^1 4t\,\mathrm{d}t$$

$$= 4\left[\frac{t^2}{2}\right]_0^1$$

$$= 4\left(\frac{1}{2} - 0\right)$$

$$= 2\ \mathrm{V}$$

Find the rms value of the given waveform



$$i = 10\,t \qquad 0 < t < 1$$

$$= 0 \qquad 1 < t < 2$$

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,\mathrm{d}t}$$

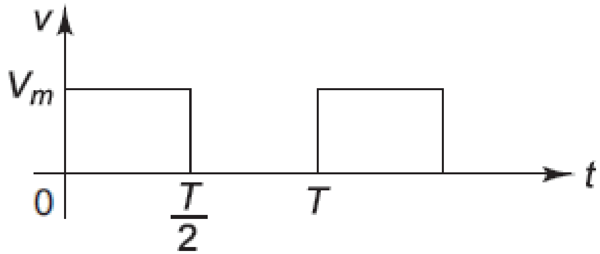
$$= \sqrt{\frac{1}{2}\left[\int_0^1 (10t)^2\,\mathrm{d}t + \int_1^2 0\,\mathrm{d}t\right]}$$

$$= \sqrt{\frac{1}{2}\times 100\left[\frac{t^3}{3}\right]_0^1}$$

$$= \sqrt{\frac{100}{2}\left[\frac{1}{3} - 0\right]}$$

$$= 4.084$$

Find the average and rms value of the waveform shown in figure.



$$v = V_m \qquad 0 < t < T/2$$

$$= 0 \qquad T/2 < t < T$$

Average value of the waveform

$$V_{\text{avg}} = \frac{1}{T}\int_0^T v(t)\,dt$$

$$= \frac{1}{T}\left[\int_0^{T/2} V_m\,dt + \int_{T/2}^{T} 0\,dt\right]$$

$$= \frac{1}{T}\int_0^{T/2} V_m\,dt$$

$$= \frac{V_m}{T}[t]_0^{T/2} \qquad = \frac{V_m}{T}\cdot\frac{T}{2}$$

$$= 0.5\ V_m$$

rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$= \sqrt{\frac{1}{T}\int_0^{T/2} V_m^2\,dt}$$

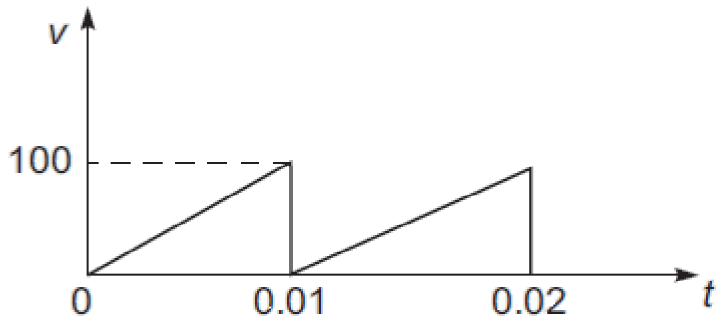
$$= \sqrt{\frac{V_m^2}{T}[t]_0^{T/2}}$$

$$= \sqrt{\frac{V_m^2}{T}\cdot\frac{T}{2}}$$

$$= \sqrt{\frac{V_m^2}{2}}$$

$$= 0.707\ V_m$$

Determine the rms value of the voltage waveform shown in figure



$$v(t) = \frac{100}{0.01}\, t = 10000\, t \qquad 0 < t < 0.01$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt}$$

$$= \sqrt{\frac{1}{0.01}\int_0^{0.01} (10000t)^2 dt}$$

$$= \sqrt{10^{10}\left[\frac{t^3}{3}\right]_0^{0.01}}$$

$$= \sqrt{10^{10}\left[\frac{(0.01)^3}{3} - 0\right]}$$

$$= 57.74 \text{ V}$$

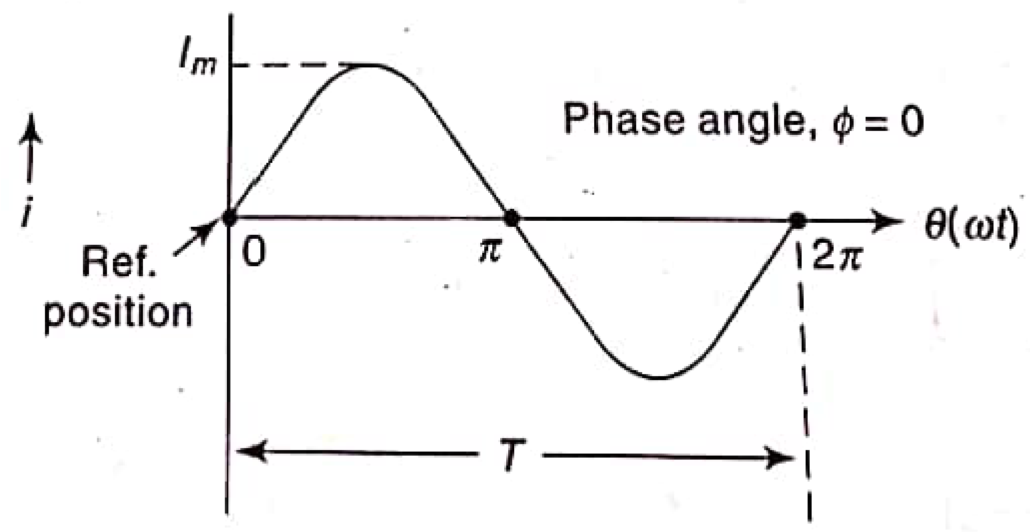
Phasor

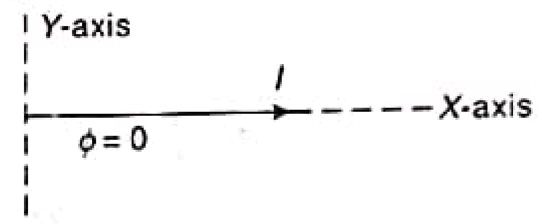
The alternating quantities are represented by phasor. A phasor is a line of definite length rotation in an anticlockwise direction at a constant angular velocity ω. The length of the phasor represents magnitude i.e. rms value of the alternating quantity and angular velocity is equal to the angular velocity of alternating quantity.

Case 1: The sinusoidal alternating current is represented by waveform.

An alternating quantity is generally referred by its rms value. As a result, length of the phasor is drawn equal to the rms value instead of maximum value.

$$I = \frac{I_m}{\sqrt{2}}$$



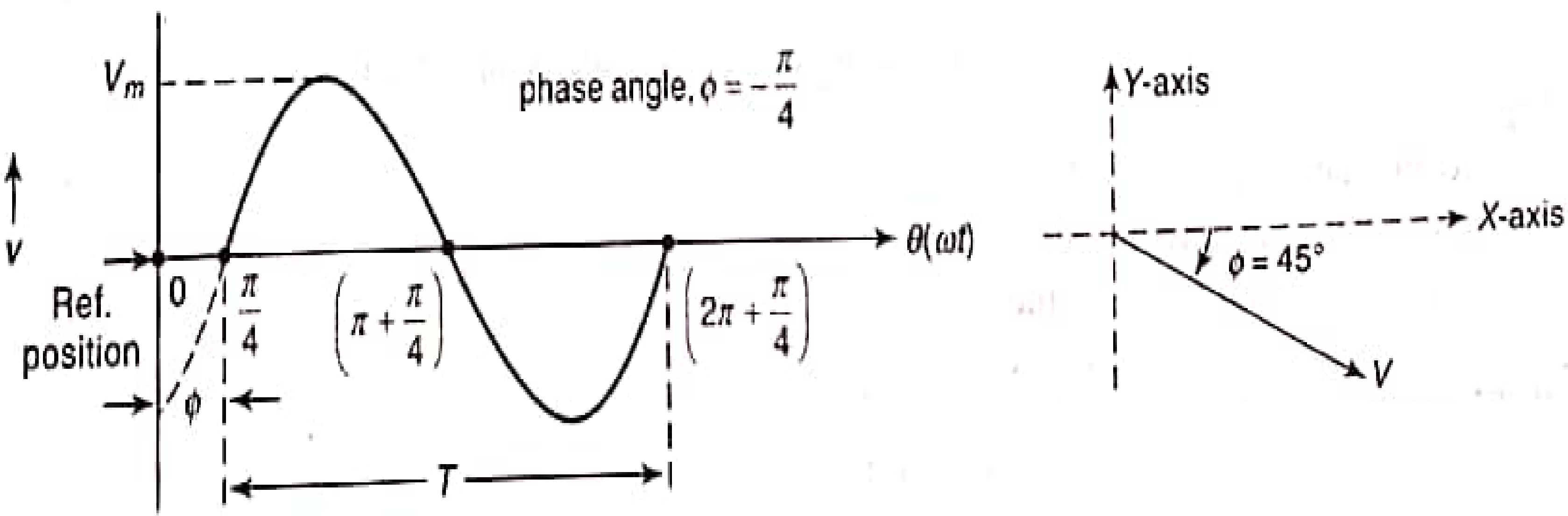


Case 2: The voltage attains its zero value (first time) after a reference position by an angle $\pi/4$ rad or $45°$

i.e. it lags behind reference. Length of the phasor is

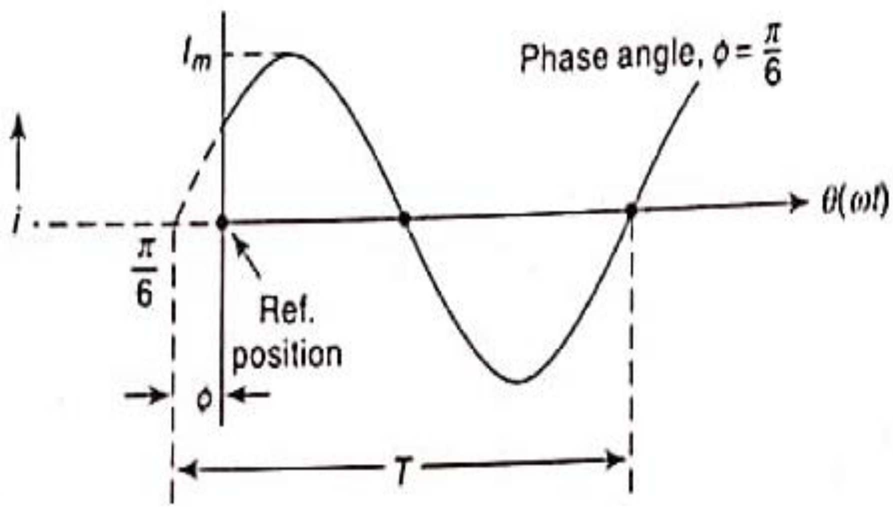
$$V = \frac{V_m}{\sqrt{2}}$$

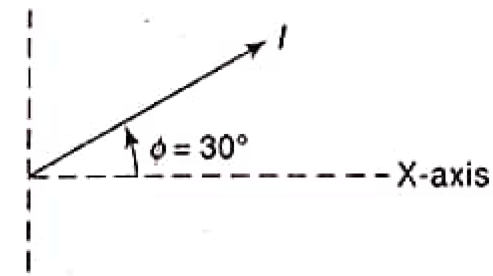
$$v = V_m \sin\left(\omega t - \frac{\pi}{4}\right) \qquad v = V_m \sin(\omega t - 45°)$$



Case 3: The sinusoidal alternating current is represented by the waveform. The current attains its zero value (first time) before a reference position by an angle $\frac{\pi}{6}$.
i.e. it leads reference. Therefore the phase angle of the current is positive. Length of the phasor is, $I = \frac{I_m}{\sqrt{2}}$

$$i = I_m \sin\left(\omega t + \frac{\pi}{6}\right) \qquad i = I_m \sin(\omega t + 30)$$





Phasor algebra

Mathematical representation of any phasor is known as phasor algebra. The phasor can be mathematically represented in two ways.

1. Rectangular form
2. Polar form

Rectangular form: It is a complex form in which operator j is used.

In rectangular form, the phasor is resolved into horizontal (x) and vertical (y) components and expressed in complex form, i.e. $\bar{I} = (x + jy)$ A

X= real part, horizontal component

Y= Imaginary part, vertical component

Magnitude of phasor, $I = \sqrt{x^2 + y^2}$

And its angle w.r.t. X-axis, , $\phi = \tan^{-1} \frac{y}{x}$.

This form is used for addition and subtraction of alternating quantities.

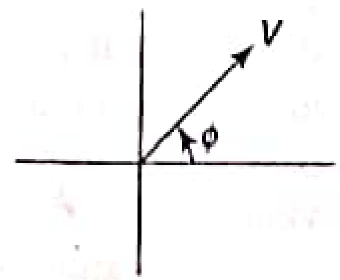
Polar form:

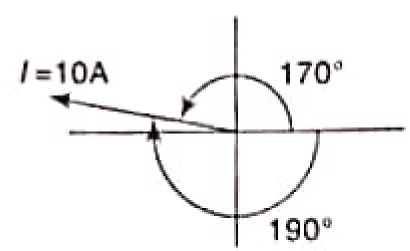
In this form current phasor is represented by $(I \angle \pm\phi)$

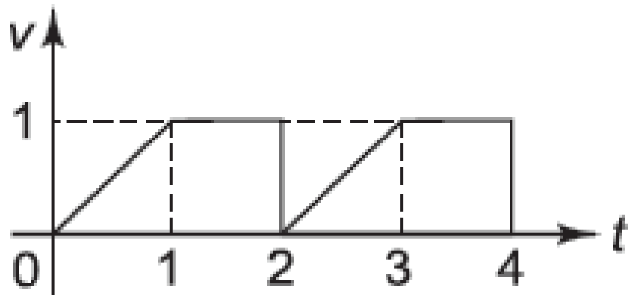
And voltage phasor is represented by $(V \angle \pm\phi)$.

$$\bar{I} = (10 \angle 170°)\ \text{A}$$

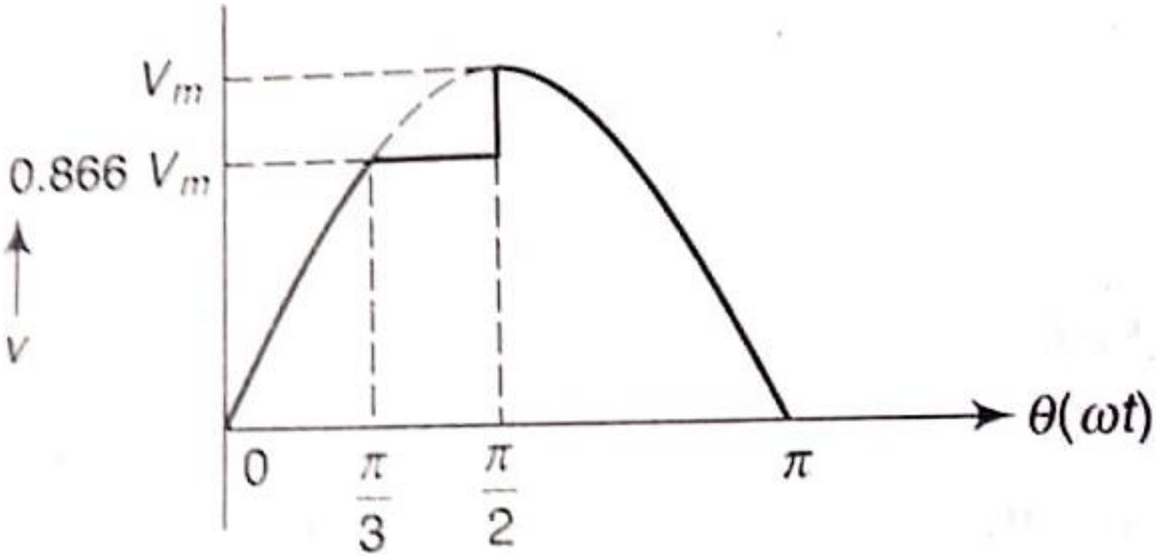
$$\bar{I} = (10 \angle -190°)\ \text{A}$$






v
1
t
0
1
2
3
4

Find the average value of the waveform shown in figure.



$$V_{\text{average}} = \frac{\int_0^{\pi} v\, d\theta}{\pi}$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi/3} v\, d\theta + \int_{\pi/3}^{\pi/2} v\, d\theta + \int_{\pi/2}^{\pi} v\, d\theta\right\}$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi/3} v\, d\theta + \int_{\pi/3}^{\pi/2} v\, d\theta + \int_{\pi/2}^{\pi} v\, d\theta\right\}$$

$$= \frac{1}{\pi}\left\{V_m[-\cos\theta]_0^{\pi/3} + 0.866\, V_m[\theta]_{\pi/3}^{\pi/2} + V_m[-\cos\theta]_{\pi/2}^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{V_m\left[-\cos\frac{\pi}{3} - (-\cos 0)\right] + 0.866\, V_m\left[\frac{\pi}{2} - \frac{\pi}{3}\right]\right.$$

$$\left. + V_m\left[-\cos\pi - \left(-\cos\frac{\pi}{2}\right)\right]\right\}$$

$$= \frac{1}{\pi}\{V_m[-0.5-(-1)] + 0.866\, V_m[1.57-1.047]$$

$$+ V_m[-(-1)-(-0)]\}$$

$$= \frac{1}{\pi}\{0.5\, V_m + 0.45\, V_m + V_m\}$$

$$= 0.621\, V_m$$

Adding and subtracting alternating quantities

Two currents i_1 and i_2 are given by the expressions $i_1 = 10 \sin\left(\omega t + \frac{\pi}{4}\right)$ and $i_2 = 8 \sin\left(\omega t - \frac{\pi}{3}\right)$.

Find (i) $i_1 + i_2$, and (ii) $i_1 - i_2$. Express the answers in the form $i = I_m \sin(\omega t \pm \phi)$.

$$i_1 = 10 \sin\left(\omega t + \frac{\pi}{4}\right) \qquad i_2 = 8 \sin\left(\omega t - \frac{\pi}{3}\right)$$

Let phasors $\bar{I}_1$ and $\bar{I}_2$ represent the alternating currents i_1 and i_2 respectively in terms of their maximum values.

Analytical method:

Resolving $\bar{I}_1$ and $\bar{I}_2$ into x- and y-components,

$$\Sigma x = 10 \cos(45°) + 8 \cos(-60°) = 11.07$$

$$\Sigma y = 10 \sin(45°) + 8 \sin(-60°) = 0.14$$

$$\text{Magnitude of } (\bar{I}_1 + \bar{I}_2) = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$= \sqrt{(11.07)^2 + (0.14)^2}$$

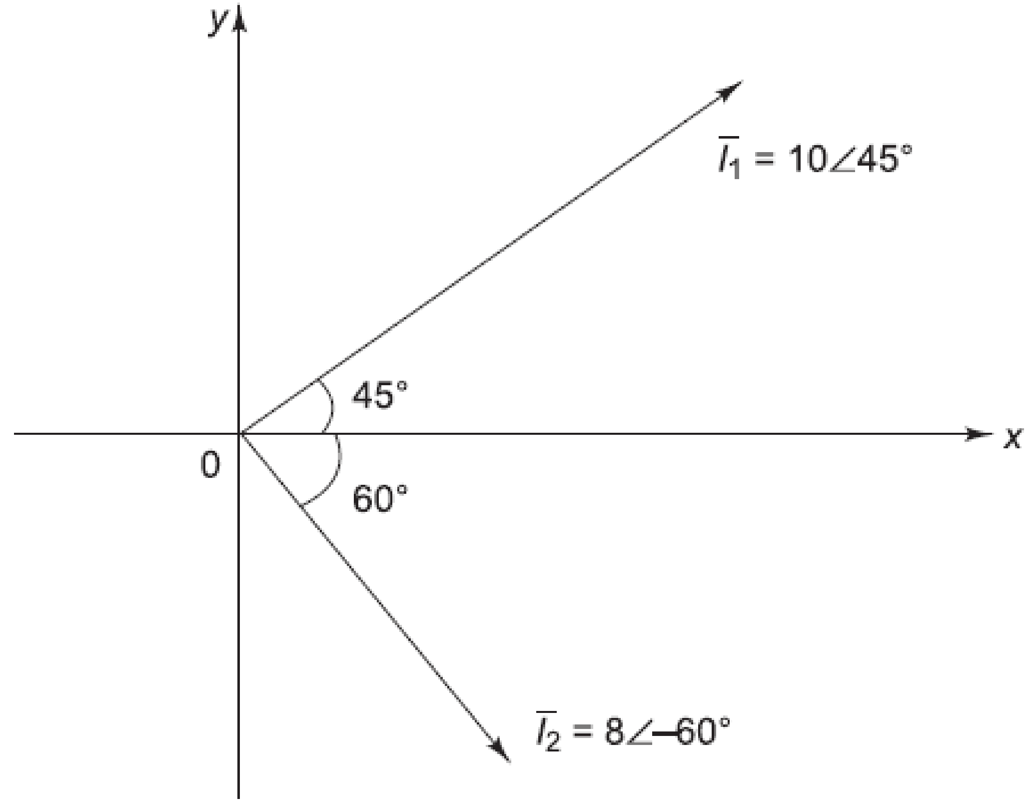
$$= 11.07 \text{ A}$$

$$\text{Phase angle } \phi = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right)$$

$$= \tan^{-1}\left(\frac{0.14}{11.07}\right)$$

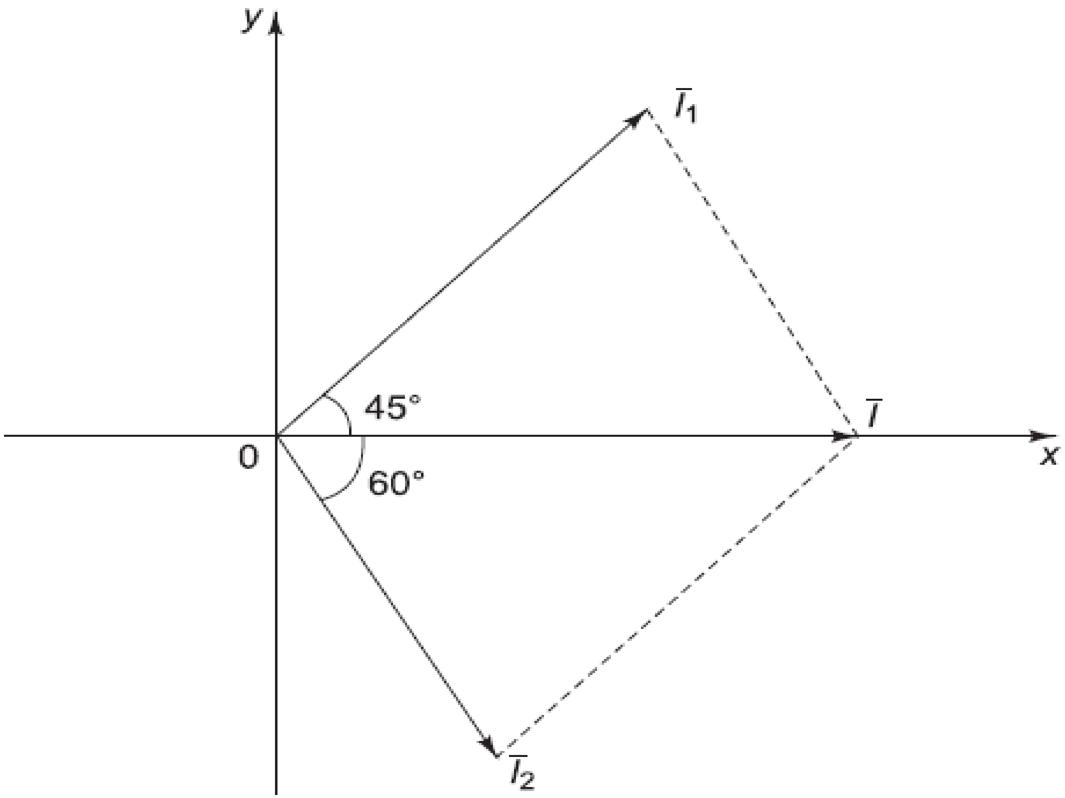
$$= 0.72°$$

$$i = i_1 + i_2 = 11.07 \sin(\omega t + 0.72°)$$



Graphical method

The phasor sum $\bar{I}_1 + \bar{I}_2$ is obtained by adding phasors $\bar{I}_1$ and $\bar{I}_2$ by the parallelogram law.



To find i1 –i2
Analytical method

Resolving $\overline{I}_1$ and $-\overline{I}_2$ into x- and y-components,

$$\Sigma x = 10 \cos(45°) - 8\cos(-60°) = 3.07$$

$$\Sigma y = 10 \sin(45°) - 8\sin(-60°) = 14$$

Magnitude of $(\overline{I}_1 - \overline{I}_2) = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$

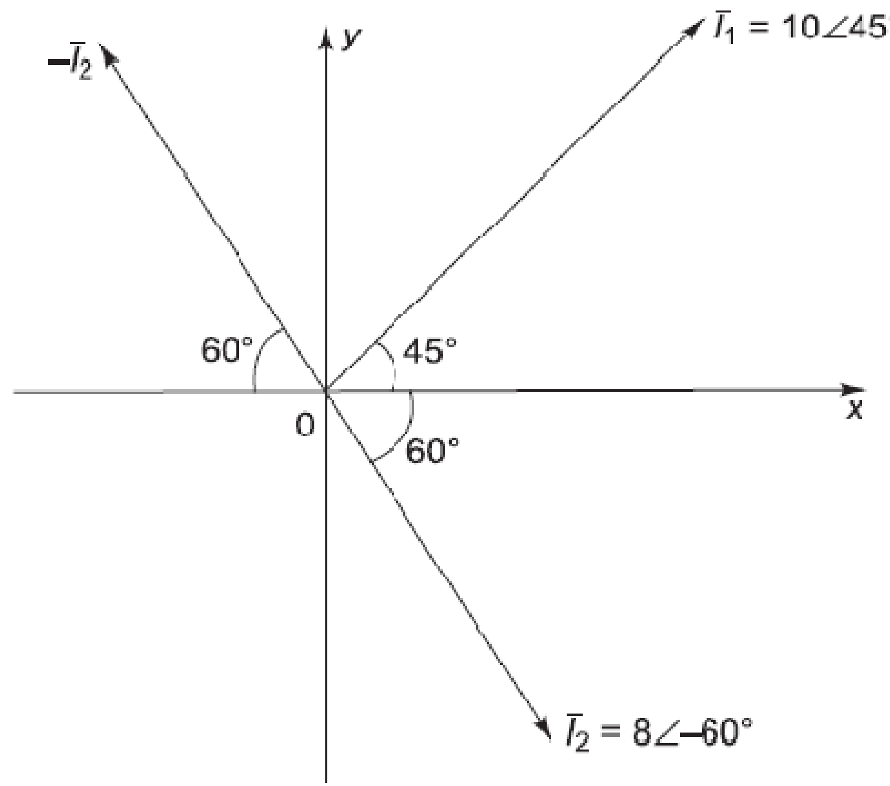
$$= \sqrt{(3.07)^2 + (14)^2}$$

$$= 14.33 \text{ A}$$

Phase angle $\phi = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right)$

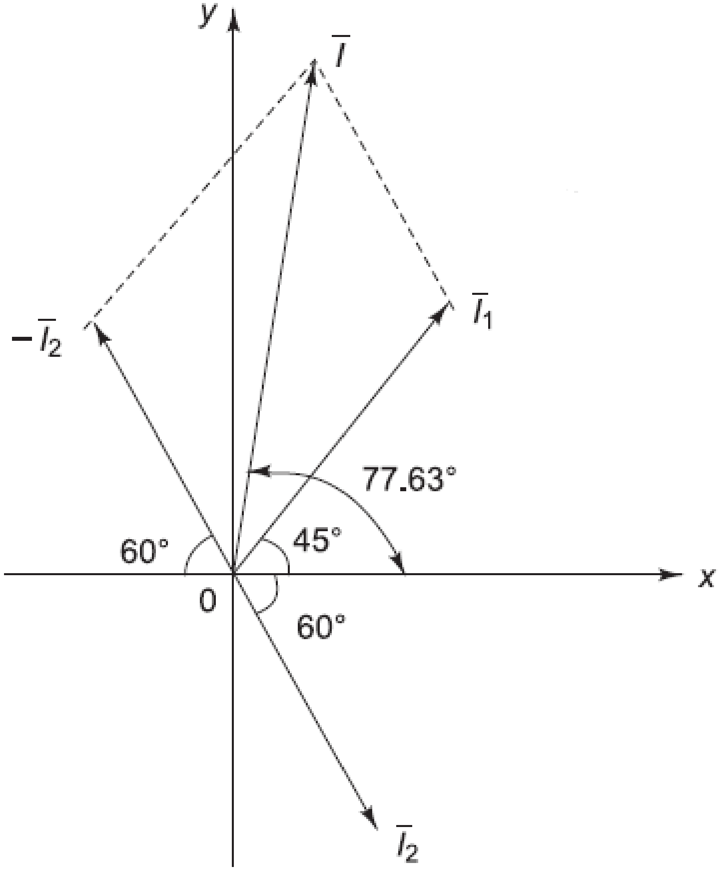
$$= \tan^{-1}\left(\frac{14}{3.07}\right) \quad = 77.63°$$

$$i = i_1 - i_2 = 14.33 \sin(\omega t + 77.63°)$$



Graphical Method

The phasor sum $\bar{I}_1 - \bar{I}_2$ is obtained by adding phasors $\bar{I}_1$ and $-\bar{I}_2$ by the parallelogram law.



Graphical method

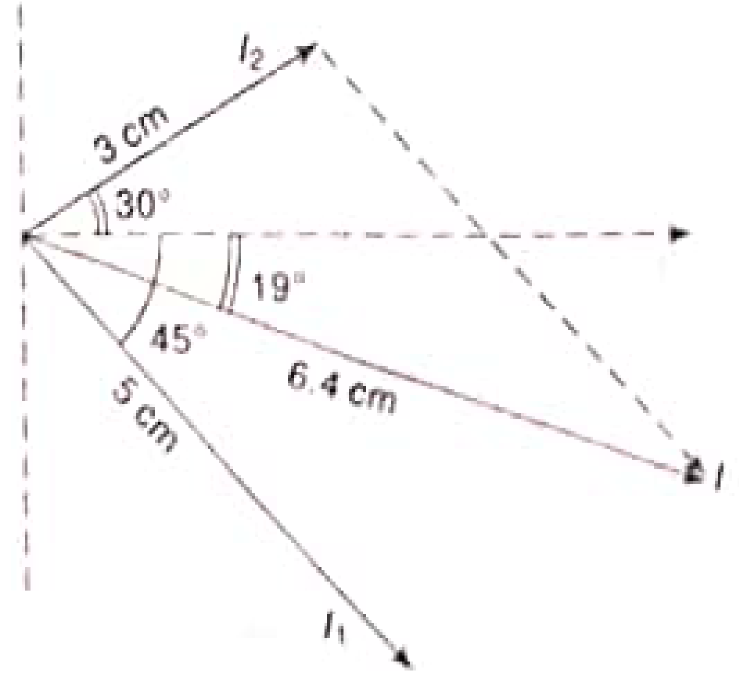
$i_1 = 7.07 \sin(\omega t - 45)$ and $i_2 = 4.24 \sin(\omega t + 30)$

The rms values are: $I_1 = \dfrac{7.07}{\sqrt{2}} = 5\text{ A}$

$I_2 = \dfrac{4.24}{\sqrt{2}} = 3\text{ A}$

$\bar{I}_1 = (5 \angle -45)\text{ A}$

$\bar{I}_2 = (3 \angle 30)\text{ A}$



The resultant I = 6.4 A

Phase angle, $\phi = -19°$

$\bar{I} = (6.4 \angle -19)\text{ A}$

$I_m = 6.4 \times \sqrt{2} = 9.05\text{ A}$

Hence resultant current is given by $i = 9.05 \sin(\omega t - 19°)$

Three voltages are represented by $v_1 = 10 \sin \omega t$, $v_2 = 20 \sin \left(\omega t - \frac{\pi}{6}\right)$ *and* $v_3 = 30 \sin \left(\omega t + \frac{\pi}{4}\right)$. *Find the magnitude and phase angle of the resultant voltage.*

$$v_1 = 10 \sin \omega t$$

$$v_2 = 20 \sin \left(\omega t - \frac{\pi}{6}\right)$$

$$v_3 = 30 \sin \left(\omega t + \frac{\pi}{4}\right)$$

Let phasors $\bar{V}_1$, $\bar{V}_2$ and $\bar{V}_3$ represent the alternating voltages v_1, v_2 and v_3 respectively in terms of their maximum values.

Resolving $\bar{V}_1$, $\bar{V}_2$ and $\bar{V}_3$ into x- and y-components,

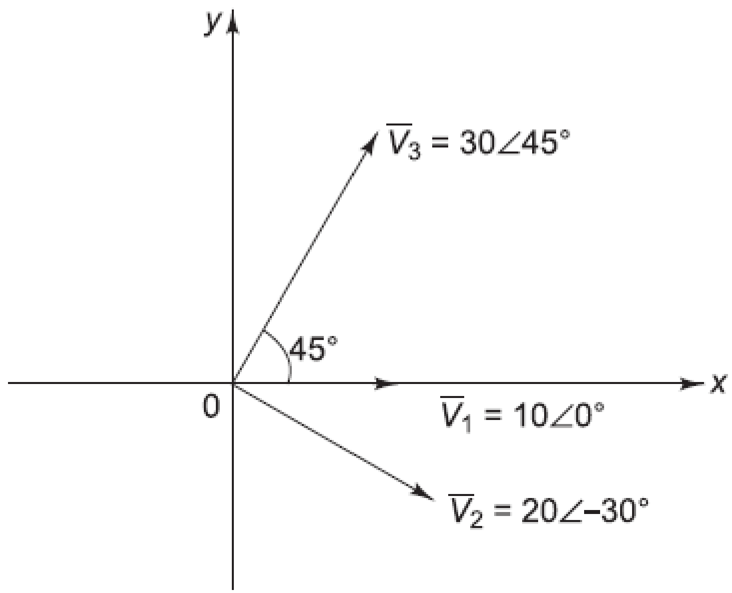
$$\Sigma x = 10 + 20 \cos(-30°) + 30 \cos(45°) = 48.53$$

$$\Sigma y = 20 \sin(-30°) + 30 \cos(45°) = 11.21$$

$$\text{Magnitude of } (\bar{V}_1 + \bar{V}_2 + \bar{V}_3) = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$= \sqrt{(48.53)^2 + (11.21)^2}$$

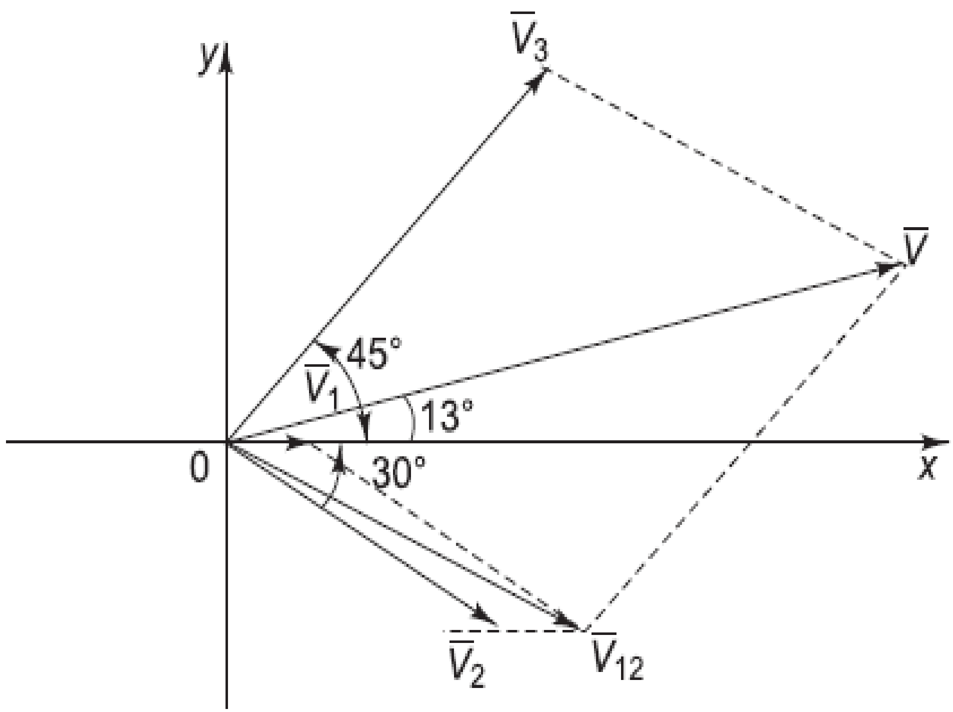
$$= 49.81 \text{ V}$$



$$
\begin{aligned}
\text{Phase angle } \phi &= \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right) \\
&= \tan^{-1}\left(\frac{11.21}{48.53}\right) \\
&= 13°
\end{aligned}
$$

$$v = 49.81 \sin(\omega t + 13°)$$

Graphical method



The instantaneous voltages across each of the four coils connected in series are given by

$$v_1 = 100 \sin \omega t, \quad v_2 = 250 \cos \omega t, \quad v_3 = 150 \sin \left(\omega t + \frac{\pi}{6}\right), \quad v_4 = 200 \sin \left(\omega t - \frac{\pi}{4}\right)$$

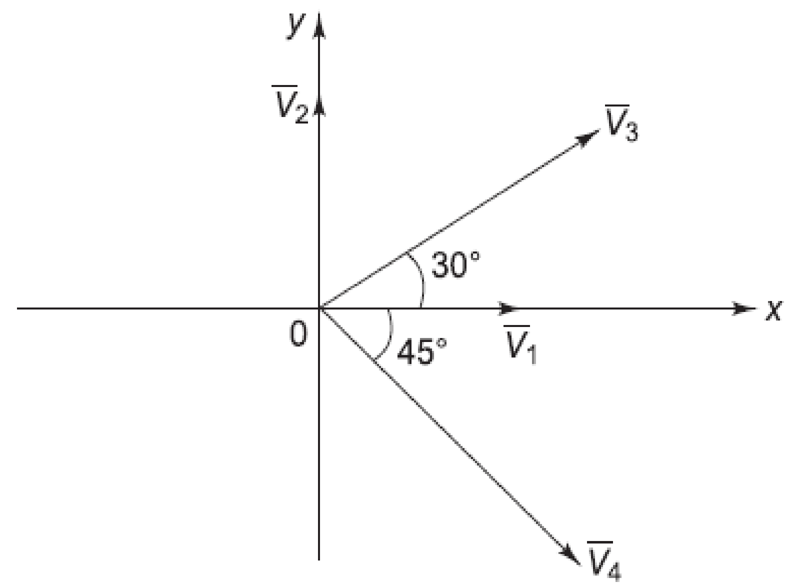
Determine the resultant voltage by analytical method.

$$v_1 = 100 \sin \omega t$$

$$v_2 = 250 \cos \omega t = 250 \sin (\omega t + 90°)$$

$$v_3 = 150 \sin \left(\omega t + \frac{\pi}{6}\right)$$

$$v_4 = 200 \sin \left(\omega t - \frac{\pi}{4}\right)$$



Let phasors $\overline{V}_1$, $\overline{V}_2$, $\overline{V}_3$ and $\overline{V}_4$ represent the instantaneous voltages v_1, v_2, v_3 and v_4 respectively in terms of their maximum values.

Resolving $\bar{V}_1$, $\bar{V}_2$, $\bar{V}_3$ and $\bar{V}_4$ into x- and y-components,

$$\Sigma x = 100 + 250 \cos(90°) + 150 \cos(30°) + 200 \cos(-45°) = 371.33$$

$$\Sigma y = 250 \sin(90°) + 150 \sin(30°) + 200 \sin(-45°) = 183.58$$

$$\text{Magnitude of } (\bar{V}_1 + \bar{V}_2 + \bar{V}_3 + \bar{V}_4) = \sqrt{(\Sigma x)^2 + (\Sigma y)^2}$$

$$= \sqrt{(371.33)^2 + (183.58)^2}$$

$$= 414.23 \text{ V}$$

$$\text{Phase angle } \phi = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right)$$

$$= \tan^{-1}\left(\frac{183.58}{371.33}\right)$$

$$= 26.31°$$

$$v = v_1 + v_2 + v_3 + v_4 = 414.23 \sin(\omega t + 26.31°)$$

By Phasor algebra

Two sinusoidal currents are given as

$$i_1 = 10\sqrt{2}\ \sin \omega t,\ i_2 = 20\sqrt{2}\ \sin(\omega t + 60°).$$

Find the expression for the sum of these currents.

$$i_1 = 10\sqrt{2}\ \sin \omega t$$

$$i_2 = 20\sqrt{2}\ \sin(\omega t + 60°)$$

Writing currents i_1 and i_2 in the phasor form,

$$\bar{I}_1 = \frac{10\sqrt{2}}{\sqrt{2}}\angle 0° = 10\angle 0°$$

$$\bar{I}_2 = \frac{20\sqrt{2}}{\sqrt{2}}\angle 60° = 20\angle 60°$$

$$\begin{aligned}\bar{I} &= \bar{I}_1 + \bar{I}_2 \\ &= 10\angle 0° + 20\angle 60° \\ &= 26.46\ \angle 40.89°\end{aligned}$$

$$\begin{aligned}i &= 26.46\sqrt{2}\sin(\omega t + 40.89°) \\ &= 37.42\sin(\omega t + 40.89°)\end{aligned}$$