- Name : Aaryan Sharma
- Roll Number: 16010123012
- Batch: A1
- Tutorial No.:1

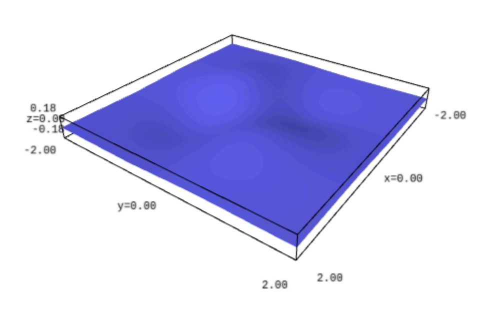
1. Plot the graph of the function f(x, y) =$xye^{-x^2-y^2}$ for $-2 \le$ x,y ≤ 2.

In [ ]:
```
var('x y')

f(x, y) = x*y*e^(-x^2 - y^2)

plot3d(f(x, y), (x, -2, 2), (y, -2, 2))
```

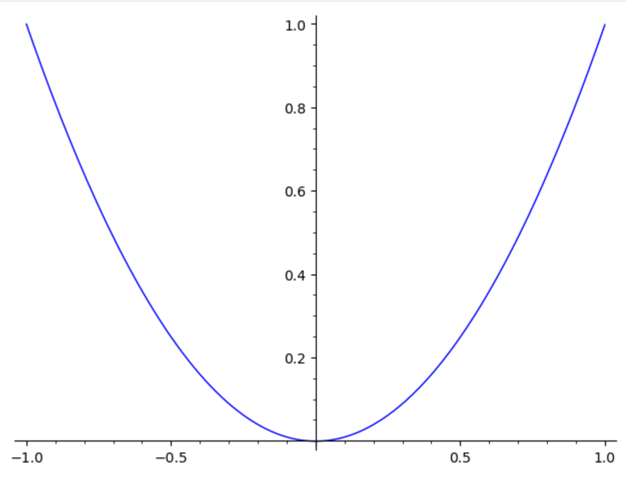
Out[ ]:



2. Find the following limits (plot graph of function):
$\lim_{x\to 0} (x^2 - \frac{2^x}{1000})$

In [ ]:
```
var('x')
f(x) = x^2 - (2^x)/1000
limit(f(x), x=0)
plot(f(x), (x, -1, 1))
```

Out[ ]:

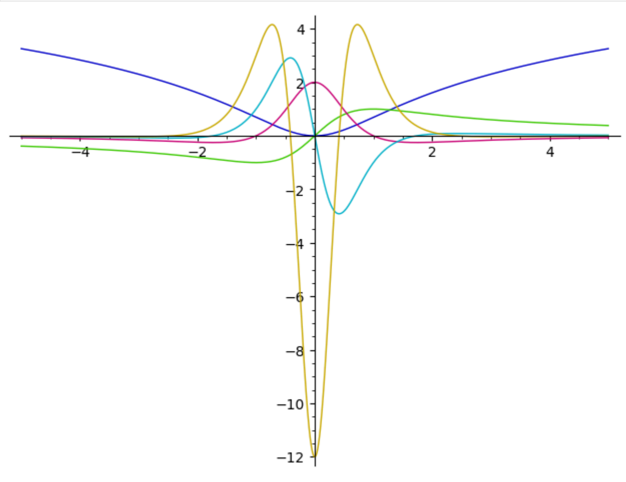


3. Find the 1st four derivative of $f(t) = \log(1 + t^2)$ and plot them along with the graph of $f(t)$.

In [ ]:
```
    var('t')
f(t) = log(1+t^2)
f1(t) = diff(f(t), t)
f2(t) = diff(f1(t), t)
f3(t) = diff(f2(t), t)
f4(t) = diff(f3(t), t)

plot([f(t), f1(t), f2(t), f3(t), f4(t)], (t, -5, 5))
```

Out[ ]:



4. What is the n-th derivative of x^x for various values of n?

In [ ]:
```
var('x, n')
f(x) = x^x
print(derivative(f(x), x, n))
```

```
0
```

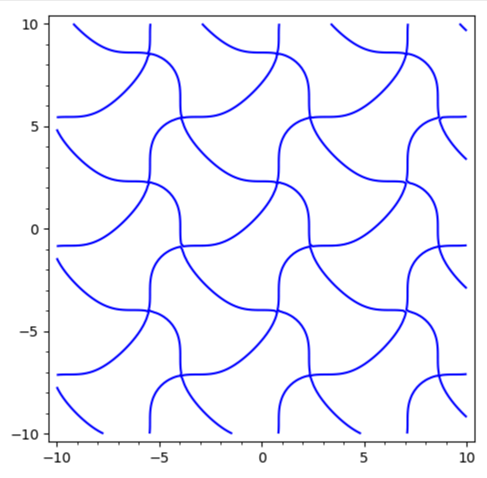
In [ ]:
```
var('x')
f(x) = x^x
n = int(input("Enter the value of n: "))
for i in range(1, n+1):
    f(x) = diff(f(x), x)
    print(f"Derivative {i}: {f(x)}")
```

```
Enter the value of n:  5
Derivative 1: x^x*(log(x) + 1)
Derivative 2: x^x*(log(x) + 1)^2 + x^x/x
Derivative 3: x^x*(log(x) + 1)^3 + 3*x^x*(log(x) + 1)/x - x^x/x^2
Derivative 4: x^x*(log(x) + 1)^4 + 6*x^x*(log(x) + 1)^2/x - 4*x^x*(log(x) + 1)/x^2 + 3*x^x/x^2 + 2*x^x/x^3
Derivative 5: x^x*(log(x) + 1)^5 + 10*x^x*(log(x) + 1)^3/x - 10*x^x*(log(x) + 1)^2/x^2 + 15*x^x*(log(x) + 1)/x^2 + 10*x^x*(log(x) + 1)/x^3 - 10*x^x/x^3 - 6*x^x/x^4
```

5. Consider the function implicitly defined $\cos(x - \sin(y)) = \sin(y - sinx)$.
i) Plot the curve represented by the given function.
ii) Find $\frac{dy}{dx}$ and $\frac{d^2y}{d^2x}$.

In [ ]:
```
var('x y')
f(x, y) = cos(x - sin(y)) - sin(y - sin(x))
implicit_plot(f(x, y) == 0, (x, -10, 10), (y, -10, 10))
```

Out[ ]:



In [ ]:
```
#  dy/dx
dg = f.diff(y)
dxg = f.diff(x)
dydx = -dg/dxg
# d^2y/dx^2
d2ydx2 = (dydx.diff(x)).subs(diff(y, x) == dydx)

print(dydx)
print(d2ydx2)
```

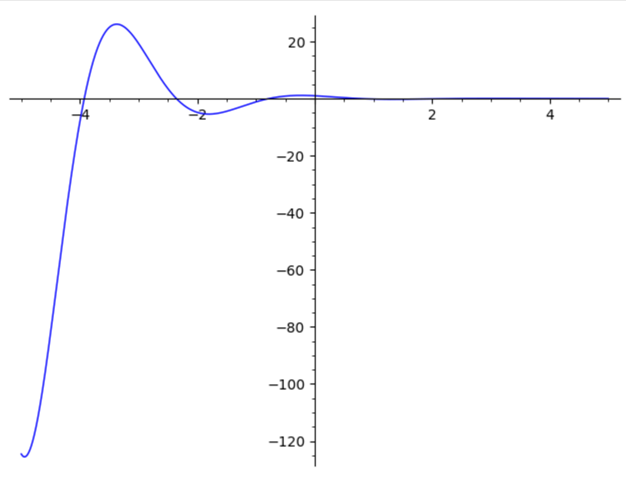
```
(x, y) |--> -(cos(y)*sin(x - sin(y)) - cos(y - sin(x)))/(cos(x)*cos(y - sin(x)) - sin(x - sin(y)))
(x, y) |--> -(cos(x - sin(y))*cos(y) - cos(x)*sin(y - sin(x)))/(cos(x)*cos(y - sin(x)) - sin(x - sin(y))) + (cos(x)^2*sin(y - sin(x)) - cos(y - sin(x))*sin(x) - cos(x - sin(y)))*(cos(y)*sin(x - sin(y)) - cos(y - sin(x)))/(cos(x)*cos(y - sin(x)) - sin(x - sin(y)))^2
```

6. Consider $f(x) = e^{-x}cos2x$.
(i) Plot the graph of the function along with Taylor's polynomial of degree 1, 2, 3, 6, 7, 9, 10.
(ii) Use sage interacts to create interactive plot to plot Taylor's polynomial along with the curve

In [ ]:
```
var('x')
f(x) = e^(-x)*cos(2*x)

# Plot the graph of the function
plot(f(x), (x, -5, 5))
```

Out[ ]:



In [ ]:
```
var('x')
f(x) = e^(-x)*cos(2*x)

T1(x) = f(0) + f.diff(x)(0)*x
T2(x) = T1(x) + f.diff(x, 2)(0)*x^2/2
T3(x) = T2(x) + f.diff(x, 3)(0)*x^3/6
T6(x) = f(0) + sum(f.diff(x, i)(0)*x^i/factorial(i) for i in range(1, 7))
T7(x) = T6(x) + f.diff(x, 7)(0)*x^7/factorial(7)
T9(x) = f(0) + sum(f.diff(x, i)(0)*x^i/factorial(i) for i in range(1, 10))
T10(x) = T9(x) + f.diff(x, 10)(0)*x^10/factorial(10)


plot(T1(x), (x, -5, 5), color='red')
plot(T2(x), (x, -5, 5), color='green')
plot(T3(x), (x, -5, 5), color='blue')
plot(T6(x), (x, -5, 5), color='orange')
plot(T7(x), (x, -5, 5), color='purple')
plot(T9(x), (x, -5, 5), color='pink')
plot(T10(x), (x, -5, 5), color='brown')


@interact
def _(deg=(1..10)):
    T = f(0) + sum(f.diff(x, i)(0)*x^i/factorial(i) for i in range(1, deg+1))
    show(plot(f(x), (x, -5, 5)) + plot(T, (x, -5, 5), color='red'))
```

```
Interactive function <function _ at 0x7faac892f920> with 1 widget
  deg: SelectionSlider(description='deg', options=(1, 2, 3, 4, 5, 6, 7, 8, 9, 10), value=1)
```

7. Evaluate the following indefinite integrals.
i) $\int \frac{-4}{\sqrt{1-x^2}} dx$
ii) $\int sin^5 x dx$

In [ ]:
```
var('x')
f(x) = -4/sqrt(1-x^2)
integrate(f(x), x)
```

Out[ ]: `-4*arcsin(x)`

In [ ]:
```
var('x')
f(x) = sin(x)^5
integral(f(x), x)
```

Out[ ]: `-1/5*cos(x)^5 + 2/3*cos(x)^3 - cos(x)`

8 Evaluate the following definite integrals.
(i) $\int_1^4 \frac{3x}{\sqrt{3x-1}} dx$
(ii) $\int_{\pi/3}^{\pi/2} \frac{1}{1+sinx-cosx} dx$

In [ ]:
```
var('x')
f(x) = 3*x/sqrt(3*x-1)
integral(f(x), x, 1, 4)
```

Out[ ]: `28/9*sqrt(11) - 10/9*sqrt(2)`

In [ ]:
```
var('x')
f(x) = 1/(1+sin(x)-cos(x))
integral(f(x), x, pi/3, pi/2)
```

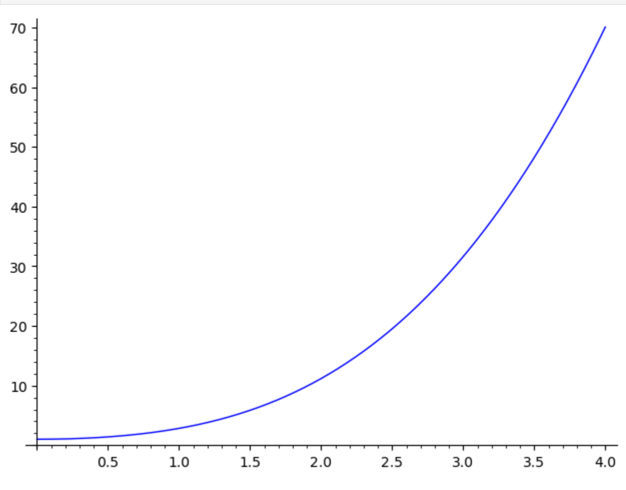
Out[ ]: `1/2*log(3) - log(2) + log(1/3*sqrt(3) + 1)`

9 Graph the curve $y = (1 + x^2)^{3/2}$ for $0 \le$ x ≤ 4 and hence find its arc length.

In [ ]:
```
var('x')
f(x) = (1+x^2)^(3/2)


plot(f(x), (x, 0, 4))
```

Out[ ]:



In [ ]:
```
arc_length = integral(sqrt(1 + diff(f(x), x)^2), x, 0, 4)
print(arc_length)
```

```
integrate(sqrt(9*(x^2 + 1)*x^2 + 1), x, 0, 4)
```

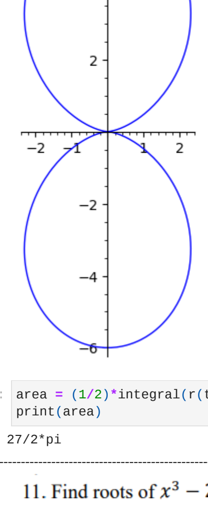
10 Find the area that the curve r = 3(1 − cos 2θ), 0 ≤ θ ≤ 2π encloses.

In [ ]:
```
var('theta')
r(theta) = 3*(1 - cos(2*theta))


x(theta) = r(theta)*cos(theta)
y(theta) = r(theta)*sin(theta)


parametric_plot((x(theta), y(theta)), (theta, 0, 2*pi))
```

Out[ ]:



In [ ]:
```
area = (1/2)*integral(r(theta)^2, theta, 0, 2*pi)
print(area)
```

```
27/2*pi
```

11. Find roots of $x^3 - 2x^2 - 5x + 6 = 0$ for x.

In [ ]:
```
x = var('x')
f(x) = x^3 - 2*x^2 - 5*x + 6
roots = solve(f(x) == 0, x)
print(roots)
```

```
[
x == 3,
x == -2,
x == 1
]
```

12. Solve the system of nonlinear equations $x^2 + y^2 = 4$ and $y = x^2 - 2$ for x and y.

In [ ]:
```
var('x y')
eq1 = x^2 + y^2 == 4
eq2 = y == x^2 - 2
solution = solve([eq1, eq2], x, y)
print(solution)
```

```
[
[x == -sqrt(3), y == 1],
[x == sqrt(3), y == 1],
```

```
[x == 0, y == -2]
]
```

In [ ]: