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## Batch: A1

## TUT5

### LAPLACE TRANSFORM:

**Q.1 Find the Laplace Transform of the following functions:**

In [3]:

```
t, s = var('t s')
f = t^3 * cos(2*t)
laplace_transform = laplace(f, t, s)
show(laplace_transform)
```

$$\frac{48\,s^4}{(s^2+4)^4} - \frac{48\,s^2}{(s^2+4)^3} + \frac{6}{(s^2+4)^2}$$

In [4]:

```
t, s = var('t', 's')
f = (e**(2*t) - e**(3*t))/t
laplace_transform = laplace(f, t, s)
show(laplace_transform)
```

$$\log\left(\frac{s-3}{s-2}\right)$$

In [6]:

```
t,s =var('t s')
f= exp(-5 * t) * sin(3*t)
show(f.laplace(t,s))
```

$$\frac{3}{s^2+10\,s+34}$$

**Q.2 Find the Inverse Laplace Transform of the following Functions:**

In [7]:

```
s = var('s')
F = 1 / (s^4 + 13*s^2 + 36)
inverse_laplace(F(s),s,t)
show(inverse_laplace(F(s),s,t))
```

$$-\frac{1}{15}\sin(3\,t) + \frac{1}{10}\sin(2\,t)$$

In [8]:

```
s = var('s')
F = (s + s^2) / ((s^2 + 1) * (s^2 + 2*s + 2))
show(inverse_laplace(F(s),s,t))
```

$$-\frac{1}{5}\,(3\,\cos(t) - \sin(t))e^{(-t)} + \frac{3}{5}\cos(t) + \frac{1}{5}\sin(t)$$

**Q.3 Solve the following differential equation using Laplace Transform:**

In [9]:

```
s,t = var('s t')
x=function('x')(t)
de=diff(x,t,t)-diff(x,t)-2*x== 20*sin(2*t)
show(desolve_laplace(de,x,ics=[0,1,2]))
```

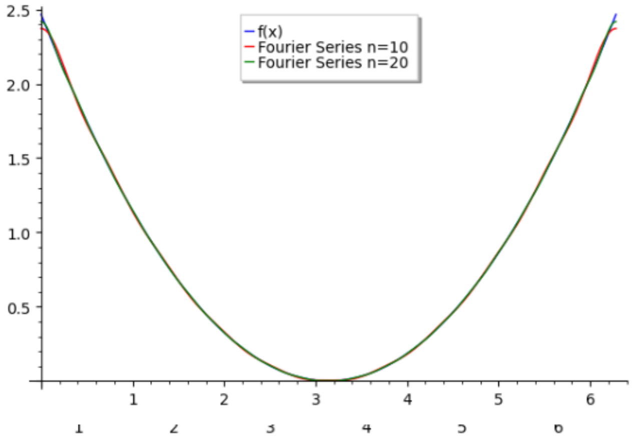
$$\cos(2\,t) + \frac{8}{3}\,e^{(2\,t)} - \frac{8}{3}\,e^{(-t)} - 3\,\sin(2\,t)$$

### FOURIER SERIES:

**Q.1 Find all the Fourier Coefficients and Fourier Series for the following functions. Also plot the graph of the function and the Fourier series:**

**(i) f(x)= ((π-x)/2)^2 in (0,2π) for n=10 and n=20**

In [11]:
```
var('x n')
L = pi
f(x) = ((pi - x)/2)^2
a0 = (1/L) * integrate(f(x), x, 0, 2*pi)
an = (1/L) * integrate(f(x) * cos(n*pi*x/L), x, 0, 2*pi)
bn = (1/L) * integrate(f(x) * sin(n*pi*x/L), x, 0, 2*pi)
s1=a0/2 + sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,10)
s2=a0/2 + sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,20)
p1 = plot(f(x), (x, 0, 2*L), color="blue", legend_label="f(x)")
p2 = plot(s1, (x, 0, 2*L), color="red", legend_label="Fourier Series n=10")
p3 = plot(s2, (x, 0, 2*L), color="green", legend_label="Fourier Series n=20")
(p1 + p2 + p3).show()
show(a0)
show(an)
show(bn)
print("Fourier series for n=10 is \n")
show(s1)
print("Fourier series for n=20 is \n")
show(s2)
```




$\frac{1}{6}\pi^2$

$\frac{1}{n^2}$

$0$

```
Fourier series for n=10 is
```

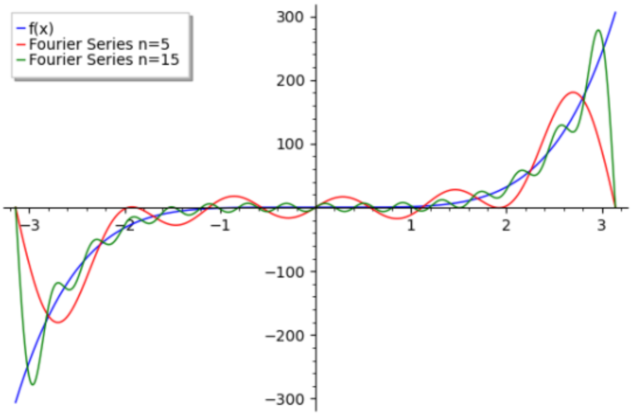
$$\frac{1}{12}\pi^2 + \frac{1}{100}\cos(10x) + \frac{1}{81}\cos(9x) + \frac{1}{64}\cos(8x) + \frac{1}{49}\cos(7x) + \frac{1}{36}\cos(6x) + \frac{1}{25}\cos(5x) + \frac{1}{16}\cos(4x) + \frac{1}{9}\cos(3x) + \frac{1}{4}\cos(2x) + \cos(x)$$

```
Fourier series for n=20 is
```

$$\frac{1}{12}\pi^2 + \frac{1}{400}\cos(20x) + \frac{1}{361}\cos(19x) + \frac{1}{324}\cos(18x) + \frac{1}{289}\cos(17x) + \frac{1}{256}\cos(16x) + \frac{1}{225}\cos(15x) + \frac{1}{196}\cos(14x) + \frac{1}{169}\cos(13x) + \frac{1}{144}\cos(12x) + \frac{1}{121}\cos(11x) + \frac{1}{100}\cos(10x) + \frac{1}{81}\cos(9x) + \frac{1}{64}\cos(8x) + \frac{1}{49}\cos(7x) + \frac{1}{36}\cos(6x) + \frac{1}{25}\cos(5x) + \frac{1}{16}\cos(4x) + \frac{1}{9}\cos(3x) + \frac{1}{4}\cos(2x) + \cos(x)$$

**(ii) f(x)=x^5 in (-π,π) for n=5 and n=15**

In [12]:
```
var('x n')
L = pi
f(x) = x^5
a0 = (1/L) * integrate(f(x), x, -L, L)
an = (1/L) * integrate(f(x) * cos(n*pi*x/L), x, -L, L)
bn = (1/L) * integrate(f(x) * sin(n*pi*x/L), x, -L, L)
s1= a0/2+sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,5)
s2= a0/2+sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,15)
p1 = plot(f(x), (x, -L, L), color="blue", legend_label="f(x)")
p2 = plot(s1, (x, -L, L), color="red", legend_label="Fourier Series n=5")
p3 = plot(s2, (x, -L, L), color="green", legend_label="Fourier Series n=15")
(p1 + p2 + p3).show()
show(a0)
show(an)
show(bn)
print("Fourier series for n=5 is \n")
show(s1)
print("Fourier series for n=15 is \n")
show(s2)
```




0

0

$$-\frac{2\left(120\,\pi + \pi^5 n^4 - 20\,\pi^3 n^2\right)(-1)^n}{\pi n^5}$$

```
Fourier series for n=5 is
```

$$\frac{2}{625}\left(125\,\pi^4 - 100\,\pi^2 + 24\right)\sin(5\,x) - \frac{1}{64}\left(32\,\pi^4 - 40\,\pi^2 + 15\right)\sin(4\,x) + \frac{2}{81}\left(27\,\pi^4 - 60\,\pi^2 + 40\right)\sin(3\,x) - \frac{1}{2}\left(2\,\pi^4 - 10\,\pi^2 + 15\right)\sin(2\,x) + 2\left(\pi^4 - 20\,\pi^2 + 120\right)\sin(x)$$

```
Fourier series for n=15 is
```

$$\frac{2}{50625}\left(3375\,\pi^4 - 300\,\pi^2 + 8\right)\sin(15\,x) - \frac{1}{33614}\left(4802\,\pi^4 - 490\,\pi^2 + 15\right)\sin(14\,x) + \frac{2}{371293}\left(28561\,\pi^4 - 3380\,\pi^2 + 120\right)\sin(13\,x) - \frac{1}{5184}\left(864\,\pi^4 - 120\,\pi^2 + 5\right)\sin(12\,x) + \frac{2}{161051}\left(14641\,\pi^4 - 2420\,\pi^2 + 120\right)\sin(11\,x) - \frac{1}{1250}\left(250\,\pi^4 - 50\,\pi^2 + 3\right)\sin(10\,x) + \frac{2}{19683}\left(2187\,\pi^4 - 540\,\pi^2 + 40\right)\sin(9\,x) - \frac{1}{2048}\left(512\,\pi^4 - 160\,\pi^2 + 15\right)\sin(8\,x) + \frac{2}{16807}\left(2401\,\pi^4 - 980\,\pi^2 + 120\right)\sin(7\,x) - \frac{1}{162}\left(54\,\pi^4 - 30\,\pi^2 + 5\right)\sin(6\,x) + \frac{2}{625}\left(125\,\pi^4 - 100\,\pi^2 + 24\right)\sin(5\,x) - \frac{1}{64}\left(32\,\pi^4 - 40\,\pi^2 + 15\right)\sin(4\,x) + \frac{2}{81}\left(27\,\pi^4 - 60\,\pi^2 + 40\right)\sin(3\,x) - \frac{1}{2}\left(2\,\pi^4 - 10\,\pi^2 + 15\right)\sin(2\,x) + 2\left(\pi^4 - 20\,\pi^2 + 120\right)\sin(x)$$

**Q.2 Find the Half range cosine series for f(x)= x 0<x<2 for n=20 . Also plot the graph of the function and the cosine series.**

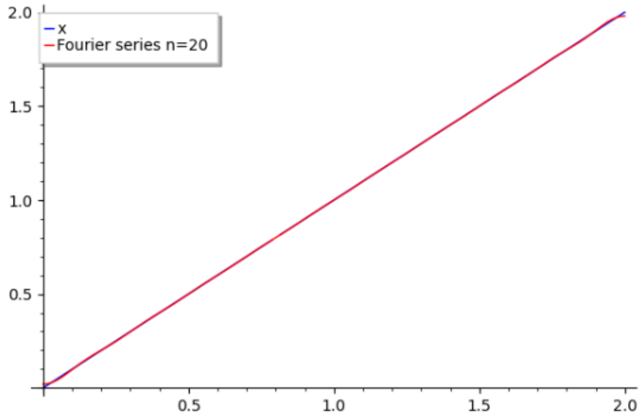
In [13]:
```
var('x')
var('n')
assume(n,'integer')
L = 2
f(x) = x
a0 = (2/L) * integrate(f(x), x, 0, 2)
an = (2/L) * integrate(f(x) * cos(n * pi * x / L), x, 0, 2)
S=a0/2 + sum(an*cos(n*pi*x/L),n,1,20)
show(a0)
show(an)
show(S)
plot(f(x),0,L,legend_label="x") + plot(S,0,L,color = "red",legend_label="Fourier series n=20")
```

$$2$$

$$\frac{4\,(-1)^n}{\pi^2 n^2} - \frac{4}{\pi^2 n^2}$$

$$-\frac{8\left(586396035225\,\cos\left(\frac{19}{2}\,\pi x\right) + 732487781025\,\cos\left(\frac{17}{2}\,\pi x\right) + 940839860961\,\cos\left(\frac{15}{2}\,\pi x\right) + 1252597448025\,\cos\left(\frac{13}{2}\,\pi x\right) + 1749495609225\,\cos\left(\frac{11}{2}\,\pi x\right) + 2613444058225\,\cos\left(\frac{9}{2}\,\pi x\right) + 4320183035025\,\cos\left(\frac{7}{2}\,\pi x\right) + 8467558748649\,\cos\left(\frac{5}{2}\,\pi x\right) + 23520996524025\,\cos\left(\frac{3}{2}\,\pi x\right) + 211688968716225\,\cos\left(\frac{1}{2}\,\pi x\right)\right)}{211688968716225\,\pi^2} + 1$$

Out[13]:



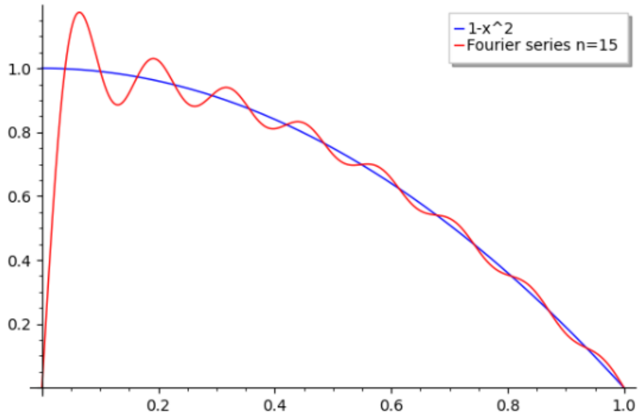

**Q.3 Find the Half range sine series for f(x)= 1-x^2 in (0,1) for n=15 . Also plot the graph of the function and the sine series.**

In [14]:
```
var('x')
var('n')
assume(n,'integer')
L = 1
f(x)= 1-x^2
bn=(2/L)*integrate(f(x)*sin(n*pi*x/L),x,0,L)
s= sum(bn*sin(n*pi*x/L),n,1,15)
show(bn)
show(s)
plot(f(x),0,L,legend_label="1-x^2") + plot(s,0,L,color = "red",legend_label="Fourier series n=15")
```

$$\frac{2\left(\pi^2 n^2+2\right)}{\pi^3 n^3}-\frac{4(-1)^n}{\pi^3 n^3}$$

$$\frac{\begin{array}{l}52227799123500\,\pi^2\sin(14\,\pi x)+60932432310750\,\pi^2\sin(12\,\pi x)+73118918772900\,\pi^2\sin(10\,\pi x)+91398648466125\,\pi^2\sin(8\,\pi x)\\+121864864621500\,\pi^2\sin(6\,\pi x)+182797296932250\,\pi^2\sin(4\,\pi x)+365594593864500\,\pi^2\sin(2\,\pi x)+216648648216\left(225\,\pi^2+4\right)\sin(15\,\pi x)\\+332812557000\left(169\,\pi^2+4\right)\sin(13\,\pi x)+549353259000\left(121\,\pi^2+4\right)\sin(11\,\pi x)+1003003001000\left(81\,\pi^2+4\right)\sin(9\,\pi x)+213174690300\\\left(49\,\pi^2+4\right)\sin(7\,\pi x)+5849513501832\left(25\,\pi^2+4\right)\sin(5\,\pi x)+27081081027000\left(9\,\pi^2+4\right)\sin(3\,\pi x)+731189187729000\left(\pi^2+4\right)\sin(\pi x)\end{array}}{365594593864500\,\pi^3}$$

Out[14]:




**Q.4 Find the Fourier series (n=15) , a10 and b15 for f(x)=x(π-x) in (-π,π).**

In [15]:

```
var('x n')
assume(n,'integer')
f(x)=x*(pi-x)
L=pi
a0=1/L*integrate(f,x,-L,L)
an=1/L*integrate(f*cos(n*pi*x/L),x,-L,L)
bn=1/L*integrate(f*sin(n*pi*x/L),x,-L,L)
s=a0/2+sum(an*cos(n*pi*x/L)+bn*sin(n*pi*x/L),n,1,15)
f1=an.substitute(n=10)
f2=bn.substitute(n=15)
show("value of a0: ",a0)
show("value of an: ",an)
show("value of bn: ",bn)
show("sum of fourier series upto n=15:  ",s)
show("value of a10: ",f1)
show("value of b15: ",f2)
```

value of a0: $-\frac{2}{3}\pi^2$

value of an: $-\frac{4(-1)^n}{n^2}$

value of bn: $-\frac{2\left(\frac{(\pi^2n^2-1)(-1)^n}{n^3}+\frac{(-1)^n}{n^3}\right)}{\pi}$

sum of fourier series upto n=15: $-\frac{1}{3}\pi^2+\frac{2}{15}\pi\sin(15\,x)-\frac{1}{7}\pi\sin(14\,x)+\frac{2}{13}\pi\sin(13\,x)-\frac{1}{6}\pi\sin(12\,x)+\frac{2}{11}\pi\sin(11\,x)-\frac{1}{5}\pi\sin(10\,x)+\frac{2}{9}\pi\sin(9\,x)-\frac{1}{4}\pi\sin(8\,x)+\frac{2}{7}\pi\sin(7\,x)-\frac{1}{3}\pi\sin(6\,x)+\frac{2}{5}\pi\sin(5\,x)-\frac{1}{2}\pi\sin(4\,x)+\frac{2}{3}\pi\sin(3\,x)-\pi\sin(2\,x)+2\,\pi\sin(x)+\frac{4}{225}\cos(15\,x)-\frac{1}{49}\cos(14\,x)+\frac{4}{169}\cos(13\,x)-\frac{1}{36}\cos(12\,x)+\frac{4}{121}\cos(11\,x)-\frac{1}{25}\cos(10\,x)+\frac{4}{81}\cos(9\,x)-\frac{1}{16}\cos(8\,x)+\frac{4}{49}\cos(7\,x)-\frac{1}{9}\cos(6\,x)+\frac{4}{25}\cos(5\,x)-\frac{1}{4}\cos(4\,x)+\frac{4}{9}\cos(3\,x)-\cos(2\,x)+4\cos(x)$

value of a10: $-\frac{1}{25}$

value of b15: $\frac{2}{15}\pi$