

Gauss's theorem in differential and Integral form:

Gauss's thm.

$$\phi_E = \frac{q}{\epsilon} \rightarrow ①$$

$$\therefore \phi_E = \int_{\text{Surface}} \vec{E} \cdot d\vec{s} \rightarrow ②$$

$$S = \frac{dq}{d\vartheta} \Rightarrow dq = S d\vartheta$$

$$q = \int_{\text{Volume}} dq = \int_{\text{Vol}} S d\vartheta \rightarrow ③$$

∴ from ①, ② and ③

$$\int_{\text{Surface}} \vec{E} \cdot d\vec{s} = \frac{1}{\epsilon} \int_{\text{Vol}} S d\vartheta \quad \therefore \epsilon \vec{E} = \vec{D}$$

$$\boxed{\int_{\text{Surface}} \vec{D} \cdot d\vec{s} = \int_{\text{Vol}} S d\vartheta} \Rightarrow \text{Integral form.}$$

using fundamental thm. of div.

$$\int_{\text{Surface}} \vec{D} \cdot d\vec{s} = \int_{\text{Vol}} (\vec{\nabla} \cdot \vec{D}) d\vartheta$$

$$\therefore \int_{\text{Vol}} (\vec{\nabla} \cdot \vec{D}) d\vartheta = \int_{\text{Vol}} S d\vartheta$$

$$\therefore \boxed{\vec{\nabla} \cdot \vec{D} = S} \Rightarrow \text{Differential form}$$

Electric Potential : It is a scalar quantity used to measure strength of a charge at a given point.

It is defined as, work done to bring unit +ve charge from ∞ to the given point.

It is also defined as a quantity whose rate of change in any direction is the electric intensity in that direction.

$$E = - \frac{dV}{dx} \quad \text{along } x\text{-axis}$$

$$\vec{E} = - \left(\hat{i} \frac{\partial V}{\partial x} + \hat{j} \frac{\partial V}{\partial y} + \hat{k} \frac{\partial V}{\partial z} \right) \quad (\text{in 3D})$$

$$\boxed{\vec{E} = - \vec{\nabla} V}$$

$$\therefore \vec{E} = - \frac{dV}{dr} \hat{r}$$

$$dV = - \vec{E} \cdot d\vec{r}$$

$$V = - \int \vec{E} \cdot d\vec{r}$$

The electric potential difference between two points 'a' and 'b'

$$V(b) - V(a) = - \int_a^b \vec{E} \cdot d\vec{r}$$

Magnetic field :-

Magnetic field is defined as a space in which a moving charge experiences a velocity dependent force.

The science of time-independent magnetic fields caused by steady currents is known as magnetostatics.

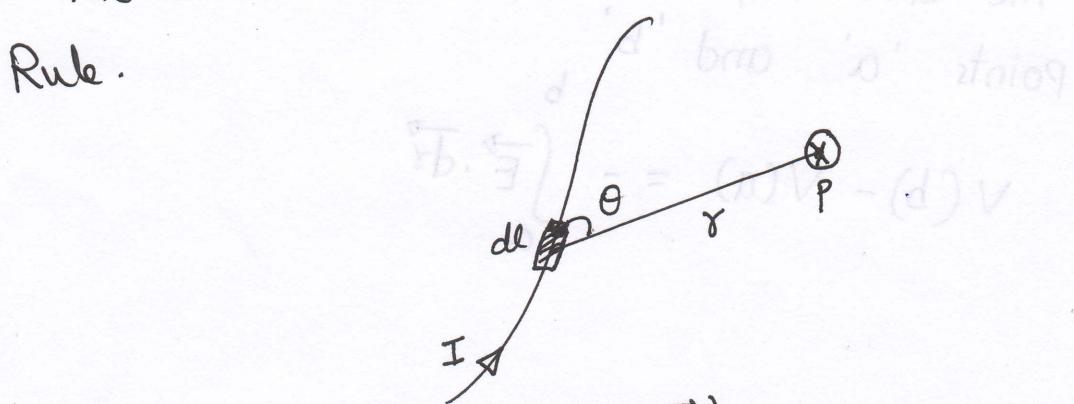
In 1819, Oersted observed that a current carrying wire produces magnetic field around it. This phenomenon is called Magnetic Effect of electric current.

Biot-Savart's law

for length element dl , carrying current I , the magnetic induction $d\vec{B}$

$$\overrightarrow{d\vec{B}} = \left(\frac{\mu_0}{4\pi} \right) I \left(\frac{dl \times \vec{r}}{r^3} \right)$$

The direction of $\overrightarrow{d\vec{B}}$ is given by Right Hand Rule.



$$\overrightarrow{B} = \int \overrightarrow{d\vec{B}} = \int \left(\frac{\mu_0}{4\pi} \right) \frac{I (dl \times \vec{r})}{r^3}$$

Ampere's law in Integral and Differential form

Ampere's law

$$\oint_{\text{line}} \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\therefore I = \int_{\text{Surface}} \vec{J} \cdot d\vec{s}$$

$$\therefore \oint_{\text{line}} \vec{B} \cdot d\vec{l} = \mu_0 \int_{\text{Surface}} \vec{J} \cdot d\vec{s}$$

$$\boxed{\oint_{\text{line}} \vec{H} \cdot d\vec{l} = \int_{\text{Surface}} \vec{J} \cdot d\vec{s}} \Rightarrow \text{Integral form}$$

From fundamental thm. of curl

$$\int_{\text{line}} \vec{H} \cdot d\vec{l} = \int_{\text{Surface}} (\vec{\nabla} \times \vec{H}) \cdot d\vec{s}$$

$$\therefore \int_{\text{Surface}} (\vec{\nabla} \times \vec{H}) \cdot d\vec{s} = \int_{\text{Surface}} \vec{J} \cdot d\vec{s}$$

\Rightarrow Differential form

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J}}$$

Gauss' thm in Magnetism

$$\phi_M = \oint_{\text{Surface}} \vec{B} \cdot d\vec{s}$$

\therefore lines of mag. field have neither begining or ending

$$\therefore \int_{\text{Surface}} \vec{B} \cdot d\vec{s} = 0$$

$$\int_{\text{Vol}} (\vec{\nabla} \cdot \vec{B}) d\vec{v} = 0$$

Vol

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$$

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Faraday's law in Integral and Differential form:

$$e = - \frac{d\phi_M}{dt} \rightarrow ①$$

$$e = \oint_{\text{line}} \vec{E} \cdot d\vec{l} \rightarrow ②$$

$$\phi_M = \iint_{\text{Surface}} \vec{B} \cdot d\vec{s} \rightarrow ③$$

$$\therefore \boxed{\oint_{\text{line}} \vec{E} \cdot d\vec{l} = - \iint_{\text{Surface}} \frac{d\vec{B}}{dt} \cdot d\vec{s}} \Rightarrow \text{Integral form}$$

from stoke's thm

$$\oint_{\text{line}} \vec{E} \cdot d\vec{l} = \iint_{\text{Surface}} (\nabla \times \vec{E}) \cdot d\vec{s}$$

$$\therefore \iint_{\text{Surface}} (\nabla \times \vec{E}) \cdot d\vec{s} = - \frac{\partial}{\partial t} \iint_{\text{Surface}} \vec{B} \cdot d\vec{s}$$

$$\therefore \boxed{\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}} \Rightarrow \text{Differential form}$$

Displacement current: From continuity eqⁿ

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\nabla \cdot \vec{J} = - \frac{\partial \rho}{\partial t} \rightarrow ①$$

Ampere's law is, $\nabla \times \vec{H} = \vec{J}$

Taking div. of both sides

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J}$$

$$0 = \nabla \cdot \vec{J}$$

$$\nabla \cdot \vec{J} = 0 \rightarrow ②$$

but $\bar{\nabla} \cdot \bar{J} \neq 0$ according to continuity eq'.

Maxwell modified Ampere's law by adding time varying electric field.

$$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}_D \rightarrow ③$$

J_D is called displacement current density

Taking div. of eq' ③

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{J}_D \rightarrow ④$$

$$\therefore \vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = 0$$

$$\therefore \vec{\nabla} \cdot \vec{J} = - \frac{\partial \vec{E}}{\partial t}$$

from ④

$$\therefore 0 = - \frac{\partial \vec{E}}{\partial t} + \vec{\nabla} \cdot \vec{J}_D$$

$$\therefore \vec{\nabla} \cdot \vec{J}_D = \frac{\partial \vec{E}}{\partial t} \rightarrow ⑤$$

$$\therefore \vec{\nabla} \cdot \vec{D} = \vec{S}$$

$$\vec{\nabla} \cdot \vec{J}_D = \frac{\partial}{\partial t} (\vec{\nabla} \cdot \vec{D})$$

$$\vec{\nabla} \cdot \vec{J}_D = \vec{\nabla} \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\vec{J}_D = \frac{\partial \vec{D}}{\partial t} \rightarrow ⑥$$

∴ from ③, modified Ampere's law is

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

Maxwell's equations:

The field equations which govern the time-varying electric and magnetic fields are now written as

(A) Differential form:

- (i) Gauss's law $\nabla \cdot \vec{D} = \rho$
- (ii) Gauss's law for magnetism, $\nabla \cdot \vec{B} = 0$
- (iii) Faraday's law, $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
- (iv) Ampere's law $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

(B) Integral form:

- (i) $\int_{\text{surface}} \vec{D} \cdot d\vec{s} = \int_{\text{vol}} \rho dV$
- (ii) $\oint_{\text{surface}} \vec{B} \cdot d\vec{s} = 0$
- (iii) $\int_{\text{line}} \vec{E} \cdot d\vec{l} = - \int_{\text{surface}} \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$
- (iv) $\int_{\text{line}} \vec{H} \cdot d\vec{l} = \int_{\text{surface}} \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{s}$

Physical Significance:

- (1) Maxwell's first equation shows that the total electric flux density \vec{D} through the surface enclosing a volume is equal to the charge density ρ within the volume. It means charge distribution generates a steady electric field.
- (2) Maxwell's second equation tells us that the net mag. flux through a closed surface is zero. It implies that mag. ^{mono-} poles do not exist.
- (3) The third equation shows that the emf around a closed path is equal to the time derivative of mag. flux density

through the surface bounded by the path. It means an electric field can also be generated by a time-varying mag. field.

④ Fourth equation shows that the magneto-motive force around a closed path is equal to conduction current plus time-derivative of electric flux density through any surface bounded by the path. It also shows that the mag. field is generated by time-varying electric field.

The Wave Equation: for free space $\sigma=0$ and $\vec{J}=0$. Maxwell's equations for free space can be written as

$$\nabla \cdot \vec{D} = 0$$

$$\therefore \vec{B} = \epsilon_0 \vec{E}$$

$$\nabla \cdot \vec{E} = 0 \rightarrow ①$$

$$\nabla \cdot \vec{B} = 0 \rightarrow ②$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow ③$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0}$$

$$\nabla \times \vec{B} = \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \rightarrow ④$$

Taking curl of eqⁿ ③, we get

$$\nabla \times (\nabla \times \vec{E}) = -\nabla \left(\frac{\partial \vec{B}}{\partial t} \right) = -\frac{\partial}{\partial t} (\nabla \times \vec{B}) \rightarrow ⑤$$

Sub ④ in ⑤, we get

$$\nabla \times (\nabla \times \vec{E}) = -\frac{\partial}{\partial t} \left(\mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} \rightarrow ⑥$$

$$\text{but } \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \vec{\nabla}^2 \vec{E}$$

$$\vec{\nabla}^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} \rightarrow ⑦$$

Similarly for mag. field

$$\vec{\nabla}^2 \vec{B} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} \rightarrow ⑧$$

Eqs ⑦ and ⑧ are wave equations. Any function satisfying such an eqn describes a wave. The square root of quantity is the reciprocal of the coeff. of time derivative that gives phase velocity.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

\therefore It indicates that em waves propagate with velocity

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

sub. values of μ_0 and ϵ_0

$$\frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.9 \times 10^{-12}}} = 3.0 \times 10^8 \text{ m/s}$$

$$\frac{1}{\sqrt{\mu_0 \epsilon_0}} = c \text{ (speed of light)}$$

The emergence of speed of light from em wave is great achievement of Maxwell's theory. Maxwell predicted that em disturbance should propagate in free space with a speed equal to speed of light hence light waves are em in nature.