CAPSTONE PROJECT

Simulation study of effect of void on Field Distribution in underground power cables

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1. ***Introduction***

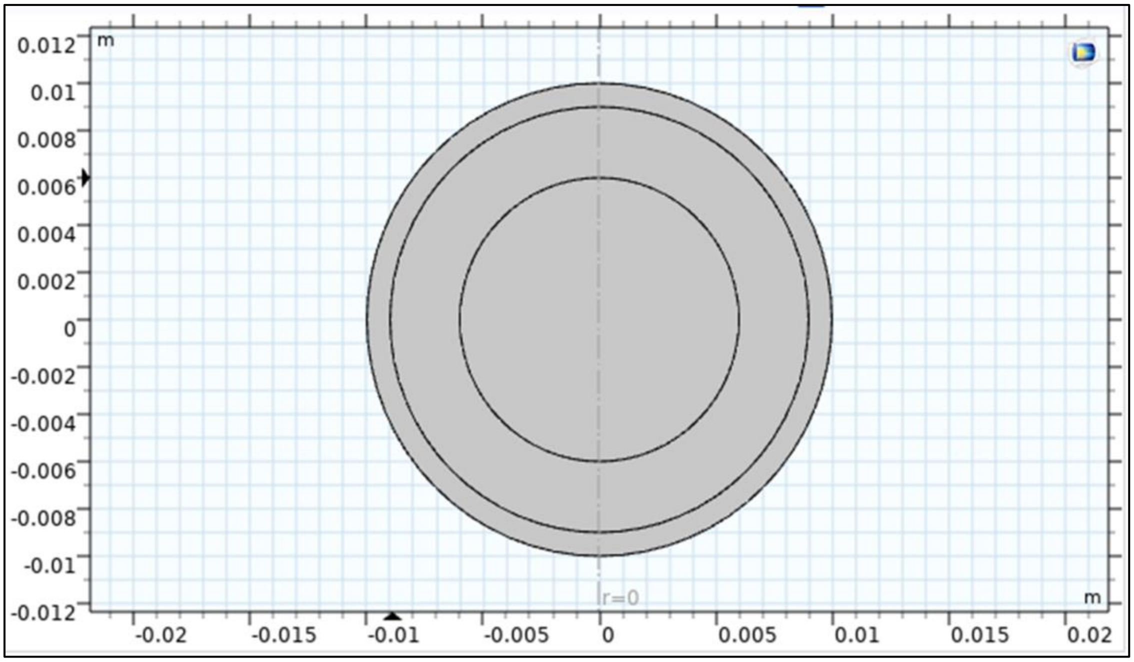
Coaxial cables are vital for high-voltage energy transmission, but defects, material imperfections, and thermal stresses can impact their performance. This study investigates the electric field and thermal behavior of a coaxial cable with four components: conductor, insulator, aluminum shield, and void. By using numerical and analytical methods, it examines the effects of void shape, position, and material non-linearities on field and temperature distributions, providing insights to enhance cable reliability and safety.

2. ***Objectives of the Work:***

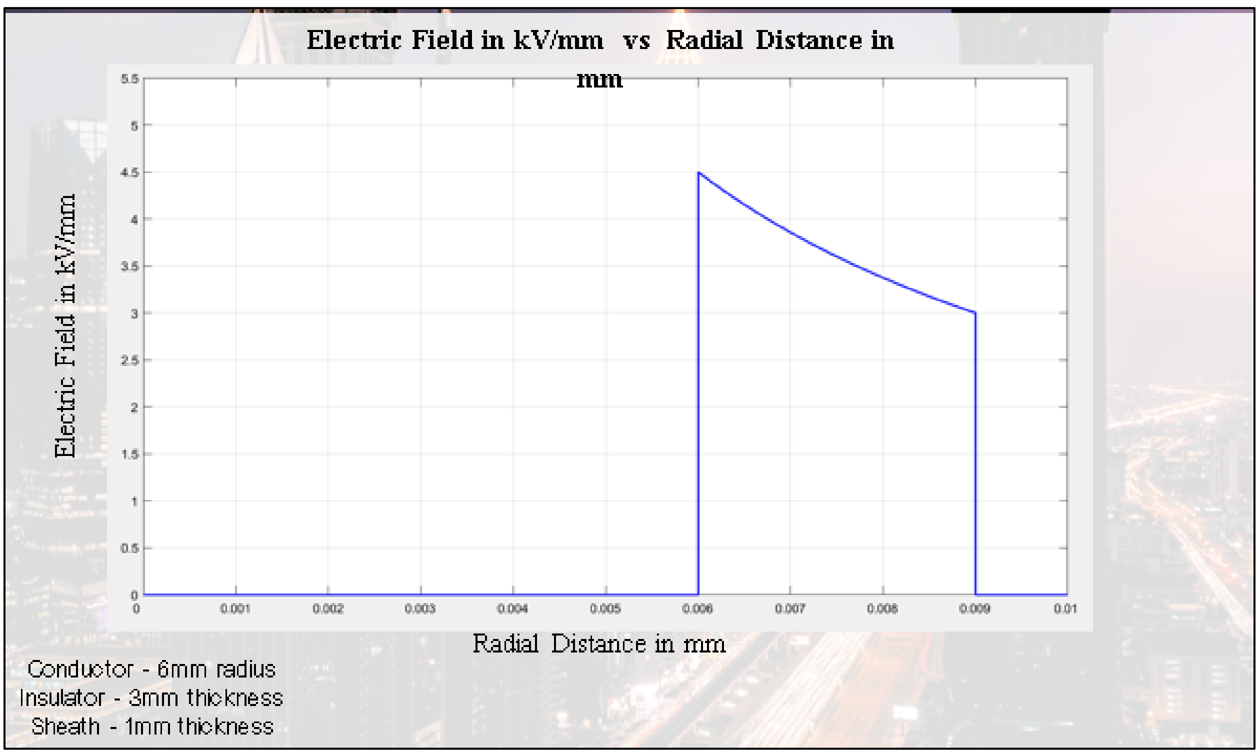
1. Electric Field Analysis: Investigate the influence of circular and elliptical voids at different positions (near and away from the conductor) on the electric field distribution.
2. Thermal Coupling: Analyze the thermal behavior of the cable under electrical stresses using coupled thermal-electrical models, addressing areas of high stress.
3. Numerical and Analytical Approaches: Employ Laplace's equation for numerical modeling and a simplified analytical three-capacitor series model to validate field behavior under an applied voltage of 11 kV.
4. Material Non-linearities: Integrate non-linear formulas for electrical permittivity and thermal conductivity to account for realistic material responses under varying field and temperature conditions.
5. Performance Comparison: Evaluate the electric field and thermal behavior along the radial axis for the different void scenarios, highlighting critical areas prone to failure.

3. Simulation/programming details and circuits

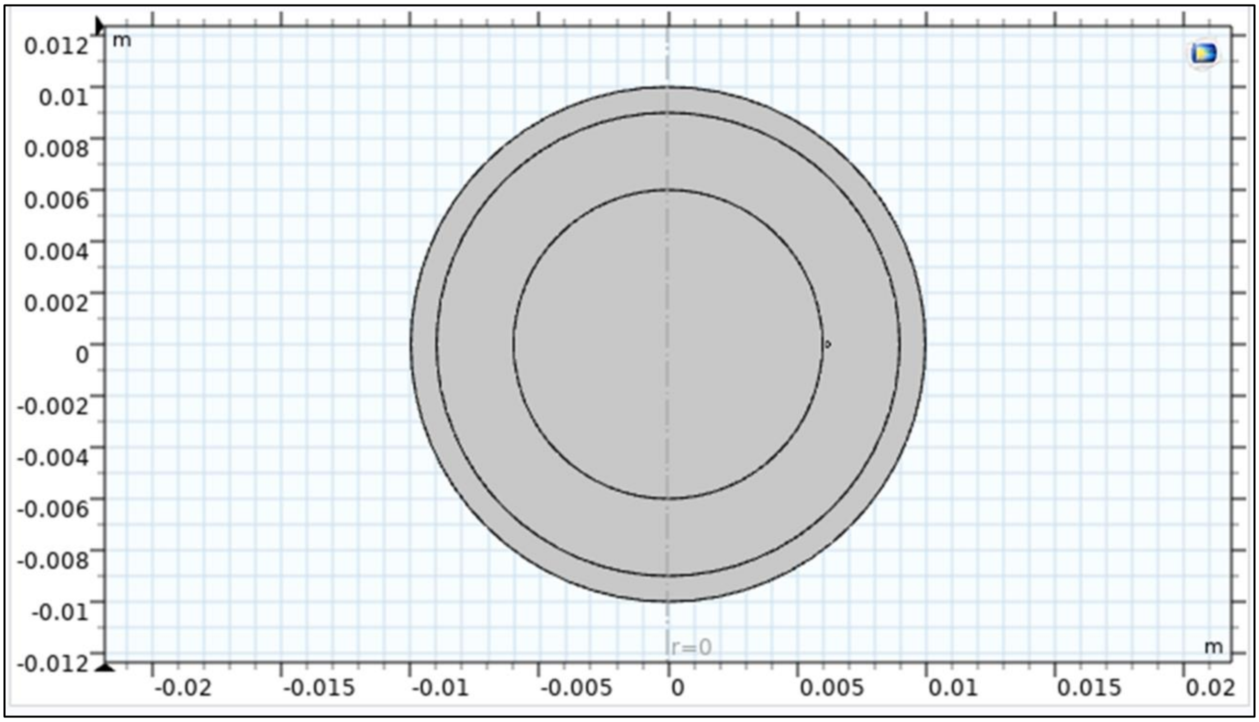
Ideal cable : With No Void



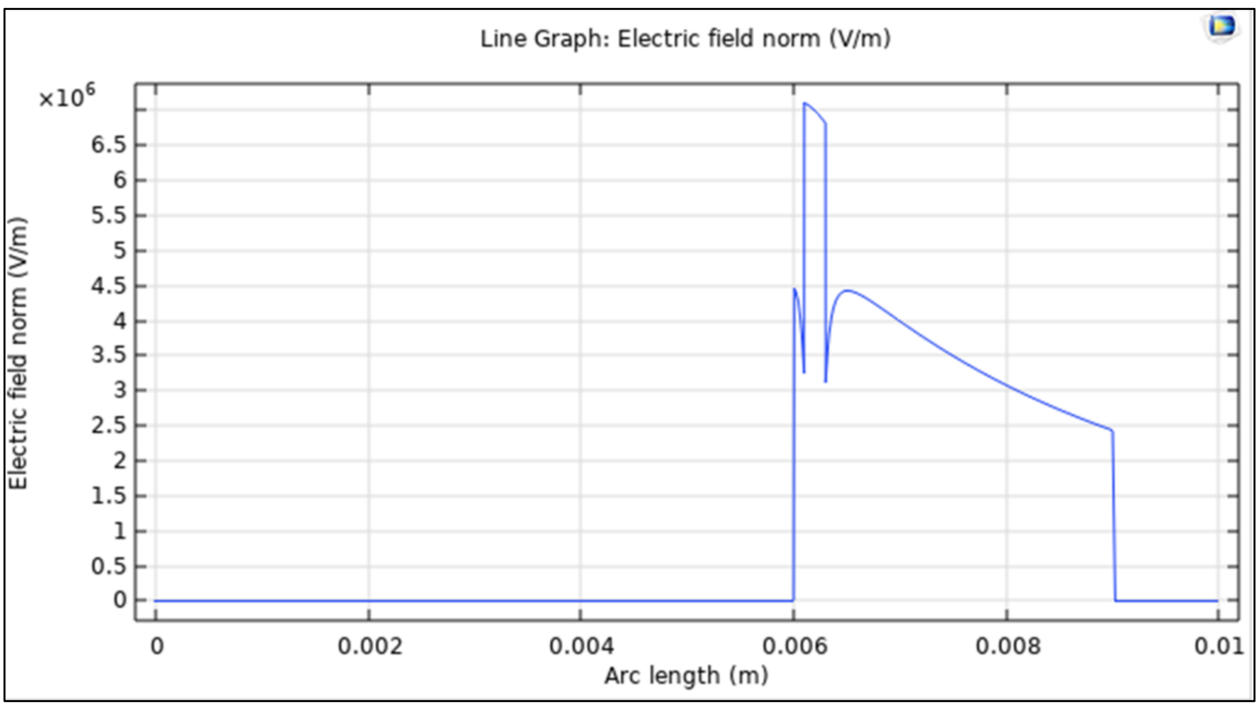
Electric Field:



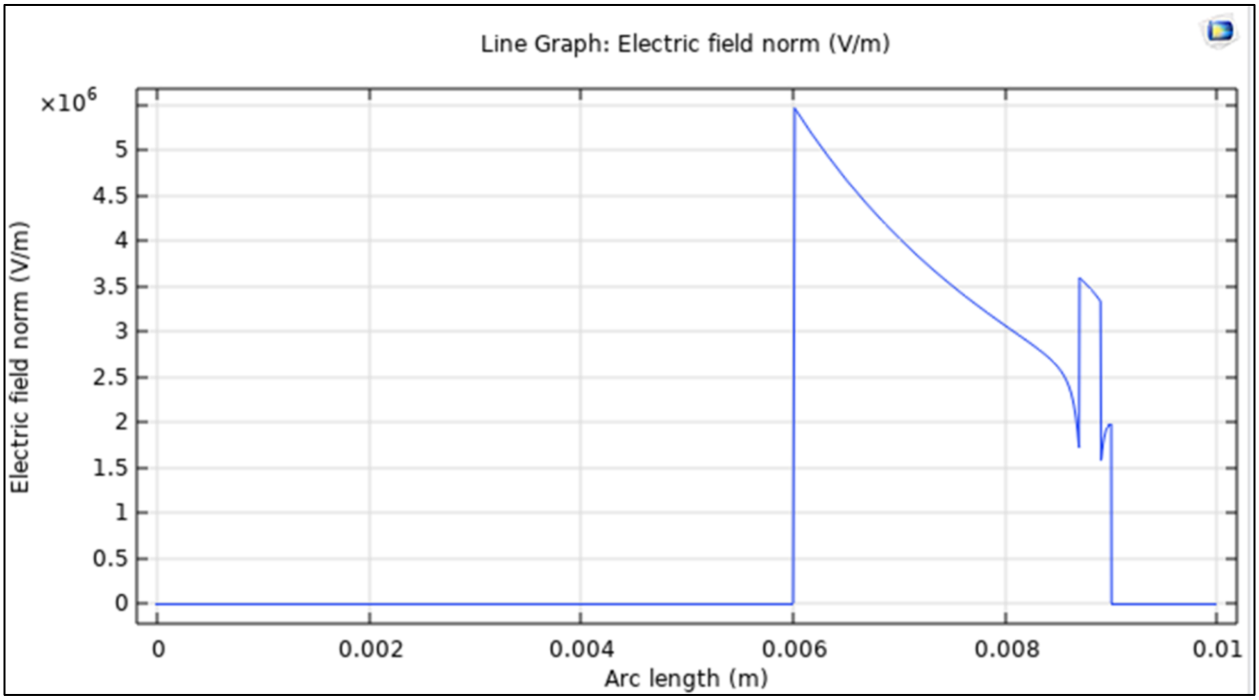
With circular void:



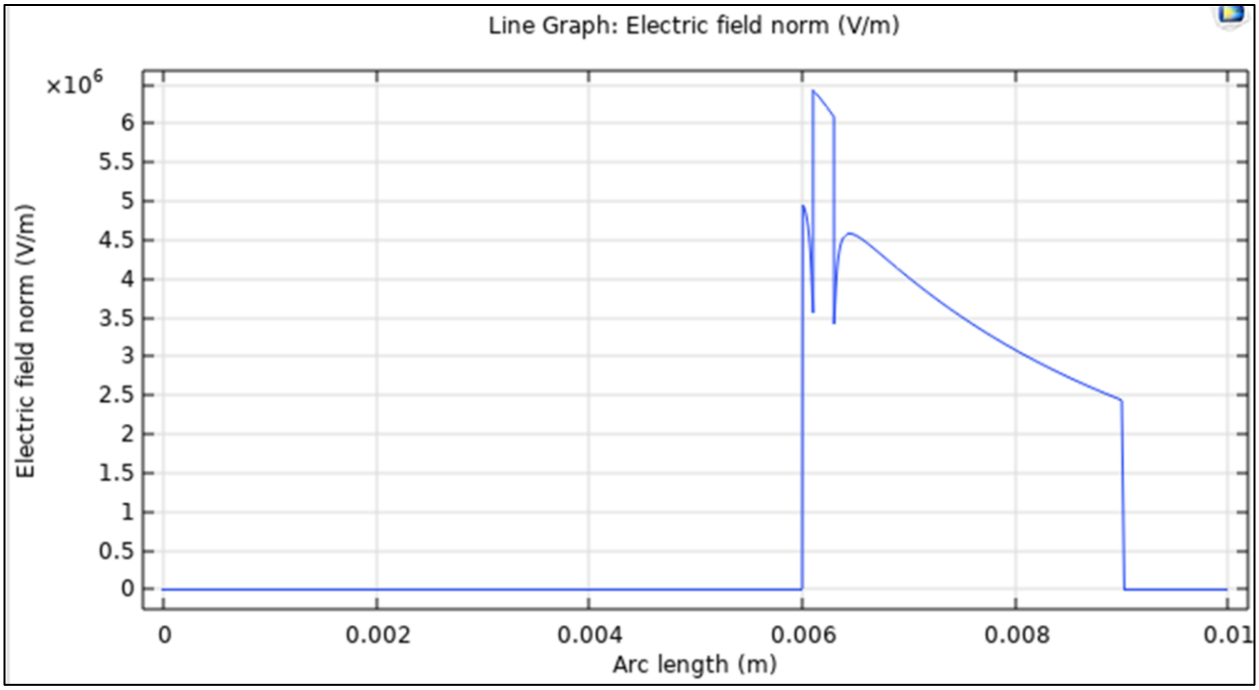
Electric Field plot when Circular Void near Conductor:



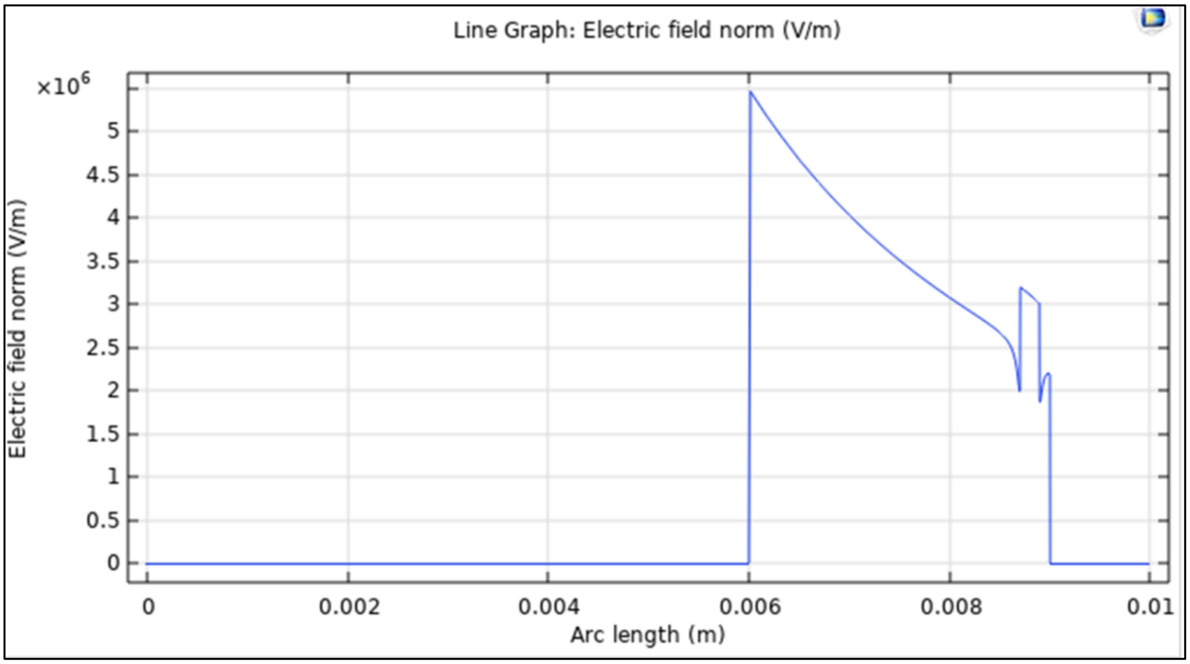
Electric Field when Circular Void away from Conductor:



Electric Field when elliptical void is near Conductor:



Electric Field when elliptical void is away from Conductor:



Comparison:

- No Void (Baseline): The electric field is smooth and uniform, serving as the ideal case for insulation performance with no risk of failure.
- Circular Void Near Conductor: Sharp field intensification occurs due to the high natural field strength near the conductor, increasing localized stress and partial discharge risk.
- Circular Void Away from Conductor: The field disturbance is less severe than near the conductor, with stress spreading outward into the insulation, reducing peak intensities but still impacting insulation stability.
- Elliptical Void Near Conductor: The elongated geometry causes even higher field intensification compared to the circular void near the conductor, with sharper peaks at the edges of the ellipse, posing the greatest risk of failure.
- Elliptical Void Away from Conductor: The disturbance spreads over a broader region, creating moderate peaks that are more significant than the circular void away from the conductor, but less critical than the near-conductor elliptical case.

Analytical method:

Poisson Equation for Cylindrical Coordinates

$$r\frac{d^2V}{dr^2} + \frac{dV}{dr} = 0$$

Dividing by r (assuming $r \neq 0$):

$$\frac{d^2V}{dr^2} + \frac{1}{r}\frac{dV}{dr} = 0$$

The general solution to this differential equation is:

$$V(r) = A\ln(r) + B$$

where A and B are constants.

Boundary Conditions:

V(r = 6 mm) = 11 kV

V(r = 9 mm) = 0

Final Expressions for Voltage and Electric Field

Voltage $V(r)$:

$$V(r) = A\ln(r) + B = 27.13\ln(r) + 127.77$$

Electric Field $E(r)$:

The electric field $E(r)$ is given by:

$$E(r) = -\frac{dV}{dr} = -\frac{A}{r} = -\frac{27.13}{r}$$

Theoretical Values

$$E = \frac{27.13}{r}$$

At $r = 6$:

$$E = \frac{27.13}{6} \approx 4.5217$$

At $r = 9$:

$$E = \frac{27.13}{9} \approx 3.0144$$

Practical Values

r = 6mm : E = 4.5 kV/mm
r= 9mm : E = 3 kV/mm

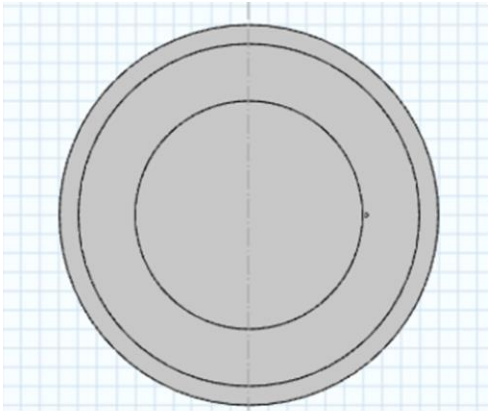
At $r = 6$:

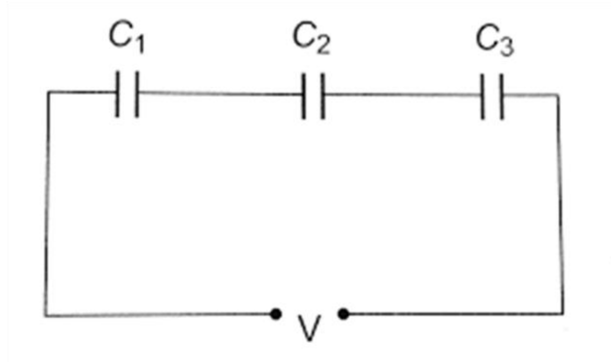
$$\text{Percentage Error} = \left|\frac{4.5 - 4.5217}{4.5}\right| \times 100 \approx \frac{0.0217}{4.5} \times 100 \approx 0.4822\%$$

At $r = 9$:

$$\text{Percentage Error} = \left|\frac{3 - 3.0144}{3}\right| \times 100 \approx \frac{0.0144}{3} \times 100 \approx 0.48\%$$

Analysing Cable as three series Capacitors





Radius of the conductor: $R_1 = 6 \text{ mm}$

Radius of the void interface: R_2 (somewhere in the void region)

Radius of the insulator: $R_3 = 9 \text{ mm}$

Relative permittivities:

- ϵ_c: Relative permittivity of conductor region (often high for conductors)
- ϵ_v: Relative permittivity of void
- ϵ_i: Relative permittivity of insulator

Boundary voltages:

- Voltage at the conductor boundary: $V(R_1) = V_1 = 11 \text{ kV}$
- Voltage at the insulator boundary: $V(R_3) = V_3 = 0 \text{ V}$

The capacitance C between two cylindrical surfaces with radii R_a and R_b and relative permittivity ϵ_r is:

$$C = \frac{2\pi\epsilon_0\epsilon_r}{\ln(R_b/R_a)}$$

where ϵ_0 is the permittivity of free space.

So we have:

- Capacitance of the conductor-void region: $C_{cv} = \frac{2\pi\epsilon_0\epsilon_c}{\ln(R_2/R_1)}$
- Capacitance of the void: $C_v = \frac{2\pi\epsilon_0\epsilon_v}{\ln(R_3/R_2)}$
- Capacitance of the insulator: $C_{vi} = \frac{2\pi\epsilon_0\epsilon_i}{\ln(R_3/R_2)}$

The total capacitance C_{total} for these capacitors in series is:

$$\frac{1}{C_{total}} = \frac{1}{C_{cv}} + \frac{1}{C_v} + \frac{1}{C_{vi}}$$

This simplifies to:

$$C_{total} = \frac{1}{\frac{\ln\left(\frac{R_2}{R_1}\right)}{2\pi\epsilon_0\epsilon_c} + \frac{\ln\left(\frac{R_3}{R_2}\right)}{2\pi\epsilon_0\epsilon_v} + \frac{\ln\left(\frac{R_3}{R_2}\right)}{2\pi\epsilon_0\epsilon_i}}$$

Voltage across the conductor-void region V_{cv}:

$$V_{cv} = V_1 \cdot \frac{C_{total}}{C_{cv}}$$

Voltage across the void V_v:

$$V_v = V_1 \cdot \frac{C_{total}}{C_v}$$

Voltage across the insulator V_{vi}:

$$V_{vi} = V_1 \cdot \frac{C_{total}}{C_{vi}}$$

$$V_{cv} = V_1 \cdot \frac{\ln\left(\frac{R_2}{R_1}\right)}{\ln\left(\frac{R_2}{R_1}\right) + \frac{\epsilon_c}{\epsilon_v}\ln\left(\frac{R_3}{R_2}\right) + \frac{\epsilon_c}{\epsilon_i}\ln\left(\frac{R_3}{R_2}\right)}$$

$$V_v = V_1 \cdot \frac{\frac{\epsilon_v}{\epsilon_c}\ln\left(\frac{R_3}{R_2}\right)}{\ln\left(\frac{R_2}{R_1}\right) + \frac{\epsilon_c}{\epsilon_v}\ln\left(\frac{R_3}{R_2}\right) + \frac{\epsilon_c}{\epsilon_i}\ln\left(\frac{R_3}{R_2}\right)}$$

$$V_{vi} = V_1 \cdot \frac{\frac{\epsilon_i}{\epsilon_c}\ln\left(\frac{R_3}{R_2}\right)}{\ln\left(\frac{R_2}{R_1}\right) + \frac{\epsilon_c}{\epsilon_v}\ln\left(\frac{R_3}{R_2}\right) + \frac{\epsilon_c}{\epsilon_i}\ln\left(\frac{R_3}{R_2}\right)}$$

- **Voltage across the conductor-void region 8.01kV**
- **Voltage across the void 2.08kV**
- **Voltage across the insulator 0.91 kV**

Newton's Forward Difference Method

Radial Distance x	Voltage $V(x)$	First Difference ΔV	Second Difference $\Delta^2 V$	Third Difference $\Delta^3 V$
6.000	11.00	-2.99	-2.94	6.79
7.000	8.01	-5.93	3.85	
7.001	2.08	-2.08		
9.000	0.00			

$$V(x) = V(x_0) + \frac{(x-x_0)}{1!}\Delta V_0 + \frac{(x-x_0)(x-x_1)}{2!}\Delta^2 V_0 + \frac{(x-x_0)(x-x_1)(x-x_2)}{3!}\Delta^3 V_0 + \dots$$

Forward Differences:

First differences ΔV:

- $\Delta V_1 = -2.99$
- $\Delta V_2 = -5.93$
- $\Delta V_3 = -2.08$

Second differences $\Delta^2 V$:

- $\Delta^2 V_1 = -2.94$
- $\Delta^2 V_2 = 3.85$

Third difference $\Delta^3 V$:

- $\Delta^3 V_1 = 6.79$

$$V(x) = 11.00 - 2.99(x-6) - 1.47(x-6)(x-7) + 1.13(x-6)(x-7)(x-7.001) + \ldots$$

where : x0 = 6 , x1 = 7, x2 = 7.001 and x3 = 9

$$E(r) = -\frac{dV(r)}{dr}$$

$$E(r) = -1.67r^2 + 24.32r - 81.34$$

$$E(6) \approx 4.46\,\mathrm{kV/mm}$$
$$E(7) \approx 7.07\,\mathrm{kV/mm}$$
$$E(9) \approx 2.27\,\mathrm{kV/mm}$$

$$\text{Percentage Difference} = \left| \frac{\text{Simulated Value} - \text{Theoretical Value}}{\text{Theoretical Value}} \right| \times 100$$

Given Values:

- **Theoretical values:**
  - $E(6) = 4.46\,\mathrm{kV/mm}$
  - $E(7) = 7.07\,\mathrm{kV/mm}$
  - $E(9) = 2.27\,\mathrm{kV/mm}$
- **Simulated values:**
  - $E(6) = 4.5\,\mathrm{kV/mm}$
  - $E(7) = 7.2\,\mathrm{kV/mm}$
  - $E(9) = 2.25\,\mathrm{kV/mm}$

1. At $r = 6$:

$$\text{Percentage Difference} = \left| \frac{4.5 - 4.46}{4.46} \right| \times 100 = \left| \frac{0.04}{4.46} \right| \times 100 \approx 0.9\%$$

2. At $r = 7$:

$$\text{Percentage Difference} = \left| \frac{7.2 - 7.07}{7.07} \right| \times 100 = \left| \frac{0.13}{7.07} \right| \times 100 \approx 1.84\%$$

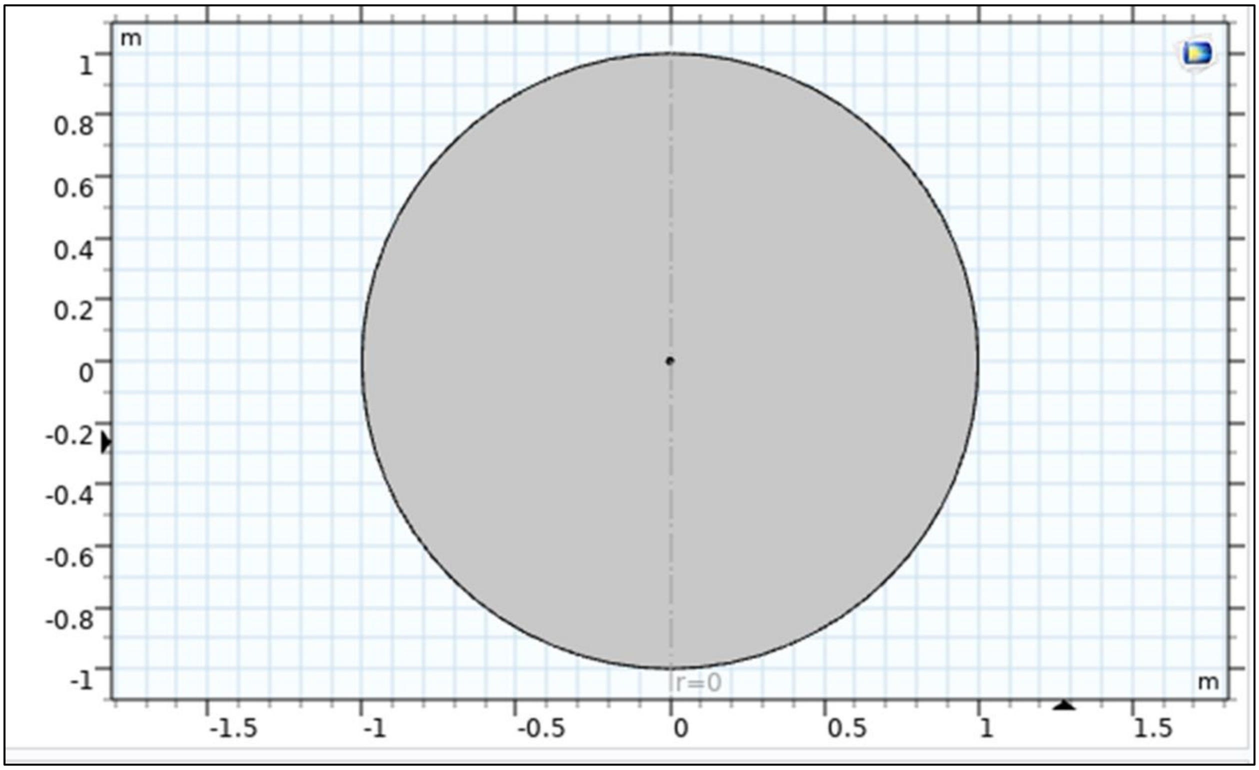
3. At $r = 9$:

$$\text{Percentage Difference} = \left| \frac{2.25 - 2.27}{2.27} \right| \times 100 = \left| \frac{-0.02}{2.27} \right| \times 100 \approx 0.88\%$$

Summary of Percentage Differences:

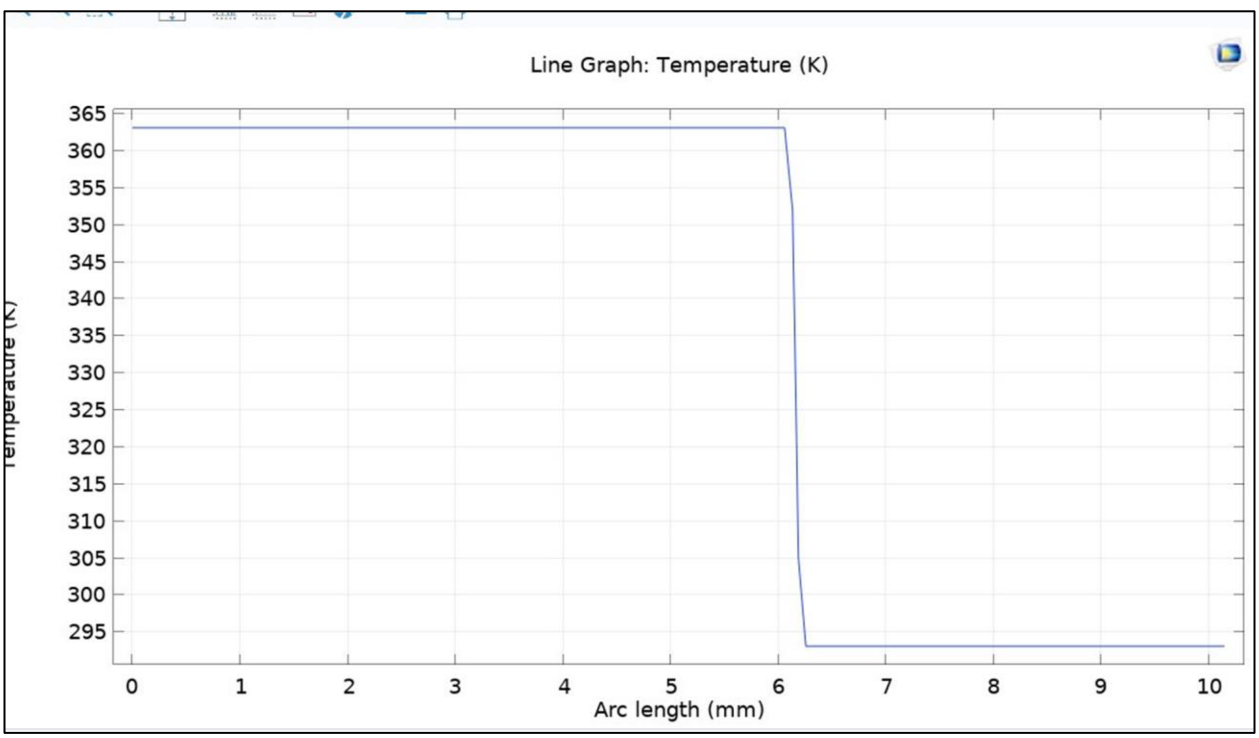
- **At $r = 6$:** 0.9%
- **At $r = 7$:** 1.84%
- **At $r = 9$:** 0.88%

Thermal Analysis of Cable:



Soil upto a radius of 1m

Temprature (K) vs Radial length (mm):



Analytical Method:

Poisson Equation for Cylindrical Coordinates

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$T(r) = C_1 \ln(r) + C_2$$

$$T(r) = -13.68 \ln(r) + 293.15$$

Using Boundary Conditions:

T (r = 6mm) = 363.15K
T (r = 1m) = 293.15 K

	Theoretical	**Practical**
T (r = 6mm)	**363.15**	**363**
T (r = 1m)	**293.15**	**293**

$$\text{Percentage Difference} = \frac{\text{Theoretical} - \text{Practical}}{\text{Practical}} \times 100$$

At $r = 1\,\text{m}$:

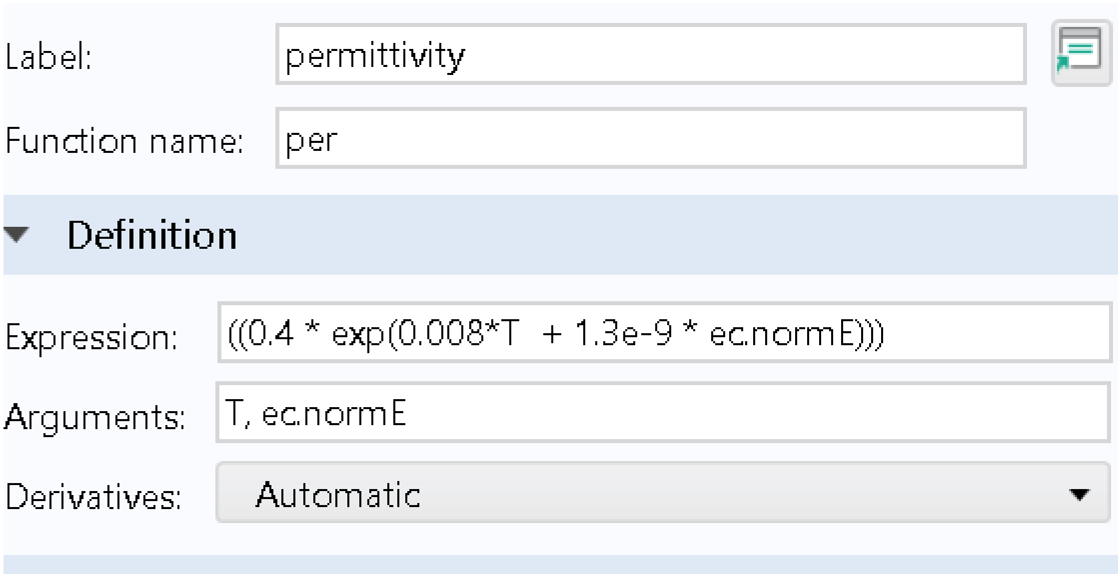
$$\text{Percentage Difference} = \frac{293.15 - 293}{293} \times 100 = \frac{0.15}{293} \times 100 \approx 0.0512\%$$

At $r = 0.006\,\text{m}$:

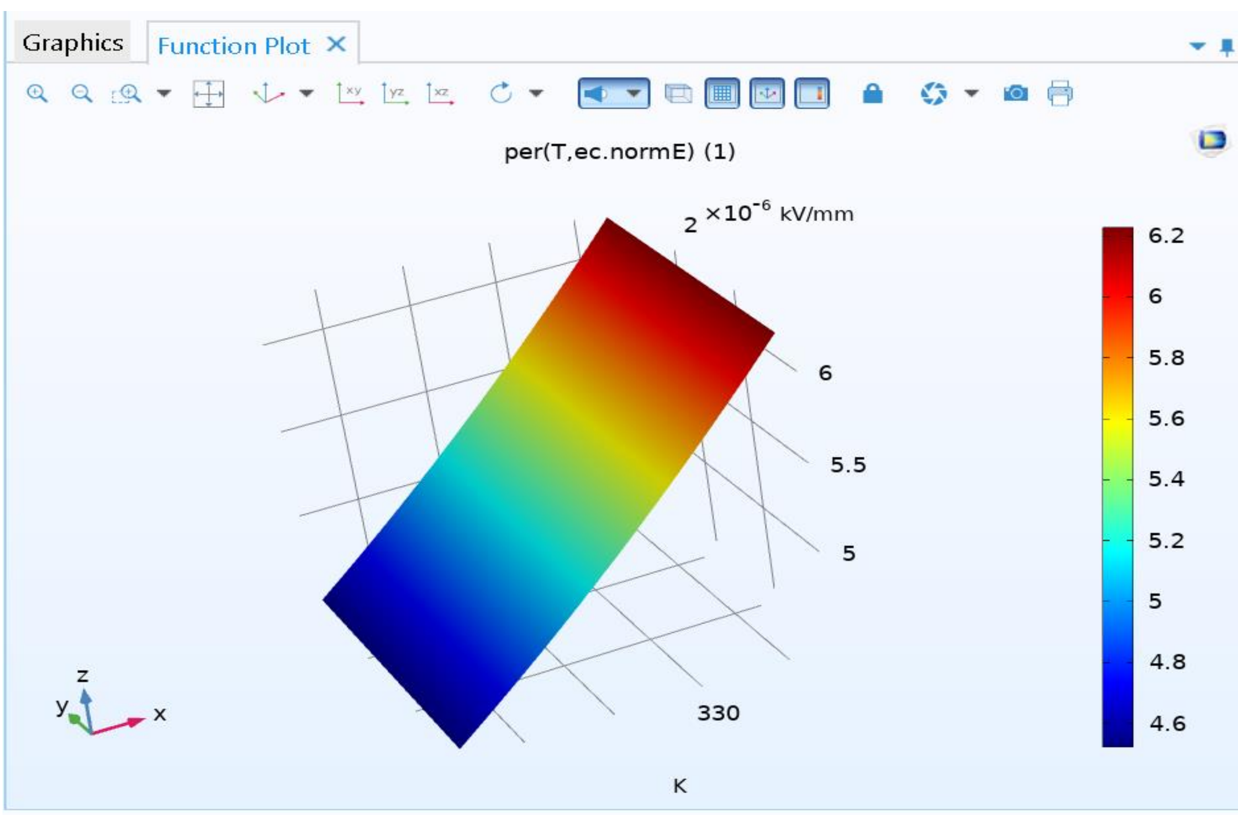
$$\text{Percentage Difference} = \frac{363.15 - 363}{363} \times 100 = \frac{0.15}{363} \times 100 \approx 0.0413\%$$

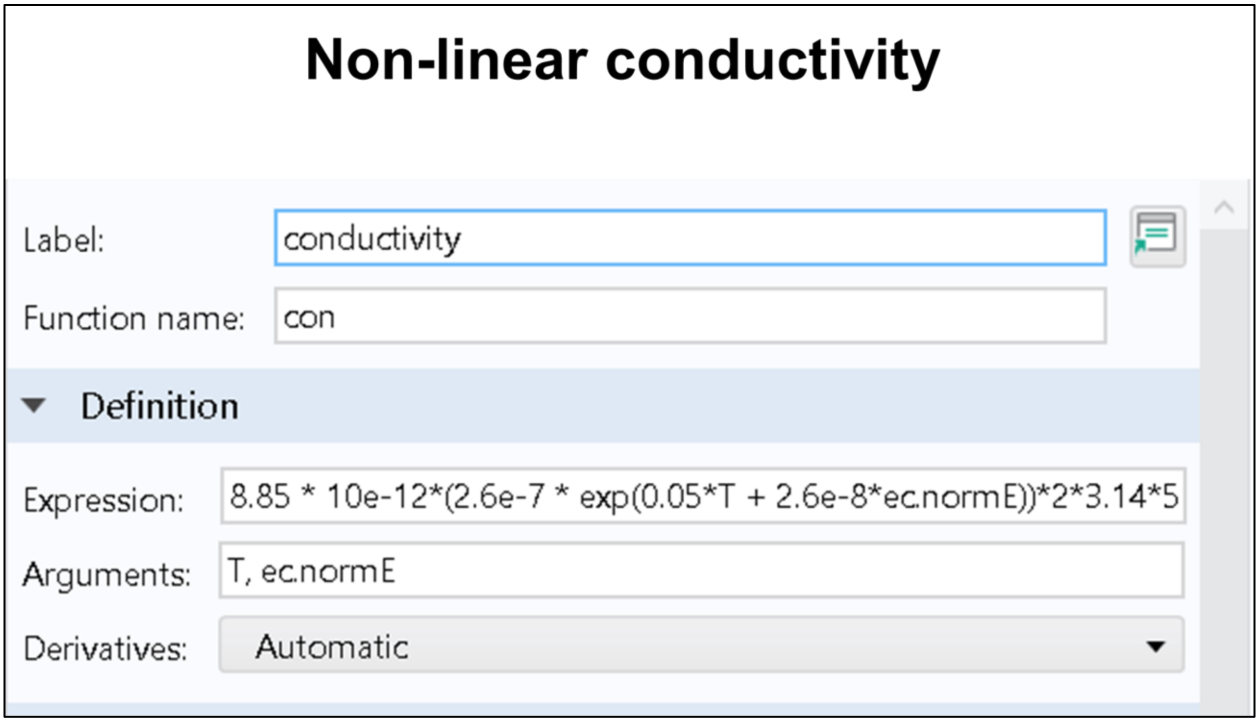
Non linear method

Non-linear permittivity

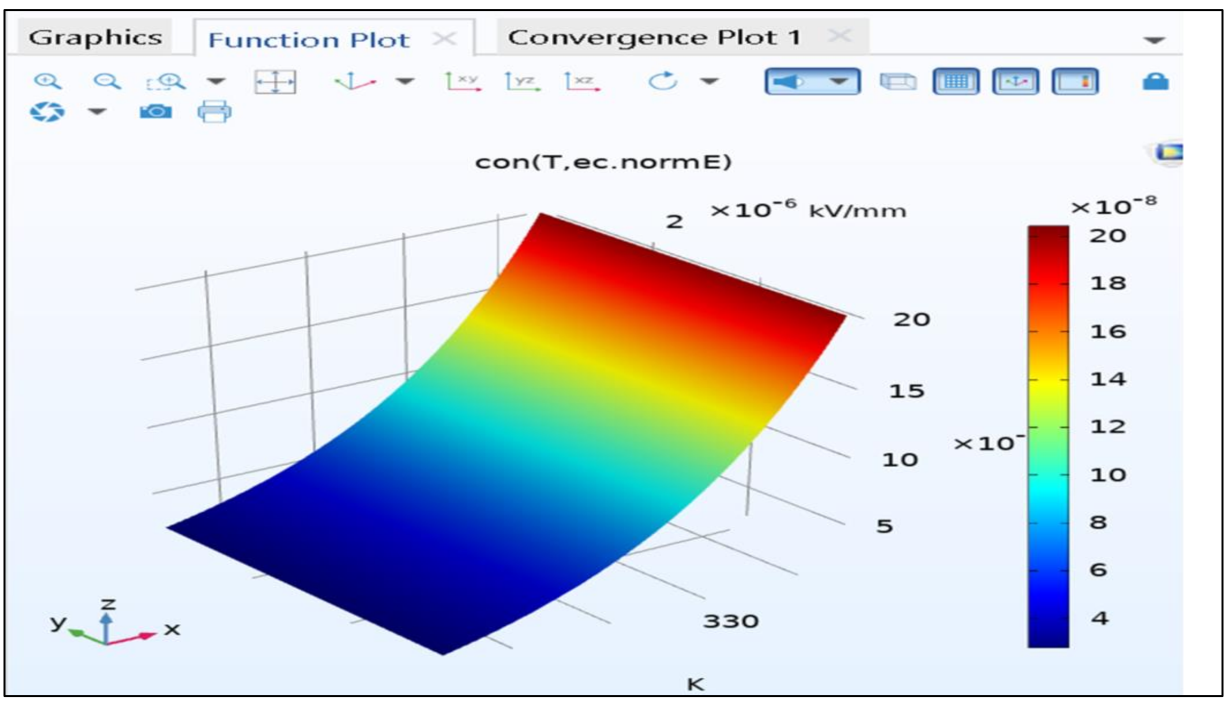

Label: permittivity
Function name: per
Definition
Expression: ((0.4 * exp(0.008*T + 1.3e-9 * ec.normE)))
Arguments: T, ec.normE
Derivatives: Automatic

Non Linear permittivity v/s radial distance:



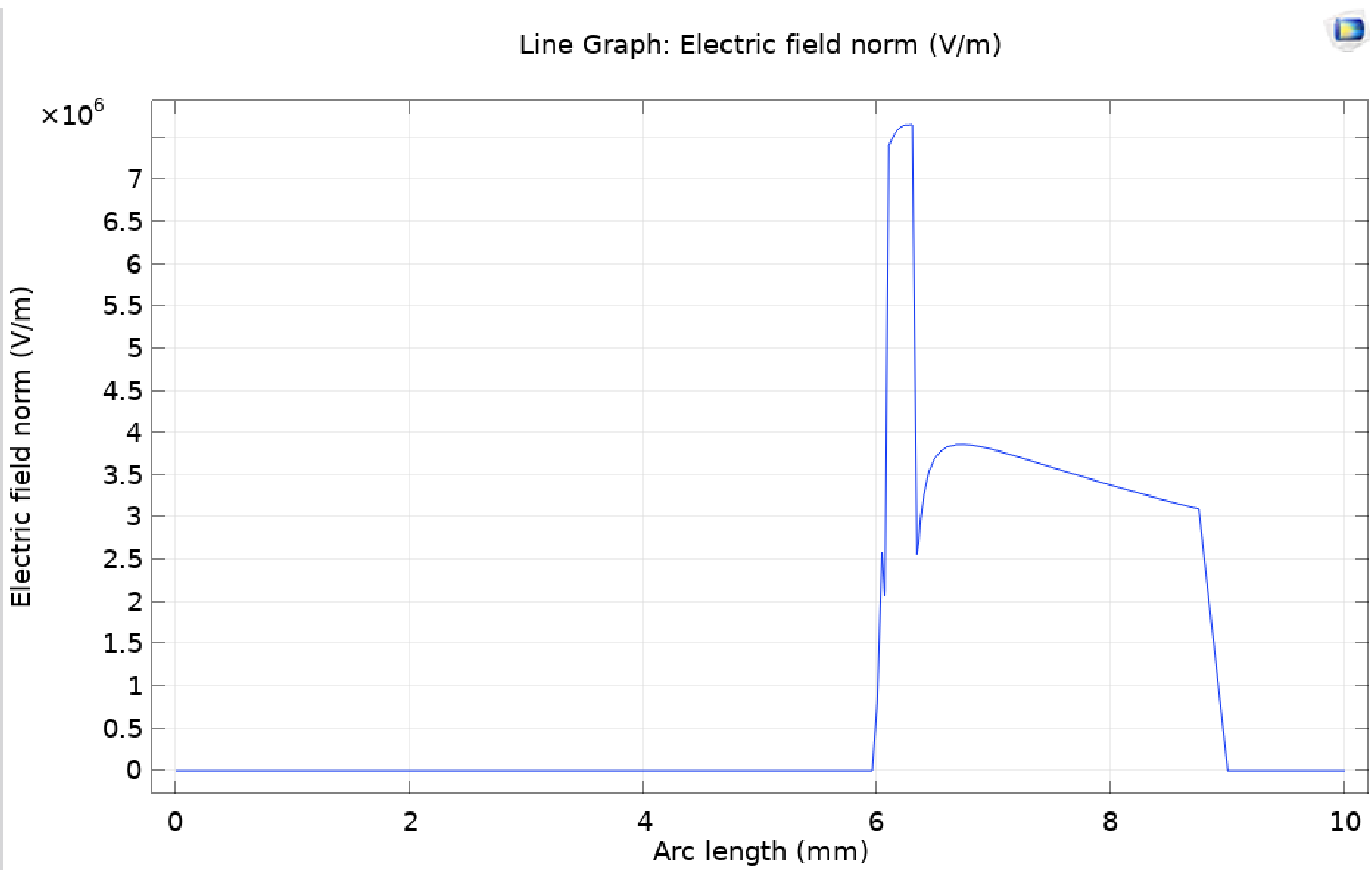


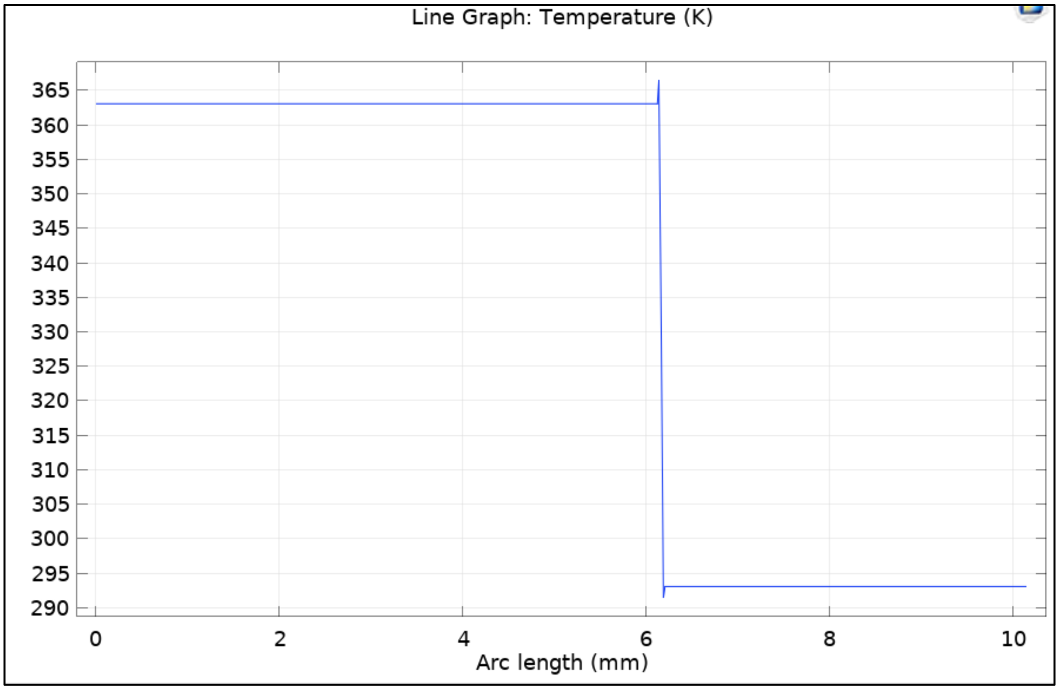
Non Linear conductivity v/s radial distance:



New parameters of insulator

	Property	Variable	Value	Unit	Property group
☑	Electrical conductivity	sigma...	con(T,ec....	S/m	Basic
☑	Relative permittivity	epsilo...	per(T,ec....	1	Basic
☑	Density	rho	920	kg/m^3	Basic
☑	Thermal conductivity	k_iso ;...	0.4	W/(m·K)	Basic
☑	Heat capacity at constant pres...	Cp	2400	J/(kg·K)	Basic





4. Results:

1. Peak Electric Field Intensity: The peak value is higher. Reason: Nonlinear permittivity and conductivity likely result in localized intensification of the electric field due to feedback effects where permittivity and conductivity depend on the field magnitude. This causes sharper gradients and peaks.

2. Electric Field Distribution:

- Linear Case: The transition in the field magnitude around the void is smoother, with less pronounced jumps and a more uniform decrease after the peak.
- Nonlinear Case: The field shows a steeper rise near the void and a more abrupt decay after the peak.
- Reason: The nonlinear properties can cause abrupt changes in the local field distribution as the material responds differently based on the field strength.

3. Behavior Near the Void:

- Linear Case: The field shows a relatively symmetrical profile near the void.

- Nonlinear Case: The field's asymmetry is more pronounced, likely due to nonlinearity creating regions of higher field amplification.

4. Decay Rate:

- Linear Case: The field decays steadily and follows a predictable pattern, consistent with linear material behavior.
- Nonlinear Case: The decay is less steady, possibly indicating that the nonlinear material properties introduce nonuniformity in the redistribution of the field.
- Reason: Nonlinear effects lead to complex interactions between permittivity, conductivity, and the field magnitude, altering the decay characteristics.

5. Impact on Material Stress:

- The nonlinear case generates a higher peak and more localized stresses within the material, which might lead to faster degradation or breakdown at the void.
- The linear case distributes stress more evenly, reducing the risk of localized failure.

5. Inferences until now:

This comprehensive project successfully explored the behavior of a coaxial cable system under various conditions, integrating electric field analysis, thermal coupling, and numerical methods to gain insight into the impact of defects and non-linear material properties on cable performance.

1. Electric Field Analysis: The inclusion of voids (circular and elliptical) demonstrated their significant impact on field distribution, especially near the conductor. The elliptical voids caused the highest electric field intensification, highlighting the critical role of defect geometry and positioning in insulation failure mechanisms.

2. Thermal Coupling and Numerical Methods: Thermal analysis using Laplace's equation provided insights into the heat distribution and its interaction with

electric fields. This coupling revealed areas of high thermal stress coinciding with regions of electric field intensification, validating the importance of thermal-electrical interplay.

3. Analytical Capacitor Model: The three-capacitor series model effectively simplified the study of void effects and provided theoretical verification for the numerical results. This analytical approach helped establish a direct connection between the applied voltage (11 kV) and the electric field disturbances caused by defects.

4. Non-linear Properties Analysis: The implementation of non-linear permittivity and thermal conductivity formulas enhanced the realism of the simulations, capturing the cable's material responses under varying field and thermal conditions. The results revealed a significant variation in both field intensity and temperature along the radial axis, emphasizing the importance of accurate material characterization.

6. Future Work:

1. 3D Analysis: Transitioning from 2D to 3D simulations will provide a more comprehensive understanding of field and thermal distributions, especially for non-symmetric voids and complex geometries, enabling a realistic assessment of cable performance.

2. Material Aging and Degradation: Exploring how insulation material properties degrade over time under combined electrical and thermal stresses will offer long-term predictions for cable life expectancy.

3. Multi-Physics Coupling: Extending the analysis to include mechanical stresses (e.g., from thermal expansion) and electromagnetic effects will provide a holistic evaluation of cable durability.