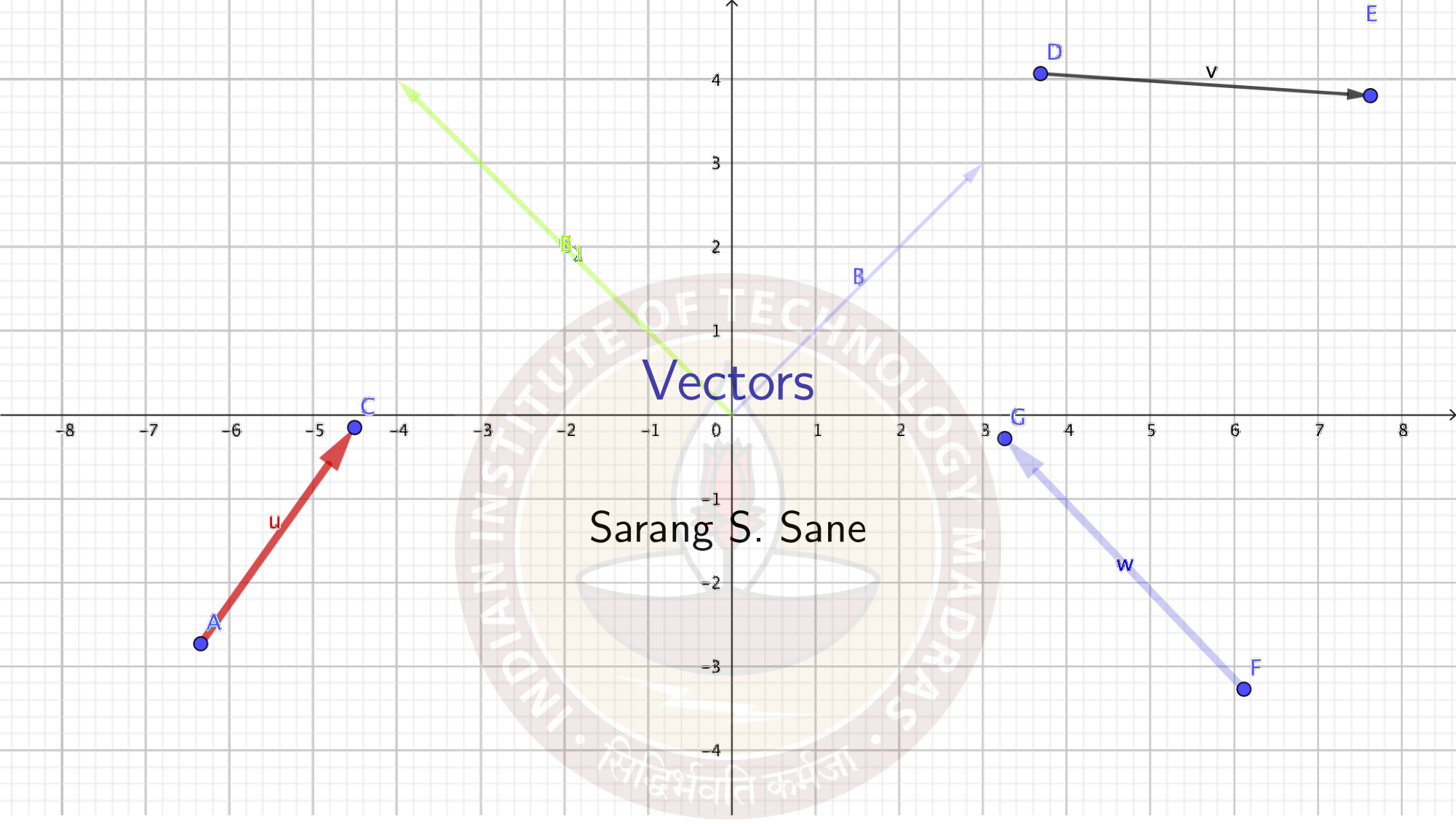
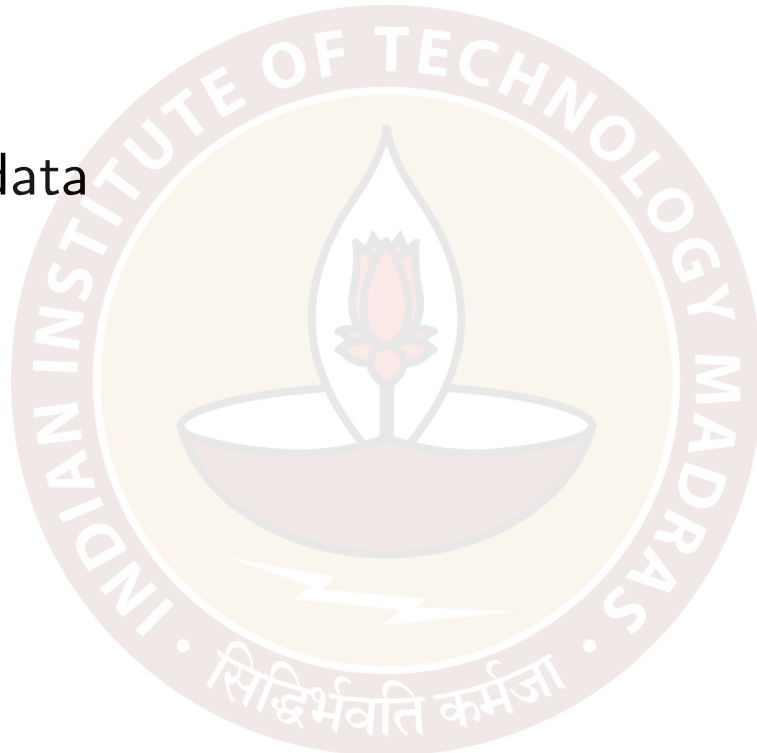



# Vectors

Sarang S. Sane

# Contents

- Vectors and data



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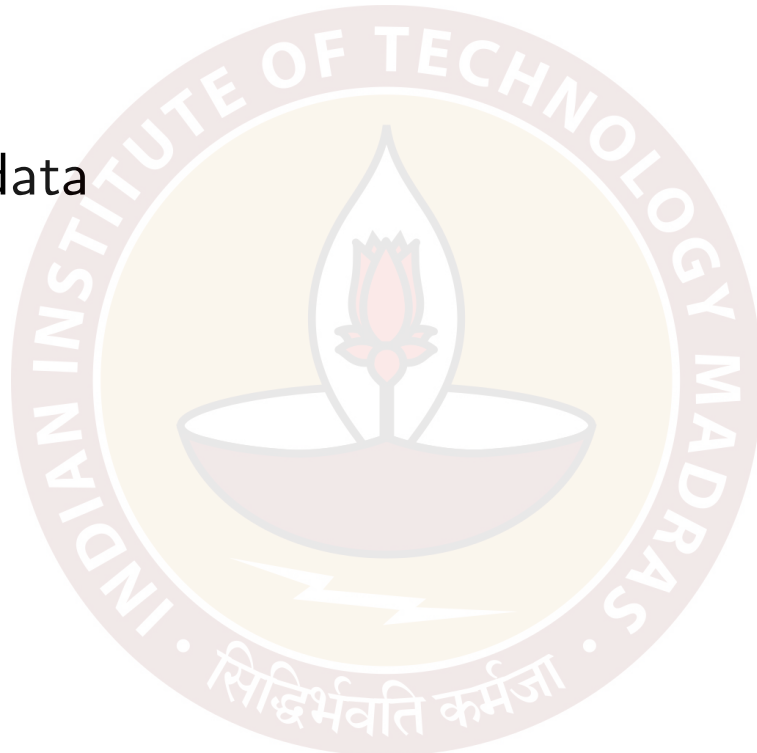

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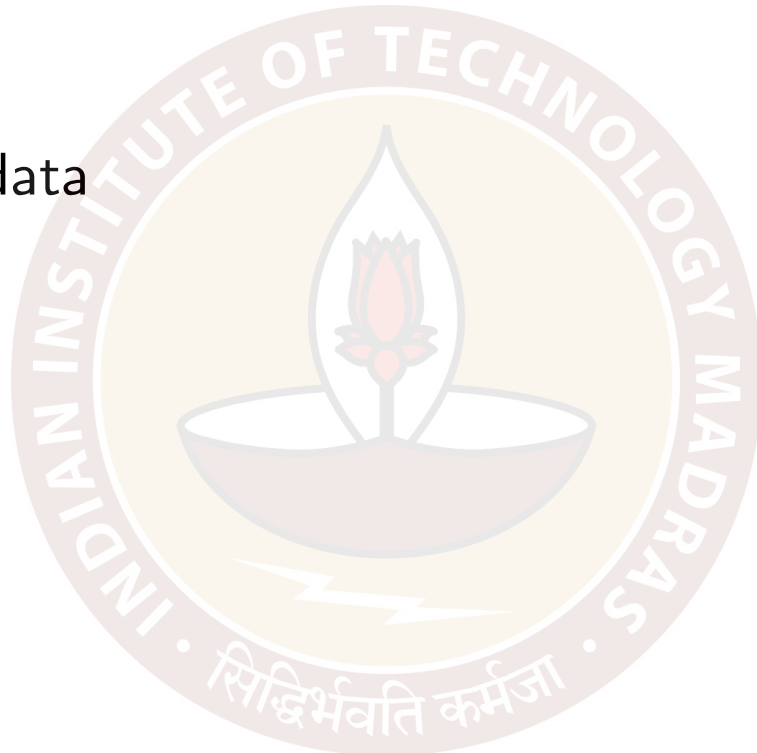

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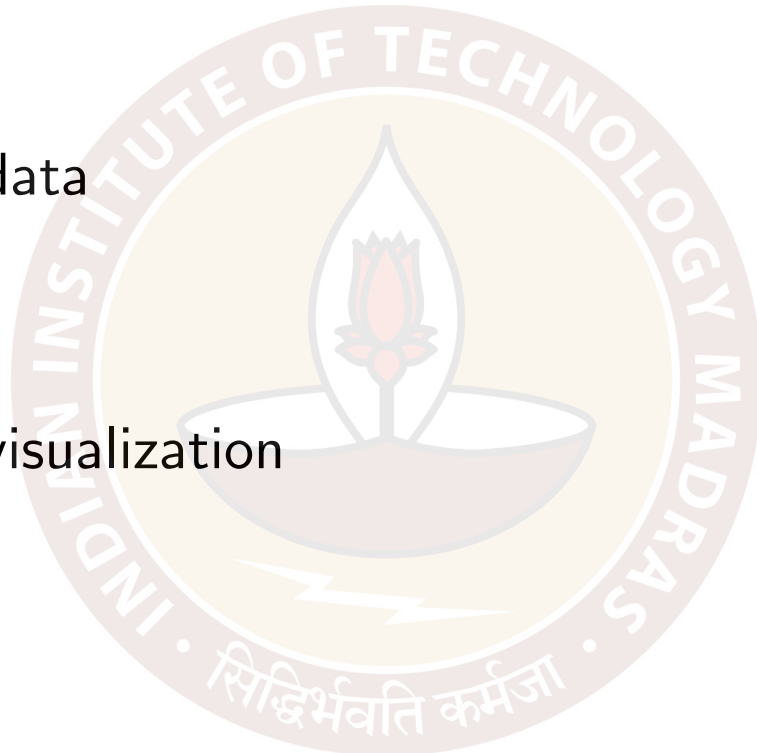

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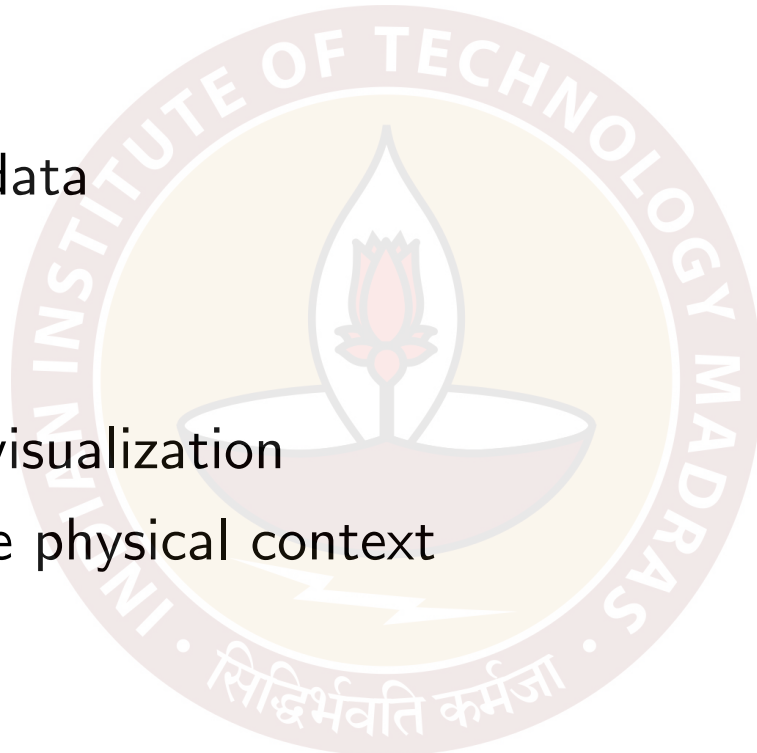

- Vectors and data
- Why vectors?
- Examples
- Vectors and visualization
- Vectors in the physical context

# Vectors and data

Often we encounter data in a table. For example :

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| Financial Year | Gross Domestic Product (in Rs. Cr) at 2004-05 Prices | Agriculture & Allied Services (in Rs. Cr.) at 2004-05 Prices | Agriculture (in Rs. Cr.) at 2004-05 Prices | Industry (in Rs. Cr.) at 2004-05 Prices | Mining and Quarrying (in Rs. Cr.) at 2004-05 Prices | Manufacturing (in Rs. Cr.) at 2004-05 Prices | Services (in Rs. Cr.) at 2004-05 Prices |
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India's GDP data from 2000-01 to 2012-3 with sector wise break-ups

# Vectors and Data

Another example :

# Vectors and Data

Another example :

| vs Teams | V.Kohli | M.S.Dhoni | R.Sharma | K.L.Rahul | S.Dhawan |
|---|---|---|---|---|---|
| Australia | 54.57 | 44.86 | 61.33 | 45.75 | 45.80 |
| England | 45.30 | 46.84 | 50.44 | 6.60 | 32.45 |
| New Zealand | 59.91 | 49.47 | 33.47 | 68.33 | 32.72 |
| South Africa | 64.35 | 31.92 | 33.30 | 26.00 | 49.87 |
| Sri Lanka | 60.00 | 64.40 | 46.25 | 34.75 | 70.21 |
| Pakistan | 48.72 | 53.52 | 51.42 | 57.00 | 54.28 |

Team-wise batting averages

# Vectors and data (Contd.)

A vector can be thought of as a list. In the context of the above examples, vectors could be columns or rows.

| | | | | | | | |
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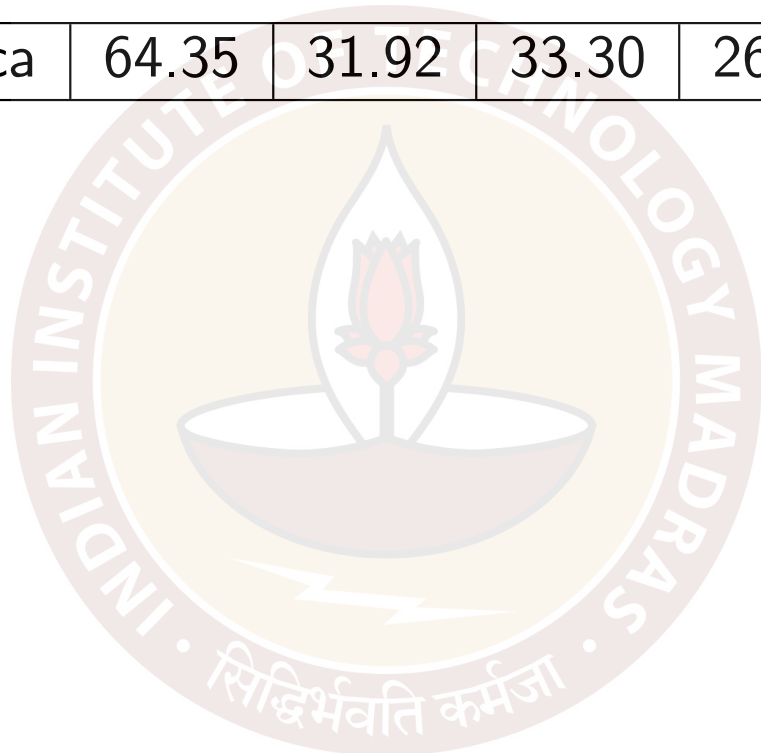

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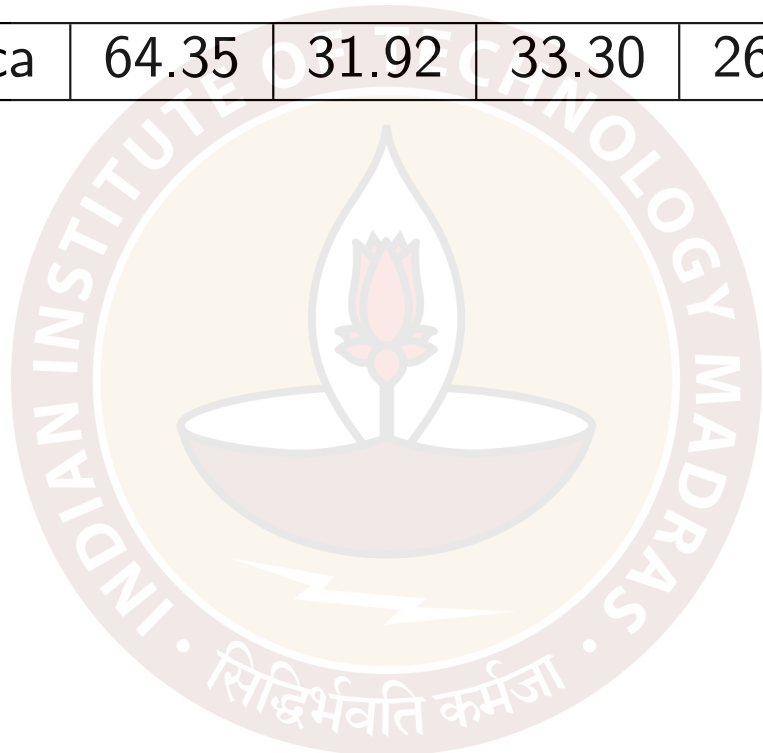

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(64.35, 31.92, 33.3, 26, 49.87) ← row vector OR row matrix

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column vector OR column matrix

$$\begin{bmatrix} 54.57 \\ 45.3 \\ 59.91 \\ 64.35 \\ 60 \\ 48.72 \end{bmatrix}$$

| V.Kohli |
|---|
| 54.57 |
| 45.30 |
| 59.91 |
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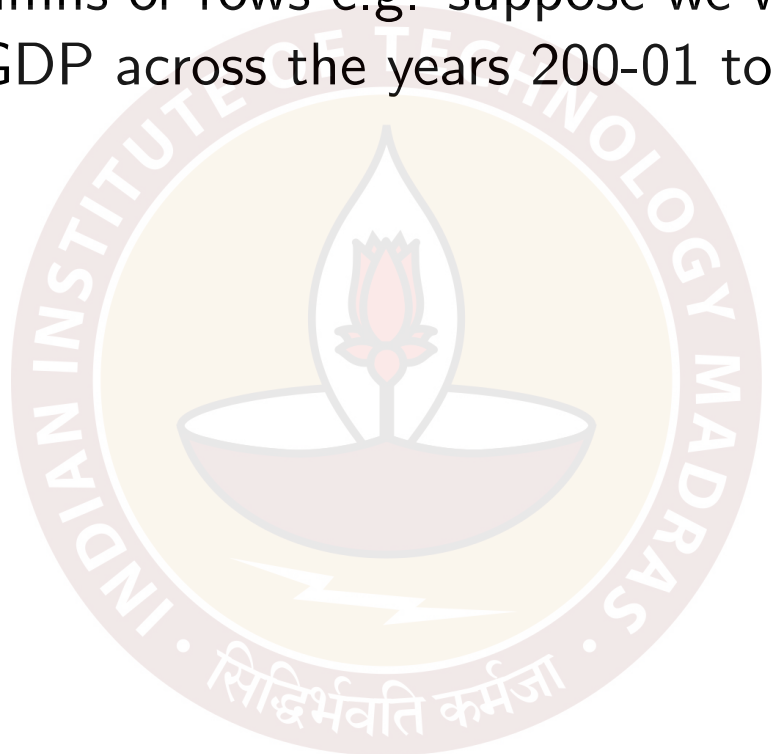
# Why vectors?

Vectors can be used to perform arithmetic operations on lists such as the table columns or rows e.g. suppose we want the average sectoral GDP across the years 200-01 to 2009-10.



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$\frac{1}{10} \times \text{Total}$ $\frac{1}{10} \times \text{Total}$ $\frac{1}{10} \times \text{Total}$ $= \frac{1}{10} \times (\quad)$

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| Items | Arun | Neela | Total |
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|  Dal in kg | 2 | + 6 | = 8 |

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|  Rice in kg | 3 | 5 | 8 |
|  Dal in kg | 2 | 6 | 8 |

Then the vectors $(3, 2)$ for Arun and $(5, 6)$ for Neela represent their demands. We can add these vectors to get $(3, 2) + (5, 6) = (8, 8)$. This says that together they have to buy 8 kg rice and 8 kg dal.

## Example 2

**Stock taking in a grocery shop :**

| Items | In stock | Buyer A | Buyer B | Buyer C | New stock |
|---|---|---|---|---|---|
|  Rice in kg | 150 | − 8 | − 12 | − 3 | + 100 |
|  Dal in kg | 50 | − 8 | − 5 | − 2 | + 75 |
|  Oil in Litres | 35 | − 4 | − 7 | − 5 | + 30 |
|  Biscuits in packets | 70 | − 10 | − 10 | − 5 | + 80 |
|   Soap Bars | 25 | − 4 | − 2 | − 1 | + 30 |

# Example 2 contd. : addition of vectors

Taking stock of the items in the grocery shop can be done easily using vector representation :

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$$(150, 50, 35, 70, 25) + (-8, -8, -4, -10 - 4) + (-12, -5, -7, -10, -2) + (-3, -2, -5, -5, -1) + (100, 75, 30, 80, 30)$$

## Example 2 contd. : addition of vectors

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Note that we add corresponding entries of the vectors. This is an example of addition of vectors.

# Example 2 contd. : scalar multiplication

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$$(8, 8, 4, 10, 4) + (8, 8, 4, 10, 4) = (16, 16, 8, 20, 8) = 2(8, 8, 4, 10, 4)$$

$$= (2\times 8, 2\times 8, 2\times 4, 2\times 10, 2\times 8)$$

## Example 2 contd.: Scalar multiplication

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Multiplying a vector by a scalar (i.e. all its entries if it is a list) is called scalar multiplication.

$$c(v_1, v_2, \ldots, v_n) = (cv_1, cv_2, cv_3, \ldots, cv_n).$$

# Visualization of vectors in $\mathbb{R}^2$

$$\text{Point } (a, b) \equiv \text{Vector } (a, b) \equiv a\hat{\mathbf{i}} + b\hat{\mathbf{j}}.$$

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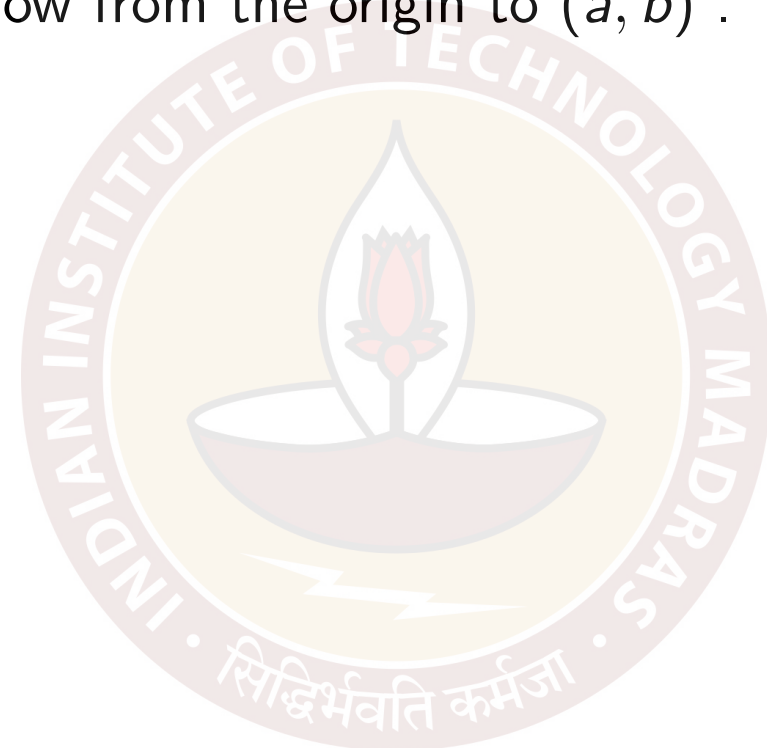

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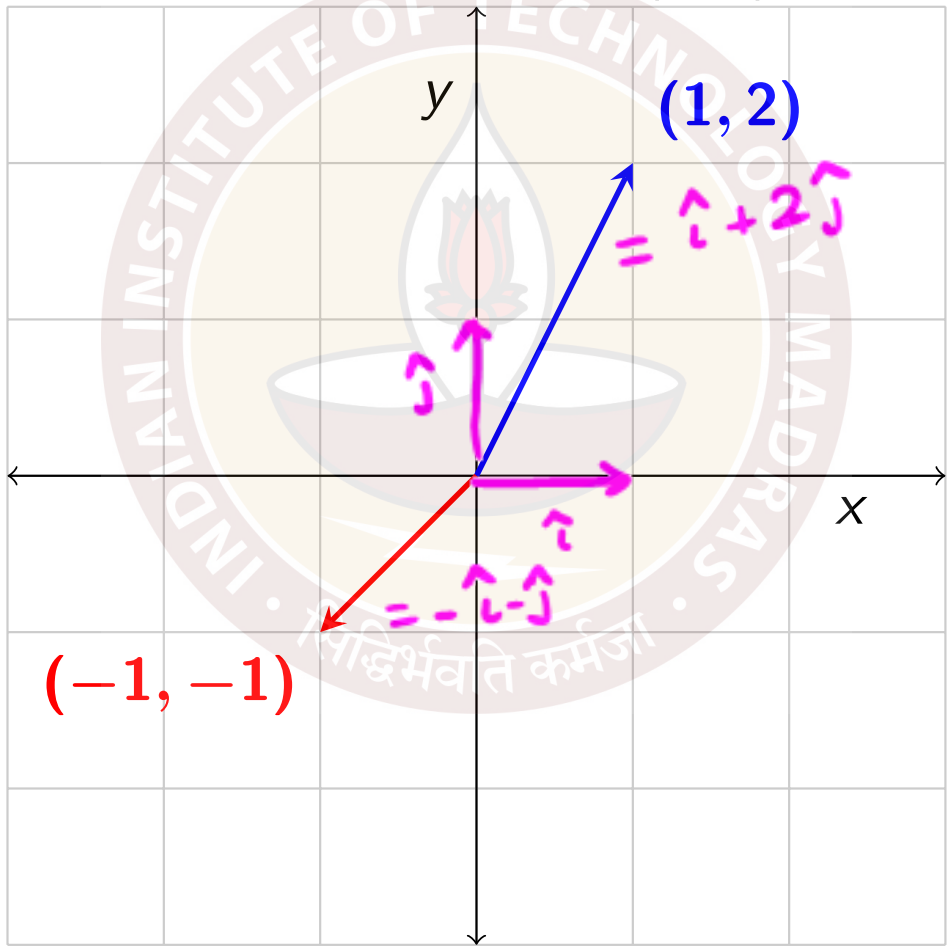

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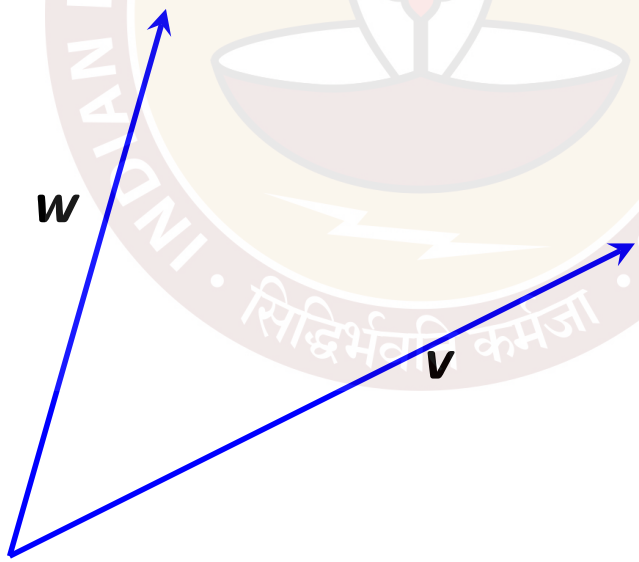

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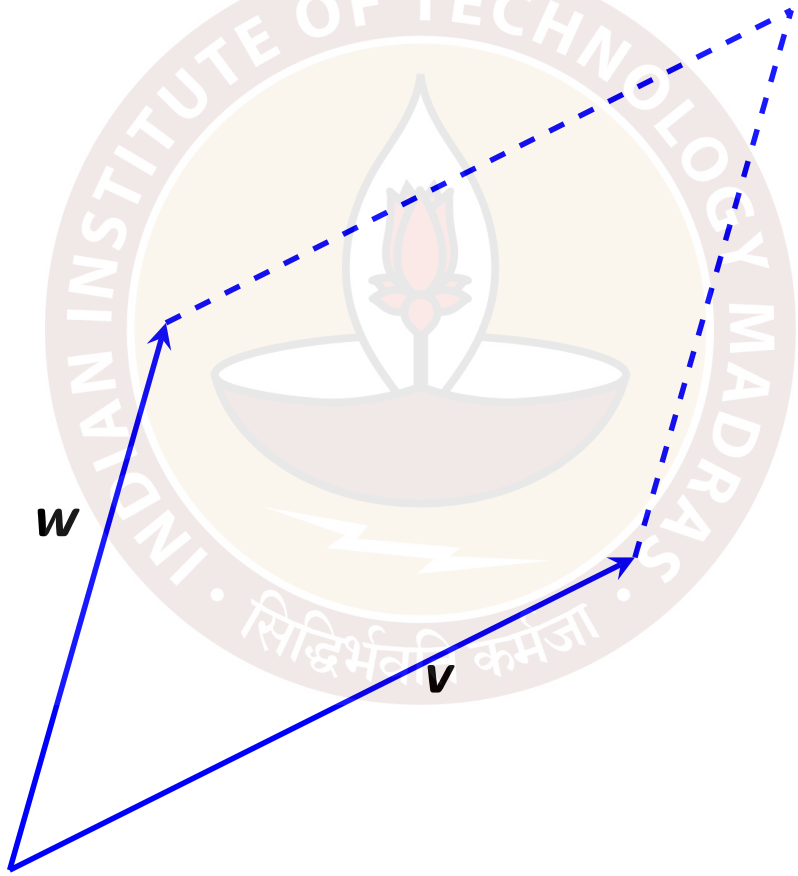

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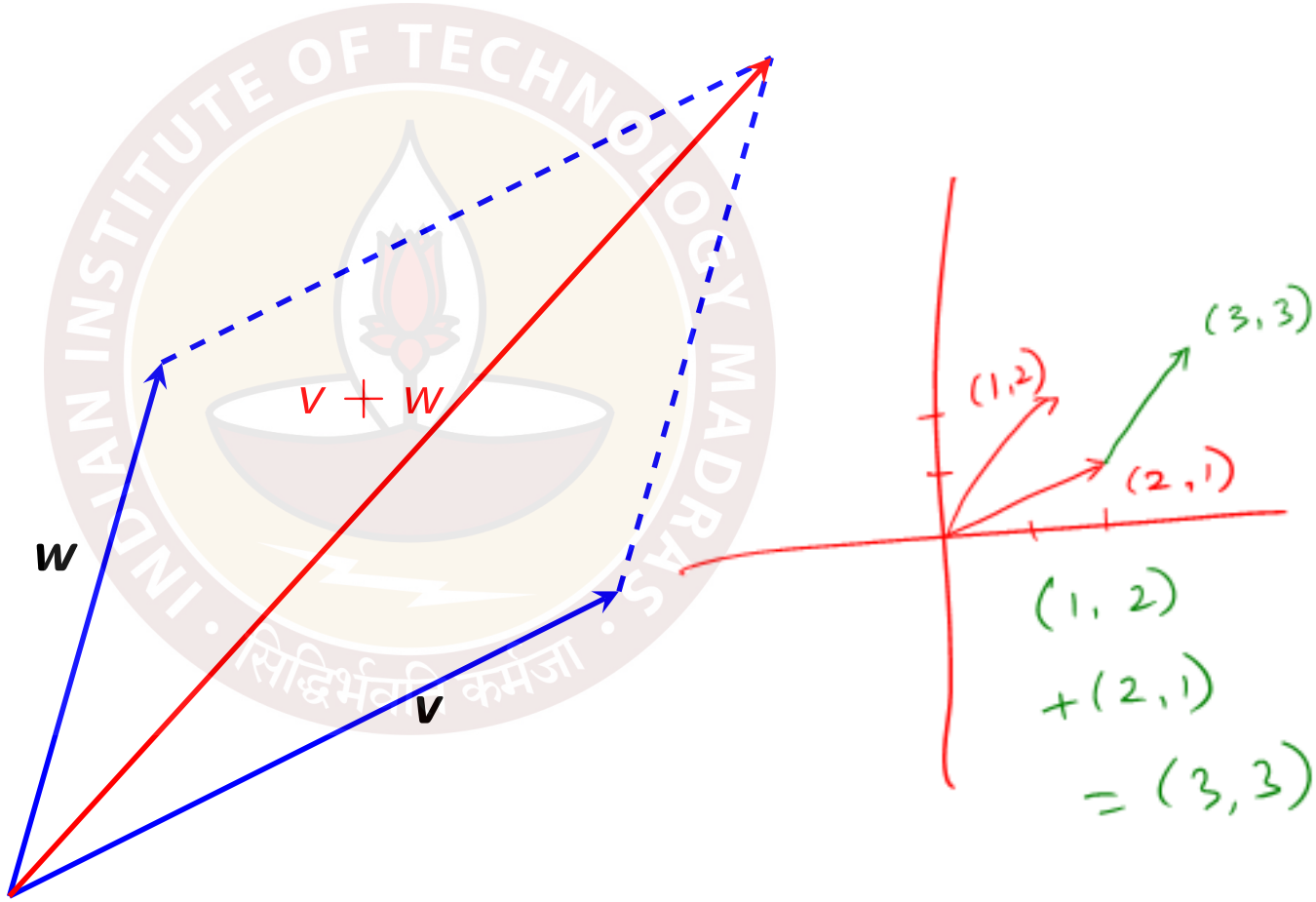
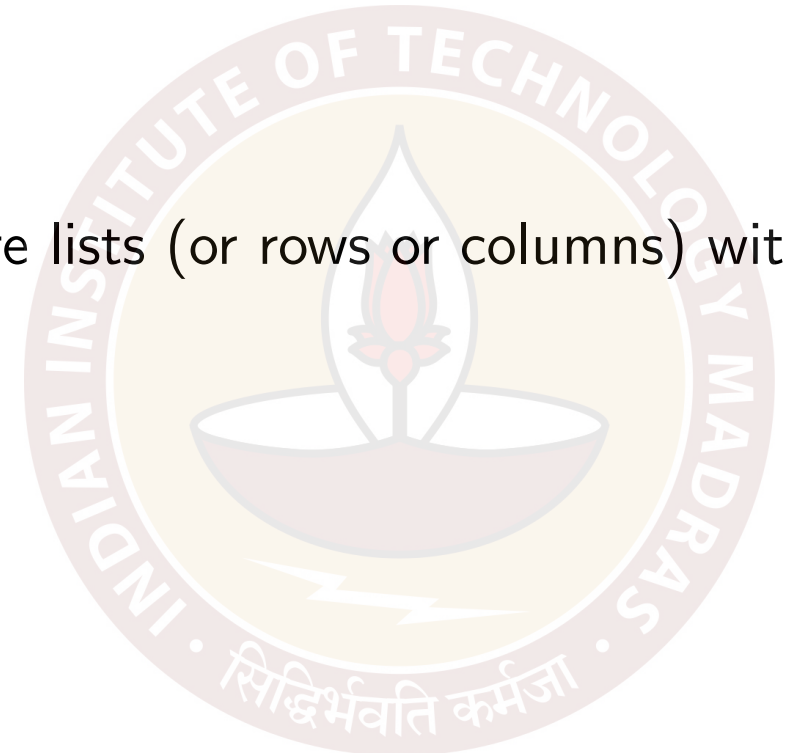


# Vectors in $\mathbb{R}^n$

Vectors in $\mathbb{R}^n$ are lists (or rows or columns) with $n$ real entries.

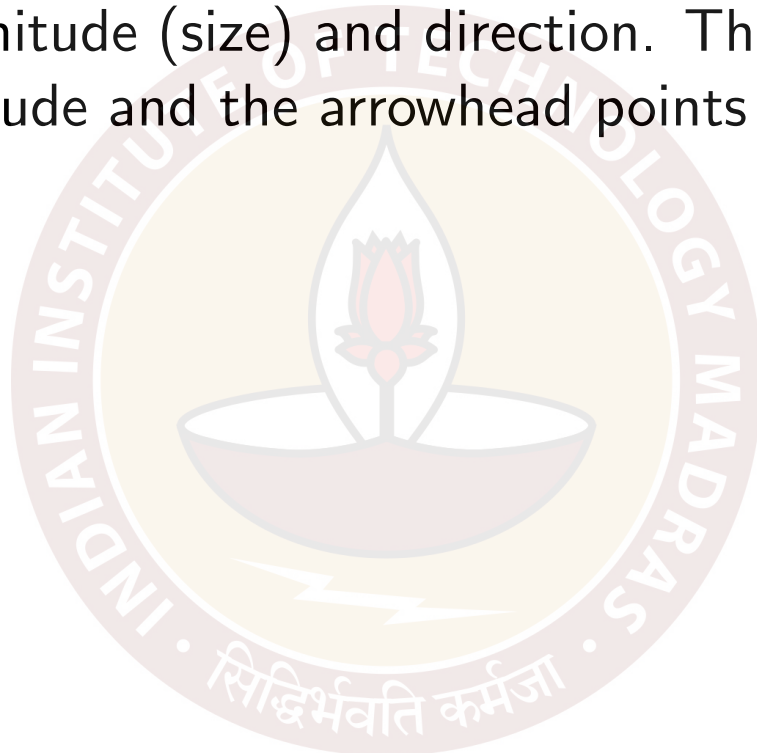


Vectors in $\mathbb{R}^n$ are lists (or rows or columns) with $n$ real entries.

Vectors with $n$ entries $\equiv$ Vectors in $\mathbb{R}^n \equiv$ Points in $\mathbb{R}^n$.

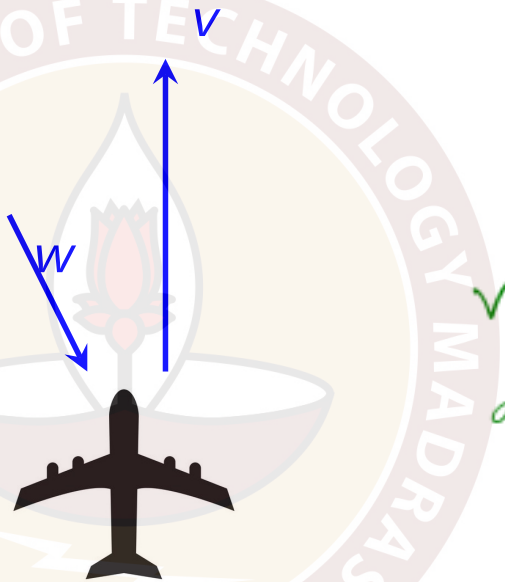
# Vectors in the physical context

A vector has magnitude (size) and direction. The length of the line shows its magnitude and the arrowhead points in the direction.



A vector has magnitude (size) and direction. The length of the line shows its magnitude and the arrowhead points in the direction.

Example: A plane is flying towards the north and wind is blowing from the North-West.




$v+w$ is the direction in which the plane moves.

$v=$ velocity of the flight and $w=$ velocity of the wind

Some examples of vectors which appear in physics :

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- Velocity

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- Velocity
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FOR THIS COURSE REMEMBER THAT VECTORS MEAN ROWS OR COLUMNS OF NUMBERS.


# Thank you