Machine Learning Foundations

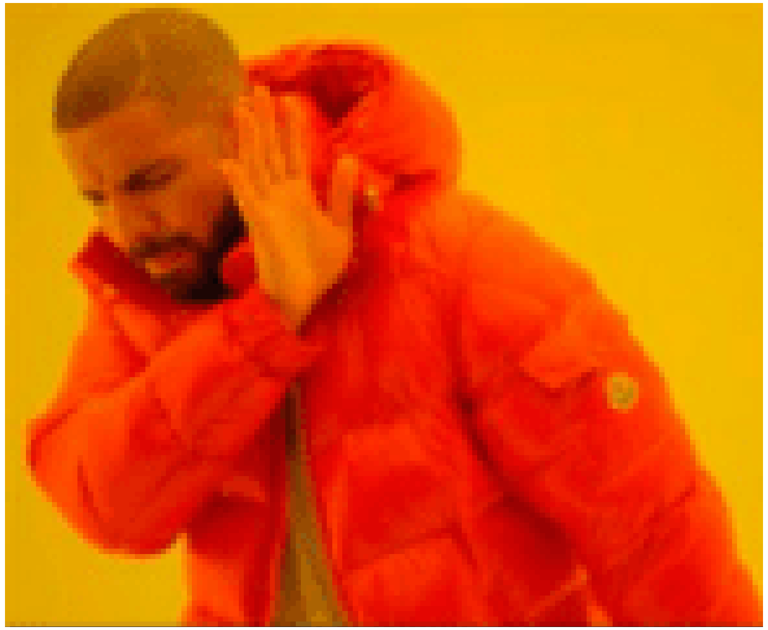
Tutorial - Week5

Arun Prakash A

$$I = U\Sigma V^T$$

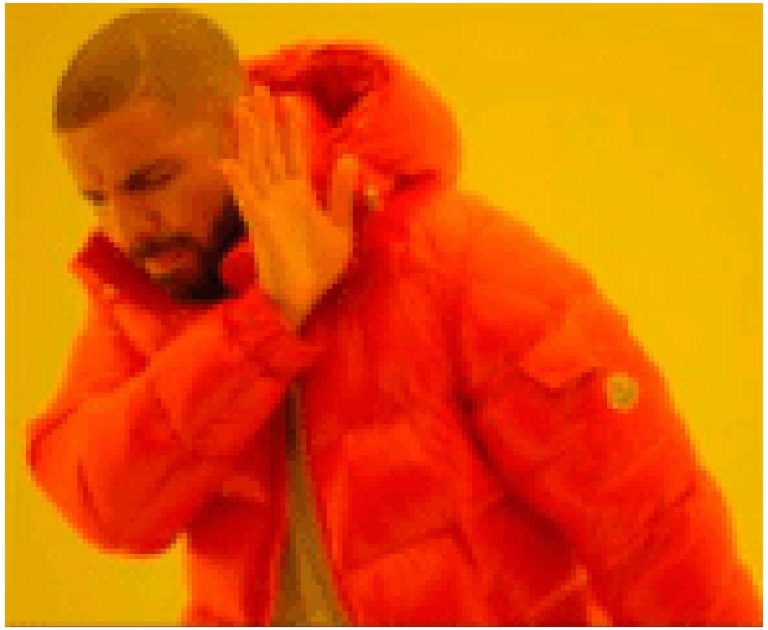
$$I = \sum_{i=1}^{k} \sigma_i u_i v_i^T \qquad\qquad I = U \Sigma V^T$$

Our Mind



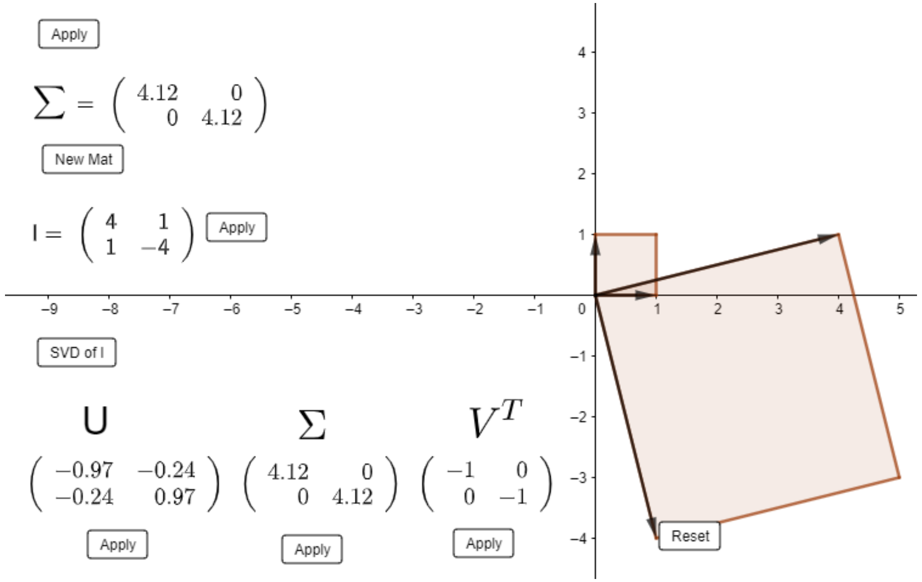
$$I = \sum_{i=1}^{k} \sigma_i u_i v_i^T \qquad\qquad I = U\Sigma V^T$$

Our Mind

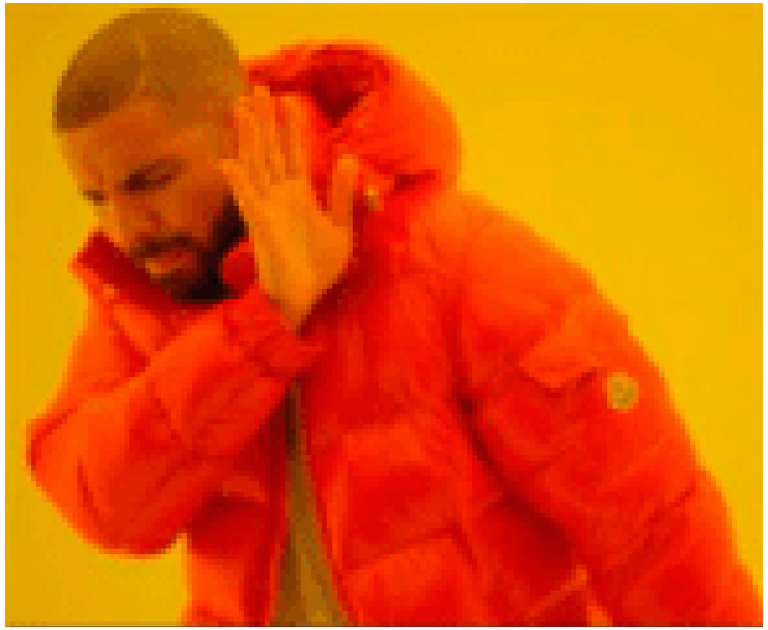


$$I = \sum_{i=1}^{k} \sigma_i u_i v_i^T$$

$$I = U \Sigma V^T$$

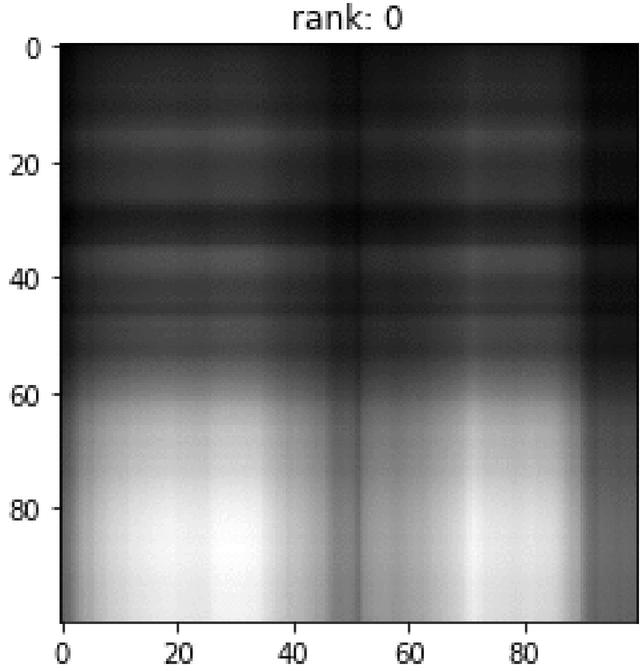


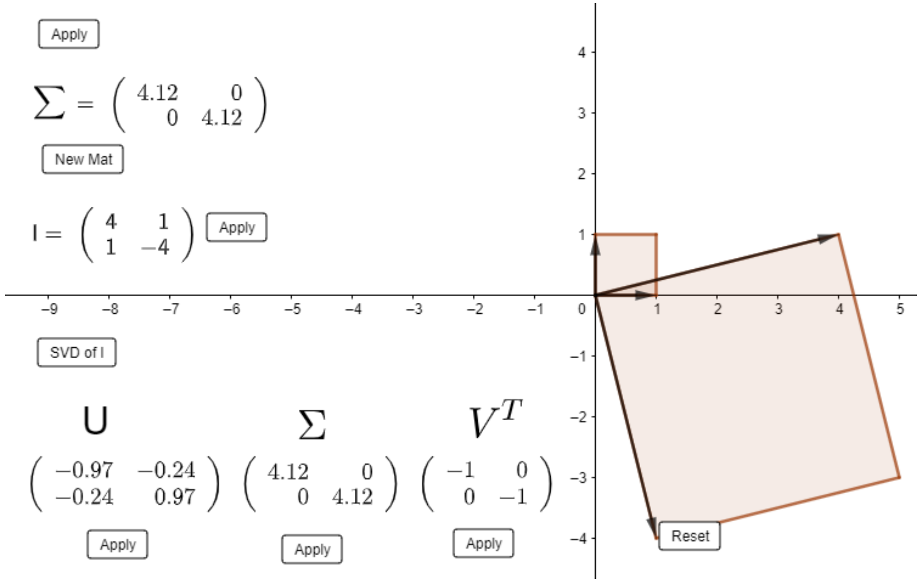
Our Mind



$$I = \sum_{i=1}^{k} \sigma_i u_i v_i^T$$

$$I = U \Sigma V^T$$

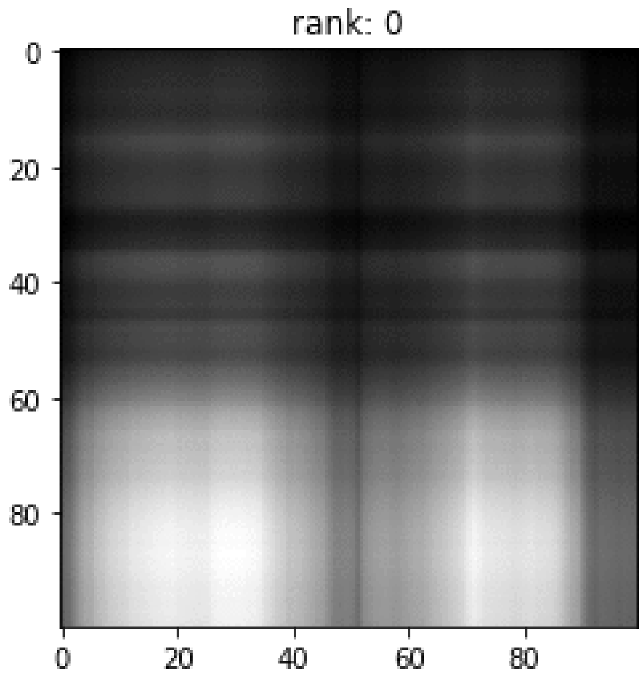


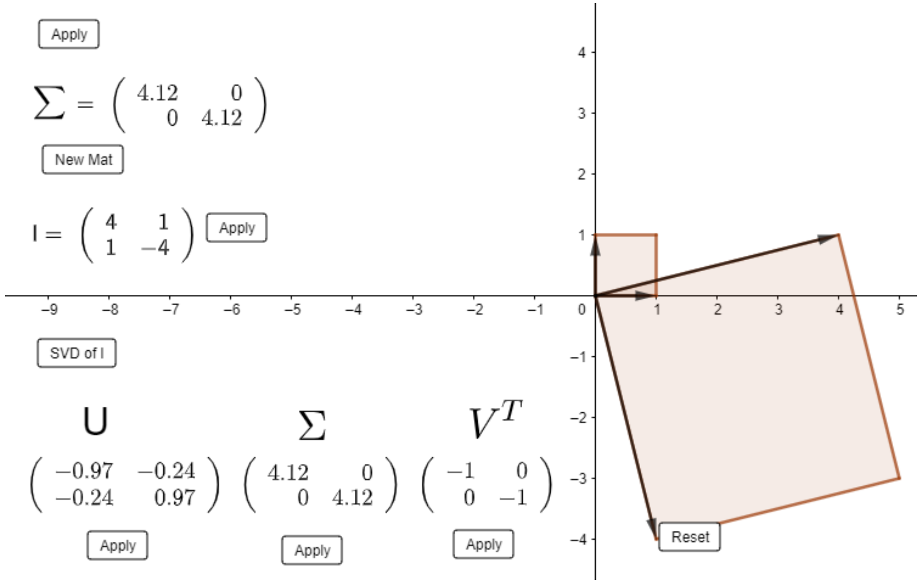


Our Mind

$$I = \sum_{i=1}^{k} \sigma_i u_i v_i^T$$

$$I = U\Sigma V^T$$





Let's play a game

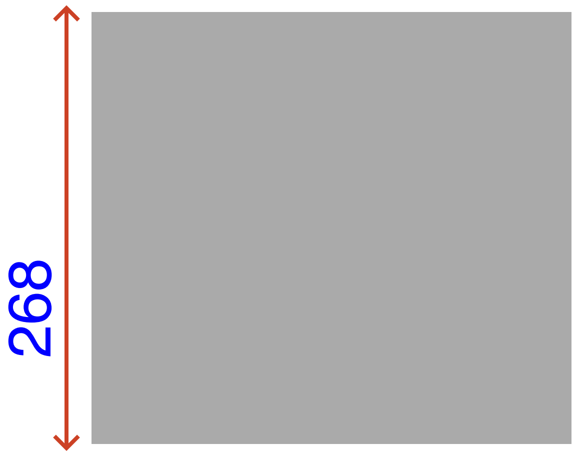
Let's play a game

Let's play a game

Image

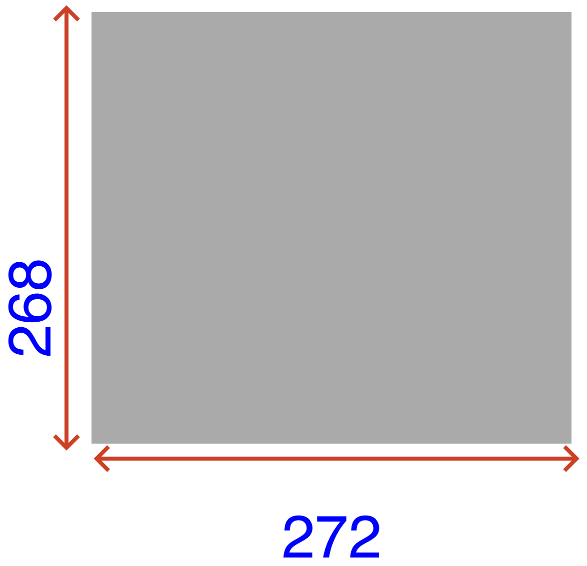
Let's play a game

Image



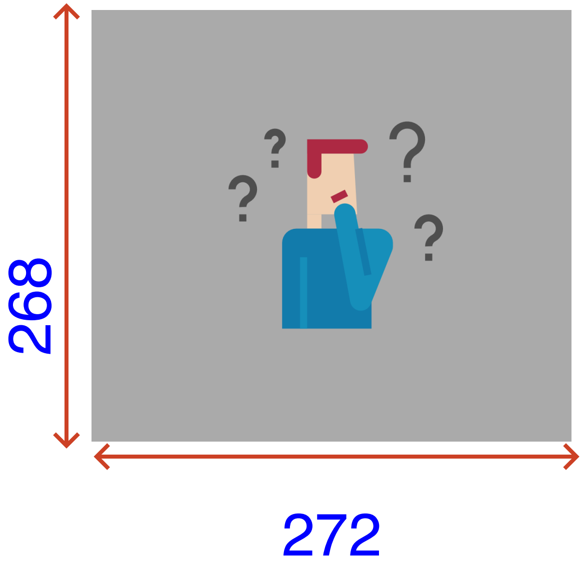
Let's play a game

Image



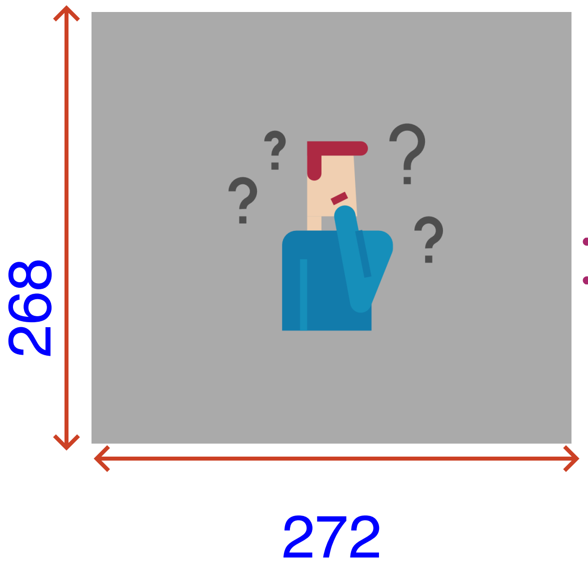
Let's play a game

Image



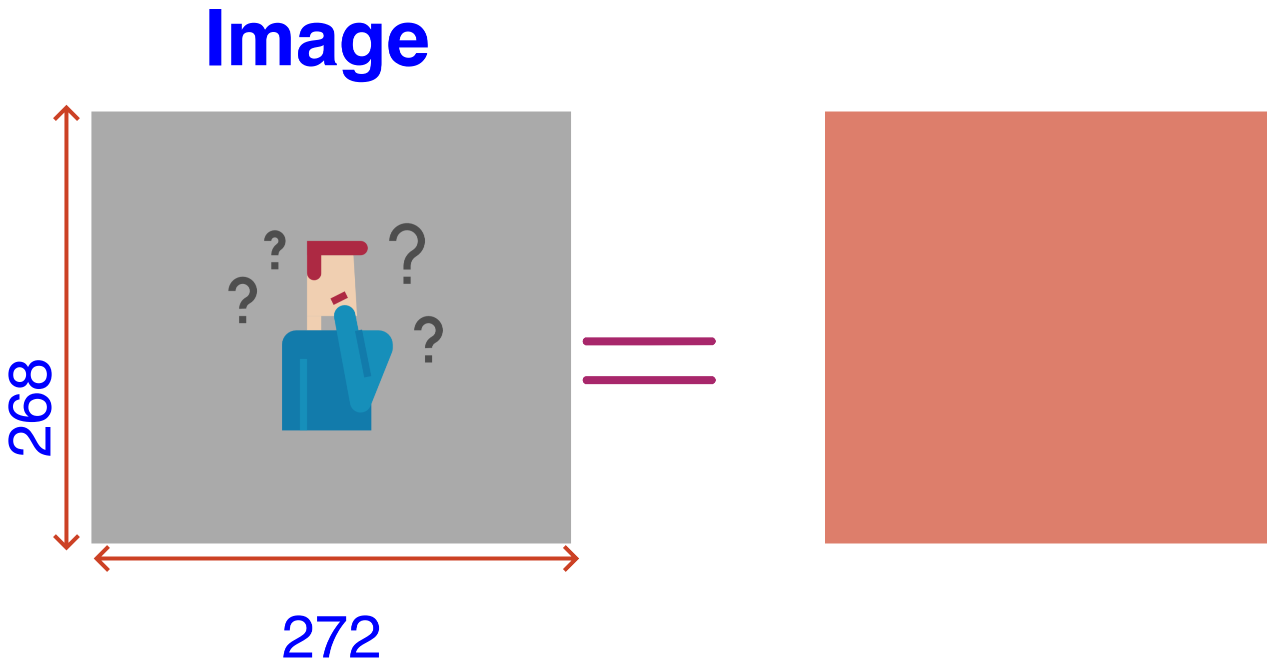
Let's play a game

Image



=

Let's play a game

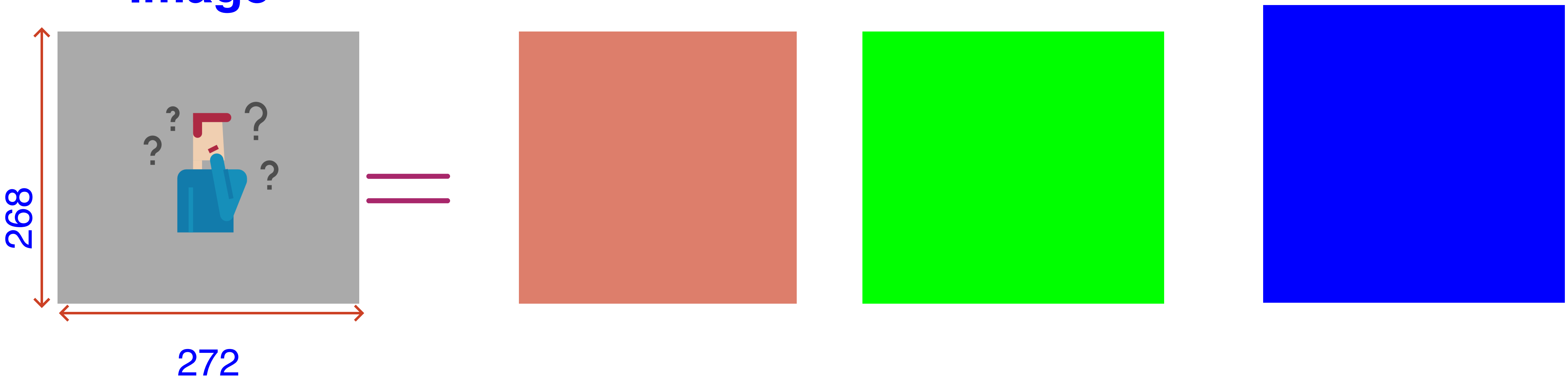


Let's play a game

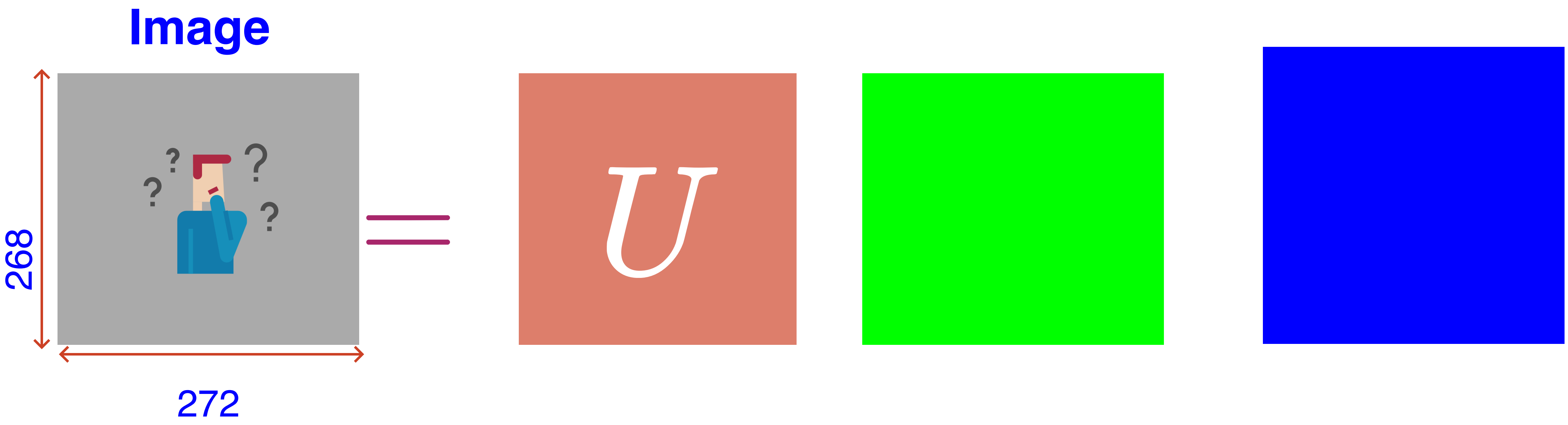
Image

Let's play a game

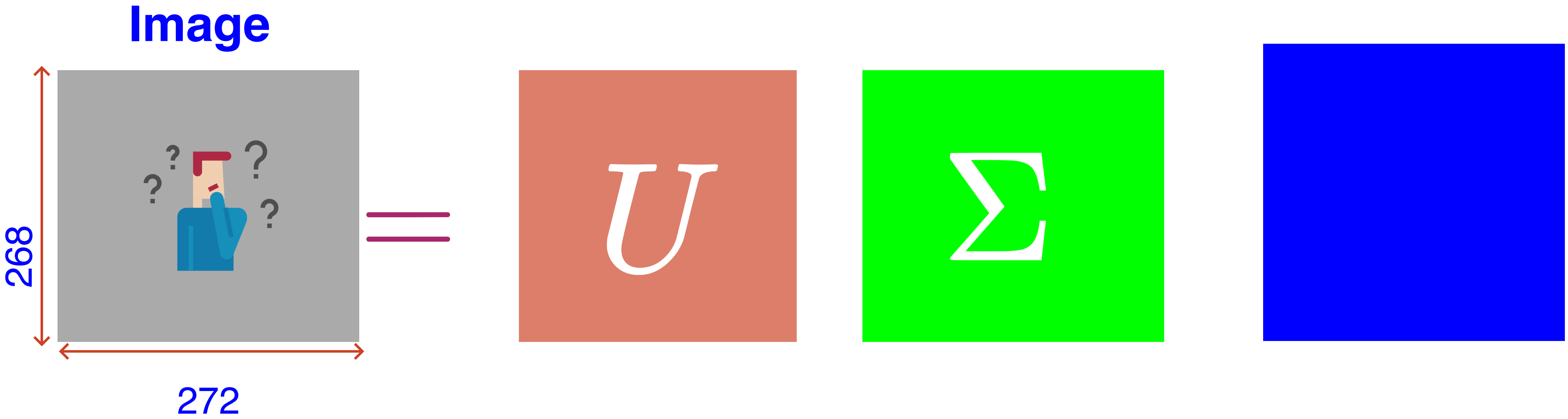
Image



Let's play a game



Let's play a game



Let's play a game

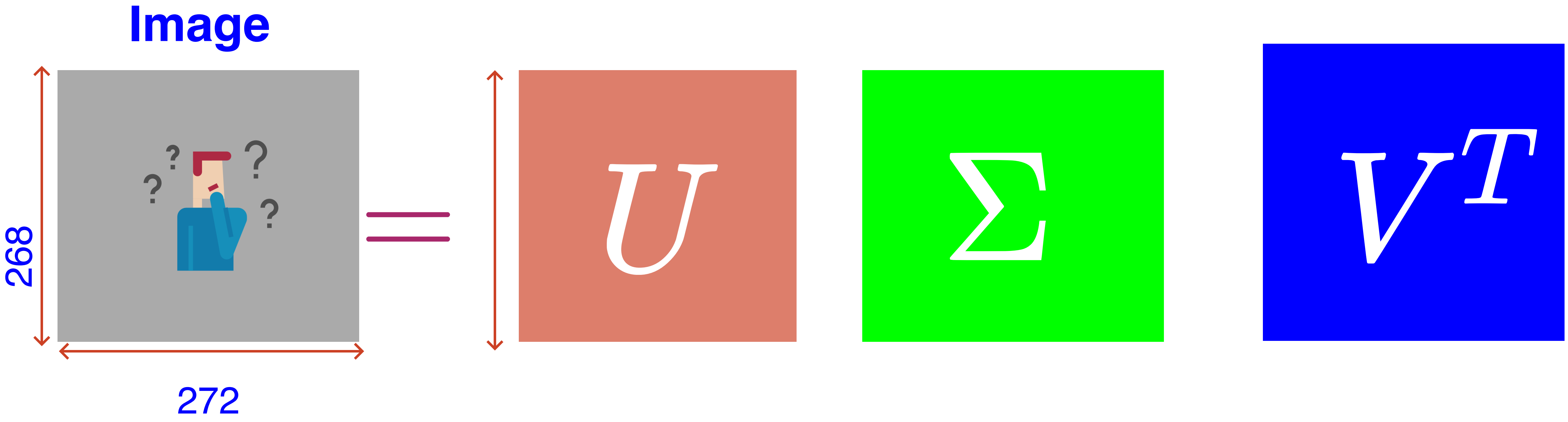
Image

268

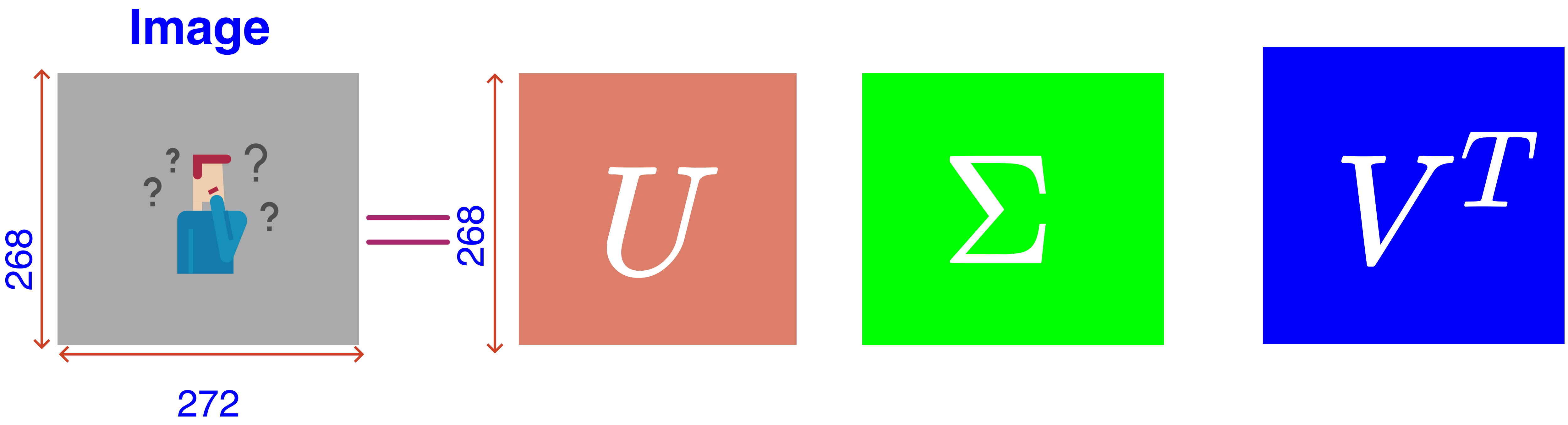
272

$$= U \quad \Sigma \quad V^T$$

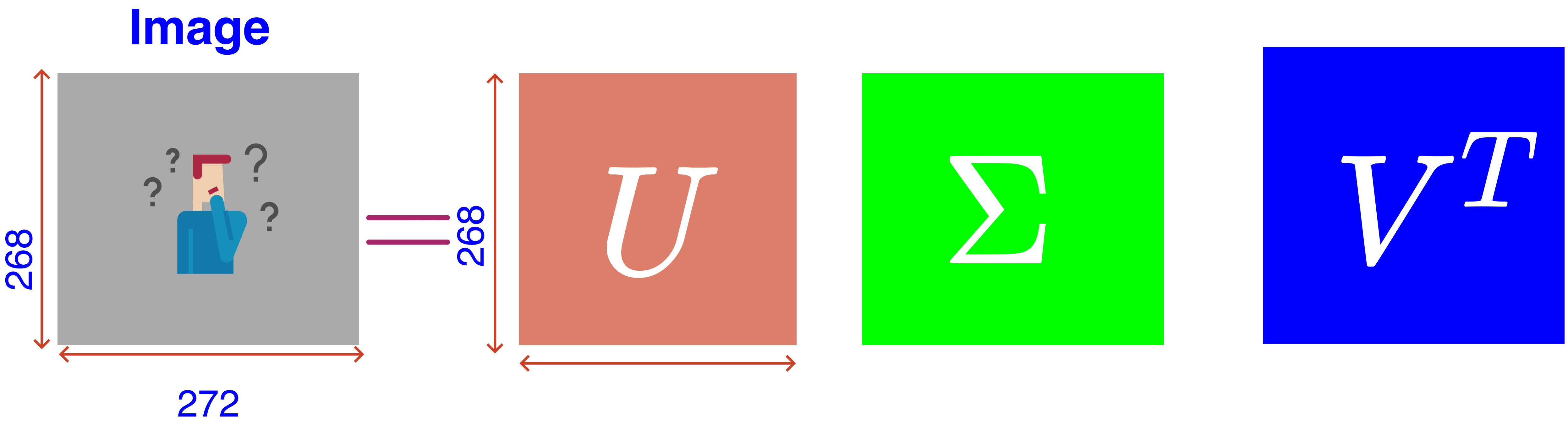
Let's play a game



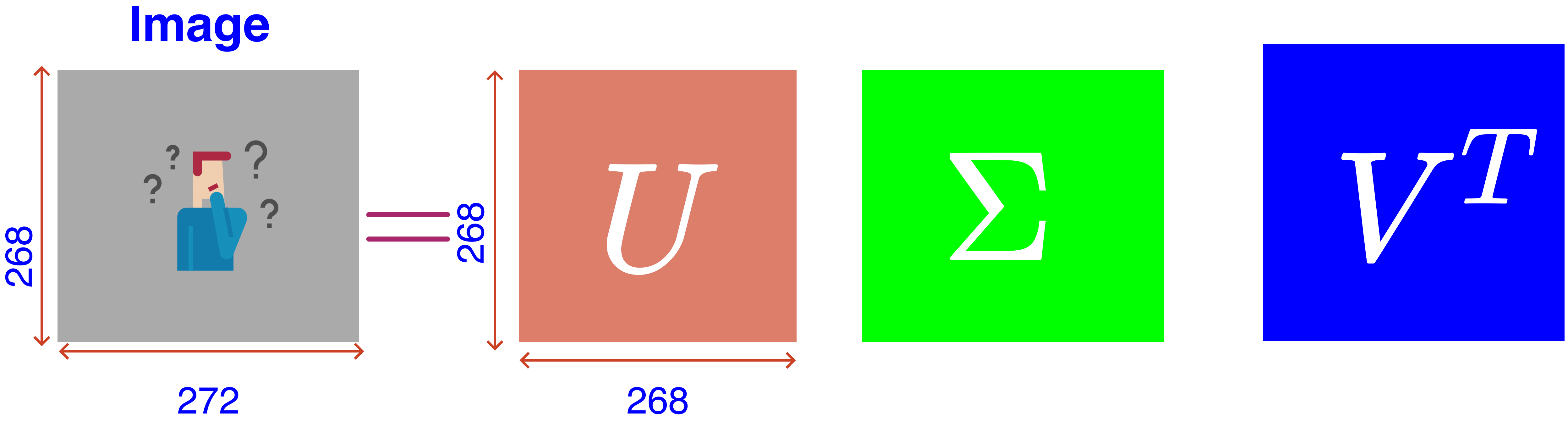
Let's play a game



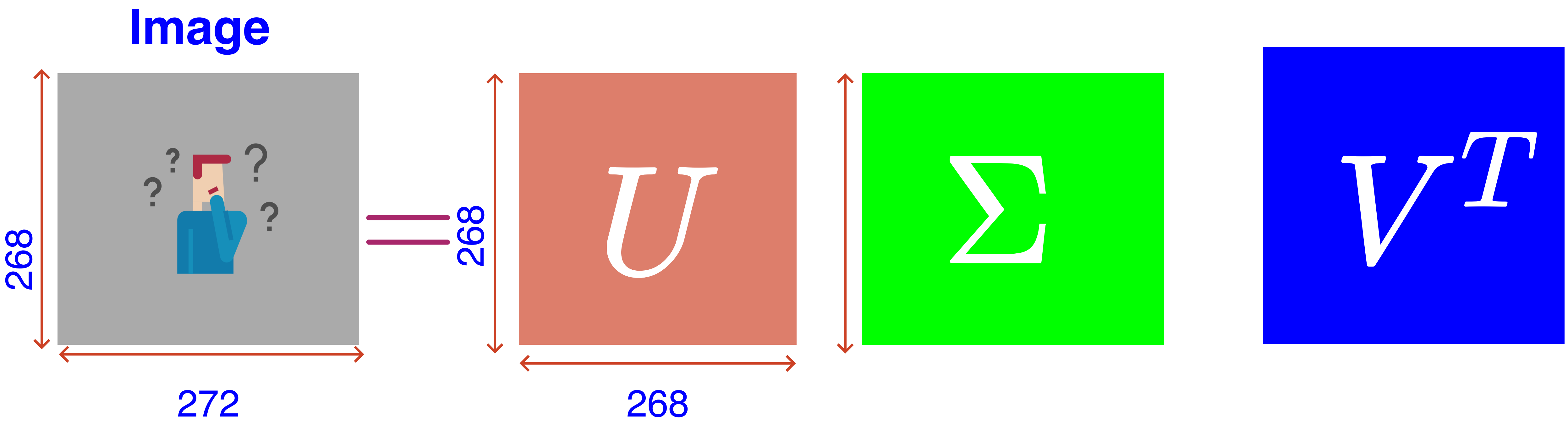
Let's play a game



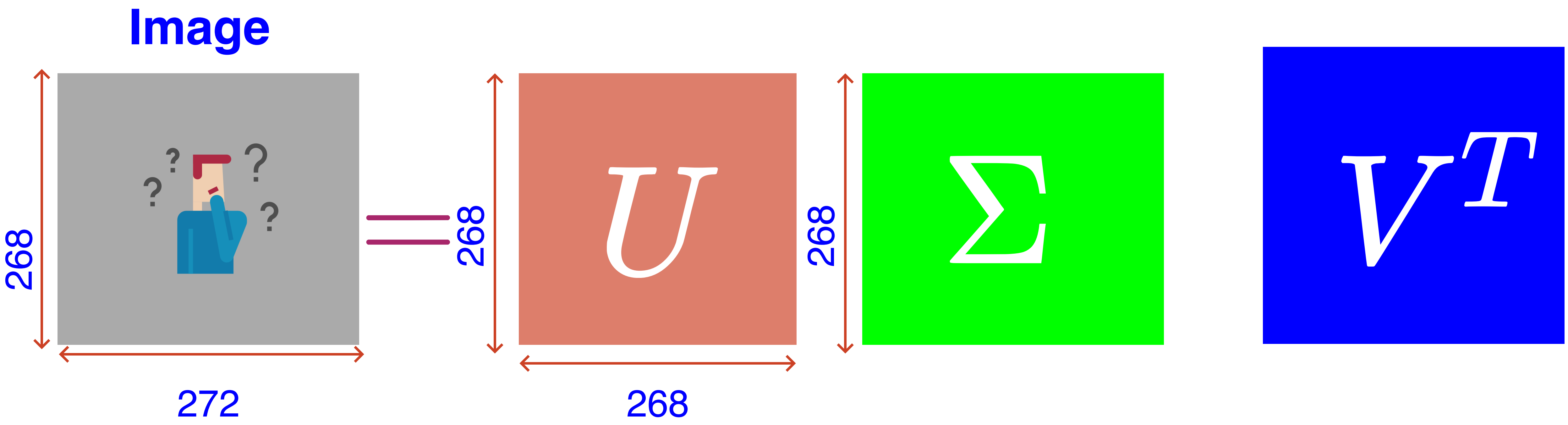
Let's play a game



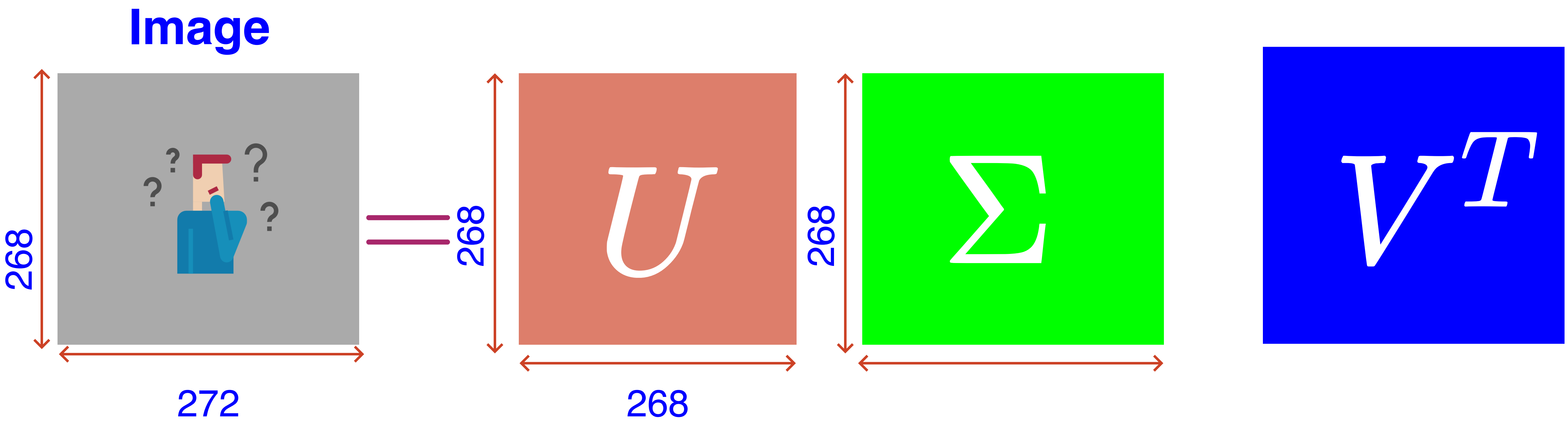
Let's play a game



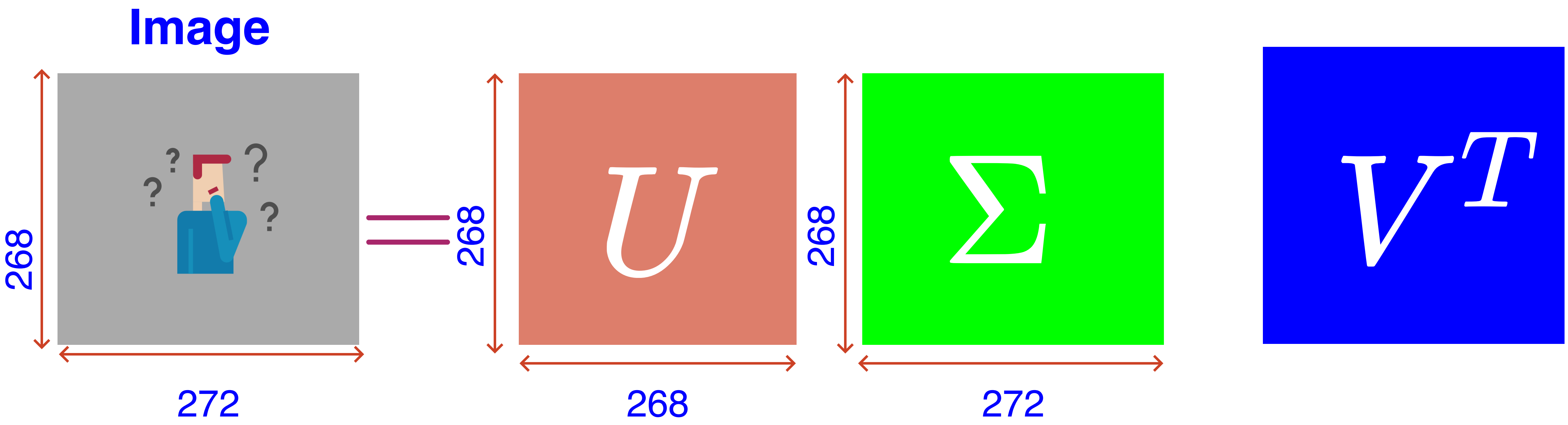
Let's play a game



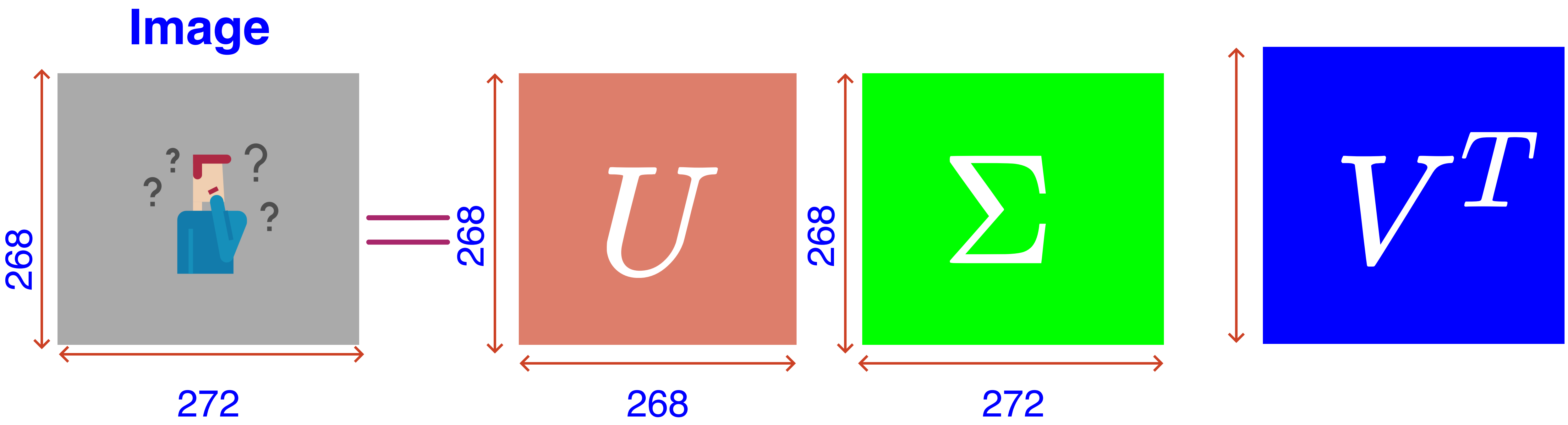
Let's play a game



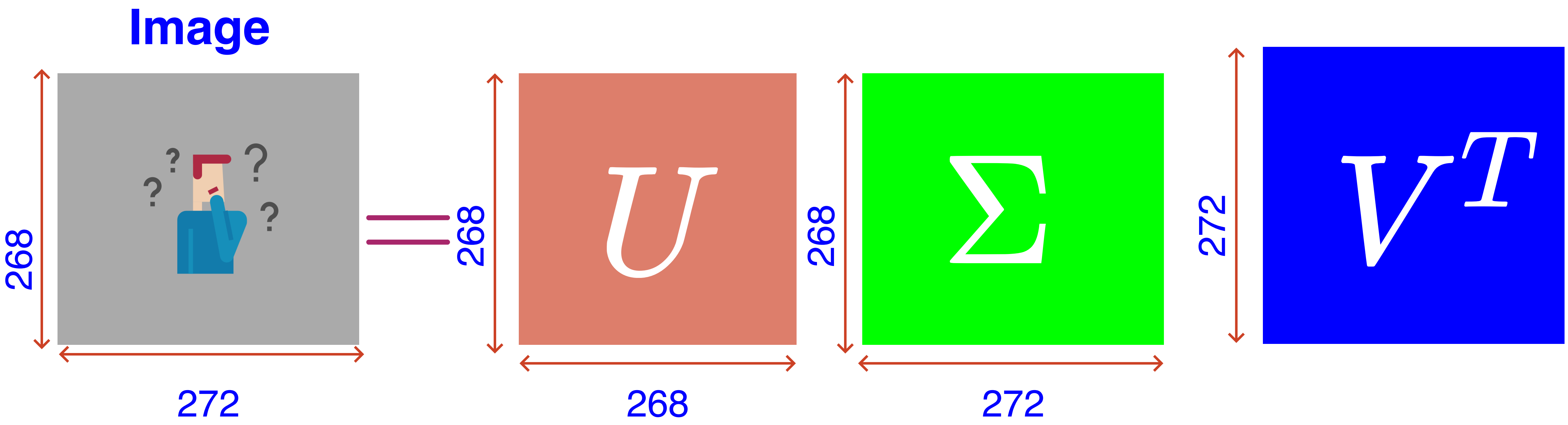
Let's play a game



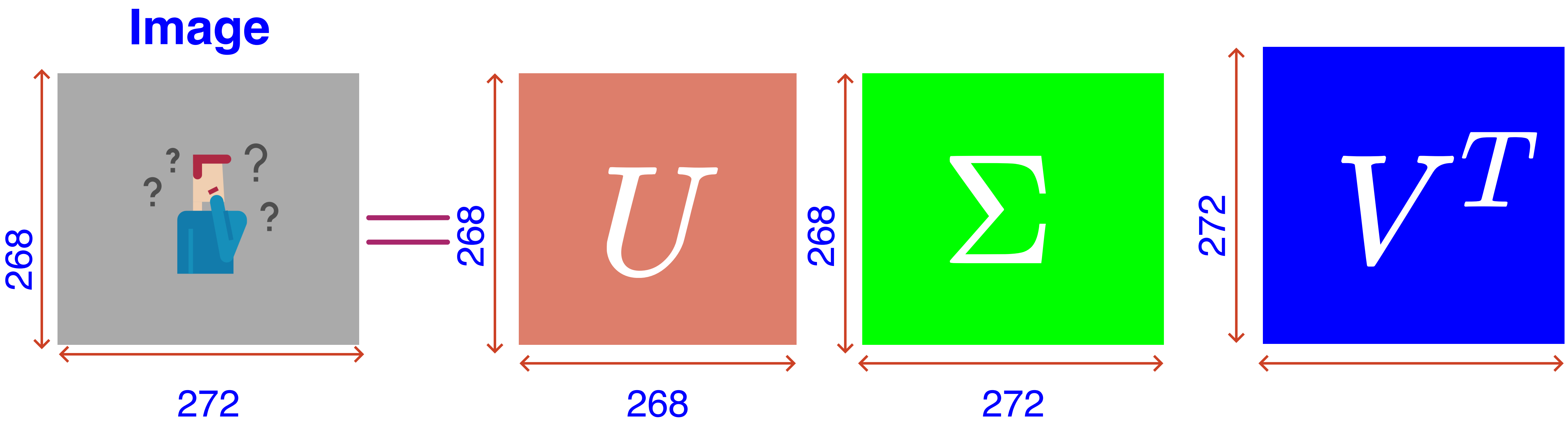
Let's play a game



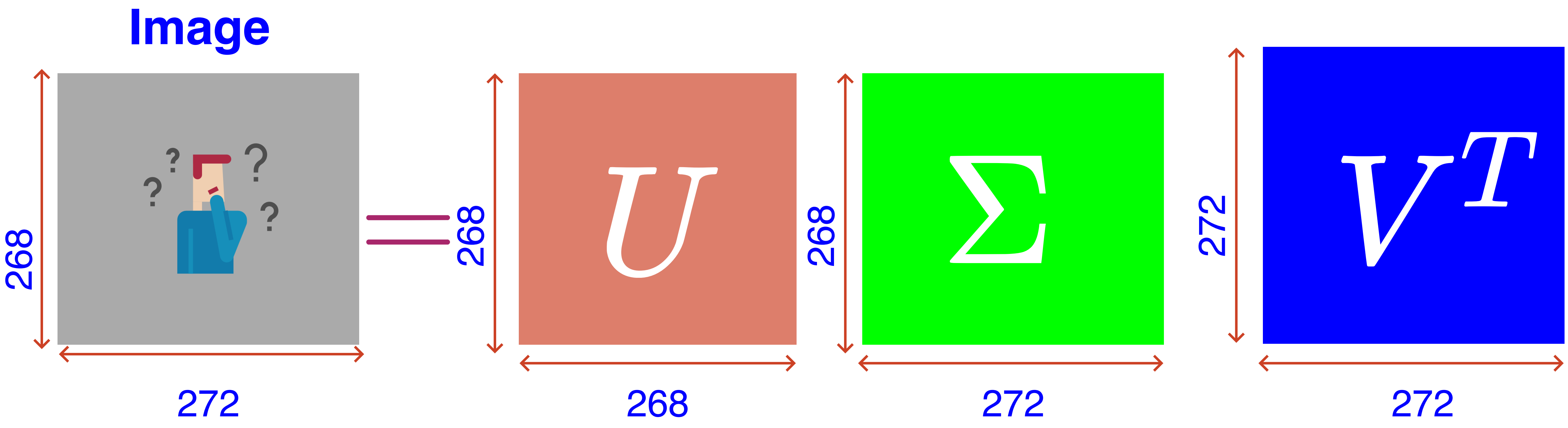
Let's play a game



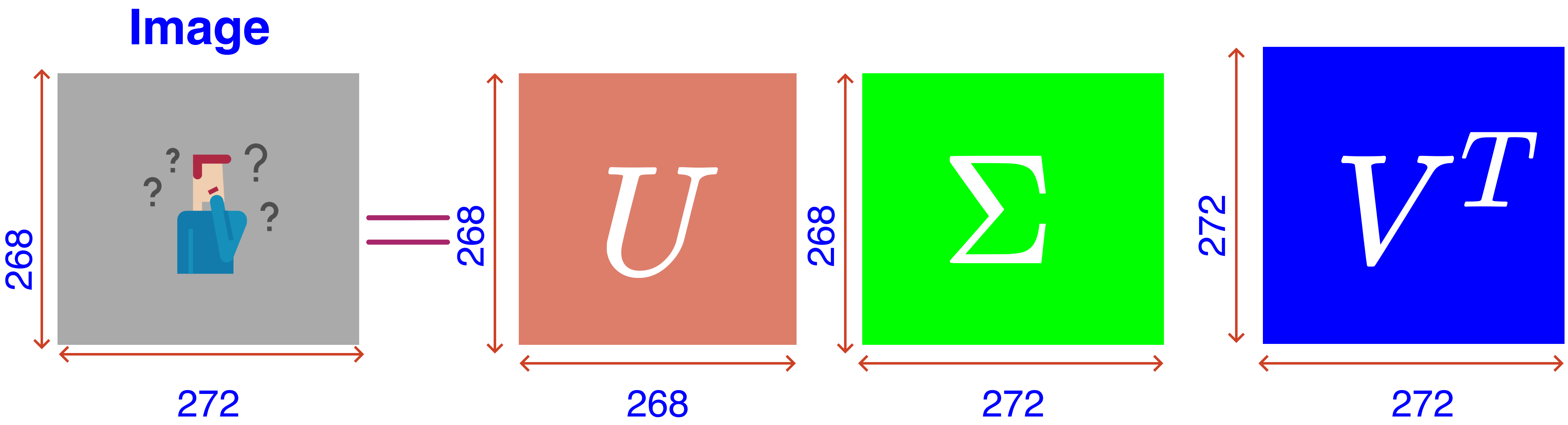
Let's play a game



Let's play a game

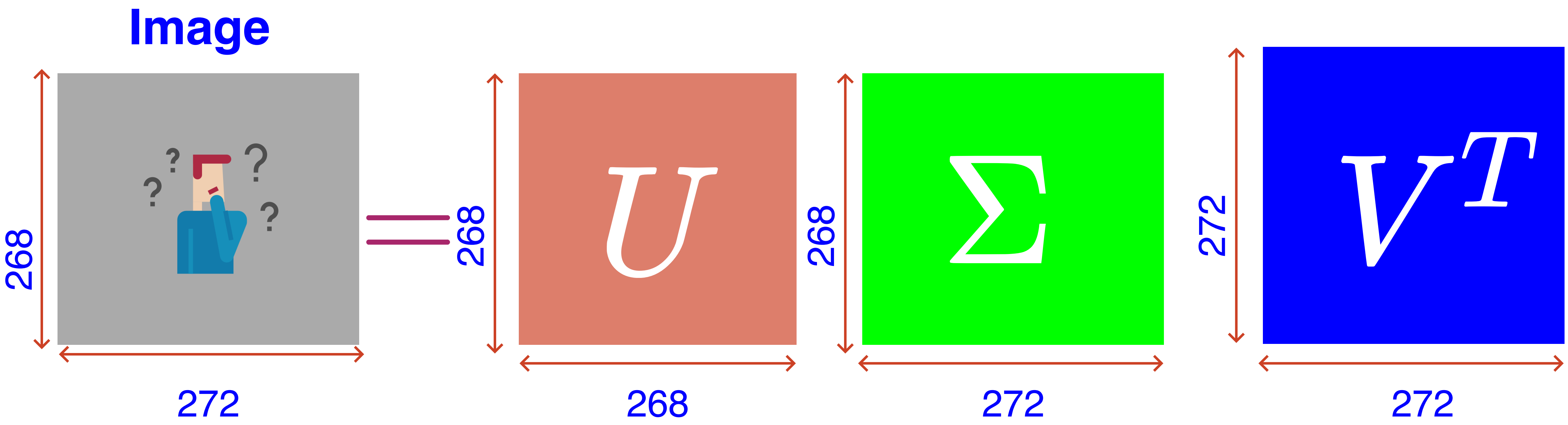


Let's play a game



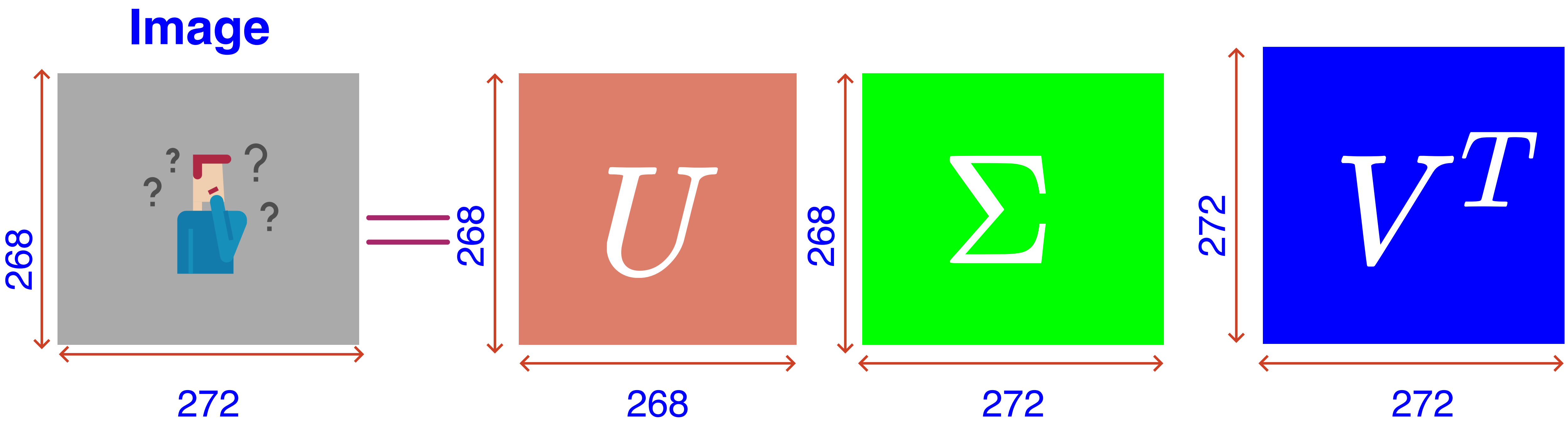
- I am going to show you a sequence of images, one after another, that contains something.

Let's play a game



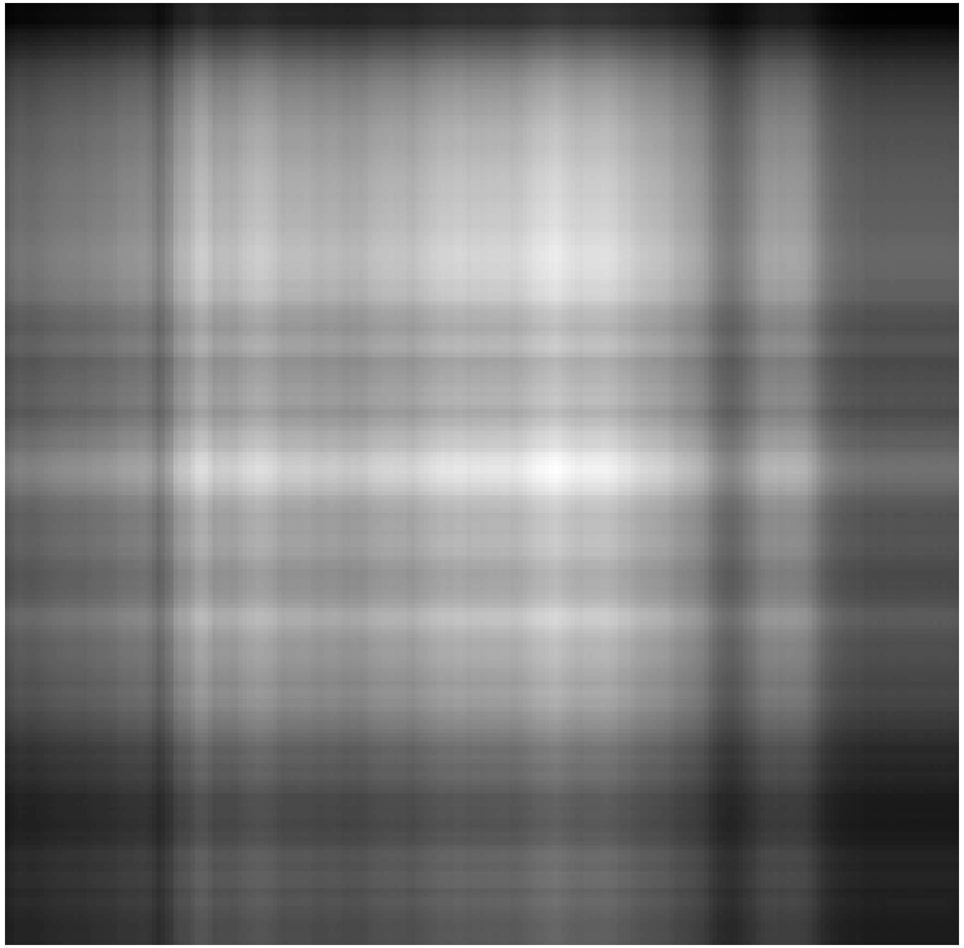
- I am going to show you a sequence of images, one after another, that contains something.
- Task : Recognise the "thing" in the images. (Note down the sequence number)

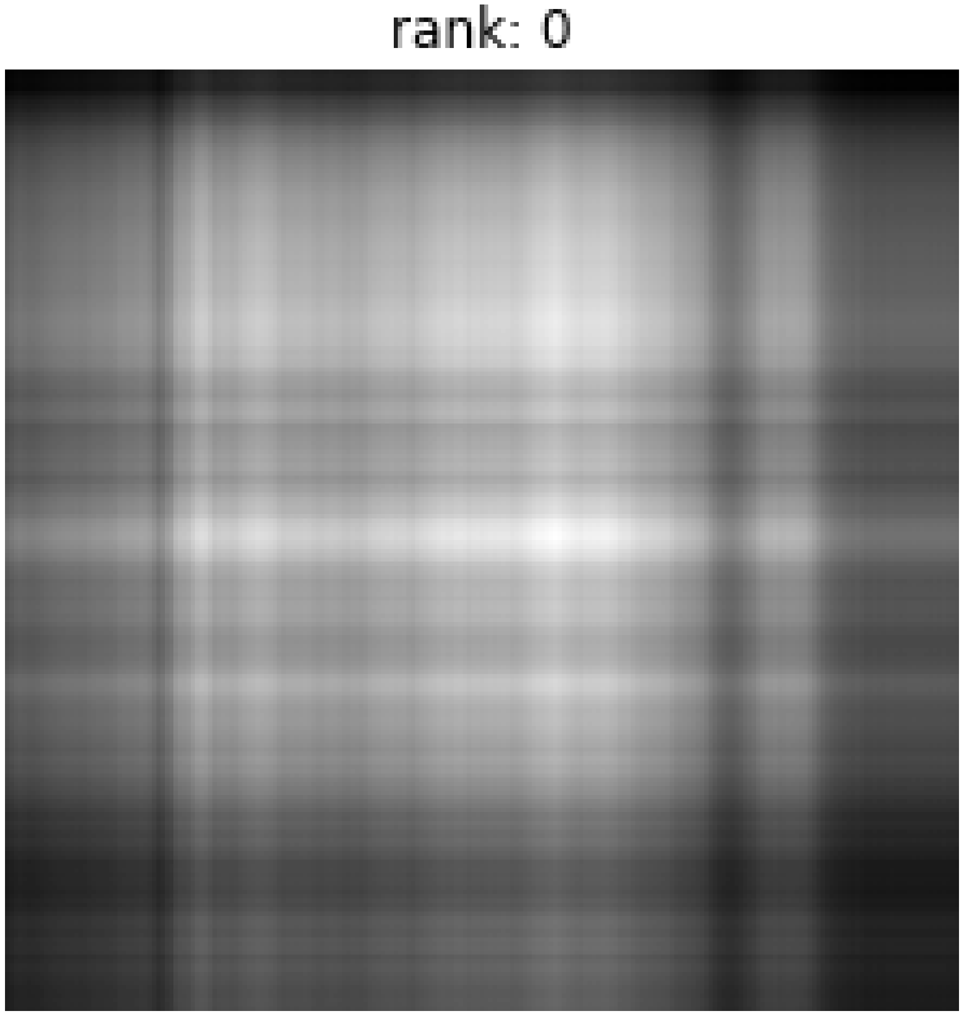
Let's play a game



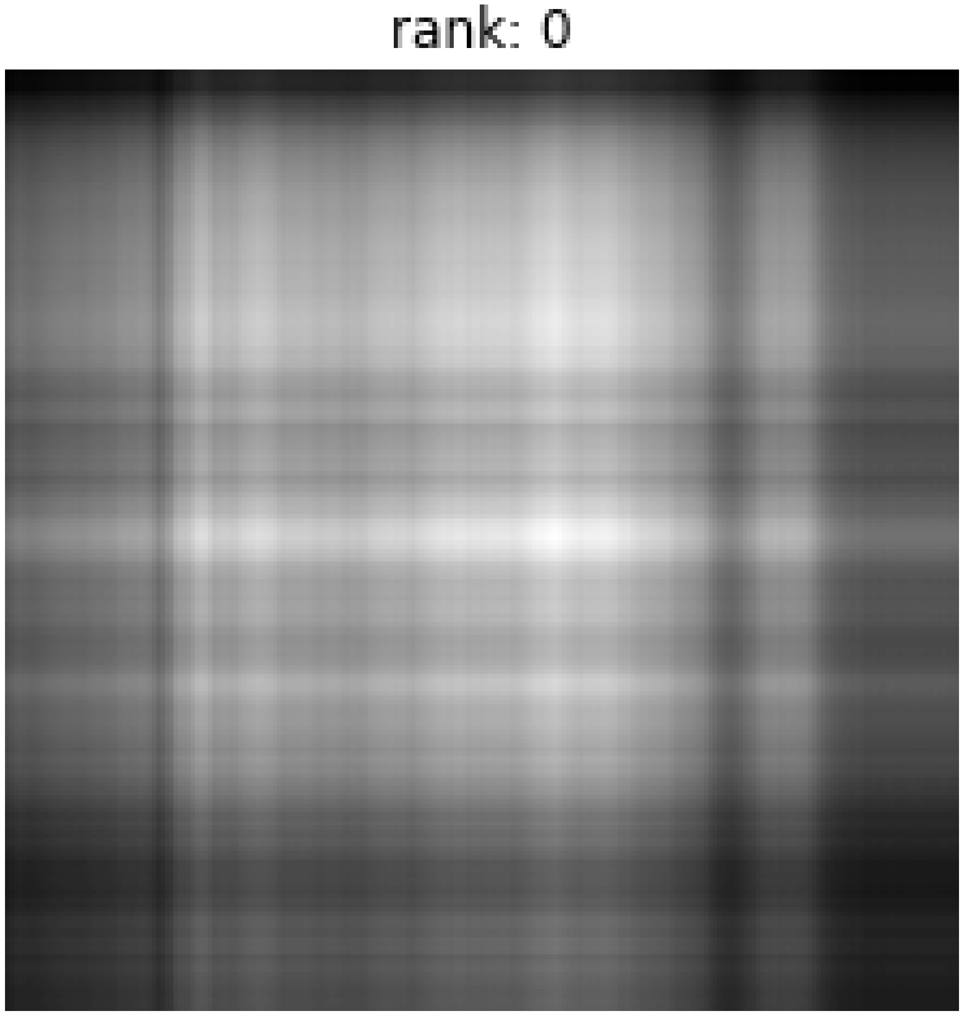
- I am going to show you a sequence of images, one after another, that contains something.
- Task : Recognise the "thing" in the images. (Note down the sequence number)
- Let's go

rank: 0





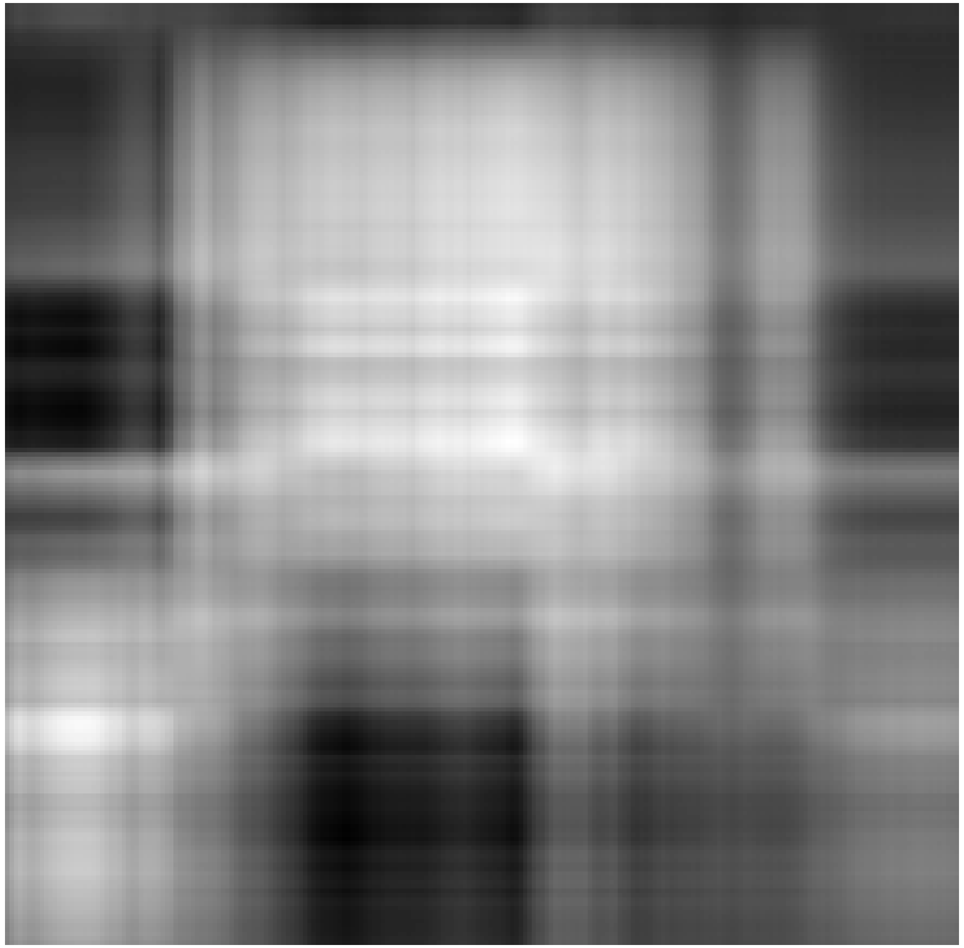
My bad: Add +1 to the rank



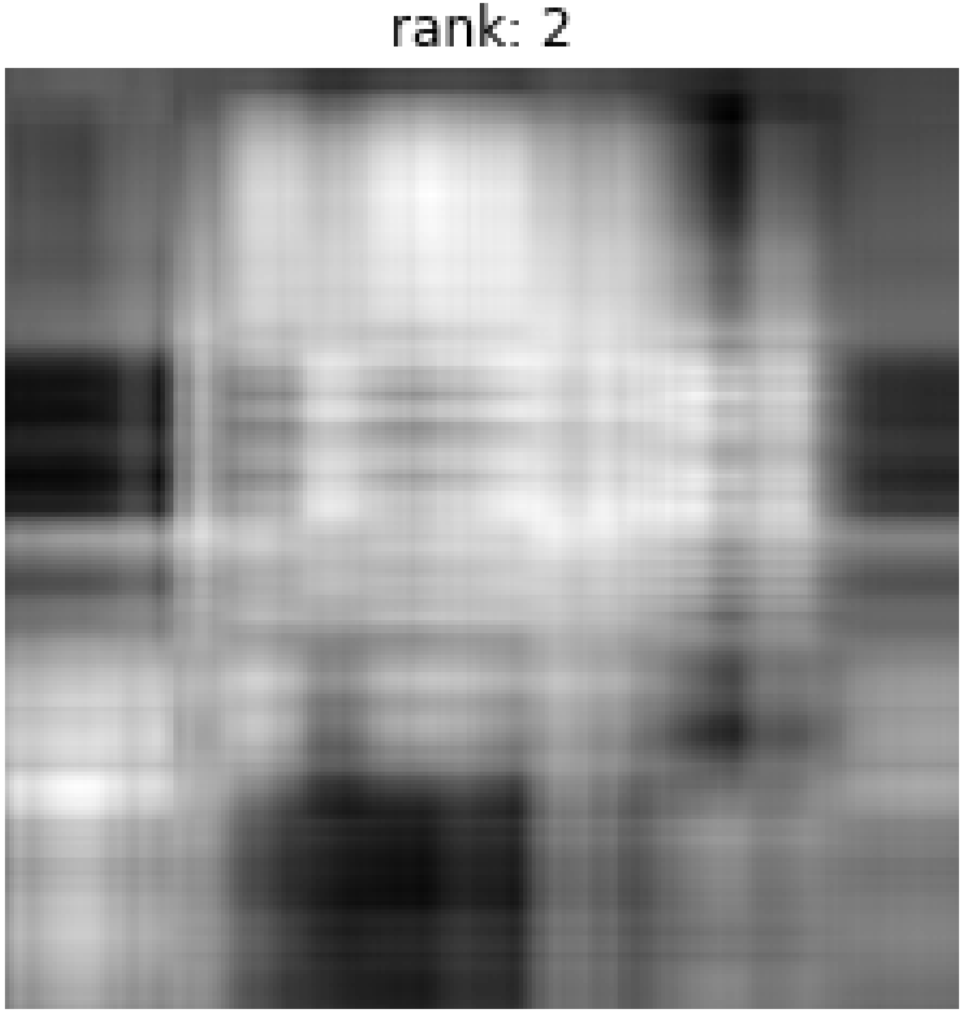
What do you perceive?

My bad: Add +1 to the rank

rank: 1

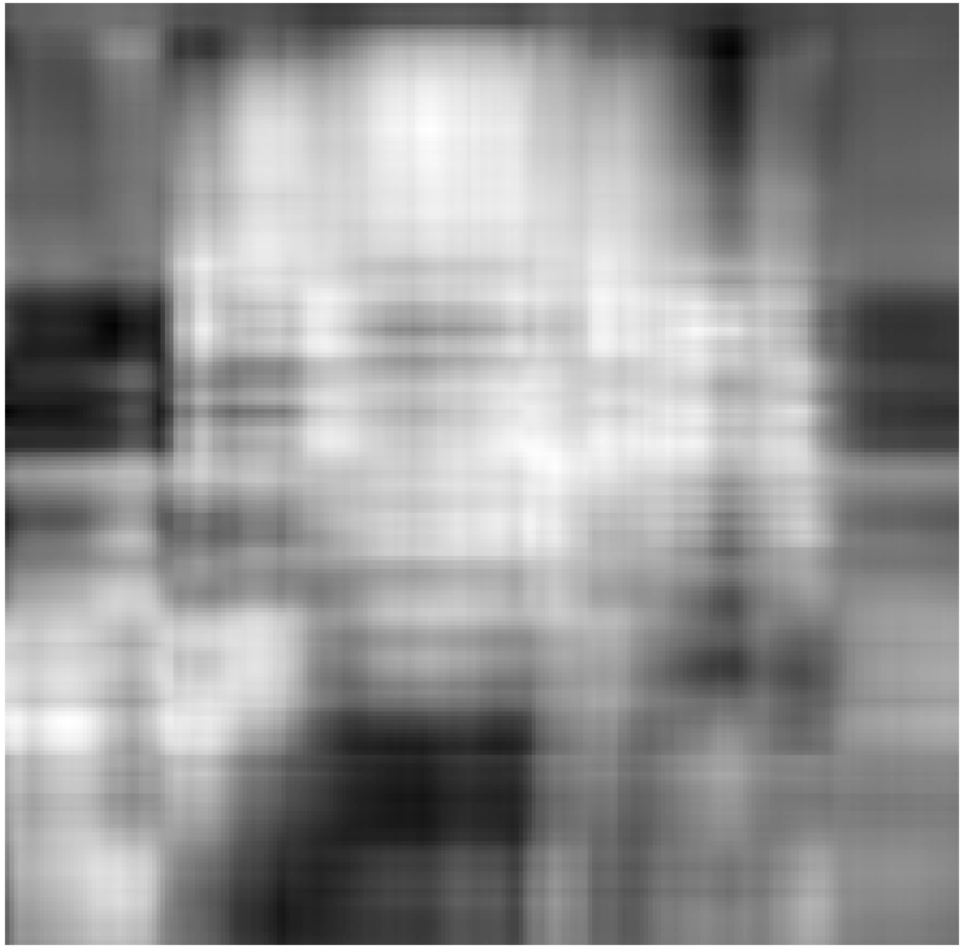


What do you perceive?

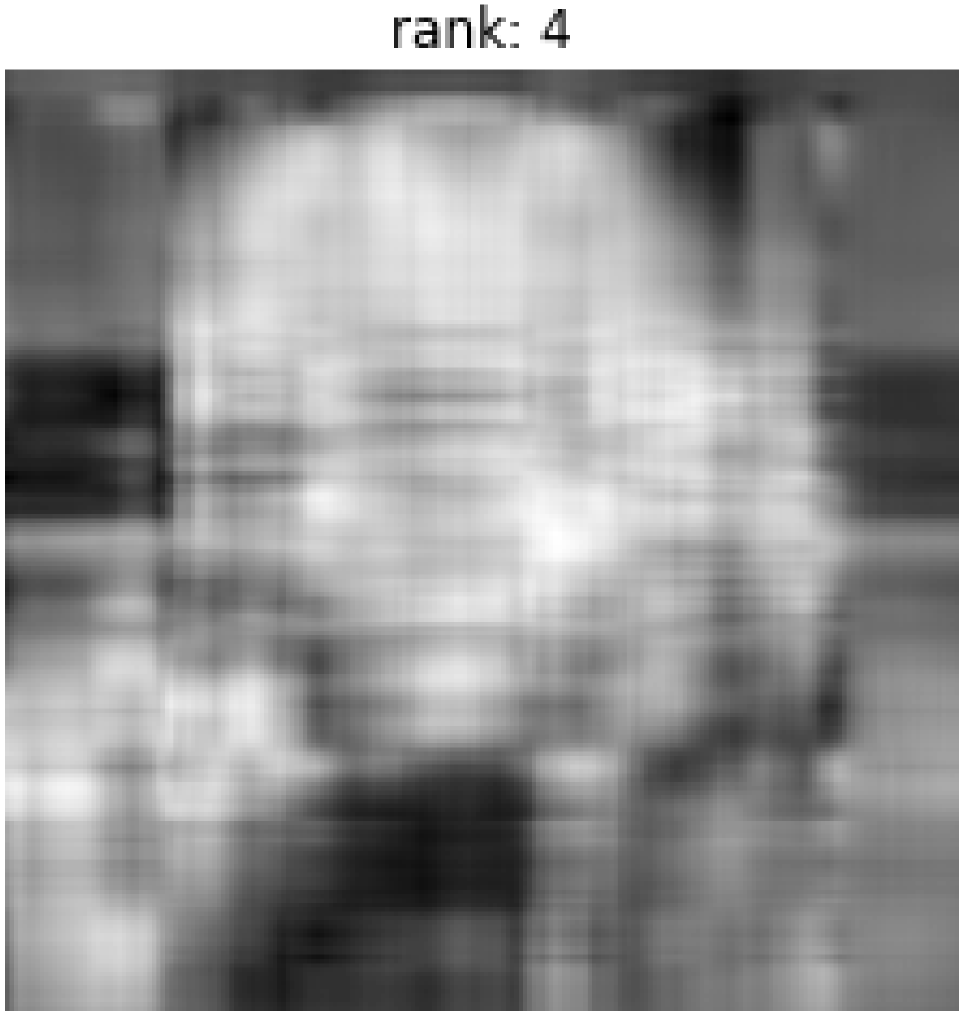


What do you perceive?

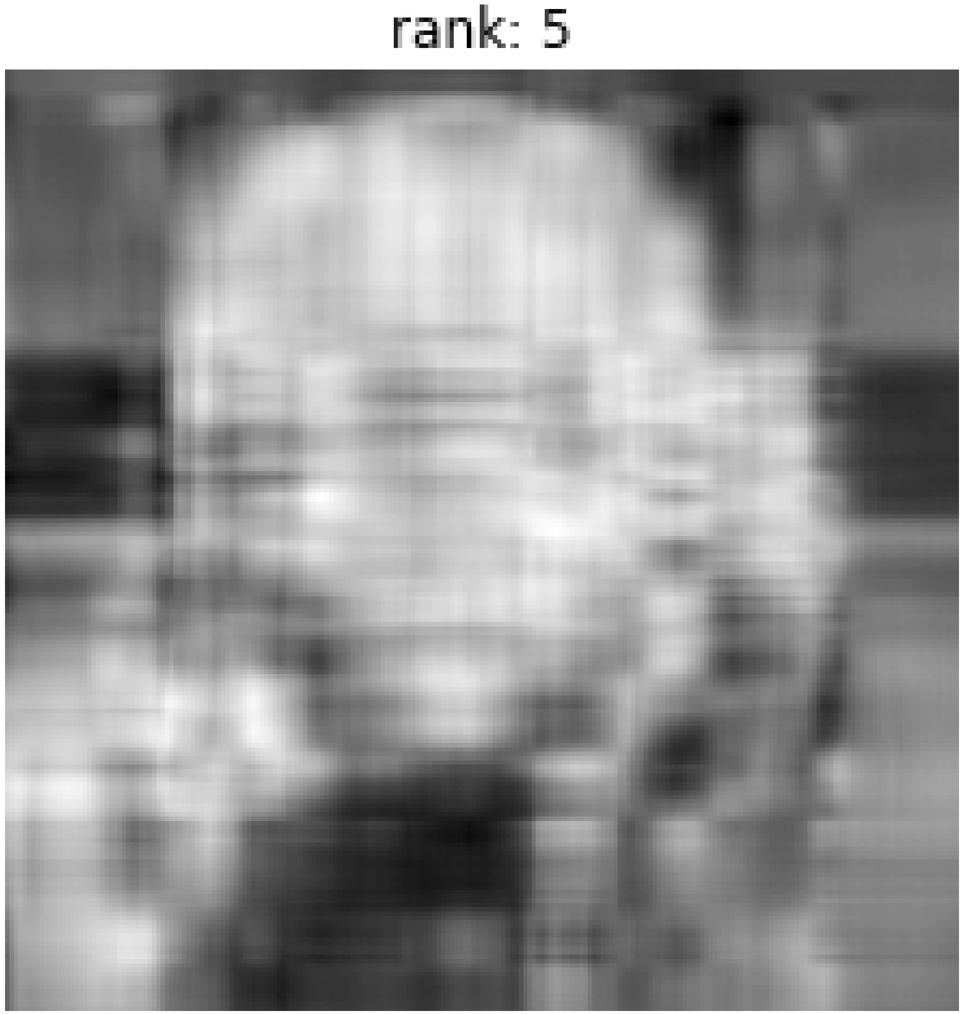
rank: 3



What do you perceive?



What do you perceive?



What do you perceive?

rank: 6


What do you perceive?

What do you perceive?

rank: 8

What do you perceive?

rank: 9

What do you perceive?

Recognise the man in the picture

Recognise the man in the picture

Recognise the man in the picture

rank: 12

Recognise the man in the picture

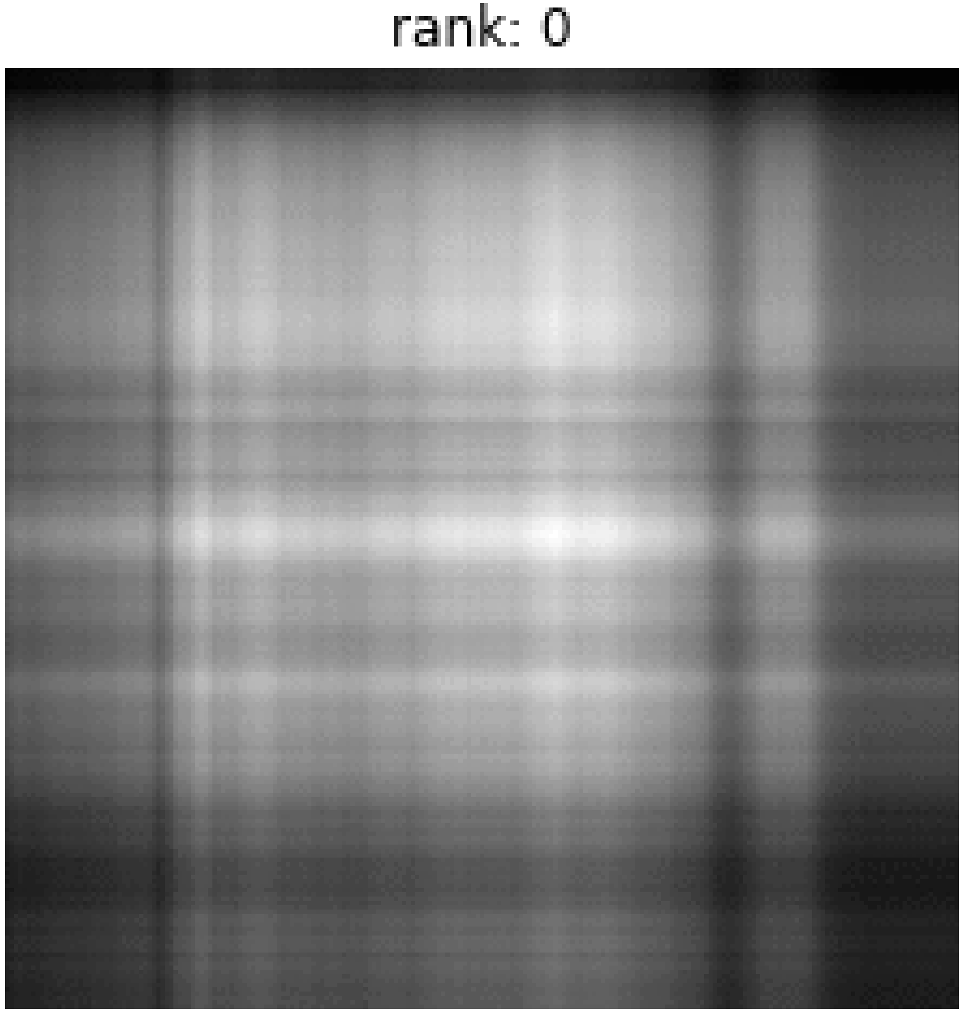
Recognise the man in the picture

rank: 14

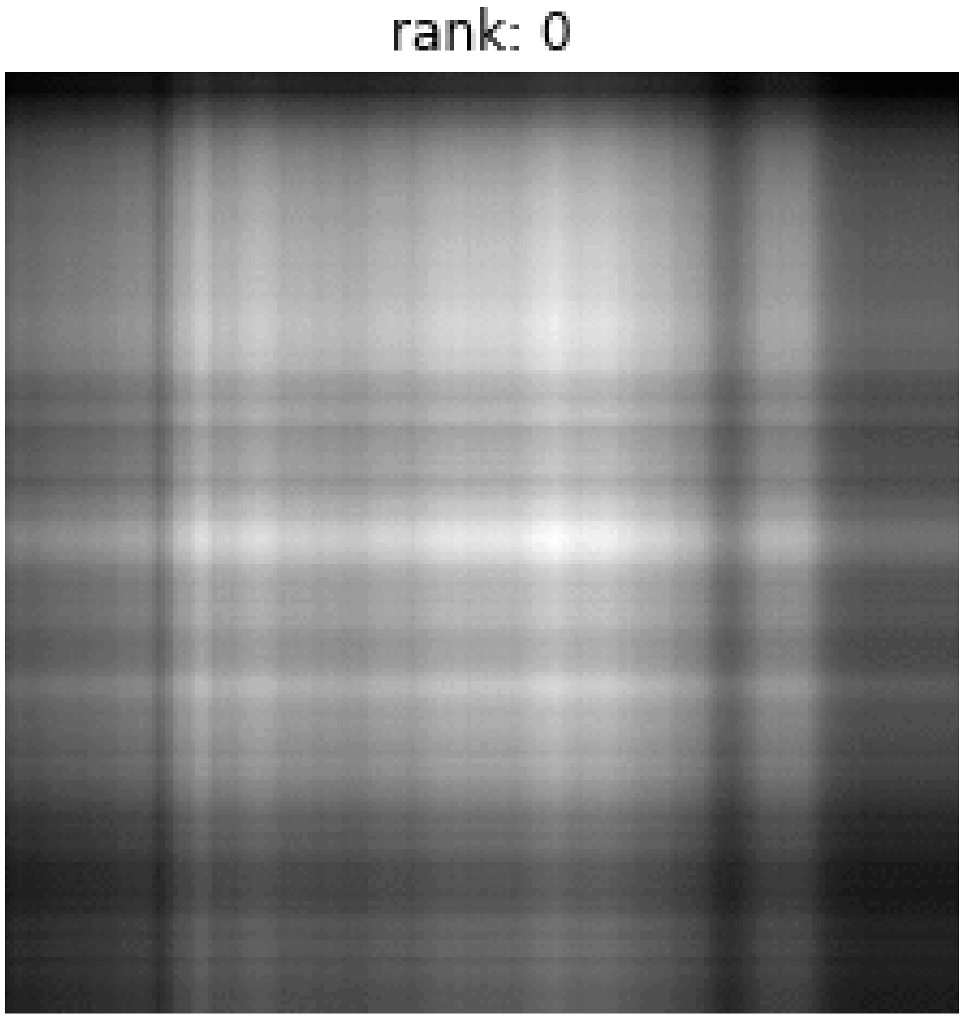
Recognise the man in the picture

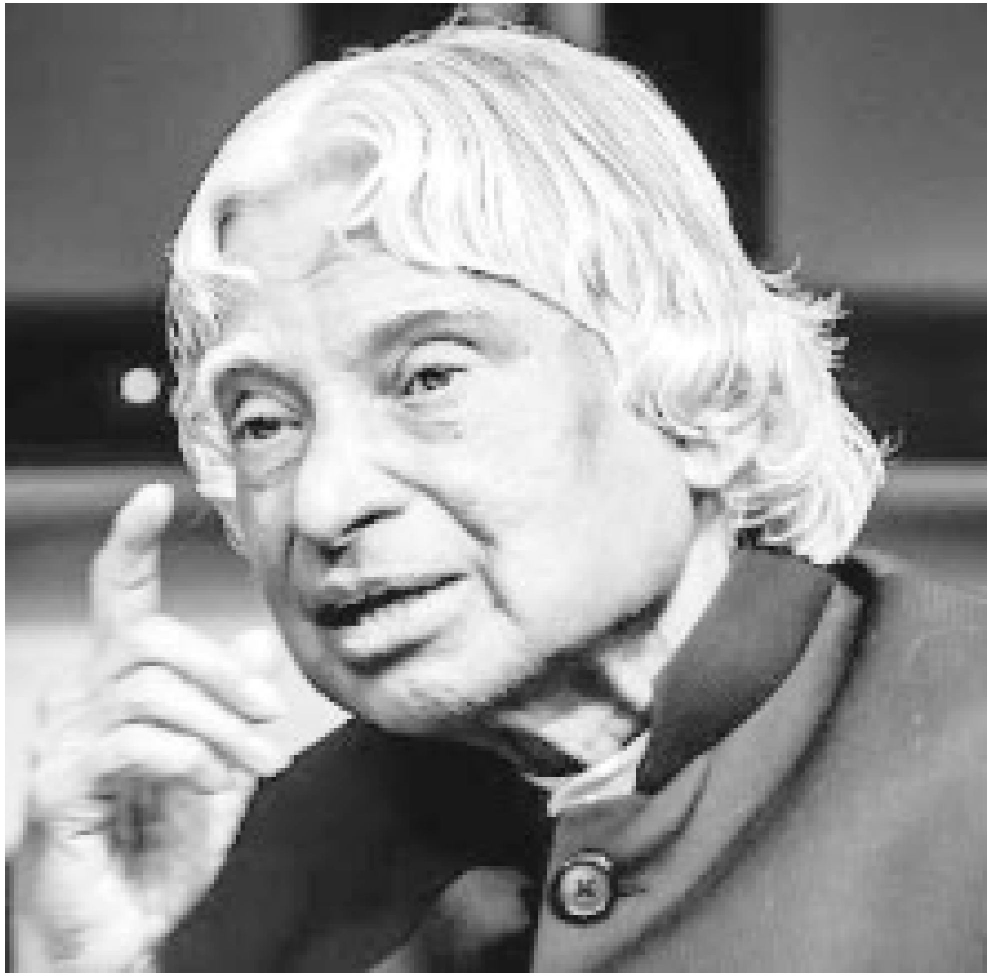
rank: 15

Rank (k =50) Approximation

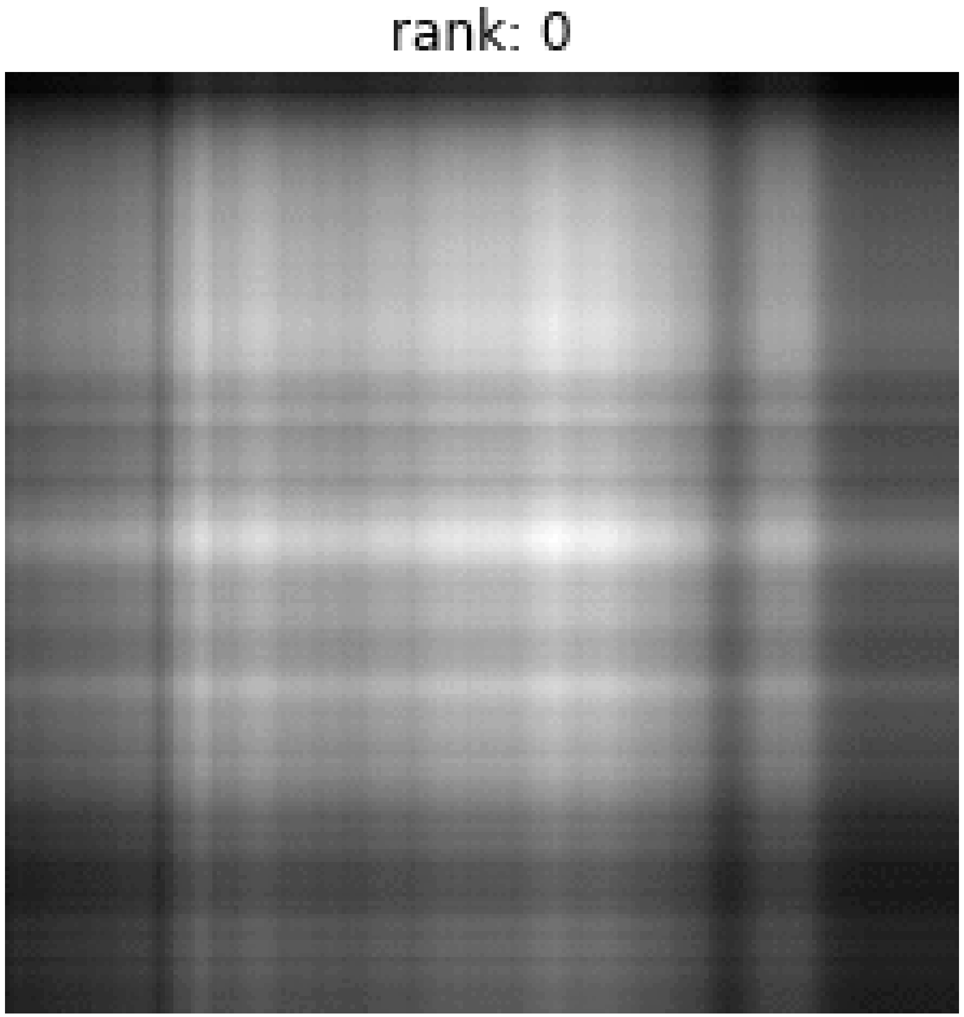


Rank (k =50) Approximation

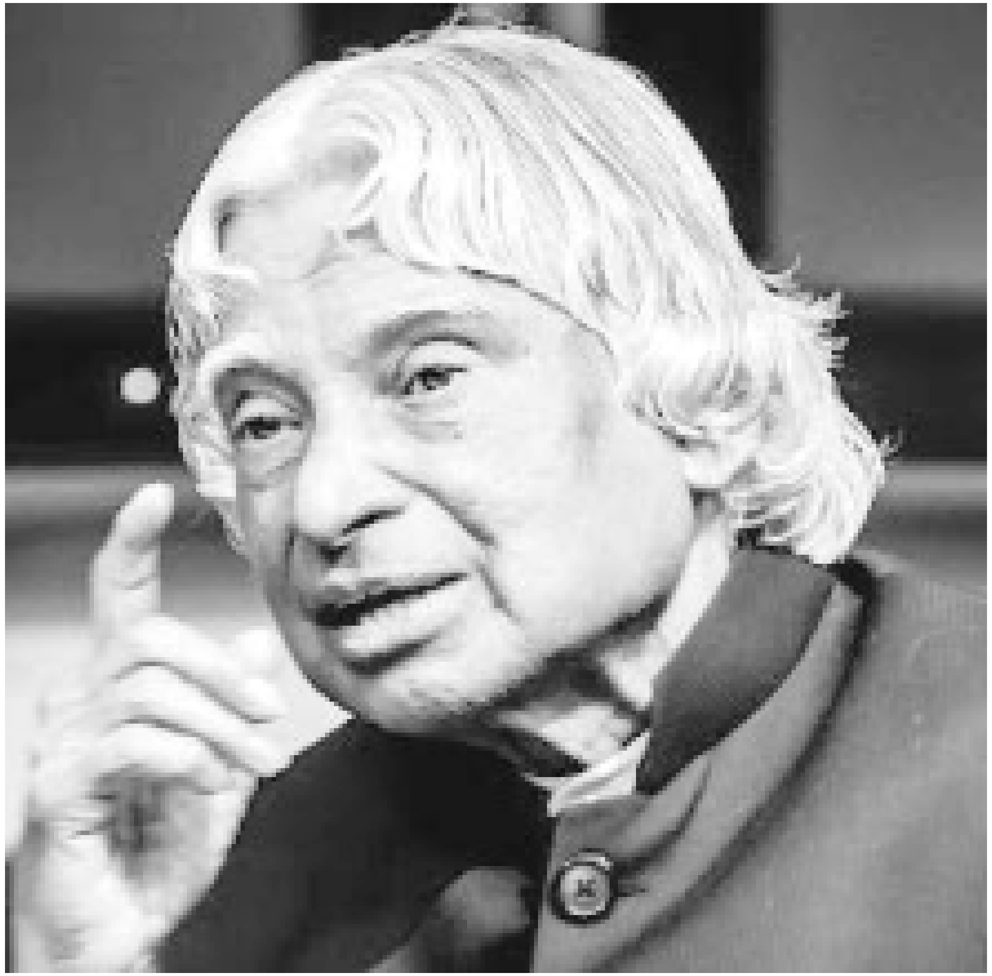




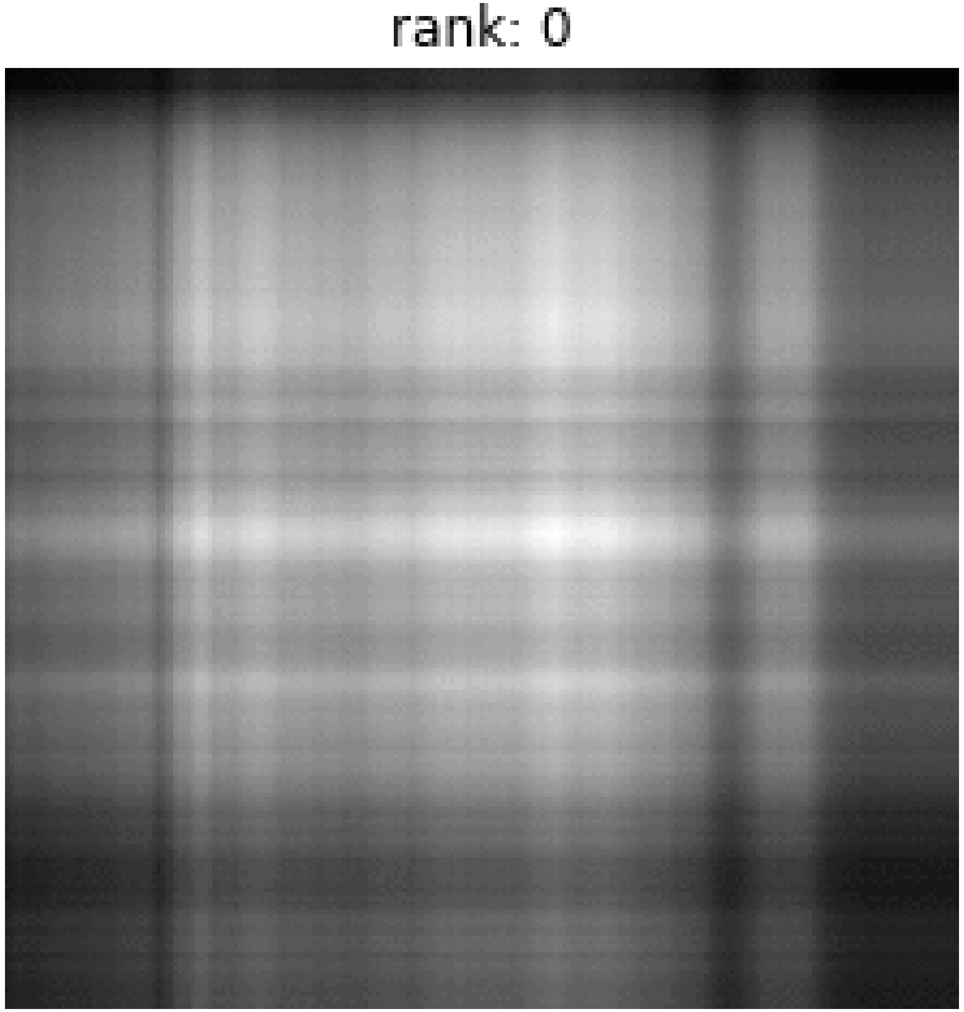
Rank (k =50) Approximation



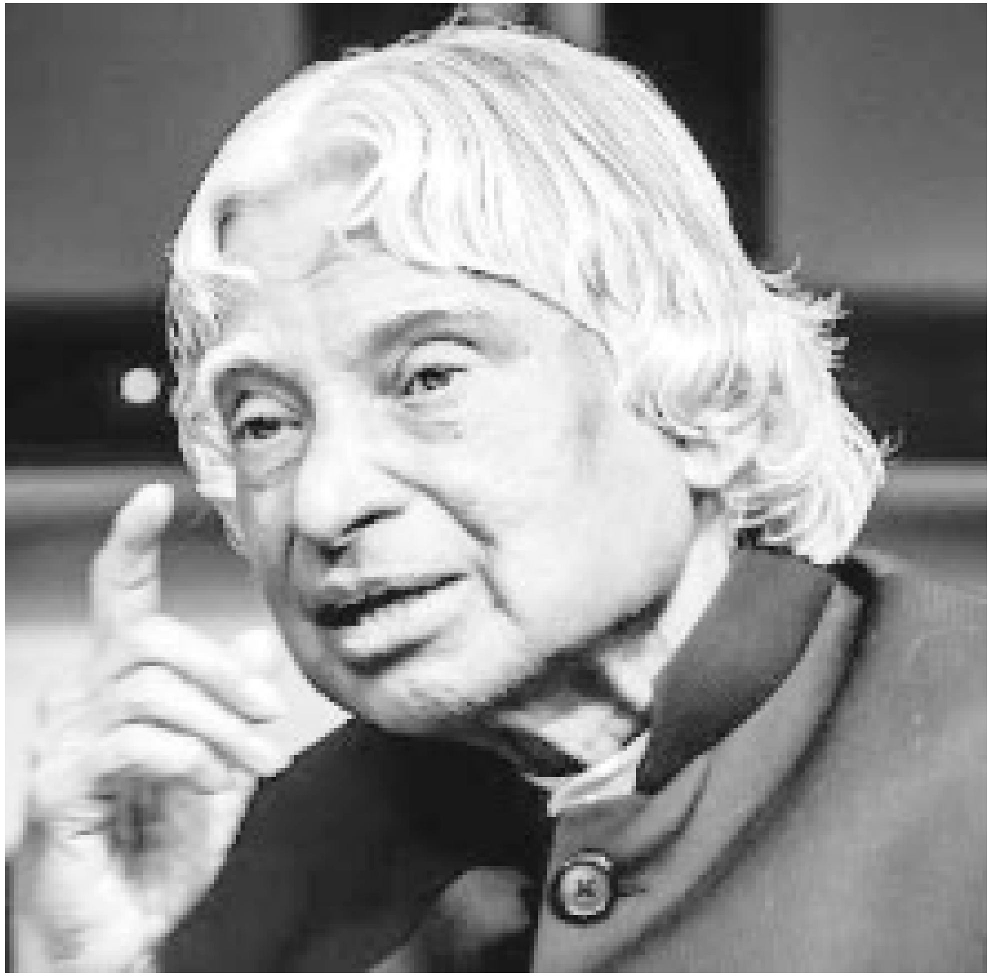
Of course, we lost some resolution (details)!



Rank (k =50) Approximation



Of course, we lost some resolution (details)!



But not the gross information!

Image compression

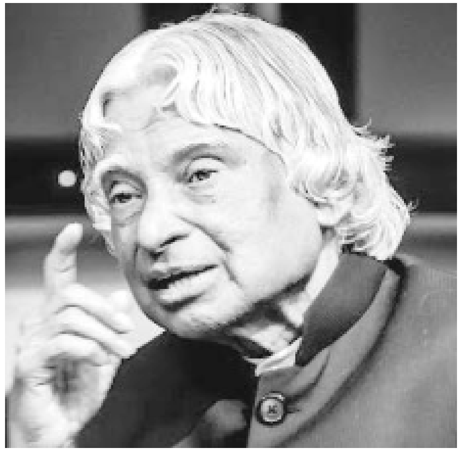


Image compression

268

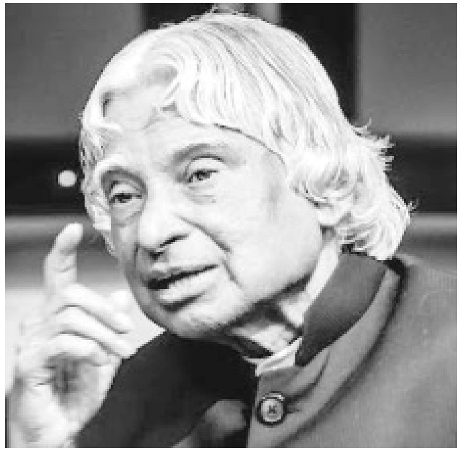


Image compression

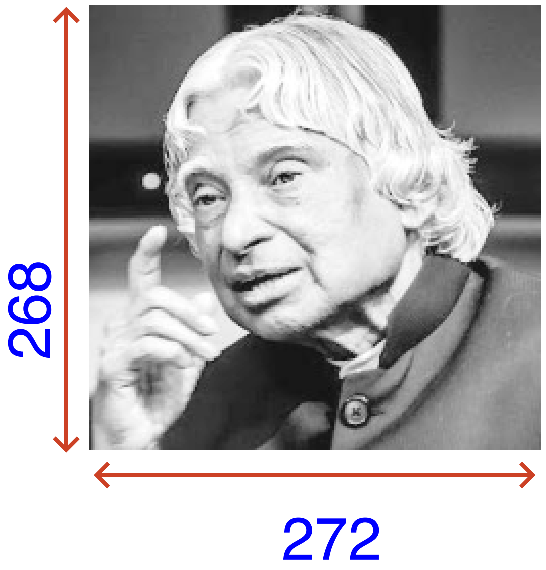
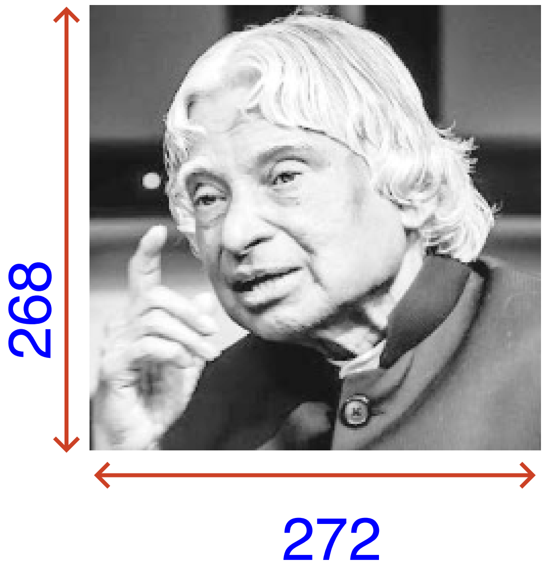


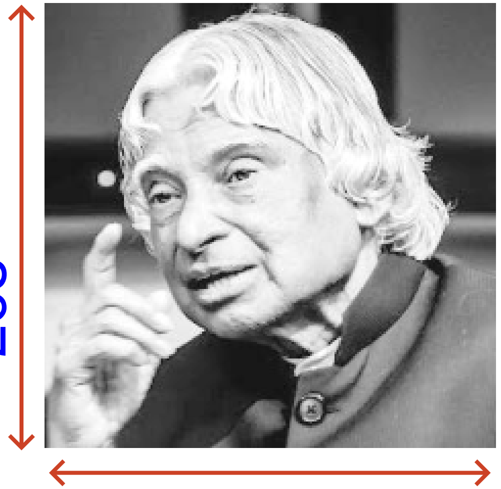
Image compression



$=$

Image compression

268



272

=

Image compression

Image compression

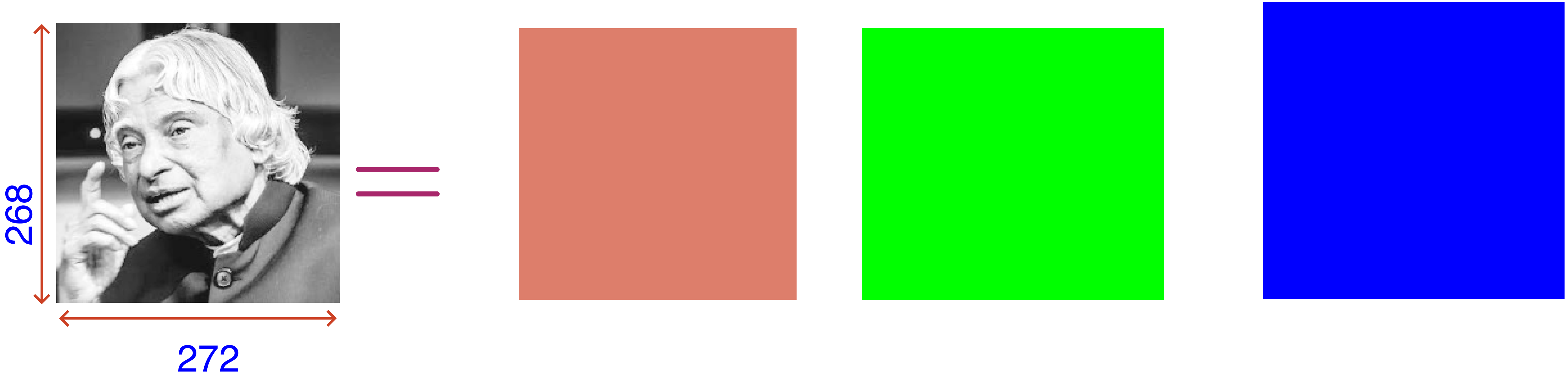
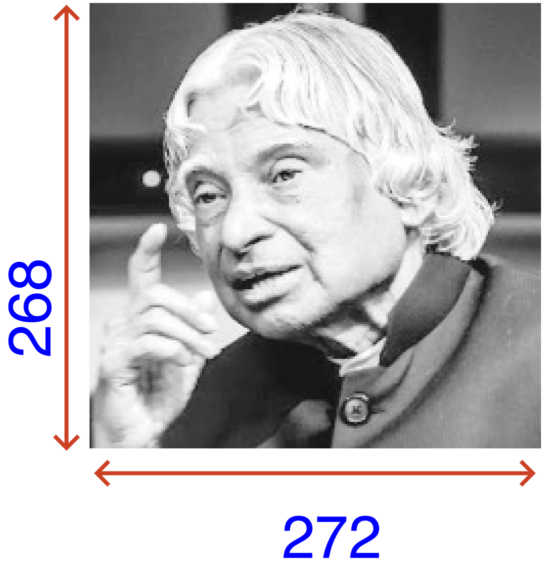
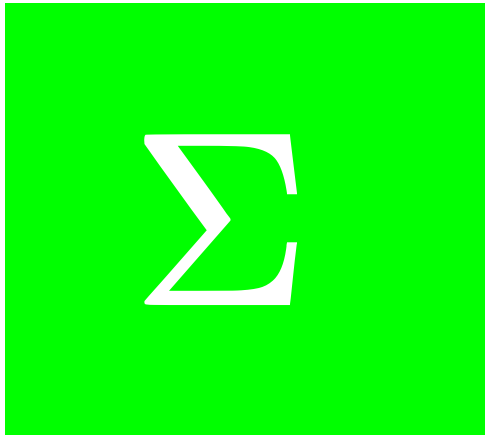


Image compression

Image compression



=



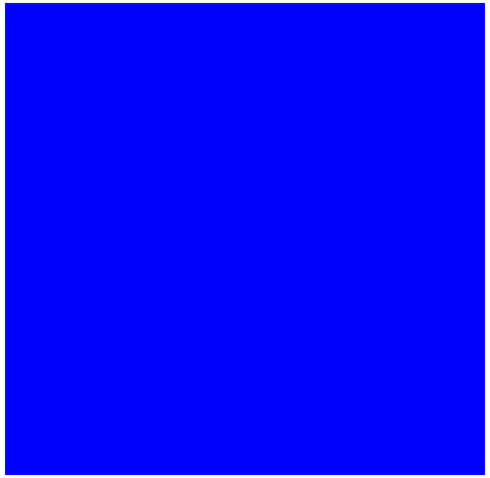
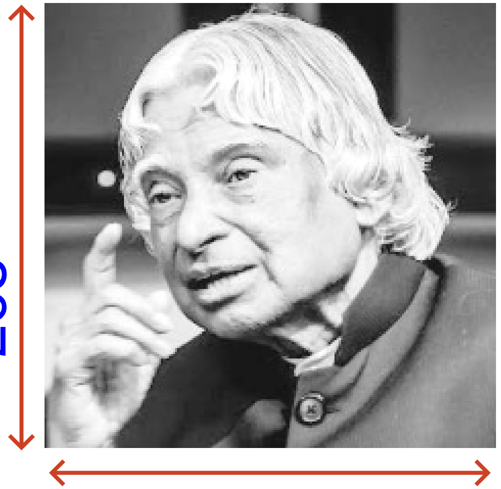


Image compression

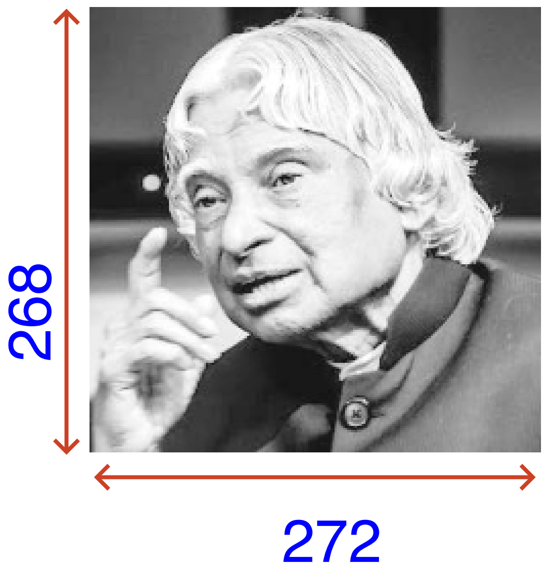


268

272

$$= \; U \; \Sigma \; V^T$$

Image compression



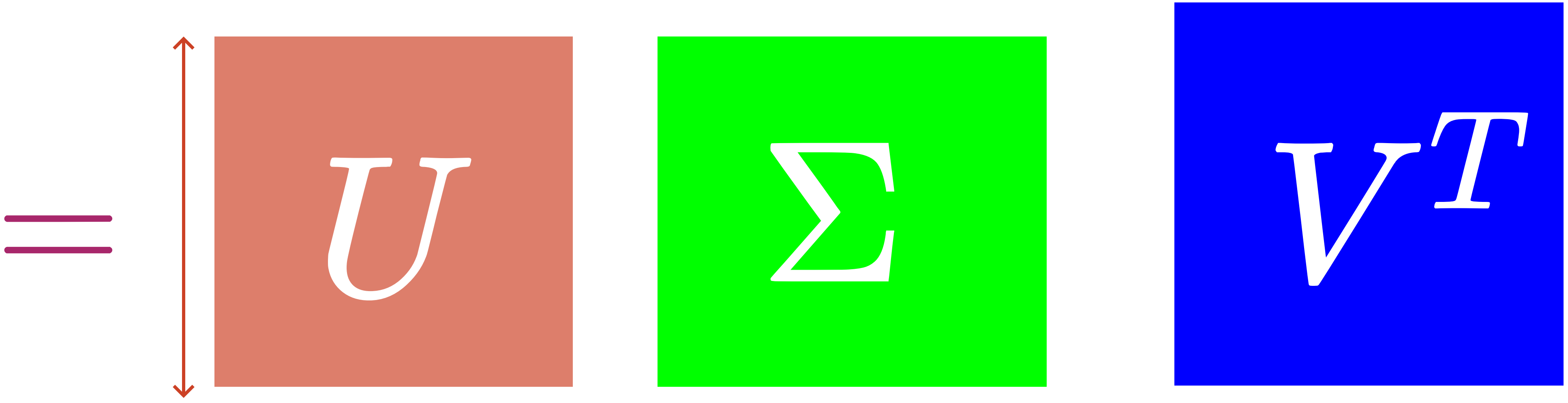


Image compression

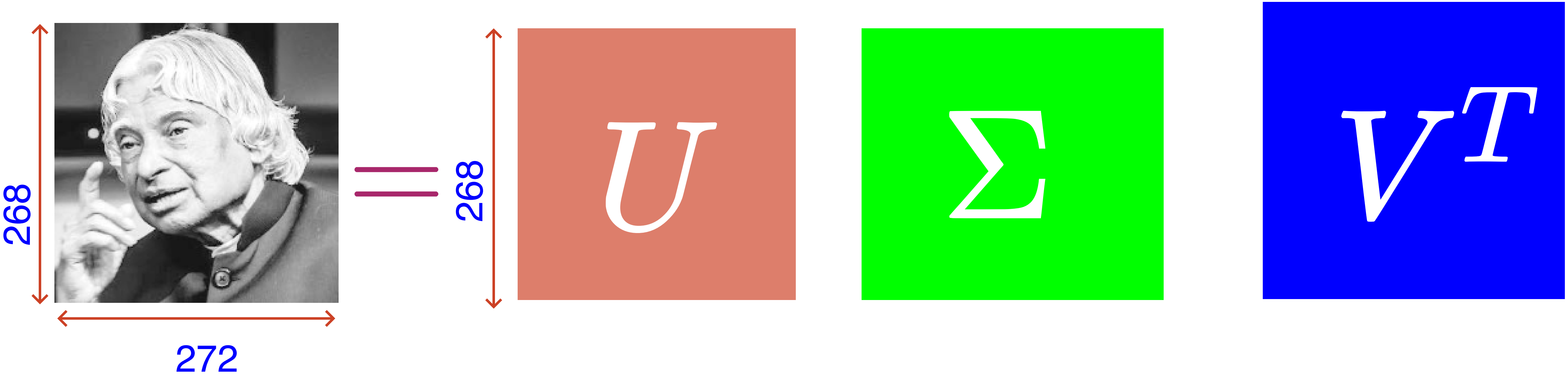


Image compression

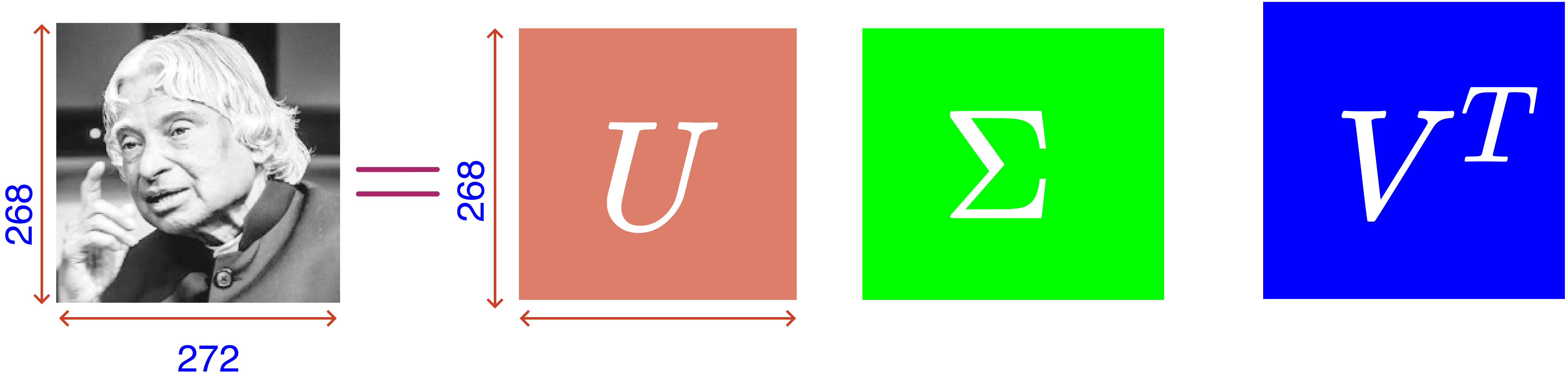


Image compression

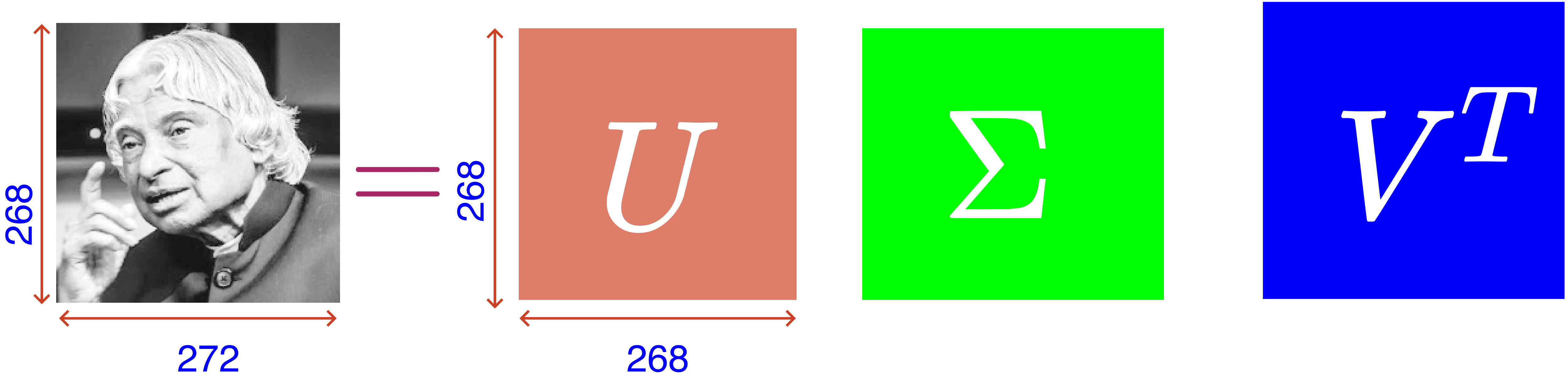


Image compression

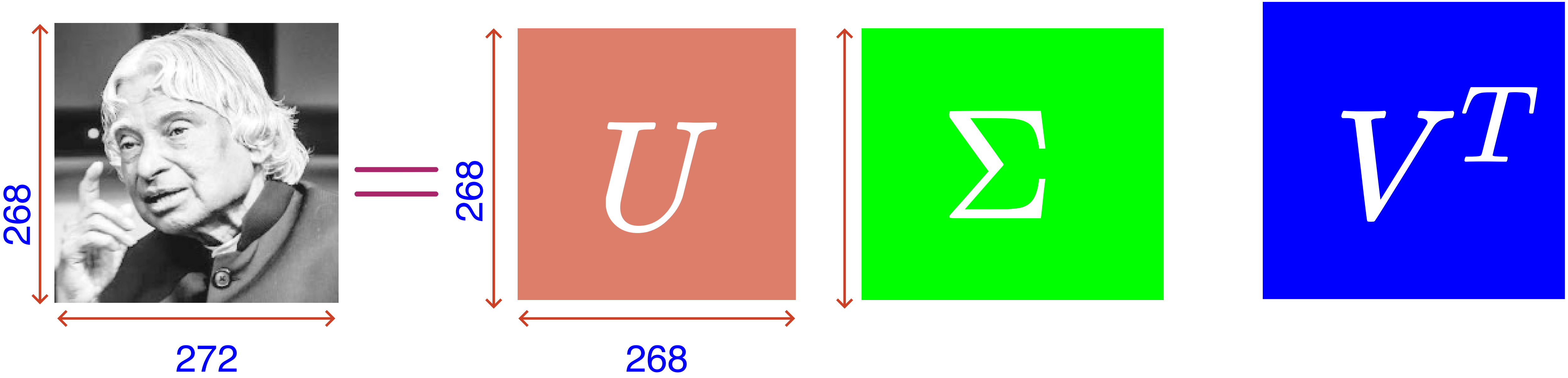


Image compression

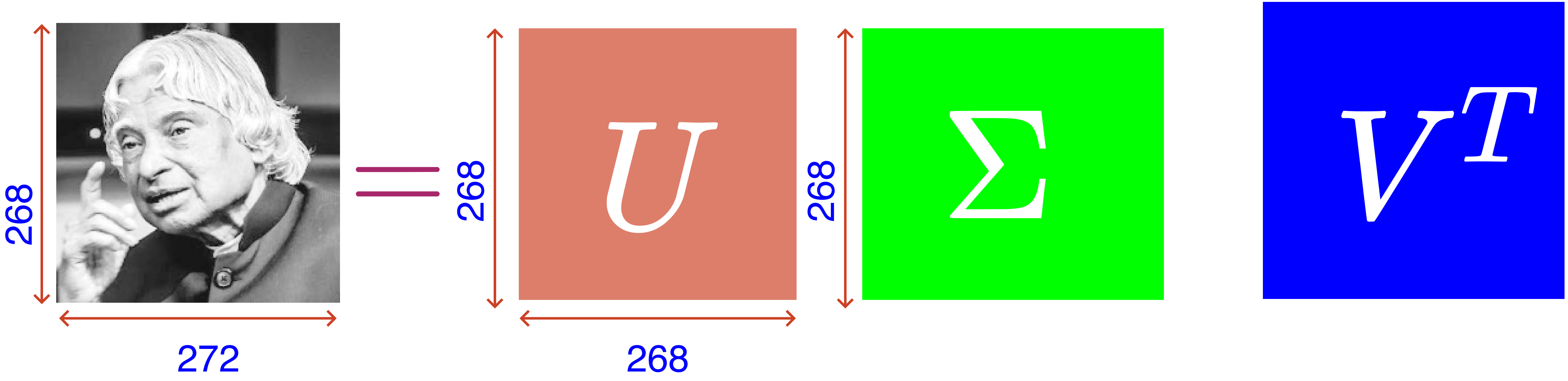


Image compression

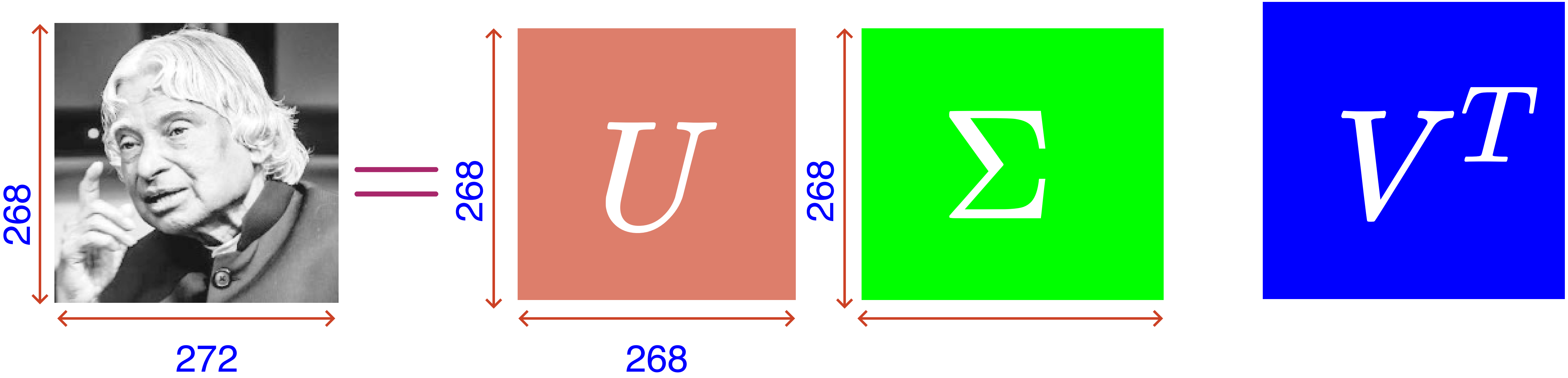


Image compression

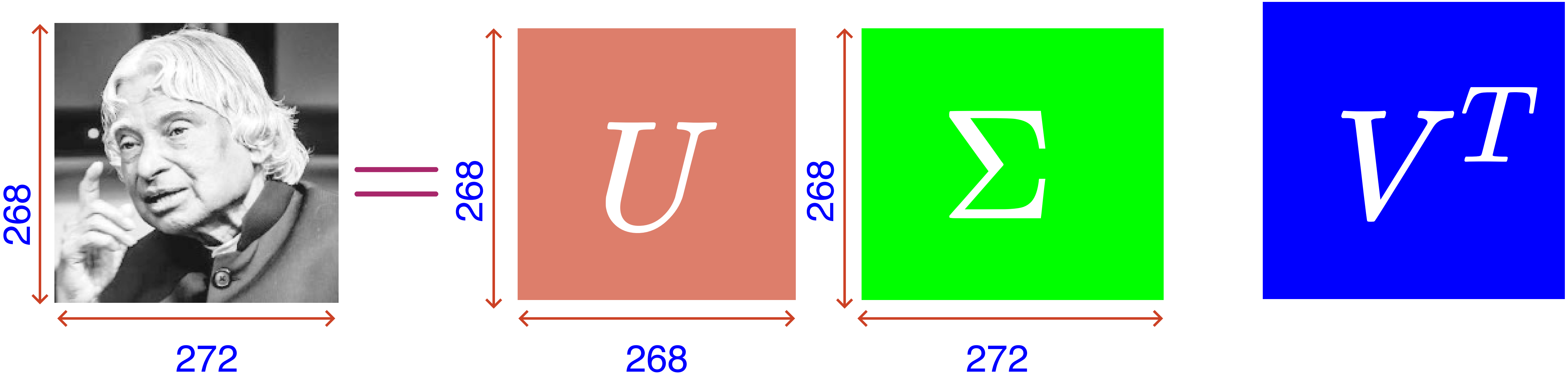


Image compression

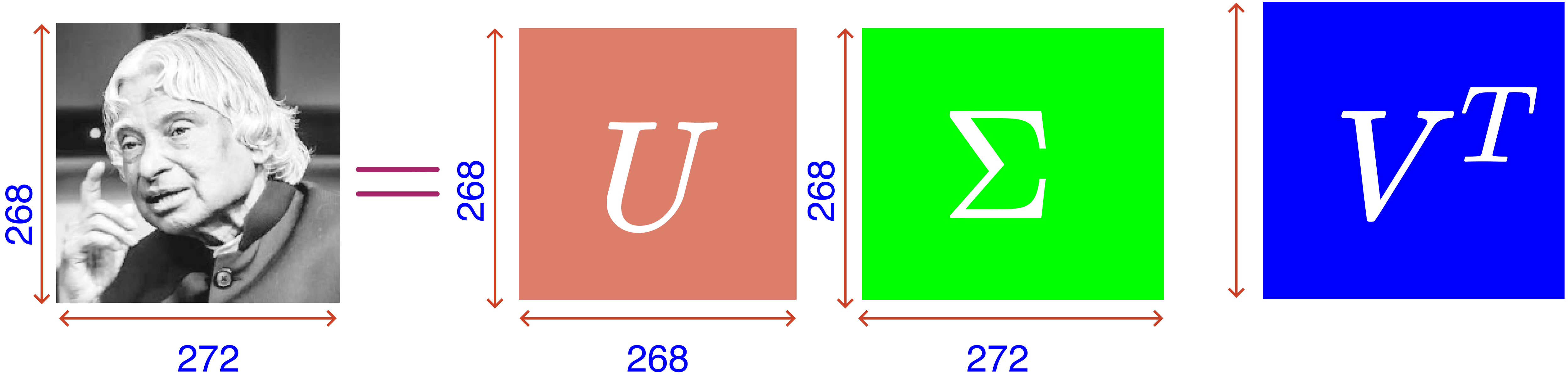


Image compression

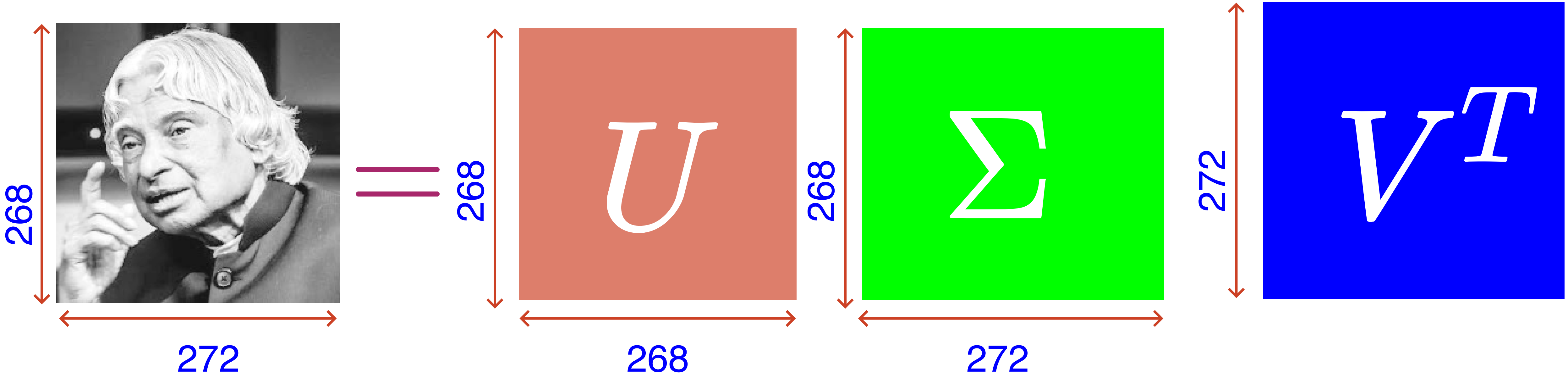


Image compression

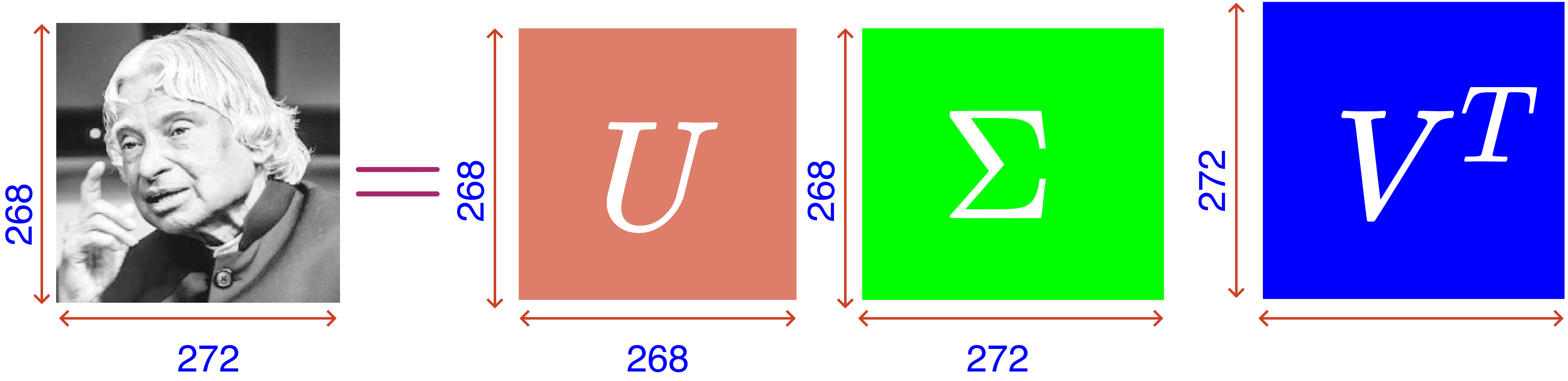


Image compression

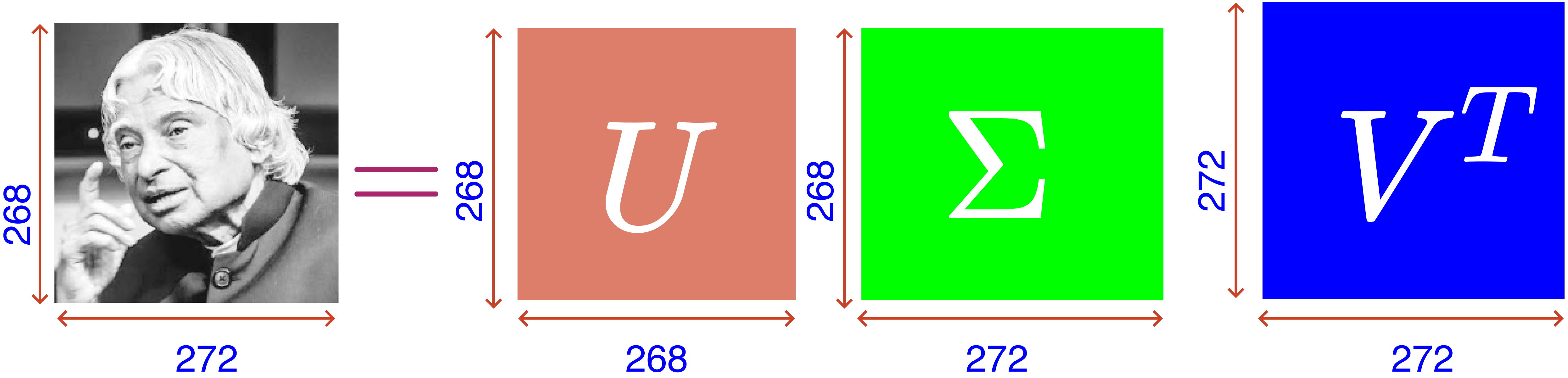


Image compression

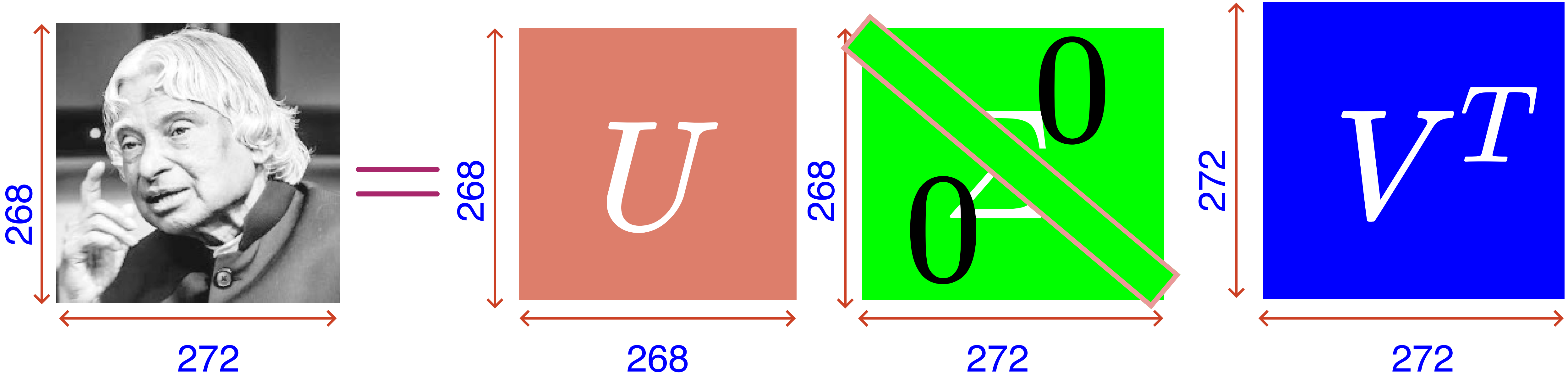


Image compression

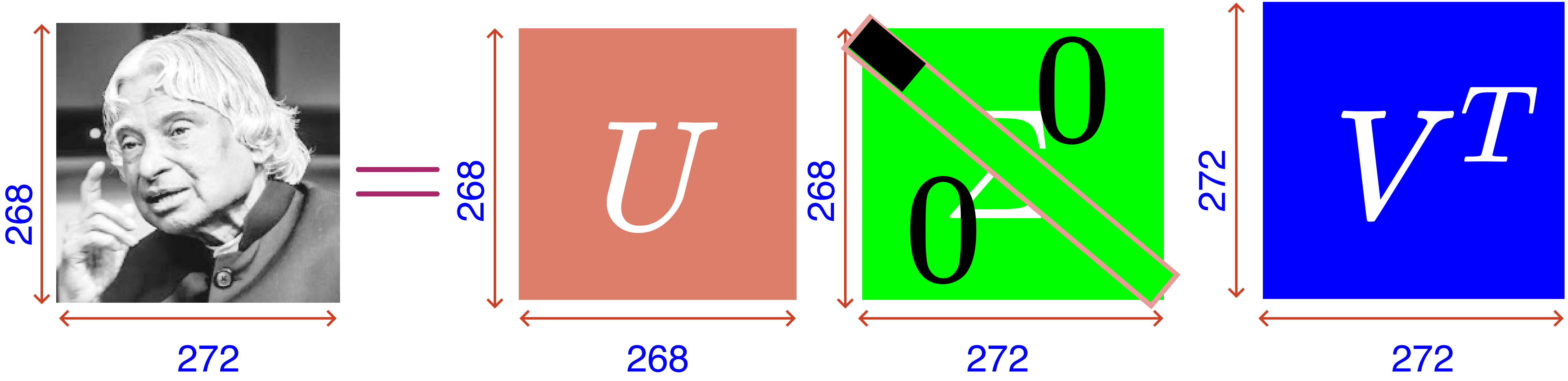
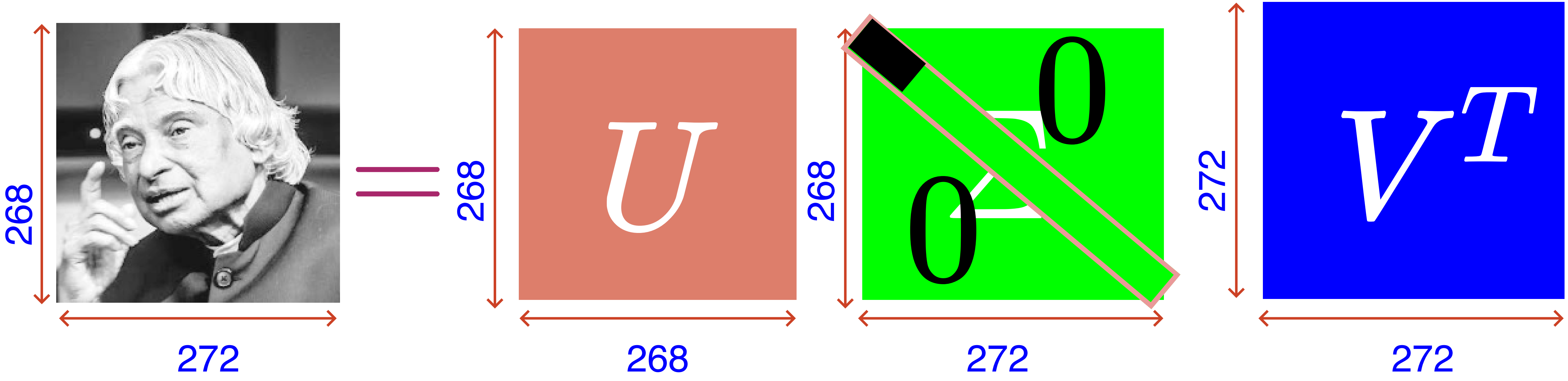
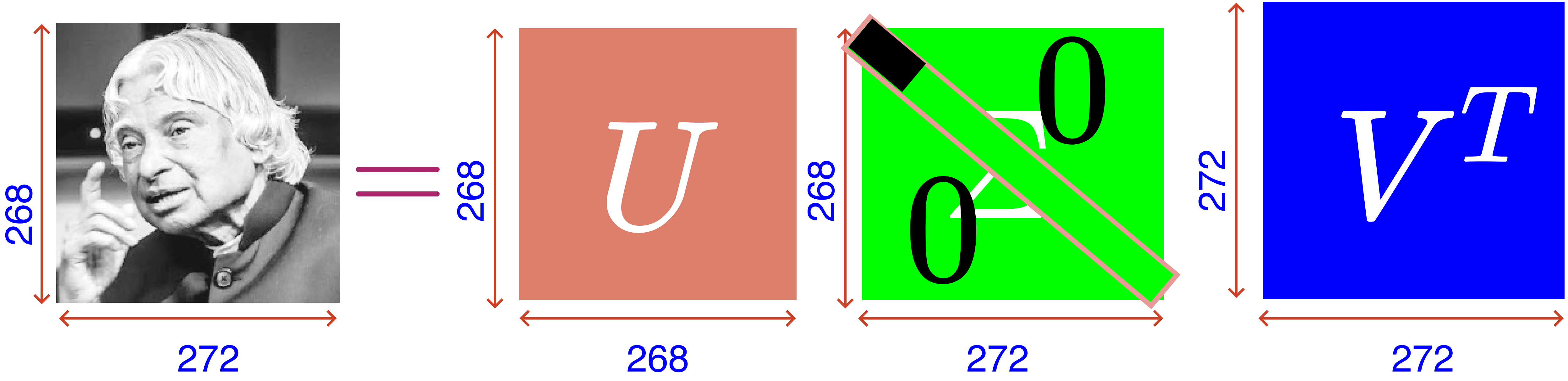


Image compression



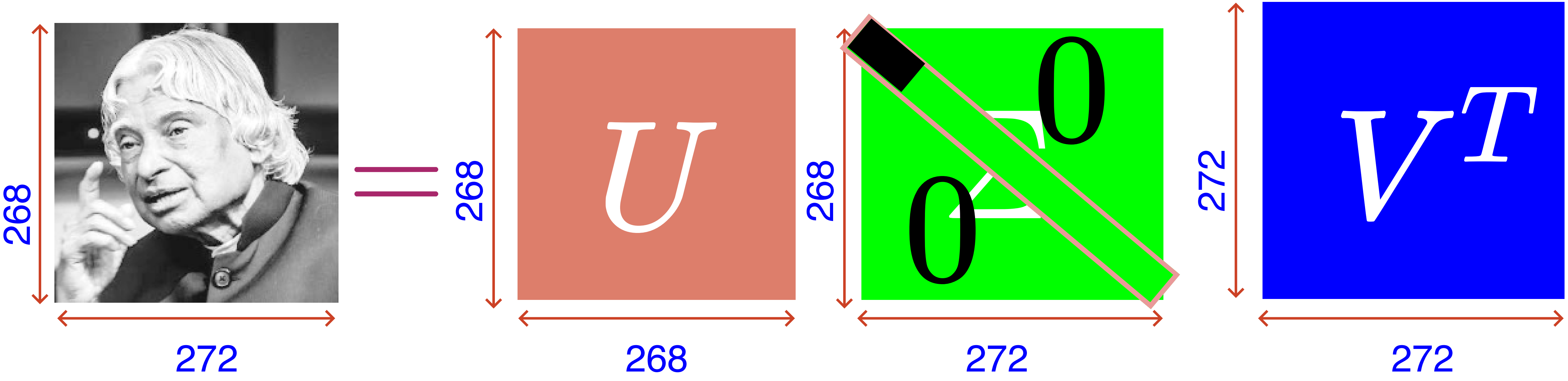
$$I = \sum_{i=1}^{k} \sigma_i u_i v_i^T$$

Image compression



$$I = \sum_{i=1}^{k=15} \sigma_i u_i v_i^T$$

Image compression

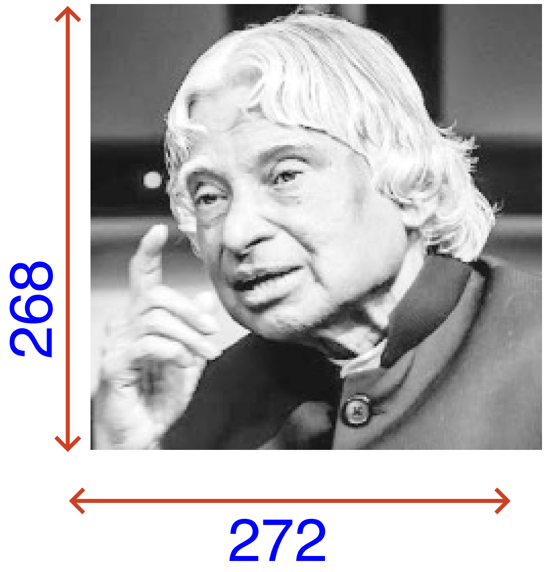


$$k = 15$$

$$I = \sum_{i=1}^{k} \sigma_i u_i v_i^T \qquad \sigma_i = 0, i > 15$$

Original Image : (Without Compression)

Original Image : (Without Compression)



Number of elements to be stored:

268*272 = 72,896

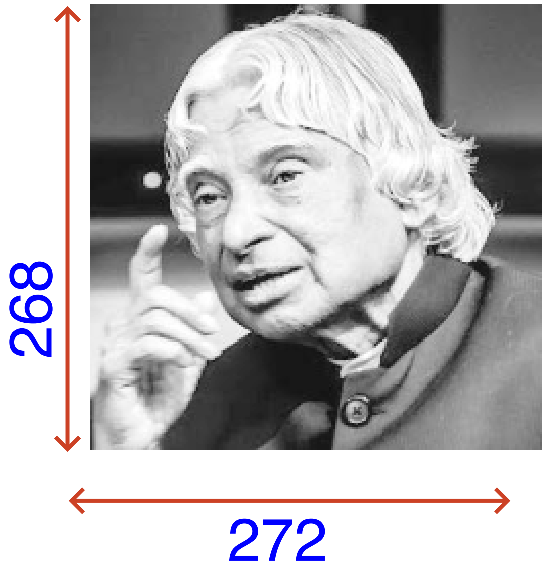
Original Image : (Without Compression)

Number of elements to be stored:

268*272 = 72,896

Approx. Image :(or Compressed)

Original Image : (Without Compression)



Number of elements to be stored:

268*272 = 72,896

Approx. Image :(or Compressed)

Original Image : (Without Compression)

Number of elements to be stored:

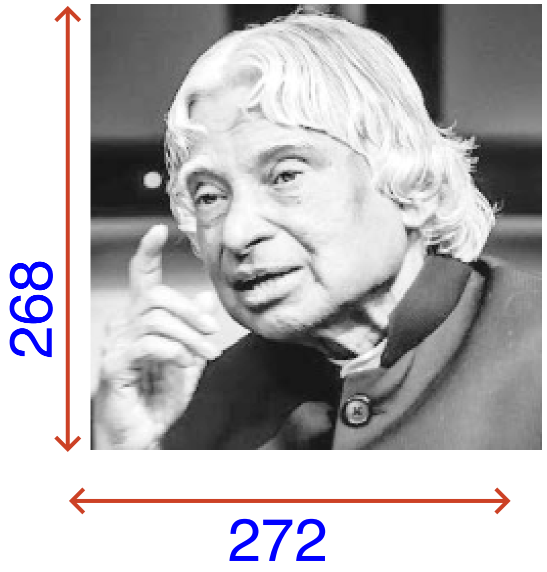
268*272 = 72,896

Approx. Image :(or Compressed)

rank: 15

- For U : 268*15 = 4020

Original Image : (Without Compression)



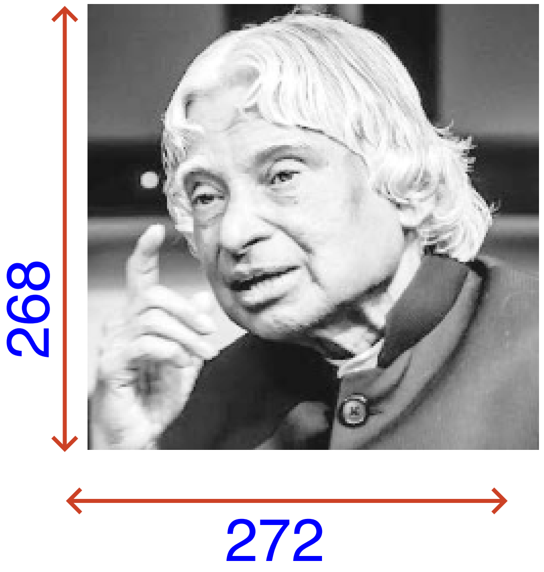
Number of elements to be stored:

268*272 = 72,896

Approx. Image :(or Compressed)

- For U : 268*15 = 4020
- For Σ: k =15

Original Image : (Without Compression)



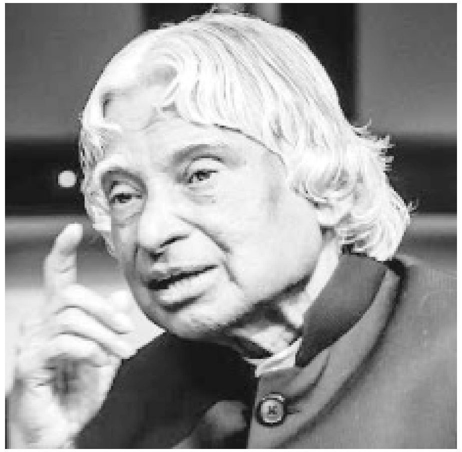
Number of elements to be stored:

268*272 = 72,896

Approx. Image :(or Compressed)

- For U : 268*15 = 4020
- For Σ: k =15
- For V^T : 15*272 = 4080

Original Image : (Without Compression)



268

272

Number of elements to be stored:

268*272 = 72,896

Approx. Image :(or Compressed)

rank: 15

- For U : 268*15 = 4020
- For Σ: k =15
- For V^T : 15*272 = 4080
- Total = 8115

Original Image : (Without Compression)

Number of elements to be stored:

268*272 = 72,896

Approx. Image :(or Compressed)

- For U : 268*15 = 4020
- For Σ: k =15
- For V^T : 15*272 = 4080
- Total = 8115
- ~= 9 times reduction in required memory for storage

Well, but, hmm..How does it work?

Well, but, hmm..How does it work?

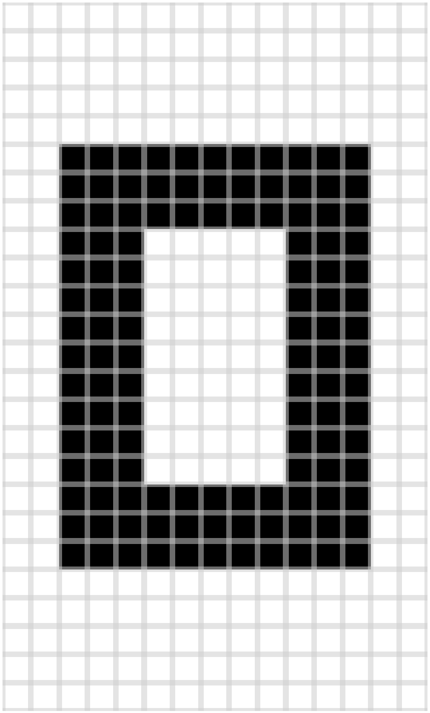


Image Source: http://www.ams.org/publicoutreach/feature-column/fcarc-svd

Well, but, hmm..How does it work?

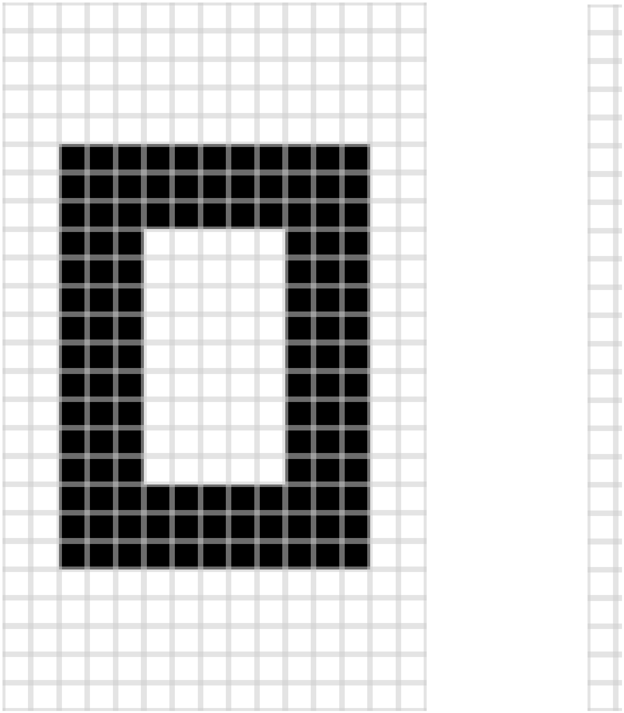


Image Source: http://www.ams.org/publicoutreach/feature-column/fcarc-svd

Well, but, hmm..How does it work?

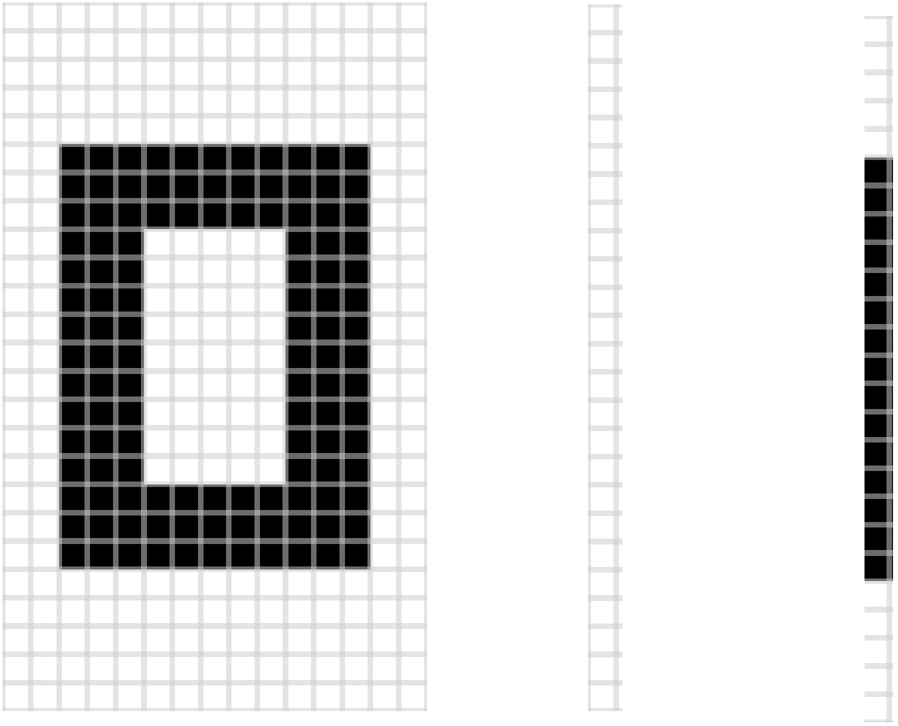


Image Source: http://www.ams.org/publicoutreach/feature-column/fcarc-svd

Well, but, hmm..How does it work?

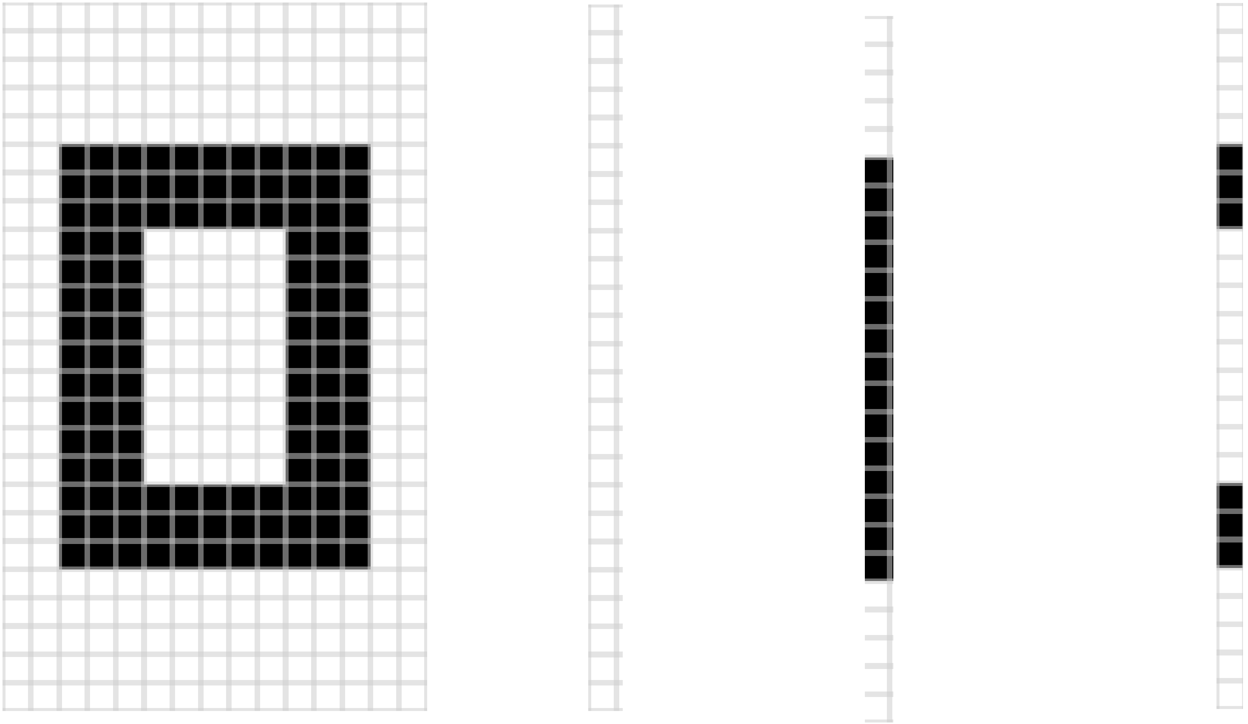


Image Source: http://www.ams.org/publicoutreach/feature-column/fcarc-svd

Well, but, hmm..How does it work?

The image is a linear combination of

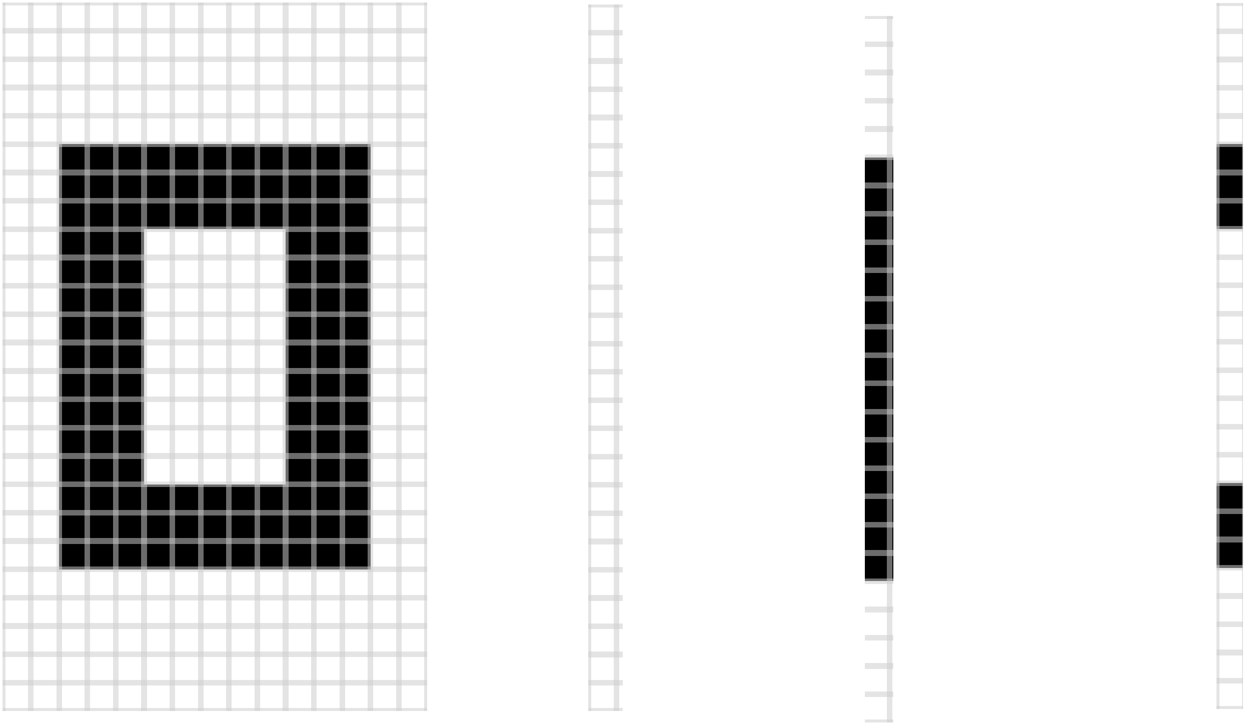
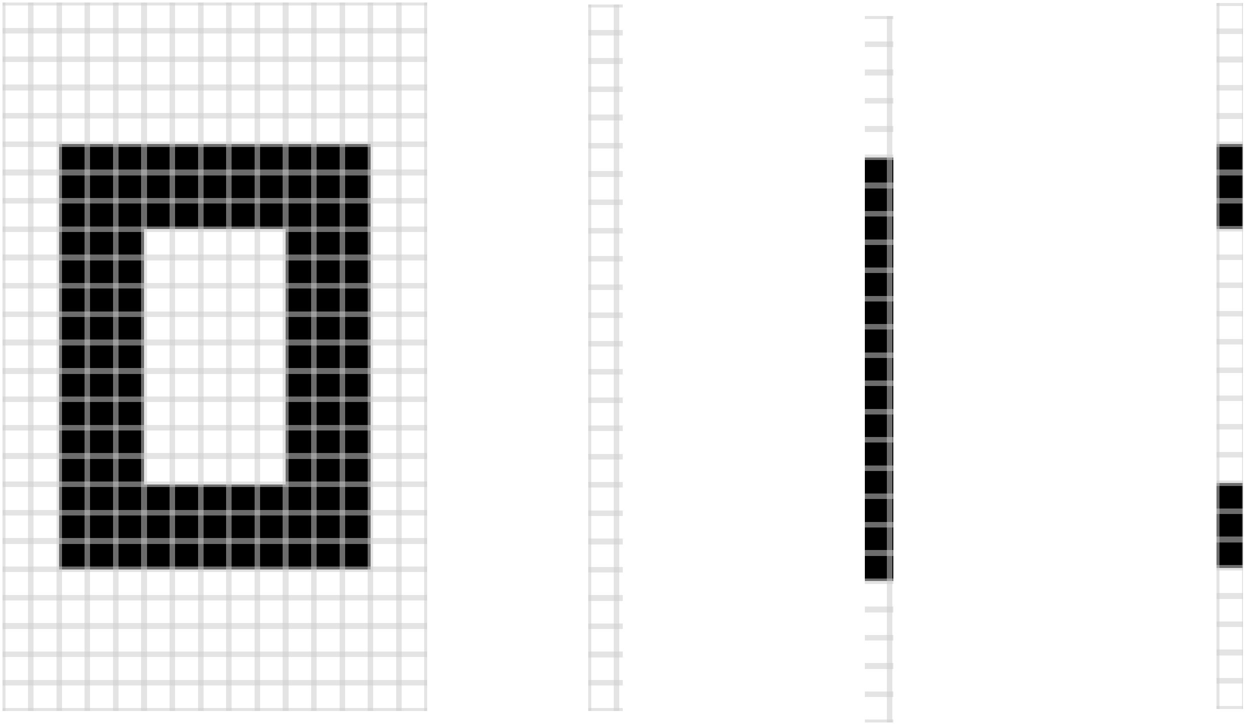


Image Source: http://www.ams.org/publicoutreach/feature-column/fcarc-svd

Well, but, hmm..How does it work?

The image is a linear combination of

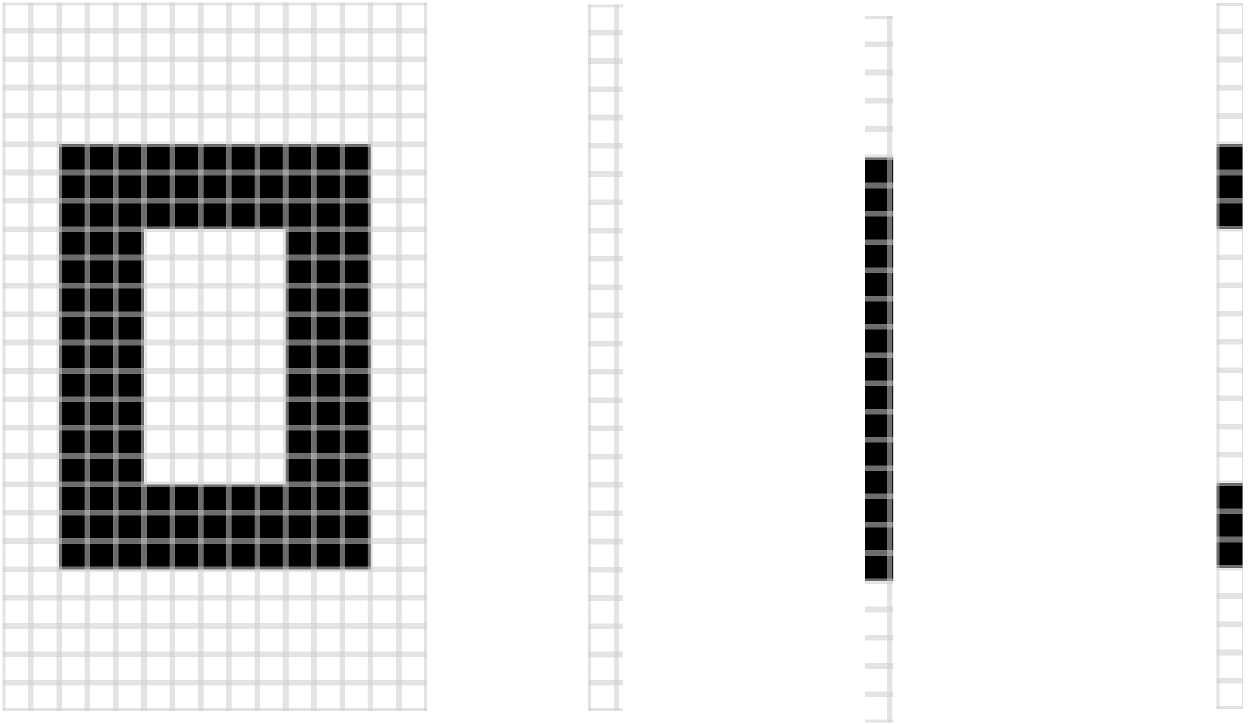


$$\sigma_1 = 14.7$$

Image Source: http://www.ams.org/publicoutreach/feature-column/fcarc-svd

Well, but, hmm..How does it work?

The image is a linear combination of



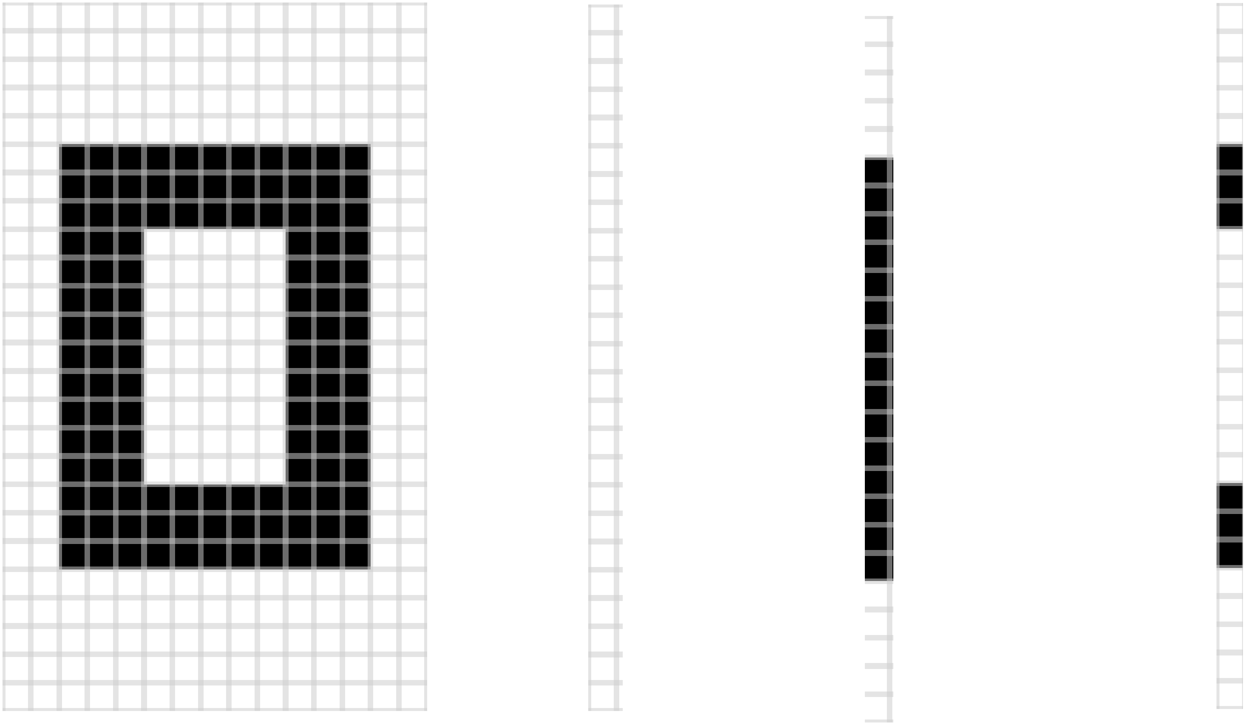
$$\sigma_1 = 14.7$$

$$\sigma_2 = 5.22$$

Image Source: http://www.ams.org/publicoutreach/feature-column/fcarc-svd

Well, but, hmm..How does it work?

The image is a linear combination of



$\sigma_1 = 14.7$

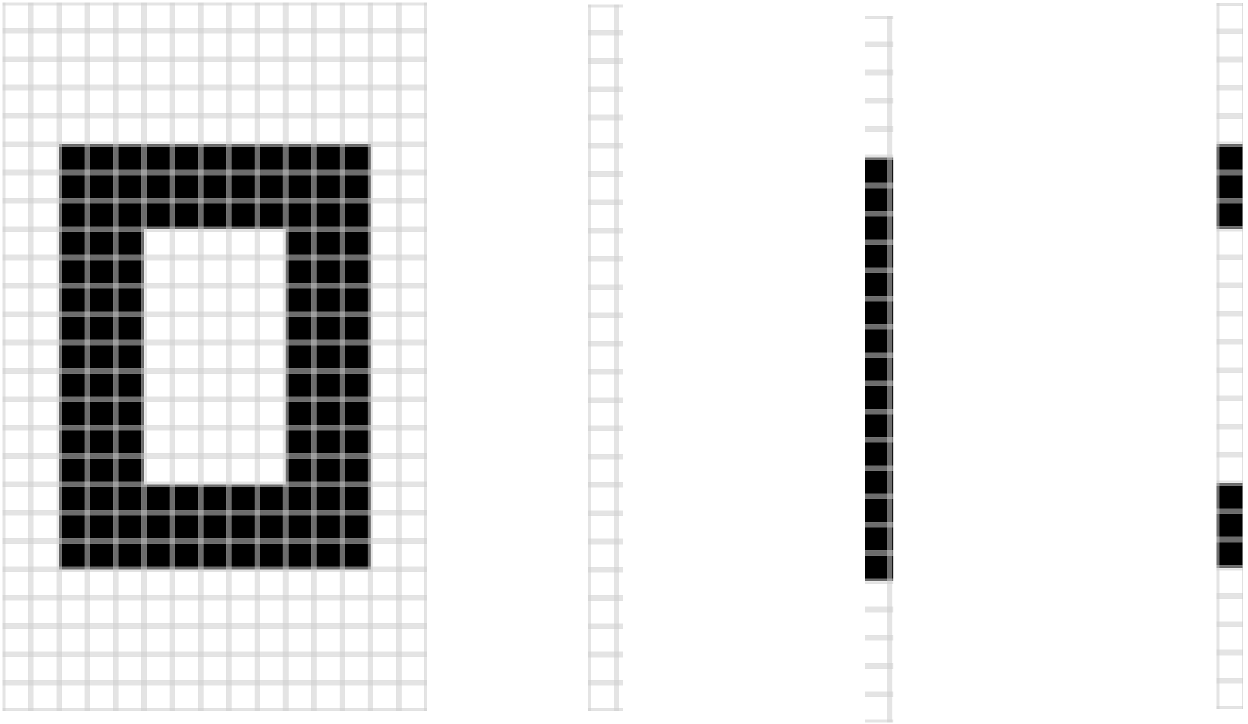
$\sigma_2 = 5.22$

$\sigma_3 = 3.31$

Image Source: http://www.ams.org/publicoutreach/feature-column/fcarc-svd

Well, but, hmm..How does it work?

The image is a linear combination of



$\sigma_1 = 14.7$

$\sigma_2 = 5.22$

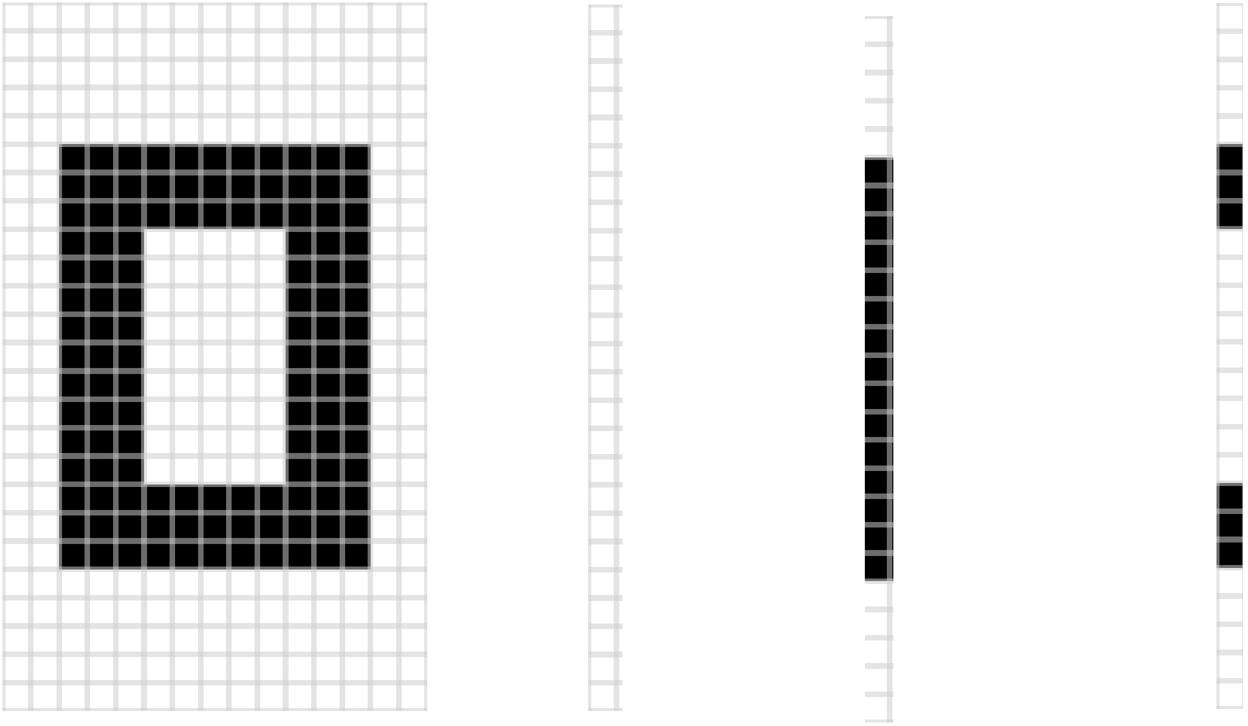
$\sigma_3 = 3.31$

$\sigma_i = 0, i > 3$

Image Source: http://www.ams.org/publicoutreach/feature-column/fcarc-svd

Well, but, hmm..How does it work?

The image is a linear combination of



$$\sigma_1 = 14.7$$

$$\sigma_2 = 5.22$$

$$\sigma_3 = 3.31$$

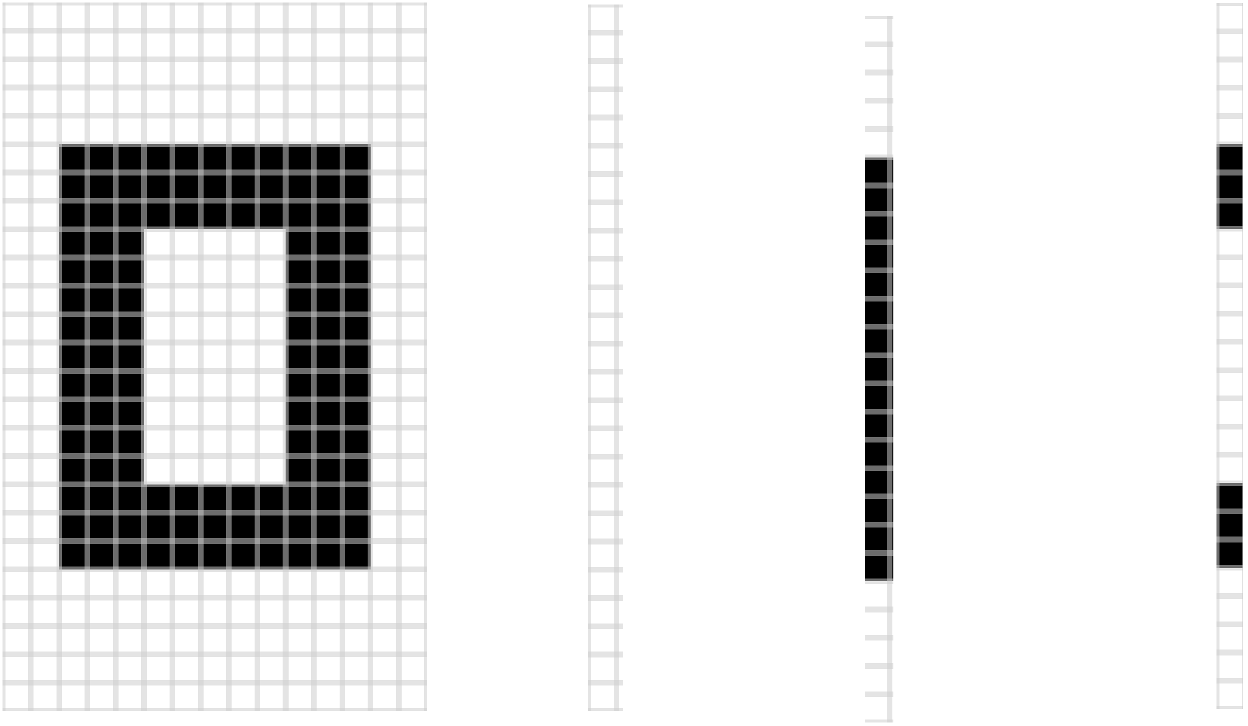
$$\sigma_i = 0, i > 3$$

It is zero by deafult!

Image Source: http://www.ams.org/publicoutreach/feature-column/fcarc-svd

Well, but, hmm..How does it work?

The image is a linear combination of



$$\sigma_1 = 14.7$$

$$\sigma_2 = 5.22$$

$$\sigma_3 = 3.31$$

$$\sigma_i = 0, i > 3$$

It is zero by deafult!

Removes redundancy!

Image Source: http://www.ams.org/publicoutreach/feature-column/fcarc-svd

Tutorial - Week5

Geometric Interpretation of SVD

Arun Prakash A

SVD

SVD

$$I = U\Sigma V^T$$

SVD

$$I = U\Sigma V^T$$

SVD

$$I = U\Sigma V^T$$

Diagonal Matrix

SVD

$$I = U\Sigma V^T$$

Diagonal Matrix

- What happens if diagonal matrices act on a set of vectors in the canonical (standard) basis?

SVD

$$I = U\Sigma V^T$$

Diagonal Matrix

- What happens if diagonal matrices act on a set of vectors in the canonical (standard) basis?
- Let us **see** it in $\mathbb{R}^2$ with help of Geogebra applet :-)

Diagonal Matrices

https://www.geogebra.org/material/iframe/id/nhksajgq/width/700/height/625/border/888888/sfsb/true/smb/false/stb/false/stbh/false/ai/false/asb/false/sri/false/rc/false/ld/false/sdz/true/ctl/false

Diagonal Matrices

https://www.geogebra.org/material/iframe/id/nhksajgq/width/700/height/625/border/888888/sfsb/true/smb/false/stb/false/stbh/false/ai/false/asb/false/sri/false/rc/false/ld/false/sdz/true/ctl/false

Diagonal matrices preserves the direction of orthogonal vectors!

Why?

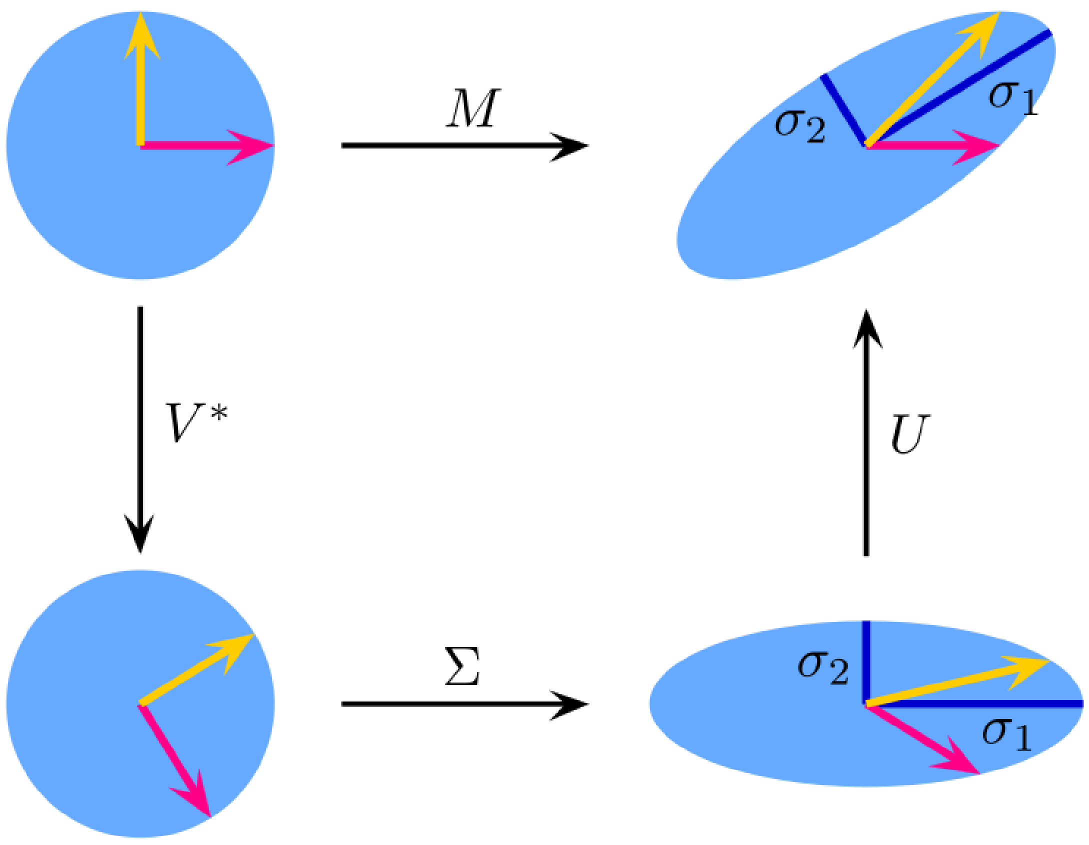
Similar Matrices

https://www.geogebra.org/material/iframe/id/dgwbf7db/width/1020/height/500/border/888888/sfsb/true/smb/false/stb/false/stbh/false/ai/false/asb/false/sri/false/rc/false/ld/false/sdz/true/ctl/false

Geometry of SVD

https://www.geogebra.org/material/iframe/id/dgwbf7db/width/1020/height/500/border/888888/sfsb/true/smb/false/stb/false/stbh/false/ai/false/asb/false/sri/false/rc/false/ld/false/sdz/true/ctl/false

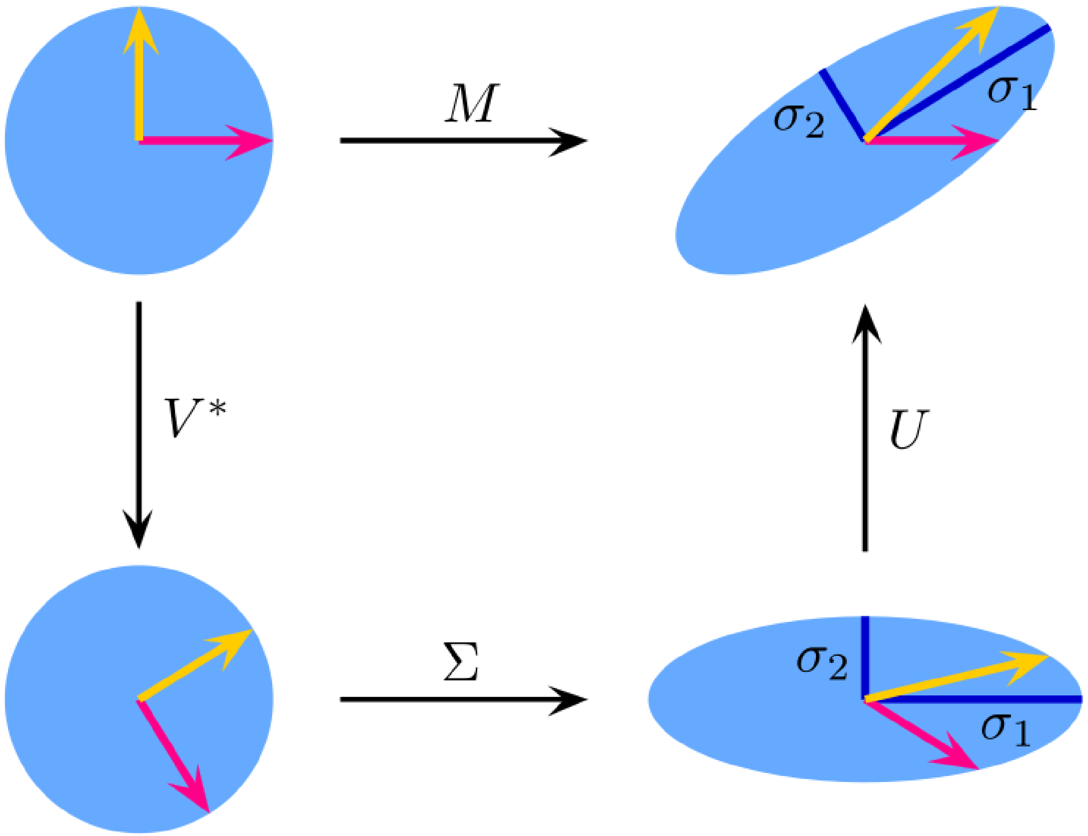
Geometry of SVD



$$M = U \cdot \Sigma \cdot V^*$$

Source: Wikipedia

Geometry of SVD

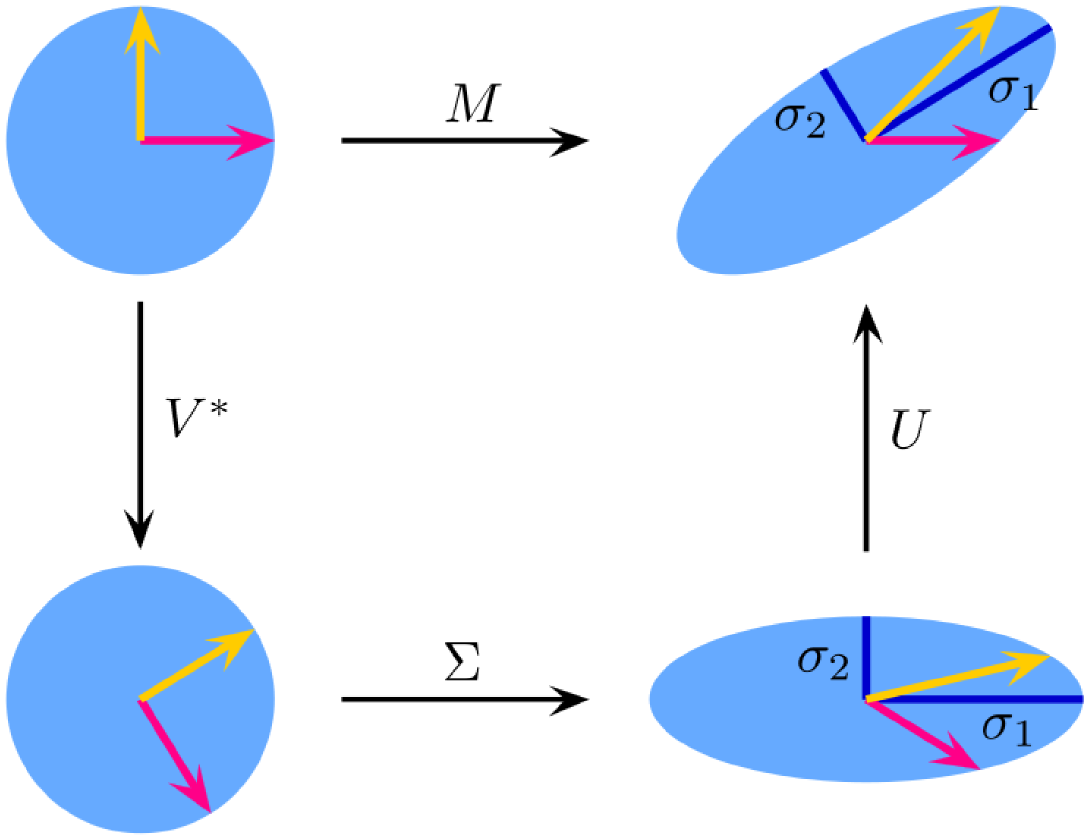


$$M = U \cdot \Sigma \cdot V^*$$

A quick summary:

Source: Wikipedia

Geometry of SVD



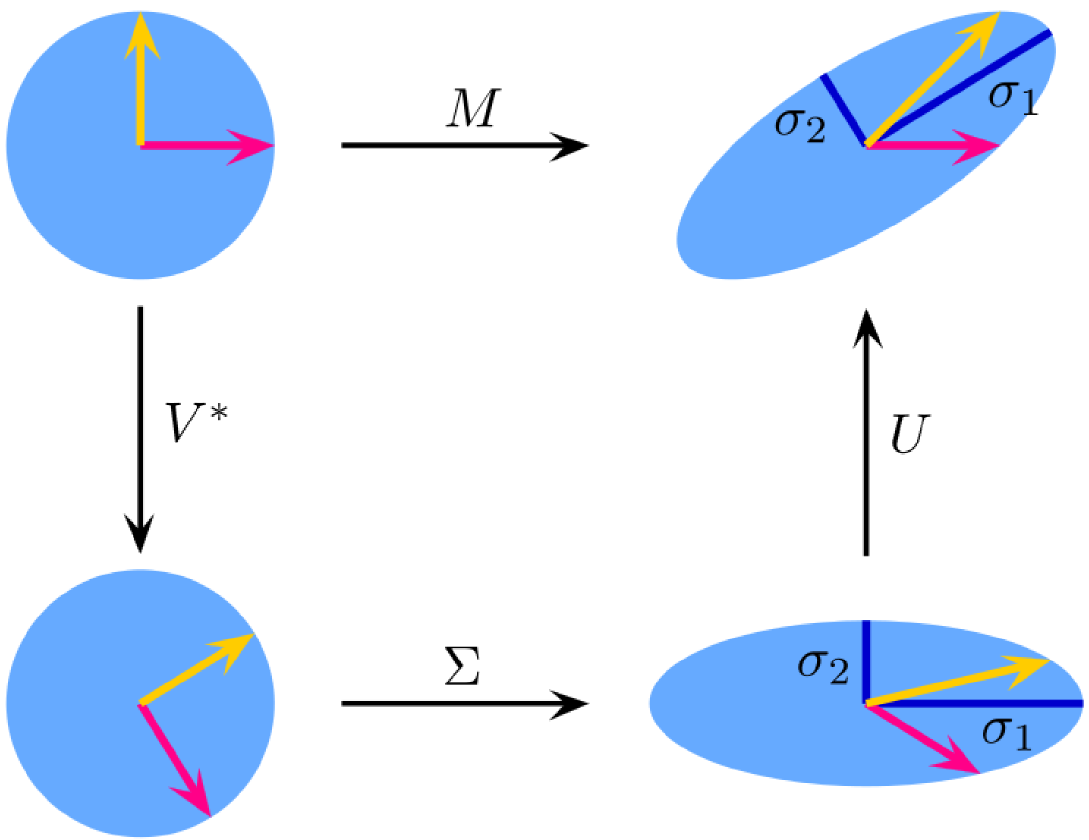
$$M = U \cdot \Sigma \cdot V^*$$

Source: Wikipedia

A quick summary:

- V^T Rotates disk D and basis e_1, e_2

Geometry of SVD



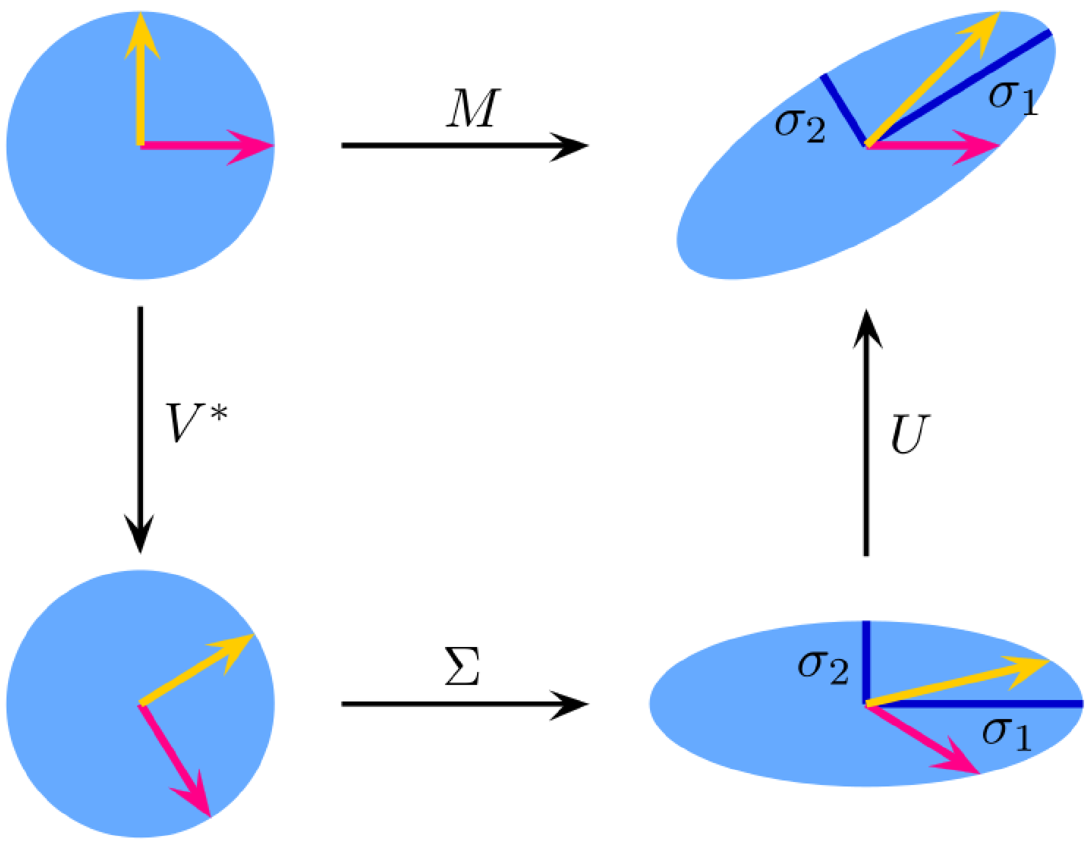
$$M = U \cdot \Sigma \cdot V^*$$

A quick summary:

- V^T Rotates disk D and basis e_1, e_2
- Σ scales the rotated disk D and σ_1, σ_2 are semi-major and semi-minor axis of an ellipse (hyper-ellipse)

Source: Wikipedia

Geometry of SVD



$$M = U \cdot \Sigma \cdot V^*$$

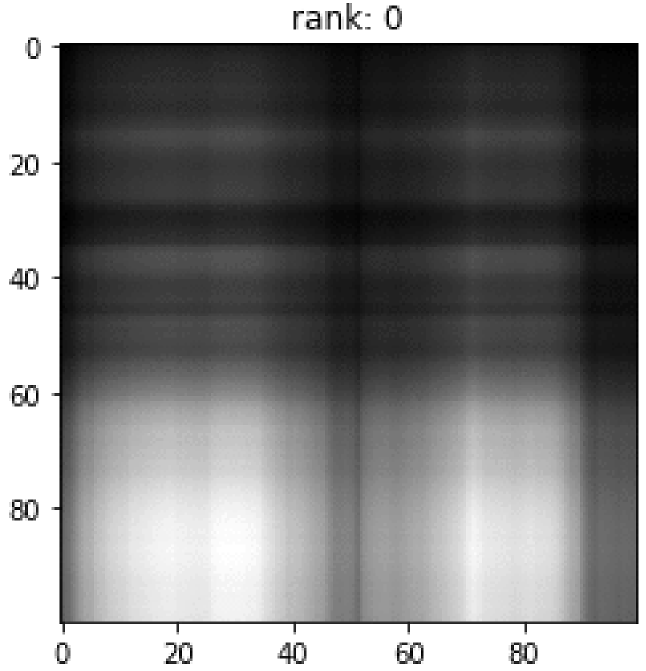
A quick summary:

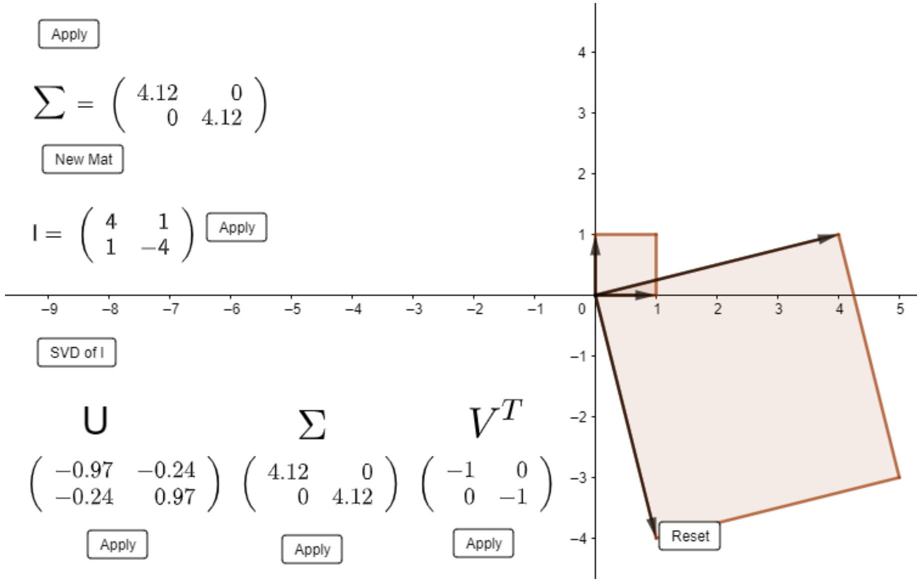
- V^T Rotates disk D and basis e_1, e_2
- Σ scales the rotated disk D and σ_1, σ_2 are semi-major and semi-minor axis of an ellipse (hyper-ellipse)
- U rotates the ellipse.

Source: Wikipedia

$$I = \sum_{i=1}^{k} \sigma_i u_i v_i^T$$

$$I = U \Sigma V^T$$





Tutorial - Week5

Some questions to think and solve

Arun Prakash A

High Dimensional Visualization

High Dimensional Visualization

"To deal with hyper-planes in a 14-dimensional space, visualize a 3-D space and say 'fourteen' to yourself very loudly. Everyone does it.

High Dimensional Visualization

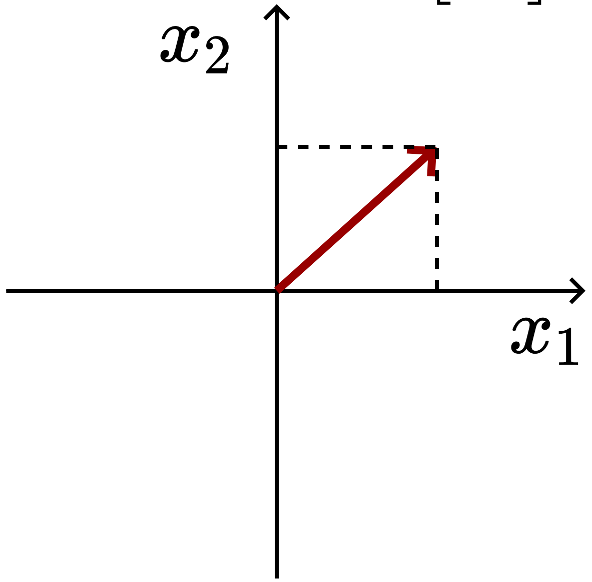
"To deal with hyper-planes in a 14-dimensional space, visualize a 3-D space and say 'fourteen' to yourself very loudly. Everyone does it.

Geoffrey Hinton

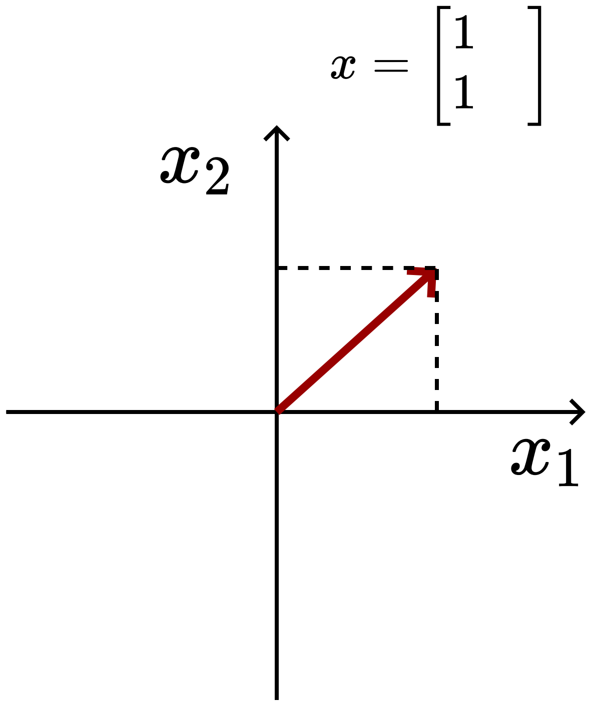
1. Is it possible to visualize complex vectors $x_i \in \mathbb{C}^2$ geometrically as we do for real vectors ?. Pause the video and think about it.

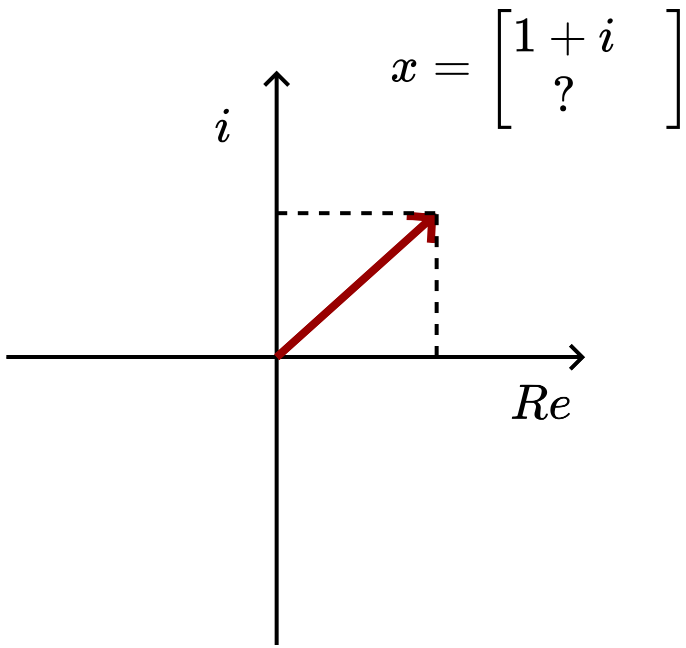
1. Is it possible to visualize complex vectors $x_i \in \mathbb{C}^2$ geometrically as we do for real vectors ?. Pause the video and think about it.

$$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



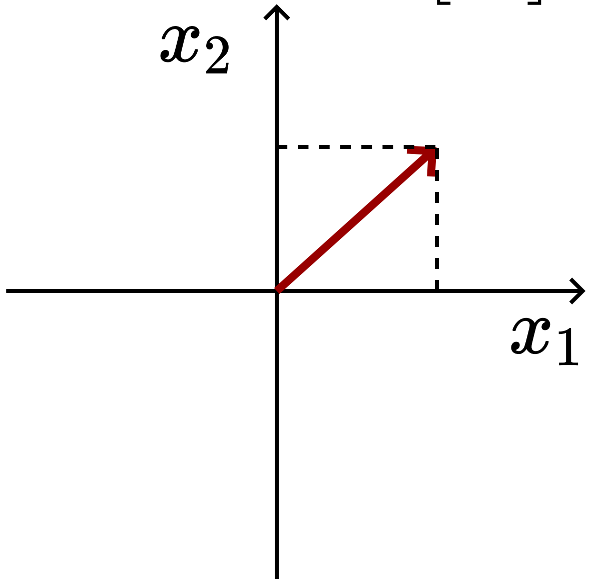
1. Is it possible to visualize complex vectors $x_i \in \mathbb{C}^2$ geometrically as we do for real vectors ?. Pause the video and think about it.



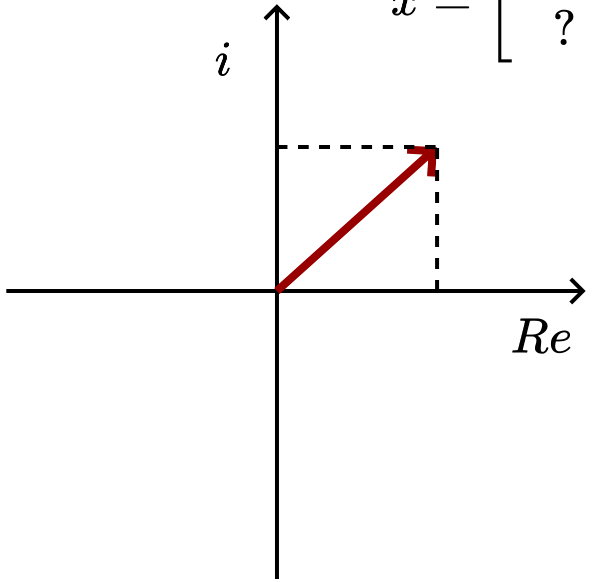


1. Is it possible to visualize complex vectors $x_i \in \mathbb{C}^2$ geometrically as we do for real vectors ?. Pause the video and think about it.

$$x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



$$x = \begin{bmatrix} 1+i \\ ? \end{bmatrix}$$



We need **4** dimensions to visualize a vector from $\mathbb{C}^2$

Do complex matrices find any real world applications?

Do complex matrices find any real world applications?

Is that just an abstract mathematical stuff?

Do complex matrices find any real world applications?

Is that just an abstract mathematical stuff?

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & i \end{bmatrix}$$

Do complex matrices find any real world applications?

Is that just an abstract mathematical stuff?

Discrete Fourier Transform (DFT)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & i \end{bmatrix}$$

Do complex matrices find any real world applications?

Is that just an abstract mathematical stuff?

Discrete Fourier Transform (DFT)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & i \end{bmatrix}$$

Do complex matrices find any real world applications?

Is that just an abstract mathematical stuff?

Discrete Fourier Transform
(DFT)

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & i \end{bmatrix}$$

Countless Applications in signal processing, Digital communication, Speech processing ...

2.Compute the inner product between two vectors $x = \begin{bmatrix} 3-2i \\ -2+i \\ -4-3i \end{bmatrix}$ and $y = \begin{bmatrix} -2+4i \\ 5-i \\ -2i \end{bmatrix}$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

2.Compute the inner product between two vectors $x = \begin{bmatrix} 3-2i \\ -2+i \\ -4-3i \end{bmatrix}$ and $y = \begin{bmatrix} -2+4i \\ 5-i \\ -2i \end{bmatrix}$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$x \cdot y = x^* y = \bar{x}^T y$$

2.Compute the inner product between two vectors $x = \begin{bmatrix} 3-2i \\ -2+i \\ -4-3i \end{bmatrix}$ and $y = \begin{bmatrix} -2+4i \\ 5-i \\ -2i \end{bmatrix}$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$x \cdot y = x^* y = \bar{x}^T y$$

$$(3+2i) \times (-2+4i)$$

2.Compute the inner product between two vectors $x = \begin{bmatrix} 3-2i \\ -2+i \\ -4-3i \end{bmatrix}$ and $y = \begin{bmatrix} -2+4i \\ 5-i \\ -2i \end{bmatrix}$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$x \cdot y = x^* y = \bar{x}^T y$$

$(3+2i) \times (-2+4i)$ = $-14+8i$

2.Compute the inner product between two vectors $x = \begin{bmatrix} 3-2i \\ -2+i \\ -4-3i \end{bmatrix}$ and $y = \begin{bmatrix} -2+4i \\ 5-i \\ -2i \end{bmatrix}$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$x \cdot y = x^* y = \bar{x}^T y$$

$(3+2i) \times (-2+4i)$ = $-14+8i$

$(-2-i) \times (5-i)$

2.Compute the inner product between two vectors $x = \begin{bmatrix} 3-2i \\ -2+i \\ -4-3i \end{bmatrix}$ and $y = \begin{bmatrix} -2+4i \\ 5-i \\ -2i \end{bmatrix}$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$x \cdot y = x^* y = \bar{x}^T y$$

$(3+2i) \times (-2+4i)$ = $-14+8i$

$(-2-i) \times (5-i)$ = $-11-3i$

2.Compute the inner product between two vectors $x = \begin{bmatrix} 3-2i \\ -2+i \\ -4-3i \end{bmatrix}$ and $y = \begin{bmatrix} -2+4i \\ 5-i \\ -2i \end{bmatrix}$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$x \cdot y = x^* y = \bar{x}^T y$$

$(3+2i) \times (-2+4i)$ = $-14+8i$

$(-2-i) \times (5-i)$ = $-11-3i$

$(-4+3i) \times (-2i)$

2.Compute the inner product between two vectors $x = \begin{bmatrix} 3-2i \\ -2+i \\ -4-3i \end{bmatrix}$ and $y = \begin{bmatrix} -2+4i \\ 5-i \\ -2i \end{bmatrix}$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$x \cdot y = x^* y = \bar{x}^T y$$

$(3+2i) \times (-2+4i)$ $= -14 + 8i$

$(-2-i) \times (5-i)$ $= -11 - 3i$

$(-4+3i) \times (-2i)$ $= 6 + 8i$

2.Compute the inner product between two vectors $x = \begin{bmatrix} 3-2i \\ -2+i \\ -4-3i \end{bmatrix}$ and $y = \begin{bmatrix} -2+4i \\ 5-i \\ -2i \end{bmatrix}$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$x \cdot y = x^* y = \bar{x}^T y$$

$(3+2i) \times (-2+4i)$ = $-14+8i$

$(-2-i) \times (5-i)$ = $-11-3i$

$(-4+3i) \times (-2i)$ = $6+8i$

2.Compute the inner product between two vectors $x = \begin{bmatrix} 3-2i \\ -2+i \\ -4-3i \end{bmatrix}$ and $y = \begin{bmatrix} -2+4i \\ 5-i \\ -2i \end{bmatrix}$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$x \cdot y = x^* y = \bar{x}^T y$$

$(3+2i) \times (-2+4i)$ = $-14+8i$

$(-2-i) \times (5-i)$ = $-11-3i$

$(-4+3i) \times (-2i)$ = $6+8i$

2.Compute the inner product between two vectors $x = \begin{bmatrix} 3-2i \\ -2+i \\ -4-3i \end{bmatrix}$ and $y = \begin{bmatrix} -2+4i \\ 5-i \\ -2i \end{bmatrix}$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$x \cdot y = x^* y = \bar{x}^T y$$

$(3+2i) \times (-2+4i)$ = $-14+8i$

$(-2-i) \times (5-i)$ = $-11-3i$

$(-4+3i) \times (-2i)$ = $6+8i$

2.Compute the inner product between two vectors $x = \begin{bmatrix} 3-2i \\ -2+i \\ -4-3i \end{bmatrix}$ and $y = \begin{bmatrix} -2+4i \\ 5-i \\ -2i \end{bmatrix}$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$x \cdot y = x^* y = \bar{x}^T y$$

$(3+2i) \times (-2+4i)$ = $-14+8i$

$(-2-i) \times (5-i)$ = $-11-3i$

$(-4+3i) \times (-2i)$ = $6+8i$

Σ

2.Compute the inner product between two vectors $x = \begin{bmatrix} 3-2i \\ -2+i \\ -4-3i \end{bmatrix}$ and $y = \begin{bmatrix} -2+4i \\ 5-i \\ -2i \end{bmatrix}$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$x \cdot y = x^* y = \bar{x}^T y$$

$(3+2i) \times (-2+4i)$ = $-14+8i$

$(-2-i) \times (5-i)$ = $-11-3i$

$(-4+3i) \times (-2i)$ = $6+8i$

Σ

2.Compute the inner product between two vectors $x = \begin{bmatrix} 3-2i \\ -2+i \\ -4-3i \end{bmatrix}$ and $y = \begin{bmatrix} -2+4i \\ 5-i \\ -2i \end{bmatrix}$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$x \cdot y = x^* y = \bar{x}^T y$$

$(3+2i) \times (-2+4i)$ = $-14+8i$

$(-2-i) \times (5-i)$ = $-11-3i$

$(-4+3i) \times (-2i)$ = $6+8i$

Σ $\rightarrow$ $-19+13i$

2.Compute the inner product between two vectors $x = \begin{bmatrix} 3-2i \\ -2+i \\ -4-3i \end{bmatrix}$ and $y = \begin{bmatrix} -2+4i \\ 5-i \\ -2i \end{bmatrix}$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$x \cdot y = x^* y = \bar{x}^T y$$

$(3+2i) \times (-2+4i)$ = $-14+8i$

$(-2-i) \times (5-i)$ = $-11-3i$ $\quad \Sigma \quad -19+13i$

$(-4+3i) \times (-2i)$ = $6+8i$

$$y \cdot x = y^* x = \bar{y}^T x$$

2.Compute the inner product between two vectors $x = [3 - 2i, -2 + i, -4 - \quad 3i]^T$ and $y = [-2 + 4i, 5 - i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

2.Compute the inner product between two vectors $x = [3 - 2i, -2 + i, -4 - \quad 3i]^T$ and $y = [-2 + 4i, 5 - i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

2.Compute the inner product between two vectors $x = [3-2i, -2+i, -4- \quad 3i]^T$ and $y = [-2+4i, 5-i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$$(3-2i) \times (-2-4i)$$

2.Compute the inner product between two vectors $x = [3-2i, -2+i, -4-\quad 3i]^T$ and $y = [-2+4i, 5-i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$(3-2i) \times (-2-4i)$ = $-14-8i$

2.Compute the inner product between two vectors $x = [3 - 2i, -2 + i, -4 - \quad 3i]^T$ and $y = [-2 + 4i, 5 - i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$(3 - 2i) \times (-2 - 4i)$ = $-14 - 8i$

$(-2 + i) \times (5 + i)$

2.Compute the inner product between two vectors $x = [3-2i, -2+i, -4-\quad 3i]^T$ and $y = [-2+4i, 5-i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$(3-2i) \times (-2-4i)$ = $-14-8i$

$(-2+i) \times (5+i)$ = $-11+3i$

2.Compute the inner product between two vectors $x = [3 - 2i, -2 + i, -4 - \quad 3i]^T$ and $y = [-2 + 4i, 5 - i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$(3 - 2i) \times (-2 - 4i)$ = $-14 - 8i$

$(-2 + i) \times (5 + i)$ = $-11 + 3i$

$(-4 - 3i) \times (2i)$

2.Compute the inner product between two vectors $x = [3 - 2i, -2 + i, -4 - \quad 3i]^T$ and $y = [-2 + 4i, 5 - i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$(3 - 2i) \times (-2 - 4i)$ = $-14 - 8i$

$(-2 + i) \times (5 + i)$ = $-11 + 3i$

$(-4 - 3i) \times (2i)$ = $6 - 8i$

2.Compute the inner product between two vectors $x = [3-2i, -2+i, -4- \quad 3i]^T$ and $y = [-2+4i, 5-i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$(3-2i) \times (-2-4i)$ = $-14-8i$

$(-2+i) \times (5+i)$ = $-11+3i$

$(-4-3i) \times (2i)$ = $6-8i$

2.Compute the inner product between two vectors $x = [3-2i, -2+i, -4-\quad 3i]^T$ and $y = [-2+4i, 5-i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$(3-2i) \times (-2-4i)$ = $-14-8i$

$(-2+i) \times (5+i)$ = $-11+3i$

$(-4-3i) \times (2i)$ = $6-8i$

2.Compute the inner product between two vectors $x = [3 - 2i, -2 + i, -4 - \quad 3i]^T$ and $y = [-2 + 4i, 5 - i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$(3 - 2i) \times (-2 - 4i)$ = $-14 - 8i$

$(-2 + i) \times (5 + i)$ = $-11 + 3i$

$(-4 - 3i) \times (2i)$ = $6 - 8i$

2.Compute the inner product between two vectors $x = [3 - 2i, -2 + i, -4 - \quad 3i]^T$ and $y = [-2 + 4i, 5 - i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$(3 - 2i) \times (-2 - 4i)$ $= -14 - 8i$

$(-2 + i) \times (5 + i)$ $= -11 + 3i$

$(-4 - 3i) \times (2i)$ $= 6 - 8i$

Σ

2.Compute the inner product between two vectors $x = [3 - 2i, -2 + i, -4 - 3i]^T$ and $y = [-2 + 4i, 5 - i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$(3 - 2i) \times (-2 - 4i)$ = $-14 - 8i$

$(-2 + i) \times (5 + i)$ = $-11 + 3i$

$(-4 - 3i) \times (2i)$ = $6 - 8i$

Σ

2.Compute the inner product between two vectors $x = [3 - 2i, -2 + i, -4 - 3i]^T$ and $y = [-2 + 4i, 5 - i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$$(3 - 2i) \times (-2 - 4i) = -14 - 8i$$

$$(-2 + i) \times (5 + i) = -11 + 3i$$

$$(-4 - 3i) \times (2i) = 6 - 8i$$

$$\Sigma \rightarrow -19 - 13i$$

2.Compute the inner product between two vectors $x = [3 - 2i, -2 + i, -4 - \quad 3i]^T$ and $y = [-2 + 4i, 5 - i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$$(3 - 2i) \times (-2 - 4i) = -14 - 8i$$

$$(-2 + i) \times (5 + i) = -11 + 3i$$

$$(-4 - 3i) \times (2i) = 6 - 8i$$

$$\Sigma \rightarrow -19 - 13i$$

$$y \cdot x = y^* x = \bar{y}^T x$$

2.Compute the inner product between two vectors $x = [3 - 2i, -2 + i, -4 - \quad 3i]^T$ and $y = [-2 + 4i, 5 - i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$(3 - 2i) \times (-2 - 4i)$ = $-14 - 8i$

$(-2 + i) \times (5 + i)$ = $-11 + 3i$ $\rightarrow \Sigma \rightarrow$ $-19 - 13i$

$(-4 - 3i) \times (2i)$ = $6 - 8i$

$$y \cdot x = y^* x = \bar{y}^T x$$

$$\overline{x \cdot y}$$

2.Compute the inner product between two vectors $x = [3 - 2i, -2 + i, -4 - \quad 3i]^T$ and $y = [-2 + 4i, 5 - i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$(3 - 2i) \times (-2 - 4i)$ = $-14 - 8i$

$(-2 + i) \times (5 + i)$ = $-11 + 3i$

$(-4 - 3i) \times (2i)$ = $6 - 8i$

$$\Sigma \rightarrow -19 - 13i$$

$$y \cdot x = y^* x = \bar{y}^T x$$

$$\overline{x \cdot y}$$

$$x \cdot y \neq y \cdot x$$

2.Compute the inner product between two vectors $x = [3-2i, -2+i, -4- \quad 3i]^T$ and $y = [-2+4i, 5-i, -2i]^T$ and verify whether they are commutative (i.e. $x \cdot y = y \cdot x$)

$$y \cdot x = y^* x = \bar{y}^T x$$

$(3-2i) \times (-2-4i)$ = $-14-8i$

$(-2+i) \times (5+i)$ = $-11+3i$

$(-4-3i) \times (2i)$ = $6-8i$

Σ

$-19-13i$

$$y \cdot x = y^* x = \bar{y}^T x$$

$$\overline{x \cdot y}$$

Is it always true?

$$x \cdot y \neq y \cdot x$$

3. Prove that $x \cdot y = \overline{y \cdot x}$ where $x \in \mathbb{C}^n$ and $y \in \mathbb{C}^n$

3. Prove that $x \cdot y = \overline{y \cdot x}$ where $x \in \mathbb{C}^n$ and $y \in \mathbb{C}^n$

$$\overline{y \cdot x} = \overline{\bar{y}x}$$

3. Prove that $x \cdot y = \overline{y \cdot x}$ where $x \in \mathbb{C}^n$ and $y \in \mathbb{C}^n$

$$\overline{y \cdot x} = \overline{\bar{y}x}$$

$$= \overline{\bar{y_1}x_1} + \overline{\bar{y_2}x_2} + \cdots + \overline{\bar{y_n}x_n}$$

3. Prove that $x \cdot y = \overline{y \cdot x}$ where $x \in \mathbb{C}^n$ and $y \in \mathbb{C}^n$

$$\overline{y \cdot x} = \overline{\bar{y}x}$$

$$= \overline{\bar{y_1}x_1} + \overline{\bar{y_2}x_2} + \cdots + \overline{\bar{y_n}x_n}$$

$$= y_1\bar{x_1} + y_2\bar{x_2} + \cdots + y_n\bar{x_n}$$

3. Prove that $x \cdot y = \overline{y \cdot x}$ where $x \in \mathbb{C}^n$ and $y \in \mathbb{C}^n$

$$\overline{y \cdot x} = \overline{\bar{y}x}$$

$$= \overline{\bar{y_1}x_1} + \overline{\bar{y_2}x_2} + \cdots + \overline{\bar{y_n}x_n}$$

$$= y_1\bar{x_1} + y_2\bar{x_2} + \cdots + y_n\bar{x_n}$$

$$= \bar{x}_1 y_1 + \bar{x}_2 y_2 + \cdots + \bar{x}_n y_n$$

3. Prove that $x \cdot y = \overline{y \cdot x}$ where $x \in \mathbb{C}^n$ and $y \in \mathbb{C}^n$

$$\overline{y \cdot x} = \overline{\bar{y}x}$$

$$= \overline{\bar{y_1}x_1} + \overline{\bar{y_2}x_2} + \cdots + \overline{\bar{y_n}x_n}$$

$$= y_1\bar{x_1} + y_2\bar{x_2} + \cdots + y_n\bar{x_n}$$

$$= \bar{x_1}y_1 + \bar{x_2}y_2 + \cdots + \bar{x_n}y_n$$

$$= \ x \cdot y$$

4. Can we use inner product to compute (cosine) angle between two complex vectors, like we do for real vectors?

More details on it : Angles in complex vector space

4. Can we use inner product to compute (cosine) angle between two complex vectors, like we do for real vectors?

No!. Not Always

More details on it : Angles in complex vector space

4. Can we use inner product to compute (cosine) angle between two complex vectors, like we do for real vectors?

No!. Not Always

Let us reason why?

More details on it : Angles in complex vector space

4. Can we use inner product to compute (cosine) angle between two complex vectors, like we do for real vectors?

No!. Not Always

Let us reason why?

$$\frac{x \cdot y}{||x||\ ||y||}$$

More details on it : Angles in complex vector space

4. Can we use inner product to compute (cosine) angle between two complex vectors, like we do for real vectors?

No!. Not Always

Let us reason why?

But some authors prefers

$$\frac{x \cdot y}{||x|| \ ||y||}$$

More details on it : Angles in complex vector space

4. Can we use inner product to compute (cosine) angle between two complex vectors, like we do for real vectors?

No!. Not Always

Let us reason why?

$$\frac{x \cdot y}{||x|| \; ||y||}$$

But some authors prefers

$$\frac{Re(x \cdot y)}{||x|| \; ||y||}$$

More details on it : Angles in complex vector space

5. Consider the matrix $A = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix}$. Find the complex eigenvector for the eigenvalue $\lambda = 8$

5. Consider the matrix $A = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix}$. Find the complex eigenvector for the eigenvalue $\lambda = 8$

$$N[A - \lambda I] = \begin{bmatrix} -6 & 3-3i \\ 3+3i & -3 \end{bmatrix}$$

5. Consider the matrix $A = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix}$. Find the complex eigenvector for the eigenvalue $\lambda = 8$

$$N[A-\lambda I] = \begin{bmatrix} -6 & 3-3i \\ 3+3i & -3 \end{bmatrix}$$

$$R_2 = R_2 + \tfrac{1}{2}(1+i)R_1$$

5. Consider the matrix $A = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix}$. Find the complex eigenvector for the eigenvalue $\lambda = 8$

$$N[A - \lambda I] = \begin{bmatrix} -6 & 3-3i \\ 3+3i & -3 \end{bmatrix}$$

$$R_2 = R_2 + \tfrac{1}{2}(1+i)R_1$$

$$= \begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix}$$

5. Consider the matrix $A = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix}$. Find the complex eigenvector for the eigenvalue $\lambda = 8$

$$N[A - \lambda I] = \begin{bmatrix} -6 & 3-3i \\ 3+3i & -3 \end{bmatrix}$$

$$R_2 = R_2 + \tfrac{1}{2}(1+i)R_1$$

$$= \begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

5. Consider the matrix $A = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix}$. Find the complex eigenvector for the eigenvalue $\lambda = 8$

$$N[A - \lambda I] = \begin{bmatrix} -6 & 3-3i \\ 3+3i & -3 \end{bmatrix}$$

$$R_2 = R_2 + \tfrac{1}{2}(1+i)R_1$$

$$= \begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-6x_1 + (3-3i)x_2 = 0$$

5. Consider the matrix $A = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix}$. Find the complex eigenvector for the eigenvalue $\lambda = 8$

$$N[A-\lambda I] = \begin{bmatrix} -6 & 3-3i \\ 3+3i & -3 \end{bmatrix}$$

$$R_2 = R_2 + \tfrac{1}{2}(1+i)R_1$$

$$= \begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-6x_1 + (3-3i)x_2 = 0$$

$$-2x_1 + (1-1i)x_2 = 0$$

5. Consider the matrix $A = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix}$. Find the complex eigenvector for the eigenvalue $\lambda = 8$

$$N[A - \lambda I] = \begin{bmatrix} -6 & 3-3i \\ 3+3i & -3 \end{bmatrix}$$

$$R_2 = R_2 + \tfrac{1}{2}(1+i)R_1$$

$$= \begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-6x_1 + (3-3i)x_2 = 0$$

$$-2x_1 + (1-1i)x_2 = 0$$

$$2x_1 = (1-1i)x_2$$

5. Consider the matrix $A = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix}$. Find the complex eigenvector for the eigenvalue $\lambda = 8$

$$N[A - \lambda I] = \begin{bmatrix} -6 & 3-3i \\ 3+3i & -3 \end{bmatrix}$$

$$x_1 = 1$$

$$R_2 = R_2 + \tfrac{1}{2}(1+i)R_1$$

$$= \begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-6x_1 + (3-3i)x_2 = 0$$

$$-2x_1 + (1-1i)x_2 = 0$$

$$2x_1 = (1-1i)x_2$$

5. Consider the matrix $A = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix}$. Find the complex eigenvector for the eigenvalue $\lambda = 8$

$$N[A - \lambda I] = \begin{bmatrix} -6 & 3-3i \\ 3+3i & -3 \end{bmatrix}$$

$$R_2 = R_2 + \tfrac{1}{2}(1+i)R_1$$

$$= \begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-6x_1 + (3-3i)x_2 = 0$$

$$-2x_1 + (1-1i)x_2 = 0$$

$$2x_1 = (1-1i)x_2$$

$$x_1 = 1$$

$$x_2 = \tfrac{2}{1-1i}$$

5. Consider the matrix $A = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix}$. Find the complex eigenvector for the eigenvalue $\lambda = 8$

$$N[A - \lambda I] = \begin{bmatrix} -6 & 3-3i \\ 3+3i & -3 \end{bmatrix}$$

$$R_2 = R_2 + \tfrac{1}{2}(1+i)R_1$$

$$= \begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-6x_1 + (3-3i)x_2 = 0$$

$$-2x_1 + (1-1i)x_2 = 0$$

$$2x_1 = (1-1i)x_2$$

$$x_1 = 1$$

$$x_2 = \tfrac{2}{1-1i}$$

$$x_2 = \tfrac{2}{1-1i} = \tfrac{2}{1-1i}\tfrac{1+1i}{1+1i}$$

5. Consider the matrix $A = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix}$. Find the complex eigenvector for the eigenvalue $\lambda = 8$

$$N[A - \lambda I] = \begin{bmatrix} -6 & 3-3i \\ 3+3i & -3 \end{bmatrix}$$

$$R_2 = R_2 + \tfrac{1}{2}(1+i)R_1$$

$$= \begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-6x_1 + (3-3i)x_2 = 0$$

$$-2x_1 + (1-1i)x_2 = 0$$

$$2x_1 = (1-1i)x_2$$

$$x_1 = 1$$

$$x_2 = \tfrac{2}{1-1i}$$

$$x_2 = \tfrac{2}{1-1i} = \tfrac{2}{1-1i}\tfrac{1+1i}{1+1i}$$

$$x_2 = 1 + 1i$$

5. Consider the matrix $A = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix}$. Find the complex eigenvector for the eigenvalue $\lambda = 8$

$$N[A - \lambda I] = \begin{bmatrix} -6 & 3-3i \\ 3+3i & -3 \end{bmatrix}$$

$$R_2 = R_2 + \tfrac{1}{2}(1+i)R_1$$

$$= \begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 & 3-3i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-6x_1 + (3-3i)x_2 = 0$$

$$-2x_1 + (1-1i)x_2 = 0$$

$$2x_1 = (1-1i)x_2$$

$$x_1 = 1$$

$$x_2 = \frac{2}{1-1i}$$

$$x_2 = \frac{2}{1-1i} = \frac{2}{1-1i}\frac{1+1i}{1+1i}$$

$$x_2 = 1 + 1i$$

$$\therefore x = \begin{bmatrix} 1 \\ 1+1i \end{bmatrix}$$

6. Let $U = \begin{bmatrix} cos(t) & -sin(t) \\ sin(t) & cos(t) \end{bmatrix}$, show that the matrix U is unitary.

6. Let $U = \begin{bmatrix} cos(t) & -sin(t) \\ sin(t) & cos(t) \end{bmatrix}$, show that the matrix U is unitary.

$$U = \begin{bmatrix} cos(t) & -sin(t) \\ sin(t) & cos(t) \end{bmatrix}$$

6. Let $U = \begin{bmatrix} cos(t) & -sin(t) \\ sin(t) & cos(t) \end{bmatrix}$, show that the matrix U is unitary.

$$U = \begin{bmatrix} cos(t) & -sin(t) \\ sin(t) & cos(t) \end{bmatrix} \qquad U^T = \begin{bmatrix} cos(t) & sin(t) \\ -sin(t) & cos(t) \end{bmatrix}$$

6. Let $U = \begin{bmatrix} cos(t) & -sin(t) \\ sin(t) & cos(t) \end{bmatrix}$, show that the matrix U is unitary.

$$U = \begin{bmatrix} cos(t) & -sin(t) \\ sin(t) & cos(t) \end{bmatrix} \qquad U^T = \begin{bmatrix} cos(t) & sin(t) \\ -sin(t) & cos(t) \end{bmatrix}$$

$$U * U^T = \begin{bmatrix} cos^2(t) + sin^2(t) & cos(t)sin(t) - sin(t)cos(t) \\ sin(t)cos(t) - cos(t)sin(t) & sin^2(t) + cos^2(t) \end{bmatrix}$$

6. Let $U = \begin{bmatrix} cos(t) & -sin(t) \\ sin(t) & cos(t) \end{bmatrix}$, show that the matrix U is unitary.

$$U = \begin{bmatrix} cos(t) & -sin(t) \\ sin(t) & cos(t) \end{bmatrix} \qquad U^T = \begin{bmatrix} cos(t) & sin(t) \\ -sin(t) & cos(t) \end{bmatrix}$$

$$U * U^T = \begin{bmatrix} cos^2(t) + sin^2(t) & cos(t)sin(t) - sin(t)cos(t) \\ sin(t)cos(t) - cos(t)sin(t) & sin^2(t) + cos^2(t) \end{bmatrix}$$

$$U * U^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

7. We know that $U = \begin{bmatrix} cos(t) & -sin(t) \\ sin(t) & cos(t) \end{bmatrix}$,is unitary.Let us take a vector $x \in R^2$ and see what happens when it get transformed by the U.

https://www.geogebra.org/material/iframe/id/ynztugm7/width/700/height/500/border/888888/sfsb/true/smb/false/stb/false/stbh/false/ai/false/asb/false/sri/false/rc/false/ld/false/sdz/true/ctl/false