Systems of Linear Equations

Sarang S. Sane

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- What is its relation with matrices?

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- What is its relation with matrices?
- How many solutions can it have?

Example

Items	Buyer A	Buyer B	Buyer C
 Rice in Kg	8	12	3
 Dal in Kg	8	5	2
 Oil in Liter	4	7	5

Example Contd.

Suppose A paid $Rs.1960$, B paid $Rs.2215$ and C paid $Rs.1135$.

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$$8x + 8y + 4z = 1960$$
$$12x + 5y + 7z = 2215$$
$$3x + 2y + 5z = 1135$$

$4x + 4y + 2z = 980$

$12x + 12y + 6z = 2940$

$12x + 8y + 20z = 4 \times 1135$

$7y - z = 725$

$3y + 13z = 2325$

$3y + 13(7y - 725) = 2325$

$\Rightarrow z = 7y - 725$

$94y = 11750 \Rightarrow y = 125 \Rightarrow z = 150$

$\Rightarrow x = 45$

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Linear Equations

A linear equation is an equation of the form

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$$

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Example

2x+3y+5z=-9, where x, y, z are variables and $2, 3, 5$ are the coefficients.

System of Linear Equations

A system of linear equations is a collection of one or more linear equations involving the same set of variables For example,

$$\begin{aligned} 3x + 2y + z &= 6 \\ x - \frac{1}{2}y + \frac{2}{3}z &= \frac{7}{6} \\ 4x + 6y - 10z &= 0 \end{aligned}$$

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$3 \times 1 + 2 \times 1 + 1 = 6.$

$1 - \frac{1}{2} \times 1 + \frac{2}{3} \times 1 = \frac{1}{2} + \frac{2}{3} = 7/6.$

$4 \times 1 + 6 \times 1 - 10 \times 1 = 0.$

is a system of three equations in the three variables x, y, z. A solution to a linear system is an assignment of values to the variables such that all the equations are simultaneously satisfied. A solution to the system above is given by

$$x = 1, y = 1, z = 1$$

General Form of System of Linear Equations

A general system of m linear equations with n unknowns can be written as

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &= b_2 \\
&\ldots \\
&\ldots \\
a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &= b_m
\end{aligned}
$$

i^{th} eqn. $a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n = b_n$.

Matrix Representation

The system of linear equations is equivalent to a matrix equation of the form

$$Ax = b$$

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$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \dots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

coefficient matrix

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$n \times 1$

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The example we mentioned above

$$\begin{aligned} 3x + 2y + z &= 6 \\ x - \frac{1}{2}y + \frac{2}{3}z &= \frac{7}{6} \\ 4x + 6y - 10z &= 0 \end{aligned}$$

can be represented as $Ax = b$, where

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A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & -\frac{1}{2} & \frac{2}{3} \\ 4 & 6 & -10 \end{bmatrix}
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can be represented as $Ax = b$, where

$$
A = \begin{bmatrix} 3 & 2 & 1 \\ 1 & -\frac{1}{2} & \frac{2}{3} \\ 4 & 6 & -10 \end{bmatrix} \qquad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad b = \begin{bmatrix} 6 \\ \frac{7}{6} \\ 0 \end{bmatrix}
$$

$Ax = b$

The first example :

$$8x + 8y + 4z = 1960$$
$$12x + 5y + 7z = 2215$$
$$3x + 2y + 5z = 1135$$

$$A = \begin{bmatrix} 8 & 8 & 4 \\ 12 & 5 & 7 \\ 3 & 2 & 5 \end{bmatrix} \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times 1} \quad b = \begin{bmatrix} 1960 \\ 2215 \\ 1135 \end{bmatrix}$$

$$A\underline{x} = b$$

Solutions to a linear system

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1) The system has infinitely many solutions. ∞

2) The system has a single unique solution. 1

3) The system has no solution. 0

Example of infinitely many Solutions

Items	Buyer A	Buyer B
 Rice in Kg	2	4
 Dal in Kg	1	2

Example of infinitely many solutions (Contd.)

Suppose A paid $Rs.215$, B paid $Rs.430$. We want to find the price of each items using this data. Suppose price of Rice is $Rs.x$ per kg., price of dal is $Rs.y$ per kg. Hence we have the following system of linear equations:

Example of infinitely many solutions (Contd.)

Suppose A paid $Rs.215$, B paid $Rs.430$. We want to find the price of each items using this data. Suppose price of Rice is $Rs.x$ per kg., price of dal is $Rs.y$ per kg. Hence we have the following system of linear equations:

$$
\begin{aligned}
2x + y &= 215 \\
4x + 2y &= 430
\end{aligned}
$$

Example of infinitely many solutions (Contd.)

Suppose A paid $Rs.215$, B paid $Rs.430$. We want to find the price of each items using this data. Suppose price of Rice is $Rs.x$ per kg., price of dal is $Rs.y$ per kg. Hence we have the following system of linear equations:

$$2x + y = 215$$
$$4x + 2y = 430$$

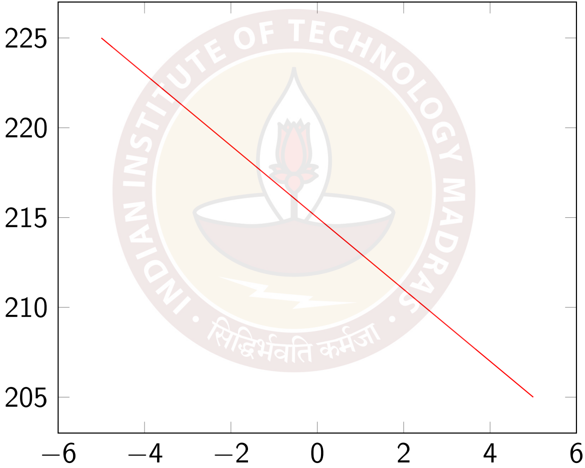
There are infinitely many x and y satisfying both the equations.

$x = 0, \; y = 215$

$x = \frac{215}{2} = 107.5 \quad , y = 0$

Example of infinitely many solutions (Contd.)

Both the equations represents the same straight line in the two dimensional plane $\mathbb{R}^2$.



Every point on this line is a solution.

Example of a system of equations with no solution

Suppose A and B bought the same amount of items as in the previous example. But for some reason the seller gave a discount to B.

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Suppose A and B bought the same amount of items as in the previous example. But for some reason the seller gave a discount to B. Suppose A paid $Rs.215$ and B paid $Rs.400$. Now after returning home they decided to find out the price of each item by solving the linear system of equations as before. Suppose price of rice is $Rs.x$ per kg., price of dal is $Rs.y$ per kg. Hence we have the following system of linear equations:

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$$2x + y = 215$$
$$4x + 2y = 400$$

$4x + 2y = 430$ $\Rightarrow$ $400 = 430$

$4x + 2y = 400$

Example of a system of equations with no solution

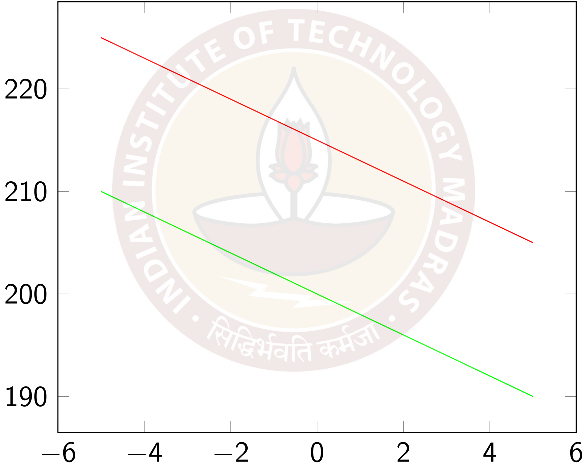
Suppose A and B bought the same amount of items as in the previous example. But for some reason the seller gave a discount to B. Suppose A paid $Rs.215$ and B paid $Rs.400$. Now after returning home they decided to find out the price of each item by solving the linear system of equations as before. Suppose price of rice is $Rs.x$ per kg., price of dal is $Rs.y$ per kg. Hence we have the following system of linear equations:

$$2x + y = 215$$
$$4x + 2y = 400$$

But in this case there are no solution of this system of equations.

Example of a system of equations with no solution

The equations represents two parallel straight lines in the two dimensional plane $\mathbb{R}^2$.



Example of a system with a unique solution

Items	Buyer A	Buyer B
 Rice in Kg	2	3
 Dal in Kg	1	1

Example with a unique solution

Suppose A paid $Rs.215$, B paid $Rs.260$. We want to find the price of each items using this data. Suppose price of Rice is $Rs.x$ per kg., price of dal is $Rs.y$ per kg. Hence we have the following system of linear equations:

Example with a unique solution

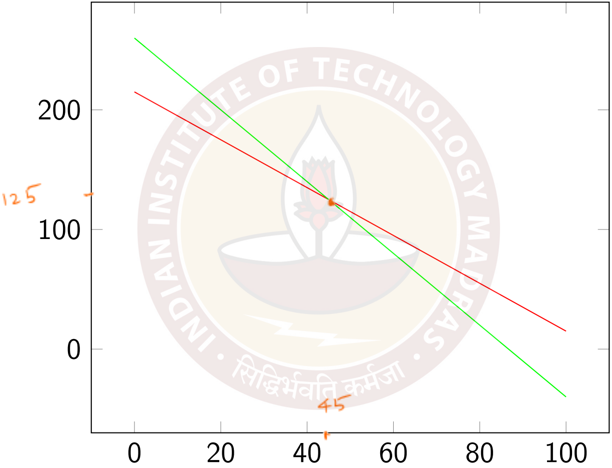
Suppose A paid $Rs.215$, B paid $Rs.260$. We want to find the price of each items using this data. Suppose price of Rice is $Rs.x$ per kg., price of dal is $Rs.y$ per kg. Hence we have the following system of linear equations:

$$2x + y = 215$$
$$3x + y = 260$$

$\Rightarrow x = 45$

$\Rightarrow y = 215 - 90 = 125$

Example with unique solution



Thank you