L.5.1 Permutations & Combinations - Basic principles of counting:

+ variable { Categorical / Numerical }
+ Association btwn variables
  (i) both categorical
  (ii) " Numerical (covariances, correlation)
  (iii) One is categorical & Numerical

+ permutation → counting with order
+ combination → " without order

eg: 1 Buying clothes.

+ gift card → buy Shirt/pant. (either of 2)

Shirt	pant
Yellow	Black
Blue	Blue
Green	Brown
Red	

→ How many different ways can I use my card?

4 → shirt
3 → pant

(If shirt no pant & vice versa) (Actions are dependent on each)

∴ total choices are 4+3 = 7.

Addition rule of counting:

+ If an action A (shirt) can occur in n_1 (4) different ways, another action B (pant) can occur in n_2 (3) different ways, then the total number of occurrence of the actions A or B is n_1+n_2.

eg: 2 Choose either a pant or a shirt (last eg!)

Now: Change to one shirt & one pant. (~~No of~~

Q: No. of choices?

Ans: (Y Bla, Y Blu, Y Brow, = 3

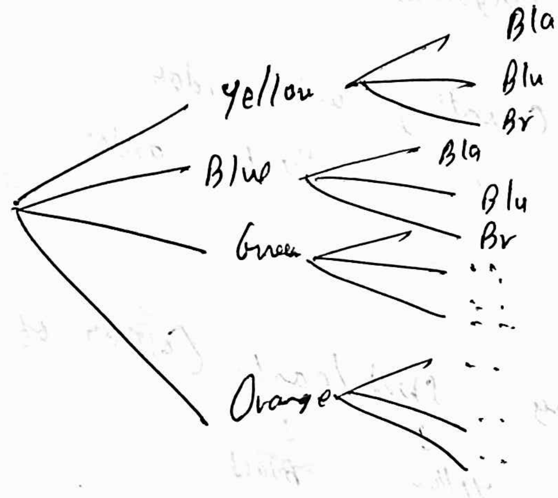
B " B " B " → 3

G " G " G " = 3

O " O " O " = 3

4 4 4 12

Tree



Matching Shirt & pants & Shoes:

4 Shirts 3 Pant 2 Shoes

Black

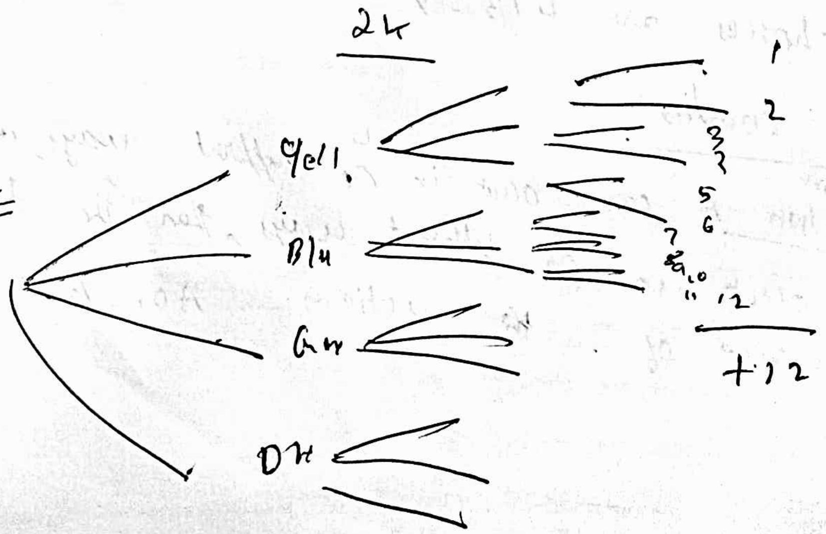
Brow.

↳ 12 pairs + Black ↓ 2

12 pairs + Brow

24

Tree



= 24 ways

Multiplication rule of Counting:

- If an action A can occur in n_1 diff ways, another action B can occur in n_2 diff ways then total no. of occurance of Actions A & B together is $n_1 \times n_2$.

(A or B) => $n_1 + n_2$

- Suppose that r actions are to be performed in a definite order. Further suppose that there are n_1 possibilities for the first actions & that corresponding to each of these possibilities are n_2 possibilities for second action and so on. Then there are $n_1 \times n_2 \times \ldots \times n_r$ possibilities altogether for the r actions

Example - 2 Application: Creating Alpha-numeric code.

Alpha numeric password:

- Create 6 digit alpha numeric letters followed by 4 numbers
  - have first two letters
  - Repetition allowed.

    No of ways - $26 \times 26 \times 10 \times 10 \times 10 \times 10 = 6{,}760{,}000$
  - Repetition not allowed.

    No. of ways - $26 \times 25 \times 10 \times 9 \times 8 \times 7 = 3{,}276{,}000$

Section summary:

- Addition rule of counting ($N_1 + N_2$ => A or B
- Multiplication " " $N_1 \times N_2$ => A & B.

Permutations & Combinations - Factorials

Lecture 5.2

Example - 3 Order of finishes in a race.

- There are 8 athletes, 100 m race, What are possible ways the athletes can finish the race. (assuming no ties)

Position →	1	2	3	4	5	6	7	8
→	P1	P2	P3	P4	P8	P7	P6	P5
(No. of choices)	8	7	6	5	4	3	2	1

∴ possible ways $= 8\times7\times6\times5\times4\times3\times2\times1$

(ie) First place - any of 8, second → any of remaining 7 & so on
last → only one

Factorial

Definition

Product of first n positive integers (counting numbers) is called n factorial & is denoted by $n!$. In symbols

$$n! = n\times(n-1)\times\cdots\cdot 1$$

Remark

By convention $\boxed{0! = 1}$ & $1! = 1$

∴ we represent $8\times7\times6\times5\times4\times3\times2\times1 \Rightarrow 8!$

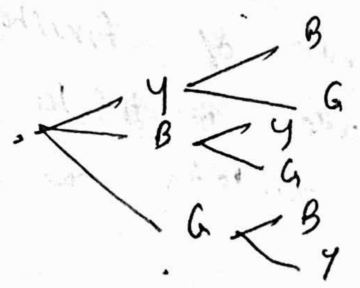
Eg:4 choosing shirts (any order of 3 shirts chosen without repetition)

	1	2	3
1	Y	B	G
2	Y	G	B
3	B	Y	G
4	B	G	Y
5	G	Y	B
6	G	B	Y

$\Rightarrow n = 3$

$n! = 3\times2\times1 = 6$

(Yellow, Blue, Green)
↓
possible order of choosing


$=6.$

Example 5

(1) $5! = 5\times4\times3\times2\times1 = 120$

(2) Observe $5! = 5\times4! \Rightarrow 5\times4\times3!$.

$$6! = 6\times\underbrace{5\times4\times3\times2\times1}$$

$$= 6\times5! \Rightarrow 6\times5\times4!$$

$$n! = n\times((n-1)\times\ldots 1)$$

$$\boxed{n! = n\times(n-1)!}$$

(ie) in general

(3) Observe $5! = 5\times4! = 5\times4\times3!$

• in general, for $i \leq n$, we have

$$\boxed{n! = n\times(n-1)\ldots\times(n-i+1)\times(n-i)!}$$

(eg) $n=5$ $i \leq n$

$i=1$
$i=2$
$i=3$
$i=4$
$i=5$

$5! = n\times4!$

$i=1$ $5\times4!$

$i=2$ $= 5\times4\times3!$

$i=3$ $= 5\times4\times3\times2!$

$i=4$ $= 5\times4\times3\times2\times1$

Example 6 : Simplifying expressions

(1) $\frac{6!}{3!} = \frac{6\times5\times4\times\overbrace{3\times2\times1}^{3!}}{3!} = 6\times5\times4 = 120$

(2) $\frac{6!\times5!}{3!\times4!} = 6\times5\times4 \cdot \times \frac{5\times\cancel{4!}}{\cancel{4!}} = 6\times5\times4\times5 = 600$.

(3) Express $25\times24\times23$ in terms of factorials

$$\frac{25\times24\times23\times22\times21\times\ldots 1}{22\times21\times\ldots} = \frac{25!}{22!}$$

Section Summary.

- Introduced factorial notation
- Simplifying expressions

L-5.3 Permutations and Combinations - Permutations: Distinct objects.

Permutation:

A ~ an ordered arrangement of all or some of n objects

(0) ordered arrangement

n-objects (Distinct)

$n=3$ | A | B | C | (here distinct) (no repetition)

(or)

$n=3$ A | A | B $\Rightarrow$ 2 Red one yellow (not distinct)

Example:

Take A - B - C → Possible arrangement - taking all at a time

All (i) 1 2 3 → 6 arrangements (all.)

	1	2	3
1	A	B	C
2	A	C	B
3	B	A	C
4	B	C	A
5	C	A	B
6	C	B	A

Some

(ii) A, B, C → Possible arrangement - taking two at a time

AB] same
BA

BC] same
CB

AC] same
CA

$n=3$, 2 at a time

↓

$\boxed{6 \text{ arrangements}}$

Example - 4

Take A, B, C, D → Possible arrangements taking all at a time

$n=4$ A B C D

$7DC$.

$7n$ ← $\boxed{A}$ B C D → 6 ⇒ $4 \times 3 \times 2 \times 1 = \underline{\underline{24}}$

B A C D → 6

C A B D → 6

D . A B C → 6

24 × ~~4~~

Example

Take A, B, C, D → Possible arrangement, two at a time

AB	BA
AC	CA
AD	DA
BC	CB
BD	DB
CD	DC

$\boxed{12}$ possible arrangement

$A < B, C, D$ (= 12)

$B < A, C, D$

$C < A, B, D$

$D < A, B, C$

Permutation formula :

No of possible permutations of r objects:, from a collection of n distinct objects given by formula.

$r \le n$,

$n = 3$ ABC
$r = 3$

$n=3$ ABC
$r=2$

$n=4$ ABCD
$r=4$

$n=4$ ABCD
$r=2$.

$\square \quad \square \quad \square \dots\dots \quad \square$

$1 \quad 2 \quad 3 \quad\quad r$

$n \times n-1 \times n-2 \dots \times \dots n-(r-1)$ as denoted by ${}^nP_r$

$${}^nP_r = \frac{n \times (n-1) \times (n-2) \dots (n-r+1) \times (n-r)(n-r-1) \dots \times 1}{(n-r)(n-r-1) \dots \times 1}$$

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

Repetition is not allowed

• Special cases.

① $^nP_0 = \frac{n!}{(n-0)!} = \frac{n!}{n!} = 1$ → There is only one ordered arrangement of 0 objects.

② $^nP_1 = \frac{n!}{(n-1)!} = n$ → There are n ways of choosing one object.

③ $^nP_n = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$ No repetition → ~~AAB~~

* Take ABC → Possible arrangement all at a time

$n=3, \quad r=3$

$^nP_n = n!$

A B C
A C B
B A C
B C A
C A B
C B A

} 6.

$n = 3$
$3! = 6.$

Case: $n=3$, $r=3$, $^nP_r = \frac{n!}{(n-r)!} = \frac{3!}{0!} = 6.$

Example

ABC → 2 at a time.

$n=3, \quad r=2 \quad ^nP_r = \frac{n!}{(n-r)!} = \frac{3!}{1!} = 6.$

Take ABCD → all at a time

$n=4, \quad r=4, \quad ^nP_r = \frac{n!}{(n-r)!} = \frac{4!}{0!} = \frac{4\times3\times2\times1}{1}$

$= 24.$

1 2 3 4

$6+6+6+6 = 24$

Take ABCD → 2 at a time.

$n=4, r=2 \quad ^nP_r = \frac{4!}{2!} = 4\times3 = 12$

① ②
A 3
B 3
C 3
D 3 = 12

Application:

- Committee of 8 persons in how many ways can we choose a chairman and vice chairman assuming one person can hold not more than one position

C	V.C.
A	7
B	7
C	7
D	7
E	7
F	
G	
H	

$\boxed{7 \times 8}$

$n = 8$.
$r = 2$.
No. of ways.

$= {}^nP_r = \frac{n!}{(n-r)!} = \frac{8!}{(8-2)!} = \cancel{8 \times 7 \times 6 \times 5 \times 4 \times 3}$

$\frac{8!}{6!} = 8 \times 7 = 56.$

Example

~~no. of~~ 4 digit number — formed using 1, 2, 3, 4, 5 no repeat.

1 2 3 4
1
2
3
4
5

$n = 5$, $r = 4$

${}^nP_r = \frac{5!}{(5-4)!} = 5 \times 4 \times 3 \times 2 = 120$

- How many of these will be even?

□ □ □ | last $\boxed{2}$
□ □ □ | $\boxed{4}$

using nos → (i) 1, 3, 4, 5.
$n = 4$
$r = 3$
left ⇒ n.

since last digit is fixed, $r = 3$.

${}^4P_3 = \frac{4!}{(4-3)!} = 4 \times 3 \times 2 = 24$

using nos → (ii) 1, 2, 3, 5 → $n = 4$, $r = 3$

${}^4P_3 = 24$

∴ No. of even = 48.

Example

6 ppl. go to cinema. sit in 10 rowed seats (No. of ways they can sit)

(i) $n = 10$, $r = 6$ ${}^{10}P_6 = 1,51,200$ (sit anywhere)

(ii) all empty seats next to each other (together)

$\boxed{\text{Note } r \leq n}$

~~n =~~ 7 cases
$r = 6$

${}^7P_6 = 5040$

Taking all empty chair as a single case (or) chair

1 2 3 4	6.
1	1 2 3 4	5
2	1 2 3 4	4.

7 cases

Eg: Take ABC → all at a time – repetition allowed

A	A	A
A	A	B
A	A	C
A	B	B
A	B	C
A	C	A
A	B	A
A	C	A
A	C	B
A	C	C
⋮		

9
+
9
+
9
27

$3 \times 3 \times 3$

$\square$	$\square$	$\square$
A	A	A
B	B	B
C	C	C

$= 27$.

(multiplication rule)

$n \to$ no. of distinct object (A B C)

$r \to$ How many times repeated $= \square\ \square\ \square$
(or)
No. of combinations contained $\boxed{1}\ \boxed{2}\ \boxed{3}$

Eg.

ABC → two at a time

A	A	A	B	B	B	C	C	C
A	B	C	A	B	C	A	B	C

$\underbrace{\qquad\qquad\qquad\qquad}_{9}$

Permutation formula:

1	2	3		r
$\square$	$\square$	$\square$	....	$\square$
n	n	n		n

$= \boxed{n^r} \to$ repetition

$n \to$ distinct objects

$r \to$ taking r objects from n, $r \le n$

The no. of possible permutations of r objects from a collection of n distinct object when repetition is allowed is given by the formula

$n \times n \times \dots \times n$.

and is denoted by n^r.

Eg:

ABC → all at a time $n = 3$, $r = 3$, $n^r = 3^3 = 27$

ABC → two at a time $n = 3$, $r = 2 = 3^2 = 9$

Summary

(i) No. repeats → $^nP_r = \frac{n!}{(n-r)!}$

(ii) allowed → n^r

Lecture-5-4 : Permutations & combinations: Permutations : objects not distinct

- Suppose we want to arrange 'DATA' →

[DAT] → 3 distinct

DA_1TA_2 , $n=4$, $n!$ $4!$

$= 4 \times 3 \times 2 = 24$

DA_1TA_2
DA_2TA_1 } → DATA

A_1TA_2D
A_2TA_1D } → ATAD

(ie) for every 2 arrangement I have one arrangement same

(ie) $\frac{24}{2} = 12$ arrangements possible

Example: Rearranging letters.

- Suppose we want to rearrang the letters in the word. "DATA". How many ways?
- There are

$$\frac{4!}{2!} = 12$$

Permutation formula:

No. of permutations of n objects when p of them are of one kind and rest distinct is equal to

$$\frac{n!}{p!}$$

- Suppose we want to rearrange letters in word "STATISTICS". How many ways it can be done

$S_1T_1AT_2IS_2T_3I_1CS_3$

S - 3
T - 3
A - 1
I - 2
C - 1
10

$10!$, $n=10$

$p_1 = 3$ of first kind - S
$p_2 = 3$ " 2nd " - T
$p_3 = 1$ " 3rd " - A
$p_4 = 2$ " 4th " - I
$p_5 = 1$ " 5th " - C

permutation formula:

- The no. of permutations of n objects where p_1 of one kind p_2 is of 2nd kind & so on p_k of k^{th} kind is given by

$$\frac{n!}{p_1!\,p_2!\ldots p_k!}$$

$$\frac{n=10}{3\,3\,1\,2\,1} = \frac{10!}{3!\,3!\,1!\,2!\,1!} = 50400$$

Section Summary:

① No of permutations of n objects when p of them are of one kind & rest distinct is equal to $\frac{n!}{p!}$

② The no. of permutations of n objects where p_1 is of one kind p_2 is of 2nd kind, and so on p_k of k^{th} kind is given by $\frac{n!}{p_1!\,p_2!\ldots p_k!}$

Example:

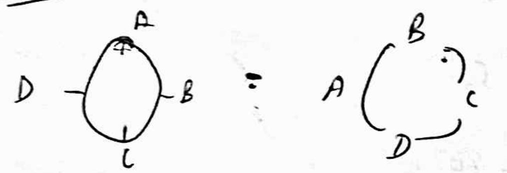
So far we saw in linear arrangement

Now

Arrange in a circle

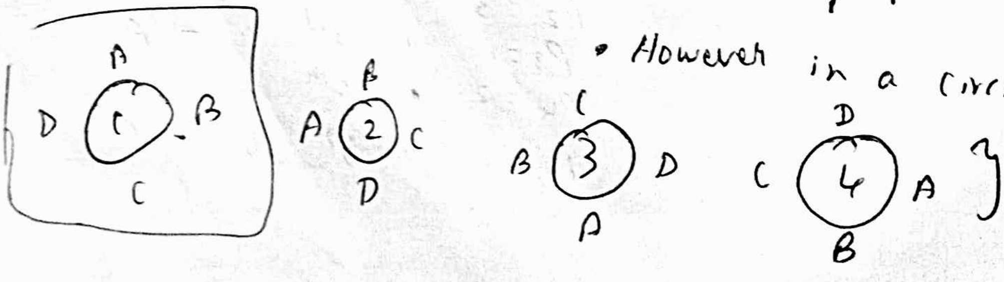
- How many ways can 4 ppl sit in a round table?
- We consider two cases; each selection is called a combination of 3 different objects taken 2 at a time.
  - Clockwise & anticlockwise are diff
  - Clockwise & " are same

ABCD



ABCD ⟹ BCDA

(i) Clock & anticlockwise are different.
- linear permutations of ABCD.
- ABCD, BCDA, CDAB, DABC are diff when people are seated in a row.
- However in a circle



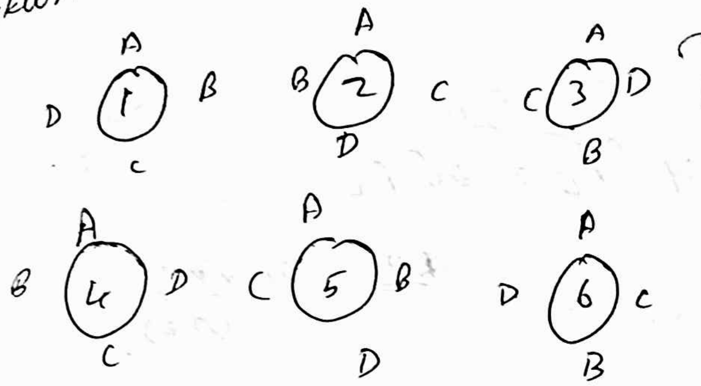
} all are same

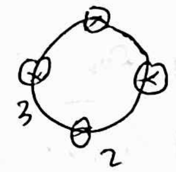
6 possible linear arrangements

ABCD   BACD
ABDC   BADC
ACBD   BCAD
ACDB   BCDA
ADBC   BDAC
ADCB   BDCA

→ distinct in linear

Clockwise & anticlockwise are different.



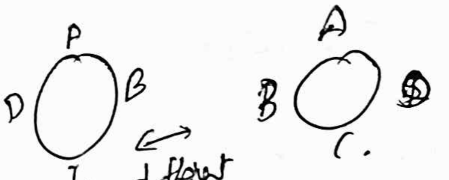


6

1 can be filled with three available alphabets (i.e.) $3! = 6$.

- In case of circular & clock & anti different we have one fixed. i.e., & consider others as combo of 3 at a time. without repetition

$$A \Rightarrow \begin{pmatrix} BCD \\ CDB \\ DBC \\ DCB \\ BDC \\ CBD \end{pmatrix} = 6 \text{ (i.e.) } (n-1)!$$



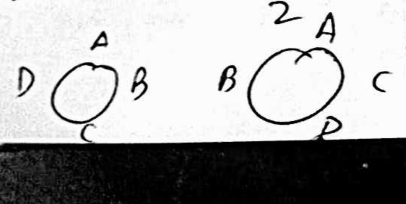
different
But mirror image

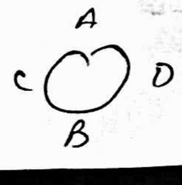
∴ No. of possible circular permutations of n objects if the clockwise & anticlockwise are different $\Rightarrow \underline{(n-1)!}$

& the rational behind this is I fix one of the objects, the other n minus 1 objects can be arranged among themselves in n minus 1 way and this is the same for any of those objects

∴ There are $(n-1)!$ ways of arranging circular permutations of n ~~diff~~ distinct objects (Clockwise & anticlockwise are different)

* (When clockwise & anticlockwise are same) → the number of ways n-distinct objects can be arranged in a circle is equal to $\dfrac{(n-1)!}{2}$





$\Rightarrow \dfrac{(n-1)!}{2}$

Eg:

linear permutation.

arrange 'n' objects in a row, choose r objects from these 'n' & arrange in a linear row.

$^nP_r = \frac{n!}{(n-r)!}$ (without repetition), with repetition it is n^r

eg: solving for n.

(i) • Find value of n if $^nP_4 = 20\,^nP_2$

LHS: $\frac{n!}{(n-4)!}$ = RHS $20 \times \frac{n!}{(n-2)!}$

$(n-2)! = 20 \times (n-4)!$

$(n-2)(n-3)\cancel{(n-4)!} = 20 \times \cancel{(n-4)!}$

$(n-2)(n-3) = 20$

$n^2 - 5n - 14 = \cancel{20}\ 0$. $(n+2)(n-7) = 0$

we get, $n = -2$ or $\boxed{n=7}$

impossible

$\therefore \boxed{n=7}$

(ii) $\frac{^nP_4}{^{n-1}P_4} = 5/3$

LHS $\frac{n!}{(n-4)!} \Big/ \frac{(n-1)!}{(n-5)!}$

$\frac{n!}{(n-4)!} \times \frac{(n-5)!}{(n-1)!}$

$\frac{n \times \cancel{(n-1)!}\ \cancel{(n-5)!}}{(n-4)\cancel{(n-5)!}\cancel{(n-1)!}} \Rightarrow \frac{n}{n-4} = 5/3$

$3n = 5n - 20$

$2n = 20$

$\boxed{n=10}$

(iii) Solving r.

$$^5P_r = 2 \times {}^6P_{r-1}$$

LHS $\quad {}^5P_r = \frac{5!}{(5-r)!}$

RHS $\quad {}^6P_{r-1} = \frac{6!}{(6-(r-1))!} = \frac{6!}{(7-r)!}$

$$\frac{5!}{(5-r)!} = \frac{6!}{(7-r)!} \times 2. \longrightarrow (1)$$

$$(7-r)! = (7-r) \times (7-1-r) \times (7-2-r)!$$

$$(7-r)! = (7-r)(6-r)(5-r)!$$

(1) can be reexpressed as

$$\frac{5!}{(5-r)!} = \frac{2 \times 6 \times 5!}{(7-r)(6-r)(5-r)!}$$

$$(7-r)(6-r) = 12$$

$$r^2 - 13r + 42 - 12 = 0$$

$$r^2 - 13r + 30 = 0$$

$$(r-3)(r-10) = 0$$

give $\boxed{r = 10 \text{ (or) } 3.}$

Topic Summary:

(1) permutations when n objects are distinct
   1.1 → repetition not allowed, $^nP_r = \frac{n!}{(n-r)!}$
   1.2 → " allowed (n^r)

(2) permutation when objects are not destinct, P_1, P_2, P_n, $\frac{n!}{P_1! P_2! .. P_n!}$

(3) Circular Permutation
   3.1 → clock & anticlockwise different $(n-1)!$
   3.2 → " " same $\frac{(n-1)!}{2}$

(4). Solve for r & n using permutation formula

Permutations:

* we saw how to arrange n objects (or) r objects out of n objects.
* Order was important, AB was different from BA
⇒ * How can we choose 2 out of 3, (or) 3 out of 10.
In this order is not important AB is same as B

→ Combinations : No. of ways we select r from n.

5.5: Permutations and Combinations: Combinations:

Permutations ⇒ ordered arrangement

Combination:

eg: How many way we select two students from a group of three students? (No orders)

A, B, C

① AB
② AC
③ BC

(AB) (BA)

No use in order. both are same
No meaning

* Each selection is called a combination of 3 different objects taken 2 at a time.

3 → 1, 1   3 ↓   (n) ↓ (r)

Example:

A, B, C - Possible combinations - taking two at a time,

First Place	Second Place
A	B
A	C
B	C

FP SP

① AB < A B, B A

② AC < A C, C A

③ BC < B C, C B

6.

No. of combination × 2! = No. of permutations
3

$^nP_r = {}^3P_2 = \frac{3!}{(3-2)!} = 6.$

Tutorial

$(n) \to (r) \to$ pair

$\to 1 \to r!$

$2 \to r!$

Combination: Notation & formula

- In general, each combinations of r objects from n objects can give rise to $r!$ arrangements.
- The no. of possible combinations of r objects from a collection of n distinct objects is donoted by ^nC_r and is given by. $^nC_r \Rightarrow \underbrace{^nC_r \cdot r!}_{} = \underset{\text{Total no. of permutations}}{^nP_r}$

n choose r.

- In general, each combination of r objects from n objects can give rise to $r!$ arrangements

$$\boxed{^nC_r = \frac{n!}{r!(n-r)!}}$$

$$^nC_r = \frac{^nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

$$^3C_2 = \frac{3!}{2!1!} = 3$$

- Another common notation is $\binom{n}{r}$ which is also referred to as the binomial coefficient $\binom{n}{r}$, (or) ^nC_r

<u>Some useful results.</u>

① $^nC_r = \dfrac{n!}{r!(n-r)!} \Leftrightarrow \dfrac{n!}{(n-r)!r!}$ $= {}^nC_{(n-r)} = \dfrac{n!}{\cancel{(n-(n-r))}!}$

$= \dfrac{n!}{(n-r)! \cdot (n-(n-r))!}$

$= \dfrac{n!}{(n-r)! \times r!}$

(ie) selecting r objects from n objects is same as rejecting $n-r$ objects from n objects

② $^nC_n = 1$ & $^nC_0 = 1$ for all values of n

$^nC_n = \dfrac{n!}{n!(n-n)!} = \dfrac{n!}{n!0!} = 1$, $\quad ^nC_0 = {}^nC_{n-0} = {}^nC_n = 1$

$= \dfrac{n!}{0!(n-0)!} = \dfrac{n!}{n!} = 1$

(3) ${}^{n}C_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r \; : \; 1 \le r \le n$

choose 3 of 5

A B C D E

$n = 5$

$r = 3$

A is part — A is not a part of 3 (solution) & choose 3 from 4

$$\binom{5}{3} = \text{fix } A \binom{4}{2} + \binom{4}{3}$$

(i.e) Here fix any one of n objects & looked at the situation that "r objects that has n" and "r does not have"

r has n — choose r-1 from remaining n-1 — $\binom{n-1}{r-1}$ + $\binom{n-1}{r}$ → remaining (n-1), because eliminating 'n', & choose r — r does not have n

(i.e) ${}^{n}C_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$

Example:

Choosing questions in an exam

① Exam, 12 questions — Part I (7 ques), Part II (5 ques) $\}$ = 12 ques

attend total of 8 questions, selecting 3 atleast from each.

∴ In how many ways can a student select the questions

	P I	P II		
8	3	5	→ ${}^7C_3 \, {}^5C_5$ +	$= \frac{7!}{3!4!} \frac{5!}{5!0!} = 35$
	4	4	→ ${}^7C_4 \, {}^5C_4$ +	$= \frac{7!}{4!3!} \frac{5!}{4!1!} = 35 \times 5 = 175$
	5	3	→ ${}^7C_5 \, {}^5C_3$	$= \frac{7!}{5!2!} \frac{5!}{3!2!} = 21 \times 10 = 210$

$\frac{5!}{3!2!} =$

$\boxed{\text{Total} = 420}$

Example: Game of cards:

Club – ♧
spade – ♤
heart – ♡
Diamond – ◇

4 Suits. – $13 \times 4 = 52$ cards

26 Black, 26 Red.

+ Choose 4 from 52 cards.

$$^{52}C_4 = \frac{52!}{4!\,48!} = 270725$$

+ All four cards of same suit.

$$^4C_1 \times {}^{13}C_4 = \frac{4!}{3!} \times \frac{13!}{4!\,9!} = 2860$$

First choose which suit

+ Cards are of same colour

$$^2C_1 \times {}^{26}C_4 = 2 \times \frac{26!}{4!\,22!} = 29900$$

Example: Choosing a cricket team.

+ Select eleven from 17 players only 5 can bowl, requirement is cricket team of 11 must include exactly 4 bowlers? How many ways can the selection be done?

17 → 5 bowl (4) B, 12 no bowl (7 NB)

choose 11 → exactly 4 bowlers

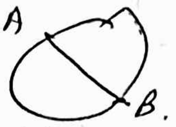
$$^5C_4 \; {}^{12}C_7 = 5 \times \frac{12!}{7!\,5!} = 3960 \text{ ways}$$

include exactly 4,

Example : Drawing lines in a circle

- Given n points on a circle, how many lines can be drawn connecting those points?

- $n = 2$ points, one line can be drawn


A B.

$2C_2 = \frac{2!}{2!0!} =$

line segment: AB

- $n = 3$ points three lines can be drawn


A B C

$\Rightarrow AB, AC, BC \quad 3C_2 = \frac{3!}{2!1!} = 3$

- In general, given n points, number of line segments that can be drawn connecting the point is $^nC_2$.

Section Summary :

① Notation & formula for selecting r objects from n objects $\quad {}^nC_r = \frac{n!}{r!(n-r)!}$

② Some useful combinatorial identities

Lecture 5.6 Permutations & Combinations Application

* Important to distinguish b/wn situations involving combinations & situations involving permutations

- Permutation – "order matters", Combination "Order doesn't matter"

$n = 3 \quad A \quad B \quad C$

$r = 2$

Permutation	Combination
AB, BA	AB
AC, CA	AC
BC, CB	BC

Example Finishing a race.

- 8 athletes 100m race

(1) How many diff ways can Gold, Silver, Bronze medals

A B C D E F G H

1 - A	D	Gold
2 - D	A	Silver
3 - F	F	Bronze

They are differe

∴ order matter

⇒ Permutation

(2) How many diff ways can you choose the top three athletes to proceed to the next round in the competition?

A B C

B C A

ABC → go to next round

So no order need. → (e) combination

Soln

(i) Order important, Hence permutation

$n = 8$, $r = 3$ ⇒ $^8P_3 = \frac{8!}{5!} = 8 \times 7 \times 6 = 336$ ways

(ii) No. of order not important ∴ Combination

$n = 8$, $r = 3$, $^8C_3 = \frac{8!}{5!\,3!} = \frac{336}{6} = 56$

$$\boxed{\begin{array}{l} 56 \times r! = {}^nP_r \\ 56 \times 3! = 336 \end{array}}$$ → verified

Example: Selecting a team:

- 40 students

(1) Choose two leaders?

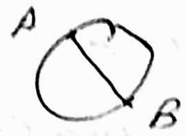
(2) Choose Captain & Vice captain → having order

Soln (1) $^{40}C_2 = 780$ ways

(2) $^{40}P_2 = 1560$

Example Draw lines in a circle:

- n points on a circle, how many lines can be drawn connecting those points?

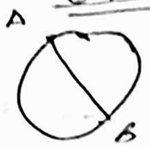


$^nC_2 \to$ given n points, we can join using $^nC_2$, total no. of lines drawn through n points. The lie on a circle.

eg: If A & B were two location on this point and, going from A to B, to B to A, which was a different case.

$\overline{AB}$, $\overline{BA}$ → direction matters (i.e.) order matters

Combination



AB = BA

no order

⇓

$n = {}^nC_2$

Permutation

Order matters

$^nC_2 \times 2! = {}^nP_2$

↓

because every line is giving two directed line

Section Summary:

- Dist. permutation & combination
  - Eg. of situations where permutation is applied, combination is applied.

1. Basic principle of counting: Addition, Multiplication
2. Factorial $(n!)$ → Simplified Expression
3. Permutation ↳ Distinct objects 'r' (repeat & non repeats)
   ↳ Object not distinct. $^nP_r = \frac{n!}{(n-r)!}$
4. Combination $^nC_r = \frac{n!}{r!(n-r)!}$

   $\boxed{^nC_r \cdot r! = {}^nP_r}$
5. Distinguish Permutation (or) Combination

week-6

Lec-6.1 Probability Basic definitions

Objectives

1. Uncertainty & random experiment
2. sample spaces, events of random experiments
3. Simple event & compound events.
4. Basic laws of probability
5. Probability of events & use of tree diagram to compute probabilities
6. conditional probability, i.e., find probability of an event given another event has occurred.
7. Distinguish independent & dependent events

Random Experiment _ Sample Space_ Event

* Venn diagram

Conditional Probability & Bayes Theorem

Intro

eg:
- => 50% chance india will win a toss
- => May guess is 'A' is the right choice
- => 30%. chance of rain tomorrow
- => Party ABC will probably win the next election

uncertainty
↓
Not sure of outcome

-> we routinely see or hear claims as the ones mentioned above.

-> To draw valid inference about a population from a sample one needs to know how likely it is that certain event will occur under various circumstances

Discriptive —Probability→ Inferential

drawn

can join

only lines

to that

Random experiment

Def: experiment

Any process that produces an observation or outcome

eg: A B

Controlled experiment
Add sugar
&
Same output

* Toss coin (A & B)

Cannot expect same outcome.

* Random experiment:

experiment whose outcome is not predictable with certainty (Toss coin)

Remark

Although outcome not known in advance, let us suppose that the set of all possible outcomes is known

Eg: of Random experiment

(i) * Guess answer to a 4 options MCQ

outcome: A B C D

(ii) * Order of finish in race with 6 students.

A, B, C, D, E, F

All possible permutation of ABCDEF.

1 A
2 B
3 C
4 D
5 E
6 F

(iii) Tossing 2 coins & noting the outcomes

outcome: HH, HT, TH, TT

(iv) Ex: measure lifetime (in hours) of a bulb

Outcome 0 (or) 1 hour 2 (or) 3 hr ... & so on

(v) Throwing a dart on a sq. unit & note the point where it lands.

O/C: Any point in the square (Assuming the dart lands within the square)

(i) ABCD → 4 choices ⇒ discrete
(ii) 1, 2, 3. .... → unique but infinite
(v) point is countable but many.

Sample Space:
Def: (Ω (or, S)) → collection of all basic outcomes
① Mutually exclusive: only one basic possible outcome that can occur must be
② exhaustive: One basic outcome can occur must occur

Die, S = {1, 2, 3, 4, 5, 6} → die
S = {H, T} → Coin

eg: of sample space:
set of all outcomes:
(i) S = {A, B, C, D}
(ii) Order of finish in a race with 6 students
A, B, C, D, E, F
S = {A-F, ABDCFE, ... 36! permutations

(iii) Two coins:
S = {HH, HT, TH, TT}
(iv) Measuring lifetime (hours) of a bulb:
S = {x: 0 ≤ x ≤ ~~∞~~}
(v) Dart throw on a unit square & not where it lands.

Section Summary
① Random experiment
② Sample space:
S = {Basic outcomes of a Random experiment}

Lecture - 6.2 Probability Events

Events:
Def: → a collection of basic outcomes.
→ an event is a subset of sample space
→ an event has occured if the outcome is in subset

Toss: $S = \{H, T\}$

Roll a die $S = \{1, 2, 3, 4, 5, 6\}$ → collection

Basic outcome: what I have when I roll a die.

$E = \{1, 3, 5\}$, $E = \{2, 4, 6\}$

subset of total sample space

collection of ~~total~~ basic outcomes.

Toss a twice, $S = \{HH, TT, HT, TH\}$.

$B = \{HH\}$.

event also.

Eg:

(i) Guessing answers to a 4 MCQ.

Ans: A; $E = \{A\}$

(ii) Order of finish in a race with 6 students -

A, B, C, D, E, F

Event: "A" finish the race first

$E = \{ABCDEF, ADCDFE, ABDCFE \dots\}$.

$\boxed{A}$ $\underbrace{\boxed{B}\boxed{C}\boxed{D}\boxed{E}\boxed{F}}_{5!\text{ ways.}}$

fixed

(iii) Tossing two coins & noting the outcomes

event: Head on the first toss $E = \{HH, HT\}$

(iv) Measure lifetimes

event: lifetime is less than (or) equal to four hours

$E = \{x : 0 \leq x \leq 4\}$

Event → $\boxed{\text{SET}}$

Set operations ← union, intersection, Complement

Union of events:

S = Sample space

$E, F \subseteq S$

these are event.

→ For any 2 events, E and F, we define the new event $E \cup F$ called the union of events E and F, to consist of all outcomes that are in E or in F or in both E and F.

$\underbrace{E \cup F}_{\text{event}}$ = all outcomes in E (or) F (or) Both

→ The event $E \cup F$ will occur if either E or F occurs

Eg: Union of events.

→ Experiment: Guessing answers to a four option multiple choice question:

Event:

→ answer is A; $E_1 = \{A\}$

" " B; $E_2 = \{B\}$

" " A or B; $E_3 = E_1 = E_1 \cup E_2 = \{A, B\}$.

→ Experiment: Order of finish in a race with 6 students. A, B, C, D, E, F.

→ A finishes first

$E_1 = \{ABC\ldots,\ ABCDFE\ldots\ldots\ AFEDBC\}$ 5!

→ B comes second

$E_2 = \{ABCDEF,\ CBADEF\ \ldots\ldots\}$ 5!

→ A comes first or B comes second.

$E_1 \cup E_2 =$

$\{A\,BCDEF,\ ABCDFE,\ ABDCFE\ldots\ \ldots\}$

Here B comes first, Not included.

Ex:

→ Tossing 2 coins and noting the outcomes.

→ head on first $E_1 = \{HH, HT\}$

→ head on second toss $E_2 = \{HH, TH\}$

→ head on 1st or 2nd toss $E_1 \cup E_2 = \{HH, HT, TH\}$

Intersection of events:

→ For given, E & F, $E \cap F$ consist of all outcomes that are in E and F.

→ $E \cap F$ if both E and F occurs.

Eg: Order of finish in race 6 students

→ A finish first

$F_1 = \{ABCDEF, ABCDFE, \ldots\}$

→ B comes second.

$E_2 = \{A, B, CD, EF; ABCDFE, \ldots\}$

→ A comes first and B comes second

$E_1 \cap E_2 = \{ABCDEF, ABCDFE, ABDCFE, \ldots ABDCFE\}$

4!

Experiment : Tossing 2 coins and noting the outcomes event.

(i) head on the first toss $E_1 = \{HH, HT\}$

(ii) " " second " $E_2 = \{HH, HT\}$

(iii) head on both toss $E_1 \cap E_2 = \{HH\}$

Null event & disjoint event:

$S = \{HH, HT, TH, TH\}$

E_1 → Head in first toss

E_2 → ~~Head in~~ Tail in first toss.

} cannot happen together

$S = \{A, B, C, D\}$ → MCQ, If only one option is correct we can't have 2 at a time

Def: Null event

An event without any outcomes the null event and designate it as ϕ.

Def: disjoint:

If intersection of E and F is the null event, then since E and F cannot simultaneously occurs, we say that E and F are disjoint or mutually exclusive

$$E \cap F = \phi.$$

eg: $S = \{H, T\}$

$E = \{H\}$

$F = \{T\}$

$E \cap F = \phi$. → If I get Head, we can't get tail and vice versa.

Eg: of Null event

Experiment: Guessing answers to a 4 option MCQ.

Event.

→ Answer is A: $E_1 = \{A\}$

" " B : $E_2 = \{B\}$

" " A & B : $E_3 = E_1 \cap E_2 = \phi$.

- we say events E_1 & E_2 are mutually exclusive (or disjoint). Occurence of E_1 disallows occurence of E_2. In other words if my A (B) is my guess then B (A) cannot be my guess.

Complement of an event:-

$S = \{H, T\}$.

$E_1 = \{H\}$ $\quad S \setminus E = \{T\} \text{(or)} = E^c$

Def: Consist of all outcomes in sample spaces that are not in E.

Toss a coin , $S = \{HH, TH, HT, TT\}$

Atleast one is Head. $E = \{HH, HT, TH\}$ → atleast one is a tail.

$E^c = \{TT\}$ → Both are Tail

Ex: Toss a coin once

$S = \{H, T\}$

E_1 → head $E_1 = \{H\}$

E_2 → tail $E_2 = \{T\}$

Event E_2 is complement of E_1 i.e., $E_2 = E_1^c$

* Tossing 2 coins & noting the outcomes

→ Sample space $S = \{HH, HT, TH, TT\}$

→ Event: Head on first toss $E_1 = \{HH, HT\}$

$E_1^c = \{TT, TH\}$.

Note: → E^c will occur if and only if E does not occur.

→ The complement of sample space is the null set, that is $S^c = \phi$.

Subsets.

Given two events E & F, if all outcomes in E are also in F, Then we say E is contained in F or E is a subset of F & $E \subset F$.

Ex: Tossing Two coins & noting the outcomes

$S = \{HH, TH, HT, TT\}$

$F = \{HH, HT\}$ → head first toss

$E = \{HH\}$ → head in both.

$E \subset F$

Lec- 6.3 venn diagram.

deck = 52 cards

Club → 4 clubs → 4 suit × 13 cards = 52.

Suit → 13 cards.

face cards → J, Q, K.

Exper.: Randomly select one card from deck. of 52 cards

Sample Space S = { collection of all 52 cards }.

Application: playing cards contd.

⇒ Describe event → card selected is king of hearts

$F = \{ K♡ \}$

⇒ E → card selected is a king.

$F = \{ 1 \times 4 \}$

⇒ E → card selected is a heart

$G = \{ 1 \times 13 \} = 13$ outcomes.

⇒ $F \cup G$ → either heart or king.

= 16 possible outcomes.

⇒ A king & A Heart.

F → K

G → Heart

$F \cap G$ only one card. = E.

Playing cards :

⇒ Let H be event of selecting an Ace. Ace c

E → A heart king.

F → a King

G → a heart

$F \cap G$ ⇒ a king and a heart

= E

H → selecting an Ace. (4)

H = { Ace club
A. heart
A - diamond
A - clover }

H = { A spade A club. A Heart A. diamond }

G → having a heart.

H → {(Ace) . . .

G → {(A ♡) 2 ♡. . . }

H & G are not mutually exclusive

* G & H →

I → {select a Queen} }
F → {king ♡, k spade . . . } } No common outcome ∴ ~~not~~ mutually exclusive

Venn diagram :

A graphical representation that is useful for illustrating logical relations among events in the venn diagram

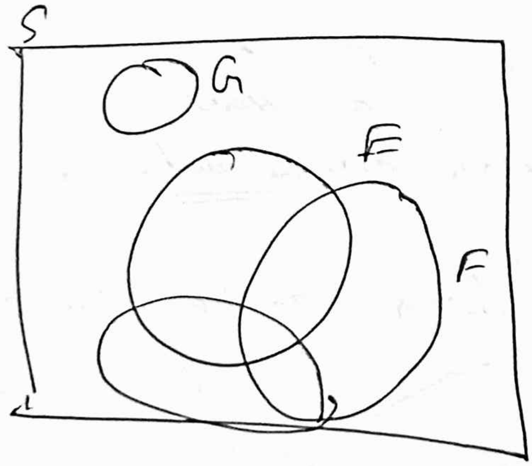
Sample space :

Event E: Union, Intersection, Complement

Representation of Sample Space :

consists of all possible outcomes and is represented by a large rectangle

* event is a subset of S.



S = {HH, HT, TH, TT}. } same
S = {HH, TH, TT, HT}.

E = {HH, HT}.

F → {TH, HH}.

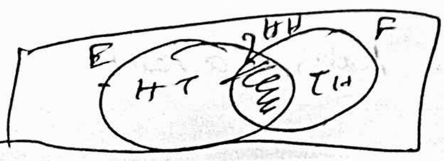
G = {TH, TT}

Representation of event: union & intersection :

⇒ Representation of event:

⇒ $E \cup F$ is entire shaded region

⇒ $E \cap F$ is the shaded in blue region



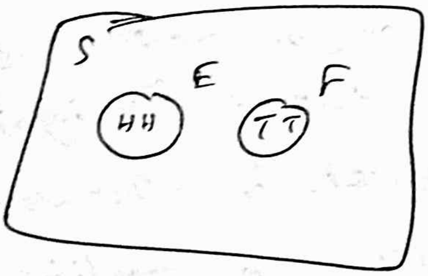
S = {HH, TH, HT, TT}
E = {HH, HT} F = {TH, HH}.

disjoint events:

$S = \{HH, THHT; TT\}$

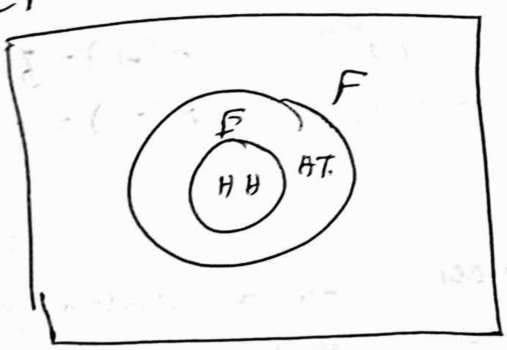
$E = \{HH\}$

$F = \{TT\}$



Subsets:

$E \subset F$



$S = \{HH, HT, TH, TT\}$

$F = \{HH, HT\}$

$E = \{HH\}$.

set of all possible basic outcomes of R.E. $E \subseteq S.E$ ϕ

Topic Summary:
1. Introduced random experiment, sample space, event
2. Notion of union, Intersection, complement of events.
3. Representation of sample space, events, using venn diagram.

L6.4 Properties of probability:

3 Main interpretation of probability:

classical approach (Apriori (or) theoretical): Let S be the sample space of a random experiment in which there are n equally likely outcomes, and the event E consists of exactly m of these outcomes, then we say the probability of the Event E is m/n and represent it as $P(E) = \frac{m}{n}$

6 sided dice (1-6)
Fair coin
H T

Roll a die.
$S = \{1, 2, 3, 4, 5, 6\}$
Event, E = an even number = $\{2, 4, 6\}$. no. of outcomes in the event are 3.

So, the probability of $S = \{1, 2, 3, 4, 5, 6\}$ $E = \{2, 4, 6\}$.

$P(E) = P(\text{rolling an even number}) = \frac{m}{n} = \frac{3}{6} = \frac{1}{2}$.

This approach assumes all the outcomes are equally likely.

(2) Relative frequency (A posteriori or, empirical) : The probability of an event in an experiment is the proportion (or fraction) of times the event occurs in a very long (theoretically infinite) series of independent repetitions of experiment. In other words if $n(E)$ is the no. of times E occurs in n repetitions of the experiment, $P(E) = \lim_{n \to \infty} \frac{n(E)}{n}$

eg: toss a coin.

Trial	1	2	3	4	. . .					10
output	H	H	T	H	T	T	H	T	T	H

$n(H) = 5$

$n(T) = 5$

(3) Subjective → assigns a "best guess" by a person, making the statement of the chances, that the event will happen. The probability measures an individual's degree of belief in the event

Probability Axioms :

Consider S. Suppose that for each event E, there is a number, denoted $P(E)$ and called the probability event E, that is in accord with the following three properties (Axioms)

Sample Space - S

$E \subset S$

|

$P(E)$

(i) For any event E, the probability of E is a number between 0 and 1. (ie) $0 \le P(E) \le 1$.

(ii) The probability of $S = 1$. $P(S) = 1$. (ie) outcome of any random exp will be an element of sample 'S' with probability 1.

(iii) For a sequence of mutually exclusive (disjoint) events

E_1, E_2

$$\boxed{P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)}$$

Probability of union of disjoint events

The third property can be stated as:

The Probability of union of disjoint events is equal to the sum of the probability of these events.

eg: E_1 & E_2 are disjoint.

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

In other words if E_1 & E_2 cannot occur simultaneously, then the probability that the outcome of the experiment is contained in either of E_1 or E_2 is equal to sum of probability that it is in E_1 and the probability that it is in E_2

eg: $S = \{1, 2, 3, 4, 5, 6\}$ → die

$$\left.\begin{array}{l} E_1 \to \text{odd} = \{1, 3, 5\} \\ E_2 \to \text{even} = \{2, 4, 6\} \end{array}\right\} \text{disjoint}$$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) = P(S) = 1$$

General properties of probability:

Properties 1, 2, 3 can be used to establish some general results concerning probabilities

(1) Probability of complement of an event: $P(E^c) = 1 - P(E)$

⇒ (1) E & E^c → Disjoint / mutually exclusive

⇒ (2) $E \cup E^c = S$

⇒ (3) $P(E \cup E^c) = P(E) + P(E^c)$ → Axiom 3.

RHS - $P(S) = 1$

$$P(E) + P(E^c) = 1$$
$$P(E^c) = 1 - P(E)$$

① Probability of complement of an event: $P(E^c) = 1 - P(E)$

E and E^c are disjoint. Also $E \cup E^c = S$

⇒ E and E^c are disjoint.

⇒ Apply property 3 to LHS $P(E \cup E^c) = P(E) + P(E^c)$

⇒ Apply property 2 to RHS $P(S) = 1$.

→ Equating both we get

$$P(E \cup E^c) = P(E) + P(E^c) = P(S) = 1$$

Hence $P(E^c) = 1 - P(E)$.

② ϕ - Null event → no outcome

$P(\phi) = 0$.

→ $S^c = \phi$.

→ Apply the above property, $P(S^c) = 1 - P(S)$

⇒ Apply property 2; $P(S) = 1$

⇒ Hence $P(\phi) = 0$.

<u>Addition rule of Probability:</u>

The following formula relates to probability of the union of events E_1 and E_2 which are not necessarily disjoint to $P(E_1)$, $P(E_2)$ and the probability of intersection of E_1 and E_2 → It is called as addition rule of probability.

For events E_1 & E_2

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2).$$

eg: E_1 = King of ♡ - 1

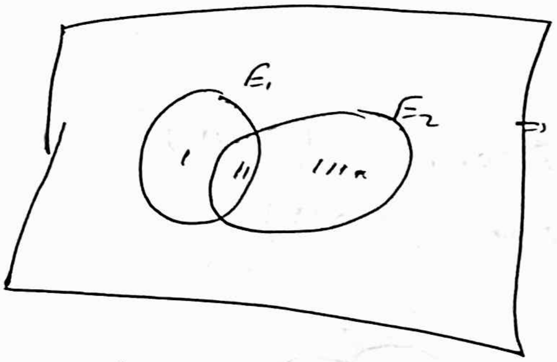
E_2 = hearts → 13

E_3 = King - 4.

what is P, if I have either a heart or a king. (not exclusive)

Proof of addition rule:

S



$\Rightarrow E_1 \cup E_2 = I \cup II \cup III$

$E_1 = I \cup II$

$E_2 = II \cup III$

$E_1 \cap E_2 = II$

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$E_1 \cup E = \underbrace{I \cup II \cup III}$$

3 disjoint sets

$$E_1 = I \cup II$$

$$P(E_1 \cup E_2) = P(I \cup II \cup III) \rightarrow P(I) + P(II) + P(III) \rightarrow (1)$$

$$P(E_1) = P(I \cup II) \rightarrow P(I) + P(II). \rightarrow (2)$$

$$P(E_2) = P(II \cup III) \rightarrow P(II + P(III) \rightarrow (3)$$

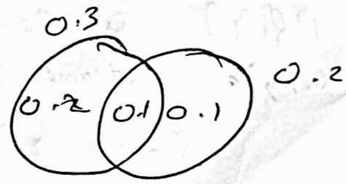
$$P(E_1 \cap E_2) = P(II) \longrightarrow (4).$$

$$P(III) = P(E_2 - P(II)$$

$$P(E_1 \cup E_2) = P(E_1 + P(E_2) - P(E_1 \cap E_2)$$

Lec. 6.5 – Probability – Applications

eg: shopping shirts & pants.



shirt – 0.3 = $P(S)$

pant = 0.2 = $P(P)$

both = 0.1 = $P(S \cap P)$

∴ neither a shirt nor a pant? $(S \cup P)^c$ $P(S \cup P)' = 1 - 0.4 = 0.6.$

either a pant or shirt = $P(S \cup P)$.

$$P(S \cup P) = P(S) + P(P) - P(S \cap P) = 0.3 + 0.2 - 0.1 = 0.4$$

$$P(S \cup P)^c = 1 - 0.4 = \boxed{0.6}$$

eg: Subject grades

$P(S) = 0.4$

$P(M) = 0.6$.

$\cancel{S \cap M} = 0.86$

$P(S \cup M)$

(i) Does not receive A in S (or) M. $= (S \cup M)^c$

(ii) Receive A in both S & M $= (S \cap M)$

Ans:

$S \cup M = 0.4 + 0.6 - S \cap M$

$\cancel{0.86} =$

$S \cap M = 0.4 + 0.6 - 0.86$

(ii) $S \cap M = 0.14$ →

(i) 0.14

L-6.6 → Equally likely outcomes:

+ it is natural to assume that each outcome in the sample space S is equally likely to occur.

+ S → consists of N outcomes, say $S = \{1, 2, \dots N\}$.

$\{1\}$ $\{2\}$ ... $\{N\}$

$S = \{H, T\}$

$\{H\}$ $\{T\}$.

$S = \{1, 2, 3, 4, 5, 6\}$

$\{1\}$ $\{2\}$ $\{3\}$ $\{4\}$ $\{5\}$ $\{6\}$

+ $P(\{i\})$ is the probability of event consisting of the single outcome i.

$S = \{1, 2\}$ $E_1 = \{1\}$ $E_2 = \{2\}$

$E_1 \cup E_2 = S$ $P(S) = 1$. (Axiom-2)

$P(E_1) = P(E_2)$ → Equally likely outcomes

Axiom - (3)

$P(E_1) \cup P(E_2) = P(E_1) + P(E_2)$.

$= P(E) + P(E) = 2P(E)$

$2P(E) = 1$

$P(E) = 1/2$

$P(E_1) = P(E_2) = 1/2$

$E_1 = \{1\}, \quad E_2 = \{2\} \quad E_3 = \{3\}$

$P(E) = 1/3.$

$P(E_1) = P(E_2) = P(E_3) = 1/3.$

$\Rightarrow \cancel{P(E)} \quad \boxed{P(E) = 1/n}$

$A = \{1, 2, 3\} \qquad P(A)$

$A = \{1\} \cup \{2\} \cup \{3\}$

$P(A) = P[\{1\}] + P\{2\} + P(\{3\}) \quad \cup \{5\}$

$= 1/N + 1/N + 1/N = 3/N. + 1/N = \frac{4}{N}.$

$\Rightarrow$ foregoing implies that the probability of any event A is equal to the proportion of the outcomes in the sample space that is in A.

(ie) $P(A) = \frac{\text{No. of outcomes in } S \text{ that are in } A}{N}$

eg: Rolling a dice.

→ Experiment: Roll a fair dice.

⇒ Sample Space; $S = \{1, 2, 3, 4, 5, 6\}$ → any one of these outcomes are equally likely to happen.

$P(E_i) = 1/6 \qquad P(\{1\}) = P(\{2\}) = P(\{6\}) = 1/6.$

→ Define A to be the event the outcome is odd $A = \{1, 3, 5\}.$

$P(A) = 3/6 = 1/2$

$P(A) = P(E_1) + P(E_2) + P(E_5) = 1/6 + 1/6 + 1/6 = 1/2, \quad \frac{n(A)}{N} = \frac{3}{6} = \frac{1}{2}$

$A = E_1 \cup E_2 \cup E_3$

$P(A) = 1/2$

⇒ Let B be the event that the outcome is greater than 4.

B = {5, 6}.

A = {1, 3, 5}
B = {5, 6}

$P(B) = (E_5 \cup E_6) P = 1/6 + 1/6 = 2/6 = \frac{1}{3}$.

$\frac{n(B)}{N} = \frac{2}{6} = 1/3$.

⇒ Let C be event that the outcome is either odd or greater than 4.

$P(C) = P(A \cup B) = P(A) + P(B) - P(A \cap B) = 3/6 + 2/6 - 1/6 = 4/6$

C = Either odd or > 4. (Not mutually exclusive)

$A \cup B$ = {1, 3, 5}
{5, 6}

Eg: Playing cards:

Probability card is either red or a queen?

S = 52, each is equally likely. = 1/52.

R → Red = 26 ⇒ $P(R) = \frac{26}{52} = 1/2$

Q → $P(Q) = 4/52 = 1/13$.

$P(R \cup Q)$ Not mutually exclusive

$= P(R) + P(Q) - P(R \cap Q)$

$= \frac{26}{52} + \frac{4}{52} - \frac{2}{52}$

$= \frac{28}{52} = \frac{7}{13}$

~~Eg~~ Section Summary:

(1) Interpretation of probability — Classical, Frequency, Subjective → when outcomes "Equally likely".

(2) Probability axioms — $0 \le P(E) \le 1$, $P(S) = 1$, $P(\cup) = \sum_{i=1}^{\infty} P(E)$ — Not necessarily disjoint

(3) Addition rule of probability.

$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$. (E).