Course: Machine learning Foundations

Solve with Instructor

WEEK 3
Fundamental spaces, projections, least squares

IIT Madras
ONLINE DEGREE

Problem 1

For a matrix of size $m \times n$ with $n > m$, there can be n pivots.

1. True
2. False

Problem 2

What will be the column spaces of these particular matrices?

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$$

a) C(A) is a line

b) C(A) is a plane

c) C(B) is a line

d) C(B) is a plane

1. (a) and (c)
2. (a) and (d)
3. (b) and (c)
4. (b) and (d)

Problem 3

Find the nullspaces of A, B, C

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} A \\ 2A \end{bmatrix} \qquad \boldsymbol{C} = \begin{bmatrix} A & 2A \end{bmatrix}$$

Problem 4

What is the reduced row echelon form of the matrix **A**?

$$A = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 2 \\ 2 & 3 & 1 & 5 \end{bmatrix}$$

a) $\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Problem 5

If a 7 by 9 matrix **A** has rank 5, what are the dimensions of the four subspaces [dim(C(**A**)), dim(R(**A**)), dim(N(**A**)), dim(N($\mathbf{A}^T$))]?

1. [5,5,4,2]
2. [5,5,2,4]
3. [7,7,4,2]
4. [7,7,2,4]

Problem 6

For a matrix $\mathbf{A}$ of size m by n,

(a) $\mathbf{A}$ and $\mathbf{A}^{\mathbf{T}}$ have the same number of pivots.

(b) $\mathbf{A}$ and $\mathbf{A}^{\mathbf{T}}$ have the same nullspace.

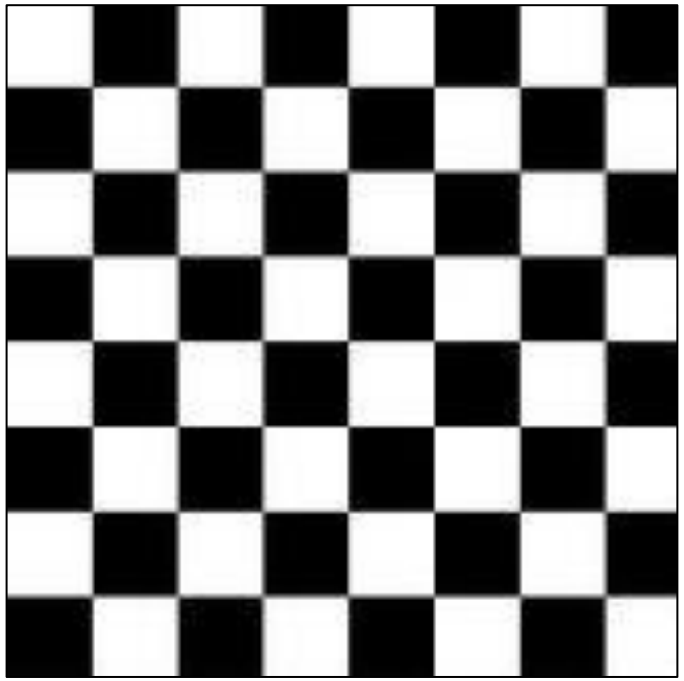
Choose the correct option.

1. (a) – True (b) – False
2. (a) – False (b) – True
3. (a) – True (b) – True
4. (a) – False (b) – False

Problem 7

Find the rank of the 8 by 8 checkerboard matrix.

(0 – Black and 1 – White)



Problem 8

$$A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix}$$

Can we express $b = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$ as a linear combination of the columns of matrix **A**?

Problem 9

Imagine it's a clear day and the Sun is shining down upon the Earth.

Assume the vector $\vec{v} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ is the ground and the vector $\vec{u} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is a stick with one endpoint on the ground and one endpoint in the air. If the shadow of the stick is cast onto the ground, find the length of the shadow.

Problem 10

These pairs of vectors are independent. Are they orthogonal or orthonormal?

a) $\begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$ and $\begin{bmatrix} 0.4 \\ -0.3 \end{bmatrix}$

b) $\begin{bmatrix} cos\theta \\ sin\theta \end{bmatrix}$ and $\begin{bmatrix} -sin\theta \\ cos\theta \end{bmatrix}$

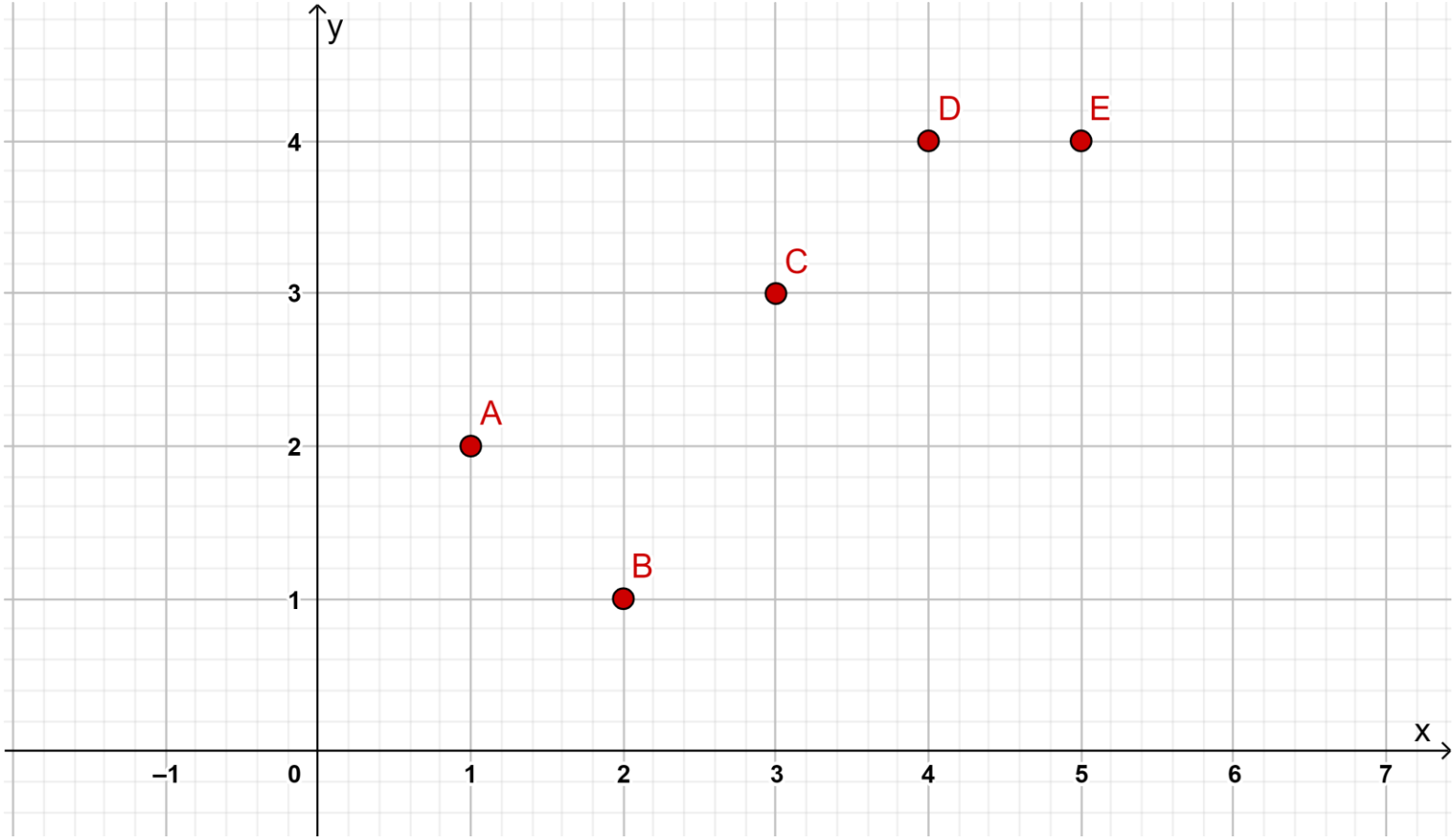
(Type your choice for a and b)

Problem 11

Find the projection matrix for $a = \begin{bmatrix} -1 \\ 3 \\ -2 \\ 1 \end{bmatrix}$ and use it to obtain the projection of $b = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ onto a and compute the length of error vector, e.

Problem 12

How can you represent these points in the 2-D space?



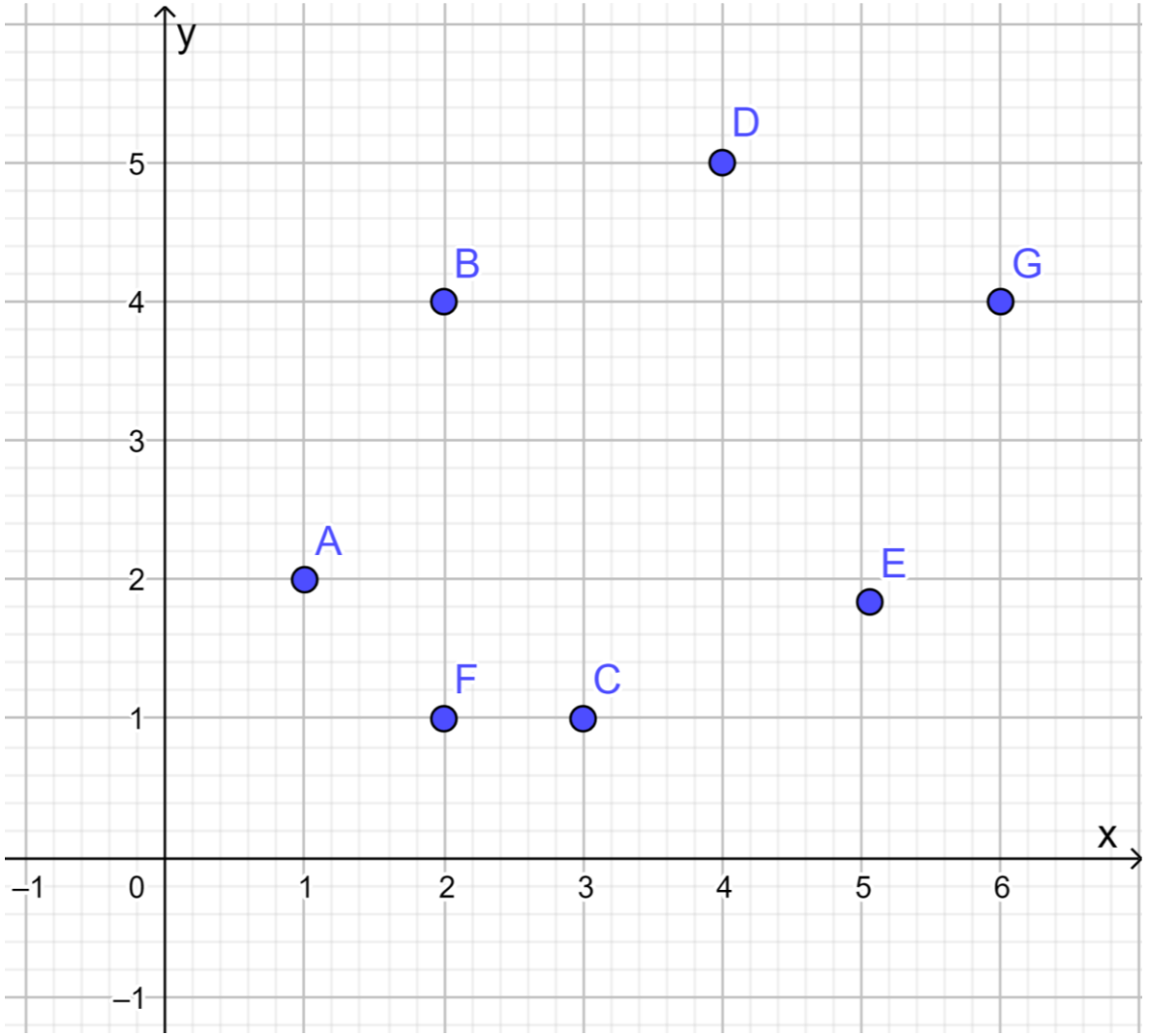
Problem 13

Consider two lines:

(a) A line with zero slope and y-intercept of 2.5.

(b) A line with zero slope and y-intercept of 3.5.

Which of these lines give the minimal error to fit the data points?



Thank You