PRACTICE PROBLEMS FOR APPLIED PROBABILITY AND STATISTICS

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Exercise Sheets

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EXERCISE 1

BASIC CONCEPTS OF PROBABILITY

1. Two events A and B have the following probabilities: $P[A] = 0.5$, $P[B] = 0.6$, and $P[\bar{A} \cap \bar{B}] = 0.25$. Find the value of $P[A \cap B]$.

2. Two events A and B have the following probabilities: $P[A] = 0.4$, $P[B] = 0.5$, and $P[A \cap B] = 0.3$. Calculate the following:
(a) $P[A \cup B]$
(b) $P[A \cap \bar{B}]$
(c) $P[\bar{A} \cup \bar{B}]$

3. If P(A) = 0.3, P (B) = 0.5 and P($A \cup B$) = 0.6, calculate: **(a)** P($A \cap B$) **(b)** P(B | A).

4. Suppose that A and B are mutually exclusive events for which P(A) = 0.3, P(B) = 0.5. What is the probability that
(a) either A and B occurs?
(b) A occurs but B does not?
(c) both A and B occur?

5. A dish contains 8 red jellybeans, 5 yellow jellybeans, 3 black jellybeans, and 4 pink jellybeans. If a jellybean is selected at random, find the probability that it is
(a) A red jellybean
(b) A black or pink jellybean
(c) Not yellow
(d) An orange jellybean.

6. A card is selected at random from a deck of 52 cards. Find the probability that it is a 6 or a diamond.

7. The probability that a student owns a computer is 0.92, and the probability that a student owns an automobile is 0.53. If the probability that a student owns both a computer and an automobile is 0.49, find the probability that the student owns a computer or an automobile.

8. A statistics class for engineers consists of 25 industrial, 10 mechanical, 10 electrical, and 8 civil engineering students. If a person is randomly selected by the instructor to answer a question, find the probability that the student chosen is **(a)** an industrial engineering major and **(b)** a civil engineering or an electrical engineering major.

9. A card is drawn at random from an ordinary deck of 52 playing cards. Find the probability that it is **(a)** an ace, **(b)** a jack of hearts, **(c)** a three of clubs or a six of diamonds, **(d)** a heart, **(e)** any suit except hearts, **(f)** a ten or a spade, **(g)** neither a four nor a club.

10. Sixty percent of the students at a certain school wear neither a ring nor a necklace. Twenty percent wear a ring and 30 percent wear a necklace. If one of the students is chosen randomly, what is the probability that this student is wearing
(a) a ring or a necklace?

(b) a ring and a necklace?

11. Ms. Perez figures that there is a 30 percent chance that her company will set up a branch office in Phoenix. If it does, she is 60 percent certain that she will be made manager of this new operation. What is the probability that Perez will be a Phoenix branch office manager?

12. A bag contains eight red balls, four green balls, and eight yellow balls. A ball is drawn at random from the bag, and it is not a red ball. What is the probability that it is a green ball?

13. Of a group of 50 Year 11 students, 32 study Art and 30 study Graphics. Each student studies at least one of these subjects.

(a) What is the probability that a randomly selected student studies Art only?

(b) Find the probability that a student selected at random from the group studies Graphics, given that the student studies Art.

14. A batch of 100 manufactured components is checked by an inspector who examines 10 components selected at random. If none of the 10 components is defective, the inspector accepts the whole batch. Otherwise, the batch is subjected to further inspection. What is the probability that a batch containing 10 defective components will be accepted?

15. The Applied Probability professor gave the class a set of 12 review problems and told them that the midterm exam would consist of 6 of the 12 problems selected at random. If Lidya memorized the solutions to 8 of the 12 problems but could not solve any of the other 4 problems, what is the probability that she got 4 or more problems correct in the exam?

16. One bag contains 4 white balls and 3 black balls, and a second bag contains 3 white balls and 5 black balls. One ball is drawn from the first bag and placed unseen in the second bag. What is the probability that a ball now drawn from the second bag is black?

17. A small town has one fire engine and one ambulance available for emergencies. The probability that the fire engine is available when needed is 0.98, and the probability that the ambulance is available when called is 0.92. In the event of an injury resulting from a burning building, find the probability that both the ambulance and the fire engine will be available, assuming they operate independently.

18. A university has twice as many undergraduate students as graduate students. Twenty five percent of the graduate students live on campus, and10% of the undergraduate students live on campus.

(a) If a student is chosen at random from the student population, what is the probability that the student is an undergraduate student living on campus?

(b) If a student living on campus is chosen at random, what is the probability that the student is a graduate student?

19. A certain manufacturer produces cars at two factories labeled A and B. Ten percent of the cars produced at factory A are found to be defective, while 5% of the cars produced at factory B are defective. If factory A produces 100,000 cars per year and factory B produces 50,000 cars per year, compute the following:

(a) The probability of purchasing a defective car from the manufacturer

(b) If a car purchased from the manufacturer is defective, what is the probability that it came from factory A?

20. A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 30%, 20%, and 50% of the products, respectively. The defect rate is different for the three procedures as follows:

$P(D|P_1) = 0.01$, $P(D|P_2) = 0.03$, $P(D|P_3) = 0.02$,

where $P(D|P_j)$ is the probability of a defective product, given plan j. If a random product was observed and found to be defective, which plan was most likely used and thus responsible?

21. A company producing electric relays has three manufacturing plants producing 50, 30, and 20 percent, respectively, of its product. Suppose that the probabilities that a relay manufactured by these plants is defective are 0.02, 0.05, and 0.01, respectively.

(a) If a relay is selected at random from the output of the company, what is the probability that it is defective?

(b) If a relay selected at random is found to be defective, what is the probability that it was manufactured by plant 2?

22. Stores A, B, C have 50, 75, and 100 employees, and, respectively, 50, 60, and 70 percent of these are women. Resignations are equally likely among all employees, regardless of sex. One employee resigns and this is a woman. What is the probability that she works in store C?

Answers

1.	0.35	**2.**	0.6; 0.1; 0.7	**3.**	0.2; 0.667
4.	0.8; 0.3; 0	**5.**	0.4; 0.35; 0.75; 0	**6.**	4/13
7.	0.96	**8.**	25/53; 18/53	**9.**	1/13; 1/26; 1/4; 3/4; 4/13; 9/13
10.	0.4; 0.1	**11.**	0.18	**12.**	0.3333
13.	0.4; 0.375	**14.**	0.33048	**15.**	8/11
16.	38/63	**17.**	0.9016	**18.**	0.067; 0.556
19.	0.0833; 0.8	**20.**	0.019; 0.1579; 0.3158; 0.5263	**21.**	0.027; 0.556
22.	0.622; 0.5				

EXERCISE 2

RANDOM VARIABLES

1. Let the random variable K denote the number of heads in four flips of a fair coin.

(a) Plot the graph of $p_K(k)$.
(b) What is $P[K \geq 3]$?
(c) What is $P[2 \leq K \leq 4]$?

2. A shipment of 20 similar laptop computers to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these computers, find the probability distribution for the number of defectives.

3. A student got a summer job at a bank, and his assignment was to model the number of customers who arrive at the bank. The student observed that the number of customers K that arrive over a given hour had the PMF

$$p_K(k) = \begin{cases} \dfrac{\lambda^k e^{-\lambda}}{k!} & k = 0, 1, 2, \dots \dots . \\ 0 & otherwise \end{cases}$$

(a) Show that $p_K(k)$ is a proper PMF.
(b) What is $P[K > 1]$?
(c) What is $P[2 \leq K \leq 4]$?

4. Consider the function

$$g(x) = \begin{cases} c & a \leq x \leq b \\ 0 & otherwise \end{cases}$$

(a) For what value of c is $g(x)$ a legitimate PDF?
(b) Find the CDF of the random variable X with the above PDF.

5. Consider the function

$$f(x) = \begin{cases} 2x & 0 \leq x \leq b \\ 0 & otherwise \end{cases}$$

(a) For what value of b is $f(x)$ a legitimate PDF?
(b) Find the CDF of the random variable X with the above PDF.

6. A random variable X has the following PDF, where K > 0:

$$f_X(x) = \begin{cases} 0 & x < 1 \\ K(x-1) & 1 \leq x \leq 2 \\ K(3-x) & 2 \leq x \leq 3 \\ 0 & x > 3 \end{cases}$$

(a) What is the value of K?
(b) Sketch $f_X(x)$.

(c) What is the CDF of X?
(d) What is $P[1 \le X \le 2]$?

7. The PDF of a random variable X is given by

$$f_X(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 \le x \le 2 \\ 0 & otherwise \end{cases}$$

(a) Find the CDF of X.
(b) Find $P[0.2 < X < 0.8]$.
(c) Find $P[0.6 < X < 1.2]$.

8. A random variable X has the density function $f(x) = c/(x^2 + 1)$, where $-\infty < x < \infty$. **(*a*)** Find the value of the constant c. **(b)** Find the probability that X^2 lies between 1/3 and 1. **(c)** Find the distribution function corresponding to the density function.

9. The distribution function for a random variable X is

$$F_X(x) = \begin{cases} 1 - e^{-2x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

Find **(a)** the density function, **(b)** the probability that $X > 2$, and **(c)** the probability that $-3 < X \le 4$.

10. A random variable X has the CDF

$$F_X(x) = \begin{cases} 0 & x < -1 \\ A(1 + x) & -1 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

(a) What is the value of A?
(b) With the above value of A, what is $P[X > 1/4]$?
(c) With the above value of A, what is $P[-0.5 \le X \le 0.5]$?

11. In a lottery there are 200 prizes of \$5, 20 prizes of \$25, and 5 prizes of \$100. Assuming that 10,000 tickets are to be issued and sold, what is a fair price to pay for a ticket?

12. Find the expected value and variance of a discrete random variable K whose PMF is given by

$$p_K(k) = \frac{5^k e^{-5}}{k!} \qquad k = 0, 1, 2, \dots \dots$$

13. A discrete random variable has probability function $f(x) = 1/2^x$ where $x = 1, 2, \dots$. Find **(a)** the mode, **(b)** the median, and **(c)** compare them with the mean.

14. A continuous random variable X has probability density given by

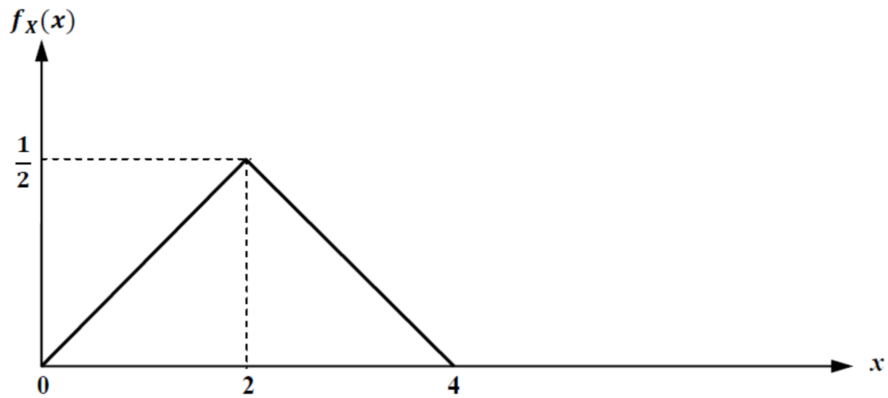
$$f_X(x) = \begin{cases} 2e^{-2x} & x > 0 \\ 0 & x \le 0 \end{cases}$$

(a) Find (i) $E(X)$, (ii) $E(X^2)$.
(b) Find (i) the variance, (ii) the standard deviation for the random variable of X.

15. Suppose the random variable X has the PDF $f_X(x) = ax^3$, $0 < x < 1$.

(a) What is the value of a?
(b) What is the expected value of X?
(c) What is the variance of X?
(d) What is the value of m so that $P[X \leq m] = 1/2$?

16. Find the mean and variance of the random variable whose triangular PDF is given in **Figure**.



17. A random variable X has the CDF

$$F_X(x) = \begin{cases} 0 & x < 1 \\ 0.5(x-1) & 1 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

(a) What is the PDF of X?
(b) What is the expected value of X?
(c) What is the variance of X?

18. The PDF of a random variable X is given by $f_X(x) = 4x\,(9 - x^2)\,/81$, $0 \leq x \leq 3$. Find the mean, variance, and third moment of X.

19. The PDF of a random variable X is given by

$$f(x) = \begin{cases} 0.1 & 30 \leq x \leq 40 \\ 0 & otherwise \end{cases}$$

Find the conditional expected value of X, given that $X \leq 35$.

20. The density function of a continuous random variable X is

$$f(x) = \begin{cases} 4x(1-x^2) & 0 \leq x \leq 1 \\ 0 & otherwise \end{cases}$$

(a) Find the mode. **(b)** Find the median. **(c)** Compare mode, median, and mean.

Answers

1. 5/16; 11/16 **2.** 68/95; 51/190; 3/190

3. $1 - [e^{-\lambda}(1+\lambda)]; e^{-\lambda}\left(\frac{\lambda^2}{2} + \frac{\lambda^3}{6} + \frac{\lambda^4}{24}\right)$ **4.** $\frac{1}{b-a}; \frac{x-a}{b-a}$

5. $1; x^2$ **6.** $1; \frac{x^2}{2} - x + \frac{1}{2}; 3x - \frac{x^2}{2} - \frac{7}{2}; 1/2$ **7.** $\frac{x^2}{2}; 2x - \frac{x^2}{2} - 1; 0.3; 0.5$

8.	$\frac{1}{\pi}; \frac{1}{6}$	**9.**	$2e^{-2x}$; 0.018316; 0.99966	**10.**	$\frac{1}{2}$; 0.375; 0.5
11.	$0.20	**12.**	5; 5	**13.**	2; 1; 1.5
14.	$\frac{1}{2}; \frac{1}{2}; \frac{1}{4}; \frac{1}{2}$	**15.**	4; $\frac{4}{5}; \frac{2}{75}$; 0.8409	**16.**	2; 0.667
17.	0.5; 2; 0.3333	**18.**	8/5; 11/25; 215/35	**19.**	32.5
20.	0.5774; 0.5412; 0.533				

EXERCISE 3

JOINT RANDOM VARIABLES

1. A fair coin is tossed four times. Let X denote the number of heads obtained in the first two tosses, and let Y denote the number of heads obtained in the last two tosses.

(a) Find the joint PMF of X and Y.

(b) Show that X and Y are independent random variables.

2. The joint PMF of two random variables X and Y is given by

$$p_{xy}(x,y) = \begin{cases} k(2x+y) & x = 1,2; y = 1,2 \\ 0 & otherwise \end{cases}$$

where k is a constant.

(a) What is the value of k?

(b) Find the marginal PMFs of X and Y.

(c) Are X and Y independent?

3. The joint probability function of two discrete random variables X and Y is given by $f(x, y) = c(2x + y)$, where x and y can assume all integers such that $0 \leq x \leq 2$, $0 \leq y \leq 3$, and $f(x, y) = 0$ otherwise.

(a) Find the value of the constant c.

(b) Find $P(X = 2, Y = 1)$.

(c) Find $P(X \geq 1, Y \leq 2)$.

(d) Find the marginal probability functions (i) of X and (ii) of Y for the random variables.

(e) Show that the random variables X and Y are dependent.

4. Two discrete random variables X and Y have the joint PMF given by

$$p_{xy}(x,y) = \begin{cases} 0.2 & x = 1, y = 1 \\ 0.1 & x = 1, y = 2 \\ 0.1 & x = 2, y = 1 \\ 0.2 & x = 2, y = 2 \\ 0.1 & x = 3, y = 1 \\ 0.3 & x = 3, y = 2 \end{cases}$$

Determine the following:

(a) the marginal PMFs of X and Y (i.e., $p_X(x)$ and $p_Y(y)$)

(b) the conditional PMF of X given Y, $p_{X|Y}(x|y)$

(c) whether X and Y are independent.

5. The joint CDF of two discrete random variables X and Y is given as follows:

$$F_{xy}(x,y) = \begin{cases} \frac{1}{8} & x = 1, y = 1 \\ \frac{5}{8} & x = 1, y = 2 \\ \frac{1}{4} & x = 2, y = 1 \\ 1 & x = 2, y = 2 \end{cases}$$

Determine the following:

(a) Joint PMF of *X* and *Y*

(b) Marginal PMF of *X*

(c) Marginal PMF of *Y*.

6. Determine if random variables *X* and *Y* are independent when their joint PDF is given by

$$f_{xy}(x,y) = \begin{cases} 2e^{-(x+y)} & 0 \le x \le y; 0 \le y < \infty \\ 0 & otherwise \end{cases}$$

7. A privately owned business operates both a drive-in facility and a walk-in facility. On a randomly selected day, let *X* and *Y* , respectively, be the proportions of the time that the drive-in and the walk-in facilities are in use, and suppose that the joint density function of these random variables is

$$f_{xy}(x,y) = \begin{cases} \frac{2}{5}(2x+3y) & 0 \le x \le 1; 0 \le y \le 1 \\ 0 & otherwise \end{cases}$$

(a) Verify $f(x, y)$ is legitimate joint pdf .

(b) Find $P[(X, Y) \in A]$, where $A = \{(x, y) \mid 0 < x < 1/2 , 1/4 < y < 1/2\}$.

8. Two random variables *X* and *Y* have the joint PDF given by

$$f_{xy}(x,y) = \begin{cases} ke^{-(2x+3y)} & x \ge 0; y \ge 0 \\ 0 & otherwise \end{cases}$$

Determine the following:

(a) the value of the constant *k* that makes $f_{XY}(x, y)$ a true joint PDF

(b) the marginal PDFs of *X* and *Y*

(c) $P[X < 1, Y < 0.5]$.

9. Two random variables *X* and *Y* have the joint PDF given by

$$f_{xy}(x,y) = \begin{cases} k(1-x^2y) & 0 \le x \le 1; 0 \le y \le 1 \\ 0 & otherwise \end{cases}$$

Determine the following:

(a) the value of the constant *k* that makes $f_{XY}(x, y)$ a true joint PDF

(b) the conditional PDFs of *X* given *Y*, $f_{X|Y}(x|y)$, and *Y* given *X*, $f_{Y|X}(y|x)$.

10. The joint CDF of two continuous random variables *X* and *Y* is given by

$$F_{XY}(x,y) = \begin{cases} 1 - e^{-ax} - e^{-by} + e^{-(ax+by)} & x > 0; y > 0 \\ 0 & otherwise \end{cases}$$

(a) Find the marginal PDFs of X and Y

(b) Show why or why not X and Y are independent.

11. The joint density for the random variables (X, Y), where X is the unit temperature change and Y is the proportion of spectrum shift that a certain atomic particle produces, is

$$f_{xy}(x,y) = \begin{cases} 10xy^2 & 0 < x < y < 1 \\ 0 & otherwise \end{cases}$$

(a) the marginal PDFs of X and Y and the conditional density $f(y|x)$.

(b) Find the probability that the spectrum shifts more than half of the total observations, given that the temperature is increased by 0.25 unit.

12. Given the joint density function

$$f_{xy}(x,y) = \begin{cases} \frac{x}{4}(1+3y^2) & 0 < x < 2; 0 < y < 1 \\ 0 & otherwise \end{cases}$$

find $f_X(x)$ and $f_Y(y)$, $f(x|y)$, and evaluate $P(1/4 < X < 1/2 | Y = 1/3)$.

13. If X and Y have the joint density function

$$f_{xy}(x,y) = \begin{cases} \frac{3}{4} + xy & 0 < x < 1; 0 < y < 1 \\ 0 & otherwise \end{cases}$$

find $f(y|x)$.

14. Suppose that the random variables X and Y have a joint density function given by

$$f_{xy}(x,y) = \begin{cases} c(2x+y) & 2 \le x \le 6; 0 \le y \le 5 \\ 0 & otherwise \end{cases}$$

Find **(a)** the constant c, **(b)** the marginal distribution functions for X and Y, **(c)** the marginal density functions for X and Y, **(d)** $P(3 < X < 4, Y > 2)$, **(e)** $P(X > 3)$, **(f)** the joint distribution function, **(g)** whether X and Y are independent.

15. Two discrete random variables X and Y have the joint PMF given by

$$p_{xy}(x,y) = \begin{cases} 0 & x=-1, y=0 \\ \frac{1}{3} & x=-1, y=1 \\ \frac{1}{3} & x=0, y=0 \\ 0 & x=0, y=1 \\ 0 & x=1, y=0 \\ \frac{1}{3} & x=1, y=1 \end{cases}$$

(a) Are *X* and *Y* independent?
(b) What is the covariance of *X* and *Y*?

16. Two events A and B are such that P[A] = ¼, P[B|A] = ½ and P[A|B] = ¼. Let the random variable X be defined such that $X = 1$ if event A occurs and $X = 0$ if event A does not occur. Similarly, let the random variables Y be defined such that $Y = 1$ if event B occurs and $Y = 0$ if event B does not occur.

(a) Find $E[X]$ and the variance of X.
(b) Find $E[Y]$ and the variance of Y.
(c) Find ρ_{XY} and determine whether or not X and Y are uncorrelated.

17. Compute the conditional mean $E[X|Y = y]$ if the joint PDF of *X* and *Y* is given by

$$f_{xy}(x,y) = \begin{cases} \frac{e^{-(x/y)}e^{-y}}{y} & x \geq 0; y \geq 0 \\ 0 & otherwise \end{cases}$$

18. The joint PDF of the random variables *X* and *Y* is defined as follows:

$$f_{xy}(x,y) = \begin{cases} 25e^{-5y} & 0 \leq x \leq 0.2; y > 0 \\ 0 & otherwise \end{cases}$$

(a) Find the marginal PDFs of *X* and *Y*.
(b) What is the covariance of *X* and *Y*?

EXERCISE 4

SPECIAL PROBABILITY DISTRIBUTIONS

Binomial Distribution

1. Each sample of water has a 10% chance of containing a particular organic pollutant. Assume that the samples are independent with regard to the presence of the pollutant. Find the probability that in the next 18 samples, exactly 2 contain the pollutant.

2. An archer hits the bull's eye 80% of the time. If he shoots 5 arrows, find the probability that he will get 4 bull's eyes.

3. A homeowner has just installed 20 light bulbs in a new home. Suppose that each has a probability 0.2 of functioning more than three months. What is the probability that at least five of these function more than three months? What is the average number of bulbs the homeowner has to replace in three months?

4. The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that (a) at least 10 survive, (b) from 3 to 8 survive, and (c) exactly 5 survive?

5. Find the probability that in five tosses of a fair die, a 3 will appear (a) twice, (b) at most once, (c) at least two times.

6. Find the probability that in a family of 4 children there will be (a) at least 1 boy, (b) at least 1 boy and at least 1 girl. Assume that the probability of a male birth is 1/2.

7. If 20% of the bolts produced by a machine are defective, determine the probability that out of 4 bolts chosen at random, (a) 1, (b) 0, (c) less than 2, bolts will be defective.

Geometric Distribution

8. The probability that a wafer contains a large particle of contamination is 0.01. If it is assumed that the wafers are independent, what is the probability that exactly 125 wafers need to be analyzed before a large particle is detected?

9. A bag contains six blue balls and four red balls. Balls are randomly drawn from the bag, one at a time, until a red ball is obtained. If we assume that each drawn ball is replaced before the next one is drawn, what is the probability that the experiment stops after exactly five balls have been drawn?

Hypergeometric Distribution

10. A certain library has a collection of 10 books on probability theory. Six of these books were written by American authors and four were written by foreign authors.

(a) If I randomly select one of these books, what is the probability that it was written by an American author?

(b) If I select five of these books at random, what is the probability that two of them were written by American authors and three of them were written by foreign authors?

11. A box contains 6 blue marbles and 4 red marbles. An experiment is performed in which a marble is chosen at random and its color observed, but the marble is not replaced. Find the probability that after 5 trials of the experiment, 3 blue marbles will have been chosen.

Poisson Distribution

12. For the case of the thin copper wire, suppose that the number of flaws follows a Poisson distribution with a mean of 2.3 flaws per millimeter. Determine the probability of exactly 2 flaws in 1 millimeter of wire.

13. Messages arrive at a switchboard in a Poisson manner at an average rate of six per hour. Find the probability for each of the following events:

(a) Exactly two messages arrive within one hour.

(b) No message arrives within one hour.

(c) At least three messages arrive within one hour.

14. Ten is the average number of oil tankers arriving each day at a certain port. The facilities at the port can handle at most 15 tankers per day. What is the probability that on a given day tankers have to be turned away?

15. Ten percent of the tools produced in a certain manufacturing process turn out to be defective. Find the probability that in a sample of 10 tools chosen at random, exactly 2 will be defective.

Exponential Distribution

16. Suppose that a number of miles that a car can run before its battery wears out is exponentially distributed with an average value of 10,000 miles. If a person desires to take a 5,000-mile trip, what is the probability that she will be able to complete her trip without having to replace her car battery? What can be said when the distribution is not exponential?

17. Assume that the length of phone calls made at a particular telephone booth is exponentially distributed with a mean of 3 minutes. If you arrive at the telephone booth just as Chris was about to make a call, find the following:

(a) The probability that you will wait more than 5 minutes before Chris is done with the call.

(b) The probability that Chris' call will last between 2 minutes and 6 minutes.

18. The life of a particular brand of batteries is exponentially distributed with a mean of 4 weeks. You just replaced the battery in your gadget with the particular brand.

(a) What is the probability that the battery life exceeds 2 weeks?

(b) Given that the battery has lasted 6 weeks, what is the probability that it will last at least another 5 weeks?

Uniform Distribution

19. If X is uniformly distributed over the interval [0, 10], compute the probability that **(a)** $2 < X < 9$, **(b)** $1 < X < 4$, **(c)** $X < 5$, **(d)** $X > 6$.

20. Let the continuous random variable X denote the current measured in a thin copper wire in milliamperes. Assume that the range of X is [0, 20 mA], and assume that the probability density function of X is $f(x) = 0.04, \quad 0 \leq x \leq 20$. What is the probability that a measurement of current is between 5 and 10 milliamperes?

21. Suppose that a large conference room at a certain company can be reserved for no more than 4 hours. Both long and short conferences occur quite often. In fact, it can be assumed that the length X of a conference has a uniform distribution on the interval [0, 4].

(a) What is the probability density function?

(b) What is the probability that any given conference lasts at least 3 hours?

Normal Distribution

22. The average life of a certain brand of automobile tires is 24,000 miles under normal driving conditions. The standard deviation is 2000 miles, and the variable is approximately normally distributed. For a randomly selected tire, find the probability that it will last between 21,800 miles and 25,400 miles.

23. The average time it takes college freshmen to complete a reasoning skills test is 24 minutes. The standard deviation is 5 minutes. If a randomly selected freshman takes the exam, find the probability that he or she takes more than 32 minutes to complete the test. Assume the variable is normally distributed.

24. The scores on a national achievement exam are normally distributed with a mean of 500 and a standard deviation of 100. If a student is selected at random, find the probability that the student scored below 680.

25. Owing to many independent error sources, the length of a manufactured machine part is normally distributed with μ = 11 cm and σ = 2 cm. If specifications require that the length be between 10.6 cm and 11.2 cm, what proportion of the manufactured parts will be rejected on average?

26. A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assuming that the resistance follows a normal distribution and can be measured to any degree of accuracy, what percentage of resistors will have a resistance exceeding 43 ohms?

EXERCISE 5

APPLIED STATISTICS

1. A random sample of size 81 is taken from a population that has a mean of 24 and variance 324. Use the central limit theorem to determine the probability that the sample mean lies between 23.9 and 24.2.

2. According to the U.S. Department of Agriculture's World Livestock Situation, the country with the greatest per capita consumption of pork is Denmark. In 1994, the amount of pork consumed by a person residing in Denmark had a mean value of 147 pounds with a standard deviation of 62 pounds. If a random sample of 25 Danes is chosen, approximate the probability that the average amount of pork consumed by the members of this group in 1994 exceeded 150 pounds.

3. From past experience it is known that the weights of salmon grown at a commercial hatchery are normal with a mean that varies from season to season but with a standard deviation that remains fixed at 0.3 pounds. If we want to be 95 percent certain that our estimate of the present season's mean weight of a salmon is correct to within ± 0.1 pounds, how large a sample is needed?

4. A large number of light bulbs was turned on continuously to determine the average number of days a bulb can last. The study revealed that the average lifetime of a bulb is 120 days with a standard deviation of 10 days. If the lifetimes are assumed to be independent normal random variables, find the confidence limits for a confidence level of 90% on the sample mean that is computed from a sample size of (a) 100 and (b) 25.

5. A random sample of 50 of the 200 electrical engineering students' grades in applied probability showed a mean of 75% and a standard deviation of 10%.
   (a) What are the 95% confidence limits for the estimate of the mean of the 200 grades?
   (b) What are the 99% confidence limits for the estimate of the mean of the 200 grades?
   (c) With what confidence can we say that the mean of all the 200 grades is 75±1?

6. The mean of the grades of 36 freshmen is used to estimate the true average grade for the freshman class. If μ is the true mean, what is the probability that the estimate differs from the true mean by 3.6 marks if the standard deviation is known to be 24?

7. What is the increase in sample size that is required to increase the confidence level of a given confidence interval for a normal random variable from 90% to 99.9%?

8. The mean lifetime of a sample of 100 lightbulbs produced by Lighting Systems Corporation is computed to be 1570 hours with a standard deviation of 120 hours. If the president of the company claims that the mean lifetime $E[X]$ of all the lightbulbs produced by the company is

1600 hours, test the hypothesis that $E[X]$ is less than 1600 hours using a level of significance of (a) 0.05 and (b) 0.01.

9. A college provost claimed that 60% of the freshmen at his school receive their degrees within four years. A curious analyst followed the progress of a particular freshman class with 36 students and found that only 15 of the students received their degrees at the end of their fourth year. Determine whether this particular class performed worse than previous classes at a level of significance of (a) 0.05 and (b) 0.01.

10. An equipment manufacturing company claimed that at least 95% of the equipment it supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 of them did not meet the specifications. Determine whether the company's claim is legitimate at a level of significance of (a) 0.01 and (b) 0.05.

11. A company wants to know with a 95% level of confidence if it can claim that the boxes of detergent that it sells contain more than 500 grams of detergent each. From past experience the company knows that the amount of detergent in the boxes is normally distributed with a standard deviation of 75 grams. A worker takes a random sample of 100 boxes and finds that the average amount of detergent in a box is 510 grams. Should the company make the claim?

12. A government agency received many consumers' complaints that boxes of cereal sold by a company contain less than the advertised weight of 20 oz of cereal with a standard deviation of 5 oz. To check the consumers' complaints, the agency bought 36 boxes of the cereal and found that the average weight of cereal was 18 oz. If the amount of cereal in the boxes is normally distributed, test the consumers' complaint at the 95% level of confidence.

13. Data were collected for a random variable Y as a function of another random variable X. The recorded (x, y) pairs are as follows: (1, 11), (3, 12), (4, 14), (6, 15), (8, 17), (9, 18), and (11, 19).

(a) Plot the scatter diagram for these data.
(b) Find the linear regression line of y on x that best fits these data.
(c) Estimate the value of y when $x = 20$.

14. The ages x and systolic blood pressures y of 12 people are shown in the following table:

Age (x)	56	42	72	36	63	47	55	49	38	42	68	60
Blood Pressure (y)	147	125	160	118	149	128	150	145	115	140	152	155

(a) Find the linear least-squares regression line of y on x.
(b) Estimate the blood pressure of a person whose age is 45 years.

15. The following table shows a random sample of 12 couples who stated the number x of children they planned to have at the time of their marriage and the number y of actual children they have.

Couple	1	2	3	4	5	6	7	8	9	10	11	12
Planned Number of Children (x)	3	3	0	2	2	3	0	3	2	1	3	2
Actual Number of Children (y)	4	3	0	4	4	3	0	4	3	1	3	1

(a) Find the linear least-squares regression line of *y* on *x*.

(b) Estimate the number of children that a couple who had planned to have 5 children actually had.