INDIAN INSTITUTE OF TECHNOLOGY MADRAS

सिद्धिर्भवति कर्मजा

IIT Madras

ONLINE DEGREE

Statistics for Data Science - 1
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Week - 5 Tutorial - 2

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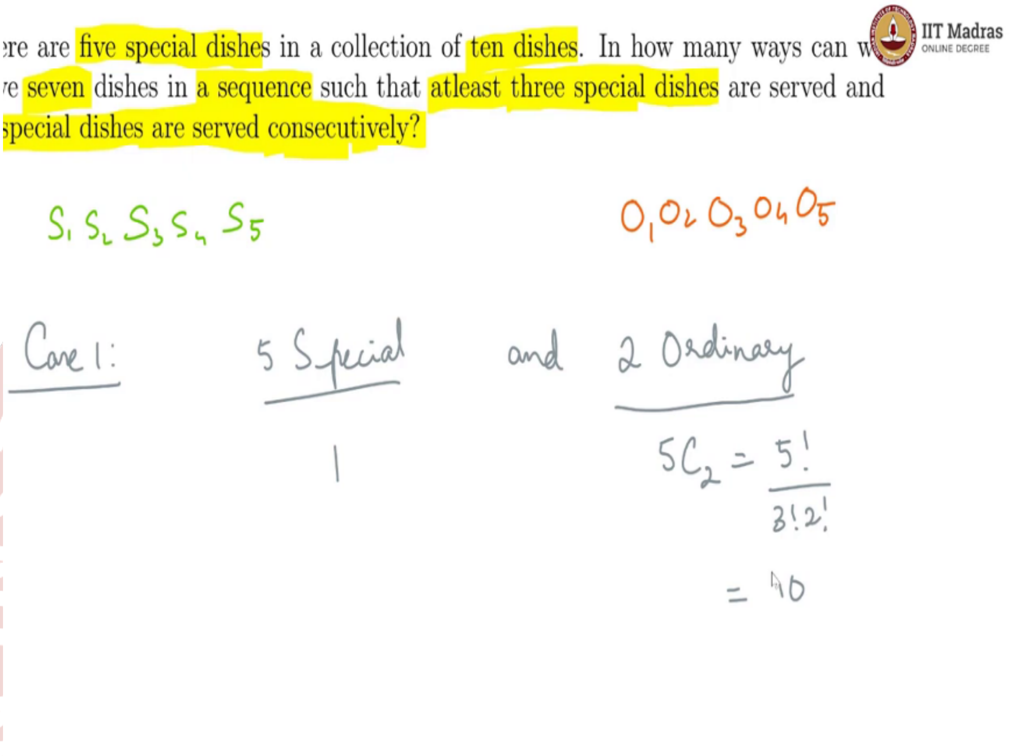

In this question, they are saying there are 5 special dishes in a collection of 10 dishes. So there are 5 special dishes, let us call them S_1, S_2, S_3, S_4, and S_5 as special. And there are ordinary dishes which are O_1, O_2, O_3, O_4, and O_5.

So these are the dishes and now, we have to serve these dishes; serve 7 dishes. Out of these 10, we have to pick 7 and serve them in a sequence. So we are looking at permutations, we are not looking at combinations. And they are saying at least 3; at least 3 special dishes should be served.

So of the 7, you can have 3 special dishes or 4 special dishes or all 5 special dishes. And all special dishes are served consecutively, that is, they are served together; there is no serving of an ordinary dish in the middle of the special dishes sequence.

So we consider the 3 cases which are case 1, let us consider the easiest one which is 5 special and 2 ordinary. Before we look at the permutations, the ordering, let us see how many ways we can pick these dishes in the first place without any sequencing. So there is only 1 way to pick the 5 special dishes because there are only 5 special dishes.

And here, you can do $^5C_2$ which is $\frac{5!}{2!\times 3!}$which gives us 10. So in 10 ways, we pick 2 ordinary dishes and in 1 way, we pick 5 special dishes.

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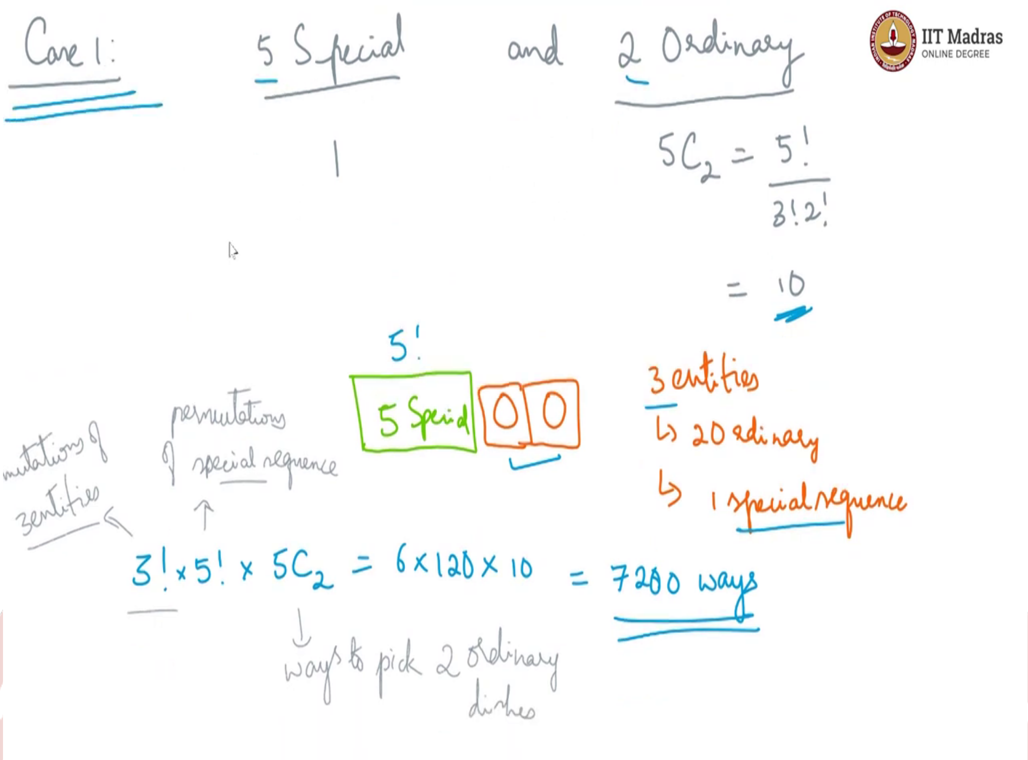

Now we have our 7 with us. However, these 5 put together, they make 1 block; 5 special. So other than that, we have 2 ordinaries. So in effect, it is like we are dealing with 3 entities which is basically 2 ordinary and the 1 sequence of 5 special dishes.

So now, these 3 entities can be permuted in 3! ways. And within them, our special sequence has 5! permutations within itself. So each of our 3 entity permutations gets 5! permutations. And then the picking of these 2 ordinary dishes happens in 10 ways. So we further multiply this by 10, which is basically ${}^5C_2$. And this should be our answer for the case 1.

So let us make sure we understand these terms. This 3! is to account for permutations of the 3 entities. And these 5 factorials are the permutations of the special sequence. So for every permutation of these 3 entities, you get 5! permutations of the special sequence. And for every such permutation of 3 entities, we have to pick, this is the number of ways to pick 2 ordinary dishes.

So for every such permutation, we have ${}^5C_2$ options of picking the ordinary dishes. So all put together, this is the answer we get for case 1, which gives us 3! is 6 into 5! is 120 into ${}^5C_2$ is 10. So we get 7200 ways. This is for case 1 where you have 5 special and 2 ordinary.

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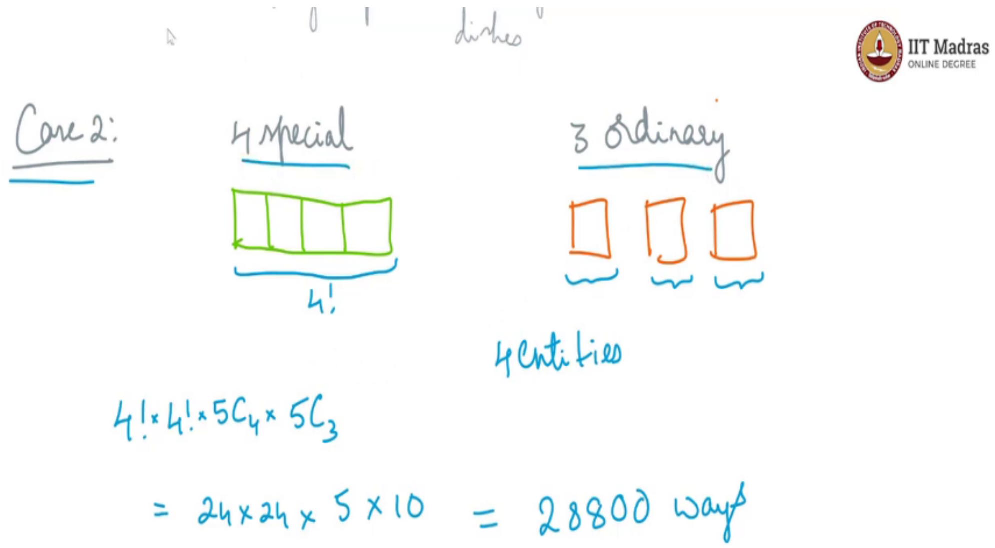

Now, let us look at case 2 which is 4 special. So these 4, we get in a sequence. And 3 ordinary, these we get as independent entities. So now, we have as, by our previous logic, we have 4 entities; these 4 special together is 1 entity, and then each of them is, each of the ordinaries is 1 entity.

So since you have 4 entities, we will have 4! permutations. So every permutation of our 4 entities also should account for the 4! permutations within the sequence. And now, we also have to account for how many ways we can pick 4 special items from 5. So we have ${}^{5}C_{4}$.

And how many ways we are picking 3 ordinary items from 5, which will give us ${}^{5}C_{3}$. So this is going to give us $24 \times 24 \times 5 \times 10$. So that is 28,800 ways for case 2, which is 4 special and 3 ordinary.

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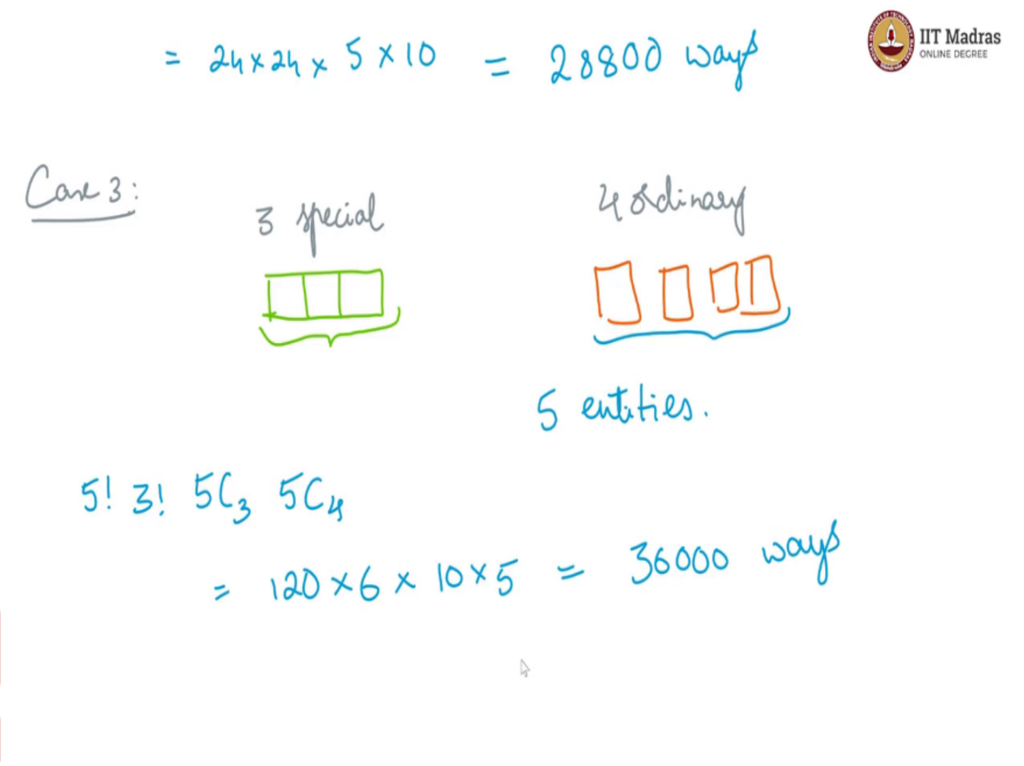

That leaves us with case 3, which is 3 special and 4 ordinary. So these 3 special, they come together like this as a special sequence, whereas the 4 ordinary gives us 4 independent entities.

So this time, we have 5 entities leading to 5! permutations of these entities. And again, each of these permutations will have to consider the permutations within the special sequence, therefore, that is 3!.

And now, we look at how many ways we can pick these 3 from 5. So that is ${}^5C_3$. And again, picking these ordinary ones from 5 that is ${}^5C_4$. So this will give us 120 × 6 × 5 × 10 which is equal to 36,000 ways.

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$= 120 \times 6 \times 10 \times 5 =$

$$
\begin{array}{r}
{\scriptstyle 2\ 1} \\
36000 \\
+\ 28800 \\
+\ \ 7200 \\
\hline
72000
\end{array}
\quad \underline{\text{ways}}
$$

So all put together, we are getting 36,000 for case 3, plus 28,800 for case 2, plus 7,200 for case 1. So that gives us 72,000 ways in all.