SWI—WEEK 11

1. The number of hours Messi spends each day practicing in ground is modelled by the continuous random variable X, with p.d.f. $f(x)$ defined by

$$f_X(x) = \begin{cases} a(x-1)(6-x) & \text{for } 1 < x < 6 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that Messi will practice between 3 and 5 hours in ground on a randomly selected day.

Solution:
We know that $\int_{-\infty}^{\infty} f(x)dx = 1$

Solving above equation taking required $f(x)$, value of a can be calculated. i.e $a = \dfrac{6}{125}$

Then calculate $P(2 \leq X \leq 5) = \int_2^5 f(x)dx$

2. The amount of time a student takes to solve a question is uniformly distributed with an average time of 12 minutes and variance $\frac{1}{3}$. Find the value of $P(10 < X < 12.5)$

Solution:

Solution:
Given random variable is uniformly distributed from $[a, b]$.

$$E[X] = \frac{b+a}{2} = 12 = b + a = 24$$

$$Var(X) = \frac{(b-a)^2}{12} = \frac{1}{3}$$

$$\Rightarrow (b-a)^2 = 4$$

$$\Rightarrow b - a = 2$$

Solving both equation, $a = 11$ and $b = 13$
Now, $P(10 < X < 12.5)$= Area under pdf between 10 to 12.5
Area $= \frac{1}{b-a} \times (12.5 - 10) = \frac{1}{2} \times (12.5 - 10) = 0.5 \times 1.5 = 0.75$

3. The number of days ahead travelers purchase their airline tickets is exponentially distributed with average amount of time equal to 28 days. If there is 80% chance that a traveler will purchase the ticket fewer than d days in advance, then what is the value of d? Write the answer to nearest digit integer.

Answer: 45

Solution :

Here for exponential distribution,

$E[X] = \frac{1}{\lambda} = 28$

$\Rightarrow \lambda = \frac{1}{28}$

Given, $P(X \leq d) = 0.8$

$\Rightarrow 1 - e^{-\lambda \times d} = 0.8$

$\Rightarrow e^{-\lambda \times d} = 0.2$

Taking log both sides,

$\Rightarrow -\lambda \times d = \ln 0.2$

Substituting the value of λ in above equation, $d = 45$

4. If the mean and variance of an exponential distribution are $\frac{1}{\lambda}$ and $\frac{1}{\lambda^2}$ respectively,then for which condition variance will be greater than the mean :

(a) $\lambda > 1$

(b) $0 < \lambda < 1$

(c) $\lambda = 1$

(d) None of the above.

Answer: b

Solution:

We know, for an exponential distribution,

$$\text{Var X} = \frac{1}{\lambda^2}$$

and

$$E[X] = \frac{1}{\lambda^2}$$

To show,

$$\text{Var}(X) > E[X]$$
$$\Rightarrow \frac{1}{\lambda^2} > \frac{1}{\lambda}$$

The above condition will be true whenever $0 < \lambda < 1$ happens. Hence option B is correct.

5. (1 point) The lifetime of a light bulb is exponentially distributed with a mean life of 18 months. If there are 60% chances that a light bulb will last at most t months, then what is the value of t?

1. $18 \ln 0.4$
2. $18 \ln 2.5$
3. $\frac{1}{18} \ln 0.4$
4. $\frac{1}{18} \ln 2.5$

Solution:

Given mean of exponential random variable (life of light bulb)=18 months.

$\Rightarrow \frac{1}{\lambda} = 18, \Rightarrow \lambda = \frac{1}{18}$.

Given $P(X \leq t) = 0.6$

$\Rightarrow 1 - e^{-\lambda t} = 0.6$

$\Rightarrow e^{-\lambda t} = 0.4$

$\lambda t = \ln 2.5$

$\frac{1}{18} t = \ln 2.5$

$t = 18 \ln 2.5$

Hence option b is correct.

6. (1 point) Let X be uniformly distributed over $[a, b]$ with $E[X] = 5$ and $E[X^2] = 28$, then what is the value of $b - a$?

Answer: 6

Solution:

Solution:

For Uniform distribution, we know

$E(X) = \frac{(b+a)}{2}$ and $E(X^2) = \frac{(b^2+a^2+ab)}{3}$

Given random variable is uniformly distributed from $[a, b]$.

$$V(X) = E[X^2] - (E[X])^2$$

$$Var(X) = \frac{(b-a)^2}{12} = 28 - 5 \times 5$$

$$\Rightarrow (b-a)^2 = 36$$

$$\Rightarrow b - a = 6$$

8. If X is a random variable with the expected value of 5 and the variance of 1, then the expected value of X^2 is

solution:

solution:

$Var(X) = 1, \; E(X) = 5$

$E(X^2) = Var(X) + (E(X))^2 = 1 + 5^2 \; E(X^2) = 26$

9. Let X and Y be continuous random variables with joint density

$$f_{XY}(x,y) \begin{cases} cxy & \text{for } 0 < x < 2,\ 1 < y < 3 \\ 0 & \text{otherwise} \end{cases}$$

Calculate the value of c

10. Suppose that random variable X is uniformly distributed between 0 and 10. Then find $P(X + \frac{10}{X} \geq 7)$. (Write answer upto two decimal places)

Answer: 0.7

Solve this quadratic equation, $X + \frac{10}{X} \geq 7$