

Contents

- ▶ What is a matrix?



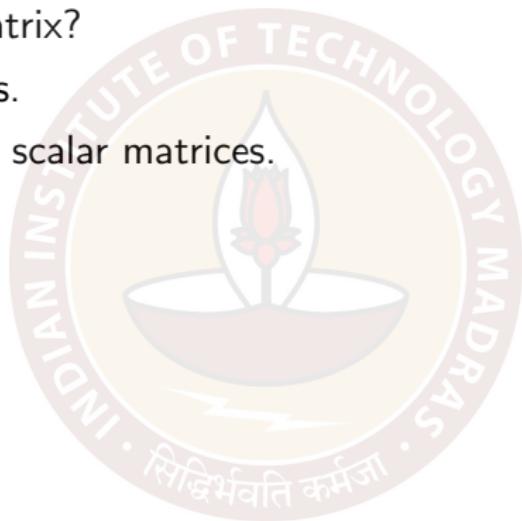
Contents

- ▶ What is a matrix?
- ▶ Related terms.



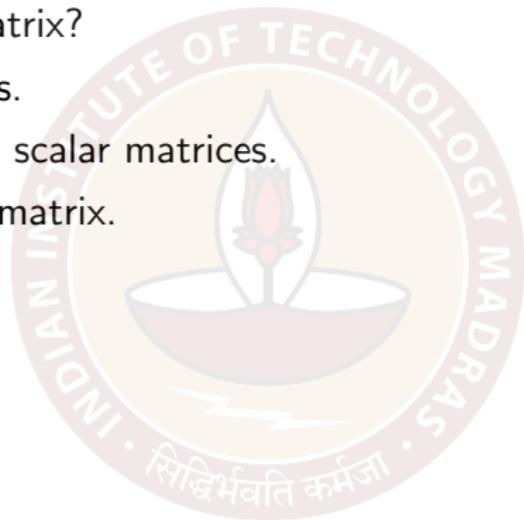
Contents

- ▶ What is a matrix?
- ▶ Related terms.
- ▶ Diagonal and scalar matrices.



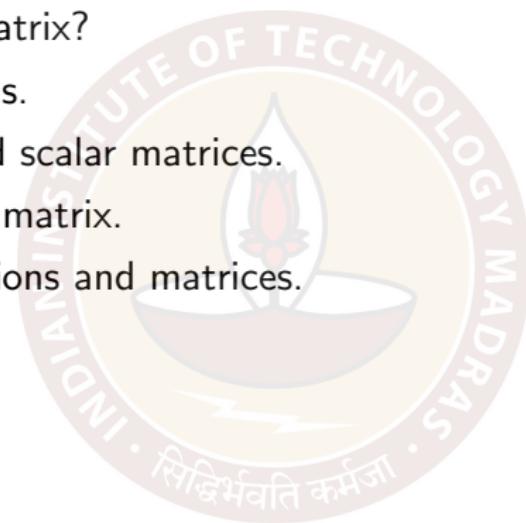
Contents

- ▶ What is a matrix?
- ▶ Related terms.
- ▶ Diagonal and scalar matrices.
- ▶ The identity matrix.



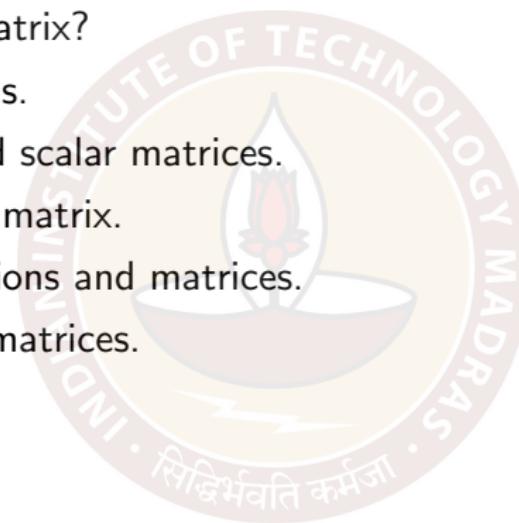
Contents

- ▶ What is a matrix?
- ▶ Related terms.
- ▶ Diagonal and scalar matrices.
- ▶ The identity matrix.
- ▶ Linear equations and matrices.



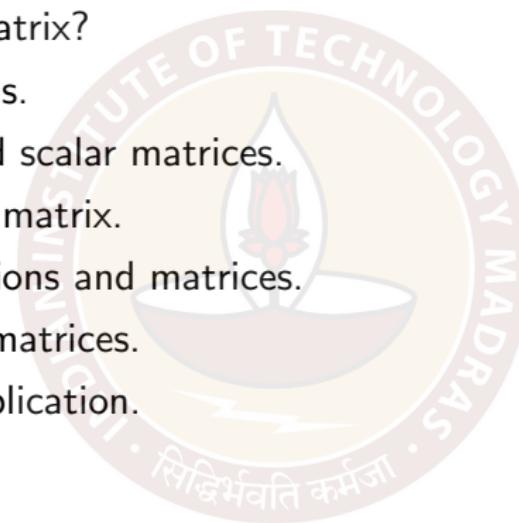
Contents

- ▶ What is a matrix?
- ▶ Related terms.
- ▶ Diagonal and scalar matrices.
- ▶ The identity matrix.
- ▶ Linear equations and matrices.
- ▶ Addition of matrices.



Contents

- ▶ What is a matrix?
- ▶ Related terms.
- ▶ Diagonal and scalar matrices.
- ▶ The identity matrix.
- ▶ Linear equations and matrices.
- ▶ Addition of matrices.
- ▶ Scalar multiplication.

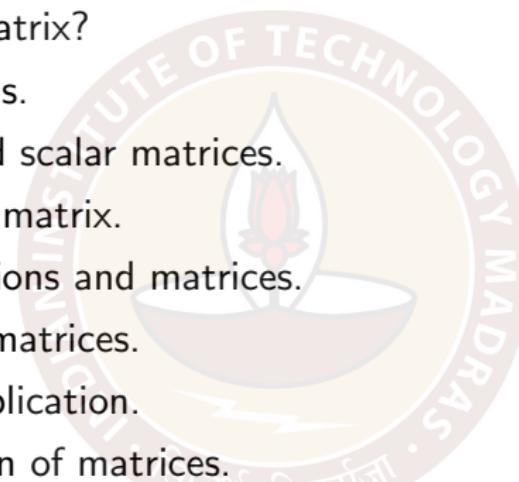


Contents

- ▶ What is a matrix?
- ▶ Related terms.
- ▶ Diagonal and scalar matrices.
- ▶ The identity matrix.
- ▶ Linear equations and matrices.
- ▶ Addition of matrices.
- ▶ Scalar multiplication.
- ▶ Multiplication of matrices.

Contents

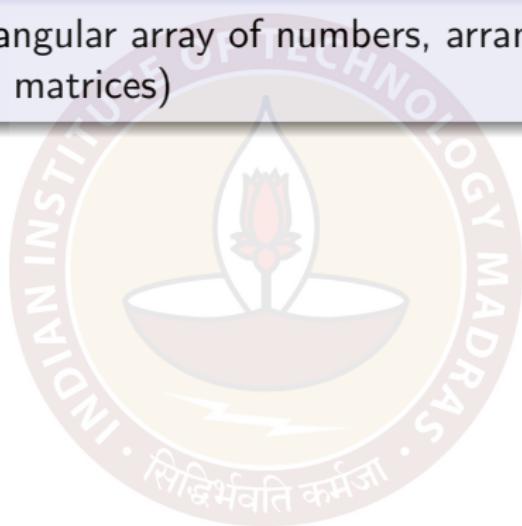
- ▶ What is a matrix?
- ▶ Related terms.
- ▶ Diagonal and scalar matrices.
- ▶ The identity matrix.
- ▶ Linear equations and matrices.
- ▶ Addition of matrices.
- ▶ Scalar multiplication.
- ▶ Multiplication of matrices.
- ▶ Properties of matrix addition and multiplication.



What is a matrix?

Definition

A matrix is a rectangular array of numbers, arranged in rows and columns. (plural : matrices)



What is a matrix?

Definition

A matrix is a rectangular array of numbers, arranged in rows and columns. (plural : matrices)

Example:
$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}_{2 \times 3}$$

This is a 2×3 matrix (2 rows and 3 columns).

What is a matrix?

Definition

A matrix is a rectangular array of numbers, arranged in rows and columns. (plural : matrices)

Example:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}_{2 \times 3}$$

This is a 2×3 matrix (2 rows and 3 columns).

- ▶ An $m \times n$ matrix has m rows and n columns.

Definition

A matrix is a rectangular array of numbers, arranged in rows and columns. (plural : matrices)

Example:

1	2	3
2	3	4

2×3

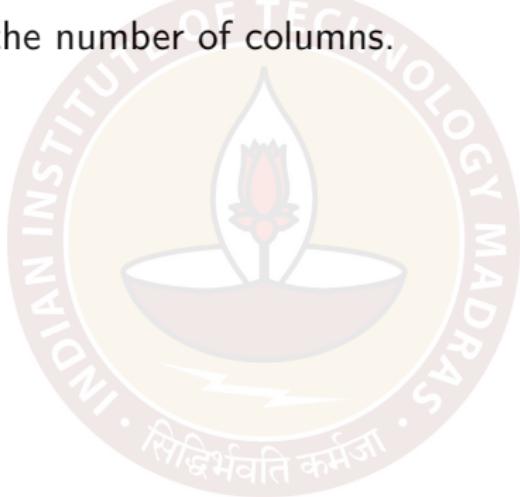
This is a 2×3 matrix (2 rows and 3 columns).

$(1, 2)$ -th entry of this matrix is 2 .
 $(2, 3)$ -th entry of " " is 4 .

- An $m \times n$ matrix has m rows and n columns.
- (i, j) -th entry of a matrix is the entry occurring in the i -th row and j -th column.

Square matrices

- ▶ A square matrix is a matrix in which the number of rows is the same as the number of columns.



- A square matrix is a matrix in which the number of rows is the same as the number of columns.

Example:

$$\begin{bmatrix} 0.3 & 0 & -7 \\ 2.8 & 0 & 1 \\ 0 & -2.5 & -1 \end{bmatrix}_{3 \times 3}$$

This is a 3×3 matrix (3 rows and 3 columns).

(2, 3)-th entry of this matrix is 1.

Square matrices

- ▶ A square matrix is a matrix in which the number of rows is the same as the number of columns.

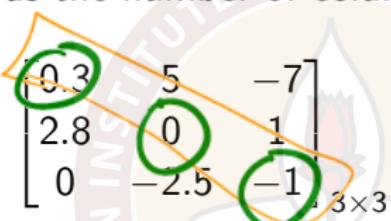
Example:
$$\begin{bmatrix} 0.3 & 5 & -7 \\ 2.8 & 0 & 1 \\ 0 & -2.5 & -1 \end{bmatrix}_{3 \times 3}$$

This is a 3×3 matrix (3 rows and 3 columns).

- ▶ The i -th diagonal entry of a square matrix is the (i, i) -th entry.

- ▶ A square matrix is a matrix in which the number of rows is the same as the number of columns.

Example:

$$\begin{bmatrix} 0.3 & 5 & -7 \\ 2.8 & 0 & 1 \\ 0 & -2.5 & -1 \end{bmatrix}_{3 \times 3}$$


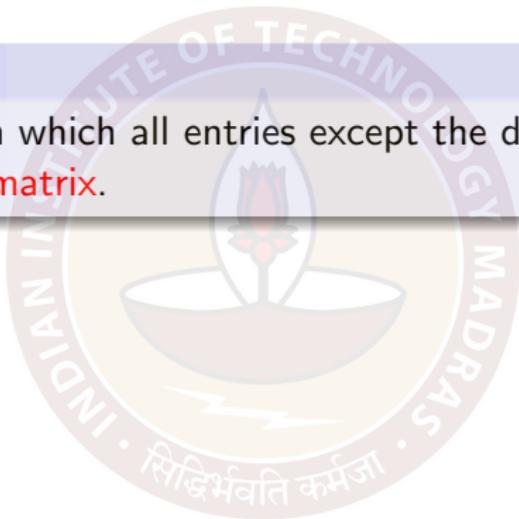
This is a 3×3 matrix (3 rows and 3 columns).

- ▶ The i -th diagonal entry of a square matrix is the (i, i) -th entry.
- ▶ The diagonal of a square matrix is the set of diagonal entries.

Diagonal Matrices

Definition

A square matrix in which all entries except the diagonal are 0 is called a **diagonal matrix**.



Definition

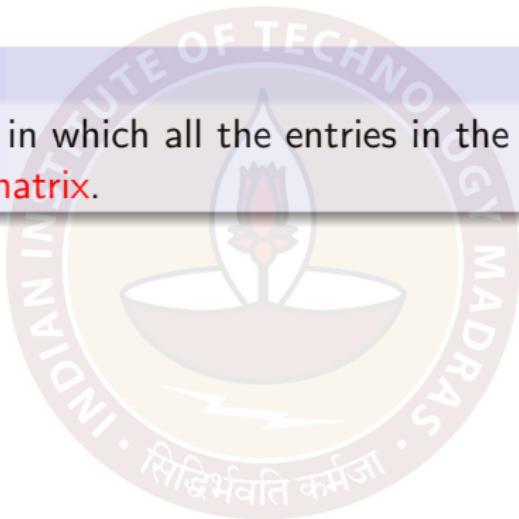
A square matrix in which all entries except the diagonal are 0 is called a **diagonal matrix**.

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 4.2 \end{bmatrix}$$

Scalar Matrices

Definition

A diagonal matrix in which all the entries in the diagonal are equal is called a **scalar matrix**.



Definition

A diagonal matrix in which all the entries in the diagonal are equal is called a **scalar matrix**.

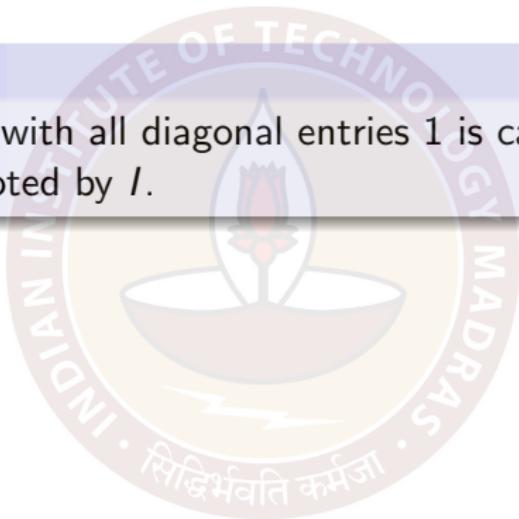
$$S = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

3 × 3

The identity matrix

Definition

The scalar matrix with all diagonal entries 1 is called the **identity matrix** and is denoted by I .



Definition

The scalar matrix with all diagonal entries 1 is called the **identity matrix** and is denoted by I .

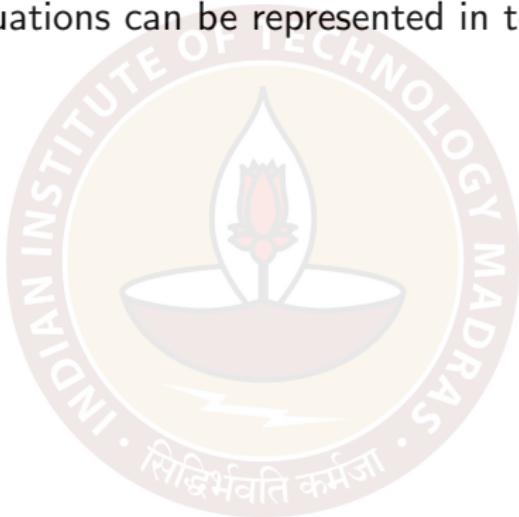
$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3×3

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Linear equations and matrices

A set of linear equations can be represented in terms of matrices.



A set of linear equations can be represented in terms of matrices.

Example

$$\begin{aligned}3x + 4y &= 5 \\4x + 6y &= 10\end{aligned}$$

can be represented by the matrix

$$\left[\begin{array}{cc|c} 3 & 4 & 5 \\ 4 & 6 & 10 \end{array} \right] \quad 2 \times 3$$

$$\left[\begin{array}{cc|c} 3 & 4 & 5 \\ 4 & 6 & 10 \end{array} \right]$$

Example:

$$\begin{bmatrix} 1 \\ 0.6 \\ 4 \end{bmatrix}_{3 \times 2} + \begin{bmatrix} 0 \\ 0.6 \\ 2.5 \end{bmatrix}_{3 \times 2} = \begin{bmatrix} 1 \\ 1.2 \\ 6.5 \end{bmatrix}_{3 \times 2} + \begin{bmatrix} 16 \\ 0 \\ 2.1 \end{bmatrix}_{3 \times 2}$$

The diagram shows the addition of two 3x2 matrices. The first matrix has circled entries 1 (green) and 7 (purple). The second matrix has circled entries 0 (green) and -7 (purple). The result matrix has circled entries 16 (purple) and 0 (purple).

Addition of matrices

Example:

$$\begin{bmatrix} 1 & 9 \\ 0.6 & 7 \\ 4 & 1.5 \end{bmatrix} + \begin{bmatrix} 0 & 7 \\ 0.6 & -7 \\ 2.5 & 0.6 \end{bmatrix} = \begin{bmatrix} 1 & 16 \\ 1.2 & 0 \\ 6.5 & 2.1 \end{bmatrix}$$

Definition

The sum of two $m \times n$ matrix A and B is calculated entrywise :
the (i,j) -th entry of the matrix $A + B$ is the sum of (i,j) -th entry
of A and (i,j) -th entry of B

$$(A + B)_{ij} = A_{ij} + B_{ij}$$

Example:
$$\begin{bmatrix} 1 & 9 \\ 0.6 & 7 \\ 4 & 1.5 \end{bmatrix} + \begin{bmatrix} 0 & 7 \\ 0.6 & -7 \\ 2.5 & 0.6 \end{bmatrix} = \begin{bmatrix} 1 & 16 \\ 1.2 & 0 \\ 6.5 & 2.1 \end{bmatrix}$$

Definition

The sum of two $m \times n$ matrix A and B is calculated entrywise :
the (i,j) -th entry of the matrix $A + B$ is the sum of (i,j) -th entry
of A and (i,j) -th entry of B

$$(A + B)_{ij} = A_{ij} + B_{ij}$$

Example:
$$\begin{bmatrix} 1/2 & -3/4 & 3 \end{bmatrix}_{1 \times 3} + \begin{bmatrix} 2 & -3 & -1 \end{bmatrix}_{1 \times 3} = \begin{bmatrix} 5/2 & -15/4 & 2 \end{bmatrix}_{1 \times 3}$$

Scalar multiplication (Multiplying a matrix by a number)

Example: $3 \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 12 & 15 & 18 \end{bmatrix}$

The scalar 3 is multiplied by each element of the matrix. The resulting matrix has dimensions 3×3 .

$$\begin{bmatrix} 3 \times 1 & 3 \times 2 & 3 \times 3 \\ 3 \times 4 & 3 \times 5 & 3 \times 6 \end{bmatrix}$$

Scalar multiplication (Multiplying a matrix by a number)

Example: $3 \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 9 \\ 12 & 15 & 18 \end{bmatrix}$

Definition

The product of a matrix A with a number c is denoted by cA and the (i,j) -th entry of cA is product of (i,j) -th entry of A with the number c .

$$(cA)_{ij} = c(A_{ij})$$

Matrix multiplication (multiplying two matrices)

Example:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix}_{2 \times 3} = \begin{bmatrix} 7 & 10 & 13 \\ 15 & 22 & 29 \end{bmatrix}_{2 \times 3}$$

$$1 \times 1 + 2 \times 3 = 1 + 6 = 7$$

(1,1)-th

$$3 \times 2 + 4 \times 4 = 6 + 16 = 22$$

(2,2)-th

$$1 \times 3 + 2 \times 5 = 3 + 10 = 13$$

(1,3)-th

No. of columns in the first matrix
= No. of rows in the second matrix

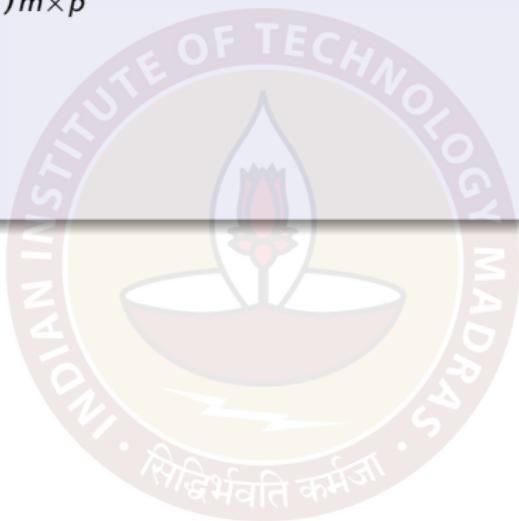
Example:
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2} \cdot \begin{bmatrix} 5 \\ 6 \end{bmatrix}_{2 \times 1} = \begin{bmatrix} 17 \\ 39 \end{bmatrix}_{2 \times 1}$$

$$1 \times 5 + 2 \times 6 = 5 + 12 = 17$$
$$3 \times 5 + 4 \times 6 = 15 + 24 = 39.$$

Matrix multiplication (multiplying two matrices)

Definition

$$A_{m \times n} B_{n \times p} = (AB)_{m \times p}$$



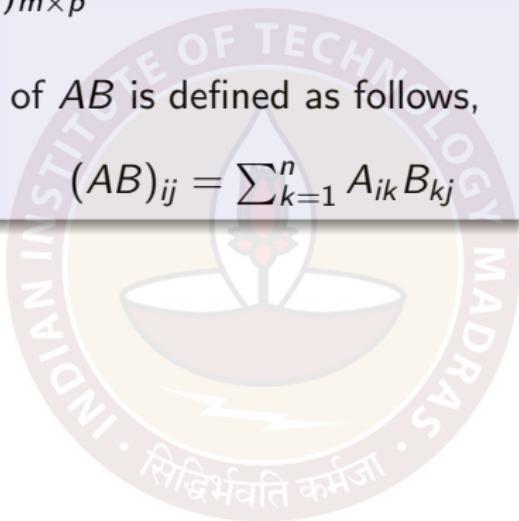
Matrix multiplication (multiplying two matrices)

Definition

$$A_{m \times n} B_{n \times p} = (AB)_{m \times p}$$

The (i, j) -th entry of AB is defined as follows,

$$(AB)_{ij} = \sum_{k=1}^n A_{ik} B_{kj}$$



Matrix multiplication (multiplying two matrices)

Definition

$$A_{m \times n} B_{n \times p} = (AB)_{m \times p}$$

The (i, j) -th entry of AB is defined as follows,

$$(AB)_{ij} = \sum_{k=1}^n A_{ik} B_{kj}$$

Remark

Multiplication of matrices A and B is defined only when the number of columns of A is the same as the number of rows of B .

Definition

$$A_{m \times n} B_{n \times p} = (AB)_{m \times p}$$

The (i, j) -th entry of AB is defined as follows,

$$(AB)_{ij} = \sum_{k=1}^n A_{ik} B_{kj}$$

Remark

Multiplication of matrices A and B is defined only when the number of columns of A is the same as the number of rows of B .

Example: $[1 \ 2 \ 3]_{1 \times 3} \begin{bmatrix} 2 & 0.8 \\ 5 & 0.7 \\ 1/2 & -2 \end{bmatrix}_{3 \times 2} = [13.5 \ -3.8]_{1 \times 2}$

$$\begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} c & 2c \\ 3c & 4c \\ 5c & 6c \end{bmatrix} = c \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

3 × 3 3 × 2 3 × 2

Multiplication by special matrices

$$\begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} c & 2c \\ 3c & 4c \\ 5c & 6c \end{bmatrix} = c \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Scalar multiplication by c = multiplication by scalar matrix cl .

$$\begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} = \begin{bmatrix} c & 2c \\ 3c & 4c \\ 5c & 6c \end{bmatrix} = c \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Scalar multiplication by c = multiplication by scalar matrix cI .

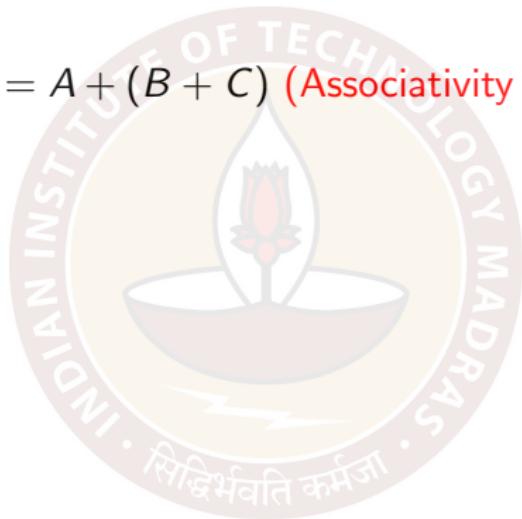
$$IA_{3 \times 3} = A_{3 \times 3} = A_{3 \times 3}I$$

$$IA_{3 \times n} = A_{3 \times n}$$

$$A_{m \times 3}I = A_{m \times 3}$$

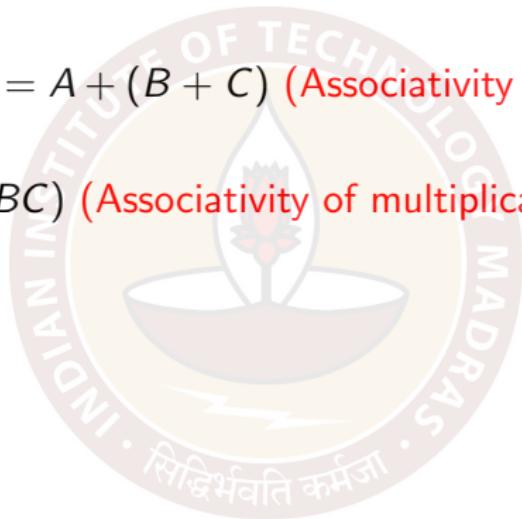
Properties of matrix addition and multiplication

- ▶ $(A + B) + C = A + (B + C)$ (Associativity of addition)



Properties of matrix addition and multiplication

- ▶ $(A + B) + C = A + (B + C)$ (Associativity of addition)
- ▶ $(AB)C = A(BC)$ (Associativity of multiplication)



Properties of matrix addition and multiplication

- ▶ $(A + B) + C = A + (B + C)$ (Associativity of addition)
- ▶ $(AB)C = A(BC)$ (Associativity of multiplication)
- ▶ $A + B = B + A$ (Commutativity of addition)

- ▶ $(A + B) + C = A + (B + C)$ (Associativity of addition)
- ▶ $(AB)C = A(BC)$ (Associativity of multiplication)
- ▶ $A + B = B + A$ (Commutativity of addition)
- ▶ In general $AB \neq BA$ (assuming both make sense)

$$\begin{bmatrix} : & : & : \end{bmatrix} \begin{bmatrix} : \\ : \end{bmatrix}$$

~~$$\begin{bmatrix} : & : & : \end{bmatrix} \begin{bmatrix} : \\ : \end{bmatrix}$$~~

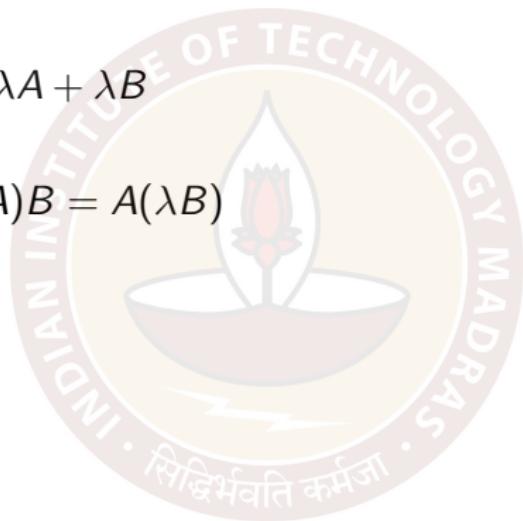
$\begin{bmatrix} : & : & : \end{bmatrix}_{3 \times 1} \quad \begin{bmatrix} : \\ : \end{bmatrix}_{2 \times 3}$

- ▶ $\lambda(A + B) = \lambda A + \lambda B$ λ is a scalar



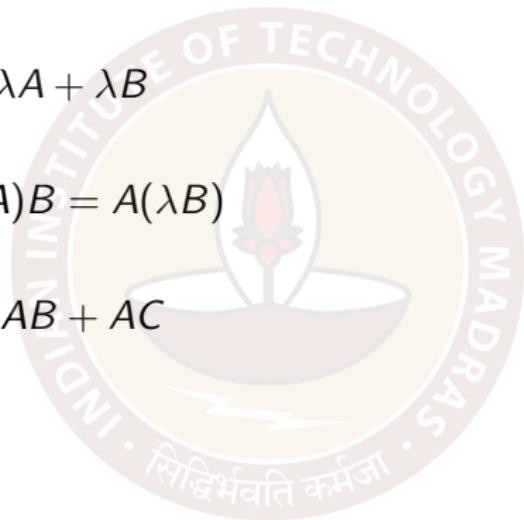
Properties of matrix addition and multiplication

- ▶ $\lambda(A + B) = \lambda A + \lambda B$
- ▶ $\lambda(AB) = (\lambda A)B = A(\lambda B)$



Properties of matrix addition and multiplication

- ▶ $\lambda(A + B) = \lambda A + \lambda B$
- ▶ $\lambda(AB) = (\lambda A)B = A(\lambda B)$
- ▶ $A(B + C) = AB + AC$



- ▶ $\lambda(A + B) = \lambda A + \lambda B$
- ▶ $\lambda(AB) = (\lambda A)B = A(\lambda B)$
- ▶ $A(B + C) = AB + AC$
- ▶ $(A + B)C = AC + BC$

$$\begin{aligned} & 2 \left[\begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right] \\ &= 2 \begin{bmatrix} 1 \\ 4 \end{bmatrix} + 2 \begin{bmatrix} 3 \\ 4 \end{bmatrix} \end{aligned}$$

