Matrices

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Contents

- What is a matrix?

Contents

- What is a matrix?
- Related terms.

Contents

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- Diagonal and scalar matrices.

Contents

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- Related terms.
- Diagonal and scalar matrices.
- The identity matrix.

Contents

- What is a matrix?
- Related terms.
- Diagonal and scalar matrices.
- The identity matrix.
- Linear equations and matrices.

Contents

- What is a matrix?
- Related terms.
- Diagonal and scalar matrices.
- The identity matrix.
- Linear equations and matrices.
- Addition of matrices.

Contents

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- Related terms.
- Diagonal and scalar matrices.
- The identity matrix.
- Linear equations and matrices.
- Addition of matrices.
- Scalar multiplication.

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- Related terms.
- Diagonal and scalar matrices.
- The identity matrix.
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- Scalar multiplication.
- Multiplication of matrices.

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- What is a matrix?
- Related terms.
- Diagonal and scalar matrices.
- The identity matrix.
- Linear equations and matrices.
- Addition of matrices.
- Scalar multiplication.
- Multiplication of matrices.
- Properties of matrix addition and multiplication.

What is a matrix?

Definition

A matrix is a rectangular array of numbers, arranged in rows and columns. (plural : matrices)

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$$\text{Example: } \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}_{2\times 3}$$

This is a 2×3 matrix (2 rows and 3 columns).

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Example: $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}_{2\times 3}$

This is a 2×3 matrix (2 rows and 3 columns).

(1,2)-th entry of this matrix is 2.
(2,3)-th entry of " " is 4.

- An $m \times n$ matrix has m rows and n columns.
- (i, j)-th entry of a matrix is the entry occuring in the i-th row and j-th column.

Square matrices

- A square matrix is a matrix in which the number of rows is the same as the number of columns.

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Example: $\begin{bmatrix} 0.3 & 5 & -7 \\ 2.8 & 0 & 1 \\ 0 & -2.5 & -1 \end{bmatrix}_{3\times 3}$

This is a 3×3 matrix (3 rows and 3 columns).

(2, 3)-th entry of this matrix is 1.

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- The diagonal of a square matrix is the set of diagonal entries.

Diagonal Matrices

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$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 4.2 \end{bmatrix}$$

Scalar Matrices

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A diagonal matrix in which all the entries in the diagonal are equal is called a scalar matrix.

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$$S = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}_{3 \times 3}$$

The identity matrix

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The scalar matrix with all diagonal entries 1 is called the identity matrix and is denoted by I.

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$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3×3

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Linear equations and matrices

A set of linear equations can be represented in terms of matrices.

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Example

$$3x + 4y = 5$$
$$4x + 6y = 10$$

can be represented by the matrix

$$\left[\begin{array}{cc|c} 3 & 4 & 5 \\ 4 & 6 & 10 \end{array}\right] \quad 2 \times 3$$

$$\begin{bmatrix} 3 & 4 & 5 \\ 4 & 6 & 10 \end{bmatrix}$$

Addition of matrices

$$\text{Example: } \begin{bmatrix} 1 & 9 \\ 0.6 & 7 \\ 4 & 1.5 \end{bmatrix} + \begin{bmatrix} 0 & 7 \\ 0.6 & -7 \\ 2.5 & 0.6 \end{bmatrix} = \begin{bmatrix} 1 & 16 \\ 1.2 & 0 \\ 6.5 & 2.1 \end{bmatrix}$$

3×2 3×2 3×2

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Definition

The sum of two $m \times n$ matrix A and B is calculated entrywise : the (i, j)-th entry of the matrix $A + B$ is the sum of (i, j)-th entry of A and (i, j)-th entry of B

$$(A + B)_{ij} = A_{ij} + B_{ij}$$

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Example: $\begin{bmatrix} 1/2 & -3/4 & 3 \end{bmatrix} + \begin{bmatrix} 2 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 5/2 & -15/4 & 2 \end{bmatrix}$

1×3 1×3 1×3

Scalar multiplication (Multiplying a matrix by a number)

Example: $3\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix} = \begin{bmatrix}3 & 6 & 9\\ 12 & 15 & 18\end{bmatrix}$

$$= \begin{bmatrix}3\times 1 & 3\times 2 & 3\times 3\\ 3\times 4 & 3\times 5 & 3\times 6\end{bmatrix}$$

Scalar multiplication (Multiplying a matrix by a number)

$$\text{Example: } 3\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix} = \begin{bmatrix}3 & 6 & 9\\12 & 15 & 18\end{bmatrix}$$

Definition

The product of a matrix A with a number c is denoted by cA and the (i,j)-th entry of cA is product of (i,j)-th entry of A with the number c.

$$(cA)_{ij} = c(A_{ij})$$

Matrix multiplication (multiplying two matrices)

Example: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix}_{2\times 3} = \begin{bmatrix} 7 & 10 & 13 \\ 15 & 22 & 29 \end{bmatrix}_{2\times 3}$

$1\times 1 + 2\times 3 = 1+6 = 7$ $\quad (1,1)$-th

$3\times 2 + 4\times 4 = 6+16 = 22$ $\quad (2,2)$-th

$1\times 3 + 2\times 5 = 3+10 = 13$ $\quad (1,3)$-th

Matrix multiplication (multiplying two matrices)

No. of columns in the first matrix = No. of rows in the second matrix

Example: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times 2} \begin{bmatrix} 5 \\ 6 \end{bmatrix}_{2\times 1} = \begin{bmatrix} 17 \\ 39 \end{bmatrix}_{2\times 1}$

$1\times 5 + 2\times 6 = 5+12 = 17$

$3\times 5 + 4\times 6 = 15+24 = 39.$

Matrix multiplication (multiplying two matrices)

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$A_{m\times n}B_{n\times p} = (AB)_{m\times p}$

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The (i,j)-th entry of AB is defined as follows,

$$(AB)_{ij} = \sum_{k=1}^{n} A_{ik}B_{kj}$$

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Remark

Multiplication of matrices A and B is defined only when the number of columns of A is the same as the number of rows of B.

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Remark

Multiplication of matrices A and B is defined only when the number of columns of A is the same as the number of rows of B.

Example: $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}_{1\times 3} \begin{bmatrix} 2 & 0.8 \\ 5 & 0.7 \\ 1/2 & -2 \end{bmatrix}_{3\times 2} = \begin{bmatrix} 13.5 & -3.8 \end{bmatrix}_{1\times 2}$

Multiplication by special matrices

$$\begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix}_{3\times 3} \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}_{3\times 2} = \begin{bmatrix} c & 2c \\ 3c & 4c \\ 5c & 6c \end{bmatrix}_{3\times 2} = c \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

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Scalar multiplication by c = multiplication by scalar matrix cI.

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Scalar multiplication by c = multiplication by scalar matrix cI.

$$IA_{3\times 3} = A_{3\times 3} = A_{3\times 3}I$$
$$IA_{3\times n} = A_{3\times n}$$
$$A_{m\times 3}I = A_{m\times 3}$$

Properties of matrix addition and multiplication

- $(A + B) + C = A + (B + C)$ (Associativity of addition)

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- $(AB)C = A(BC)$ (Associativity of multiplication)
- $A+B = B+A$ (Commutativity of addition)

Properties of matrix addition and multiplication

- $(A + B) + C = A + (B + C)$ (Associativity of addition)
- $(AB)C = A(BC)$ (Associativity of multiplication)
- $A + B = B + A$ (Commutativity of addition)
- In general $AB \neq BA$ (assuming both make sense)

Properties of matrix addition and multiplication

- $\lambda(A+B) = \lambda A + \lambda B$

λ is a scalar

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- $A(B + C) = AB + AC$
- $(A + B)C = AC + BC$

$$2\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix}\right) = 2\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

Thank you