

Course: Machine learning Foundations

Solve with Instructor

WEEK 9



IIT Madras
ONLINE DEGREE

Problem 1

Which of the following sets are convex, closed and bounded?

$$A = \{(x, y) \in \mathbb{R}^2; x^2 + y^2 \leq 4\}$$

$$B = \{(x, y) \in \mathbb{R}^2; y = 2x + 4\}$$

$$C = \{(x, y) \in \mathbb{R}^2; x + y \leq 2, x \geq 0, y \geq 0\}$$

$$D = \{(x, y) \in \mathbb{R}^2; y > x^2\}$$

$$E = \{(x, y) \in \mathbb{R}^2; xy > 1, x \geq 0, y \geq 0\}$$

$$F = \{(x, y) \in \mathbb{R}^2; xy > 1, x \leq 0, y \leq 0\}$$



Solution

□ <https://www.geogebra.org/m/kuk45ab4>

Problem 2

Let $f(x) = 3 + x^2 + x$ and $g(x) = e^{-x}$. What can you say about their composition $h_1 = f \circ g$ and $h_2 = g \circ f$?

- A. h_1 is convex and h_2 is concave
- B. h_1 is concave and h_2 is convex
- C. h_1 and h_2 is convex
- D. h_1 and h_2 is concave



Solution

□ <https://www.geogebra.org/m/esqcd4he>

Problem 3

Consider the following problem.

Maximize $x + y$ subject to the constraints

$$x \geq 0, \quad y \geq 0, \quad -3x + 2y \leq -1, \quad x - y \leq 2.$$

Is the feasible region bounded or unbounded?

Solution

□ <https://www.geogebra.org/m/fggdwdsc>

Problem 4

□ Given below is a set of data points and their labels.

x	y
[1,2]	2.5
[2,1.5]	3
[2,3]	5
[3,4]	7

Perform linear regression on this data set. Let initial w at $t=1$ be $[0.1, 0.1]^T$. Using the gradient descent update equation with a learning rate as 0.1, compute the value of w at $t = 2$.

Solution

- $\nabla w = (X^T X)(X^T y)$
- $X^T X = \begin{bmatrix} 18 & 23 \\ 23 & 31.25 \end{bmatrix}$
- $X^T y = \begin{bmatrix} 39.5 \\ 52.5 \end{bmatrix}$
- $w^2 = w^1 - \eta \nabla w$
- $w^2 = \begin{bmatrix} 3.64 \\ 4.807 \end{bmatrix}$



Thank You