# Graphs and tangents

Sarang S. Sane

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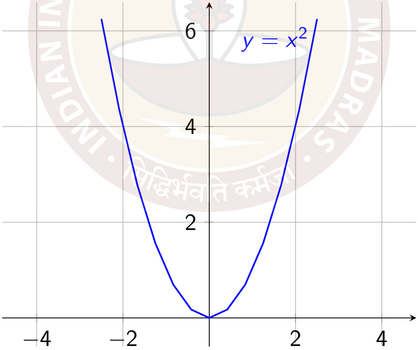
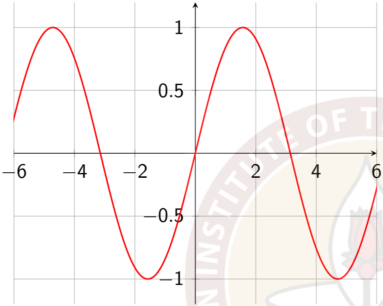
Example :



# Graphs of functions : recall the trigonometric functions




Graph of $y = sin(x)$

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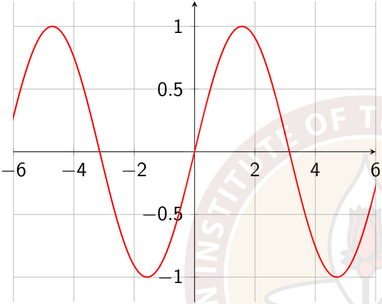
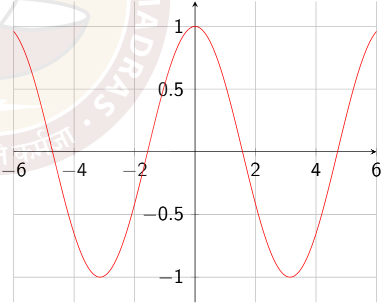
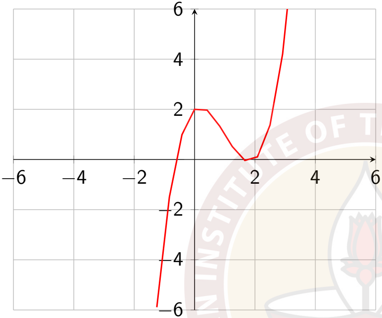



Graph of $y = sin(x)$

Graph of $y = cos(x)$



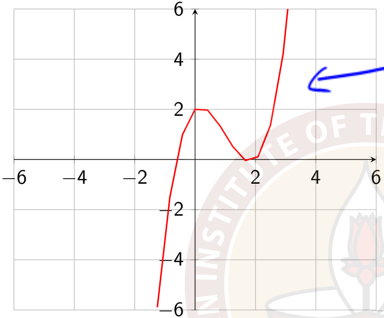
# Graphs of functions : more examples




$$\Gamma(x^3 - 3x^2 + x + 2)$$

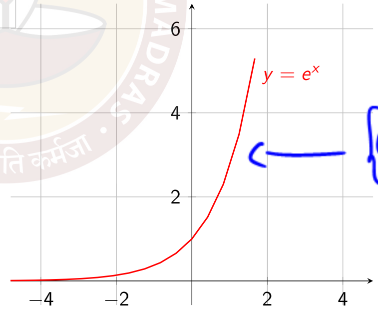
# Graphs of functions : more examples

$\{(x, x^3-3x^2+x+2) \mid x \in \mathbb{R}\}$



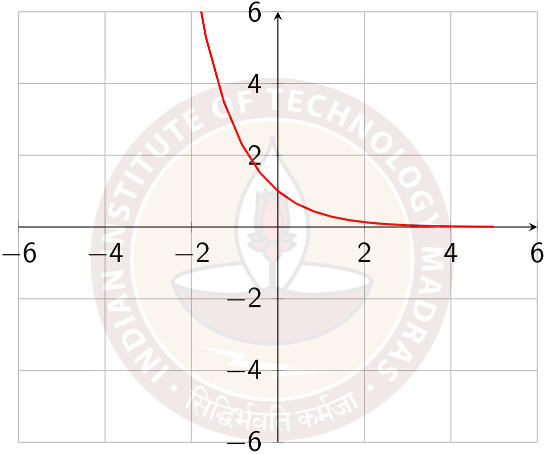

$\Gamma(x^3 - 3x^2 + x + 2)$

Graph of $y = e^x$



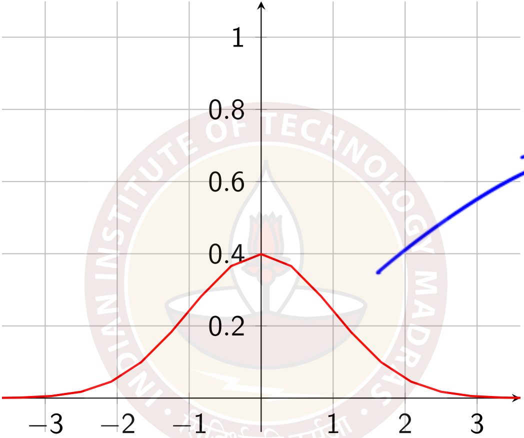

$\{(x, e^x) \mid x \in \mathbb{R}\}$

# Graphs of functions : exponential decay




Graph of $y = e^{-x}$

# Graphs of functions : the normal distribution




$\{(x, \frac{1}{\sqrt{2\pi}} e^{-x^2/2}) \mid x \in \mathbb{R}\}$

Graph of $y = \dfrac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

# Curves

A curve is a figure that is obtained as the path of a moving point.

# Curves : Visual example

# Curves (contd.)

A curve can be thought of as a figure obtained by bending a line at various places.

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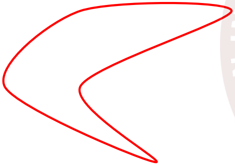

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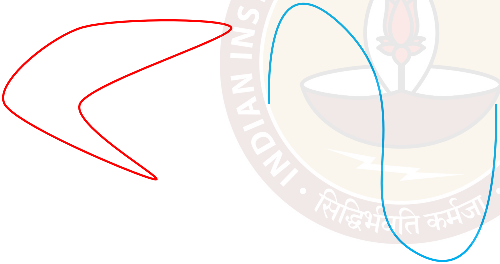

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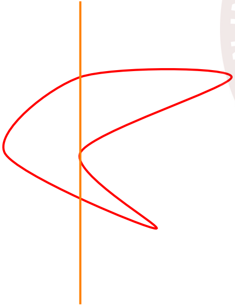

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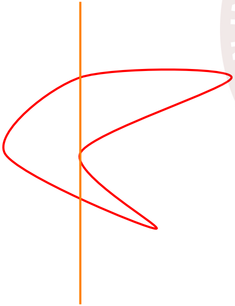

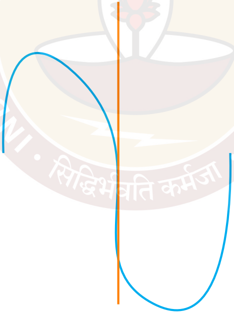

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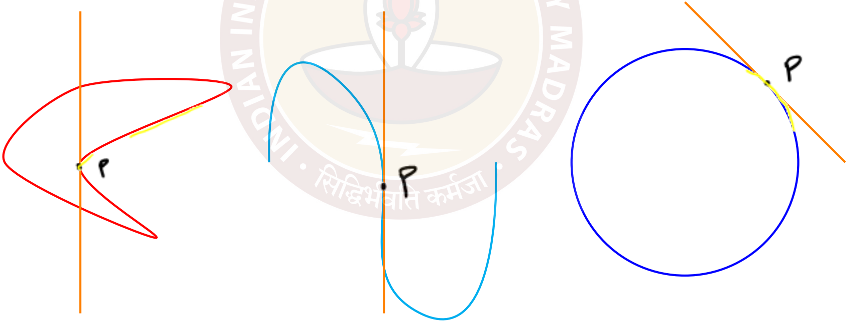

# Tangent lines : some means of identification

Often, a tangent line to a curve $C$ at a point $p$ (on $C$) has the property that it passes through the point $p$ but does not intersect the curve $C$ in any other point *close to* the point $p$, and lines parallel and close to it either do not intersect $C$ *close to* $p$, or intersect the curve $C$ in two (or more) points close to $p$.

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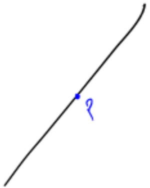


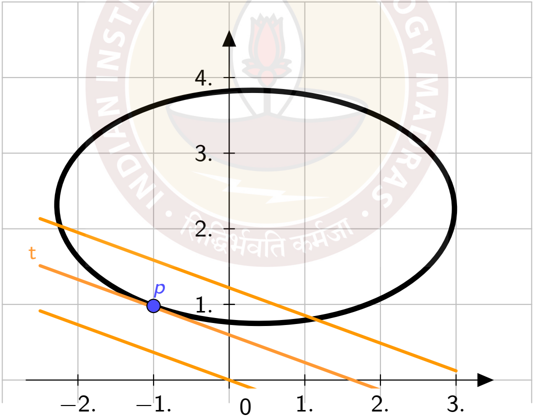


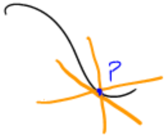

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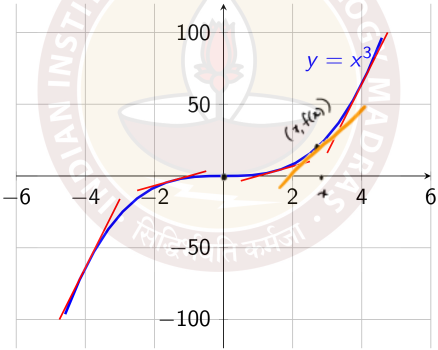
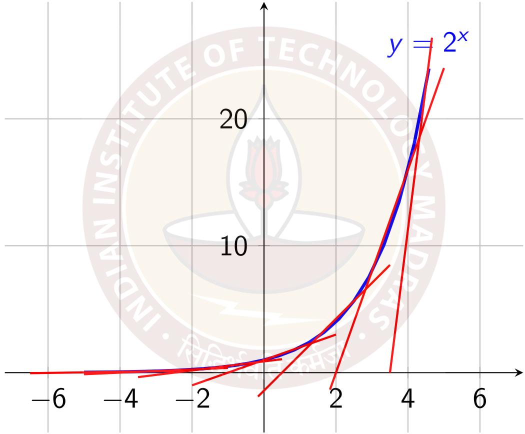


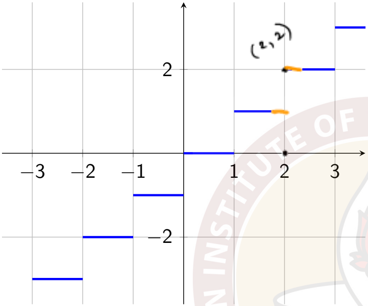

$f(x) = x^3$

Figure: Tangent lines for $y = x^3$

# Tangent line for $y = 2^x$



# Examples advising caution




$\Gamma\left(\lfloor x \rfloor\right)$

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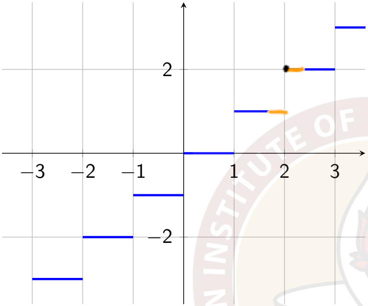
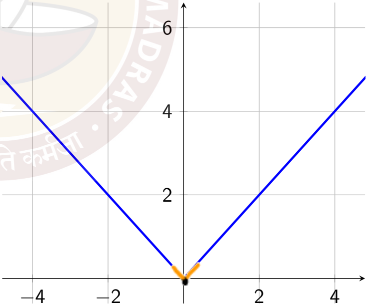



$\Gamma(\lfloor x \rfloor)$

Graph of $y = |x|$



# Thank you