Quantum Key Distribution: from Principles to Practicalities

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(February 1, 2008)

We review the main protocols for key distribution based on principles of quantum mechanics, describing the general underlying ideas, discussing implementation requirements and pointing out directions of current experiments. The issue of security is addressed both from a principal and real-life point of view.

I. PRINCIPLES

The desire and necessity to transmit secret messages from one person to another is probably as old as the capability of human beings to communicate. Cryptography is the art to encode a text in such a way that a spy (or eavesdropper) can get as little information as possible about it, and only the authorized receiver can decode it perfectly. The methods to perform this task have been improved over thousands of years. An important class of today's schemes are public-key crypto-systems [14], in which mutually inverse transformations are used for encoding and decoding. The instruction for encoding is made public, and safety relies on the high complexity of the inverse transformation (factorization of large prime numbers). In principle this system could be broken, though, by faster algorithms (see Shor's algorithm in quantum computation).

The only crypto-system that has been proven to be safe is using a random key which is only known to the sender and the receiver. The recipe for the sender is to translate the text with a look-up table into a sequence of 0's and 1's, e.g., A→ 00001, B → 00011, etc., (this translation alone is fairly easy to decipher by an enemy) and then to add modulo 2 the random key (a random sequence of 0's and 1's), which needs to be of the same length as the message.

The result is that letters which were the same in the original message are encoded into completely uncorrelated strings. Only the receiver can decode the message by adding again the secret key. This method is only safe, though, if the key is used just once, otherwise consecutive messages reveal information about the messages.[1] Therefore this type of protocol is also labeled with the key word "one-time pad", because in the second world war the key would be written on a sheet torn from a pad.

Unfortunately, the problem of secrecy is hereby only shifted to the problem of distributing the key in a safe way to the receiver. In principle, a spy can get hold of the key, copy it and send it on to the receiver. This is the point where quantum physics enters the stage: if the key distribution makes use of quantum states (this is possible in different ways which will be explained in detail in the following) the spy cannot measure them without disturbing them. Thus principles of quantum mechanics can help to make the key distribution safe. Often this young research area is, slightly misleading, also referred to as quantum cryptography (for an introduction see [6]).

In the modern communication society there is widespread need of secure transmission of secret information (e.g. credit card numbers, passwords). Therefore, a practical realization of these ideas is certainly very desirable, and some experimental results have indeed already been achieved. We will summarize the occurring problems and solutions for some of them and point out the open questions.

Let us list the main ingredients of quantum mechanics which allow for different protocols of secure key distribution - these will be explained in more detail in the following chapter:

- *non-orthogonal states cannot be distinguished perfectly*
  A quantum mechanical two-state system cannot only be in the state $|0\rangle$ or $|1\rangle$, but more generally in a linear superposition $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ with complex coefficients α and β satisfying $|\alpha|^2 + |\beta|^2 = 1$. Due to the laws of quantum mechanics, it is impossible to distinguish reliably between

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[1] By adding two messages encoded with the same key one obtains the sum of the two original messages. This narrows down the possible combinations and reveals a considerable amount of information to an eavesdropper.

$$|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle \quad \text{and}$$
$$|\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle \tag{1}$$

unless the state overlap is $\langle\psi_1|\psi_2\rangle = 0$, i.e. the states are orthogonal.

- *no-cloning theorem*
  It is impossible, due to linearity and unitarity of quantum mechanics, to create perfect copies of an unknown quantum state [33]. Thus a spy is not able to produce perfect copies of a quantum state in transit in order to measure it, while sending on the original.

- *entanglement (quantum correlation)*
  Two or more quantum systems can be correlated or entangled. An entangled state cannot be written as a direct product of the subsystems. The singlet of two spin-$\frac{1}{2}$-systems is an example of a maximally entangled state:

$$|\psi^-\rangle = \tfrac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \quad . \tag{2}$$

  (The four maximally entangled states of two spin-$\frac{1}{2}$-systems are called Bell states.) If a measurement is done on one of these two quantum systems (in any basis), the result will be 0 or 1 with equal probability. The state of the other system is anti-correlated, i.e. if the first system collapsed into 0, the second collapses into 1 and vice versa. Without any measurement, though, none of the two systems *is* in a fixed state.

- *causality and superposition*
  Causality is *not* an ingredient of non-relativistic quantum mechanics. Nevertheless it is mentioned in this list of principles because together with the superposition principle it can be used for secure key distribution: if the two terms of which a superposition consists are sent with a time delay relative to each other, such that they are not causally connected, the eavesdropper cannot spy on them.

II. CONCRETE PROTOCOLS

In this chapter we explain different approaches to the task of establishing a common secret key between two parties. The sender of the key is usually called Alice and the receiver Bob. Here we will assume that no enemy (usually called Eve) is present. In chapter III we will then discuss how a spy can gain some information on the key.

We can distinguish the following main three classes of protocols.

1) *BB84 class:*
   In 1984 Bennett and Brassard suggested a quantum cryptographic protocol that relies on the use of non-orthogonal states [4]. It is often referred to as BB84. There have been several ideas for variations of this protocol which will for this review be included in the BB84-class.

   – *BB84:*
   In the BB84 protocol [4] Alice sends randomly one of the four quantum states

$$\begin{aligned} &|0\rangle \quad , \\ &|1\rangle \quad , \\ &|\bar{0}\rangle = \tfrac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \quad , \\ &|\bar{1}\rangle = \tfrac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \quad , \end{aligned} \tag{3}$$

   with equal probability. Here the states $|0\rangle$ and $|\bar{0}\rangle$ represent bit value '0', the states $|1\rangle$ and $|\bar{1}\rangle$ stand for bit value '1'. The first two states in equation (3) correspond to a spin-$\frac{1}{2}$-particle being polarized in positive or negative z-direction, the last two to polarization in positive or negative x-direction. This can be graphically visualized as in figure 1. (All figures in connection with the protocols show directions corresponding to polarization vectors of spin-$\frac{1}{2}$-particles.)

   The states in eq. (3) can also be represented by linearly polarized photons: the first two states then correspond to vertically and horizontally polarized photons, the last two to polarization angles 45^o and 135^o with respect to the vertical axis.

   When Bob receives a state from Alice, he chooses randomly either the x- or the z-basis for making a measurement. His result will always be either $|0\rangle$ or $|1\rangle$. But only in the cases where he picked the "right"

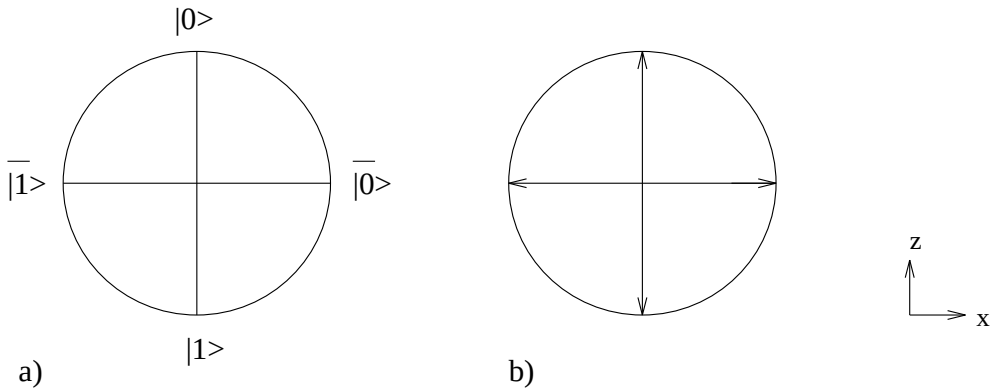

FIG. 1. Directions corresponding to polarization of a spin-$\frac{1}{2}$-particle for the BB84 protocol: a) ensemble of states Alice sends, b) Bob's directions of measurement. Note that orthogonal states point in opposite directions, see e.g. $|\,0\rangle$ and $|\,1\rangle$, which point in $+z$ and $-z$ direction, respectively.

basis, i.e. the one which Alice used, is his result correlated with the bit Alice sent. If, e.g., Alice sent $|\,\bar{0}\rangle$, but Bob measures along the z-direction, he will find either $|\,0\rangle$ or $|\,1\rangle$ with equal probability. After Alice sent and Bob measured the necessary number of states, Alice phones Bob (or uses some other "classical" channel) and tells him when she used which basis. They throw away the cases in which they used different bases, and thus have established a secret key. This key is called the *sifted key*.

– *B92:*
In this protocol by Bennett [2] Alice chooses between two non-orthogonal states to be sent to Bob. It was shown that in principle *any* two non-orthogonal states of a quantum system can be used for quantum key distribution. Let $|\,u_0\rangle$ and $|\,u_1\rangle$ be the two non-orthogonal states which represent the bit values 0 and 1, see figure 2.

Bob makes a measurement with a set of so-called POVM's (positive operator valued measurements), which gives as result either "$|\,u_0\rangle$" or "$|\,u_1\rangle$" or "I don't know" (see, e.g., [30]). For example, if Alice sends $|\,u_0\rangle$, Bob will either find $|\,u_0\rangle$ or an inconclusive result, but never $|\,u_1\rangle$. They can then use the public channel to discard inconclusive results, thus arriving at a correlated string of bits.

In practice the two non-orthogonal states can be realized by two low-intensity coherent states (note that two different coherent states are never exactly orthogonal, and for low intensities they become significantly non-orthogonal). An additional strong reference pulse is used in order to enhance security of the protocol (see section IV A).

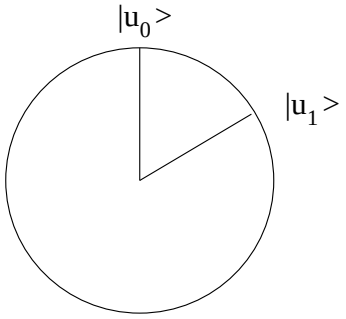

FIG. 2. B92 protocol: two non-orthogonal states.

– *4+2 protocol:*
The protocol described in [22] combines ideas from BB84 and B92: as in BB84 Alice chooses between two

different bases (so the number of possible states to send is 4), and as in B92 the two states within a basis, representing bit '0' and '1', are non-orthogonal. As in B92, a strong reference pulse is used.

Thus, this protocol corresponds to realizing BB84 with coherent states and a strong reference pulse.

– *Six state protocol:*
In the six state protocol [10,1] Alice enlarges her ensemble of quantum states she sends across to Bob, using in addition to the four states in BB84 the states

$$|\,\bar{\bar{0}}\rangle = \tfrac{1}{\sqrt{2}}(|\,0\rangle + i|\,1\rangle) \quad \text{and}$$
$$|\,\bar{\bar{1}}\rangle = \tfrac{1}{\sqrt{2}}(|\,0\rangle - i|\,1\rangle)\ , \tag{4}$$

which describe a spin-$\frac{1}{2}$-particle polarized in positive or negative y-direction. (In the case of photons, these states represent circular polarization.) The six states are shown in figure 3.

Thus, Alice sends a state randomly polarized in positive or negative x-, y-, or z-direction to Bob, who measures randomly in the $x-, y-$ or z-basis. As in BB84 they communicate over a public channel and keep only those cases in which their basis was the same.

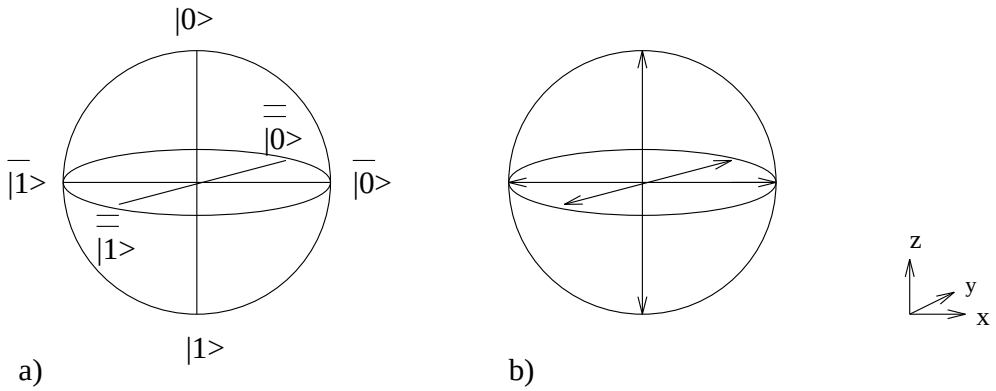

FIG. 3. Six state protocol: a)ensemble of states Alice sends, b)Bob's directions of measurement.

2) *Ekert scheme:*
In the key distribution scheme designed by Ekert [15] Alice and Bob are sharing a number of maximally entangled states consisting of two two-state systems, such that each of them has hold of one of the two correlated systems. Let us indicate this by labeling the singlet with indices A and B:

$$|\,\psi^-\rangle = \tfrac{1}{\sqrt{2}}(|\,0\rangle_A|\,1\rangle_B - |\,1\rangle_A|\,0\rangle_B) \quad . \tag{5}$$

They store their entangled states until they decide to establish the key, then Alice chooses randomly one of the three measurement directions indicated in figure 4 whereas Bob chooses a set of directions rotated by 45^o.

They use again just those cases in which their measurement directions were the same. Only then their results are correlated. The runs where they used different directions can be used to test the Bell inequality and thus find out whether anybody has interfered with their systems.

3) *Goldenberg/Vaidman class:*
The idea of this class of protocols is to use a superposition of states, which arrive at different times at Bob's site.

– *Goldenberg/Vaidman:*
The scheme described in [19] uses two orthogonal states, $|\,\Psi_0\rangle$ and $|\,\Psi_1\rangle$, to represent bits '0' and '1', given by

$$|\,\Psi_0\rangle = \tfrac{1}{\sqrt{2}}(|\,a\rangle + |\,b\rangle) \quad ,$$
$$|\,\Psi_1\rangle = \tfrac{1}{\sqrt{2}}(|\,a\rangle - |\,b\rangle) \quad , \tag{6}$$

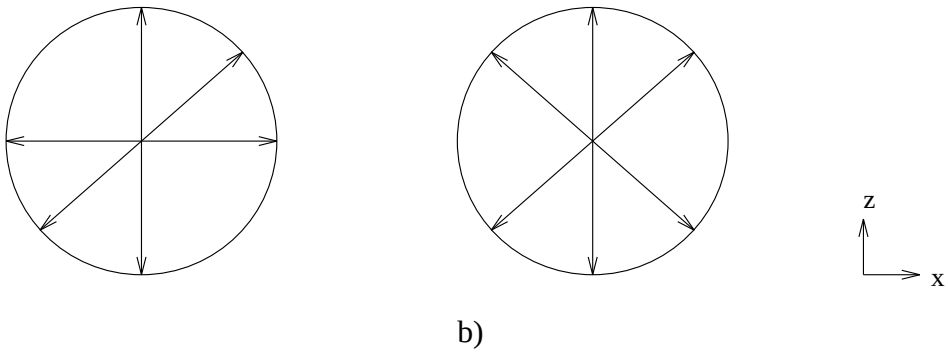

FIG. 4. Ekert protocol: a)Alice's directions of measurement, b)Bob's directions of measurement.

where $|\,a\rangle$ and $|\,b\rangle$ are localized normalized wavepackets which are sent from Alice to Bob along two channels of different 'length': wavepacket $|\,b\rangle$ is delayed for some fixed time until $|\,a\rangle$ has already reached Bob. This can for example be achieved by using an interferometer with one short and one long arm. Bob has to wait with the readout of the superposition until both $|\,a\rangle$ and $|\,b\rangle$ have reached him. In order to make it impossible for a spy to do her job, the times at which the wavepacket $|\,a\rangle$ is sent, have to be random. The advantage of using orthogonal states is that in principle there is no waste of photons.

– *Koashi/Imoto:*
The authors of [23] show how to circumvent the necessity of random timing by making the interferometer asymmetric, i.e. by using beamsplitters that do not have equal transmittivity and reflectivity. This means that the amplitudes in eq. (6) change to

$$\begin{aligned} |\,\Psi_0\rangle &= -i\sqrt{R}|\,a\rangle + \sqrt{T}|\,b\rangle \quad , \\ |\,\Psi_1\rangle &= \quad \sqrt{T}|\,a\rangle - i\sqrt{R}|\,b\rangle \quad . \end{aligned} \tag{7}$$

The different amplitudes for $|\,a\rangle$ deprive Eve of the possibility (given she knows the sending times) to use the simple strategy to send Bob a dummy $|\,a\rangle$ and later, after learning the phase, to send him $\pm|\,b\rangle$.

III. SECURITY

Due to the principles of quantum mechanics described above, it is impossible for the spy Eve to gain *perfect* knowledge of the quantum state sent from Alice to Bob. Nevertheless, she can acquire *some* knowledge. Without interaction of a spy, each two-level quantum system carries 1 bit of information (commonly called qubit) from Alice to Bob. When Eve gets hold of part of this information, she cannot prevent causing a disturbance to the state arriving at Bob's side, and thus introducing a non-zero error rate. In principle, Bob can find out about this error rate and thus about the existence of a spy by communicating with Alice.

The source for Eve's knowledge are measurements performed on the signals (quantum states). The simplest eavesdropping attack for Eve would be to measure each signal just as Bob would do, and then to resend a signal to Bob which corresponds to the measurement result.

However, quantum mechanics allows more general measurements than these simple projection measurements. Eve can bring an auxiliary quantum system (a probe) in contact with the signal so that they interact, and then perform a projection measurement on the auxiliary system to draw some information about the signal from it. All measurements, including the simple projection measurements, can be described in this fashion [20,30]. Another opportunity arises for Eve: she might delay the measurement of the auxiliary system until she learns more about the signal during public discussion. An example for useful information is the signal set from which a signal has been drawn. More involved strategies within quantum mechanics correlate measurements of several signals, thereby attacking the key as a whole rather than the individual components. This scenario is referred to as *coherent* eavesdropping. A simpler class is that of *collective* eavesdropping where to each signal an individual probe is attached just as in the individual attack. These probes, however, now can be read out together in a coherent process.

As mentioned above, in the ideal case we are always able to identify an eavesdropping activity by the occurrence of errors in the transmission. In a real world this becomes a tricky issue. We will always have some detector noise,

misalignments of detectors and so on. It should be pointed out that we cannot even in principle distinguish errors due to noise from errors due to eavesdropping activity. We therefore assume that all errors are due to eavesdropping. An other issue, not discussed here, is that of statistics. Eavesdroppers can be lucky: they create errors only on average, so in any specific realization the error rate might be zero (with probability exponentially small in the key length, of course). We are guided by the idea that a small error rate, for example 1 %, implies that an eavesdropper was not very active, while a big error rate is the signature of a serious eavesdropping attempt. But what is the meaning of "small" and "big"?

From an information theoretic point of view, the natural measure of "knowledge" about some signal is the Shannon information. It is measured in bits and can be defined for any two parties, the sender of the signal and the observer (receiver). In general terms, the knowledge of the observer consists of obtained measurement results and any additional gathered knowledge, like the announced basis of signals in the BB84 protocol. All this knowledge will be denoted by M.

From the receiver's point of view there will be an a-priori $p(x)$ and an a-posteriori $p(x|M)$ probability distribution for the signal x. The knowledge M will turn up with probability $q(M)$. The Shannon information can now be defined as the *expected* change in entropy of the two probability distributions. It is therefore given by

$$I = -\sum_x p(x)\log_2 p(x) + \sum_M q(M)\sum_x p(x|M)\log_2 p(x|M) \quad . \tag{8}$$

For a binary channel with equal a-priori probabilities for the two signals the Shannon information can be expressed in terms of the error probability e with which the signals are received. It is given (in bits per signal) by

$$I[e] = 1 + e\log_2 e + (1-e)\log_2(1-e) \; . \tag{9}$$

This is the Shannon information, per element of the sifted key, between Alice and Bob, I_{AB}, with the observed error rate e of the channel. On the other hand we will use the information I_E, generally given by equation (8), which Eve obtains on the key where M then represents her measurement results and all the information exchange between Alice and Bob over the public channel.

Another proposed measure of Eve's knowledge is the probability that the eavesdropper guesses the correct key given her knowledge about it.

A fundamental difference between classical cryptography and the use of a one-time pad together with quantum key distribution is that the former one is vulnerable to technological improvements (faster computers and algorithms) and therefore has to be designed to keep the secret secure against improvements which occur during the whole period of time in which the secrecy is required. Quantum key distribution, on the other hand, needs to be designed to be secure only against technology available at the time (and location) of the quantum part of key distribution. Therefore it makes sense to give the estimates of the Shannon information for various scenarios. They differ by the technology available to Eve. Examples for potential improvement of Eve's knowledge are the ability to perform delayed measurements (needs physical storage of auxiliary quantum systems), the availability of quantum channels superior to those used by Alice and Bob (for example in form of an optical fibre which is less lossy and noisy), and the ability to perform coherent eavesdropping attacks (needs ability to manipulate and store coherently several quantum systems).

Let us now quote some results on maximal information leakage to the eavesdropper. They are valid under the assumption of ideal BB84 signal states, for example single photons.

It has been shown that the simple intercept-resend strategy leads for the BB84 protocol to an average error rate of 25 % while it yields at best 0.5 bit of information per signal [16,21]. The optimal probability of a correct guess would be 75% in that case.

Bounds on the obtainable Shannon information for eavesdropping on single bits can be found in the literature for different protocols. Fuchs et al. give bounds for the BB84 [17] and the B92 protocol [18]. A bound for the six state protocol was obtained in [10]. These bounds are illustrated in figure 5. Note the trade-off between Eve's information gain and the disturbance she causes: more information for Eve means higher error rate for Bob. For reasonably low error rates Eve's maximal information is smallest in the six-state protocol, as it uses the biggest ensemble of input states.

Bounds for the Shannon information in more general attacks are studied in [12] for BB84 and [1] for the six-state protocol.

The important result from these estimates is that even for small error rates the eavesdropper might be in possession of information about the key at a level deemed dangerous for secure communication. For example, at an observed error rate of 1% we find that an eavesdropper might have gained up to 0.024 bit of Shannon information per bit of key even for the six-state protocol. This is far too high to allow the direct use of the obtained key for encryption. Instead, one uses the tool of privacy amplification [5] (see following section) to extract a short secure key from the long insecure key.

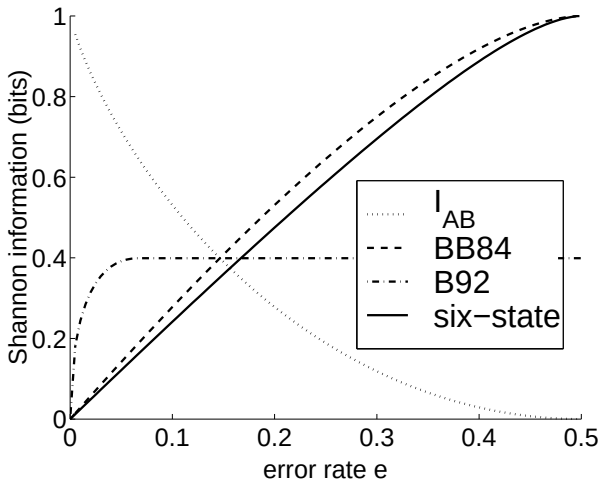

FIG. 5. Maximal mutual information I_E on the sifted key shared between Alice and Eve as function of Bob's error rate e for the protocols BB84 [17] and the six-state protocol [10] together with the the mutual information between Alice and Bob given by the curve I_{AB}. The graph for the B92 protocol [18] with state overlap of $1/\sqrt{2}$ displays I_E for the raw key and not for the sifted key.

One of the advantages of the Ekert scheme is that by storing the states at both ends of the transmission line and coherent manipulation on each side between the accumulated states the performance of the key distribution could be enhanced. This technique is called *quantum privacy amplification* [13] and effectively gives a new, shorter key with lower error rate.

IV. ELEMENTS FOR REALISTIC IMPLEMENTATIONS

In the previous section we have seen that the Shannon information available to an eavesdropper about the *sifted key* (that is the key directly after the key exchange) is too high to allow secret communication directly. Fortunately, it is possible to process this key with help of a purely classical protocol in order to distill a new, shorter key from the sifted key which exponentially approximates a secret key. We present the procedure here in a form which is valid only if Eve's activity is restricted to attacks on individual signals (as opposed to coherent or collective attacks). However, the steps executed in a quantum key distribution apparatus are the same in the general case, only the reasoning behind them changes, as indicated below.

More details about the full protocol to deal with restricted attacks in a realistic scenario can be found in [26]. Here we will concentrate only on the main points. An important point for practical realization is that in a realistic protocol no ideal public channel exists which can be overheard but not changed by an eavesdropper. This property of a channel can only be approximated by using an open channel where messages will be *authenticated* by means of a small secret key shared before the start of the communication. Only this method ensures that Alice and Bob do not fall victim to the *separate world attack*, known as well as *man in the middle attack*. In this attack an eavesdropper cuts the quantum and the classical channel dividing the world into two parts. One of these parts contains Alice, and Eve pretends to her to be Bob and vice versa in the other part. Alice and Bob unknowingly never communicate directly with each other. Only authentication by means of previous shared secret knowledge can counteract to this attack. In this view quantum key distribution will grow a large secret key from a small seed secret key. A by-product of this changed scenario is that we are free to use shared secret bits in intermediate states to enhance or make clearer the performance of the protocol.

The first step in that direction is *error correction*. Alice and Bob exchange redundant information over the public channel to reconcile their versions of the key. Obviously, the amount of exchanged redundant information has to be kept as small as possible, since the information flow to Eve has to be taken account of. (One possibility is to encode it using part of the initially shared secret key.) What is the minimum amount of exchanged redundant bits? A correctly received binary string of length n_{sif} carries exactly n_{sif} bits of Shannon information. On the other hand, if Bob received this key with an error rate e then he is in possession of $n_{sif}I_{AB}$ bits only. He, therefore, has to get hold of the

difference of $n_{sif} - n_{sif} I_{AB}[e]$ bits of information. Since the public channel can be made error free[2] the information per signal sent there is the ideal 1 bit, so for each bit of information missing, Alice has to send on average one signal. Therefore the minimum amount n_{min} of bits to be exchanged is given by the Shannon bound,

$$n_{min} = -n_{sif} \left(e \log_2 e + (1-e) \log_2 (1-e) \right) \ . \tag{10}$$

The best known practical protocol is that of Brassard and Salvail [8]. It uses an interactive information exchange between the two sides. The requirements for a good error correction protocol are to be as close as possible to the minimum number of exchanged bits given by the Shannon bound and a success rate of correction as high as possible. In contrast to a standard problem in error correction the channel used for transmission of the redundant bits can be assumed to be error free, which allows for improved, specialized error correction schemes.

Starting from the reconciled key, Alice and Bob now use *privacy amplification* [5] to establish a secret key. The idea behind privacy amplification is to hash the reconciled key of length n_{rec} into a shorter key of length n_{fin} using random hashing. An example for hashing is to take parity bits of random subsets of the reconciled key to form the new key. In general, we shorten the reconciled key by the fraction τ_1 and then by additional n_S bits to a final key length of $n_{fin} = (1-\tau_1) n_{rec} - n_S$. As shown by Bennett et al. [5] Eve's Shannon information on the final key is bounded by

$$I_E^{final} \leq \log_2 (2^{-n_S} + 1) \approx \frac{2^{-n_S}}{\ln 2} \ . \tag{11}$$

A consequence is that I_{final} can be made exponentially small by means of the number of security bits n_S.

The central quantity in this context is the collision probability P_{coll}, and the fraction τ_1 is given by $\tau_1 = 1 + \frac{1}{n_{rec}} \log P_{coll}$. Here P_{coll} is a measure of the a posteriori probability distribution P_{post} of the reconciled key conditioned on all information available to the eavesdropper. It is defined by the relation $P_{coll} = \sum_x \left(P_{post} \right)^2$ where the sum is taken over all reconciled keys. For security against eavesdropping strategies attacking individual signals only it is essential to find an upper bound on the collision probability. Bounds for P_{coll} and expressions for τ_1 for the BB84 protocol are given in [25,32,26], for the B92 protocol in [32] and for the six-state protocol in [1].

With these results it is possible to calculate the optimal rate at which one can extract secure bits from the sifted key. We assume error correction at the Shannon bound of equation (10) and encryption of the redundant bits. Then the balance between new secure bits being created and old secure bits being used up gives an average creation rate per bit of the sifted key of

$$R_{corr} = I_{AB}[e] - \tau_1[e] \tag{12}$$

if we use error correction, and

$$R_{del} = I_{AB}[e] - \tau_1[e](1-e) - e \tag{13}$$

if we discard errors from the key. To obtain the creation rate of secure bits as a fraction of the sent quantum signals we have to multiply R_{corr} and R_{del} by a factor 1/2 for the B92 and the BB84 protocol, and by 1/3 for the six-state protocol. A direct comparison for the resulting rates in case of discarded errors is made in figure 6. The results show that the restriction to eavesdropping attacks on individual signals allows secure quantum key distribution with existing experiments. The tolerable error rates, leading to positive rates, are 4%, 10.5%, and 12% for the three protocols respectively. The six-state protocol gives the lowest gain for error rates below $\approx 0.65\%$ while it becomes superior to the BB84 protocol for error rates bigger than approximately 8%. Though tolerable error rates are achievable with present day experiments, some work still has to be done to cope with the signal states which are not the ideal one-photon states (see section IV A).

For more general strategies than those measuring individual signals the presented way of error correction, estimation of collision probability, and privacy amplification is no longer valid since in that case Eve might make use of the knowledge of the particular hashing function (choice of random subsets for parity bits) to optimize her measurements. Instead, one has to directly estimate the Shannon information on the final key. This has been done for a wide class of *collective attacks* in [7], while bounds in the most general case are obtained in [29,24]. The proof given in [29] leads

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[2] Any information sent through the public channel can be put into code words, using any error correction scheme, to protect it against errors. This encoding into codewords does not change the amount of Shannon information contained, and one codeword can be regarded as one signal.

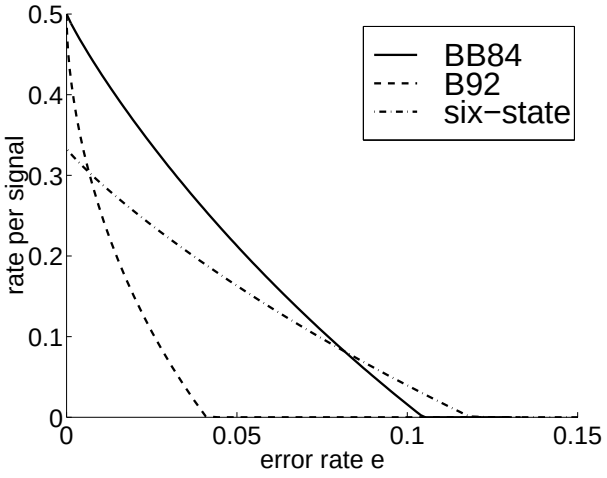

FIG. 6. The rate $\frac{1}{2}R_{del}$ for the B92 protocol with overlap $1/\sqrt{2}$ between the two signal states has been calculated using the results by Slutsky et al. [32]. The rate $\frac{1}{2}R_{del}$ for the BB84 protocol is obtained with the estimates from [32,26] and the estimates leading to the rate $\frac{1}{3}R_{del}$ for the six-state protocol are taken from [1].

to a maximal tolerated error rate of circa 7 %. The proof of [24] uses the Ekert scheme in connection with [13] to tolerate higher error rates, as mentioned in the discussion of the Ekert scheme, but it needs local operations operating with an error rate below the threshold set for fault tolerant quantum computing.

It is important to note that the key generated by quantum key distribution is different from the key assumed in the one-time pad or as seed for the Data Encryption Standard. These keys are assumed to be absolutely secure and certainly shared between Alice and Bob. The key established in quantum key distribution does not carry those absolute attributes. It is not absolute secure. Instead, we can make a statement about it of the following form: With probability $1-\alpha$ an eavesdropper has less Shannon information than a tolerated value I_E^{tol} on that key (secrecy) and it is shared between Alice and Bob with probability $1-\beta$. The two probabilities α and β can be made arbitrary small (on cost of the key rate) as long as the initial error rate is below the cut-off rate mentioned above for the different scenarios. To our knowledge, this subtle difference between the key properties assumed in applications and the key properties resulting from quantum key distribution has not been explored sufficiently yet. Especially, it would be interesting to explore what values for I_E^{tol}, α and β are required for applications.

A. Problems and practicalities

All current implementations of quantum key distribution make use of quantum optical methods. In this context we will discuss realization issues important for the security aspect without going into technical details. The problem of realizing quantum cryptography consists of three parts: realization of the signal states, transportation of the signals to Bob and an efficient measurement of the signals.

The simplest choice of signal states, from the theoretical point of view, are single photons with the polarization as carrier of the signal. However, at present we do not have a source which would give us single photons on demand. Instead, one uses weak laser pulses. On average, each pulse contains typically 0.1 photons. The photon number distribution is such that most pulses contain no photon, around 10% contain one photon and 1% contain more than one photon. The pulses containing more than one photon endanger security of transmission, since an eavesdropper could split off one photon and extract the full information about the signal later on without causing any disturbance of the channel. This has to be taken account of when calculating the amount by which the key is shortened during privacy amplification. The transmission is totally insecure if the number of received signals is smaller than the number of multiple-photon signals sent.

One of the big problems in quantum key distribution is loss of signals in the fibre. It has been shown that strong loss in the transmission going together with multi-photon components of the signal states renders key distribution in all key distribution schemes insecure unless a strong reference pulse is used [22]. This strong reference pulse is an original part of the B92 protocol [2]. It fights the problem that the eavesdropper has means to suppress a signal without causing errors by sending a vacuum state to Bob. A strong reference pulse, however, makes sure that no such state exists.

To keep the error rate low, the set-up should be stable under influence of the environment. In the case of polarization

based cryptography the main error source is cross talk between the two polarization modes and a random (classical) rotation of the polarization along the propagation direction of the fibre. Here the proposals of the BB84 or B92 type are easier to implement than the time separated ideas of Goldenberg/Vaidman and Koashi/Imoto. In the first group the signal travels from Alice to Bob and is influenced by the environment as an entity, while in the second group we have two parts of a signal interacting with two different environments. We therefore cannot expect the error rates of the second group to be as good as the 1% error rates of the first group. This is the reason why no experimental realization of the second group has been tackled yet.

For the detection schemes we find that it poses a problem to lower the amplitude of coherent states below a certain point in order to improve the single-photon approximation. Bob's detectors will give false alarm (dark counts) with a fixed probability proportional to the time the detector is gated. Using weaker pulses will increase the number of dark counts with respect to the real counts, which effectively increases the error rate because a dark count will give a random measurement result.

One of the advantages of the Ekert scheme is that is allows to use quantum privacy amplification, thereby giving a new raw key with lower error rate than the original key. This allows to go below the cut-off rate for the tolerated error rate even with a noisy channel. However, the necessary storing and manipulation devices are not available at present.

B. Experiments

Quantum key distribution was implemented for the first time by Bennett et al. in a demonstration set-up [3]. The transfer of the signals took place over 32 cm of free air with (incoherent) faint pulses. Experimental demonstrations of the BB84 protocol near to commercial realizations are reported by the group at British Telecom by Marand and Townsend [28]. Over a distance up to 30 km they achieved an error rate of 1.5–4 % with an average photon number per pulse of 0.1–0.2 photons.

Several experiments have been done implementing an approximate B92 protocol. In these experiments the strong reference pulse of the original scheme is omitted, thereby using the idea of two non-orthogonal pulses only. It is known that this omission renders the scheme more insecure. Best results regarding low error rate are achieved here by the group in Geneva. They achieve error rates of about 0.5–1.35 % over distances of 23 km with an average photon number of 0.1–0.2 [34]. An initial problem of their scheme to give a low key rate only has now been resolved. Other schemes in free-space key distribution and over fibre are reported by the Los Alamos group, going over 40 km in fibre and 1 km in free space [11]. Variations of the Ekert scheme have been implemented by Rarity et al. [31].

V. OPEN QUESTIONS AND SUMMARY

From the point of fundamental physics the most interesting question is to show security against the most general coherent eavesdropping attack on single photon signals. This has been achieved by Mayer [29] and by Lo and Chau [24]. From the practical point of view these proofs are not relevant yet since they do not deal with realistic situations. For this one would need the use of efficient error correction methods, the ability to cope with large losses and with realistic error rates and, finally, the extension to realistic signals like dim coherent states or photons from parametric downconversion.

For practical purposes it makes sense to restrict eavesdropping strategies to attacks on individual signals. For this scenario workable schemes for single-photon states have been presented in [32,26]. The extension to realistic signal states has been achieved recently [27].

The experimental groups will have to look for set-ups improving the rate at which the key is generated. It is essential to keep in mind that it is not the aim to minimize the error rate but to maximize the key rate!

The questions directed to the audience dealing with the classical part of quantum key distribution are: a) what is a good goal for the security of the final key? b) How good does it have to be? (In terms of I_E^{tol}, α and β as introduced in section IV.) c) What is the optimal reconciliation protocol in these circumstances?

In summary, quantum key distribution is a truly interdisciplinary topic in quantum information. It brings together cryptologists, classical information scientists, and experimental and theoretical physicists. At present, there are physical systems which already produce sifted keys at a reasonable rate with a low error rate. Although the implementation is not ideal, theoretical work should soon show in which scenario it is possible to extract secure keys from that. To optimize procedures more work in error correction etc. is needed. After realizing the nature of security of the final key, we need more input about the specific requirements for applications - as quantum key distribution has already passed the first threshold towards implementation.

ACKNOWLEDGMENTS

The authors took benefit from the 1998 quantum information workshops at ISI (Italy) and Benasque Center for Physics (Spain) and wish to thank their organizers and Elsag–Bailey for support. DB acknowledges support by the European TMR Research Network ERP-4061PL95-1412 and by Deutsche Forschungsgemeinschaft under grant SFB 407, and NL by the Academy of Finland.

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