SOME MORE TERMS RELATED TO FUZZY SETS

ALPHA-CUT

- An α-cut or α-level set of a fuzzy set $A \subseteq X$ is an ORDINARY SET $A_\alpha \subseteq X$, such that:
  $A_\alpha = \{\mu_A(x) \geq \alpha, \ \forall x \in X\}$.

  Consider $X = \{1, 2, 3\}$ and set A

  $A = 0.3/1 + 0.5/2 + 1/3$

  then $A_{0.5} = \{2, 3\}$,

  $A_{0.1} = \{1, 2, 3\}$,

  $A_1 = \{3\}$

FUZZY SET NORMALITY

- A fuzzy subset of X is called **normal** if there exists at least one element $x \in X$ such that $\mu_A(x) = 1$.
- A fuzzy subset that is not normal is called **subnormal**.
- All crisp subsets except for the null set are normal. In fuzzy set theory, the concept of nullness essentially generalises to subnormality.
- The **height** of a fuzzy subset *A* is the large membership grade of an element in *A*

  $h(A) = \max_x(\mu_A(x))$

FUZZY SETS CORE AND SUPPORT

- Assume *A* is a fuzzy subset of *X*:

- the **support** of *A* is the crisp subset of *X* consisting of all elements with membership grade:

$$supp(A) = \{x \mid \mu_A(x) > 0 \text{ and } x \in X\}$$

- the **core** of *A* is the crisp subset of *X* consisting of all elements with membership grade:

$$core(A) = \{x \mid \mu_A(x) = 1 \text{ and } x \in X\}$$

FUZZY SET MATH OPERATIONS

- $aA = \{a\mu_A(x),\ \forall x \in X\}$

  Let a =0.5, and

  A = {0.5/a, 0.3/b, 0.2/c, 1/d}

  then

  A^a = {0.25/a, 0.15/b, 0.1/c, 0.5/d}

- $A^a = \{\mu_A(x)^a,\ \forall x \in X\}$

  Let a =2, and

  A = {0.5/a, 0.3/b, 0.2/c, 1/d}

  then

  A^a = {0.25/a, 0.09/b, 0.04/c, 1/d}

FUZZY SETS EXAMPLES

- Consider two fuzzy subsets of the set X,

  X = {a, b, c, d, e }

  referred to as A and B

  A = {1/a, 0.3/b, 0.2/c 0.8/d, 0/e}

  and

  B = {0.6/a, 0.9/b, 0.1/c, 0.3/d, 0.2/e}

FUZZY SETS EXAMPLES

- Support:

  $supp(A)$ = {a, b, c, d }

  $supp(B)$ = {a, b, c, d, e }

- Core:

  $core(A)$ = {a}

  $core(B)$ = {o}

- Cardinality:

  $card(A) = 1+0.3+0.2+0.8+0 = 2.3$

  $card(B) = 0.6+0.9+0.1+0.3+0.2 = 2.1$

FUZZY SETS EXAMPLES

- Complement:

  A = {1/a, 0.3/b, 0.2/c 0.8/d, 0/e}

  $\neg A$ = {0/a, 0.7/b, 0.8/c 0.2/d, 1/e}

- Union:

  $A \cup B$ = {1/a, 0.9/b, 0.2/c, 0.8/d, 0.2/e}

- Intersection:

  $A \cap B$ = {0.6/a, 0.3/b, 0.1/c, 0.3/d, 0/e}

FUZZY SETS EXAMPLES

- aA:

  for a=0.5

  aA = {0.5/a, 0.15/b, 0.1/c, 0.4/d, 0/e}

- A^a:

  for a=2

  A^a = {1/a, 0.09/b, 0.04/c, 0.64/d, 0/e}

- a-cut:

  $A_{0.2}$ = {a, b, c, d}

  $A_{0.3}$ = {a, b, d}

  $A_{0.8}$ = {a, d}

  A_1 = {a}

FUZZY RULES

- In 1973, Lotfi Zadeh published his second most influential paper. This paper outlined a new approach to analysis of complex systems, in which Zadeh suggested capturing human knowledge in **fuzzy rules**.

- A fuzzy rule can be defined as a conditional statement in the form:

  IF x is A

  THEN y is B

- where x and y are linguistic variables; and A and B are linguistic values determined by fuzzy sets on the universe of discourses X and Y, respectively.

CLASSICAL VS FUZZY RULES

- A classical IF-THEN rule uses binary logic, for example,

Rule: 1	Rule: 2
IF speed is > 100	IF speed is < 40
THEN stopping_distance is long	THEN stopping_distance is short

- The variable speed can have any numerical value between 0 and 220 km/h, but the linguistic variable stopping_distance can take either value long or short. In other words, classical rules are expressed in the black-and-white language of Boolean logic.

CLASSICAL VS FUZZY RULES

- We can also represent the stopping distance rules in a fuzzy form:

Rule: 1	Rule: 2
IF speed is fast	IF speed is slow
THEN stopping_distance is long	THEN stopping_distance is short

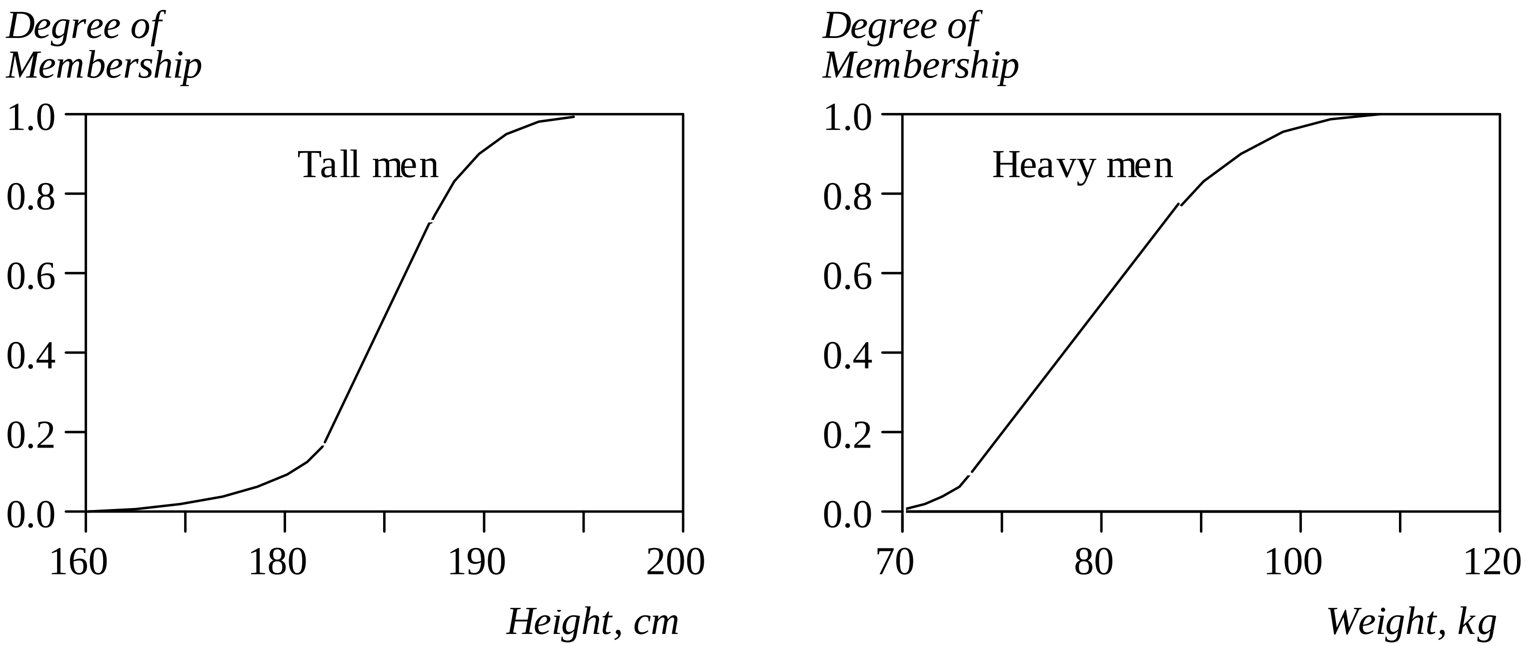
- In fuzzy rules, the linguistic variable speed also has the range (the universe of discourse) between 0 and 220 km/h, but this range includes fuzzy sets, such as slow, medium and fast. The universe of discourse of the linguistic variable stopping_distance can be between 0 and 300 m and may include such fuzzy sets as short, medium and long.

CLASSICAL VS FUZZY RULES

- Fuzzy rules relate fuzzy sets.

- In a fuzzy system, all rules fire to some extent, or in other words they fire partially. If the antecedent is true to some degree of membership, then the consequent is also true to that same degree.

FIRING FUZZY RULES

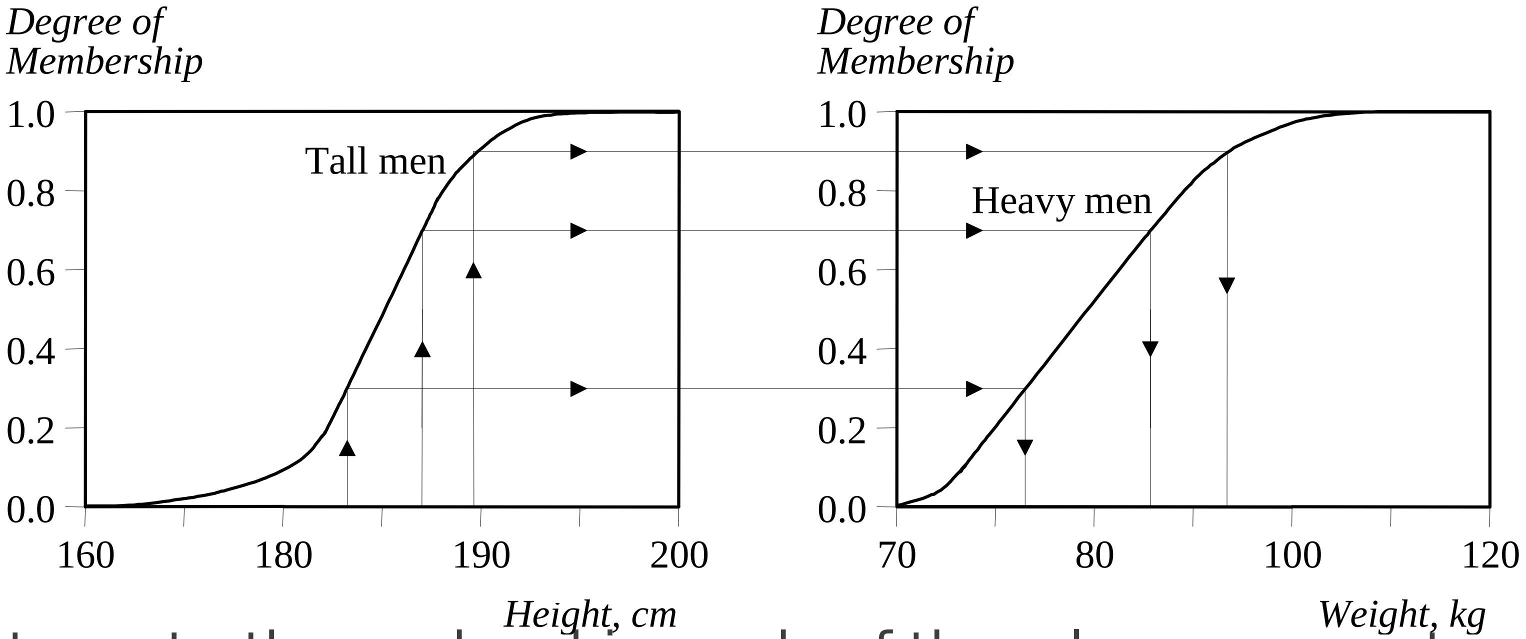


- These fuzzy sets provide the basis for a weight estimation model. The model is based on a relationship between a man's height and his weight:

  IF height is tall

  THEN weight is heavy

FIRING FUZZY RULES



- The value of the output or a truth membership grade of the rule consequent can be estimated directly from a corresponding truth membership grade in the antecedent. This form of fuzzy inference uses a method called **monotonic selection**.

FIRING FUZZY RULES

- A fuzzy rule can have multiple antecedents, for example:

  IF project_duration is long
  AND project_staffing is large
  AND project_funding is inadequate
  THEN risk is high

  IF service is excellent
  OR food is delicious
  THEN tip is generous

- The consequent of a fuzzy rule can also include multiple parts, for instance:

  IF temperature is hot
  THEN hot_water is reduced;
  cold_water is increased

FUZZY SETS EXAMPLE

- Air-conditioning involves the delivery of air which can be warmed or cooled and have its humidity raised or lowered.
- An air-conditioner is an apparatus for controlling, especially lowering, the temperature and humidity of an enclosed space. An air-conditioner typically has a fan which blows/cools/circulates fresh air and has cooler and the cooler is under thermostatic control. Generally, the amount of air being compressed is proportional to the ambient temperature.
- Consider Johnny’s air-conditioner which has five control switches: COLD, COOL, PLEASANT, WARM and HOT. The corresponding speeds of the motor controlling the fan on the air-conditioner has the graduations: MINIMAL, SLOW, MEDIUM, FAST and BLAST.

FUZZY SETS EXAMPLE

- The rules governing the air-conditioner are as follows:

RULE 1:
IF TEMP is COLD THEN SPEED is MINIMAL

RULE 2:
IF TEMP is COOL THEN SPEED is SLOW

RULE 3:
IF TEMP is PLEASANT THEN SPEED is MEDIUM

RULE 4:
IF TEMP is WARM THEN SPEED is FAST

RULE 5:
IF TEMP is HOT THEN SPEED is BLAST

The **temperature** graduations are related to Johnny's perception of ambient temperatures.

where:

Y : *temp* value belongs to the set ($0<\mu_A(x)<1$)

Y* : *temp* value is the ideal member to the set ($\mu_A(x)=1$)

N : temp value is not a member of the set ($\mu_A(x)=0$)

Temp (^{0}C).	COLD	COOL	PLEASANT	WARM	HOT
0	Y*	N	N	N	N
5	Y	Y	N	N	N
10	N	Y	N	N	N
12.5	N	Y*	N	N	N
15	N	Y	N	N	N
17.5	N	N	Y*	N	N
20	N	N	N	Y	N
22.5	N	N	N	Y*	N
25	N	N	N	Y	N
27.5	N	N	N	N	Y
30	N	N	N	N	Y*

Johnny's perception of the **speed** of the motor is as follows:

where:

Y : *temp* value belongs to the set ($0<\mu_A(x)<1$)

Y* : *temp* value is the ideal member to the set ($\mu_A(x)=1$)

N : temp value is not a member of the set ($\mu_A(x)=0$)

Rev/sec (RPM)	MINIMAL	SLOW	MEDIUM	FAST	BLAST
0	Y*	N	N	N	N
10	Y	N	N	N	N
20	Y	Y	N	N	N
30	N	Y*	N	N	N
40	N	Y	N	N	N
50	N	N	Y*	N	N
60	N	N	N	Y	N
70	N	N	N	Y*	N
80	N	N	N	Y	Y
90	N	N	N	N	Y
100	N	N	N	N	Y*

FUZZY SETS EXAMPLE

- The analytically expressed membership for the reference fuzzy subsets for the **temperature** are:

- COLD:

  for $0 \leq t \leq 10$ $\mu_{COLD}(t) = - t / 10 + 1$

- SLOW:

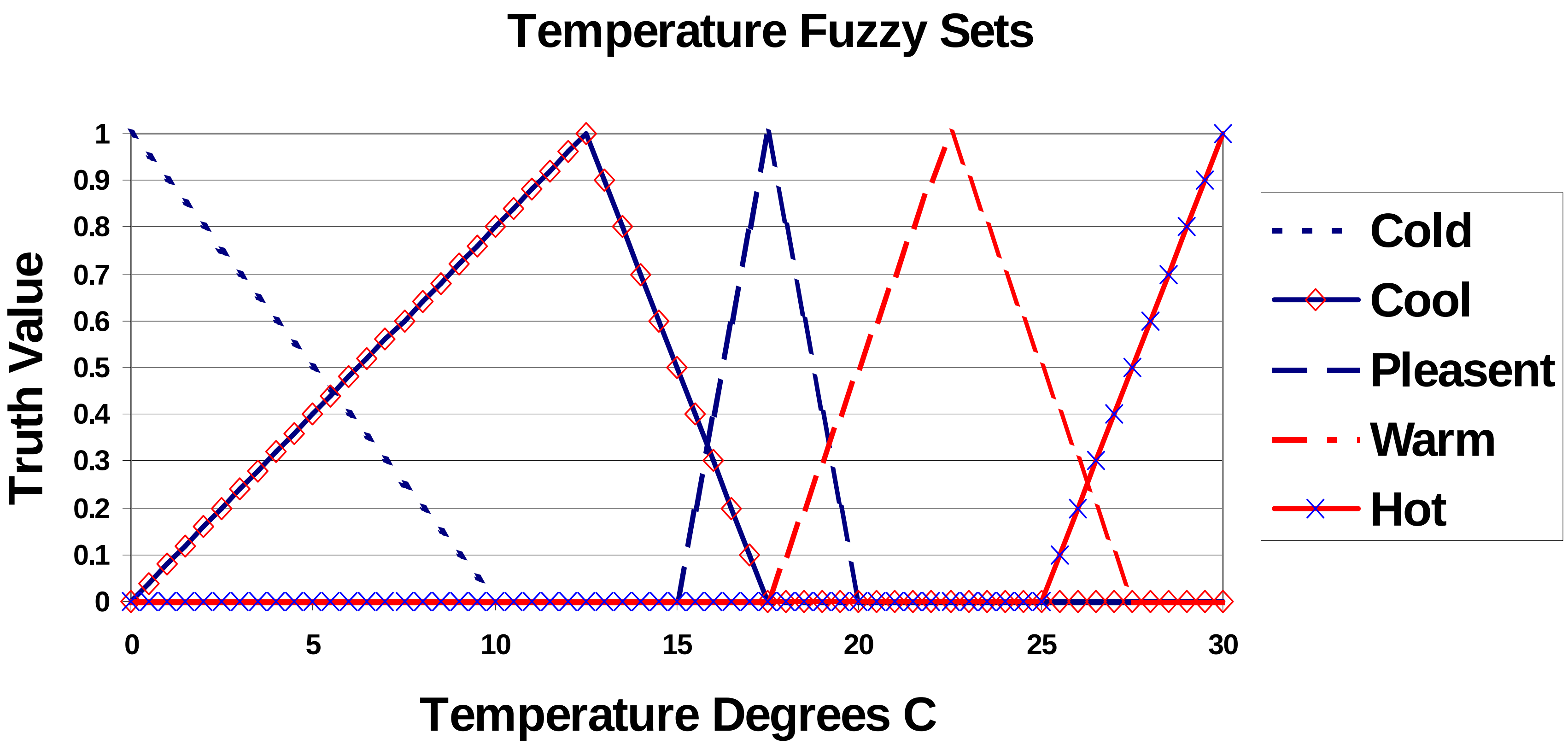
  for $0 \leq t \leq 12.5$ $\mu_{SLOW}(t) = t / 12.5$

  for $12.5 \leq t \leq 17.5$ $\mu_{SLOW}(t) = - t / 5 + 3.5$

- etc… all based on the linear equation:

  $y = ax + b$

FUZZY SETS EXAMPLE



FUZZY SETS EXAMPLE

- The analytically expressed membership for the reference fuzzy subsets for the **temperature** are:

- MINIMAL:

  for $0 \leq v \leq 30$ $\mu_{COLD}(t) = -v / 30 + 1$

- SLOW:

  for $10 \leq v \leq 30$ $\mu_{SLOW}(t) = v / 20 - 0.5$

  for $30 \leq v \leq 50$ $\mu_{SLOW}(t) = -v / 20 + 2.5$

- etc… all based on the linear equation:

  $y = ax + b$

FUZZY SETS EXAMPLE