Expected Value:

Average : $\dfrac{\sum_{i=1}^{n} x_i}{n}$

Population: 3, 3, 3, 4, 5

Average: $\dfrac{3+3+3+4+5}{5} = 3.6$
$(\bar{x})$.

or $\dfrac{3(3)+4(1)+5(1)}{5}$

or, $\dfrac{3}{5}\times 3+\dfrac{1}{5}\times 4+\dfrac{1}{5}\times 5$

or $0.6\times 3+0.2\times 4+0.2\times 5$

probabilities.

$$E[x] = \sum_{\forall x \in X} x\,p(x).$$

Die roll

x	1	2	3	4	5	6
$p(x)$	1/6	1/6	1/6	1/6	1/6	1/6

$$E[X] = \sum_{i=1}^{6} x_i \, p(x_i)$$

$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} \; \ldots \; 6 \times \frac{1}{6}$$

$$= 3.5$$

Say, we roll a die 10 times

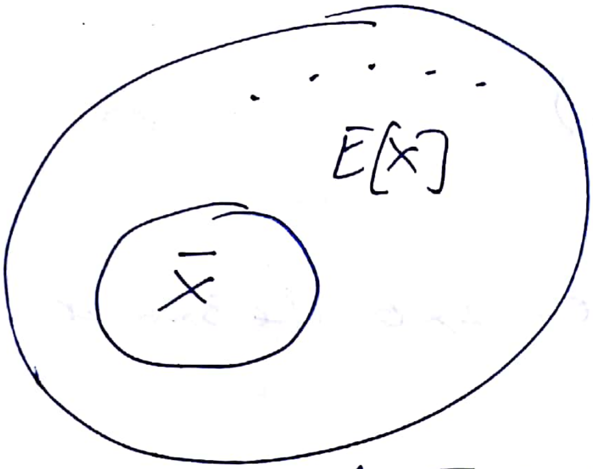
5, 2, 6, 2, 2, 1, 2, 3, 6, 1.

$\bar{X} = 3$.

In this case, $E[x] \neq \bar{X}$

why?

If the no. of rolls is large, then average will converge to the Expected value



Graphical Intuition.

E.g. 2 X = # of pushups in any workout session

X	$p(x)$
0	0.1
1	0.15
2	0.4
3	0.25
4	0.1

$E[X] = ?$

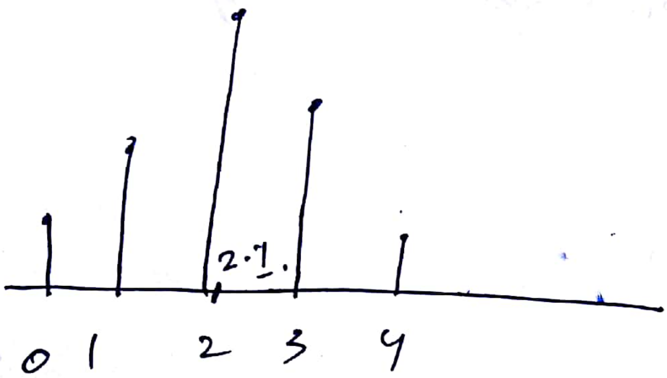
Note: Expected value is the weighted outcome.

~~of event.~~

$$E[x] = \sum_{\forall x} x\,p(x)$$

$$= 0\times 0.1 + 1\times 0.15 + 2\times 0.4 + 3\times 0.25 + 4\times 0.1$$

$$= 2.1.$$

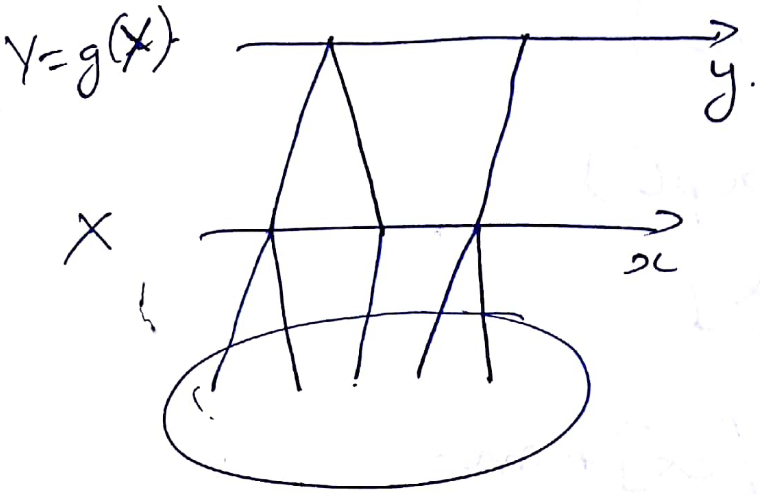


Properties

1) If $Y = g(x)$

$$E[Y] = \sum_{x} g(x)\,p_X(x)$$

③



2) $E[c] = c.$

$E[3].$

It is a RV that always takes 3

$\Rightarrow p = 1.$

$\Rightarrow E[3] = \sum_x x\, p(x)$

$= 3 \times 1 = 3.$

3) $E[cX] = c\,E[X].$

M - Money in dollar

M' - " " INR

$E[60M] = 60 \times E[M].$

or, $E[cX] = \sum_{x} g(x)\, p_X(x)$

$= \sum \alpha x p(x)$

$= \alpha \sum x p(x)$

$= \alpha E[x]$.

3) $E[cX+m] = E[cx]+m$.

$= cE[X]+m$.

Proof on next sheet.

Question

X	PMF
0	0.2
1	0.5
2	0.3.

Calculate $E[X]$, $E[X^2]$, $E[X^3]$.

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Proof of 3

$$E[aX+b] = \sum_{x} (ax+b)\, p(x)$$

$$= \sum_{x} ax\, p(x) + \sum_{x} b\, p(x)$$

$$= a \sum_{x} x\, p(x) + b \sum_{x} p(x)$$

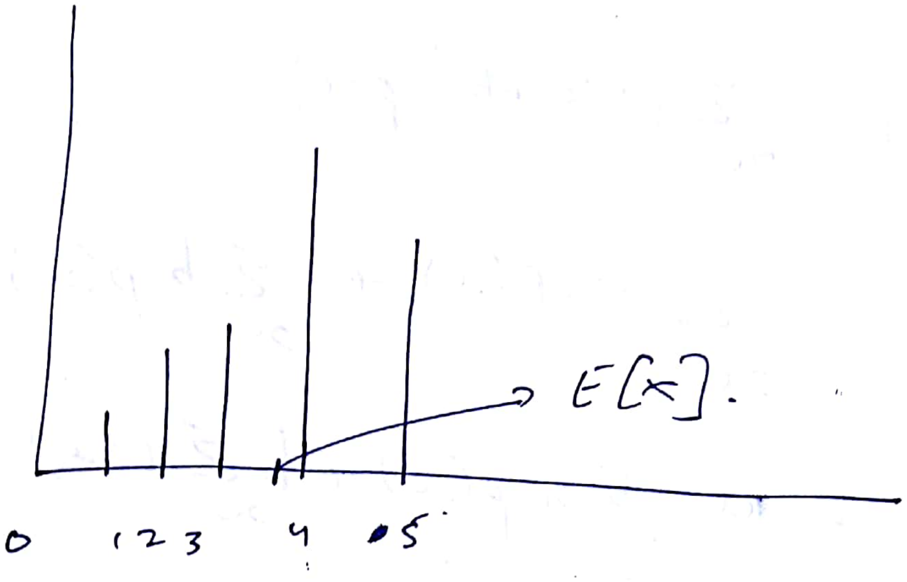
$$= a\, E[X] + b.$$

E.g. 3 Feedback

1 – very bad

5 – very satisfied.

Rating	Freq.	p	$x\,p(x)$
1	– 5	0.046	0.046
2	– 10	0.093	0.186
3	– 11	0.102	0.306
4	– 44	0.407.	1.628
5	– 38	0.351.	1.755
	108	1.	3.7.



Variance.

R.V. number.

$$\mathrm{var}(X) = E[(X - E[X])^2]$$

Variance gives an idea of how far away from the mean we expect to be on an average.

Variance : Emphasis on outliers.

Why

$(0.1)^2 = 0.01$ larger values are penalized.

$(1.1)^2 = 1.21$

5

Proof

$$var(x) = E[(X - p\,E[x])^2] = E[x^2] - (E[x])^2$$

let's assume.

$$g(x) = (x - E[x])^2.$$

$$E[(X - E[x])^2] = \sum[(x - E[x])^2 \times p(x)]$$

$$= \sum(x^2 + (E[x])^2 - 2x\,E[x])\,p(x)$$

$$= \sum x^2 p(x) + \cancel{\sum(E[x])^2 p(x)}$$

$$\sum((E[x])^2 - 2x\,E[x])\,p(x).$$

or, $\sum x^2 p(x) + \sum(E[x])^2 \times p(x) - \sum 2x\,E[x]\,p(x)$

or $\sum x^2 p(x) + (E[x])^2 \sum p(x) - 2\,E[x] \sum x\,p(x)$

or $E[x^2] + (E[x])^2 - 2(E[x])^2$

$\Rightarrow E[x^2] - (E[x])^2.$

Properties of variance.

i) $Var[x] \geq 0$

ii) $Var(x+b) = Var(x)$.

iii) $Var(ax+b) = a^2 Var(x)$.

proof.

$$Var[ax+b] = E[(ax+b - E[ax+b])^2]$$

$$= E[(ax+\cancel{b} - aE[x] - \cancel{b})^2]$$

$$= E[a^2(x - E[x])^2]$$

$$= a^2 E[(x - E[x])^2].$$

$$= a^2 var(x).$$

Question $Var(b) = ?$

e.g. $Var(9) = ?$

It is zero.