PRACTICE PROBLEMS BASED ON SET ASSOCIATIVE MAPPING-

(**Ref.** https://www.gatevidyalay.com/set-associative-mapping-practice-problems/)

Problem-01:

Consider a 2-way set associative mapped cache of size 16 KB with block size 256 bytes. The size of main memory is 128 KB. Find-

- 1. Number of bits in tag
- 2. Tag directory size

Solution-

Given-

- Set size = 2
- Cache memory size = 16 KB
- Block size = Frame size = Line size = 256 bytes
- Main memory size = 128 KB

We consider that the memory is byte addressable.

Number of Bits in Physical Address-

We have,

Size of main memory

- = 128 KB
- $= 2^{17}$ bytes

Thus, Number of bits in physical address = 17 bits



Number of Bits in Block Offset-

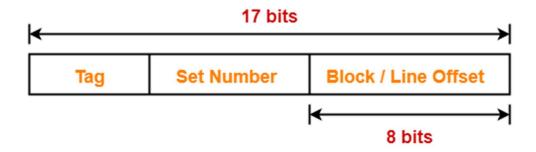
We have,

Block size

= 256 bytes

= 2⁸ bytes

Thus, Number of bits in block offset = 8 bits



Number of Lines in Cache-

Total number of lines in cache

- = Cache size / Line size
- = 16 KB / 256 bytes
- = 2^{14} bytes / 2^8 bytes
- = 64 lines

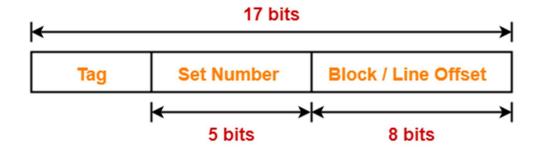
Thus, Number of lines in cache = 64 lines

Number of Sets in Cache-

Total number of sets in cache

- = Total number of lines in cache / Set size
- = 64 / 2
- = 32 sets
- = 2⁵ sets

Thus, Number of bits in set number = 5 bits

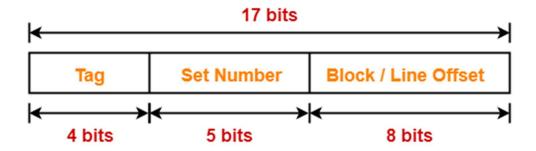


Number of Bits in Tag-

Number of bits in tag

- = Number of bits in physical address (Number of bits in set number + Number of bits in block offset)
- = 17 bits (5 bits + 8 bits)
- = 17 bits 13 bits
- = 4 bits

Thus, Number of bits in tag = 4 bits



Tag Directory Size-

Tag directory size

- = Number of tags x Tag size
- = Number of lines in cache x Number of bits in tag
- $= 64 \times 4 \text{ bits}$
- = 256 bits
- = 32 bytes

Thus, size of tag directory = 32 bytes

Also Read-Practice Problems On Direct Mapping

Problem-02:

Consider a 8-way set associative mapped cache of size 512 KB with block size 1 KB. There are 7 bits in the tag. Find-

- 1. Size of main memory
- 2. Tag directory size

Solution-

Given-

- Set size = 8
- Cache memory size = 512 KB
- Block size = Frame size = Line size = 1 KB
- Number of bits in tag = 7 bits

We consider that the memory is byte addressable.

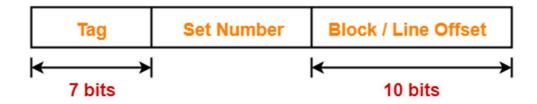
Number of Bits in Block Offset-

We have,

Block size

- = 1 KB
- $= 2^{10}$ bytes

Thus, Number of bits in block offset = 10 bits



Number of Lines in Cache-

Total number of lines in cache

- = Cache size / Line size
- = 512 KB / 1 KB
- = 512 lines

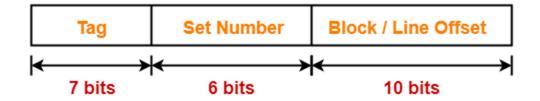
Thus, Number of lines in cache = 512 lines

Number of Sets in Cache-

Total number of sets in cache

- = Total number of lines in cache / Set size
- = 512 / 8
- = 64 sets
- $= 2^6 \text{ sets}$

Thus, Number of bits in set number = 6 bits



Number of Bits in Physical Address-

Number of bits in physical address

- = Number of bits in tag + Number of bits in set number + Number of bits in block offset
- = 7 bits + 6 bits + 10 bits
- = 23 bits

Thus, Number of bits in physical address = 23 bits

Size of Main Memory-

We have,

Number of bits in physical address = 23 bits

Thus, Size of main memory

- $= 2^{23}$ bytes
- = 8 MB

Tag Directory Size-

Tag directory size

- = Number of tags x Tag size
- = Number of lines in cache x Number of bits in tag
- $= 512 \times 7 \text{ bits}$
- = 3584 bits
- = 448 bytes

Thus, size of tag directory = 448 bytes

Problem-03:

Consider a 4-way set associative mapped cache with block size 4 KB. The size of main memory is 16 GB and there are 10 bits in the tag. Find-

- 1. Size of cache memory
- 2. Tag directory size

Solution-

Given-

- Set size = 4
- Block size = Frame size = Line size = 4 KB
- Main memory size = 16 GB
- Number of bits in tag = 10 bits

We consider that the memory is byte addressable.

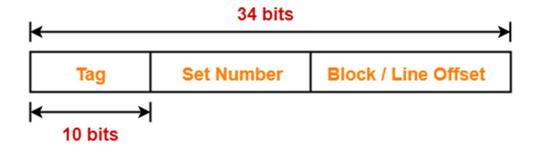
Number of Bits in Physical Address-

We have,

Size of main memory

- = 16 GB
- $= 2^{34}$ bytes

Thus, Number of bits in physical address = 34 bits



Number of Bits in Block Offset-

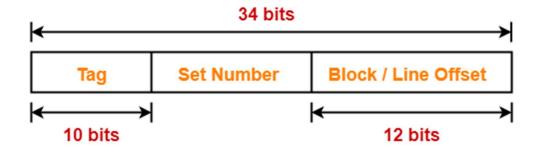
We have,

Block size

= 4 KB

 $= 2^{12}$ bytes

Thus, Number of bits in block offset = 12 bits



Number of Bits in Set Number-

Number of bits in set number

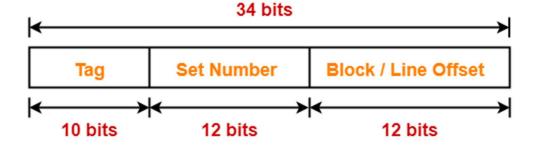
= Number of bits in physical address – (Number of bits in tag + Number of bits in block offset)

= 34 bits - (10 bits + 12 bits)

= 34 bits - 22 bits

= 12 bits

Thus, Number of bits in set number = 12 bits



Number of Sets in Cache-

We have-

Number of bits in set number = 12 bits

Thus, Total number of sets in cache = 2^{12} sets

Number of Lines in Cache-

We have-

Total number of sets in cache = 2^{12} sets

Each set contains 4 lines

Thus,

Total number of lines in cache

- = Total number of sets in cache x Number of lines in each set
- $= 2^{12} \times 4 \text{ lines}$
- = 2¹⁴ lines

Size of Cache Memory-

Size of cache memory

- = Total number of lines in cache x Line size
- $= 2^{14} \times 4 \text{ KB}$

- $= 2^{16} \text{ KB}$
- = 64 MB

Thus, Size of cache memory = 64 MB

Tag Directory Size-

Tag directory size

- = Number of tags x Tag size
- = Number of lines in cache x Number of bits in tag
- $= 2^{14} \times 10 \text{ bits}$
- = 163840 bits
- = 20480 bytes
- = 20 KB

Thus, size of tag directory = 20 KB

Also Read-Practice Problems On Fully Associative Mapping

Problem-04:

Consider a 8-way set associative mapped cache. The size of cache memory is 512 KB and there are 10 bits in the tag. Find the size of main memory.

Solution-

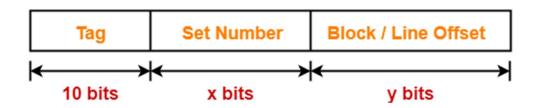
Given-

- Set size = 8
- Cache memory size = 512 KB
- Number of bits in tag = 10 bits

We consider that the memory is byte addressable.

Let-

- Number of bits in set number field = x bits
- Number of bits in block offset field = y bits



Sum of Number Of Bits Of Set Number Field And Block Offset Field-

We have.

Cache memory size = Number of sets in cache x Number of lines in one set x Line size Now, substituting the values, we get-

512 KB =
$$2^{x}$$
 x 8 x 2^{y} bytes
 2^{19} bytes = 2^{3+x+y} bytes

$$19 = 3 + x + y$$

$$x + y = 19 - 3$$

$$x + y = 16$$

Number of Bits in Physical Address-

Number of bits in physical address

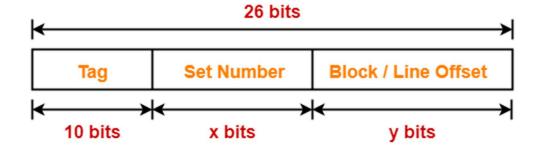
= Number of bits in tag + Number of bits in set number + Number of bits in block offset

= 10 bits +
$$x$$
 bits + y bits

= 10 bits +
$$(x + y)$$
 bits

= 26 bits

Thus, Number of bits in physical address = 26 bits



Size of Main Memory-

We have,

Number of bits in physical address = 26 bits

Thus, Size of main memory

 $= 2^{26}$ bytes

= 64 MB

Thus, size of main memory = 64 MB

Problem-05:

Consider a 4-way set associative mapped cache. The size of main memory is 64 MB and there are 10 bits in the tag. Find the size of cache memory.

Solution-

Given-

- Set size = 4
- Main memory size = 64 MB
- Number of bits in tag = 10 bits

We consider that the memory is byte addressable.

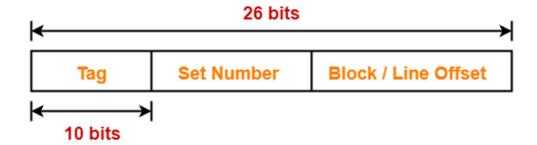
Number of Bits in Physical Address-

We have,

Size of main memory

- = 64 MB
- $= 2^{26}$ bytes

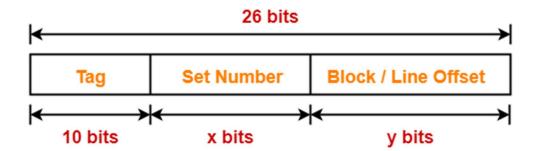
Thus, Number of bits in physical address = 26 bits



Sum Of Number Of Bits Of Set Number Field And Block Offset Field-

Let-

- Number of bits in set number field = x bits
- Number of bits in block offset field = y bits



Then, Number of bits in physical address

= Number of bits in tag + Number of bits in set number + Number of bits in block offset

So, we have-

26 bits = 10 bits +
$$x$$
 bits + y bits

$$26 = 10 + (x + y)$$

$$x + y = 26 - 10$$

$$x + y = 16$$

Thus, Sum of number of bits of set number field and block offset field = 16 bits

Size of Cache Memory-

Cache memory size

- = Number of sets in cache x Number of lines in one set x Line size
- $= 2^{x} x 4 x 2^{y}$ bytes
- = 2^{2+x+y} bytes
- $= 2^{2+16}$ bytes
- $= 2^{18}$ bytes
- = 256 KB