Department of Mathematics, Bennett University
Engineering Calculus (EMAT101L)
Tutorial Sheet 2

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1. Let $\{a_n\}$ be a sequence of real numbers. Define the sequence $\{s_n\}$ by $s_n = \frac{1}{n}\sum_{i=1}^{n} a_i$. If $\{a_n\}$ is monotone and bounded show that $\{s_n\}$ is also monotone and bounded.

2. Show that the following sequence is bounded below by $\frac{1}{2}$ and nonincreasing
$$s_1 = 1, s_{n+1} = \frac{1}{3}(s_n + 1),\ n \geq 1.$$

3. If $a_1 > 0$ and for $n \geq 1$, $a_{n+1} = \frac{1}{2}\left(a_n + \frac{2}{a_n}\right)$, then show that the sequence $\{a_n\}$ is nonincreasing and bounded below. Also, find the limit.

4. Find the limit superior and the limit inferior for the the following sequences:
   (a) $a_n = \sin\frac{n\pi}{3}$, $n \in \mathbb{N}$.
   (b) $a_n = n(1 + (-1)^n)$, $n \in \mathbb{N}$.
   (c) $a_n = \frac{(-1)^n}{n^2}$, $n \in \mathbb{N}$.

5. For $|x| < 1$ and a fixed positive integer m, prove that
$$\lim_{n\to\infty} \frac{m(m-1)(m-2)....(m-n+1)}{n!}x^n = 0.$$

6. Let $x_n = \frac{n^2}{n^3+n+1} + \frac{n^2}{n^3+n+2} + \cdots + \frac{n^2}{n^3+2n}$. Then show that $\lim_{n\to\infty} x_n = 1$.