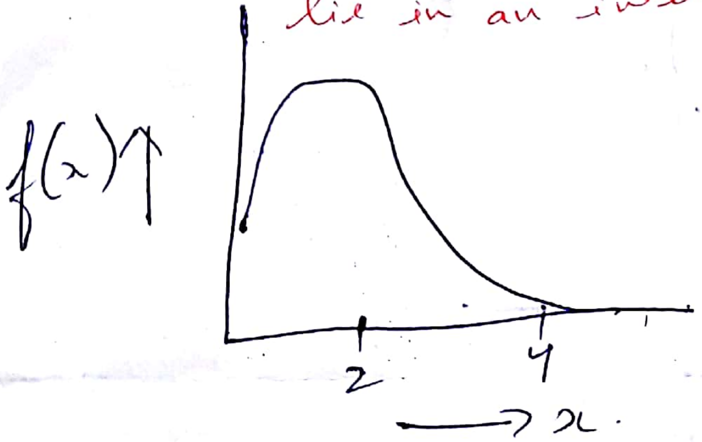
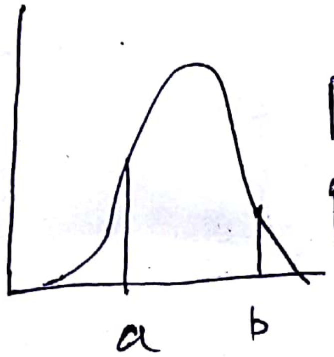
Continous RVs : They can take on an infinite number of values corresponding to every value in the interval.

Here to model probabilities, we use

PDF : It is a function whose value gives the ~~...~~ likelihood that the value of a RV would lie in an interval.



$f(x)$ = ht. of the curve at x.



$P(a < X < b)$ = Area under the curve.
$P(a \leq X \leq b)$

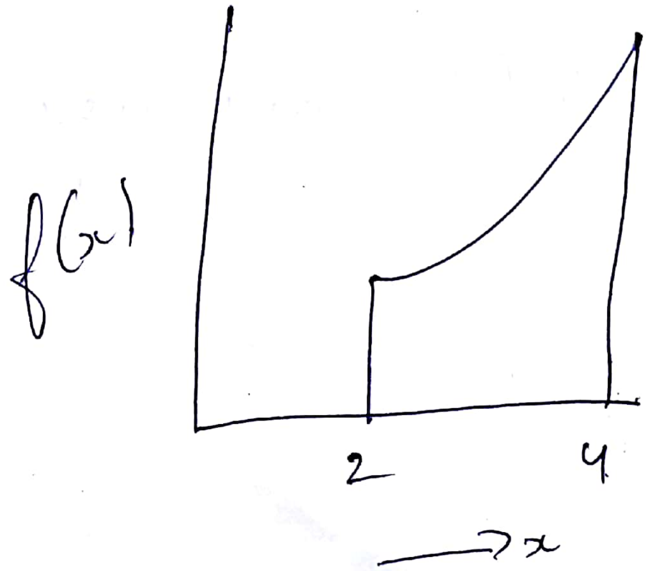
$f(x) \geq 0$

~~Area~~ $\int_{-\infty}^{\infty} f(x) = 1.$

E.g.

$$f(x) = \frac{1}{60}x^3 \quad \text{for } 2 \le x \le 4$$

$$P(X > 3) = \int_3^4 f(x)\,dx = 0.729.$$



$$E[X] = \int_{-\infty}^{\infty} x f(x)\,dx.$$

$$= \int_2^4 x \times \frac{1}{60} x^3 \, dx$$

$$= \frac{1}{60}\left[\frac{x^5}{5}\right]_2^4.$$

$$= \frac{992}{300}$$

Continous RV

①

Recall in discrete case, we use PMF.

$$P(X=x).$$

In continous case, we use PDF.

$$P(a \leq X \leq b) = \int_a^b f_X(x)\,dx.$$

By property of continous spaces

$$P(X=a)=0.$$

$$2k+1$$

Think of it as:

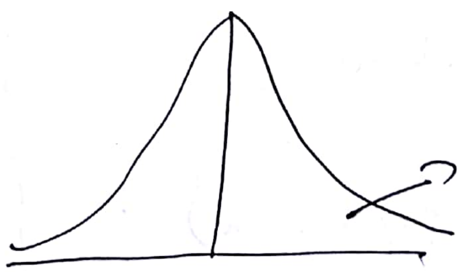
$$(2k+1)$$

$$\int_a^a f(x)\,dx = 0. \qquad 2(k+1)+1.$$

$$2k+3.$$

$$f(x) \geq 0$$

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1.$$



$$\int_{-\infty}^{\infty} f(x) = 1$$

$\longrightarrow x.$

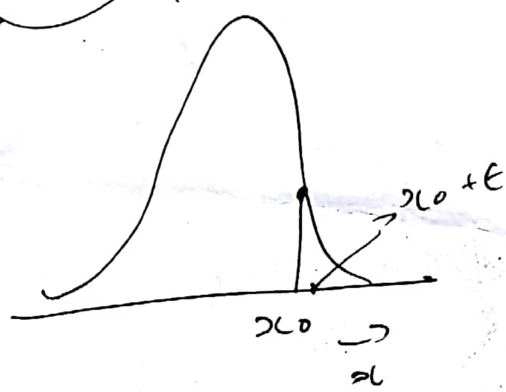
$\Rightarrow$ Probability is nothing but area under the curve

Question $f_x(x = x_0) = 0$ The probability

However, probability density is

$$f(x_0) = \int^{x_0+\epsilon} f(x)\, dx.$$ ϵ is very small.

X_0



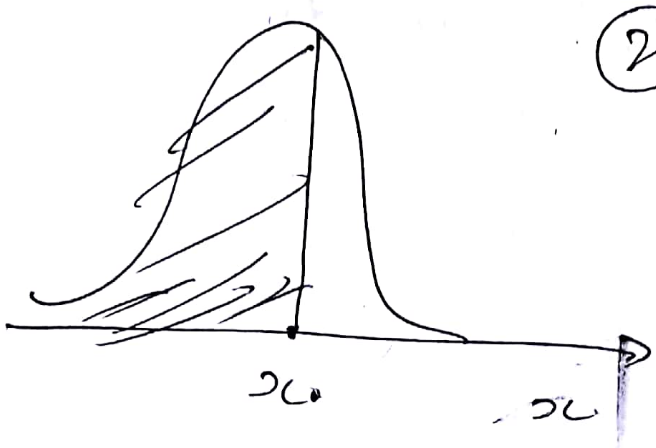
$\rightarrow$ Here $f(x_0)$ now becomes density.

We call it as probability density.

CDF

By definition

$$F_x(x) = P(X \le x) = \int_{-\infty}^{x} f_x(x)\, dx.$$



Shaded region is now the CDF.

$$P(a \le X < b) = \int_a^b f(x)\,dx$$

$$= F(b) - F(a).$$

Expectation

Discrete

$$E[X] = \sum_{\forall x} x\,p(x)$$

Continuous case

$$E[X] = \int_{-\infty}^{\infty} x\, f_X(x)\,dx.$$

$$\mathrm{Var}(X) = \cancel{\int_{-\infty}^{\infty} (x - E[X])^2 f_X(x)\,dx}\; E[(X - E[X])^2].$$

$$E[g(x)] = \int_{-\infty}^{\infty} g(x)\, f_X(x)\, dx$$

Uniform Distribution (a, b)

say we want to pick a random point in $[a, b]$. All points equally likely.

$$f(x) = \begin{cases} c & a \leq x \leq b \\ 0 & \text{otherwise.} \end{cases}$$

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1.$$

For uniform dist.

$$\int_a^b c\, dx = 1.$$

$$\Rightarrow c = \frac{1}{b-a}.$$