Department of Mathematics, Bennett University
Engineering Calculus (EMAT101L)
Tutorial Sheet 4

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1. Show that each of the following limits does not exist:

   $(a)\ \lim_{x\to 0}\cos\left(\frac{1}{x}\right), \quad (b)\ \lim_{x\to 0}\frac{1}{x}, \quad (c)\ \lim_{x\to a}\sin\left(\frac{1}{(x-a)^{1/k}}\right),\ k\geq 1.$

2. Determine the points and nature of discontinuity of the following functions:

   $(a)\ \frac{1}{x-2}-3x, \ \ (b)\ \frac{\cos x}{x-\pi/2}, \ \ (c)\ \frac{\sqrt{x^4+1}}{1+\sin^2 x}.$

3. Find the asymptotes of the graph of the following:

   $(a)\ \frac{x^2-3}{2x-4}, \quad (b)\ \frac{x^2-4}{x-1}, \quad (c)\ \frac{x^3+1}{x}.$

4. Determine which of the following functions are uniformly continuous in the interval mentioned:

   $(a)\ e^{x^2}\sin x^2, \ \ (0,1), \quad (b)\ |\sin x|, \ \ [0,\infty), \quad (c)\ \sqrt{x}\sin x, \ (0,1).$

5. Determine if the following equations admits solutions in the interval mentioned.

   $(a)\ x^5-3x^2=-1, \ [0,1], \ \ (b)\ \sin^2 x-2\cos x=-1, \ \left[0,\frac{\pi}{2}\right], \ \ (c)\ \sin x=\frac{2x-1}{x+2}, \ [0,\pi].$

6. (a) Give an example of a function which is discontinuous at every point of $\mathbb{R}$.

   (b) Give an example of a function which is continuous only at one point.

   (c) Give an example of a function which is continuous only at the integers.

   (d) Give an example of a function which is discontinuous at every rational point, but continuous at every irrational point.

   (e) Give an example of a function which is continuous everywhere.