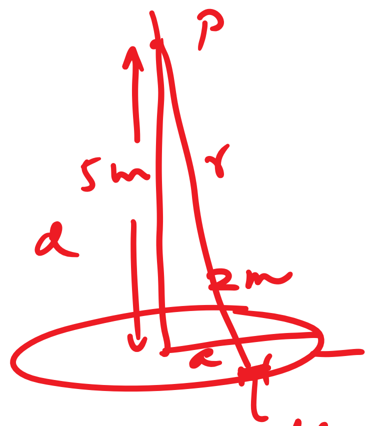
Live simply so that others can simply live

Mahatma Gandhi



$Q = 50\,nC$

$\lambda\, dl$

$$dV = \frac{\lambda\, dl}{4\pi\epsilon_0 r} = \frac{\lambda\, dl}{4\pi\epsilon_0 \sqrt{a^2 + d^2}}$$

$$\vec{E} = -\nabla V$$

$V(x, y, z)$

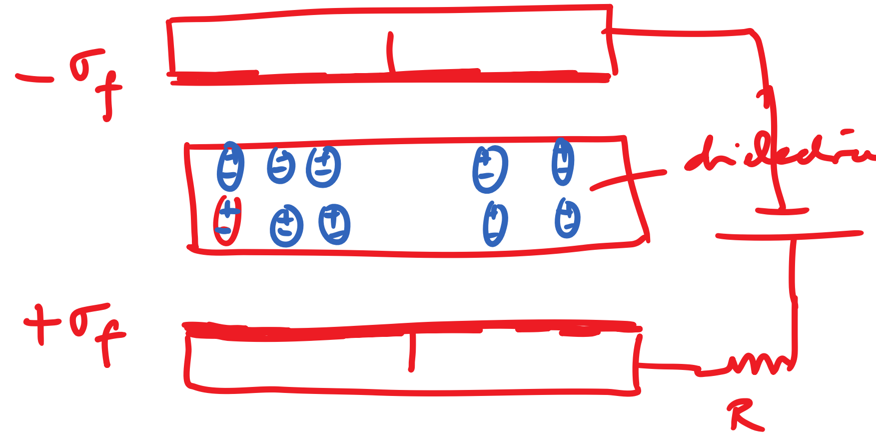
$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \times \vec{E} = 0$$

$$\vec{E} = -\nabla V$$

$\vec{P}$: Polarization

Dipole moment per unit volume



$\vec{P} \equiv$

Bound Volume charge density $\quad \rho_b = -\nabla \cdot \vec{P}$

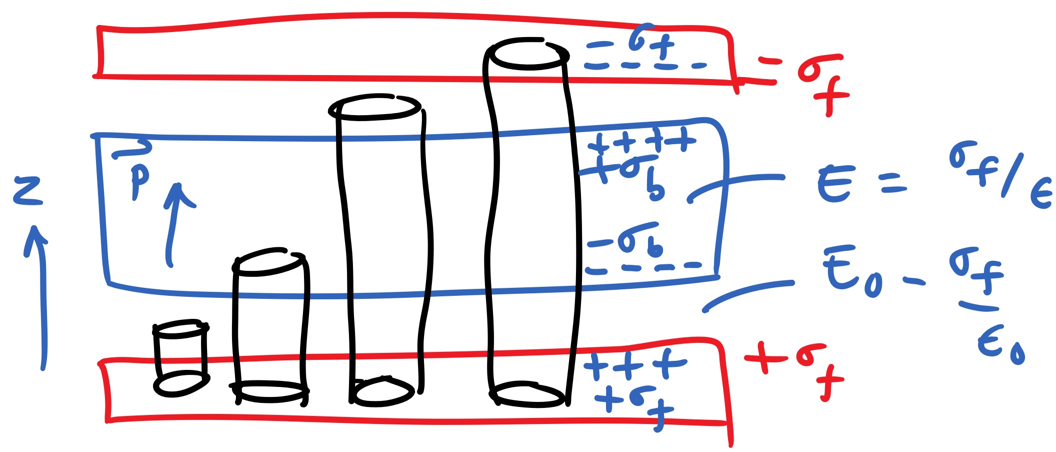
Bound Surface charge density $\quad \sigma_b = \vec{P} \cdot \hat{n}$

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$: DISPLACEMENT VECTOR

$$\boxed{\vec{\nabla} \cdot \vec{D} = \rho_f}$$

$$\boxed{\oint \vec{D} \cdot d\vec{a} = Q_{f,enc}}$$

$\vec{P} = \epsilon_0 \chi \vec{E}$



$\vec{D} = \sigma_f \hat{z}$ in all regions between the capacitor plates

In free space $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

$$= \epsilon_0 \vec{E}$$

$$\boxed{\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{\sigma_f}{\epsilon_0} \hat{z}}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$= \epsilon_0 \vec{E} + \epsilon_0 \chi \vec{E}$$

$$= \epsilon_0 (1+\chi) \vec{E}$$

$$= \epsilon \vec{E}$$

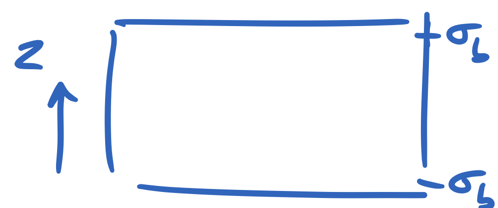
$\epsilon = \epsilon_0 (1+\chi)$
Permittivity of the dielectric

Within the dielectric

$$\boxed{\vec{E} = \frac{\vec{D}}{\epsilon} = \frac{\sigma_f}{\epsilon} \hat{z}}$$

$$\vec{E} = \vec{E}_0 + \vec{E}_b$$

$$= \frac{\sigma_f}{\epsilon_0} \hat{z} - \frac{\sigma_b}{\epsilon_0} \hat{z}$$



$$\vec{P} = \epsilon_0 \chi \vec{E}$$

$$\sigma_b = \vec{P} \cdot \hat{n} = \epsilon_0 \chi E$$

$$\vec{E} = \frac{\sigma_f}{\epsilon_0} \hat{z} - \chi \vec{E}$$

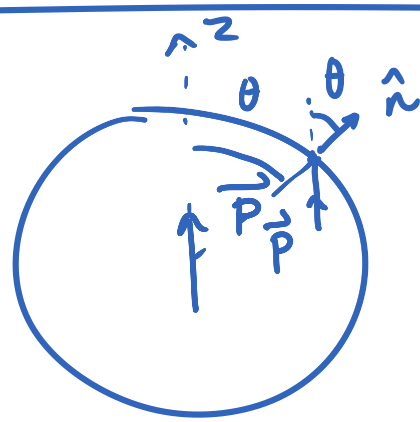
$$(1+\chi) \vec{E} = \frac{\sigma_f}{\epsilon_0} \hat{z} \quad \Rightarrow \quad \vec{E} = \frac{\sigma_f}{\epsilon_0 (1+\chi)} \hat{z}$$

$$= \frac{\sigma_f}{\epsilon} \hat{z}$$

$$\sigma_b = P = \epsilon_0 \chi \underbrace{\frac{\sigma_f}{\epsilon}}_{|\vec{E}|} = \frac{\cancel{\epsilon_0} \chi}{\cancel{\epsilon_0}(1+\chi)} \sigma_f$$

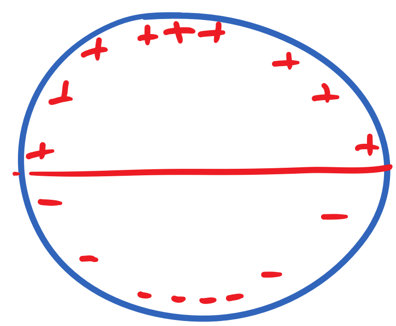
$$\boxed{\sigma_b = \frac{\chi}{1+\chi} \sigma_f}$$

Dielectric sphere with uniform polarization $\overline{P}$



$$\rho_b = -\vec{\nabla} \cdot \vec{P} = 0$$

$$\sigma_b = \vec{P} \cdot \hat{n} = P \cos\theta$$



V of uniformly polarized sphere ?

$\propto \vec{E}$

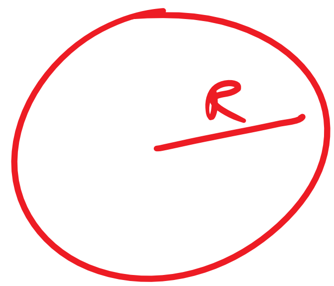
$$V = \frac{1}{4\pi\epsilon_0} \iiint_{\text{Volume}} \frac{\vec{P} \cdot (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} d\tau$$

$$= \frac{1}{4\pi\epsilon_0} \vec{P} \cdot \underbrace{\iiint_{\text{Sphere}} \frac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} d\tau}$$

Uniformly charged sphere with charge density ρ

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \iiint_{\text{sphere}} \frac{\rho\,(\vec{r}-\vec{r}')\,d\tau}{|\vec{r}-\vec{r}'|^3}$$

$$= \frac{1}{4\pi\epsilon_0}\,\rho \iiint \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,d\tau$$

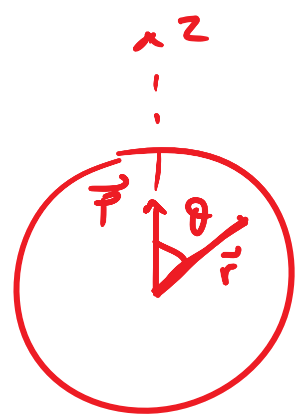


$$\vec{E} = \frac{\rho\,\vec{r}}{3\epsilon_0} \qquad r < R$$

<u>$r < R$</u>

$$\frac{1}{4\pi\cancel{\epsilon_0}}\,\cancel{\rho} \iiint \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,d\tau = \frac{\cancel{\rho}\,\vec{r}}{3\cancel{\epsilon_0}}$$

$$\iiint \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}\,d\tau = \frac{4\pi\vec{r}}{3} \qquad r < R$$



$$V = \frac{1}{4\pi\epsilon_0}\,\frac{4\pi}{3}\,\vec{P}\cdot\vec{r}$$

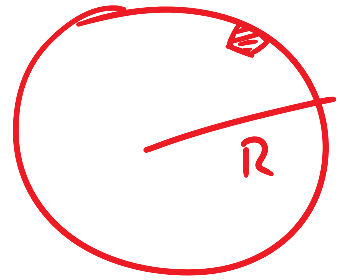
$$= \frac{\vec{P}\cdot\vec{r}}{3\epsilon_0} = \frac{P r \cos\theta}{3\epsilon_0}$$

$$\boxed{V = \frac{Pz}{3\epsilon_0}} \qquad r < R$$

$$\vec{E} = -\vec{\nabla} V = -\frac{P}{3\epsilon_0}\hat{z} = -\frac{\vec{P}}{3\epsilon_0}$$

$$\sigma = P\cos\theta$$

$$\vec{E} = -\frac{\vec{P}}{3\epsilon_0} \quad r<R$$



$$Q = \int_0^{\pi}\int_0^{2\pi} \sigma R^2 \sin\theta \, d\theta \, d\phi \qquad dA = R^2 \sin\theta \, d\theta \, d\phi$$

$$= \; ??$$