Nodal Analysis and Mesh Analysis

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Nodal Analysis and Mesh Analysis

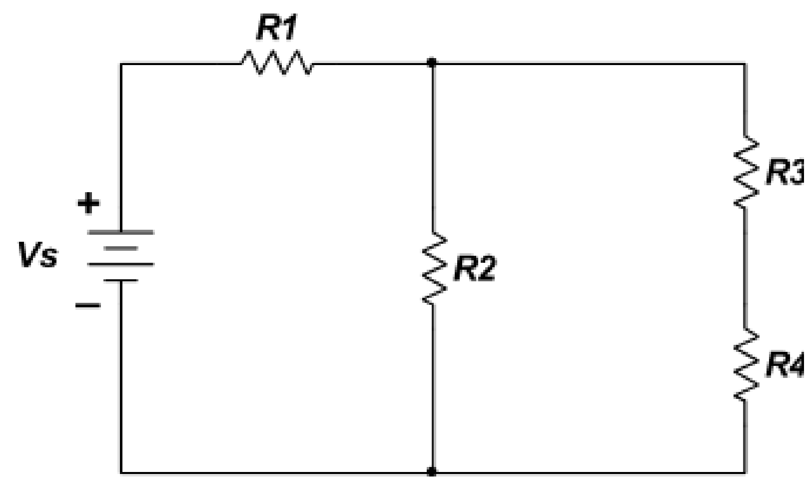
- Two powerful methods used to analyse a given circuit
- Based on systematic application of Kirchhoff's laws
- Can be used to automate circuit analysis
- Applications
  - Design of large circuits (Integrated Circuits (ICs) or chips)
  - Board design

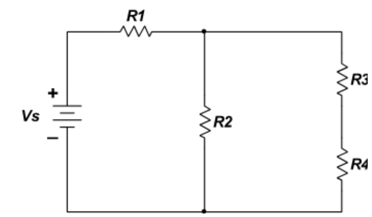
Nodal Analysis

- A voltage is always defined as the potential difference between two points.
- The voltage at a certain point of a circuit means that the voltage is measured between the point under consideration and some other point in the circuit.
- In most cases that reference point is referred to as ground.
- The node method or the node voltage method or nodal analysis, is a very powerful approach for circuit analysis
- Nodal analysis based on the application of KCL, KVL and Ohm’s law.

Nodal Analysis

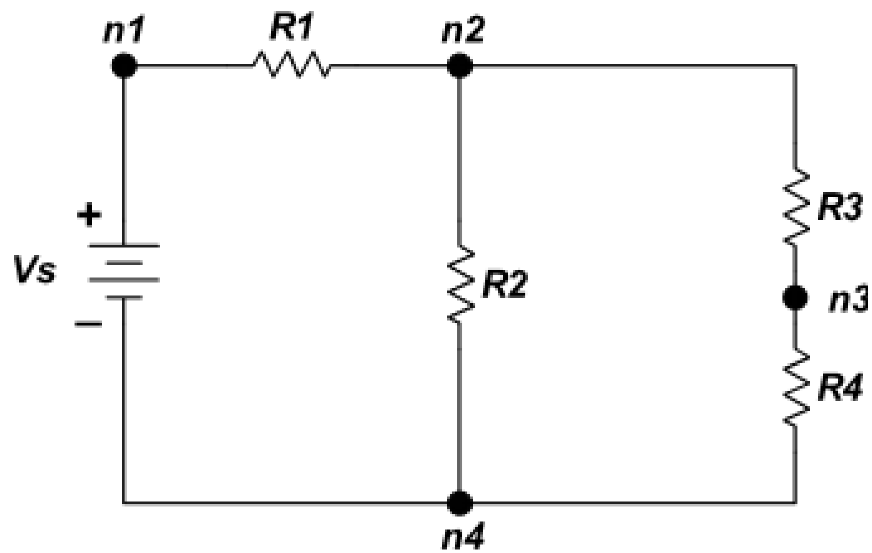
- Consider a typical circuit shown in the figure
- The procedure for analyzing a circuit with the node method is based on the few steps.

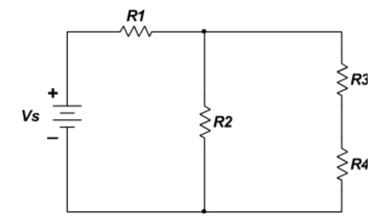




Nodal Analysis

1) Label all circuit parameters and separately list the unknown parameters and the known.
2) Identify all nodes of the circuit.



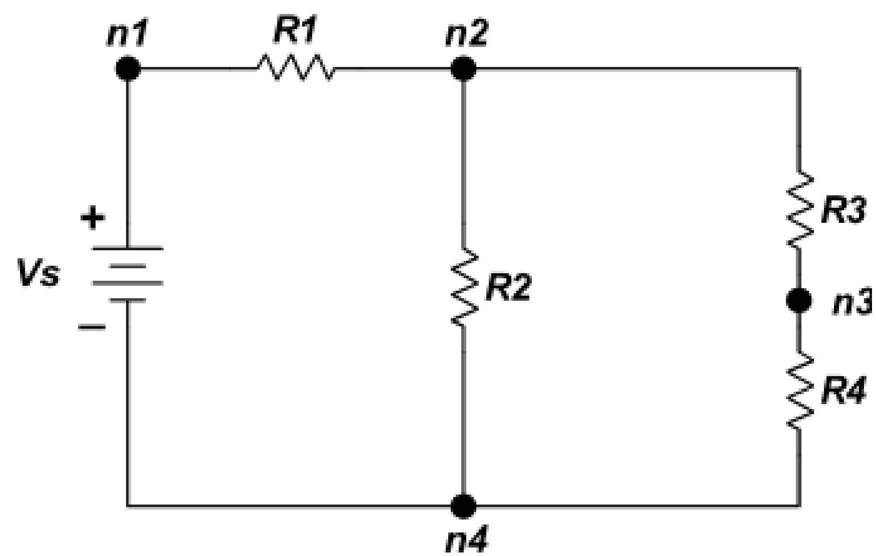


Nodal Analysis

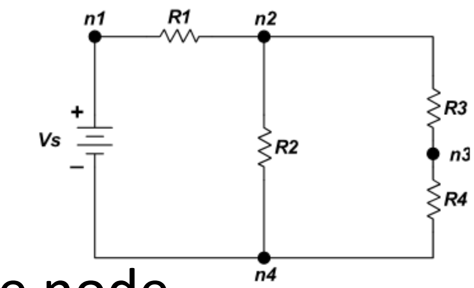
3) Select a node as the reference node.
   a) This reference node is called as the ground
   b) Assign a potential of 0 Volts to the ground
   c) All other voltages in the circuit are measured with respect to the reference node (ground node).

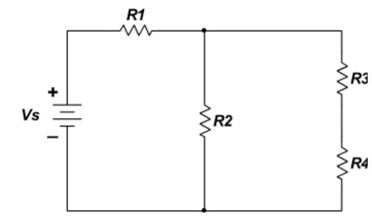
How to select a reference node:
A useful reference node makes the problem easier to understand and solve.
General guidelines to select the reference node.

1. Largest number of elements connected to it.
2. One which is connected to the maximum number of voltage sources.

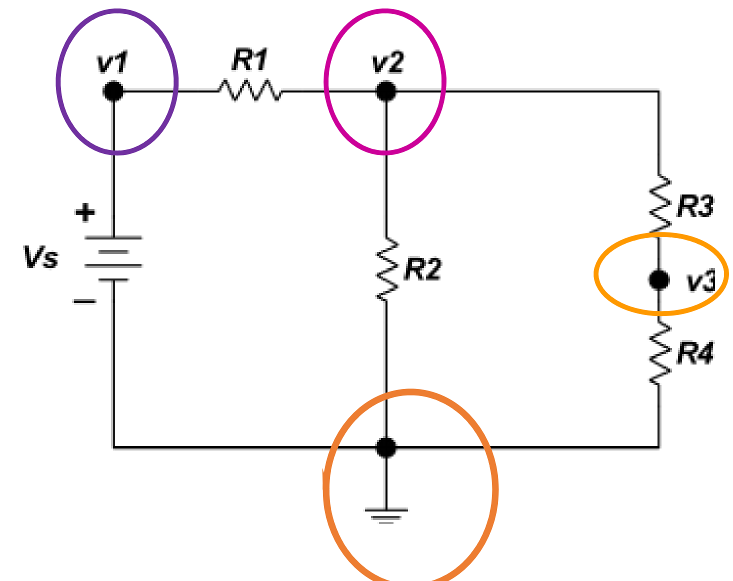


Nodal Analysis



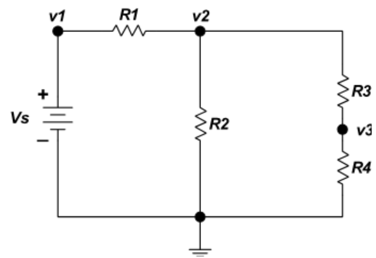


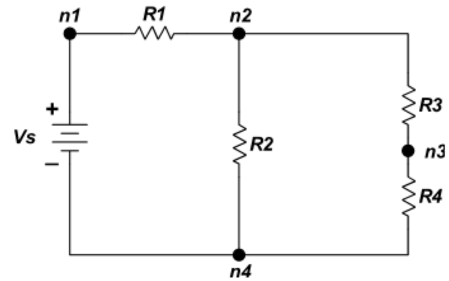
4) For the circuit shown, ***n4*** is the reference node
5) The reference node is assigned voltage 0 V indicated by ground symbol.
6) Label the voltages at all other nodes (node voltages are labelled ***v1***, ***v2***, ***v3***).

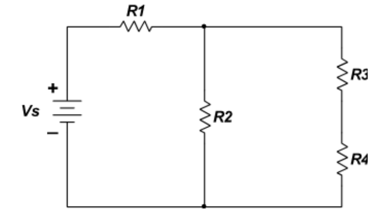


Node n4 is replaced with ground symbol

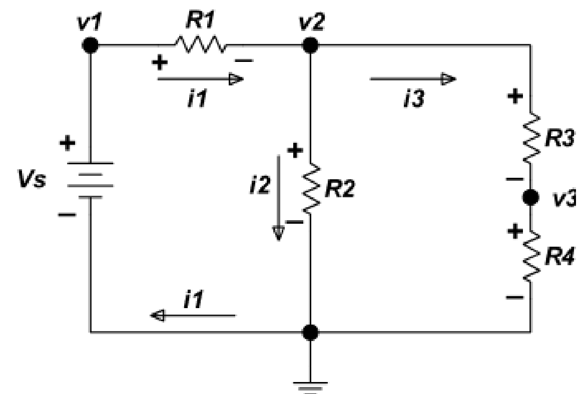
Nodal Analysis



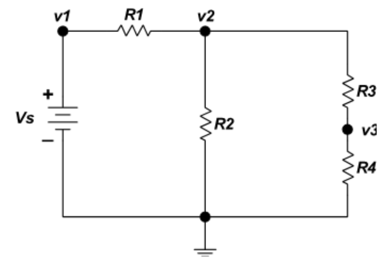


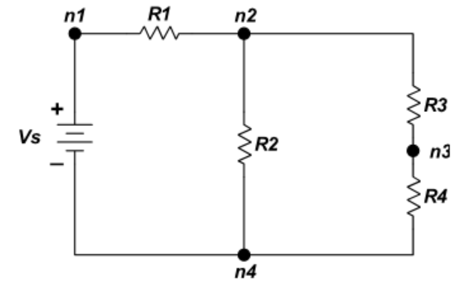


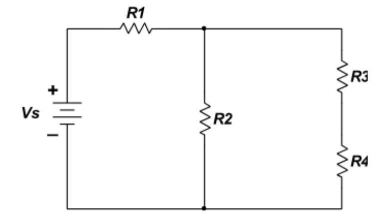
7) Assign polarities and current direction



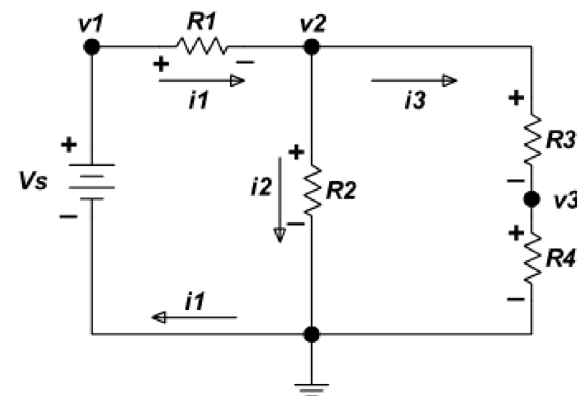
Nodal Analysis



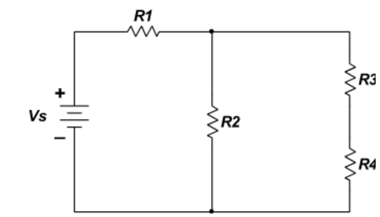




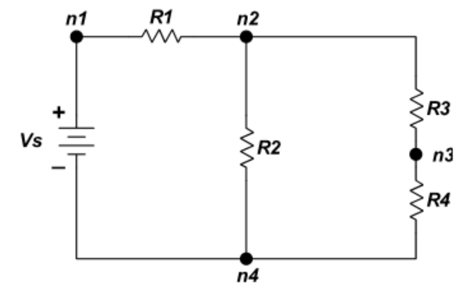
7) Assign polarities and current direction



Nodal Analysis



8) Apply KCL at each node and express the branch currents in terms of the node voltages.



a) At node ***n1***, v1 = Vs.
b) KCL at node ***n2*** associated with voltage v2 results in

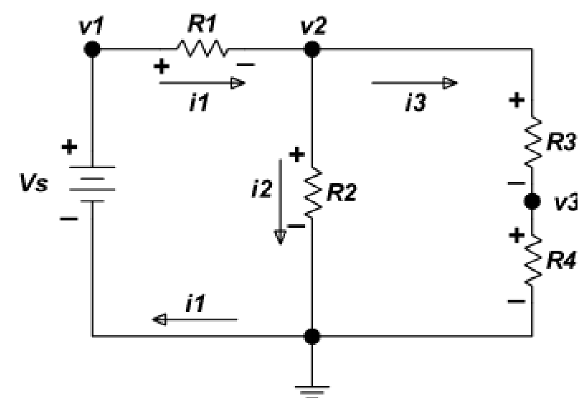
$$i_1 = i_2 + i_3$$

c) Currents i_1, i_2 and i_3 can be expressed in terms of voltages v1, v2 and v3 as

$$i_1 = \frac{V_s - v_2}{R_1}$$

$$i_2 = \frac{v_2}{R_2}$$

$$i_3 = \frac{v_2 - v_3}{R_3}$$



Nodal Analysis

d) By combining the above equations from 8(b) and 8(c)

$$\frac{V_s - v_2}{R_1} - \frac{v_2}{R_2} - \frac{v_2 - v_3}{R_3} = 0$$

e) Re-writing the above equation as a liner function of unknown voltages v2 and v3 results in

$$v_2\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - v_3\frac{1}{R_3} = V_s\frac{1}{R_1}$$

f) KCL at node n3 (voltage v3) results in

$$\frac{v_2 - v_3}{R_3} - \frac{v_3}{R_4} = 0 \Rightarrow -v_2\frac{1}{R_3} + v_3\left(\frac{1}{R_3} + \frac{1}{R_4}\right) = 0$$

Nodal Analysis

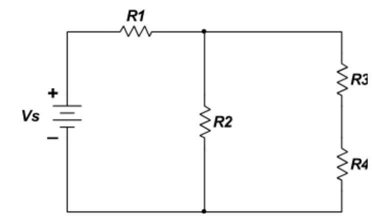
g) Next step is write them in the form of a matrix (this example might be easy to solve without writing matrix).

$$v_2\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)-v_3\frac{1}{R_3} = V_s\frac{1}{R_1}$$

$$-v_2\frac{1}{R_3} + v_3\left(\frac{1}{R_3}+\frac{1}{R_4}\right) = 0$$

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4} \end{bmatrix}\begin{bmatrix} v_2 \\ v_3 \end{bmatrix}=\begin{bmatrix} V_S \\ 0 \end{bmatrix}$$

Nodal Analysis

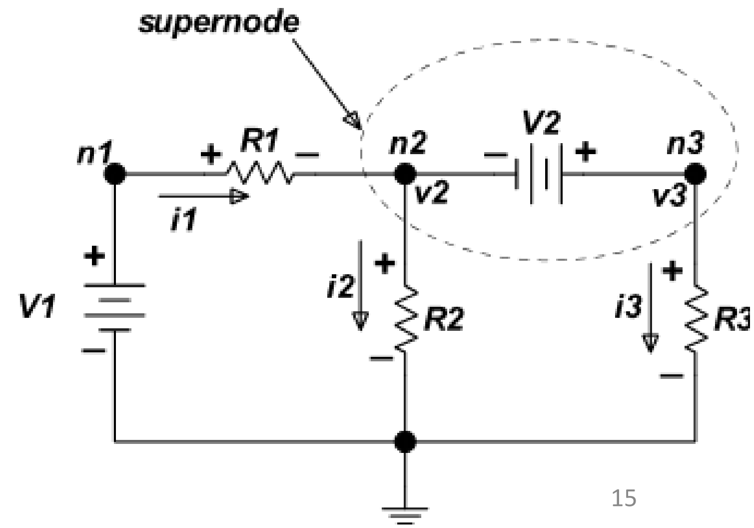


9) Solve the resulting simultaneous equations for the node voltages.
10) Once the node voltages are known, the branch currents are obtained by using Ohm's law.

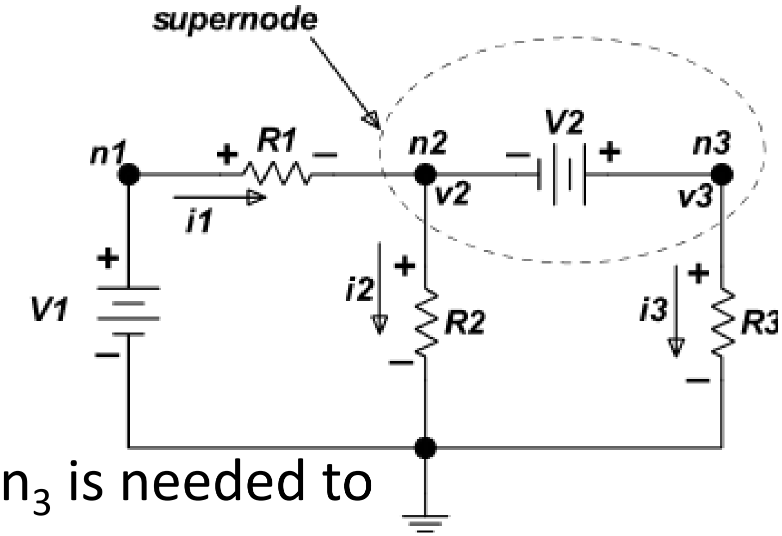
Nodal analysis with floating voltage sources

- A voltage source which is not connected to the reference node is called as a floating voltage source
  - Special care must be taken while analyzing the circuit
- The voltage source V_2 is not connected to the reference node and thus it is a **floating voltage** source.

❖ Part of the circuit enclosed by the dotted ellipse is called a supernode.
❖ KCL is applicable at supernode.
❖ Supernode introduces an extra constraint.



Nodal analysis with floating voltage sources



- Applying KCL at supernode results in

$$i_1 = i_2 + i_3$$

- Writing in terms of node voltages,

$$\frac{V_1 - v_2}{R_1} = \frac{v_2}{R_2} + \frac{v_3}{R_3}$$

- The relation between node voltages n_2 and n_3 is needed to completely define the problem
- The constraint is

$$V_2 = v_3 - v_2$$

Nodal analysis with floating voltage sources

- Combining the equations results in

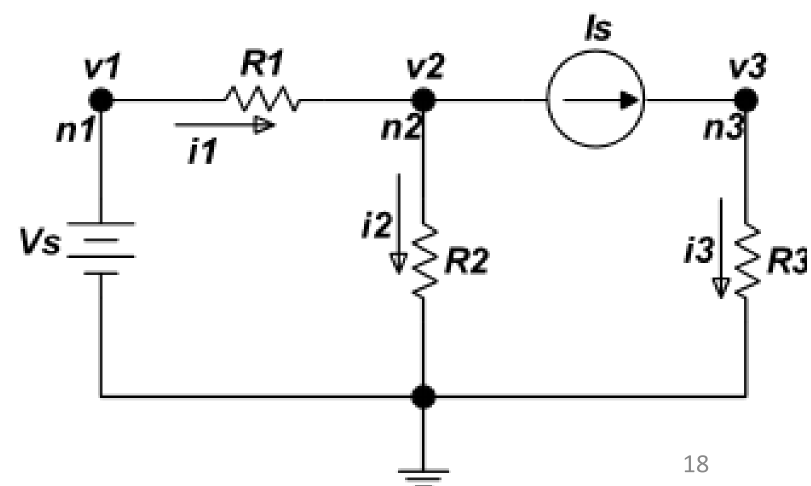
$$\boxed{v_2 = \frac{\dfrac{V_1}{R_1} - \dfrac{V_2}{R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}}$$

$$\boxed{v_3 = \frac{\dfrac{V_1}{R_1} - \dfrac{V_2}{R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} - V_2}$$

Nodal Analysis: Circuit with voltage and current source

- After initial steps of the nodal method, apply KCL to the designated nodes.
- The current source $\mathbf{I_s}$ constraints the current i3 such that i3= $\mathbf{I_s}$
- At node **n1**, Vs = v1.
- KCL at node **n2** results in

$$i_1 = i_2 + i_3 = i_2 + I_s$$



Nodal Analysis: Circuit with voltage and current source

- Applying Ohm's law results in

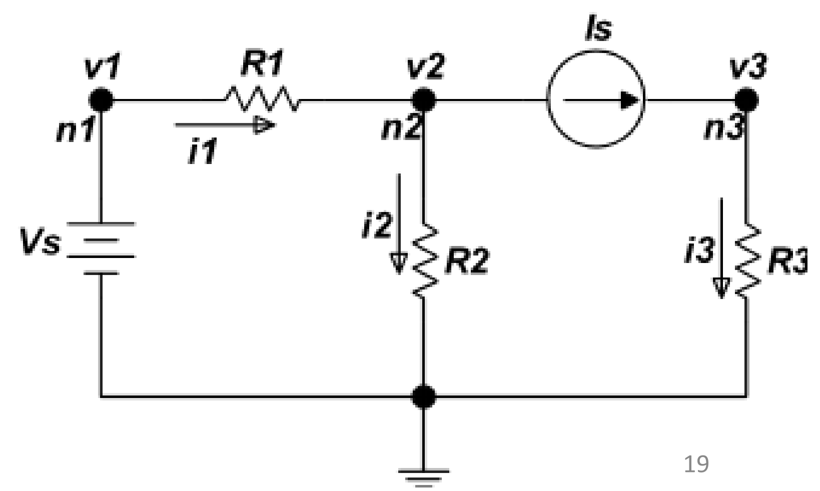
$$\frac{V_s - v_2}{R_1} = \frac{v_2}{R_2} + \frac{v_3}{R_3}$$

- The current source results in a constraint given by

$$v_3 = I_s R_3$$

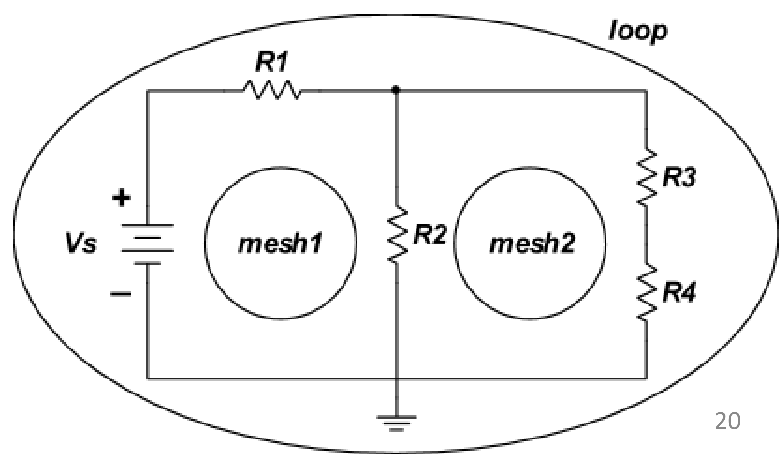
- Combining above results yields in

$$v_2 = \frac{\frac{V_S}{R_1} - I_S R_3}{\frac{1}{R_1} + \frac{1}{R_2}}$$



The Mesh Method

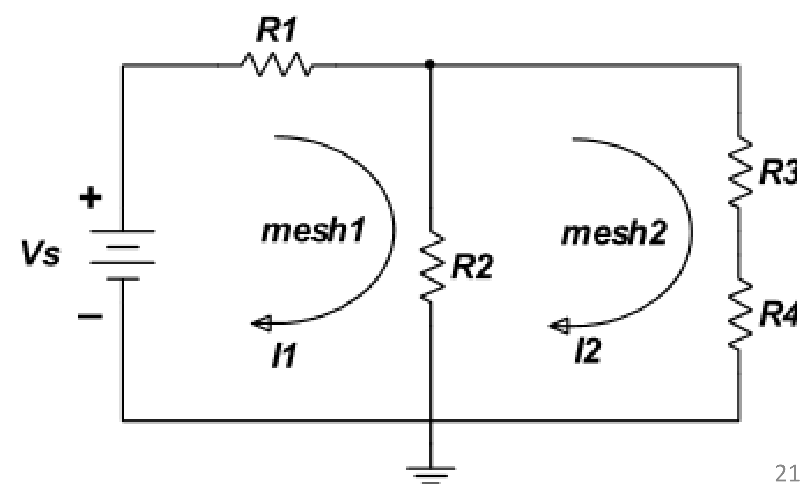
- Mesh method uses mesh currents as the circuit variables
- Method is similar to Nodal analysis
- Primarily KVL is used
- A mesh is defined as a loop which does not contain any other loops



The Mesh Method

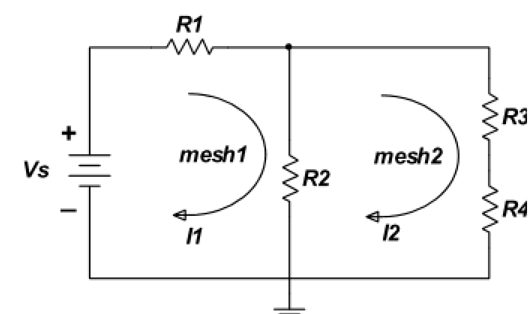
1) Label all circuit parameters and separately list the unknown parameters and the known.
2) Identify all nodes of the circuit.
3) Assign mesh currents and label polarities.

- The direction of the mesh currents I_1 and I_2 is taken in clockwise direction
- Definition for the current direction is arbitrary
- Helps if consistency is maintained in defining these current directions (easy to debug).



The Mesh Method

- In certain parts of the mesh, the branch current and mesh current can be same.
- The branch of the circuit containing resistor R2 is shared by the two meshes and thus the branch current
  - The current flowing through R2 is the difference of the two mesh currents.
  - Mesh current are indicated by using the symbol **I** and the symbol **i** is used to indicate the branch currents.

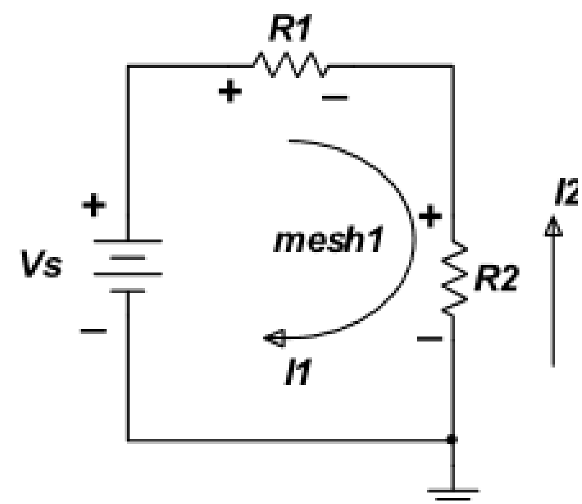


$$I2\left(R3+R4\right)+\left(I2-I1\right)R2=0$$

The Mesh Method

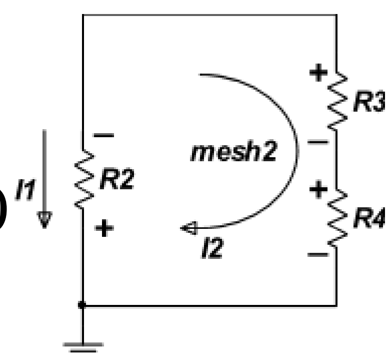
4) Apply KVL at each mesh and express the voltages in terms of the mesh currents.
   a) Consider mesh 1
   b) Care must be taken to carry all the information of the shared branches.
   c) The direction of mesh current I2 on the shared branch is from mesh 2.
   d) Applying KVL to mesh 1

$$I_1R_1+\left(I_1-I_2\right)R_2-V_s=0$$



   e) Considering mesh 2
   f) Apply KVL

$$I_2\left(R_3+R_4\right)+\left(I_2-I_1\right)R_2=0$$



The Mesh Method

5) Solve the resulting simultaneous equations for the mesh currents.

$$I_1\left(R_1 + R_2\right) - I_2 R_2 = V_s$$
$$-I_1 R_2 + I_2\left(R_2 + R_3 + R_4\right) = 0$$

a) Writing the equations in matrix for

$$\begin{bmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 + R_4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \end{bmatrix}$$

The Mesh Method

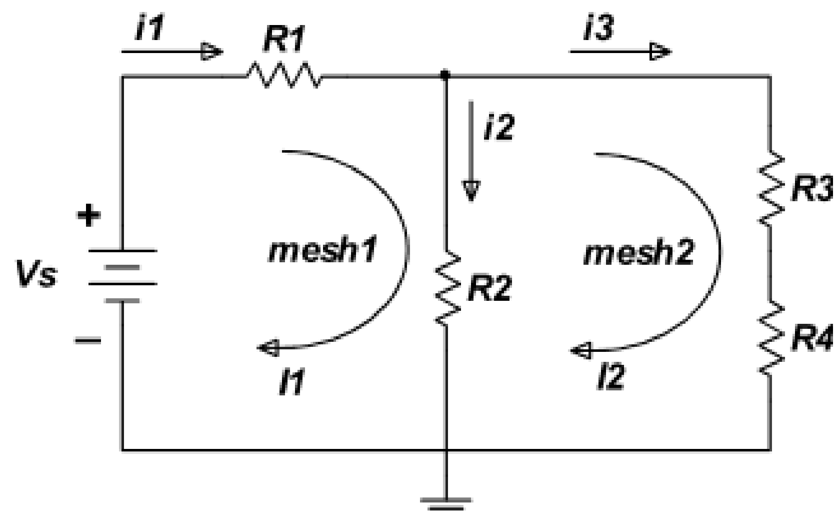
7) As the mesh currents are known, the voltages may be obtained from Ohm's law.

8) The branch currents can be found as

$$i_1 = I_1$$

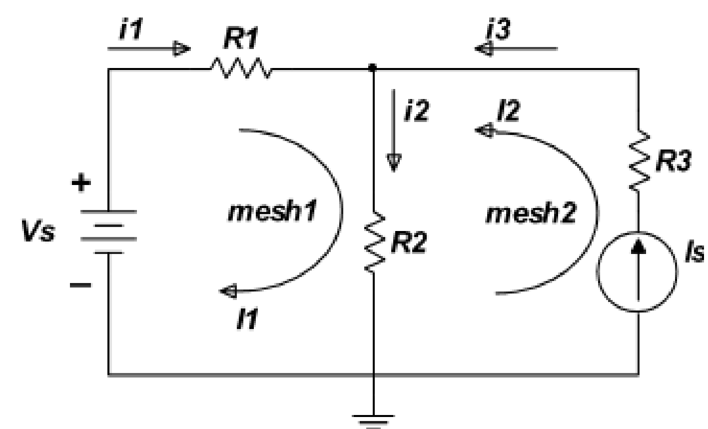
$$i_2 = I_1 - I_2$$

$$i_3 = I_2$$



The Mesh Method: With Current Sources

- The mesh current of the mesh containing the current source is equal to the current of the current source: i.e. $\mathbf{I_2 = I_s}$
- In defining the direction of the mesh current use the direction of the current ad defined by the current source (direction that of Is).
- In this example, the branch current $i_3 = I_s$.



The Mesh Method: With Current Sources

- Applying KVL around mesh1

$$I_1 R_1 + \left(I_1 + I_S\right) R_2 = V_s$$

- Solving for I1,

$$I_1 = \frac{V_S - I_S R_2}{R_1 + R_2}$$