Department of Mathematics
Bennett University
EMAT101L: July-December, 2018
Tutorial Sheet-1 (Multivariable Calculus)

1) For each of the following sets in their mentioned spaces, find out whether the given set is (i) open, (ii) closed, (iii) bounded.

   (a) Space $= \mathbb{R}^2$, $S = \{(x,y) \in \mathbb{R}^2 : \; xy > 0\}$.
   (b) Space $= \mathbb{R}^2$, $S = \{(x,y) \in \mathbb{R}^2 : \; x^2 + y^2 \leq 1 \text{ and } \; y \geq 0\}$.
   (c) Space $= \mathbb{R}^2$, $S = \{(x,y) \in \mathbb{R}^2 : \; y < 1\}$.
   (d) Space $= \mathbb{R}^3$, $S = \left\{(\frac{1}{k}, k.0) \in \mathbb{R}^3 : \; k \in \mathbb{N}\right\}$.

2) Use the $\epsilon - \delta$ definition to show that the following function is continuous at $(0,0)$.

$$f(x,y) = \begin{cases} \frac{4xy^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

3) Find the limit of the following function at $(0,0)$ using polar coordinates.

$$f(x,y) = \begin{cases} \frac{x^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

4) Examine if the limit as $(x,y) \to (0,0)$ exists:

$$(i)\; f(x,y) = \begin{cases} \frac{x^3+y^3}{x^2-y^2} & \text{if } x \neq \pm y \\ 0 & \text{if } x = \pm y. \end{cases} \quad (ii)\; \frac{\sin(xy)}{x^2+y^2} \quad (iii)\; xy\frac{(x^2-y^2)}{x^2+y^2}.$$

5) Examine the continuity of $f : \mathbb{R}^2 \to \mathbb{R}$ at $(0,0)$:

$$(i)\; f(x,y) = \begin{cases} \frac{\sin^2(x-y)}{|x|+|y|} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases} \quad (ii)\; f(x,y) = \begin{cases} \frac{x^2y}{x^4+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

6) For the functions $f : \mathbb{R}^2 \to \mathbb{R}$ given below which of the following limits exist:

$$\lim_{(x,y)\to(0,0)} f(x,y), \quad \lim_{x\to 0}\lim_{y\to 0} f(x,y), \quad \lim_{y\to 0}\lim_{x\to 0} f(x,y).$$

$$(i) f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases} \quad (ii)\; f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} + y \sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } \; x = 0. \end{cases}$$

$$(iii)\; f(x,y) = \begin{cases} x + y\sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } \; x = 0. \end{cases}$$

$$(iv)\; f(x,y) = \begin{cases} \frac{x^2y^2}{x^2y^2+(x-y)^2} & \text{when } x^2y^2 + (x-y)^2 \neq 0 \\ 0 & \text{otherwise.} \end{cases}$$