Tutorial Sheet 5

Eigenvalues/ Eigenvectors

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1. Let $A = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}$. Then

   (a) Find all the eigenvalues and eigenvectors of A.

   (b) Find A^{-1}, using Cayley Hamilton Theorem .

   (c) Calculate algebraic multiplicity and geometric multiplicity of eigenvalues.

   (d) Is A diagonaizable? If yes, find P such that $D = P^{-1}AP$. Also calculate A^k.

   (e) If $f(t) = t^3 + 5t - 4$. Then compute $f(A)$.

2. Repeat the excercise 1 for the following matrices:

$$(i)A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 3 \\ -1 & 0 & 3 \end{pmatrix} \quad (ii)A = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad (iii)A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 0 \\ 0 & 2 & 1 \end{pmatrix} \quad (iv)E = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

3. All the eigenvalues of $n \times n$ Hermitian/Symmetric matrix A are real.

4. The matrix A has $(1, 0, 1)^t$ and $(1, 1, 0)^t$ as eigenvectors, both with eigenvalue 4, and its trace is 2. Find the determinant and characteristic polynomial of A.

5. The matrix A has characteristic polynomial $t(t+1)(t-2)$. What is the characteristic polynomial of A^3.

6. For an $n \times n$ matrix A, prove the following:

   (a) A and A^t have the same set of eigenvalues.

   (b) If $\lambda \neq 0$ is an eigenvalue of an invertible matrix A then $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.

   (c) If λ is an eigenvalue of A then λ^k is an eigenvalue of A^k for any natural k.

   (d) If A and B are $n \times n$ matrices with A nonsingular then BA^{-1} and $A^{-1}B$ have the same set of eigenvalues.

   (e) Similar matrices have the same characteristic polynomial.
   In each case, what can you say about the eigenvectors?

7. Construct a nondiagonal 2×2 matrix that is diagonalizable but not invertible.