

Tutorial 10 Solution

Q1) Let S denote the set of students who participate in the singing competition and D denote the set of students who participate in the dancing competition.

Given that $|S \cup D| = 70$ and $|S| = 50$

$|D|$ = No. of students who get a chance to perform during the annual function.

$|S \cap D|$ = No. of students who get 10 additional points in general proficiency.

Given that $|(S \cap D)'| = 30$, $|S \cap D| = 70 - 30 = 40$

We know that $|S \cup D| = |S| + |D| - |S \cap D|$.

$$\text{Hence, } |D| = 70 - 50 + 40 = 60$$

Thus, the number of students who get the chance to perform during the annual function but do not get additional point is

$$|D| - |S \cap D| = 60 - 40 = 20 \quad \text{Ans.}$$

Q2) In Caesar cipher, the alphabets A to Z are represented by numbers 0 to 25.

Every alphabet of the given text needs to be replaced using the given equation

$$c = f(p) = p + 3 \pmod{26}$$

B O O K

1 14 14 10

After applying the function.

4 17 17 13

E R R N

BOOK \rightarrow ERRN Ans.

Similarly, PARK \rightarrow SDUN Ans.

Q3) The stated problem can be solved using Chinese Remainder Theorem.

The equations are as follows:-

$$x \equiv 3 \pmod{4}$$

$$x \equiv 2 \pmod{5}$$

$$x \equiv 4 \pmod{7}$$

Here, let x denote the minimum no. of pens in the bag.

Here, $m_1 = 4$, $m_2 = 5$, $m_3 = 7$, $a_1 = 3$, $a_2 = 2$, $a_3 = 4$

$$M = 4 \times 5 \times 7 = 140$$

$$\text{Now, } M_1 = \frac{140}{4} = 35$$

$$M_1^{-1} = 35 \times (?) \equiv 1 \pmod{4} \\ = 3$$

$$M_2 = \frac{140}{5} = 28$$

$$M_2^{-1} = 28 \times (?) \equiv 1 \pmod{5} \\ = 2$$

$$M_3 = \frac{140}{7} = 20$$

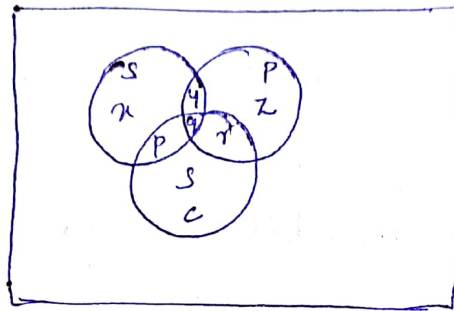
$$M_3^{-1} = 20 \times (?) \equiv 1 \pmod{7} \\ = 6$$

$$\therefore x = (3 \times 35 \times 3 + 2 \times 28 \times 2 + 4 \times 20 \times 6) \pmod{140}$$

(2)

$$\begin{aligned}
 x &= (315 + 112 + 480) \bmod 140 \\
 &= 907 \bmod 140 \\
 &= \underline{15} \text{ } \underline{67} \text{ Ans.}
 \end{aligned}$$

Q5) The problem can be understood with the help of the Venn Diagram



In this Venn Diagram, the sets S, P, and C represent the sets of persons who like singing, playing and cooking, respectively. The variable written inside a portion shows the number of persons in that subset.

Given that $x = 35$, $z = 20$, $s = 25$, $y + q = 10$, $q + r = 15$,

$p + q = 8$ and $q = 5$. Solving these equations, we get

$p = 3$, $r = 12$ and $y = 5$.

(a) No. of people who like singing $= x + y + p + q = 35 + 5 + 3 + 5 = 48$.

(c) No. of people who like playing $= y + z + q + r = 5 + 20 + 5 + 12 = 42$.

(b) No. of people who like cooking $= s + r + p + q = 25 + 12 + 3 + 5 = 45$.

(d) No. of people included in the survey $= x + y + z + s + p + q + r = 35 + 5 + 20 + 25 + 3 + 5 + 12 = 105$.

(3)

Q4a)

$$2x \equiv 3 \pmod{5}$$

$$a=2, b=3, m=5$$

$$\text{let } d = \gcd(2, 5)$$

$$d=1$$

\therefore There is one solution.

$$2x \equiv 3 \pmod{5}$$

$$\text{let } x=1 \Rightarrow 2 \pmod{5} = 3 \pmod{5} \quad \times$$

$$\text{let } x=2 \Rightarrow 4 \pmod{5} = 3 \pmod{5} \quad \times$$

$$\text{let } x=3 \Rightarrow 6 \pmod{5} = 1 \quad \times$$

$$\text{let } x=4 \Rightarrow 8 \pmod{5} = 3 \pmod{5} \quad \checkmark$$

$$\text{so } x=4.$$

$$b) 3x \equiv 2 \pmod{8}$$

$$a=3 \quad b=2 \quad c=8$$

$$\text{let } d = \gcd(3, 8) \Rightarrow d=1$$

\therefore There is one solution

$$3x \equiv 2 \pmod{8}$$

$$\text{let } x=1 \Rightarrow 3 \pmod{8} = 2 \pmod{8} \quad \times$$

$$\text{let } x=2 \Rightarrow 6 \pmod{8} = 6 \pmod{8} \quad \times$$

$$\text{let } x=3 \Rightarrow 9 \pmod{8} = 1 \quad \times$$

$$\text{let } x=4 \Rightarrow 12 \pmod{8} = 4 \quad \times$$

$$\text{let } x=5 \Rightarrow 15 \pmod{8} = 7 \quad \times$$

$$\text{let } x=6 \Rightarrow 18 \pmod{8} = 2 \quad \checkmark$$

$$\text{so } x=6.$$