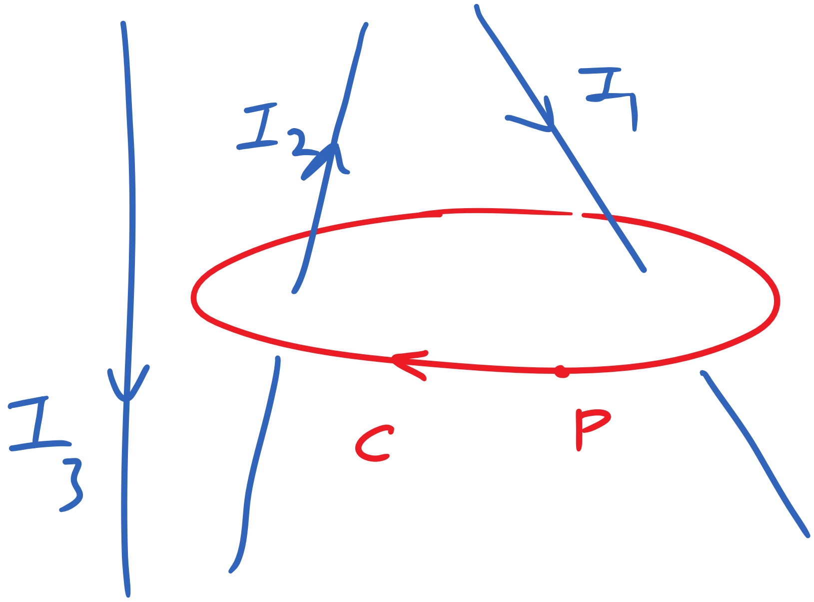
Problem

a) $\oint_C \vec{B} \cdot \vec{dl} = ?$

b) Magnetic field at P on C depends on I_3

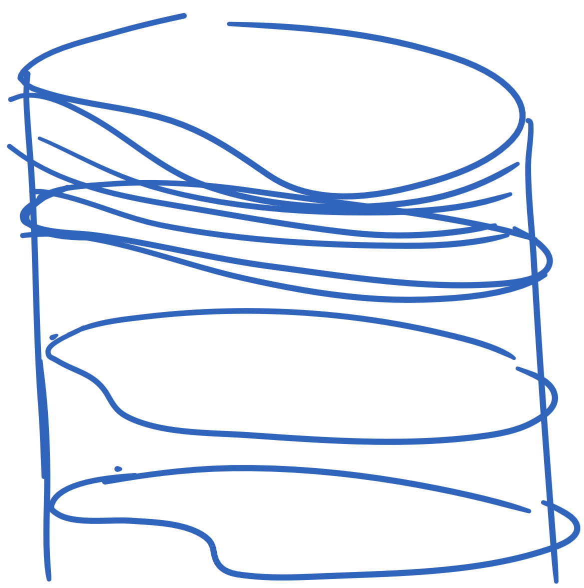
YES / NO

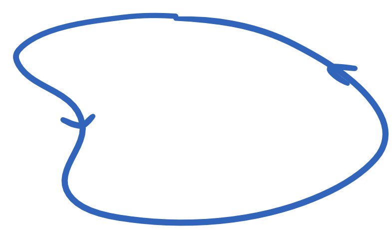


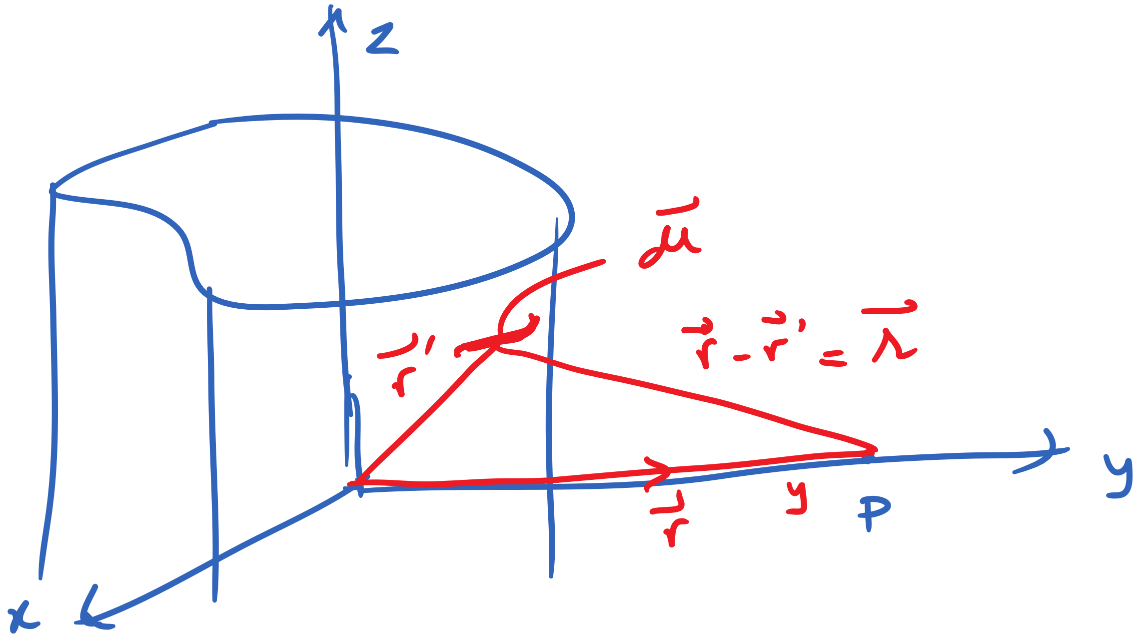
a) $\oint \vec{B} \cdot \vec{dl} = \mu_0 (I_1 - I_2)$

b)

SOLENOID







$$d\vec{B} = \frac{\mu_0}{4\pi} I \, \frac{\vec{dl} \times \vec{\mathscr{r}}}{\mathscr{r}^3}$$

$$\vec{r} = y\,\hat{y}$$

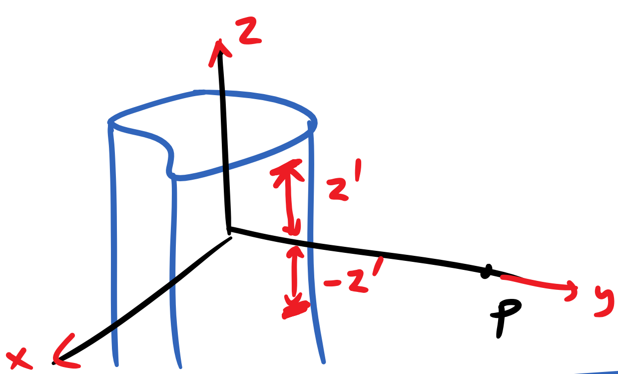
$$\vec{r}' = x'\hat{x} + y'\hat{y} + z'\hat{z}$$

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}' = -x'\,\hat{x} + (y - y')\,\hat{y} - z'\,\hat{z}$$

$$\vec{dl} = dx\,\hat{x} + dy\,\hat{y}$$

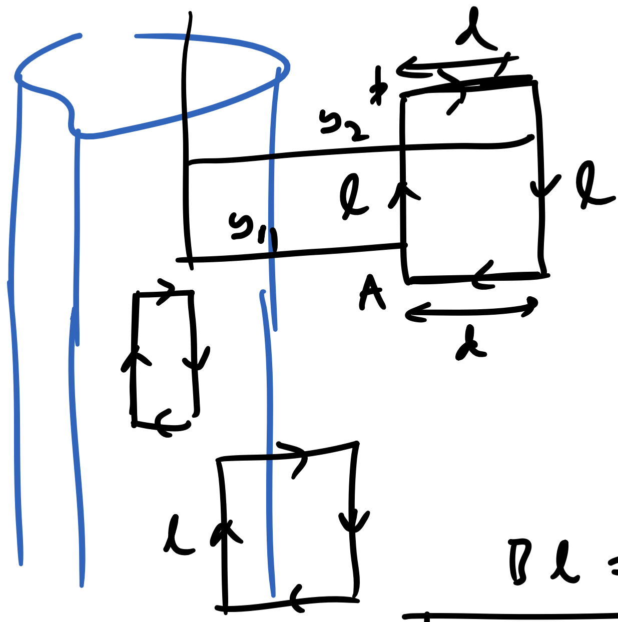
$$\vec{dl} \times \vec{\mathscr{r}} = (dx\,\hat{x} + dy\,\hat{y}) \times \left[-x'\hat{x} + (y - y')\hat{y} - z'\hat{z}\right]$$

$$= (y - y')\,dx\,\hat{z} + \underbrace{z'\,dx\,\hat{y}} + x'\,dy\,\hat{z} - \underbrace{z'\,dy\,\hat{x}}$$



$$d\vec{B} = \frac{\mu_0 I}{4\pi} \, \frac{\vec{dl} \times \vec{\mathscr{r}}}{\mathscr{r}^3}$$

$$\boxed{\vec{B} \to \hat{z}}$$



$$\oint \vec{B}\cdot d\vec{l} = B(y_1)\,l - B(y_2)\,l = 0$$

$$B(y_1) = B(y_2)$$

$$B\,l = \mu_0 N I l$$

$$\boxed{\vec{B} = \mu_0 N I \,\hat{z}}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$\nabla\cdot\vec{B} = 0$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{l}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\int \frac{\vec{I}\times\vec{r}}{r^3}\,dl$$

Volume Current

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}\times\vec{r}}{r^3}\,d\tau' \qquad \vec{r} = \vec{r} - \vec{r}\,'$$

Surface current

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{K}\times\vec{r}}{r^3}\,dA'$$

$$\boxed{\begin{array}{l} \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{B} = \mu_0 \vec{J} \end{array}}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \int_{Vol} \frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \, d\tau'$$

$$\nabla \cdot \vec{B} = \frac{\mu_0}{4\pi} \int \nabla \cdot \left(\frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \right) d\tau'$$

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{B})$$

$$\nabla \cdot \left(\vec{J}(\vec{r}\,') \times \frac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \right) = \frac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \cdot \cancel{\nabla \times \vec{J}(\vec{r}\,')} - \vec{J}(\vec{r}\,') \cdot \underbrace{\nabla \times \left(\frac{\vec{r} - \vec{r}\,'}{|\vec{r} - \vec{r}\,'|^3} \right)}_{= 0}$$

$$\nabla \times \left(\frac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \right) = \nabla \times \nabla \left(\frac{1}{|\vec{r} - \vec{r}\,'|} \right) = 0$$

$$\boxed{\nabla \cdot \vec{B} = 0}$$

$$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J}}$$

Magnetostatics

$$\int (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_0 \int \vec{J} \cdot d\vec{A}$$

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 I}$$

$$\nabla \cdot \vec{B} = 0 \qquad \oint \vec{B} \cdot d\vec{A} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} \qquad \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$\int (\nabla \cdot \vec{B})\, d\tau = \oint \vec{B} \cdot d\vec{A} = 0$$

$$\vec{J} = \frac{I}{\pi R^2} \hat{z}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$B \cdot 2\pi r = \mu_0 \frac{I}{\pi R^2} \cdot \pi r^2$$

$$\boxed{\vec{B} = \frac{\mu_0 I r}{2\pi R^2} \hat{\phi}}$$

$$\vec{\nabla} \cdot \vec{B} = 0 ; \qquad \nabla \times \vec{B} = \mu_0 \frac{I}{\pi R^2} \hat{z} = \mu_0 \vec{J} \quad ; r < R$$

$$\nabla \cdot \vec{B} = 0 \quad ; \qquad \nabla \times \vec{B} = 0 \quad ; \qquad r > R$$