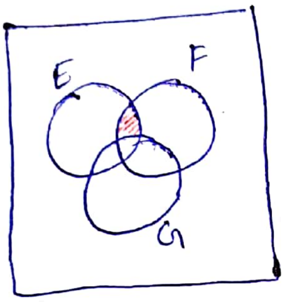
Tutorial Solution - 06

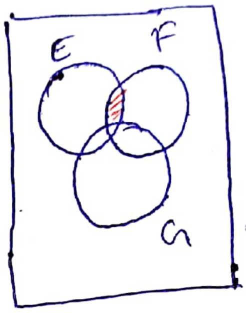
Q1) E, F and G are three finite sets where

$X = (E \cap F) - (F \cap G)$ and

$Y = (E - (E \cap G)) - (E - F)$

Using Venn Diagram, X and Y can be represented as

X 

Y 

Therefore, the answer is (c) $X = Y$.

Q2) Let U denote the universal set, P → Set of students who took programming language, D → Set of students who took data structures and C → Set of students who took computer organization.

Therefore,

$U = 200$

$n(P) = 125$, $n(D) = 85$, $n(C) = 65$.

$n(P \cap D) = 50$, $n(D \cap C) = 35$, $n(P \cap C) = 30$

$n(P \cap D \cap C) = 15$

$n(P \cup D \cup C) = 125 + 85 + 65 - 50 - 35 - 30 + 15$
$= 175$

$\therefore n(P \cup D \cup C)^c = 200 - 175 = \underline{\underline{25}}$ Ans.

Q3) Universal set $X = \{a, b, c, d, e\}$

$\tilde{A} = \{(1, a), (0.3, b), (0.2, c), (0.8, d), (0, e)\}$

$\tilde{B} = \{(0.6, a), (0.9, b), (0.1, c), (0.3, d), (0.2, e)\}$

(a) $Supp(\tilde{A}) = \{a, b, c, d\}$

$Supp(\tilde{B}) = \{a, b, c, d, e\}$

(b) $Core(\tilde{A}) = \{a\}$

$Core(\tilde{B}) = \phi$

(c) $n(\tilde{A}) = 1 + 0.3 + 0.2 + 0.8 + 0 = 2.3$

$n(\tilde{B}) = 0.6 + 0.9 + 0.1 + 0.3 + 0.2 = 2.1$

(d) $\neg(\tilde{A}) = \{(0, a), (0.7, b), (0.8, c), (0.2, d), (1, e)\}$

$\neg(\tilde{B}) = \{(0.4, a), (0.1, b), (0.9, c), (0.7, d), (0.8, e)\}$

(e) $\tilde{A} \cup \tilde{B} = \{(1, a), (0.9, b), (0.2, c), (0.8, d), (0.2, e)\}$

(f) $\tilde{A} \cap \tilde{B} = \{(0.6, a), (0.3, b), (0.1, c), (0.3, d), (0, e)\}$

(g) $a\tilde{A} = \{(0.5, a), (0.15, b), (0.1, c), (0.4, d), (0, e)\}$

when $a = 0.5$

$a\tilde{B} = \{(0.3, b), (0.45, b), (0.05, c), (0.15, d), (0.1, e)\}$

For $a = 2$

(h) $\tilde{A}^a = \{(1,a), (0.09, b), (0.04, c), (0.64, d), (0, e)\}$

$\tilde{B}^a = \{(0.36, a), (0.81, b), (0.01, c), (0.09, d), (0.04, e)\}$

(i) $\tilde{A}_{0.3} = \{a, b, d\}$, $\tilde{A}_{0.9} = \{a\}$

$\tilde{B}_{0.3} = \{a, b, d\}$, $\tilde{B}_{0.9} = \{b\}$

(j) $h(\tilde{A}) = 1$, $h(\tilde{B}) = 0.9$

(k) $\tilde{A}$ is a normal fuzzy set.

Q4) (a) $A = \{1, 2\}$ $\therefore n(A) = 2$

(b) $A = \{1, 2, 3\}$ $\therefore n(A) = 3$

(c) $A = \{1, 2\}$ $\therefore n(A) = 2$

Q5) (a) Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let (x, y) be any element of $A \times (B \cap C)$. Then,

$(x, y) \in A \times (B \cap C) \Rightarrow x \in A$ and $y \in (B \cap C)$

$\Rightarrow x \in A$ and $(y \in B$ and $y \in C)$

$\Rightarrow (x \in A$ and $y \in B)$ and $(x \in A$ and $y \in C)$

$\Rightarrow (x, y) \in A \times B$ and $(x, y) \in A \times C$

$\Rightarrow (x, y) \in (A \times B) \cap (A \times C)$

$\therefore A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ — (1)

Now, let R.H.S = $(x,y) \in (A\times B) \cap (A\times C)$

$\Rightarrow (x,y) \in A\times B$ and $(x,y) \in A\times C$

$\Rightarrow (x\in A, y\in B)$ and $(x\in A, y\in C)$

$\Rightarrow x\in A$ and $(y\in B$ and $y\in C)$

$\Rightarrow x\in A$ and $y\in B\cap C$

$\Rightarrow (x,y) \in A\times (B\cap C)$

Hence, $(A\times B) \cap (A\times C) \subseteq A\times (B\cap C)$ — ②

From ① and ②, we get

$$\boxed{A\times (B\cap C) = (A\times B) \cap (A\times C)}$$

⑥ Prove that $A-B = A\cap \overline{B}$

L.H.S = Let $x \in A-B$ then,

$x \in A-B \Rightarrow x\in A$ and $x \notin B$

$\Rightarrow x\in A$ and $x\in \overline{B}$

$\Rightarrow x\in A\cap \overline{B}$

$A-B \subseteq A\cap \overline{B}$ — ①

R.H.S = Let $x\in A\cap\overline{B} \Rightarrow x\in A$ and $x\in \overline{B}$

$\Rightarrow x\in A$ and $x\notin B$

$\Rightarrow x\in A-B$

$A\cap \overline{B} \subseteq A-B$ — ②

Hence from ① and ② $A-B = A\cap \overline{B}$

(c) Prove $A-(B\cap C) = (A-B)\cup(A-C)$

L.H.S = Let $x \in A-(B\cap C) \Rightarrow x \in A$ and $x \notin (B\cap C)$

$\Rightarrow x \in A$ and $(x \notin B$ or $x \notin C)$

$\Rightarrow (x \in A$ and $x \notin B)$ or $(x \in A$ and $x \notin C)$

$\Rightarrow x \in (A-B)$ or $x \in (A-C)$

$\Rightarrow x \in (A-B)\cup(A-C)$

So, $A-(B\cap C) \subseteq (A-B)\cup(A-C)$ — 1

R.H.S = Let $x \in (A-B)\cup(A-C)$

$\Rightarrow (x \in A$ and $x \notin B)$ or $(x \in A$ and $x \notin C)$

$\Rightarrow (x \in A$ and $x \notin (B\cap C))$

$\Rightarrow x \in A-(B\cap C)$

So, $(A-B)\cup(AC) \subseteq A-(B\cap C)$ — 2

Hence, from 1 and 2 $A-(B\cap C) = (A-B)\cup(A-C)$

Q6) $S_1 = \{1,2,3\}$

$S_2 = \{x \mid x^2-2x+1=0\} = \{1\}$ $(\because \{x \mid (x-1)^2=0\})$

$S_3 = \{x \mid x^3-6x^2+11x-6=0\} = \{1,2,3\}$

$(\because \{x \mid (x-1)(x-2)(x-3)=0\})$

From the above calculation, we can see that

$\boxed{S_1 = S_3}$ Ans.

Q7) $A = \{1,2,3\}$, $B = \{4,5\}$, $C = \{1,2,3,4,5\}$

(a) $A \times B = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5)\}$

(b) $C \times B = \{(1,4), (1,5), (2,4), (2,5), (3,4), (3,5), (4,4), (4,5), (5,4), (5,5)\}$

(c) $B \times B = \{(4,4), (4,5), (5,4), (5,5)\}$

Prove that $(C \times B) - (A \times B) = (B \times B)$

$L.H.S = (C \times B) - (A \times B) = \{(4,4), (4,5), (5,4), (5,5)\}$

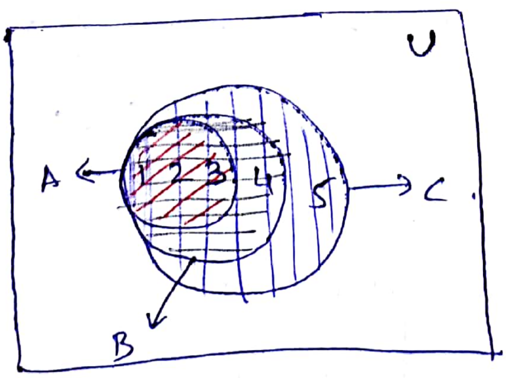
$R.H.S = (B \times B) = \{(4,4), (4,5), (5,4), (5,5)\}$

$\therefore$ L.H.S = R.H.S. Hence Proved

Q8) Let $A = \{1,2,3\}$, $B = \{1,2,3,4\}$, $C = \{1,2,3,4,5\}$

Here, $A \subseteq B$ and $B \subseteq C$.

$\therefore$ Since all the elements of A are also present in C, we can say that $A \subseteq C$.



From, the venn diagram, it is clear that if $A \subseteq B$, $B \subseteq C$, then $A \subseteq C$.