I learned very early the difference between knowing the name of something & knowing something

Richard Feynman

VECTOR CALCULUS

$$f(x) = x\,\underbrace{e^{-x^2/2a^2}}_{\text{GAUSSIAN}}$$

Positions of maxima or minima.

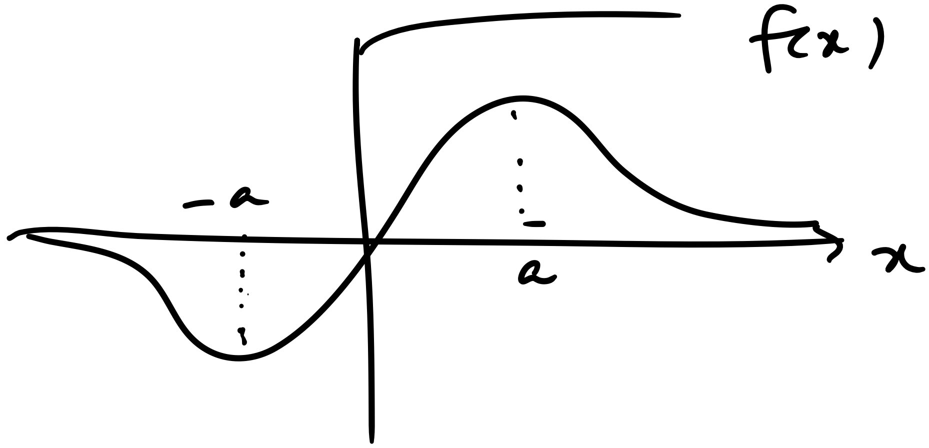
$$\frac{df}{dx} = 0 \qquad x = +a \;\&\; -a$$

Maximum at $x = +a$

Minimum at $x = -a$

$$\frac{d^2f}{dx^2} > 0 \qquad \text{Minimum}$$

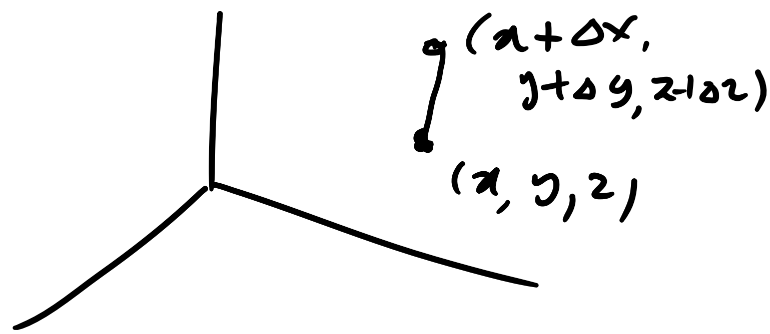
$$\frac{d^2f}{dx^2} < 0 \qquad \text{Maximum}$$



$x\, e^{-x^2/2a^2}$

GRADIENT

$f(x, y, z)$



$$f(x+\Delta x, y+\Delta y, z+\Delta z) \approx f(x, y, z)$$

$$\Delta f = f(x+\Delta x, y+\Delta y, z+\Delta z) - f(x, y, z)$$

$$\boxed{\Delta f = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial z}\Delta z}$$

$$= \left(\hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}\right) \cdot \underbrace{\left(\hat{x}\,\Delta x + \hat{y}\,\Delta y + \hat{z}\,\Delta z\right)}_{\Delta\vec{r}}$$

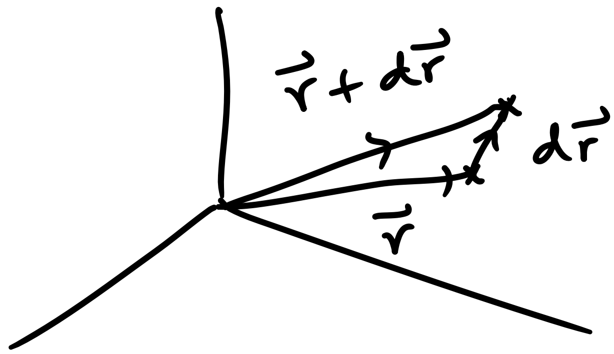
$$= \nabla f \cdot \Delta\vec{r}$$

$\vec{\nabla} f$ = gradient of f

$$= \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}$$

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$: VECTOR OPERATOR

$$\boxed{df = \nabla f \cdot d\vec{r}}$$



∇f : Point along normal to surfaces of constant f

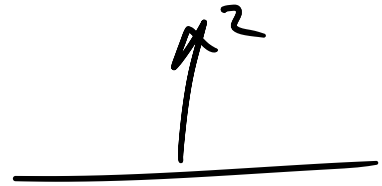
$|\nabla f|$: Represents maximum rate of increase of f

$$\frac{df}{dr} = |\nabla f| \quad \text{maximum}$$

$$df = \nabla f \cdot d\vec{r} = |\nabla f|\, dr \cos\theta$$

θ : angle between $\vec{\nabla} f$ & $d\vec{r}$

Example



① $V(x,y,z) = mgz$

$$\nabla V = \hat{x}\frac{\partial V}{\partial x} + \hat{y}\frac{\partial V}{\partial y} + \hat{z}\frac{\partial V}{\partial z} = mg\,\hat{z}$$

② $f(x,y,z) = \sqrt{x^2+y^2+z^2}$

$$\nabla f = \frac{1}{\sqrt{x^2+y^2+z^2}}\left(x\hat{x} + y\hat{y} + z\hat{z}\right)$$

$$= \frac{\vec{r}}{|\vec{r}|} = \hat{r}$$

$$\nabla f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}$$

$$= \hat{x}\frac{1}{\cancel{2}\sqrt{}} \times \cancel{2}x + \ldots$$

③ $f(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

$$\nabla f = -\frac{\hat{r}}{r^2} = -\frac{\vec{r}}{r^3}$$

④ $\vec{\nabla} f \neq f\,\vec{\nabla}$

$\vec{\nabla}$: DEL OPERATOR

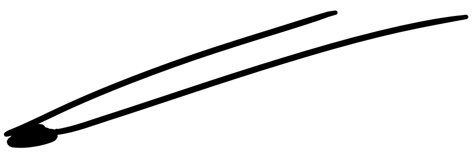
⑤

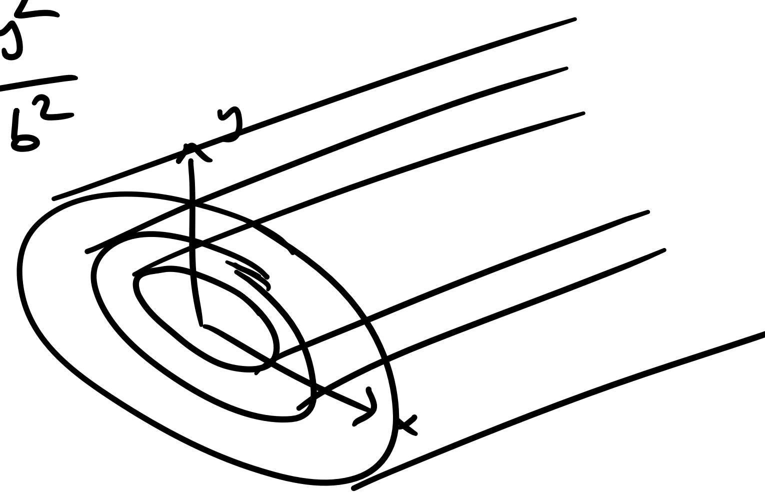
$$\int_a^b df = f(b) - f(a) = \int_a^b \nabla f \cdot d\vec{r}$$

$$\oint df = 0 \Rightarrow \boxed{\oint \nabla f \cdot d\vec{r} = 0}$$

⑥

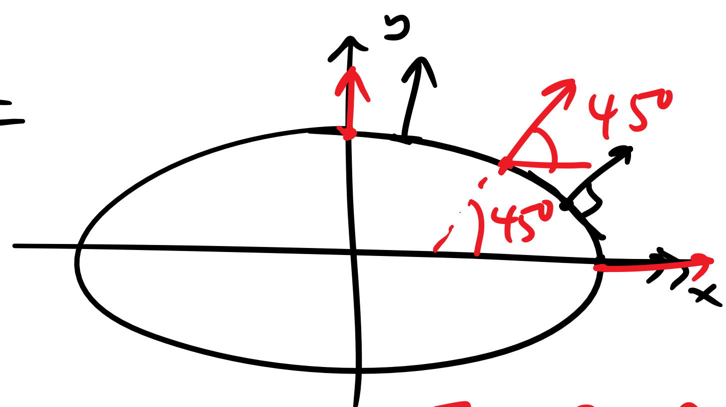
$$f(x, y, z) = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$





$$\nabla f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}$$

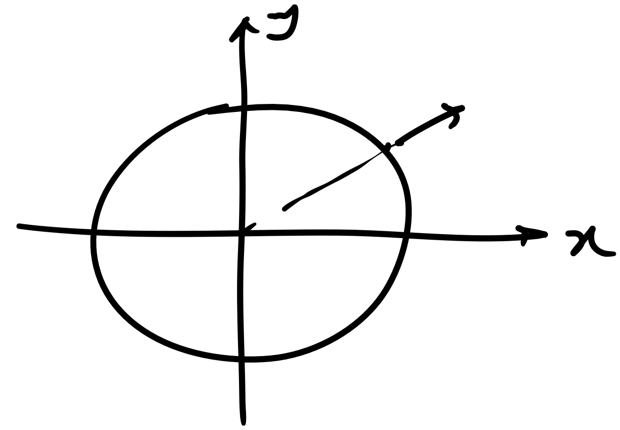
$$= \underline{\underline{\frac{2x\,\hat{x}}{a^2} + \frac{2y\,\hat{y}}{b^2}}}$$



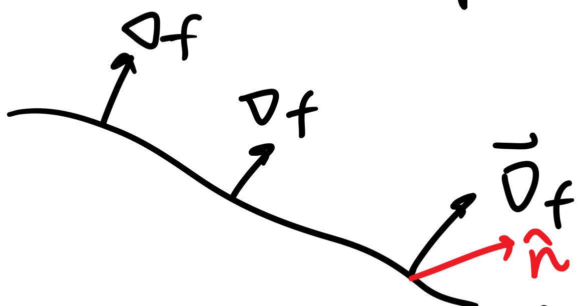
$$\nabla f = \frac{2x}{a^2}\hat{x}$$

$a = b$

$$\nabla f = \frac{2(x\hat{x} + y\hat{y})}{a^2}$$



$\vec{\nabla} f$: Direction normal to $f =$ constant surface



$\vec{\nabla} f \cdot \hat{n}$: represents rate of change of f along $\hat{n}$ direction

$$\nabla f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z} \qquad \text{Cartesian}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi} \qquad \text{Spherical}$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z} \qquad \text{Cylindrical}$$

DIVERGENCE & CURL

$(\nabla \cdot \vec{F})$ $(\nabla \times \vec{F})$