Independence

Defn: Events A and B are independent if

$$P(A \cap B) = P(A) \times P(B)$$

Disjoint: Completely different: If A occurs B cannot occur.

Note: Mistake Independence $\neq$ Disjoint.

A, B, C are independent if

$P(A \cap B) = P(A) \times P(B)$, $P(A \cap C) = P(A) \times P(C)$;

$P(B \cap C) = P(B) \times P(C)$ and $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$.

General form:

$$P(A_1 \ldots A_n) = P(A_1 \cap A_2) \ldots P(A_1 \cap A_2 \ldots A_n)$$

~~If~~ In case of independence

$$P(A, B) = P(A)\, P(B)$$

Example.

1) At least one 6 with 6 dice

2) At least two 6's " 12 dice

Assumption: All rolls are independent.

Case 1: $P(\text{At least one six}) = 1 - P(\text{exactly no six})$

$$= 1 - \frac{5}{6} \times \frac{5}{6} \cdots \frac{5}{6}$$

$$= 1 - \left(\frac{5}{6}\right)^6 \approx 0.665$$

Case 2: $P(\text{At least two six}) = 1 - P(\text{exactly no six}) - P(\text{exactly one six})$

$$= 1 - \left(\frac{5}{6}\right)^{12} - 12 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^{11}$$

$$= 0.659$$

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Then: At least three sixes with 18 dice

$$= 1 - \left(\frac{5}{6}\right)^{18} - 18 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^{17} - {}^{18}C_2 \times \left(\frac{1}{6}\right)^{2} \times \left(\frac{5}{6}\right)^{16}$$

1 − (no six) − (one six) − (two six).

$$\text{or, } 1 - \sum_{i=0}^{2} \binom{18}{i} \left(\frac{1}{6}\right)^{i} \left(\frac{5}{6}\right)^{18-i}$$

New Topic: Conditional Probability.

Central idea of science, life: You have belief about something.

Question: How do you update your believes in light of evidence?

We have uncertainty. How to update your beliefs.

Here conditioning helps.

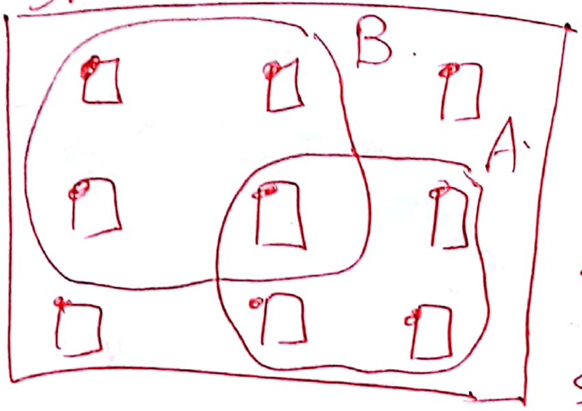
$P(A|B) = P(A)$ if A are B independent

If A and B are not independent

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \; , \text{ if } P(B) > 0.$$

Example

Showroom: 9 Phone



Constraint / Total max = 1 kg.

Some phones are heavy

Some ,, ,, light.

$P(A|B) = ?$

Now, we know phones in B are chosen.

Now the D^r $P(B)$ is [illegible] renormalization

E.g.

If $A = B$, then

$$P(A|B) = P(B|B) = \frac{P(B \cap B)}{P(B)} = \frac{\cancel{P(B)}}{\cancel{P(B)}} = 1.$$

The denominator helps in limiting the sample space to the world and the elements in B.

~~Intuition Building~~

For the previous problem

$P(A) = 4/9 = $ (4 pebbles out of 9)

$P(B) = 4/9$

$P(A \cap B) = \frac{1}{9}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{9}}{4/9} = \frac{1}{4}.$$

EXTRA.

10 coins atleast ~~B~~ 2 heads

$P(H) = 0.4 \quad ; \quad P(T) = 0.6$

$$1 - \sum_{i=0}^{2} 10 \times (0.4)^{i} \times (0.6)^{10-i}$$

$1 - (0.6)^{10}$ ← 10 coins atleast 1 head.

$1 - (0.6)^{10} - 10 \times 0.4 \times (0.6)^{9}$ ← 10 coins atleast 2 head

1 − no head − exactly one head.

Intuition 2 :

Parents 2 kids

$P(GG \mid \text{At least one girl})$

original sample space

BG, BB, GG, GB

we know

Condition : "At least one is a girl

$\Rightarrow$ Sample space = GG, BG, GB

$\Rightarrow P(GG \mid 1G) = 1/3$

Another way:

$P(GG) = 1/4$

$$P(\text{At least one girl}) = 1 - \underbrace{P(\text{no girl})}_{=BB}$$

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(\underbrace{GG}_{GG} \cap \underbrace{\text{At least one girl}}_{GG,\,BG,\,GB}) = \frac{1}{4}$$

$$\Rightarrow P(GG \mid \text{At least one girl}) = \frac{\frac{1}{\cancel{4}}}{\frac{3}{\cancel{4}}} = \frac{1}{3}$$