Solution – Tut sheet 5

① X = time taken to install a certain hardware.

$E(X) = \mu$ = unknown. [$\mu \to$ mean installation time.]

$Var(X) = \sigma^2 = 5^2$.

95% C.I. of μ = ?

$(1-\alpha)100 = 95 \Rightarrow \alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025$.

$$95\% \text{ C.I. of } \mu = \left(\bar{X} - Z_{\alpha/2}\frac{\sigma}{\sqrt{N}},\ \bar{X} + Z_{\alpha/2}\frac{\sigma}{\sqrt{N}}\right)$$

$$= \left(42 - Z_{0.025} \times \frac{5}{\sqrt{64}},\ 42 + Z_{0.025}\frac{5}{\sqrt{64}}\right)$$

$$= (40.8,\ 43.2)$$

[From Z-table $Z_{0.025} = 1.96$]

② Population variance is unknown.

$1-\alpha = 0.9 \Rightarrow \alpha = 0.1 \Rightarrow \frac{\alpha}{2} = 0.05$.

$$90\% \text{ C.I. of } \mu = \left(\bar{X} - t_{N-1,\frac{\alpha}{2}}\frac{S}{\sqrt{N}},\ \bar{X} + t_{N-1,\frac{\alpha}{2}}\frac{S}{\sqrt{N}}\right)$$

$$= \left(71492 - t_{39,0.05}\frac{28}{\sqrt{40}},\ 71492 + t_{39,0.05} \times \frac{28}{\sqrt{40}}\right)$$

$$= (71484.7,\ 71499.3)$$

[$t(n) \approx Z$ for $n \geq 30$.
so $t_{39,0.05} = Z_{0.05} = 1.64$]

(3) $N = 9$, $\bar{x} = \dfrac{5 + 8.5 + 12 + \dots + 10.5}{9}$

$$S = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2}$$

95% C.I. of $\mu = \left(\bar{x} - t_{N-1, \frac{\alpha}{2}} \frac{S}{\sqrt{N}}, \bar{x} + t_{N-1; \frac{\alpha}{2}} \frac{S}{\sqrt{N}} \right)$

$= (6.63, 11.37)$

(4) X = No. of concurrent users after the equipment upgrade.

$E(X) = \mu \rightarrow$ unknown.

$Var(X) = \sigma^2 = 800^2$.

$H_0 : \mu = 5000$

$H_1 : \mu > 5000$

$\bar{X} = 5200$, $N = 100$, $\alpha = 0.05$

$$z^* = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{N}} = \frac{5200 - 5000}{800 / \sqrt{100}} = 2.5$$

$z_\alpha = z_{0.05} = 1.64$

Since $z^* > z_\alpha$, we reject H_0.

(5) $H_0 : \mu = 1800$

$H_1 : \mu > 1800$

$\sigma^2 = 100^2$, $N = 50$.

$\bar{X} = 1850$, $\alpha = 0.01$

$$z^* = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{N}}$$

$= \cancel{10}$

$$z^* = \frac{1850-1800}{100/\sqrt{50}} = \frac{\sqrt{50}}{2} = 3.53$$

$$z_\alpha = z_{0.01} = 2.32$$

Hypothesis testing results in accepting H_1.
That is given sample supports the claim that the breaking strength of the cable has increased.

(6) X = axle diameter of engine part
$\mu = E(X)$ = mean axle diameter.

$H_0 : \mu = 0.7$
$H_1 : \mu \neq 0.7$

$N = 10$, $\bar{X} = 0.742$, $s = 0.04$, $\alpha = 0.05$

$$t^* = \frac{\bar{X}-\mu_0}{s/\sqrt{N}} = \frac{0.742-0.7}{0.04/\sqrt{10}} = 3.32$$

$$t_{N-1,\frac{\alpha}{2}} = t_{9, 0.025} = 2.262$$

As $t^* > t_{N-1,\frac{\alpha}{2}}$, so we reject H_0 (that is accept H_1.)

(7) $N = 50$, $\bar{x} = 304.6$

$\mu =$ mean expenditure on internet per year

$H_0 : \mu = 325$ $\qquad H_1 : \mu \neq 325$

Given that $\sigma = 101.5$, $\alpha = 0.05$.

$$z^* = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{N}} = \frac{304.6 - 325}{101.5/\sqrt{50}} = -1.42$$

$$z_{\alpha/2} = z_{0.025} = 1.96$$

Here $\cancel{z^*}$ $-1.96 < -1.42$.

That is $-z_{\alpha/2} < z^*$.

So we reject H_1.

Claim is rejected.

Test is:
Accept H_1 if $z^* > z_{\alpha/2}$ or $z^* < -z_{\alpha/2}$.
Otherwise if $-z_{\alpha/2} < z^* < z_{\alpha/2}$ reject H_1.

(8) Because of large samples, we can consider that both the populations are normally distributed.

$$X \sim N(\mu_X, \sigma_X^2) \quad \& \quad Y \sim N(\mu_Y, \sigma_Y^2)$$

It is asked to test statistically if there is any difference between μ_X and μ_Y under the condition that $\sigma_X^2 = \sigma_Y^2 = 25^2$.

$H_0: \mu_x = \mu_y$

$H_1: \mu_x \neq \mu_y$ $\Rightarrow$ This can be written as $H_0: \mu_x - \mu_y = 0$

$H_1: \mu_x - \mu_y \neq 0$

Z-test

$$z^* = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_x^2}{N} + \frac{\sigma_y^2}{M}}} = \frac{675 - 680}{\sqrt{\frac{25^2}{1000} + \frac{25^2}{2000}}} = -5.164$$

$Z_{\alpha/2} = Z_{0.025} = 1.96$.

As $z^* < -Z_{\alpha/2}$, so we accept H_1.

$(-5.164 < -1.96)$

(9) X_1 = weight of item produced by process 1.

$\mu_1 = E(X_1)$

X_2 = weight of item produced by process 2.

$\mu_2 = E(X_2)$

$H_0: \mu_1 = \mu_2$ $\Rightarrow$ $H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 \neq \mu_2$ $\quad$ $H_1: \mu_1 - \mu_2 \neq 0$

σ_1^2 and σ_2^2 are given. $[\sigma_1^2 = \sigma_2^2 = 13^2]$

$N_1 = 250$, $\bar{X} = 120$

$N_2 = 400$, $\bar{Y} = 124$

$\alpha = 0.05$

Z-test : $$z^* = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}}} = \frac{120 - 124}{\sqrt{\frac{13^2}{250} + \frac{13^2}{400}}} = -3.82$$

$z_{\alpha/2} = z_{0.025} = 1.96$

As $z^* < -z_{\alpha/2}$ $(-3.82 < -1.96)$, so we accept H_1.

(10) X = life of electric bulb of type I.

$\mu_X = E(X) =$ average life ---

Y = life of electric bulb of type II.

$\mu_Y = E(Y) =$ average life ---

Given that

$N = 8$, $\bar{X} = 1234$, $S_X = 36$

$M = 7$, $\bar{Y} = 1036$, $S_Y = 40$

$H_0 : \mu_X \leq \mu_Y$ $\Rightarrow$ $H_0 : \mu_X - \mu_Y \leq 0$

$H_1 : \mu_X > \mu_Y$ $\quad$ $H_1 : \mu_X - \mu_Y > 0$

$\alpha = .1$ (10%)

As true standard deviations are not given so statistic will be t-statistic.

$$t^* = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{S_X^2}{N} + \frac{S_Y^2}{M}}} = \frac{1234 - 1036}{\sqrt{\frac{36^2}{8} + \frac{40^2}{7}}} = 10.82$$

$t_{N+M-2,\alpha} = t_{8+7-2,\,0.1} = t_{13,\,0.1} = 1.35$ (From t-table).

As $t^* > t_{N+M-2,\alpha}$, so we accept H_1.