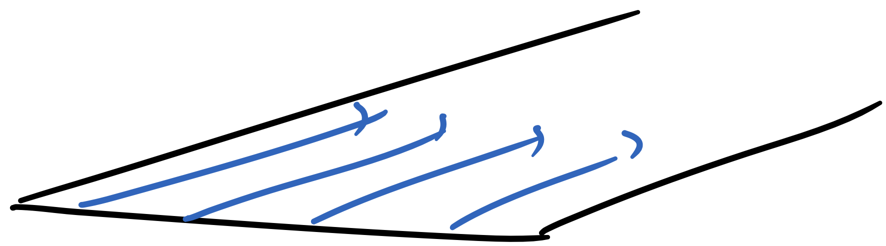
Our brain is a decent scientist
but an outstanding lawyer

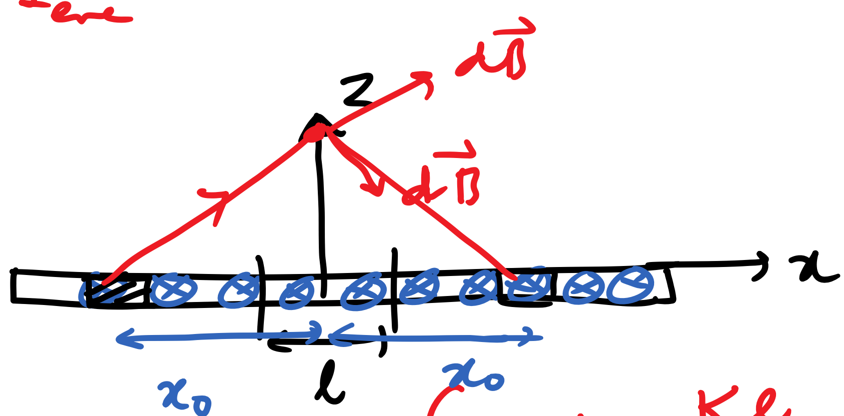
MLODINOW
(SUBLIMINAL)

Surface Current



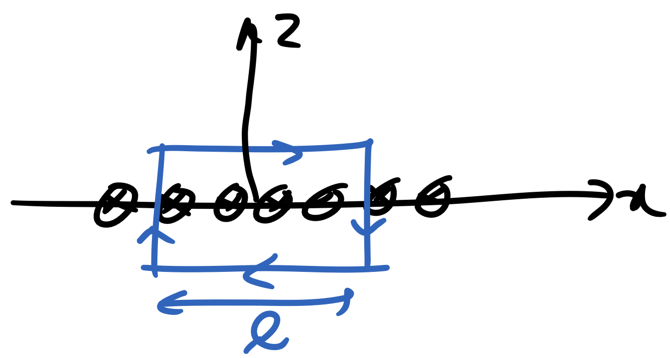
$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enc}$$

$\vec{K}$



Current = Kl

$\vec{B} \rightarrow \hat{x}$



$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enc}$$

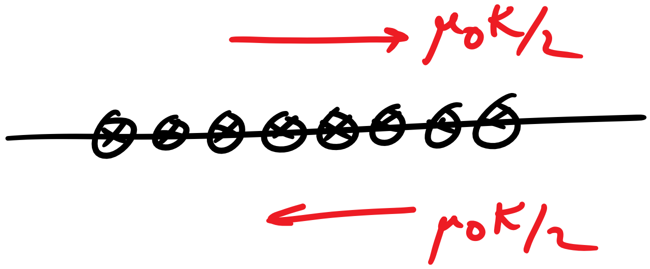
$$Bl + Bl = \mu_0 K l$$

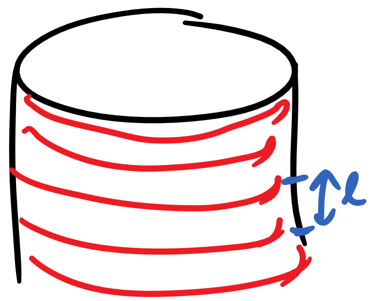
$$B = \frac{\mu_0 K}{2}$$

$$\boxed{\vec{B} = \frac{\mu_0 K}{2} \hat{x}}$$

$z > 0$

$$\vec{B} = -\frac{\mu_0 K}{2}\hat{x} \qquad z<0$$





N turns/unit length

I : Current

$NIl = Kl$

$$\boxed{K = NI}$$

$$\boxed{\begin{aligned} \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{B} &= \mu_0 \vec{J} \end{aligned}}$$

$\nabla \cdot (\nabla \times \vec{F}) = 0$ for all $\vec{F}$

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

$\vec{A}$: VECTOR POTENTIAL

$$\vec B = \frac{\mu_0}{4\pi} \int_{\text{Volume}} \frac{\vec J \times (\vec r - \vec r')}{|\vec r - \vec r'|^3}\, d\tau'$$

$$\frac{(\vec r - \vec r')}{|\vec r - \vec r'|^3} = -\nabla\left(\frac{1}{|\vec r - \vec r'|}\right)$$

$$\vec B = -\frac{\mu_0}{4\pi} \int \vec J \times \nabla\left(\frac{1}{|\vec r - \vec r'|}\right) d\tau'$$

$$\boxed{\nabla \times (f\vec F) = f\,\nabla \times \vec F - \vec F \times \nabla f}$$

$$\nabla \times \left(\frac{\vec J}{|\vec r - \vec r'|}\right) = \frac{1}{|\vec r - \vec r'|}\underbrace{\nabla \times \vec J(\vec r')}_{=0} - \vec J \times \nabla\left(\frac{1}{|\vec r - \vec r'|}\right)$$

$$\vec B = \frac{\mu_0}{4\pi} \int \nabla \times \left(\frac{\vec J}{|\vec r - \vec r'|}\right) d\tau'$$

$$= \nabla \times \underbrace{\left[\frac{\mu_0}{4\pi} \int \frac{\vec J}{|\vec r - \vec r'|}\, d\tau'\right]}$$

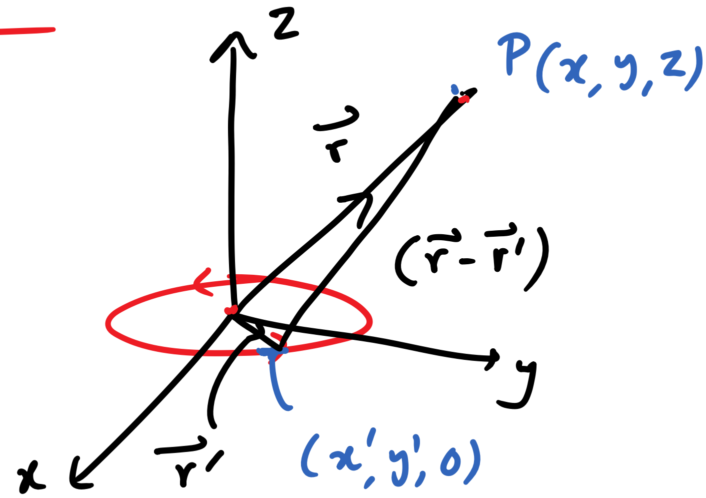
$$\boxed{\vec A = \frac{\mu_0}{4\pi} \int \frac{\vec J}{|\vec r - \vec r'|}\, d\tau'}$$ Volume current

$$\boxed{\vec A = \frac{\mu_0}{4\pi} \int \frac{\vec I}{|\vec r - \vec r'|}\, dl'}$$ Line

$$\boxed{\vec A = \frac{\mu_0}{4\pi} \int \frac{\vec K}{|\vec r - \vec r'|}\, dA'}$$ Surface

MAGNETIC DIPOLE

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi}\oint \frac{I\,\vec{dl}'}{|\vec{r}-\vec{r}'|}}$$



$$\vec{r} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$$

$$\vec{r}' = x'\,\hat{x} + y'\,\hat{y}$$

$$(\vec{r}-\vec{r}') = (x-x')\,\hat{x} + (y-y')\,\hat{y} + z\,\hat{z}$$

$$\vec{dl}' = \hat{x}\,dx' + \hat{y}\,dy'$$

$$\vec{A} = \frac{\mu_0 I}{4\pi}\oint \frac{(\hat{x}\,dx' + \hat{y}\,dy')}{|\vec{r}-\vec{r}'|}$$

$$|\vec{r}-\vec{r}'| = \left[(x-x')^2 + (y-y')^2 + z^2\right]^{1/2}$$

$$= \left[x^2 + y^2 + z^2 - 2xx' - 2yy' + x'^2 + y'^2\right]^{1/2}$$

$$\simeq \left[r^2 - 2xx' - 2yy'\right]^{1/2}$$

$$\frac{1}{|\vec{r}-\vec{r}'|} \simeq \frac{1}{\left[r^2 - 2xx' - 2yy'\right]^{1/2}} = \frac{1}{r}\left[1 - \frac{(2xx' + 2yy')}{r^2}\right]^{-1/2}$$

$$= \frac{1}{r}\left[1 + \frac{(xx' + yy')}{r^2}\right]$$

$$\vec{A} = \frac{\mu_0 I}{4\pi} \oint \frac{\vec{dl}'}{|\vec{r} - \vec{r}'|}$$

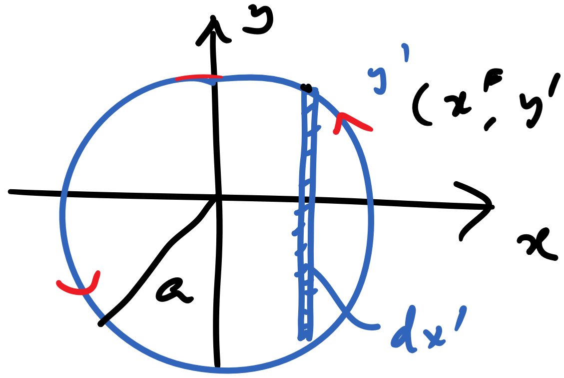
$$= \frac{\mu_0 I}{4\pi} \left[\frac{1}{r} \left[\underbrace{\oint x' dx'}_{=0} + \underbrace{\oint y' dy'}_{=0} \right] + \frac{1}{r^3} \oint (xx' + yy')(\hat{x}\, dx' + \hat{y}\, dy') \right]$$

$$\oint y' dy' = \int_{y_1}^{y_1} y' dy' = 0$$

$$\hat{x}\, x \cancel{\oint x' dx'} + \hat{x}\, y \oint y' dx' + \hat{y}\, x \oint x' dy' + \hat{y}\, y \cancel{\oint y' dy'}$$

$$\oint y' dx' = -\pi a^2$$

$$\oint x' dy' = +\pi a^2$$



$$\boxed{\vec{A} = \frac{\mu_0 I \pi a^2}{4\pi r^3} \left[x\hat{y} - y\hat{x} \right]}$$