Electrostatic Potential

$$\nabla \times \vec{E} = 0$$

$$\nabla \cdot \vec{E} = \rho / \epsilon_0$$

$$\vec{E} = -\nabla V$$

Q. Consider a uniformly charged sphere of radius R carrying a charge Q. What is the value of

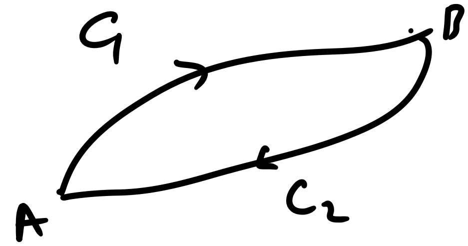
$\nabla \cdot \vec{E}$ at $r = R/2$ & $r = 2R$?

a) $r = R/2$ $\quad \nabla \cdot \vec{E} = \cancel{0}$; $\cancel{\dfrac{Q}{8\pi\epsilon_0 R^2}}$; $\dfrac{Q}{\frac{4}{3}\pi R^3 \epsilon_0}$ ✓

b) $r = 2R$ $\quad \nabla \cdot \vec{E} = 0$ ✓ ;

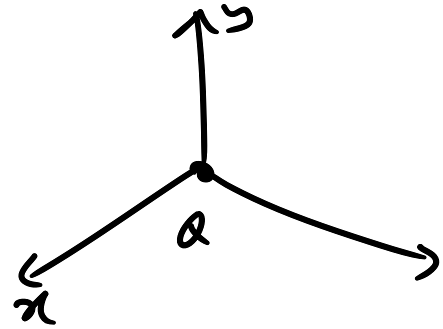
$$\nabla \times \vec{E} \Big|_{r = R/2} = 0$$

$\oint \vec{E} \cdot \vec{dl} = 0$ Conservative force

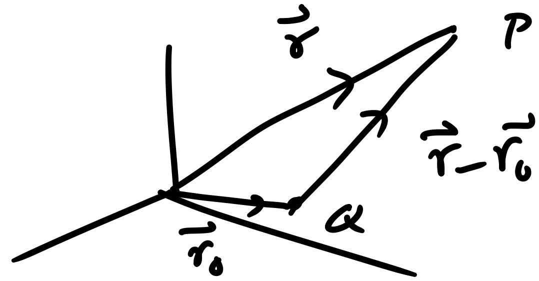


Point charge

$$V = \frac{Q}{4\pi\epsilon_0 r}$$



$$V(x,y,z) = \frac{Q}{4\pi\epsilon_0 |\vec{r} - \vec{r}_0|}$$

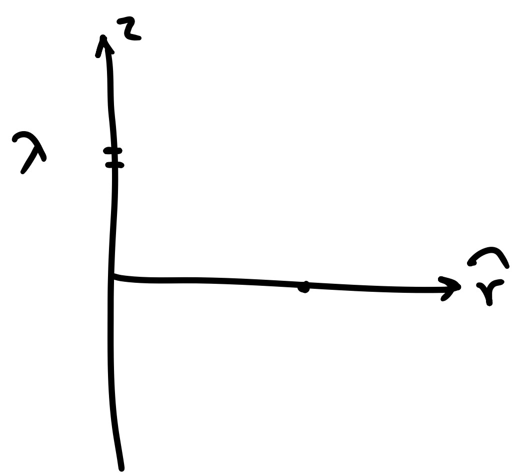


$$\vec{\mathfrak{r}} = \vec{r} - \vec{r}_0$$

$$V = \frac{Q}{4\pi\epsilon_0 \mathfrak{r}}$$

$$\vec{E} = -\nabla V$$

Example



- Line charge
- Cylindrical Coordinate system

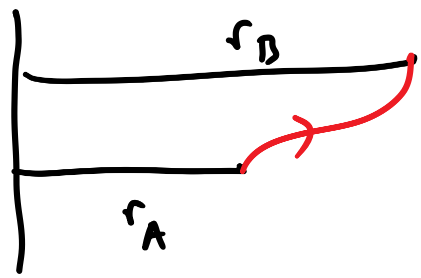
$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$$

$$V = -\int_{r_A}^{r_B} \vec{E} \cdot \vec{dl}$$

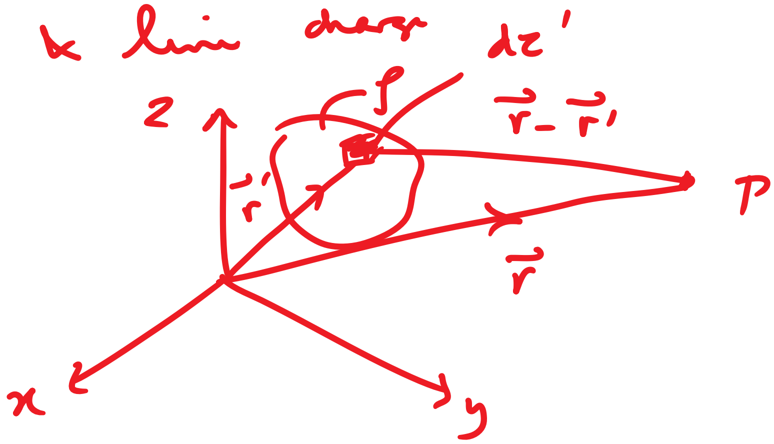
$$\boxed{\vec{dl} = \hat{r}\, dr + r\, d\varphi\, \hat{\varphi} + dz\, \hat{z}}$$

$$= -\int_{r_A}^{r_B} \frac{\lambda}{2\pi\epsilon_0 r}\, dr$$

$$= -\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_B}{r_A}\right) = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_A}{r_B}\right)$$



Volume, Surface & line charge



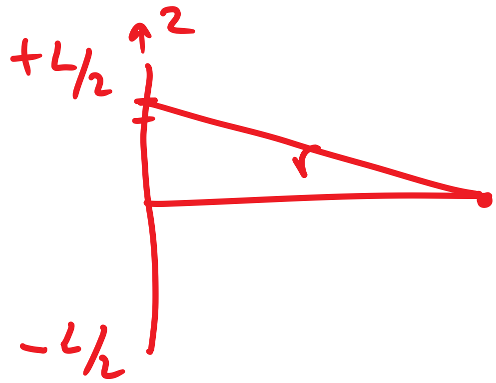
$$dV = \frac{\rho(\vec{r}')\, d\tau'}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|}$$

$$\boxed{V = \frac{1}{4\pi\epsilon_0} \iiint \frac{\rho(\vec{r}')\, d\tau'}{|\vec{r} - \vec{r}'|}}$$

$$\boxed{V = \frac{1}{4\pi\epsilon_0} \iint \frac{\sigma(\vec{r}')\, da'}{|\vec{r} - \vec{r}'|}}$$

1 $$\boxed{V = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda\, dl'}{|\vec{r}-\vec{r}'|}}$$

$V_A - V_B = ?$



$\nabla\cdot\vec{E} = \rho/\epsilon_0$

$\vec{E} = -\nabla V$

$\nabla\cdot(\nabla V) = -\rho/\epsilon_0$

$$\boxed{\nabla^2 V = -\rho/\epsilon_0}$$

LAPLACIAN

$$\nabla^2 = \vec{\nabla}\cdot\vec{\nabla} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

PIERRE SIMON LAPLACE

POISSON'S EQUATION

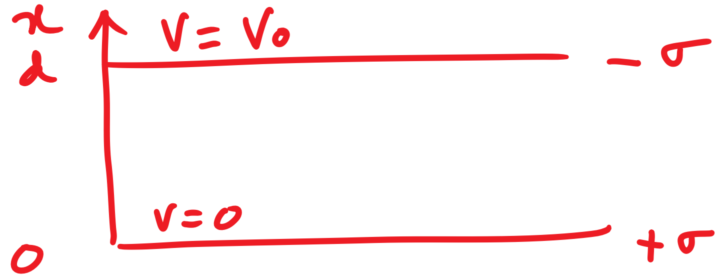
POISSON

$$\boxed{\nabla^2 V = 0}$$

LAPLACE EQUATION

$$\boxed{\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0}$$

○ Parallel plate Capacitor



$V(x)$

$$\frac{d^2V}{dx^2} = 0$$

$$\frac{d}{dx}\left(\frac{dV}{dx}\right) = 0$$

$V(x=0) = 0$

$V(x=d) = V_0$

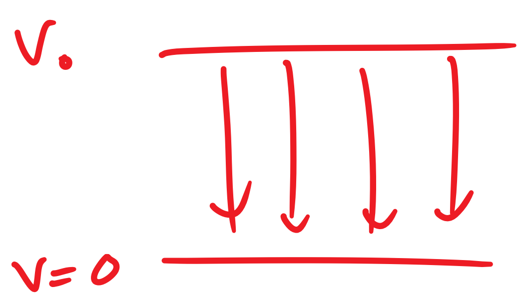
$$\frac{dV}{dx} = C_1$$

$$\boxed{V(x) = C_1 x + C_2}$$

$V(0) = C_2 = 0$

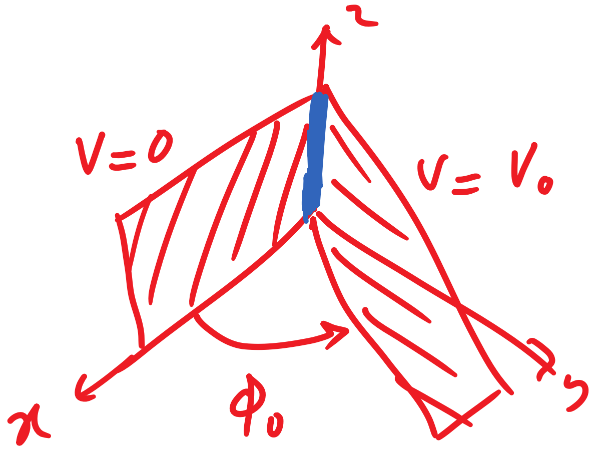
$V(x=d) = C_1 d = V_0 \Rightarrow C_1 = \frac{V_0}{d}$

$$\boxed{V(x) = \frac{V_0 x}{d}}$$



$$\vec{E} = -\nabla V = -\frac{V_0}{d}$$

$V(\phi)$



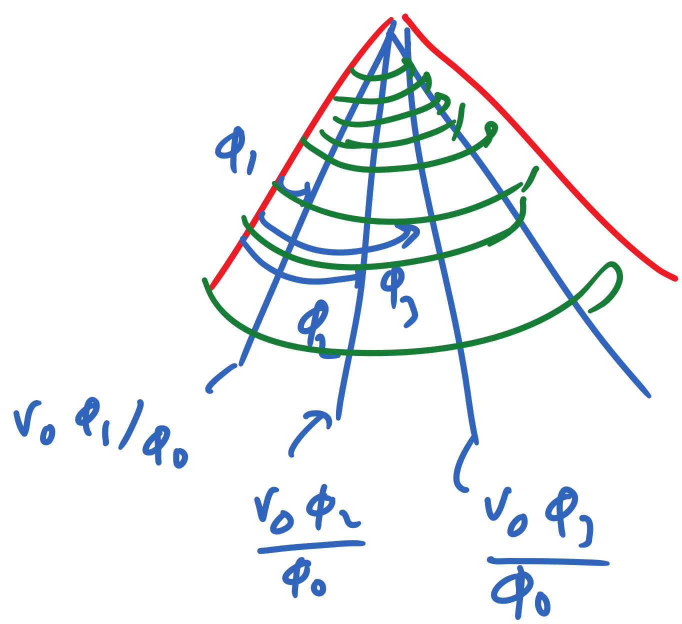
$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\frac{d^2 V}{d\phi^2} = 0 \qquad r \neq 0$$

$$V = C_1 \phi + C_2$$

$$V(\phi = 0) = C_2 = 0$$
$$V(\phi = \phi_0) = C_1 \phi_0 = V_0$$

$$\boxed{V = \frac{V_0 \phi}{\phi_0}}$$



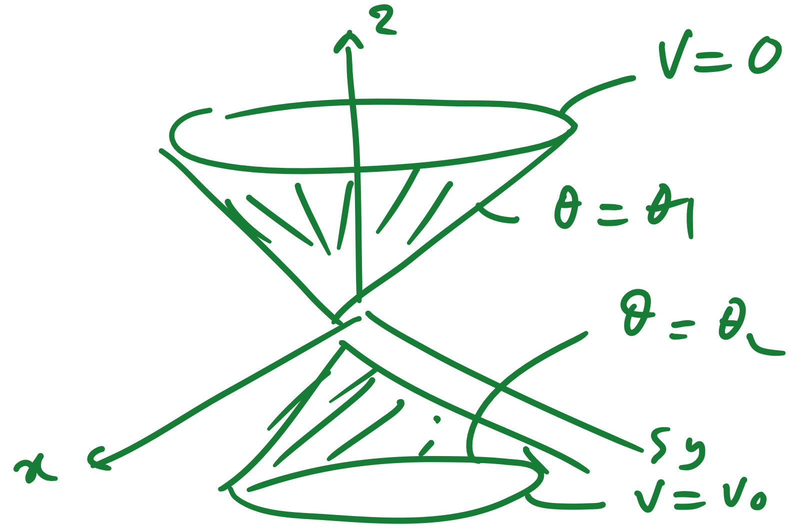
$$\vec{E} = -\nabla V$$

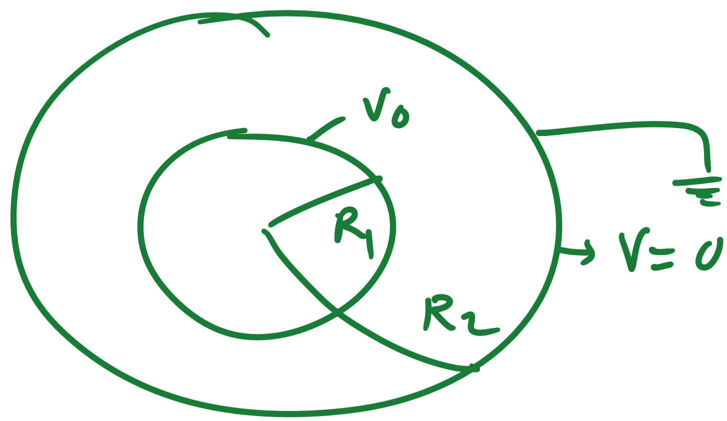
$$\nabla V = \frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \phi}\hat{\phi} + \frac{\partial V}{\partial z}\hat{z}$$

$$\boxed{\vec{E} = -\frac{1}{r}\frac{V_0}{\phi_0}\hat{\phi}}$$

Spherical Polar

θ = Constant





$\nabla^2 V = 0$ Laplace's equation