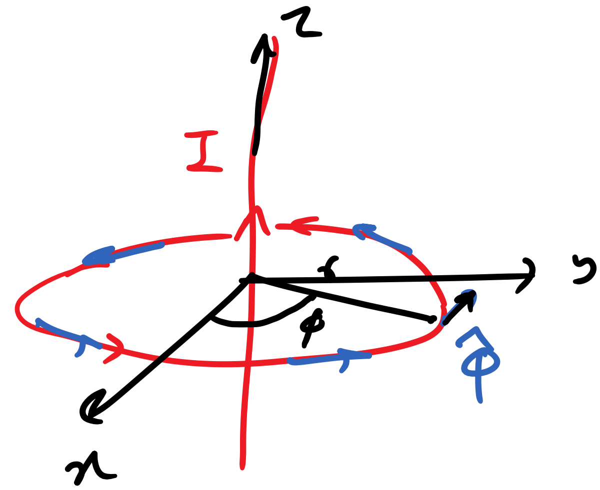
Divergence



$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\varphi}$$

$$r = (x^2+y^2)^{1/2}$$

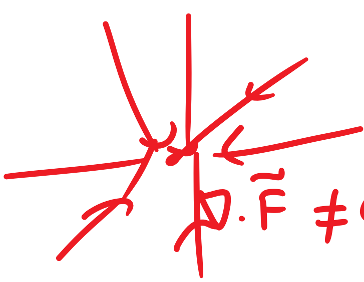
$$\hat{\varphi} = -\sin\varphi\,\hat{x} + \cos\varphi\,\hat{y}$$

$$x = r\cos\varphi \; ; \quad y = r\sin\varphi$$

$$\hat{\varphi} = -\frac{y\,\hat{x}}{r} + \frac{x\,\hat{y}}{r} = \frac{1}{r}(-y\,\hat{x} + x\,\hat{y})$$

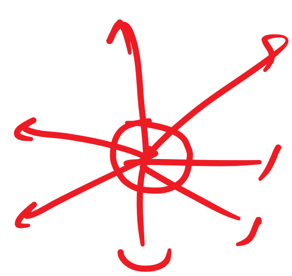
$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi(x^2+y^2)}(-y\,\hat{x} + x\,\hat{y})}$$

$$\nabla\cdot\vec{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$



$\nabla\cdot F < 0$

$\nabla\cdot\vec{F} \neq 0$



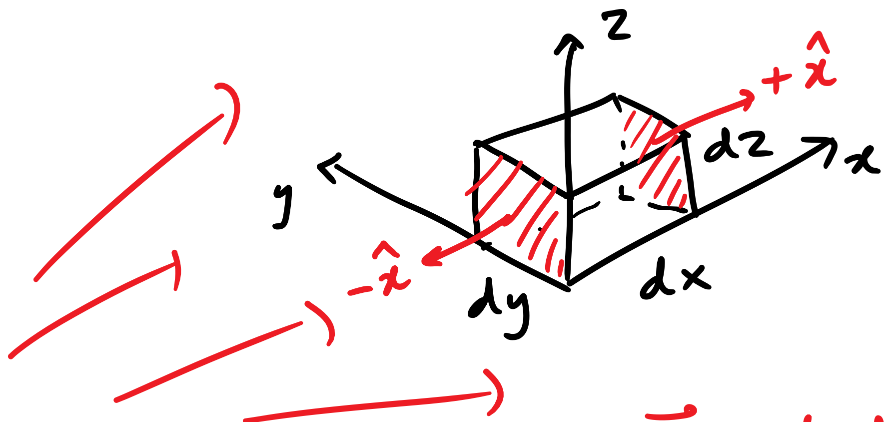
$\nabla\cdot F > 0$

$$\vec{F} = xy\,\hat{x} + 2yz\,\hat{y} + 3xz\,\hat{z}$$

$$\nabla\cdot\vec{F} = y + 2z + 3x \neq 0$$

$$F_2 = x^2\,\hat{x} + 3xz^2\,\hat{y} - 2xz\,\hat{z}$$

$$\nabla\cdot\vec{F_2} = 0$$

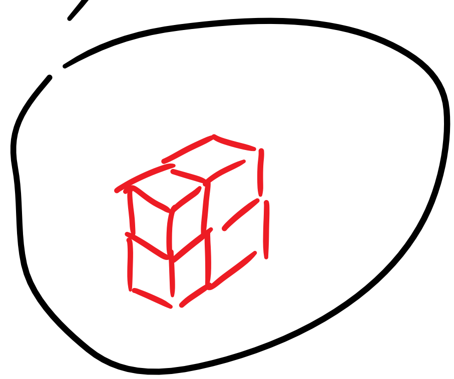


$$d\vec{S_1} = -dy\,dz\,\hat{x}$$

$$\vec{F}\cdot d\vec{S_1}$$

$$\nabla\cdot\vec{F}\;dx\,dy\,dz = \oiint \vec{F}\cdot d\vec{A}$$

$$\vec{\nabla}\cdot\vec{F} = \lim_{\Delta V\to 0}\left(\frac{\oiint \vec{F}\cdot d\vec{A}}{\Delta V}\right)$$



$$\boxed{\iiint \nabla\cdot\vec{F}\, dV = \oiint \vec{F}\cdot d\vec{A}}$$

GAUSS's Theorem

$dV = dx\,dy\,dz$ Volume element

$d\vec{A} = dA\,\hat{n}$ = Surface element

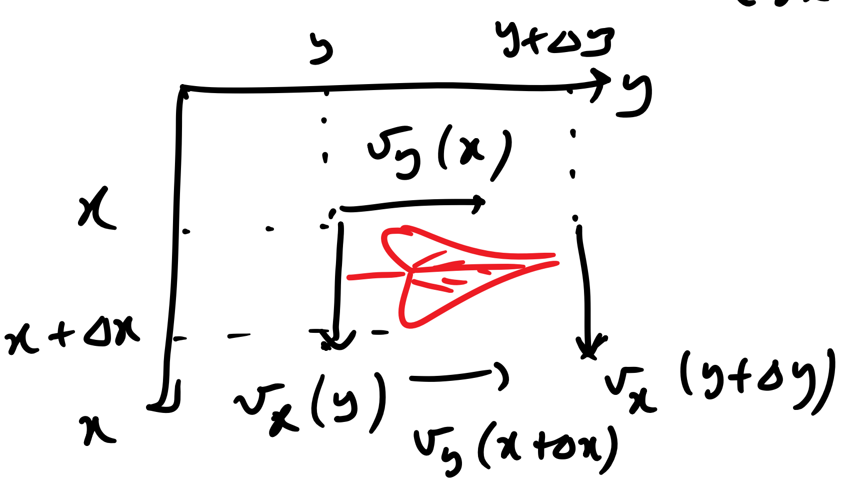
Divergence theorem

CURL

$\nabla\times\vec{F}$: Curl of $\vec{F}$

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right) + \hat{y}\left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right) + \hat{z}\left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)$$



$$v_x(y+\Delta y) - v_x(y) = v_x(y) + \frac{\partial v_x}{\partial y}\Delta y - v_x(y)$$

$$= \frac{\partial v_x}{\partial y}\Delta y$$

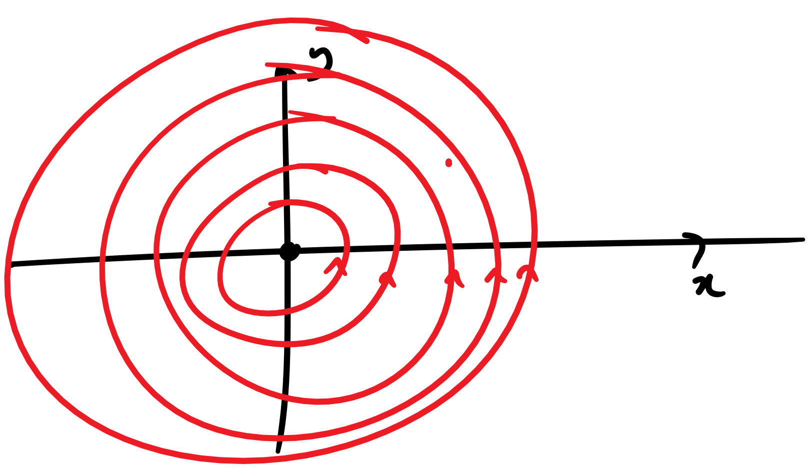
$$v_y(x+\Delta x) - v_y(x) = \frac{\partial v_y}{\partial x}\cdot\Delta x$$

$$\boxed{\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)}$$

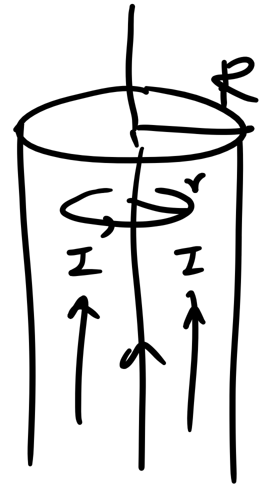
$$\vec{B} = \frac{\mu_0 I}{2\pi(x^2+y^2)}(-y\hat{x} + x\hat{y}) \checkmark$$

$r \neq 0$

$$\nabla \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -\frac{y}{x^2+y^2} & \frac{x}{x^2+y^2} & 0 \end{vmatrix} = 0$$



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$



$$2\pi r \cdot B = \mu_0 \frac{I}{\cancel{\pi} R^2}\cdot \cancel{\pi} r^2$$

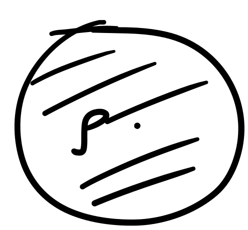
$$\boxed{\vec{B} = \frac{\mu_0 I r}{2\pi R^2}\hat{\phi}}$$

$$\nabla\times\vec{B} = \frac{\mu_0 I}{\pi R^2}\hat{z} = \mu_0 \vec{J} \qquad \vec{J} = \frac{I}{\pi R^2}\hat{z}$$

$$\boxed{\nabla\times\vec{B} = \mu_0 \vec{J}}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{q\,\vec{r}}{4\pi\varepsilon_0 r^3} = \frac{q\,(x\hat{x}+y\hat{y}+z\hat{z})}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}}$$

$$\nabla\times\vec{E} = \ ? \qquad (r\neq 0)$$



$$\boxed{\iint_{\text{surface}} (\nabla \times \vec{F}) \cdot d\vec{A} = \oint_{\text{line}} \vec{F} \cdot d\vec{l}}$$

STOKES THEOREM

$$\boxed{\iiint \nabla \cdot \vec{F} \, dV = \oiint \vec{F} \cdot d\vec{A}}$$

GAUSS'S THEOREM