Quiz - 2

(1) Let m be a positive integer with $m > 1$. Show that the relation

$$R = \{(a,b) \mid a \equiv b \pmod{m}\}$$

is an equivalence relation on the set of integers (2 marks)

(Note: Here, $a \equiv b \pmod{m}$ is read as 'a' is congruent to 'b' modulo 'm' and its equivalent form $(a-b)$ is divisible by 'm')

(2) Shanu is looking for a plot to construct his office whose cost is low around 30 lakhs and which is within a distance of 5 kms from his house. Represent the given conditions using membership functions. Also, determine which of the following plot is more suitable for being bought by Shanu: (2 marks)

(i) Plot A: Cost - 50 lakhs and Distance from house - 2 kms.

(ii) Plot B: Cost - 28 lakhs and Distance from house - 6.8 kms.

(3) Let $A_n = \{i \in Z : i \text{ is divisible by } n\}$ be a set where $n \in N$. Compute the following (1 mark)

(i) $A_3 \cap A_7$ (ii) $A_3 \cup A_7$

(4) Determine which of the following sets are null sets: (1 mark)

(i) $A = \{x \mid 3x - 2 = 0, x \in Q\}$

(ii) $B = \{x \mid 30x - 59 = 0, x \in N\}$

(5) If $A = \{4, 5, 7, 8, 10\}$, $B = \{4, 5, 9\}$ and $C = \{1, 4, 6, 9\}$, then verify that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (1 mark)

Find (a) $A - \phi = ?$ (b) $\phi - A = ?$ (1 mark)

(7) If $A = \{+, -\}$, find A^2. (1 mark)

(8) How many proper subsets of $\{1,2,3,4,5\}$ contain the numbers 2 and 5. How many of them also do not contain the number 3? (1 mark)

(9) If $A = \{1,2,3,4,8\}$ and $B = \{2,4,6,7\}$ then compute $A \Delta B$. (2 marks)

(10) Below is the list of relations among people. For each of the following relations, state whether the relation is reflexive, symmetric, antisymmetric or transitive: (1 mark)

(a) xRy stands for x is a child of y

(b) xRy stands for x is a spouse of y.

(11) Compute the cartesian product of $A = \{0,1\}$, $B = \{1,2\}$ and $C = \{0,1,2\}$. (1 mark)

(Note:- $A \times B \times C$ would contain ordered triples of the form (a,b,c) where $a \in A$, $b \in B$ and $c \in C$).

(12) In a survey 1000 households, washing machines, vacuum cleaners and refrigerators were counted. Each house had at least one of these appliances, 400 had no refrigerator, 380 no vacuum cleaner and 542 no washing machine, 294 had both a vacuum cleaner and a washing machine, 277 both a refrigerator and a vacuum cleaner, 190 both a refrigerator and a washing machine. How many households had all the three appliances? How many had only a vacuum cleaner? (2 marks)

Let $A = \{1, 2, \ldots, 10\}$ and R and S be binary relations in A defined by

mRn if $(m-n)$ is divisible by 3

mSn if $(m-n)$ is divisible by 4.

Compute $\sim(R \cup S)$. (2 marks)

(14) Let $A = B = \{1, 2, 3, 4\}$. $f = \{(1,4), (2,1), (3,2), (4,3)\}$ and $g = \{(1,2), (2,3), (3,4), (4,1)\}$ be two functions.

Verify that f and g are invertible. (1 mark)

(15) (a) Determine whether or not the following relation is a function with domain $\{1,2,3,4\}$. If ~~any~~ the relation is not a function, explain why? (1 mark)

$R_1 = \{(1,2), (2,3), (4,2)\}$

(b) Determine whether or not the following relation is a function. If yes, compute its range.

$R_2 = \{(x, y) \mid x, y \in Z, y = x^2 + 7\}$ which is a relation from Z to Z.