MAXWELL'S EQUATIONS

$$\nabla \cdot \vec{E} = \rho/\epsilon_0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$\vec{J}_D = \epsilon_0 \dfrac{\partial \vec{E}}{\partial t}$ Displacement Current density

$\vec{J}_D = \epsilon \dfrac{\partial \vec{E}}{\partial t}$ (medium)

Free Space

$\rho = 0$; $\vec{J} = 0$

$$\nabla \cdot \vec{E} = 0 \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{B} = \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \times (\nabla \times \vec{E}) = -\nabla \times \frac{\partial \vec{B}}{\partial t}$$

$$= -\frac{\partial}{\partial t}\left(\nabla \times \vec{B}\right)$$

$$= -\frac{\partial}{\partial t}\left(\epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\nabla \times (\nabla \times \vec{E}) = -\epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

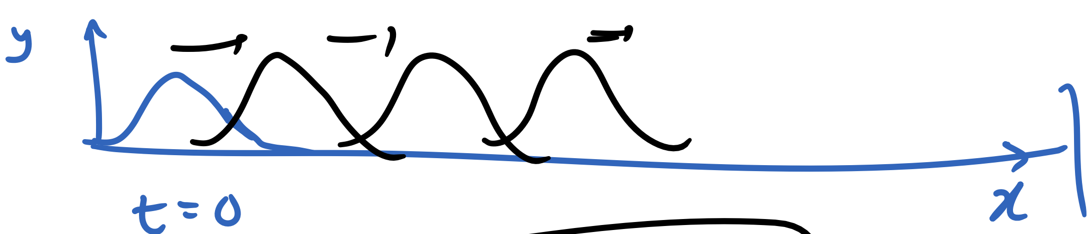
$$\boxed{\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}}$$

$$= -\nabla^2 \vec{E} \qquad (\nabla \cdot \vec{E} = 0)$$

$$\boxed{\nabla^2 \vec{E} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

$$\boxed{\frac{\partial^2}{\partial x^2}\vec{E} + \frac{\partial^2}{\partial y^2}\vec{E} + \frac{\partial^2}{\partial z^2}\vec{E} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

Three dimensional WAVE EQUATION



$$\boxed{\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}}$$

$$v = \sqrt{\frac{T}{m}}$$

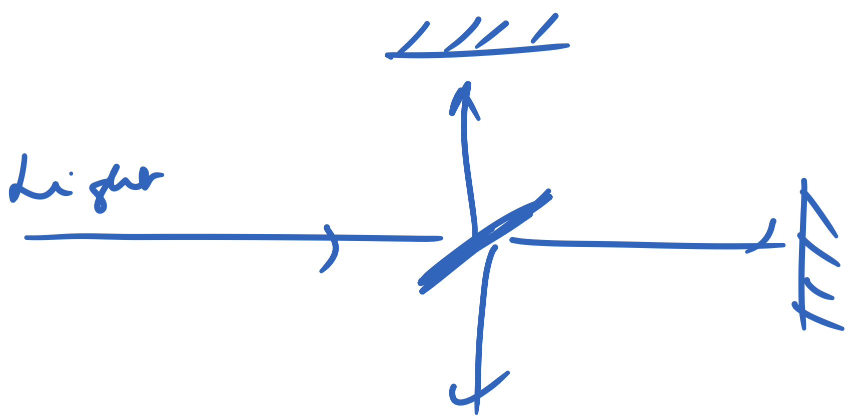
mass/length

$$\boxed{c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}}$$ Speed of EM Waves

H. HERTZ (1888)

$c = 299,792,458$ m/s
$\simeq 3 \times 10^8$ m/s

MICHELSON & MORLEY
EXPERIMENT



$$\boxed{\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}}$$ One dimensional Wave Equation

d' ALEMBERT (1717 - 1783)

$f(x - vt)$

$\sin \alpha(x - vt)$

$(x - vt)^2$

$e^{-(x-vt)^2/\beta^2}$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

$(x - vt) = \zeta$ ZETA

$$\frac{\partial \zeta}{\partial x} = 1$$

$$\frac{\partial \zeta}{\partial t} = -v$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial \zeta} \cdot \frac{\partial \zeta}{\partial x} = \frac{\partial f}{\partial \zeta}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial \zeta}\left(\frac{\partial f}{\partial \zeta}\right)\frac{\partial \zeta}{\partial x} = \frac{d^2 f}{d\zeta^2}$$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial \zeta} \cdot \frac{\partial \zeta}{\partial t} = -v \frac{df}{d\zeta}$$

$$\frac{\partial^2 f}{\partial t^2} = v^2 \frac{d^2 f}{d\zeta^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 f}{\partial t^2}$$

$f(x - vt)$

$g(x + vt)$

$$\sin(kx - \omega t) = \sin k\left(x - \frac{\omega}{k} t\right)$$

$$= \sin k\,\underline{\underline{(x - vt)}}$$