Exponential Distribution.

$X \sim Exp(\lambda)$.

$PDF = \lambda e^{-\lambda x}$ $\quad x > 0$

0 $\quad$ otherwise.

$CDF = \int_0^x \lambda e^{-\lambda x} dx$.

$= 1 - e^{-\lambda x}$.

$E[X] = \frac{1}{\lambda}$, $Var[X] = \frac{1}{\lambda^2}$.

$Y \sim \lambda X$

$CDF(Y) = 1 - e^{-y}$.

$PDF(Y) = e^{-y}$

Proof: $X \sim Exp(\lambda)$.

let's look at CDFs

$$CDF(Y) = P(Y \leq y)$$

$$= P(\lambda X \leq y)$$

$$= P\left(X \leq \frac{y}{\lambda}\right)$$

$$= 1 - e^{-y}.$$

To get PDF, we differentiate the CDF,

$$\Rightarrow PDF(Y) = e^{-y}.$$

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Memoryless Property

$$P(X \geq s+t \mid X \geq s) = P(X \geq t).$$

$$P(X \geq s) = 1 - P(X \leq s)$$

$$= 1 - \cancel{e}(1 - e^{-\lambda s})$$

$$= e^{-\lambda s}$$

Now, $$P(X \geq s+t \mid X \geq s) = \frac{P(X \geq s+t \cap X \geq s)}{P(X \geq s)}$$

$\therefore$ s+t implies X is greater than s $$= \frac{P(X \geq s+t)}{P(X \geq s)}$$

$$= \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}}$$

$$= e^{-\lambda t}.$$