Acc Application of prob & stats

- Finance
- Gambling
- Image recognition

Probability started (Basic) as a consequence of gambling.

Sample Space :
(set of all possible outcomes in an experiment
anything.

An event is the subset of the sample space

Prob : $\dfrac{\#\text{ of favourable outcomes}}{\#\text{ of possible outcomes}}$

e.g. 2 coins
HH, HT, TH, TT

$P(HH) = 1/4$

21

13

13x

Assumption: All entries are equally likely.

Counting: Multiplication rule

Experiment 1 — $E1$ outcomes

" 2 $E2$ "

" 3 $E3$ "

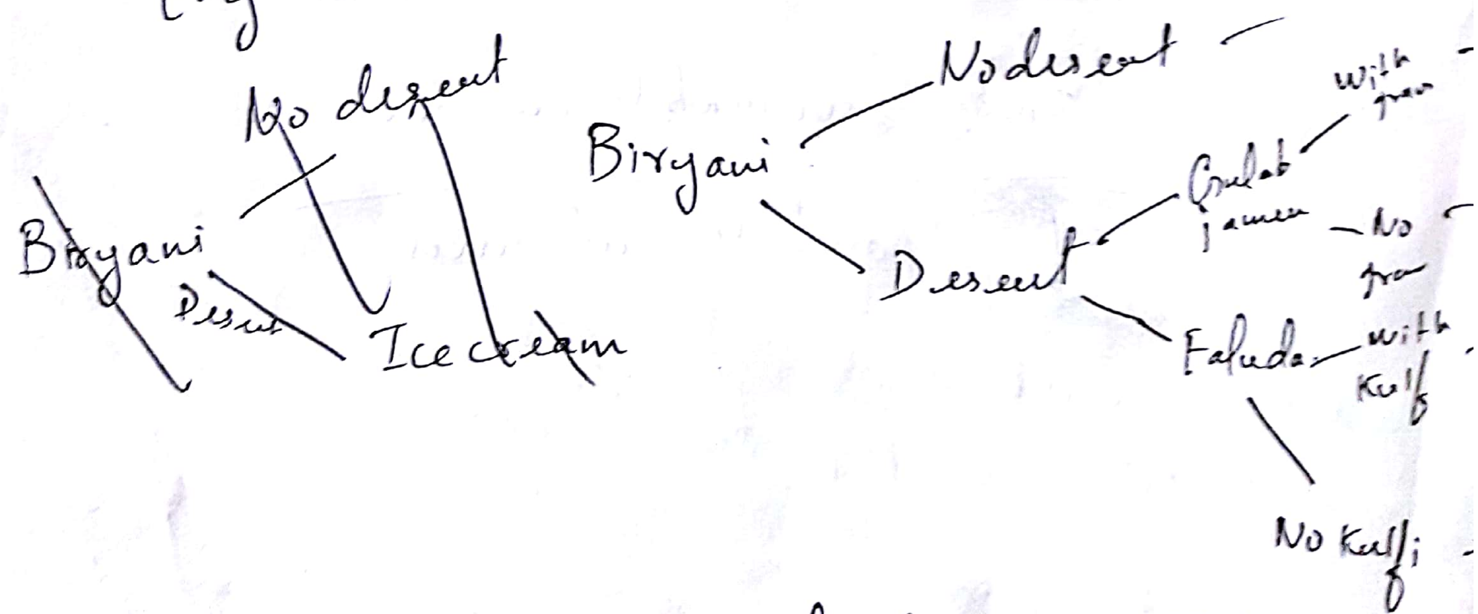
" 4 $E4$ "

:

E n En "

Total outcomes $= E1 \times E2 \ldots \times En$.

E.g.



Total outcomes $= 5$.

Birthday Problem : Group - k people

Prob 2 people have same birthday?

Assumption: 365 days

All days are equally likely.

Independent birthdays.

if $k > 365$ $p = 1$.

↳ two people have same birthday.

if $k \leq 365$

at $k = 23$ ($p = 50\%$)

$$P(\text{no match}) = \frac{365 \times 364 \cdots (365-k+1)}{365^k}$$

$$P(\text{match}) = \begin{cases} 50.7\%, & \text{if } k = 23 \\ 97\%, & k = 50 \\ 99.99\%, & k = \cancel{85}\ 100 \end{cases}$$

Example

23 people

↳ We need two people

$= \binom{23}{2} = \frac{23 \times 22}{2} = 253.$

253 pair (good chances).

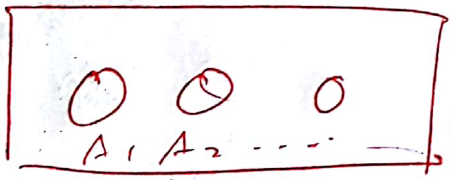
Axioms

1) $P(\emptyset) = 0$; $P(S) = 1$

2) $P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$

$A_1, A_2, A \ldots$ are disjoint

S.



Properties

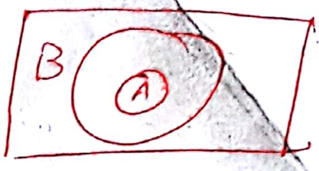
1) $P(A^c) = 1 - P(A)$.

$1 = P(S) = P(A \cup A^c)$

$1 = P(A) + P(A^c)$

or $P(A) = 1 - P(A^c)$.

2) If $A \subseteq B$, then $P(A) \le P(B)$.



$B = A \cup (B \cap A^c)$.

or $P(B) = P(A) + P(B \cap A^c) \geq P(A)$.

3) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ — (1)

$P(A \cup B) = P(A \cup (B \cap A^c))$

$= P(A) + P(B \cap A^c)$. — (2)

To prove

$P(B) - P(A \cap B) = P(B \cap A^c)$

or $P(B) = P(B \cap A^c) + P(A \cap B)$.

$B \cap A^c$, $A \cap B$ are disjoint

union B.

$\Rightarrow P(B) = P(B \cap A^c) + P(A \cap B)$

Inclusion - Exclusion Principle. 4

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

general formula

$$P(A_1 \cup A_2 \ldots A_n) = \sum_{j=1}^{n} P(A_j) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) + \ldots \ldots (-1)^{n+1} P(A_1 \ldots A_n).$$

Proof by interpretation. (story proof).

1) $^{n}C_{k} = {}^{n}C_{n-k}$.

2) $n\binom{n-1}{k-1} = k\binom{n}{k}$

→ Same thing in different ways

We select k people from n; with 1 designated as the leader.

3) $\binom{m+n}{k} = \sum_{j=0}^{k}\binom{m}{j}\binom{n}{k-j}$.

Definition 2 of prob: A prob sample consists of S and P, where S is a sample space, and P, a function takes an event $A \subseteq S$ as an input and returns $P(A) \in [0,1]$ as output. such that

1) $P(\emptyset) = \emptyset$ (or 0); $P(S) = 1$.

2) $P(\bigcup_{i=1}^{\infty} A_n) = \sum_{n=1}^{\infty} P(A_n)$ if $A_1 \dots A_n$ are disjoint (non-overlapping)