Free Space

$$\nabla \cdot \vec{E} = 0$$

$\rho = 0$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$\vec{J} = 0$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \underbrace{\epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}}$$

$$\boxed{\nabla^2 \vec{E} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

Three dimensional wave equation

$$\boxed{\nabla^2 \vec{B} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2}}$$

$$\vec{E} = \vec{E}_0 \sin(kz - \omega t)$$

$$\underbrace{\frac{\partial^2 \vec{E}}{\partial x^2}}_{= 0} + \underbrace{\frac{\partial^2 \vec{E}}{\partial y^2}}_{= 0} + \frac{\partial^2 \vec{E}}{\partial z^2} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\frac{\partial^2 \vec{E}}{\partial z^2} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{E} = \vec{E}_0 \sin(kz - \omega t)$$

$$-k^2 \cancel{\vec{E}_0} \sin(\cancel{kz-\omega t}) = \epsilon_0 \mu_0 (-\omega^2) \cancel{\vec{E}_0} \sin(\cancel{kz-\omega t})$$

$$\boxed{k^2 = \epsilon_0 \mu_0 \omega^2}$$

$$k = \sqrt{\epsilon_0 \mu_0}\, \omega$$

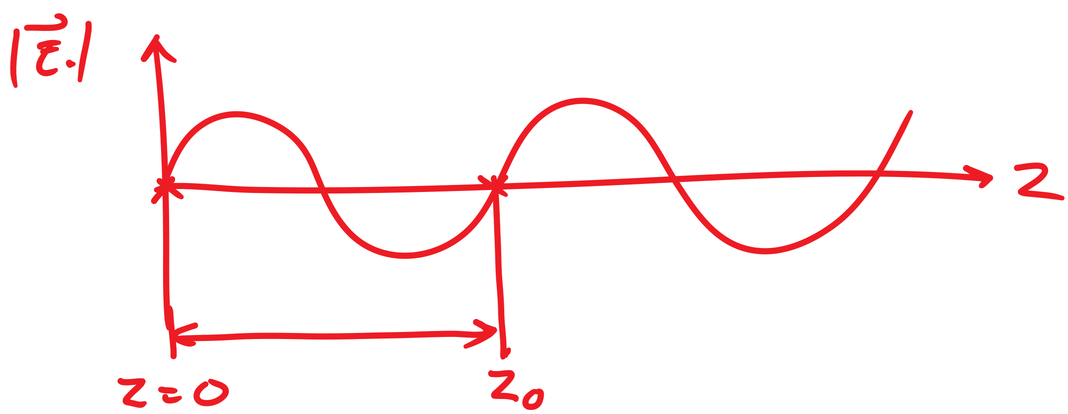
$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

$$\Rightarrow \quad \boxed{\omega = ck}$$

$$\vec{E} = \vec{E}_0 \sin(kz - \omega t)$$

<u>$t=0$</u>

$$\vec{E} = \vec{E}_0 \sin kz$$

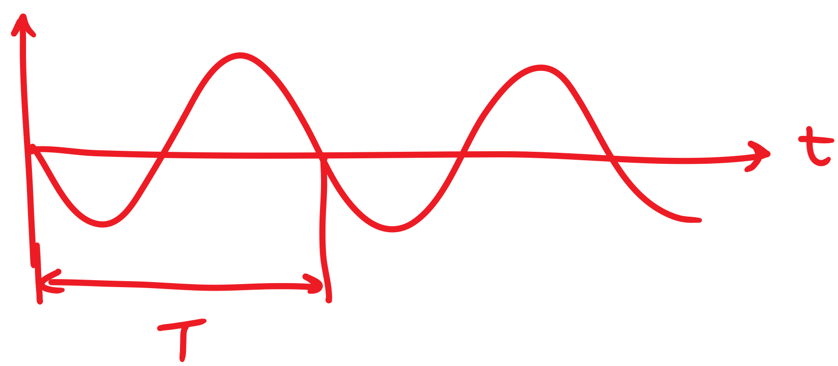


$$\sin kz_0 = 0 \quad \Rightarrow \quad kz_0 = 2\pi$$

$$\lambda = \frac{2\pi}{k} \quad \Rightarrow \quad \boxed{k = \frac{2\pi}{\lambda}}$$

$z=0$

$$\vec{E} = \vec{E}_0 \sin(-\omega t)$$
$$= -E_0 \sin \omega t$$



$$\omega T = 2\pi \Rightarrow \omega = \frac{2\pi}{T} = 2\pi\nu$$

$$\vec{E} = \vec{E}_0 \sin\left[\underbrace{\frac{2\pi}{\lambda} z}_{kz} - \underbrace{2\pi\nu t}_{\omega t}\right]$$

$$= \vec{E}_0 \sin\left[2\pi\left(\frac{z}{\lambda} - \nu t\right)\right]$$

$$\omega = ck \; ; \quad \omega = 2\pi\nu \; ; \quad k = \frac{2\pi}{\lambda}$$

$$2\pi\nu = c \cdot \frac{2\pi}{\lambda} \Rightarrow \boxed{c = \nu\lambda}$$

Visible $\quad 400\,nm < \lambda < 750\,nm$

$$\lambda = 500\,nm = 500 \times 10^{-9}\,m$$

$$\nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{5\times10^{-7}} = 6\times10^{14}\,Hz$$

$$\vec{E} = \vec{E}_0 \sin(kz - \omega t)$$

$$k = \frac{2\pi}{\lambda} ; \qquad \omega = 2\pi\nu ; \qquad \omega = ck$$

$$c = \nu\lambda$$

$$\nabla \cdot \vec{E} = 0$$

$$\vec{E} = (E_{0x}\hat{x} + E_{0y}\hat{y} + E_{0z}\hat{z}) \sin(kz - \omega t)$$

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

$$E_{0z}\, k \cos(kz - \omega t) = 0$$

$$\Rightarrow \boxed{E_{0z} = 0}$$

$$\vec{E} = \underline{\underline{\hat{x}}}\, E_0 \sin(kz - \omega t)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{B} = \underset{\overset{=}{?}}{\vec{B}_0} \sin(kz - \omega t)$$

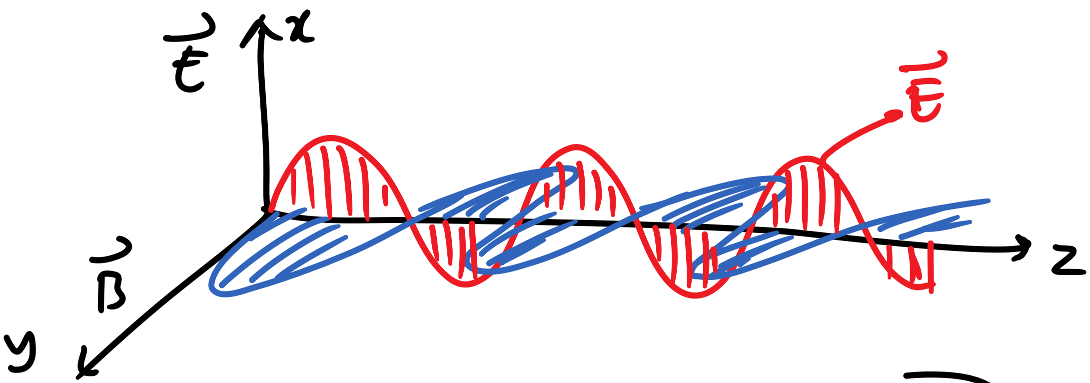
$$\frac{\partial \vec{B}}{\partial t} = \vec{B}_0 (-\omega) \cos(kz - \omega t)$$

$$\nabla \times \vec{E} = E_0\, \hat{y}\, k \cos(kz - \omega t)$$

$$E_0 \hat{y} k = \vec{B}_0 \omega$$

$$\vec{B}_0 = \hat{y} \frac{E_0 k}{\omega} = \hat{y} \frac{E_0}{c}$$

$\omega = ck$



$[\vec{E}, \vec{B}, z$ (Prop direction)$]$

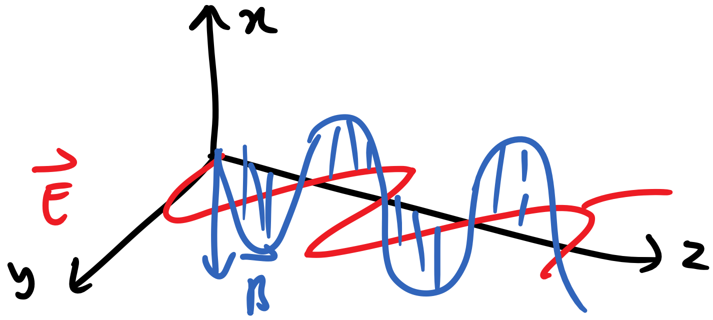
$$\boxed{\begin{aligned} \vec{E} &= \hat{x} E_0 \sin(kz - \omega t) \\ \vec{B} &= \hat{y} \frac{E_0}{c} \sin(kz - \omega t) \end{aligned}}$$

$\omega = ck$

$\lambda = 1\,\mu m$; $\vec{E}$ along $\hat{y}$, Prop along z

$$\vec{E} = \hat{y} E_0 \sin(2\pi \times 10^6 z - 6\pi \times 10^{14} t) \qquad k = 2\pi \times 10^6\, m^{-1}$$

$$\vec{B} = -\hat{x} \frac{E_0}{c} \sin(2\pi \times 10^6 z - 6\pi \times 10^{14} t)$$



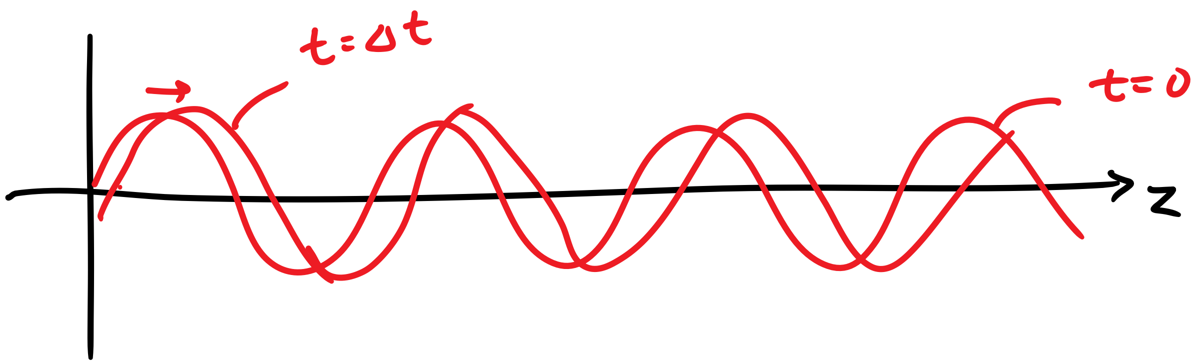
ε $\quad \vec{E} \Rightarrow \hat{y}$

$\vec{B} \Rightarrow -\hat{x}$

k : Propagation Vector $\frac{2\pi}{\lambda}$ (Length)$^{-1}$

ω : Angular frequency $= 2\pi\nu$ (Time)$^{-1}$

$c \cong 3 \times 10^{8}$ m/s PHASE VELOCITY



$\vec{E} = E_0 \hat{x} \sin(kz)$ $\qquad t=0$

$\vec{E} = E_0 \hat{x} \sin(kz - \omega \Delta t)$