Experiment No. 4: RC Filter Design

AIM: Study of Low Pass and High Pass Characteristics of RC Filter Circuit

a) Study the transfer function and phase shift of a low pass RC filter network.
b) Study the transfer function and phase shift of a high pass RC filter network.

APPARATUS REQUIRED: Bread-Board, Digital Multimeter, Digital Oscilloscope, Resistors, Capacitors, Connecting Wires, Testing probes, CRO probes, Function Generator

THEORY: Filter is an essential component for any communication system. A filter or a filter circuit is nothing but a frequency selective circuit which reject the undesired frequency band and pass the desired frequency band. Frequency-selective or filter circuits pass only those input signals to the output that is in the desired range of frequencies (called passband). The amplitude of signals outside this range of frequencies (called stop band) is reduced (ideally reduced to zero). The frequency between the pass and stop bands is called the cut-off frequency (ω_c). Typically, in these circuits, the input and output currents are kept to a small value, and as such, the current transfer function is not an important parameter. The main parameter is the voltage transfer function in the frequency domain, $H_V(j\omega) = \frac{V_o}{V_i}$. Subscript "$V$" of $H_V(j\omega)$ is dropped. As $H(j\omega)$ is a complex number, it has both a magnitude and a phase. Filter circuits, thus, in general introduce a phase, the difference between the input and output signals.

LOW AND HIGH PASS FILTER:

A low pass filter (LPF) attenuates or rejects all high-frequency signals and passes only low-frequency signals below its characteristic frequency called as cut-off frequency, (ω_c). The transfer function of an ideal low-pass fillter is shown in Fig. 4.1. A high pass filter (HPF), is the exact opposite of the LPF circuit. It attenuates or rejects all low-frequency signals and passes only high-frequency signals above its cut-off frequency, (ω_c). The transfer function of an indal high-pass filter is shown in Fig. 4.1. In reality, the pass band and stop band of a filter are not clearly defined, as $|H(j\omega)|$ varies continuously from its maximum towards zero. The cut-off frequency is, therefore, defined as the frequency at which $|H(j\omega)|$ is reduced to $\frac{1}{\sqrt{2}}$ or 0.7 of its maximum value. The cut-off frequency corresponds to signal power being reduced by $\frac{1}{2}$ as $P \propto V^2$. More realistic transfer function of a low-pass filter and an high-pass filter are shown in Fig. 4.2.

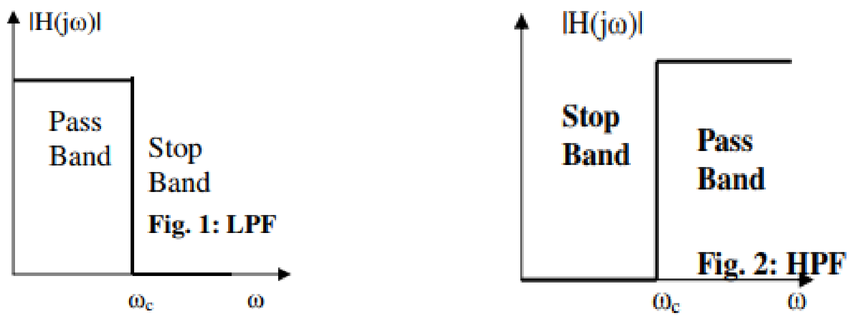


Fig. 4.1: Transfer function of ideal low and high pass filter

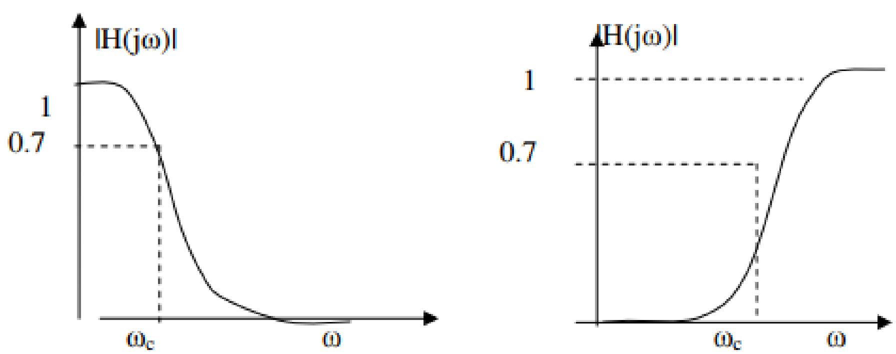


Fig. 4.2: Transfer function of practical low and high pass filter

RC Filter: The simplest passive filter circuit can be made by connecting a single resistor and a single capacitor in series across an input signal, (V_i) with the output signal, (V_O) taken from the junction of these two components. Depending on which way around we connect the resistor and the capacitor with regards to the output signal determines the type of filter construction resulting in either a Low Pass or a High Pass Filter. As there are two passive components within this type of filter design the output signal has an amplitude smaller than its corresponding input signal, therefore passive RC filters attenuate the signal and have a gain of less than one, (unity).

Low-Pass RC Filter: A RC circuit shown in Fig. 4.3 acts as a low-pass filter. For no load resistance (output is an open circuit, R → ∞), output voltage V_O, is given by

$$V_O = \frac{\left(\frac{1}{j\omega C}\right)}{R + \left(\frac{1}{j\omega C}\right)} V_i = \frac{1}{1 + j\left(\omega RC\right)} V_i \tag{1}$$

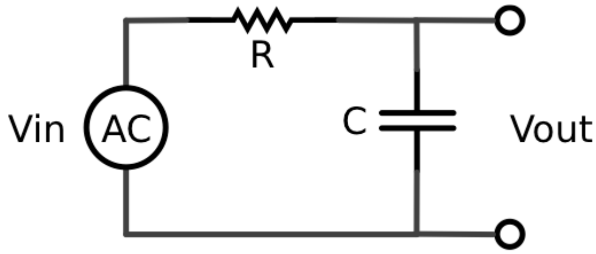


Fig. 4.3: Low-Pass RC Circuit

The transfer function $|H(j\omega)|$ is given by,

$$\left|H(j\omega)\right| = \left|\frac{V_O}{V_i}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}} \tag{2}$$

When ω→0,

$$\left|H(j\omega C)\right|_{\omega\to 0} = 1 \tag{3}$$

For $\omega = \omega_c$,

$$\left|H(j\omega)\right|_{\omega\to\omega_C} = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+(\omega RC)^2}} \Rightarrow \omega_C = \frac{1}{RC} \tag{4}$$

Thus, the transfer function can be re-written as,

$$H(j\omega) = \frac{1}{1+j\dfrac{\omega}{\omega_C}} \Rightarrow \left|H(j\omega)\right| = \frac{1}{\sqrt{1+\left(\dfrac{\omega}{\omega_C}\right)^2}} \tag{5}$$

The phase difference, Φ is given by,

$$\Phi = -\tan^{-1}\left(\frac{\omega}{\omega_C}\right) \tag{6}$$

High-Pass RC Filter: A series RC circuit shown in Fig.4.4 acts as a high-pass filter.

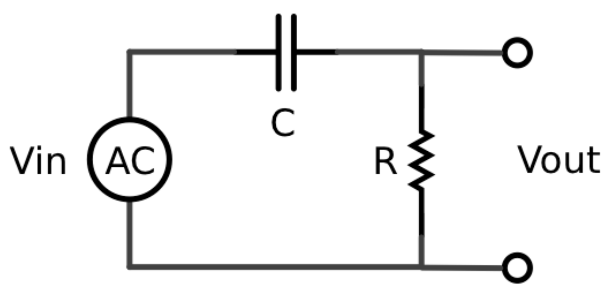


Fig. 5: High-Pass RC Circuit

For no load resistance (open circuit output voltage), the output voltage, V_O, is given by

$$V_O = \frac{R}{R + \left(\frac{1}{j\omega C} \right)} V_i \tag{7}$$

The transfer function is given by

$$H(j\omega) = \frac{V_O}{V_i} = \frac{1}{1 - j\left(\frac{1}{\omega RC} \right)} \tag{8}$$

Thus,

$$\left| H(j\omega) \right| = \left| \frac{V_O}{V_i} \right| = \frac{1}{\sqrt{1 + \left(\frac{1}{\omega RC} \right)^2}} \tag{9}$$

For $\omega = \omega_c$,

$$\left| H(j\omega) \right| = \frac{1}{\sqrt{2}} \Rightarrow \omega_C = \frac{1}{RC} \tag{10}$$

The transfer function can be re-written as

$$\begin{aligned} H(j\omega) &= \frac{1}{1 - j\left(\frac{\omega_C}{\omega} \right)} \\ \left| H(j\omega) \right| &= \frac{1}{\sqrt{1 + \left(\frac{\omega_C}{\omega} \right)^2}} \end{aligned} \tag{11}$$

The phase difference, Φ, is given by

$$\Phi = -tan^{-1}\left(\frac{\omega_C}{\omega} \right) \tag{12}$$

PROCEDURE:

1. Begin lab by familiarizing yourself with the function generator and oscilloscope.

2. Read and measure the values of R and C.

3. Using the oscilloscope, set the function generator to produce a sine wave. Note the value of the amplitude. The amplitude of the input signal must remain constant during the experiment. This signal serves as the input.

4. Set up the low/high pass RC filter on the breadboard as shown in the circuit diagram. Use the function generator to apply a sine wave signal to the input. Use the dual trace oscilloscope to observe both V_i and V_O. Make sure that the two oscilloscope probes have their grounds connected to the function generator ground.

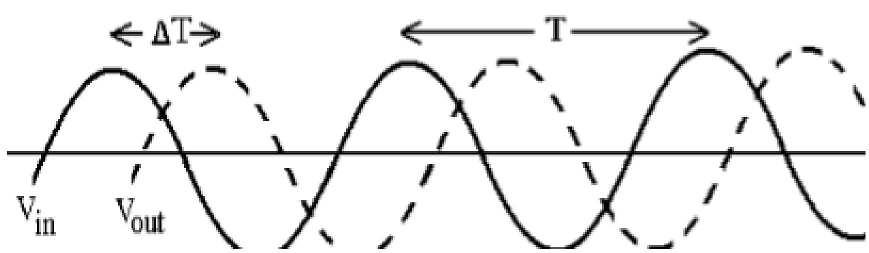
5. For several frequencies between 20 Hz and 20 kHz (the audio frequency range) measure the peak-to-peak amplitude of the output signal. Check often to make sure that the amplitude of input signal V_i remains at the set value and that VOLTS/DIV dials are in their calibrated positions. Take enough data (at least up to 10 times the cut-off frequency, for low pass and down to 1/10 times cut-off frequency, for high pass filter) to make sure that your data recording is complete. If needed use the STOP button of the oscilloscope at the desired frequency to acquire data.

6. From your measurements determine the transfer function

$$|H(j\omega)| = \left|\frac{V_O}{V_i}\right| = \frac{V_O}{V_i} \tag{13}$$

Compute the transfer function using eq. (5) or eq. (11) for an LPF and HPF respectively.

1. For each listed frequency, measure the phase shift with a proper sign as shown in the diagram below.



2. The phase shift angle in degrees is

$$\emptyset = \left(\frac{\Delta T}{T}\right) X\ 360°$$

3. Compute the phase shift angle for each frequency for low/high pass filter.

OBSERVATIONS:

R = _____________, C = ___________

I. Low Pass Filter: V_i (pp) = ______

$$f_c = \frac{1}{2\pi RC} = \_\_\_\_\_\_\_\_\_\_\_\_$$

(a) Observation Table for $|H(j\omega)|$

S.no.	Frequency (k Hz) (ω =2πf)	$\frac{f}{f_c}$	Vo (PP) volts	$\|H(j\omega)\| = \frac{V_o(pp)}{V_i(pp)}$	$\|H(j\omega)\| = \frac{1}{\sqrt{1+\left(\frac{f}{f_c}\right)^2}}$
1	0.5	0.3125	9.5	0.95	
2	1	0.625	8.56	0.856	
3	2		6.5	0.65	
4	3		5.0	0.50	
5	4		3.9	0.39	
6	5		3.3	0.33	
7	6		2.72	0.272	

(b) Observation Table phase angle

S.no.	Frequency (kHz) (ω =2πf)	$\frac{\omega}{\omega_c}$	ΔT (ms)	T (ms)	$\emptyset = \left(\frac{\Delta T}{T}\right) X\, 360°$ (deg)	$Phase, \emptyset = -\tan^{-1}\left(\frac{\omega}{\omega_c}\right)$
1						
2						
3						
4						
5						
6						

II. High Pass Filter: V_i (pp) = ____________________

$$\omega_c = \frac{1}{RC} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

(c) Observation Table for $|H(j\omega)|$

S.no.	Frequency (kHz) (ω =2πf)	$\frac{\omega_c}{\omega}$	$V_O(pp)$ Volt	$\lvert H(j\omega)\rvert = \frac{V_o(pp)}{V_i(pp)}$	$\lvert H(j\omega)\rvert_{dB}$	$\lvert H(j\omega)\rvert = \frac{1}{\sqrt{1+\left(\frac{\omega_c}{\omega}\right)^2}}$
1						
2						
3						
4						
5						
6						

(d) Observation Table for phase angle

S.no.	Frequency (kHz) (ω =2πf)	$\frac{\omega_c}{\omega}$	ΔT (ms)	T (ms)	$\emptyset = \left(\frac{\Delta T}{T}\right) X\ 360°$ (deg)	$Phase, \emptyset = -\tan^{-1}\left(\frac{\omega_c}{\omega}\right)$
1						
2						
3						
4						
5						
6						

Graphs: Trace and study bode plots of |H(jω)|dB and Φ versus f(×2π) in a semi-log format for an RC LPF and HPF. Determine the cut-off frequency from the graph. Also, estimate the frequency roll-off for each filter

RESULTS: