Department of Mathematics, Bennett University
Engineering Calculus (EMAT101L)
Tutorial Sheet 1

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1. Find the infimum and supremum of the following sets:

   (a) $S = \{1 + (-1)^n : n \in \mathbb{N}\}$.

   (b) $S = \{\frac{(-1)^n}{n^2} : n \in \mathbb{N}\}$.

   (c) $S = \{\sin\left(\frac{n\pi}{3}\right) : n \in \mathbb{N}\}$.

   (d) $S = \{\frac{1}{n+m} : n, m \in \mathbb{N}\}$.

2. Let S be a non-empty subset of $\mathbb{R}$ and α be a real number. If $\alpha = \sup S$, then show that $s \leq \alpha$ for all $s \in S$ and for any $\epsilon > 0$, there is some $s_0 \in S$ such that $\alpha - \epsilon < s_0$.

3. If $x + \epsilon > y$ holds for each $\epsilon > 0$, then show that $x \geq y$.

4. If for any $\epsilon > 0$, $|x - y| < \epsilon$, then show that $x = y$.

5. Use the Archimedean property to show that $\cap_{n\in\mathbb{N}}(-\frac{1}{n}, \frac{1}{n}) = \{0\}$.

6. Let $r \in \mathbb{R}$. Prove that there exists a sequence $\{x_n\}$ of rational numbers such that $\lim\limits_{n\to\infty} x_n = r$.

7. By the definition of convergence, prove the limit of the following:

   (a) $\lim\limits_{n\to\infty} \frac{2n}{2+n} = 2$.

   (b) $\lim\limits_{n\to\infty} \frac{5}{1+n^2} = 0$.

   (c) For $p > 0$, show that $\lim\limits_{n\to\infty} \dfrac{1}{n^p} = 0$.

   (d) For $n \in \mathbb{N}$, let $a_n = \frac{9}{10} + \frac{9}{10^2} + \cdots + \frac{9}{10^n}$. Then show that $\lim\limits_{n\to\infty} a_n = 1$.

8. Let $a, a_1 \in \mathbb{R}$. For $n \geq 1$, the sequence a_n is defined by $2a_{n+1} + a_n = a$. Show that the sequence converges and its limit is $\dfrac{a}{3}$.

9. Prove or disprove (true or false) the following:

   (a) The sequence $\left\{\sum\limits_{i=1}^{n} \frac{1}{\sqrt{n^2+i}}\right\}$ converges to 1.

   (b) The sequence $\sum\limits_{k=0}^{n} \frac{1}{(n+k)^2}$ converges to 0.

   (c) If $0 < a < b$, $\lim\limits_{n\to\infty} \left(\frac{a^{n+1}+b^{n+1}}{a^n+b^n}\right) = a$.

10. If $\{a_n\}$ is a bounded sequence and $\{b_n\}$ is another sequence which converges to 0, show that the product sequence also converges to 0. What can you say about the product sequence, if $\{b_n\}$ converges, but to a non-zero point?