Quiz Solution

① To show that R is an equivalence relation, we must prove it is:- (2 marks)

(a) Reflexive:- $a-a$ is divisible by m.

Since, $a-a=0$ and $0 = 0 \times m$.

$\therefore$ $a \equiv a \pmod{m}$ and the relation is reflexive.

(b) Symmetric:- Suppose $a \equiv b \pmod{m}$, then $(a-b)$ is divisible by m. So, $a-b = km$ where k is an integer.

It follows that $(b-a) = (-k)m$ so, $b \equiv a \pmod{m}$ and the relation is symmetric.

(c) Transitive:- Suppose $a \equiv b \pmod{m}$ and $b \equiv c \pmod{m}$.

So, m divides both $(a-b)$ and $(b-c)$ such that

$(a-b) = km$ -(1) and $(b-c) = nm$ -(2) where k and n are integers.

Adding (1) and (2), we get

$a-b+b-c = km + nm$

$\Rightarrow a-c = m(k+n)$

Thus, $a \equiv c \pmod{m}$

$\therefore$ The relation is transitive.

Hence, $a \equiv b \pmod{m}$ is an equivalence relation

$$\mu_{Cost}(x) = \begin{cases} 1 & \text{if } x \leq 30 \text{ lakh.} \\ \frac{30}{x} & \text{if } x > 30 \text{ lakh.} \end{cases}$$ (2 marks)

$$\mu_{dist}(y) = \begin{cases} 1 & \text{if } y \leq 5 \text{ kms} \\ \frac{5}{x} & \text{if } y > 5 \text{ kms.} \end{cases}$$

Cost = $\{(A, 0.6), (B, 1)\}$

Distance = $\{(A, 1), (B, 0.7)\}$

The best choice would be the plot which has a higher membership degree in the intersection set of cost and distance.

$\therefore$ $C \cap D = \{(A, 0.6), (B, 0.7)\}$.

$\therefore$ Plot B is a better choice.

③ $A_n = \{ i \in Z : i \text{ is divisible by } n\}$ be a set where $n \in N$. (1 mark)

(i) $A_3 \cap A_7 = \{ i \in Z : i \text{ is divisible by both 3 and 7}\}$

$= \{ i \in Z : i \text{ is divisible by } 21\}$

$= \{0, 21, 42, 63, \ldots\}$

(ii) $A_3 \cup A_7 = \{ i \in Z : i \text{ is divisible by 3 or 7}\}$

$= \{0, 3, 7, 6, 9, 14, 21 \ldots\}$.

④ (i) $A = \{x \mid 3x - 2 = 0, x \in Q\}$ (1 mark)

$x = 1, 2, \ldots$

$3 \times 1 - 2 \neq 0$

$\therefore$ Null set.

(ii) $B = \{x \mid 30x - 59 = 0, x \in N\}$

$x = 1, \quad 1^3 - 1 = 0$

$\therefore$ It is not a null set.

$A = \{4, 5, 7, 8, 10\}$, $B = \{4, 5, 9\}$ and $C = \{1, 4, 6, 9\}$

$A \cap (B \cup C) = A \cap \{1, 4, 5, 6, 9\} = \{4, 5\}$ (1 mark).

$(A \cap B) = \{4, 5\}$ $(A \cap B) \cup (A \cap C) = \{4, 5\}$.

$(A \cap C) = \{4\}$

$\therefore A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ Hence Proved

(6) (a) $A - \emptyset = A$ (b) $\emptyset - A = \emptyset$ (1 mark)

(7) If $A = \{+, -\}$, then $A^2 = A \times A = \{(+, +), (+, -), (-, +), (-, -)\}$ (1 mark)

(8) $2^3 - 1 = 7$, $2^2 = 4$ (1 mark)

(9) $A = \{1, 2, 3, 4, 8\}$ and $B = \{2, 4, 6, 7\}$

$A \cup B = \{1, 2, 3, 4, 6, 7, 8\}$, $A \cap B = \{2, 4\}$ (2 marks)

$A \Delta B = A \cup B - A \cap B = \{1, 3, 6, 7, 8\}$

(10) (a) Antisymmetric (b) Symmetric (1 mark)

(11) $A \times B \times C = \{(0,1,0), (0,1,1), (0,1,2), (0,2,0), (0,2,1), (0,2,2),$
$(1,1,0), (1,1,1), (1,1,2), (1,2,0), (1,2,1), (1,2,2)\}$
(1 mark)

(12) Let A, B, C denote the sets of households having a washing machine, a vaccum cleaner and a refrigerator respectively.

Hence, given that:
$n(A \cup B \cup C) = 1000$, $n(C') = 400$, $n(B') = 380$,
$n(A') = 542$, $n(A \cap B) = 294$, $n(B \cap C) = 277$, $n(A \cap C) = 190$

But, $C \cap C' = \phi$ and $n(C \cap C') = 0$

$\therefore$ $n(C) + n(C') = n(U) = 1000$

$\therefore$ $n(C) + 400 = 1000$

$\therefore$ $\boxed{n(C) = 600}$

Similarly, $\boxed{n(A) = 458}$ and $\boxed{n(B) = 620}$

$\therefore$ From $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$

$- n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$1000 = 458 + 620 + 600 - 294 - 190 - 277 +$

$n(A \cap B \cap C)$

or $\boxed{n(A \cap B \cap C) = 83}$ Ans.

This gives the number of households having all the appliances.

Now, $(B \cap C' \cap A')$ is set of all households which have a vaccum cleaner but no washing machine or refrigerator.

Hence, we need to compute $n(B \cap C' \cap A')$.

But, $(B \cap C' \cap A') \cup (B' \cap C' \cap A') = C' \cap A'$

We know, there is no houshold which has no appliances

ie, $n(B' \cap C' \cap A') = 0$

$\therefore$ $n(B \cap C' \cap A') = n(C' \cap A') = n(U) - n(A \cup C)$

$= n(U) - \{n(A) + n(C) - n(A \cap C)\}$

$= 1000 - 458 - 600 + 190 = 132.$

$\boxed{n(B \cap C' \cap A') = 132}$ Ans

This gives the number of households having refrigerator only.

Given mRn if $(m-n)$ is divisible by 3

mSn if $(m-n)$ is divisible by 4.

$\therefore \; m(R \cup S)n$ if $(m-n)$ is divisible by 3 or is divisible by 4.

$\therefore$ $\boxed{m \sim (R \cup S) n \text{ if } (m-n) \text{ is not divisible by 3 and not divisible by 4.}}$ Ans

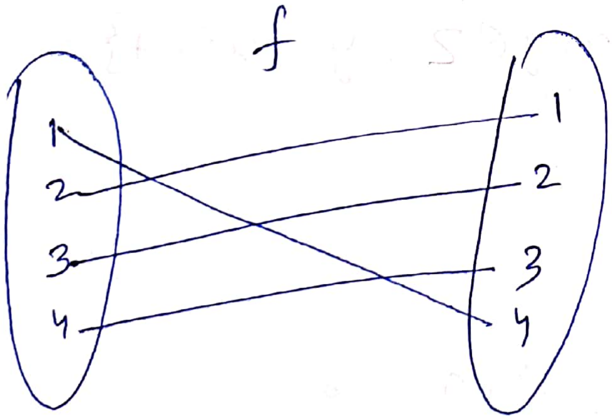
(2 marks)

(14) Given are two functions 'f' and 'g' defined on set $A = B = \{1,2,3,4\}$. (1 mark)

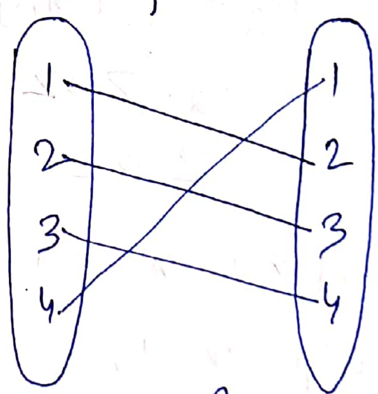
These two functions will be invertible if $\boxed{f^{-1} = g}$

and $\boxed{g^{-1} = f}$.

f



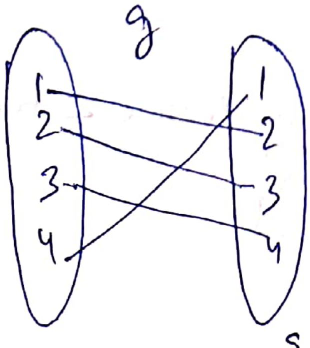
f^{-1}



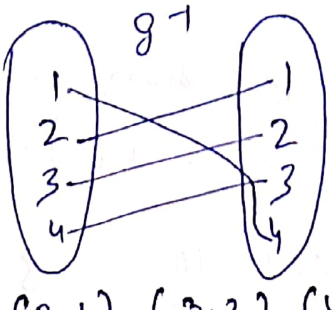
$\therefore \; f^{-1} = \{(1,2), (2,3), (3,4), (4,1)\}$.

So, $\boxed{f^{-1} = g}$ verified.

g



g^{-1}



$\therefore \; g^{-1} = \{(1,4), (2,1), (3,2), (4,3)\}$

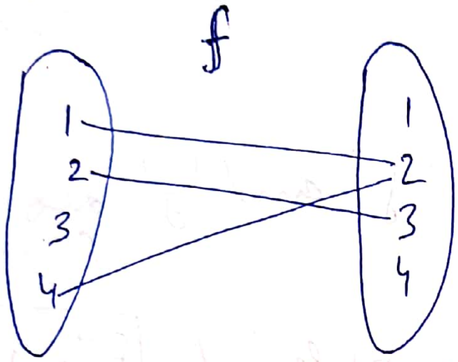
So, $\boxed{g^{-1} = f}$ verified.

$\therefore$ f and g are invertible.

(15) (a) Domain = $\{1, 2, 3, 4\}$. (0.5 + 0.5 = 1 mark)

$R_1 = \{(1,2), (2,3), (4,2)\}$.

R_1 as a function f.



$\therefore$ R_1 is not a function because there is no image for the element 3 in the domain. Ans.

(b) $R_2 = \{(x, y) \mid x, y \in Z,\ y = x^2 + 7\}$

$R_2 : Z \to Z$.

$\therefore$ $F : Z \to Z$.

$y = f(x) = x^2 + 7$.

Putting $x = 1$, $y = 1^2 + 7 = 8$.

$x = 2$, $y = 2^2 + 7 = 4 + 7 = 11$

$x = 3$, $y = 3^2 + 7 = 9 + 7 = 16$

$\vdots$

Therefore, R_2 is a function, since for each $x \in Z$ there is a unique y given by $\boxed{y = x^2 + 7}$.

Range = $\{8, 11, 16 \ldots\}$ Ans.