Electromagnetic induction

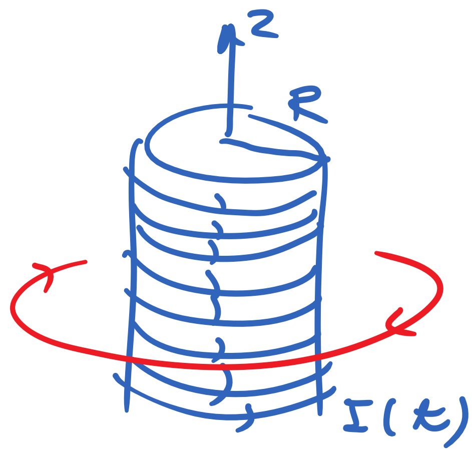
$$\boxed{\mathcal{E} = \oint_C \vec{E}\cdot\vec{dl} = -\frac{\partial \Phi_m}{\partial t} = -\frac{\partial}{\partial t}\int_A \vec{B}\cdot\vec{da}}$$

$$\boxed{\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

Example

$\vec{B} = \mu_0 n I \hat{z}$ inside

$= 0$ outside



$\rho = 0$

$\nabla\cdot\vec{E} = 0$

$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

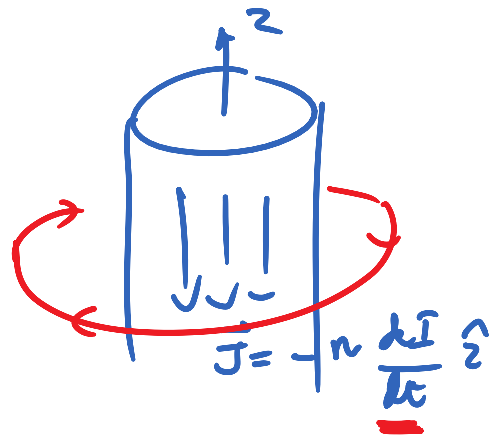
$\nabla\cdot\vec{B} = 0$

$\nabla\times\vec{B} = \mu_0\vec{J}$

$$\boxed{\begin{aligned}\nabla\times\vec{E} &= -\mu_0 n \frac{dI}{dt}\hat{z} \quad & r<R \\ &= 0 & r>R \\ \nabla\cdot\vec{E} &= 0\end{aligned}}$$

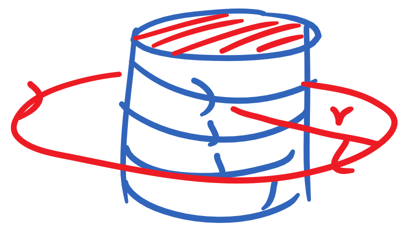
$$\vec{J} = -n\frac{dI}{dt}\hat{z} \qquad r<R$$

$$= 0 \qquad r>R$$



$$\nabla\cdot\vec{B} = 0 \quad \& \quad \nabla\times\vec{B} = \mu_0\vec{J}$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a}$$



$r>R$

$$2\pi r E = -\frac{d}{dt}\left(\pi R^2 \mu_0 n I\right)$$

$$E = -\frac{\pi R^2 \mu_0 n}{2\pi r}\frac{dI}{dt}$$

$$\boxed{\vec{E} = -\frac{\mu_0 n R^2}{2r}\frac{dI}{dt}\hat{\varphi}}$$

$r<R$

$$\boxed{\vec{E} = -\frac{\mu_0 n r}{2}\frac{dI}{dt}\hat{\varphi}}$$

$$\boxed{\begin{aligned} &\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_0} \\ &\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \\ &\nabla\cdot\vec{B} = 0 \\ &\nabla\times\vec{B} = \mu_0\vec{J} \end{aligned}}$$

$$\nabla\cdot(\nabla\times\vec{E}) = -\frac{\partial}{\partial t}(\nabla\cdot\vec{B}) = 0$$

$$\nabla\cdot(\nabla\times\vec{B}) = \mu_0\,\nabla\cdot\vec{J}$$

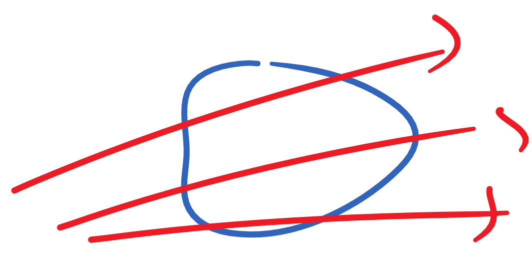
$$0 = \mu_0\,\nabla\cdot\vec{J} \quad *$$

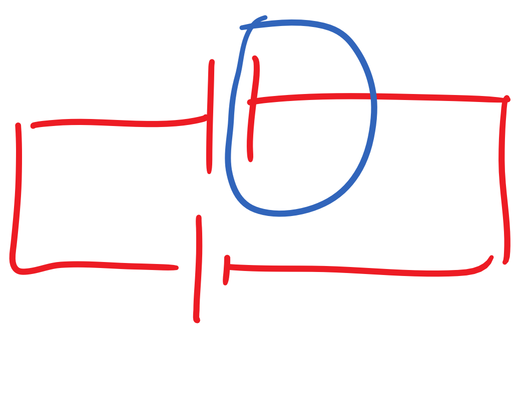
$$\boxed{\nabla\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0}$$

Continuity equation

$$\oint\vec{J}\cdot d\vec{a} = \int\nabla\cdot\vec{J}\,d\tau$$

$$= 0 \quad *$$

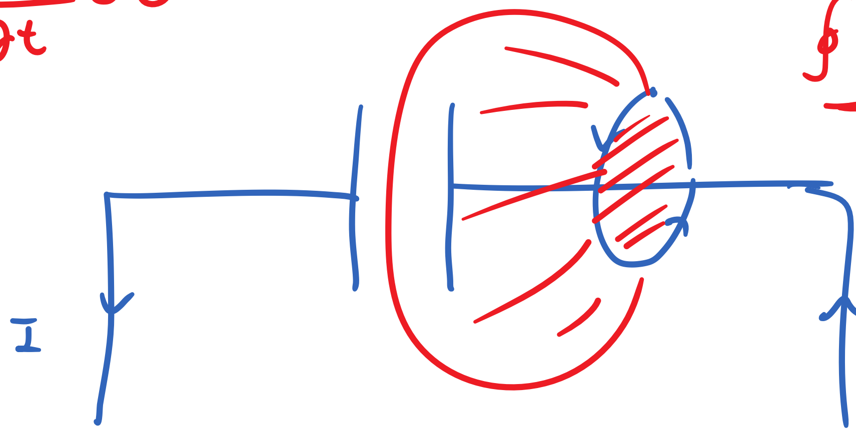




DISPLACEMENT CURRENT

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$



$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enc}$$

$$= \mu_0 \int \vec{J} \cdot \vec{da}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \Rightarrow \rho = \epsilon_0 \nabla \cdot \vec{E}$$

$$\nabla \cdot \vec{J} + \frac{\partial}{\partial t}\left(\epsilon_0 \nabla \cdot \vec{E}\right) = 0 \Rightarrow \nabla \cdot \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right) = 0$$

$$\boxed{\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t}\right)}$$

$$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

$$\epsilon_0 \frac{\partial \vec{E}}{\partial t} \Rightarrow \text{Displacement Current}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

MAXWELL'S EQUATIONS