Data Link Layer

Error Detection and Correction

Error detection and correction

- Data can be corrupted during transmission.
- Some applications require that errors be detected and corrected.

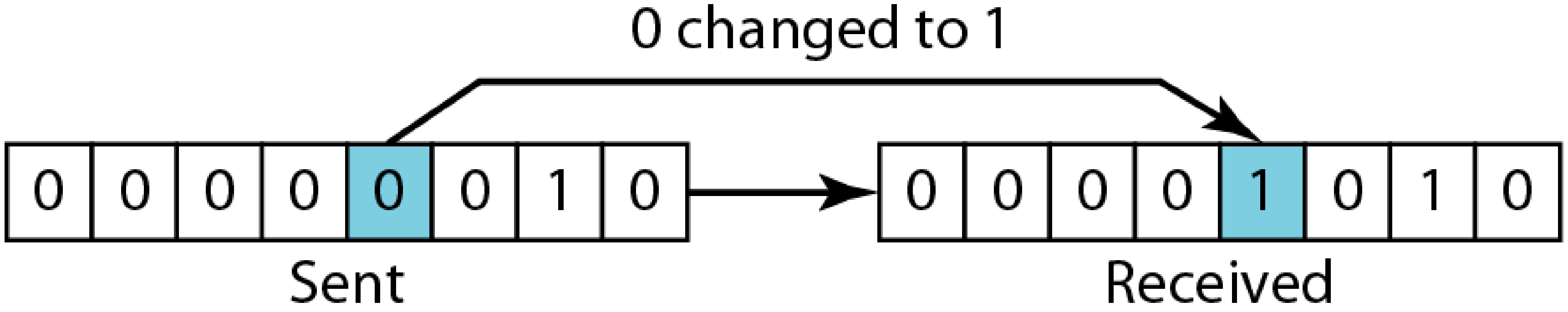
Types of Errors

- Single-bit error
- Burst error

- To detect or correct errors, we need to send extra (redundant) bits with data.
- These Redundant bits are added by sender and removed by the receiver
- Redundancy is achieved through various coding schemes

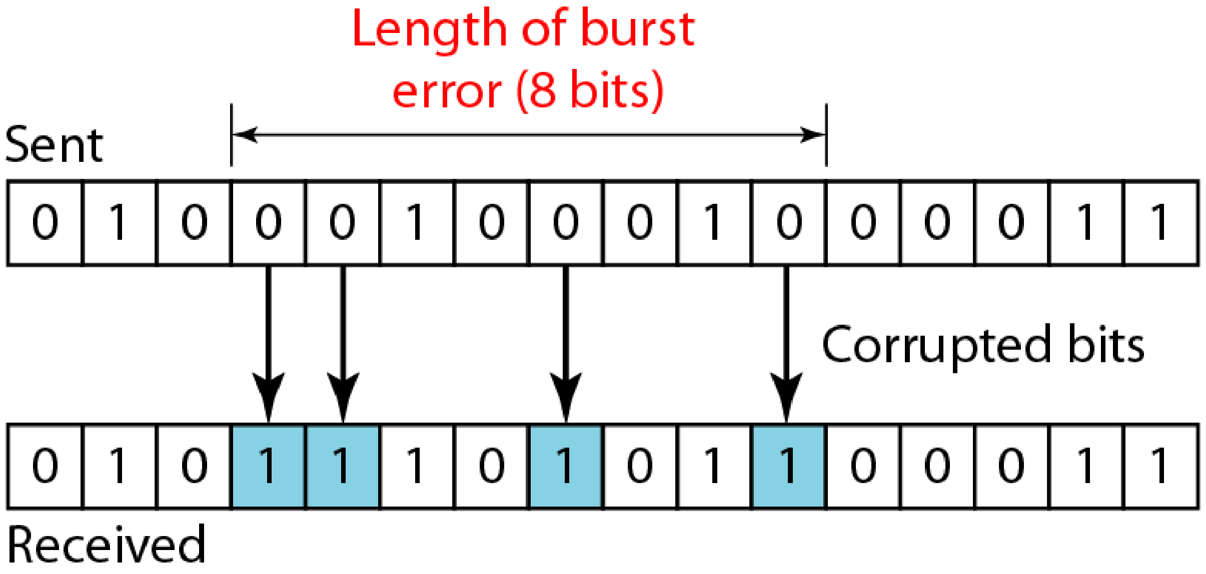
Single-bit error

In a single-bit error, only 1 bit in the data unit has changed.



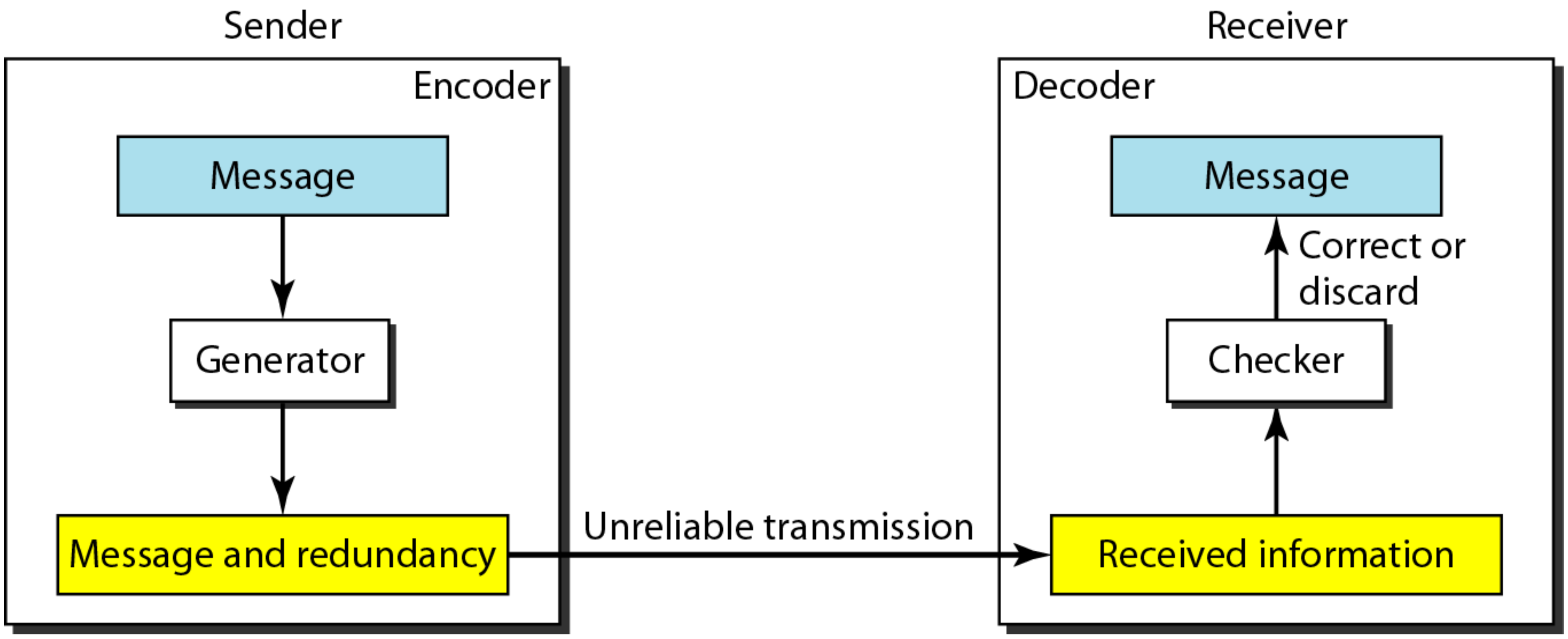
Burst error

A burst error means that 2 or more bits in the data unit have changed.



- A burst error is more likely to occur than a single-bit error.
- The duration of noise is normally longer than the duration of 1 bit, which means that when noise affects data, it affects a set of bits.
- The number of bits affected depends on the data rate and duration of noise.

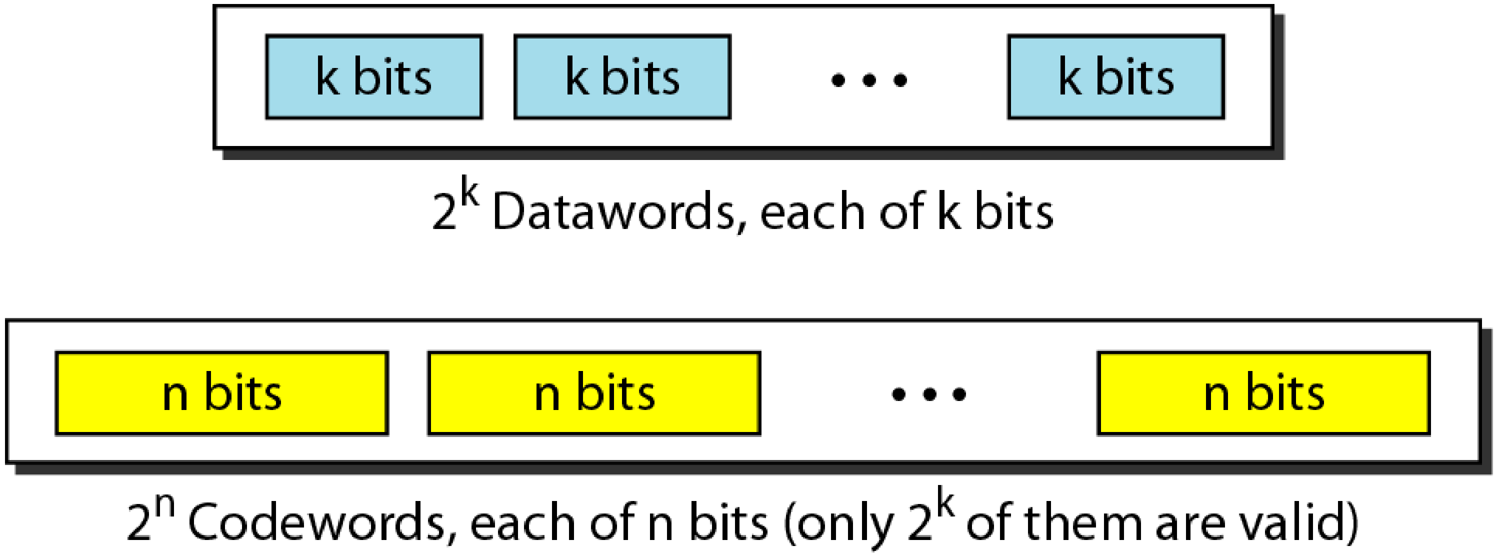
The structure of encoder and decoder



BLOCK CODING

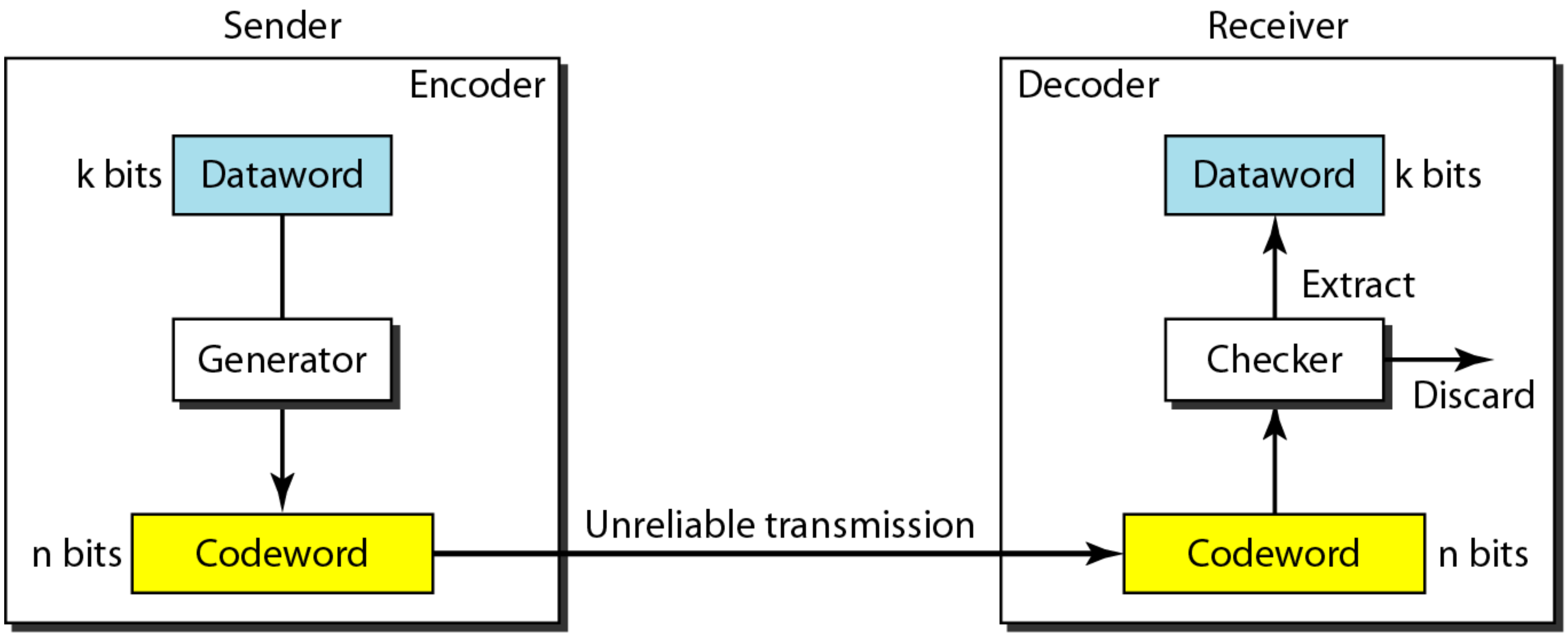
- *In block coding, message is divided into blocks, each of k bits, called datawords.*
- *We add r redundant bits to each block to make the length n = k + r.*
- *The resulting n-bit blocks are called codewords.*

Datawords and codewords



Example:- 4B/5B block coding

- *Process of error detection in block coding*



Example: single bit error detection

Let us assume that k = 2 and n = 3.

Assume the sender encodes the dataword 01 as 011 and sends it to the receiver. Consider the following cases:

- The receiver receives 011. It is a valid codeword. The receiver extracts the dataword 01 from it.
- The codeword is corrupted during transmission, and 111 is received. This is not a valid codeword and is discarded.
- The codeword is corrupted during transmission, and 000 is received. This is a valid codeword. The receiver incorrectly extracts the dataword 00.

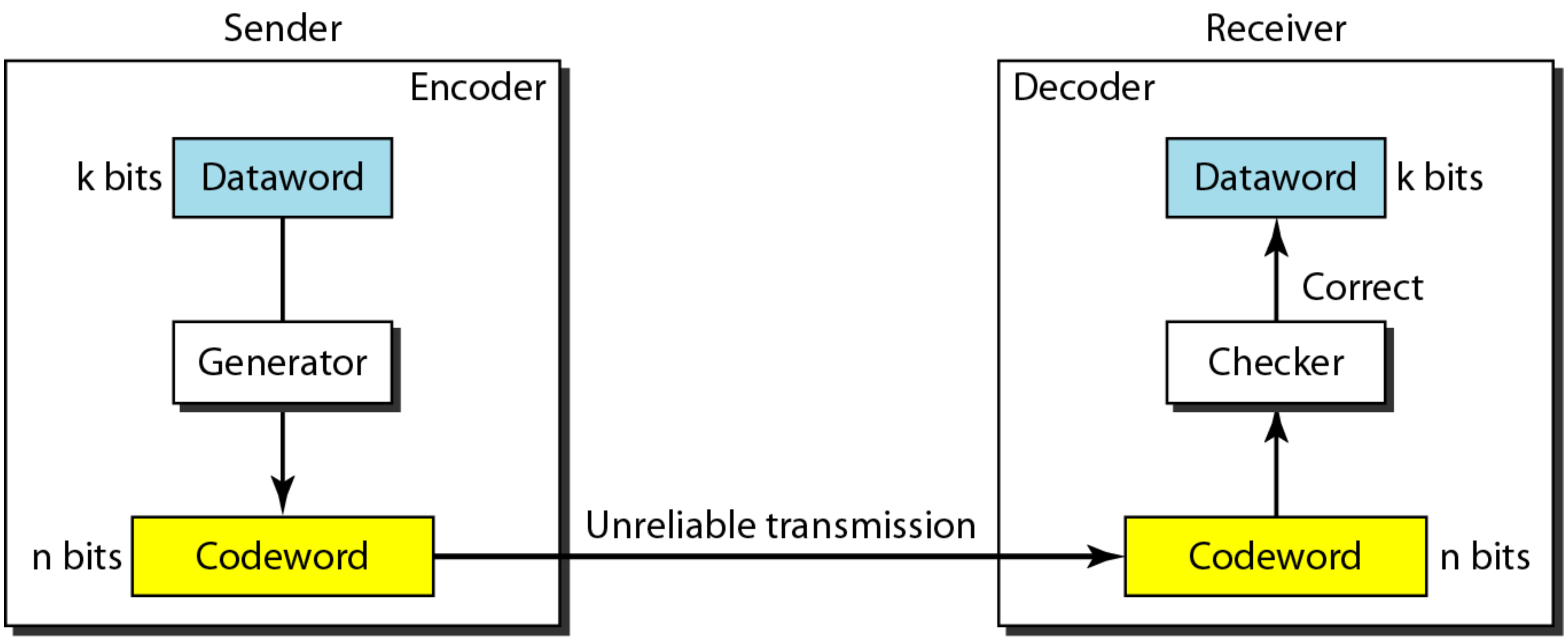
Two corrupted bits have made the error undetectable.

Datawords	Codewords
00	000
01	011
10	101
11	110

List of datawords and codewords.

Error correction

Structure of encoder and decoder



Example: *Add more redundant bits.*

- *3 redundant bits added to the 2-bit dataword to make 5-bit codewords.*
- *Assume the dataword is 01. The sender creates the codeword 01011. The codeword is corrupted during transmission, and 01001 is received.*
- *First, the receiver finds that the received codeword is not in the table. This means an error has occurred.*

Dataword	*Codeword*
00	00000
01	01011
10	10101
11	11110

A code for error correction

The receiver, assuming that there is only 1 bit corrupted, uses the following strategy to guess the correct dataword.

- *Comparing the received codeword with the first codeword in the table (01001 versus 00000), there are two different bits.*
- *By the same reasoning, the original codeword cannot be the third or fourth one in the table.*
- *The original codeword must be the second one in the table because this is the only one that differs from the received codeword by 1 bit. The receiver replaces 01001 with 01011 and consults the table to find the dataword 01.*

XORing of two single bits or two words

$0 \oplus 0 = 0 \qquad 1 \oplus 1 = 0$

a. Two bits are the same, the result is 0.

$0 \oplus 1 = 1 \qquad 1 \oplus 0 = 1$

b. Two bits are different, the result is 1.

$$
\begin{array}{cccccc}
 & 1 & 0 & 1 & 1 & 0 \\
\oplus & 1 & 1 & 1 & 0 & 0 \\
\hline
 & 0 & 1 & 0 & 1 & 0
\end{array}
$$

c. Result of XORing two patterns

Hamming distance

The Hamming distance between two words is the number of differences between corresponding bits.

Find the Hamming distance between two pairs of words.

1. *The Hamming distance d(000, 011) is 2 because*

$$\boxed{000 \oplus 011 \text{ is } 011 \text{ (two 1s)}}$$

2. *The Hamming distance d(10101, 11110) is 3 because*

$$\boxed{10101 \oplus 11110 \text{ is } 01011 \text{ (three 1s)}}$$

- The minimum Hamming distance is the smallest Hamming distance between all possible pairs in a set of words.

Find the minimum Hamming distance of the coding scheme in table shown in previous slide

Solution

We first find all Hamming distances.

$d(000, 011) = 2$	$d(000, 101) = 2$	$d(000, 110) = 2$	$d(011, 101) = 2$
$d(011, 110) = 2$	$d(101, 110) = 2$		

The d_{min} in this case is 2.

Question

Find the minimum Hamming distance of the coding scheme

Solution

We first find all the Hamming distances.

$d(00000, 01011) = 3$	$d(00000, 10101) = 3$	$d(00000, 11110) = 4$
$d(01011, 10101) = 4$	$d(01011, 11110) = 3$	$d(10101, 11110) = 3$

The d_{min} in this case is 3.

To guarantee the detection of up to s errors in all cases, the minimum Hamming distance in a block code must be $d_{min} = s + 1$.