

Department of Mathematics, Bennett University
EMAT203L (Probability and Statistics)
Tutorial sheet 3

1. Let X be a random variable such that $P(X > \frac{1}{2}) = \frac{7}{8}$ and following PDF:

$$f_X(x) = \begin{cases} ax & 0 \leq x < 1 \\ b - x & 1 \leq x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

Find values of a and b . Find distribution function of X and draw its graph.

2. Let X be a random variable such that $P(X > 2) = \frac{7}{8}$, and its PDF is given as follows:

$$f_X(x) = \begin{cases} 0 & x < 1 \\ \frac{x}{4} + a & 1 \leq x < 3 \\ -\frac{x}{4} + b & 3 \leq x < 5 \\ 0 & 5 \leq x. \end{cases}$$

Find values of a and b . Find distribution function of X and draw its graph.

3. Let X be a random variable such that $E(X) = -1$, $\text{Variance}(X) = 2$ and following PDF:

$$f_X(x) = \begin{cases} ax & -1 \leq x < 1 \\ bx + c & 1 \leq x < 2 \\ 0 & \text{otherwise.} \end{cases}$$

Find values of a , b and c . Find distribution function of X and draw its graph.

4. What is the expected value of uniformly distributed random variable X whose PDF is given as follows:

$$f_X(x) = \begin{cases} \frac{1}{5} & 2 \leq x \leq 7, \\ 0 & \text{otherwise.} \end{cases}$$

5. Let number of customers in a day arriving in a bank (say X) follows Poisson distribution with λ as 50 customers per day and pmf as follows:

$$P(X = k) = \frac{e^{-\lambda} \lambda^k}{k!}, \quad k = 0, 1, 2, \dots$$

- (a) What is the average number of customers in the bank?
- (b) What is the probability that on any given day, there is no customer in the bank?
- (c) What is the probability that number of customers in a day is always greater than or equals to 5.

6. The life of a light bulb in months is denoted by a random variable X with following PDF (Exponential distribution):

$$f_X(x) = 0.25e^{-0.25x}, \quad x > 0.$$

- (a) What is the average life of light bulb?
- (b) What is the probability that the life of light bulb will be more than 2 months?
- (c) What is the probability that the life of light bulb will be less than 45 days?
- (d) Given that the bulb was working for last 1 month, what is the probability that it will be working for next 3 months?
- (e) The 10 pieces of the bulb are put under observation independently for 6 months. Let Y denotes the number of working bulbs after 6 months of inspection.
Find $P(Y = 3)$? What is distribution function and PMF of random variable Y ?
- (f) The 5 pieces of the bulb are put in a series and wired together. (A series system works if all of its component are working.) Let Z denotes the life of this series system in months.
Find $P(Z \leq 2.5)$. What is distribution function and PDF of random variable Z ?