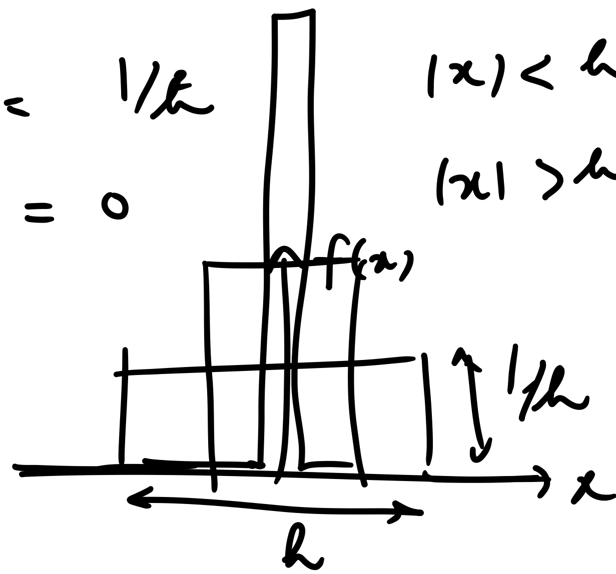
Yesterday I was clever, so I wanted to change the world. Today I am wise, so I am changing myself.

Jalaluddin Rumi

DIRAC DELTA FUNCTION

$$f(x) = 1/h \qquad |x| < h/2$$
$$= 0 \qquad |x| > h/2$$

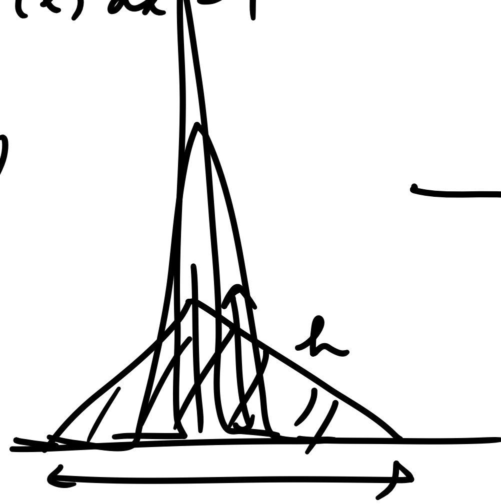


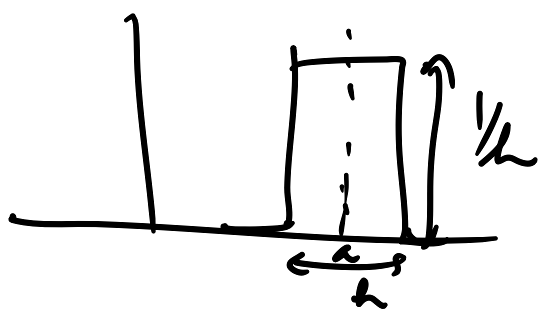
$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\delta(x) = 0 \quad \text{for} \quad x \neq 0$$

$$\& \quad \int_{-\infty}^{\infty} \delta(x)\,dx = 1$$

$\delta(x-a)$





$$f(x)\,\delta(x-a) = 0 \quad \text{for} \quad x \neq 0$$

$$\int_{-\infty}^{\infty} \underbrace{f(x)\,\delta(x-a)}\,dx = \int_{-\infty}^{\infty} f(a)\,\delta(x-a)\,dx$$

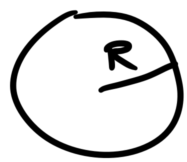
$$= f(a) \int_{-\infty}^{\infty} \delta(x-a)\,dx$$

$$= f(a)$$

$$\delta^3(\vec{r}) = \delta(x)\,\delta(y)\,\delta(z)$$

Point charge at origin Q

$$\rho = \frac{Q}{\frac{4\pi}{3} R^3}$$



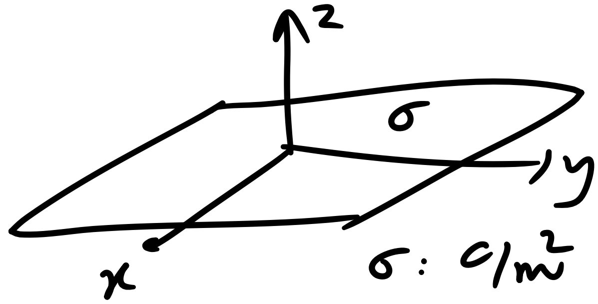
$$\boxed{\rho(\vec{r}) = Q\,\delta^3(\vec{r})}$$

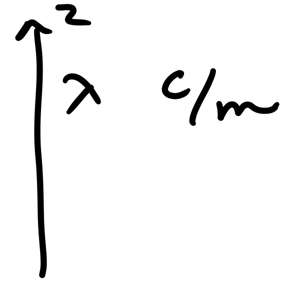
$$\iiint \rho(\vec{r})\,d\tau = \iiint Q\,\delta^3(\vec{r})\,d\tau$$

$$= Q \iiint \delta^3(\vec{r})\,d\tau$$

$$= Q$$

Prob: σ & λ in terms of Dirac delta functions.



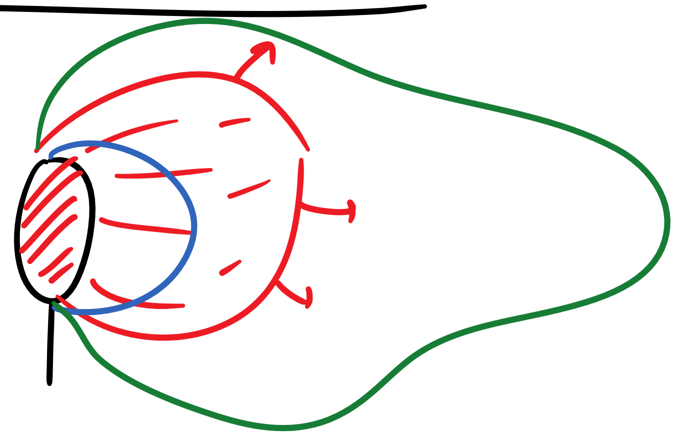


$$\boxed{\iint_S (\nabla\times\vec{F})\cdot d\vec{A} = \oint_L \vec{F}\cdot d\vec{l}}$$

STOKE'S THEOREM

$$\boxed{\iiint_V \nabla\cdot F \, dV = \oiint_S \vec{F}\cdot d\vec{A}}$$

GAUSS'S THEOREM



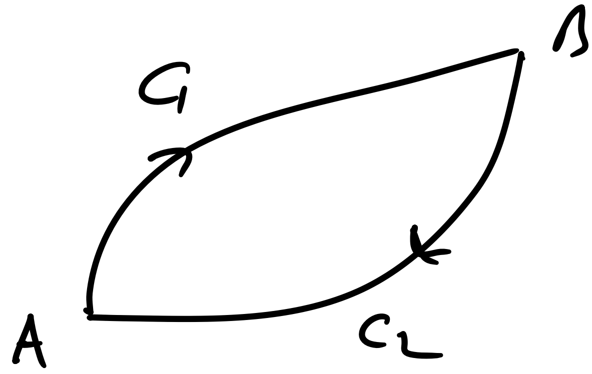
$$\nabla\times\vec{E} = 0$$

$$\nabla\times(\nabla V) = 0$$

$\Rightarrow$ $\vec{E}$ can be expressed as a gradient of a scalar function

$$\iint (\nabla \times \vec{E}) \cdot d\vec{A} = \oint \vec{E} \cdot \vec{dl}$$

$$\boxed{\oint \vec{E} \cdot \vec{dl} = 0}$$

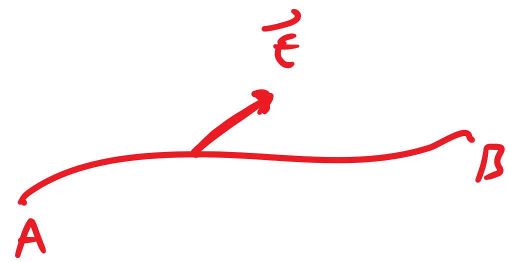


$$\int_{A\,(C_1)}^{B} \vec{E} \cdot \vec{dl} + \int_{B\,(C_2)}^{A} \vec{E} \cdot \vec{dl} = 0$$

$$\int_{A\,(C_1)}^{B} \vec{E} \cdot \vec{dl} = -\int_{B\,(C_2)}^{A} \vec{E} \cdot \vec{dl} = +\int_{A\,(C_2)}^{B} \vec{E} \cdot \vec{dl}$$

CONSERVATIVE FORCE

Electrostatic Potential



$\vec{E}$

$$W = \int_B^A (-\vec{E}) \cdot \vec{dl} = -\int_B^A \vec{E} \cdot \vec{dl}$$

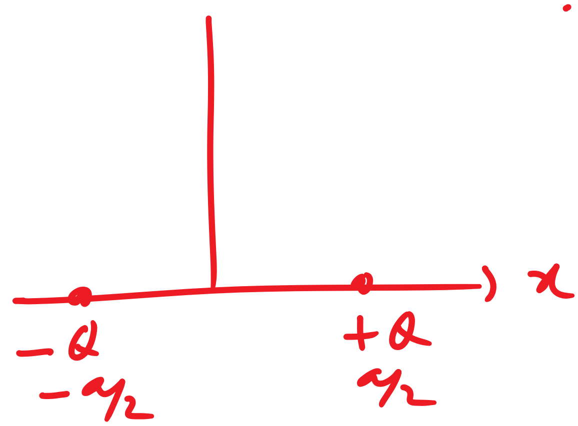
$$\nabla \times \vec{E} = 0 \quad : \quad \vec{E} = -\nabla V(x, y, z)$$

$$W = \int_B^A \vec{\nabla} V \cdot \vec{dl} = V(A) - V(B)$$

$$\Delta V = V(A) - V(B) = -\int_B^A \vec{E}\cdot\vec{dl}$$

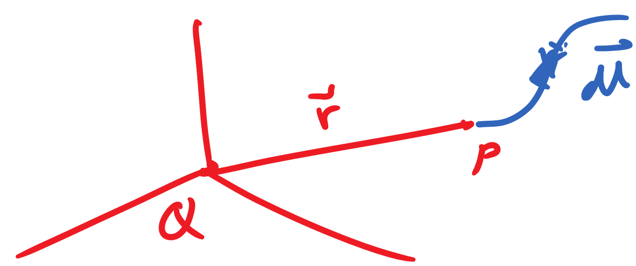
V : Electrostatic Potential

$$\boxed{\vec{E} = -\nabla V}$$



Potential due to a point charge

$$\boxed{V(\vec{r}) = \frac{Q}{4\pi\epsilon_0 r}}$$



$$V(P) = -\int_\infty^P \vec{E}\cdot\vec{dl}$$

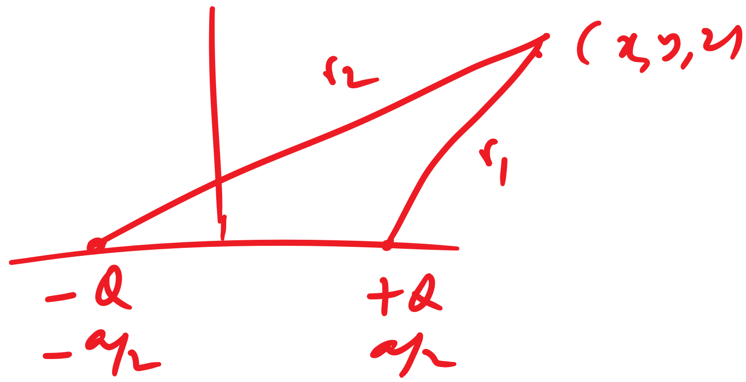
$$\vec{E} = \frac{Q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2} \; ; \qquad \vec{dl} = \hat{r}\,dr + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

$$\oint \vec{dl} = 0 \; ; \qquad \oint dl = \text{Perimeter}$$

$$\vec{E}\cdot\vec{dl} = \frac{Q}{4\pi\epsilon_0 r^2}(\hat{r}\cdot\vec{dl}) = \frac{Q}{4\pi\epsilon_0 r^2}\,dr$$

$$V(P) = -\frac{Q}{4\pi\epsilon_0}\int_{\infty}^{P}\frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0 r}$$

$$\vec{E} = -\nabla V = -\nabla\left(\frac{Q}{4\pi\epsilon_0 r}\right) = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

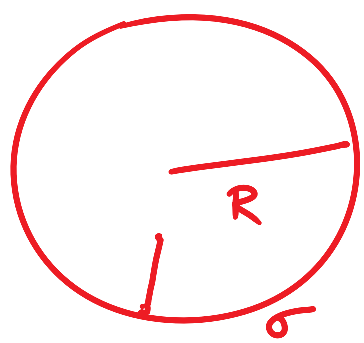


$$V = \frac{Q}{4\pi\epsilon_0 r_1} - \frac{Q}{4\pi\epsilon_0 r_2}$$

$$\boxed{\vec{E} = -\nabla V}$$

<u>Example</u>

Charge $Q = 4\pi R^2 \sigma$

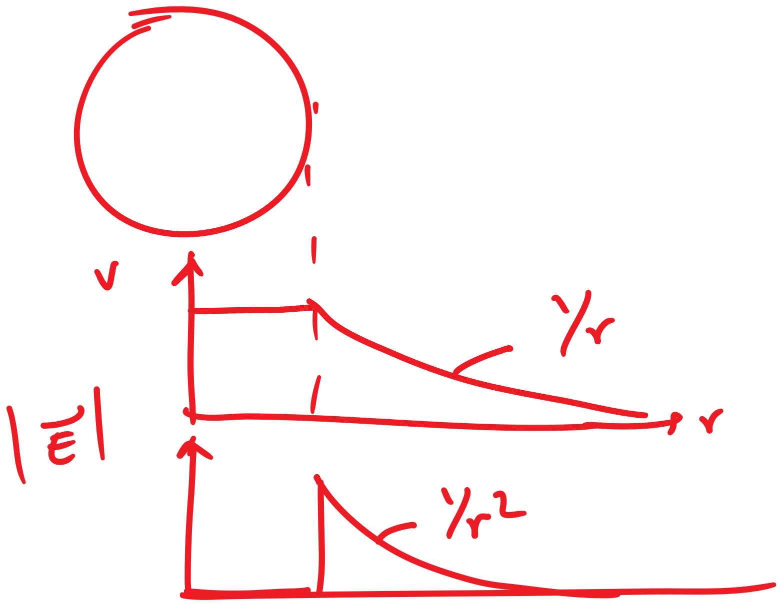


$$\vec{E} = 0 \qquad r < R$$

$$= \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \qquad r > R$$

$$V(r) = \frac{Q}{4\pi\epsilon_0 r} \qquad r > R$$

$$= \frac{Q}{4\pi\epsilon_0 R} \qquad r < R$$

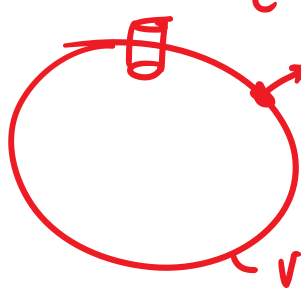


$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2}\hat{r} \quad \text{at} \quad r = R$$

$$V = \frac{Q}{4\pi\epsilon_0 R} \qquad r = R$$

$$= \frac{4\pi R^2 \sigma}{4\pi\epsilon_0 R} = \frac{\sigma R}{\epsilon_0}$$

$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{r}$$



$$V = \frac{\sigma R}{\epsilon_0}$$

$$\frac{\sigma_1 R_1}{\epsilon_0} = \frac{\sigma_2 R_2}{\epsilon_0}$$

$$\sigma_1 R_1 = \sigma_2 R_2$$

$$R_1 > R_2 \Rightarrow \sigma_2 > \sigma_1$$

$$\boxed{\begin{aligned}\vec{\nabla}\cdot\vec{E} &= \rho/\epsilon_0\\ \nabla\times\vec{E} &= 0\\ \vec{E} &= -\nabla V\end{aligned}}$$

$$\nabla\cdot(\nabla V) = -\frac{\rho}{\epsilon_0}$$

$$\left(\hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}+\hat{z}\frac{\partial}{\partial z}\right)\left(\hat{x}\frac{\partial V}{\partial x}+\hat{y}\frac{\partial V}{\partial y}+\hat{z}\frac{\partial V}{\partial z}\right) = -\frac{\rho}{\epsilon_0}$$

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2} = -\frac{\rho}{\epsilon_0}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) = \frac{\partial^2 V}{\partial x^2}$$

$$\boxed{\nabla^2 V = -\rho/\epsilon_0}$$ POISSON'S EQUATION

$$\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$ LAPLACIAN OPERATOR

$$\boxed{\nabla^2 V = 0}$$ LAPLACE EQUATION