Name of student: .....

Batch No:..... **Enrollment No.** .....

COURSE NAME: LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS

B.TECH TUTORIAL QUIZ-1 FALL SEMESTER 2017-18

COURSE CODE : EMAT102L MAX. TIME: 20 min

COURSE CREDIT: 3-0-0 MAX. MARKS: 10

1. Solve $3x + 4y = 2$, $y = 7$ in $\mathbb{Z}_5$. [2]
   Solution: $y = 7 = 2$ in $\mathbb{Z}_5$.
   $x = 4 \times 3^{-1} = 8 = 3$ in $\mathbb{Z}_5$.

2. Find the rank of matrix and augmented matrix, row echelon form, reduced row echelon form, free variable, leading variable of the following system

$$x_1 + x_4 = 2, \quad x_2 + x_3 = 5, \quad x_2 + 2x_3 = 0, \quad x_2 + 2x_4 = 2b, \; b \in \mathbb{R}.$$

   Also Solve it.
   Solution: The augmented matrix is given by

$$[A|b] = \begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 1 & 0 & 5 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 1 & 0 & 2 & 2b \end{bmatrix}$$

   Row echelon form of $[A|b] = \begin{bmatrix} 1 & 0 & 0 & 1 & 2 \\ 0 & 1 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 2 & 2b-5 \end{bmatrix}$

   Leading variable= x_1, x_2, x_3, x_4.
   No **free Variable**.
   $Rank(A) = 4$, $Rank([A|b]) = 4$.

   Row reduced echelon form of $[A|b] = \begin{bmatrix} 1 & 0 & 0 & 0 & 7-b \\ 0 & 1 & 0 & 0 & 10 \\ 0 & 0 & 1 & 0 & -5 \\ 0 & 0 & 0 & 1 & b-5 \end{bmatrix}$

   The values are $x_1 = 7 - b$, $x_2 = 10$, $x_3 = -5$, $x_4 = b - 5$.

3. Solve the system of equations over $\mathbb{Z}_5$

$$\begin{aligned} x + y &= 3 \\ 2x &= 3 \end{aligned}$$

Solution: $x = 3 \times 2^{-1} = 3 \times 3 = 9 = 4$ in $\mathbb{Z}_5$.
$y = -1 = 4$ in $\mathbb{Z}_5$.

4. Find determinant of the following matrix over $\mathbb{Z}_7$

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{bmatrix}$$

Solution: $det(M) = 6$.

5. Find row Echelon form, rank of the following matrix

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 6 & 4 \end{bmatrix}$$

Solution: Row echolon form of $M = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -1 \\ 0 & 0 & -5 \end{bmatrix}$

$Rank(A) = 3$.

6. Find determinant of the following matrix over $\mathbb{Z}_7$

$$M = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Solution: $det(M) = 48 = 6$ in $\mathbb{Z}_7$.

7. Solve the system of equations over $\mathbb{Z}_7$

$$\begin{aligned} x + y &= 5 \\ 2x &= 4 \end{aligned}$$

Solution: $x = 2$ in $\mathbb{Z}_7$.
$y = 3$ in $\mathbb{Z}_7$.

8. Find row Echelon form, rank of the following matrix

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

Solution: Row echolon form of $M = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$Rank(A) = 2$.

9. Solve $3x + y = 4,\ y = -2$ in $\mathbb{Z}_7$. [2]
Solution: $y = -2 = 5$ in $\mathbb{Z}_7$.
$x = 6 \times 3^{-1} = 6.5 = 30 = 2$ in $\mathbb{Z}_7$.

10. For the following system of linear equations [8]

$$\begin{aligned} x - 2y + 3t &= 2 \\ 2x + y + z + t &= -4 \\ 4x - 3y + z + 7t &= 8 \end{aligned}$$

Find its row Echelon form, row reduced Echelon form, rank of the corresponding matrix, leading variables, free variables and solution of the system.
Solution: The augmented matrix is given by

$$[A|b] = \begin{bmatrix} 1 & -2 & 0 & 3 & 2 \\ 2 & 1 & 1 & 1 & -4 \\ 4 & -3 & 1 & 7 & 8 \end{bmatrix}$$

Row echelon form of $[A|b] = \begin{bmatrix} 1 & -2 & 0 & 3 & 2 \\ 0 & 5 & 1 & -5 & -8 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix}$

Here $Rank(A) = 2$, $Rank([A|b]) = 3$.
Therefore, the given system has no solution.
Leading variables$= x, y$ but there is no **free variable**.
Row reduced echelon form of $[A|b] = \begin{bmatrix} 1 & 0 & \frac{2}{5} & 1 & 0 \\ 0 & 1 & \frac{1}{5} & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

11. Solve $2x_1 + x_2 = 2, x_2 = 4$ in $\mathbb{Z}_5$. [2]
Solution: $x_2 = 4$ in $\mathbb{Z}_5$.
$x_1 = 3 \times 2^{-1} = 3.3 = 9 = 4$ in $\mathbb{Z}_5$.

12. For the following system of linear equations [8]

$$\begin{aligned} x+y+z &= 3 \\ x+2y+z &= 4 \\ 2x+3y+2z &= 7 \end{aligned}$$

Find its row Echelon form, row reduced Echelon form, rank of the corresponding matrix, leading variables, free variables and solution of the system.
Solution: The augmented matrix is given by

$$[A|b] = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 1 & 4 \\ 2 & 3 & 2 & 7 \end{bmatrix}$$

Row echelon form of $[A|b] = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Leading variables= x, y and **free Variable**=z.
$Rank(A) = Rank([A|b]) = 2$.
Thus the given system has infinitely many solutions.
Row reduced echelon form of $[A|b] = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
The values are $x = 2 - k$, $y = 1$, $z = k$, where $k \in \mathbb{R}$.

13. Solve the following equation over $\mathbb{Z}_3$

$$2x = 1.$$

Solution:

$$x = 2.$$

14. Consider the system of equations

$$\begin{aligned} x+2y-z &= 0 \\ 4x+2y+6z &= 2 \\ x-4y+9z &= 2 \end{aligned}$$

Write row Echelon form, row reduced Echelon form, rank of corresponding matrix, leading variables, free variables and solution of the system.

Solution: Row Echelon form-

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -3 & 5 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row reduced Echelon form-

$$\begin{bmatrix} 1 & 0 & \frac{7}{3} & \frac{2}{5} \\ 0 & 1 & \frac{-5}{3} & \frac{-1}{5} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank is 2, leading variables are x, y, free variable is z, system has infinite solution which is $x = \frac{2}{3} - \frac{t}{3}$, $y = \frac{-1}{3} + \frac{5t}{3}$, $z = t$ where t is in $\mathbb{R}$.

15. Solve the following equation over $\mathbb{Z}_5$

$$3x = 4.$$

Solution:

$$x = 3.$$

16. Consider the system of equations

$$\begin{aligned} x + 2y - z &= 0 \\ 2x + y + 3z &= 1 \\ x - 4y + 9z &= 2 \end{aligned}$$

Write row Echelon form, row reduced Echelon form, rank of corresponding matrix, leading variables, free variables and solution of the system.

Solution:
Row Echelon form-

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -3 & 5 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row reduced Echelon form-

$$\begin{bmatrix} 1 & 0 & \frac{7}{3} & \frac{2}{5} \\ 0 & 1 & \frac{-5}{3} & \frac{-1}{5} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank is 2, leading variables are x, y, free variable is z, system has infinite solution which is $x = \frac{2}{3} - \frac{t}{3}$, $y = \frac{-1}{3} + \frac{5t}{3}$, $z = t$, where t is in $\mathbb{R}$.