EECE105L TUTORIAL SHEET-08 SOLUTIONS ①

1)

a) $f(t)$ lags $g(t)$ by 10^0

b) $f(t)$ leads $g(t)$ by 100^0

c) $f(t) = 10\sin(\omega t - 20^0)$

$g(t) = 10\cos(\omega t + \cancel{30}80)^0$

$= 10\sin(\omega t + 90 + 80)$

$= 10\sin(\omega t + 1\cancel{70}^0)$

$g(t)$ leads $f(t) = 190^0$

d) $f(t) = -10\sin(\omega t + 20^0)$

$= 10\sin(\omega t + 20 + 180^0)$

$= 10\sin(\omega t + 200^0)$

$g(t) = 10\sin(\omega t - 80^0)$

$f(t)$ leads $g(t)$ by 280^0

e) $f(t) = 10\sin(\omega t - 20^0)$

$g(t) = -10\sin(\omega t + 80^0) = 10\sin(\omega t + 180^0 + 80^0)$

$g(t)$ leads 280^0 by $f(t) \approx \cancel{260}$

f) $f(t) = -A\sin(\omega t + \theta) = A\sin(\omega t + \theta + \pi)$

$g(t) = B\cos(\omega t - \phi) = B\sin(\omega t - \phi + \pi/2)$

phase = $\theta + \pi - (-\phi + \pi/2) = (\theta + \phi) - \pi/2$

②

If $(\theta+\phi) > \pi/2$ f(t) leads g(t) by $(\theta+\phi)-\pi/2$

If $\theta+\phi < \pi/2$ f(t) lags g(t) by $(\theta+\phi)-\pi/2$

if $\theta+\phi = \pi/2$ f(t) and g(t) are in phase

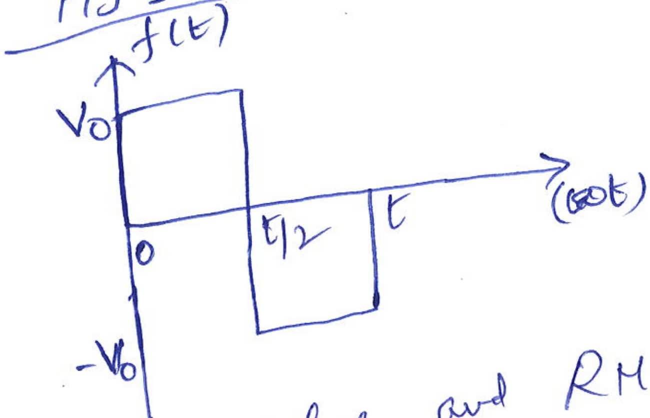
g) phase $(\theta-\phi)$

If $\theta=\phi \Rightarrow$ Then f(t) g(t) are in phase

if $\theta > \phi$ f(t) leads g(t) by $(\theta-\phi)$

if $\theta < \phi$ g(t) leads f(t) by $|(\theta-\phi)|$

2. Fig. 1



Average value and RMS value over one period.

$$\text{Average value} = \frac{V_0\cdot\left(\frac{t}{2}-0\right)+(-V_0)\left(t-\frac{t}{2}\right)}{t-0} = 0$$

$$\text{RMS value } (V_{RMS}) = \sqrt{\frac{V_0^2\left(\frac{t}{2}-0\right)+(-V_0)^2\left(t-\frac{t}{2}\right)}{t-0}}$$

$$= \sqrt{\frac{(V_0^2+V_0^2)t}{2t}} = \sqrt{2}\,V_0$$

Peak value $(V_p) = V_0$ $\quad V_{RMS} = \sqrt{2}\,V_p$

peak amplitude $= V_0$

peak to peak value (V_{pp}) $= V_{Max} - V_{Min}$

$= V_0 - (-V_0) = 2V_0$

$= 2V_p$

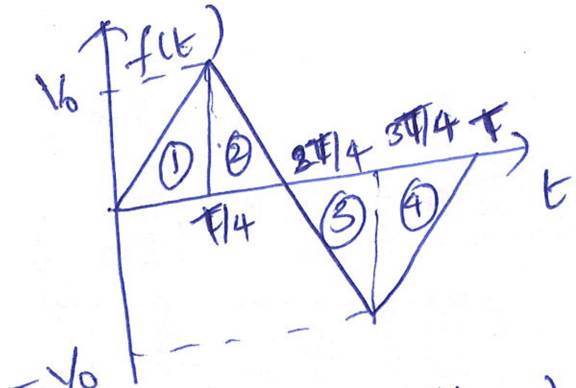
Average value and RMS over half period

Average Value $= \dfrac{V_0 \cdot \left(\frac{t}{2} - 0\right)}{\frac{t}{2} - 0} = V_0$ (+ve half cycle)

Average value $= \dfrac{-V_0\left(t - \frac{t}{2}\right)}{t - \frac{t}{2}} = -V_0$ (-ve half cycle)

RMS value $= \sqrt{\dfrac{(V_0)^2\left(\frac{t}{2} - 0\right)}{\left(\frac{t}{2} - 0\right)}} = V_0$

Fig (2)



Average value $= \dfrac{\frac{1}{2}V_0\left(\frac{t}{4} - 0\right) + \frac{1}{2}V_0\left(\frac{t}{2} - \frac{t}{4}\right) + \frac{1}{2}(-V_0)\left(\frac{3t}{4} - \frac{t}{2}\right) + \frac{1}{2}(-V_0)\left(t - \frac{3t}{4}\right)}{t - 0}$

$= 0$

Region-I

$f(t) = mt + c$

@ $t=0$ $f(t)=0$ —(1) at $t=\frac{T}{4}$, $f(t)=V_0$ —(2)

from (1) & (2) $c=0$ $\quad V_0 = m\cdot\frac{t}{4}$ $\quad m=\frac{4V_0}{T}$

$f_1(t) = \frac{4V_0 t}{T}$ — (A)

Region-II

$f_2(t) = mt + C$

$f_2(t) = V_0$ at $t = T/4$; $f(t) = 0$ at $t = T/2$

and $f_1(t) = f_2(t)$ at $t = T/4$

$V_0 = m\cdot\frac{T}{4} + C = \frac{mT}{4} - \frac{mT}{2} = -\frac{mT}{4} = V_0$

at $t=\frac{T}{2}$ $\quad 0 = m\cdot\frac{T}{2} + C \Rightarrow C = -\frac{mT}{2}$

$m = -\frac{4V_0}{T}$ $\qquad C = -\left(-\frac{4V_0}{T}\right)\cdot\frac{T}{2} = 2V_0$

$f_2(t) = -\frac{4V_0}{T}t + 2V_0$ — (B)

Region-III

$f_3(t) = mt + c$ $\qquad f_3(t) = f_2(t)$ at $t=\frac{T}{2}$

@ $t=\frac{T}{2}$, $\quad 0 = m\cdot\frac{T}{2} + c \Rightarrow c = -\frac{mT}{2}$

@ $t = \frac{3T}{4}$ $\quad f_3(t) = -V_0 = m\cdot\frac{3T}{4} + c$ $\quad \boxed{m = -\frac{4V_0}{\cancel{3}T}}$

$C = -\left(-\frac{4}{\cancel{3}}\frac{V_0}{T}\right)\frac{T}{2} = \frac{2}{\cancel{3}}V_0$ $\qquad C = 2V_0$

$f_3(t) = -\frac{4}{\cancel{3}}\frac{V_0}{T}t + \frac{2}{\cancel{3}}V_0$ — (C)

Region-IV

$f_4(t) = mt + c$ $\qquad f_4(t) = f_3(t)$ at $t = \frac{3T}{4}$

@ $t = \frac{3T}{4}$, $\quad -V_0 = m\cdot\frac{3T}{4} + c$

@ $t = T$ $\quad 0 = mT + c \Rightarrow c = -mT$

$$-V_0 = \frac{m \cdot 3T}{4} - mT \Rightarrow -V_0 = \frac{mT}{4}$$

$$\Rightarrow m = \frac{-4V_0}{T}$$

$$C = -\frac{4V_0}{T} \cdot T = -4V_0$$

$$f_4(t) = \frac{4V_0}{T}t - 4V_0 \quad \text{— (D)}$$

Average Value $= \dfrac{\int_0^{T/4} f_1(t)\,dt + \int_{T/4}^{T/2} f_2(t)\,dt + \int_{T/2}^{3T/4} f_3(t)\,dt + \int_{3T/4}^{T} f_4(t)\,dt}{T-0}$

$$\int_0^{T/4} f_1(t)\,dt = \frac{4V_0}{T}\frac{t^2}{2} = \frac{V_0 T}{8}$$

$$\int_{T/4}^{T/2} f_2(t)\,dt = -\frac{4V_0}{T}\cdot\frac{t^2}{2} + 2V_0 t \Big|_{T/4}^{T/2}$$

$$= V_0 T\left[-\frac{3}{8} + \frac{1}{2}\right] = \frac{1}{8} V_0 T$$

$$\int_{T/2}^{3T/4} f_3(t)\,dt = -\frac{4}{\cancel{8}}\frac{V_0}{T}\frac{t^2}{2} + \frac{2}{\cancel{8}} V_0 t \Big|_{T/2}^{3T/4}$$

$$= \frac{2V_0 T}{3}\left[-\frac{2}{\cancel{8}}\left(\left(\frac{3}{4}\right)^2 - \left(\frac{1}{2}\right)^2\right) + \left(\frac{3}{4} - \frac{1}{2}\right)\right]$$

$$\frac{2V_0 T}{\cancel{8}}\left[\frac{-5}{\cancel{8}} + \frac{1}{4}\right] = \frac{-V_0 T}{8}$$

$$\int_{3T/4}^{T} f_4(t)\,dt = 4V_0\left[\frac{t^2}{2T} - t\right]_{3T/4}^{T}$$

$$= 4V_0 T\left[\frac{1}{2}\cdot\frac{7}{16} - \frac{1}{4}\right] = -\frac{1}{8} V_0 T$$

Average value: $\dfrac{\frac{1}{8}V_0T + \frac{1}{8}V_0T - \frac{1}{8}V_0T - \frac{1}{8}V_0T}{T-0} = 0$

RMS value:

$$\int_0^{T/4} f_1^2(t)\,dt = \frac{16V_0^2}{T^2}\frac{t^3}{3}\Big|_0^{T/4} = \frac{V_0^2 T}{12}$$

$$\int_{T/4}^{T/2} f_2^2(t)\,dt = \int_{T/4}^{T/2}\left(\frac{16V_0^2}{T^2}t^2 + 4V_0^2 - \frac{16V_0^2}{T^2}t\right)dt$$

$$= \frac{16V_0^2}{T^2}\cdot\frac{t^3}{3} + 4V_0^2 t - \frac{16V_0^2}{T^2}\cdot\frac{t^2}{2}\Big|_{T/4}^{T/2}$$

$$= V_0^2T\left[\frac{7}{12} + 4\left(\frac{1}{2}-\frac{1}{4}\right) - \frac{16}{2}\left(\frac{3}{16}\right)\right] = \frac{V_0^2 T}{12}$$

$$\cancel{\int_{T/2}^{3T/4} f_3^2(t)\,dt = \int_{T/2}^{3T/4}\left(\frac{16}{9}\frac{V_0^2}{T^2}t^2 + \frac{4}{9}V_0^2 - \frac{16}{9}\frac{V_0^2}{T}t\right)dt}$$

$$\cancel{= \frac{4}{9}V_0^2\left[\frac{4}{T^2}\cdot\frac{t^3}{3} + t - \frac{4}{T}\frac{t^2}{2}\right]\Big|_{T/2}^{3T/4} = \frac{V_0^2 T}{12}}$$

$$\int_{T/2}^{3T/4} f_3^2(t)\,dt = \int_{T/2}^{3T/4}\left(\frac{16V_0^2}{T^2}t^2 + 4V_0^2 - \frac{16V_0^2}{T^2}t\right)dt$$

$$= \frac{V_0^2 T}{12}$$

$$\int_{3T/4}^{T} f_4^2(t)\,dt = \int_{3T/4}^{T}\left[\frac{16V_0^2}{T^2}t^2 + 16V_0 - \frac{32V_0^2}{T}t\right]dt$$

$$= \frac{V_0^2 T}{12}$$

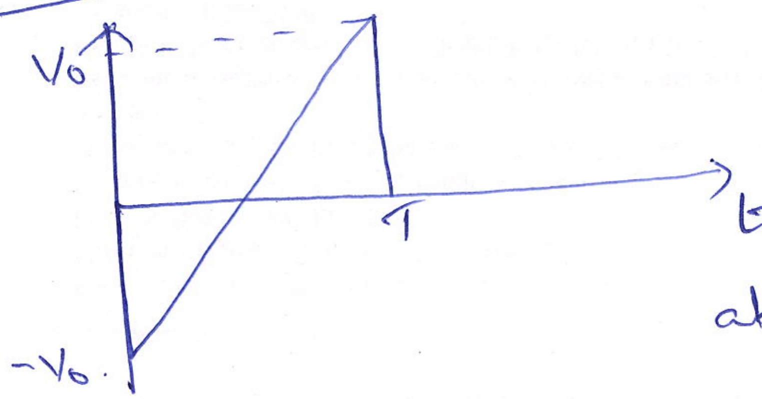
$$\text{RMS value} = \sqrt{\frac{4 \cdot \frac{V_0^2 T}{12}}{T}} = \frac{V_0}{\sqrt{3}} = V_{RMS}$$

Peak value $V_p = V_0$ $\qquad V_{RMS} = \frac{V_p}{\sqrt{3}}$

Peak to peak value $V_{pp} = 2V_0 = 2V_p$

Peak amplitude $= V_0$

Fig. 3



$f(t) = mt + C$

at $t = 0$ $f(t) = -V_0$

$-V_0 = m \cdot 0 + C$

$C = -V_0$

at $t = T$, $f(t) = V_0$

$V_0 = m \cdot T - V_0$

$m = \frac{2V_0}{T}$

$$f(t) = \frac{2V_0}{T} \cdot t - V_0$$

$$\text{Average value} = \int_0^T \left(\frac{2V_0}{T} \cdot t - V_0 \right) dt$$

$$= \frac{V_0}{T} t^2 - V_0 t \Big|_0^T = 0$$

$$\text{RMS value} \; \int_0^T f^2(t)\,dt = \int_0^T \left[\left(\frac{4V_0^2}{T^2} t^2 \right) + V_0^2 - \frac{4V_0^2 t}{T} \right] dt$$

$$= \frac{4V_0^2}{T^2}\cdot\frac{t^3}{3} + V_0^2 t - \frac{4V_0^2}{T}\frac{t^2}{2}\Big|_0^T = \frac{V_0^2}{3T}$$

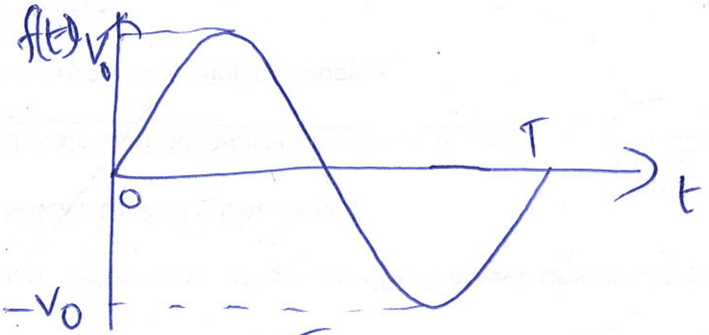
$$V_{RMS} = \sqrt{\frac{\frac{V_0^2}{3T\cdot T}}{T}} = \frac{V_0}{\sqrt{3}}$$

$V_p = V_0$

$V_{pp} = 2V_0 = V_p$

Peek amplitude $= V_0$

Fig. 4



$\omega = 2\pi f$

$f = 1/T$

Average value $V_0 \int_0^T \sin(\omega t)\,dt = V_0 \int_0^T \sin(2\pi f t)\,dt = 0$

$$\text{RMS value} = \sqrt{\frac{V_0^2 \int_0^T \sin^2(2\pi f t)\,dt}{T}} = \frac{V_0}{\sqrt{2}}$$

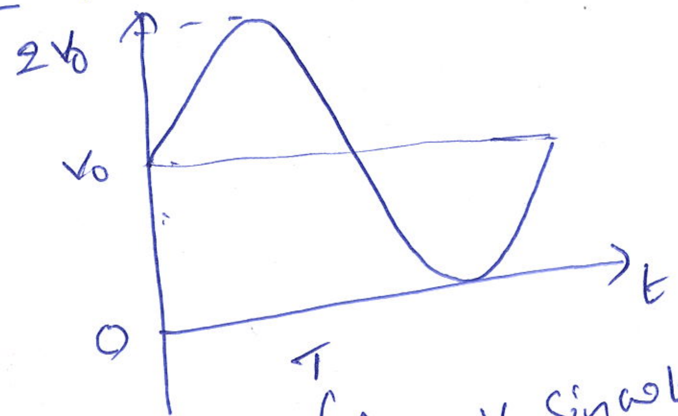
$$\int_0^T \sin^2 \omega t\,dt = \frac{1}{2}\int_0^T (1-\cos 2\omega t)\,dt = \frac{1}{2}\left[t - \frac{\sin 4\pi f t}{2\pi f}\right]_0^T = T/2$$

$V_{pp} = 2V_0$ $\quad V_p = V_0$ $\quad V_{RMS} = \frac{V_p}{\sqrt{2}}$

Peek amplitude $= V_0$

3.

Fig. 5



$f(t) = V_0 + V_0 \sin \omega t$

$$V_{Average} = \frac{\int_0^T (V_0 + V_0 \sin \omega t)\,dt}{T-0}$$

$$= \frac{V_0 t + \frac{V_0}{\omega t}(-\cos \omega t)\Big|_0^T}{T-0} = V_0$$

$$V_{RMS} = \sqrt{\frac{\int_0^T (V_0 + V_0 \sin \omega t)^2\,dt}{T-0}}$$

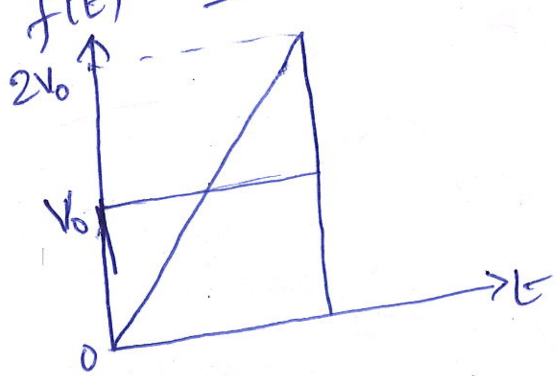
$$= \sqrt{\frac{\int_0^T (V_0^2 + 2V_0^2 \sin \omega t + V_0^2 \sin^2 \omega t)\,dt}{T-0}}$$

$$= \sqrt{\frac{V_0^2 t\Big|_0^T + 0 + V_0^2 \int_0^T \frac{1}{2}(1 - \cos 2\omega t)\,dt}{T}}$$

$$= \sqrt{\frac{V_0^2 T + \frac{V_0^2}{2}T}{T}} = \sqrt{\frac{3}{2}}\,V_0$$

Peak Value $= V_0$ $\quad V_{PP} = 2V_0$ $\quad$ Peak Amplitude $= V_0$

fig.6



$$f(t) = \frac{2V_0}{T} t - V_0 + V_0$$

$$= \frac{2V_0}{T} t$$

$$\text{Average value} = \frac{\frac{2V_0}{T}\int_0^T t\,dt}{T-0} = \frac{2V_0}{T^2} \frac{t^2}{2}\Big|_0^T$$

$$= V_0$$

$$\text{RMS value} = \sqrt{\frac{\int_0^T \frac{4V_0^2}{T^2} t^2 dt}{T-0}} = \sqrt{\frac{4V_0^2}{T^3} \frac{t^3}{3}\Big|_0^T}$$

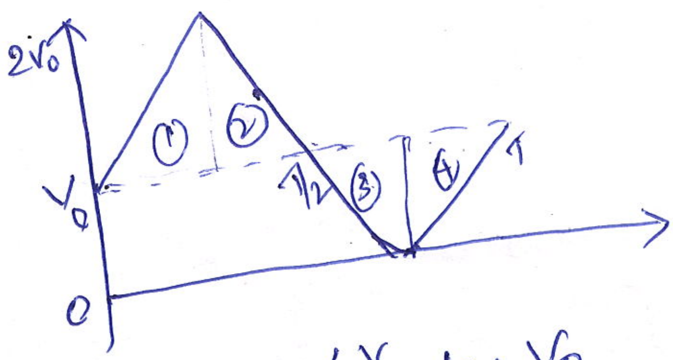
$$= \frac{2}{\sqrt{3}} V_0$$

Peak value $= 2V_0$

$V_{pp} = 2V_0$

Peak amplitude $= V_0$

Fig.7



Region ① $\quad f_1(t) = \frac{4V_0}{T} t + V_0$

Region ② $\quad f_2(t) = -\frac{4V_0}{T} t + 3V_0$

Region ③ $\quad f_3(t) = -\frac{4V_0}{T} t + 3V_0$

Region ④ $\quad f_4(t) \quad \frac{4V_0}{T} t - 3V_0$

$$\text{Average} = \frac{\int_0^{T/4}\left(\frac{4V_0}{T}t+V_0\right)dt + \int_{T/4}^{T/2}\left(-\frac{4V_0}{T}t+3V_0\right)dt + \int_{T/2}^{3T/4}\left(-\frac{4V_0}{T}t+3V_0\right)dt + \int_{3T/4}^{T}\left(\frac{4V_0}{T}t-3V_0\right)dt}{T-0}$$

$$= \frac{\frac{4V_0}{T}\frac{t^2}{2}\Big|_0^{T/4} + V_0 t\Big|_0^{T/4} + \left(-\frac{4V_0}{T}\frac{t^2}{2}+3V_0t\right)\Big|_{T/4}^{T/2} + \left(-\frac{4V_0}{T}\frac{t^2}{2}+3V_0t\right)\Big|_{T/2}^{3T/4} + \left(\frac{4V_0}{T}\frac{t^2}{2}-3V_0t\right)\Big|_{3T/4}^{T}}{T}$$

$$= 2V_0$$

RMS Value

$$\int_0^{T/4} f_1^2(t)\,dt = \int_0^{T/4}\left(\frac{4V_0}{T}t+V_0\right)^2 dt = \frac{7}{12}V_0^2 T$$

$$\int_{T/4}^{T/2} f_2(t)\,dt = \frac{\cancel{241}}{\cancel{864}}V_0^2 T\ \frac{7}{12}$$

$$\int_{T/2}^{3T/4} f_3(t)\,dt = \frac{1}{12}V_0^2 T$$

$$\int_{3T/4}^{T} f_4(t)\,dt = \frac{1}{12}V_0^2 T$$

$$V_{RMS} = \sqrt{\frac{\left(\frac{7}{12}+\frac{7}{12}+\frac{1}{12}+\frac{1}{12}\right)V_0^2 T}{T}}$$

$$= \frac{2}{\sqrt{3}}V_0$$

Fig. 8

Average: $\frac{V_0 \cdot T/2 + V_0 \cdot T/2}{T-0} = V_0$.

RMS: $\sqrt{\frac{V_0^2 \cdot T/2 + V_0^2 \cdot \frac{T}{2}}{T-0}} = V_0$

4)

Fig. 9

Average Value = $\frac{0 \times (5-0) + 6 \times (10-5) + 3 \times (20-10) + (-3)(30-20) + 0 \cdot 5}{35-0}$

$= \frac{0 + 30 + 30 - 30}{35} = \frac{30}{35} = \frac{6}{7}$ V

RMS: $\sqrt{\frac{0 + 6^2 \times 5 + 9 \times 10 + 9 \times 10 + 0}{35}}$

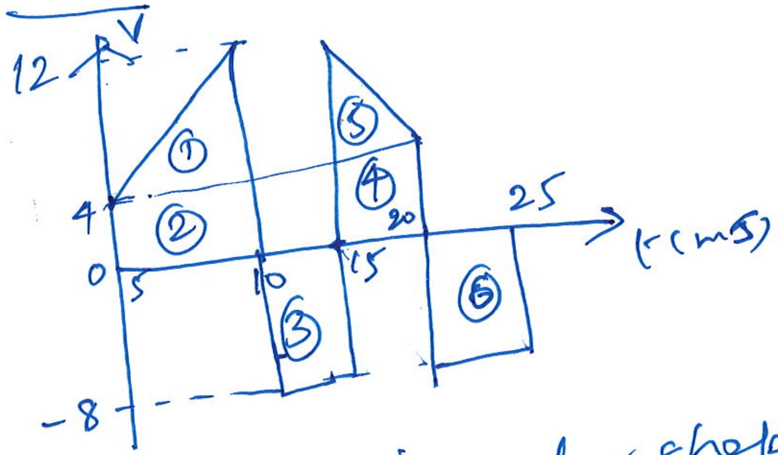
$V_{RMS} = 3.2$ V

Peak value = 6 V

$V_{pp} = 9$ V

Peak amplitude = $6 - \frac{6}{7} = \frac{36}{7}$ V

Fig. 10



(1) & (2) Region has a triangular shape. Boundary is defined by a line $y = mx + c$ (or) more precisely,

$$V_1(t) = mt + C \quad \text{— (1)}$$

at $t = 5$, $V(t) = 4$ at $t = 10$ $V(t) = 12$

Substituting in (1)

$$\left.\begin{aligned} 12 &= 10m + C \\ 4 &= 5m + C \end{aligned}\right\} \Rightarrow 5m = 8, \quad m = 1.6 \text{ V/ms}$$

$$12 = 16 + C \Rightarrow C = -4 \text{ V}$$

So (1) can be re-written as $V(t) = 1.6t - 4$

Region 5 and (4)

$$V_2(t) = mt + C$$

at $t = 15$, $V(t) = 12$

$t = 20$ $V(t) = 4$

$$12 = 15m + c$$
$$4 = 20m + c$$
$$-5m = 8 \Rightarrow m = -1.6 \text{ V/ms}$$

$12 = -24 + C \Rightarrow C = 36\,V$

$V_2(t) = -1.6t + 36 \quad — ②$

$$\text{Average value} = \frac{\int_{t_1}^{t_2} V_1(t)\,dt + V_3(t_3 - t_2) + \int_{t_3}^{t_4} V_2(t)\,dt + V_4(t_5 - t_4)}{t_5 - t_1}$$

$$\int_{t_1}^{t_2} V_1(t)\,dt = \int_5^{10} (1.6t - 4)\,dt$$

$$= 0.8t^2 - 4t \Big|_5^{10}$$

$$= 0.8(10^2 - 5^2) - 4(10 - 5) = 40\,V$$

$$\int_{t_3}^{t_4} V_2(t)\,dt = \int_{15}^{20} (-1.6t + 36)\,dt$$

$$= -0.8t^2 + 36t \Big|_{15}^{20}$$

$$= -0.8(20^2 - 15^2) + 36(20 - 15)$$

$$= -140 + 180 = 40\,V$$

$V_3(t_3 - t_2) = -8(15 - 10) = -40\,V$

$V_4(t_5 - t_4) = -8(25 - 20) = -40\,V$

$$\text{Average value} = \frac{40 - 40 + 40 - 40}{25 - 5} = 0\,V$$

RMS Value:

$$\left[\frac{\int_{t_1}^{t_2} V_1^2(t)\,dt + V_3^2(t_3-t_2) + \int_{t_3}^{t_4} V_2^2(t)\,dt + V_4^2(t_5-t_4)}{t_5-t_1}\right]^{1/2}$$

$$\int_{t_1}^{t_2} V_1^2(t)\,dt = \int_{t_1}^{t_2} (1.6t-4)^2\,dt$$

$$= \int_5^{10} \left[(1.6)^2 t^2 - 12.8t + 16\right] dt$$

$$\left. \underset{(746.67)}{2.56\frac{t^3}{3}} - \underset{(480)}{12.8\frac{t^2}{2}} + \underset{80}{16t} \right|_5^{10} = \cancel{296.67}\ 346.67$$

$$V_3^2(t_3-t_2) = (-8)^2 \times (15-10) = 320$$

$$\int_{t_3}^{t_4} V_2^2(t)\,dt = \int_{15}^{20} (-1.6t+36)^2\,dt = \int_{15}^{20} (2.56t^2 - 115.2t + 1296)\,dt$$

$$= \left. 2.56\frac{t^3}{3} - 115.2\frac{t^2}{2} + 1296t \right|_{15}^{20}$$

$$3946.67 - 10080 + 6480 = 346.67$$

$$V_4^2(t_5-t_4) = (-8)^2(25-20) = 320$$

$$\text{RMS Value} = \left[\frac{346.67+320+346.67+320}{25-5}\right]^{1/2}$$

$$= 8.16\ \text{V}$$

Peak value = 12 V

peak to peak value = 12 − (−8) = 20 V

Peak amplitude = 12 − 0 = 12 V