Tutorial Sheet 4

Vector Space, Basis And Dimension

---

1. Consider $P_2(\mathbb{R})$, the vector space of all polynomials of degree less than or equal to 2 with coefficients from $\mathbb{R}$. The set $\{1-x, 1+x, x^2\}$ is a basis of $P_2(\mathbb{R})$.

2. Let $S = \{(1,0,0,2,3),(0,1,1,0,0),(1,1,1,2,3)\}$. Then find the basis of $L(S)$ and extend it to the basis of $\mathbb{R}^5$.

3. Recall the vector space $P_4(\mathbb{R})$. Is the set,

$$W = \{p(x) \in P_4(\mathbb{R}) : p(-1) = p(1) = 0\}$$

   a subspace of $P_4(\mathbb{R})$? If yes, find its dimension.

4. Let $V = \{(x,y,z,w) \in R^4 : x+y-z+w = 0, x+y+z+w = 0\}$ and $W = \{(x,y,z,w) \in \mathbb{R}^4 : x-y-z+w = 0, x+2y-w = 0\}$ be two subspaces of $\mathbb{R}^4$. Find bases and dimensions of V, W, $V \cap W$ and $V+W$.

5. Show that the set of $n \times n$ upper triangular real matrices is a subspace of $\mathbb{R}^{n\times n}$. Find a basis and its dimension.

6. Suppose U and W are subspaces of $\mathbb{R}^8$ such that $dimU = 3$, $dimW = 5$, and $U + W = \mathbb{R}^8$. Prove that $U \cap W = \{0\}$.