Department of Mathematics, Bennett University
EMAT102L, Tutorial Sheet 7
Ordinary Differential Equations

---

1. Determine the order and degree of the following differential equations. Also, state whether they are linear or nonlinear:
(a) $\frac{dy}{dx}+8x\left(\frac{dy}{dx}\right)^2 = x^2y$ (b) $xy = \sqrt{x}\left(\frac{dy}{dx}\right)+k/\left(\frac{dy}{dx}\right)$ (c) $\frac{dy}{dx}+x^2y = xe^x$
(d) $\frac{d^7x}{dt^7} + \left(\frac{d^5x}{dt^5}\right)\left(\frac{d^3x}{dt^3}\right) + x = t$ (e) $\left(\frac{dr}{ds}\right)^4 = \sqrt{\frac{d^2r}{ds^2}+1}$ (f) $\frac{dx}{dy} + \sqrt{x} = 0$

2. Show that $y = a\cos(mx + b)$ is a solution of the differential equation $\frac{d^2y}{dx^2} + m^2y = 0$.

3. For each of the following families of curves, find a differential equation (of least order) for which each member of the family is a solution.
(a) $\{y = c_1e^x + c_2e^{-3x} : c_1, c_2 \in \mathbb{R}\}$ (b) $\{y = x\sin(x + c) : c \in \mathbb{R}\}$

4. Find the solution of the initial value problem $ydy = xdx,\ y(0) = \beta$, where $\beta \in \mathbb{R}$.

5. Consider the equation $y'(x) = cy(x), 0 < x < \infty$, where c is a real constant. Then
   (a) Show that if ϕ is any solution and $\psi(x) = \phi(x)e^{-cx}$, then $\psi(x)$ is a constant.
   (b) If $c < 0$, then show that every solution tends to zero as $x \to \infty$.

6. What can you say about the solution of the differential equation $\left|\frac{d^2y}{dx^2}\right| + \left|\frac{dy}{dx}\right| + y^2 + 2 = 0$?

7. Consider the differential equation $\frac{dy}{dx} = y^4 + 6$.
(a) Show that there exist no constant solutions of the above differential equation.
(b) Is it possible for the solution curve to have any relative extrema?

8. The population of a certain country is known to increase at a rate proportional to the number of people presently living in the country. If after two years, the population has doubled, and after three years the population is 20,000, estimate the number of people initially living in the country.

9. Solve the following initial value problems:
(a) $\frac{dy}{dx} = (1 + y^2)\tan x, \quad y(0) = \sqrt{3}$ (b) $\frac{dy}{d\theta} = y\sin\theta,\ y(\pi) = -3$

10. Solve the following ODEs:
(a) $(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$ (b) $\left(x\tan\frac{y}{x} + y\right)dx - xdy = 0$
(c) $\frac{dy}{dx} = \frac{4x + 6y + 5}{3y + 2x + 4}$ (d) $\frac{dy}{dx} = \frac{x + 2y - 3}{2x + y - 3}$