Indicator variables

$$I(E) = 1 \quad \text{if } E \text{ occurs}$$
$$= 0 \quad \text{otherwise.}$$

$$I_{x<a} = 0$$

$$I_{x \geq a} = 1.$$

$$E[I_{x \geq a}] = 0 \times P(x<a) + 1(P(x \geq a)$$

$$a \times I_{x \geq a} = a \leq X$$

$$aE[I_{x \geq a}] \leq E[x].$$

$$\text{or } aP(X \geq a) \leq E[x].$$

Markov Inequality

Weak law of large numbers

For any $c > 0$, $P(|\bar{x} - \mu| > c) \to 0$

as $n \to \infty$

Proof $P(|\bar{x} - \mu| > c) \leq \dfrac{Var(\bar{x})}{c^2}$

$$\leq \frac{Var\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)}{c^2}$$

$$\leq \frac{\frac{1}{n^2} Var\left(\sum_{i=1}^{n} X_i\right)}{c^2}$$

$$\leq \frac{1}{n^2} \frac{n \times \sigma^2}{c^2} = \frac{\sigma^2}{nc^2} \to 0$$

as

Lt. $n \to \infty$

Chebyshev Inequality.

$$P(|x-\mu| \geq c) \leq \frac{\sigma^2}{c^2}$$

Proof: $P(|x-\mu| \geq c) = P((x-\mu)^2 \geq c^2)$

or $P((x-\mu)^2 \geq c^2) \leq \frac{\cancel{Var(X)}\, E[(X-\mu)^2]}{c^2}$

By markov inequality

or $\quad$ " $\quad \leq \frac{\sigma^2}{c^2}$.

Markov Inequality.

$$P(X \geq a) \leq \frac{E[X]}{a}$$

Central limit theorem
and
law of large numbers.

Let's assume that we have $X_1, X_2 \ldots X_n \ldots$

They are i.i.d. with some μ and Var σ^2.

$$\overline{X_n} = \frac{1}{n}\sum_{i=1}^{n} X_i \quad \text{(Sample mean)}$$

Now $\overline{X_n} \to \mu$ for $n \to \infty$ with Pr. $\to 1$ (convergence).

Example

$X_j \sim N(\mu, \sigma^2)$, then

$$\frac{X_1 + X_2 \ldots X_n}{n} \to \mu \text{ with prob } 1.$$