Name of student: ....

Batch No:.......... **Enrollment No.** ....

COURSE NAME: LINEAR ALGEBRA AND ORDINARY DIFFERENTIAL EQUATIONS

B.TECH **TUTORIAL QUIZ-3** **FALL SEMESTER 2018-19**

COURSE CODE : EMAT102L MAX. TIME: 30 min

COURSE CREDIT: 3-1-0 MAX. MARKS: 10

1. Let A be 3×3 matrix with real entries such that $det(A)$ is 6 and trace of A is 0. If $det(A+I) = 0$. Find all the eigenvalues of A. [1]

Solution: Since $det(A+I)=0 \Rightarrow \lambda_1 = -1$

$trace(A)=0 \Rightarrow \lambda_1 + \lambda_2 + \lambda_3 = 0 \Rightarrow \boxed{\lambda_2 + \lambda_3 = 1}$ — (1)

$det A = 6 \Rightarrow \lambda_1 \lambda_2 \lambda_3 = 6 \Rightarrow \boxed{\lambda_2 \lambda_3 = -6}$ — (2)

Solving (1) & (2), we obtain $\lambda_2 = -2, \lambda_3 = 3$.

2. Every invertible matrix is diagonalizable. Justify your answer. [1]

Solution: In general it is not true.

$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$. eigenvalues of $A = 1, 1 \Rightarrow$ A.M of $1 = 2$

$det A \neq 0 \Rightarrow A$ is invertible.

But G.M of $1 = 1$ as, $(A - \lambda I)X = 0 \Rightarrow (A-I)X = 0 \Rightarrow \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x_2 = 0, \; x_1 = t$

$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

3. Find the eigenvalues and eigenvectors of matrix $A = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$. Also calculate the eigenvalues of $(A-I)^2$. [3]

Solution:

Eigenvalues of $A = 2, 2$

eigenvalues of $A - I = 2-1, 2-1 = 1, 1$

eigenvalues of $(A-I)^2 = 1^2, 1^2 = 1, 1$

Eigenvectors of A corresponding to $2 = \{X : (A-2I)X = 0\}$

$= \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} : \begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

$= \left\{ \begin{bmatrix} t \\ 0 \end{bmatrix} \; x_2 = 0, \; x_1 = t \right\}$

$= t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

4. Consider $\mathbb{R}^3$ with standard inner product. Let $u = (1, 0, 1)$, $v = (2, -1, 0)$. Then calculate $\cos\theta$. [2]

Solution:

$$\cos\theta = \frac{\langle u, v\rangle}{\|u\| \|v\|} = \frac{u \cdot v}{\sqrt{\langle u, u\rangle}\sqrt{\langle v, v\rangle}}$$

$u \cdot v = 2$

Thus $\cos\theta = \frac{2}{\sqrt{2}\sqrt{5}} = \sqrt{2/5}$

$\langle u, u\rangle = u \cdot u = 1 + 1 = 2$

$\langle v, v\rangle = 4 + 1 = 5$

5. Let V be an inner product space. Let W be a non-empty set. Then [2]

$$W^\perp = \{v \in V : \langle v, w\rangle = 0 \text{ for all } w \in W\}.$$

(a) If $W = \{(x, y, z) \in \mathbb{R}^3 : x + y = 0\}$. Then find $W^\perp$ with respect to the standard inner product. Also find basis and dimension of W, $W^\perp$.

Solution: $W = \{(x, y, z) \in \mathbb{R}^3 : x + y = 0\}$

$= \{(x, -x, z) \in \mathbb{R}^3\}$

$= \{x(1, -1, 0) + z(0, 0, 1)\} = \text{span}\{(1, -1, 0), (0, 0, 1)\}$

$\dim W = 2$, Basis$(W) = \{(1, -1, 0), (0, 0, 1)\}$

$$W^\perp = \left\{(x, y, z) \in \mathbb{R}^3 : \begin{array}{l}\langle (x, y, z), (1, -1, 0)\rangle = 0 \\ \langle (x, y, z), (0, 0, 1)\rangle = 0\end{array}\right\}$$

$= \{(x, y, z) \in \mathbb{R}^3 : x - y = 0 \quad z = 0\}$

$x = y$

$= \{(x, x, 0)\} = \{x(1, 1, 0)\}$. Basis $W^\perp = \{(1, 1, 0)\}$

$\dim W^\perp = 1$

6. If $\langle u, v\rangle = 4 - i$. Then calculate $\langle (2 + i)u, (2 + i)v\rangle$. [1]

Solution:

$\langle (2+i)u, (2+i)v\rangle = (2+i)\overline{(2+i)}\langle u, v\rangle$

$= (2+i)(2-i)(4-i)$

$= 5(4-i) = 20 - 5i$