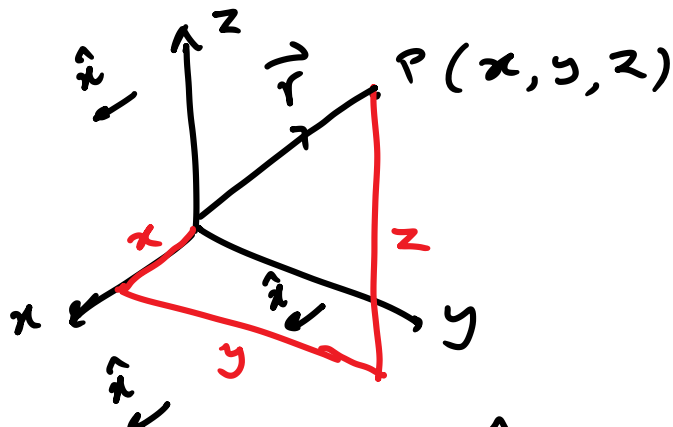
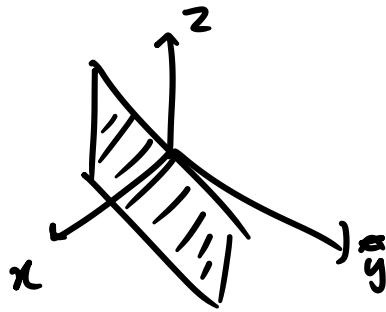


Spherical Polar Coordinates

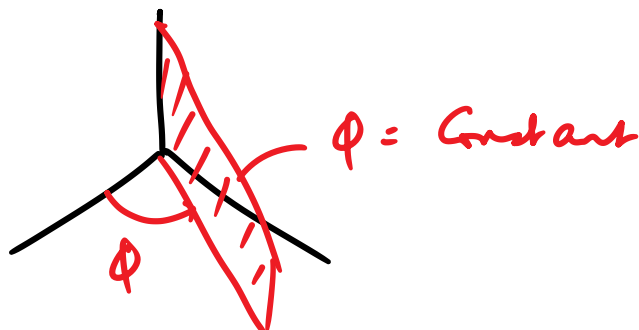
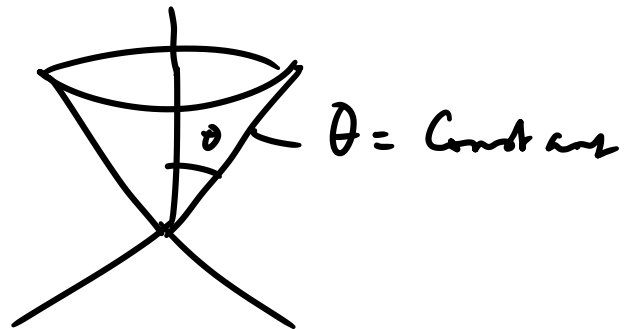
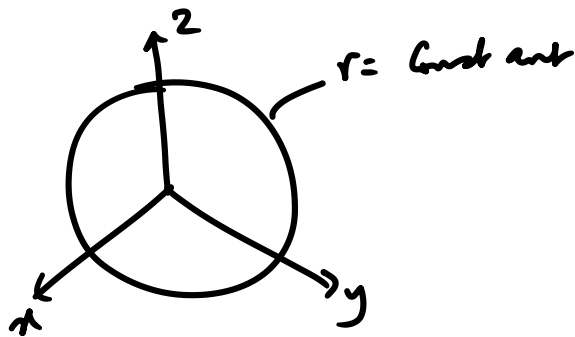


$$\begin{aligned}\vec{r} &= \hat{i}x + \hat{j}y + \hat{k}z \\ &= \hat{x}x + \hat{y}y + \hat{z}z\end{aligned}$$

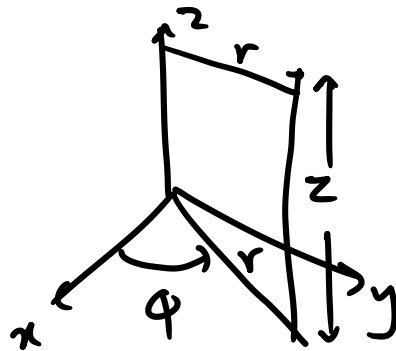
$$\vec{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$$



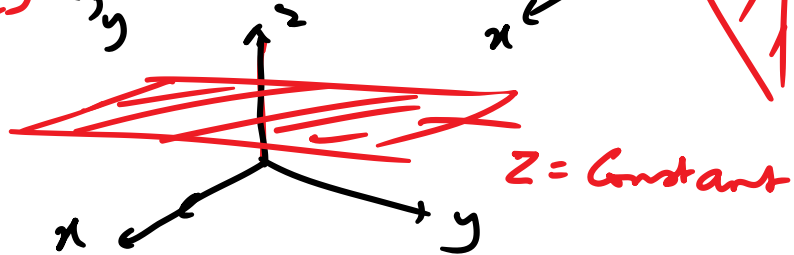
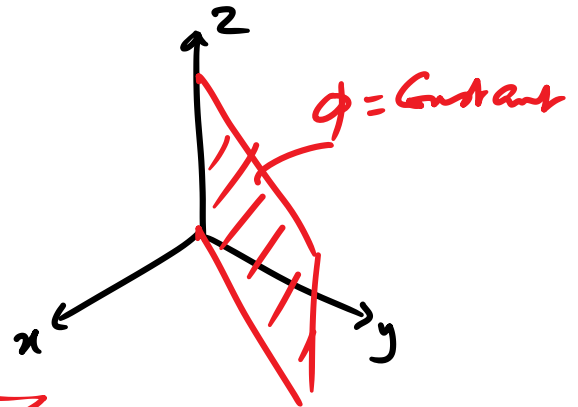
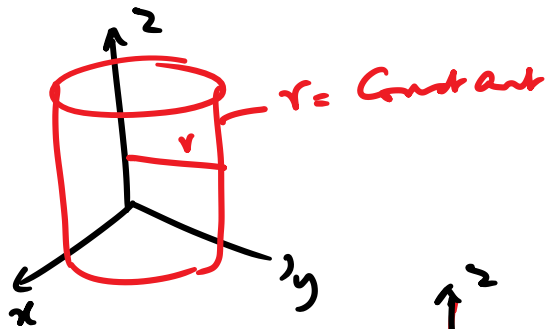
Spherical Polar Coordinate System



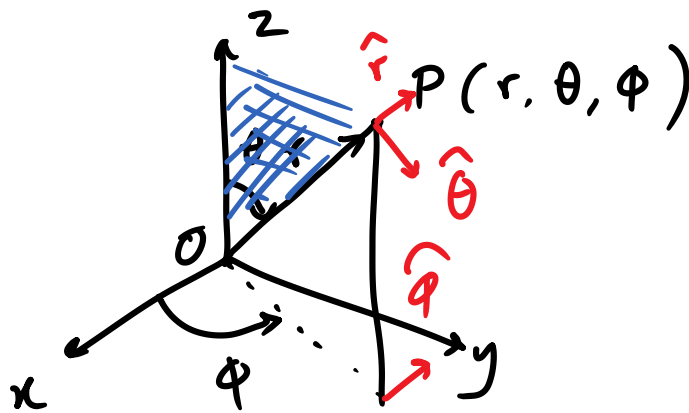
Cylindrical Coordinate System



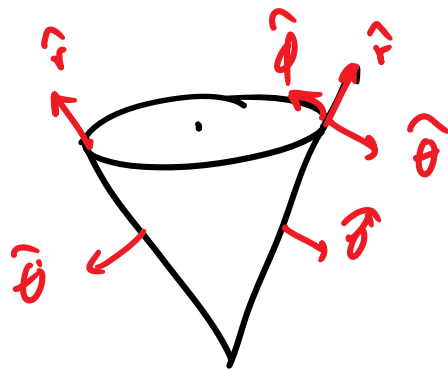
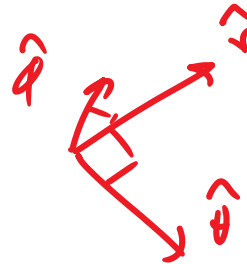
(r, ϕ, z)



Spherical polar Coordinates

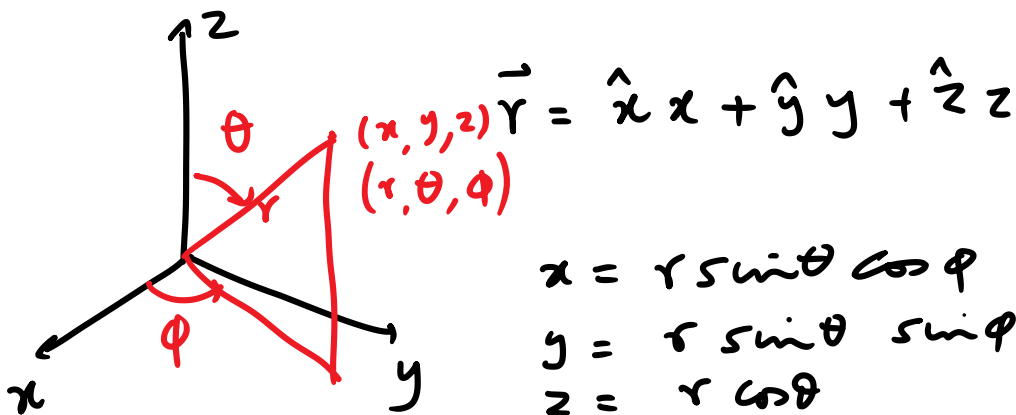


$\hat{r}, \hat{\theta}, \hat{\phi}$



$$\begin{aligned} \hat{r} \cdot \hat{\theta} &= 0 & \hat{r} \times \hat{\theta} &= \hat{\phi} \\ \hat{r} \cdot \hat{\phi} &= 0 & \hat{\theta} \times \hat{\phi} &= \hat{r} \\ \hat{\theta} \cdot \hat{\phi} &= 0 & \hat{\phi} \times \hat{r} &= \hat{\theta} \end{aligned}$$

$$\hat{r}(r, \theta, \phi); \hat{\theta}(r, \theta, \phi); \hat{\phi}(r, \theta, \phi)$$



$$\begin{aligned} x &= r \sin \theta \cos \phi \\ y &= r \sin \theta \sin \phi \\ z &= r \cos \theta \end{aligned}$$

$$\begin{aligned} \vec{r} &= \hat{x} r \sin \theta \cos \phi + \hat{y} r \sin \theta \sin \phi + \hat{z} r \cos \theta \\ &= r (\hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta) \end{aligned}$$

$$\hat{r} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta$$

$$\boxed{\vec{r} = r \hat{r}}$$

$$\vec{r} = x \hat{x} + y \hat{y} + z \hat{z}$$

$$d\vec{r} = dx \hat{x} + \hat{y} dy + \hat{z} dz$$

$$\vec{r} = r \hat{r}$$

$$d\vec{r} = d(r \hat{r}) = \hat{r} dr + r d\hat{r}$$

$$d\hat{r}(\theta, \phi) = \frac{\partial \hat{r}}{\partial \theta} d\theta + \frac{\partial \hat{r}}{\partial \phi} d\phi$$

$$\hat{r} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta$$

$$\begin{aligned} \frac{\partial \hat{r}}{\partial \theta} &= \hat{x} \cos \theta \cos \phi + \hat{y} \cos \theta \sin \phi - \hat{z} \sin \theta \\ &= \hat{\theta} \end{aligned}$$

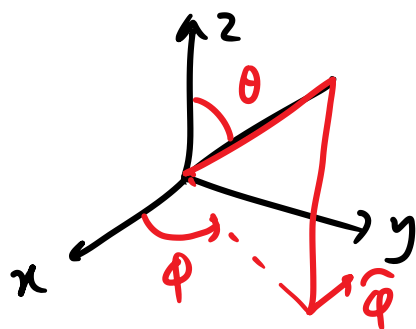
$$\hat{\theta} = \begin{matrix} & \alpha & \beta & \gamma & \delta \\ & \epsilon & \zeta & \eta & \theta \\ & \iota & \kappa & \lambda & \mu \\ & \nu & \xi & \omicron & \pi \\ & \rho & \sigma & \tau & \upsilon \\ & \phi & \psi & \chi & \omega \end{matrix}$$

$$\hat{\theta} = \alpha \hat{x} + \beta \hat{y} + \gamma \hat{z}$$

$$\alpha = \hat{\theta} \cdot \hat{x}, \quad \beta = \hat{\theta} \cdot \hat{y}; \quad \gamma = \hat{\theta} \cdot \hat{z}$$

$$\hat{r} = \hat{x} \sin \theta \cos \phi + \hat{y} \sin \theta \sin \phi + \hat{z} \cos \theta$$

$$\begin{aligned} \frac{\partial \hat{r}}{\partial \phi} &= -\hat{x} \sin \theta \sin \phi + \hat{y} \sin \theta \cos \phi \\ &= \sin \theta (-\hat{x} \sin \phi + \hat{y} \cos \phi) \end{aligned}$$



$$\hat{\phi} = -\hat{x} \sin \phi + \hat{y} \cos \phi$$

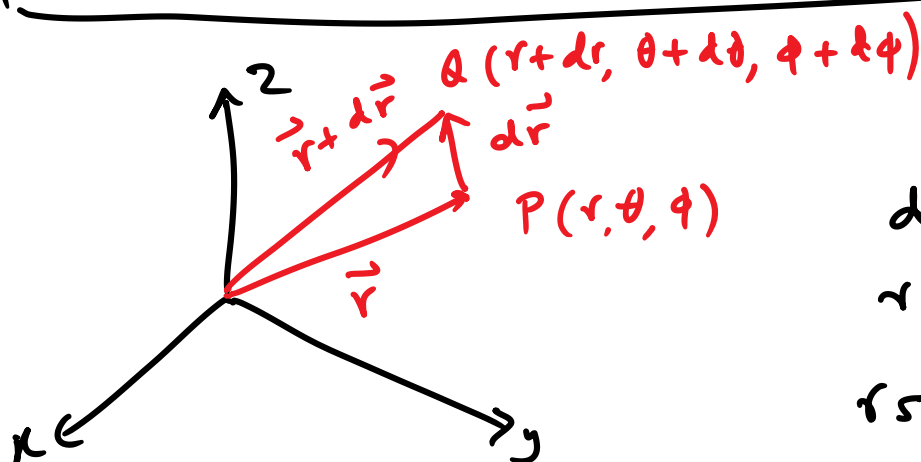
$$\frac{\partial \hat{r}}{\partial \phi} = \sin \theta \hat{\phi}$$

$$\vec{r} = r \hat{r}$$

$$d\vec{r} = \hat{r} dr + r d\hat{r}$$

$$= \hat{r} dr + r \left(\frac{\partial \hat{r}}{\partial \theta} d\theta + \frac{\partial \hat{r}}{\partial \phi} d\phi \right)$$

$$d\vec{r} = \hat{r} dr + r d\theta \hat{\theta} + r \sin \theta d\phi \hat{\phi}$$



$$\begin{aligned} & dr \\ & r d\theta \\ & r \sin \theta d\phi \end{aligned}$$

