BENNETT
UNIVERSITY
A TIMES GROUP INITIATIVE

Network Theorems

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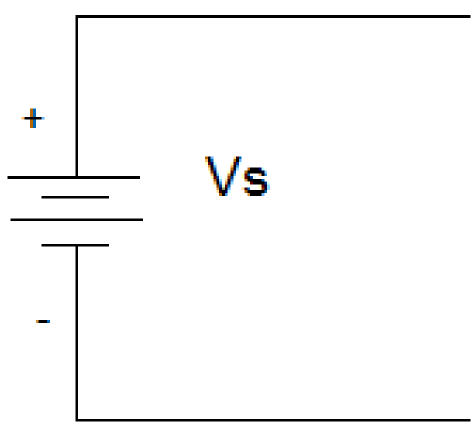
Voltage and Current Sources

Voltage Sources

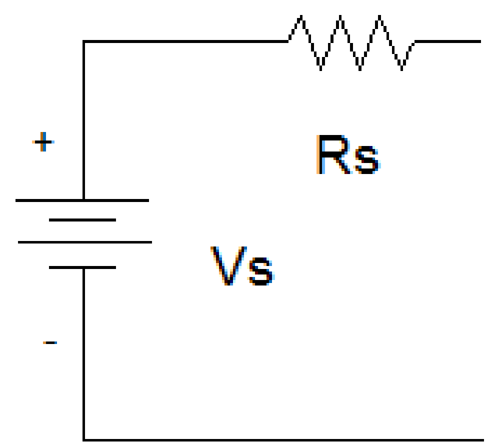
- Ideal

  - An ideal voltage source has zero internal resistance.
  - It can produce as much current as is needed to provide power to the rest of the circuit.



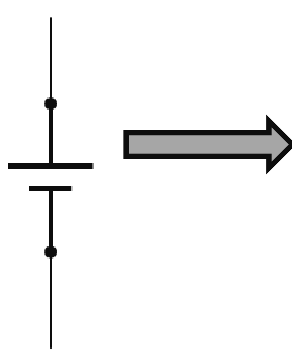
- Real

  - A real voltage sources is modeled as an ideal voltage source in series with a resistor.
  - There are limits to the current and output voltage from the source.

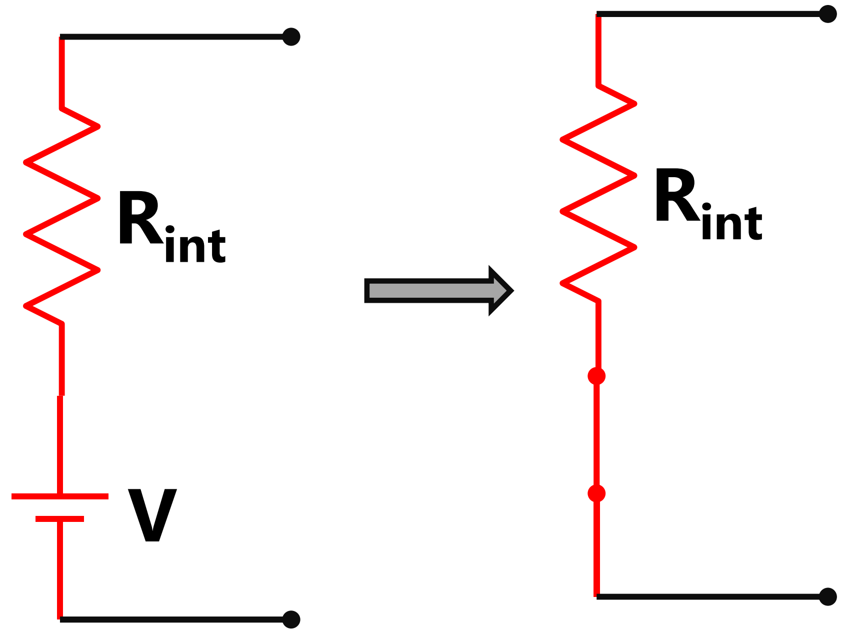


Voltage Source

- Voltage sources are modelled as

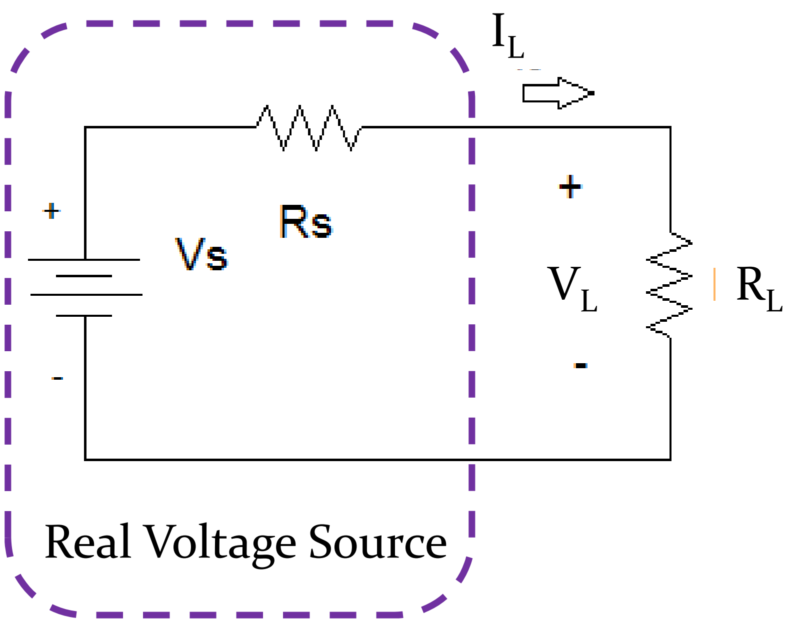


Ideal Voltage source



Non-ideal Voltage source

Limitations of Real Voltage Source



$$V_L = \frac{R_L}{R_L + R_S} V_S$$

$$I_L = V_L / R_L$$

Voltage Source Limitations

- $R_L = 0$

$$V_L = 0\text{V}$$
$$I_{L\max} = V_S / R_S$$
$$P_L = 0\text{W}$$

- $R_L = \infty$

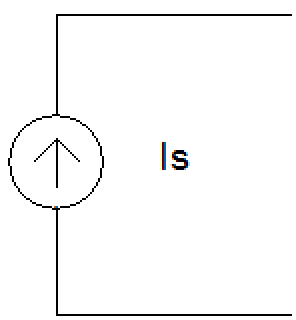
$$V_L = V_S$$
$$I_{L\min} = 0\text{A}$$
$$\text{P}_L = 0\text{W}$$

Current Sources

- Ideal

› An ideal current source has no internal resistance.

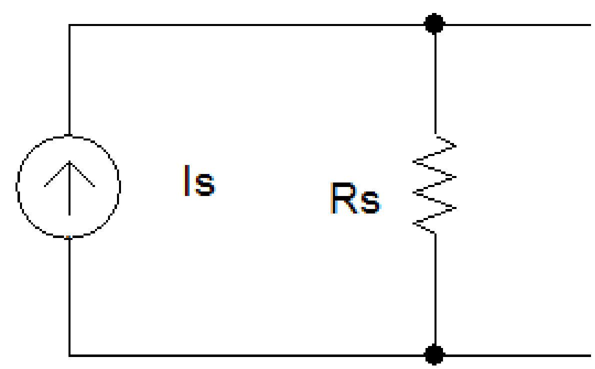
› It can produce as much voltage as is needed to provide power to the rest of circut



- Real

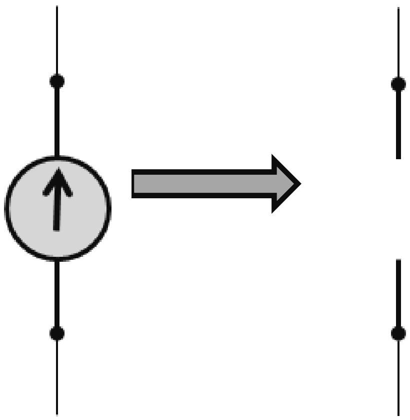
› A real current sources is modeled as an ideal current source in parallel with a resistor.

› Limitations on the maximum voltage and current.

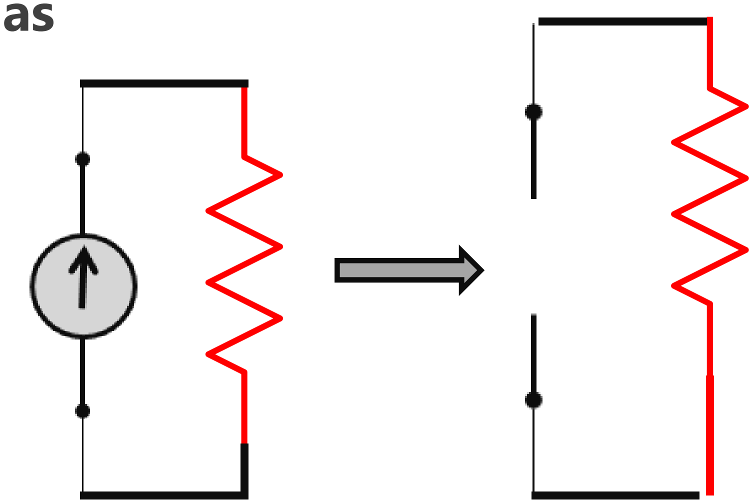


Current Source

- Current sources are modelled as



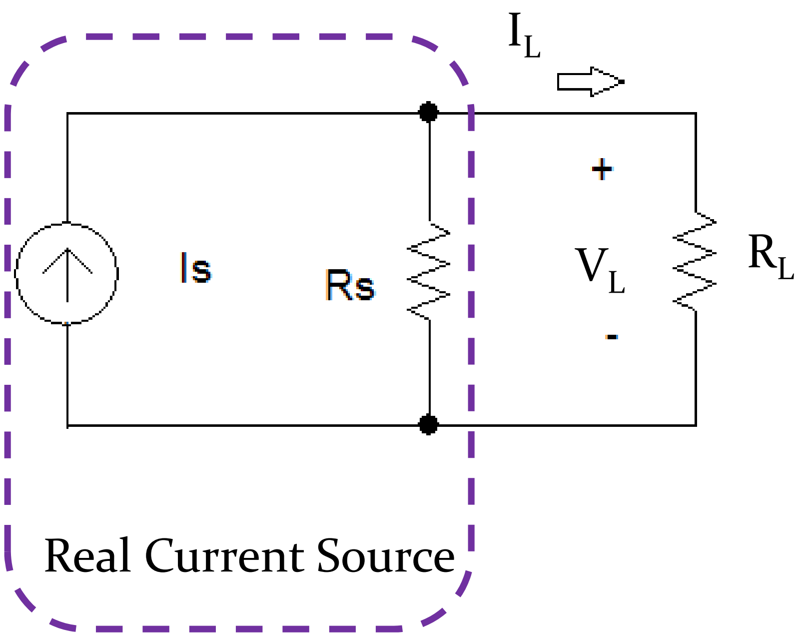
Ideal current source



Non-ideal current source

Limitations of Real Current Source

- Appear as the resistance of the load on the source approaches Rs.



$$I_L = \frac{R_S}{R_L + R_S} I_S$$

$$V_L = I_L R_L$$

Current Source Limitations ….

-

$R_L = 0\Omega$

- $R_L = \infty\Omega$

$$I_L = I_S$$

$$V_{L\min} = 0\text{V}$$

$$P_L = 0\text{W}$$

$$I_L = 0\text{A}$$

$$V_{L\max} = I_S R_S$$

$$\text{P}_\text{L} = 0\text{W}$$

Electronic Response

- For a real voltage source, what is the voltage across the load resistor when $R_s = R_L$
- For a real current source, what is the current through the load resistor when $R_s = R_L$

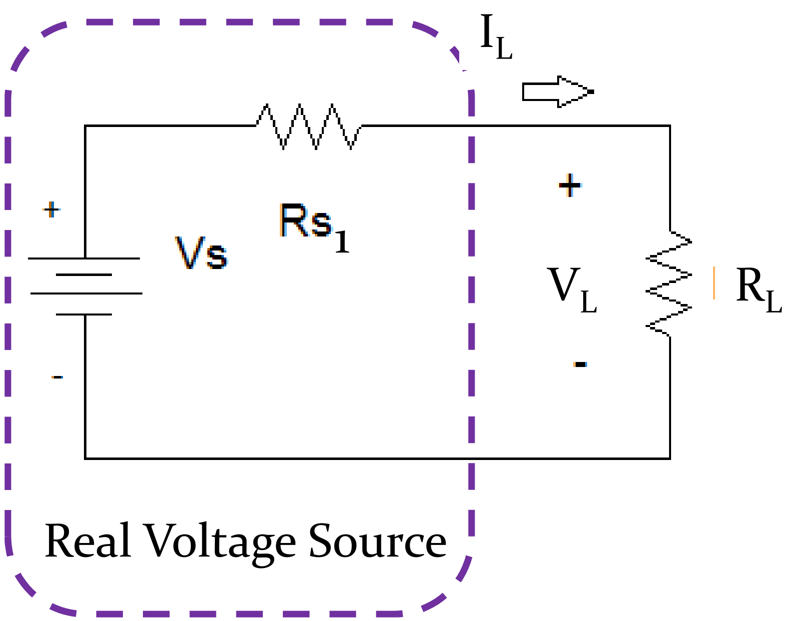
Equivalence

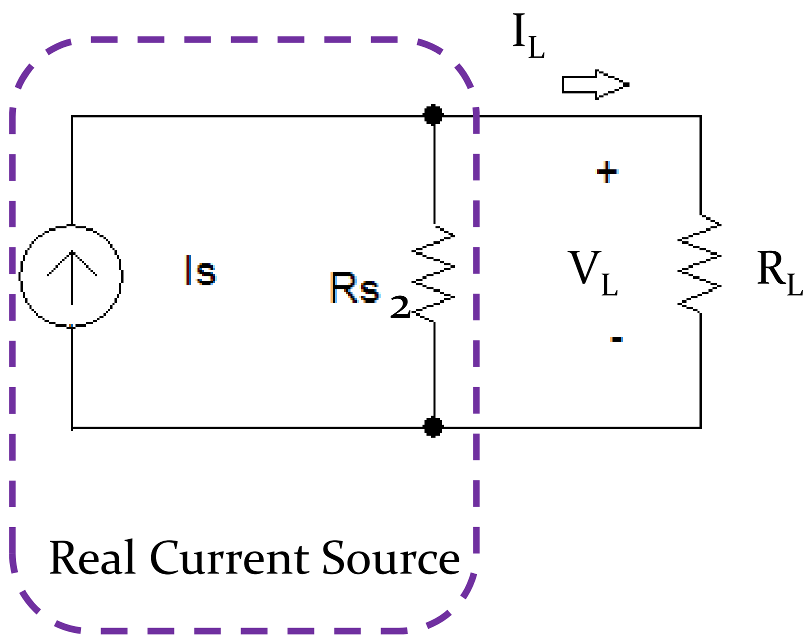
- An equivalent circuit is one in which the i-v characteristics are identical to that of the original circuit.
  - The magnitude and sign of the voltage and current at a particular measurement point are the same in the two circuits.

Equivalent Circuits

- R_L in both circuits must be identical.

I_L and V_L in the left circuit = I_L and V_L on the left

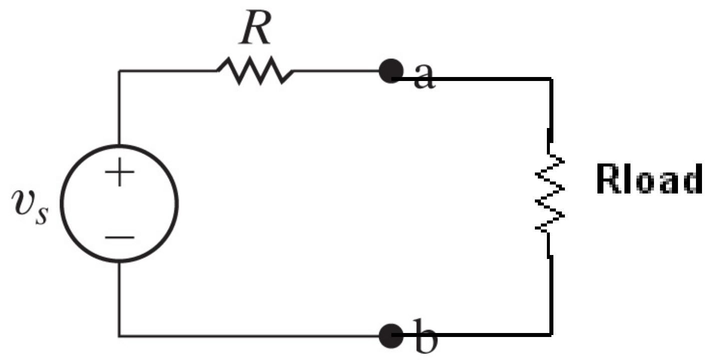




How to make them equivalent

- Attach a load resistor (the same one) to each circuit, and ensure that each circuit delivers the same voltage and current to the load.

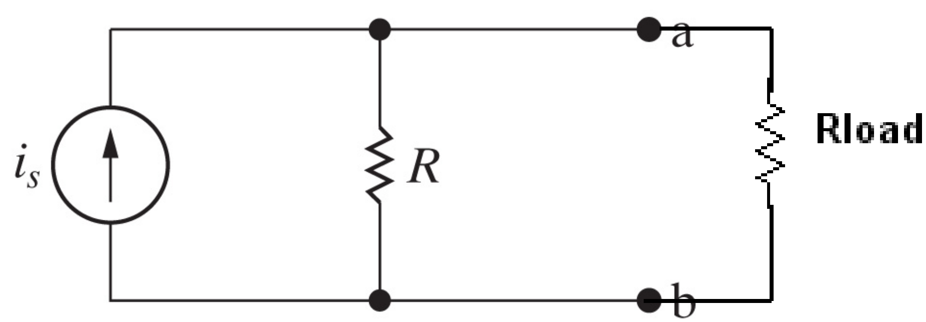
How to make them equivalent



(a)

$$i_L = \frac{v_s}{R + R_{load}}$$

⇕



(b)

$$i_L = \frac{R}{R + R_{load}} i_s$$

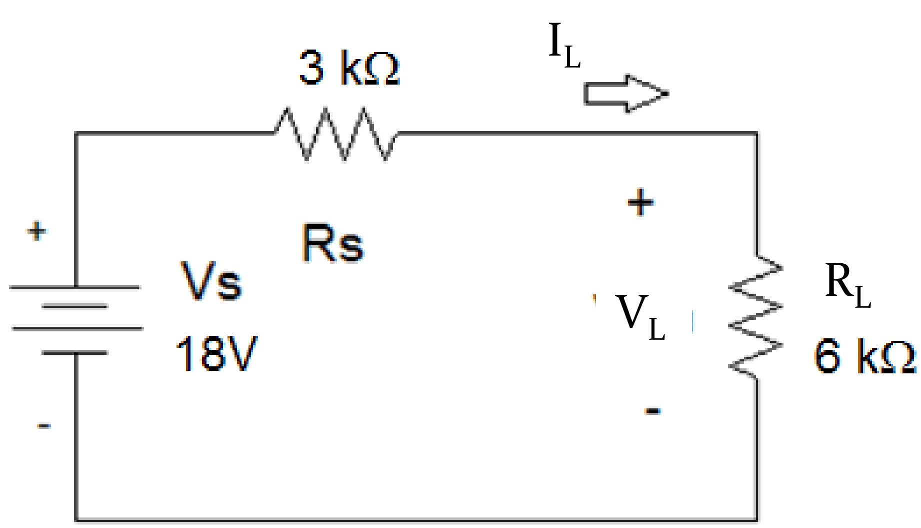
Equating the two currents

$$i_L = \frac{v_s}{R + R_{load}} = \frac{R}{R + R_{load}} i_s$$

$$\boxed{\begin{aligned} i_s &= \frac{v_s}{R} \\ v_s &= i_s R \end{aligned}}$$

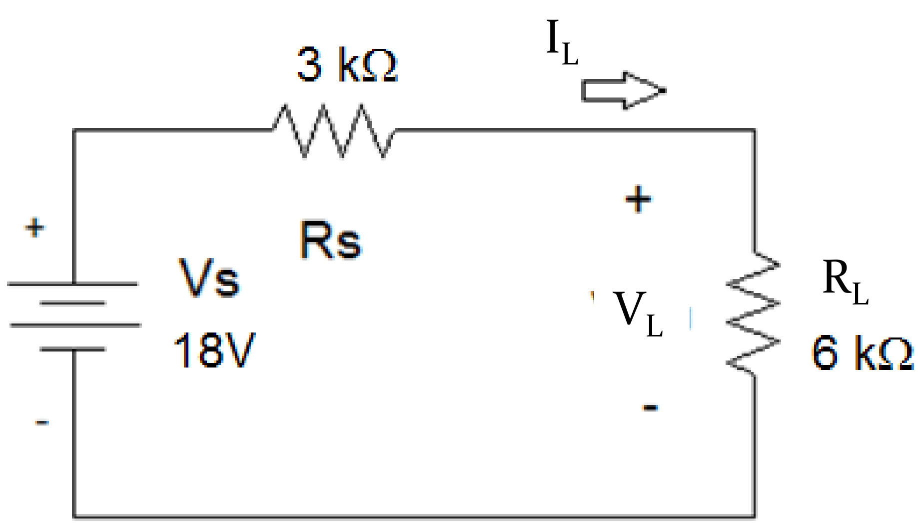
Example #1

- Find an equivalent current source to replace Vs and Rs in the circuit below.



Example #1 (con't)

- Find I_L and V_L.



$$V_L = \frac{R_L}{R_L + R_S} V_S$$

$$V_L = \frac{6k\Omega}{6k\Omega + 3k\Omega} 18V = 12V$$

$$I_L = V_L / R_L$$

$$I_L = 12V / 6k\Omega = 2mA$$

$$P_{Vs} = P_L + P_{Rs}$$

$$P_{Vs} = 12V(2mA) + (18V - 12V)(2mA)$$

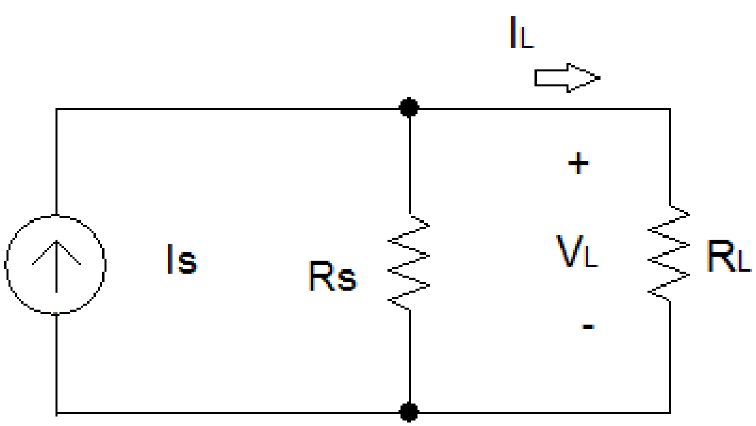
$$P_{Vs} = 36mW$$

Example #1 (con't)

If R_S = 3kΩ

$$I_S = \frac{R_L + R_S}{R_S} I_L$$

$$I_S = \frac{6k\Omega + 3k\Omega}{3k\Omega} 2mA = 6mA$$



$$V_L = V_{Is} = I_L R_L = 12V$$

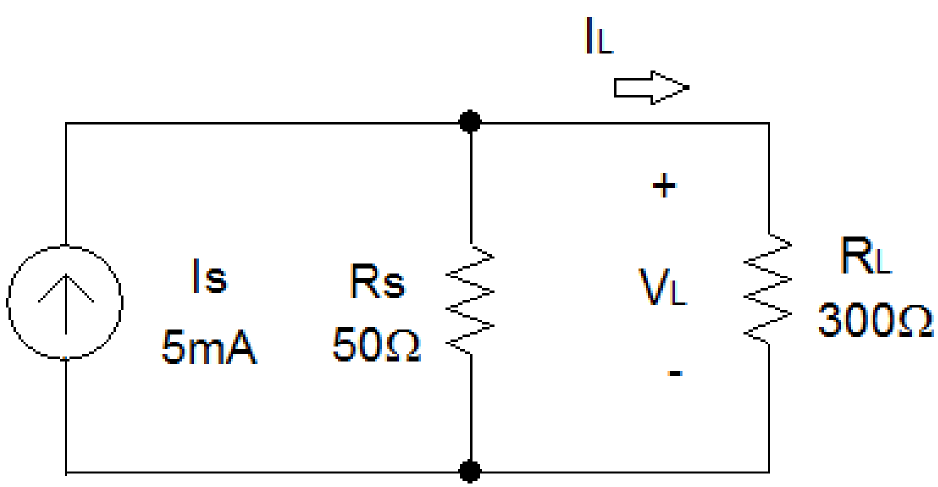
$$P_{Is} = P_L + P_{Rs} = V_L I_L + V_{Rs} I_{Rs}$$

$$P_{Is} = 12V(2mA) + 12V(6mA - 2mA)$$

$$P_{Is} = 72mW$$

Example #2

- Find an equivalent voltage source to replace Is and Rs in the circuit below.



Example #2 (con't)

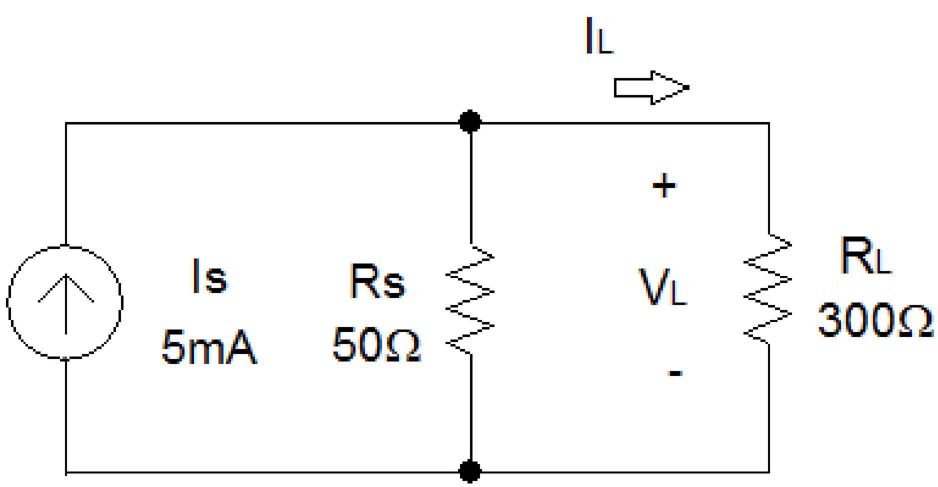
- Find I_L and V_L.

$$I_L = \frac{50\,\Omega}{300\,\Omega + 50\,\Omega} I_S$$

$$I_L = 0.714\ mA$$

$$V_L = I_L R_L$$

$$V_L = 0.714\ mA\,(300\,\Omega) = 0.214\ V$$



$$P_{Vs} = P_L + P_{Rs}$$

$$P_{Vs} = 0.214\ V\,(0.714\ mA) + 0.214\ V\,(5\,mA - 0.714\ mA)$$

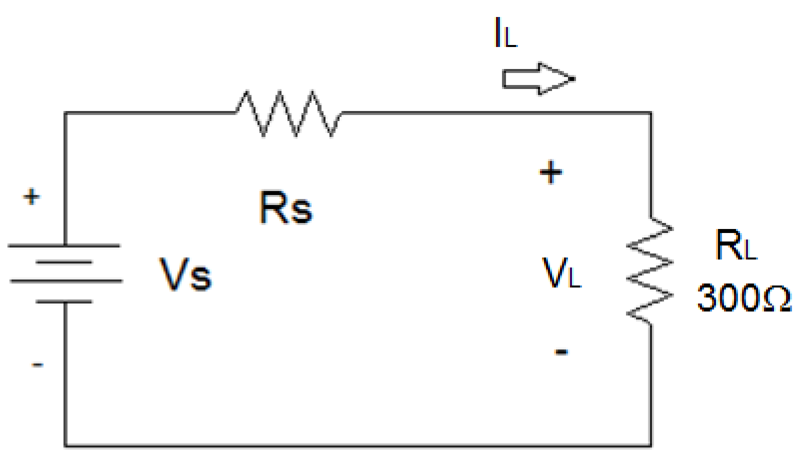
$$P_{Vs} = 1.07\ mW$$

Example #2 (con't)

If R_S = 50Ω

$$V_S = \frac{R_L + R_S}{R_L} V_L$$

$$V_S = \frac{300\Omega + 50\Omega}{300\Omega} 0.214V = 0.25V$$



$$I_L = I_{Vs} = V_L / R_L = 0.714mA$$

$$P_{Vs} = P_L + P_{Rs} = V_L I_L + V_{Rs} I_{Rs}$$

$$P_{Vs} = 0.214V(0.714A)$$

$$+ (0.25V - 0.214V)(0.714mA)$$

$$P_{Vs} = 0.179mW$$

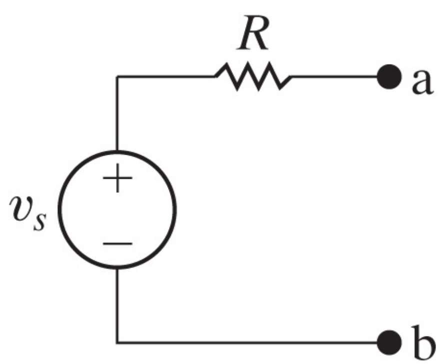
Summary

- An equivalent circuit is a circuit where the voltage across and the current flowing through a load R_L are identical.
  - As the shunt resistor in a real current source decreases in magnitude, the current produced by the ideal current source must increase.
  - As the series resistor in a real voltage source increases in magnitude, the voltage produced by the ideal voltage source must increase.
    - The power dissipated by R_L is 50% of the power produced by the ideal source when $R_L = R_S$.

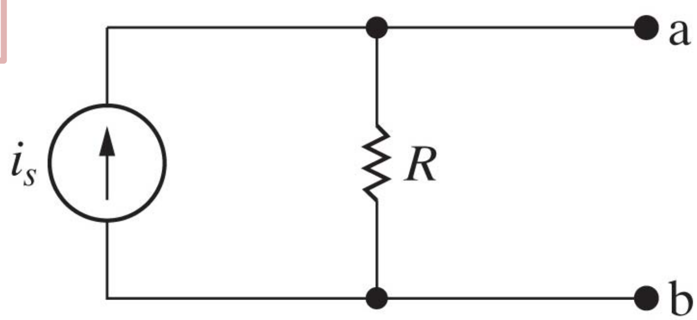
Source Transformation

Source transformation

- Convert the voltage source in series with R into an equivalent current source in parallel with the same resistance and vice-versa.



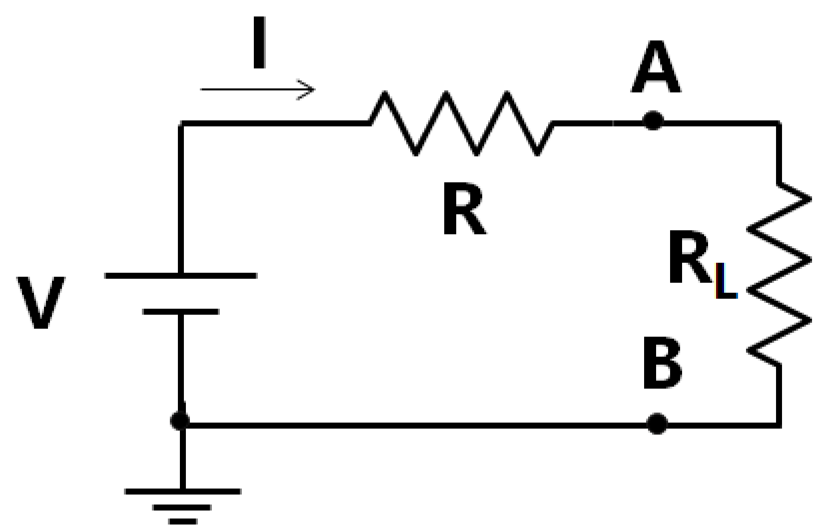
$$v_s = i_s R$$



Maximum Power Transfer Theorem

Maximum Power Transfer Theorem

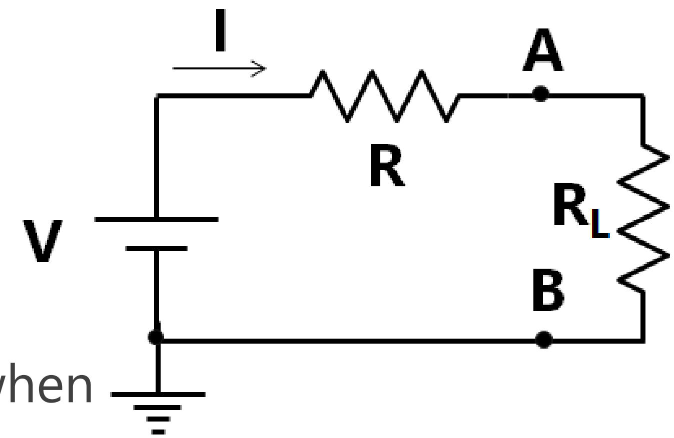
- In all practical cases, energy sources have non-zero internal resistance. Thus, there are losses inherent in any real source.
- The aim of an energy source is to provide power to a load.
- Given a circuit with a known resistance, what is the resistance of the load that will result in the maximum power being delivered to the load?
- Consider the following circuit



Maximum Power Transfer Theorem

- The power delivered to the load (absorbed by R_L) is

$$P = I^2 R_L = \left(\frac{V}{R + R_L} \right)^2 R_L$$



- The power delivered to load is maximum when

$$\frac{\partial P}{\partial R_L} = 0$$

$$\frac{\partial P}{\partial R_L} = V^2 \left[\frac{1}{(R + R_L)^2} - 2R_L \frac{1}{(R + R_L)^3} \right] = 0$$

Maximum Power Transfer Theorem

$$\frac{\partial P}{\partial R_L} = V^2 \left[\frac{1}{(R+R_L)^2} - 2R_L \frac{1}{(R+R_L)^3} \right] = 0$$

$$R + R_L = 2R_L \qquad \boxed{R_L = R}$$

- Thus, maximum power transfer takes place when the resistance of the load equals is equal to the resistance of the circuit

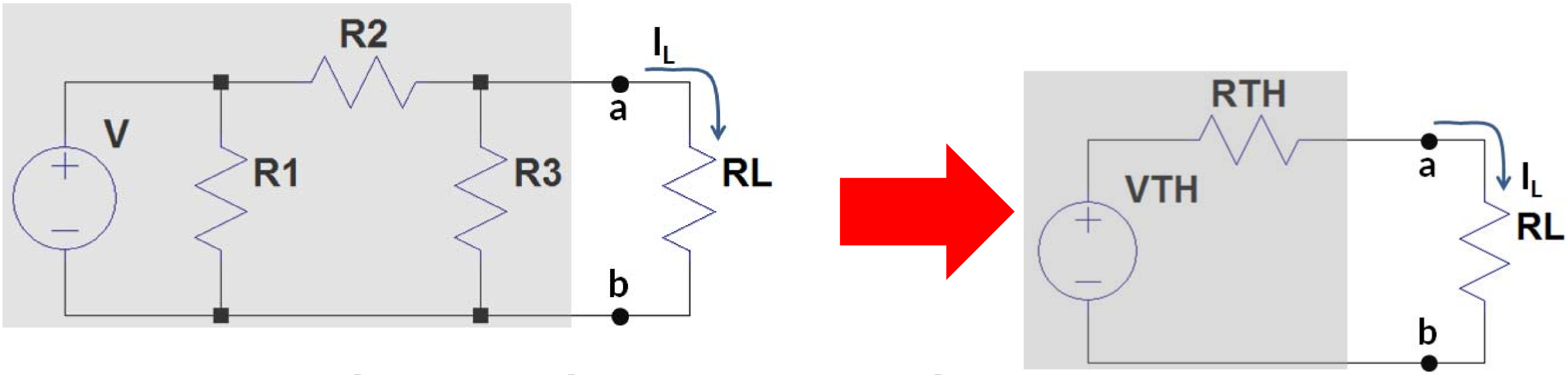
$$P_{\max} = \left(\frac{V}{R+R_L} \right)^2 R_L \Big|_{R_L = R} = \frac{V^2}{4R}$$

- At best, one-half of the power is dissipated in the resistance of the circuit and the other one-half in the load.

Thevenin’s Theorem

Thevenin’s Theorem

- **Thevenin’s theorem**: ***Any two-terminal, linear bilateral dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor***



- ***By using Thevenin’s theorem, an equivalence at the terminals is established — the internal construction and characteristics of the original network and the Thevenin equivalent are usually entirely different***

To Find R_{TH} and V_{TH}

1) Remove that portion of the network across which the Thevenin equivalent circuit is to be found. It requires that the load resistor R_L be temporarily removed from the network.

2) Mark the terminals of the remaining two-terminal network.

3) Calculate R_{TH} by first setting all sources as described:
   a) Voltage sources are replaced by short circuits
   b) Current sources by open circuits
   c) Find the resultant resistance between the two marked terminals. Note: If the internal resistance of the voltage and/or current sources is included in the original network, the internal resistances need to be retained.

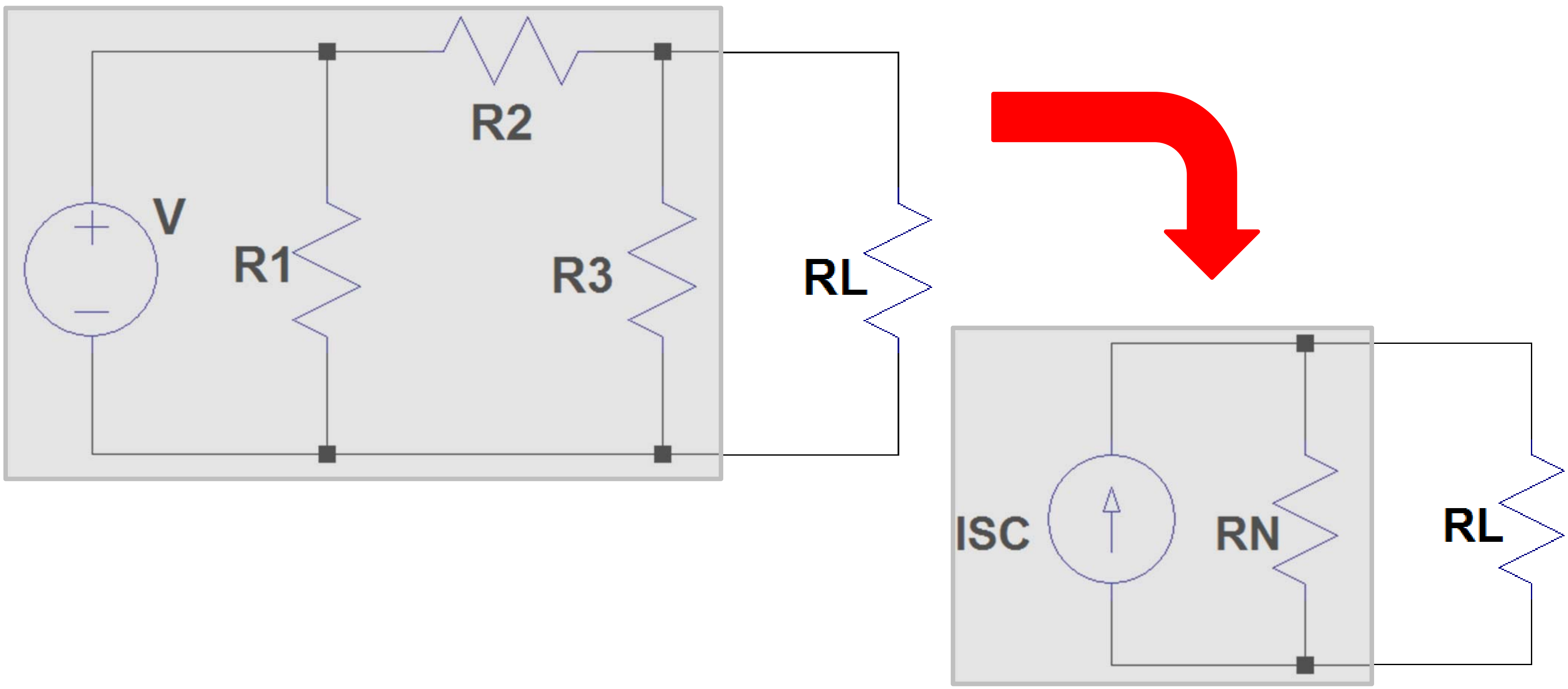
To Find R_{TH} and V_{TH}

4) Calculate V_{TH} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals.
5) Draw the Thevenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Norton Theorem

Norton’s Theorem

- Norton’s Theorem: It is possible to simplify any linear circuit to an equivalent circuit with just a single current source and parallel resistance connected to a load.



To Find R_N and V_N

1) Remove that portion of the network across which the Thevenin equivalent circuit is to be found. It requires that the load resistor R_L be temporarily removed from the network.

2) Mark the terminals of the remaining two-terminal network.

3) Calculate R_{TH} by first setting all sources as described:
    a) Voltage sources are replaced by short circuits
    b) Current sources by open circuits
    c) Find the resultant resistance between the two marked terminals. Note: If the internal resistance of the voltage and/or current sources is included in the original network, the internal resistances need to be retained.

To Find R_N and V_N

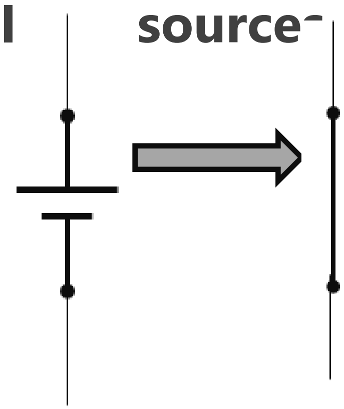
4) Calculate I_N by first returning all sources to their original position and then finding the short-circuit current between the marked terminals.

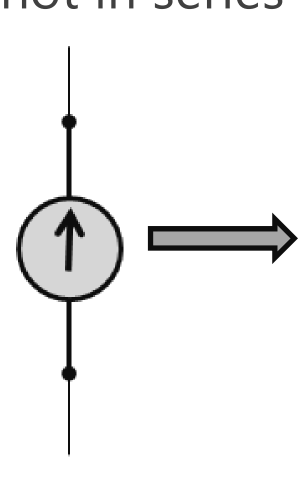
5) *Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.*

Superposition Theorem

Superposition Theorem

- **Superposition theorem**: ***The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produced independently by each source.***
- The **superposition theorem,** is useful in finding solutions to he networks with two or more sources that are not in series or parallel.
- **I sources are modelled as**



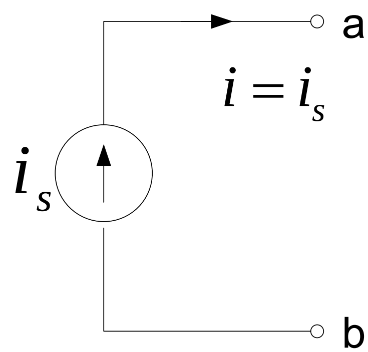


Superposition Principle

- *The **superposition principle** states that the voltage across (or the current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.*
- Because the circuit is linear we can find the response of the circuit to each source acting alone, and then add them up to find the response of the circuit to all sources acting together. This is known as the **superposition principle**.

Turning sources off

Current source:

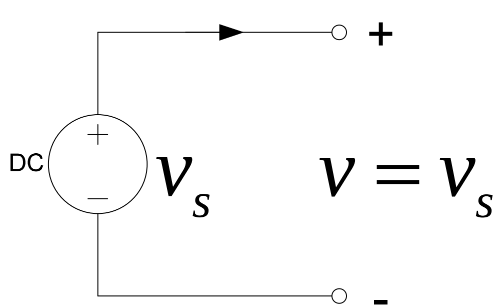


We replace it by a current source where

$$i_s \equiv 0$$

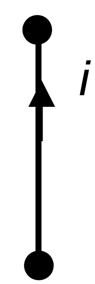
An open-circuit

Voltage source:



We replace it by a voltage source where

$$v_s \equiv 0$$



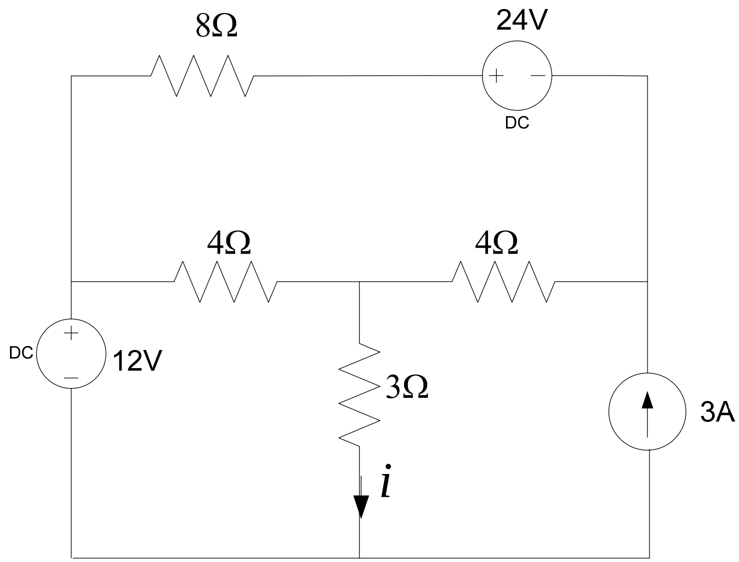
An short-circuit

Steps in Applying the Superposition Principle

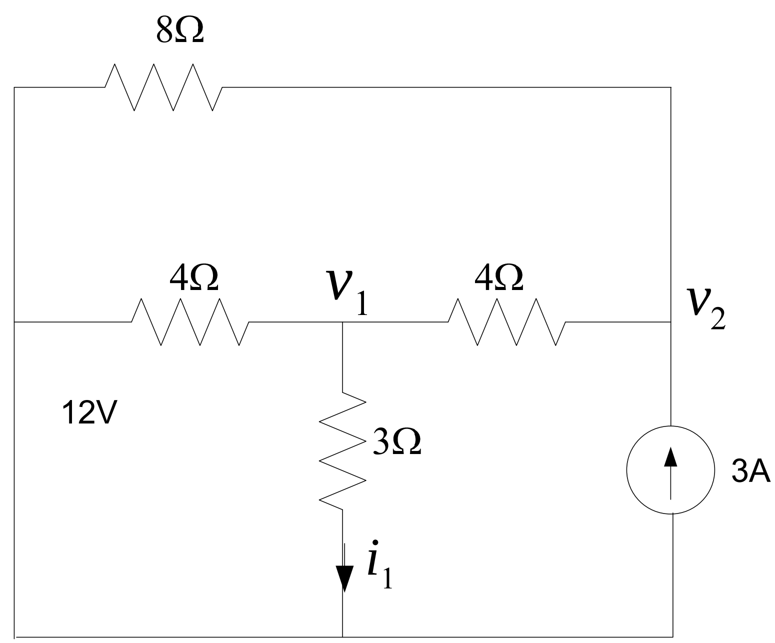
1. Turn off all independent sources except one. Find the output (voltage or current) due to the active source.
2. Repeat step 1 for each of the other independent sources.
3. Find the total output by adding algebraically all of the results found in steps 1 & 2 above.

In some cases, but certainly not all, superposition can simplify the analysis.

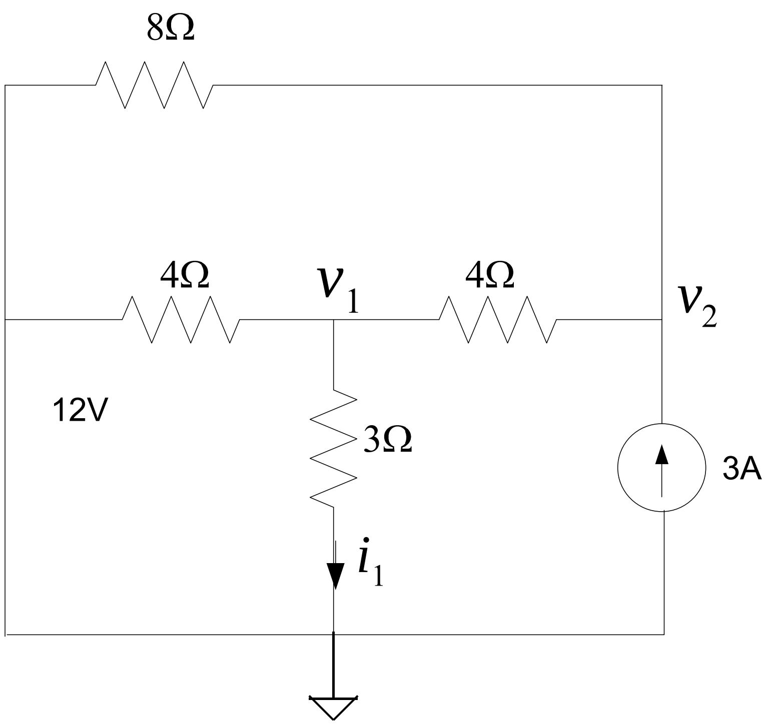
Example: In the circuit below, find the current *i* by superposition



Turn off the two voltage sources (replace by short circuits).



$$\begin{pmatrix} 1/4+1/3+1/4 & -1/4 \\ -1/4 & 1/4+1/8 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$



$$i_1 = 1$$

Node v1 $\quad \frac{1}{4}v_1 + \frac{1}{3}v_1 - \frac{1}{4}(v_1 - v_2) = 0$

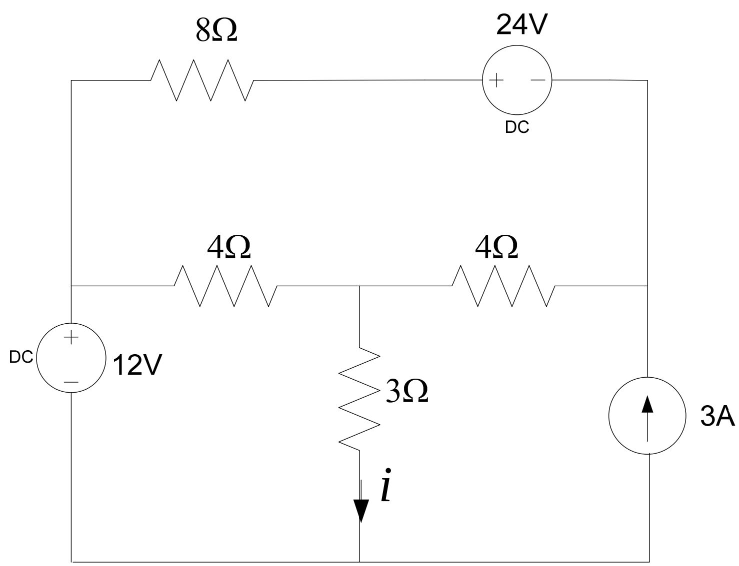
Node v2 $\quad -\frac{1}{4}v_1 + \frac{3}{8}v_2 = 3$

$$v_2 = \frac{10}{3}v_1$$

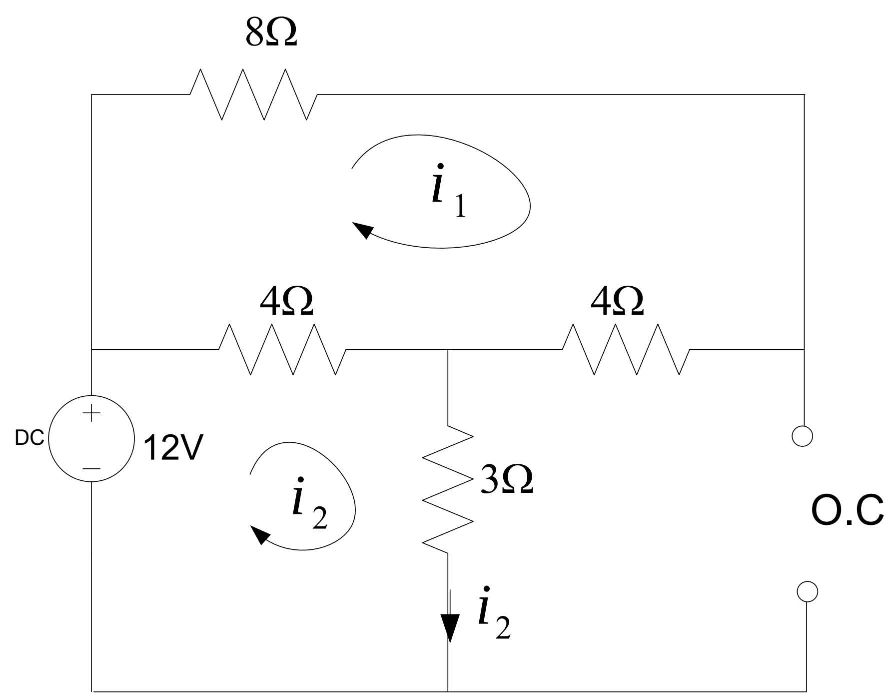
$$v_1\left(\frac{10}{8} - \frac{2}{8}\right) = 3$$

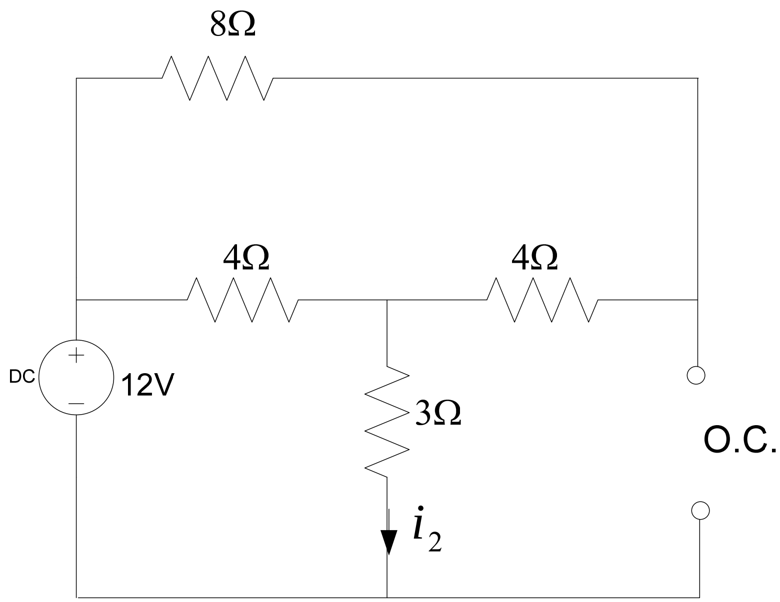
$$v_1 = 3$$

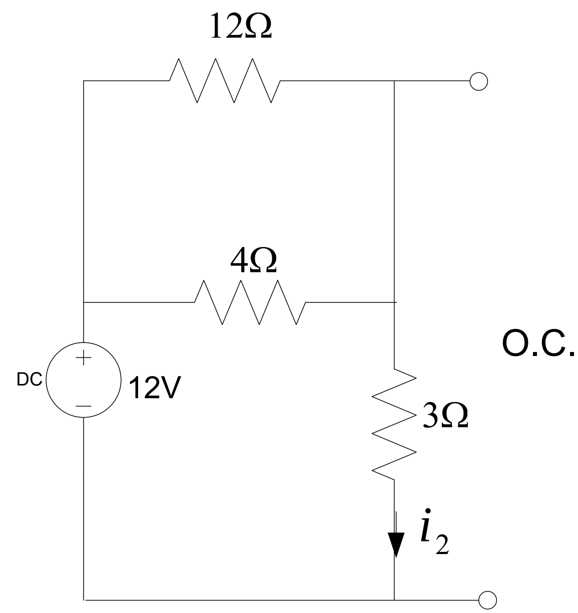
Example: In the circuit below, find the current *i* by superposition



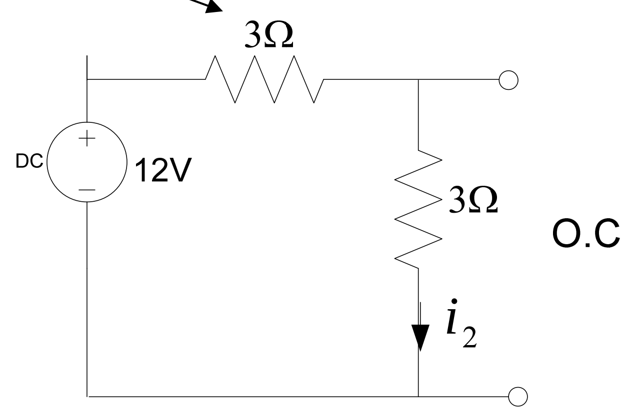
Turn off the 24V & 3A sources:





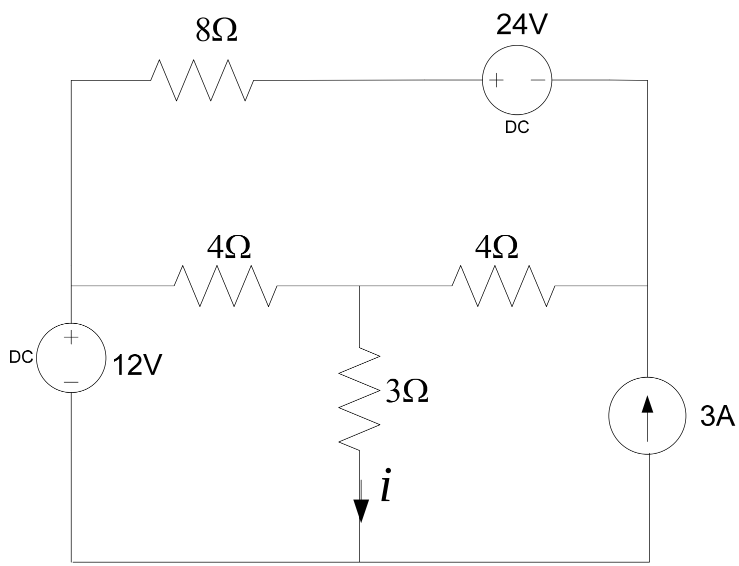


$$\frac{12\times 4}{16}=3$$

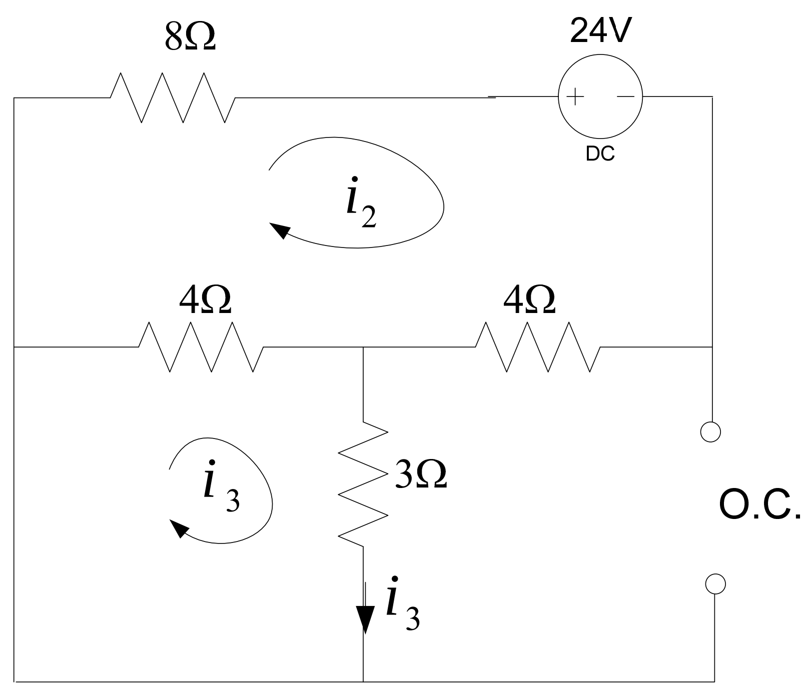


$$i_2=\frac{12}{6}=2$$

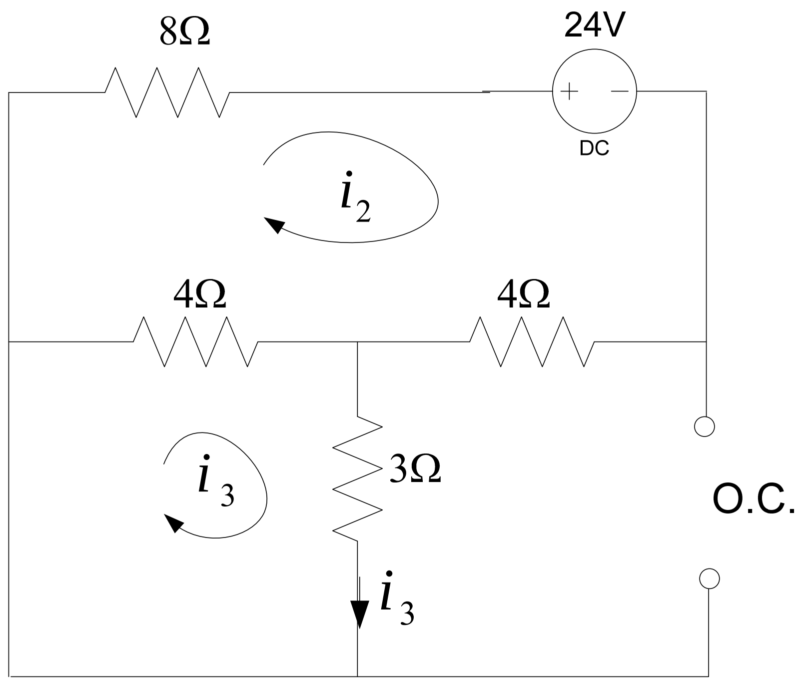
Example: In the circuit below, find the current *i* by superposition



Turn off the 3A & 12V sources:



$$\begin{pmatrix} 4+8+4 & -4 \\ -4 & 4+3 \end{pmatrix}\begin{pmatrix} i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} -24 \\ 0 \end{pmatrix}$$



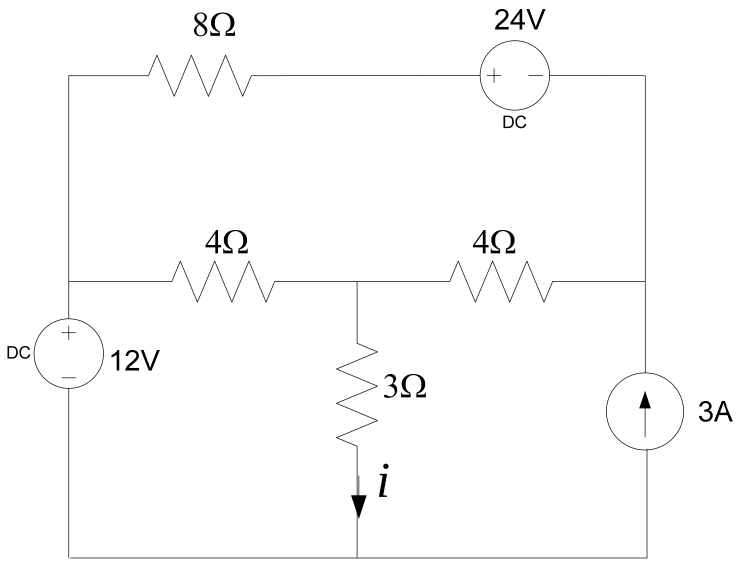
$$16i_2 - 4i_3 = -24$$

$$-4i_2 + 7i_3 = 0$$

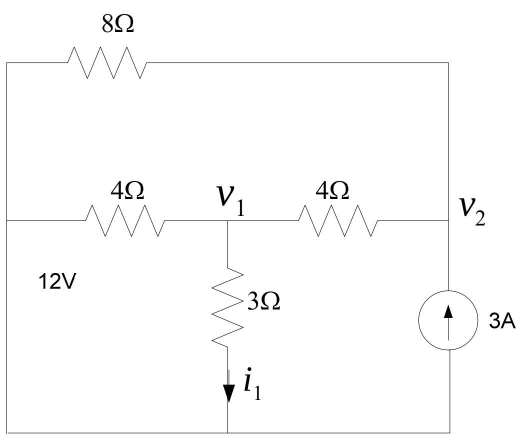
$$i_2 = \frac{7}{4} i_3$$

$$i_3\left(28 - 4\right) = -24$$

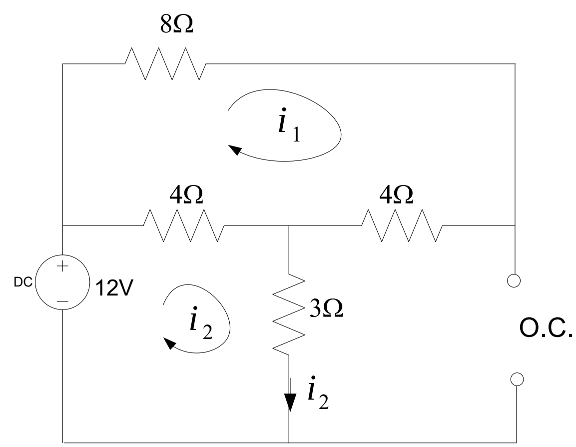
$$i_3 = -1$$



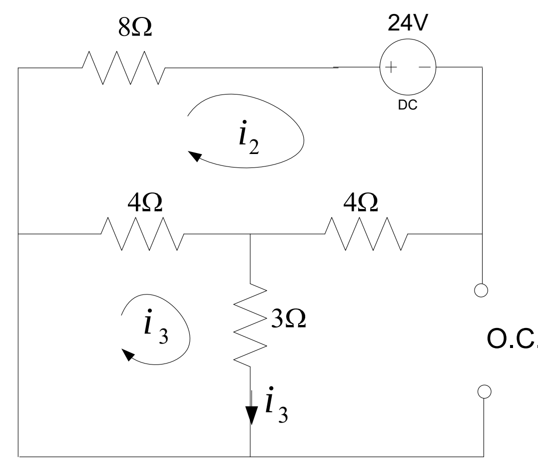
$$i = i_1 + i_2 + i_3 = 1\text{A} + 2\text{A} - 1\text{A} = 2\text{A}$$



$$i_1 = 1$$



$$i_2 = 2$$



$$i_3 = -1$$