Name of student: ..........................................................

Batch No:.......... Enrollment No. ....................

COURSE NAME: LINEAR ALGEBRA AND ORDINARY DIFFERENTIAL EQUATIONS

B.TECH- QUIZ TEST

FALL SEMESTER 2018-19

COURSE CODE : EMAT102L

MAX. TIME: 30 min

COURSE CREDIT: 3-1-0

MAX. MARKS: 10

1. Find the orthogonal trajectory of $x^2 = \frac{a}{2}e^{2y}$, where $a$ is an arbitrary constant. [2]

**Solution:** Given family of curves is $x^2 = \frac{a}{2}e^{2y}$ —— ①

Diff. ① w.r.t. '$x$', we get $2x = \frac{a}{2}.2e^{2y}\frac{dy}{dx} = ae^{2y}.\frac{dy}{dx}$

$$\Rightarrow a = \frac{2x}{e^{2y}}.\frac{1}{\frac{dy}{dx}}$$

From ①, we get $x^2 = \frac{2x}{2e^{2y}}.\frac{1}{\frac{dy}{dx}}.e^{2y} = \frac{x}{\frac{dy}{dx}}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x}$$

Replacing $\frac{dy}{dx}$ by $\frac{-1}{\frac{dy}{dx}}$, we get

$$\frac{-1}{\frac{dy}{dx}} = \frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = -x$$

$$\Rightarrow \boxed{y = -\frac{x^2}{2} + C} \text{ Ans}$$

2. Solve the differential equation $(-2xy + \sin x)dx - (x^2 + \cos y)dy = 0$. —— ① [2]

**Solution:**

Here $M = -2xy + \sin x$, $N = -x^2 - \cos y$

$$\Rightarrow \frac{\partial M}{\partial y} = -2x \qquad \frac{\partial N}{\partial x} = -2x$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \Rightarrow \text{① is an exact DE.}$$

The solution of ① is given by

$$\int_{y=\text{Constant}} M\,dx + \int (\text{terms of } N \text{ not containing } x)\,dy = C$$

$$\Rightarrow \int_{y=\text{Constant}} (-2xy + \sin x)dx + \int (-\cos y)\,dy = C$$

$$\Rightarrow -\frac{2x^2}{2}y - \cos x - \sin y = C \Rightarrow \boxed{x^2y + \cos x + \sin y = C_1} \text{ Ans}$$

$(C_1 = -C)$

3. Suppose $y = f(x)$ is a solution of the differential equation $\frac{dy}{dx} = y(a - by)$, where $a$ and $b$ are positive constants. Find the intervals on the $y$-axis for which the function $y = f(x)$ is strictly increasing, without solving the differential equation. [2]

**Solution:** Using the given DE, the solution $y=f(x)$ is strictly increasing if

$$\frac{dy}{dx} > 0 \Rightarrow y(a-by) > 0$$

Case-I: $y > 0$ and $a - by > 0$

$\Rightarrow y > 0$ and $y < \frac{a}{b}$

$\Rightarrow \boxed{0 < y < \frac{a}{b}}$

Case-II: $y < 0$ and $a - by < 0$

$\Rightarrow y < 0$ and $y > \frac{a}{b}$

which is not possible as a and b are positive constants.

Hence, the interval on the $y$-axis for which the solution $y=f(x)$ is strictly increasing is $0 < y < \frac{a}{b}$.

4. Discuss the existence and uniqueness of the following initial value problem in $R: |x| \leq 1, |y| \leq 1$.

$$\frac{dy}{dx} = y^{1/3} + x,\ y(0) = 0 \quad \text{——— (1)}$$

**Solution:** Here $f(x,y) = y^{1/3} + x$, which is continuous in $R$.

Also, $|f(x,y)| = |y^{1/3}+x| \leq |y|^{1/3} + |x| \leq (1)^{1/3} + (1) = 2 = M$

By Picard's existence theorem, $\exists$ solution of (1) in $|x| \leq h$, where

$h = \min.\left(a, \frac{b}{M}\right) = \min.\left(1, \frac{1}{2}\right) = \frac{1}{2}$

$\Rightarrow |x| \leq \frac{1}{2}$

But $f(x,y)$ doesnot satisfy Lipschitz condition as $\frac{|f(x,y_1) - f(x,0)|}{|y_1 - 0|} = \frac{|y_1^{1/3} + x - x|}{|y_1|} = \frac{1}{y_1^{2/3}}$ [for $y_1>0$ and $y_2=0$]

which is unbounded in the neighbourhood of 0.

$\Rightarrow f(x,y)$ doesnot satisfy Lipschitz condition.

$\Rightarrow$ uniqueness of the solution is not guaranteed.

5. Let $y = \phi(x)$ and $y = \psi(x)$ be the solutions of $y'' - 2xy' + (\sin x^2)y = 0$ such that $\phi(0) = 1, \phi'(0) = 1$ and $\psi(0) = 1, \psi'(0) = 2$. Then find the value of Wronskian $W(\phi, \psi)$ at $x = 1$. [2]

**Solution:**

As $W(\phi,\psi)(x) = c\, e^{-\int \frac{a_1(x)}{a_0(x)}dx} = c\, e^{-\int -2x\, dx} = c\, e^{x^2}$

$\Rightarrow W(\phi,\psi)(0) = ce^0 = c$

$\Rightarrow \begin{vmatrix} \phi(0) & \psi(0) \\ \phi'(0) & \psi'(0) \end{vmatrix} = c$ $\quad \left[\because W(y_1,y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}\right]$

$\Rightarrow \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = c$

$\Rightarrow (2-1) = c \Rightarrow c = 1.$

Thus $\boxed{W(\phi,\psi)(x) = e^{x^2}}$

$\Rightarrow \boxed{W(\phi,\psi)(1) = e^1 = e}$

Ans.

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