**Department of Mathematics, Bennett University**
**Engineering Calculus (EMAT101L)**
**Assignment**

---

1. Let $\{a_n\}$ be a sequence of real numbers such that $\lim_{n\to\infty}\left|a_n+3\left(\frac{n}{n+1}\right)^n\right|^{\frac{1}{n}}=\frac{2}{3}$. Determine $\lim_{n\to\infty} a_n$.

2. Let $f:[0,1]\to\mathbb{R}$ be a differentiable function such that $f(0)=0$ and $f(1)=1$. Show that there exist $x,y\in(0,1)$ with $x\neq y$ such that $\frac{1}{f'(x)}+\frac{1}{f'(y)}=2$.

3. Evaluate: $\lim_{x\to\infty}\left[(x+1)^{\frac{x+2}{x+1}}-x^{\frac{x+1}{x}}\right]$.

4. Compute the sums of the following series

   (a) $\sum_{n=1}^{\infty}\sqrt{n+2}-2\sqrt{n+1}+\sqrt{n}$ $\qquad$ (b) $\sum_{n=1}^{\infty}\frac{2}{n^3+3n^2+2n}$.

5. Give examples of series such that

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=1$$

   but (a) the series diverges, (b) the series conditionally converges, (c) the series absolutely converges.

6. Determine all real values of $p$ for which the integral $\int_0^{\infty}\frac{x^{p-1}}{1+x}\,dx$ is convergent.