Complex Numbers

EECE105L

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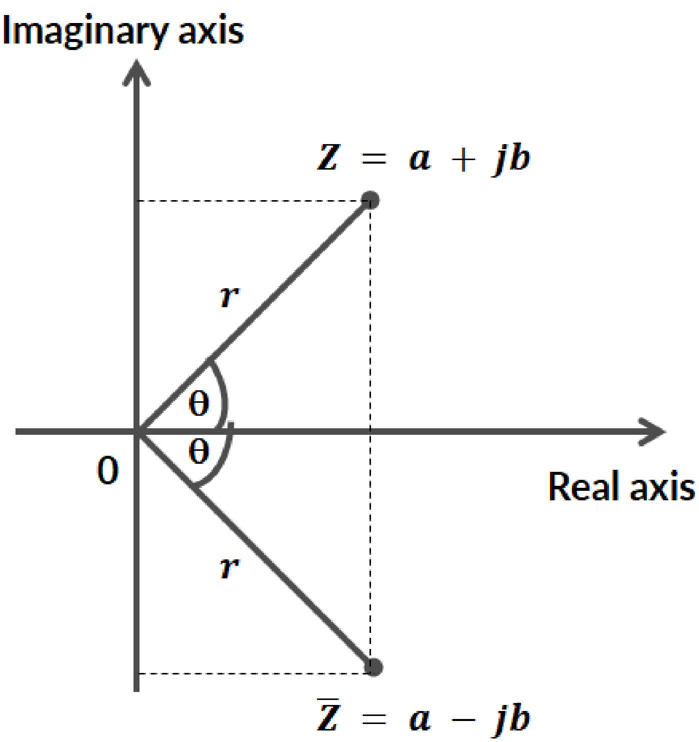
Cartesian Representation of Complex Numbers

- Let $Z = a + jb$ be a complex number
- $\boldsymbol{a}$ is known as real part and $\boldsymbol{b}$ as imaginary part
- $\boldsymbol{j}$ is defined as $j = \sqrt{-1}$
- Complex conjugate of Z is $\bar{Z} = a - jb$
- Modulus of complex number is defined as $|Z| = \sqrt{Z\bar{Z}} = \sqrt{a^2 + b^2}$

Polar Representation of Complex Number

- A complex number $Z = a + jb$ can be represented in a polar form
- In polar form, $Z = a + jb = re^{j\theta}$, where $\mathrm{r} = |Z| = \sqrt{a^2 + b^2}$
- $\theta = \tan^{-1}\left(\frac{b}{a}\right)$
- $\bar{Z} = a - jb = re^{-j\theta}$
- $|Z| = \sqrt{Z\bar{Z}} = \sqrt{re^{j\theta}re^{-j\theta}} = \sqrt{r^2} = r$



Simplifying Complex Numbers: Cartesian Representation

- Let $Z_1 = a + jb$ and $Z_2 = c + jd$
- $Z_1 + Z_2 = (a + c) + j(b + d)$
- $Z_1 - Z_2 = (a - c) + j(b - d)$
- $Z_1 \times Z_2 = (ac - bd) + j(bc + ad)$
- $\frac{Z_1}{Z_2} = \frac{(ac+bd)+ j(bc-ad)}{c^2+d^2}$
- Note: It is easy to add or subtract complex number in Cartesian representation

Simplifying Complex Numbers: Polar Representation

- Let $Z_1 = r_1 e^{j\theta_1}$ and $Z_2 = r_2 e^{j\theta_2}$
- $Z_1 \times Z_2 = r_1 r_2 e^{j(\theta_1+\theta_2)}$
- $\frac{Z_1}{Z_2} = \frac{r}{r_2} e^{j(\theta_1-\theta_2)}$
- Note: It is easy to multiply or divide complex number using Polar representation

Impedance

- Consider a resistance whose resistance is R. Then in complex form, the impedance is $Z_R = Re^{j0}$
- Consider a capacitor whose capacitance is C. Then in complex form, the impedance, $Z_C = \frac{1}{j\omega C} = -jX_C = X_C e^{-j\frac{\pi}{2}}, X_C = \frac{1}{\omega C}$
- Consider a inductor whose inductance is L. Then in complex form, the impedance, $Z_L = j\omega L = jX_L = X_L e^{j\frac{\pi}{2}}, X_L = \omega L$

Sin and Cos terms in Polar Representation

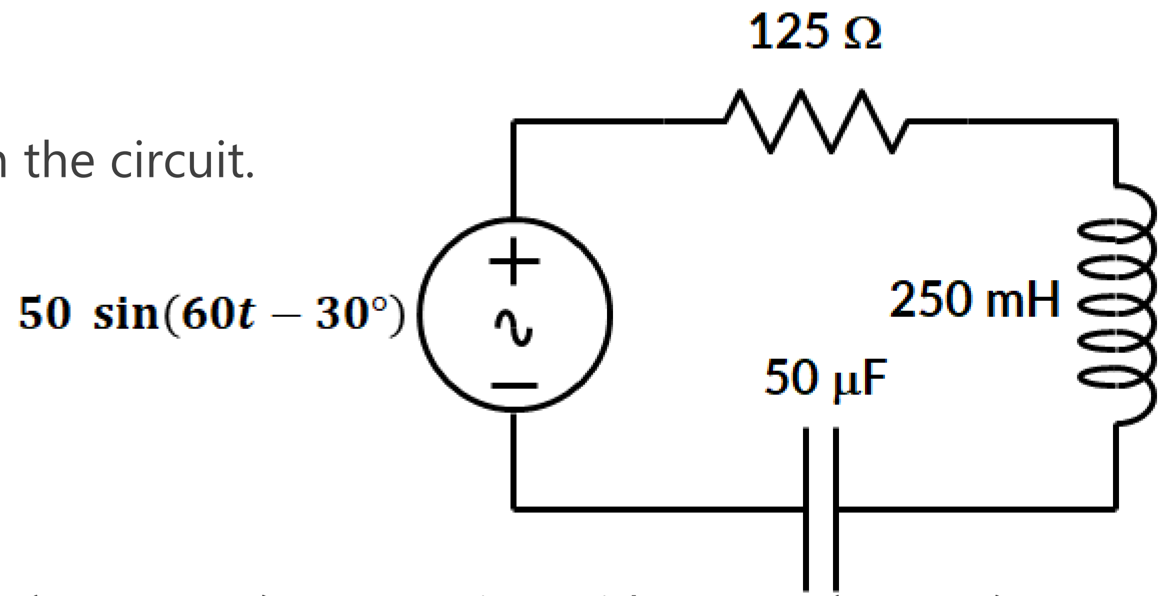
- $V = V_0 \sin(\omega t)$ can be written as $V = Im(V_0 e^{j\omega t})$. Here, ***Im*** indicates imaginary part of the complex number
- Similarly, $V = V_0 \cos(\omega t)$ can be written as $V = Re(V_0 e^{j\omega t})$. Here ***Re*** indicates real part of the complex number.

Shorthand Representation

- Let $Z = a + jb. Z = re^{j\theta}$
- In short-hand notation, Z is written as $\mathrm{Z} = r\angle\theta$

Example 1

- Find the current through the circuit.



- **Solution:**
- From the voltage, $\mathbf{50\ \sin(60}\boldsymbol{t}\mathbf{-30°)}$, comparing with $\boldsymbol{V_0}\ \mathbf{\sin(}\boldsymbol{\omega t-\theta}\mathbf{)}$
  - ❖ V_0 = 50 V, ω = 60 rad/sec, θ = 30^0.
- $X_L = \omega L = 15$ ⍰. $X_C = \frac{1}{\omega C} = 333.33\ \Omega$.
- $Z_L = jX_L = j15\ \Omega\ \ Z_C = -jX_c = -j333.33\ \Omega$. $\mathrm{Z_R} = 125\ \Omega$
- $Z = 125 - j318.33\ \Omega$, $\mathrm{Z} = 341.99\ \mathrm{e}^{-\mathrm{j}68.56^0}\Omega = 314.99\angle - 68.56^0$
- $\mathrm{I} = \frac{V}{Z} = \frac{50\angle{-30^0}}{314.99\ \angle{-68.56^0}} = 0.146\ \angle 38.56^0 A$