Department of Mathematics, Bennett University
Engineering Calculus (EMAT101L)
Tutorial Sheet 8

---

1. Discuss the convergence/divergence of improper integrals of first kind

$$(a) \int_0^\infty e^{-x}\cos x\ dx, \quad (b) \int_1^\infty \frac{dx}{x^2(1+e^x)}, \quad (c) \int_1^\infty \frac{x+1}{\sqrt{x^3}}\ dx, \quad (d) \int_0^\infty \frac{dx}{x^2+\sqrt{x}}.$$

2. Discuss the convergence/divergence of improper integrals of second kind

$$(a) \int_1^2 \frac{\sqrt{x}}{\ln x}\ dx, \quad (b) \int_0^1 \frac{\sin(x^2)}{\sqrt{x}}\ dx, \quad (c) \int_1^{\pi/2} \frac{\tan x}{x^{3/2}}\ dx, \quad (d) \int_2^3 \frac{\log x}{\sqrt{|2-x|}}\ dx.$$

3. Discuss the convergence/divergence of improper integrals

$$(a) \int_0^\infty x^{-\frac{1}{2}} e^{x^2} dx, \quad (b) \int_0^\infty \frac{1+x}{1+x^3}\ dx.$$

4. Show the following:

$$(a) \int_0^\infty e^{-tx}\frac{\sin x}{x}\ dx = \frac{\pi}{2} - \arctan t, \quad (b) \int_0^1 \frac{x^t-1}{\ln x}\ dx = \ln(1+t).$$

5. Using Beta and Gamma functions, evaluate the following:

$$(a) \int_0^\infty e^{-x^2} dx, \quad (b) \int_0^{\pi/2} \sqrt{\tan x}\ dx, \quad (c) \int_0^1 x^m \left(\log\left(\frac{1}{x}\right)\right)^n dx, \quad (d) \int_0^{\pi/2} \sin^4\theta\ \cos^6\theta\ d\theta.$$