Tutorial - 7 Solution

① Asymmetric:-

$\forall a \forall b (aRb \rightarrow (b,a) \notin R)$

To prove:-

$(\forall a \forall b (aRb \rightarrow (b,a) \notin R)) \rightarrow (\forall a \forall b ((aRb \wedge bRa) \rightarrow a=b))$

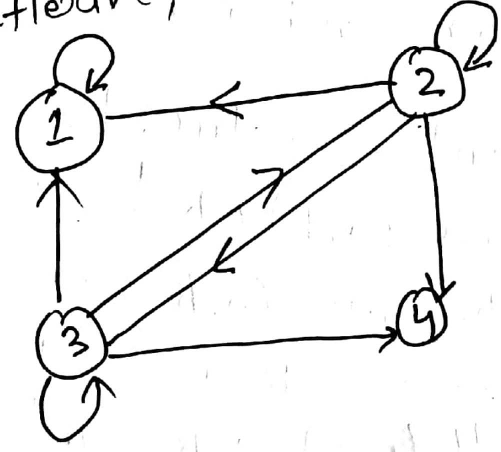
proof:-

1) Assume R is asymmetric
2) $\forall a \forall b ((a,b) \notin R \vee (b,a) \notin R)$ (step 1 and by defn)
3) $\forall a \forall b ((aRb \wedge bRa) \longrightarrow a=b)$ implication's premise is false.
4) Therefore, asymmetry implies antisymmetry.

② Let $x, y, z \in N$ such that xRy and yRz, so there are integers m, n such that $y = mx$ and $z = ny$. Thus there $z = (mn)x$, so x divides z and xRz. Thus the relation is transitive.

③ (a) false
(b) false.

④ (b) Neither reflexive, nor irreflexive but transitive.



(5) Let n be the no. of element in a set

maximum no. of relations $= 2^{n^2}$

(6) Let $x \in Z$. Then $x-x=0$ and 0 is divisible by 6

Therefore, xRx for all $x \in Z$

Hence, R is reflexive.

Again $xRy \Rightarrow (x-y)$ is divisible by 6

$\Rightarrow -(x-y)$ is divisible by 6

$\Rightarrow (y-x)$ is divisible by 6

$\Rightarrow yRx$.

Hence R is symmetric

xRy and $xRz \Rightarrow (x-y)$ is divisible by 6

and $(y-z)$ is divisible by 6.

$\Rightarrow [(x-y)+(y-z)]$ is divisible by 6

$\Rightarrow (x-z)$ is divisible by 6.

$\Rightarrow xRz$

R is transitive.

Thus R is an equivalence relation.

(7) Suppose R is antisymmetric. Let $(a,b) \in R \cap R^{-1}$.
Then $(a,b) \in R$ and $(a,b) \in R^{-1}$. Again $(a,b) \in R^{-1}$
implies $(b,a) \in R$. Thus $(a,b) \in R$ and also $(b,a) \in R$
Hence, $b=a$ because R is antisymmetric. This
is true for all $(a,b) \in R \cap R^{-1}$. Hence every elemen

of $R \cap R^{-1}$ is of the form (a,a) where $a \in A$, therefore $R \cap R^{-1} \subseteq I_A$.

Conversely, suppose $R \cap R^{-1} \subseteq I_A$. Let $(a,b) \in A \times A$ such that $(a,b) \in R$ and $(b,a) \in R$, i.e. $(a,b) \in R$ and $(a,b) \in R^{-1}$. Then $(a,b) \in R \cap R^{-1}$. Since $R \cap R^{-1} \subseteq I_A$, it follows that $b = a$. Hence R is antisymmetric.

8)

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

$R = \{(1,a), (1,c), (2,c), (3,a), (4,b)\}$

$R^{-1} = \{(a,1), (c,1), (c,2), (a,3) (b,4)\}$

(i) $R \circ R^{-1} = \{(1,1), (2,2), (1,3) (1,2), (3,3), (3,1), (2,1), (4,4)\}$

it is symmetric.

(ii) $R^{-1} \circ R = \{(a,a), (a,c), (c,a), (c,c), (b,b)\}$.

it is Symmetric.

(iii) Reflexive $= \{(a,a) (b,b) (c,c)\}$

$= \{(1,1) (2,2) (3,3) (4,4)\}$

transitive $= \{ \{(1,3) (1,2), (2,1) (1,3), (1,1)\}\}$

$\{(a,c) (c,c)\}$

$\therefore$ It is equivalence relations.

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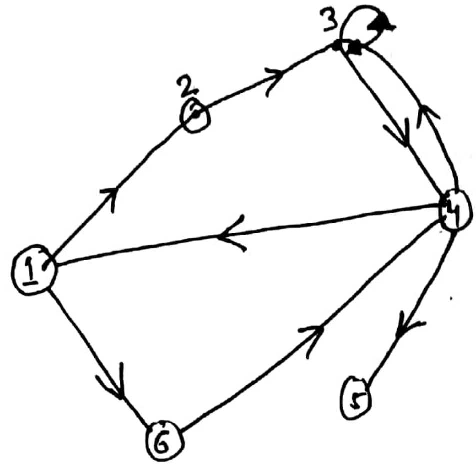
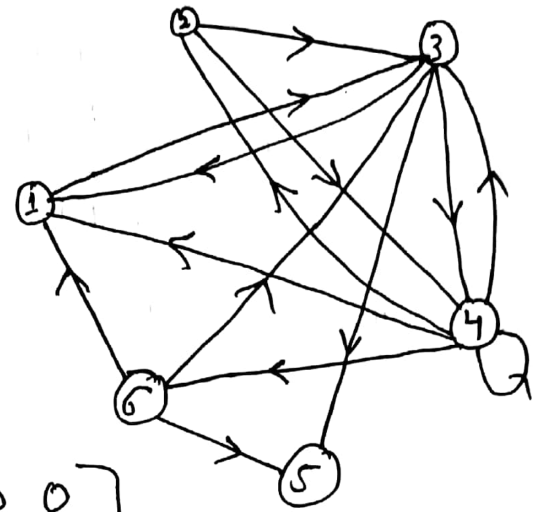
9)

R



R^2



$$M_{R^2} = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

10) $R^{(r)} = \{(a,a),(b,b),(c,c),(d,d),(a,b),(b,c),(d,c),(d,a),(a,d)\}$

$R^{(s)} = \{(a,b),(b,a),(b,c),(c,b),(d,c),(c,d),(d,a),(a,d),(d,d)\}$

$R^{*} = \{(a,a),(a,b),(a,c),(a,d),(b,c),(d,a),(d,b),(d,c),(d,d)\}$

11) $\underline{\underline{2^{n^2-n}}}$