# Vector potential

Magnetic dipole

$\vec{m} = I \cdot \vec{a}$


$\vec{a} = a \cdot \hat{z}$

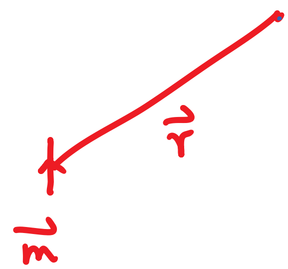
$\nabla \cdot \vec{B} = 0$

$\vec{B} = \nabla \times \vec{A}$

$\vec{A}$ : Vector potential

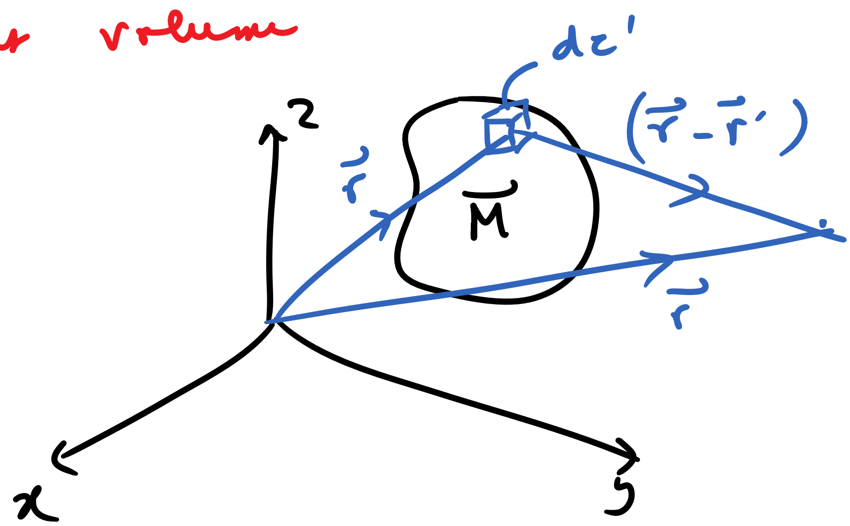
$\nabla \cdot \vec{A} = 0$

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{r}}{r^3}}$$




MAGNETIZATION

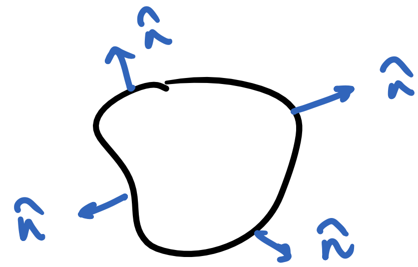
$\vec{M}$ = Magnetic dipole moment per unit volume




$$d\vec{A} = \frac{\mu_0}{4\pi} \frac{(\vec{M}\, d\tau') \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

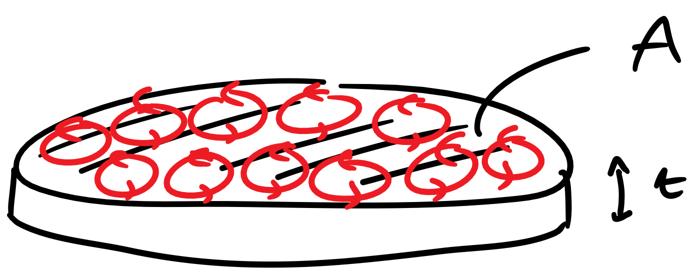
$$\boxed{\vec{A} = \frac{\mu_0}{4\pi} \int_{\text{Volume}} \frac{\vec{M} \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \, d\tau'}$$

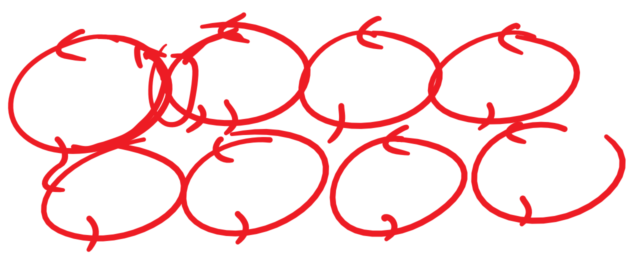
$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\nabla' \times \vec{M}}{|\vec{r} - \vec{r}\,'|} \, d\tau' + \frac{\mu_0}{4\pi} \int \frac{\vec{M} \times \hat{n}}{|\vec{r} - \vec{r}\,'|} \, da'$$

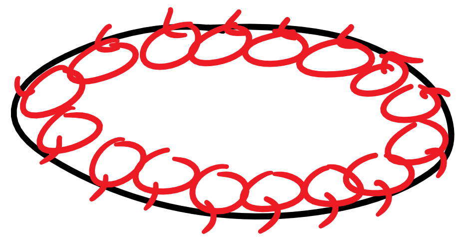



$\vec{J}_b = \nabla \times \vec{M}$ Bound Volume Current density

$\vec{K}_b = \vec{M} \times \hat{n}$ Bound Surface current density





$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

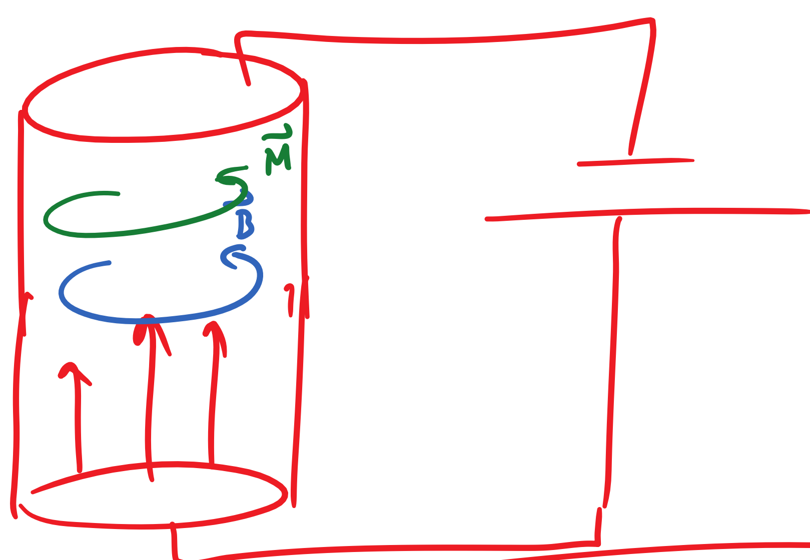
$$\nabla \times \left(\frac{\vec{B}}{\mu_0}\right) = \vec{J} = \vec{J}_f + \vec{J}_b = \vec{J}_f + \nabla \times \vec{M}$$

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right) = \vec{J}_f$$

$$\boxed{\vec{H} \equiv \frac{\vec{B}}{\mu_0} - \vec{M}}$$

$$\boxed{\nabla \times \vec{H} = \vec{J}_f}$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$




$$\boxed{\nabla \times \vec{H} = \vec{J}_f}$$

$$\boxed{\oint \vec{H} \cdot d\vec{l} = I_{fenc}}$$

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

$$\vec{M} = \chi_m \vec{H}$$

$\chi_m$: Magnetic Susceptibility

$$\vec{B} = \mu_0(1+\chi_m)\vec{H}$$

$$\mu = \mu_0(1+\chi_m)$$

$\mu$: Permeability of the medium

$$\boxed{\vec{B} = \mu\vec{H}}$$

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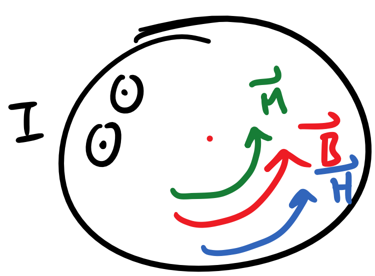
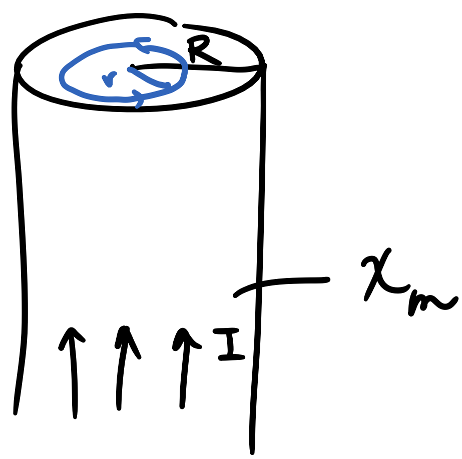
$$\oint \vec{H}\cdot d\vec{l} = I_{free}$$




$$2\pi r \cdot H = \frac{I}{\cancel{\pi} R^2}\cdot \cancel{\pi} r^2$$

$$\boxed{\vec{H} = \frac{I}{2\pi R^2} r\,\hat{\varphi}} \qquad r < R$$

$$\boxed{\vec{H} = \frac{I}{2\pi r}\,\hat{\varphi}} \qquad r > R$$

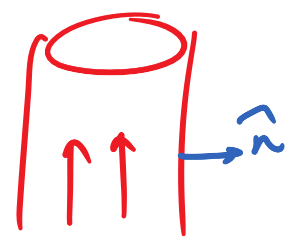
$$\vec{B} = \mu \vec{H} = \frac{\mu I}{2\pi R^2} r \hat{\varphi} \qquad r < R$$

$$= \mu_0 \vec{H} = \frac{\mu_0 I}{2\pi r} \hat{\varphi} \qquad r > R$$

$$\vec{M} = \chi_m \vec{H} = \frac{\chi_m I}{2\pi R^2} r \hat{\varphi} \qquad r < R$$

$$= 0 \qquad r > R$$

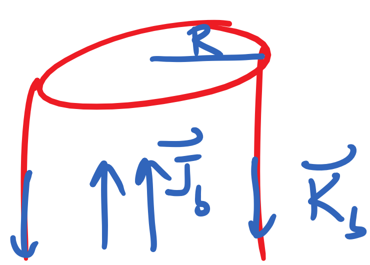
$$\vec{K}_b = \vec{M} \times \hat{n}\Big|_{r=R} = \frac{\chi_m I}{2\pi R^2} R(\hat{\varphi} \times \hat{r})$$




$(\hat{r}, \hat{\varphi}, \hat{z})$

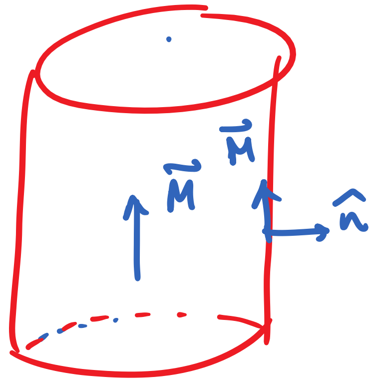
$$\boxed{\vec{K}_b = -\frac{\chi_m I}{2\pi R} \hat{z}}$$

$$\boxed{\vec{J}_b = \nabla \times \vec{M} = \frac{\chi_m I}{\pi R^2} \hat{z}}$$




$$2\pi R K_b = -\chi_m I$$

$$\pi R^2 J_b = \chi_m I$$




Uniformly magnetized

$\vec{J}_b = 0$

$\vec{K}_b =$