Department of Mathematics, Bennett University
Engineering Calculus (EMAT101L)
Tutorial Sheet 7

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1. Determine which of the following functions defined on $[0,1]$ are integrable

   $(a)\ f(x) = \begin{cases} 1, & x < 1 \\ 2, & x = 1 \end{cases}$ $\quad (b)\ f(x) = \begin{cases} 1+x, & x \in \mathbb{Q} \\ 1-x, & x \notin \mathbb{Q} \end{cases}$

   $(c)\ f(x) = \begin{cases} \sin x, & x = \frac{1}{n}, n \in \mathbb{N} \\ \cos x, & \text{otherwise} \end{cases}$

   $(d)\ f(x) = \begin{cases} x[x], & 0 \le x \le 5 \\ 0, & x = 0 \end{cases}$ where $[x]$ is integral value of x.

2. (a) Show that the function f defined by

   $$f(x) = \begin{cases} \frac{1}{2^n}, & \frac{1}{2^{n+1}} < x \le \frac{1}{2^n}, n = 0, 1, 2, \cdots \\ 0, & x = 0, \end{cases}$$

   is integrable on $[0,1]$.

   (b) Suppose f and g are two bounded functions on $[a,b]$ such that $f(x) = g(x)$ except a finite number of points x in [a,b]. If g is integrable on $[a,b]$ then prove that f is integrable on $[a,b]$ and $\int_a^b f = \int_a^b g$.

   (c) Suppose f and g are continuous functions on $[a,b]$ and $\int_a^b f = \int_a^b g$. Then show that there exists $x \in [a,b]$ such that $f(x) = g(x)$.

3. Prove that

   (a) $\lim\limits_{n\to\infty} \left(\dfrac{1}{n+1} + \dfrac{1}{n+2} + \ldots + \dfrac{1}{2n} \right) = \log_e 2.$

   (b) $\lim\limits_{n\to\infty} \dfrac{1}{n} \left[\sin\dfrac{\pi}{n} + \sin\dfrac{2\pi}{n} + \ldots + \sin\dfrac{n\pi}{n} \right] = \dfrac{2}{\pi}.$

4. Using the fundamental theorem, evaluate $\displaystyle\int_0^1 \left(2x \sin\frac{1}{x} - \cos\frac{1}{x} \right) dx.$

5. Find the error in the $\int_a^b f'(x)dx = f(b) - f(a)$ for the following functions:

   (a) $f(x) = x^2 \cos\left(\frac{\pi}{x^2}\right)$ if $0 < x \le 1$, $f(0) = 0$. (b) $f(x) = -\frac{1}{x-1}$ in $[0,2]$.

   (c) $f(x) = 2\sqrt{x}$ in $[0,1]$.