Department of Mathematics, Bennett University
EMAT102L, Tutorial Sheet 8
Ordinary Differential Equations

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1. Find all real valued C^1 solutions $y(x)$ of the differential equation $xy'(x) + y(x) = x,\ x \in (-1, 1)$.

2. Under what conditions, the following differential equations are exact?
   (a) $[h(x) + g(y)]dx + [f(x) + k(y)]dy = 0$ (b) $(x^3 + xy^2)dx + (ax^2y + bxy^2)dy = 0$
   (c) $\left(\frac{1}{x^2} + \frac{1}{y^2}\right) dx + \left(\frac{cx+1}{y^3}\right) dy = 0$

3. Examine the following differential equations for exactness. Solve them by finding appropriate integrating factors if necessary:
   (a) $(\sin x \tan y + 1)dx - \cos x \sec^2 y dy = 0$. (b) $e^x dx + (e^x \cot y + 2y \csc y)dy = 0$.
   (c) $(3xy + y^2)dx + (x^2 + xy)dy = 0$. (d) $ydx + (2x - ye^y)dy = 0$.

4. Suppose $M(x, y)dx + N(x, y)dy = 0$ has an integrating factor $\mu(x, y)$ such that $df = \mu M dx + \mu N dy$ is an exact differential. Show that the equation has an infinite number of integrating factors by demonstrating that the product $\mu G(f)$, where G is an arbitrary continuous function from $\mathbb{R}$ to $\mathbb{R}$, is also an integrating factor.

5. Solve the following linear/reducible to linear ODEs:
   (a) $(x + 2y^3)\frac{dy}{dx} = y$ (b) $(1 + y^2) + (x - e^{-\tan^{-1} y})\frac{dy}{dx} = 0$
   (c) $x\frac{dy}{dx} + y = x^2y^2$ (d) $y^{1/2}\frac{dy}{dx} + y^{3/2} = 1, y(0) = 4$.

6. (a) Find the orthogonal trajectories to the family of curves $x^2 + y^2 = cx$.
   (b) Find the value of n such that the curves $x^n + y^n = c$ are orthogonal trajectories of the family $y = \frac{x}{1 - c_1x}$.
   (c) Show that the family of parabolas $y^2 = 2cx + c^2$ is self-orthogonal.

7. Suppose $P(x)$ is continuous on some closed interval I and a is a number in I. What can be said about the existence and uniqueness of the solution of initial value problem $y' + P(x)y = 0; y(a) = 0$(Without Soving)?

8. Verify whether the following functions satisfy Lipschitz condition or not on the given sets R.
   (i) $f(x, y) = x^3 \sin y$ on $R : |x| \leq 2, -\infty < y < \infty$.
   (ii) $f(x, y) = y^{1/3}$ on $R : |x| \leq 1, |y| \leq 1$.
   (iii) $f(x, y) = x^2 + y, |x| \leq 1, |y| < \infty$.

9. Discuss the existence and uniqueness of solution for the following initial value problems (IVP) in the region $R : |x| \leq 1, |y| \leq 1$.

   (a) $\dfrac{dy}{dx} = 3y^{2/3},\ y(0) = 0.$ (b) $\dfrac{dy}{dx} = x^2 + y^2,\ y(0) = 0.$

   (c) $\dfrac{dy}{dx} = \sin x \cos y + xy^2,\ y(0) = 0.$

10. For the following initial value problems, compute the first three iterates using Picard's iteration method.

    (i) $y' = x^2 + y^2 - 1,\ y(0) = 1.$ (ii) $y' = 1 + 2y^2,\ y(0) = 0.$