$$f\left(\frac{x \mid \mu, \sigma^2}{\theta}\right) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$L(\theta \mid X) = f(x_1, x_2 \ldots x_n \mid \theta)$$

$$= \prod_{i=1}^{n} f(x_i \mid \theta)$$

$$= \prod_{i=1}^{n} \frac{1}{2\pi\sigma^2} e^{-\left(\frac{x_i-\mu}{2\sigma}\right)^2}$$

$$= \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} \times \prod_{i=1}^{n} e^{-\left(\frac{x_i-\mu}{2\sigma}\right)^2}$$

$$\operatorname{Log}(l(\theta \mid x)) = \frac{-n}{2} \log(2\pi\sigma^2) + \log e\left(-\left(\frac{(x_1-\mu)^2 + (x_2-\mu)^2 + \cdots (x_n-\mu)^2}{2\sigma^2}\right)\right)$$

$$= \frac{-n}{2} \log(2\pi\sigma^2) - \frac{\sum_{i=1}^{n} (x_i-\mu)^2}{2\sigma^2}$$

$$\frac{\partial \log(L(\theta|x))}{\partial \mu} = 0$$

$$0 - \left(\frac{\Sigma x_i - n\mu}{\sigma^2}\right) = 0$$

$$\Rightarrow \mu = \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\frac{\partial \log(L(\theta|x))}{\partial \sigma} = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu)^2$$

For Regression

$$y_i \sim N(\cancel{\theta^T x}\ X^T\theta, \sigma^2).$$

$y_i$'s are iid

$$L(\theta; X|y) = P(y_1 \dots y_n | X_1 \dots X_n ; \theta)$$

$$= \prod_{i=1}^{n} p(y_i|x_i;\theta)$$

$$= \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{\left(-\frac{\Sigma(y_i - x_i\theta^T)^2}{2\sigma^2}\right)}$$

$$= (2\pi\sigma^2)^{-n/2} e^{-\frac{1}{2\sigma^2}(Y-X\theta)^T(Y-X\theta)}.$$

$$\log(l(\theta : X|Y) = \frac{-n}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}(Y-X\theta)^T(Y-X\theta).$$

$$\frac{\partial \log(L(\cdot))}{\partial \theta} = 0$$

$$0 - \frac{1}{2\sigma^2}\left[0 - 2X^TY + X^TX\theta\right]$$

$$\Rightarrow \theta = (X^TX)^{-1}X^TY.$$

Note

$$\sum_{i=1}^{n} a_i^2 = A^TA.$$