Department of Mathematics, Bennett University
Engineering Calculus (EMAT101L)
Tutorial Sheet 5

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1. Determine if the following functions are differentiable at 0. Find $f'(0)$ if exists

$(a)\ f(x) = \begin{cases} x, & x \in \mathbb{Q} \\ \sin x, & x \notin \mathbb{Q}. \end{cases}$ $(b)\ f(x) = \begin{cases} \sqrt{x}\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$

$(c)\ f(x) = \begin{cases} x^2\cos\frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$ $(d)\ f(x) = \begin{cases} e^{-\frac{1}{x^2}}, & x \neq 0 \\ 0, & x = 0. \end{cases}$

$(e)\ f(x) = \begin{cases} x\cos\frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$ $(f)\ f(x) = e^{-|x|}.$

2. Determine if f' is continuous at 0 for the following functions:

$(a)\ f(x) = \begin{cases} x^3\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$ $(b)\ f(x) = \begin{cases} x^2\cos\frac{1}{x}, & x \neq 0 \\ 0, & x = 0. \end{cases}$

$(c)\ f(x) = \begin{cases} x^2\ln\frac{1}{|x|}, & x \neq 0 \\ 0, & x = 0. \end{cases}$

3. When a circular plate of metal is heated in an oven, its radius increases at the rate of 0.01cm/min. At what rate the plate's area increasing when the radius is 50 cm?

4. Let f be a continuous on $[a, b]$ and differentiable at every point in (a, b). Suppose there exists $c \in \mathbb{R}$ such that $f'(x) = c$ for $x \in (a, b)$. Then there exists $k \in \mathbb{R}$ such that $f(x) = cx + k$ for all $x \in [a, b]$.

5. Prove that if f, g are differentiable on $\mathbb{R}$, $f'(x) \leq g'(x)$ on $\mathbb{R}$ and $f(0) = g(0)$, then $f(x) \leq g(x)$ for $x \geq 0$.

6. Let $f : [0, 1] \to \mathbb{R}$ be differentiable, $f(\frac{1}{2}) = \frac{1}{2}$ and $0 < \alpha < 1$. Suppose $|f'(x)| \leq \alpha$ for all $x \in [0, 1]$. Show that $|f(x)| < 1$ for all $x \in [0, 1]$.

7. Evaluate the following limits:

$(a)\ \lim_{x\to 0} \dfrac{e^x - (1+x)}{x^2}, \quad (b)\ \lim_{t\to 0} \dfrac{1 - \cos t - (t^2/2)}{t^4}, \quad (c)\ \lim_{x\to\infty} x^2(e^{-1/x^2} - 1).$