Name of student: ..........................................................

Batch No:..........     Enrollment No. ....................

COURSE NAME: LINEAR ALGEBRA AND ORDINARY DIFFERENTIAL EQUATIONS

B.TECH     TUTORIAL QUIZ-2     SPRING SEMESTER 2018-19

COURSE CODE : EMAT102L     MAX. TIME: 20 min

COURSE CREDIT: 3-1-0     MAX. MARKS: 10

1. Check whether $W = \{(a_1, a_2, a_3) \in \mathbb{R}^3 : a_1^2 + a_2^2 + a_3^2 \leq 1\}$ is a subspace of $\mathbb{R}^3$ or not? [3]

Not a subspace as for $\alpha = 4 \in \mathbb{R}$ and $x = (\frac{1}{2}, 0, 0) \in W$,

$\alpha x = 4(\frac{1}{2}, 0, 0) = (2, 0, 0) \notin W$     $[\because 2^2 + 0 + 0 = 4 > 1]$

$\Rightarrow \alpha x \notin W$ for $\alpha = 4 \in \mathbb{R}$, $x = (\frac{1}{2}, 0, 0) \in W$

$\Rightarrow W$ is not a subspace of $\mathbb{R}^3$.

2. Find the basis and dimension of subspace $W = \{(x, y, z, w) \in \mathbb{R}^4 : w - z = y - x\}$ of $\mathbb{R}^4$ and extend it to form the basis of $\mathbb{R}^4$. [5]

Here $W = \{(x, y, z, w) \in \mathbb{R}^4 : w - z = y - x\}$

$= \{(x, y, z, y - x + z) \in \mathbb{R}^4 : x, y, z \in \mathbb{R}\}$

$= \{x(1, 0, 0, -1) + y(0, 1, 0, 1) + z(0, 0, 1, 1) : x, y, z \in \mathbb{R}\}$

$\Rightarrow W = \text{span}\{(1, 0, 0, -1), (0, 1, 0, 1), (0, 0, 1, 1)\}$.

Also, $\{(1,0,0,-1), (0,1,0,1), (0,0,1,1)\}$ is linearly independent.

$\Rightarrow \{(1,0,0,-1), (0,1,0,1), (0,0,1,1)\}$ is a basis of $W$.

To extend the basis $\{(1,0,0,-1), (0,1,0,1), (0,0,1,1)\}$ of $W$ to form the basis of $\mathbb{R}^4$, we need one more vector as $\dim \mathbb{R}^4 = 4$.

Let us take that vector as $(0, 0, 0, 1)$, then

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ is in the row echelon form.

$\Rightarrow$ basis of $\mathbb{R}^4 = \{(1,0,0,-1), (0,1,0,1), (0,0,1,1), (0,0,0,1)\}$.

Ans.

3. Is $\mathbb{R}^2$ with vector addition and scalar multiplication defined as [2]

$$(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2), \quad \lambda(x_1, x_2) = (\lambda x_1, 0)$$

a vector space?

Not a vector space as $1 \cdot (x_1, x_2) \neq (x_1, x_2)$.

Here $1 \cdot (x_1, x_2) = (x_1, 0) \neq (x_1, x_2)$

for $(x_1, x_2) \in \mathbb{R}^2$.

or If we take $x = (x_1, x_2) = (1, 2) \in \mathbb{R}^2$, then

$1 \cdot x = 1 \cdot (1, 2) = (1, 0) \neq (1, 2) = x$

$\Rightarrow$ $1 \cdot x \neq x$ for $x = (1, 2) \in \mathbb{R}^2$.

$\Rightarrow$ Not a vector space.