MAGNETOSTATICS

Magnetic field $\vec{B}$

Oersted in 1820

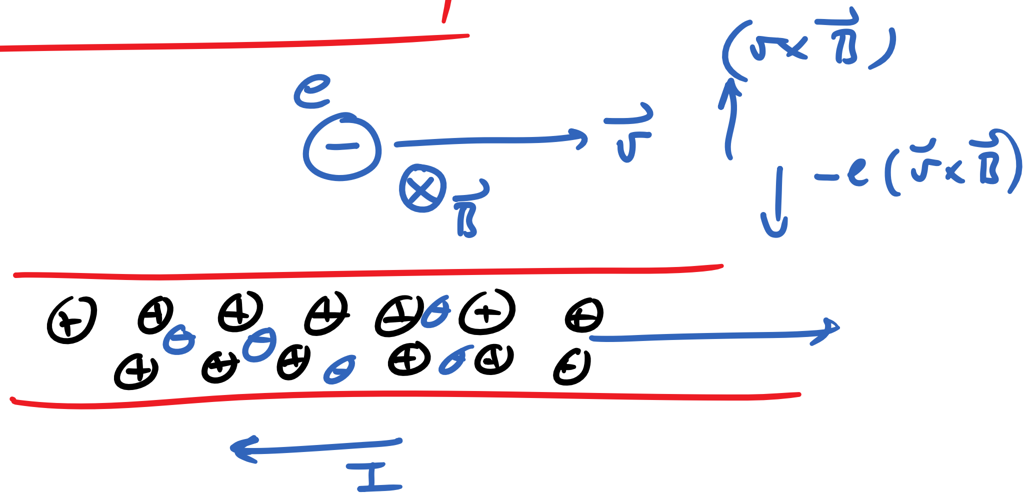
Ampere

Michael Faraday 1831

LORENTZ FORCE

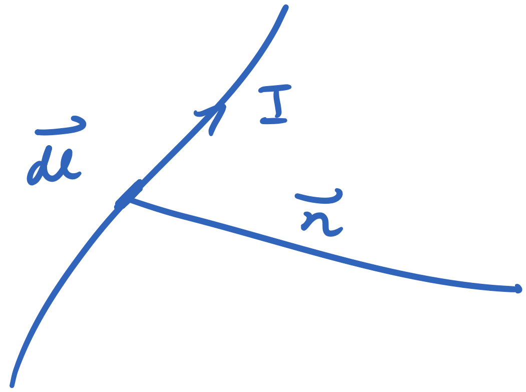
$$\vec{F}_B = q(\vec{v} \times \vec{B})$$

$$\boxed{\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B})}$$



BIOT-SAVART LAW

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} I \frac{\vec{dl}\times\vec{r}}{r^3}}$$



μ_0: Permeability of free space

$= 4\pi\times10^{-7}$ SI units

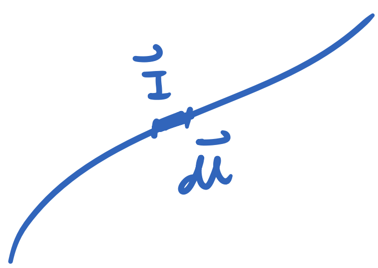
TESLA $\qquad 1\,T = 1\,NA/m \qquad \dfrac{F}{q\cdot v}$

$= 10^4$ GAUSS

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\int I \frac{\vec{dl}\times\vec{r}}{r^3}$$

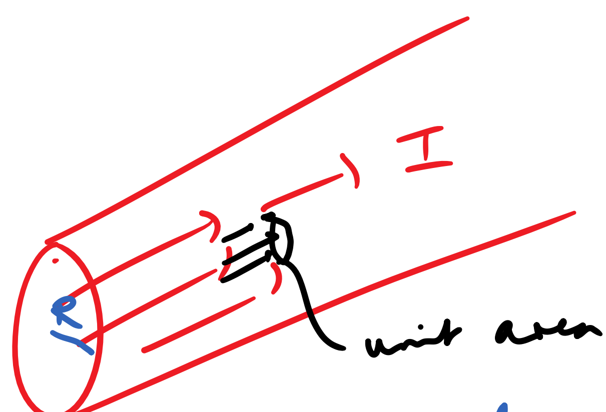
I: Line Current

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{I}\times\vec{r}}{r^3}\, dl$$



VOLUME current

Surface currents

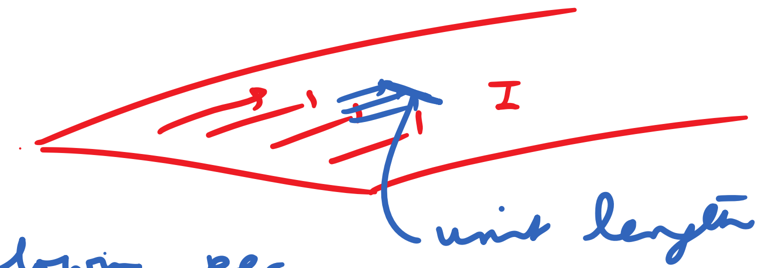


$\vec{J}$: Volume current density

Current flowing per unit area $\perp$ to the flow

$$J = \frac{I}{\pi R^2} \quad A/m^2$$

Surface current density



$\vec{K}$ = Current flowing per unit length $\perp$ to the flow

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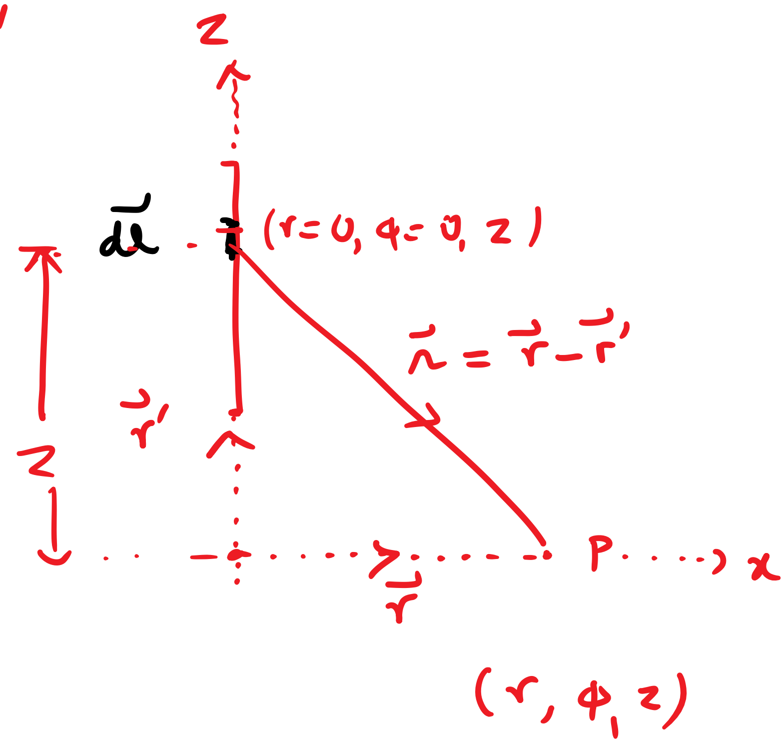
BIOT SAVART LAW

Example



$$d\vec{B} = \frac{\mu_0}{4\pi} I \frac{d\vec{l} \times \vec{\mathscr{r}}}{\mathscr{r}^3}$$

$$d\vec{l} = dz\,\hat{z}$$

$$\vec{r} = r\,\hat{r}$$

$$\vec{r}' = z\,\hat{z}$$

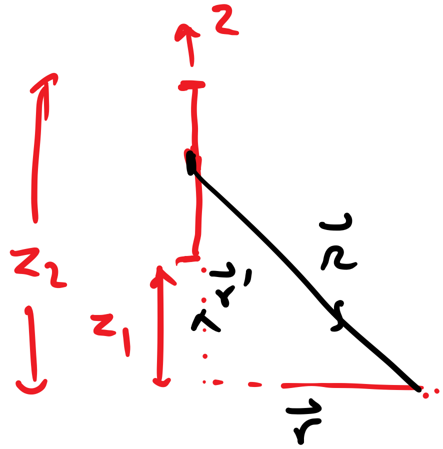
$$d\vec{l} \times \vec{\mathscr{r}} = dz\,\hat{z} \times (r\hat{r} - z\hat{z})$$

$$= r\,dz\,\hat{z} \times \hat{r} - z\,dz\,\hat{z} \times \hat{z}$$

$$= r\,dz\,\hat{\phi} - 0$$

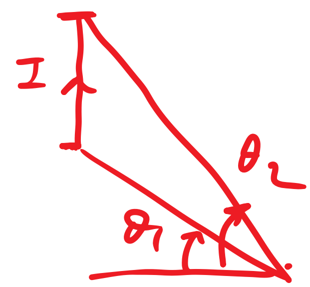
$$\mathscr{r}^2 = |\vec{r} - \vec{r}'|^2 = (r^2 + z^2)$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{r\,dz\,\hat{\varphi}}{(r^2+z^2)^{3/2}}$$



$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_{z_1}^{z_2} \frac{r}{(r^2+z^2)^{3/2}}\,dz\,\hat{\varphi}$$

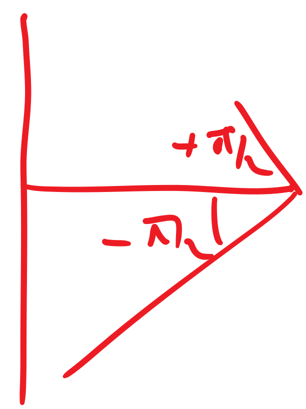
$$= \frac{\mu_0 I r\,\hat{\varphi}}{4\pi} \int_{z_1}^{z_2} \frac{dz}{(r^2+z^2)^{3/2}}$$

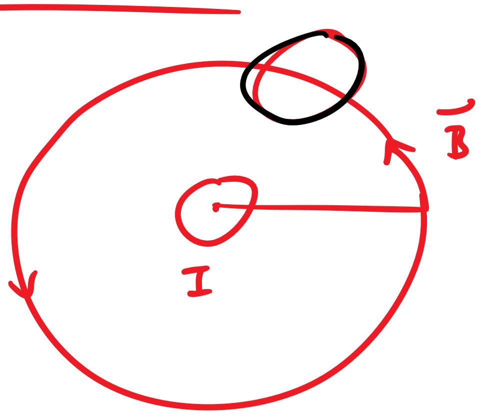


$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi r}\left(\sin\theta_2 - \sin\theta_1\right)\hat{\varphi}}$$

Infinitely long wire

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\varphi}}$$



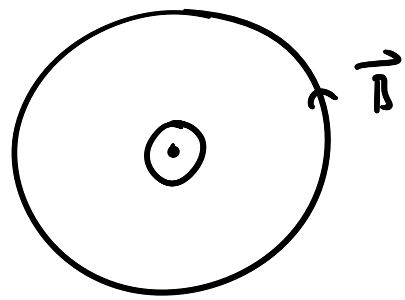


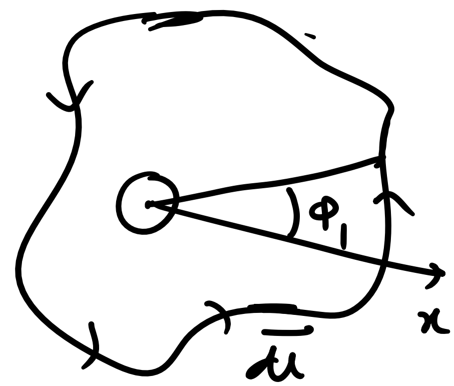
$$\boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

$$\nabla\times\vec{B} = \left(\cancel{\frac{1}{r}\frac{\partial B_z}{\partial\varphi}} - \cancel{\frac{\partial B_\varphi}{\partial z}}\right)\hat{r} + \left(\cancel{\frac{\partial B_r}{\partial z}} - \cancel{\frac{\partial B_z}{\partial r}}\right)\hat{\varphi} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rB_\varphi) - \cancel{\frac{\partial B_r}{\partial\varphi}}\right)\hat{\ }$$

$$\nabla \times \vec{B} = 0$$

$$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J}}$$





$$\oint \vec{B} \cdot \vec{dl} = \oint \frac{\mu_0 I}{2\pi r} (\hat{\varphi} \cdot \vec{dl})$$

$$\vec{dl} = dr\,\hat{r} + r\,d\varphi\,\hat{\varphi} + dz\,\hat{z}$$

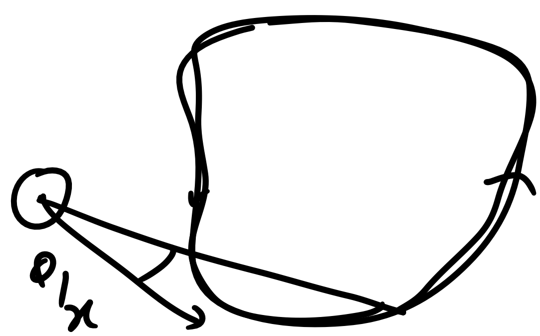
$$\boxed{\varphi_1 \text{ to } \varphi_1 + 2\pi}$$

$$\hat{\varphi} \cdot \vec{dl} = r\,d\varphi$$

$$\oint \vec{B} \cdot \vec{dl} = \frac{\mu_0 I}{2\pi} \oint d\varphi$$

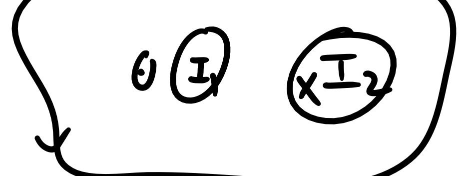
$$= \mu_0 I$$

$$\oint \vec{B} \cdot \vec{dl} = 0$$



φ_1 to φ_1

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enc}}$$



$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 (I_1 - I_2)}$$