Department of Mathematics, Bennett University
Quiz-1 Examination, Fall Semester 2018-19

Name of student: .... Enrolment No: ....

Course Code :	EMAT101L	Max. Time	: 50 Minutes
Course Name :	Engineering Calculus	Max. Marks	: 10

1) Examine whether $\lim_{x \to 0} \sin(\frac{1}{x})$ exists. [2]

2) If $0 < a < 1$, then show that $\lim_{n \to \infty} a^n = 0$. [2]

3) Examine whether the series $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n(n-1)}}$ is convergent or divergent. [2]

4) Find the values of $x \in \mathbb{R}$ for which the series $\sum_{n=1}^{\infty} \frac{x^n}{n}$ converges or diverges. [2]

5) Check the convergence of the following sequences: [1 + 1 = 2]

(a) $\left\{\frac{n^3}{2^n}\right\}$ (b) $\left\{\frac{3^n}{n^4}\right\}$

Space for Answers

Solution 1 : Consider the sequences $\{x_n\} = \left\{\frac{1}{n\pi}\right\}$, $\{y_n\} = \left\{\frac{1}{2n\pi + \frac{\pi}{2}}\right\}$.

Then $x_n \to 0$, $y_n \to 0$ as $n \to \infty$, and

$$\sin\left(\frac{1}{x_n}\right) = \sin(n\pi) \to 0 \text{ as } n \to \infty$$

$$\sin\left(\frac{1}{y_n}\right) = \sin\left(2n\pi + \frac{\pi}{2}\right) \to 1 \text{ as } n \to \infty.$$

Therefore, $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ doesnot exist.

Solution 2 : Since $0 < a < 1$, we can write $a = \frac{1}{1+b}$ for some $b > 0$.

Also, we have $(1+b)^n \geq 1 + nb$.

$$\therefore \quad 0 < a^n = \frac{1}{(1+b)^n} \leq \frac{1}{1+nb} < \frac{1}{nb}.$$

So, by Sandwich Theorem, we conclude that $\lim\limits_{n\to\infty} a^n = 0$.

Solution 3: Since $\dfrac{1}{\sqrt{n(n-1)}} > \dfrac{1}{n} > 0 \quad \forall\, n \geq 2$

and $\sum\limits_{n=1}^{\infty} \frac{1}{n}$ is divergent.

So, by comparison test, $\sum\limits_{n=2}^{\infty} \frac{1}{\sqrt{n(n-1)}}$ is divergent.

Solution 4: Let $a_n = \dfrac{x^n}{n}$, Then

$$\lim_{n\to\infty} |a_n|^{\frac{1}{n}} = \lim_{n\to\infty} \left|\frac{x^n}{n}\right|^{\frac{1}{n}} = \lim_{n\to\infty} \frac{|x|}{n^{1/n}} = |x|.$$

Thus, the series converges for $|x|<1$ and diverges for $|x|>1$.

Solution 5 (a):

Let $a_n = \dfrac{n^3}{2^n}$. Then $\lim\limits_{n\to\infty} a_n^{1/n} = \lim\limits_{n\to\infty} \left(\dfrac{n^3}{2^n}\right)^{1/n}$

$$= \lim_{n\to\infty} \frac{n^{3/n}}{2} = \frac{1}{2} < 1.$$

$\therefore \left\{\dfrac{n^3}{2^n}\right\}$ converges to 0.

(b) Let $a_n = \dfrac{3^n}{n^4}$,

Then, $\lim\limits_{n\to\infty} a_n^{1/n} = \lim\limits_{n\to\infty} \left(\dfrac{3^n}{n^4}\right)^{1/n} = \lim\limits_{n\to\infty} \dfrac{3}{n^{4/n}} = 3 > 1.$

$\therefore \left\{\dfrac{3^n}{n^4}\right\}$ is divergent.