Department of Mathematics, Bennett University
EMAT102L, Tutorial Sheet 9
Ordinary Differential Equations

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1. Determine the largest interval in which the given initial value problem is certain to have a unique solution.
   (a) $x^2y'' + 4y = x,\ y(1) = 1,\ y'(1) = 2.$
   (b) $(x-3)y'' - 3xy' + 4y = \sin x,\ y(-2) = 2,\ y'(-2) = 1.$

2. Consider the differential equation $x^2y'' - 4xy' + 6y = 0$.
   (a) Verify that the functions $y_1(x) = x^3$ and $y_2(x) = x^2|x|$ are linearly independent solutions of the given differential equation on $(-\infty, \infty)$.
   (b) Show that y_1 and y_2 are linearly dependent on $(-\infty, 0)$ and on $(0, \infty)$.
   (c) Although y_1 and y_2 are linearly independent, show that $W(y_1, y_2) = 0$ for all $x \in (-\infty, \infty)$. Does this violate the fact that $W(y_1, y_2) = 0$ for every $x \in (-\infty, \infty)$ implies y_1 and y_2 are linearly dependent?

3. Verify that $y_1(x) = e^x$ and $y_2(x) = xe^x$ are solutions of $y'' - 2y' + y = 0$ for $x \in \mathbb{R}$. Do they constitute a fundamental set of solutions?

4. If y_1 and y_2 are linearly independent solutions of $xy'' + 2y' + xe^xy = 0, x \in (0, \infty)$ and if $W(y_1, y_2)(1) = 2$, find the value of $W(y_1, y_2)(5)$.

5. Find the second linearly independent solution of the following problems using the method of reduction of order. Hence find the general solution.
   (a) $(2x+1)y'' - 4(x+1)y' + 4y = 0,\ \ y_1 = e^{2x}$
   (b) $9y'' - 12y' + 4y = 0,\ y_1 = e^{\frac{2x}{3}}$
   (c) $x^2y'' - xy' + 2y = 0,\ y_1 = xsin(lnx).$
   (d) $(1-x^2)y'' + 2xy' = 0,\ y_1 = 1.$
   (e) $(x-1)y'' - xy' + y = 0,\ y_1 = e^x.$

6. Find the general solution of the following differential equations.
   (a) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + y = 0.$ (b) $\dfrac{d^5y}{dx^5} - 2\dfrac{d^4y}{dx^4} + \dfrac{d^3y}{dx^3} = 0.$
   (c) $\dfrac{d^3y}{dx^3} - \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - y = 0.$

7. Find a linear differential equation with constant coefficients and of order 3 which admits the following solutions.
   (a) $\cos x$, $\sin x$ and e^{-3x} (b) e^x, e^{2x}, e^{3x}

8. Find the condition on λ for which all the solutions of $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} - \lambda y = 0$ tends to zero as $x \to \infty$.

9. Find the particular solution of the following differential equations using the method of undetermined coefficients and hence find the general solution.
   (a) $y'' + 2y' + 2y = 4e^x \sin x$. (b) $y'' + y = 2\sin x + \sin 2x$.
   (c) $y''' - y'' + y' - y = x^2$.

10. By using the method of variation of parameters, find the general solution of the following differential equations.
    (a) $y'' + y = \sec x$. (b) $y'' + 4y = 3\text{cosec}2x$.
    (c) $y'' + 6y' + 9y = \dfrac{e^{-3x}}{x^3}$.