Tutorial - 6 Solutions

Q1 Fractional Knapsack problem

(a) Bag size = 14

item	A	B	C	D	E	F
profit:	7	8	14	5	10	15
size	2	1	5	2	4	3

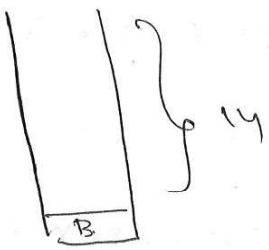
Step 1: Compute score = profit / size for each item.

item	A	B	C	D	E	F
score	7/2 = 3.5	8/1 = 8	14/5 = 2.8	5/2 = 2.5	10/4 = 2.5	15/3 = 5

∴ Pick the items into bag in the order of non-increasing scores

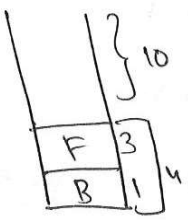
(i.e) B F A C D E (you can pick E first then D).

Step 2: Initially bag is empty
∴ free space = 14

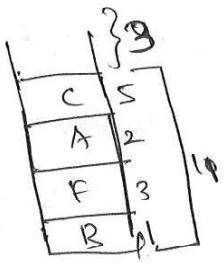


① First, pick B, it occupies 1 unit of space.
∴ free space in the bag = 13

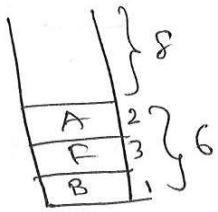
② Then, pick F,



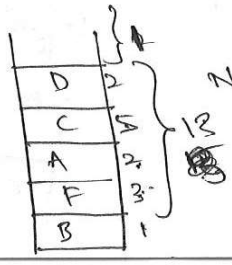
⇒ pick C



③ pick A



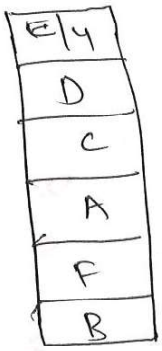
pick D



Next, we cannot pick E completely, as only 1 unit of free space in the bag

and E size is 4 units.

⇒ So just pick just $1/4^{th}$ of item E.



⇒ profit = Sum of profits of B, F, A, C, D and $1/4^{th}$ profit of E.

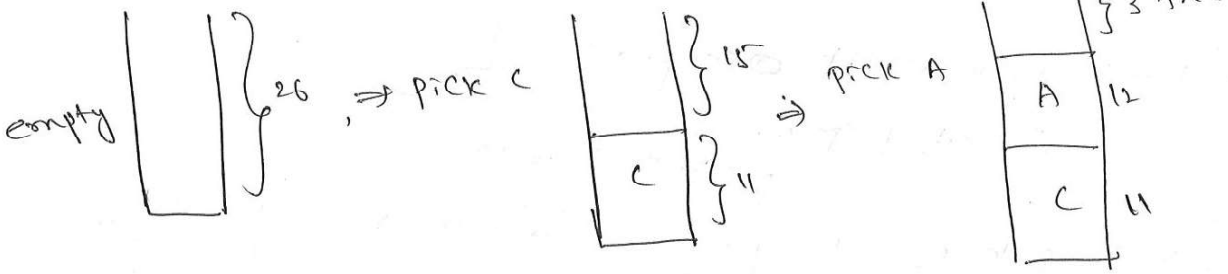
$= 8 + 15 + 7 + 14 + 5 + 10/4$

$= 51.5$

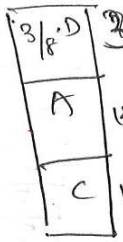
(b) Bag size = 26

item	A	B	C	D	E
profit	24	13	23	15	16
size	12	7	11	8	9
Score = profit/size	24/12 = 2	13/7 = 1.85	23/11 = 2.09	15/8 = 1.875	16/9 = 1.77

⇒ Pick the items in the order C, A, D, B, and E



Then pick $3/8^{th}$ portion of D



profit = profit of C + profit of A + $\frac{3}{8}$ × profit of D

$= 23 + 24 + \frac{3}{8} \times 15$

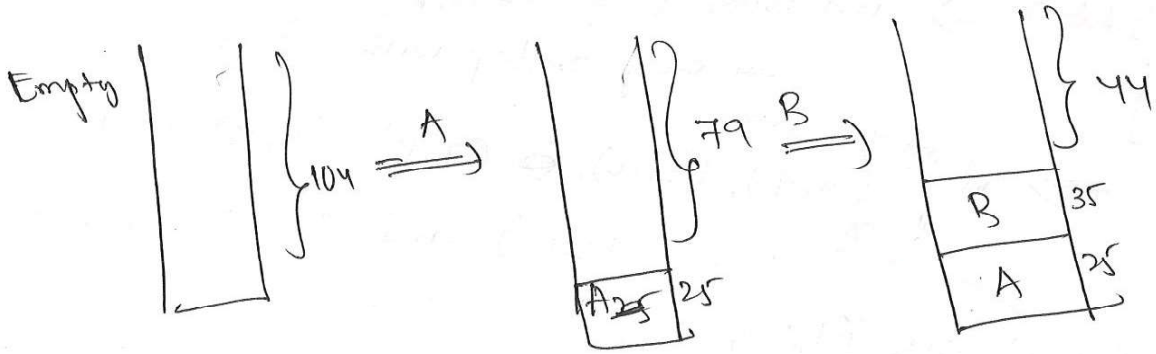
$= 52.625$

(c). Bag size = 104

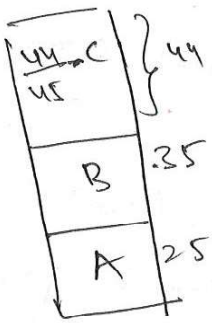
item	A	B	C	D	E	F	G	H
profit	350	400	450	20	70	8	5	5
size	25	35	45	5	25	3	2	2
score =	350/25 = 14	400/35 = 11.42	450/45 = 10	20/5 = 4	70/25 = 2.8	8/3 = 2.66	5/2 = 2.5	5/2 = 2.5

⇒ pick the items in the order

A, B, C, D, E, F, G, and H



C can't be picked completely



⇒ profit = profit(A) + profit(B) + $\frac{44}{45}$ × profit(C)

= 350 + 400 + $\frac{44}{45}$ × 450

= 350 + 400 + 440 = 1190.

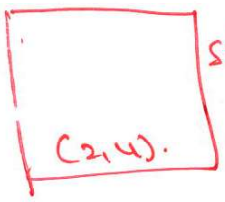
Q(2)

(a) (1,5), (2,4), (2,6), (5,10), (4,7), (9,14), (8,16), (2,10), (4,12), (7,11), (9,12), and (10,14)

- Sort the intervals w.r.to finish/end time (~~y-coordinate~~)

⇒ (2,4), (1,5), (2,6), (4,7), (5,10), (2,10), (7,11), (4,12), (9,12), (9,14), (10,14), and (8,16)

⟹ pick (2,4) into Solution



⇒ intervals (1,5), (2,6), (2,10) cannot be in the solution as they overlap with (2,4).

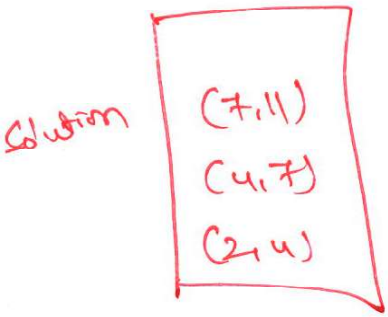
⟹ only intervals from (4,7), (5,10), (7,11), (4,12), (9,12), (9,14), (10,14) and (8,16) can be in Soln

pick (4,7) into solution

⇒ intervals (5,10), (4,12), cannot be in the Solution

⟹ (7,11), (9,12), (9,14), (10,14), and (8,16) only can be part of the solution

pick (7,11). into solution



⟹ (9,12), (9,14), (10,14), and (8,16) cannot be in the Solut.

⟹ No new intervals add to the Solution

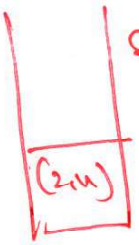
⟹ optimum Solution = { (2,4), (4,7), (7,11) }

(b) (2,4), (2,6), (3,7), (3,5), (2,5), (7,10), (3,8), (4,11), (5,9), (9,11), (6,9), (7,12), and (4,12).

→ Sort the interval w.r.to. finish/end point.

⇒ (2,4), (3,5), (2,5), (2,6) (3,7), (3,8), (6,9), (5,9), (7,10), (4,11), (9,11), (7,12), and (4,12)

pick (2,4) into solution



⇒ (2,5), (2,6), (3,7), (3,8), (3,5), cannot be in the solut

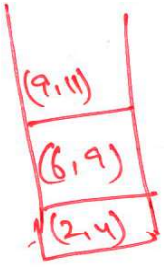
⇒ only intervals can belong to solution are (6,9), (5,9), (7,10), (4,11), (9,11), (7,12) and (4,12)

pick (6,9) into solution

⇒ (5,9), (7,10), (4,11) (4,12) cannot be in the solut

⇒ only possible intervals in the solut (9,11) and (7,12)

pick (9,11) into solution



⇒ (7,12) cannot be in the solution

⇒ No more intervals can add to the solution

⇒ Solution = {(2,4), (6,9), (9,11)}

(3) (a)

Shortest job first

order of the schedule = 3, 2, 5, 1, and 4

job	processing time	start time	end time	deadline	lateness
3	1	0	1	8	0
2	2	1	3	7	0
5	2	3	5	10	0
1	3	5	8	4	4
4	4	8	12	8	4

∴ Max. lateness = 4

optimal : Earliest deadline first

order of the job : 1, 2, 3, 4, and 5

job	processing time	start time	end time	deadline	lateness
1	3	0	3	4	0
2	2	3	5	7	0
3	1	5	6	8	0
4	4	6	10	8	2
5	2	10	12	10	2

∴ max. lateness = 2

(b) Shortest job first

order: 3, 1, 4, 2, 6, 5

job	processing time	Start time	end time	deadline	lateness
3	2	0	2	3	0
1	4	2	6	10	0
4	6	6	12	10	2
2	7	12	19	7	12
6	8	19	27	12	15
5	10	27	37	8	29.

∴ max. lateness = 29

Optimal: Earliest Deadline first (EDF)

order of the jobs : 3, 2, 5, 1, 4, 6.

job	processing time	Start time	end time	deadline	lateness
3	2	0	2	3	0
2	7	2	9	7	2
5	10	9	19	8	11
1	4	19	23	10	13
4	6	23	29	10	19
6	8	29	37	12	25

∴ max. lateness = 25

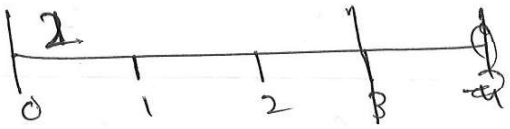
Q4) (a)

job	1	2	3	4	5
deadline	2	1	3	2	1
profit	6	10	2	4	2

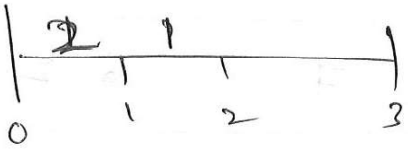
∴ Sort the jobs in the non-~~decreasing~~ increasing order of profit.

⇒ 2, 1, 4, 3, 5.

job 2 deadline is 1

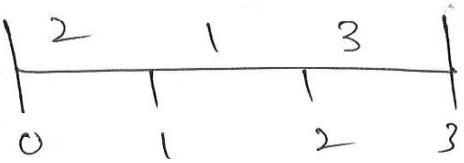


job 1 deadline is 2



job 4 deadline is 2, ~~but~~ not possible to schedule

job 3 deadline is 3



⇒ profit = 10 + 6 + 2 = 18

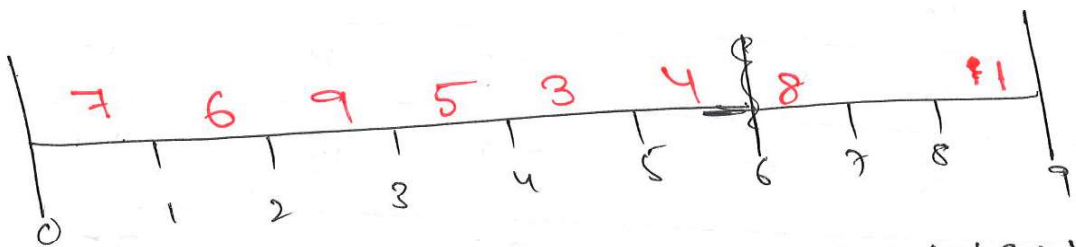
(b)

job	1	2	3	4	5	6	7	8	9	10
deadline	9	2	5	7	4	2	5	7	4	3
profit	15	2	18	1	25	20	8	10	12	5

⇒ Sort the jobs in the non-increasing profits.

⇒ 5, 6, 3, 1, 9, 8, 7, 10, 2, and 4.
(10, 2 ×)

Ans:



profit = ~~18~~ 25 + 20 + 18 + 15 + 12 + 10 + 8 + 1

= 109