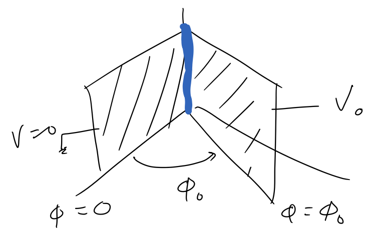
If we are sad, the moon appears sad!

Thich Nhat Hanh



$\nabla^2 V = 0$ Laplace's equation

$V = 0$ at $\phi = 0$
$V = V_0$ at $\phi = \phi_0$

$V(\phi)$

$$\boxed{V = \frac{V_0 \phi}{\phi_0}}$$

$$\boxed{\vec{E} = -\nabla V}$$

$$\boxed{\nabla^2 V = 0}$$ Laplace's equation

$$\boxed{\nabla^2 V = -\frac{\rho}{\epsilon_0}}$$ POISSON's Equation

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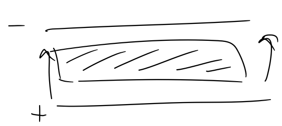
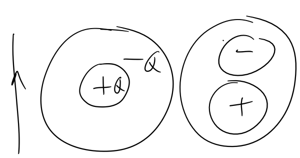
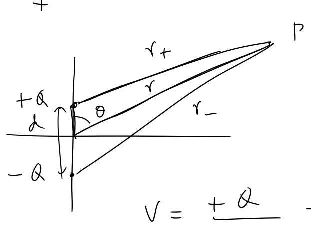
Electric dipole







$$V = \frac{+Q}{4\pi\epsilon_0 r_+} - \frac{Q}{4\pi\epsilon_0 r_-}$$

$$r_+^2 = r^2 + \left(\frac{d}{2}\right)^2 - 2 \cdot r \cdot \frac{d}{2}\cos\theta$$
$$= r^2 + \frac{d^2}{4} - rd\cos\theta$$
$$r_-^2 = r^2 + \frac{d^2}{4} + rd\cos\theta$$

$r >\!\!> d$

$$\frac{1}{r_+} = \frac{1}{\left[r^2 + \frac{d^2}{4} - rd\cos\theta\right]^{1/2}}$$
$$= \frac{1}{r}\left[1 - \frac{d}{r}\cos\theta + \frac{d^2}{4r^2}\right]^{-1/2}$$
$$= \frac{1}{r}\left[1 + \frac{d}{2r}\cos\theta + \cdots\right]$$

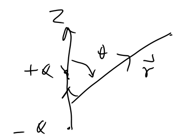
$$\frac{1}{r_-} = \frac{1}{r}\left[1 - \frac{d}{2r}\cos\theta + \cdots\right]$$

$$V = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$$

$$= \frac{Q}{4\pi\epsilon_0}\cdot\frac{d}{r^2}\cos\theta$$

$$\boxed{V = \frac{Qd\cos\theta}{4\pi\epsilon_0 r^2}}$$

Potential due to a dipole



Dipole moment $\vec p = Qd\,\hat z$

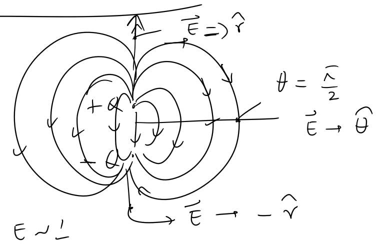
$$\boxed{V = \frac{\vec p\cdot\hat r}{4\pi\epsilon_0 r^2}} = \frac{\vec p\cdot\vec r}{4\pi\epsilon_0 r^3}$$

$$V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2}$$

$$\vec E = -\nabla V = -\left(\frac{\partial V}{\partial r}\hat r + \frac{1}{r}\frac{\partial V}{\partial\theta}\hat\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\hat\phi\right)$$

$$= -\left(\frac{p\cos\theta}{4\pi\epsilon_0}\frac{(-2)}{r^3}\hat r + \frac{1}{r}\cdot\frac{p(-\sin\theta)}{4\pi\epsilon_0 r^2}\hat\theta + 0\right)$$

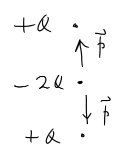
$$\boxed{\vec E = \frac{p}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat r + \sin\theta\,\hat\theta\right)}$$



Monopole $V \sim \frac{1}{r}$, $E \sim \frac{1}{r^2}$

Dipole $V \sim \frac{1}{r^2}$, $E \sim \frac{1}{r^3}$

Quadrupole $V \sim \frac{1}{r^3}$, $E \sim \frac{1}{r^4}$

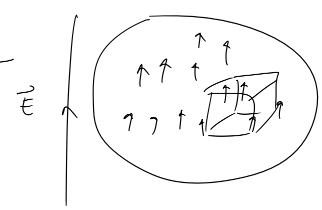


DIELECTRICS

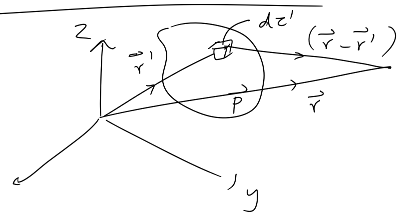
Dipole moment per unit volume

$\Rightarrow$ POLARIZATION

$$\vec P = \frac{\sum \vec p_i}{\text{Volume}}$$



Electric field due to a polarized dielectric



$$dV = \frac{\vec P\, d\tau' \cdot (\vec r - \vec r')}{4\pi\epsilon_0 |\vec r - \vec r'|^3}$$

$$V = \frac{1}{4\pi\epsilon_0}\iiint_{\text{Volume}} \vec P\cdot\frac{(\vec r - \vec r')}{|\vec r - \vec r'|^3}\, d\tau'$$

$$\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = -\nabla\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)$$

$$\nabla\left(\frac{1}{\mathscr{r}}\right) = -\frac{\vec{\mathscr{r}}}{\mathscr{r}^3} \qquad \vec{\mathscr{r}} = \vec{r}-\vec{r}'$$

$$\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$V = \frac{1}{4\pi\epsilon_0}\iiint \vec{P}\cdot\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) d\tau'$$

$$\boxed{\nabla\cdot(\vec{P}f) = \vec{P}\cdot\nabla f + f\,\nabla\cdot\vec{P}}$$

$$\nabla'\cdot\left(\frac{\vec{P}}{|\vec{r}-\vec{r}'|}\right) = \underbrace{\vec{P}\cdot\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)} + \frac{1}{|\vec{r}-\vec{r}'|}\nabla'\cdot\vec{P}$$

$$\vec{P}\cdot\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right) = \nabla'\cdot\left(\frac{\vec{P}}{|\vec{r}-\vec{r}'|}\right) - \frac{\nabla'\cdot\vec{P}}{|\vec{r}-\vec{r}'|}$$

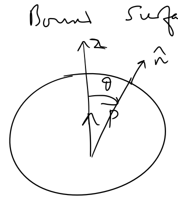
$$V = \frac{1}{4\pi\epsilon_0}\iiint \nabla'\cdot\left(\frac{\vec{P}}{|\vec{r}-\vec{r}'|}\right)d\tau' - \frac{1}{4\pi\epsilon_0}\iiint \frac{\vec{\nabla}\cdot\vec{P}}{|\vec{r}-\vec{r}'|}d\tau'$$

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\oint_S \frac{\vec{P}\cdot\hat{n}\,dA'}{|\vec{r}-\vec{r}'|} + \frac{1}{4\pi\epsilon_0}\iiint_V \frac{[-\nabla'\cdot\vec{P}]}{|\vec{r}-\vec{r}'|}d\tau'}$$

$$V = \frac{1}{4\pi\epsilon_0}\iiint \frac{\rho\,d\tau}{|\vec{r}-\vec{r}'|} \quad ; \quad V = \frac{1}{4\pi\epsilon_0}\oint \frac{\sigma\,dA}{|\vec{r}-\vec{r}'|}$$

$\rho_b = -\vec{\nabla}\cdot\vec{P}$: Bound Volume charge density

$\sigma_b = \vec{P}\cdot\hat{n}$: Bound Surface charge density



$\rho_b = 0$

$\sigma_b = \vec{P}\cdot\hat{r} = P\cos\theta$