The best way to get a good idea
is to get a lot of them

Linus PAULING

DIVERGENCE

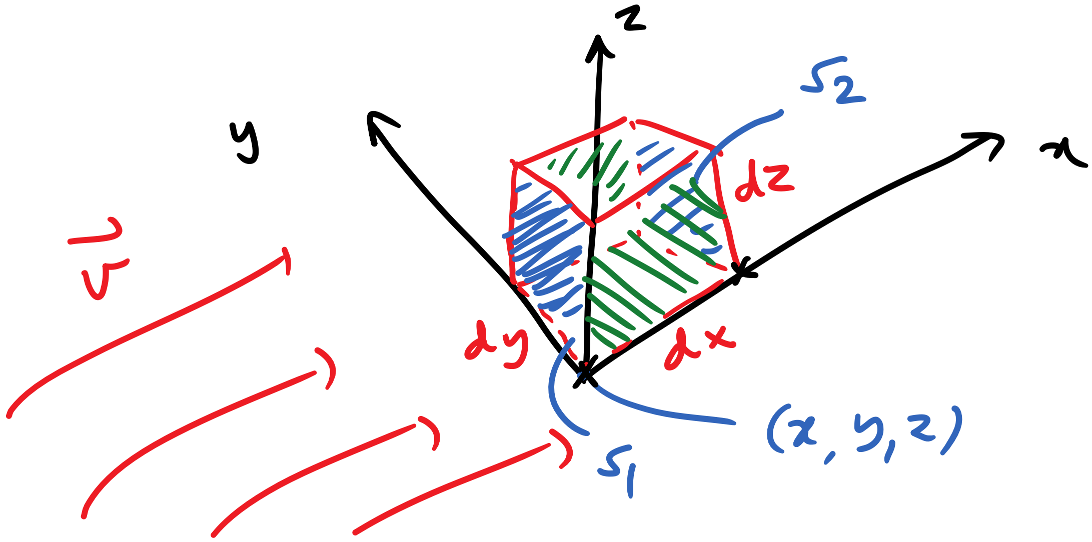
$$\vec{\nabla} f = \hat{x}\frac{\partial f}{\partial x} + \hat{y}\frac{\partial f}{\partial y} + \hat{z}\frac{\partial f}{\partial z}$$

$\vec{\nabla}$: DEL OPERATOR

$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

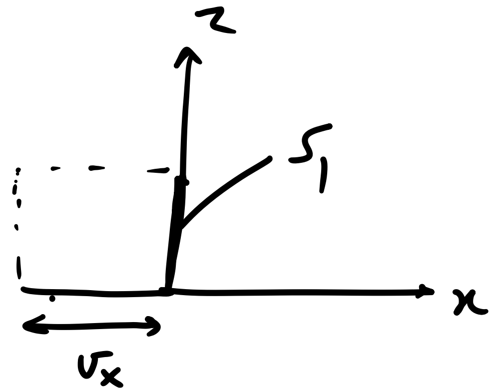
$$\vec{\nabla}\cdot\vec{F} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot\left(\hat{x}F_x + \hat{y}F_y + \hat{z}F_z\right)$$

$$\boxed{\nabla\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}}$$ DIVERGENCE OF $\vec{F}$



Mass of fluid entering through S_1 in time dt

=



$$\rho(v_x\,dy\,dz)\,dt$$

$$\underbrace{(\rho v_x)_x\,dy\,dz\,dt}$$

Mass of fluid leaving the volume in time dt

$$\underbrace{(\rho v_x)_{x+dx}\,dy\,dz\,dt}$$

Net mass of fluid leaving the volume along S_1 & S_2 in time dt

$$\left[(\rho v_x)_{x+dx} - (\rho v_x)_x\right]\underline{\underline{dy\,dz}}\,dt$$

$$(\rho v_x)_{x+dx} = (\rho v_x)_x + \frac{\partial}{\partial x}(\rho v_x)\,dx$$

$\frac{\partial}{\partial x}(\rho v_x)\, dx\, dy\, dz\, dt$: Surface $\perp$ x

$\frac{\partial}{\partial y}(\rho v_y)\, dx\, dy\, dz\, dt$: Surface $\perp$ y

$\frac{\partial}{\partial z}(\rho v_z)\, dx\, dy\, dz\, dt$: Surface $\perp$ z

Net mass of fluid leaving the volume in dt

$$\left[\frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z)\right] dx\, dy\, dz\, dt$$

$$= \vec{\nabla}\cdot(\rho\vec{v})\, dx\, dy\, dz\, dt$$

$$= -\frac{\partial}{\partial t}(\rho\, dx\, dz\, dy)\, dt$$

$$\boxed{\nabla\cdot(\rho\vec{v}) = -\frac{\partial \rho}{\partial t}}$$

CONTINUITY EQUATION

Incompressible fluid $\rho =$ Constant

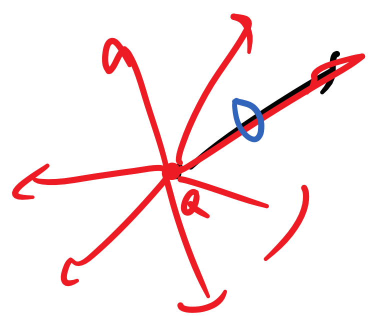
$$\boxed{\nabla\cdot\vec{v} = 0}$$

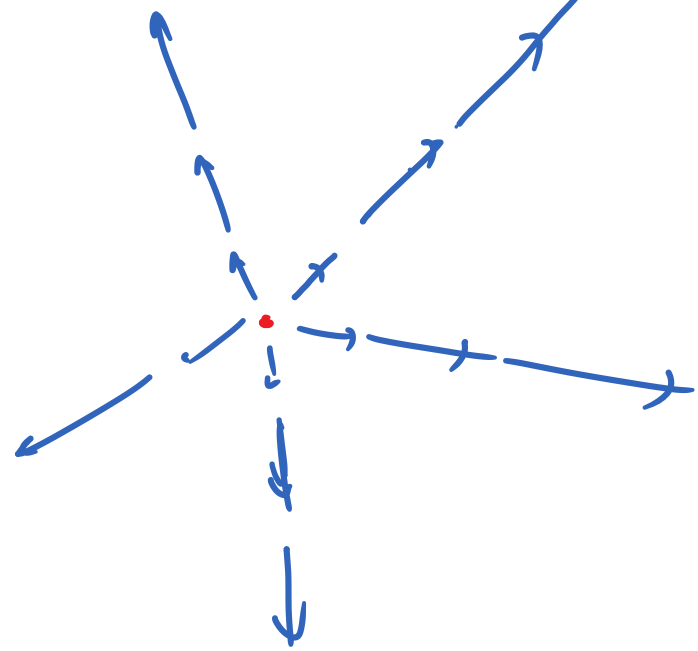
Example

① $\vec{F} = \vec{r} = (\hat{x}x + \hat{y}y + \hat{z}z)$

$$\nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z}$$

$$= 3$$





Ex

$$\vec{F} = A\frac{\hat{r}}{r^2} = A\frac{\vec{r}}{r^3}$$

$$= \frac{A}{(x^2+y^2+z^2)^{3/2}}(\hat{x}x + \hat{y}y + \hat{z}z)$$

$$\frac{\partial F_x}{\partial x} = A\frac{\partial}{\partial x}\left(\frac{x}{(x^2+y^2+z^2)^{3/2}}\right)$$

$$= A\left[\frac{1}{(x^2+y^2+z^2)^{3/2}} - \frac{3}{2}\frac{x\ (2x)}{(x^2+y^2+z^2)^{5/2}}\right]$$

$$= A\left[\frac{1}{(x^2+y^2+z^2)^{3/2}} - \frac{3x^2}{(x^2+y^2+z^2)^{5/2}}\right]$$

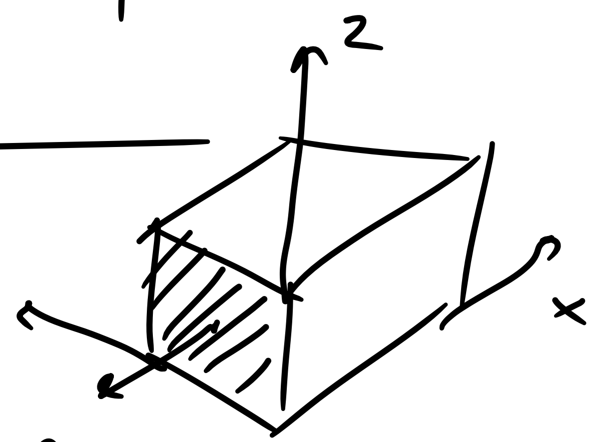
$$\vec{\nabla} \cdot \vec{F} = 0 \qquad (r \neq 0)$$

<u>Example</u>

$$\vec{F} = \hat{x}\, y$$

$$\nabla \cdot \vec{F} = 0$$

$$\boxed{\vec{\nabla} \cdot \vec{B} = 0}$$ Magnetic field



$$(\rho v_x)_x \, dy\, dz$$

$$-(\rho \vec{v}) \cdot d\vec{A}$$

$$-\hat{x}\, dy\, dz$$

$$\boxed{\iiint \nabla \cdot \vec{F}\, dx\, dy\, dz = \oiint \vec{F} \cdot d\vec{A}}$$ GAUSS's THEOREM