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A TIMES GROUP INITIATIVE

Passive Circuit Elements

Rama Komaragiri

Passive Circuit Elements

- Circuit components (or circuit elements) which cannot control current by means of another electrical signal are called passive devices
- Example: Resistors, Capacitors and Inductors

Capacitors – Energy Storage Devices

Objective of Lecture

Explain the construction of a capacitor and how charge is stored.

Explain several types of capacitors

To calculate the relationship between charge, voltage, and capacitance

To calculate charging and discharging time of a capacitor

Find the relation between voltage, current, and capacitance; power; and energy

Evaluating Equivalent capacitance when a set of capacitors are in series and in parallel

Impedance of a capacitor

Capacitor

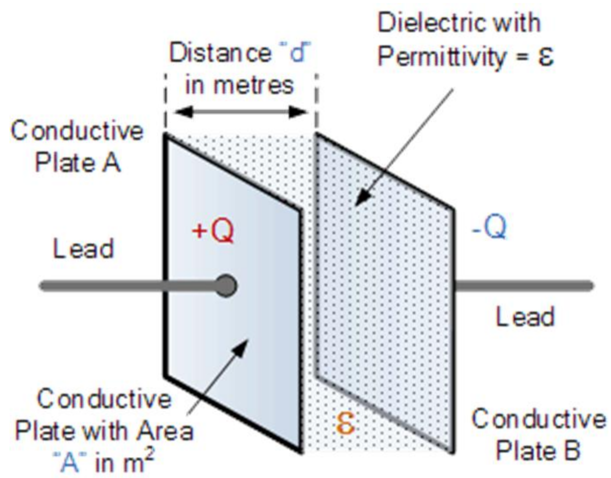
- Two conductive plates separated by an insulator (or dielectric) forms a capacitor.
  - Commonly illustrated as two parallel metal plates separated by a distance, d.

$$C = \frac{\varepsilon A}{d}$$

where $\varepsilon = \varepsilon_r \varepsilon_o$

ε_r is the relative dielectric constant

ε_o is the vacuum permittivity

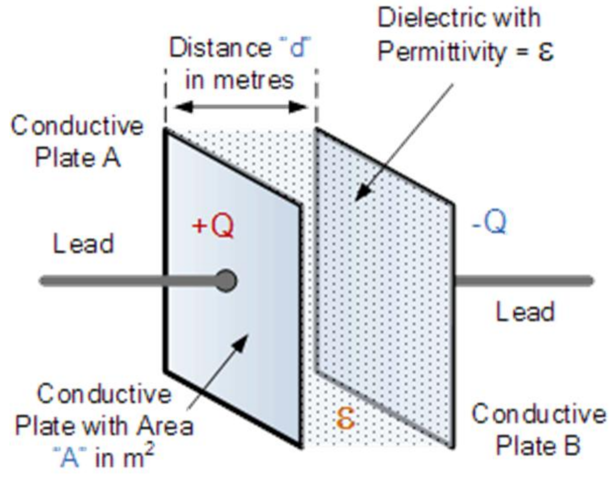


http://www.electronics-tutorials.ws/capacitor/cap1a.gif

Effect of Dimensions

- Capacitance increases with
  - increasing surface area of the plates,
  - decreasing spacing between plates, and
  - increasing the relative dielectric constant of the insulator between the two plates.

$$C = \frac{\varepsilon A}{d}$$



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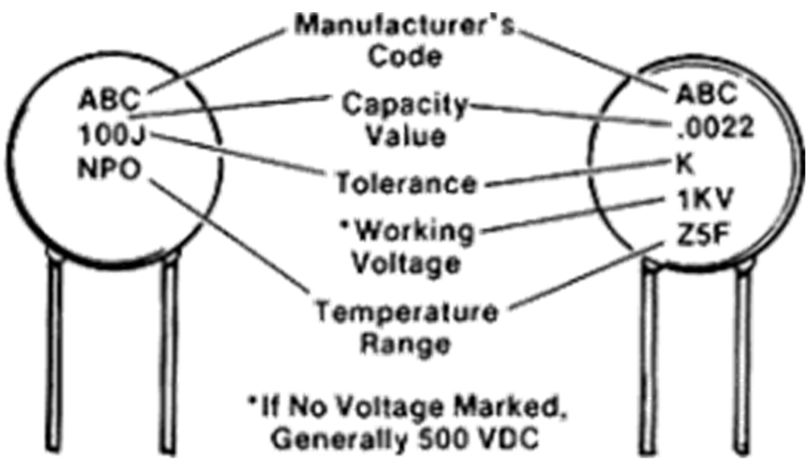
Types of Capacitors

- Fixed Capacitors
  - Nonpolarized
    - May be connected into circuit with either terminal of capacitor connected to the high voltage side of the circuit.
      - Insulator: Paper, Mica, Ceramic, Polymer
  - Electrolytic
    - The negative terminal must always be at a lower voltage than the positive terminal
      - Plates or Electrodes: Aluminum, Tantalum

Nonpolarized

- Difficult to make nonpolarized capacitors that store a large amount of charge or operate at high voltages.
  - Tolerance on capacitance values is very large
    - can be as high as ±50%



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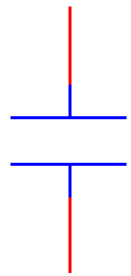
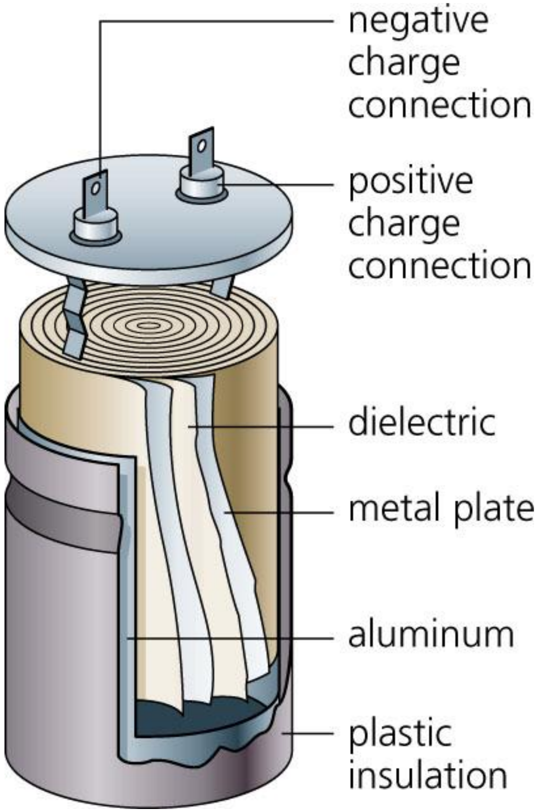
Circuit Symbol

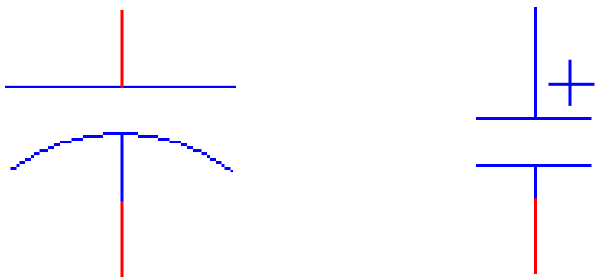


Nonpolarized

- Cross section



Circuit Symbol



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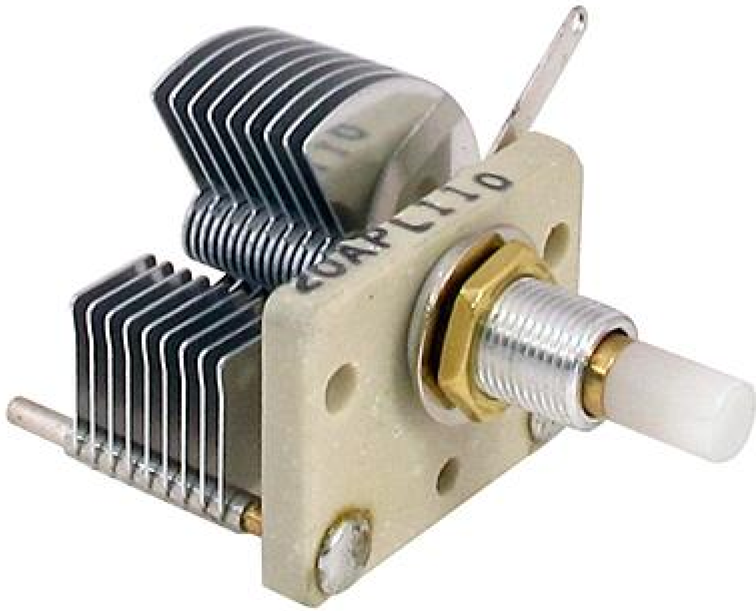
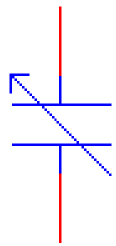
Variable Capacitors

- Cross-section area of capacitor plate is changed as one set of plates are rotated with respect to the other.



Circuit Symbol



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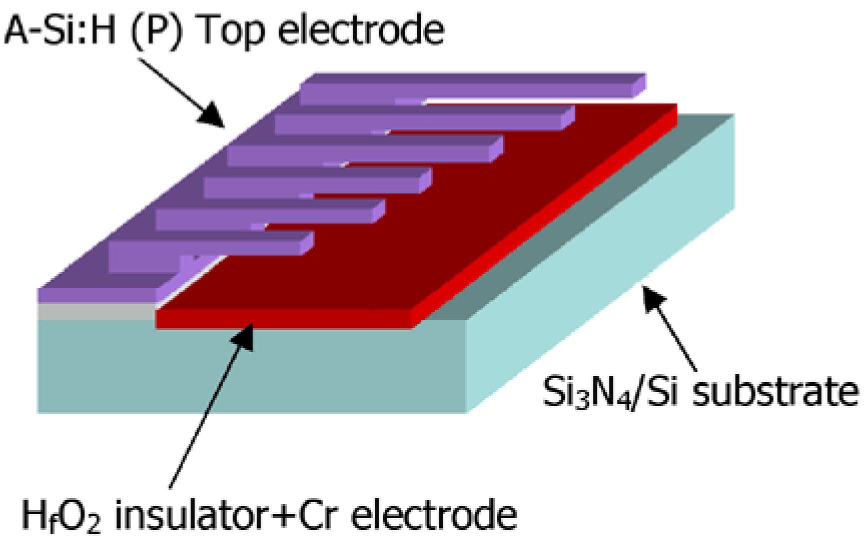
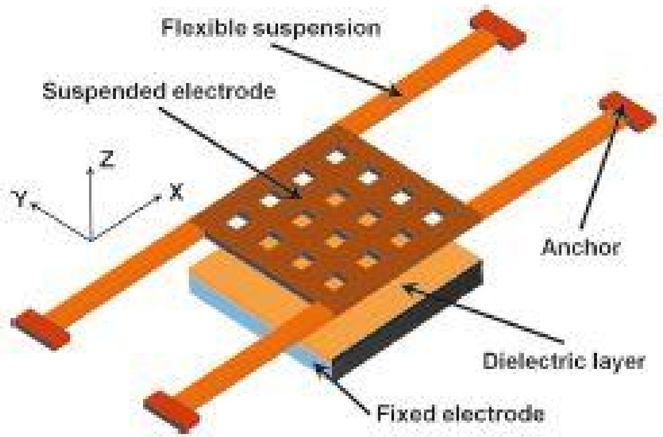
MEMS Capacitor

- MEMS (Microelectromechanical system)
  - Can be a variable capacitor by changing the distance between electrodes.
  - Use in sensing applications as well as in RF electronics.





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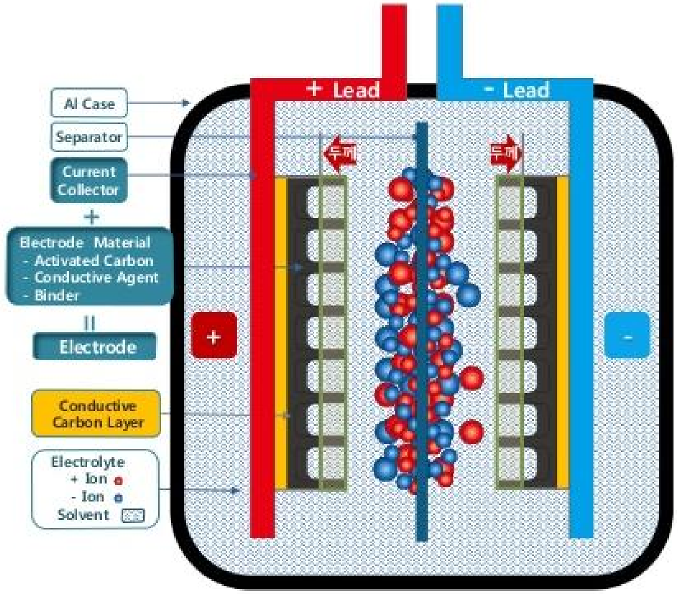
http://www.silvaco.com/tech_lib_TCAD/simulationstandard/2005/aug/a3/a3.html

Electric Double Layer Capacitor

- Also known as a supercapacitor or ultracapacitor
  - Used in high voltage/high current applications.
    - Energy storage for alternate energy systems.



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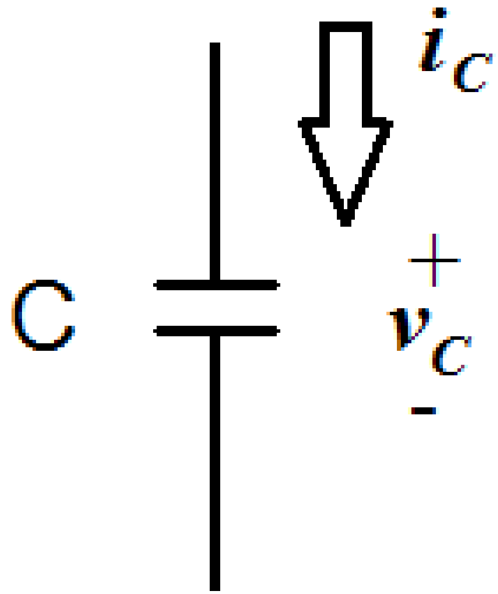
Electrical Properties of a Capacitor

- Acts like an open circuit at steady state when connected to a d.c. voltage or current source.
- Voltage on a capacitor must be continuous
  - There are no abrupt changes to the voltage, but there may be discontinuities in the current.
- An ideal capacitor does not dissipate energy, it uses power when charging energy and returns power when discharging energy.

Sign Conventions

The sign convention used with a capacitor is the same as for a power dissipating device.

- When current flows into the positive side of the voltage across the capacitor, it is positive and the capacitor is dissipating power.
- When the capacitor releases energy back into the circuit, the sign of the current will be negative.



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Current-Voltage Relationships

$$q = Cv_C$$

$$i_C = \frac{dq}{dt}$$

$$i_C = C\frac{dv_C}{dt}$$

$$v_C = \frac{1}{C}\int_{t_o}^{t_1} i_C dt$$

- Let $$v_c = V_0 sin(\omega t)$$

$$i_c = C\frac{d}{dt}(V_0 sin(\omega t))$$

$$= \omega CV_0 cos(\omega t)$$

$$= \omega CV_0 \sin\left(\frac{\pi}{2} - \omega t\right)$$

- Current through capacitor leads voltage across capacitor by $\frac{\pi}{2}$
- What is the significance of above statement?

Current-Voltage Relationships

$$i_C = C\frac{dv_C}{dt}$$

$$v_C = \frac{1}{C}\int_{t_o}^{t_1} i_C dt$$

- Let $v_c = V_0 e^{j\omega t}$

$$i_c = C\frac{d}{dt}\left(V_0 e^{j\omega t}\right) = j\omega C V_0 e^{j\omega t}$$

- Re-writing the equation (similar to that of V= IR)

$$v_c = \frac{1}{j\omega C} i_c = -jX_c i_C = Z_c i_c, X_c = \frac{1}{\omega C}$$

- X_C is called as reactance (in ohm) of the capacitor
  - When $\omega = 0$, $X_C = \infty$, means reactance is infinite → Capacitor blocks DC
  - When $\omega = \infty$, $X_C = 0$, means capacitor behaves like a short at higher frequencies
  - Frequency dependent electrical behavior of capacitance on circuit

Energy Storage

- Charge is stored on the plates of the capacitor.

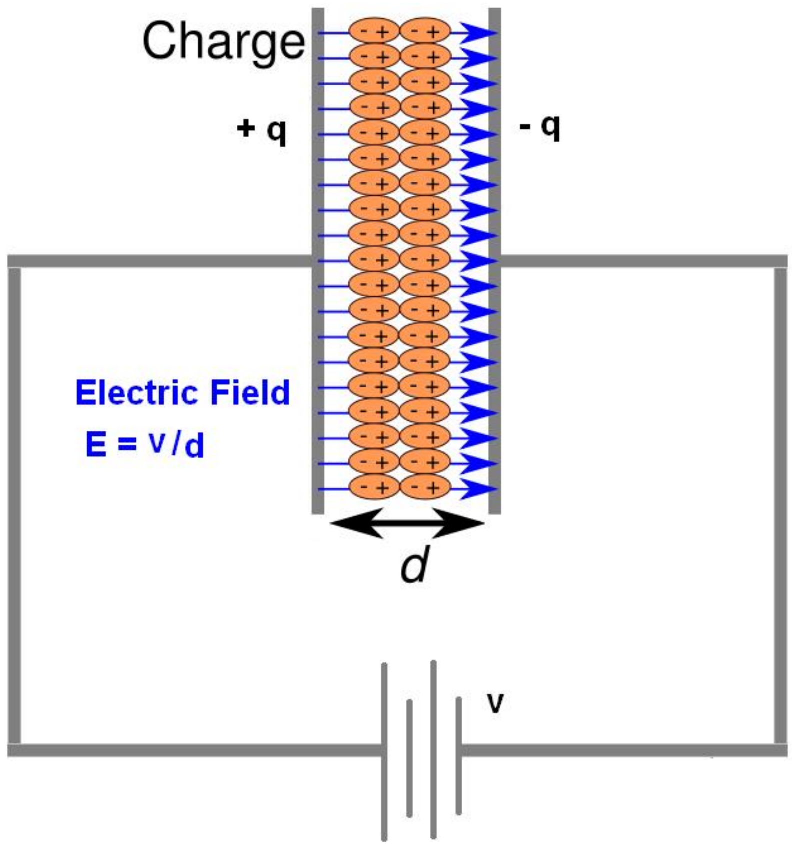
Equation:

$$Q = CV$$

Units:

Farad = Coulomb/Voltage

Farad is abbreviated as F



Power and Energy

- P is power and E is energy
- An ideal capacitor stores energy when charged and gives away energy while discharging

$$p_C = i_C v_C$$

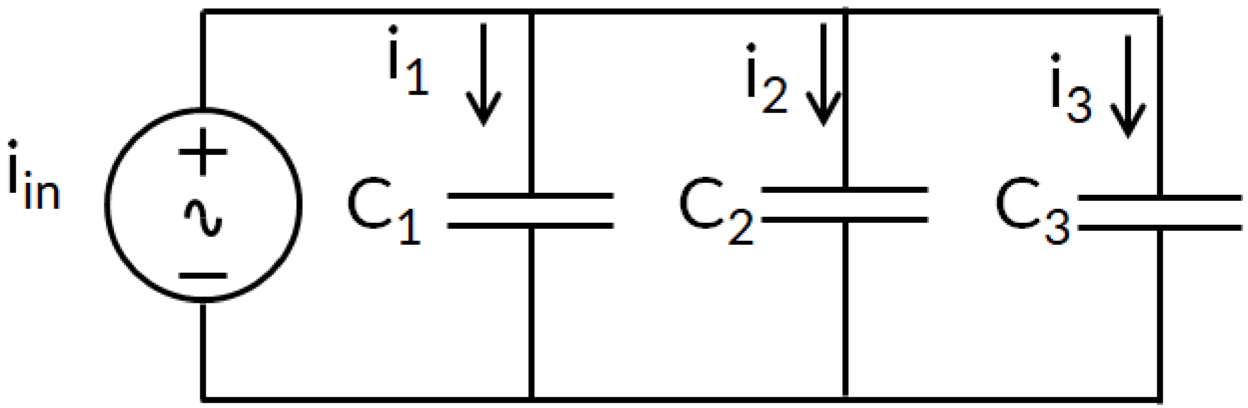
$$p_C = C v_C \frac{dv_C}{dt}$$

$$w_C = \frac{1}{2} C {v_C}^2$$

$$w_C = \frac{q^2}{2C}$$

Capacitors in Parallel

- Consider capacitors connected in parallel configuration
  - Voltage across the capacitors is equal
  - Current divides



- Writing KCL,

$$i_{in} = i_1 + i_2 + i_3$$

- Noting the current voltage relation for a capacitor as $i = C\dfrac{dv}{dt}$

C_{eq} for Capacitors in Parallel

- If C_{eq} is the net capacitance, then $i_{in} = C_{eq}\dfrac{dv}{dt}$
- Writing current voltage relations for individual capacitances as

$$i_1 = C_1\frac{dv}{dt},\ i_2 = C_2\frac{dv}{dt},\ i_3 = C_3\frac{dv}{dt}$$

i

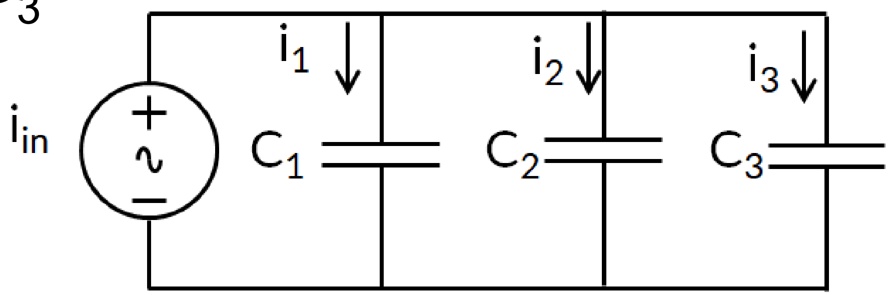
- Substituting into KCL,

$$i_{in} = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt}$$

- Thus

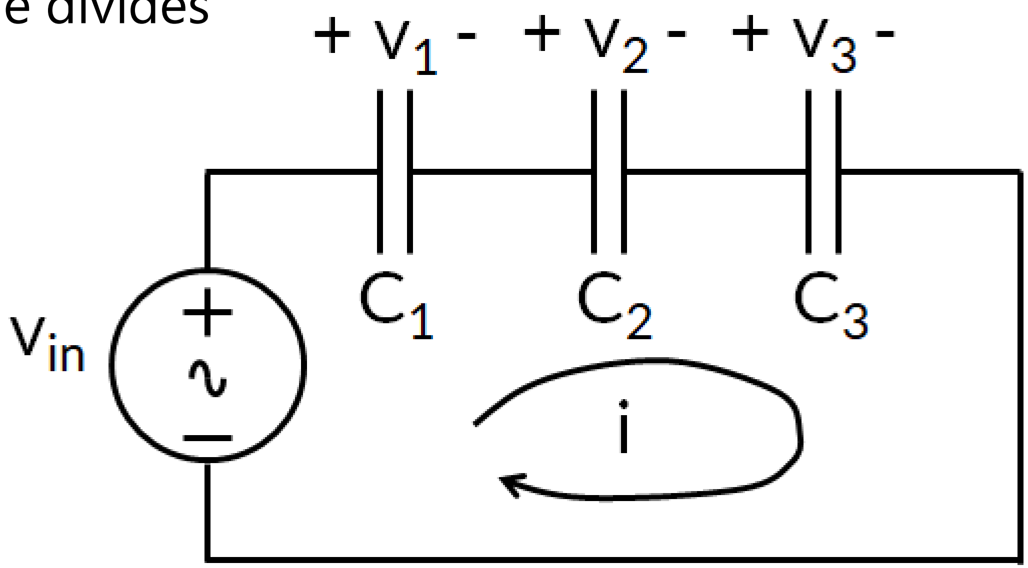
$$C_{eq} = C_1 + C_2 + C_3$$

$$C_{eq} = \sum_{p=1}^{m} C_p$$



Capacitors in Series

- Consider capacitors connected in series configuration
  - Current through the capacitors is same
  - Voltage divides



- Applying KVL

$$V_{in} = V_1 + V_2 + V_3$$

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C_{eq} for Capacitors in Series

- Noting the relation between voltage across the capacitor and current through a capacitor as

$$v_1 = \frac{1}{C_1}\int_{t_o}^{t_1} i\,dt,\ v_2 = \frac{1}{C_2}\int_{t_o}^{t_1} i\,dt,\ v_3 = \frac{1}{C_3}\int_{t_o}^{t_1} i\,dt$$

- If C_{eq} is the total capacitance, then

$$v_{in} = \frac{1}{C_{eq}}\int_{t_o}^{t_1} i\,dt$$

- Substituting back into KVL

$$v_{in} = \frac{1}{C_1}\int_{t_o}^{t_1} i\,dt + \frac{1}{C_2}\int_{t_o}^{t_1} i\,dt + \frac{1}{C_3}\int_{t_o}^{t_1} i\,dt$$

- Thus,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$C_{eq} = \left[\sum_{s=1}^{n} \frac{1}{C_s}\right]^{-1}$$

General Equations for C_{eq}

› Parallel Combination

- If m capacitors are in parallel, then

$$C_{eq} = \sum_{p=1}^{m} C_p$$

- Series Combination

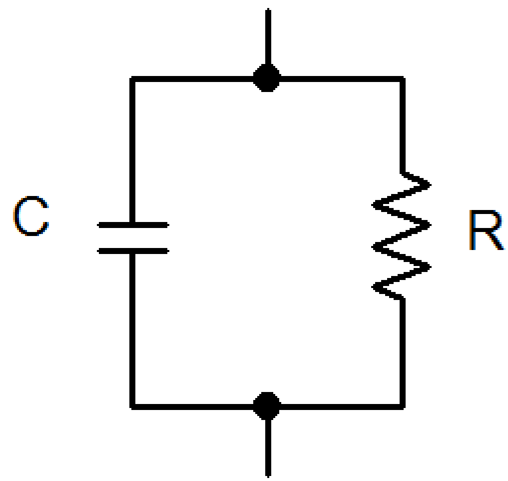
- If n capacitors are in series, then:

i

$$C_{eq} = \left[\sum_{s=1}^{n} \frac{1}{C_s} \right]^{-1}$$

Properties of a Real Capacitor

- A real capacitor does dissipate energy due leakage of charge through its insulator.
  – Real capacitor is modeled by keeping a resistor in parallel with an ideal capacitor.



Charging a capacitor

- One of the functions of capacitor is storing charge (and thus energy)
- Capacitor has an ability to store charge when a potential difference is applied across the capacitor plates
- Energy is stored in the electric field between positive and negative plates.
- When a voltage is applied across a capacitor, current flows into the capacitor plates and develops a potential difference across the capacitor
- With time, the potential difference between the battery and the capacitor become smaller and the flow rate of electrons (thus current flow) reduces
- The charging process continues until the capacitor becomes fully charged.
- The charging current follows an exponential curve.

Charging a capacitor

- To start with, it is easy to store charge in the capacitor.
- As more charge is stored on the plates of the capacitor, it becomes increasingly difficult to place additional charge on the plates due to Coulombic repulsion
- When the initial charge on the capacitor plates is small, as charge is stored on the capacitor plates, the voltage across the capacitor increases rapidly
- As the charge on the capacitor plates increases, it becomes difficult to add extra charges on plate and voltage across the capacitor increases more slowly
- Coulombic repulsion from the charge already on the plates creates an opposing force to limit the addition of more charge on the plates
- The charging current follows an exponential curve.

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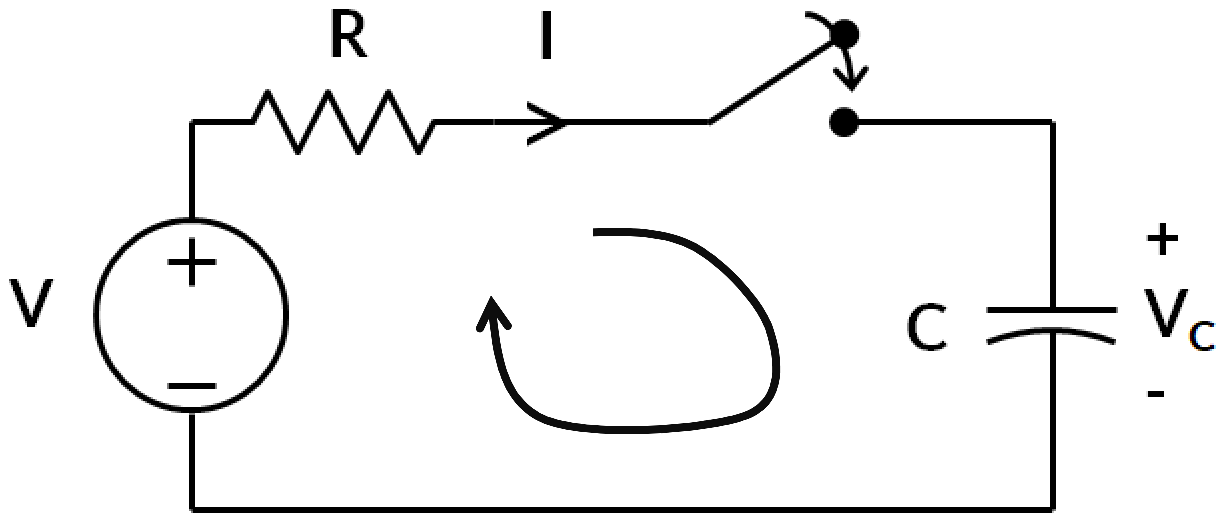
Charging a capacitor

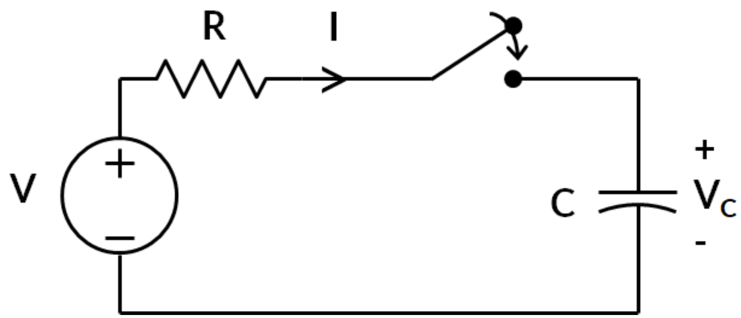
- Consider the circuit shown in figure



- Apply KVL $$V = V_R + V_C$$

- Noting that $C = \dfrac{q}{V_C} \Rightarrow V_c = \dfrac{q}{C}$ and $V_R = Ri_c$

Charging a capacitor



$$V = iR + \frac{q}{C}$$

$$VC = RC\frac{dq}{dt} + q$$

$$VC - q = RC\frac{dq}{dt}$$

$$\int \frac{dt}{RC} = \int \frac{dq}{VC-q} \qquad \int \frac{dt}{RC} = -\int \frac{d(VC-q)}{VC-q}$$

$$C_1 + \frac{t}{RC} = -\ln(VC-q)$$

C_1: Integration constant

$$VC - q = e^{c_1 - \frac{t}{RC}}$$

$$VC - q = C_2 e^{-\frac{t}{RC}}$$

C_2: Integration constant, ($e^{C_1} = C_2$)

Charging a capacitor

- Using the initial boundary condition: at time t = 0, when the capacitor is not initially uncharged, q = 0

$$VC - q = C_2 e^{-\frac{t}{RC}} \Rightarrow C_2 = VC$$

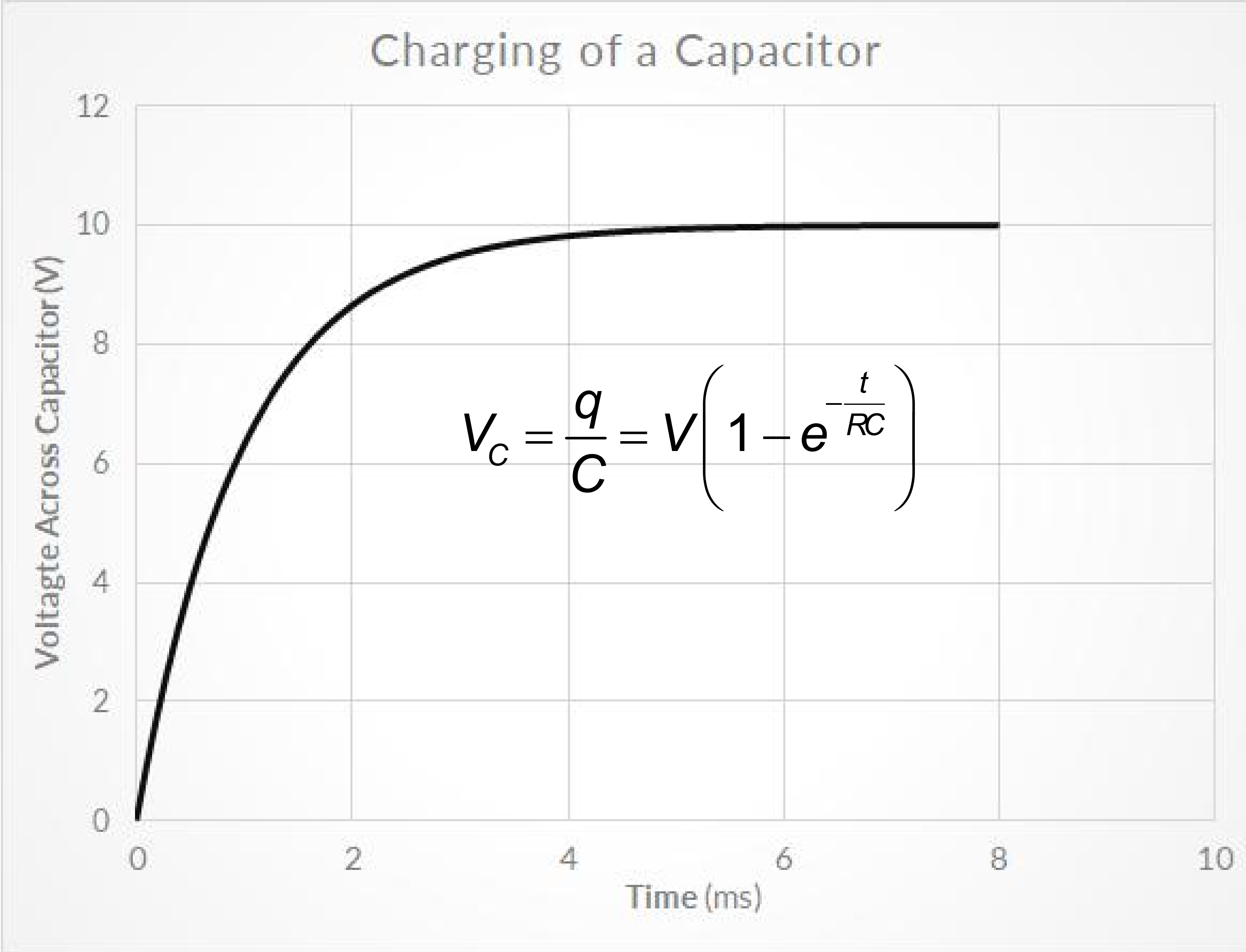
$$VC - q = VCe^{-\frac{t}{RC}} \qquad q = VC\left(1 - e^{-\frac{t}{RC}}\right)$$

$$V_C = \frac{q}{C} = V\left(1 - e^{-\frac{t}{RC}}\right) \Rightarrow \boxed{V_C = V\left(1 - e^{-\frac{t}{RC}}\right)}$$

$$i = \frac{dq}{dt} = VC\frac{d}{dt}\left(1 - e^{-\frac{t}{RC}}\right) \quad i = VC\left(0 - e^{-\frac{t}{RC}} \times -\frac{1}{RC}\right) \Rightarrow \boxed{i = \frac{V}{R}e^{-\frac{t}{RC}}}$$

Note: C_2 indicates total charge in the system, not stored in the capacitor

Charging of a capacitor



Time Constant

- The rate of charging is determined by the charging equation determined by the RC constant in the exponential term.

$$V_C = \frac{q}{C} = V\left(1 - e^{-t/RC}\right)$$

- The term RC is termed the **time constant (most famously RC time constant)** since it affects the rate of charge. Mathematically, this is the time taken for the capacitor to reach 0.632 of the fully charged value.

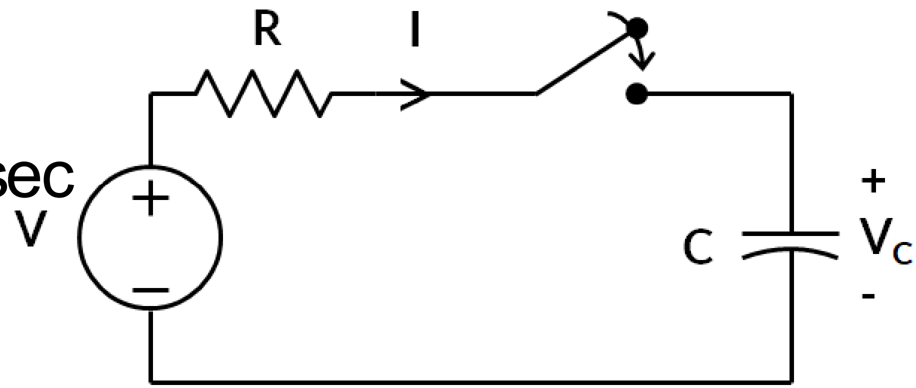
Time Constant

- According to the charging equation, theoretically, capacitors takes infinite time to charge completely.
- For all practical purposes, it is assumed that a capacitor can be charged completely in only five times of the time constant, meaning the capacitor is said fully charged after $5\times$ RC.
- After 5 time constant, q, V_c and current will be over 99% ($1-e^{-5} = 0.9932$) to their final values.

Example: Charging a Capacitor

- An uncharged capacitor of capacitance 400 μF is connected to a 50 V D.C. supply in series along with a current limiting resistor of 5 kΩ. Calculate

i. The voltage of the capacitor at the end of 0.4 seconds of charging
ii. The charging current at the end of 0.4 seconds
iii. The time taken for the capacitor to be charged to 30 volts.



i. Using the charge formula,

$$\tau = RC = 200\times10^{-6}\times5\times10^{3} = 2\,\text{sec}$$

$$V_C = \frac{q}{C} = V\left(1-e^{-\frac{t}{\tau}}\right)$$

$$V_C = 50\left(1-e^{-\frac{0.4}{2}}\right) = 9.06\,V$$

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Example: Charging a Capacitor

ii) The charging current at the end of 0.4 seconds

$$i_c = \frac{V}{R} e^{-\frac{t}{RC}} \qquad i_c = \frac{50}{2000} e^{-\frac{0.4}{2}} = 4.53\, mA$$

iii) To charge the capacitor to 30 Volts, using the formula,

$$V_C = \frac{q}{C} = V\left(1 - e^{-\frac{t}{\tau}}\right) \qquad 30 = 50\left(1 - e^{-\frac{t}{2}}\right)$$

$$1 - 0.6 = e^{-\frac{t}{2}} \qquad -\frac{t}{2} = \ln(0.4) = -0.916$$

$$t = 1.83\, s$$

Charging an initially charged Capacitor

- The ability to add charge to a capacitor depends on:
  - the amount of charge already on the plates of the capacitor and the force (voltage) driving the charge towards the plates (i.e., current)
- If at the start of charging, the capacitor is charged to a voltage of V_1 Volts,
- Then the initial condition gets modified as at t = 0, $q = CV_1$
- Thus, applying boundary condition

$$VC - CV_1 = C_2 e^{-\frac{t}{RC}}$$

$$C_2 = C(V - V_1)$$

$$VC - q = C(V - V_1)\ e^{-\frac{t}{RC}}$$

$$q = VC(1 - e^{-\frac{t}{RC}}) + V_1 C e^{-\frac{t}{RC}}$$

Charging an initially charged Capacitor

$$V_C = \frac{q}{C} = V - Ve^{-\frac{t}{RC}} + V_1 e^{-\frac{t}{RC}}$$

$$V_C = V\left(1 - e^{-\frac{t}{RC}}\right) + V_1 e^{-\frac{t}{RC}}$$

$$i_c = \frac{dq}{dt} = VC\frac{d}{dt}\left(1 - e^{-\frac{t}{RC}}\right) + V_1 C\frac{d}{dt}\left(e^{-\frac{t}{RC}}\right)$$

$$i_c = \frac{V}{R}e^{-\frac{t}{RC}} - \frac{V_1}{R}e^{-\frac{t}{RC}} \quad \Rightarrow \quad i_c(t) = \frac{(V - V_1)}{R}e^{-\frac{t}{RC}}$$

Example: Charging an initially charged Capacitor

- A capacitor of capacitance 400 μF is initially charged to 40 V is connected to a 80 V D.C. supply in series along with a current limiting resistor of 5 kΩ. Calculate the voltage of the capacitor at the end of 0.4 seconds of charging.
- **Solution:** Using the formula for capacitor with an initial voltage,

$$V_C = V + \left(V_1 - V\right)e^{-\frac{t}{RC}}$$

$$\tau = RC = 2\sec$$

$$V_C = 80 + \left(40 - 80\right)e^{-\frac{0.4}{2}} = 47.25\,V$$

Discharging a Capacitor

- Coulombic repulsion between charges already existing the plates creates a force that lets charges to discharge out of the capacitor once the voltage on the charge in the capacitor is decreased
- Coulombic repulsion decreases as charge more charge is removed from the capacitor plates.
- Initially, voltage across the capacitor decreases rapidly as charge is removed from the plates
- As more and more charge is removed, voltage across the capacitor decreases more slowly as it becomes difficult to force the remaining charge out of the capacitor.

Discharging a capacitor

- Applying KVL

$$V_R + V_C = 0$$

$$0 = i \times R + \frac{q}{C}$$

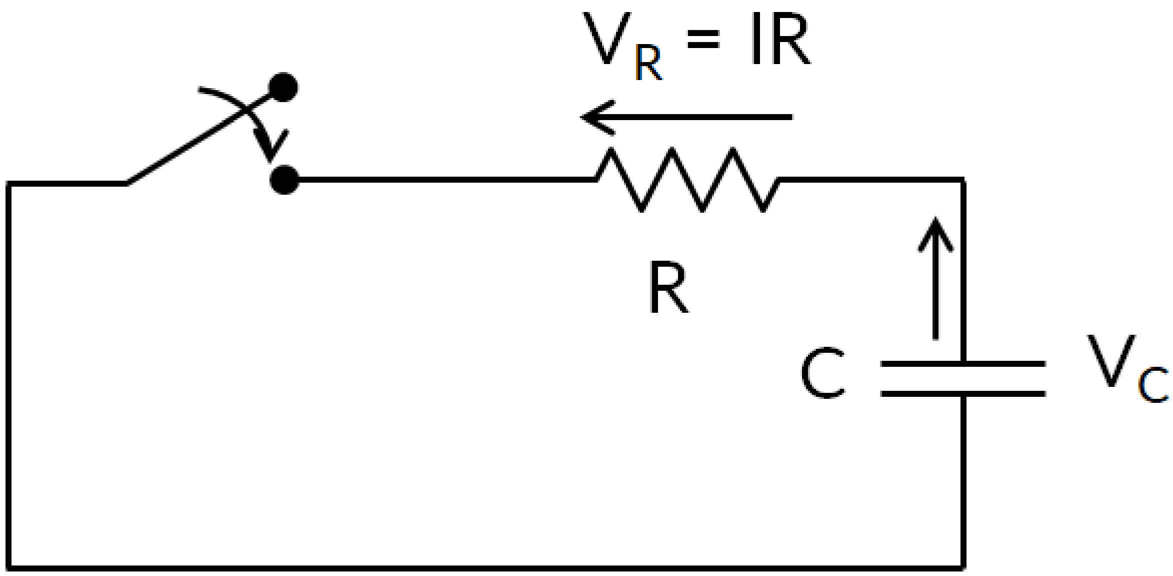
$$-\frac{q}{C} = i \times R$$

$$-\int \frac{dt}{RC} = \int \frac{dq}{q}$$

$$-\frac{t}{RC} + C_1 = \ln q$$

$$q = e^{-t/RC + C_1}$$

$$q = C_2 e^{-\frac{t}{RC}}$$



C_1: integration constant

C_2: integration constant

Discharging a capacitor

Substitute boundary condition, at t = 0,

Voltage across C = V,   q = VC

C_2 = VC

$$q = VCe^{-\frac{t}{RC}}$$

$$V_C = \frac{q}{C} = Ve^{-\frac{t}{RC}}$$

$$i = \frac{dq}{dt} = VC \times -\frac{1}{RC} e^{-\frac{t}{RC}} \qquad i = -\frac{V}{R} e^{-\frac{t}{RC}}$$

(Note that the negative sign indicate that the current is opposite to the charging current's direction)

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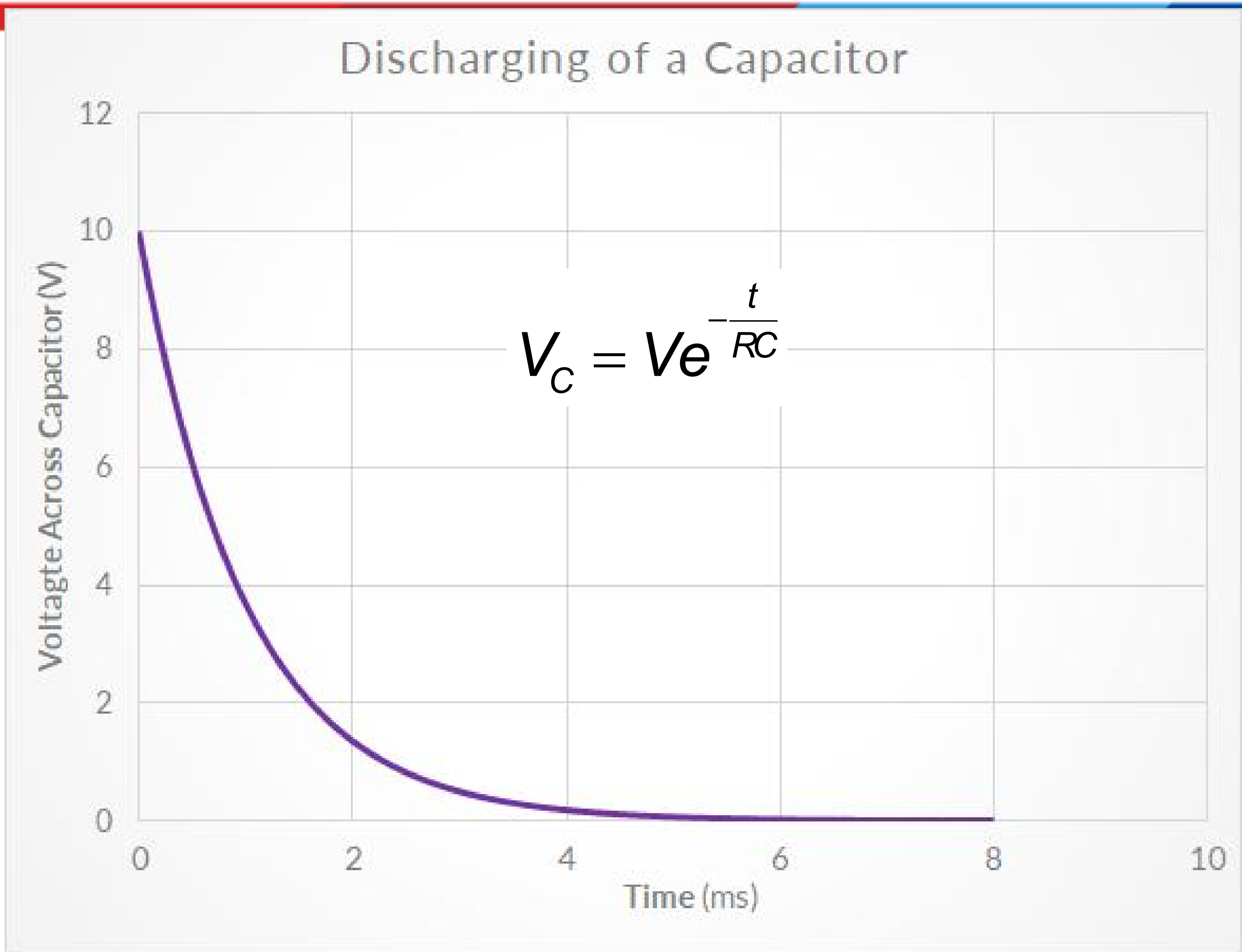
Discharging a capacitor

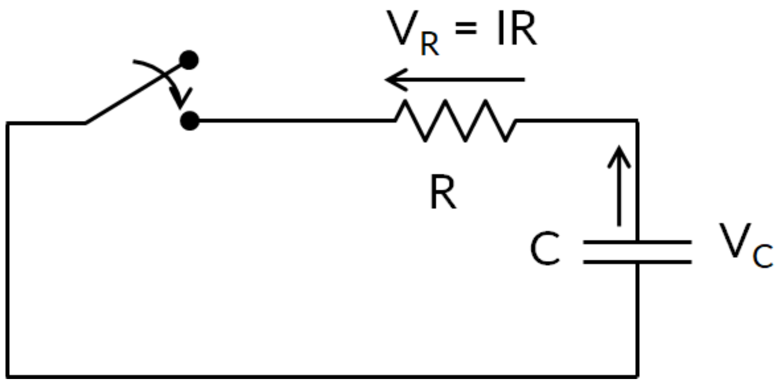


Example: Discharging a capacitor

- A 2 µF capacitor previously charged to 40 Volts is to be discharged through a resistance of 5 kΩ. Find the voltage across the terminals of the capacitor at the end of 18 ms.
- **Solution:** Using the formula for a discharging capacitor,

$$V_C = Ve^{-\frac{t}{RC}} \qquad \tau = RC = 10\,ms$$

$$V_C = 40e^{-\frac{18}{10}} = 6.61\,V$$



Example: Discharging a capacitor

- A 1000 μ F capacitor previously charged to 80 Volts is

to be discharged through a resistance of 20 kΩ. Find the voltage across the terminals of the capacitor at the end of 15 seconds.

Summary

- Capacitors are energy storage devices.
- Capacitor stores energy in electric field
- A capacitor act like an open circuit at steady state when a DC voltage or current has been applied.
- Capacitor behaves like a open circuit at very low frequencies and as a short circuit at higher frequencies
- Reactance of a capacitor decreases as frequency of operation increases
- In a capacitor, current leads voltage by a phase $\pi/2$ (voltage or lags current by a phase $\pi/2$)

Summary

- The voltage across a capacitor must be a continuous function; the current flowing through a capacitor can be discontinuous.

$$i_C = C\frac{dv_C}{dt} \qquad v_C = \frac{1}{C}\int_{t_o}^{t_1} i_C\,dt$$

- RC time constant is a time constant in which the charge on capacitor can increase or decrease by a factor e

- The equations for equivalent capacitance for

capacitors in parallel

$$C_{eq} = \sum_{p=1}^{P} C_P$$

capacitors in series

$$C_{eq} = \left[\sum_{s=1}^{S} \frac{1}{C_s}\right]^{-1}$$

Inductors

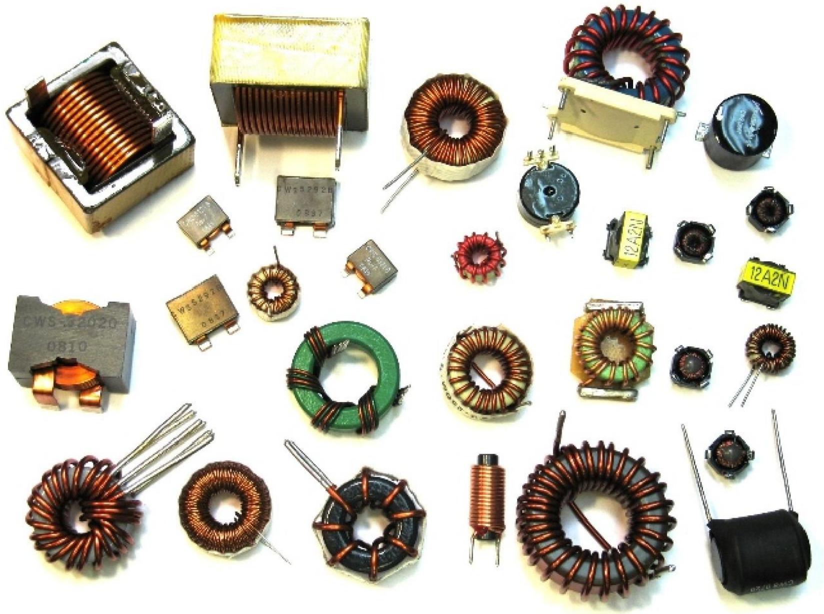
Energy Storage Devices

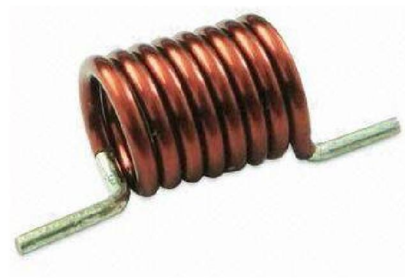
Objective of Lecture

- Describe
  - The construction of an inductor
  - How energy is stored in an inductor
  - The electrical properties of an inductor
    - Relationship between voltage, current, and inductance; power; and energy
  - Equivalent inductance when a set of inductors are in series and in parallel

Inductors

- Generally formed by a coil of conducting wire
  - Conducting wire is usually wrapped around a solid core.
  - In the absence of a core, the inductor is said to have an ‘air core’.



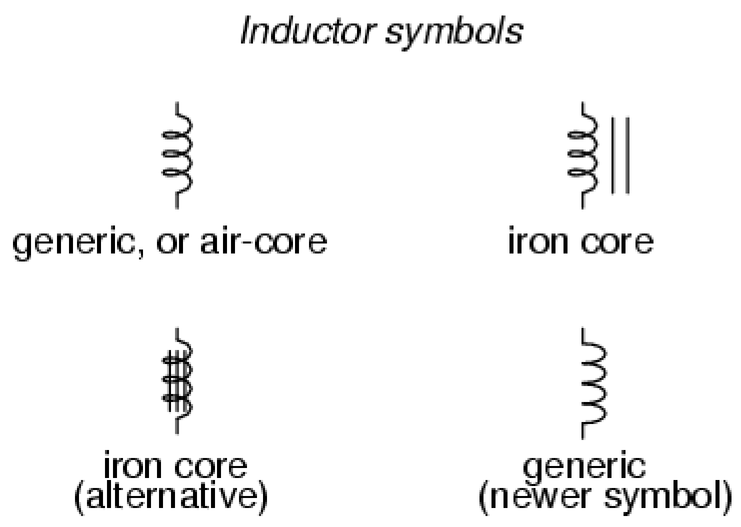


https://is.alicdn.com/img/pb/859/856/326/326856859_550.jpg
https://www.autodesk.com/products/eagle/blog/wp-content/uploads/2017/06/Inductors-Group.jpg

Circuit Symbols

- L , inductance, has the units of Heny (H)

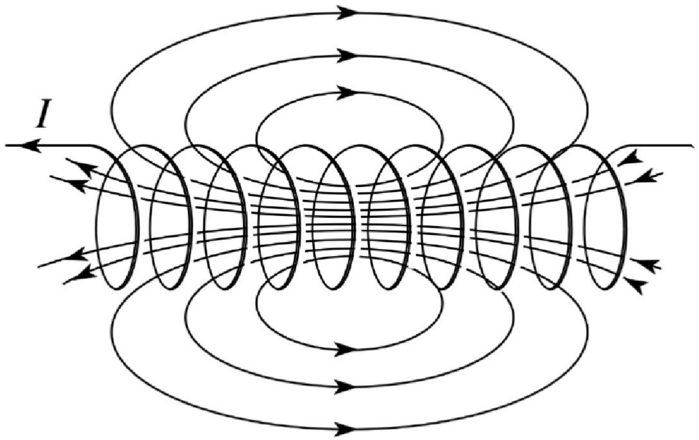
$$1\ H = 1\ \frac{Vs}{A}$$

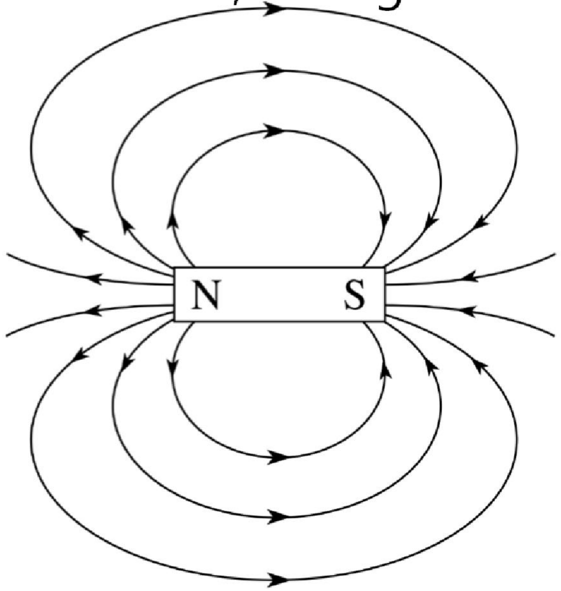


https://www.ibiblio.org/kuphaldt/electricCircuits/DC/00355.png

Inductor

- Inductor stores energy in an magnetic field created by an electric current flowing through it
  - Inductor opposes change (chokes) in current flowing through a conductor.
    - Current through an inductor is continuous; voltage can be discontinuous.



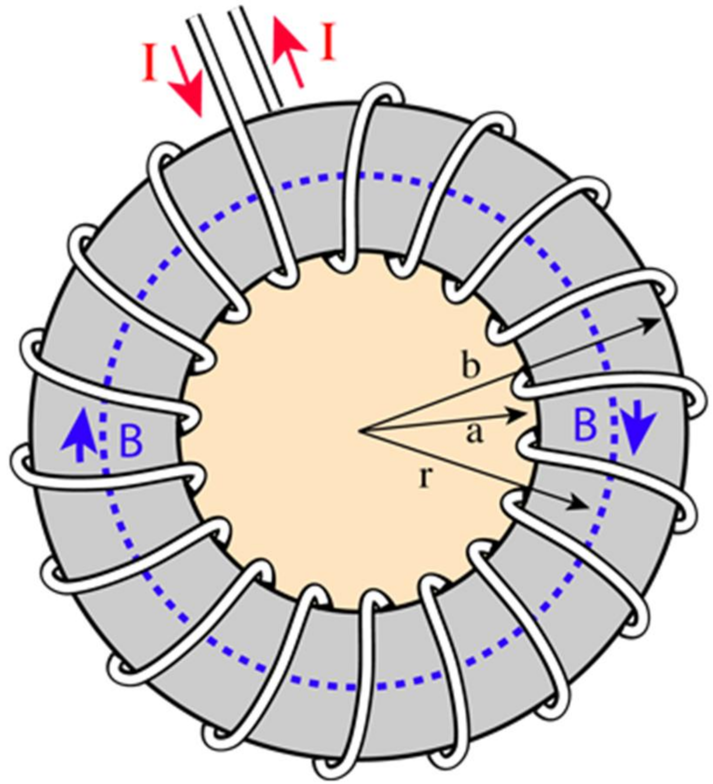


http://www.physics.brocku.ca/PPLATO/h-flap/phys4_4f_4.png

Calculations of L

For a solenoid (toroidal inductor)

$$L = \frac{N^2 \mu A}{\ell} = \frac{N^2 \mu_r \mu_o A}{\ell}$$



N is the number of turns of wire

A is the cross-sectional area of the toroid in m^2.

μ_r is the relative permeability of the core material

μ_o is the vacuum permeability ($4\pi \times 10^{-7}$ H/m)

ℓ is the length of the wire used to wrap the toroid in meters

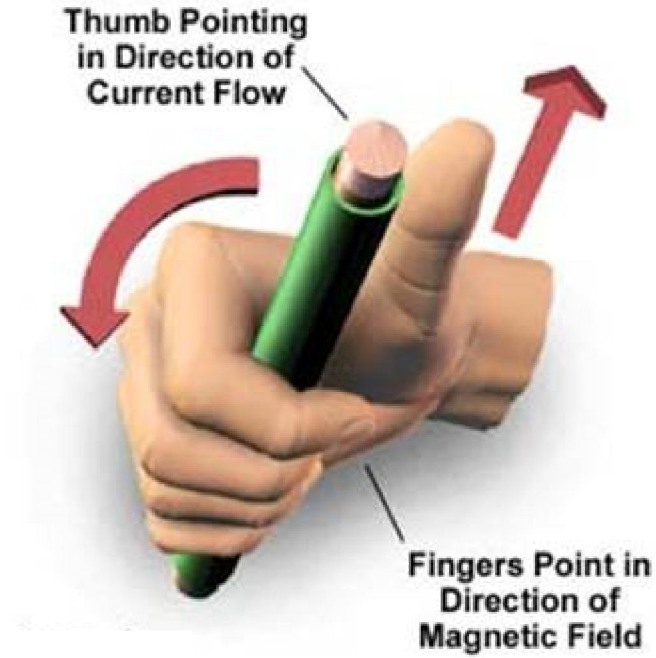
http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/imgmag/tor.png

Wire

- A bare wire has its share of inductance

$$L = \ell\left[\ln\left(4\frac{\ell}{d}\right) - 1\right]\left(2x10^{-7}\right)H$$

d is the diameter of the wire in meters.

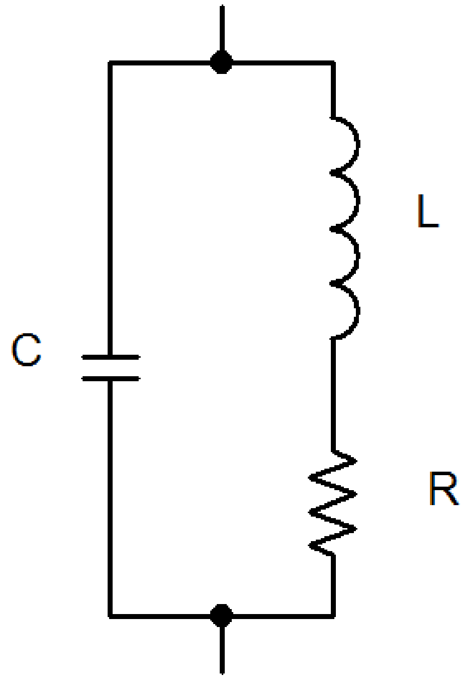


Properties of an Inductor

- Inductor acts like an short circuit in steady state
- Current through an inductor must be continuous, meaning there are no abrupt changes to the current but there can be abrupt changes in the voltage across an inductor.
- No energy or power is dissipated by an ideal inductor. Ideal inductor absorbs energy or power from the circuit when storing energy and restores energy into circuit while discharging

Properties of a Real Inductor

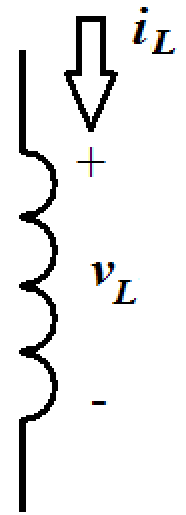
- Due to resistive losses and capacitive coupling between turns of wire



Sign Convention

- When current flows into the positive side of the voltage across the inductor, the current is positive and the inductor is dissipating power

- When an inductor releases energy back into the circuit, the sign of the current is negative.



Current - Voltage Relationships

$$v_L = L\frac{di}{dt}$$

$$i_L = \frac{1}{L}\int_{t_o}^{t_1} v_L dt$$

- Let

$$i_L = I_0 sin(\omega t)$$

$$v_L = L\frac{d}{dt}(I_0 sin(\omega t))$$

$$= \omega LI_0 \sin(\omega t)$$

$$= \omega LI_0 \sin\left(\frac{\pi}{2} - \omega t\right)$$

- Voltage across inductor leads current through inductor by $\frac{\pi}{2}$
- What is the significance of above statement?

Current-Voltage Relationships

- Let $$i_L = I_0 e^{j\omega t}$$

$$v_L = L\frac{d}{dt}\left(I_0 e^{j\omega t}\right) = j\omega L I_0 e^{j\omega t}$$

$$v_L = L\frac{di}{dt}$$

$$i_L = \frac{1}{L}\int_{t_o}^{t_1} v_L dt$$

- Re-writing the equation (similar to that of V= IR)

$$v_L = j\omega L i_L = jX_L i_L = Z_L i_c, X_L = \omega L$$

- X_L is called as reluctance (in ohm) of an inductor
  - When $\omega = 0$, $X_L = 0$, means reactance is zero → inductor behaves like a short at DC
  - When $\omega = \infty$, $X_L = \infty$, means inductor behaves like a open circuit at higher frequencies
  - Frequency dependent electrical behavior of inductance on circuit

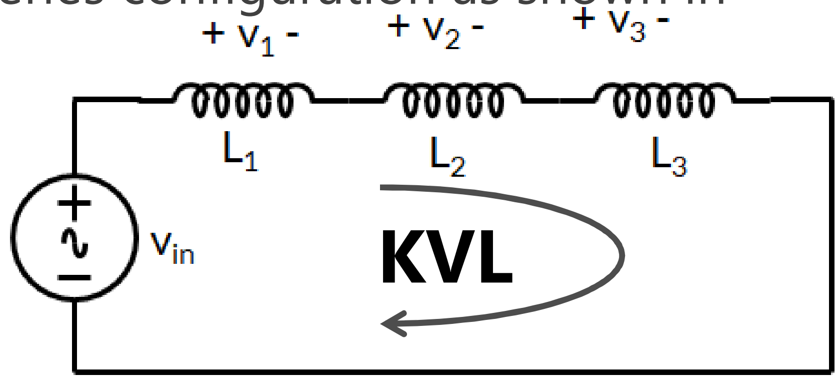
Power and Energy

$$p_L = v_L i_L = L i_L \int_{t_o}^{t_1} i_L dt$$

$$w = \int_{t_o}^{t_1} L \frac{di_L}{dt} i_L dt = L \int_{t_o}^{t_1} i_L di_L$$

Inductors in Series

- Consider inductors connected in a series configuration as shown in the circuit



- Applying KVL

$$V_{in} = V_1 + V_2 + V_3$$

- Noting the relation between voltage across the inductor and current through inductor, $V_1 = L_1 \frac{di}{dt}, V_2 = L_2 \frac{di}{dt}, V_3 = L_3 \frac{di}{dt}$
- If L_{eq} is the total inductance of the circuit, then

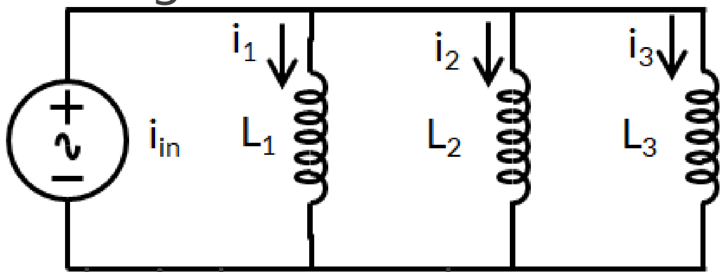
$$V_{in} = L_{eq} \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3$$

$$L_{eq} = \sum_{s=1}^{S} L_s$$

Inductors in Parallel

- Consider inductors connected in a parallel configuration as shown in the circuit



- Applying KCL $\quad i_{in} = i_1 + i_2 + i_3$
- Noting the relation between voltage across the inductor and current through inductor,

$$i_1 = \frac{1}{L_1}\int_{t_o}^{t_1} v\,dt,\; i_2 = \frac{1}{L_2}\int_{t_o}^{t_1} v\,dt,\; i_3 = \frac{1}{L_3}\int_{t_o}^{t_1} v\,dt$$

- If L_{eq} is the total inductance of the circuit, then

$$i_{in} = \frac{1}{L_{eq}}\int_{t_o}^{t_1} v\,dt = \frac{1}{L_1}\int_{t_o}^{t_1} v\,dt + \frac{1}{L_2}\int_{t_o}^{t_1} v\,dt + \frac{1}{L_3}\int_{t_o}^{t_1} v\,dt$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \qquad L_{eq} = \left[\sum_{p=1}^{n} \frac{1}{L_p}\right]^{-1}$$

General Equations for L_{eq}

- Series Combination
- If m inductors are in series, then

$$L_{eq} = \sum_{s=1}^{S} L_s$$

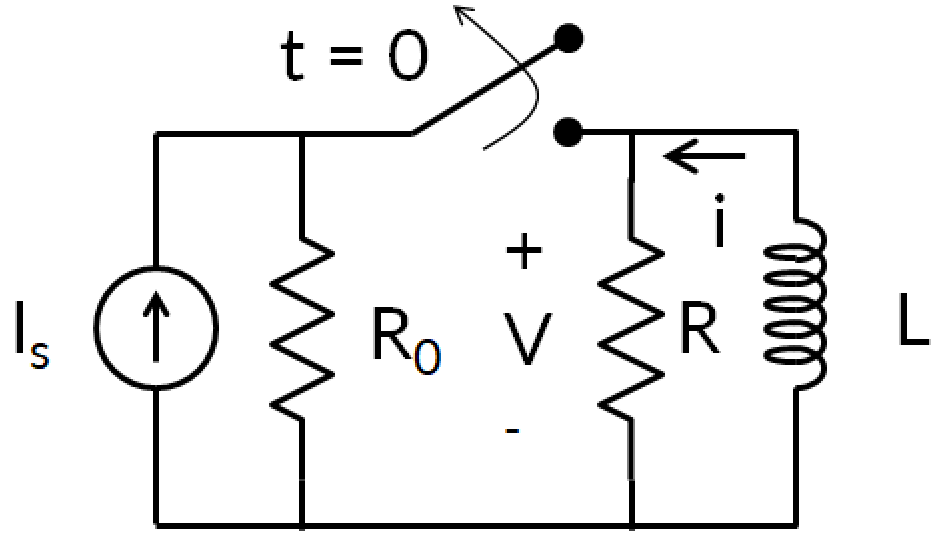
- Parallel Combination
- If n inductors are in parallel, then:

i

$$L_{eq} = \left[\sum_{p=1}^{n} \frac{1}{L_p} \right]^{-1}$$

Current through an Inductor

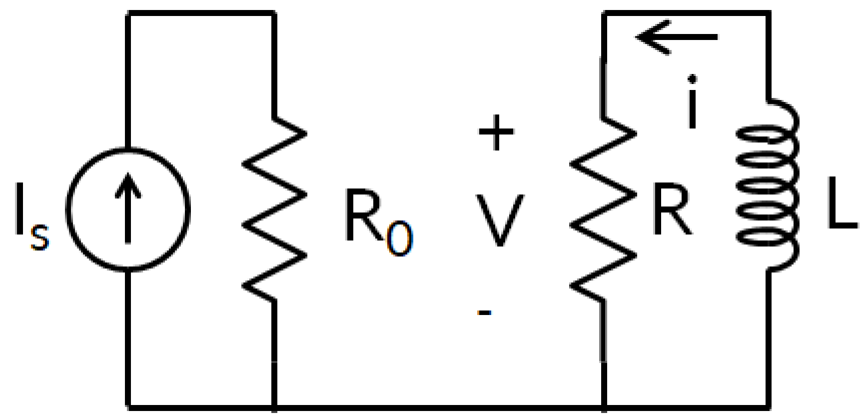
- Consider the following circuit, for which the switch is closed for *t<0*, and at *t = 0*, the switch is opened.



- The dc voltage V across the resistor R, supplies a constant current to the RL circuit

Current through an Inductor

- For $t \leq 0$, $i(t) = I_0$
- For $t \geq 0$, the circuit reduce to



- Note:
  - 0^-: is used to denote the time just prior to switching.
  - $0^+$is used to denote the time immediately after switching.

Discharge current through an Inductor

- Applying KVL to the circuit:
- $v(t)+Ri(t)=0$ $\rightarrow L\dfrac{di(t)}{dt}+Ri(t)=0$
- $L\dfrac{di(t)}{dt}=-Ri(t)$ $\rightarrow \dfrac{di(t)}{i(t)}=-\dfrac{R}{L}dt$
- Thus, the current is given by

$$\ln\frac{i(t)}{i(0)}=-\frac{R}{L}t$$

Discharge Current through an Inductor

- Finally, the discharge current through the inductor (de-energizing the inductor through a current) is given by

$$i(t) = i(0)e^{-\frac{R}{L}t} = I_0 e^{-\frac{R}{L}t}$$

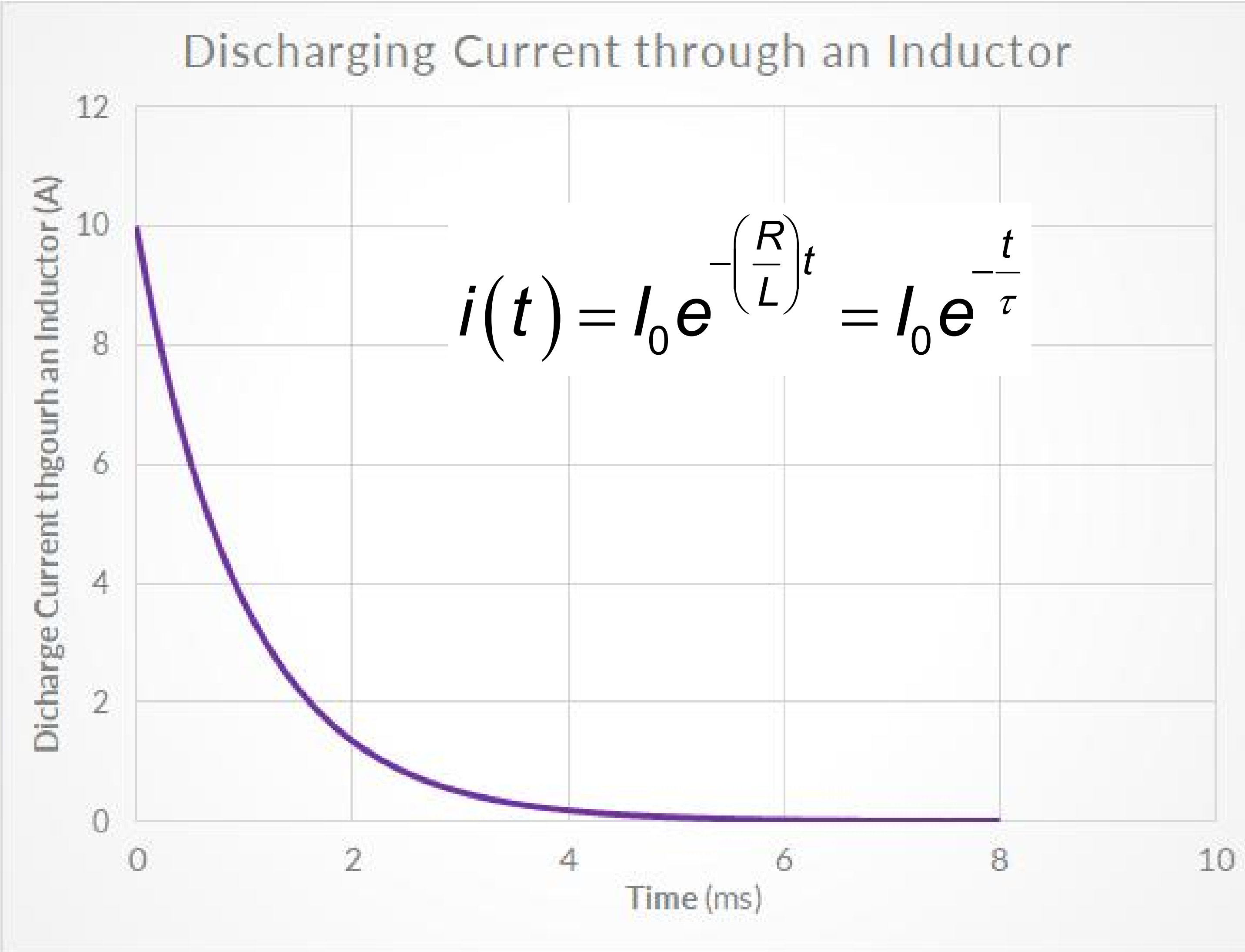
- The voltage v(t) across the resistor R is given by

$$v(t) = i(t)R = I_0 \mathrm{R} e^{-\left(\frac{R}{L}\right)t}$$

- Time constant τ is given by

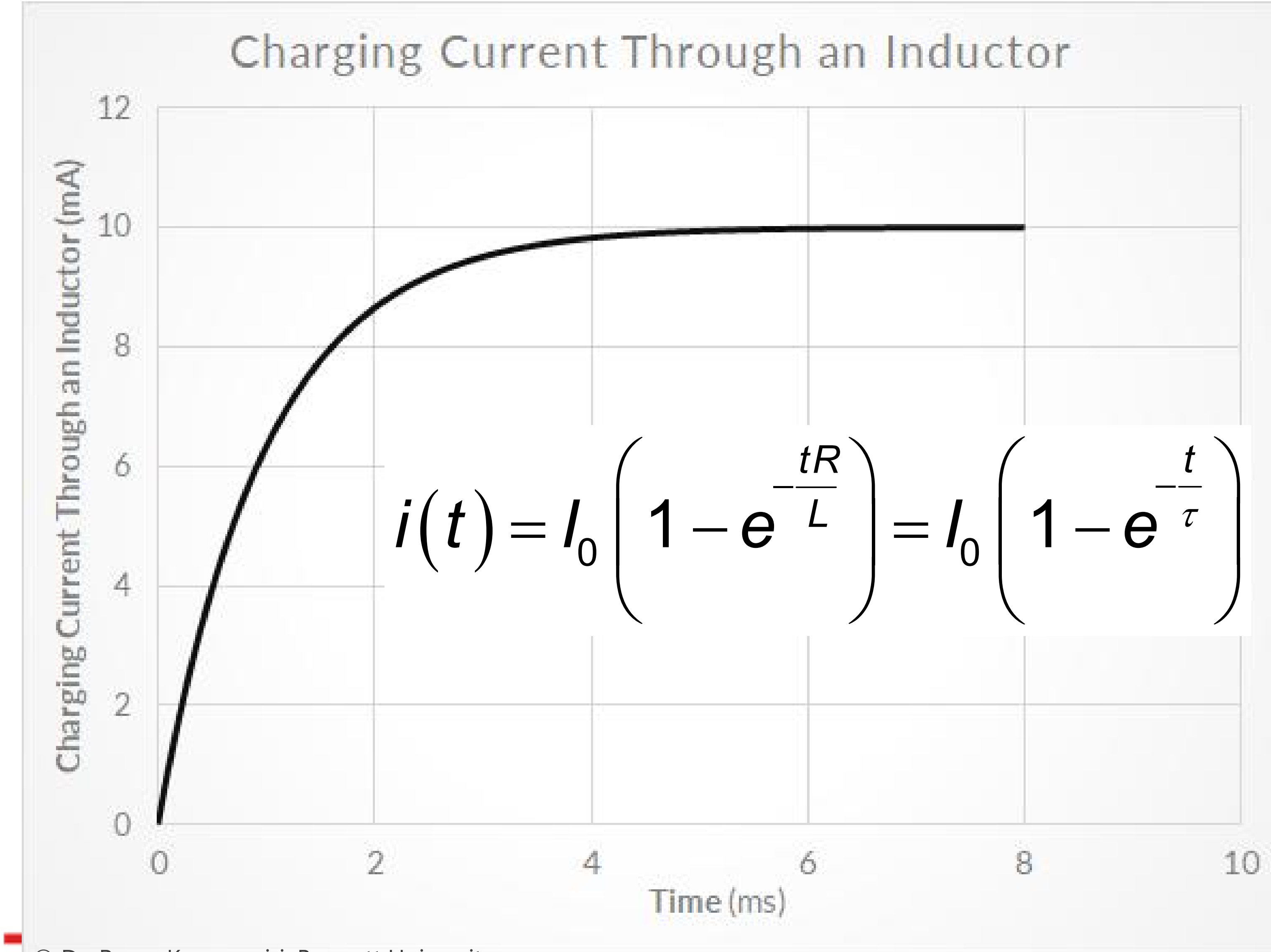
$$\tau = \frac{L}{R}$$

Discharge through Inductor



$$i(t) = I_0 e^{-\left(\frac{R}{L}\right)t} = I_0 e^{-\frac{t}{\tau}}$$

Charging Current through an Inductor



$$i(t) = I_0\left(1 - e^{-\frac{tR}{L}}\right) = I_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

Summary

- Inductor is a energy storage device
- Inductors store energy in a magnetic field
- An ideal inductor acts like a short circuit at low frequencies and open circuit at higher frequencies
- Reluctance of an inductor increases as frequency increases
- In an inductor, voltage leads current by a phase ${}^{\pi}/_{2}$ (or current lags voltage by a phase ${}^{\pi}/_{2}$)
- The current through an inductor must be continuous; the voltage across an inductor can be discontinuous.

Summary

- Inductors store energy in a magnetic field
- An ideal inductor acts like a short circuit at low frequencies and open circuit at higher frequencies
- Reluctance of an inductor increases as frequency increases
- The current through an inductor must be continuous; the voltage across an inductor can be discontinuous.
- The equation for equivalent inductance for

inductors in series

$$\boxed{L_{eq} = \sum_{s=1}^{m} L_s}$$

inductors in parallel

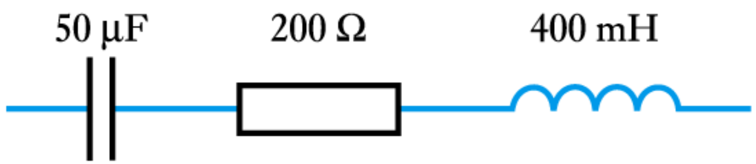
$$\boxed{L_{eq} = \left[\sum_{p=1}^{n} \frac{1}{L_p} \right]^{-1}}$$

Impedance Calculation

Example 1

- Calculate impedance of the following circuit at a frequency of 50 Hz.
- At 50Hz, the angular frequency $\omega = 2\pi f = 2\times\pi\times 50 = 314$ rad/s

$$Z_c = -jX_C = \frac{1}{j\omega C} = \frac{1}{j314\times 50\times 10^{-6}F} = -j63.7\,\Omega$$



$$Z_L = jX_L = j\omega L = j314\times 400\times 10^{-3} = j125.6\,\Omega$$

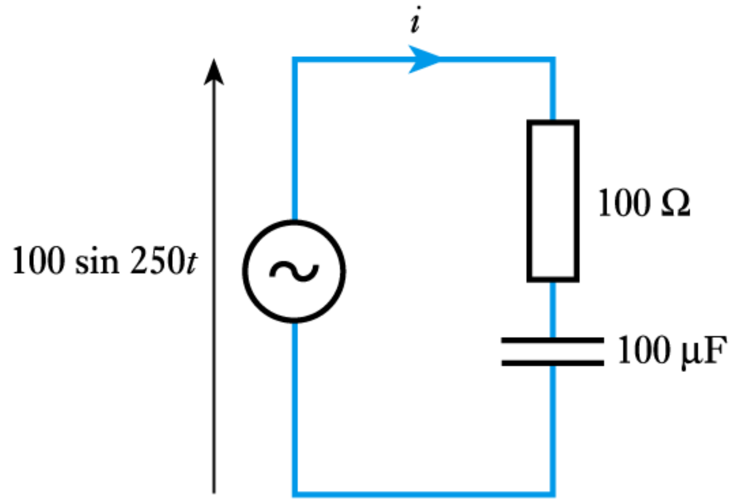
- The capacitor, resistor and inductor are in series,

$$\mathbf{Z} = \mathbf{Z_C} + \mathbf{Z_R} + \mathbf{Z_L} = R + \mathrm{j}\left(X_L - X_C\right) = R + \mathrm{j}\left(\omega L - \frac{1}{\omega C}\right)$$

$$\mathbf{Z} = 200 + \mathrm{j}\left(314\times 400\times 10^{-3} - \frac{1}{314\times 50\times 10^{-6}}\right) = \left(200 + \mathrm{j}62\right)\ \Omega$$

Example 2

- Calculate the current *i* through the circuit.
- **Solution:** $i = \dfrac{V}{Z}$
- ω = 250 rad/sec



$$\mathbf{Z} = R - jX_C = R - j\frac{1}{\omega C}$$

$$\mathbf{Z} = 100 - j\frac{1}{250 \times 10^{-4}} = 100 - j40\,\Omega$$

$$|\mathbf{Z}| = \sqrt{100^2 + 40^2} = 107.7\ \Omega$$

$$\angle\mathbf{Z} = \tan^{-1}\frac{-40}{100} = -21.8^\circ$$

$$\mathbf{Z} = 107.7\angle -21.8^\circ$$

$$i = \frac{V}{\mathbf{Z}} = \frac{100\angle 0}{107.7\angle -21.8} = 0.93\angle 21.8^\circ$$