Department of Mathematics
Bennett University
EMAT101L: July-December, 2018
Tutorial Sheet-3 (Multivariable Calculus)

1) Evaluate the following iterated integrals:
(i) $\int_0^{\ln 2}\int_1^{\ln 5} e^{2x+y}\,dy\,dx$

(ii) $\iint_{\mathcal{R}} \frac{\sqrt{x}}{y^2}\,dA$ $\qquad \mathcal{R}:\ 0 \le x \le 4$ and $1 \le y \le 2$

(iii) $\iint_{\mathcal{R}} y\,\sin(x+y)\,dA$ $\qquad \mathcal{R}:\ [-\pi, 0] \times [0, \pi]$

(iv) $\iint_{\mathcal{R}} \frac{y}{1+x^2y^2}\,dA$ $\qquad \mathcal{R}:\ [0,1] \times [0,1]$

Answers: $(i)\ \frac{3}{2}(5-e)$, $\quad (ii)\ \frac{8}{3}$, $\quad (iii)\ 4$, $\quad (iv)\ \frac{\pi}{4} - \frac{1}{2}\ln 2$.

2) Write an iterated integral for $\iint_{\mathcal{R}} dA$ over the following region $\mathcal{R}$ using both vertically and horizontally simple regions:
(i) Bounded by $x = 0$, $y = 1$ and $y = \tan x$.
(ii)Bounded by $x = 0$, $y = 0$, $y = 1$ and $y = \ln x$.

3) Use the given transformations to transform the integrals and evaluate them:

(a) $u = 3x + 2y, v = x + 4y$ and $I = \displaystyle\iint_R (3x^2 + 14xy + 8y^2)\,dA$ where R is the region in the first quadrant bounded by the lines $y + \frac{3}{2}x = 1, y + \frac{3}{2}x = 3, y + \frac{1}{4}x = 0$, and $y + \frac{1}{4}x = 1$.

(b) $u = x + 2y, v = x - y$ and $I = \displaystyle\int_0^{2/3}\int_y^{2-2y} (x + 2y)e^{(y-x)}dA$

(c) $u = xy,\ v = x^2 - y^2$ and $I = \displaystyle\iint_R (x^2 + y^2)dA$, where R is the region bounded by $xy = 1, xy = 2, x^2 - y^2 = 1$ and $x^2 - y^2 = 2$.

4) Find the area of the following:

(a) The region lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$ in the first quadrant.

(b) The region common to the interiors of the cardioids $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$.