Name of student: ....

Batch No:.......... **Enrollment No.** ....

COURSE NAME: LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS

B.TECH **TUTORIAL QUIZ-2** **FALL SEMESTER 2018-19**

COURSE CODE : EMAT102L MAX. TIME: 20 min

COURSE CREDIT: 3-1-0 MAX. MARKS: 10

1. Let $\mathbb{V} = \mathbb{R}^3(\mathbb{R})$ be a vector space and $S = \{(1,1,0), (0,-1,1), (1,0,1)\}$. Find the condition on a, b, c such that $(a,b,c) \in span(S)$. [2]
   Solution:

2. Let $\mathbb{V}$ be the set of real numbers. Is $\mathbb{V}$ a vector space over the set of complex numbers($\mathbb{C}$). Give explanation to justify your answer. [2]
   Solution:

3. Let $\mathbb{V} = \{(x_1, \ldots\ldots, x_{10}) \in \mathbb{R}^{10} | x_1 = x_2 = 2x_3, x_7 - x_8 - x_9 - x_{10} = 0\}$. Then find the $\dim\mathbb{V}$. [3]

Solution:

4. Suppose $\mathbb{U}$ and $\mathbb{W}$ are distinct four-dimensional subspaces of a vector space $\mathbb{V}$, where $\dim\mathbb{V} = 6$. Find the possible dimensions of $\mathbb{U} \cap \mathbb{W}$. [3]

Solution:

Solutions - Tutorial Quiz Test 2

Sol^n 1

$$V = \mathbb{R}^3(\mathbb{R})$$

$$S = \{(1,1,0), (0,-1,1), (1,0,1)\}$$

$\because$ $(a,b,c) \in L(S)$

$$\Rightarrow \quad (a,b,c) = \alpha(1,1,0) + \beta(0,-1,1) + \gamma(1,0,1)$$

$$(a,b,c) = (\alpha+\gamma, \alpha-\beta, \beta+\gamma)$$

$$a = \alpha+\gamma \quad \text{—— (1)}$$

$$b = \alpha-\beta \quad \text{—— (2)}$$

$$c = \beta+\gamma \quad \text{—— (3)}.$$

On adding equ^n (2) & (3).

$$b+c = \alpha+\gamma$$

from equ^n (1).

$$\boxed{b+c = a}$$

Sol^n 2

V is not a vector space over $\mathbb{C}$.

$\because$ $\alpha = i$, $x \in \mathbb{R}$

but $\alpha x = ix \notin \mathbb{R}$.

Sol^n 3

$$V = \{(x_1, x_2, x_3, \ldots, x_{10}) \in \mathbb{R}^{10};\ x_1 = x_2 = 2x_3,\ x_7 - x_8 - x_9 - x_{10} = 0\}$$

$$= \{(2x_3, 2x_3, x_3, x_4, x_5, x_6, x_8+x_9+x_{10}, x_8, x_9, x_{10}) \in \mathbb{R}^{10};\ x_3, x_4, x_5, x_6, x_8, x_9, x_{10} \in \mathbb{R}\}$$

$$= \{x_3\underbrace{(2,2,1,0,0,0,0,0,0,0)}_{a_1} + x_4\underbrace{(0,0,0,1,0,0,0,0,0,0)}_{a_2}$$
$$+ x_5\underbrace{(0,0,0,0,1,0,0,0,0,0)}_{a_3} + x_6\underbrace{(0,0,0,0,0,1,0,0,0,0)}_{a_4}$$
$$+ x_8\underbrace{(0,0,0,0,0,0,1,1,0,0)}_{a_5} + x_9\underbrace{(0,0,0,0,0,0,1,0,1,0)}_{a_6}$$
$$+ x_{10}\underbrace{(0,0,0,0,0,0,1,0,0,1)}_{a_7};\ x_3, x_4, x_5, x_6, x_8, x_9, x_{10} \in \mathbb{R}\}$$

$V = \text{span}\{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$

and $\quad a_i \;\; \forall \;\; 1 \le i \le 7$ are L.I. vectors.

$\Rightarrow \quad \dim V = 7.$

Sol^n 4 $\quad$ Since U, W are distinct subspaces of V.

$$\Rightarrow \quad U \subset U+W$$
$$W \subset U+W$$

$\Rightarrow \quad \dim(U+W) > 4 \qquad [\because \dim U = \dim W = 4]$

Since $U+W$ is a subspace of V.

$\Rightarrow \quad \dim(U+W) \le 6 \qquad [\because \dim V = 6]$

So we have two cases.

(i) $\dim(U+W) = 5$

(ii) $\dim(U+W) = 6$

Now when $\dim(U+W) = 5$

$$\dim(U \cap W) = 4+4-5 = 3$$

when $\dim(U+W) = 6$

$$\dim(U \cap W) = 4+4-6 = 2$$