# Set Theory

# What is a set?

- A set is a group of “objects”
  - People in a class: { Alice, Bob, Chris }
  - Classes offered by a department: { CS 101, CS 202, … }
  - Colors of a rainbow: { red, orange, yellow, green, blue, purple }
  - States of matter { solid, liquid, gas, plasma }
  - States in the US: { Alabama, Alaska, Virginia, … }
  - Sets can contain non-related elements: { 3, a, red, Virginia }

- Although a set can contain (almost) anything, we will most often use sets of numbers
  - All positive numbers less than or equal to 5: {1, 2, 3, 4, 5}
  - A few selected real numbers: { 2.1, π, 0, -6.32, e }

# Set properties 1

- Order does not matter
  - We often write them in order because it is easier for humans to understand it that way
  - {1, 2, 3, 4, 5} is equivalent to {3, 5, 2, 4, 1}

- Sets are notated with curly brackets

# Set properties 2

- Sets do not have duplicate elements
  - Consider the set of vowels in the alphabet.
    - It makes no sense to list them as {a, a, a, e, i, o, o, o, o, o, u}
    - What we really want is just {a, e, i, o, u}
  - Consider the list of students in this class
    - Again, it does not make sense to list somebody twice
- Note that a list is like a set, but order does matter and duplicate elements are allowed
  - We won’t be studying lists much in this class

# Specifying a set 1

- Sets are usually represented by a capital letter (A, B, S, etc.)
- Elements are usually represented by an italic lower-case letter (*a*, *x*, *y*, etc.)
- Easiest way to specify a set is to list all the elements: A = {1, 2, 3, 4, 5}
  - Not always feasible for large or infinite sets

# Specifying a set 2

- Can use an ellipsis (…): B = {0, 1, 2, 3, …}
  - Can cause confusion. Consider the set C = {3, 5, 7, …}. What comes next?
  - If the set is all odd integers greater than 2, it is 9
  - If the set is all prime numbers greater than 2, it is 11

- Can use set-builder notation
  - D = $\{x : x \text{ is prime and } x > 2\}$
  - E = $\{x : x \text{ is odd and } x > 2\}$
  - The colon (:) means “such that”
  - Thus, set D is read (in English) as: “all elements $x$ such that $x$ is prime and $x$ is greater than 2”