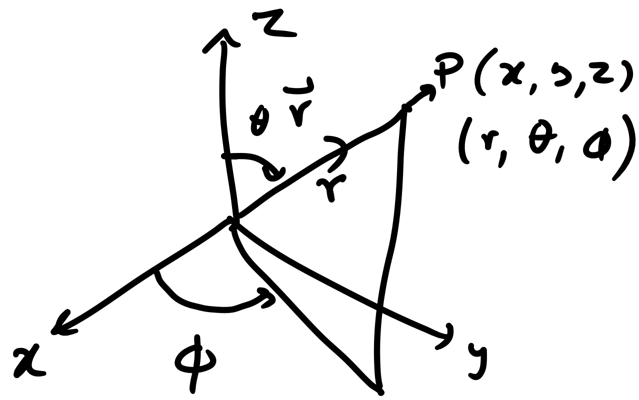
Liking & disliking are conditioned habits.

E. Easwaran

Coordinate Systems

Spherical Polar



$$0 \leqslant r \leqslant \infty$$

$$0 \leqslant \theta \leqslant \pi$$

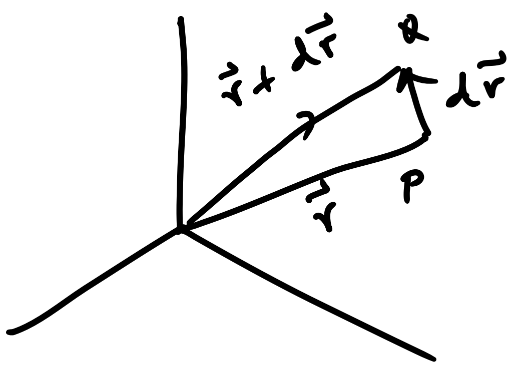
$$0 \leqslant \phi \leqslant 2\pi$$

$$\vec{r} = \hat{x}\, x + \hat{y}\, y + \hat{z}\, z$$

$$d\vec{r} = \hat{x}\, dx + \hat{y}\, dy + \hat{z}\, dz$$

$$\vec{r} = \hat{r}\, r$$

$$\boxed{d\vec{r} = \hat{r}\, dr + r\, d\theta\, \hat{\theta} + r \sin\theta\, d\phi\, \hat{\phi}}$$



$$\vec{v} = \frac{d\vec{r}}{dt} = \hat{r}\frac{dr}{dt} + r\frac{d\theta}{dt}\hat{\theta} + r\sin\theta\frac{d\phi}{dt}\hat{\phi}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2 + r\sin\theta\,\dot{\varphi}^2\right)\hat{r}$$
$$+ \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\varphi}^2\sin\theta\cos\theta\right)\hat{\theta}$$
$$+ \left(r\sin\theta\,\ddot{\varphi} + 2r\dot{\theta}\dot{\varphi}\cos\theta + 2\dot{r}\dot{\varphi}\sin\theta\right)\hat{\varphi}$$

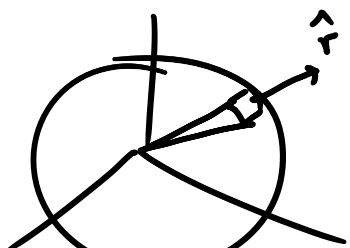
$\phi =$ constant $\Rightarrow$ $\dot{\varphi} = 0$ & $\ddot{\varphi} = 0$

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta}$$

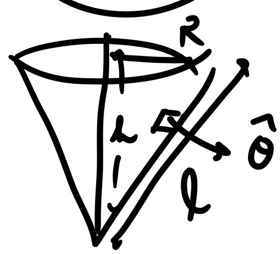
Centripetal   CORIOLIS

– Length elements $\quad dr, \; r\,d\theta, \; r\sin\theta\,d\phi$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad \hat{r} \quad\quad \hat{\theta} \quad\quad \hat{\phi}$



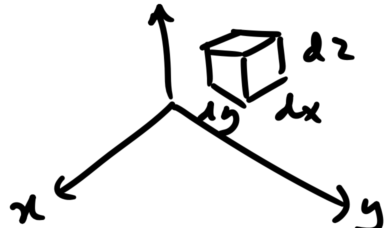
$$d\vec{A} = r^2\sin\theta\,d\theta\,d\varphi\,\hat{r}$$



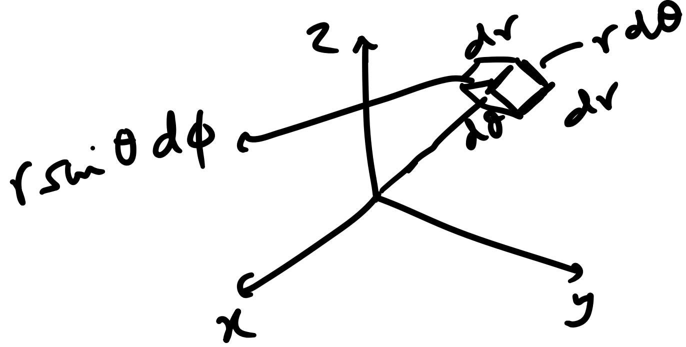
$$d\vec{A} = r\sin\theta\,dr\,d\varphi\,\hat{\theta}$$

$$A = \int_0^{l}\int_0^{2\pi} r\sin\theta\,dr\,d\varphi$$
$$= \pi l^2\sin\theta = \pi R\sqrt{h^2 + R^2}$$

Volume element = $dx\,dy\,dz$

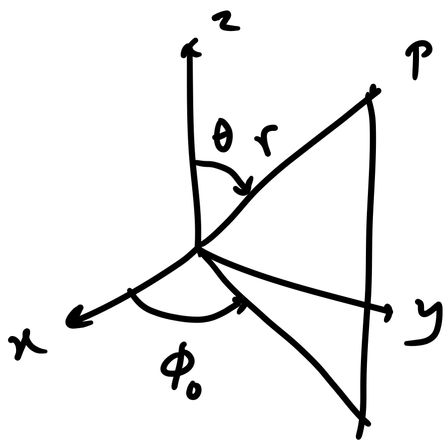


Volume element: $dV = r^2\sin\theta\,dr\,d\theta\,d\varphi$



$$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$$
$$\hat{\theta} = \cos\theta\cos\phi\,\hat{i} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$$
$$\hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

Q: Invert the equations & obtain $\hat{x}$, $\hat{y}$ & $\hat{z}$ in terms of $\hat{r}$, $\hat{\theta}$ & $\hat{\phi}$



$P(x_0, y_0, z_0)$
(r_0, θ_0, ϕ_0)

$$r_0 = \sqrt{x_0^2 + y_0^2 + z_0^2}$$

$$r_0\cos\theta = z_0$$

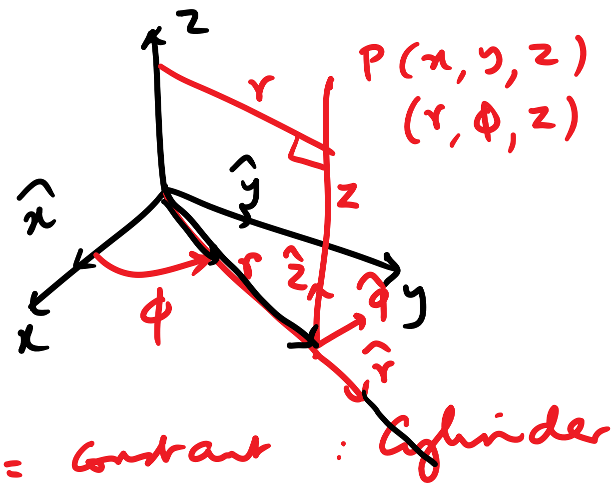
$$\theta_0 = \cos^{-1}\left(\frac{z_0}{\sqrt{x_0^2+y_0^2+z_0^2}}\right)$$

$$x_0 = r_0\sin\theta_0\cos\phi_0$$
$$y_0 = r_0\sin\theta_0\sin\phi_0$$

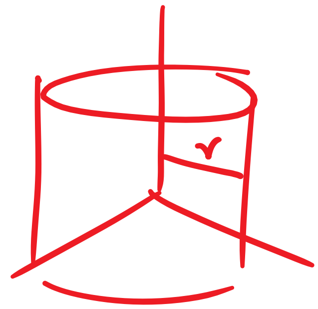
$$\tan\phi_0 = \frac{y_0}{x_0} \Rightarrow \phi_0 = \tan^{-1}\left(\frac{y_0}{x_0}\right)$$

Cylindrical Coordinate system

Cylindrical Coordinate system



r = constant : Cylinder



$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\vec{r}.\hat{x} = x = r\cos\phi$$

$$\vec{r}\cdot\hat{y} = y = r\sin\phi$$

$$\vec{r} = r\cos\phi\,\hat{x} + r\sin\phi\,\hat{y} + z\hat{z}$$

$$= r(\hat{x}\cos\phi + \hat{y}\sin\phi) + z\hat{z}$$

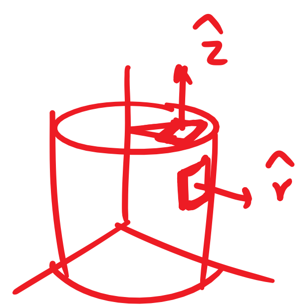
$$\boxed{\vec{r} = r\hat{r} + z\hat{z}}$$

$$d\vec{r} = \hat{r}\,dr + r\,d\hat{r} + \hat{z}\,dz$$

$$\boxed{d\vec{r} = \hat{r}\,dr + r\,d\phi\,\hat{\phi} + \hat{z}\,dz}$$

$$d\hat{r} = \cancel{\frac{\partial\hat{r}}{\partial r}}dr + \frac{\partial\hat{r}}{\partial\phi}d\phi + \cancel{\frac{\partial\hat{r}}{\partial z}}dz$$

$$= \hat{\phi}\,d\phi$$



Length elements: dr, $r\,d\phi$, dz

Area elements: $r\,d\phi\,dz$ $\hat{r}$

$dr\,dz$ $\hat{\phi}$

$r\,dr\,d\phi$ $\hat{z}$

Volume element: $r\,dr\,d\varphi\,dz$

VECTOR CALCULUS

$f(x)$: $\dfrac{df}{dx}$

GRADIENT : $f(x,y,z)$

$$\boxed{df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz}$$