Department of Mathematics, Bennett University
Engineering Calculus (EMAT101L)
Solutions for Tutorial Sheet 6

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1. (a) $\sin x = 1 - \frac{(x-\frac{\pi}{2})^2}{2!} + \frac{(x-\frac{\pi}{2})^4}{4!}$.

   (b) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5}$.

2. Remainder term here is $\frac{x^4 \sin c}{4!}$. So find δ such that $|\frac{x^4}{4!}| < 5 \times 10^{-4}$ in $|x| < \delta$. As for $|x| < \frac{3}{10}$, $|\frac{x^4}{4!}| < 5 \times 10^{-4}$, thus $\frac{3}{10}$ is the required δ.

3. $|R_n(x)| = \left|\frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}\right| < \frac{x^4}{4!} < \frac{1}{2^4 4!}$.

4. $|R_1(x)| = \left|\frac{f^{(2)}(c)}{2!}x^2\right| < \frac{x^2}{2! \times 4(1+c)^{\frac{3}{2}}} < \frac{(0.01)^2}{8}$.

5. (a) $R = 1$ and interval of convergence$=(1, 3]$.

   (b) $R = 5^{\frac{2}{3}}$ and interval of convergence$=(-2 - 5^{\frac{2}{3}}, -2 + 5^{\frac{2}{3}})$.

   (c) $R = \frac{1}{2}$ and interval of convergence$=(-\frac{1}{2}, \frac{1}{2})$.

6. (a) $\frac{1}{1+x} = \frac{d}{dx}\log(1+x) = \frac{d}{dx}(x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots) = 1 - x + x^2 - x^3 + \cdots$. Also $R = 1$.

   (b) $\sinh x = \frac{d}{dx}\left(\frac{e^x + e^{-x}}{2}\right)$. Now write down the series of $\frac{e^x+e^{-x}}{2}$ and differentiate.

   (c) $\cos^{-1} x = \int \frac{-1}{\sqrt{(1-x^2)}}dx$. Then write down the series of $\frac{-1}{\sqrt{(1-x^2)}}$ and integrate.