Solution - Tutorial sheet 1

1. Sample space $\Omega = \{(1,1), (1,2), \ldots, (1,6), (2,1), (2,2) \ldots\}$

Event A = Sum of numbers occurred in two dices is 4.

$= \{(1,3), (2,2), (3,1)\}$.

So

$$P(A) = \frac{n(A)}{n(\Omega)}$$

[As all the sample points are equally likely to occur.]

$$= \frac{3}{36} = \frac{1}{12}$$

2. (a) P(ticket drawn has number 2 or 4)

$= P\left(\text{ticket with no. 2 or 4} / \text{Urn 1}\right) P\left(\text{Urn 1 is chosen}\right) + P\left(\text{ticket with no. 2 or 4} / \text{Urn 2}\right) P(\text{Urn 2})$

$$= \frac{2}{4} \times \frac{1}{2} + \frac{2}{6} \times \frac{1}{2} = \frac{5}{12}$$

(b) Answer $= \frac{1}{8}$

(c) Answer $= \frac{5}{24}$

(Similar to part (a))

3. P(target is hit atleast once) = P(Jim hits or Bill hits)
$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad A \quad\quad\quad\quad B$

$= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

As A & B are independent events –

$$= P(A) + P(B) - P(A)P(B)$$

$$= 0.8 + 0.7 - 0.8 \times 0.7$$

$$= 0.94$$

4 Given that $P(A \cap B) = P(A)P(B)$

a
$$P(A \cap B^c) = P(A \cap (\Omega - B))$$

$$= P(A \cap \Omega - A \cap B)$$

$$= P(A - A \cap B)$$

$$A = A \cap \Omega = A \cap (B \cup B^c)$$

$$= (A \cap B) \cup (A \cap B^c)$$

$$P(A) = P(A \cap B) + P(A \cap B^c)$$ [$A \cap B$ & $A \cap B^c$ are disjoint sets.]

$$\Rightarrow P(A \cap B^c) = P(A) - P(A \cap B)$$

$$= P(A) - P(A)P(B)$$

$$= P(A)[1 - P(B)]$$

$$= P(A)P(B^c)$$

b Similar to part a.

Use $B = B \cap (A \cup A^c)$

(c) $A^c = A^c \cap \Omega$

$= A^c \cap (B \cup B^c)$

$= (A^c \cap B) \cup (A^c \cap B^c)$

$\Rightarrow P(A^c) = P(A^c \cap B) + P(A^c \cap B^c)$

$\Rightarrow P(A^c \cap B^c) = P(A^c) - P(A^c \cap B)$

$= P(A^c) - P(A^c)P(B)$ [using part (b)]

$= P(A^c)[1 - P(B)]$

$= P(A^c)\, P(B^c)$

5 (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)P(B)$

$= \frac{1}{3} + \frac{3}{4} - \frac{1}{3} \times \frac{3}{4} = \frac{5}{6}$

(b) $P(A / A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)}$

$= \frac{P(A)}{P(A \cup B)} = \frac{1/3}{5/6} = 2/5$

(c) Answer $= \frac{9}{10}$. (Similar to part (b))

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6 Consider following events:

R = the selected voter is from Republican.

D = " " " Democrates.

I = " " " Independent.

Also assume that

A = a selected voter in the region opposes military spending.

Thus

$$P(A) = P(A/R)P(R) + P(A/D)P(D) + P(A/I)P(I)$$

$$= .4 \times .6 + .65 \times .3 + .55 \times .1$$

$$= 0.49$$

7 (a) Consider events:

A = the selected IC is defective.

B1 = the selected IC is from supplier A.

B2 = " " " B.

B3 = " " " C.

(a) $$P(A) = P(A/B_1)P(B_1) + P(A/B_2)P(B_2) + P(A/B_3)P(B_3)$$

$$= 0.05 \times \frac{1000}{6000} + 0.1 \times \frac{2000}{6000} + 0.1 \times \frac{3000}{6000}$$

$$= 0.09167$$

(b) P(IC is from supplier A / IC is defective)

$= P(B_1/A)$

$= \dfrac{P(B_1 \cap A)}{P(A)}$

$= \dfrac{P(A/B_1)\,P(B_1)}{P(A)}$

$= \dfrac{0.05 \times \frac{1000}{6000}}{0.09167}$

$= 0.0909$

(8) Consider events –

A = aircraft is defective.

B = equipment is showing defects in the aircraft.

The asked probability is $P(A/B)$.

$P(A/B) = \dfrac{P(B/A)\,P(A)}{P(B)}$ [Baye's theorem]

Given that

$P(B/A) = 0.95$, $P(B/A^c) = 0.01$, $P(A) = 0.02$

$\Rightarrow P(A^c) = 1 - 0.02 = 0.98$

By total probability theorem —

$$P(B) = P(B/A)P(A) + P(B/A^c)P(A^c)$$

$$= 0.95 \times 0.02 + 0.01 \times 0.98$$

$$= 0.0288$$

So

$$P(A/B) = \frac{0.95 \times 0.02}{0.0288} \approx 0.66.$$

9 Similar to questions 6 & 7.

Answers → a 0.36

b 0.625