BENNETT UNIVERSITY
A TIMES GROUP INITIATIVE

Series and Parallel Connections

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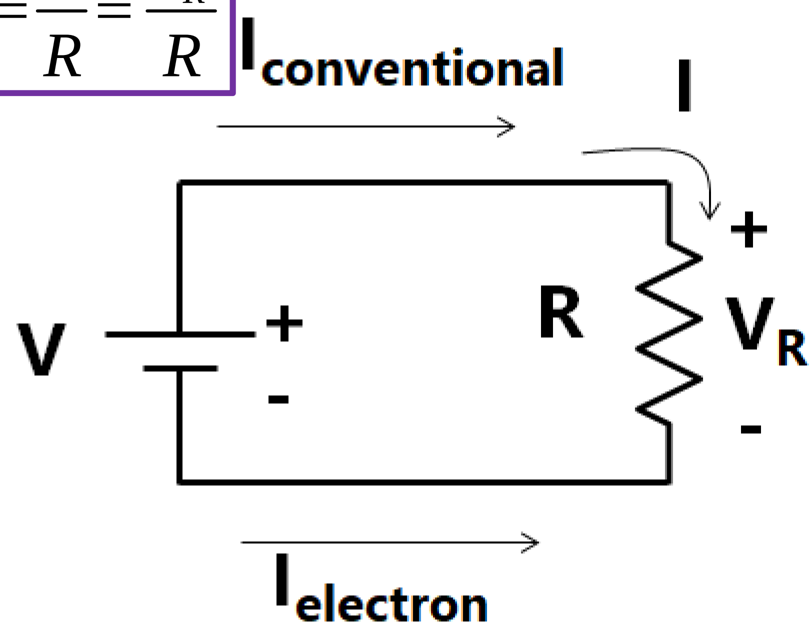
Direction of Current Flow: Voltage Source

- If the wires is an ideal conductor (having no opposition to current flow), then the potential difference across the resistor R (V_R) is equal to the supply voltage (V)
- By convention, the direction of conventional current flow and electron current flow are opposite

$$\boxed{I = \frac{V}{R} = \frac{V_R}{R}}$$

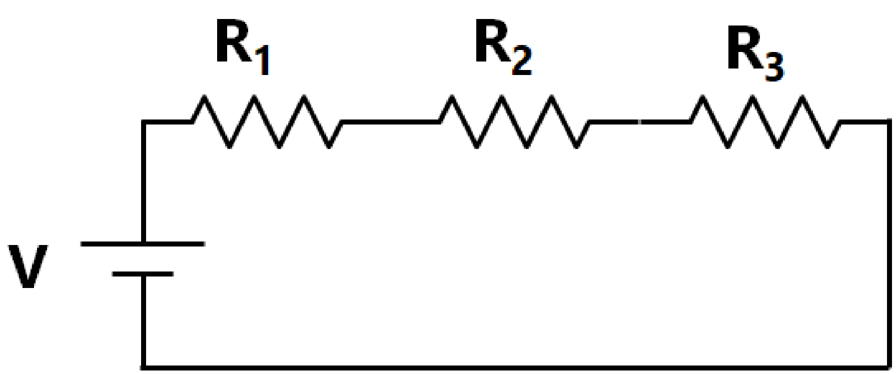
- Following the conventional current flow,
- a rise in the potential across the battery (- to +)
- a drop in the potential across the resistor (+ to -)



Series Connections

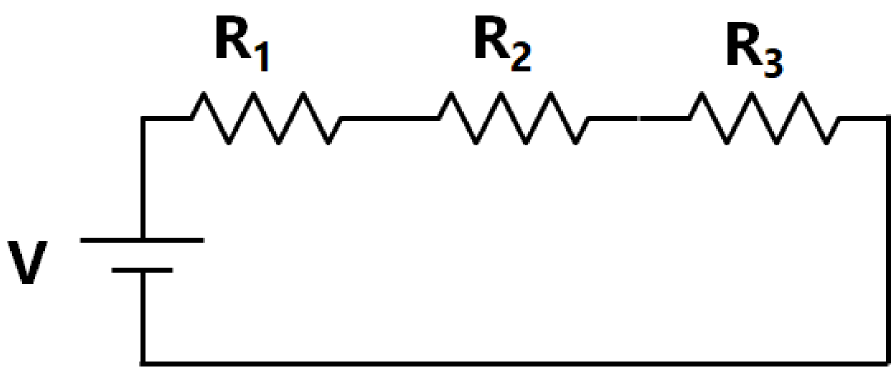
Series Connections

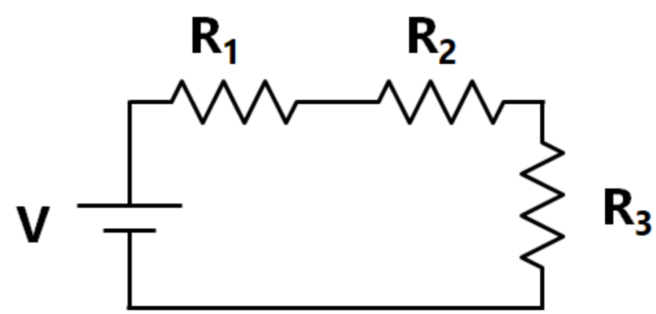
- Circuit is an arrangement of components (or circuit elements) that results in a continuous flow of charge, or current, through the configuration
- Series Configuration: The current is same in the every point in the circuit

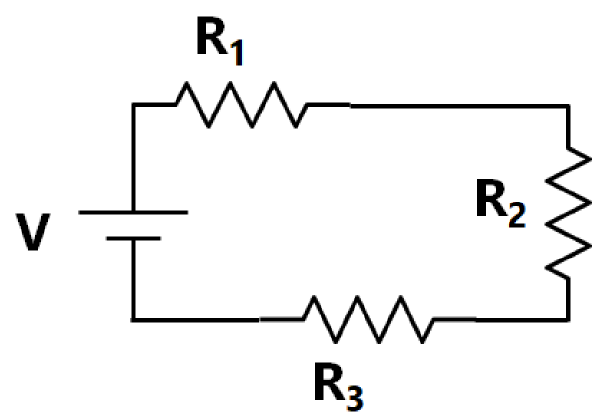


Resistors in Connected in Series

- Series Configuration: The current is same in the every point in the circuit
  - In a circuit, if two elements are in series, the current must be same
  - If the currents are same in two adjoining circuit branches, the elements may or may not be in series

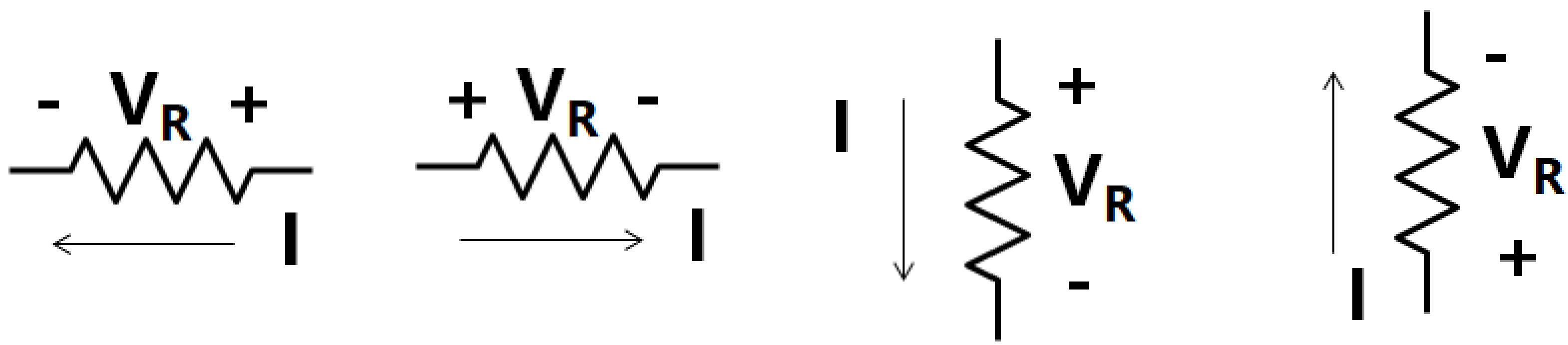






Direction of Current in a Resistor

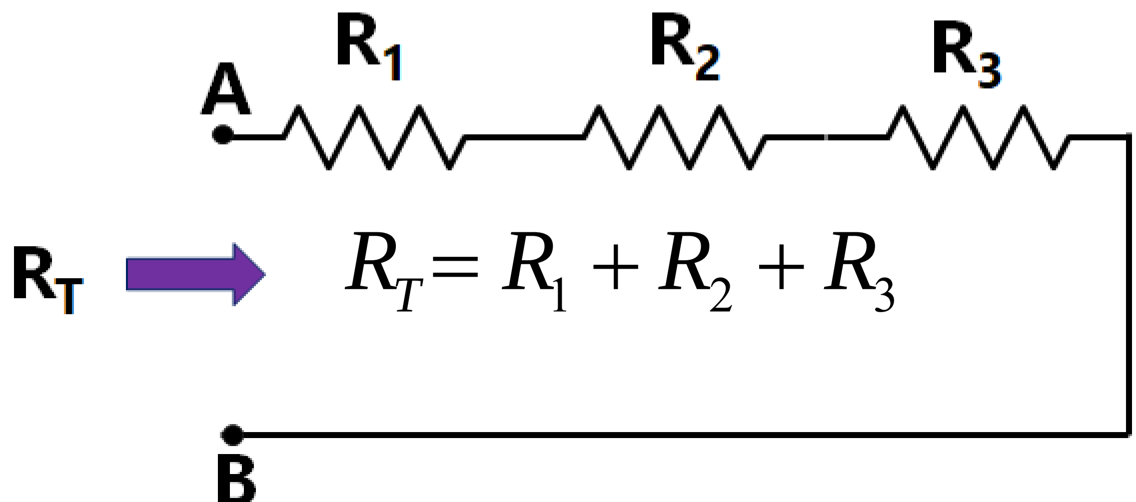
- The polarity of the voltage across a resistor is determined by the direction of current
- Current entering a resistor creates a voltage drop with polarity as shown



- **The sign of current flow in a resistor when current flows from positive potential to negative potential is negative**

Resistors in Connected in Series

- The total resistance of a series configuration is the sum of the individual resistances as seen between the terminals “A” and “B”

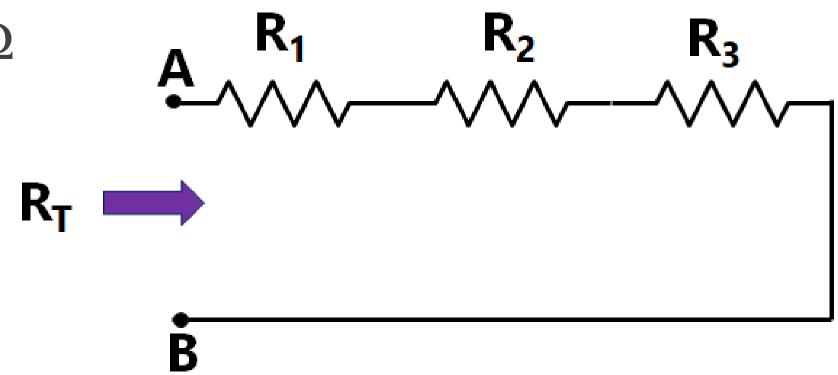


$$R_T = R_1 + R_2 + R_3$$

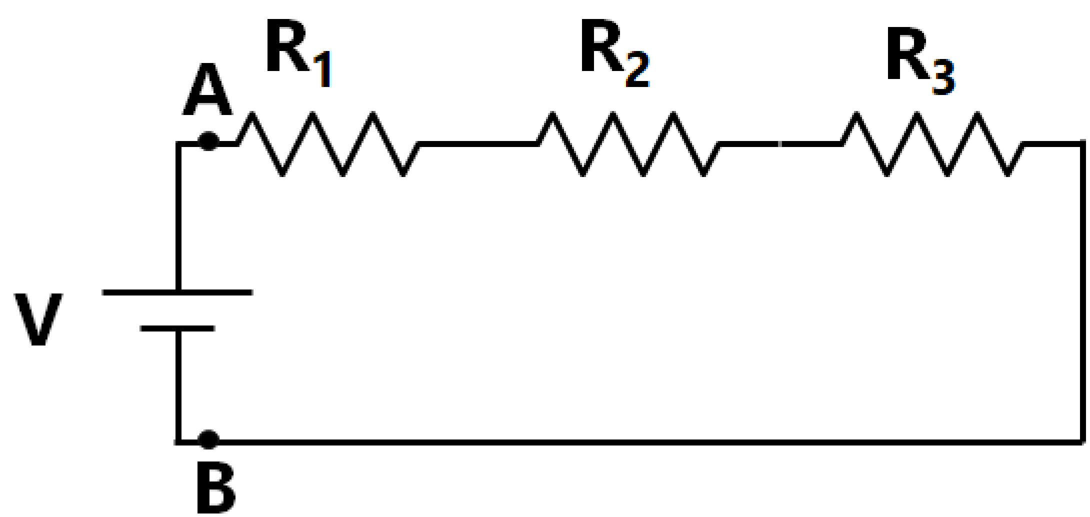
- The largest resistor in the series combination will have most impact on the total resistance
- More resistors in series combination, the greater the resistance, no matter what the value of the resistor is

Resistors in Connected in Series

- Let R_1 = 1 kΩ, R_2 = 5 kΩ, R_3 = 1.5 kΩ
- The total resistance R_T = 7.5 kΩ.
- Let V = 15 V
- Current **I** through the circuit is

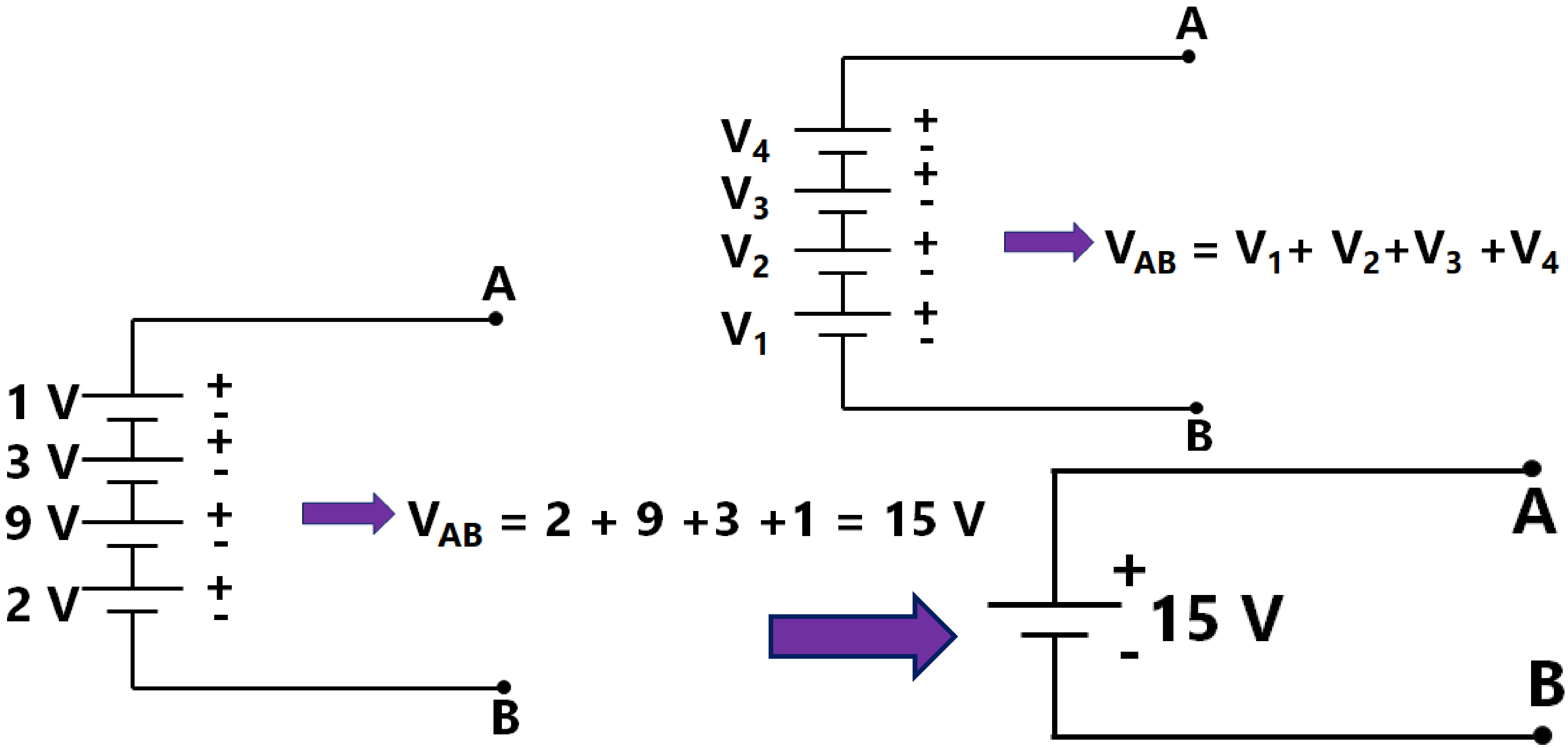


$$I = \frac{V}{R_T} = \frac{15V}{7.5k\Omega} = 2\,mA$$

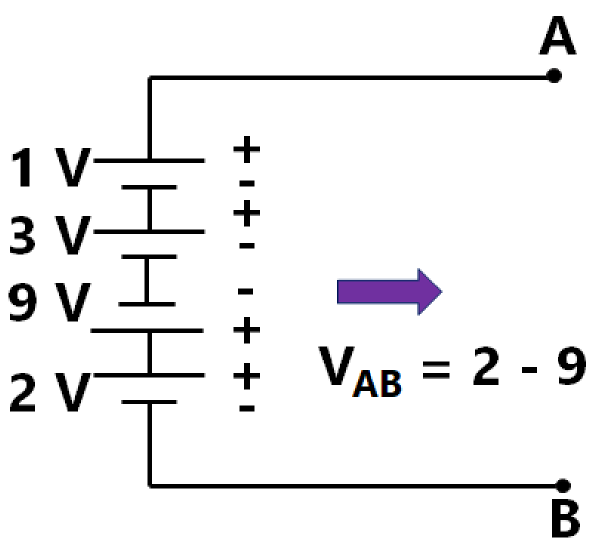


Voltage Sources Connected in Series

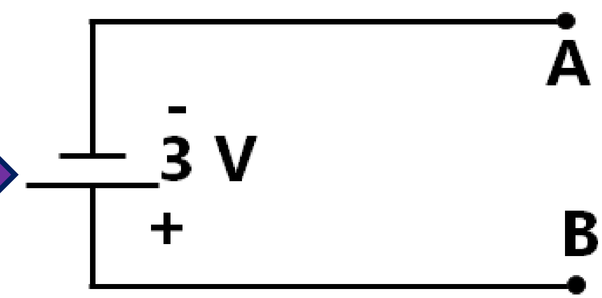
- The net voltage is obtained by summing up the voltages with same polarity and subtracting voltages with opposite polarity



Voltage Sources Connected in Series

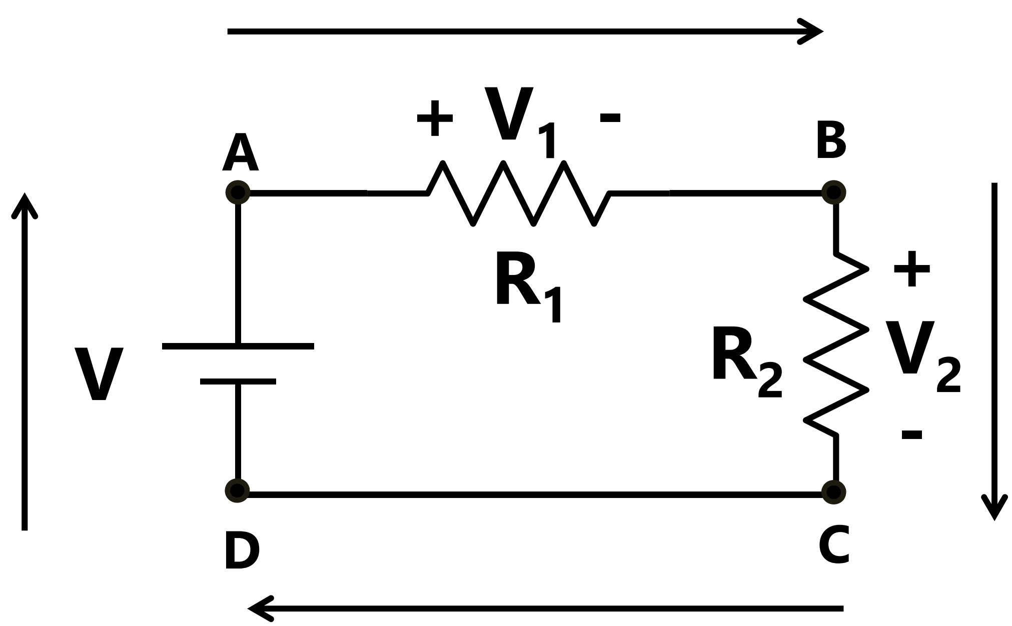


$$V_{AB} = 2 - 9 + 3 + 1 = -3\ \text{V}$$



Kirchhoff's Voltage Law

- A definition of closed path is required by starting at appoint and ending at the same point thus forming a closed loop
- **Kirchhoff's Voltage Law (KVL):** The algebraic sum of the potential raises and drops around a closed path (closed loop) is zero



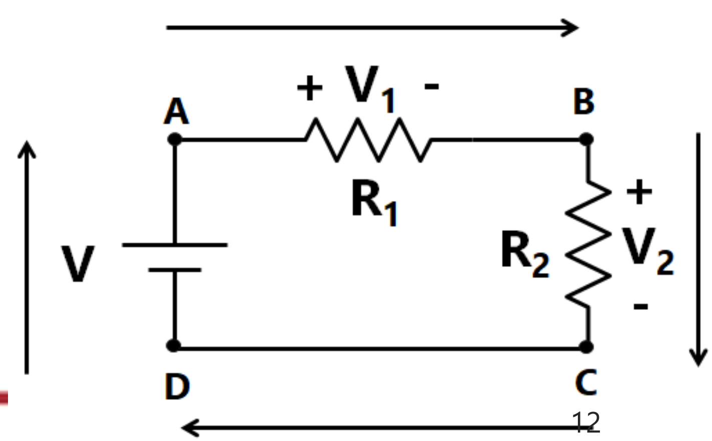
Kirchhoff's Voltage Law

- Writing out the voltages in sequence by noting the sign convention,
  - V: with positive sign
  - V_1, V_2: with negative sign
- By applying KVL,
  - Loop: "A" → "B"→ "C"→ "D" → "A"

$$+V - V_1 - V_2 = 0$$

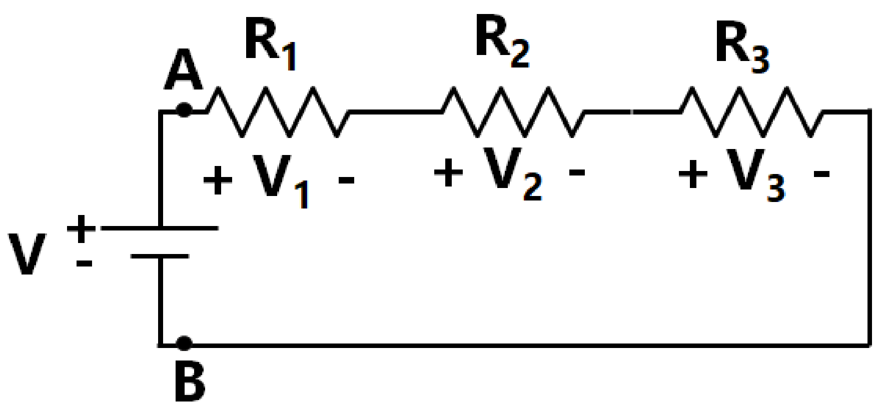
$$V = V_1 + V_2$$

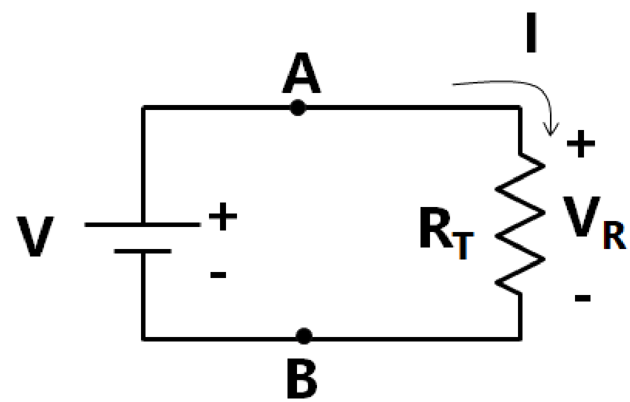
$$\sum_{raises} V = \sum_{drops} V$$



Resistances Connected in Series-Voltage Divider Rule

- To calculate the voltage drop across the each resistance in a series circuit





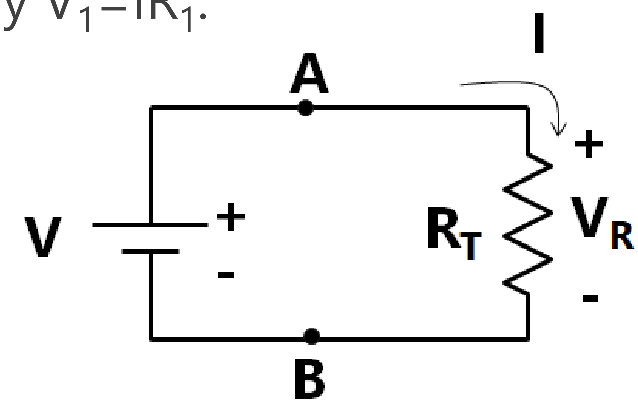
$$R_T = R_1 + R_2 + R_3$$

- Current through the circuit *I* is given by

$$I = \frac{V}{R_T} = \frac{V_R}{R_T}$$

Resistances Connected in Series-Voltage Divider Rule

- Voltage drop V_1 across resistor R_1 is given by $V_1=IR_1$.
- Similarly, $V_2=IR_2$, $V_3=IR_3$
- $IR_1 + IR_2 + IR_3 = IR_T = V$
- Thus, voltage drop across R_1 is given by



$$V_1 = IR_1 = \frac{V}{R_T} R_1 = \frac{R_1}{R_T} V$$

$$R_T = R_1 + R_2 + R_3$$

- Similarly,

$$V_2 = IR_2 = \frac{V}{R_T} R_2 = \frac{R_2}{R_T} V \qquad V_3 = IR_3 = \frac{V}{R_T} R_3 = \frac{R_3}{R_T} V$$

- Thus,

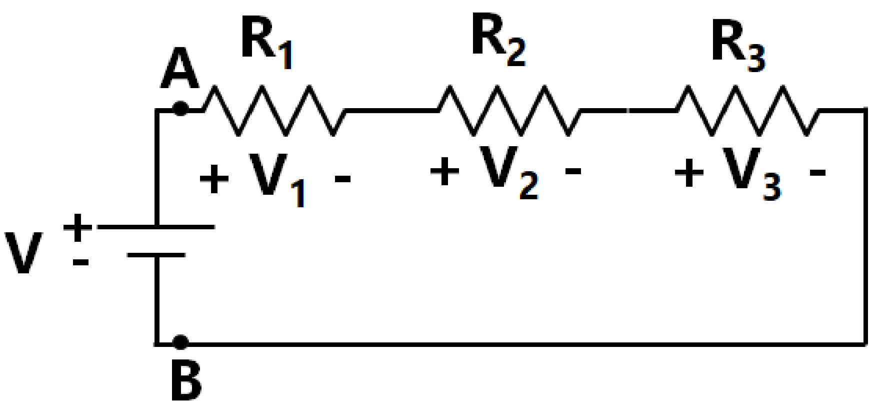
$$V = IR_1 + IR_2 + IR_3 = \left(\frac{R_1}{R_T} + \frac{R_2}{R_T} + \frac{R_3}{R_T} \right) V$$

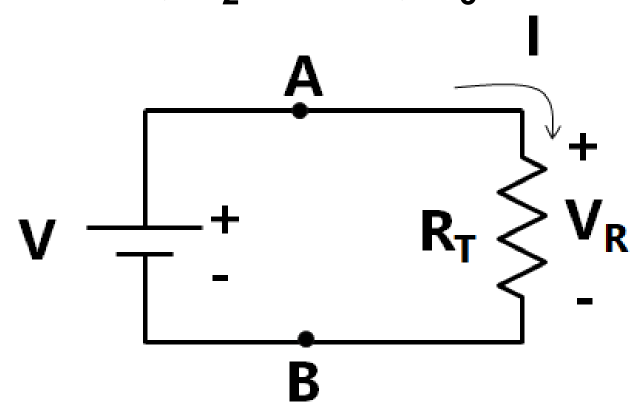
The voltage drop is maximum across the largest resistor in the series combination

Resistances Connected in Series

- To calculate the voltage drop across the each resistance in a series circuit

R_1 = 1 kΩ, R_2 = 5 kΩ, R_3 = 1.5 kΩ





$$I = \frac{V}{R_T} = \frac{15V}{7.5k\Omega} = 2\,mA$$

$$V_1 = IR_1 = 2V,\ V_2 = IR_2 = 10V, V_3 = IR_3 = 3V$$

$$R_2\,(5\,k\Omega) > R_3\,(1.5\,k\Omega) > R_1\,(1\,k\Omega)$$

$$V_2\left(10\ V\right) > V_3\left(3\ V\right) > V_1\left(2\ V\right)$$

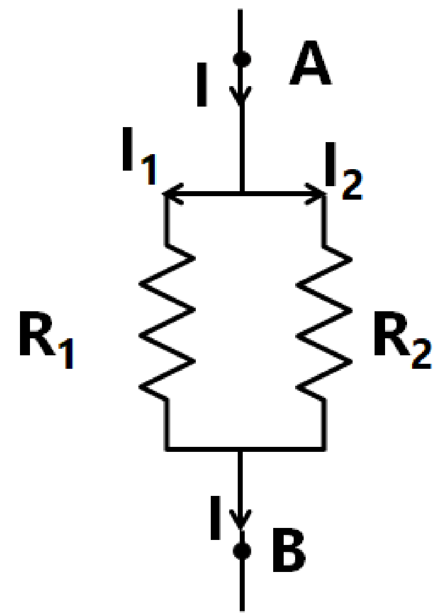
$$V_R = V_1 + V_2 + V_3 = 2 + 10 + 3 = 15V = V$$

Parallel Connections

Parallel Connections

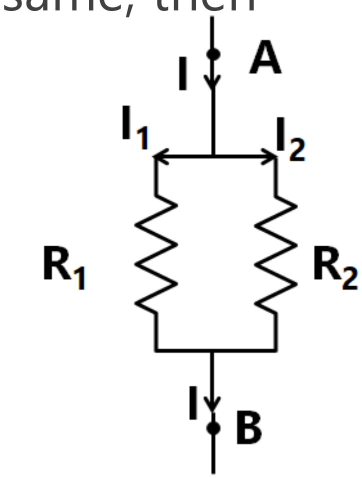
- Two elements or circuits or branches are in **parallel** if they have tw**o points in common**
- The total current before branching is sum of currents in the individual branches

$$I = I_1 + I_2$$



Parallel Connections

- Two elements or circuits or branches are in **parallel** if they have tw**o points in common**
- Voltage across parallel elements is always same
- If two elements are in parallel the voltage across them must be same.
- If the voltage across two neighbouring elements is the same, then the two elements may not be in parallel



Equivalent Resistance of a Parallel Combination

- Let **I** be the total current and $\mathbf{V_{AB}}$ be the resistance between points **A** and **B** and $\mathbf{R_T}$ be the equivalent resistance between points **A** and **B**
- The total current **I** is given

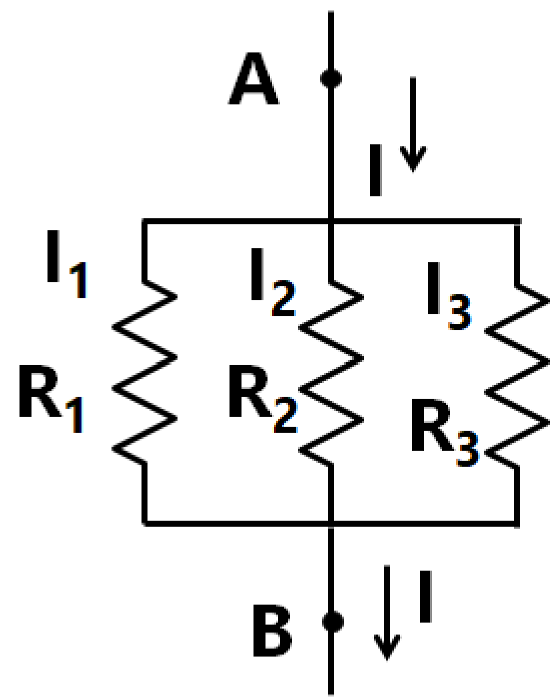
$$I = \frac{V_{AB}}{R_T}$$

- The total current $\mathbf{I = I_1 + I_2 + I_3}$
- Current through resistance R_1 is given by

$$I_1 = \frac{V_{AB}}{R_1}$$

- Similarly,

$$I_2 = \frac{V_{AB}}{R_2} \qquad I_3 = \frac{V_{AB}}{R_3}$$

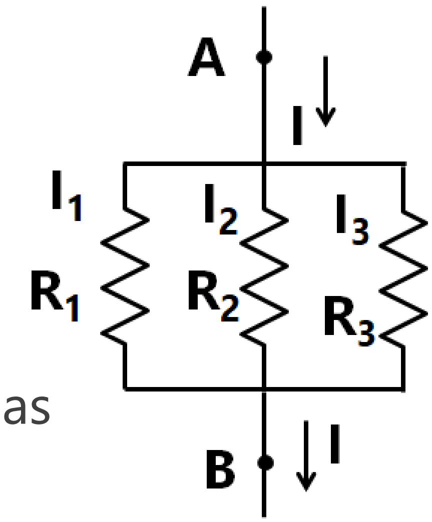


Equivalent Resistance of a Parallel Combination

- Thus

$$I = \frac{V_{AB}}{R_T} = I_1 + I_2 + I_3 = \frac{V_{AB}}{R_1} + \frac{V_{AB}}{R_2} + \frac{V_{AB}}{R_3}$$

$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

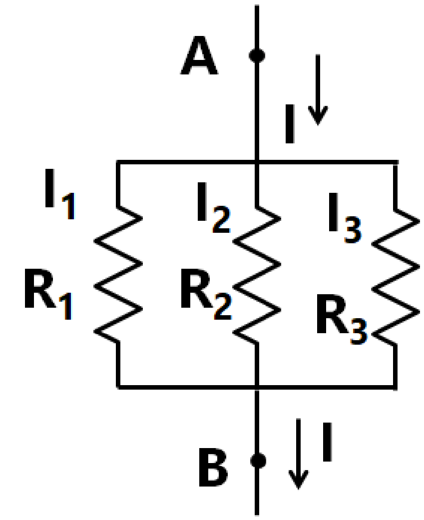


- The large the value of resistance is, less the effect it has

Current Division

- Current I_1 through resistance R_1 is given by

$$I_1 = \frac{V_{AB}}{R_1}$$

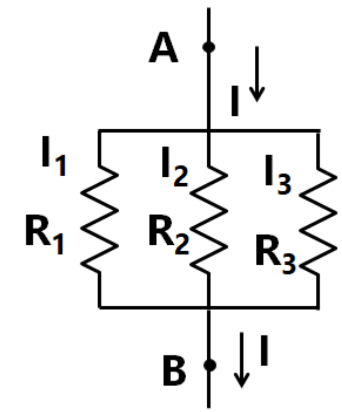


- Substituting $$I = \frac{V_{AB}}{R_T} \Rightarrow I_1 = \frac{IR_T}{R_1}$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \Rightarrow R_T = \frac{R_1 R_2 R_3}{R_2 R_3 + R_3 R_1 + R_1 R_2}$$

- Thus, $$I_1 = \frac{R_2 R_3}{R_2 R_3 + R_3 R_1 + R_1 R_2} I$$

Current Division



$$I_1 = \frac{R_2R_3}{R_2R_3 + R_3R_1 + R_1R_2} I$$

$$I_2 = \frac{R_3R_1}{R_2R_3 + R_3R_1 + R_1R_2} I$$

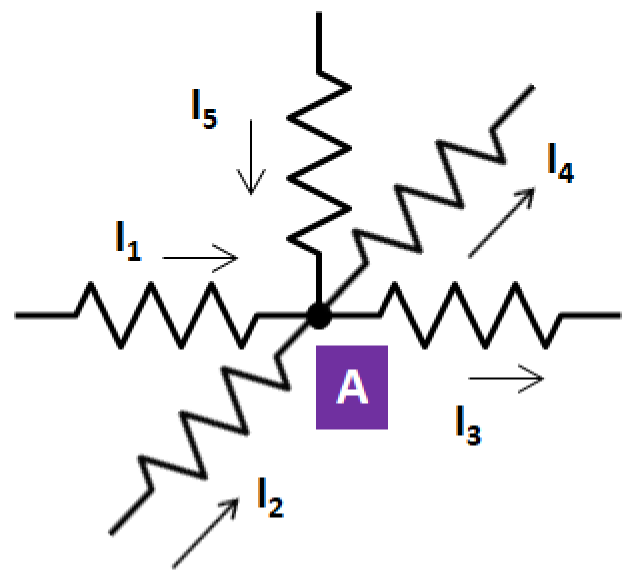
$$I_3 = \frac{R_1R_2}{R_2R_3 + R_3R_1 + R_1R_2} I$$

- The branch with largest resistance in a parallel combination has maximum current

Kirchhoff's Current Law

- The algebraic sum of currents entering and leaving a junction or region of a network is zero
- Let I_i be the currents that enter in to a junction and let I_o be currents leaving a junction. Then **KCL** states

$$\boxed{\Sigma I_i = \Sigma I_o}$$



- I_1, I_2 and I_5 are entering junction or node "**A**"
- I_3 and I_4 are leaving junction or node "**A**"
- Thus, as per KCL, $I_1 + I_2 + I_5 = I_3 + I_4$