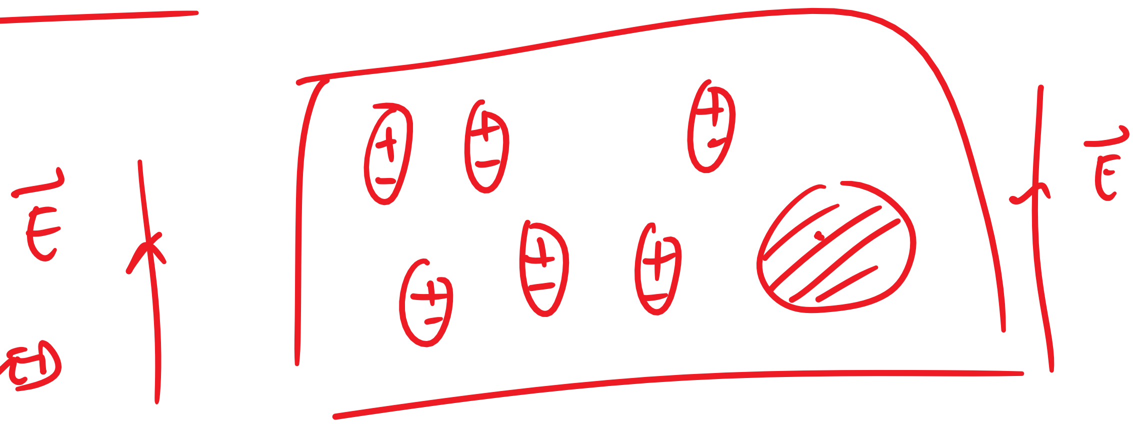
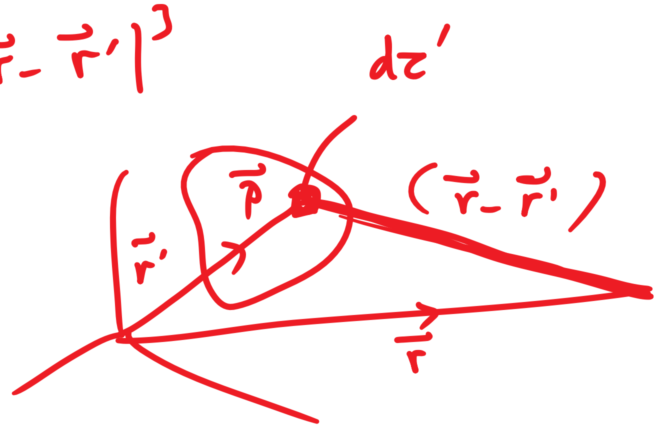
Dielectrics



POLARIZED

$\vec{P}$: dipole moment / unit volume

$$V = \frac{1}{4\pi\epsilon_0}\iiint_{\text{Volume}} \frac{\vec{P}\cdot(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3}\, d\tau'$$



$$= V_{vol} + V_{surf}$$

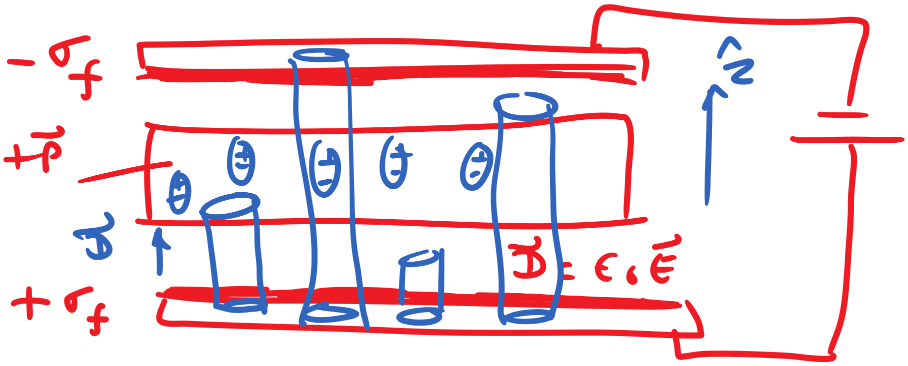
$$\boxed{\begin{aligned} \rho_b &= -\vec{\nabla}\cdot\vec{P} \\ \sigma_b &= \vec{P}\cdot\hat{n} \end{aligned}}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\Downarrow$$

$$\vec{\nabla}\cdot\vec{D} = \rho_f$$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P}$$



$$\boxed{\vec{D} = \epsilon_0\vec{E} + \vec{P}}$$

$$\boxed{\vec{P} = \epsilon_0\chi\vec{E}}$$

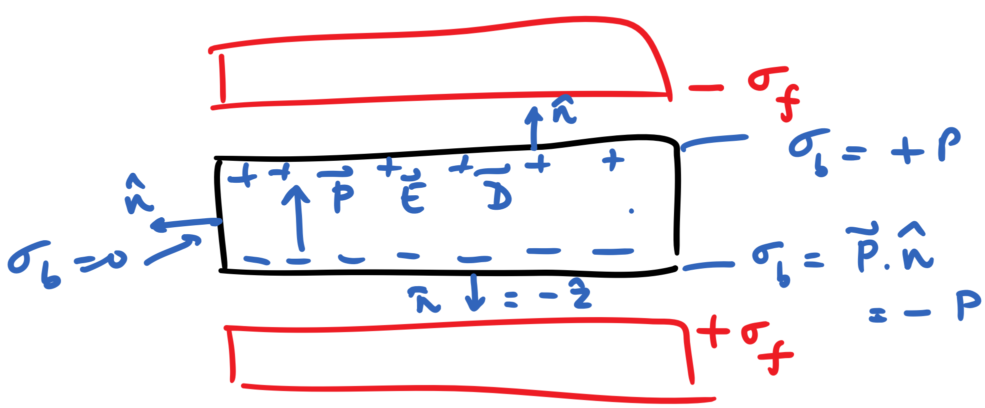
$$\boxed{\oint\!\!\oint \vec{D}\cdot d\vec{a} = Q_{f\,enc}}$$

$$\vec{D} = \sigma_f$$ in air

in dielectric

$$\vec{E}_{air} = \frac{\vec{D}_{air}}{\epsilon_0} = \frac{\sigma_f}{\epsilon_0}\hat{z}$$

$$\vec{E}_{diel} = \frac{\vec{D}_{diel}}{\epsilon} = \frac{\sigma_f}{\epsilon}\hat{z} = \frac{\sigma_f}{\epsilon_0 K}\hat{z}$$

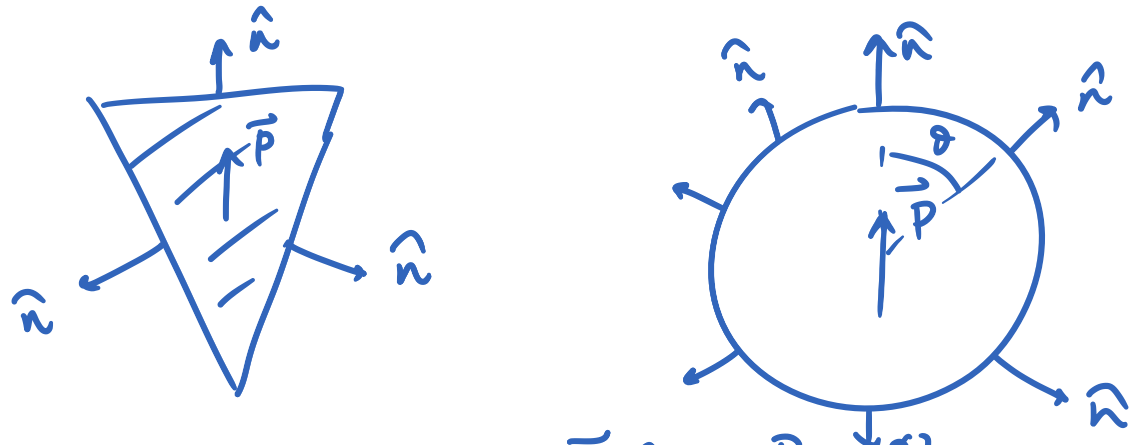


$$\vec{E}_{die} = \vec{E}_0 - \frac{\sigma_b}{\epsilon_0}$$

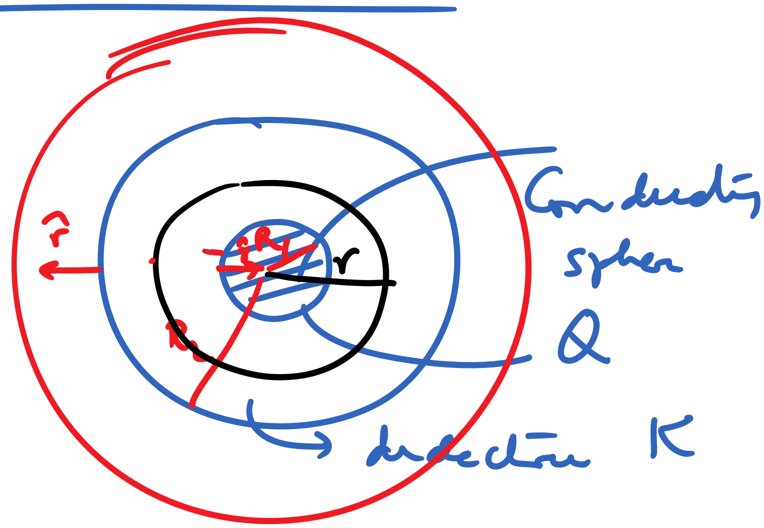
$$E_{diel} = \frac{\sigma_f}{\epsilon_0} - \frac{\sigma_b}{\epsilon_0}$$

$$\sigma_b = P = \epsilon_0\chi E_{diel}$$

1



$\sigma_b = \vec{P}\cdot\hat{n} = P\cos\theta$



$$\oint \vec{D}\cdot d\vec{a} = Q_{free}$$

$\vec{D} \rightarrow \hat{r}$

$\vec{D} = \hat{r}\, D(r)$

$\vec{D} = 0, \quad \vec{E} = 0 \qquad r < R_1$

$D \cdot 4\pi r^2 = Q_f = Q$

$$\boxed{\vec{D} = \frac{Q}{4\pi r^2}\hat{r}} \qquad R_1 < r < R_2$$

$$\boxed{\vec{D} = \frac{Q}{4\pi r^2}\hat{r}} \qquad r > R_2$$

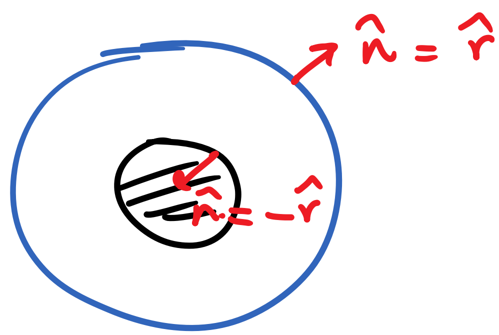
For $\quad R_1 < r < R_2$
$$\boxed{\vec{E} = \frac{\vec{D}}{\epsilon} = \frac{Q}{4\pi\epsilon r^2}\hat{r}}$$

$r > R_2$
$$\boxed{\vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}}$$

1 $\vec{P} = \epsilon_0 (K-1) \vec{E} = \dfrac{\epsilon_0 (K-1) Q}{4\pi \epsilon r^2} \hat{r}$

$$\rho_b = -\vec{\nabla}\cdot\vec{P} = 0$$

$\sigma_b \Big|_{r=R_1} = -\vec{P}\cdot\hat{r}$ $\qquad\qquad$ $\sigma_b \Big|_{r=R_2}$



$\vec{P} = A\,\hat{r}$

$\hat{n} = -\hat{r}$

$\sigma_b = \vec{P}\cdot\hat{n}$

$= A\,\hat{r}\cdot(-\hat{r})$

$= -A$

$$\boxed{\sigma_b = -\frac{\epsilon_0 (K-1) Q}{4\pi \epsilon R_1^2}} \quad \text{at } R_1$$

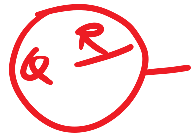
$$\boxed{\sigma_b = +\frac{\epsilon_0 (K-1) Q}{4\pi \epsilon R_2^2}} \quad \text{at } R_2$$

$$\vec{P} = P_0 \frac{\hat{r}}{r^2}$$

$$\vec{\nabla}\cdot\vec{P} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 P_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, P_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}P_\phi$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{P_0}{r^2}\right) + 0 + 0 = 0$$

PS3
Q-8



$90 \times 10^{3}\, V$

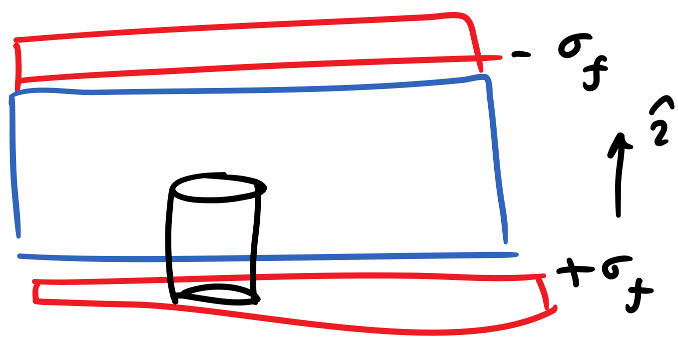
$$E_B = 30 \times 10^{3}\, V/cm = 3 \times 10^{6}\, V/m$$

$$V = \frac{Q}{4\pi\epsilon_0 R}$$

$$E = \frac{Q}{4\pi\epsilon_0 R^2} = \frac{V}{R} < 3 \times 10^{6}$$

$$R > \frac{V}{3 \times 10^{6}} = \frac{9 \times 10^{4}}{3 \times 10^{6}}\, m = 3 \times 10^{-2}\, m$$

$$\epsilon = \epsilon_1 + \frac{(\epsilon_2 - \epsilon_1)x}{d}$$

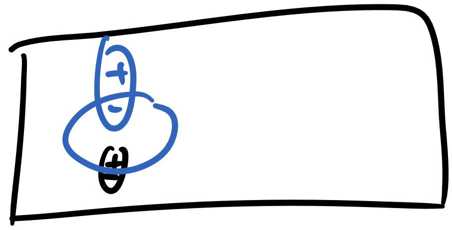


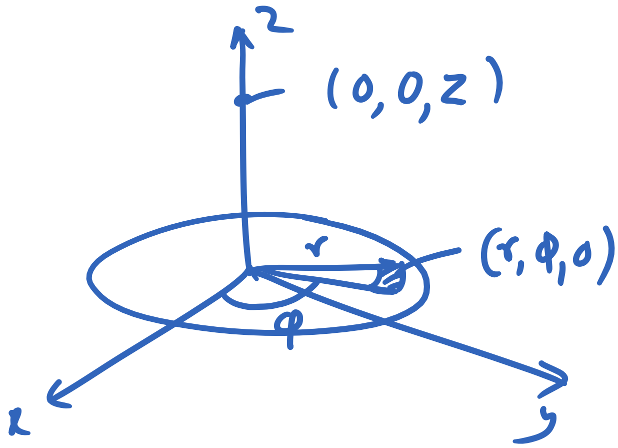
$$D = \sigma_f \qquad \oiint \vec{D} \cdot d\vec{a} = Q_{f\,enc}$$

$$\vec{E} = \frac{\vec{D}}{\epsilon} = \frac{\sigma_f}{\left[\epsilon_1 + \frac{(\epsilon_2 - \epsilon_1)}{d} x\right]} \hat{z}$$

$$V = -\int_A^B \vec{E} \cdot d\vec{l} = \int_0^d \frac{\sigma_f}{\left[\epsilon_1 + \frac{(\epsilon_2 - \epsilon_1)}{d} x\right]} dx$$

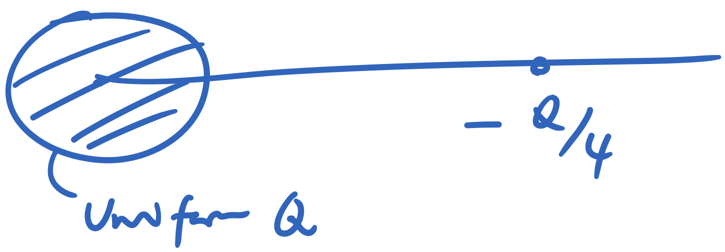
$$\vec{P} = \epsilon_0 \chi \vec{E} = \cancel{\epsilon \vec{E}}\ \epsilon_0 (K-1)\vec{E}$$
$$= (\epsilon_0 K - \epsilon_0)\vec{E} = (\epsilon - \epsilon_0)\vec{E}$$
$$= \vec{D} - \epsilon_0 \vec{E}$$

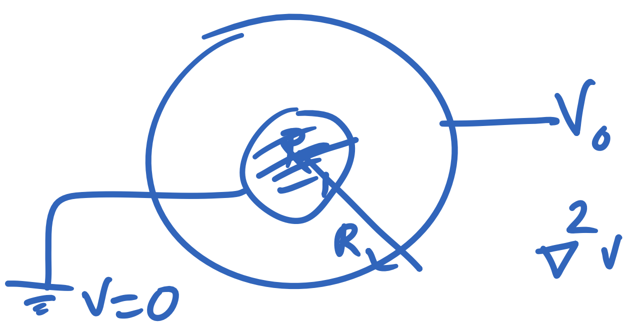




(r, ϕ, z)

$r\,dr\,d\phi$





$\nabla^2 V = 0$

$V(r)$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$$

$$\frac{d}{dr}\left(r^2\frac{dV}{dr}\right)=0$$

$$r^2\frac{dV}{dr}=C_1$$

$$\frac{dV}{dr}=\frac{C_1}{r^2}$$

$$\boxed{V=-\frac{C_1}{r}+C_2}$$