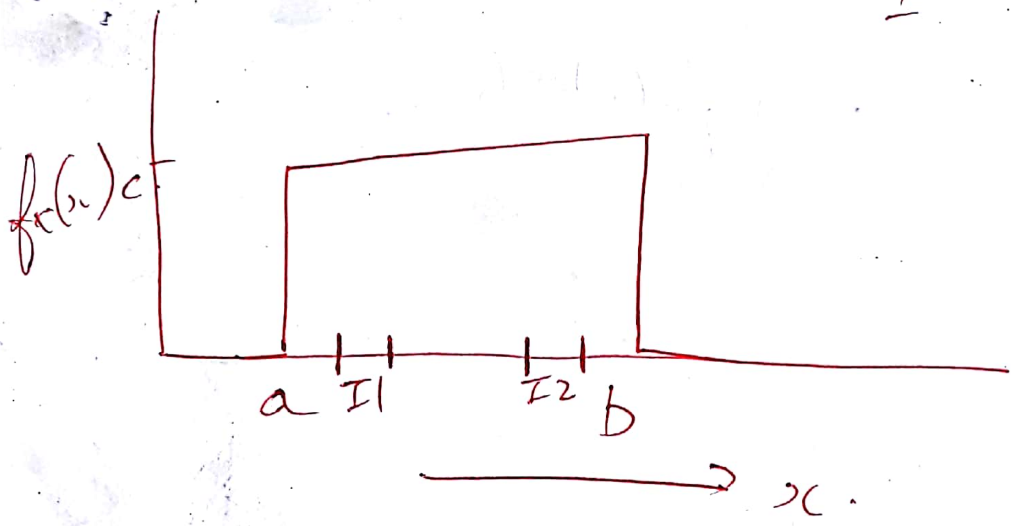
I



if $\operatorname{len}(I1) = \operatorname{len}(I2)$
& $P(I1) = P(I2)$.

Ig

$$E[X] = \int_a^b x \cdot \frac{1}{b-a}\, dx = \frac{b^2 - a^2}{(b-a)2} = \frac{a+b}{2}$$

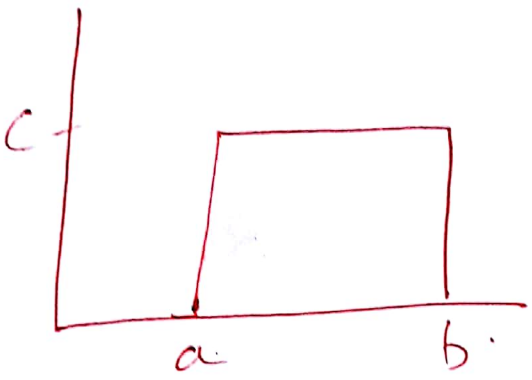
or use common sense; $E[X]$ is like center of gravity (mid value)

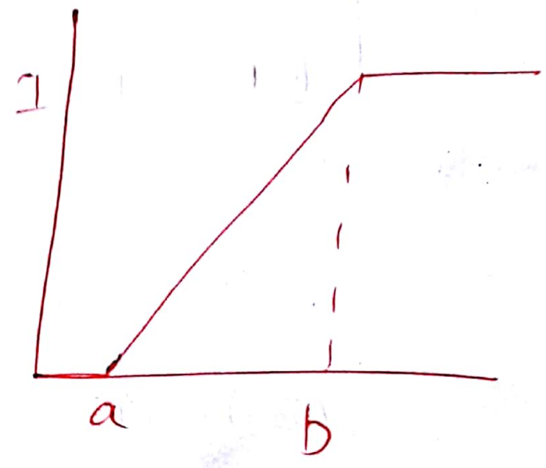
$$\Rightarrow E[X] = \frac{a+b}{2}$$

$$\operatorname{Var}(X) = \frac{(b-a)^2}{12}$$

CDF:

$$F_X(x) = P(X \le x) = \int_{-\infty}^{x} f_X(x)\,dx.$$





$$f_X(x) = \frac{d F_X(x)}{dx}.$$

Gaussian (Normal) Distribution.

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} = \underbrace{N(0,1)}_{\text{Standard Normal Distribution}}$$

$E[X] = 0.$

$Var[X] = 1.$

$$E[x] = \int_{-\infty}^{\infty} x\,p(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x\, e^{-x^2/2}\, dx. \quad \text{II}$$

$$x^2/2 = v$$

$$x\,dx = dv.$$

$$\Rightarrow E[x] = \frac{1}{\sqrt{2\pi}} \int e^{-v}\, dv.$$

$$= \frac{1}{\sqrt{2\pi}} \left[-e^{-v}\right].$$

$$= \frac{1}{\sqrt{2\pi}} \left[-e^{-x^2/2}\right]_{-\infty}^{\infty}$$

$$= 0.$$

Now, Normal distribution that are not std centered at 0; say at μ.

Gaussian Distribution :

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$X \sim N(\mu, \sigma^2)$

$E[X] = \mu$

$Var[X] = \sigma^2$.

Suppose

$\alpha = aX + b$

We know $E[X] = \mu$; $Var[X] = \sigma^2$

$\Rightarrow E[\alpha] = a\mu + b$

and $Var[\alpha] = a^2\sigma^2$.

Moreover, $\alpha \sim N(a\mu + b, a^2\sigma^2)$

IV

$x \to \infty$

III.

$f(x) = ax^2 \quad 0 \leq X \leq 3.$

$= 0 \quad$ otherwise.

$P(1 < X < 2).$

We know $\int_{-\infty}^{\infty} f_x(x)\, dx = 1$

$\int_{-\infty}^{\infty} ax^2\, dx = 1$

$a \int_0^3 x^2\, dx$

$a \left. \frac{x^3}{3} \right|_0^3 = 1.$

$\Rightarrow a = 1/9.$

$- P(X=1) - P(X=2).$

$P(1 < X < 2) = \int_1^2 ax^2\, dx = \frac{1}{9} \left. \frac{x^3}{3} \right|_1^2 = \frac{1}{9} \times \frac{8-1}{3}$

$= \frac{7}{27}.$

$X \sim N(\mu, \sigma^2)$

$$E[X] = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \quad - \text{(A)}$$

First, say we compute $E[X]$, where $X \sim N(0,1)$

$$E[X] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x \, e^{-x^2/2} dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[-e^{-x^2/2} \right]_{-\infty}^{\infty}$$

$$= 0. \quad - \text{(B)}$$

Now, lets go back to (A)

$$\frac{x-\mu}{\sigma} = y \Rightarrow x = y\sigma + \mu.$$

$$\Rightarrow dy = \frac{1}{\sigma} \times dx \Rightarrow dx = \sigma \, dy.$$

A becomes
$$\int_{-\infty}^{\infty} (\mu + \sigma y) \frac{1}{\sqrt{2\pi} \cancel{\sigma}} \times e^{-y^2/2} \times \cancel{\sigma} \, dy$$

V

PDF = Sum = 1   0 (From (B))

$$= \mu \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2}\, dy + \sigma \int_{-\infty}^{\infty} y \times \frac{1}{\sqrt{2\pi}} e^{-y^2/2}\, dy$$

$$= \mu \times 1 + \sigma \times 0$$

$$= \mu.$$

65% − 1 SD $(\mu+\sigma, \mu-\sigma)$

95% − 2 SD $(\mu-2\sigma, \mu+2\sigma)$

99.7% − 3 SD $(\mu-3\sigma, \mu+3\sigma)$

$$CDF = \frac{1}{2}\left[1 + erf\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right]$$

$$erf(x) = \frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-t^2}\, dt.$$