PROPOSITIONAL LOGIC

SECTION 1.1: PROPOSITIONAL LOGIC

proposition : true = T (or 1) or false = F (or 0) (binary logic)

- ‘the moon is made of green cheese’
- ‘ go to town!’ X - imperative
- ‘What time is it?’ X – interrogative

propositional variables: P, Q, R, S, . . .

New Propositions from old: calculus of propositions - relate new propositions to old using TRUTH TABLES

- Logical operators: unary, binary

- Unary:
  - Negation

- Binary
  - Conjunction
  - Disjunction
  - Exclusive OR
  - Implication
  - Biconditional

Unary Operators

- Negation

‘not’
Symbol: ¬

Truth Table

P	¬ P
F(0)	T(1)
T(1)	F(0)

Example: P: I am going to town

¬ P: I am not going to town;
It is not the case that I am going to town;
I ain’t goin’.

- Binary Operators
  - Conjunction: 'and'

    Symbol: ˄

Truth Table

P	Q	P ˄ Q
0	0	0
0	1	0
1	0	0
1	1	1

Example: P - 'I am going to town'

Q - 'It is going to rain'

P ˄ Q: 'I am going to town and it is going to rain.'

Note: Both P and Q must be true!!!!!

- Binary
  - Disjunction: *inclusive* ‘or’

    Symbol: ˅

Example: P - ‘I am going to town’

Q - ‘It is going to rain’

P ˅ Q: ‘I am going to town or it is going to rain.’

Truth Table:

P	Q	P ˅ Q
0	0	0
0	1	1
1	0	1
1	1	1

Note: Only one of P and Q must be true. Hence, the *inclusive* nature.

- Binary
  - Exclusive OR: Symbol ⊕

Example:

P - ‘I am going to town’

Q - ‘It is going to rain’

Truth Table

P	Q	P ⊕ Q
0	0	0
0	1	1
1	0	1
1	1	0

P ⊕ Q: ‘Either I am going to town or it is going to rain.’

Note: Only one of P and Q must be true.

- Binary

  - Implication: ‘If…then…’
    Symbol:

Example:
P - ‘I am going to town’
Q - ‘It is going to rain’

P Q: ‘If I am going to town then it is going to rain.’

Truth Table

P	Q	P Q
0	0	1
0	1	1
1	0	0
1	1	1

- Implication (cont.)

Equivalent forms:

If P, then Q

P implies Q

If P, Q

P only if Q

P is a sufficient condition for Q

Q if P

Q whenever P

Q is a necessary condition for P

Note: The implication is false <u>only when P is true and Q is false</u>!

There is no causality implied here!

‘If the moon is made of green cheese then I have more money than Bill Gates’ (T)

‘If the moon is made of green cheese then I’m on welfare’ (T)

‘If 1+1=3 then your grandma wears combat boots’ (T)

‘If I’m wealthy then the moon is not made of green cheese.’ (T)

‘If I’m not wealthy then the moon is not made of green cheese.’ (T)

Terminology:

P = premise, hypothesis, antecedent

Q = conclusion, consequence

More terminology:

$Q \rightarrow P$ is the CONVERSE of $P \rightarrow Q$

$\neg Q \rightarrow \neg P$ is the CONTRAPOSITIVE of $P \rightarrow Q$

Example:

Find the converse and contrapositive of the following statement:

R: ‘Raining tomorrow is a sufficient condition for my not going to town.’

Step 1: Assign propositional variables to component propositions

P: It will rain tomorrow

Q: I will not go to town

Step 2: Symbolize the assertion

R: P□Q

Step 3: Symbolize the converse

Q□P

Step 4: Convert the symbols back into words

‘If I don’t go to town then it will rain tomorrow’

or

‘Raining tomorrow is a necessary condition for my not going to town.’

or

‘My not going to town is a sufficient condition for it raining tomorrow.’

- Binary

    - Biconditional: ‘if and only if’, ‘iff’

      Symbol: ↔

Truth Table

P	Q	P ↔ Q
0	0	1
0	1	0
1	0	0
1	1	1

Example: P - ‘I am going to town’, Q - ‘It is going to rain’

P ↔ Q: ‘I am going to town if and only if it is going to rain.’

Note: Both P and Q must have the same truth value.

- Imprecision of the natural language:

  ‘If you finish your meal then you can have dessert’

- Breaking assertions into component propositions - look for the logical operators!

Example: ‘If I go to Harry’s or go to the country I will not go shopping.’

P: I go to Harry’s
Q: I go to the country
R: I will go shopping

If......P......or.....Q.....then....not.....R

$(P \vee Q) \rightarrow \neg R$

Constructing a truth table:

- one column for each propositional variable
- one for the compound proposition
- count in binary
- n propositional variables = 2^n rows

You may find it easier to include columns for propositions which themselves are component propositions.

Truth Table

P	**Q**	**R**	**(P V Q)□¬R**
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

CHAPTER 1: (PART 2): THE FOUNDATIONS: LOGIC AND PROOFS

PROPOSITIONAL EQUIVALENCE

PROPOSITIONAL EQUIVALENCES

- A tautology is a proposition which is always true .

  Classic Example: $P \vee \neg P$

- A contradiction is a proposition which is always false .

  Classic Example: $P \wedge \neg P$

- A contingency is a proposition which neither a tautology nor a contradiction.

  Example: (P V Q) □ ¬R

PROPOSITIONAL EQUIVALENCES (CONT.)

- Two propositions P and Q are logically equivalent if $P \leftrightarrow Q$ is a tautology. We write:

$$P \Leftrightarrow Q$$

PROPOSITIONAL EQUIVALENCES (CONT.)

- Example:

$$(P \rightarrow Q) \wedge (Q \rightarrow P) \Leftrightarrow (P \leftrightarrow Q)$$

- Proof:
  - The left side and the right side must have the same truth values independent of the truth value of the component propositions.
  - To show a proposition is not a tautology: use an abbreviated truth table
    - try to find a counter example or to disprove the assertion.
    - search for a case where the proposition is false

PROPOSITIONAL EQUIVALENCES (CONT.)

- Case 1: Try left side false, right side true

Left side false: only one of $P \rightarrow Q$ or $Q \rightarrow P$ need be false.

1a. Assume $P \rightarrow Q = F$.

Then $P = T$, $Q = F$. But then right side $P \leftrightarrow Q = F$. Wrong guess.

1b. Try $Q \rightarrow P = F$. Then $Q = T$, $P = F$. Then $P \leftrightarrow Q = F$. Another wrong guess.

PROPOSITIONAL EQUIVALENCES

- Case 2. Try left side true, right side false

If right side is false, P and Q cannot have the same truth value.

2a. Assume P =T, Q = F.

Then P□Q = F and the conjunction must be false so the left side cannot be true in this case. Another wrong guess.

2b. Assume Q = T, P = F.

Again the left side cannot be true. We have exhausted all possibilities and not found a counterexample. The two propositions must be logically equivalent.

Note: Because of this equivalence, if and only if or iff is also stated as is a necessary and sufficient condition for.