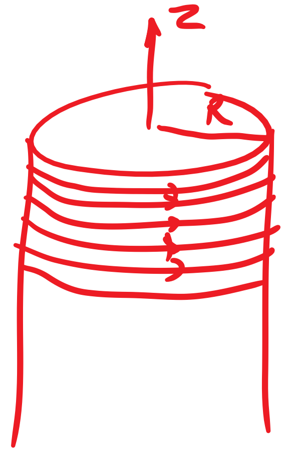
Vector potential due to a solenoid

$$\nabla \cdot \vec{A} = 0$$

$$\nabla \times \vec{A} = \vec{B}$$

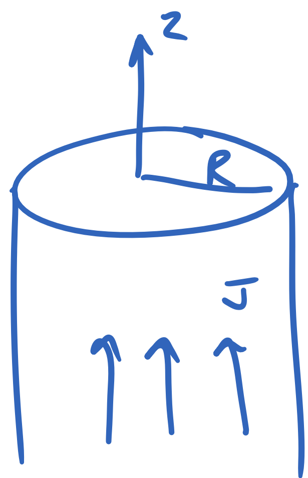


$$\vec{B} = \mu_0 n I \hat{z} \; ; \quad r < R$$

$$= 0 \; ; \quad r > R$$

$$\boxed{\begin{aligned} \nabla \times \vec{A} &= \mu_0 n I \hat{z} \; ; \quad r < R \\ &= 0 \; ; \quad r > R \\ \& \; \nabla \cdot \vec{A} &= 0 \end{aligned}}$$

$$\boxed{\begin{aligned} \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{B} &= \mu_0 J \hat{z} \quad r < R \\ &= 0 \quad r > R \end{aligned}}$$



$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enc}$$

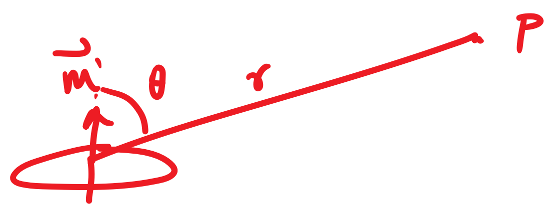
$r < R$ $\quad 2\cancel{\pi}\cancel{r} B = \mu_0 J . \cancel{\pi} r^{\cancel{2}} \Rightarrow \vec{B} = \dfrac{\mu_0 J r}{2} \hat{\phi}$

$r > R$ $\quad 2\cancel{\pi} r B = \mu_0 J . \cancel{\pi} R^2 \Rightarrow \vec{B} = \dfrac{\mu_0 J R^2}{2 r} \hat{\phi}$

$$\boxed{\vec{A} = \frac{\mu_0 n I r}{2}\hat{\phi} \quad r<R \qquad = \frac{\mu_0 n I R^2}{2r}\hat{\phi} \quad r>R}$$

Vector potential due to a dipole

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{r}}{r^2}$$



$$\vec{B} = \nabla\times\vec{A}$$

Magnetic fields produced by media

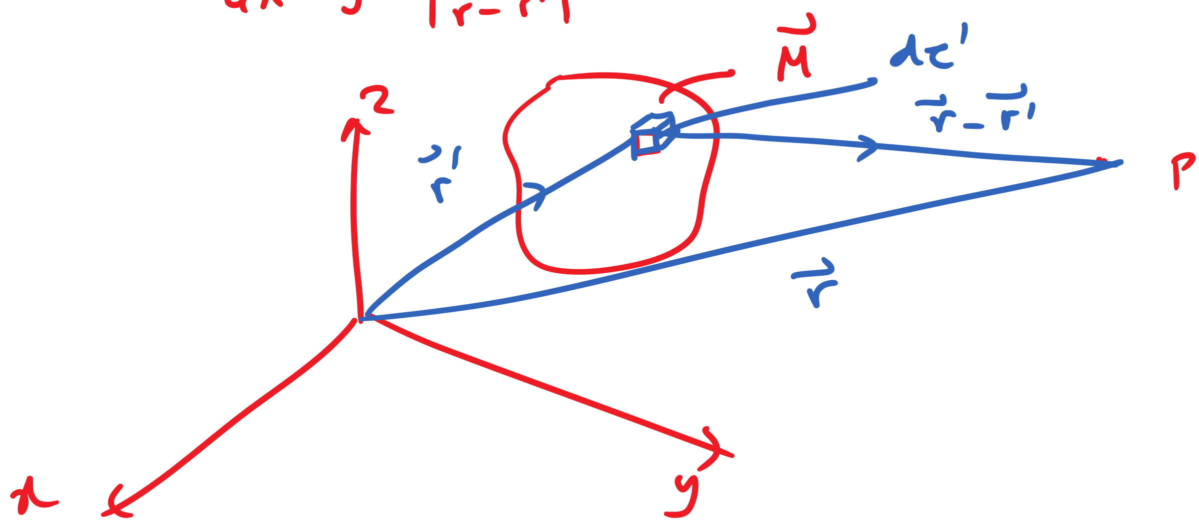
MAGNETIZATION

$\vec{M}$ = Magnetic dipole moment per unit volume

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{I}\, dl'}{|\vec{r}-\vec{r}'|}$$ Line Current

$$= \frac{\mu_0}{4\pi} \int \frac{\vec{K}\, da'}{|\vec{r}-\vec{r}'|}$$ Surface Current

$$= \frac{\mu_0}{4\pi} \int \frac{\vec{J}\, d\tau'}{|\vec{r}-\vec{r}'|}$$ Volume Current



$$d\vec{A} = \frac{\mu_0}{4\pi} \frac{(\vec{M}\, d\tau') \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{M} \times (\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} d\tau'$$

$$\vec{M} \times \frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3} = \vec{M} \times \nabla' \left(\frac{1}{|\vec{r}-\vec{r}'|} \right)$$

$\nabla' \rightarrow \vec{r}'$

$$\nabla' \times \left(\frac{\vec{M}}{|\vec{r}-\vec{r}'|} \right) = \frac{1}{|\vec{r}-\vec{r}'|} \nabla' \times \vec{M} - \underbrace{\vec{M} \times \nabla' \left(\frac{1}{|\vec{r}-\vec{r}'|} \right)}$$

$$\vec{M} \times \nabla' \left(\frac{1}{|\vec{r}-\vec{r}'|} \right) = \frac{1}{|\vec{r}-\vec{r}'|} \vec{\nabla}' \times \vec{M} - \nabla' \times \left(\frac{\vec{M}}{|\vec{r}-\vec{r}'|} \right)$$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{\nabla}' \times \vec{M}}{|\vec{r}-\vec{r}'|} d\tau' - \frac{\mu_0}{4\pi} \int \nabla' \times \left(\frac{\vec{M}}{|\vec{r}-\vec{r}'|} \right) d\tau'$$

$$\boxed{\int_{\text{Vol}} (\nabla \times \vec{F}) \, d\tau' = - \oint_{\text{Surf}} \vec{F} \times d\vec{a}'}$$

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{\nabla}' \times \vec{M}}{|\vec{r}-\vec{r}'|} d\tau' + \frac{\mu_0}{4\pi} \int \frac{\vec{M} \times \hat{n} \; da'}{|\vec{r}-\vec{r}'|}}$$

Volume Current density

$$\underline{\vec{J}_b = \nabla \times \vec{M}}$$

Surface Current density

$$\vec{K}_b = \vec{M} \times \hat{n}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$= \mu_0 (\vec{J}_f + \vec{J}_b)$$

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} \right) = \vec{J}_f + \nabla \times \vec{M}$$

$$\nabla \times \left(\frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J}_f$$

$$\vec{H} \equiv \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\boxed{\nabla \times \vec{H} = \vec{J}_f}$$

Ampere's law

$$\boxed{\oint \vec{H} \cdot d\vec{l} = I_{f\,enc}}$$

$$\vec{B} = \mu_0 (\vec{H} + \vec{M})$$

$$\vec{M} = \chi_m \vec{H}$$

χ_m: Magnetic Susceptibility

$$\vec{B} = \mu_0 (1 + \chi_m) \vec{H}$$

$$\boxed{\vec{B} = \mu \vec{H}}$$

$$\mu = \mu_0 (1 + \chi_m)$$

μ: Permeability of the medium

$\mu_0 = 4\pi \times 10^{-7}$ SI units

$$\vec{K}_b = \vec{M} \times \hat{n} \quad \& \quad \vec{J}_b = \nabla \times \vec{M}$$