Filter Circuits

EECE105L

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Filters

Need of Filters

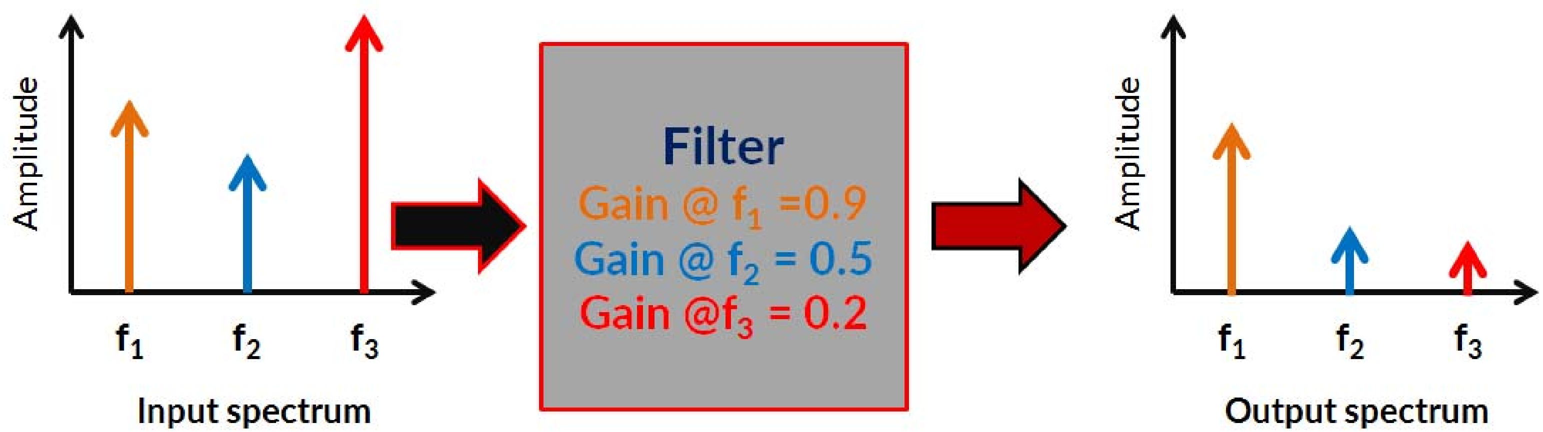
- Circuits that perform signal processing functions are *filters*
- Filters are specifically used to
  - remove unwanted frequency components from the signal
  - to enhance wanted ones
  - or both.
- Filters are essential building blocks in many systems,
  - Example: used in communication and instrumentation systems
- A common need for filter circuits is in high-performance stereo systems, where certain ranges of audio frequencies need to be amplified or suppressed for best sound quality and power efficiency

What a Filter Does?

- Filter is an electrical network that modifies the amplitude and phase characteristics of a signal with respect to frequency
- A filter will not add any extra frequency component or delete any frequency component.
- A filter changes the relative amplitudes of various frequency components and their phase relationships.
- In electronic systems, filters are useful in emphasizing signals in certain frequency ranges and reject signals in other frequency ranges.
- A filter has a gain which is dependent on signal frequency.

What a Filter Does?

- Consider a signal with frequency f_1 is contaminated with unwanted signals at f_2 and f_3
- Consider when the signal is passed through a circuit that has very low gain at f_2 and f_3 when compared to f_1
- While the useful signal component (f_1) remains, the undesired signal components (f_2 and f_3) are removed



What a Filter Does?

- Relative to signal at frequency f_1, once the signals at frequencies f_2 and f_3 are sufficiently attenuated, the performance of the filter is considered to be satisfactory
- Different types of filters
  - **Low-pass filter:** low frequencies are passed, high frequencies are attenuated.
  - **High-pass filter:** high frequencies are passed, low frequencies are attenuated.
  - **Band-pass filter:** only frequencies in a frequency band are passed.
  - **Band-stop filter or band-reject filter:** only frequencies in a frequency band are attenuated.
  - **Notch filter:** rejects just one specific frequency - an extreme band-stop filter.
  - **Comb filter**: has multiple regularly spaced narrow pass-bands giving the band-form the appearance of a comb.
  - **All-pass filter:** all frequencies are passed, but the phase of the output is modified.

Study of Filter

- Filter is characterized by two important observations
  - Transfer Function or Transfer Characteristics: A mathematical function describing the output response of a filter system to the input or stimulus
    - Transfer function in filters is studied as a frequency response
  - Phase response: How the phase of filter changes with frequency
- The order of the filter is decided by the order of the differential equation that need to be solved.
  - 1st order differential equation- 1st order filter
  - 2nd order differential equation- 2nd order filter
- Important properties of filter
  - 3 dB Frequency or cut-off frequency

Filter - Important Characteristics

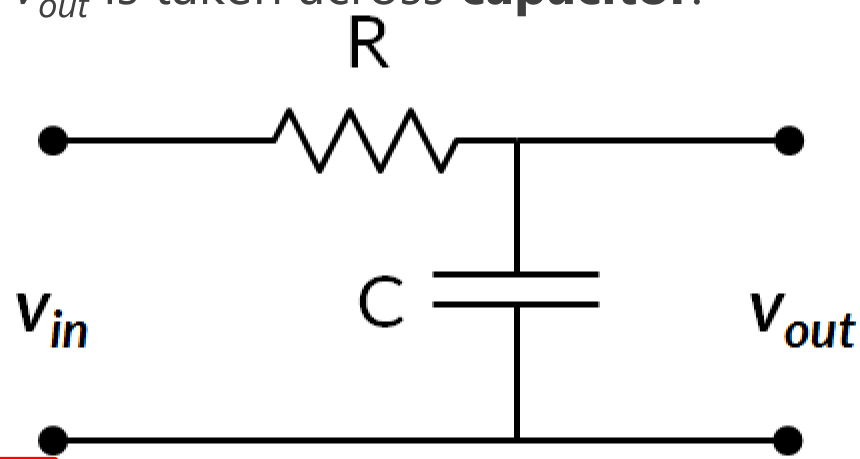
- Transfer function ($H(\omega)$): is a mathematical representation of a filter which describes the relation between input and output.
- Cut-off Frequency (f_c) or 3-dB frequency: The frequency at which the transfer function becomes half.
- Transfer Characteristics: A plot between $H(\omega)$ Vs. ω.
- Phasor diagrams: Plots the phase of gain at different frequencies.

RC Filters

RC Low Pass Filter

RC Low Pass Filter – Transfer Function (H(ω))

- Low pass filter passes low frequency signals and attenuates high frequency signals
- Consider an input signal v_{in}. **Output** v_{out} is taken across **capacitor**.
- Transfer function **H(ω)**:

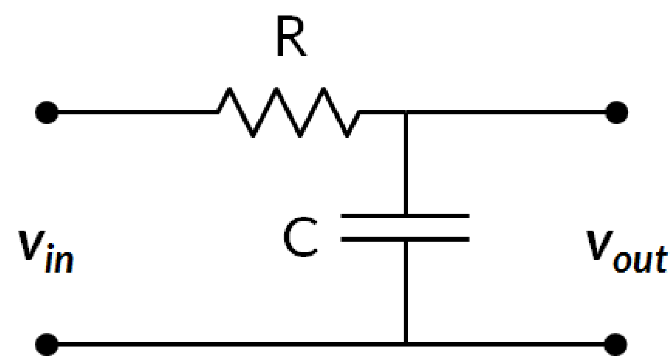


$$H(\omega)=\frac{V_{out}}{V_{in}}=\frac{V_C}{V_R+V_C}$$

$$\boxed{H(\omega)=\frac{V_C}{V_R+V_C}=\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}=\frac{1}{1+j\omega RC}}$$

RC Low Pass Filter – Transfer Function (H(ω))

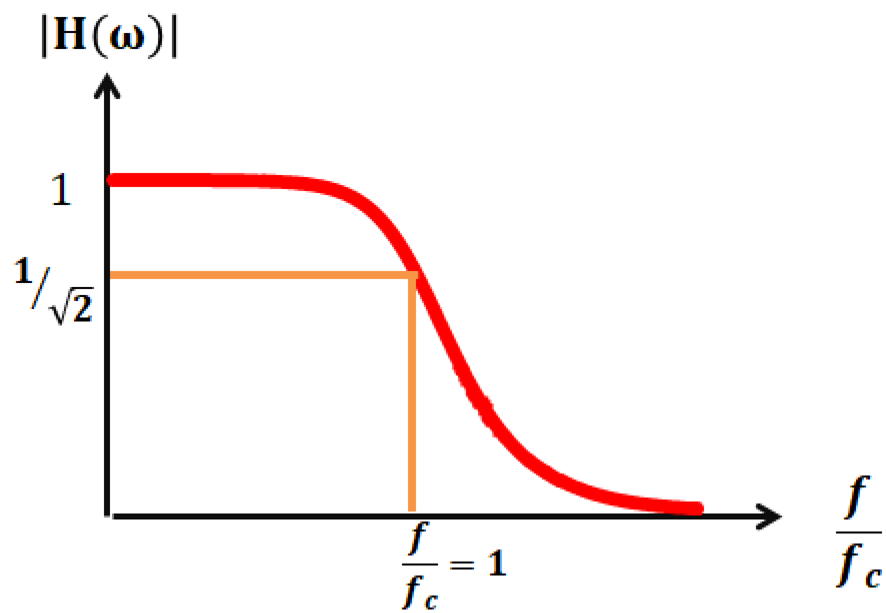
- Transfer function **H(ω)**:



$$H(\omega)=\frac{V_C}{V_R+V_C}=\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}=\frac{1}{1+j\omega RC}$$

$$H(\omega)=\frac{1}{1+j\omega RC}$$

$$\left|H(\omega)\right|=\frac{1}{\sqrt{1+(\omega RC)^2}}$$



RC Low Pass Filter – Cut-off Frequency (f_c)

- Cut-off Frequency (f_c) or 3-dB frequency is the frequency at which transfer function is ½.

$$\left|H(\omega)\right| = \frac{1}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{2}} \Rightarrow \omega_c = \frac{1}{RC}$$

$$\boxed{f_c = \frac{1}{2\pi RC}}$$

$$\boxed{\left|H(\omega)\right| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}} = \frac{1}{\sqrt{1+\left(\frac{f}{f_c}\right)^2}}}$$

- For example, if R = 160 Ω, C = 1 μF, then cut-off frequency (f_c) of a low pass filter is 1 kHz.

RC Low Pass Filter – Phase Angle (ϕ)

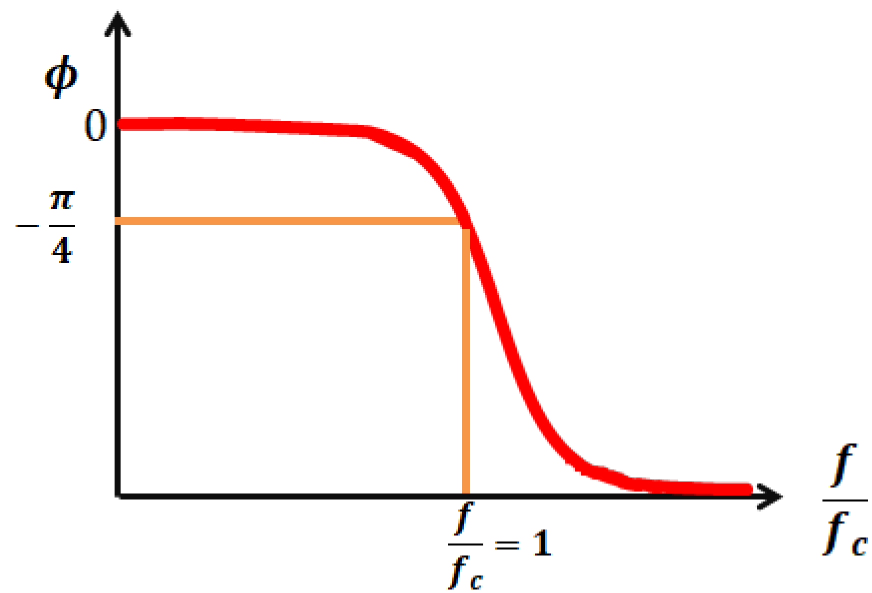
- Rewriting the Transfer function,

$$H(\omega) = \frac{1}{1 + j\omega RC} = \frac{1 - j\omega RC}{1 + (\omega RC)^2} = |H(\omega)| \angle \phi$$

- Phase angle (ϕ)

$$\phi = tan^{-1}\left(\frac{-\omega RC}{1}\right) = -tan^{-1}(\omega RC)$$

$$\boxed{\phi = -tan^{-1}\left(\frac{\omega}{\omega_c}\right) = -tan^{-1}\left(\frac{f}{f_c}\right)}$$



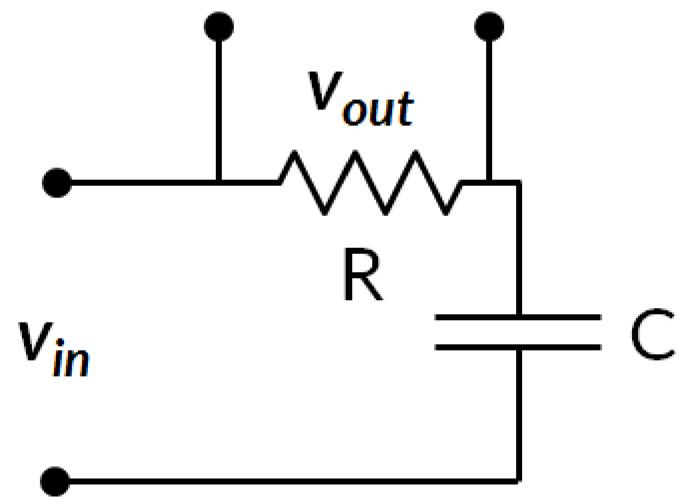
RC Filters

RC High Pass Filter

RC High Pass Filter – Transfer Function (H(ω))

- High pass filter passes high frequency signals and attenuates low frequency signals
- Consider an input signal v_{in}. **Output** v_{out} is taken across **resistor**.
- Transfer function **H(ω)**:

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{V_R}{V_R + V_C}$$

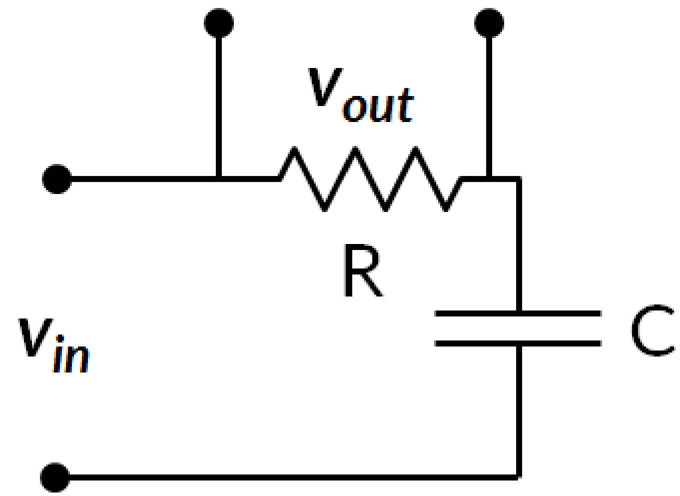


$$\boxed{H(\omega) = \frac{V_R}{V_R + V_C} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}}$$

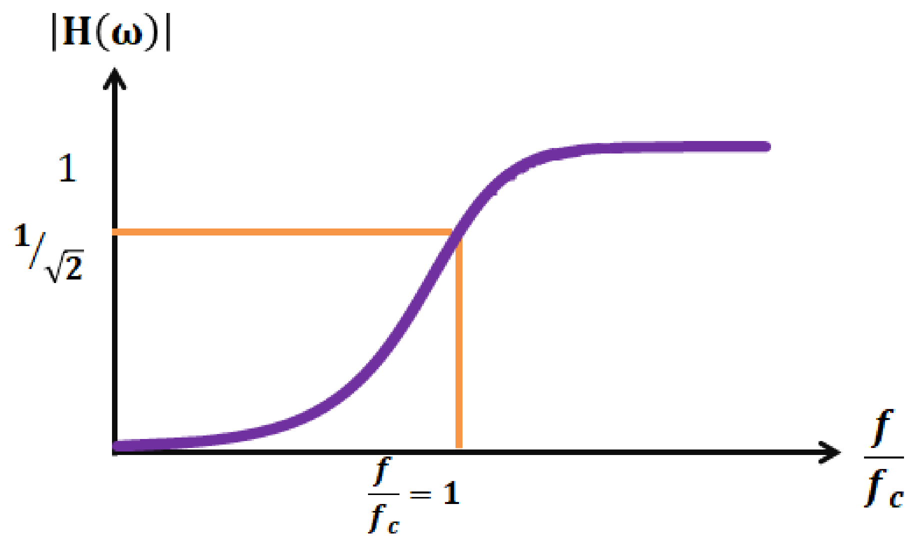
RC High Pass Filter - Transfer Function (H(ω))

- Transfer function **H(ω)**:

$$H(\omega)=\frac{V_R}{V_R+V_C}=\frac{R}{R+\frac{1}{j\omega C}}=\frac{j\omega RC}{1+j\omega RC}$$



$$\boxed{\left|H(\omega)\right|=\frac{\omega RC}{\sqrt{1+(\omega RC)^2}}}$$



RC High Pass Filter – Cut-off Frequency (f_c)

- Cut-off Frequency (f_c) or 3-dB frequency is the frequency at which transfer function is ½.

$$\left|H(\omega)\right| = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{2}} \Rightarrow \omega_c = \frac{1}{RC}$$

$$\boxed{f_c = \frac{1}{2\pi RC}}$$

$$\left|H(\omega)\right| = \frac{1}{\sqrt{1+\left(\frac{\omega_c}{\omega}\right)^2}} = \frac{1}{\sqrt{1+\left(\frac{f_c}{f}\right)^2}}$$

- For example, if R = 160 Ω, C = 1 μF, then cut-off frequency (f_c) of a high-pass filter is 1 kHz.

RC High Pass Filter – Phase Angle (ϕ)

- Re-writing transfer function

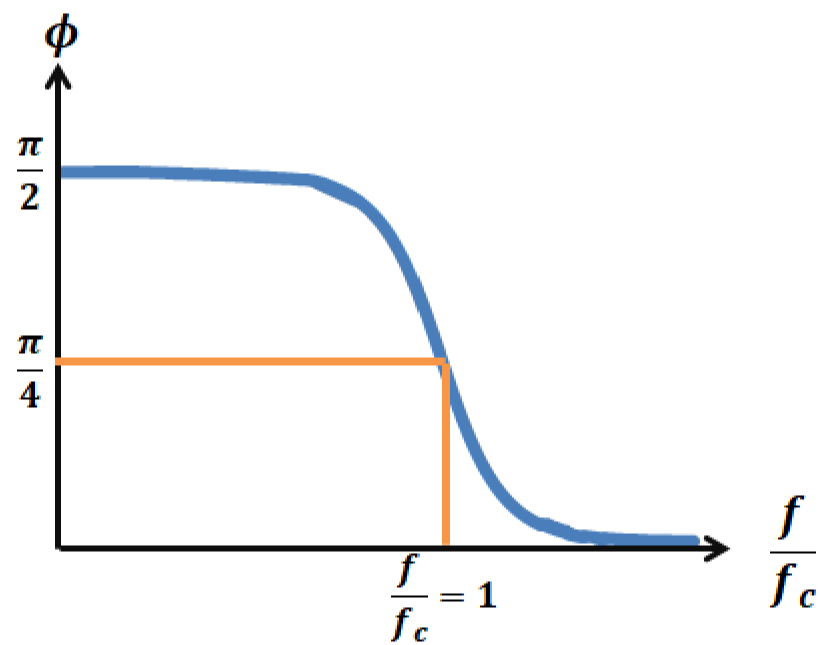
$$H(\omega) = \frac{j\omega RC}{1 + j\omega RC}\,\frac{1 - j\omega RC}{1 - j\omega RC} = \frac{(\omega RC)^2 + j\omega RC}{1 + (\omega RC)^2} = |H(\omega)|\angle\phi$$

- Phase angle (ϕ)

$$\phi = \tan^{-1}\left(\frac{\omega RC}{(\omega RC)^2}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

$$= \tan^{-1}\left(\frac{\omega_c}{\omega}\right) = \tan^{-1}\left(\frac{f_c}{f}\right)$$

$$\boxed{\phi = \tan^{-1}\left(\frac{f_c}{f}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{f}{f_c}\right)}$$



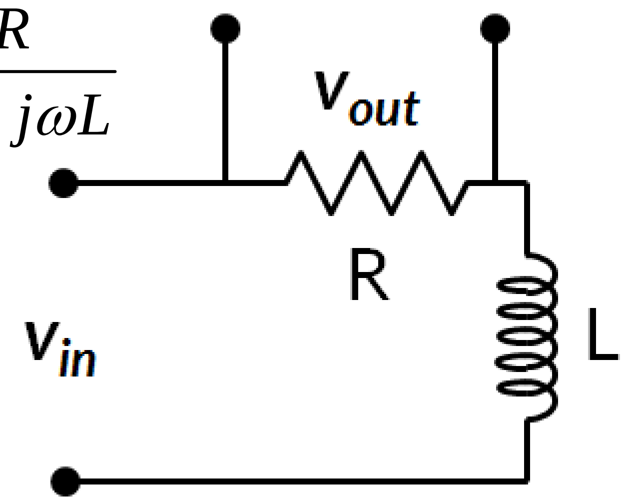
RL Filters

RL Low Pass Filter

RL Low Pass Filter – Transfer Function (H(ω))

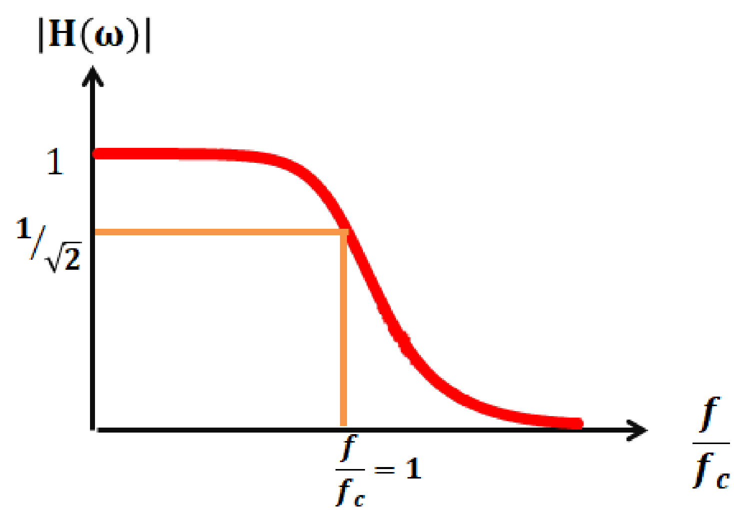
- Transfer function

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{V_R}{V_R + V_L} = \frac{R}{R + j\omega L}$$



$$\boxed{|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}}}$$

$$\boxed{H(\omega) = \frac{1}{1 + \frac{j\omega L}{R}}}$$



RL Low Pass Filter – Cut-off Frequency (f_c)

- Cut-off frequency (fc)

$$\left|H(\omega)\right| = \frac{1}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}} = \frac{1}{\sqrt{2}} \Rightarrow \omega_c = \frac{R}{L}$$

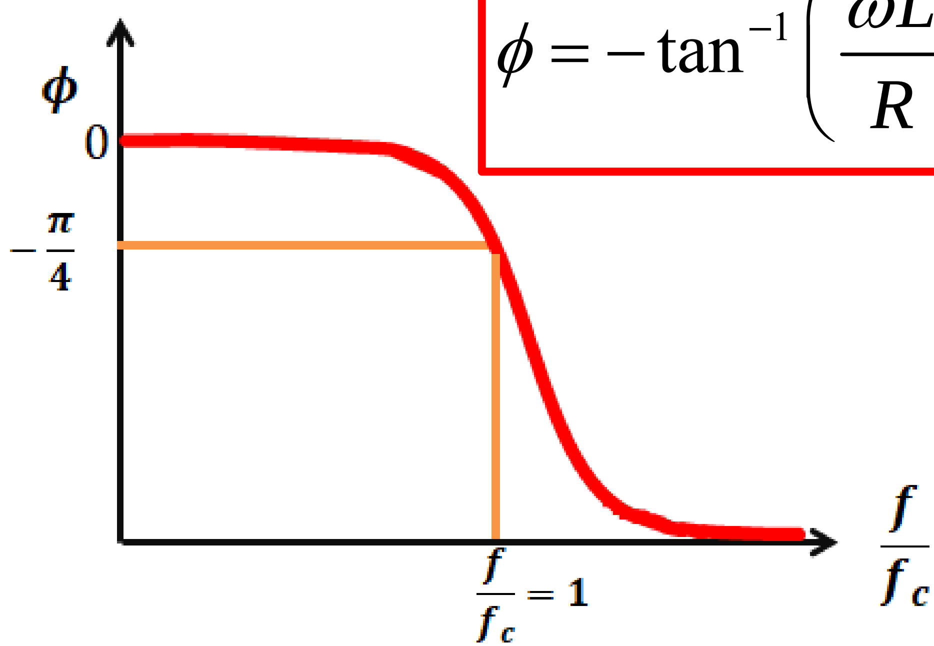
$$\boxed{f_c = \frac{R}{2\pi L}}$$

RL Low Pass Filter – Phase Angle (ϕ)

- Transfer function is

$$H(\omega)=\frac{1}{1+\frac{j\omega L}{R}}=\frac{1-\frac{j\omega L}{R}}{1+\left(\frac{\omega L}{R}\right)^2}=\left|H(\omega)\right|\angle\phi$$

$$\boxed{\phi=-\tan^{-1}\left(\frac{\omega L}{R}\right)=-\tan^{-1}\left(\frac{\omega}{\omega_c}\right)=-\tan^{-1}\left(\frac{f}{f_c}\right)}$$



RL Filters

RL High Pass Filter

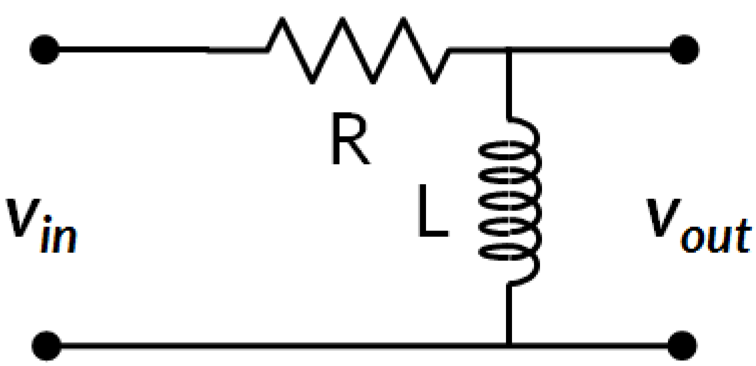
RL High Pass Filter – Transfer Function (H(ω))

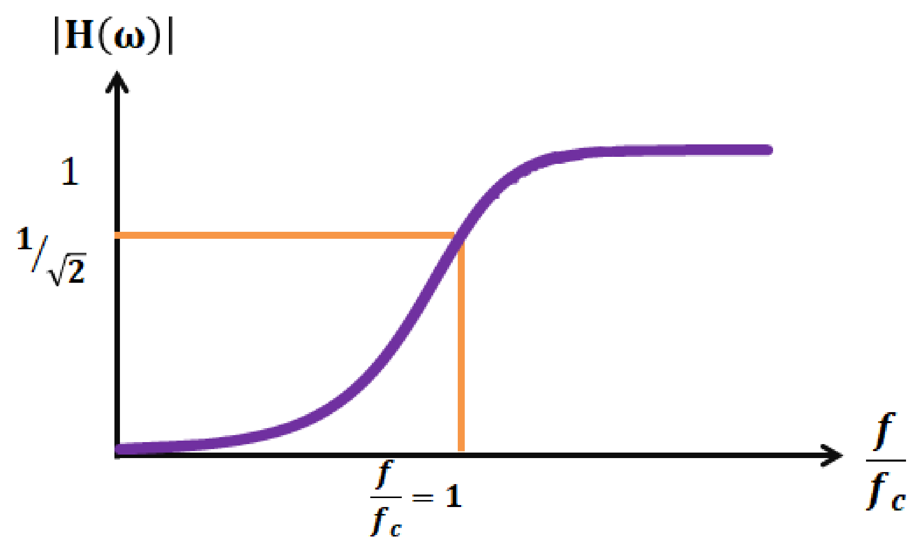
- Transfer function

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{V_L}{V_R + V_L} = \frac{j\omega L}{R + j\omega L}$$

$$\boxed{H(\omega) = \frac{1}{1 + \frac{R}{j\omega L}}}$$

$$\boxed{\left|H(\omega)\right| = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega L}\right)^2}}}$$





RL High Pass Filter – Cut-off Frequency (f_c)

- Cut-off frequency (fc)

$$|H(\omega)| = \frac{1}{\sqrt{1+\left(\frac{R}{\omega L}\right)^2}} = \frac{1}{\sqrt{2}} \Rightarrow \omega_c = \frac{R}{L}$$

$$\boxed{f_c = \frac{R}{2\pi L}}$$

RL High Pass Filter – Phase Angle (ϕ)

- Transfer function is

$$H(\omega)=\frac{1}{1+\frac{R}{j\omega L}}=\frac{1-\frac{R}{j\omega L}}{1+\left(\frac{R}{\omega L}\right)^2}=\left|H(\omega)\right|\angle\phi$$

$$\boxed{\phi=\tan^{-1}\left(\frac{R}{\omega L}\right)=\tan^{-1}\left(\frac{\omega_c}{\omega}\right)=\tan^{-1}\left(\frac{f_c}{f}\right)=\frac{\pi}{2}-\tan^{-1}\left(\frac{f}{f_c}\right)}$$