① (a) $P(X \in \{2,5\}) = P(X=2) + P(X=5)$

$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

(b) Mean of X $= E(X) = 1 \times \frac{1}{6} + 2 \times \frac{1}{4} + 5 \times \frac{1}{4} + 7 \times \frac{1}{3}$

$= \frac{51}{12} = 4.25$

Variance of X $= Var(X)$

$= E(X^2) - (E(X))^2$

$= E(X^2) - \left(\frac{51}{12}\right)^2$

$= \left[1^2 \times \frac{1}{6} + 2^2 \times \frac{1}{4} + 5^2 \times \frac{1}{4} + 7^2 \times \frac{1}{3}\right] - \left(\frac{51}{12}\right)^2$

$= \frac{285}{12} - \left(\frac{51}{12}\right)^2$

$= 5.6875$

② We know that sum of p_k's is 1.

$\Rightarrow p_1 + p_2 + p_3 + p_4 + p_5 + p_6 = 1$

$0.1 + k + 0.2 + 2k + 0.3 + k = 1$

$$4k + 0.6 = 1$$

$$4k = 1 - 0.6$$

$$4k = 0.4$$

$$k = \frac{0.4}{4} \Rightarrow \boxed{k = 0.1}$$

So we have

x_i	-2	-1	0	1	2	3
p_i	$.1$	$.1$	$.2$	$.2$	$.3$	$.1$

Mean $= E(X) = (-2)(.1) + (-1) \times (.1) + 0 \times (0.2)$
$+ 1 \times (0.2) + 2 \times (0.3) + 3 \times (0.1)$

$= 0.8$

Variance $= Var(X)$.

$= E(X^2) - (E(X))^2$

$= \cancel{(-2)^2} E(X^2) - (0.8)^2$

$= [(-2)^2 \times 0.1 + (-1)^2 \times 0.2 + 0^2 \times (0.2)$
$+ 1^2 \times (0.2) + 2^2 \times (0.3) + 3^2 \times (0.1)]$
$- (0.8)^2$

$= 2.8 - .64$

$= 1.16$

3 a $P(7.5 \le X \le 8.3) = \int_{7.5}^{8.3} f_X(x)\,dx$

$$= \int_{7.5}^{8.3} \left(\frac{2}{51}x\right) dx$$

$$= \frac{2}{51}\int_{7.5}^{8.3} x\,dx.$$

$$= \frac{2}{51}\left[\frac{x^2}{2}\right]_{x=7.5}^{x=8.3}$$

$$= \frac{2}{51}\left[\frac{8.3^2}{2} - \frac{7.5^2}{2}\right]$$

$$= \frac{2}{51}\left[\frac{12.64}{2}\right]$$

$$\approx 0.248$$

$P(X \le 9.2) = \int_{7}^{9.2} f_X(x)\,dx$

$$= \frac{2}{51}\int_{7}^{9.2} x\,dx.$$

$$= \frac{2}{51}\left[\frac{9.2^2 - 7^2}{2}\right] \approx 0.699$$

$$P(X = 8.58) = 0$$

$$P(X \geq 8) = \int_{8}^{10} f_X(x)\,dx$$

$$= \int_{8}^{10} \frac{2}{51} x\,dx$$

$$= \frac{2}{51}\left[\frac{10^2 - 8^2}{2}\right]$$

$$\approx 0.706$$

$$P(X > 8) = \cancel{P(X}\int_{8}^{10} f_X(x)\,dx$$

$$\approx 0.706$$

$$P(X \leq 8) = 1 - P(X > 8)$$

$$= 1 - 0.706$$

$$= 0.294$$

(b) Mean $= E(X) = \int_{a}^{b} x f_X(x)\,dx$

$$= \int_{7}^{10} x \times \left(\frac{2}{51} x\right) dx$$

$$= \frac{2}{51} \int_7^{10} x^2 dx.$$

$$= \frac{2}{51} \left[\frac{x^3}{3} \right]_{x=7}^{x=10}$$

$$= \frac{2}{51} \left[\frac{10^3 - 7^3}{3} \right]$$

$$\simeq - 8.59$$

$$\text{Variance} = Var(X)$$

$$= E(X^2) - (E(X))^2$$

$$= E(X^2) - (-8.59)^2$$

$$= \int_a^b (x^2 f_X(x)) dx - 73.79$$

$$= \int_7^{10} \left(x^2 \frac{2}{51} x\right) dx - 73.79$$

$$= \frac{2}{51} \int_7^{10} x^3 dx - 73.79$$

$$= \frac{2}{51} \left[\frac{10^4 - 7^4}{4} \right] - 73.79$$

$$= 0.71$$

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(4) (a) $P(X > 0.7) = \int_{0.7}^{1} (3x^2)\, dx$

$= 3\left[\frac{x^3}{3}\right]_{0.7}^{1}$

$= 0.657$

(b) Mean $= E(X) = \int_0^1 (x \times 3x^2)\, dx$

$= 3\int_0^1 x^3\, dx$

$= \frac{3}{4}$

Variance $= E(X^2) - \left(\frac{3}{4}\right)^2$

$= \int_0^1 x^2 \times 3x^2\, dx - \frac{9}{16}$

$= 3\int_0^1 x^4\, dx - \frac{9}{16}$

$= \frac{3}{5} - \frac{9}{16}$

$= \frac{3}{80}$

(c) Sol. $P(X \le a) = P(X > a)$

$$\Rightarrow \int_0^a f_X(x)\,dx = \int_a^1 f_X(x)\,dx$$

$$\Rightarrow \int_0^a (3x^2)\,dx = \int_a^1 (3x^2)\,dx$$

$$\Rightarrow \cancel{3}\int_0^a x^2\,dx = \cancel{3}\int_a^1 x^2\,dx$$

$$\Rightarrow \int_0^a x^2\,dx = \int_a^1 x^2\,dx$$

$$\Rightarrow \left[\frac{x^3}{3}\right]_{x=0}^{x=a} = \left[\frac{x^3}{3}\right]_{x=a}^{x=1}$$

$$\Rightarrow \frac{a^3 - 0}{\cancel{3}} = \frac{1^3 - a^3}{\cancel{3}}$$

$$\Rightarrow a^3 = 1 - a^3$$

$$\Rightarrow 2a^3 = 1$$

$$\Rightarrow a^3 = \frac{1}{2}$$

$$\Rightarrow a = \left(\frac{1}{2}\right)^{1/3}$$

$$= 0.7937$$

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(d) $P(X > b) = 0.35$

$$\int_b^1 f_X(x)\,dx = 0.35$$

$$\int_b^1 (3x^2)\,dx = 0.35$$

$$\cancel{3}\left[\frac{1^3 - b^3}{\cancel{3}}\right] = 0.35$$

$$1 - b^3 = 0.35$$

$$b^3 = 1 - 0.35 = 0.65$$

$$b = (0.65)^{1/3}$$

$$b = 0.8242$$

(5) (a) Standard deviation is square root of variance.

$Var(X) = \sigma_X^2 = 4$, $Var(Y) = \sigma_Y^2 = 7$

$E(X) = \mu_X = -2$, $E(Y) = \mu_Y = 3$

Standard deviation of $X = \sigma_x = \sqrt{4} = 2$

" " " $Y = \sigma_y = \sqrt{7} = 2.6$

(b) Mean of $2X = E(2X)$

$= 2E(X)$

$= 2 \times (-2)$

$= -4$

Mean of $-3X = E(-3X)$

$= -3E(X)$

$= (-3) \times (-2)$

$= 6$

$Var(2X) = 2^2 \, Var(X) = 2^2 \times 4 = 16$

$Var(-3X) = (-3)^2 \, Var(X) = 9 \times 4 = 36$

(c) $E(X+7) = E(X) + 7 = -2 + 7 = 5$

$E(-3X+5) = -3E(X) + 5 = -3 \times (-2) + 5 = 11$

$Var(X+7) = Var(X) = 4$

$Var(-3X+5) = Var(-3X) = (-3)^2 \, Var(X)$

$= 9 \times 4 = 36$

(d) $E(2X+3Y+7) = 2E(X) + 3E(Y) + 7$

$= 2\times(-2) + 3\times(3) + 7$

$= -4 + 9 + 7$

$= 12$

$Var(2X+3Y+7) = 2^2 Var(X) + 3^2 Var(Y)$

$= 2^2 \times 4 + 3^2 \times 7$

$= 4\times 4 + 9\times 7$

$= 79$

$E(-3X+5Y) = (-3)E(X) + 5E(Y)$

$= (-3)(-2) + 5\times(3)$

$= 21$

$Var(-3X+5Y) = (-3)^2 Var(X) + 5^2 Var(Y)$

$= 9\times 4 + 25\times 7$

$= 211$

(6) Consider following random variable

X = no. of heads in 10 tosses.

Here we can see that

X is Binomially distributed,

$X \sim B(10, \frac{1}{2})$ where

$n = 10$ and $p = \frac{1}{2}$ (probability of head in one trial)

The asked probability is $P(X=4)$.

$$P(X=4) = {}^{10}C_4 \left(\frac{1}{2}\right)^4 \left(1-\frac{1}{2}\right)^{10-4} \quad \left[{}^{n}C_{x} p^{x} (1-p)^{n-x}\right]$$

$$= \frac{10!}{6!\,4!} \times \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^6$$

(8) Let X = no. of customers coming to a bank in a day.

Given that $X \sim P(8)$.

$$P(X=0) = \frac{e^{-8} 8^0}{0!} = e^{-8}$$

$$\approx 0.0003355$$