The more a thing tends to be permanent, the more it tends to be lifeless.

Alan Watts
ZEN

VECTOR POTENTIAL

$\nabla \cdot \vec{B} = 0 \Rightarrow \vec{B} = \nabla \times \vec{A}$

$\vec{A}$: Vector Potential

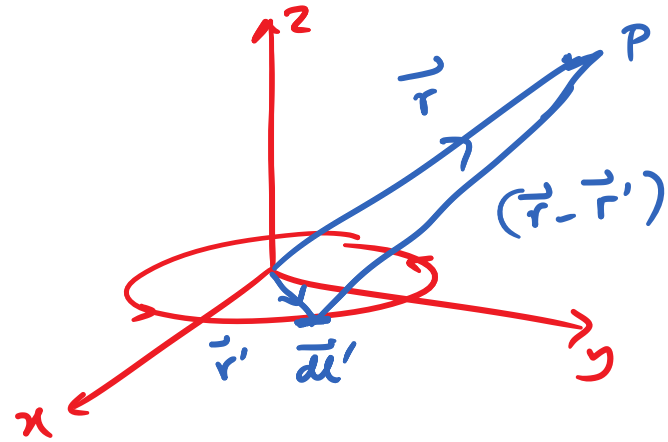
$$\boxed{\nabla \cdot \vec{A} = 0}$$ COULOMB GAUGE

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}\, d\tau'}{|\vec{r}-\vec{r}'|}$$

$$= \frac{\mu_0}{4\pi} \int \frac{\vec{K}\, da'}{|\vec{r}-\vec{r}'|}$$

$$= \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{l}'}{|\vec{r}-\vec{r}'|} = \frac{\mu_0}{4\pi} \int \frac{\vec{I}\, dl'}{|\vec{r}-\vec{r}'|}$$

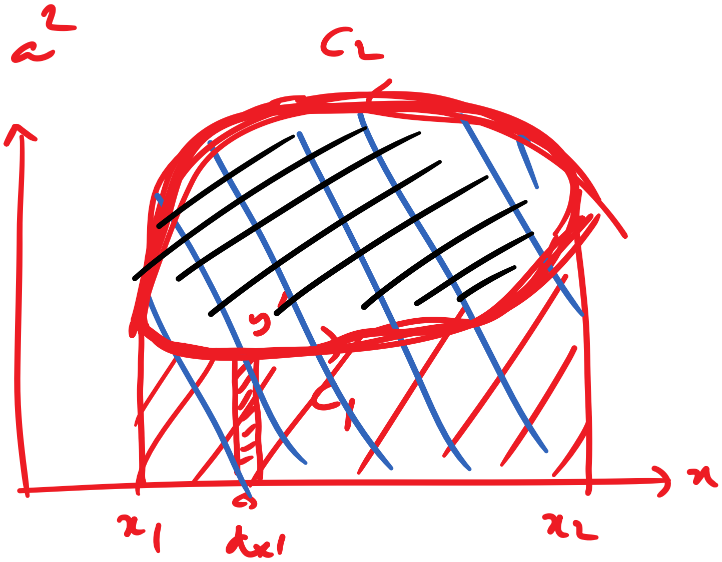
Vector potential due to a circular loop



$$\vec{A} \simeq \frac{\mu_0}{4\pi r^3}\left[x\,\hat{y}\underbrace{\oint x'\,dy'}_{+\pi a^2} + y\,\hat{x}\underbrace{\oint y'\,dx'}_{-\pi a^2} \right]$$

$$\oint x'\,dy' = \pi a^2$$

$$\oint y'\,dx' = -\pi a^2$$



$$\oint y'\,dx' = \int_{x_1,\,C_1}^{x_2} y'\,dx' + \int_{x_2,\,C_2}^{x_1} y'\,dx'$$

$$= \int_{x_1,\,C_1}^{x_2} y'\,dx' - \int_{x_1,\,C_2}^{x_2} y'\,dx'$$

$$\int_{x_1,\,C_1}^{x_2} y'\,dx' =$$

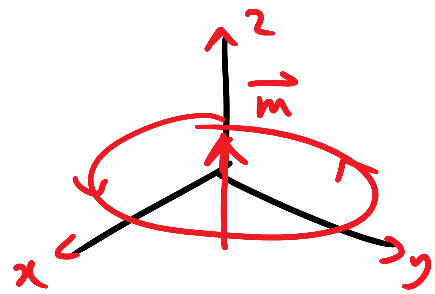
= − Area enclosed by the loop

$$\vec{A} = \frac{\mu_0 I \pi a^2}{4\pi r^3}\left(x\hat{y} - y\hat{x}\right)$$

Magnetic dipole moment

$$\vec{m} = \text{Current} \times \text{Vector area of loop}$$

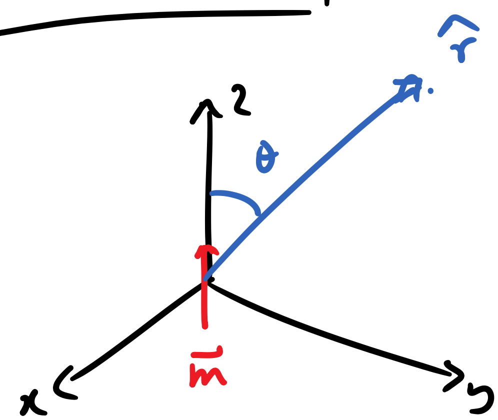
$$= I \times \pi a^2 \,\hat{z}$$



$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\vec{m} \times \vec{r} = I\pi a^2 \times \left(x\hat{y} - y\hat{x}\right)$$

$$\boxed{\vec{A} = \frac{\mu_0}{4\pi r^3}\,\vec{m}\times\vec{r} = \frac{\mu_0}{4\pi r^2}\left(\vec{m}\times\hat{r}\right)}$$



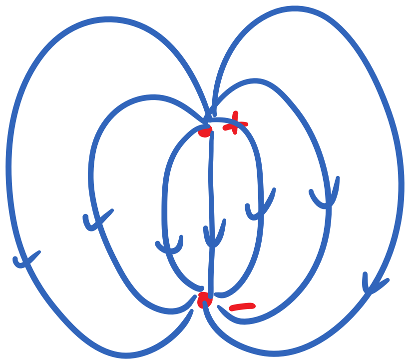
$$\vec{B} = \nabla \times \vec{A}$$

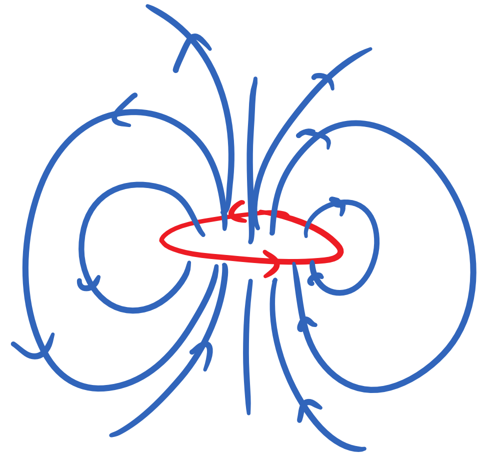
$$\vec{A} = \frac{\mu_0}{4\pi r^2}\, m \sin\theta \,\hat{\varphi}$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\boxed{\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)}$$ Magnetic dipole

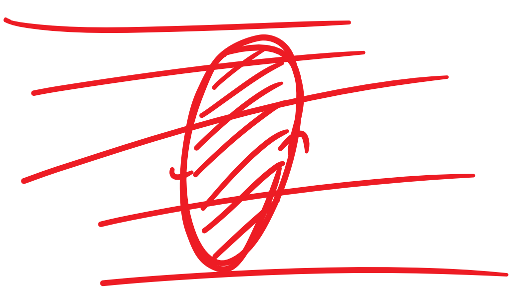
$$\boxed{\vec{E} = \frac{p}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)}$$ Electric dipole





Vector potential of a solenoid

$$\int \vec{B}\cdot d\vec{a} = \int (\nabla\times\vec{A})\cdot d\vec{a} = \oint \vec{A}\cdot d\vec{l}$$



$\nabla\cdot\vec{B} = 0$

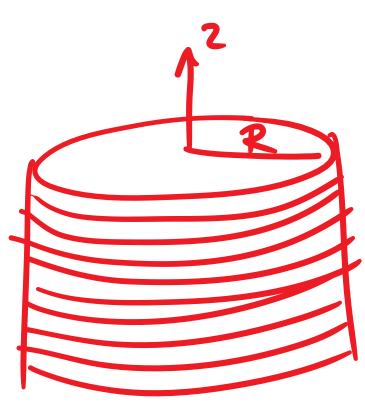
$\nabla\times\vec{B} = \mu_0\vec{J}$

$\nabla\cdot\vec{A} = 0$

$\nabla\times\vec{A} = \vec{B}$

$\nabla\times\vec{A} = \vec{B}$

$\nabla\cdot\vec{A} = 0$

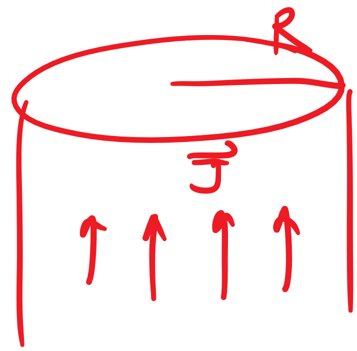


$\vec{B} = \mu_0 n I\,\hat{z}$ $\quad r < R$

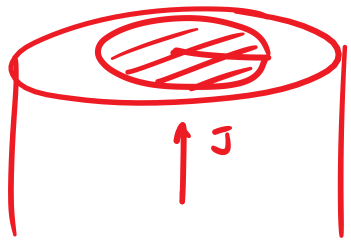
$= 0$ $\quad r > R$

$\nabla \times \vec{B} = \mu_0 \vec{J}$

$\nabla \cdot \vec{B} = 0$



$$\vec{J} = \mu_0 n I \hat{z} \qquad r < R$$
$$= 0 \qquad r > R$$



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$2\pi r B = \mu_0 \pi r^2 J$$

$$\boxed{\vec{B} = \frac{\mu_0 J r}{2} \hat{\varphi}} \quad r < R$$

$r > R \qquad 2\pi r B = \mu_0 \pi R^2 J$

$$\boxed{\vec{B} = \frac{\mu_0 R^2 J}{2r} \hat{\varphi}} \quad r > R$$

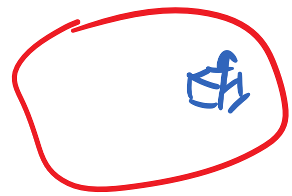
$$\boxed{\begin{aligned} r<R \qquad \vec{A} &= \frac{\mu_0 n I r}{2} \hat{\varphi} \qquad r < R \\ &= \frac{\mu_0 n I}{2r} \hat{\varphi} \qquad r > R \end{aligned}}$$

$$\int \vec{B} \cdot d\vec{a} = \oint \vec{A} \cdot d\vec{l}$$

Magnetic fields in presence of matter

MAGNETIZATION $\vec{M}$

$\vec{M}$ = Magnetic dipole moment per unit volume



$\vec{K}_b$: Bound surface current density

$\vec{J}_b$: Bound volume current density

$$\boxed{\begin{aligned} \vec{K}_b &= \vec{M} \times \hat{n} \\ \vec{J}_b &= \nabla \times \vec{M} \end{aligned}}$$