Tutorial-7 Solutions

Q(1) Weighted Interval Selection / Activity Scheduling problem

(a)

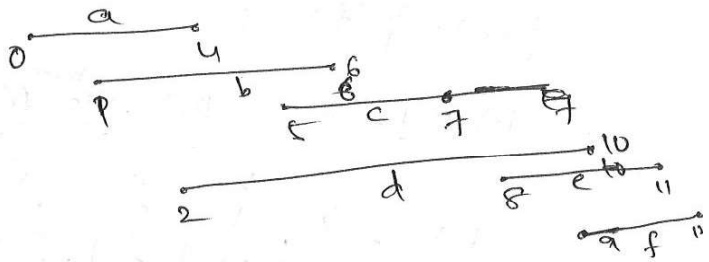
Interval/activity	a	b	c	d	e	f
Startpoint	0	1	5	2	8	9
endpoint	4	6	7	10	11	12
value(v)	2	4	4	7	2	1

(1) Sort the intervals w.r.to finish. / endpoint (in non-decreasing order)

1 2 3 4 5 6
a, b, c, d, e, f

(2) Define $p(j) = i$, such that i is the largest index $i < j$, and intervals i and j do not overlap, if such i does not exist, then $p(j) = 0$.

$p(1) = p(a) = 0$
$p(2) = p(b) = 0$
$p(3) = p(c) = 1$
$p(4) = p(d) = 0$
$p(5) = p(e) = 3$
$p(6) = p(f) = 3$



$M[0] = 0$ & $M[j] = \max\{M[j-1], v_j + M[p(j)]\}$ for $j = 1, \ldots, 6$.

	0	1	2	3	4	5	6
M	0	2	4	6	7	8	8

Since $M[6] == M[5]$, interval 6 (f) is not in the solution

$M[5] != M[4]$, interval e is in the solution, However, the interval 4 is not in the solution since d & e intersect

$M[3] != M[2]$ (d) $\Rightarrow$ c in the solution, So b is not in the solution, finally, a is also in the solution.

Sol: {a, c, e}

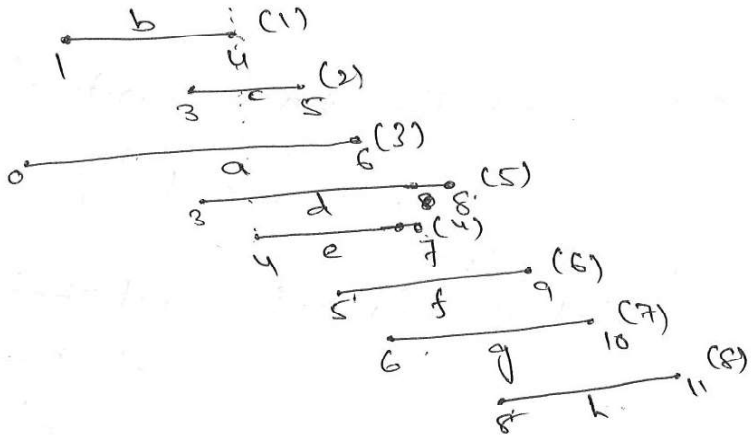
(b)

Interval	a	b	c	d	e	f	g	h
Startpoint	0	1	3	3	4	5	6	8
endpoint	6	4	5	8	7	9	10	11
value	2	1	1	3	4	1	1	2

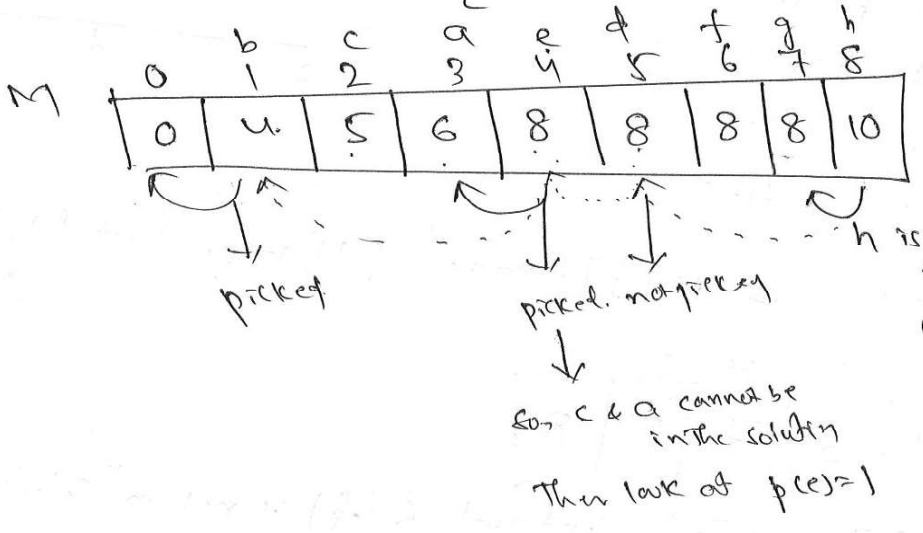
- Sort the intervals w.r.to endpoint, non-decreasing order.

b, c, a, e, d, f, g, h.
1 2 3 4 5 6 7 8

- $p(1) = p(b) = 0$,
  $p(2) = p(c) = 0$
  $p(3) = p(a) = 0$
  $p(4) = p(e) = 1$
  $p(5) = p(d) = 0$
  $p(6) = p(f) = 2$
  $p(7) = p(g) = 3$
  $p(8) = p(h) = 5$



$M[0] = 0$, $M[j] = \max\{M[j-1], v_j + M[p(j)]\}$



∴ Solution: {b, e, h} with value = 10

0-1 knapsack problem

Q(2) (a) Knapsack size = 8

item	A	B	C	D	E	F
profit v_i	7	8	14	5	10	15
size s_i	2	1	5	2	4	3

Table V[ ][ ], with 7 rows and 9 columns. (size of knapsack)

	0	1	2	3	4	5	6	7	8
No item 0	0	0	0	0	0	0	0	0	0
A 1	0	0	7	7	7	7	7	7	7
B 2	0	8	8	15	15	15	15	15	15
C 3	0	8	8	15	15	15	22	22	29
D 4	0	8	8	15	15	17	22	22	29
E 5	0	8	8	15	15	18	22	25	29
F 6	0	8	8	15	23	23	30	30	33

(items: rows 1–6)

$V[0][w] = 0$ for all w & $V[i][0] = 0$ for all i

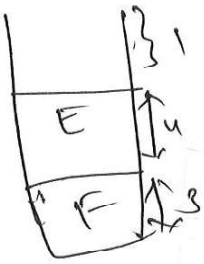
$$V[i][w] = \max\{V[i-1][w],\ v_i + V[i-1][w-s_i]\} \quad \text{if } w \le s_i$$

∴ M[6][8] = 33. So, the optimal value = 33.

∴ M[6][8] > M[5][8] ⟹ item 6 (F) is picked.

5 { [ ] } 8 ⟹ So, only size 5 is free in knapsack, so go to the
3 { [F] }

cell V[5][5] ⟹ V[5][5] > V[4][5] ⟹ item 5 (E) is picked



$\Rightarrow$ free space = 1.

$\Rightarrow$ go to the cell $V[4][1]$

$V[4][1] = V[3][1]$, so item 4 is not picked

$\Rightarrow$ ~~$V[3][1] = V[2][1]$~~. $\Rightarrow$ item 3 is also not picked

$\Rightarrow$ $V[2][1] > V[1][1] \Rightarrow$ item 2 $\in (B)$ is picked

& Bag is full.

$\Rightarrow$ optimum set of items

$= \{B, E, F\}$ with

Value = 33.

Q(2) (b) Knapsack size = 12

item	A	B	C	D	E
profit v	24	13	23.	15	16
sizes	5	6	4	8	7

V[ ][ ].

w (size of Knapsack)

items	0	1	2	3	4	5	6	7	8	9.	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
A 1	0	0	0	0	0	24	24	24	24	24	24	24	24
B 2	0	0	0	0	0	24	24	24	24	24	24	24	24
C 3	0	0	0	0	23	24	24	24	24	47	47	47	47
D 4	0	0	0	0	23	24	24	24	24	47	47	47	47
E 5	0	0	0	0	23	24	24	24	24	47	47	47	47

$V[0][w] = 0, \quad V[i][0] = 0$

$V[i][w] = \max \{ V[i-1][w], \; v_i + V[i-1][w - s_i] \}$ if $w \le s_i$

$V[5][12] = 47 \Rightarrow$ optimum value $= 47$.

$\Rightarrow V[5][12] = V[4][12] = V[3][12] \Rightarrow$ items E (5) & D (4) are not picked into the solution

$V[3][12] > V[2][12] \Rightarrow$ item 3 (C) is picked into the solution

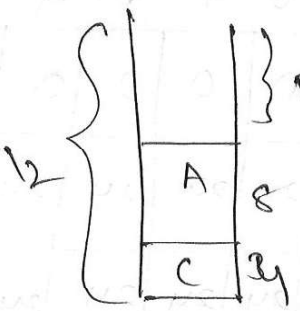
& 8 (8 above C (4) in bag of 12) ∴ Bag free space = 8 and need to fill with items from {1, 2}

⇒ Then look at cell V[2][8].

$V[2][8] = 24$

$V[2][8] == V[1][8]$, ⇒ item 2 (B) is not picked into the solution

⇒ But $V[1][8] \neq V[0][8]$ ⇒ item 1 (A) is picked



Bag is not full, but no items left to pick

⇒ Solution = {A, C} with profit = 47