Solutions - Tutorial Test 2

Ques-1 Let $C([-2,2])$ be the set of all real valued continuous functions on the interval $[-2,2]$. Let

$$W = \{ f \in C[-2,2] : f(\tfrac{1}{5}) = f(\tfrac{1}{6}) = 0 \}.$$

Check whether W is a subspace of $C([-2,2])$ or not.

Solution: (i) 0 (zero function) $\in W$

because $0(\tfrac{1}{5}) = 0(\tfrac{1}{6}) = 0$ (Here $0 \to$ zero function

(ii) Let $f \in W$, then $f(\tfrac{1}{5}) = f(\tfrac{1}{6}) = 0$.

and $g \in W$, then $g(\tfrac{1}{5}) = g(\tfrac{1}{6}) = 0$.

Let $\alpha, \beta \in F$, then

$$(\alpha f + \beta g)(\tfrac{1}{5}) = \alpha f(\tfrac{1}{5}) + \beta g(\tfrac{1}{5}) = \alpha \cdot 0 + \beta \cdot 0 = 0$$

and $$(\alpha f + \beta g)(\tfrac{1}{6}) = \alpha f(\tfrac{1}{6}) + \beta g(\tfrac{1}{6}) = \alpha \cdot 0 + \beta \cdot 0 = 0$$

$\Rightarrow \alpha f + \beta g \in W$

$\Rightarrow W$ is a subspace of $C([-2,2])$.

Ques-2 Write the matrix $E = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}$ as a linear combination of the matrices $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$.

Solution: Let $E = \alpha \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + \beta \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} + \gamma \begin{bmatrix} 0 & 2 \\ 0 & -1 \end{bmatrix}$,

$\alpha, \beta, \gamma \in \mathbb{R}$

then $\begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} \alpha & \alpha+2\gamma \\ \alpha+\beta & \beta-\gamma \end{bmatrix}$

$\Rightarrow$ $\alpha = 3,$

$\alpha+2\gamma = 1$

$\alpha+\beta = 1$

$\beta - \gamma = -1$

Solving the above equations, we get $\alpha=3$, $\beta=-2$, $\gamma=-1$.

Thus, $\boxed{E = 3A - 2B - C}$ Ans.

Ques-3 Let V be the vector space of all 3X3 matrices over the real field IR and

$$W = \{A \in M_{3\times3}(\mathbb{R}) : \text{trace}(A) = 0\}$$

is a subspace of V. Find the basis and dimension of W.

Solution! Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \in W$,

$\Rightarrow$ where $\text{trace}(A) = a_{11} + a_{22} + a_{33} = 0$

$\Rightarrow$ $a_{33} = -a_{11} - a_{22}$

$\Rightarrow$ $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & -a_{11}-a_{22} \end{bmatrix}$

$$= a_{11} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}}_{A_1} + a_{12} \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{A_2} + a_{13} \underbrace{\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{A_3}$$

$$+ a_{21} \underset{A_4}{\begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}} + a_{22} \underset{A_5}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}} + a_{23} \underset{A_6}{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}$$

$$+ a_{31}\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}}_{A_7} + a_{32}\underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{A_8}$$

$$\Rightarrow A = a_{11}A_1 + a_{12}A_2 + a_{13}A_3 + a_{21}A_4 + a_{22}A_5 + a_{23}A_6 + a_{31}A_7 + a_{32}A_8 .$$

$\Rightarrow W = \text{span}\{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8\}$

Also, $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8$ are linearly independent.

$\Rightarrow \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8\}$ forms the basis of W and $\dim W = 8$.

Ques 4 Find the basis and dimension of $\mathbb{C}^2(\mathbb{R})$.

Solution: $\mathbb{C}^2 = \{(a+ib, c+id) : a, b, c, d \in \mathbb{R}\}$.

Let $(a+ib, c+id) \in \mathbb{C}^2$

$\Rightarrow (a+ib, c+id) = a(1,0) + b(i,0) + c(0,1) + d(0,i)$, where $a, b, c, d \in \mathbb{R}$.

Also, $\{(1,0), (i,0), (0,1), (0,i)\}$ is linearly independent.

$\Rightarrow \{(1,0), (i,0), (0,1), (0,i)\}$ forms basis of $\mathbb{C}^2(\mathbb{R})$ and dimension $\mathbb{C}^2(\mathbb{R}) = 4$.