Department of Mathematics, Bennett University
Engineering Calculus (EMAT101L)
Tutorial Sheet 3

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1. Show that the following series diverges

   $(a)\ \sum_{n=1}^{\infty} n^{\frac{1}{n}}, \quad (b)\ \sum_{n=1}^{\infty} \left(1+\frac{x}{n}\right)^n, \quad (c)\ \sum_{n=1}^{\infty} \log\left(\frac{n+1}{n}\right).$

2. If $0 \le a_n \le 1$ $(n \ge 0)$ and if $0 \le x < 1$, then prove that $\sum_{n=1}^{\infty} a_n x^n$ converges.

3. Determine which of the following series converges/diverges:

   $(a)\sum_{n=1}^{\infty} \frac{\log n}{n^{3/2}}, \quad (b)\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}}, \quad (c)\sum_{n=1}^{\infty} \frac{\sqrt[n]{n}}{n^2}.$

4. Determine which of the following series converges/diverges:

   $(a)\ \sum_{n=1}^{\infty} \frac{n^{\sqrt{2}}}{2^n}, \quad (b)\ \sum_{n=1}^{\infty} \frac{n!}{10^n}, \quad (c)\ \sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}.$

5. Determine which of the following series converges absolutely/conditionally:

   $(a)\ \sum_{n=1}^{\infty} (-1)^n \frac{1}{n}, \quad (b)\ \sum_{n=1}^{\infty} (-1)^n \frac{\sin nx}{n^2}, \quad (c)\ \sum_{n=2}^{\infty} (-1)^n \frac{1}{\log n}.$

6. Find the value of x for which the following series converges:

   $(a)\ \sum_{n=0}^{\infty} (n+1+2^n)x^n, \quad (b)\ \sum_{n=0}^{\infty} \frac{n!x^n}{n^n}, \quad (c)\ \sum_{n=1}^{\infty} \frac{n^{n^2}}{(n+1)^{n^2}}(x-1)^n.$

7. Test the convergence of the infinite series:

   $(a)\ \sum_{n=0}^{\infty} \sin\left(\frac{\pi}{2^n}\right), \quad (b)\ \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{1}{\sqrt{n}}\right).$