Introduction to Discrete Mathematical Structures

Introduction

- Discrete Mathematics is a branch of mathematics involving discrete elements that uses algebra and arithmetic. It is increasingly being applied in the practical fields of mathematics and computer science. It is a very good tool for improving reasoning and problem-solving capabilities.

- Types of Mathematics: Mathematics can be broadly classified into two categories –
  - Continuous Mathematics
  - Discrete Mathematics.

- **Continuous Mathematics** is based upon continuous number line or the real numbers. It is characterized by the fact that between any two numbers, there are almost always an infinite set of numbers. For example, a function in continuous mathematics can be plotted in a smooth curve without breaks.
- **Discrete Mathematics**, on the other hand, involves distinct values; i.e. between any two points, there are a countable number of points. For example, if we have a finite set of objects, the function can be defined as a list of ordered pairs having these objects, and can be presented as a complete list of those pairs.

Why to study DMS?

- In computer science basic mathematical concept is used.
- It helps to understand other subjects in CS
- Main aim is to think in mathematical manner.

Areas in which discrete mathematics concepts are applied

- Formal Languages (computer languages)
- Compiler Design
- Data Structures
- Computability
- Automata Theory
- Algorithm Design
- Relational Database Theory
- Complexity Theory (counting)

Example (counting):

- The Traveling Salesman Problem

Important in

- circuit design
- many other CS problems

Given:

- n cities $c_1, c_2, \dots, c_n$
- distance between city i and j, d_{ij}

Find the shortest tour.

Assume a very fast PC:

1 flop = 1 nanosecond

= 10^{-9} sec.

= 1,000,000,000 ops/sec

= 1 GHz.

A tour requires n-1 additions. How many different tours?

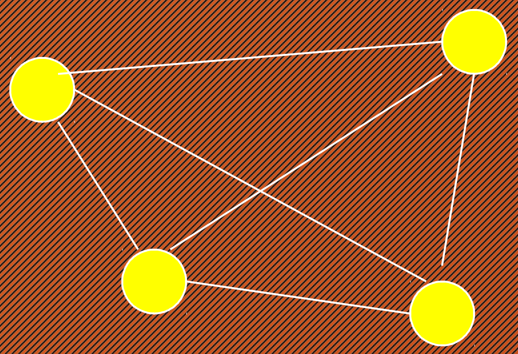
Choose the first city n ways,

the second city n-1 ways,

the third city n-2 ways,

etc.

tours = n (n-1) (n-2) . . . .(2) (1) = n! (*Combinations*)



Total number of additions = n(n-1)! (*Rule of Product*)

If n=8, T(n) = 8•7! = 40,320 flops < 1/3 second.

HOWEVER . . . .

If n=50, T(n) = 50•49!

= $3.04\ 10^{64}$

= $3.04\ 10^{55}$ seconds

= $5.0\ 10^{53}$ minutes

= $8.0\ 10^{51}$ hours

= $3.0\ 10^{50}$ days

= $4.0\ 10^{49}$ weeks

= $7.0\ 10^{47}$ years.

...a long time. You'll be an old person (dead) before it's finished!

There are some problems for which we do not know if efficient algorithms exist to solve them!