Tutorial – 7

1) Prove with the help of an example that every asymmetric relation is also an antisymmetric relation i.e. ($Asymmetric\ Relation \subseteq Antisymmetric\ Relation$).

2) Let R be a relation on a set of positive integers defined as xRy if and only if $x|y$ that is x divides y. Determine whether the relation R is transitive or not.

3) Determine whether the following are true or false:
"If a relation is"
(a) Symmetric and Transitive ⇒ Reflexive
(b) Irreflexive and Symmetric ⇒ Transitive

4) The binary relation $R = \{(1,1), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4)\}$on a set $A = \{1, 2, 3, 4\}$ is
(a) Reflexive, symmetric and transitive
(b) Neither reflexive, nor irreflexive but transitive
(c) Irreflexive, symmetric and transitive
(d) Irreflexive and antisymmetric

5) Compute the maximum number of relations possible from a set.

6) If a relation R on a set of integers Z is define as

$$R = \{(x, y) | x \in Z, y \in Z, (x - y)\ is\ divisible\ by\ 6\ \}$$

Then prove that R is an equivalence relation.

7) Let R be a relation on A. Prove that "R is antisymmetric if and only if $R \cap R^{-1} \subseteq I_A$".

8) Let R be the relation from $S = \{1,2,3,4\}$ to $T = \{a, b, c\}$ represented as

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

(a) Show that RoR^{-1} is a symmetric relation on S.
(b) Show that $R^{-1}oR$ is a symmetric relation on T.
(c) Are the relation RoR^{-1} and $R^{-1}oR$ equivalent relation?

9) A relation R defined on $A = \{1,2,3,4,5\}$ as

$$R = \{(1,2), (1,6), (2,3), (3,3), (3,4), (4,1), (4,3), (4,5), (6,4)\}$$

Draw the graph of R and R^2 and hence find M_{R^2}.

10) Consider the relation R defined on A as follows:
$R = \{(a, b), (b, c), (d, c), (d, a), (a, d), (d, d)\}$ on $A = \{a, b, c, d\}$
Find the reflexive, transitive and symmetric closure of R.

11) Let a set X contain n elements. How many reflexive relations will there be on X?