Tutorial Solution – 08

1. (a) f is not a function because the element b of A does not have an image in B.

(b) f is not a function because the element c of A does not have a unique image in B.

(c) f is a function because every element of A has a unique image in B.

(d) f is a function because every element in A has a unique image in B.

2. We have to prove this function is both injective and surjective.

If f(x1) = f(x2), then 2x1–3=2x2–3 and it implies that x1=x2.

Hence, f is injective.

Here, 2x–3=y

So, x=(y+5)/3 which belongs to R and f(x) = y.

Hence, f is surjective.

Since f is both surjective and injective, we can say f is bijective.

3. The function f is invertible because it is a one-to-one correspondence. The inverse function f -1 reverses the correspondence given by f, so f -1(1) = c, f -1(2) = a, and f -1(3) = b.

4.(c) From the definition of the sum and product of functions, it follows that (f1 + f2)(x) = f1 (x) + f2(x) = x 2 + (x – x2) = X and (f1 f2)(x)=x2 (x-x2) =x3- x4 When f is a function from a set A to a set B, the image of a subset of A can also be defined.

5. (a) The composition of f and g is given by f(g(x)) which is equal to 2(3x + 4) + 1.

6. Thus, the correct option is A.

7. a) The conversion formula r = (π /180)* d stipulates r as a function of d. The opposite conversion formula d = (180/ π)* r specifies the same relationship between r and d, only this time from the other direction, so it defines a different function. In both cases, the domain and co-domain can be taken to be the set R of real numbers.

b) Since a definite amount of money is either owed to or by each person with a social security number, P is a function from the set of social security numbers to the set of positive and negative numbers given to two decimal places. P is a function even though there is no set formula for calculating P(x); in fact, while a complicated equation might be devised, it would be of no use whatsoever as a formula. It only summarizes a discrete set of data; it would not cover any new cases that arise. On the other hand, if x indicated taxable income instead of social security numbers, a usable formula could be found, though it would not be a single equation. Currently, the tax percentage changes as income increases; there are different levels of taxation (formulas) depending on income level.

8. No domain is given; we will assume it is the largest set of real numbers for which the formula makes sense, namely, R − {−3}. To prove that f is one-to-one, we will show that the pre-images of two equal images are themselves equal. So suppose x1/ x1 + 3 = x2 /x2 + 3. Cross-multiplying and simplifying yields x1 = x2. Thus, f is one-to-one. f is obviously onto its range by definition. To determine the range, we must see what y-values result from real x-values different from 3.

Suppose y = x/ x + 3.

All such y are real numbers, but not all real numbers can be put into this form. To see which ones can, we will solve this equation for x. This will show explicitly which x-values in the domain, if any, can generate a given y-value. Cross-multiplying, xy + 3y = x, so x(1 − y) = 3y. Hence x = 3y/ 1 − y, which is defined for all y except 1. So, y = 1 has no pre-image x. However, given any real number y is not equal to 1, these x-values will produce it: (3y/ 1 – y)/ (3y /1 – y) + 3 = 3y /3y + (3 − 3y) = 3y /3 = y. Thus, the only y-value that must be excluded from R is 1. This would give 1 = x x + 3 , leading to 3 = 0, which is false. Therefore, the range of f is R − {1}. Note that here y = 1 is the equation of the asymptote for the function f.