काल करे तो आज कर, आज करे तो अब
पल मे परलय होयेगी, बहुरी करेगा कब

कबीर

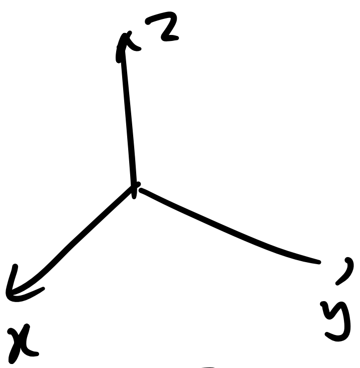
(1440–1518)

Mystic poet, Musician

Hindu saint, Sufi

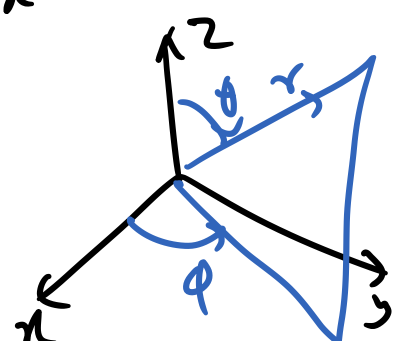
---

① Coordinate Systems



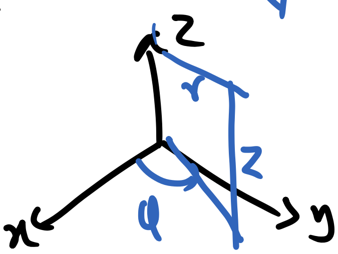
$$(\hat{x}, \hat{y}, \hat{z})$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$



$$(\hat{r}, \hat{\theta}, \hat{\varphi})$$

$$\vec{r} = r\hat{r}$$



$$(\hat{r}, \hat{\phi}, \hat{z})$$

$$\vec{r} = r\hat{r} + z\hat{z}$$

② Vector operators

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$$

$$\nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\int_A^B \nabla f \cdot \vec{dl} = f(B) - f(A)$$

$$\int_V \nabla \cdot \vec{F}\, dV = \oint_S \vec{F} \cdot d\vec{a}$$ GAUSS's Theorem

$$\int_S \nabla \times \vec{F} \cdot d\vec{a} = \oint_{\text{Line}} \vec{F} \cdot \vec{dl}$$ Stokes' Theorem

(3) Electrostatics

$\vec{E}$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$$\boxed{\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}}$$

$$\oint \vec{E} \cdot d\vec{l} = 0 \Rightarrow \boxed{\nabla \times \vec{E} = 0}$$

$$\boxed{\vec{E} = -\nabla V}$$

V: Electrostatic Potential

$$\vec{E} = \int \frac{\rho(\vec{r}\,')\,(\vec{r} - \vec{r}\,')}{4\pi\epsilon_0 |\vec{r} - \vec{r}\,'|^3}\, dV' \qquad \text{Volume}$$

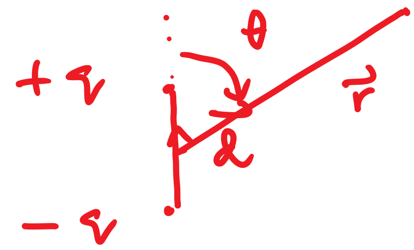
$$= \int \frac{\sigma(\vec{r}\,')\,(\vec{r} - \vec{r}\,')}{4\pi\epsilon_0 |\vec{r} - \vec{r}\,'|^3}\, da' \qquad \text{Area}$$

$\nabla^2 V = 0$ Laplace equation

$\nabla^2 V = -\frac{\rho}{\epsilon_0}$ Poisson's equation

Electric dipole

$$\vec{p} = q\,\vec{d}$$



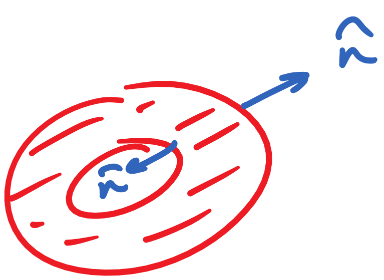
$$V = \frac{p\cos\theta}{4\pi\epsilon_0 r^2} = \frac{\vec{p}\cdot\vec{r}}{4\pi\epsilon_0 r^3}$$

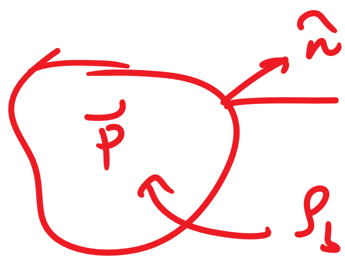
Dielectrics

$\vec{P}$: Polarization

Bound Volume charge density $\rho_b = -\nabla\cdot\vec{P}$

Bound surface charge density $\sigma_b = \vec{P}\cdot\hat{n}$





$\sigma_b = \vec{P}\cdot\hat{n}$

$\rho_b = -\vec{\nabla}\cdot\vec{P}$

$$\vec{D} = \epsilon_0\vec{E} + \vec{P}$$

$$\boxed{\nabla\cdot\vec{D} = \rho_f \qquad \oint \vec{D}\cdot d\vec{a} = Q_{f\,enc}}$$

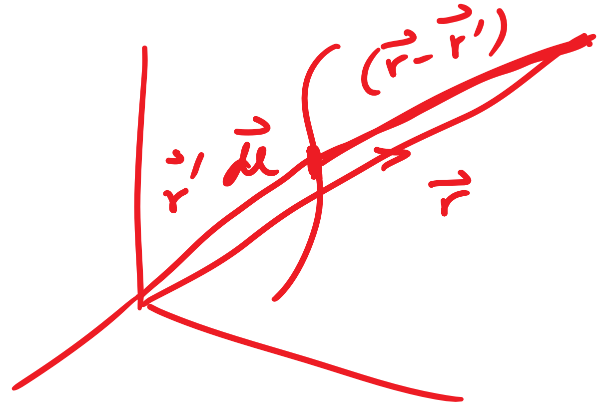
$$\vec{P} = \epsilon_0\chi\vec{E}$$

χ : Electric Susceptibility

$$\vec{D} = \epsilon_0(1+\chi)\,\vec{E}$$
$$= \epsilon\,\vec{E}$$

Magnetostatics

Lorentz force $\quad \vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$



$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I\, d\vec{l} \times \vec{\mathscr{r}}}{\mathscr{r}^3}$$

$\underline{\underline{\vec{\mathscr{r}}}} = \vec{r} - \vec{r}'$

$$= \frac{\mu_0}{4\pi} \int \frac{\vec{J} \times \vec{\mathscr{r}}}{\mathscr{r}^3}\, dV'$$

$$= \frac{\mu_0}{4\pi} \int \frac{\vec{K} \times \vec{\mathscr{r}}}{\mathscr{r}^3}\, da'$$

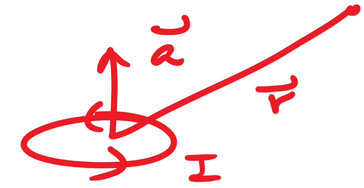
$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \mu_0 \vec{J}$

$\oint \vec{B} \cdot d\vec{a} = 0 \qquad \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

$\boxed{\vec{B} = \nabla \times \vec{A}} \qquad \vec{A}$: Vector Potential

Magnetic dipole

$$\vec{m} = I\vec{a}$$



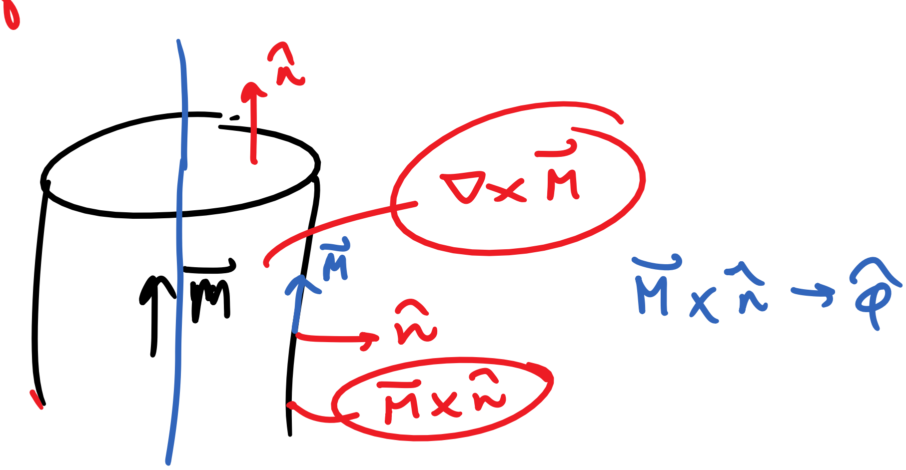
$$\boxed{\vec{A} = \frac{\mu_0}{4\pi r^2}\left(\vec{m}\times\hat{r}\right)}$$

Materials

$\vec{M}$: Magnetization

$\equiv$ Bound volume current density $\vec{J}_b = \nabla\times\vec{M}$

Bound surface current density $\vec{K}_b = \vec{M}\times\hat{n}$



$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$\vec{M} = \chi_m \vec{H}$

χ_m : Magnetic Susceptibility

$$\vec{B} = \mu_0(1+\chi_m)\vec{H} = \mu \vec{H} \qquad \mu = \mu_0(1+\chi_m)$$

$$\boxed{\nabla \times \vec{H} = \vec{J}_f} \qquad \boxed{\oint \vec{H} \cdot \vec{dl} = I_{f\,enc}}$$

Electromagnetic Induction

$$\boxed{\mathcal{E} = \oint \vec{E} \cdot \vec{dl} = -\frac{d}{dt}\int \vec{B} \cdot \vec{da}}$$

emf

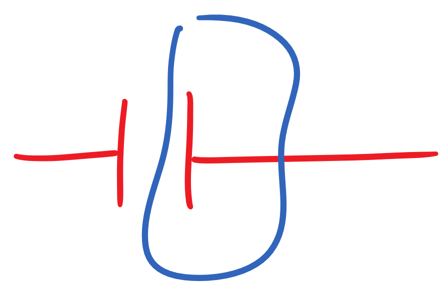
$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$



$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\nabla \times \vec{B} = \mu_0 (\vec{J} + \vec{J}_D) = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

Displacement Current density

$$\vec{J}_D = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Maxwells equations

$$\boxed{\begin{aligned} \nabla \cdot \vec{E} &= \rho/\epsilon_0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{B} &= \mu_0 \vec{J} + \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \end{aligned}}$$

Free space

$$\rho = 0, \quad \vec{J} = 0$$

$$\boxed{\begin{aligned} \nabla \cdot \vec{E} &= 0 & \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} & \nabla \times \vec{B} &= \epsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \end{aligned}}$$

Wave equation

$$\boxed{\nabla^2 \vec{E} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \simeq 3 \times 10^8 \text{ m/s}$$

$$\boxed{\frac{\partial^2 \vec{E}}{\partial z^2} = \epsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2}}$$

$\vec{E} = \vec{E}_0 \sin(kz - \omega t) \longrightarrow$ Wave along $+z$ direction

$$k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi\nu$$

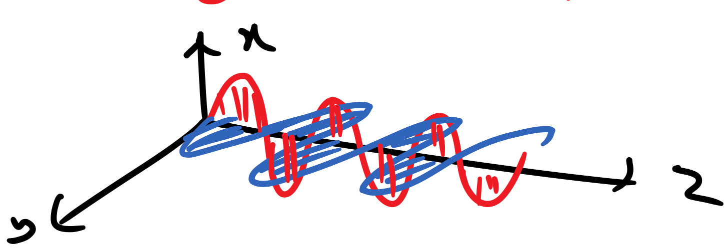
$$c = \nu\lambda$$

$$\vec{E} = \vec{E}_0 \sin\left(\frac{2\pi}{\lambda} z - 2\pi\nu t\right)$$

$\vec{E}_0 \perp z$-direction

$$\vec{E} = E_0 \hat{x} \sin\left(\frac{2\pi}{\lambda} z - 2\pi\nu t\right)$$

$$\vec{B} = \frac{E_0}{c} \hat{y} \sin\left(\frac{2\pi}{\lambda} z - 2\pi\nu t\right)$$



$(\vec{E}, \vec{B}$, Propagation direction$) \equiv$ RH Coordinate System

---

Boundary Condition

- Tangential Component of $\vec{E}$ = Continuous

  Perpendicular component of $\vec{B}$ = Continuous