Solution - Tutorial sheet 4

(1) Given that $X \sim N(12, 16)$.

(a) From standard normal distribution table (Z-table):

$P(X \geq 20) = 0.0228$

$P(X > 20) = 0.0228$

$P(X \leq 20) = 1 - P(X > 20) = 0.9772$

$$P(0 \leq X \leq 12) = P(X \leq 12) - P(X \leq 0)$$
$$= [1 - P(X>12)] - [1 - P(X>0)]$$
$$= P(X>0) - P(X>12)$$

[From table]

$$= 0.5 - 0.0013$$
$$= .4987$$

(b) $P(Z > z_{0.57}) = 0.57$

$P(Z < z_{0.57}) = 1 - \cancel{0.5} P(Z > z_{0.57})$

$= 1 - .57 = .43$

$\Rightarrow P(Z > -\cancel{0.5} z_{0.57}) = 0.43$

From Z-table

$-z_{0.57} = 0.18 \Rightarrow \boxed{z_{0.57} = -0.18}$

(c) From Z-table

$P(Z > 0.71) = 0.24 \Rightarrow \boxed{a = 0.71}$

(2) $\underline{X = Z}$

$P(-0.75 < Z < 0.75) = 0.42$

$\Rightarrow P(Z < 0.75) - P(Z < -0.75) = 0.42$

$\Rightarrow [1 - P(Z > 0.75)] - P(Z > 0.75) = 0.42$

$\Rightarrow 1 - 2P(Z > 0.75) = 0.42$

$\Rightarrow P(Z > 0.75) = \frac{1 - 0.42}{2} = 0.29$

(3) Not in syllabus.

(4) X – yield of one-acre plot.

Given that $X \sim N(662, 32^2)$

(a) Let $p = P(X > 700) = P\left(\frac{X - 662}{32} > \frac{700 - 662}{32}\right)$

$= P(Z > 1.19)$

$= .117$

$\underline{\text{Ans}}$ 1000×0.117

≈ 117 plots

[Binomial$(1000, .117)$

Average $= np$

$= 1000 \times 0.117$)

(b) $p = P(X < 650) = P\left(\frac{X - 662}{32} < \frac{650}{32}\right) = \underline{.352}$

$\underline{\text{Ans}}$ $1000 \times .352$

$= 352$ plots.

(5) X = marks obtained by a student
$\sim N(65, 5^2)$.

$p = P(X > 70) = P\left(\frac{X-65}{5} > \frac{70-65}{5}\right) = P(Z > 1)$
$= 0.1587$

Ans $^3C_2 (0.1587)^2 (1-0.1587)^{3-2}$
$= 0.06357$

(6) $2X_1 - X_2 + X_3 + 7 \sim N(\mu, \sigma^2)$

where

$\mu = E(2X_1 - X_2 + X_3 + 7)$
$= 2E(X_1) - E(X_2) + E(X_3) + 7$
$= 2 \times 0 - 1 + (-2) + 7$
$= 4$

$\sigma^2 = Var(2X_1 - X_2 + X_3 + 7)$
$= 2^2 Var(X_1) + (-1)^2 Var(X_2) + Var(X_3)$
$= 2^2 \times 5 + (-1)^2 \times 1 + 9$
$= 30.$

(7) let $A = 2X + Y \sim N(19, 4 \times 9 + 16) \equiv N(19, 52)$

$B = 4X - 3Y \sim N(4 \times 6 - 3 \times 7, 4^2 \times 9 + (-3)^2 \times 16)$
$\equiv N(3, 288)$

P.T.O.

Thus the given question is to find λ such that $P(A \leq \lambda) = P(B \geq 4\lambda)$

$$\Rightarrow P\left(\frac{A-19}{\sqrt{52}} \leq \frac{\lambda-19}{\sqrt{52}}\right) = P\left(\frac{B-3}{\sqrt{288}} \geq \frac{4\lambda-3}{\sqrt{288}}\right)$$

$$\Rightarrow P\left(Z \leq \frac{\lambda-19}{\sqrt{52}}\right) = P\left(Z \geq \frac{4\lambda-3}{\sqrt{288}}\right)$$

$$\Rightarrow \frac{\lambda-19}{\sqrt{52}} = -\left(\frac{4\lambda-3}{\sqrt{288}}\right)$$

Solving for $\lambda \Rightarrow \lambda = 7.51$

(8) Sol If $X_1 \sim B(n_1, p)$ & $X_2 \sim B(n_2, p)$ independent then

$$X_1 + X_2 \sim B(n_1+n_2, p).$$

So here $X+Y \sim B(10+15, \frac{1}{3}) \equiv B(25, \frac{1}{3})$

Mean $= 25 \times \frac{1}{3}$

Variance $= 25 \times \frac{1}{3} \times \frac{2}{3}$.

(9) $X+Y \sim B(6+4, \frac{1}{2})$

$\sim B(10, \frac{1}{2})$

10

X = No. of calls between 10–11 a.m.

$\sim$ Poisson(2)

Y = No. of calls between 11 am – 12 noon

$\sim$ Poisson(6)

$P(X+Y>5) = \cancel{1-P(}$

$= \cancel{\sum_{i=6}^{\infty} P(X+Y}$

$= 1 - P(X+Y \leq 5)$ $\quad [X+Y \sim \text{Poisson}(8)]$

$= 1 - \sum_{k=0}^{5} \frac{e^{-8}\, 8^{k}}{k!}$

$= 0.8088.$