Amdahl's Law

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<u>What</u> is Amdahl’s Law?

Amdahl's law is an expression used to **find the maximum expected improvement S to an overall system when only part of the system (f_E) is improved by a factor f_I**. Amdahl’s Law is often used in parallel computing to predict the theoretical maximum speedup using multiple processors.

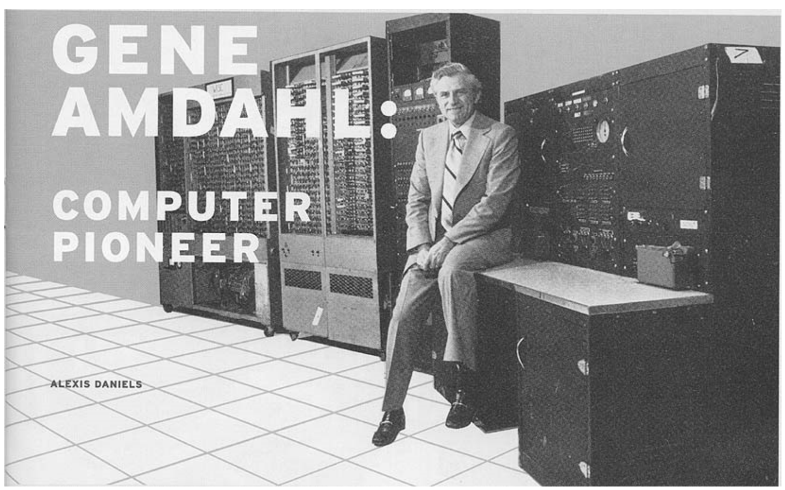
<u>Why</u> Do We Use Amdahl’s Law?

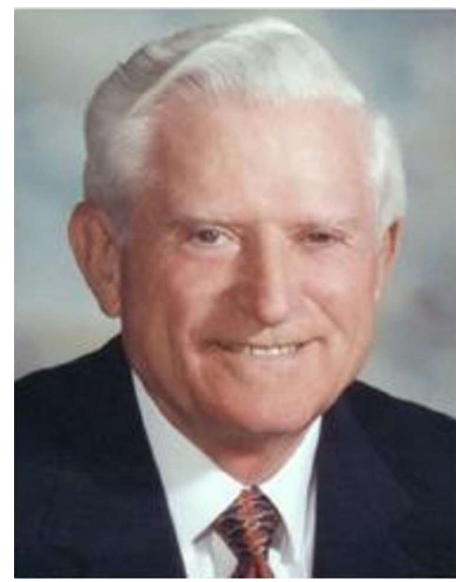
➔ ESTIMATE SYSTEM PERFORMANCE…

➔ Performance Improvement Problems!!

Who Was Gene Amdahl?

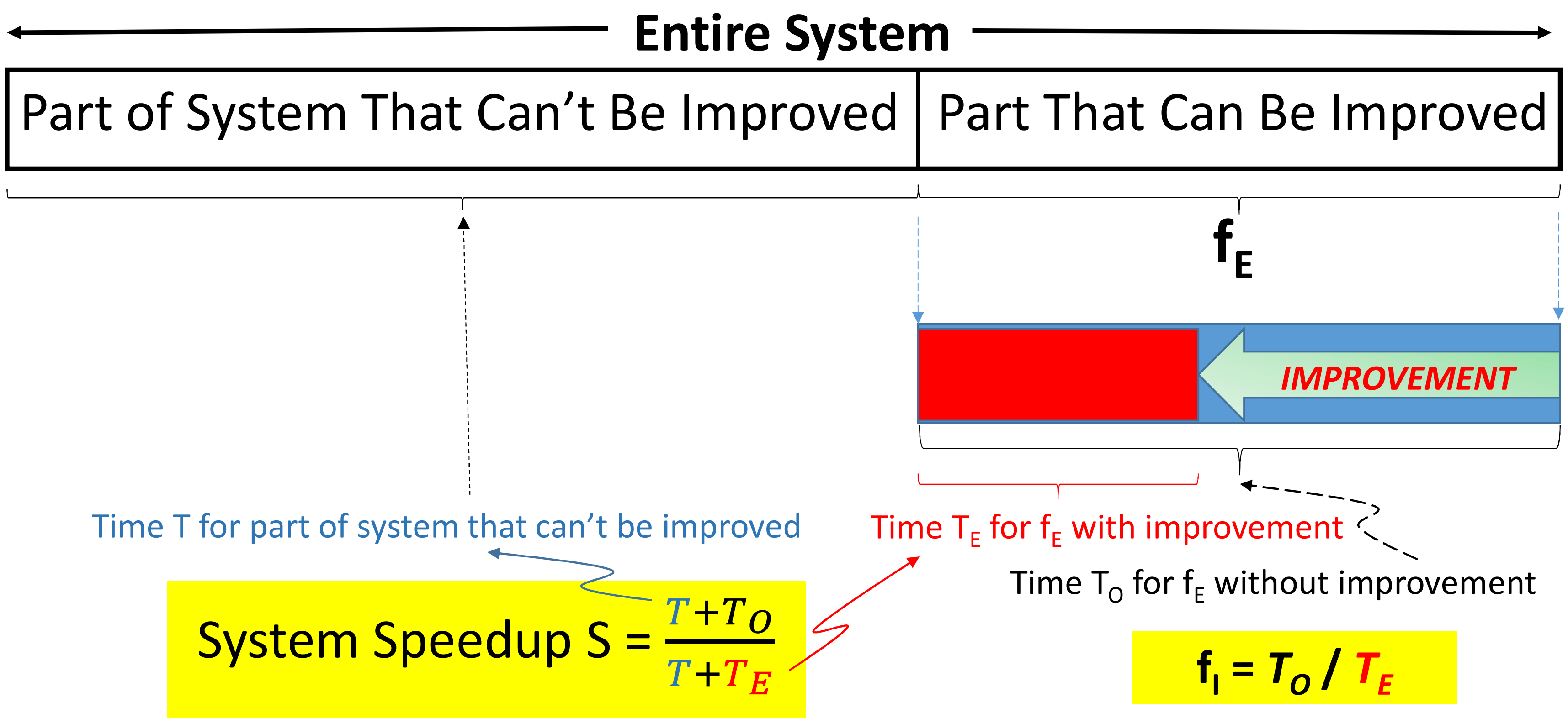
Gene Myron Amdahl (November 16, 1922 – November 10, 2015) was an American computer architect and high-tech entrepreneur, chiefly known for his work on mainframe computers at IBM and later his own companies, especially Amdahl Corporation. He formulated Amdahl's Law…





What Kinds of Problems Do We Solve With Amdahl's Law?

➔ Performance Improvement Problems!!



<u>How</u> Do We Solve Amdahl's Law Problems?

Amdahl's Law Equation:

$$\underset{(S)}{\text{Speedup}} = \frac{1}{(1 - \underset{(f_E)}{\text{fraction enhanced}}) + (\underset{(f_E)}{\text{fraction enhanced}} / \underset{(f_I)}{\text{factor of improvement}})}$$

Divide Problem Space Into Three Cases:

1. **Given:** f_E and f_I **Find:** S
2. **Given:** S and f_E **Find:** f_I
3. **Given:** S and f_I **Find:** f_E

How Do We Solve Amdahl's Law Problems?

Case 1: Given: f_E and f_I **Find:** S

If we know f_E and f_I, then we use the Speedup equation (above) to determine S.

Example: Let a program have 40 percent of its code enhanced (so $f_E = 0.4$) to run 2.3 times faster (so $f_I = 2.3$). What is the overall system speedup S?

Step 1: Setup the equation: $S = ((1 - f_E) + (f_E / f_I))^{-1}$

Step 2: Plug in values & solve $S = ((1 - 0.4) + (0.4 / 2.3))^{-1}$

$= (0.6 + 0.174)^{-1} = 1 / 0.774$

$= \mathbf{1.292}$

How Do We Solve Amdahl's Law Problems?

Case 2: Given: S and f_E **Find:** f_I

Example: Let a program have 40 percent of its code enhanced (so $f_E = 0.4$) to yield a system speedup 4.3 times faster (so $S = 4.3$). What is the factor of improvement f_I of the portion enhanced?

Case #1:

Can we do this? In other words, let's determine if by enhancing 40 percent of the system, it is possible to make the system go 4.3 times faster …

Step 1: Assume the limit, where f_I = infinity, so $S = ((1 - f_E) + (f_E / f_I))^{-1} \rightarrow S = 1 / (1 - f_E)$

Step 2: Plug in values & solve $S = ((1 - 0.4))^{-1} = 1 / 0.6 = \mathbf{1.67}$.

Step 3: So S = 1.67 is the **maximum possible speedup**, and we cannot achieve S = 4.3 !!

Oops, that one didn't work so well … let's try another example

How Do We Solve Amdahl's Law Problems?

Case 2: Given: S and f_E **Find:** f_I

A different case: Let's determine if by enhancing 40 percent of the system, it is possible to make the system go 1.3 times faster …

Step 1: Assume the limit, where f_I = infinity, so $S = ((1 - f_E) + (f_E / f_I))^{-1} \rightarrow S = 1 / (1 - f_E)$

Step 2: Plug in values & solve $S = ((1 - 0.4))^{-1} = 1 / 0.6 = \mathbf{1.67}$.

Step 3: So $S = 1.67$ is the **maximum possible speedup**, and we can achieve $S = 1.3$!!

Step 4: Solve speedup equation for f_I :

$$1/S = (1 - f_E) + (f_E / f_I) \quad \text{[invert both sides]}$$

$$1/S - (1 - f_E) = f_E / f_I \quad \text{[subtract } (1 - f_E)\text{]}$$

$$(1/S - (1 - f_E))^{-1} = f_I / f_E \quad \text{[invert both sides]}$$

$$f_E \cdot (1/S - (1 - f_E))^{-1} = f_I \quad \text{[multiply by } f_E\text{]}$$

Step 5: Plug in values & solve:

$$f_I = f_E \cdot (1/S - (1 - f_E))^{-1}$$

$$= 0.4 \cdot (1/1.3 - (1 - 0.4))^{-1}$$

$$= 0.4 / (0.769 - 0.6) = \mathbf{2.367}$$

Step 6: Check your work: $S = ((1 - f_E) + (f_E / f_I))^{-1} = (0.6 + (0.4/2.367))^{-1} = 1.3$ ✓

How Do We Solve Amdahl's Law Problems?

Case 3: Given: S and f_I **Find:** f_E

Example: Let a program have a portion f_E of its code enhanced to run 4 times faster (so $f_I = 4$), to yield a system speedup 3.3 times faster (so $S = 3.3$). What is the fraction enhanced (f_E)?

Step 1: *Can this be done*? Assuming f_I = infinity, $S = 3.3 = ((1 - f_E))^{-1}$ so minimum $f_E = 0.697$

Yes, this can be done for maximum f_I, so let's solve the equation to determine actual f_E

Step 2: Solve speedup equation for f_E :

$S = ((1 - f_E) + (f_E / f_I))^{-1}$	[state the equation]
$3.3 = ((1 - f_E) + (f_E / 4))^{-1}$	[plug in values]
$(1 - f_E) + f_E/4 = 1/3.3 = 0.303$	[invert both sides]
$1 - 0.75f_E = 0.303$	[regroup]
$0.75f_E = 1 - 0.303 = 0.697$	[commutativity]
$f_E = 0.697 / 0.75 = \mathbf{0.929}$	[divide by 0.75]

Step 3: Check your work: $S = ((1 - f_E) + (f_E / f_I))^{-1} = (0.071 + (0.929/4))^{-1} = 3.3$ ✓