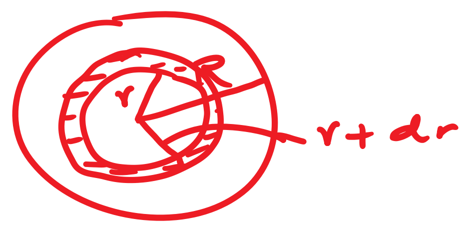
The chief beauty of time is that you cannot waste it in advance.

ARNOLD BENNETT

(How to live 24 hrs a day)

$$\rho = \rho_0 + \alpha r$$



$$\frac{4\pi}{3}(r+dr)^3 - \frac{4\pi}{3}r^3$$

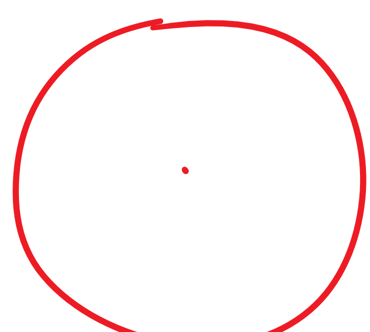
$$= \frac{4\pi}{3}\left(r^3 + 3r^2 dr\right) - \frac{4\pi}{3}r^3 = 4\pi r^2 dr$$

$$dQ = \rho(r)\, 4\pi r^2 dr$$

$$Q = \int_0^R \rho(r)\, 4\pi r^2 dr$$

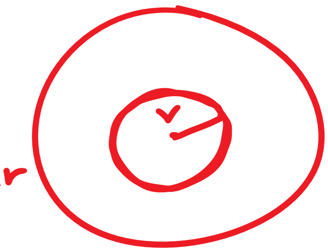
$$= 4\pi \int_0^R (\rho_0 + \alpha r)\, r^2 dr$$

$$\vec{E}(r) \rightarrow \hat{r}$$



$r < R$

$$\oint\!\!\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

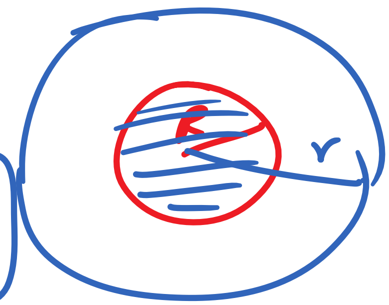


$$E\ 4\pi r^2 = \frac{1}{\epsilon_0} 4\pi \int_0^r (\rho_0 + \alpha r) r^2 dr$$

$$= \frac{4\pi}{\epsilon_0}\left[\rho \frac{r^3}{3} + \alpha \frac{r^4}{4}\right]$$

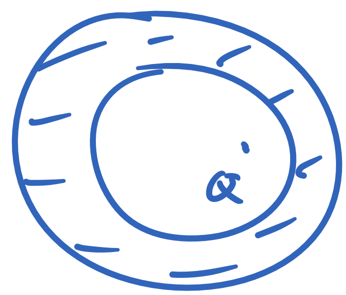
$r > R$

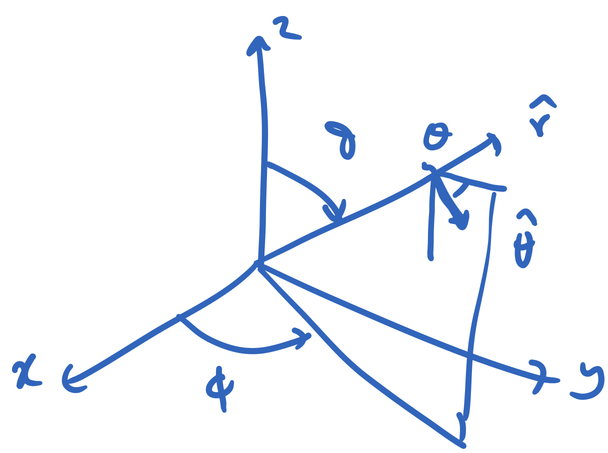
$$E \cdot 4\pi r^2 = \frac{Q_{tr}}{\epsilon_0}$$



$$= \frac{1}{\epsilon_0} 4\pi \left[\frac{\rho R^3}{3} + \frac{\alpha R^4}{4}\right]$$

Q5 (c)





$$\hat{\theta} = \alpha \hat{x} + \beta \hat{y} + \gamma \hat{z}$$

$$\hat{\theta}\cdot\hat{x} = \alpha = \cos\theta \cos\varphi$$

$\hat{r}$ $\hat{\theta}$ $\hat{\varphi}$

$\hat{x} = \hat{r} \quad \hat{\theta} \quad \hat{\varphi}$

$\hat{y} = \hat{r} \quad \hat{\theta} \quad \hat{\varphi}$

$\hat{z} = \hat{r} \quad \hat{\theta} \quad \hat{\varphi}$

$$\begin{pmatrix}\hat{r}\\ \hat{\theta}\\ \hat{\varphi}\end{pmatrix} = \begin{pmatrix}A_1 & A_2 & A_3\\ B_1 & B_2 & B_3\\ C_1 & C_2 & C_3\end{pmatrix}\begin{pmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{pmatrix}$$

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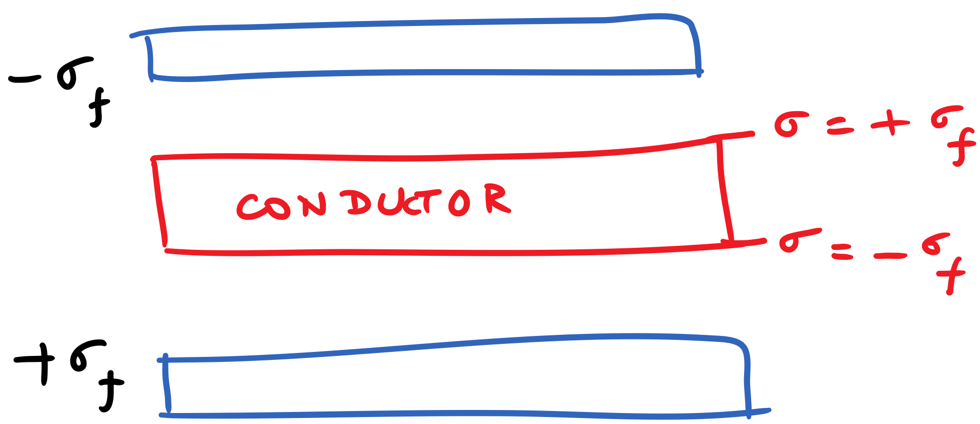
iii) $\vec{\nabla}\cdot\vec{D}$ at A = $\vec{\nabla}\cdot\vec{D} = \rho_f$

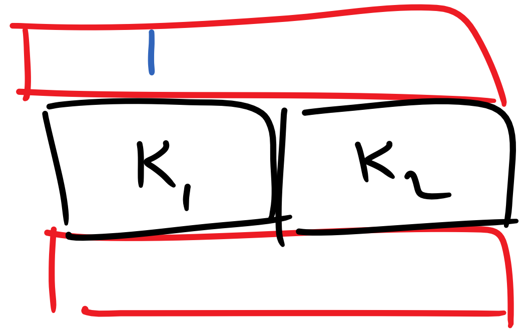
$\vec{A}\,(\nabla\cdot\vec{B})$ $\qquad$ $(\vec{A}\cdot\vec{\nabla})\,\vec{B}$

$$\left(A_x\hat{x} + A_y\hat{y} + A_z\hat{z}\right)\cdot\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\vec{B}$$

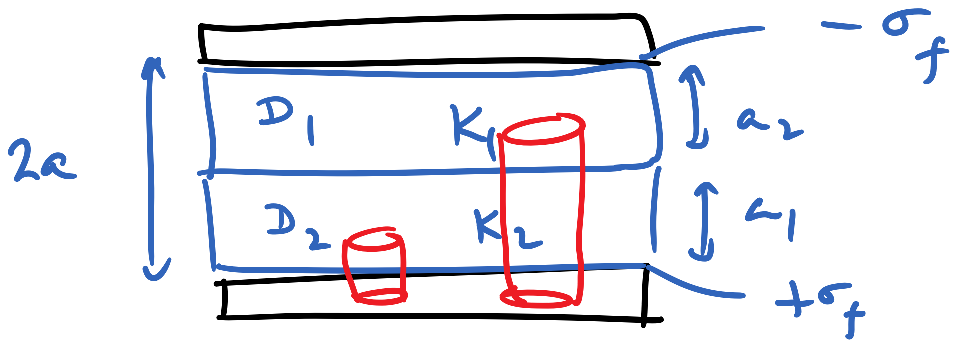
$$= \left(A_x\frac{\partial}{\partial x} + A_y\frac{\partial}{\partial y} + A_z\frac{\partial}{\partial z}\right)\vec{B}$$

$$\vec{A}\left(\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}\right)$$





Q-3



$D_1 = \sigma_f \qquad D_2 = \sigma_f \qquad \oint \vec{D}\cdot d\vec{A} = Q_{f\,enc}$

$E_1 = \dfrac{\sigma_f}{\epsilon_0 K_1} \qquad E_2 = \dfrac{\sigma_f}{\epsilon_0 K_2}$

$\vec{D} = \epsilon_0 K \vec{E}$
$= \epsilon \vec{E}$

$V = E_1 a_1 + E_2 a_2$

$P_1 = \epsilon_0 \chi_1 E_1 = \epsilon_0 (K_1 - 1) E_1 = \epsilon_0 (K_1 - 1) \dfrac{\sigma_f}{\epsilon_0 K_1}$

$= \dfrac{(K_1 - 1)}{K_1} \sigma_f$

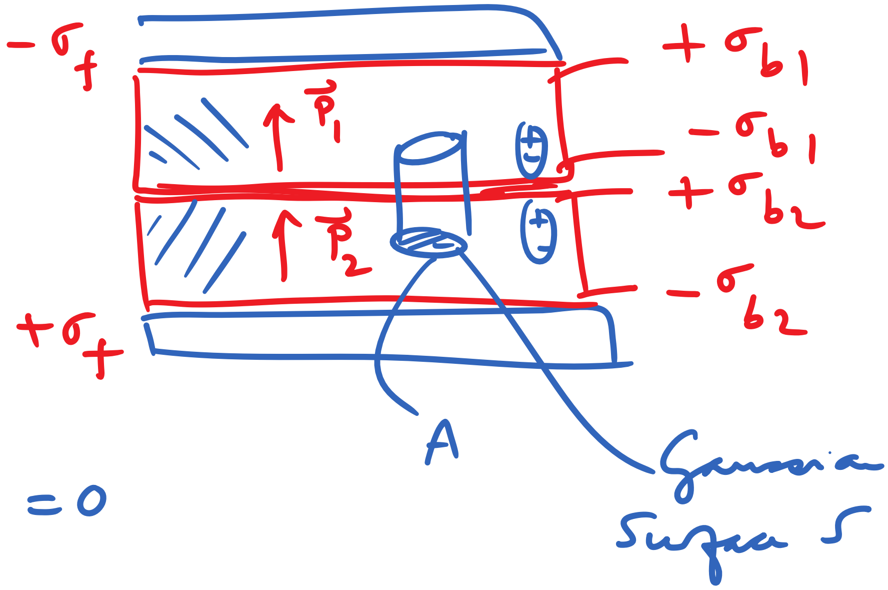
$P_2 = \dfrac{(K_2 - 1)}{K_2} \sigma_f$

$$\boxed{K = 1 + \chi}$$

$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \epsilon_0 \vec{E} + \epsilon_0 \chi \vec{E}$
$= \epsilon_0 (1 + \chi) \vec{E} = \epsilon_0 K \vec{E}$

$$\vec{P} = \epsilon_0 \chi \vec{E}$$

$$\vec{D} = \epsilon \vec{E}$$
$$= \epsilon_0 (1+\chi) \vec{E}$$



$$\oiint_S \vec{D} \cdot d\vec{a} = 0$$

$$\oiint \vec{E} \cdot d\vec{a} = \frac{(\sigma_{b_2} - \sigma_{b_1}) A}{\epsilon_0}$$

$$\vec{V} = r(2 + \sin^2\varphi)\hat{r} + r \sin\varphi \cos\varphi \, \hat{\varphi} + 3z\hat{z}$$

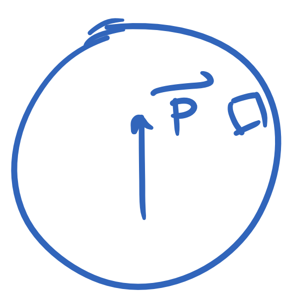
$$\vec{\nabla} \cdot \vec{V} = \frac{1}{r}\frac{\partial}{\partial r}\left[r^2 (2 + \sin^2\varphi)\right] + \frac{1}{r}\frac{\partial}{\partial \varphi}(r \sin\varphi \cos\varphi) + \frac{\partial}{\partial z}(3z)$$

$$= \frac{1}{\cancel{r}} 2\cancel{r} (2 + \sin^2\varphi) + (\cos^2\varphi - \sin^2\varphi) + 3$$

$$= 4 + 1 + 3 = 8$$

$$\vec{\nabla} \cdot \vec{F} = \frac{1}{r}\frac{\partial}{\partial r}(r F_r) + \frac{1}{r}\frac{\partial}{\partial \varphi}(F_\varphi) + \frac{\partial F_z}{\partial z}$$

$$\rho_b = \vec{\nabla} \cdot \vec{P} = 0$$
$$\sigma_b = \vec{P} \cdot \hat{n} = P \cos\theta$$



$$Q_L = \oiint \sigma\, r^2 \sin\vartheta \, d\vartheta \, d\varphi = \rho R^2 \int_0^{\pi} \sin\vartheta \cos\vartheta \, d\vartheta \int_0^{2\pi} d\varphi$$