Random Variable: It is a function from a Sample Space (of an experiment) to a real value.

→ Denoted by capital letter, e.g. X.

$X : f : S \to \mathbb{R}$.

$x : x \in \mathbb{R}$.

E.g. Coin toss

$S = \{H, T\}$

X = # no. of heads

$x = 0 \quad \{T\}$

$x = 1 \quad \{H\}$

2 coin tosses

$S = \{HT, TT, HH, TH\}$

$X = 0 \quad : S = \{TT\} \qquad p = 1/4$

$x = 1 \quad : \{HT, TH\} \qquad p = 1/2$

$x = 2 \quad : \{HH\} \qquad p = 1/4$.

Pair of dice

$$S = \begin{Bmatrix} (1,1) & \cdots\cdots & (1,6) \\ (6,1) & & (6,6) \end{Bmatrix}$$

X = diff b/w two die faces

$X = 0$: $S = \{(1,1)\ (2,2)\ (3,3) \ldots (6,6)\}$

$X = 1$: $S = \{(1,2)\ (2,1)\ (3,4)\ (4,3)\ (5,4)\ (4,5)\ (6,5)\ (5,6)\ (2,3)\ (3,2)\}$

$X = 2$: $S = \{(1,3)\ (3,1)\ (2,4)\ (4,2) \ldots (4,6)\ (6,4)\}$

$X = 5 = (1,6)\ (6,1)$

Discrete RV → They are countable
→ They can be large.

Continous RV → They are not countable
They lies in a range.

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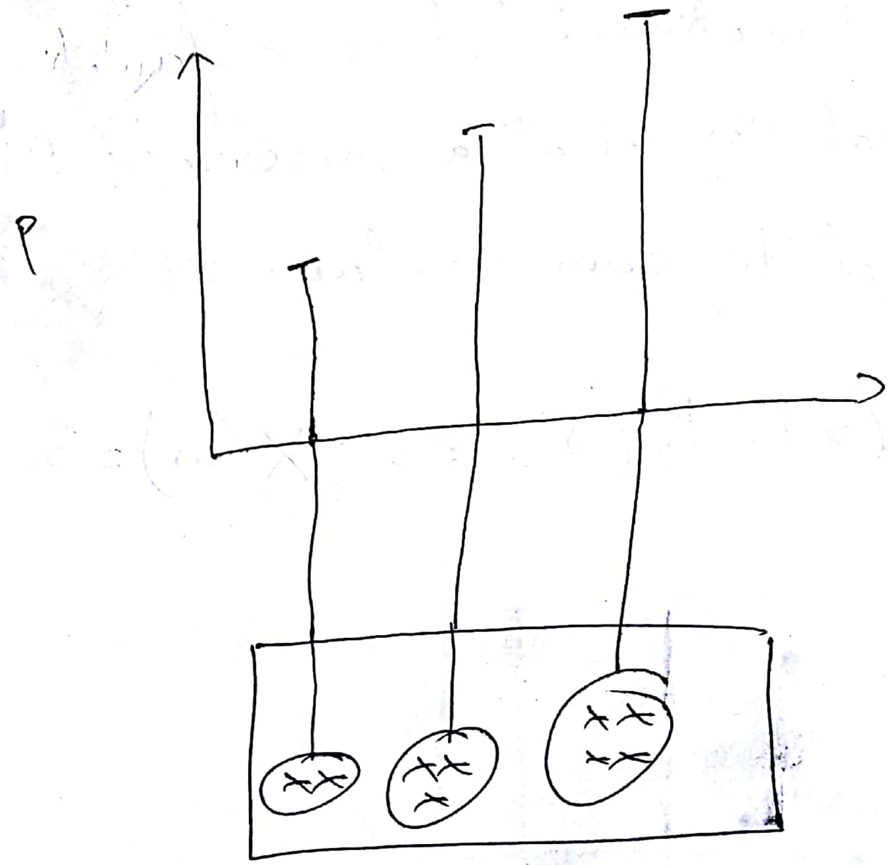
Now, let's go hiae·

Graphical Representation.



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Revist 2 coin problem

$S = \{(HH) (TT) (HT) (TH)\}$

X = head

$X = 0$ $\quad p(X=0) = 1/4$

$x = 1$ $\quad p(x=1) = 1/2$

$x = 2$ $\quad p(x=2) = 1/4$

~~properties~~

~~$p(X=x)$~~

Probability Mass Function: It is a function that gives the probability that a discrete RV is exactly equal to some value.

$$P_X(x) \text{ or } f_X(x) = P_r(\{\omega \in S : X(\omega) = x\}).$$

properties

1) $f_X(x_i) \geq 0$

2) $\sum_{x_i} f_X(x_i) = 1.$

Now, let's go bias.

$p(H) = p$

$n = 4.$

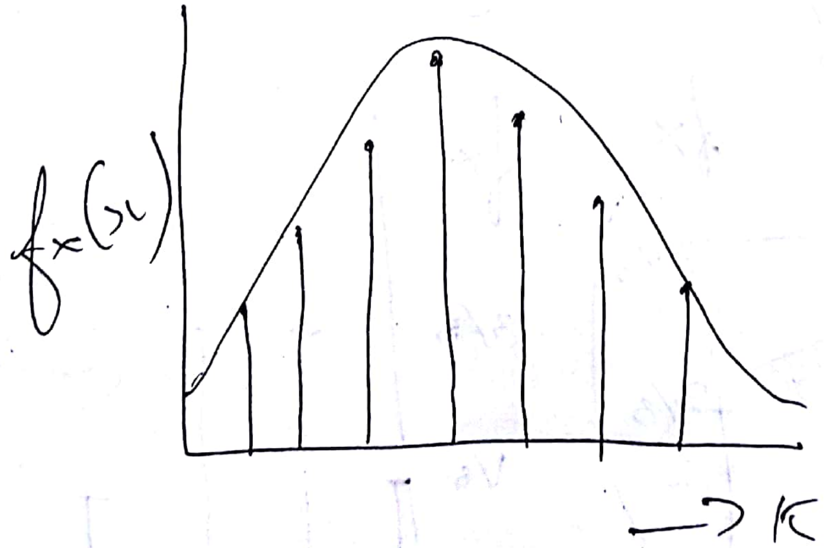
$$p_x(2) = p(HHTT) + p(HTHT) + p(HTTH) + p(THHT) + p(THTH) + p(TTHH)$$

$$= 6p^2(1-p)^2.$$

$$= \binom{4}{2} p^2 (1-p)^2$$

In general.

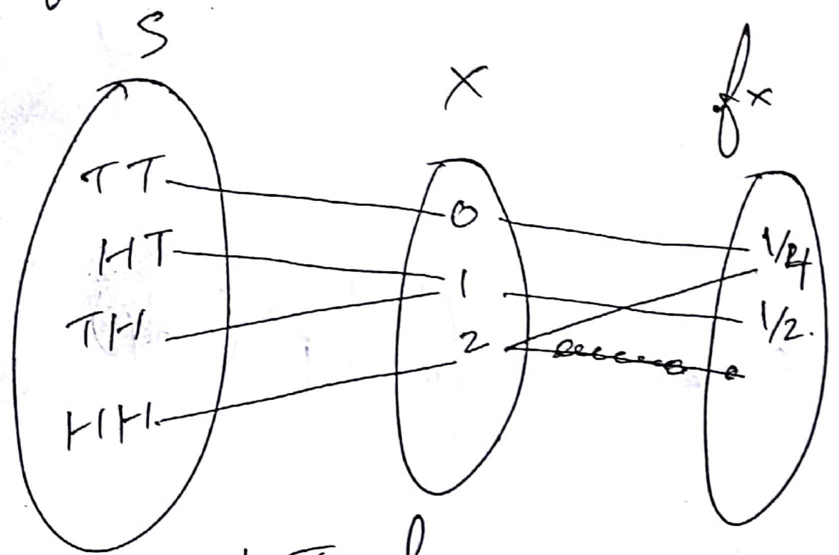
$$\cancel{p_x} \; f_x(k) \text{ or } P_x(x=k) = \binom{n}{k} p^k (1-p)^{n-k}.$$



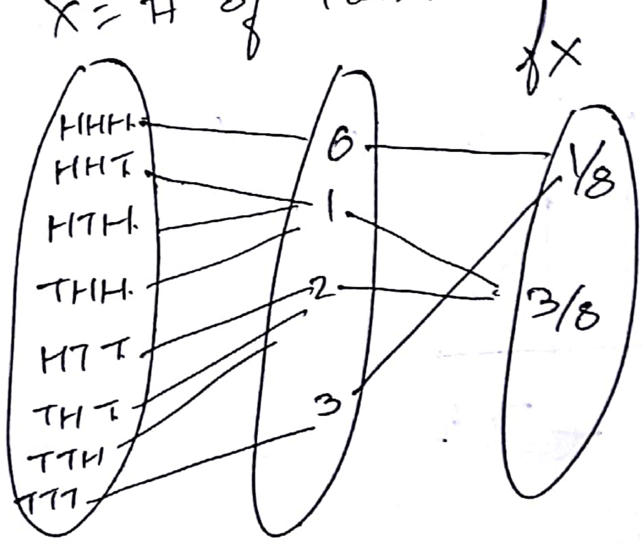
X	0	1	2
$f_X(x)$	1/4	1/2	1/4

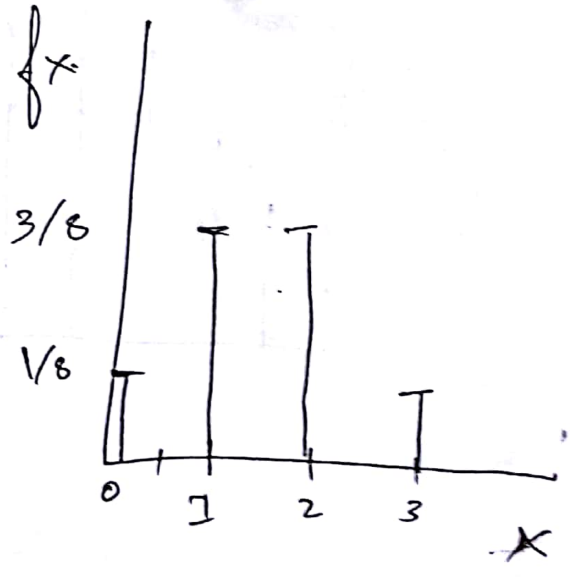
$$\sum_{x_i} f_X(x_i) = f_X(0) + f_X(1) + f_X(2)$$
$$= 1/4 + 1/2 + 1/4 = 1.$$

3) $f_X(x) = \Pr(\{\omega \in S : X(\omega) = x\})$.



E.g. $X = \#$ of Tail





Cumulative Distribution Function: It is a function evaluated at x, that gives the probability that X will take on a value that is less than x.

Mathematically.

$$F_X(x) = P(X \leq x).$$

for an interval.

$\cancel{F_X(x) = F_X(b)}$

$$P(a < X \leq b) = F_X(b) - F_X(a)$$

$a < b$

If X is a discrete RV.

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} P_X(X = x_i).$$

$$= \sum_{x_i \leq x} p(x_i)$$

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E.g. 2 coin tosses. $X = \#$ of heads

$F_X(0) = f_X(0) = 1/4$. (TT).

$F_X(1) = P(X \leq 1)$

$= P(X=0) + P(X=1) = P(TT) + P(TH) + P(HT)$

$= 1/4 + 1/2 = 3/4$.

$F_X(2) = P(X \leq 2)$

$= P(X=0) + P(X=1) + P(X=2)$

$= P(TT) + P(TH) + P(HT) + P(HH)$

$= 1/4 + 1/4 + 1/4 + 1/4 = 1$.

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 1/4 & 0 \leq x < 1 \\ 3/4 & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$