Name of student: ..........................................................

Batch No:.......... Enrollment No. ....................

COURSE NAME: LINEAR ALGEBRA AND DIFFERENTIAL EQUATIONS

B.TECH TUTORIAL QUIZ-4 FALL SEMESTER 2018-19

COURSE CODE : EMAT102L MAX. TIME: 15 min

COURSE CREDIT: 3-1-0 MAX. MARKS: 10

1. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation which is defined as $T((x, y, z)) = (2x + y, y + 2z)$. Then find the basis and dimension of null space and range space of $T$. [5]

**Solution:**

$$T((x,y,z)) = (2x+y, y+2z)$$

$$\text{Null space }(T) = \{(x,y,z) : T((x,y,z)) = (0,0)\}$$

$$= \{(x,y,z) : (2x+y, y+2z) = (0,0)\}$$

$$= \left\{(x,y,z) : \begin{array}{l} 2x+y=0 \\ y+2z=0 \end{array}\right\}$$

$$= \left\{(x,y,z) : \begin{array}{l} y=-2x \\ y=-2z \end{array} \Rightarrow x = z = \frac{-y}{2}\right\}$$

$$= \{(-y/2, y, -y/2) : y \in \mathbb{R}\}$$

$$= \{y(-1/2, 1, -1/2) : y \in \mathbb{R}\} = \text{span}\{\underbrace{(-1/2, 1, -1/2)}_{v_1}\}$$

$v_1$ is linearly independent and span null space $(T)$.

$\therefore$ $\dim(\text{Null}(T)) = 1$, Basis $= \{(-1/2, 1, -1/2)\}$.

$$\text{Range Space }(T) = \{(2x+y, y+2z) : x,y,z \in \mathbb{R}\}$$

$$= \{x(2,0) + y(1,1) + z(0,1) : x,y,z \in \mathbb{R}\}$$

$$= \text{span}\{\underbrace{(1,1)}_{v_1}, \underbrace{(0,1)}_{v_2}\}.$$

Scanned by CamScanner

$\{v_1, v_2\}$ are linearly independent and form a basis for $R(T)$.   . Basis $R(T) = \{(0,1), (1,1)\}$.

$\dim R(T) = 2$

2. Is $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined as $T((x, y, z)) = (x + 5, y)$ a linear transformation? [2]

**Solution:**

No, $T$ is not a linear Transformation.

because $T((0,0,0)) = (5,0) \neq (0,0)$

OR

$X = (1,0,0)$ $\quad$ $T(X+Y) = (8,0)$

$Y = (2,0,0)$ $\quad$ $T(X) + T(Y) = (6,0) + (7,0) = (13,0) \neq T(X+Y)$

$X+Y = (3,0,0)$

3. If $V$ and $W$ are finite-dimensional vector spaces such that $dim(V) > dim(W)$, then no linear map from $V$ to $W$ is injective. [3]

**Solution:** suppose $T: V \to W$ is a linear map, which is injective

Then, $\boxed{\dim V = \dim(\text{Null}(T)) + \dim((R(T))}$ — (1)

Since $T$ is injective $\Rightarrow \boxed{\dim(\text{Null}(T)) = 0}$ — (2)

(1) & (2)

$\Rightarrow \boxed{\dim V = \dim(R(T))}$ — (3)

Also, we know that $\boxed{\dim(R(T)) < \dim W}$ — (4)

(3) & (4) $\Rightarrow$ $\dim V < \dim W$, which is a contradiction by given condition.



Scanned by CamScanner