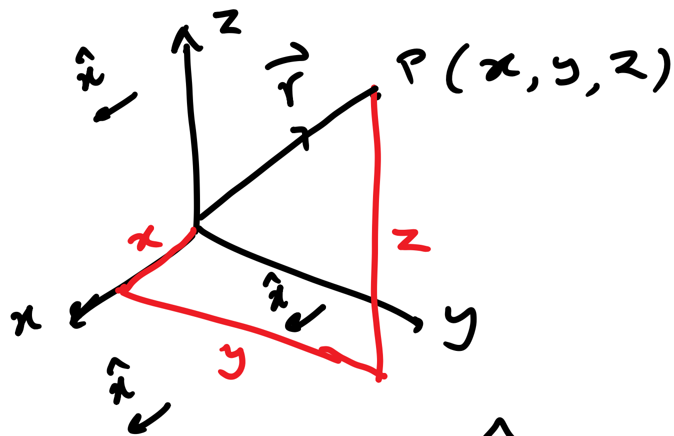
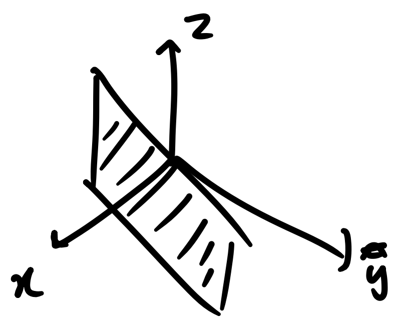
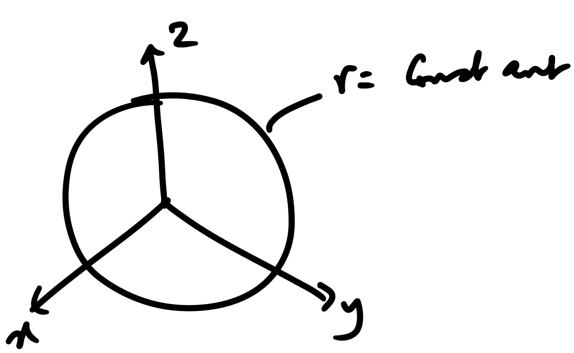
Spherical Polar Coordinates

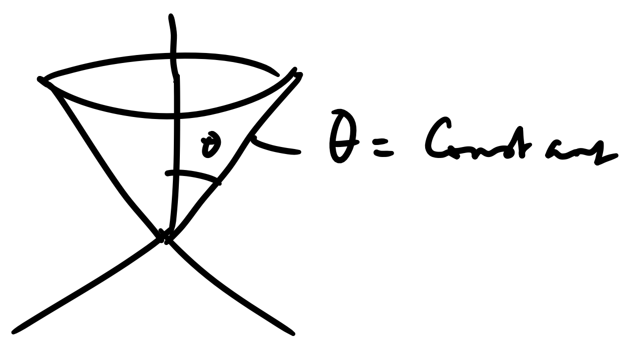


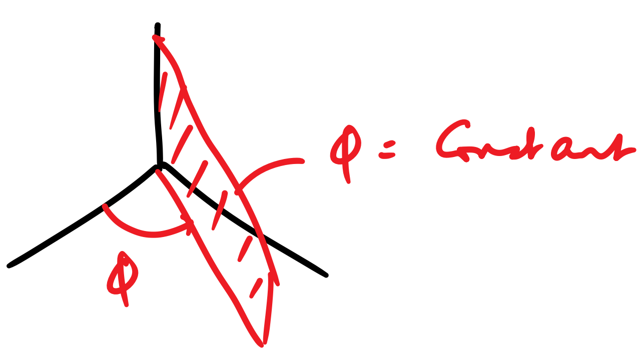
$$\vec{r} = \hat{i}\,x + \hat{j}\,y + \hat{k}\,z$$
$$= \hat{x}\,x + \hat{y}\,y + \hat{z}\,z$$
$$\vec{A} = \hat{x}\,A_x + \hat{y}\,A_y + \hat{z}\,A_z$$



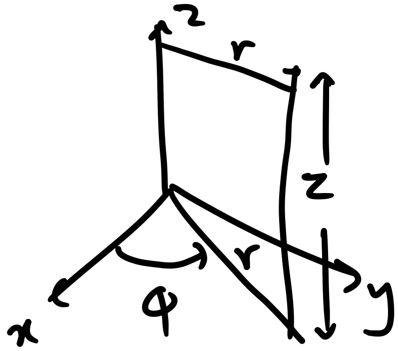
Spherical Polar Coordinate System



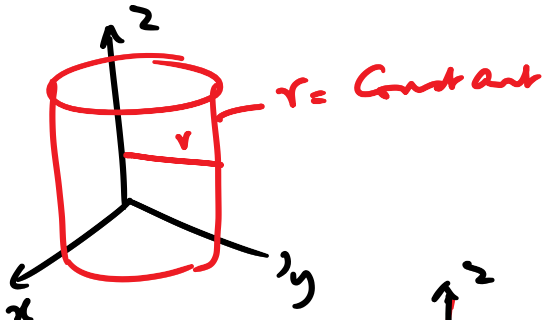


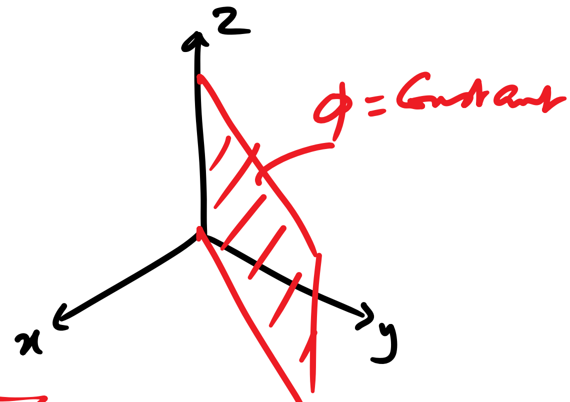


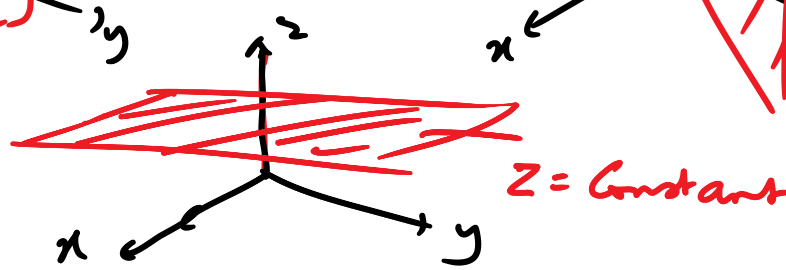
Cylindrical Coordinate System



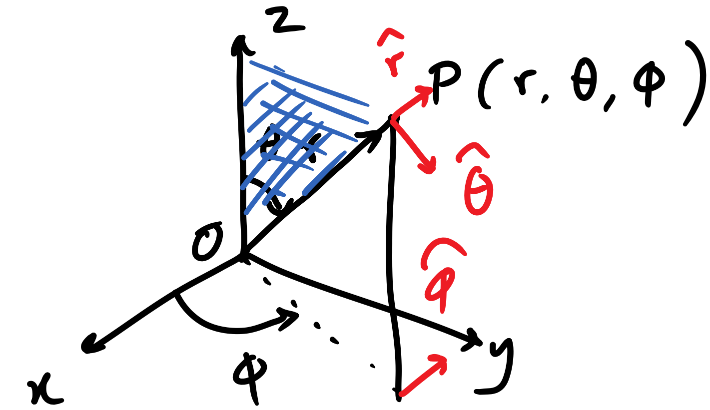
(r, ϕ, z)



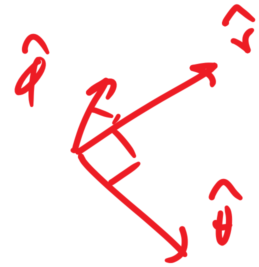


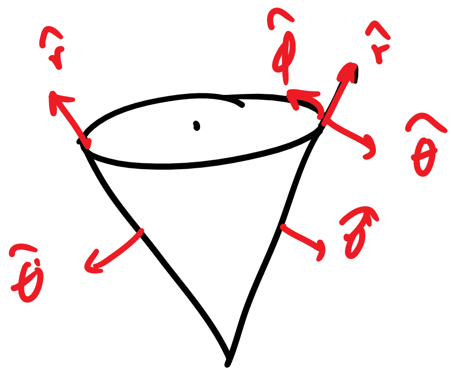


Spherical polar coordinates



$\hat{r}, \hat{\theta}, \hat{\phi}$



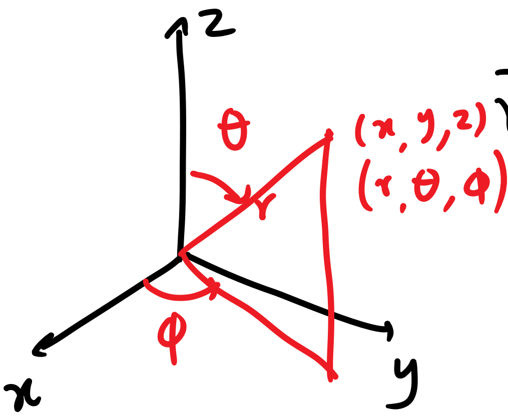


$\hat{r}\cdot\hat{\theta} = 0 \qquad \hat{r}\times\hat{\theta} = \hat{\phi}$

$\hat{r}\cdot\hat{\phi} = 0 \qquad \hat{\theta}\times\hat{\phi} = \hat{r}$

$\hat{\theta}\cdot\hat{\phi} = 0 \qquad \hat{\phi}\times\hat{r} = \hat{\theta}$

$\hat{r}(r,\theta,\phi)\,;\quad \hat{\theta}(r,\theta,\phi)\,;\quad \hat{\phi}(r,\theta,\hat{\phi})$



$\vec{r} = \hat{x}\,x + \hat{y}\,y + \hat{z}\,z$

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$$\vec{r} = \hat{x}\,r\sin\theta\cos\phi + \hat{y}\,r\sin\theta\sin\phi + \hat{z}\,r\cos\theta$$
$$= r\,(\hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta)$$

$$\hat{r} = \hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta$$

$$\boxed{\vec{r} = r\,\hat{r}}$$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$d\vec{r} = dx\,\hat{x} + \hat{y}\,dy + \hat{z}\,dz$$

$$\vec{r} = r\,\hat{r}$$

$$d\vec{r} = d(r\hat{r}) = \hat{r}\,dr + r\,d\hat{r}$$

$$d\hat{r}(\theta,\varphi) = \frac{\partial\hat{r}}{\partial\theta}d\theta + \frac{\partial\hat{r}}{\partial\varphi}d\varphi$$

$$\hat{r} = \hat{x}\sin\theta\cos\varphi + \hat{y}\sin\theta\sin\varphi + \hat{z}\cos\theta$$

$$\frac{\partial\hat{r}}{\partial\theta} = \hat{x}\cos\theta\cos\varphi + \hat{y}\cos\theta\sin\varphi - \hat{z}\sin\theta$$

$$= \hat{\theta}$$

$$\hat{\theta} =$$

α β γ δ

ϵ ζ η θ

ι κ λ μ

ν ξ o π

ρ σ τ υ

ϕ ψ χ ω

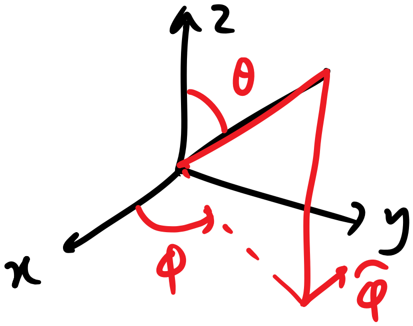
$$\hat{\theta} = \alpha\hat{x} + \beta\hat{y} + \gamma\hat{z}$$

$$\alpha = \hat{\theta}\cdot\hat{x}, \quad \beta = \hat{\theta}\cdot\hat{y}; \quad \gamma = \hat{\theta}\cdot\hat{z}$$

$$\hat{r} = \hat{x}\sin\theta\cos\varphi + \hat{y}\sin\theta\sin\varphi + \hat{z}\cos\theta$$

$$\frac{\partial\hat{r}}{\partial\varphi} = -\hat{x}\sin\theta\sin\varphi + \hat{y}\sin\theta\cos\varphi$$

$$= \sin\theta\,(-\hat{x}\sin\varphi + \hat{y}\cos\varphi)$$



$$\hat{\phi} = -\hat{x}\sin\phi + \hat{y}\cos\phi$$

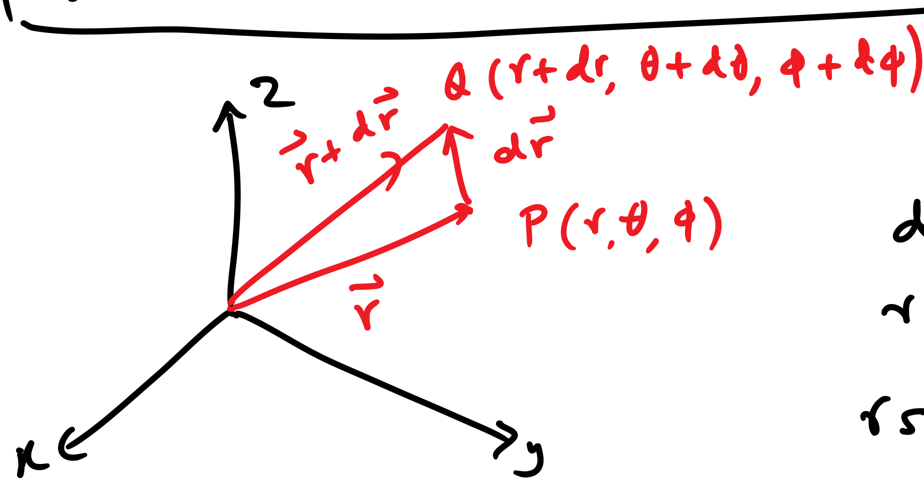
$$\frac{\partial \hat{r}}{\partial \phi} = \sin\theta\,\hat{\phi}$$

$$\vec{r} = r\,\hat{r}$$

$$d\vec{r} = \hat{r}\,dr + r\,d\hat{r}$$

$$= \hat{r}\,dr + r\left(\frac{\partial \hat{r}}{\partial \theta}d\theta + \frac{\partial \hat{r}}{\partial \phi}d\phi\right)$$

$$\boxed{d\vec{r} = \hat{r}\,dr + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}}$$



dr

$r\,d\theta$

$r\sin\theta\,d\phi$