

Merge Sort

Merge Sort is a **Divide and Conquer** sorting algorithm.

It works by:

1. **Dividing** the array into two halves,
2. **Recursively sorting** each half, and then
3. **Merging** the two sorted halves back together.

Think of it like splitting a messy pile of papers into smaller piles, sorting each one, and then combining them neatly.

♦ **Algorithmic Steps (pseudocode)**

Algorithm: MergeSort(A, left, right)

Input: Array A with indices from left to right

Output: Sorted array A in ascending order

1. if $\text{left} < \text{right}$ then
2. $\text{mid} \leftarrow (\text{left} + \text{right}) / 2$
3. MergeSort(A, left, mid) // Sort left half
4. MergeSort(A, mid + 1, right) // Sort right half
5. Merge(A, left, mid, right) // Merge the two sorted halves
6. end if

Merge procedure (for step 5):

Algorithm: Merge(A, left, mid, right)

1. Create two temporary arrays:

$L[] \leftarrow A[\text{left}..\text{mid}]$

$R[] \leftarrow A[\text{mid}+1..\text{right}]$

2. $i \leftarrow 0, j \leftarrow 0, k \leftarrow \text{left}$

3. while $i < \text{length}(L)$ and $j < \text{length}(R)$ do

4. if $L[i] \leq R[j]$ then

5. $A[k] \leftarrow L[i]$

6. $i \leftarrow i + 1$

7. else

8. $A[k] \leftarrow R[j]$

9. $j \leftarrow j + 1$

10. $k \leftarrow k + 1$

11. end while

12. Copy any remaining elements of $L[]$ (if any)

13. Copy any remaining elements of $R[]$ (if any)

♦ Implementation in C++

```
#include <iostream>

using namespace std;

// Function to merge two halves

void merge(int arr[], int left, int mid, int right) {

    int n1 = mid - left + 1;

    int n2 = right - mid;

    // Temporary arrays

    int L[n1], R[n2];

    // Copy data to temp arrays

    for (int i = 0; i < n1; i++)

        L[i] = arr[left + i];

    for (int j = 0; j < n2; j++)

        R[j] = arr[mid + 1 + j];

    // Merge the temp arrays back into arr[left..right]

    int i = 0, j = 0, k = left;

    while (i < n1 && j < n2) {
```

```
if (L[i] <= R[j]) {

    arr[k] = L[i];

    i++;

} else {

    arr[k] = R[j];

    j++;

}

k++;

}

// Copy remaining elements of L[], if any

while (i < n1) {

    arr[k] = L[i];

    i++;

    k++;

}

// Copy remaining elements of R[], if any

while (j < n2) {

    arr[k] = R[j];

    j++;

    k++;

}
```

```

    }

}

// Recursive Merge Sort

void mergeSort(int arr[], int left, int right) {

    if (left < right) {

        int mid = left + (right - left) / 2;

        mergeSort(arr, left, mid);    // Sort first half

        mergeSort(arr, mid + 1, right); // Sort second half

        merge(arr, left, mid, right); // Merge the sorted halves

    }

}

// Driver code

int main() {

    int arr[] = {38, 27, 43, 3, 9, 82, 10};

    int n = sizeof(arr) / sizeof(arr[0]);

    mergeSort(arr, 0, n - 1);

```

```

cout << "Sorted array: ";

for (int i = 0; i < n; i++)

    cout << arr[i] << " ";

cout << endl;


return 0;
}

```

♦ Time & Space Complexity

Case	Time Complexity
Best	$O(n \log n)$
Average	$O(n \log n)$
Worst	$O(n \log n)$

Space Complexity: $O(n)$ - uses temporary arrays during merging.

Stable? ☒ Yes

Adaptive? ☐ No - still divides even if already sorted.

