EE655: Computer Vision & Deep Learning

Lecture 17

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Credits: Prof. Shree K. Nayar

Image Stitching

Image 1

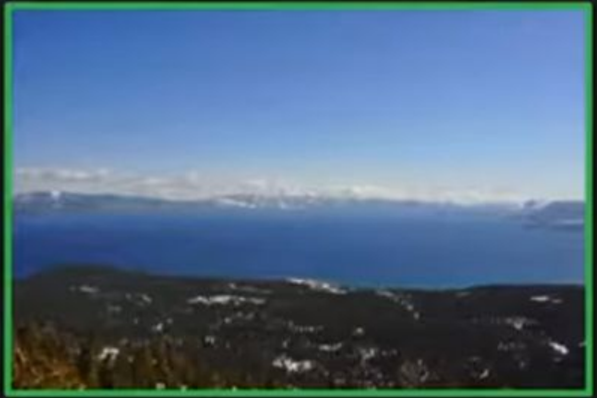


Image 2

Image 3

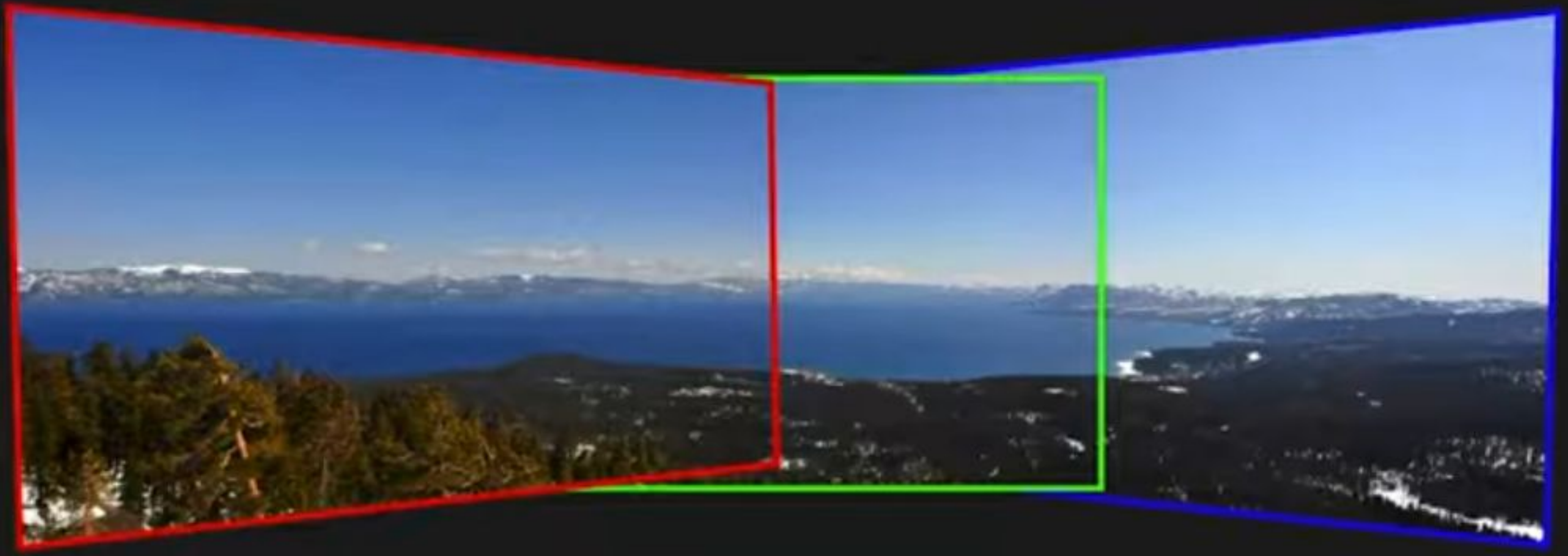
How would you align these images?

Image Stitching

Find corresponding points
(using feature detectors like SIFT)

Image Stitching

Find geometric relationship between the images

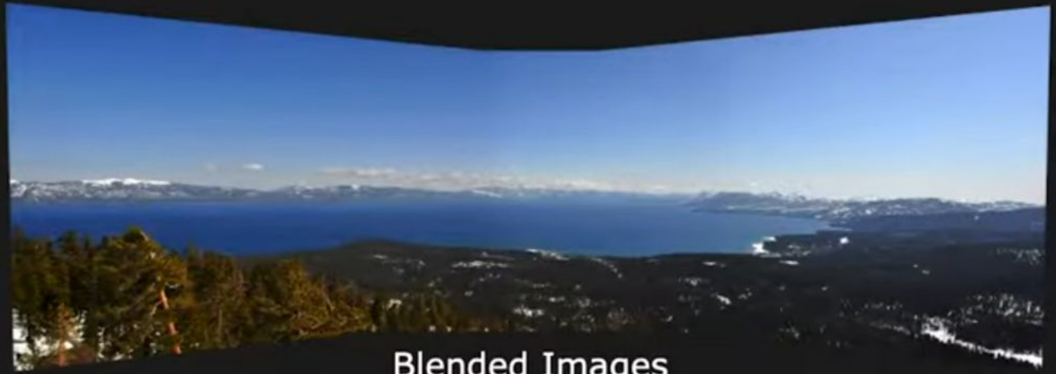


Warp images so that corresponding points align

K. Nayar

Image Stitching

Overlaid Aligned Images



Blended Images

Blend images to remove hard seams

Image Stitching

Combine multiple photos to create a larger photo

Topics:

(1) 2x2 Image Transformations

(2) 3x3 Image Transformations

(3) Computing Homography

(4) Dealing with Outliers: RANSAC

Image Manipulation

Image Filtering: Change range (brightness)

$$g(x, y) = T_r(f(x, y))$$

$$\rightarrow \boxed{T_r} \rightarrow$$

Image Warping: Change domain (location)

$$g(x, y) = f\left(T_d(x, y)\right)$$

Transformation T_d is a coordinate changing operator

$$\rightarrow \boxed{T_d} \rightarrow$$

Global Warping/Transformation

Translation

Rotation

Scaling and Aspect

$$\boxed{g(x,y) = f\big(T(x,y)\big)}$$

Affine

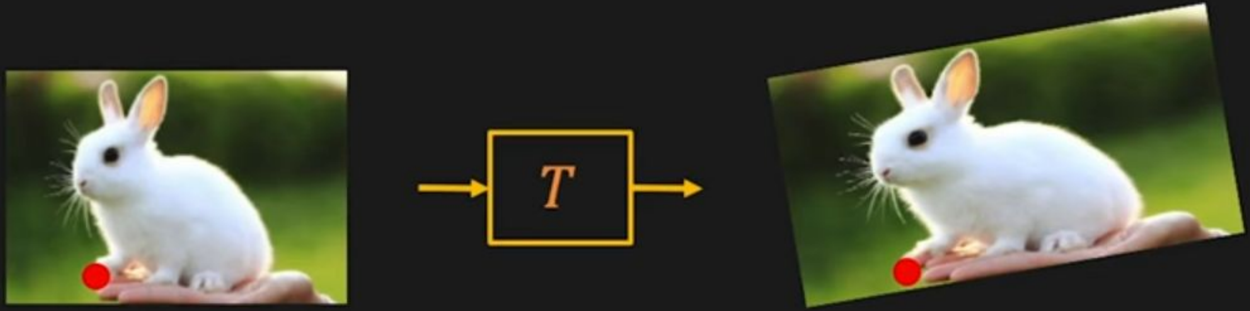
Projective

Barrel

Transformation T is the same over entire domain

Often can be described by just a few parameters

2x2 Linear Transformations

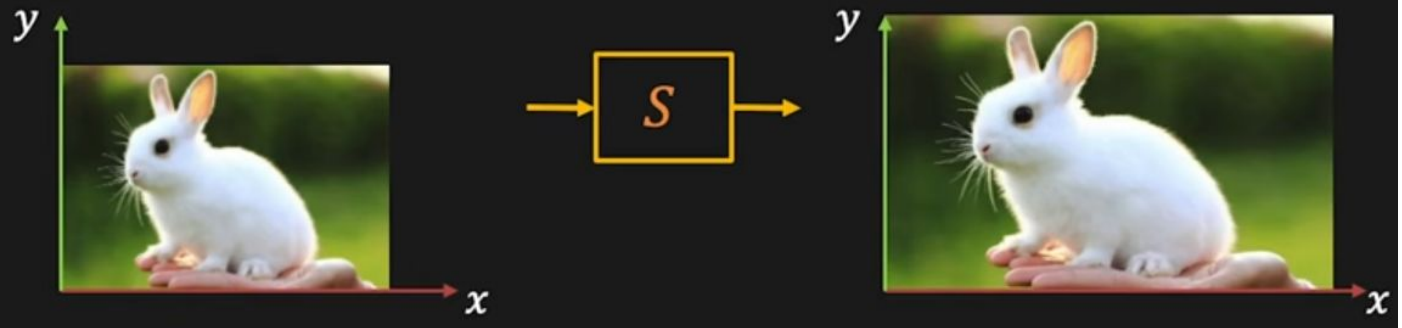


$\mathbf{p}_1 = (x_1, y_1)$ $\qquad$ $\mathbf{p}_2 = (x_2, y_2)$

T can be represented by a matrix.

$$\mathbf{p}_2 = T\mathbf{p}_1 \qquad \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = T \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \qquad \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Scaling (Stretching or Squishing)

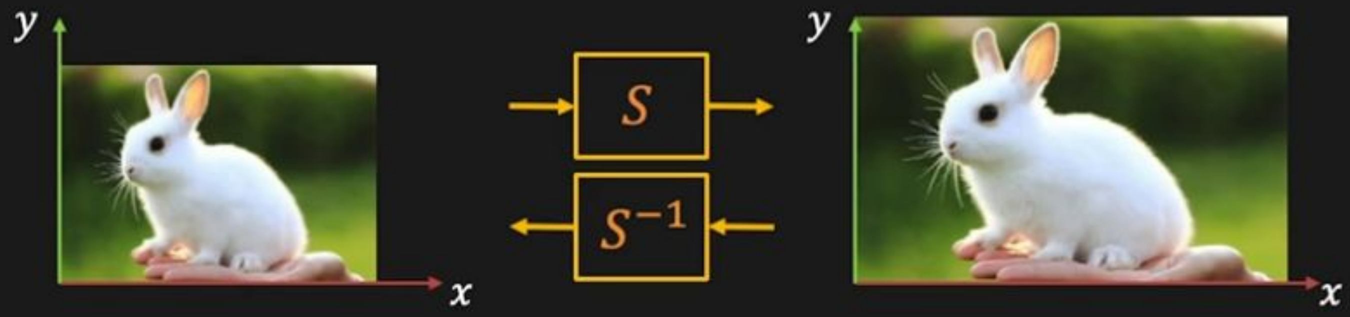


Forward:

$$x_2 = ax_1 \qquad y_2 = by_1$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = S \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Scaling (Stretching or Squishing)



Forward:

$$x_2 = a x_1 \qquad y_2 = b y_1$$

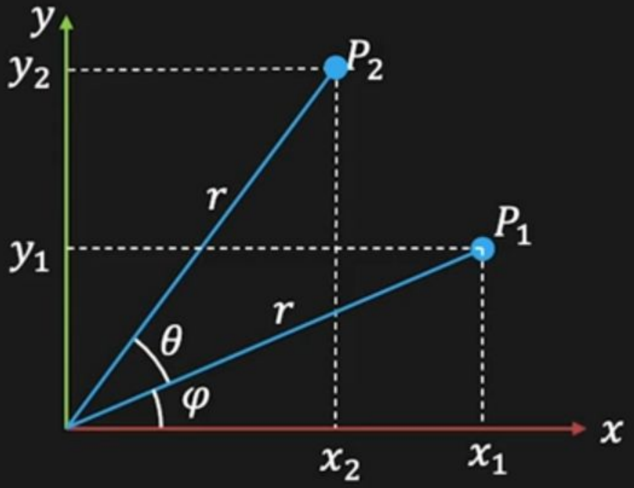
$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = S \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Inverse:

$$x_1 = \frac{1}{a} x_2 \qquad y_1 = \frac{1}{b} y_2$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = S^{-1} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/b \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

2D Rotation



$$x_1 = r\cos(\varphi)$$

$$y_1 = r\sin(\varphi)$$

$$x_2 = r\cos(\varphi + \theta)$$

$$x_2 = r\cos\varphi\cos\theta - r\sin\varphi\sin\theta$$

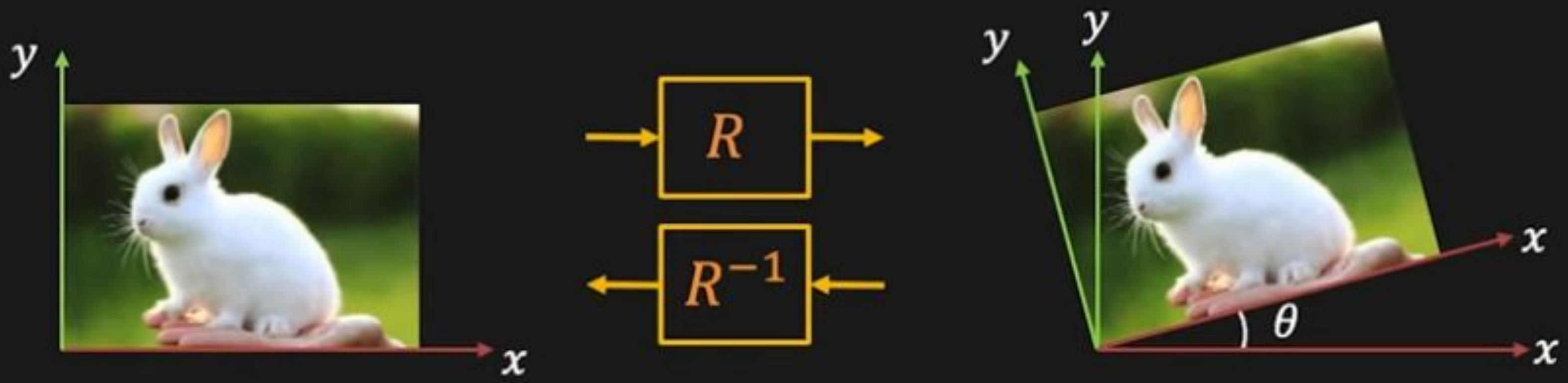
$$x_2 = x_1\cos\theta - y_1\sin\theta$$

$$y_2 = r\sin(\varphi + \theta)$$

$$y_2 = r\cos\varphi\sin\theta + r\sin\varphi\cos\theta$$

$$y_2 = x_1\sin\theta + y_1\cos\theta$$

Rotation



Forward:

$$x_2 = x_1 cos\theta - y_1 sin\theta$$

$$y_2 = x_1 sin\theta + y_1 cos\theta$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = R \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

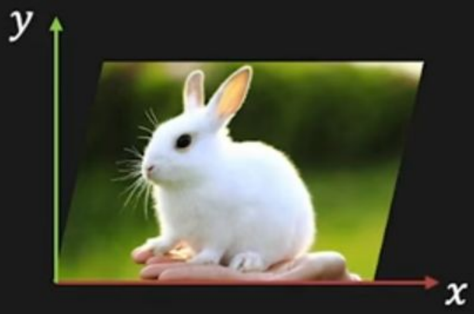
Inverse:

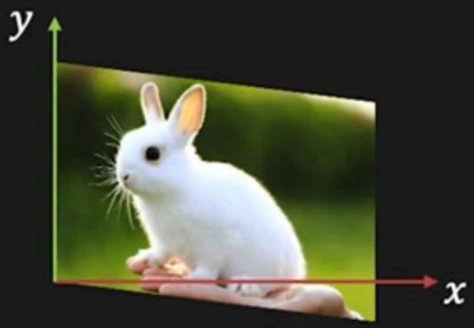
$$x_1 = x_2 cos\theta + y_2 sin\theta$$

$$y_1 = -x_2 sin\theta + y_2 cos\theta$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = R^{-1} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

Skew





Horizontal Skew:

$$x_2 = x_1 + m_x y_1$$

$$y_2 = y_1$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = S_x \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 & m_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

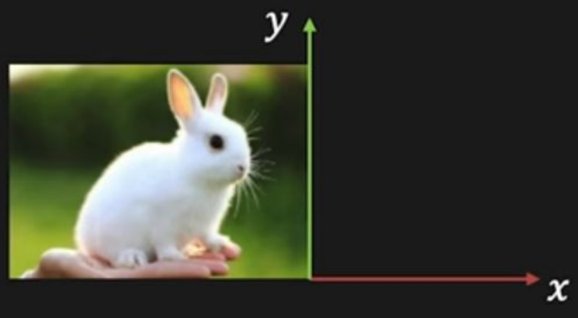
Vertical Skew:

$$x_2 = x_1$$

$$y_2 = m_y x_1 + y_1$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = S_x \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ m_y & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Mirror



Mirror about Y-axis:

$$x_2 = -x_1$$
$$y_2 = y_1$$

$$M_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Mirror about line $y = x$:

$$x_2 = y_1$$
$$y_2 = x_1$$

$$M_{xy} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

2x2 Matrix Transformations

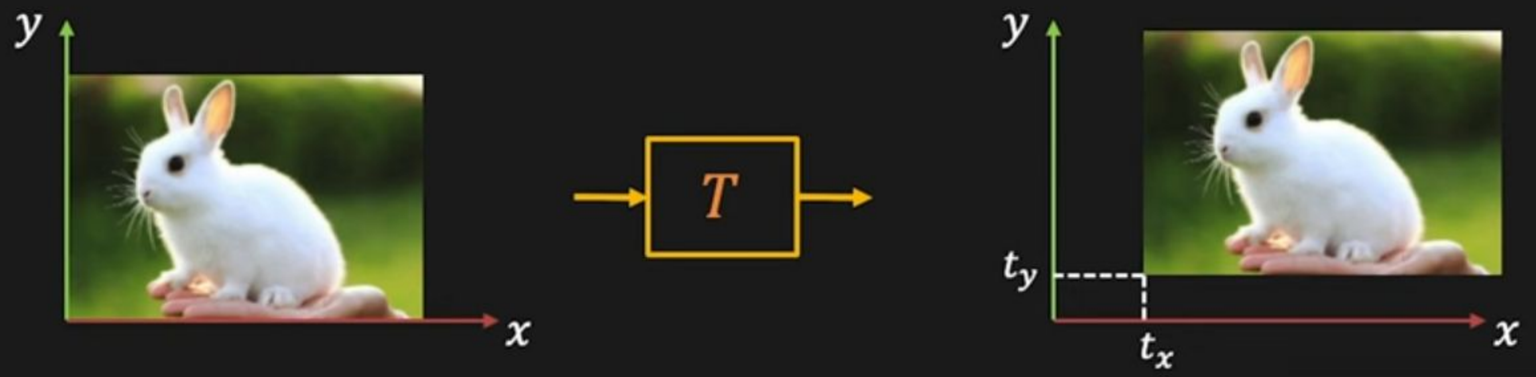
Any transformation of the form:

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

- Origin maps to the origin
- Lines map to lines
- Parallel lines remain parallel
- Closed under composition

$$\left.\begin{aligned} \mathbf{p}_2 &= T_{21}\mathbf{p}_1 \\ \mathbf{p}_3 &= T_{32}\mathbf{p}_2 \\ \mathbf{p}_3 &= T_{31}\mathbf{p}_1 \end{aligned}\right\} \mathbf{p}_3 = T_{32}\mathbf{p}_2 = T_{32}T_{21}\mathbf{p}_1 \Rightarrow T_{31} = T_{32}T_{21}$$

Translation



$$x_2 = x_1 + t_x \qquad y_2 = y_1 + t_y$$

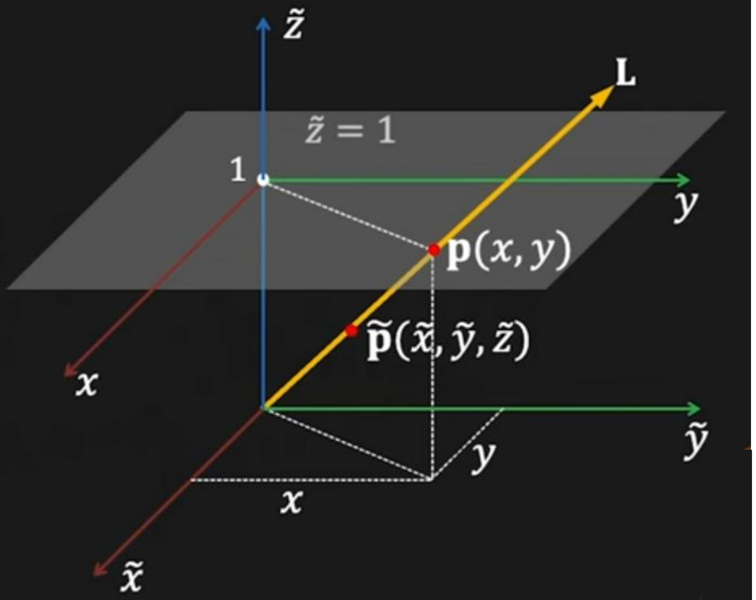
Can translation be expressed as a 2x2 matrix? No.

Homogenous Coordinates

The homogenous representation of a 2D point $\mathbf{p} = (x, y)$ is a 3D point $\tilde{\mathbf{p}} = (\tilde{x}, \tilde{y}, \tilde{z})$. The third coordinate $\tilde{z} \neq 0$ is fictitious such that:

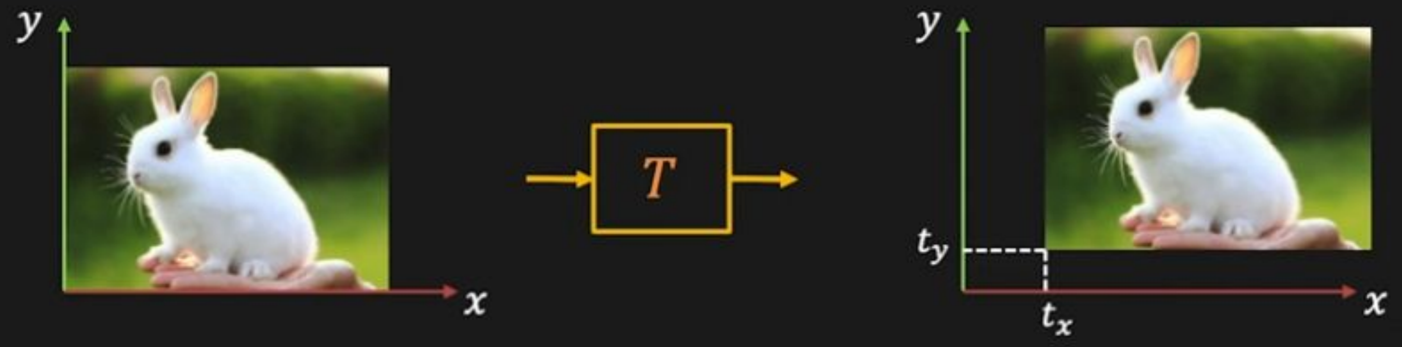
$$x = \frac{\tilde{x}}{\tilde{z}} \qquad y = \frac{\tilde{y}}{\tilde{z}}$$

$$\boxed{\mathbf{p} \equiv \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{z}x \\ \tilde{z}y \\ \tilde{z} \end{bmatrix} \equiv \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = \tilde{\mathbf{p}}}$$



Every point on line $\mathbf{L}$ (except origin) represents the homogenous coordinate of $\mathbf{p}(x, y)$

Translation



$$x_2 = x_1 + t_x \qquad y_2 = y_1 + t_y$$

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Scaling, Rotation, Skew, Translation

$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Scaling

$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} 1 & m_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Skew

$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Translation

$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Rotation

Affine Transformation

Any transformation of the form:

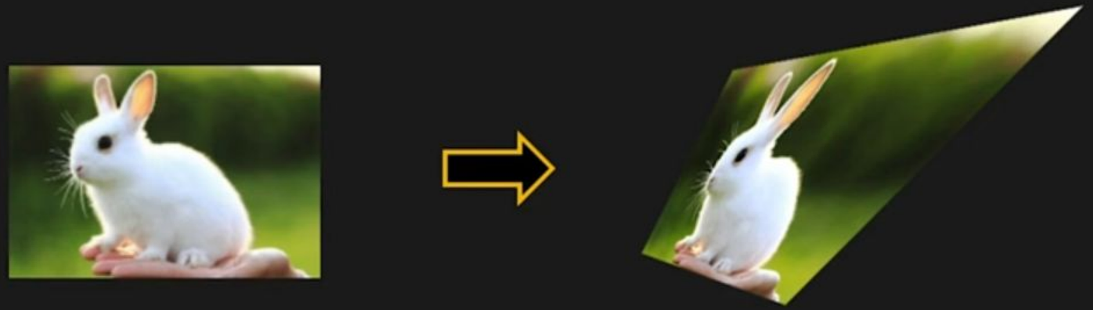
$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

- Origin does not necessarily map to the origin
- Lines map to lines
- Parallel lines remain parallel
- Closed under composition

Projective Transformation

Any transformation of the form:

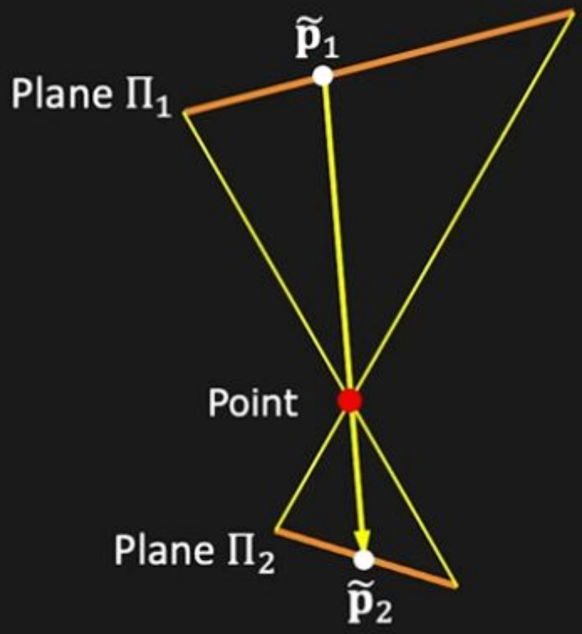
$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix} \qquad \widetilde{\mathbf{p}}_2 = H\widetilde{\mathbf{p}}_1$$



Also called Homography

Projective Transformation

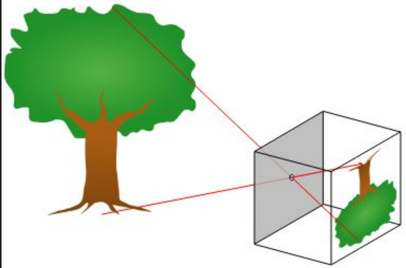
Mapping of one plane to another through a point



$$\tilde{\mathbf{p}}_2 = H\tilde{\mathbf{p}}_1$$

$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

Same as imaging a plane through a pinhole



Projective Transformation

Homography can only be defined up to a scale.

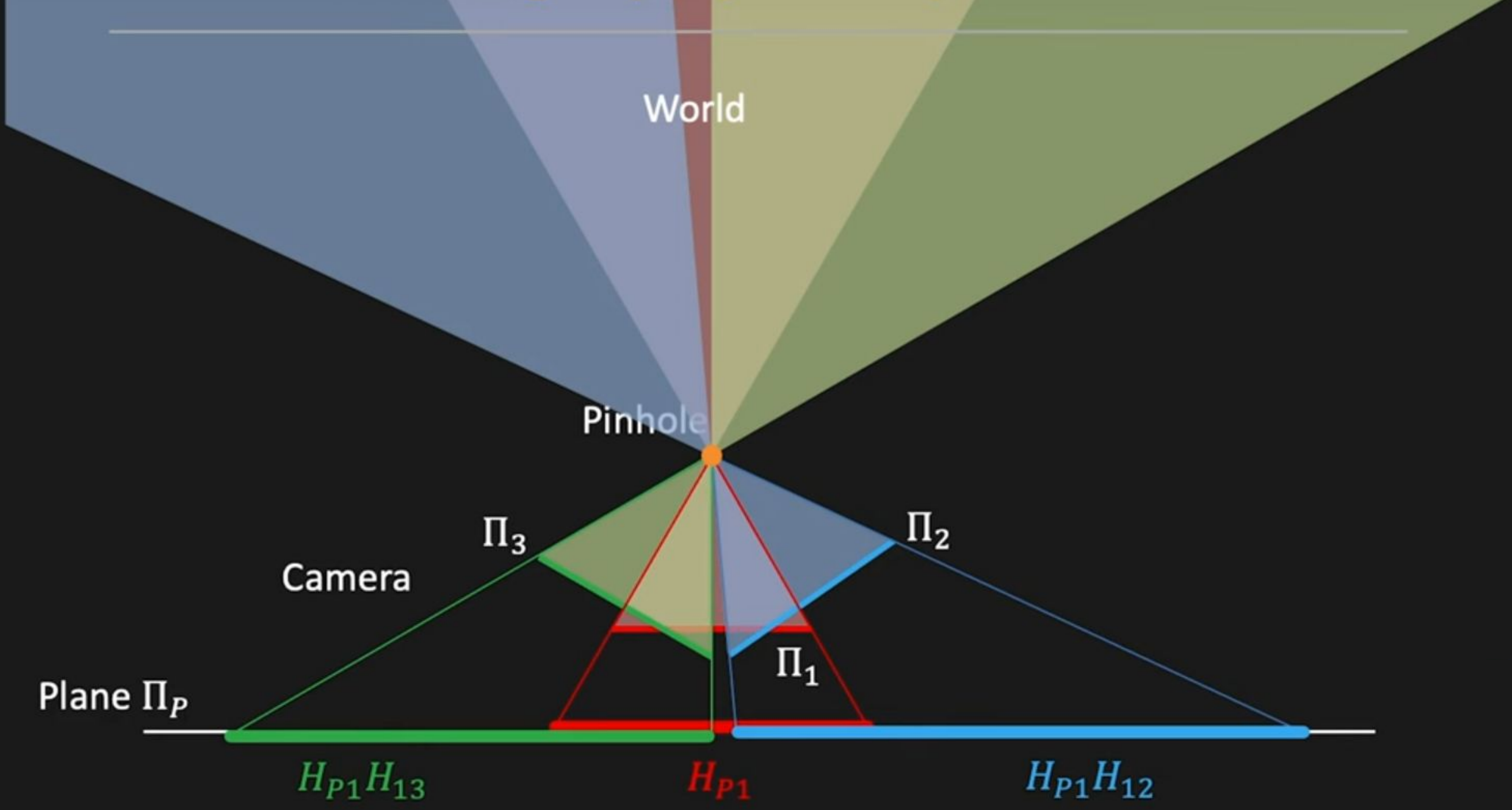
$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} \equiv k \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

If we fix scale such that $\sqrt{\Sigma(h_{ij})^2} = 1$ then 8 free parameters

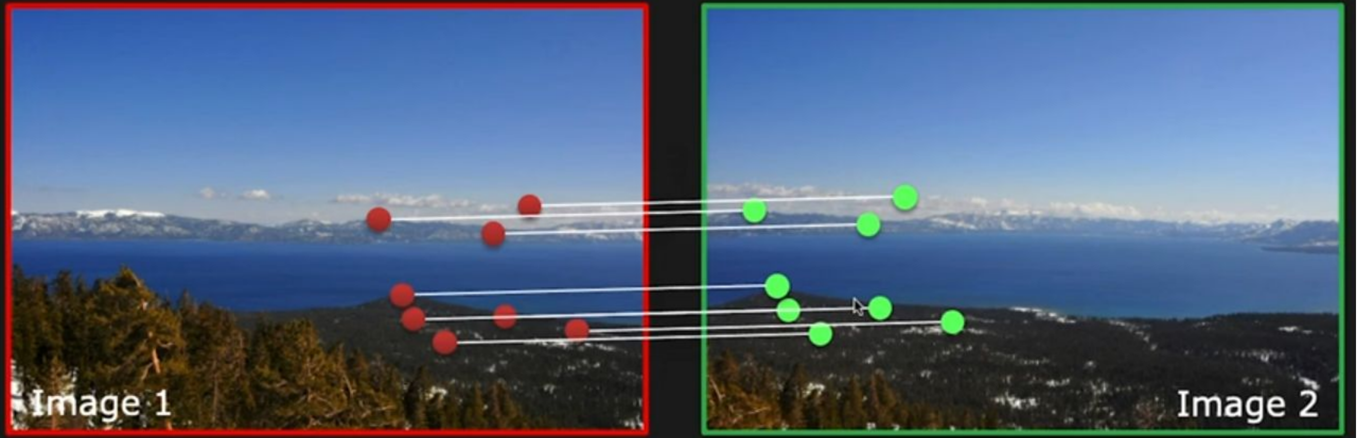
- Origin does not necessarily map to the origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Closed under composition

Parallel tracks need not appear parallel in the image

Homography Composition



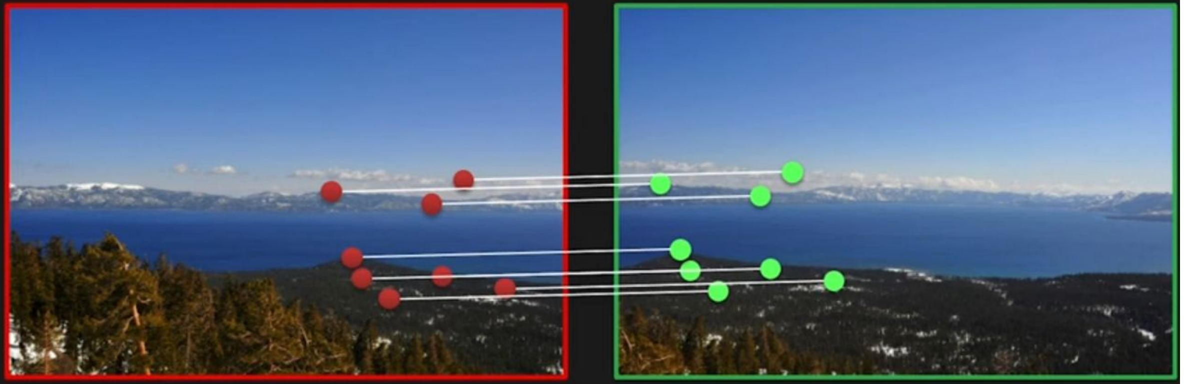
Computing Homography



Given a set of matching features/points between images 1 and 2, find the homography H that best "agrees" with the matches.

The scene points should lie on a plane, or be distant (plane at infinity), or imaged from the same point.

Computing Homography



Source Image

Destination Image

$$\begin{bmatrix} x_d \\ y_d \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_d \\ \tilde{y}_d \\ \tilde{z}_d \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_s \\ y_s \\ 1 \end{bmatrix}$$

How many unknowns? 9 ...But 8 degrees of freedom

How many minimum pairs of matching points? 4

Computing Homography

For a given pair i of corresponding points:

$$x_d{}^{(i)} = \frac{\tilde{x}_d{}^{(i)}}{\tilde{z}_d{}^{(i)}} = \frac{h_{11}x_s^{(i)} + h_{12}y_s^{(i)} + h_{13}}{h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}}$$

$$\Rightarrow$$

$$y_d{}^{(i)} = \frac{\tilde{y}_d{}^{(i)}}{\tilde{z}_d{}^{(i)}} = \frac{h_{21}x_s^{(i)} + h_{22}y_s^{(i)} + h_{23}}{h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}}$$

Rearranging the terms:

$$x_d{}^{(i)}\left(h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}\right) = h_{11}x_s^{(i)} + h_{12}y_s^{(i)} + h_{13}$$

$$\Rightarrow$$

$$y_d{}^{(i)}\left(h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}\right) = h_{21}x_s^{(i)} + h_{22}y_s^{(i)} + h_{23}$$

Computing Homography

$$x_d{}^{(i)}\left(h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}\right) = h_{11}x_s^{(i)} + h_{12}y_s^{(i)} + h_{13}$$

$$y_d{}^{(i)}\left(h_{31}x_s^{(i)} + h_{32}y_s^{(i)} + h_{33}\right) = h_{21}x_s^{(i)} + h_{22}y_s^{(i)} + h_{23}$$

Rearranging the terms and writing as linear equation:

$$\underbrace{\boxed{\begin{bmatrix} x_s^{(i)} & y_s^{(i)} & 1 & 0 & 0 & 0 & -x_d^{(i)}x_s^{(i)} & -x_d^{(i)}y_s^{(i)} & -x_d^{(i)} \\ 0 & 0 & 0 & x_s^{(i)} & y_s^{(i)} & 1 & -y_d^{(i)}x_s^{(i)} & -y_d^{(i)}y_s^{(i)} & -y_d^{(i)} \end{bmatrix}}}_{\text{(Known)}} \underbrace{\boxed{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}}}_{\substack{\mathbf{h} \\ \text{(Unknown)}}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Computing Homography

Combining the equations for all corresponding points:

$$\underbrace{\begin{bmatrix} x_s^{(1)} & y_s^{(1)} & 1 & 0 & 0 & 0 & -x_d^{(1)}x_s^{(1)} & -x_d^{(1)}y_s^{(1)} & -x_d^{(1)} \\ 0 & 0 & 0 & x_s^{(1)} & y_s^{(1)} & 1 & -y_d^{(1)}x_s^{(1)} & -y_d^{(1)}y_s^{(1)} & -y_d^{(1)} \\ & & & & \vdots & & & & \\ x_s^{(i)} & y_s^{(i)} & 1 & 0 & 0 & 0 & -x_d^{(i)}x_s^{(i)} & -x_d^{(i)}y_s^{(i)} & -x_d^{(i)} \\ 0 & 0 & 0 & x_s^{(i)} & y_s^{(i)} & 1 & -y_d^{(i)}x_s^{(i)} & -y_d^{(i)}y_s^{(i)} & -y_d^{(i)} \\ & & & & \vdots & & & & \\ x_s^{(n)} & y_s^{(n)} & 1 & 0 & 0 & 0 & -x_d^{(n)}x_s^{(n)} & -x_d^{(n)}y_s^{(n)} & -x_d^{(n)} \\ 0 & 0 & 0 & x_s^{(n)} & y_s^{(n)} & 1 & -y_d^{(n)}x_s^{(n)} & -y_d^{(n)}y_s^{(n)} & -y_d^{(n)} \end{bmatrix}}_{\substack{A \\ \text{(Known)}}} \underbrace{\begin{bmatrix} h_{11} \\ h_{12} \\ h_{13} \\ h_{21} \\ h_{22} \\ h_{23} \\ h_{31} \\ h_{32} \\ h_{33} \end{bmatrix}}_{\substack{\mathbf{h} \\ \text{(Unknown)}}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

Solve for $\mathbf{h}$: $\boxed{A\,\mathbf{h} = \mathbf{0}}$ such that $\|\mathbf{h}\|^2 = 1$

Constrained Least Squares

Solve for $\mathbf{h}$: $\boxed{A\,\mathbf{h} = \mathbf{0}}$ such that $||\mathbf{h}||^2 = 1$

Define least squares problem:

$$\min_{\mathbf{h}} \; ||A\mathbf{h}||^2 \;\; \text{such that } ||\mathbf{h}||^2 = 1$$

We know that:

$$||A\mathbf{h}||^2 = (A\mathbf{h})^T(A\mathbf{h}) = \mathbf{h}^T A^T A\mathbf{h} \quad \text{and} \quad ||\mathbf{h}||^2 = \mathbf{h}^T\mathbf{h} = 1$$

$$\boxed{\min_{\mathbf{h}}(\mathbf{h}^T A^T A\mathbf{h}) \;\; \text{such that } \; \mathbf{h}^T\mathbf{h} = 1}$$

Constrained Least Squares

$$\min_{\mathbf{h}}(\mathbf{h}^T A^T A\mathbf{h}) \text{ such that } \mathbf{h}^T\mathbf{h} = 1$$

Define Loss function $L(\mathbf{h}, \lambda)$:

$$L(\mathbf{h}, \lambda) = \mathbf{h}^T A^T A\mathbf{h} - \lambda(\mathbf{h}^T\mathbf{h} - 1)$$

Taking derivatives of $L(\mathbf{h}, \lambda)$ w.r.t $\mathbf{h}$: $2A^T A\mathbf{h} - 2\lambda\mathbf{h} = \mathbf{0}$

$$\boxed{A^T A\mathbf{h} = \lambda\mathbf{h}}$$ Eigenvalue Problem

Eigenvector $\mathbf{h}$ with smallest eigenvalue λ of matrix $A^T A$ minimizes the loss function $L(\mathbf{h})$.

Matlab: `eig(A'*A)` returns eigenvalues and vectors of $A^T A$

References

https://www.youtube.com/watch?v=J1DwQzab6Jg&list=PL2zRqk16wsdp8KbDfHKvPYNGF2L-zQASc&index=1

https://www.youtube.com/watch?v=K2XLXlyPqCA&list=PL2zRqk16wsdp8KbDfHKvPYNGF2L-zQASc&index=2

https://www.youtube.com/watch?v=B8kMB6Hv2eI&list=PL2zRqk16wsdp8KbDfHKvPYNGF2L-zQASc&index=3

https://www.youtube.com/watch?v=l_qjO4cM74o&list=PL2zRqk16wsdp8KbDfHKvPYNGF2L-zQASc&index=4