EE655: Computer Vision & Deep Learning

Lecture 03

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Lecture Outline

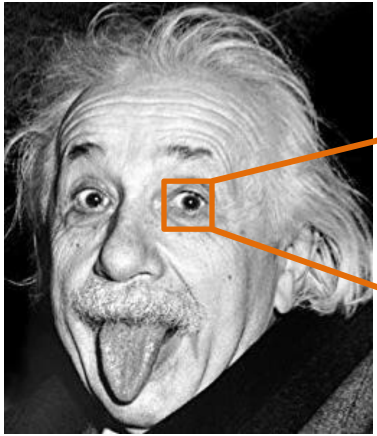
Convolution

Image Gradient

Edge

Image Spatial Filtering ~ Correlation

$$I_f(i,j) = \sum_{k,l} I(i+k, j+l)\, z(k,l)$$



$$\begin{array}{|c|c|c|c|c|} \hline 2 & 1 & 4 & 4 & 7 \\ \hline 1 & 2 & 2 & 3 & 6 \\ \hline 3 & 3 & 5 & 8 & 9 \\ \hline 5 & 2 & 2 & 6 & 7 \\ \hline 8 & 3 & 2 & 1 & 3 \\ \hline \end{array} \otimes \begin{array}{|c|c|c|} \hline a & b & c \\ \hline d & e & f \\ \hline g & h & i \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|} \hline & & & & \\ \hline & y & & & \\ \hline & & & & \\ \hline & & & & \\ \hline & & & & \\ \hline \end{array}$$

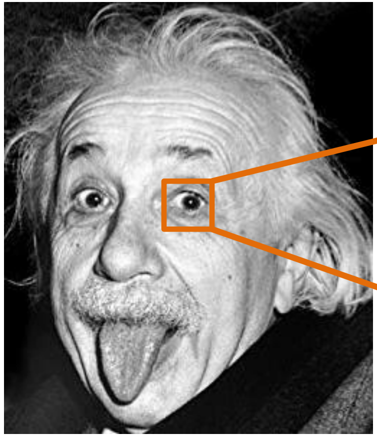
I — Image

z — Filter

I_f — Filtered image

$$y = 2a + b + 4c + d + 2e + 2f + 3g + 3h + 5i$$

Boundary

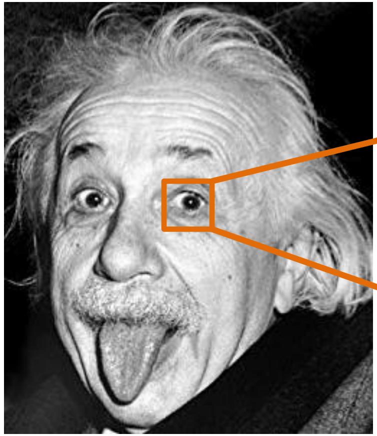


$$\underset{I}{\begin{bmatrix}2&1&4&4&7\\1&2&2&3&6\\3&3&5&8&9\\5&2&2&6&7\\8&3&2&1&3\end{bmatrix}} \otimes \underset{z}{\begin{bmatrix}a&b&c\\d&e&f\\g&h&i\end{bmatrix}} = \underset{I_f}{\begin{bmatrix}y&&&&\\&&&&\\&&&&\\&&&&\\&&&&\end{bmatrix}}$$

Image | Filter | Filtered image

$$y = ?$$

Boundary



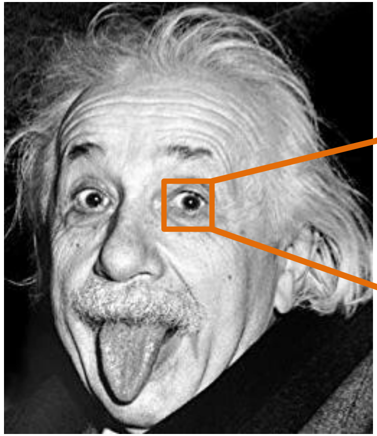
$$
\begin{matrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 1 & 4 & 4 & 7 & 0 \\
0 & 1 & 2 & 2 & 3 & 6 & 0 \\
0 & 3 & 3 & 5 & 8 & 9 & 0 \\
0 & 5 & 2 & 2 & 6 & 7 & 0 \\
0 & 8 & 3 & 2 & 1 & 3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{matrix}
\otimes
\begin{bmatrix}
a & b & c \\
d & e & f \\
g & h & i
\end{bmatrix}
=
\begin{bmatrix}
y & & & & \\
 & & & & \\
 & & & & \\
 & & & & \\
 & & & &
\end{bmatrix}
$$

I Image | z Filter | I_f Filtered image

Zero padding

$$y = 2e + f + h + 2i$$

Boundary

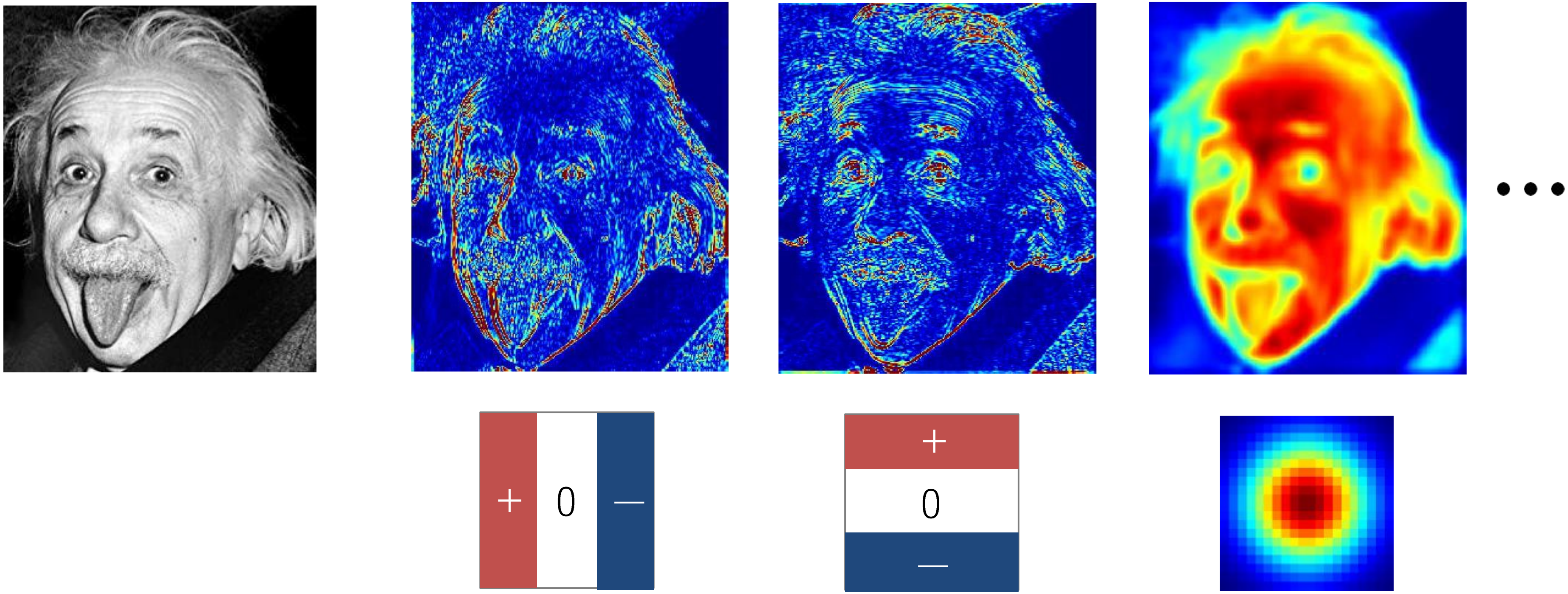


$$\underset{\substack{I \\ \text{Image}}}{\begin{array}{ccccccc} 0&0&0&0&0&0&0\\ 0&2&1&4&4&7&0\\ 0&1&2&2&3&6&0\\ 0&3&3&5&8&9&0\\ 0&5&2&2&6&7&0\\ 0&8&3&2&1&3&0\\ 0&0&0&0&0&0&0 \end{array}} \otimes \underset{\substack{z \\ \text{Filter}}}{\begin{array}{|c|c|c|}\hline a&b&c\\\hline d&e&f\\\hline g&h&i\\\hline\end{array}} = \underset{\substack{I_f \\ \text{Filtered image}}}{\begin{array}{|c|c|c|c|c|}\hline \; & \; & \; & \; & \; \\\hline & & & & \\\hline & & & & \\\hline & & & & y \\\hline & & & & \\\hline\end{array}}$$

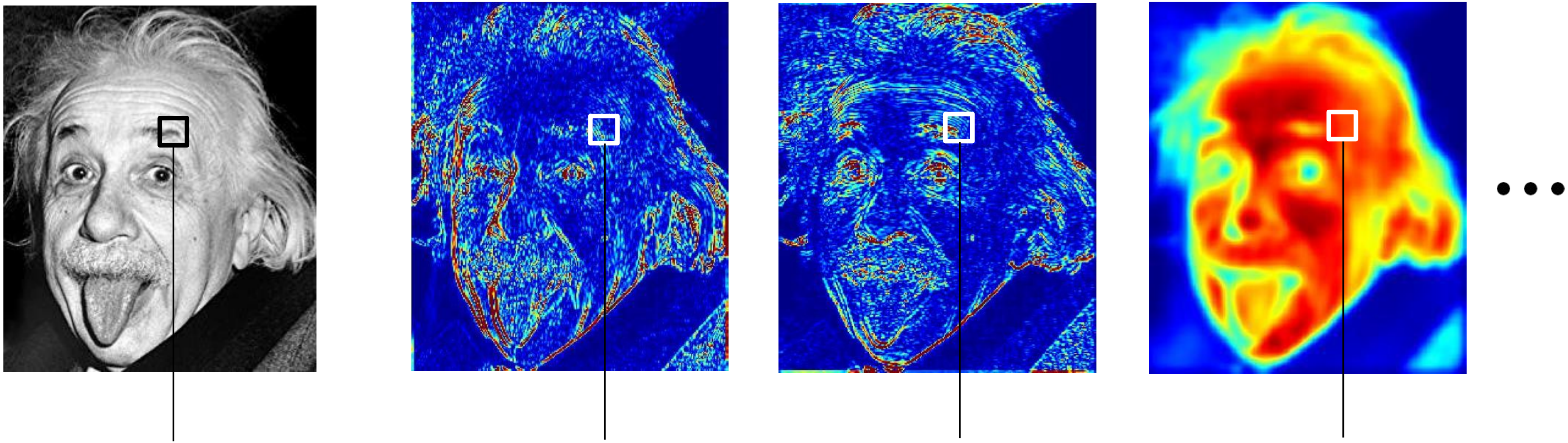
Zero padding

$$y = 8a + 9b + 6d + 7e + g + 3h$$

Filtering as Feature Extraction



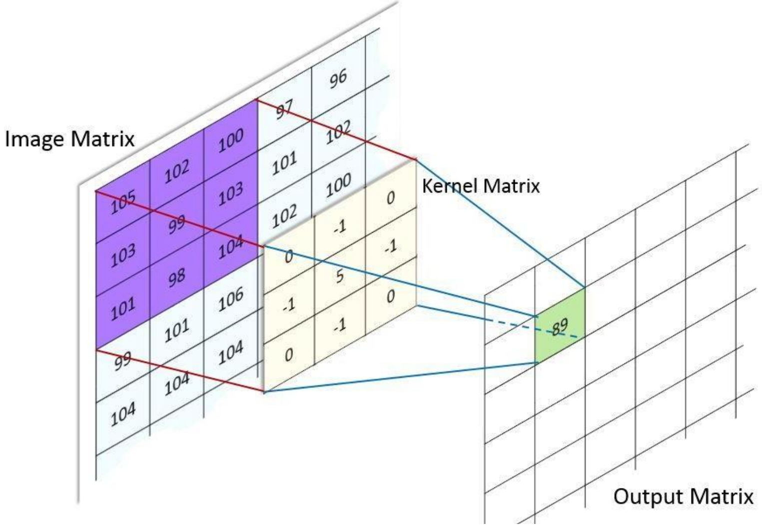
Filtering as Feature Extraction

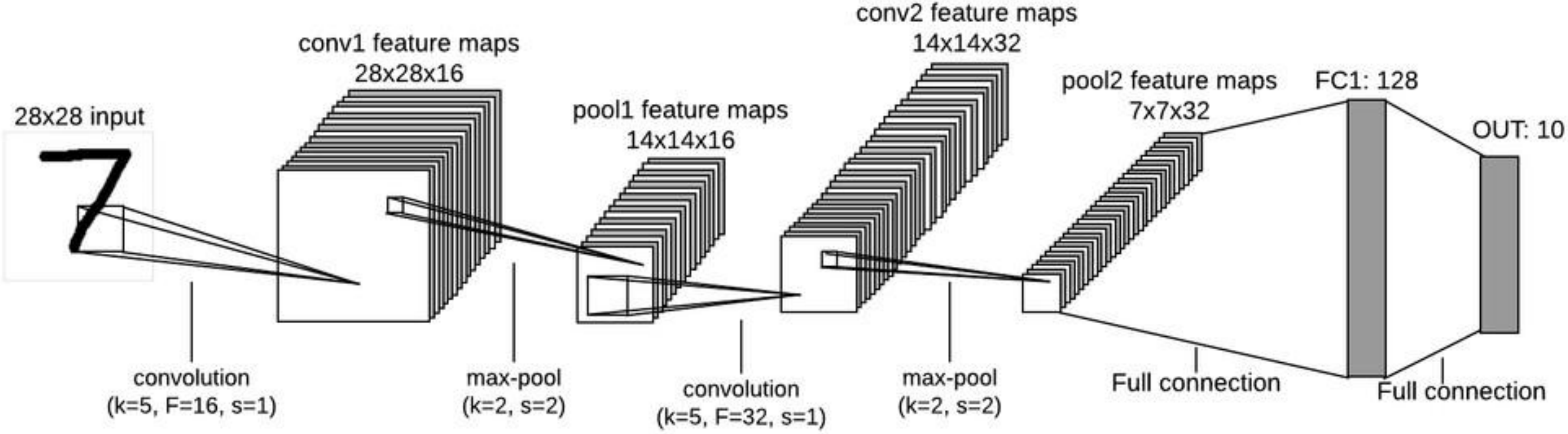


$I(x)=221$ $f_1(x) = 0.3$ $f_2(x) = -0.1$ $f_3(x) = 0.9$ …

Pixel-level features

Convolutional Neural Network



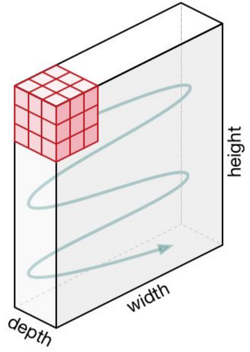


For color images,
we use 3-D Filters

-1	0	1
-1	0	1
-1	0	1

-1	0	1
-1	0	1
-1	0	1

-1	0	1
-1	0	1
-1	0	1



CORRELATION VS. CONVOLUTION

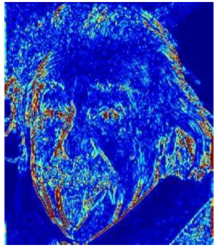
Image correlation:

$$I \otimes z = I$$

$$\sum_{k,l} I(i+k, j+l) z(k,l) = I_f(i,j)$$

2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$$\otimes \ \boxed{\mathsf{F}} \ =$$



Correlation vs. Convolution

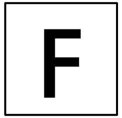
Image correlation:

$$I \otimes z = I_f$$

$$\sum_{k,l} I(i+k, j+l) z(k,l) = I_f(i,j)$$

2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$\otimes$



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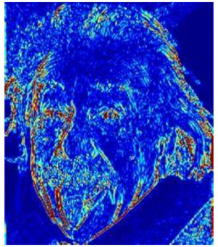


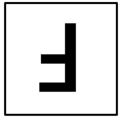
Image convolution:

$$I * z = J$$

$$\sum_{k,l} I(i-k, j-l) z(k,l) = J(i,j)$$

2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$\otimes$



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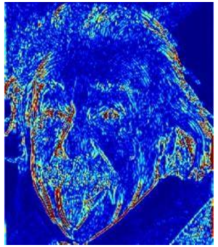
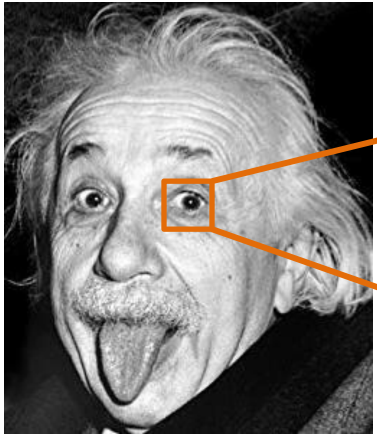


Flip the filter in both dimension (bottom to top, right to left)

Correlation vs. Convolution



Correlation

2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$\otimes$

a	b	c
d	e	f
g	h	i

$=$

	y			

$$y = 2a + b + 4c + d + 2e + 2f + 3g + 3h + 5i$$

Convolution

2	1	4	4	7
1	2	2	3	6
3	3	5	8	9
5	2	2	6	7
8	3	2	1	3

$\otimes$

Flipped Filter

i	h	g
f	e	d
c	b	a

$=$

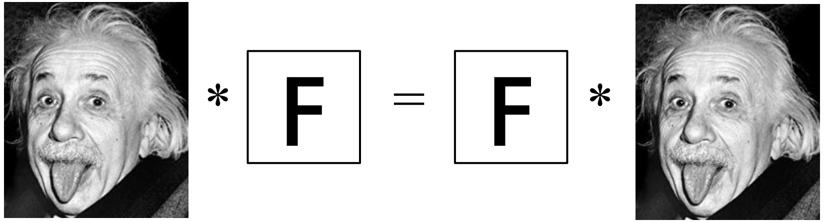
	z			

$$z = 2i + h + 4g + f + 2e + 2d + 3c + 3b + 5a$$

Properties of Convolution

Commutative:

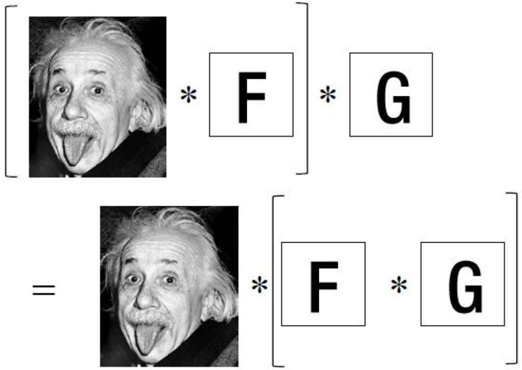
$$f * g = g * f$$



A filter can be filtered by an image.

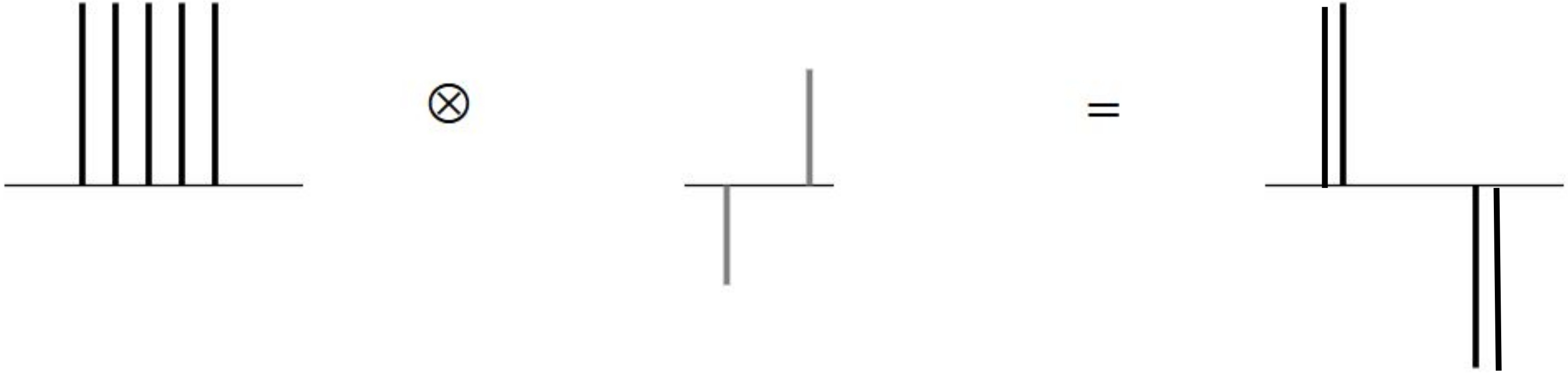
Associative:

$$(f * g) * h = f * (g * h)$$



A composition of convolutions can be pre-computed.

DIFFERENTIATION

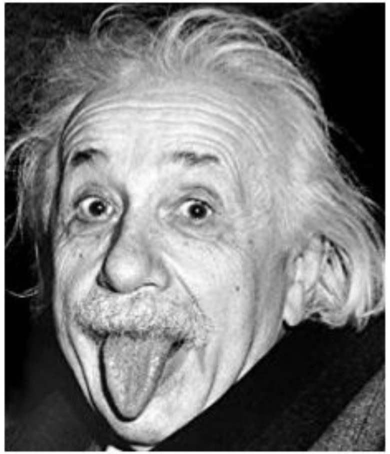


$$\frac{df}{du} = \lim_{h \to 0} \frac{f(u+h) - f(u-h)}{2h} \longrightarrow I \otimes z = \frac{\partial I}{\partial u}$$

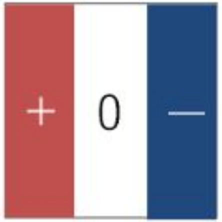
In discrete domain such as an image, h=1 (the smallest change)

So, 0.5*f(u+1)+0*f(u)+(-0.5)*f(u-1).
The filter turns out to be [-0.5,0,0.5], which can be scaled to [-1,0,1]

IMAGE DIFFERENTIATION

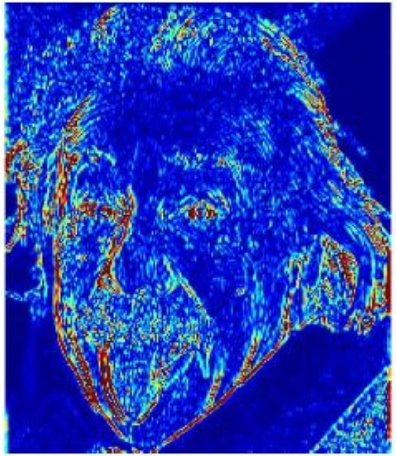


$\otimes$



$$I \otimes z = \frac{\partial I}{\partial u}$$

$=$

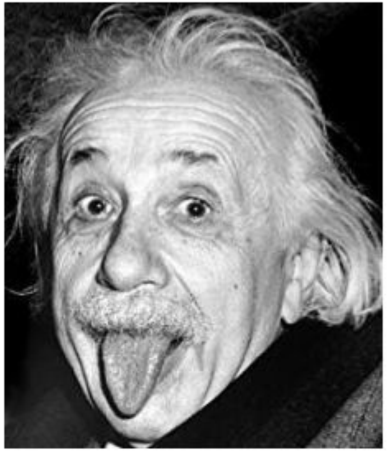


Differentiation

+1 0 -1
+1 0 -1
+1 0 -1

Prewitt Filter

Image Differentiation

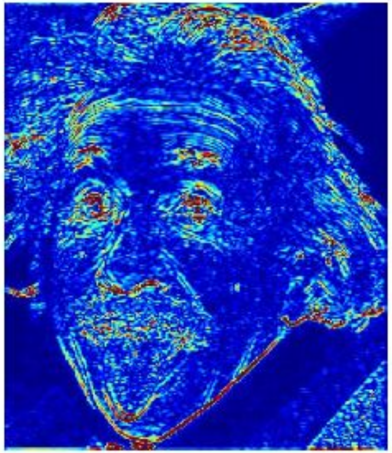


$\otimes$

$$I \otimes z = \frac{\partial I}{\partial v}$$

$$\begin{array}{ccc} +1 & +1 & +1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{array}$$

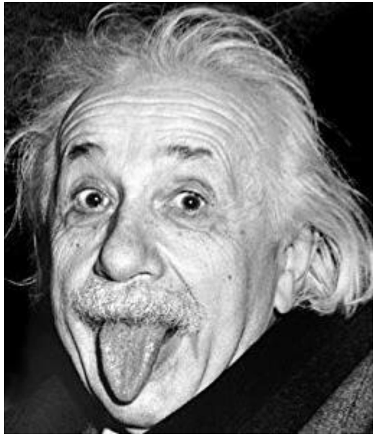
$=$

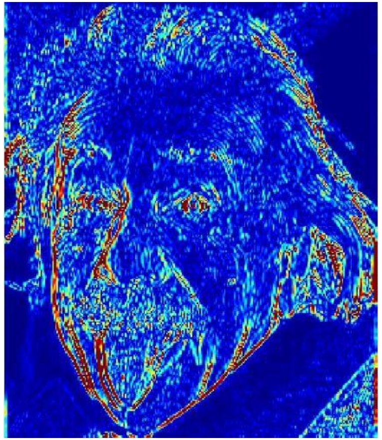


Differentiation

Prewitt Filter

RECALL: EDGE RESPONSE





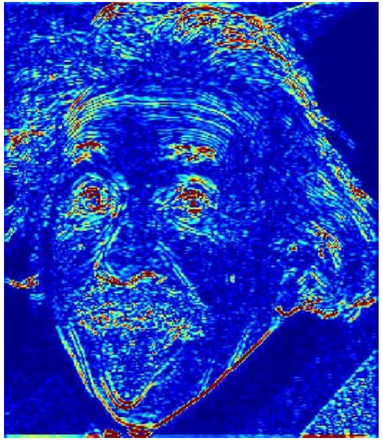
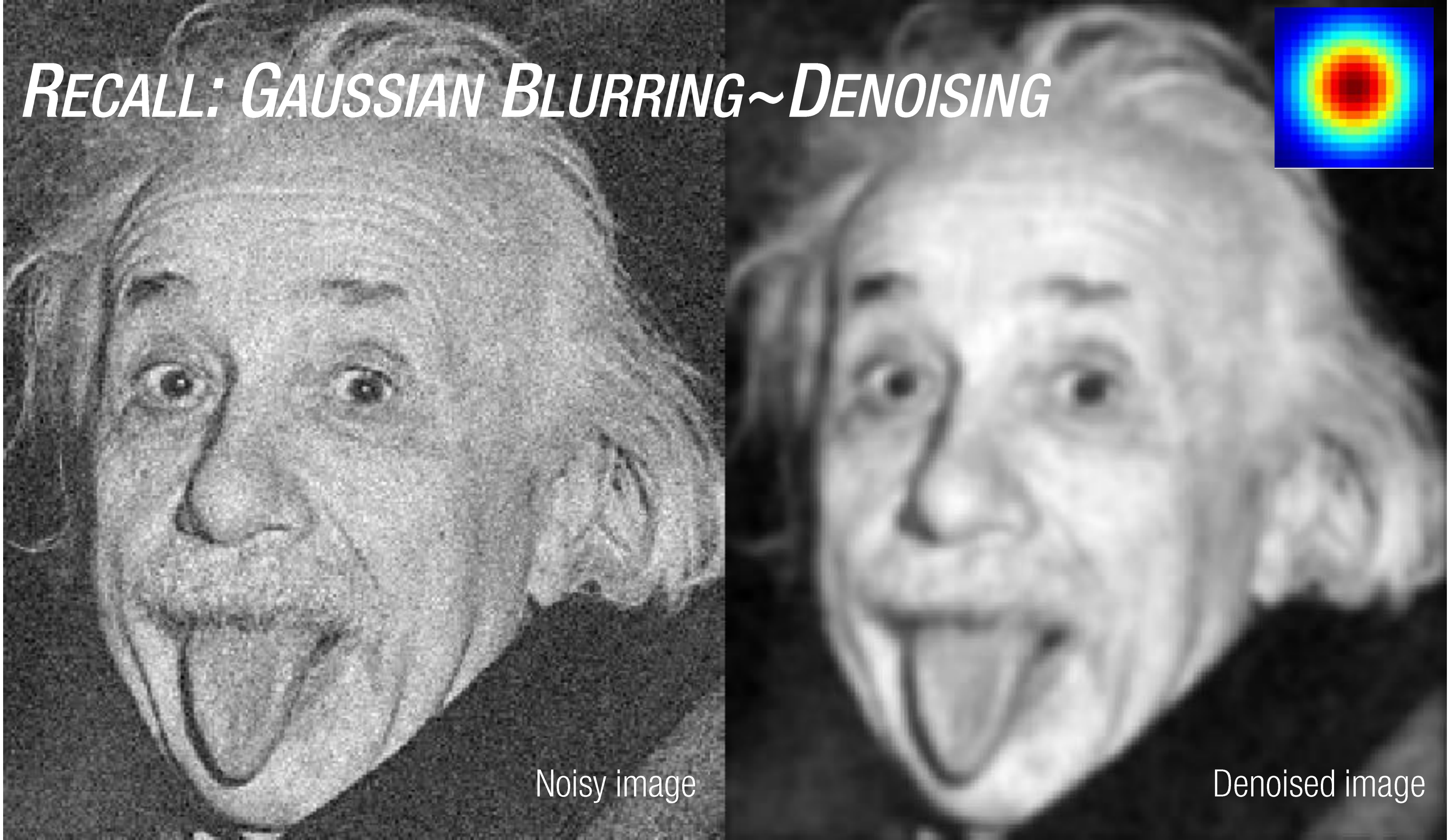


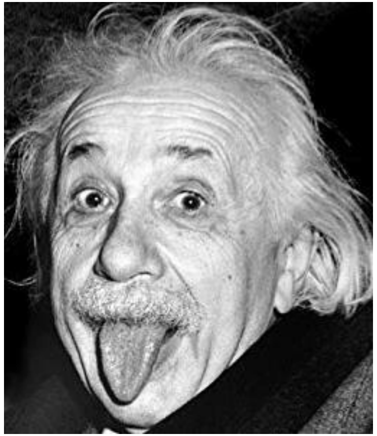
Image differentiation can be used for edge detection but it is VERY noisy.

How to detect edge reliably?

Recall: Gaussian Blurring~Denoising



STRATEGY: DENOISE AND DIFFERENTIATE



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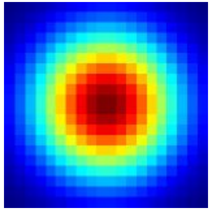

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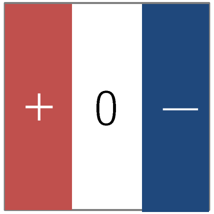

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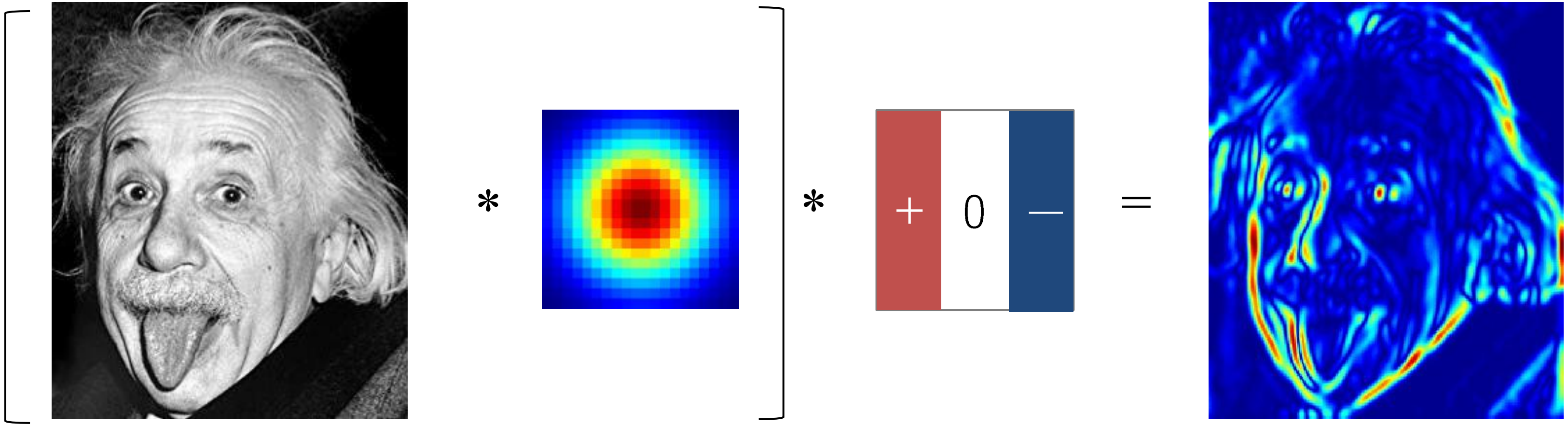
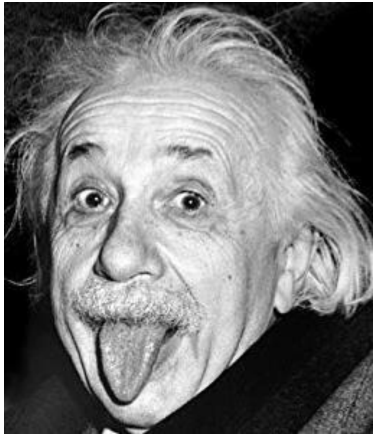
Strategy: Denoise and Differentiate



Associativity



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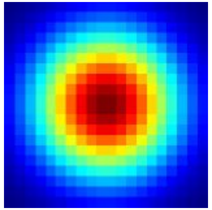

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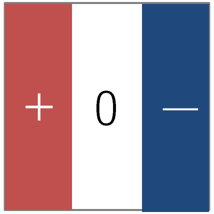

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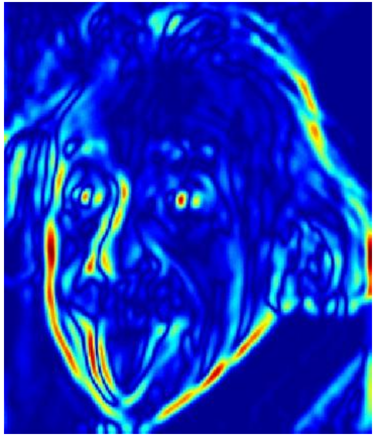
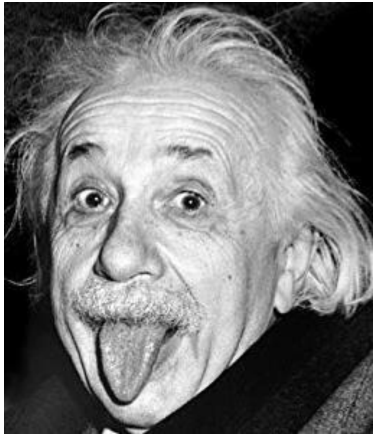


$$\frac{1}{2\pi\sigma^{2}} e^{-\frac{k^2+l^2}{2\sigma^2}}$$

$$\frac{\partial}{\partial u}$$

Commutativity



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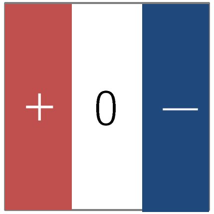
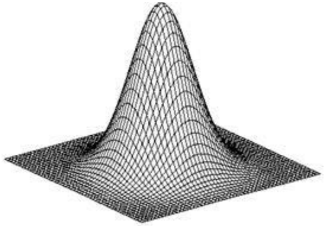
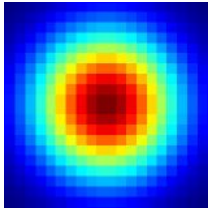


$$\frac{\partial}{\partial u}$$

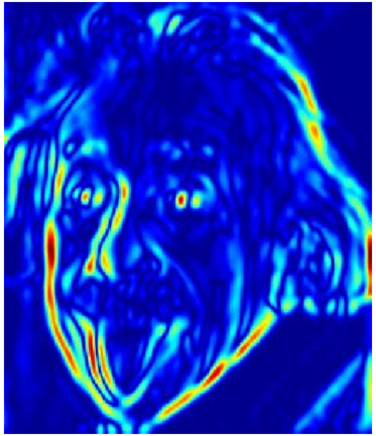
$*$



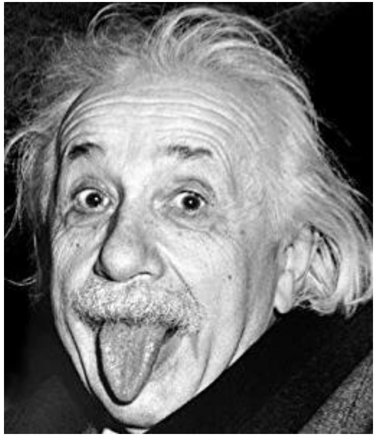


$$\frac{1}{2\pi\sigma^2} e^{-\frac{k^2+l^2}{2\sigma^2}}$$

$\Big] =$



Sobel Filter



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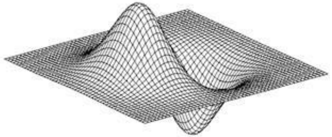
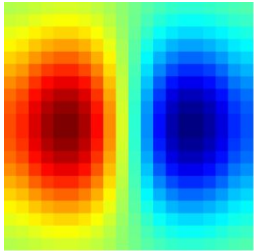
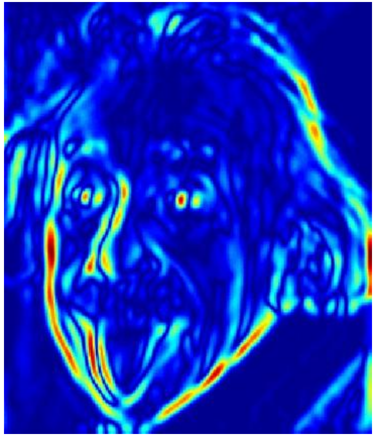




$$\frac{\partial}{\partial u}\frac{1}{2\pi\sigma^2}e^{-\frac{k^2+l^2}{2\sigma^2}}$$

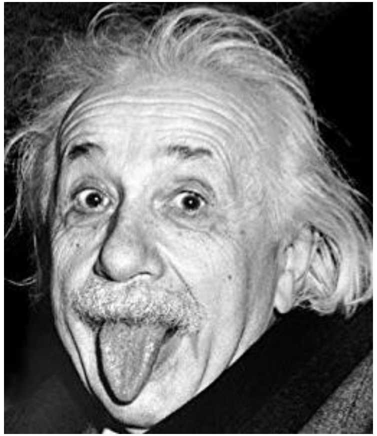
$=$



Sobel filter: derivative of Gaussian filter, e.g.,

$$\begin{matrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{matrix}$$

Sobel Filter



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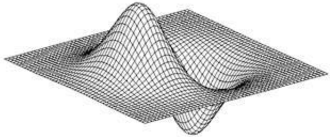
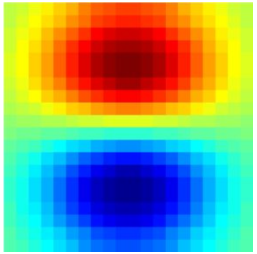
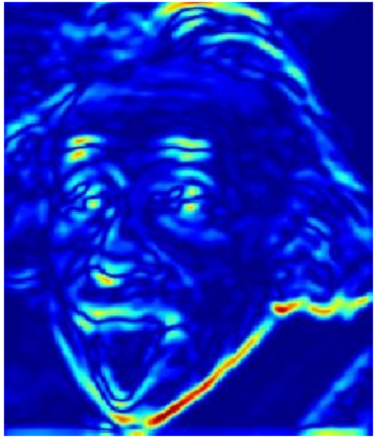




$$\frac{\partial}{\partial v}\frac{1}{2\pi\sigma^{2}}e^{-\frac{k^{2}+l^{2}}{2\sigma^{2}}}$$

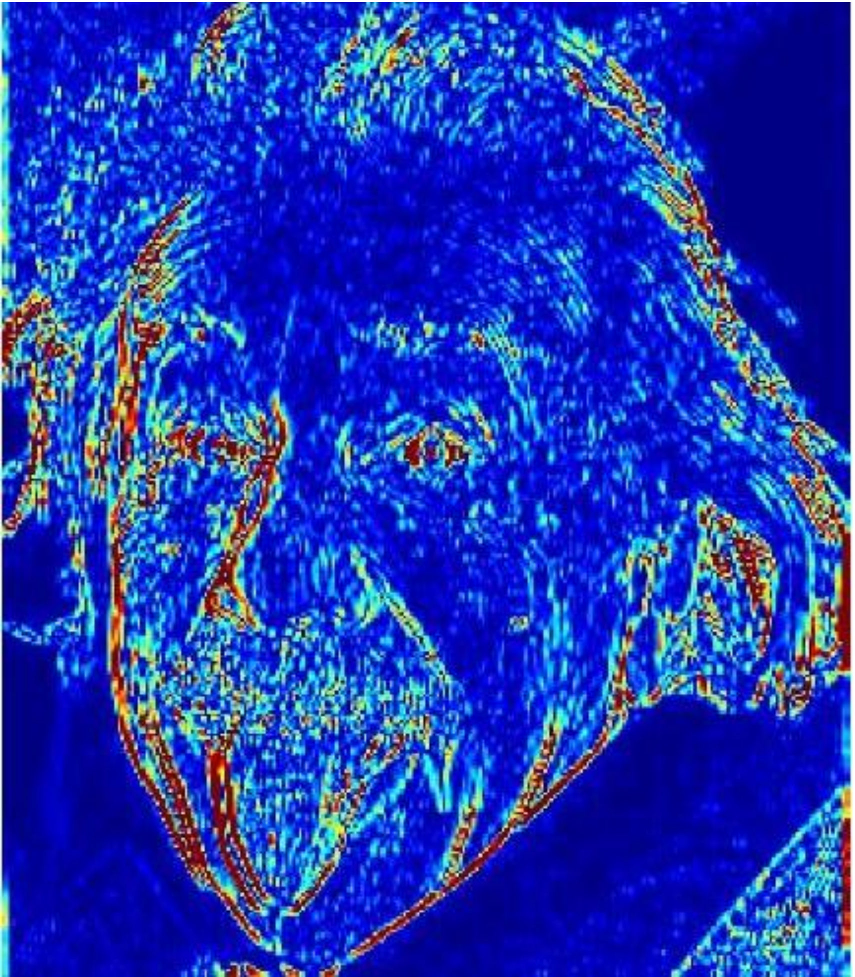
$=$

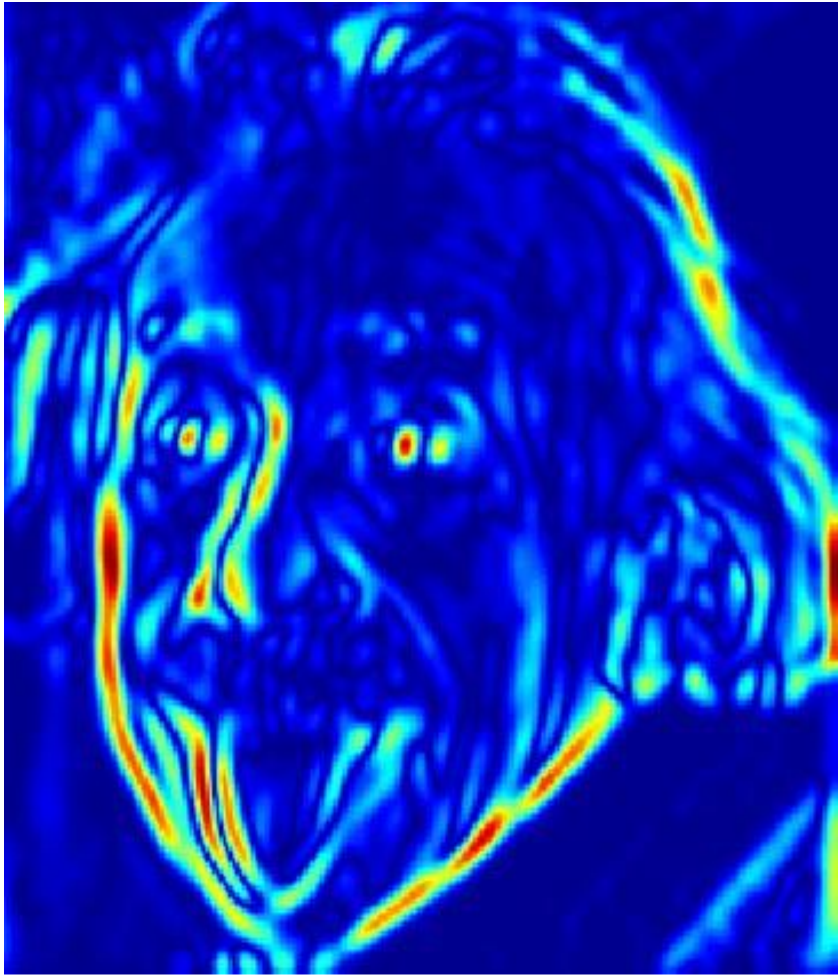


Sobel filter: derivative of Gaussian filter, e.g.,

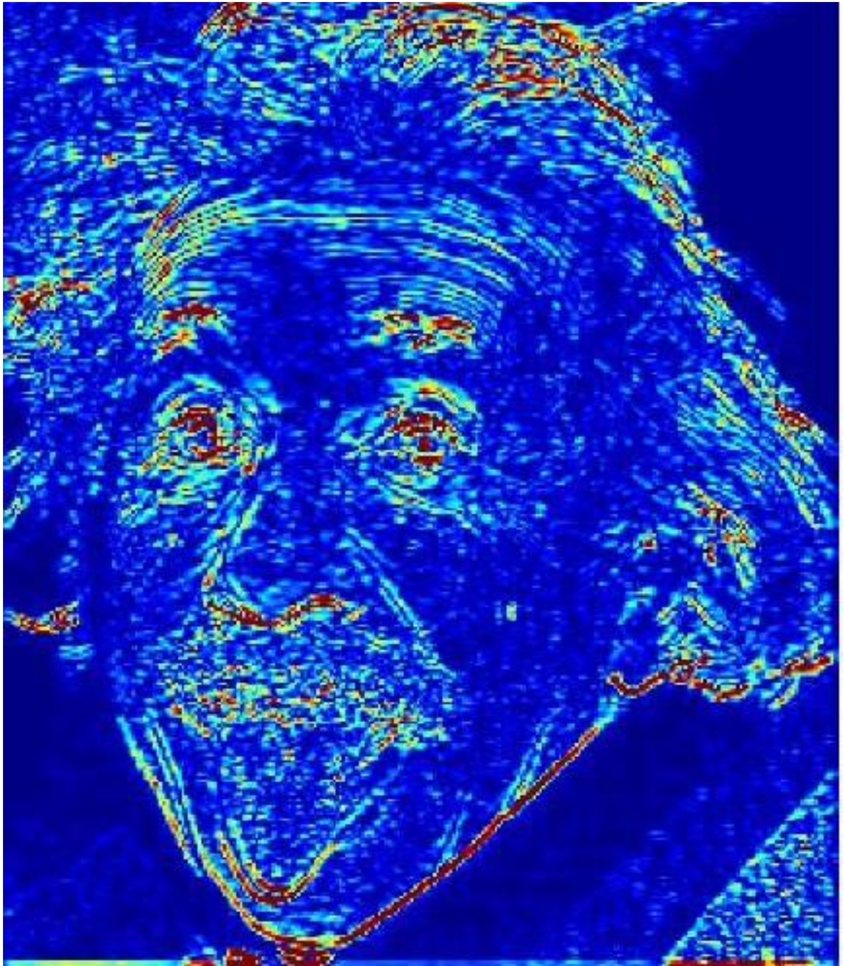
$$\begin{array}{ccc} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{array}$$

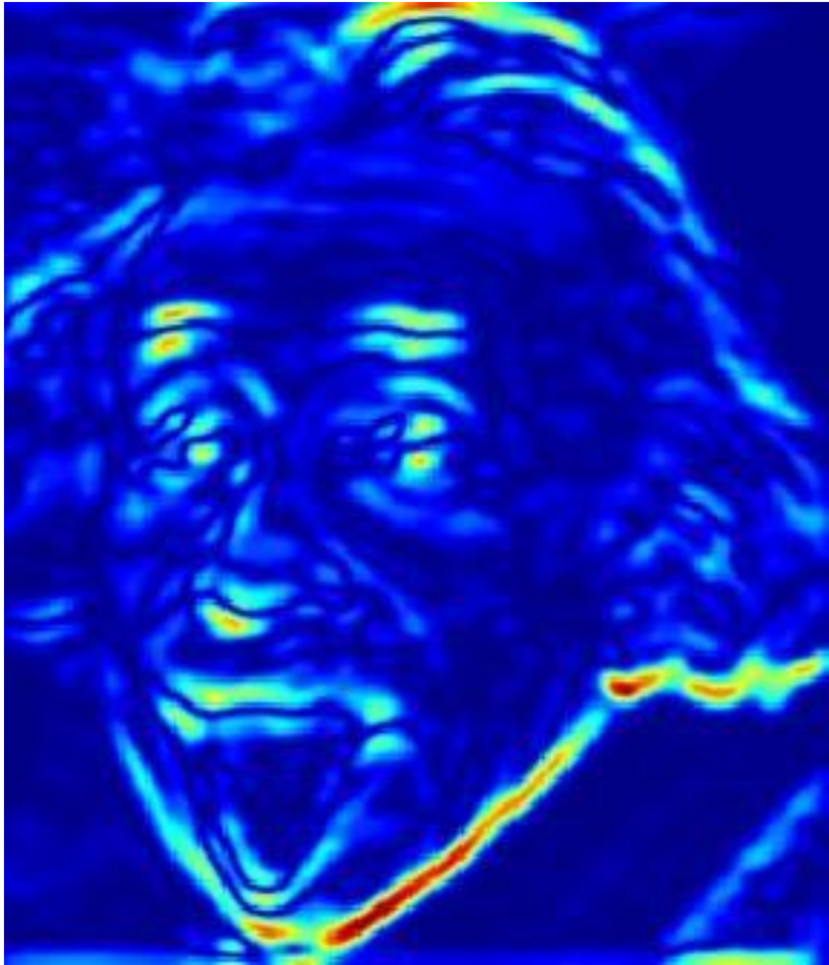
Prewitt vs Sobel





Prewitt vs Sobel





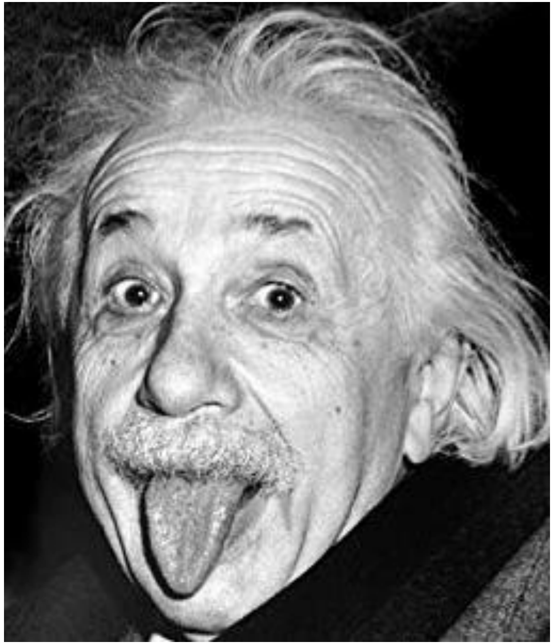
Lecture Outline

Convolution

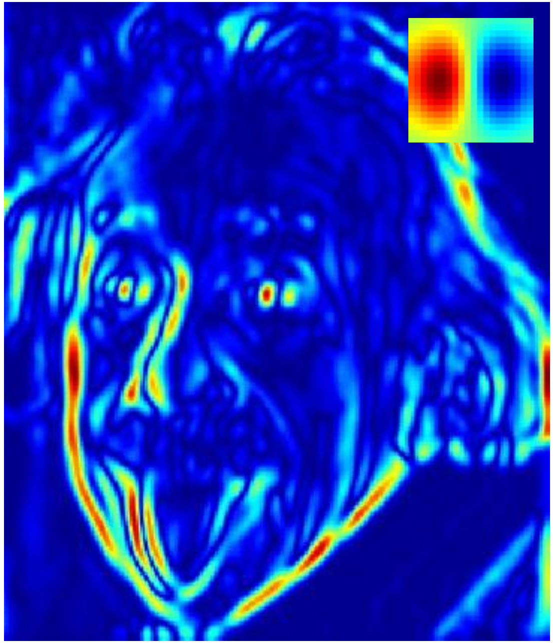
Image Gradient

Edge

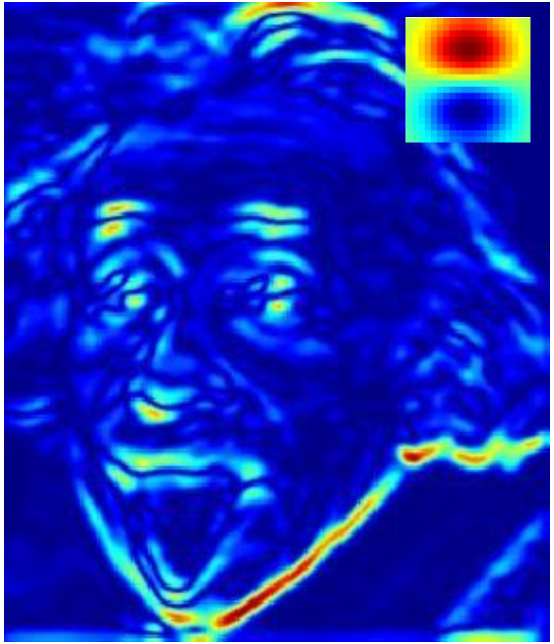
Image Partial Differential



I

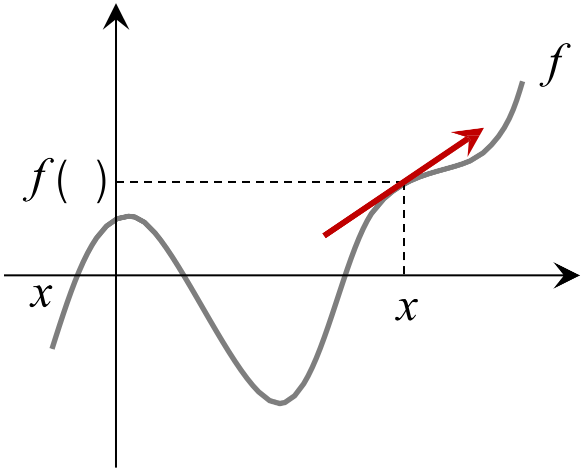


$\frac{\partial I}{\partial u}$

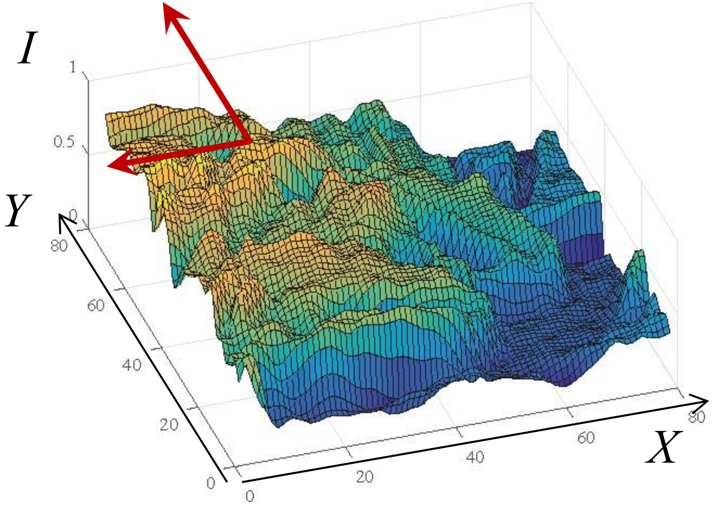


$\frac{\partial I}{\partial v}$

IMAGE GRADIENT



$$\frac{df(x)}{dx}$$



$$\nabla I = \frac{\partial I(x,y)}{\partial x}\mathbf{i} + \frac{\partial I(x,y)}{\partial y}\mathbf{j}$$

Gradient

def) a multivariate generalization of the derivative.

Image Gradient

$$\nabla I = \frac{\partial I(x,y)}{\partial x}\mathbf{i} + \frac{\partial I(x,y)}{\partial y}\mathbf{j}$$

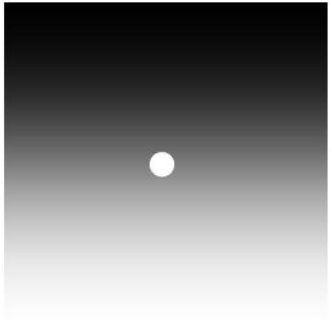
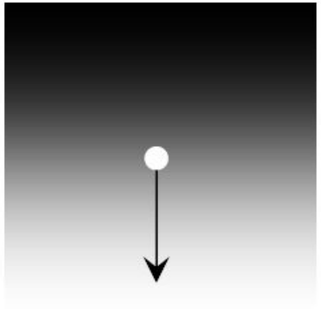
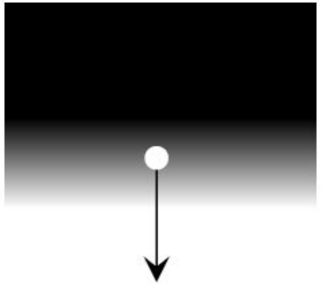


Image Gradient

$$\nabla I = \frac{\partial I(x,y)}{\partial x}\mathbf{i} + \frac{\partial I(x,y)}{\partial y}\mathbf{j}$$

Magnitude of gradient is proportional to the greatest rate of change





$$c_1 < c_2$$

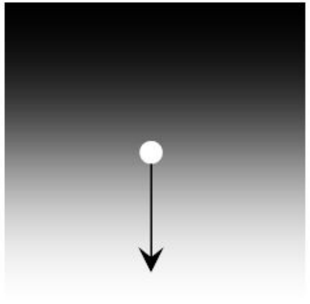
$$\frac{\partial I}{\partial u} = 0 \quad \frac{\partial I}{\partial v} = c_1$$

$$\frac{\partial I}{\partial u} = 0 \quad \frac{\partial I}{\partial v} = c_2$$

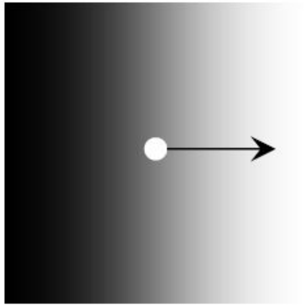
Image Gradient

$$\nabla I = \frac{\partial I(x,y)}{\partial x}\mathbf{i} + \frac{\partial I(x,y)}{\partial y}\mathbf{j}$$

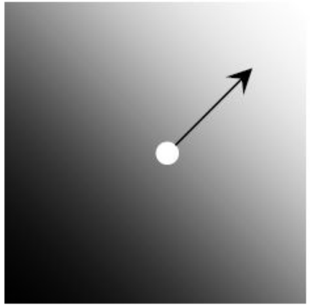
Direction of the gradient is towards the greatest rate of change (increase)



$$\frac{\partial I}{\partial u} = 0 \quad \frac{\partial I}{\partial v} = c_1$$

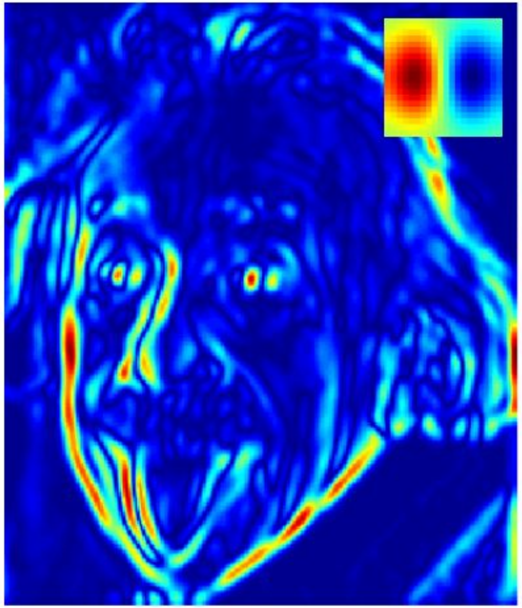


$$\frac{\partial I}{\partial u} = c_2 \quad \frac{\partial I}{\partial v} = 0$$

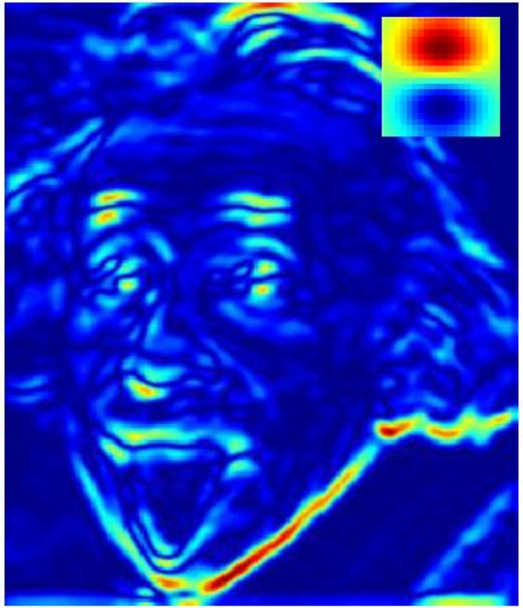


$$\frac{\partial I}{\partial u} = c_3 \quad \frac{\partial I}{\partial v} = c_4$$

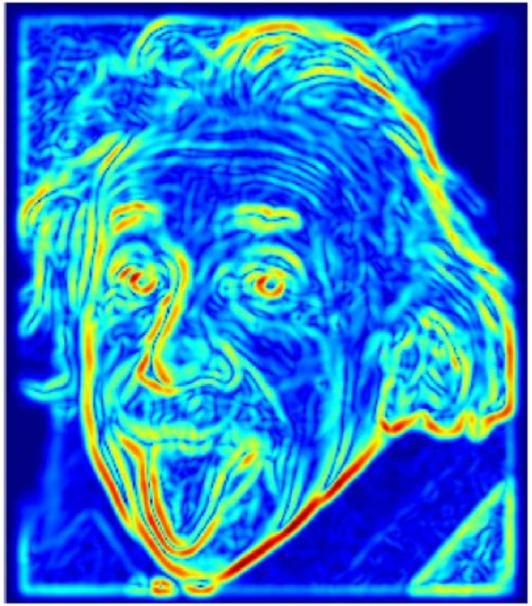
Image Gradient Magnitude



$$\frac{\partial I}{\partial u}$$

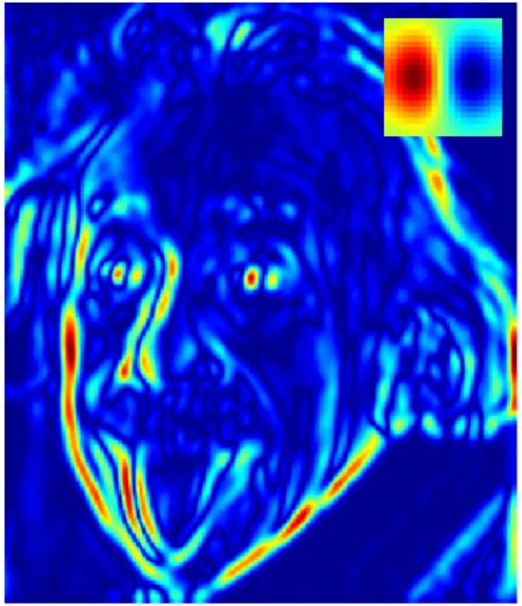


$$\frac{\partial I}{\partial v}$$

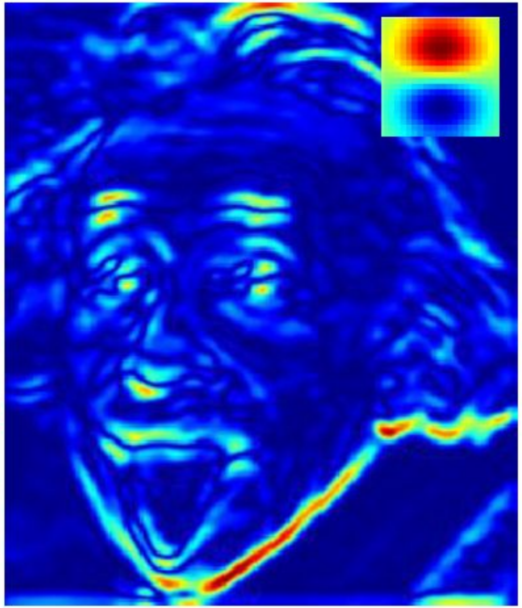


$$\|\nabla I\| = \sqrt{\left(\frac{\partial I}{\partial u}\right)^2 + \left(\frac{\partial I}{\partial v}\right)^2}$$

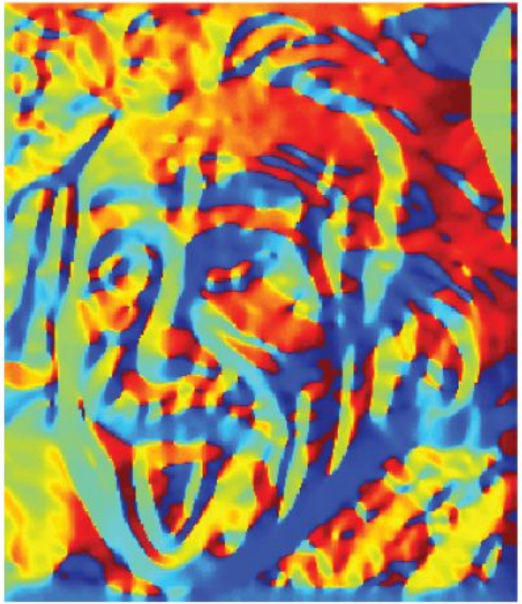
Image Gradient Direction



$$\frac{\partial I}{\partial u}$$



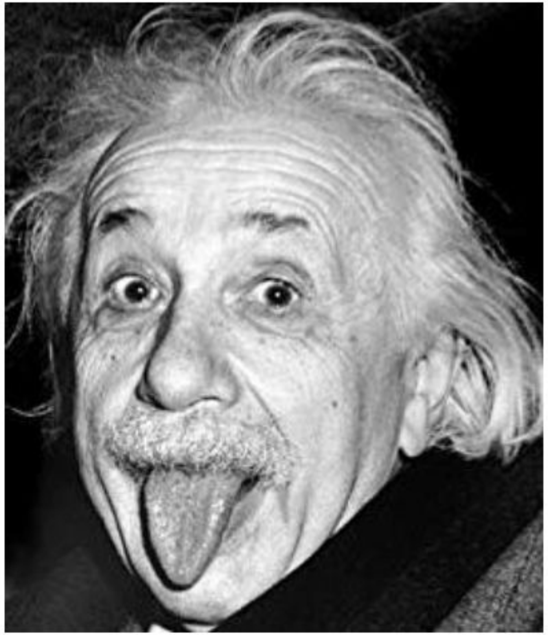
$$\frac{\partial I}{\partial v}$$



$$\measuredangle \nabla I = \mathrm{atan2}\left(\frac{\partial I}{\partial v}, \frac{\partial I}{\partial u}\right)$$

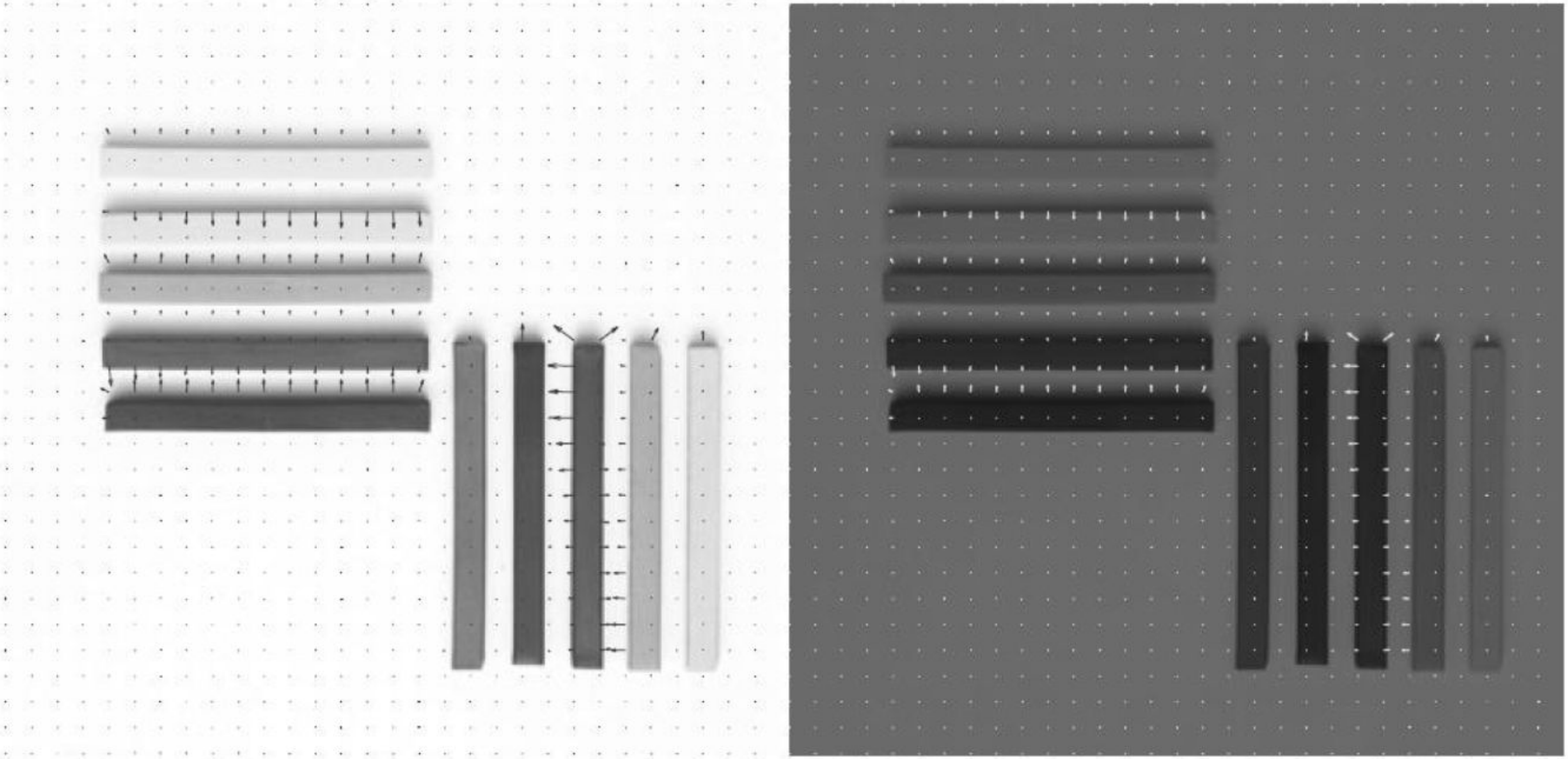
MATLAB

IMAGE GRADIENT DIRECTION



Angle with gradient magnitude thresholding

ILLUMINATION INVARIANT GRADIENT

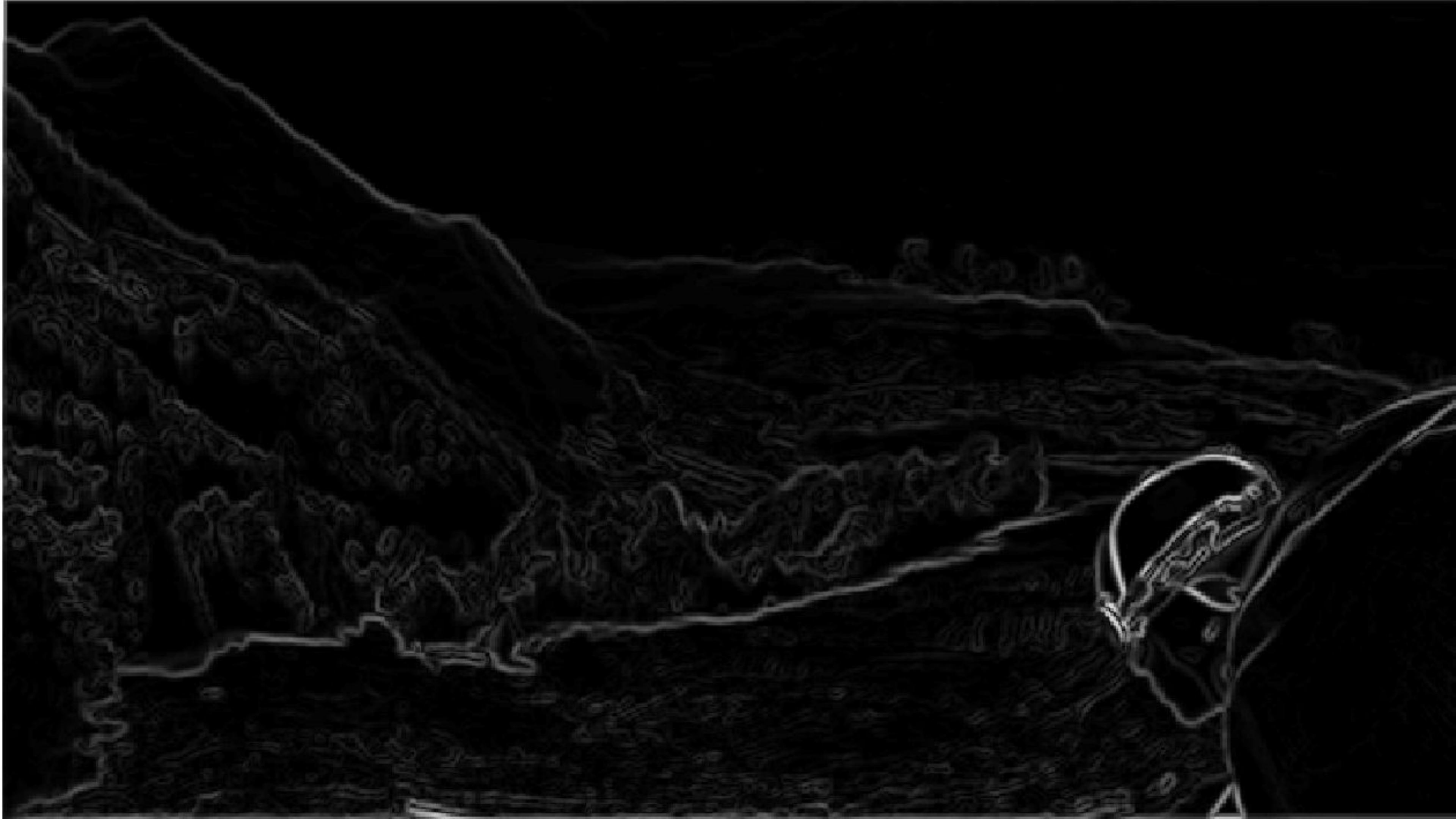


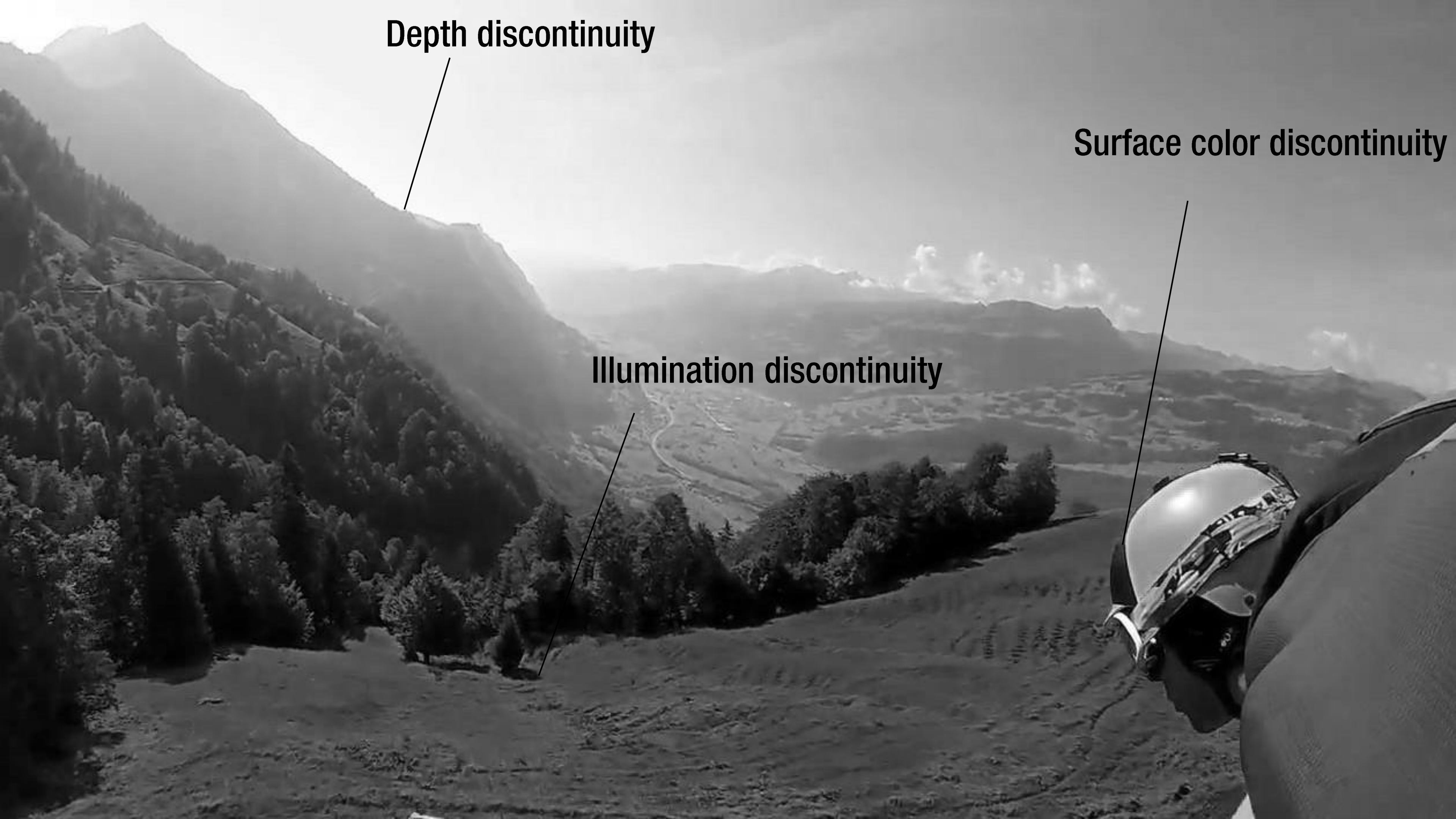
Lecture Outline

Convolution

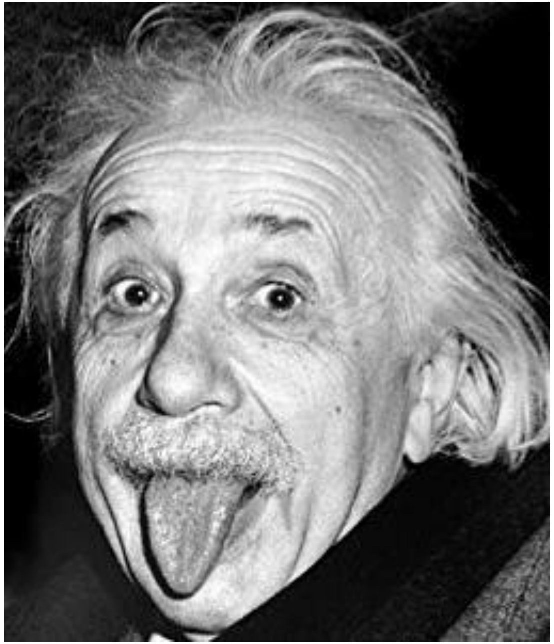
Image Gradient

Edge

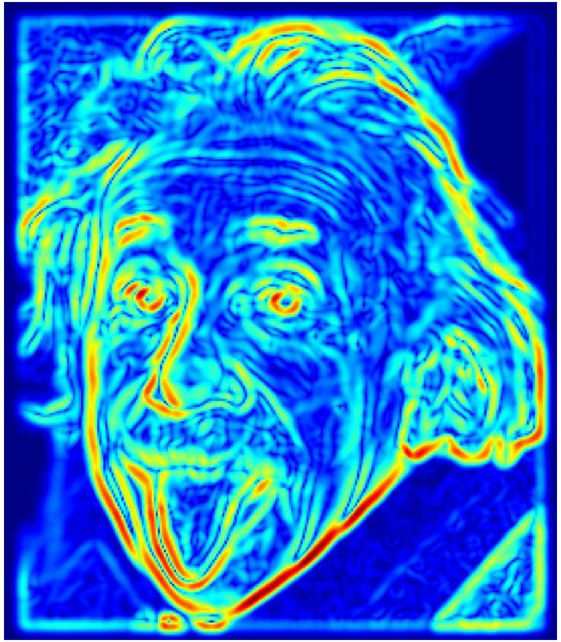



Depth discontinuity
Surface color discontinuity
Illumination discontinuity

EDGE DETECTION: WOULD MAGNITUDE THRESHOLDING WORK?

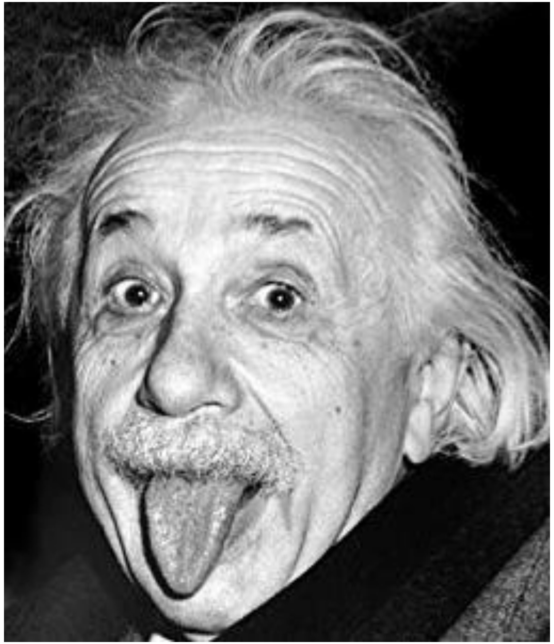


I



$\|\nabla I\| > \varepsilon$

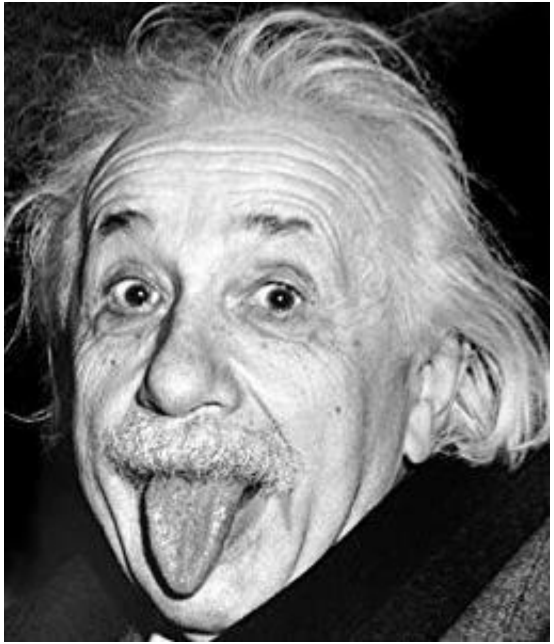
EDGE DETECTION: WOULD MAGNITUDE THRESHOLDING WORK?



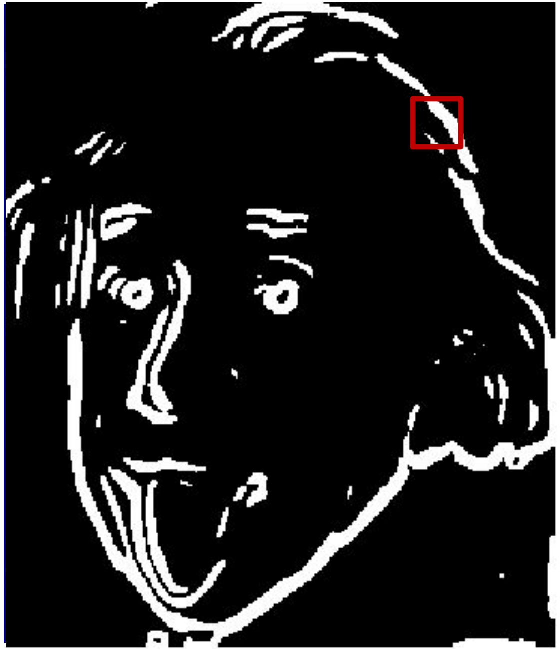
I

$\|\nabla I\| > \varepsilon$

EDGE DETECTION: WOULD MAGNITUDE THRESHOLDING WORK?

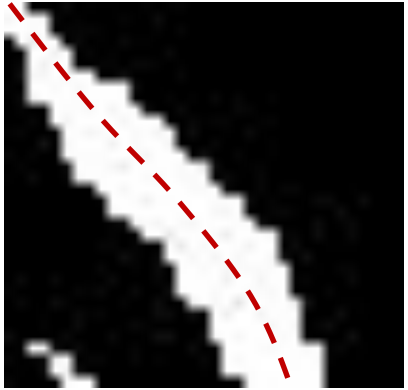


I

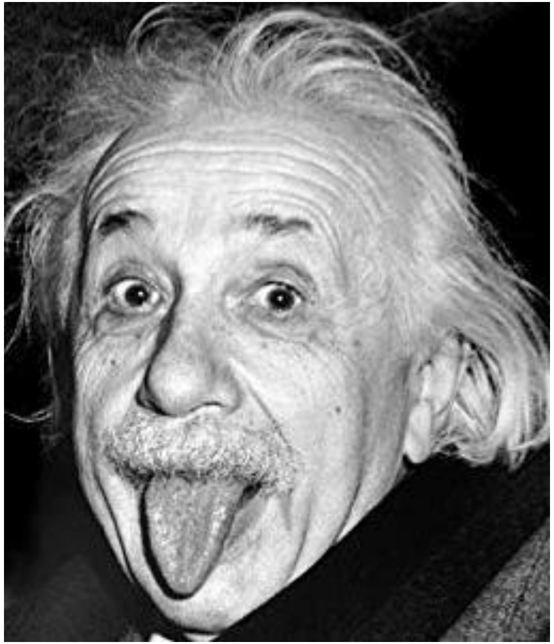


$\|\nabla I\| > \varepsilon$

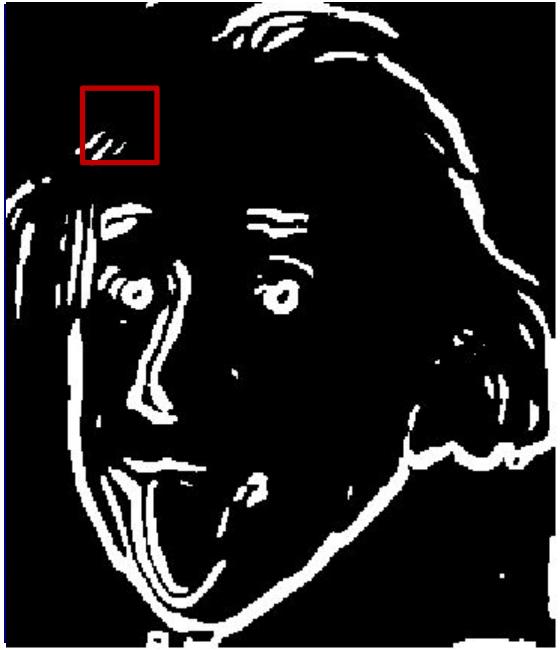
Non-locality



EDGE DETECTION: WOULD MAGNITUDE THRESHOLDING WORK?



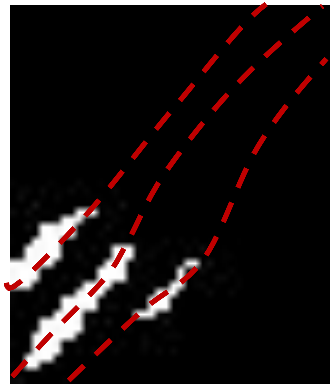
$$I$$



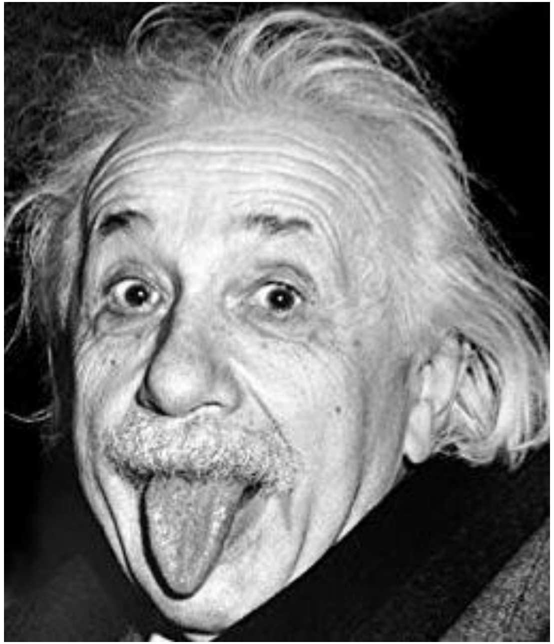
$$\|\nabla I\| > \varepsilon$$

Non-locality

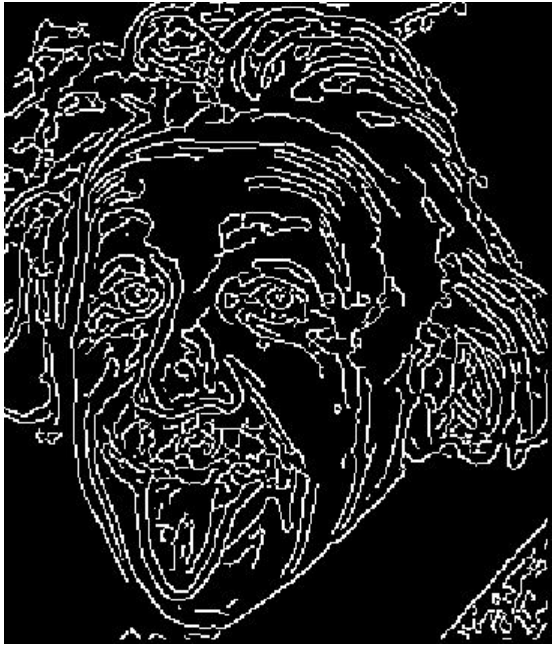
Discontinuity



CANNY EDGE DETECTOR



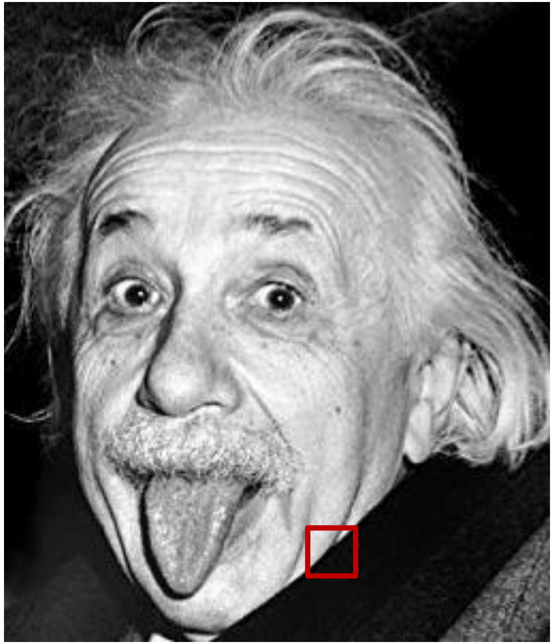
I



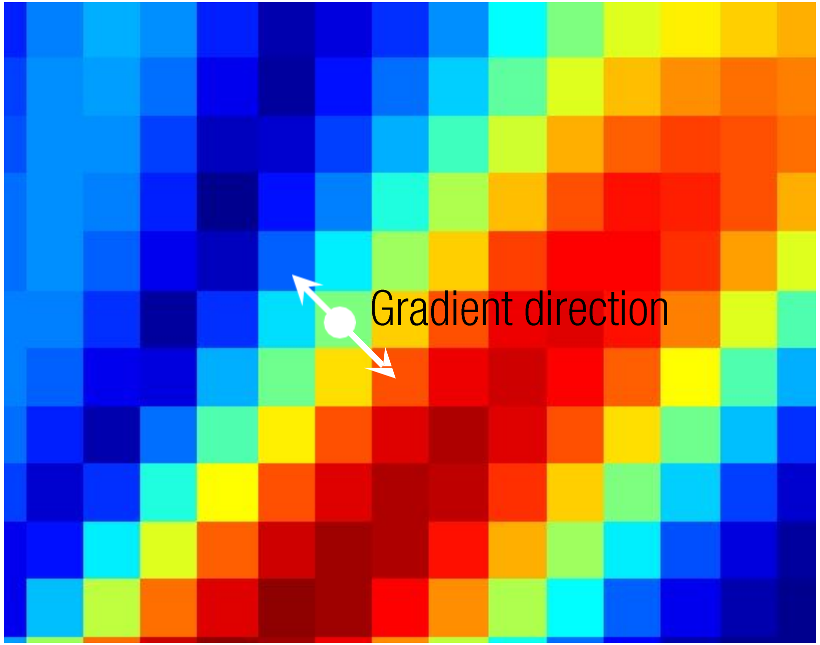
John F. Canny, UC Berkely

Canny, *A Computational Approach to Edge Detection*, TPAMI (1986)

Edge Localization: Non-maximum Supression



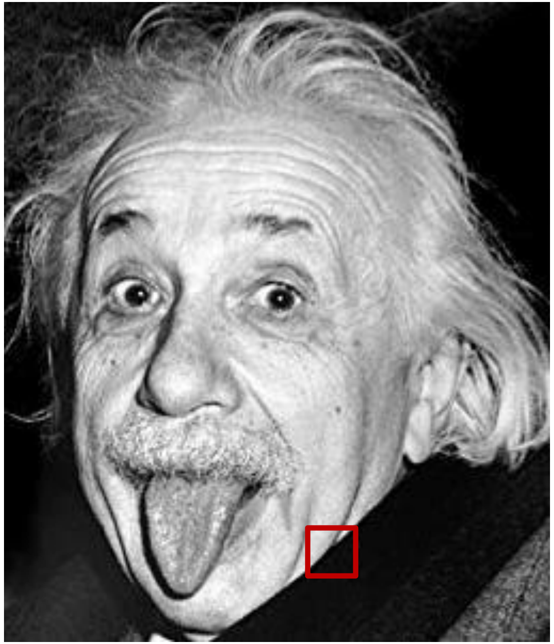
I



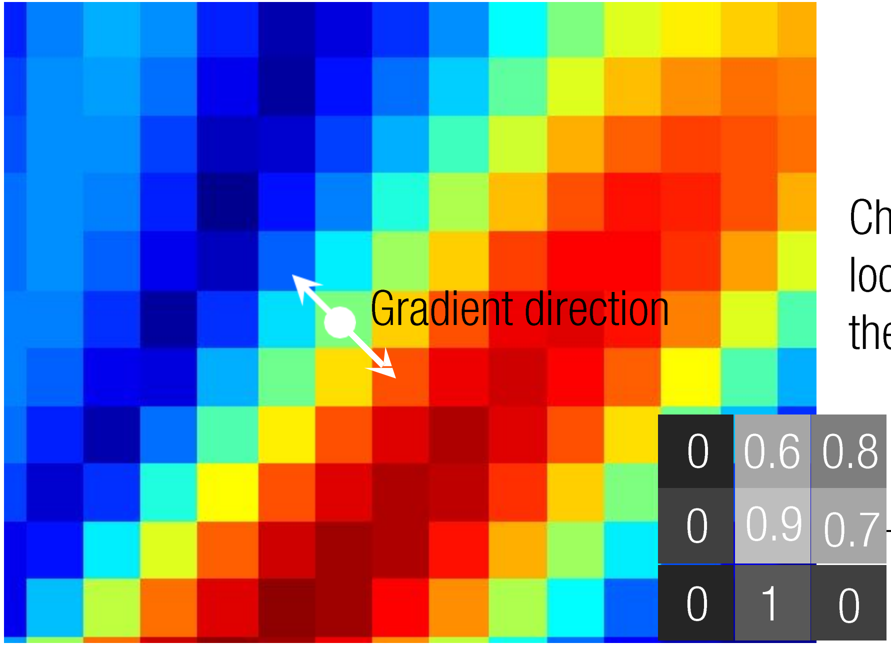
Edge response

Check if the pixel is local maximum along the gradient direction

Edge Localization: Non-maximum Supression

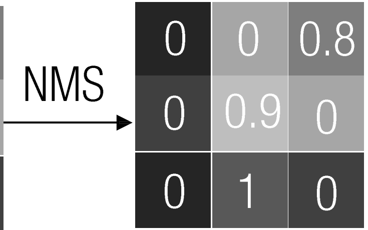


I

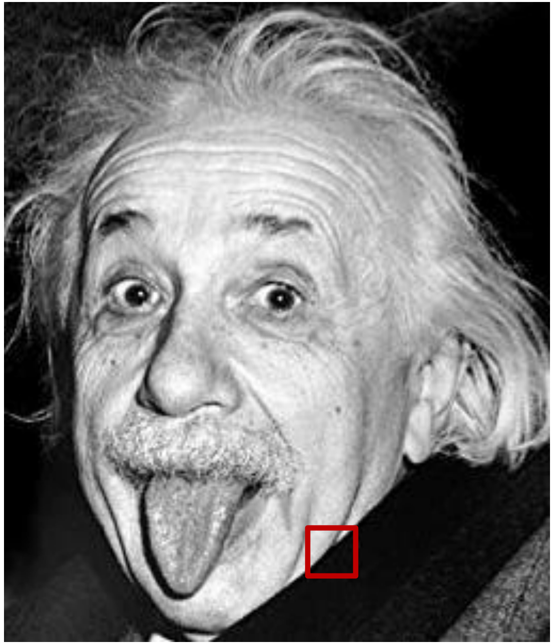


Edge response

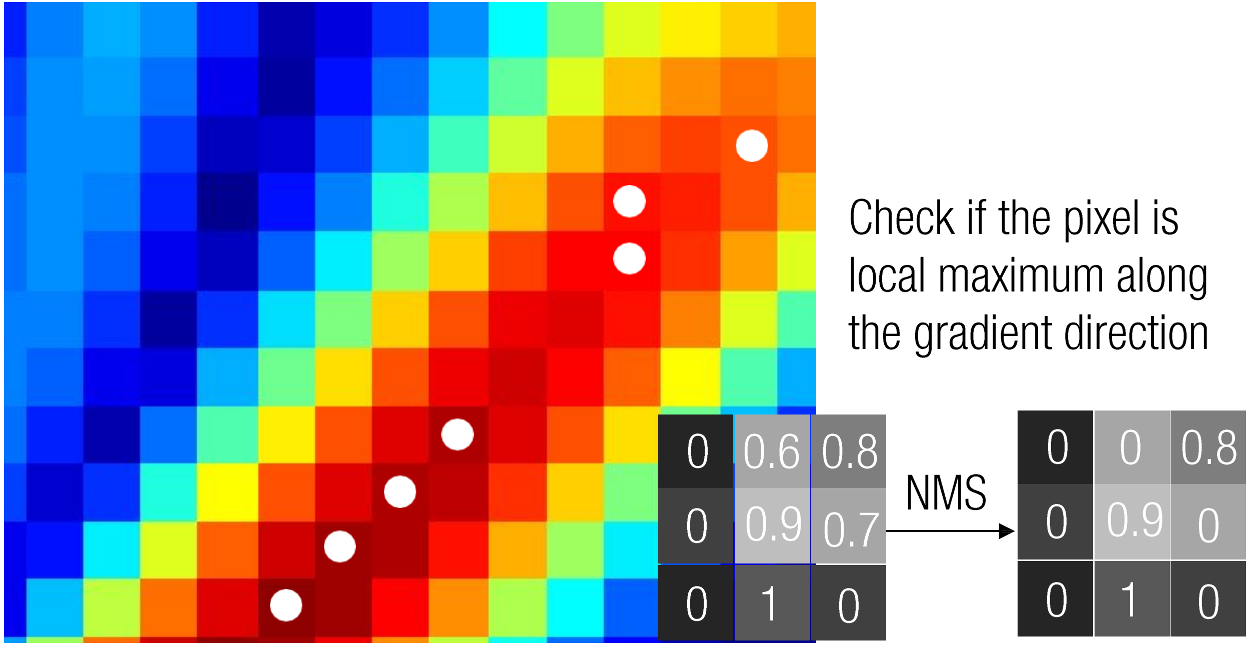
Check if the pixel is local maximum along the gradient direction



EDGE LOCALIZATION: NON-MAXIMUM SUPRESSION



I



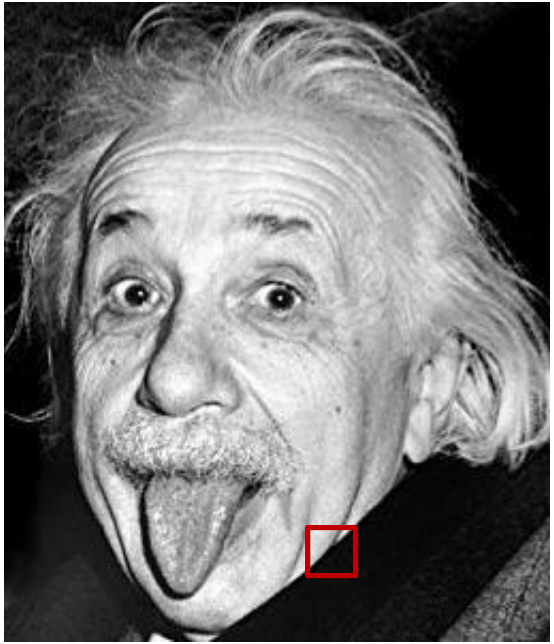
Edge response

Check if the pixel is local maximum along the gradient direction

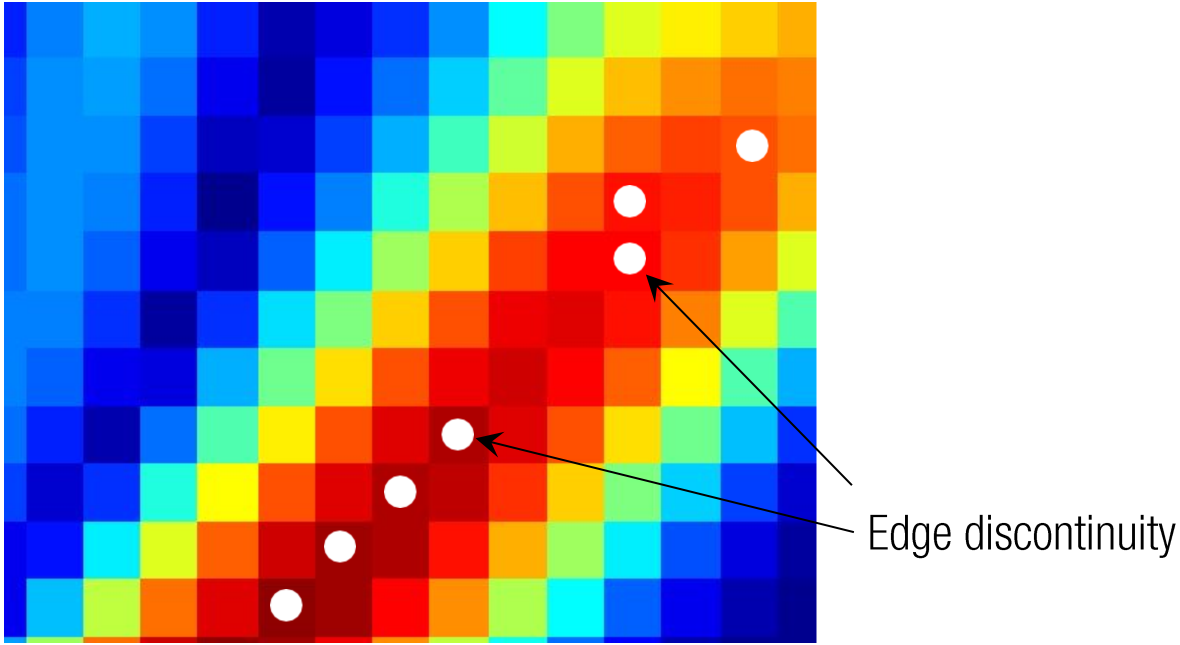
Thresholding

Localized edge

EDGE LINKING

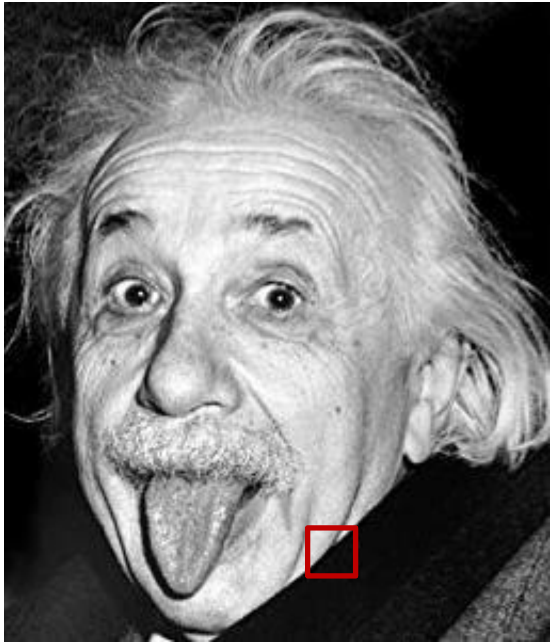


I



Edge response

EDGE LINKING



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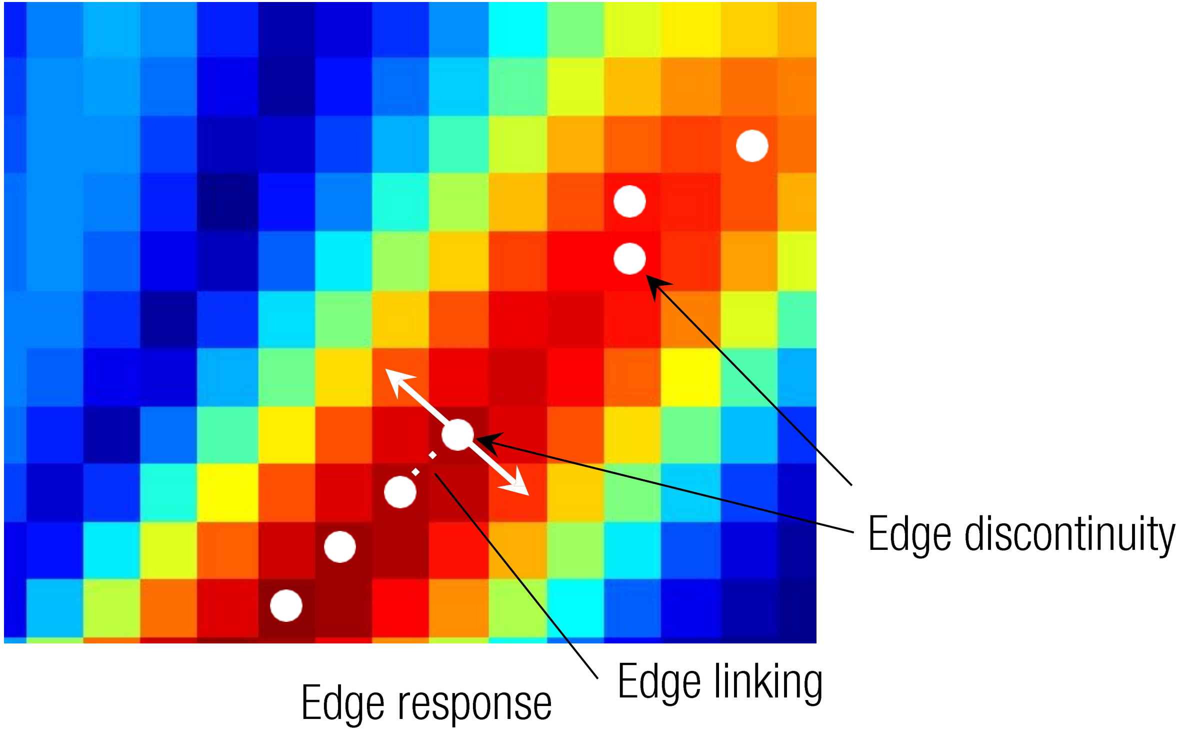
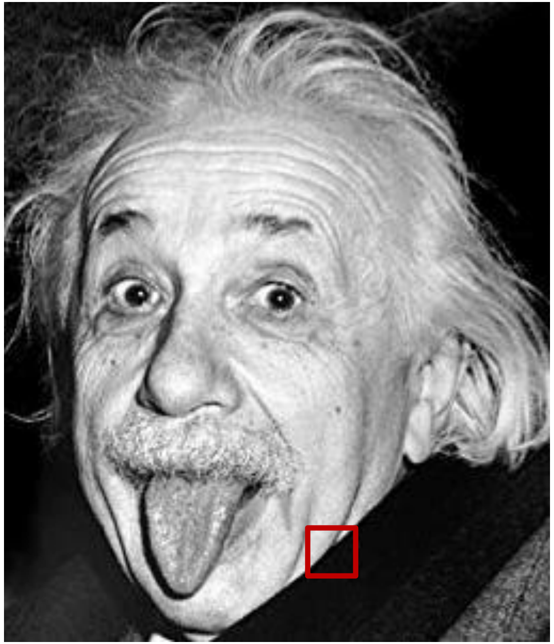
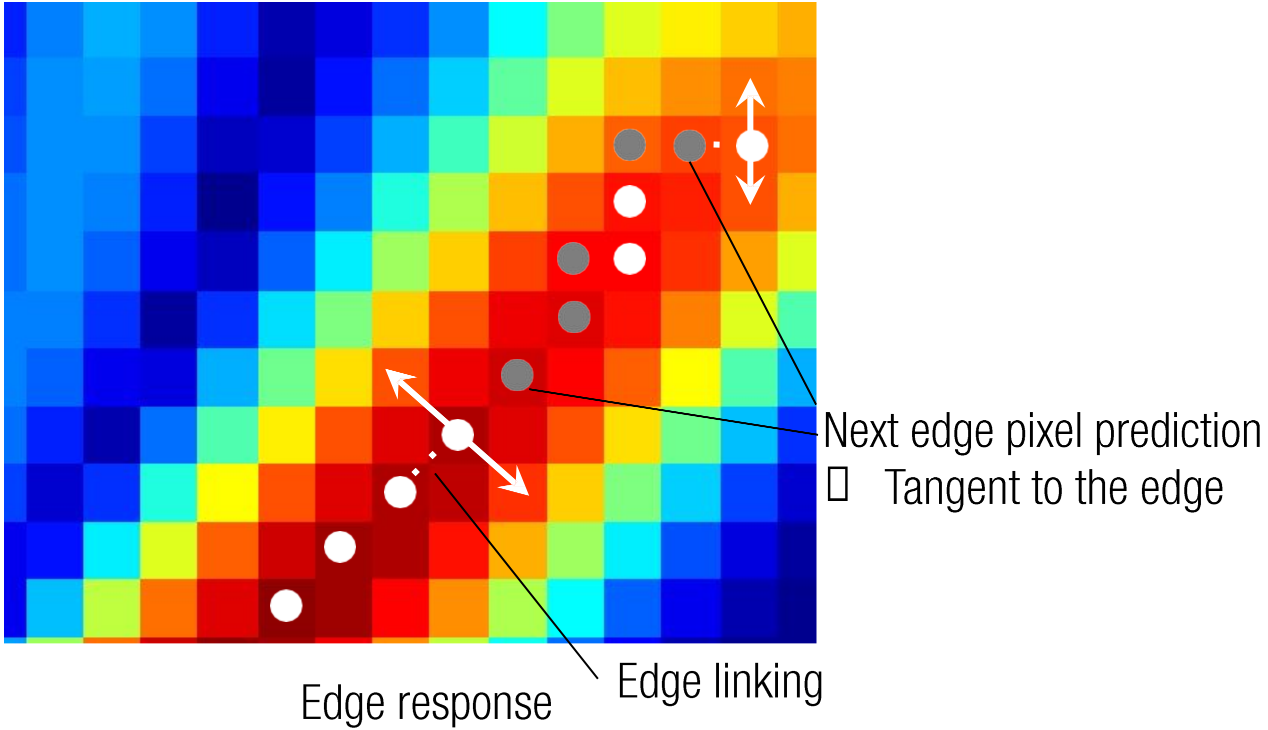


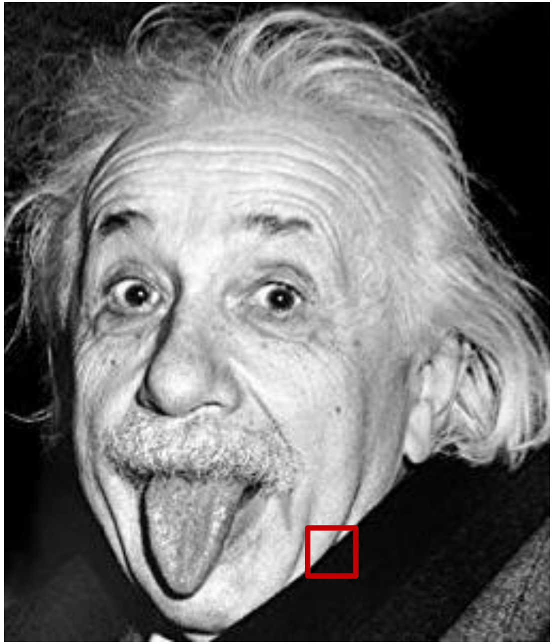
EDGE PREDICTION



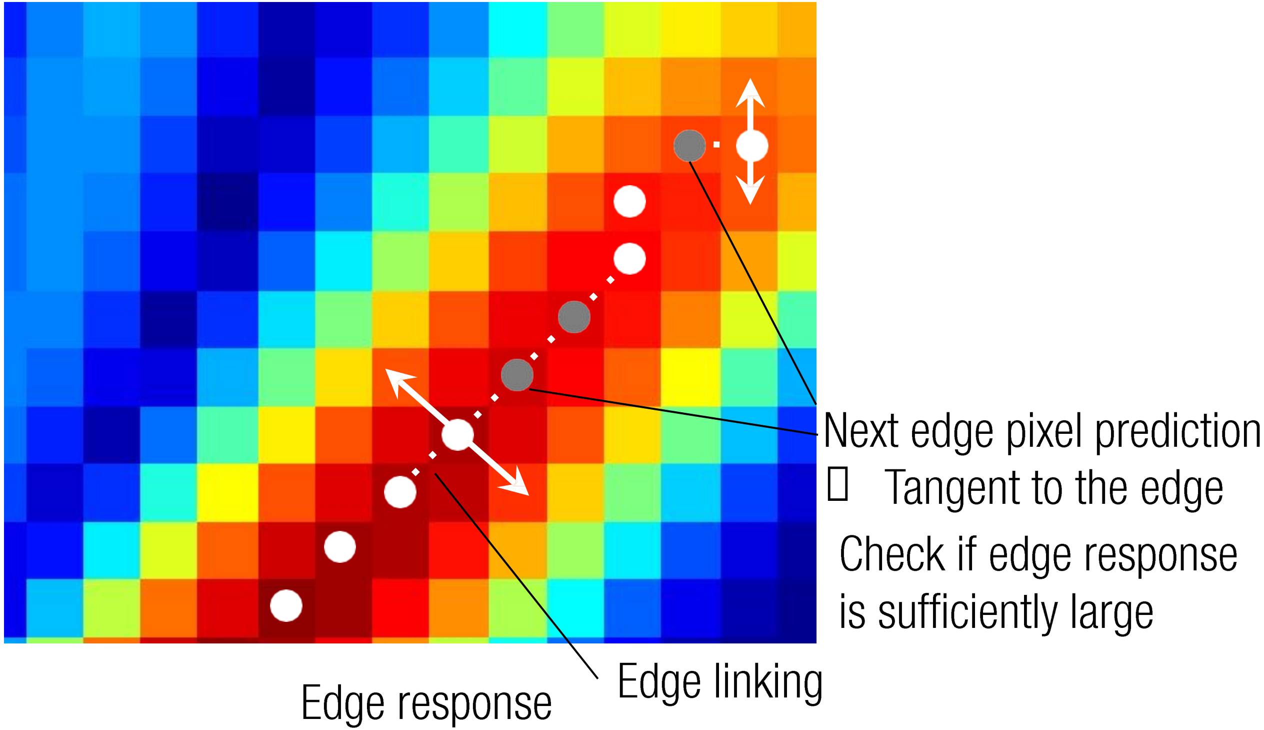
I



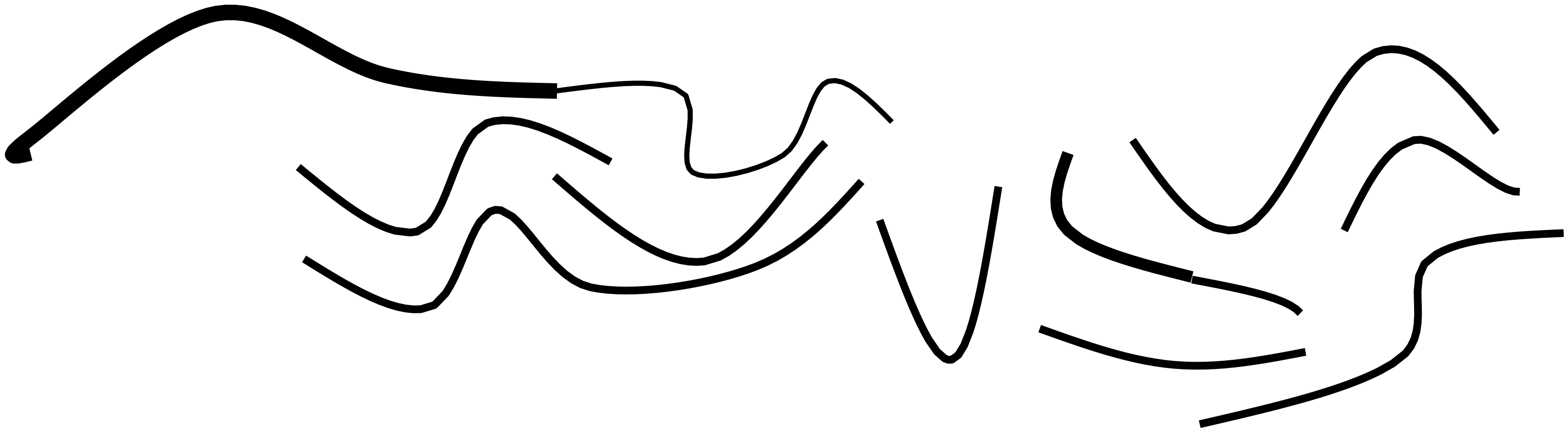
EDGE PREDICTION: HYSTERESIS



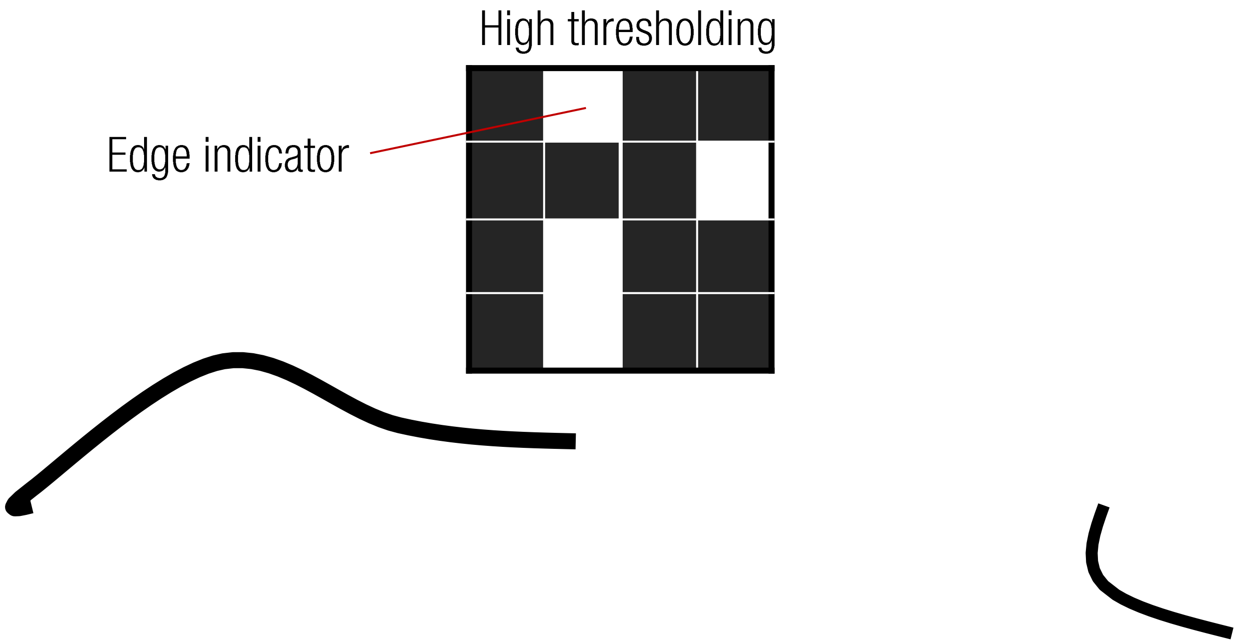
I



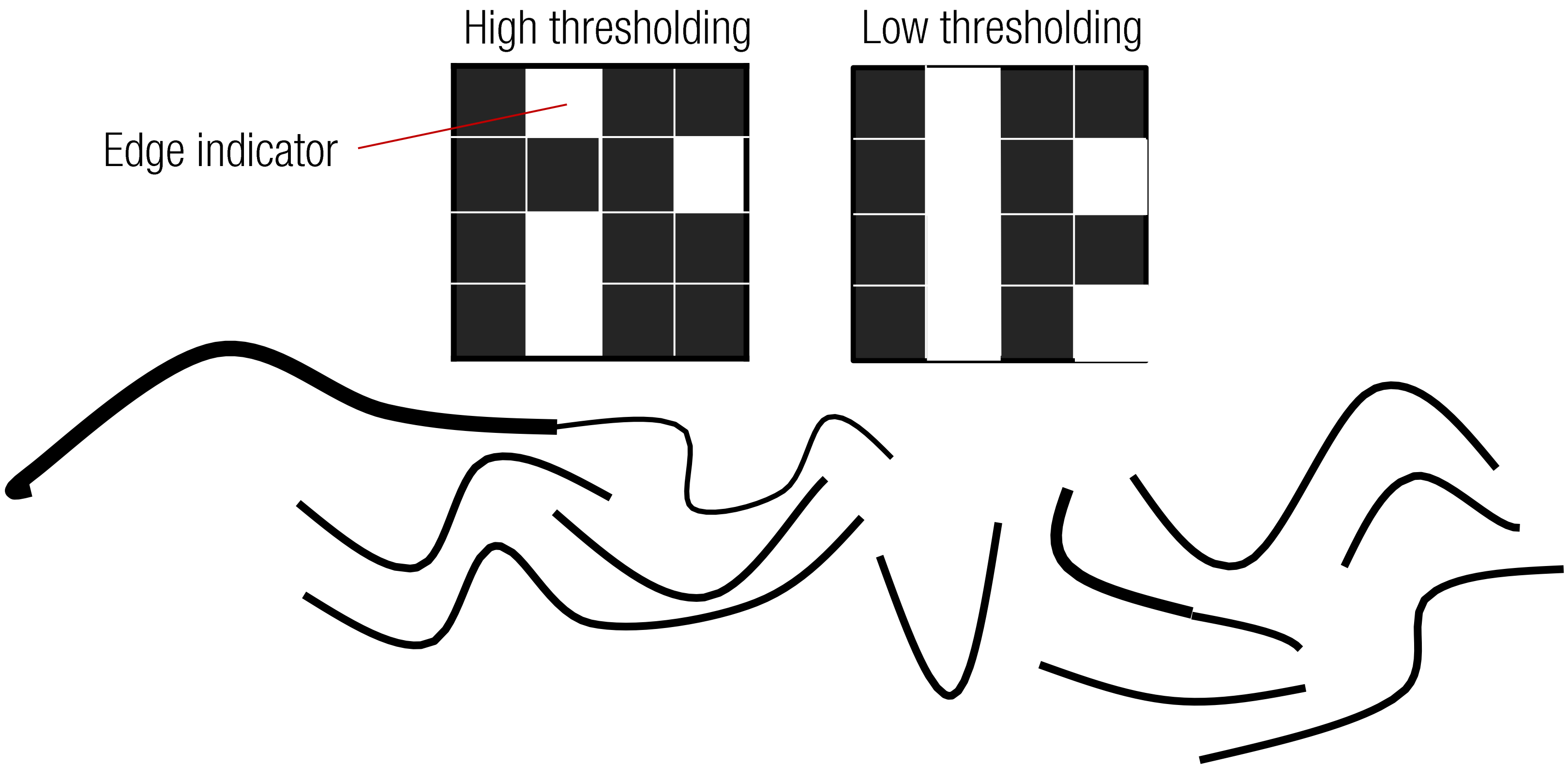
EDGE LINKING: HYSTERESIS



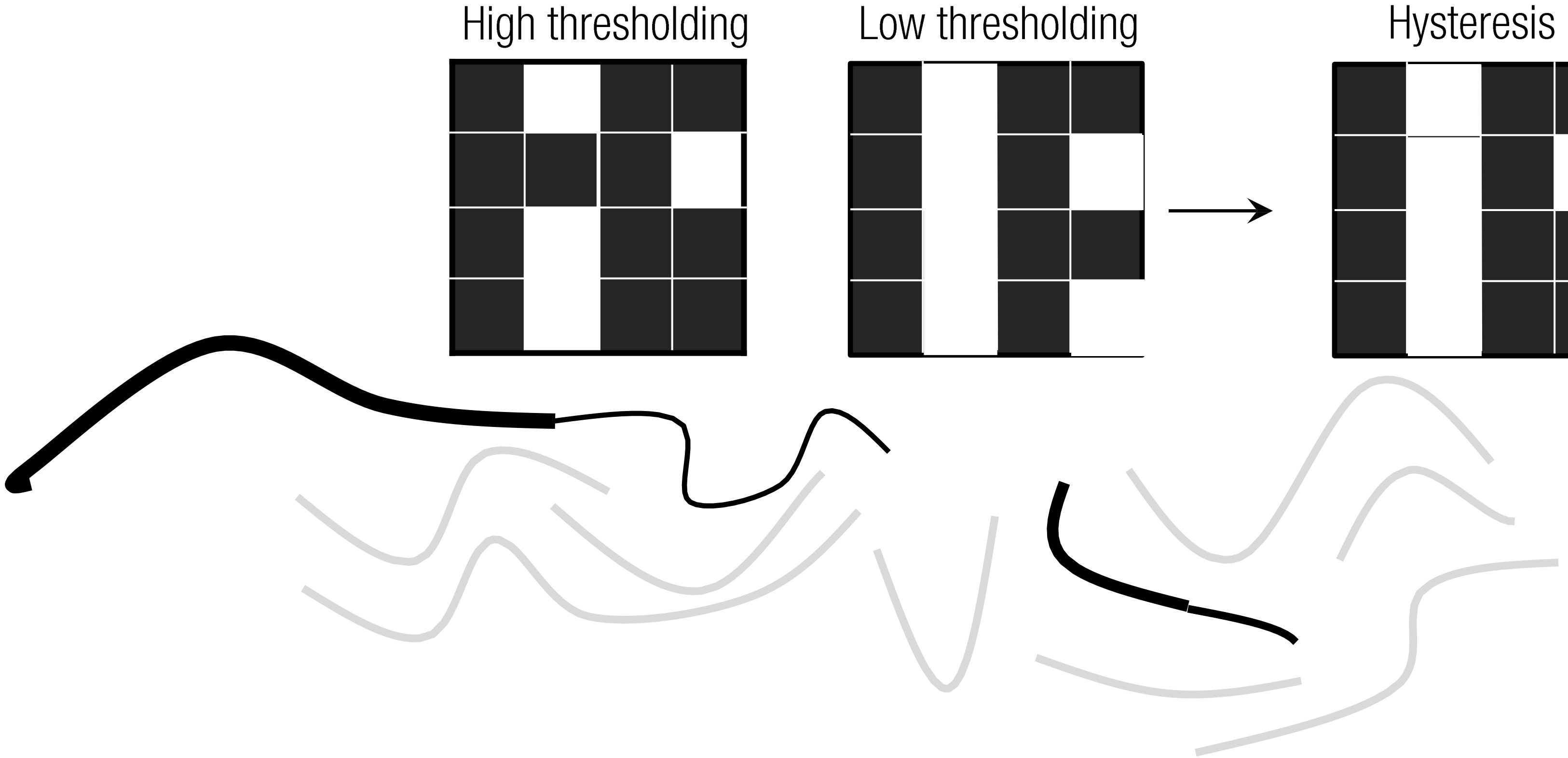
Edge Linking: Hysteresis



Edge Linking: Hysteresis



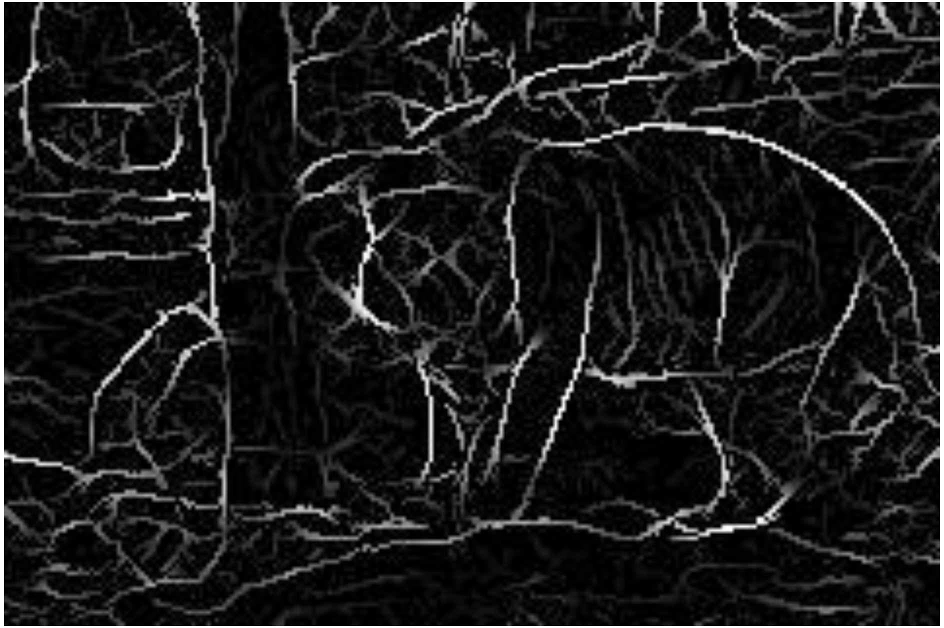
EDGE LINKING: HYSTERESIS



Localized edge

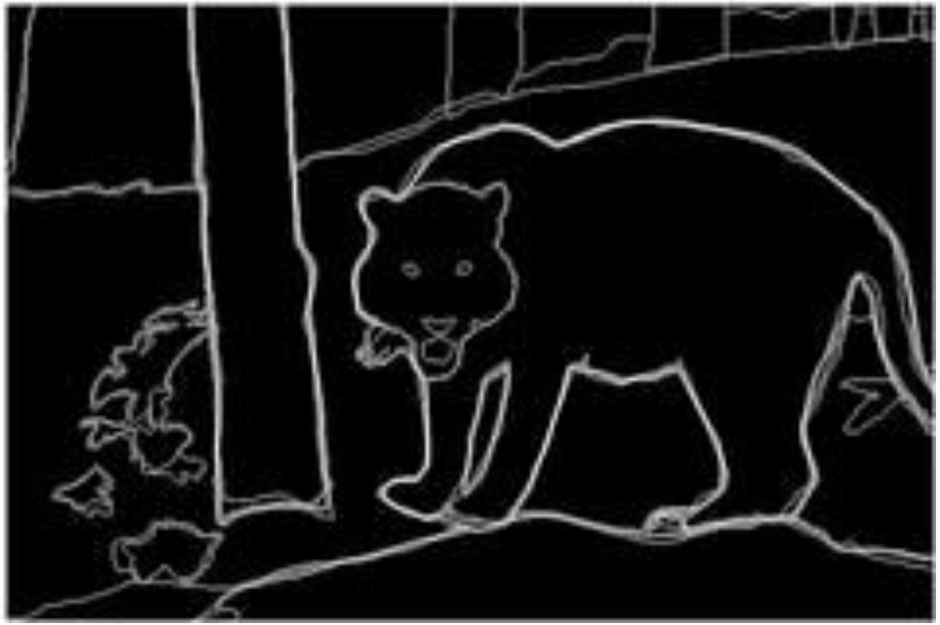
Edge linking

IS EDGE DETECTOR SOLVED?



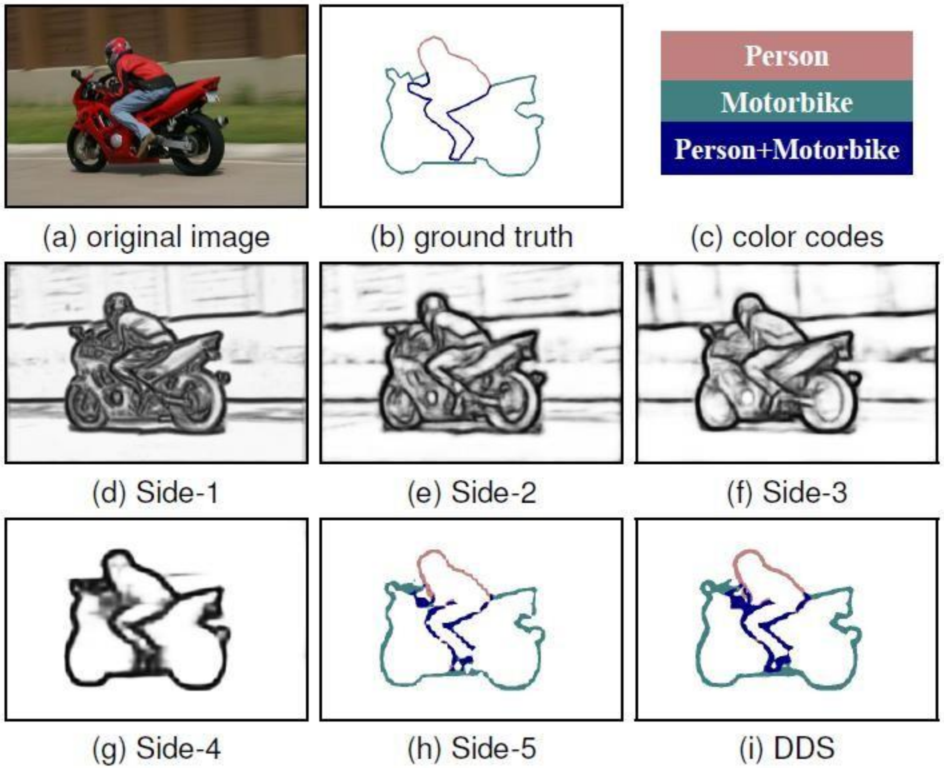
Canny edges

IS EDGE DETECTOR SOLVED?



Human perceived edges

SEMANTIC EDGES



Liu et al., “Semantic Edge Detection with Diverse Deep Supervision”, TPAMI (2018)

Liu et al., “Semantic Edge Detection with Diverse Deep Supervision”, TPAMI (2018)