Homework-3: Sigmoid Function and Its Derivative (EE655)

Aaryan Tiwari
Roll Number: 230023

Plot the sigmoid function and its derivative programmatically.

1 Proof of the Derivative of Sigmoid Function

The sigmoid function is defined as:

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

To differentiate, we use the quotient rule. Let:

$$S(x) = \sigma(x) = \frac{1}{1+e^{-x}}$$

Taking the derivative:

$$S'(x) = \frac{d}{dx}\left(\frac{1}{1+e^{-x}}\right)$$

Using the chain rule:

$$S'(x) = \frac{0(1+e^{-x}) - 1(-e^{-x})(-1)}{(1+e^{-x})^2}$$

$$S'(x) = \frac{e^{-x}}{(1+e^{-x})^2}$$

Since:

$$\sigma(x) = \frac{1}{1+e^{-x}}$$

Rewriting e^{-x} as:

$$e^{-x} = \frac{1}{\sigma(x)} - 1$$

$$S'(x) = \sigma(x)(1-\sigma(x))$$

Thus, we have proved that:

$$\frac{d}{dx}\sigma(x) = \sigma(x)(1-\sigma(x))$$

2 Python Code and Output Plot

Below is the Python code used to plot both the sigmoid function and its derivative:

```python
import numpy as np
import matplotlib.pyplot as plt

def sigmoid(x):
    return 1 / (1 + np.exp(-x))

def sigmoid_der(x):
    return sigmoid(x) * (1 - sigmoid(x))

x = np.linspace(-10, 10, 400)
y = sigmoid(x)
y_derivative = sigmoid_der(x)

plt.figure(figsize=(8, 5))
plt.plot(x, y, label="Sigmoid Function", color='blue')
plt.plot(x, y_derivative, label="Sigmoid Derivative", color='red', linestyle="dashed")
plt.axhline(0, color='black', linewidth=0.5)
plt.axvline(0, color='black', linewidth=0.5)
plt.legend()
plt.xlabel("x")
plt.ylabel("y")
plt.title("Sigmoid Function and Its Derivative")
plt.grid()
plt.show()
```

The output of the code is shown below: