ENGS 105 Problem 5

Hyperthermia, or elevated tissue temperature, has been proposed as a cancer therapy and one approach for inducing hyperthermia that is presently under investigation is microwave heating. A simple model of hyperthermic treatment can be developed by making a transverse slice through the body section containing the tumor. We wish to compute the heating rate and subsequent redistribution of thermal energy due to heat conduction and blood flow in the body. The patient anatomy to be considered is illustrated below in Figure 1.

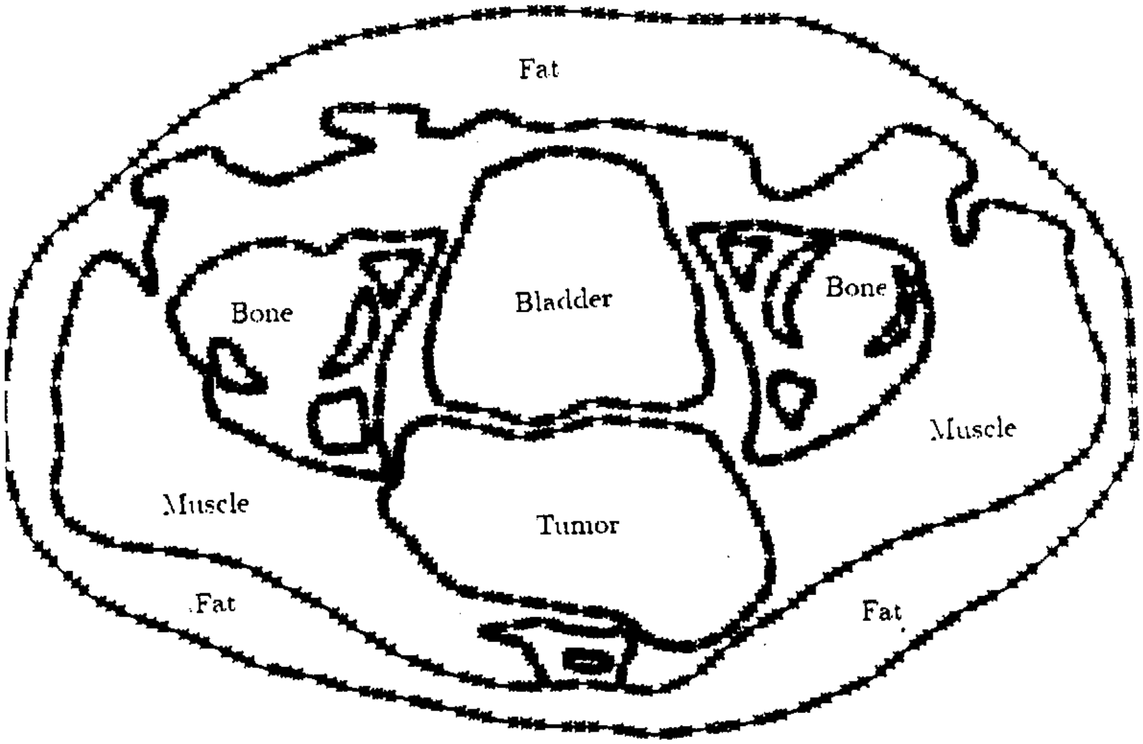


Figure 1. Patient cross-sectional geometry.

In the steady-state, heat transfer in tissue can be described by the "bioheat" equation

$$\nabla \cdot \kappa \nabla T - mT + \sigma |E|^2 = 0$$

where

κ is the thermal conductivity of tissue $(watt/m/^oC)$
m is the blood perfusion coefficient of tissue $(watt/m^3/^oC)$
$\sigma|E|^2$ is the heating rate distribution produced in the body $(watt/m^3)$
T is the scaled tissue temperature such that $T = 0$ is normal body temperature (^oC).

The boundary condition around the body surface is assumed to be of the form

$$-\kappa \nabla T \cdot \hat{\mathbf{n}} = h(T - T_a)$$

where

$\hat{\mathbf{n}}$ is the outward-pointing normal vector to the boundary
h is a heat transfer coefficient between the skin and the surrounding air ($watt/m^2/^oC$)
T_a is the ambient temperature of the surrounding air (^oC).

Your job is to solve this problem on the finite element patient model described below. Report contours of the steady-state temperature distribution reached through solution of the governing PDE (be sure to label at least some of the contour values. You may do this by hand if necessary). After having computed to solution to the "baseline" case described below, consider what happens if the blood flow in the bladder is increased to *one-half that of muscle.* Which produces a better hyperthermia treatment in terms of elevating tumor tissue above body temperature?
If you have difficulties developing your code, you can maximize your credit by obtaining correct solutions to simpler problems. Simplifications you may wish to consider would be to (i) make the boundary conditions Type I with $T = 0$, (ii) eliminate quadrilaterals from the mesh by dividing them into triangles.

Problem Notes :

The Grid: A finite element grid of the problem domain has been provided and can be copied from the course webpage under ES105 Library. Click on data/ and download the files described below. Four files are available for your use: (1) **npeltr4.dat** is the node file. Each line in the file has three entries – the global node #, the x-coordinate, and the y-coordinate; (2) **epeltr4.dat** is the element file. Each line has 6 entries – the element #, the global node # 's of the 4 nodes comprising the element, and a material property #; (3) **bpeltr4.dat** is the Type III boundary file. Each line contains 7 entries – an index, the global node # where a Type III condition is to be enforced, a "3" which indicates the boundary condition type is III, the two neighboring boundary nodes on either side of the specified node, the heat transfer coefficient h, and the ambient temperature T_a. (4) **ppelt4.dat** is the heating rate file. Each line contains 3 entries – an index, the element number, the heating rate (i.e. $\sigma|E|^2$) for that element. The heat generation distribution is assumed to be constant on a given element, but different in different elements. **Note that all the files contain commas which separate the various entries on each line in a particular file.**

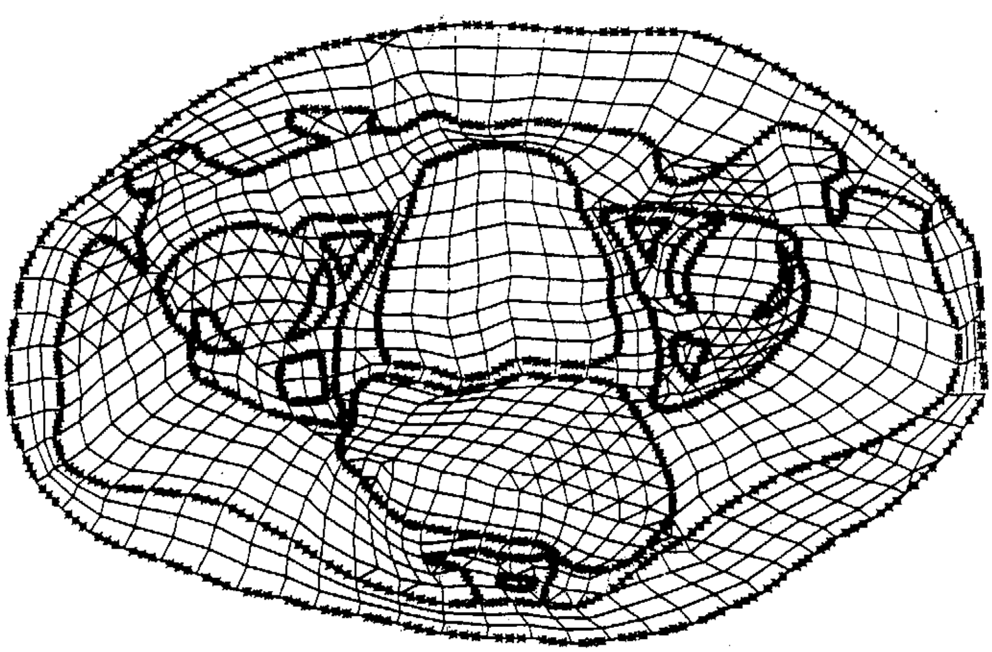
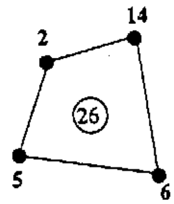
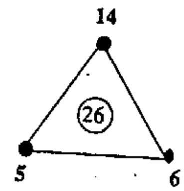


Figure 2. Finite element grid of the problem domain

The element file contains both triangular and quadrilateral elements (seë grid, Fig 2) below. The convention for distinguishing between the two types of elements is to make local node # 3 equal to local node # 4 for a triangle. For example, the two elements shown below would appear in the element file list as follows



Incidence list convention: 26. 5. 6. 14. 2



Incidence list convention: 26. 5. 6. 14. 14

This data structure provides an easy way to make a general code which can handle both types of elements. When you enter an element, decide which type it is and branch to one of two separate element matrix calculations, then return to a common global matrix assembly.

Tissue Properties: The tissue properties are constant within a given tissue type. The mesh is composed of 6 different regions, thermally, which can be identified in the element list by the element material property number. The following Table gives the material property number and the properties that change from region to region:

Tissue	Material#	κ	$c_t \times \rho_t$	m
Fat	3	0.210	$2.12E+6$	200
Muscle	4	0.642	$3.72E+6$	2001.4
Bone	5	0.436	$2.25E+6$	0.0
Bladder	6	0.561	$3.98E+6$	0.0
Marrow	7	0.515	$4.35E+6$	1482.5
Tumor	8	0.642	$3.72E+6$	0.0