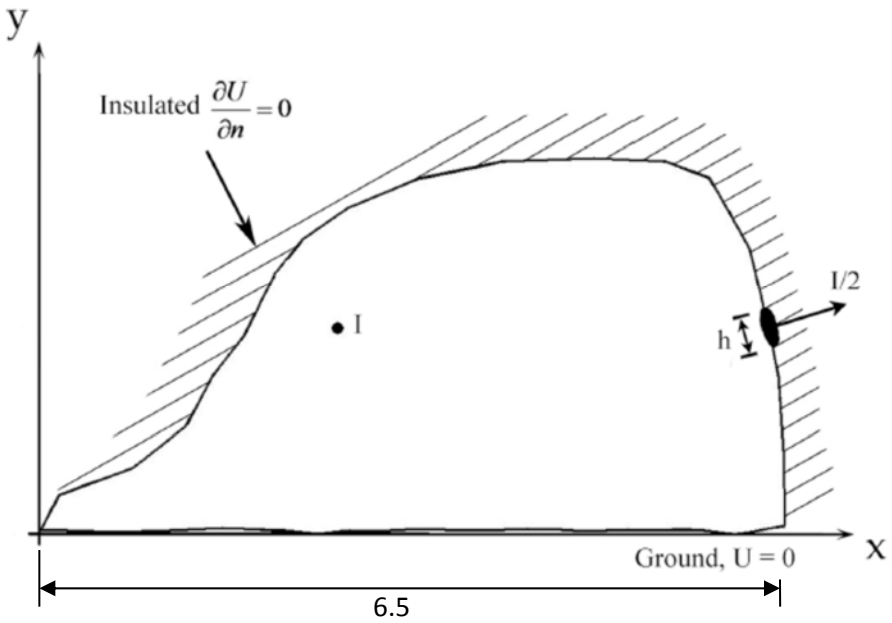
Problem 44 – Current in a plate.

A wire carrying current *I* is welded to a thin conducting plate of variable thickness. One-half of the current is removed through an electrode of length h attached to the boundary: this current is uniformly distributed along h. One edge of the plate is grounded as shown: the balance of the boundary is insulated. The governing PDE for the potential ϕ is

$$\nabla \cdot t\nabla\phi = -\frac{I}{\sigma}\delta(x_0, y_0),$$

where *t* is thickness and σ is conductivity.



Use the mesh 'hw44', details of the location of the wire and electrode are given in 'hw44.README'.

a) Sketch the solution, then solve for ϕ, use $\frac{I}{\sigma} = 1$ and $t = 1 + \frac{x}{10} + \frac{y}{5}$. Plot the equipotentials. Use the banded 'SOLVE' subroutine.

b) Compute and plot element values of the current vector by Ohm's law: $\frac{1}{\sigma} i = -\nabla\phi$
Optional: Compute and plot nodal values of *i* using a Galerkin approach.

c) Resolve part a using LU with LAPACK. Confirm that your answers are the same as above.

d) Compute the covariance of the potential solution given the following error models. Plot contours of the square root of the diagonals of the covariance matrix.
   1. Current Input I varies by 50%, plus or minus.
   2. Current Output I/2 varies by 40%, plus or minus.
   3. Ground potential varies by .5 volts, plus or minus, with three different correlation scale lengths: 0, 2, and 5. (Notice that the physical length scale of the ground, on the picture, is 6.5. Do *not* assume that the mesh coordinates in the file are reported in the same units.)
   4. Combine all three, assuming each source of variability is independent. In this composite plot, use the length scale 2 for the ground variability.