CS641

Modern Cryptology

Indian Institute of Technology, Kanpur

Due by: March 10, 2021 23:55 PM

Mid Semester Examination

Max Marks: 50

Instructions.

- Solutions should be mandatorily LaTeXed using the template shared and submitted through GradeScope before time. Mention Group Numbers and member names in solutions (refer template instructions).
- Clearly express solutions avoiding unnecessary details. Everything discussed in class is not required to be proved again. And anything non-trivial must be proved.
- Write the solutions on your own. Acknowledge the source wherever required. Keep in my mind department's Anti-Cheating Policy.

Question 1. (15 marks) Consider a variant of DES algorithm in which the S-box S1 is changed as follows:

> For every six bit input α, the following property holds: $\text{S1}(\alpha) = \text{S1}(\alpha \oplus 001100) \oplus 1111$.

All other S-boxes and operations remain the same. Design an algorithm to break four rounds of this variant. In order to get any credit, your algorithm must make use of the changed behavior of S1.

Question 2. (15 marks) The SUBSET-SUM problem is defined as follows:

> Given $(a_1, \ldots, a_n) \in \mathbb{Z}^n$ and $m \in \mathbb{Z}$, find $(b_1, \ldots, b_n) \in \{0, 1\}^n$ such that $\sum_{i=1}^{n} a_i b_i = m$ if it exists.

This problem is believed to be a hard-to-solve problem in general. Consider a hypothetical scenario where Anubha and Braj have access to a fast method of solving SUBSET-SUM problem. They use the following method to exchange a secret key of AES:

> Anubha generates an $n = 128$ bit secret key k. She then chooses n positive integers $a_1, \ldots, a_n$ such that $a_i > \sum_{1 \leq j < i} a_j$. She computes $m = \sum_{i=1}^{n} a_i k_i$ and sends $(a_1, a_2, \ldots, a_n, m)$ to Braj, where k_i is ith bit of k. Upon receiving numbers $(a_1, a_2, \ldots, a_n, m)$, Braj solves the SUBSET-SUM problem to extract the key k.

Show that an attacker Ela does not need to solve SUBSET-SUM problem to retrieve the key k from $(a_1, a_2, \ldots, a_n, m)$.

Question 3. (20 marks) Having falied to arrive at a secret key as above, Anubha and Braj try another method. Let G be the group of $n \times n$ invertible matrices over field F, $n = 128$. Let $a, b, g \in G$ such that $ab \neq ba$. The group G and the elements a, b, g are publicly known. Anubha and Braj wish to create a shared secret key as follows:

> Anubha chooses integers ℓ, m randomly with $1 < \ell, m \leq 2^n$, and sends $u = a^\ell g b^m$ to Braj. Braj chooses integers r, s randomly with $1 < r, s \leq 2^n$, and sends $v = a^r g b^s$ to Anubha. Anubha computes $k_a = a^\ell v b^m = a^{\ell+r} g b^{m+s}$. Braj computes $k_b = a^r u b^s = a^{\ell+r} g b^{m+s}$. The secret key is thus $k = k_a = k_b$.

Show that even this attempt fails as Ela can find k using u and v.

Hint: Show that Ela can

1. find elements x and y such that $xa = ax$, $yb = by$, and $u = xgy$,
2. use x, y, and v to compute k.