MATH-4101(3)-001 **Assignment 1** Due: Jan 20, 2022 (before class)

1. Express the following in the form of $a + ib \in \mathbb{C}$.

   (a) $(1-i)^4$;

   (b) $\frac{8-3i}{(4+3i)(3-21)}$;

   (c) $\left(\dfrac{1+i}{1-i}\right)^n$ for $n \in \mathbb{N}$;

   (d) $z + (1/z), z \neq 0, z \in \mathbb{C}$;

   (e) $\left(\frac{1+z}{1-z}\right)$ for $z \neq 1$.

2. Prove the parallelogram law:

$$|z+w|^2 + |z-w|^2 = 2(|z|^2 + |w|^2).$$

3. If $|\alpha| < 1$, show that $|z| \leq 1$ if and only if $\left|\dfrac{z-\alpha}{1-\bar{\alpha}z}\right| \leq 1$, with equality if and only if $|z| = 1$.

4. Prove that the equation of a circle in the complex plane is given by

$$z\bar{z} + a\bar{z} + \bar{a}z + b = 0, \qquad a \in \mathbb{C}, b \in \mathbb{R} \text{ and } |a|^2 > b.$$

   Hint: The circle with centre at α and radius r is described by $|z-\alpha| = r$. Draw a picture.

5. Give a geometric description of the set of points z satisfying the following equations or inequalities: (Draw pictures)

   (a) $\operatorname{Im} z > 0$

   (b) $|z-i| < |z+i|$

   (c) $1 < |z| < 2$

   (d) $|z-3+2i| < 5$

   (e) $\operatorname{Re}[(1+i)z] > 0$

   (f) $\operatorname{Re} z = |z-4|$

   (g) $|z|^2 > z + \bar{z}$