Assignment 3 - Mathematics-I
Departments: ECE, CSE, ENE
Pondicherry University

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This assignment covers the syllabus for our second internal: Sections 13.1 - 13.10 for the rest of Unit II and Sections 5.6 - 5.16 for Unit III (Textbook: Engineering Mathematics by Bali and Manish Goyal, 9th edition and Lecture notes for annihilator method)

1. Solve the following LDEs by annihilator method, or inverse differential method.
   (a) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} - 3y = x^2e^{3x} + \sin 2x.$
   (b) $(D^2 - 6D + 9)y = 25x^2e^{2x}\sin x$
2. Solve $(D^2 - 2D + 1)y = \dfrac{e^x}{x^2+1}$ using the method of variation of parameters.
3. Solve $(2x+1)^2\dfrac{d^2y}{dx^2} - 6(2x+1)\dfrac{dy}{dx} + 16y = 4\cos\log(1+2x).$
4. If $u = x^2\tan^{-1}\dfrac{y}{x} - y^2\tan^{-1}\dfrac{x}{y}$, show that $\frac{\partial^2 u}{\partial x\partial y} = \frac{x^2-y^2}{x^2+y^2}$.
5. If $x = e^u\tan v, y = e^u\sec v$, find the value of $\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right)\cdot\left(x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}\right)$
   (Hint: express u and v or a function of u or v as a homogenous function in x and y.)
6. If $u = \tan^{-1}(\frac{y}{x^2})$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 2u\cos^2 u.$
7. Let $u = f(x,y)$ and $x = x(s,t) = \frac{1}{2}s(e^t + e^{-t}), y = \frac{1}{2}s(e^t - e^{-t})$. Show that
$$u_{xx} - u_{yy} = u_{ss} + \frac{1}{s}u_s - \frac{1}{s^2}u_{tt}.$$
8. Expand $e^x\cos y$ at $(0,0)$ using Taylor's theorem.