Assignment 1 - BSMT 331 - Mathematical Modelling
Integrated M.Sc (Math) - Pondicherry University

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1. A dairy farmer owns a cow that just calved. The cow's daily milk yield declines linearly over the lactation period:

   - At $t = 0$ (just after peak), the cow produces 12 litres per day.
   - Milk production decreases steadily to 0 liters per day on $t = 300$ days.

   The farmer sells milk at a price p (₹/litre), which fluctates seasonally due to market and weather conditions. The daily feeding and maintenance cost is ₹250 per day.

   (a) Write the milk yield function $m(t)$ (litres/day) and the farmer's profit function $P(t)$ in which the latter would contain the parameter p. (Hint: You have to find the total milk produced up to time t for the revenue calculation)

   (b) When should the farmer stop keeping the cow in milk production to maximize profit. Denote it by $t^*(p)$.

   (c) Suppose milk prices fluctuate as follows:

Situation	Milk price p (₹/litre)
Normal Market	35
Summer drought	40
Monsoon surplus	30

   Compute $t^*(p)$ for each scenario.

   (d) Conduct a sensitivity analysis on this parameter p. Consider both the optimal time to stop and the resulting profit when the market is normal.

   Now, additionally:

   - The government gives a milk subsidy of ₹2 per litre.
   - The farmer considers the time value of money, discounting future revenue at an annual interest rate of 5% (assume 365 days per year, continuous discounting).

   Without discounting: Total revenue = Price × total litres. With discounting: revenue from milk on day t is reduced by factor e^{-rt}. So,

   $$\text{price value of milk at day } t = (p + \text{subsidy}) \cdot m(t) \cdot e^{-rt}.$$

   (e) Recalculate the optimal stopping time $t^*(p)$ and perform a sensitivity analysis on the subsidy. Take $p = 35$.

2. (a) Find the maximum and minimum of $f(x, y, z) = x^3 + y^3 + z^3$ subject to the constraints $x^2 + y^2 + z^2 = 1$ and $x + y + z = 1$

   (b) Let $f(x, y) = x^2 + 2x + y^2$. Find the minimum value of f on $|x| + |y| = 4$ by Lagrange multipliers. Why can't this method be used to find the maximum value of f?

   (c) Let $A : \mathbb{R}^n \to \mathbb{R}^m$ be a linear map. Show that the function $f(x) = \|Ax\|$ has a maximum value on the unit sphere $S = \{x \in \mathbb{R}^n : \|x\| = 1\}$.

3. The Department of Fisheries, Government of Puducherry, is studying the sustainability of a sardine fishery operating off the coast of Pondicherry (near Veerampattinam and Ariyankuppam).

   Marine scientists estimate that in the absence of fishing, the fish biomass x (measured in tonnes) evolves according to the logistic growth:

$$\frac{dx}{dt} = 0.4x\left(1 - \frac{x}{1000}\right)$$

   where:

   - x : fish biomass (tonnes)
   - 0.4 is the intrinsic growth rate per year
   - 1000 tonnes is the carrying capacity of the coastal ecosystem.

   Fishing is introduced with harvesting effort E, and catch is proportional to both population and effort:

$$H = qEx$$

   where:

   - E : fishing effort (measured in boat-days per year)
   - $q = 0.01$ is the catchability co-efficient

   Fish are sold in the Pondicherry market at ₹1 lakh per tonne. The cost of operating fishing boats is ₹8 lakh per boat-day.

   Thus the full model becomes:

$$\frac{dx}{dt} = 0.4x\left(1 - \frac{x}{1000}\right) - 0.01Ex$$

   The Fisheries Department wants to determine:

   - The long-run fish stock level x
   - The fishing Effort E

   so as to maximize sustainable annual profit, subject to the condition that the fish population remains constant over time (i.e., biological growth equals harvest).

   (a) Formulate the problem as a constrained optimisation problem in the variables x and E.

   (b) Solve for the profit-maximising stock level and fishing effort.

   (c) Compare your answer with the stock level that maximizes biological growth alone (Maximum Sustainable Yield).

   (d) Intrepret the Lagrange multiplier economically in the context of Ponicherry's fisheries policy.