1. Discuss the convergence of the sequences briefly whose n-th terms are:

   (a) i^n/n^2

   (b) $i(n!/n^n)$

   (c) $(-1)^n + i(n+1)/n$

   (d) $(1 + \frac{4}{n})^n + 7ni/(n+4)$

   (e) $\cos n\pi + i$

   (f) $z^n, |z| < 1$ (Assume $\lim_n a^n = 0$ for real $0 < a < 1$)

2. If $\lim_n z_n = z$, then show that $\lim_n |z_n| = |z|$. Is the converse true?

3. Find which of the following series are absolutely convergent:

   (a) $\sum\limits_{n=1}^{\infty} i^n/n^2$

   (b) $\sum\limits_{n=1}^{\infty} \frac{(-1)^{n+1} \log n}{n}$

   (c) $\sum\limits_{n=1}^{\infty} \frac{(1+i)^n}{n}$

   (d) $\sum\limits_{n=1}^{\infty} (n+1) \frac{(1+i)^n}{n!}$

4. Determine all $z \in \mathbb{C}$ for which the series are absolutely convergent:

   (a) $\sum\limits_{n=1}^{\infty} \frac{z^n}{n^2}$

   (b) $\sum\limits_{n=1}^{\infty} \frac{1}{2^n} \left(\frac{1}{z+1} \right)^n$

   (c) $\sum\limits_{n=1}^{\infty} \frac{1}{n!} \left(\frac{1}{z} \right)^n$

   (d) $\sum\limits_{n=1}^{\infty} \left(\frac{z-1}{z+1} \right)^n$

5. Recall the fact about the multiplication of convergent series mentioned in the class. Given two series $\sum_n a_n$ and $\sum_n b_n$, their *Cauchy product* is a series $\sum_n c_n$ where $c_n := \sum_{k=0}^{n} a_k b_{n-k}$. It is motivated by the product of polynomials and power series. For instance, if we let $p(z) := \sum_{k=0}^{m} a_k z^k$ and $q(z) := \sum_{k=0}^{n} b_k z^k$, then the product of polynomials is given by $pq(z) = \sum_{r=0}^{m+n} c_r z^r$, where $c_r := \sum_{k+l=r} a_k b_l$.

   (a) Show that the Cauchy product of the series for $\exp(z)$ and $\exp(w)$ converges to $\exp(z + w)$ for $z, w \in \mathbb{C}$. (The word you are searching for is Binomial theorem)

(b) Deduce that $\left(\sum_{n=0}^{\infty} \frac{z^n}{n!}\right)\left(\sum_{n=0}^{\infty} \frac{(-z)^n}{n!}\right) = 1$ for all z. In particular, $\exp(z) \neq 0$ and $\exp(z)^{-1} = \exp(-z)$ for any $z \in \mathbb{C}$.

(c) Show that $\exp(\bar{z}) = \overline{\exp(z)}$. Deduce that if x is real then $|\exp(ix)| = 1$.