MATH-4101(3)-001 **Assignment 2** Due: Jan 27, 2022 (before class)

1. Simplify
   (a) $(1+i\sqrt{3})^{99}$
   (b) $(-\sqrt{3}+3i)^{31}$

2. (a) Prove that for $z \in \mathbb{C}, z \neq 1$ we have $1+z+\cdots+z^n = \dfrac{1-z^{n+1}}{1-z}$, for any $n \in \mathbb{N}$. Hence deduce that

$$\frac{\alpha^n-\beta^n}{\alpha-\beta} = \alpha^{n-1}+\alpha^{n-2}\beta+\cdots+\alpha\beta^{n-2}+\beta^{n-1}, \qquad \text{for } \alpha \neq \beta.$$

   Hint: Take $z=\alpha/\beta$ in the first part, if $\beta \neq 0$. If $\beta = 0$, the result is obvious.

   (b) Use part a) to show that

$$\sum_{k=0}^{n} \cos(k\theta) = \frac{1}{2}+\frac{\sin(n+\frac{1}{2}\theta)}{2\sin(\theta/2)} \quad \& \quad \sum_{k=0}^{n} \sin(k\theta) = \frac{\cos(\theta/2)-\cos(\frac{n+1}{2}\theta)}{2\sin(\theta/2)}.$$

   (c) Let $\omega := \operatorname{cis}(2\pi/n)$ for an integer $n>1$. Show that $\sum_{k=1}^{n}\omega^k = 0$. Why is this geometrically clear if n is even?

   Observe that the n-th roots of unity other than 1 satisfy the cyclotomic equation $z^{n-1}+z^{n-2}+\cdots+z+1=0$.

3. (a) Solve the equation $z^3+8i=0$, where $z \in \mathbb{C}$.
   (b) Find all cube roots of $\sqrt{3}+i$.