

1 Teaching Statement

1.1 Teaching Philosophy

Tell me, and I forget. Teach me, and I remember. Involve me, and I learn.

This often-quoted saying reminds us that education is not just about transferring information—it's about transforming experience. As educators, we hold the responsibility and privilege to shape how students first encounter knowledge. These moments are formative, especially when the learning environment recognizes and affirms the diverse identities students bring into the room.

My aim is to make mathematical ideas come alive—not simply as technical content, but as a space where students feel intellectually and personally included. I want the teaching materials to become transparent, allowing students to see and feel mathematics, to find wonder beyond the symbols and syntax, and to connect to the subject on their own terms. I strive to foster learning environments where all students—regardless of their prior preparation, cultural background, race, gender identity, or ability—can experience mathematical thinking as an act of agency, creativity, and belonging.

Wonder, when cultivated intentionally, becomes a quiet mode of resistance against the alienation that students from marginalized communities often feel in formal STEM spaces. By honoring the questions students carry with them—especially those shaped by different ways of seeing the world—I encourage them to participate as co-creators in the learning process. The personal connection students make with abstract or quantitative ideas is not a luxury; it is the foundation of meaningful, enduring, and just learning.

I believe building trust precedes building understanding. To this end, I prioritize kindness over cleverness. I work to dismantle perceived hierarchies in the classroom by fostering relationships rooted in respect, openness, and authenticity. In the first weeks of class, I make it a point to engage students in low-stakes discussions that draw from their prior knowledge and cultural touchpoints. These conversations often reveal the richness of their thinking and set the tone for an environment where mistakes are welcomed as part of intellectual risk-taking.

Incorporating organized board work, structured lectures, accessibility outside of class, well-selected assignments, reassuring words during exams, and lively discussions about mathematical concepts can greatly motivate and boost students' confidence in using mathematics effectively. Teaching, for me, is an evolving practice grounded in empathy and justice. I am committed to refining my pedagogy to confront and counteract educational inequities. It is my hope that students leave my classes not only with conceptual clarity but also with the confidence that their voice matters in mathematics and beyond.

1.2 Teaching Practices

1.2.1 In the Classroom

Setting the Stage for Inclusion

I approach my first classes with a sense of curiosity and care. I recognize that students bring with them a rich tapestry of identities—linguistic, cultural, racial, neurodivergent, and more—and that these shape how they experience mathematics. My early interactions are designed to create a psychologically safe and socially welcoming space. I use low-barrier engagement techniques such as conversational games or real-world analogies that resonate across backgrounds, involving 70–80% of the class. I emphasize that educators are bridges—not barriers—to learning. This not only builds rapport but signals to students from historically excluded groups that their presence and participation are valued from the outset.

I also send a welcome email or announcement that includes not just the course theme, but also accessibility details, land acknowledgments (where applicable), and my commitment to inclusive and respectful dialogue. By acknowledging the broader socio-educational context of our classroom, I invite students to see themselves as active participants in a shared learning community.

Grounding Abstractions in Experience

I use examples that are carefully selected not only for conceptual clarity but also for cultural relevance. In my Applied Math for Business and Administration course, I adapted financial examples such as TFSA and RRSP to reflect local Canadian contexts. I now extend this practice to include community-based or historically underrepresented economic narratives—such as cooperatives, microfinance initiatives, or informal economies—that better reflect the lived experiences of a broader range of students.

In my Complex Analysis course, we developed metaphors like the “cardamom pod” or “cinnamon bark” trick—accessible, culturally textured analogies that help students extract insight from dense theoretical material. These metaphors were memorable not because they were whimsical, but because they gave students culturally

resonant footholds in abstract terrain. Students across varied mathematical backgrounds appreciated and reused these tools, demonstrating how inclusive language can support deep learning.

Strategies for Inclusive and Meaningful Learning

Schema Theory - Honoring What Students Already Know: I begin by asking, “What does this remind you of?”—not as a rhetorical gesture, but as a way of validating the diverse intellectual and cultural schemas students bring to the classroom. This helps demystify the subject and makes students feel their existing knowledge matters. These small gestures can be powerful for students from underrepresented or non-traditional academic paths.

Zone of Proximal Development - Encouraging Collaborative Growth: I position myself as a co-thinker and guide, making room for students to articulate partial understandings, question dominant solutions, or construct alternate pathways. For students who feel alienated by formal mathematics language, this approach legitimizes their ways of reasoning and helps them build confidence from where they are.

Social Constructivism - Valuing All Voices: Embracing the principles of social constructivism and inclusive pedagogy, I promote collaborative learning by creating structured opportunities where all students—including those who are introverted, multilingual, or from historically excluded groups—can engage meaningfully (tools like Mentimeter). I remain mindful of participation dynamics and encourage contributions in varied forms (spoken, written, visual), ensuring equitable space for voices that may be marginalized in traditional discussions.

Scaffolding - Supporting Varied Learning Journeys: I follow a progressive release of responsibility (“I do, we do, you do together, you do alone”), while offering differentiated supports like optional guided notes, varied problem formats, and targeted challenges. These practices aim to remove structural barriers without compromising intellectual rigor.

Heuristics and History: I draw on historical and cross-cultural narratives in mathematics, such as the development of continuity or number systems across civilizations. When we discuss foundational definitions—like Bolzano’s $\varepsilon - \delta$ formulation—I include references to lesser-known contributors or prior cultural techniques. This not only deepens conceptual understanding but also helps decolonize students’ view of where mathematics comes from.

1.2.2 Outside the classroom

Accessible Support and Dialogue

I hold office hours not just as a time for clarification but as a space for relational learning. I check in with students on how they’re doing, ask how the material connects with their personal or disciplinary interests, and use those moments to adjust my instruction. I make it clear that needing help is not a deficit but a natural and necessary part of learning. To support accessibility, I offer multiple formats for support—written clarifications, group sessions, or one-on-one check-ins—and ensure that students from different time zones or caregiving responsibilities are accommodated.

Equitable and Transparent Evaluation

Assessment is where inequities can be most deeply felt, so I design assignments that progressively build on skills and allow students multiple modes of demonstration. I use clear rubrics, scaffolded problem-solving steps, and revision opportunities to support equity of outcome. Where platforms like WebAssign are used, I ensure they don’t assume uniform digital access or speed. In feedback, I aim for comments that validate effort and offer concrete, compassionate suggestions for improvement—particularly for students whose experiences of education have included marginalization or systemic bias.

Inclusive Lecture Materials

I create lecture notes that speak to different learning styles—using diagrams, layered examples, and exploratory prompts. When recording or sharing notes, I use accessible formats (e.g., readable PDFs, captions) and offer summaries for students with processing difficulties. These notes are not static; they evolve with classroom dynamics and include questions raised by students, reflections on disciplinary relevance, and cross-field connections. In one diverse Complex Analysis course, the mix of students—across year levels and programs—necessitated adaptive content. I curated and supplemented our readings to meet their collective needs, allowing flexibility while maintaining mathematical depth.

1.3 Professional Development

Pedagogical Training and Reflective Growth

In Winter 2019, I participated in the Graduate Student Instructor Training Seminar at the University of Waterloo—an intensive, term-long engagement focused on post-secondary mathematics pedagogy. We examined best practices in evaluating student-written proofs, evidence-based approaches to active learning, and strategies for fostering metacognition among math students. The seminar's format included formal presentations, critical readings, and collaborative roundtable discussions, all of which encouraged deep reflection on the broader aims of teaching beyond content delivery.

A transformative component of this seminar was its practical emphasis: participants delivered three mini-lectures simulating the beginning stages of a course, with peer and faculty observation and feedback. These sessions not only offered constructive criticism on technical delivery but also insights into inclusive communication, student engagement, and classroom climate. Through this process, I became acutely aware of how minor adjustments in tone, pacing, or framing can significantly impact students' sense of belonging and intellectual safety.

Micro-credentials at Conestoga College

In 2023, I completed three micro-credentials through Conestoga College's Teaching and Learning department, each of which contributed meaningfully to my inclusive teaching practice.

- **Understanding Outcomes-Based Education (OBE) and Curriculum** This course focused on aligning learning outcomes with backward design and measurable assessments. I engaged in a course outline quality audit using Bloom's Taxonomy, but also approached outcomes with a lens toward *accessibility and fairness*. I reflected on how traditional course objectives can unintentionally obscure the learning needs of multilingual or first-generation students, and revised them for clarity, cultural neutrality, and transparency.
- **Active Learning leading to assessment** This module emphasized classroom management and assessment in diverse environments. We explored the didactic triangle, Schema theory, Zone of Proximal Development, and intercultural sensitivity strategies. I paid close attention to ways in which class participation can be broadened to include multiple communication styles and where formative feedback can act as a corrective against systemic disparities. We also discussed the importance of academic integrity without punitive assumptions, particularly for students navigating unfamiliar academic norms.
- **Conestoga's Learning Management System** This credential trained me in designing and communicating course expectations in a clear, accessible, and inclusive manner using the eConestoga platform. I learned to implement the Essential Elements checklist not just as a compliance tool but as a means to ensure digital equity—for example, by ensuring content is screen-reader compatible, using plain language instructions, and providing asynchronous access wherever possible.

Ongoing Commitment to Inclusive Mathematics Education

These professional development experiences have reshaped my teaching as a dynamic, responsive, and ethically grounded practice. They have made me increasingly aware of how educational structures—ranging from syllabi to evaluation tools—can either reinforce or disrupt existing inequities. As such, I actively seek to refine my approach using resources from communities focused on anti-racist, decolonial, and inclusive STEM education. I am committed to ongoing reflection and growth in this area, recognizing that inclusive teaching is not a fixed endpoint but an evolving journey that must respond to new challenges, perspectives, and student voices.

1.4 Piazza demonstration

A demonstration of guiding a student in an online discussion platform called Piazza.

The screenshot shows a Piazza question titled "Question about Taylor's theorem". The question asks, "How do we get the formula for Taylor's theorem??". It has a status of "good question" and 0 answers. Below the question, there is a note: "the instructors' answer, where instructors collectively construct a single answer". A detailed answer is provided: "Fix $x \in I$ such that $x \neq a$. We are concerned about finding a formula for $R_{n,a}(x) = f(x) - T_{n,a}(x)$. The following calculation is completely in spirit with how we figured out Taylor's polynomials in the first place. Remember again, that our a and x are fixed values. Now create a new polynomial $p(t)$ such that $p(a) = f(a)$, $p^{(k)}(a) = f^{(k)}(a)$ for $1 \leq k \leq n$, and $p(x) = f(x)$. To find this polynomial, start a follow-up discussion. We will continue from there. The proof of this is quite involving, so in order to understand this, we will have to do quite an amount of work!" The answer has 53 views and was updated 6 hours ago by [redacted] and [redacted].

followup discussions, for lingering questions and comments

Resolved Unresolved [@808_f1](#)

(Anon. Scale to classmates) 1 day ago
isn't it Taylor polynomials $T_{k,a}(x)$, where $1 \leq k \leq n$?
helpful! | 0

1 day ago
Yes, up until n^{th} -degree, $p(t)$ should retain the n^{th} -degree Taylor polynomial. So $p(t) = T_{n,a}(t) + \text{something}$. Now furthermore, we want $p(t)$ to satisfy $p(x) = f(x)$ and this something shouldn't disturb what we already have $p(a) = f(a)$, $p^{(k)}(a) = f^{(k)}(a)$ for $1 \leq k \leq n$. So can you guess what should be the form of this something?
good comment | 0

Anon. Scale to classmates) 1 day ago
something will be the minus $T_{k,a}$ where $k=n-1$, is that correct?
helpful! | 0

24 hours ago
No, that's incorrect. What should this something be if we want $p(a) = f(a)$? Note that we already have $T_{n,a}(a) = f(a)$.
good comment | 0

Anon. Scale to classmates) 24 hours ago
wait, I'm confused, isn't $p(a)=f(a)$ already? in the first question I answered?
Actions ▾

24 hours ago
We have got only part of $p(t)$, that part is $T_{n,a}(t)$. Unfortunately, at our fixed x , $T_{n,a}(x) \neq f(x)$. So we need to add something more to our $p(t)$. This is ongoing. We haven't figured out $p(t)$ completely yet.
So far, our $p(t) = f(a) + f^{(1)}(a)(t-a) + \frac{f^{(2)}(a)}{2!}(t-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(t-a)^n + \text{something}(t)$, right? So if we want $p(a) = f(a)$, what should this something be at a ?
good comment | 0

23 hours ago
 a and x are fixed. For the sake of clarity, we want our one particular polynomial $p(t)$ to satisfy all the following conditions: $p(a) = f(a)$, $p^{(k)}(a) = f^{(k)}(a)$ for $1 \leq k \leq n$, and $p(x) = f(x)$.
good comment | 0

Anon. Scale to classmates) 22 hours ago
if $p(t) = f(a) + f^{(1)}(a)(t-a) + \dots + \text{something}(t)$, then if we sub $t = a$, $p(a) = f(a)$ since $(a-a) = 0$, do we need $\text{something}(t)$ in this case?
helpful! | 0

22 hours ago
It's $p(a) = f(a) + \text{something}$ at a . So this something has to vanish at $t = a$. If that's the case, what form should it be?
good comment | 0

Anon. Scale to classmates) 21 hours ago
it has to include $(t-a)$
helpful! | 0

21 hours ago
Yes, now, for any constant $K \in \mathbb{R}$, consider $p(t) = T_{n,a}(t) + K(t-a)^{n+1}$. That something must be have a factor of atleast $(n+1)$ -th power of $(t-a)$, otherwise, it would disturb the conditions: $p(a) = f(a)$, $p^{(k)}(a) = f^{(k)}(a)$ for $1 \leq k \leq n$. (Let me know if this doesn't make sense)!
Now we only need that $p(t)$ to satisfy $p(x) = f(x)$. So what should be K such that this is guaranteed?
good comment | 0

Anon. Scale to classmates) 21 hours ago
I think I get the first part. Is like if we want $p^{(n)}(a) = f^{(n)}(a)$, we need to have $K(t-a)^{n+1}$ to make sure it won't disturb $p^{(K)}(a) = f^{(k)}(a)$, for $1 < k < n$, because $(a-a)=0$, thus it won't affect. But does that mean $n+2, n+3, \dots$ will also work?
helpful! | 0

21 hours ago
Yes, $n+2, n+3, \dots$ will also work! But our hypothesis is that f is $n+1$ -times differentiable. Find K . We are almost there to find a function to which we would apply Rolle's theorem or MVT.
good comment | 0

Anon. Scale to classmates) 21 hours ago
is it $\frac{f^{(n+1)}(t)}{(n+1)!}$?
helpful! | 0

20 hours ago
Substitute $t = x$ in $p(t) = T_{n,a}(t) + K(t-a)^{n+1}$ and equate it to $f(x)$. Get an expression for K . You will understand why we fix x such that $x \neq a$.
good comment | 0

Anon. Scale to classmates) 20 hours ago
because if $x=a$, $(x-a)^{n+1} = 0$ and we can't have 0 in denominator,
helpful! | 0

20 hours ago
yes, so what is K ?
good comment | 0

Anon. Scale to classmates) 20 hours ago
$$\frac{f(x) - T_{n,a}(x)}{(x-a)^{n+1}}$$

helpful! | 0

 20 hours ago
Excellent! All of this above struggle is to put that Taylor's remainder $R_{n,a}(x) = f(x) - T_{n,a}(x)$ as a coefficient of a polynomial because one of the ways to get to a coefficient of a polynomial is through their derivatives of appropriate order!

So our $p(t) = T_{n,a}(t) + \frac{R_{n,a}(x)}{(x-a)^{n+1}}(t-a)^{n+1}$ satisfies $p(a) = f(a)$ and $p(x) = f(x)$. WLOG, if we assume $x > a$, then if we consider $g(t) = f(t) - p(t)$ on the interval $[a, x]$, then $g(a) = g(x) = 0$. So what do we get by Rolle's theorem? Furthermore, what is $g^{(k)}(a)$ for $1 \leq k \leq n$?

[good comment](#) | 0

 (Anon. Scale to classmates) 20 hours ago
by Rolle's theorem there's $c \in [a,x]$ s.t $g'(c)=0$.

Since we assume $f^{(k)}(a) = p^{(k)}(a)$
 $g^{(k)}(a) = 0$

I don't think I understand the connection between Rolle's theorem and $g^{(k)}(a)$

[helpful!](#) | 0

 20 hours ago
Okay, instead of c , I will call it c_1 . So we get $c_1 \in (a, x)$ such that $g^{(1)}(c_1) = 0$. Now consider $g^{(1)}(t)$ on the interval $[a, c_1]$. As per our construction of $p(t)$, $g^{(1)}(a) = g^{(1)}(c_1) = 0$. Furthermore, $g^{(1)}$ is continuous on $[a, c_1]$ and differentiable on (a, c_1) . So, again by Rolle's theorem, what do we get? If we keep continuing, what do we get?

[good comment](#) | 0

 (Anon. Scale to classmates) 20 hours ago
we will get $g^{(k)}(c_k) = 0$ for $1 < k < n$

[helpful!](#) | 0

 20 hours ago
Wonderful. Our ultimate step is here: Now $g^{(n)}$ is continuous on $[a, c_n]$ and differentiable on (a, c_n) . So we get a $c \in (a, c_n) \subset (a, x)$ such that $g^{(n+1)}(c) = 0$.

Now, compute $g^{(n+1)}(c)$ in terms of $f^{(n+1)}$ and $p^{(n+1)}$, and let me know what do you get.

[good comment](#) | 0

 (Anon. Scale to classmates) 18 hours ago
as we defined $g(t) = f(t) - p(t)$, $g^{(n+1)}(c) = f^{(n+1)}(c) + g^{(n+1)}(c)$

[helpful!](#) | 0

 18 hours ago
Not right! $g^{(n+1)}(c) = f^{(n+1)}(c) - p^{(n+1)}(c)$! Now, you need to just compute $p^{(n+1)}(c)$ and equate $f^{(n+1)}(c) - p^{(n+1)}(c)$ to 0!

[good comment](#) | 0

 17 hours ago
It may be useful to know what is m^{th} order derivative of a polynomial of degree m ?

[good comment](#) | 0

 (Anon. Scale to classmates) 17 hours ago
my bad, that's the typo $g^{(n+1)}(c) = \dots$

$p^{(n+1)}(c) = 0 = K?$

[helpful!](#) | 0

 7 hours ago
It is not equal to zero and it is not just equal to K . You are missing a constant! I edited the previous comment for possible clarification.

[good comment](#) | 0

 (Anon. Scale to classmates) 17 hours ago
 $K * (n + 1)!$

[helpful!](#) | 0

 17 hours ago
Yes. You have proved Taylor's theorem. Congratulations. Now since we have $g^{(n+1)}(c) = f^{(n+1)}(c) - p^{(n+1)}(c) = 0$, $f^{(n+1)}(c) = p^{(n+1)}(c) = K(n + 1)!$. Rearranging and plugging the value for K you got earlier, you get $\frac{f(x) - T_{n,a}(x)}{(x-a)^{n+1}} = K = \frac{f^{(n+1)}(c)}{(n+1)!}$. With one more step of rearranging, you have Taylor's theorem!

[good comment](#) | 0

 (Anon. Scale to classmates) 17 hours ago
so $f(x) - T_{n,a}(x) = \frac{f^{(n+1)}(c)}{(n+1)!} (x-a)^{n+1}$

Thank you!!!!!! my brain is burning rn. I'll try to go through the process again by myself.

[helpful!](#) | 0

 17 hours ago
You are welcome! Yes, go through the process, and later if you can, post it as a student's answer from which I will know that you understand.

[good comment](#) | 1

From the followup-discussion below, I gather the details as follows:

- $p(t) = f(a) + f'(a)(t-a) + \frac{f''(a)}{2!}(t-a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(t-a)^n + \underbrace{\frac{R_{n,a}(x)}{(x-a)^{n+1}}}_{\text{This is a constant as } x \text{ is fixed}}(t-a)^{n+1}$.
- Rewritten $p(t) = T_{n,a}(t) + \frac{R_{n,a}(x)}{(x-a)^{n+1}}(t-a)^{n+1}$.
- By construction, $p(t)$ is such that $p(a) = f(a), p^{(k)}(a) = f^{(k)}(a)$ for $1 \leq k \leq n$, and $p(x) = f(x)$. Also $p^{(n+1)}(t) = \frac{R_{n,a}(x)}{(x-a)^{n+1}}(n+1)!$
- Consider $g(t) = f(t) - p(t)$. Then g is $n + 1$ -times differentiable on I such that $g(a) = g(x) = 0, g^{(k)}(a) = 0$ for $1 \leq k \leq n$.
- By Rolle's theorem, we get $c_1 \in (a, x)$ such that $g'(c_1) = 0$. By repeated application of Rolle's theorem, we get $c_k \in (a, c_{k-1}) \subset (a, x)$ such that $g^{(k)}(c_k) = 0$ for $2 \leq k \leq n$.
- Ultimately, apply Rolle's theorem to $g^{(n)}(t)$ on the interval $[a, c_n]$ and get $c \in (a, c_n) \subset (a, x)$ such that $g^{(n+1)}(c) = 0$.
- Now, $g^{(n+1)}(c) = 0 \Rightarrow f^{(n+1)}(c) - p^{(n+1)}(c) = 0$, and hence $f^{(n+1)}(c) = p^{(n+1)}(c) = \frac{R_{n,a}(x)}{(x-a)^{n+1}}(n+1)!$ Rearranging, we get $R_{n,a}(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$.

On Continuity of $f(x) = x^2$ at a point:

[question](#)

stop following **71 views**

Actions ▾

Max value of x vs min value of x for function limits

For this limit here, why do we select the maximum value that x can be and not the minimum? From my intuitive understanding selecting a maximum value for x creates a leads to delta being smaller, so it would more accurately represent at what point away from a that the distance from the function to the limit is less than epsilon, but by selecting a minimum value for x (in this case 2), this would no longer be guaranteed since you are making delta bigger, and thus increasing the domain that x could be under (which might lead to a value being outside the bounds of $(L - \epsilon, L + \epsilon)$). Is my intuition correct? Also is there a way to represent this idea graphically because I can't come up with one.

$$2) \text{ Prove } \lim_{x \rightarrow 5} x^2 = 25.$$

Proof: Let $\epsilon > 0$ be given. Let $\delta = \min\{1, \frac{\epsilon}{11}\}$
Then, if $0 < |x-5| < \delta$, we get

$$|x^2 - 25| = |x-5||x+5|$$

First,
say $\delta \leq 1$,
let's find an
upper bound on
 $|x+5|$!

Since $|x-5| < \delta \leq 1$, $4 < x < 6$,

$$\text{so } |x+5| \leq |6+5| = 11.$$

$$\begin{aligned} \text{So we get } |x^2 - 25| &= |x+5||x-5| \\ &\leq 11|x-5| \\ &< 11\delta \\ &\leq 11\frac{\epsilon}{11} \\ &= \epsilon \end{aligned}$$

Aside
want
 $|x-5| < \epsilon$
 $\rightarrow |x-5| < \frac{\epsilon}{11}$

as desired

QED

exam

[Edit](#) good question | 0

Updated 3 days ago by (Anon. Gear to classmates)

the students' answer, where students collectively construct a single answer

Actions ▾

We find the maximum value of $|x+5|$ so that we can determine what value it must surely be less than. To do so, we maximize x. This way, with the restriction that $|x-5| < 1$, we can simplify $|x+5| \leq 11$ in our string of inequalities, which lets us get rid of the $|x+5|$ factor which prevented us from getting $|x^2 - 25| < \epsilon$. If we picked the minimum x value, we would have $|x+5| \leq |4+5|$ which is not true in the case that x is 5 for example. I hope I interpreted your question right

[Edit](#) good answer | 0

Updated 3 days ago by (Anon. Gear to classmates)



the instructors' answer, where instructors collectively construct a single answer

Actions ▾

Your intuition is right and this intuition is very specific to these kinds of functions. The following discussion could possibly give you a way to understand this graphically and refines that intuition beyond the max or min considerations!

Remember why we are doing all of this in the first place. We want to find a $\delta > 0$ for a given $\varepsilon > 0$ such that $|x - 5| < \delta \Rightarrow |x^2 - 25| < \varepsilon$. Let us investigate (different manifestations of) $|x^2 - 25| < \varepsilon$:

$$|x^2 - 25| < \varepsilon \iff |x - 5||x + 5| < \varepsilon \iff |x - 5| < \frac{\varepsilon}{|x + 5|}$$

So if we find a $\delta > 0$ (independent of x , but dependant on ε and 5, the point of interest) that sits in-between $|x - 5|$ and $\frac{\varepsilon}{|x + 5|}$, we are done, right?

Because if $|x - 5| < \delta < \frac{\varepsilon}{|x + 5|}$, then for sure $|x - 5| < \frac{\varepsilon}{|x + 5|}$ and hence $|x^2 - 25| < \varepsilon$.

This is a representative example of how to show the continuity of a function at a point. Even in my grad school, in order to show the continuity of some weird function (on unexplainable space to you) at a point, I had to remember how we prove the continuity of x^2 because this very subtly makes use of the concept of continuity being a "local" phenomenon.

Being a "local" phenomenon means only what happens near our point of interest is relevant. So while we are concerned about continuity at $x = 5$, we don't need to know what is happening at $x = 0$, or 100, or -5 . So we shall restrict our analysis to a small interval around $x = 5$ such as when $x \in (0, 10)$, $(2, 8)$, or $(4, 6)$. This restriction matters in how we are going to choose δ . In order to save you from this mattering restriction, we traditionally consider $\delta \leq 1$, meaning an interval of points such that $|x - 5| < 1$.

Now if $|x - 5| < 1$, then $|x + 5| < 11$. So, with consideration of this restriction, we can find a $\delta > 0$ (independent of x , but dependant on ε and 5, the point of interest) that sits in-between $|x - 5|$ and $\frac{\varepsilon}{|x + 5|}$. When $|x - 5| < 1$, we get $\frac{\varepsilon}{11} < \frac{\varepsilon}{|x + 5|}$. That's why $\delta = \min\{1, \frac{\varepsilon}{11}\}$ works! (Conceptual question: Where is the dependency of δ on 5 here?)

Graphically, what does this mean is: this open interval $|x - 5| < \delta$ is an subinterval in $\left(5 - \frac{\varepsilon}{|x + 5|}, 5 + \frac{\varepsilon}{|x + 5|}\right)$ for each $x \in (4, 6)$.

If you had chosen the minimum value or any other value of $|x + 5|$ when $x \in (4, 6)$, there is no apparent way to relate $|x - 5|$ and $\frac{\varepsilon}{|x + 5|}$ meaningfully!

Aside: if one chooses their local restriction such that $|x - 5| < 2$, then $|x + 5| < 12$. And $\delta = \min\{2, \frac{\varepsilon}{12}\}$ would work in this case. There are infinitely many δ you can choose for this to work!

Instead, if we were to figure out the continuity of $\frac{x-5}{x+5}$ at $x = 5$, here we would want to find a $\delta > 0$ such that $|x - 5| < \delta \Rightarrow \left|\frac{x-5}{x+5}\right| < \varepsilon$, right?

That means, if we find a $\delta > 0$ such that $|x - 5| < \delta < \varepsilon|x + 5|$, we would be done! Again restrict $|x - 5| < 1$. Then $9 < |x + 5|$. Can you tell what δ will work here?

Edit good answer | 0

Updated 60 minutes ago by [redacted]