Assignment 2 - Mathematics-I Selected Solutions
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1. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ 4 & -4 & 3 \end{bmatrix}$.

   (a) Diagonalize A by similarity transformation M and verify that $M^{-1}AM = D$, where D is the diagonal matrix with eigenvalues of A as diagonal entries.

   (b) Compute the value of $(A^6 - 4A^5 - A^4 + 16A^3 - 17A^2 + A + 2I)$, using Cayley-Hamilton theorem.

2. Using Cayley-Hamilton Theorem, find the eigenvalues of A and, hence, find A^n (n, a positive integer), given $A = \begin{bmatrix} 6 & 2 \\ -2 & 1 \end{bmatrix}$. Verify for A^3.

3. Reduce $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_1x_3$ into canonical form by orthogonal reduction and find the rank, signature, index, and nature of the quadratic form. Express the change of variables employed by the orthogonal transformation explicitly.

   Eigenvalues of the matrix of the given quadratic form $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ are $2, 2, 8$.

   As $A - 2I = \begin{pmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{pmatrix} \sim \begin{pmatrix} 2 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow 2x_1 - x_2 + x_3 = 0,$ letting $x_1 = k, x_2 = l$, we get that $(k, l, l - 2k) = k\,(1, 0, -2) + l\,(0, 1, 1)$ solves $(A - 2I)X = 0$. Since we are interested in orthogonal reduction and the vectors $(1, 0, -2), (0, 1, 1)$ are not orthogonal, we have to find a vector of the form $(k, l, l - 2k)$ that is orthogonal to one of the above vectors, say $(0, 1, 1)$, i.e., We want k, l s.t $(k, l, l-2k).(0, 1, 1) = 0$ i.e., $l + l - 2k = 0 \Rightarrow k = l$. Particularly, choose $k = l = 1$. Then $(1, 1, -1)$ satisfies $(A - 2I)X = 0$ and is orthogonal to $(0, 1, 1)$. Let $X_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and $X_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ be the orthogonal eigenvectors corresponding to the eigenvalue $\lambda = 2$!

   $$\text{As } A - 8I = \begin{pmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow k \overbrace{\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}}^{X_3} \text{ solves } (A - 8I)X = 0.$$

   Therefore, modal matrix $M = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ and normalised modal matrix $N = \begin{bmatrix} 0 & \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \end{bmatrix}$.

   Now N is an orthogonal matrix with $NAN^T = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ and the transformation

$$X = NY: \begin{array}{ll} x_1 & = \frac{y_2}{\sqrt{3}} + \frac{2y_3}{\sqrt{6}} \\ x_2 & = \frac{y_1}{\sqrt{2}} + \frac{y_2}{\sqrt{3}} - \frac{y_3}{\sqrt{6}} \\ x_3 & = \frac{y_1}{\sqrt{2}} - \frac{y_2}{\sqrt{3}} + \frac{y_3}{6} \end{array}$$ orthogonally reduces the given quadratic form to

$$2y_1^2 + 2y_2^2 + 8y_3^2.$$

Furthermore, $r = 3, p = 3, s = 2p - r = 3$ and since $r = p = 3 = \#$no. of variables, the given QF is positive definite.

4. Solve the following differential equations:

   (a) $\tan y \dfrac{dy}{dx} + \tan x = \cos y \cos^3 x$ (CORRECTION)

   (b) $\dfrac{dy}{dx} - xy + y^3 e^{-x^2} = 0$

5. Check if the following DE's are exact, if not, find a suitable integrating factor and solve them:

   (a) $(x^2 y - 2xy^2)dx - (x^3 - 3x^2 y)dy = 0$

   (b) $(e^y + xe^y)dx + xe^y dy = 0$