AutoML: Evaluation

Background: Statistical Hypotheses Tests

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Background: statistical hypothesis tests

- When we have a lot of data, we need to summarize it
  - But we already saw that summarization hides a lot of data
  - Ideally, we want to draw high-level conclusions
    (e.g., "A outperforms B on datasets of type X")

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  - But we already saw that summarization hides a lot of data
  - Ideally, we want to draw high-level conclusions (e.g., "A outperforms B on datasets of type X")
- Problem: we only have a finite number of observations
  - Can we attribute observed performance differences to chance?
  - Are we reasonably sure that a claim we make is reproducible?

  $\leadsto$ Statistical tests can help

Statistical hypothesis testing

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	Truly not guilty	Truly guilty
Found not guilty	Acquittal	Type II Error
Found guilty	Type I Error	Conviction

$\rightsquigarrow$ We want to minimize Type I error!

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8. If $p < \alpha$, reject null hypothesis in favor of alternative hypothesis
   - If $p > \alpha$, it doesn't tell you anything about the null hypothesis!

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- Let's say we observed IQ values x_i of 9 students in the class:
  - $\{x_1, \ldots, x_9\} = \{116, 128, 125, 119, 89, 99, 105, 116, 118\}$.
  - The sample mean is $\bar{x} = 112.8$
  - Does this data support the claim?

Example continued

- Distribution of the test statistic
  - Under H_0, we know that each $x_i \sim \mathcal{N}(100, 15)$
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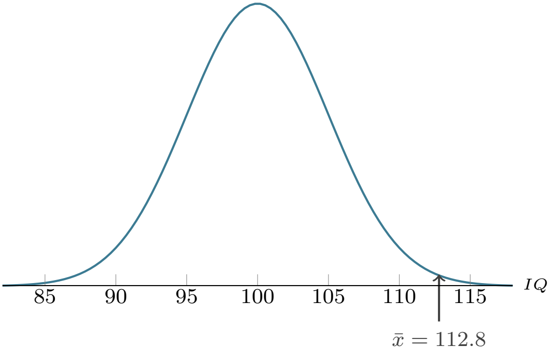
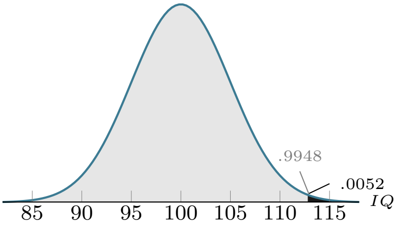
  - The test statistic that we measure is the sample mean $\bar{x} = \frac{1}{9}\sum_{i=1}^{9} x_i$
  - Under H_0, the distribution of $\bar{x}$ is $\mathcal{N}(100, 15/\sqrt{9})$
    - Our observation $\bar{x} = 112.8$ is quite extreme under that distribution



General principle



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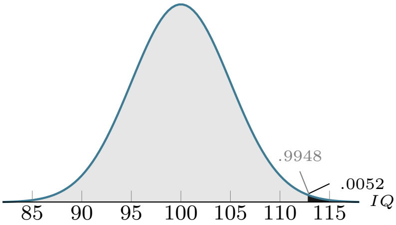

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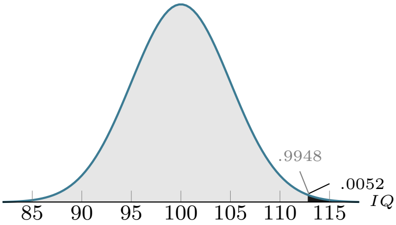

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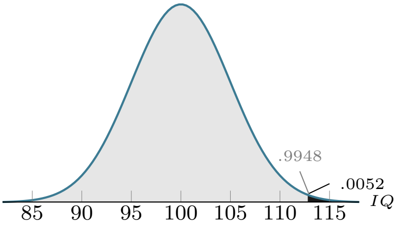

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- With $\alpha = 0.01$, would we reject H_0 in this case?

Summary of example

- We just used a so-called Z-test
- H_0: $\mu = \mu_0$, H_1: $\mu > \mu_0$
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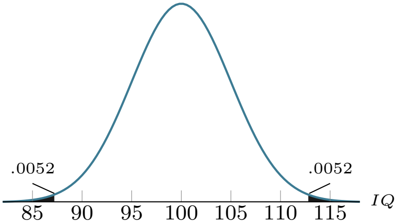
- Equivalent: compute the Z-statistic: $Z = (\bar{x} - \mu_0)/s$ and evaluate cumulative distribution $\Phi(Z)$ of Z under $\mathcal{N}(0, 1)$
  - There are standard tables to look up $\Phi(Z)$ for different values of Z
  - Nowadays, there are standard libraries to compute $\Phi(Z)$

Two-sided tests



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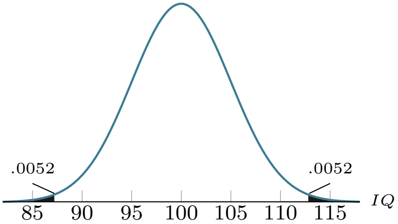

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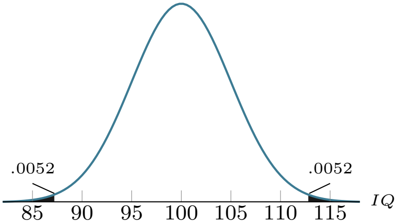

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  - Our data is often far from normally-distributed
    - $\leadsto$ E.g., exponential runtime distributions of optimizers
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- Beware (ii): if you use cross-validation, observations are not independent (you cannot apply statistical tests that assume independence)