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Ques 2 Defining the dynamic table's behaviour

When inserting an element into the table, if the table is full, the table doubles in size.

Doubling the size of the table takes $O(n)$ time, where n is the current size of the table.

After resizing the new element is inserted into the table.

i) Aggregate Method:

In this method we calculate the total time for n insertions and divide it by n to find average time per insertion.

Let n_i be the size of the table when inserting ith element.

Initially $n_1 = 1$

when resizing $n_i = 2 \times n_{i-1}$

The time taken to insert an element into a table of size n is proportional to n.

$T_a \rightarrow$ amortized time per insertion.

$\boxed{T_a = \frac{T}{n}}$ $\rightarrow$ total time for n insertion.

The first insertion takes $O(1)$ time (without resizing).

The second insertion takes $O(1)$ time (resizing needed doubling the size)

The third insertion takes $O(2)$ time (resizing, doubling the size)

⋮

The k^{th} insertion takes $O(2^{k-1})$ time (resizing, doubling size $k-1$ times

⋮

The n^{th} insertion takes $O(2^{k-1})$ times

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$T = O(1) + O(1) + O(2) + O(4) + \cdots + O(2^{k-1})$

$T = O(2^0) + O(2^1) + O(2^2) + O(2^3) + \cdots + O(2^{k-1})$

This is geometric series with common ratio $= 2$

Sum of geometric series :-

$T = O(2^{k} - 1) = O(2^{\log_2 n} - 1) = O(n-1)$ $\boxed{= O(n)}$

Thus, $\boxed{T_a = O(1)}$

ii) Accounting method

In this method, we assign each insertion a "charge" that covers the cost of insertion and resizing.

Charge $O(1) \longrightarrow$ for each insertion operation
Charge $O(n) \longrightarrow$ for each resizing operation.

C_i be the total cost of inserting the i^{th} element.

$A_i = C_i + C_{i-1}$

Where A_i is the amortized cost per insertion.

$C_i = O(1)$ for i^{th} insertion.
$C_{i-1} = O(n)$ for i^{th} resizing operation.

The total charge T for n insertion can be calculated as,

$$T = \underbrace{n \times O(1)}_{\text{Cost of n insertions}} + \underbrace{\frac{n}{2} \times O(n)}_{\text{Cost of resizing operations.}}$$

$$T = O(n) + O\left(\frac{n^2}{2}\right) = O(n^2)$$

The amortized cost per operation is then;

$$\boxed{T_a = \frac{T}{n} = O(n)}$$