

Team Members

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Question 4a

$$\text{Let } y = w_0 + w_1 e^{-x_1} + w_2 x_1 + w_3 x_1 x_2$$

$$\text{Loss function} = \text{SSE} = L$$

$$= \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$= \sum_{i=1}^n (y_i - \hat{w}_0 - \hat{w}_1 e^{-x_1} - \hat{w}_2 x_1 - \hat{w}_3 x_1 x_2)^2$$

Taking derivative with all 4 weights

$$\frac{\partial}{\partial \hat{w}_0} (L) = 2 * \sum_{i=1}^n (y_i - \hat{w}_0 - \hat{w}_1 e^{-x_1} - \hat{w}_2 x_1 - \hat{w}_3 x_1 x_2) * -1$$

$$= -2 \sum_{i=1}^n (y_i - \hat{y}_i)$$

$$\frac{\partial}{\partial \hat{w}_1} (L) = 2 * \sum_{i=1}^n (y_i - \hat{w}_0 - \hat{w}_1 e^{-x_1} - \hat{w}_2 x_1 - \hat{w}_3 x_1 x_2) * -e^{-x_1}$$

$$= -2 \sum_{i=1}^n (y_i - \hat{y}_i) e^{-x_1}$$

$$\frac{\partial}{\partial \hat{w}_2} (L) = 2 * \sum_{i=1}^n (y_i - \hat{w}_0 - \hat{w}_1 e^{-x_1} - \hat{w}_2 x_1 - \hat{w}_3 x_1 x_2) * -x_1$$

$$= -2 \sum_{i=1}^n (y_i - \hat{y}_i) x_1$$

$$\frac{\partial(L)}{\partial(\hat{w}_3)} = 2 * \sum_{i=1}^n (y_i - \hat{w}_0 - \hat{w}_1 e^{-x_1} - \hat{w}_2 x_1 - \hat{w}_3 x_1 x_2) * -x_1 x_2$$

$$= -2 \sum_{i=1}^n (y_i - \hat{y}_i) x_1 x_2$$

Now we upgrade the new weights using these gradient

$$w_0(\text{new}) = w_0 - \eta * \frac{\partial}{\partial w_0} (L)$$

$$= w_0 - \eta (-2 \sum_{i=1}^n (y_i - \hat{y}_i))$$

$$w_1(\text{new}) = w_1 - \eta (-2 \sum_{i=1}^n (y_i - \hat{y}_i) e^{-x_1})$$

$$w_2(\text{new}) = w_2 - \eta (-2 \sum_{i=1}^n (y_i - \hat{y}_i) x_1)$$

$$w_3(\text{new}) = w_3 - \eta (-2 \sum_{i=1}^n (y_i - \hat{y}_i) x_1 x_2)$$