

TOPIC 2 LINEAR PROGRAMMING – SHORT TERM FINANCING



Short Term Financing

- Corporations routinely face the problem of financing shortterm cash commitments
- Linear programming can help in figuring out an optimal combination of financial assets to meet these commitments
- The decision variables depend on the list of assets to choose from and the list of liabilities to meet
- The objective is usually to minimize the net present/future value of meeting all the commitments
 - Or maximize money left over at the end of the time horizon
- The primary constraints are that all cash inflows and outflows for each time period should be sufficient to meet liabilities



Short Term Financing

• Consider the following short term financing problem (given in 1000's):

Month	Jan	Feb	Mar	Apr	May	Jun
Net cash flow	-150	-100	200	-200	50	300

- The available sources of funds are
 - a line of credit from your bank, up to \$100k, 1% interest per month
 - in Jan-Mar, you can issue a 3-month commercial paper bearing an interest of 2% for the 3-month period
 - excess funds can be reinvested at a rate of 0.3% per month



Class Participation

- Spend a few minutes thinking about what aspects of this problem are important
- How would you write equations to solve it?



 x_i is money borrowed from bank at time periods 1-5, y_i is money borrowed from bond investors at time periods 1-3 Original Constraints

Jan:
$$x_1+1_1-150 \ge 0$$
 $x_1+y_1 \ge 150$

Feb: $x_2+1_2+1.003(x_1+y_1-150)-1.01x_1 \ge 100$

March: $x_3+y_3+1.003(x_1+y_2+1.003(x_1+y_1-150)-1.01x_1-100)-1.01x_1 \ge -20$

April: $x_4+1.003(...)-1.01x_3-1.02y_1 \ge 200$
 x_1+y_2



Add slack

Jun:
$$X_1 + Y_1 - Z_1 = 150$$

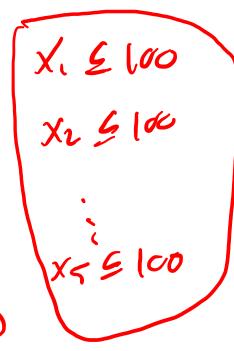
Feb: $X_2 + Y_2 + 1.003 Z_1 - 1.01 X_1 - Z_2 = 100$

Mark: $X_3 + Y_3 + 1.003 Z_2 - 1.01 X_2 - Z_3 = -200$

April: $X_4 + 1.003 Z_3 - 1.01 X_3 - 1.02 Y_1 - Z_4 = 200$

May: $X_5 + 1.003 Z_4 - 1.01 X_4 - 1.02 Y_2 - Z_5 = 50$

June: $1.003 Z_5 - 1.01 X_5 - 1.02 Y_3 - Z_6 = -300$





Objective

min 6.01 (all bank loages) + 0.06 x (all bond bans)

0.01 (all bank loages) + 0.06 x (all bond bans)

0.01 (all bank loages) + 0.06 x (all bond bans)

max bapak account value in June

max 0 x, 10 x2 - " toxx + 0 y, + 0 y2 to y3 + 02, " + 1 26

Max c^TX s, t. Ax = b LB \(\times \times \) \(\times \) UB



Constraint Matrix

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Standard form



Short Term Financing

- Decision Variables
 - $x_1, ..., x_5$: total amount draw from line of credit in each month
 - $y_1, ..., y_3$: total amount of commercial paper issued in each month
 - $z_1, ..., z_6$: excess funds in each month
- Objective maximize the cash balance at the end of six months
 - $\max z_6$
- Constraints
 - Meet monthly liabilities
 - $x_i \le 100$
 - $x_i, y_i, z_i \ge 0$



The LP

- Choose $x_1, ..., x_5, y_1, ..., y_3, z_1, ..., z_6$
- To maximize z_6
- Such that:

$$x_1 + y_1 - z_1 = 150$$

$$x_2 + y_2 - 1.01x_1 + 1.003z_1 - z_2 = 100$$

$$x_3 + y_3 - 1.01x_2 + 1.003z_2 - z_3 = -200$$

$$x_4 - 1.02y_1 - 1.01x_3 + 1.003z_3 - z_4 = 200$$

$$x_5 - 1.02y_2 - 1.01x_4 + 1.003z_4 - z_5 = -50$$

$$-1.02y_3 - 1.01x_5 + 1.003z_5 - z_6 = -300$$

$$x_i, y_i, z_i \ge 0$$

$$x_i \le 100$$



The Optimal Solution

	Jan	Feb	Mar	Apr	May	Jun
Bank		50.98				
Bonds	150	49.02	203.43			
Excess Funds			351.94			92.47
Liability	-150	-100	200	-200	50	300





Slack Variables

- Sometimes it makes sense to modify an inequality constraint
 - Add a slack variable and change inequality to equality
- If you need to use the difference between the left and right side of an inequality you can eliminate a lot of tedious algebra
- Inside the simplex method, the algorithm actually creates slack variables for all the constraints anyway
 - You need not worry about extra computational cost