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Question 2 :→

$P(C_1|x) = \mu = \dfrac{e^{\beta \cdot x}}{1+e^{\beta \cdot x}}$

$\beta_0 = -7$, $\beta_1 = 2$, $\beta_2 = 0.001$

X_1 = high school, X_2 = SAT score

$P(\text{completing graduate school} \mid X_1 = 3.4, X_2 = 1500) = \dfrac{e^{-7+3.4\times2+0.001\times1500}}{1+e^{-7+3.4\times2+0.001\times1500}}$

$= \dfrac{e^{1.3}}{1+e^{1.3}} = \dfrac{3.669}{4.669} = 0.785$

$P(\text{completing graduate school} \mid X_1 = 3.7, X_2 = 1500) = \dfrac{e^{-7+3.7*2+1500*0.001}}{1+e^{-7+3.7*2+0.001*1500}}$

$= \dfrac{e^{1.9}}{1+e^{1.9}} = \dfrac{6.685}{7.685} = 0.869$

Probability increased $= 0.869 - 0.785 = 0.084$

Question 1

		Predicted Class		
		C_1	C_2	Reject
True class	C_1	0	r	c
	C_2	s	0	c

Let $f(x) = P(C_1|x)$

a) Expected Loss when x is labelled C_1

$= 0 * f(x) + (1-f(x)) * s$

$= s - f(x) * s$

$= -s * f(x) + s$

→ Hence a decreasing function of $f(x)$

Expected Loss when x is labelled C_2

$= r * f(x) + (1-f(x)) * 0$

$= r * f(x)$

Hence an increasing function of $f(x)$

b) Expected loss when labelled C_1

$= -s * f(x) + s$

$= (1-f(x)) * s$

Since, $0 \le 1 - f(x) < 1$

$0 \le s * (1-f(x)) \le s$ ($\because$ As s is a positive no.) —(i)

Expected Loss when x labelled C_2

$= r * f(x)$

Since $0 \le f(x) \le 1$,

$= 0 \le r * f(x) \le r$ (as r is a +ve no) —(ii)

Expected Loss when x is Rejected

$= c * P(\text{Reject}|x) + c * (1 - P(\text{Reject}|x))$

Since $c = 0$, the entire term $= 0$ —(iii)

From the equations (i),(ii),(ii) we can find that decision which minimizes the expected loss is to reject all instances of x

c) $r = 5$, $s = 2$

$EC_1 = r f(x) < c$

$5 f(x) < c$

$EC_2 = s(1 - f(x)) < c$

$2(1 - f(x)) < c$

$5 f(x) = 2 - 2 f(x)$

$f(x) = 2/7 = 0.285$

$5 f(x) = 5 * 0.285 = \underline{1.428} \Rightarrow$ Minimum value of c for no x to be rejected

d) $r = 7, s = 4, c = 3$

$c_1 = s(1 - f(x)) < r f(x)$

$s(1 - f(x)) < c$

$4(1 - f(x)) < 7 f(x)$

$f(x) > 4/11$

$4(1 - f(x)) < 3$

$f(x) > 1 - 3/4$

$f(x) > 1/4$

$f(x) \Rightarrow 1 > f(x) > 4/11$

c_2 $r f(x) < s(1 - f(x))$

$r f(x) < c$

$7 f(x) < s(1 - f(x))$

$4 f(x) < 4 - 4 f(x)$

$f(x) < 4/11$

$r f(x) < c$

$7 f(x) < 3$

$f(x) < 3/7$

$0 < f(x) < 4/11$

Reject $\Rightarrow$

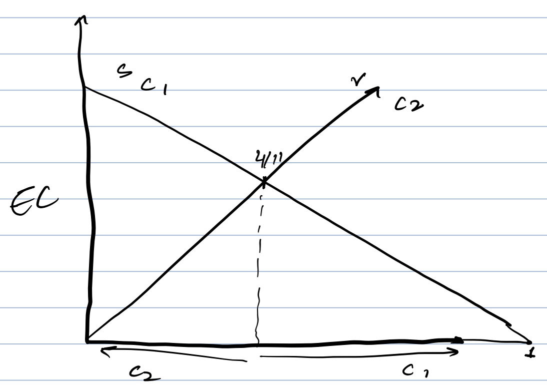
$r f(x) < c$

$f(x) < 3/7$

$s(1 - f(x)) < c$

$f(x) > 1/4$

$1/4 < f(x) < 3/7$



Question 4

$S=1$	$S=0$
0.001	0.999

Season → Rain ← A.P

AP=0	AP=1
0.998	0.002

$P(\text{season} = 1) = 0.001$

$P(AP = 1) = 0.002$

$P(\text{Rain} = 1 | \text{season} = 0, AP = 0) = 0.001$

$P(R = 1 | S = 0, AP = 1) = 0.29$

$P(R = 1 | S = 1, AP = 0) = 0.94$

	Rain=0	Rain 1
S=0, AP=0	0.999	0.001

$S=0, AP=1$	0.71	0.29
$S=1, AP=0$	0.06	0.94
$S=1, AP=1$	0.05	0.95

Umbrella

	$U=0$	$U=1$
$R=0$	0.95	0.05
$R=1$	0.1	0.9

$P(R=1 \mid S=1, AP=1) = 0.95$
$P(U=1 \mid R=1) = 0.9$
$P(U=1 \mid R=0) = 0.05$

a) $P(U=0) = P(U=0 \mid R=1) * P(R=1) + P(U=0 \mid R=0) * P(R=0)$

$= 0.1 * P(R=1) + 0.95 * P(R=0)$

$P(R=1) = P(R=1 \mid S=0, AP=0) * P(S=0 \cap AP=0) + P(R=1 \mid S=0, AP=1) * P(S=0 \cap AP=1)$
$+ P(R=1 \mid S=1, AP=0) * P(S=1 \cap AP=0) + P(R=1 \mid S=1, AP=1) * P(S=1 \cap AP=1)$
$= 0.001 * 0.999 * 0.998 + 0.29 * 0.999 * 0.002 + 0.94 * 0.001 * 0.998$
$+ 0.95 * 0.001 * 0.002$
$= 0.000997 + 0.000579 + 0.000938 + 0.0000019$
$= 0.0025$

$P(R=0) = P(R=0 \mid S=0, AP=0) * P(S=0 \cap AP=0) + P(R=0 \mid S=0, AP=1) * P(S=0 \cap AP=1)$
$+ P(R=0 \mid S=1, AP=0) * P(S=1 \cap AP=0) + P(R=0 \mid S=1, AP=1) * P(S=1 \cap AP=1)$
$= 0.999 * 0.999 * 0.998 + 0.71 * 0.999 * 0.002 + 0.06 * 0.001 * 0.998$
$+ 0.05 * 0.001 * 0.002$
$= 0.996 + 0.00141 + 0.000059 + 0.0000007$
$= 0.9974$

$P(U=0) = 0.1 * 0.0025 + 0.95 * 0.9974$
$= 0.00025 + 0.94753$
$= 0.94778$

b) $P(R=1, A=0) = P(R \mid S, A') * P(S, A') + P(R \mid S', A') P(S', A')$
$= 0.94 * 0.001 * 0.998 + 0.001 * 0.999 * 0.998$
$= 0.001935$

c) $P(S=1 \mid R=1) = \dfrac{P(S=1, AP=0, R=1) + P(S=1, AP=1, R=1)}{P(R=1)}$ (calculated Above)

$= \dfrac{0.000938 + 0.000002}{0.002516}$

$= 0.373551$