Sample - 001101011110110100111001101100l

Sample size - 30

Population - 100

$$P(x) = \begin{cases} q & \text{for } x=0 \\ 1-q & \text{for } x=1 \end{cases}$$

Since it is a Yes No question

$$P(x) \begin{cases} 0.5 & \text{for } x=0 \\ 0.5 & \text{for } x=1 \end{cases}$$

No. of times 1 occured - 17

No of times 0 occurred - 13

1 Likelihood function for q

Let n be total no. of observation

Let a be number of observation for $x=0$

Let q be the probability of $x=0$

$$L(p=q \mid n,a) = \frac{n!}{a!\,(n-a)!} (q)^{a} (1-q)^{n-a}$$

2. Maximum Likelihood Estimator

→ To find maximum likelihood,

we take derivative of Likelihood function

Q2 $\ln L(q|n,a) = \ln\left[\frac{(n!)}{a!(n-a)!} q^a (1-q)^{n-a}\right]$

$$= \ln\left(\frac{n!}{a!(n-a)!}\right) + \ln(q^a) + \ln\left[(1-q)^{n-a}\right]$$

$$= \ln\left(\frac{n!}{a!(n-a)!}\right) + a \cdot \ln(q) + (n-a)\ln(1-q)$$

Taking derivative with respect to q

$$\frac{d \ln L(q|n,a)}{dq} = 0 + \frac{a}{q} + \frac{(n-a)(-1)}{1-q}$$

$$= 0 + \frac{a}{q} - \frac{(n-a)}{(1-q)} \quad \text{—(i)}$$

To find peak, equate (i) = 0

$$\frac{a}{q} - \frac{n-a}{(1-q)} = 0$$

Multiply both side by $q(1-q)$

$$a(1-q) - (n-a)q = 0$$

$$a - aq - nq + aq = 0$$

$$a - nq = 0$$

$\boxed{q = \frac{a}{n}}$ → Maximum Likelihood for q where there a successes in n trials

~~∵ Maxi~~

Q3 Maximum likelihood for q when

no of success of q = 13

$n = 30$

$\therefore$ MLE of $q = \frac{13}{30} = 0.433$

Q.4 for Bionomial distribution

Mean = $E(a) = nq$

Variance = $nq(1-q)$
(Var(a)

$$E(MLE(q)) = E\left(\frac{a}{n}\right)$$

$$= \frac{1}{n} E(a)$$

$$= \frac{1}{n}(nq)$$

$$= q$$