JPMC Quant Challenge - SiliconLobby [Hiten & Arkadeep]

Question 2

30 September 2018

Question 2 [Quant]

Solution

Given Stochastic process is similar to that used in Cox–Ingersoll–Ross model:

$$dX_t = a(\theta - X_t)\,dt + \sigma\sqrt{X_t}\,dW_t$$

where =Wt is a Wiener process (modelling the random market risk factor) and a, θ, and σ, are the parameters.

Hence we can use the results derived for CIR here.

1. E[X_T]

Using CIR Model Result

$E[X_T | X_0] = X_0 e^{(-at)} + \theta (1 - e^{(-at)})$

Code:

```
#1 answer
answer1 <- X0 * exp(-a*t) + theta*(1-exp(-a*t))
```

2. E[max(X_T - 100,0)]

Method:

Here we use MonteCarlo simulation to trace path of the stochastic process.

1) ***sim*** represents the the no. of simulations we use

2) We start by initialising our paths by given $\mathbf{X_0}$ and then **add** Δ**X** to it for each Δt = (t/breaks) for t = 2 where ΔX can be calculated as

$$\Delta X = a(\theta - X) + \sigma\sqrt{X}\Delta W$$

3) Iteratively perform these vector operations to get X_T.

4) Evaluate *mean(max(X_T-100,0))* over sample of generated X_Ts and report it as the expectation.

Alternative Method:

The distribution of future values of a CIR process can be computed in closed form:

$$X_T = \frac{Y}{2c}$$

where $c = \frac{2a}{(1 - e^{(-aT)})\sigma^2}$ and Y is is a non-central Chi-Squared distribution with $\frac{4ab}{\sigma^2}$ degrees of freedom and non-centrality parameter $2cX_0e^{-aT}$

Instead of simulating the Xt paths we can directly generate random numbers X_T by generating r.v. given by distribution above.

Code:

```
#2 answer
sim = 10^5
vec<- rep(X0,sim)
breaks = 100
for(i in seq(1,breaks)){
  x<-rnorm(sim)
  vec<- vec+a*(theta - vec)*(t/breaks) + sigma*sqrt(vec)*x*sqrt(t/breaks);
}
for(i in seq(1,length(vec))){
  vec[i] = max(vec[i]-100,0)
}
answer2<-mean(vec)
```

3. $\partial E[\max(X_T - 100,0)]/\partial\sigma$

Let $Q = \partial E[\max(X_T - 100,0)]/\partial\sigma$,

To evaluate Q, we performed simulations similar to how we did in part 2. But this time we took two different sigma varying by small quantity “del”. Both simulation took same random variables all that varied was sigma. Since the differential where condition is not followed is 0, taking mean on all corresponding values of

$H = (f_{\sigma+del} - f_{\sigma})/del$ if both $f_{\sigma+del}$ and f_{σ} are >100

$= 0$ otherwise

gives us the required derivative.

Code:

```
#3 answer
del<-10^-7
sigma1<-sigma+del
vec<- rep(X0,sim)
vec1<-rep(X0,sim)
for(i in seq(1,breaks)){
  x<-rnorm(sim)
  vec<- vec+a*(theta - vec)*(t/breaks) + sigma*sqrt(vec)*x*sqrt(t/breaks);
  vec1<- vec1+ a*(theta - vec1)*(t/breaks) + sigma1*sqrt(vec1)*x*sqrt(t/breaks);
}
vega <- 0
for(i in seq(1,sim)){
  if(vec[i]>100&&vec1[i]>100){
    vega <- vega+((vec1[i]-vec[i])/del)
  }
}
answer3<-vega/sim
```