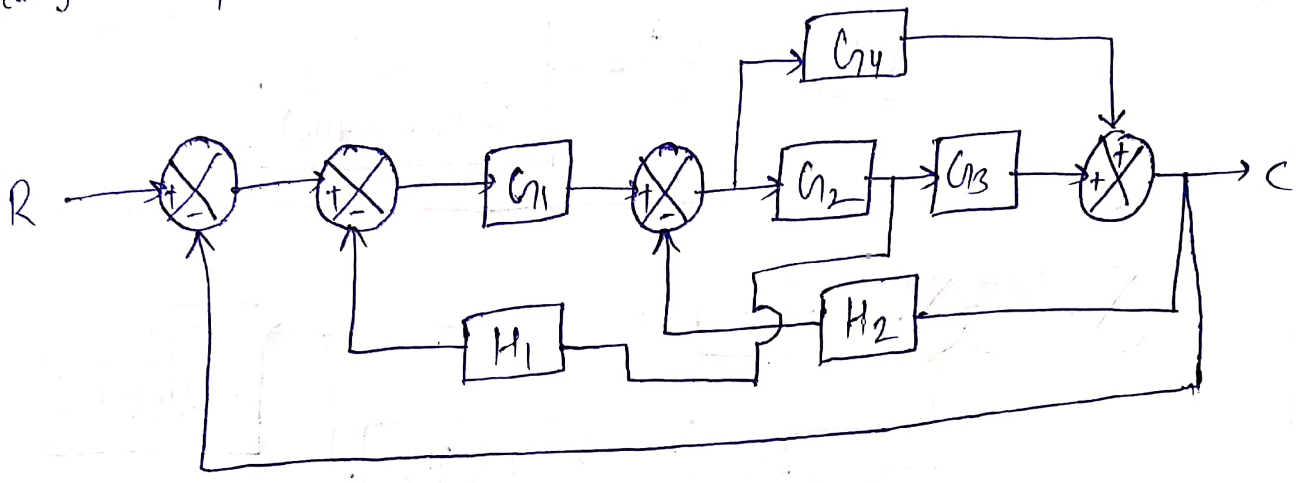
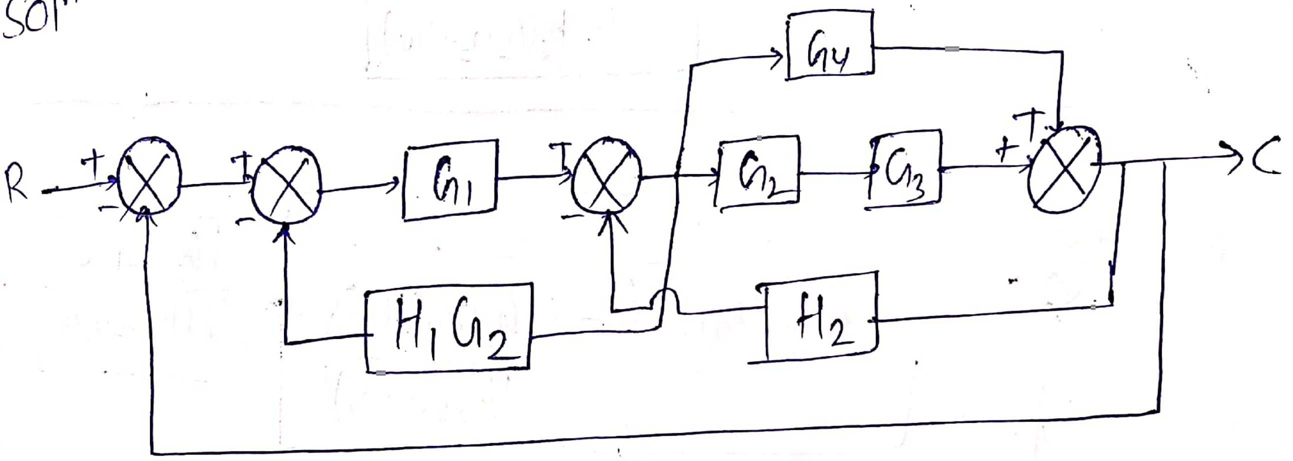
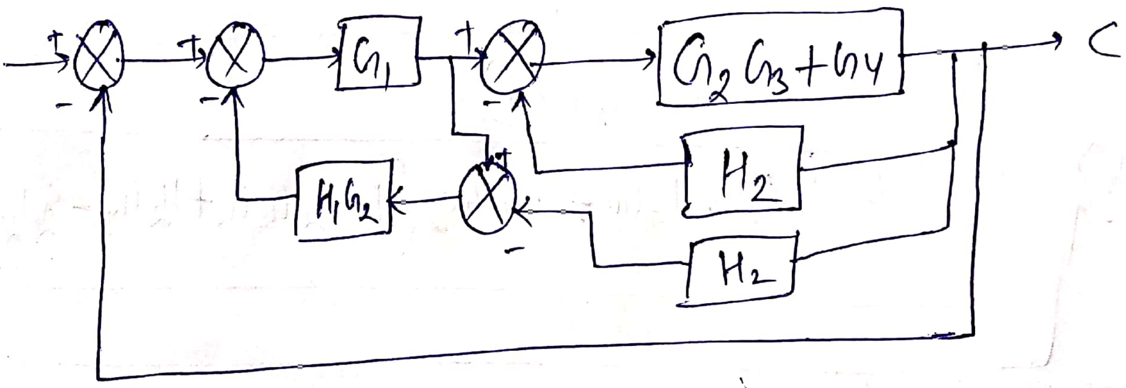
1. Find the transfer function for the system whose block diagram represented as belao:

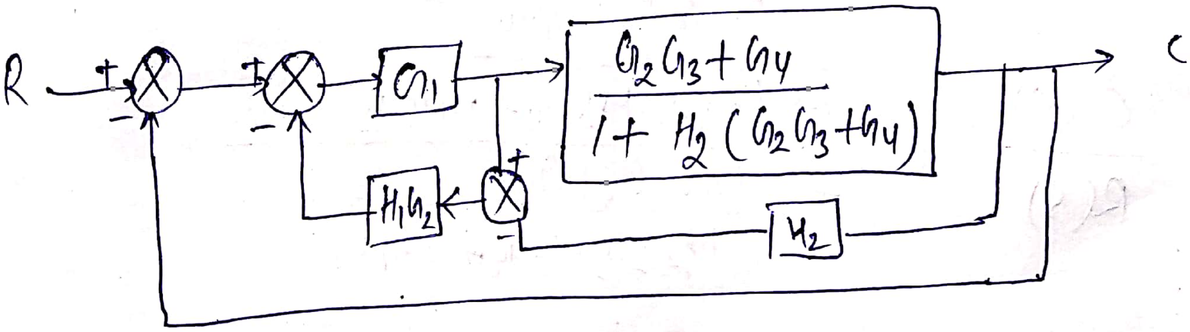
(a)

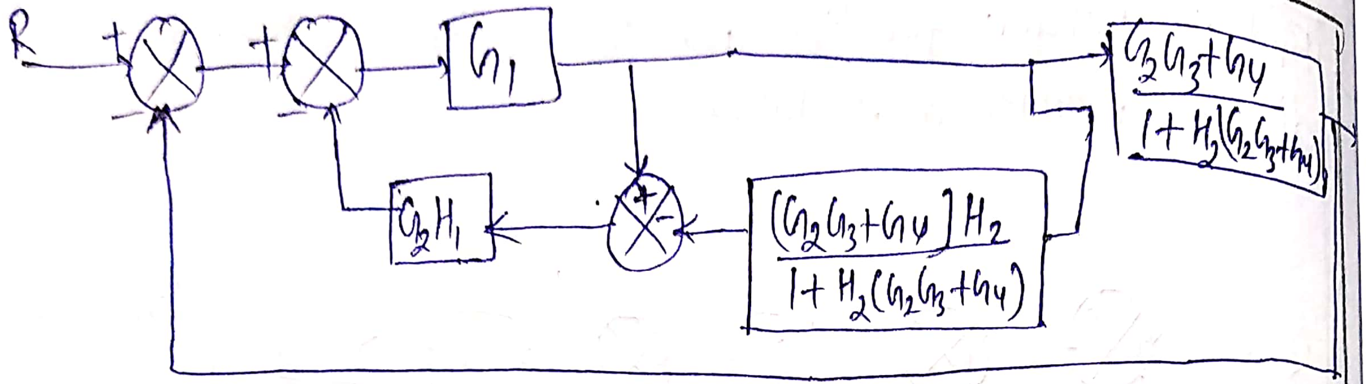


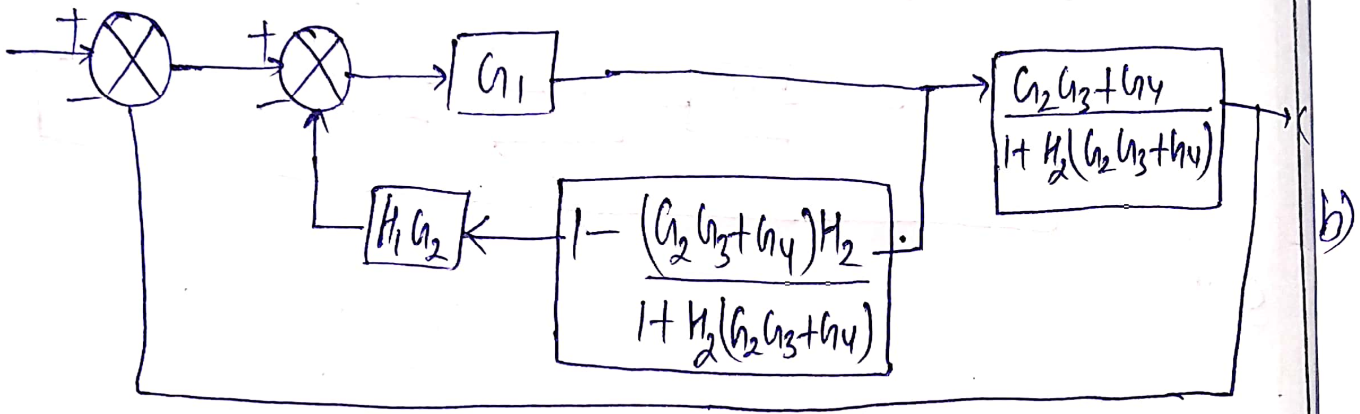
Soln

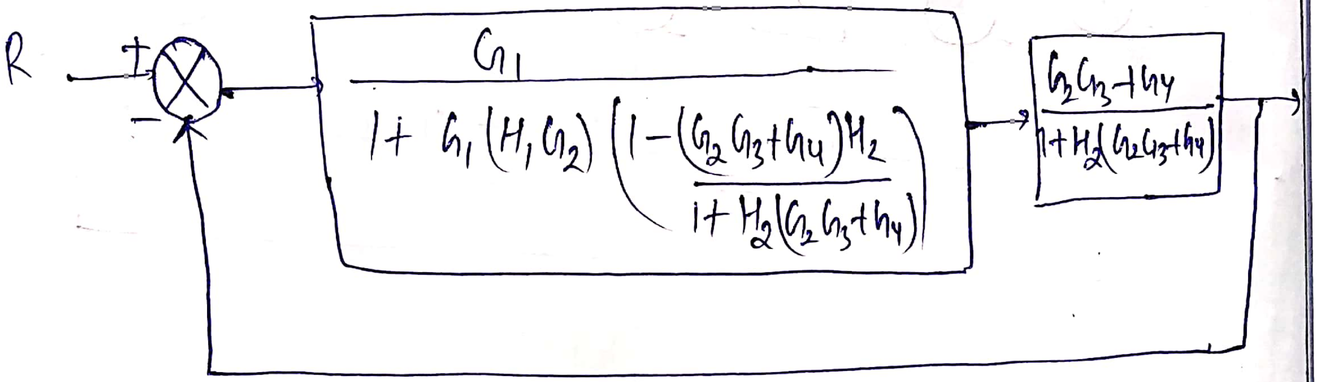


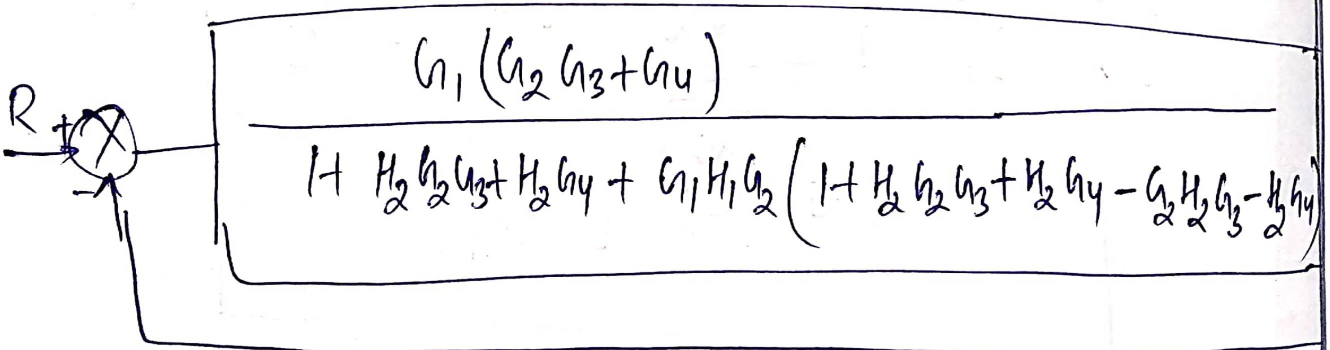












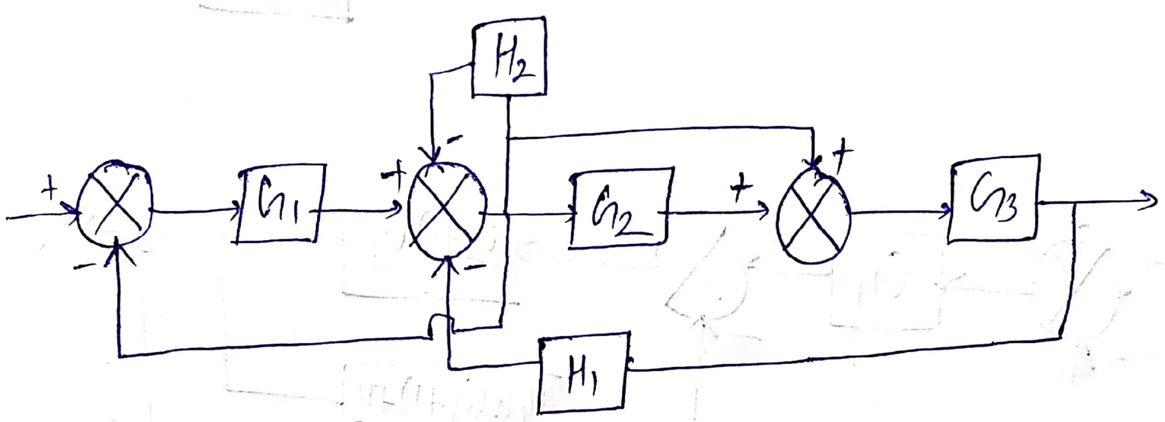
∴ T.F is

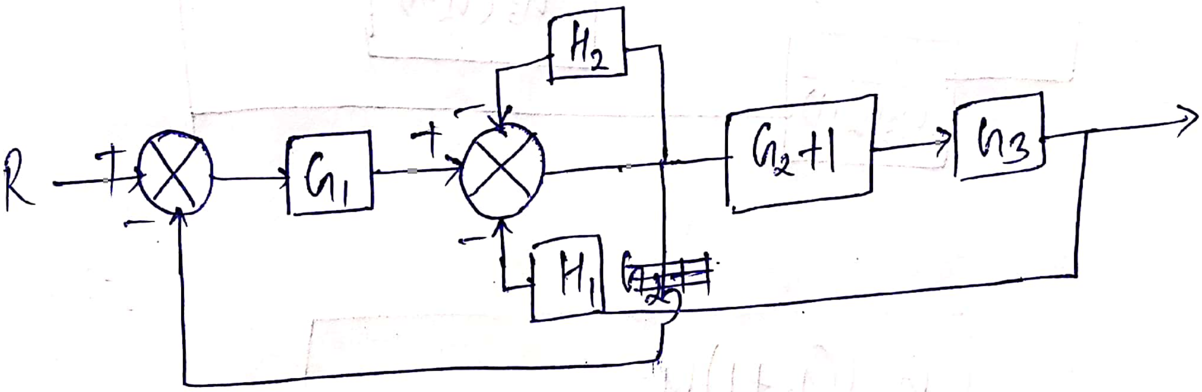
$$\frac{C(S)}{R(S)} = \frac{\dfrac{G_1G_2G_3+G_1G_4}{1+H_2G_2G_3+H_2G_4+G_1H_1G_2}}{1+\dfrac{G_1G_2G_3+G_1G_4}{1+H_2G_2G_3+H_2G_4+G_1H_1G_2}}$$

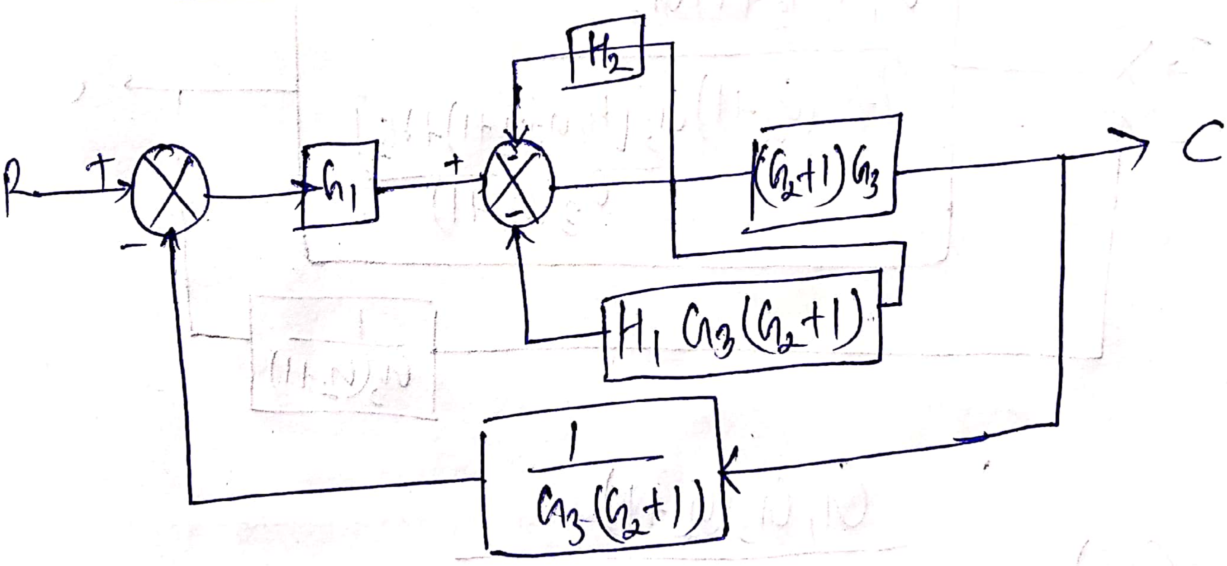
$$\frac{C(S)}{R(S)} = \frac{G_1G_2G_3+G_1G_4}{1+H_2G_2G_3+H_1G_1G_2+H_2G_4+G_1G_2G_3+G_1G_4}$$

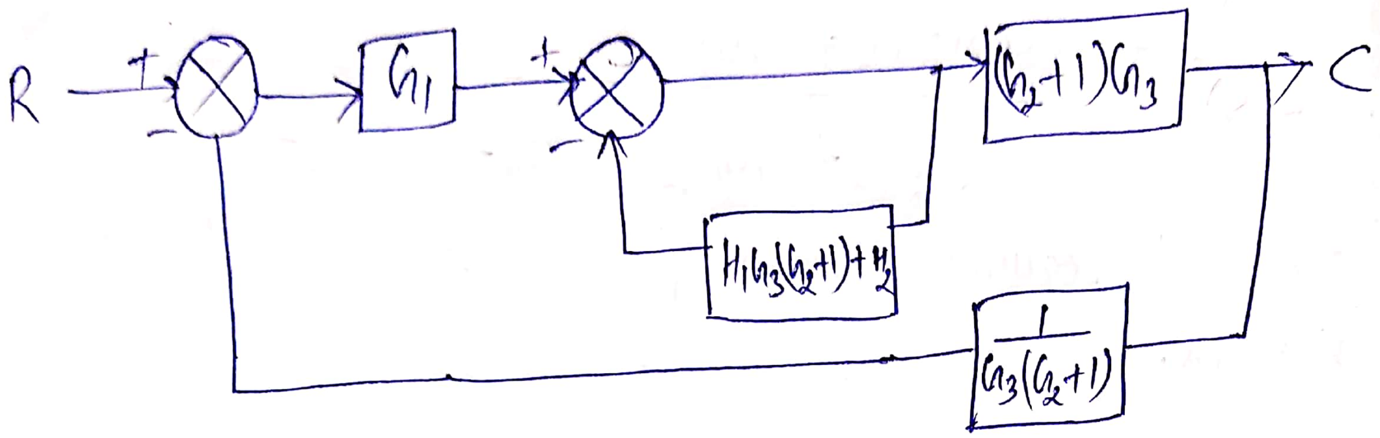
which is required transfer function of above block diagram.

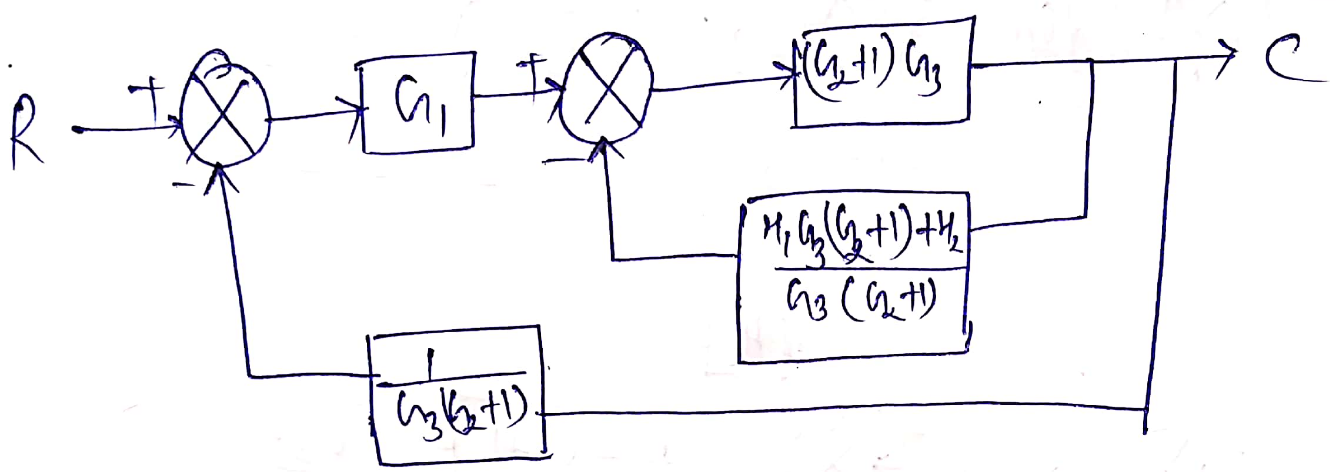
b)

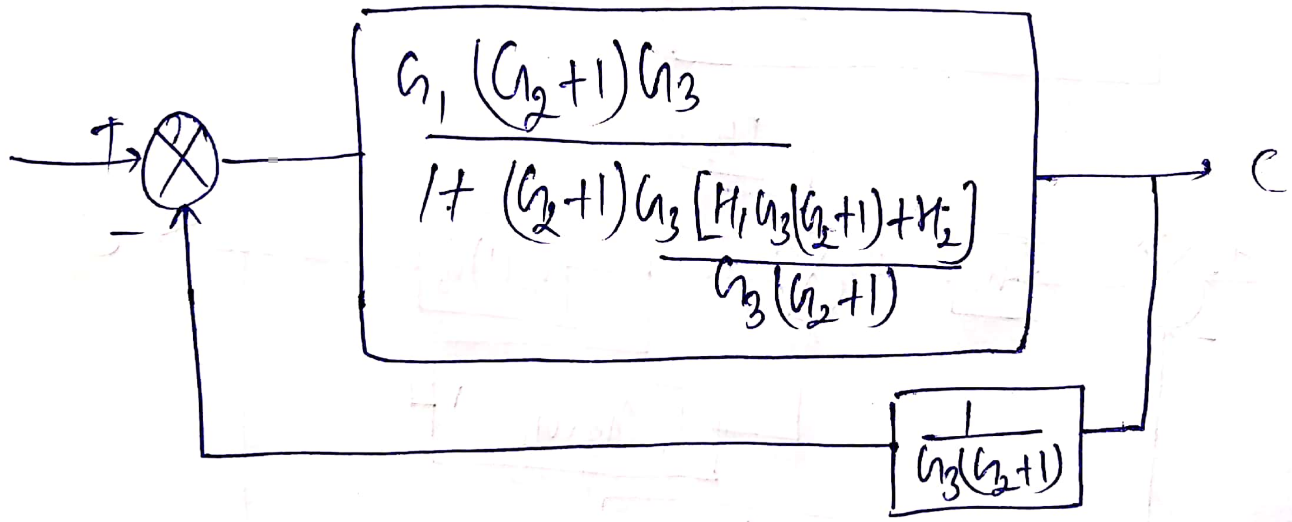








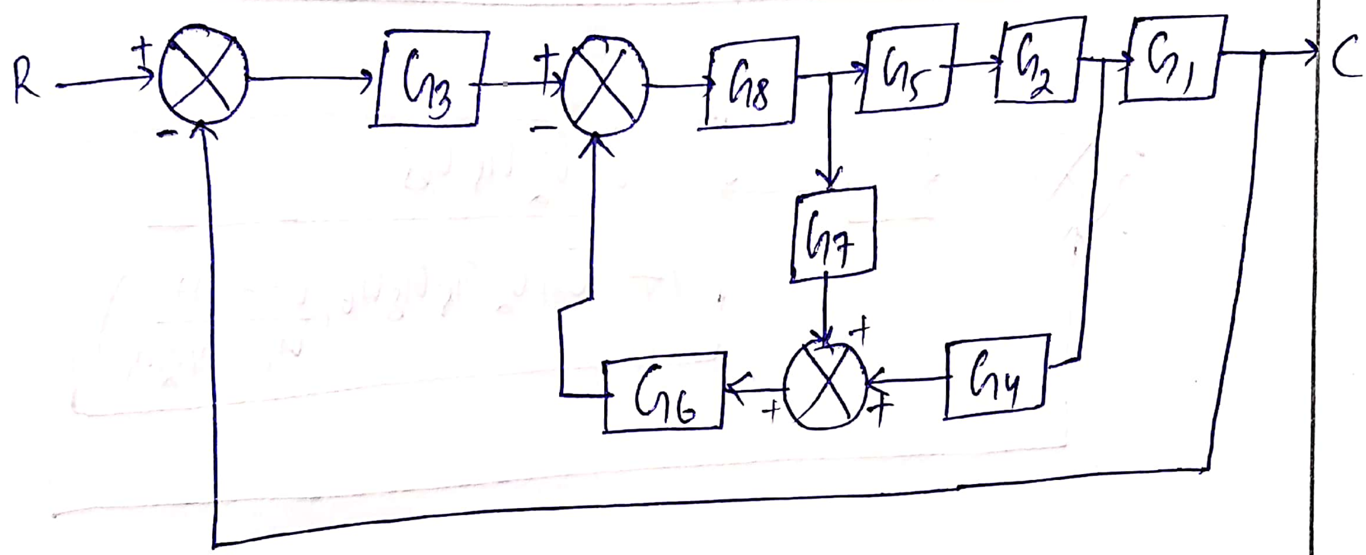


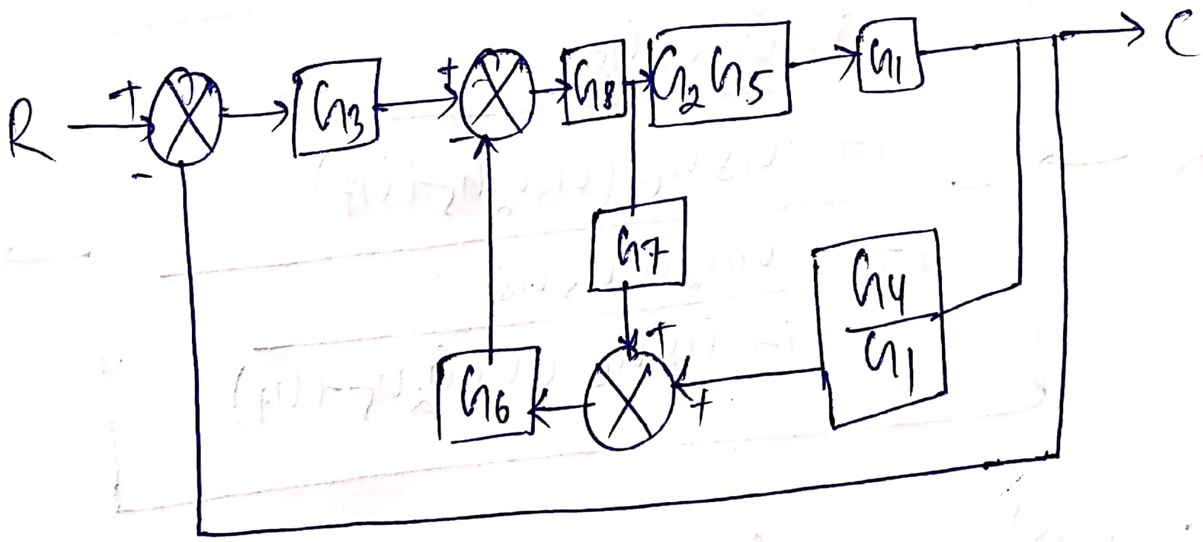


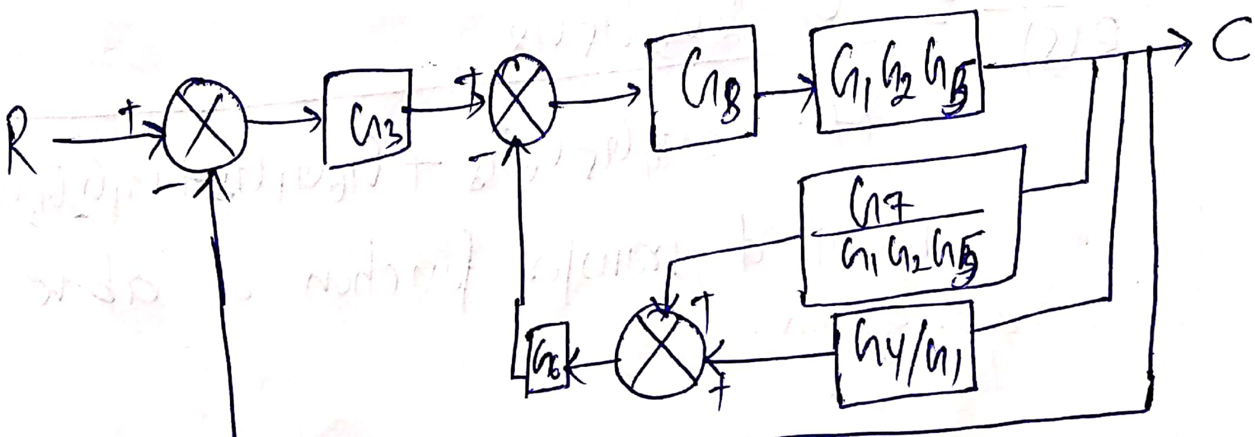
$$\frac{C(S)}{R(S)} = \frac{\dfrac{G_1 G_3 (G_2+1)}{1 + H_2 + H_1 G_2 G_3 + H_1 G_3}}{1 + \dfrac{1}{G_3(G_2+1)} \times \dfrac{G_1 G_3 (G_2+1)}{1 + H_2 + H_1 G_3 + H_1 G_2 G_3}}$$

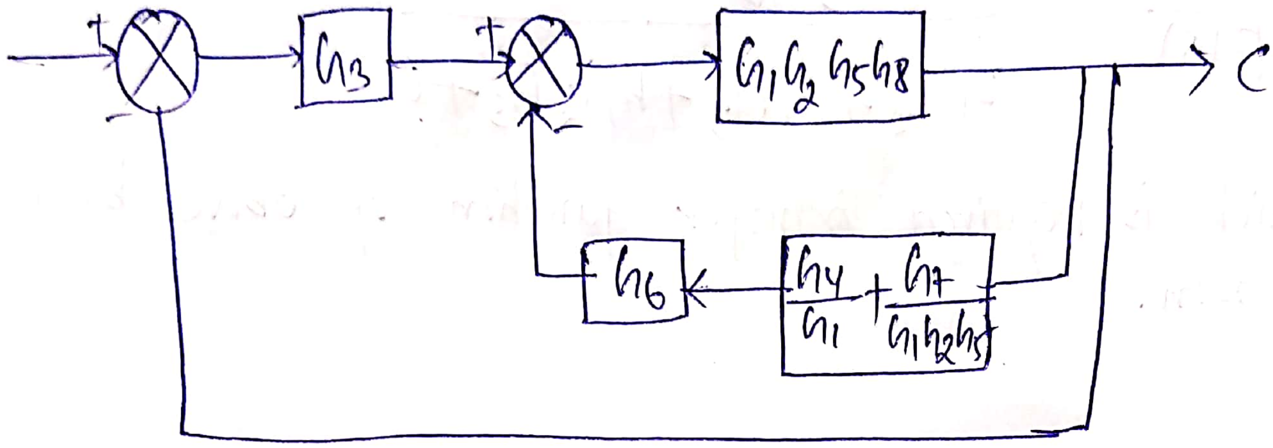
$$\frac{C(S)}{R(S)} = \frac{G_1 G_3 (G_2 + 1)}{1 + H_2 + H_1 G_3 + H_1 G_2 G_3 + G_1}$$

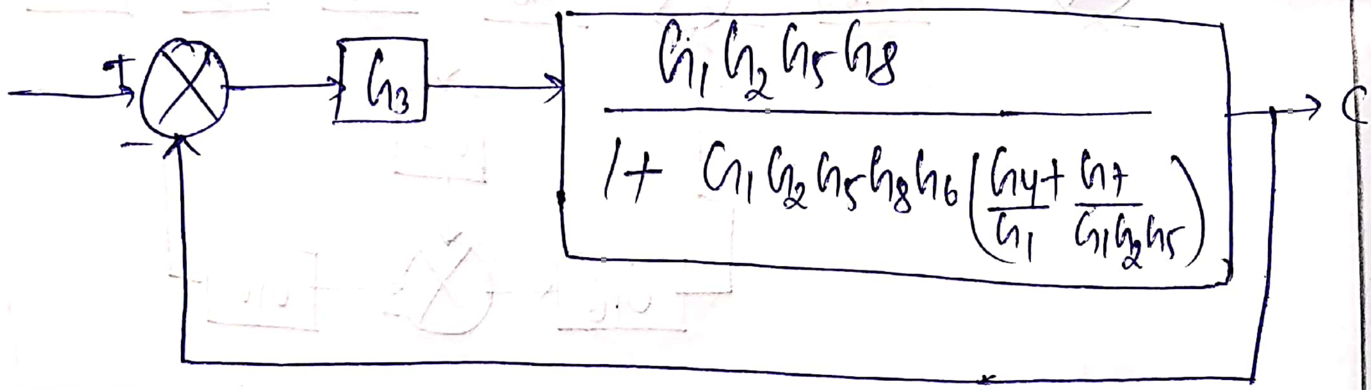
which is required transfer function of above block diagram.

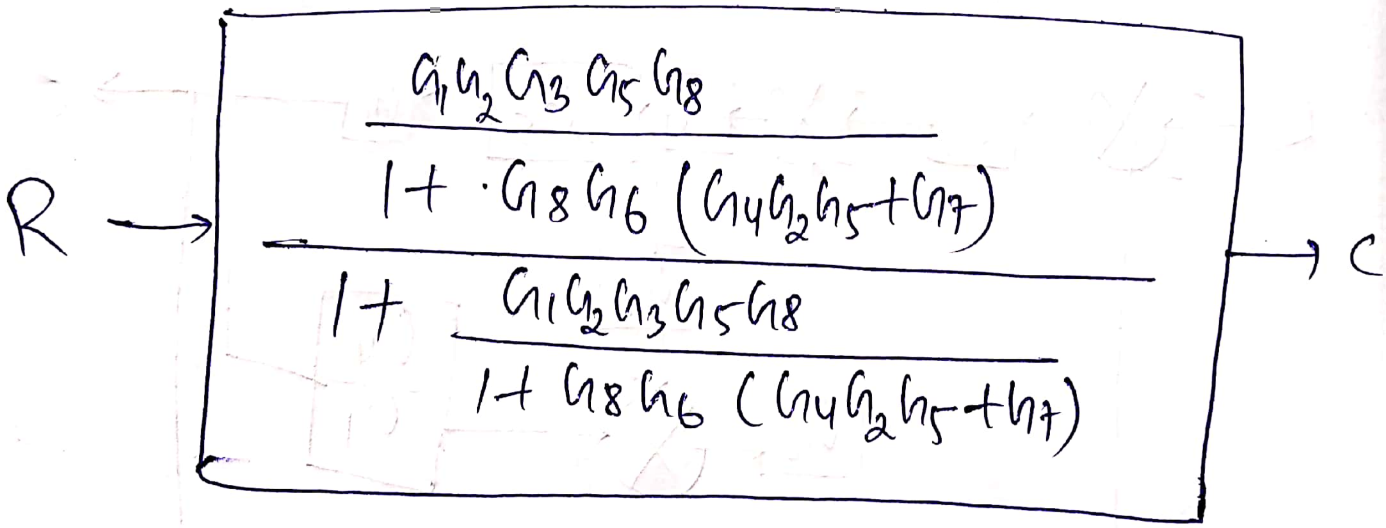
5) 







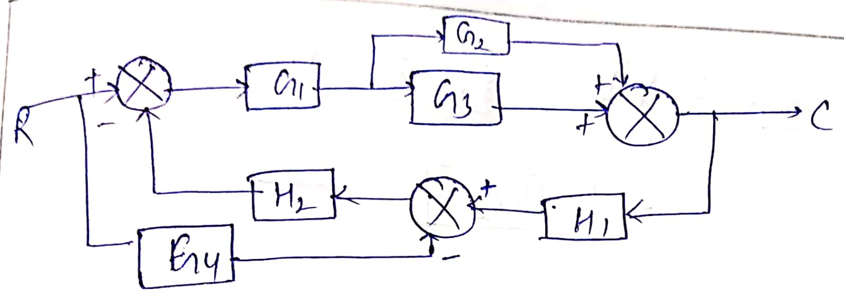




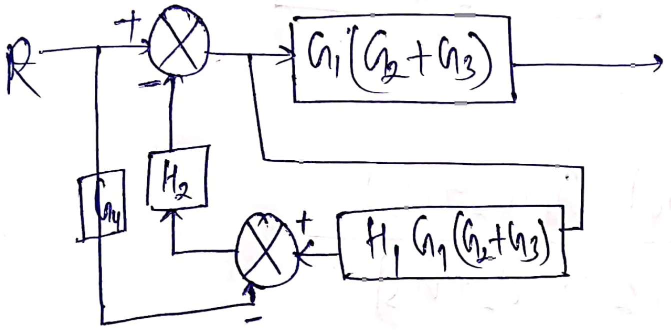
$$\therefore \frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_5G_8}{1 + G_2G_4G_5G_6G_8 + G_6G_7G_8 + G_1G_2G_3G_5G_8}$$

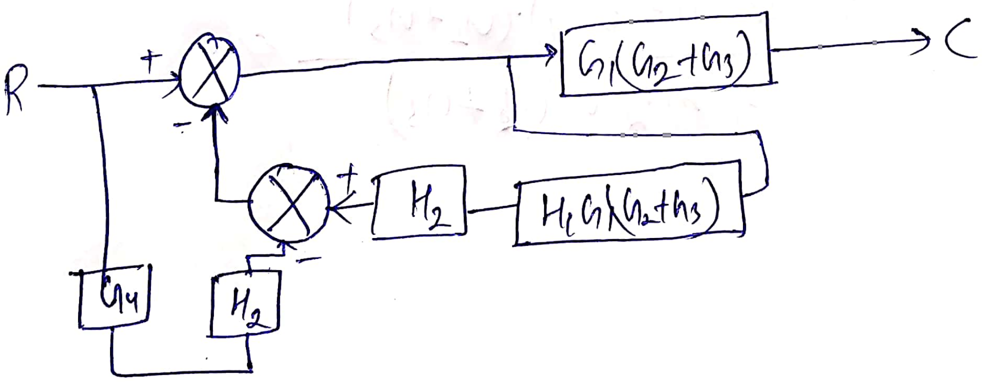
which is required transfer function of above block diagram.

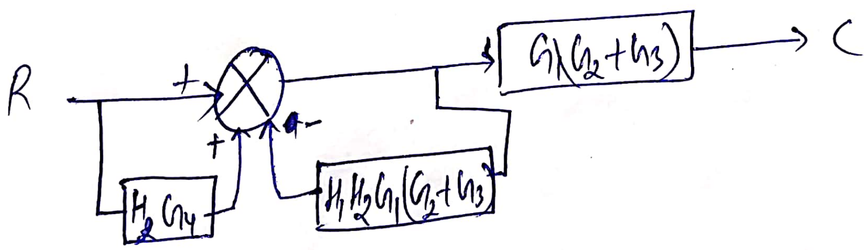
d)

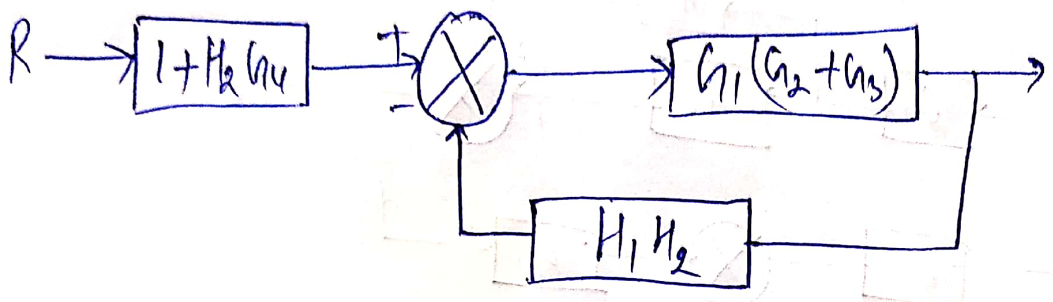

R
+
−
G_1
G_2
G_3
+
+
C
H_2
+
H_1
G_4
−

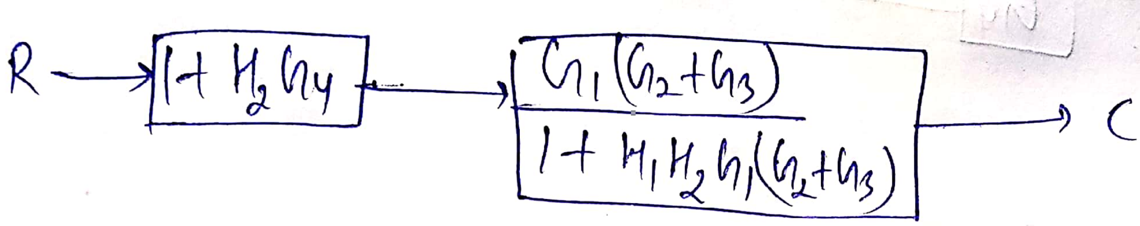
Sol^n


R
+
−
$G_1(G_2+G_3)$
C
H_2
G_4
+
$H_1 G_1(G_2+G_3)$
−


R
+
−
$G_1(G_2+G_3)$
C
+
H_2
$H_1 G_1(G_2+G_3)$
−
G_4
H_2


R
+
+
−
$G_1(G_2+G_3)$
C
$H_2 G_4$
$H_1 H_2 G_1(G_2+G_3)$



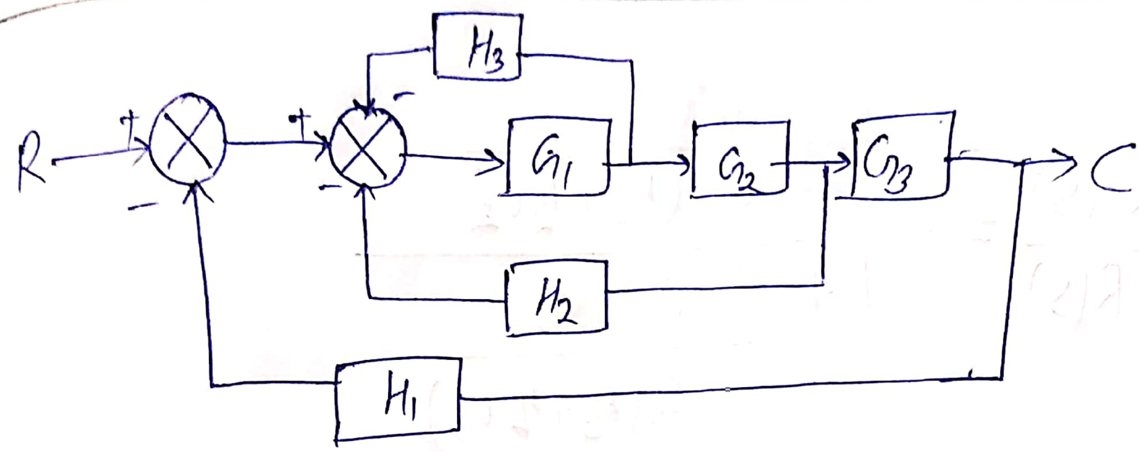


Now,

$$\frac{C(s)}{R(s)} = \frac{(1+H_2G_4)\; G_1(G_2+G_3)}{1+H_1H_2G_1(G_2+G_3)}$$

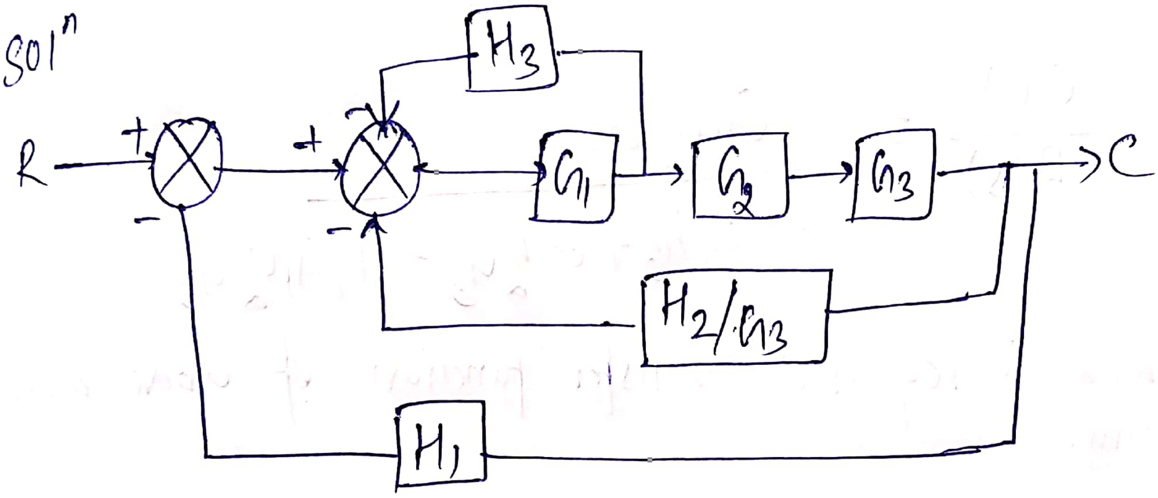
$$= \frac{G_1\,(1+H_2G_4)\cdot(G_2+G_3)}{1+H_1H_2G_1(G_2+G_3)} \;//$$

(e)

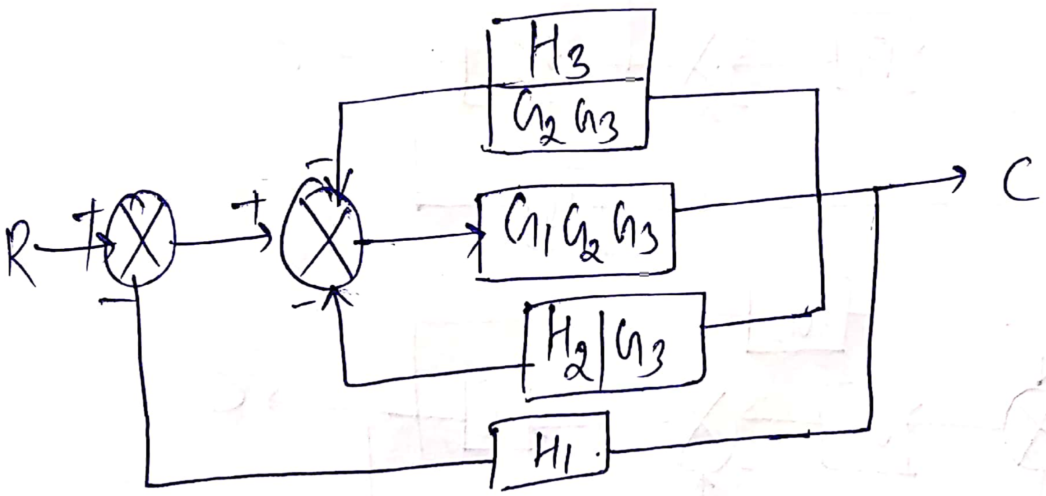


R
+
-
H_3
G_1
G_2
G_3
C
H_2
H_1

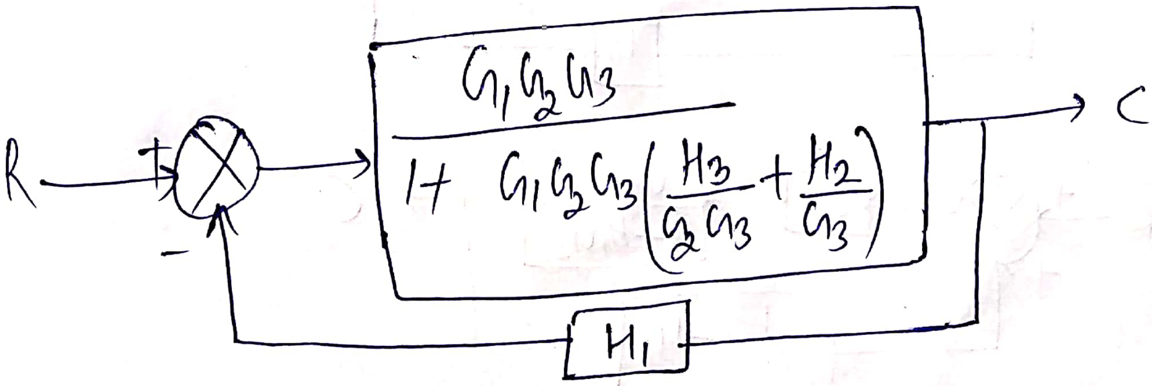
Sol^n



R
+
-
H_3
G_1
G_2
G_3
C
H_2/G_3
H_1



R
+
-
$\frac{H_3}{G_2 G_3}$
$G_1 G_2 G_3$
C
$H_2|G_3$
H_1



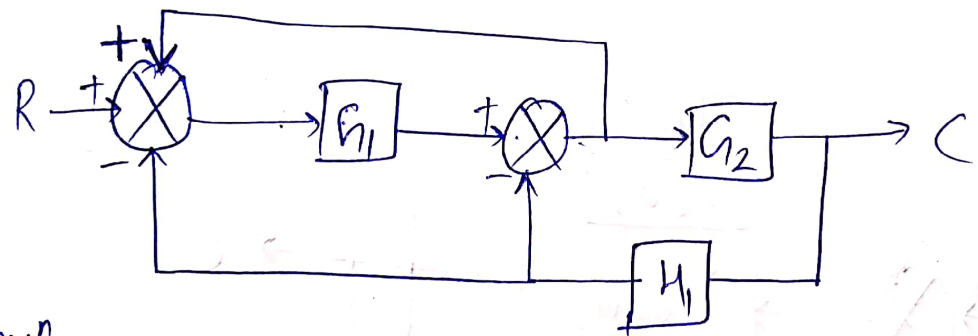
R
+
-
$\frac{G_1 G_2 G_3}{1+ G_1 G_2 G_3\left(\frac{H_3}{G_2 G_3}+\frac{H_2}{G_3}\right)}$
C
H_1

$$\frac{C(S)}{R(S)} = \frac{\dfrac{G_1 G_2 G_3}{1 + G_1 (H_3 + H_2 G_2)}}{1 + \dfrac{H_1 G_1 G_2 G_3}{1 + G_1 (H_3 + H_2 G_3)}}$$

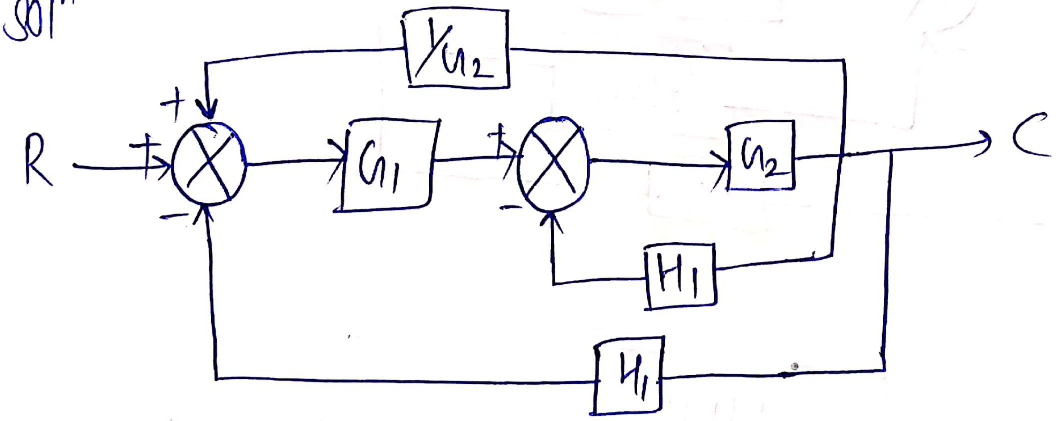
or, $$\frac{C(S)}{R(S)} = \frac{G_1 G_2 G_3}{1 + G_1 H_3 + G_1 H_2 G_3 + H_1 G_1 G_2 G_3}$$

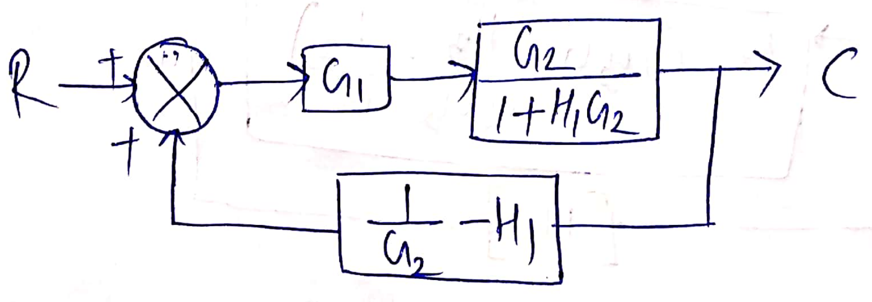
Which is required transfer function of above block diagram.

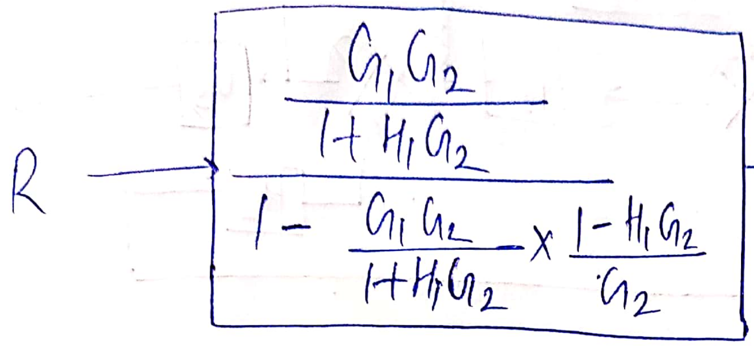
f.



sol^n







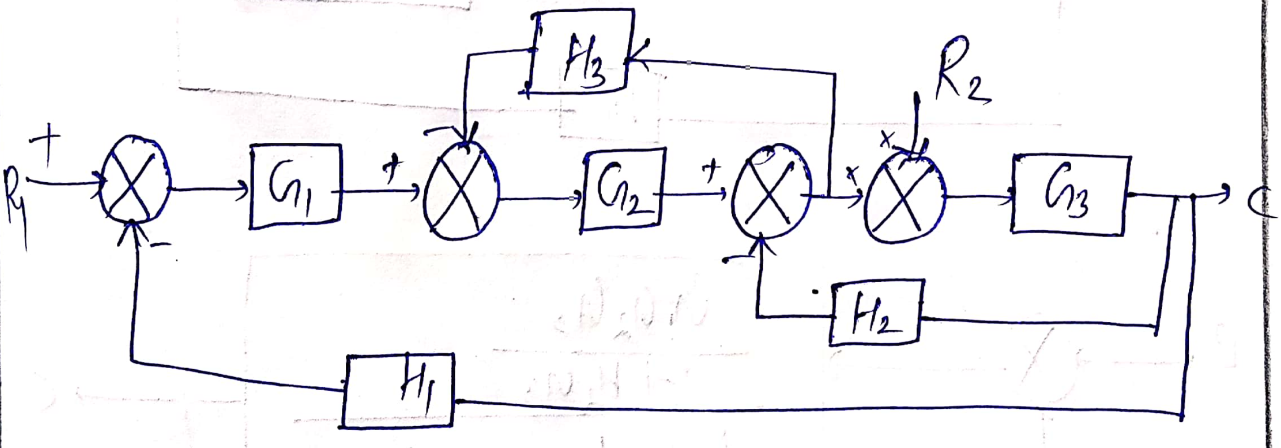
$$\frac{\dfrac{G_1 G_2}{1+H_1 G_2}}{1-\dfrac{G_1 G_2}{1+H_1 G_2}\times\dfrac{1-H_1 G_2}{G_2}}$$

$$\frac{C(S)}{R(S)} = \frac{G_1 G_2 \cdot G_2}{G_2(1+H_1 G_2) - G_1 G_2(1-H_1 G_2)}$$

$$\therefore \frac{C(S)}{R(S)} = \frac{G_1 G_2}{1+H_1 G_2 - G_1 + G_1 H_1 G_2}$$

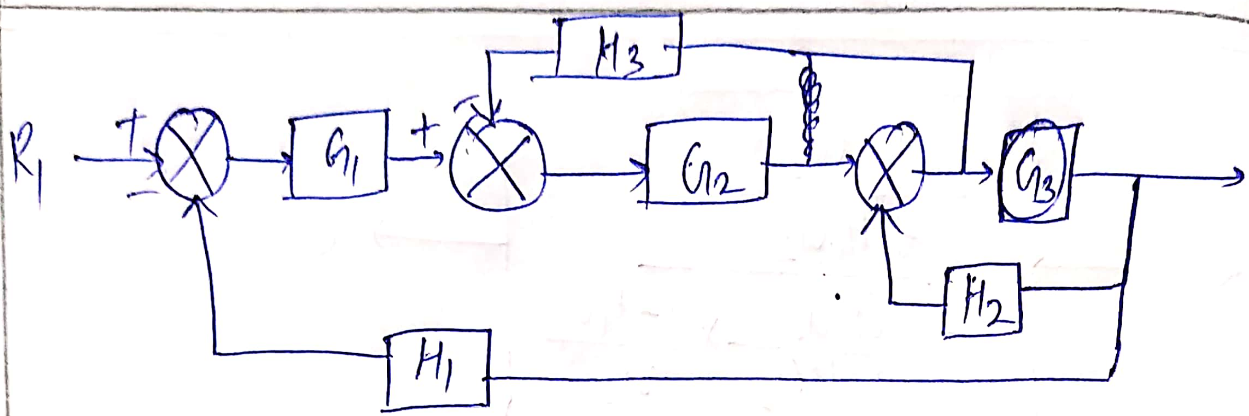
which is required transfer function of above block diagram.

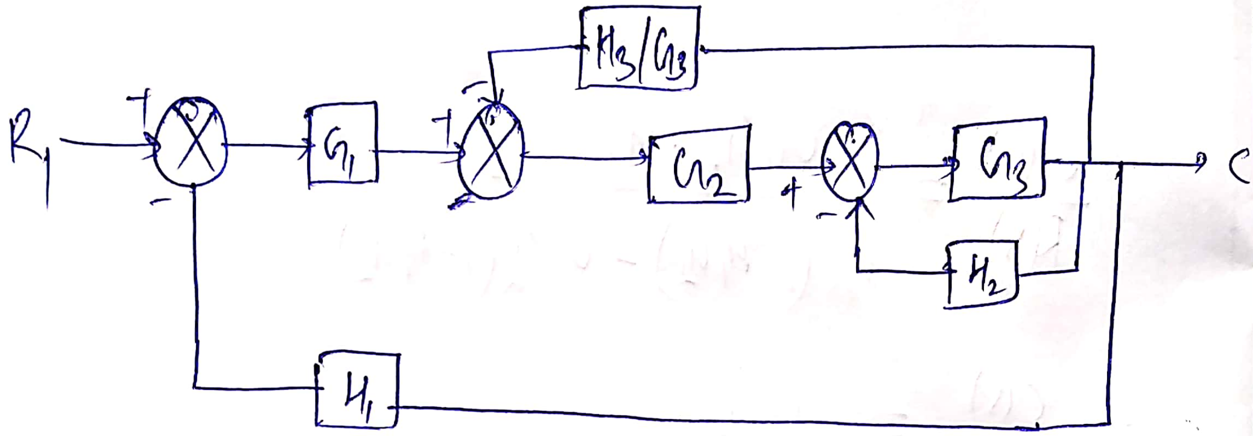
g.

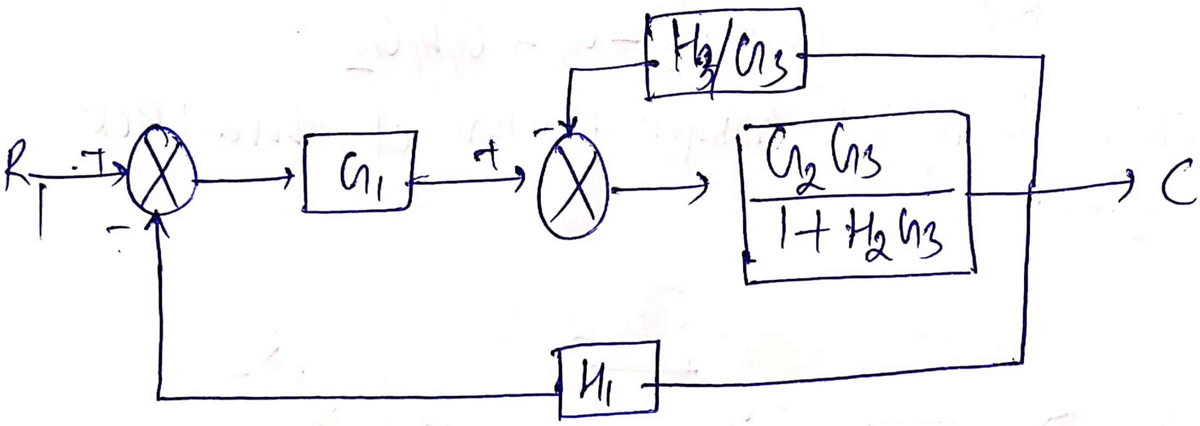


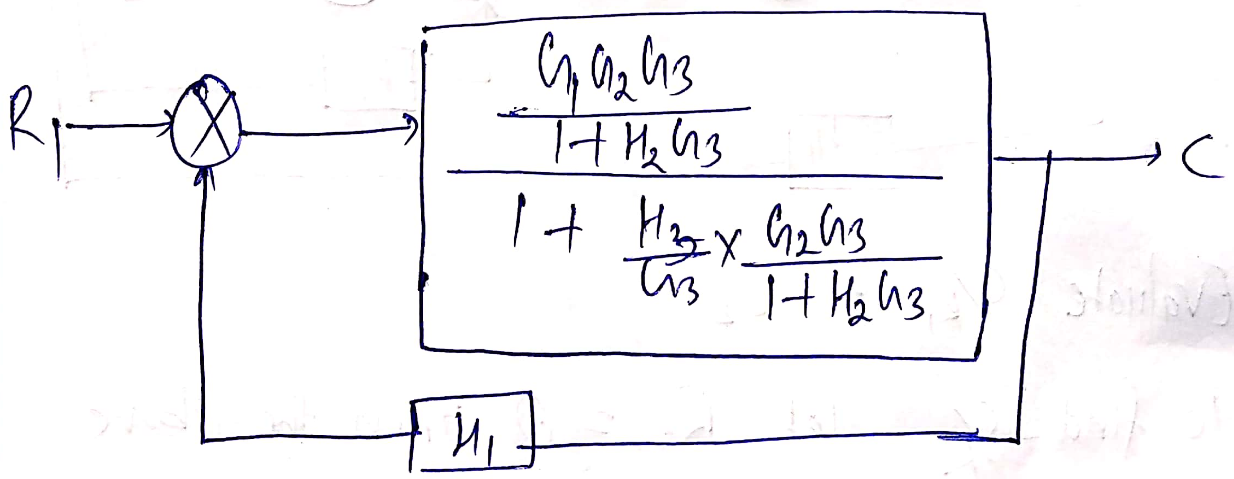
Evaluate C/R_1 & C/R_2

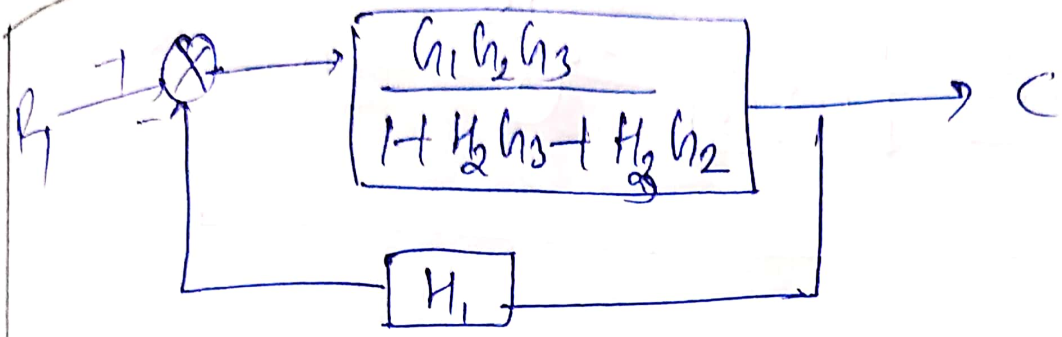
To find C/R_1, let $R_2 = 0$ then the above block diagram can be reduced as


R
+
-
G1
+
-
H3
G2
G3
H2
H1


R
+
-
G1
+
-
H3/G3
G2
+
-
G3
H2
H1
C

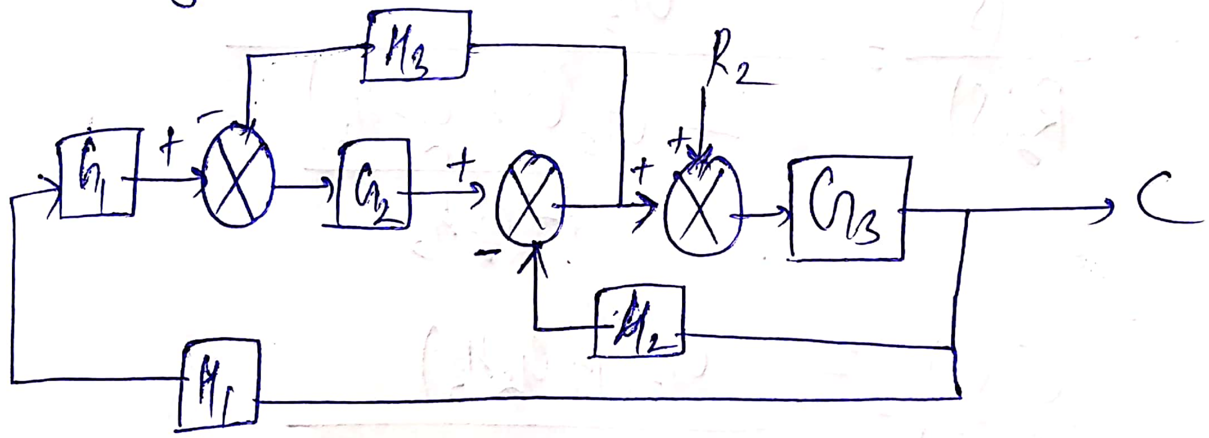

R
+
-
G1
+
-
H3/G3
G2 G3 / 1 + H2 G3
H1
C

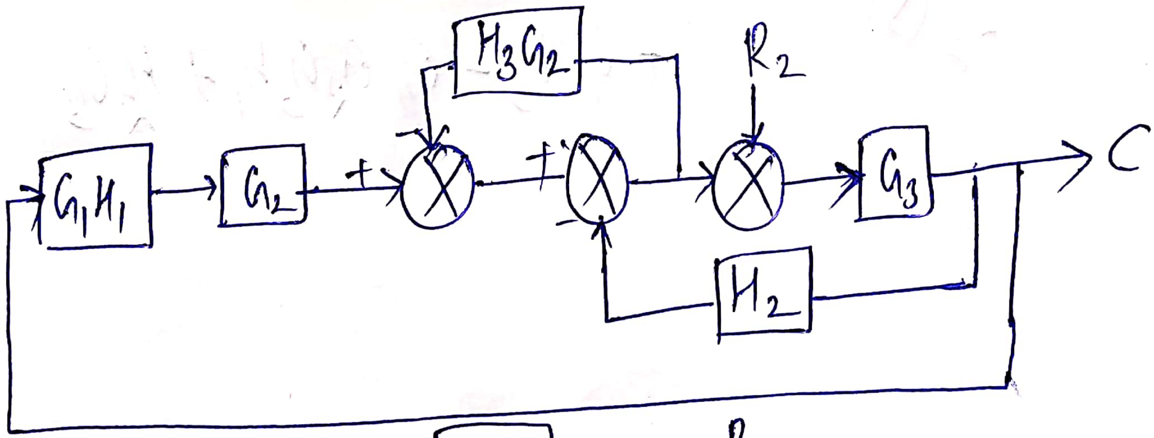

R
G1 G2 G3 / 1 + H2 G3
1 + H3/G3 x G2 G3 / 1 + H2 G3
H1
C

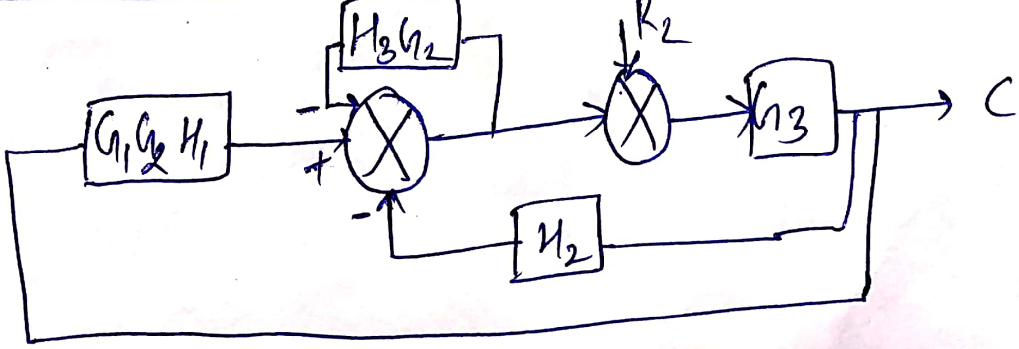


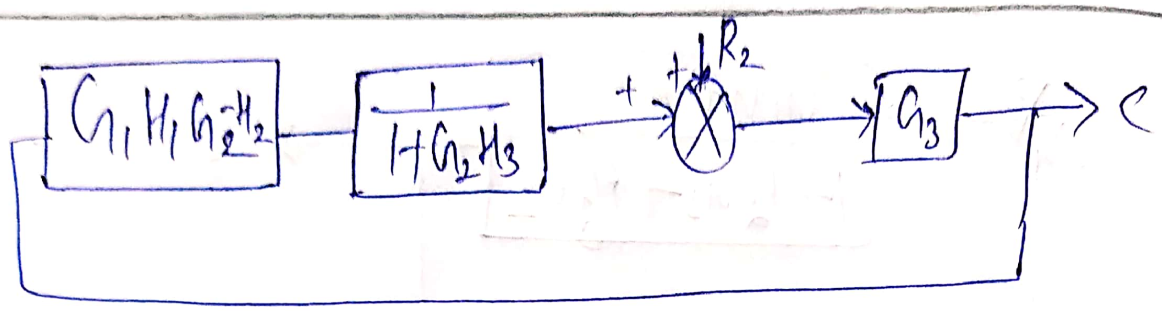
$$\therefore \frac{C(s)}{R_1(s)} = \frac{G_1 G_2 G_3}{1+H_2G_3+H_3G_2+G_1G_2G_3H_1} \quad \#$$

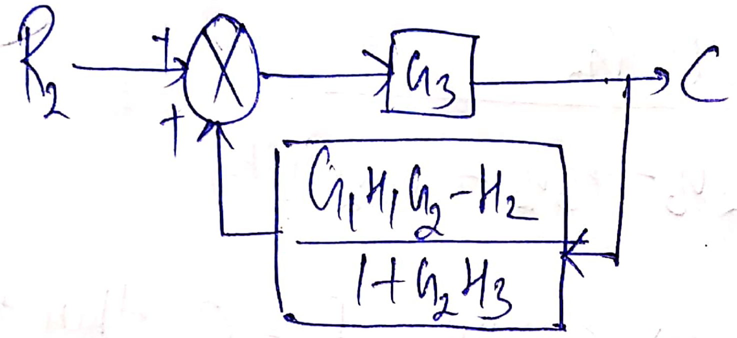
To find $\frac{C}{R_2}$ let us assume $R_1 = 0$ then the block diagram becomes.







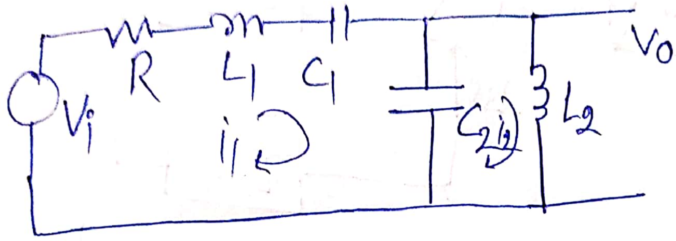




$$\therefore \frac{C(S)}{R_2(S)} = \frac{G_3}{1 - \frac{G_3(G_1H_1G_2-H_2)}{1+G_2H_3}}$$

$$\therefore \frac{C(S)}{R_2(S)} = \frac{G_3(1+G_2H_3)}{1+G_2H_3 - G_1G_2G_3H_1 + H_2G_3}$$

1) From the electrical circuit shown below, construct the block diagram.



Also obtain transfer function $\frac{V_o(S)}{V_i(S)}$ using block reduction technique & also from differential equation.

Soln

We have,

$$V_i - V_o = R i_1 + L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int (i_1 - i_2)\, dt$$

L.T

$$V_1(S) - V_o(S) = \left(R + L_1 S + \frac{1}{C_1 S}\right) I_1$$

$$I_1(S) = \frac{V_1(S) - V_o(S)}{R + L_1 S + \frac{1}{C_1 S}}$$

Again,

$$V_o = \frac{1}{C_2}\int (i_1 - i_2)\, dt$$

L.T

or, $$V_o = \frac{I_1(S) - I_2(S)}{C_2 S}$$

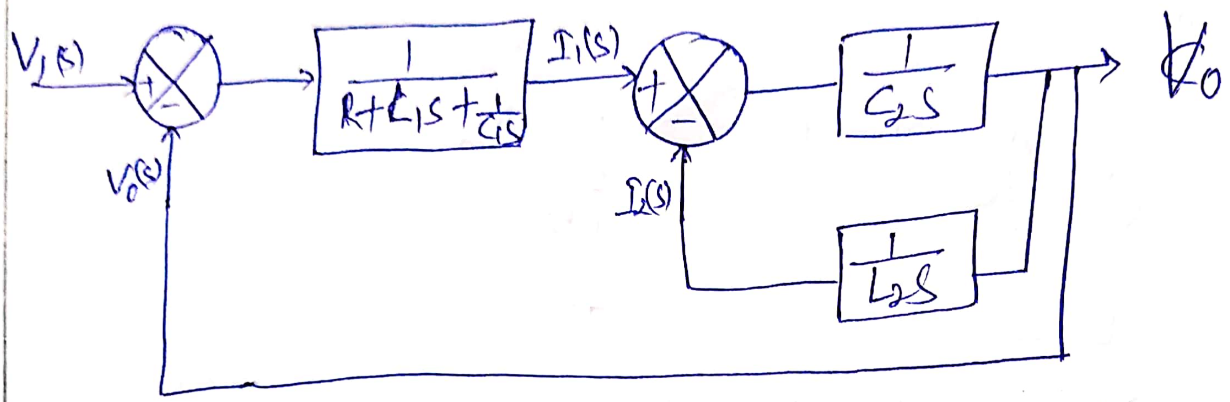
Again,

$$V_o = L_2 \frac{di_2}{dt}$$

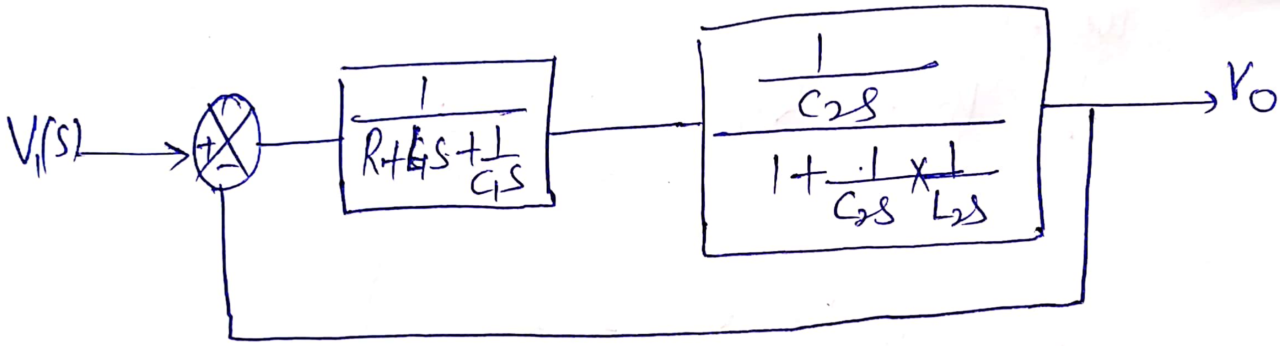
L.T

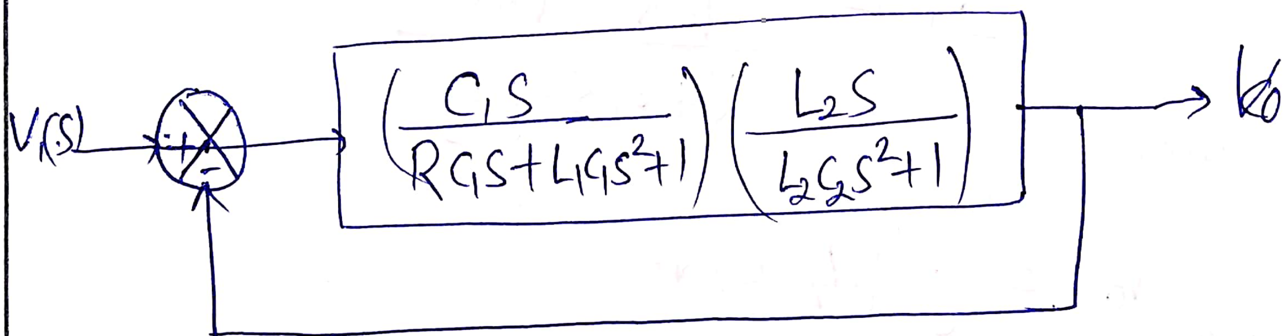
$$I_2(S) = \frac{V_o}{L_2 S}$$

Now block diagram can be drawn as



Now, transfer function can be calculated by block diagram reduction method as





Now,

$$\frac{V_o(s)}{V_i(s)} = \frac{\dfrac{C_1L_2s^2}{(RC_1s+L_1C_1s^2+1)(1+L_2C_2s^2)}}{1+\dfrac{C_1L_2s^2}{(RC_1s+L_1C_1s^2+1)(1+L_2C_2s^2)}}$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{C_1L_2s^2}{(RC_1s+L_1C_1s^2+1)(1+L_2C_2s^2)+C_1L_2s^2}$$

Now,
By differential Equation,
For 1st loop,

$$V_i = R + L\frac{di_1}{dt} + \frac{1}{C_2}\int(i_1 - i_2)\,dt + \frac{1}{C_1}\int i_1 dt \quad \text{--- (1)}$$

Again,

$$0 = L_2\frac{di_2}{dt} + \frac{1}{C_2}\int(i_2 - i_1)\,dt \quad \text{--- (2)}$$

Again,

$$V_o = L_2\frac{di_2}{dt} \quad \text{--- (3)}$$

L.T of eq^n (3) is

$$V_o(S) = L_2 S\, I_2(S)$$

$$\therefore I_2 = \frac{V_o}{L_2 S} \quad \text{--- (4)}$$

L.T of eq^n (2) is

$$0 = L_2 S\, I_2 + \frac{I_2 - I_1}{C_2 S}$$

$$0 = \frac{V_o L_2 S}{L_2 S} + \frac{V_o}{C_2 S^2 L_2} - \frac{I_1}{C_2 S}$$

or, $$I_1 = V_o\left(\frac{C_2 L_2 S^2 + 1}{L_2 S}\right) \quad \text{--- (5)}$$

L.T of eq^n (1) is

$$V_1 = RI_1 + L_1 s I_1 + \frac{I_1}{C_1 s} + \frac{I_1 - I_2}{C_2 s}$$

From eq^n (4) & (5), we get,

$$V_1 = \left(R + L_1 s + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right) \frac{V_0 (1 + C_2 L_2 s^2)}{L_2 s} - \frac{1}{C_2 s} \frac{V_0}{L_2 s}$$

$$\frac{V_1}{V_0} = \frac{(RC_1C_2 s + L_1 C_1 C_2 s^2 + C_1 + C_2)(1 + C_2 L_2 s^2)}{L_2 C_1 C_2 s^2} - \frac{1}{C_2 L_2 s^2}$$

$$\frac{V_1}{V_0} = \frac{(RC_1C_2 s + L_1 C_1 C_2 s^2 + C_1 + C_2)(1 + C_2 L_2 s^2) - C_1}{C_1 C_2 L_2 s^2}$$

or, $$\frac{V_0}{V_1} = \frac{C_1 C_2 L_2 s^2}{RC_1C_2 s + L_1 C_1 C_2 s^2 + \cancel{C_1} + C_2 + RC_1 C_2^2 L_2 s^3 + L_1 C_1 C_2^2 L_2 s^4 + C_1 C_2 L_2 s^2 + C_2^2 L_2 s^2 - \cancel{C_1}}$$

$$= \frac{C_1 L_2 s^2}{RC_1 s + L_1 C_1 s^2 + 1 + RC_1 C_2 L_2 s^3 + L_1 C_1 C_2 L_2 s^4 + C_1 L_2 s^2 + C_2 L_2 s^2}$$

$$= \frac{C_1 L_2 s^2}{(RC_1 s + L_1 C_1 s^2 + 1)1 + L_2 C_2 s^2 (RC_1 s + L_1 C_1 s^2 + 1) + C_1 L_2 s^2}$$

$$\therefore \frac{V_0(s)}{V_1(s)} = \frac{C_1 L_2 s^2}{(RC_1 s + L_1 C_1 s^2 + 1)(1 + L_2 C_2 s^2) + C_1 L_2 s^2}$$

which is equal.

1. A function F(s) is given as

$$F(s) = \frac{4(s+2)(s+3)}{s(s+1)(s+4)}$$

Calculate the initial value f(0) for the given function.

Sol^n Use partial fraction

$$F(s) = \frac{4(s+2)(s+3)}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}$$

$$\text{or, } 4(s^2+5s+6) = A(s^2+5s+4) + Bs^2 + 4Bs + Cs^2 + Cs$$

$$\text{or, } 4s^2 + 20s + 24 = (A+B+C)s^2 + (5A+4B+C)s + 4A$$

Equating like terms,

$$24 = 4A$$
$$A = 6$$

$$5A + 4B + C = 20 \qquad A + B + C = 4$$
$$4B + C = -10 \qquad B + C = -2$$

$$B = -8/3 \qquad C = 2/3.$$

Now,

$$F(s) = \frac{6}{s} - \frac{8}{3(s+1)} + \frac{2}{3(s+4)}$$

Now, Use laplace inverse, we get

$$\mathcal{L}^{-1}F(s) = f(t) = \mathcal{L}^{-1}\left[\frac{6}{s} - \frac{8}{3(s+1)} + \frac{2}{3(s+4)}\right]$$

$\therefore f(t) = 6 - \frac{8}{3}e^{-t} + \frac{2}{3}e^{-4t}$

$$f(0) = 6 - \frac{8}{3}e^{-0} + \frac{2}{3}e^{-0}$$

$$= 6 - 8/3 + \frac{2}{3}$$

$$= \frac{18-8+2}{3} = 4.$$

2. A function $F(s)$ is given as

$$F(s) = \frac{4(s+2)(s+3)}{s(s+1)(s+4)}$$

Calculate the final value $f(t)$ for the given function,

Solⁿ

From the solution of Q. N. 1, we have get

$$f(t) = 6 - \frac{8}{3}e^{-t} + \frac{2}{3}e^{-4t}.$$

Obtain the laplace Inverse of

$$G(s) = \frac{s^3+5s^2+9s+7}{(s+1)(s+2)}$$

sol^n

By using partial fraction,

$$\frac{s^3+5s^2+9s+7}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{s+2}$$

or, $s^3+5s^2+9s+7 = As+2A+Bs+B$

ie $A+B=9$ & $2A+B=7$

Solving, we get

$A=-2$

$$G(s) = \frac{s^2(s+2)+3s(s+2)+3(s+2)+1}{(s+1)(s+2)}$$

$$= \frac{(s^2+3s+3)(s+2)}{(s+1)(s+2)} + \frac{1}{(s+1)(s+2)}$$

$$= \frac{s(s+1)+2(s+1)}{s+1} + \frac{1}{s+1} + \frac{1}{(s+1)s+2}$$

$$= s+2 + \frac{s+2+1}{(s+1)(s+2)}$$

$$= s+2 + \frac{s+3}{(s+1)(s+2)}$$

Now,

$$\frac{s+3}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

or, $s+3 = As + 2A + Bs + B$

Equating the coefficient of like terms,

$A + B = 1 \qquad A = 1 - B$

$2A + B = 3 \qquad A = 2$

$2 - 2B + B = 3$

$B = -1$

$$\therefore G(s) = s + 2 + \frac{2}{s+1} - \frac{1}{s+2}$$

Now, Inverse laplace transform

$$\mathcal{L}^{-1} G(s) = g(t) = \mathcal{L}^{-1}\left(s + 2 + \frac{2}{s+1} - \frac{1}{s+2}\right)$$

$$= \delta'(t) + 2\delta(t) + 2e^{-t} - e^{-2t}$$

$\because \mathcal{L}^{-1}(s) = \delta'(t)$

$\because \mathcal{L}^{-1} 1 = \delta(t)$ #

1. Obtain the laplace transform of the following differential equation given by

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 3x = 4$$

a) Initial conditions $x'(0) = 3$ & $x(0) = 2$

b) zero initial conditions.

Solⁿ

a) we have,

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 3x = 4 \quad -①$$

L.T of eqⁿ ① is

$$s^2X - s\,x(0) - x'(0) + 2[sX - x(0)] + 3\cdot X = \frac{4}{s}$$

where $X = \mathcal{L}^{-1}x = X(s)$

$$\therefore s^2X - 2s - 3 + 2sX - 4 + 3X = \frac{4}{s}$$

or, $X(s^2+2s+3) = \frac{4}{s} + 2s + 7$

or, $X(s) = \dfrac{4+2s^2+7s}{s(s^2+2s+3)}$

b) If all initial conditions are zero then,

$$s^2X + 2sX + 3X = \frac{4}{s}$$

$$\therefore X(S) = \frac{4}{S(S^2+2S+3)} \quad \#$$

5. Find the inverse laplace transform.

$$F(S) = \frac{2s+12}{s^2+2s+5}$$

Use the partial fraction

$$\cancel{\frac{2s+12}{s^2+2s+5}} = \cancel{A+B}\ \cancel{2s+12}$$

$$= \frac{2s+12}{s^2+2s+1+4}$$

$$= \frac{2s+12}{(s+1)^2+2^2}$$

$$= \frac{2(s+1)}{(s+1)^2+2^2} + \frac{10}{(s+1)^2+2^2}$$

Now,

$$\mathcal{L}^{-1}F(s) = f(t) = \mathcal{L}^{-1}\frac{2(s+1)}{(s+1)^2+2^2} + \mathcal{L}^{-1}\frac{10\times 2}{(s+1)^2+2^2}\times\frac{1}{2}$$

$$= 2e^{-t}\cos 2t + 10e^{-t}\cdot\tfrac{1}{2}\cdot\sin 2t$$

$$= 2e^{-t}\cos 2t + 5e^{-t}\sin 2t$$

$$= e^{-t}(2\cos 2t + 5\sin 2t) \quad \#$$

For the transfer function

$$G(s) = \frac{1}{2} \frac{(s^2+4)(1+2.5s)}{(s^2+2)(1+0.5s)}$$

Find poles and zeros in s-plane and determine the value of the transfer function at $s=2$.

Soln

$$G(s) = \frac{(s^2+4)(1+2.5s)}{2(s^2+2)(1+0.5s)}$$

$$= \frac{[s^2-(2i)^2]\left(\frac{1}{2.5}+s\right)\times 2.5}{\cancel{2}[s^2-(\sqrt{2}i)^2]\left(\frac{1}{0.5}+s\right)\times \cancel{0.5}}$$

$$= \frac{2.5(s+(2i))(s-2i)\left[s-\left(\frac{-1}{2.5}\right)\right]}{(s-\sqrt{2}i)(s+(\sqrt{2}i))[s-(-2)]}$$

$\therefore$ Zeros $= -2i, 2i, -\frac{1}{2.5}$

Poles $= \sqrt{2}i, -\sqrt{2}i, -2$.

7. Obtain the partial fraction expression of F(S) given below

$$F(S) = \frac{S^2+2S+3}{(S+1)^3}$$

By using partial fractions

$$F(S) = \frac{S^2+2S+3}{(S+1)^3} = \frac{A}{S+1} + \frac{B}{(S+1)^2} + \frac{C}{(S+1)^3}$$

or, $S^2+2S+3 = AS^2+2AS+A+BS+B+C$

Equating the coefficient of like terms,

$\therefore A+B+C=3 \qquad 2A+B=2, \; A=1$

$C=2 \qquad \therefore B=0$

$$\therefore F(S) = \frac{1}{S+1} + \frac{2}{(S+1)^3}$$

8. Find the solution x(t) of the differential equation.

$\ddot{x}+3\dot{x}+2x=0, \quad x(0)=a \;\&\; \dot{x}(0)=b.$

Laplace transform of above differential equation is

$\mathcal{L}\ddot{x} + \mathcal{L}3\dot{x} + \mathcal{L}2x = 0.$

or, $S^2X - S\,x(0) - x'(0) + 3[SX - x(0)] + 2X = 0$

or, $X(S^2+3S+2) - aS - b - 3a = 0$

$$\therefore X(S) = \frac{aS+b+3a}{S^2+3S+2}$$ #

Again,

$$X(s) = \frac{as+b+3a}{(s+1)(s+2)}$$

Use partial fraction

$$X(s) = \frac{as+b+3a}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

or, $as+b+3a = As+2A+Bs+B$

we get,

$2A+B = b+3a$ $\quad$ $A+B=a$.

$2a-2B+B = b+3a$ $\quad$ $A = a-B$

$B = -a-b$ $\quad$ $A = 2a+b$

$$\therefore X(s) = \frac{2a+b}{s+1} - \frac{a+b}{s+2}$$

Now, laplace inverse,

$$x(t) = \mathcal{L}^{-1}\frac{2a+b}{s+1} - \mathcal{L}^{-1}\frac{a+b}{s+2}$$

$$= (2a+b)e^{-t} - (a+b)e^{-2t} \quad \#$$

9: Find the solution $x(t)$ of the differential equations,

$\ddot{x} + 2\dot{x} + 5x = 3 \quad x(0) = 0 \quad \& \quad \dot{x}(0) = 0.$

Sol^n laplace transform of above differential equation is

$$s^2X + 2sX + 5X = \frac{3}{s}$$

or, $$X(s) = \frac{3}{s(s^2+2s+5)}$$

Use partial fraction,

$$X(s) = \frac{3}{s(s^2+2s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+5}$$

$$3 = As^2 + 2As + 5A + Bs^2 + Cs$$

we get,

$A + B = 0 \quad 2A + C = 0 \quad 5A = 3$

$B = -3/5 \quad C = -\frac{6}{5} \quad A = 3/5$

$$\therefore X(s) = \frac{3}{5s} - \frac{3(s+2)}{5(s^2+2s+5)}$$

$$= \frac{3}{5s} - \frac{3(s+2)}{5[(s+1)^2+2^2]}$$

$$X(s) = \frac{3}{5s} - \frac{3(s+1)}{5[(s+1)^2+2^2]} - \frac{3}{5}\frac{1}{[(s+1)^2+2^2]}$$

Now, inverse is

$$x(t) = \frac{3}{5}\mathcal{L}^{-1}\frac{1}{s} - \frac{3}{5}\mathcal{L}^{-1}\frac{s+1}{(s+1)^2+2^2} - \frac{3}{5\times 2}\mathcal{L}^{-1}\frac{2}{(s+1)^2+2^2}$$

$$= \frac{3}{5}\cdot 1 - \frac{3}{5}e^{-t}\cos 2t - \frac{3}{10}e^{-t}\sin 2t$$

$$= \frac{3}{5}\left[1 - e^{-t}\cos 2t - \frac{e^{-t}\sin 2t}{2}\right] \quad \#$$

). Find the laplace transform of f(t) defined by,

$$f(t) = 0 \qquad \text{for } t < 0$$

$$= te^{-3t}, \quad \text{for } t \geq 0$$

Soln

$$\mathcal{L}\, f(t) = \int_0^{\infty} e^{-st}\, t \cdot e^{-3t}\, dt + \int_{-\infty}^{0} 0\, dt$$

$$F(s) = \int_0^{\infty} t\, e^{-t(3+s)}\, dt$$

$$= t\int_0^{\infty} e^{-t(3+s)}\, dt - \int_0^{\infty} 1 \cdot \int e^{-t(3+s)}\, dt$$

$$= t \cdot \frac{e^{-t(3+s)}}{-(3+s)}\Bigg|_0^{\infty} - \frac{e^{-t(3+s)}}{(3+s)^2}\Bigg|_0^{\infty}$$

$$F(s) = 0 - \left(0 - \frac{1}{(3+s)^2}\right) = \frac{1}{(3+s)^2}$$

11. What is the laplace transform of

$$f(t) = 0 \quad \text{for } t < 0$$
$$= \sin(\omega t + \theta) \quad \text{for } t \geq 0.$$

where θ is a constant?

Soln

$$\mathcal{L}f(t) = F(s) = \int_{-\infty}^{0} e^{-st}\, 0\, dt + \int_0^\infty e^{-st} \sin(\omega t + \theta)\, dt$$

$$= \sin(\omega t + \theta)\int_0^\infty e^{-st} dt - \int_0^\infty \cos(\omega t + \theta)\, \omega \int e^{-st} dt\, dt$$

$$= \sin(\omega t + \theta) \cdot \frac{e^{-st}}{(-s)}\Big|_0^\infty - \omega \cdot \int_0^\infty \cos(\omega t + \theta) \cdot \frac{e^{-st}}{-s} dt$$

$$= 0 - \frac{\sin(\theta) \cdot 1}{(-s)} + \frac{\omega}{s}\int_0^\infty \cos(\omega t + \theta) \times e^{-st} dt$$

$$= \frac{\sin(\theta)}{s} + \frac{\omega}{s}\cos(\omega t + \theta)\frac{e^{-st}}{(-s)}\Big|_0^\infty + \frac{\omega}{s}\int_0^\infty \sin(\omega t + \theta)\frac{e^{-st}}{(-s)}dt$$

$$= \frac{\sin\theta}{s} + \frac{\omega\cos\theta}{s^2} + \frac{\omega^2}{(-s)^2}\int_0^\infty \sin(\omega t + \theta) e^{-st} dt$$

or, $$F(s)\left(1 + \frac{\omega^2}{s^2}\right) = \frac{s\sin\theta + \omega\cos\theta}{s^2}$$

or, $$F(s) = \frac{s\sin\theta + \omega\cos\theta}{s^2 + \omega^2}$$ #

Find the initial value of $\frac{df}{dt}$ when the Laplace transform of $f(t)$ is given by. $F(s) = \mathcal{L} f(t) = \frac{2s+1}{s^2+s+1}$.

Soln

$$F(s) = \frac{2s+1}{s^2+s+1}$$

$$= \frac{2s+1}{s^2+2s\cdot\frac{1}{2}+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}$$

$$= \frac{2s+1}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$F(s) = \frac{2\left(s+\frac{1}{2}\right)}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

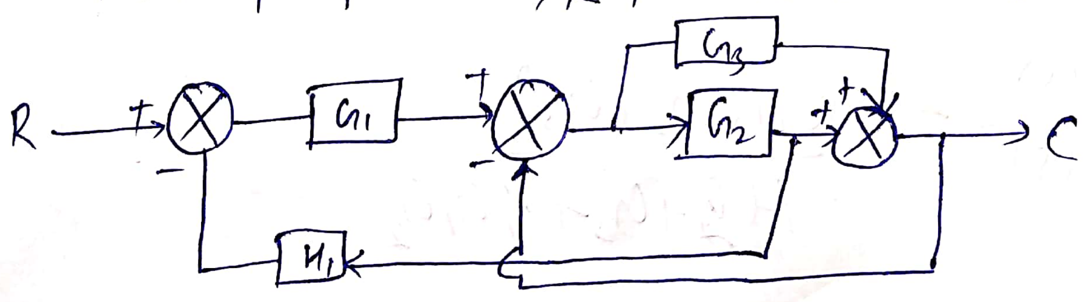
Take laplace inverse, we get

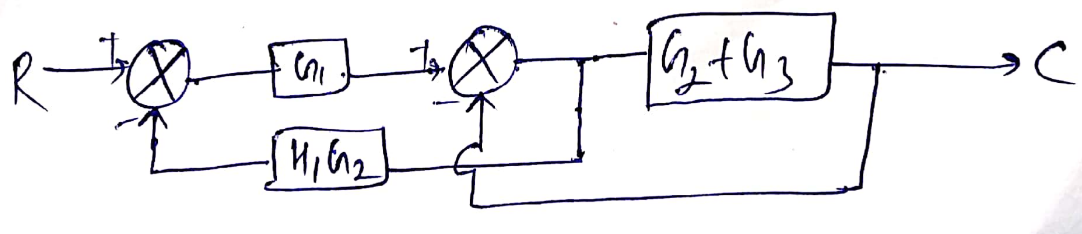
$$f(t) = 2e^{-\frac{1}{2}t}.\cos\frac{\sqrt{3}}{2}t$$

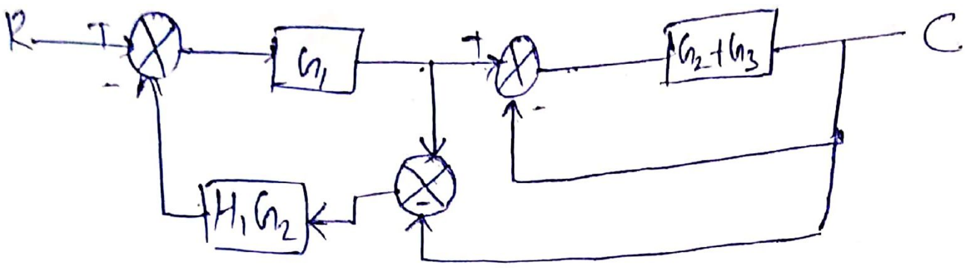
$$\frac{df(t)}{dt} = 2e^{-\frac{1}{2}t}\left(-\sin\frac{\sqrt{3}}{2}t\right)\times\frac{\sqrt{3}}{2} + 2\cos\frac{\sqrt{3}}{2}t.e^{-\frac{1}{2}t}.\left(-\frac{1}{2}\right)$$

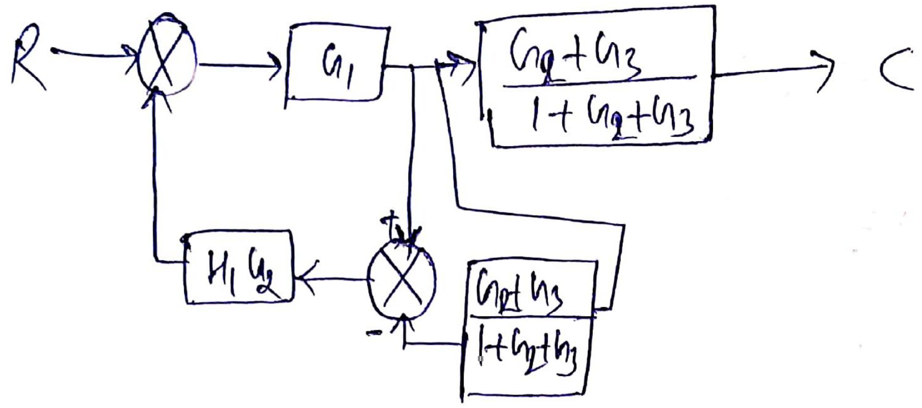
$$= -e^{-\frac{1}{2}t}\left[\sqrt{3}\sin\frac{\sqrt{3}}{2}t + \cos\frac{\sqrt{3}}{2}t\right] \quad \#$$

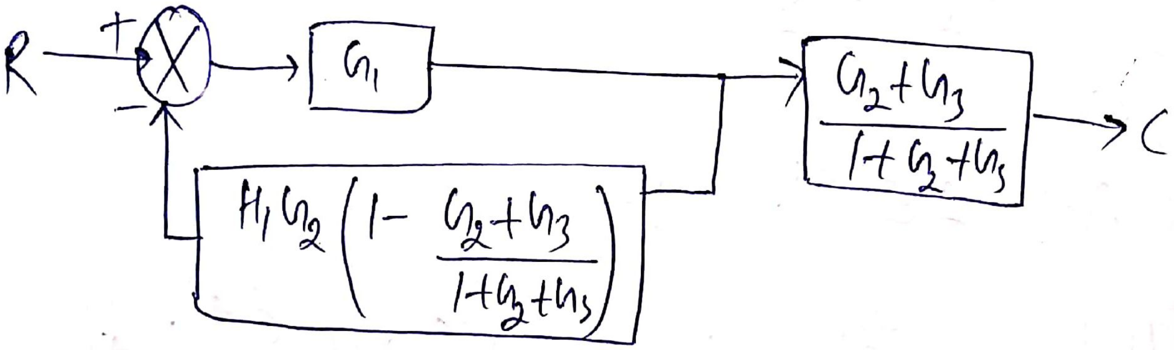
Obtain the transfer function C/R from the block diagram given below.

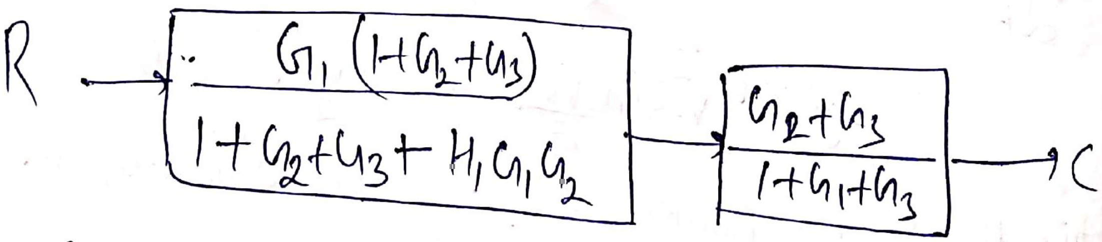






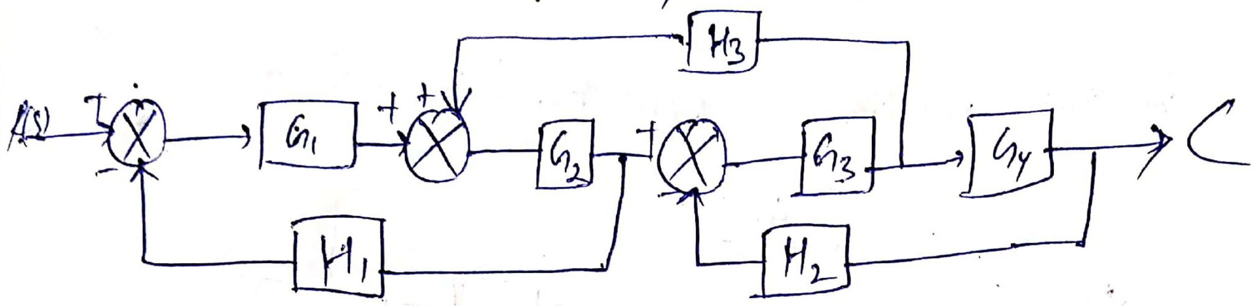


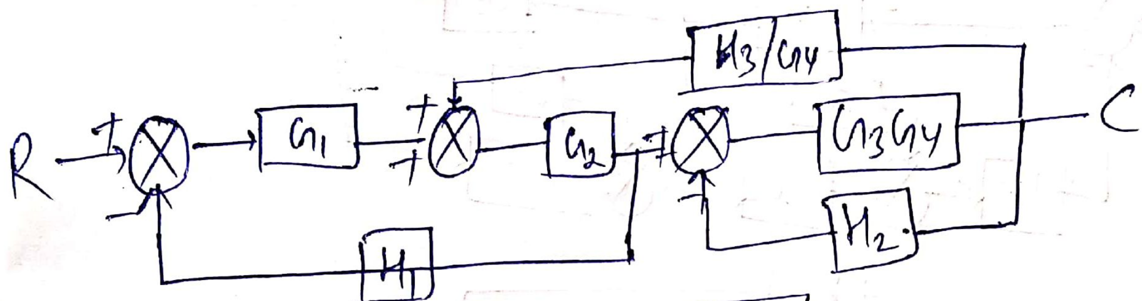


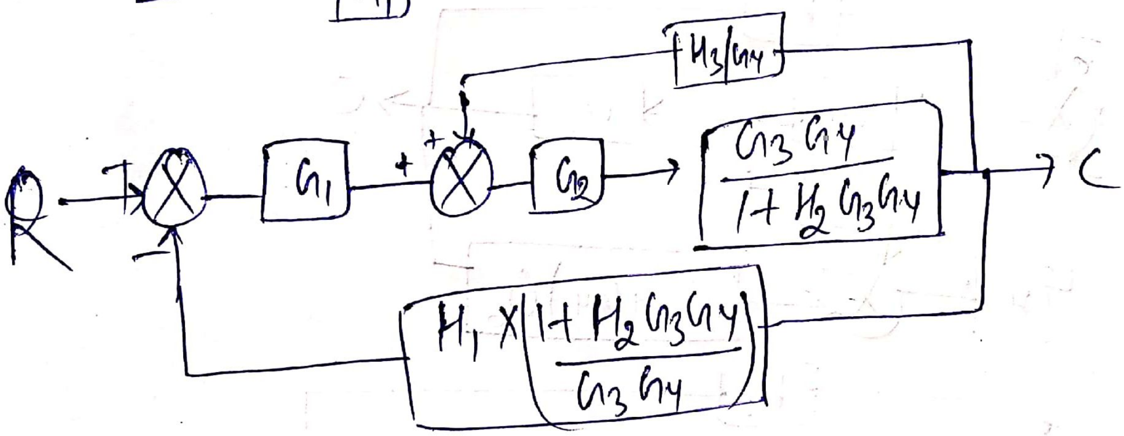


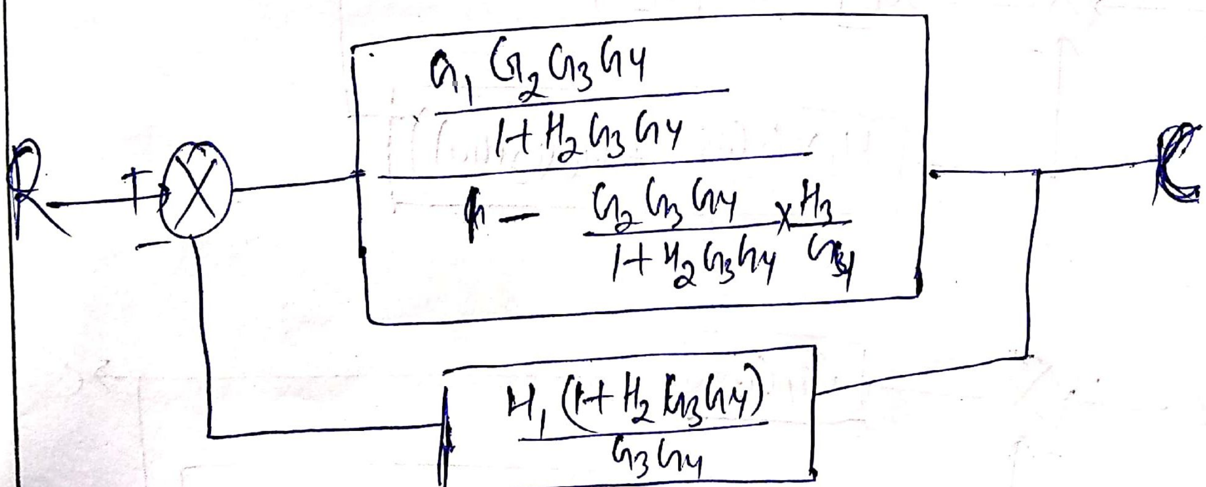
$$\therefore \frac{C(s)}{R(s)} = \frac{G_1(G_2+G_3)}{1+G_2+G_3+H_1G_1G_2}$$

Simplify the block diagram of the figure given below then obtain the Transfer function C(s)/R(s)





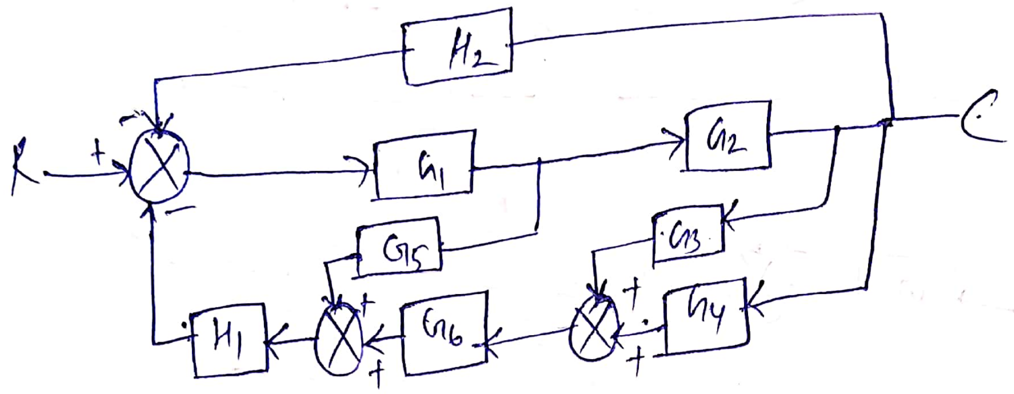


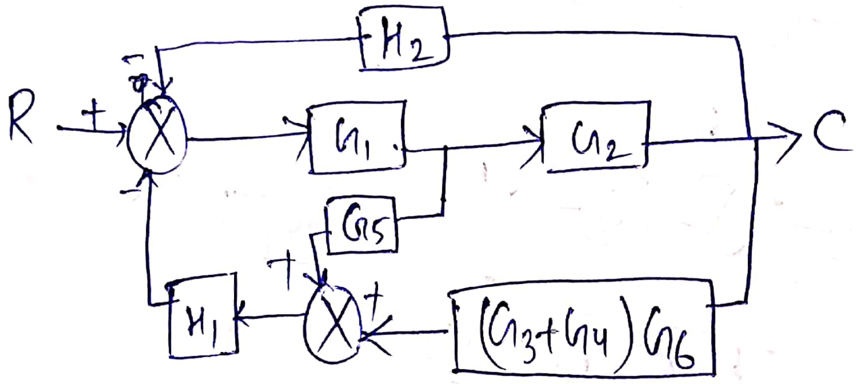


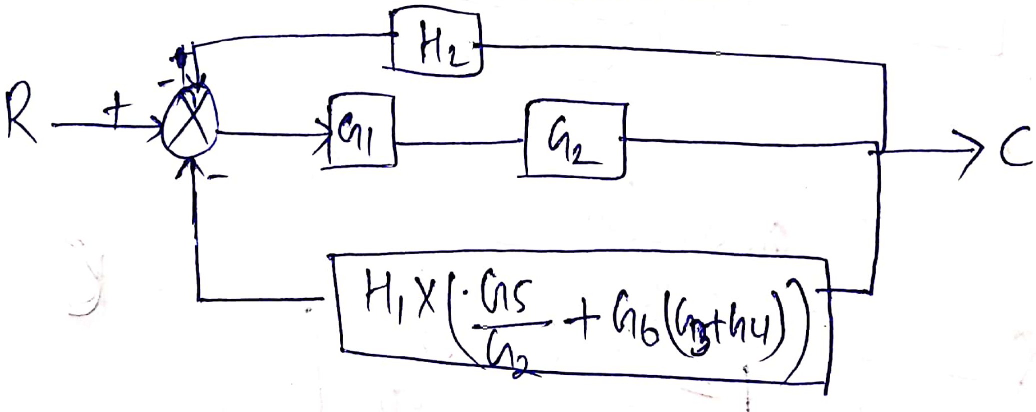
$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2G_3G_4}{1+H_2G_3G_4 - H_3G_2G_3}}{1+\dfrac{G_1G_2G_3G_4}{1+H_2G_3G_4 - H_3G_2G_3} \times \dfrac{H_1(1+H_2G_3G_4)}{G_3G_4}}$$

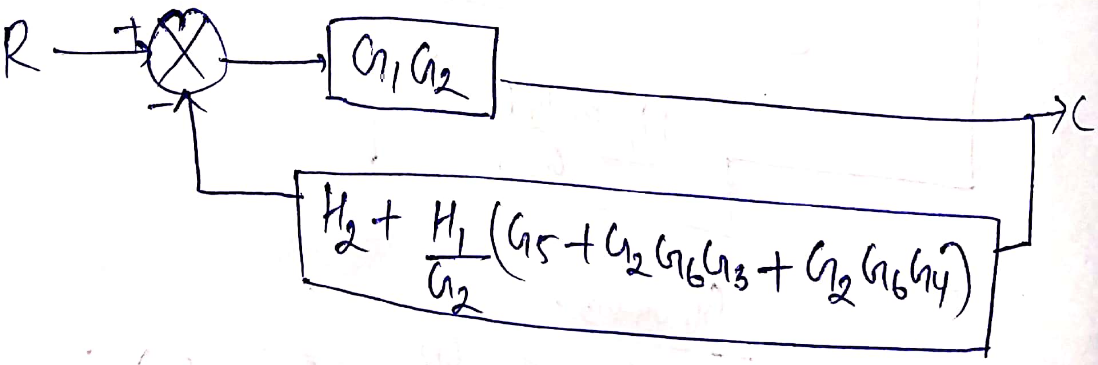
$$\therefore \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + H_2 G_3 G_4 - H_3 G_2 G_3 + G_1 G_2 H_1 (1 + G_3 G_4 H_2)}$$

Q5. Obtain the transfer function C/R from the block diagram. #





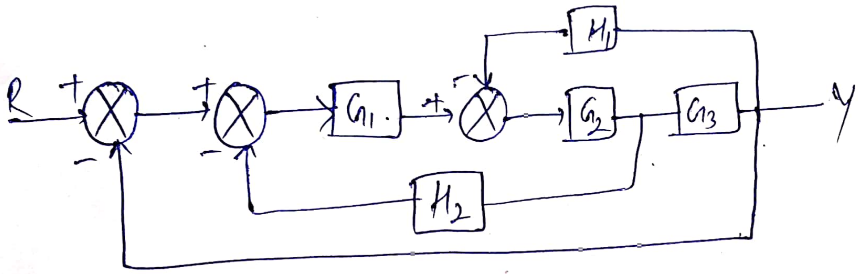


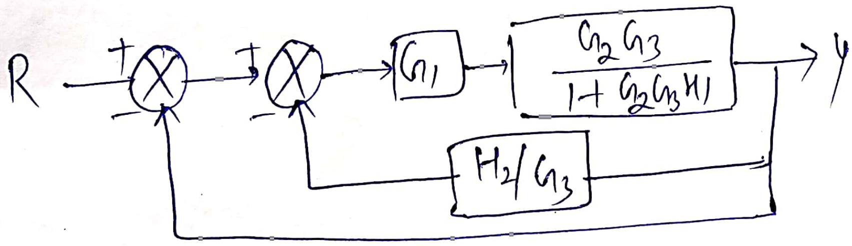


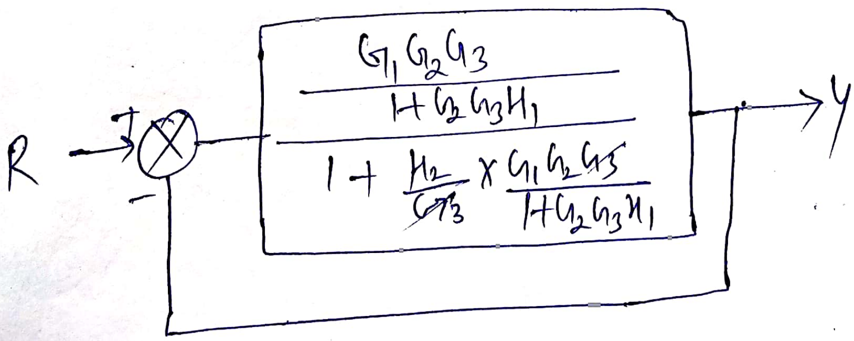
$$\therefore \frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + \frac{\cancel{G_1} G_1 G_2 \times [H_2 G_2 + H_1 (G_5 + G_3 G_6 G_3 + G_3 G_6 G_4)]}{\cancel{G_2}}}$$

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2}{1 + G_1 G_2 H_2 + G_1 H_1 (G_5 + G_2 G_6 G_3 + G_2 G_6 G_4)} \quad \#$$

6. Obtain the transfer function Y/R from the block diagram given below.







$$\therefore \frac{Y(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + G_2 G_3 H_1 + G_1 G_2 H_2 + G_1 G_2 G_3} \quad \#$$