1. What is control system? How are Control systems classified in different types. Explain.

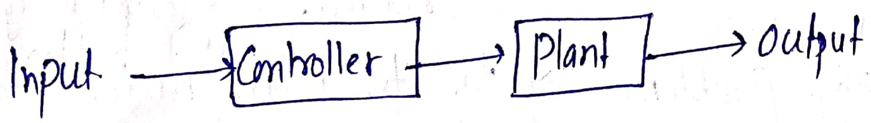
⇒ Control system is the means by which any quantity of interest in a system can be varied or kept constant. It is a set of mechanical or electrical devices that regulates other devices or systems by way of control loops. Typically, Control systems are computerized. Control systems have played a centeral role in the development and advancement of modern technology and civilization. Practically, every aspect of our day-to-day life is affected less or more by some control system. A bathroom toilet tank, a refrigerater, an air conditioner, a geezer, an automatic iron, etc are all control system.

There are two main types of control system. They are

(i) Open loop control system

Any physical system which does not automatically correct for variations in it's output is called an open loop system. Eg: traffic light control.

Block diagram:

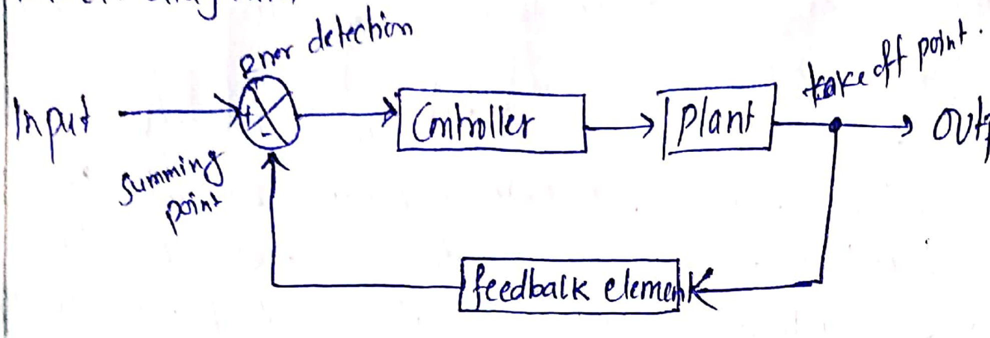


ii) closed loop control system

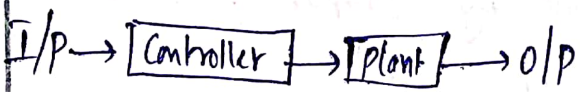
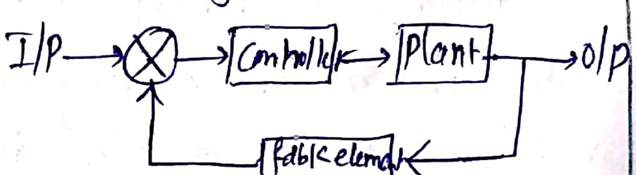
A system that maintains a pescribed relationship between the output and the reference input by comparing them & using the difference as means of control.

Eg: room tempr controlled system.

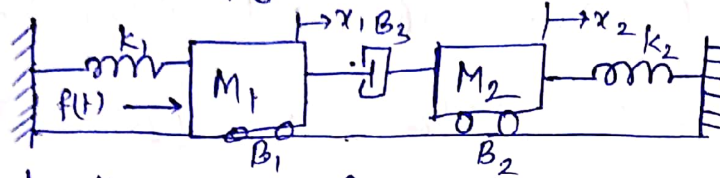
Block diagram



2. Compare open loop and Closed loop Control system.

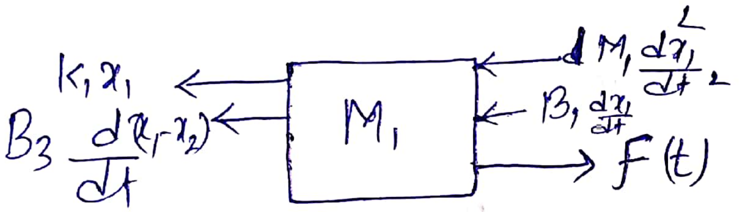
Open loop Control System	Closed loop control system.
* Any Physical system which does not automatically correct for variations in its output is Called an open loop system.	* A system that maintains a prescribed relationship between the output & the reference input by comparing them and using the difference as means of Control called closed loop Control system.
* Simple in construction and design.	* They are Complicated to design.
* Any change in output cannot be corrected automatically & less accurate.	* Highly accurate as any error arising is corrected due to presence of feedback system.
* There is no any problem related to stability in open loop Control system.	* Stability is the major problem and more care is needed to design a stable Closed loop system.
* Block diagram 	* Block diagram 
* Eg: Traffic light control, Electric hand drier, automatic Washing machine etc.	* Eg: Room tempr Controlled system, Automatic electric Iron, water level Controller, etc.

8. Write the differential equations describing the dynamics of system shown in figure.



(i) Also find the value of $X_2(s)/F(s)$ for the system.
(ii) Find equivalent electrical system. Use f-V analogy & f-i analogy.

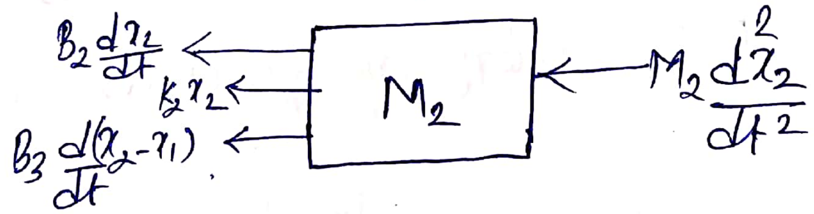
For mass M_1



The differential equation is given as

$$f(t) = M_1 \frac{d^2x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1 x_1 + B_3 \frac{d(x_1 - x_2)}{dt} \quad \text{--- (1)}$$

For Mass M_2



$$0 = M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + K_2 x_2 + B_3 \frac{d(x_2 - x_1)}{dt} \quad \text{--- (2)}$$

(i) The laplace transform eq^n of eq^n ① & ② are:

$$F(s) = M_1 s^2 X_1 + B_1 s X_1 + K_1 X_1 + B_3 s (X_1 - X_2)$$

$$= (M_1 s^2 + B_1 s + B_3 s + K) X_1(s) - B_3 s X_2(s) \quad \text{——③}$$

and

$$0 = M_2 s^2 X_2 + B_2 s X_2 + K_2 X_2 + B_3 s (X_2 - X_1)$$

$$\text{or, } X_1(s) = \frac{(M_2 s^2 + B_2 s + K_2 + B_3 s) X_2(s)}{B_3 s} \quad \text{——④}$$

from eq^n ③ & ④, we get,

$$F(s) = [M_1 s^2 + (B_1 + B_3) s + K_1] \frac{[M_2 s^2 + (B_2 + B_3) s + K_2] X_2(s)}{B_3 s}$$

$$B_3 s X_2$$

$$\therefore \frac{X_2(s)}{F(s)} = \frac{B_3 s}{[M_1 s^2 + (B_1 + B_3) s + K_1][M_2 s^2 + (B_2 + B_3) s + K_2] - (B_3 s)^2}$$

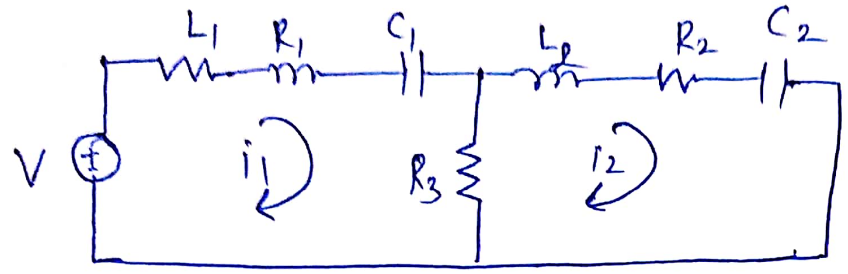
(ii) Now, the F·V and F·I analogy are given as
from eq^n ①

$$V = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int i_1 dt + R_3 (i_1 - i_2)$$

& from eq^n ②,

$$0 = L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2} \int i_2 dt + R_3 (i_2 - i_1)$$

Equivalen electrical system by F-V analogy is given as



For F-I analogy,

from eq^n ①,

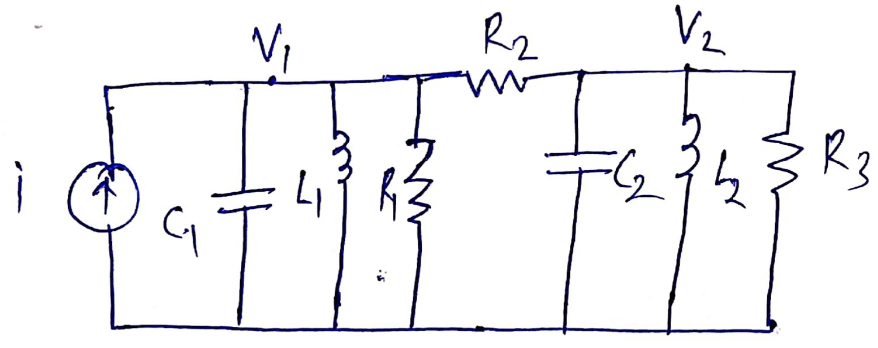
$$i = C_1\frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{1}{L}\int V_1\,dt + \frac{(V_1-V_2)}{R_3}$$

from eq^n ⑪,

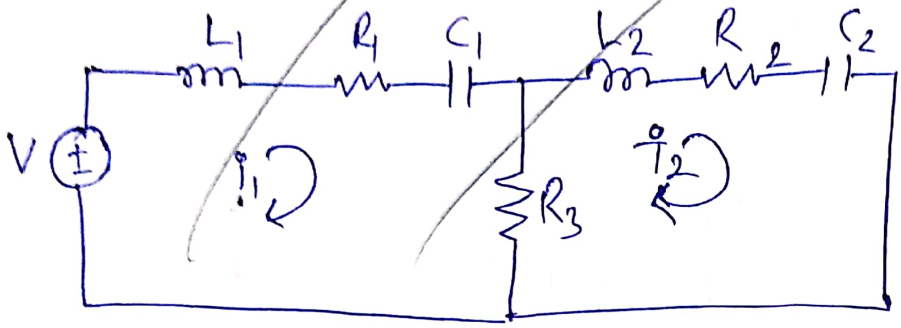
$$0 = C_2\frac{dV_2}{dt} + \frac{1}{R_2}V_2 + \frac{1}{L_2}\int V_2\,dt + \frac{1}{R_3}(V_2-V_1)$$

~~from eq^n (111)~~

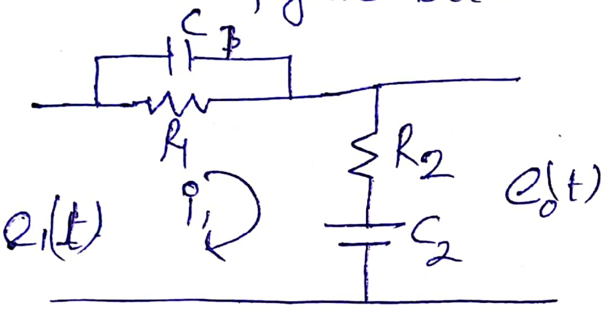
Now, Equivalent electrical system by F-I analogy is given as



Equivalent electrical system is given as



4. Determine the transfer function for the electrical circuit shown in the figure below.



Sol^n

$$e_i(S) = \left[\left(\frac{1}{SC_1} \| R_1\right) + R_2 + \frac{1}{C_2S}\right] I_1(S)$$

$$e_i(S) = \left[\frac{R_1/C_1S}{R_1 + \frac{1}{C_1S}} + R_2 + \frac{1}{C_2S}\right] I_2(S) \quad \text{--- (I)}$$

Again, we have,

$$e_o(S) = \left(R_2 + \frac{1}{C_2S}\right) I_1(S)$$

$$I_1(S) = \frac{e_o(S)}{\left(R_2 + \frac{1}{C_2S}\right)} \quad \text{--- (II)}$$

Now, from eq^n (I) & (II), we get

$$e_1(S) = \left(\frac{R_1}{R_1C_1S+1} + R_2 + \frac{1}{C_2S}\right)\frac{E_0(S)}{\left(R_2+\frac{1}{C_2S}\right)}$$

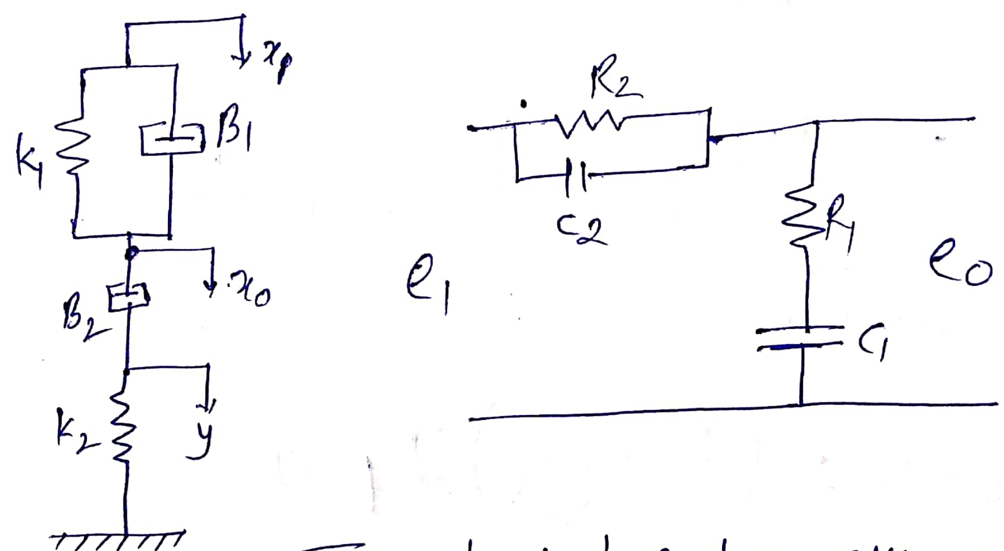
or,
$$\frac{e_0(s)}{e_1(S)} = \frac{\left(R_2+\frac{1}{C_2S}\right)}{\left(\frac{R_1}{R_1C_1S+1} + R_2 + \frac{1}{C_2S}\right)}$$

$$= \frac{R_2C_2S+1}{\frac{R_1C_2S}{R_1C_1S+1} + R_2C_2S+1}$$

$$\therefore \frac{e_0(S)}{e_1(S)} = \frac{(R_2C_2S+1)(R_1C_1S+1)}{R_1C_2S + (1+R_1C_1S)(1+R_2C_2S)}$$

which is required transfer function.

5. Show that the system shown below are analogous system.



For mechanical system assume,
$x_1 \rightarrow$ input
$x_0 \rightarrow$ output.

Soln

At x_0,

$$b_2\left(\frac{dx_0}{dt}-\frac{dy}{dt}\right)+b_1\frac{d}{dt}(x_0-x_1)+k_1(x_0-x_1)=0 \quad \text{---(I)}$$

Again,

At y,

$$k_2 y+b_2\frac{d}{dt}(y-x_0)=0 \quad \text{---(2)}$$

Laplace transform of above eqn (1) is

$$B_2(SX_0-SY_0)+B_1(SX_0-SX_1)+K_1(X_0-X_1)=0 \quad \text{---(III)}$$

Laplace transform of eqn (II) is

$$K_2 Y+B_2 S(Y-X_0)=0$$

or, $$Y=\frac{B_2 S\, X_0(S)}{K_2+B_2 S} \quad \text{---(IV)}$$

From eqn (III) & (IV) we get,

$$B_2\left(SX_0-\frac{S^2 B_2 X_0}{K_2+B_2 S}\right)+B_1(SX_0-SX_1)+K_1(X_0-X_1)=0$$

or, $$\left(B_2 S-\frac{B_2^2 S^2}{K_2+B_2 S}+B_1 S+K_1\right)X_0=X_1(B_1 S+K_1)$$

$$\therefore \frac{X_0}{X_1}=\frac{(B_1 S+K_1)}{\left(B_2 S-\frac{B_2^2 S^2}{K_2+B_2 S}+B_1 S+K_1\right)} \quad \text{---(V)}$$

~~Laplace transform of eqn (v)~~

F.V ~~& F.I~~ anlogous of eqn (v) is

$$\frac{E_0(s)}{E_1(s)} = \frac{R_1 S + \frac{1}{C_1}}{R_2 S - \frac{R_2^2 S^2}{1/C_2 + R_2 S} + R_1 S + \frac{1}{C_1}}$$

$$= \frac{(R_1 C_1 S + 1)/C_1 \times (1/C_2 + R_2 S)}{\left(R_2 S + R_1 S + \frac{1}{C_1}\right)\left(\frac{1}{C_2} + R_2 S\right) - R_2^2 S^2}$$

$$= \frac{(R_1 C_1 S + 1)(R_2 C_2 S + 1)}{(R_1 C_1 S + R_2 C_1 S + 1)(R_2 C_2 S + 1) - R_2^2 C_1 C_2 S^2}$$

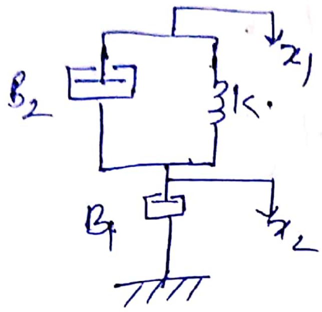
$$= \frac{(R_1 C_1 S + 1)(R_2 C_2 S + 1)}{R_2 C_1 S (R_2 C_2 S + 1) + (R_1 C_1 S + 1)(R_2 C_2 S + 1) - R_2^2 C_1 C_2 S^2}$$

$$= \frac{(R_1 C_1 S + 1)(R_2 C_2 S + 1)}{(R_1 C_1 S + 1)(R_2 C_2 S + 1) + \cancel{R_2^2 C_1 C_2 S^2} + R_2 C_1 S - \cancel{R_2^2 C_1 C_2 S^2}}$$

$$= \frac{(R_2 C_1 S + 1)(R_2 C_2 S + 1)}{(R_1 C_1 S + 1)(R_2 C_2 S + 1) + R_2 C_1 S}$$

From Q.N. (4) also same equation was produced. So the above electrical diagram is equivalent to given mechanical system.

6. Draw electrical analogous (F-V analogy) for the system given below.



At x_2,

$$B_1 \frac{dx_2}{dt} + K(x_2 - x_1) + B_2 \frac{d(x_2 - x_1)}{dt} = 0 \quad \text{— (1)}$$

~~Electrical equivalent or F-V & F-I analogous eq^n of eq^n (1) is given as~~

Laplace transform of eq^n (1) is

$$B_1 S X_2 + K(X_2 - X_1) + B_2 S (X_2 - X_1) = 0.$$

or, $B_1 S X_2 + K X_2 + B_2 S X_2 = K X_1 + B_2 S X_1$

or, $$\frac{X_2}{X_1} = \frac{K + B_2 S}{B_1 S + K + B_2 S} \quad \text{— (II)}$$

Now, F.V ~~& F-I~~ analogous of eq^n (II) is

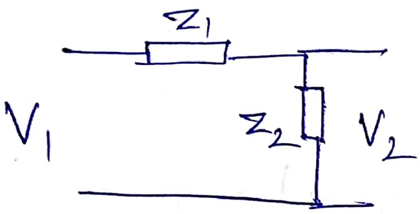
$$\frac{E_2(S)}{E_1(S)} = \frac{\frac{1}{C} + R_2 S}{R_1 S + R_2 S + 1/C} \quad \text{— (III)}$$

$$\frac{E_2(S)}{E_1(S)} = \frac{R_2 C S}{S R_1 C + R_2 C S + 1}$$

From eqⁿ (iii),

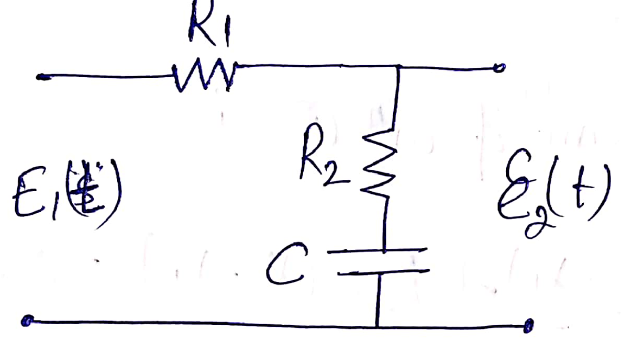
$$\frac{E_2(s)}{E_1(s)} = \frac{\frac{1}{cs} + R_2}{R_1 + R_2 + \frac{1}{cs}}$$

If we have,

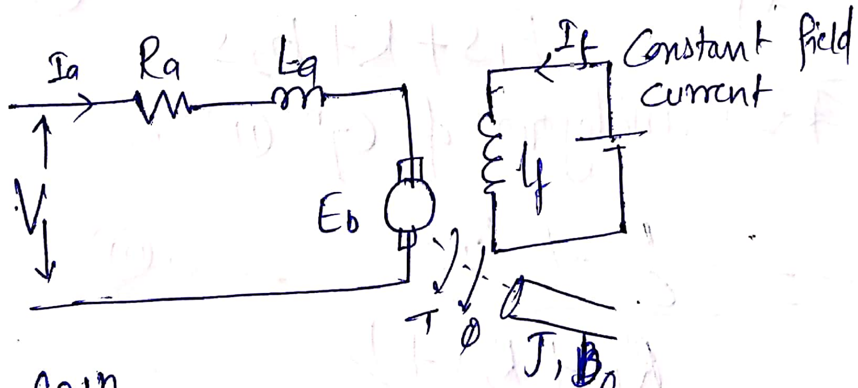


Then ⇒ $\frac{V_2}{V_1} = \frac{Z_2}{Z_1+Z_2}$

On the same way, we get the required electrical system as



7. Find out the transfer function of armature control dc motor shaon in figure :



Soln

The air flux is given as

$\phi \propto i_f$

$\phi = k_f i_f$ where k_f is constant,

Torque developed is given as

$T_m = k_i k_f i_f i_a$ when k_i is a constant.

$T_m = k_T i_a$ where k_T is known as torque constant

Now,

$$e_b = k_b \frac{d\theta}{dt} \quad \text{— (I)}$$

Again,

$$L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a \quad \text{— (II)}$$

$$J\frac{d^2\theta}{dt^2} + b_o \frac{d\theta}{dt} \cancel{=} T_m = k_T i_a \quad \text{— (III)}$$

Now,
Transfer function can be calculated as below.
First find the laplace transform of above all equations, (I), (II) & (III),

$E_b = k_b s \theta$ $\quad$ k_b is back emf constant

$$(L_a s + R_a) I_a = E_a - E_b$$

$$(Js^2 + b_o s)\theta = K_T I_a$$

Solving above equation, we get,

$$I_a = \frac{(Js^2 + b_o s)\theta}{k_T}$$

$$(L_a s + R_a) \times \frac{(Js^2 + b_0 s)\,\theta}{K_T} = E_a - K_b s\theta$$

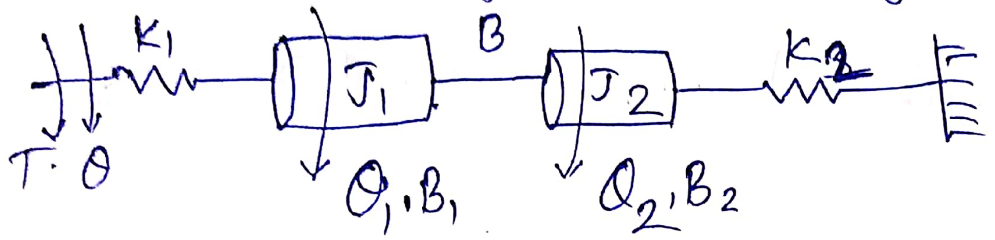
$$\left[\frac{(sL_a + R_a)(Js^2 + b_0 s)}{K_T} + K_b s\right]\theta(s) = E_a(s)$$

or, $$\frac{\theta(s)}{E_a(s)} = \frac{K_T}{(sL_a + R_a)s.(Js + b_0) + K_b K_T s}$$

or, $$\frac{\theta(s)}{E_a(s)} = \frac{K_T}{s\left[(sL_a + R_a)(Js + b_0) + K_b K_T\right]}$$

#

For the mechanical system shown in figure below:



Write differential Equations.

i) Draw equivalent mechanical network.

Obtain transfer function $\frac{\theta_1(s)}{T(s)}$.

Draw analogous electrical network (T - i analogy).

Sol^n

At θ,

$$T = K_1(\theta - \theta_1) \quad \text{—} \; (1)$$

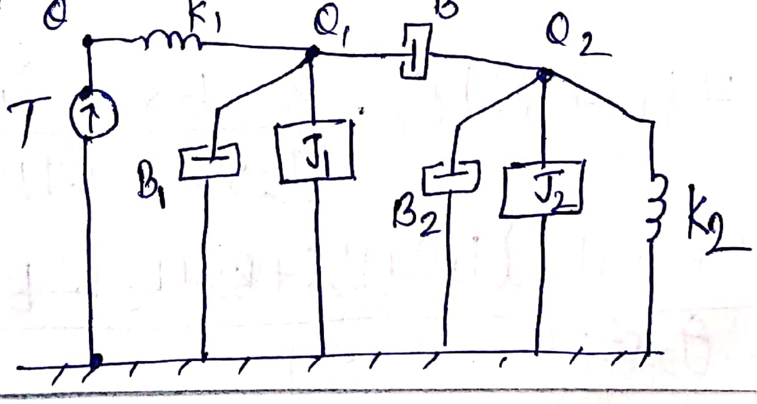
At θ_1,

$$0 = K_1(\theta_1 - \theta) + J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + B \frac{d(\theta_1 - \theta_2)}{dt} \; \text{—} \; (2)$$

At θ_2,

$$0 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + B \frac{d(\theta_2 - \theta_1)}{dt} + K_2\theta_2 \; \text{—} \; (3)$$

The equivalent mechanical network is given as



(iii) L.T of eq^n 1 is

$$T(S) = K(\theta - \theta_1) \quad \text{--- (4)} \quad \text{ie } \theta = \frac{T(S) + K\theta_1}{K}$$

LT of eq^n 2 is

$$0 = J_1 S^2 \theta_1 + B_1 S \theta_1 + BS(\theta_1 - \theta_2) + K(\theta_1 - \theta)$$

From eq^n 4,

$$0 = (J_1 S^2 + B_1 S + BS)\theta_1 - BS\theta_2 + K\left(\theta_1 - \frac{T(S) + K\theta_1}{K}\right)$$

$$= (J_1 S^2 + B_1 S + BS)\theta_1 - BS\theta_2 + K\theta_1 - T(S) - K\theta_1$$

or, $$T(S) = (J_1 S^2 + B_1 S + BS)\theta_1 - BS\theta_2 \quad \text{--- (5)}$$

L.T. of eq^n 3 is

$$0 = J_2 S^2 \theta_2 + B_2 S \theta_2 + BS(\theta_2 - \theta_1) + K_2 \theta_2$$

$$0 = (J_2 S^2 + B_2 S + BS + K_2)\theta_2 - BS\theta_1$$

$$\theta_2 = \frac{BS \quad \theta_1}{J_2 S^2 + B_2 S + BS + K_2} \quad \text{--- (6)}$$

From eq^n 5 & 6, we get,

$$T(S) = (J_1 S^2 + B_1 S + BS)\theta_1 - \frac{BS \cdot BS \quad \theta_1}{J_2 S^2 + B_2 S + BS + K_2}$$

$$T(S) = \left[\frac{(J_1 S^2 + B_1 S + BS)(J_2 S^2 + B_2 S + BS + K_2) - B^2 S^2}{(J_2 S^2 + B_2 S + BS + K_2)}\right]\theta_1(S)$$

i.e. $$\frac{\theta_1(S)}{T(S)} = \frac{J_2 S^2 + B_2 S + BS + K_2}{(J_1 S^2 + B_1 S + BS)(J_2 S^2 + B_2 S + BS + K_2) - B^2 S^2}$$

which is required transfer function.

) T-i analogy is given as

from eqn ①,

$$V = \frac{1}{C_1}\int (i - i_1)\,dt$$

from eqn ②,

$$0 = L_1 \frac{di_1}{dt} + R_1 i_1 + R(i_1 - i_2) + \frac{1}{C_1}\int (i_1 - i)\,dt$$

from eqn ③,

$$0 = L_2 \frac{di_2}{dt} + R_2 i_2 + R(i_2 - i_1) + \frac{1}{C_2}\int i_2\,dt$$

from above eqn. in electrical system above mechanical is equivalent as